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ACI DESIGN HANDBOOK

Design of Structural Reinforced Concrete Elements in Accordance with the Strength Design Method of ACI 318-95

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P.O. Box 9094
Farmington Hills, Michigan
48333-9094
TEL: 248-848-3800
FAX: 248-848-3801

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ACI Design Handbook

Design of Structural Reinforced Concrete Elements in Accordance with the Strength Design Method of ACI 318-95

Reported by ACI Committee 340

Mohsen A. Issa, Chairman

Husam A. Omar, Secretary

Param D. Bhat
William W. Bintzer
Patrick J. Creegan
Om P. Dixit
Noel J. Everard
Richard Furlong*
Moussa A. Issa
James S. Lai
Douglas D. Lee
S. Ali Mirza
Edward G. Nawy
William E. Rushing, Jr.
Charles G. Salmon
Murat Saatcioglu
Sudhakar P. Verma

*Consulting Member

The ACI Design Handbook is intended for use by individuals having a general familiarity with the strength design method and with "Building Code Requirements for Reinforced Concrete (ACI 318-95)." This publication provides information for the engineering design and analysis of beams, one-way slabs, brackets, footings, pile caps, columns, two-way slabs, and seismic design.

Information is presented on three sections: Design Aids, Design Examples, and Commentary on Design Aids. The Design Examples illustrate the use of the Design Aids, which are tables and graphs intended to eliminate routine and repetitious calculations. The Commentary explains the analytical basis for the Design Aids.

Keywords: anchorage (structural); axial loads; bars; beams (supports); bending; bending moments; biaxial loads; brackets; buckling; columns (supports); concrete construction; concrete piles; concrete slabs; connections; cracking (fracturing); deflection; flanges; flexural strength; footings; frames; load factors; loads (forces); long columns; moments of inertia; pile caps; reinforced concrete; reinforcing steels; shear strength; slenderness ratio; spiral columns; splicing; stiffness; strength analysis; structural analysis; structural design; T-beams; tension; torsion.

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AMERICAN CONCRETE INSTITUTE
P.O. Box 9094
Farmington Hills, MI 48331-9094
Phone: 248/848-3700 Fax: 248/848-3701

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ACI 318-95

*Strength Reduction Factors**

FLEXURE, WITHOUT AXIAL LOAD		*0.90*
AXIAL TENSION OR AXIAL TENSION WITH FLEXURE		*0.90*
AXIAL COMPRESSION OR AXIAL COMPRESSION WITH FLEXURE	*Members with spiral reinforcement conforming to Section 10.9.3*	*0.75***
	Other reinforced members	*0.70***
	Members in high seismic zones with factored axial compressive forces exceeding ($A_g f'_c/10$) if transverse reinforcement does not conform to Section 21.4.4	*0.50*
SHEAR AND TORSION		*0.85*
SHEAR AND TORSION (in regions of high seismic risk)	*Nominal shear strength of the member is less than the nominal shear corresponding to the development of the nominal flexural strength of the member*	*0.60*
	Shear in joints of buildings	*0.85*
*BEARING ON CONCRETE****		*0.70*
FLEXURE IN PLAIN CONCRETE		*0.65*

* *Design strength provided by a member shall be taken as the nominal strength, calculated from the design aids given in this handbook, multiplied by the appropriate strength reduction factor ϕ. Alternate strength reduction ϕ factors for use with ASCE 7 load factors are included in ACI 318-95, Appendix C.*

** *See ACI 318-95 Section 9.3.2.2 or Appendix B for adjustment to these ϕ values for low levels of axial compression.*

*** *Also see ACI 318-95 Section 18.13.*

FOREWORD

The ACI Design Handbook is intended for use by persons having a general familiarity with the strength design method and with "Building Code Requirements for Structural Concrete (ACI 318-95)."

This volume presents information for the engineering design and analysis of beams, slabs, brackets, footings, pile caps, columns, two-way slabs, and seismic design.

SECTIONS

Design Aids are tables and graphs intended to save the designer the effort of repeatedly performing routine calculations. All Design Aids apply to concrete having f'_c ranging between 3 and 12 ksi with Grade 40, 60, and 75 steel reinforcement depending on the type of the structural member. A note at the bottom of each Design Aid indicates which Design Example illustrates the use of the table or graph.

Design Examples illustrate the use of the Design Aids (but are not intended to show how to design a structure).

Commentary on Design Aids gives the basis for the Design Aids.

For judicious application and optimum efficiency, users of this handbook should first acquaint themselves with the Commentary. Design Examples will help verify procedures and results. It is not, however, the objective of the handbook to teach the novice how to design in reinforced concrete. Readers are expected to be competent in design before attempting to use this handbook.

ACI COMMITTEE 340 AND ITS WORK

ACI Committee 340, Design Aids for Building Codes, was organized in 1958 for the purpose of preparing a handbook that would simplify the design of reinforced concrete structural elements using the strength design method in accordance with the ACI Building Code. The handbook was prepared in two volumes: Volume 1 covering beams, one-way-action-slabs, footings, and other members except columns; and Volume 2 treating columns only. Volume 1 was issued in 1967; Volume 2 was ready first and published in 1964. Both volumes were in accordance with the 1963 version of the ACI Building Code.

In 1971 when ACI 318-71 was issued, Committee 340 was charged with revising the existing handbook material to bring it into accordance with that code. The resulting second edition of Volume 1, which incorporated some material on columns, was issued in 1973 and a second, corrected printing was published in 1974.

The August 1977 ACI Journal carried "Step-by-Step Design Procedures in Accordance with the Strength Design Method of ACI 318-71," subsequently published as ACI Committee Report 340.3R-77.

While ACI 318-77 was being prepared for publication in the fall of 1977, Committee 340 was revising the handbook volumes accordingly. The resulting new edition of Volume 2 on columns was published in May 1978. In late 1978, a supplement to the Strength Design Handbook dealing with two-way-action slabs and entitled *Slab Design in Accordance with ACI 318-77* was published.

The third edition of Volume 1, published in 1981, contained two divisions: Division I dealt with beams, one-way slabs, brackets, footings, and pile caps and incorporated the Step-by-Step Design Procedures—all material being updated to ACI 318-77. Division II consisted of the two-way slab design supplement.

The fourth edition of Volume 1 was published in 1984. It was the revised edition of the third edition to conform to ACI 318-83, including new flow charts for design of members in flexure. Design of Two-Way Slabs was published as a supplement to Volume 1 in 1985.

The fifth edition of Volume 1 was formatted in the same manner as the fourth edition. Design of Two-Way Slabs (Supplement to Volume 1) was made a separate volume, Volume 3 of the ACI Design Handbook.

This edition is developed in accordance with ACI 318-95. This version of the code was prepared in a format to correspond to a strength reduction factor of 1.0. This format was used in order to make the handbook more useful for international use.

HANDBOOK USER'S COMMENTS

ACI Committee 340 welcomes suggestions from users of this handbook on how to make future printings more useful. Comments should be directed to ACI Committee 340, American Concrete Institute, P.O. Box 9094, Farmington Hills, Michigan 48333.

ACKNOWLEDGMENTS

Many individuals and organizations have contributed to the preparation of this book, giving their time and effort to the preparation of charts, tables, examples, and computer programs, as well as undertaking critical review of the manuscript. Although it is practical to acknowledge individually all of these contributions, they are nonetheless greatly appreciated, for such efforts have contributed materially to the quality of this handbook.

The leadership and work of the committee chairman, Mohsen A. Issa, are greatly appreciated for which a large portion of the handbook development process is attributed to. The work of every committee member involved in the development of the New Design Handbook is appreciated. Also appreciated are the efforts of Dr. Issa's graduate students, Alfred A. Yousif and Stanislav Dekic.

Grateful acknowledgment is made of computer time contributed by the University of Illinois at Chicago.

NOTATION

This notation section defines symbols used in this volume covering beams, one-way slabs, footings, pile caps, columns, two-way slabs, and their reinforcement. Words in parentheses such as "(Flexure)" or "(Shear)" indicate portions of this handbook in which symbol is used.

a = depth of equivalent rectangular stress block, in. (Flexure)
= length for column section considered rigid (one half slab thickness) or length of rigid column section at beam end (square column with boxed capital), in. (Two-way slabs)
= factor for computing K_t, in^{-1}

a_{bal} = depth of equivalent rectangular stress block for balanced conditions, in. (Flexure)

a_c = immediate deflection at midspan, in. (Deflection)

a_d = immediate deflection due to dead load, in.

a_{d+l} = immediate deflection due to dead load and live load, in.

a_l = immediate deflection due to live load, in.

a_n = $f_y(1 - 0.59\omega) / 12000$, ft-kip / in^3, a coefficient for computing reinforcement area A_s. (Flexure)

a_n' = $f_y(1 - d'/d) / 12000$, ft-kip / in^3, a coefficient for computing reinforcement area A'_s (Flexure)

a_n'' = $\dfrac{87000}{12000}\left(1-\dfrac{d}{d}\right)\left(1-\dfrac{d/d}{c/d}\right)-\dfrac{0.85}{12000}f_c'\left(1-\dfrac{d}{d}\right)$,
a coefficient for computing reinforcement area A'_s when compression reinforcement does not yield. (Flexure)

a_{nf} = $f_y(1 - h_f / 2d) / 12000$, ft-kip / in^3, a coefficient for evaluating flange effects on moment in T-beams (Flexure)

A = any area, in^2
= $b_w t / n$ = effective tension area of concrete for crack control, in^2 per bar (Reinforcement)

A' = $b_w t / n'$ = effective tension area of concrete for crack control in case bundled bars are used, in^2, per bar bundle (Reinforcement)

A_1 = loaded area, in^2

A_2 = the area of the lowest base of the largest frustum of pyramid, cone, or tapered wedge contained wholly within the support and having for its upper base the loaded area, and having side slopes of 1 vertical to 2 horizontal, in^2

A_b = area of individual bar, in^2. (Reinforcement)

A_c = area of concrete at cross section considered, in^2 (Flexure)
= area of critical shear section = $b_o d$ (Two-way slabs)

= area of core of spirally reinforced column measured to outside diameter of spiral

A_{cp} = area enclosed by outside perimeter of concrete cross section

A_o = gross area enclosed by shear flow path, in^2

A_{cw} = minimum area of tension reinforcement A_{sw} to keep neutral axis low enough for compression reinforcement to reach yield strain under factored load conditions, in^2 (Flexure)

A_f = area of reinforcement in brackets or corbel resisting factored moment $[V_u a + N_{uc}(h\text{-}d)]$, in^2

A_g = gross section area of column cross section, in^2 (Shear)

A_h = area of shear reinforcement parallel to flexural tension reinforcement, in^2 (Shear)

A_n = area of tension reinforcement to resist force N_{uc} on brackets, in^2 (Shear)

A_{oh} = area enclosed by centerline of the outermost closed transverse torsional reinforcement, in^2

A_s = area of non-prestressed tension reinforcement, in^2 (Shear, Two-way slabs)

A'_s = area of compression reinforcement, in^2 (Flexure)

$A_{s,min}$ = minimum amount of flexural reinforcement, in^2

A_{sf} = area of tension reinforcement in tension zone required to counterbalance compressive force in overhanging portion of flanges in flanged section, in^2 (Flexure)

A_{sa} = area of bar or wire from which spiral is formed, in^2 (Columns)

A_{st} = total area of longitudinal reinforcement in cross section, in^2

A_{sw} = area of tension reinforcement required to counterbalance compressive force in web or steam of flanged section, or in concrete alone in beams reinforced in compression, in^2 (Flexure)

A_{s1} = area of larger bars in a bundle, in^2 (Reinforcement)
= area of tension reinforcement required under factored load conditions for a rectangular beam with tension reinforcement only, in^2 (Flexure)
= area of steel per ft of slab width, in^2 (Two-way slabs)

A_{s2} = area of smaller bars in the bundle, in^2 (Reinforcement)
= area of tension reinforcement required under factored load conditions to counterbalance compressive force contributed by compressive reinforcement, in^2 (Flexure)

A_{s3} = maximum area of tension reinforcement at which depth of stress block a will be equal to or smaller then flange thickness h_f (Flexure)

A_t = area of one leg of a closed stirrup resisting torsion, within a distance s, in^2 (Shear)

A_{tr} = total cross section area of all transverse reinforcement which is within the spacing s and which crosses the potential plane of splitting through the reinforcement being developed, in^2

A_v = total area of web reinforcement in tension within distance s, measured in direction parallel to longitudinal reinforcement, in^2 (Shear)

A_{vf} = area of shear friction reinforcement, in^2

b = width of compression face of member, in. (Flexure)
= overall cross section dimension of rectangular column, in. (Columns)

b' = capital depth measured from lower surface of slab (Two-way slabs)

b_0 = perimeter of critical section for two-way shear, in.

b_1 = width of the critical section defined in 11.12.1.2(a) measured in the direction of the span for which moments are determined, in.

b_2 = width of the critical section defined in 11.12.1.2(a) measured in the direction perpendicular to b_1
= ratio of long side to short side of concentrated load or reaction area
= width of column transverse to direction of applied moment (= c_2 when there is no capital), in.

b_{dp} = size of square drop panel, ft (Two-way slabs)

b_w = web width, in.
= width of beam stem, in. (Two-way slabs)

B_n = nominal bearing strength of loaded area

c = spacing or cover dimension, in.
= distance from extreme compression fiber to neutral axis

c_1 = size of rectangular or equivalent rectangular column, capital, or bracket measured in the direction of the span for which moments are being determined, in.

c_2 = size of rectangular or equivalent rectangular column, capital, or bracket measured transverse to the direction of the span for which moments are being determined, in.

c_{cl} = clear concrete cover to surface of outer layer of reinforcement, in. (Two-way slabs)

C = compression force, kips (Flexure)
= torsion constant, see Eq. (13-7) (Two-way slabs)

C_c = compression force in concrete, kips (Flexure)

C_m = factor relating actual moment diagram to an equivalent uniform moment diagram (For members braced against sidesway and without transverse loads between supports, $C_m = 0.6 + 0.4(M_1/M_2)$ but not less than 0.4. For all other cases, C_m shall be taken as 1.0.)

C_s = compression force in reinforcement, kips (Flexure)

d = distance from the extreme compression fiber to centroid of tension reinforcement, in. (Flexure, Two-way slabs)

d' = distance from the extreme compression fiber to centroid of compression reinforcement, in. (Flexure)

d_b = nominal diameter of bar, in. (Reinforcement)

d_{be} = equivalent diameter for bundled bars, in. (Reinforcement)

d_{b1} = diameter of a reinforcing bar closest to concrete extreme tensile surface, in. (Two-way slabs)

d_{b1}, d_{b2} = diameters of bars in bundles with two different sizes, in. (Reinforcement)

d_c = distance from extreme tensile surface to center of closest tensile reinforcing bar, in. (Reinforcement, Two-way slabs)

d'_c = distance from extreme tensile fiber to center of gravity of closest bundle or layer of bundles, in. (Reinforcement)

d_{dp} = distance from extreme compression fiber to centroid of tension reinforcement of drop panel, in.

d_s, d_{sp} = nominal diameter of stirrups, in. (Shear, Columns)

d_s = distance from extreme tensile fiber to centroid of tension reinforcement = $t/2$, in. (Reinforcement)

D = dead loads, or their relative internal moments and forces

e = eccentricity of axial load at end of beam, measured from centerline of beam, in. (Flexure)

e' = eccentricity of axial load at end of member, measured from centroid of the tension reinforcement, calculated by conventional methods of frame analysis, in. (Flexure)

e_x = eccentricity along x-axis, in. (Columns)

e_y = eccentricity along y-axis, in. (Columns)

E_c = modulus of elasticity of concrete

$$= 0.033\,w^{1.5}\sqrt{f'_c}\ \text{, ksi}$$

E_{cb} = modulus of elasticity of beam concrete, ksi (Two-way slabs)

E_{cc} = modulus of elasticity of column concrete, ksi (Two-way slabs)

E_{cs} = modulus of elasticity of slab concrete, ksi (Two-way slabs)

E_s = modulus of elasticity of steel reinforcement (29000 ksi) (Flexure)

EI = flexural stiffness of cross section for frame analysis, k-in^2 (Flexure)

= flexural stiffness of compression member, k-in^2 (Flexure)

f_c' = specified compressive strength of concrete, psi

f_{ct} = average tensile splitting strength of light weight aggregate concrete, psi

f_r = $7.5\sqrt{f_c'}$, modulus of rupture of concrete, psi

f_s = calculated tension stress in reinforcement at service loads, ksi (Reinforcement, Two-way slabs)

f'_s = calculated stress in reinforcement in compression, $E_s \epsilon'_s \le f_y$, psi (Flexure)

f_y = specified yield strength of nonprestressed reinforcement, psi (Flexure, Two-way slabs)

f_{yv} = yield strength of closed transverse torsional reinforcement

f_{yl} = yield strength of longitudinal torsional reinforcement

F = flexural coefficient = $\frac{b\,d^2}{12000}$ or M_n/K_n (Flexure)

h = overall thickness of section or thickness of member (Beams, One-way slabs, Two-way slabs)

= diameter of round column or side of a rectangular column, in (Columns)

h_o = diameter of round column, in.

h_c = pier or column dimension parallel to investigated direction (= c_l when there is no capital, for Two-way slabs), in. (Two-way slabs)

h_{core} = core diameter of spiral column = outside column dimension minus cover, in. (Columns)

h_{dp} = total thickness of drop panel (slab thickness plus drop), in. (Two-way slabs)

h_f = effective thickness of a column for slenderness considerations, in.

h_f = flange thickness, in. (Flexure, Deflection)

h_s = thickness of slab, in. (Two-way slabs)

$h_{s(d)}$ = minimum thickness of slab, governed by deflection requirements, in. (Two-way slabs)

$h_{s(s)}$ = minimum thickness of slab, governed by shear requirements, in. (Two-way slabs)

I = moment of inertia of section resisting externally applied loads, in^4 (Shear)

I_c = moment of inertia of gross section of column, in^4 (Columns)

I_{cr} = moment of inertia of cracked section transformed to concrete, in^4 (Deflection)

I_b = moment of inertia about centroidal axis of gross section of beam (including part of adjacent slab section as defined in ACI 318-95, Section 13.2.4), in^4 (Two-way slabs)

I_g = moment of inertia of gross concrete section about the centroidal axis, neglecting reinforcement, in^4 (Deflection)

I_{gt} = gross moment of inertia of T-section, in^4 (Deflection)

I_{se} = moment of inertia of reinforcement about centroidal axis of member cross section, in. (Columns)

J_c = property of assumed critical section analogous to polar moment of inertia (Two-way slabs)

j_f = $(d - 0.5\,h_f) / d$, ratio of lever arm between flange centroid and centroid of tension reinforcement to effective depth d of a section (Flexure)

j_n = $(d - 0.5\,a) / d$, ratio of lever arm between centroid of compression rectangular stress block and tension reinforcement to effective depth d of a rectangular section (Flexure)

k = moment coefficient for flexural members (Flexure)

= steel strength factor used in evaluation of $h_{s(d)}$ (Two-way slabs)

= effective length factor for compression members (Columns)

k_c = column stiffness coefficient (Two-way slabs)

k_s = flexural stiffness coefficient (Two-way slabs)

k_1 = perimeter shear stress factor, in^{-2} (Two-way slabs)

k_2 = moment-shear transfer stress factor, in^{-2} ft^{-1} (Two-way slabs)

k_3 = moment-shear transfer stress factor, in^{-2} ft^{-1} (Two-way slabs)

k_2' = moment-shear transfer stress factor for square column or capital, in^{-2} ft^{-1} (Two-way slabs)

k_3' = moment-shear transfer stress factor for square column or capital, in^{-2} ft^{-1} (Two-way slabs)

K = fracture coefficient used in crack width determination to obtain maximum allowable spacing of reinforcement in two way slabs and plates (Two-way slabs)

= a constant relating to EI and having the same units as EI

K_{a1} = $1728\,\ell_2 / 48E_c$ = coefficient for immediate deflection of beam (Deflection)

K_{a2} = $a_c b / \delta_c\, w$ = coefficient for approximate immediate deflection of beam (Deflection)

K_{a3} = coefficient relating moment at midspan to deflection at midspan (Deflection)

K_{crt} = $\frac{f_r}{12000} \times \frac{h^2}{6}$, coefficient for computing cracking moment of T-section (Deflection)

K_{i1} = I_{cr} / bd^3, coefficient for moment of inertia of cracked rectangular sections with tension reinforcement only (Deflection)

K_{i2} = I_{cr} / bd^3, coefficient for moment of inertia of cracked rectangular sections with compression reinforcement, or T-beams (Deflection)

K_{i3} = I_e / I_g, coefficient for effective moment of inertia (Deflection)

K_{i4} = $I_{gt} / (b_w h^3 / 12)$, coefficient for gross moment of inertia of T-beams (Deflections)

K_n = $12000\ M_n / bd^2 = f_c'\ \omega\ (1 - 0.59\ \omega)$, strength coefficient of resistance, psi (Flexure)

K_{nf} = $\dfrac{0.85 f_c'}{12000}\left(\dfrac{b}{b_w} - 1\right)$, coefficient for computing reinforcement area A_{sf}, psi (Flexure)

K_t = torsional stiffness of transverse torsional member; moment per unit of rotation =

$$\frac{9 E_{cs} C}{l_2 \left(1 - \frac{c_2}{l_2}\right)^3} \text{ (Two-way slabs)}$$

K_{tr} = transverse reinforcement index = $\dfrac{A_{tr} f_{yt}}{1500\ sn}$

= $\dfrac{(b_w - 3.5)(h - 3.5)}{12} \alpha_t$, coefficient for design of torsion reinforcement (Shear)

K_u = strength coefficient for resistance = $M_u/F = 12{,}000\ M_u / (bd^2) = f_c'\omega(1 - 0.59\omega)$

K_v = ρ_{vf} divided by the reinforcement ratio for shear friction reinforcement perpendicular to shear plane (Shear)

= $A_v f_y$, shear coefficient for stirrups (Shear)

K_{v4} = $[6.5 - 5.1\ (N_{uc} / V_u)^{1/2}][1+(64 + 160\ (N_{uc} / V_u)^{3/2} \rho]\ 0.5$ (Shear)

K_{v5} = $2.0 - a / d$ (Shear)

ΣK = Combined flexural stiffness of slab and column (Two-way slabs)

ℓ = span length, ft or in. (Reinforcement, Shear)

= span length of beam, center-to-center of supports (Two-way slabs)

= width of slab strip used to calculate α (Two-way slabs)

= span length of beam or slab, as defined in ACI 318-95, Section 8.7, in. (Columns)

ℓ_1 to ℓ_5 = minimum spans required for bar development depending on type of span and support and percentage of bars extended into support, ft and in. (Reinforcement)

ℓ_1 = length of slab span in the direction in which moments are being determined, measured center-to-center supports (Two-way slabs)

ℓ_2 = length of slab span transverse to ℓ_1, measured center-to-center supports (Two-way slabs)

= width of interior design frame (transverse to ℓ_1), measured from center line to center line of adjacent slab panels (ACI 318-95, Section 13.6.2.3) (Two-way slabs)

= width of exterior design frame , measured from center line to center line of adjacent slab panels (ACI 318-95, Section 13.6.2.3) (Two-way slabs)

ℓ_a = embedment length at support or at point of inflection, in. (Reinforcement)

= average of ℓ_ℓ or ℓ_s (Two-way slabs)

ℓ_c = length of compression member in a frame, measured from center to center of joints in the frame

= vertical distance between supports, in.

= height of column

ℓ_d = development length, in. (Reinforcement)

ℓ_d' = usable (available) anchorage length, in. (Reinforcement)

ℓ_{db} = basic development length of straight bars, in as specified in Sections 12.2.2 and 12.3.2 of ACI 318-95 (Reinforcement)

ℓ_{dh} = development length of hooked bars, to exterior face of bar at the bend, in. (Reinforcement)

ℓ_{hb} = basic development length of standard hook in tension, in. (Reinforcement)

ℓ_ℓ = longer of ℓ_1 or width of design frame ℓ_2

ℓ_n = clear span measured face to face of supports (Reinforcement)

= clear span measured face to face of supports or face to face of beams in slabs with beams (Two-way slabs)

ℓ_s = shorter of ℓ_1 or width of design frame ℓ_2

ℓ_u = unsupported height of column (Two-way slabs)

ℓ_v = length of shearhead arm from centroid of concentrated load or reaction, in. (Shear)

L = live loads, or their related internal moments and forces (Two-way slabs)

= magnified factored moment to be used for design of column (Columns)

m = distance from exterior face of edge panel to center of exterior column (Two-way slabs)

M = fixed-end moment coefficient (Two-way slabs)

M_1 = smaller factored end moment on compression member, positive if member is bent in single curvature, negative if bent in double curvature, kip-ft (Columns)

M_{1ns} = factored end moment on compression member at the end at which M_1 acts, due to loads that cause no appreciable side sway, calculated using a first order elastic frame analysis

= unbalanced moment at support, in direction of span for which moments are being determined (Two-way slabs)

M_{1s} = factored end moment on compression member at the end at which M_1 acts, due to loads that cause appreciable side sway, calculated using a first order elastic frame analysis

M_2 = larger factored end moment on compression member, always positive (Columns)

= unbalanced moment perpendicular to M_1 (Two-way slabs)

$M_{2,min}$ = minimum value of M_2

M_{2ns} = factored end moment on compression member at the end at which M_2 acts, due to loads that cause no appreciable side sway, calculated using a first order elastic frame analysis

M_{2s} = factored end moment on compression member at the end at which M_2 acts, due to loads that cause appreciable side sway, calculated using a first order elastic frame analysis

$-M_i$ = factored negative moment at interior column (except first interior column), kip-ft (Two-way slabs)

$+M_i$ = factored positive moment at interior span , kip-ft (Two-way slabs)

$-M_e$ = factored negative moment at exterior support, kip-ft (Two-way slabs)

$-M_{ie}$ = factored negative moment at first interior support, kip-ft (Two-way slabs)

$+M_m$ = factored positive moment at midspan of exterior span, kip-ft (Two-way slabs)

M_a = maximum moment in member at stage for which deflection is being computed, in-lb

M_c = moment at center of beam or a moment value related to the deflection, kip-ft (Deflection)

M_{cr} = cracking moment of gross concrete section $= I_g f_r / y_t$, in-ft (Deflection)

M_d = moment due to dead load, kip-ft (Deflection)

$M_{d+\ell}$ = moment due to dead and live load, kip-ft (Deflection)

M_ℓ = moment due to live load, kip-ft (Deflection)

= reinforcing spacing grid index for crack control (Two-way slabs)

M_n = nominal moment strength of section, kip-ft (Flexure)

M_{nc} = nominal moment strength of section with compression and tension reinforcement, kip-ft (Flexure)

M_{nf} = nominal moment strength of overhanging flanges of T-beam, kip-ft (Flexure)

M_{nw} = nominal moment strength of rectangular beam (or web of T-beam) when reinforced for tension only, kip-ft (Flexure)

M_{n1} = nominal moment strength of a cross section before compression reinforcement and extra tension reinforcement are added = M_n - M_{n2}, kip-ft (Flexure)

M_{n2} = that portion of M_n assigned to compression reinforcement or flange regions of I and T-sections, kip-ft (Flexure)

M_0 = total factored static moment (Flexure)

M_s = moment at point of zero shear (Shear)

= moment due to loads causing appreciable sway

M_{s1} = moment at left support, for deflection, kip-ft (Deflection)

M_{s2} = moment at right support, for deflection, kip-ft (Deflection)

M_u = applied factored moment at section, kip-ft (Flexure, Two-way slabs)

M_{us} = factored moment acting on section if axial force N_u is considered to act at centroid of tension reinforcement, kip-ft (Flexure)

M_{nx} = nominal moment strength about x-axis (Columns)

M_{ny} = nominal moment strength about y-axis (Columns)

M_{nox} = equivalent uniaxial moment strength about x-axis (Columns)

M_{noy} = equivalent uniaxial moment strength about y-axis (Columns)

M_{ux} = factored moment about x-axis (Columns)

M_{uy} = factored moment about y-axis (Columns)

M_w = service wind load moment, kip-ft (Columns)

n = modular ratio = E_s / E_c (Deflection)

= equivalent number of bars = tensile reinforcement area over largest bar area, for crack control (Reinforcement)

= number of longitudinal torsion bars (Shear)

= number of bar diameters between center and perimeter of bend (Reinforcement)

= number of bars in flexural tension reinforcement (Reinforcement)

n' = equivalent number of bar bundles = projected surface of bundle over actual surface of bundle (Reinforcement)

N_s = compressive force on reinforcement in a cross section, kip-ft (Flexure)

N_u = factored axial load normal to cross section occurring simultaneously with V_u - to be taken as positive for compression, negative for tension, and to include effects of tension due to shrinkage and creep, kips (Shear)

N_{uc} = factored tensile force on bracket or corbel acting simultaneously with V_u, kips (Shear)

P = service concentrated load on beam, kips (Deflection)

P_b = nominal axial load strength (balanced strain conditions), kips (Columns)

P_c = critical axial load, kips (Columns)

P_d = service axial dead load, kips (Columns)

P_ℓ = service axial live load, kips (Columns)

P_n = nominal axial load strength at given eccentricity, kips (Columns)

P_{ni} = approximation of nominal axial load strength at eccentricities e_x and e_y, kips (Columns)

$P_{nx}*$ = nominal axial load strength for eccentricity e_y along x axis only, x-axis being axis of bending, kips (Columns)

$P_{ny}*$ = nominal axial load strength for eccentricity e_x along y axis only, y-axis being axis of bending, kips (Columns)

P_o = nominal axial load strength at zero eccentricity, kips (Columns)

P_u = factored axial load at given eccentricity, kips (Columns)

P_{ux} = factored axial load for eccentricity e_y, along y-axis only, kips (Flexure)

P_{ux} = factored axial load for eccentricity e_x, along x-axis only ≤ ϕP_{ny}, kips (Flexure)

P_{uy} = factored axial load for eccentricity e_y, along y-axis only ≤ ϕP_{nx}, kips (Flexure)

p_{cp} = outside perimeter of cross-section A_{cp}, in.

p_h = perimeter of center line of outermost closed transverse torsional reinforcement, in.

q_s = stability index for story

r = radius of gyration of cross section of compressive member

s = center to center spacing of bars, in.
= center to center spacing of web reinforcement, in. (Shear)
= maximum spacing of transverse reinforcement within ℓ_d center to center, in.

s_l = required stirrup distance, ft (Shear)

s_1, s_2 = center to center spacing of reinforcement in either direction "1" or "2", in. (Shear)

s_l = reinforcement spacing measured in direction of span for which moments and crack control are being analyzed, in. (Two-way slabs)

s_2 = reinforcement spacing measured perpendicular to spanwise direction of span for which moments and crack control are being analyzed, in. (Two-way slabs)

s_b, S_b = clear spacing between bars or bundles of bars, in. (Reinforcement)

S = elastic section modulus of section, in^3
= pitch of spiral, center to center of bar (Columns)

t = thickness of tension area for crack control, in. (Reinforcement)
= thickness of wall of hollow section, in.

T = load caused by the cumulative effect of temperature, creep, shrinkage, differential settlement, and temperature
= tension force on reinforcement, kips (Flexure)

T_s = nominal torsional moment strength provided by torsion reinforcement (Reinforcement)

T_u = factored torsional moment at section (Shear)

u = beam width factor in ratio u / h_s, used in calculation of α_f; $u = b$ for interior beam; $u = 2b$ for edge beam (Two-way slabs)

v_c = $(V_c / b_w d)$, nominal shear stress carried by concrete, psi (Shear)

v_{cw} = shear stress at diagonal cracking due to all factored loads, when such cracking is result of excessive principal tensile stresses in web, psi (Shear)

v_n = $(V_n / b_w d)$, nominal shear stress, psi (Shear)
= (V_n / bd), nominal shear stress, psi (Two-way slabs)

v_s = $(V_s / b_w d)$, nominal shear stress carried by reinforcement, psi (Shear)

V_c = nominal shear strength attributable to concrete, kips (Shear, Two-way slabs)

V_s = nominal shear strength attributable to shear reinforcement, kips (Shear)

V_n = nominal shear strength at section, kips (Shear)

V_u = factored shear force, kips (Shear)
= factored horizontal shear in story
= factored perimeter shear force on critical shear section (Two-way slabs)

V = factored shear force caused by wall supported slab (Two-way slabs)

w = crack width, in. (Reinforcement, Two-way slabs)
= pattern loading unit load, psf (Two-way slabs)

w_c = unit weight of concrete, pcf

w_d = uniformly distributed factored dead load, kips per ft (Two-way slabs), or kips per in

w_l = uniformly distributed factored live load, kips per ft (Two-way slabs), or kips per in

w_{max} = maximum tolerable crack width for type of exposure, in. (Two-way slabs)

w_{mech} = mechanical load per unit area, psf (Two-way slabs)

w_s = superimposed dead load, psf (total dead load not including self weight of slab, Two-way slabs)

w_u = factored load per unit length of beam (Flexure)
= factored load per unit area, psf; = (typically) $1.4\, w_d + 1.7\, w_l$ (Two-way slabs)

x = variable distance
= shorter overall dimension of rectangular part of section, in. (Two-way Slabs)

x = distance between centroid of column and centroid of shear section, in. (Two-way slabs)

x_{bc} = minimum clear spacing between bundled bars, in. (Reinforcement)

x_e = distance from extreme tensile fiber to neutral axis, in. (Deflection)

y = variable distance
= longer overall dimension of rectangular part of section, in. (Shear, Two-way slabs)

= centroidal distance from bottom of bundled bars, in. (Reinforcement)

y_t = distance from centroidal axis of gross section, neglecting reinforcement, to extreme fiber in tension, in.

z = quantity limiting distribution of flexural reinforcement (Reinforcement)

α = angle between shear reinforcement and longitudinal axis of member, degrees (Shear)

= bar location factor

= relative beam stiffness; ratio of flexural stiffness of beam section to flexural stiffness of a width of slab bounded laterally by center lines of adjacent panels (if any) on each side of beam = $(E_{cb}I_b) / (E_{cs}I_s)$ (Two-way slabs)

α_1 = α in direction ℓ_1

α_2 = α in direction ℓ_2

α_b = b/b_w(Deflection)

α_s = constant used to compute V_c in nonprestressed slabs

α_m = average value of α for all beams on edges of slab panel (Two-way slabs)

β = ratio of distance between extreme tensile fiber and neutral axis to distance between neutral axis and centroid of tensile reinforcement , x_e / x_c (Reinforcement)

= coating factor

= ratio of long to short clear spans (ℓ_n) of a slab panel (Two-way slabs)

= biaxial bending design constant = constant portion of uniaxial factored moment strengths M_{nox} and M_{noy} which may be permitted to act simultaneously on the column cross section (Columns)

β_1 = a coefficient relating depth of equivalent rectangular stress block to depth from compression face to neutral axis = 0.85 for $f_c' \leq 4.0$ ksi and $0.85 - 0.05(f_c' - 4.0)$ for $f_c' > 4.0$ ksi, ($\beta_1 \geq 0.65$), (Flexure)

β_a = ratio of dead load per unit area to live load per unit area (in each case without load factors) (Two-way slabs)

β_c = design coefficient for deflection $= m\rho'/n\rho$; $= (n-1)\rho'/n\rho$; $= (b/b_w - 1)h_f/dn\rho$ (Deflection)

= ratio of long side to short side of concentrated load or reaction area (Shear, Two-way slabs)

β_d = ratio of maximum factored axial dead load to maximum total factored axial load, where the load is due to gravity effects only in the calculation for P_c in Eq. (10.7), or ratio of the maximum factored sustained lateral load to the maximum total factored lateral load in the story in the calculation for P_c in Eq. (10.8) (Columns)

β_f = ratio between long and short center-to-center spans (ℓ_ℓ / ℓ_s) (Two-way slabs)

β_h = h_f / h (Deflection)

β_t = ratio of torsional stiffness of edge beam to flexural stiffness of a width of slab equal to span length of beam, center-to-center of supports = $(E_{cb}C) / (E_{cs}I_s)$ (Two-way slabs)

β_u = ratio of c to d (Flexure)

β_v = $\sin\alpha + \cos\alpha$ for inclined stirrups (Shear)

γ = bar size factor

= factor in calculating slab thickness required by deflection; = the value of the denominator of Eq. (9-11), or (9-13) divided by 1000 (Two-way slabs)

= ratio of distance between centroid of outer rows of bars and thickness of cross section, in the direction of bending (Columns)

γ_f = fraction of unbalanced moment transferred to column by flexure, Eq. (13-1) (Two-way slabs)

γ_p = moment at point of zero shear to simple span maximum moment (Shear)

γ_v = fraction of unbalanced moment transferred by eccentricity of shear at slab-column connection; $= 1 - \gamma_f$ (Two-way slabs)

δ = coefficient depending on type of span and degree of reinforcement (Deflection)

δ_b = moment magnification factor for columns braced against sidesway (Columns)

δ_s = moment magnification factor for frames not braced against sidesway, to reflect lateral drift resulting from lateral and gravity loads (Columns)

ϵ = unit strain, in. / in. (Flexure)

ϵ_c = unit strain in concrete (Flexure)

ϵ_s = unit strain in tension reinforcement (Flexure)

ϵ'_s = unit strain in compression reinforcement (Flexure)

ϵ_y = f_y / E_s nominal yield strain of reinforcement (Flexure)

θ = angle of compression diagonals in truss analogy for torsion

λ = lightweight aggregate concrete factor. When lightweight concrete is used $f_{ct}^2/(6.7^2 f_c')$. When normal weight concrete is used 1.0 (Shear)

= ratio of M_n with compression reinforcement to M_n without compression reinforcement (Columns)

= multiplier for additional long-time deflection, equals to ratio of creep and shrinkage deflection to immediate deflection due to sustained loads (Deflection)

λ_m = a coefficient relating development length to minimum required span length

μ = coefficient of friction

ξ = time-dependent factor for sustained load (Deflection)

ξ = dimensionless constant used in computing I_g and I_{se} (Columns)

ρ = tension reinforcement ratio = A_s / bd (Flexure)

ρ' = compression reinforcement ratio= A'_s / bd (Flexure)

ρ_b = reinforcement ratio producing balanced conditions (Flexure)

ρ_{bc} = balanced percentage of reinforcement for a section with compression reinforcement (Flexure)

ρ_g = A_{st} / A_g = ratio of total reinforcement area to cross-sectional area of column (Columns)

ρ_f = $A_{sf} / b_w d$ (Flexure)

ρ_{tl} = active steel ratio = $A_{st} / (24 d_c)$ (Two-way slabs)

ρ_{vf} = reinforcement ratio for shear friction reinforcement (Shear)

ϕ = strength reduction factor as defined in Section 9.3 of ACI 318-95

μ_e, μ_{ie}, μ_m = factors used in distribution of moment in an exterior span (Two-way slabs)

ω = coefficient indicating relative strength of reinforcement and concrete in member
= $\rho f_y / f_c'$ (Flexure, Two-way slabs)

ψ = ratio of sum of stiffness $\Sigma(I / \ell_c)$ of compression members in a plane at one end of a compression member

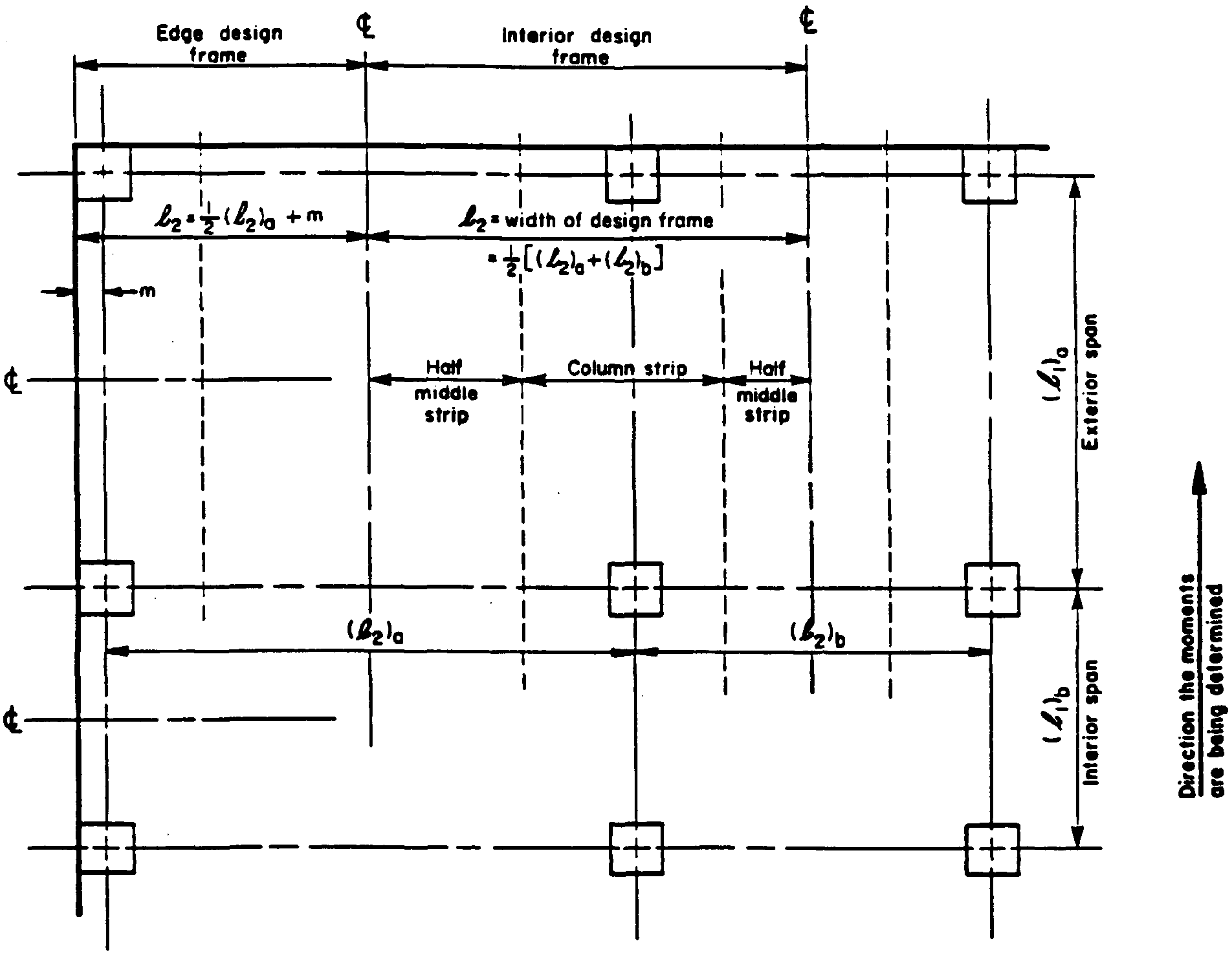


Fig. 1—Notation for slabs

DESIGN AIDS

FLEXURE

FLEXURE 1 - Reinforcement ratios and a_n for quick approximate design of rectangular beams with no compression reinforcement

Reference: ACI 318-95 Sections 9.3.2, 10.2, 10.3.1-10.3.3, and 10.5.1 and ACI 318R-95 Section 10.3.1

$$A_s = \frac{M_n}{a_n d} \quad \textit{where } M_n \textit{ is in kip-ft and } d \textit{ in in.}$$

$$a_n = \frac{f_y\left(1 - \frac{a}{2d}\right)}{12{,}000} \qquad M_n \geq \frac{M_u}{\phi}$$

$$\textit{Taking } \left(1 - \frac{a}{2d}\right) = 0.89 \textit{ (which is a typical value)} \Rightarrow a_n = \frac{0.89 f_y}{12{,}000}$$

$$\rho_{min} = \frac{3\sqrt{f'_c}}{f_y} \geq \frac{200}{f_y}, \quad \rho_{max} = \frac{0.75\beta_1\left(0.85 f'_c\right)}{f_y}\left(\frac{87{,}000}{87{,}000 + f_y}\right), \quad \textit{preferred } \rho \cong 0.5\rho_{max}$$

f'_c	3,000 psi $\beta_1 = 0.85$	4,000 psi $\beta_1 = 0.85$	5,000 psi $\beta_1 = 0.80$	6,000 psi $\beta_1 = 0.75$
f_y = 40,000 psi				
a_n	2.97	2.97	2.97	2.97
ρ_{min}	0.0050	0.0050	0.0053	0.0058
preferred ρ	0.0139	0.0186	0.0218	0.0246
ρ_{max}	0.0278	0.0371	0.0437	0.0491
f_y = 60,000 psi				
a_n	4.45	4.45	4.45	4.45
ρ_{min}	0.0033	0.0033	0.0035	0.0039
preferred ρ	0.0080	0.0107	0.0126	0.0141
ρ_{max}	0.0160	0.0214	0.0252	0.0283
f_y = 75,000 psi				
a_n	5.56	5.56	5.56	5.56
ρ_{min}	0.0027	0.0027	0.0028	0.0031
preferred ρ	0.0058	0.0078	0.0091	0.0103
ρ_{max}	0.0116	0.0155	0.0183	0.0205

For use of this Design Aid, see Flexure Example 1

FLEXURE 2.1 - Nominal strength coefficients for design of rectangular beams with tension reinforcement only, f'_c = 3000 psi

Reference: ACI 318-95 Sections 9.3.2, 10.2, and 10.3.1-10.3.3 and ACI 318R-95 Section 10.3.1

$M_n \geq M_u/\phi$

$M_n = K_n F$, ft-kips

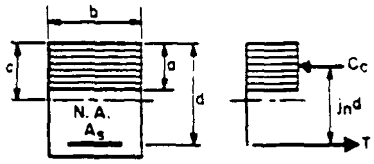
where $K_n = f'_c \omega j_n$

$\omega = \rho f_y / f'_c$

and $F = bd^2/12{,}000$ (from FLEXURE 5)

Also, $M_n = A_s d a_n$ (A_s in in.^{2})

where $a_n = f_y j_n / 12{,}000$



$c/d = 1.18(\omega/\beta_1)$

$a/d = \beta_1(c/d)$

$\beta_1 = 0.85$

$j_n = 1 - (a/2d)$

$= 1 - \omega/1.7$

		f'_c = 3000 psi								
		f_y = 40,000 psi		f_y = 60,000 psi		f_y = 75,000 psi				
ω	K_n	ρ^*	a_n	ρ^*	a_n	ρ^*	a_n	c/d	a/d	j_n
0.020	59	0.0015	3.29	0.0010	4.94	0.0008	6.18	0.028	0.024	0.988
0.030	88	0.0023	3.27	0.0015	4.91	0.0012	6.14	0.042	0.035	0.982
0.040	117	0.0030	3.25	0.0020	4.88	0.0016	6.10	0.056	0.047	0.976
0.050	146	0.0038	3.24	0.0025	4.85	0.0020	6.07	0.069	0.059	0.971
0.060	174	0.0045	3.22	0.0030	4.82	0.0024	6.03	0.083	0.071	0.965
0.070	201	0.0053	3.20	0.0035	4.79	0.0028	5.99	0.097	0.083	0.959
0.080	229	0.0060	3.18	0.0040	4.76	0.0032	5.96	0.111	0.094	0.953
0.090	256	0.0068	3.16	0.0045	4.74	0.0036	5.92	0.125	0.106	0.947
0.100	282	0.0075	3.14	0.0050	4.71	0.0040	5.88	0.139	0.118	0.941
0.110	309	0.0083	3.12	0.0055	4.68	0.0044	5.85	0.153	0.130	0.935
0.120	335	0.0090	3.10	0.0060	4.65	0.0048	5.81	0.167	0.142	0.929
0.130	360	0.0097	3.08	0.0065	4.62	0.0052	5.77	0.180	0.153	0.924
0.140	385	0.0105	3.06	0.0070	4.59	0.0056	5.74	0.194	0.165	0.918
0.150	410	0.0113	3.04	0.0075	4.56	0.0060	5.70	0.208	0.177	0.912
0.160	435	0.0120	3.02	0.0080	4.53	0.0064	5.66	0.222	0.189	0.906
0.170	459	0.0128	3.00	0.0085	4.50	0.0068	5.63	0.236	0.201	0.900
0.180	483	0.0135	2.98	0.0090	4.47	0.0072	5.59	0.250	0.212	0.894
0.190	506	0.0143	2.96	0.0095	4.44	0.0076	5.55	0.264	0.224	0.888
0.200	529	0.0150	2.94	0.0100	4.41	0.0080	5.51	0.278	0.236	0.882
0.210	552	0.0158	2.92	0.0105	4.38	0.0084	5.48	0.292	0.248	0.876
0.220	575	0.0165	2.90	0.0110	4.35	0.0088	5.44	0.305	0.260	0.871
0.230	597	0.0173	2.88	0.0115	4.32	0.0092	5.40	0.319	0.271	0.865
0.240	618	0.0180	2.86	0.0120	4.29	0.0096	5.37	0.333	0.283	0.859
0.250	640	0.0188	2.84	0.0125	4.26	0.0100	5.33	0.347	0.295	0.853
0.260	661	0.0195	2.82	0.0130	4.24	0.0104	5.29	0.361	0.307	0.847
0.270	681	0.0203	2.80	0.0135	4.21	0.0108	5.26	0.375	0.319	0.841
0.280	702	0.0210	2.78	0.0140	4.18	0.0112	5.22	0.389	0.330	0.835
0.290	722	0.0218	2.76	0.0145	4.15	0.0116	5.18	0.403	0.342	0.829
0.300	741	0.0225	2.75	0.0150	4.12			0.416	0.354	0.824
0.310	760	0.0233	2.73	0.0155	4.09			0.430	0.366	0.818
0.320	779	0.0240	2.71	0.0160	4.06			0.444	0.378	0.812
0.330	798	0.0248	2.69					0.458	0.389	0.806
0.340	816	0.0255	2.67					0.472	0.401	0.800
0.350	834	0.0263	2.65					0.486	0.413	0.794
0.360	851	0.0270	2.63					0.500	0.425	0.788
0.370	868	0.0278	2.61					0.514	0.437	0.782
ρ_{max}		0.0278		0.0160		0.0116				

* Values of ρ above light rule are less than ρ_{min}; $\rho_{min} = 3\sqrt{f'_c}/f_y \geq 200/f_y$ as provided in Section 10.5.1 of ACI 318-95

For use of this Design Aid, see Flexure Examples 1, 2, 3 and 4

FLEXURE 2.2 - Nominal strength coefficients for design of rectangular beams with tension reinforcement only, f'_c = 4000 psi

Reference: ACI 318-95 Sections 9.3.2, 10.2, and 10.3.1-10.3.3 and ACI 318R-95 Section 10.3.1

$M_n \geq M_u/\phi$

$M_n = K_n F$, ft-kips

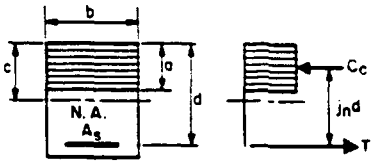
where $K_n = f'_c \omega j_n$

$\omega = \rho f_y / f'_c$

and $F = bd^2/12{,}000$ (from FLEXURE 5)

Also, $M_n = A_s d a_n$ (A_s in in.^{2})

where $a_n = f_y j_n / 12{,}000$



$c/d = 1.18(\omega/\beta_1)$

$a/d = \beta_1(c/d)$

$\beta_1 = 0.85$

$j_n = 1 - (a/2d)$

$= 1 - \omega/1.7$

		f'_c = 4000 psi								
		f_y = 40,000 psi		f_y = 60,000 psi		f_y = 75,000 psi				
ω	K_n	ρ^*	a_n	ρ^*	a_n	ρ^*	a_n	c/d	a/d	j_n
0.020	79	0.0020	3.29	0.0013	4.94	0.0011	6.18	0.028	0.024	0.988
0.030	118	0.0030	3.27	0.0020	4.91	0.0016	6.14	0.042	0.035	0.982
0.040	156	0.0040	3.25	0.0027	4.88	0.0021	6.10	0.056	0.047	0.976
0.050	194	0.0050	3.24	0.0033	4.85	0.0027	6.07	0.069	0.059	0.971
0.060	232	0.0060	3.22	0.0040	4.82	0.0032	6.03	0.083	0.071	0.965
0.070	268	0.0070	3.20	0.0047	4.79	0.0037	5.99	0.097	0.083	0.959
0.080	305	0.0080	3.18	0.0053	4.76	0.0043	5.96	0.111	0.094	0.953
0.090	341	0.0090	3.16	0.0060	4.74	0.0048	5.92	0.125	0.106	0.947
0.100	376	0.0100	3.14	0.0067	4.71	0.0053	5.88	0.139	0.118	0.941
0.110	412	0.0110	3.12	0.0073	4.68	0.0059	5.85	0.153	0.130	0.935
0.120	446	0.0120	3.10	0.0080	4.65	0.0064	5.81	0.167	0.142	0.929
0.130	480	0.0130	3.08	0.0087	4.62	0.0069	5.77	0.180	0.153	0.924
0.140	514	0.0140	3.06	0.0093	4.59	0.0075	5.74	0.194	0.165	0.918
0.150	547	0.0150	3.04	0.0100	4.56	0.0080	5.70	0.208	0.177	0.912
0.160	580	0.0160	3.02	0.0107	4.53	0.0085	5.66	0.222	0.189	0.906
0.170	612	0.0170	3.00	0.0113	4.50	0.0091	5.63	0.236	0.201	0.900
0.180	644	0.0180	2.98	0.0120	4.47	0.0096	5.59	0.250	0.212	0.894
0.190	675	0.0190	2.96	0.0127	4.44	0.0101	5.55	0.264	0.224	0.888
0.200	706	0.0200	2.94	0.0133	4.41	0.0107	5.51	0.278	0.236	0.882
0.210	736	0.0210	2.92	0.0140	4.38	0.0112	5.48	0.292	0.248	0.876
0.220	766	0.0220	2.90	0.0147	4.35	0.0117	5.44	0.305	0.260	0.871
0.230	796	0.0230	2.88	0.0153	4.32	0.0123	5.40	0.319	0.271	0.865
0.240	824	0.0240	2.86	0.0160	4.29	0.0128	5.37	0.333	0.283	0.859
0.250	853	0.0250	2.84	0.0167	4.26	0.0133	5.33	0.347	0.295	0.853
0.260	881	0.0260	2.82	0.0173	4.24	0.0139	5.29	0.361	0.307	0.847
0.270	908	0.0270	2.80	0.0180	4.21	0.0144	5.26	0.375	0.319	0.841
0.280	936	0.0280	2.78	0.0187	4.18	0.0149	5.22	0.389	0.330	0.835
0.290	962	0.0290	2.76	0.0193	4.15	0.0155	5.18	0.403	0.342	0.829
0.300	988	0.0300	2.75	0.0200	4.12			0.416	0.354	0.824
0.310	1014	0.0310	2.73	0.0207	4.09			0.430	0.366	0.818
0.320	1039	0.0320	2.71	0.0213	4.06			0.444	0.378	0.812
0.330	1064	0.0330	2.69					0.458	0.389	0.806
0.340	1088	0.0340	2.67					0.472	0.401	0.800
0.350	1112	0.0350	2.65					0.486	0.413	0.794
0.360	1135	0.0360	2.63					0.500	0.425	0.788
0.370	1158	0.0370	2.61					0.514	0.437	0.782
ρ_{max}		0.0371		0.0214		0.0155				

* Values of ρ above light rule are less than ρ_{min}; $\rho_{min} = 3\sqrt{f'_c}/f_y \geq 200/f_y$ as provided in Section 10.5.1 of ACI 318-95

For use of this Design Aid, see Flexure Examples 1, 2, 3 and 4

FLEXURE 2.3 - Nominal strength coefficients for design of rectangular beams with tension reinforcement only, f'_c = 5000 psi

Reference: ACI 318-95 Sections 9.3.2, 10.2, and 10.3.1-10.3.3 and ACI 318R-95 Section 10.3.1

$M_n \geq M_u/\phi$

$M_n = K_n F$, ft-kips

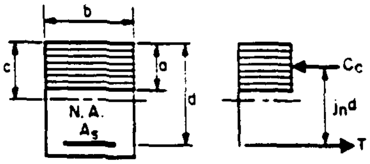
where $K_n = f'_c \omega j_n$

$\omega = \rho f_y / f'_c$

and $F = bd^2/12{,}000$ (from FLEXURE 5)

Also, $M_n = A_s d a_n$ (A_s in in.^{2})

where $a_n = f_y j_n / 12{,}000$



$c/d = 1.18(\omega/\beta_1)$

$a/d = \beta_1(c/d)$

$\beta_1 = 0.80$

$j_n = 1 - (a/2d)$

$= 1 - \omega/1.7$

		f'_c = 5000 psi								
		f_y = 40,000 psi		f_y = 60,000 psi		f_y = 75,000 psi				
ω	K_n	ρ*	a_n	ρ*	a_n	ρ*	a_n	c/d	a/d	j_n
0.020	99	0.0025	3.29	0.0017	4.94	0.0013	6.18	0.030	0.024	0.988
0.030	147	0.0038	3.27	0.0025	4.91	0.0020	6.14	0.044	0.035	0.982
0.040	195	0.0050	3.25	0.0033	4.88	0.0027	6.10	0.059	0.047	0.976
0.050	243	0.0063	3.24	0.0042	4.85	0.0033	6.07	0.074	0.059	0.971
0.060	289	0.0075	3.22	0.0050	4.82	0.0040	6.03	0.089	0.071	0.965
0.070	336	0.0088	3.20	0.0058	4.79	0.0047	5.99	0.103	0.083	0.959
0.080	381	0.0100	3.18	0.0067	4.76	0.0053	5.96	0.118	0.094	0.953
0.090	426	0.0113	3.16	0.0075	4.74	0.0060	5.92	0.133	0.106	0.947
0.100	471	0.0125	3.14	0.0083	4.71	0.0067	5.88	0.147	0.118	0.941
0.110	514	0.0138	3.12	0.0092	4.68	0.0073	5.85	0.162	0.130	0.935
0.120	558	0.0150	3.10	0.0100	4.65	0.0080	5.81	0.177	0.142	0.929
0.130	600	0.0163	3.08	0.0108	4.62	0.0087	5.77	0.192	0.153	0.924
0.140	642	0.0175	3.06	0.0117	4.59	0.0093	5.74	0.206	0.165	0.918
0.150	684	0.0188	3.04	0.0125	4.56	0.0100	5.70	0.221	0.177	0.912
0.160	725	0.0200	3.02	0.0133	4.53	0.0107	5.66	0.236	0.189	0.906
0.170	765	0.0213	3.00	0.0142	4.50	0.0113	5.63	0.251	0.201	0.900
0.180	805	0.0225	2.98	0.0150	4.47	0.0120	5.59	0.266	0.212	0.894
0.190	844	0.0238	2.96	0.0158	4.44	0.0127	5.55	0.280	0.224	0.888
0.200	882	0.0250	2.94	0.0167	4.41	0.0133	5.51	0.295	0.236	0.882
0.210	920	0.0263	2.92	0.0175	4.38	0.0140	5.48	0.310	0.248	0.876
0.220	958	0.0275	2.90	0.0183	4.35	0.0147	5.44	0.325	0.260	0.871
0.230	994	0.0288	2.88	0.0192	4.32	0.0153	5.40	0.339	0.271	0.865
0.240	1031	0.0300	2.86	0.0200	4.29	0.0160	5.37	0.354	0.283	0.859
0.250	1066	0.0313	2.84	0.0208	4.26	0.0167	5.33	0.369	0.295	0.853
0.260	1101	0.0325	2.82	0.0217	4.24	0.0173	5.29	0.384	0.307	0.847
0.270	1136	0.0338	2.80	0.0225	4.21	0.0180	5.26	0.398	0.319	0.841
0.280	1169	0.0350	2.78	0.0233	4.18			0.413	0.330	0.835
0.290	1203	0.0363	2.76	0.0242	4.15			0.428	0.342	0.829
0.300	1235	0.0375	2.75	0.0250	4.12			0.443	0.354	0.824
0.310	1267	0.0388	2.73					0.457	0.366	0.818
0.320	1299	0.0400	2.71					0.472	0.378	0.812
0.330	1330	0.0413	2.69					0.487	0.389	0.806
0.340	1360	0.0425	2.67					0.502	0.401	0.800
0.350	1390							0.516	0.413	0.794
0.360	1419							0.531	0.425	0.788
0.370	1447							0.546	0.437	0.782
ρ_{max}		0.0437		0.0252		0.0183				

* Values of ρ above light rule are less than ρ_{min}; $\rho_{min} = 3\sqrt{f'_c}/f_y \geq 200/f_y$ as provided in Section 10.5.1 of ACI 318-95

For use of this Design Aid, see Flexure Examples 1, 2, 3 and 4

FLEXURE 2.4 - Nominal strength coefficients for design of rectangular beams with tension reinforcement only, f'_c = 6000 psi

Reference: ACI 318-95 Sections 9.3.2, 10.2, and 10.3.1-10.3.3 and ACI 318R-95 Section 10.3.1

$M_n \geq M_u/\phi$

$M_n = K_n F$, ft-kips

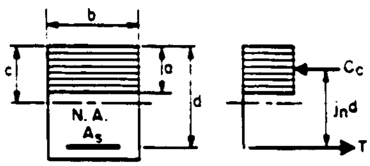
where $K_n = f'_c \omega j_n$

$\omega = \rho f_y / f'_c$

and $F = bd^2/12{,}000$ (from FLEXURE 5)

Also, $M_n = A_s d a_n$ (A_s in in.^{2})

where $a_n = f_y j_n / 12{,}000$



$c/d = 1.18(\omega/\beta_1)$

$a/d = \beta_1(c/d)$

$\beta_1 = 0.75$

$j_n = 1 - (a/2d)$

$= 1 - \omega/1.7$

		f'_c = 6000 psi								
		f_y = 40,000 psi		f_y = 60,000 psi		f_y = 75,000 psi				
ω	K_n	ρ^*	a_n	ρ^*	a_n	ρ^*	a_n	c/d	a/d	j_n
0.020	119	0.0030	3.29	0.0020	4.94	0.0016	6.18	0.031	0.024	0.988
0.030	177	0.0045	3.27	0.0030	4.91	0.0024	6.14	0.047	0.035	0.982
0.040	234	0.0060	3.25	0.0040	4.88	0.0032	6.10	0.063	0.047	0.976
0.050	291	0.0075	3.24	0.0050	4.85	0.0040	6.07	0.079	0.059	0.971
0.060	347	0.0090	3.22	0.0060	4.82	0.0048	6.03	0.094	0.071	0.965
0.070	403	0.0105	3.20	0.0070	4.79	0.0056	5.99	0.110	0.083	0.959
0.080	457	0.0120	3.18	0.0080	4.76	0.0064	5.96	0.126	0.094	0.953
0.090	511	0.0135	3.16	0.0090	4.74	0.0072	5.92	0.142	0.106	0.947
0.100	565	0.0150	3.14	0.0100	4.71	0.0080	5.88	0.157	0.118	0.941
0.110	617	0.0165	3.12	0.0110	4.68	0.0088	5.85	0.173	0.130	0.935
0.120	669	0.0180	3.10	0.0120	4.65	0.0096	5.81	0.189	0.142	0.929
0.130	720	0.0195	3.08	0.0130	4.62	0.0104	5.77	0.205	0.153	0.924
0.140	771	0.0210	3.06	0.0140	4.59	0.0112	5.74	0.220	0.165	0.918
0.150	821	0.0225	3.04	0.0150	4.56	0.0120	5.70	0.236	0.177	0.912
0.160	870	0.0240	3.02	0.0160	4.53	0.0128	5.66	0.252	0.189	0.906
0.170	918	0.0255	3.00	0.0170	4.50	0.0136	5.63	0.267	0.201	0.900
0.180	966	0.0270	2.98	0.0180	4.47	0.0144	5.59	0.283	0.212	0.894
0.190	1013	0.0285	2.96	0.0190	4.44	0.0152	5.55	0.299	0.224	0.888
0.200	1059	0.0300	2.94	0.0200	4.41	0.0160	5.51	0.315	0.236	0.882
0.210	1104	0.0315	2.92	0.0210	4.38	0.0168	5.48	0.330	0.248	0.876
0.220	1149	0.0330	2.90	0.0220	4.35	0.0176	5.44	0.346	0.260	0.871
0.230	1193	0.0345	2.88	0.0230	4.32	0.0184	5.40	0.362	0.271	0.865
0.240	1237	0.0360	2.86	0.0240	4.29	0.0192	5.37	0.378	0.283	0.859
0.250	1279	0.0375	2.84	0.0250	4.26	0.0200	5.33	0.393	0.295	0.853
0.260	1321	0.0390	2.82	0.0260	4.24			0.409	0.307	0.847
0.270	1363	0.0405	2.80	0.0270	4.21			0.425	0.319	0.841
0.280	1403	0.0420	2.78	0.0280	4.18			0.441	0.330	0.835
0.290	1443	0.0435	2.76					0.456	0.342	0.829
0.300	1482	0.0450	2.75					0.472	0.354	0.824
0.310	1521	0.0465	2.73					0.488	0.366	0.818
0.320	1559	0.0480	2.71					0.503	0.378	0.812
0.330	1596							0.519	0.389	0.806
0.340	1632							0.535	0.401	0.800
0.350	1668							0.551	0.413	0.794
0.360	1703							0.566	0.425	0.788
0.370	1737							0.582	0.437	0.782
ρ_{max}		0.0491		0.0283		0.0205				

* Values of ρ above light rule are less than ρ_{min}; $\rho_{min} = 3\sqrt{f'_c}/f_y \geq 200/f_y$ as provided in Section 10.5.1 of ACI 318-95

For use of this Design Aid, see Flexure Examples 1, 2, 3 and 4

FLEXURE 3.1—Nominal strength coefficients for rectangular beams with compression reinforcement in which $f'_s = f_y$, and for flanged sections with $h_f < a$; f'_c = 3000 and 4000 psi.

Reference: ACI 318-95 Sections 9.3.2, 10.2, and 10.3.1-10.3.4, and ACI 318R-95 Section 10.3.1-10.3.3.

For a rectangular beam with compression reinforcement in which $f\sigma_c = f_y$:

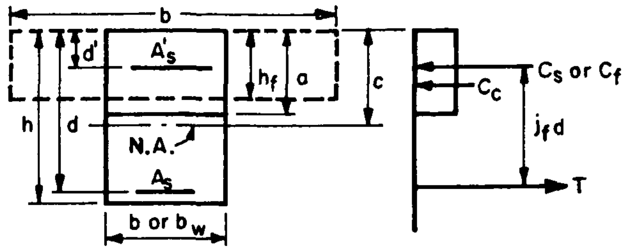
$$\rho - \rho' \geq 0.85\beta_1 \frac{f'_c}{f_y}\frac{d'}{d}\frac{87,000}{87,000 - f_y}$$

and $M_{n2} = A_{s2}da'_n$, kip-ft

where $a'_n = \frac{f_y}{12,000}\left(1 - \frac{d'}{d}\right)$

$A_s = A_{s1} + A_{s2}$

$\rho = A_s/b_d$, $M_n \geq M_u/\phi$



For a flanged section with $h_f < a$:

$M_{n2} = A_{sf}da_{nf}$, kip-ft

where $A_{sf} = K_{nf}j_f b_w h_f/a_{nf}$, in.^{2}

$$K_{nf} = \frac{0.85f'_c}{12,000}\left(\frac{b}{b_w} - 1\right)$$

(K_{nf} from FLEXURE 3.3)

$$a_{nf} = \frac{f_y}{12,000}\left(1 - \frac{h_f}{2d}\right)$$

$j_f = 1 - h_f/2d$

$A_s = A_{sw} + A_{sf}$

f'_c	3000						4000							
f_y	40,000		60,000		75,000		40,000		60,000		75,000			
d'/d	ρ - ρ'	a'_n or a_{nf}	ρ - ρ'	a'_n or a_{nf}	ρ - ρ'	a'_n or a_{nf}	ρ - ρ'	a'_n or a_{nf}	ρ - ρ'	a'_n or a_{nf}	ρ - ρ'	a'_n or a_{nf}	h_f/d	j_f
0.01	0.0010	3.30	0.0012	4.95	0.0021	6.19	0.0013	3.30	0.0016	4.95	0.0028	6.19	0.02	0.99
0.02	0.0020	3.27	0.0023	4.90	0.0042	6.13	0.0027	3.27	0.0031	4.90	0.0056	6.13	0.04	0.98
0.03	0.0030	3.23	0.0035	4.85	0.0063	6.06	0.0040	3.23	0.0047	4.85	0.0084	6.06	0.06	0.97
0.04	0.0040	3.20	0.0047	4.80	0.0084	6.00	0.0053	3.20	0.0062	4.80	0.0112	6.00	0.08	0.96
0.05	0.0050	3.17	0.0058	4.75	0.0105	5.94	0.0067	3.17	0.0078	4.75	0.0140	5.94	0.10	0.95
0.06	0.0060	3.13	0.0070	4.70	0.0126	5.88	0.0080	3.13	0.0093	4.70	0.0168	5.88	0.12	0.94
0.07	0.0070	3.10	0.0081	4.65	0.0147	5.81	0.0094	3.10	0.0109	4.65	0.0196	5.81	0.14	0.93
0.08	0.0080	3.07	0.0093	4.60		5.75	0.0107	3.07	0.0124	4.60		5.75	0.16	0.92
0.09	0.0090	3.03	0.0105	4.55		5.69	0.0120	3.03	0.0140	4.55		5.69	0.18	0.91
0.10	0.0100	3.00	0.0116	4.50		5.63	0.0134	3.00	0.0155	4.50		5.63	0.20	0.90
0.11	0.0110	2.97	0.0128	4.45		5.56	0.0147	2.97	0.0171	4.45		5.56	0.22	0.89
0.12	0.0120	2.93	0.0140	4.40		5.50	0.0160	2.93	0.0186	4.40		5.50	0.24	0.88
0.13	0.0130	2.90	0.0151	4.35		5.44	0.0174	2.90	0.0202	4.35		5.44	0.26	0.87
0.14	0.0140	2.87	0.0163	4.30		5.38	0.0187	2.87	0.0217	4.30		5.38	0.28	0.86
0.15	0.0150	2.83	0.0175	4.25		5.31	0.0201	2.83	0.0233	4.25		5.31	0.30	0.85
0.16	0.0160	2.80	0.0186	4.20		5.25	0.0214	2.80	0.0248	4.20		5.25	0.32	0.84
0.17	0.0171	2.77	0.0198	4.15		5.19	0.0227	2.77	0.0264	4.15		5.19	0.34	0.83
0.18	0.0181	2.73	0.0210	4.10		5.13	0.0241	2.73	0.0279	4.10		5.13	0.36	0.82
0.19	0.0191	2.70		4.05		5.06	0.0254	2070		4.05		5.06	0.38	0.81
0.20	0.0201	2.67		4.00		5.00	0.0267	2.67		4.00		5.00	0.40	0.80
0.21	0.0211	2.63		3.95		4.94	0.0281	2.63		3.95		4.94	0.42	0.79
0.22	0.0221	2.60		3.90		4.88	0.0294	2.60		3.90		4.88	0.44	0.78
0.23	0.0231	2.57		3.85		4.81	0.0308	2.57		3.85		4.81	0.46	0.77
0.24	0.0241	2.53		3.80		4.75	0.0321	2.53		3.80		4.75	0.48	0.76
0.25	0.0251	2.50		3.75		4.69	0.0334	2.50		3.75		4.69	0.50	0.75
0.26	0.0261	2.47		3.70		4.63	0.0348	2.47		3.70		4.63	0.52	0.74
0.27	0.0271	2.43		3.65		4.56	0.0361	2.43		3.65		4.56	0.54	0.73
0.28	0.0281	2.40		3.60		4.50	0.0374	2.40		3.60		4.50	0.56	0.72
0.29	0.0291	2.37		3.55		4.44	0.0388	2.37		3.55		4.44	0.58	0.71
0.30	0.0301	2.33		3.50		4.37	0.0401	2.33		3.50		4.37	0.60	0.70
0.31	0.0311	2.30		3.45		4.31	0.0415	2.30		3.45		4.31	0.62	0.69
0.32	0.0321	2.27		3.40		4.25	0.0428	2.27		3.40		4.25	0.64	0.68
0.33	0.0331	2.23		3.35		4.19	0.0441	2.23		3.35		4.19	0.66	0.67
0.34	0.0341	2.20		3.30		4.12	0.0455	2.20		3.30		4.12	0.68	0.66
0.35	0.0351	2.17		3.25		4.06	0.0468	2.17		3.25		4.06	0.70	0.65
0.36	0.0361	2.13		3.20		4.00	0.0481	2.13		3.20		4.00	0.72	0.64
0.37	0.0371	2.10		3.15		3.94	0.0495	2.10		3.15		3.94	0.74	0.63

For use of this Design Aid, see Flexure Example 6

FLEXURE 3.2—Nominal strength coefficients for rectangular beams with compression reinforcement in which $f'_s = f_y$ and for flanged sections with $h_f < a$; $f'_c = 5000$ and 6000 psi.

Reference: ACI 318-95 Sections 9.3.2, 10.2, and 10.3.1-10.3.4, and ACI 318R-95 Section 10.3.1-10.3.3.

For a rectangular beam with compression reinforcement in which $f'_c = f_y$:

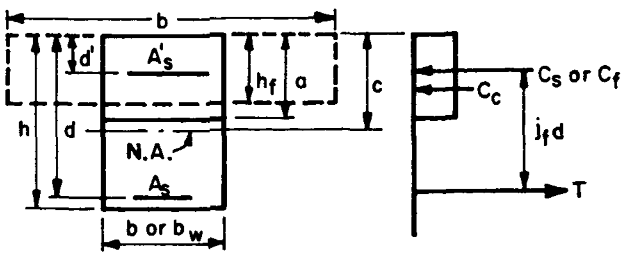
$$\rho - \rho' \geq 0.85\beta_1 \frac{f'_c d'}{f_y d} \frac{87,000}{87,000 - f_y}$$

and $M_{n2} = A_{s2} d a'_n$, kip-ft

where $a'_n = \frac{f_y}{12,000}\left(1 - \frac{d'}{d}\right)$

$A_s = A_{s1} + A_{s2}$

$\rho = A_s/b_d$, $M_n \geq M_u/\phi$



For a flanged section with $h_f < a$:

$M_{n2} = A_{sf} d a_{nf}$, kip-ft

where $A_{sf} = K_{nf} j_f b_w h_f / a_{nf}$, in.^{2}

$$K_{nf} = \frac{0.85 f'_c}{12,000}\left(\frac{b}{b_w} - 1\right)$$

(K_{nf} from FLEXURE 3.3)

$$a_{nf} = \frac{f_y}{12,000}\left(1 - \frac{h_f}{2d}\right)$$

$j_f = 1 - h_f/2d$

$A_s = A_{sw} + A_{sf}$

f'_c	5000						6000							
f_y	40,000		60,000		75,000		40,000		60,000		75,000			
d'/d	$\rho - \rho'$	a'_n or a_{nf}	$\rho - \rho'$	a'_n or a_{nf}	$\rho - \rho'$	a'_n or a_{nf}	$\rho - \rho'$	a'_n or a_{nf}	$\rho - \rho'$	a'_n or a_{nf}	$\rho - \rho'$	a'_n or a_{nf}	h_f/d	j_f
0.01	0.0016	3.30	0.0018	4.95	0.0033	6.19	0.0018	3.30	0.0021	4.95	0.0037	6.19	0.02	0.99
0.02	0.0031	3.27	0.0037	4.90	0.0066	6.13	0.0035	3.27	0.0041	4.90	0.0074	6.13	0.04	0.98
0.03	0.0047	3.23	0.0055	4.85	0.0099	6.06	0.0053	3.23	0.0062	4.85	0.0111	6.06	0.06	0.97
0.04	0.0063	3.20	0.0073	4.80	0.0131	6.00	0.0071	3.20	0.0082	4.80	0.0148	6.00	0.08	0.96
0.05	0.0079	3.17	0.0091	4.75	0.0164	5.94	0.0089	3.17	0.0103	4.75	0.0185	5.94	0.10	0.95
0.06	0.0094	3.13	0.0110	4.70	0.0197	5.88	0.0106	3.13	0.0123	4.70	0.0222	5.88	0.12	0.94
0.07	0.0110	3.10	0.0128	4.65	0.0230	5.81	0.0124	3.10	0.0144	4.65	0.0259	5.81	0.14	0.93
0.08	0.0126	3.07	0.0146	4.60		5.75	0.0142	3.07	0.0164	4.60		5.75	0.16	0.92
0.09	0.0142	3.03	0.0164	4.55		5.69	0.0159	3.03	0.0185	4.55		5.69	0.18	0.91
0.10	0.0157	3.00	0.0183	4.50		5.63	0.0177	3.00	0.0205	4.50		5.63	0.20	0.90
0.11	0.0173	2.97	0.0201	4.45		5.56	0.0195	2.97	0.0226	4.45		5.56	0.22	0.89
0.12	0.0189	2.93	0.0219	4.40		5.50	0.0212	2.93	0.0246	4.40		5.50	0.24	0.88
0.13	0.0205	2.90	0.0237	4.35		5.44	0.0230	2.90	0.0267	4.35		5.44	0.26	0.87
0.14	0.0220	2.87	0.0256	4.30		5.38	0.0248	2.87	0.0288	4.30		5.38	0.28	0.86
0.15	0.0236	2.83	0.0274	4.25		5.31	0.0266	2.83	0.0308	4.25		5.31	0.30	0.85
0.16	0.0252	2.80	0.0292	4.20		5.25	0.0283	2.80	0.0329	4.20		5.25	0.32	0.84
0.17	0.0267	2.77	0.0310	4.15		5.19	0.0301	2.77	0.0349	4.15		5.19	0.34	0.83
0.18	0.0283	2.73	0.0329	4.10		5.13	0.0319	2.73	0.0370	4.10		5.13	0.36	0.82
0.19	0.0299	2.70		4.05		5.06	0.0336	2.70		4.05		5.06	0.38	0.81
0.20	0.0315	2.67		4.00		5.00	0.0354	2.67		4.00		5.00	0.40	0.80
0.21	0.0330	2.63		3.95		4.94	0.0372	2.63		3.95		4.94	0.42	0.79
0.22	0.0346	2.60		3.90		4.88	0.0389	2.60		3.90		4.88	0.44	0.78
0.23	0.0362	2.57		3.85		4.81	0.0407	2.57		3.85		4.81	0.46	0.77
0.24	0.0378	2.53		3.80		4.75	0.0425	2.53		3.80		4.75	0.48	0.76
0.25	0.0393	2.50		3.75		4.69	0.0443	2.50		3.75		4.69	0.50	0.75
0.26	0.0409	2.47		3.70		4.63	0.0460	2.47		3.70		4.63	0.52	0.74
0.27	0.0425	2.43		3.65		4.56	0.0478	2.43		3.65		4.56	0.54	0.73
0.28	0.0441	2.40		3.60		4.50	0.0496	2.40		3.60		4.50	0.56	0.72
0.29	0.0456	2.37		3.55		4.44	0.0513	2.37		3.55		4.44	0.58	0.71
0.30	0.0472	2.33		3.50		4.37	0.0531	2.33		3.50		4.37	0.60	0.70
0.31	0.0488	2.30		3.45		4.31	0.0549	2.30		3.45		4.31	0.62	0.69
0.32	0.0503	2.27		3.40		4.25	0.0566	2.27		3.40		4.25	0.64	0.68
0.33	0.0519	2.23		3.35		4.19	0.0584	2.23		3.35		4.19	0.66	0.67
0.34	0.0535	2.20		3.30		4.12	0.0602	2.20		3.30		4.12	0.68	0.66
0.35	0.0551	2.17		3.25		4.06	0.0620	2.17		3.25		4.06	0.70	0.65
0.36	0.0566	2.13		3.20		4.00	0.0637	2.13		3.20		4.00	0.72	0.64
0.37	0.0582	2.10		3.15		3.94	0.0655	2.10		3.15		3.94	0.74	0.63

For use of this Design Aid, see Flexure Example 6

FLEXURE 3.3 - Coefficient K_{nf} for use in computing A_{sf} for a flanged section with $h_f < a$

Reference: ACI 318-95 Sections 9.3.2, 10.2, and 10.3.1-10.3.4 and ACI 318R-95 Section 10.3.1-10.3.3

$$M_n \geq \frac{M_u}{\phi} \qquad A_{sf} = \frac{(K_{nf})(j_f)(b_w)(h_f)}{a_{nf}}\text{, in.}^2 \quad \text{where } K_{nf} = \frac{0.85 f'_c}{12{,}000}\left(\frac{b}{b_w} - 1\right)$$

$$j_f = 1 - h_f/2d,$$

$$\text{and } a_{nf} = \frac{f_y}{12{,}000}\left(1 - \frac{h_f}{2d}\right)$$

(j_f and a_{nf} from FLEXURE 3.1 and 3.2)

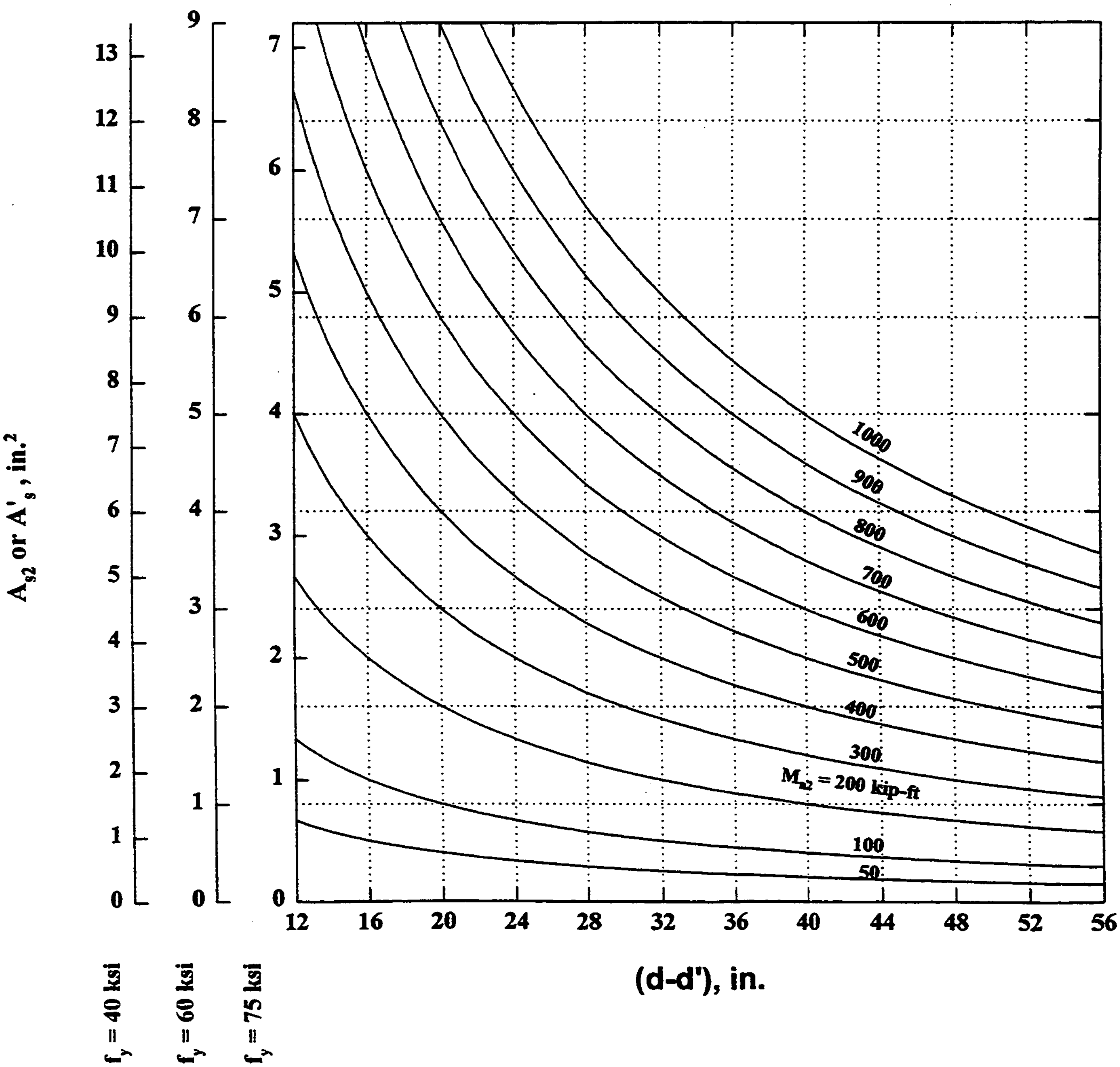
	K_{nf}			
	f'_c, psi			
b/b_w	3000	4000	5000	6000
2.0	0.213	0.283	0.354	0.425
2.2	0.255	0.340	0.425	0.510
2.4	0.298	0.397	0.496	0.595
2.6	0.340	0.453	0.567	0.680
2.8	0.383	0.510	0.638	0.765
3.0	0.425	0.567	0.708	0.850
3.2	0.468	0.623	0.779	0.935
3.4	0.510	0.680	0.850	1.020
3.6	0.553	0.737	0.921	1.105
3.8	0.595	0.793	0.992	1.190
4.0	0.638	0.850	1.063	1.275
4.2	0.680	0.907	1.133	1.360
4.4	0.723	0.963	1.204	1.445
4.6	0.765	1.020	1.275	1.530
4.8	0.808	1.077	1.346	1.615
5.0	0.850	1.133	1.417	1.700
5.2	0.893	1.190	1.488	1.785
5.4	0.935	1.247	1.558	1.870
5.6	0.978	1.303	1.629	1.955
5.8	1.020	1.360	1.700	2.040
6.0	1.063	1.417	1.771	2.125
6.2	1.105	1.473	1.842	2.210
6.4	1.148	1.530	1.913	2.295
6.6	1.190	1.587	1.983	2.380
6.8	1.233	1.643	2.054	2.465
7.0	1.275	1.700	2.125	2.550
7.2	1.318	1.757	2.196	2.635
7.4	1.360	1.813	2.267	2.720
7.6	1.403	1.870	2.338	2.805
7.8	1.445	1.927	2.408	2.890
8.0	1.488	1.983	2.479	2.975
8.2	1.530	2.040	2.550	3.060
8.4	1.573	2.097	2.621	3.145
8.6	1.615	2.153	2.692	3.230
8.8	1.658	2.210	2.763	3.315
9.0	1.700	2.267	2.833	3.400
9.2	1.743	2.323	2.904	3.485
9.4	1.785	2.380	2.975	3.570
9.6	1.828	2.437	3.046	3.655
9.8	1.870	2.493	3.117	3.740
10.0	1.912	2.550	3.187	3.825

FLEXURE 4 - Nominal strength M_{n2} for compression reinforcement in which $f'_s = f_y$

Reference: ACI 318-95 Sections 9.3.2, 10.2, and 10.3.1-10.3.4 and ACI 318R-95 Section 10.3.1-10.3.3

$$M_n \geq \frac{M_u}{\phi} \qquad A_s' \approx A_{s2} \quad \frac{12{,}000M_{n2}}{f_y(d \quad d')}\ ,\ in.^2$$

Note: To take into account the effect of the displaced concrete, multiply the value of A'_s obtained from the graph by $f_y/(f_y - 0.85f'_c)$.

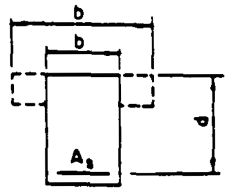


For use of this Design Aid, see Flexure Example 6

FLEXURE 5 - Coefficient F for use in calculating nominal strengths M_n, M_{nl}, and M_{nw}

M_n (or M_{nl} or M_{nw}) = K_nF

where $M_n \geq M_u/\phi$, K_n is from FLEXURE 2, and $F = bd^2/12000$



d	b = 4.0	5.0	6.0	7.0	7.5	8.0	9.0	9.5	10.0	11.0	12.0	13.0	15.0	17.0	19.0	21.0	23.0	25.0	30.0	36.0	42.0	48.0
5.0	0.008	0.010	0.013	0.015	0.016	0.017	0.019	0.020	0.021	0.023	0.025	0.027	0.031	0.035	0.040	0.044	0.048	0.052	0.063	0.075	0.088	0.100
5.5	0.010	0.013	0.015	0.018	0.019	0.020	0.023	0.024	0.025	0.028	0.030	0.033	0.038	0.043	0.048	0.053	0.058	0.063	0.076	0.091	0.106	0.121
6.0	0.012	0.015	0.018	0.021	0.023	0.024	0.027	0.029	0.030	0.033	0.036	0.039	0.045	0.051	0.057	0.063	0.069	0.075	0.090	0.108	0.126	0.144
6.5	0.014	0.018	0.021	0.025	0.026	0.028	0.032	0.033	0.035	0.039	0.042	0.046	0.053	0.060	0.067	0.074	0.081	0.088	0.106	0.127	0.148	0.169
7.0	0.016	0.020	0.025	0.029	0.031	0.033	0.037	0.039	0.041	0.045	0.049	0.053	0.061	0.069	0.078	0.086	0.094	0.102	0.123	0.147	0.172	0.196
7.5	0.019	0.023	0.028	0.033	0.035	0.038	0.042	0.045	0.047	0.052	0.056	0.061	0.070	0.080	0.089	0.098	0.108	0.117	0.141	0.169	0.197	0.225
8.0	0.021	0.027	0.032	0.037	0.040	0.043	0.048	0.051	0.053	0.059	0.064	0.069	0.080	0.091	0.101	0.112	0.123	0.133	0.160	0.192	0.224	0.256
8.5	0.024	0.030	0.036	0.042	0.045	0.048	0.054	0.057	0.060	0.066	0.072	0.078	0.090	0.102	0.114	0.126	0.138	0.151	0.181	0.217	0.253	0.289
9.0	0.027	0.034	0.041	0.047	0.051	0.054	0.061	0.064	0.068	0.074	0.081	0.088	0.101	0.115	0.128	0.142	0.155	0.169	0.203	0.243	0.284	0.324
9.5	0.030	0.038	0.045	0.053	0.056	0.060	0.068	0.071	0.075	0.083	0.090	0.098	0.113	0.128	0.143	0.158	0.173	0.188	0.226	0.271	0.316	0.361
10.0	0.033	0.042	0.050	0.058	0.063	0.067	0.075	0.079	0.083	0.092	0.100	0.108	0.125	0.142	0.158	0.175	0.192	0.208	0.250	0.300	0.350	0.400
10.5	0.037	0.046	0.055	0.064	0.069	0.074	0.083	0.087	0.092	0.101	0.110	0.119	0.138	0.156	0.175	0.193	0.211	0.230	0.276	0.331	0.386	0.441
11.0	0.040	0.050	0.061	0.071	0.076	0.081	0.091	0.096	0.101	0.111	0.121	0.131	0.151	0.171	0.192	0.212	0.232	0.252	0.303	0.363	0.424	0.484
11.5	0.044	0.055	0.066	0.077	0.083	0.088	0.099	0.105	0.110	0.121	0.132	0.143	0.165	0.187	0.209	0.231	0.253	0.276	0.331	0.397	0.463	0.529
12.0	0.048	0.060	0.072	0.084	0.090	0.096	0.108	0.114	0.120	0.132	0.144	0.156	0.180	0.204	0.228	0.252	0.276	0.300	0.360	0.432	0.504	0.576
12.5	0.052	0.065	0.078	0.091	0.098	0.104	0.117	0.124	0.130	0.143	0.156	0.169	0.195	0.221	0.247	0.273	0.299	0.326	0.391	0.469	0.547	0.625
13.0	0.056	0.070	0.085	0.099	0.106	0.113	0.127	0.134	0.141	0.155	0.169	0.183	0.211	0.239	0.268	0.296	0.324	0.352	0.423	0.507	0.592	0.676
13.5	0.061	0.076	0.091	0.106	0.114	0.122	0.137	0.144	0.152	0.167	0.182	0.197	0.228	0.258	0.289	0.319	0.349	0.380	0.456	0.547	0.638	0.729
14.0	0.065	0.082	0.098	0.114	0.123	0.131	0.147	0.155	0.163	0.180	0.196	0.212	0.245	0.278	0.310	0.343	0.376	0.408	0.490	0.588	0.686	0.784
14.5	0.070	0.088	0.105	0.123	0.131	0.140	0.158	0.166	0.175	0.193	0.210	0.228	0.263	0.298	0.333	0.368	0.403	0.438	0.526	0.631	0.736	0.841
15.0	0.075	0.094	0.113	0.131	0.141	0.150	0.169	0.178	0.188	0.206	0.225	0.244	0.281	0.319	0.356	0.394	0.431	0.469	0.563	0.675	0.788	0.900
15.5	0.080	0.100	0.120	0.140	0.150	0.160	0.180	0.190	0.200	0.220	0.240	0.260	0.300	0.340	0.380	0.420	0.460	0.501	0.601	0.721	0.841	0.961
16.0	0.085	0.107	0.128	0.149	0.160	0.171	0.192	0.203	0.213	0.235	0.256	0.277	0.320	0.363	0.405	0.448	0.491	0.533	0.640	0.768	0.896	1.02
16.5	0.091	0.113	0.136	0.159	0.170	0.182	0.204	0.216	0.227	0.250	0.272	0.295	0.340	0.386	0.431	0.476	0.522	0.567	0.681	0.817	0.953	1.09
17.0		0.120	0.145	0.169	0.181	0.193	0.217	0.229	0.241	0.265	0.289	0.313	0.361	0.409	0.458	0.506	0.554	0.602	0.723	0.867	1.01	1.16
17.5		0.128	0.153	0.179	0.191	0.204	0.230	0.242	0.255	0.281	0.306	0.332	0.383	0.434	0.485	0.536	0.587	0.638	0.766	0.919	1.07	1.23
18.0		0.135	0.162	0.189	0.203	0.216	0.243	0.257	0.270	0.297	0.324	0.351	0.405	0.459	0.513	0.567	0.621	0.675	0.810	0.972	1.13	1.30
18.5		0.143	0.171	0.200	0.214	0.228	0.257	0.271	0.285	0.314	0.342	0.371	0.428	0.485	0.542	0.599	0.656	0.713	0.856	1.03	1.20	1.37
19.0			0.181	0.211	0.226	0.241	0.271	0.286	0.301	0.331	0.361	0.391	0.451	0.511	0.572	0.632	0.692	0.752	0.903	1.08	1.26	1.44
20.0			0.200	0.233	0.250	0.267	0.300	0.317	0.333	0.367	0.400	0.433	0.500	0.567	0.633	0.700	0.767	0.833	1.00	1.20	1.40	1.60
21.0			0.221	0.257	0.276	0.294	0.331	0.349	0.368	0.404	0.441	0.478	0.551	0.625	0.698	0.772	0.845	0.919	1.10	1.32	1.54	1.76
22.0			0.242	0.282	0.303	0.323	0.363	0.383	0.403	0.444	0.484	0.524	0.605	0.686	0.766	0.847	0.928	1.01	1.21	1.45	1.69	1.94
23.0				0.309	0.331	0.353	0.397	0.419	0.441	0.485	0.529	0.573	0.661	0.749	0.838	0.926	1.01	1.10	1.32	1.59	1.85	2.12
24.0				0.336	0.360	0.384	0.432	0.456	0.480	0.528	0.576	0.624	0.720	0.816	0.912	1.01	1.10	1.20	1.44	1.73	2.02	2.30
25.0				0.365	0.391	0.417	0.469	0.495	0.521	0.573	0.625	0.677	0.781	0.885	0.990	1.09	1.20	1.30	1.56	1.88	2.19	2.50
26.0				0.394	0.423	0.451	0.507	0.535	0.563	0.620	0.676	0.732	0.845	0.958	1.07	1.18	1.30	1.41	1.69	2.03	2.37	2.70
27.0					0.456	0.486	0.547	0.577	0.608	0.668	0.729	0.790	0.911	1.03	1.15	1.28	1.40	1.52	1.82	2.19	2.55	2.92
28.0					0.490	0.523	0.588	0.621	0.653	0.719	0.784	0.849	0.980	1.11	1.24	1.37	1.50	1.63	1.96	2.35	2.74	3.14
29.0					0.526	0.561	0.631	0.666	0.701	0.771	0.841	0.911	1.05	1.19	1.33	1.47	1.61	1.75	2.10	2.52	2.94	3.36
30.0					0.563	0.600	0.675	0.713	0.750	0.825	0.900	0.975	1.13	1.28	1.43	1.58	1.73	1.88	2.25	2.70	3.15	3.60
31.0						0.641	0.721	0.761	0.801	0.881	0.961	1.04	1.20	1.36	1.52	1.68	1.84	2.00	2.40	2.88	3.36	3.84
32.0						0.683	0.768	0.811	0.853	0.939	1.02	1.11	1.28	1.45	1.62	1.79	1.96	2.13	2.56	3.07	3.58	4.10
33.0						0.726	0.817	0.862	0.908	0.998	1.09	1.18	1.36	1.54	1.72	1.91	2.09	2.27	2.72	3.27	3.81	4.36
34.0						0.771	0.867	0.915	0.963	1.06	1.16	1.25	1.45	1.64	1.83	2.02	2.22	2.41	2.89	3.47	4.05	4.62
36.0							0.972	1.03	1.08	1.19	1.30	1.40	1.62	1.84	2.05	2.27	2.48	2.70	3.24	3.89	4.54	5.18
38.0							1.08	1.14	1.20	1.32	1.44	1.56	1.81	2.05	2.29	2.53	2.77	3.01	3.61	4.33	5.05	5.78
40.0							1.20	1.27	1.33	1.47	1.60	1.73	2.00	2.27	2.53	2.80	3.07	3.33	4.00	4.80	5.60	6.40
42.0							1.32	1.40	1.47	1.62	1.76	1.91	2.21	2.50	2.79	3.09	3.38	3.68	4.41	5.29	6.17	7.06
44.0								1.53	1.61	1.77	1.94	2.10	2.42	2.74	3.07	3.39	3.71	4.03	4.84	5.81	6.78	7.74
46.0								1.68	1.76	1.94	2.12	2.29	2.65	3.00	3.35	3.70	4.06	4.41	5.29	6.35	7.41	8.46
48.0								1.82	1.92	2.11	2.30	2.50	2.88	3.26	3.65	4.03	4.42	4.80	5.76	6.91	8.06	9.22
50.0								1.98	2.08	2.29	2.50	2.71	3.13	3.54	3.96	4.38	4.79	5.21	6.25	7.50	8.75	10.0
52.0									2.25	2.48	2.70	2.93	3.38	3.83	4.28	4.73	5.18	5.63	6.76	8.11	9.46	10.8
54.0									2.43	2.67	2.92	3.16	3.65	4.13	4.62	5.10	5.59	6.08	7.29	8.75	10.2	11.7
56.0									2.61	2.87	3.14	3.40	3.92	4.44	4.97	5.49	6.01	6.53	7.84	9.41	11.0	12.5
58.0									2.80	3.08	3.36	3.64	4.21	4.77	5.33	5.89	6.45	7.01	8.41	10.1	11.8	13.5
60.0										3.30	3.60	3.90	4.50	5.10	5.70	6.30	6.90	7.50	9.00	10.8	12.6	14.4
64.0										3.75	4.10	4.44	5.12	5.80	6.49	7.17	7.85	8.53	10.2	12.3	14.3	16.4
68.0										4.24	4.62	5.01	5.78	6.55	7.32	8.09	8.86	9.63	11.6	13.9	16.2	18.5
72.0										4.75	5.18	5.62	6.48	7.34	8.21	9.07	9.94	10.8	13.0	15.6	18.1	20.7

For use of this Design Aid, see Flexure Examples 4 and 6

FLEXURE 6.1.1 - Nominal strength M_n for slab sections 12 in. wide

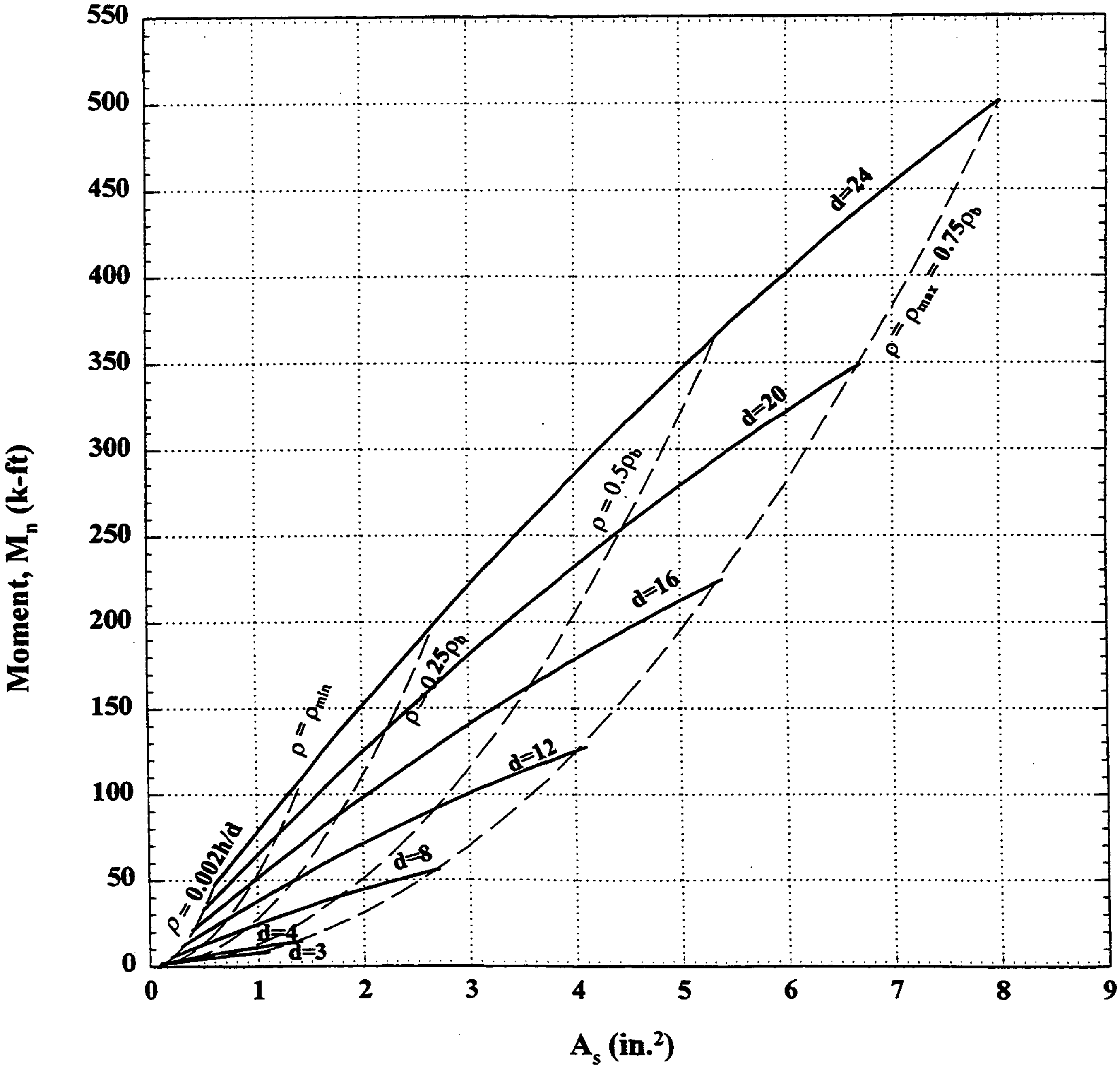
Reference: ACI 318-95 Sections 7.12, 8.4.1, 8.4.3, 9.3.2, 10.2, 10.3.1-10.3.3, 10.5.1 and 10.5.3 and ACI 318R-95 Sections 10.3.1 and 10.3.2

$f'_c = 3000$ psi

$f_y = 40{,}000$ psi

$M_n = 3.33 A_s d - 2.18 A_s^2$, k-ft

$$M_n \geq \frac{M_u}{\phi}$$



FLEXURE 6.1.2 - Nominal strength M_n for slab sections 12 in. wide

Reference: ACI 318-95 Sections 7.12, 8.4.1, 8.4.3, 9.3.2, 10.2, 10.3.1-10.3.3, 10.5.1 and 10.5.3 and ACI 318R-95 Sections 10.3.1 and 10.3.2

$f'_c = 3000$ psi $\qquad M_n = 3.33A_sd - 2.18A_s^2$, k-ft

$f_y = 40{,}000$ psi $\qquad M_n \geq \dfrac{M_u}{\phi}$

Shrinkage & Temperature reinforcement = $0.002bh$

$$A_{smax} = \frac{0.75\beta_1(0.85f'_c)}{f_y}\left(\frac{87000}{87000 + f_y}\right)bd = \frac{0.75(0.85)(0.85)(3)}{40000}\left(\frac{87000}{87000 + 40000}\right)(12)d$$

				M_n (Nominal Moment, k-ft)						
d	As min	As max	As	d=3	d=4	d=8	d=12	d=16	d=20	d=24
3	0.096	1.00	0.10	0.94						
4	0.120	1.34	0.12	1.17	1.57					
5	0.144	1.67	0.15	1.45	1.95					
6	0.168	2.00	0.20	1.91	2.58	5.25				
7	0.192	2.34	0.25	2.36	3.20	6.53	9.86	13.20		
8	0.216	2.67	0.50	4.46	6.12	12.79	19.46	26.12	32.79	39.46
9	0.240	3.01	0.75	6.27	8.77	18.77	28.77	38.77	48.77	58.77
10	0.264	3.34	1.00	7.82	11.15	24.49	37.82	51.15	64.49	77.82
11	0.288	3.67	1.25		13.26	29.93	46.60	63.26	79.93	96.60
12	0.312	4.01	1.50		15.10	35.10	55.10	75.10	95.10	115.10
13	0.336	4.34	1.75			39.99	63.33	86.66	109.99	133.33
14	0.360	4.68	2.00			44.62	71.29	97.95	124.62	151.29
15	0.384	5.01	2.25			48.97	78.97	108.97	138.97	168.97
16	0.408	5.35	2.50			53.05	86.38	119.72	153.05	186.38
17	0.432	5.68	2.75			56.86	93.52	130.19	166.86	203.52
18	0.456	6.01	3.00				100.39	140.39	180.39	220.39
19	0.480	6.35	3.25				106.99	150.32	193.65	236.99
20	0.504	6.68	3.50				113.31	159.98	206.64	253.31
21	0.528	7.02	3.75				119.36	169.36	219.36	269.36
22	0.552	7.35	4.00				125.14	178.47	231.81	285.14
23	0.576	7.68	4.25				130.65	187.31	243.98	300.65
24	0.600	8.02	4.50					195.88	255.88	315.88
			4.75					204.18	267.51	330.84
			5.00					212.20	278.87	345.53
			5.25					219.95	289.95	359.95
			5.50					227.43	300.76	374.10
			5.75						311.30	387.97
			6.00						321.57	401.57
			6.25						331.56	414.90
			6.50						341.29	427.95
			6.75						350.74	440.74
			7.00							453.25
			7.25							465.48
			7.50							477.45
			7.75							489.14
			8.00							500.57
			8.02							501.47

FLEXURE 6.2.1 - Nominal strength M_n for slab sections 12 in. wide

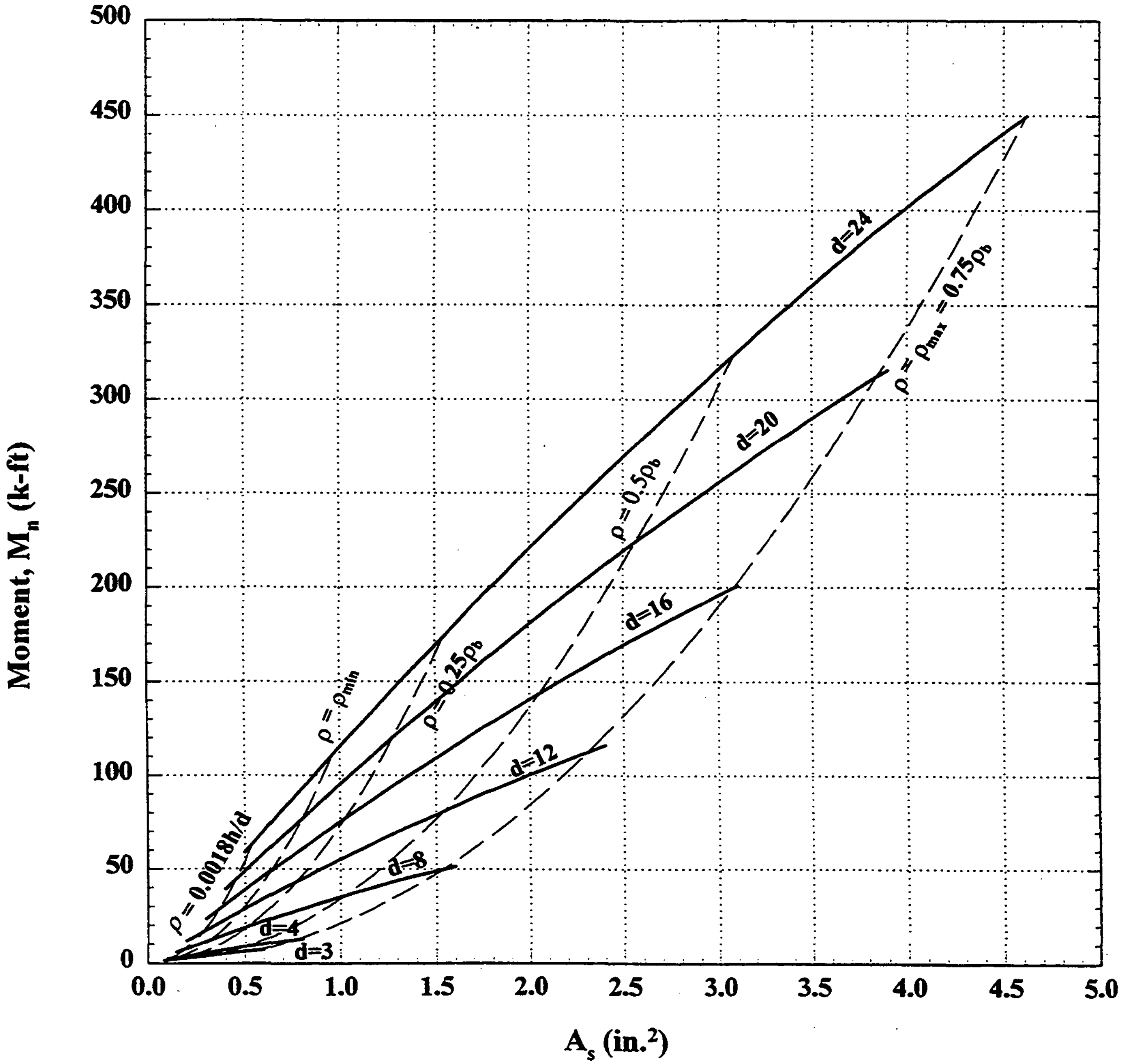
Reference: ACI 318-95 Sections 7.12, 8.4.1, 8.4.3, 9.3.2, 10.2, 10.3.1-10.3.3, 10.5.1 and 10.5.3 and ACI 318R-95 Sections 10.3.1 and 10.3.2

$f'_c = 3000$ psi

$f_y = 60{,}000$ psi

$M_n = 5.0A_sd - 4.90A_s^2$, k-ft

$$M_n \geq \frac{M_u}{\phi}$$



FLEXURE 6.2.2 - Nominal strength M_n for slab sections 12 in. wide

Reference: ACI 318-95 Sections 7.12, 8.4.1, 8.4.3, 9.3.2, 10.2, 10.3.1-10.3.3, 10.5.1 and 10.5.3 and ACI 318R-95 Sections 10.3.1 and 10.3.2

$f'_c = 3000$ psi

$f_y = 60{,}000$ psi

$M_n = 5.0A_s d - 4.90A_s^2$, k-ft

$$M_n \geq \frac{M_u}{\phi}$$

Shrinkage & Temperature reinforcement = 0.0018*bh*

$$A_{smax} = \frac{0.75\beta_1(0.85f'_c)}{f_y}\left(\frac{87000}{87000+f_y}\right)bd = \frac{0.75(0.85)(0.85)(3)}{60000}\left(\frac{87000}{87000+60000}\right)(12)d$$

				M_n (Nominal Moment, k-ft)						
d	As min	As max	As	d=3	d=4	d=8	d=12	d=16	d=20	d=24
3	0.086	0.58	0.09	1.25						
4	0.108	0.77	0.11	1.56	2.10					
5	0.130	0.96	0.13	1.87	2.52					
6	0.151	1.15	0.15	2.14	2.89	5.89				
7	0.173	1.35	0.20	2.80	3.80	7.80	11.80	15.80		
8	0.194	1.54	0.40	5.22	7.22	15.22	23.22	31.22	39.22	47.22
9	0.216	1.73	0.60	7.24	10.24	22.24	34.24	46.24	39.22	70.24
10	0.238	1.92	0.80		12.86	28.86	44.86	60.86	51.94	92.86
11	0.259	2.12	1.00			35.10	55.10	75.10	64.49	115.10
12	0.281	2.31	1.20			40.94	64.94	88.94	76.86	136.94
13	0.302	2.50	1.40			46.39	74.39	102.39	89.06	158.39
14	0.324	2.69	1.60			51.45	83.45	115.45	101.09	179.45
15	0.346	2.89	1.80				92.12	128.12	112.94	200.12
16	0.367	3.08	2.00				100.39	140.39	124.62	220.39
17	0.389	3.27	2.20				108.27	152.27	136.12	240.27
18	0.410	3.46	2.40				115.76	163.76	147.45	259.76
19	0.432	3.66	2.60					174.86	158.61	278.86
20	0.454	3.85	2.80					185.57	169.59	297.57
21	0.475	4.04	3.00					195.88	180.39	315.88
22	0.497	4.23	3.20					205.80	191.02	333.80
23	0.518	4.43	3.40						201.48	351.33
24	0.540	4.62	3.60						296.47	368.47
			3.80						309.22	385.22
			4.00						321.57	401.57
			4.20							417.53
			4.40							433.10
			4.60							448.27
			4.62							449.77

FLEXURE 6.3.1 - Nominal strength M_n for slab sections 12 in. wide

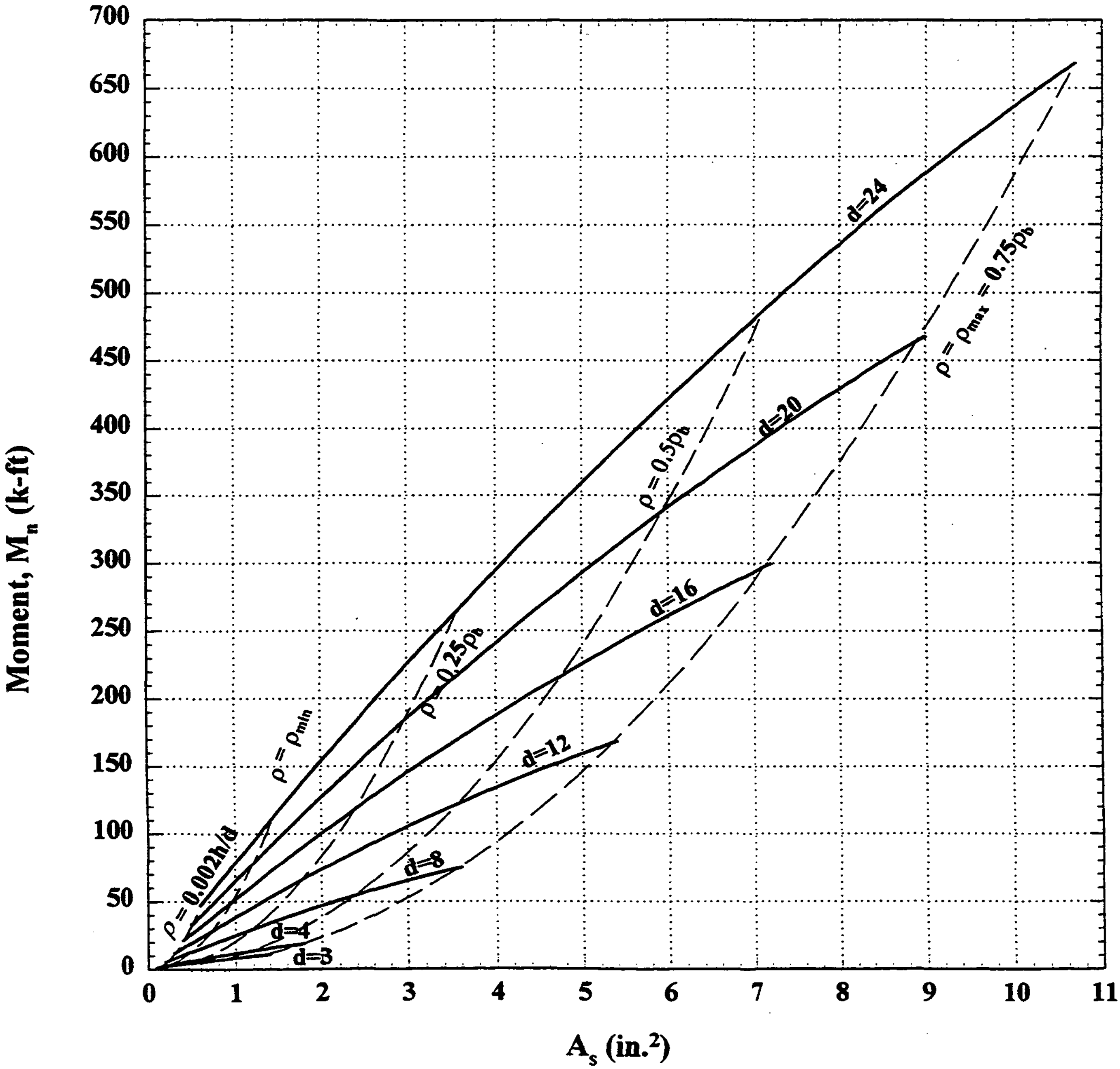
Reference: ACI 318-95 Sections 7.12, 8.4.1, 8.4.3, 9.3.2, 10.2, 10.3.1-10.3.3, 10.5.1 and 10.5.3 and ACI 318R-95 Sections 10.3.1 and 10.3.2

$f'_c = 4000$ psi

$f_y = 40{,}000$ psi

$M_n = 3.33A_s d - 1.63A_s^2$, k-ft

$$M_n \geq \frac{M_u}{\phi}$$



FLEXURE 6.3.2 - Nominal strength M_n for slab sections 12 in. wide

Reference: ACI 318-95 Sections 7.12, 8.4.1, 8.4.3, 9.3.2, 10.2, 10.3.1-10.3.3, 10.5.1 and 10.5.3 and ACI 318R-95 Sections 10.3.1 and 10.3.2

$f'_c = 4000$ psi $M_n = 3.33A_s d - 1.63A_s^2$, k-ft

$f_y = 40{,}000$ psi $M_n \geq \dfrac{M_u}{\phi}$

Shrinkage & Temperature reinforcement = 0.002*bh*

$$A_{smax} = \frac{0.75\beta_1(0.85f'_c)}{f_y}\left(\frac{87000}{87000 + f_y}\right)bd = \frac{0.75(0.85)(0.85)(4)}{40000}\left(\frac{87000}{87000 + 40000}\right)(12)d$$

				M_n (Nominal Moment, k-ft)						
d	As min	As max	As	d=3	d=4	d=8	d=12	d=16	d=20	d=24
3	0.096	1.34	0.10	0.94						
4	0.120	1.78	0.12	1.18	1.58					
5	0.144	2.23	0.15	1.46	1.96					
6	0.168	2.67	0.20	1.93	2.60	5.27				
7	0.192	3.12	0.25	2.40	3.23	6.56	9.90	13.23		
8	0.216	3.56	0.50	4.59	6.26	12.92	19.59	26.26	32.92	39.59
9	0.240	4.01	0.75	6.58	9.08	19.08	29.08	39.08	49.08	59.08
10	0.264	4.45	1.00	8.37	11.70	25.03	38.37	51.70	65.03	78.37
11	0.288	4.90	1.40	10.80	15.46	34.13	52.80	71.46	90.13	108.80
12	0.312	5.35	1.80		18.71	42.71	66.71	90.71	114.71	138.71
13	0.336	5.79	2.20			50.76	80.09	109.42	138.76	168.09
14	0.360	6.24	2.60			58.29	92.95	127.62	162.29	196.95
15	0.384	6.68	3.00			65.29	105.29	145.29	185.29	225.29
16	0.408	7.13	3.40			71.78	117.11	162.44	207.78	253.11
17	0.432	7.57	3.80			77.74	128.41	179.07	229.74	280.41
18	0.456	8.02	4.20				139.18	195.18	251.18	307.18
19	0.480	8.46	4.60				149.42	210.76	272.09	333.42
20	0.504	8.91	5.00				159.15	225.82	292.48	359.15
21	0.528	9.35	5.40				168.35	240.35	312.35	384.35
22	0.552	9.80	5.80					254.37	331.70	409.03
23	0.576	10.25	6.20					267.86	350.52	433.19
24	0.600	10.69	6.60					280.82	368.82	456.82
			7.00					293.27	386.60	479.93
			7.40					305.19	403.86	502.52
			7.80						420.59	524.59
			8.20						436.80	546.13
			8.60						452.48	567.15
			9.00						467.65	587.65
			9.40							607.62
			9.80							627.07
			10.20							646.00
			10.60							664.41
			10.69							668.47

FLEXURE 6.4.1 - Nominal strength M_n for slab sections 12 in. wide

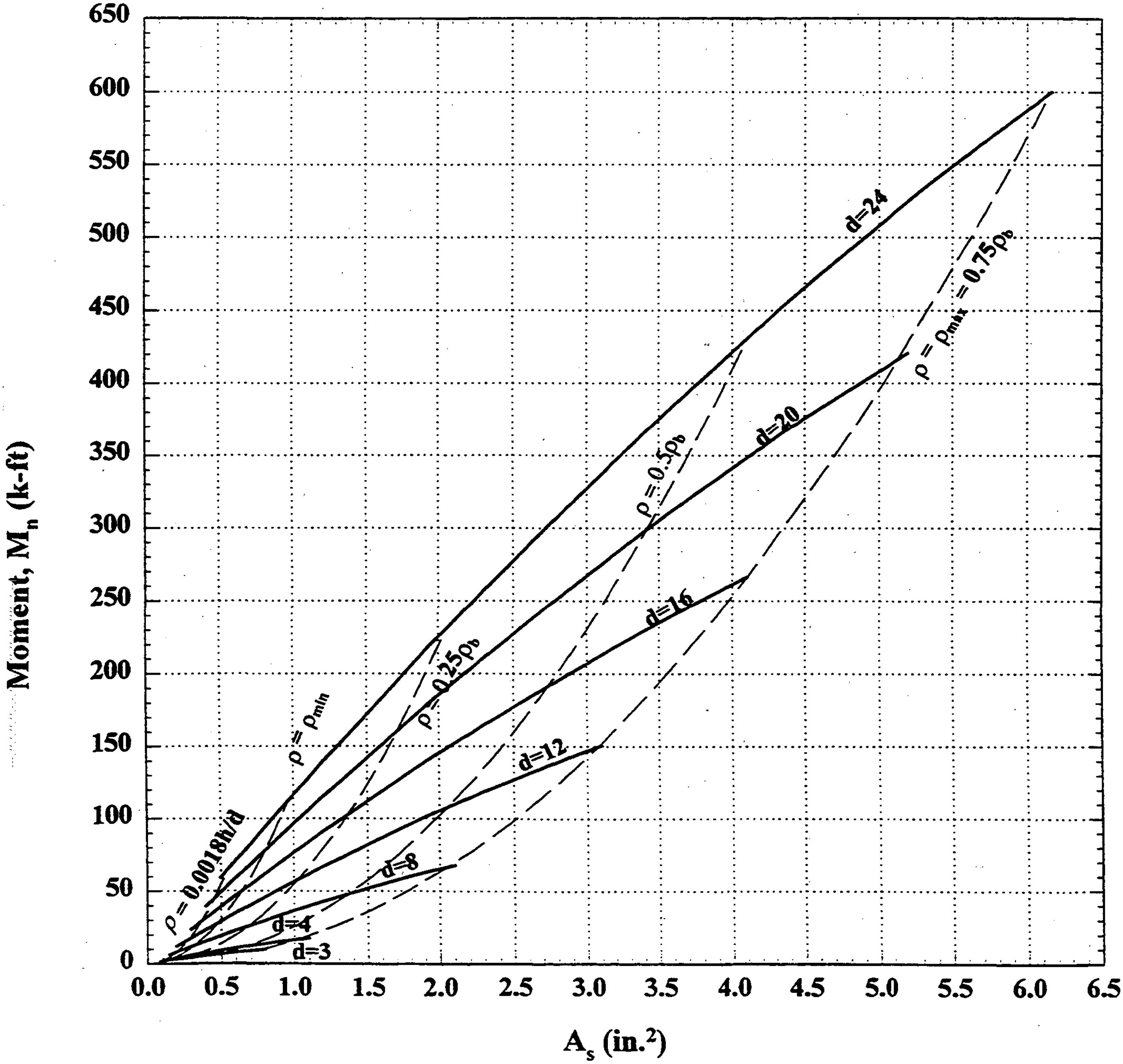
Reference: ACI 318-95 Sections 7.12, 8.4.1, 8.4.3, 9.3.2, 10.2, 10.3.1-10.3.3, 10.5.1 and 10.5.3 and ACI 318R-95 Sections 10.3.1 and 10.3.2

$f'_c = 4000$ psi

$f_y = 60{,}000$ psi

$M_n = 5.0A_sd - 3.68A_s^2$, k-ft

$$M_n \geq \frac{M_u}{\phi}$$



FLEXURE 6.4.2—Nominal strength M_n for slab sections 12 in. wide.

Reference: ACI 318-95 Sections 7.1.2, 8.4.1, 8.4.3, 9.3.2, 10.2, 10.3.1-10.3.3, 10.5.1, 10.5.3, and ACI 318R-95 Sections 10.3.1 and 10.3.2.

$f'_c = 4000$ psi

$f_y = 60{,}000$ psi

$$M_n = 5.0A_s d - 3.68A_s^2 \text{, k-ft}$$

$$M_n \geq \frac{M_u}{\phi}$$

Shrinkage and temperature reinforcement = $0.0018bh$

$$A_{smax} = \frac{0.75\beta_1(0.85f'_c)}{f_y}\left(\frac{87,000}{87,000+f_y}\right)bd = \frac{0.75(0.85)(0.85)(4)}{60,000}\left(\frac{87,000}{87,000+60,000}\right)(12)d$$

				M_n (Nominal Moment, k-ft)						
d	A_s *min*	A_s *max*	A_s	$d = 3$	$d = 4$	$d = 8$	$d = 12$	$d = 16$	$d = 20$	$d = 24$
3	0.086	0.77	0.09	1.26						
4	0.108	1.03	0.11	1.58	2.12					
5	0.130	1.28	0.13	1.89	2.54					
6	0.151	1.54	0.15	2.17	2.92	5.92				
7	0.173	1.80	0.20	2.85	3.85	7.85	11.85	15.85		
8	0.194	2.05	0.40	5.41	7.41	15.41	23.41	31.41	39.41	47.41
9	0.216	2.31	0.60	7.68	10.68	22.68	34.68	46.68	58.68	70.68
10	0.238	2.57	0.80	9.65	13.65	29.65	45.65	61.65	77.65	93.65
11	0.259	2.82	1.00		16.32	36.32	56.32	76.32	96.32	116.32
12	0.281	3.08	1.20		18.71	42.71	66.71	90.71	114.71	138.71
13	0.302	3.34	1.40			48.79	76.79	104.79	132.79	160.79
14	0.324	3.59	1.60			54.59	86.59	118.59	150.59	182.59
15	0.346	3.85	1.80			60.09	96.09	132.09	168.09	204.09
16	0.367	4.10	2.00			65.29	105.29	145.29	185.29	225.29
17	0.389	4.36	2.20			70.21	114.21	158.21	202.21	246.21
18	0.410	4.62	2.40				122.82	170.82	218.82	266.82
19	0.432	4.87	2.60				131.15	183.15	235.15	287.15
20	0.454	5.13	2.80				139.18	195.18	251.18	307.18
21	0.475	5.39	3.00				146.91	206.91	266.91	326.91
22	0.497	5.64	3.20				154.35	218.35	282.35	346.35
23	0.518	5.90	3.40					229.50	297.50	365.50
24	0.540	6.16	3.60					240.35	312.35	384.35
			3.80					250.91	326.91	402.91
			4.00					261.18	341.18	421.18
			4.20					271.15	355.15	439.15
			4.40						368.82	456.82
			4.60						382.21	474.21
			4.80						395.29	491.29
			5.00						408.09	508.09
			5.20						420.59	524.59
			5.40							540.79
			5.60							556.71
			5.80							572.32
			6.00							587.65
			6.16							599.69

FLEXURE 6.5.1 - Nominal strength M_n for slab sections 12 in. wide

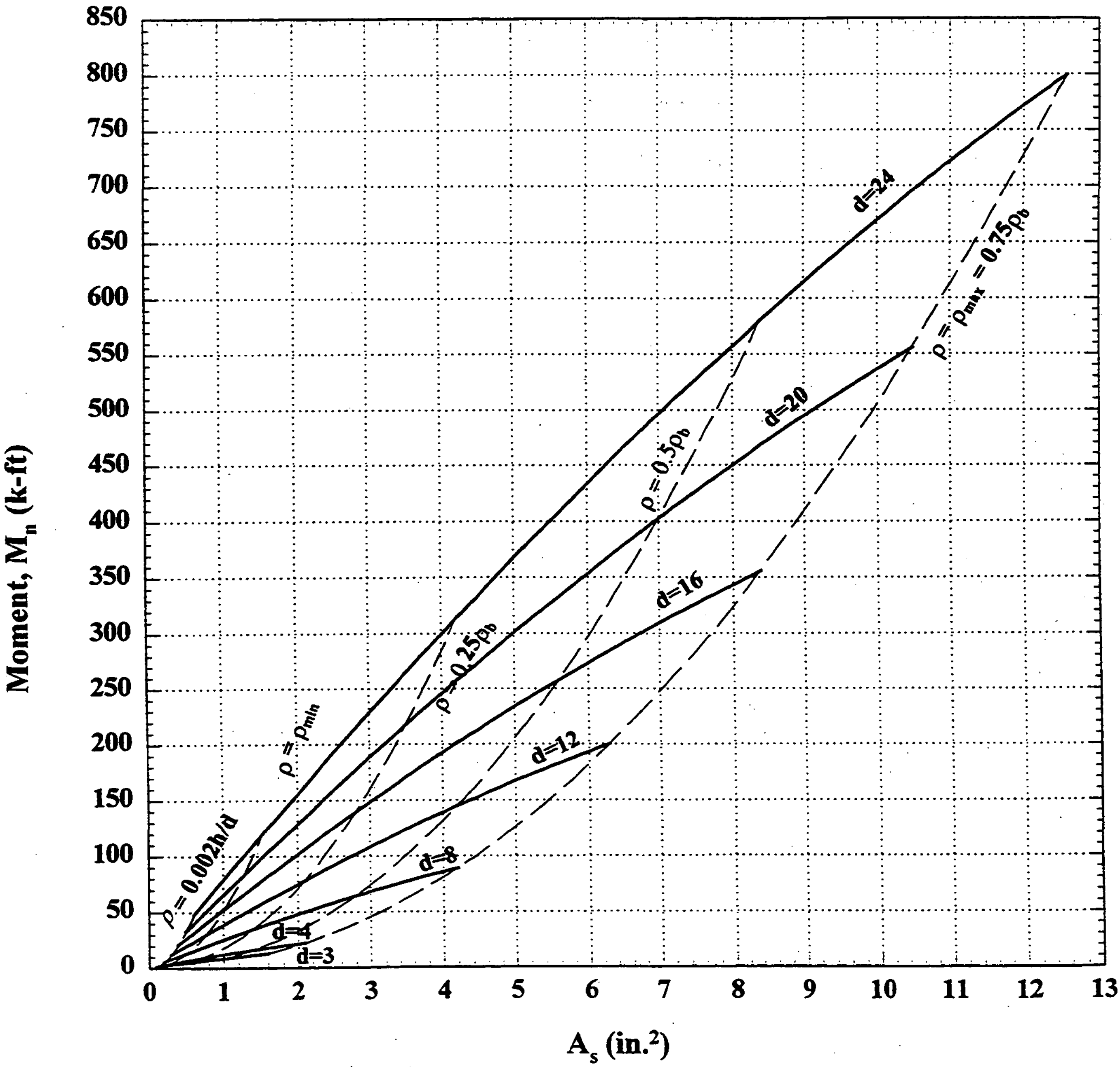
Reference: ACI 318-95 Sections 7.12, 8.4.1, 8.4.3, 9.3.2, 10.2, 10.3.1-10.3.3, 10.5.1 and 10.5.3 and ACI 318R-95 Sections 10.3.1 and 10.3.2

$f'_c = 5000$ psi

$f_y = 40,000$ psi

$M_n = 3.33A_sd - 1.31A_s^2$, k-ft

$$M_n \geq \frac{M_u}{\phi}$$



FLEXURE 6.5.2 - Nominal strength M_n for slab sections 12 in. wide

Reference: ACI 318-95 Sections 7.12, 8.4.1, 8.4.3, 9.3.2, 10.2, 10.3.1-10.3.3, 10.5.1 and 10.5.3 and ACI 318R-95 Sections 10.3.1 and 10.3.2

$f'_c = 5000$ psi $M_n = 3.33A_s d - 1.31A_s^2$, k-ft

$f_y = 40{,}000$ psi $M_n \geq \dfrac{M_u}{\phi}$

Shrinkage & Temperature reinforcement = 0.002*bh*

$$A_{smax} = \frac{0.75\beta_1(0.85f'_c)}{f_y}\left(\frac{87000}{87000+f_y}\right)bd = \frac{0.75(0.80)(0.85)(5)}{40000}\left(\frac{87000}{87000+40000}\right)(12)d$$

				M_n (Nominal Moment, k-ft)						
d	As min	As max	As	d=3	d=4	d=8	d=12	d=16	d=20	d=24
3	0.096	1:57	0.10	0.95						
4	0.120	2.10	0.12	1.18	1.58					
5	0.144	2.62	0.15	1.47	1.97					
6	0.168	3.14	0.20	1.95	2.61	5.28				
7	0.192	3.67	0.25	2.42	3.25	6.58	9.92	13.25		
8	0.216	4.19	0.50	4.67	6.34	13.01	19.67	26.34	33.01	39.67
9	0.240	4.72	0.75	6.76	9.26	19.26	29.26	39.26	49.26	59.26
10	0.264	5.24	1.00	8.69	12.03	25.36	38.69	52.03	65.36	78.69
11	0.288	5.76	1.40	11.44	16.10	34.77	53.44	72.10	90.77	109.44
12	0.312	6.29	1.80	13.76	19.76	43.76	67.76	91.76	115.76	139.76
13	0.336	6.81	2.20		23.01	52.34	81.67	111.01	140.34	169.67
14	0.360	7.34	2.60			60.50	95.16	129.83	164.50	199.16
15	0.384	7.86	3.00			68.24	108.24	148.24	188.24	228.24
16	0.408	8.38	3.40			75.56	120.89	166.22	211.56	256.89
17	0.432	8.91	3.80			82.46	133.12	183.79	234.46	285.12
18	0.456	9.43	4.20			88.94	144.94	200.94	256.94	312.94
19	0.480	9.96	4.60				156.34	217.67	279.01	340.34
20	0.504	10.48	5.00				167.32	233.99	300.65	367.32
21	0.528	11.01	5.40				177.88	249.88	321.88	393.88
22	0.552	11.53	5.80				188.03	265.36	342.69	420.03
23	0.576	12.05	6.20				197.75	280.42	363.08	445.75
24	0.600	12.58	6.60				207.06	295.06	383.06	471.06
			7.00					309.28	402.61	495.95
			7.40					323.08	421.75	520.42
			7.80					336.47	440.47	544.47
			8.20					349.44	458.77	568.10
			8.60					361.99	476.65	591.32
			9.00						494.12	614.12
			9.40						511.16	636.50
			9.80						527.79	658.46
			10.20						544.00	680.00
			10.60						559.79	701.12
			11.00							721.83
			11.40							742.12
			11.80							761.99
			12.20							781.44
			12.58							799.53

FLEXURE 6.6.1 - Nominal strength M_n for slab sections 12 in. wide

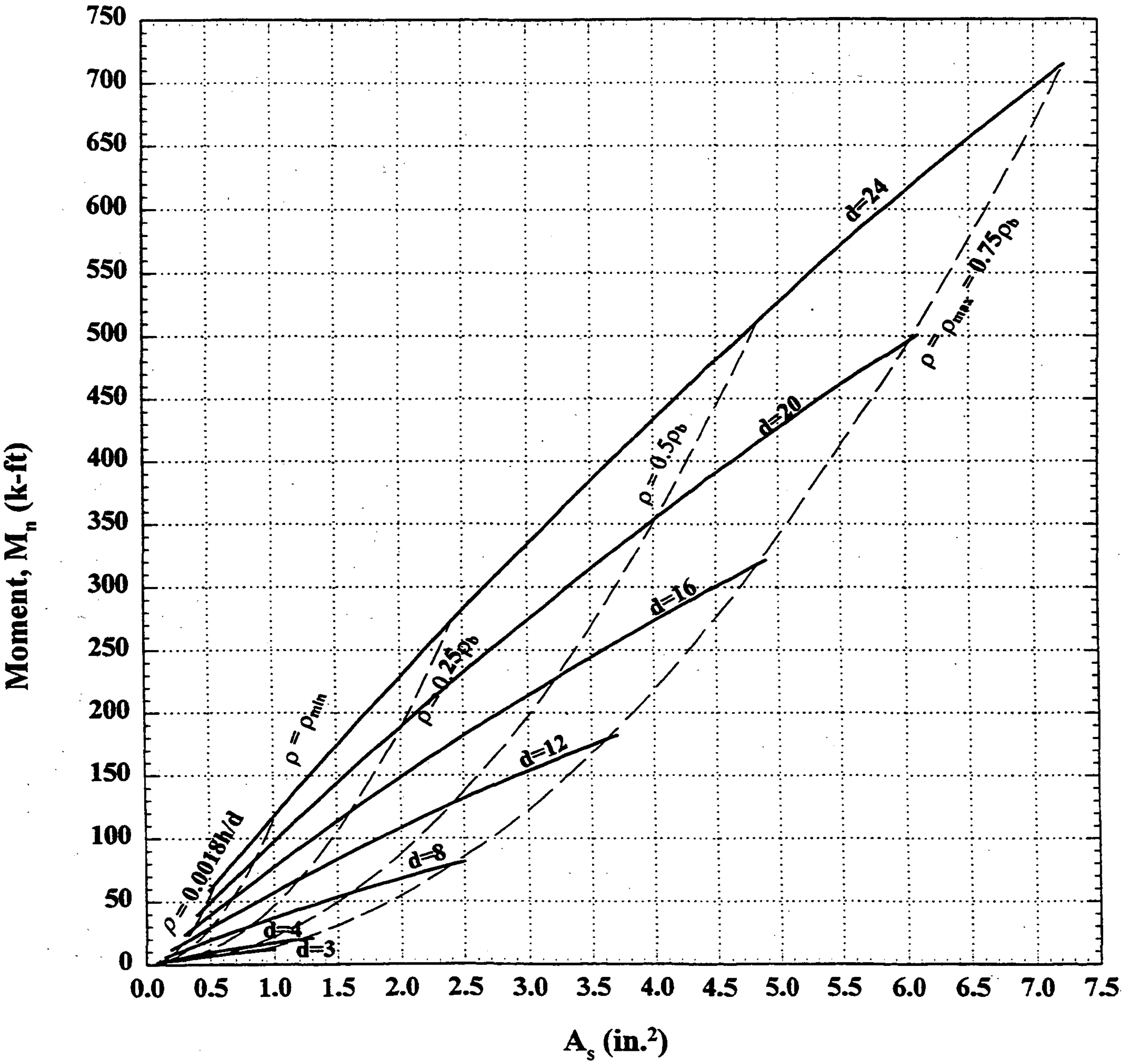
Reference: ACI 318-95 Sections 7.12, 8.4.1, 8.4.3, 9.3.2, 10.2, 10.3.1-10.3.3, 10.5.1 and 10.5.3 and ACI 318R-95 Sections 10.3.1 and 10.3.2

$f'_c = 5000$ psi

$f_y = 60{,}000$ psi

$M_n = 5.0A_s d - 2.94A_s^2$, k-ft

$$M_n \geq \frac{M_u}{\phi}$$



FLEXURE 6.6.2 - Nominal strength M_n for slab sections 12 in. wide

Reference: ACI 318-95 Sections 7.12, 8.4.1, 8.4.3, 9.3.2, 10.2, 10.3.1-10.3.3, 10.5.1 and 10.5.3 and ACI 318R-95 Sections 10.3.1 and 10.3.2

$f'_c = 5000$ psi

$f_y = 60{,}000$ psi

$$M_n = 5.0A_s d - 2.94A_s^2\text{, k-ft}$$

$$M_n \geq \frac{M_u}{\phi}$$

Shrinkage & Temperature reinforcement = $0.0018bh$

$$A_{smax} = \frac{0.75\beta_1(0.85f'_c)}{f_y}\left(\frac{87000}{87000+f_y}\right)bd = \frac{0.75(0.80)(0.85)(5)}{60000}\left(\frac{87000}{87000+60000}\right)(12)d$$

				M_n (Nominal Moment, k-ft)						
d	As min	As max	As	d=3	d=4	d=8	d=12	d=16	d=20	d=24
3	0.086	0.91	0.09	1.27						
4	0.108	1.21	0.11	1.59	2.13	4.29				
5	0.130	1.51	0.20	2.88	3.88	7.88	11.88	15.88		
6	0.151	1.81	0.40	5.53	7.53	15.53	23.53	31.53	39.53	47.53
7	0.173	2.11	0.60	7.94	10.94	22.94	34.94	46.94	58.94	70.94
8	0.194	2.41	0.80	10.12	14.12	30.12	46.12	62.12	78.12	94.12
9	0.216	2.72	1.00	12.06	17.06	37.06	57.06	77.06	97.06	117.06
10	0.238	3.02	1.20		19.76	43.76	67.76	91.76	115.76	139.76
11	0.259	3.32	1.40		22.24	50.24	78.24	106.24	134.24	162.24
12	0.281	3.62	1.60			56.47	88.47	120.47	152.47	184.47
13	0.302	3.92	1.80			62.47	98.47	134.47	170.47	206.47
14	0.324	4.23	2.00			68.24	108.24	148.24	188.24	228.24
15	0.346	4.53	2.20			73.76	117.76	161.76	205.76	249.76
16	0.367	4.83	2.40			79.06	127.06	175.06	223.06	271.06
17	0.389	5.13	2.60			84.12	136.12	188.12	240.12	292.12
18	0.410	5.43	2.80				144.94	200.94	256.94	312.94
19	0.432	5.73	3.00				153.53	213.53	273.53	333.53
20	0.454	6.04	3.20				161.88	225.88	289.88	353.88
21	0.475	6.34	3.40				170.00	238.00	306.00	374.00
22	0.497	6.64	3.60				177.88	249.88	321.88	393.88
23	0.518	6.94	3.80				185.53	261.53	337.53	413.53
24	0.540	7.24	4.00					272.94	352.94	432.94
			4.20					284.12	368.12	452.12
			4.40					295.06	383.06	471.06
			4.60					305.76	397.76	489.76
			4.80					316.24	412.24	508.24
			5.00					326.47	426.47	526.47
			5.20						440.47	544.47
			5.40						454.24	562.24
			5.60						467.76	579.76
			5.80						481.06	597.06
			6.00						494.12	614.12
			6.20						506.94	630.94
			6.40							647.53
			6.60							663.88
			6.80							680.00
			7.00							695.88
			7.20							711.53
			7.24							714.63

FLEXURE 6.7.1 - Nominal strength M_n for slab sections 12 in. wide

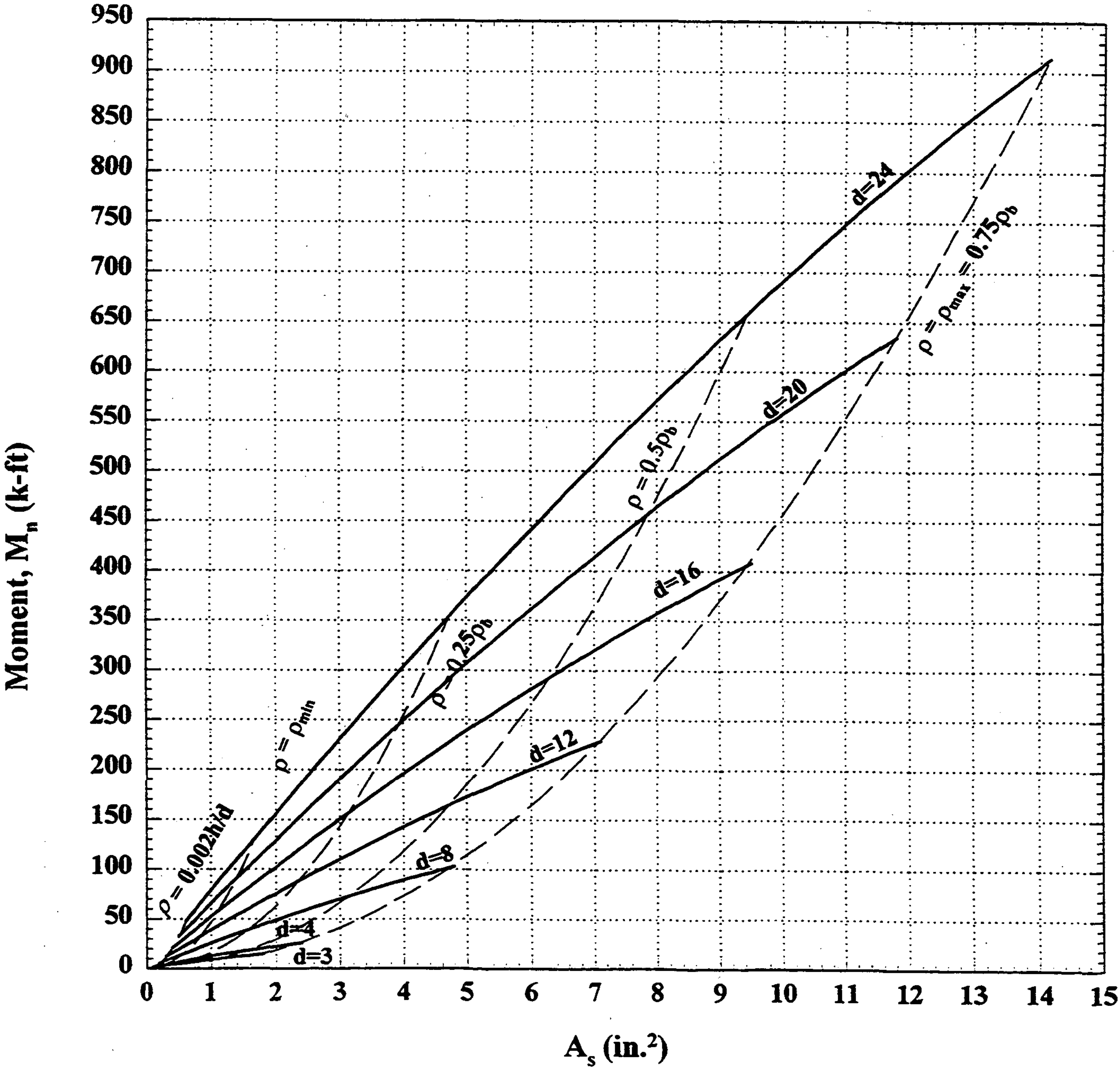
Reference: ACI 318-95 Sections 7.12, 8.4.1, 8.4.3, 9.3.2, 10.2, 10.3.1-10.3.3, 10.5.1 and 10.5.3 and ACI 318R-95 Sections 10.3.1 and 10.3.2

$f'_c = 6000$ psi

$f_y = 40{,}000$ psi

$M_n = 3.33A_s d - 1.09A_s^2$, k-ft

$$M_n \geq \frac{M_u}{\phi}$$



FLEXURE 6.7.2 - Nominal strength M_n for slab sections 12 in. wide

Reference: ACI 318-95 Sections 7.12, 8.4.1, 8.4.3, 9.3.2, 10.2, 10.3.1-10.3.3, 10.5.1 and 10.5.3 and ACI 318R-95 Sections 10.3.1 and 10.3.2

$f'_c = 6000$ psi $\qquad M_n = 3.33A_sd - 1.09A_s^2$, k-ft

$f_y = 40{,}000$ psi $\qquad M_n \geq \dfrac{M_u}{\phi}$

Shrinkage & Temperature reinforcement = 0.002*bh*

$$A_{smax} = \frac{0.75\beta_1(0.85f'_c)}{f_y}\left(\frac{87000}{87000+f_y}\right)bd = \frac{0.75(0.75)(0.85)(6)}{40000}\left(\frac{87000}{87000+40000}\right)(12)d$$

				M_n (Nominal Moment, k-ft)						
d	As min	As max	As	d=3	d=4	d=8	d=12	d=16	d=20	d=24
3	0.096	1.77	0.10	0.95						
4	0.120	2.36	0.12	1.18	1.58					
5	0.144	2.95	0.15	1.48	1.98					
6	0.168	3.54	0.20	1.96	2.62	5.29	7.96	10.62	13.29	15.96
7	0.192	4.13	0.60	5.61	7.61	15.61	23.61	31.61	39.61	47.61
8	0.216	4.72	1.00	8.91	12.24	25.58	38.91	52.24	65.58	78.91
9	0.240	5.31	1.40	11.86	16.53	35.20	53.86	72.53	91.20	109.86
10	0.264	5.90	1.80	14.47	20.47	44.47	68.47	92.47	116.47	140.47
11	0.288	6.49	2.20		24.06	53.39	82.73	112.06	141.39	170.73
12	0.312	7.07	2.60		27.30	61.97	96.64	131.30	165.97	200.64
13	0.336	7.66	3.00			70.20	110.20	150.20	190.20	230.20
14	0.360	8.25	3.40			78.07	123.41	168.74	214.07	259.41
15	0.384	8.84	3.80			85.60	136.27	186.94	237.60	288.27
16	0.408	9.43	4.20			92.78	148.78	204.78	260.78	316.78
17	0.432	10.02	4.60			99.62	160.95	222.28	283.62	344.95
18	0.456	10.61	5.00			106.10	172.77	239.43	306.10	372.77
19	0.480	11.20	5.40				184.24	256.24	328.24	400.24
20	0.504	11.79	5.80				195.36	272.69	350.02	427.36
21	0.528	12.38	6.20				206.13	288.79	371.46	454.13
22	0.552	12.97	6.60				216.55	304.55	392.55	480.55
23	0.576	13.56	7.00				226.62	319.96	413.29	506.62
24	0.600	14.15	7.40				236.35	335.02	433.68	532.35
			7.80					349.73	453.73	557.73
			8.20					364.09	473.42	582.75
			8.60					378.10	492.77	607.43
			9.00					391.76	511.76	631.76
			9.40					405.08	530.41	655.75
			9.80					418.05	548.71	679.38
			10.20						566.67	702.67
			10.60						584.27	725.60
			11.00						601.53	748.19
			11.40						618.43	770.43
			11.80						634.99	792.32
			12.20							813.86
			12.60							835.06
			13.00							855.90
			13.40							876.40
			13.80							896.55
			14.15							913.89

FLEXURE 6.8.1 - Nominal strength M_n for slab sections 12 in. wide

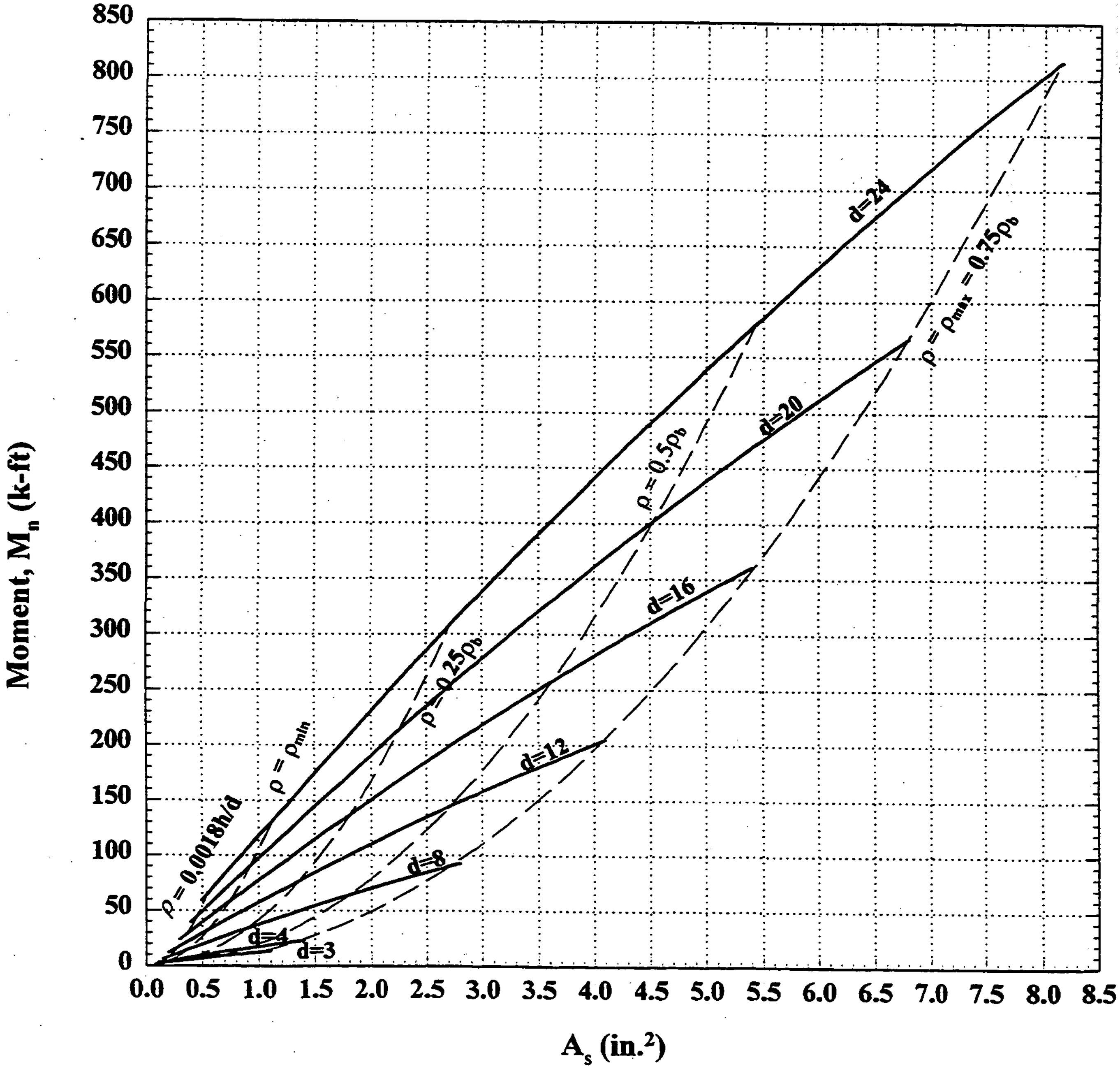
Reference: ACI 318-95 Sections 7.12, 8.4.1, 8.4.3, 9.3.2, 10.2, 10.3.1-10.3.3, 10.5.1 and 10.5.3 and ACI 318R-95 Sections 10.3.1 and 10.3.2

$f'_c = 6000$ psi

$f_y = 60{,}000$ psi

$M_n = 5.0A_s d - 2.45A_s^2$, k-ft

$M_n \geq \dfrac{M_u}{\phi}$



FLEXURE 6.8.2 - Nominal strength M_n for slab sections 12 in. wide

Reference: ACI 318-95 Sections 7.12, 8.4.1, 8.4.3, 9.3.2, 10.2, 10.3.1-10.3.3, 10.5.1 and 10.5.3 and ACI 318R-95 Sections 10.3.1 and 10.3.2

f'_c = 6000 psi

f_y = 60,000 psi

$$M_n = 5.0A_s d - 2.45A_s^2\text{, k-ft}$$

$$M_n \geq \frac{M_u}{\phi}$$

Shrinkage & Temperature reinforcement = 0.0018*bh*

$$A_{smax} = \frac{0.75\beta_1(0.85f'_c)}{f_y}\left(\frac{87000}{87000 + f_y}\right)bd = \frac{0.75(0.75)(0.85)(6)}{60000}\left(\frac{87000}{87000 + 60000}\right)(12)d$$

				M_n *(Nominal Moment, k-ft)*						
d	As min	As max	As	d=3	d=4	d=8	d=12	d=16	d=20	d=24
3	0.086	1.02	0.09	1.27						
4	0.108	1.36	0.11	1.59	2.13					
5	0.130	1.70	0.13	1.91	2.56					
6	0.151	2.04	0.15	2.19	2.94	5.94				
7	0.173	2.38	0.20	2.90	3.90	7.90				
8	0.194	2.72	0.25	3.60	4.85	9.85	14.85	19.85	24.85	
9	0.216	3.06	0.50	6.89	9.39	19.39	29.39	39.39	49.39	59.39
10	0.238	3.40	0.75	9.87	13.62	28.62	43.62	58.62	73.62	88.62
11	0.259	3.74	1.00	12.55	17.55	37.55	57.55	77.55	97.55	117.55
12	0.281	4.07	1.25	14.92	21.17	46.17	71.17	96.17	121.17	146.17
13	0.302	4.41	1.50		24.49	54.49	84.49	114.49	144.49	174.49
14	0.324	4.75	1.75			62.49	97.49	132.49	167.49	202.49
15	0.346	5.09	2.00			70.20	110.20	150.20	190.20	230.20
16	0.367	5.43	2.25			77.59	122.59	167.59	212.59	257.59
17	0.389	5.77	2.50			84.68	134.68	184.68	234.68	284.68
18	0.410	6.11	2.75			91.46	146.46	201.46	256.46	311.46
19	0.432	6.45	3.00				157.94	217.94	277.94	337.94
20	0.454	6.79	3.25				169.11	234.11	299.11	364.11
21	0.475	7.13	3.50				179.98	249.98	319.98	389.98
22	0.497	7.47	3.75				190.53	265.53	340.53	415.53
23	0.518	7.81	4.00				200.78	280.78	360.78	440.78
24	0.540	8.15	4.25				210.73	295.73	380.73	465.73
			4.50					310.37	400.37	490.37
			4.75					324.70	419.70	514.70
			5.00					338.73	438.73	538.73
			5.25					352.44	457.44	562.44
			5.50					365.86	475.86	585.86
			5.75						493.96	608.96
			6.00						511.76	631.76
			6.25						529.26	654.26
			6.50						546.45	676.45
			6.75						563.33	698.33
			7.00						579.90	719.90
			7.25							741.17
			7.50							762.13
			7.75							782.79
			8.00							803.14
			8.15							815.20

REINFORCEMENT

REINFORCEMENT 1 - Nominal cross section area, weight, and nominal diameter of ASTM standard reinforcing bars

BAR SIZE DESIGNATION	NOMINAL CROSS SECTION AREA (sq. in.)	WEIGHT (lb/ft)	NOMINAL DIAMETER (in.)	NOMINAL PERIMETER (in.)
#3	0.11	0.376	0.375	1.18
#4	0.20	0.668	0.500	1.57
#5	0.31	1.043	0.625	1.96
#6	0.44	1.502	0.750	2.36
#7	0.60	2.044	0.875	2.75
#8	0.79	2.670	1.000	3.14
#9	1.00	3.400	1.128	3.54
#10	1.27	4.303	1.270	3.99
#11	1.56	5.313	1.410	4.43
#14	2.25	7.650	1.693	5.32
#18	4.00	13.600	2.257	7.09

Note: The nominal dimensions of a deformed bar are equivalent to those of a plain bar having the same mass per foot as the deformed bars.

REINFORCEMENT 2—Cross section areas for various combinations of bars.

Areas A_s (or A'_s), sq. in.

Columns headed 0, 5 contain data for bars of one size in groups of one to ten.

Columns headed 1, 2, 3, 4, 5 contain data for bars of two sizes with one to five of each size.

			0	5		1	2	3	4	5		1	2	3	4	5		1	2	3	4	5
	1		0.20	1.20		0.31	0.42	0.53	0.64	0.75												
	2		0.40	1.40		0.51	0.62	0.73	0.84	0.95												
#4	3	#4	0.60	1.60	#3	0.71	0.82	0.93	1.04	1.15												
	4		0.80	1.80		0.91	1.02	1.13	1.24	1.35												
	5		1.00	2.00		1.11	1.22	1.33	1.44	1.55												
	1		0.31	1.86		0.51	0.71	0.91	1.11	1.31		0.42	0.53	0.64	0.75	0.86						
	2		0.62	2.17		0.82	1.02	1.22	1.42	1.62		0.73	0.84	0.95	1.06	1.17						
#5	3	#5	0.93	2.48	#4	1.13	1.33	1.53	1.73	1.93	#3	1.04	1.15	1.26	1.37	1.48						
	4		1.24	2.79		1.44	1.64	1.84	2.04	2.24		1.35	1.46	1.57	1.68	1.79						
	5		1.55	3.10		1.75	1.95	2.15	2.35	2.55		1.66	1.77	1.88	1.99	2.10						
	1		0.44	2.64		0.75	1.06	1.37	1.68	1.99		0.64	0.84	1.04	1.24	1.44		0.55	0.66	0.77	0.88	0.99
	2		0.88	3.08		1.19	1.50	1.81	2.12	2.43		1.08	1.28	1.48	1.68	1.88		0.99	1.10	1.21	1.32	1.43
#6	3	#6	1.32	3.52	#5	1.63	1.94	2.25	2.56	2.87	#4	1.52	1.72	1.92	2.12	2.32	#3	1.43	1.54	1.65	1.76	1.87
	4		1.76	3.96		2.07	2.38	2.69	3.00	3.31		1.96	2.16	2.36	2.56	2.76		1.87	1.98	2.09	2.20	2.31
	5		2.20	4.40		2.51	2.82	3.13	3.44	3.75		2.40	2.60	2.80	3.00	3.20		2.31	2.42	2.53	2.64	2.75
	1		0.60	3.60		1.04	1.48	1.92	2.36	2.80		0.91	1.22	1.53	1.84	2.15		0.80	1.00	1.20	1.40	1.60
	2		1.20	4.20		1.64	2.08	2.52	2.96	3.40		1.51	1.82	2.13	2.44	2.75		1.40	1.60	1.80	2.00	2.20
#7	3	#7	1.80	4.80	#6	2.24	2.68	3.12	3.56	4.00	#5	2.11	2.42	2.73	3.04	3.35	#4	2.00	2.20	2.40	2.60	2.80
	4		2.40	5.40		2.84	3.28	3.72	4.16	4.60		2.71	3.02	3.33	3.64	3.95		2.60	2.80	3.00	3.20	3.40
	5		3.00	6.00		3.44	3.88	4.32	4.76	5.20		3.31	3.62	3.93	4.24	4.55		3.20	3.40	3.60	3.80	4.00
	1		0.79	4.74		1.39	1.99	2.59	3.19	3.79		1.23	1.67	2.11	2.55	2.99		1.10	1.41	1.72	2.03	2.34
	2		1.58	5.53		2.18	2.78	3.38	3.98	4.58		2.02	2.46	2.90	3.34	3.78		1.89	2.20	2.51	2.82	3.13
#8	3	#8	2.37	6.32	#7	2.97	3.57	4.17	4.77	5.37	#6	2.81	3.25	3.69	4.13	4.57	#5	2.68	2.99	3.30	3.61	3.92
	4		3.16	7.11		3.76	4.36	4.96	5.56	6.16		3.60	4.04	4.48	4.92	5.36		3.47	3.78	4.09	4.40	4.71
	5		3.95	7.90		4.55	5.15	5.75	6.35	6.95		4.39	4.83	5.27	5.71	6.15		4.26	4.57	4.88	5.19	5.50
	1		1.00	6.00		1.79	2.58	3.37	4.16	4.95		1.60	2.20	2.80	3.40	4.00		1.44	1.88	2.32	2.76	3.20
	2		2.00	7.00		2.79	3.58	4.37	5.16	5.95		2.60	3.20	3.80	4.40	5.00		2.44	2.88	3.32	3.76	4.20
#9	3	#9	3.00	8.00	#8	3.79	4.58	5.37	6.16	6.95	#7	3.60	4.20	4.80	5.40	6.00	#6	3.44	3.88	4.32	4.76	5.20
	4		4.00	9.00		4.79	5.58	6.37	7.16	7.95		4.60	5.20	5.80	6.40	7.00		4.44	4.88	5.32	5.76	6.20
	5		5.00	10.00		5.79	6.58	7.37	8.16	8.95		5.60	6.20	6.80	7.40	8.00		5.44	5.88	6.32	6.76	7.20
	1		1.27	7.62		2.27	3.27	4.27	5.27	6.27		2.06	2.85	3.64	4.43	5.22		1.87	2.47	3.07	3.67	4.27
	2		2.54	8.89		3.54	4.54	5.54	6.54	7.54		3.33	4.12	4.91	5.70	6.49		3.14	3.74	4.34	4.94	5.54
#10	3	#10	3.81	10.16	#9	4.81	5.81	6.81	7.81	8.81	#8	4.60	5.39	6.18	6.97	7.76	#7	4.41	5.01	5.61	6.21	6.81
	4		5.08	11.43		6.08	7.08	8.08	9.08	10.08		5.87	6.66	7.45	8.24	9.03		5.68	6.28	6.88	7.48	8.08
	5		6.35	12.70		7.35	8.35	9.35	10.35	11.35		7.14	7.93	8.72	9.51	10.30		6.95	7.55	8.15	8.75	9.35

For use of this Design Aid, see Flexure Example 2.

(continued)

			0	5		1	2	3	4	5		1	2	3	4	5		1	2	3	4	5
#11	1	#11	1.56	9.36	#10	2.83	4.10	5.37	6.64	7.91	#9	2.56	3.56	4.56	5.56	6.56	#8	2.35	3.14	3.93	4.72	5.51
	2		3.12	10.92		4.39	5.66	6.93	8.20	9.47		4.12	5.12	6.12	7.12	8.12		3.91	4.70	5.49	6.28	7.07
	3		4.68	12.48		5.95	7.22	8.49	9.76	11.03		5.68	6.68	7.68	8.68	9.68		5.47	6.26	7.05	7.84	8.63
	4		6.24	14.04		7.51	8.78	10.05	11.32	12.59		7.24	8.24	9.24	10.24	11.24		7.03	7.82	8.61	9.40	10.19
	5		7.80	15.60		9.07	10.34	11.61	12.88	14.15		8.80	9.80	10.80	11.80	12.80		8.59	9.38	10.17	10.96	11.75
#14	1	#14	2.25	13.50	#11	3.81	5.37	6.93	8.49	10.05	#10	3.52	4.79	6.06	7.33	8.60	#9	3.25	4.25	5.25	6.25	7.25
	2		4.50	15.75		6.06	7.62	9.18	10.74	12.30		5.77	7.04	8.31	9.58	10.85		5.50	6.50	7.50	8.50	9.50
	3		6.75	18.00		8.31	9.87	11.43	12.99	14.55		8.02	9.29	10.56	11.83	13.10		7.75	8.75	9.75	10.75	11.75
	4		9.00	20.25		10.58	12.12	13.68	15.24	16.80		10.27	11.54	12.81	14.08	15.35		10.00	11.00	12.00	13.00	14.00
	5		11.25	22.50		12.81	14.37	15.93	17.49	19.05		12.52	13.79	15.06	16.33	17.60		12.25	13.25	14.25	15.25	16.25
#18	1	#18	4.00	24.00	#14	6.25	8.50	10.75	13.00	15.25	#11	5.56	7.12	8.68	10.24	11.80	#10	5.27	6.54	7.81	9.08	10.35
	2		8.00	28.00		10.25	12.50	14.75	17.00	19.25		9.56	11.12	12.68	14.24	15.80		9.27	10.54	11.81	13.08	14.35
	3		12.00	32.00		14.25	16.50	18.75	21.00	23.25		13.56	15.12	16.68	18.24	19.80		13.27	14.54	15.81	17.08	18.35
	4		16.00	36.00		18.25	20.50	22.75	25.00	27.25		17.56	19.12	20.68	22.24	23.80		17.27	18.54	19.81	21.08	22.35
	5		20.00	40.00		22.25	24.50	26.75	29.00	31.25		21.56	23.12	24.68	26.24	27.80		21.27	22.54	23.81	25.08	26.35

Example 1: Find the area of 2 #5 bars: Go down the first column to "#5" and read, under the "0" column, A = 0.62 sq in.

Example 2: Find the area of 8 #5 bars: Go down the first column to "#5" and read, under the first "5" column (5 + 3), A = 2.48 sq in.

Example 3: Find the area of 2 #7 + 3 #6 bars: Go down the first column to "#7" and proceed horizontally on the "2" line until the "3" column of the #6 group and read A = 2.52 sq in.

Example 4: Find the area of 3 #8 + 4 #6 bars: Go down the first column to "#8" and horizontally on the "3" line until the "4" column of the #6 group and read A = 4.13 sq in.

This table does not apply for combination of more than two sizes.

This table does not apply for more than 10 bars of one size, or five bars each of two sizes.

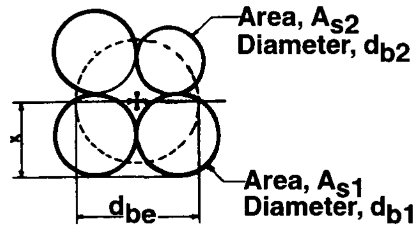
REINFORCEMENT 3—Properties of bundled bars

Reference: ACI 318-89 Section 7.6.6.5.

Equivalent diameter, $d_{be} = \sqrt{\frac{4}{\pi} A_s}$

Centroidal distance, x

Centroid of group, ✢



Bars				Combination of bars								
				Equivalent diameters d_{be}, in.								
#8	1.42	1.74	2.01	#7	#6	1.15	1.37	1.45	1.56	1.63	1.69
9	1.60	1.95	2.26	7	5	1.08	1.25	1.39	1.40	1.52	1.64
10	1.80	2.20	2.54	8	7	1.33	1.59	1.67	1.82	1.88	1.94
11	1.99	2.44	2.82	8	6	1.25	1.46	1.60	1.64	1.77	1.89
				9	8	1.51	1.81	1.88	2.07	2.13	2.20
				9	7	1.43	1.67	1.82	1.89	2.02	2.14
				10	9	1.70	2.04	2.12	2.33	2.40	2.47
				10	8	1.62	1.90	2.06	2.15	2.29	2.42
				11	10	1.90	2.28	2.36	2.61	2.68	2.75
				11	9	1.81	2.13	2.29	2.41	2.55	2.69
				Centroidal distance x, from bottom of bundle, in.								
#4	0.25	0.39	0.50	#4	#3	0.23	0.38	0.33	0.43	0.40	0.46
5	0.31	0.49	0.62	5	4	0.29	0.47	0.43	0.55	0.53	0.58
6	0.37	0.59	0.75	5	3	0.28	0.46	0.37	0.49	0.44	0.55
				6	5	0.35	0.57	0.53	0.67	0.66	0.71
7	0.44	0.69	0.87	6	4	0.34	0.55	0.47	0.61	0.57	0.67
8	0.50	0.79	1.00								
9	0.56	0.89	1.13	7	6	0.41	0.67	0.62	0.80	0.78	0.83
				7	5	0.39	0.65	0.56	0.73	0.69	0.79
10	0.63	1.00	1.27	8	7	0.47	0.77	0.72	0.93	0.90	0.95
11	0.70	1.11	1.41	8	6	0.46	0.75	0.66	0.86	0.81	0.92
				9	8	0.54	0.86	0.82	1.05	1.03	1.08
				9	7	0.52	0.85	0.75	0.98	0.94	1.04
				10	9	0.60	0.98	0.92	1.19	1.16	1.22
				10	8	0.58	0.95	0.86	1.11	1.07	1.18
				11	10	0.67	1.08	1.03	1.32	1.31	1.36
				11	9	0.65	1.06	0.96	1.24	1.20	1.31

Example: Find the equivalent diameter of a single bar for 4 #9 bars. For 4 #9 bars, read d_{be} = 2.26 in., and the centroidal distance x equals 1.13 in.

REINFORCEMENT 4 - Sectional properties and areas of plain and deformed welded wire reinforcement

Reference: Welded Wire Fabric Manual of Standards Practice, 3rd edition, 1979, p. 18, published by Wire Reinforcement Institute, Inc., 301 E. Sardusky St., Findlay, OH 45640

Wire Size Number		Nominal Diameter, in.	Nominal Weight, lb/ft	A_s, in.²/ft Center to center spacing, in.						
Deformed	Plain			2	3	4	6	8	10	12
D45	W45	0.757	1.530	2.70	1.80	1.35	0.90	0.675	0.54	0.45
D31	W31	0.628	1.054	1.86	1.24	0.93	0.62	0.465	0.372	0.31
D30	W30	0.618	1.020	1.80	1.20	0.90	0.60	0.45	0.360	0.30
D28	W28	0.597	0.952	1.68	1.12	0.84	0.56	0.42	0.336	0.28
D26	W26	0.575	0.934	1.56	1.04	0.78	0.52	0.39	0.312	0.26
D24	W24	0.553	0.816	1.44	0.96	0.72	0.48	0.36	0.288	0.24
D22	W22	0.529	0.748	1.32	0.88	0.66	0.44	0.33	0.264	0.22
D20	W20	0.505	0.680	1.20	0.80	0.60	0.40	0.30	0.24	0.20
D18	W18	0.479	0.612	1.08	0.72	0.54	0.36	0.27	0.216	0.18
D16	W16	0.451	0.544	0.96	0.64	0.48	0.32	0.24	0.192	0.16
D14	W14	0.422	0.476	0.84	0.56	0.42	0.28	0.21	0.168	0.14
D12	W12	0.391	0.408	0.72	0.48	0.36	0.24	0.18	0.144	0.12
D11	W11	0.374	0.374	0.66	0.44	0.33	0.22	0.165	0.132	0.11
	W10.5	0.366	0.357	0.63	0.42	0.315	0.21	0.158	0.126	0.105
D10	W10	0.357	0.340	0.60	0.40	0.30	0.20	0.15	0.12	0.10
	W9.5	0.348	0.323	0.57	0.38	0.285	0.19	0.143	0.114	0.095
D9	W9	0.338	0.306	0.54	0.36	0.27	0.18	0.135	0.108	0.09
	W8.5	0.329	0.289	0.51	0.34	0.255	0.17	0.128	0.102	0.085
D8	W8	0.319	0.272	0.48	0.32	0.24	0.16	0.12	0.096	0.08
	W7.5	0.309	0.255	0.45	0.30	0.225	0.15	0.113	0.09	0.075
D7	W7	0.299	0.238	0.42	0.28	0.21	0.14	0.105	0.084	0.07
	W6.5	0.288	0.221	0.39	0.26	0.195	0.13	0.098	0.078	0.065
D6	W6	0.275	0.204	0.36	0.24	0.18	0.12	0.09	0.072	0.06
	W5.5	0.255	0.187	0.33	0.22	0.165	0.11	0.083	0.066	0.055
D5	W5	0.252	0.170	0.30	0.20	0.15	0.10	0.075	0.06	0.05
	W4.5	0.239	0.153	0.27	0.18	0.135	0.09	0.068	0.054	0.045
D4	W4	0.226	0.136	0.24	0.16	0.12	0.08	0.06	0.048	0.04
	W3.5	0.211	0.119	0.21	0.14	0.105	0.07	0.053	0.042	0.035
	W3	0.195	0.102	0.18	0.12	0.09	0.06	0.045	0.036	0.03
	W2.9	0.192	0.098	0.174	0.116	0.087	0.058	0.043	0.035	0.029
	W2.5	0.173	0.085	0.15	0.10	0.075	0.05	0.038	0.03	0.025
	W2	0.150	0.068	0.12	0.08	0.06	0.04	0.03	0.024	0.02
	W1.5	0.138	0.051	0.09	0.06	0.045	0.03	0.023	0.018	0.015
	W1.4	0.134	0.049	0.084	0.056	0.042	0.028	0.021	0.017	0.014

Note: Wire sizes other than those listed below wire size number W22 or D22 including larger sizes may be produced provided the quality required is sufficient to justify manufacture

Example: A fabric of W6 wires spaced 6 in. center to center has a cross-sectional area of 0.12 in.²/ft of width.

Table (with mirror revisions) reprinted by permission of the Wire Reinforcement Institute, Inc.

REINFORCEMENT 5 - Specifications and properties of wire and welded wire reinforcement

Reference: *Welded Wire Reinforcement Manual of Standard Practice*, 3rd edition, 1979, p. 8, published by Wire Reinforcement Institute, Inc., 301 E. Sardusky St., Findlay, OH 45640; ACI 318-95 Sections 3.5.3.4, 3.5.3.5, and 3.5.3.6

REINFORCEMENT 5.1 - Specifications* covering welded wire reinforcement

U.S. Specification	Canadian Standard	Title
ASTM A 82	CSA G 30.3	Steel Wire, Plain, for Concrete Reinforcement
ASTM A 185	CSA G 30.5	Steel Welded Wire Reinforcement, Plain, for Concrete Reinforcement
ASTM A 496	CSA G 30.14	Steel Wire, Deformed, for Concrete Reinforcement
ASTM A 497	CSA G 30.15	Steel Welded Wire Reinforcement, Deformed, for Concrete Reinforcement

*The American Society for Testing and Materials (ASTM) publishes specifications for the wire used to manufacture reinforcement and for both smooth and deformed wire reinforcement. The Canadian Standards Association (CSA) publishes identical standards with identical titles for use in Canada. These are considered to be governing specifications for both wire and welded wire reinforcement. Some governmental agencies have special specifications which will control if cited

REINFORCEMENT 5.2 - Minimum requirements of steel wires in welded wire reinforcement

Types of wire	Wire size	Minimum tensile strength, psi	Minimum yield strength,* psi	Weld shear, strength,† psi	Maximum yield strength, psi
Welded plain wire reinforcement ASTM A 185, SCA G 30.15	W 0.5 through W 31	75,000	65,000	35,000	80,000
Welded deformed wire reinforcement ASTM A 185, SCA G 30.15	W 4 through D 31	80,000	70,000	35,000	80,000

*The yield strength values shown are ASTM requirements for minimum yield strength measured at strain of 0.005 in./in. ACI 318-95 Section 3.5.3 states that yield strength values greater than 60,000 psi may be used, provided they are measured at strain of 0.35 percent if yield strength specified in the design exceeds 60,000 psi. Higher yield strength welded wire reinforcement is available and can be used when compliance with ACI 318-95 is certified.

†The values shown are the ASTM requirements for weld shear strength, which contributes to the bonding and anchorage of the wire reinforcement in concrete. A maximum size differential of wires being welded together is maintained to assure adequate weld shear strength. For plain wires, the smaller wire must be not smaller than wire size W 1.2 and have an area of larger wire. Typical examples of the maximum wire size differential are:

Larger wire size	Smaller wire size
W20 (MW 130)	W8 (MW 52)
W14 (MW 90)	W5.5 (MW 36)

For deformed wires, the smaller wire must be not smaller than the wire size D4 and have an area of 40 percent or more of the area of the larger wire.

REINFORCEMENT 6—Common stock styles of welded wire fabric.

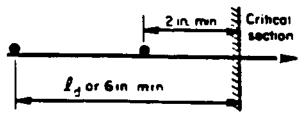
Reference: *Welded Wire Fabric Manual of Standard Practice*, 3rd edition, 1979, p. 7, published by Wire Reinforcement Institute, Inc., 301 E. Sardusky St., Findlay, OH 45640; *CRSI Manual of Standard Practice*, 24th edition, 1986, p. 2-3, published by concrete Reinforcing Steel Institute, 933 N. Plum Grove Rd., Schaumberg, IL 60173-4758.

Style designation		Steel area, sq in. per ft			
New designation (by W-number)	Old designation (by steel wire gage)	Longit.	Trans.	Approximate weight, lb/100 sq ft	Metric style designation
Rolls					
6 x6—W1.4 x W1.4	6 x 6—10 x 10	0.028	0.028	21	152 x 152 MW9.1 x MW9.1
6 x6—W2.0 x W2.0	6 x 6—8 x 8*	0.040	0.040	29	152 x 152 MW13.3 x MW13.3
6 x6—W2.9 x W2.9	6 x 6—6 x 6	0.58	0.58	42	152 x 152 MW18.7 x MW18.7
6 x6—W4.0 x W4.0	6 x 6—4 x 4	0.80	0.80	58	152 x 152 MW25.8 x MW25.8
4 x 4—W1.4 x W1.4	4 x4—10 x 10	0.42	0.42	31	102 x 102 MW9.1 x MW9.1
4 x 4—W2.0 x W2.0	4 x 4—8 x 8*	0.060	0.060	43	102 x 102 MW13.3 x MW13.3
4 x 4—W2.9 x W2.9	4 x 4—6 x 6	0.087	0.087	62	102 x 102 MW18.7 x MW18.7
4 x 4—W4.0 x W4.0	4 x 4—4 x 4	0.120	0.120	86	102 x 102 MW25.8 x MW25.8
Sheets					
6 x6—W2.9 x W2.9	6 x 6—6 x 6	0.058	0.058	42	152 x 152 MW18.7 x MW18.7
6 x6—W4.0 x W4.0	6 x 6—4 x 4	0.080	0.080	58	152 x 152 MW25.8 x MW25.8
6 x6—W5.5 x W5.5	6 x 6—2 x 2†	0.110	0.110	80	152 x 152 MW34.9 x MW34.9
4 x 4—W4.0 x W4.0	4 x4—4 x 4	0.120	0.120	86	102 x 102 MW25.8 x MW25.8

*Exact W-number size for 8 gage is W2.1.
†Exact W-number size for 2 gage is W5.4.

REINFORCEMENT 7.1.1 - Typical development and splice length, *in*, for welded *plain* wire reinforcement; f_y = 60,000 psi; f_c' = 3000 psi

Reference: Welded Wire Reinforcement Manual of Standard Practice, 3rd edition, 1979, p. 20, published by Wire Reinforcement Institute, Inc., 301 E. Sardusky St., Findlay, OH 45640; ACI 318-95, Sections 12.8 and 12.19

WIRES TO BE DEVELOPED OR SPLICED						 Overlap between outermost cross wires of each fabric sheet shall be not less than a) one spacing of cross wires plus 2 in., b) 1.5 ℓ_d, or c) 6 in			
		Development length when cross-wire spacing is, in.				Splice length when cross-wire spacing is, in.			
Aw	Spacing, s_w, in.	4	6	8	12	4	6	8	12
W1.4	4	6	8	10	14	6	8	10	14
to	6	6	8	10	14	6	8	10	14
W5	12	6	8	10	14	6	8	10	14
W6	4	6	8	10	14	7	8	10	14
	6	6	8	10	14	6	8	10	14
	12	6	8	10	14	6	8	10	14
W7	4	6	8	10	14	8	8	10	14
	6	6	8	10	14	6	8	10	14
	12	6	8	10	14	6	8	10	14
W8	4	6	8	10	14	9	9	10	14
	6	6	8	10	14	6	8	10	14
	12	6	8	10	14	6	8	10	14
W9	4	6	8	10	14	10	10	10	14
	6	6	8	10	14	7	8	10	14
	12	6	8	10	14	6	8	10	14
W10	4	6	8	10	14	12	12	12	14
	6	6	8	10	14	8	8	10	14
	12	6	8	10	14	6	8	10	14
W12	4	8	9	10	14	14	14	14	14
	6	6	8	10	14	9	9	10	14
	12	6	8	10	14	6	8	10	14
W14	4	9	9	10	14	16	16	16	16
	6	6	8	10	14	11	11	11	14
	12	6	8	10	14	6	8	10	14
W16	4	10	10	10	14	18	18	18	18
	6	7	8	10	14	12	12	12	14
	12	6	8	10	14	6	8	10	14
W18	4	11	11	11	14	20	20	20	20
	6	8	8	10	14	14	14	14	14
	12	6	8	10	14	7	8	10	14
W20	4	12	12	12	14	23	23	23	23
	6	8	8	10	14	15	15	15	15
	12	6	8	10	14	8	8	10	14
W31	4	23	23	23	23	34	34	34	34
	6	15	15	15	15	23	23	23	23
	12	8	8	10	14	12	12	12	14

Development lengths include the factor 0.8 allowed by ACI 318-95 section 12.2.3.4 but have not been modified by other factors given in Sections 12.2.3, 12.2.4, and 12.2.5.

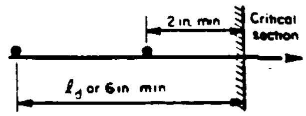
Splice length in this table are those required when area of reinforcement provided is less than twice that required by analysis at splice location.

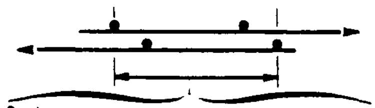
When area of reinforcement provided is at least twice that required by analysis, see ACI 318-95, Section 12.9.2.

Reprinted (with minor revisions) from Manual of Standard Practice, Wire Reinforcement Institute, Inc.

REINFORCEMENT 7.1.2 - Typical development and splice length, *in*, for welded *plain* wire reinforcement; f_y = 60,000 psi; f_c' = 4000 psi

Reference: Welded Wire Reinforcement Manual of Standard Practice, 3rd edition, 1979, p. 20, published by Wire Reinforcement Institute, Inc., 301 E. Sardusky St., Findlay, OH 45640; ACI 318-95, Sections 12.8 and 12.19





Overlap between outermost cross wires of each fabric sheet shall be not less than a) one spacing of cross wires plus 2 in., b) 1.5 ℓ_d, or c) 6 in.

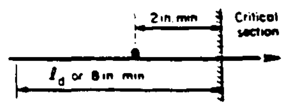
WIRES TO BE DEVELOPED OR SPLICED		Development length when cross-wire spacing is, in.				Splice length when cross-wire spacing is, in.			
Aw	Spacing, s_w, in.	4	6	8	12	4	6	8	12
W1.4	4	6	8	10	14	6	8	10	14
to	6	6	8	10	14	6	8	10	14
W5	12	6	8	10	14	6	8	10	14
W6	4	6	8	10	14	6	8	10	14
	6	6	8	10	14	6	8	10	14
	12	6	8	10	14	6	8	10	14
W7	4	6	8	10	14	7	8	10	14
	6	6	8	10	14	6	8	10	14
	12	6	8	10	14	6	8	10	14
W8	4	6	8	10	14	8	8	10	14
	6	6	8	10	14	6	8	10	14
	12	6	8	10	14	6	8	10	14
W9	4	6	8	10	14	9	10	10	14
	6	6	8	10	14	6	8	10	14
	12	6	8	10	14	6	8	10	14
W10	4	7	8	10	14	10	10	10	14
	6	6	8	10	14	7	8	10	14
	12	6	8	10	14	6	8	10	14
W12	4	8	8	10	14	12	12	12	14
	6	6	8	10	14	8	8	10	14
	12	6	8	10	14	6	8	10	14
W14	4	9	9	10	14	14	14	14	14
	6	6	8	10	14	9	9	10	14
	12	6	8	10	14	6	8	10	14
W16	4	11	11	11	14	16	16	16	16
	6	7	8	10	14	11	11	11	14
	12	6	8	10	14	6	8	10	14
W18	4	12	12	12	14	18	18	18	18
	6	8	8	10	14	12	12	12	14
	12	6	8	10	14	6	8	10	14
W20	4	13	13	13	14	20	20	20	20
	6	9	9	10	14	13	13	13	14
	12	6	8	10	14	8	8	10	14

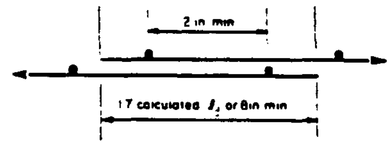
Development lengths include the factor 0.8 allowed by ACI 318-95 section 12.2.3.4 but have not been modified by other factors given in Sections 12.2.3, 12.2.4, and 12.2.5.
Splice length in this table are those required when area of reinforcement provided is less than twice that required by analysis at splice location. When area of reinforcement provided is at least twice that required by analysis, see ACI 318-95, Section 12.9.2.

Reprinted (with minor revisions) from Manual of Standard Practice, Wire Reinforcement Institute, Inc.

REINFORCEMENT 7.2.1 - Typical development and splice length, *in*, for welded *deformed* wire reinforcement; f_y = 60,000 psi; f_c' = 3000 psi

Reference: ACI 318-95, Sections 12.2, 12.7, and 12.18



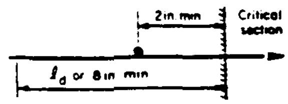
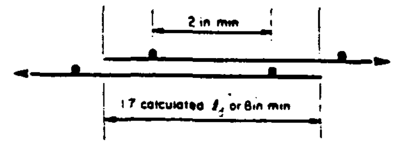


WIRES TO BE DEVELOPED OR SPLICED		Development length when overhang is, in.				Splice length when overhang is, in.			
Aw	Spacing, s_w, in.	0"	3"	4"	6"	0"	4"	6"	12"
D4	4	8	8	8	8	8	8	8	14
	6	8	8	8	8	8	8	8	14
	12	8	8	8	8	8	8	8	14
D5	4	8	8	8	8	8	8	8	14
	6	8	8	8	8	8	8	8	14
	12	8	8	8	8	8	8	8	14
D6	4	8	8	8	8	9	9	9	14
	6	8	8	8	8	9	9	9	14
	12	8	8	8	8	9	9	9	14
D7	4	8	8	8	8	9	9	9	14
	6	8	8	8	8	9	9	9	14
	12	8	8	8	8	9	9	9	14
D8	4	8	8	8	8	10	10	10	14
	6	8	8	8	8	10	10	10	14
	12	8	8	8	8	10	10	10	14
D9	4	8	8	8	8	11	11	11	14
	6	8	8	8	8	11	11	11	14
	12	8	8	8	8	11	11	11	14
D10	4	8	8	8	8	13	13	13	14
	6	8	8	8	8	11	11	11	14
	12	8	8	8	8	11	11	11	14
D12	4	8	8	8	8	15	15	15	15
	6	8	8	8	8	12	12	12	14
	12	8	8	8	8	12	12	12	14
D14	4	8	8	8	8	17	17	17	17
	6	8	8	8	8	13	13	13	14
	12	8	8	8	8	13	13	13	14
D16	4	10	10	10	10	20	20	20	20
	6	8	8	8	8	14	14	14	14
	12	8	8	8	8	14	14	14	14
D18	4	11	11	11	11	22	22	22	22
	6	8	8	8	8	15	15	15	15
	12	8	8	8	8	15	15	15	15
D20	4	12	12	12	12	25	25	25	25
	6	8	8	8	8	17	17	17	17
	12	8	8	8	8	16	16	16	16

Development lengths include the factor 0.8 allowed by ACI 318-95 section 12.2.3.4 but have not been modified by other factors given in Sections 12.2.3, 12.2.4, and 12.2.5.

REINFORCEMENT 7.2.2 - Typical development and splice length, *in*, for welded *deformed* wire reinforcement; f_y = 60,000 psi; f_c' = 4000 psi

Reference: ACI 318-95, Sections 12.2, 12.7, and 12.18

WIRES TO BE DEVELOPED OR SPLICED									
		Development length when overhang is, in.				Splice length when overhang is, in.			
Aw	Spacing, s_w, in.	0"	3"	4"	6"	0"	6"	8"	12"
D4	4	8	8	8	8	8	8	10	14
	6	8	8	8	8	8	8	10	14
	12	8	8	8	8	8	8	10	14
D5	4	8	8	8	8	8	8	10	14
	6	8	8	8	8	8	8	10	14
	12	8	8	8	8	8	8	10	14
D6	4	8	8	8	8	8	8	10	14
	6	8	8	8	8	8	8	10	14
	12	8	8	8	8	8	8	10	14
D7	4	8	8	8	8	8	8	10	14
	6	8	8	8	8	8	8	10	14
	12	8	8	8	8	8	8	10	14
D8	4	8	8	8	8	8	8	10	14
	6	8	8	8	8	8	8	10	14
	12	8	8	8	8	8	8	10	14
D9	4	8	8	8	8	8	8	10	14
	6	8	8	8	8	8	8	10	14
	12	8	8	8	8	8	8	10	14
D10	4	8	8	8	8	8	8	10	14
	6	8	8	8	8	8	8	10	14
	12	8	8	8	8	8	8	10	14
D12	4	8	8	8	8	8	8	10	14
	6	8	8	8	8	8	8	10	14
	12	8	8	8	8	8	8	10	14
D14	4	8	8	8	8	9	9	10	14
	6	8	8	8	8	9	9	10	14
	12	8	8	8	8	9	9	10	14
D16	4	8	8	8	8	9	9	10	14
	6	8	8	8	8	9	9	10	14
	12	8	8	8	8	9	9	10	14
D18	4	8	8	8	8	10	10	10	14
	6	8	8	8	8	10	10	10	14
	12	8	8	8	8	10	10	10	14
D20	4	8	8	8	8	10	10	10	14
	6	8	8	8	8	10	10	10	14
	12	8	8	8	8	10	10	10	14

Development lengths include the factor 0.8 allowed by ACI 318-95 section 12.2.3.4 but have not been modified by other factors given in Sections 12.2.3, 12.2.4, and 12.2.5.

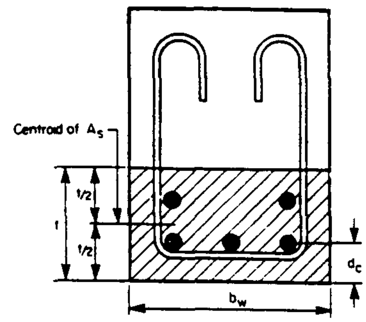
REINFORCEMENT 8.1 - Maximum *A* values per bar for crack control in beams and slabs

Reference: ACI 318-95 Section 10.6.4 and ACI 318R-95 Section 10.6.4

For interior exposure: $\text{Max } A = \frac{1}{d_c}\left(\frac{175}{0.6f_y}\right)^3$

For exterior exposure: $\text{Max } A = \frac{1}{d_c}\left(\frac{145}{0.6f_y}\right)^3$

where f_y is in ksi



d_c	f_y = 60 ksi		f_y = 75 ksi	
	Interior exposure	Exterior exposure	Interior exposure	Exterior exposure
1.00	114.9	65.3	58.8	33.5
1.25	91.9	52.3	47.1	26.8
1.50	76.6	43.6	39.2	22.3
1.75	65.6	37.3	33.6	19.1
2.00	57.4	32.7	29.4	16.7
2.25	51.1	29.0	26.1	14.9
2.50	45.9	26.1	23.5	13.4
2.75	41.8	23.8	21.4	12.2
3.00	38.3	21.8	19.6	11.2
3.25	35.3	20.1	18.1	10.3
3.50	32.8	18.7	16.8	9.6
3.75	30.6	17.4	15.7	8.9
4.00	28.7	16.3	14.7	8.4
4.25	27.0	15.4	13.8	7.9
4.50	25.5	14.5	13.1	7.4
4.75	24.2	13.8	12.4	7.0
5.00	23.0	13.1	11.8	6.7

Note: Where actual f_s value is used instead of $f_s = 0.6f_y$, the table value shall be multiplied by $0.216(f_y/f_s)^3$. The ratio $(b_w t/n) \leq A$ where n is the number of bars of the same diameter. If the reinforcement consists of several sizes, total A_s/Area of largest bar = n.

For use of this Design Aid, see Reinforcement Examples 1, 2 and 3.

REINFORCEMENT 8.2–Beam web size and reinforcement required for crack control

Reference: ACI 318-95 Section 10.6.4 and ACI 318R-95 Section 10.6.4

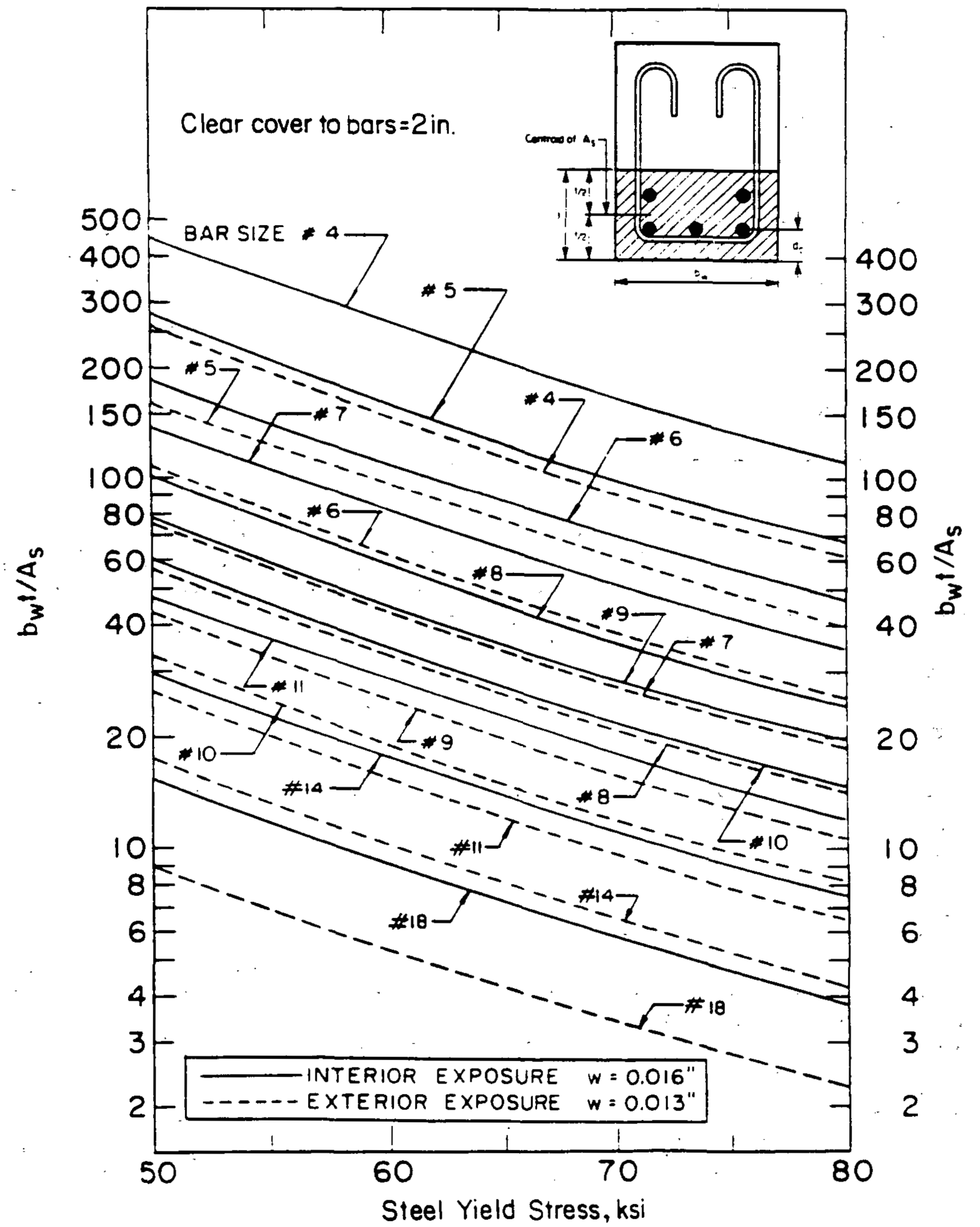
$$\frac{b_w t}{A_s} = \frac{1}{A_b d_c}\left(\frac{z}{f_s}\right)^3$$

where z = 175 for interior exposure

= 145 for exterior exposure

$f_s = 0.6 f_y$

and #4 stirrup assumed



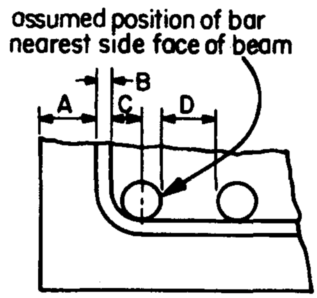
Note: The value $b_w t/A_s$ from this chart × area of one bar = A value in REINFORCEMENT 8.1.

For use of this Design Aid, see Reinforcement Examples 1, 2 and 3.

REINFORCEMENT 9—Minimum beam web widths required for two or more bars in one layer for cast-in-place nonprestressed concrete.

Reference: ACI 318-89, Sections 7.2.2, 7.6.1, 7.7.1(c), and AASHTO* Standard Specifications for Highway Bridges (16th edition, 1996) Division I, Sections 8.17.3.1, 8.21.1, 8.22.1, 8.23.2.2, and Table 8.23.2.1.

Minimum beam width = $2(A + B + C) + (n - 1)(D + d_b)$ where $A + B + C - 1/2\ d_b \geq 2.0$ in. cover required for longitudinal bars and these assumptions are made:



For ACI 318

B = 0.375 in. for #3 stirrups
= 0.500 in. for #4 stirrups

D = 1 d_b
≥ 1 in.
≥ 1 1/3 nominal aggregate size

For both ACI and AASHTO

A = 1 1/2 in. concrete cover to stirrup
B = 0.635 in. for #5 stirrups
= 0.750 in. for #6 stirrups

C = stirrup bend radius of 2 stirrup bar diameter for #5 and smaller stirrups
= stirrup bend radius of 3 stirrup bar diameters for #6 stirrups
≥ 1/2 d_b of longitudinal bars

For AASHTO

B = 0.375 in. for #3 stirrups (minimum stirrup size for #10 and smaller longitudinal bars)
= 0.500 in. for #4 stirrups (minimum stirrup size for #11 and larger longitudinal bars)

D = 1 1/2 d_b
≥ 1 in.
≥ 1 1/2 nominal aggregate size

	ACI 318-95 3/4-in. aggregate interior exposure #3 stirrups		ACI 318-95 1-in. aggregate interior exposure #3 stirrups		AASHTO requirements cast-in-place concrete 1-in. aggregate exposed to earth or weather	
Bar size	Minimum web width for 2 bars, in.	Increment for each added bar, in.	Minimum web width for 2 bars, in.	Increment for each added bar, in.	Minimum web width for 2 bars, in.	Increment for each added bar, in.
#4	6 3/4	1 1/2	7 1/8	1 7/8	7.25	2.000
#5	6 7/8	1 5/8	7 1/4	2	7.37	2.125
#6	7	1 3/4	7 3/8	2 1/8	7.50	2.250
#7	7 1/8	1 7/8	7 1/2	2 1/4	7.62	2.375
#8	7 1/4	2	7 5/8	2 3/8	7.75	2.500
#9	7 1/2	2 1/4	7 3/4	2 1/2	8.32	2.820
#10	7 7/8	2 1/2	7 7/8	2 5/8	8.68	3.175
#11	8 1/8	2 7/8	8 1/8	2 7/8	9.52	3.525
#14	8 7/8	3 3/8	8 7/8	3 3/8	10.23	4.232
#18	10 1/2	4 1/2	10 1/2	4 1/2	11.90	5.642

Notes

1. **Stirrups**: For stirrups larger than those used for table above, increase web width by the following amounts (in.):

Source	Main reinforcement size	#4 stirrup	#5 stirrup	#6 stirrup
ACI requirements	#4 through #11	3/4	1 1/2	2 1/4
	#14	1/2	1 1/4	2
	#18	1/4	3/4	1 1/2
AASHTO requirements	#4 through #10	0.75	1.50	2.25
	#11 through #14	—	0.75	1.50
	#18	—	0.49	1.24

2. **ACI cover requirements**: For exterior exposure with use of #6 or larger stirrups, add 1 in. to web width.

3. **AASHTO cover requirements**: For interior exposure, 1/2 in. may be deducted from beam widths.

4. **Bars of different sizes**: For beams with bars of two or more sizes, determine from table the beam web width required for the given number of largest size bars; then add the indicated increments for each smaller bar.

5. **Example**: Find the minimum web width for a beam reinforced with two #8 bars; a beam reinforced with three #8 bars; a beam reinforced with three #9 and two #6 bars.

	2 #8	3 #8	3 #9 + 2 #6
ACI (3/4 in. aggregate)	7 1/4	9 1/4	13 1/4
ACI (1 in. aggregate)	7 5/8	10	14 1/2
AASHTO	7.75	10.25	15.64

*AASHTO = American Association of State Highway and Transportation Officials, 444 North Capitol St., N. W., Suite 225, Washington, D. C. 20001, U. S. A.

For use of this Design Aid, see Flexure Example 1.

REINFORCEMENT 10 - Minimum beam web widths for various bar combinations (interior exposure)

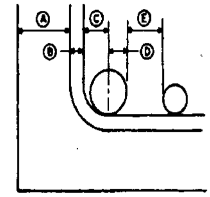
Reference: ACI 318-95 Sections 7.2.2, 7.6.1, and 7.7.1 (c)

Columns at left headed 1-5 and 6-10 bars are for minimum web width b_w of beam having bars of one size only. Remaining columns are for combinations of 1 to 5 bars of each of two sizes.

Calculated values of beam web width b_w rounded upward to nearest half inch.

Where bars of two sizes are used, larger bar(s) assumed to be placed along outside face(s) of beam.

Aggregate size assumed ≤ 3/4 in.



A = clear cover of 1-1/2 in.
B = 3/4 in. diameter of #3 stirrups
C = for #11 and smaller bars: twice diameter of #3 stirrups; for #14 and #18 bars: 1/2 diameter of bar
D = 1/2 diameter of larger bar
E = 1/2 spacing for larger bar plus 1/2 spacing for smaller bar (spacing is d_b for #9 and larger bars, 1 in. for #8 and smaller bars)

No. of bars	Bar size	ACI min. b_w, in.: 1 to 5 bars	ACI min. b_w, in.: 6 to 10 bars	Size of smaller bars	ACI min. b_w, in. — No. of smaller bars: 1	2	3	4	5	Size of smaller bars	ACI min. b_w, in. — No. of smaller bars: 1	2	3	4	5	Size of smaller bars	ACI min. b_w, in. — No. of smaller bars: 1	2	3	4	5
1		5.5	12.5																		
2		7.0	13.5																		
3	#3	8.0	15.0																		
4		9.5	16.5																		
5		11.0	18.0																		
1		5.5	13.0		7.0	8.5	9.5	11.0	12.5												
2		7.0	14.5		8.5	9.5	11.0	12.5	14.0												
3	#4	8.5	16.0	#3	10.0	11.0	12.5	14.0	15.5												
4		10.0	17.5		11.5	12.5	14.0	15.5	17.0												
5		11.5	19.0		13.0	14.0	15.5	17.0	18.5												
1		5.5	13.5		7.0	8.5	10.0	11.5	13.0		7.0	8.5	9.5	11.0	12.5						
2		7.0	15.0		8.5	10.0	11.5	13.0	14.5		8.5	10.0	11.0	12.5	14.0						
3	#5	8.5	17.0	#4	10.0	11.5	13.0	14.5	16.0	#3	10.0	11.5	13.0	14.0	15.5						
4		10.5	18.5		12.0	13.5	15.0	16.5	18.0		11.5	13.0	14.5	16.0	17.0						
5		12.0	20.0		13.5	15.0	16.5	18.0	19.5		13.5	14.5	16.5	17.5	19.0						
1		5.5	14.0		7.0	9.0	10.5	12.0	13.5		7.0	8.5	10.0	11.5	13.0		7.0	8.5	10.0	11.0	12.5
2		7.0	16.0		9.0	10.5	12.0	13.5	15.5		8.5	10.0	11.5	13.0	14.5		8.5	10.0	11.5	12.5	14.0
3	#6	9.0	17.5	#5	10.5	12.0	14.0	15.5	17.0	#4	10.5	12.0	13.5	15.0	16.5	#3	10.5	11.5	13.0	14.5	16.0
4		10.5	19.5		12.5	14.0	15.5	17.0	19.0		12.0	13.5	15.0	16.5	18.0		12.0	13.5	15.0	16.0	17.5
5		12.5	21.0		14.0	15.5	17.5	19.0	20.5		14.0	15.5	17.0	18.5	20.0		14.0	15.0	16.5	18.0	19.5
1		5.5	15.0		7.5	9.0	11.0	12.5	14.5		7.0	9.0	10.5	12.0	13.5		7.0	8.5	10.0	11.5	13.0
2		7.5	16.5		9.0	11.0	12.5	14.5	16.0		9.0	10.5	12.0	14.0	15.5		9.0	10.5	12.0	13.5	15.0
3	#7	9.0	18.5	#6	11.0	12.5	14.5	16.0	18.0	#5	11.0	12.5	14.0	15.5	17.5	#4	10.5	12.0	13.5	15.0	16.5
4		11.0	20.5		13.0	14.5	16.5	18.0	20.0		12.5	14.5	16.0	17.5	19.0		12.5	14.0	15.5	17.0	18.5
5		13.0	22.5		14.5	16.5	18.0	20.0	21.5		14.5	16.0	18.0	19.5	21.0		14.5	16.0	17.5	19.0	20.5
1		5.5	15.5		7.5	9.5	11.0	13.0	15.0		7.5	9.0	11.0	12.5	14.5		7.5	9.0	10.5	12.0	14.0
2		7.5	17.5		9.5	11.0	13.0	15.0	17.0		9.0	11.0	12.5	14.5	16.0		9.0	10.5	12.5	14.0	15.5
3	#8	9.5	19.5	#7	11.5	13.0	15.0	17.0	19.0	#6	11.0	13.0	14.5	16.5	18.0	#5	11.0	12.5	14.5	16.0	17.5
4		11.5	21.5		13.5	15.0	17.0	19.0	21.0		13.0	15.0	16.5	18.5	20.0		13.0	14.5	16.5	18.0	19.5
5		13.5	23.5		15.5	17.0	19.0	21.0	23.0		15.0	17.0	18.5	20.5	22.0		15.0	16.5	18.5	20.0	21.5
1		5.5	17.0		7.5	9.5	11.5	13.5	15.5		7.5	9.5	11.5	13.0	15.0		7.5	9.0	11.0	12.5	14.5
2		8.0	19.0		10.0	12.0	14.0	16.0	18.0		9.5	11.5	13.5	15.5	17.0		9.0	11.5	13.0	15.0	16.5
3	#9	10.0	21.5	#8	12.0	14.0	16.0	18.0	20.0	#7	12.0	14.0	15.5	17.5	19.5	#6	12.0	13.5	15.5	17.0	19.0
4		12.5	23.5		14.5	16.5	18.5	20.5	22.5		14.0	16.0	18.0	20.0	21.5		14.0	16.0	17.5	19.5	21.0
5		14.5	26.0		16.5	18.5	20.5	22.5	24.5		16.5	18.5	20.0	22.0	24.0		16.5	18.0	20.0	21.5	23.5
1		5.5	18.0		8.0	10.0	12.5	14.5	17.0		8.0	10.0	12.0	14.0	16.0		7.5	9.5	11.5	13.5	15.0
2		8.0	20.5		10.5	12.5	15.0	17.0	19.5		10.0	12.0	14.0	16.0	18.0		10.0	12.0	13.5	15.5	17.5
3	#10	10.5	23.5	#9	13.0	15.0	17.5	19.5	22.0	#8	12.5	14.5	16.5	18.5	20.5	#7	12.5	14.5	16.0	18.0	20.0
4		13.0	26.0		15.5	17.5	20.0	22.0	24.5		15.0	17.0	19.0	21.0	23.0		15.0	17.0	18.5	20.5	22.5
5		15.5	28.5		18.0	20.0	22.5	24.5	27.0		17.5	19.5	21.5	23.5	25.5		17.5	19.5	21.5	23.0	25.0
1		5.5	19.5		8.0	10.5	13.0	15.5	18.0		8.0	10.5	12.5	15.0	17.0		8.0	10.0	12.0	14.0	16.0
2		8.5	22.5		11.0	13.5	16.0	18.5	21.0		10.5	13.0	15.0	17.5	19.5		10.5	12.5	14.5	16.5	18.5
3	#11	11.0	25.0	#10	13.5	16.0	19.0	21.5	24.0	#9	13.5	15.5	18.0	20.0	22.5	#8	13.0	15.0	17.0	19.0	21.0
4		14.0	28.0		16.5	19.0	21.5	24.0	26.5		16.0	18.5	20.5	23.0	25.0		16.0	18.0	20.0	22.0	24.0
5		17.0	31.0		19.5	22.0	24.5	27.0	29.5		19.0	21.5	23.5	26.0	28.0		19.0	21.0	23.0	25.0	27.0
1		5.5	22.5		8.5	11.5	14.5	17.0	20.0		8.5	11.0	13.5	16.0	18.5		8.5	10.5	13.0	15.0	17.5
2		9.0	26.0		12.0	14.5	17.5	20.5	23.0		11.5	14.0	16.5	19.0	22.0		11.5	13.5	16.0	18.0	20.5
3	#14	12.5	29.5	#11	15.5	18.0	21.0	23.5	26.5	#10	15.0	17.5	20.0	22.5	25.0	#9	14.5	17.0	19.0	21.5	23.5
4		16.0	33.0		18.5	21.5	24.5	27.0	30.0		18.5	21.0	23.5	26.0	28.5		18.0	20.5	22.5	25.0	27.0
5		19.0	36.0		22.0	25.0	27.5	30.5	33.5		22.0	24.5	27.0	29.5	32.0		21.5	23.5	26.0	28.5	30.5
1		6.5	29.0		10.0	13.5	16.5	20.0	23.5		9.5	12.5	15.0	18.0	21.0		9.5	12.0	14.5	17.0	19.5
2		11.0	33.5		14.0	17.5	21.0	24.5	27.5		13.5	16.5	19.0	22.0	25.0		13.5	16.0	18.5	21.0	23.5
3	#18	15.5	38.0	#14	18.5	22.0	25.5	29.0	32.0	#11	18.0	21.0	23.5	26.5	29.5	#10	18.0	20.5	23.0	25.5	28.0
4		20.0	42.5		23.0	26.5	30.0	33.5	36.5		22.5	25.5	28.5	31.0	34.0		22.5	25.0	27.5	30.0	32.5
5		24.5	47.0		27.5	31.0	34.5	38.0	41.0		27.0	30.0	33.0	35.5	38.5		27.0	29.5	32.0	34.5	37.0

Examples: For 2 #6 bars, minimum b_w = 7.0 in. For 8 #6 bars, minimum b_w =17.5 in. For 2 #7 bars plus 3 #6 bars, minimum b_w = 12.5 in. For 3 #6 bars plus 5 #4 bars, minimum b_w = 16.5 in.

REINFORCEMENT 11—Maximum web width b_w per bar for single bars used as flexural tension reinforcement in beam webs and slabs, as required for crack control provisions

Reference: ACI 318-95 Sections 7.6.1, 7.6.2, 7.6.5, 7.7.1, and 10.6.4 and Commentary on Section 10.6.4.

For bars in one layer:

$$\frac{Max\ b_w}{n} = \frac{\left(\frac{z}{0.6f_y}\right)^3}{2d_c^{\ 2}}$$

For bars in three layers:

$$\frac{Max\ b_w}{n} = \frac{\left(\frac{z}{0.6f_y}\right)^3}{2d_c(3 + 1.5d_b)}$$

For bars in two layers:

$$\frac{Max\ b_w}{n} = \frac{\left(\frac{z}{0.6f_y}\right)^3}{d_c(5 + 2d_b)}$$

Exposure	No. of layers	f_y, ksi	Max b_w per bar, in. Bar size #4	#5	#6	#7	#8	#9	#10	#11	#14	#18
Interior z=175 (for 2 in. clear cover over primary flexural reinforcement)	1 layer	60	11.35	10.74	10.18	9.67	9.19	8.74	8.27	7.85	7.09	5.87
	2 layers*	60	8.51	7.95	7.44	6.98	6.57	6.18	5.78	5.43	4.81	(3.86)‡
	3 layers*	60	6.81	6.31	5.86	5.46	5.11	4.77	4.45	4.15	3.64	(2.88)‡
Exterior z = 145 (for 2 in. clear cover over primary flexural reinforcement)	1 layer	60	6.45	6.11	5.79	5.50	5.23	4.97	4.71	4.46	4.03	(3.34)‡
	2 layers*	60	4.84	4.52	4.23	3.97	3.73	3.51	3.29	3.09	(2.74)‡	(2.20)‡
	3 layers*	60	3.87	3.59	3.33	3.11	2.90	2.72	2.53	(2.36)‡	(2.07)‡	(1.64)‡

*Space between layers of bars is assumed to be 1 in.; bars in a vertical row have the same diameter.

†Primary flexural reinforcement in *slabs* (other than concrete joist construction) shall not be spaced farther apart than three times the slab thickness, nor 18 in., according to ACI 318-95, Section 7.6.5.

‡Parenthesis around value indicate case where minimum beam web width requirement exceeds maximum permitted by ACI 318-95—that is, where maximum width per bar as limited by crack control provisions is less than the increment for each bar as given in REINFORCEMENT 9 for 3/4 in. aggregate.

For use of this Design Aid, see Reinforcement Examples 1 and 4.

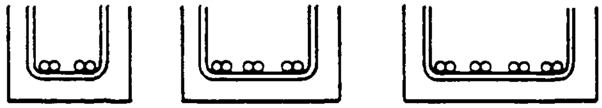
REINFORCEMENT 12–Minimum beam web widths b_w for various combinations of bundled bars (interior exposure)

Reference: ACI318-95 Sections 7.2.2, 7.6.6.1, 7.6.6.2, 7.6.6.3, 7.6.6.5, and 7.7.1

Calculated values of beam web width b_w rounded upward to nearest half inch.

Assumptions:

- aggregate size ≤3/4 in.
- clear cover of 1½ in.
- #3 stirrups



2 bundles   3 bundles   4 bundles

Each bundle may contain two, three, or four bars.

Minimum beam web width b_w, in.*			
Bar size			
Two bundles			
#8	10.0	10.0	10.5
#9	10.5	11.0	11.0
#10	11.0	11.5	12.0
#11	11.5	12.0	12.5
Three bundles			
#8	13.5	14.0	14.5
#9	14.5	15.0	15.5
#10	15.5	16.0	17.0
#11	16.5	17.5	18.0
Four bundles			
#8	17.0	17.5	18.5
#9	18.0	19.0	20.0
#10	20.0	21.0	22.0
#11	21.5	22.5	24.0

*For beams conforming to AASHTO specifications, add 1 in. to tabulated beam web width.

Example: For 2 bundles of 3 #10, minimum b_w = 11.5 in.

REINFORCEMENT 13—Maximum web width b_w per bundle, as required for crack control provisions for bars of one size in one layer

Reference: ACI 318-95 Sections 7.6, 7.7.1, and 10.6.4; and "Crack Control in Beams Reinforced with Bundled Bars Using ACI 318-71," Edward G. Nawy, ACI JOURNAL, *Proceedings* V. 69, No. 10, Oct. 1972, pp. 637-639.

$$\frac{b_w}{n} = \frac{k\left(\frac{z}{f_s}\right)^3 (\text{ no. of bars per bundle})}{2(d_c')^2}\text{, in. per bundle}$$

where $k =$ 0.815 and $d'_c = 2 + 0.5\,d_b$, in., for two bars per bundle

$k =$ 0.650 and $d'_c = 2 + 0.788\,d_b$, in., for three bars per bundle

$k =$ 0.570 and $d'_c = 2 + d_b$, in., for four bars per bundle

$z =$ 175 for interior exposure

$z =$ 145 for exterior exposure

$f_s =$ $0.6 f_y$, ksi

Exposure	Bar arrangement	f_y, ksi	Maximum web width b_w per bundle, in. per bundle			
			Bar size			
			#8	#9	#10	#11
Interior $z = 175$ (for 2 in. clear cover over main reinforcement)	 $d'_c = 2 + 0.5d_b$, in.	60	15.0	14.2	13.5	12.8
	 $d'_c = 2 + 0.788d_b$, in.	60	14.4	13.4	12.4	11.6
	 $d'_c = 2 + d_b$, in.	60	14.6	13.4	12.2	19.5 11.3
Exterior $z = 145$ (for 2 in. clear cover over main reinforcement)	 $d'_c = 2 + 0.5d_b$, in.	60	8.5	8.1	7.7	7.3
	 $d'_c = 2 + 0.788d_b$, in.	60	8.2	7.6	7.1	6.6
	 $d'_c = 2 + d_b$, in.	60	8.3	7.6	7.0	6.4

REINFORCEMENT 14–Bar selection table for beams

Reference: ACI 318-95 Sections 3.3, 7.6.1, 7.6.2, 7.6.6.5, 7.7.1, and 10.6.4; 1977*** AASHTO Articles 1.5.4(B)(2), 1.5.5(A), (C), and (E), and 1.5.6(A) and (B)

A_s, sq in.	Quantity and size of bars	Arrangement*	Minimum b_w, in.,†‡ meeting ACI requirements (interior exposure, #3 stirrups, 3/4 in. aggregate)	Minimum b_w, in.,†§ meeting AASHTO requirements (exterior exposure, #3 stirrups, 1 in. aggregate)	Maximum b_w, in.,** meeting ACI crack control requirements: Interior exposure f_y = 60 ksi	Maximum b_w, in.,** meeting ACI crack control requirements: Exterior exposure f_y = 60 ksi
0.40	2 #4	1L	7.0	7.5	22.5	13.0
0.60	3 #4	1L	8.5	9.5	34.0	19.5
0.62	2 #5	1L	7.0	7.5	21.5	12.0
0.80	4 #4	1L	10.0	11.5	45.5	26.0
0.88	2 #6	1L	7.0	7.5	20.5	11.5
0.93	3 #5	1L	8.5	9.5	32.0	18.5
1.00	5 #4	1L	11.5	13.5	57.0	32.0
1.08	2 #6 + 1 #4	1L	8.5	9.5	25.5	15.0
1.20	2 #7	1L	7.5	8.0	19.5	11.0
1.24	4 #5	1L	10.5	12.0	43.0	24.5
1.32	3 #6	1L	9.0	10.0	30.5	17.5
1.40	2 #7 + 1 #4	1L	9.0	10.0	23.0	13.0
1.55	5 #5	1L	12.0	14.0	53.5	30.5
1.58	2 #8	1L	7.5	8.0	18.5	10.5
1.64	2 #7 + 1 #6	1L	9.0	10.0	27.0	15.5
1.76	4 #6	1L	10.5	12.0	40.5	23.0
1.76	4 #6	2L	7.0	7.5	30.0	17.0
1.80	3 #7	1L	9.0	10.0	29.0	16.5
1.86	6 #5	1L	13.5	16.0	64.5	36.5
1.86	6 #5	2L	8.5	9.5	47.5	27.0
2.00	2 #9	1L	8.0	8.5	17.5	10.0
2.12	2 #6 + 4 #5	1L	13.5	16.0	51.0	29.0
2.12	2 #6 + 4 #5	2L	9.0	10.0	38.0	21.5
2.20	5 #6	1L	12.5	14.5	51.0	29.0
2.20	5 #6	2L	9.0	10.0	37.0	21.0
2.31	2 #9 + 1 #5	1L	10.0	10.5	20.5	12.0
2.37	3 #8	1L	9.5	10.5	27.5	15.5
2.40	4 #7	1L	11.0	12.5	38.5	22.0
2.40	4 #7	2L	7.5	8.0	28.0	16.0
2.54	2 #10	1L	8.0	9.0	16.5	9.5
2.65	6 #6	1L	14.0	16.5	61.0	35.0
2.64	6 #6	2L	9.0	10.0	44.5	25.5
2.78	2 #8 + 2 #7	1L	11.0	12.5	33.0	19.0
2.78	2 #8 + 2 #7	2L	7.5	8.0	24.0	14.0

(continued)

*1L = one layer of bars; 2L = two layers of bars; 3L = three layers of bars; 2b = two bundles of bars; 3b = three bundles of bars; 4b = four bundles of bars. Width of bundle is taken as $2d_b$.

†Table values rounded upward to nearest 1/2 in.

‡Table values taken from REINFORCEMENT 10 and 12. For stirrups larger than #3, see Note 2 of REINFORCEMENT 9. For exterior exposure, see Note 3 of REINFORCEMENT 9.

§For individual bars, table values calculated from values in REINFORCEMENT 9. For bundled bars, table values calculated on the basis of concrete cover of 1½ in. to stirrup [AASHTO 1.5.6(A)]. Aggregate size assumed to be ≤1 in. for #8 and smaller bars and $\leq d_b$ for #9 and larger bars. For stirrups larger than #3, see Note 2 of REINFORCEMENT 9. For interior exposure, see Note 3 of REINFORCEMENT 9.

**Table values calculated on the basis of 2 in. concrete cover to main reinforcement. For single bars of one size, table values calculated from REINFORCEMENT 11. For bundled bars of one size, table values calculated from REINFORCEMENT 13. For combinations of bar sizes, table values calculated on the basis of d_c = distance from extreme tension fiber to *centroid* of lowest layer of flexural reinforcement.

***1977 AASHTO with 1983 Interim Specifications

For use of this Design Aid, see Reinforcement Examples 1, 3 and 4

A_s, sq in.	Quantity and size of bars	Arrangement*	Minimum b_w, in.,†‡ meeting ACI requirements (interior exposure, #3 stirrups, 3/4 in. aggregate)	Minimum b_w, in.,†§ meeting AASHTO requirements (exterior exposure, #3 stirrups, 1 in. aggregate)	Maximum b_w, in.,** meeting ACI crack control requirements: Interior exposure f_y = 60 ksi	Maximum b_w, in.,** meeting ACI crack control requirements: Exterior exposure f_y = 60 ksi
3.00	3 #9	1L	10.0	11.0	26.0	15.0
3.00	5 #7	1L	13.0	15.0	48.5	27.5
3.00	5 #7	2L	9.0	10.0	35.0	20.0
3.12	2 #11	1L	8.5	9.0	15.5	9.0
3.16	4 #8	1L	11.5	13.0	37.0	21.0
3.16	4 #8	2b	10.0	10.5	30.0	17.0
3.16	4 #8	2L	7.5	8.0	26.5	15.0
3.38	2 #8 + 3 #7	1L	13.0	15.0	40.0	23.0
3.38	2 #8 + 3 #7	2L	9.5	10.5	31.0	17.5
3.60	6 #7	1L	15.0	17.5	58.0	33.0
3.60	6 #7	2L	9.0	10.0	42.0	24.0
3.81	3 #10	1L	10.5	12.0	25.0	14.0
3.95	5 #8	1L	13.5	15.5	46.0	26.0
3.95	5 #8	2L	9.5	10.5	33.0	18.5
4.00	4 #9	1L	12.5	14.0	35.0	20.0
4.00	4 #9	2b	10.5	11.5	28.5	16.0
4.00	4 #9	2L	8.0	8.5	24.5	14.0
4.12	2 #11 + 1 #9	1L	10.5	12.0	21.0	12.0
4.12	2 #10 + 2 #8	1L	12.0	13.5	28.0	16.0
4.12	2 #10 + 2 #8	2L	8.0	8.5	20.5	11.5
4.34	2 #10 + 3 #7	1L	13.5	16.0	30.0	17.0
4.34	2 #10 + 3 #7	2L	9.0	10.0	23.5	13.5
4.40	10 #6	1L	21.0	25.5	102.0	58.0
4.40	10 #6	2L	12.5	14.5	74.5	42.5
4.50	2 #14	1L	9.0††	10.0	14.0	—
4.68	3 #11	1L	11.0	12.5	23.5	13.5
4.74	6 #8	1L	15.5	18.0	55.0	31.5
4.74	6 #8	2b	10.0	11.0	29.0	16.5
4.74	6 #8	2L	9.5	10.5	39.5	22.5
4.80	8 #7	2L	11.0	12.5	56.0	32.0
5.00	5 #9	1L	14.5	17.0	44.0	25.0
5.00	5 #9	2L	10.0	11.0	31.0	17.5
5.08	4 #10	1L	13.0	15.0	33.0	19.0
5.08	4 #10	2b	11.0	12.0	27.0	15.5
5.08	4 #10	2L	8.0	8.5	23.0	13.0
5.40	9 #7	2L	13.0	15.0	63.0	36.0
5.40	9 #7	3L	9.0	10.0	49.0	28.0

(continued)

*1L = one layer of bars; 2L = two layers of bars; 3L = three layers of bars; 2b = two bundles of bars; 3b = three bundles of bars; 4b = four bundles of bars. Width of bundle is taken as $2d_b$.

†Table values rounded upward to nearest 1/2 in.

‡Table values taken from REINFORCEMENT 10 and 12. For stirrups larger than #3, see Note 2 of REINFORCEMENT 9. For exterior exposure, see Note 3 of REINFORCEMENT 9.

§For individual bars, table values calculated from values in REINFORCEMENT 9. For bundled bars, table values calculated on the basis of concrete cover of 1 1/2 in. to stirrup [AASHTO 1.5.6(A)]. Aggregate size assumed to be ≤1 in. for #8 and smaller bars and $\le d_b$ for #9 and larger bars. For stirrups larger than #3, see Note 2 of REINFORCEMENT 9. For interior exposure, see Note 3 of REINFORCEMENT 9.

**Table values calculated on the basis of 2 in. concrete cover to main reinforcement. For single bars of one size, table values calculated from REINFORCEMENT 11. For bundled bars of one size, table values calculated from REINFORCEMENT 13. For combinations of bar sizes, table values calculated on the basis of d_c = distance from extreme tension fiber to *centroid* of lowest layer of flexural reinforcement.

††Exceeds maximum beam web width meeting crack control provisions for exterior exposure.

A_s, sq in.	Quantity and size of bars	Arrangement*	Minimum b_w, in.,†‡ meeting ACI requirements (interior exposure, #3 stirrups, 3/4 in. aggregate)	Minimum b_w, in.,†§ meeting AASHTO requirements (exterior exposure, #3 stirrups, 1 in. aggregate)	Maximum b_w, in.,** meeting ACI crack control requirements: Interior exposure f_y = 60 ksi	Maximum b_w, in.,** meeting ACI crack control requirements: Exterior exposure f_y = 60 ksi
5.66	2 #11 + 2 #10	1L	13.5	15.5	29.0	16.5
5.66	2 #11 + 2 #10	2L	8.5	9.0	20.5	11.5
6.00	6 #9	1L	17.0	19.5	52.5	30.0
6.00	6 #9	2b	11.0	12.0	27.0	15.0
6.00	6 #9	2L	10.0	11.0	37.0	21.0
6.24	4 #11	1L	14.0	16.0	31.5	18.0
6.24	4 #11	2b	11.5	13.0	25.5	14.5
6.24	4 #11	2L	8.5	9.0	22.0	12.5
6.32	8 #8	2L	11.5	13.0	52.5	30.0
6.32	8 #8	2b	10.5	11.5	29.0	16.5
6.35	5 #10	1L	15.5	18.0	41.5	23.5
6.35	5 #10	2L	10.5	12.0	29.0	16.5
6.75	3 #14	1L	12.5††	14.5	21.5	—
7.11	9 #8	2L	13.5	15.5	59.0	33.5
7.11	9 #8	3b	14.0	15.5	43.0	24.5
7.11	9 #8	3L	9.5	10.5	46.0	26.0
7.51	4 #11 + 1 #10	1L	17.0	19.5	38.0	21.5
7.51	4 #11 + 1 #10	2L	11.0	12.5	28.0	16.0
7.62	6 #10	1L	18.0	21.5	49.5	28.5
7.62	6 #10	2L	10.5	12.0	34.5	19.5
7.62	6 #10	2b	11.5	12.5	25.0	14.0
7.80	5 #11	1L	17.0	20.0	39.0	22.5
7.80	5 #11	2L	11.0	12.5	27.0	15.5
7.90	10 #8	2L	13.5	15.5	65.5	37.5
7.90	10 #8	3L	11.5	13.0	51.0	29.0
8.00	2 #18	1L	11.0††	12.0	11.5	—
8.00	8 #9	2L	12.5	14.0	49.5	28.0
8.00	8 #9	2b	11.0	12.5	27.0	15.0
8.50	2 #14 + 4 #9	1L	18.0††	21.5	29.5	—
8.50	2 #14 + 4 #9	2L	11.5	13.0	21.5	12.0
9.00	4 #14	1L	16.0	18.5	28.5	16.0
9.00	4 #14	2L	9.0	10.0	19.0	11.0
9.00	9 #9	2L	14.5	17.0	55.5	31.5
9.00	9 #9	3b	15.0	17.0	40.0	23.0
9.00	9 #9	3L	10.0	11.0	43.0	24.5

(continued)

*1L = one layer of bars; 2L = two layers of bars; 3L = three layers of bars; 2b = two bundles of bars; 3b = three bundles of bars; 4b = four bundles of bars. Width of bundle is taken as $2d_b$.

†Table values rounded upward to nearest 1/2 in.

‡Table values taken from REINFORCEMENT 10 and 12. For stirrups larger than #3, see Note 2 of REINFORCEMENT 9. For exterior exposure, see Note 3 of REINFORCEMENT 9.

§For individual bars, table values calculated from values in REINFORCEMENT 9. For bundled bars, table values calculated on the basis of concrete cover of 1 1/2 in. to stirrup [AASHTO 1.5.6(A)]. Aggregate size assumed to be ≤1 in. for #8 and smaller bars and $\leq d_b$ for #9 and larger bars. For stirrups larger than #3, see Note 2 of REINFORCEMENT 9. For interior exposure, see Note 3 of REINFORCEMENT 9.

**Table values calculated on the basis of 2 in. concrete cover to main reinforcement. For single bars of one size, table values calculated from REINFORCEMENT 11. For bundled bars of one size, table values calculated from REINFORCEMENT 13. For combinations of bar sizes, table values calculated on the basis of d_c = distance from extreme tension fiber to *centroid* of lowest layer of flexural reinforcement.

††Exceeds maximum beam web width meeting crack control provisions for exterior exposure.

A_s, sq in.	Quantity and size of bars	Arrangement*	Minimum b_w, in.,†‡ meeting ACI requirements (interior exposure, #3 stirrups, 3/4 in. aggregate)	Minimum b_w, in.,†§ meeting AASHTO requirements (exterior exposure, #3 stirrups, 1 in. aggregate)	Maximum b_w, in.,** meeting ACI crack control requirements: Interior exposure f_y = 60 ksi	Maximum b_w, in.,** meeting ACI crack control requirements: Exterior exposure f_y = 60 ksi
9.36	6 #11	1L	19.5	23.5	47.0	27.0
9.36	6 #11	2b	12.0	13.5	23.0	13.0
9.36	6 #11	2L	11.0	12.5	32.5	18.5
9.48	12 #8	4b	17.5	20.5	57.5	33.0
9.48	12 #8	2L	15.5	18.0	79.0	44.5
9.48	12 #8	3b	14.5	16.5	44.0	25.0
9.48	12 #8	3L	11.5	13.0	61.5	35.0
10.00	10 #9	2L	14.5	17.0	62.0	35.0
10.00	10 #9	3L	12.5	14.0	47.5	27.0
10.16	8 #10	2L	13.0	15.0	46.0	26.5
10.16	8 #10	2b	12.0	13.0	29.0	16.5
10.25	2 #18 + 1 #14	1L	14.0††	16.5	15.5	—
10.80	5 #11 + 3 #9	2L	15.5	18.5	41.5	23.5
10.80	5 #11 + 3 #9	3L	11.0	12.5	32.5	18.5
11.25	5 #14	1L	19.0	23.0	35.5	20.0
11.25	5 #14	2L	12.5	14.5	24.0	13.5
11.43	9 #10	3b	16.0	18.5	37.0	21.5
11.43	9 #10	2L	15.5	18.0	52.0	29.5
11.43	9 #10	3L	10.5	12.0	40.0	23.0
12.00	3 #18	1L	15.5*	18.0	17.5	—
12.00	12 #9	4b	19.0	22.0	53.5	30.5
12.00	12 #9	2L	17.0	19.5	74.0	42.0
12.00	12 #9	3b	15.5	18.0	40.0	23.0
12.00	12 #9	3L	12.5	14.0	57.0	32.5
12.48	8 #11	2L	14.0	16.0	43.5	24.5
12.48	8 #11	2b	12.5	14.0	22.5	13.0
12.70	10 #10	2L	15.5	18.0	58.0	33.0
13.50	6 #14	1L	22.5	27.0	42.5	24.0
13.50	6 #14	2L	12.5	14.5	29.0	16.5
14.04	9 #11	3b	17.5	20.0	35.0	20.0
14.04	9 #11	2L	17.0	20.0	49.0	28.0
14.04	9 #11	3L	11.0	12.5	37.5	21.0
15.12	3 #18 + 2 #11	1L	21.0	25.0	23.5	—
15.12	3 #18 + 2 #11	2L	15.5	18.0	18.5	—
15.24	12 #10	4b	21.0	24.5	49.5	28.5
15.24	12 #10	2L	18.0	21.5	69.5	39.5
15.24	12 #10	3b	17.0	19.5	36.5	21.0
15.24	12 #10	3L	13.0	15.0	53.5	30.5
15.60	10 #11	2L	17.0	20.0	54.5	31.0

(continued)

*1L = one layer of bars; 2L = two layers of bars; 3L = three layers of bars; 2b = two bundles of bars; 3b = three bundles of bars; 4b = four bundles of bars. Width of bundle is taken as $2d_b$.

†Table values rounded upward to nearest 1/2 in.

‡Table values taken from REINFORCEMENT 10 and 12. For stirrups larger than #3, see Note 2 of REINFORCEMENT 9. For exterior exposure, see Note 3 of REINFORCEMENT 9.

§For individual bars, table values calculated from values in REINFORCEMENT 9. For bundled bars, table values calculated on the basis of concrete cover of 1 1/2 in. to stirrup [AASHTO 1.5.6(A)]. Aggregate size assumed to be ≤1 in. for #8 and smaller bars and $\leq d_b$ for #9 and larger bars. For stirrups larger than #3, see Note 2 of REINFORCEMENT 9. For interior exposure, see Note 3 of REINFORCEMENT 9.

**Table values calculated on the basis of 2 in. concrete cover to main reinforcement. For single bars of one size, table values calculated from REINFORCEMENT 11. For bundled bars of one size, table values calculated from REINFORCEMENT 13. For combinations of bar sizes, table values calculated on the basis of d_c = distance from extreme tension fiber to *centroid* of lowest layer of flexural reinforcement.

††Exceeds maximum beam web width meeting crack control provisions for exterior exposure.

A_s, sq in.	Quantity and size of bars	Arrangement*	Minimum b_w, in.,†‡ meeting ACI requirements (interior exposure, #3 stirrups, 3/4 in. aggregate)	Minimum b_w, in.,†§ meeting AASHTO requirements (exterior exposure, #3 stirrups, 1 in. aggregate)	Maximum b_w, in.,** meeting ACI crack control requirements: Interior exposure f_y = 60 ksi	Maximum b_w, in.,** meeting ACI crack control requirements: Exterior exposure f_y = 60 ksi
16.00	4 #18	1L	20.0††	23.5	23.5	—
16.00	4 #18	2L	11.0††	12.0	15.5	—
16.50	3 #18 + 2 #14	1L	22.0††	26.0	25.5	—
16.50	3 #18 + 2 #14	2L	15.5††	18.0	19.5	—
18.00	8 #14	2L	16.0	18.5	38.5	22.0
18.00	8 #14	3L	12.5	14.5	29.0	16.5
18.72	12 #11	2L	19.5	23.5	65.0	37.0
18.72	12 #11	4b	22.5	26.5	46.5	26.5
18.72	12 #11	3b	18.0	21.0	34.0	19.0
18.72	12 #11	3L	14.0	16.0	50.0	28.5
20.00	5 #18	1L	24.5††	29.0	29.5	—
20.00	5 #18	2L	15.5††	18.0	19.5	—
20.25	9 #14	2L	19.0	23.0	43.5	24.5
20.25	9 #14	3L	12.5	14.5	33.0	19.0
21.00	3 #18 + 4 #14	3L	15.5††	18.0	20.0	—
21.12	8 #14 + 2 #11	2L	19.0	23.0	46.5	26.5
21.12	8 #14 + 2 #11	3L	16.0	18.5	36.0	20.5
22.50	10 #14	2L	19.0	23.0	48.0	27.5
22.50	10 #14	3L	16.0	18.5	36.5	21.0
24.00	6 #18	1L	29.0††	34.5	35.0	—
24.00	6 #18	2L	15.5††	18.0	23.0	—
25.00	4 #18 + 4 #14	2L	20.0††	23.5	27.5	—
25.00	4 #18 + 4 #14	3L	15.5††	16.5	22.0	—
27.00	12 #14	2L	22.5	27.0	57.5	33.0
27.00	12 #14	3L	16.0	18.5	43.5	25.0
28.00	7 #18	2L	20.0††	23.5	27.0	—
28.00	7 #18	3L	15.5††	18.0	20.0	—

*1L = one layer of bars; 2L = two layers of bars; 3L = three layers of bars; 2b = two bundles of bars; 3b = three bundles of bars; 4b = four bundles of bars. Width of bundle is taken as $2d_b$.

†Table values rounded upward to nearest 1/2 in.

‡Table values taken from REINFORCEMENT 10 and 12. For stirrups larger than #3, see Note 2 of REINFORCEMENT 9. For exterior exposure, see Note 3 of REINFORCEMENT 9.

§For individual bars, table values calculated from values in REINFORCEMENT 9. For bundled bars, table values calculated on the basis of concrete cover of 1½ in. to stirrup [AASHTO 1.5.6(A)]. Aggregate size assumed to be ≤1 in. for #8 and smaller bars and $\leq d_b$ for #9 and larger bars. For stirrups larger than #3, see Note 2 of REINFORCEMENT 9. For interior exposure, see Note 3 of REINFORCEMENT 9.

**Table values calculated on the basis of 2 in. concrete cover to main reinforcement. For single bars of one size, table values calculated from REINFORCEMENT 11. For bundled bars of one size, table values calculated from REINFORCEMENT 13. For combinations of bar sizes, table values calculated on the basis of d_c = distance from extreme tension fiber to *centroid* of lowest layer of flexural reinforcement.

††Exceeds maximum beam web width meeting crack control provisions for exterior exposure.

REINFORCEMENT 15 - Areas of bars in a section 1 ft wide

Spacing, in.	Cross section area of bar, As (or As'), sq. in.											Spacing, in.
	Bar size											
	#3	#4	#5	#6	#7	#8	#9	#10	#11	#14	#18	
4.0	0.33	0.60	0.93	1.32	1.80	2.37	3.00	3.81	4.68			4.0
4.5	0.29	0.53	0.83	1.17	1.60	2.11	2.67	3.39	4.16	6.00		4.5
5.0	0.26	0.48	0.74	1.06	1.44	1.90	2.40	3.05	3.74	5.40	9.60	5.0
5.5	0.24	0.44	0.68	0.96	1.31	1.72	2.18	2.77	3.40	4.91	8.73	5.5
6.0	0.22	0.40	0.62	0.88	1.20	1.58	2.00	2.54	3.12	4.50	8.00	6.0
6.5	0.20	0.37	0.57	0.81	1.11	1.46	1.85	2.34	2.88	4.15	7.38	6.5
7.0	0.19	0.34	0.53	0.75	1.03	1.35	1.71	2.18	2.67	3.86	6.86	7.0
7.5	0.18	0.32	0.50	0.70	0.96	1.26	1.60	2.03	2.50	3.60	6.40	7.5
8.0	0.17	0.30	0.47	0.66	0.90	1.19	1.50	1.91	2.34	3.38	6.00	8.0
8.5	0.16	0.28	0.44	0.62	0.85	1.12	1.41	1.79	2.20	3.18	5.65	8.5
9.0	0.15	0.27	0.41	0.59	0.80	1.05	1.33	1.69	2.08	3.00	5.33	9.0
9.5	0.14	0.25	0.39	0.56	0.76	1.00	1.26	1.60	1.97	2.84	5.05	9.5
10.0	0.13	0.24	0.37	0.53	0.72	0.95	1.20	1.52	1.87	2.70	4.80	10.0
10.5	0.13	0.23	0.35	0.50	0.69	0.90	1.14	1.45	1.78	2.57	4.57	10.5
11.0	0.12	0.22	0.34	0.48	0.65	0.86	1.09	1.39	1.70	2.45	4.36	11.0
11.5	0.11	0.21	0.32	0.46	0.63	0.82	1.04	1.33	1.63	2.35	4.17	11.5
12.0	0.11	0.20	0.31	0.44	0.60	0.79	1.00	1.27	1.56	2.25	4.00	12.0
13.0	0.10	0.18	0.29	0.41	0.55	0.73	0.92	1.17	1.44	2.08	3.69	13.0
14.0	0.09	0.17	0.27	0.38	0.51	0.68	0.86	1.09	1.34	1.93	3.43	14.0
15.0	0.09	0.16	0.25	0.35	0.48	0.63	0.80	1.02	1.25	1.80	3.20	15.0
16.0	0.08	0.15	0.23	0.33	0.45	0.59	0.75	0.95	1.17	1.69	3.00	16.0
17.0	0.08	0.14	0.22	0.31	0.42	0.56	0.71	0.90	1.10	1.59	2.82	17.0
18.0	0.07	0.13	0.21	0.29	0.40	0.53	0.67	0.85	1.04	1.50	2.67	18.0

Example: #9 bars spaced 7½ in. apart provide 1.60 in.^{2}/ft of section width.

For use of this Design Aid, see Flexure Example 3, and 4.

REINFORCEMENT 16 - Maximum bar spacing for single bars in one row for one-way slabs

Reference: ACI 318-95 Sections 7.6.1, 7.6.5, 7.7.1, and 10.6.4 and ACI 318R-95 on Sections 7.7.1 and 10.6.4

$$\frac{Max\ b_w}{n} = \frac{\left(\dfrac{\dfrac{1.2}{\beta} z}{0.6 f_y}\right)^3}{2(clear\ cover + 0.5d_b)^2}$$

Exposure	f_y, ksi	Maximum center-to-center bar spacing, in.*										
		Bar size										
		#3	#4	#5	#6	#7	#8	#9	#10	#11	#14	#18
Interior z = 175	60	18†	18†	18†	18†	18†	18†	18†	18†	18†	7	6
Exterior z = 145	60	18†	18†	18†	18†	18†	18†	17	15	14	4	–‡

* Except where noted, table values are governed by crack control provisions of ACI 318-95, Section 10.6.4 and are based on:

For #3-#11 bars: ¾ in. cover and $\beta = 1.25$
For #14 and #18 bars: 1½ in. cover and $\beta = 1.35$

† Maximum spacing is governed by provisions in ACI 318-95, Section 7.6.5 that primary flexural reinforcement shall not be spaced farther apart than three times slab thickness, nor 18 in.

‡ Calculated maximum spacing of 3.32 in. satisfying crack control provision of ACI 318-95, Section 10.6.4 is less than minimum spacing of $2d_b$ (=4.514 in.) required by ACI 318-95, Section 7.6.1.

For use of this Design Aid, see Reinforcement Example 2.

REINFORCEMENT 17–Basic development length ratios of bars in tension

References: ACI 318-95 Sections 12.2.2 and 12.2.4

$$\text{Development length ratios, } \frac{l_d}{d_b} = \alpha\ \beta\ \lambda \left(\frac{l_d}{d_b}\right)_{BASIC}$$

Basic Development Length Ratios, $(\ell_d/d_b)_{BASIC}$													
Bars	Category \ f_y	40 ksi				60 ksi				75 ksi			
	f'_c	3 ksi	4 ksi	5 ksi	6 ksi	3 ksi	4 ksi	5 ksi	6 ksi	3 ksi	4 ksi	5 ksi	6 ksi
#3 ~ #6	I	29	25	23	21	44	38	34	31	55	47	42	39
	II	44	38	34	31	66	57	51	46	82	71	64	58
#7 ~ #18	I	37	32	28	26	55	47	42	39	68	59	53	48
	II	55	47	42	39	82	71	64	58	103	89	80	73

Notes: 1. See category chart for Categories I and II

2. α = Bar location factor, 1.3 for top bars; 1.0 for other bars

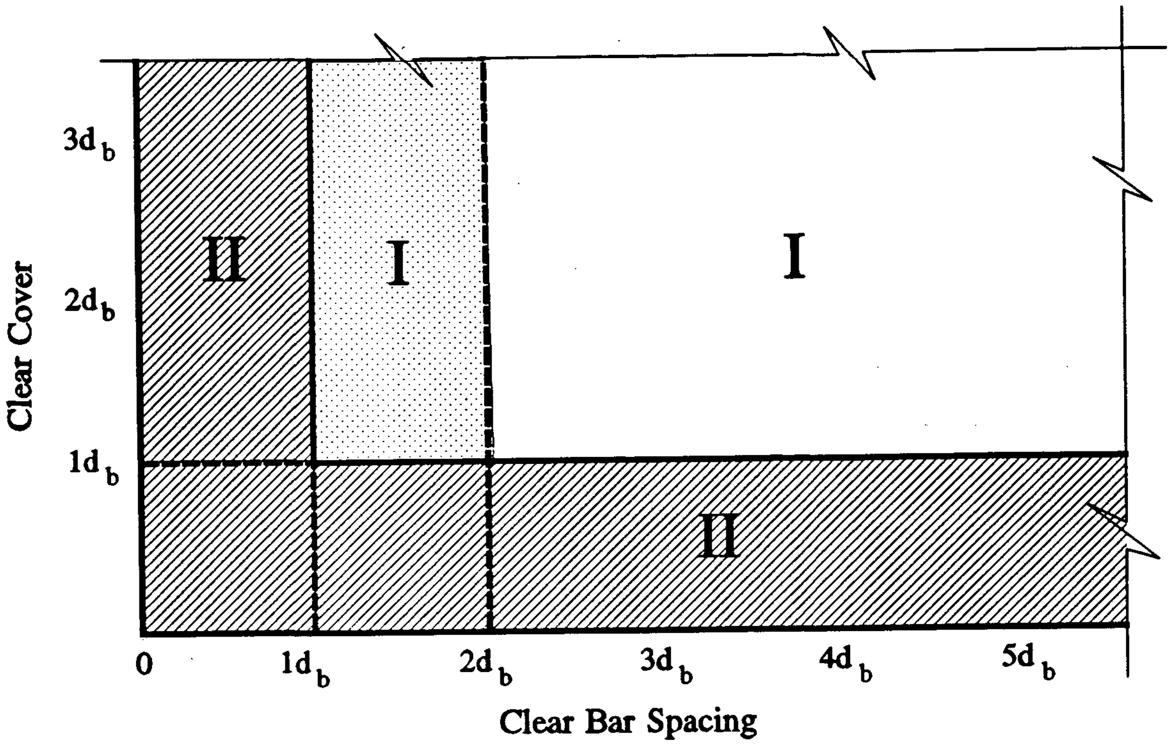
β = Coating factor
1.5 = Epoxy-coated bars with cover < $3d_b$ or clear spacing < $6d_b$
1.2 = All other epoxy-coated bars
1.0 = Uncoated bars

λ = Lightweight aggregate concrete factor, 1.3 for lightweight concrete; 1.0 for normal weight concrete

3. Minimum spacing $\ell_d \geq 12"$

For use of this Design Aid, see Reinforcement Examples 6, 8, and 9

REINFORCEMENT 17-Basic development length ratios of bars in tension (cont'd)



CATEGORY CHART

Category I:
- Clear spacing ≥ d_b
- Clear cover ≥ d_b
- Code minimum stirrups or ties throughout ℓ_d

or

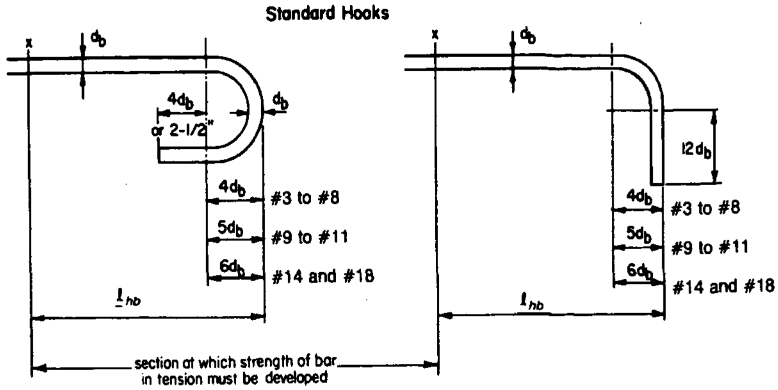
- Clear spacing ≥ $2d_b$
- Clear cover ≥ d_b

Category II: All other cases

For use of this Design Aid, see Reinforcement Examples 6, 8, and 9

REINFORCEMENT 18.1 —Basic development length l_{hb} of standard hooks in tension

Reference: ACI 318-95, Sections 7.1 and 12.5.1-12.5.3



Development length, $l_{dh} = \alpha l_{hb} \geq 8_{db}$, ≥6 in., where α represents modifiers from Note 1 below and l_{hb} is basic development length of standard hooks in tension

$$= 1200 d_b \left(\frac{f_y}{60,000}\right) / \sqrt{f_c'} \text{ in.}$$

		Basic development length l_{hb}, in., of standard hooks in tension								
	f_y	40 ksi				60 ksi				
Bar size	f_c' / d_b in.	3 ksi	4 ksi	5 ksi	6 ksi	3 ksi	4 ksi	5 ksi	6 ksi	$8d_b$ in.
#3	0.375	5.5	4.7	4.2	3.9	8.2	7.1	6.4	5.8	3
#4	0.500	7.3	6.3	5.7	5.2	11.0	9.5	8.5	7.8	4
#5	0.625	9.1	7.9	7.1	6.5	13.7	11.9	10.6	9.7	5
#6	0.750	11.0	9.5	8.5	7.7	16.4	14.2	12.7	11.6	6
#7	0.875	12.8	11.1	9.9	9.0	19.2	16.6	14.9	13.6	7
#8	1.000	14.6	12.6	11.3	10.3	21.9	19.0	17.0	15.5	8
#9	1.128	16.5	14.3	12.8	11.6	24.7	21.4	19.1	17.5	9
#10	1.270	18.5	16.1	14.4	13.1	27.8	24.1	21.6	19.7	10
#11	1.410	20.6	17.8	16.0	14.6	30.9	26.8	23.9	21.8	11
#14	1.693	—	—	—	—	37.1	32.1	28.7	26.2	14
#18	2.257	—	—	—	—	49.5	42.8	38.3	35.0	18

Note 1: To compute development length l_{dh} for a standard hook in tension, multiply basic development length l_{hb} from table above by applicable modification factors:

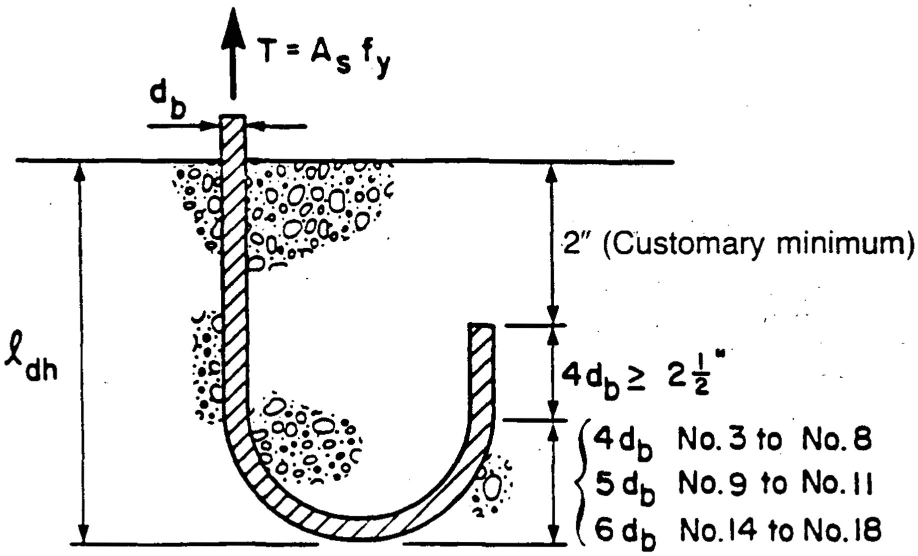
- α = 0.7 for #11 and smaller bars with side cover normal to plane of hook not less than 2 1/2 in.; and for 90 deg hook, cover on bar extension beyond hook not less than 2 in.
- α = 0.8 for #11 and smaller bars with hook enclosed vertically or horizontally within ties or stirrup ties spaced along the full development length not greater than $3d_b$, where d_b is diameter of hooked bar
- α = 1.3 for lightweight concrete
- α = A_s required / A_s provided

Note 2: Values of basic development length l_{hb} above the heavy line are less than the minimum development length of 6 in. Development length l_{dh} (basic development length l_{hb} multiplied by the applicable modification factors) shall not be less than $8d_b$, nor less than 6 in., whichever is greater.

For use of this Design Aid, see Reinforcement Example 10.

REINFORCEMENT 18.2–Minimum embedment lengths to provide 2 in. cover to tail of standard 180-degree end hook

Reference: ACI 318-95, Sections 7.1.1 and 7.2.1



bar size	#3	#4	#5	#6	#7	#8	#9	#10	#11	#14	#18
l_{dh}	6"	7"	7"	8"	9"	10"	13"	14"	15"	19"	25"

Example: Find minimum embedment length l_{dh} which will provide 2 in. cover over the tail of a standard 180-degree end hook in a #8 bar: For #8 bar, read l_{dh} = 10 in.

REINFORCEMENT 19.1—Maximum size of positive moment reinforcement bars satisfying $\ell_d = (M_n/V_u) + \ell_a$ [Eq. (12.2) of ACI 318-89] for various span lengths; f_y = 40,000 psi

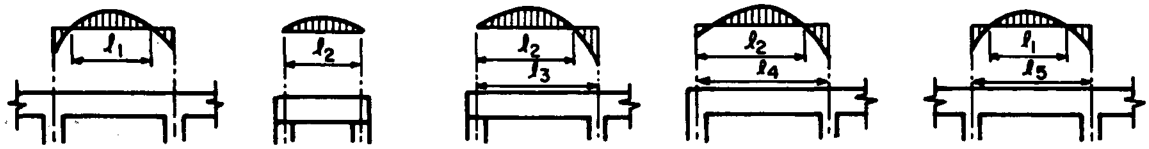
l_1 is span between points of inflection in a beam in an interior bay of a continuous span.

l_2 is span between points of zero moment in a span in which ends of positive moment reinforcement are confined by a compressive reaction — as for a simply supported span.

l_3 is span length in an exterior bay of a continuous span in which the discontinuous end of the span is *unrestrained.*

l_4 is span length in an exterior bay of a continuous span in which the discontinuous end of the span is *restrained.*

l_5 is span length in an interior bay of a continuous span.



f'_c, psi	Bar size no.	**Development length ℓ_d, in.	l_1* All	l_1* Through bars 1/2	l_1* Through bars 1/3	l_2† All	l_2† Through bars 1/2	l_2† Through bars 1/3	l_3‡ All	l_3‡ Through bars 1/2	l_3‡ Through bars 1/3	l_3‡ Through bars 1/4	l_4‡ All	l_4‡ Through bars 1/2	l_4‡ Through bars 1/3	l_4‡ Through bars 1/4	l_5‡ All	l_5‡ Through bars 1/2	l_5‡ Through bars 1/3	l_5‡ Through bars 1/4
3000	4	14.7	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
	5	18.3	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
	6	22.0	4-9	8-6	11-9	5-9	10-9	15-6	4-9	7-9	8-9	8-9	5-3	9-9	11-9	12-6	5-9	7-9	8-3	9-6
	7	32.0	6-6	13-9	19-3	7-9	15-6	22-9	7-9	10-6	11-9	12-9	8-3	13-9	15-9	17-9	7-9	11-6	12-9	13-9
	8	36.6	7-6	14-9	22-6	8-6	17-9	25-9	8-9	11-6	13-9	14-9	10-9	15-3	18-9	20-9	9-3	12-9	14-3	15-9
	9	41.1	8-6	16-9	25-6	9-9	19-9	28-9	9-9	13-6	15-9	16-9	11-9	17-9	20-9	22-9	10-6	14-3	16-6	17-9
	10	45.7	9-6	18-9	27-6	10-9	21-3	31-9	10-6	14-6	16-9	18-6	12-6	18-9	22-9	25-9	11-9	15-9	17-6	19-9
	11	50.3	10-9	20-3	30-9	11-3	23-6	35-9	11-3	15-6	18-9	20-3	13-3	20-6	24-9	27-9	12-9	17-3	19-9	21-9
4000	4	12.7	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
	5	15.9	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
	6	19.0	3-3	6-3	9-6	4-9	9-6	13-3	3-9	6-9	7-3	7-9	4-3	8-3	9-3	10-3	4-9	6-9	7-6	8-9
	7	27.7	5-9	10-9	15-9	6-9	13-9	19-6	6-9	8-6	10-3	11-9	7-9	11-9	13-9	15-6	6-6	9-9	10-9	11-6
	8	31.7	6-9	12-9	17-6	7-9	14-6	22-3	7-3	10-3	11-6	12-6	8-6	13-3	15-9	17-6	7-9	10-9	12-6	13-6
	9	35.6	6-9	13-6	20-9	8-9	16-6	24-3	7-6	11-9	13-9	14-6	9-3	14-9	17-3	19-6	8-6	12-9	14-9	15-3
	10	39.6	7-6	14-9	22-6	9-9	18-6	27-9	8-9	12-9	14-9	16-9	10-9	16-9	19-9	21-6	9-9	13-9	15-3	16-3
	11	43.5	8-6	16-9	24-9	10-9	20-3	30-9	9-9	13-6	16-3	17-3	11-9	18-6	21-9	24-6	10-3	14-9	17-6	18-9
5000	4	12.1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
	5	14.2	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
	6	17.0	2-9	5-9	7-9	4-3	8-6	12-6	3-3	5-6	6-6	7-6	3-6	6-9	8-6	9-9	3-3	5-9	6-3	7-9
	7	24.8	4-6	8-9	13-6	5-9	11-6	17-3	5-9	7-6	9-6	10-9	5-9	10-9	12-6	13-3	6-9	8-3	9-6	10-9
	8	28.3	5-9	10-9	14-6	6-6	13-9	19-6	6-6	9-9	10-9	11-6	6-6	11-9	14-3	15-6	7-3	9-6	11-9	12-9
	9	31.9	5-3	11-6	16-6	7-6	14-9	22-9	6-9	10-9	11-6	12-6	7-3	13-3	15-3	17-9	7-6	10-9	12-3	13-6
	10	35.4	6-6	12-9	18-6	8-3	16-3	24-3	7-9	11-3	13-9	14-3	8-9	14-6	17-9	19-9	8-9	12-3	13-9	15-3
	11	38.9	6-9	13-9	20-6	9-9	18-9	27-9	8-3	12-3	14-3	15-3	9-9	16-9	19-9	21-3	9-9	13-6	15-3	16-9
6000	4	12.1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
	5	13.0	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
	6	15.5	2-9	4-3	6-9	3-3	7-3	11-3	2-9	4-6	5-9	6-6	2-6	5-9	7-9	8-6	3-9	5-9	6-6	6-3
	7	22.6	3-9	7-6	11-3	5-9	10-6	15-3	4-6	7-6	8-9	9-3	5-6	9-9	11-9	12-9	5-6	7-6	9-6	9-9
	8	25.9	4-9	8-9	12-9	6-9	12-6	18-9	5-9	8-6	9-3	10-9	5-9	10-9	12-9	14-3	6-6	8-9	10-6	11-3
	9	29.1	4-3	9-6	14-6	6-9	13-3	20-9	5-3	9-9	10-3	11-3	6-9	12-9	14-9	16-6	6-3	10-9	11-6	12-6
	10	32.3	5-6	10-9	15-3	7-9	15-3	22-9	6-6	10-9	12-6	13-9	7-3	13-6	16-9	17-9	7-3	11-9	12-6	13-9
	11	35.6	5-9	11-3	17-9	8-6	16-9	24-6	6-9	11-9	13-6	14-9	7-6	14-3	17-9	19-9	8-9	12-9	14-6	15-9

**l_d = 1.0 × (Basic tension development length)

= $d_b \alpha B \lambda / 25 \sqrt{f'_c}$ for bars ≤ #6 ≥ 12 in.

= $d_b \alpha B \lambda / 20 \sqrt{f'_c}$ for bars ≥ #7 ≥ 12 in.

†Values assume bars are confined by a compressive reaction at the simply supported ends of the beam and bar embedment terminates at the point of zero moment.

‡Values assume embedment past the points of zero moment equal to the larger of the effective depth *d* or 12 bar diameters, but such embedment is not assumed greater than the distance to the end of the span. The effective depth used for establishing the assumed embedment was taken at the minimum thickness *h* from Table 9.5(a) of ACI-318-95. The distance from the inflection points to the ends of a span was taken as 0.15 of the span for continuous ends at interior supports, and 0.10 of the span at the exterior restrained supports.

REINFORCEMENT 19.2—Maximum size of positive moment reinforcement bars satisfying $\ell_d = (M_n/V_u) + \ell_a$ [Eq. (12.2) of ACI 318-89] for various span lengths; f_y = 40,000 psi

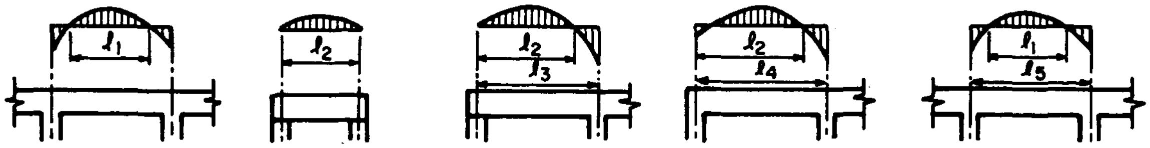
l_1 is span between points of inflection in a beam in an interior bay of a continuous span.

l_2 is span between points of zero moment in a span in which ends of positive moment reinforcement are confined by a compressive reaction—as for a simply supported span.

l_3 is span length in an exterior bay of a continuous span in which the discontinuous end of the span is *unrestrained.*

l_4 is span length in an exterior bay of a continuous span in which the discontinuous end of the span is *restrained.*

l_5 is span length in an interior bay of a continuous span.



f'_c, psi	Bar size no.	**Development length ℓ_d, in.	l_1* Through bars All	l_1* 1/2	l_1* 1/3	l_2† Through bars All	l_2† 1/2	l_2† 1/3	l_3‡ Through bars All	l_3‡ 1/2	l_3‡ 1/3	l_3‡ 1/4	l_4‡ Through bars All	l_4‡ 1/2	l_4‡ 1/3	l_4‡ 1/4	l_5‡ Through bars All	l_5‡ 1/2	l_5‡ 1/3	l_5‡ 1/4
3000	4	22.0	5-6	9-9	14-6	5-9	10-9	15-6	4-9	7-9	8-9	8-9	6-3	9-9	11-9	12-6	5-9	7-9	8-3	9-6
	5	27.4	6-9	12-9	18-6	6-9	12-9	19-9	6-3	8-6	10-9	11-3	7-9	11-6	13-9	15-9	6-9	9-9	10-9	11-3
	6	32.9	7-3	14-3	21-6	7-3	15-6	23-9	7-3	10-9	12-9	13-6	9-9	13-6	16-3	18-9	8-6	11-9	13-3	14-9
	7	48.0	11-9	22-9	33-9	11-9	22-9	33-9	10-6	15-9	17-9	19-6	13-6	19-3	23-3	26-9	11-3	16-9	18-3	20-6
	8	54.8	13-3	25-6	38-6	12-9	25-9	38-9	12-9	17-6	20-9	22-3	15-9	22-9	27-6	30-9	13-9	18-6	21-9	23-9
	9	61.7	14-6	29-9	43-3	14-9	28-9	42-6	13-9	19-3	22-6	24-	16-9	25-3	30-9	34-9	15-9	21-3	24-9	26-3
	10	68.5	16-9	32-6	48-9	16-9	31-9	24-6	15-6	21-9	25-6	27-9	18-9	28-9	33-3	37-9	16-9	23-9	26-9	28-9
	11	75.4	17.9	35-9	53-9	17-9	35-9	52-6	16-3	23-3	27-6	30-3	20-3	31-6	37-9	41-9	18-9	25-6	29-6	31-3
	14	95.9	22-9	45-6	67-9	22-9	44-6	66-9	20-6	30-9	35-3	38-6	26-9	39-9	47-9	52-3	23-3	32-6	37-9	40-6
	18	123.3	29-9	58-6	86-9	28-9	57-3	85-9	26-6	38-9	45-9	49-6	33-6	50-3	60-6	67-3	30-9	41-3	48-6	51-3
4000	4	19.0	4-9	8-9	11-9	4-9	9-6	13-3	4-6	6-9	7-3	7-9	5-9	8-3	9-3	10-3	4-3	6-9	7-6	8-9
	5	23.8	5-9	10-3	14-9	5-9	11-9	16-9	5-9	7-9	8-9	9-9	6-3	9-6	11-3	13-6	5-9	8-3	9-9	10-9
	6	28.5	6-3	11-6	17-9	6-9	13-9	19-9	6-9	9-6	10-9	11-6	7-6	11-9	14-3	15-9	7-9	9-3	11-9	12-6
	7	41.6	9-9	18-9	28-6	9-3	19-3	29-6	6-9	8-6	10-3	11-9	7-9	11-9	13-9	15-6	6-6	9-9	10-9	11-6
	8	47.5	10-9	21-6	32-3	11-9	22-3	33-9	10-9	15-3	17-9	19-9	13-9	19-9	23-3	26-6	11-9	16-9	18-9	20-9
	9	53.4	12-9	24-6	36-3	12-9	24-9	37-9	11-6	16-3	19-6	21-6	14-3	22-9	26-3	29-6	13-9	18-9	20-6	22-9
	10	59.3	13-9	26-6	40-9	13-6	27-9	41-3	13-9	18-6	21-6	23-9	16-6	24-9	29-3	32-6	14-3	20-9	23-9	25-6
	11	65.3	11-9	29-3	44-9	15-9	30-9	45-6	14-3	20-6	24-3	26-3	17-6	26-9	32-3	26-6	16-6	22-9	25-9	27-6
	14	83.1	18-6	37-9	56-3	19-3	38-3	57-3	18-3	26-9	30-9	33-3	22-3	34-9	41-3	45-9	20-9	28-9	32-3	35-9
	18	106.8	24-3	48-6	72-9	24-9	49-9	74-6	23-9	33-6	39-9	42-6	29-9	43-3	52-9	58-9	26-9	36-6	41-9	45-3
5000	4	17.0	3-9	6-6	10-3	4-3	8-6	12-6	3-3	5-6	6-6	7-6	4-9	7-6	8-6	9-9	4-6	5-9	6-3	7-9
	5	21.3	4-9	8-3	12-9	5-9	10-9	14-9	4-3	6-6	7-9	8-9	5-3	8-3	10-6	11-9	5-9	7-9	8-9	9-9
	6	25.5	5-6	10-9	15-6	6-9	12-6	17-9	5-3	8-3	9-6	10-9	6-3	10-9	12-6	14-9	6-9	8-3	10-9	10-6
	7	37.2	8-3	16-3	24-3	8-9	17-9	25-9	8-6	11-9	13-6	15-9	10-6	15-3	18-6	20-9	9-9	12-3	14-9	15-9
	8	42.5	9-9	18-6	27-3	10-9	19-6	29-9	9-3	13-6	15-9	17-3	11-9	17-3	21-9	23-6	10-3	14-9	16-9	18-9
	9	47.8	10-6	20-9	31-3	11-6	22-9	33-3	10-9	15-3	17-9	19-6	13-9	19-3	23-9	26-6	11-9	16-3	18-3	20-6
	10	53.1	11-3	23-3	34-6	12-3	24-3	37-3	11-6	16-9	19-9	21-9	14-9	21-3	26-3	29-6	13-9	18-6	20-9	22.3
	11	58.4	12-9	25-9	37-6	13-9	27-6	40-3	12-3	18-9	21-3	23-3	16-3	24-3	28-6	32-3	14-6	19-9	22-6	24-9
	14	74.3	16-9	32-9	48-9	17-9	34-9	51-6	16-6	23-3	27-9	29-3	20-9	30-3	36-6	40-9	18-6	25-3	29-6	31-9
	18	95.5	20-9	41-6	61-9	22-	44-3	66-9	20-6	29-6	35-6	38-9	26-6	39-9	47-9	52-6	23-9	32-9	37-3	40-6
6000	4	15.5	3-6	5-9	8-3	3-3	7-3	11-3	3-6	5-9	5-9	6-6	4-9	6-3	7-9	8-6	4-3	5-9	6-6	6-3
	5	19.4	3-9	7-9	10-9	4-9	9-9	13-6	4-6	6-6	7-9	7-9	5-3	8-3	9-9	10-9	4-6	6-6	7-6	8-9
	6	23.3	4-6	8-9	13-6	5-9	11-3	16-9	5-6	7-9	8-9	9-6	5-9	9-3	11-6	13-3	5-9	8-3	9-6	10-3
	7	33.9	7-6	14-9	21-3	8-3	15-3	23-3	7-9	10-9	12-6	13-3	9-9	14-6	16-3	18-6	8-6	11-3	13-9	14-9
	8	38.8	8-6	16-9	24-3	9-6	18-9	27-6	8-6	12-9	14-6	15-6	10-9	16-3	19-9	21-3	9-6	13-9	15-6	16-3
	9	43.6	9-6	18-9	27-6	10-9	20-9	30-6	9-9	13-9	16-9	17-9	12-6	18-9	21-6	24-3	10-9	14-3	17-9	18-6
	10	48.5	10-6	20-9	30-6	11-3	22-6	33-9	10-9	15-9	17-9	19-9	13-3	20-6	24-9	26-9	11-9	16-9	19-6	20-6
	11	53.3	11-6	22-9	33-6	12-9	24-6	37-9	11-3	16-9	19-9	21-9	14-9	22-3	26-6	29-9	13-9	18-6	20-9	22-3
	14	67.8	14-9	28-3	42-9	15-9	31-9	47-9	14-9	21-9	24-6	27-3	18-9	27-6	33-3	37-6	16-9	23-3	26-6	28-3
	18	87.2	18-9	36-6	54-3	20-9	40-3	60-9	19-9	27-3	32-3	34-9	23-9	35-3	43-6	48-9	21-9	29-6	34-6	36-9

**l_d = 1.0 × (Basic tension development length)
= $d_b\alpha B\lambda/25\ \sqrt{f'_c}$ for bars ≤ #6 ≥ 12 in.
= $d_b\alpha B\lambda/20\ \sqrt{f'_c}$ for bars ≥ #7 ≥ 12 in.

†Values assume bars are confined by a compressive reaction at the simply supported ends of the beam and bar embedment terminates at the point of zero moment.

‡Values assume embedment past the points of zero moment equal to the larger of the effective depth d or 12 bar diameters, but such embedment is not assumed greater than the distance to the end of the span. The effective depth used for establishing the assumed embedment was taken at the minimum thickness h from Table 9.5(a) of ACI-318-95. The distance from the inflection points to the ends of a span was taken as 0.15 of the span for continuous ends at interior supports, and 0.10 of the span at the exterior restrained supports.

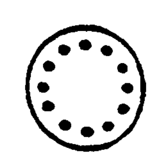
REINFORCEMENT 20.1— Maximum allowable spiral pitch *s*, in., for *circular* spiral columns

References: ANSI A.38.1; ACI 318-95 Sections 3.3.3(c), 7.10.4.3, 10.0. and 10.9.3

Note 1—Tabulated values of spiral pitch are compatible with 1-in. maximum size aggregate unless marked with an asterisk.

Note 2—For spirally reinforced circular columns, industry practice is to specify column diameter in even, whole inches and pitch in increments of one-quarter inch.

Note 3—Spirals recommended as most economical are listed in "Table 2. Recommended Standard Spirals for Circular Columns" of Appendix B of the "Manual of Standard Practice," CRSI Publication MSP-1-97 issued by the Concrete Reinforcing Steel Institute, Schaumberg, IL. In the table below, boxes enclose the regions in which spiral pitch for even-inch column diameters coincides with CRSI recommended spiral size and pitch.



Column size *h*, in.	Core diameter in.	C4-60 columns			C5-60 columns			C6-60 columns			C8-60 columns		
		Spiral size			Spiral size			Spiral size			Spiral size		
		#3	#4	#5	#3	#4	#5	#3	#4	#5	#3	#4	#5
12	9	2	3-1/2§	3-1/2§	1-1/2*	2-3/4§	3-1/2§	†	2-1/4§	3-1/2§	†	1-3/4§	2-3/4§
13	10	2	3-1/2§	3-1/2§	1-1/2*	3§	3-1/2§	†	2-1/4§	3-1/2§	†	1-3/4§	2-3/4§
14	11	2	3-1/2§	3-1/2§	1-1/2*	3§	3-1/2§	†	2-1/4§	3-1/2§	†	1-3/4§	2-3/4§
15	12	2	3-1/2	3-1/2§	1-1/2*	3	3-1/2§	†	2-1/2	3-1/2§	†	1-3/4*	2-3/4§
16	13	2	3-1/2	3-1/2§	1-1/2*	3	3-1/2§	†	2-1/2	3-1/2§	†	1-3/4*	2-3/4§
17	14	2	3-1/2	3-1/2§	1-1/2*	3	3-1/2§	†	2-1/2	3-1/2§	†	1-3/4*	2-3/4§
18	15	2	3-1/2	3-1/2	1-1/2*	3	3-1/2	†	2-1/2	3-1/2	†	1-3/4*	3
19	16	2	3-1/2	3-1/2	1-3/4	3	3-1/2	†	2-1/2	3-1/2	†	2	3
20	17	2	3-1/2	3-1/2	1-3/4	3	3-1/2	†	2-1/2	3-1/2	†	2	3
21	18	2	3-1/2	3-1/2	1-3/4	3	3-1/2	†	2-1/2	3-1/2	†	2	3
22	19	2	3-1/2	3-1/2	1-3/4	3	3-1/2	†	2-1/2	3-1/2	†	2	3
23	20	2	3-1/2	3-1/2	1-3/4	3	3-1/2	†	2-1/2	3-1/2	†	2	3
24	21	2	3-1/2	3-1/2	1-3/4	3	3-1/2	†	2-1/2	3-1/2	†	2	3
25	22	2-1/4	3-1/2	3-1/2	1-3/4	3	3-1/2	1-1/2*	2-1/2	3-1/2	†	2	3
26	23	2-1/4	3-1/2	3-1/2	1-3/4	3-1/4	3-1/2	1-1/2*	2-3/4	3-1/2	†	2	3
27	24	2-1/4	3-1/2	3-1/2	1-3/4	3-1/4	3-1/2	1-1/2*	2-3/4	3-1/2	†	2	3
28	25	2-1/4	3-1/2	3-1/2	1-3/4	3-1/4	3-1/2	1-1/2*	2-3/4	3-1/2	†	2	3
29	26	2-1/4	3-1/2	3-1/2	1-3/4	3-1/4	3-1/2	1-1/2*	2-3/4	3-1/2	†	2	3
30	27	2-1/4	3-1/2	3-1/2	1-3/4	3-1/4	3-1/2	1-1/2*	2-3/4	3-1/2	†	2	3
31	28	2-1/4	3-1/2	3-1/2	1-3/4	3-1/4	3-1/2	1-1/2*	2-3/4	3-1/2	†	2	3
32	29	2-1/4	3-1/2	3-1/2	1-3/4	3-1/4	3-1/2	1-1/2*	2-3/4	3-1/2	†	2	3
33	30	2-1/4	3-1/2	3-1/2	1-3/4	3-1/4	3-1/2	1-1/2*	2-3/4	3-1/2	†	2	3
34	31	2-1/4	3-1/2	3-1/2	1-3/4	3-1/4	3-1/2	1-1/2*	2-3/4	3-1/2	†	2	3
35	32	2-1/4	3-1/2	3-1/2	1-3/4	3-1/4	3-1/2	1-1/2*	2-3/4	3-1/2	†	2	3
36	33	2-1/4	3-1/2	3-1/2	1-3/4	3-1/4	3-1/2	1-1/2*	2-3/4	3-1/2	†	2	3
37	34	2-1/4	3-1/2	3-1/2	1-3/4	3-1/4	3-1/2	1-1/2*	2-3/4	3-1/2	†	2	3
38	35	2-1/4	3-1/2	3-1/2	1-3/4	3-1/4	3-1/2	1-1/2*	2-3/4	3-1/2	†	2	3-1/4
39	36	2-1/4	3-1/2	3-1/2	1-3/4	3-1/4	3-1/2	1-1/2*	2-3/4	3-1/2	†	2	3-1/4
40	37	2-1/4	3-1/2	3-1/2	1-3/4	3-1/4	3-1/2	1-1/2*	2-3/4	3-1/2	†	2	3-1/4
41	38	2-1/4	3-1/2	3-1/2	1-3/4	3-1/4	3-1/2	1-1/2*	2-3/4	3-1/2	†	2	3-1/4
42	39	2-1/4	3-1/2	3-1/2	1-3/4	3-1/4	3-1/2	1-1/2*	2-3/4	3-1/2	†	2	3-1/4
43	40	2-1/4	3-1/2	3-1/2	1-3/4	3-1/4	3-1/2	1-1/2*	2-3/4	3-1/2	†	2	3-1/4
44	41	2-1/4	3-1/2	3-1/2	1-3/4	3-1/4	3-1/2	1-1/2*	2-3/4	3-1/2	†	2	3-1/4
45	42	2-1/4	3-1/2	3-1/2	1-3/4	3-1/4	3-1/2	1-1/2*	2-3/4	3-1/2	†	2	3-1/4
46	43	2-1/4	3-1/2	3-1/2	1-3/4	3-1/4	3-1/2	1-1/2*	2-3/4	3-1/2	†	2	3-1/4
47	44	2-1/4	3-1/2	3-1/2	1-3/4	3-1/4	3-1/2	1-1/2*	3-3/4	3-1/2	†	2	3-1/4
48	45	2-1/4	3-1/2	3-1/2	1-3/4	3-1/4	3-1/2	1-1/2*	2-3/4	3-1/2	†	2	3-1/4
49	46	2-1/4	3-1/2	3-1/2	1-3/4	3-1/4	3-1/2	1-1/2*	2-3/4	3-1/2	†	2	3-1/4
50	47	2-1/4	3-1/2	3-1/2	1-3/4	3-1/4	3-1/2	1-1/2*	2-3/4	3-1/2	†	2	3-1/4

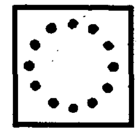
*With this spiral pitch, use aggregate no larger than 3/4 in. normal size (Section 3.3.2 of ACI 318-95).

†Calculated maximum allowable pitch provides clear spacing less than allowable minimum of 1 in. (Section 7.12.2 of ACI 318-95).

§Customarily available fabricating equipment limits the maximum core diameter to 12 in. for #4 spirals and 15 in. for #5 spirals. Smaller core diameters may be fabricated by special equipment.

REINFORCEMENT 20.2—Maximum allowable spiral pitch s, in., for square columns

References: ANSI A.38.1; ACI 318-95 Sections 3.3.3(c), 7.10.4.3, 10.0 and 10.9.3



Note: Tabulated values of spiral pitch are compatible with 1-in. maximum size aggregate unless married with an asterisk.

Column size *h*, in.	Core diameter, in.	S4-60 columns Spiral sizes		S5-60 columns Spiral sizes		S6-60 columns Spiral sizes
		#4	#5	#4	#5	#5
12	9	2§	3-1/4§	1-3/4*§	2-1/2§	2§
13	10	2§	3-1/4§	1-3/4*§	2-1/2§	2§
14	11	2§	3-1/4§	1-1/2*§	2-1/2§	2§
15	12	2	3-1/4§	1-1/2*	2-1/2§	2§
16	13	2	3-1/4§	1-1/2*	2-1/2§	2§
17	14	2	3§	1-1/2*	2-1/2§	2§
18	15	2	3	1-1/2*	2-1/2	2
19	16	2	3	1-1/2*	2-1/2	2
20	17	2	3	1-1/2*	2-1/4	2
21	18	1-3/4*	3	1-1/2*	2-1/4	2
22	19	1-3/4*	2-3/4	1-1/2*	2-1/4	1-3/4*
23	20	1-3/4*	2-3/4	1-1/2*	2-1/4	1-3/4*
24	21	1-3/4*	2-3/4	†	2-1/4	1-3/4*
25	22	1-3/4*	2-3/4	†	2-1/4	1-3/4*
26	23	1-3/4*	2-3/4	†	2	1-3/4*
27	24	1-3/4*	2-3/4	†	2	1-3/4*
28	25	1-3/4*	2-1/2	†	2	1-3/4*
29	26	1-1/2*	2-1/2	†	2	1-3/4*
30	27	1-1/2*	2-1/2	†	2	†
31	28	1-1/2*	2-1/2	†	2	†
32	29	1-1/2*	2-1/2	†	2	†
33	30	1-1/2*	2-1/2	†	2	†
34	31	1-1/2*	2-1/4	†	1-3/4*	†
35	32	1-1/2*	2-1/4	†	1-3/4*	†
36	33	1-1/2*	2-1/4	†	1-3/4*	†
37	34	1-1/2*	2-1/4	†	1-3/4*	†
38	35	1-1/2*	2-1/4	†	1-3/4*	†
39	36	†	2-1/4	†	1-3/4*	†
40	37	†	2-1/4	†	1-3/4*	†
41	38	†	2	†	1-3/4*	†
42	39	†	2	†	1-3/4*	†
43	40	†	2	†	†	†
44	41	†	2	†	†	†
45	42	†	2	†	†	†
46	43	†	2	†	†	†
47	44	†	2	†	†	†
48	45	†	2	†	†	†
49	46	†	2	†	†	†
50	47	†	1-3/4*	†	†	†

*With this spiral pitch, use aggregate no larger than 3/4 in. normal size (Section 3.3.2 of ACI 318-95).

†Calculated maximum allowable pitch provides clear spacing less than allowable maximum of 1 in. (Section 7.10.4.3 of ACI 318-95).

‡No values are calculated for #3 spirals with S4-60, S5-60, or S6-60 columns, and for #4 spirals with S6-60 columns, because for all column sizes listed, maximum allowable pitch provides clear spacing less than allowable minimum of 1 in. (Section 7.10.4.3 of ACI 318-95).

§Customarily available fabricating equipment limits the maximum core diameter to 12 in. for #4 spirals and 15 in. for #5 spirals. Smaller core diameters may be fabricated by special equipment.

REINFORCEMENT 20.3-Recommended minimum number of spacers with various spiral sizes and column sizes

Reference: ACI 318R-95 Section 7.10.4

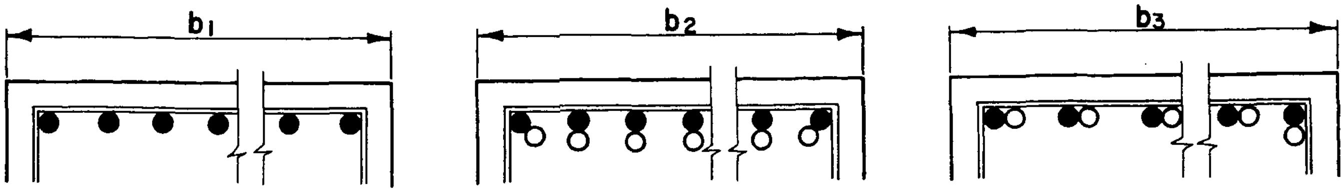
Note: ACI 318-95 Section 7.10.4.9 requires that "Spirals shall be held in firmly in place and true to line." But the 1995 code does not require that this be accomplished by installation of spacers. However, ACI 318-R95 advices when spacers are used, minimums in the table below may be used for guidance.

Spiral size	Column size *h*, in.			
	$h < 20$	$20 \le h \le 24$	$24 < h \le 30$	$h > 30$
#3	2	3	3	4
#4	2	3	3	4
#5	3	3	4	4

REINFORCEMENT 21—Minimum face dimension *b*, in., of rectangular tied columns accommodating various numbers of bars *n* per face

References: ACI 318-95 Sections 3.3.2, 7.6.3, 7.6.4, 7.7.1(c), 7.10.5.1, 12.14.2.1, and 15.8.2.3

Design conditions: 1.5 in. cover between tie and outer surface of column, #4 ties, clear distance between bars of 1.5 in. for #5-#8 bars and $1.5d$, for #9-#18 bars, column face dimensions rounded upward to nearest 0.5 in. Required bend diameters of ties and deformation of bars are neglected.



Bearing splices | Normal lap splices | Tangential lap splices

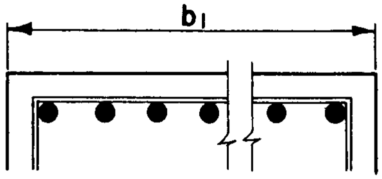
Bar size*		Number of bars per face (between splices), *n*/face													
		2	3	4	5	6	7	8	9	10	11	12	13	14	*n*
#5	b_1, in.	7.0	9.0	11.0	13.5	15.5	17.5	19.5	22.0	24.0	26.0	28.0	30.5	32.5	$2.125n + 2.500$
	b_2, in.	8.0	10.0	12.0	14.0	16.5	18.5	20.5	22.5	25.0	27.0	29.0	31.0	33.5	$2.125n + 3.366$
	b_3, in.	7.5	10.5	13.0	16.0	18.5	21.5	24.0	27.0	29.5	32.5	35.0	38.0	40.5	$2.750n + 1.875$
	A_{st}, in.^{2}	0.62	0.93	1.24	1.55	1.86	2.17	2.48	2.79	3.10	3.41	3.72	4.03	4.34	$0.31n$
#6	b_1, in.	7.0	9.5	11.5	14.0	16.0	18.5	20.5	23.0	25.0	27.5	29.5	32.0	34.0	$2.250n + 2.500$
	b_2, in.	8.5	10.5	13.0	15.0	17.5	19.5	22.0	24.0	26.5	28.5	31.0	33.0	35.5	$2.250n + 3.538$
	b_3, in.	8.0	11.0	14.0	17.0	20.0	23.0	26.0	29.0	32.0	35.0	38.0	41.0	44.0	$3.000n + 1.750$
	A_{st}, in.^{2}	0.88	1.32	1.76	2.20	2.64	3.08	3.52	3.96	4.40	4.84	5.28	5.72	6.16	$0.44n$
#7	b_1, in.	7.5	10.0	12.0	14.5	17.0	19.5	21.5	24.0	26.5	29.0	31.0	33.5	36.0	$2.375n + 2.500$
	b_2, in.	8.5	11.0	13.5	16.0	18.0	20.5	23.0	25.5	27.5	30.0	32.5	35.0	37.0	$2.375n + 3.710$
	b_3, in.	8.5	11.5	15.0	18.0	21.05	24.5	28.0	31.0	34.5	37.5	41.0	44.0	47.5	$3.250n + 1.625$
	A_{st}, in.^{2}	1.20	1.80	2.40	3.00	3.60	4.20	4.80	5.40	6.00	6.60	7.20	7.80	8.40	$0.60n$
#8	b_1, in.	7.5	10.0	12.5	15.0	17.5	20.0	22.5	25.0	27.5	30.0	32.5	35.0	37.5	$2.500n + 2.500$
	b_2, in.	9.0	11.5	14.0	16.5	19.0	21.5	24.0	26.5	29.0	31.5	34.0	36.5	39.0	$2.500n + 3.880$
	b_3, in.	8.5	12.0	15.5	19.0	22.5	26.0	29.5	33.0	36.5	40.0	43.5	47.0	50.5	$3.500n + 1.500$
	A_{st}, in.^{2}	1.58	2.37	3.16	3.95	4.74	5.53	6.32	7.11	7.90	8.69	9.48	10.27	11.06	$0.79n$
#9	b_1, in.	8.0	11.0	14.0	16.5	19.5	22.5	25.0	28.0	31.0	33.5	36.5	39.0	42.0	$2.820n + 2.308$
	b_2, in.	10.0	12.5	15.5	18.0	21.0	24.0	26.5	29.5	32.5	35.0	38.0	41.0	43.5	$2.820n + 3.865$
	b_3, in.	9.5	13.5	17.0	21.0	25.0	29.0	33.0	37.0	41.0	45.0	49.0	53.0	56.5	$3.948n + 1.180$
	A_{st}, in.^{2}	2.00	3.00	4.00	5.00	6.00	7.00	8.00	9.00	10.00	11.00	12.00	13.00	14.00	$1.00n$
#10	b_1, in.	8.5	12.0	15.0	18.0	21.5	24.5	27.5	31.0	34.0	37.5	40.5	43.5	47.0	$3.175n + 2.095$
	b_2, in.	10.5	13.5	17.0	20.0	23.0	26.5	29.5	32.5	36.0	39.0	42.0	45.5	48.5	$3.175n + 3.848$
	b_3, in.	10.0	14.5	19.0	23.5	27.5	32.0	36.5	41.0	45.5	50.0	54.5	59.0	63.5	$4.445n + 0.825$
	A_{st}, in.^{2}	2.54	3.81	5.08	6.35	7.62	8.89	10.16	11.43	12.70	13.97	15.24	16.51	17.78	$1.27n$
#11	b_1, in.	9.0	12.5	16.0	19.5	23.5	27.0	30.5	34.0	37.5	41.0	44.5	48.0	51.5	$3.525n + 1.885$
	b_2, in.	11.0	14.5	18.0	21.5	25.0	28.5	32.0	36.0	39.5	43.0	46.5	50.0	53.5	$3.525n + 3.831$
	b_3, in.	10.5	15.5	20.5	25.5	30.5	35.5	40.0	45.0	50.0	55.0	60.0	65.0	70.0	$4.935n + 0.475$
	A_{st}, in.^{2}	3.12	4.68	6.24	7.80	9.36	10.92	12.48	14.04	15.60	17.16	18.72	20.28	21.84	$1.56n$

*For bars larger than #11, lap splices shall not be used except: 1) at footings, #14 and #18 longitudinal bars, in compression only, may be lap-spliced with dowels; and 2) #14 and #18 bars, in compression only, may be lap-spliced to #11 and smaller bars.

REINFORCEMENT 22.1.1—Maximum number of bars n_{max} that can be accommodated in square columns having bars equally distributed on four faces using *bearing* splices

References: ACI 318-95 Sections 3.3.2, 7.6.3, 7.7.1(c), 7.10.5.1, 10.9.1, and 10.9.2

Design conditions: 1.5 in. cover between ties and outer surface of column; #4 ties; clear distance between longitudinal bars of 1.5 in. for #5-#8 bars and 1.5 times nominal bar diameter for #9-#18 bars; aggregate not larger than 3/4 minimum clear spacing between bars; minimum of four bars per column: ρ_g no less than 0.01 and no greater than 0.08. For other tie sizes and cover, use formula in Commentary on Reinforcement 22. Required bend diameters of ties and deformation of bars are neglected.



Bearing splices

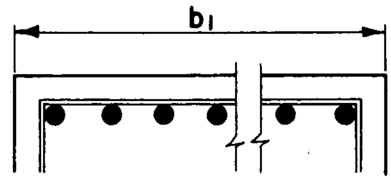
b_1, in.	A_g, in.^{2}		Bar size								
			#5	#6	#7	#8	#9	#10	#11	#14	#18
		n_{max}	8	8	8	8	4	4	4	*	—
10	100	A_{st}	2.48	3.52	4.80	6.32	4.00	5.08	6.24		
		ρ_g	0.0248	0.0352	0.0480	0.0632	0.0400	0.0508	0.0624		
		n_{max}	12	8	8	8	8	4	4	4	—
11	121	A_{st}	3.72	3.52	4.80	6.32	8.00	5.08	6.24	9.00	
		ρ_g	0.0307	0.0291	0.0397	0.0522	0.0661	0.0420	0.0516	0.0744	
		n_{max}	12	12	12	8	8	8	4	4	*
12	144	A_{st}	3.72	5.28	7.20	6.32	8.00	10.16	6.24	9.00	
		ρ_g	0.0258	0.0367	0.0500	0.0439	0.0556	0.0706	0.0433	0.0625	
		n_{max}	12	12	12	12	8	8	8	4	*
13	169	A_{st}	3.72	5.28	7.20	9.48	8.00	10.16	12.48	9.00	
		ρ_g	0.0220	0.0312	0.0426	0.0561	0.0473	0.0601	0.0738	0.0533	
		n_{max}	16	16	12	12	12	8	8	4	*
14	196	A_{st}	4.96	7.04	7.20	9.48	12.00	10.16	12.48	9.00	
		ρ_g	0.0253	0.0359	0.0367	0.0484	0.0612	0.0518	0.0637	0.0459	
		n_{max}	16	16	16	16	12	12	8	8	4
15	225	A_{st}	4.96	7.04	9.60	12.64	12.00	15.24	12.48	18.00	16.00
		ρ_g	0.0220	0.0313	0.0427	0.0562	0.0533	0.0677	0.0555	0.0800	0.0711
		n_{max}	20	20	16	16	12	12	12	8	4
16	256	A_{st}	6.20	8.80	9.60	12.64	12.00	15.24	18.72	18.00	16.00
		ρ_g	0.0242	0.0344	0.0375	0.0494	0.0469	0.0595	0.0731	0.0703	0.0625
		n_{max}	20	20	20	16	16	12	12	8	4
17	289	A_{st}	6.20	8.80	12.00	12.64	16.00	15.24	18.72	18.00	16.00
		ρ_g	0.0215	0.0304	0.0415	0.0437	0.0554	0.0527	0.0648	0.0623	0.0554
		n_{max}	24	20	20	20	16	16	12	8	4
18	324	A_{st}	7.44	8.80	12.00	15.80	16.00	20.32	18.72	18.00	16.00
		ρ_g	0.0230	0.0272	0.0370	0.0488	0.0494	0.0627	0.0578	0.0556	0.0494
		n_{max}	24	24	20	20	16	16	12	12	4
19	361	A_{st}	7.44	10.56	12.00	15.80	16.00	20.32	18.72	27.00	16.00
		ρ_g	0.0206	0.0293	0.0332	0.0438	0.0443	0.0563	0.0519	0.0748	0.0443

*ρ_g exceeds 0.08 with four bars.

REINFORCEMENT 22.1.2—Maximum number of bars n_{max} that can be accommodated in square columns having bars equally distributed on four faces using *bearing* splices

References: ACI 318-95 Sections 3.3.2, 7.6.3, 7.7.1(c), 7.10.5.1, 10.9.1, and 10.9.2

Design conditions: 1.5 in. cover between ties and outer surface of column, #4 ties, clear distance between longitudinal bars of 1.5 in. for #5-#8 bars and 1.5 times nominal bar diameter for #9-#18 bars: aggregate not larger than 3/4 minimum clear spacing between bars; minimum of four bars per column: ρ_g no less than 0.01 and no greater than 0.08. For other tie sizes and cover, use formula in Commentary on Reinforcement 22. Required bend diameters of ties and deformation of bars are neglected.



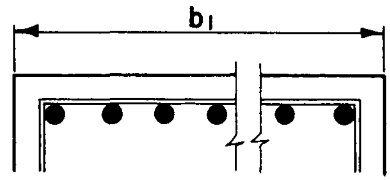
Bearing splices

b_1, in.	A_g, in.^{2}		Bar size								
			#5	#6	#7	#8	#9	#10	#11	#14	#18
		n_{max}	28	24	24	24	20	16	16	12	8
20	400	A_{st}	8.68	10.56	14.40	18.96	20.00	20.32	24.96	27.00	32.00
		ρ_g	0.0217	0.0264	0.0360	0.0474	0.0500	0.0508	0.0624	0.0675	0.08
		n_{max}	28	28	24	24	20	16	16	12	8
21	441	A_{st}	8.68	12.32	14.40	18.96	20.00	20.32	24.96	27.00	32.00
		ρ_g	0.0197	0.0279	0.0327	0.0430	0.0454	0.0461	0.0566	0.0612	0.0726
		n_{max}	32	28	28	24	20	20	16	12	8
22	484	A_{st}	9.92	12.32	16.80	18.96	20.00	25.40	24.96	27.00	32.00
		ρ_g	0.0205	0.0255	0.0347	0.0392	0.0413	0.0525	0.0516	0.0558	0.0661
		n_{max}	32	32	28	28	24	20	16	16	8
23	529	A_{st}	9.92	14.08	16.80	22.12	24.00	25.40	24.96	36.00	32.00
		ρ_g	0.0188	0.0266	0.0318	0.0418	0.0454	0.0480	0.0472	0.0681	0.0605
		n_{max}	36	32	32	28	24	20	20	16	8
24	576	A_{st}	11.16	14.08	19.20	22.12	24.00	25.40	31.20	36.00	32.00
		ρ_g	0.0194	0.0244	0.0333	0.0384	0.0417	0.0441	0.0542	0.0625	0.0556
		n_{max}	36	36	32	32	28	24	20	16	12
25	625	A_{st}	11.16	15.84	19.20	25.28	28.00	30.48	31.20	36.00	48.00
		ρ_g	0.0179	0.0253	0.0307	0.0404	0.0448	0.0488	0.0499	0.0576	0.0768
		n_{max}	40	36	32	32	28	24	20	16	12
26	676	A_{st}	12.40	15.84	19.20	25.28	28.00	30.48	31.20	36.00	48.00
		ρ_g	0.0183	0.0234	0.0284	0.0374	0.0414	0.0451	0.0462	0.0533	0.0710
		n_{max}	40	36	36	32	28	24	24	20	12
27	729	A_{st}	12.40	15.84	21.60	25.28	28.00	30.48	37.44	45.00	48.00
		ρ_g	0.0170	0.0217	0.0296	0.0347	0.0384	0.0418	0.0514	0.0617	0.0658
		n_{max}	44	40	36	36	32	28	24	20	12
28	784	A_{st}	13.64	17.60	21.60	28.44	32.00	35.56	37.44	45.00	48.00
		ρ_g	0.0174	0.0224	0.0276	0.0363	0.0408	0.0454	0.0478	0.0574	0.0612
		n_{max}	44	40	40	36	32	28	24	20	16
29	841	A_{st}	13.64	17.60	24.00	28.44	32.00	35.56	37.44	45.00	48.00
		ρ_g	0.0162	0.0209	0.0285	0.0338	0.0380	0.0422	0.0445	0.0535	0.0761

REINFORCEMENT 22.1.3—Maximum number of bars n_{max} that can be accommodated in square columns having bars equally distributed on four faces using *bearing* splices

References: ACI 318-95 Sections 3.3.2, 7.6.3, 7.7.1(c), 7.10.5.1, 10.9.1, and 10.9.2

Design conditions: 1.5 in. cover between ties and outer surface of column, #4 ties, clear distance between longitudinal bars of 1.5 in. for #5-#8 bars and 1.5 times nominal bar diameter for #9-#18 bars: aggregate not larger than 3/4 minimum clear spacing between bars; minimum of four bars per column: ρ_g no less than 0.01 and no greater than 0.08. For other tie sizes and cover, use formula in Commentary on Reinforcement 22. Required bend diameters of ties and deformation of bars are neglected.



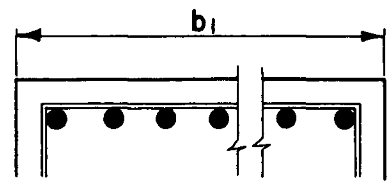
Bearing splices

			Bar size								
b_1, in.	A_g, in.^{2}		#5	#6	#7	#8	#9	#10	#11	#14	#18
		n_{max}	44	44	40	40	32	28	24	20	16
30	900	A_{st}	13.64	19.36	24.00	31.60	32.00	35.56	37.44	45.00	64.00
		ρ_g	0.0152	0.0215	0.0267	0.0351	0.0356	0.0395	0.0416	0.0500	0.0711
		n_{max}	48	44	44	40	36	32	28	20	16
31	961	A_{st}	14.88	19.36	26.40	31.60	36.00	40.64	43.68	45.00	64.00
		ρ_g	0.0155	0.0201	0.0275	0.0329	0.0375	0.0423	0.0455	0.0468	0.0666
		n_{max}	48	48	44	40	36	32	28	24	16
32	1024	A_{st}	14.88	21.12	26.40	31.60	36.00	40.64	43.68	54.00	64.00
		ρ_g	0.0145	0.0206	0.0258	0.0309	0.0352	0.0397	0.0427	0.0527	0.0625
		n_{max}	52	48	44	44	36	32	28	24	16
33	1089	A_{st}	16.12	21.12	26.40	34.76	36.00	40.64	43.68	54.00	64.00
		ρ_g	0.0148	0.0194	0.0242	0.0319	0.0331	0.0373	0.0401	0.0496	0.0588
		n_{max}	52	52	48	44	40	36	32	24	16
34	1156	A_{st}	16.12	22.88	28.80	34.76	40.00	45.72	49.92	54.00	64.00
		ρ_g	0.0139	0.0198	0.0249	0.0301	0.0346	0.0396	0.0432	0.0467	0.0554
		n_{max}	56	52	48	48	40	36	32	24	20
35	1225	A_{st}	17.36	22.88	28.80	37.92	40.00	45.72	49.92	54.00	80.00
		ρ_g	0.0142	0.0187	0.0235	0.0310	0.0327	0.0373	0.0408	0.0441	0.0653
		n_{max}	56	52	52	48	40	36	32	28	20
36	1296	A_{st}	17.36	22.88	31.20	37.92	40.00	45.72	49.92	63.00	80.00
		ρ_g	0.0134	0.0177	0.0241	0.0293	0.0309	0.0353	0.0385	0.0486	0.0617
		n_{max}	60	56	52	48	44	36	32	28	20
37	1369	A_{st}	18.60	24.64	31.20	37.92	44.00	45.72	49.92	63.00	80.00
		ρ_g	0.0136	0.0180	0.0228	0.0277	0.0321	0.0334	0.0365	0.0460	0.0584
		n_{max}	60	56	52	52	44	40	36	28	20
38	1444	A_{st}	19.84	26.40	33.60	41.08	49.00	50.80	56.16	63.00	80.00
		ρ_g	0.0129	0.0171	0.0216	0.0284	0.0305	0.0352	0.0389	0.0436	0.0554
		n_{max}	64	60	56	52	48	40	36	28	20
39	1521	A_{st}	18.60	26.64	31.20	41.08	44.00	50.80	56.16	63.00	80.00
		ρ_g	0.0130	0.0174	0.0221	0.0270	0.0316	0.0334	0.0369	0.0414	0.0526
		n_{max}	64	60	56	56	48	40	36	32	20
40	1600	A_{st}	19.84	26.40	33.60	44.24	48.00	50.80	56.16	72.00	80.00
		ρ_g	0.0124	0.0165	0.0210	0.0276	0.0300	0.0318	0.0351	0.0450	0.0500

REINFORCEMENT 22.1.4—Maximum number of bars n_{max} that can be accommodated in square columns having bars equally distributed on four faces using *bearing* splices

References: ACI 318-95 Sections 3.3.2, 7.6.3, 7.7.1(c), 7.10.5.1, 10.9.1, and 10.9.2

Design conditions: 1.5 in. cover between ties and outer surface of column, #4 ties, clear distance between longitudinal bars of 1.5 in. for #5-#8 bars and 1.5 times nominal bar diameter for #9-#18 bars: aggregate not larger than 3/4 minimum clear spacing between bars; minimum of four bars per column: ρ_g no less than 0.01 and no greater than 0.08. For other tie sizes and cover, use formula in Commentary on Reinforcement 22. Required bend diameters of ties and deformation of bars are neglected.



Bearing splices

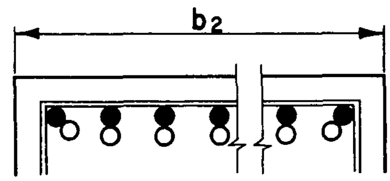
b_1, in.	A_g, in.^{2}		Bar size								
			#5	#6	#7	#8	#9	#10	#11	#14	#18
		n_{max}	68	64	60	56	48	44	40	32	24
41	1681	A_{st}	21.08	28.16	36.00	44.24	48.00	55.88	62.40	72.00	96.00
		ρ_g	0.0125	0.0168	0.0214	0.0263	0.0286	0.0332	0.0371	0.0428	0.0571
		n_{max}	68	64	60	56	52	44	40	32	24
42	1764	A_{st}	21.08	28.16	36.00	44.24	52.00	55.88	62.40	72.00	96.00
		ρ_g	0.0120	0.0160	0.0204	0.0251	0.0295	0.0317	0.0354	0.0408	0.0544
		n_{max}	72	68	64	60	52	44	40	32	24
43	1849	A_{st}	22.32	29.92	38.40	47.40	52.00	55.88	62.40	72.00	96.00
		ρ_g	0.0121	0.0162	0.0208	0.0256	0.0281	0.0302	0.0337	0.0389	0.0519
		n_{max}	72	68	64	60	52	48	40	36	24
44	1936	A_{st}	22.32	29.92	38.40	47.40	52.00	60.96	62.40	81.00	96.00
		ρ_g	0.0115	0.0155	0.0198	0.0245	0.0269	0.0315	0.0322	0.0418	0.0496
		n_{max}	76	68	64	64	56	48	44	36	24
45	2025	A_{st}	23.56	29.92	38.40	50.56	56.00	60.96	65.64	81.00	96.00
		ρ_g	0.0116	0.0148	0.0190	0.0250	0.0277	0.0301	0.0339	0.0400	0.0474
		n_{max}	76	72	68	64	56	48	44	36	28
46	2116	A_{st}	23.56	31.68	40.80	50.56	56.00	60.96	68.64	81.00	112.00
		ρ_g	0.0113	0.0150	0.0193	0.0239	0.0265	0.0288	0.0324	0.0383	0.0529
		n_{max}	76	72	68	64	56	52	44	36	28
47	2209	A_{st}	23.56	31.68	40.80	50.56	56.00	66.04	68.64	81.00	112.00
		ρ_g	0.0107	0.0143	0.0185	0.0229	0.0254	0.0299	0.0311	0.0367	0.0507
		n_{max}	80	76	72	68	60	52	48	36	28
48	2304	A_{st}	24.80	33.44	43.20	53.72	60.00	66.04	74.88	81.00	112.0
		ρ_g	0.0108	0.0145	0.0188	0.0233	0.0260	0.0287	0.0325	0.0352	0.0486
		n_{max}	80	76	72	68	60	52	48	40	28
49	2401	A_{st}	24.80	33.44	43.20	53.72	60.00	66.04	74.88	81.00	112.00
		ρ_g	0.0103	0.0139	0.0180	0.0224	0.0250	0.0275	0.0312	0.0375	0.0466
		n_{max}	84	80	76	72	60	56	48	40	28
50	2500	A_{st}	26.04	35.20	45.60	56.88	60.00	71.12	74.88	90.00	112.00
		ρ_g	0.0104	0.0141	0.0182	0.0228	0.0240	0.0284	0.0300	0.0360	0.0448

REINFORCEMENT 22.2.1—Maximum number of bars n_{max} that can be accommodated in square columns having bars equally distributed on four faces using *normal lap* splices

References: ACI 318-95 Sections 3.3.2, 7.6.3, 7.6.4, 7.7.1(c), 7.10.5.1, 10.9.1, 10.9.2, 12.14.2.1, 12.16.2, and 15.8.2.3

Design conditions: 1.5 in. cover between ties and outer surface of column, #4 ties; clear distance between longitudinal bars of 1.5 in. for #5-#8 bars and 1.5 times nominal bar diameter for #9-#18 bars; aggregate not larger than 3/4 minimum clear spacing between bars; minimum of four bars per column; ρ_g no less than 0.01 and no greater than 0.08. For other tie sizes and cover, use formula in Commentary on Reinforcement 22. Required bend diameters of ties and deformation of bars are neglected.

Note: Lap splices should not be used for bars larger than #11, except: 1) at footings, #14 and #18 longitudinal bars, in compression only, may be lap spliced with dowels; and 2) #14 and #18 bars, in compression only, may be lap spliced to #11 and smaller bars.



Normal lap splices

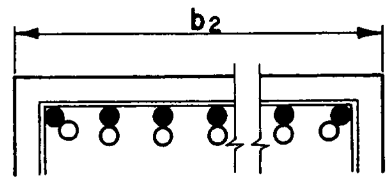
b_2, in.	A_g, in.^{2}		Bar size						
			#5	#6	#7	#8	#9	#10	#11
		n_{max}	8	4	4	4	4	—	—
10	100	A_{st}	2.48	1.76	2.40	3.16	4.00		
		ρ_g	0.0248	0.0176	0.0240	0.0316	0.0400		
		n_{max}	8	8	8	4	4	4	4
11	121	A_{st}	2.48	3.52	4.80	3.16	4.00	5.08	6.24
		ρ_g	0.0205	0.0291	0.0397	0.0261	0.0331	0.0420	0.0516
		n_{max}	12	8	8	8	4	4	4
12	144	A_{st}	3.72	3.52	4.80	6.32	4.00	5.08	6.24
		ρ_g	0.0258	0.0244	0.0333	0.0439	0.0278	0.0353	0.0433
		n_{max}	12	12	8	8	8	4	4
13	169	A_{st}	3.72	5.28	4.80	6.32	8.00	5.08	6.24
		ρ_g	0.0220	0.0312	0.0284	0.0374	0.0473	0.0301	0.0369
		n_{max}	16	12	12	12	8	8	4
14	196	A_{st}	4.96	5.28	7.20	9.48	8.00	10.16	6.24
		ρ_g	0.0253	0.0269	0.0367	0.0484	0.0408	0.0518	0.0318
		n_{max}	16	16	12	12	8	8	8
15	225	A_{st}	4.96	7.04	7.20	9.48	8.00	10.16	12.48
		ρ_g	0.0220	0.0313	0.0320	0.0421	0.0356	0.0452	0.0555
		n_{max}	16	16	16	12	12	8	8
16	256	A_{st}	4.96	7.04	9.60	9.48	12.00	10.16	12.48
		ρ_g	0.0194	0.0275	0.0375	0.0370	0.0469	0.0397	0.0488
		n_{max}	20	16	16	16	12	12	8
17	289	A_{st}	6.20	7.04	9.60	12.64	12.00	15.24	12.48
		ρ_g	0.0215	0.0244	0.0332	0.0437	0.0415	0.0527	0.0432
		n_{max}	20	20	20	16	16	12	12
18	324	A_{st}	6.20	8.80	12.00	12.64	16.00	15.24	18.72
		ρ_g	0.0191	0.0272	0.0370	0.0390	0.0494	0.0470	0.0578
		n_{max}	24	20	20	20	16	12	12
19	361	A_{st}	7.44	8.80	12.00	15.80	16.00	15.24	18.72
		ρ_g	0.0206	0.0244	0.0332	0.0438	0.0443	0.0422	0.0519

REINFORCEMENT 22.2.2—Maximum number of bars n_{max} that can be accommodated in square columns having bars equally distributed on four faces using *normal lap* splices

References: ACI 318-95 Sections 3.3.2, 7.6.3, 7.6.4, 7.7.1(c), 7.10.5.1, 10.9.1, 10.9.2, 12.14.2.1, 12.16.2, and 15.8.2.3

Design conditions: 1.5 in. cover between ties and outer surface of column; #4 ties; clear distance between longitudinal bars of 1.5 in. for #5-#8 bars and 1.5 times nominal bar diameter for #9-#18 bars; aggregate not larger than 3/4 minimum clear spacing between bars; minimum of four bars per column; ρ_g no less than 0.01 and no greater than 0.08. For other tie sizes and cover, use formula in Commentary on Reinforcement 22. Required bend diameters of ties and deformation of bars are neglected.

Note: Lap splices should not be used for bars larger than #11, except: 1) at footings, #14 and #18 longitudinal bars, in compression only, may be lap spliced with dowels; and 2) #14 and #18 bars, in compression only, may be lap spliced to #11 and smaller bars.



Normal lap splices

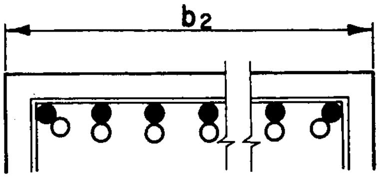
b_2, in.	A_g, in.^{2}		Bar size						
			#5	#6	#7	#8	#9	#10	#11
		n_{max}	24	24	20	20	16	16	12
20	400	A_{st}	7.44	10.56	12.00	15.80	16.00	20.32	18.72
		ρ_g	0.0186	0.0264	0.0300	0.0395	0.0400	0.0508	0.0468
		n_{max}	28	24	24	20	20	16	12
21	441	A_{st}	8.68	10.56	14.40	15.80	20.00	20.32	18.72
		ρ_g	0.0197	0.0239	0.0327	0.0358	0.0454	0.0461	0.0425
		n_{max}	28	28	24	24	20	16	16
22	484	A_{st}	8.68	12.32	14.40	18.96	20.00	20.32	24.96
		ρ_g	0.0179	0.0254	0.0298	0.0392	0.0413	0.0420	0.0516
		n_{max}	32	28	28	24	20	20	16
23	529	A_{st}	9.92	12.32	16.80	18.96	20.00	25.40	24.96
		ρ_g	0.0188	0.0233	0.0318	0.0358	0.0378	0.0480	0.0472
		n_{max}	32	32	28	28	24	20	16
24	576	A_{st}	9.92	14.08	16.80	22.12	24.00	25.40	24.96
		ρ_g	0.0172	0.0244	0.0292	0.0384	0.0417	0.0441	0.0433
		n_{max}	36	32	28	28	24	20	20
25	625	A_{st}	11.16	14.08	16.80	22.12	24.00	25.40	31.20
		ρ_g	0.0179	0.0225	0.0269	0.0354	0.0384	0.0406	0.0499
		n_{max}	36	32	32	28	24	20	20
26	676	A_{st}	11.16	14.08	19.20	22.12	24.00	25.40	31.20
		ρ_g	0.0165	0.0208	0.0284	0.0327	0.0355	0.0376	0.0462
		n_{max}	40	36	32	32	28	24	20
27	729	A_{st}	12.40	15.84	19.20	25.28	28.00	30.48	31.20
		ρ_g	0.0170	0.0217	0.0263	0.0347	0.0384	0.0418	0.0428
		n_{max}	40	36	36	32	28	24	20
28	784	A_{st}	13.64	15.84	21.60	25.28	28.00	30.48	31.20
		ρ_g	0.0174	0.0202	0.0276	0.0322	0.0357	0.0389	0.0398
		n_{max}	44	40	36	36	28	24	24
29	841	A_{st}	13.64	17.60	21.60	28.44	28.00	30.48	37.44
		ρ_g	0.0162	0.0209	0.0257	0.0338	0.0333	0.0362	0.0445

REINFORCEMENT 22.2.3—Maximum number of bars n_{max} that can be accommodated in square columns having bars equally distributed on four faces using *normal lap* splices

References: ACI 318-95 Sections 3.3.2, 7.6.3, 7.6.4, 7.7.1(c), 7.10.5.1, 10.9.1, 10.9.2, 12.14.2.1, 12.16.2, and 15.8.2.3

Design conditions: 1.5 in. cover between ties and outer surface of column; #4 ties; clear distance between longitudinal bars of 1.5 in. for #5-#8 bars and 1.5 times nominal bar diameter for #9-#18 bars; aggregate not larger than 3/4 minimum clear spacing between bars; minimum of four bars per column; ρ_g no less than 0.01 and no greater than 0.08. For other tie sizes and cover, use formula in Commentary on Reinforcement 22. Required bend diameters of ties and deformation of bars are neglected.

Note: Lap splices should not be used for bars larger than #11, except: 1) at footings, #14 and #18 longitudinal bars, in compression only, may be lap spliced with dowels; and 2) #14 and #18 bars, in compression only, may be lap spliced to #11 and smaller bars.



Normal lap splices

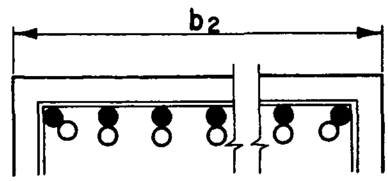
			Bar size						
b_2, in.	A_g, in.^{2}		#5	#6	#7	#8	#9	#10	#11
		n_{max}	44	40	40	36	32	28	24
30	900	A_{st}	13.64	17.60	24.00	28.44	32.00	35.56	37.44
		ρ_g	0.0152	0.0196	0.0267	0.0316	0.0356	0.0395	0.0416
		n_{max}	48	44	40	36	32	28	24
31	961	A_{st}	14.88	19.36	24.00	28.44	32.00	35.56	37.44
		ρ_g	0.0155	0.0201	0.0250	0.0296	0.0333	0.0370	0.0390
		n_{max}	48	44	40	40	32	28	28
32	1024	A_{st}	14.88	19.36	24.00	31.60	32.00	35.56	43.68
		ρ_g	0.0145	0.0189	0.0234	0.0309	0.0312	0.0347	0.0427
		n_{max}	48	48	44	40	36	32	28
33	1089	A_{st}	14.88	21.12	26.40	31.60	36.00	40.64	43.68
		ρ_g	0.0137	0.0194	0.0242	0.0290	0.0331	0.0373	0.0401
		n_{max}	52	48	44	44	36	32	28
34	1156	A_{st}	16.12	21.12	26.40	34.76	36.00	40.64	43.68
		ρ_g	0.0139	0.0183	0.0228	0.0301	0.0311	0.0352	0.0378
		n_{max}	52	48	48	44	40	32	28
35	1225	A_{st}	16.12	21.12	28.80	34.76	40.00	40.64	43.68
		ρ_g	0.0132	0.0172	0.0235	0.0284	0.0327	0.0332	0.0357
		n_{max}	56	52	48	44	40	36	32
36	1296	A_{st}	17.36	22.88	28.80	34.76	40.00	45.72	49.92
		ρ_g	0.0134	0.0177	0.0222	0.0268	0.0309	0.0353	0.0385
		n_{max}	56	52	52	48	40	36	32
37	1369	A_{st}	17.36	22.88	31.20	37.92	40.00	45.72	49.92
		ρ_g	0.0127	0.0167	0.0228	0.0277	0.0292	0.0334	0.0365
		n_{max}	60	56	52	48	44	36	32
38	1444	A_{st}	18.60	24.64	31.20	37.92	44.00	45.72	49.92
		ρ_g	0.0129	0.0171	0.0216	0.0263	0.0305	0.0317	0.0346
		n_{max}	60	56	52	52	44	40	32
39	1521	A_{st}	18.60	24.64	31.20	41.08	44.00	50.80	49.92
		ρ_g	0.0122	0.0162	0.0205	0.0270	0.0289	0.0334	0.0328

REINFORCEMENT 22.2.4—Maximum number of bars n_{max} that can be accommodated in square columns having bars equally distributed on four faces using *normal lap* splices

References: ACI 318-95 Sections 3.3.2, 7.6.3, 7.6.4, 7.7.1(c), 7.10.5.1, 10.9.1, 10.9.2, 12.14.2.1, 12.16.2, and 15.8.2.3

Design conditions: 1.5 in. cover between ties and outer surface of column; #4 ties; clear distance between longitudinal bars of 1.5 in. for #5-#8 bars and 1.5 times nominal bar diameter for #9-#18 bars; aggregate not larger than 3/4 minimum clear spacing between bars; minimum of four bars per column; ρ_g no less than 0.01 and no greater than 0.08. For other tie sizes and cover, use formula in Commentary on Reinforcement 22. Required bend diameters of ties and deformation of bars are neglected.

Note: Lap splices should not be used for bars larger than #11, except: 1) at footings, #14 and #18 longitudinal bars, in compression only, may be lap spliced with dowels; and 2) #14 and #18 bars, in compression only, may be lap spliced to #11 and smaller bars.



Normal lap splices

b_2, in.	A_g, in.^{2}		Bar size						
			#5	#6	#7	#8	#9	#10	#11
		n_{max}	64	60	56	52	44	40	36
40	1600	A_{st}	19.84	26.40	33.60	41.08	44.00	50.80	56.16
		ρ_g	0.0124	0.0165	0.0210	0.0257	0.0275	0.0318	0.0351
		n_{max}	64	60	56	52	48	40	36
41	1681	A_{st}	19.84	26.40	33.60	41.08	48.00	50.80	56.16
		ρ_g	0.0118	0.0157	0.0200	0.0244	0.0286	0.0302	0.0334
		n_{max}	68	64	60	56	48	44	36
42	1764	A_{st}	21.08	28.16	36.00	44.24	48.00	55.88	56.16
		ρ_g	0.0120	0.0160	0.0204	0.0251	0.0272	0.0317	0.0318
		n_{max}	68	64	60	56	48	44	40
43	1849	A_{st}	21.08	28.16	36.00	44.24	48.00	55.88	62.40
		ρ_g	0.0114	0.0152	0.0195	0.0239	0.0260	0.0302	0.0337
		n_{max}	72	64	60	60	52	44	40
44	1936	A_{st}	22.32	28.16	36.00	47.40	52.00	55.88	62.40
		ρ_g	0.0115	0.0145	0.0186	0.0245	0.0269	0.0289	0.0322
		n_{max}	72	68	64	60	52	44	40
45	2025	A_{st}	22.32	29.92	38.40	47.40	52.00	55.88	62.40
		ρ_g	0.0110	0.0148	0.0190	0.0234	0.0257	0.0276	0.0308
		n_{max}	76	68	64	60	52	48	40
46	2116	A_{st}	23.56	29.92	38.40	47.40	52.00	60.96	62.40
		ρ_g	0.0111	0.0141	0.0181	0.0224	0.0246	0.0288	0.0295
		n_{max}	76	72	68	64	56	48	44
47	2209	A_{st}	23.56	31.68	40.80	50.56	56.00	60.96	68.64
		ρ_g	0.0107	0.0143	0.0185	0.0229	0.0254	0.0276	0.0311
		n_{max}	80	72	68	64	56	48	44
48	2304	A_{st}	24.80	31.68	40.80	50.56	56.00	60.96	68.64
		ρ_g	0.0108	0.0138	0.0177	0.0219	0.0243	0.0265	0.0298
		n_{max}	80	76	72	68	60	52	44
49	2401	A_{st}	24.80	31.68	40.80	50.56	56.00	60.96	68.64
		ρ_g	0.0103	0.1039	0.0180	0.0224	0.0250	0.0275	0.0286
		n_{max}	*	76	72	68	60	52	48
50	2500	A_{st}		33.44	43.20	53.72	60.00	66.04	74.88
		ρ_g		0.0134	0.0173	0.0215	0.0240	0.0264	0.0300

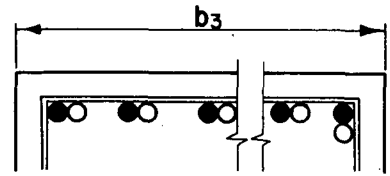
*ρ_g is less than 0.01 with maximum number of bars that can be accommodated.

REINFORCEMENT 22.3.1—Maximum number of bars n_{max} that can be accommodated in square columns having bars equally distributed on four faces using *normal lap* splices

References: ACI 318-95 Sections 3.3.2, 7.6.3, 7.6.4, 7.7.1(c), 7.10.5.1, 10.9.1, 10.9.2, 12.14.2.1, 12.16.2, and 15.8.2.3

Design conditions: 1.5 in. cover between ties and outer surface of column; #4 ties; clear distance between longitudinal bars of 1.5 in. for #5-#8 bars and 1.5 times nominal bar diameter for #9-#18 bars; aggregate not larger than 3/4 minimum clear spacing between bars; minimum of four bars per column; ρ_g no less than 0.01 and no greater than 0.08. For other tie sizes and cover, use formula in Commentary on Reinforcement 22. Required bend diameters of ties and deformation of bars are neglected.

Note: Lap splices should not be used for bars larger than #11, except: 1) at footings, #14 and #18 longitudinal bars, in compression only, may be lap spliced with dowels; and 2) #14 and #18 bars, in compression only, may be lap spliced to #11 and smaller bars.



Tangential lap splices

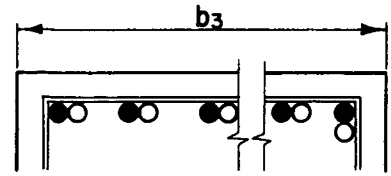
b_3, in.	A_g, in.^{2}		Bar size #5	#6	#7	#8	#9	#10	#11
		n_{max}	4	4	4	4	4	4	—
10	100	A_{st}	1.24	1.76	2.40	3.16	4.00	5.08	
		ρ_g	0.0124	0.0176	0.0240	0.0316	0.0400	0.0508	
		n_{max}	8	8	4	4	4	4	4
11	121	A_{st}	2.48	3.52	2.40	3.16	4.00	5.08	6.24
		ρ_g	0.0205	0.0291	0.0198	0.0261	0.0331	0.0420	0.0516
		n_{max}	8	8	8	8	4	4	4
12	144	A_{st}	2.48	3.52	4.80	6.32	4.00	5.08	6.24
		ρ_g	0.0172	0.0244	0.0333	0.0439	0.0278	0.0353	0.0433
		n_{max}	12	8	8	8	4	4	4
13	169	A_{st}	3.72	3.52	4.80	6.32	4.00	5.08	6.24
		ρ_g	0.0220	0.0208	0.0284	0.0374	0.0237	0.0301	0.0369
		n_{max}	12	12	8	8	8	4	4
14	196	A_{st}	3.72	5.28	4.80	6.32	8.00	5.08	6.24
		ρ_g	0.0190	0.0269	0.0245	0.0322	0.0408	0.0259	0.0318
		n_{max}	12	12	12	8	8	8	4
15	225	A_{st}	3.72	5.28	7.20	6.32	8.00	10.16	6.24
		ρ_g	0.0165	0.0235	0.0320	0.0281	0.0356	0.0452	0.0277
		n_{max}	16	12	12	12	8	8	8
16	256	A_{st}	4.96	5.28	7.20	9.48	8.00	10.16	12.48
		ρ_g	0.0194	0.0206	0.0281	0.0370	0.0312	0.0397	0.0488
		n_{max}	16	16	12	12	12	8	8
17	289	A_{st}	4.96	7.04	7.20	9.48	12.00	10.16	12.48
		ρ_g	0.0172	0.0244	0.0249	0.0328	0.0415	0.0352	0.0432
		n_{max}	16	16	16	12	12	8	8
18	324	A_{st}	4.96	7.04	9.60	9.48	12.00	10.16	12.48
		ρ_g	0.0153	0.0217	0.0296	0.0293	0.0370	0.0314	0.0385
		n_{max}	20	16	16	16	12	12	8
19	361	A_{st}	6.20	7.04	9.60	12.64	12.00	15.24	12.48
		ρ_g	0.0172	0.0195	0.0266	0.0350	0.0332	0.0422	0.0346

REINFORCEMENT 22.3.2—Maximum number of bars n_{max} that can be accommodated in square columns having bars equally distributed on four faces using *normal lap* splices

References: ACI 318-95 Sections 3.3.2, 7.6.3, 7.6.4, 7.7.1(c), 7.10.5.1, 10.9.1, 10.9.2, 12.14.2.1, 12.16.2, and 15.8.2.3

Design conditions: 1.5 in. cover between ties and outer surface of column; #4 ties; clear distance between longitudinal bars of 1.5 in. for #5-#8 bars and 1.5 times nominal bar diameter for #9-#18 bars; aggregate not larger than 3/4 minimum clear spacing between bars; minimum of four bars per column; ρ_g no less than 0.01 and no greater than 0.08. For other tie sizes and cover, use formula in Commentary on Reinforcement 22. Required bend diameters of ties and deformation of bars are neglected.

Note: Lap splices should not be used for bars larger than #11, except: 1) at footings, #14 and #18 longitudinal bars, in compression only, may be lap spliced with dowels; and 2) #14 and #18 bars, in compression only, may be lap spliced to #11 and smaller bars.



Tangential lap splices

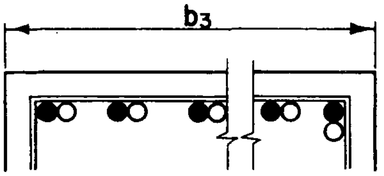
b_3, in.	A_g, in.^{2}		Bar size						
			#5	#6	#7	#8	#9	#10	#11
		n_{max}	20	20	16	16	12	12	8
20	400	A_{st}	6.20	8.80	9.60	12.64	12.00	15.24	12.48
		ρ_g	0.0155	0.0220	0.0240	0.0316	0.0300	0.0381	0.0312
		n_{max}	20	20	16	16	16	12	12
21	441	A_{st}	6.20	8.80	9.60	12.64	16.00	15.24	18.72
		ρ_g	0.0141	0.0200	0.0218	0.0287	0.0363	0.0346	0.0424
		n_{max}	24	20	20	16	16	12	12
22	484	A_{st}	7.44	8.80	12.00	12.64	16.00	15.24	18.72
		ρ_g	0.0154	0.0182	0.0248	0.0261	0.0331	0.0315	0.0387
		n_{max}	24	24	20	20	16	12	12
23	529	A_{st}	7.44	10.56	12.00	15.80	16.00	15.24	18.72
		ρ_g	0.0141	0.0200	0.0227	0.0299	0.0302	0.0288	0.0354
		n_{max}	28	24	20	20	16	16	12
24	576	A_{st}	8.68	10.56	12.00	15.80	16.00	20.32	18.72
		ρ_g	0.0151	0.0183	0.0208	0.0274	0.0278	0.0353	0.0325
		n_{max}	28	24	24	20	20	16	12
25	625	A_{st}	8.68	10.56	14.40	15.80	20.00	20.32	18.72
		ρ_g	0.0139	0.0169	0.0230	0.0253	0.0320	0.0325	0.0300
		n_{max}	28	28	24	24	20	16	16
26	676	A_{st}	8.68	12.32	14.40	18.96	20.00	20.32	24.96
		ρ_g	0.0128	0.0182	0.0213	0.0280	0.0296	0.0301	0.0369
		n_{max}	32	28	24	24	20	16	16
27	729	A_{st}	9.92	12.32	14.40	18.96	20.00	20.32	24.96
		ρ_g	0.0136	0.0169	0.0198	0.0260	0.0274	0.0279	0.0342
		n_{max}	32	28	28	24	20	20	16
28	784	A_{st}	9.92	12.32	16.80	18.96	20.00	25.40	24.96
		ρ_g	0.0127	0.0157	0.0214	0.0242	0.0255	0.0324	0.0318
		n_{max}	32	32	28	24	24	20	16
29	841	A_{st}	9.92	14.08	16.80	18.96	24.00	25.40	24.96
		ρ_g	0.0118	0.0167	0.0200	0.0225	0.0285	0.0302	0.0297

REINFORCEMENT 22.3.3—Maximum number of bars n_{max} that can be accommodated in square columns having bars equally distributed on four faces using *normal lap* splices

References: ACI 318-95 Sections 3.3.2, 7.6.3, 7.6.4, 7.7.1(c), 7.10.5.1, 10.9.1, 10.9.2, 12.14.2.1, 12.16.2, and 15.8.2.3

Design conditions: 1.5 in. cover between ties and outer surface of column; #4 ties; clear distance between longitudinal bars of 1.5 in. for #5-#8 bars and 1.5 times nominal bar diameter for #9-#18 bars; aggregate not larger than 3/4 minimum clear spacing between bars; minimum of four bars per column; ρ_g no less than 0.01 and no greater than 0.08. For other tie sizes and cover, use formula in Commentary on Reinforcement 22. Required bend diameters of ties and deformation of bars are neglected.

Note: Lap splices should not be used for bars larger than #11, except: 1) at footings, #14 and #18 longitudinal bars, in compression only, may be lap spliced with dowels; and 2) #14 and #18 bars, in compression only, may be lap spliced to #11 and smaller bars.



Tangential lap splices

b_3, in.	A_g, in.^{2}		Bar size						
			#5	#6	#7	#8	#9	#10	#11
		n_{max}	36	32	28	28	24	20	16
30	900	A_{st}	11.16	14.08	16.80	22.12	24.00	25.40	24.96
		ρ_g	0.0124	0.0156	0.0187	0.0246	0.0267	0.0282	0.0277
		n_{max}	36	32	32	28	24	20	20
31	961	A_{st}	11.16	14.08	19.20	22.12	24.00	25.40	31.20
		ρ_g	0.0116	0.0147	0.0200	0.0230	0.0250	0.0264	0.0325
		n_{max}	36	36	32	28	24	24	20
32	1024	A_{st}	11.16	15.84	19.20	22.12	24.00	30.48	31.20
		ρ_g	0.0109	0.0155	0.0188	0.0216	0.0234	0.0298	0.0305
		n_{max}	40	36	32	32	28	24	20
33	1089	A_{st}	12.40	15.84	19.20	25.28	28.00	30.48	31.20
		ρ_g	0.0114	0.0145	0.0176	0.0232	0.0257	0.0280	0.0287
		n_{max}	40	36	32	32	28	24	20
34	1156	A_{st}	12.40	15.84	19.20	25.28	28.00	30.48	31.20
		ρ_g	0.0107	0.0137	0.0166	0.0219	0.0242	0.0264	0.0270
		n_{max}	44	40	36	32	28	24	20
35	1225	A_{st}	13.64	17.60	21.60	25.28	28.00	30.48	31.20
		ρ_g	0.0111	0.0144	0.0176	0.0206	0.0229	0.0249	0.0255
		n_{max}	44	40	36	32	28	24	24
36	1296	A_{st}	13.64	17.60	21.60	25.28	28.00	30.48	37.44
		ρ_g	0.0105	0.0136	0.0167	0.0195	0.0216	0.0235	0.0289
		n_{max}	44	40	36	36	32	28	24
37	1369	A_{st}	13.64	17.60	21.60	28.44	32.00	35.56	37.44
		ρ_g	0.0100	0.0129	0.0158	0.0208	0.0234	0.0260	0.0273
		n_{max}	48	44	40	36	32	28	24
38	1444	A_{st}	14.88	19.36	24.00	28.44	32.00	35.56	37.44
		ρ_g	0.0103	0.0134	0.0166	0.0197	0.0222	0.0246	0.0259
		n_{max}	*	44	40	36	32	28	24
39	1521	A_{st}		19.36	24.00	28.44	32.00	35.56	37.44
		ρ_g		0.0127	0.0158	0.0187	0.0210	0.0234	0.0246

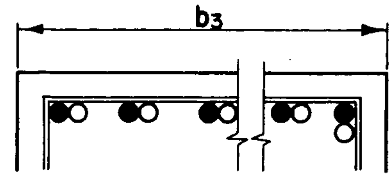
*ρ_g is less than 0.01 with maximum number of bars that can be accommodated.

REINFORCEMENT 22.3.4—Maximum number of bars n_{max} that can be accommodated in square columns having bars equally distributed on four faces using *normal lap* splices

References: ACI 318-95 Sections 3.3.2, 7.6.3, 7.6.4, 7.7.1(c), 7.10.5.1, 10.9.1, 10.9.2, 12.14.2.1, 12.16.2, and 15.8.2.3

Design conditions: 1.5 in. cover between ties and outer surface of column; #4 ties; clear distance between longitudinal bars of 1.5 in. for #5-#8 bars and 1.5 times nominal bar diameter for #9-#18 bars; aggregate not larger than 3/4 minimum clear spacing between bars; minimum of four bars per column; ρ_g no less than 0.01 and no greater than 0.08. For other tie sizes and cover, use formula in Commentary on Reinforcement 22. Required bend diameters of ties and deformation of bars are neglected.

Note: Lap splices should not be used for bars larger than #11, except: 1) at footings, #14 and #18 longitudinal bars, in compression only, may be lap spliced with dowels; and 2) #14 and #18 bars, in compression only, may be lap spliced to #11 and smaller bars.



Tangential lap splices

b_3, in.	A_g, in.^{2}		Bar size #5	#6	#7	#8	#9	#10	#11
		n_{max}	*	44	40	40	32	28	28
40	1600	A_{st}		19.36	24.00	31.60	32.00	35.56	43.68
		ρ_g		0.0121	0.0150	0.0198	0.0200	0.0222	0.0272
		n_{max}	*	48	44	40	36	32	28
41	1681	A_{st}		21.12	26.40	31.60	36.00	40.64	43.68
		ρ_g		0.0126	0.0157	0.0188	0.0214	0.0242	0.0260
		n_{max}	*	48	44	40	36	32	28
42	1764	A_{st}		21.12	26.40	31.60	36.00	40.64	43.68
		ρ_g		0.0120	0.0150	0.0179	0.0204	0.0230	0.0248
		n_{max}	*	48	44	40	36	32	28
43	1849	A_{st}		21.12	26.40	31.60	36.00	40.64	43.68
		ρ_g		0.0114	0.0143	0.0171	0.0195	0.0220	0.0236
		n_{max}	*	52	48	44	36	32	28
44	1936	A_{st}		22.88	28.80	34.76	36.00	40.64	43.68
		ρ_g		0.0118	0.0149	0.0180	0.0186	0.0210	0.0226
		n_{max}	*	52	48	44	40	32	32
45	2025	A_{st}		22.88	28.80	34.76	40.00	40.64	49.92
		ρ_g		0.0113	0.0142	0.0172	0.0198	0.0201	0.0247
		n_{max}	*	52	48	44	40	36	32
46	2116	A_{st}		22.88	28.80	34.76	40.00	45.72	49.92
		ρ_g		0.0108	0.0136	0.0164	0.0189	0.0216	0.0236
		n_{max}	*	56	48	48	40	36	32
47	2209	A_{st}		24.64	28.80	37.92	40.00	45.72	49.92
		ρ_g		0.0112	0.0130	0.0172	0.0181	0.0207	0.0226
		n_{max}	*	56	52	48	44	36	32
48	2304	A_{st}		24.64	28.80	37.92	44.00	45.72	49.92
		ρ_g		0.0103	0.0135	0.0165	0.0179	0.0190	0.0217
		n_{max}	*	56	52	48	44	36	32
49	2401	A_{st}		24.64	31.20	37.92	44.00	45.72	49.92
		ρ_g		0.0107	0.0135	0.0165	0.0174	0.0198	0.0217
		n_{max}	*	60	52	48	44	40	36
50	2500	A_{st}		26.40	31.20	37.92	44.00	50.80	56.16
		ρ_g		0.0106	0.0125	0.0152	0.0176	0.0203	0.0225

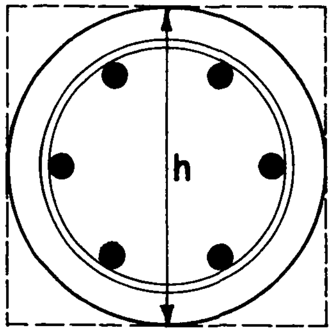
*ρ_g is less than 0.01 with maximum number of bars that can be accommodated.

REINFORCEMENT 23.1.1—Maximum number of bars n_{max} that can be accommodated in columns having bars arranged in a circle using *bearing* splices

References: ACI 318-95 Sections 3.3.2, 7.6.3, 7.7.1(c), 7.10.4.2, 10.9.1, and 10.9.2

Design conditions: 1.5 in. cover between spirals or ties and outer surface of column; #4 spirals or ties; clear distance between longitudinal bars of 1.5 in. for #5-#8 bars and 1.5 times nominal bar diameter for #9-#18 bars; aggregate not larger than 3/4 minimum clear spacing between bars; minimum number of bars is four for bars within circular ties, and six for bars enclosed by spirals; ρ_g no less than 0.01 and no greater than 0.08. For other tie sizes and cover, use formula in Commentary on Reinforcement 23.

Note: Values of n_{max} and A_{st} in this table apply to both circular and square columns of face dimension h having bars arranged in a circle; tabulated values of A_g and ρ_g apply only to circular columns. For square columns, multiply table value of A_g by $4/\pi$ and multiply table value of ρ_g by $\pi/4$.



Bearing splices

h, in.	A_g, in.^{2}		Bar size #5	#6	#7	#8	#9*	#10*	#11*	#14*	#18*
		n_{max}	7	7	6	6	5	4	—	—	—
10	78.5	A_{st}	2.17	3.08	3.60	4.74	5.00	5.08			
		ρ_g	0.0276	0.0392	0.0459	0.0604	0.0637	0.0647			
		n_{max}	9	8	7	7	6	5	4	—	—
11	95	A_{st}	2.79	3.52	4.20	5.53	6.00	6.35	6.24		
		ρ_g	0.0294	0.0371	0.0442	0.0582	0.0632	0.0668	0.0657		
		n_{max}	10	9	9	8	7	6	5	4	—
12	113	A_{st}	3.10	3.96	5.40	6.32	7.00	7.62	7.80	9.00	
		ρ_g	0.0274	0.0350	0.0478	0.0559	0.0619	0.0674	0.0690	0.0796	
		n_{max}	12	11	10	9	8	7	6	4†	—
13	133	A_{st}	3.72	4.84	6.00	7.11	8.00	8.89	9.36	9.00	
		ρ_g	0.0280	0.0364	0.0451	0.0535	0.0602	0.0668	0.0704	0.0677	
		n_{max}	13	12	11	11	9	8	7	5	—
14	154	A_{st}	4.03	5.28	6.60	8.69	9.00	10.16	10.92	11.25	
		ρ_g	0.262	0.0343	0.0429	0.0564	0.0584	0.0660	0.0709	0.0731	
		n_{max}	15	14	13	12	10	9	8	6	—
15	177	A_{st}	4.65	6.16	7.80	9.48	10.00	11.43	12.48	13.50	
		ρ_g	0.0263	0.0348	0.0440	0.0536	0.0565	0.0646	0.0705	0.0763	
		n_{max}	16	15	14	13	11	10	9	7	4†
16	201	A_{st}	4.96	6.60	8.40	10.27	11.00	12.70	14.04	15.75	16.00
		ρ_g	0.0247	0.0328	0.0418	0.0511	0.0547	0.0632	0.0699	0.0784	0.0796
		n_{max}	18	17	15	14	13	11	10	8	4†
17	227	A_{st}	5.58	7.48	9.00	11.06	13.00	13.97	15.60	18.00	16.00
		ρ_g	0.0246	0.0330	0.0396	0.0487	0.0573	0.0615	0.0687	0.0793	0.0705
		n_{max}	19	18	17	16	14	12	11	8	5†
18	254	A_{st}	5.89	7.92	10.20	12.64	14.00	15.24	17.16	18.00	20.00
		ρ_g	0.0232	0.0312	0.0402	0.0498	0.0551	0.0600	0.0676	0.0709	0.0787
		n_{max}	21	19	18	17	15	13	11	9	5†
19	284	A_{st}	6.51	8.36	10.80	13.43	15.00	16.51	17.16	20.25	20.00
		ρ_g	0.0229	0.0294	0.0380	0.0473	0.0528	0.0581	0.0604	0.0713	0.0704

*Entries above the stepped line are for circular tied columns only.

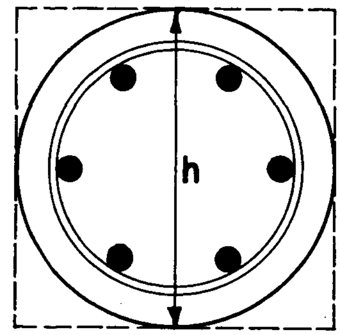
†Maximum number of bars governed by $\rho_g \le 0.08$ rather than by bar spacing.

REINFORCEMENT 23.1.2—Maximum number of bars n_{max} that can be accommodated in columns having bars arranged in a circle using *bearing* splices

References: ACI 318-95 Sections 3.3.2, 7.6.3, 7.7.1(c), 7.10.4.2, 10.9.1, and 10.9.2

Design conditions: 1.5 in. cover between spirals or ties and outer surface of column; #4 spirals or ties; clear distance between longitudinal bars of 1.5 in. for #5-#8 bars and 1.5 times nominal bar diameter for #9-#18 bars; aggregate not larger than 3/4 minimum clear spacing between bars; minimum number of bars is four for bars within circular ties, and six for bars enclosed by spirals; ρ_g no less than 0.01 and no greater than 0.08. For other tie sizes and cover, use formula in Commentary on Reinforcement 23.

Note: Values of n_{max} and A_{st} in this table apply to both circular and square columns of face dimension h having bars arranged in a circle; tabulated values of A_g and ρ_g apply only to circular columns. For square columns, multiply table value of A_g by $4/\pi$ and multiply table value of ρ_g by $\pi/4$.



Bearing splices

h, in.	A_g, in.^{2}		Bar size								
			#5	#6	#7	#8	#9	#10	#11	#14	#18
		n_{max}	22	21	19	18	16	14	12	10	6†
20	314	A_{st}	6.82	9.24	11.40	14.22	16.00	17.78	18.72	22.50	24.00
		ρ_g	0.0217	0.0294	0.0363	0.0453	0.0510	0.0566	0.0596	0.0717	0.0764
		n_{max}	24	22	21	20	17	15	13	11	6†
21	346	A_{st}	7.44	9.68	12.60	15.80	17.00	19.05	20.28	24.75	24.00
		ρ_g	0.0215	0.0280	0.0364	0.0457	0.0491	0.0551	0.0586	0.0715	0.0694
		n_{max}	25	24	22	21	18	16	14	11	7†
22	380	A_{st}	7.75	10.56	13.20	16.59	18.00	20.32	21.84	24.75	28.00
		ρ_g	0.0204	0.0278	0.0347	0.0437	0.0474	0.0535	0.0575	0.0651	0.0737
		n_{max}	27	25	23	22	19	17	15	12	8†
23	415	A_{st}	8.37	11.00	13.80	17.38	19.00	21.59	23.40	27.00	32.00
		ρ_g	0.0202	0.0265	0.0333	0.0419	0.0458	0.0520	0.0564	0.0651	0.0771
		n_{max}	28	26	25	23	20	18	16	13	9
24	452	A_{st}	8.68	11.44	15.00	18.17	20.00	22.86	24.96	29.25	36.00
		ρ_g	0.0192	0.0253	0.0332	0.0402	0.0442	0.0506	0.0552	0.0647	0.0796
		n_{max}	30	28	26	25	22	19	17	14	9†
25	491	A_{st}	9.30	12.32	15.60	19.75	22.00	24.13	26.52	31.50	36.00
		ρ_g	0.0189	0.0251	0.0318	0.0402	0.0448	0.0491	0.0540	0.0642	0.0733
		n_{max}	31	29	27	26	23	20	18	14	10
26	531	A_{st}	9.61	12.76	16.20	20.54	23.00	25.40	28.08	31.50	40.00
		ρ_g	0.0181	0.0240	0.0305	0.0387	0.0433	0.0478	0.0529	0.0593	0.0753
		n_{max}	33	31	29	27	24	21	19	15	11
27	573	A_{st}	10.23	13.64	17.40	21.33	24.00	26.67	29.64	33.75	44.00
		ρ_g	0.0179	0.0238	0.0304	0.0372	0.0419	0.0465	0.0517	0.0589	0.0768
		n_{max}	34	32	30	28	25	22	20	16	11
28	616	A_{st}	10.54	14.08	18.00	22.12	25.00	27.94	31.20	36.00	44.00
		ρ_g	0.0171	0.0229	0.0292	0.0359	0.0406	0.0454	0.0506	0.0584	0.0714
		n_{max}	36	33	31	30	26	23	20	17	12
29	661	A_{st}	11.16	14.52	18.60	23.70	26.00	29.21	31.20	38.25	48.00
		ρ_g	0.0169	0.0220	0.0281	0.0359	0.0393	0.0442	0.0472	0.0579	0.0726

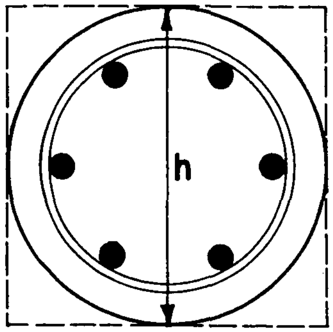
†Maximum number of bars governed by $\rho_g \le 0.08$ rather than by bar spacing.

REINFORCEMENT 23.1.3—Maximum number of bars n_{max} that can be accommodated in columns having bars arranged in a circle using *bearing* splices

References: ACI 318-95 Sections 3.3.2, 7.6.3, 7.7.1(c), 7.10.4.2, 10.9.1, and 10.9.2

Design conditions: 1.5 in. cover between spirals or ties and outer surface of column; #4 spirals or ties; clear distance between longitudinal bars of 1.5 in. for #5-#8 bars and 1.5 times nominal bar diameter for #9-#18 bars; aggregate not larger than 3/4 minimum clear spacing between bars; minimum number of bars is four for bars within circular ties, and six for bars enclosed by spirals; ρ_g no less than 0.01 and no greater than 0.08. For other tie sizes and cover, use formula in Commentary on Reinforcement 23.

Note: Values of n_{max} and A_{st} in this table apply to both circular and square columns of face dimension h having bars arranged in a circle; tabulated values of A_g and ρ_g apply only to circular columns. For square columns, multiply table value of A_g by $4/\pi$ and multiply table value of ρ_g by $\pi/4$.



Bearing splices

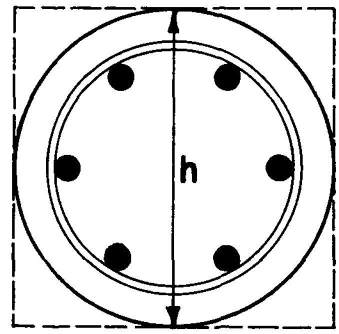
h, in.	A_g, in.[2]		Bar size #5	#6	#7	#8	#9	#10	#11	#14	#18
		n_{max}	37	35	33	31	27	24	21	17	13
30	707	A_{st}	11.47	15.40	19.80	24.49	27.00	30.48	32.76	38.25	52.00
		ρ_g	0.0162	0.0218	0.0280	0.0346	0.0382	0.0431	0.0463	0.0541	0.0736
		n_{max}	38	36	34	32	28	25	22	18	13
31	755	A_{st}	11.78	15.84	20.40	25.28	28.00	31.75	34.32	40.50	52.00
		ρ_g	0.0156	0.0210	0.0270	0.0335	0.0371	0.0421	0.0455	0.0536	0.0689
		n_{max}	40	38	35	33	29	26	23	19	14
32	804	A_{st}	12.40	16.72	21.00	26.07	29.00	33.02	35.88	42.75	56.00
		ρ_g	0.0154	0.0208	0.0261	0.0324	0.0361	0.0411	0.0446	0.0532	0.0697
		n_{max}	41	39	37	35	31	27	24	20	14
33	855	A_{st}	12.71	17.16	22.20	27.65	31.00	34.29	37.44	45.00	56.00
		ρ_g	0.0149	0.0201	0.0260	0.0323	0.0363	0.0401	0.0438	0.0526	0.0655
		n_{max}	43	40	38	36	32	28	25	20	15
34	908	A_{st}	13.33	17.60	22.80	28.44	32.00	35.56	39.00	45.00	60.00
		ρ_g	0.0147	0.0194	0.0251	0.0313	0.0352	0.0392	0.0430	0.0496	0.0661
		n_{max}	44	42	39	37	33	29	26	21	15
35	962	A_{st}	13.64	18.48	23.40	29.23	33.00	36.83	40.56	47.25	60.00
		ρ_g	0.0142	0.0192	0.0243	0.0304	0.0343	0.0383	0.0422	0.0491	0.0624
		n_{max}	46	43	41	38	34	30	27	22	16
36	1018	A_{st}	14.26	18.92	24.60	30.02	34.00	38.10	42.12	49.50	64.00
		ρ_g	0.0140	0.0186	0.0242	0.0295	0.0334	0.0374	0.0414	0.0486	0.0629
		n_{max}	47	45	42	40	35	31	28	23	16
37	1075	A_{st}	14.57	19.80	25.20	31.60	35.00	39.37	43.68	51.75	64.00
		ρ_g	0.0136	0.0184	0.0234	0.0294	0.0326	0.0366	0.0406	0.0481	0.0595
		n_{max}	49	46	43	41	36	32	28	23	17
38	1134	A_{st}	15.19	20.24	25.80	32.39	36.00	40.64	43.68	51.75	68.00
		ρ_g	0.0134	0.0178	0.0228	0.0286	0.0317	0.0358	0.0385	0.0448	0.0600
		n_{max}	50	47	45	42	37	33	29	24	18
39	1195	A_{st}	15.50	20.68	27.00	33.18	37.00	41.91	45.24	54.00	72.00
		ρ_g	0.0130	0.0173	0.0226	0.0278	0.0310	0.0351	0.0379	0.0452	0.0603

REINFORCEMENT 23.1.4—Maximum number of bars n_{max} that can be accommodated in columns having bars arranged in a circle using *bearing* splices

References: ACI 318-95 Sections 3.3.2, 7.6.3, 7.7.1(c), 7.10.4.2, 10.9.1, and 10.9.2

Design conditions: 1.5 in. cover between spirals or ties and outer surface of column; #4 spirals or ties; clear distance between longitudinal bars of 1.5 in. for #5-#8 bars and 1.5 times nominal bar diameter for #9-#18 bars; aggregate not larger than 3/4 minimum clear spacing between bars; minimum number of bars is four for bars within circular ties, and six for bars enclosed by spirals; ρ_g no less than 0.01 and no greater than 0.08. For other tie sizes and cover, use formula in Commentary on Reinforcement 23.

Note: Values of n_{max} and A_{st} in this table apply to both circular and square columns of face dimension h having bars arranged in a circle; tabulated values of A_g and ρ_g apply only to circular columns. For square columns, multiply table value of A_g by $4/\pi$ and multiply table value of ρ_g by $\pi/4$.



Bearing splices

h, in.	A_g, in.^{2}		#5	#6	#7	#8	#9	#10	#11	#14	#18
			Bar size								
		n_{max}	52	49	46	43	38	34	30	25	18
40	1257	A_{st}	16.12	21.56	27.60	33.97	38.00	43.18	46.80	56.25	72.00
		ρ_g	0.0128	0.0171	0.0220	0.0270	0.0302	0.0344	0.0372	0.0447	0.0573
		n_{max}	53	50	47	45	39	35	31	26	19
41	1320	A_{st}	16.43	22.00	28.20	35.55	39.00	44.45	48.36	58.50	76.00
		ρ_g	0.0124	0.0167	0.0214	0.0269	0.0295	0.0337	0.0366	0.0443	0.0576
		n_{max}	55	51	49	46	41	36	32	26	19
42	1385	A_{st}	17.05	22.44	29.40	36.34	41.00	45.72	49.92	58.50	76.00
		ρ_g	0.0123	0.0162	0.0212	0.0262	0.0296	0.0330	0.0360	0.0422	0.0549
		n_{max}	56	53	50	47	42	37	33	27	20
43	1452	A_{st}	17.36	23.32	30.00	37.13	42.00	46.99	51.48	60.75	80.00
		ρ_g	0.0120	0.0161	0.0207	0.0256	0.0289	0.0324	0.0355	0.0418	0.0551
		n_{max}	58	54	51	48	43	38	34	28	20
44	1521	A_{st}	17.98	23.76	30.60	37.92	43.00	48.26	53.04	63.00	80.00
		ρ_g	0.0118	0.0156	0.0201	0.0249	0.0283	0.0317	0.0349	0.0414	0.0526
		n_{max}	59	56	53	50	44	39	35	29	21
45	1590	A_{st}	18.29	24.64	31.80	39.50	44.00	49.53	54.60	65.25	84.00
		ρ_g	0.0115	0.0155	0.0200	0.0248	0.0277	0.0312	0.0343	0.0410	0.0528
		n_{max}	61	57	54	51	45	40	36	29	22
46	1662	A_{st}	18.91	25.08	32.40	40.29	45.00	50.80	56.16	65.25	88.00
		ρ_g	0.0114	0.0151	0.0195	0.0242	0.0271	0.0306	0.0338	0.0393	0.0529
		n_{max}	62	58	55	52	46	41	37	30	22
47	1735	A_{st}	19.22	25.52	33.00	41.08	46.00	52.07	57.72	67.50	88.00
		ρ_g	0.0111	0.0147	0.0190	0.0237	0.0265	0.0300	0.0333	0.0389	0.0507
		n_{max}	64	60	57	54	47	42	37	31	23
48	1810	A_{st}	19.84	26.40	34.20	42.66	47.00	53.34	57.72	69.75	92.00
		ρ_g	0.0110	0.0146	0.0189	0.0236	0.0260	0.0295	0.0319	0.0385	0.0508
		n_{max}	65	61	58	55	48	43	38	32	23
49	1886	A_{st}	20.15	26.84	34.80	43.45	48.00	54.61	59.28	72.00	92.00
		ρ_g	0.0107	0.0142	0.0185	0.0230	0.0255	0.0290	0.0314	0.0382	0.0488
		n_{max}	67	63	59	56	49	44	39	32	24
50	1963	A_{st}	20.77	27.72	35.40	44.24	49.00	55.88	60.84	72.00	96.00
		ρ_g	0.0106	0.0141	0.0180	0.0225	0.0250	0.0285	0.0310	0.0367	0.0489

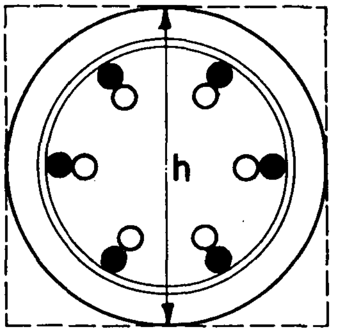
REINFORCEMENT 23.2.1—Maximum number of bars n_{max} that can be accommodated in columns having bars arranged in a circle using *normal lap* splices

References: ACI 318-95 Sections 3.3.2, 7.6.3, 7.6.4, 7.7.1(c), 7.10.4.2, 10.9.1, 10.9.2, 12.14.2.1, 12.16.2, and 15.8.2.3

Design conditions: 1.5 in. cover between spirals or ties and outer surface of column; #4 spirals or ties; clear distance between longitudinal bars of 1.5 in. for #5-#8 bars and 1.5 times nominal bar diameter for #9-#18 bars; aggregate not larger than 3/4 minimum clear spacing between bars; minimum number of bars is four for bars within circular ties, and six for bars enclosed by spirals; ρ_g no less than 0.01 and no greater than 0.08. For other tie sizes and cover, use formula in Commentary on Reinforcement 23.

Note 1: Values of n_{max} and A_{st} in this table apply to both circular and square columns of face dimension h having bars arranged in a circle; tabulated values of A_g and ρ_g apply only to circular columns. For square columns, multiply table value of A_g by $4/\pi$ and multiply table value of ρ_g by $\pi/4$.

Note 2: Lap splices should not be used for bars larger than #11, except: 1) at footings, #14 and #18 longitudinal bars, in compression only, may be lap spliced with dowels; and 2) #14 and #18 bars, in compression only, may be lap spliced up to #11 and smaller bars.



Normal lap splices

			Bar size						
h, in.	A_g, in.^{2}		#5	#6*	#7*	#8*	#9*	#10*	#11*
		n_{max}	6	4	4	—	—	—	—
10	78.5	A_{st}	1.86	1.76	2.4				
		ρ_g	0.0237	0.0224	0.0306				
		n_{max}	7	6	5	4	—	—	—
11	95	A_{st}	2.17	2.64	3.00	3.16			
		ρ_g	0.0228	0.0278	0.0316	0.0333			
		n_{max}	8	7	6	6	4	—	—
12	113	A_{st}	2.48	3.08	3.60	4.74	4.00		
		ρ_g	0.0219	0.0273	0.0319	0.0419	0.0354		
		n_{max}	10	9	8	7	5	4	—
13	133	A_{st}	3.10	3.96	4.80	5.53	5.00	5.08	—
		ρ_g	0.0233	0.0298	0.0361	0.0416	0.0376	0.0382	
		n_{max}	11	10	9	8	7	5	4
14	154	A_{st}	3.41	4.40	5.40	6.32	7.00	6.35	6.24
		ρ_g	0.0221	0.0286	0.0351	0.0410	0.0455	0.0412	0.0405
		n_{max}	13	12	10	9	8	6	5
15	177	A_{st}	4.03	5.28	6.00	7.11	8.00	7.62	7.80
		ρ_g	0.0228	0.0298	0.0339	0.0402	0.0452	0.0431	0.0441
		n_{max}	14	13	12	11	9	7	6
16	201	A_{st}	4.34	5.72	7.20	8.69	9.00	8.89	9.36
		ρ_g	0.0216	0.0285	0.0358	0.0432	0.0448	0.0442	0.0466
		n_{max}	16	14	13	12	10	8	7
17	227	A_{st}	4.96	6.16	7.80	9.48	10.00	10.16	10.92
		ρ_g	0.0219	0.0271	0.0344	0.0418	0.0441	0.0448	0.0481
		n_{max}	17	16	14	13	11	9	8
18	254	A_{st}	5.27	7.04	8.40	10.27	11.00	11.43	12.48
		ρ_g	0.0207	0.277	0.0331	0.0404	0.0433	0.0450	0.0491
		n_{max}	19	17	16	14	12	10	9
19	284	A_{st}	5.89	7.48	9.60	11.06	12.00	12.70	14.04
		ρ_g	0.207	0.0263	0.0338	0.0389	0.0423	0.0447	0.0494

*Entries above the stepped line are for circular tied columns only.

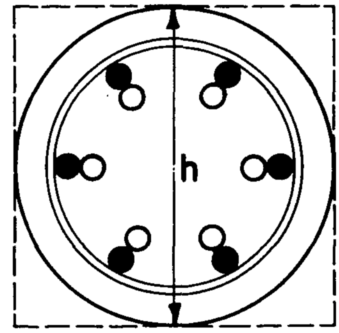
REINFORCEMENT 23.2.2—Maximum number of bars n_{max} that can be accommodated in columns having bars arranged in a circle using *normal lap* splices

References: ACI 318-95 Sections 3.3.2, 7.6.3, 7.6.4, 7.7.1(c), 7.10.4.2, 10.9.1, 10.9.2, 12.14.2.1, 12.16.2, and 15.8.2.3

Design conditions: 1.5 in. cover between spirals or ties and outer surface of column; #4 spirals or ties; clear distance between longitudinal bars of 1.5 in. for #5-#8 bars and 1.5 times nominal bar diameter for #9-#18 bars; aggregate not larger than 3/4 minimum clear spacing between bars; minimum number of bars is four for bars within circular ties, and six for bars enclosed by spirals; ρ_g no less than 0.01 and no greater than 0.08. For other tie sizes and cover, use formula in Commentary on Reinforcement 23.

Note 1: Values of n_{max} and A_{st} in this table apply to both circular and square columns of face dimension h having bars arranged in a circle: tabulated values of A_g and ρ_g apply only to circular columns. For square columns, multiply table value of A_g by $4/\pi$ and multiply table value of ρ_g by $\pi/4$.

Note 2: Lap splices should not be used for bars larger than #11, except: 1) at footings, #14 and #18 longitudinal bars, in compression only, may be lap spliced with dowels; and 2) #14 and #18 bars, in compression only, may be lap spliced up to #11 and smaller bars.



Normal lap splices

			Bar size						
h, in.	A_g, in.^{2}		#5	#6	#7	#8	#9	#10	#11
		n_{max}	20	19	17	16	13	11	10
20	314	A_{st}	6.20	8.36	10.20	12.64	13.00	13.97	15.60
		ρ_g	0.0197	0.0266	0.0325	0.0403	0.0414	0.0445	0.0497
		n_{max}	22	20	18	17	15	12	11
21	346	A_{st}	6.82	8.80	10.80	13.43	15.00	15.24	17.16
		ρ_g	0.0197	0.0254	0.0312	0.0388	0.0434	0.0440	0.0496
		n_{max}	23	21	20	18	16	13	12
22	380	A_{st}	7.13	9.24	12.00	14.22	16.00	16.51	18.72
		ρ_g	0.0188	0.0243	0.0316	0.0374	0.0421	0.0434	0.0493
		n_{max}	25	23	21	20	17	14	13
23	415	A_{st}	7.75	10.012	12.60	15.80	17.00	17.78	20.28
		ρ_g	0.0187	0.0244	0.0304	0.0381	0.0410	0.0428	0.0489
		n_{max}	26	24	22	21	18	15	13
24	452	A_{st}	8.06	10.56	13.20	16.59	18.00	19.05	20.28
		ρ_g	0.0178	0.0234	0.0292	0.0367	0.0398	0.0421	0.0449
		n_{max}	28	26	24	22	19	16	14
25	491	A_{st}	8.68	11.44	14.40	17.38	19.00	20.32	21.84
		ρ_g	0.0177	0.0233	0.0293	0.0354	0.0387	0.0414	0.0445
		n_{max}	29	27	25	23	20	17	15
26	531	A_{st}	8.99	11.88	15.00	18.17	20.00	21.59	23.40
		ρ_g	0.0169	0.0224	0.0282	0.0342	0.0377	0.0407	0.0441
		n_{max}	31	28	26	25	21	18	16
27	573	A_{st}	9.61	12.32	15.60	19.75	21.00	22.86	24.96
		ρ_g	0.0168	0.0215	0.0272	0.0345	0.0366	0.0399	0.0436
		n_{max}	32	30	28	26	22	19	17
28	616	A_{st}	9.92	13.20	16.80	20.54	22.00	24.13	26.52
		ρ_g	0.0161	0.0214	0.0273	0.0333	0.0357	0.0392	0.0431
		n_{max}	34	31	29	27	24	20	18
29	661	A_{st}	10.54	13.64	17.40	21.33	24.00	25.40	28.08
		ρ_g	0.0159	0.0206	0.0263	0.0323	0.0363	0.0384	0.0425

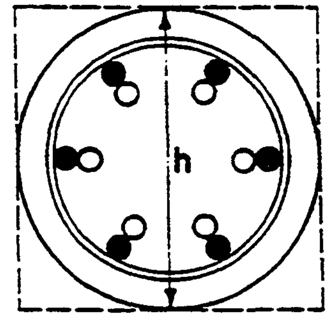
REINFORCEMENT 23.2.3—Maximum number of bars n_{max} that can be accommodated in columns having bars arranged in a circle using *normal lap* splices

References: ACI 318-95 Sections 3.3.2, 7.6.3, 7.6.4, 7.7.1(c), 7.10.4.2, 10.9.1, 10.9.2, 12.14.2.1, 12.16.2, and 15.8.2.3

Design conditions: 1.5 in. cover between spirals or ties and outer surface of column; #4 spirals or ties; clear distance between longitudinal bars of 1.5 in. for #5-#8 bars and 1.5 times nominal bar diameter for #9-#18 bars; aggregate not larger than 3/4 minimum clear spacing between bars: minimum number of bars is four for bars within circular ties and six for bars enclosed by spirals; ρ_g no less than 0.01 and no greater than 0.08. For other tie sizes and cover, use formula in Commentary on Reinforcement 23.

Note 1: Values of n_{max} and A_{st} in this table apply to both circular and square columns of face dimension h having bars arranged in a circle; tabulated values of A_g and ρ_g apply only to circular columns. For square columns, multiply table value of A_g by $4/\pi$ and multiply table value of ρ_g by $\pi/4$.

Note 2: Lap splices should not be used for bars larger than #11, except at 1) footings, #14 and #18 longitudinal bars, in compression only, may be lap spliced with dowels, and 2) #14 and #18 bars, in compression only, may be lap spliced to #11 and smaller bars.



Normal lap splices

h, in.	A_g, in.^{2}		Bar size #5	#6	#7	#8	#9	#10	#11
		n_{max}	35	33	30	28	25	21	19
30	707	A_{st}	10.85	14.52	18.00	22.12	25.00	26.67	29.64
		ρ_g	0.0153	0.0205	0.0255	0.0313	0.0354	0.0377	0.0419
		n_{max}	37	34	32	30	26	22	20
31	755	A_{st}	11.47	14.96	19.20	23.70	26.00	27.94	31.20
		ρ_g	0.0152	0.0198	0.0254	0.0314	0.0344	0.0370	0.0413
		n_{max}	38	35	33	31	27	23	21
32	804	A_{st}	11.78	15.40	19.80	24.49	27.00	29.21	32.76
		ρ_g	0.0147	0.0192	0.0246	0.0305	0.0336	0.0363	0.0407
		n_{max}	40	37	34	32	28	24	22
33	855	A_{st}	12.40	16.28	20.40	25.28	28.00	30.48	34.32
		ρ_g	0.0145	0.0190	0.0239	0.0296	0.0327	0.0356	0.0401
		n_{max}	41	38	36	33	29	25	22
34	908	A_{st}	12.71	16.72	21.60	26.07	29.00	31.75	34.32
		ρ_g	0.0140	0.0184	0.0238	0.0287	0.0319	0.0350	0.0378
		n_{max}	43	40	37	35	30	26	23
35	962	A_{st}	13.33	17.60	22.20	27.65	30.00	33.02	35.88
		ρ_g	0.0139	0.0183	0.0231	0.0287	0.0312	0.0343	0.0373
		n_{max}	44	41	38	36	31	27	24
36	1018	A_{st}	13.64	18.04	22.80	28.44	31.00	34.29	37.44
		ρ_g	0.0134	0.0177	0.0224	0.0279	0.0305	0.0337	0.0368
		n_{max}	46	42	40	37	32	28	25
37	1075	A_{st}	14.26	18.48	24.00	29.23	32.00	35.56	39.00
		ρ_g	0.0133	0.0172	0.0223	0.0272	0.0298	0.0331	0.0363
		n_{max}	47	44	41	38	34	29	26
38	1134	A_{st}	14.57	19.36	24.60	30.02	34.00	36.83	40.56
		ρ_g	0.0128	0.0171	0.0217	0.0265	0.0300	0.0325	0.0358
		n_{max}	48	45	42	40	35	30	27
39	1195	A_{st}	14.88	19.80	25.20	31.60	35.00	38.10	42.12
		ρ_g	0.0125	0.0166	0.0211	0.0264	0.0293	0.0319	0.0352

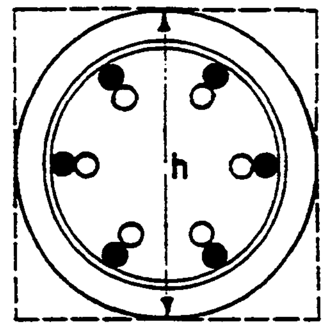
REINFORCEMENT 23.2.4—Maximum number of bars n_{max} that can be accommodated in columns having bars arranged in a circle using *normal lap* splices

References: ACI 318-95 Sections 3.3.2, 7.6.3, 7.6.4, 7.7.1(c), 7.10.4.2, 10.9.1, 10.9.2, 12.14.2.1, 12.16.2, and 15.8.2.3

Design conditions: 1.5 in. cover between spirals or ties and outer surface of column; #4 spirals or ties; clear distance between longitudinal bars of 1.5 in. for #5-#8 bars and 1.5 times nominal bar diameter for #9-#18 bars: aggregate not larger than 3/4 minimum clear spacing between bars; minimum number of bars is four for bars within circular ties and six for bars enclosed by spirals; ρ_g no less than 0.01 and no greater than 0.08. For other tie sizes and cover, use formula in Commentary on Reinforcement 23.

Note 1: Values of n_{max} and A_{st} in this table apply to both circular and square columns of face dimension h having bars arranged in a circle: tabulated values of A_g and ρ_g apply only to circular columns. For square columns, multiply table value of A_g by $4/\pi$ and multiply table value of ρ_g by $\pi/4$.

Note 2: Lap splices should not be used for bars larger than #11, except 1) at footings, #14 and #18 longitudinal bars, in compression only, may be lap spliced with dowels, and 2) #14 and #18 bars, in compression only, may be lap spliced to #11 and smaller bars.



Normal lap splices

h, in.	A_g, in.²		Bar size #5	#6	#7	#8	#9	#10	#11
		n_{max}	50	47	44	41	36	31	28
40	1257	A_{st}	15.50	20.68	26.40	32.39	36.00	39.37	43.68
		ρ_g	0.0123	0.0165	0.0210	0.0258	0.0286	0.0313	0.0347Rj
		n_{max}	51	48	45	42	37	32	29
41	1320	A_{st}	15.81	21.12	27.00	33.18	37.00	40.64	45.24
		ρ_g	0.0120	0.0160	0.0205	0.0251	0.0280	0.0308	0.0343
		n_{max}	53	49	46	43	38	33	30
42	1385	A_{st}	16.43	21.56	27.60	33.97	38.00	41.91	46.80
		ρ_g	0.0119	0.0156	0.0199	0.0245	0.0274	0.0303	0.0338
		n_{max}	54	51	48	45	39	34	30
43	1452	A_{st}	16.74	22.44	28.80	35.55	39.00	43.18	46.80
		ρ_g	0.0115	0.0155	0.0198	0.0245	0.0269	0.0297	0.0322
		n_{max}	56	52	49	46	40	35	31
44	1521	A_{st}	17.36	22.88	29.40	36.34	40.00	44.45	48.36
		ρ_g	0.0114	0.0150	0.0193	0.0239	0.0263	0.0292	0.0318
		n_{max}	57	54	50	47	41	36	32
45	1590	A_{st}	17.67	23.76	30.00	37.13	41.00	45.72	49.92
		ρ_g	0.0111	0.0149	0.0189	0.0234	0.0258	0.0288	0.0314
		n_{max}	59	55	52	48	42	37	33
46	1662	A_{st}	18.29	24.20	31.20	37.92	42.00	46.99	51.48
		ρ_g	0.0110	0.0146	0.0188	0.0228	0.0253	0.0283	0.0310
		n_{max}	60	56	53	50	44	38	34
47	1735	A_{st}	18.60	24.64	31.80	39.50	44.00	48.26	53.04
		ρ_g	0.0107	0.0142	0.0183	0.0228	0.0254	0.0278	0.0306
		n_{max}	62	58	54	51	45	39	35
48	1810	A_{st}	19.22	25.52	32.40	40.29	45.00	49.53	54.60
		ρ_g	0.0106	0.0141	0.0179	0.0223	0.0249	0.0274	0.0302
		n_{max}	63	59	56	52	46	40	36
49	1886	A_{st}	19.53	25.96	33.60	41.08	46.00	50.80	56.16
		ρ_g	0.0104	0.0138	0.0178	0.0218	0.0244	0.0269	0.0298
		n_{max}	65	61	57	54	47	41	37
50	1963	A_{st}	20.15	26.84	34.20	42.66	47.00	52.07	57.72
		ρ_g	0.0103	0.0137	0.0174	0.0217	0.0239	0.0265	0.0294

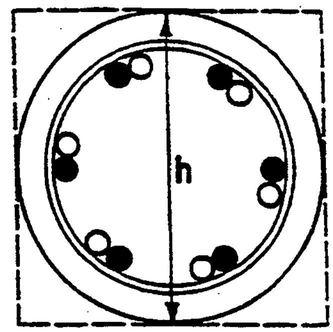
REINFORCEMENT 23.3.1—Maximum number of bars n_{max} that can be accommodated in columns having bars arranged in a circle using *tangential lap* splices

References: ACI 318-95 Sections 3.3.2, 7.6.3, 7.64, 7.7.1(c), 7.10.4.2, 10.9.1, 10.9.2, 12.14.2.1, 12.16.2, and 15.8.2.3

Design conditions: 1.5 in. cover between spirals or ties and outer surface of column; #4 spirals or ties; clear distance between longitudinal bars of 1.5 in. for #5-#8 bars and 1.5 times nominal bar diameter for #9-#18 bars: aggregate not larger than 3/4 minimum clear spacing between bars; minimum number of bars is four for bars within circular ties and six for bars enclosed by spirals: ρg no less than 0.01 and no greater than 0.08. for other tie sizes and cover, use formula in Commentary on Reinforcement 23.

Note 1: Values of n_{max} and A_{st} in this table apply to both circular and square columns of face dimension h having bars arranged in a circle; tabulated values of A_g and ρ_g apply only to circular columns. For square columns, multiply table value of A_g by $4/\pi$ and multiply table value of ρ_g by $\pi/4$.

Note 2: Lap splices should not be used for bars larger than #11, except 1) at footings, #14 and #18 longitudinal bars, in compression only may be lap spliced with dowels, and 2) #14 and #18 bars, in compression only, may be lap spliced to #11 and smaller bars.



Tangential lap splices

h, in.	A_g, in.^{2}		#5	#6*	#7*	#8*	#9*	#10*	#11*
		n_{max}	6	5	4	4	—	—	—
10	78.5	A_{st}	1.86	2.20	2.40	3.16			
		ρ_g	0.0237	0.0280	0.0306	0.0403			
		n_{max}	7	6	5	5	4	—	—
11	95.0	A_{st}	2.17	2.64	3.00	3.95	4.00		
		ρ_g	0.0228	0.0278	0.0316	0.0416	0.0421		
		n_{max}	8	7	6	6	5	4	4
12	113	A_{st}	2.48	3.08	3.60	4.74	5.00	5.08	6.24
		ρ_g	0.0219	0.0273	0.0319	0.0419	0.0442	0.0450	0.0552
		n_{max}	9	8	7	7	6	5	4
13	133	A_{st}	2.79	3.52	4.20	5.53	6.00	6.35	6.24
		ρ_g	0.0210	0.0265	0.0316	0.0416	0.0451	0.0477	0.0469
		n_{max}	10	9	8	8	6	6	5
14	154	A_{st}	3.10	3.96	4.80	6.32	6.00	7.62	7.80
		ρ_g	0.0201	0.0257	0.0312	0.0410	0.0390	0.0495	0.0506
		n_{max}	11	10	9	8	7	6	6
15	177	A_{st}	3.41	4.40	5.40	6.32	7.00	7.62	9.36
		ρ_g	0.0193	0.0249	0.0305	0.0357	0.0395	0.0431	0.0529
		n_{max}	12	11	10	9	8	7	6
16	201	A_{st}	3.72	4.84	6.00	7.11	8.00	8.89	9.36
		ρ_g	0.0185	0.0241	0.0299	0.0354	0.0398	0.0442	0.0466
		n_{max}	14	12	11	10	9	8	7
17	227	A_{st}	4.34	5.28	6.60	7.90	9.00	10.16	10.92
		ρ_g	0.0191	0.0233	0.0291	0.0348	0.0396	0.0448	0.0481
		n_{max}	15	13	12	11	10	8	7
18	254	A_{st}	4.65	5.72	7.20	8.69	10.00	10.16	10.92
		ρ_g	0.0183	0.0225	0.0283	0.0342	0.0394	0.0400	0.0430
		n_{max}	16	14	13	12	11	9	8
19	284	A_{st}	4.96	6.16	7.80	9.48	11.00	11.43	12.48
		ρ_g	0.0175	0.0217	0.0275	0.0334	0.0387	0.0402	0.0439

*Entries above the stepped line are for circular tied columns only.

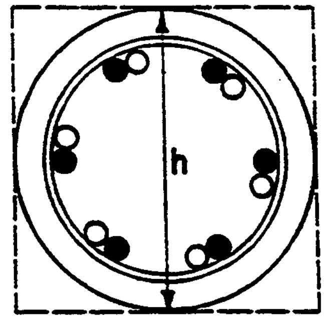
REINFORCEMENT 23.3.2—Maximum number of bars n_{max} that can be accommodated in columns having bars arranged in a circle using *tangential lap* splices

References: ACI 318-95 Sections 3.3.2, 7.6.3, 7.6.4, 7.7.1(c), 7.10.4.2, 10.9.1, 10.9.2, 12.14.2.1, 12.16.2, and 15.8.2.3.

Design conditions: 1.5 in. cover between spirals or ties and outer surface of column; #4 spirals or ties; clear distance between longitudinal bars of 1.5 in. for #5-#8 bars and 1.5 times nominal bar diameter for #9-#18 bars; aggregate not larger than 3/4 minimum clear spacing between bars: minimum number of bars is four for bars within circular ties and six for bars enclosed by spirals; ρ_g no less than 0.01 and no greater than 0.08. For other tie sizes and cover, use formula in Commentary on Reinforcement 23.

Note 1: Values of n_{max} and A_{st} in this table apply to both circular and square columns of face dimension h having bars arranged in a circle; tabulated values of A_g and ρ_g apply only to circular columns. For square columns, multiply table value of A_g by $4/\pi$ and multiply table value of ρ_g by $\pi/4$.

Note 2: Lap splices should not be used for bars larger than #11, except 1) at footings, #14 and #18 longitudinal bars, in compression only, may be lap spliced with dowels, and 2) #14 and #18 bars, in compression only, may be lap spliced to #11 and smaller bars.



Tangential lap splices

h, in.	A_g, in.^{2}		Bar size #5	#6	#7	#8	#9	#10	#11
		n_{max}	17	15	14	13	11	10	9
20	314	A_{st}	5.27	6.60	8.40	10.27	11.00	12.70	14.04
		ρ_g	0.0168	0.0210	0.0268	0.0327	0.0350	0.0404	0.0447
		n_{max}	18	16	15	14	12	11	9
21	346	A_{st}	5.58	7.04	9.00	11.06	12.00	13.97	14.04
		ρ_g	0.0161	0.0203	0.0260	0.0320	0.0347	0.0404	0.0406
		n_{max}	19	18	16	15	13	11	10
22	380	A_{st}	5.89	7.92	9.60	11.85	13.00	13.97	15.60
		ρ_g	0.0155	0.0208	0.0253	0.0312	0.0342	0.0368	0.0411
		n_{max}	20	19	17	16	14	12	11
23	415	A_{st}	6.20	8.36	10.20	12.64	14.00	15.24	17.16
		ρ_g	0.0149	0.0201	0.0246	0.0305	0.0337	0.0367	0.0413
		n_{max}	22	20	18	17	15	13	11
24	452	A_{st}	6.82	8.80	10.80	13.43	15.00	16.51	17.16
		ρ_g	0.0151	0.0195	0.0239	0.0297	0.0332	0.0365	0.0380
		n_{max}	23	21	19	17	15	13	12
25	491	A_{st}	7.13	9.24	11.40	13.43	15.00	16.51	18.72
		ρ_g	0.0145	0.0188	0.0232	0.0274	0.0305	0.0336	0.0381
		n_{max}	24	22	20	18	16	14	13
26	531	A_{st}	7.44	9.68	12.00	14.22	16.00	17.78	20.28
		ρ_g	0.0140	0.0182	0.0226	0.0268	0.0301	0.0335	0.0382
		n_{max}	25	23	21	19	17	15	13
27	573	A_{st}	7.75	10.12	12.60	15.01	17.00	19.05	20.28
		ρ_g	0.0135	0.0177	0.0220	0.0262	0.0297	0.0332	0.0354
		n_{max}	26	24	22	20	18	16	14
28	616	A_{st}	8.06	10.56	13.20	15.80	18.00	20.32	21.84
		ρ_g	0.0131	0.0171	0.0214	0.0256	0.0292	0.0330	0.0355
		n_{max}	27	25	23	21	18	16	15
29	661	A_{st}	8.37	11.00	13.80	16.59	18.00	20.32	23.40
		ρ_g	0.0127	0.0166	0.0209	0.0251	0.0272	0.0307	0.0354

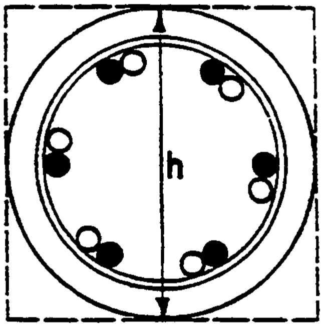
REINFORCEMENT 23.3.3—Maximum number of bars n_{max} that can be accommodated in square columns having bars arranged in a circle using *tangential lap* splices

References: ACI 318-95 Sections 3.3.2, 7.6.3, 7.6.4, 7.7.1(c), 7.10.4.2, 10.9.1, 10.9.2, 12.14.2.1, 12.16.2, and 15.8.2.3

Design conditions: 1.5 in. cover between spirals or ties and outer surface of column; #4 spirals or ties; clear distance between longitudinal bars of 1.5 in. for #5-#8 bars and 1.5 times nominal bar diameter for #9-#18 bars; aggregate not larger than 3/4 minimum clear spacing between bars; minimum number of bars is four for bars within circular ties and six for bars enclosed by spirals; ρ_g no less than 0.01 and no greater than 0.08. For other tie sizes and cover, use formula in Commentary on Reinforcement 23.

Note 1: Values of n_{max} and A_{st} in this table apply to both circular and square columns of face dimension h having bars arranged in a circle; tabulated values of A_g and ρ_g apply only to circular columns. For square columns, multiply table value of A_g by $4/\pi$ and multiply table value of ρ_g by $\pi/4$.

Note 2: Lap splices should not be used for bars larger than #11, except 1) at footings, #14 and #18 longitudinal bars, in compression only, may be lap spliced with dowels, and 2) #14 and #18 bars, in compression only, may be lap spliced to #11 and smaller bars.



Tangential lap splices

h, in.	A_g, in.^{2}		Bar size #5	#6	#7	#8	#9	#10	#11
		n_{max}	28	26	24	22	19	17	15
30	707	A_{st}	8.68	11.44	14.40	17.38	19.00	21.59	23.40
		ρ_g	0.0123	0.0162	0.0204	0.0246	0.0269	0.0305	0.0331
		n_{max}	30	27	25	23	20	18	16
31	755	A_{st}	9.30	11.88	15.00	18.17	20.00	22.86	24.96
		ρ_g	0.0123	0.0157	0.0199	0.0241	0.0265	0.0303	0.0331
		n_{max}	31	28	26	24	21	18	16
32	804	A_{st}	9.61	12.32	15.60	18.96	21.00	22.86	24.96
		ρ_g	0.0120	0.0153	0.0194	0.0236	0.0261	0.0284	0.0310
		n_{max}	32	29	27	25	22	19	17
33	855	A_{st}	9.92	12.76	16.20	19.75	22.00	24.13	26.52
		ρ_g	0.0116	0.0149	0.0189	0.0231	0.0257	0.0282	0.0310
		n_{max}	33	30	28	26	22	20	18
34	908	A_{st}	10.23	13.20	16.80	20.54	22.00	25.40	28.08
		ρ_g	0.0113	0.0145	0.0185	0.0226	0.0242	0.0280	0.0309
		n_{max}	34	31	29	26	23	21	18
35	962	A_{st}	10.52	13.64	17.40	20.54	23.00	26.67	28.08
		ρ_g	0.0110	0.0142	0.0181	0.0214	0.0239	0.0277	0.0292
		n_{max}	35	32	30	27	24	21	19
36	1018	A_{st}	10.85	14.08	18.00	21.33	24.00	26.67	29.64
		ρ_g	0.0107	0.0138	0.0177	0.0210	0.0236	0.0262	0.0291
		n_{max}	36	33	31	28	25	22	20
37	1075	A_{st}	11.16	14.52	18.60	22.12	25.00	27.94	31.20
		ρ_g	0.0114	0.0135	0.0173	0.0206	0.0233	0.0260	0.0290
		n_{max}	38	34	31	29	26	23	20
38	1134	A_{st}	11.78	14.96	18.60	22.91	26.00	29.21	31.20
		ρ_g	0.0104	0.0132	0.0164	0.0202	0.0229	0.0258	0.0275
		n_{max}	39	35	32	30	26	23	21
39	1195	A_{st}	12.09	15.40	19.20	23.70	26.00	29.21	32.76
		ρ_g	0.0101	0.0129	0.0161	0.0198	0.0218	0.0244	0.0274

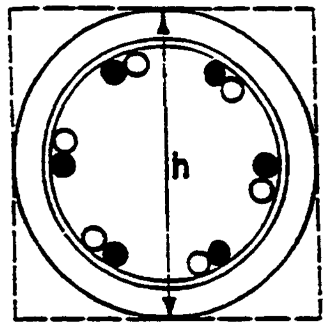
REINFORCEMENT 23.3.4—Maximum number of bars n_{max} that can be accommodated in columns having bars arranged in a circle using *tangential lap* splices

References: ACI 318-95 Sections 3.3.2, 7.6.3, 7.6.4, 7.7.1(c), 7.10.4.2, 10.9.1, 10.9.2, 12.14.2.1, 12.16.2, and 15.8.2.3

Design conditions: 1.5 in. cover between spirals or ties and outer surface of column; #4 spirals or ties; clear distance between longitudinal bars of 1.5 in. for #5-#8 bars and 1.5 times nominal bar diameter for #9-#18 bars; aggregate not larger than 3/4 minimum clear spacing between bars; minimum number of bars is four for bars within circular ties and six for bars enclosed by spirals: ρ_g no less than 0.01 and no greater than 0.08. For other tie sizes and cover, use formula in Commentary on Reinforcement 23.

Note 1: Values of n_{max} and A_g in this table apply to both circular and square columns of face dimension h having bars arranged in a circle; tabulated values of A_g and ρ_g apply only to circular columns. For square columns, multiply table value of A_g by $4/\pi$ and multiply table value of ρ_g by $\pi/4$.

Note 2: Lap splices should not be used for bars larger than #11, except 1) at footings, #14 and #18 longitudinal bars, in compression only, may be lap spliced with dowels, and 2) #14 and #18 bars, in compression only, may be lap spliced to #11 and smaller bars.



Tangential lap slices

h, in.	A_g, in.^{2}		Bar size #5	#6	#7	#8	#9	#10	#11
		n_{max}	*	36	33	31	27	24	21
40	1257	A_{st}		15.84	19.80	24.49	27.00	30.48	32.76
		ρ_g		0.0126	0.0158	0.0195	0.0215	0.0242	0.0261
		n_{max}	*	37	34	32	28	25	22
41	1320	A_{st}		16.28	20.40	25.28	28.00	31.75	34.32
		ρ_g		0.0123	0.0155	0.0192	0.0212	0.0241	0.0260
		n_{max}	*	38	35	33	29	25	23
42	1385	A_{st}		16.72	21.00	26.07	29.00	31.75	35.88
		ρ_g		0.0121	0.0152	0.0188	0.0209	0.0229	0.0259
		n_{max}	*	40	36	34	30	26	23
43	1452	A_{st}		17.60	21.60	26.86	30.00	33.02	35.88
		ρ_g		0.0121	0.0149	0.0185	0.0207	0.0227	0.0247
		n_{max}	*	41	37	34	30	27	24
44	1521	A_{st}		18.04	22.20	26.86	30.00	34.29	37.44
		ρ_g		0.0119	0.0146	0.0177	0.0197	0.0225	0.0246
		n_{max}	*	42	38	35	31	28	25
45	1590	A_{st}		18.48	22.80	27.65	31.00	35.56	39.00
		ρ_g		0.0116	0.0143	0.0174	0.0195	0.0224	0.0245
		n_{max}	*	43	39	36	32	28	25
46	1662	A_{st}		18.92	23.40	28.44	32.00	35.56	39.00
		ρ_g		0.0114	0.0141	0.0171	0.0193	0.0214	0.0235
		n_{max}	*	44	40	37	33	29	26
47	1735	A_{st}		19.36	24.00	29.23	33.00	36.83	40.56
		ρ_g		0.0112	0.0138	0.0168	0.0190	0.0212	0.0234
		n_{max}	*	45	41	38	34	30	27
48	1810	A_{st}		19.80	24.60	30.02	34.00	38.10	42.12
		ρ_g		0.0109	0.0136	0.0166	0.0188	0.0210	0.0233
		n_{max}	*	46	42	39	34	30	27
49	1886	A_{st}		20.24	25.20	30.81	34.00	38.10	42.12
		ρ_g		0.0107	0.0134	0.0163	0.0180	0.0202	0.0223
		n_{max}	*	47	43	40	35	31	28
50	1963	A_{st}		20.68	25.80	31.60	35.00	39.37	43.68
		ρ_g		0.0105	0.0131	0.0161	0.0178	0.0201	0.0223

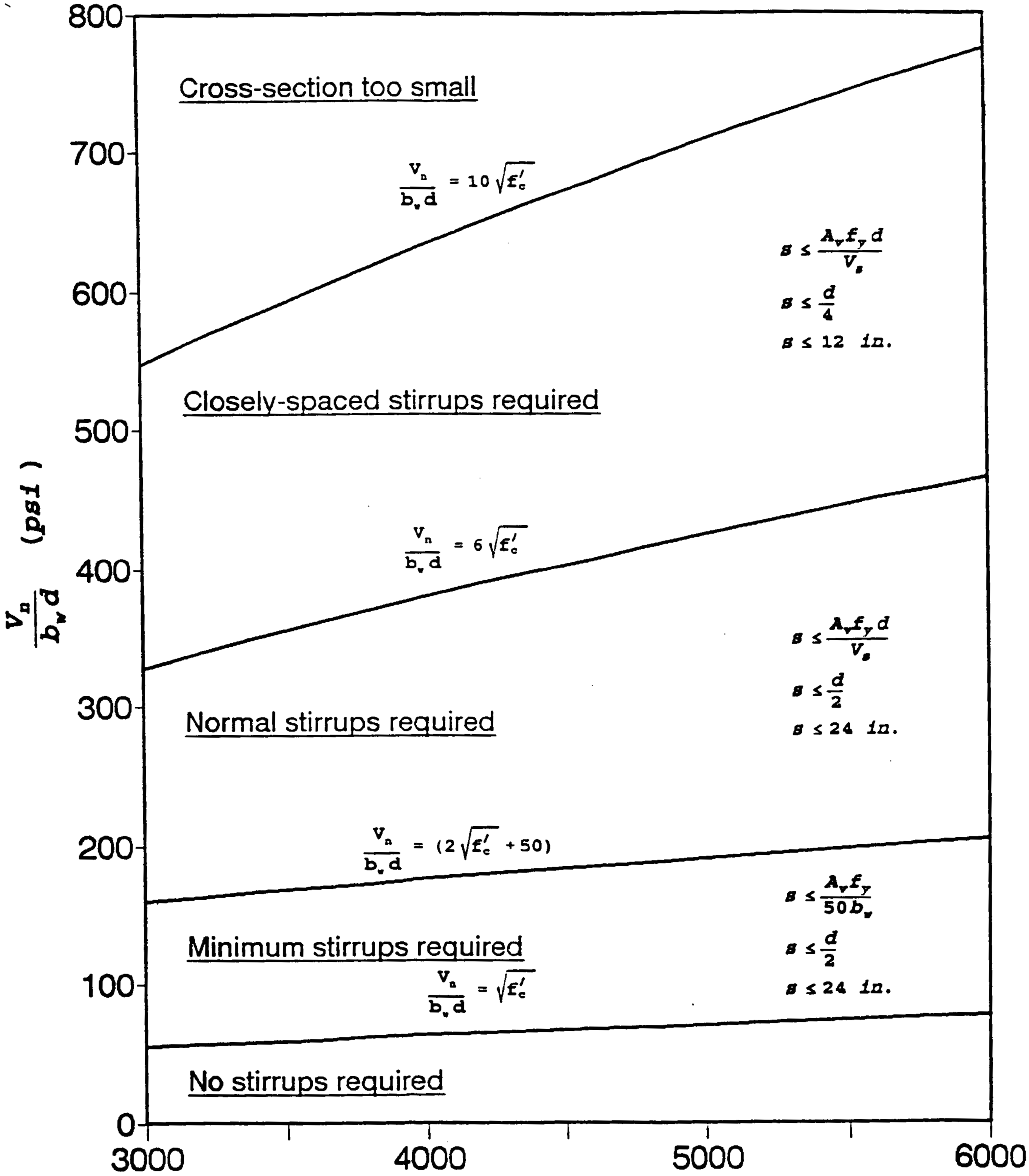
*$\rho_g < 0.01$

SHEAR

SHEAR 1 Stirrup Design Requirements for Nonprestressed Beams with Vertical Stirrups and Normal-Weight Concrete Subjected to Flexure and Shear Only

REFERENCE : ACI 318-95 Sections 11.1.1, 11.3.1.1, 11.5.4, 11.5.5.1, 11.5.5.3, 11.5.6.2, and 11.5.6.8

$$V_n \geq \frac{V_u}{\phi}$$

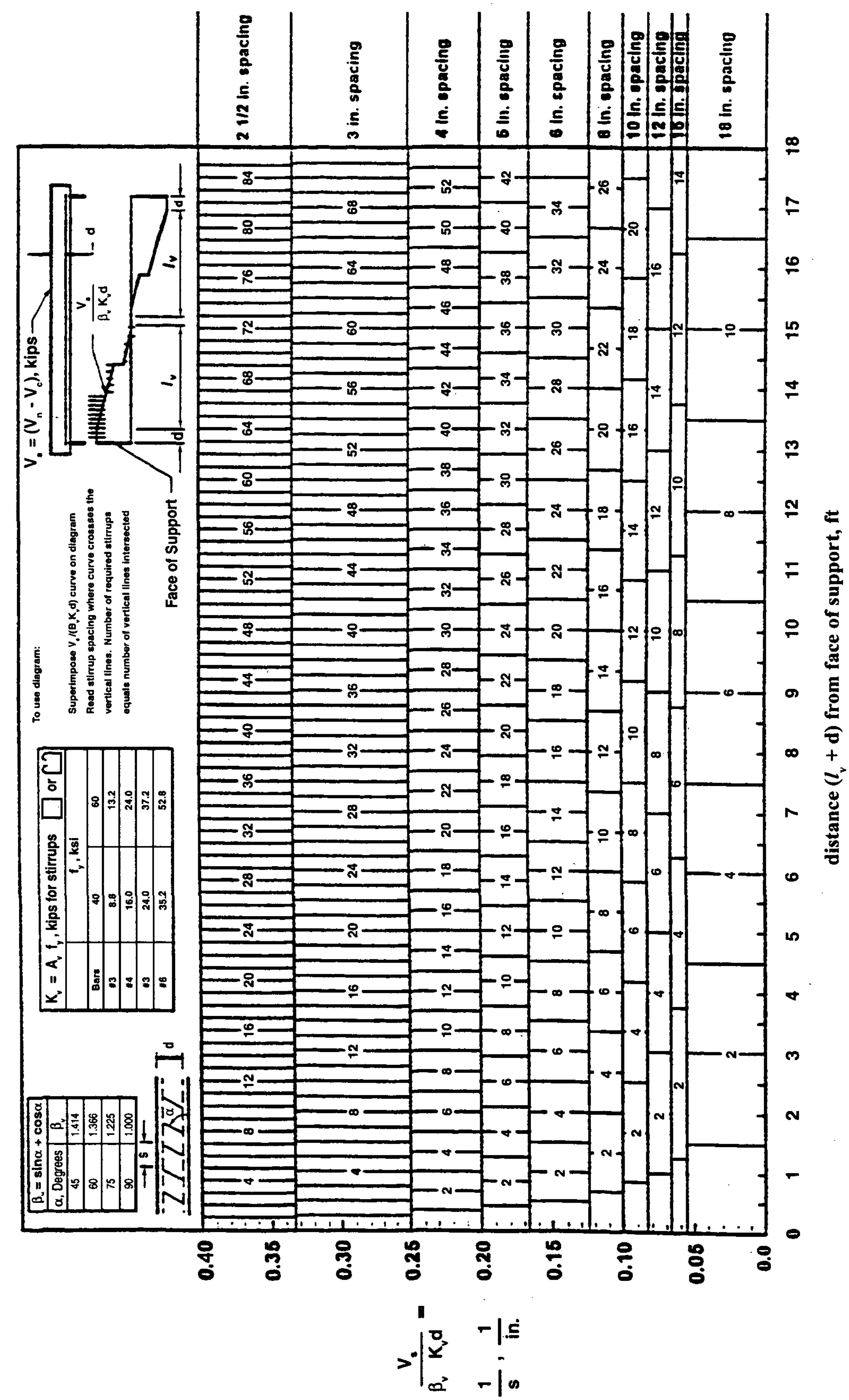


For use of this Design Aid, see Shear Examples 1, 4, 5, and 6.

SHEAR 2 - Diagram for selecting spacing of stirrups

Reference: ACI 318-95 Section 11.5.6.3

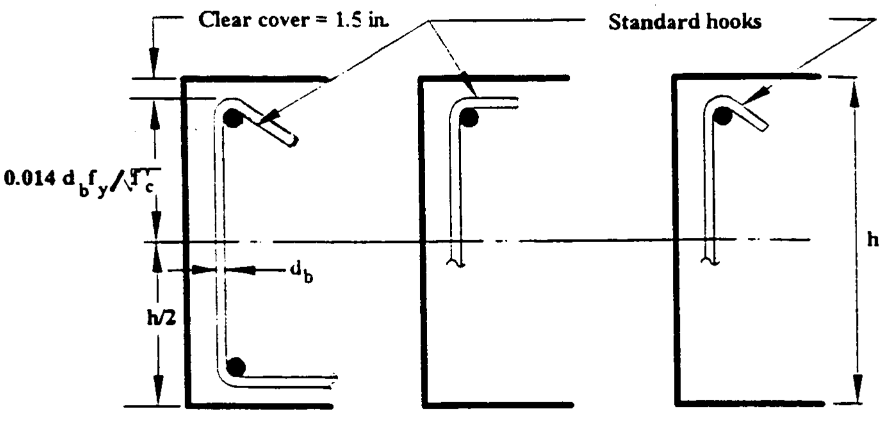
$$V_n \geq \frac{V_u}{\phi}$$



For use of this Design Aid, see Shear Examples 5, 6, and 7.

SHEAR 3 - Minimum beam height to provide embedment required for #6, #7, and #8 vertical stirrups with f_y = 60,000 psi

Reference: ACI 318-95 Section 12.13.2.2



$$h = 2\left(\frac{0.014\, d_b\, f_y}{\sqrt{f'_c}} + 1.5\right) \text{(in.)}$$

f'_c psi	*Minimum beam height h, in. f_y = 60,000 psi Bar size of stirrup #6	#7	#8
3000	26.0	29.8	33.7
4000	22.9	26.2	29.6
5000	20.8	23.8	26.8
6000	19.3	22.0	24.7

* Table values are for beams with 1.5 in. cover. For cover greater than 1.5 in. add 2(cover - 1.5) to table values.

Example: Determine whether a 24-in.-high beam of 4000 psi concrete will provide sufficient embedment for Grade 60 #6 vertical stirrups.

Solution: For f_y = 60,000 psi, f'_c = 4000 psi, and #6 stirrups, read minimum beam height which is 22.9 in. Therefore, the 24-in.-high beam will provide sufficient embedment.

SHEAR 4.1 - Design shear strength V_s of U-stirrups; f_y = 40 ksi

Reference: ACI 318-95 Sections 11.5.5.3 and 11.5.6.2

$V_s = V_n - V_c = A_v f_y (d/s)$

$$V_n \geq \frac{V_u}{\phi}$$

Maximum $b_w = A_v f_y / 50s$

Stirrup size	s, in / d, in	2	3	4	5	6	7	8	9	10	11	12	14	16	18	20
		V_s, kips														
#3 stirrup*	8	35	23	18												
	10	44	29	22	18											
	12	53	35	26	21	18										
	14	62	41	31	25	21	18									
	16	70	47	35	28	23	20	18								
	18	79	53	40	32	26	23	20	18							
	20	88	59	44	35	29	25	22	20	18						
	22	97	65	48	39	32	28	24	22	19	19					
	24	106	70	53	42	35	30	26	23	21	19	18				
	26	114	76	57	46	38	33	29	25	23	21	19				
	28	123	82	62	49	41	35	31	27	25	22	21	18			
	30	132	88	66	53	44	38	33	29	26	24	22	19			
	32	141	94	70	56	47	40	35	31	28	26	23	20	18		
	34	150	100	75	60	50	43	37	33	30	27	25	21	19		
	36	158	106	79	63	53	45	40	35	32	29	26	23	20	18	
	38	167	111	84	67	56	48	42	37	33	30	28	24	21	19	
	40	176	117	88	70	59	50	44	39	35	32	29	25	22	20	18
	max b_w, in	88	59	44	35	29	25	22	20	18	16	15	13	11	10	9
#4 stirrup*	8	64	43	32												
	10	80	53	40	32											
	12	96	64	48	38	32										
	14	112	75	56	45	37	32									
	16	128	85	64	51	43	37	32								
	18	144	96	72	58	48	41	36	32							
	20	160	107	80	64	53	46	40	36	32						
	22	176	117	88	70	59	50	44	39	35	32					
	24	192	128	96	77	64	55	48	43	38	35	32				
	26	208	139	104	83	69	59	52	46	42	38	35				
	28	224	149	112	90	75	64	56	50	45	41	37	32			
	30	240	160	120	96	80	69	60	53	48	44	40	34			
	32	256	171	128	102	85	73	64	57	51	47	43	37	32		
	34	272	181	136	109	91	78	68	60	54	49	45	39	34		
	36	288	192	144	115	96	82	72	64	58	52	48	41	36	32	
	38	304	203	152	122	101	87	76	68	61	55	51	43	38	34	
	40	320	213	160	128	107	91	80	71	64	58	53	46	40	36	32
	max b_w, in	160	107	80	64	53	46	40	36	32	29	27	23	20	18	16

Maximum spacing = d/2 (for values above the stepped line in each table; printed once in each table)

*All stirrups must be anchored in accordance with ACI 318-95 Section 12.13.2

SHEAR 4.2 - Design shear strength V_s of U-stirrups; f_y = 60 ksi

Reference: ACI 318-95 Sections 11.5.5.3 and 11.5.6.2

$V_s = V_n - V_c = A_v f_y (d/s)$

$$V_n \geq \frac{V_u}{\phi}$$

Maximum $b_w = A_v f_y / 50s$

V_s, kips

Stirrup size	d, in \ s, in	2	3	4	5	6	7	8	9	10	11	12	14	16	18	20
#3 stirrup*	8	53	35	26												
	10	66	44	33	26											
	12	79	53	40	32	26										
	14	92	62	46	37	31	26									
	16	106	70	53	42	35	30	26								
	18	119	79	59	48	40	34	30	26							
	20	132	88	66	53	44	38	33	29	26						
	22	145	97	73	58	48	41	36	32	29	26					
	24	158	106	79	63	53	45	40	35	32	29	26				
	26	172	114	86	69	57	49	43	38	34	31	29				
	28	185	123	92	74	62	53	46	41	37	34	31	26			
	30	198	132	99	79	66	57	50	44	40	36	33	28			
	32	211	141	106	84	70	60	53	47	42	38	35	30	26		
	34	224	150	112	90	75	64	56	50	45	41	37	32	28		
	36	238	158	119	95	79	68	59	53	48	43	40	34	30	26	
	38	251	167	125	100	84	72	63	56	50	46	42	36	31	28	
	40	264	176	132	106	88	75	66	59	53	48	44	38	33	29	26
	max b_w, in	132	88	66	53	44	38	33	29	26	24	22	19	17	15	13

Maximum spacing = d/2

Stirrup size	d, in \ s, in	2	3	4	5	6	7	8	9	10	11	12	14	16	18	20
#4 stirrup*	8	96	64	48												
	10	120	80	60	48											
	12	144	96	72	58	48										
	14	168	112	84	67	56	48									
	16	192	128	96	77	64	55	48								
	18	216	144	108	86	72	62	54	48							
	20	240	160	120	96	80	69	60	53	48						
	22	264	176	132	106	88	75	66	59	53	48					
	24	288	192	144	115	96	82	72	64	58	52	48				
	26	312	208	156	125	104	89	78	69	62	57	52				
	28	336	224	168	134	112	96	84	75	67	61	56	48			
	30	360	240	180	144	120	103	90	80	72	65	60	51			
	32	384	256	192	154	128	110	96	85	77	70	64	55	48		
	34	408	272	204	163	136	117	102	91	82	74	68	58	51		
	36	432	288	216	173	144	123	108	96	86	79	72	62	54	48	
	38	456	304	228	182	152	130	114	101	91	83	76	65	57	51	
	40	480	320	240	192	160	137	120	107	96	87	80	69	60	53	48
	max b_w, in	240	160	120	96	80	69	60	53	48	44	40	34	30	27	24

Maximum spacing = d/2

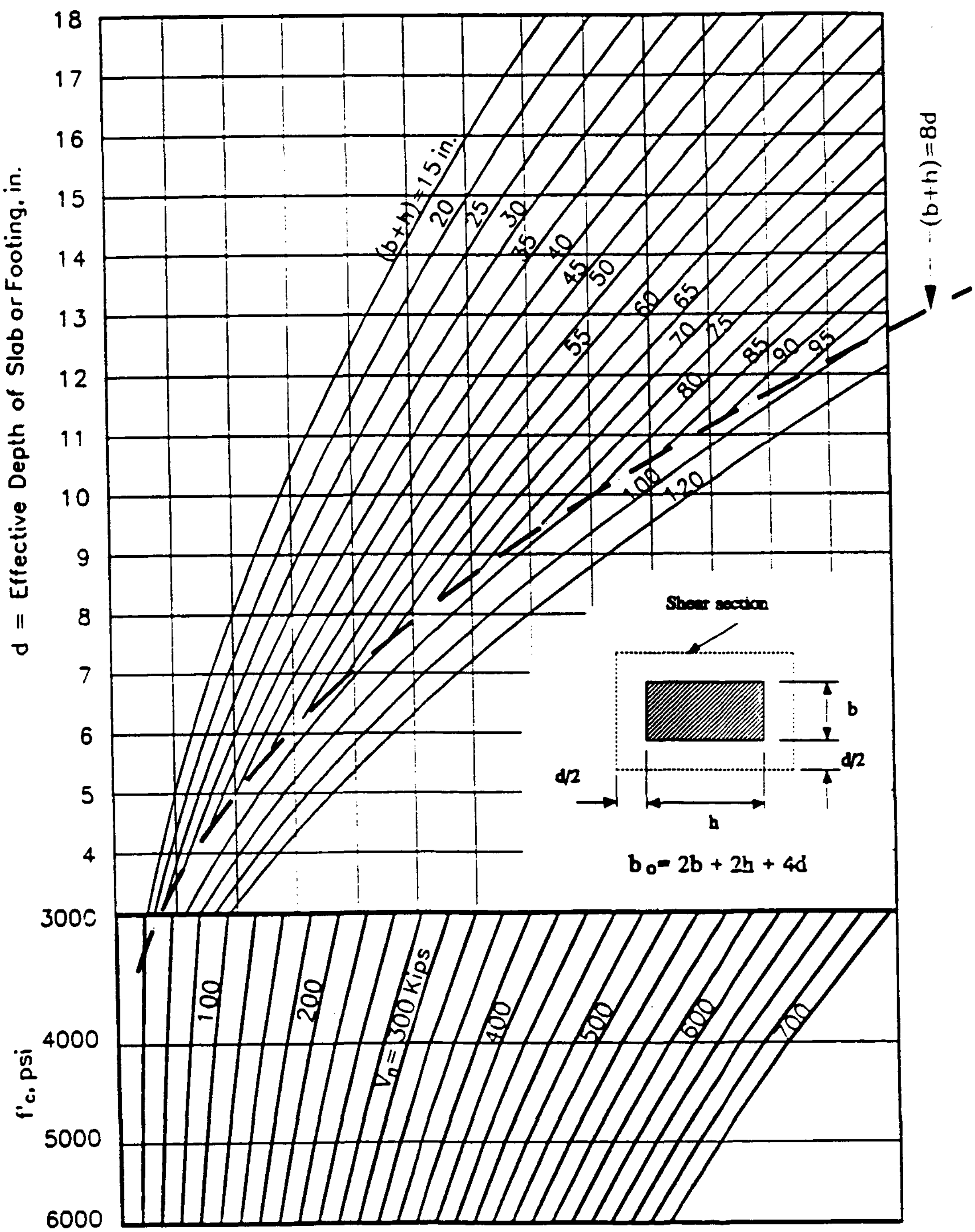
*All stirrups must be anchored in accordance with ACI 318-95 Section 12.13.2

SHEAR 5.1 - Effective depth of footings and slabs required to provide perimeter shear strength at an interior rectangular column (α_s = 40) for which $\beta_c = h/b \le 2$.

Reference: ACI 318-95, Sections 11.12.1.2, 11.12.1.3, and 11.12.2.1

$$V_n = V_c \geq V_u / \phi$$

$$V_c = \left(2 + \frac{4}{\beta_c}\right)\sqrt{f_c'}\, b_o d \quad (11\text{-}36), \quad V_c = \left(\frac{\alpha_s d}{b_o} + 2\right)\sqrt{f_c'}\, b_o d \quad (11\text{-}37), \quad V_c = 4\sqrt{f_c'}\, b_o d \quad (11\text{-}38)$$



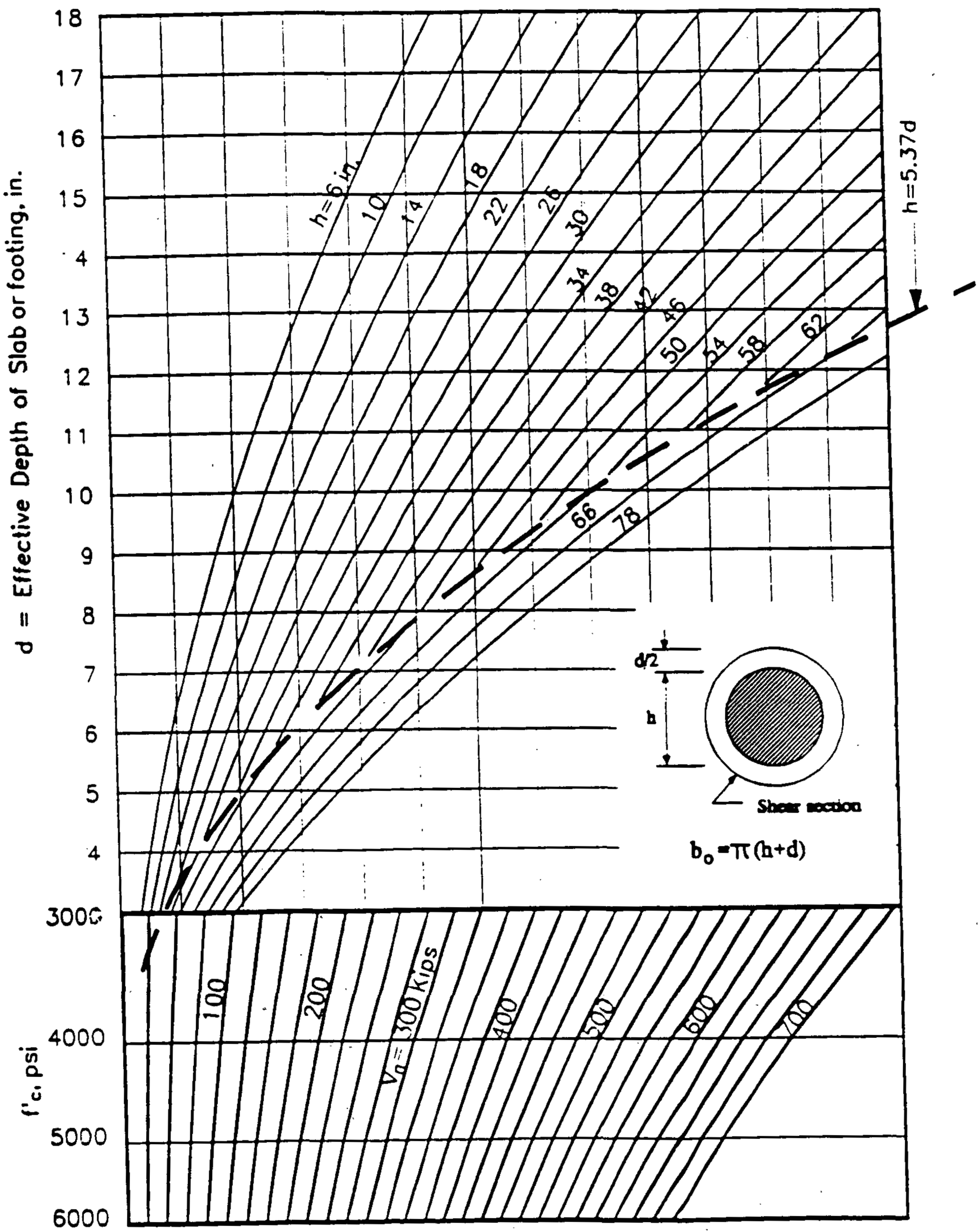
For use of this Design Aid, see Shear Example 8.

SHEAR 5.2 - Effective depth of footings and slabs required to provide perimeter shear strength at an interior circular column ($\alpha_s = 40$).

Reference: ACI 318-95, Sections 11.12.1.2, and 11.12.2.1

$$V_n = V_c \geq V_u / \phi$$

$$V_c = \left(\frac{\alpha_s d}{b_o} + 2\right)\sqrt{f'_c}\, b_o d \quad (11\text{-}37), \quad V_c = 4\sqrt{f'_c}\, b_o d \quad (11\text{-}38)$$

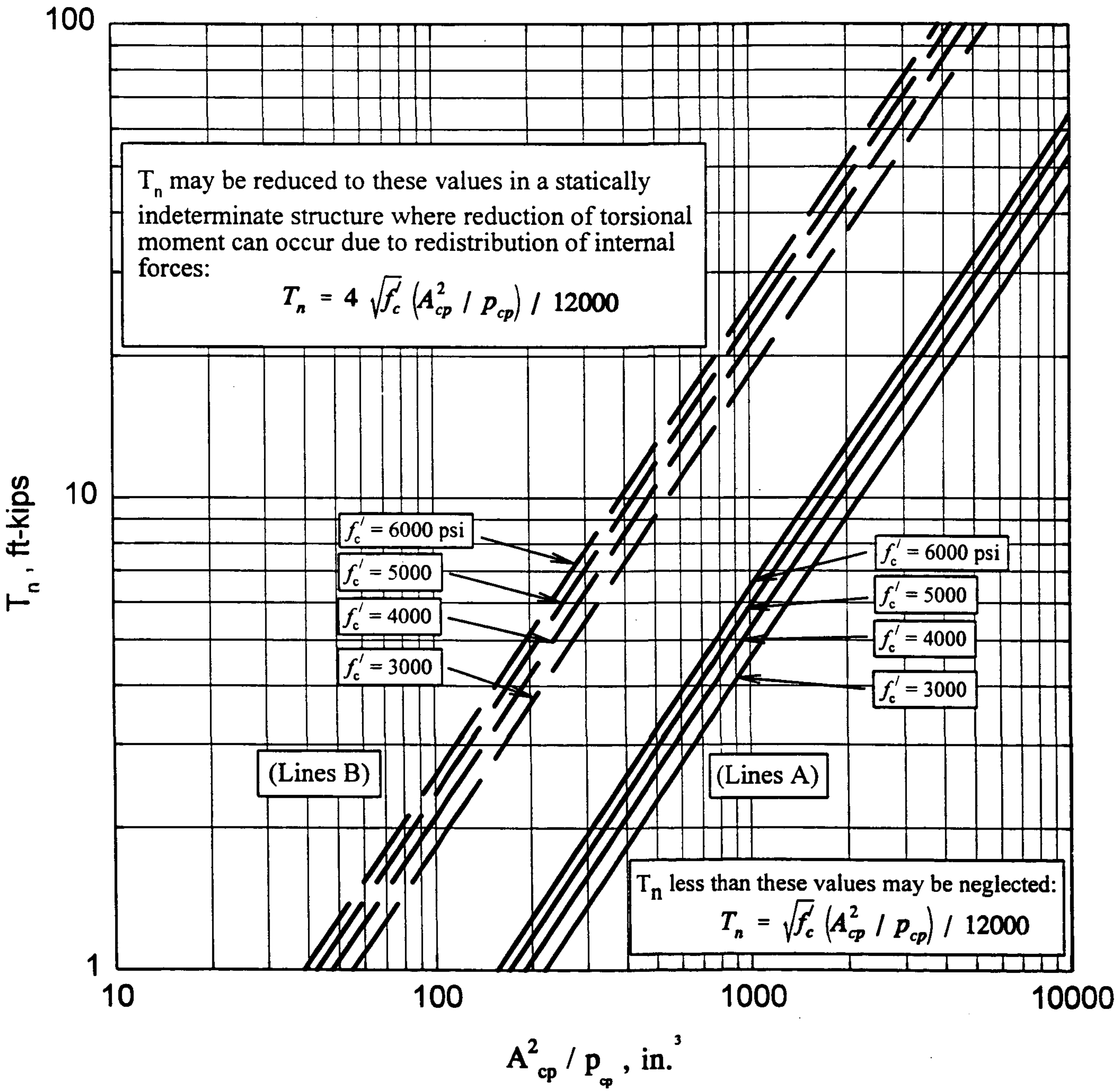


For use of this Design Aid, see Shear Example 9.

SHEAR 6 - Maximum nominal torsional moment T_n that may be neglected (Lines A) (ACI 318-95 Section 11.6.1), and
Maximum nominal torsional moment T_n required for statically indeterminate torsion (Lines B) (ACI 318-95 Section 11.6.2)

Reference: ACI 318-95, Sections 11.6.1 and 11.6.2

$$T_n \geq T_u / \phi$$



For use of this Design Aid, see Shear Examples 10 and 11.

SHEAR 7.1.1 – Values of K_{vs} (k/in.) and K_{vc} (kips), f'_c = 3,000 psi

References: ACI 318-95 Section 11.6

$$K_{vs} = f_{yv} d \ \ (k/in.) \quad \& \quad K_{vc} = \frac{\sqrt{f_c'}\, b_w d}{500} \ \ (kips) \, , \quad d = h - 2.5 \ \ (in.)$$

h (in.)	K_{vs}, for f_{yv}		Beam width, b (in.)										
	40	60	10	12	14	16	18	20	22	24	26	28	30
10	300	450	8.22	9.86	11.50	13.15	14.79	16.43	18.07	19.72	21.36	23.00	24.65
12	380	570	10.41	12.49	14.57	16.65	18.73	20.81	22.89	24.98	27.06	29.14	31.22
14	460	690	12.60	15.12	17.64	20.16	22.68	25.20	27.71	30.23	32.75	35.27	37.79
16	540	810	14.79	17.75	20.70	23.66	26.62	29.58	32.53	35.49	38.45	41.41	44.37
18	620	930	16.98	20.38	23.77	27.17	30.56	33.96	37.35	40.75	44.15	47.54	50.94
20	700	1050	19.17	23.00	26.84	30.67	34.51	38.34	42.17	46.01	49.84	53.68	57.51
22	780	1170	21.36	25.63	29.91	34.18	38.45	42.72	46.99	51.27	55.54	59.81	64.08
24	860	1290	23.55	28.26	32.97	37.68	42.39	47.10	51.81	56.52	61.24	65.95	70.66
26	940	1410	25.74	30.89	36.04	41.19	46.34	51.49	56.63	61.78	66.93	72.08	77.23
28	1020	1530	27.93	33.52	39.11	44.69	50.28	55.87	61.45	67.04	72.63	78.21	83.80
30	1100	1650	30.12	36.15	42.17	48.20	54.22	60.25	66.27	72.30	78.32	84.35	90.37
32	1180	1770	32.32	38.78	45.24	51.71	58.17	64.63	71.09	77.56	84.02	90.48	96.95
34	1260	1890	34.51	41.41	48.31	55.21	62.11	69.01	75.91	82.82	89.72	96.62	103.52
36	1340	2010	36.70	44.04	51.38	58.72	66.06	73.39	80.73	88.07	95.41	102.75	110.09
38	1420	2130	38.89	46.67	54.44	62.22	70.00	77.78	85.55	93.33	101.11	108.89	116.66
40	1500	2250	41.08	49.30	57.51	65.73	73.94	82.16	90.37	98.59	106.81	115.02	123.24
42	1580	2370	43.27	51.92	60.58	69.23	77.89	86.54	95.19	103.85	112.50	121.16	129.81
44	1660	2490	45.46	54.55	63.65	72.74	81.83	90.92	100.01	109.11	118.20	127.29	136.38
46	1740	2610	47.65	57.18	66.71	76.24	85.77	95.30	104.83	114.36	123.89	133.43	142.96
48	1820	2730	49.84	59.81	69.78	79.75	89.72	99.69	109.65	119.62	129.59	139.56	149.53
50	1900	2850	52.03	62.44	72.85	83.25	93.66	104.07	114.47	124.88	135.29	145.69	156.10

SHEAR 7.1.2 – Values of K_{vs} (k/in.) and K_{vc} (kips), f'_c = 4,000 psi

References: ACI 318-95 Section 11.6

$$K_{vs} = f_{yv} d \ \ (k/in.) \quad \& \quad K_{vc} = \frac{\sqrt{f_c'}\, b_w d}{500} \ \ (kips) \, , \quad d = h - 2.5 \ \ (in.)$$

h (in.)	K_{vs}, for f_{yv}		Beam width, b (in.)										
	40	60	10	12	14	16	18	20	22	24	26	28	30
10	300	450	9.49	11.38	13.28	15.18	17.08	18.97	20.87	22.77	24.67	26.56	28.46
12	380	570	12.02	14.42	16.82	19.23	21.63	24.03	26.44	28.84	31.24	33.65	36.05
14	460	690	14.55	17.46	20.37	23.27	26.18	29.09	32.00	34.91	37.82	40.73	43.64
16	540	810	17.08	20.49	23.91	27.32	30.74	34.15	37.57	40.98	44.40	47.81	51.23
18	620	930	19.61	23.53	27.45	31.37	35.29	39.21	43.13	47.05	50.98	54.90	58.82
20	700	1050	22.14	26.56	30.99	35.42	39.84	44.27	48.70	53.13	57.55	61.98	66.41
22	780	1170	24.67	29.60	34.53	39.47	44.40	49.33	54.26	59.20	64.13	69.06	74.00
24	860	1290	27.20	32.63	38.07	43.51	48.95	54.39	59.83	65.27	70.71	76.15	81.59
26	940	1410	29.73	35.67	41.62	47.56	53.51	59.45	65.40	71.34	77.29	83.23	89.18
28	1020	1530	32.26	38.71	45.16	51.61	58.06	64.51	70.96	77.41	83.86	90.31	96.77
30	1100	1650	34.79	41.74	48.70	55.66	62.61	69.57	76.53	83.48	90.44	97.40	104.36
32	1180	1770	37.31	44.78	52.24	59.70	67.17	74.63	82.09	89.56	97.02	104.48	111.94
34	1260	1890	39.84	47.81	55.78	63.75	71.72	79.69	87.66	95.63	103.60	111.57	119.53
36	1340	2010	42.37	50.85	59.32	67.80	76.27	84.75	93.22	101.70	110.17	118.65	127.12
38	1420	2130	44.90	53.89	62.87	71.85	80.83	89.81	98.79	107.77	116.75	125.73	134.71
40	1500	2250	47.43	56.92	66.41	75.89	85.38	94.87	104.36	113.84	123.33	132.82	142.30
42	1580	2370	49.96	59.96	69.95	79.94	89.94	99.93	109.92	119.91	129.91	139.90	149.89
44	1660	2490	52.49	62.99	73.49	83.99	94.49	104.99	115.49	125.99	136.48	146.98	157.48
46	1740	2610	55.02	66.03	77.03	88.04	99.04	110.05	121.05	132.06	143.06	154.07	165.07
48	1820	2730	57.55	69.06	80.57	92.09	103.60	115.11	126.62	138.13	149.64	161.15	172.66
50	1900	2850	60.08	72.10	84.12	96.13	108.15	120.17	132.18	144.20	156.22	168.23	180.25

SHEAR 7.1.3 – Values of K_{vs} (k/in.) and K_{vc} (kips), f'_c = 5,000 psi

References: ACI 318-95 Section 11.6

$$K_{vs} = f_y d \quad (k/in.) \quad \& \quad K_{vc} = \frac{\sqrt{f'_c}\, b_w d}{500} \quad (kips)\,, \quad d = h - 2.5 \quad (in.)$$

	K_{vs}, for f_{yv}		Beam width, b (in.)										
h (in.)	40	60	10	12	14	16	18	20	22	24	26	28	30
10	300	450	10.61	12.73	14.85	16.97	19.09	21.21	23.33	25.46	27.58	29.70	31.82
12	380	570	13.44	16.12	18.81	21.50	24.18	26.87	29.56	32.24	34.93	37.62	40.31
14	460	690	16.26	19.52	22.77	26.02	29.27	32.53	35.78	39.03	42.28	45.54	48.79
16	540	810	19.09	22.91	26.73	30.55	34.37	38.18	42.00	45.82	49.64	53.46	57.28
18	620	930	21.92	26.30	30.69	35.07	39.46	43.84	48.22	52.61	56.99	61.38	65.76
20	700	1050	24.75	29.70	34.65	39.60	44.55	49.50	54.45	59.40	64.35	69.30	74.25
22	780	1170	27.58	33.09	38.61	44.12	49.64	55.15	60.67	66.19	71.70	77.22	82.73
24	860	1290	30.41	36.49	42.57	48.65	54.73	60.81	66.89	72.97	79.05	85.14	91.22
26	940	1410	33.23	39.88	46.53	53.17	59.82	66.47	73.11	79.76	86.41	93.06	99.70
28	1020	1530	36.06	43.27	50.49	57.70	64.91	72.12	79.34	86.55	93.76	100.97	108.19
30	1100	1650	38.89	46.67	54.45	62.23	70.00	77.78	85.56	93.34	101.12	108.89	116.67
32	1180	1770	41.72	50.06	58.41	66.75	75.09	83.44	91.78	100.13	108.47	116.81	125.16
34	1260	1890	44.55	53.46	62.37	71.28	80.19	89.10	98.00	106.91	115.82	124.73	133.64
36	1340	2010	47.38	56.85	66.33	75.80	85.28	94.75	104.23	113.70	123.18	132.65	142.13
38	1420	2130	50.20	60.25	70.29	80.33	90.37	100.41	110.45	120.49	130.53	140.57	150.61
40	1500	2250	53.03	63.64	74.25	84.85	95.46	106.07	116.67	127.28	137.89	148.49	159.10
42	1580	2370	55.86	67.03	78.21	89.38	100.55	111.72	122.90	134.07	145.24	156.41	167.58
44	1660	2490	58.69	70.43	82.17	93.90	105.64	117.38	129.12	140.86	152.59	164.33	176.07
46	1740	2610	61.52	73.82	86.13	98.43	110.73	123.04	135.34	147.64	159.95	172.25	184.55
48	1820	2730	64.35	77.22	90.09	102.95	115.82	128.69	141.56	154.43	167.30	180.17	193.04
50	1900	2850	67.18	80.61	94.05	107.48	120.92	134.35	147.79	161.22	174.66	188.09	201.53

SHEAR 7.1.4 – Values of K_{vs} (k/in.) and K_{vc} (kips), f'_c = 6,000 psi

References: ACI 318-95 Section 11.6

$$K_{vs} = f_y d \quad (k/in.) \quad \& \quad K_{vc} = \frac{\sqrt{f'_c}\, b_w d}{500} \quad (kips)\,, \quad d = h - 2.5 \quad (in.)$$

	K_{vs}, for f_{yv}		Beam width, b (in.)										
h (in.)	40	60	10	12	14	16	18	20	22	24	26	28	30
10	300	450	11.62	13.94	16.27	18.59	20.91	23.24	25.56	27.89	30.21	32.53	34.86
12	380	570	14.72	17.66	20.60	23.55	26.49	29.43	32.38	35.32	38.27	41.21	44.15
14	460	690	17.82	21.38	24.94	28.51	32.07	35.63	39.19	42.76	46.32	49.88	53.45
16	540	810	20.91	25.10	29.28	33.46	37.65	41.83	46.01	50.19	54.38	58.56	62.74
18	620	930	24.01	28.81	33.62	38.42	43.22	48.02	52.83	57.63	62.43	67.23	72.04
20	700	1050	27.11	32.53	37.96	43.38	48.80	54.22	59.64	65.07	70.49	75.91	81.33
22	780	1170	30.21	36.25	42.29	48.33	54.38	60.42	66.46	72.50	78.54	84.59	90.63
24	860	1290	33.31	39.97	46.63	53.29	59.95	66.62	73.28	79.94	86.60	93.26	99.92
26	940	1410	36.41	43.69	50.97	58.25	65.53	72.81	80.09	87.37	94.66	101.94	109.22
28	1020	1530	39.50	47.41	55.31	63.21	71.11	79.01	86.91	94.81	102.71	110.61	118.51
30	1100	1650	42.60	51.12	59.64	68.16	76.69	85.21	93.73	102.25	110.77	119.29	127.81
32	1180	1770	45.70	54.84	63.98	73.12	82.26	91.40	100.54	109.68	118.82	127.96	137.10
34	1260	1890	48.80	58.56	68.32	78.08	87.84	97.60	107.36	117.12	126.88	136.64	146.40
36	1340	2010	51.90	62.28	72.66	83.04	93.42	103.80	114.18	124.56	134.93	145.31	155.69
38	1420	2130	55.00	66.00	76.99	87.99	98.99	109.99	120.99	131.99	142.99	153.99	164.99
40	1500	2250	58.09	69.71	81.33	92.95	104.57	116.19	127.81	139.43	151.05	162.67	174.28
42	1580	2370	61.19	73.43	85.67	97.91	110.15	122.39	134.62	146.86	159.10	171.34	183.58
44	1660	2490	64.29	77.15	90.01	102.87	115.72	128.58	141.44	154.30	167.16	180.02	192.87
46	1740	2610	67.39	80.87	94.35	107.82	121.30	134.78	148.26	161.74	175.21	188.69	202.17
48	1820	2730	70.49	84.59	98.68	112.78	126.88	140.98	155.07	169.17	183.27	197.37	211.46
50	1900	2850	73.59	88.30	103.02	117.74	132.46	147.17	161.89	176.61	191.33	206.04	220.76

SHEAR 7.2.1 – Values of K_{tcr} (ft-k), f'_c = 3,000 psi

References: ACI 318-95 Section 11.6

$$K_{tcr} = \frac{\sqrt{f'_c}\; A_{cp}^2/p_{cp}}{3000} \quad (ft\text{-}k)\,, \quad A_{cp} = bh \quad (in.^2)\,, \quad p_{cp} = 2(b+h) \quad (in.)$$

	Beam width, b (in.)										
h (in.)	10	12	14	16	18	20	22	24	26	28	30
10	4.56	5.98	7.46	8.99	10.56	12.17	13.81	15.47	17.14	18.83	20.54
12	5.98	7.89	9.91	12.02	14.20	16.43	18.71	21.03	23.38	25.76	28.17
14	7.46	9.91	12.52	15.27	18.12	21.05	24.06	27.12	30.24	33.40	36.60
16	8.99	12.02	15.27	18.70	22.27	25.97	29.77	33.65	37.61	41.64	45.72
18	10.56	14.20	18.12	22.27	26.62	31.13	35.79	40.56	45.44	50.41	55.46
20	12.17	16.43	21.05	25.97	31.13	36.51	42.08	47.80	53.66	59.64	65.73
22	13.81	18.71	24.06	29.77	35.79	42.08	48.60	55.32	62.22	69.28	76.47
24	15.47	21.03	27.12	33.65	40.56	47.80	55.32	63.10	71.09	79.28	87.64
26	17.14	23.38	30.24	37.61	45.44	53.66	62.22	71.09	80.22	89.59	99.18
28	18.83	25.76	33.40	41.64	50.41	59.64	69.28	79.28	89.59	100.20	111.06
30	20.54	28.17	36.60	45.72	55.46	65.73	76.47	87.64	99.18	111.06	123.24
32	22.26	30.59	39.83	49.85	60.57	71.91	83.78	96.15	108.95	122.14	135.69
34	23.98	33.03	43.09	54.03	65.75	78.17	91.21	104.80	118.89	133.44	148.40
36	25.72	35.49	46.38	58.24	70.98	84.51	98.73	113.58	128.99	144.93	161.33
38	27.46	37.96	49.69	62.49	76.27	90.91	106.33	122.46	139.23	156.58	174.47
40	29.21	40.45	53.01	66.77	81.59	97.37	114.02	131.45	149.60	168.40	187.79
42	30.97	42.94	56.36	71.08	86.96	103.89	121.78	140.54	160.08	180.35	201.29
44	32.73	45.45	59.72	75.41	92.36	110.46	129.60	149.70	170.67	192.44	214.94
46	34.49	47.96	63.10	79.76	97.79	117.07	137.49	158.95	181.36	204.65	228.75
48	36.26	50.48	66.49	84.13	103.25	123.72	145.43	168.26	192.14	216.97	242.68
50	38.04	53.01	69.89	88.52	108.74	130.41	153.41	177.64	202.99	229.39	256.74

SHEAR 7.2.2 – Values of K_{tcr} (ft-k), f'_c = 4,000 psi

References: ACI 318-95 Section 11.6

$$K_{tcr} = \frac{\sqrt{f'_c}\; A_{cp}^2/p_{cp}}{3000} \quad (ft\text{-}k)\,, \quad A_{cp} = bh \quad (in.^2)\,, \quad p_{cp} = 2(b+h) \quad (in.)$$

	Beam width, b (in.)										
h (in.)	10	12	14	16	18	20	22	24	26	28	30
10	5.27	6.90	8.61	10.38	12.20	14.05	15.94	17.86	19.79	21.75	23.72
12	6.90	9.11	11.44	13.88	16.39	18.97	21.61	24.29	27.00	29.75	32.53
14	8.61	11.44	14.46	17.63	20.92	24.31	27.78	31.32	34.92	38.57	42.26
16	10.38	13.88	17.63	21.59	25.71	29.98	34.37	38.86	43.43	48.08	52.80
18	12.20	16.39	20.92	25.71	30.74	35.95	41.32	46.84	52.47	58.21	64.04
20	14.05	18.97	24.31	29.98	35.95	42.16	48.59	55.20	61.96	68.87	75.89
22	15.94	21.61	27.78	34.37	41.32	48.59	56.12	63.88	71.85	80.00	88.30
24	17.86	24.29	31.32	38.86	46.84	55.20	63.88	72.86	82.09	91.54	101.19
26	19.79	27.00	34.92	43.43	52.47	61.96	71.85	82.09	92.63	103.45	114.52
28	21.75	29.75	38.57	48.08	58.21	68.87	80.00	91.54	103.45	115.70	128.24
30	23.72	32.53	42.26	52.80	64.04	75.89	88.30	101.19	114.52	128.24	142.30
32	25.70	35.33	45.99	57.57	69.94	83.03	96.75	111.02	125.80	141.04	156.69
34	27.69	38.15	49.76	62.39	75.92	90.26	105.32	121.01	137.29	154.09	171.36
36	29.70	40.98	53.55	67.25	81.97	97.58	114.00	131.15	148.95	167.35	186.29
38	31.71	43.84	57.37	72.16	88.06	104.97	122.78	141.41	160.77	180.81	201.46
40	33.73	46.70	61.22	77.10	94.21	112.44	131.66	151.79	172.74	194.45	216.84
42	35.76	49.58	65.08	82.07	100.41	119.96	140.62	162.28	184.85	208.25	232.43
44	37.79	52.48	68.96	87.07	106.64	127.55	149.65	172.86	197.08	222.21	248.20
46	39.83	55.38	72.86	92.10	112.92	135.18	158.76	183.53	209.42	236.31	264.13
48	41.87	58.29	76.78	97.15	119.22	142.86	167.92	194.29	221.86	250.53	280.23
50	43.92	61.21	80.70	102.22	125.56	150.58	177.15	205.12	234.40	264.87	296.46

SHEAR 7.2.3 – Values of K_{tcr} (ft-k), f'_c = 5,000 psi

References: ACI 318-95 Section 11.6

$$K_{tcr} = \frac{\sqrt{f'_c}\ A_{cp}^2/p_{cp}}{3000}\ (ft\text{-}k)\ ,\quad A_{cp} = bh\ (in.^2)\ ,\quad p_{cp} = 2(b+h)\ (in.)$$

h (in.)	Beam width, b (in.)										
	10	12	14	16	18	20	22	24	26	28	30
10	5.89	7.71	9.62	11.60	13.64	15.71	17.82	19.97	22.13	24.31	26.52
12	7.71	10.18	12.79	15.52	18.33	21.21	24.16	27.15	30.19	33.26	36.37
14	9.62	12.79	16.17	19.71	23.39	27.18	31.06	35.01	39.04	43.12	47.25
16	11.60	15.52	19.71	24.14	28.75	33.52	38.43	43.44	48.56	53.76	59.03
18	13.64	18.33	23.39	28.75	34.37	40.19	46.20	52.37	58.66	65.08	71.59
20	15.71	21.21	27.18	33.52	40.19	47.14	54.32	61.71	69.28	77.00	84.85
22	17.82	24.16	31.06	38.43	46.20	54.32	62.74	71.42	80.33	89.44	98.72
24	19.97	27.15	35.01	43.44	52.37	61.71	71.42	81.46	91.78	102.35	113.14
26	22.13	30.19	39.04	48.56	58.66	69.28	80.33	91.78	103.57	115.67	128.04
28	24.31	33.26	43.12	53.76	65.08	77.00	89.44	102.35	115.67	129.35	143.37
30	26.52	36.37	47.25	59.03	71.59	84.85	98.72	113.14	128.04	143.37	159.10
32	28.73	39.50	51.42	64.36	78.20	92.83	108.16	124.13	140.65	157.69	175.18
34	30.96	42.65	55.63	69.75	84.89	100.92	117.75	135.30	153.49	172.27	191.58
36	33.20	45.82	59.87	75.19	91.64	109.10	127.45	146.63	166.53	187.10	208.28
38	35.45	49.01	64.14	80.68	98.46	117.36	137.28	158.10	179.75	202.15	225.23
40	37.71	52.22	68.44	86.20	105.33	125.71	147.20	169.71	193.13	217.40	242.44
42	39.98	55.44	72.76	91.76	112.26	134.12	157.22	181.43	206.67	232.84	259.86
44	42.25	58.67	77.10	97.35	119.23	142.60	167.32	193.26	220.34	248.44	277.49
46	44.53	61.91	81.46	102.97	126.25	151.14	177.49	205.20	234.13	264.20	295.31
48	46.82	65.17	85.84	108.61	133.30	159.72	187.74	217.22	248.05	280.10	313.30
50	49.10	68.43	90.23	114.28	140.38	168.36	198.06	229.33	262.06	296.14	331.46

SHEAR 7.2.4 – Values of K_{tcr} (ft-k), f'_c = 6,000 psi

References: ACI 318-95 Section 11.6

$$K_{tcr} = \frac{\sqrt{f'_c}\ A_{cp}^2/p_{cp}}{3000}\ (ft\text{-}k)\ ,\quad A_{cp} = bh\ (in.^2)\ ,\quad p_{cp} = 2(b+h)\ (in.)$$

h (in.)	Beam width, b (in.)										
	10	12	14	16	18	20	22	24	26	28	30
10	6.45	8.45	10.54	12.71	14.94	17.21	19.53	21.87	24.24	26.64	29.05
12	8.45	11.15	14.01	17.00	20.08	23.24	26.46	29.74	33.07	36.44	39.84
14	10.54	14.01	17.71	21.59	25.62	29.77	34.02	38.35	42.76	47.23	51.76
16	12.71	17.00	21.59	26.44	31.49	36.72	42.09	47.59	53.19	58.89	64.66
18	14.94	20.08	25.62	31.49	37.65	44.03	50.61	57.36	64.26	71.29	78.43
20	17.21	23.24	29.77	36.72	44.03	51.64	59.51	67.60	75.89	84.34	92.95
22	19.53	26.46	34.02	42.09	50.61	59.51	68.73	78.24	88.00	97.98	108.15
24	21.87	29.74	38.35	47.59	57.36	67.60	78.24	89.23	100.54	112.11	123.94
26	24.24	33.07	42.76	53.19	64.26	75.89	88.00	100.54	113.45	126.70	140.26
28	26.64	36.44	47.23	58.89	71.29	84.34	97.98	112.11	126.70	141.70	157.06
30	29.05	39.84	51.76	64.66	78.43	92.95	108.15	123.94	140.26	157.06	174.28
32	31.48	43.26	56.33	70.51	85.66	101.69	118.49	135.97	154.08	172.74	191.90
34	33.92	46.72	60.94	76.41	92.99	110.55	128.99	148.21	168.14	188.72	209.87
36	36.37	50.19	65.59	82.37	100.39	119.51	139.62	160.62	182.43	204.96	228.15
38	38.84	53.69	70.27	88.38	107.86	128.57	150.38	173.19	196.91	221.44	246.73
40	41.31	57.20	74.97	94.43	115.39	137.71	161.25	185.90	211.57	238.15	265.58
42	43.79	60.73	79.71	100.52	122.97	146.92	172.22	198.75	226.39	255.06	284.66
44	46.28	64.27	84.46	106.64	130.61	156.21	183.29	211.71	241.37	272.15	303.98
46	48.78	67.82	89.24	112.79	138.29	165.56	194.44	224.78	256.48	289.42	323.50
48	51.28	71.39	94.03	118.98	146.02	174.97	205.66	237.96	271.72	306.84	343.21
50	53.79	74.96	98.84	125.19	153.78	184.43	216.96	251.22	287.08	324.40	363.09

SHEAR 7.3.1 – Values of K_{ts} (ft-k/in.), f_{yv} = 40,000 psi

References: ACI 318-95 Section 11.6

$$K_{ts} = \frac{A_o f_{yv} \cot\theta}{12} \;(ft\text{-}k/in.)\;, \quad A_o = 0.85(b - 3.5)(h - 3.5) \;(in.^2)\;, \quad \theta = 45°$$

concrete cover to centerline of stirrups = 1.75 *in.*

h (in.)	Beam width, b (in.)										
	10	12	14	16	18	20	22	24	26	28	30
10	119.70	156.53	193.36	230.19	267.03	303.86	340.69	377.52	414.35	451.18	488.01
12	156.53	204.70	252.86	301.02	349.19	397.35	445.51	493.68	541.84	590.01	638.17
14	193.36	252.86	312.36	371.85	431.35	490.85	550.34	609.84	669.33	728.83	788.33
16	230.19	301.02	371.85	442.68	513.51	584.34	655.17	726.00	796.83	867.66	938.48
18	267.03	349.19	431.35	513.51	595.67	677.83	760.00	842.16	924.32	1006.48	1088.64
20	303.86	397.35	490.85	584.34	677.83	771.33	864.82	958.32	1051.81	1145.31	1238.80
22	340.69	445.51	550.34	655.17	760.00	864.82	969.65	1074.48	1179.30	1284.13	1388.96
24	377.52	493.68	609.84	726.00	842.16	958.32	1074.48	1190.64	1306.80	1422.96	1539.12
26	414.35	541.84	669.33	796.83	924.32	1051.81	1179.30	1306.80	1434.29	1561.78	1689.27
28	451.18	590.01	728.83	867.66	1006.48	1145.31	1284.13	1422.96	1561.78	1700.61	1839.43
30	488.01	638.17	788.33	938.48	1088.64	1238.80	1388.96	1539.12	1689.27	1839.43	1989.59
32	524.84	686.33	847.82	1009.31	1170.80	1332.29	1493.78	1655.27	1816.77	1978.26	2139.75
34	561.67	734.50	907.32	1080.14	1252.97	1425.79	1598.61	1771.43	1944.26	2117.08	2289.90
36	598.51	782.66	966.82	1150.97	1335.13	1519.28	1703.44	1887.59	2071.75	2255.91	2440.06
38	635.34	830.82	1026.31	1221.80	1417.29	1612.78	1808.27	2003.75	2199.24	2394.73	2590.22
40	672.17	878.99	1085.81	1292.63	1499.45	1706.27	1913.09	2119.91	2326.73	2533.56	2740.38
42	709.00	927.15	1145.31	1363.46	1581.61	1799.77	2017.92	2236.07	2454.23	2672.38	2890.53
44	745.83	975.32	1204.80	1434.29	1663.77	1893.26	2122.75	2352.23	2581.72	2811.21	3040.69
46	782.66	1023.48	1264.30	1505.12	1745.94	1986.75	2227.57	2468.39	2709.21	2950.03	3190.85
48	819.49	1071.64	1323.80	1575.95	1828.10	2080.25	2332.40	2584.55	2836.70	3088.86	3341.01
50	856.32	1119.81	1383.29	1646.78	1910.26	2173.74	2437.23	2700.71	2964.20	3227.68	3491.16

SHEAR 7.3.2 – Values of K_{ts} (ft-k/in.), f_{yv} = 60,000 psi

References: ACI 318-95 Section 11.6

$$K_{ts} = \frac{A_o f_{yv} \cot\theta}{12} \;(ft\text{-}k/in.)\;, \quad A_o = 0.85(b - 3.5)(h - 3.5) \;(in.^2)\;, \quad \theta = 45°$$

concrete cover to centerline of stirrups = 1.75 *in.*

h (in.)	Beam width, b (in.)										
	10	12	14	16	18	20	22	24	26	28	30
10	179.55	234.80	290.04	345.29	400.54	455.78	511.03	566.28	621.52	676.77	732.02
12	234.80	307.04	379.29	451.54	523.78	596.03	668.27	740.52	812.76	885.01	957.25
14	290.04	379.29	468.53	557.78	647.02	736.27	825.51	914.76	1004.00	1093.25	1182.49
16	345.29	451.54	557.78	664.02	770.27	876.51	982.75	1089.00	1195.24	1301.48	1407.73
18	400.54	523.78	647.02	770.27	893.51	1016.75	1139.99	1263.24	1386.48	1509.72	1632.96
20	455.78	596.03	736.27	876.51	1016.75	1156.99	1297.23	1437.48	1577.72	1717.96	1858.20
22	511.03	668.27	825.51	982.75	1139.99	1297.23	1454.47	1611.72	1768.96	1926.20	2083.44
24	566.28	740.52	914.76	1089.00	1263.24	1437.48	1611.72	1785.95	1960.19	2134.43	2308.67
26	621.52	812.76	1004.00	1195.24	1386.48	1577.72	1768.96	1960.19	2151.43	2342.67	2533.91
28	676.77	885.01	1093.25	1301.48	1509.72	1717.96	1926.20	2134.43	2342.67	2550.91	2759.15
30	732.02	957.25	1182.49	1407.73	1632.96	1858.20	2083.44	2308.67	2533.91	2759.15	2984.38
32	787.26	1029.50	1271.74	1513.97	1756.21	1998.44	2240.68	2482.91	2725.15	2967.38	3209.62
34	842.51	1101.75	1360.98	1620.21	1879.45	2138.68	2397.92	2657.15	2916.39	3175.62	3434.85
36	897.76	1173.99	1450.22	1726.46	2002.69	2278.92	2555.16	2831.39	3107.62	3383.86	3660.09
38	953.00	1246.24	1539.47	1832.70	2125.93	2419.17	2712.40	3005.63	3298.86	3592.10	3885.33
40	1008.25	1318.48	1628.71	1938.95	2249.18	2559.41	2869.64	3179.87	3490.10	3800.33	4110.56
42	1063.50	1390.73	1717.96	2045.19	2372.42	2699.65	3026.88	3354.11	3681.34	4008.57	4335.80
44	1118.74	1462.97	1807.20	2151.43	2495.66	2839.89	3184.12	3528.35	3872.58	4216.81	4561.04
46	1173.99	1535.22	1896.45	2257.68	2618.90	2980.13	3341.36	3702.59	4063.82	4425.05	4786.27
48	1229.24	1607.47	1985.69	2363.92	2742.15	3120.37	3498.60	3876.83	4255.06	4633.28	5011.51
50	1284.48	1679.71	2074.94	2470.16	2865.39	3260.62	3655.84	4051.07	4446.29	4841.52	5236.75

SHEAR 7.4.1 – Values of K_t (ft-k), f'_c = 3,000 psi

References: ACI 318-95 Section 11.6

$$K_t = \frac{17\, A_{oh}^2 \sqrt{f'_c}}{12000\, p_h} \quad (ft\text{-}k)\ , \quad A_{oh} = (b - 3.5)(h - 3.5) \quad (in.^2)\ , \quad p_h = 2(b + h - 7) \quad (in.)$$

concrete cover to centerline of stirrups = 1.75 *in.*

h (in.)	Beam width, b (in.)										
	10	12	14	16	18	20	22	24	26	28	30
10	5.33	7.90	10.63	13.48	16.41	19.40	22.44	25.51	28.61	31.74	34.88
12	7.90	11.91	16.27	20.86	25.62	30.53	35.53	40.62	45.78	50.99	56.24
14	10.63	16.27	22.46	29.06	35.97	43.13	50.48	57.99	65.62	73.36	81.18
16	13.48	20.86	29.06	37.89	47.21	56.91	66.93	77.20	87.68	98.34	109.16
18	16.41	25.62	35.97	47.21	59.14	71.64	84.60	97.94	111.61	125.55	139.71
20	19.40	30.53	43.13	56.91	71.64	87.14	103.29	119.97	137.11	154.64	172.50
22	22.44	35.53	50.48	66.93	84.60	103.29	122.82	143.08	163.95	185.36	207.21
24	25.51	40.62	57.99	77.20	97.94	119.97	143.08	167.12	191.96	217.48	243.61
26	28.61	45.78	65.62	87.68	111.61	137.11	163.95	191.96	220.96	250.84	281.49
28	31.74	50.99	73.36	98.34	125.55	154.64	185.36	217.48	250.84	285.28	320.67
30	34.88	56.24	81.18	109.16	139.71	172.50	207.21	243.61	281.49	320.67	361.00
32	38.04	61.54	89.08	120.09	154.08	190.65	229.47	270.27	312.81	356.90	402.36
34	41.21	66.86	97.05	131.14	168.62	209.06	252.08	297.40	344.74	393.88	444.65
36	44.39	72.21	105.07	142.29	183.32	227.69	275.00	324.94	377.20	431.54	487.76
38	47.59	77.59	113.14	153.52	198.14	246.51	298.20	352.84	410.13	469.80	531.62
40	50.79	82.99	121.25	164.82	213.08	265.51	321.64	381.08	443.50	508.61	576.15
42	53.99	88.40	129.39	176.19	228.13	284.66	345.29	409.61	477.26	547.91	621.30
44	57.21	93.83	137.57	187.61	243.27	303.95	369.15	438.42	511.37	587.66	667.00
46	60.42	99.28	145.77	199.08	258.49	323.36	393.18	467.46	545.79	627.82	713.21
48	63.65	104.73	154.00	210.60	273.78	342.89	417.37	496.72	580.51	668.35	759.89
50	66.87	110.20	162.26	222.16	289.14	362.52	441.71	526.18	615.49	709.22	807.00

SHEAR 7.4.2 – Values of K_t (ft-k), f'_c = 4,000 psi

References: ACI 318-95 Section 11.6

$$K_t = \frac{17\, A_{oh}^2 \sqrt{f'_c}}{12000\, p_h} \quad (ft\text{-}k)\ , \quad A_{oh} = (b - 3.5)(h - 3.5) \quad (in.^2)\ , \quad p_h = 2(b + h - 7) \quad (in.)$$

concrete cover to centerline of stirrups = 1.75 *in.*

h (in.)	Beam width, b (in.)										
	10	12	14	16	18	20	22	24	26	28	30
10	6.15	9.12	12.28	15.57	18.95	22.40	25.91	29.46	33.04	36.65	40.28
12	9.12	13.76	18.78	24.08	29.59	35.25	41.03	46.90	52.86	58.87	64.94
14	12.28	18.78	25.93	33.55	41.54	49.80	58.29	66.96	75.77	84.71	93.74
16	15.57	24.08	33.55	43.75	54.51	65.71	77.28	89.14	101.25	113.56	126.04
18	18.95	29.59	41.54	54.51	68.29	82.72	97.69	113.09	128.87	144.97	161.33
20	22.40	35.25	49.80	65.71	82.72	100.62	119.26	138.53	158.32	178.56	199.19
22	25.91	41.03	58.29	77.28	97.69	119.26	141.83	165.22	189.32	214.03	239.27
24	29.46	46.90	66.96	89.14	113.09	138.53	165.22	192.97	221.65	251.13	281.30
26	33.04	52.86	75.77	101.25	128.87	158.32	189.32	221.65	255.14	289.65	325.03
28	36.65	58.87	84.71	113.56	144.97	178.56	214.03	251.13	289.65	329.41	370.27
30	40.28	64.94	93.74	126.04	161.33	199.19	239.27	281.30	325.03	370.27	416.85
32	43.93	71.05	102.87	138.67	177.92	220.15	264.97	312.08	361.20	412.11	464.61
34	47.59	77.20	112.06	151.43	194.71	241.40	291.08	343.40	398.07	454.82	513.43
36	51.26	83.39	121.32	164.30	211.68	262.91	317.55	375.20	435.55	498.30	563.21
38	54.95	89.59	130.64	177.27	228.79	284.65	344.33	407.43	473.58	542.48	613.86
40	58.64	95.82	140.00	190.32	246.05	306.58	371.39	440.03	512.11	587.29	665.28
42	62.35	102.08	149.41	203.44	263.42	328.70	398.71	472.98	551.09	632.68	717.41
44	66.06	108.35	158.85	216.63	280.90	350.97	426.25	506.24	590.48	678.57	770.18
46	69.77	114.63	168.32	229.88	298.47	373.39	454.00	539.77	630.23	724.94	823.55
48	73.49	120.93	177.83	243.18	316.13	395.94	481.94	573.56	670.31	771.74	877.45
50	77.22	127.25	187.36	256.53	333.87	418.60	510.04	607.58	710.71	818.93	931.84

SHEAR 7.4.3 – Values of K_t (ft-k), f'_c = 5,000 psi

References: ACI 318-95 Section 11.6

$$K_t = \frac{17\, A_{oh}^2 \sqrt{f'_c}}{12000\, p_h} \;(ft\text{-}k)\,, \quad A_{oh} = (b - 3.5)(h - 3.5)\;(in.^2)\,, \quad p_h = 2(b + h - 7)\;(in.)$$

concrete cover to centerline of stirrups = 1.75 *in.*

h (in.)	Beam width, b (in.)										
	10	12	14	16	18	20	22	24	26	28	30
10	6.88	10.19	13.72	17.40	21.19	25.05	28.97	32.94	36.94	40.98	45.03
12	10.19	15.38	21.00	26.93	33.08	39.41	45.87	52.44	59.10	65.82	72.61
14	13.72	21.00	28.99	37.51	46.44	55.68	65.17	74.86	84.71	94.70	104.81
16	17.40	26.93	37.51	48.91	60.94	73.47	86.40	99.66	113.20	126.96	140.92
18	21.19	33.08	46.44	60.94	76.35	92.48	109.22	126.44	144.09	162.08	180.37
20	25.05	39.41	55.68	73.47	92.48	112.50	133.34	154.88	177.01	199.64	222.70
22	28.97	45.87	65.17	86.40	109.22	133.34	158.57	184.72	211.66	239.29	267.51
24	32.94	52.44	74.86	99.66	126.44	154.88	184.72	215.75	247.81	280.77	314.50
26	36.94	59.10	84.71	113.20	144.09	177.01	211.66	247.81	285.26	323.83	363.40
28	40.98	65.82	94.70	126.96	162.08	199.64	239.29	280.77	323.83	368.29	413.98
30	45.03	72.61	104.81	140.92	180.37	222.70	267.51	314.50	363.40	413.98	466.05
32	49.11	79.44	115.01	155.04	198.92	246.13	296.25	348.92	403.84	460.75	519.45
34	53.20	86.32	125.29	169.31	217.69	269.89	325.44	383.94	445.05	508.50	574.04
36	57.31	93.23	135.64	183.69	236.66	293.94	355.03	419.49	486.96	557.12	629.69
38	61.43	100.17	146.06	198.19	255.80	318.24	384.97	455.52	529.48	606.51	686.31
40	65.56	107.14	156.53	212.78	275.09	342.77	415.23	491.97	572.56	656.61	743.81
42	69.70	114.13	167.04	227.45	294.51	367.49	445.77	528.81	616.14	707.35	802.09
44	73.85	121.14	177.60	242.20	314.06	392.40	476.57	565.99	660.17	758.67	861.09
46	78.01	128.16	188.19	257.01	333.70	417.46	507.59	603.49	704.62	810.51	920.75
48	82.17	135.21	198.82	271.89	353.45	442.67	538.82	641.26	749.43	862.83	981.02
50	86.33	142.27	209.48	286.81	373.28	468.01	570.24	679.30	794.59	915.59	1041.83

SHEAR 7.4.4 – Values of K_t (ft-k), f'_c = 6,000 psi

References: ACI 318-95 Section 11.6

$$K_t = \frac{17\, A_{oh}^2 \sqrt{f'_c}}{12000\, p_h} \;(ft\text{-}k)\,, \quad A_{oh} = (b - 3.5)(h - 3.5)\;(in.^2)\,, \quad p_h = 2(b + h - 7)\;(in.)$$

concrete cover to centerline of stirrups = 1.75 *in.*

h (in.)	Beam width, b (in.)										
	10	12	14	16	18	20	22	24	26	28	30
10	7.53	11.17	15.03	19.06	23.21	27.44	31.74	36.08	40.47	44.89	49.33
12	11.17	16.85	23.00	29.50	36.24	43.17	50.25	57.45	64.74	72.11	79.54
14	15.03	23.00	31.76	41.09	50.87	61.00	71.39	82.00	92.80	103.74	114.81
16	19.06	29.50	41.09	53.58	66.76	80.48	94.65	109.18	124.00	139.08	154.37
18	23.21	36.24	50.87	66.76	83.63	101.31	119.64	138.51	157.84	177.55	197.59
20	27.44	43.17	61.00	80.48	101.31	123.24	146.07	169.66	193.90	218.69	243.95
22	31.74	50.25	71.39	94.65	119.64	146.07	173.70	202.35	231.87	262.13	293.05
24	36.08	57.45	82.00	109.18	138.51	169.66	202.35	236.34	271.47	307.57	344.52
26	40.47	64.74	92.80	124.00	157.84	193.90	231.87	271.47	312.49	354.74	398.08
28	44.89	72.11	103.74	139.08	177.55	218.69	262.13	307.57	354.74	403.44	453.49
30	49.33	79.54	114.81	154.37	197.59	243.95	293.05	344.52	398.08	453.49	510.53
32	53.80	87.02	125.98	169.84	217.91	269.62	324.53	382.22	442.38	504.73	569.03
34	58.28	94.56	137.25	185.47	238.47	295.65	356.50	420.58	487.53	557.03	628.83
36	62.78	102.13	148.59	201.23	259.25	322.00	388.91	459.53	533.44	610.29	689.79
38	67.30	109.73	160.00	217.11	280.22	348.62	421.72	499.00	580.02	664.40	751.82
40	71.82	117.36	171.47	233.09	301.35	375.48	454.86	538.93	627.21	719.29	814.80
42	76.36	125.02	182.99	249.16	322.62	402.57	488.32	579.28	674.95	774.87	878.64
44	80.90	132.70	194.55	265.32	344.03	429.85	522.05	620.01	723.18	831.08	943.28
46	85.45	140.40	206.16	281.55	365.55	457.31	556.04	661.09	771.87	887.87	1008.63
48	90.01	148.11	217.80	297.84	387.18	484.92	590.25	702.47	820.96	945.18	1074.65
50	94.57	155.85	229.47	314.19	408.91	512.68	624.67	744.14	870.43	1002.98	1141.27

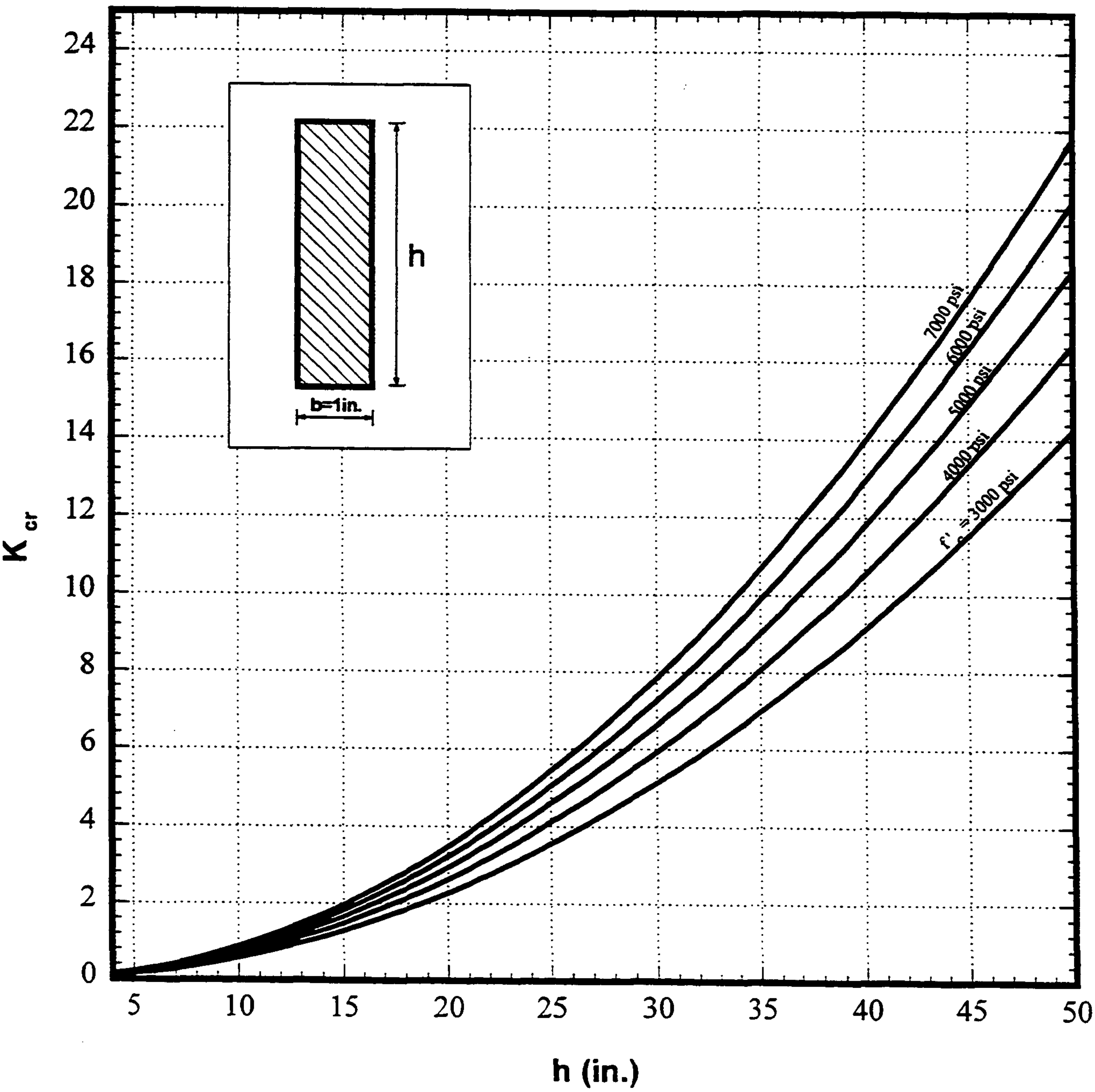
DEFLECTION

DEFLECTION 1.1-Cracking moment M_{cr} for rectangular sections

Reference: ACI 318-95 Section 9.5.2.3

$$K_{cr} = (f_r/12{,}000)(h^2/6)$$
$$= [(7.5\sqrt{f_c'})(h^2)]/72{,}000$$
$$= (h^2\sqrt{f_c'})/9{,}600 \text{ , kip-ft per in. of width}$$

$M_{cr} = bK_{cr}$, kip-ft for normal weight concrete
$M_{cr} = bK_{cr}$ (0.85), kip-ft for "sand-lightweight" concrete
$M_{cr} = bK_{cr}$ (0.75), kip-ft for "all-lightweight" concrete
For flanged section, $M_{cr} = b_w K_{cr} K_{crt}$; obtain K_{crt} from DEFLECTION 1.2 or 1.3



For use of this Design Aid, see Deflection Examples 1,5, and 8.

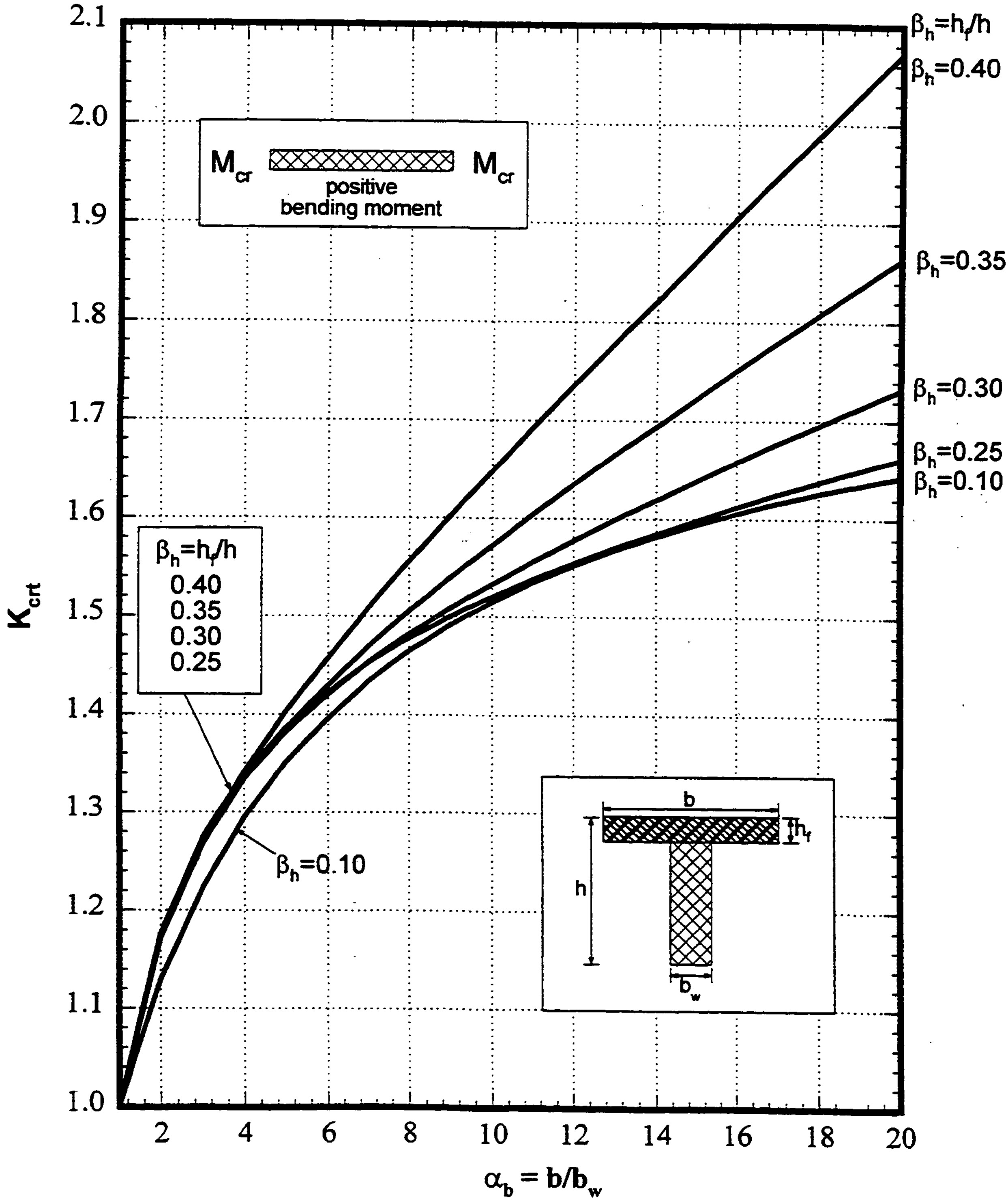
DEFLECTION 1.2-Cracking moment M_{cr} for T or L sections with tension at the bottom (positive moment)

Reference: ACI 318-95 Section 9.5.2.3

$$M_{cr} = \frac{f_r b_w h^2}{72{,}000} K_{crt} = b_w K_{cr} K_{crt}$$

$$K_{crt} = \frac{1+(\alpha_b-1)\beta_h[4-6\beta_h+4\beta_h^2+(\alpha_b-1)\beta_h^3]}{1+(\alpha_b-1)\beta_h(2-\beta_h)}$$

Obtain K_{cr} from DEFLECTION 1.1



For use of this Design Aid, see Deflection Example 8.

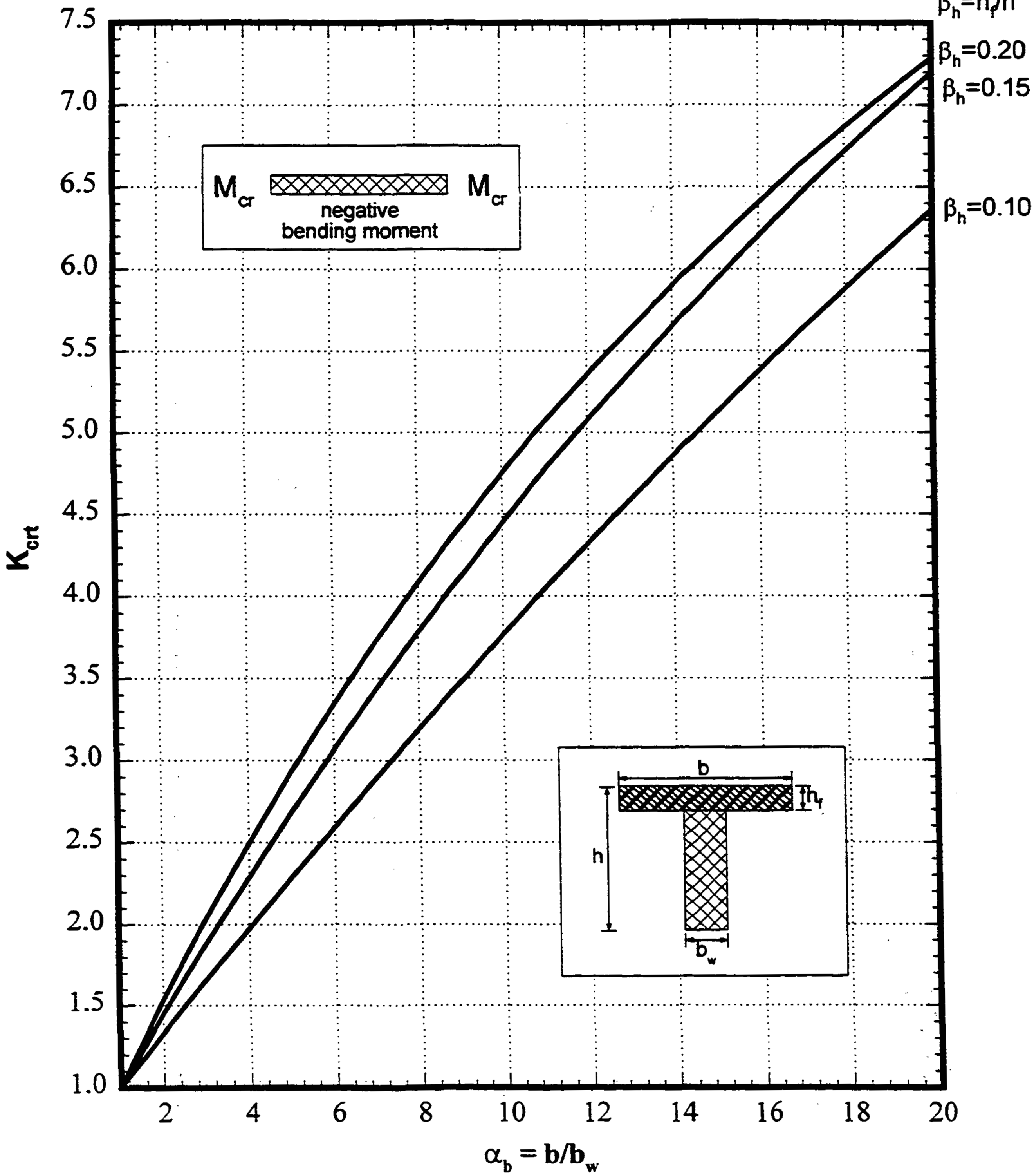
DEFLECTION 1.3.1-Cracking moment M_{cr} for T or L sections with tension at the top (negative moment); β_h = 0.1, 0.15 and 0.2

Reference: ACI 318-95 Section 9.5.2.3

$$M_{cr} = \frac{f_r b_w h^2}{72{,}000} K_{crt} = b_w K_{cr} K_{crt}$$

$$K_{crt} = \frac{1+(\alpha_b-1)\beta_h[4-6\beta_h+4\beta_h^2+(\alpha_b-1)\beta_h^3]}{1+(\alpha_b-1)\beta_h(2-\beta_h)}$$

Obtain K_{cr} from DEFLECTION 1.1



For use of this Design Aid, see Deflection Example 8.

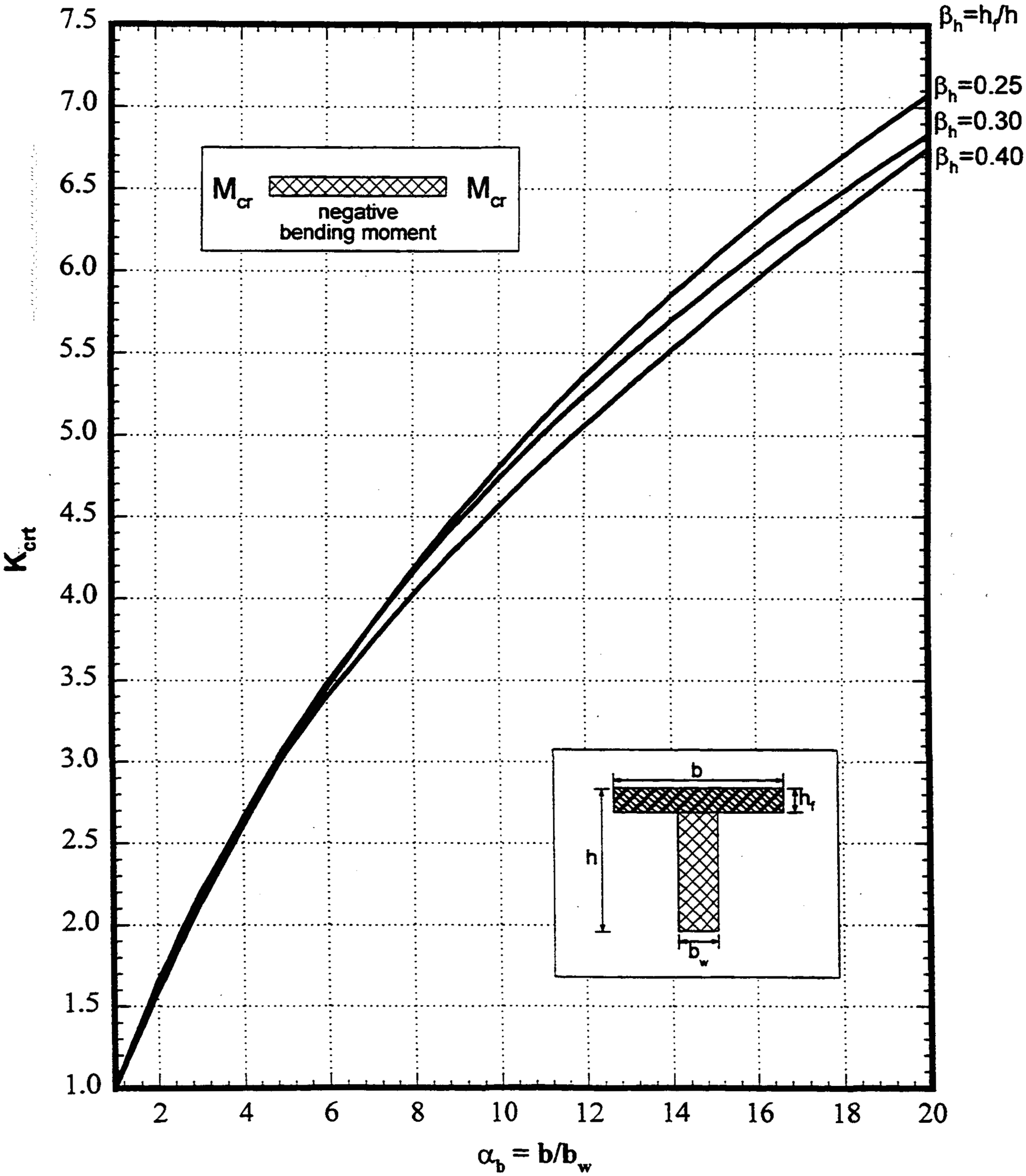
DEFLECTION 1.3.2-Cracking moment M_{cr} for T or L sections with tension at the top (negative moment); β_h = 0.25, 0.30 and 0.40

Reference: ACI 318-95 Section 9.5.2.3

$$M_{cr} = \frac{f_r b_w h^2}{72{,}000} K_{crt} = b_w K_{cr} K_{crt}$$

$$K_{crt} = \frac{1+(\alpha_b-1)\beta_h[4-6\beta_h+4\beta_h^2+(\alpha_b-1)\beta_h^3]}{1+(\alpha_b-1)\beta_h(2-\beta_h)}$$

Obtain K_{cr} from DEFLECTION 1.1



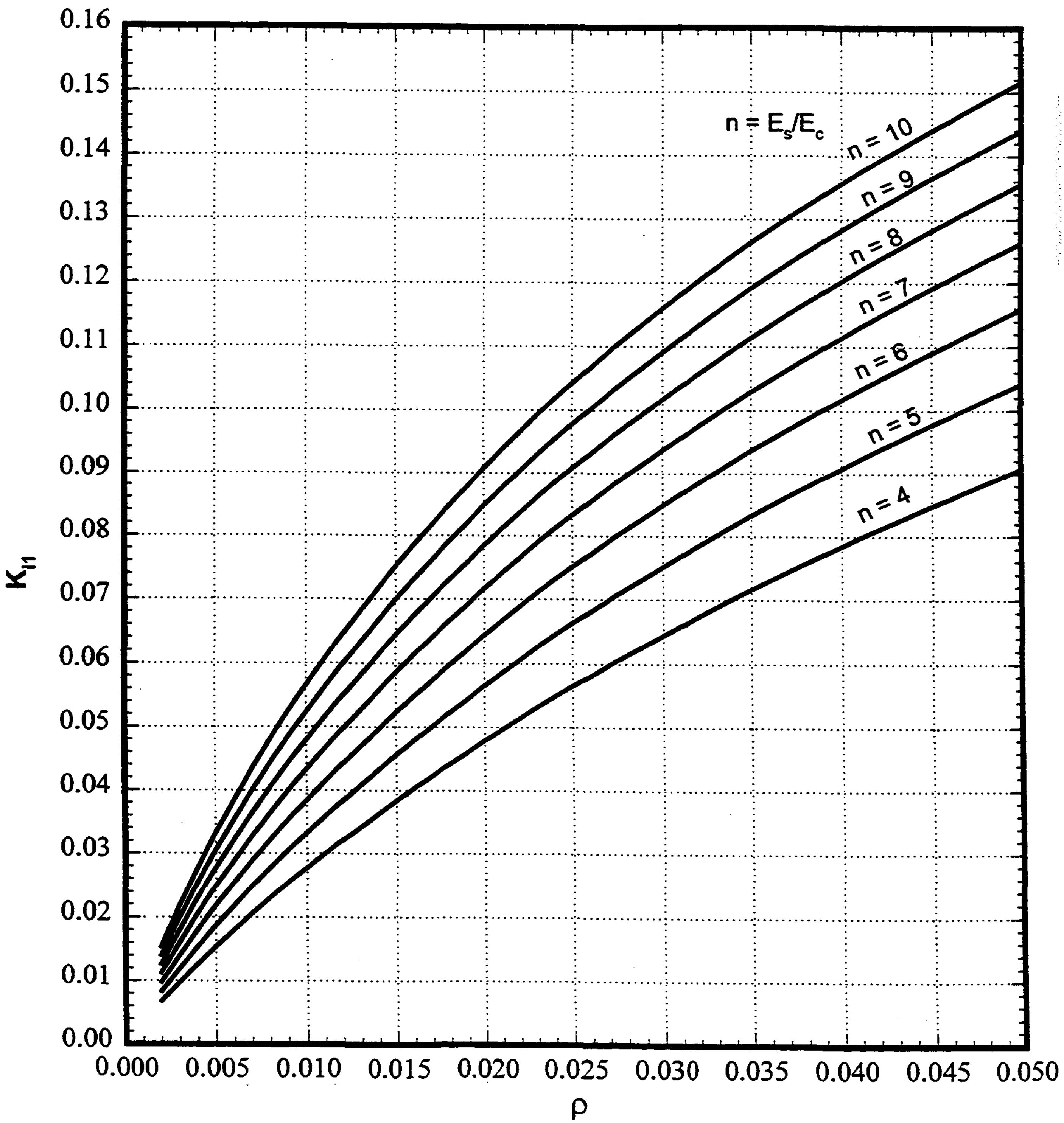
For use of this Design Aid, see Deflection Example 8.

DEFLECTION 2-Cracked section moment of inertia I_{cr} for rectangular sections with tension reinforcement only

Reference: ACI 318-95 Section 10.2

$$I_{cr} = K_{i1}\, bd^3$$

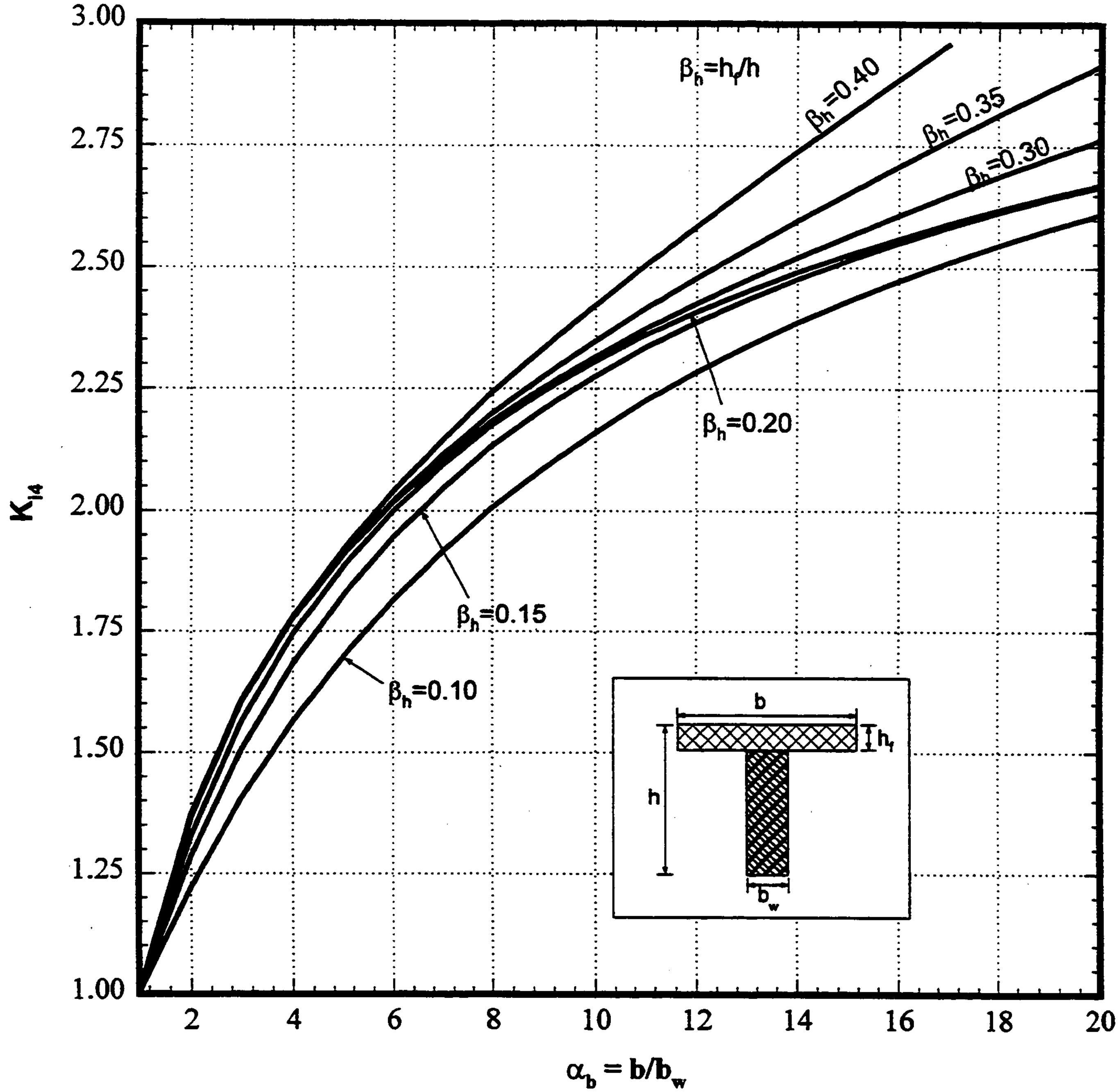
$$K_{i1} = \frac{(c/d)^3}{3} + \rho n[1 - (2c/d) + (c/d)^2]$$



For use of this Design Aid, see Deflection Examples 1 and 5.

DEFLECTION 3 - Gross moment of inertia for I_g of T-section

$$I_g = K_{i4}\,(b_w h^3 / 12) \qquad K_{i4} = 1 + (\alpha_b - 1)\beta_h^3 + \frac{3(1-\beta_h)^2(\beta_h)(\alpha_b - 1)}{1 + \beta_h(\alpha_b - 1)}$$

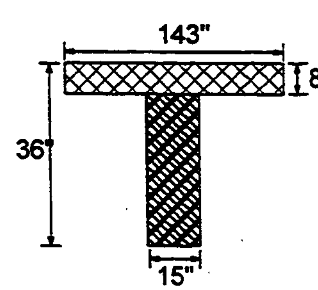


Example: For the T-beam shown, find the moment of inertia I_g :

$\alpha_b = b / b_w = 143/15 = 9.53$

$\beta_h = h_f / h = 8/36 = 0.22$

Interpolating between the curves for β_h = 0.20 and 0.30, read $\mathbf{K_{i4} = 2.28}$



$I_g = K_{i4}\,(b_w h^3 / 12) = 2.28[15(36)^3/12] =$ **133,000 in^4.**

DEFLECTION 4.1—Cracked-section moment of inertia I_{cr} for rectangular sections with compression steel, or T-sections (values of K_{i2}); for β_c from 0.1 through 0.9.

Reference: ACI 318-95 Section 10.2

$$I_{cr} = K_{i2} b_w d^3$$

$$K_{i2} = \left[\frac{(c/d)^3}{3} + pn\{1-(2c/d)+(c/d)^2\} + \rho n \beta_c\left\{(c/d)^2 - 2c/d\,\frac{d'}{d} + \left(\frac{d'}{d}\right)^2\right\}\right]$$

where for rectangular sections, $\beta_c = (n-1)\rho'/(\rho n)$, and for T-sections, $\beta_c = \left(\frac{b}{b_w} - 1\right) h_f/(d\rho_w n)$

		K_{i2}														
	d'/d or	ρn ($\rho_w n$ for T-sections)														
β_c	$h_f/2d$	0.02	0.04	0.06	0.08	0.10	0.12	0.14	0.16	0.18	0.20	0.22	0.24	0.26	0.28	0.30
0.1	0.02	0.015	0.028	0.039	0.049	0.058	0.066	0.074	0.081	0.088	0.095	0.101	0.107	0.113	0.118	0.123
0.1	0.10	0.015	0.028	0.039	0.048	0.057	0.065	0.073	0.080	0.087	0.093	0.099	0.105	0.111	0.116	0.121
0.1	0.20	0.015	0.028	0.038	0.048	0.057	0.065	0.072	0.079	0.086	0.092	0.098	0.104	0.109	0.114	0.119
0.1	0.30	0.015	0.028	0.038	0.048	0.057	0.064	0.072	0.079	0.085	0.091	0.097	0.103	0.108	0.113	0.117
0.1	0.40	0.015	0.028	0.038	0.048	0.057	0.064	0.072	0.078	0.085	0.091	0.096	0.102	0.107	0.112	0.116
0.2	0.02	0.015	0.028	0.039	0.049	0.059	0.067	0.076	0.084	0.091	0.098	0.105	0.112	0.118	0.124	0.130
0.2	0.10	0.015	0.028	0.039	0.049	0.058	0.066	0.074	0.082	0.089	0.096	0.102	0.109	0.115	0.121	0.126
0.2	0.20	0.015	0.028	0.038	0.048	0.057	0.065	0.073	0.080	0.087	0.093	0.100	0.106	0.111	0.117	0.122
0.2	0.30	0.015	0.028	0.038	0.048	0.057	0.065	0.072	0.079	0.086	0.092	0.098	0.103	0.109	0.114	0.119
0.2	0.40	0.016	0.028	0.039	0.048	0.057	0.064	0.072	0.079	0.085	0.091	0.097	0.102	0.107	0.112	0.117
0.3	0.02	0.016	0.028	0.040	0.050	0.060	0.069	0.078	0.086	0.094	0.102	0.109	0.116	0.123	0.130	0.137
0.3	0.10	0.015	0.028	0.039	0.049	0.058	0.067	0.075	0.083	0.091	0.098	0.105	0.112	0.118	0.125	0.131
0.3	0.20	0.015	0.028	0.039	0.048	0.057	0.066	0.073	0.081	0.088	0.095	0.101	0.107	0.113	0.119	0.125
0.3	0.30	0.015	0.028	0.038	0.048	0.057	0.065	0.072	0.079	0.086	0.092	0.098	0.104	0.110	0.115	0.120
0.3	0.40	0.016	0.028	0.039	0.048	0.057	0.064	0.072	0.079	0.085	0.091	0.097	0.102	0.107	0.112	0.117
0.4	0.02	0.016	0.028	0.040	0.051	0.061	0.070	0.079	0.088	0.097	0.105	0.113	0.121	0.128	0.136	0.143
0.4	0.10	0.015	0.028	0.039	0.049	0.059	0.068	0.076	0.085	0.093	0.100	0.108	0.115	0.122	0.129	0.135
0.4	0.20	0.015	0.028	0.039	0.048	0.057	0.066	0.074	0.081	0.089	0.096	0.102	0.109	0.115	0.121	0.127
0.4	0.30	0.016	0.028	0.038	0.048	0.057	0.065	0.072	0.079	0.086	0.093	0.099	0.105	0.111	0.116	0.121
0.4	0.40	0.016	0.028	0.039	0.048	0.057	0.064	0.072	0.079	0.085	0.091	0.097	0.102	0.108	0.113	0.118
0.5	0.02	0.016	0.029	0.040	0.051	0.062	0.072	0.081	0.090	0.099	0.108	0.116	0.125	0.133	0.141	0.149
0.5	0.10	0.015	0.028	0.039	0.050	0.060	0.069	0.078	0.086	0.094	0.102	0.110	0.118	0.125	0.132	0.139
0.5	0.20	0.015	0.028	0.039	0.048	0.058	0.066	0.074	0.082	0.090	0.097	0.104	0.110	0.117	0.123	0.130
0.5	0.30	0.016	0.028	0.038	0.048	0.057	0.065	0.072	0.080	0.087	0.093	0.099	0.105	0.111	0.117	0.123
0.5	0.40	0.016	0.028	0.039	0.048	0.057	0.064	0.072	0.079	0.085	0.091	0.097	0.103	0.108	0.113	0.118
0.6	0.02	0.016	0.029	0.041	0.052	0.063	0.073	0.083	0.092	0.101	0.111	0.120	0.128	0.137	0.146	0.154
0.6	0.10	0.015	0.028	0.040	0.050	0.060	0.069	0.079	0.087	0.096	0.104	0.112	0.120	0.128	0.136	0.143
0.6	0.20	0.015	0.028	0.039	0.049	0.058	0.067	0.075	0.083	0.090	0.098	0.105	0.112	0.119	0.125	0.132
0.6	0.30	0.016	0.028	0.038	0.048	0.057	0.065	0.073	0.080	0.087	0.093	0.100	0.106	0.112	0.118	0.124
0.6	0.40	0.016	0.028	0.039	0.048	0.057	0.064	0.072	0.079	0.085	0.091	0.097	0.103	0.108	0.113	0.116
0.7	0.02	0.016	0.029	0.041	0.053	0.063	0.074	0.084	0.094	0.104	0.113	0.123	0.132	0.141	0.150	0.159
0.7	0.10	0.015	0.028	0.040	0.050	0.061	0.070	0.080	0.089	0.097	0.106	0.114	0.123	0.131	0.139	0.147
0.7	0.20	0.015	0.028	0.039	0.049	0.058	0.067	0.075	0.083	0.091	0.099	0.106	0.113	0.120	0.127	0.134
0.7	0.30	0.016	0.028	0.038	0.048	0.057	0.065	0.073	0.080	0.87	0.094	0.100	0.107	0.113	0.119	0.125
0.7	0.40	0.016	0.028	0.039	0.048	0.057	0.064	0.072	0.079	0.085	0.091	0.097	0.103	0.108	0.114	0.119
0.8	0.02	0.016	0.029	0.041	0.053	0.064	0.075	0.086	0.096	0.106	0.116	0.126	0.135	0.145	0.154	0.164
0.8	0.10	0.015	0.028	0.040	0.051	0.061	0.071	0.080	0.090	0.099	0.108	0.116	0.125	0.133	0.142	0.150
0.8	0.20	0.015	0.028	0.039	0.049	0.058	0.067	0.076	0.084	0.092	0.100	0.107	0.115	0.122	0.129	0.136
0.8	0.30	0.016	0.028	0.038	0.048	0.057	0.065	0.073	0.080	0.087	0.094	0.101	0.107	0.114	0.120	0.126
0.8	0.40	0.016	0.028	0.039	0.048	0.057	0.064	0.072	0.079	0.085	0.091	0.097	0.103	0.108	0.114	0.119
0.9	0.02	0.016	0.029	0.042	0.054	0.065	0.076	0.087	0.098	0.108	0.118	0.128	0.139	0.149	0.158	0.168
0.9	0.10	0.016	0.028	0.040	0.051	0.062	0.072	0.081	0.091	0.100	0.109	0.118	0.127	0.136	0.145	0.153
0.9	0.20	0.015	0.028	0.039	0.049	0.058	0.067	0.076	0.084	0.093	0.101	0.108	0.116	0.123	0.131	0.138
0.9	0.30	0.016	0.028	0.038	0.048	0.057	0.065	0.073	0.080	0.088	0.095	0.101	0.108	0.114	0.121	0.127
0.9	0.40	0.016	0.028	0.039	0.048	0.057	0.064	0.072	0.079	0.085	0.091	0.097	0.103	0.109	0.114	0.119

For use of this Design Aid, see Deflection Examples 3, 4, and 5.

DEFLECTION 4.2—Cracked-section moment of inertia I_{cr} for rectangular sections with compression steel, or T-sections (values of K_{i2}); for β_c from 1.0 through 5.0.

Reference: ACI 318-95 Section 10.2

$$I_{cr} = K_{12} b_w d^3$$

$$K_{i2} = \left[\frac{(c/d)^3}{3} + \rho n\{1 - (2c/d) + (c/d)^2\} + \rho n \beta_c\left\{(c/d)^2 - 2c/d\,\frac{d'}{d} + \left(\frac{d'}{d}\right)^2\right\}\right]$$

where for rectangular sections, $\beta_c = (n-1)\rho'/(\rho n)$, and for T-sections, $\beta_c = \left(\frac{b}{b_w} - 1\right) h_f/(d\rho_w n)$

		K_{i2}														
	d'/d or $h_f/2d$	ρn ($\rho_w n$ for T-sections)														
β_c		0.02	0.04	0.06	0.08	0.10	0.12	0.14	0.16	0.18	0.20	0.22	0.24	0.26	0.28	0.30
1.0	0.02	0.016	0.029	0.042	0.054	0.066	0.077	0.088	0.099	0.110	0.121	0.131	0.142	0.152	0.152	0.172
1.0	0.10	0.016	0.028	0.040	0.051	0.062	0.072	0.082	0.092	0.101	0.111	0.120	0.129	0.138	0.147	0.156
1.0	0.20	0.015	0.028	0.039	0.049	0.059	0.068	0.076	0.085	0.093	0.101	0.109	0.117	0.125	0.132	0.140
1.0	0.30	0.016	0.028	0.038	0.048	0.057	0.065	0.073	0.081	0.088	0.095	0.102	0.108	0.115	0.121	0.128
1.0	0.40	0.016	0.029	0.039	0.048	0.057	0.064	0.072	0.079	0.085	0.091	0.097	0.103	0.109	0.114	0.120
1.5	0.02	0.016	0.030	0.044	0.057	0.069	0.082	0.094	0.106	0.118	0.130	0.142	0.154	0.166	0.178	0.190
1.5	0.10	0.016	0.029	0.041	0.053	0.064	0.075	0.086	0.096	0.107	0.117	0.128	0.138	0.148	0.159	0.169
1.5	0.20	0.015	0.028	0.039	0.049	0.059	0.069	0.078	0.087	0.096	0.105	0.114	0.122	0.131	0.139	0.147
1.5	0.30	0.016	0.028	0.038	0.048	0.057	0.065	0.073	0.081	0.089	0.096	0.103	0.111	0.118	0.124	0.131
1.5	0.40	0.017	0.029	0.039	0.048	0.057	0.064	0.072	0.079	0.085	0.092	0.098	0.104	0.110	0.115	0.121
2.0	0.02	0.016	0.031	0.045	0.059	0.072	0.085	0.099	0.112	0.125	0.138	0.151	0.164	0.177	0.190	0.203
2.0	0.10	0.016	0.029	0.042	0.054	0.066	0.077	0.089	0.100	0.112	0.123	0.134	0.145	0.156	0.167	0.178
2.0	0.20	0.015	0.028	0.039	0.050	0.060	0.070	0.080	0.089	0.098	0.108	0.117	0.126	0.135	0.144	0.153
2.0	0.30	0.016	0.028	0.038	0.048	0.057	0.065	0.079	0.082	0.090	0.097	0.105	0.112	0.120	0.127	0.134
2.0	0.40	0.017	0.029	0.039	0.048	0.057	0.064	0.072	0.079	0.085	0.092	0.098	0.104	0.110	0.116	0.122
2.5	0.02	0.016	0.031	0.046	0.060	0.074	0.088	0.102	0.116	0.130	0.144	0.158	0.172	0.186	0.199	0.213
2.5	0.10	0.016	0.029	0.042	0.055	0.067	0.079	0.091	0.103	0.115	0.127	0.139	0.151	0.162	0.174	0.186
2.5	0.20	0.015	0.028	0.039	0.050	0.061	0.071	0.081	0.091	0.100	0.110	0.120	0.129	0.139	0.148	0.158
2.5	0.30	0.016	0.028	0.038	0.048	0.057	0.066	0.074	0.082	0.090	0.098	0.106	0.114	0.121	0.129	0.136
2.5	0.40	0.017	0.029	0.040	0.049	0.057	0.064	0.072	0.079	0.085	0.092	0.098	0.104	0.111	0.117	0.123
3.0	0.02	0.017	0.032	0.047	0.062	0.076	0.091	0.106	0.120	0.135	0.149	0.164	0.178	0.193	0.207	0.221
3.0	0.10	0.016	0.029	0.043	0.056	0.068	0.081	0.093	0.106	0.118	0.130	0.143	0.155	0.167	0.180	0.192
3.0	0.20	0.015	0.028	0.039	0.050	0.061	0.071	0.082	0.092	0.102	0.112	0.122	0.132	0.142	0.151	0.161
3.0	0.30	0.016	0.028	0.038	0.048	0.057	0.066	0.074	0.083	0.091	0.099	0.107	0.115	0.123	0.131	0.138
3.0	0.40	0.018	0.030	0.040	0.049	0.057	0.064	0.072	0.079	0.085	0.092	0.098	0.105	0.111	0.117	0.123
3.5	0.02	0.017	0.032	0.048	0.063	0.078	0.093	0.108	0.123	0.138	0.153	0.168	0.183	0.198	0.213	0.228
3.5	0.10	0.016	0.030	0.043	0.056	0.069	0.082	0.095	0.108	0.121	0.133	0.146	0.159	0.171	0.184	0.197
3.5	0.20	0.015	0.028	0.039	0.051	0.061	0.072	0.082	0.093	0.103	0.113	0.124	0.134	0.144	0.154	0.164
3.5	0.30	0.016	0.028	0.038	0.048	0.057	0.066	0.075	0.083	0.091	0.100	0.108	0.116	0.124	0.132	0.140
3.5	0.40	0.018	0.030	0.040	0.049	0.057	0.064	0.072	0.079	0.085	0.092	0.099	0.105	0.111	0.118	0.124
4.0	0.02	0.017	0.033	0.048	0.064	0.079	0.095	0.110	0.126	0.141	0.157	0.172	0.187	0.203	0.218	0.234
4.0	0.10	0.016	0.030	0.043	0.057	0.070	0.083	0.096	0.110	0.123	0.136	0.149	0.162	0.175	0.188	0.201
4.0	0.20	0.015	0.028	0.040	0.051	0.062	0.073	0.083	0.094	0.104	0.115	0.125	0.136	0.146	0.157	0.167
4.0	0.30	0.016	0.028	0.038	0.048	0.057	0.066	0.075	0.083	0.092	0.100	0.108	0.117	0.125	0.133	0.141
4.0	0.40	0.018	0.030	0.040	0.049	0.057	0.064	0.072	0.079	0.086	0.092	0.099	0.105	0.112	0.118	0.124
4.5	0.02	0.017	0.033	0.049	0.065	0.081	0.096	0.112	0.128	0.144	0.159	0.175	0.191	0.207	0.222	0.238
4.5	0.10	0.016	0.030	0.044	0.057	0.071	0.084	0.098	0.111	0.124	0.138	0.151	0.164	0.178	0.191	0.204
4.5	0.20	0.015	0.028	0.040	0.051	0.062	0.073	0.084	0.095	0.105	0.116	0.127	0.137	0.148	0.158	0.169
4.5	0.30	0.016	0.028	0.038	0.048	0.057	0.066	0.075	0.084	0.092	0.101	0.109	0.117	0.126	0.134	0.142
4.5	0.40	0.018	0.030	0.040	0.049	0.057	0.064	0.072	0.079	0.086	0.092	0.099	0.105	0.112	0.118	0.124
5.0	0.02	0.017	0.033	0.050	0.066	0.082	0.098	0.114	0.130	0.146	0.162	0.178	0.194	0.210	0.226	0.242
5.0	0.10	0.016	0.030	0.044	0.058	0.072	0.085	0.099	0.112	0.126	0.140	0.153	0.167	0.180	0.194	0.207
5.0	0.20	0.015	0.028	0.040	0.051	0.062	0.073	0.084	0.095	0.106	0.117	0.128	0.139	0.149	0.160	0.171
5.0	0.30	0.016	0.028	0.038	0.048	0.057	0.066	0.075	0.084	0.093	0.101	0.110	0.118	0.126	0.135	0.143
5.0	0.40	0.019	0.031	0.040	0.049	0.057	0.064	0.072	0.079	0.086	0.092	0.099	0.106	0.112	0.118	0.125

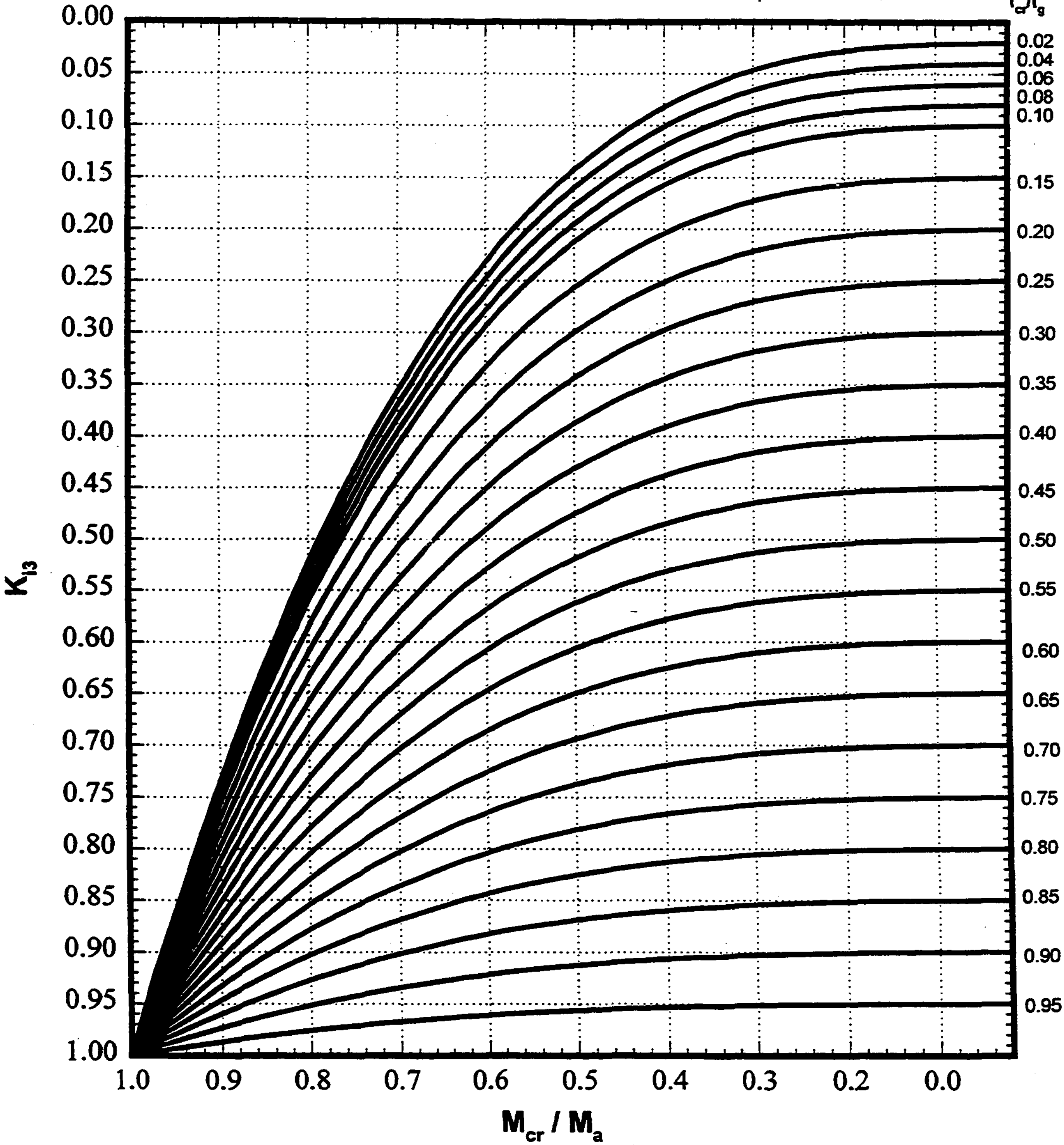
For use of this Design Aid, see Deflection Examples 3, 4, and 5.

DEFLECTION 5.1-Effective moment of inertia I_e (values of K_{i3})

Reference: ACI 318-95 Section 9.5.2.3

$$I_e = K_{i3} I_g$$

$$K_{i3} = \left(\frac{M_{cr}}{M_a}\right)^3 + \left[1 - \left(\frac{M_{cr}}{M_a}\right)^3\right]\left(\frac{I_{cr}}{I_g}\right)$$



Note: For M_{cr} see DEFLECTION 1.

For I_{cr} see DEFLECTION 2 or 4.

$I_g = bh^3/12$ or $K_{i4}(b_w h^3/12)$

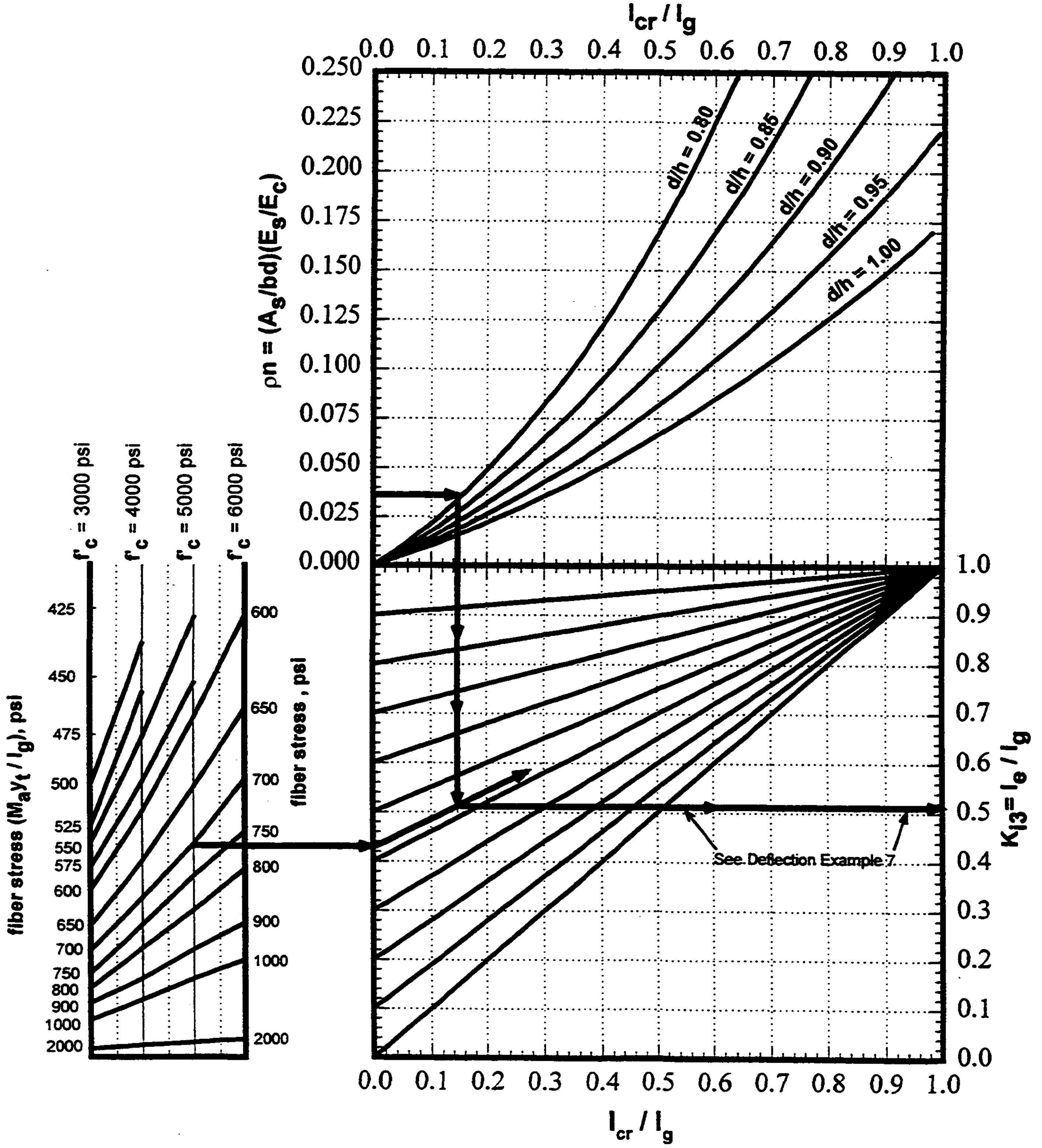
see DEFLECTION 3.

For use of this Design Aid, see Deflection Examples 1,2, and 5.

DEFLECTION 5.2-Effective moment of inertia I_e for rectangular sections with tension reinforcement only (values of K_{i3})

Reference: ACI 318-95 Section 9.5.2.3

$$I_e = K_{i3}\, I_g; \text{ where } K_{i3} = \left(\frac{M_{cr}}{M_a}\right)^3 + \left[1 - \left(\frac{M_{cr}}{M_a}\right)^3\right]\left(\frac{I_{cr}}{I_g}\right)$$



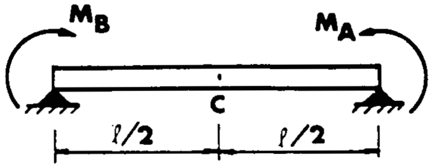
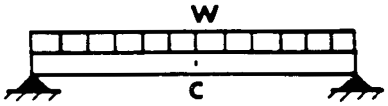
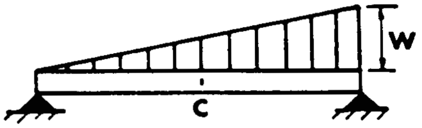
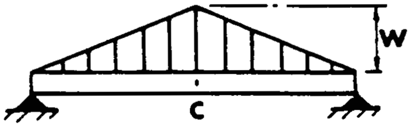
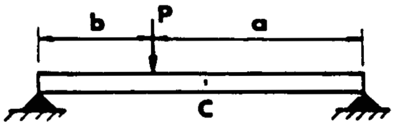
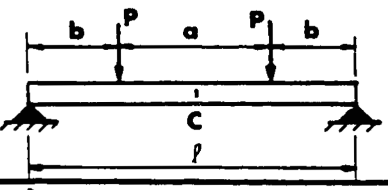
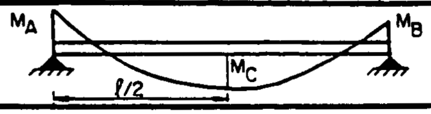
For use of this Design Aid, see Deflection Examples 7.

DEFLECTION 6.1-Coefficient K_{a3} and typical M_c formulas for calculating immediate deflection of flexural members

Reference: ACI 318-95 Sections 8.3 and 9.5.1

$$a_c = \frac{\Sigma(K_{a3} M_c)}{I_e} K_{a1}, \text{ in.}$$

where K_{a1} is from DEFLECTION 6.2
and I_e is from DEFLECTION 5.1 or 5.2

Case	Condition	M_c, kip-ft	K_{a3}
1		$\frac{M_A + M_B}{2}$	6.0
2		$\frac{w\ell^2}{8}$	5.0
3		$\frac{w\ell^2}{16}$	5.0
4		$\frac{w\ell^2}{12}$	4.8
5		$\frac{Pb}{2}$	Cases 5 and 6 b/ℓ — K_{a3} 0.125 — 5.875 0.200 — 5.680 0.250 — 5.500 0.333 — 5.111 0.400 — 4.720 0.500 — 4.000
6		Pb	
7		$M_C - 0.1(M_A + M_B)$	5.0

Note: Use service loads, not factored loads, in calculating deflections.

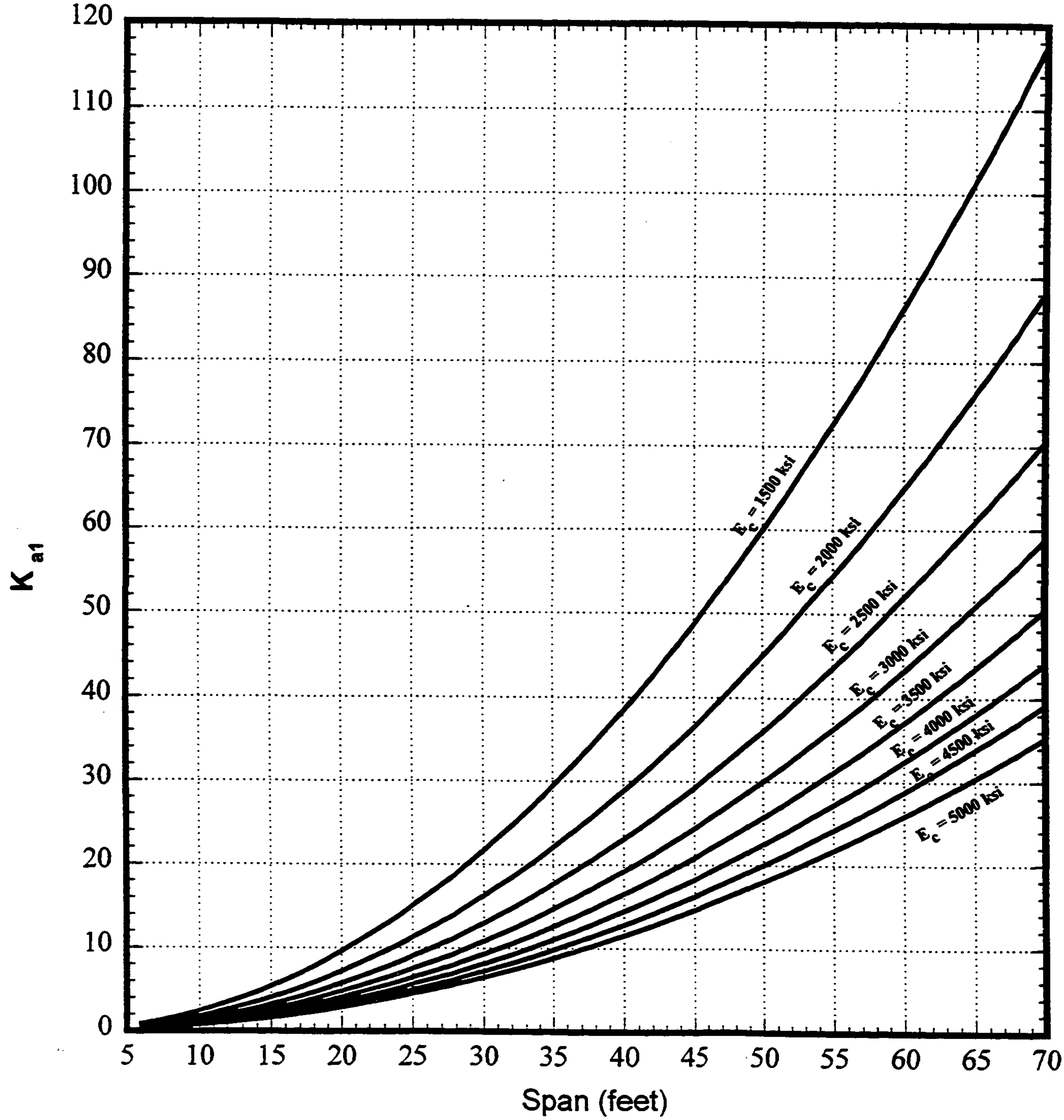
For use of this Design Aid, see Deflection Examples 2 and 5.

DEFLECTION 6.2-Coefficient K_{a1} for calculating immediate deflection of flexural members

Reference: ACI 318-95 Sections 8.3 and 8.5

$$K_{a1} = \frac{(1728)(\ell)^2}{48E_c} \qquad a_c = \frac{\Sigma(K_{a3} M_c)}{I_e} K_{a1}, \text{ in.}$$

where K_{a3} and M_c are from DEFLECTION 6.1 and
I_e is from DEFLECTION 5.1 or 5.2 and
E_c is from DEFLECTION 9



For use of this Design Aid, see Deflection Examples 2 and 5.

DEFLECTION 7—Values of K_{a2} and δ_c for use in $a_c = (K_{a2}\delta_c)(w/b)$ (immediate deflection by the approximate method)

Reference: ACI 318-95 Sections 8.3 and 9.5.2.3

$$a_c = (K_{a2}\delta_c)\,(w/b) \qquad K_{a2} = \frac{K_{a1}}{I_g} \qquad \delta_c = (I_g/I_e)\,\Sigma(K_{a3}M_c)$$

span, ft	K_{a2} — Thickness h, in.																										
	4	5	6	7	8	9	10	12	14	16	18	20	22	24	26	28	30	32	34	36	38	40	42	44	46	48	50
5	1.34	0.68	0.40	0.25	0.17	0.12	0.09	0.05	0.03	0.02	0.01	0.01	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
6	2.77	1.42	0.82	0.52	0.35	0.24	0.18	0.10	0.06	0.04	0.03	0.02	0.02	0.01	0.01	0.01	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
7	5.14	2.63	1.52	0.96	0.64	0.45	0.33	0.19	0.12	0.08	0.06	0.04	0.03	0.02	0.02	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.00	0.00	0.00	0.00	0.00
8	8.75	4.49	2.60	1.63	1.10	0.77	0.56	0.32	0.20	0.14	0.10	0.07	0.05	0.04	0.03	0.03	0.02	0.02	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.00
9	14.03	7.18	4.16	2.62	1.75	1.23	0.90	0.52	0.33	0.22	0.15	0.11	0.08	0.06	0.05	0.04	0.03	0.03	0.02	0.02	0.02	0.01	0.01	0.01	0.01	0.01	0.01
10	21.39	10.95	6.34	3.99	2.67	1.88	1.37	0.79	0.50	0.33	0.23	0.17	0.13	0.10	0.08	0.06	0.05	0.04	0.03	0.03	0.02	0.02	0.02	0.02	0.01	0.01	0.01
11	31.31	16.03	9.28	5.84	3.91	2.75	2.00	1.16	0.73	0.49	0.34	0.25	0.19	0.14	0.11	0.09	0.07	0.06	0.05	0.04	0.04	0.03	0.03	0.02	0.02	0.02	0.02
12	44.35	22.71	13.14	8.28	5.54	3.89	2.84	1.64	1.03	0.69	0.49	0.35	0.27	0.21	0.16	0.13	0.11	0.09	0.07	0.06	0.05	0.04	0.04	0.03	0.03	0.03	0.02
13	61.09	31.28	18.10	11.40	7.64	5.36	3.91	2.26	1.42	0.95	0.67	0.49	0.37	0.28	0.22	0.18	0.14	0.12	0.10	0.08	0.07	0.06	0.05	0.05	0.04	0.04	0.03
14	82.17	42.07	24.35	15.33	10.27	7.21	5.26	3.04	1.92	1.28	0.90	0.55	0.49	0.38	0.30	0.24	0.19	0.16	0.13	0.11	0.10	0.08	0.07	0.05	0.05	0.05	0.04
15	108.28	55.44	32.08	20.20	13.53	9.51	6.93	4.01	2.53	1.69	1.19	0.87	0.65	0.50	0.39	0.32	0.26	0.21	0.18	0.15	0.13	0.11	0.09	0.08	0.07	0.06	0.06
16	140.17	71.77	41.53	26.15	17.52	12.31	8.97	5.19	3.27	2.19	1.54	1.12	0.84	0.65	0.51	0.41	0.33	0.27	0.23	0.19	0.16	0.14	0.12	0.11	0.09	0.08	0.07
17	178.64	91.46	52.93	33.33	22.33	15.68	11.43	6.62	4.17	2.79	1.96	1.43	1.07	0.83	0.65	0.52	0.42	0.35	0.29	0.25	0.21	0.18	0.15	0.13	0.12	0.10	0.09
18	224.53	114.96	66.53	41.89	28.07	19.71	14.37	8.32	5.24	3.51	2.46	1.80	1.35	1.04	0.82	0.65	0.53	0.44	0.37	0.31	0.26	0.22	0.19	0.17	0.15	0.13	0.11
19	278.74	142.71	82.59	52.01	34.84	24.47	17.84	10.32	6.50	4.36	3.06	2.23	1.68	1.29	1.01	0.81	0.66	0.54	0.45	0.38	0.33	0.28	0.24	0.21	0.18	0.16	0.14
20	342.21	175.21	101.40	83.85	42.78	30.04	21.90	12.67	7.98	5.35	3.76	2.74	2.06	1.58	1.25	1.00	0.81	0.67	0.56	0.47	0.40	0.34	0.30	0.26	0.23	0.20	0.18
21	415.96	212.97	123.25	77.61	52.00	36.52	26.62	15.41	9.70	6.50	4.56	3.33	2.50	1.93	1.51	1.21	0.99	0.81	0.68	0.57	0.49	0.42	0.36	0.31	0.27	0.24	0.21
22	501.03	256.53	148.45	93.49	62.63	43.99	32.07	18.56	11.69	7.83	5.50	4.01	3.01	2.32	1.82	1.46	1.19	0.98	0.82	0.69	0.58	0.50	0.43	0.38	0.33	0.29	0.26
23	598.53	306.45	177.34	111.68	74.82	52.55	38.31	22.17	13.96	9.35	6.57	4.79	3.60	2.77	2.18	1.74	1.42	1.17	0.97	0.82	0.70	0.60	0.52	0.45	0.39	0.35	0.31
24	709.61	363.32	210.26	132.41	88.70	82.30	45.42	26.28	16.55	11.09	7.79	5.68	4.27	3.29	2.58	2.07	1.68	1.39	1.16	0.97	0.83	0.71	0.61	0.53	0.47	0.41	0.36
25	835.48	427.77	247.55	155.89	104.44	73.35	53.47	30.94	19.49	13.05	9.17	6.68	5.02	3.87	3.04	2.44	1.98	1.63	1.36	1.15	0.97	0.84	0.72	0.63	0.55	0.48	0.43
26	977.40	500.43	289.60	182.37	122.17	85.81	62.55	36.20	22.80	15.27	10.73	7.82	5.87	4.52	3.56	2.85	2.32	1.91	1.59	1.34	1.14	0.98	0.84	0.73	0.64	0.57	0.50
27		581.97	336.79	212.09	142.08	99.79	72.75	42.10	26.51	17.76	12.47	9.09	6.83	5.26	4.14	3.31	2.69	2.22	1.85	1.56	1.33	1.14	0.96	0.85	0.75	0.66	0.58
28		673.10	389.53	245.30	164.33	115.41	84.14	48.69	30.66	20.54	14.43	10.52	7.90	6.09	4.79	3.83	3.12	2.57	2.14	1.80	1.53	1.31	1.14	0.99	0.6	0.76	0.67
29		774.53	448.22	282.26	189.09	132.81	96.82	56.03	35.28	23.64	16.60	12.10	9.09	7.00	5.51	4.41	3.59	2.95	2.45	2.08	1.76	1.51	1.31	1.14	0.99	0.88	0.77
30		887.02	513.32	323.26	216.56	152.09	110.88	84.17	40.41	27.07	19.01	13.85	10.41	8.02	6.31	5.05	4.11	3.38	2.82	2.36	2.02	1.73	1.50	1.30	1.14	0.00	0.89
31			585.26	368.56	248.91	173.41	126.42	73.16	48.07	30.86	21.68	15.80	11.87	9.14	7.19	5.76	4.68	3.86	3.22	2.71	2.30	1.98	1.71	1.48	1.30	1.14	1.01
32			664.51	418.47	280.34	196.89	143.53	83.06	52.31	35.04	24.61	17.94	13.48	10.38	8.17	6.54	5.32	4.38	3.65	3.08	2.62	2.24	1.94	1.68	1.47	1.30	1.15
33			751.55	473.28	317.06	222.68	162.34	93.94	59.16	39.63	27.84	20.29	15.25	11.74	9.24	7.40	6.01	4.95	4.13	3.48	2.96	2.54	2.19	1.91	1.67	1.47	1.30
34			846.87	533.31	357.28	250.93	182.92	105.86	66.66	44.66	31.37	22.87	17.18	13.23	10.41	8.33	6.77	5.58	4.86	3.92	3.33	2.86	2.47	2.15	1.88	1.86	1.46
35			950.99	596.87	401.20	281.77	205.41	118.87	74.86	50.15	35.22	25.68	19.29	14.95	11.69	9.36	7.61	6.27	5.23	4.40	3.74	3.21	2.77	2.41	2.11	1.96	1.64
36				670.31	449.05	315.38	229.91	133.05	83.79	56.13	39.42	28.74	21.59	16.53	13.06	10.47	8.52	7.02	5.85	4.93	4.19	3.59	3.10	2.70	2.36	2.08	1.84
37				747.95	501.07	351.91	256.55	148.46	93.49	62.63	43.99	32.07	24.09	18.56	14.60	11.69	9.50	7.83	6.53	5.50	4.68	4.01	3.46	3.01	2.64	2.32	2.05
38				832.14	557.47	391.53	285.42	165.18	104.02	89.68	48.94	35.68	26.81	20.65	16.24	13.00	10.57	8.71	7.26	6.12	5.20	4.46	3.85	3.35	2.93	2.58	2.28
39				923.25	618.51	434.40	316.68	183.26	115.41	77.31	54.30	39.58	29.74	22.91	18.02	14.43	11.73	9.66	8.06	6.79	5.77	4.95	4.27	3.72	3.25	2.86	2.53
40					684.43	480.70	350.43	202.79	127.71	85.55	60.09	43.80	32.91	25.35	19.94	15.96	12.98	10.69	8.92	7.51	6.39	5.48	4.73	4.11	3.60	3.17	2.80
41					755.48	530.60	386.81	223.85	140.96	94.43	66.32	48.35	36.33	27.98	22.01	17.62	14.33	11.80	9.84	8.29	7.05	6.04	5.22	4.54	3.97	3.50	3.09
42					831.93	584.29	425.95	246.60	155.23	103.99	73.04	53.24	40.00	30.81	24.23	19.40	15.78	13.00	10.84	9.13	7.76	6.66	5.75	5.00	4.38	3.85	3.41
43					914.03	641.95	467.98	270.82	170.55	114.25	80.24	58.50	43.95	33.85	26.63	21.32	17.33	14.28	11.91	10.03	8.53	7.31	6.32	5.49	4.81	4.23	3.74
44						703.79	513.06	296.91	186.98	125.26	87.97	64.13	48.18	37.11	29.19	23.37	19.00	15.66	13.05	11.00	9.35	8.02	6.93	6.02	5.27	4.64	4.10
45						769.98	561.32	324.84	204.56	137.04	96.25	70.16	52.72	40.60	31.94	25.57	20.79	17.13	14.28	12.03	10.23	8.77	7.58	6.59	5.77	5.08	4.49
46						840.74	612.90	354.59	223.36	149.63	105.09	76.61	57.56	44.34	34.87	27.92	22.70	18.70	15.59	13.14	11.17	9.58	8.27	7.19	6.30	5.54	4.90
47						916.26	667.96	386.55	243.42	163.08	114.53	83.49	62.73	48.32	38.00	30.43	24.74	20.38	18.99	14.32	12.17	10.44	9.02	7.84	6.86	6.04	5.34
48						996.77	726.64	420.51	264.81	177.40	124.60	90.83	68.24	52.56	41.34	33.10	26.91	22.18	18.49	15.57	13.24	11.35	9.81	8.53	7.47	6.57	5.81
49							789.12	456.67	287.58	192.66	135.31	98.64	74.11	57.08	44.90	35.95	29.23	24.08	20.08	16.91	14.38	12.33	10.65	9.26	8.11	7.14	6.31
50							855.53	495.10	311.78	208.87	146.70	106.94	80.35	61.89	48.68	38.97	31.69	26.11	21.77	18.34	15.59	13.37	11.55	10.04	8.79	7.74	6.84
51							926.06	535.91	337.48	226.09	158.79	115.76	86.97	66.99	52.69	42.19	34.30	28.26	23.56	19.85	16.88	14.47	12.50	10.87	9.51	8.37	7.41
52								579.20	364.74	244.35	171.61	125.11	93.99	72.40	56.94	45.59	37.07	30.54	25.46	21.45	18.24	15.64	13.51	11.75	10.28	9.05	8.01
53								625.05	393.62	263.69	185.20	135.01	101.44	78.13	61.45	49.20	40.00	32.96	27.48	23.15	19.68	16.88	14.58	12.68	11.10	9.77	8.64
54								673.58	424.18	284.17	199.58	145.49	109.31	84.20	66.22	53.02	43.11	35.52	29.61	24.95	21.21	18.19	15.71	13.66	11.96	10.52	9.31
55								724.88	456.48	305.81	214.78	156.57	117.64	90.61	71.27	57.06	46.39	38.23	31.87	26.85	22.83	19.57	16.91	14.70	12.87	11.33	10.02
56								779.05	490.60	328.66	230.83	168.27	125.43	97.38	76.59	61.32	49.86	41.08	34.25	28.85	24.53	21.03	18.17	15.80	13.83	12.17	10.77
57								836.21	526.59	352.77	247.76	180.62	135.70	104.53	82.21	65.82	53.52	44.10	36.76	30.97	26.33	22.58	19.50	16.96	14.85	13.07	11.56
58								896.45	564.53	378.19	265.61	193.63	145.48	112.06	88.14	70.57	57.37	47.27	39.41	33.20	28.23	24.20	20.91	18.18	15.91	14.01	12.39
59								959.89	604.48	404.95	284.41	207.34	155.77	119.99	94.37	75.56	61.43	50.62	42.20	35.55	30.23	25.92	22.39	19.47	17.04	15.00	13.27
60									646.51	433.11	304.19	221.75	166.61	128.33	100.94	80.81	65.70	54.14	45.14	38.02	32.33	27.72	23.94	20.83	18.23	16.04	14.19

Note: Table values are based on normal weight concrete with $f'_c = 3000$ psi. For different concrete qualities, multiply K_{a2} by the following factor:

f'_c, psi	w_c, pcf	multiplier
3000	145	1
	115	1.45
	90	2.05
4000	145	0.9
	115	0.25
	90	1.75
5000	145	0.8
	115	1.1
	90	1.6
6000	145	0.7
	115	1
	90	1.45

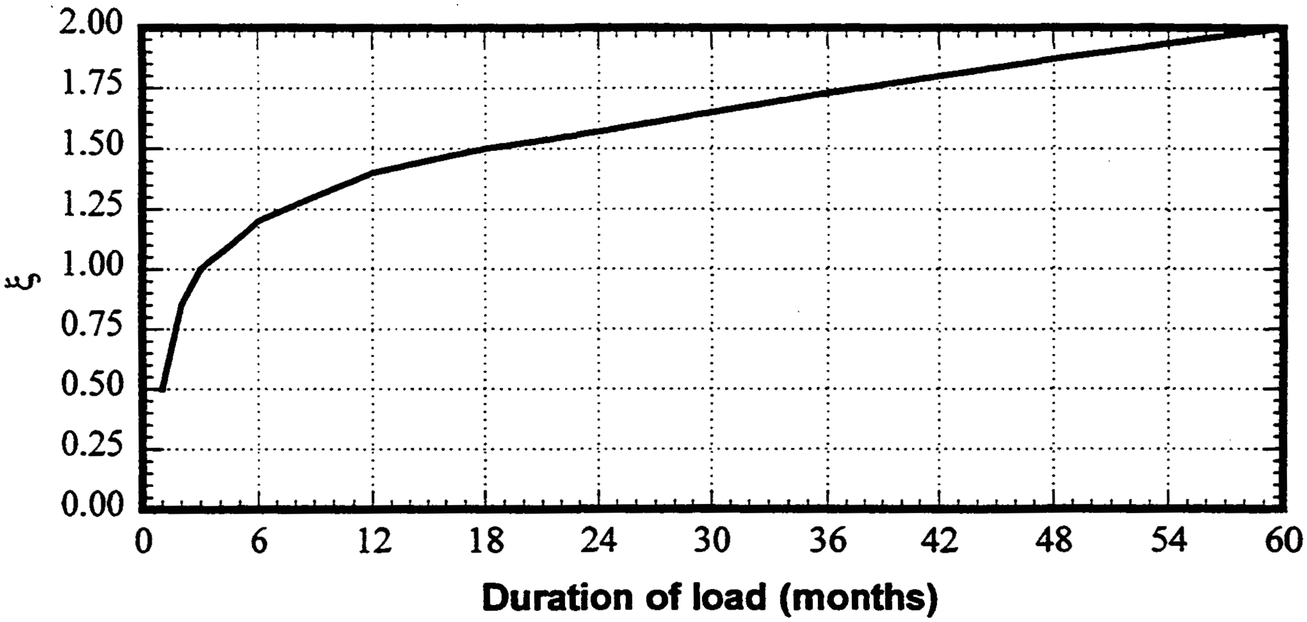
Type of span	δ_c — Degree of reinforcement: $\rho > 0.60\rho_{bal}$	$\rho < 0.60\rho_{bal}$
Simple span	0.62	1.03
End span with discontinuous end unrestrained	0.42	0.7
End span framed into beam or column	0.31	0.51
Interior span	0.25	0.42

For use of this design Aid, see Deflection Example 6.

DEFLECTION 8-Creep and shrinkage deflection (additional long-time deflection) due to sustained loads

Reference: ACI 318-95 Section 9.5.2.5

creep and shrinkage deflection = $\lambda(a_c)$;where $\lambda = \frac{\xi}{1 + 50\rho'}$



Time	Values of ξ per ACI 318-95 Section 9.5.2.5
3 months	1.0
6 months	1.2
12 months	1.4
5 years or more	2.0

Example: For a flexural member with $\rho' = 0.005$ the immediate deflection under sustained load is 0.5 in.
Find the long term deflection after 3 months and after 5 years?

After 3 months, deflection = $0.5 + \frac{1}{1 + 50(0.005)}(0.5) = 0.90$ in.

After 5 years, deflection = $0.5 + \frac{2}{1 + 50(0.005)}(0.5) = 1.30$ in.

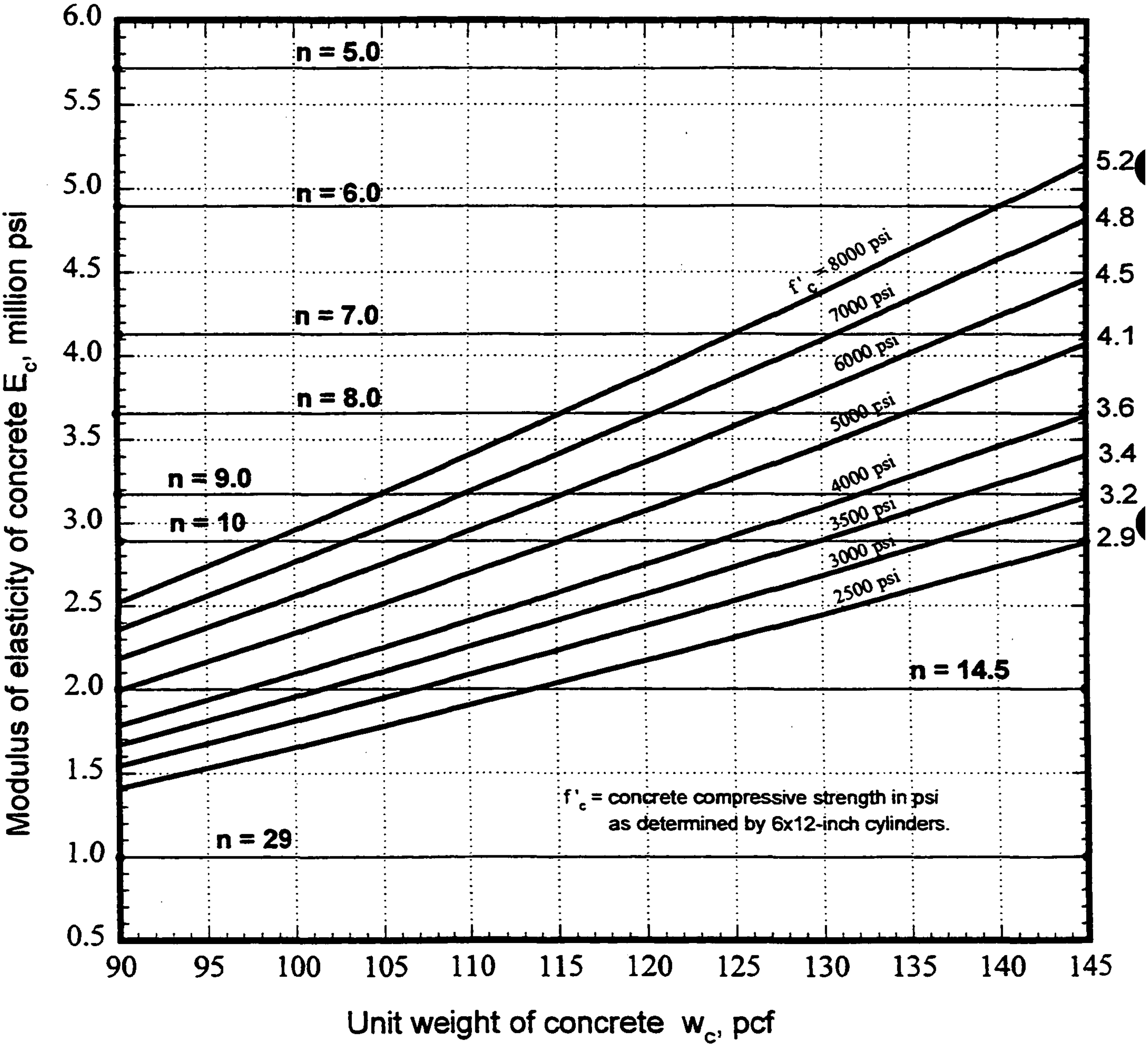
DEFLECTION 9-Modulus of elasticity E_c for various concrete strengths

Reference: ACI 318-95 Sections 8.5.1 and 8.5.2

$$E_c = 33 w_c^{1.5} \sqrt{f'_c}$$

$n = E_s / E_c$, where E_s = 29,000,000 psi

Note: For practical use, n may be taken as the nearest whole number.



Example: For sand-lightweight concrete w_c = 120 pcf, and f'_c = 5000 psi, find the modulus of elasticity E_c and the modular ratio, "n"

Find where w_c = 120 on the horizontal axis and proceed vertically

upward to f'_c = 5000. Then proceed horizontally to the left,

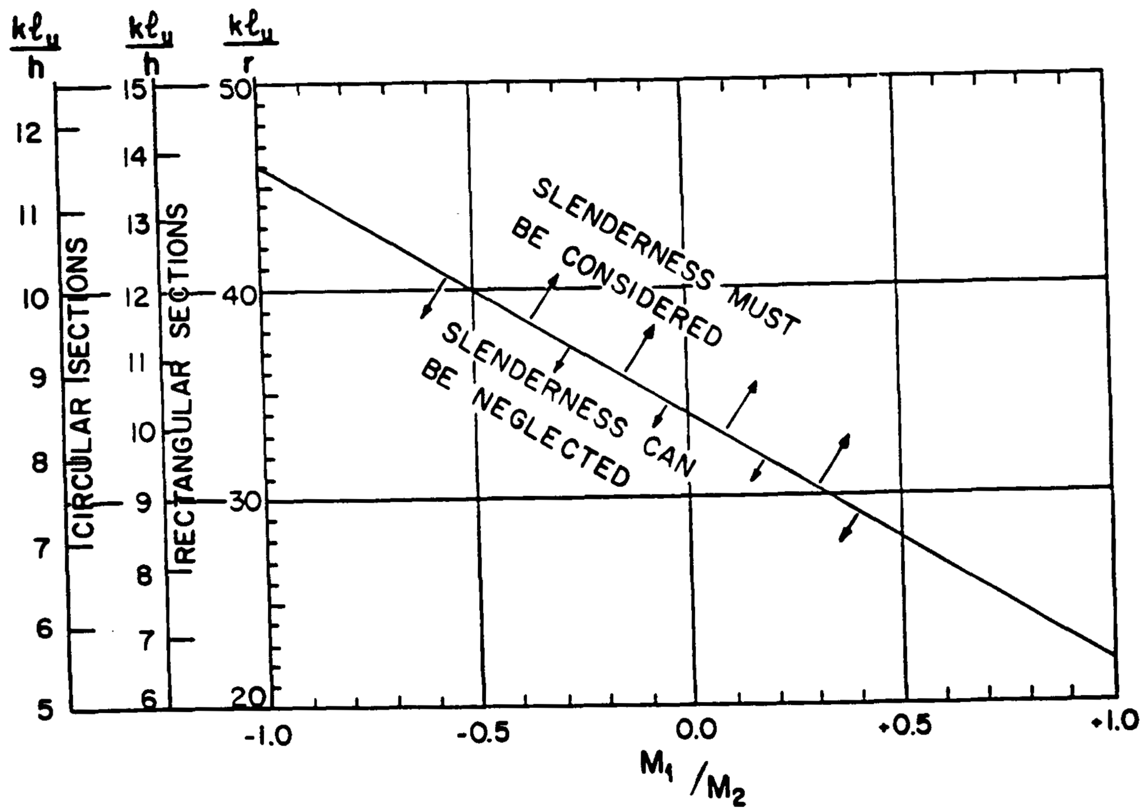
and read E_c = 3.1 million psi and n = 9.5 (use n = 9)

COLUMNS

COLUMNS 1—Slenderness ratios* $k\ell_u/r$ and $k\ell_u/h$ below which effects of slenderness may be neglected for columns braced against sidesway

References: ACI 318-95 Sections 10.11.3 and 10.11.4.1

M_1/M_2	-1.0	-0.8	-0.6	-0.4	-0.2	0.0	+0.2	+0.4	+0.6	+0.8	+1.0
COLUMN MOMENT DIAGRAM											
GENERAL $k\ell_u/r$	46.0	43.6	41.2	38.8	36.4	34.0	31.6	29.2	26.8	24.4	22.0
RECTANGLES $k\ell_u/h$	13.8	13.1	12.4	11.6	10.9	10.2	9.5	8.8	8.0	7.3	6.6
CIRCLES $k\ell_u/h$	11.5	10.9	10.3	9.7	9.1	8.5	7.9	7.3	6.7	6.1	5.5

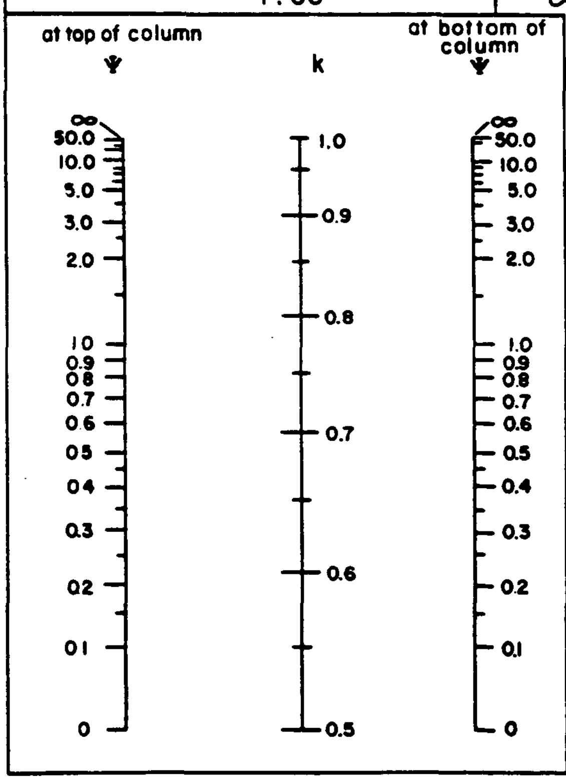


*Where $kl_u/r < (34 - 12 M_1/M_2)$, slenderness effects may be neglected as provided in Section 10.11.4.1 of ACI 318-95.

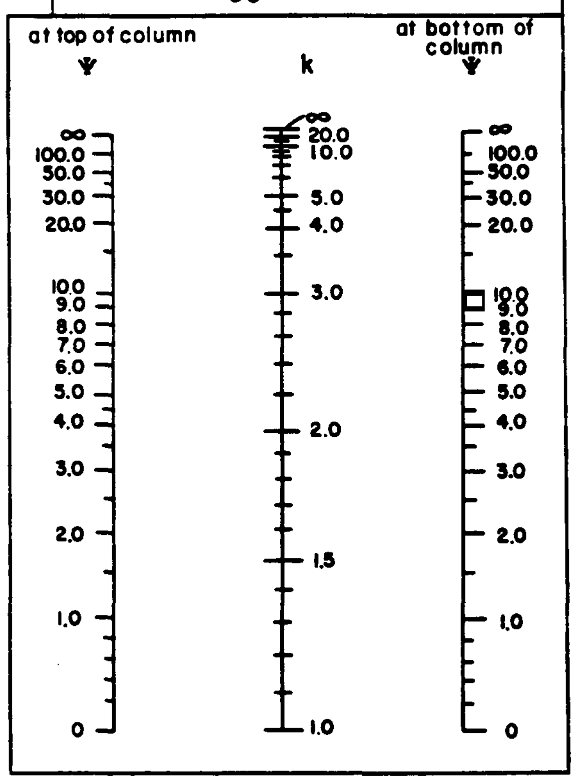
COLUMNS 2—Effective length factor *k* for columns in braced and nonbraced frames

References: ACI 318R-95 Section 10.11.2; Thomas C. Kavanagh, "Effective Length of Framed Columns," *Transactions*, ASCE, **127** (1962). Part II, 81-101. For detailed derivation, see *Steel Structures: Design and Behavior*, by C. G. Salmon and J. E. Johnson, 2nd Ed., Harper & Row Publishers, New York, 1980, pp. 843-851.

BRACED FRAMES $\Psi = \frac{-2k}{\pi} \tan \frac{\pi}{2k}$ k	Ψ	UNBRACED FRAMES $\Psi = \frac{6k}{\pi} \cot \frac{\pi}{2k}$ k
0.50	0.00	1.00
0.61	0.25	1.08
0.69	0.50	1.16
0.74	0.75	1.24
0.77	1.00	1.32
0.82	1.50	1.46
0.86	2.00	1.59
0.88	2.50	1.71
0.89	3.00	1.82
0.92	4.00	2.03
0.93	5.00	2.23
0.96	10.00	3.01
1.00	∞	∞



BRACED FRAMES

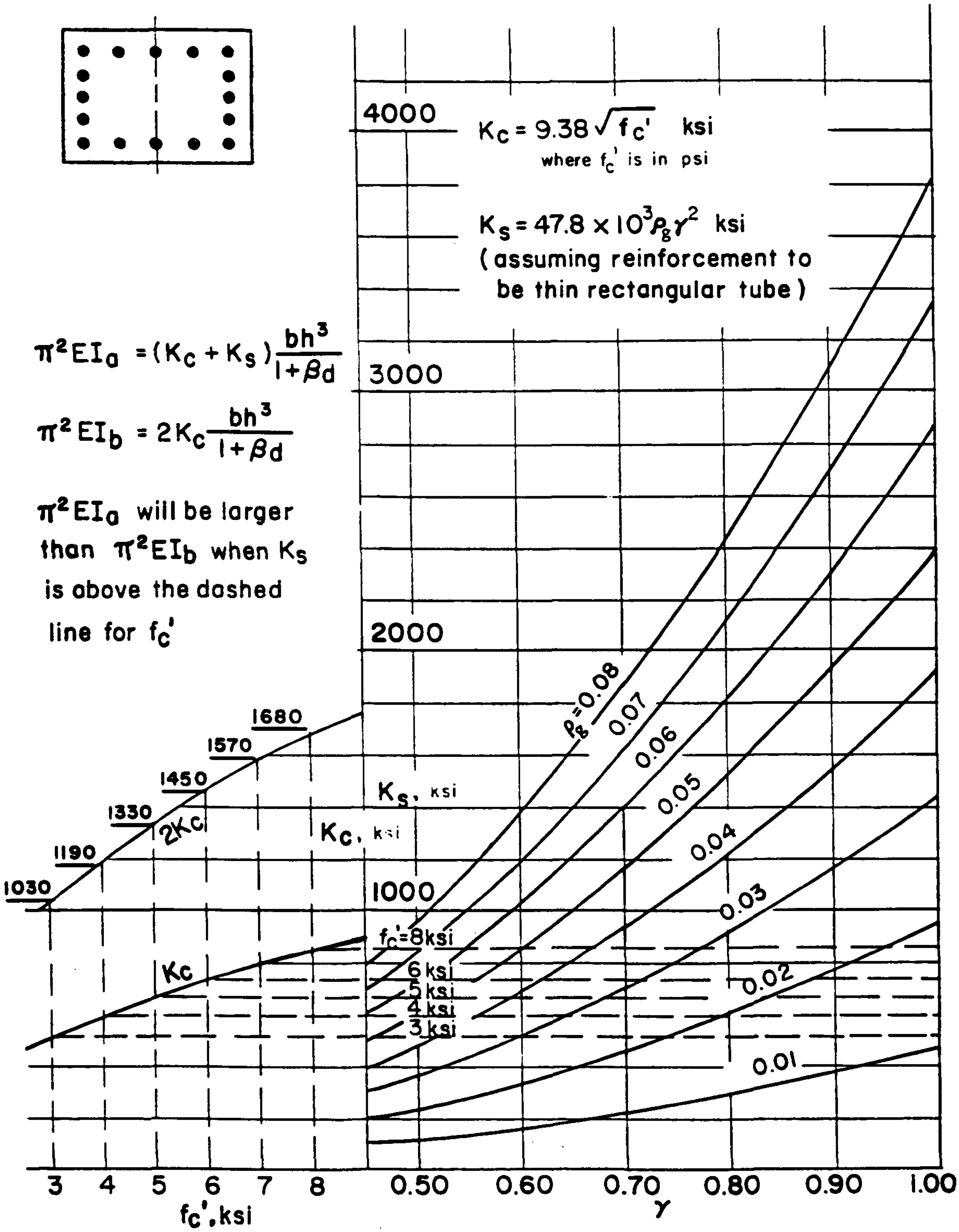


UNBRACED FRAMES

Ψ = relative column stiffness = ratio of $\Sigma(EI/l_c)$ of column to $\Sigma(EI/l)$ of beams, in a plane at one end of a column.

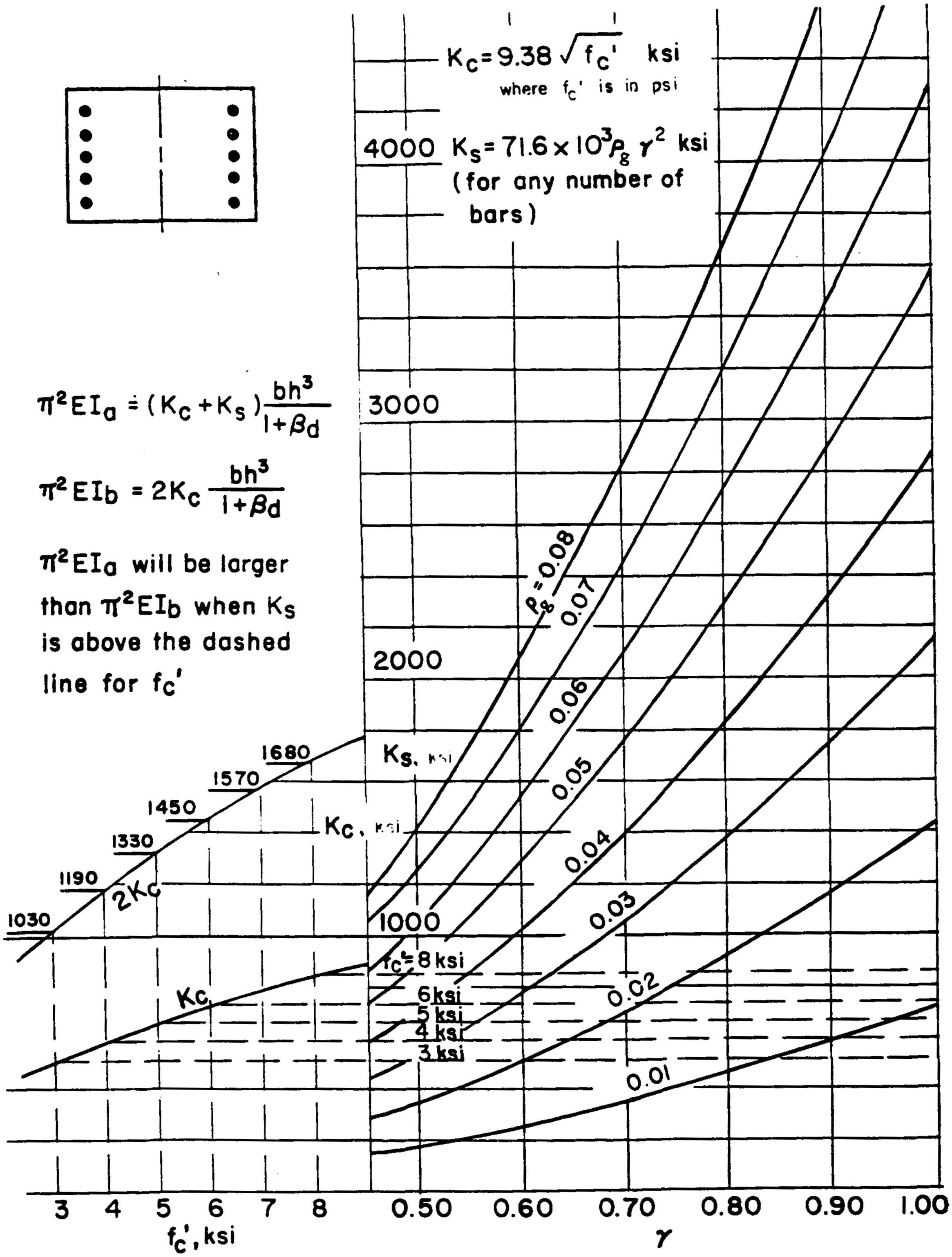
COLUMNS 3.1—Factors K_c and K_s for computing flexural stiffness term ($\pi^2 EI$) for rectangular tied columns with steel on four faces

References: ACI 318-95 Sections 8.5.1, 8.5.2, and 10.11.5.2



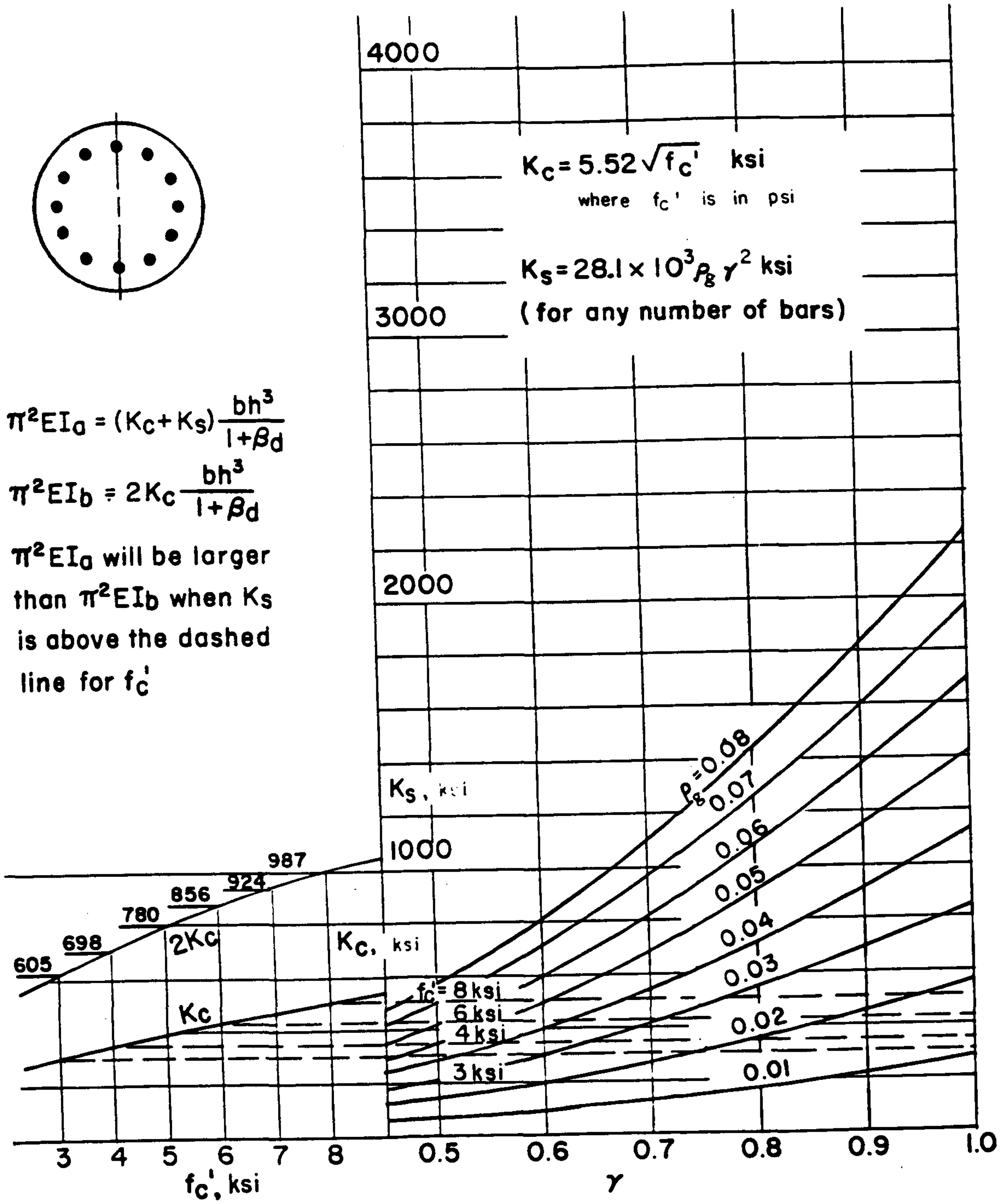
COLUMNS 3.2—Factors K_c and K_s for computing flexural stiffness term ($\pi^2 EI$) for rectangular tied columns with steel on two end faces

References: ACI 318-95 Sections 8.5.1, 8.5.2, and 10.11.5.2



COLUMNS 3.3—Factors K_c and K_s for computing flexural stiffness term ($\pi^2 EI$) for circular spiral columns

References: ACI 318-95 Sections 8.5.1, 8.5.2, and 10.11.5.2



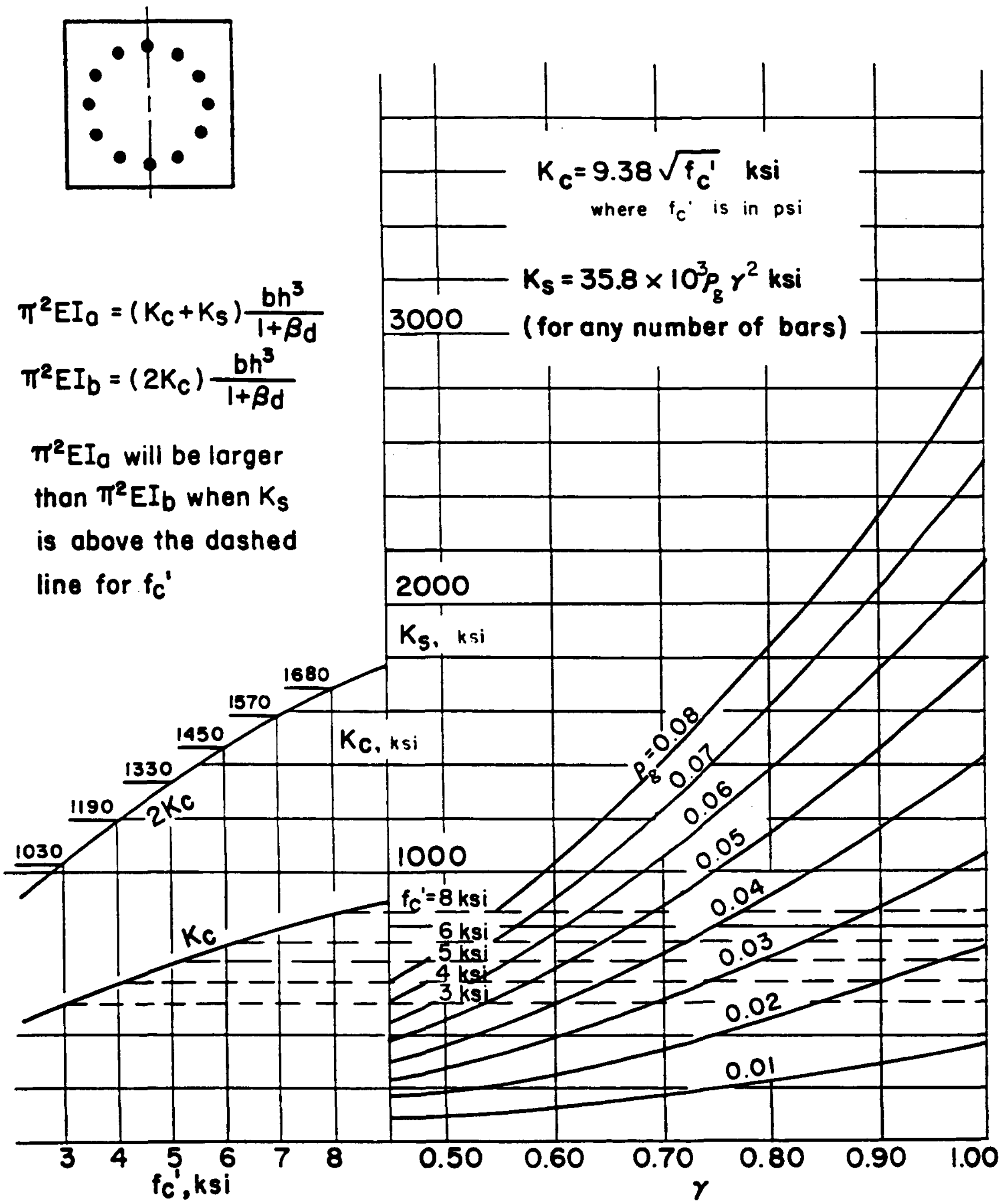
COLUMNS 3.4—Factors K_c and K_s for computing flexural stiffness term ($\pi^2 EI$) for square spiral columns

References: ACI 318-95 Sections 8.5.1, 8.5.2, and 10.11.5.2

$$\pi^2 EI_a = (K_c + K_s)\frac{bh^3}{1+\beta_d}$$

$$\pi^2 EI_b = (2K_c)\frac{bh^3}{1+\beta_d}$$

$\pi^2 EI_a$ will be larger than $\pi^2 EI_b$ when K_s is above the dashed line for f_c'



COLUMNS 4.1—Values of $[(E_cI_g)/2.5] \times 10^{-5}$ for computing flexural stiffness *EI* of cracked sections of rectangular and circular columns—f'_c = 3 ksi

References: "Building Code Requirements for Structural Concrete-ACI 318" Chapters 8 and 10.

$$EI = \frac{(E_cI_g)/2.5}{1+\beta_d} = \frac{\text{table value} \times 10^5}{1+\beta_d} \text{kip-in.}^2$$

Note—This table is for concrete for which f'_c = 3 ksi, w_c = 145 pcf, and E_c = 3122 ksi. For concrete of different f'_c and E_c, multiply table value by E_c/3122 for use in computing *EI*. $0 < \beta_d < 1$.

h, in.	Rectangular columns																		Circular columns
	b, in.																		
	6	8	10	12	14	16	18	20	22	24	26	28	30	32	31	36	42	48	
6	1	2	2	3	3	4	4	4	5	5	6	6	7	7	8	8	9	11	1
8	3	4	5	6	7	9	10	11	12	13	14	15	16	17	18	19	22	26	3
10	6	8	10	12	15	17	19	21	23	25	27	29	31	33	35	37	44	50	6
12	11	14	18	22	25	29	32	36	40	43	47	50	54	58	61	65	76	86	13
14	17	23	29	34	40	46	51	57	63	69	74	80	86	91	97	103	120	137	24
16	26	34	43	51	60	68	77	85	94	102	111	119	128	136	145	153	179	205	40
18	36	49	61	73	85	97	109	121	134	146	158	170	182	194	206	218	255	291	64
20	50	67	83	100	117	133	150	167	183	200	216	233	250	266	283	300	350	400	98
22	66	89	111	133	155	177	199	222	244	266	288	310	332	355	377	399	465	532	144
24	86	115	144	173	201	230	259	288	316	345	374	403	432	460	489	518	604	691	203
26	110	146	183	219	256	293	329	366	402	439	476	512	549	585	622	658	768	878	280
28	137	183	228	274	320	366	411	457	503	548	594	640	685	731	777	822	959	1097	377
30	169	225	281	337	393	450	506	562	618	674	731	787	843	899	955	1012	1180	1349	497
32	205	273	341	409	477	546	614	682	750	818	887	955	1023	1091	1159	1228	1432	1637	643
34	245	327	409	491	573	654	736	818	900	982	1063	1145	1227	1309	1391	1472	1718	1963	819
36	291	358	486	583	680	777	874	971	1068	1165	1262	1360	1457	1554	1651	1748	2039	2331	1030
42	463	617	771	925	1079	1234	1388	1542	1696	1850	2005	2159	2313	2467	2621	2776	3238	3701	1907
48	691	921	1151	1381	1611	1841	2072	2302	2532	2762	2992	3223	3453	3683	3913	4143	4834	5524	3254

COLUMNS 4.2—Values of $[(E_cI_g)/2.5] \times 10^{-5}$ for computing flexural stiffness *EI* of cracked sections of rectangular and circular columns—f'_c = 4 ksi

References: "Building Code Requirements for Structural Concrete-ACI 318" Chapters 8 and 10.

$$EI = \frac{(E_cI_g)/2.5}{1+\beta_d} = \frac{\text{table value} \times 10^5}{1+\beta_d} \text{kip-in.}^2$$

Note—This table is for concrete for which f'_c = 4 ksi, w_c = 145 pcf, and E_c = 3605 ksi. For concrete of different f'_c and E_c, multiply table value by E_c/3605 for use in computing *EI*. $0 < \beta_d < 1$.

	Rectangular columns																		
	b, in.																		
h, in.	6	8	10	12	14	16	18	20	22	24	26	28	30	32	34	36	42	48	Circular columns
6	2	2	3	3	4	4	5	5	6	6	7	7	8	8	9	9	11	12	1
8	4	5	6	7	9	10	11	12	14	15	16	17	18	20	21	22	26	30	3
10	7	10	12	14	17	19	22	24	26	29	31	34	36	38	41	43	50	58	7
12	12	17	21	25	29	33	37	42	46	50	54	58	62	66	71	75	87	100	15
14	20	26	33	40	46	53	59	66	73	79	86	92	99	106	112	119	138	158	27
16	30	39	49	59	69	79	89	98	108	118	128	138	148	158	167	177	207	236	46
18	42	56	70	84	98	112	126	140	154	168	182	196	210	224	238	252	294	336	74
20	58	77	96	115	135	154	173	192	211	231	250	269	288	308	327	346	404	461	113
22	77	102	128	154	179	205	230	256	281	307	333	358	384	409	435	461	537	614	166
24	100	133	166	199	233	266	299	332	365	399	432	465	498	532	565	598	698	797	235
26	127	169	211	253	296	338	380	422	465	507	549	591	634	676	718	760	887	1014	323
28	158	211	264	317	369	422	475	528	580	633	686	739	791	844	897	950	1108	1266	435
30	195	260	324	389	454	519	584	649	714	779	844	908	973	1038	1103	1168	1363	1557	573
32	236	315	394	473	551	630	709	788	866	945	1024	1103	1181	1260	1339	1418	1654	1890	742
34	283	378	472	567	661	756	850	945	1039	1134	1228	1322	1417	1511	1606	1700	1984	2267	946
36	336	449	561	673	785	897	1009	1121	1233	1346	1458	1570	1682	1794	1906	2018	2355	2691	1189
42	534	712	890	1068	1246	1424	1603	1781	1959	2137	2315	2493	2671	2849	3027	3205	3739	4273	2203
48	797	1063	1329	1595	1861	2126	2392	2658	2924	3189	3455	3721	3987	4523	4518	4784	5582	6379	3758

COLUMNS 4.3—Values of $[(E_cI_g)/2.5] \times 10^{-5}$ for computing flexural stiffness EI of cracked sections of rectangular and circular columns—$f_c' = 5$ ksi

References: "Building Code Requirements for Structural Concrete—ACI 318" Chapters 8 and 10.

$$EI = \frac{(E_cI_g)/2.5}{1+\beta_d} = \frac{\text{table value} \times 10^5}{1+\beta_d} \text{kip-in.}^2$$

Note—This table is for concrete for which $f_c' = 5$ ksi, $w_c = 145$ pcf, and $E_c = 4031$ ksi. For concrete of different f_c' and E_c, multiply table value by $E_c/4031$ for use in computing EI. $0 < \beta_d < 1$.

h, in.	Rectangular columns																		Circular columns
	b, in.																		
	6	8	10	12	14	16	18	20	22	24	26	28	30	32	34	36	42	48	
6	2	2	3	3	4	5	5	6	6	7	8	8	9	9	10	10	12	14	1
8	4	6	7	8	10	11	12	14	15	17	18	19	21	22	23	25	29	33	3
10	8	11	13	16	19	21	24	27	30	32	35	38	40	43	46	48	56	64	8
12	14	19	23	28	33	37	42	46	51	56	60	65	70	74	79	84	98	111	16
14	22	29	37	44	52	59	66	74	81	88	96	103	111	118	125	133	155	177	30
16	33	44	55	66	77	88	99	110	121	132	143	154	165	176	187	198	231	264	52
18	47	63	78	94	110	125	141	157	172	188	204	219	235	251	266	282	329	376	83
20	64	86	107	129	150	172	193	215	236	258	279	301	322	344	365	387	451	516	127
22	86	114	143	172	200	229	258	286	315	343	372	401	429	458	486	515	601	687	185
24	111	149	186	223	260	297	334	371	409	446	483	520	557	594	631	669	780	891	263
26	142	189	236	283	331	378	425	472	519	567	614	661	708	756	803	850	992	1133	362
28	177	236	295	354	413	472	531	590	649	708	767	826	885	944	1003	1062	1239	1416	486
30	218	290	363	435	508	580	653	725	798	871	943	1016	1088	1161	1233	1306	1524	1741	641
32	264	352	440	528	616	704	792	880	969	1057	1145	1233	1321	1409	1497	1585	1849	2113	830
34	317	422	528	634	739	845	950	1056	1162	1267	1373	1479	1584	1690	1795	1901	2218	2535	1058
36	376	501	627	752	878	1003	1128	1254	1379	1504	1630	1755	1880	2006	2131	2257	2633	3009	1329
42	597	796	995	1194	1394	1593	1792	1991	2190	2389	2588	2787	2986	3185	3384	3583	4181	4778	2463
48	891	1189	1486	1783	2080	2377	2674	2972	3269	3566	3863	4160	4457	4755	5052	5349	6240	7132	4201

COLUMNS 4.4—Values of $[(E_c I_g)/2.5] \times 10^{-5}$ for computing flexural stiffness EI of cracked sections of rectangular and circular columns—f'_c = 6 ksi

References: "Building Code Requirements for Structural Concrete—ACI 318" Chapters 8 and 10.

$$EI = \frac{(E_c I_g)/2.5}{1 + \beta_d} = \frac{\text{table value} \times 10^5}{1 + \beta_d} \text{kip-in.}^2$$

Note—This table is for concrete for which f'_c = 6 ksi, w_c = 145 pcf, and E_c = 4415 ksi. For concrete of different f'_c and E_c, multiply table value by E_c/4415 for use in computing EI. $0 < \beta_d < 1$.

h, in.	Rectangular columns, b, in.																		Circular columns
	6	8	10	12	14	16	18	20	22	24	26	28	30	32	34	36	42	48	
6	2	3	3	4	4	5	6	6	7	8	8	9	10	10	11	11	13	15	1
8	5	6	8	9	11	12	14	15	17	18	20	21	23	24	26	27	32	36	4
10	9	12	15	18	21	24	26	29	32	35	38	41	44	47	50	53	62	71	9
12	15	20	25	31	36	41	46	51	56	61	66	71	76	81	86	92	107	122	18
14	24	32	40	48	57	65	73	81	89	97	105	113	121	129	137	145	170	194	33
16	36	48	60	72	84	96	109	121	133	145	157	169	181	193	205	217	253	289	57
18	51	69	86	103	120	137	154	172	189	206	223	240	257	275	292	309	360	412	91
20	71	94	118	141	165	188	212	235	259	283	306	330	353	377	400	424	495	565	139
22	94	125	157	188	219	251	282	313	345	376	407	439	470	501	533	564	658	752	203
24	122	163	203	244	285	326	366	407	448	488	529	570	610	651	692	732	855	977	288
26	155	207	259	310	362	414	466	517	569	621	673	724	776	828	879	931	1086	1242	396
28	194	258	323	388	452	517	582	646	711	775	840	905	969	1034	1098	1163	1357	1551	533
30	238	318	397	477	556	636	715	795	874	954	1033	1113	1192	1272	1351	1431	1669	1907	702
32	289	386	482	579	675	772	868	965	1061	1157	1254	1350	1447	1543	1640	1736	2025	2315	909
34	347	463	578	694	810	926	1041	1157	1273	1388	1504	1620	1735	1851	1967	2082	2429	2777	1159
36	412	549	687	824	961	1099	1236	1373	1511	1648	1785	1923	2060	2197	2335	2472	2884	3296	1456
42	654	872	1090	1308	1527	1745	1963	2181	2399	2617	2835	3053	3271	3489	3707	3925	4580	5324	2698
48	977	1302	1628	1953	2279	2604	2930	3255	3581	3906	4232	4557	4883	5208	5534	5859	6836	7813	4602

COLUMNS 4.5—Values of $[(E_cI_g)/2.5] \times 10^{-5}$ for computing flexural stiffness EI of cracked sections of rectangular and circular columns—$f_c' = 9$ ksi

References: "Building Code Requirements for Structural Concrete—ACI 318" Chapters 8 and 10.

$$EI = \frac{(E_cI_g)/2.5}{1+\beta_d} = \frac{\text{table value} \times 10^5}{1+\beta_d} \text{kip-in.}^2$$

Note—This table is for concrete for which $f_c' = 9$ ksi, $w_c = 145$ pcf, and $E_c = 5407$ ksi. For concrete of different f_c' and E_c, multiply table value by $E_c/5407$ for use in computing EI. $0 < \beta_d < 1$.

	Rectangular columns																		
	b, in.																		
h, in.	6	8	10	12	14	16	18	20	22	24	26	28	30	32	34	36	42	48	Circular columns
6	2	3	4	5	5	6	7	8	9	9	10	11	12	12	13	14	16	19	1
8	6	7	9	11	13	15	17	18	20	22	24	26	28	30	31	33	39	44	4
10	11	14	18	22	25	29	32	36	40	43	47	50	54	58	61	65	16	87	11
12	19	25	31	37	44	50	56	62	69	75	81	87	93	100	106	112	131	150	22
14	30	40	49	59	69	79	89	99	109	119	129	138	148	158	168	178	208	237	41
16	44	59	74	89	103	118	133	148	162	177	192	207	221	226	251	266	310	354	70
18	63	84	105	126	147	168	189	210	231	252	273	294	315	336	357	378	442	505	111
20	87	115	144	173	202	231	260	288	317	346	375	404	433	461	490	519	606	692	170
22	115	154	192	230	269	307	345	384	422	461	499	537	576	614	653	691	806	921	249
24	150	199	249	299	349	399	449	498	548	598	648	698	748	797	847	897	1047	1196	352
26	190	253	317	380	444	507	570	634	697	760	824	887	950	1014	1077	1141	1331	1521	485
28	237	317	396	475	554	633	712	791	871	950	1029	1108	1187	1266	1345	1424	1662	1899	653
30	292	389	487	584	681	779	876	973	1071	1168	1265	1363	1460	1557	1655	1752	2044	2336	860
32	354	473	591	709	827	945	1063	1181	1299	1418	1536	1654	1772	1890	2008	2126	2481	2835	1113
34	425	567	708	850	992	1134	1275	1417	1559	1700	1842	1984	2125	2267	2409	2550	2976	3401	1419
36	505	673	841	1009	1177	1346	1514	1682	1850	2018	2187	2355	2523	2691	2859	3028	3532	4037	1783
42	801	1068	1335	1603	1870	2137	2404	2671	2938	3205	3472	3739	4006	4273	4540	4808	5609	6410	3304
48	1196	1595	1993	2392	2791	3189	3588	3987	4386	4784	5183	5582	5980	6379	6778	7176	8372	9568	5636

COLUMNS 4.6—Values of $[(E_c I_g)/2.5] \times 10^{-5}$ for computing flexural stiffness EI of cracked sections of rectangular and circular columns—$f_c' = 12$ ksi

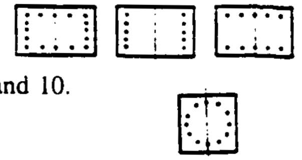
References: "Building Code Requirements for Structural Concrete-ACI 318" Chapters 8 and 10.

$$EI = \frac{(E_c I_g)/2.5}{1 + \beta_d} = \frac{\text{table value} \times 10^5}{1 + \beta_d} \text{kip-in.}^2$$

Note—This table is for concrete for which $f_c' = 12$ ksi, $w_c = 145$ pcf, and $E_c = 6244$ ksi. For concrete of different f_c' and E_c, multiply table value by $E_c/6244$ for use in computing EI. $0 < \beta_d < 1$.

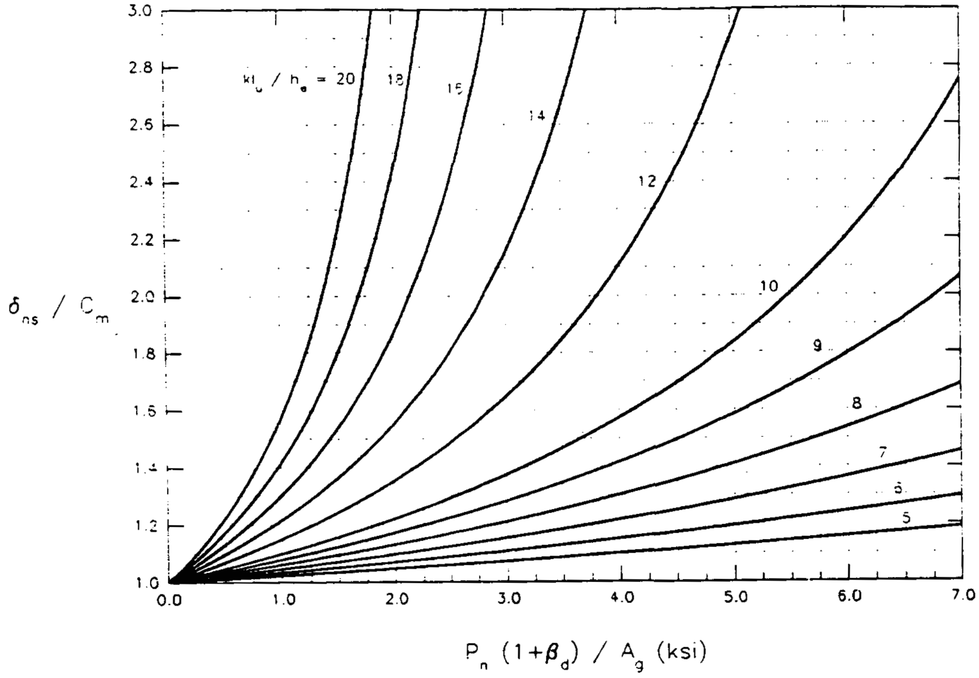
	Rectangular columns																		
	b, in.																		
h, in.	6	8	10	12	14	16	18	20	22	24	26	28	30	32	34	36	42	48	Circular columns
6	3	4	4	5	6	7	8	9	10	11	12	13	13	14	15	16	19	22	2
8	6	9	11	13	15	17	19	21	23	26	28	30	32	34	36	38	45	51	5
10	12	17	21	25	29	33	37	42	46	50	54	58	62	67	71	75	87	100	12
12	22	29	36	43	50	58	65	72	79	86	94	101	108	115	122	129	151	173	25
14	34	46	57	69	80	91	103	114	126	137	148	160	171	183	194	206	240	274	47
16	51	68	85	102	119	136	153	171	188	205	222	239	256	273	290	307	358	409	80
18	73	97	121	146	170	194	218	243	267	291	316	340	364	388	413	437	510	583	129
20	100	133	167	200	233	266	300	333	366	400	433	466	500	533	566	599	699	799	196
22	133	177	222	266	310	355	399	443	488	532	576	621	665	709	754	798	931	1064	287
24	173	230	288	345	403	460	518	575	633	691	748	806	863	921	978	1036	1208	1381	407
26	219	293	366	439	512	585	658	732	805	878	951	1024	1097	1171	1244	1317	1536	1756	560
28	274	366	457	548	640	731	822	914	1005	1097	1188	1279	1371	1462	1553	1645	1919	2193	754
30	337	450	562	674	787	899	1012	1124	1236	1349	1461	1573	1686	1798	1911	2023	2360	2697	993
32	409	546	682	818	955	1091	1228	1364	1500	1637	1773	1910	2046	2182	2319	2455	2864	3274	1286
34	491	654	818	982	1145	1309	1472	1636	1800	1963	2127	2291	2454	2618	2781	2945	3436	3927	1638
36	583	777	971	1165	1360	1554	1748	1942	2136	2331	2525	2719	2913	3107	3302	3496	4079	4661	2059
42	925	1234	1542	1850	2159	2467	2776	3084	3392	3701	4009	4318	4626	4934	5243	5551	6477	7402	3815
48	1381	1841	2302	2762	3223	3683	4143	4604	5064	5545	5985	6445	6905	7366	7826	8286	9668	11049	6508

COLUMNS 5.1–Moment magnifier term δ_{ns}/C_m for rectangular tied columns and square columns with steel arranged in a circle–$f_c' = 3$ ksi

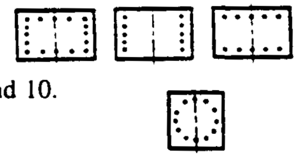


References: "Building Code Requirements for Structural Concrete-ACI 318" Chapters 8, 9 and 10.

$$\frac{\delta_{ns}}{C_m} = \frac{1}{1 - (0.000910)\left(\dfrac{P_n(1+\beta_d)}{A_g}\right)\left(\dfrac{kl_u}{h_e}\right)^2}$$

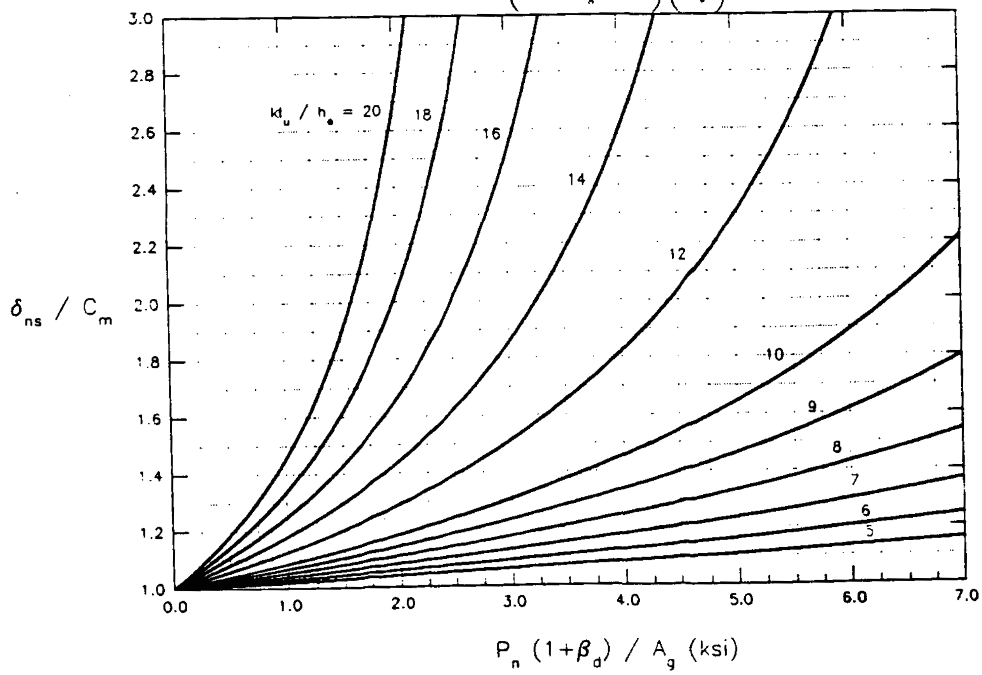


COLUMNS 5.2–Moment magnifier term δ_{ns}/C_m for rectangular tied columns and square columns with steel arranged in a circle–$f_c' = 4$ ksi

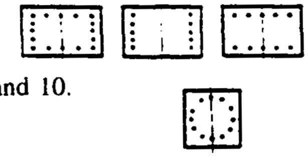


References: "Building Code Requirements for Structural Concrete-ACI 318" Chapters 8, 9 and 10.

$$\frac{\delta_{ns}}{C_m} = \frac{1}{1 - (0.000784)\left(\dfrac{P_n(1+\beta_d)}{A_g}\right)\left(\dfrac{kl_u}{h_e}\right)^2}$$

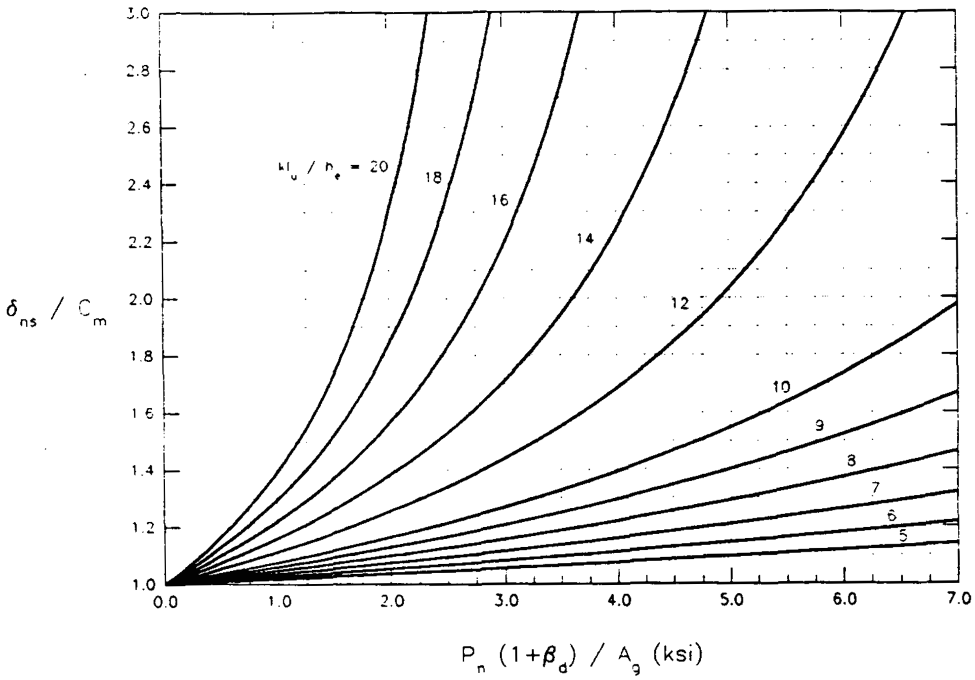


COLUMNS 5.3–Moment magnifier term δ_{ns}/C_m for rectangular tied columns and square columns with steel arranged in a circle–f'_c = 5 ksi

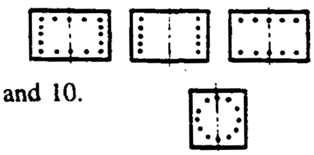


References: "Building Code Requirements for Structural Concrete-ACI 318" Chapters 8, 9 and 10.

$$\frac{\delta_{ns}}{C_m} = \frac{1}{1 - (0.000707)\left(\frac{P_n\,(1 + \beta_d)}{A_g}\right)\left(\frac{kl_u}{h_e}\right)^2}$$

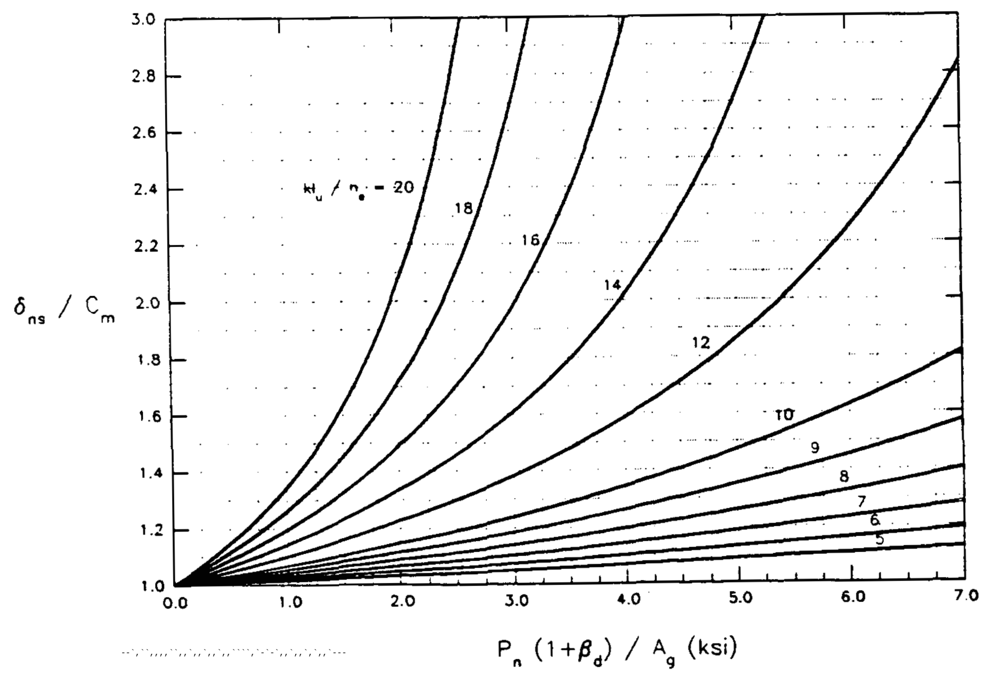


COLUMNS 5.4–Moment magnifier term δ_{ns}/C_m for rectangular tied columns and square columns with steel arranged in a circle–f'_c = 6 ksi

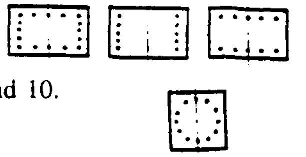


References: "Building Code Requirements for Structural Concrete-ACI 318" Chapters 8, 9 and 10.

$$\frac{\delta_{ns}}{C_m} = \frac{1}{1 - (0.000643)\left(\frac{P_n\,(1 + \beta_d)}{A_g}\right)\left(\frac{kl_u}{h_e}\right)^2}$$

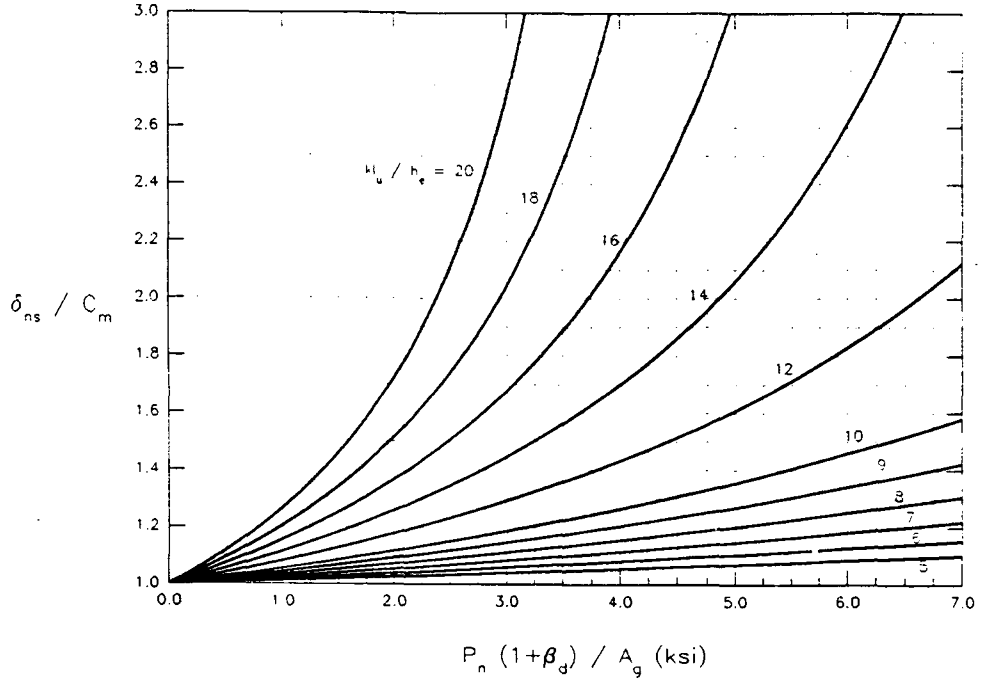


COLUMNS 5.5—Moment magnifier term δ_{ns}/C_m for rectangular tied columns and square columns with steel arranged in a circle—f'_c = 9 ksi

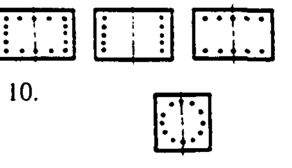


References: "Building Code Requirements for Structural Concrete-ACI 318" Chapters 8, 9 and 10.

$$\frac{\delta_{ns}}{C_m} = \frac{1}{1 - (0.000525)\left(\frac{P_n(1+\beta_d)}{A_g}\right)\left(\frac{kl_u}{h_e}\right)^2}$$

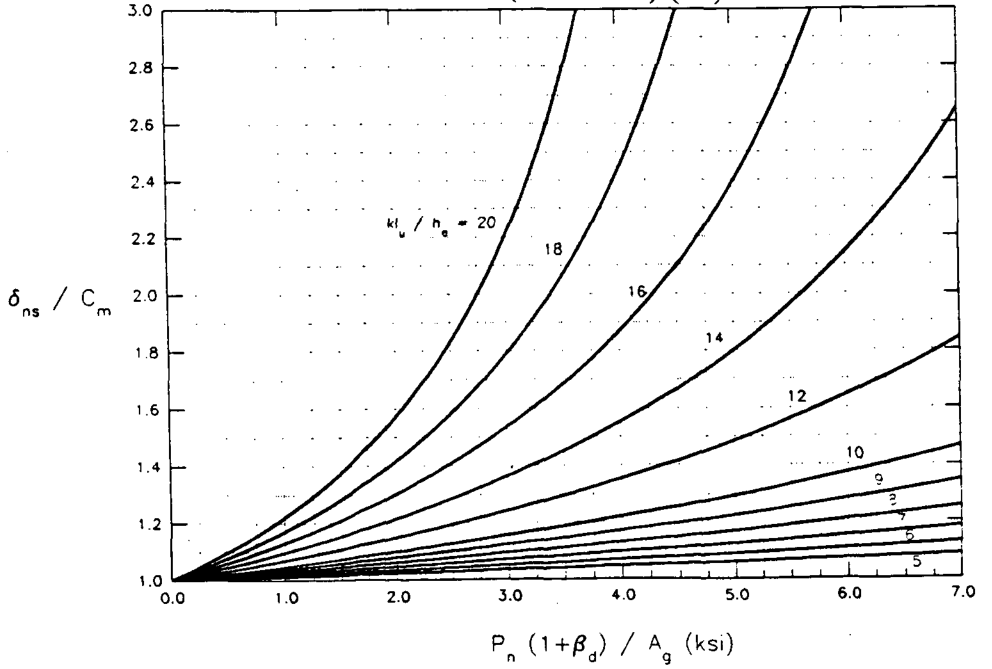


COLUMNS 5.6—Moment magnifier term δ_{ns}/C_m for rectangular tied columns and square columns with steel arranged in a circle—f'_c = 12 ksi



References: "Building Code Requirements for Structural Concrete-ACI 318" Chapters 8, 9 and 10.

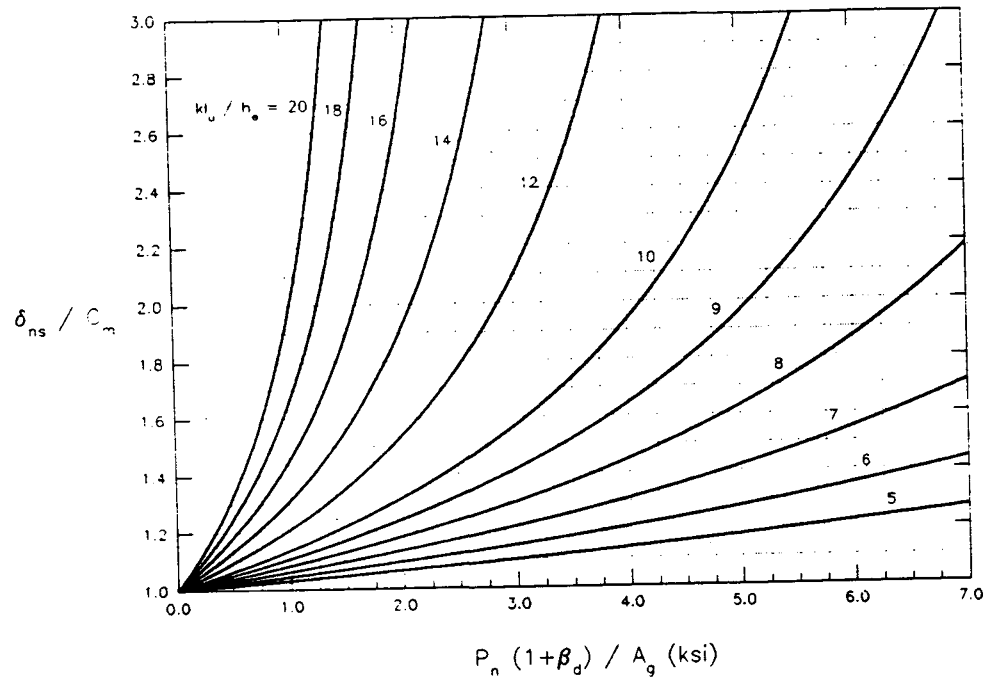
$$\frac{\delta_{ns}}{C_m} = \frac{1}{1 - (0.000454)\left(\frac{P_n(1+\beta_d)}{A_g}\right)\left(\frac{kl_u}{h_e}\right)^2}$$



COLUMNS 5.7—Moment magnifier term δ_{ns}/C_m for circular columns—f'_c = 3 ksi

References: "Building Code Requirements for Structural Concrete-ACI 318" Chapters 8, 9 and 10.

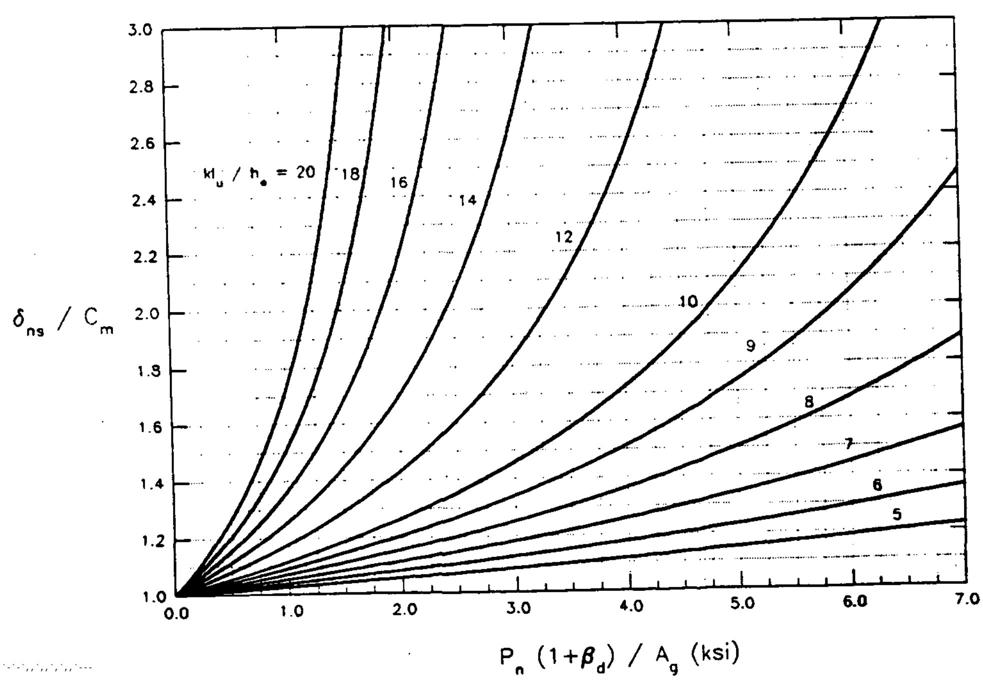
$$\frac{\delta_{ns}}{C_m} = \frac{1}{1 - (0.00121)\left(\frac{P_n(1+\beta_d)}{A_g}\right)\left(\frac{kl_u}{h_e}\right)^2}$$



COLUMNS 5.8—Moment magnifier term δ_{ns}/C_m for circular columns—f'_c = 4 ksi

References: "Building Code Requirements for Structural Concrete-ACI 318' Chapters 8, 9 and 10.

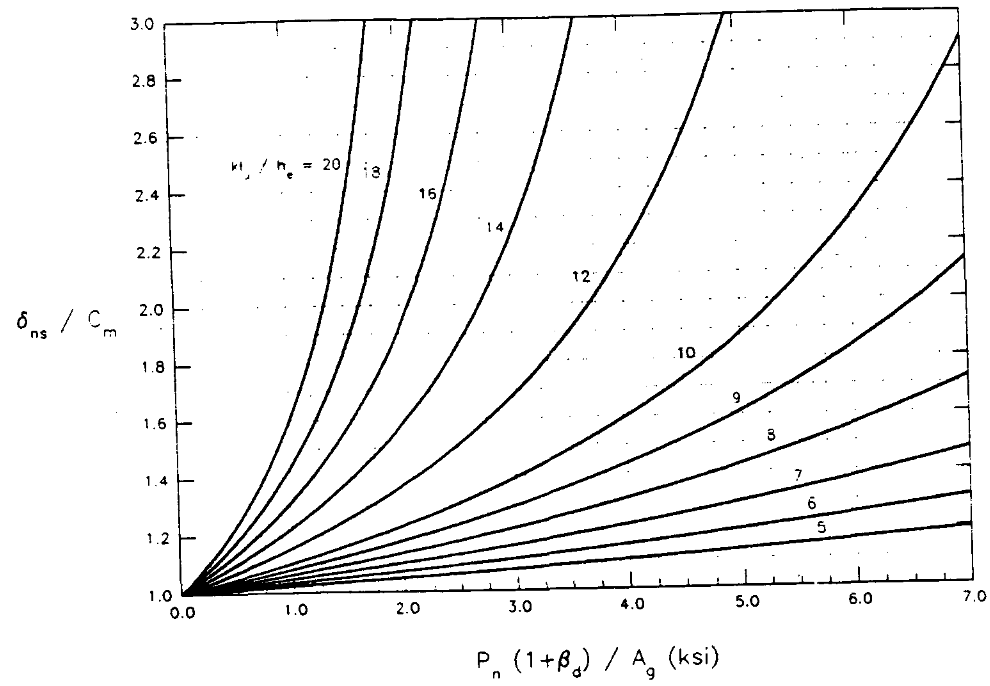
$$\frac{\delta_{ns}}{C_m} = \frac{1}{1 - (0.00105)\left(\frac{P_n(1+\beta_d)}{A_g}\right)\left(\frac{kl_u}{h_e}\right)^2}$$



COLUMNS 5.9—Moment magnifier term δ_{ns}/C_m for circular columns—f'_c = 5 ksi

References: "Building Code Requirements for Structural Concrete-ACI 318" Chapters 8, 9 and 10.

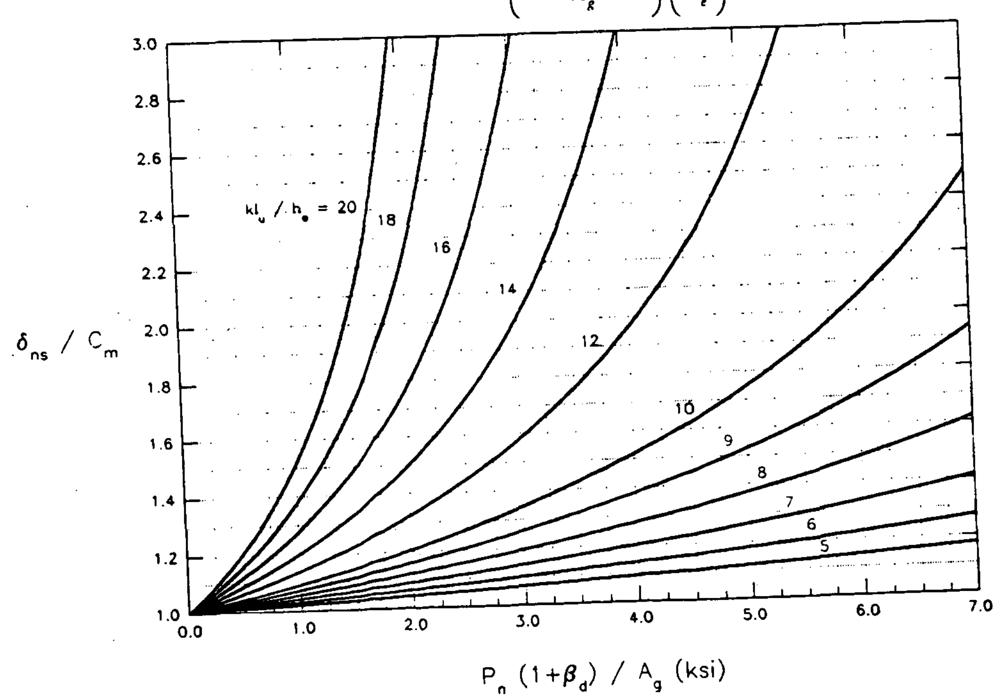
$$\frac{\delta_{ns}}{C_m} = \frac{1}{1 - (0.000938)\left(\frac{P_n\,(1+\beta_d)}{A_g}\right)\left(\frac{kl_u}{h_e}\right)^2}$$



COLUMNS 5.10—Moment magnifier term δ_{ns}/C_m for circular columns—f'_c = 6 ksi

References: "Building Code Requirements for Structural Concrete-ACI 318" Chapters 8, 9 and 10.

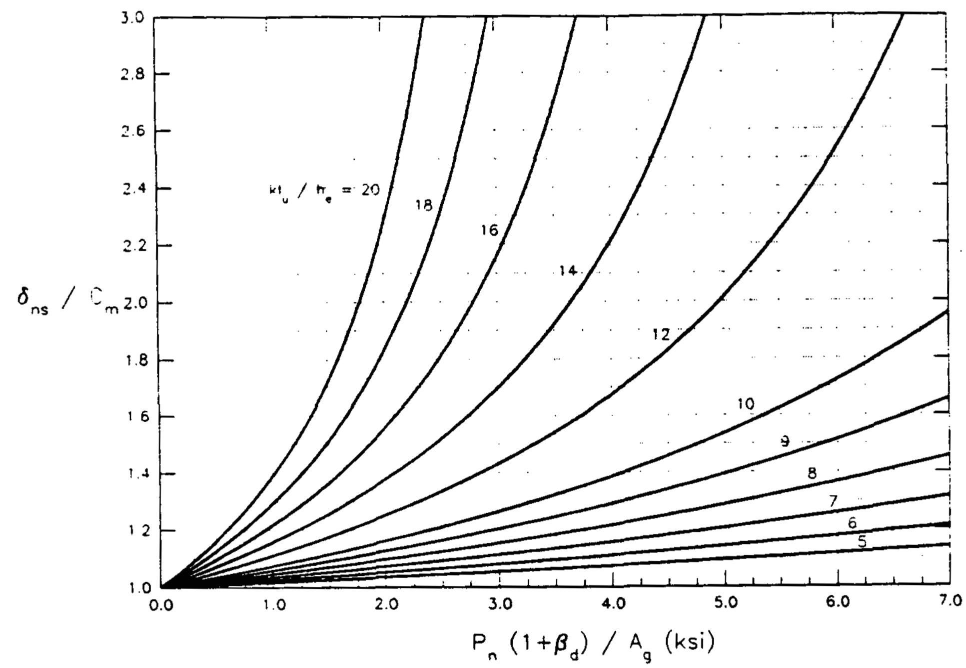
$$\frac{\delta_{ns}}{C_m} = \frac{1}{1 - (0.000854)\left(\frac{P_n\,(1+\beta_d)}{A_g}\right)\left(\frac{kl_u}{h_e}\right)^2}$$



COLUMNS 5.11—Moment magnifier term δ_{ns}/C_m for circular columns—f'_c = 9 ksi

References: "Building Code Requirements for Structural Concrete-ACI 318" Chapters 8, 9 and 10.

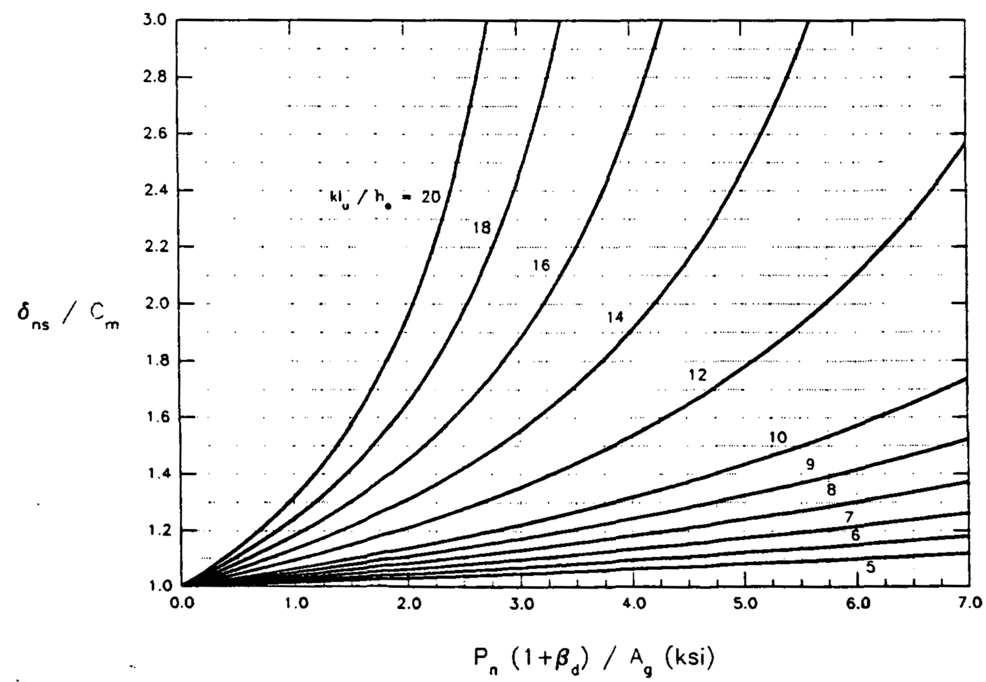
$$\frac{\delta_{ns}}{C_m} = \frac{1}{1 - (0.000699)\left(\frac{P_n(1+\beta_d)}{A_g}\right)\left(\frac{kl_u}{h_e}\right)^2}$$



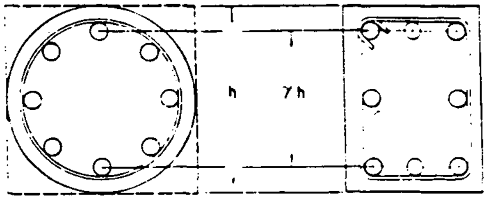
COLUMNS 5.12—Moment magnifier term δ_{ns}/C_m for circular columns—f'_c = 12 ksi

References: "Building Code Requirements for Structural Concrete-ACI 318" Chapters 8, 9 and 10.

$$\frac{\delta_{ns}}{C_m} = \frac{1}{1 - (0.000606)\left(\frac{P_n(1+\beta_d)}{A_g}\right)\left(\frac{kl_u}{h_e}\right)^2}$$



COLUMNS 6.1—Values of γ for column cross sections—For #3 and #4 ties or spirals



γ = ratio of distance between centroids of outer rows of bars and thickness of cross section, in the direction of bending

$$= \frac{h - (2 \times \text{cover}) - (2 \times \text{tie thickness}) - d_b}{h}$$

#3 ties or spirals*

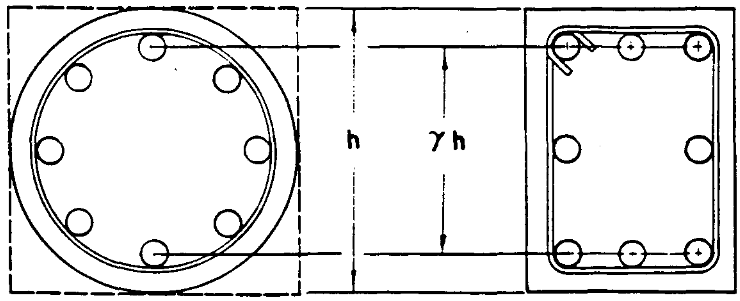
h, in.	1 ½ in. cover: Bar size #6	#7	#8	#9	#10	#11†	#14†	#18†	2 in. cover: Bar size #6	#7	#8	#9	#10	#11†	#14†	#18†	3 in. cover: Bar size #6	#7	#8	#9	#10	#11†	#14†	#18†
8	0.44	0.42	0.41	0.39	—	—	—	—	0.31	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
9	0.50	0.49	0.47	0.46	0.44	—	—	—	0.39	0.38	0.36	0.35	—	—	—	—	—	—	—	—	—	—	—	—
10	0.55	0.54	0.52	0.51	0.50	—	—	—	0.45	0.44	0.43	0.41	0.40	—	—	—	0.25	—	—	—	—	—	—	—
11	0.59	0.58	0.57	0.56	0.54	—	—	—	0.50	0.49	0.48	0.47	0.45	—	—	—	0.32	0.31	0.30	0.28	—	—	—	—
12	0.62	0.61	0.60	0.59	0.58	—	—	—	0.54	0.53	0.52	0.51	0.50	—	—	—	0.38	0.36	0.35	0.33	0.33	—	—	—
13	0.65	0.64	0.63	0.62	0.61	0.60	—	—	0.58	0.57	0.56	0.55	0.54	0.53	—	—	0.42	0.41	0.40	0.39	0.38	—	—	—
14	0.68	0.67	0.66	0.65	0.64	0.63	—	—	0.61	0.60	0.59	0.58	0.57	0.56	—	—	0.46	0.46	0.45	0.44	0.43	—	—	—
15	0.70	0.69	0.68	0.68	0.67	0.66	0.64	—	0.63	0.62	0.62	0.61	0.60	0.59	0.57	—	0.50	0.49	0.48	0.47	0.47	0.46	—	—
16	0.72	0.71	0.70	0.70	0.69	0.68	0.66	—	0.66	0.65	0.64	0.63	0.62	0.61	0.60	—	0.53	0.52	0.52	0.51	0.50	0.49	—	—
17	0.74	0.73	0.72	0.71	0.70	0.70	0.68	—	0.68	0.67	0.66	0.65	0.65	0.64	0.62	—	0.56	0.55	0.54	0.54	0.53	0.52	0.50	—
18	0.75	0.74	0.74	0.73	0.72	0.71	0.70	—	0.69	0.69	0.68	0.67	0.67	0.66	0.64	—	0.58	0.58	0.57	0.56	0.56	0.55	0.53	—
19	0.76	0.76	0.75	0.74	0.74	0.73	0.71	—	0.71	0.70	0.70	0.69	0.68	0.68	0.66	—	0.61	0.60	0.59	0.59	0.58	0.57	0.56	—
20	0.78	0.77	0.76	0.76	0.75	0.74	0.73	0.70	0.72	0.72	0.71	0.71	0.70	0.69	0.68	—	0.62	0.62	0.61	0.61	0.60	0.59	0.58	—
21	0.79	0.78	0.77	0.77	0.76	0.75	0.74	0.71	0.74	0.73	0.73	0.72	0.71	0.69	0.69	0.67	0.64	0.64	0.63	0.62	0.62	0.61	0.60	—
22	0.80	0.79	0.78	0.78	0.77	0.77	0.75	0.73	0.75	0.74	0.74	0.73	0.73	0.72	0.71	0.68	0.66	0.65	0.65	0.64	0.64	0.63	0.62	—
23	0.80	0.80	0.79	0.79	0.78	0.78	0.76	0.74	0.76	0.76	0.75	0.74	0.74	0.73	0.72	0.70	0.67	0.67	0.66	0.66	0.65	0.65	0.63	0.61
24	0.81	0.81	0.80	0.80	0.79	0.78	0.77	0.75	0.77	0.76	0.76	0.76	0.75	0.74	0.73	0.71	0.69	0.68	0.68	0.67	0.67	0.66	0.65	0.62

#4 ties or spirals

h, in.	1 ½ in. cover: Bar size #6	#7	#8	#9	#10	#11†	#14†	#18†	2 in. cover: Bar size #6	#7	#8	#9	#10	#11†	#14†	#18†	3 in. cover: Bar size #6	#7	#8	#9	#10	#11†	#14†	#18†
10	0.52	0.51	0.50	0.49	0.47	0.46	—	—	0.42	0.41	0.40	0.39	0.37	0.36	—	—	0.22	—	—	—	—	—	—	—
11	0.57	0.56	0.55	0.53	0.52	0.51	0.48	—	0.48	0.47	0.45	0.44	0.43	0.42	0.39	—	0.30	0.28	0.27	0.26	—	—	—	—
12	0.60	0.59	0.58	0.57	0.56	0.55	0.53	—	0.52	0.51	0.50	0.49	0.48	0.47	0.43	—	0.35	0.34	0.33	0.32	0.31	0.30	—	—
13	0.63	0.62	0.62	0.61	0.59	0.58	0.56	—	0.56	0.55	0.54	0.53	0.52	0.51	0.49	—	0.40	0.39	0.38	0.37	0.36	0.35	0.33	—
14	0.66	0.65	0.64	0.63	0.62	0.61	0.59	—	0.59	0.58	0.57	0.56	0.55	0.54	0.52	—	0.45	0.44	0.43	0.42	0.41	0.40	0.38	—
15	0.68	0.68	0.67	0.66	0.65	0.64	0.62	0.58	0.62	0.61	0.60	0.59	0.58	0.57	0.55	0.52	0.48	0.48	0.47	0.46	0.45	0.44	0.42	0.38
16	0.70	0.70	0.69	0.68	0.67	0.66	0.64	0.61	0.64	0.63	0.62	0.62	0.61	0.60	0.58	0.55	0.52	0.51	0.50	0.49	0.48	0.47	0.46	0.42
17	0.72	0.71	0.71	0.70	0.69	0.68	0.66	0.63	0.66	0.65	0.65	0.64	0.63	0.62	0.61	0.57	0.54	0.54	0.53	0.52	0.51	0.51	0.49	0.46
18	0.74	0.73	0.72	0.72	0.71	0.70	0.68	0.65	0.68	0.67	0.67	0.66	0.65	0.64	0.63	0.60	0.57	0.56	0.56	0.55	0.54	0.53	0.52	0.49
19	0.75	0.74	0.74	0.73	0.72	0.72	0.70	0.67	0.70	0.69	0.68	0.68	0.67	0.66	0.65	0.62	0.59	0.59	0.58	0.57	0.56	0.56	0.54	0.51
20	0.76	0.76	0.75	0.74	0.74	0.73	0.72	0.69	0.71	0.71	0.70	0.69	0.69	0.68	0.67	0.64	0.61	0.61	0.60	0.59	0.59	0.58	0.57	0.54
21	0.77	0.77	0.76	0.76	0.75	0.74	0.73	0.70	0.73	0.72	0.71	0.71	0.70	0.69	0.68	0.65	0.63	0.62	0.62	0.61	0.61	0.60	0.59	0.56
22	0.78	0.78	0.77	0.77	0.76	0.75	0.74	0.72	0.74	0.73	0.73	0.72	0.72	0.71	0.70	0.67	0.65	0.64	0.64	0.63	0.62	0.62	0.60	0.58
23	0.79	0.79	0.78	0.78	0.77	0.76	0.75	0.73	0.75	0.74	0.74	0.73	0.73	0.72	0.71	0.68	0.66	0.66	0.65	0.65	0.64	0.63	0.62	0.60
24	0.80	0.80	0.79	0.79	0.78	0.77	0.76	0.74	0.76	0.76	0.75	0.74	0.74	0.73	0.72	0.70	0.68	0.67	0.67	0.66	0.66	0.65	0.64	0.61
26	0.82	0.81	0.81	0.80	0.80	0.79	0.78	0.76	0.78	0.77	0.77	0.76	0.76	0.75	0.74	0.72	0.70	0.70	0.69	0.69	0.68	0.68	0.67	0.64
28	0.83	0.83	0.82	0.82	0.81	0.81	0.80	0.78	0.79	0.79	0.79	0.78	0.78	0.77	0.76	0.74	0.72	0.72	0.71	0.71	0.70	0.70	0.69	0.67
30	0.84	0.84	0.83	0.83	0.82	0.82	0.81	0.79	0.81	0.80	0.80	0.80	0.79	0.79	0.78	0.76	0.74	0.74	0.73	0.73	0.72	0.72	0.71	0.69
32	0.85	0.85	0.84	0.84	0.83	0.83	0.82	0.80	0.82	0.82	0.81	0.81	0.80	0.80	0.79	0.77	0.76	0.75	0.75	0.75	0.74	0.74	0.73	0.71
34	0.86	0.86	0.85	0.85	0.84	0.84	0.83	0.82	0.83	0.83	0.82	0.82	0.82	0.81	0.80	0.79	0.77	0.77	0.76	0.76	0.76	0.75	0.74	0.73
36	0.87	0.86	0.86	0.86	0.85	0.85	0.84	0.82	0.84	0.84	0.83	0.83	0.83	0.82	0.81	0.80	0.78	0.78	0.78	0.77	0.77	0.77	0.76	0.74
38	0.88	0.87	0.87	0.87	0.86	0.86	0.85	0.84	0.85	0.85	0.84	0.84	0.84	0.83	0.82	0.81	0.80	0.79	0.79	0.79	0.78	0.78	0.77	0.76
40	0.88	0.88	0.88	0.87	0.87	0.86	0.86	0.84	0.86	0.85	0.85	0.85	0.84	0.84	0.83	0.82	0.81	0.80	0.80	0.80	0.79	0.79	0.78	0.77
42	0.89	0.88	0.88	0.88	0.87	0.86	0.86	0.85	0.86	0.86	0.86	0.85	0.85	0.85	0.84	0.83	0.82	0.81	0.81	0.81	0.80	0.80	0.79	0.78
44	0.89	0.89	0.89	0.88	0.88	0.87	0.87	0.86	0.87	0.87	0.86	0.86	0.86	0.85	0.85	0.84	0.82	0.82	0.82	0.82	0.81	0.81	0.80	0.79
46	0.90	0.89	0.89	0.89	0.89	0.88	0.88	0.86	0.88	0.87	0.87	0.87	0.86	0.86	0.85	0.84	0.83	0.83	0.83	0.82	0.82	0.82	0.81	0.80
48	0.90	0.90	0.90	0.89	0.89	0.89	0.88	0.87	0.88	0.88	0.88	0.87	0.87	0.87	0.86	0.85	0.84	0.84	0.83	0.83	0.83	0.82	0.82	0.81
50	0.90	0.90	0.90	0.90	0.89	0.89	0.89	0.87	0.88	0.88	0.88	0.88	0.87	0.87	0.87	0.85	0.84	0.84	0.84	0.84	0.83	0.83	0.83	0.81

* #3 bars may not be readily available.

† #3 ties are not permitted with #11, #14, and #18 bars (Section 7.10.5.1 of ACI 318-95); however, #3 spirals may be used with these bar sizes.



γ = ratio of distance between centroids of outer rows of bars and thickness of cross section, in the direction of bending

$$= \frac{h - (2 \times \text{cover}) - (2 \times \text{tie thickness}) - d_b}{h}$$

h, in.	1 ½ in. cover								2 in. cover								3 in. cover							
	Bar size #6	#7	#8	#9	#10	#11	#14	#18	Bar size #6	#7	#8	#9	#10	#11	#14	#18	Bar size #6	#7	#8	#9	#10	#11	#14	#18
	#5 ties or spirals																							
24	0.79	0.79	0.78	0.78	0.77	0.76	0.75	0.73	0.75	0.74	0.74	0.73	0.73	0.72	0.71	0.69	0.67	0.66	0.66	0.65	0.64	0.64	0.63	0.60
26	0.81	0.80	0.80.	0.79	0.79	0.78	0.77	0.75	0.77	0.76	0.76	0.75	0.75	0.74	0.73	0.71	0.69	0.69	0.68	0.68	0.67	0.67	0.66	0.63
28	0.82	0.82	0.81	0.81	0.80	0.80	0.79	0.77	0.79	0.78	0.78	0.77	0.77	0.76	0.75	0.73	0.71	0.71	0.71	0.70	0.70	0.69	0.68	0.64
30	0.83	0.83	0.82	0.82	0.82	0.81	0.80	0.78	0.80	0.80	0.79	0.79	0.78	0.78	0.77	0.75	0.73	0.73	0.72	0.72	0.72	0.71	0.70	0.68
32	0.84	0.84	0.84	0.83	0.83	0.82	0.81	0.80	0.81	0.81	0.80	0.80	0.80	0.79	0.78	0.77	0.75	0.75	0.74	0.74	0.73	0.73	0.72	0.70
34	0.85	0.85	0.85	0.84	0.84	0.83	0.83	0.81	0.82	0.82	0.82	0.81	0.81	0.80	0.80	0.78	0.76	0.76	0.76	0.75	0.75	0.75	0.74	0.70
36	0.86	0.86	0.85	0.85	0.85	0.84	0.83	0.82	0.83	0.83	0.83	0.82	0.82	0.82	0.81	0.79	0.78	0.77	0.77	0.77	0.76	0.76	0.75	0.74
38	0.87	0.87	0.86	0.86	0.85	0.85	0.84	0.83	0.84	0.84	0.84	0.83	0.83	0.82	0.82	0.80	0.79	0.79	0.78	0.78	0.78	0.77	0.76	0.75
40	0.88	0.87	0.87	0.87	0.86	0.86	0.85	0.83	0.85	0.85	0.84	0.84	0.84	0.83	0.83	0.81	0.80	0.80	0.79	0.79	0.79	0.78	0.78	0.76
42	0.88	0.88	0.88	0.87	0.87	0.87	0.86	0.85	0.86	0.85	0.85	0.85	0.84	0.84	0.83	0.82	0.81	0.81	0.80	0.80	0.80	0.79	0.79	0.77
44	0.89	0.88	0.88	0.88	0.87	0.87	0.86	0.85	0.86	0.86	0.86	0.86	0.85	0.85	0.84	0.83	0.82	0.82	0.81	0.81	0.81	0.80	0.80	0.78
46	0.89	0.89	0.89	0.88	0.88	0.88	0.87	0.86	0.87	0.87	0.86	0.86	0.86	0.86	0.85	0.84	0.83	0.82	0.82	0.82	0.81	0.81	0.81	0.79
48	0.90	0.89	0.89	0.89	0.88	0.88	0.88	0.86	0.88	0.87	0.87	0.87	0.86	0.86	0.86	0.84	0.83	0.83	0.83	0.83	0.82	0.82	0.81	0.80
50	0.90	0.90	0.90	0.89	0.89	0.89	0.89	0.87	0.88	0.88	0.88	0.87	0.87	0.87	0.86	0.85	0.84	0.84	0.84	0.83	0.83	0.83	0.82	0.81

COLUMNS 7.1.1 - Nominal load-moment strength interaction diagram, R3-60.6

References: "Building Code Requirements for Structural Concrete-ACI 318" Chapters 9 and 10, and "Ultimate Strength Design of Reinforced Concrete Columns", ACI Special Publication SP-7, by Everard and Cohen, 1964, pp. 152-182 (with corrections).

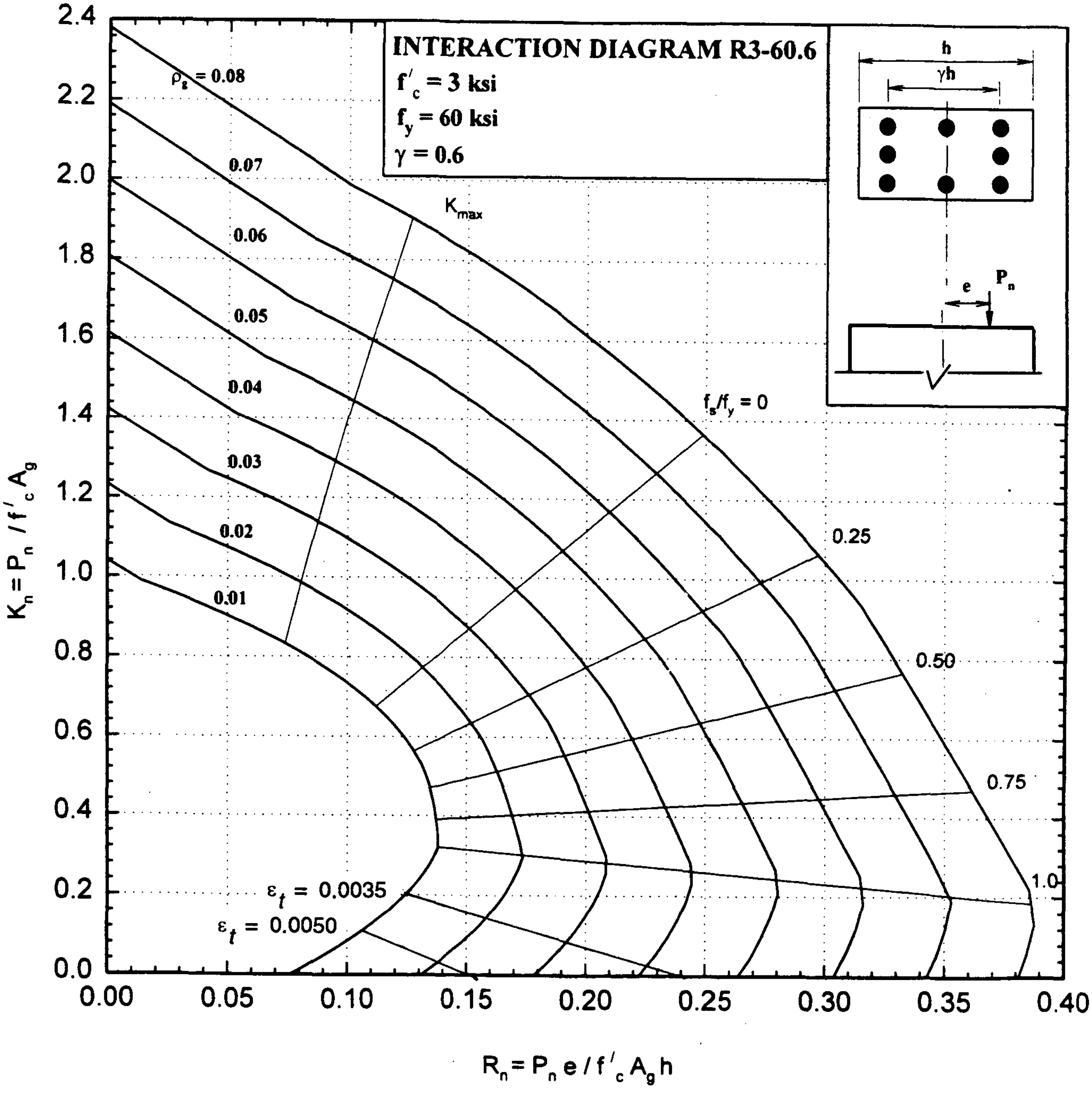


Diagram plotted using data from computer programs developed by Dr. Noel J. Everard.
Computer plot of data for diagram by Dr. Mohsen A. Issa and Alfred A. Yousif.

COLUMNS 7.1.2 - Nominal load-moment strength interaction diagram, R3-60.7

References: "Building Code Requirements for Structural Concrete-ACI 318" Chapters 9 and 10, and "Ultimate Strength Design of Reinforced Concrete Columns", ACI Special Publication SP-7, by Everard and Cohen, 1964, pp. 152-182 (with corrections).

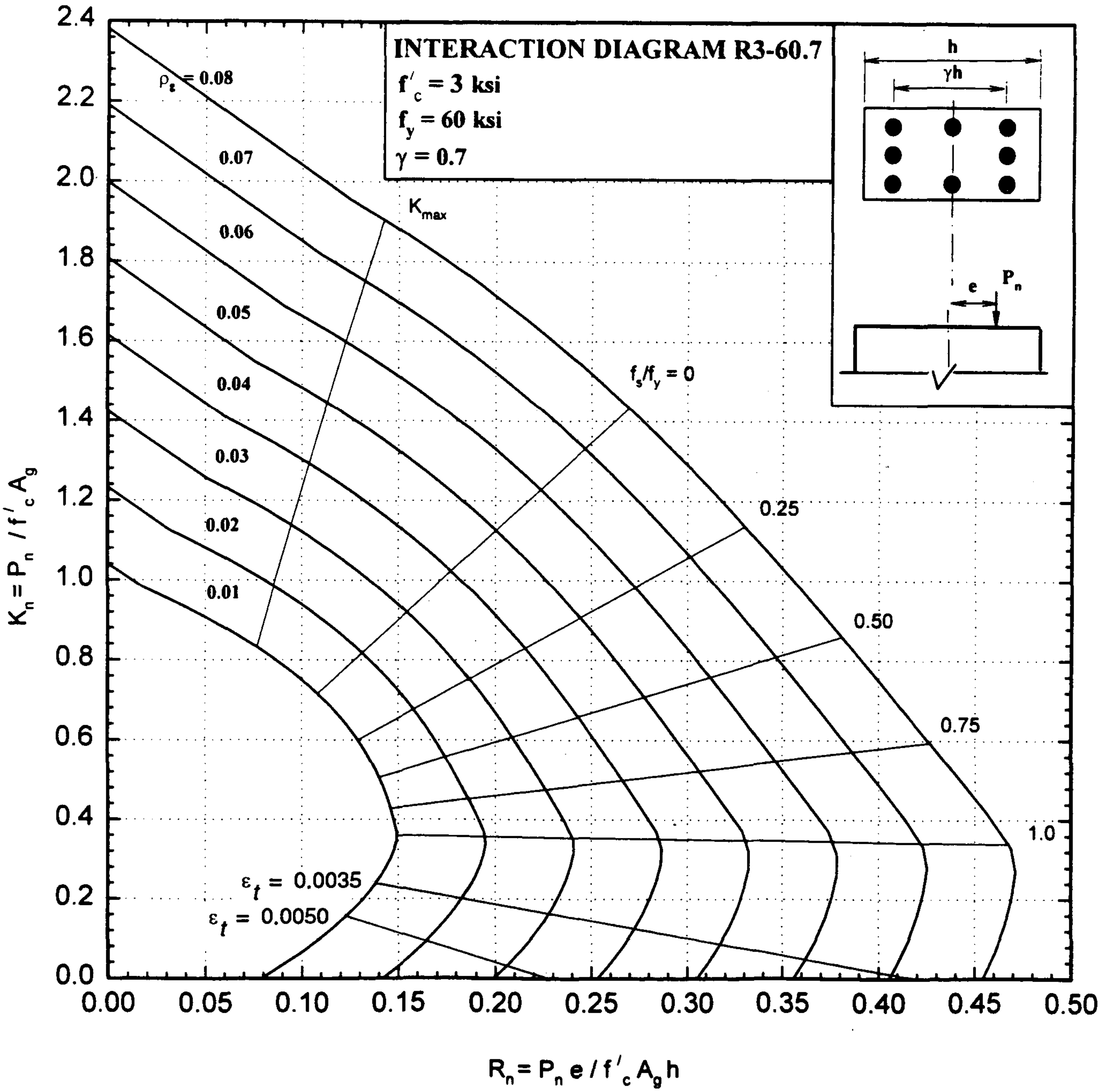


Diagram plotted using data from computer programs developed by Dr. Noel J. Everard.
Computer plot of data for diagram by Dr. Mohsen A. Issa and Alfred A. Yousif.

COLUMNS 7.1.3 - Nominal load-moment strength interaction diagram, R3-60.8

References: "Building Code Requirements for Structural Concrete-ACI 318" Chapters 9 and 10, and "Ultimate Strength Design of Reinforced Concrete Columns", ACI Special Publication SP-7, by Everard and Cohen, 1964, pp. 152-182 (with corrections).

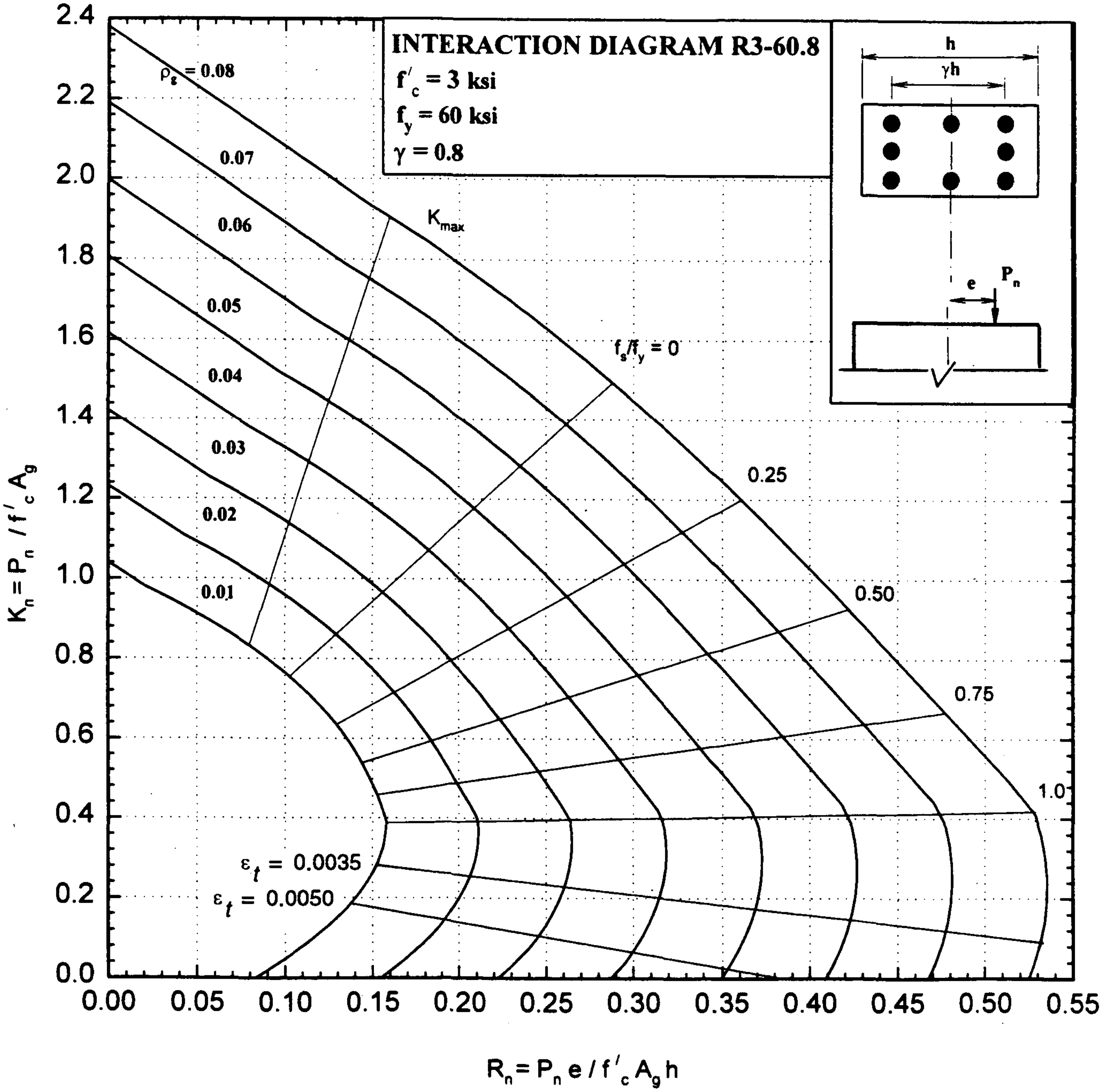


Diagram plotted using data from computer programs developed by Dr. Noel J. Everard.
Computer plot of data for diagram by Dr. Mohsen A. Issa and Alfred A. Yousif.

COLUMNS 7.1.4 - Nominal load-moment strength interaction diagram, R3-60.9

References: "Building Code Requirements for Structural Concrete-ACI 318" Chapters 9 and 10, and "Ultimate Strength Design of Reinforced Concrete Columns", ACI Special Publication SP-7, by Everard and Cohen, 1964, pp. 152-182 (with corrections).

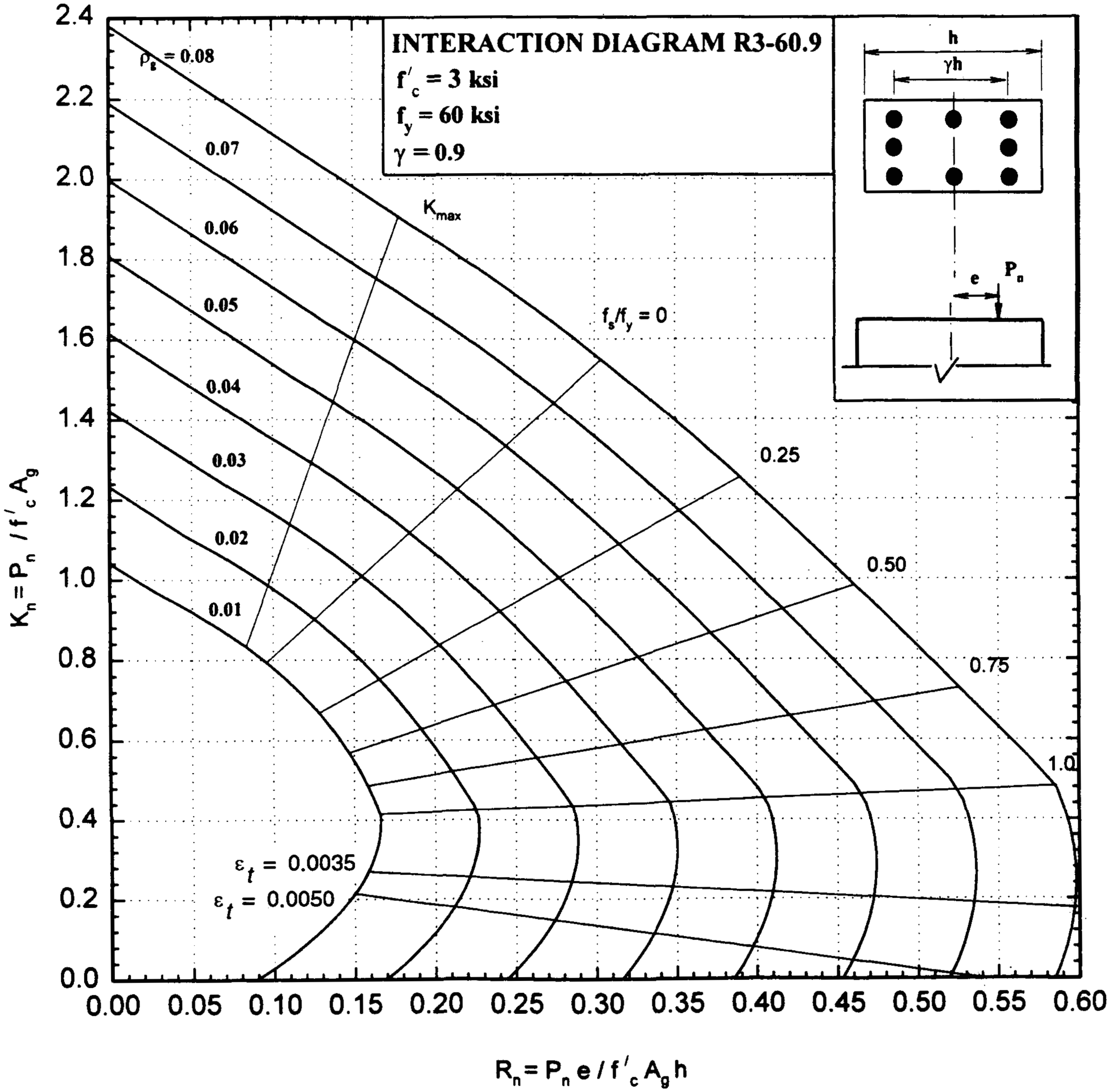


Diagram plotted using data from computer programs developed by Dr. Noel J. Everard. Computer plot of data for diagram by Dr. Mohsen A. Issa and Alfred A. Yousif.

COLUMNS 7.2.1 - Nominal load-moment strength interaction diagram, R4-60.6

References: "Building Code Requirements for Structural Concrete-ACI 318" Chapters 9 and 10, and "Ultimate Strength Design of Reinforced Concrete Columns", ACI Special Publication SP-7, by Everard and Cohen, 1964, pp. 152-182 (with corrections).

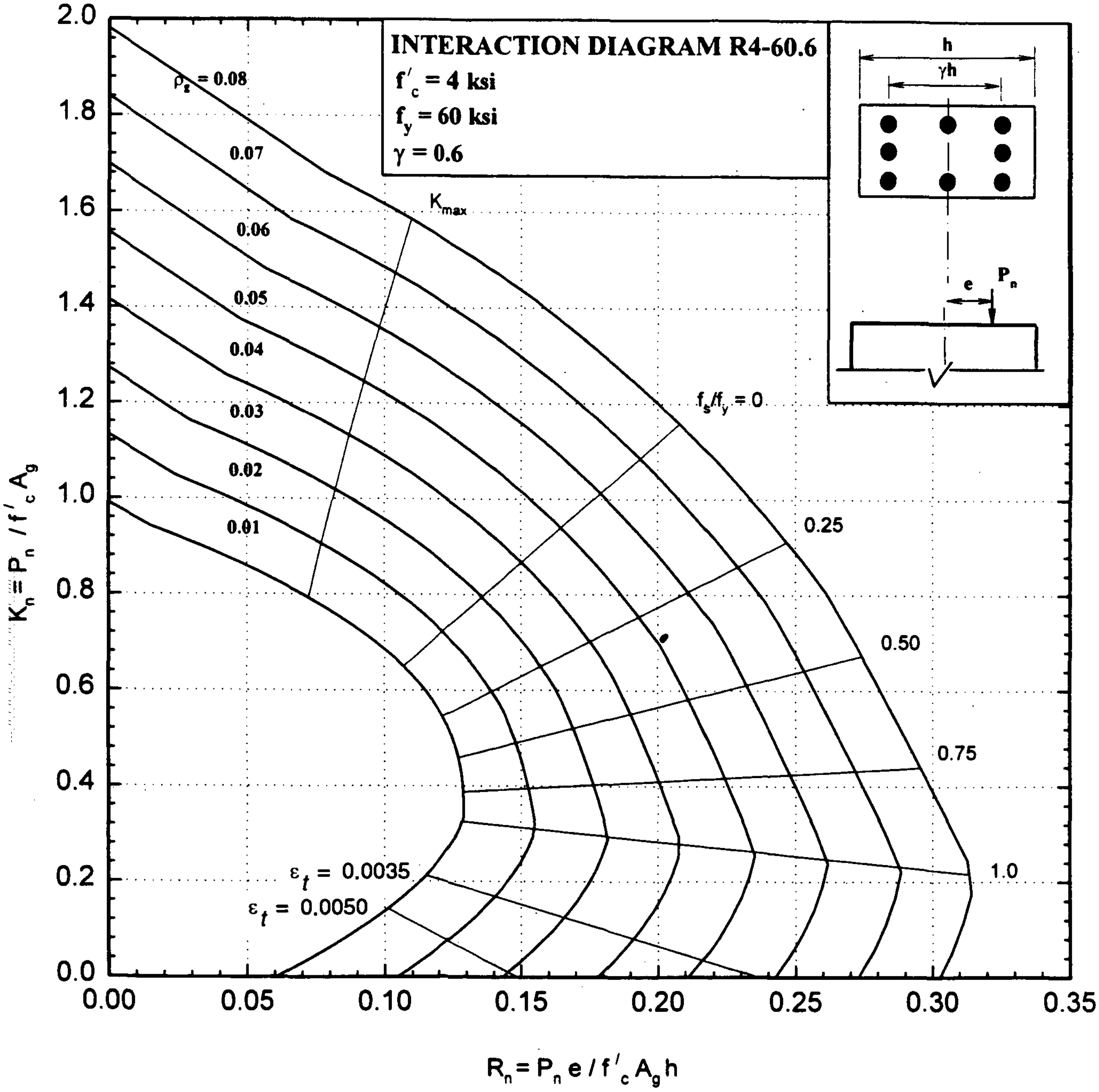


Diagram plotted using data from computer programs developed by Dr. Noel J. Everard.
Computer plot of data for diagram by Dr. Mohsen A. Issa and Alfred A. Yousif.

COLUMNS 7.2.2 - Nominal load-moment strength interaction diagram, R4-60.7

References: "Building Code Requirements for Structural Concrete-ACI 318" Chapters 9 and 10, and "Ultimate Strength Design of Reinforced Concrete Columns", ACI Special Publication SP-7, by Everard and Cohen, 1964, pp. 152-182 (with corrections).

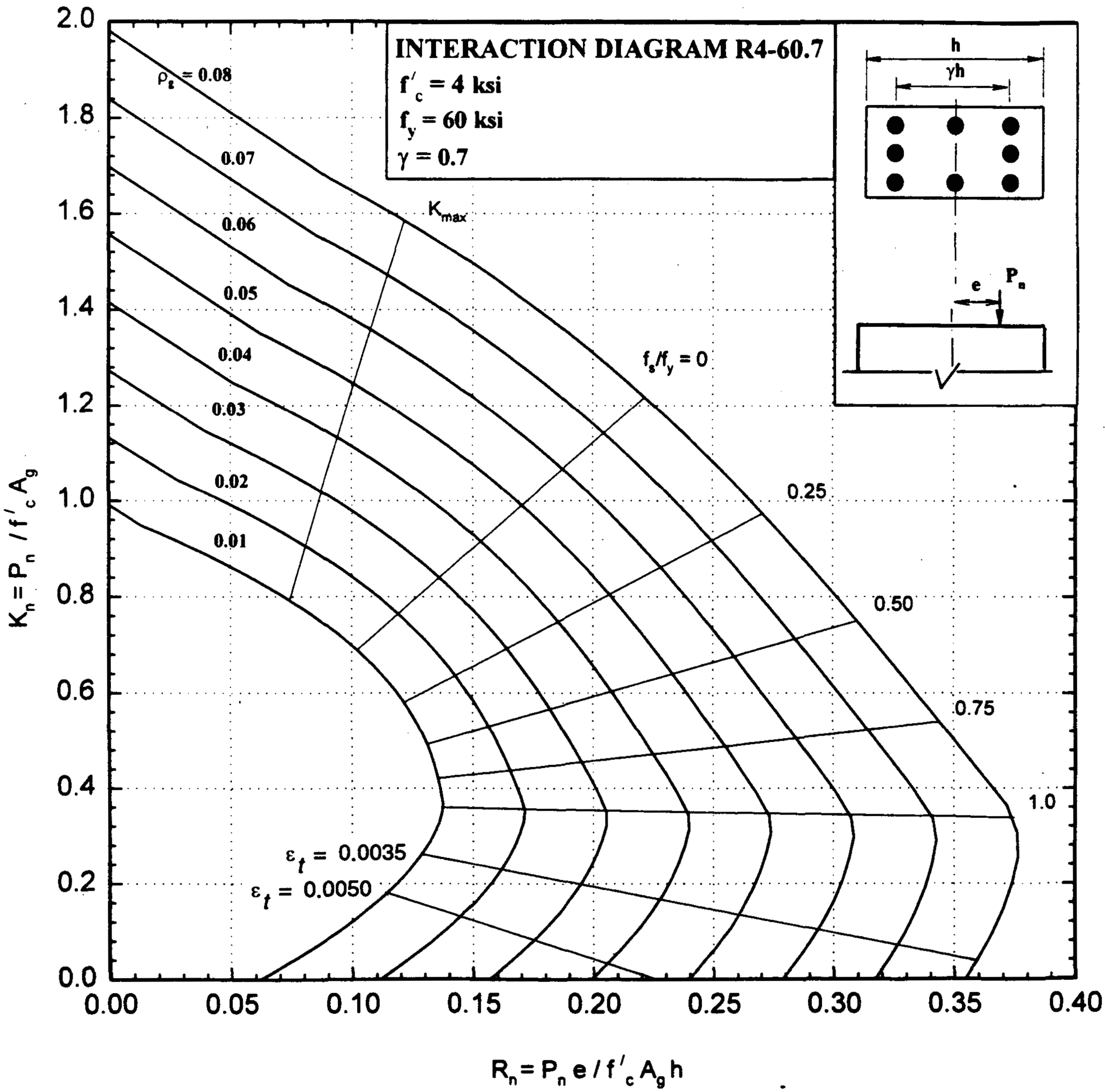


Diagram plotted using data from computer programs developed by Dr. Noel J. Everard.
Computer plot of data for diagram by Dr. Mohsen A. Issa and Alfred A. Yousif.

References: "Building Code Requirements for Structural Concrete-ACI 318" Chapters 9 and 10, and "Ultimate Strength Design of Reinforced Concrete Columns", ACI Special Publication SP-7, by Everard and Cohen, 1964, pp. 152-182 (with corrections).

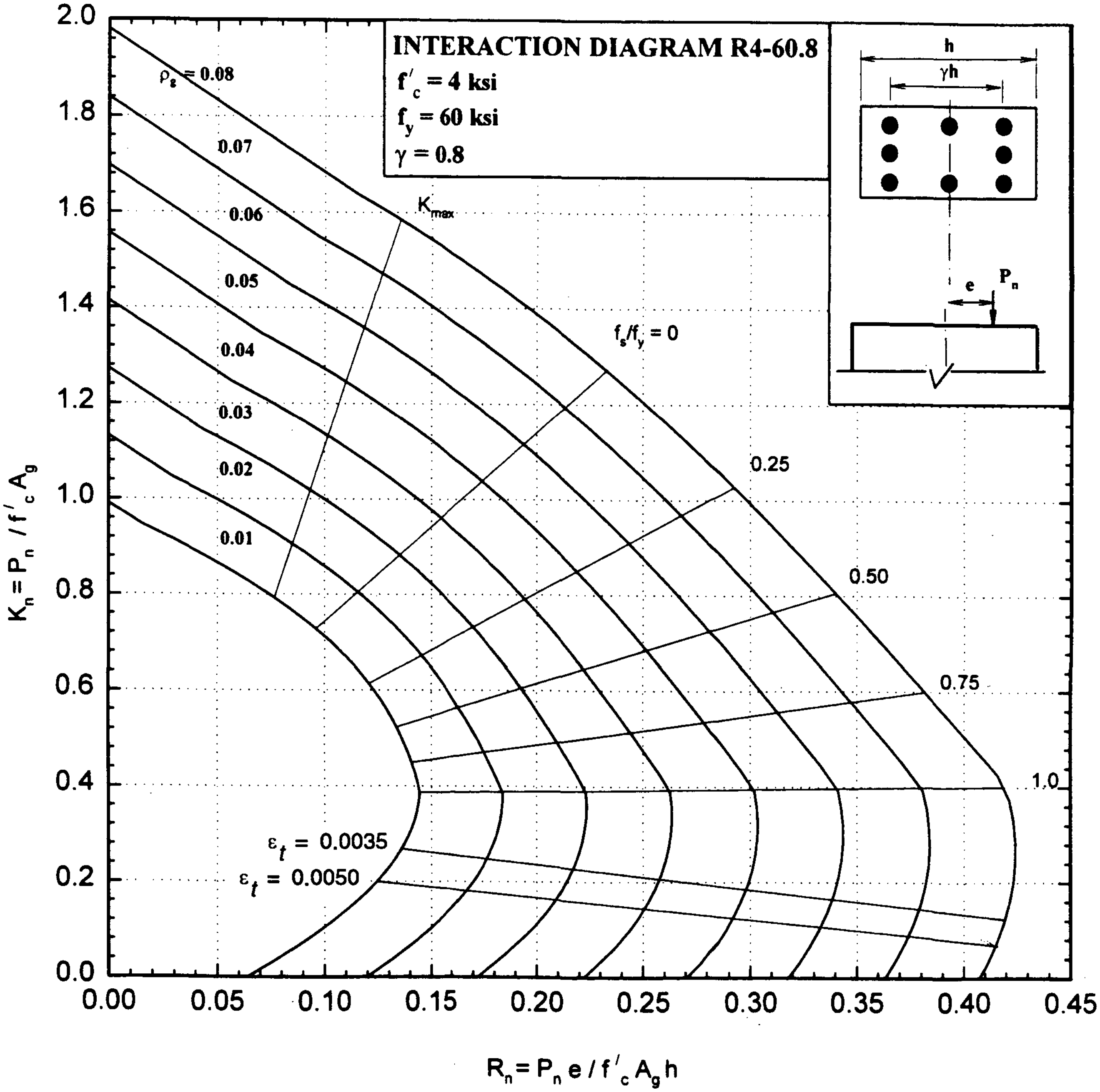


Diagram plotted using data from computer programs developed by Dr. Noel J. Everard.
Computer plot of data for diagram by Dr. Mohsen A. Issa and Alfred A. Yousif.

COLUMNS 7.2.4 - Nominal load-moment strength interaction diagram, R4-60.9

References: "Building Code Requirements for Structural Concrete-ACI 318" Chapters 9 and 10, and "Ultimate Strength Design of Reinforced Concrete Columns", ACI Special Publication SP-7, by Everard and Cohen, 1964, pp. 152-182 (with corrections).

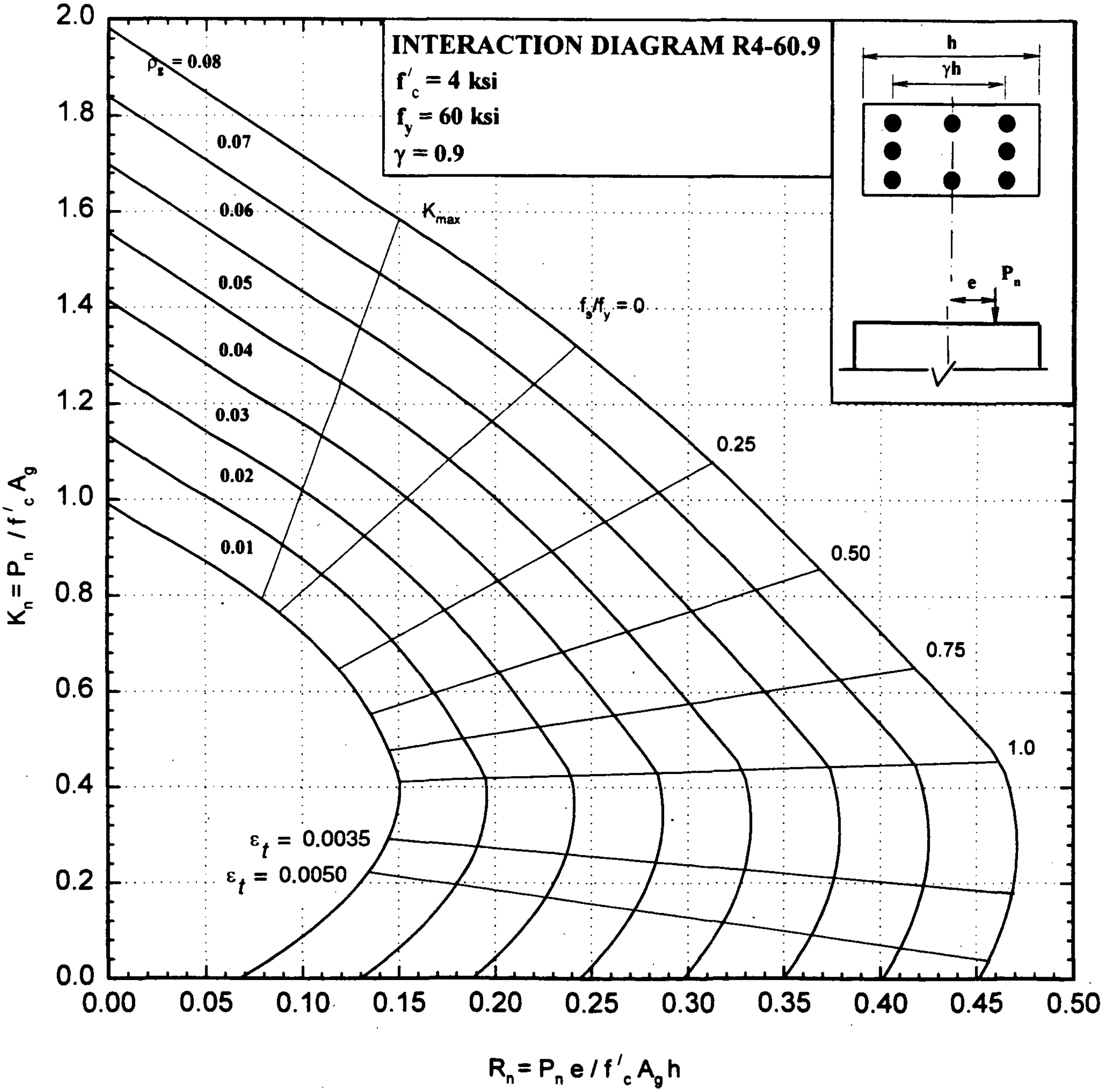


Diagram plotted using data from computer programs developed by Dr. Noel J. Everard. Computer plot of data for diagram by Dr. Mohsen A. Issa and Alfred A. Yousif.

COLUMNS 7.3.1 - Nominal load-moment strength interaction diagram, R5-60.6

References: "Building Code Requirements for Structural Concrete-ACI 318" Chapters 9 and 10, and "Ultimate Strength Design of Reinforced Concrete Columns", ACI Special Publication SP-7, by Everard and Cohen, 1964, pp. 152-182 (with corrections).

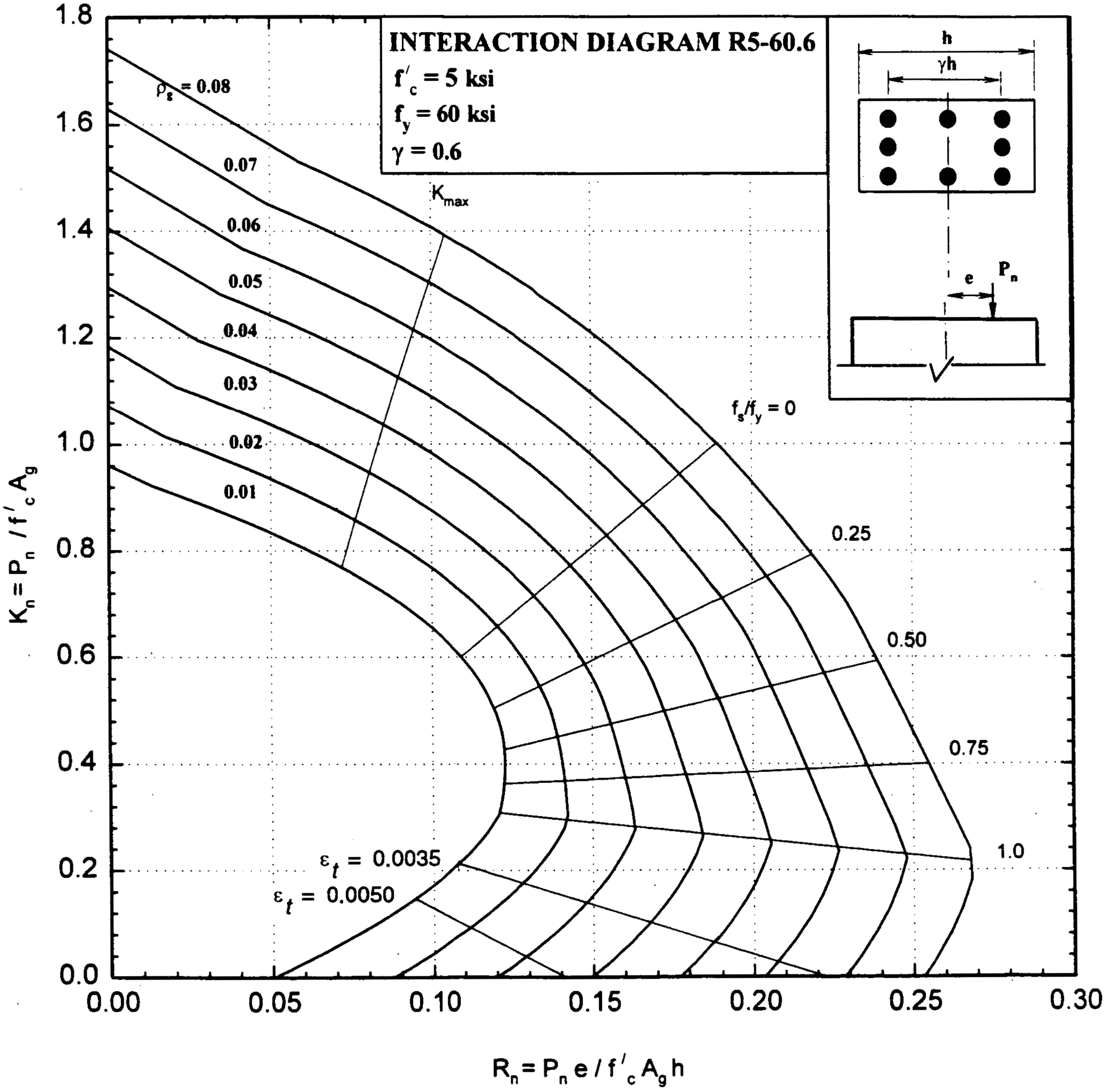


Diagram plotted using data from computer programs developed by Dr. Noel J. Everard.
Computer plot of data for diagram by Dr. Mohsen A. Issa and Alfred A. Yousif.

COLUMNS 7.3.2 - Nominal load-moment strength interaction diagram, R5-60.7

References: "Building Code Requirements for Structural Concrete-ACI 318" Chapters 9 and 10, and "Ultimate Strength Design of Reinforced Concrete Columns", ACI Special Publication SP-7, by Everard and Cohen, 1964, pp. 152-182 (with corrections).

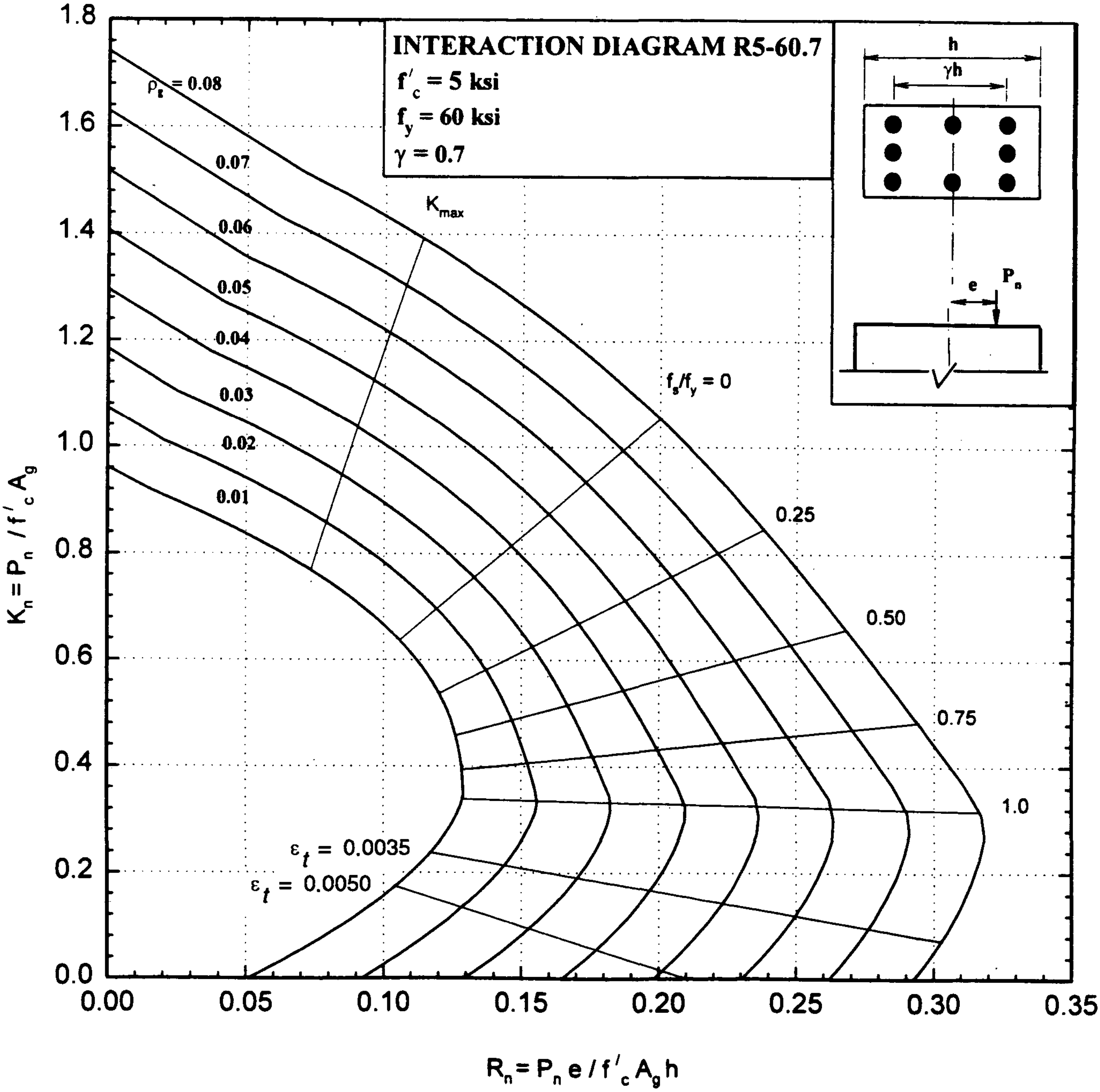


Diagram plotted using data from computer programs developed by Dr. Noel J. Everard.
Computer plot of data for diagram by Dr. Mohsen A. Issa and Alfred A. Yousif.

COLUMNS 7.3.3 - Nominal load-moment strength interaction diagram, R5-60.8

References: "Building Code Requirements for Structural Concrete-ACI 318" Chapters 9 and 10, and "Ultimate Strength Design of Reinforced Concrete Columns", ACI Special Publication SP-7, by Everard and Cohen, 1964, pp. 152-182 (with corrections).

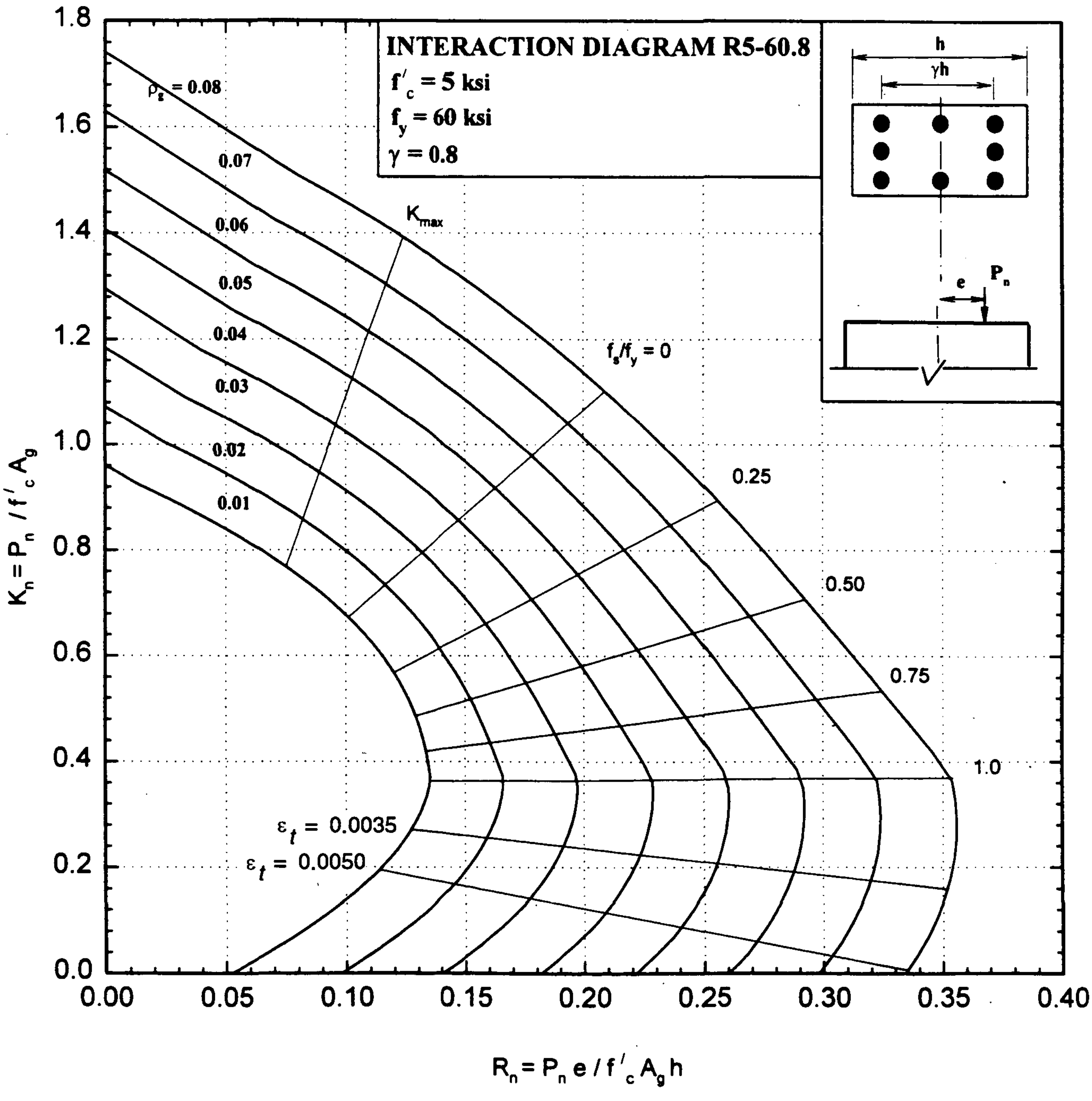


Diagram plotted using data from computer programs developed by Dr. Noel J. Everard. Computer plot of data for diagram by Dr. Mohsen A. Issa and Alfred A. Yousif.

COLUMNS 7.3.4 - Nominal load-moment strength interaction diagram, R5-60.9

References: "Building Code Requirements for Structural Concrete-ACI 318" Chapters 9 and 10, and "Ultimate Strength Design of Reinforced Concrete Columns", ACI Special Publication SP-7, by Everard and Cohen, 1964, pp. 152-182 (with corrections).

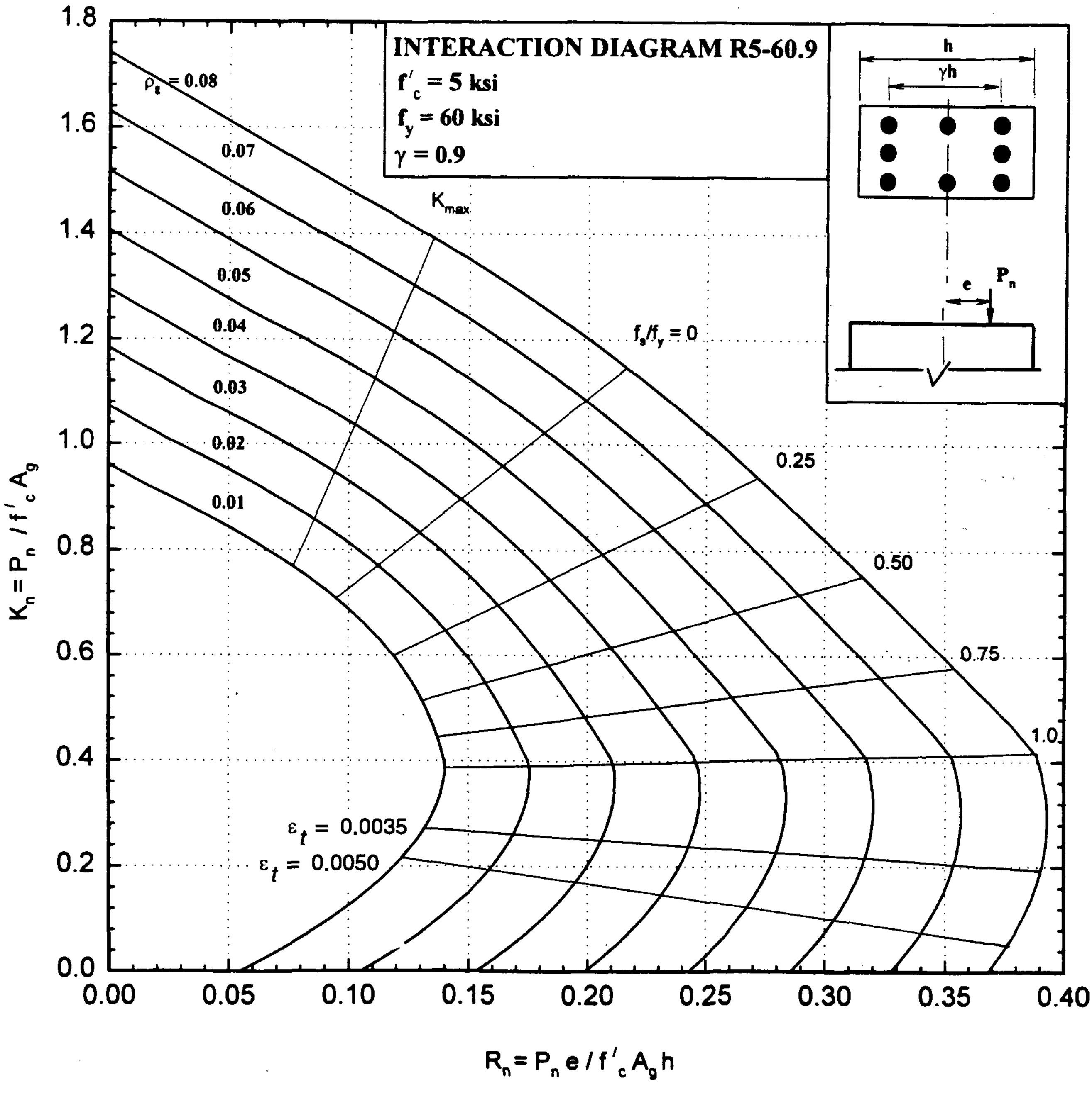


Diagram plotted using data from computer programs developed by Dr. Noel J. Everard.
Computer plot of data for diagram by Dr. Mohsen A. Issa and Alfred A. Yousif.

COLUMNS 7.4.1 - Nominal load-moment strength interaction diagram, R6-60.6

References: "Building Code Requirements for Structural Concrete-ACI 318" Chapters 9 and 10, and "Ultimate Strength Design of Reinforced Concrete Columns", ACI Special Publication SP-7, by Everard and Cohen, 1964, pp. 152-182 (with corrections).

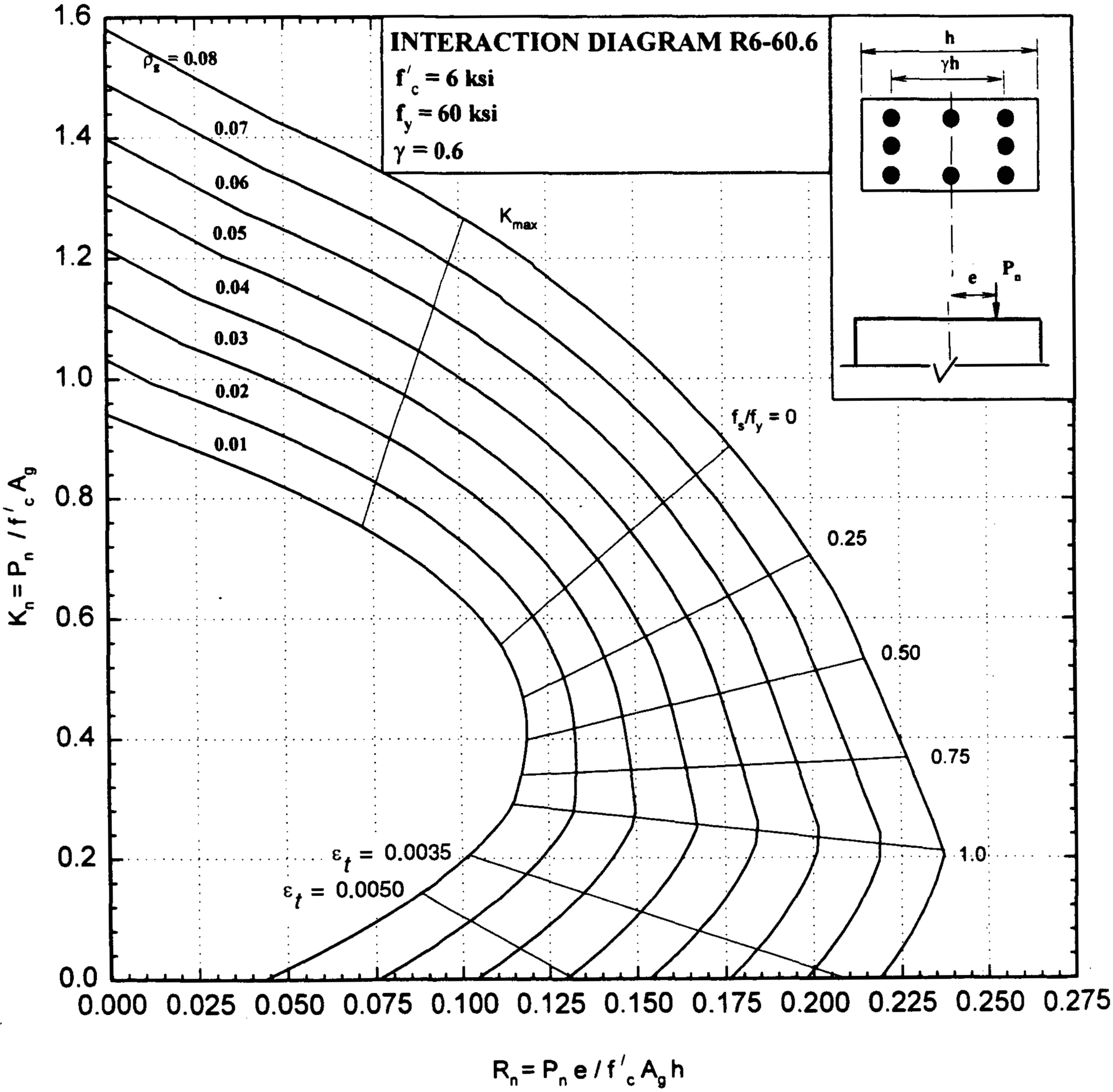


Diagram plotted using data from computer programs developed by Dr. Noel J. Everard.
Computer plot of data for diagram by Dr. Mohsen A. Issa and Alfred A. Yousif.

COLUMNS 7.4.2 - Nominal load-moment strength interaction diagram, R6-60.7

References: "Building Code Requirements for Structural Concrete-ACI 318" Chapters 9 and 10, and "Ultimate Strength Design of Reinforced Concrete Columns", ACI Special Publication SP-7, by Everard and Cohen, 1964, pp. 152-182 (with corrections).

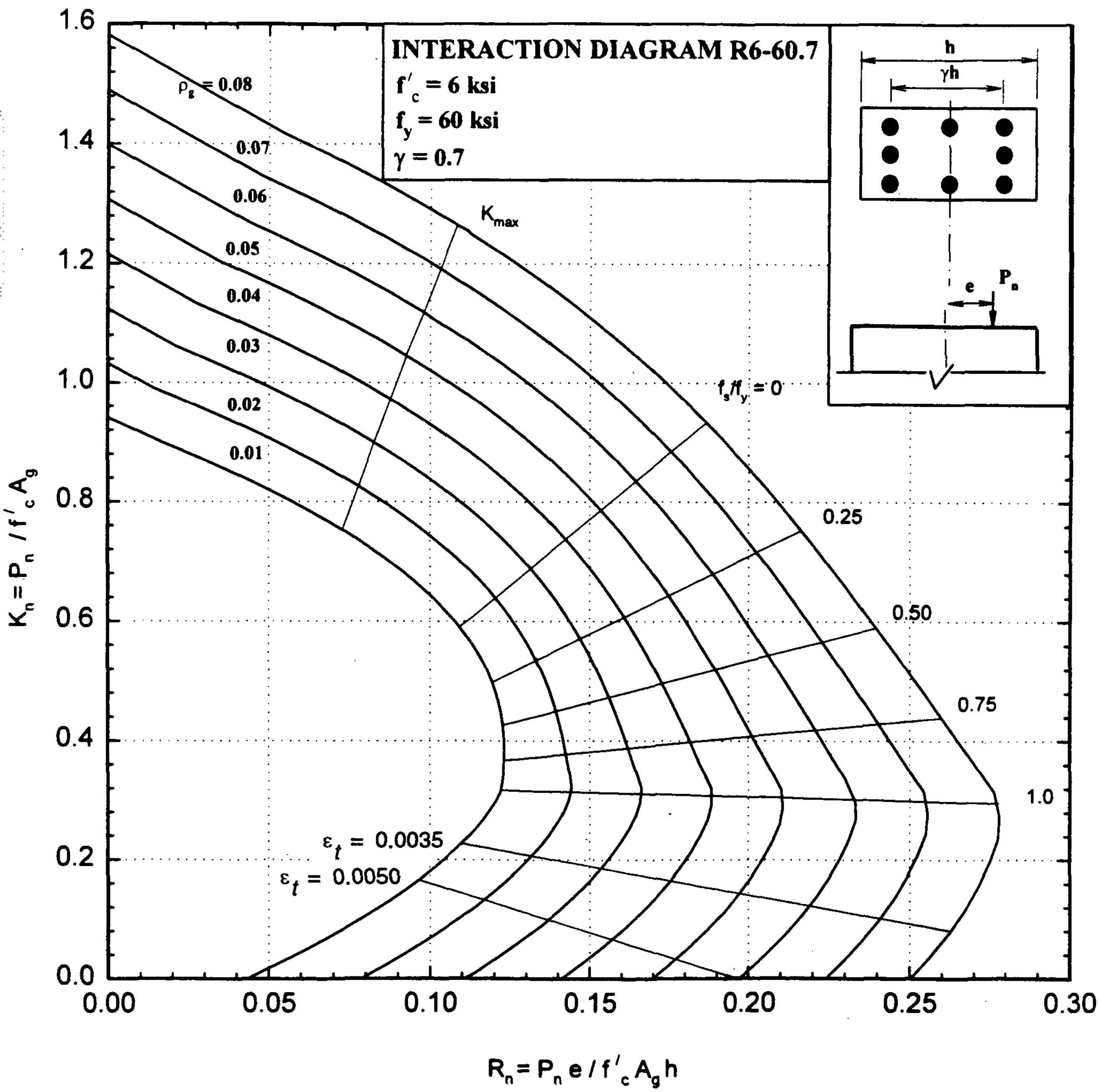


Diagram plotted using data from computer programs developed by Dr. Noel J. Everard.
Computer plot of data for diagram by Dr. Mohsen A. Issa and Alfred A. Yousif.

COLUMNS 7.4.3 - Nominal load-moment strength interaction diagram, R6-60.8

References: "Building Code Requirements for Structural Concrete-ACI 318" Chapters 9 and 10, and "Ultimate Strength Design of Reinforced Concrete Columns", ACI Special Publication SP-7, by Everard and Cohen, 1964, pp. 152-182 (with corrections).

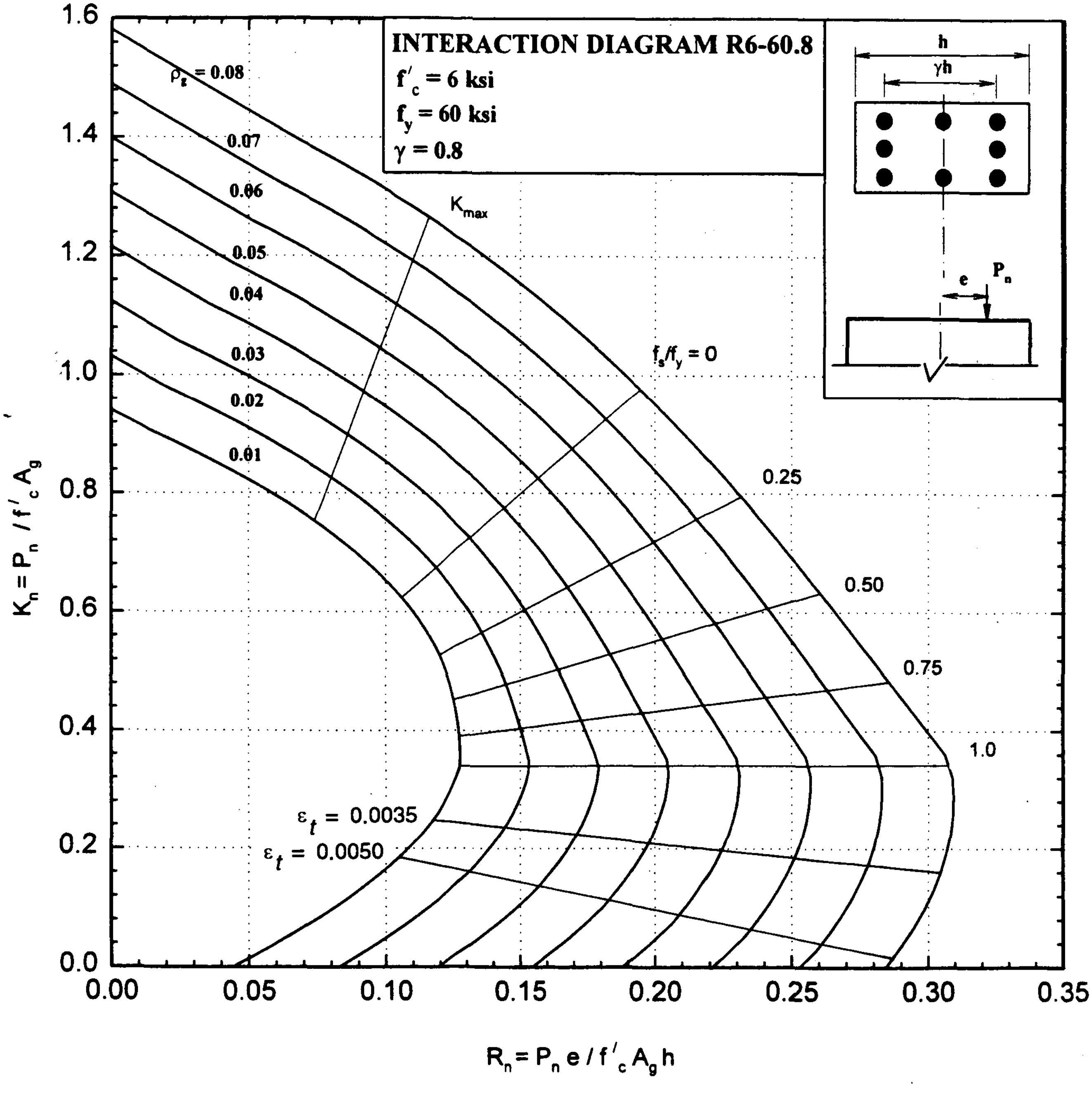


Diagram plotted using data from computer programs developed by Dr. Noel J. Everard.
Computer plot of data for diagram by Dr. Mohsen A. Issa and Alfred A. Yousif.

COLUMNS 7.4.4 - Nominal load-moment strength interaction diagram, R6-60.9

References: "“Building Code Requirements for Structural Concrete-ACI 318" Chapters 9 and 10, and "Ultimate Strength Design of Reinforced Concrete Columns", ACI Special Publication SP-7, by Everard and Cohen, 1964, pp. 152-182 (with corrections).

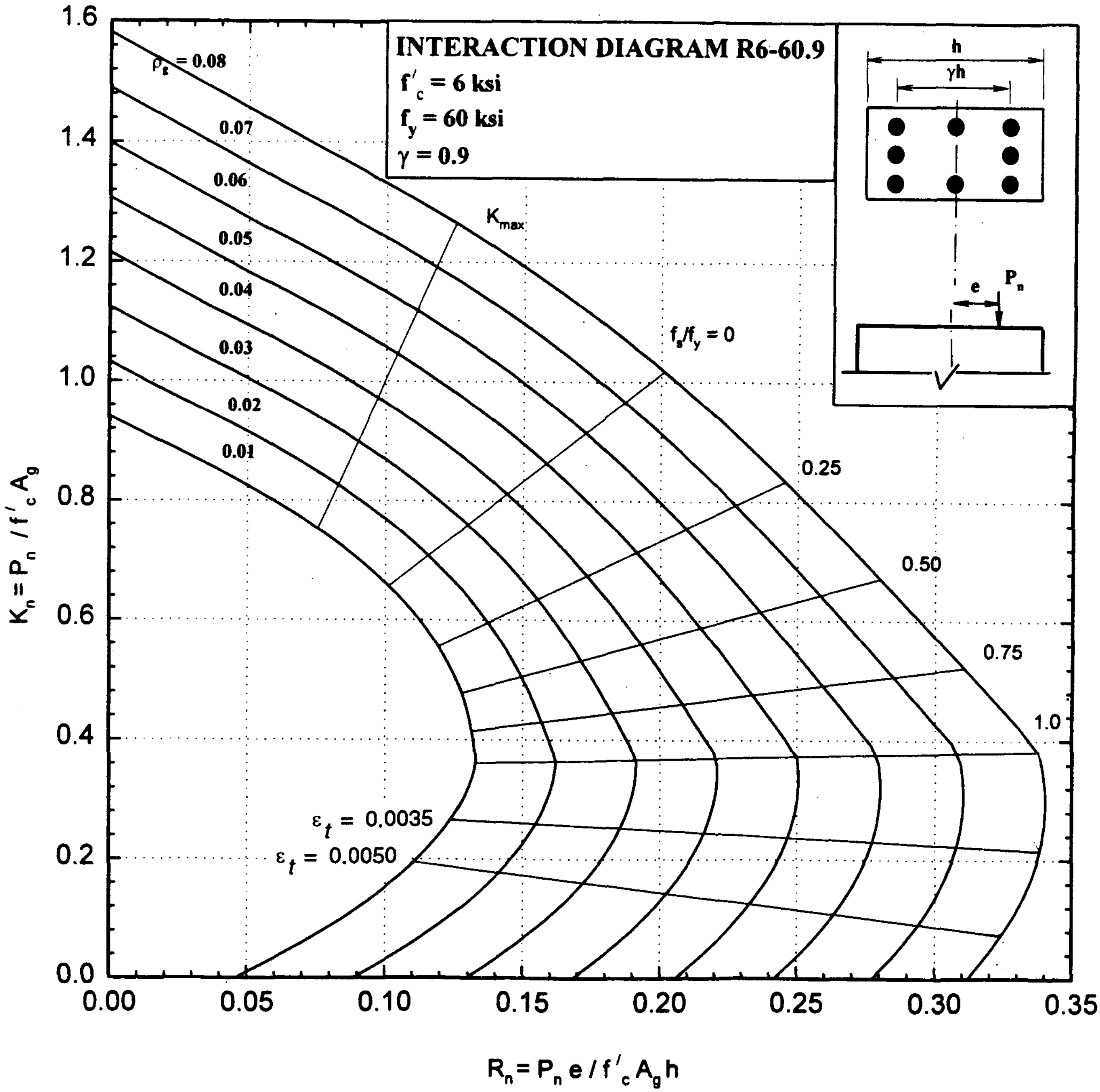


Diagram plotted using data from computer programs developed by Dr. Noel J. Everard.
Computer plot of data for diagram by Dr. Mohsen A. Issa and Alfred A. Yousif.

COLUMNS 7.5.1 - Nominal load-moment strength interaction diagram, R9-75.6

References: "Building Code Requirements for Structural Concrete-ACI 318" Chapters 9 and 10, and "Ultimate Strength Design of Reinforced Concrete Columns", ACI Special Publication SP-7, by Everard and Cohen, 1964, pp. 152-182 (with corrections).

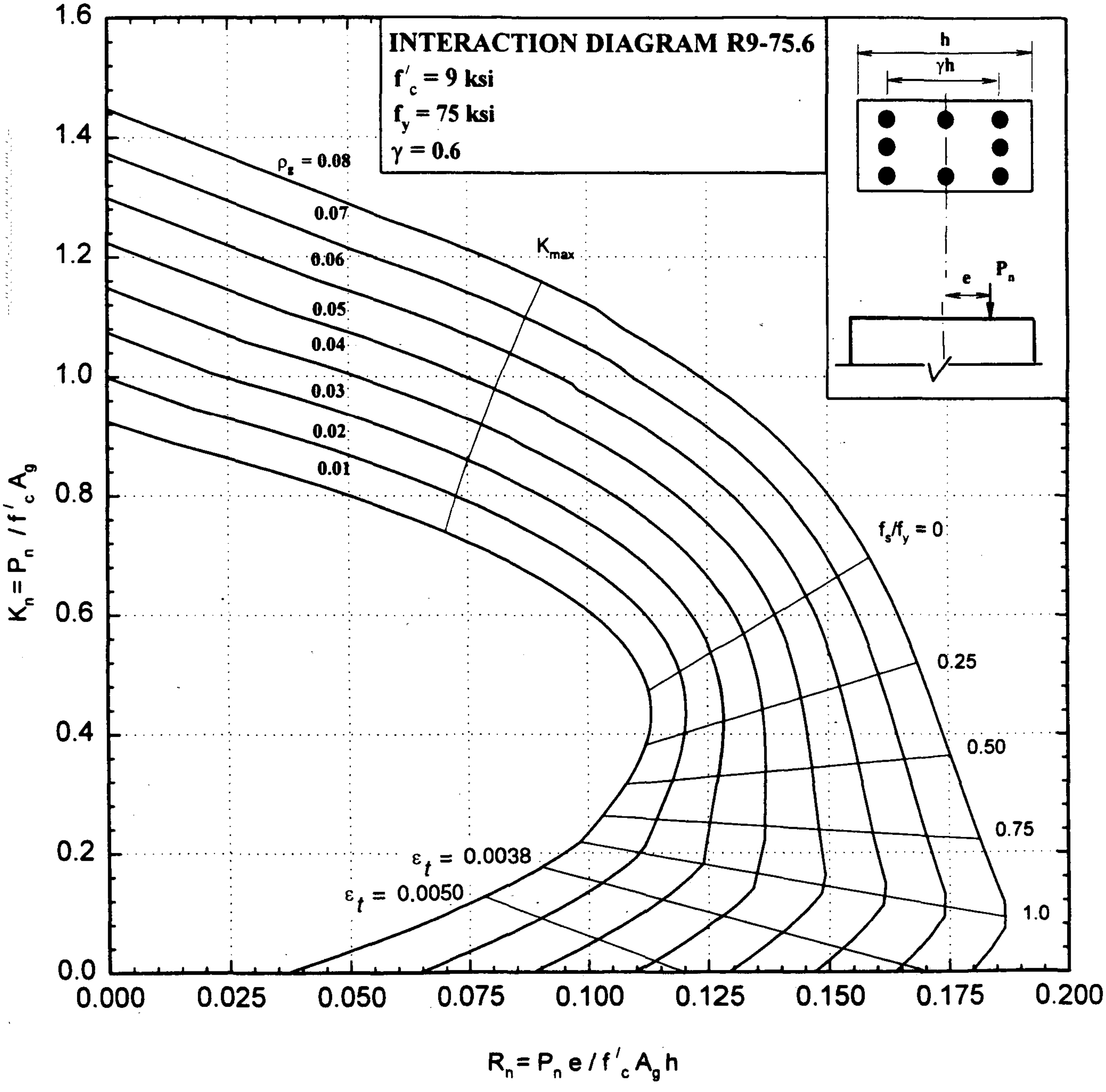


Diagram plotted using data from computer programs developed by Dr. Noel J. Everard. Computer plot of data for diagram by Dr. Mohsen A. Issa and Alfred A. Yousif.

COLUMNS 7.5.2 - Nominal load-moment strength interaction diagram, R9-75.7

References: "Building Code Requirements for Structural Concrete-ACI 318" Chapters 9 and 10, and "Ultimate Strength Design of Reinforced Concrete Columns", ACI Special Publication SP-7, by Everard and Cohen, 1964, pp. 152-182 (with corrections).

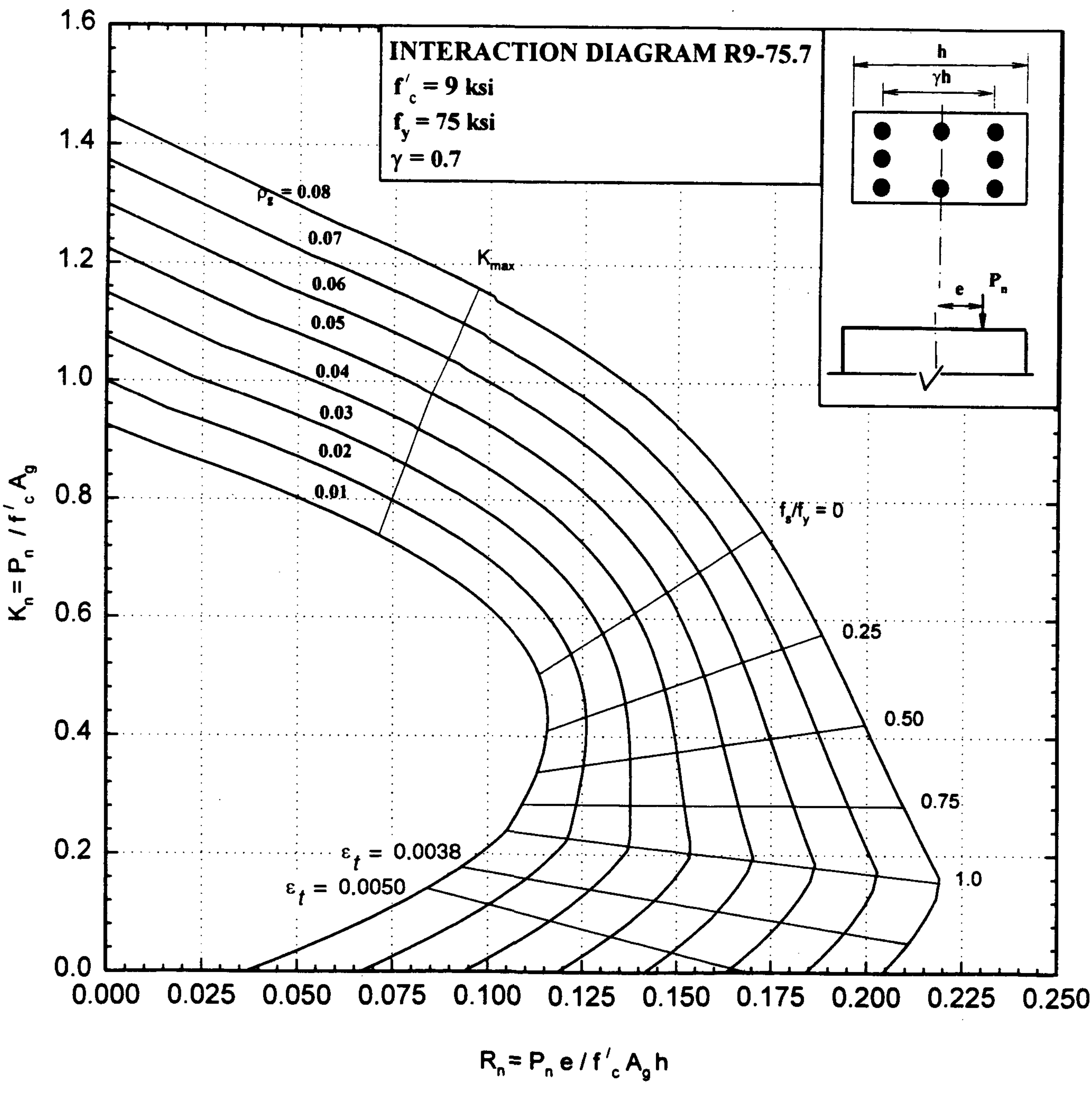


Diagram plotted using data from computer programs developed by Dr. Noel J. Everard.
Computer plot of data for diagram by Dr. Mohsen A. Issa and Alfred A. Yousif.

COLUMNS 7.5.3 - Nominal load-moment strength interaction diagram, R9-75.8

References: "Building Code Requirements for Structural Concrete-ACI 318" Chapters 9 and 10, and "Ultimate Strength Design of Reinforced Concrete Columns", ACI Special Publication SP-7, by Everard and Cohen, 1964, pp. 152-182 (with corrections).

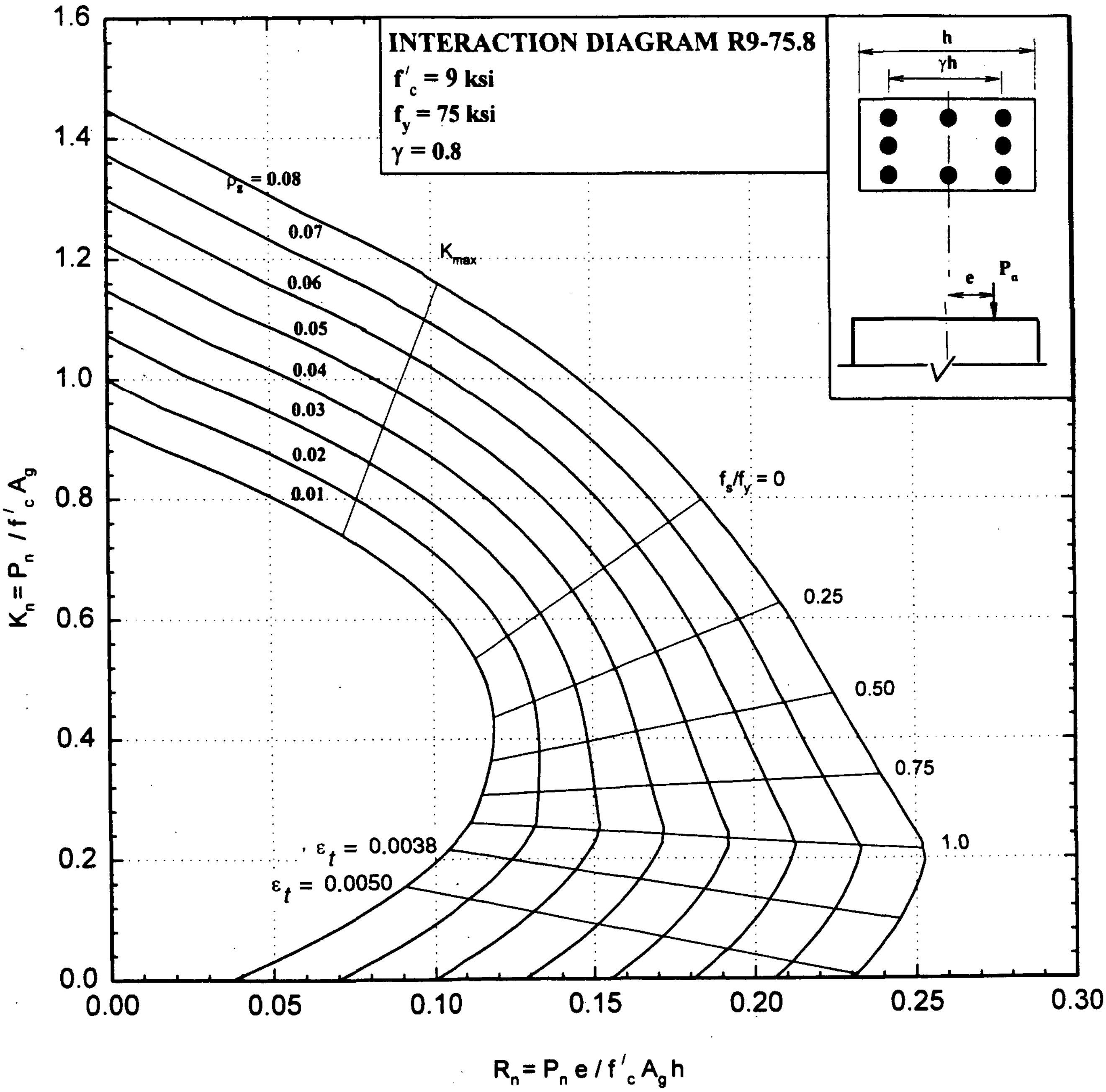


Diagram plotted using data from computer programs developed by Dr. Noel J. Everard.
Computer plot of data for diagram by Dr. Mohsen A. Issa and Alfred A. Yousif.

COLUMNS 7.5.4 - Nominal load-moment strength interaction diagram, R9-75.9

References: "Building Code Requirements for Structural Concrete-ACI 318" Chapters 9 and 10, and "Ultimate Strength Design of Reinforced Concrete Columns", ACI Special Publication SP-7, by Everard and Cohen, 1964, pp. 152-182 (with corrections).

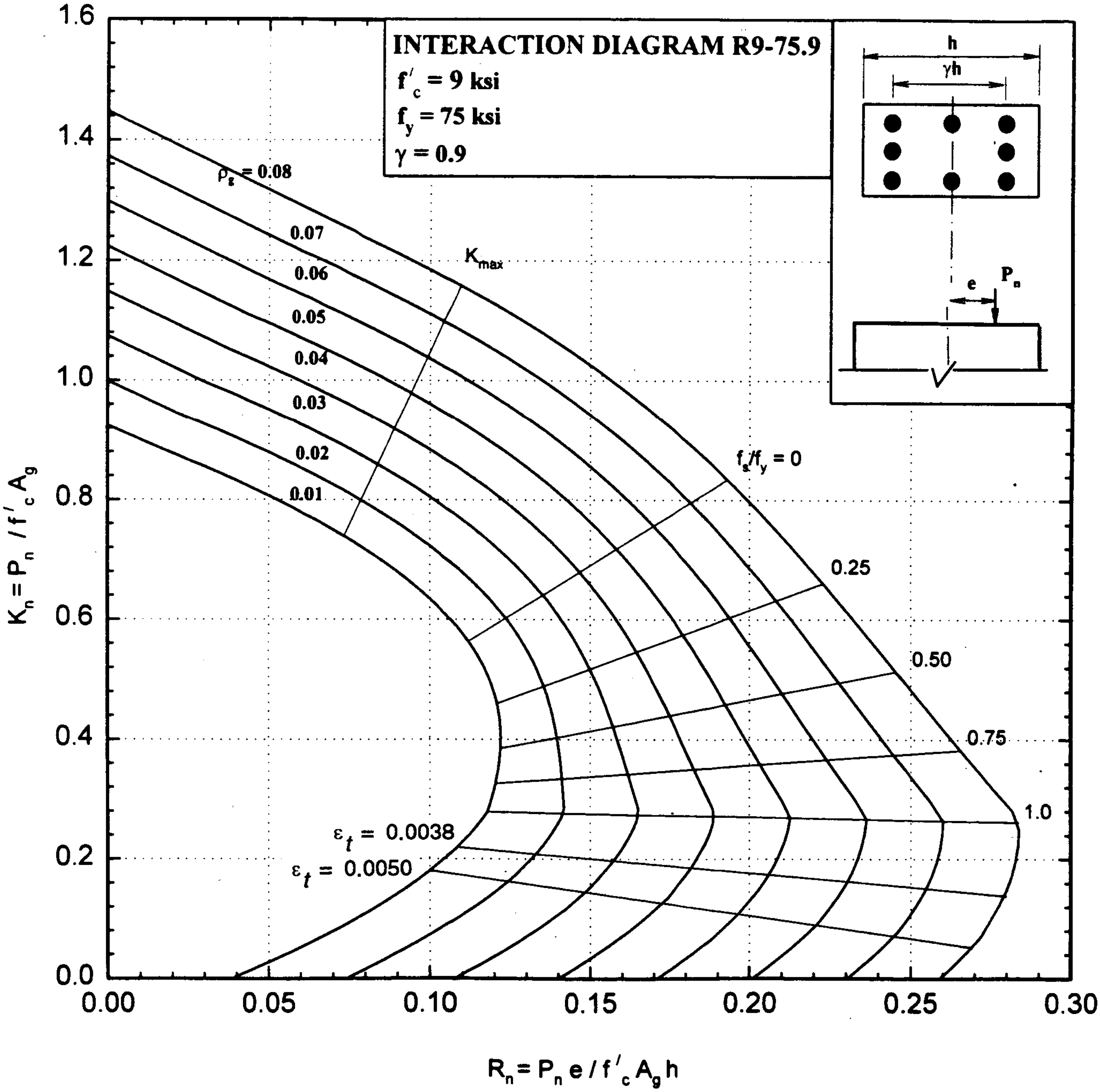


Diagram plotted using data from computer programs developed by Dr. Noel J. Everard.
Computer plot of data for diagram by Dr. Mohsen A. Issa and Alfred A. Yousif.

COLUMNS 7.6.1 - Nominal load-moment strength interaction diagram, R12-75.6

References: "Building Code Requirements for Structural Concrete-ACI 318" Chapters 9 and 10, and "Ultimate Strength Design of Reinforced Concrete Columns", ACI Special Publication SP-7, by Everard and Cohen, 1964, pp. 152-182 (with corrections).

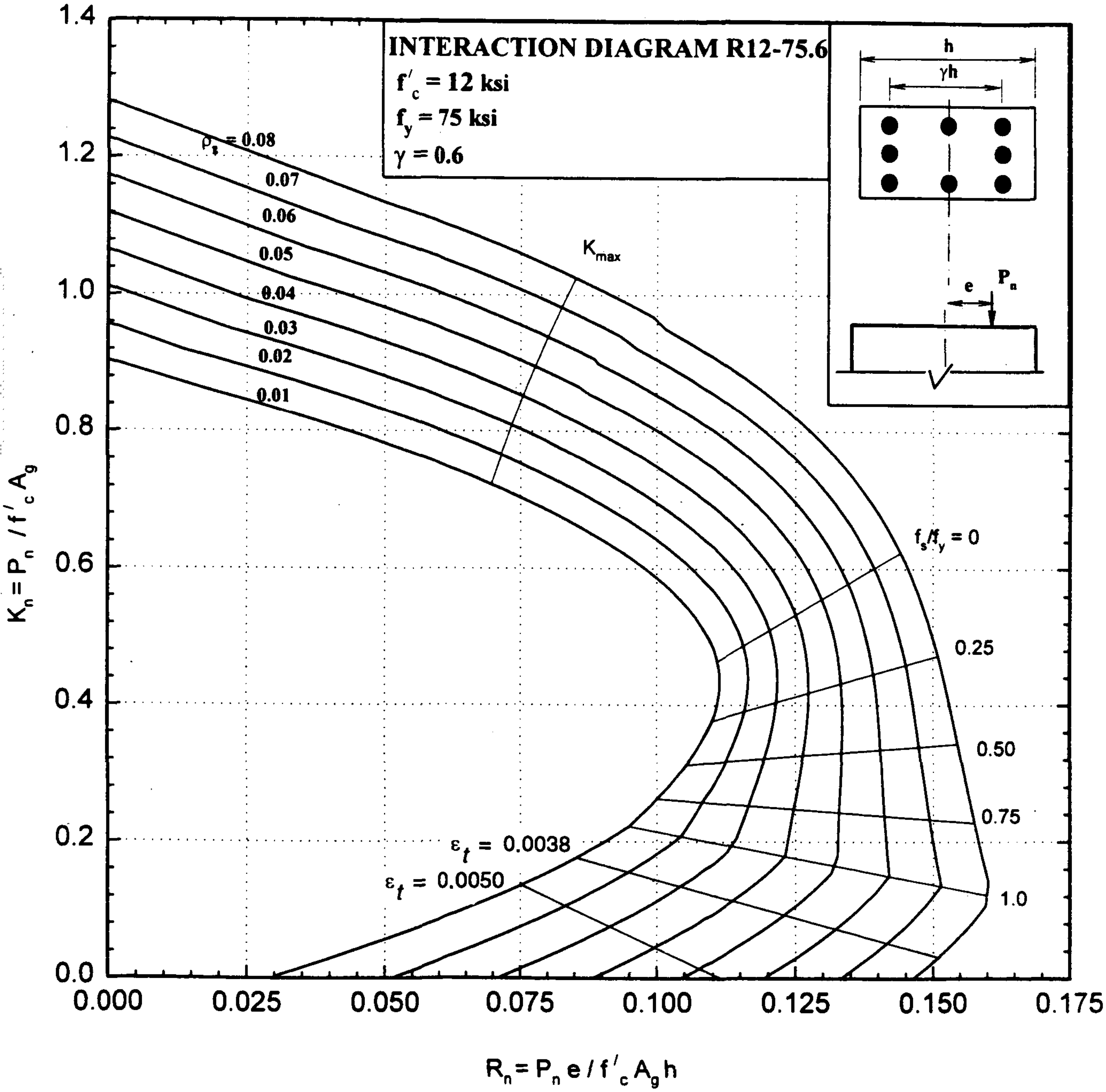


Diagram plotted using data from computer programs developed by Dr. Noel J. Everard.
Computer plot of data for diagram by Dr. Mohsen A. Issa and Alfred A. Yousif.

COLUMNS 7.6.2 - Nominal load-moment strength interaction diagram, R12-75.7

References: "Building Code Requirements for Structural Concrete-ACI 318" Chapters 9 and 10, and "Ultimate Strength Design of Reinforced Concrete Columns", ACI Special Publication SP-7, by Everard and Cohen, 1964, pp. 152-182 (with corrections).

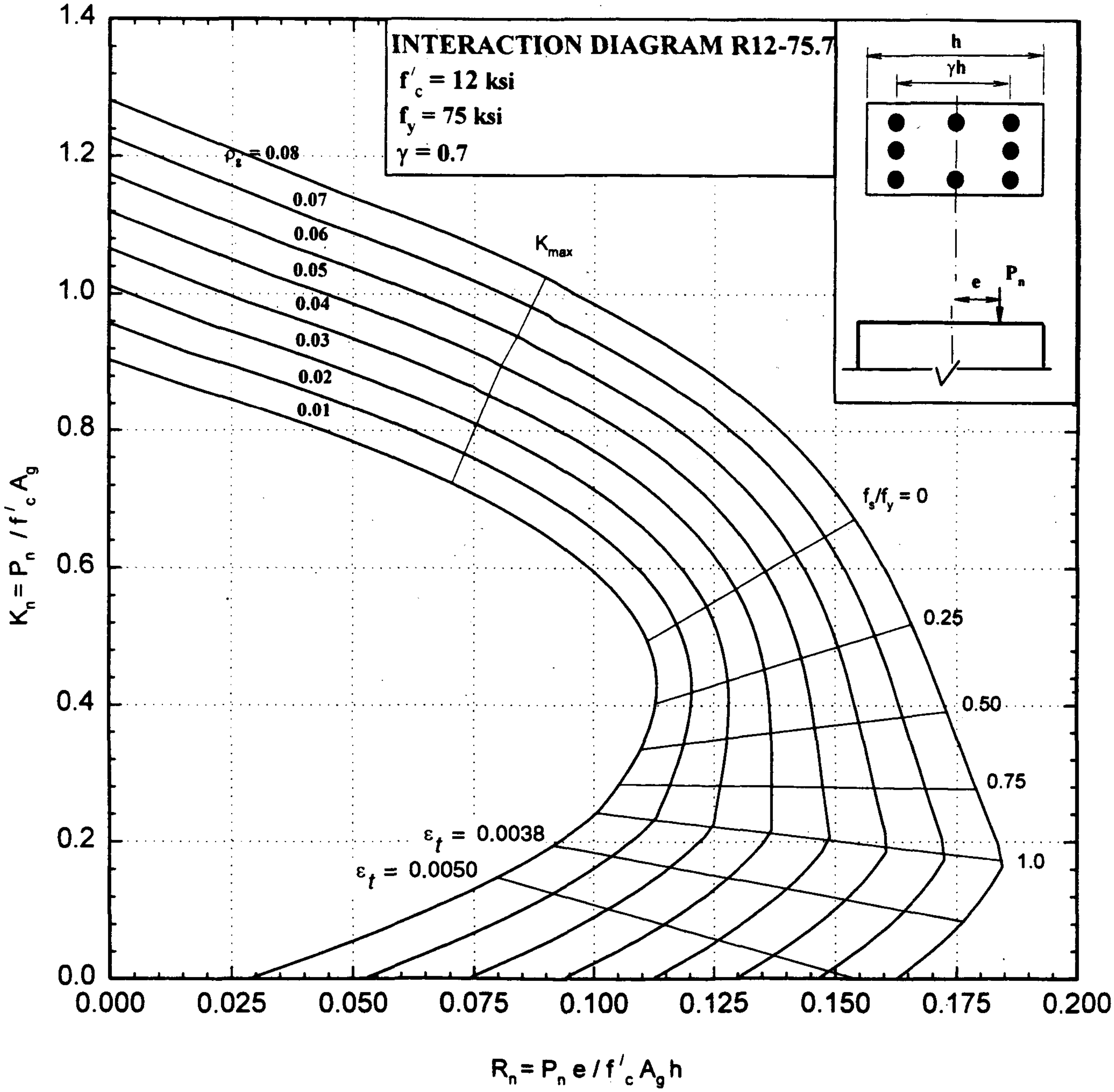


Diagram plotted using data from computer programs developed by Dr. Noel J. Everard.
Computer plot of data for diagram by Dr. Mohsen A. Issa and Alfred A. Yousif.

COLUMNS 7.6.3 - Nominal load-moment strength interaction diagram, R12-75.8

References: "Building Code Requirements for Structural Concrete-ACI 318" Chapters 9 and 10, and "Ultimate Strength Design of Reinforced Concrete Columns", ACI Special Publication SP-7, by Everard and Cohen, 1964, pp. 152-182 (with corrections).

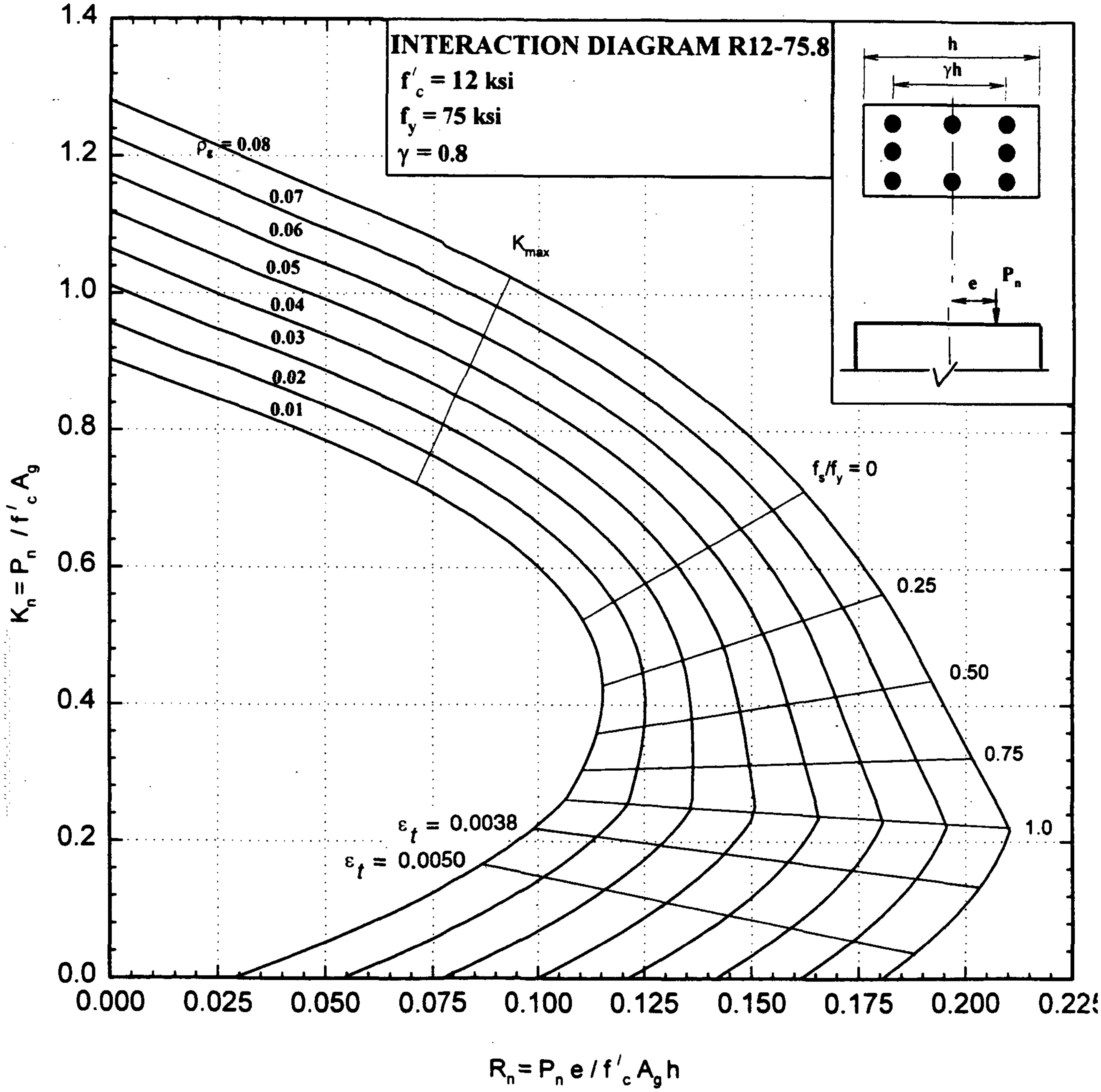


Diagram plotted using data from computer programs developed by Dr. Noel J. Everard.
Computer plot of data for diagram by Dr. Mohsen A. Issa and Alfred A. Yousif.

COLUMNS 7.6.4 - Nominal load-moment strength interaction diagram, R12-75.9

References: "Building Code Requirements for Structural Concrete-ACI 318" Chapters 9 and 10, and "Ultimate Strength Design of Reinforced Concrete Columns", ACI Special Publication SP-7, by Everard and Cohen, 1964, pp. 152-182 (with corrections).

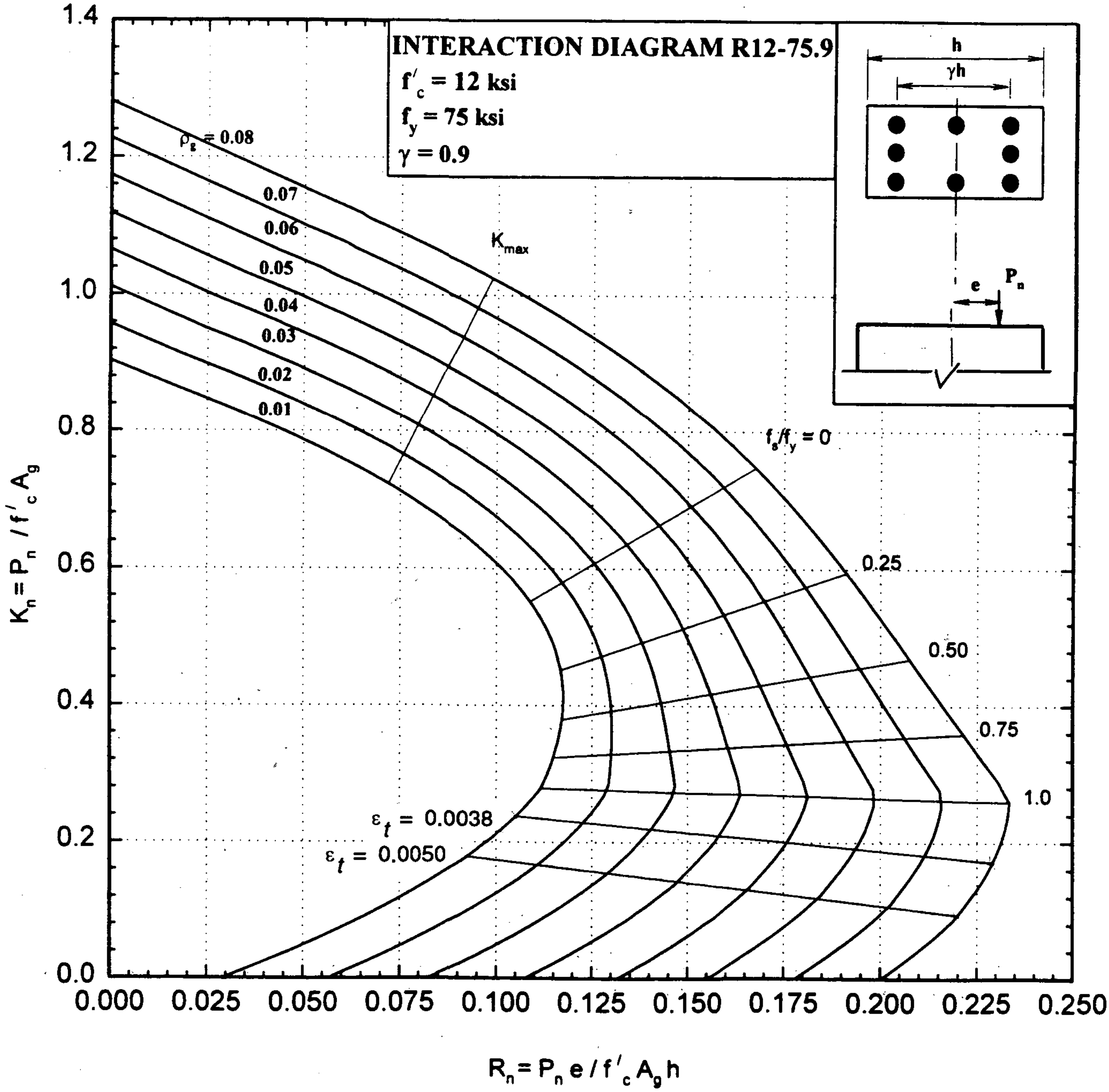


Diagram plotted using data from computer programs developed by Dr. Noel J. Everard. Computer plot of data for diagram by Dr. Mohsen A. Issa and Alfred A. Yousif.

COLUMNS 7.7.1 - Nominal load-moment strength interaction diagram, L3-60.6

References: "Building Code Requirements for Structural Concrete-ACI 318" Chapters 9 and 10, and "Ultimate Strength Design of Reinforced Concrete Columns", ACI Special Publication SP-7, by Everard and Cohen, 1964, pp. 152-182 (with corrections).

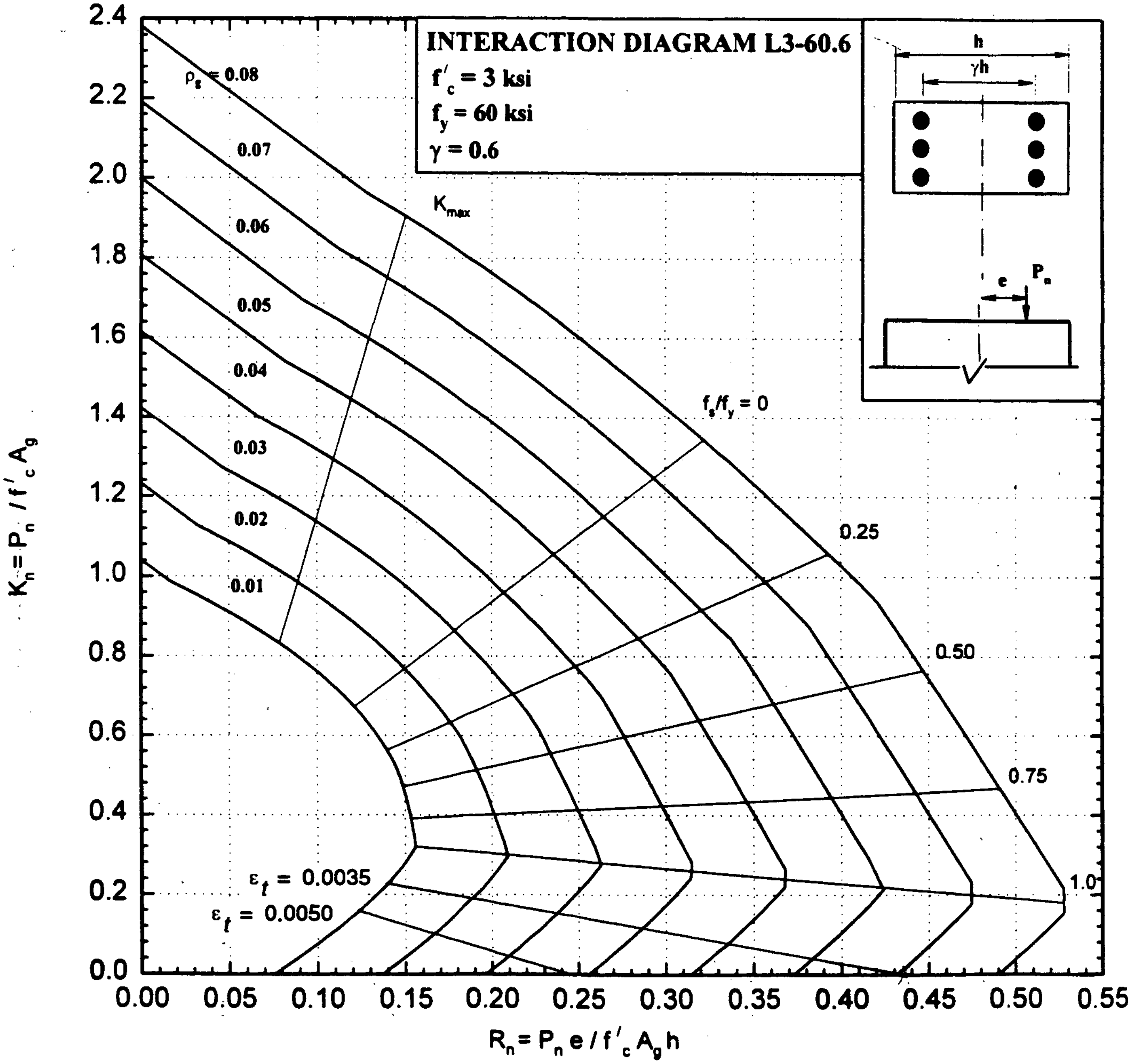


Diagram plotted using data from computer programs developed by Dr. Noel J. Everard. Computer plot of data for diagram by Dr. Mohsen A. Issa and Alfred A. Yousif.

COLUMNS 7.7.2 - Nominal load-moment strength interaction diagram, L3-60.7

References: "Building Code Requirements for Structural Concrete-ACI 318" Chapters 9 and 10, and "Ultimate Strength Design of Reinforced Concrete Columns", ACI Special Publication SP-7, by Everard and Cohen, 1964, pp. 152-182 (with corrections).

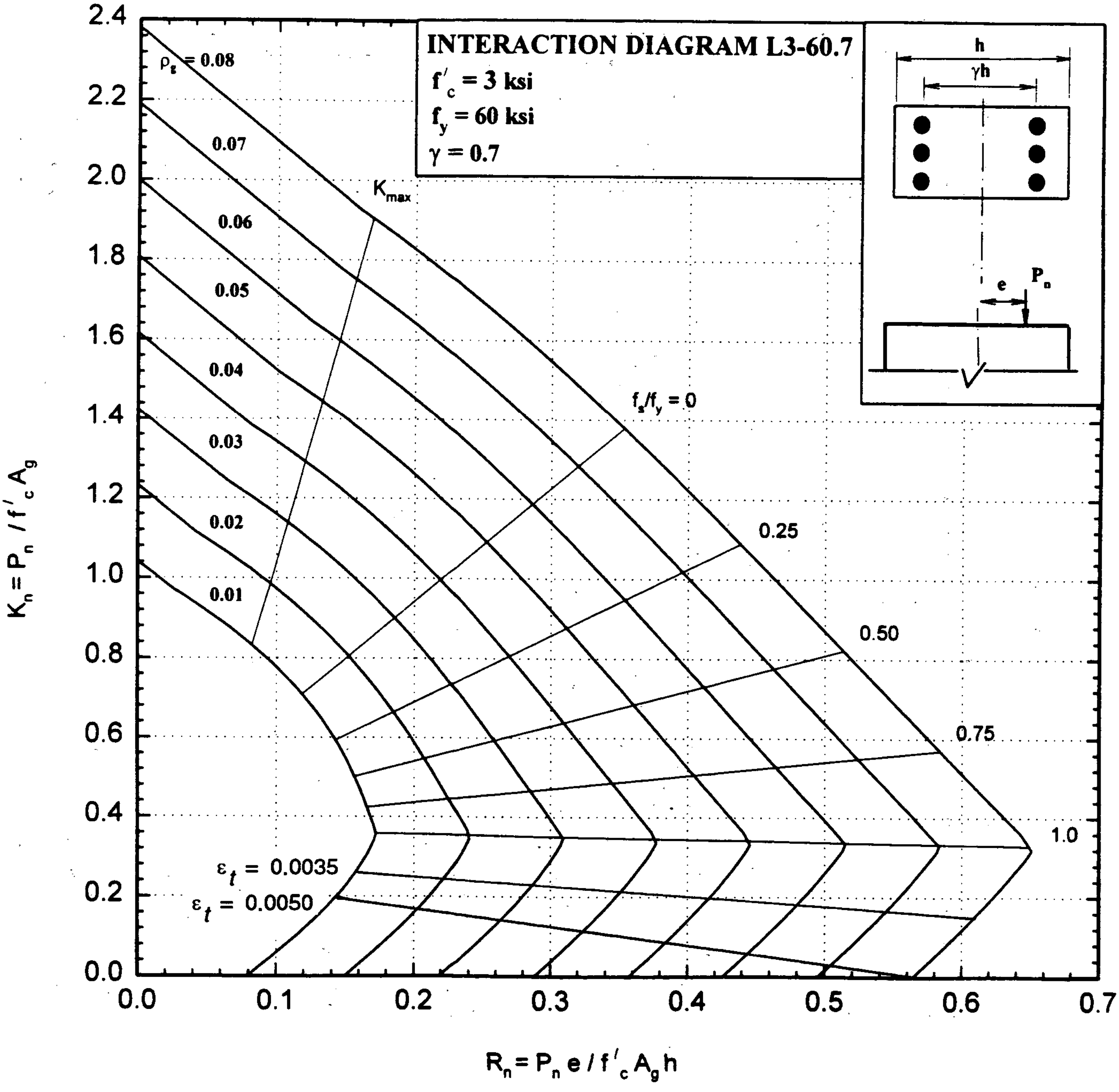


Diagram plotted using data from computer programs developed by Dr. Noel J. Everard.
Computer plot of data for diagram by Dr. Mohsen A. Issa and Alfred A. Yousif.

COLUMNS 7.7.3 - Nominal load-moment strength interaction diagram, L3-60.8

References: "Building Code Requirements for Structural Concrete-ACI 318" Chapters 9 and 10, and "Ultimate Strength Design of Reinforced Concrete Columns", ACI Special Publication SP-7, by Everard and Cohen, 1964, pp. 152-182 (with corrections).

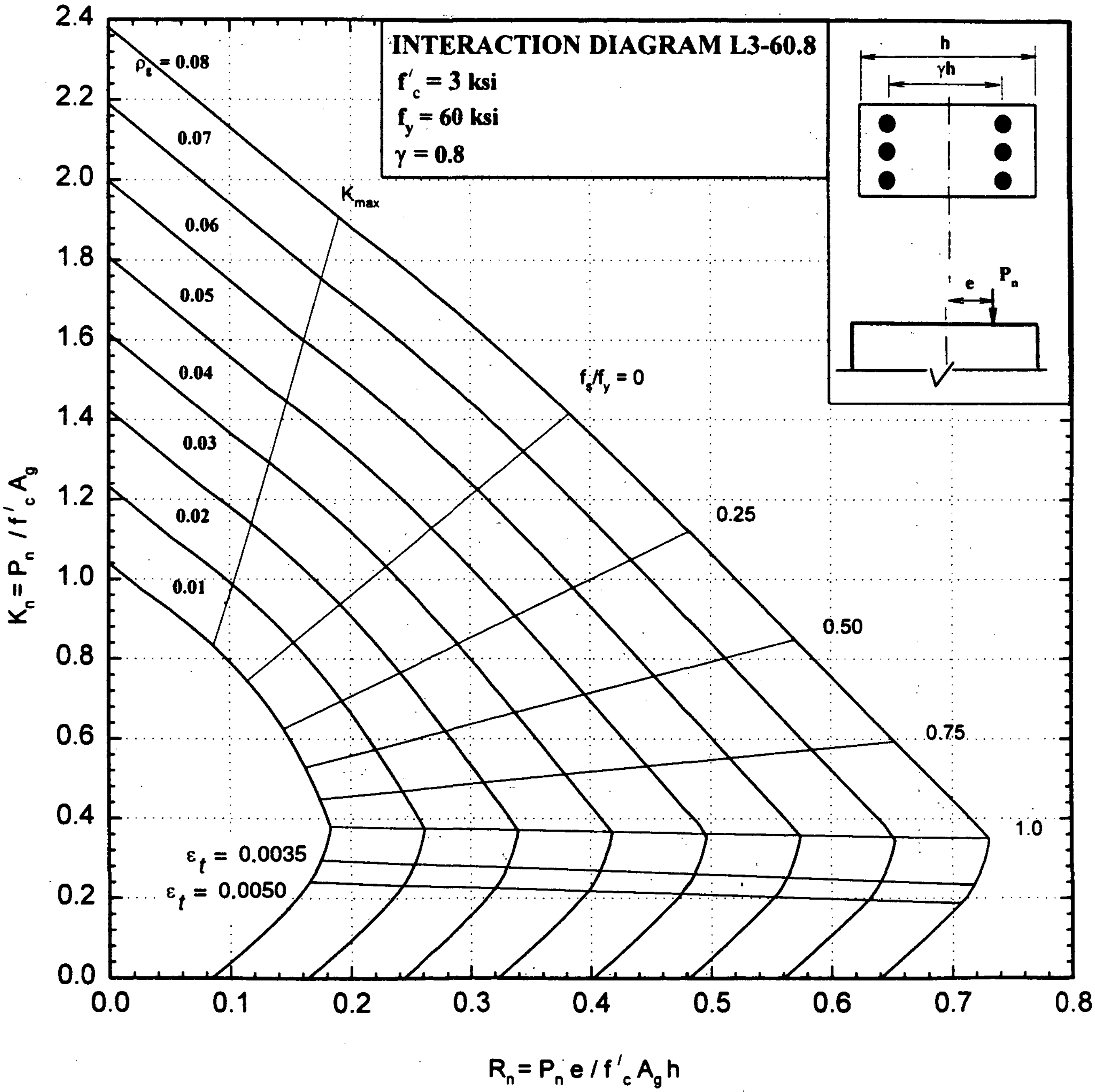


Diagram plotted using data from computer programs developed by Dr. Noel J. Everard.
Computer plot of data for diagram by Dr. Mohsen A. Issa and Alfred A. Yousif.

COLUMNS 7.7.4 - Nominal load-moment strength interaction diagram, L3-60.9

References: "Building Code Requirements for Structural Concrete-ACI 318" Chapters 9 and 10, and "Ultimate Strength Design of Reinforced Concrete Columns", ACI Special Publication SP-7, by Everard and Cohen, 1964, pp. 152-182 (with corrections).

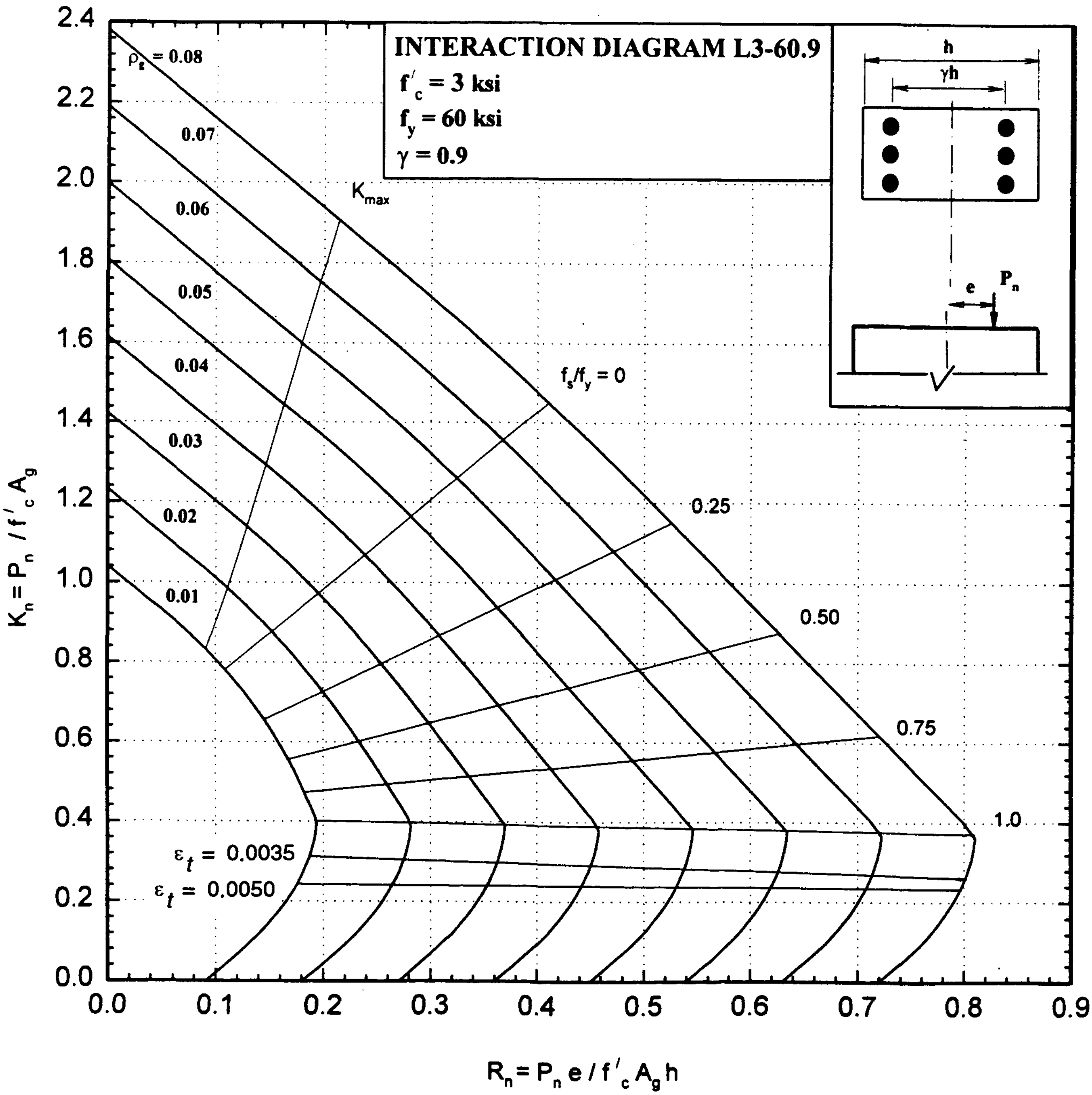


Diagram plotted using data from computer programs developed by Dr. Noel J. Everard.
Computer plot of data for diagram by Dr. Mohsen A. Issa and Alfred A. Yousif.

COLUMNS 7.8.1 - Nominal load-moment strength interaction diagram, L4-60.6

References: "Building Code Requirements for Structural Concrete-ACI 318" Chapters 9 and 10, and "Ultimate Strength Design of Reinforced Concrete Columns", ACI Special Publication SP-7, by Everard and Cohen, 1964, pp. 152-182 (with corrections).

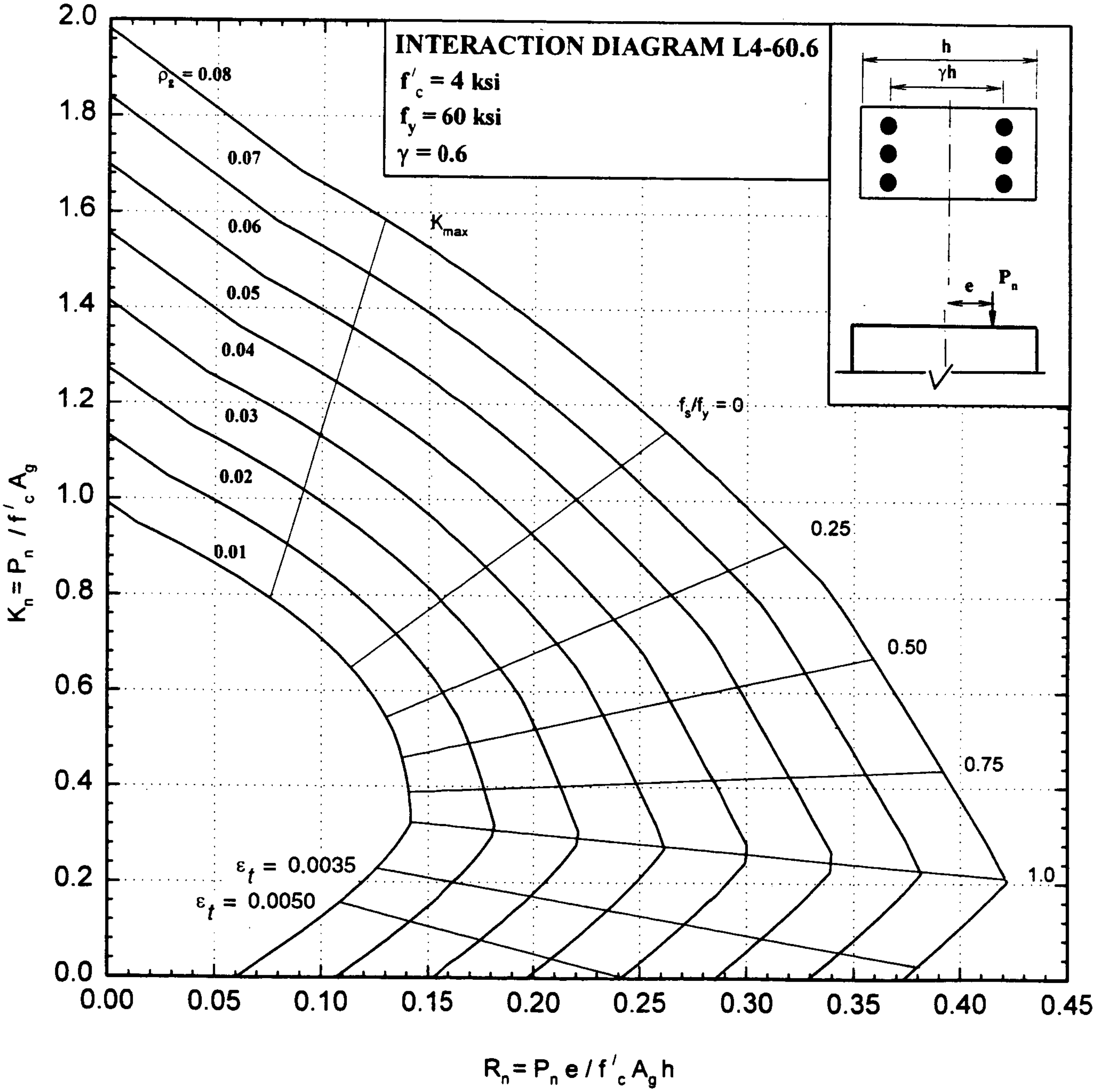


Diagram plotted using data from computer programs developed by Dr. Noel J. Everard.
Computer plot of data for diagram by Dr. Mohsen A. Issa and Alfred A. Yousif.

COLUMNS 7.8.2 - Nominal load-moment strength interaction diagram, L4-60.7

References: "Building Code Requirements for Structural Concrete-ACI 318" Chapters 9 and 10, and "Ultimate Strength Design of Reinforced Concrete Columns", ACI Special Publication SP-7, by Everard and Cohen, 1964, pp. 152-182 (with corrections).

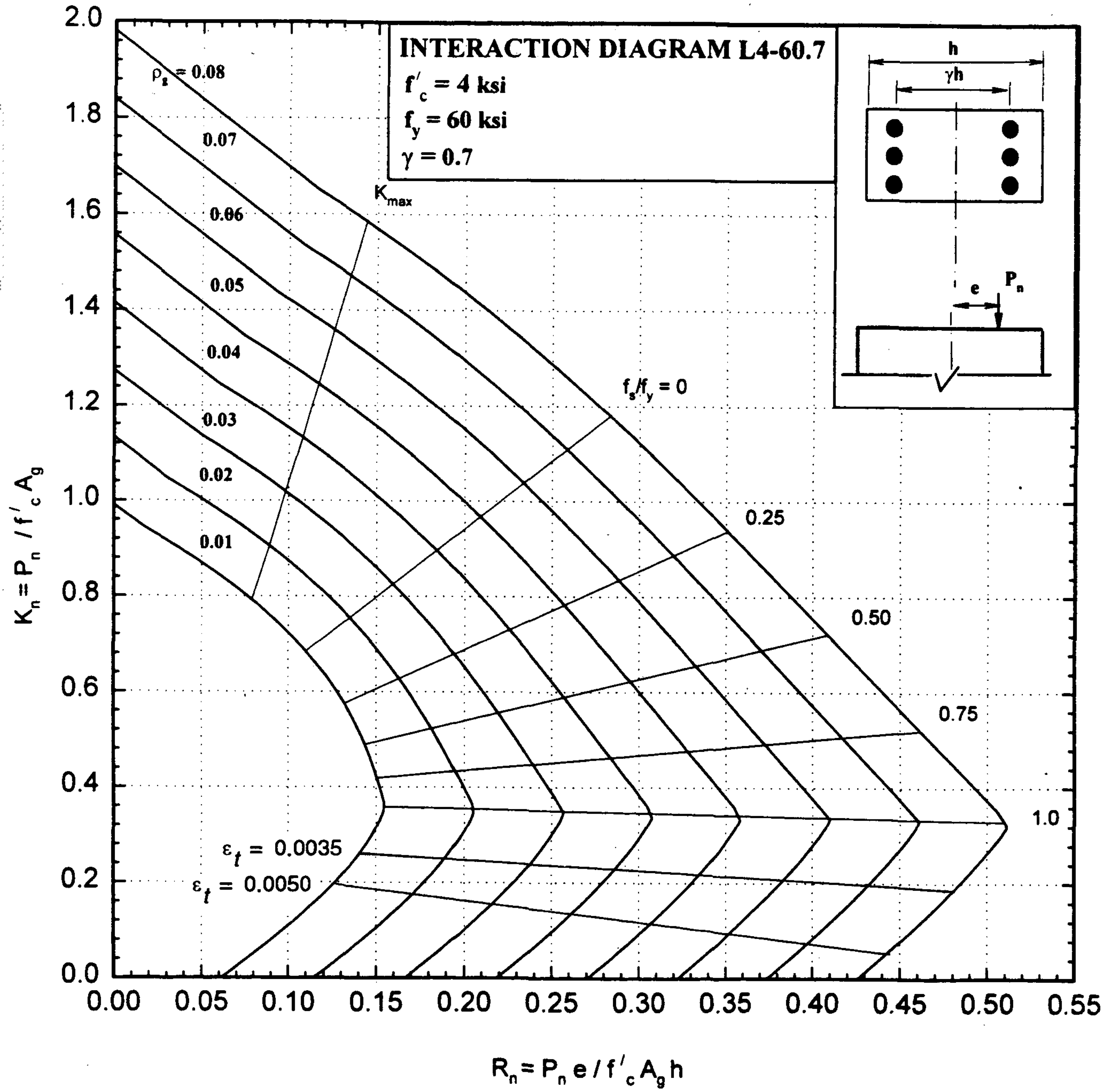


Diagram plotted using data from computer programs developed by Dr. Noel J. Everard.
Computer plot of data for diagram by Dr. Mohsen A. Issa and Alfred A. Yousif.

COLUMNS 7.8.3 - Nominal load-moment strength interaction diagram, L4-60.8

References: "Building Code Requirements for Structural Concrete-ACI 318" Chapters 9 and 10, and "Ultimate Strength Design of Reinforced Concrete Columns", ACI Special Publication SP-7, by Everard and Cohen, 1964, pp. 152-182 (with corrections).

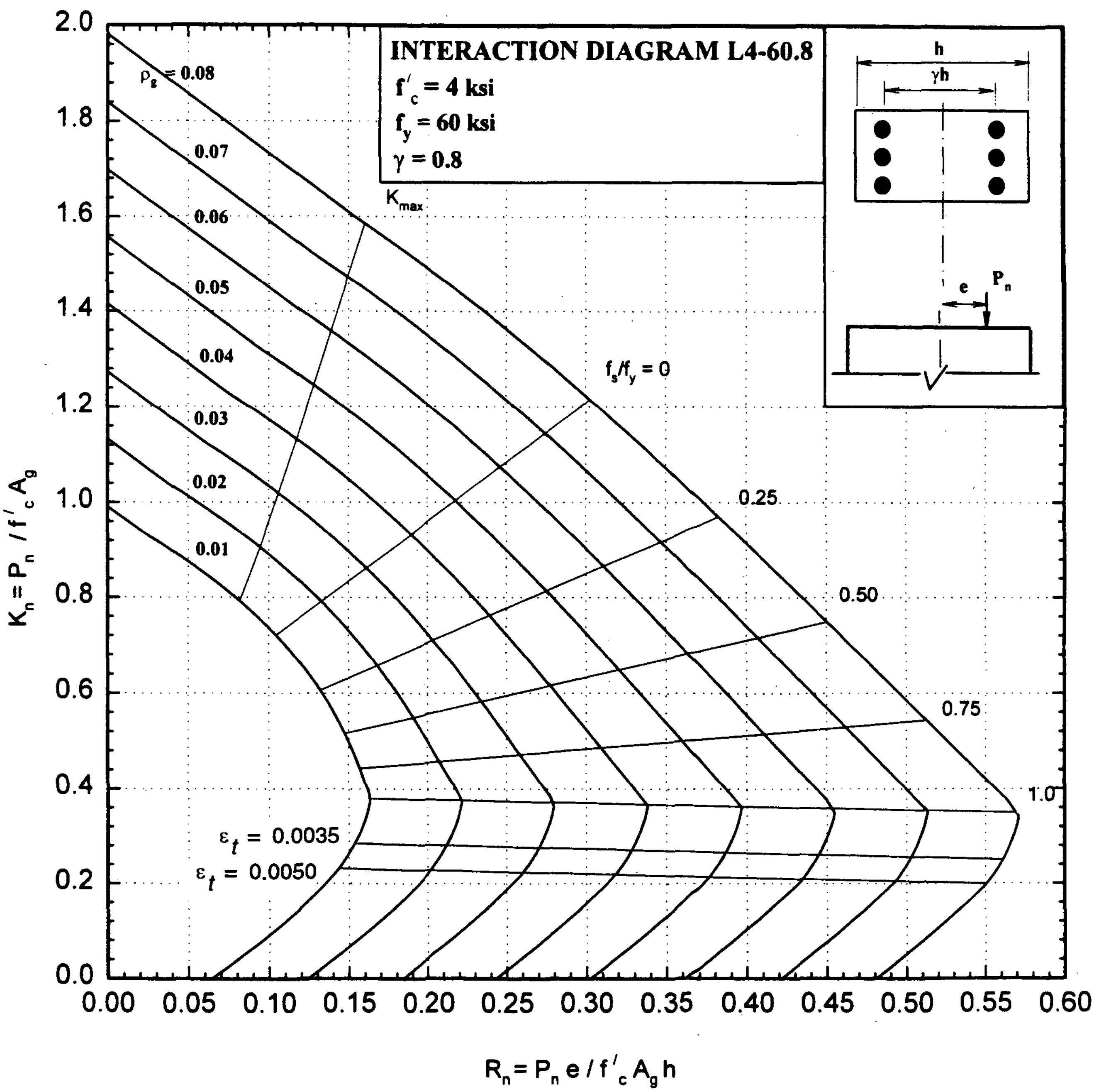


Diagram plotted using data from computer programs developed by Dr. Noel J. Everard.
Computer plot of data for diagram by Dr. Mohsen A. Issa and Alfred A. Yousif.

COLUMNS 7.8.4 - Nominal load-moment strength interaction diagram, L4-60.9

References: "Building Code Requirements for Structural Concrete-ACI 318" Chapters 9 and 10, and "Ultimate Strength Design of Reinforced Concrete Columns", ACI Special Publication SP-7, by Everard and Cohen, 1964, pp. 152-182 (with corrections).

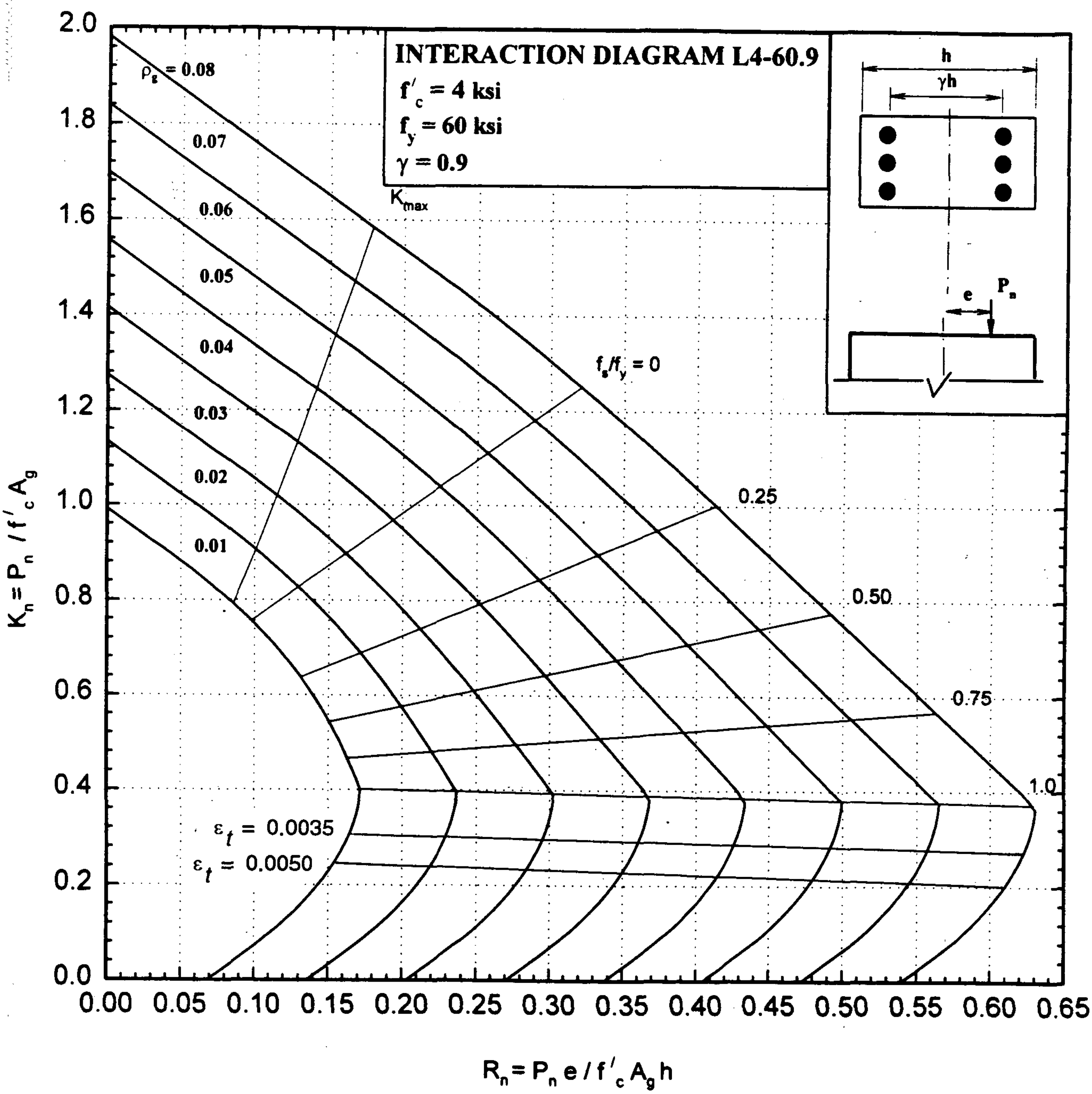


Diagram plotted using data from computer programs developed by Dr. Noel J. Everard.
Computer plot of data for diagram by Dr. Mohsen A. Issa and Alfred A. Yousif.

COLUMNS 7.9.1 - Nominal load-moment strength interaction diagram, L5-60.6

References: "Building Code Requirements for Structural Concrete-ACI 318" Chapters 9 and 10, and "Ultimate Strength Design of Reinforced Concrete Columns", ACI Special Publication SP-7, by Everard and Cohen, 1964, pp. 152-182 (with corrections).

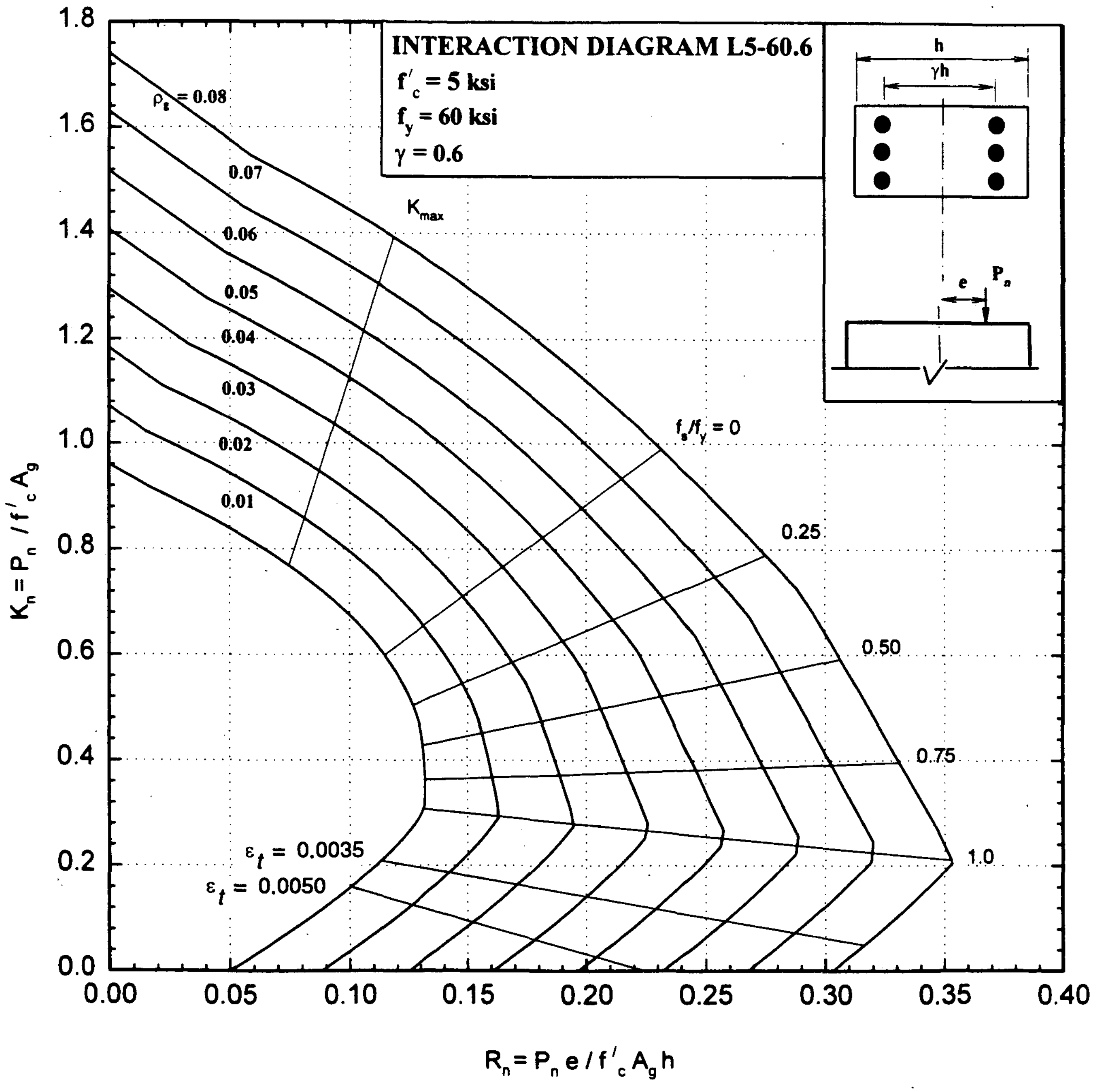


Diagram plotted using data from computer programs developed by Dr. Noel J. Everard.
Computer plot of data for diagram by Dr. Mohsen A. Issa and Alfred A. Yousif.

COLUMNS 7.9.2 - Nominal load-moment strength interaction diagram, L5-60.7

References: "Building Code Requirements for Structural Concrete-ACI 318" Chapters 9 and 10, and "Ultimate Strength Design of Reinforced Concrete Columns", ACI Special Publication SP-7, by Everard and Cohen, 1964, pp. 152-182 (with corrections).

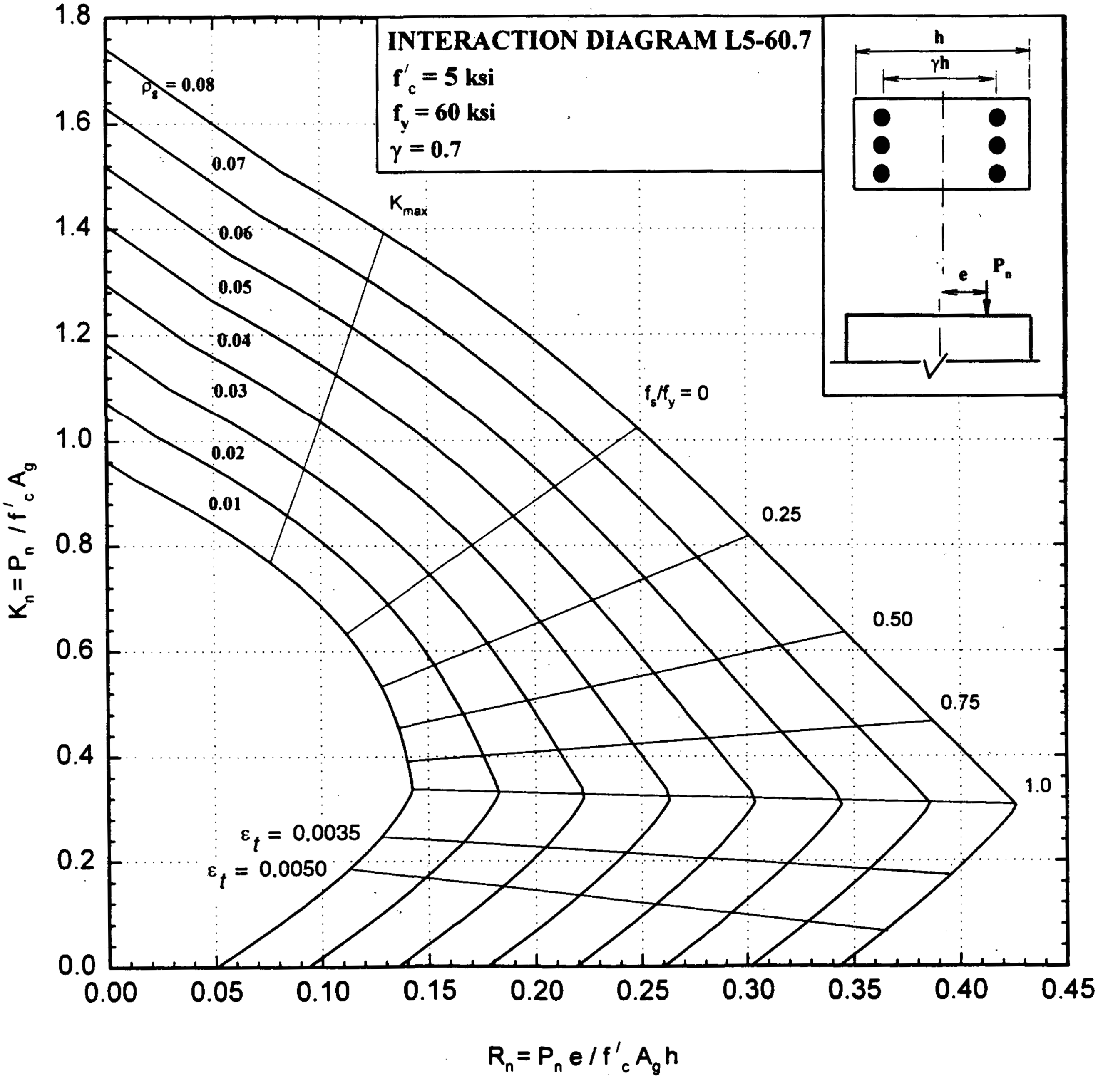


Diagram plotted using data from computer programs developed by Dr. Noel J. Everard. Computer plot of data for diagram by Dr. Mohsen A. Issa and Alfred A. Yousif.

COLUMNS 7.9.3 - Nominal load-moment strength interaction diagram, L5-60.8

References: "Building Code Requirements for Structural Concrete-ACI 318" Chapters 9 and 10, and "Ultimate Strength Design of Reinforced Concrete Columns", ACI Special Publication SP-7, by Everard and Cohen, 1964, pp. 152-182 (with corrections).

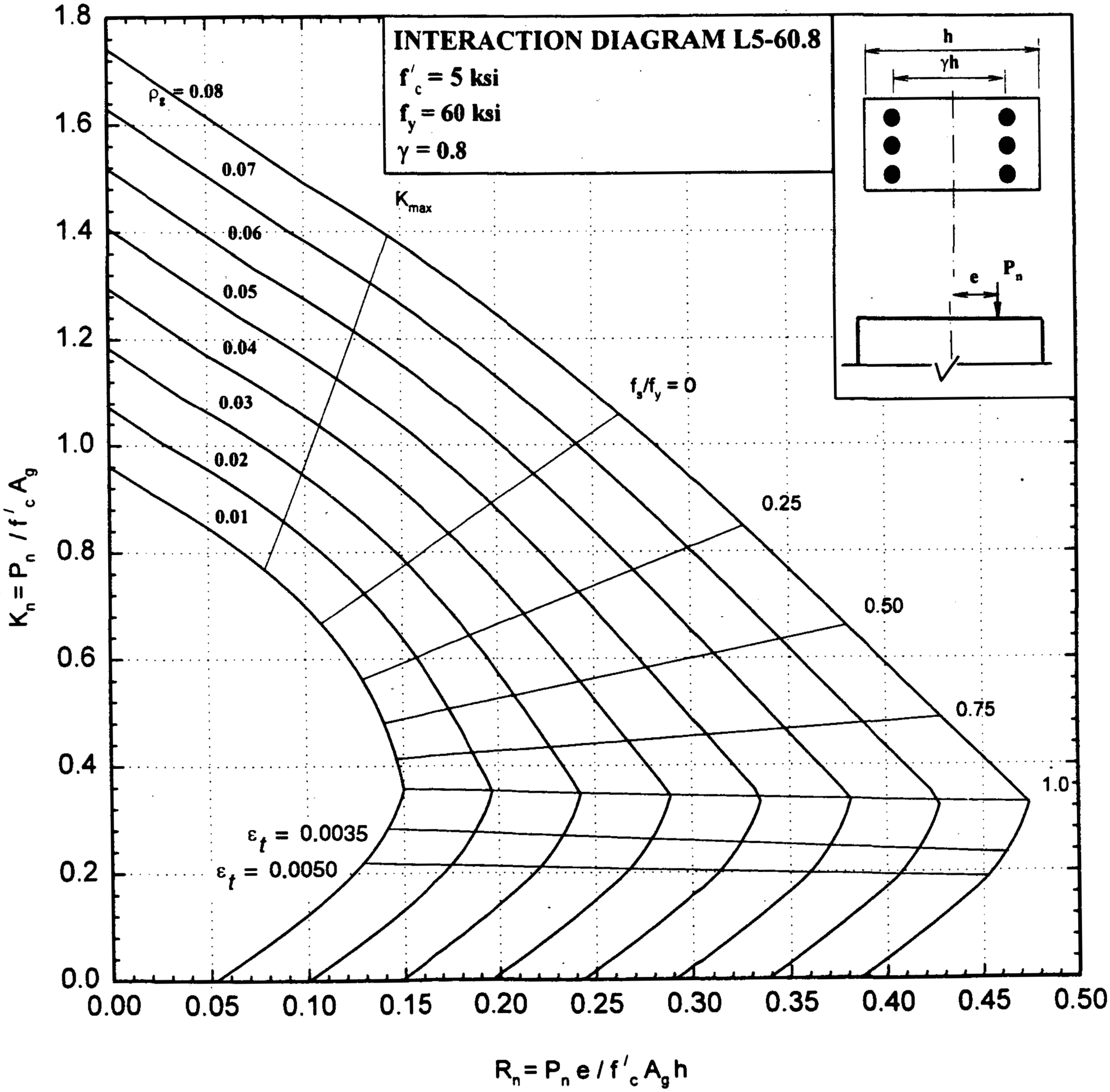


Diagram plotted using data from computer programs developed by Dr. Noel J. Everard. Computer plot of data for diagram by Dr. Mohsen A. Issa and Alfred A. Yousif.

COLUMNS 7.9.4 - Nominal load-moment strength interaction diagram, L5-60.9

References: "Building Code Requirements for Structural Concrete-ACI 318" Chapters 9 and 10, and "Ultimate Strength Design of Reinforced Concrete Columns", ACI Special Publication SP-7, by Everard and Cohen, 1964, pp. 152-182 (with corrections).

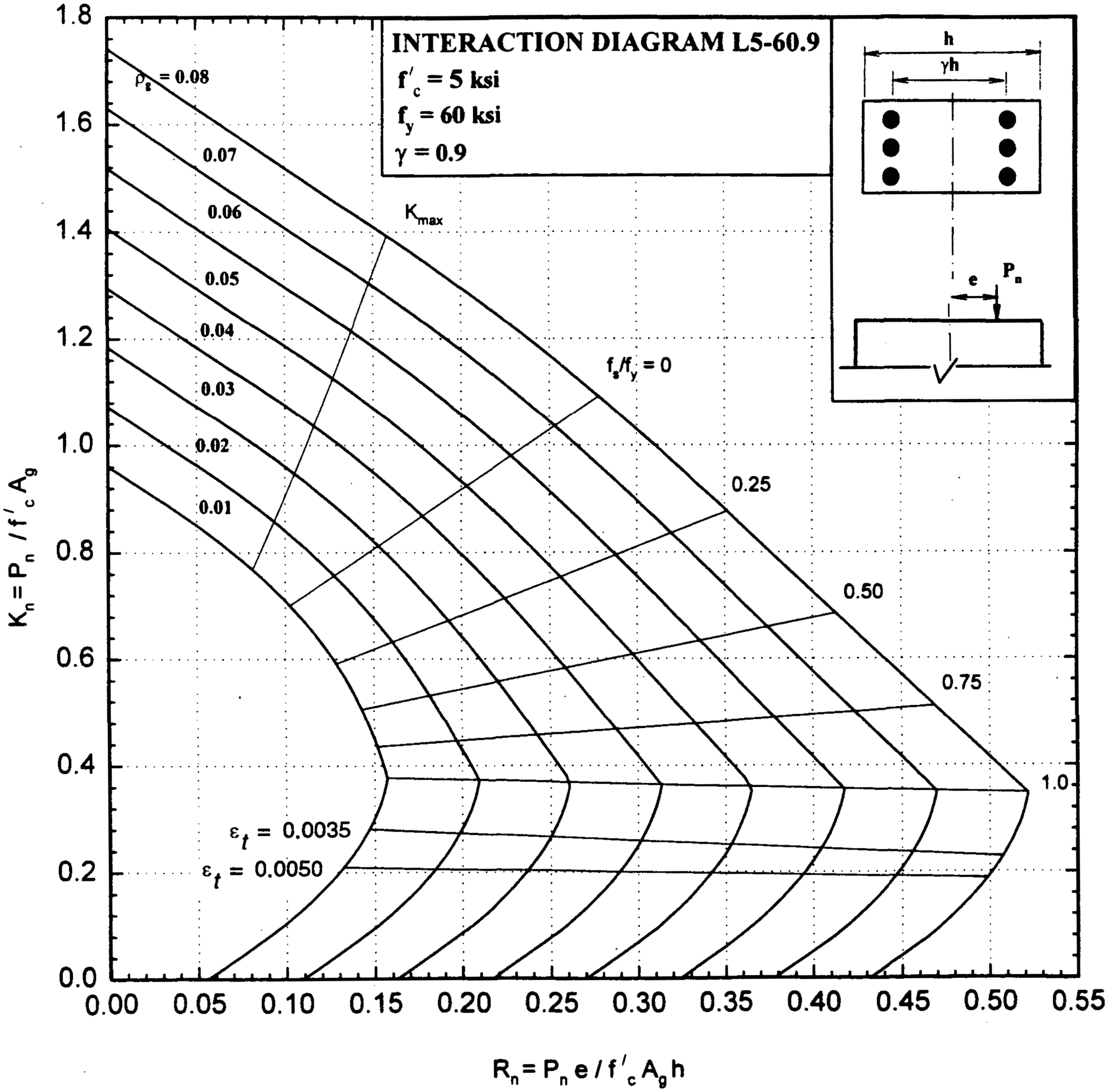


Diagram plotted using data from computer programs developed by Dr. Noel J. Everard.
Computer plot of data for diagram by Dr. Mohsen A. Issa and Alfred A. Yousif.

COLUMNS 7.10.1 - Nominal load-moment strength interaction diagram, L6-60.6

References: "Building Code Requirements for Structural Concrete-ACI 318" Chapters 9 and 10, and "Ultimate Strength Design of Reinforced Concrete Columns", ACI Special Publication SP-7, by Everard and Cohen, 1964, pp. 152-182 (with corrections).

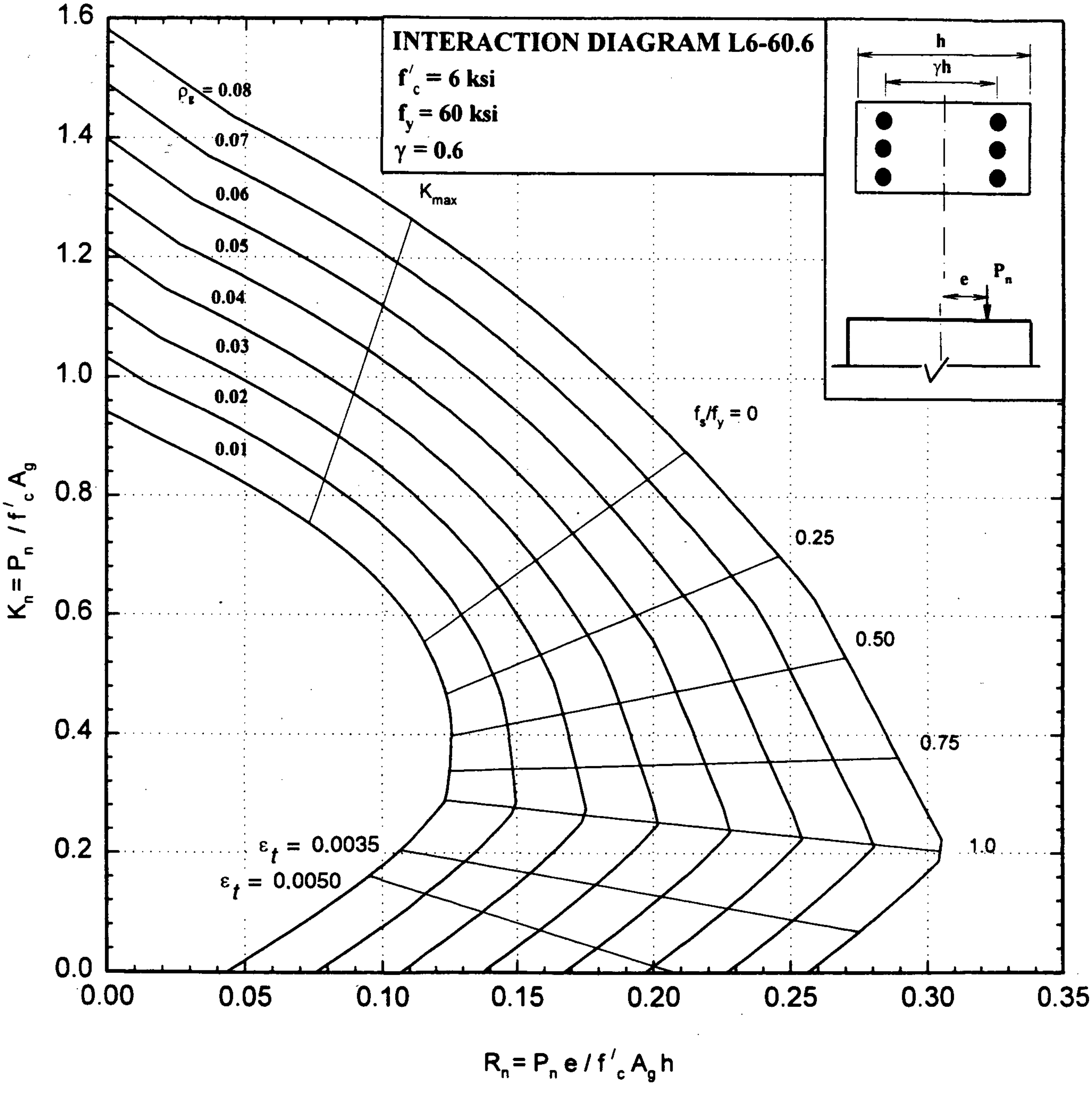


Diagram plotted using data from computer programs developed by Dr. Noel J. Everard. Computer plot of data for diagram by Dr. Mohsen A. Issa and Alfred A. Yousif.

COLUMNS 7.10.2 - Nominal load-moment strength interaction diagram, L6-60.7

References: "Building Code Requirements for Structural Concrete-ACI 318" Chapters 9 and 10, and "Ultimate Strength Design of Reinforced Concrete Columns", ACI Special Publication SP-7, by Everard and Cohen, 1964, pp. 152-182 (with corrections).

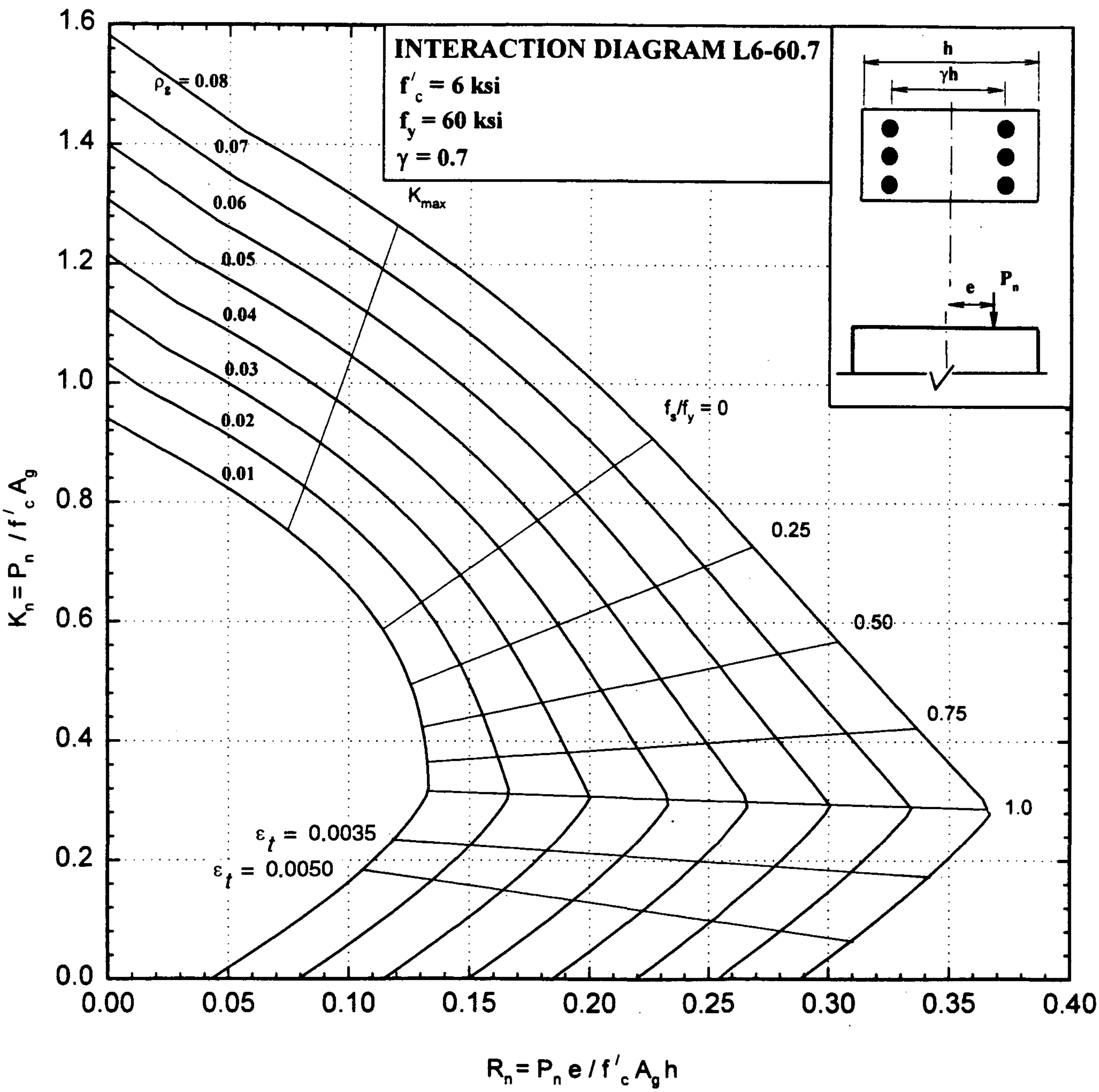


Diagram plotted using data from computer programs developed by Dr. Noel J. Everard.
Computer plot of data for diagram by Dr. Mohsen A. Issa and Alfred A. Yousif.

COLUMNS 7.10.3 - Nominal load-moment strength interaction diagram, L6-60.8

References: "Building Code Requirements for Structural Concrete-ACI 318" Chapters 9 and 10, and "Ultimate Strength Design of Reinforced Concrete Columns", ACI Special Publication SP-7, by Everard and Cohen, 1964, pp. 152-182 (with corrections).

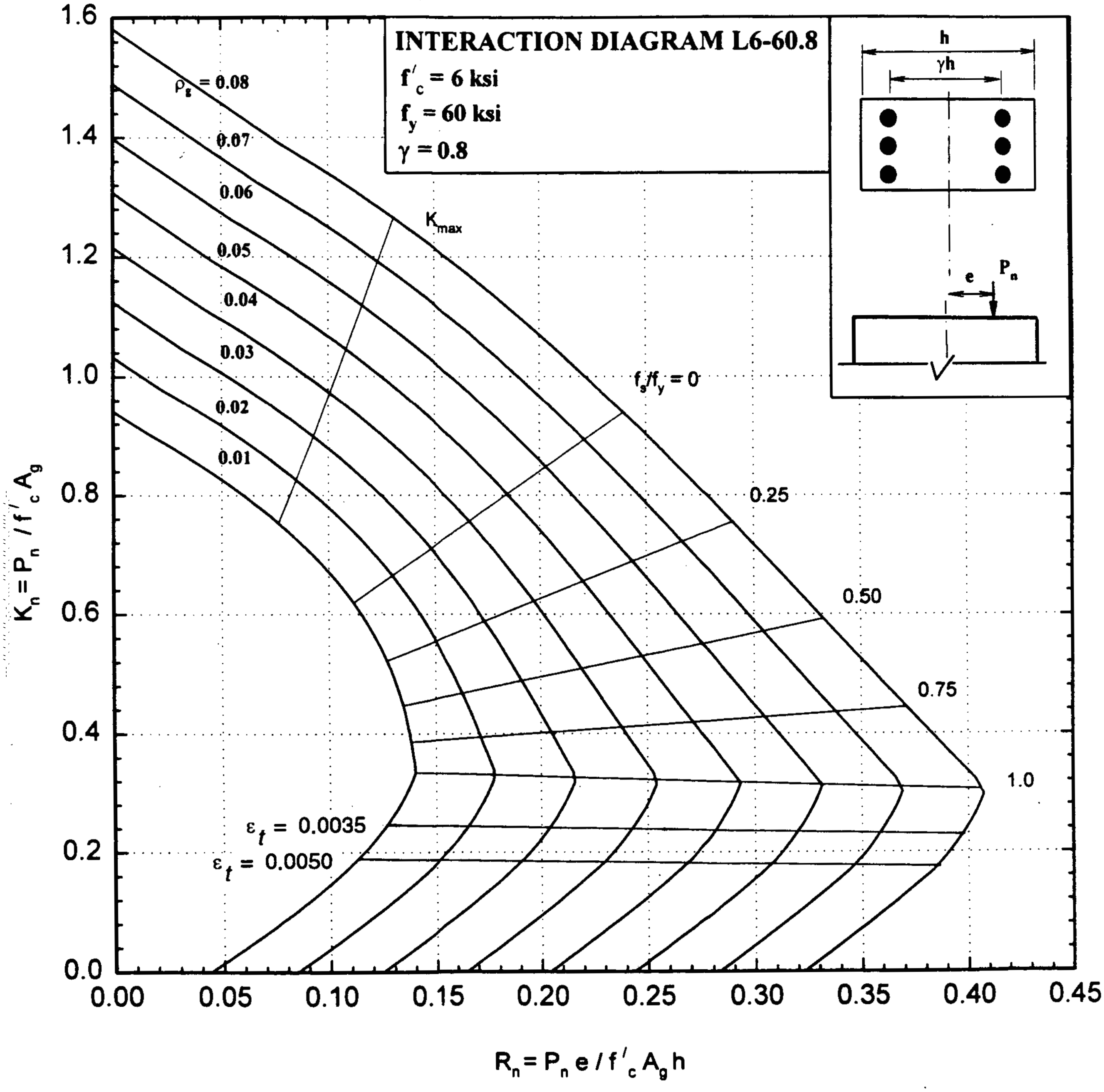


Diagram plotted using data from computer programs developed by Dr. Noel J. Everard.
Computer plot of data for diagram by Dr. Mohsen A. Issa and Alfred A. Yousif.

References: "Building Code Requirements for Structural Concrete-ACI 318" Chapters 9 and 10, and "Ultimate Strength Design of Reinforced Concrete Columns", ACI Special Publication SP-7, by Everard and Cohen, 1964, pp. 152-182 (with corrections).

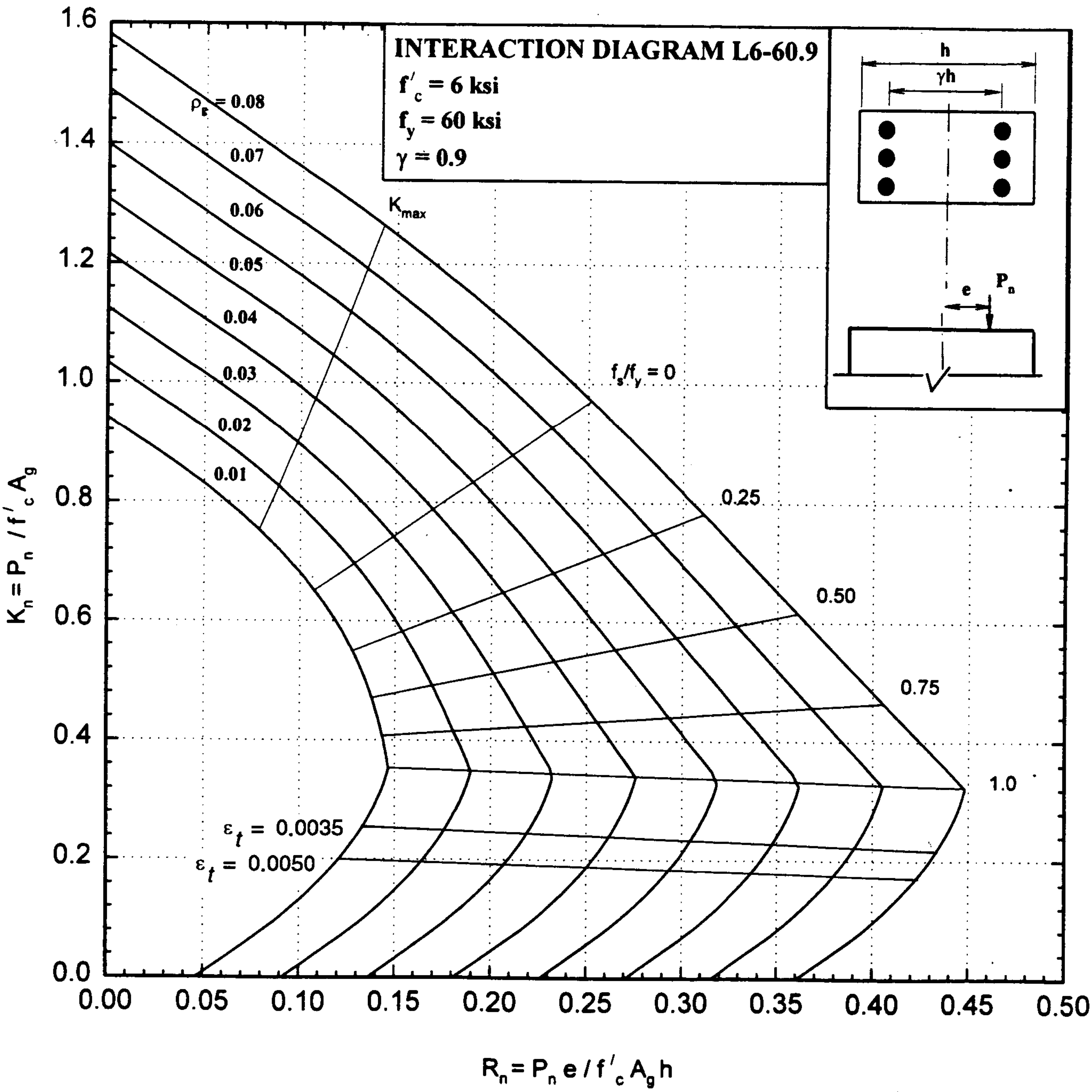


Diagram plotted using data from computer programs developed by Dr. Noel J. Everard.
Computer plot of data for diagram by Dr. Mohsen A. Issa and Alfred A. Yousif.

COLUMNS 7.11.1 - Nominal load-moment strength interaction diagram, L9-75.6

References: "Building Code Requirements for Structural Concrete-ACI 318" Chapters 9 and 10, and "Ultimate Strength Design of Reinforced Concrete Columns", ACI Special Publication SP-7, by Everard and Cohen, 1964, pp. 152-182 (with corrections).

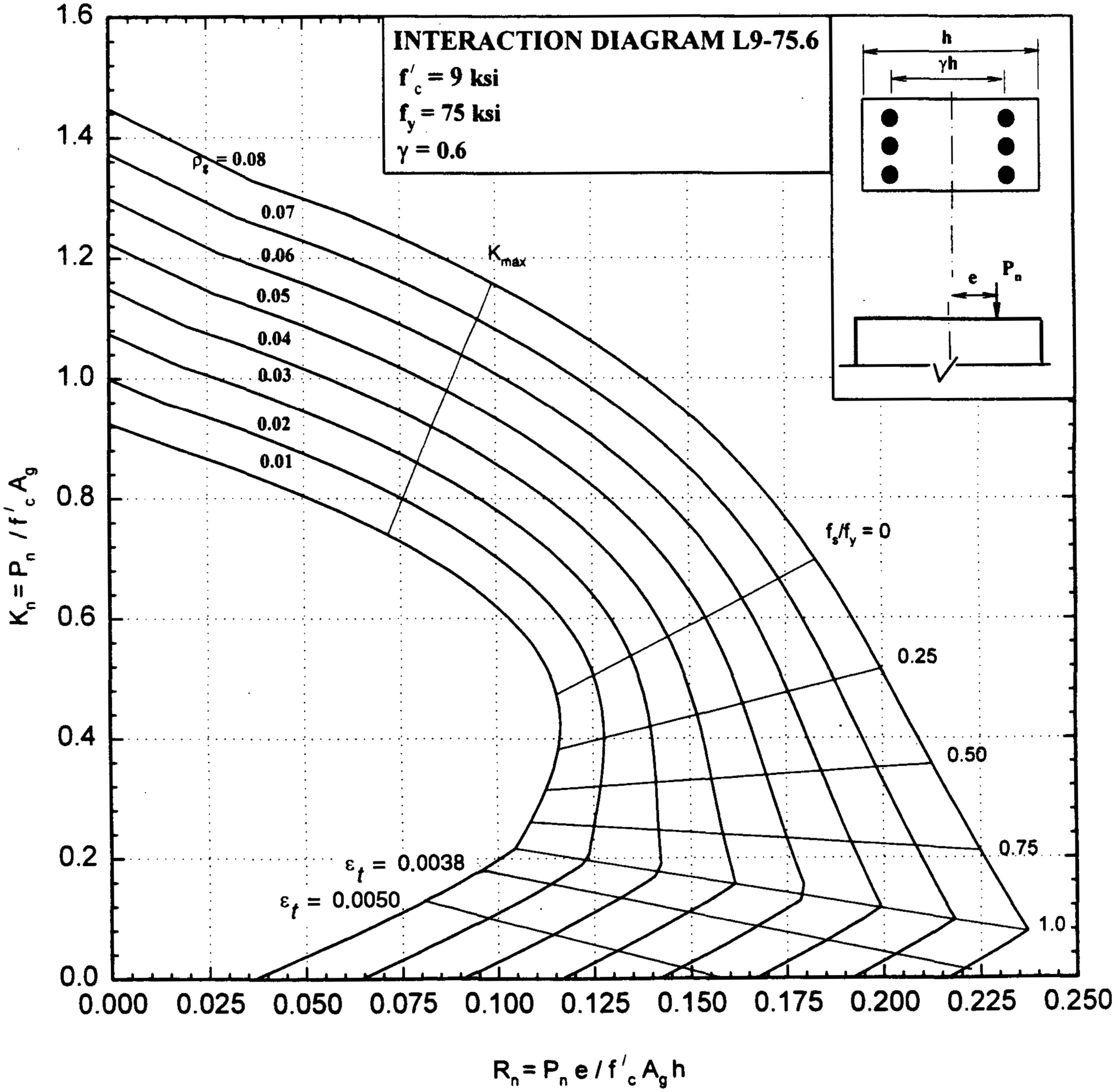


Diagram plotted using data from computer programs developed by Dr. Noel J. Everard.
Computer plot of data for diagram by Dr. Mohsen A. Issa and Alfred A. Yousif.

COLUMNS 7.11.2 - Nominal load-moment strength interaction diagram, L9-75.7

References: "Building Code Requirements for Structural Concrete-ACI 318" Chapters 9 and 10, and "Ultimate Strength Design of Reinforced Concrete Columns", ACI Special Publication SP-7, by Everard and Cohen, 1964, pp. 152-182 (with corrections).

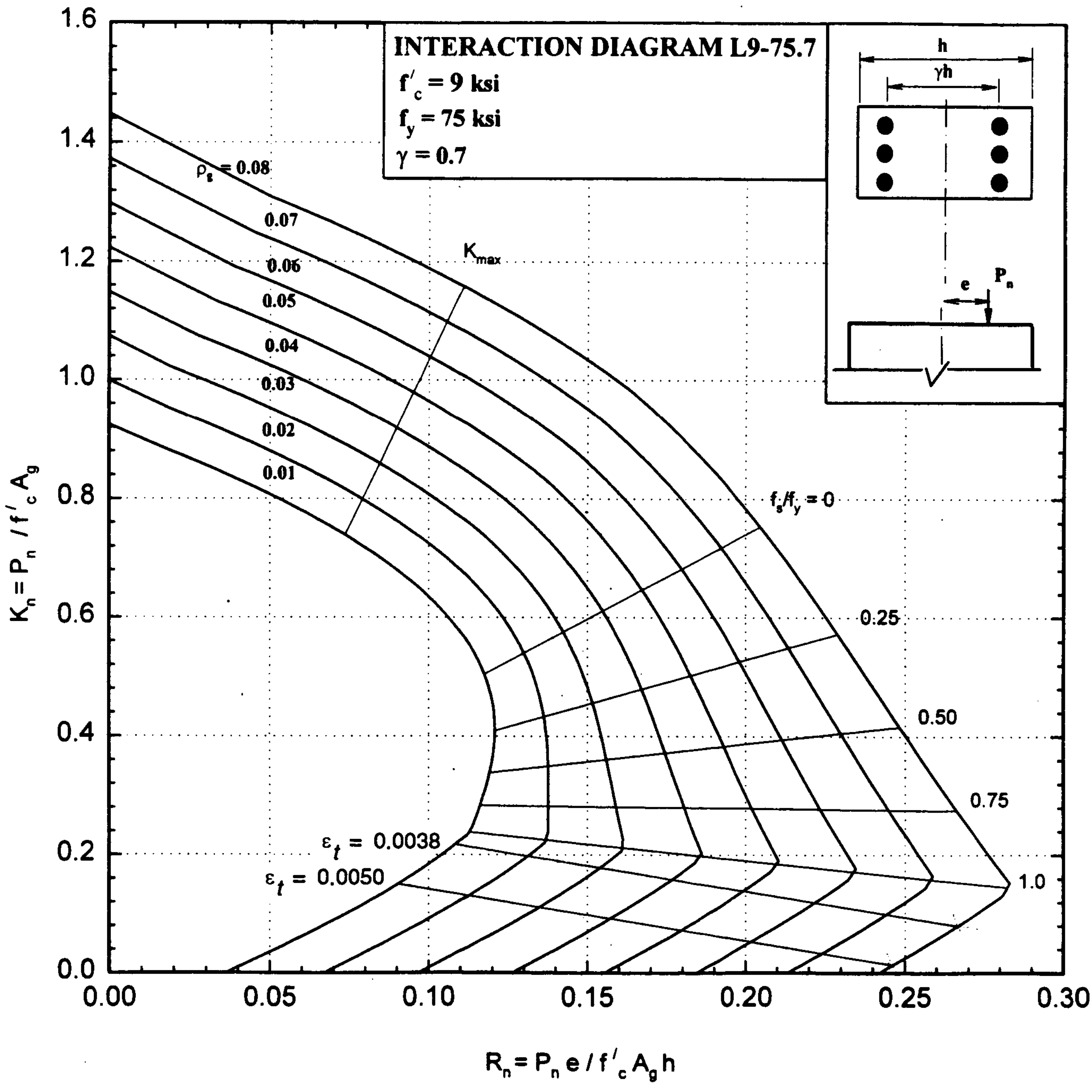


Diagram plotted using data from computer programs developed by Dr. Noel J. Everard.
Computer plot of data for diagram by Dr. Mohsen A. Issa and Alfred A. Yousif.

COLUMNS 7.11.3 - Nominal load-moment strength interaction diagram, L9-75.8

References: "Building Code Requirements for Structural Concrete-ACI 318" Chapters 9 and 10, and "Ultimate Strength Design of Reinforced Concrete Columns", ACI Special Publication SP-7, by Everard and Cohen, 1964, pp. 152-182 (with corrections).

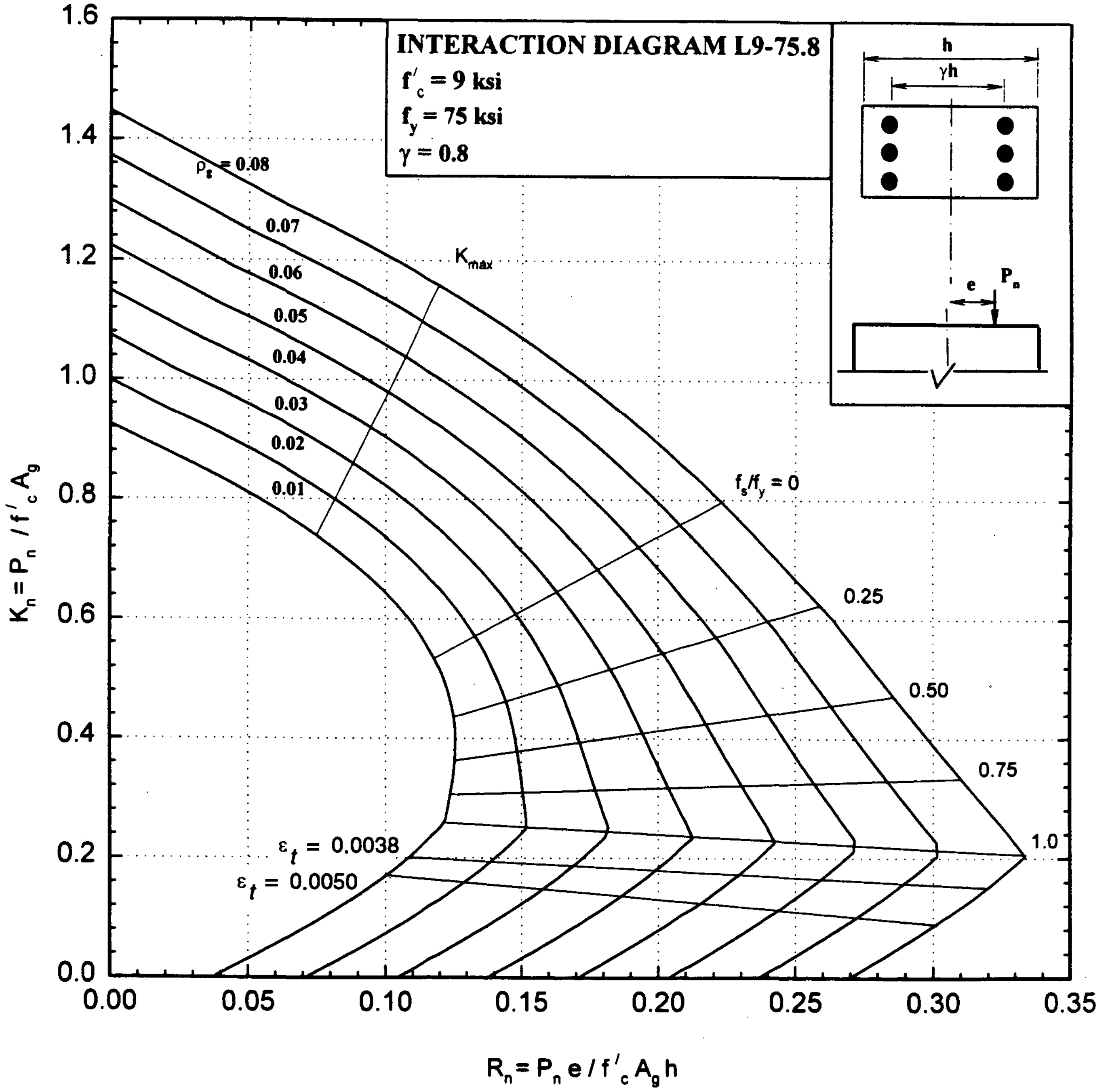


Diagram plotted using data from computer programs developed by Dr. Noel J. Everard. Computer plot of data for diagram by Dr. Mohsen A. Issa and Alfred A. Yousif.

COLUMNS 7.11.4 - Nominal load-moment strength interaction diagram, L9-75.9

References: "Building Code Requirements for Structural Concrete-ACI 318" Chapters 9 and 10, and "Ultimate Strength Design of Reinforced Concrete Columns", ACI Special Publication SP-7, by Everard and Cohen, 1964, pp. 152-182 (with corrections).

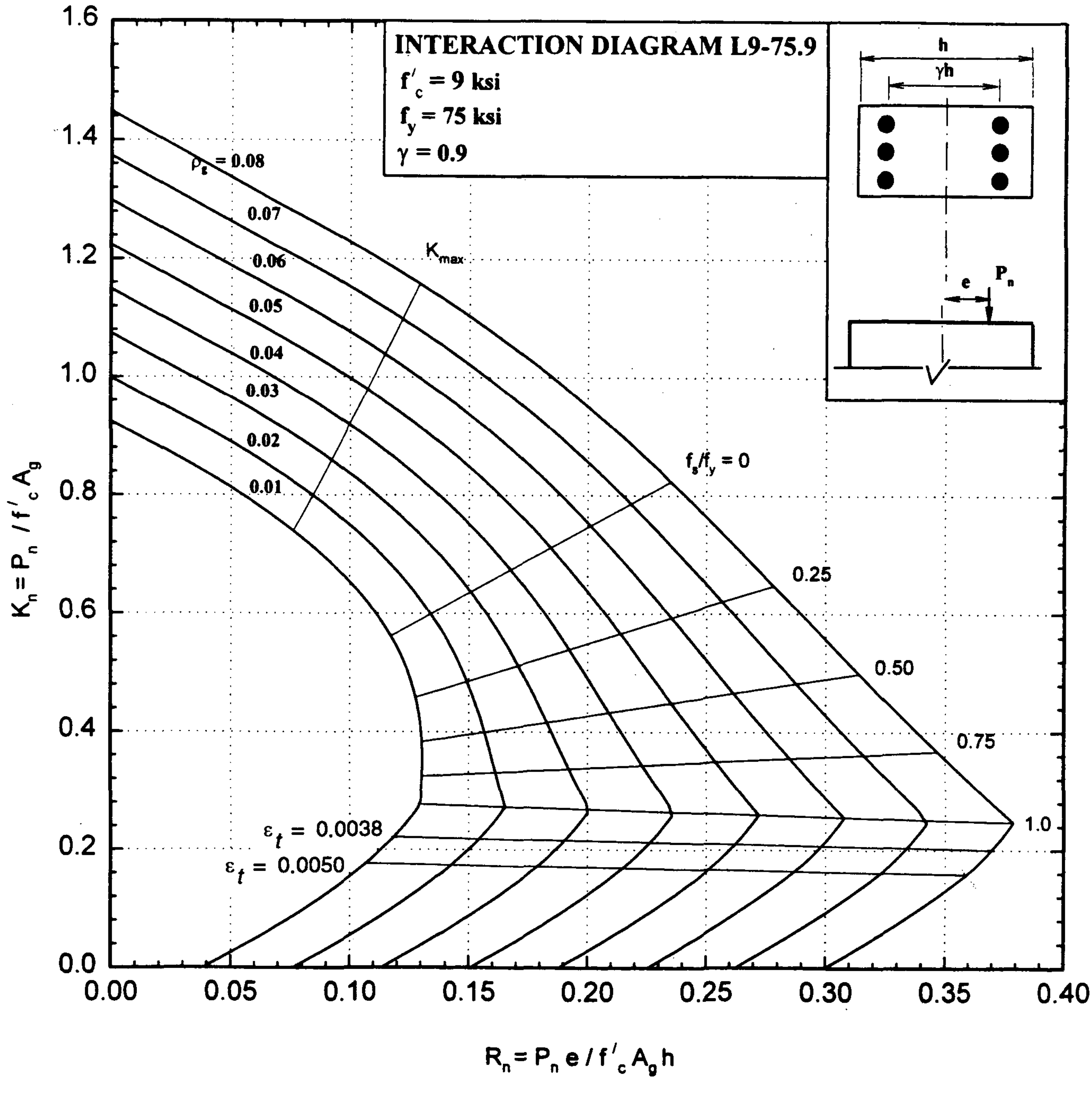


Diagram plotted using data from computer programs developed by Dr. Noel J. Everard. Computer plot of data for diagram by Dr. Mohsen A. Issa and Alfred A. Yousif.

COLUMNS 7.12.1 - Nominal load-moment strength interaction diagram, L12-75.6

References: "Building Code Requirements for Structural Concrete-ACI 318" Chapters 9 and 10, and "Ultimate Strength Design of Reinforced Concrete Columns", ACI Special Publication SP-7, by Everard and Cohen, 1964, pp. 152-182 (with corrections).

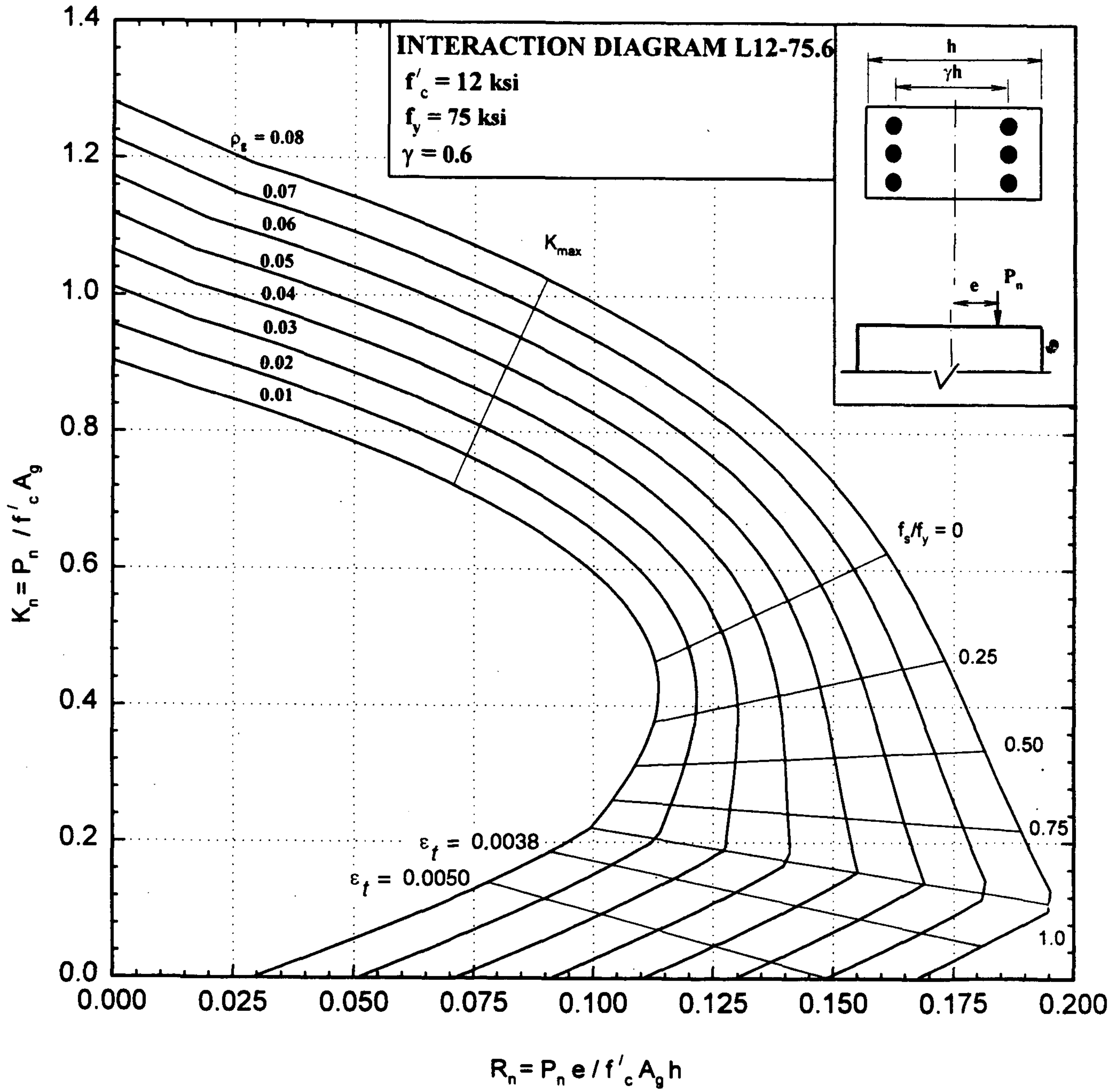


Diagram plotted using data from computer programs developed by Dr. Noel J. Everard.
Computer plot of data for diagram by Dr. Mohsen A. Issa and Alfred A. Yousif.

COLUMNS 7.12.2 - Nominal load-moment strength interaction diagram, L12-75.7

References: "Building Code Requirements for Structural Concrete-ACI 318" Chapters 9 and 10, and "Ultimate Strength Design of Reinforced Concrete Columns", ACI Special Publication SP-7, by Everard and Cohen, 1964, pp. 152-182 (with corrections).

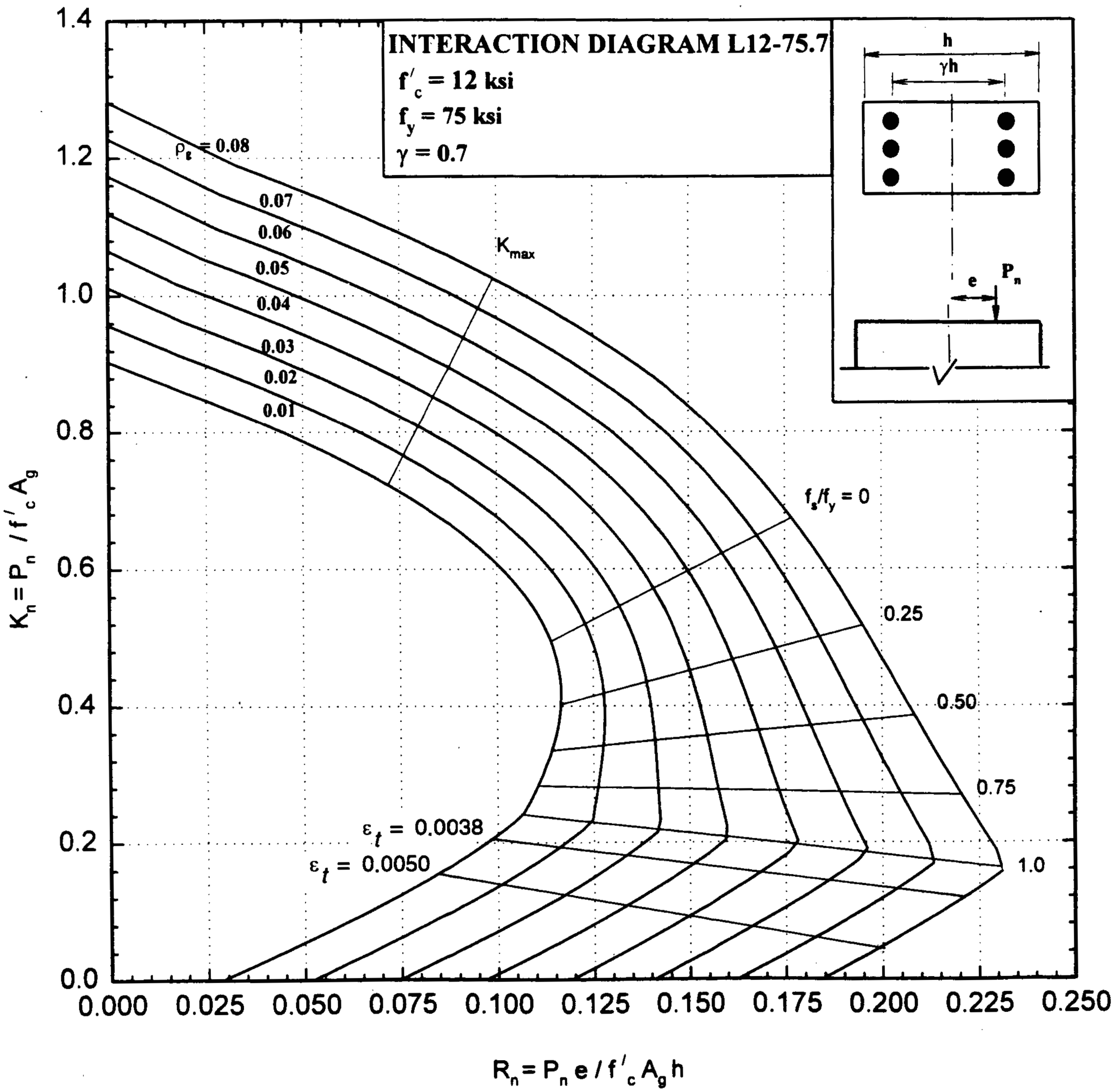


Diagram plotted using data from computer programs developed by Dr. Noel J. Everard. Computer plot of data for diagram by Dr. Mohsen A. Issa and Alfred A. Yousif.

COLUMNS 7.12.3 - Nominal load-moment strength interaction diagram, L12-75.8

References: "Building Code Requirements for Structural Concrete-ACI 318" Chapters 9 and 10, and "Ultimate Strength Design of Reinforced Concrete Columns", ACI Special Publication SP-7, by Everard and Cohen, 1964, pp. 152-182 (with corrections).

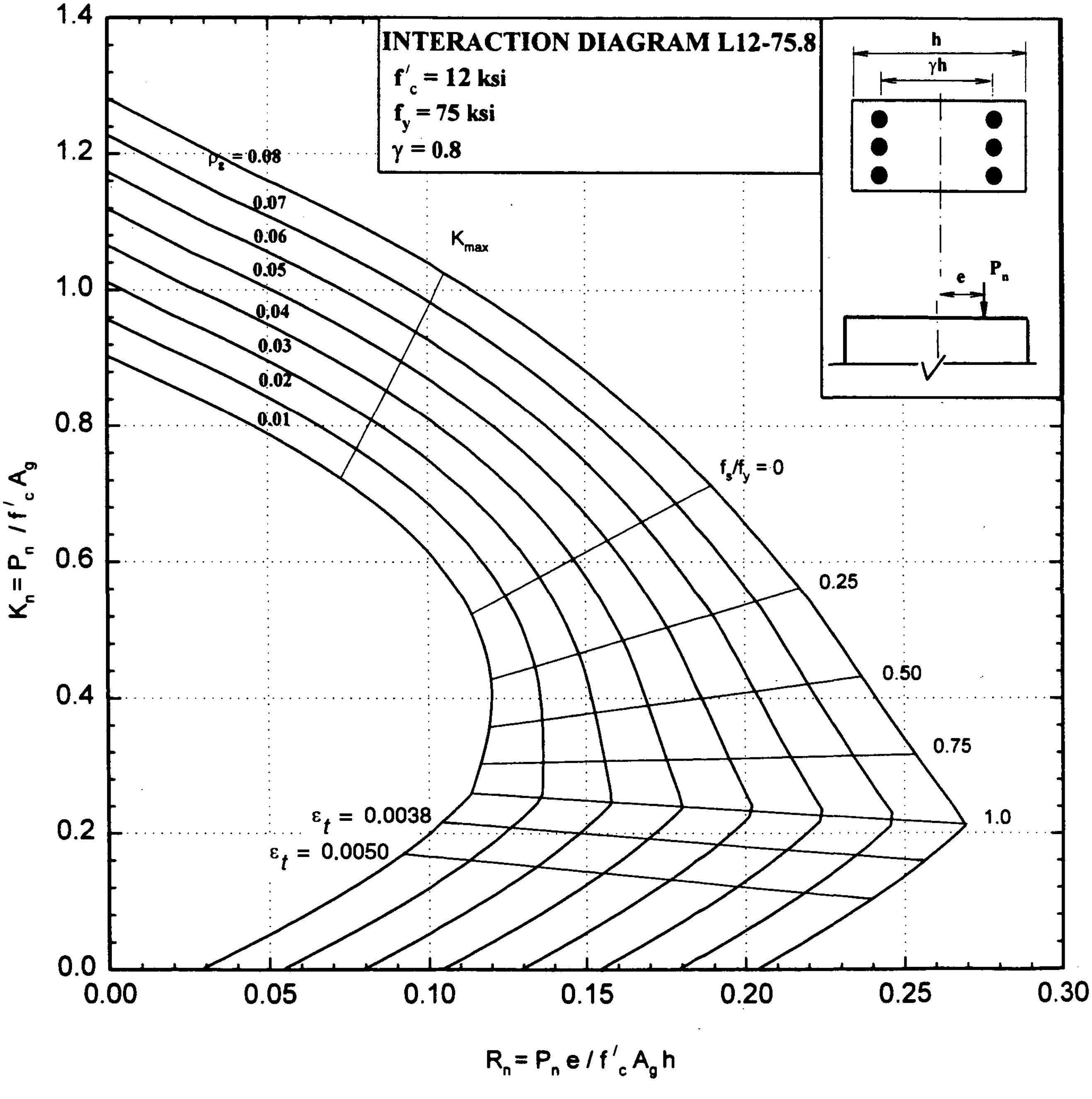


Diagram plotted using data from computer programs developed by Dr. Noel J. Everard. Computer plot of data for diagram by Dr. Mohsen A. Issa and Alfred A. Yousif.

COLUMNS 7.12.4 - Nominal load-moment strength interaction diagram, L12-75.9

References: "Building Code Requirements for Structural Concrete-ACI 318" Chapters 9 and 10, and "Ultimate Strength Design of Reinforced Concrete Columns", ACI Special Publication SP-7, by Everard and Cohen, 1964, pp. 152-182 (with corrections).

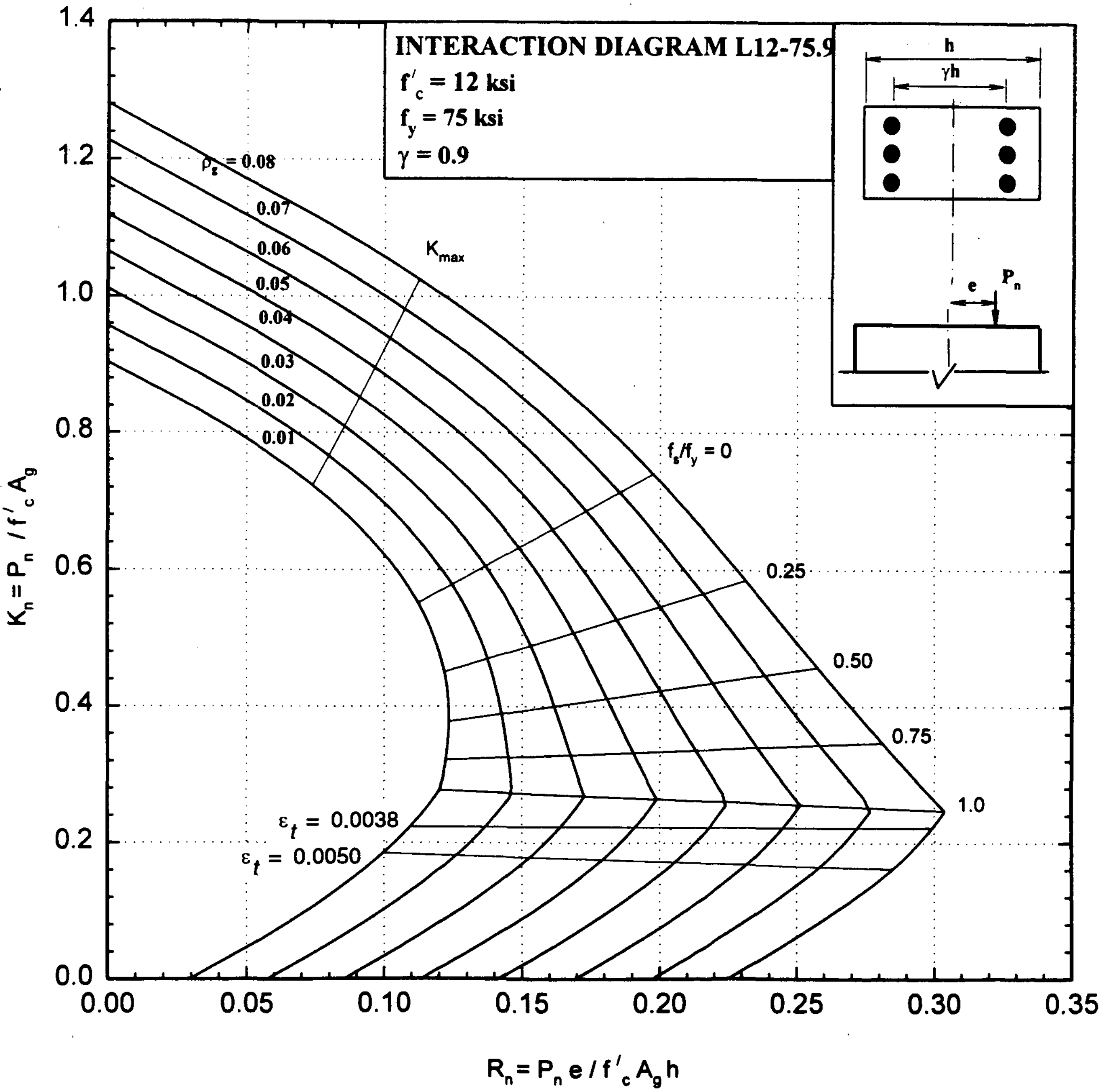


Diagram plotted using data from computer programs developed by Dr. Noel J. Everard. Computer plot of data for diagram by Dr. Mohsen A. Issa and Alfred A. Yousif.

COLUMNS 7.13.1 - Nominal load-moment strength interaction diagram, C3-60.6

References: "Building Code Requirements for Structural Concrete-ACI 318" Chapters 9 and 10, and "Ultimate Strength Design of Reinforced Concrete Columns", ACI Special Publication SP-7, by Everard and Cohen, 1964, pp. 152-182 (with corrections).

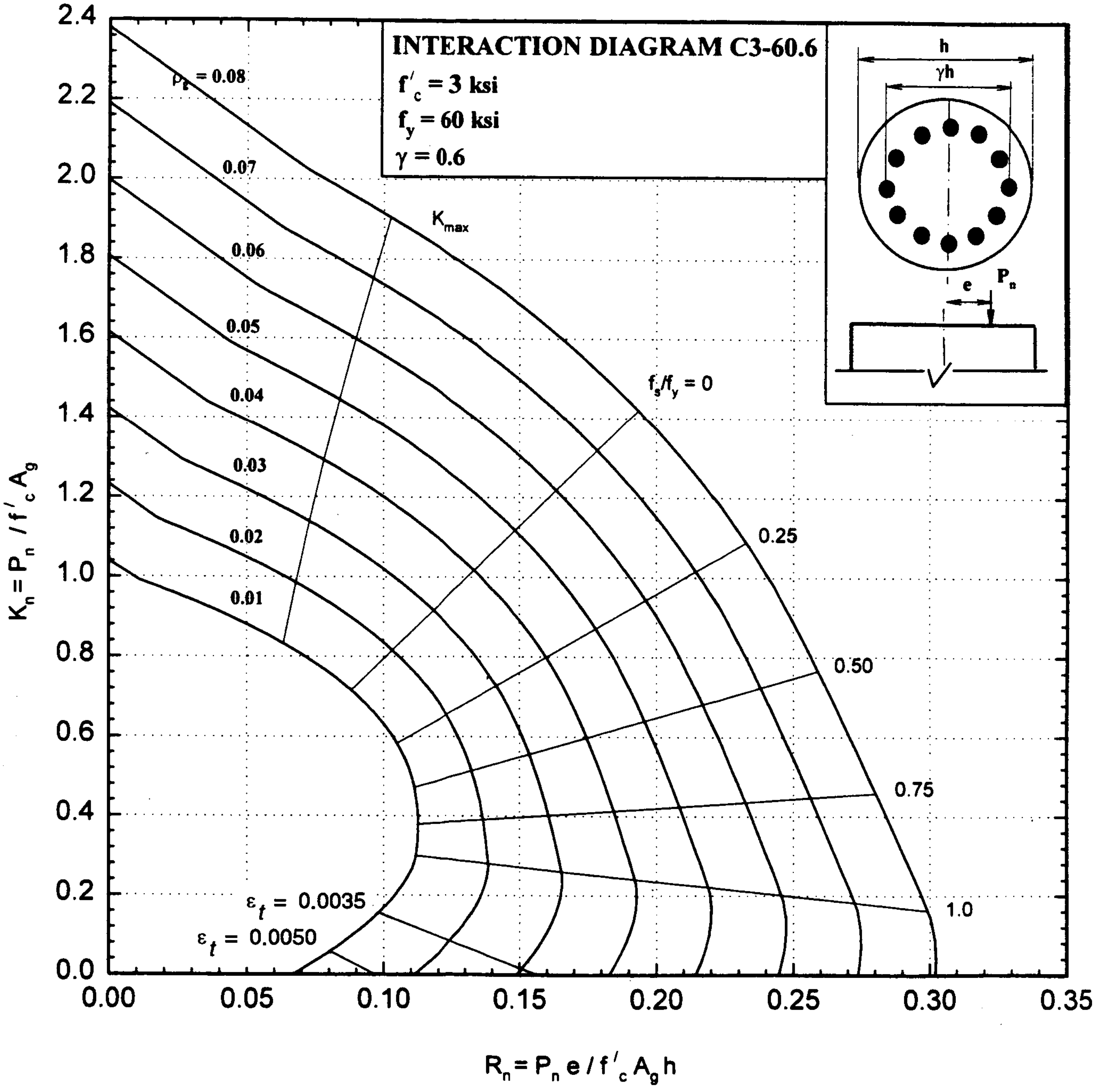


Diagram plotted using data from computer programs developed by Dr. Noel J. Everard.
Computer plot of data for diagram by Dr. Mohsen A. Issa and Alfred A. Yousif.

COLUMNS 7.13.2 - Nominal load-moment strength interaction diagram, C3-60.7

References: "Building Code Requirements for Structural Concrete-ACI 318" Chapters 9 and 10, and "Ultimate Strength Design of Reinforced Concrete Columns", ACI Special Publication SP-7, by Everard and Cohen, 1964, pp. 152-182 (with corrections).

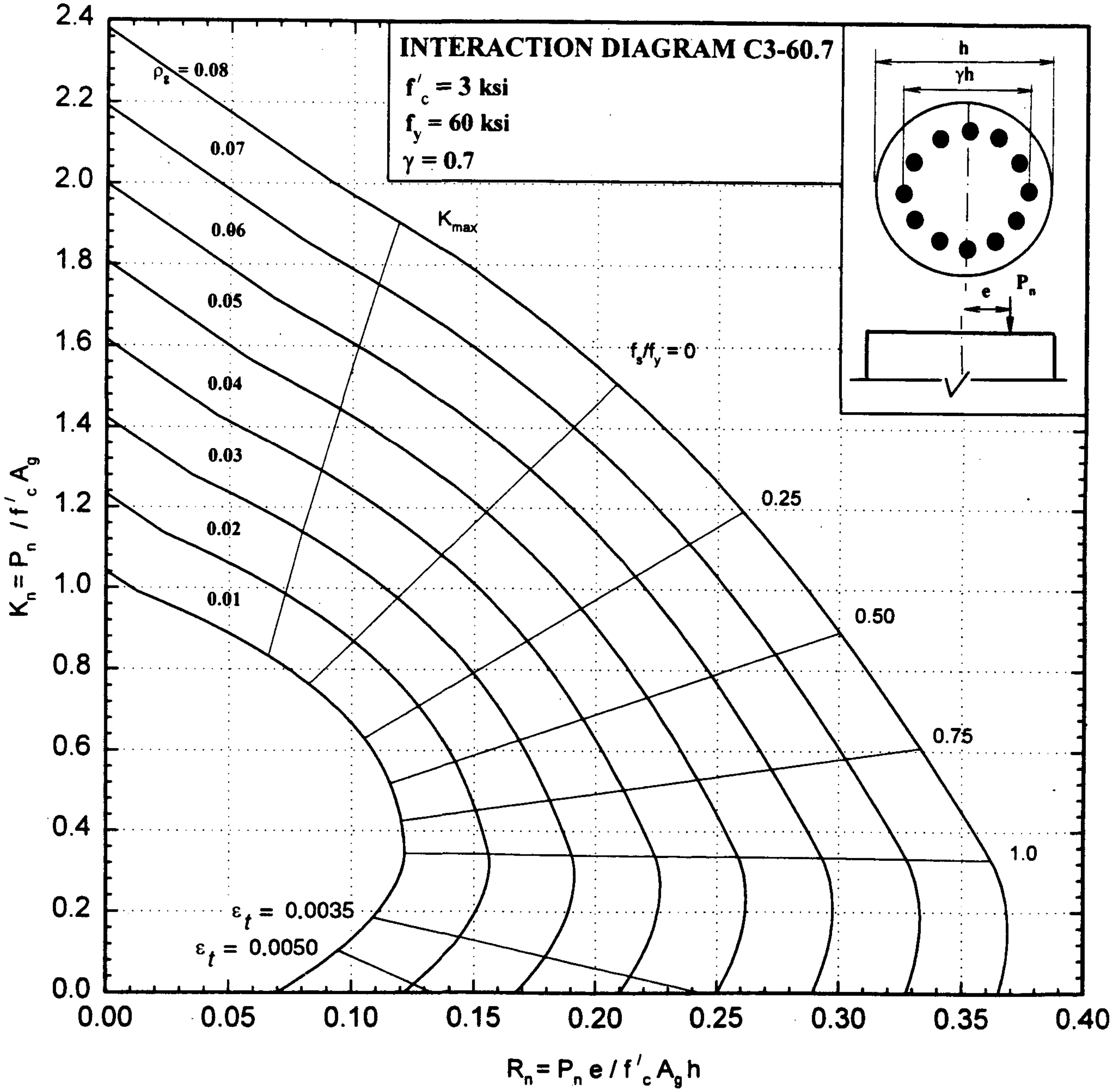


Diagram plotted using data from computer programs developed by Dr. Noel J. Everard. Computer plot of data for diagram by Dr. Mohsen A. Issa and Alfred A. Yousif.

References: "Building Code Requirements for Structural Concrete-ACI 318" Chapters 9 and 10, and "Ultimate Strength Design of Reinforced Concrete Columns", ACI Special Publication SP-7, by Everard and Cohen, 1964, pp. 152-182 (with corrections).

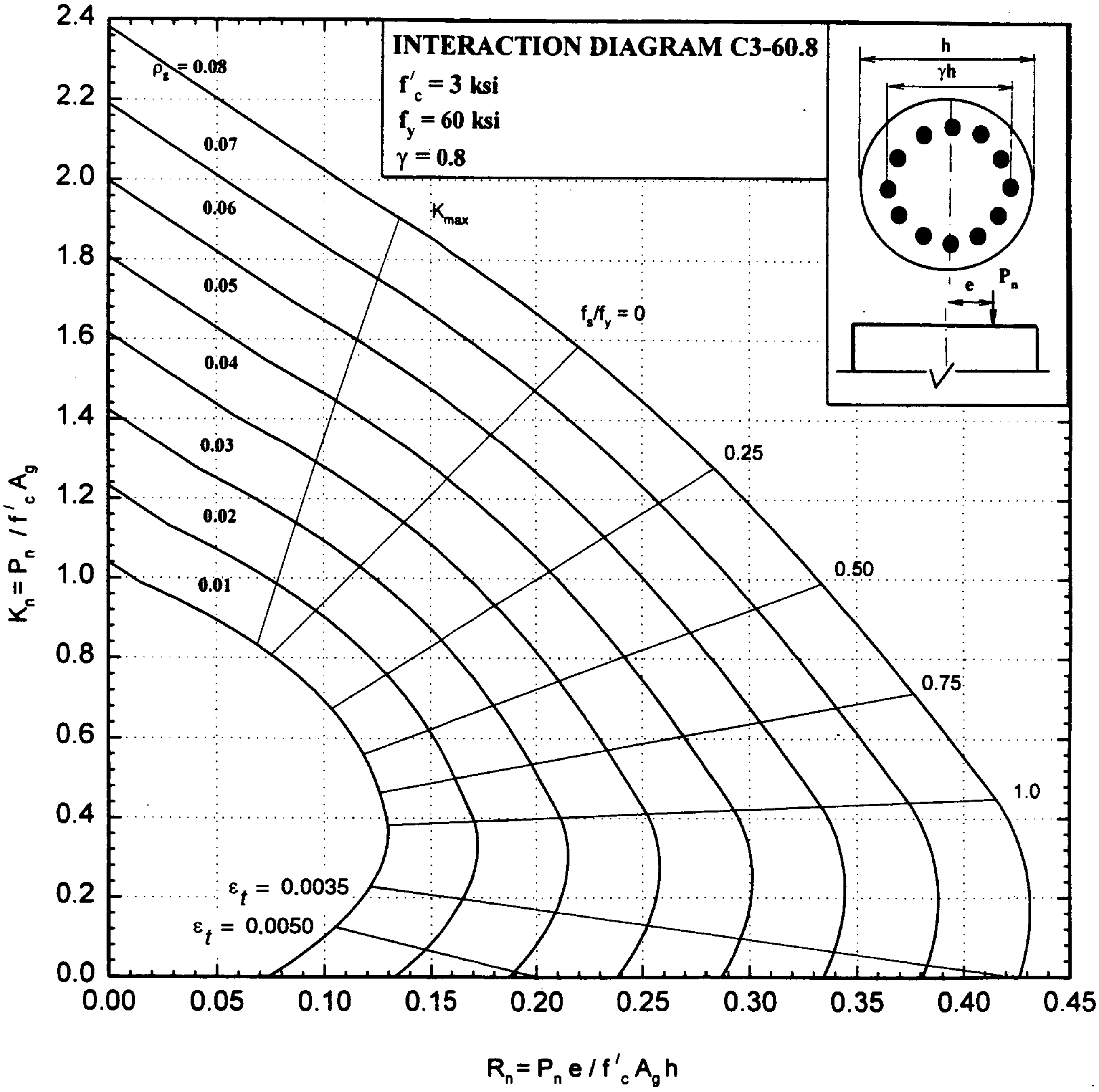


Diagram plotted using data from computer programs developed by Dr. Noel J. Everard.
Computer plot of data for diagram by Dr. Mohsen A. Issa and Alfred A. Yousif.

COLUMNS 7.13.4 - Nominal load-moment strength interaction diagram, C3-60.9

References: "Building Code Requirements for Structural Concrete-ACI 318" Chapters 9 and 10, and "Ultimate Strength Design of Reinforced Concrete Columns", ACI Special Publication SP-7, by Everard and Cohen, 1964, pp. 152-182 (with corrections).

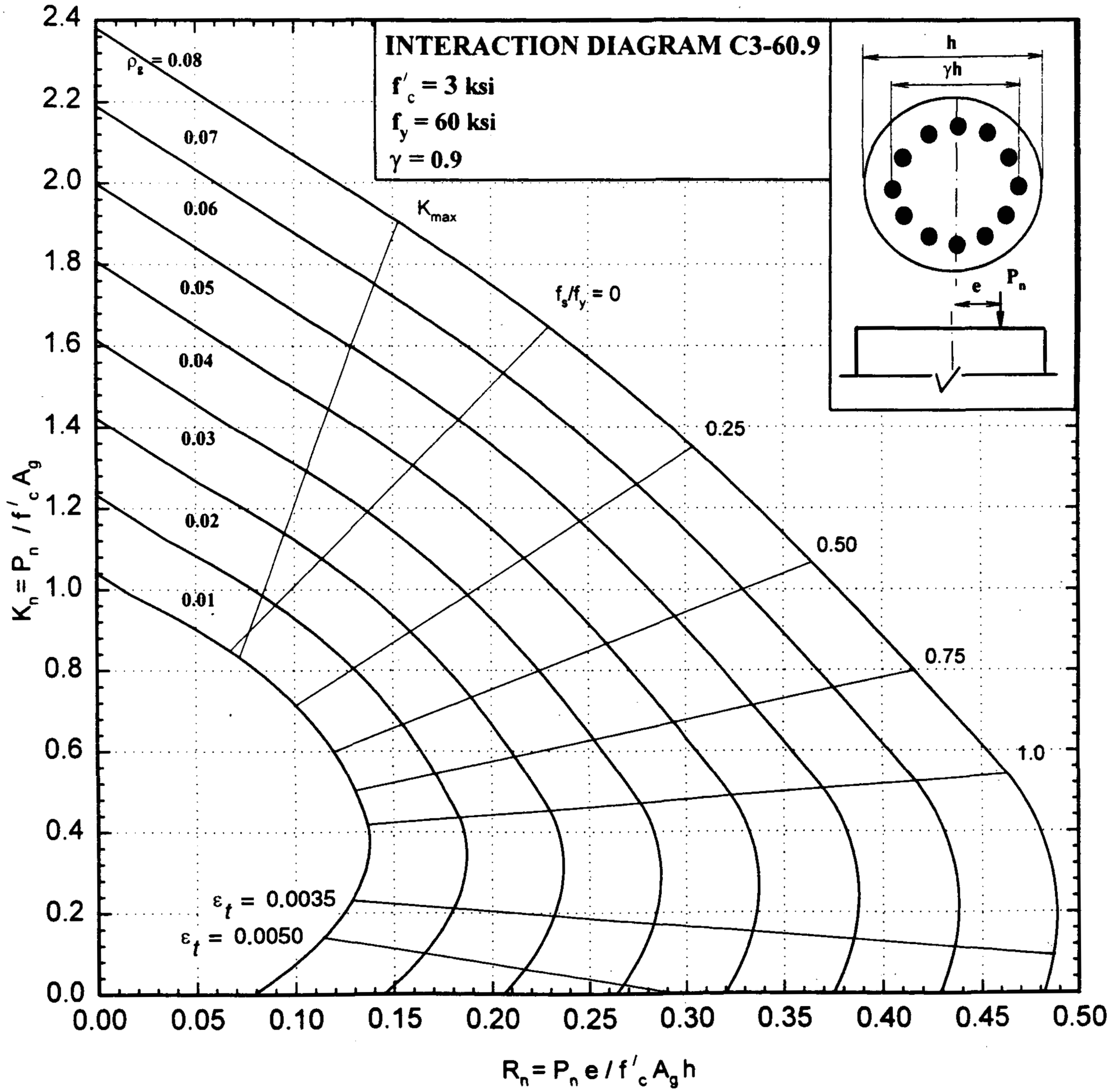


Diagram plotted using data from computer programs developed by Dr. Noel J. Everard. Computer plot of data for diagram by Dr. Mohsen A. Issa and Alfred A. Yousif.

COLUMNS 7.14.1 - Nominal load-moment strength interaction diagram, C4-60.6

References: "Building Code Requirements for Structural Concrete-ACI 318" Chapters 9 and 10, and "Ultimate Strength Design of Reinforced Concrete Columns", ACI Special Publication SP-7, by Everard and Cohen, 1964, pp. 152-182 (with corrections).

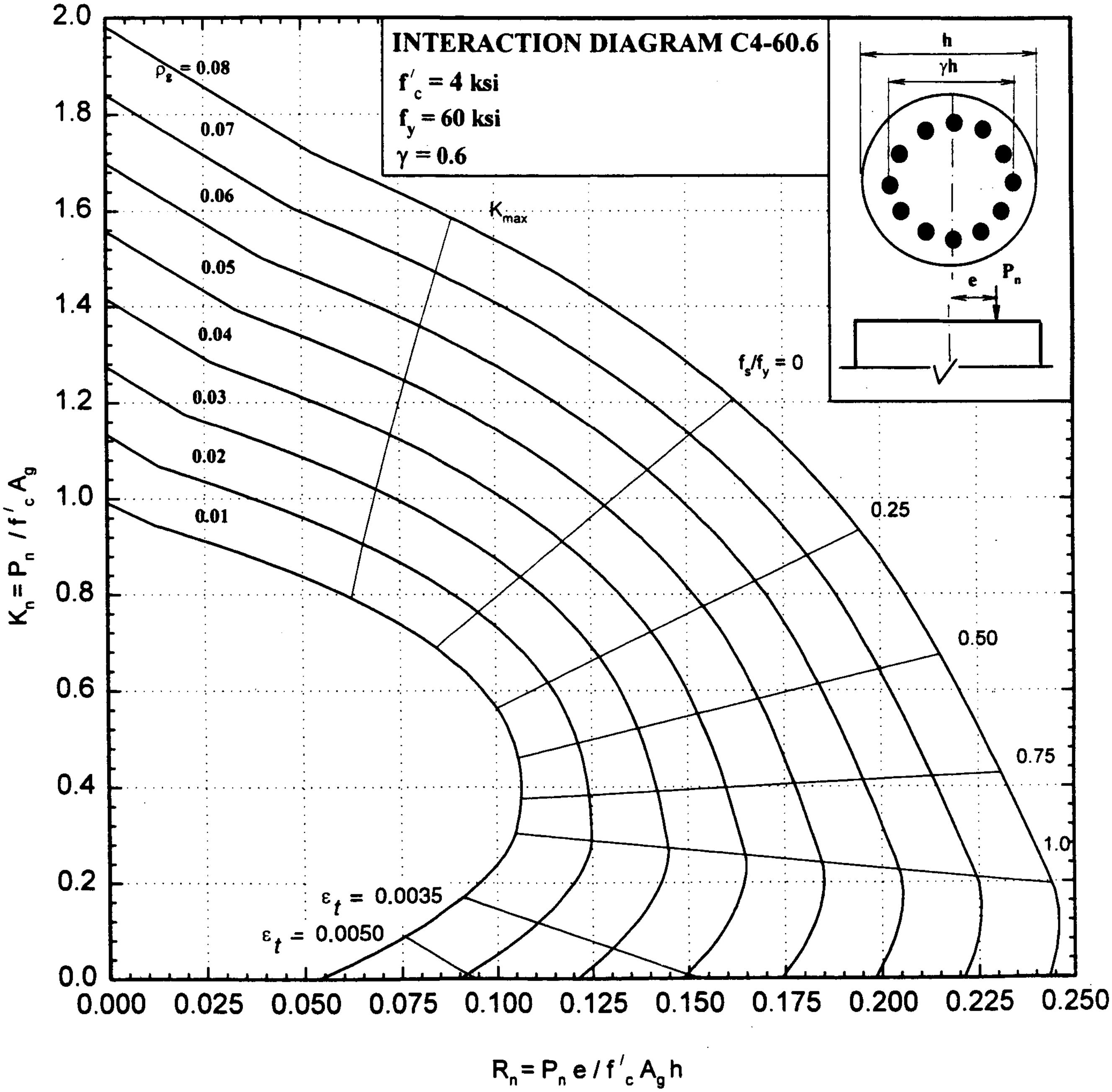


Diagram plotted using data from computer programs developed by Dr. Noel J. Everard. Computer plot of data for diagram by Dr. Mohsen A. Issa and Alfred A. Yousif.

COLUMNS 7.14.2 - Nominal load-moment strength interaction diagram, C4-60.7

References: "Building Code Requirements for Structural Concrete-ACI 318" Chapters 9 and 10, and "Ultimate Strength Design of Reinforced Concrete Columns", ACI Special Publication SP-7, by Everard and Cohen, 1964, pp. 152-182 (with corrections).

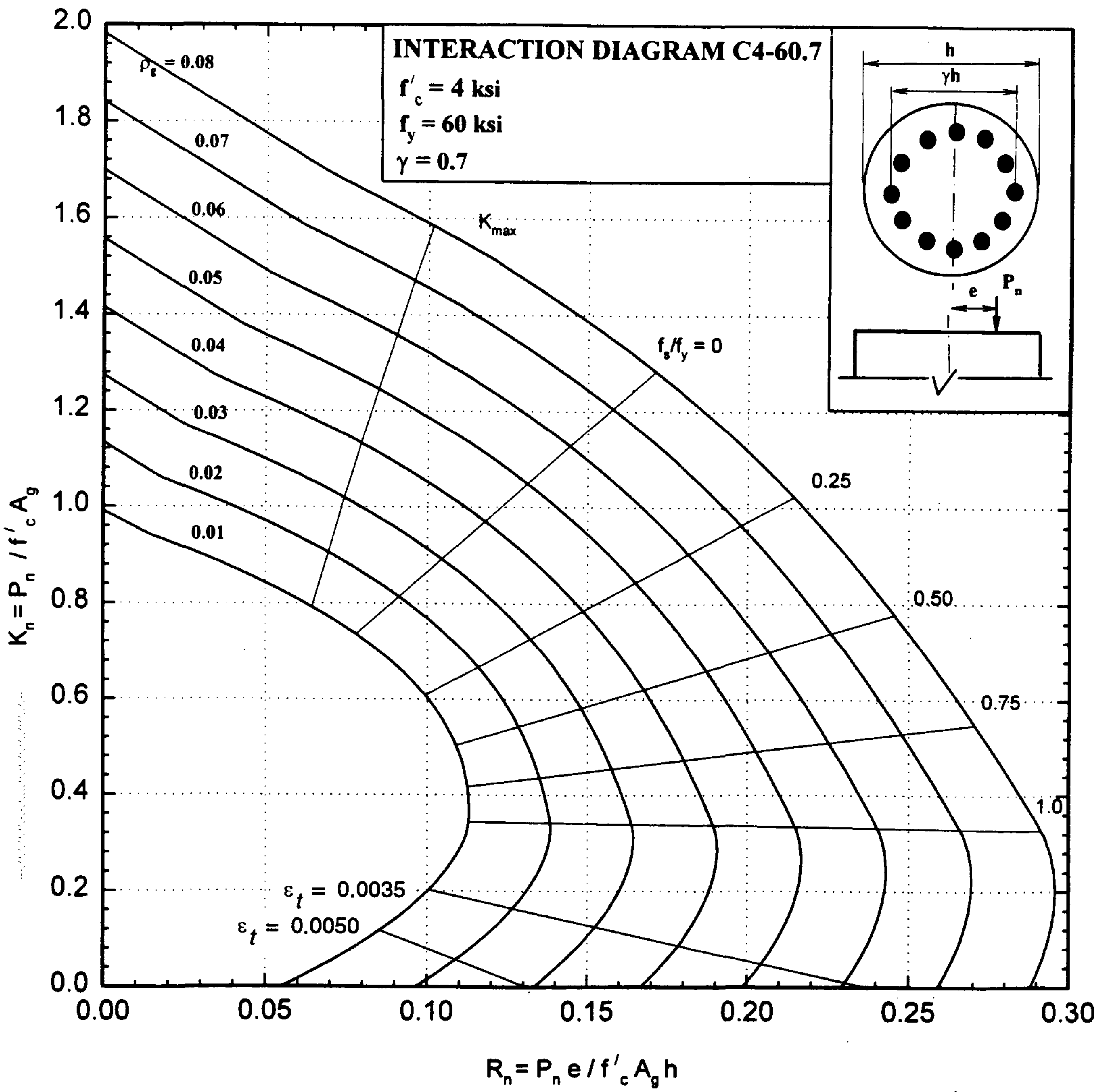


Diagram plotted using data from computer programs developed by Dr. Noel J. Everard. Computer plot of data for diagram by Dr. Mohsen A. Issa and Alfred A. Yousif.

COLUMNS 7.14.3 - Nominal load-moment strength interaction diagram, C4-60.8

References: "Building Code Requirements for Structural Concrete-ACI 318" Chapters 9 and 10, and "Ultimate Strength Design of Reinforced Concrete Columns", ACI Special Publication SP-7, by Everard and Cohen, 1964, pp. 152-182 (with corrections).

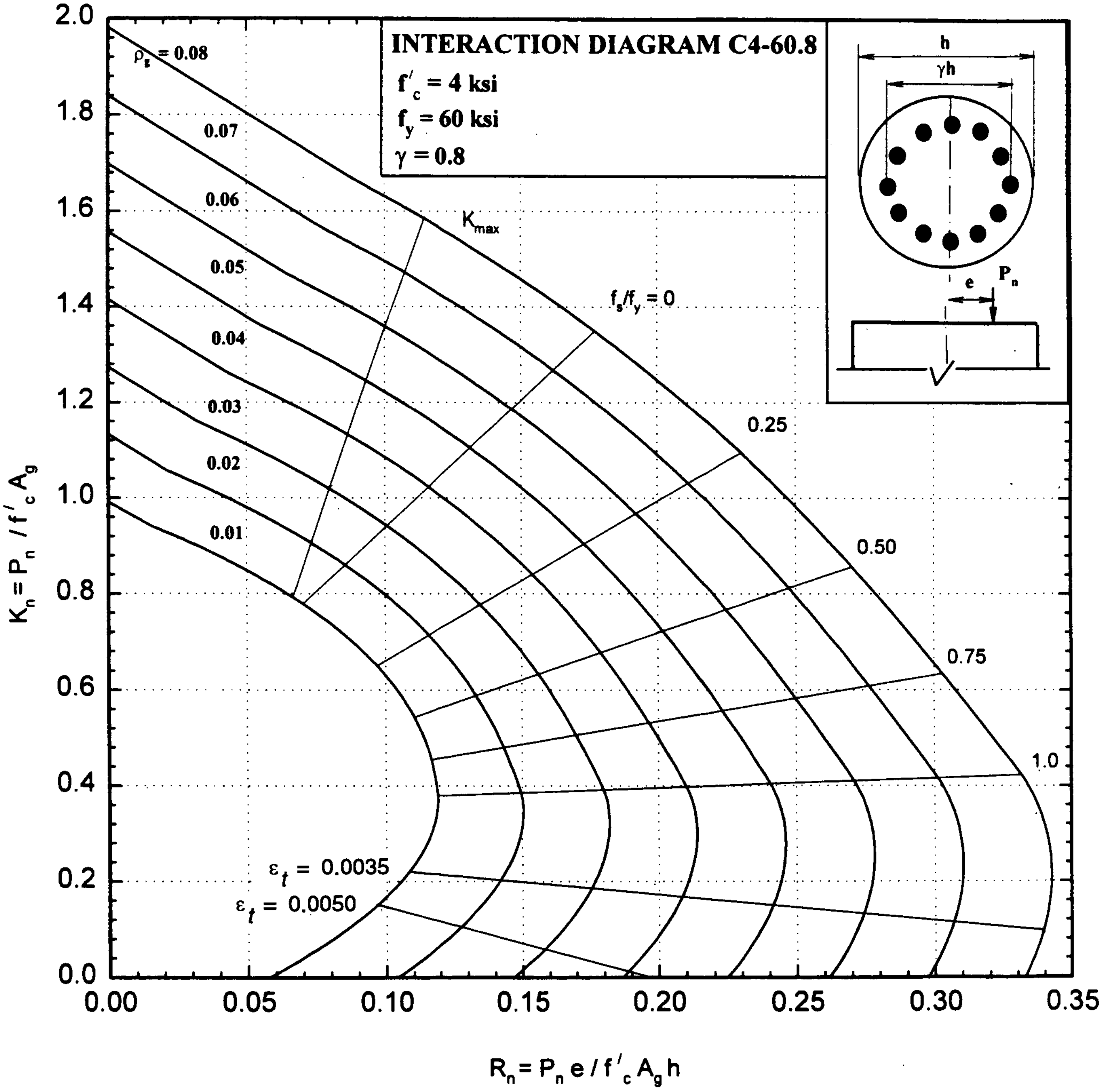


Diagram plotted using data from computer programs developed by Dr. Noel J. Everard. Computer plot of data for diagram by Dr. Mohsen A. Issa and Alfred A. Yousif.

COLUMNS 7.14.4 - Nominal load-moment strength interaction diagram, C4-60.9

References: "Building Code Requirements for Structural Concrete-ACI 318" Chapters 9 and 10, and "Ultimate Strength Design of Reinforced Concrete Columns", ACI Special Publication SP-7, by Everard and Cohen, 1964, pp. 152-182 (with corrections).

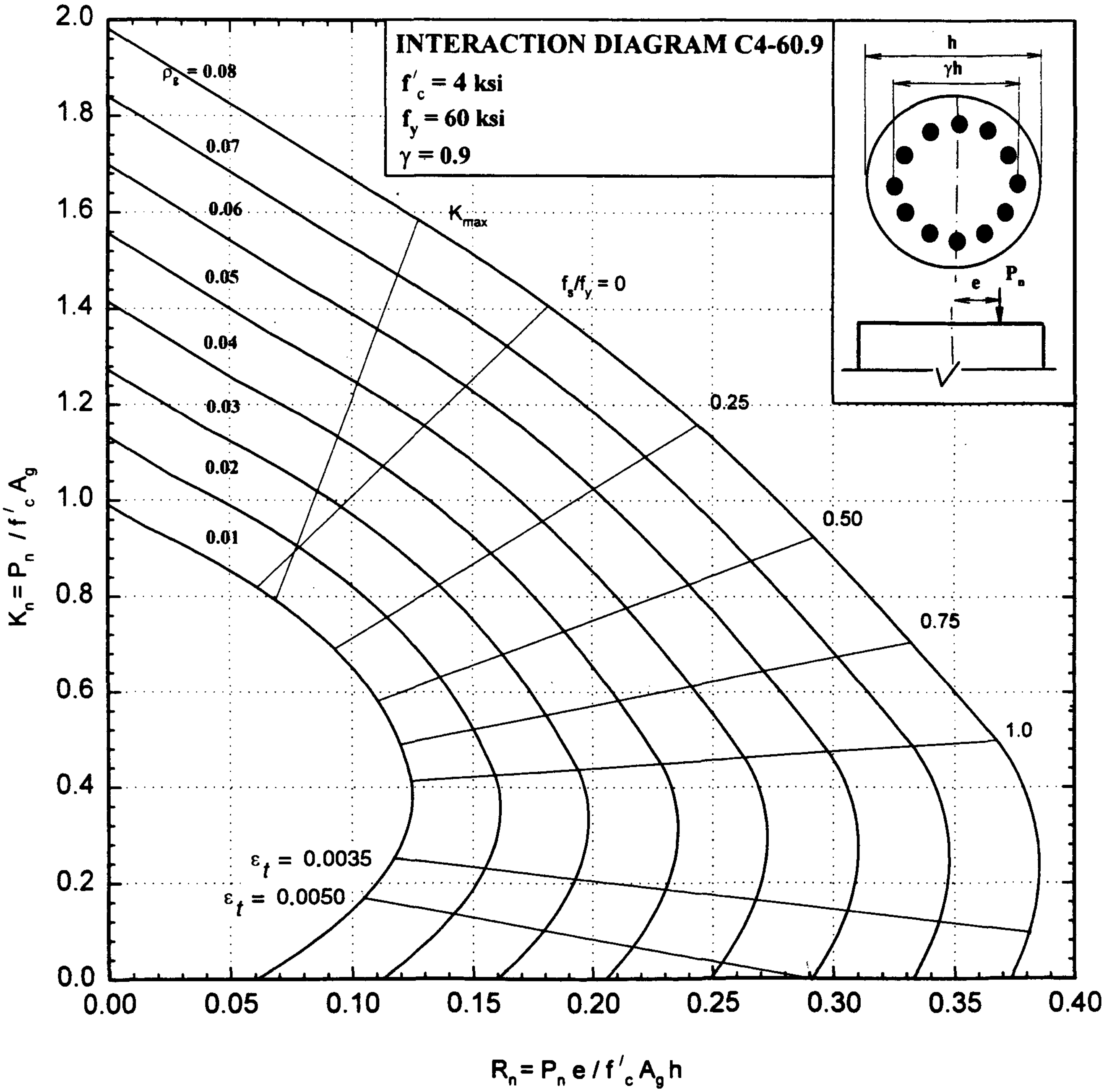


Diagram plotted using data from computer programs developed by Dr. Noel J. Everard.
Computer plot of data for diagram by Dr. Mohsen A. Issa and Alfred A. Yousif.

COLUMNS 7.15.1 - Nominal load-moment strength interaction diagram, C5-60.6

References: "Building Code Requirements for Structural Concrete-ACI 318" Chapters 9 and 10, and "Ultimate Strength Design of Reinforced Concrete Columns", ACI Special Publication SP-7, by Everard and Cohen, 1964, pp. 152-182 (with corrections).

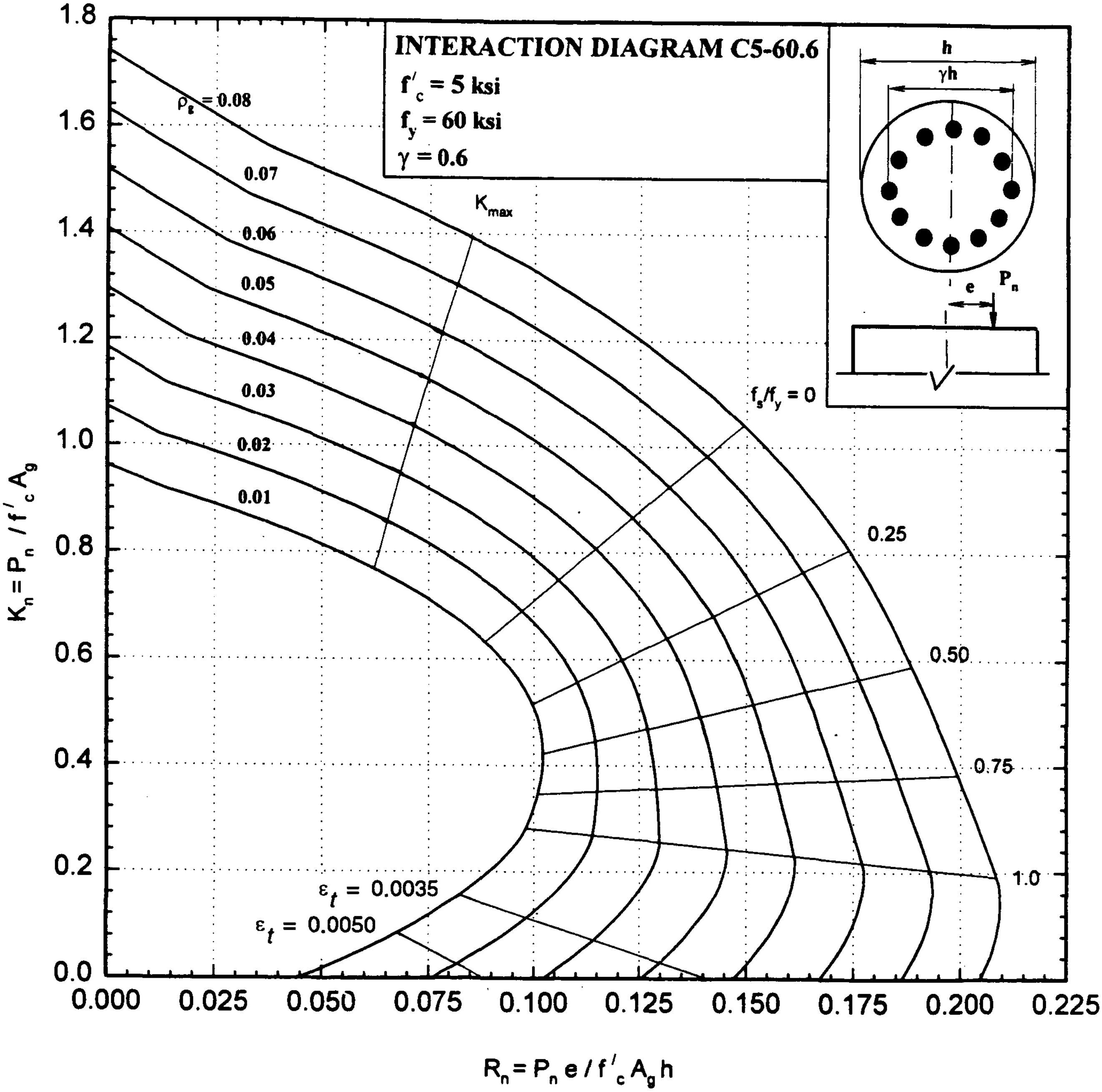


Diagram plotted using data from computer programs developed by Dr. Noel J. Everard. Computer plot of data for diagram by Dr. Mohsen A. Issa and Alfred A. Yousif.

COLUMNS 7.15.2 - Nominal load-moment strength interaction diagram, C5-60.7

References: "Building Code Requirements for Structural Concrete-ACI 318" Chapters 9 and 10, and "Ultimate Strength Design of Reinforced Concrete Columns", ACI Special Publication SP-7, by Everard and Cohen, 1964, pp. 152-182 (with corrections).

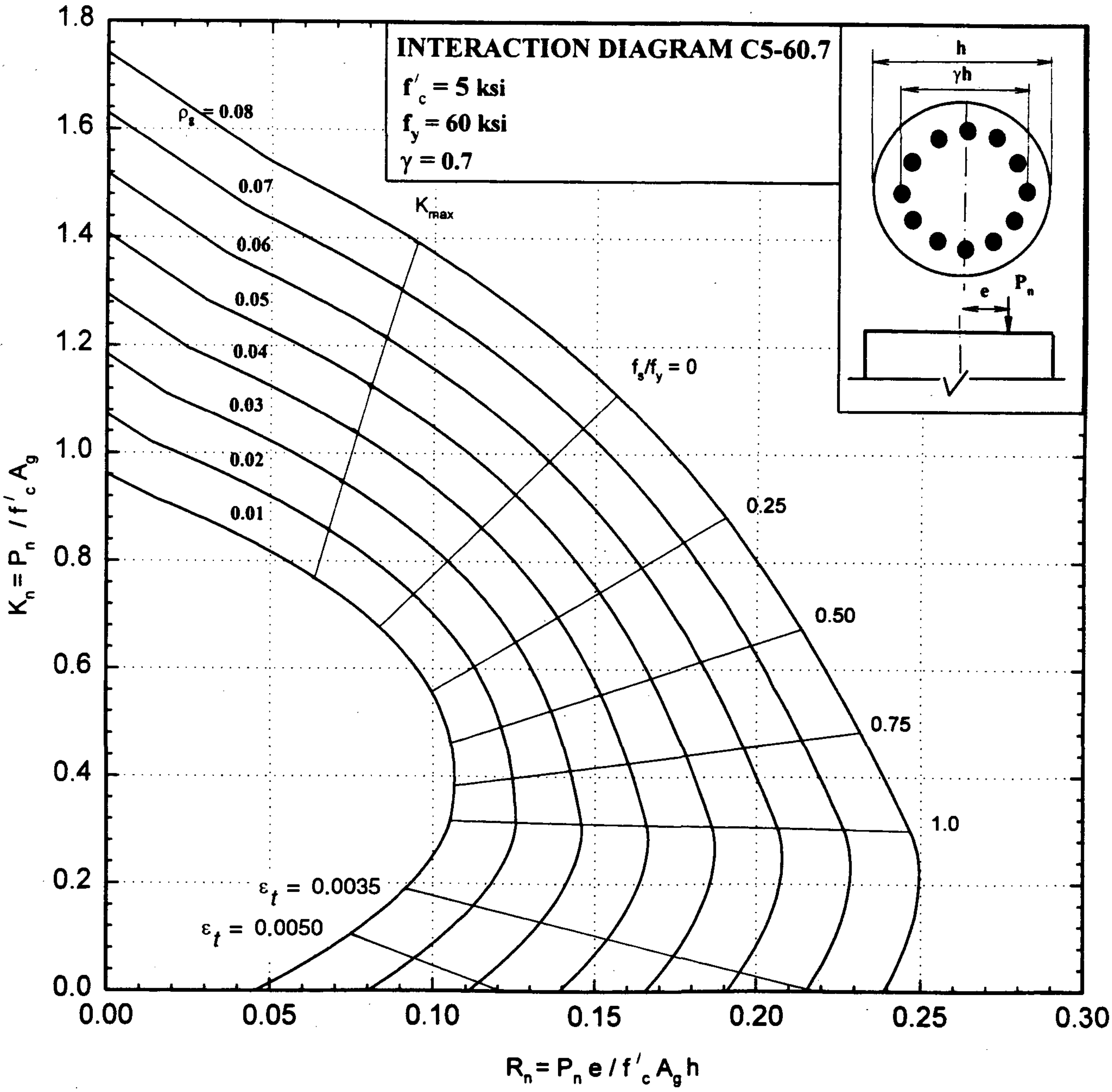


Diagram plotted using data from computer programs developed by Dr. Noel J. Everard. Computer plot of data for diagram by Dr. Mohsen A. Issa and Alfred A. Yousif.

COLUMNS 7.15.3 - Nominal load-moment strength interaction diagram, C5-60.8

References: "Building Code Requirements for Structural Concrete-ACI 318" Chapters 9 and 10, and "Ultimate Strength Design of Reinforced Concrete Columns", ACI Special Publication SP-7, by Everard and Cohen, 1964, pp. 152-182 (with corrections).

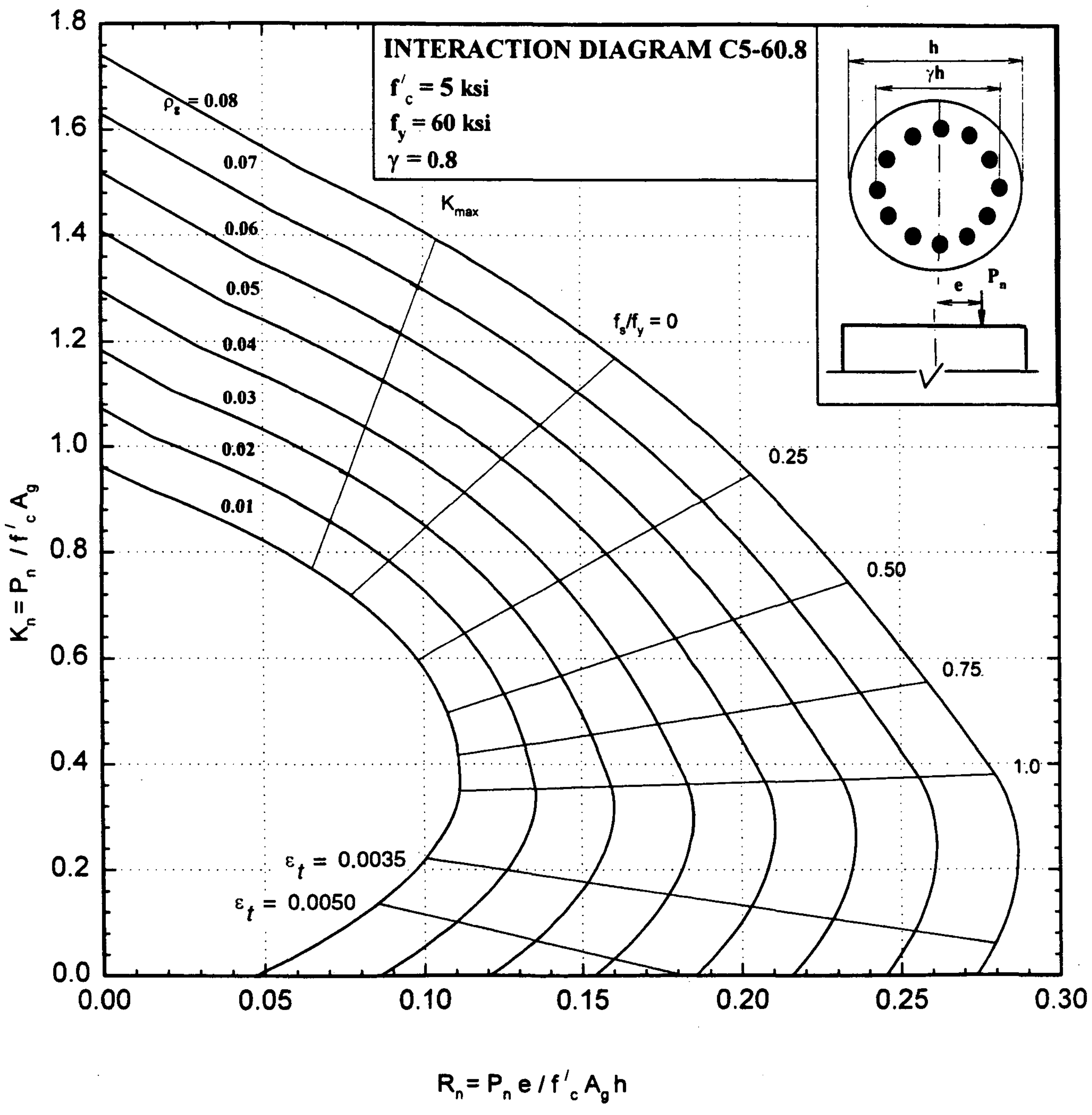


Diagram plotted using data from computer programs developed by Dr. Noel J. Everard.
Computer plot of data for diagram by Dr. Mohsen A. Issa and Alfred A. Yousif.

COLUMNS 7.15.4 - Nominal load-moment strength interaction diagram, C5-60.9

References: "Building Code Requirements for Structural Concrete-ACI 318" Chapters 9 and 10, and "Ultimate Strength Design of Reinforced Concrete Columns", ACI Special Publication SP-7, by Everard and Cohen, 1964, pp. 152-182 (with corrections).

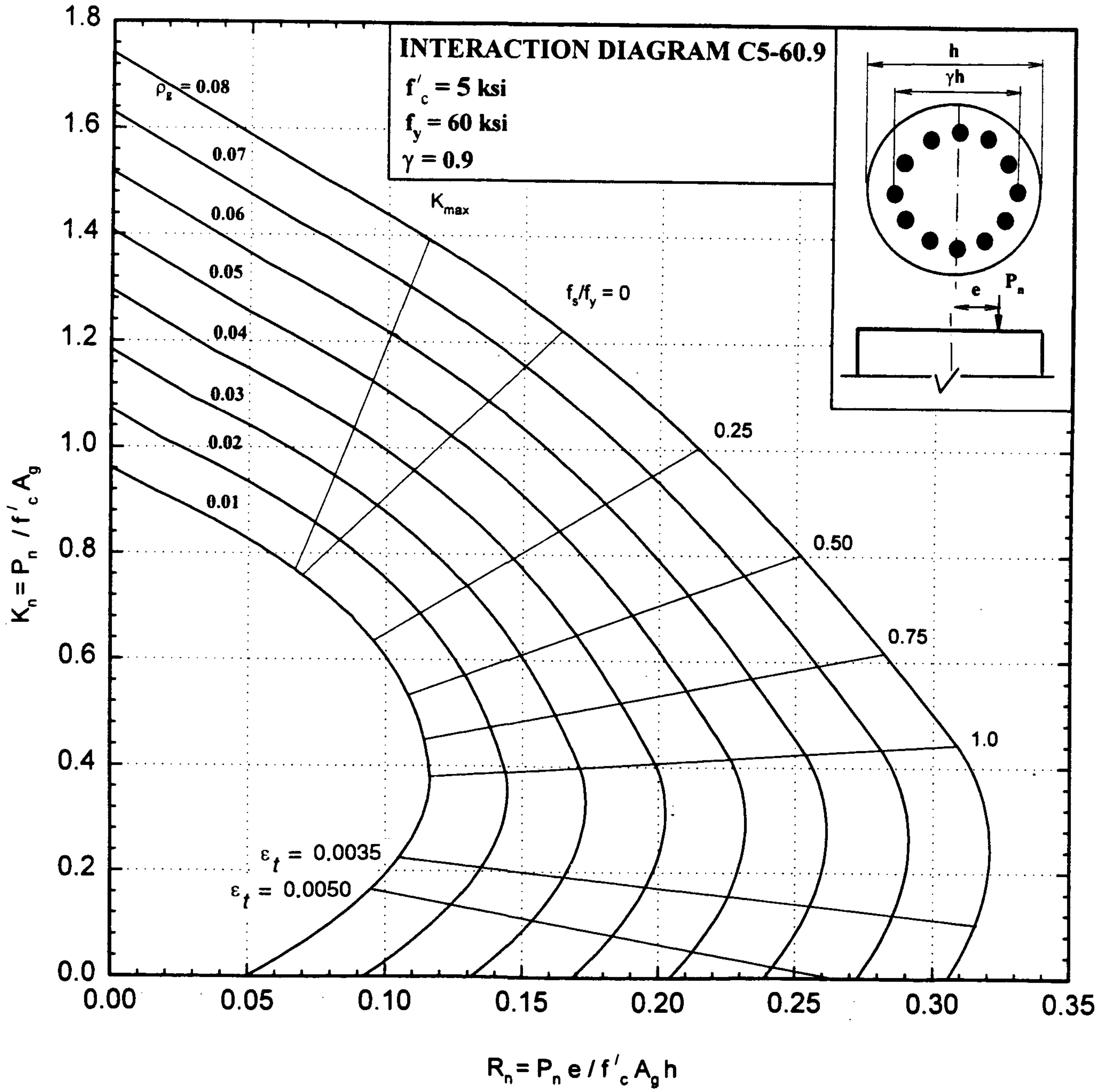


Diagram plotted using data from computer programs developed by Dr. Noel J. Everard.
Computer plot of data for diagram by Dr. Mohsen A. Issa and Alfred A. Yousif.

COLUMNS 7.16.1 - Nominal load-moment strength interaction diagram, C6-60.6

References: "Building Code Requirements for Structural Concrete-ACI 318" Chapters 9 and 10, and "Ultimate Strength Design of Reinforced Concrete Columns", ACI Special Publication SP-7, by Everard and Cohen, 1964, pp. 152-182 (with corrections).

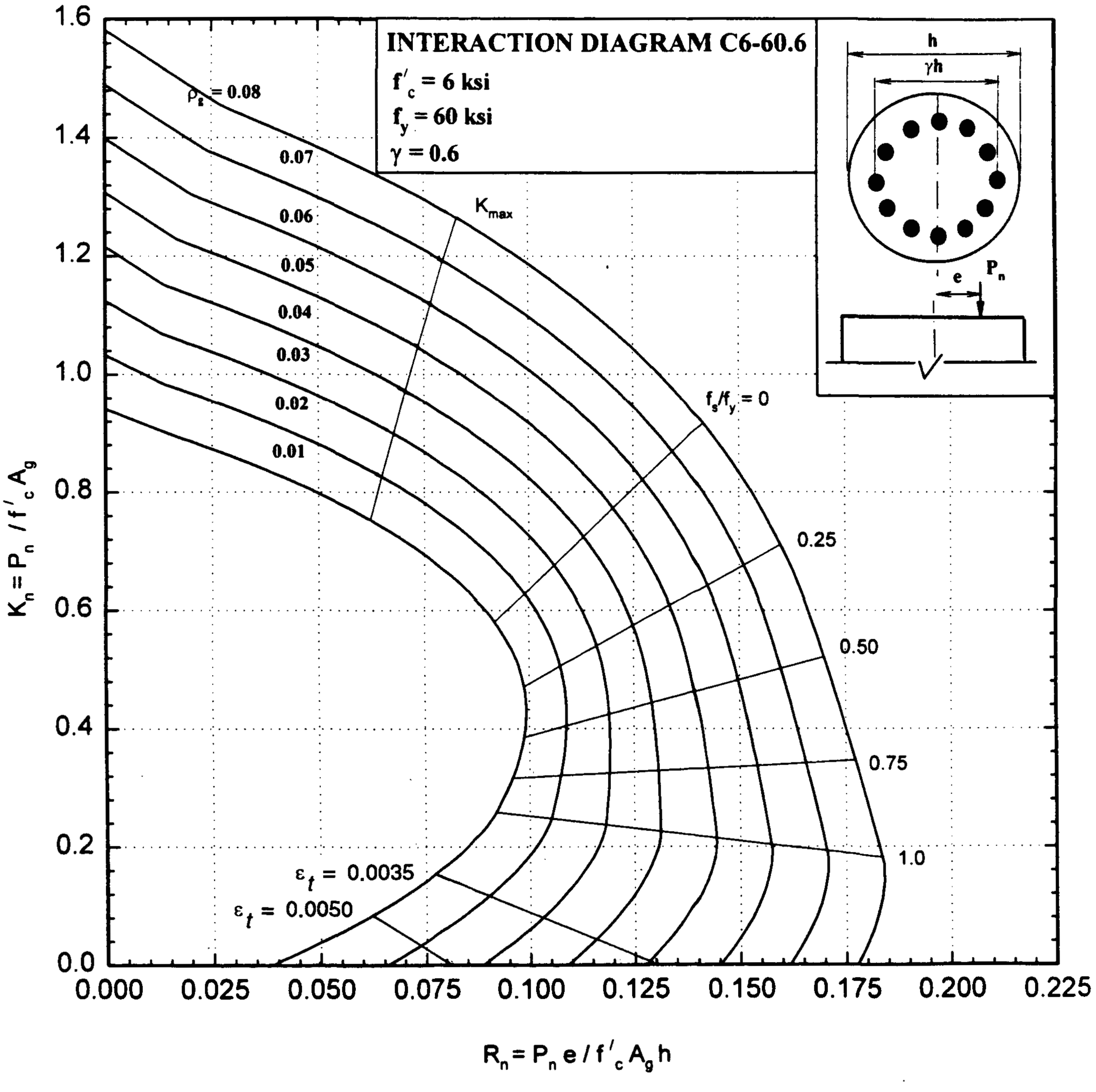


Diagram plotted using data from computer programs developed by Dr. Noel J. Everard.
Computer plot of data for diagram by Dr. Mohsen A. Issa and Alfred A. Yousif.

COLUMNS 7.16.2 - Nominal load-moment strength interaction diagram, C6-60.7

References: "Building Code Requirements for Structural Concrete-ACI 318" Chapters 9 and 10, and "Ultimate Strength Design of Reinforced Concrete Columns", ACI Special Publication SP-7, by Everard and Cohen, 1964, pp. 152-182 (with corrections).

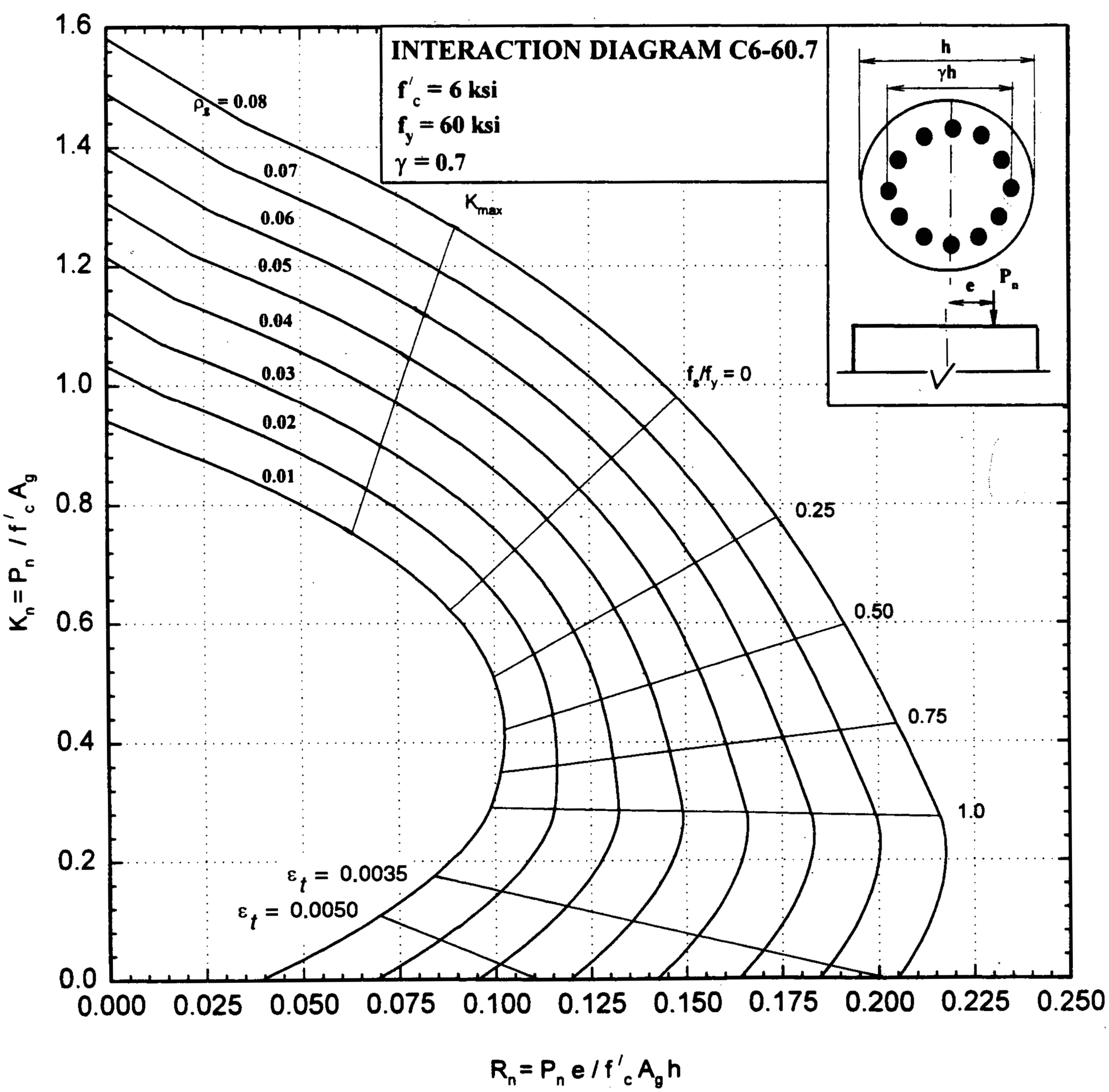


Diagram plotted using data from computer programs developed by Dr. Noel J. Everard.
Computer plot of data for diagram by Dr. Mohsen A. Issa and Alfred A. Yousif.

COLUMNS 7.16.3 - Nominal load-moment strength interaction diagram, C6-60.8

References: "Building Code Requirements for Structural Concrete-ACI 318" Chapters 9 and 10, and "Ultimate Strength Design of Reinforced Concrete Columns", ACI Special Publication SP-7, by Everard and Cohen, 1964, pp. 152-182 (with corrections).

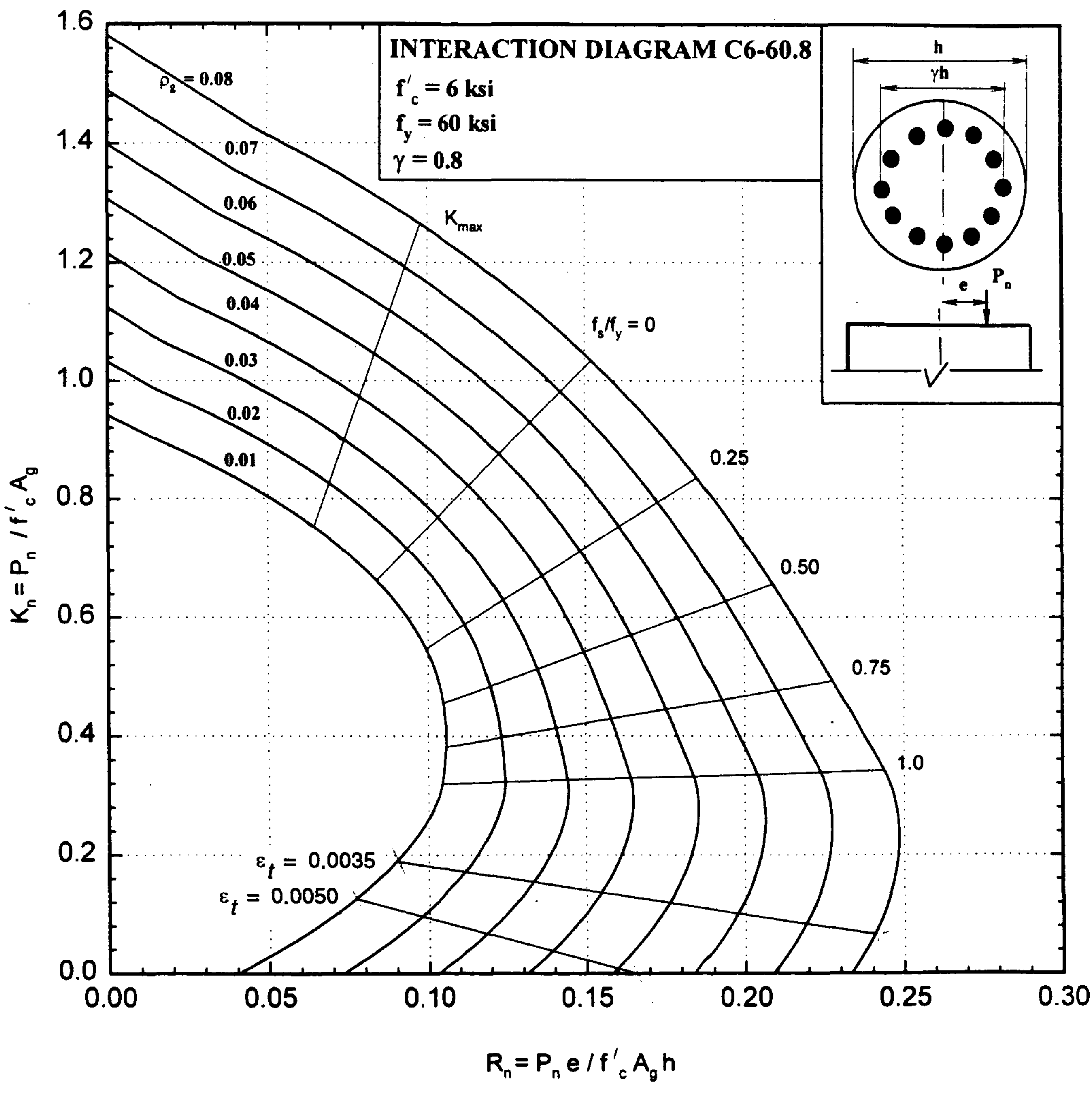


Diagram plotted using data from computer programs developed by Dr. Noel J. Everard.
Computer plot of data for diagram by Dr. Mohsen A. Issa and Alfred A. Yousif.

COLUMNS 7.16.4 - Nominal load-moment strength interaction diagram, C6-60.9

References: "Building Code Requirements for Structural Concrete-ACI 318" Chapters 9 and 10, and "Ultimate Strength Design of Reinforced Concrete Columns", ACI Special Publication SP-7, by Everard and Cohen, 1964, pp. 152-182 (with corrections).

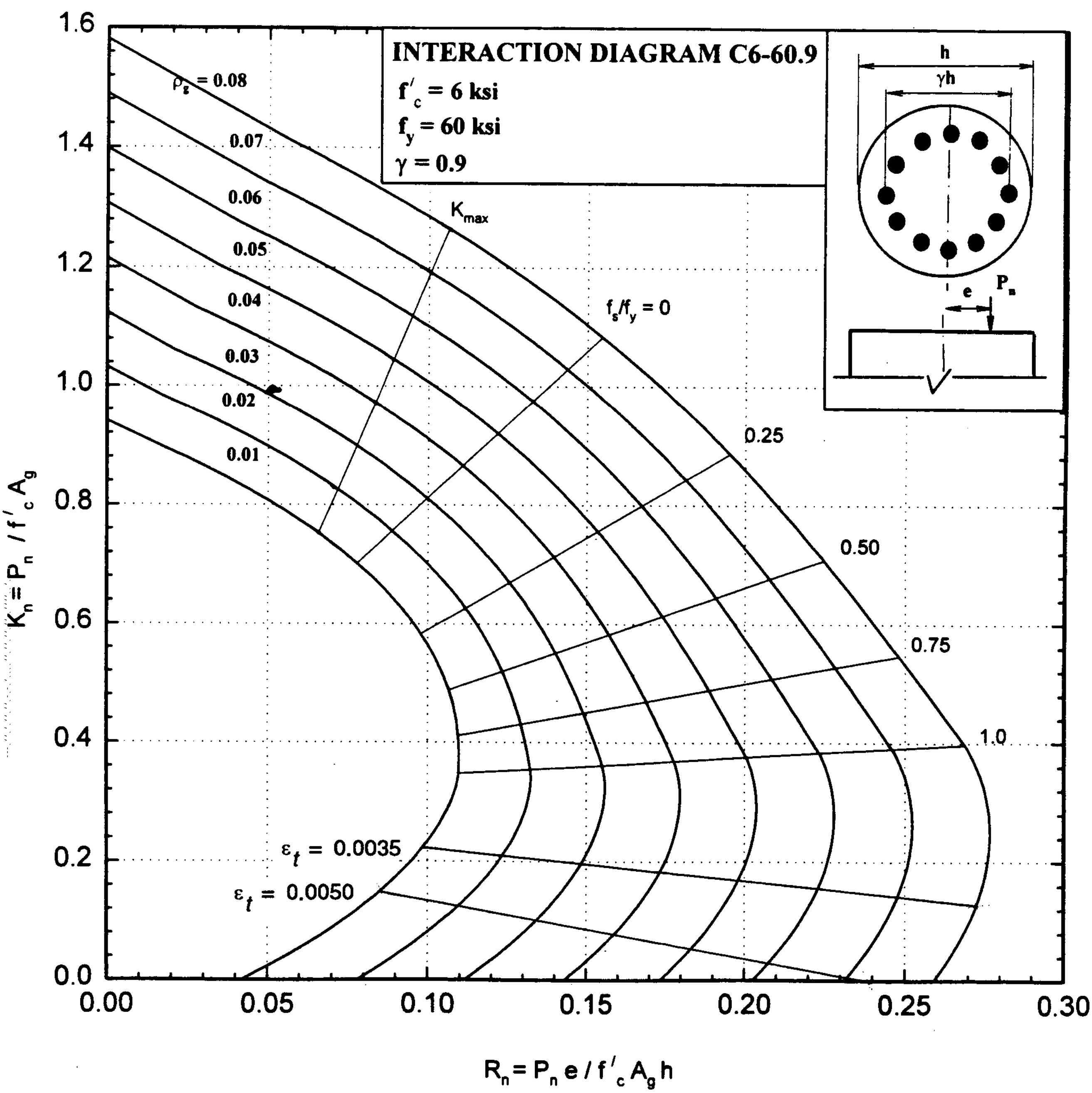


Diagram plotted using data from computer programs developed by Dr. Noel J. Everard. Computer plot of data for diagram by Dr. Mohsen A. Issa and Alfred A. Yousif.

COLUMNS 7.17.1 - Nominal load-moment strength interaction diagram, C9-75.6

References: "Building Code Requirements for Structural Concrete-ACI 318" Chapters 9 and 10, and "Ultimate Strength Design of Reinforced Concrete Columns", ACI Special Publication SP-7, by Everard and Cohen, 1964, pp. 152-182 (with corrections).

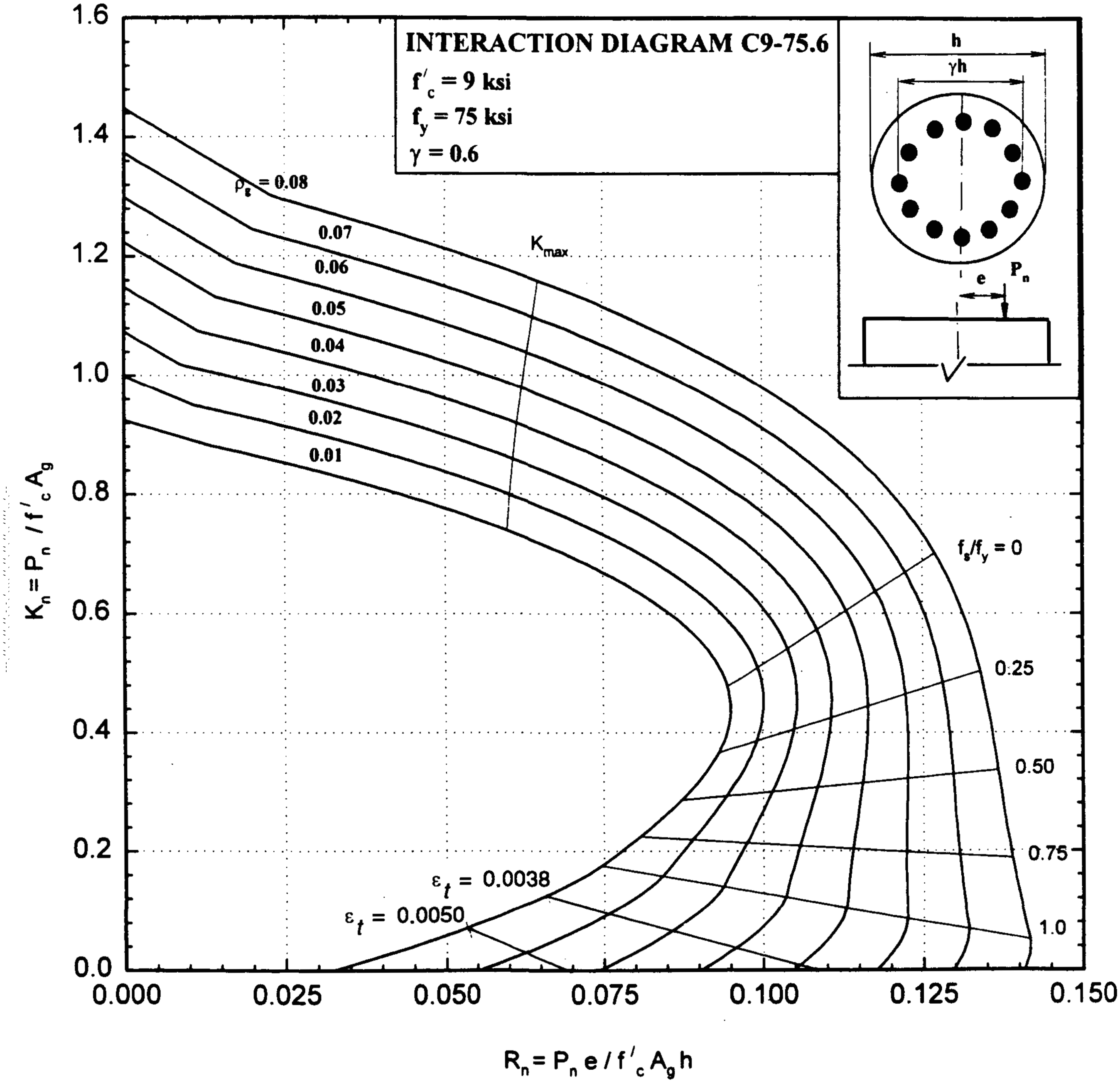


Diagram plotted using data from computer programs developed by Dr. Noel J. Everard. Computer plot of data for diagram by Dr. Mohsen A. Issa and Alfred A. Yousif.

COLUMNS 7.17.2 - Nominal load-moment strength interaction diagram, C9-75.7

References: "Building Code Requirements for Structural Concrete-ACI 318" Chapters 9 and 10, and "Ultimate Strength Design of Reinforced Concrete Columns", ACI Special Publication SP-7, by Everard and Cohen, 1964, pp. 152-182 (with corrections).

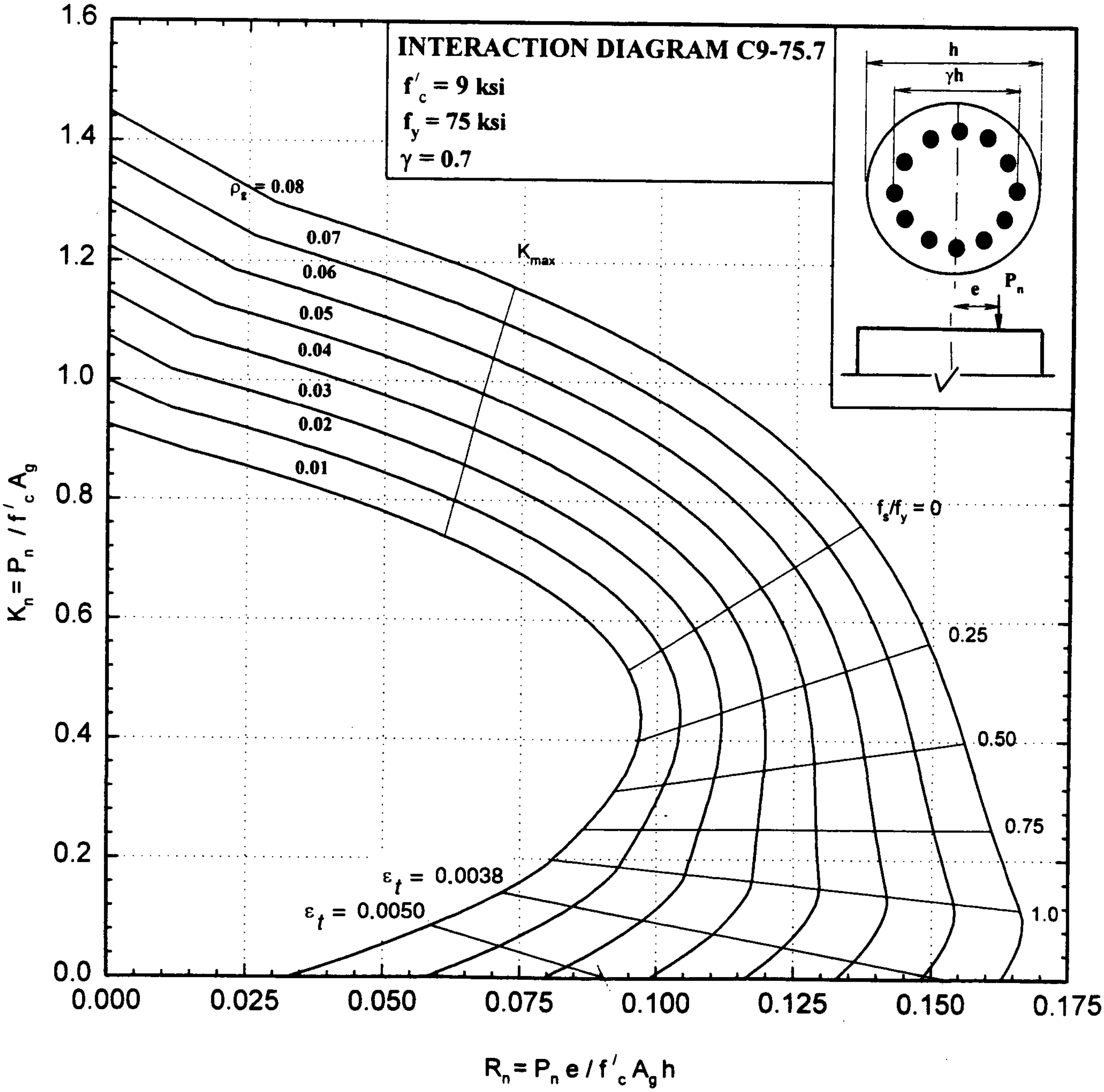


Diagram plotted using data from computer programs developed by Dr. Noel J. Everard.
Computer plot of data for diagram by Dr. Mohsen A. Issa and Alfred A. Yousif.

COLUMNS 7.17.3 - Nominal load-moment strength interaction diagram, C9-75.8

References: "Building Code Requirements for Structural Concrete-ACI 318" Chapters 9 and 10, and "Ultimate Strength Design of Reinforced Concrete Columns", ACI Special Publication SP-7, by Everard and Cohen, 1964, pp. 152-182 (with corrections).

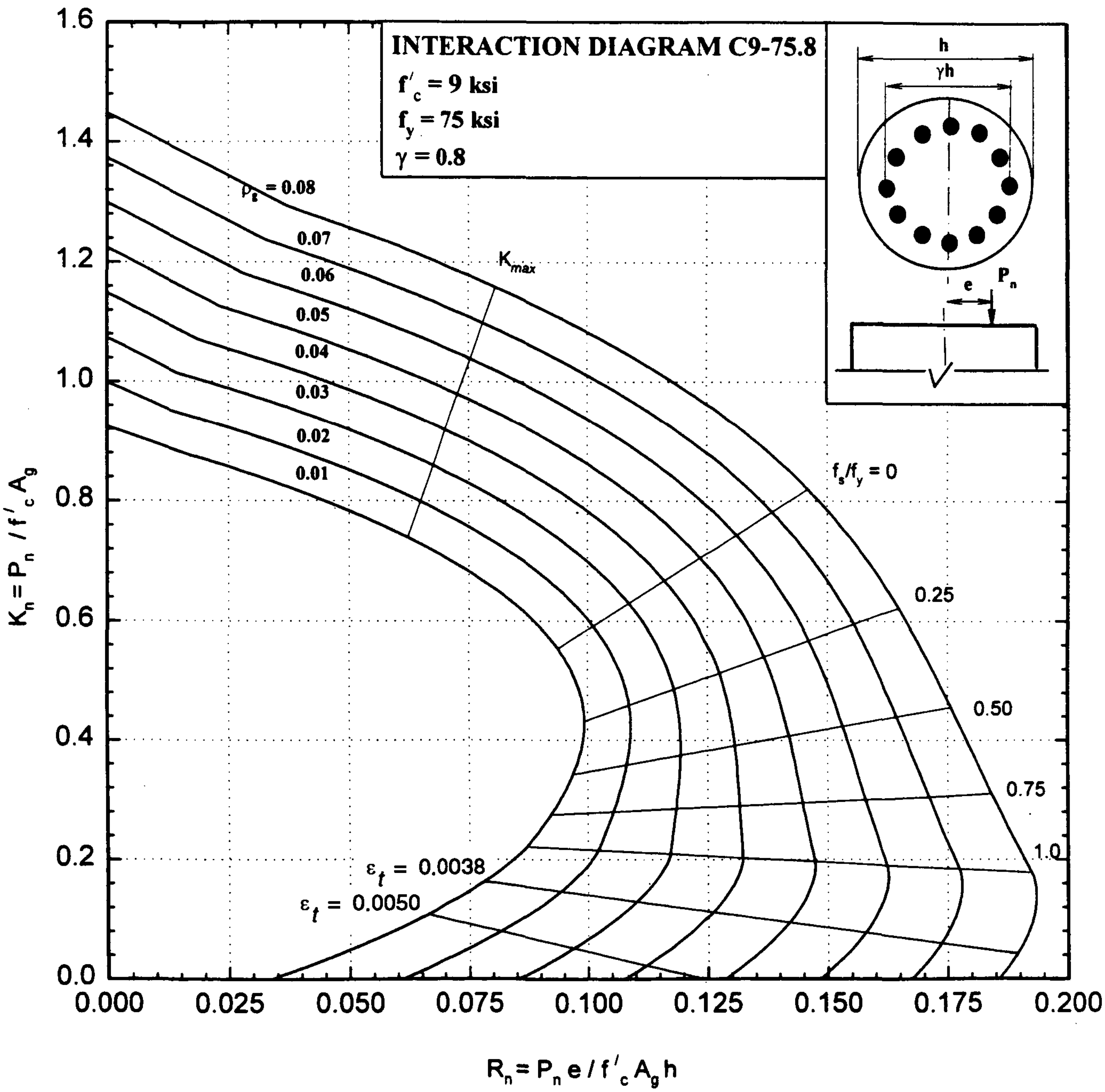


Diagram plotted using data from computer programs developed by Dr. Noel J. Everard. Computer plot of data for diagram by Dr. Mohsen A. Issa and Alfred A. Yousif.

COLUMNS 7.17.4 - Nominal load-moment strength interaction diagram, C9-75.9

References: "Building Code Requirements for Structural Concrete-ACI 318" Chapters 9 and 10, and "Ultimate Strength Design of Reinforced Concrete Columns", ACI Special Publication SP-7, by Everard and Cohen, 1964, pp. 152-182 (with corrections).

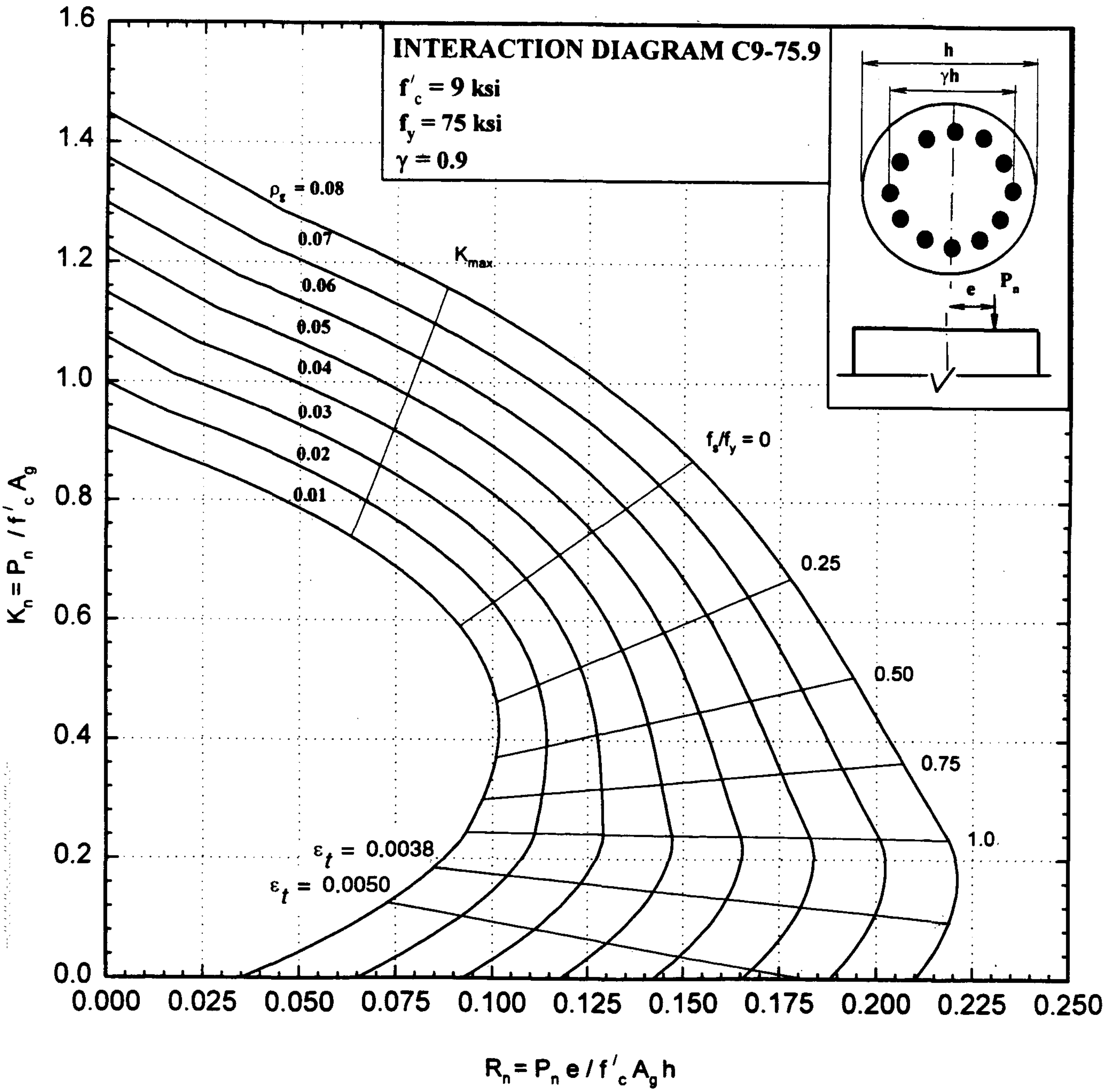


Diagram plotted using data from computer programs developed by Dr. Noel J. Everard.
Computer plot of data for diagram by Dr. Mohsen A. Issa and Alfred A. Yousif.

COLUMNS 7.18.1 - Nominal load-moment strength interaction diagram, C12-75.6

References: "Building Code Requirements for Structural Concrete-ACI 318" Chapters 9 and 10, and "Ultimate Strength Design of Reinforced Concrete Columns", ACI Special Publication SP-7, by Everard and Cohen, 1964, pp. 152-182 (with corrections).

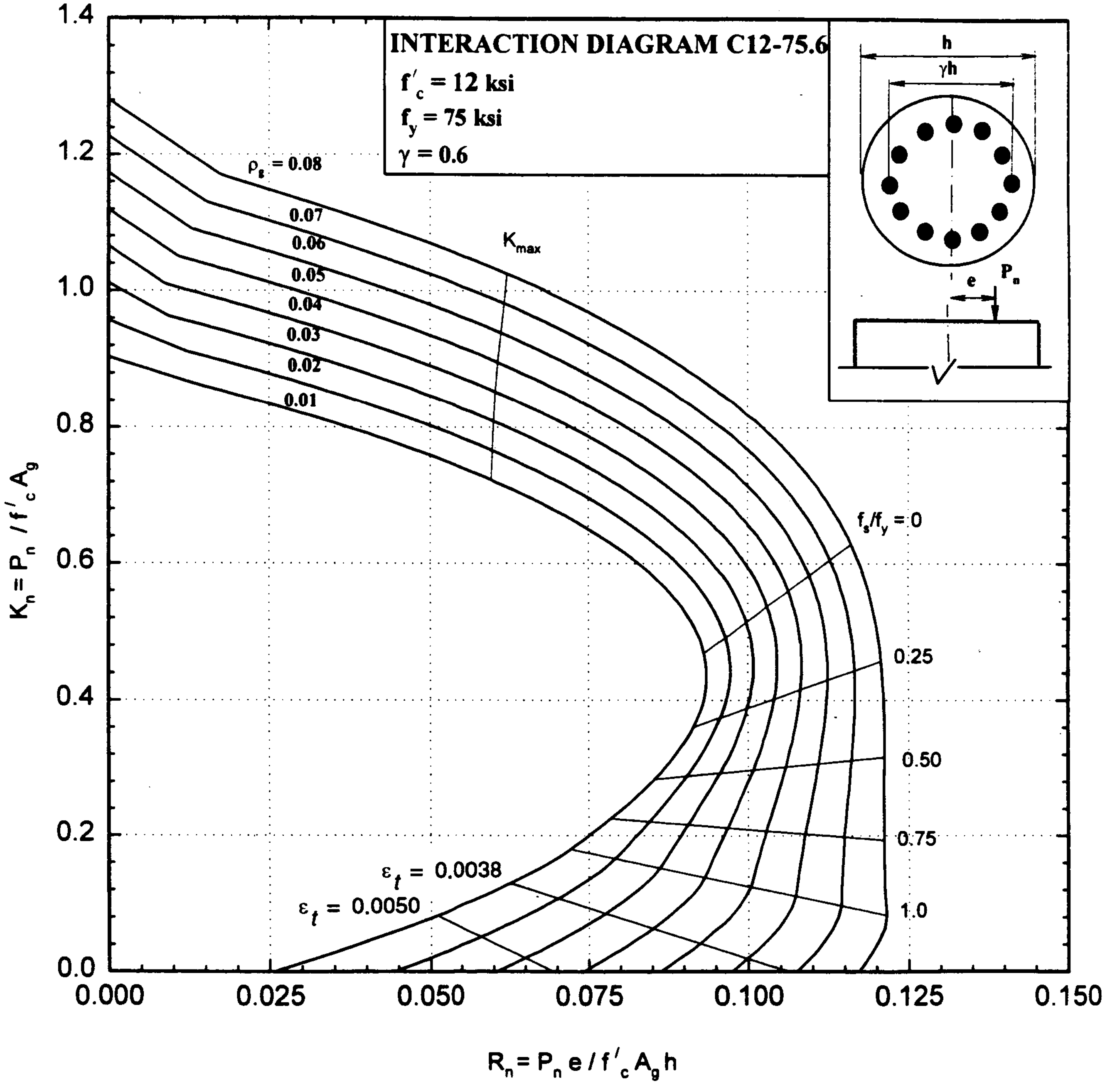


Diagram plotted using data from computer programs developed by Dr. Noel J. Everard.
Computer plot of data for diagram by Dr. Mohsen A. Issa and Alfred A. Yousif.

COLUMNS 7.18.2 - Nominal load-moment strength interaction diagram, C12-75.7

References: "Building Code Requirements for Structural Concrete-ACI 318" Chapters 9 and 10, and "Ultimate Strength Design of Reinforced Concrete Columns", ACI Special Publication SP-7, by Everard and Cohen, 1964, pp. 152-182 (with corrections).

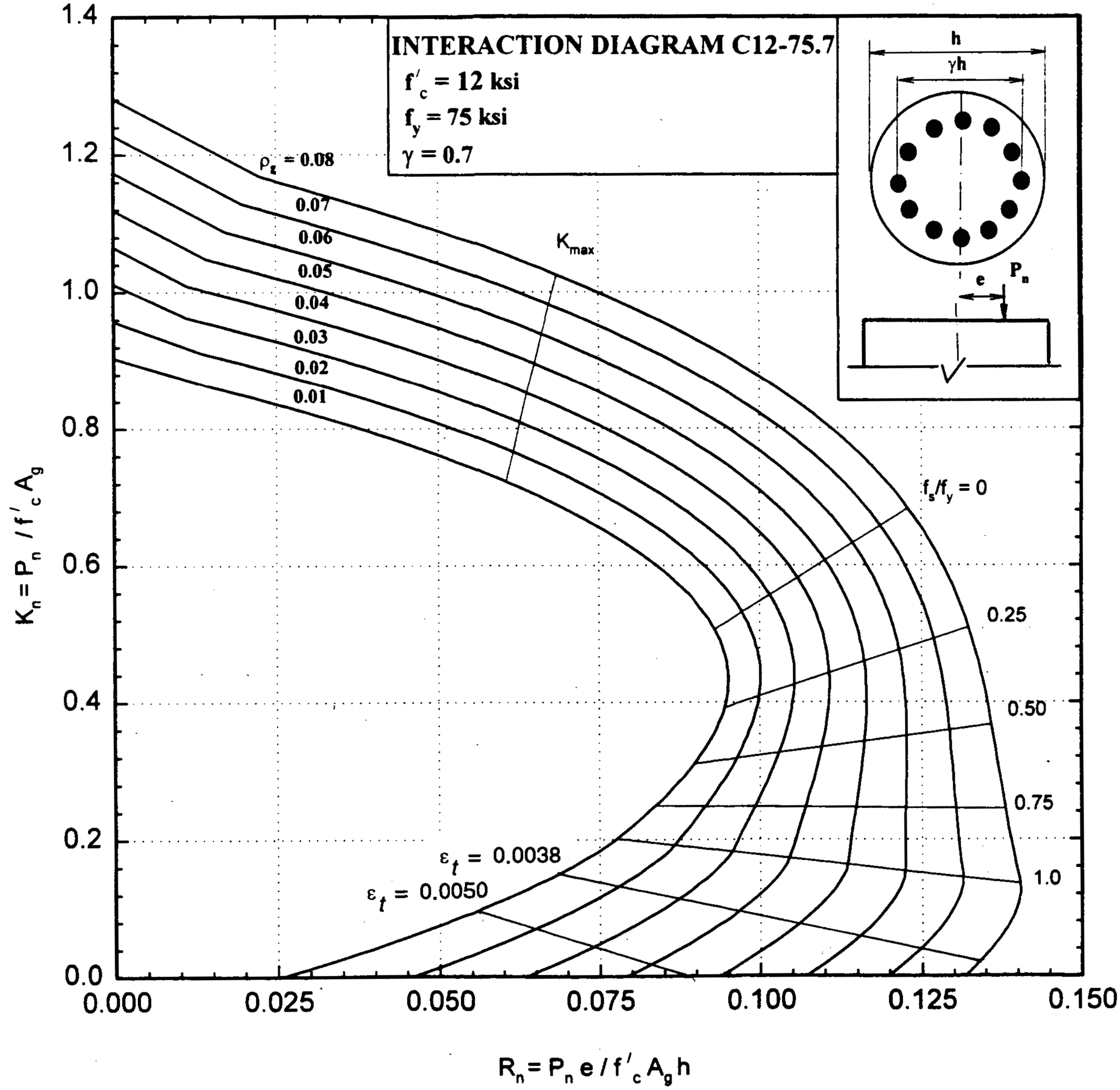


Diagram plotted using data from computer programs developed by Dr. Noel J. Everard.
Computer plot of data for diagram by Dr. Mohsen A. Issa and Alfred A. Yousif.

COLUMNS 7.18.3 - Nominal load-moment strength interaction diagram, C12-75.8

References: "Building Code Requirements for Structural Concrete-ACI 318" Chapters 9 and 10, and "Ultimate Strength Design of Reinforced Concrete Columns", ACI Special Publication SP-7, by Everard and Cohen, 1964, pp. 152-182 (with corrections).

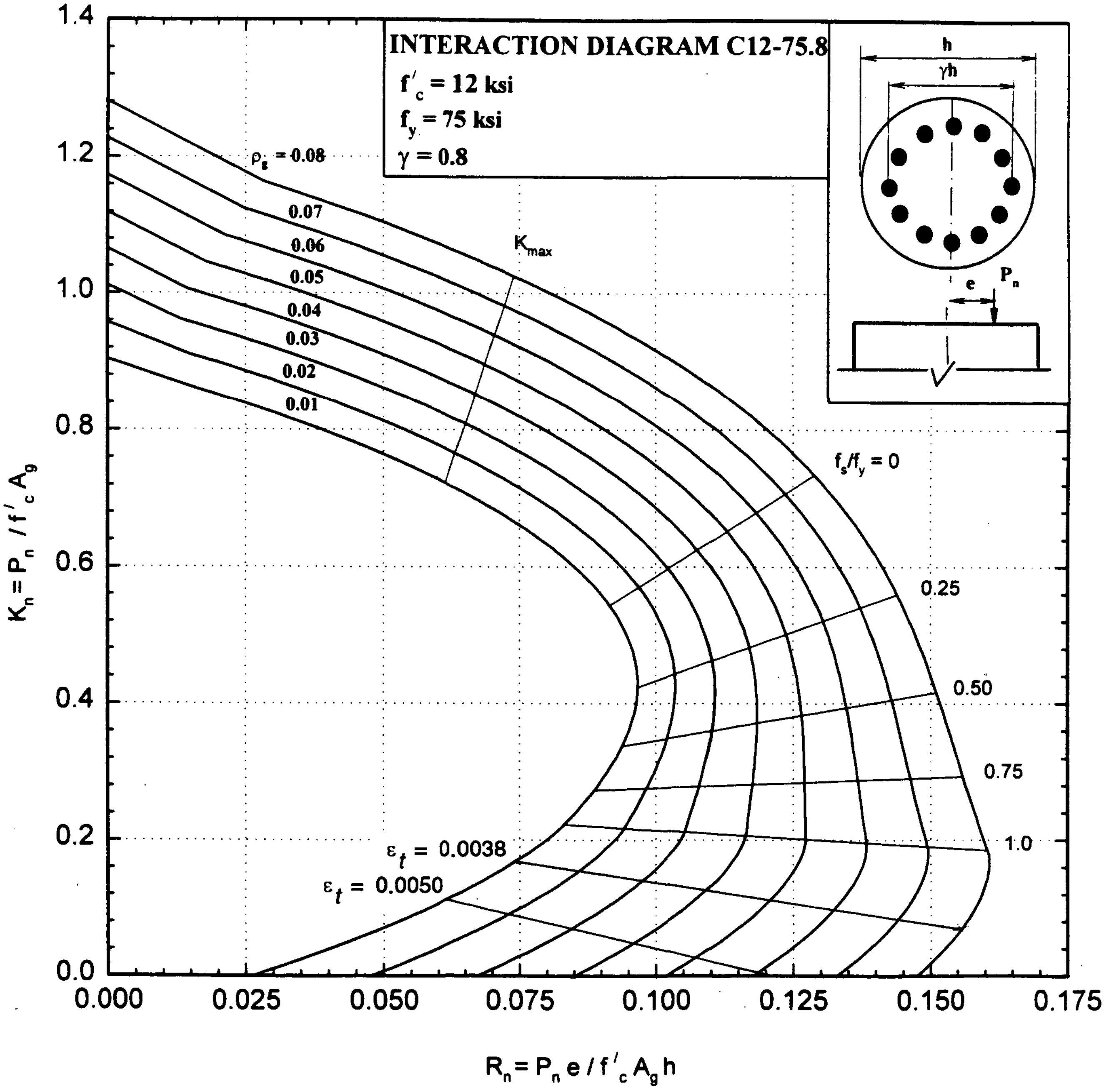


Diagram plotted using data from computer programs developed by Dr. Noel J. Everard.
Computer plot of data for diagram by Dr. Mohsen A. Issa and Alfred A. Yousif.

COLUMNS 7.18.4 - Nominal load-moment strength interaction diagram, C12-75.9

References: "Building Code Requirements for Structural Concrete-ACI 318" Chapters 9 and 10, and "Ultimate Strength Design of Reinforced Concrete Columns", ACI Special Publication SP-7, by Everard and Cohen, 1964, pp. 152-182 (with corrections).

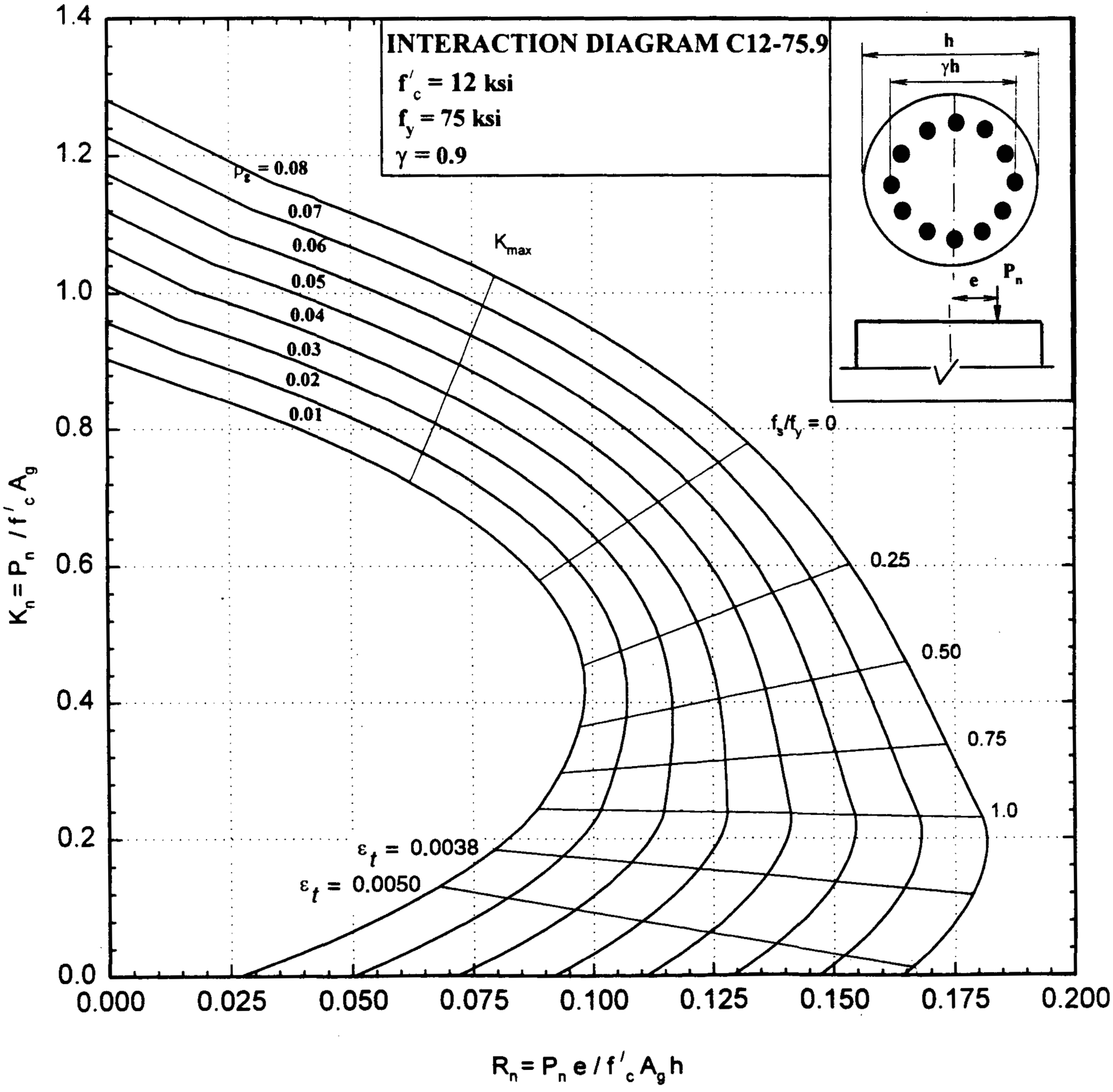


Diagram plotted using data from computer programs developed by Dr. Noel J. Everard. Computer plot of data for diagram by Dr. Mohsen A. Issa and Alfred A. Yousif.

COLUMNS 7.19.1 - Nominal load-moment strength interaction diagram, S3-60.6

References: "Building Code Requirements for Structural Concrete-ACI 318" Chapters 9 and 10, and "Ultimate Strength Design of Reinforced Concrete Columns", ACI Special Publication SP-7, by Everard and Cohen, 1964, pp. 152-182 (with corrections).

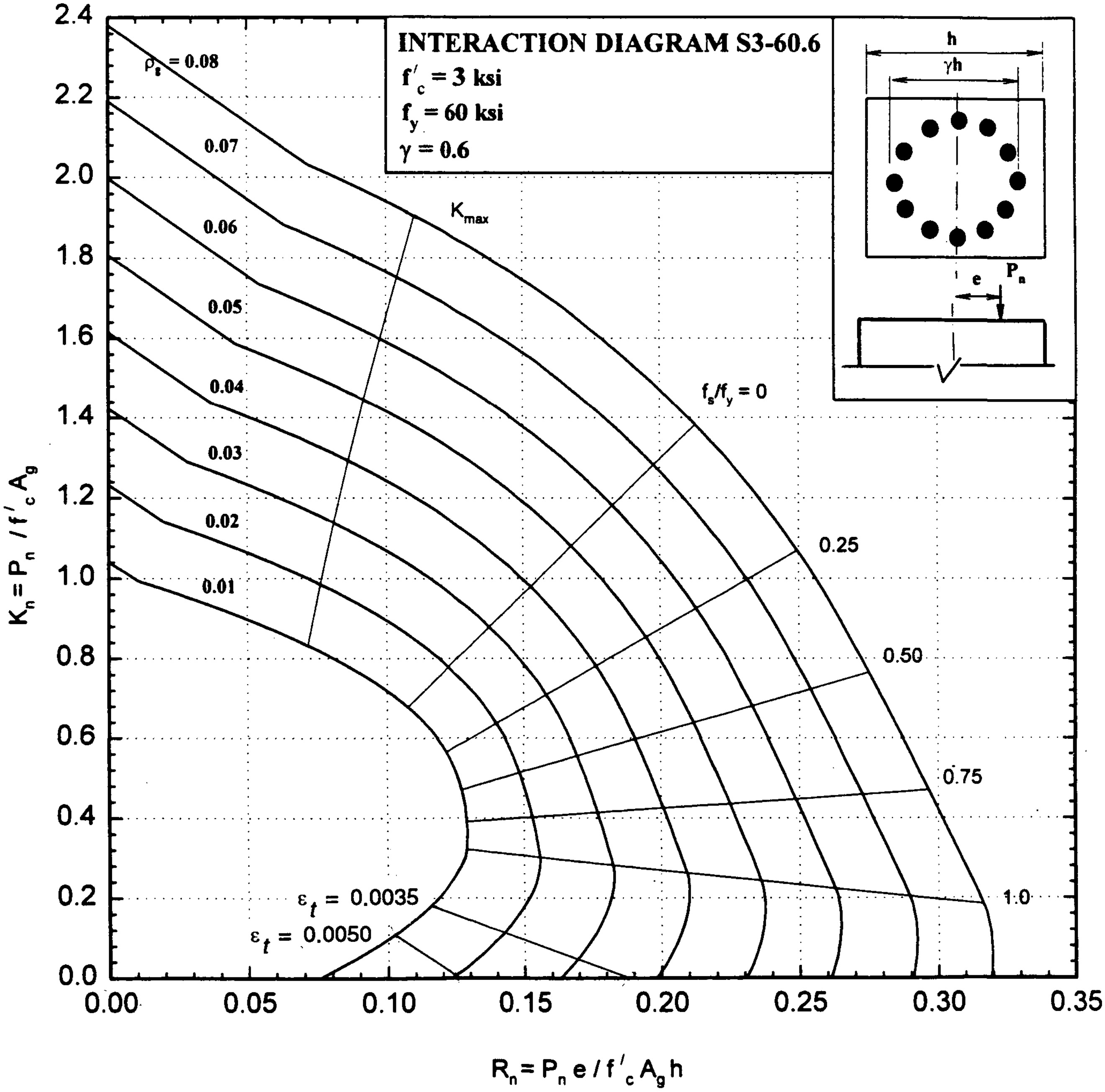


Diagram plotted using data from computer programs developed by Dr. Noel J. Everard. Computer plot of data for diagram by Dr. Mohsen A. Issa and Alfred A. Yousif.

COLUMNS 7.19.2 - Nominal load-moment strength interaction diagram, S3-60.7

References: "Building Code Requirements for Structural Concrete-ACI 318" Chapters 9 and 10, and "Ultimate Strength Design of Reinforced Concrete Columns", ACI Special Publication SP-7, by Everard and Cohen, 1964, pp. 152-182 (with corrections).

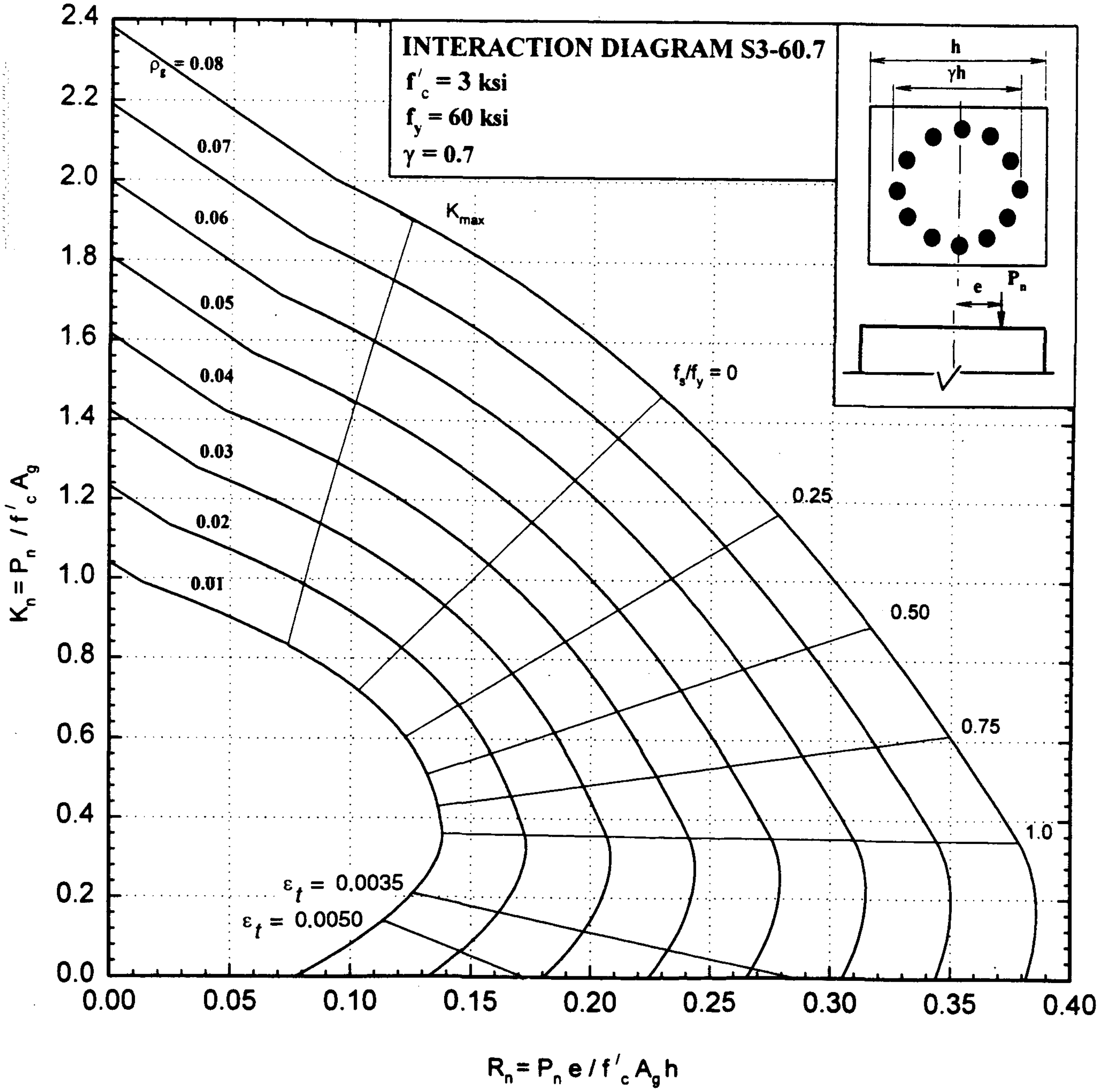


Diagram plotted using data from computer programs developed by Dr. Noel J. Everard.
Computer plot of data for diagram by Dr. Mohsen A. Issa and Alfred A. Yousif.

COLUMNS 7.19.3 - Nominal load-moment strength interaction diagram, S3-60.8

References: "Building Code Requirements for Structural Concrete-ACI 318" Chapters 9 and 10, and "Ultimate Strength Design of Reinforced Concrete Columns", ACI Special Publication SP-7, by Everard and Cohen, 1964, pp. 152-182 (with corrections).

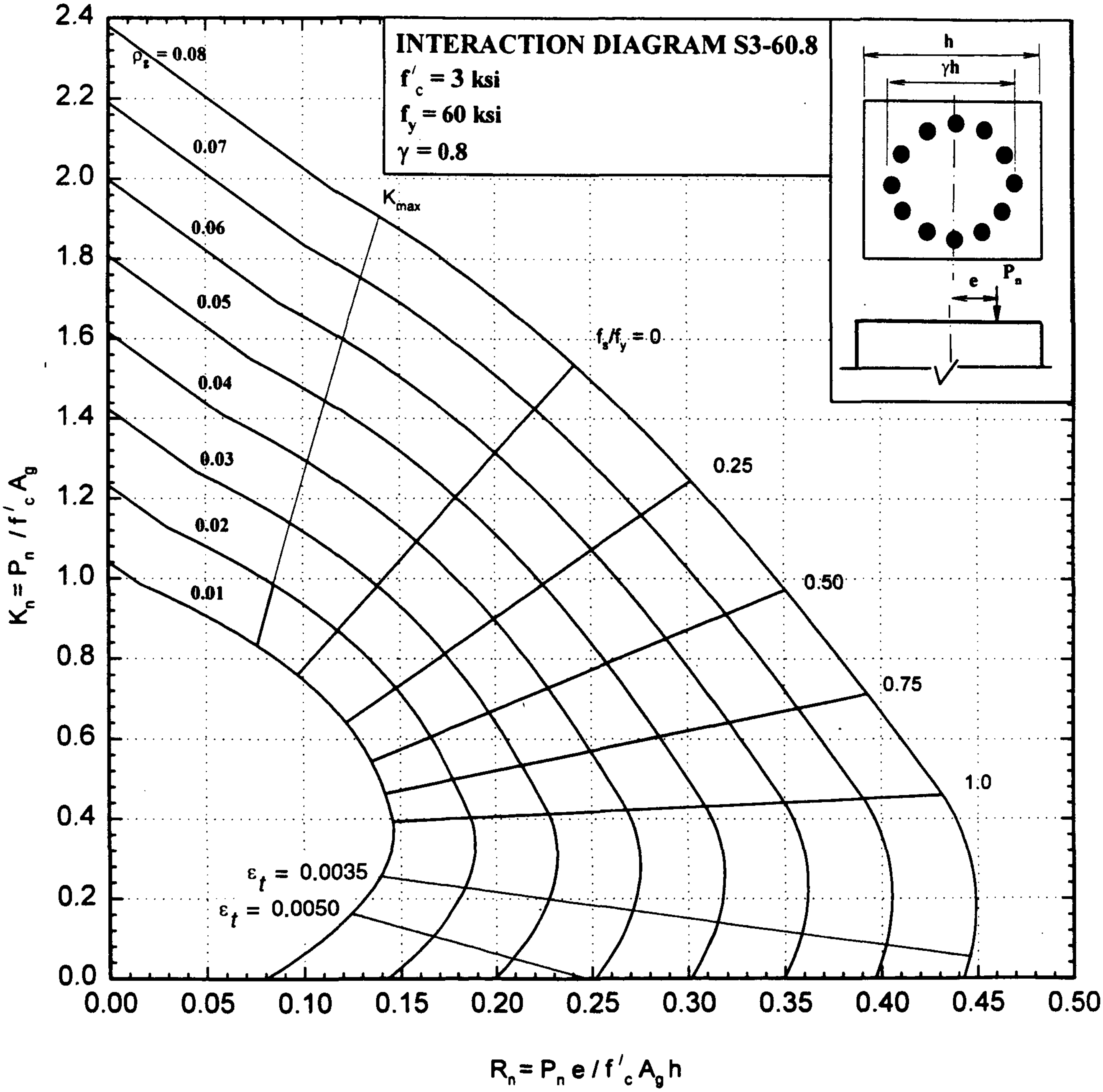


Diagram plotted using data from computer programs developed by Dr. Noel J. Everard.
Computer plot of data for diagram by Dr. Mohsen A. Issa and Alfred A. Yousif.

COLUMNS 7.19.4 - Nominal load-moment strength interaction diagram, S3-60.9

References: "Building Code Requirements for Structural Concrete-ACI 318" Chapters 9 and 10, and "Ultimate Strength Design of Reinforced Concrete Columns", ACI Special Publication SP-7, by Everard and Cohen, 1964, pp. 152-182 (with corrections).

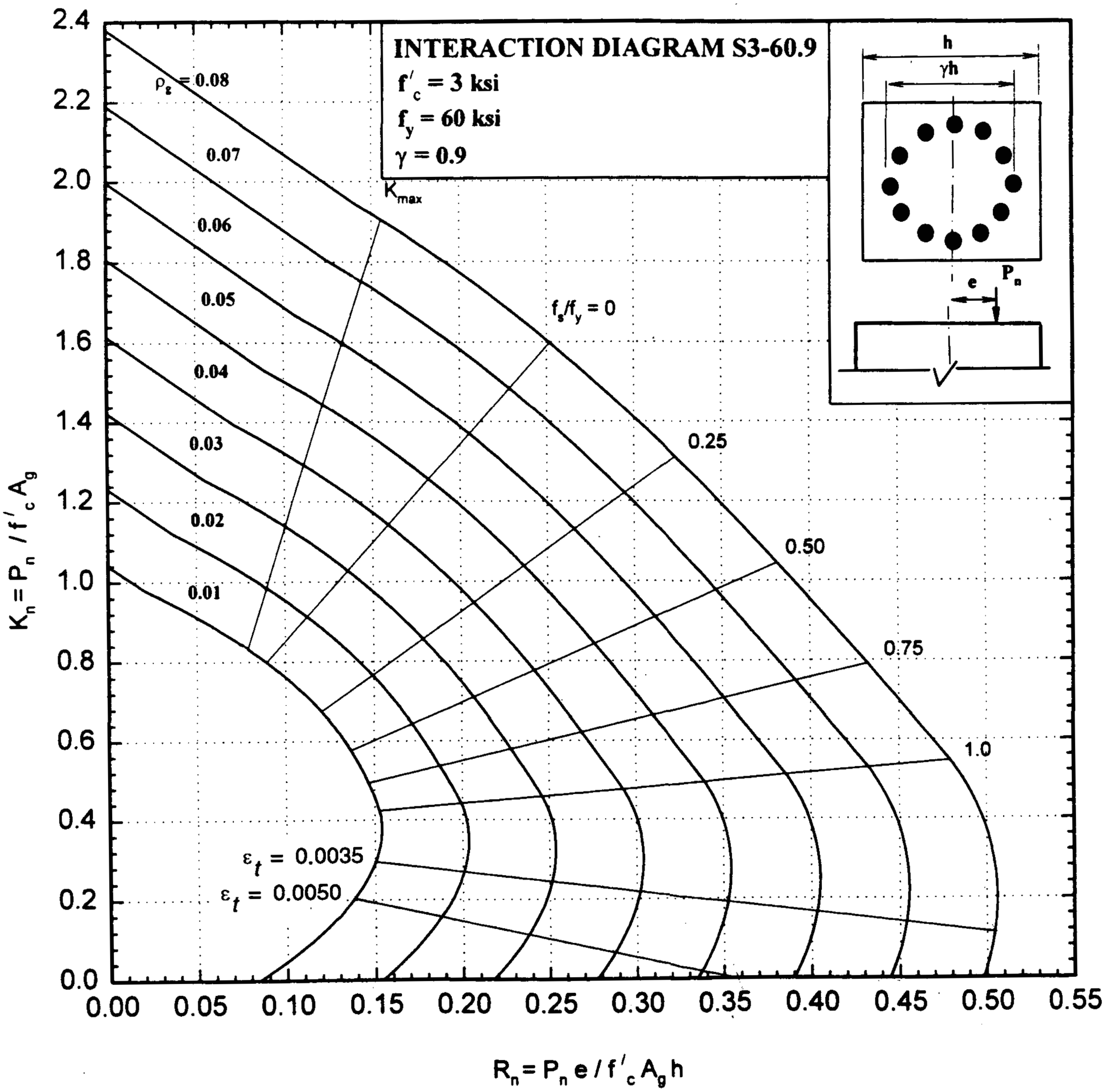


Diagram plotted using data from computer programs developed by Dr. Noel J. Everard. Computer plot of data for diagram by Dr. Mohsen A. Issa and Alfred A. Yousif.

COLUMNS 7.20.1 - Nominal load-moment strength interaction diagram, S4-60.6

References: "Building Code Requirements for Structural Concrete-ACI 318" Chapters 9 and 10, and "Ultimate Strength Design of Reinforced Concrete Columns", ACI Special Publication SP-7, by Everard and Cohen, 1964, pp. 152-182 (with corrections).

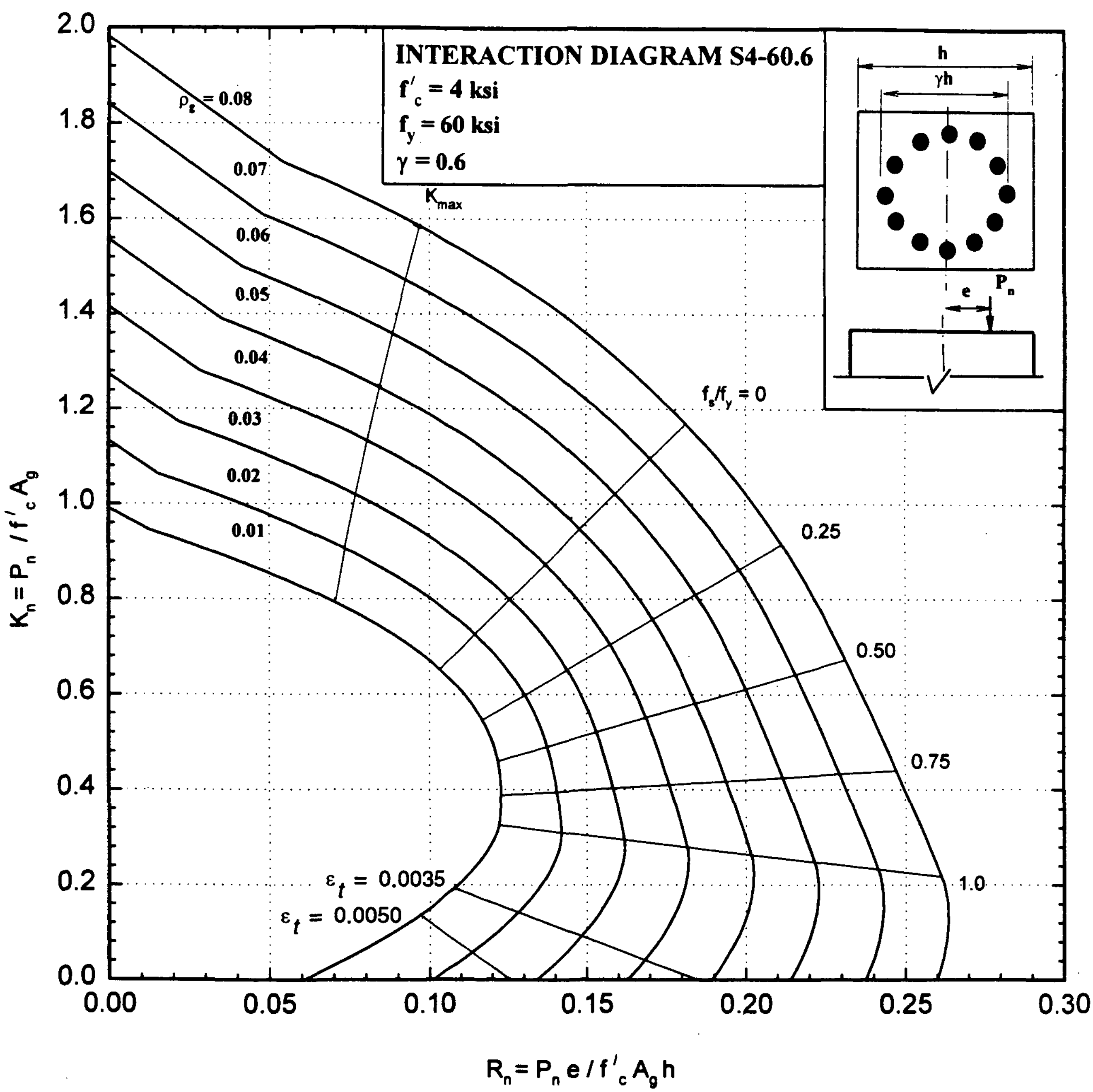


Diagram plotted using data from computer programs developed by Dr. Noel J. Everard.
Computer plot of data for diagram by Dr. Mohsen A. Issa and Alfred A. Yousif.

COLUMNS 7.20.2 - Nominal load-moment strength interaction diagram, S4-60.7

References: "Building Code Requirements for Structural Concrete-ACI 318" Chapters 9 and 10, and "Ultimate Strength Design of Reinforced Concrete Columns", ACI Special Publication SP-7, by Everard and Cohen, 1964, pp. 152-182 (with corrections).

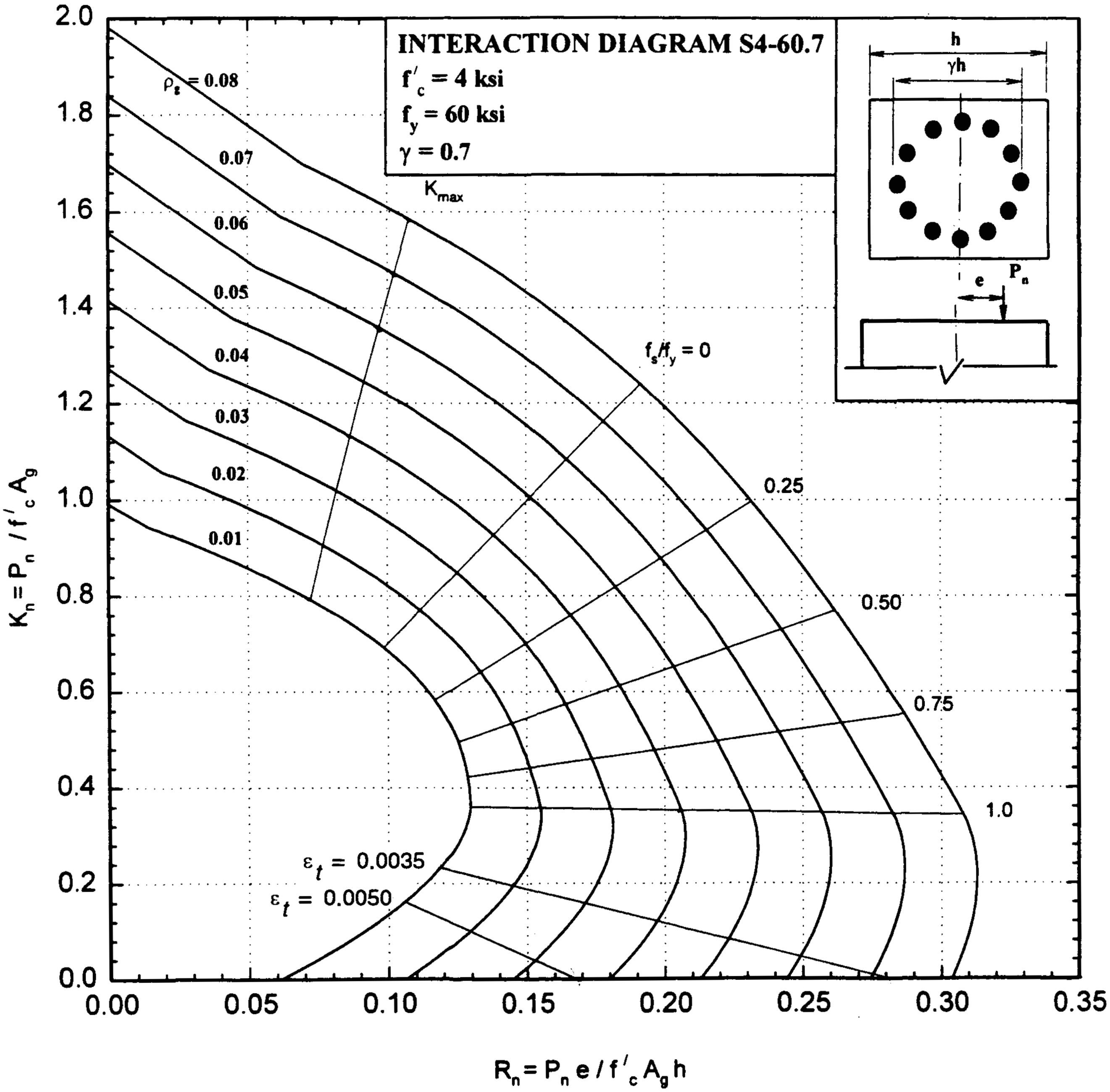


Diagram plotted using data from computer programs developed by Dr. Noel J. Everard.
Computer plot of data for diagram by Dr. Mohsen A. Issa and Alfred A. Yousif.

COLUMNS 7.20.3 - Nominal load-moment strength interaction diagram, S4-60.8

References: "Building Code Requirements for Structural Concrete-ACI 318" Chapters 9 and 10, and "Ultimate Strength Design of Reinforced Concrete Columns", ACI Special Publication SP-7, by Everard and Cohen, 1964, pp. 152-182 (with corrections).

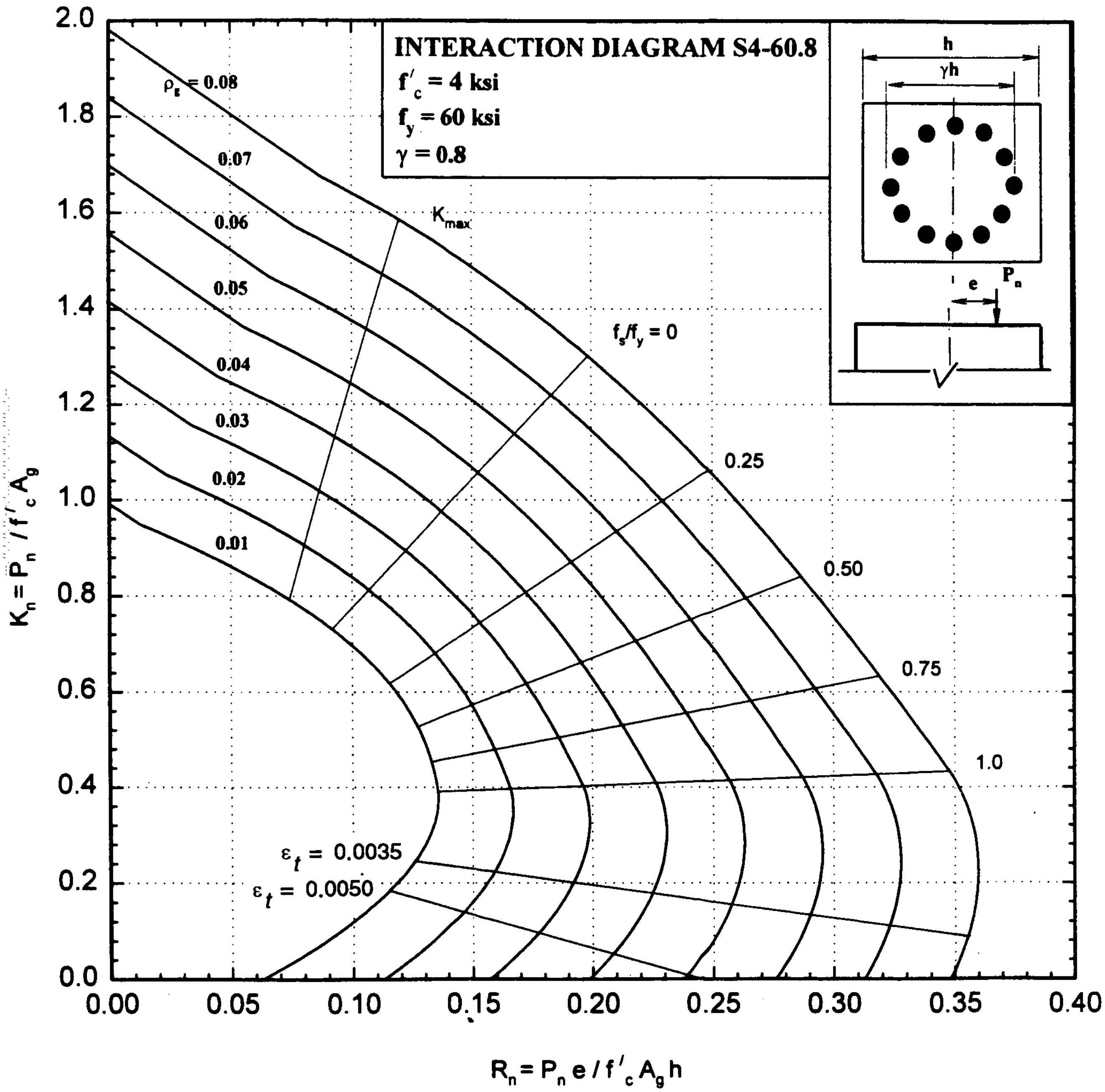


Diagram plotted using data from computer programs developed by Dr. Noel J. Everard.
Computer plot of data for diagram by Dr. Mohsen A. Issa and Alfred A. Yousif.

COLUMNS 7.20.4 - Nominal load-moment strength interaction diagram, S4-60.9

References: "Building Code Requirements for Structural Concrete-ACI 318" Chapters 9 and 10, and "Ultimate Strength Design of Reinforced Concrete Columns", ACI Special Publication SP-7, by Everard and Cohen, 1964, pp. 152-182 (with corrections).

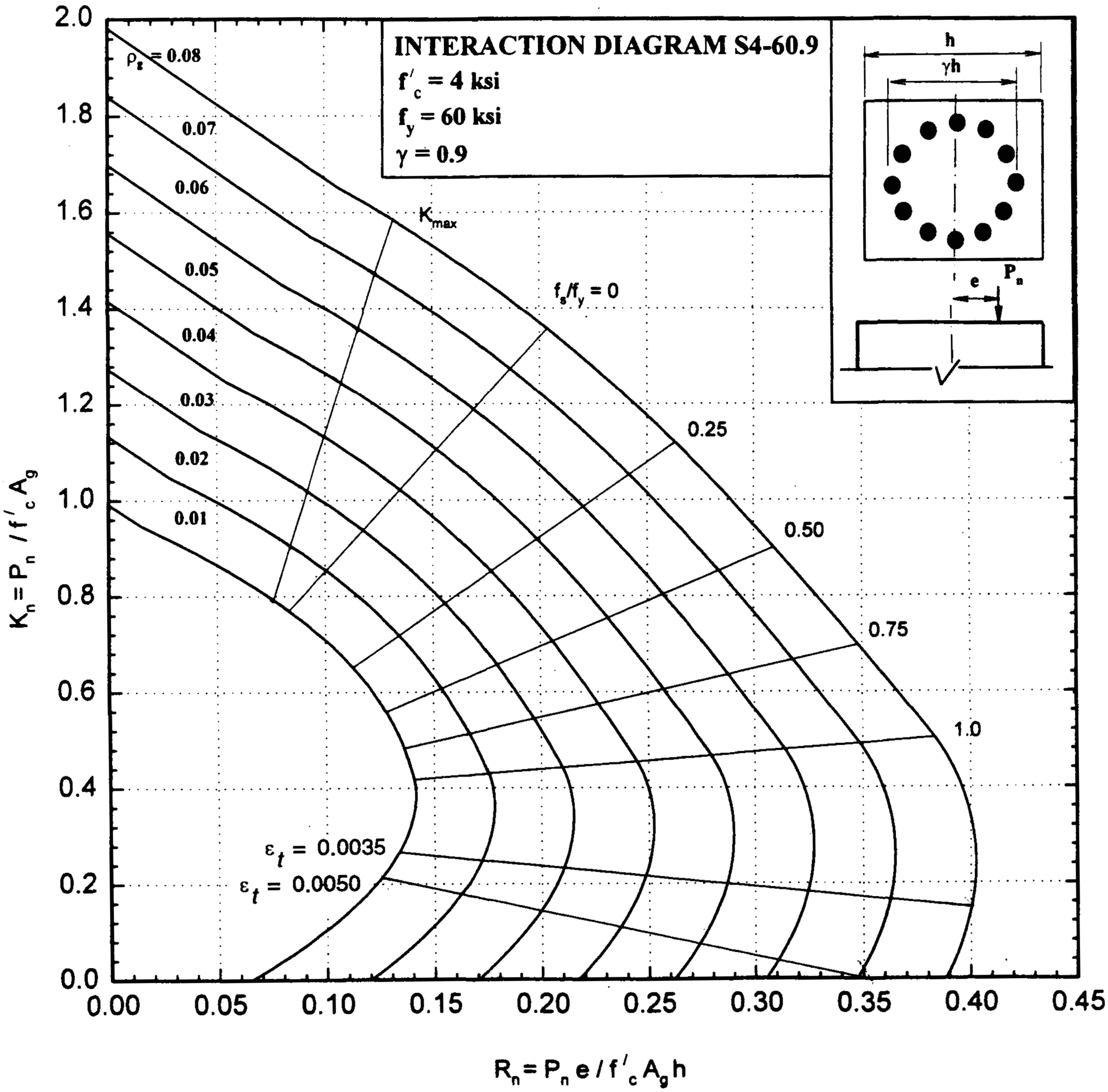


Diagram plotted using data from computer programs developed by Dr. Noel J. Everard.
Computer plot of data for diagram by Dr. Mohsen A. Issa and Alfred A. Yousif.

COLUMNS 7.21.1 - Nominal load-moment strength interaction diagram, S5-60.6

References: "Building Code Requirements for Structural Concrete-ACI 318" Chapters 9 and 10, and "Ultimate Strength Design of Reinforced Concrete Columns", ACI Special Publication SP-7, by Everard and Cohen, 1964, pp. 152-182 (with corrections).

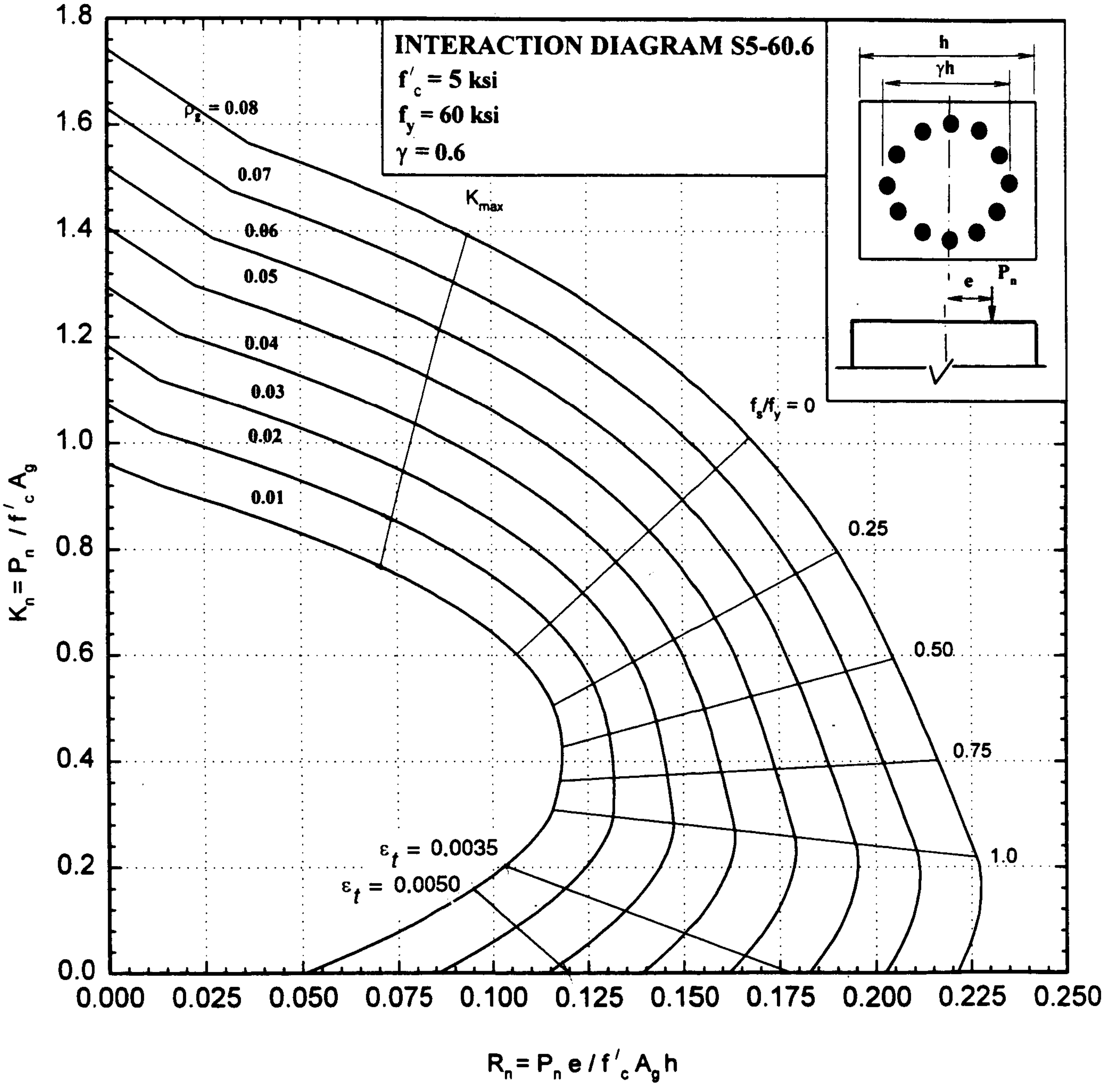


Diagram plotted using data from computer programs developed by Dr. Noel J. Everard.
Computer plot of data for diagram by Dr. Mohsen A. Issa and Alfred A. Yousif.

COLUMNS 7.21.2 - Nominal load-moment strength interaction diagram, S5-60.7

References: "Building Code Requirements for Structural Concrete-ACI 318" Chapters 9 and 10, and "Ultimate Strength Design of Reinforced Concrete Columns", ACI Special Publication SP-7, by Everard and Cohen, 1964, pp. 152-182 (with corrections).

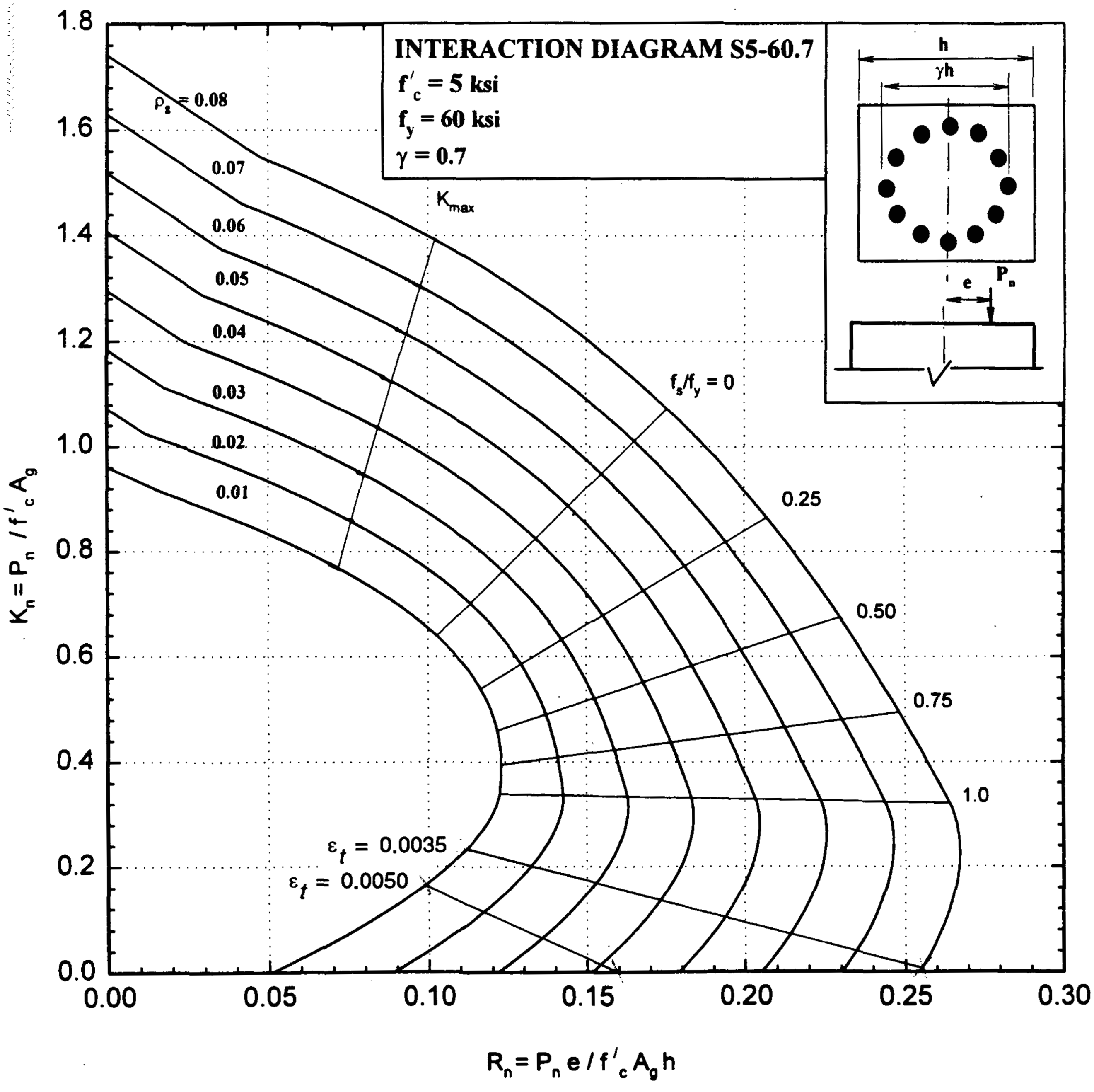


Diagram plotted using data from computer programs developed by Dr. Noel J. Everard. Computer plot of data for diagram by Dr. Mohsen A. Issa and Alfred A. Yousif.

COLUMNS 7.21.3 - Nominal load-moment strength interaction diagram, S5-60.8

References: "Building Code Requirements for Structural Concrete-ACI 318" Chapters 9 and 10, and "Ultimate Strength Design of Reinforced Concrete Columns", ACI Special Publication SP-7, by Everard and Cohen, 1964, pp. 152-182 (with corrections).

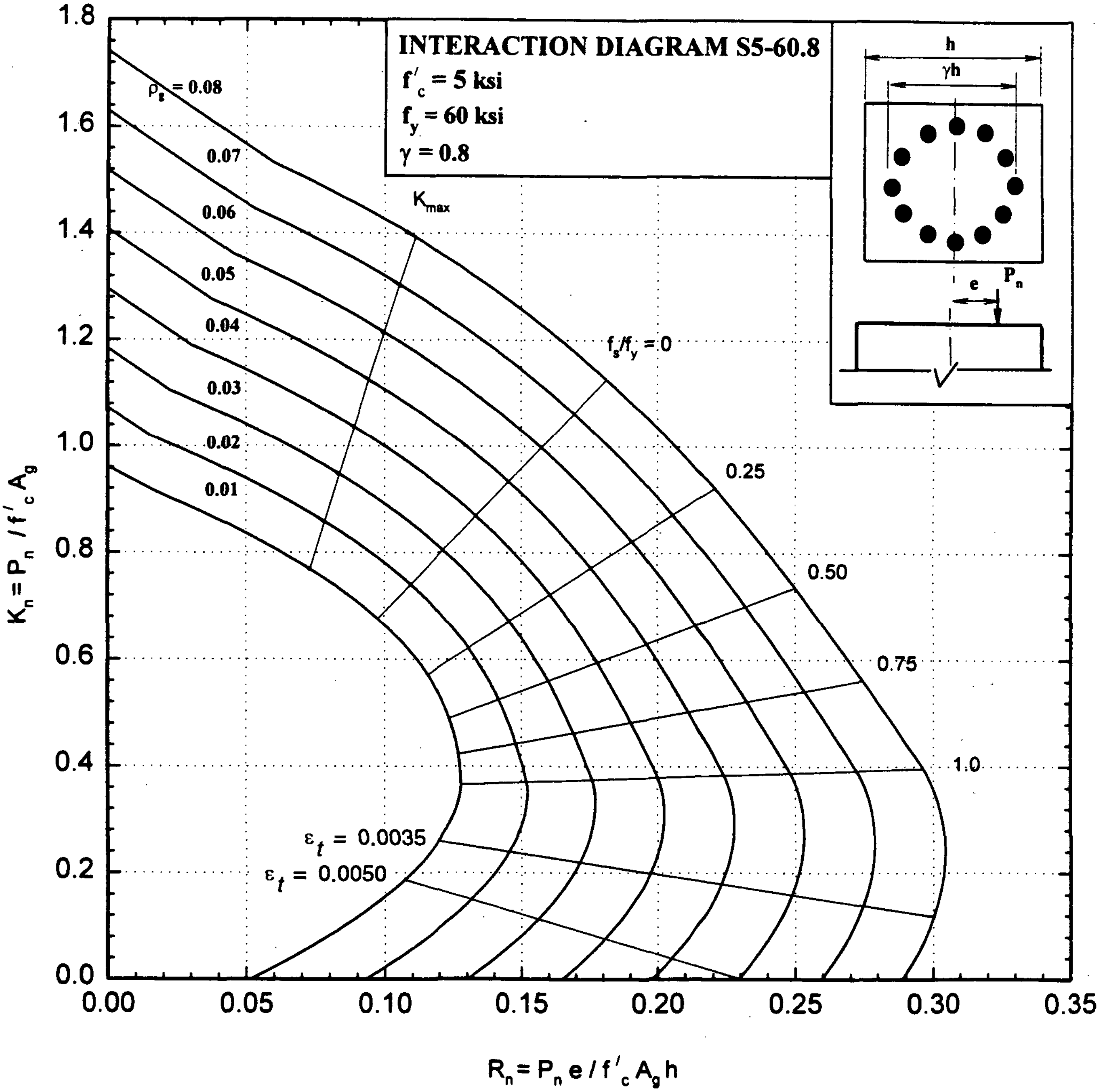


Diagram plotted using data from computer programs developed by Dr. Noel J. Everard. Computer plot of data for diagram by Dr. Mohsen A. Issa and Alfred A. Yousif.

COLUMNS 7.21.4 - Nominal load-moment strength interaction diagram, S5-60.9

References: "Building Code Requirements for Structural Concrete-ACI 318" Chapters 9 and 10, and "Ultimate Strength Design of Reinforced Concrete Columns", ACI Special Publication SP-7, by Everard and Cohen, 1964, pp. 152-182 (with corrections).

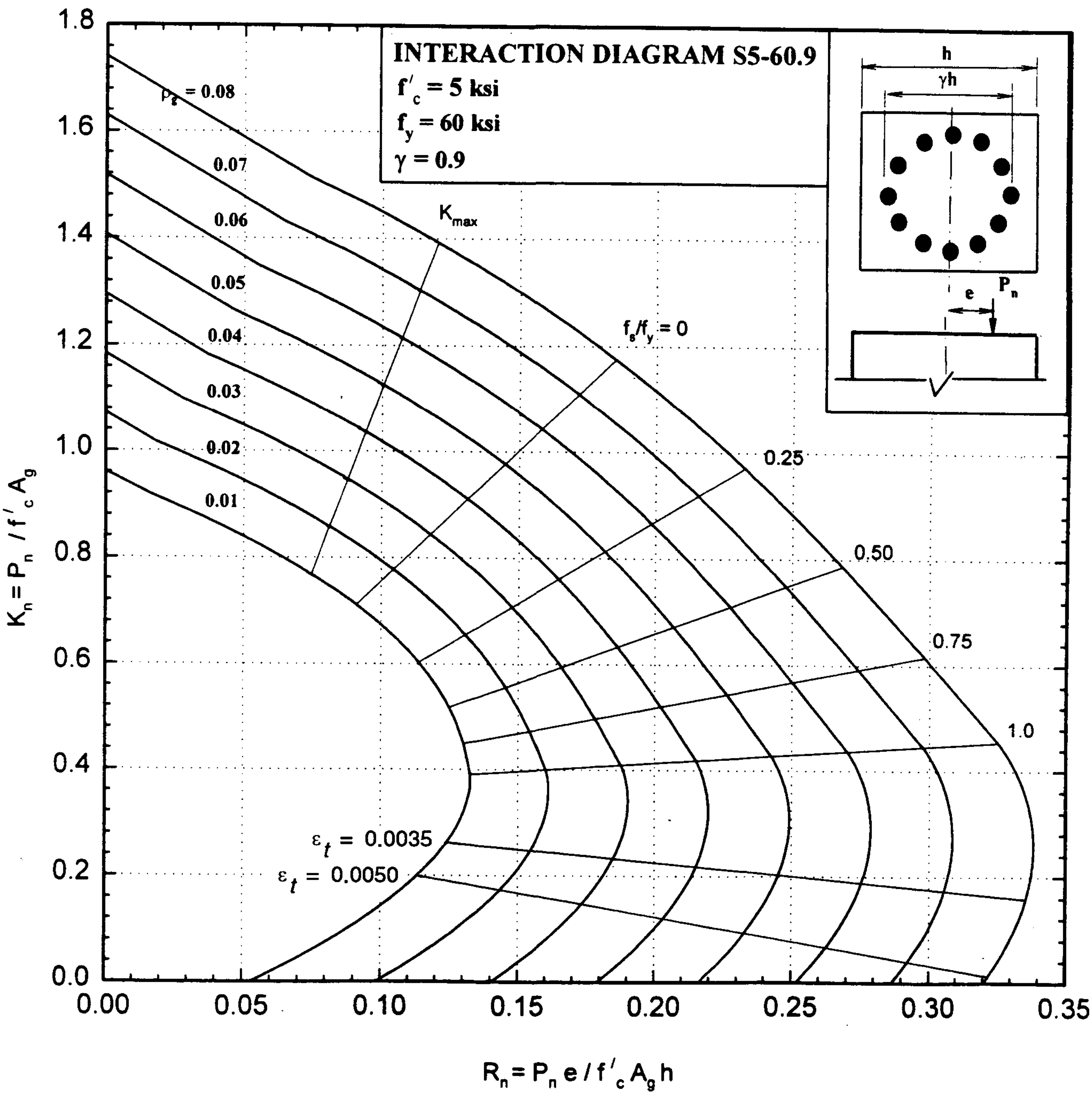


Diagram plotted using data from computer programs developed by Dr. Noel J. Everard.
Computer plot of data for diagram by Dr. Mohsen A. Issa and Alfred A. Yousif.

COLUMNS 7.22.1 - Nominal load-moment strength interaction diagram, S6-60.6

References: "Building Code Requirements for Structural Concrete-ACI 318" Chapters 9 and 10, and "Ultimate Strength Design of Reinforced Concrete Columns", ACI Special Publication SP-7, by Everard and Cohen, 1964, pp. 152-182 (with corrections).

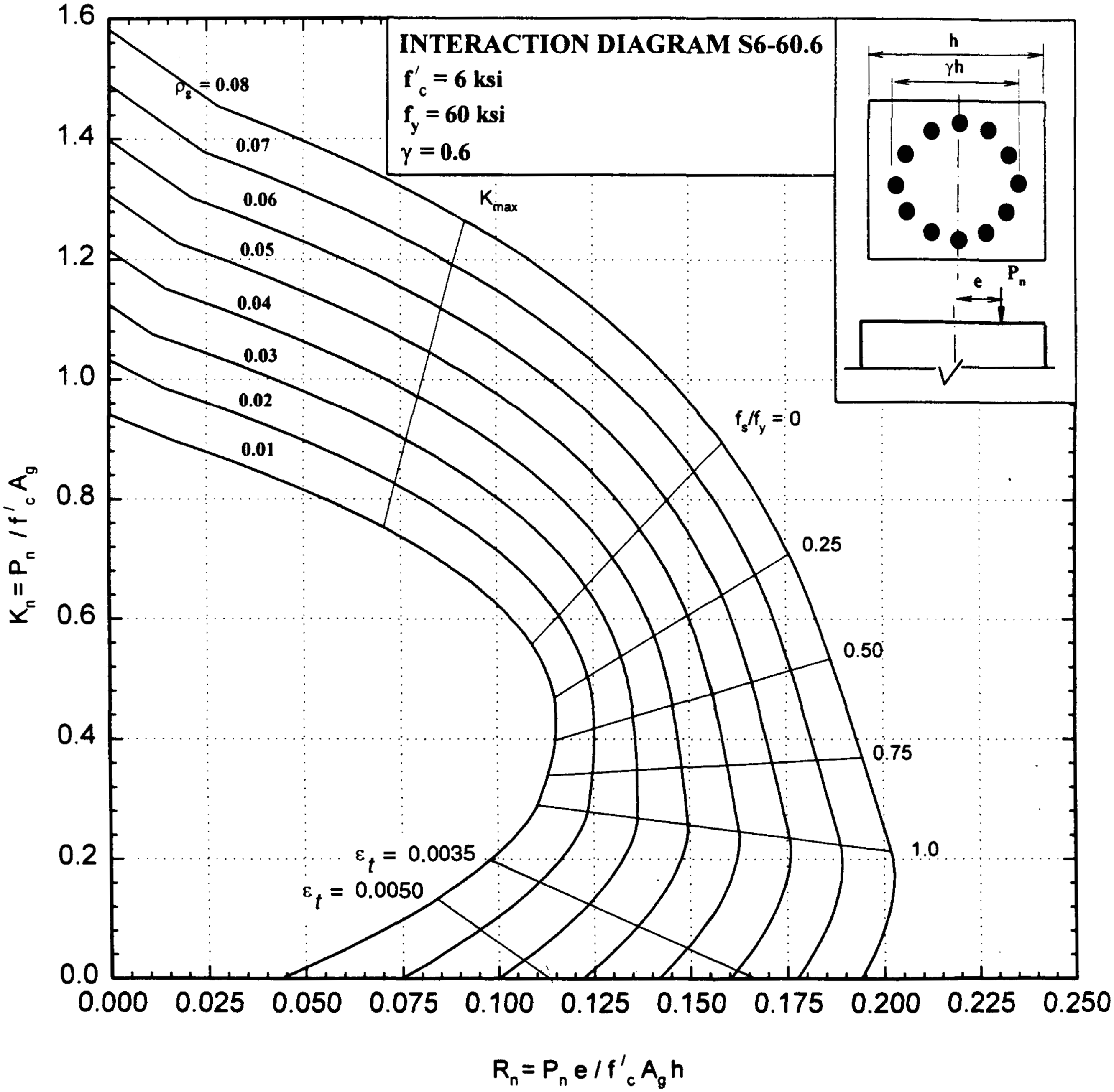


Diagram plotted using data from computer programs developed by Dr. Noel J. Everard.
Computer plot of data for diagram by Dr. Mohsen A. Issa and Alfred A. Yousif.

COLUMNS 7.22.2 - Nominal load-moment strength interaction diagram, S6-60.7

References: "Building Code Requirements for Structural Concrete-ACI 318" Chapters 9 and 10, and "Ultimate Strength Design of Reinforced Concrete Columns", ACI Special Publication SP-7, by Everard and Cohen, 1964, pp. 152-182 (with corrections).

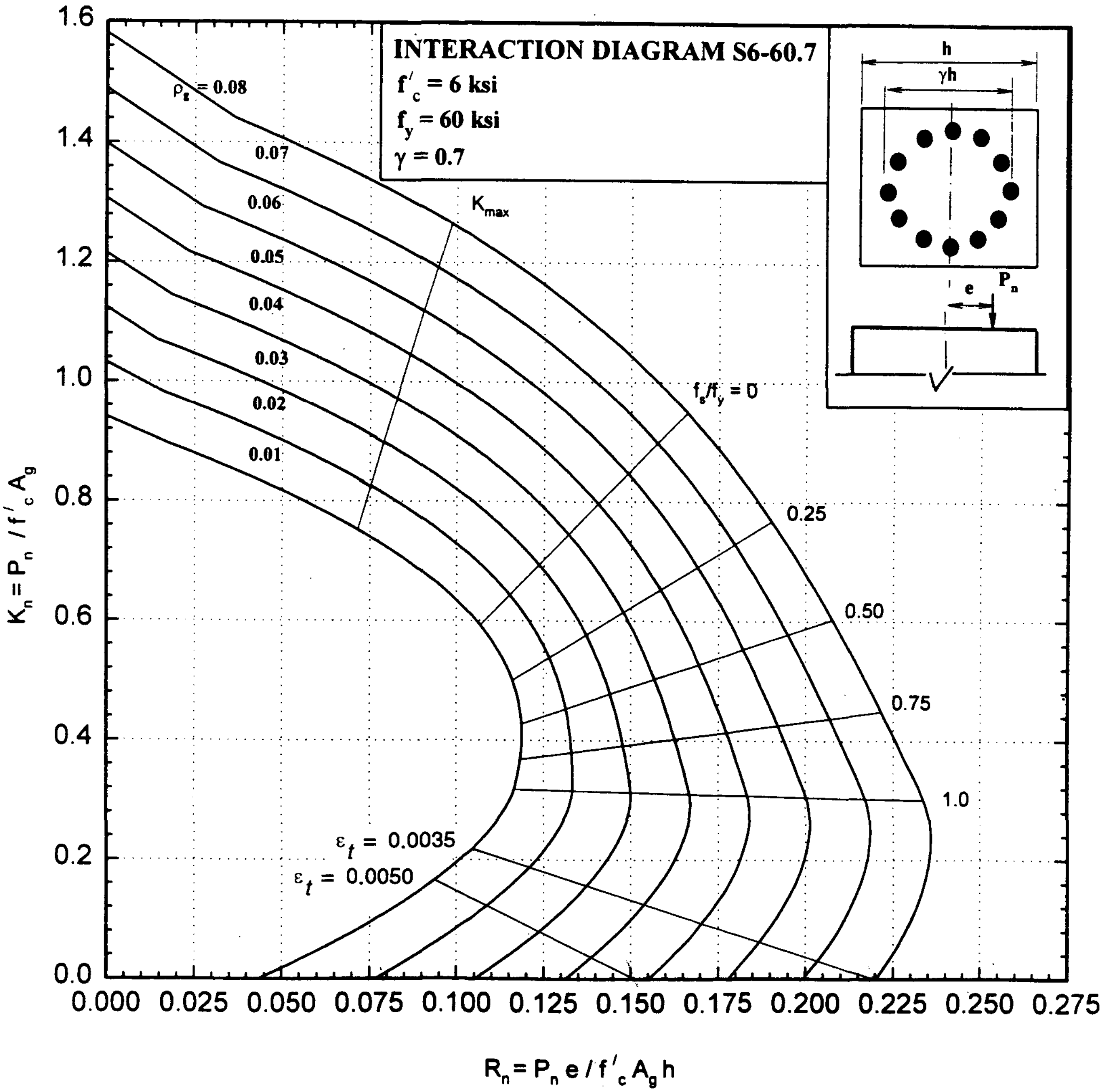


Diagram plotted using data from computer programs developed by Dr. Noel J. Everard.
Computer plot of data for diagram by Dr. Mohsen A. Issa and Alfred A. Yousif.

COLUMNS 7.22.3 - Nominal load-moment strength interaction diagram, S6-60.8

References: "Building Code Requirements for Structural Concrete-ACI 318" Chapters 9 and 10, and "Ultimate Strength Design of Reinforced Concrete Columns", ACI Special Publication SP-7, by Everard and Cohen, 1964, pp. 152-182 (with corrections).

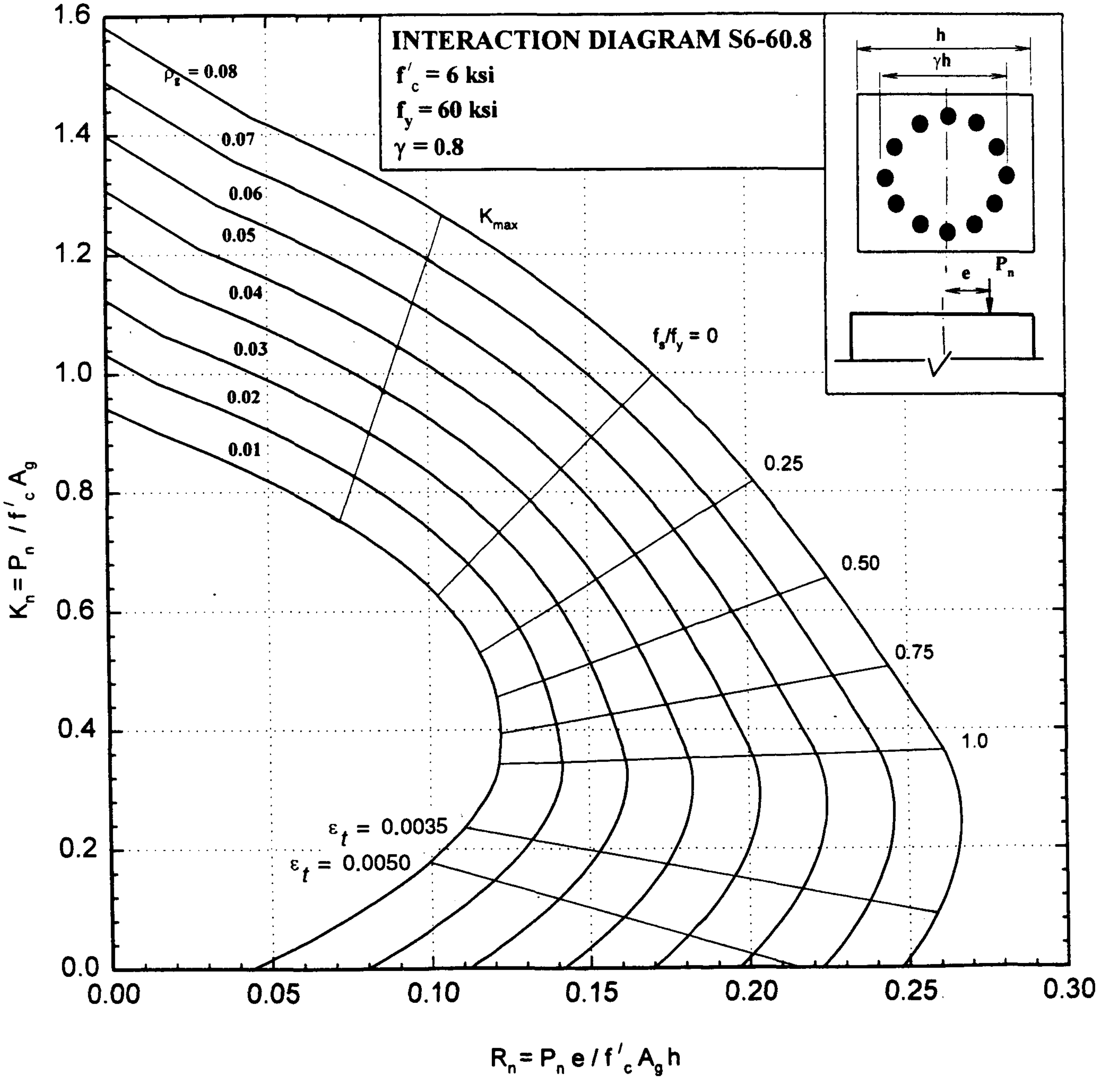


Diagram plotted using data from computer programs developed by Dr. Noel J. Everard.
Computer plot of data for diagram by Dr. Mohsen A. Issa and Alfred A. Yousif.

COLUMNS 7.22.4 - Nominal load-moment strength interaction diagram, S6-60.9

References: "Building Code Requirements for Structural Concrete-ACI 318" Chapters 9 and 10, and "Ultimate Strength Design of Reinforced Concrete Columns", ACI Special Publication SP-7, by Everard and Cohen, 1964, pp. 152-182 (with corrections).

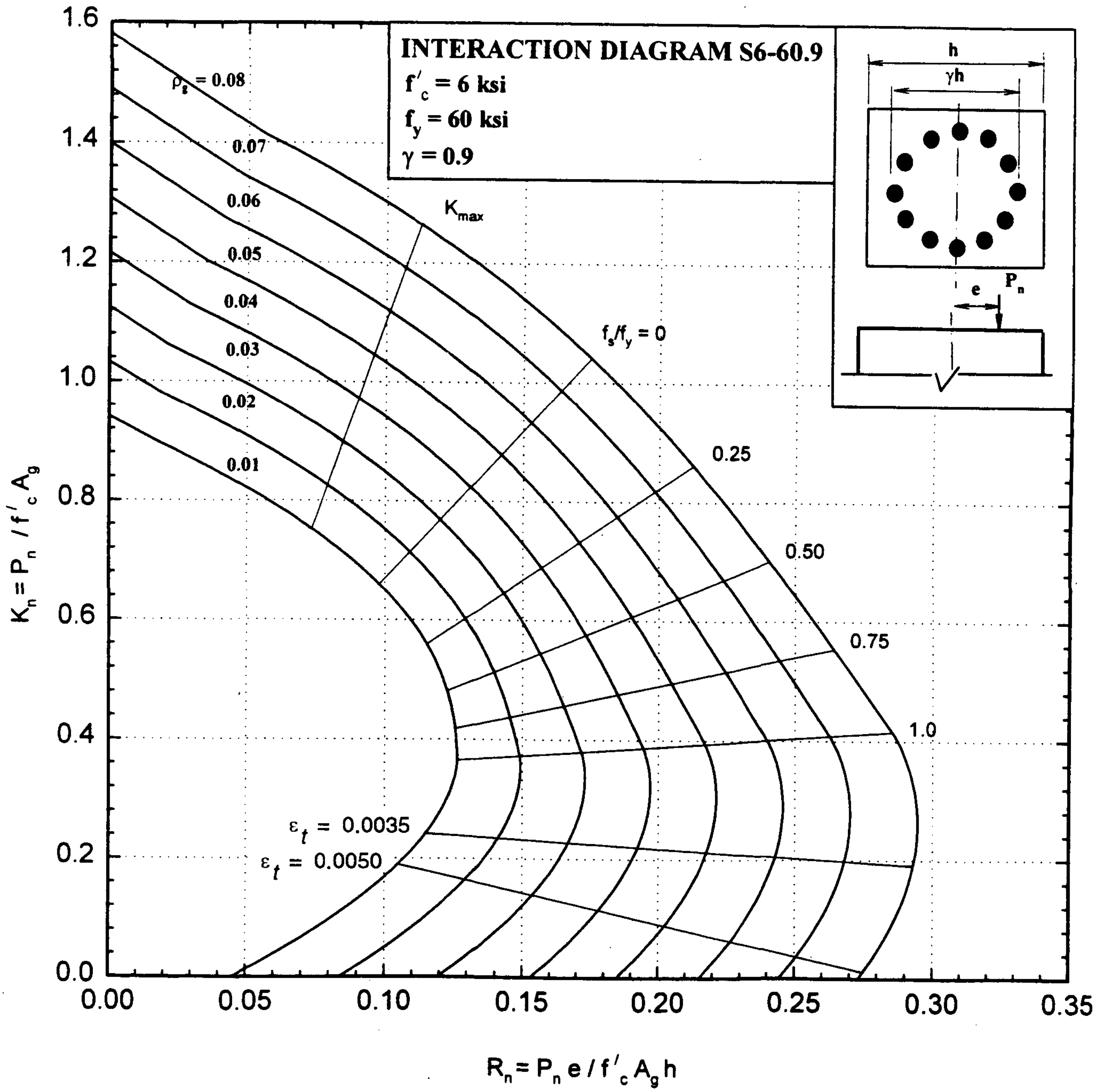


Diagram plotted using data from computer programs developed by Dr. Noel J. Everard. Computer plot of data for diagram by Dr. Mohsen A. Issa and Alfred A. Yousif.

COLUMNS 7.23.1 - Nominal load-moment strength interaction diagram, S9-75.6

References: "Building Code Requirements for Structural Concrete-ACI 318" Chapters 9 and 10, and "Ultimate Strength Design of Reinforced Concrete Columns", ACI Special Publication SP-7, by Everard and Cohen, 1964, pp. 152-182 (with corrections).

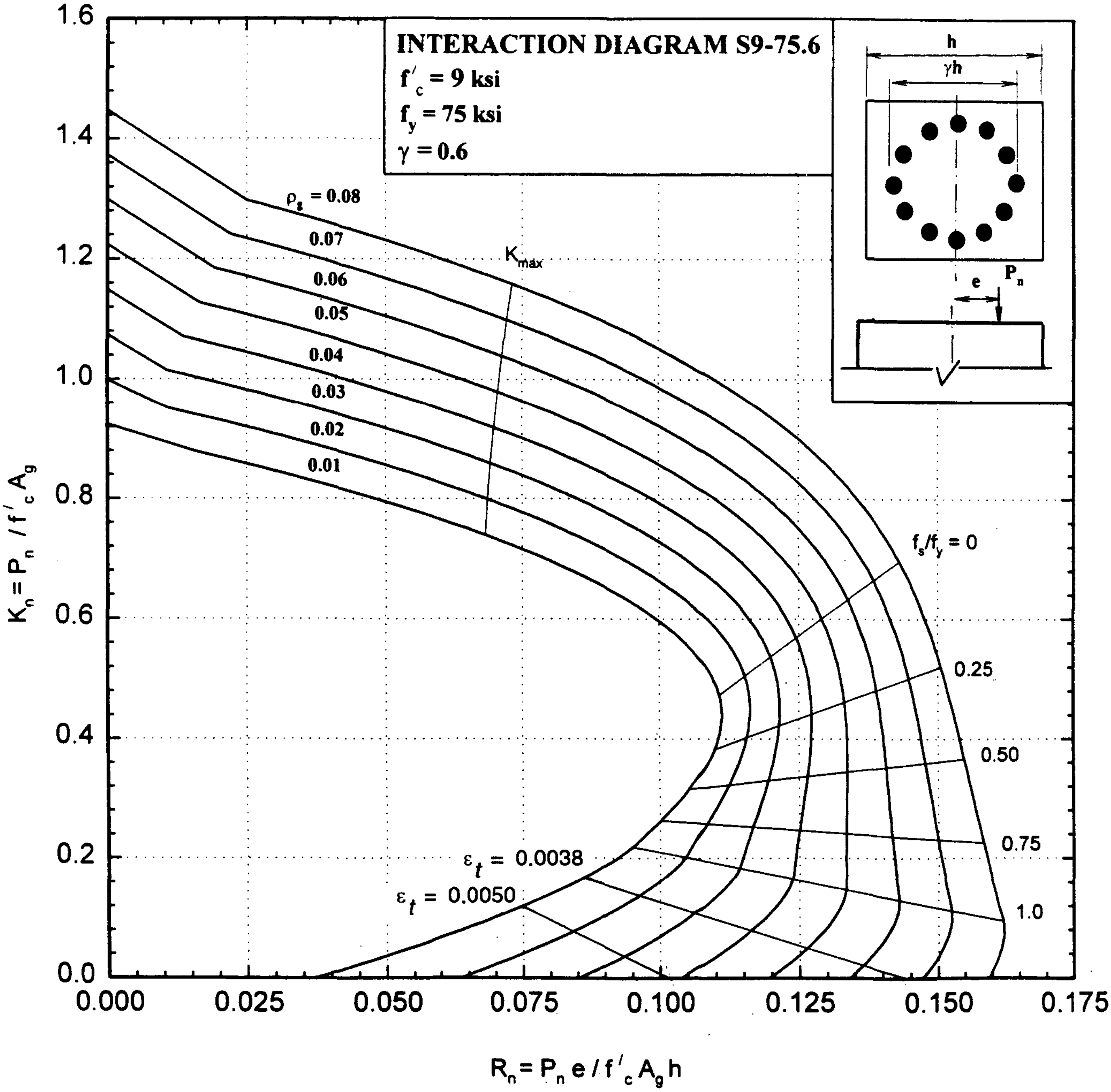


Diagram plotted using data from computer programs developed by Dr. Noel J. Everard.
Computer plot of data for diagram by Dr. Mohsen A. Issa and Alfred A. Yousif.

COLUMNS 7.23.2 - Nominal load-moment strength interaction diagram, S9-75.7

References: "Building Code Requirements for Structural Concrete-ACI 318" Chapters 9 and 10, and "Ultimate Strength Design of Reinforced Concrete Columns", ACI Special Publication SP-7, by Everard and Cohen, 1964, pp. 152-182 (with corrections).

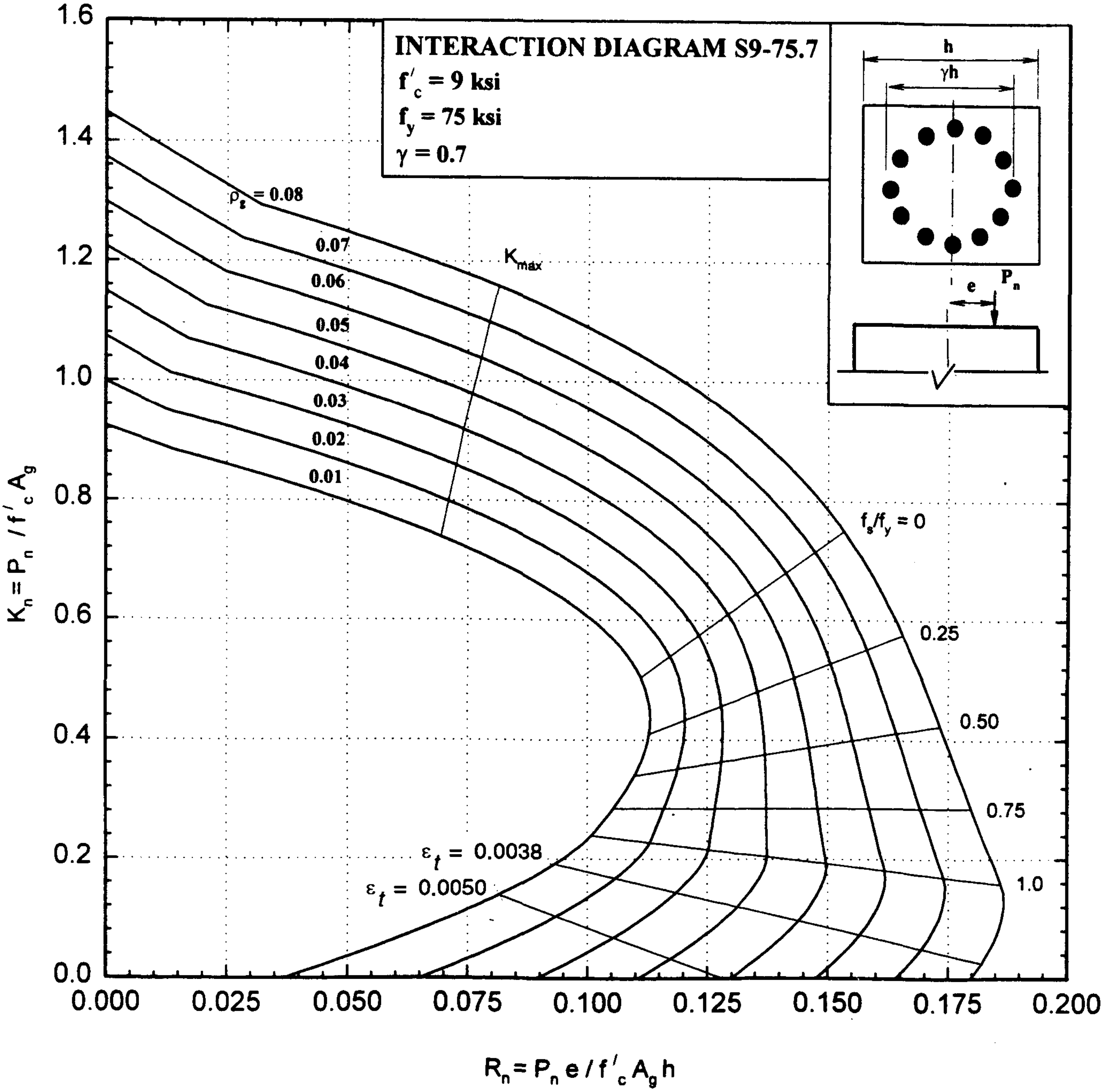


Diagram plotted using data from computer programs developed by Dr. Noel J. Everard.
Computer plot of data for diagram by Dr. Mohsen A. Issa and Alfred A. Yousif.

COLUMNS 7.23.3 - Nominal load-moment strength interaction diagram, S9-75.8

References: "Building Code Requirements for Structural Concrete-ACI 318" Chapters 9 and 10, and "Ultimate Strength Design of Reinforced Concrete Columns", ACI Special Publication SP-7, by Everard and Cohen, 1964, pp. 152-182 (with corrections).

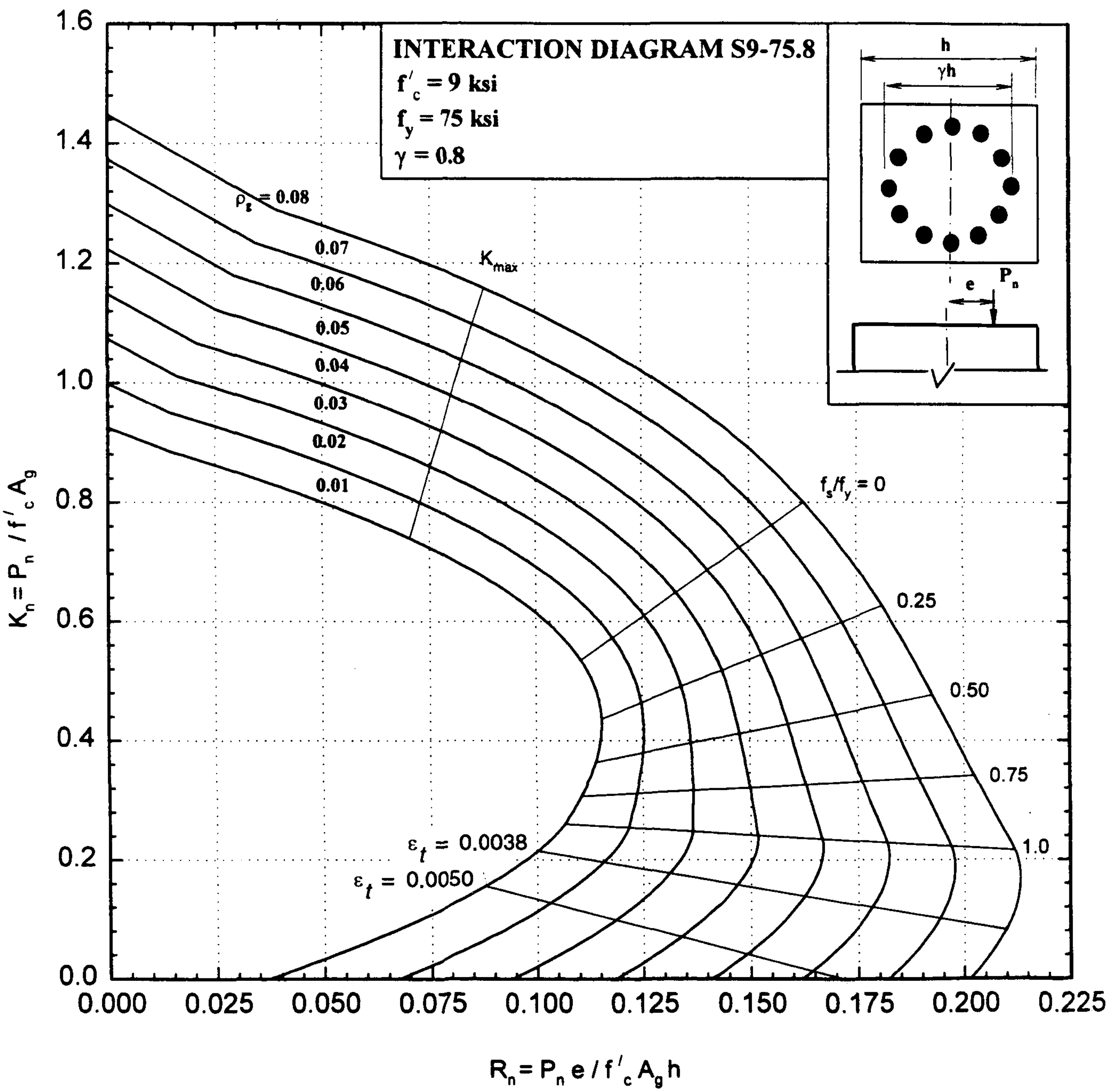


Diagram plotted using data from computer programs developed by Dr. Noel J. Everard.
Computer plot of data for diagram by Dr. Mohsen A. Issa and Alfred A. Yousif.

COLUMNS 7.23.4 - Nominal load-moment strength interaction diagram, S9-75.9

References: "Building Code Requirements for Structural Concrete-ACI 318" Chapters 9 and 10, and "Ultimate Strength Design of Reinforced Concrete Columns", ACI Special Publication SP-7, by Everard and Cohen, 1964, pp. 152-182 (with corrections).

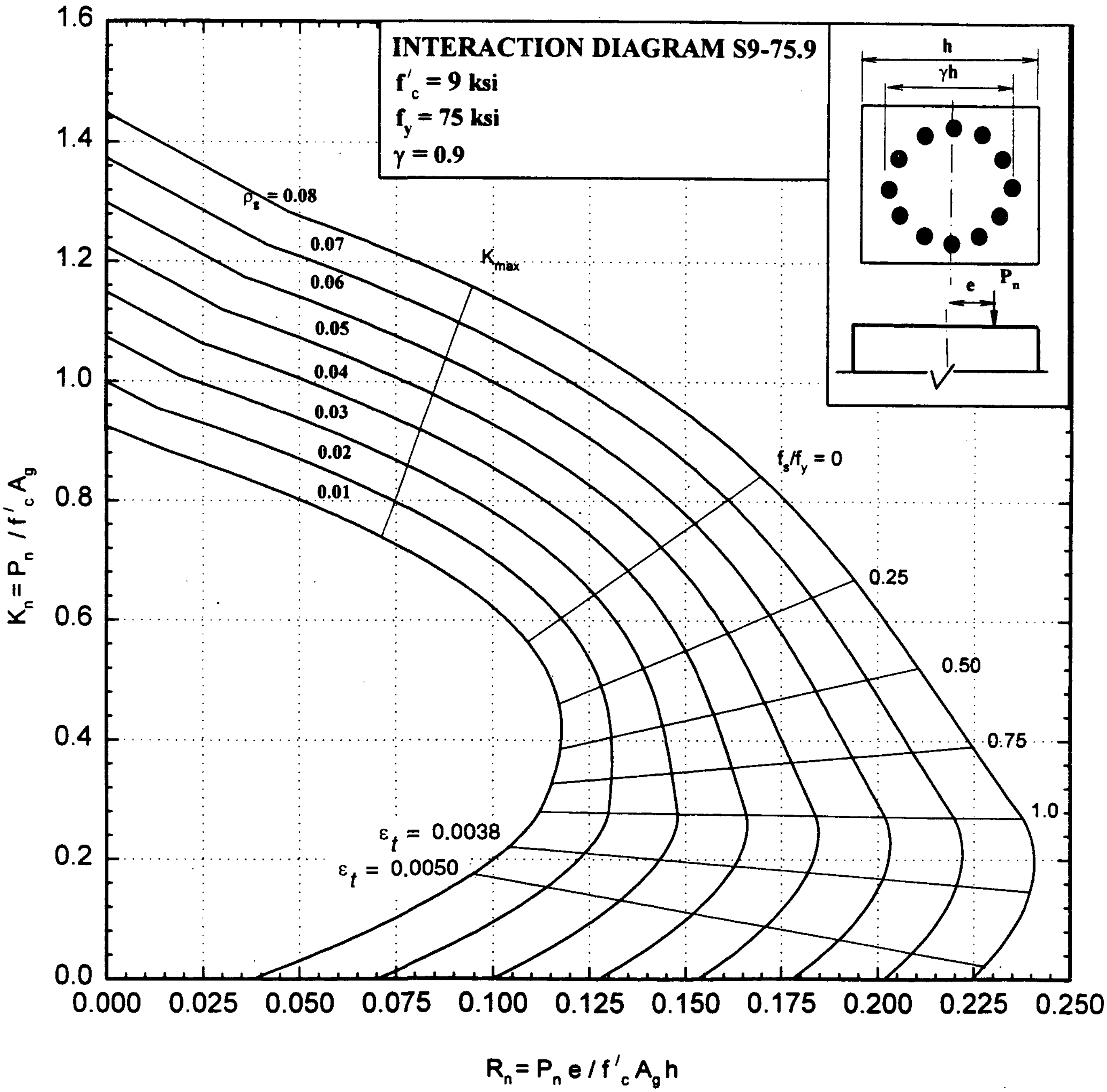


Diagram plotted using data from computer programs developed by Dr. Noel J. Everard.
Computer plot of data for diagram by Dr. Mohsen A. Issa and Alfred A. Yousif.

COLUMNS 7.24.1 - Nominal load-moment strength interaction diagram, S12-75.6

References: "Building Code Requirements for Structural Concrete-ACI 318" Chapters 9 and 10, and "Ultimate Strength Design of Reinforced Concrete Columns", ACI Special Publication SP-7, by Everard and Cohen, 1964, pp. 152-182 (with corrections).

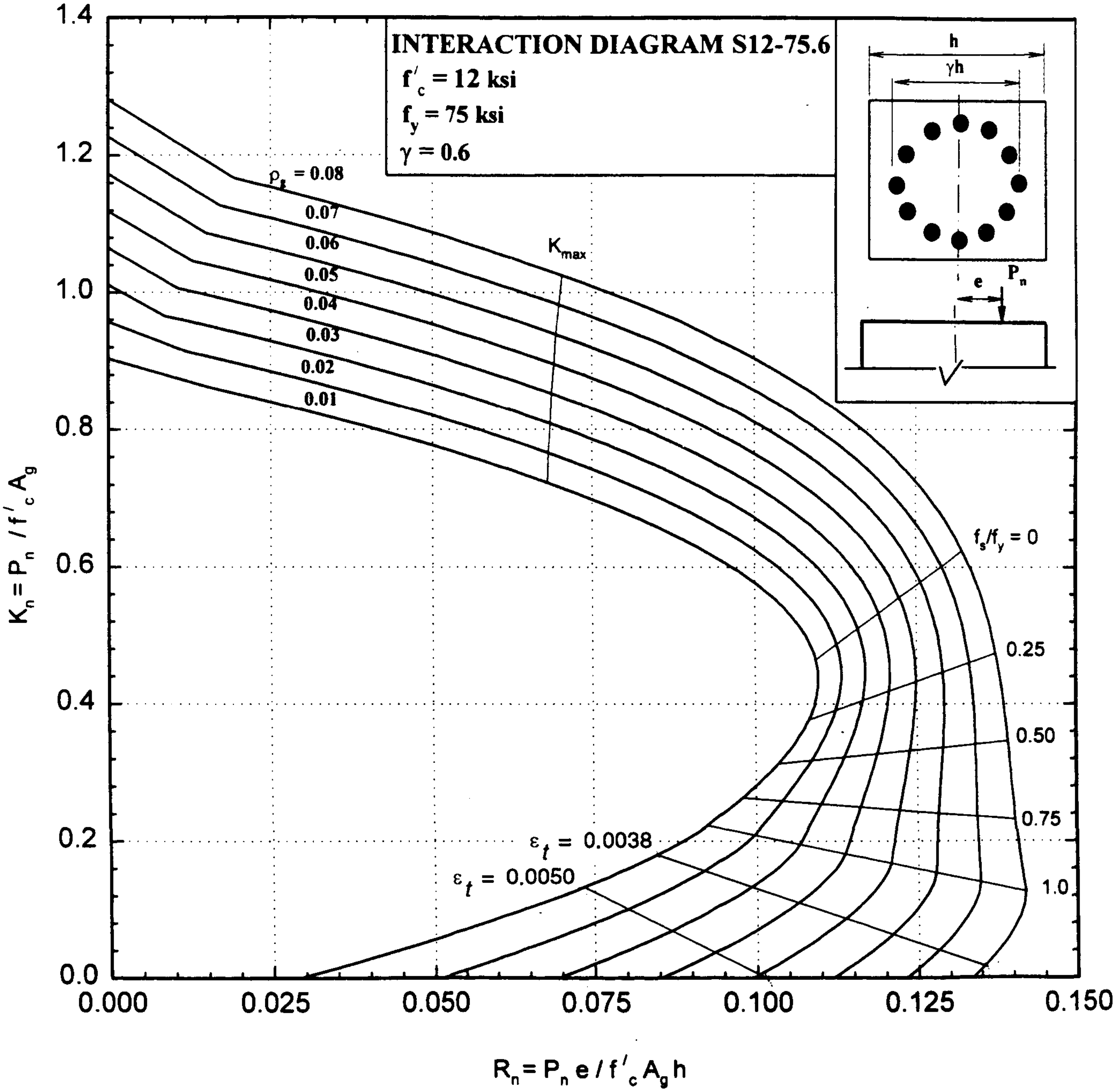


Diagram plotted using data from computer programs developed by Dr. Noel J. Everard. Computer plot of data for diagram by Dr. Mohsen A. Issa and Alfred A. Yousif.

COLUMNS 7.24.2 - Nominal load-moment strength interaction diagram, S12-75.7

References: "Building Code Requirements for Structural Concrete-ACI 318" Chapters 9 and 10, and "Ultimate Strength Design of Reinforced Concrete Columns", ACI Special Publication SP-7, by Everard and Cohen, 1964, pp. 152-182 (with corrections).

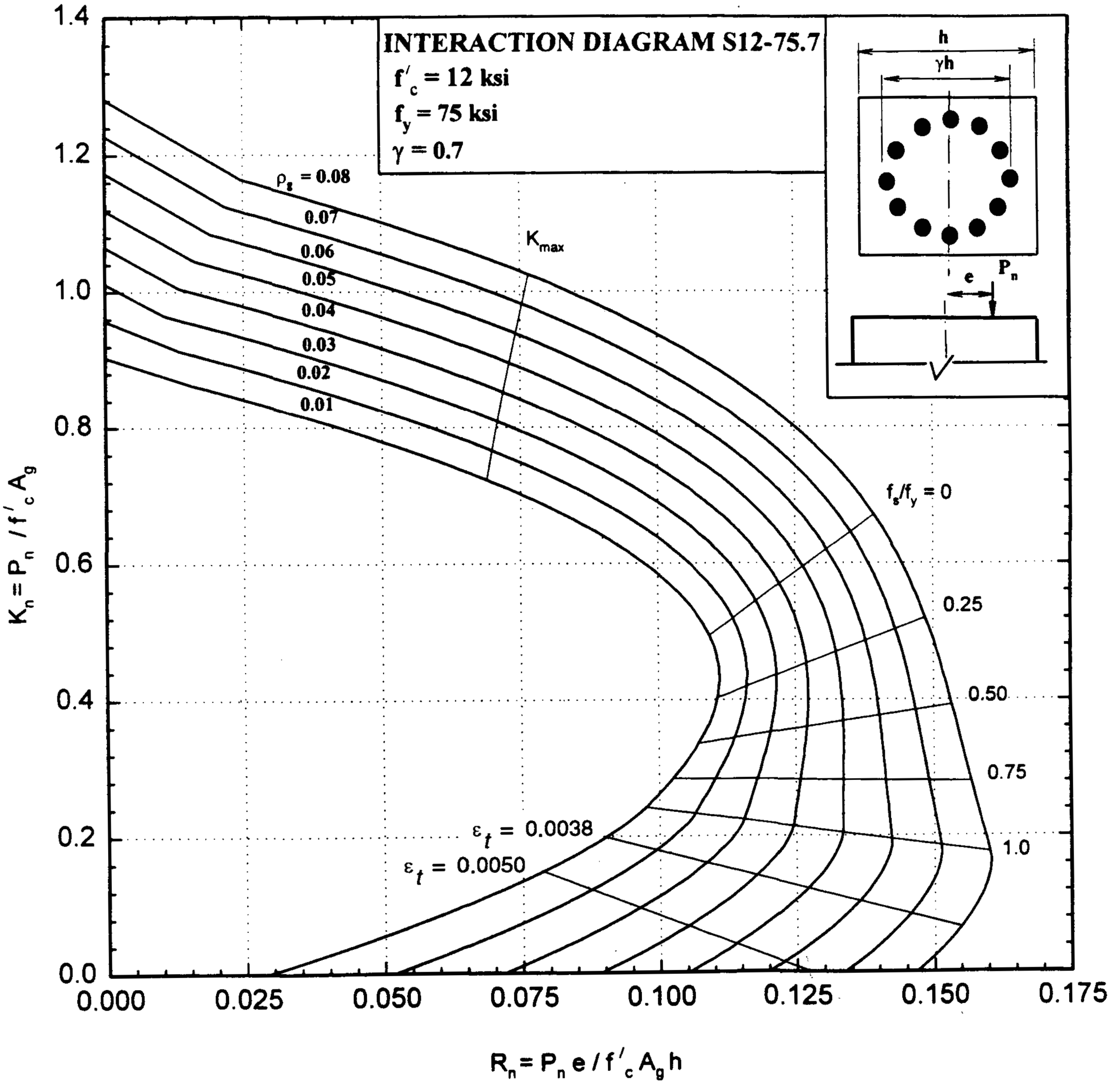


Diagram plotted using data from computer programs developed by Dr. Noel J. Everard. Computer plot of data for diagram by Dr. Mohsen A. Issa and Alfred A. Yousif.

COLUMNS 7.24.3 - Nominal load-moment strength interaction diagram, S12-75.8

References: "Building Code Requirements for Structural Concrete-ACI 318" Chapters 9 and 10, and "Ultimate Strength Design of Reinforced Concrete Columns", ACI Special Publication SP-7, by Everard and Cohen, 1964, pp. 152-182 (with corrections).

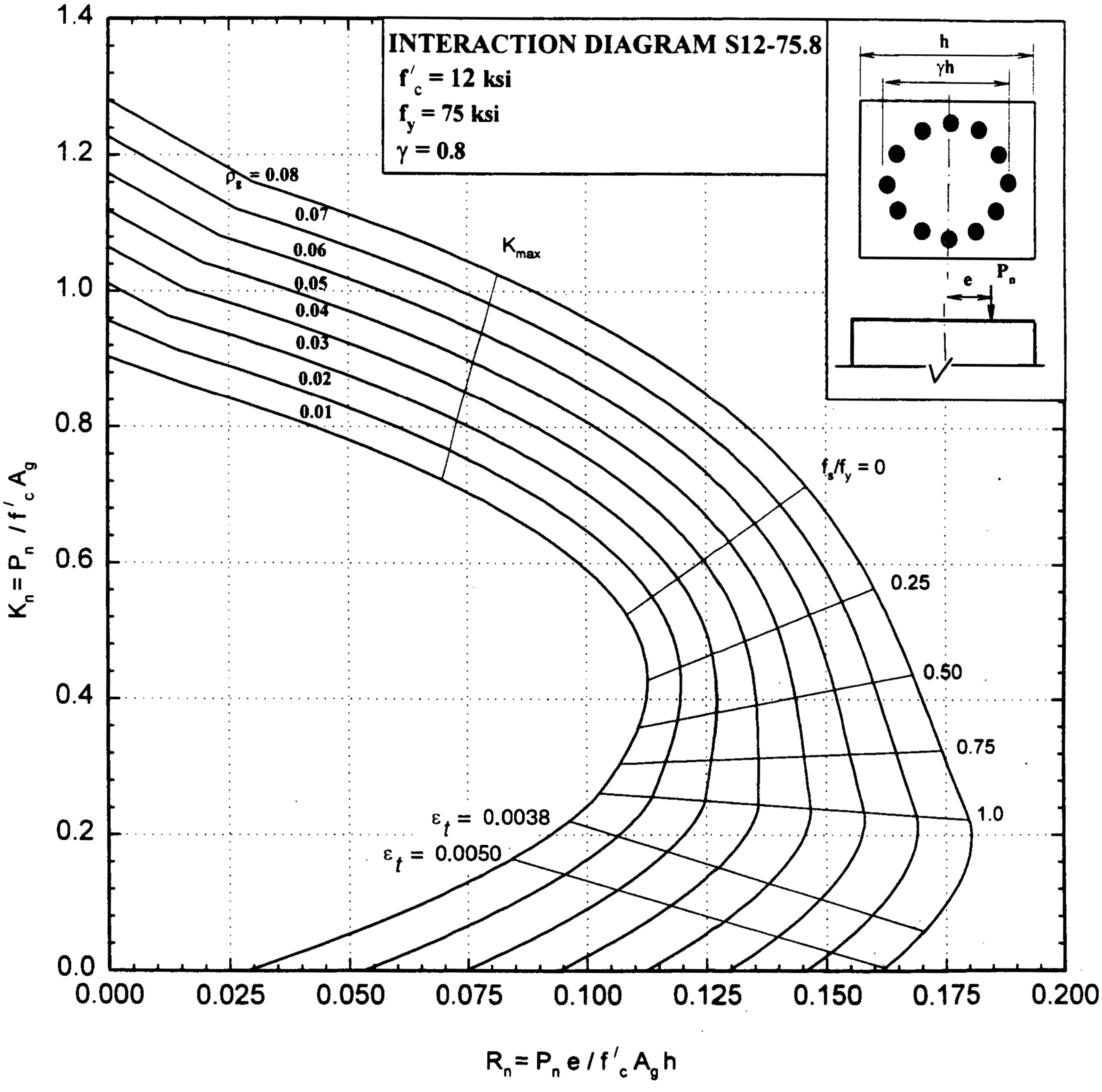


Diagram plotted using data from computer programs developed by Dr. Noel J. Everard. Computer plot of data for diagram by Dr. Mohsen A. Issa and Alfred A. Yousif.

COLUMNS 7.24.4 - Nominal load-moment strength interaction diagram, S12-75.9

References: "Building Code Requirements for Structural Concrete-ACI 318" Chapters 9 and 10, and "Ultimate Strength Design of Reinforced Concrete Columns", ACI Special Publication SP-7, by Everard and Cohen, 1964, pp. 152-182 (with corrections).

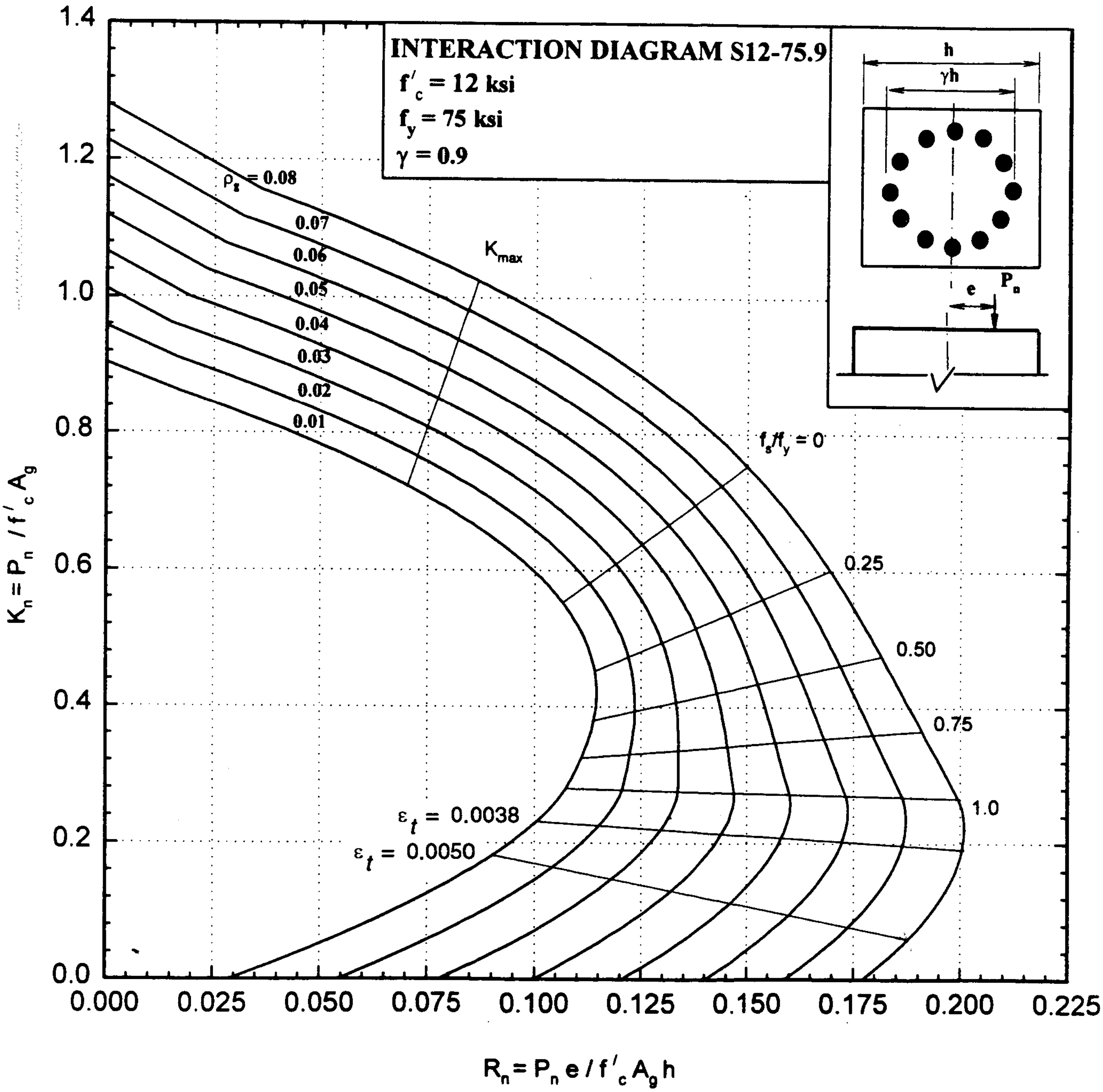
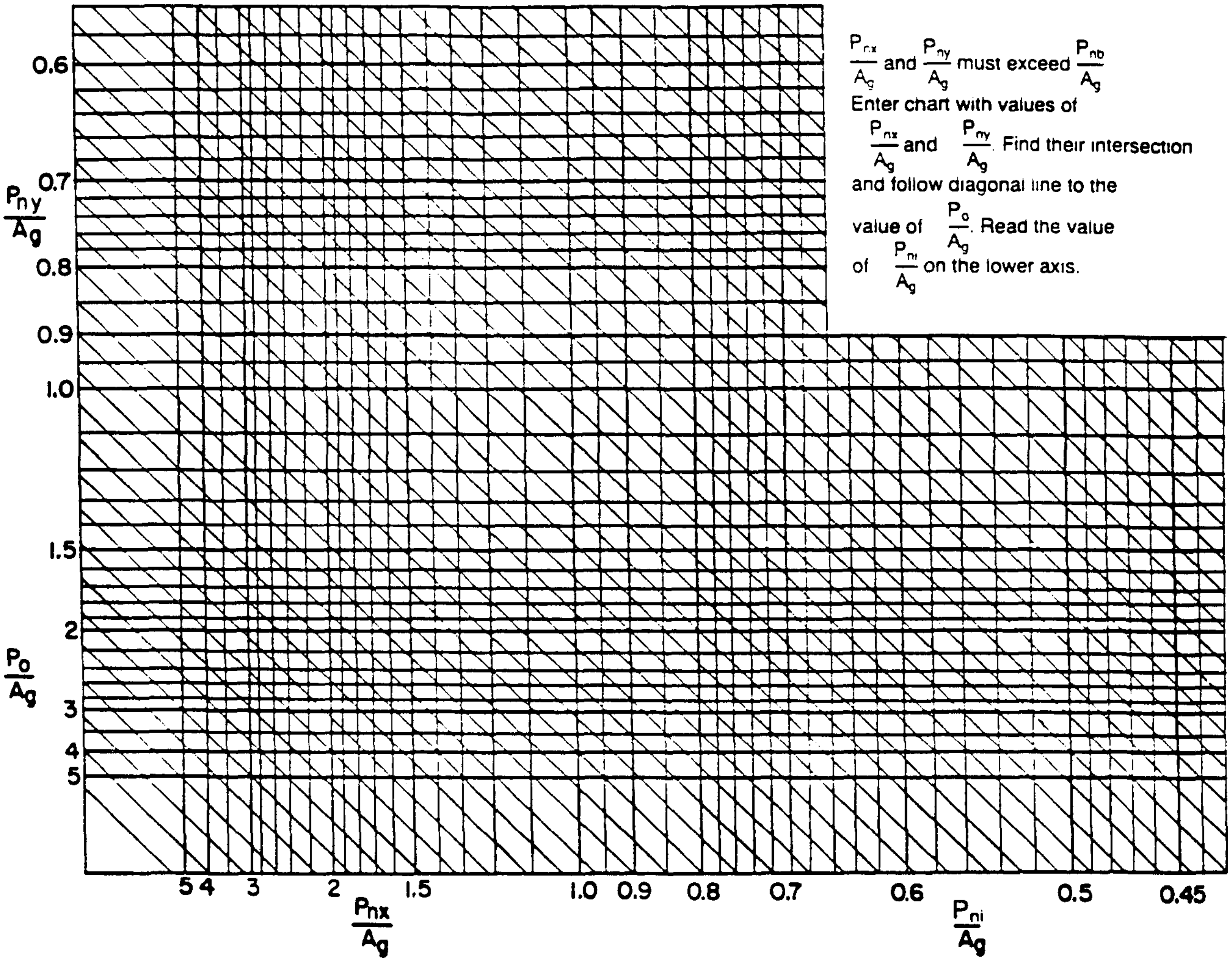


Diagram plotted using data from computer programs developed by Dr. Noel J. Everard.
Computer plot of data for diagram by Dr. Mohsen A. Issa and Alfred A. Yousif.

COLUMNS 8—Solution to reciprocal load equation for biaxial bending—P_{ni}/A_g as a function of P_{nx}/A_g, P_{ny}/A_g, and P_o/A_g

Reference: Bresler, Boris, "Design Criteria for Reinforced Columns under Axial Load and Biaxial Bending," ACI JOURNAL, *Proceedings* V. 57, No. 11, Nov., 1960, pp. 481–490

$$\frac{A_g}{P_{ni}} = \frac{A_g}{P_{nx}} + \frac{A_g}{P_{ny}} - \frac{A_g}{P_o}$$

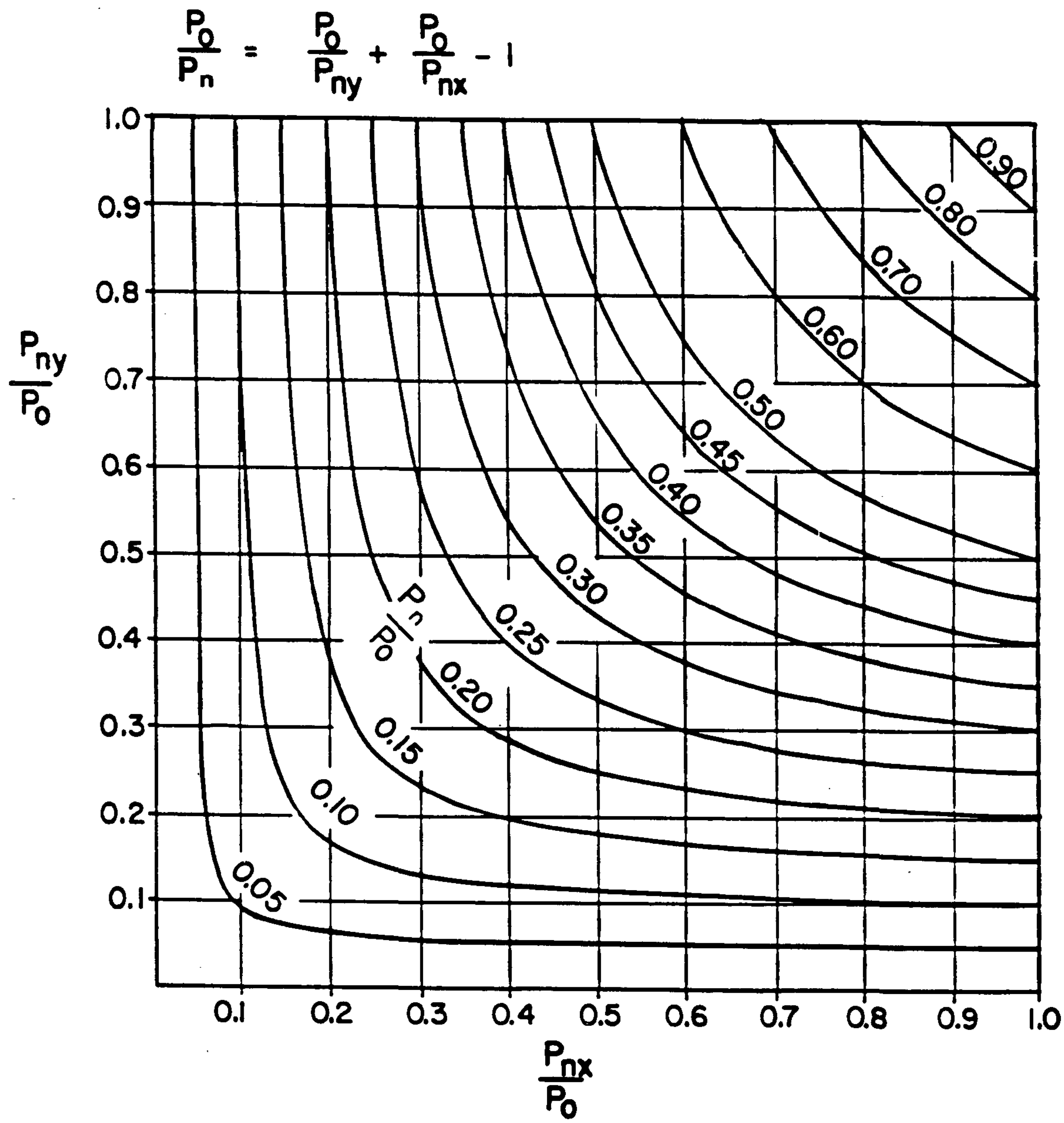


$\frac{P_{nx}}{A_g}$ and $\frac{P_{ny}}{A_g}$ must exceed $\frac{P_{nb}}{A_g}$

Enter chart with values of $\frac{P_{nx}}{A_g}$ and $\frac{P_{ny}}{A_g}$. Find their intersection and follow diagonal line to the value of $\frac{P_o}{A_g}$. Read the value of $\frac{P_{ni}}{A_g}$ on the lower axis.

COLUMNS 9 —Solution to reciprocal load equation for biaxial bending—P_n/P_o as a function of P_{nx}/P_o and P_{ny}/P_o

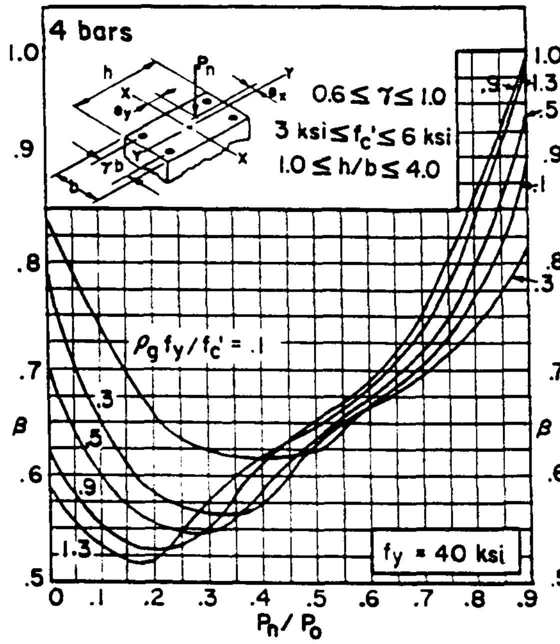
Reference: Bresler, Boris. "Design Criteria for Reinforced Columns under Axial Load and Biaxial Bending," ACI JOURNAL, *Proceedings* V. 57, No. 11, Nov., 1960, pp. 481–490



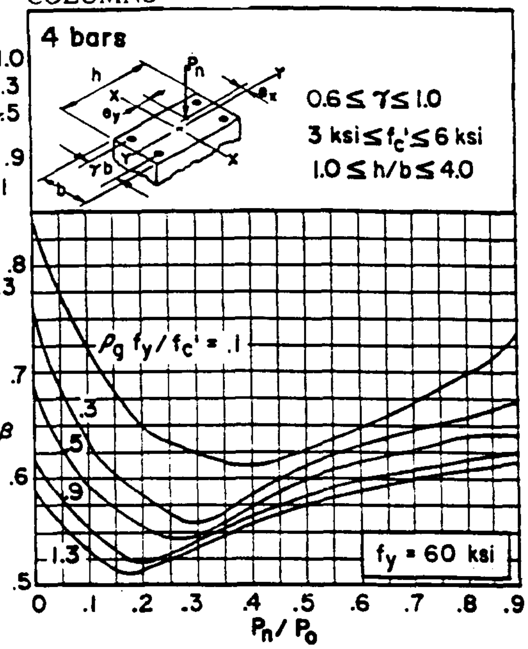
COLUMNS 10.1 —Biaxial bending design constant* β—For rectangular columns with two bars on each of four faces

Reference: Parme, Alfred, L.; Nieves, Jose M.; and Gouwens, Albert. "Capacity of Reinforced Rectangular Columns Subject to Biaxial Bending," ACI JOURNAL, *Proceedings* V. 63, No. 9, Sept. 1966, pp. 911–923

COLUMNS 10.1.1



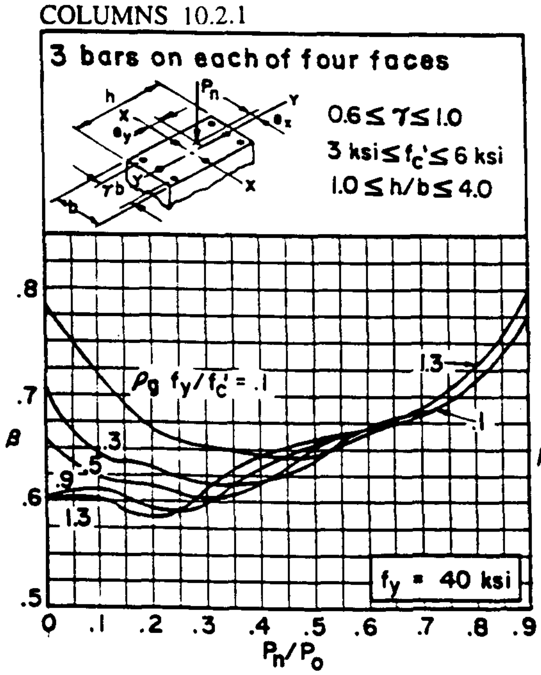
COLUMNS 10.1.2



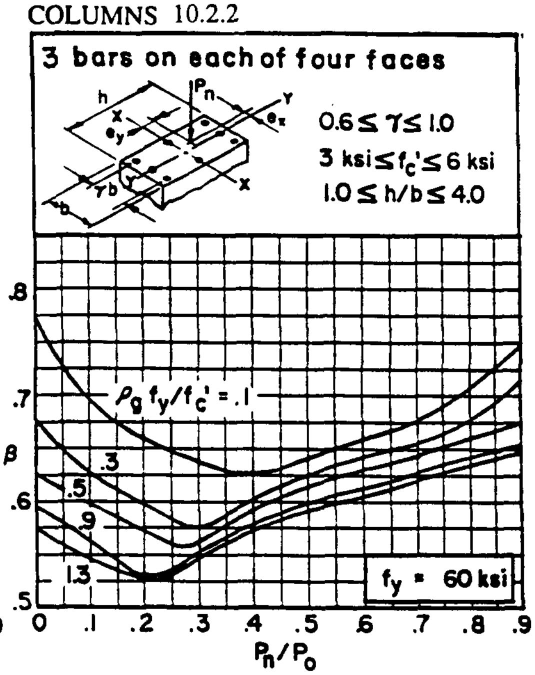
COLUMNS 10.2 —Biaxial bending design constant* β—For rectangular columns with three bars on each of four faces

Reference: same as for COLUMNS 10.1

COLUMNS 10.2.1



COLUMNS 10.2.2

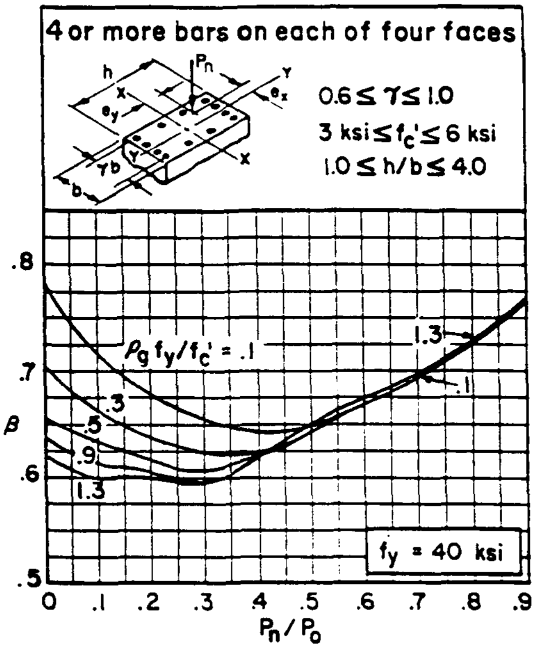


*β = constant portion of uniaxial factored moment strengths M_{nox} and M_{noy} which may be permitted to act simultaneously on the column cross section; value of β depends on P_n / P_o and properties of column material and cross section

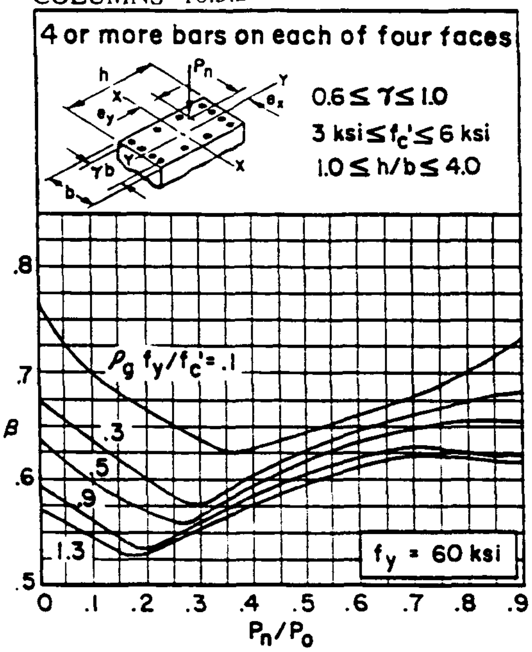
COLUMNS 10.3—Biaxial bending design constant* β—For rectangular columns with four or more bars on each of four faces

Reference: same as for COLUMNS 10.1

COLUMNS 10.3.1



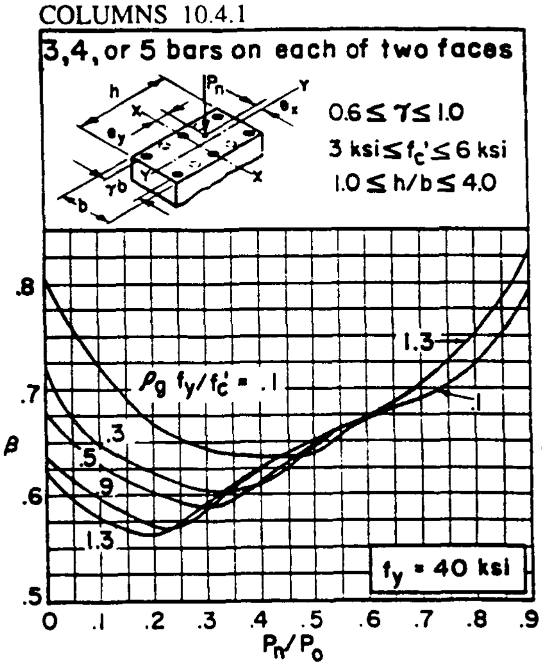
COLUMNS 10.3.2



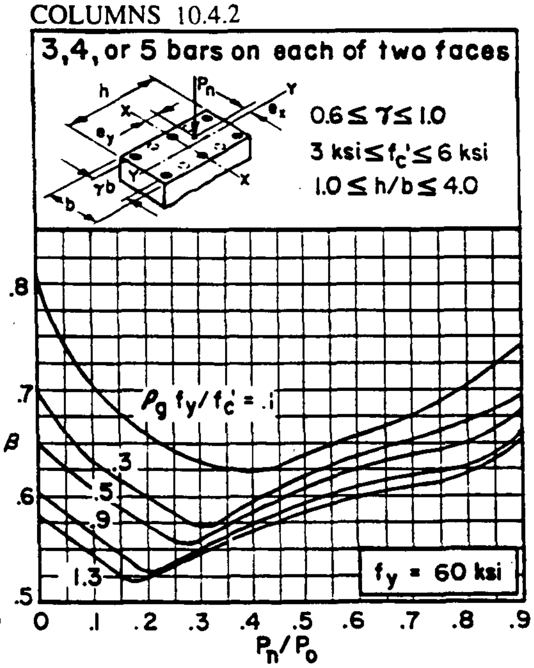
COLUMNS 10.4—Biaxial bending design constant* β—For rectangular columns with three, four, or five bars on each of two opposite faces

Reference: same as for COLUMNS 10.1

COLUMNS 10.4.1



COLUMNS 10.4.2

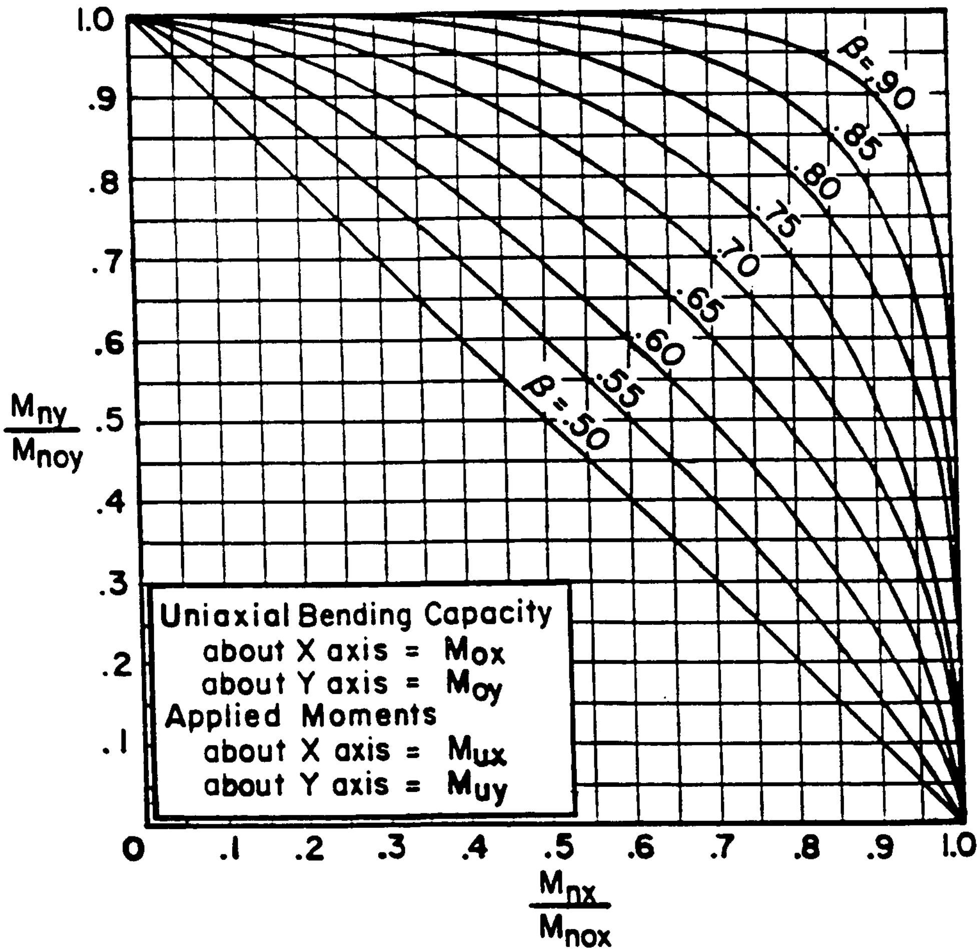


*β = constant portion of uniaxial factored moment strengths M_{nox} and M_{noy} which may be permitted to act simultaneously on the column cross section: value of β depends on P_n / P_o and properties of column material and cross section

COLUMNS 11—Biaxial moment relationship

Reference: Parme, Alfred L.; Neives, Jose M.; and Gouwens, Albert. "Capacity of Reinforced Rectangular Columns Subject to Biaxial Bending," ACI JOURNAL, *Proceedings* V. 63, No. 9, Sept., 1966, pp. 911–923

β = constant portion of uniaxial factored moment strengths M_{nox} and M_{noy} which may be permitted to act simultaneously on the column cross section; value of β depends on P_n / P_o and properties of column material and cross section



TWO-WAY SLABS

SLABS 1.1 - Minimum thickness* of slabs without interior beams

Reference: ACI 318-95 Section 9.5.3.2 and its Table 9.5 (c)

Yield stress f_y, psi Note (1)	Without drop panels Note (2)			With drop panels Note (2)		
	Exterior panels		Interior panels	Exterior panels		Interior panels
	Without edge beams	With edge beams Note (3)		Without edge beams	With edge beams Note (3)	
40,000	$\ell_n/33$	$\ell_n/36$	$\ell_n/36$	$\ell_n/36$	$\ell_n/40$	$\ell_n/40$
60,000	$\ell_n/30$	$\ell_n/33$	$\ell_n/33$	$\ell_n/33$	$\ell_n/36$	$\ell_n/36$
75,000	$\ell_n/28$	$\ell_n/31$	$\ell_n/31$	$\ell_n/31$	$\ell_n/34$	$\ell_n/34$

*Minimum thickness determined from this table shall not be taken less than 5 in. for slabs without drop panels nor 4 in. for slabs with drop panels.

(1) For values of reinforcement yield stress between 40,000, 60,000, and 75,000 psi, minimum thickness shall be obtained by linear interpolation.
(2) Drop panel is defined in articles 13.4.7.1 and 13.4.7.2.
(3) Slabs with beams between columns along exterior edges. The value of α for the edge beam shall not be less than 0.8.

SLABS 1.2.1 - Minimum Slab thickness for deflection of slabs on beams, drop panels or bands.

Reference: ACI 318-95 Section 9.5.3.3.

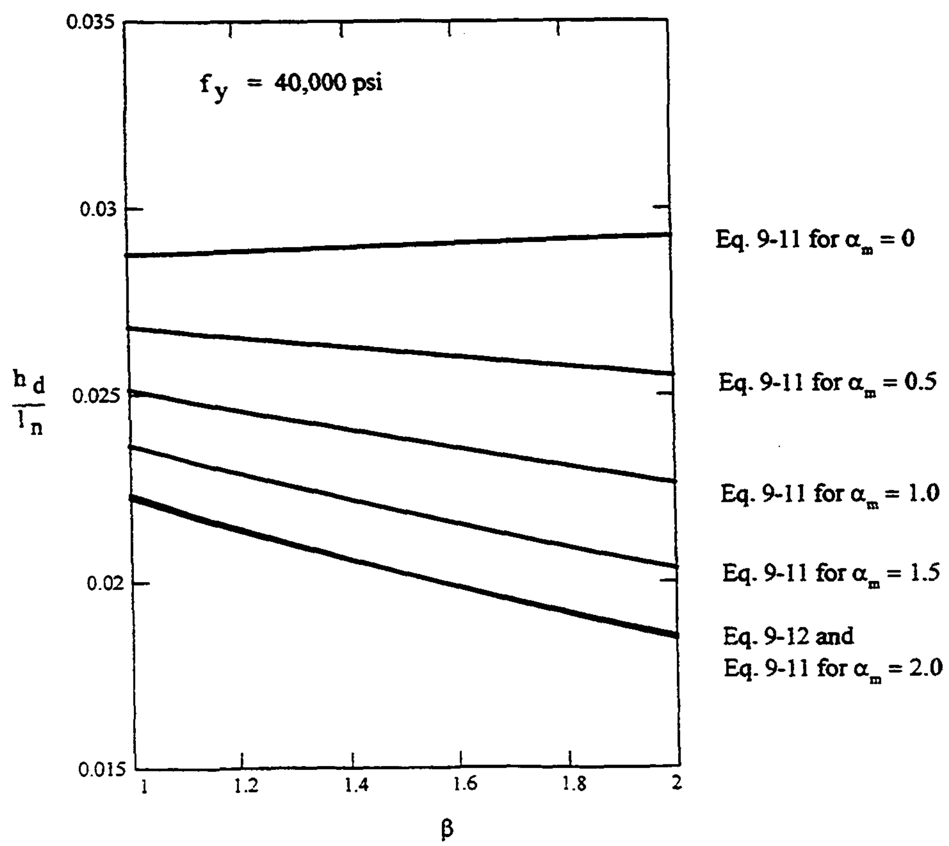
$f_y = 40{,}000$ psi

$$\frac{h_d}{l_n} = \frac{0.8 + \dfrac{f_y}{200000}}{36 + 5\cdot\beta\cdot\left[\alpha_m - 0.12\cdot\left(1 + \dfrac{1}{\beta}\right)\right]} \quad \text{for Eq. (9-11)}$$

$$= \frac{0.8 + \dfrac{f_y}{200000}}{36 + 9\cdot\beta} \quad \text{for Eq. (9-12)}$$

in which $\beta = \dfrac{l_{n(long\_direction)}}{l_{n(short\_direction)}}$

and $\alpha_m =$ average value of α (where α = ratio of flexural stiffness of beam section to flexural stiffness of a width of slab bounded laterally by centerline of adjacent panel, if any, an each side of beam) for all beams on edges of a panel.



For use of this Design Aid, see Slabs Example 1, Step 2 and Slabs Example 2, Step 2.

SLABS 1.2.2 - Minimum Slab thickness for deflection of slabs on beams, drop panels or bands.

Reference: ACI 318-95 Section 9.5.3.3.

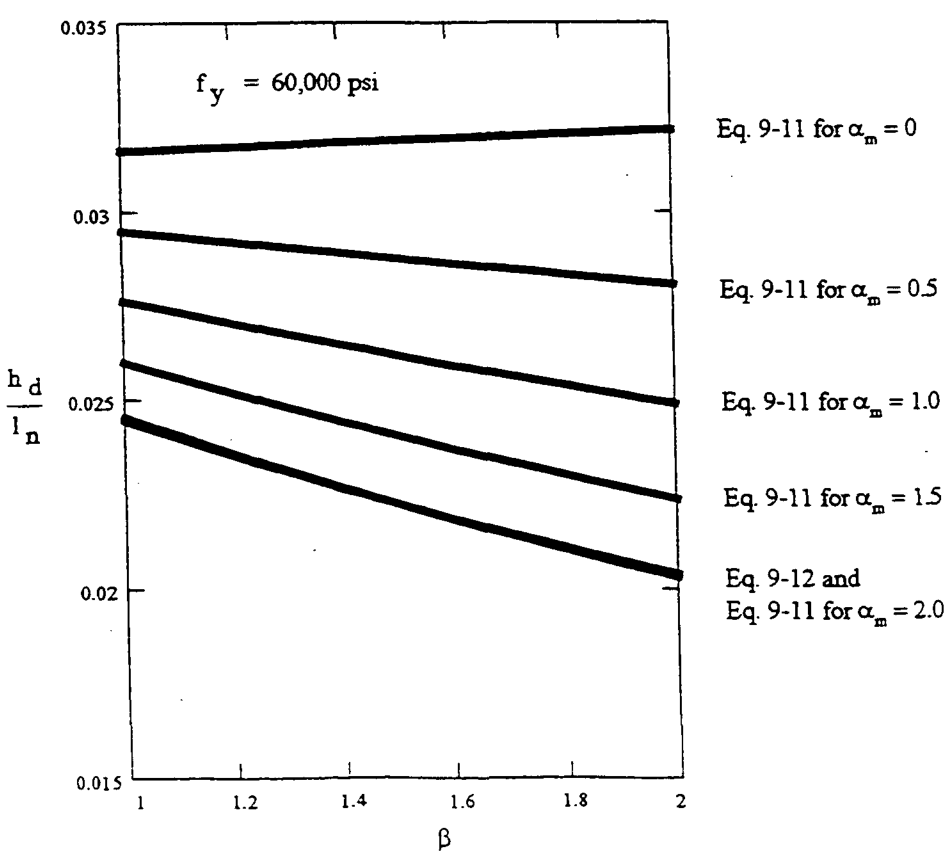
$$f_y = 60{,}000 \text{ psi}$$

$$\frac{h_d}{l_n} = \frac{0.8 + \frac{f_y}{200000}}{36 + 5 \cdot \beta \cdot \left[\alpha_m - 0.12 \cdot \left(1 + \frac{1}{\beta}\right)\right]} \qquad \text{for Eq. (9-11)}$$

$$= \frac{0.8 + \frac{f_y}{200000}}{36 + 9 \cdot \beta} \qquad \text{for Eq. (9-12)}$$

in which $\beta = \frac{l_{n(long\_direction)}}{l_{n(short\_direction)}}$

and α_m = average value of α (where α = ratio of flexural stiffness of beam section to flexural stiffness of a width of slab bounded laterally by centerline of adjacent panel, if any, an each side of beam) for all beams on edges of a panel.

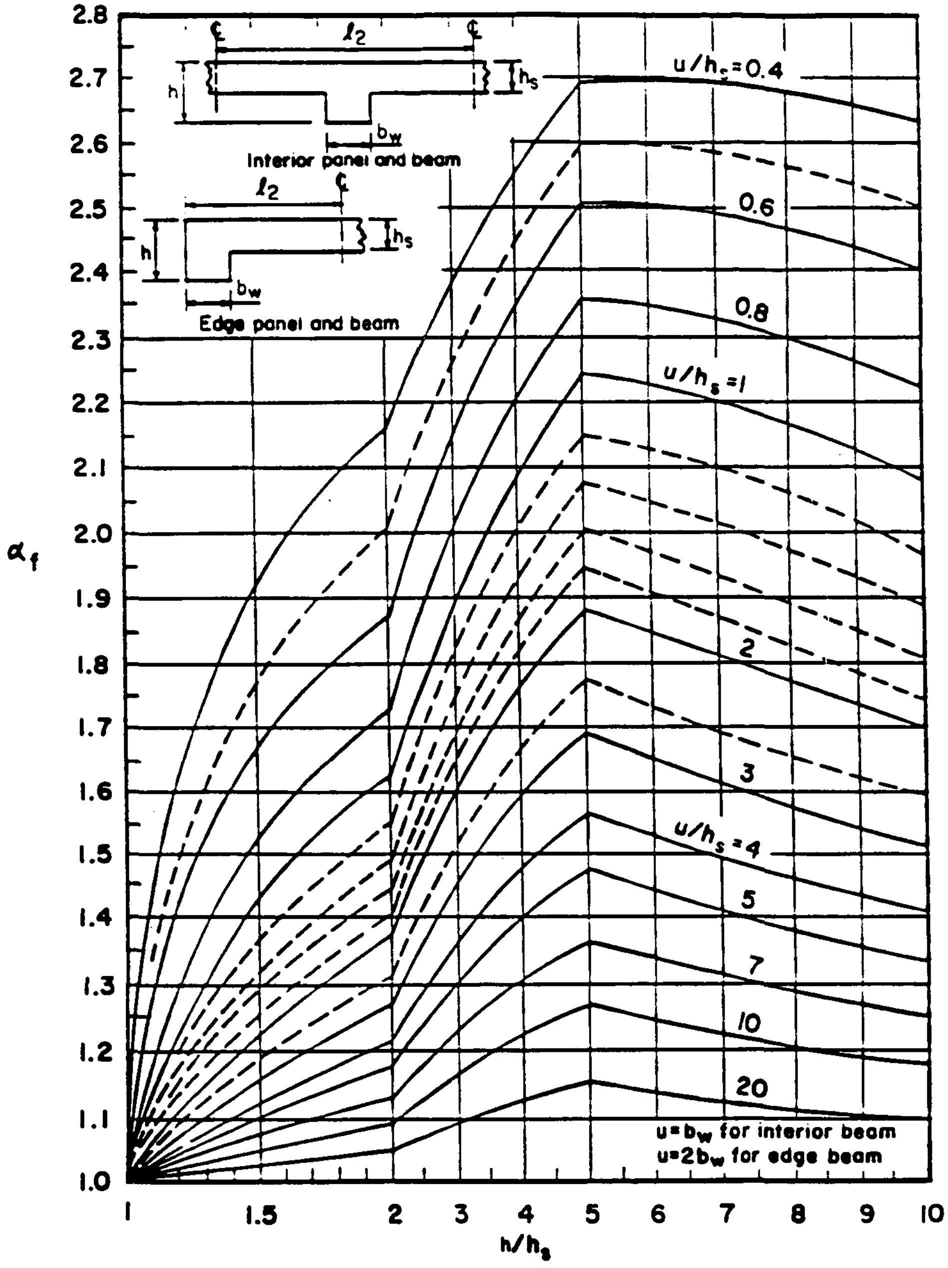


For use of this Design Aid, see Slabs Example 1, Step 2 and Slabs Example 2, Step 2.

SLABS 2—Factor α_f for calculating α

Reference: ACI 318-95 Sections 13.0 (definition of α) and 13.2.4

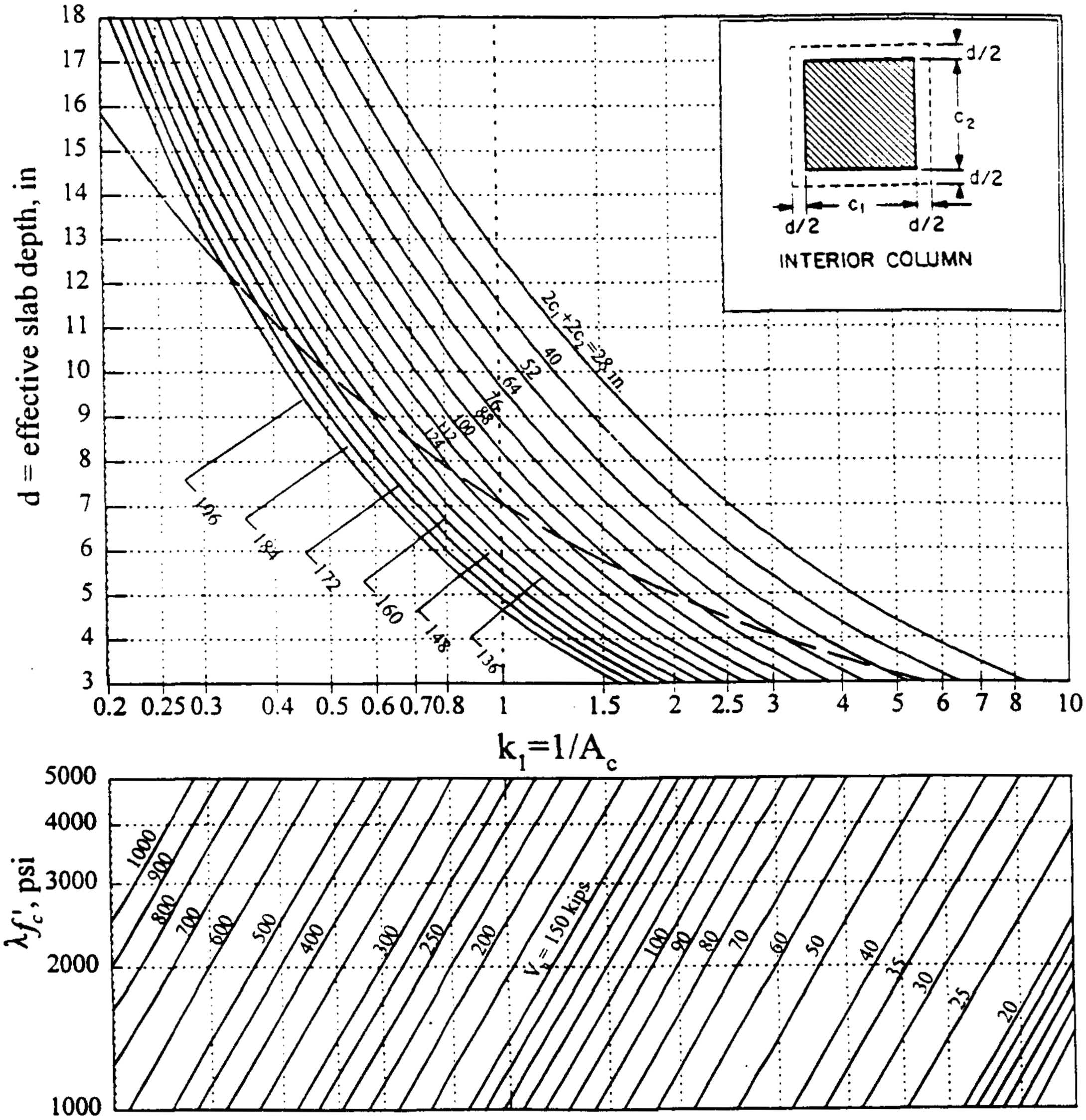
$$\alpha = \frac{E_{cb} I_b}{E_{cs} I_s} = \frac{E_{cb}}{E_{cs}} \frac{b_w}{l_2} \left[\frac{h}{h_s} \right]^3 \alpha_f$$



Note: Abrupt change in slope of curve at $h/h_s = 5$ is due to width limit of T-beam flange defined in ACI 318-95 Section 13.2.4.

SLABS 3.1 - Factor k_l for perimeter shear - Interior column

Reference: ACI 318-95 Sections 11.12.1.2 and 11.12.2



$\lambda f_c' = 1.8^2 f_{ct}^2$ for lightweight concrete ($\lambda \leq 1$)

$\lambda = 1$ for normal weight concrete

Perimeter shear stress $v_n = k_l V_n$

NOTE: 1. The broken line of the upper portion of this Design Aid indicates the limit of curves where Eq. (11.37) governs.

2.The upper portion of this Design Aid is used to obtain the factor k_l for use in the final evaluation of shear-moment capacity of the slab-column connection at the interior columns.

The curves in the upper part of this Design Aid can also ve used to obtain a trial value of effective slab depth d when only factored perimeter shear force v_u and the effective column perimeter ($2c_l + 2c_2$) are known.

The curves are based on Eq. (11.38). For the purpose of obtaining a trial value of effective depth, it may be necessary to modify k_l if Eq. (11.36) or Eq. (11.37) governs.

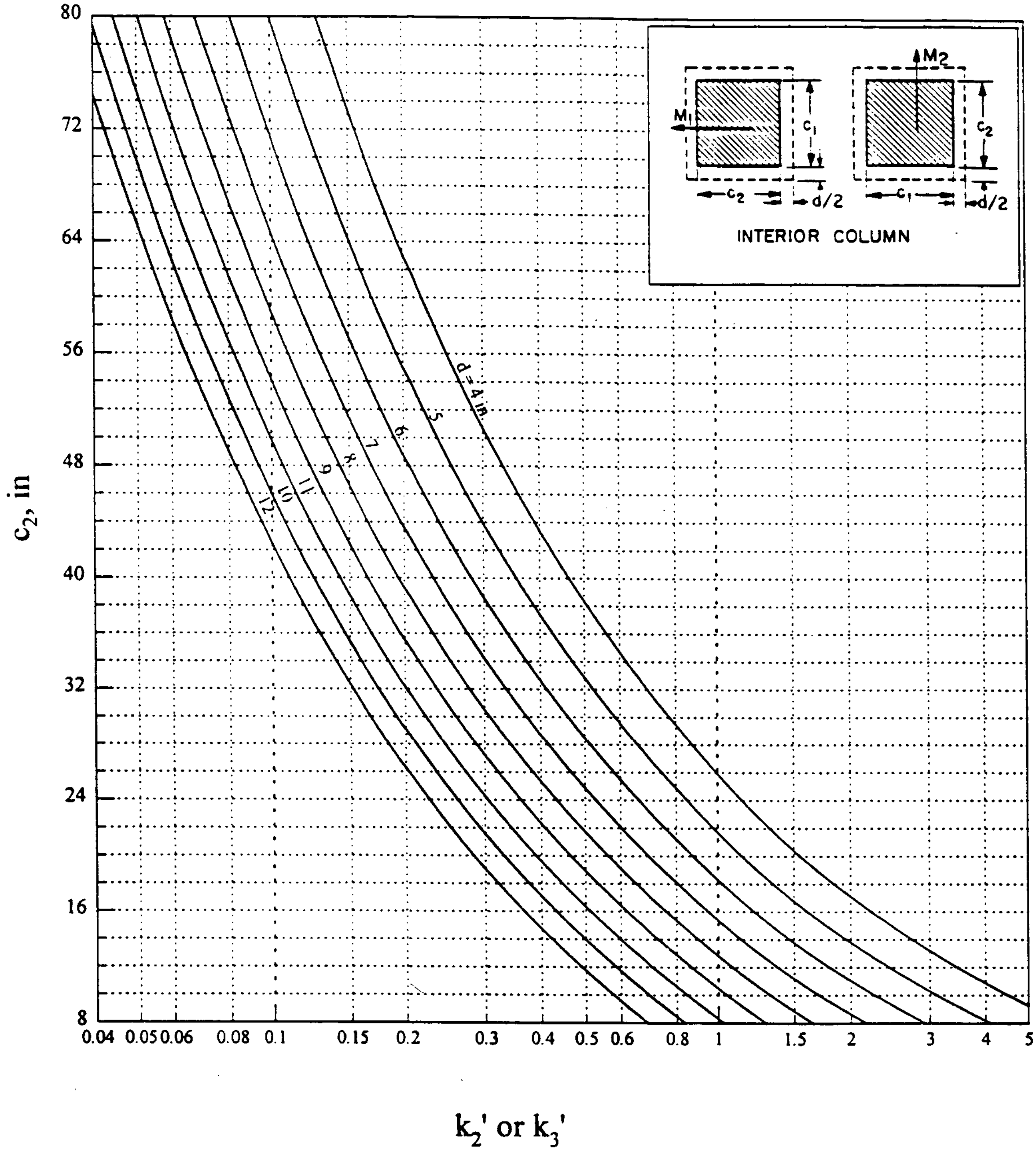
Eq. (11.38) governs if $\beta_c \leq 2$ and $b_0 / d \leq 20$. When one or both of these conditions are not satisfied, enter the graph with $\lambda f_c'$, go across to v_u, and up up to read k_l. Then,

- if $\beta_c > 2$, multiply k_l by $(0.5 + 1 / \beta_c)$
- if $b_0 / d > 20$, multiply k_l by $(10b_0 / d + 0.5)$
- if $\beta_c > 2$ and $b_0 / d \leq 20$, determine both modified values of k_l.

Enter the graph with modified value of k_l, proceed up to the effective column perimeter, and go across to read a trial value of effective slab depth.

SLABS 3.2 - Factor k_2' and k_3' for perimeter shear - Square interior column

Reference: ACI 318-95 /ACI 318R-95 Section 11.12.2

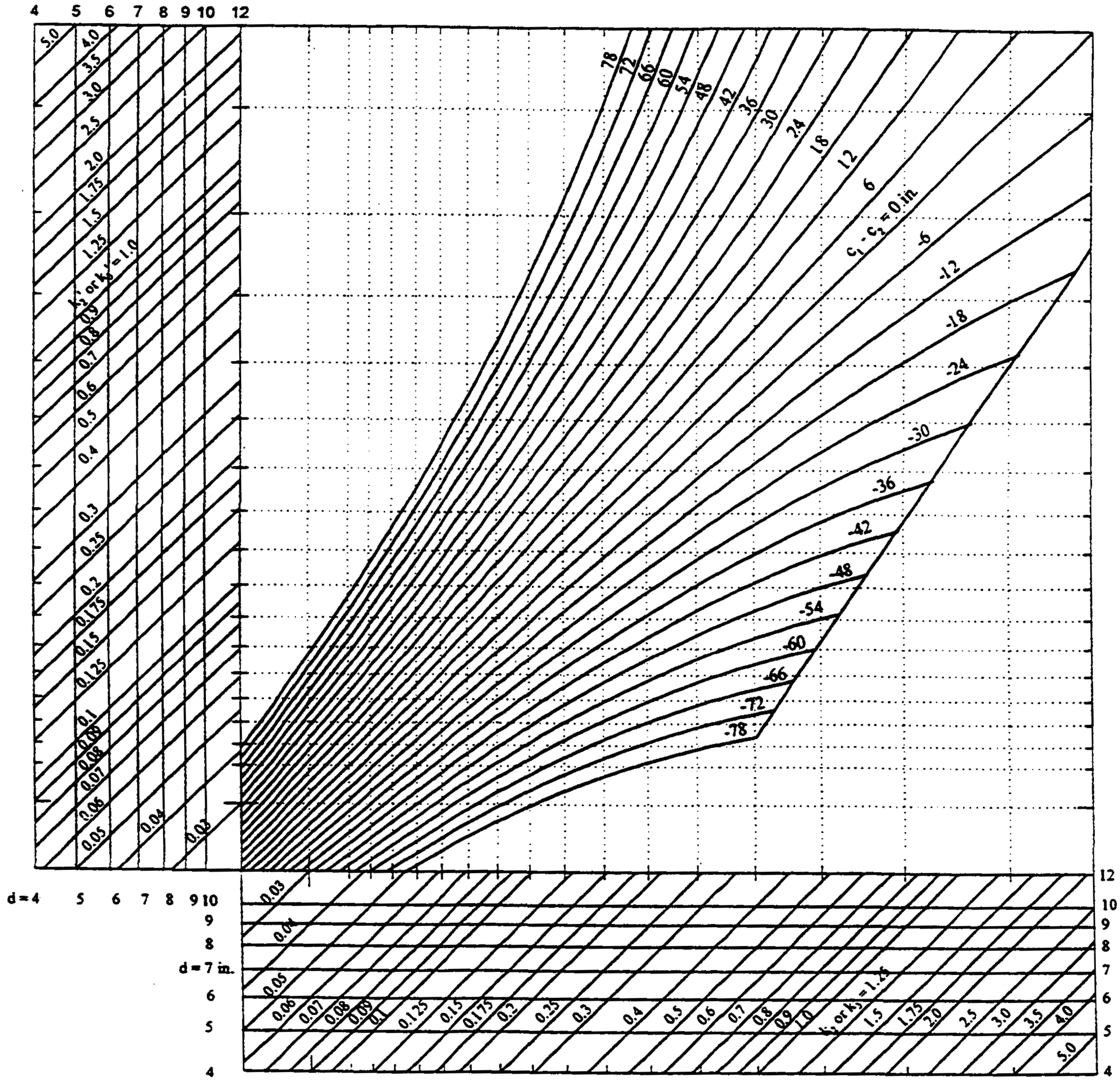


When $c_1 = c_2$, $k_2' = k_2$, $k_3' = k_3$, and shear stress due to moment-shear transfer = $k_2'M_1 + k_3'M_2$. (where shear stress is in psi and M_1 and M_2 are in ft-kips)

When $c_1 \neq c_2$, use k_2' and k_3' to find k_2 and k_3 from slabs 3.3; shear stress due to moment transfer = $k_2M_1 + k_3M_2$.

SLABS 3.3 - Factors k_2 and k_3 (corrected from k_2' and k_3') for perimeter shear - Non-square rectangular column

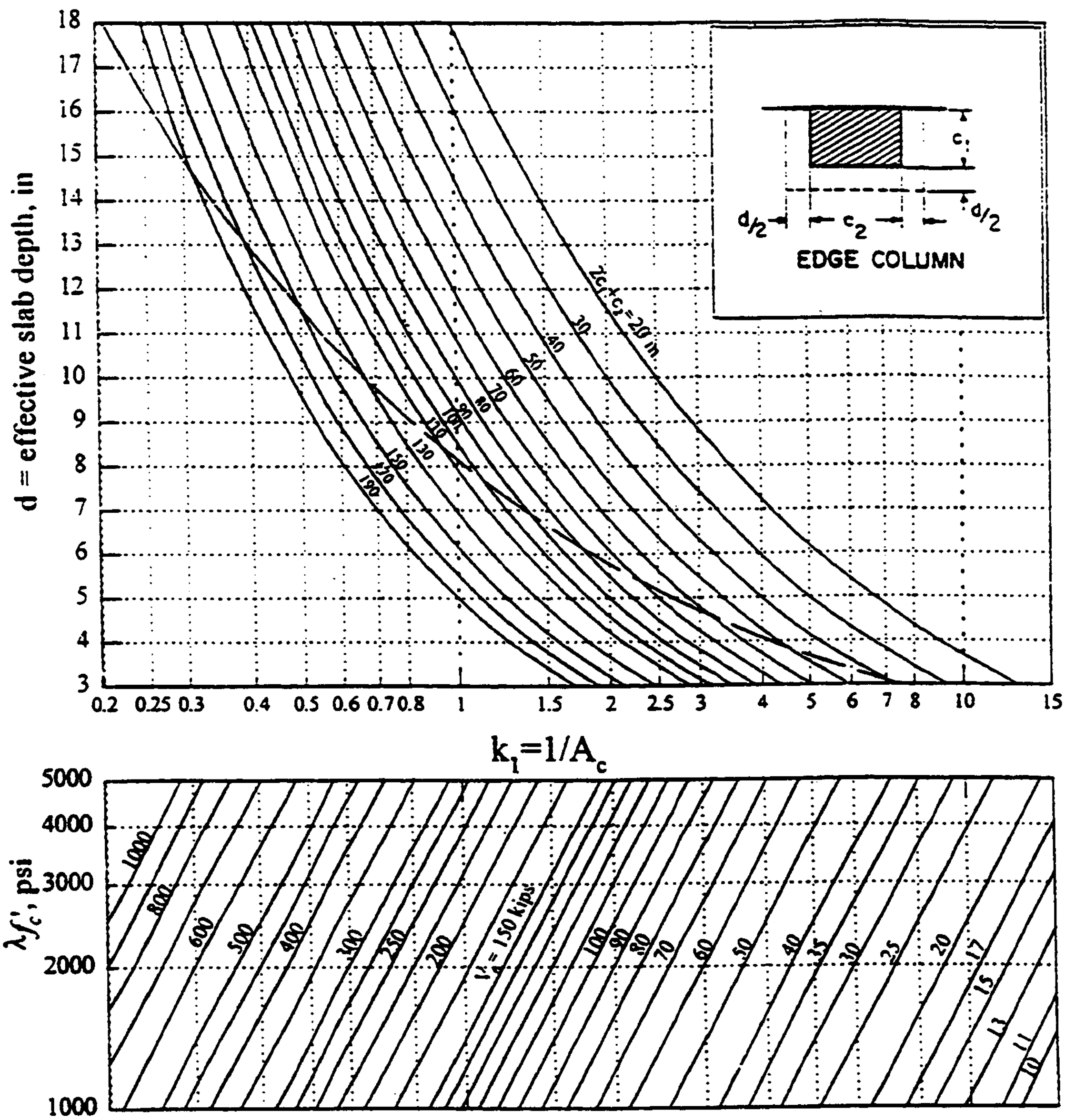
Reference: ACI 318-95/ACI 318R-95 Section 11.12.2



Shear stress due to moment-shear transfer = $k_2 M_1 + k_3 M_2$ (where shear stress is in psi and M_1 and M_2 are in ft-kips)

SLABS 3.4—Factor k_1 for perimeter shear—Edge column

Reference: ACI 318-95 Sections 11.12.1.2 and 11.12.2



$\lambda f_c' = 1.8^2 f_{ct}^2$ for lightweight concrete ($\lambda \le 1$)

$\lambda = 1$ for normal weight concrete

Perimeter shear stress $v_n = k_l V_n$

NOTE: 1. The broken line of the upper portion of this Design Aid indicates the limit of curves where Eq. (11.37) governs.

2. The upper portion of this Design Aid is used to obtain the factor k_l for use in the final evaluation of shear-moment capacity of the slab-column connection at the edge columns.

The curves in the upper part of this Design Aid can also be used to obtain a trial value of effective slab depth d when only factored perimeter shear force v_n and the effective column perimeter $(2c_1 + c_2)$ are known.

The curves are based on Eq. (11.38). For the purpose of obtaining a trial value of effective depth, it may be necessary to modify k_l if Eq. (11.36) or Eq. (11.37) governs.

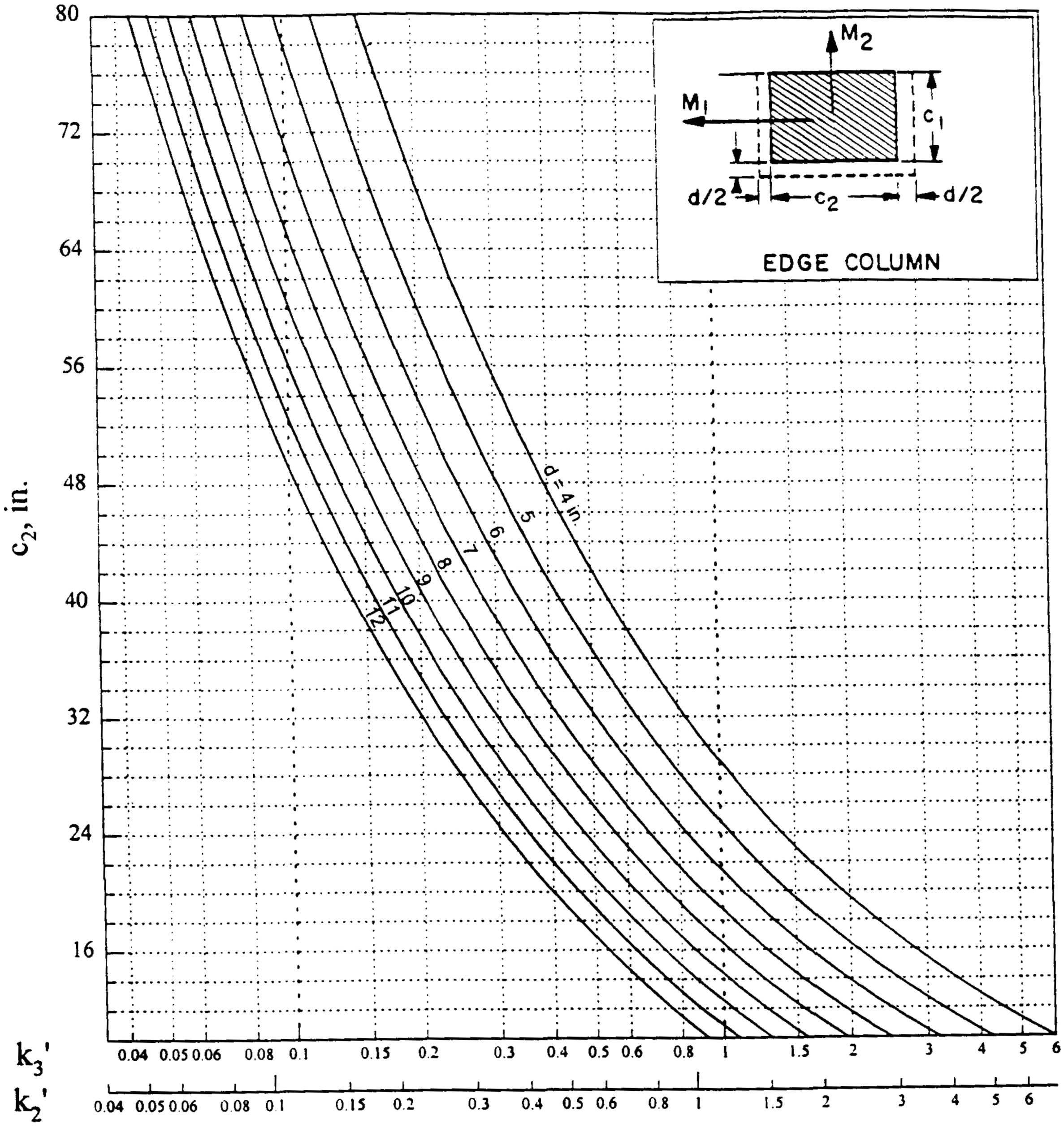
Eq. (11.38) governs if $\beta_c \le 2$ and $b_o / d \le 15$. When one or both of these conditions are not satisfied, enter the graph with $\lambda f_c'$, go across to vnu, and up to read k_l, Then

- if $\beta_c > 2$, multiply k_l by $(0.5 + 1 / \beta_c)$
- if $b_o/d > 15$, multiply k_l by $(7.5 b_o / d + 0.5)$
- if $\beta_c > 2$ and $b_o/d \le 15$, determine both modified values of k_l.

Enter the graph with modified value of k_1, proceed up to the effective column perimeter, and go across to read a trial value of effective depth.

SLABS 3.5 - Factor k_2' and k_3' for perimeter shear - Square edge column

Reference: ACI 318-95 /ACI 318R-95 Section 11.12.2

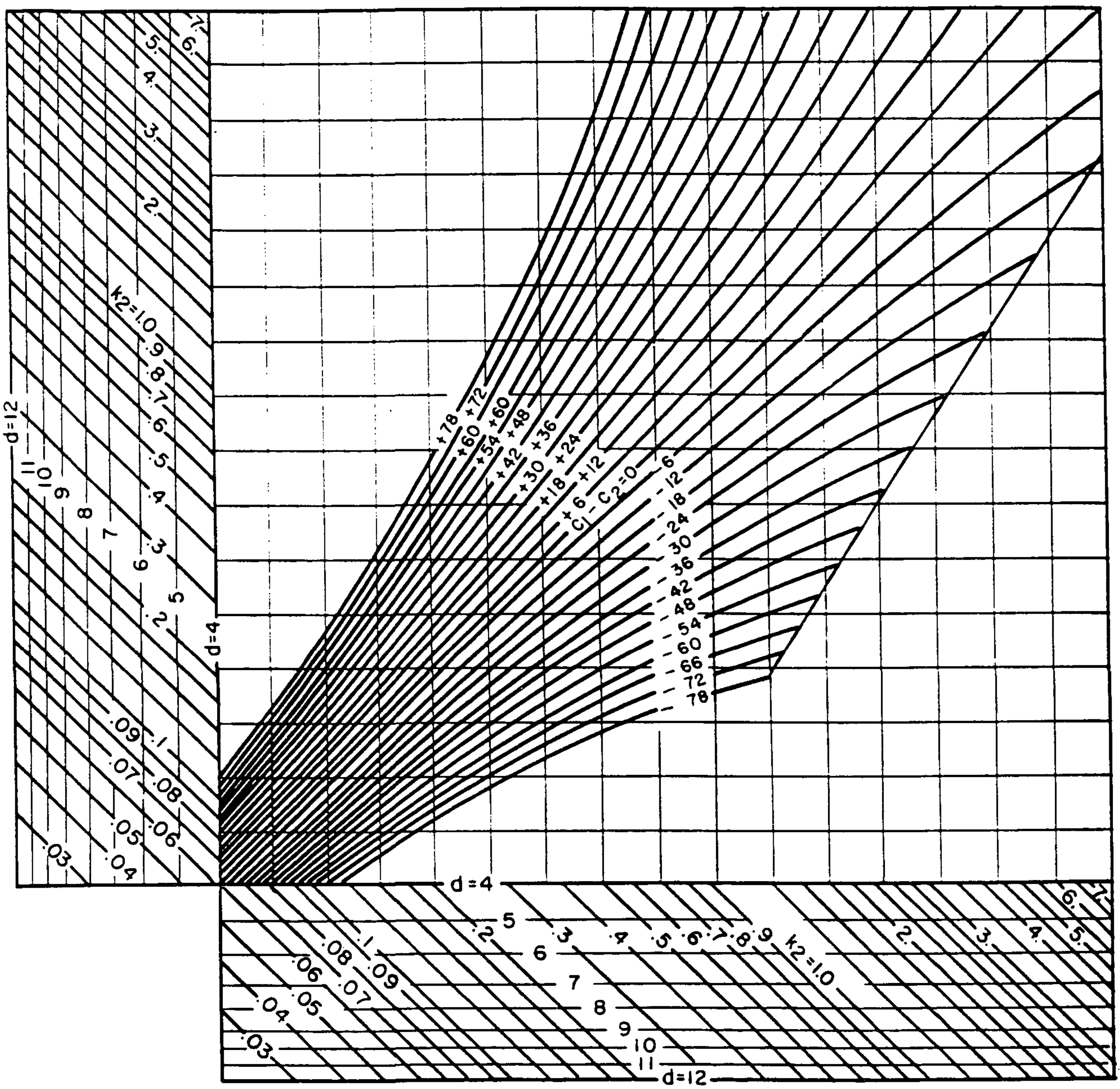


When $c_1 = c_2$, $k_2' = k_2$, $k_3' = k_3$, and shear stress due to moment-shear transfer = $k_2'M_1 + k_3'M_2$. (where shear stress is in psi and M_1 and M_2 are in ft-kips)

When $c_1 \neq c_2$, use k_2' and k_3' to find k_2 and k_3 from slabs 3.3; shear stress due to moment transfer = $k_2M_1 + k_3M_2$.

SLABS 3.6 - Factors k_2 (corrected from k_2') for perimeter shear - Non-square rectangular edge column

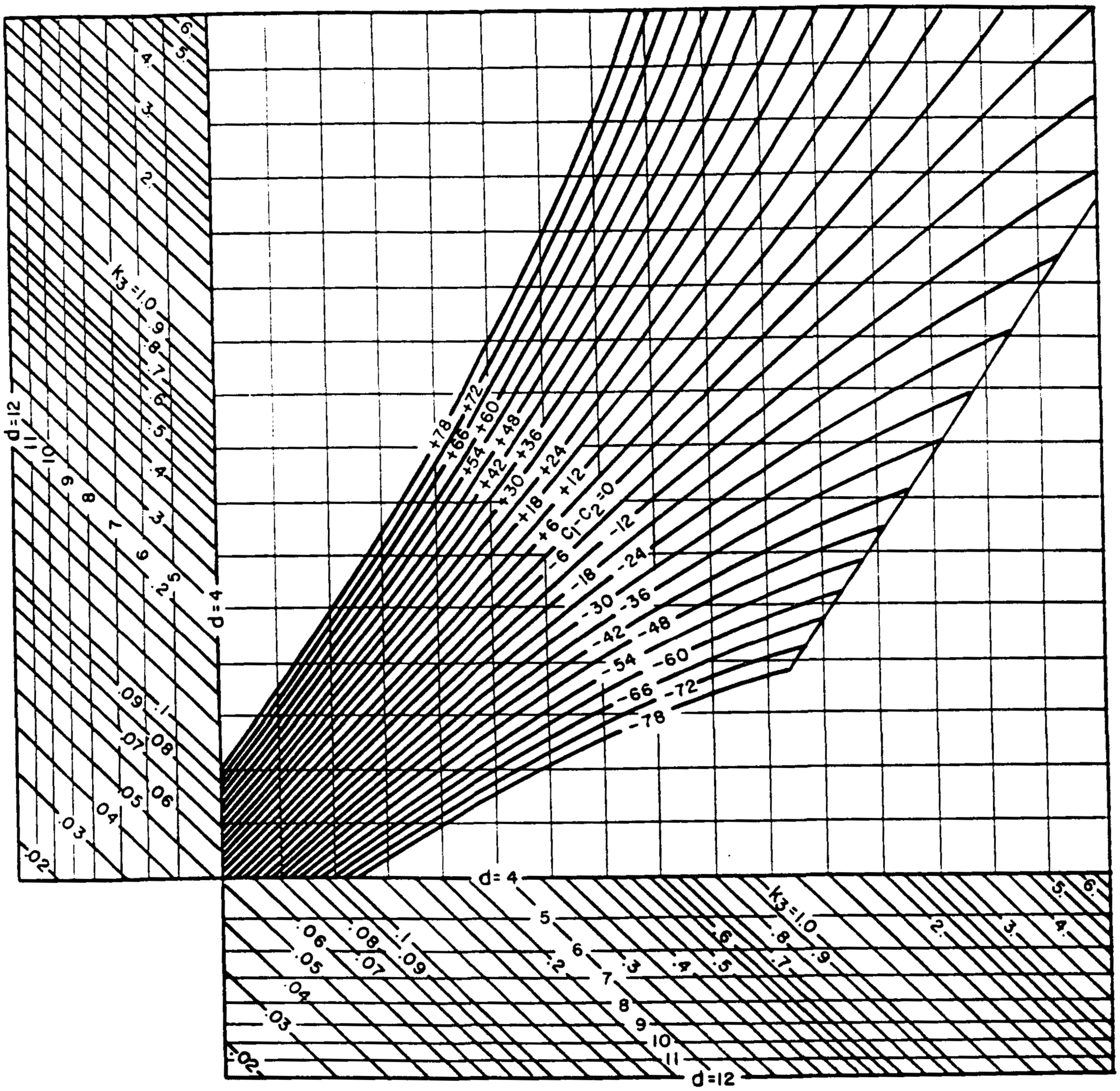
Reference: ACI 318-95/ACI 318R-95 Section 11.12.2



Shear stress due to moment-shear transfer = $k_2 M_1 + k_3 M_2$ (where shear stress is in psi and M_1 and M_2 are in ft-kips)
For M_1 and M_2 see SLABS 3.5; for k_2 see SLABS 3.7.

SLABS 3.7 - Factors k_3 (corrected from k_3') for perimeter shear - Non-square rectangular edge column

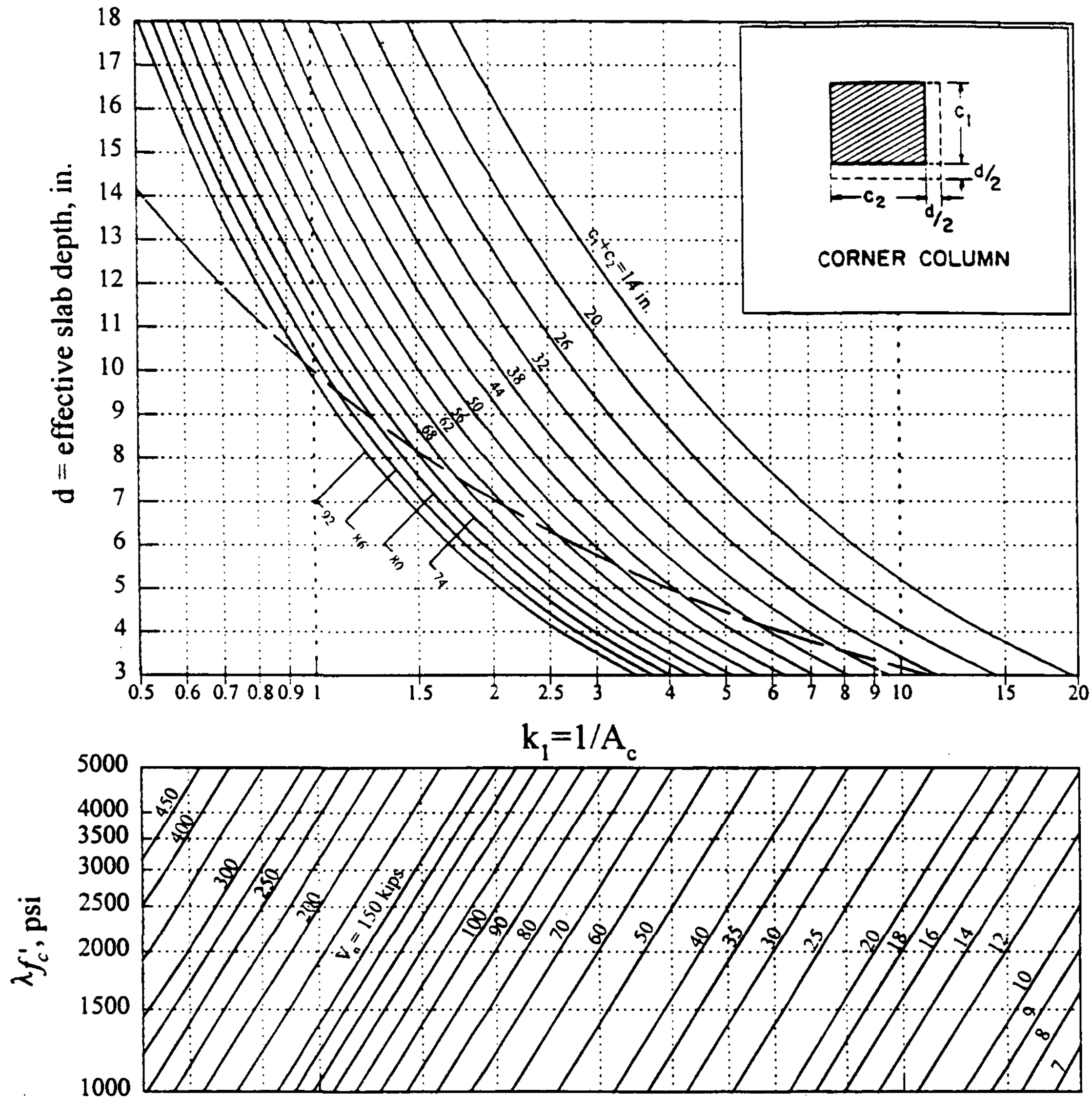
Reference: ACI 318-95/ACI 318R-95 Section 11.12.2



Shear stress due to moment-shear transfer = $k_2 M_1 + k_3 M_2$ (where shear stress is in psi and M_1 and M_2 are in ft-kips)
For M_1 and M_2 see SLABS 3.5; for k_2 see SLABS 3.6.

SLABS 3.8 - Factor k_1 for perimeter shear - Corner column

Reference: ACI 318-95 Sections 11.12.1.2 and 11.12.2



$\lambda f_c' = 1.8^2 f_{ct}^2$ for lightweight concrete ($\lambda \leq 1$)

$\lambda = 1$ for normal weight concrete

Perimeter shear stress $v_n = k_1 V_n$

NOTE: 1. The broken line of the upper portion of this Design Aid indicates the limit of curves where Eq. (11.37) governs.

2. The upper portion of this Design Aid is used to obtain the factor k_1 for use in the final evaluation of shear-moment capacity of the slab-column connection at the corner columns.

The curves in the upper part of this Design Aid can also ve used to obtain a trial value of effective slab depth d when only factored perimeter shear force v_n and the effective column perimeter ($c_1 + c_2$) are known.

The curves are based on Eq. (11.38). For the purpose of obtaining a trial value of effective depth, it may be necessary to modify k_1 if Eq. (11.36) or Eq. (11.37) governs.

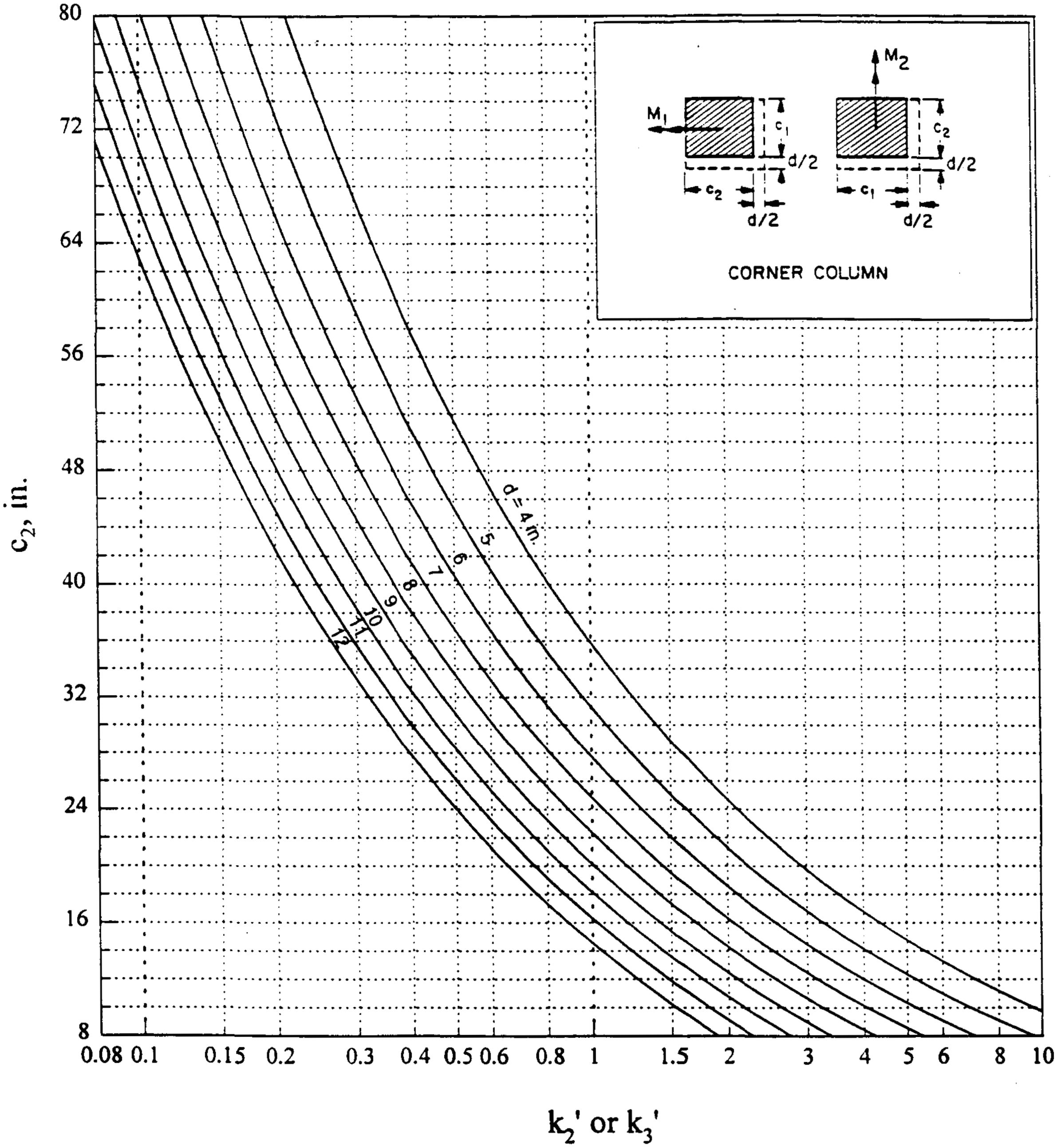
Eq. (11.38) governs if $\beta_c \leq 2$ and $b_o / d \leq 10$. When one or both of these conditions are not satisfied, enter the graph with $\lambda f_c'$, go across to v_n, and up up to read k_1. Then,

- if $\beta_c > 2$, multiply k_1 by $(0.5 + 1 / \beta_c)$
- if $b_o / d > 10$, multiply k_1 by $(5b_o / d + 0.5)$
- if $\beta_c > 2$ and $b_o / d \leq 10$, determine both modified values of k_1.

Enter the graph with modified value of k_1, proceed up to the effective column perimeter, and go across to read a trial value of effective slab depth.

SLABS 3.9 - Factors k_2' and k_3' for perimeter shear - Square corner column

Reference: ACI 318-95 /ACI 318R-95 Section 11.12.2

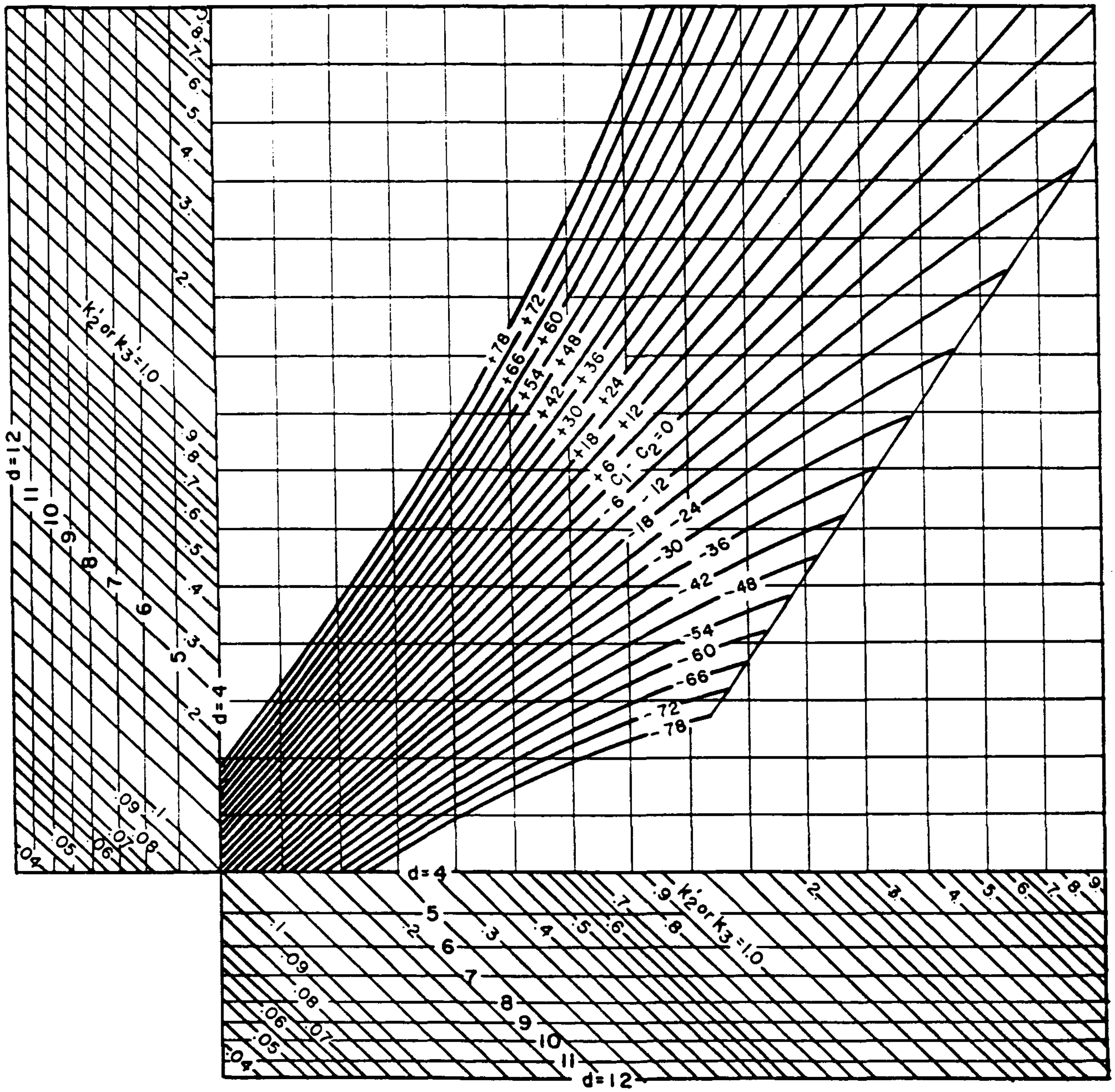


When $c_1 = c_2$, $k_2' = k_2$, $k_3' = k_3$, and shear stress due to moment-shear transfer = $k_2'M_1 + k_3'M_2$. (where shear stress is in psi and M_1 and M_2 are in ft-kips)

When $c_1 \neq c_2$, use k_2' and k_3' to find k_2 and k_3 from slabs 3.3; shear stress due to moment transfer = $k_2M_1 + k_3M_2$.

SLABS 3.10 - Factors k_2 and k_3 (corrected from k_2' and k_3') for perimeter shear - Non-square rectangular corner column

Reference: ACI 318-95/ACI 318R-95 Section 11.12.2



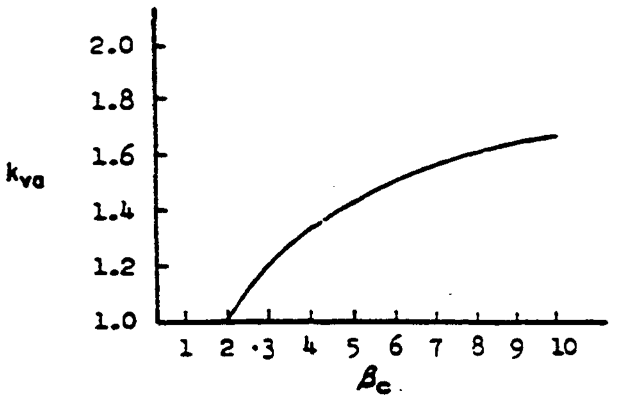
Shear stress due to moment-shear transfer = $k_2 M_1 + k_3 M_2$ (where shear stress is in psi and M_1 and M_2 are in ft-kips)

For M_1 and M_2 see SLABS 3.9.

SLABS 3.11—Correction factor *kva* to be applied to effective slab depth for column (or capital) apsect ratios greater than 2.0

Reference: ACI 318-95, Section 11.12.2.

β_c	k_{va}
2 or less	1
2.5	1.11
3.0	1.20
3.5	1.27
4.0	1.33
5.0	1.43
10.0	1.67



Apply correction factor k_{va} when $\beta_c > 2.0$.
$\beta_c = c_1/c_2$ or c_2/c_1, whichever > 1.
Multiply factor k_{va} times depth d taken from SLABS 9.1, 9.4, or 9.8.

Example: For an edge column for which $c_1 = 16$ in. and $c_2 = 48$ in., concrete is of normal weight, $f'_c = 4000$ psi, and $V_u = 200$ kips, determine effective slab depth d.

Using SLABS 9.4 for an edge column, read $d = 11.3$ in. for $\beta_c \leq 2.0$. Since $\beta_c = 48/16 = 3.0$, from SLABS 9.11 read $k_{va} = 1.20$. Therefore $d = 1.20\ (11.3) = 13.6$ in.

TWO-WAY ACTION REINFORCEMENT 1—Maximum spacing of main reinforcement for two-way action slabs and plates for crack control

(Recommended by ACI Committee 224, Cracking, but not required by ACI 318-95)
Reference: ACI 224R-95

		Maximum reinforcement spacing, in. (see Note 1)			
		Interior exposure		Exterior exposure	
		$K = 2.8 \times 10^{-5}$ $w_{max} = 0.016$ in. (See Notes 2 and 3)		$K = 2.8 \times 10^{-5}$ $w_{max} = 0.013$ in. (See Notes 2 and 3)	
		$s_1 = s_2$, in.		$s_1 = s_2$, in.	
		f_y, ksi		f_y, ksi	
Bar size	d_c, in.	50	60	50	60
#3	0.94	9	8	7	6
#4	1.00	10	9	8	7
#5	1.06	11	9	9	7
#6	1.12	12	10	10	8
#7	1.19	12	10	10	8
#8	1.25	13	11	10	9
#9	1.31	13	11	11	9
#10	1.39	14	12	11	9
#11	1.46	14	12	12	10

Note 1—Table values for bar sizes #3 to #11 are based on 3/4 in. clear concrete cover required in slabs subject to usual interior or exterior exposure. Slabs subject to alternate wetting and drying or direct leaking and runoff are not included in the table. For such exposure conditions, refer to section 4.3 of ACI 224R-89. The above maximum values are recommended for structures where flexural cracking at service load and overload conditions can be serious such as in office buildings, schools, parking garages, industrial buildings, schools, parking garages, industrial buildings, and other floors where the service live load levels exceed those in normal size apartment building panels and in all cases of adverse exposure conditions.

Note 2—Table values are for uniformly loaded square panels continuous on all four edges. For other conditions, multiply table values by:

K	Fully restrained slabs and plates	Multiply by
2.8×10^{-5}	Uniformely loaded, square	1.0
2.1×10^{-5}	At concentrated loads and columns	1.33
2.1×10^{-5}	$0.5 < l_s/l_t < 0.75$	1.33
1.6×10^{-5}	l_s/l_t	1.75

For simply supported slabs multiply spacing values by 0.65. Interpolate multiplier values for intermediate span ratio l_s/l_t values or for partial restraint at boundaries such as cases of end and corner panels of multipmanel floor systems.

Note 3

$$w_{max} = \kappa\beta f_s\sqrt{\frac{d_{b1}s_2}{\rho_{t1}}} = \kappa\beta f_s\sqrt{\frac{s_1 s_2 d_c}{d_{b1}} \times \frac{8}{\pi}}$$

= maximum crack width at tensile face of concrete, in.

f_s = actual average stress in reinforcement at service load level, or 40 percent of the design yield strength f_y (ksi).

TWO-WAY ACTION REINFORCEMENT 2 - Maximum tolerable crack widths (Recommended by ACI Committee 224, Cracking, but not required by ACI 318-95)

Reference: ACI 224R-95

Exposure condition	Tolerable crack width: w_{max}, in.
Dry air or protective membrane	0.016
Humidity, moist air, soil	0.012
Deicing chemicals	0.007
Seawater and seawater spray: wetting and drying	0.006
Water-retaining structures	0.004

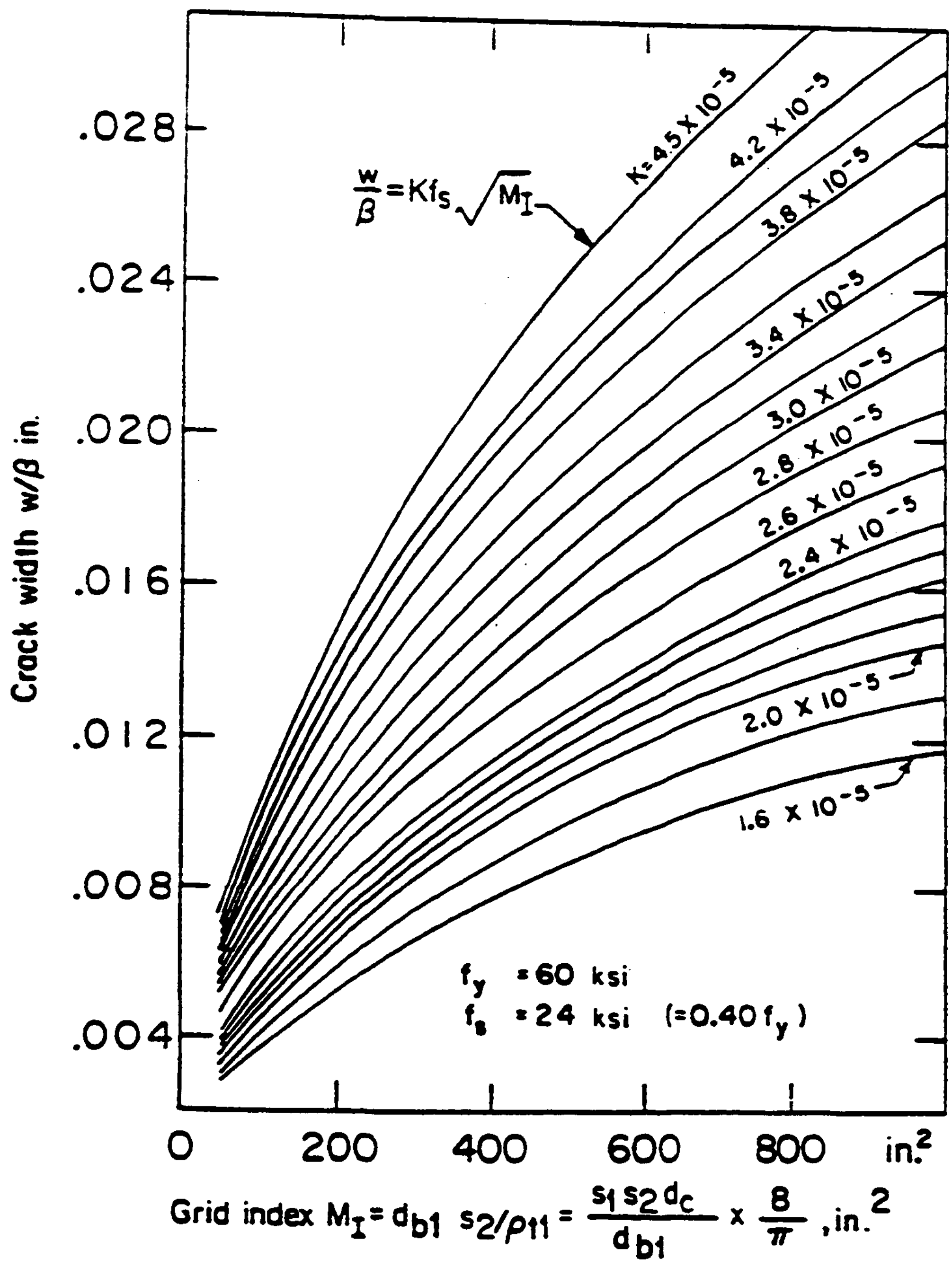
In comparison with these values, Section 10.6 of ACI 318-95 considers two sets of exposure conditions only: w = 0.016 in. for interior exposure, and w = 0.013 in. for exterior exposure.

For use of this Design Aid, see Slab Reinforcement Example.

TWO-WAY ACTION REINFORCEMENT 3—Crack widths as a function of grid index M_I in slabs and plates for any exposure condition (Recommended by ACI Committee 224, Cracking, but not required by ACI 318-95)

Reference: ACI 224R-94

$$\frac{w}{\beta} = K f_s \sqrt{M_I}\text{, where } f_s \text{ is in ksi}$$



SEISMIC

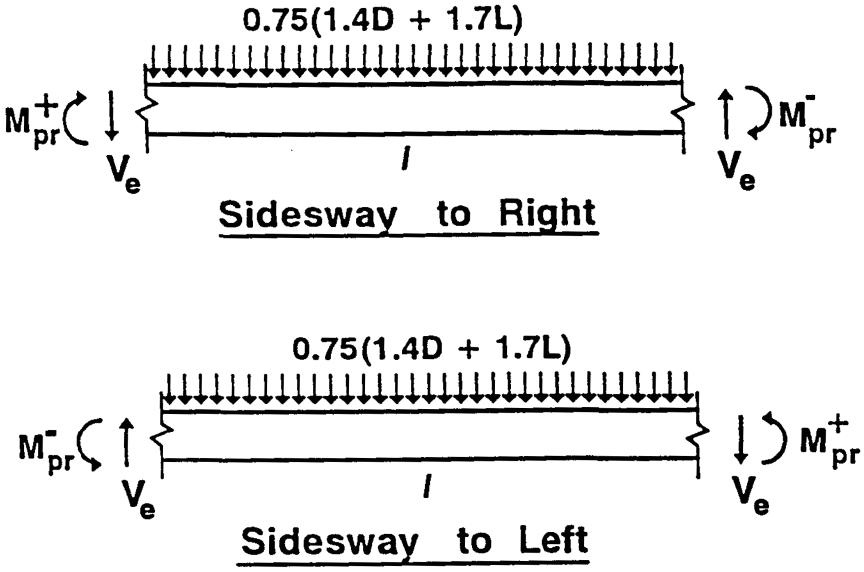
SEISMIC 1 - Probable moment strengths for beams.

Reference: ACI 318-95 Section 21.0 and 21.3.4.1

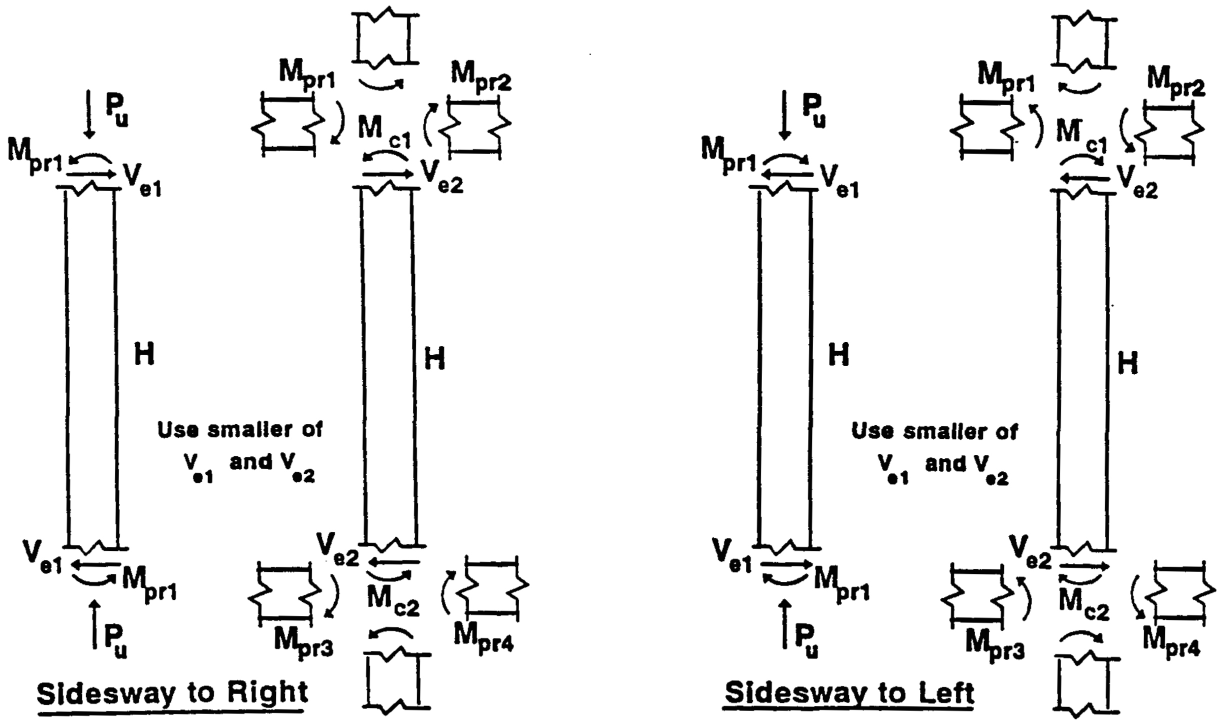
$M_{pr} = K_{pr} F$ ft-kips $F = bd^2/1200$ $\rho = A_s/bd$

ρ	$1.25f_y$ = 50,000 psi			$1.25f_y$ = 75,000 psi		
	f'_c = 4000 psi	f'_c = 6000 psi	f'_c = 8000 psi	f'_c = 4000 psi	f'_c = 6000 psi	f'_c = 8000 psi
	K_{pr} (psi)					
0.005	241	244	245	354	361	365
0.006	287	291	293	420	430	435
0.007	332	338	341	484	498	505
0.008	376	384	388	547	565	574
0.009	420	430	435	608	630	641
0.010	463	475	482	667	695	709
0.011	506	520	528	725	758	775
0.012	547	565	574	781	821	840
0.013	588	609	619	835	882	905
0.014	628	652	664	888	942	969
0.015	667	695	709	939	1001	1032
0.016	706	737	753	988	1059	1094
0.017	744	779	797	1036	1116	1155
0.018	781	821	840	1082	1171	1216
0.019	817	862	884	1126	1226	1276
0.020	853	902	926	1169	1279	1335
0.021	888	942	969	1210	1332	1393
0.022	922	981	1011		1383	1450
0.023	956	1020	1053		1433	1506
0.024	988	1059	1094		1482	1562
0.025	1020	1097	1135		1530	1616

SEISMIC 2 - Seismic design shear force, V_e, for beams and columns.
(Note: V_e shall not be less than that required by analysis of the structure)

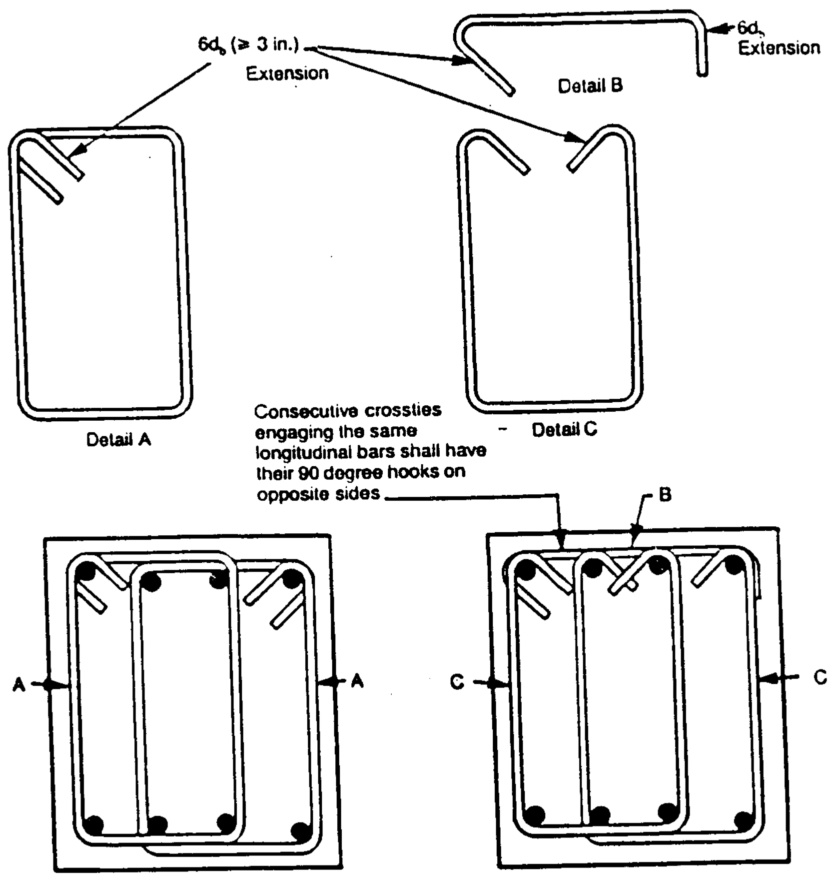


Beam Design Shear Force

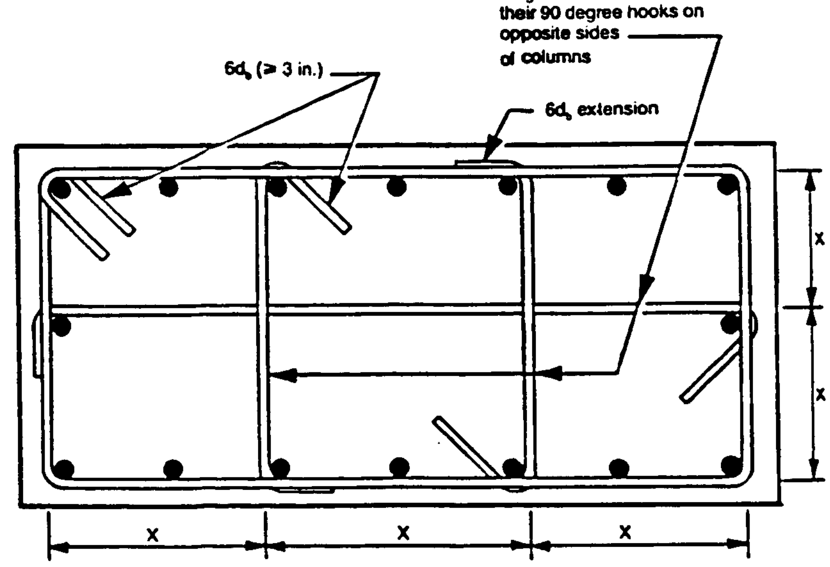


Column Design Shear Force

SEISMIC 3 - Details of transverse reinforcement for beams and columns.



Overlapping Hoops for Beams



Transverse Reinforcement for Columns

SEISMIC 4—Voumetric ratio of spiral reinforcement (ρ_s) for concrete confinement
Reference: ACI 318-95, Section 10.9.3; Eq. (10-6), (21-2).

$$P_s \geq 0.45\left(\frac{A_g}{A_c} - 1\right)\frac{f'_c}{f_{yk}} \quad \text{but} \quad \rho_s \geq 0.12\frac{f_c{}'}{f_{yh}}$$

A_g/A_c	f_{yh} = 40,000 psi			f_{yh} = 60,000 psi		
	f'_c = 4000 psi	f'_c = 6000 psi	f'_c = 8000 psi	f'_c = 4000 psi	f'_c = 6000 psi	f'_c = 8000 psi
	ρ_s					
1.1	0.012	0.018	0.024	0.008	0.012	0.016
1.2	0.012	0.018	0.024	0.008	0.012	0.016
1.3	0.014	0.020	0.027	0.009	0.014	0.018
1.4	0.018	0.027	0.036	0.012	0.018	0.024
1.5	0.023	0.034	0.045	0.015	0.023	0.030
1.6	0.027	0.041	0.054	0.018	0.027	0.036
1.7	0.032	0.047	0.063	0.021	0.032	0.042
1.8	0.036	0.054	0.072	0.024	0.036	0.048
1.9	0.041	0.061	0.081	0.027	0.041	0.054
2.0	0.045	0.068	0.090	0.030	0.045	0.060
2.1	0.050	0.074	0.099	0.033	0.050	0.066
2.2	0.054	0.081	0.108	0.036	0.054	0.072
2.3	0.058	0.088	0.117	0.039	0.058	0.078
2.4	0.063	0.094	0.126	0.042	0.063	0.084
2.5	0.067	0.101	0.135	0.045	0.067	0.090

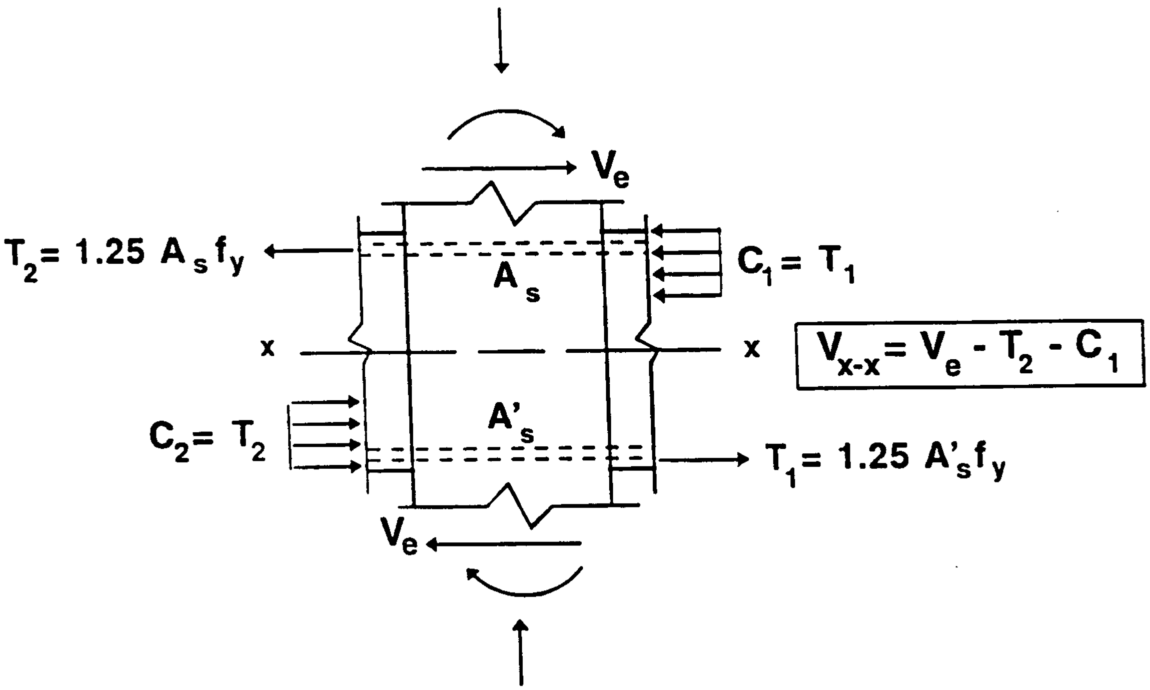
SEISMIC 5—Area ratio of rectilinear hoop reinforcement (ρ_c) for concrete confinement

Reference: ACI 318-95, Section 21.4.41, Eq. (21-3), (21-4).

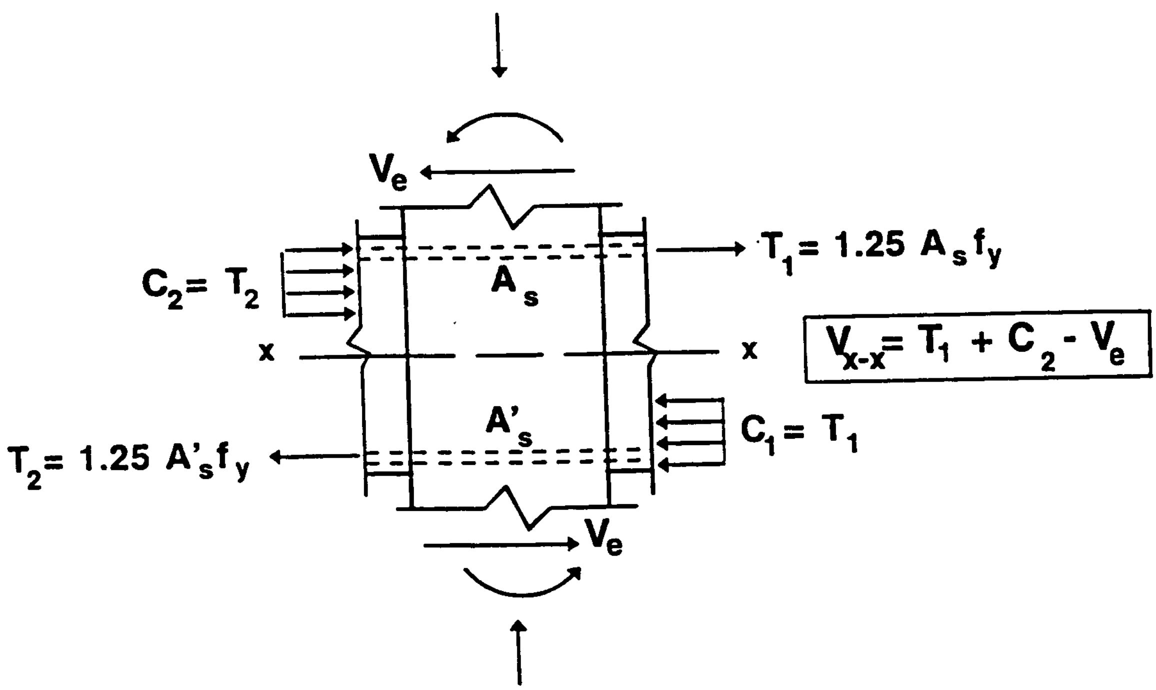
$$\rho_c = \frac{A_{sh}}{Sh_c} \geq 0.3\left(\frac{A_g}{A_{ch}} - 1\right)\frac{f'_c}{f_{yh}} \quad \text{But} \quad \rho_c \geq 0.09\frac{f'_c}{f_{yh}}$$

A_g/A_c	f_{yh} = 40,000 psi			f_{yh} = 60,000 psi		
	f'_c = 4000 psi	f'_c = 6000 psi	f'_c = 8000 psi	f'_c = 4000 psi	f'_c = 6000 psi	f'_c = 8000 psi
	ρ_s					
1.1	0.009	0.014	0.018	0.006	0.009	0.012
1.2	0.009	0.014	0.018	0.006	0.009	0.012
1.3	0.009	0.014	0.018	0.006	0.009	0.012
1.4	0.012	0.018	0.024	0.008	0.012	0.016
1.5	0.015	0.023	0.030	0.010	0.015	0.020
1.6	0.018	0.027	0.036	0.012	0.018	0.024
1.7	0.021	0.032	0.042	0.014	0.021	0.028
1.8	0.024	0.036	0.048	0.016	0.024	0.032
1.9	0.027	0.041	0.054	0.018	0.027	0.036
2.0	0.030	0.045	0.060	0.020	0.030	0.040
2.1	0.033	0.050	0.066	0.022	0.033	0.044
2.2	0.036	0.054	0.072	0.024	0.036	0.048
2.3	0.039	0.058	0.078	0.026	0.039	0.052
2.4	0.042	0.063	0.084	0.028	0.042	0.056
2.5	0.045	0.067	0.090	0.030	0.045	0.060

SEISMIC 6 - Joint shear, V_{x-x}, in an interior beam-column joint.

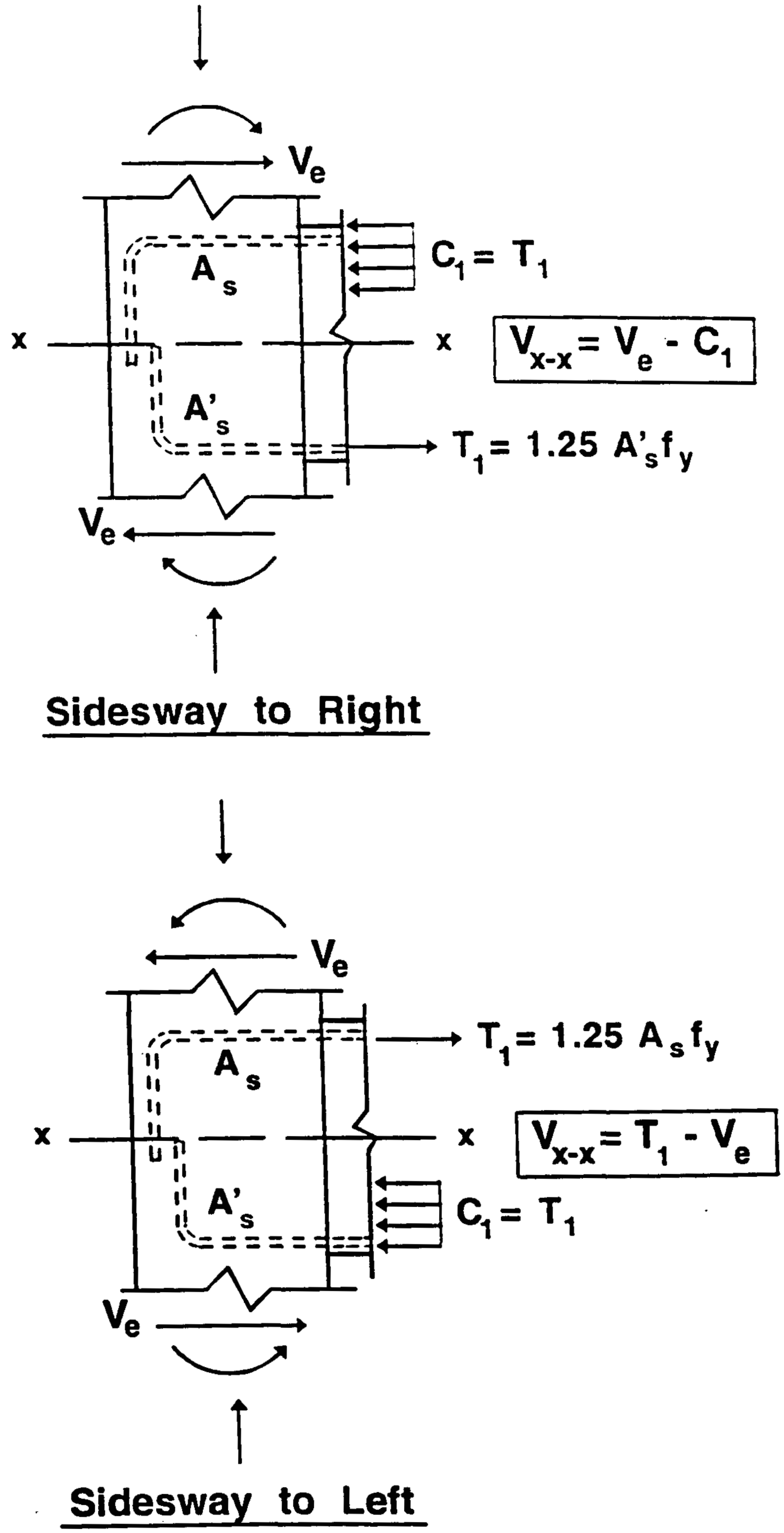


Sidesway to Right



Sidesway to Left

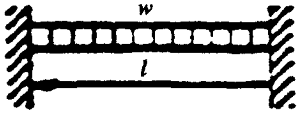
SEISMIC 7 - Joint shear, V_{x-x}, in an exterior beam-column joint.



GENERAL

GENERAL 1.1—Moments in beams with fixed ends, ft-kips

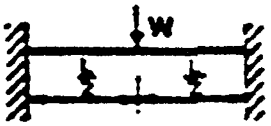
Unifrom load



Fixed-end moments: Table values $M = \frac{1}{12}wl^2$

Midspan moments: Table values × 0.5 $M = \frac{1}{24}wl^2$

Concentrated load at midspan

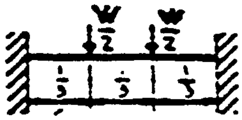


Fixed-end moments: Table values $M = \frac{1}{8}Wl$

Midspan moments: Table values × 0.5 $M = \frac{1}{8}Wl$

Span *l*	Uniform load w, kips per ft										Span *l*	Concentrated load W, kips									
	For the following loads use table values directly											For the following loads use table values directly									
	1	2	3	4	5	6	7	8	9	10		10	20	30	40	50	60	70	80	90	100
	For the following loads, use table values divided by 10											For the following loads, use table values divided by 10									
	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0		1	2	3	4	5	6	7	8	9	10
5'-0"	2.08	4.17	6.25	8.33	10.4	12.5	14.6	16.7	18.7	20.8	5'-0"	6.3	12.5	18.8	25.0	31.3	37.5	43.8	50.0	56.3	62.5
5'-3"	2.30	4.59	6.89	9.19	11.5	13.8	15.1	18.4	20.7	23.0	5'-3"	6.6	13.1	19.7	26.3	32.8	39.4	45.9	52.5	59.1	65.6
5'-6"	2.52	5.04	7.56	10.1	12.6	15.1	17.6	20.2	22.7	25.2	5'-6"	6.9	13.8	20.6	27.5	34.4	41.3	48.1	55.0	61.9	68.8
5'-9"	2.76	5.51	8.27	11.0	13.8	16.5	19.3	22.0	24.8	27.6	5'-9"	7.2	14.6	21.6	28.8	35.9	43.1	50.3	57.5	64.7	71.9
6'-0"	3.00	6.00	9.00	12.0	15.0	18.0	21.0	24.0	27.0	30.0	6'-0"	7.5	15.0	22.5	30.0	37.5	45.0	52.5	60.0	67.5	75.0
6'-3"	3.23	6.51	9.77	13.0	16.3	19.5	22.8	26.0	29.3	32.6	6'-3"	7.8	15.6	23.4	31.3	39.1	46.9	54.7	62.5	70.3	78.1
6'-6"	3.52	7.04	10.6	14.1	17.6	21.1	24.6	28.2	31.7	35.2	6'-6"	8.1	16.3	24.4	32.5	40.6	48.8	56.9	65.0	73.1	81.3
6'-9"	3.80	7.59	11.4	15.2	19.0	22.8	26.6	30.4	34.2	38.0	6'-9"	8.4	16.9	25.3	33.8	42.2	50.6	59.1	67.5	75.9	84.4
7'-0"	4.08	8.17	12.3	16.3	20.4	24.5	26.6	32.7	36.8	40.8	7'-0"	8.8	17.5	26.3	35.0	43.8	52.5	61.3	70.0	78.8	87.5
7'-3"	4.38	8.76	13.1	17.5	21.9	26.3	30.7	35.0	39.4	43.8	7'-3"	9.1	18.1	27.2	36.3	45.3	54.4	63.4	72.5	81.6	90.6
7'-6"	4.69	9.38	14.1	18.8	23.4	28.1	32.8	37.5	42.2	46.9	7'-6"	9.4	18.8	28.1	37.5	46.9	56.3	65.6	75.0	84.4	93.8
7'-9"	5.01	10.0	15.0	20.0	25.0	30.0	35.0	40.0	45.0	50.1	7'-9"	9.7	19.4	29.1	38.8	48.4	58.1	67.8	77.5	87.2	96.9
8'-0"	5.33	10.7	16.0	21.3	26.7	32.0	37.3	42.7	48.0	53.3	8'-0"	10.0	20.0	30.0	40.0	50.0	60.0	70.0	80.0	90.0	100
8'-3"	5.67	11.3	17.0	22.7	28.4	34.0	39.7	45.4	51.1	56.7	8'-3"	10.3	20.6	30.9	41.3	51.6	61.9	72.2	82.5	92.8	103
8'-6"	6.02	12.0	18.1	24.1	30.1	36.1	42.2	48.2	54.2	60.2	8'-6"	10.6	21.3	31.9	42.5	53.1	63.8	74.4	85.0	95.6	106
8'-9"	6.38	12.8	19.1	25.5	31.9	38.3	44.7	51.0	57.4	63.8	8'-9"	10.9	21.9	32.8	43.8	54.7	65.6	76.6	87.5	96.4	109
9'-0"	6.75	13.5	20.3	27.0	33.7	40.5	47.3	54.0	60.8	67.5	9'-0"	11.3	22.5	33.8	45.0	56.3	67.5	78.8	90.0	101	113
9'-3"	7.13	14.3	21.4	28.5	35.6	42.8	49.9	57.0	64.2	71.3	9'-3"	11.6	23.1	34.7	46.3	57.8	69.4	80.9	92.5	104	116
9'-6"	7.52	15.0	22.6	30.1	37.6	45.1	52.6	60.2	67.7	75.2	9'-6"	11.9	23.8	35.6	47.5	59.4	71.3	83.1	95.0	107	119
9'-9"	7.92	15.8	23.8	31.7	39.6	47.5	55.5	63.4	71.3	79.2	9'-9"	12.2	24.4	36.6	48.8	60.9	73.1	85.3	97.5	110	122
10'-0"	8.33	16.7	25.0	33.3	41.7	50.0	58.3	66.7	75.0	83.3	10'-0"	12.5	25.0	37.5	50.0	62.5	75.0	87.5	100	113	125
10'-6"	9.19	18.4	27.6	36.8	45.9	55.1	64.3	73.5	82.7	91.9	10'-6"	13.1	26.3	39.4	52.5	65.6	78.8	91.9	105	118	131
11'-0"	10.1	20.2	30.3	40.3	50.4	60.5	70.6	80.7	90.8	101	11'-0"	13.8	27.5	41.3	55.0	68.8	82.5	96.3	110	124	138
11'-6"	11.0	22.0	33.1	44.1	55.1	66.1	77.2	88.2	99.2	110	11'-6"	14.4	28.8	43.1	57.5	71.9	86.3	101	115	129	144
12'-0"	12.0	24.0	36.0	48.0	60.0	72.0	84.0	96.0	106	120	12'-0"	15.0	30.0	45.0	60.0	75.0	90.0	105	120	135	150
12'-6"	13.0	26.0	39.1	52.1	65.1	78.1	91.2	104	117	130	12'-6"	15.6	31.3	46.9	62.5	78.1	93.8	109	125	141	156
13'-0"	14.1	28.2	42.2	56.3	70.4	84.5	98.6	113	127	141	13'-0"	16.3	32.5	48.8	65.0	81.3	97.5	114	130	146	163
13'-6"	15.2	30.4	45.6	60.8	75.9	91.1	106	122	137	152	13'-6"	16.9	33.8	50.6	67.5	84.4	101	118	135	152	169
14'-0"	16.3	32.7	49.0	65.3	81.7	96.0	114	131	147	163	14'-0"	17.5	35.0	52.5	70.0	87.5	105	122	140	157	175
14'-6"	17.5	35.0	52.6	70.1	87.6	105	123	140	158	175	14'-6"	18.1	36.3	54.4	72.5	90.6	109	127	145	163	181
15'-0"	18.8	37.5	56.3	75.0	93.8	113	131	150	169	188	15'-0"	18.8	37.5	56.3	75.0	93.8	112	131	150	169	188
15'-6"	20.0	40.0	60.1	80.1	100	120	140	160	180	200	15'-6"	19.4	38.8	58.1	77.5	96.9	116	136	155	174	194
16'-0"	21.3	42.7	64.0	85.3	107	128	149	171	192	213	16'-0"	20.0	40.0	60.0	80.0	100	120	140	160	180	200
16'-6"	22.7	45.4	68.1	90.8	113	136	159	182	204	227	16'-6"	20.6	41.3	61.9	82.5	103	124	144	165	186	206
17'-0"	24.1	48.2	72.2	96.3	120	145	169	193	217	241	17'-0"	21.3	42.5	63.8	85.0	106	128	149	170	191	213
17'-6"	25.5	51.0	76.6	102	128	153	179	204	230	255	17'-6"	21.9	43.8	65.6	87.5	109	131	153	175	197	219
18'-0"	27.0	54.0	81.0	108	135	162	189	216	243	270	18'-0"	22.5	45.0	67.5	90.0	112	135	158	180	202	225
18'-6"	28.5	57.0	85.6	114	143	171	200	228	257	285	18'-6"	23.1	46.3	69.4	92.5	116	139	162	185	208	231
19'-0"	30.1	60.2	90.3	120	150	181	211	241	271	301	19'-0"	23.8	47.5	71.3	95.0	119	142	166	190	214	238
19'-6"	31.7	63.4	95.1	127	158	190	222	254	285	317	19'-6"	24.4	48.8	73.1	97.5	122	146	171	195	219	244
20'-0"	33.3	66.7	100	133	167	200	233	267	300	333	20'-0"	25.0	50.0	75.0	100	125	150	175	200	225	250
20'-6"	35.0	70.0	105	140	175	210	245	280	315	350	20'-6"	25.6	51.3	76.9	103	128	154	179	205	231	256
21'-0"	36.8	73.5	110	147	184	221	257	294	331	368	21'-0"	26.3	52.5	78.8	105	131	158	184	210	236	263
21'-6"	38.5	77.0	116	154	193	231	270	308	347	385	21'-6"	26.9	53.8	80.6	108	134	161	188	215	242	269
22'-0"	40.3	80.7	121	161	202	242	282	323	363	403	22'-0"	27.5	55.0	82.5	110	138	165	192	220	248	275
22'-6"	42.2	84.4	127	169	211	253	295	338	380	422	22'-6"	28.1	56.3	84.4	113	141	169	197	225	253	281
23'-0"	44.1	88.2	132	176	220	264	309	353	397	441	23'-0"	28.8	57.5	86.3	115	144	172	201	230	259	288
23'-6"	46.0	92.0	138	184	230	276	322	368	414	460	23'-6"	29.4	58.8	88.1	118	147	176	206	235	264	294
24'-0"	48.0	96.0	144	192	240	288	336	384	432	480	24'-0"	30.0	60.0	90.0	120	150	180	210	240	270	300
24'-6"	50.0	100	150	200	250	300	350	400	450	500	24'-6"	30.6	61.3	91.9	123	153	184	214	245	276	306
25'-0"	52.1	104	156	208	260	312	365	417	469	521	25'-0"	31.3	62.5	93.8	125	156	188	219	250	281	313
25'-6"	54.2	108	163	217	271	325	379	434	488	542	25'-6"	31.9	63.8	95.6	128	159	191	223	255	287	319
26'-0"	56.3	113	169	225	282	338	394	451	507	563	26'-0"	32.5	65.0	97.5	130	162	195	228	260	292	325
26'-6"	58.5	117	176	234	293	351	410	468	527	585	26'-6"	33.1	66.3	99.4	133	166	199	232	265	298	331
27'-0"	60.8	122	182	243	304	365	425	486	547	608	27'-0"	33.8	67.5	101	135	169	202	236	270	304	338
27'-6"	63.0	126	189	252	315	378	441	504	567	630	27'-6"	34.4	68.8	103	138	172	206	241	275	309	344
28'-0"	65.3	131	196	261	327	392	457	523	588	653	28'-0"	35.0	70.0	105	140	175	210	245	280	315	350
28'-6"	67.7	135	203	271	338	406	474	542	609	677	28'-6"	35.6	71.3	107	143	178	214	249	285	321	356
29'-0"	70.1	140	210	280	350	420	491	561	631	701	29'-0"	36.3	72.5	109	145	181	218	254	290	326	363
29'-6"	72.5	145	218	290	363	435	508	580	653	725	29'-6"	36.9	73.8	111	148	184	221	258	295	332	369
30'-0"	75.0	150	225	300	375	450	525	600	675	750	30'-0"	37.5	75.0	113	150	188	225	262	300	338	375

Concentrated loads at third-points

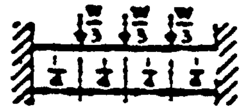


Fixed-end moments: Table values $M = \frac{1}{9}wl$

Midspan moments: Table values × 0.5 $M = \frac{1}{18}wl$

Span l	Uniform load w, kips									
	For the following loads use table values directly									
	10	20	30	40	50	60	70	80	90	100
	For the following loads, use table values divided by 10									
	1	2	3	4	5	6	7	8	9	10
5'-0"	5.6	11.1	16.7	22.2	27.8	33.3	38.9	44.4	50.0	56.6
5'-3"	5.8	11.7	17.5	23.3	29.2	35.0	40.8	46.7	52.5	58.3
5'-6"	6.1	12.2	18.3	24.4	30.6	36.7	42.8	48.9	55.0	61.1
5'-9"	6.4	12.8	19.2	25.6	31.9	38.3	44.7	51.1	57.5	63.9
6'-0"	6.7	13.3	20.0	26.7	33.3	40.0	46.7	53.3	60.0	66.7
6'-3"	6.9	13.9	20.8	27.8	34.7	41.7	48.6	56.6	62.5	69.4
6'-6"	7.2	14.4	21.7	28.9	36.1	43.3	50.6	57.8	65.0	72.2
6'-9"	7.5	15.0	22.5	30.0	37.5	45.0	52.5	60.0	67.5	75.0
7'-0"	7.8	15.6	23.3	31.1	38.9	46.7	54.4	62.2	70.0	77.8
7'-3"	8.1	16.1	24.2	32.2	40.3	48.3	56.4	64.4	72.5	80.6
7'-6"	8.3	16.7	25.0	33.3	41.7	50.0	58.3	66.7	75.0	83.3
7'-9"	8.6	17.2	25.8	34.4	43.1	51.7	60.3	68.9	77.5	86.1
8'-0"	8.9	17.8	26.7	35.6	44.4	53.3	62.2	71.1	80.0	88.9
8'-3"	9.2	18.3	27.5	36.7	45.8	56.0	64.2	73.3	82.5	91.7
8'-6"	9.4	18.9	28.3	37.8	47.2	56.7	66.1	75.6	85.0	94.4
8'-9"	9.7	19.4	29.2	38.9	48.6	58.3	68.1	77.8	87.5	97.2
9'-0"	10.0	20.0	30.0	40.0	50.0	60.0	70.0	80.0	90.0	100
9'-3"	10.3	20.6	30.8	41.1	51.4	61.7	71.9	82.2	92.5	103
9'-6"	10.6	21.1	31.7	42.2	52.8	63.3	73.9	84.4	95.0	106
9'-9"	10.8	21.7	32.5	43.3	54.2	65.0	75.8	86.7	97.5	108
10'-0"	11.1	22.2	33.3	44.4	55.6	66.7	77.8	88.9	100	111
10'-6"	11.7	23.3	35.0	46.7	58.3	70.0	81.7	93.3	105	117
11'-0"	12.2	24.4	36.7	48.9	61.1	73.3	85.6	97.8	110	122
11'-6"	12.8	25.6	38.3	51.1	63.9	76.7	89.4	102	115	128
12'-0"	13.3	26.7	40.0	53.3	66.7	80.0	93.3	107	120	133
12'-6"	13.9	27.8	41.7	55.6	69.4	83.3	97.2	111	125	139
13'-0"	14.4	28.9	43.3	57.8	72.2	86.7	101	116	130	144
13'-6"	15.0	30.0	45.0	60.0	75.0	90.0	105	120	135	150
14'-0"	15.6	31.1	46.7	62.2	77.8	93.3	109	124	140	156
14'-6"	16.1	32.2	48.3	64.4	80.6	96.7	113	129	145	161
15'-0"	16.7	33.3	50.0	66.7	83.3	100	117	133	150	167
15'-6"	17.2	34.4	51.7	68.9	86.1	103	121	138	155	172
16'-0"	17.8	35.6	53.3	71.1	88.9	107	124	142	160	178
16'-6"	18.3	36.7	55.0	73.3	91.7	110	128	147	165	183
17'-0"	18.9	37.8	56.7	75.6	94.4	113	132	151	170	189
17'-6"	19.4	38.9	58.3	77.8	97.2	117	136	156	175	194
18'-0"	20.0	40.0	60.0	80.0	100	120	140	160	180	200
18'-6"	20.6	41.1	61.7	82.2	103	123	144	164	185	206
19'-0"	21.1	42.2	63.3	84.4	106	127	148	169	190	211
19'-6"	21.7	43.3	65.0	86.7	108	130	152	173	195	217
20'-0"	22.2	44.4	66.6	88.8	111	133	156	178	200	222
20'-6"	22.8	45.6	68.3	91.1	114	137	159	182	205	228
21'-0"	23.3	46.7	70.0	93.3	117	140	163	187	210	233
21'-6"	23.9	47.8	71.7	95.6	119	143	167	191	215	239
22'-0"	24.4	48.9	73.3	97.8	122	147	171	196	220	244
22'-6"	25.0	50.0	75.0	100	125	150	175	200	225	250
23'-0"	25.6	51.1	76.7	102	128	153	179	204	230	256
23'-6"	26.1	52.2	78.3	104	131	157	183	209	235	261
24'-0"	26.7	53.3	80.0	107	133	160	187	213	240	267
24'-6"	27.2	54.4	81.7	109	136	163	191	218	245	272
25'-0"	27.8	55.6	83.3	111	139	167	194	222	250	278
25'-6"	28.3	56.7	85.0	113	142	170	198	227	255	283
26'-0"	28.9	57.8	86.7	116	144	173	202	231	260	289
26'-6"	29.4	58.9	88.3	118	147	177	206	236	265	294
27'-0"	30.0	60.0	90.0	120	150	180	210	240	270	300
27'-6"	30.6	61.1	91.7	122	153	183	214	244	275	306
28'-0"	31.1	62.2	93.3	124	156	187	218	249	280	311
28'-6"	31.7	63.3	95.0	127	158	190	220	253	285	317
29'-0"	32.2	64.4	96.7	129	161	193	226	258	290	322
29'-6"	32.8	65.6	98.3	131	164	197	229	262	295	328
30'-0"	33.3	66.7	100	133	167	200	233	267	300	333

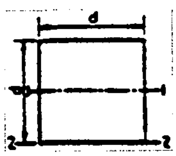
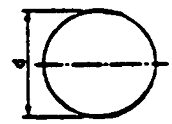
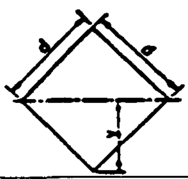
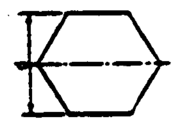
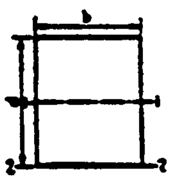
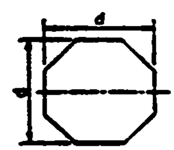
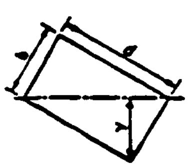
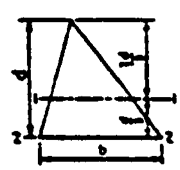
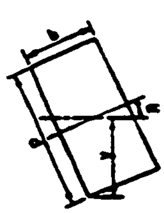
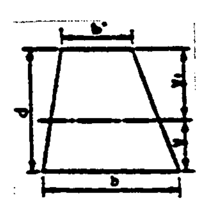
Concentrated loads at fourth-points



Fixed-end moments: Table values $M = \frac{5}{48}Wl$

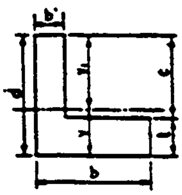
Midspan moments: Table values × 0.6 $M = \frac{3}{48}Wl$

Span l	Concentrated load W, kips									
	For the following loads use table values directly									
	10	20	30	40	50	60	70	80	90	100
	For the following loads, use table values divided by 10									
	1	2	3	4	5	6	7	8	9	10
5'-0"	5.2	10.4	15.6	20.8	26.0	31.2	36.5	41.7	46.9	52.1
5'-3"	5.5	10.9	16.4	21.9	27.3	32.8	38.3	43.8	49.2	54.7
5'-6"	5.7	11.5	17.2	22.9	28.6	34.4	40.1	45.8	51.6	57.3
5'-9"	6.0	12.0	18.0	24.0	29.9	35.6	41.9	47.9	53.9	60.0
6'-0"	6.3	12.5	18.8	25.0	31.3	37.5	43.8	50.0	56.3	62.5
6'-3"	6.5	13.0	19.5	26.0	32.6	39.1	45.6	52.1	58.6	65.1
6'-6"	6.8	13.5	20.3	27.1	33.9	40.6	47.4	54.2	60.9	67.7
6'-9"	7.0	14.1	21.1	28.1	35.2	42.2	49.2	56.3	63.3	70.3
7'-0"	7.3	14.6	21.9	29.2	36.5	43.8	51.0	58.3	65.6	72.9
7'-3"	7.6	15.1	22.7	30.2	37.8	45.3	52.9	60.4	68.0	75.5
7'-6"	7.8	15.6	23.4	31.3	39.1	46.9	54.7	62.5	70.3	78.1
7'-9"	8.1	16.1	24.2	32.3	40.4	48.4	56.5	64.6	72.7	80.7
8'-0"	8.3	16.7	25.0	33.3	41.7	50.0	58.3	66.7	75.0	83.3
8'-3"	8.6	17.2	25.8	34.4	43.0	51.6	60.2	68.7	77.3	85.9
8'-6"	8.9	17.7	26.6	35.4	44.3	53.1	62.0	70.8	79.7	88.5
8'-9"	9.1	18.2	27.3	36.5	45.6	54.7	63.8	72.9	82.0	91.1
9'-0"	9.4	18.8	28.1	37.5	46.9	56.3	65.6	75.0	84.4	93.8
9'-3"	9.6	19.3	28.9	38.5	48.2	57.8	67.4	77.1	86.7	96.4
9'-6"	9.9	19.8	29.7	39.6	49.5	59.4	69.3	79.2	89.1	99.0
9'-9"	10.2	20.3	30.5	40.6	50.8	60.9	71.1	81.2	91.4	102
10'-0"	10.4	20.8	31.3	41.7	52.1	62.5	72.9	83.3	93.8	104
10'-6"	10.9	21.9	32.8	43.8	54.7	65.6	76.6	87.5	98.4	109
11'-0"	11.5	22.9	34.4	45.8	57.3	68.7	80.2	91.7	103	115
11'-6"	12.0	24.0	35.9	47.9	59.9	71.9	83.9	95.8	108	120
12'-0"	12.5	25.0	37.5	50.0	62.5	75.0	87.5	100	113	125
12'-6"	13.0	26.0	39.1	52.1	65.1	78.1	91.1	104	117	130
13'-0"	13.5	27.1	40.6	54.2	67.7	81.3	94.8	108	122	135
13'-6"	14.1	28.1	42.2	56.3	70.3	84.4	98.4	113	127	141
14'-0"	14.6	29.2	43.7	58.3	72.9	87.5	102	117	131	146
14'-6"	15.1	30.2	45.3	60.4	75.5	90.6	106	121	136	151
15'-0"	15.6	31.3	46.9	62.5	78.1	93.8	109	125	141	156
15'-6"	16.1	32.3	48.4	64.6	80.7	96.9	113	129	145	161
16'-0"	16.7	33.3	50.0	66.7	83.3	100	117	133	150	167
16'-6"	17.2	34.4	51.6	68.8	85.9	103	120	138	155	172
17'-0"	17.7	35.4	53.1	70.8	88.5	106	124	142	159	177
17'-6"	18.2	36.5	54.7	72.9	91.1	109	128	146	164	182
18'-0"	18.8	37.5	56.3	75.0	93.8	113	131	150	169	188
18'-6"	19.3	38.5	57.8	77.1	96.4	116	135	154	173	193
19'-0"	19.8	39.6	59.4	79.2	99.0	119	139	158	178	198
19'-6"	20.3	40.6	60.9	81.3	102	122	142	163	183	203
20'-0"	20.8	41.7	62.5	83.3	104	125	146	167	187	208
20'-6"	21.4	42.7	64.1	85.4	107	128	149	171	192	214
21'-0"	21.9	43.8	65.6	87.5	109	131	153	175	197	219
21'-6"	22.4	44.8	67.2	89.6	112	134	157	179	202	224
22'-0"	22.9	45.8	68.8	91.7	115	138	160	183	206	229
22'-6"	23.4	46.9	70.3	93.8	117	141	164	188	211	234
23'-0"	24.0	47.9	71.9	95.8	120	144	168	192	216	240
23'-6"	24.5	49.0	73.4	97.9	122	147	171	196	220	245
24'-0"	25.0	50.0	75.0	100	125	150	175	200	225	250
24'-6"	25.5	51.0	76.6	102	128	153	179	204	230	255
25'-0"	26.0	52.1	78.1	104	130	156	182	208	234	260
25'-6"	26.6	53.1	79.7	106	133	159	186	212	239	266
26'-0"	27.1	54.2	81.2	108	135	162	190	217	244	271
26'-6"	27.6	55.2	82.8	110	138	166	193	221	248	276
27'-0"	28.1	56.3	84.4	112	141	169	197	225	253	281
27'-6"	28.6	57.3	85.9	115	143	172	200	229	258	286
28'-0"	29.2	58.3	87.5	117	146	175	204	233	262	292
28'-6"	29.7	59.4	89.1	119	148	178	208	238	267	297
29'-0"	30.2	60.4	90.6	121	151	181	211	242	272	302
29'-6"	30.7	61.5	92.2	123	154	184	215	246	277	307
30'-0"	31.3	62.5	93.8	125	156	188	219	250	281	313

Dash-and-dot lines are drawn through centers of gravity A = area of section; I = moment of inertia; R = radius of gyration			
	$A = d^2$ $I_1 = \frac{d^4}{12}$ $I_2 = \frac{d^4}{3}$ $R_1 = 0.2887\ d$ $R_2 = 0.57774\ d$		$A = \frac{\pi d^2}{4} = 0.7854\ d^2$ $I = \frac{v d^4}{64} = 0.0491 d^4$ $R = \frac{d}{4}$
	$A = d^2$ $y = 0.7071\ d$ $I = \frac{d^4}{12}$ $R = 0.2887d$		$A = 0.8660\ d^2$ $I = 0.060\ d^4$ $R = 0.264\ d$
	$A = bd$ $I_1 = \frac{bd^3}{12}$ $I_2 = \frac{bd^3}{3}$ $R_1 = 0.2887\ d$ $R_2 = 0.5774\ d$		$A = 0.8284\ d^2$ $I = 0.055\ d^4$ $R = 0.257\ d$
	$A = bd$ $y = \frac{bd}{\sqrt{b^2 + d^2}}$ $I = \frac{b^3 d^3}{6(b^2 + d^2)}$ $R = \frac{bd}{\sqrt{6(b^2 + d^2)}}$		$A = \frac{bd}{2}$ $I_1 = \frac{bd^3}{36}$ $I_2 = \frac{bd^3}{12}$ $R_1 = 0.236\ d$ $R_2 = 0.408\ d$
	$A = bd$ $y = \frac{b \sin\infty + d \cos\infty}{2}$ $R = \frac{\sqrt{b^2 \sin^2(\infty + d^2 \cos^2\infty)}}{12}$		$A = \frac{d}{2}(b + b')$ $y = \frac{d(2b + b')}{3(b + b')}$ $y = \frac{d(b + 2b')}{3(b + b')}$ $I = \frac{d^2(b^2 + 4bb' + b'^2)}{36(b + b')}$ $R = \frac{d}{6(b + b')}\sqrt{2(b^2 + 4bb' + b'^2)}$

Dash-and-dot lines are drawn through centers of gravity

A = area of section; I = moment of inertia; R = radius of gyration



$$A = bt + b'_c$$

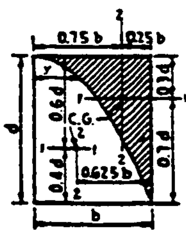
$$y = \frac{d^2b' + t^2(b-b')}{2(bt + b'_c)}$$

$$y_1 = d - y$$

$$I = \frac{b'y_1^3 + by^3 - (b-b')(y-t)^3}{3}$$

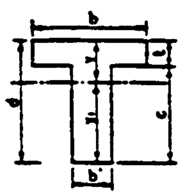
$$R = \sqrt{\frac{I}{A}}$$

Section of parabola



$$y^2 = \frac{b^2}{d}x$$

For parabola:	For compliment:
$A = \frac{2bd}{3}$	$A = \frac{bd}{3}$
$I_1 = \frac{8}{175}bd^3$	$I_1 = \frac{37}{2100}bd^3$
$I_2 = \frac{19}{480}b^3d$	$I_2 = \frac{1}{80}b^3d$



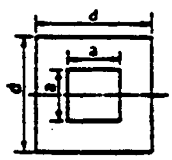
$$A = bt + b'_c$$

$$y = \frac{d^2b' + t^2(b-b')}{2(bt + b'_c)}$$

$$y_1 = d - y$$

$$I = \frac{b'y_1^3 + by^3 - (b-b')(y-t)^3}{3}$$

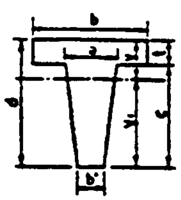
$$R = \sqrt{\frac{I}{A}}$$



$$A = d^2 - a^2$$

$$I = \frac{d^4 - a^4}{12}$$

$$R = \sqrt{\frac{d^2 + a^2}{12}}$$



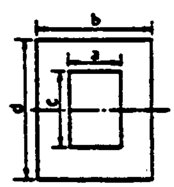
$$A = bt + \frac{c(a+b')}{2}$$

$$y = \frac{3bt^2 + 3b'c(d+t) + c(a-b')(c+3t)}{3[2bt + c(a+b')]}$$

$$y_1 = d - y$$

$$I = \frac{4bt^2 + c^3(3b' + a)}{12} - A(y-t)^2$$

$$R = \sqrt{\frac{I}{A}}$$

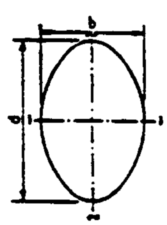


$$A = bd - ac$$

$$I = \frac{bd^3 - ac^3}{12}$$

$$R = \sqrt{\frac{bd^3 - ac^3}{12(bd - ac)}}$$

Elipse



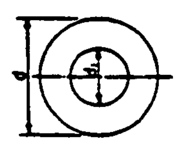
$$A = 0.7854bd$$

$$I_1 = \frac{\pi bd^3}{64} = 0.0491bd^3$$

$$I_2 = \frac{\pi b^3d}{64} = 0.0491b^3d$$

$$R_1 = \frac{d}{4}$$

$$R_2 = \frac{b}{4}$$

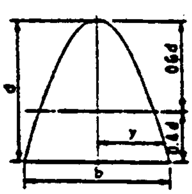


$$A = \frac{\pi(d^2 - d_1^2)}{4} = 0.7854(d^2 - d_1^2)$$

$$I = \frac{\pi(d^4 - d_1^4)}{64} = 0.0491(d^4 - d_1^4)$$

$$R = 1/4\sqrt{d^2 + d_1^2}$$

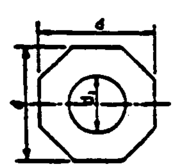
Parabola



Equation:

$$y^2 = \frac{b^2}{4d^x}$$

$$A = \frac{2bd}{3}$$



$$A = 0.8284d^2 - 0.7854d_1^2$$

$$= 0.7854(1.055d^2 - d_1^2)$$

$$I = 0.0547d^4 - 0.0491d_1^4$$

$$= 0.0491(1.115d^4 - d_1^4)$$

$$= 0.0491[(1.056)^2d^4 - d_1^4]$$

$$R = 1/4\sqrt{1.056d^2 + d_1^2}$$

Conversion Factors*

To convert from	to	multipy by †
	Length	
inch	millimeter (mm)	25.4E
foot	meter (m)	0.3048E
yard	meter (m)	0.9144E
mile (statute)	kilometer (km)	1.609
	Area	
square inch	square centimeter (cm^2)	6.452
square foot	square meter (m^2)	0.09290
square yard	square meter (m^2)	0.8361
	Volume (Capacity)	
ounce	cubic centimeter (cm^3)	29.57
gallon	cubic meter (m^3)‡	0.003785
cubic inch	cubic centimeter (cm^3)	16.4
cubic foot	cubic meter (m^3)	0.02832
cubic yard	cubic meter (m^3)‡	0.765
	Force	
kilogram-force	newton (N)	9.807
kip-force	kilonewton (kN)	4.448
pound-force	newton (N)	4.448
	Pressure or Stress (Force per Area)	
kilogram-force/square meter	pascal (Pa)	9.807
kip-force/square inch (ksi)	megapascal (MPa)	6.895
newton/square meter (N/m^2)	pascal (Pa)	1.000E
pound-force/square foot	pascal (Pa)	47.88
pound-force/square inch (psi)	pascal (Pa)	6895
	Bending Moment or Torque	
inch-pound-force	newton-meter (Nm)	0.1130
foot-pound-force	newton-meter (Nm)	1.356
meter-kilogram-force	newton-meter (Nm)	9.807
	Mass	
ounce-mass (avoirdupois)	gram (g)	28.35
pound-mass (avoirdupois)	kilogram (kg)	0.4536
ton (metric)	megagram (Mg)	1.000E
ton (short, 2000 lbm)	megagram (Mg)	0.9072
	Mass per Volume	
pound-mass/cubic foot	kilogram/cubic meter (kg/m^3)	16.02
pound-mass/cubic yard	kilogram/cubic meter (kg/m^3)	0.5933
pound-mass/gallon	kilogram/cubic meter (kg/m^3)	119.8
	Temperature§	
deg Fahrenheit (F)	deg Celsius (C)	$t_C = (t_F - 32)/1.8$
deg Celsius (C)	deg Fahrenheit (F)	$t_F = 1.8t_C + 32$

*This selected list gives practical conversion factors of units found in concrete technology. The reference source for information on SI units and more exact conversion factors is "Standard for Metric Practice" ASTM E 380. Symbols of metric units are given in parentheses.

†E indicates that the factor given is exact.

‡ One liter (cubic decimeter) equals 0.001 m^3 or 1000 cm^3.

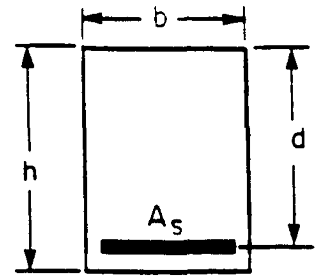
§ These equations convert one temperature reading to another and include the necessary scale corrections. To convert a difference in temperature from Fahrenheit degrees to Celsius degrees, divide by 1.8 only, i.e. a change from 70 to 88 F represents a change of 18 F deg or 18/1.8 = 10 C deg.

DESIGN EXAMPLES

FLEXURE

FLEXURE EXAMPLE 1 - Determination of tension reinforcement area for rectangular beam subject to simple bending; no compression reinforcement

For a rectangular section subject to factored bending moment M_u, determine the area of reinforcement required, with the dimensions given. Assume interior construction, not exposed to weather.



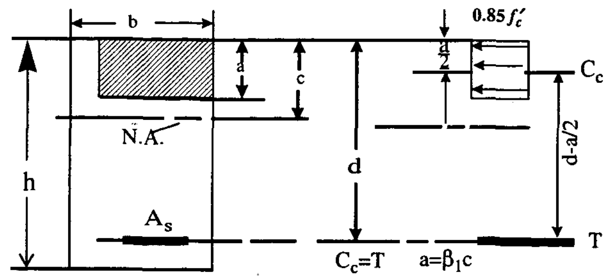
Given:

M_u = 90 ft-kips ⇒ $M_n = M_u/\phi$ = 90/0.9 = 100 ft-kips
f'_c = 4000 psi
f_y = 60,000 psi
b = 10 in.
h = 20 in.

ACI 318-95 Section	Procedure	Calculation	Design Aid
7.7.1	**Step 1A**--Estimate d by allowing for the radius of longitudinal bars, a stirrup, and clear cover. d = h - allowance	Reasonable allowance are 2.5 in. for interior exposure, 3.0 in. for exterior exposure. Estimate d = 20 - 2.5 = 17.5 in.	
Chapter 10	**Step 2A**--Determine F for the section.	For b = 10 in. and d = 17.5 in., F = 0.255	FLEXURE 5
	Step 3A--Compute $K_n = M_n/F$. Required nominal strength M_n equals M_u/ϕ.	K_n = 100/0.255 = 392	
	Step 4A--Determine ρ or a_n.	For K_n = 392, interpolate ρ = 0.0070 or a_n = 4.70	FLEXURE 2.2
	Step 5A--Compute A_s = ρbd or = $M_n/(a_n d)$	A_s = 0.0070x10x17.5 = 1.22 sq in. or = 100/(4.70 x 17.5) = 1.22 sq in.	
	Alternate procedure with FLEXURE 1 **Step 1C**--Estimated	d = h - 2.5 = 17.5 in.	
	Step 2C--Determine preferred ρ.	For f'_c = 4000 psi and f_y = 60.000 psi, read preferred ρ = 0.0107	FLEXURE 1
	Step 3C--Compute A_s = ρbd	A_s = (0.0107)(10)(17.5) = 1.87 sq in.	
	Step 4C--Check $\phi M_n \geq M_u$	$a = \dfrac{A_s f_y}{0.85 f'_c b} = \dfrac{(1.87)(60)}{(0.85)(4)(10)} = 3.3$ *in.* $\phi M_n = \phi A_s f_y(d - a/2)$ = 0.9(1.87)(60)(17.5 - 3.3/2)/12 = 133 ft-k > 90 ft-k OK	

FLEXURE EXAMPLE 2—Design of rectangular beam subject to simple bending; no compression reinforcement

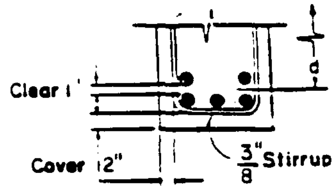
For a rectangular section subject to factored bending moment M_u, determine beam depth h and reinforcement A_s, assuming $\rho = 1/2\rho_{max}$.



Given:

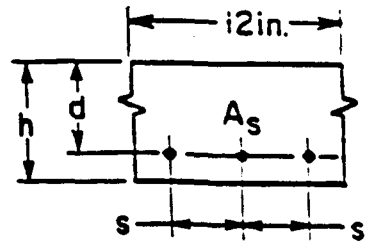
M_u = 139.5 ft-kips ⇒ $M_n = M_u/\phi$ = 139.5/0.9 = 155 ft-kips
f'_c = 4000 psi
f_y = 60,000 psi
Beam is exposed to weather

ACI 318-95 Section	Procedure	Calculation	Design Aid
10.3.3	**Step 1**—Determine beam size on the basis of the assumed reinforcement ratio.	For f'_c = 4000 psi, f_y = 60,000 psi	FLEXURE 2.2
10.3.2		$\rho_{max} = 0.75\ \rho_b = 0.0214$	
	Required $M_n = M_u/\phi$	$\rho = 0.5\rho_{max} = (0.5)(0.0214) = 0.0107$	
	Obtain coefficients a_n and K_n Compute F,	For ρ = 0.0107, read a_n = 4.53, K_n = 580	FLEXURE 2.2
	$F = \frac{M_n}{K_n}$	$F = \frac{155}{580} = 0.27$	
	Try a 10 in. width and find d	For b = 10 in. and F = 0.27, d = 18 in.	FLEXURE 5
	Step 2—Determine reinforcement (there are several convenient ways).		
	$A_s = \frac{M_n}{a_n d}$	$A_s = \frac{155}{(4.53)(18)} = 1.90$ sq in.	FLEXURE 2.2
	Or use $A_s = \rho bd$	$A_s = 0.0107 \times 10 \times 18 = 1.93$ sq in.	FLEXURE 2.2
7.7.1	**Step 3**—Select bars.	5 #6 bars provide A_s = 2.20 sq in.	REINFORCEMENT 2
7.6.1, 7.6.2	Check minimum width.	with 3 #6 in one layer, b = 9 in.	REINFORCEMENT 10
	Check distribution of reinforcement as governed by crack control.	or b_{min} = 7 + 1.75 = 8.75 < 10 in. width OK	REINFORCEMENT 9
		5 #6 bars in 2 layers, maximum width = 21.0 > 10 in.	REINFORCEMENT 14

ACI 318-95 Section	Procedure	Calculation	Design Aid
7.6.1, 7.6.2 7.7.1 3.3.3		Approximate check on clear distance between bars. $= \frac{10 - 2(2) - 2\left(\frac{3}{8}\right) - 3(0.750)}{2}$ $= 1\frac{1}{2}$ in. > 1 in. OK	
	Step 4--Determine total depth of beam. Round off upward because A_s provided is slightly less than that required.	Required d = 18.0 in. ½ clear distance = 0.5 in. Bar diameter = 0.75 in. Stirrup diameter = 0.375 in. Clear cover = 2.0 in. 21.6 in. Use h = 22 in with A_s = 2.20 sq in.	REIN-FORCE-MENT 1 REIN-FORCE-MENT 1
	Step 5--Check $\phi M_n \geq M_u$	$a = \frac{A_s f_y}{0.85 f_c' b} = \frac{(2.20)(60)}{(0.85)(4)(10)} = 3.88$ *in.* $\phi M_n = \phi A_s f_y (d - a/2)$ $= 0.9(2.20)(60)(18 - 3.88/2)/12$ $= 159.0$ ft-k > 139.5 ft-k OK	
9.5.2	**Step 6**--Check deflection, if necessary. For members supporting elements *not likely* to be damaged by large deflection, deflection must be checked if h is less than that indicated by Table 9.5(a). If the member supports element *likely* to be damaged by large deflection, deflection must be checked in all cases.	Omitted in this example.	Commentary on FLEXURE 2

FLEXURE EXAMPLE 3 - Selection of slab thickness and tension reinforcement for slab subject to simple bending; no compression reinforcement

For a slab subject to a factored bending moment M_u, determine the thickness h and the reinforcement required.



Given:

M_u = 10.8 ft-kips ⇒ $M_n = M_u/\phi$ = 10.8/0.9 = 12 ft-kips

f'_c = 3000 psi

f_y = 60,000 psi

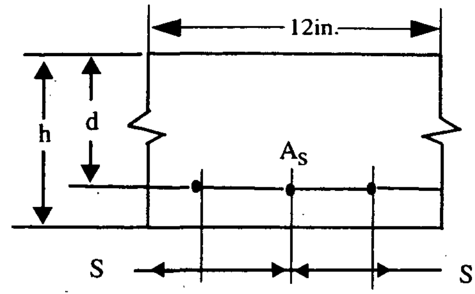
Subject to exterior exposure

ACI 318-95 Section	Procedure	Calculation	Design Aid
	Step 1--Unless a certain thickness is desired, select a trial selection such that $\rho = (3/8)\rho_b$, a value that will provide sections with good characteristics of ductility, bar placement and stiffness. Look up A_s	Required $M_n = M_u/\phi$ For $\rho = (3/8)\rho_b$ and M_n = 12 ft-k, d = 6.2 in. and A_s = 0.59 sq in. A_s = (0.008)(6.2)(12) = 0.59	FLEXURE 6.2.1 FLEXURE 1
	Step 2--Select bars and spacing. (Maximum s = 3h)	#5 @ 6 in. provides A_s = 0.62 sq in./ft, OK; d = 6.20 in.	REIN-FORCE-MENT 15
7.7.1 ACI 318-95 7.7	**Step 3**--Determine h. Slab soffits are not usually considered directly exposed to weather.	d = 6.20 Bar radius = 0.31 Clear cover = 0.75 ------- 7.26 Use 7½ in. slab Then d = 6.44 in.	REIN-FORCE-MENT 1
	Step 4--Recompute A_s required and revise bar and spacing selection if desirable.	For d = 6.44 in. and required M_n = 12 ft-kips, find required A_s = 0.40 sq in./ft #5 @ 9 in. provides A_s = 0.41 in.^{2}	FLEXURE 6.2.1
10.6.4	**Step 5**--Check distribution for crack control.	Exterior #5 @ 60,000 psi, No check required s_{max} = 18 in., so s = 9 in. OK	REIN-FORCE-MENT 16
7.12 10.5.3	**Step 6**--Check minimum reinforcement $A_{s\,min}$ = 0.0018bh. This amount is required also in the direction transverse to the main reinforcement to serve as temperature and shrinkage reinforcement.	$A_{s\,min}$ = 0.0018 x 12 x 7.5 = 0.16 sq in./ft So #5 @ 9 in. OK	
9.5.2	**Step 7**--Deflection must be checked if h is less than that indicated on Table 9.5(a), and must be checked in all cases if the member supports elements likely to be damaged by large deflection.		Commentary on FLEX-URE 2

ACI 318-95 Section	Procedure	Calculation	Design Aid
	Step 8--Check $\phi M_n \geq M_u$	$a = \frac{A_s f_y}{0.85 f_c' b} = \frac{(0.41)(60)}{(0.85)(3)(12)} = 0.80\ in.$ $\phi M_n = \phi A_s f_y (d - a/2)$ $= 0.9(0.41)(60)(6.44-0.80/2)/12$ $= 11.1$ ft-k > 10.8 ft-k OK	

FLEXURE EXAMPLE 4—Selection of slab thickness and tension reinforcement area for slab subject to simple bending; no compression reinforcement: given $\rho = 0.5\rho_b$ or slab thickness

For a given factored moment M_u per foot slab width, determine the necessary thickness h and reinforcement A_s, assuming $\rho = 0.5\rho_b$. Also, if thickness is given as 11 in., find A_s.



Given:

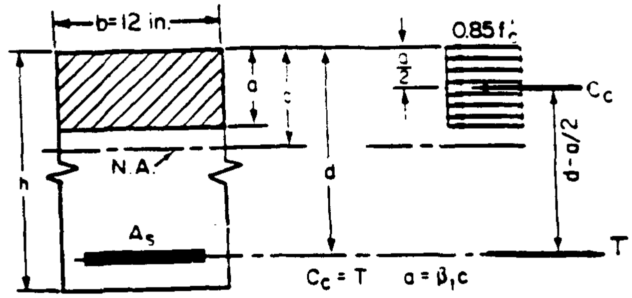
$M_u = 63$ ft-kips$\Rightarrow M_n = M_u / \phi = 63/0.9 = 70$ ft-kips
$f'_c = 4000$ psi
$f_y = 60{,}000$ psi
Not exposed to weather

ACI 318-95 Section	Procedure	Calculation	Design Aid
Chapter 10	**Method A. using FLEXTURE 6** **Step 1A**—At the line $\rho = 0.5\rho_b$, move to $M_n = M_u / \phi = 70$ ft-kips	Read $A_s = 1.68$ sq in./ft and $d = 9.7$ in.	FLEXTURE 6.4.1
	Step 2A—Select bars and spacing.	Try #8 @ 5.5 in. to obtain $A_s = 1.72$ sq in./ft	REIN-FORCE-MENT 15
	Step 3A—Check maximum spacing (crack control).	$S_{max} = 18$ in., so 5.5 is OK	REIN-FORCE-MENT 16
7.7	**Step 4A**—Determine slab thickness h. Round off h upward because A_s provided is slightly less than that required	d = 9.70 Bar radius = 0.50 Clear cover = 0.75 10.95 Use h = 11 in.	REIN-FORCE-MENT 1
10.5.3 and 7.12	**Step 5A**—Check minimum steel. For f_y = 60,000 psi, $A_s \geq 0.0018$ bh. This amount is also required in the direction transverse to the main reinforcement to serve as temperature and shrinkage reinforcement	Minimum As = 0.0018 x 12 x 11 = 0.24 sq in./ft 1.72 sq in./ft > 0.24 sq in./ft OK	
9.5.2	**Step 6A**—For members supporting elements *not likely* to be damaged by large deflection, deflection must be checked if h is less than that indicated by Table 9.5(a). If the member supports elements *likely* to be damaged by large deflection, deflection must be checked in all cases.	Omitted in this example	Commentary on FLEX-URE 2

ACI 318-95 Section	Procedure	Calculation	Design Aid
	Method B, using FLEXURE 6 and FLEXURE 2 **Step 1B**--Determine $0.75\rho_b$ from maximum value in the table, and compute $\rho = 0.5\rho_b$	$\rho_{max} = 0.75\rho_b = 0.0214$ $\rho = \frac{0.5}{0.75} \times 0.0214 = 0.0143$	FLEXURE 2.2
	Step 2B--Determine K_n.	For $\rho = 0.0143$, $K_n = 749$ and $a_n = 4.37$	FLEXURE 2.2
	Step 3B--Compute $F = M_n/K_n$.	$F = 70/749 = 0.093$	
	Step 4B--Determine d required.	For 12 in. width and F = 0.093, d = 9.7 in.	FLEXURE 5
	Step 5B--Compute $A_s = M_n/a_nd$.	$A_s = 70/(4.37 \times 9.7) = 1.65$ sq in./ft #8 @ 5.5 in. is OK A_s provided = 1.72 sq in.	REIN-FORCE-MENT 15
	Step 6B through 10B--Repeat Steps 2 through 6 of Method A.		
	Method C, if slab thickness is given as 11 in. **Step 1C**--Assume size of reinforcement and determine effective depth (three methods given) (a) Determine A_s from F, K_n, a_n, or (b) ρ from FLEXURE 2.2	Use #8 bars $d = 11.00 - 0.75 - 1.0/2 = 9.75$ $F = \frac{12x(9.75)^2}{12{,}000} = 0.095$ $K_n = 70/0.095 = 737$ $a_n = 4.38$, $\rho = 0.0140$ $A_s = \frac{M_n}{a_n d} = \frac{70}{4.38x9.75}$ $= 1.64$ *sq in./ft, or* $A_s = \rho bd = 0.0140 \times 12 \times 9.75$ $= 1.64$ sq in./ft	FLEXURE 2.2
	Step 2C through 6C--Repeat Steps 2 through 6 of Method A.		

FLEXURE EXAMPLE 5 - Selection of slab thickness (for deflection control) and tension reinforcement for slab subject to simple bending; no compression reinforcement

For a 12 in. width of slab subject to factored bending moment M_u, determine the slab thickness h and the reinforcement required. Assume continuous ends with ℓ = 29 ft.



Given:

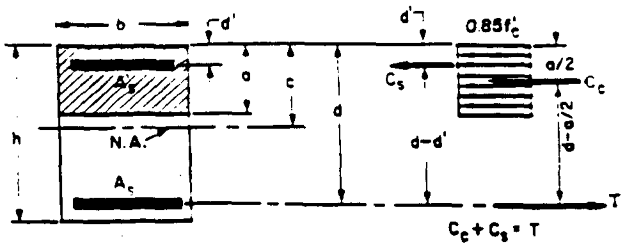
M_u = 36 ft-kips ⇒ $M_n = M_u/\phi$ = 36/0.9 = 40 ft-kips
f'_c = 4000 psi
f_y = 60,000 psi
b = 12 in.
Slab not exposed to weather

ACI 318-95 Section	Procedure	Calculation	Design Aid
9.5.1, 9.5.2 7.7.1 10.3.3, 10.5.3	**Step 1**--Select a trial section. Required $M_n = M_u/\phi$. In the absence of a desired thickness or other restrictions, slab depths can be selected for a ρ value near $0.5\rho_{max}$--which is the "preferred" ρ or typical ρ and can be read from FLEXURE 1--or, for h_{min} using Table 9.5(a). This example assumes non-structural elements are not likely to be damaged. Therefore, using Table 9.5(a), compute $h_{min} = l/28$ Allow about 1½ in. for slab concrete cover and the radius of reinforcing bars. Try d = 11 in. Find A_s (Note: For M_n = 40 ft-kips, any depth between 6 and 17 in. might be used)	h_{min} = 12 x 29/28 = 12.4 in. d = h_{min} - 1.5 = 12.4-1.5 = 10.9 in. For d = 11 in and M_n = 40 ft-k/ft, A_s = 0.80 sq in./ft	FLEXURE 1 FLEXURE 2 Commentary FLEXURE 6.4.1
7.6	**Step 2**--Select the bar size and spacing. Assume a bar size, and compute required spacing: c-c bar spacing = $12A_b/A_s$	Try #6 bars: c-c bar spacing = 12(0.44)/0.80 = 6.6 in. Use 6.0 in. spacing.	
10.6	**Step 3**--Check distribution of flexural reinforcement.	The maximum spacing for #6 bars in a one-way slab is 18 in. OK	REINFORCEMENT 16
7.7.1	**Step 4**--Compute h and round off upward because A_s provided is slightly less than that required.	h = d + bar radius + cover = 11 + 0.38 + 0.75 = 12.13 in. Use h = 12½ in.	REINFORCEMENT 1

ACI 318-95 Section	Procedure	Calculation	Design Aid
9.5.1,9.5.2	**Step 5**--For members supporting elements *not likely* to be damaged by large deflection, check deflection if thickness is less than that given in Table 9.5(a) of ACI 318-95. If the member supports elements *likely* to be damaged by large deflection, deflection must be checked in all cases.	$h = 12.5$ in. $> h_{min} = 12.4$ in.; $\therefore$ no deflection check needed [if Table 9.5(a) is applicable]	

FLEXURE EXAMPLE 6 - Determination of tension and compression reinforcement areas for rectangular beam subject to simple bending; compression reinforcement is found not to yield

For a rectangular beam of the given dimensions subject to factored bending moment M_u, determine the required tension and compression steel areas.



Given:

M_u = 189 ft-kips ⇒ $M_n = M_u/\phi$ = 189/0.9 = 210 ft-kips

f'_c = 5000 psi

f_y = 60,000 psi

b = 12 in.

d = 12 in.

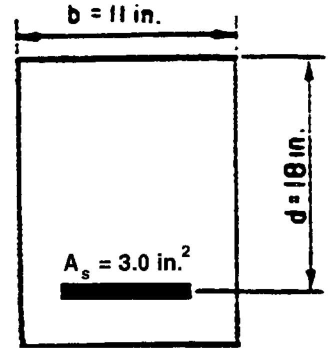
ACI 318-95 Section	Procedure	Calculation	Design Aid
ACI318R-95 10.3	**Step 1**--Determine the strength of the section when ρ_{max} is used without compression steel.	For f'_c = 5000 psi and f_y = 60,000 psi, read ρ_{max} = 0.0252, K_n = 1240, a_n = 4.11, and c/d = 0.443	FLEXURE 2.3
	Determine F	For b = 12 in. and d = 12 in., read F = 0.144	FLEXURE 5
	Compute $M_n = K_nF$	M_n = 0.144 x 1240 = 179 ft-kips < M_u/ϕ (= 210 ft-kips)	
	If this strength is less than the factored moment M_u, compression steel is required, and the computed strength M_n becomes M_{n1} and A_s becomes A_{s1}.		
	Compute $M_{n2} = M_n - M_{n1}$	M_{n2} = 210 - 179 = 31 ft-kips	
	Compute $A_{s1} = \rho_{max}bd$	A_{s1} = 0.0252 x 12 x 12 = 3.6 sq in.	
	Step 2--Determine the area A_{s2} of additional tension steel. Find a'_n.	For d'/d = 0.2, f'_c = 5000 psi, and f_y = 60,000 psi, read a'_n = 4.00	FLEXURE 3.2
	$A_{s2} = \dfrac{M_{n2}}{a'_n d}$	$A_{s2} = \dfrac{31}{4.00 \times 12} = 0.65$ sq in.	
	Compute total $A_s = A_{s1} + A_{s2}$	A_s = 3.60 + 0.65 = 4.25 sq in.	FLEXURE 4 Commentary
	Step 3--Determine the area A'_s of compression steel.	Omitted in this example	
	Compare a'_n with a_n''; if $a_n'' < a'_n$ compression steel has not yielded and a_n'' should be used in computing A'_s.		
10.2, 10.3	$A_s' = \dfrac{M_{n2}}{a_n'' d}$		

ACI 318-95 Section	Procedure	Calculation	Design Aid
10.3.3 and ACI 318R-95 Table 10.3.2	**Step 4**--Check that $\rho < \rho_{max}$.	For f'_c = 5 ksi, f_y = 60 ksi, ρ'/ρ = 0.89/4.25 = 0.21, and d'/d = 0.2, interpolate to find ρ_{max} = 0.0298 $\rho = \frac{4.25}{12 \times 12} = 0.0295 < 0.0298$ so $\rho < \rho_{max}$	
7.11.1	**Step 5**--Compression steel must be enclosed by ties or stirrups.	Omitted in this example	
9.5.2	**Step 6**--Check deflection if beam thickness is less than that given in Table 9.5(a), or if member is supporting or attached to members likely to be damaged by large deflection.	Omitted in this example	

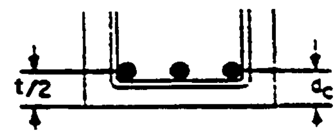
REINFORCEMENT

REINFORCEMENT EXAMPLE 1—For rectangular beams subject to simple bending, selection of reinforcement satisfying bar spacing and cover requirements and crack control provisions (using REINFORCEMENT 8.1, 8.2, or 11)

Select reinforcement for beam shown. Use one layer of bars.



Given:
f'_c = 4000 psi
f_b = 60,000 psi
b (= b_w) = 11 in.
Exterior exposure
ACI cover requirements
#4 stirrups

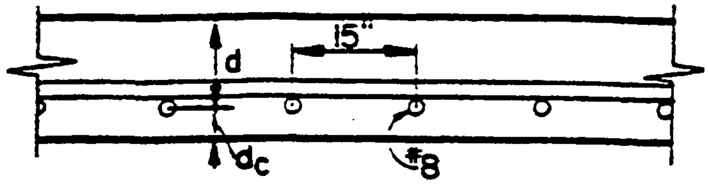


ACI 318-95 Section	Procedure	Calculation		Design Aid
	Step 1—Select trial size(s) of bars satisfying spacing and cover requirements and crack control provisions.	For A_s = 3.0 in.^{2} and one layer, read 3 #9 and 5 #7 bars		REINFORCEMENT 14
		3 #9 bars	5 #7 bars	
7.6 7.7.1(b)	Check whether given beam width ≥ minimum width satisfying bar spacing and cover requirements.	given b = 11 in. Min. b_w = [10 + 3/4 (for #4 stirrups)] = 11 in. ∴ 3 #9 bars OK	given b = 11 in. Min. b_w = [13 + 3/4 (for #4 stirrups)] = 14 in. ∴ 5 #7 bars OK	REINFORCEMENT 14
10.6.4	Check whether given beam width ≤ maximum width satisfying crack control provisions.	given b = 11 in.; max. b_w = 15 in. ∴ 3 #9 bars OK		REINFORCEMENT 14
	Step 2—Using REINFORCEMENT 8.1, verify that reinforcement selected satisfies crack control provisions.			
7.7.1(b)	d_c = 2 in. clear cover to #9 bars + $1/2d_b$, in.	$d_c = 2 + 1/2(1.128) = 2.564$ in.		REINFORCEMENT 1
		For d_c = 2.564 in.; f_y = 60,000 psi; and exterior exposure, find by interpolation A = 25.5 in.^{2}		REINFORCEMENT 8.1
10.6.4	Max. $b_w = \frac{An}{t}$			
	where $t = 2d_c$	$t = 2d_c = 2(2.564) = 5.128$ in. Max. $b_w = \frac{(25.5)(3)}{5.128} = 14.9$ in.		

ACI 318-95 Section	Procedure	Calculation	Design Aid
		given b = 11 in. < Max. b_w = 14.9 in. ∴ 3 #9 bars satisfy crack control provisions.	
10.6.4	**Step 3**—Using REINFORCEMENT 8.2, verify that reinforcement selected satisfies crack control provisions. $\text{Max. } b_w = n\frac{b_w t}{A_s}\left(\frac{A_s}{t}\right)$	For #9 bar, f_y = 60 ksi, and exterior exposure read $b_w t/A_s$ = 25 $\text{Max. } b_w = 25\left(\frac{3.0}{5.128}\right) = 14.6 \text{ in.}$ given b = 11 in. < Max. b_w = 14.6 in. ∴ 3 #9 bars satisfy crack control provisions.	REINFORCEMENT 8.2
10.6.4	**Step 4**—Using REINFORCEMENT 11, verify that reinforcement selected satisfies crack control provisions. $\text{Max. } b_w = n\frac{b_w \text{max.}}{n}$	For #9 bars, exterior exposure, one layer of bars, and f_y = 60 ksi, read b_w/n = 4.97 in. Max. b_w = 3(4.97) = 14.9 in. given b = 11 in. < Max. b_w = 14.9 in. ∴ 3 #9 bars satisfy crack control provisions.	REINFORCEMENT 11

REINFORCEMENT EXAMPLE 2—For a one-way slab, verification that reinforcement satisfies spacing and cover requirements and crack control provisions (using REINFORCEMENT 16)

Verify that bar spacing in the slab shown does not exceed maximum allowed by ACI 318-95, Section 7.6.5, and meets crack control provisions



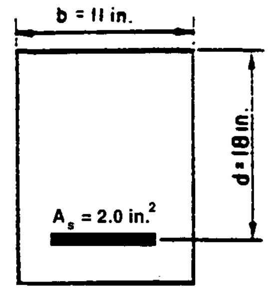
Given:
f'_c = 4000 psi
f_y = 60,000 psi
h = 7 in.
Interior exposure
β (= ratio of distances to the neutral axis from the extreme tension fiber and from the centroid of main reinforcement) = 1.25

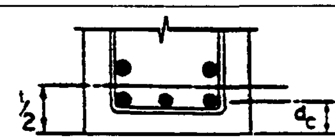
ACI 318-95 Section	Procedure	Calculation	Design Aid
7.6.5	**Step 1**—Check whether bar spacing is less than $3h$ or 18 in.	$3h = 3(7) = 21$ in. given s = 15. in. < $3h$ = 21 in. given s = 15 in. < 18 in. ∴ spacing satisfies ACI 318-95, Section 7.6.5.	
	Step 2—Using REINFORCEMENT 16, check whether bar spacing satisfies crack control provisions		
7.7.1(c)	d_c = clear cover + $(1/2)d_b$	For interior exposure and #8 bars, clear cover for slab is 3/4 in. $d_c = 3/4 + (1/2)(1.000) = 1.250$ in.	
ACI 318R-95 10.6.4	Adjust z for β associated with this slab (as opposed to $\beta = 1.2$ for beams).	For beams ($\beta = 1.20$) and $z = 175$ kips/in. For slabs $\beta = 1.25$ and $z = \frac{1.20}{1.25}(175) = 168$ kips/in.	
10.6.4	$z = f_y \sqrt[3]{d_c A}$ where $f_s = 0.6f_y$, ksi Max. $A = \frac{1}{d_c}\left(\frac{z}{0.6f_y}\right)^3$, in.^{2} Max. $A = 2d_c(s_{max})$	$A = \frac{1}{1.250}\left(\frac{168}{36}\right)^3 = 81.3$ in.^{2} Max. $s = \frac{81.3}{2(1.250)} = 32.5$ in. > 15 in. given ∴ spacing of 15 in. satisfies crack control provisions.	Commentary on REINFORCEMENT 8
7.65 10.6.4 Commentary on 10.6.4	**Step 3**—Using REINFORCEMENT 16, check whether bar spacing satisfies crack control provisions	For interior exposure, #8 bars, and f_y = 60 ksi, read Max. s = 18in. > 15 in. given ∴ spacing of 15 in. satisfies crack control provisions.	REINFORCEMENT 16

REINFORCEMENT EXAMPLE 3—For rectangular beam subjected to simple bending, selection of reinforcement (found to require two layers) satisfying bar spacing and cover requirements and crack control provisions (verified using REINFORCEMENT 8.1)

Select reinforcement satisfying bar spacing and cover requirements and crack control provisions

Given:
f_c' = 4000 psi
f_y = 60,000 psi
b (= b_w) = 11 in.
Exterior exposure
#4 stirrups



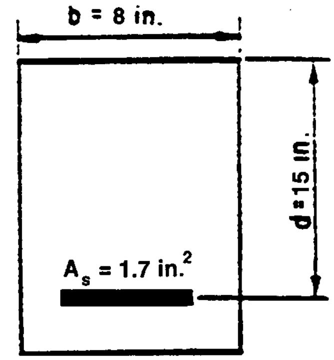
ACI 318-95 Section	Procedure	Calculation	Design Aid
	Step 1—Select trial size of bars satisfying spacing and cover requirements and crack control provisions.	For A_s = 2.0 in., read 2 #9 in one layer	REINFORCEMENT 14
7.6 7.7.1(b)	Check whether given beam width ≥ minimum beam width satisfying bar spacing and cover requirements.	For 2 #9 bars, read Min. b = 8.25 in. given b = 11 in. > Min. b = 8.25 in. ∴ 2 # 9 bars satisfy bar spacing and cover requirements.	REINFORCEMENT 9 (Note 3) REINFORCEMENT 14
	Check whether given beam width ≤ maximum beam width satisfying crack control provisions.	given b = 11 in. > Max. b = 10 in. ∴ 2 # 9 bars do not satisfy crack control provisions.	
7.6 7.7.1(b)	**Step 2**—Select another trial reinforcement in two layers.	Find that a two-layer reinforcement for A_s slightly larger than 2.0 in.^{2} is 5 #6 bars at A_s = 2.20 in.^{2} For 3 #6 in lower layer: given b = 11 in. > Min. b_w = 9.5 in. For 5 #6 in two layers: given b = 11 in. < Max. b_w = 21.0 in. ∴ 5 # 6 satisfy crack control provisions.	REINFORCEMENT 9 (Note 3) REINFORCEMENT 14
7.7.1(b)	**Step 3**—Using REINFORCEMENT 8.1, verify that reinforcement selected satisfies crack control provisions. d_c = clear cover to d_s + (1/2)d_b, in.	 d_c = 1.5 + 0.500 + (1/2)(0.750) = 2.375 in. For #6 bars, d_c = 2.375 in.; f_y = 60 ksi; and exterior exposure, find by interpolation Max. A = 27.6 in.^{2}	REINFORCEMENT 8.1

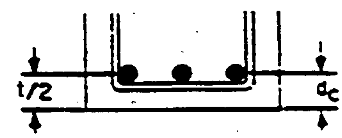
ACI 318-95 Section	Procedure	Calculation	Design Aid
7.6.2	$\frac{t}{2}$ = distance from extreme tension fiber to centroid of reinforcement Max. $b_w = \frac{An}{t}$, in.	$\frac{t}{2} = 2.375 + \frac{5-3}{5}\left(\frac{0.750}{2} + 1.00 + \frac{0.750}{2}\right) = 3.075$ in. $t = 2(3.075) = 6.15$ in. Max. $b_w = \frac{(27.6)(5)}{6.15} = 22.4$ in. given $b = 11$ in. < Max. $b_w = 22.4$ in. ∴ 5 #6 bars satisfy crack control provisions.	

REINFORCEMENT EXAMPLE 4—For rectangular beam subject to simple bending, selection of reinforcement satisfying bar spacing and cover requirements and crack control provisions (verified using REINFORCEMENT 11)

Select reinforcement satisfying bar spacing and cover requirements and crack control provisions.

Given:
f'_c = 5000 psi
f_y = 60,000 psi
b (= b_w) = 8 in.
Interior exposure
#4 stirrups



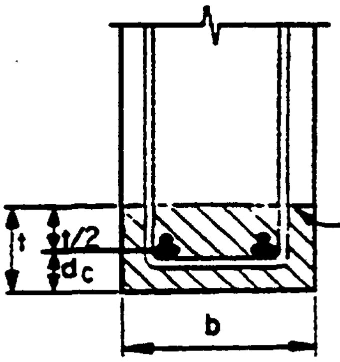


ACI 318-95 Section	Procedure	Calculation		Design Aid
	Step 1—Select trial reinforcement satisfying spacing and cover requirements and crack control provisions.	For A_s = 1.7 in.^{2}, read 4 #6 bars in one or two layers gives A_s = 1.76 in.^{2}		REINFORCEMENT 14
		4 #6 bars, 1 layer	4 #6 bars, 2 layers	
7.6.1, 7.6.2, 7.7.1(c)	Check whether given beam width ≥ minimum width satisfying bar spacing and cover requirements.	given b = 8 in. < Min. b = [10.5 + 3/4 in. for #4 stirrups] = 11.25 in. ∴ 4 #6 bars cannot be used in beam with b = 8 in.	given b = 8 in. > Min. b = [7.0 + 3/4 in for #4 stirrups] = 7.75 in. OK	REINFORCEMENT 14
10.6.4	Check whether given beam width ≤ maximum width satisfying crack control provisions.		given b = 8 in. < Max. b = 30.0 in.	REINFORCEMENT 14
		∴ use 4 #6 bars in 2 layers.		
10.6.4	**Step 2**—Using REINFORCEMENT 11, verify that reinforcement selected satisfies crack control provisions.	For interior exposure, #6 bars in two layers, and f_y = 60 ksi, read b_{wmax} = 4(7.44) = 29.76 in.		REINFORCEMENT 11
	max. $b = n\dfrac{b_{w,max}}{n}$	given b = 8 in. < max. b = 29.8 in.		
		∴ 4 #6 bars in 2 layers satisfies crack control provisions.		

REINFORCEMENT EXAMPLE 5—Determination of maximum width of a beam reinforced with bundled bars satisfying crack control provisions

Find maximum width of the beams shown which will satisfy crack control provisions.

Given:
f_y = 60,000 psi
f_s = 36,000 psi
$b\ (= b_w)$ = 11 in.
Interior exposure
Two bundles of 3 #8 bars
#4 stirrups



ACI 318-95 Section	Procedure	Calculation	Design Aid
10.6.4	**Step 1**—Calculate maximum beam width satisfying control provisions.		
7.7.1(c)	d_c' = clear cover + d_s + distance from bottom of bundle to its centroid, in.	$d_c' = 1.5 + 0.500 + 0.79 = 2.79$ in.	REINFORCEMENT 1 and 3
	t = 2(distance from extreme tension fiber to centroid of reinforcement)	$t = 2d_c' = 2(2.79) = 5.58$ in.	
10.6.4	Max. $A' = \left(\frac{z}{f_s}\right)^3\left(\frac{1}{d_c}\right)$	Max. $A' = \left(\frac{175}{36}\right)^3\left(\frac{1}{2.79}\right) = 41.2$ in.^{2} Max. $A' = 41.2 = \frac{\text{max.}b_w(5.58)}{0.650(6)}$ in.^{2} Max. $b_w = \frac{(41.2)(0.650)(6)}{5.58} = 28.8$ in.	REINFORCEMENT 13
	Step 2—Verify maximum beam width using REINFORCEMENT 13.	For #8 bars in bundles of 3 and f_y = 60,000 psi, read max. beam width per bundle = 14.4 in. for two bundles. Max. $b = 2(14.4) = 28.8$ in.	REINFORCEMENT 13

REINFORCEMENT EXAMPLE 6—Determination of development length required for positive-moment reinforcement in a continuous beam

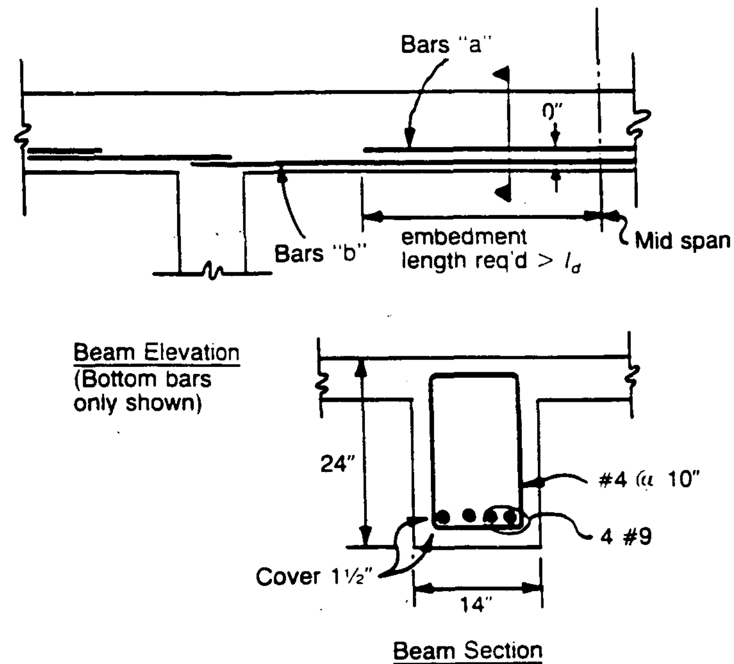
Determine development length of positive moment bars (Bars "a") of a continuous beam to check the embedment length requirement.

Given:

f'_c = 4 ksi (normal weight concrete)
f_y = 60 ksi
4 #9 Bottom bars (no excess bars)
{ Bars "a" – 2 #9
{ Bars "b" – 2 #9

Stirrups—#4 @ 10" (f_{yt} = 60 ksi)
(minimum shear reinforcement)

(Assume that ACI 318-95, Section 12.10.5 is satisfied for termination of positive Bars "a" in a tension zone.)



ACI 318-95 Section	Procedure	Calculation	Design Aid
12.2.2	**Step 1**—Check minimum stirrups, clear cover, and clear bar spacing (C_s). Determine category.	Minimum stirrups OK Clear cover, 1.5 + 0.5 = 2.0 = 1.77d_b C_s = [14 - 2(1.5 + 0.5) - 4 x 1.128]/3 = 1.829 = 1.62d_b ∴ Category I applies	REINFOR-CEMENT 17
	Step 2—Determine basic development length ratios.	$\left(\frac{l_d}{d_b}\right)_{basic} = 47$	REINFOR-CEMENT 17
12.2.4	**Step 3**—Check modifying factors.	$\alpha = \beta = \lambda = 1.0$	
12.2.2	**Step 4**—Determine final development length.	$\ell_d = 47d_b = 47 \times 1.128 = 53$ in. > 12 in. (minimum) OK	

REINFORCEMENT EXAMPLE 7—Determination of development length required for positive-moment reinforcement confined by stirrups

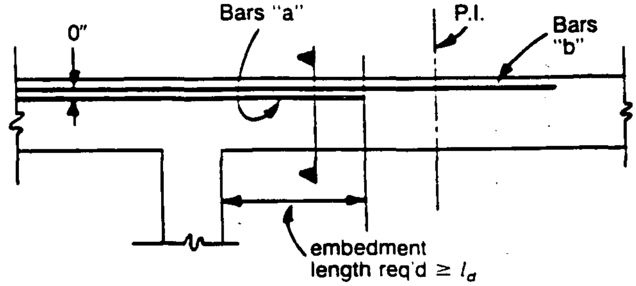
Determine a reduced tension development length of positive moment bars (Bars "a") considering confinement effect of stirrups for the continuous beam of REINFORCEMENT EXAMPLE 6, according to Section 12.2.3, ACI 318-95.

Given:
See REINFORCEMENT EXAMPLE 6 for given data.

ACI 318-95 Section	Procedure	Calculation	Design Aid
12.2.3	**Step 1**—Determine the governing c/d_b.	c/d_b (from clear cover) = 1.77 c/d_b (from clear bar spacing) = 1.62/2 = 0.81 (∴ Governs) (See step 1 of EXAMPLE 6)	
12.2.3	**Step 2**—Determine K_{tr}.	$\frac{A_{tr} f_{yt}}{1500\, sn} = \frac{2 \times 0.2 \times 60000}{1500 \times 10 \times 2} = 0.80$	
12.2.3	**Step 3**—Check $\frac{c + K_{tr}}{d_b} \le 2.5$	$\frac{c + K_{tr}}{d_b} = 0.81 + \frac{0.80}{1.128} = 1.52 < 2.5$ *OK*	
12.2.3 12.2.4	**Step 4**—Calculate development length ratios according to Section 12.2.3.	$\frac{l_d}{d_b} = \frac{3}{40} \frac{f_y}{\sqrt{f'_c}} \frac{\alpha \beta \gamma \lambda}{\left(\frac{c + K_{tr}}{d_b}\right)}$ $\alpha = \beta = \gamma = \lambda = 1.0$ $\frac{l_d}{d_b} = \frac{3}{40} \frac{60000 \times 1.0}{\sqrt{4000} \times 1.52} = 46.8$	
12.2.3	**Step 5**—Determine final development length.	$\ell_d = 46.8 d_b = 46.8 \times 1.128$ = 52.79 in. > 12 in. (minimum) OK	

REINFORCEMENT EXAMPLE 8—Determination of development length required for negative-moment reinforcement

Determine development length of negative moment bars (Bars "a") of a continuous beam to check the embedment length requirement.



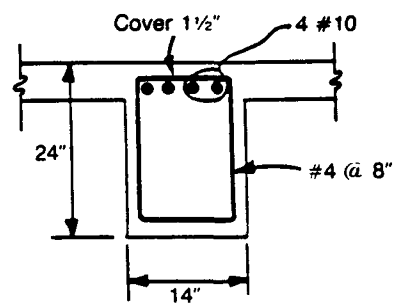
Beam Elevation (Top bars only shown)

Given:

f'_c = 4 ksi (light weight concrete)
f_y = 60 ksi
4 #10 negative moment bars (10% excess)
{ Bars "a" – 2 #10
  Bars "b" – 2 #10

Stirrups—#4 @ 8" (≥ minimum stirrups)

(Assume that ACI 318-95, Section 12.10.5 is satisfied for termination of negative Bars "a" in a tension zone.)



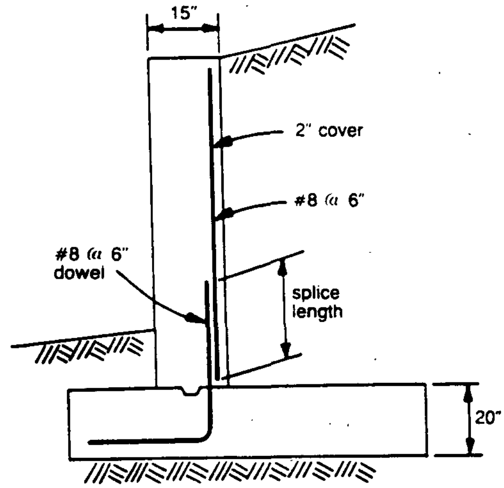
Beam Section

ACI 318-95 Section	Procedure	Calculation	Design Aid
12.2.2	**Step 1**—Check minimum stirrups, clear cover, and clear bar spacing (C_s). Determine category.	Minimum stirrups OK Clear cover, 1.5 + 0.5 = 2.0 = 1.575d_b C_s = [14 - 2(1.5 + 0.5) - 4 x 1.270]/3 = 1.64 = 1.291d_b ∴ Category I applies	REINFOR-CEMENT 17
	Step 2—Determine basic development length ratios.	$\left(\frac{l_d}{d_b}\right)_{basic} = 47$	REINFOR-CEMENT 17
12.2.4	**Step 3**—Check modifying factors.	α = 1.3 β = 1.0 λ = 1.3	REINFOR-CEMENT 17
12.2.5	**Step 4**—Check excess rebar factor.	Excess rebar factor = 1/1.10 = 0.91	
12.2.2	**Step 5**—Determine final development length.	ℓ_d = 47d_b = 1.3 x 1.0 x 1.3 x 0.91 x 47d_b = 72.28d_b = 72.28 x 1.270 = 91.8 in. > 12 in. (minimum) OK	

REINFORCEMENT EXAMPLE 9–Determination of splice length required for dowels in tension in the stem of a retaining wall

Determine tension splice length of dowel bars, spliced at one point with tension bars in retaining wall stem.

Given:
f'_c = 3 ksi (normal weight concrete)
f_y = 60 ksi
Stem and dowel bars
#8 @ 6" (15% excess)



Section—Retaining Wall
(Bars under investigation only are shown for clarity.)

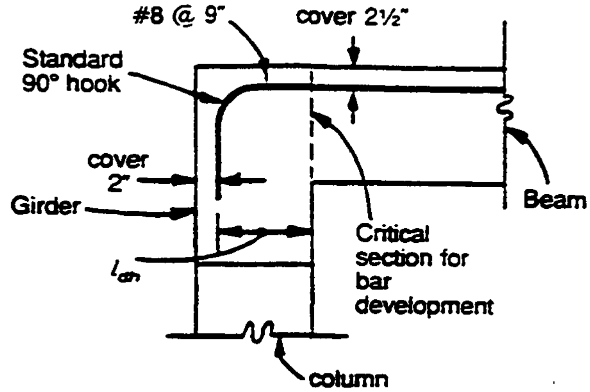
ACI 318-95 Section	Procedure	Calculation	Design Aid
12.2.2	**Step 1**–Check clear cover and clear bar spacing (C_s). Determine category.	Clear cover, 2.0 = $2d_b$ C_s = 6 - 1 = 5.0 = $5d_b$ ∴ Category I applies	REINFOR-CEMENT 17
	Step 2–Determine basic development length ratios.	$\left(\frac{l_d}{d_b}\right)_{basic} = 55$	REINFOR-CEMENT 17
12.2.4	**Step 3**–Check modifying factors.	$\alpha = \beta = \lambda = 1.0$	REINFOR-CEMENT 17
12.2.2	**Step 4**–Determine tension development length.	$\ell_d = 55d_b$ = 55 x 1.0 = 55 in. > 12 in. (minimum) OK	
12.15.1 12.15.2	**Step 5**–Determine splice class.	Class "B" splice since: (a) $A_{s\,(provided)}/A_{s\,(required)}$ = 1.15 < 2.00 (b) All bars spliced at one point	
12.15.1	**Step 6**–Determine splice length.	$\ell_s = 1.3\ell_d$ = 1.3 x 55 = 71.5 in. = 72 in.	

REINFORCEMENT EXAMPLE 10—Determination development length required for bar ending in a standard 90-deg hook

Determine a tension development length with standard 90-degree hook.

Given:

f_c' = 4000 psi (LWC)
f_y = 60,000 psi
#8 bars spaced 9 in. on centers, with at leat 3 in. clear space at edge.



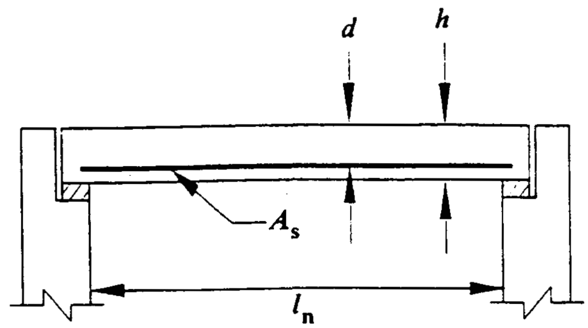
(top beam bars only shown for clarity)

ACI 318-95 Section	Procedure	Calculation	Design Aid
12.5.2	**Step 1**—Find basic development length with standard 90-degree hook l_{hb}.	For f_y = 60,000 psi; f_c' = 4000 psi; and #8 bars, read l_{hb} = 19.0 in.	REINFORCEMENT 18.1
12.5.3	**Step 2**—Determine all applicable modifiers.	From cover requirement, $\alpha_1 = 0.7$ From lightweight concrete, $\alpha_3 = 1.3$ $\therefore l_{dh} = \alpha_1\alpha_3 l_{hb}$ $= 0.7 \times 1.3 \times 19$ $= 17.3$ in.	
12.5.1	**Step 3**—Check min. development length.	l_{dh} min. = $8d_b$ = 8 in. $\therefore l_{dh} > 8$ in. $\therefore l_{dh} = 17.3 = 18$ in. (final)	

SHEAR

SHEAR EXAMPLE 1 - Design of beam for shear strength by method of ACI 318-95, Section 11.3.1

Determine the maximum factored shear V_u for which the beam shown must be designed (occurring at a section at a distance d from the face of the support) in accordance with Section 11.1.3 of ACI 318-95. Using the simplified method, determine the shear strength ϕV_c attributable to concrete. Assume normal weight concrete. There is no torsion. If stirrups are needed, determine spacing of #3 stirrups at the location where they must be the most closely spaced.



Given:

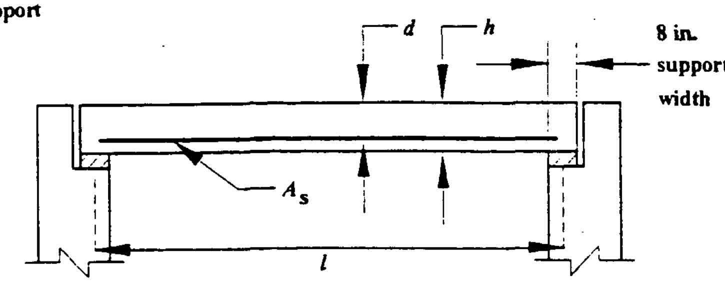
Live load = 1.0 *kips/ft*
Superimposed dead load = 0.75 *kips/ft*
f'_c = 3000 *psi*
f_y = 40,000 *psi*
b_w = 12 *in.*
d = 17 *in.*
h = 20 *in.*
A_s = 3.1 *sq in.*
l_n = 20 *ft*

ACI 318-95 Section	Procedure	Calculation	Design Aid
9.2.1	Step 1 - Determine factored load w_u. Compute beam weight Compute total dead load = beam weight + superimposed dead load Compute w_u = 1.4 (dead load) + 1.7 (live load)	*Beam weight* = $\frac{12(20)}{144}(0.15)$ = 0.25 *kips/ft* *Total dead load* = 0.25 + 0.75 = 1.00 *kips/ft* w_u = 1.4(1.00) + 1.7(1.00) = 3.1 *kips/ft*	
11.1.3.1	Step 2 - Determine V_u at distance d from face of support. *Compute* $V_u = w_u\left(\frac{l_n}{2} - d\right)$	$V_u = 3.1\left(\frac{20}{2} - \frac{17}{12}\right) = 26.6$ *kips*	
11.3.1.1 9.3.2.3	Step 3 - Determine the shear strength ϕV_c attributable to the concrete, using the simplified method. $\phi V_c = \phi(2\sqrt{f'_c}b_w d)$	$V_c = 2(\sqrt{3000})(12)(17)\frac{1}{1000} = 22.3$ *kips* ϕV_c = 0.85(22.3) = 19.0 *kips*	
11.5.5.1	Step 4 - Since $V_u > 0.5\phi V_c$, stirrups are needed.	26.6 *kips* > $\frac{19.0}{2}$	
11.1.1	Step 5 - Compute $\phi V_s = V_u - \phi V_c$.	ϕV_s = 26.6 - 19.0 = 7.6 *kips*	

ACI 318-95 Section	Procedure	Calculation	Design Aid
11.5.6.8	Step 6 - Compare with Max ϕV_s. $Max\ \phi V_s = \phi(8\sqrt{f_c'})b_w d$ $= 4\phi V_c$	Max ϕV_s = 4 (19.0) = 76.0 *kips* 76.0 *kips* > 7.6; therefore beam size is large enough	
11.5.5.3	Step 7 - Compare with Min ϕV_s Min $\phi V_s = \phi(50)b_w d$	$Min\ \phi V_s = 0.85(50)(12)(17)\frac{1}{1000}$ = 8.7 *kips* 8.7 > 7.6, therefore use ϕV_s = 8.7 *kips*	
11.5.6.2 11.5.4.1	Step 8 - Determine stirrup spacing for ϕV_s = 8.7 kips when f_y = 40,000 *psi* for the stirrup steel. $s = \frac{\phi A_v f_y d}{\phi V_s}$	For d = 17 *in.* and #3 stirrups $s = \frac{0.85(0.22)(40)(17)}{8.7}$ = 14.6 *in.* which is greater than maximum permissable $s = d/2$ = 8.5 *in.* *Use* $s = \frac{d}{2}$ = 8½ *in.*	
	Alternate Procedure Steps 1 and 2 - Same as Steps 1 and 2 above.		
11.1.1 9.3.2.3	Step 3 - Determine $V_n/(b_w d)$. *Compute* $V_n = \frac{V_u}{\phi}$ *Compute* $\frac{V_n}{b_w d}$	$V_n = \frac{26.6}{0.85}$ = 31.3 *kips* $\frac{V_n}{b_w d} = \frac{31.3(1000)}{12(17)}$ = 153 *psi*	
11.1.1 11.3.1.1 11.5.4.1 11.5.5.1 11.5.5.3 11.5.6.2 11.5.6.8	Step 4 - Determine whether stirrups are needed and, if so, determine the spacing of #3 stirrups. *Compute* $s \le \frac{A_v f_y}{50 b_w}$ $s \le \frac{d}{2}$ $s \le 24$ *in.*	For f_c' = 3000 psi and $V_n/(b_w d)$ = 153 *psi*, minimum stirrups are required, and $s \le \frac{2(0.11)(40{,}000)}{50(12)}$ = 14.7 *in.* $s \le \frac{17}{2}$ = 8.5 *in.* (*controls*) $s \le 24$ *in.* Use s = 8½ *in.*	SHEAR 1 SHEAR 1 SHEAR 1 SHEAR 1

SHEAR EXAMPLE 2 - Determination of shear strength of concrete in beam by more detailed method of Section 11.3.2

Use the detailed method to find shear strength of normal weight concrete at the distance d from the face of support for the beam shown.



Given:

$w_u = 3.1$ *kips/ft*
$l = 20$ *ft*
$b_w = 12$ *in.*
$d = 17$ *in.*
$h = 20$ *in.*
$A_s = 3.1$ *sq in.*
$f_c' = 3000$ *psi*
$f_y = 40{,}000$ *psi*

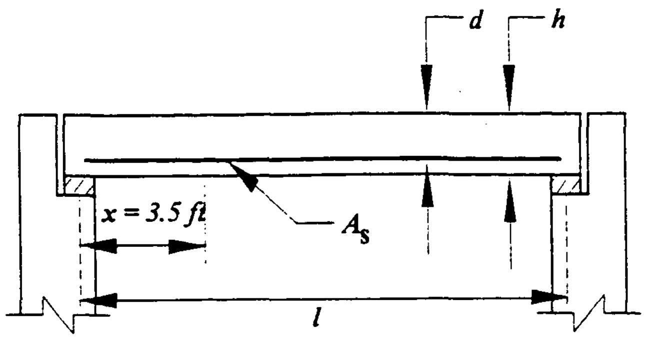
ACI 318-95 Section	Procedure	Calculation	Design Aid
11.1.3.1	Step 1 - Calculate the factored moment M_u at d from the face of support. Distance d from face equals $d + 4$ in. from center of support.	$M_u = \frac{w_u}{2}(x)(l - x)$ $= \frac{3.1}{2}\left(\frac{17+4}{12}\right)\left(20 - \frac{17+4}{12}\right)$ $= 49.5$ *ft-kips*	
	Step 2 - Calculate ρ_w.	$\rho_w = \frac{A_s}{b_w d} = \frac{3.1}{12(17)} = 0.015$	
	Step 3 - Calculate factored shear V_u at d from face of support.	$V_u = 3.1\left[10 - \frac{17+4}{12}\right]$ $= 25.6$ *kips*	
	Step 4 - Calculate $\rho_w V_u d/M_u$.	$\frac{\rho_w V_u d}{M_u} = \frac{0.015(25.6)(17)}{49.5(12)} = 0.011$	
11.3.2.1	Step 5 - Determine ϕV_c. $\phi V_c = \phi\left(1.9\sqrt{f_c'} + 2500\rho_w \frac{V_u d}{M_u}\right)b_w d$	For $f_c' = 3000$ *psi*, and $\rho_w V_u d/M_u = 0.011$, $V_c = \frac{[1.9\sqrt{3000} + 2500(0.011)](12)(17)}{1000}$ $= 26.8$ *kips* $\phi V_c = 0.85(26.8) = 22.8$ *kips*	

SHEAR EXAMPLE 3 - Determination of shear strength of concrete in beam by method of ACI 318-95 Section 11.3.1, and more detailed method of Section 11.3.2

Find the factored shear V_u and the shear strength ϕV_c for normal weight concrete beam at a point 3.5 *ft* from the centre of support.

Given:

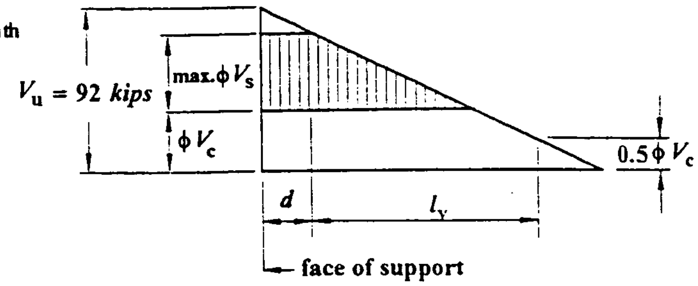
w_u = 3.1 *kips/ft*
l = 20 *ft*
b_w = 12 *in.*
d = 17 *in.*
h = 20 *in.*
A_s = 3.1 *sq in.*
f_c' = 3000 *psi*
f_y = 40,000 *psi*



ACI 318-95 Section	Procedure	Calculation	Design Aid
	Step 1 - Compute V_u at 3.5 *ft* from centre of support	$V_u = w_u\left(\frac{l}{2} - 3.5\right)$ $= 3.1\left(\frac{20}{2} - 3.5\right)$ $= 20.2$ *kips*	
11.3.1.1 9.3.2.3	Step 2 - Determine ϕV_c by simple method. $\phi V_c = \phi(2\sqrt{f_c'}\, b_w d)$	$V_c = 2(\sqrt{3000})(12)(17)\frac{1}{1000} = 22.3$ *kips* $\phi V_c = 0.85(22.3) = 19.0$ *kips*	
11.3.2.1	Step 3 - Determine ϕV_c by detailed method. Compute M_u at 3.5 *ft* from centre of support Compute $\rho_w = A_s / (b_w d)$ Compute $\rho_w V_u d / M_u$ Compute ϕV_c $V_c = \left(1.9\sqrt{f_c'} + 2500\rho_w \frac{V_u d}{M_u}\right) b_w d$	$M_u = \frac{w_u}{2}(x)(l - x)$ $= \frac{3.1}{2}(3.5)(20 - 3.5)$ $= 89.5$ *ft-kips* $\rho_w = \frac{3.1}{(12)(17)} = 0.0152$ $\frac{\rho_w V_u d}{M_u} = \frac{0.0152(20.2)17}{12(89.5)} = 0.0049$ $V_c = \frac{[1.9\sqrt{3000} + 2500(0.0049)](12)(17)}{1000}$ $= 23.7$ *kips* $\phi V_c = 0.85(23.7) = 20.2$ *kips*	

SHEAR EXAMPLE 4 - Selection of size and spacing of vertical stirrups (minimum stirrups required)

Determine the size and spacing of stirrups for a beam with the given factored shear diagram.



Given:

b_w = 20 *in.*
d = 30 *in.*
w_u = 5.11 *kips/ft*
f'_c = 4000 *psi*
f_y = 40,000 *psi*

ACI 318-95 Section	Procedure	Calculation	Design Aid
11.1.3.1	Step 1 - Determine V_u at d from face of support. $V_u = V_u$ at end $- w_u d$	$V_u = 92 - 5.11\left(\frac{30}{12}\right) = 79.2$ *kips*	
11.3.1.1 9.3.2.3	Step 2 - Determine ϕV_c $V_c = (2\sqrt{f'_c})b_w d$	$V_c = 2(\sqrt{4000})(20)(30)\frac{1}{1000} = 75.9$ *kips* $\phi V_c = 0.85(75.9) = 64.5$ *kips*	
11.1.1 9.3.2.3	Step 3 - Compute $V_n/(b_w d)$ at distance d from face of support. Compute $V_n = \frac{V_u}{\phi}$ Compute $\frac{V_n}{b_w d}$	$V_n = \frac{79.2}{0.85} = 93.2$ *kips* $\frac{V_n}{b_w d} = \frac{93.2(1000)}{20(30)} = 155$ *psi*	
11.1.1 11.3.1.1 11.5.4.1 11.5.5.1 11.5.5.3 11.5.6.2 11.5.6.8	Step 4 - Determine spacing of #3 stirrups at distance d from face of support. Compute $s \le \frac{A_v f_y}{50 b_w}$ $\le \frac{d}{2}$ ≤ 24 *in.* Try stirrups of larger diameter (#4)	For f'_c = 4000 psi and $V_n/(b_w d)$ = 155 psi, minimum stirrups are required, and $s \le \frac{(0.22)\,40{,}000}{50(20)} = 8.8$ *in.* $\le \frac{30}{2} = 15$ *in.* ≤ 24 *in.* Use #3 @ 9 *in.* wherever stirrups are required. $s \le \frac{(0.40)\,40{,}000}{50(20)} = 16$ *in.* $\le \frac{30}{2} = 15$ *in.* ≤ 24 *in.* Use #4 @ 15 *in.* wherever stirrups are required.	SHEAR 1 SHEAR 1 SHEAR 1 SHEAR 1 SHEAR 1 SHEAR 1 SHEAR 1

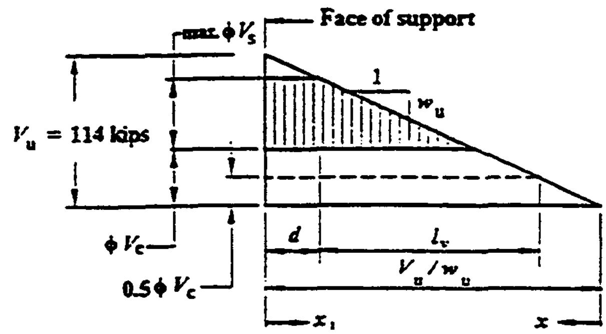
ACI 318-95 Section	Procedure	Calculation	Design Aid
11.5.5.1	Step 5—Compute length ($d + l_v$) over which stirrups are required. $$l_v = \frac{V_u at d - 0.5\phi V_c}{w_u}$$ *Note*: Some designers would use stirrups through to midspan	$$l_v = \frac{79.2 - 32.2}{5.11} = 9.2 \text{ ft}$$ $$d + l_v = \frac{30}{12} + 9.2 = 11.7 \text{ ft}$$ For #3 stirrups: First stirrup @ 4-1/2 *in.* from face, then 15 @ 9 *in.* (16 stirrups) For #4 stirrups: First stirrup @ 7-1/2 *in.* from face, then 9 @ 15 *in.* (10 stirrups)	

SHEAR EXAMPLE 5—Design of vertical stirrups for beam for which shear diagram is triangular

For the shear diagram shown, determine spacing and number of #3 stirrups.

Given:

V_a = 114 kips at face of support
W_a=7.54 kips/ft
b_w = 13 in.
d = 20 in. = 1.67 ft
f'_c = 4000 psi
f_y = 60,000 psi



ACI 318-95 Section	Procedure	Calculation	Design Aid
11.1.3.1	Step 1—At d from face of support, compute $V_u = V_{end} - w_u d$.	$V_u = 114 - 7.54(1.67) = 101.4$ kips	
11.3.1.1 9.3.2.3	Step 2—Find V_c and ϕV_c using simple procedure. $V_c = 2\sqrt{f'_c}b_w d$	$V_c = 2(\sqrt{4000})(13)(20)\frac{1}{1000} = 32.9\ kips$ $\phi V_c = 0.85(32.9) = 28.0\ kips$	
11.1.1 9.3.2.3 11.5.6.8	Step 3—Compute maximum shear to be carried by stirrups. $V_s = \left(\frac{V_u}{\phi} - V_c\right)$	$V_s = \left(\frac{101.4}{0.85} - 32.9\right) = 86.4\ kips$ $Max.\ V_s = 4(V_c) > 86.4\ kips$ Therefore, section size is OK	
11.5.5.1	Step 4—Compute distance ($d + l_v$) to location where stirrups are no longer required. $l_v = \frac{(V_u \text{ at } d - 0.5\phi V_c)}{w_u}$	$l_v = \frac{101.4 - 14.0}{7.54} = 11.6\ ft$ $d + l_v = 1.67 + 11.6 = 13.3\ ft$	
11.5.6	Step 5—Compute at distance d $\frac{V_s}{\beta_v K_v d} = \frac{1}{s}$	For #3 stirrups and f_y = 60,000 psi, read $K_v = 13.2\ kips$. $\beta_v = 1$ for vertical stirrups. $\frac{V_s}{\beta_v K_v d} = \frac{86.4}{1.0(13.2)(20)} = 0.327$	SHEAR 2
	Step 6—Determine spacing using SHEAR 2. Place straightedge to intersect ordinate of 0.327 at 1.67 ft (= d) from support and abscissa at $d + \left(\text{max. } \frac{\phi V_s}{w_u}\right) = 1.67 + \frac{0.85(86.4)}{7.54} = 11.4\ ft$	Read number of stirrups on chart and spacing at right edge, using a half spacing of 1-1/2 in. to first stirrup. Use 1 @ 1-1/2 in., 15 @ 3 in., 8 (i.e., 19-11) @ 4 in., 4 (i.e., 17-13) @ 6 in., and 6 (i.e., 16-10) @ 10 in. The 10 *in.* spacing extends to $d + l_v$ = 13.3 *ft* from support. Total stirrups = 2(34) = 68 for beam.	SHEAR 2

ACI 318-95 Section	Procedure	Calculation	Design Aid
11.5.4.3	*Note*: The maximum spacing of $d/2$ applies except where V_s exceeds $2V_c$, in which regions maximum spacing is $d/4$. Many designers consider it good practice to use stirrups at a convenient spacing not exceeding $d/2$ between l_v and the center of the beam.		
	Alternate Procedure Steps 1 and 2—Same as Steps 1 and 2 above.		
	Step 3—Determine $(d + l_v)$	As calculated in Step 4 above, $(d + l_v) = 13.3\,ft$	
11.1.1 9.3.2.3	Step 4—Compute $V_n/(b_w d)$ at distance d from support face. Compute $V_n = \frac{V_u}{\phi}$ Compute $\frac{V_n}{b_w d}$	$V_n = \frac{101.4}{0.85} = 119.3$ kips $\frac{V_n}{b_w d} = \frac{119.3(1000)}{13(20)} = 459$ psi	
11.1.1 11.3.1.1 11.5.4.1 11.5.5.1 11.5.5.3 11.5.6.2 11.5.6.8	Step 5—Determine spacing of #3 stirrups at distance d from face of support, using SHEAR 1 Compute $s \le \frac{A_v f_y d}{V_s}$ $\le \frac{d}{4}$ $\le 12\ in.$	For $f'_c = 4000\,psi$ and $V_n/(b_w d) = 459\,psi$, closely-spaced stirrups are required, and $s \le \frac{(0.22)(60)(20)}{86.4} = 3.1$ in. $\le \frac{20}{4} = 5$ in. ≤ 12 in. #3 stirrups @ 3 in. are needed near support	SHEAR 1 SHEAR 1 SHEAR 1 SHEAR 1
11.5.6.2 11.1.1 9.3.2.3	Step 6—Establish region where stirrups at maximum spacing (#3 @ 10 in.) are needed. Compute $V_s = \frac{A_v f_y d}{s}$ Compute $V_n = V_c + V_s$ Compute $V_u = \phi V_n$ Compute distance from point of zero shear force. Compute distance from face of support.	$V_s = \frac{(0.22)(60)(20)}{10} = 26.4$ kips $V_n = 32.9 + 26.4 = 59.3$ kips $V_u = 0.85(59.3) = 50.4$ kips $x = \frac{50.4}{7.54} = 6.7$ ft $x_1 = \frac{114}{7.54} - 6.7 = 8.4$ ft	

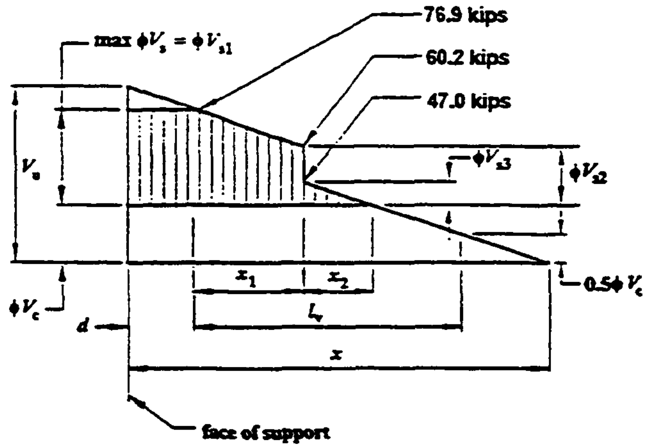
ACI 318-95 Section	Procedure	Calculation	Design Aid
11.5.5.1 11.5.5.3 11.3.1.1 9.3.2.3	Step 7—Determine whether requirement for minimum shear reinforcement is satisfied for region established in Step 6, using SHEAR 1.	$\frac{V_n}{b_w d} = \frac{59.3(1000)}{13(20)} = 228$ psi With $f_c' = 4000$ psi and $V_n/(b_w d) = 228$ psi, #3 stirrups @ 10 in. fall above the region that requires minimum stirrups. #3 stirrups @ 10 in. are needed for a distance from 8.4 ft to $(d + l_v) = 13.3$ ft from face of support.	SHEAR 1
11.5.6.2 11.1.1 9.3.2.3	Step 8—Establish region where $s = 5$ in., a spacing between 3 in. (Step 5) and 10 in. (Step 7), can be used. Compute $V_s = \frac{A_v f_y d}{s}$ Compute $V_n = V_c + V_s$ Compute $V_s = \phi V_n$ Compute distance from point of zero shear force. Compute distance from face of support.	$V_s = \frac{(0.22)(60)(20)}{5} = 52.8$ kips $V_n = 32.9 + 52.8 = 85.7$ kips $V_u = 0.85(85.7) = 72.8$ kips $x = \frac{72.8}{7.54} = 9.7$ ft $x_1 = \frac{114}{7.54} - 9.7 = 5.5$ ft #3 stirrups @ 5 in. are needed for a distance from 5.5 ft to 8.4 ft from face of support. The 5-in. spacing satisfies the upper limit on stirrup spacing. Use 1 @ 1-1/2 in., 22 @ 3 in., 7 @ 5 in., and 6 @ 10 in. Total stirrups = 2(36) = 72 for the beam.	

SHEAR EXAMPLE 6—Design of vertical stirrups for beam for which shear diagram is trapezoidal and triangular

For the factored shear diagram shown, determine the required spacing of vertical #3 stirrups, following the simplified method.

Given:

b_w = 13 in.
d = 20 in.
f'_c = 4000 psi
f_y = 60,000 psi
x_1 = 1.83 ft
x_2 = 2.26 ft
l_v = 5.75 ft

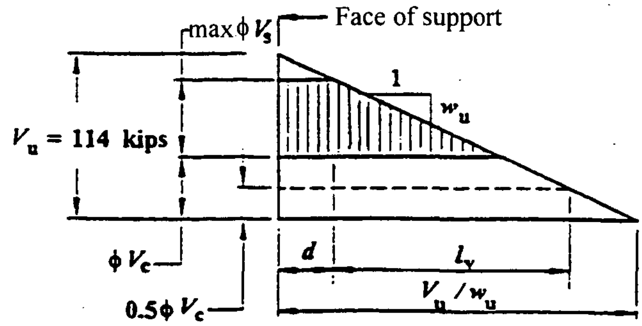


ACI 318-95 Section	Procedure	Calculation	Design Aid
11.3.1.1	Step 1—Determine V_c by simplified method. $V_c = 2\sqrt{f'_c}b_w d$	$V_c = 2(\sqrt{4000})(13)(20)\frac{1}{1000}$ = 32.9 kips	
1.1.1 9.3.2.3	Step 2—Determine shear to be carried by stirrups. $V_s = \left(\frac{V_s}{\phi} - V_c\right)$	$V_{s1} = \left(\frac{76.9}{0.85} - 32.9\right) = 57.6$ kips $V_{s2} = \left(\frac{60.2}{0.85} - 32.9\right) = 37.9$ kips $V_{s3} = \left(\frac{47.0}{0.85} - 32.9\right) = 22.4$ kips	
11.5.4.3	Step 3—Note that $V_{s1} < 2V_c$, so that $s \leq d/4$ limit does not apply.	57.6 < 2(32.9) OK	
11.1.1 9.3.2.3 11.5.4.1 11.5.5.1 11.5.5.3	Step 4—Maximum stirrup spacing (where V_u = 47.0 kips) is governed by $d/2$ or minimum stirrup area. Compute $V_n = \frac{V_u}{\phi}$ Compute $\frac{V_n}{b_w d}$	$V_n = \frac{47.0}{0.85} = 55.3$ kips $\frac{V_n}{b_w d} = \frac{55.3(1000)}{13(20)} = 213$ psi For f'_c = 4000 psi and $V_n/(b_w d)$ = 213 psi, stirrups with maximum $s = d/2$ = 10 in. are required to satisfy maximum spacing and minimum area of stirrups.	SHEAR 1

ACI 318-95 Section	Procedure	Calculation	Design Aid
11.5.6.3	Step 5—Compute $V_s/(\beta_v K_v d)$ for V_{s1}, V_{s2}, and V_{s3}. For vertical stirrups, $\beta_v = 1$	For #3 stirrups and f_y = 60,000 psi, read K_v = 13.2 kips $\frac{V_{s1}}{\beta_v K_v d} = \frac{57.6}{1(13.2)20} = 0.218$ $\frac{V_{s2}}{\beta_v K_v d} = \frac{37.9}{1(13.2)20} = 0.144$ $\frac{V_{s3}}{\beta_v K_v d} = \frac{22.4}{1(13.2)20} = 0.085$	SHEAR 2
11.1.3.1	Step 6—Plot graph of $V_s/(\beta_v K_v d)$ versus s, using tracing paper over SHEAR 2. Note that stirrups must continue at least $(l_v + d)$ = 7.42 ft from face of support. Note also that V_{s1} is constant for a distance d before it begins to decrease.	Stirrups from face of support 1 @ 2 in., 6 @ 4 in., 4 @5 in., 5 @ 10 in.	SHEAR 2

Rework Shear Example 5 using #3 stirrups inclined at 45 degrees.

Given:
V_u = 114 kips at face of support
w_u = 7.54 kips/ft
b_w = 13 in.
d = 20 in. = 1.67 ft
f_c' = 4000 psi
f_y = 60,000 psi

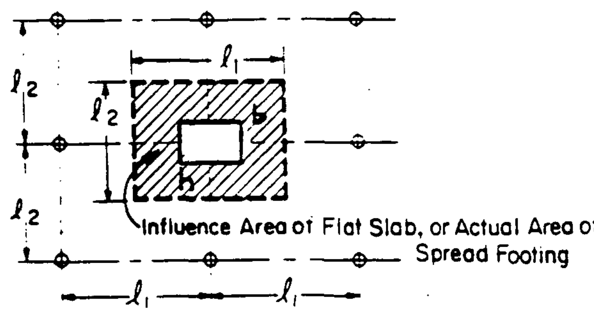


ACI 318-95 Section	Procedure	Calculation	Design Aid
11.1.3.1	Step 1—Compute V_u at d from face of support. $V_u = V_{end} - w_u d$	V_s = 114 - 7.54(1.67) = 101.4 kips	
11.3.1.1 9.3.2.3	Step 2—Compute V_c and ϕV_c using simple procedure. $V_c = 2\sqrt{f_c'}\, b_w d$	$V_c = 2(\sqrt{4000})(13)(20)\frac{1}{1000} = 32.9$ kips ϕV_c = 0.85(32.9) = 28.0 kips	
11.1.1 9.3.2.3 11.5.6.8	Step 3—Compute maximum shear to be carried by stirrups. $V_s = \left(\frac{V_u}{\phi} - V_c\right)$	$V_s = \left(\frac{101.4}{0.85} - 32.9\right) = 86.4$ kips Max $Vs = 4(V_c) > 86.4$ kips Therefore, section size is OK	
11.5.5.1	Step 4—Compute distance ($d + l_v$) to location where stirrups are no longer required. $l_v = \frac{V_u \ at\ d - 0.5\phi V_c}{w_u}$	$l_v = \frac{101.4 - 14.0}{7.54} = 11.6$ ft $d + l_v = 1.67 + 11.6 = 13.3$ ft	
11.5.6.3	Step 5—Compute at distance d, $\frac{V_s}{\beta_v K_v d} = \frac{1}{s}$	Read β_v = 1.414 for 45-deg inclined stirrups Read K_v = 13.2 *kips* $\frac{V_s}{\beta_v K_v d} = \frac{86.4}{1.414(13.2)(20)} = 0.231$	SHEAR 2
11.5.4.2 11.5.4.3	Step 6—Determine maximum spacing permitted with $V_s \leq 2V_c$ $Max.\ s = \frac{d}{2}(1 + \cot\alpha)$ Wherever $V_s > 2V_c$, half this spacing is the maximum permitted.	For α = 45 deg, $Max.\ s = d = 20\ in.$ with $V_s \leq 2\ V_c$ $Max.\ s = 10\ in.$ wherever $V_s > 2\ V_c$	

ACI 318-95 Section	Procedure	Calculation	Design Aid
	Step 7 - Place straightedge on SHEAR 2 to intersect ordinate at 0.231 and abscissa at 1.67 ft and base line at abscissa of $[d + max. (\phi V_s/w_u)] = 1.67 + 0.85(86.4)/7.54 = 11.4$ ft	Stirrups from face of support: 1 @ 2 *in.*, 12 @ 4 *in.*, 6 @ 6 *in.*, 4 @ 10 *in.*, 2 @ 18 *in.* A total of 2(1 + 12 + 6 + 4 + 2) = 50 are needed for beam.	SHEAR 2

SHEAR EXAMPLE 8 - Determination of thickness of slab (or footing) required to provide perimeter shear strength

Determine the depth required for shear strength of the flat slab shown for normal weight concrete. Assume no shear reinforcement is to be used.



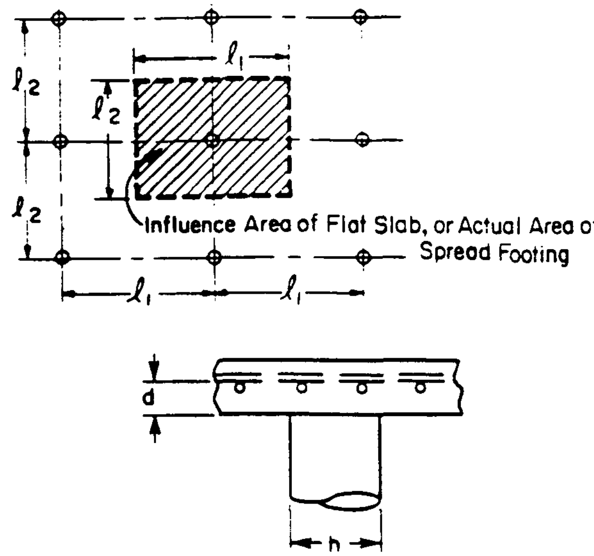
Given:

f'_c = 4000 psi
f_y = 60,000 psi
ℓ_1 = 24 ft
ℓ_2 = 20 ft
w_u = 1100 psf
h = 35 in.
b = 30 in.

ACI 318-95 Section	Procedure	Calculation	Design Aid
11.12.1.2	**Step 1**- Compute V_u to be carried by the perimeter effective section, and then find V_n. $V_u = w_u l_1 l_2 - w_u\,[(h+d)(b+d)]$ $V_n = \frac{V_u}{\phi}$ Neglect second term in step 1.	V_u = 1.100(24)(20) = 528 *kips* $V_n = \frac{528}{0.85}$ = 621 *kips*	
11.12.2	**Step 2**- Use SHEAR 5 to obtain trial d.	Enter for f'_c=4000 psi, move right to V_n=621 kips; then vertically to (b+h)=65 in., and then left to read effective d = 13.5 in.	SHEAR 5.1
	Step 3- Recheck V_u, using full equation given in step 1, and then check V_n.	$V_u = 528 - 1.100\left[\frac{(35+13.5)(30+13.5)}{144}\right]$ V_u = 512 *kips* $V_n = \frac{V_u}{\phi} = \frac{512}{0.85}$ = 602 *kips* Read d = 13.3 in.; little change from trial value.	SHEAR 5.1

SHEAR EXAMPLE 9 - Determination of thickness of slab (or footing) required to provide perimeter shear strength

Determine the depth required for shear strength of the flat slab shown for normal weight concrete. Assume no shear reinforcement is to be used.



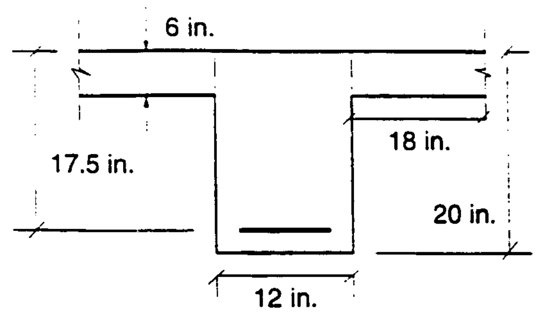
Given:

f'_c = 5000 psi
f_y = 40,000 psi
ℓ_1 = 20 ft
ℓ_2 = 18 ft
h = 34 in. diameter
w_u = 986 psf

ACI 318-95 Section	Procedure	Calculation	Design Aid
11.12.1.2	**Step 1**- Compute V_u to be carried by the perimeter effective section, and then find V_n. $$V_u = w_u l_1 l_2 - w_u\left[\frac{\pi(h+d)^2}{4}\right]$$ $$V_n = \frac{V_u}{\phi}$$ Neglect second term in step 1.	V_u = 0.986(20)(18) = 355 *kips* $$V_n = \frac{355}{0.85} = 418 \textit{ kips}$$	
11.12.2	**Step 2**- Use SHEAR 5 to obtain trial d.	Enter for f'_c=5000 psi, move right to V_n=418 kips; then vertically to h=34 in., and then left to read effective d = 10.5 in.	SHEAR 5.2
	Step 3- Recheck V_u, using full equation given in step 1, and then check V_n.	$$V_u = 355 - 0.986\left[\frac{\pi(34+10.5)^2}{(4)(144)}\right]$$ V_u = 344 *kips* $$V_n = \frac{V_u}{\phi} = \frac{344}{0.85} = 405 \textit{ kips}$$ Read d = 10.3 in.; little change from trial value.	SHEAR 5.2

SHEAR EXAMPLE 10 - Design of T-section for torsion

A T-section is unsymmetrically loaded. The required nominal torsional moment strength T_n = 5.9 ft-kips. The required nominal shear strength at the location where The required nominal torsional moment strength occurs is V_n = 59 kips. Investigate whether torsion can be neglected for design purposes.



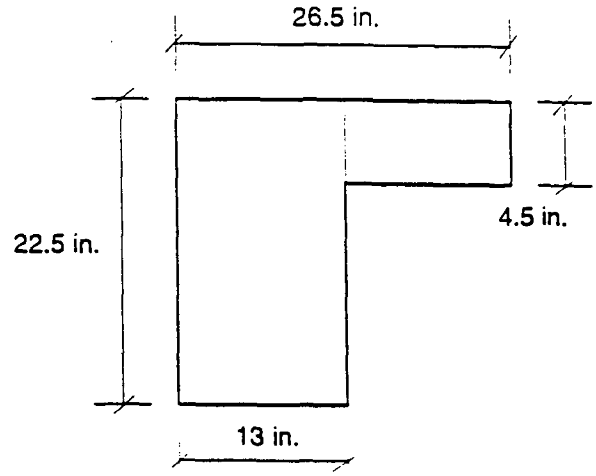
Given:

f'_c = 3000 psi
f_y = 60,000 psi

ACI 318-95 Section	Procedure	Calculation	Design Aid
11.6.1	**Step 1**- Determine the dimensional properties A_{cp} & p_{cp}	A_{cp} = 48 x 17.5 - 2 x 11.5 x 18 = 426 in.^{2} p_{cp} = 4 x 18 + 2 x (12 + 6 + 11.5) = 131 in. A_{cp}^2/p_{cp} = $(426)^2/131$ = 1385 in.^{3}	
11.6.1	**Step 2**- Determine the nominal torsional moment T_n below which torsion may be neglected. $Min\ T_n = \sqrt{f'_c}\left(\frac{A_{cp}^2}{p_{cp}}\right)$	$Min\ T_n = \sqrt{3000}\left(\frac{(426)^2}{131}\right) = 75877\ lb\text{-}in.$ = 6.3 ft-k > 5.9 ft-k Thus, torsion can be neglected.	
11.6.1	**Alternate Step 2** using SHEAR 6 - Determine nominal torsional moment T_n below which torsion may be neglected.	For A_{cp}^2/p_{cp} = 1385 and f'_c = 3000 psi, read T_n = 6.3 ft-k Therefore, torsion can be neglected.	SHEAR 6

SHEAR EXAMPLE 11 - Design of spandrel beam for torsion

Estimate the nominal torsional moment strength T_n in a spandrel beam if the restraining moment at the exterior end of slab panel (4.5-in. slab and a clear span of 12 ft) is $M = wL^2 / 24$.



Given:

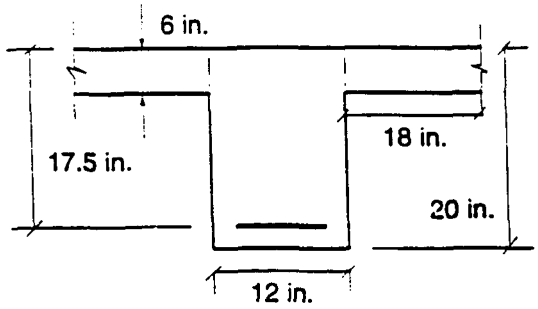
Column 18 x 18 in.
Beam 13 x 22.5 in. overall
Span = 27 ft c/c of columns
f'_c = 3000 psi
L.L. = 100 psf
D.L. = 75 psf
$w_u = 1.4(75) + 1.7(100) = 275$ psf

ACI 318-95 Section	Procedure	Calculation	Design Aid
	Step 1- Compute nominal moment $M_n = w_u L^2/24(\phi)$	$M_n = \dfrac{(0.275)(12)^2}{(0.85)(24)} = 1.94\ ft\text{-}k/ft$	
	Step 2- Compute the maximum nominal torsional moment, T_n .	T_n = 0.5 clear span of spandrel $T_n = \frac{1}{2}(27-18/12)(1.94) = 24.7\ ft\text{-}k$	
11.6.1	Alternate procedure: Step 3- Using SHEAR 6	$A_{cp} = 22.5 \times 13 + 13.5 \times 4.5$ $= 353.25$ in.^{2} $p_{cp} = 2 \times (26.5 + 22.5) = 98$ in. $A^2_{cp}/p_{cp} = (353.25)^2/98 = 1273$ in.^{3}	
11.6.1	**Step 4**- Determine maximum nominal torsional moment T_n .	For $A^2_{cp}/p_{cp} = 1273$ and $f'_c = 3$ ksi, read Max $T_n = 24$ ft-k little difference from actual calculation.	SHEAR 6

SHEAR EXAMPLE 12 - Design of T-section for torsion and flexural shear reinforcement

Repeat Shear Example 10 if the required nominal torsional moment strength T_n = 25 ft-kips. The required flexural reinforcement A_s = 2.6 in.^{2} for positive moment. Investigate the section for torsion and determine the torsion reinforcement required.

Given:
a = 4 in.
b = 12 in.
V_u = 80 kips
N_{uc} = 50 kips
f'_c = 5000 psi
f_{yv} = 60,000 psi
f_{yl} = 60,000 psi



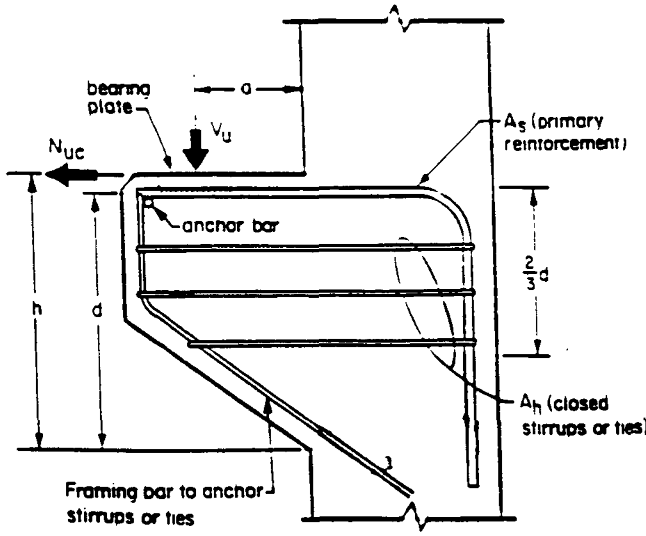
ACI 318-95 Section	Procedure	Calculation	Design Aid
11.6.1	**Step 1**-Determine whether torsion reinforcement is required	Minimum nominal torsional moment is less than the resulting maximum nominal moment (see SHEAR EXAMPLE 10), Min T_n = 6.3 ft-kips < 25 ft-kips ∴ torsion reinforcement is required.	
11.6.1	**Step 2**-Determine A_{oh} and p_h A_{oh} = [h - 2(cover + tie radius)] x [b_w - 2(cover + tie radius)] p_h = 2[h - 2(cover + tie radius)] + 2[b_w - 2(cover + tie radius)]	A_{oh} = [20 - 2(1.75)] x [12 - 2(1.75)] = 140.25 in.^{2} p_h = 2[20-2(1.75)] + 2[12 - 2(1.75)] = 50 in.	
11.6.3.1 Eq. (11-18)	**Step 3**-Determine if section has sufficient dimensions $\sqrt{\left(\frac{V_u}{b_w d}\right)^2+\left(\frac{T_u p_h}{1.7A_{oh}^2}\right)^2} \le \phi\left(\frac{V_c}{b_w d}+8\sqrt{f'_c}\right)$	$\sqrt{\left(\frac{80{,}000}{12(17.5)}\right)^2+\left(\frac{25{,}000(12)}{0.85(1.7)(140.25)^2}\right)^2}$ $= 381\ psi$ $\le 0.85\left(\frac{29{,}700}{(12)(17.5)}+8\sqrt{5000}\right)$ $= 601\ psi$ OK	
11.6.3.6 Eq. (11-21)	**Step 4**-Determine transverse reinforcement for torsion $\frac{A_t}{s} = \frac{T_n}{2\ A_o f_{yv} \cot\ \theta}$	For f_{yv} = 60 ksi, θ = 45 degrees and $A_o = 0.85A_{oh}$ $\frac{A_t}{s} = \frac{25(12)}{2(0.85)(140.25)(60)(1)}$ $= 0.021\ in.^2/in.$	

ACI 318-95 Section	Procedure	Calculation	Design Aid
11.3.1.1 Eq.(11-3) 11.5.6 Eq.(11-15)	**Step 5**-Determine A_v/s required for flexural shear $V_c = 2\sqrt{f_c'}\, b_w d$ $V_s = \dfrac{V_u}{\phi} - V_c$ $\dfrac{A_v}{s} = \dfrac{V_s}{f_y d}$	$V_c = 2\sqrt{5000}(12)(17.5) = 29.7\ kips$ $V_s = \dfrac{80}{.85} - 29.7 = 64.4\ kips$ $\dfrac{A_v}{s} = \dfrac{64.4}{(60)(17.5)} = 0.061\ in.^2/in$	
11.5.5.3 Eq.(11-13)	**Step 6**-Check minimum A_v/s for flexure $\dfrac{A_v}{s} \geq \dfrac{50\ b_w}{f_y}$	$minimum\ \dfrac{A_v}{s} = \dfrac{50(12)}{60{,}000} = 0.01\ in.^2/in$ $\leq \dfrac{A_v}{s} = 0.061\ in.^2/in.\quad OK$	
11.6.3.7 Eq. (11-22)	**Step 7**-Determine the additional longitudinal reinforcement $A_l = \dfrac{A_t}{s} p_h \left(\dfrac{f_{yv}}{f_{yl}}\right) \cot^2\theta$	$A_l = 0.021(50)(1)(1) = 1.05\ in.^2$	
11.6.5.3 Eq. (11-24)	**Step 8**-Determine the minimum total area of longitudinal torsional reinforcement $A_{l,min} = \dfrac{5\sqrt{f_c'}\, A_{cp}}{f_{yl}} - \left(\dfrac{A_t}{s}\right) p_h \dfrac{f_{yv}}{f_{yl}}$ $where\ \dfrac{A_t}{s} \geq \dfrac{25\ b_w}{f_{yv}}$	$A_t/s = 0.021 > 25(12)/60{,}000$ $= 0.005$ OK $A_{l,min} = \dfrac{5\sqrt{5000}(426)}{60{,}000}$ $- (0.021)(50)(1) = 1.46\ in.^2$	
11.6.5.2 Eq. (11-23)	**Step 9**-Determine minimum torsional reinforcement $\dfrac{A_v}{s} + \dfrac{2A_t}{s} \geq \dfrac{50\ b_w}{f_{yv}}$	$\dfrac{A_v}{s} + \dfrac{2A_t}{s} = 0.061 + 2(0.021)$ $= 0.103\ in.^2/in. > 0.01\ in.^2/in.$	
	Step 10-Determine hoop size and spacing For #4, A_v (or $2A_t$) = 0.40 in.^{2} For #5, A_v (or $2A_t$) = 0.62 in.^{2}	For #4, s = 0.40/0.103 = 3.89 in. For #5, s = 0.62/0.103 = 6.02 in.	REIN-FORCE-MENT 2

ACI 318-95 Section	Procedure	Calculation	Design Aid
11.6.6.1 11.5.4	**Step 11**-Spacing of stirrups shall not exceed the smaller of $p_h/8$ or 12 in. If $V_s < 2V_c$, $s \leq d/2$ ≤ 24 in. Otherwise, $s \leq d/4$ ≤ 12 in.	$p_h / 8 = 50 / 8 = 6.25$ in. s = 6 in. < 6.25 in. < 12 in. (OK) $V_s = 64.4$ kips > 2(29.7) = 59.4 kips d/4 = 17.5/4 = 4.4 in. < 6 in. Therefore use #4 closed stirrups @ 3.5 in. spacing	
11.6.6.2	**Step 12**-Select longitudinal steel. These bars must be distributed around perimeter at a spacing not to exceed 12 in. Thus bars are required at mid-depth. Steel required for torsion is additive to that required for flexure. Bar shall have diameter at least 1/24 of the stirrup spacing but not less than #3 bar. There shall be at least one longitudinal bar in each corner of the stirrup.	$A_l = 1.46/3 = 0.49$ in.^{2} at top, bottom, and mid-depth. Use 2 #5 at top Use 2 #5 at mid-depth $A_s = 2.6 + 0.49 = 3.09$ in.^{2} Use 4 #8 as bottom layer $d_b = .625$ in. > 3.5/24 = 0.15 in. > 0.375 in. (OK) Spacing s = (h - 2 x cover - 2 x d(stirrup) - d_b)/2 = (20 - 2 x 1.5 - 2 x 0.5 - 0.625)/2 = 7.7 in. Use s = 7.5 in. < 12 in. (OK)	REIN-FORCE-MENT 2

SHEAR EXAMPLE 13 - Design of bracket in which provision is made to prevent development of horizontal tensile force ($N_{uc} = 0$)

Check the capacity of the monolithically cast normal weight concrete bracket shown. If the proposed depth *d* is not adequate, suggest a new value for *d* and determine the required steel areas A_s and A_h. Special provisions are to be taken to insure transverse tension $N_{uc} = 0$.



Given:

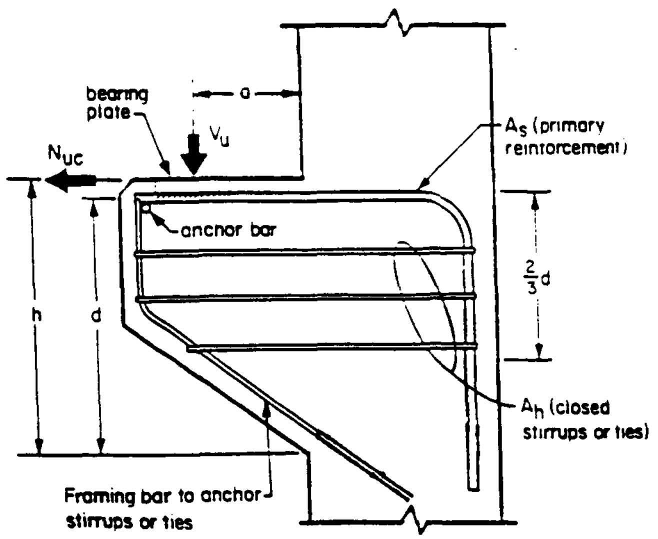
a = 4 in.
h = 12 in.
d = 10 in.
b = 12 in.
V_u = 90 kips
f'_c = 5000 psi
f_y = 60,000 psi

ACI 318-95 Section	Procedure	Calculation	Design Aid
11.9.1	**Step 1**—Check a/d	a/d = 4/10 = 0.40 < 1.0	
	Step 2—Compute required nominal shear strength V_n. $V_n = V_u/\phi$	Required V_n = 90/0.85 = 105.9 kips	
11.9.3.2.1	**Step 3**—Establish maximum nominal stress v_n	$v_n = 0.2f'_c$ = 1000 psi > 800 psi Max v_n = 800 psi	
	Step 4—Determine required concrete shear transfer area A_c and obtain required effective depth d. Required $A_c = V_n/v_n$	Required A_c = 105.9/0.800 = 132 in.^{2} Required d = A_c/b = 132/12 = 11 in. minimum ∴ d = 10 in. is not adequate; try d = 12 in.	
7.7.1(c)	**Step 5**—Estimate h by h = d + 1½ in. cover + ½d_h	h = 12 + 1½ + ½ = 14 in. ∴ try h = 14 in. instead of 12 in.	
11.7.4	**Step 6**—Determine shear-friction reinforcement. $A_{vf} = \dfrac{V_n}{f_y\,\mu}$	For normal weight concrete placed monolithically, μ = 1.4λ, λ = 1.0; therefore μ = 1.4 $A_{vf} = \dfrac{105.9}{60(1.4)} = 1.26\ in.^2$	
	Step 7—Check flexure steel	$M_u = V_ua$ = 90 x 4/12 = 30 kip-ft M_n = 30/0.85 = 35.3 kip-ft $F = \dfrac{bd^2}{12{,}000} = \dfrac{12(12)^2}{12{,}000} = 0.144$	FLEXURE 5

ACI 318-95 Section	Procedure	Calculation	Design Aid
11.9.3	Determine K_n	$K_n = M_n/F = 35.3/0.144 = 245$ interpolate to get $\rho = 0.0040$ $A_f = 0.0042(12)(12) = 0.61$ in.^{2}	FLEXURE 2.3
	Step 8—Select main tension reinforcement A_s .		
11.9.5	Min $\rho = 0.04(f'_c / f_y)$ Min A_s = Min ρ(b)(d) and	Min ρ =0.04(5000/60,000) = 0.0033 Min A_s =0.0033(12)(12)= 0.475 in.^{2}	
11.9.3.5	Min $A_s \leq A_f \leq A_{vf}/1.5$, whichever is greater	$A_f = 0.61$ in.^{2} from Step 7 Min $A_s = 1.26/1.5 = 0.84$ in.^{2} $\therefore A_s = 0.84$ in.^{2}	
11.9.4	$A_h \geq 0.50 A_s$	$A_h \geq 0.42$ in.^{2}	
11.9.6	Note: An anchor bar should be welded at exterior end of the A_s bars, and interior end must be developed into the supporting column.	For A_s at tension face, use 5 #4 for which $A_s = 1.00$ in.^{2}	REIN-FORCE-MENT 14
11.9.4		For A_h uniformly distributed over the upper two-thirds of the effective depth, use 2 #4 closed hoops so that $A_h = 0.80$ in.^{2} Locate centers of these closed hoops at 3 and 6 in. from center of main tension reinforcement.	REIN-FORCE-MENT 14

SHEAR EXAMPLE 14 - Design of bracket in which there is a horizontal tensile force N_{uc}

Design a bracket to support the loads shown. Normal weight concrete bracket is cast monolithically with supporting wall.



Given:

$a = 4$ in.
$b = 12$ in.
$V_u = 80$ kips
$N_{uc} = 50$ kips
$f_c' = 5000$ psi
$f_y = 60{,}000$ psi

ACI 318-95 Section	Procedure	Calculation	Design Aid
11.9.1 & 11.9.3.4	**Step 1**—Horizontal tension on bracket $0.2 \le \frac{N_u}{V_u} \le 1$	$0.2 < 50/80 < 1$	
11.9.3.2.1 7.7.1(c) 11.9.1	**Step 2**—Determine Min d for maximum nominal stress v_n. $v_n = 0.2f_c' \le 800$ psi	Required $V_n = V_u/\phi = 80/0.85 = 94.1$ kips $Min\ d = \frac{V_n}{bv_n} = \frac{94.1}{12(0.800)} = 9.8$ *in.* Required h= d +1½ in. cover + ½d_b Try h = 13 in. (d = 11 in. and ½d_b ≈ ½ in. a/d < 1.0	
11.9.3	**Step 3**—Required moment strength M_n	$M_n = V_n a + N_{uc}(h - d)/\phi$ $M_n = 94.1(4/12) + 50(2/12)/0.85$ $= 41.2$ kip-ft	
11.7.4	**Step 4**—Determine shear-friction reinforcement. $A_{vf} = \frac{V_n}{f_y \mu}$	For monolithic normal weight construction, $\mu = 1.4$ $A_{vf} = \frac{94.1}{60(1.4)} = 1.12$ *in.*^{2}	
11.9.3.4	**Step 5**—Transverse tension reinforcement. $A_n = \frac{N_{uc}}{\phi f_y}$ *where* $\phi = 0.85$	$A_n = \frac{50}{0.85(60)} = 0.98$ *in.*^{2}	

ACI 318-95 Section	Procedure	Calculation	Design Aid
10.3.1	**Step 6**—Flexural reinforcement	$F = \frac{bd^2}{12{,}000} = \frac{12(11)^2}{12{,}000} = 0.121$	FLEXURE 5
11.9.3.1	Determine K_n	$K_n = M_n/F = 41.2/0.121 = 340$	
		interpolate to get $\rho = 0.0059$ $A_f = 0.0059(12)(11) = 0.78$ in.^{2}	FLEXURE 2.3
	Step 7—Determine main reinforcement A_s		
11.9.5	Min $\rho = 0.04(f'_c/f_y)$ Min A_s = Min $\rho(b)(d)$	Min $\rho = 0.04(5000/60{,}000) = 0.0033$ Min $A_s = 0.0033(12)(11) = 0.44$ in.^{2}	
11.9.3.5	$A_s = 2A_{vf}/3 + A_n$ or $A_s = A_f + A_n$, whichever is greater	$A_s = 2(1.12)/3 + 0.98 = 1.73$ in.^{2} or $A_s = 0.78 + 0.98 = 1.76$ in.^{2} $\therefore A_s = 1.76$ in.^{2}	
11.9.6	**Step 8**—Select bars for A_s and hoops for A_h	For A_s, use 3 #7 for which $A_s = 1.80$ in.^{2}. Weld these bars to cross bar and to steel bearing plate.	REIN-FORCE-MENT 14
11.9.4	$A_h \geq 0.5(A_s - A_n)$	$A_h \geq 0.5(1.76 - 0.98) \geq 0.39$ $\therefore$ use 2 #3 hoops for which $A_h = 0.44$ in.^{2} Locate centers of these closed hoops at 3½ and 7 in. from center of main tension reinforcement.	

SHEAR EXAMPLE 15 - Design for shear and equilibrium torsion

Given:
f_c' = 4000 psi
f_y = 60,000 psi
b = 14 in.
h = 25 in.
Required V_u = 45 kips
Required T_u = 41.7 (k-ft)

ACI 318-95 Section	Procedure	Calculation	Design Aid
	Step 1- Determine whether shear reinforcement is required. Using SHEAR 7.1.2, read K_{vc}. If $V_u > 0.5\phi K_{vc}$, then A_v is required.	For ϕ = 0.85, ϕK_{vc} = 0.85 x 39.84 = 33.9 kips. V_u = 45 kips > $0.5\phi K_{vc}$ = 0.5 x 33.9 = 17 kips, therefore A_v is required.	SHEAR 7.1.2
11.6.1	**Step 2**- Determine whether torsion reinforcement is required. Using SHEAR 7.2.2 read K_{tcr}. If $T_u > 0.25\phi K_{tcr}$, then A_t is required.	For ϕ = 0.85, ϕK_{tcr} = 0.85 x 33.12 = 28.2 (k-ft). T_u = 41.7 (k-ft) > $0.25\phi K_{tcr}$ = 0.25 x 28.2 = 7 (k-ft), therefore A_t is required.	SHEAR 7.2.2
11.6.3.1 Eq. (11-18)	**Step 3**- Determine if section has sufficient dimensions. Using SHEAR 7.4.2, read K_t. If $\sqrt{\left(\frac{V_u}{5\phi K_{vc}}\right)^2 + \left(\frac{T_u}{\phi K_t}\right)^2} \le 1$ then the section is adequate	For ϕ = 0.85, ϕK_t = 0.85 x 71.4 = 60.6 (k-ft). $\sqrt{\left(\frac{45}{5\times 33.9}\right)^2 + \left(\frac{41.7}{60.6}\right)^2} = 0.74 < 1$ ∴ section is adequate.	SHEAR 7.1.2 7.4.2
11.6.3.6 Eq. (11-21)	**Step 4**- Determine transverse reinforcement for shear and torsion. Using SHEAR 7.1.2, and SHEAR 7.3.2 read K_{vs} and K_{ts}: $A_{vt}/s = K_{ct} = (V_u - \phi K_{vc})/\phi K_{vs} + T_u/\phi K_{ts}$, where $(V_u - \phi K_{vc}) \ge 0$	For ϕ = 0.85, ϕK_{vc} = 33.9 kips, ϕK_{vs} = 0.85 x 1350 = 1149 k/in. ϕK_{ts} = 0.85 x 959.4 = 816 (k-ft/in.) K_{ct} = (45 - 33.9)/1149 + 41.7/816 = 0.0608 in.^{2}/in.	SHEAR 7.1.2 SHEAR 7.3.2
11.6 .5 .2 Eq. (11-23)	**Step 5**- Check minimum transverse reinforcement: If $K_{ct} < 50b_w/f_{yv}$ then use $K_{ct} = 50b_w/f_{yv}$.	(50 x 14)/60,000 = 0.012 in.^{2}/in. < K_{ct} = 0.0608 in.^{2}/in. (OK).	

ACI 318-95 Section	Procedure	Calculations	Design Aid
11.6.6.1 11.5.4	**Step 6**- Compute closed stirrups spacing $s = 2A_{bar}/K_{ct}$ in. $s \le (b + h - 7)/4 \le 12$ in. If $V_s = (V_u - \phi K_{vc})/\phi < 2K_{vc}$, $s \le d/2$ ≤ 24 in.	Using #4 hoops, $A_{bar} = 0.2$ in^2; $s = (2 \times 0.2)/0.0608 = 6.58$ in. Use $s = 6.5$ in. $(b + h - 7)/4 = (14 + 25 - 7)/4$ $= 8$ in. > 6.5 in. (OK) $V_s = (45 - 33.9)/0.85 = 13.1$ kips $< 2(39.84) = 79.7$ kips; $d/2 = (25 - 2.5)/2$ $= 11.25$ in. > 6.5 in. (OK)	REIN - FORCE - MENT 1
11.6.3.7 Eq. (11-22)	**Step 7**- Determine the additional longitudinal torsional reinforcement: $A_l = (b + h - 7)(T_u/\phi K_{ts}) f_{yv}/f_{yl}$. With $\theta = 45°$.	$f_{yv} = f_{yl} = 60$ ksi, $\theta = 45°$ $A_l = (14 + 25 - 7) \times (41.7/816) \times (60/60) = 1.64$ in^2.	SHEAR 7.3.2
11.6.5.3 Eq. (11-24)	**Step 8**- Determine the minimum total area of longitudinal torsional reinforcement $$A_{l,min} = \frac{5\sqrt{f'_c}\,A_{cp}}{f_{yl}} - \left(\frac{A_t}{s}\right) p_h \frac{f_{yv}}{f_{yl}}$$ $$where\ \frac{A_t}{s} \ge \frac{25\,b_w}{f_{yv}}$$ and $A_t/s = T_u/2\phi K_{ts}$, $A_{cp} = bh$; $p_h = 2(b + h - 7)$	$A_t/s = T_u/2\phi K_{ts} = 41.7/(2 \times 816)$ $= 0.026$ in^2/in. $> (25 \times 14)/60{,}000$ $= 0.006$ in^2/in. (OK) $$A_{l,min} = \frac{5\sqrt{4000} \times 14 \times 25)}{60{,}000}$$ $$- 0.026 \times 2 \times (14 + 25 - 7)\frac{60{,}000}{60{,}000}$$ $$= 0.181 in.^2 < A_l = 1.64\ in.^2\ (OK)$$	SHEAR 7.3.2
11.6.6.2	**Step 9**- Select longitudinal steel. These bars must be distributed around perimeter at a spacing not to exceed 12 in. The steel required for torsion is additive to that required for flexure. Bar shall have diameter at least 1/24 of the stirrup spacing but not less than #3 bar. There shall be a longitudinal bar in each corner of the stirrup.	Distributing the bars in three layers gives total of 6 bars. $A_{bar} = A_l/6 = 1.64/6 = 0.27$ in^2. Use 6#5 bars. $d_{b,l} = 0.625$ in. $> s/24 = 6.5/24 = 0.27$ in. > 0.375 in. (OK) $A_l = 6 \times 0.31 = 1.86$ in^2. Spacing $s_l = (h - 2 \times \text{cover} - 2 \times d_b(\text{hoops}) - d_{b,l})/2$ $= (25 - 2 \times 1.5 - 2 \times 0.5 - 0.625)/2$ $= 10.2$ in. < 12 in. (OK)	REIN - FORCE - MENT 1

SHEAR EXAMPLE 16 - Design for shear and equilibrium torsion

Given:
f_c' = 4000 psi
f_y = 60,000 psi
b = 18 in.
h = 27 in.
Required V_u = 72.9 kips
Required T_u = 56.7 (k-ft)

ACI 318-95 Section	Procedure	Calculation	Design Aid
	Step 1- Determine whether shear reinforcement is required. Using SHEAR 7.1.2, read K_{vc}. If $V_u > 0.5\phi K_{vc}$, then A_v is required.	For $\phi = 0.85$, $\phi K_{vc} = 0.85 \times 55.8 = 47.4$ kips. $V_u = 72.9$ kips $> 0.5\phi K_{vc} = 0.5 \times 47.4$ $= 23.7$ kips, therefore A_v is required.	SHEAR 7.1.2
11.6.1	**Step 2**- Determine whether torsion reinforcement is required. Using SHEAR 7.2.2 read K_{tcr}. If $T_u > 0.25\phi K_{tcr}$, then A_t is required.	For $\phi = 0.85$, $\phi K_{tcr} = 0.85 \times 55.3 = 47$ (k-ft). $T_u = 56.7$ (k-ft) $> 0.25\phi K_{tcr}$ $= 0.25 \times 47 = 11.8$ (k-ft), therefore A_t is required.	SHEAR 7.2.2
11.6.3.1 Eq. (11-18)	**Step 3**- Determine if section has sufficient dimensions. Using SHEAR 7.4.2, read K_t. If $\sqrt{\left(\frac{V_u}{5\phi K_{vc}}\right)^2 + \left(\frac{T_u}{\phi K_t}\right)^2} \le 1$ then the section is adequate	For $\phi = 0.85$, $\phi K_t = 0.85 \times 137 = 116.4$ (k-ft). $\sqrt{\left(\frac{72.9}{5 \times 47.4}\right)^2 + \left(\frac{56.7}{116.4}\right)^2} = 0.57 < 1$ ∴ section is adequate.	SHEAR 7.1.2 7.4.2
11.6.3.6 Eq. (11-21)	**Step 4**- Determine transverse reinforcement for shear and torsion. Using SHEAR 7.1.2, and SHEAR 7.3.2 read K_{vs} and K_{ts}: $A_{vt}/s = K_{ct} = (V_u - \phi K_{vc})/\phi K_{vs} + T_u/\phi K_{ts}$, where $(V_u - \phi K_{vc}) \ge 0$	For $\phi = 0.85$, $\phi K_{vc} = 47.4$ kips, $\phi K_{vs} = 0.85 \times 1470 = 1250$ k/in. $\phi K_{ts} = 0.85 \times 1448 = 1231$ (k-ft/in.) $K_{ct} = (72.9 - 47.4)/1250 + 56.7/1231$ $= 0.0665$ in.^{2}/in.	SHEAR 7.1.2 SHEAR 7.3.2
11.6.5.2 Eq. (11-23)	**Step 5**- Check minimum transverse reinforcement: If $K_{ct} < 50b_w/f_{yv}$ then	(50 x 18)/60,000 = 0.015 in.^{2}/in. $< K_{ct}$ = 0.0665 in.^{2}/in. (OK).	

ACI 318-95 Section	Procedure	Calculation	Design Aid
11.6.6.1 11.5.4	**Step 6**- Compute closed stirrups spacing $s = 2A_{bar}/K_{ct}$ in. $s \le (b + h - 7)/4 \le 12$ in. If $V_s = (V_u - \phi K_{vc})/\phi < 2K_{vc}$, $s \le d/2$ ≤ 24 in.	Using #4 hoops, $A_{bar} = 0.2$ in^2; $s = (2 \times 0.2)/0.0665 = 6$ in. $(b + h - 7)/4 = (18 + 27 - 7)/4$ $= 9.5$ in. > 6 in. (OK) $V_s = (72.9 - 47.4)/0.85 = 30$ kips $< 2(55.8) = 111.6$ kips; $d/2 = (27 - 2.5)/2$ $= 12.25$ in. > 6 in. (OK)	REIN - FORCE - MENT 1
11.6.3.7 Eq. (11-22)	**Step 7**- Determine the additional longitudinal torsional reinforcement: $A_l = (b + h - 7)(T_u/\phi K_{ts}) f_{yv}/f_{yl}$ With $\theta = 45°$.	$f_{yv} = f_{yl} = 60$ ksi, $\theta = 45°$ $A_l = (18 + 27 - 7) \times (56.7/1231) \times (60/60) = 1.75$ in^2.	SHEAR 7.3.2
11.6.5.3 Eq. (11-24)	**Step 8**- Determine the minimum total area of longitudinal torsional reinforcement $A_{l,min} = \dfrac{5\sqrt{f'_c}\,A_{cp}}{f_{yl}} - \left(\dfrac{A_t}{s}\right) p_h \dfrac{f_{yv}}{f_{yl}}$ where $\dfrac{A_t}{s} \ge \dfrac{25\,b_w}{f_{yv}}$ and $A_t/s = T_u/2\phi K_{ts}$, $A_{cp} = bh$; $p_h = 2(b + h - 7)$	$A_t/s = T_u/2\phi K_{ts} = 56.7/(2 \times 1231)$ $= 0.023$ in^2/in. $> (25 \times 18)/60{,}000$ $= 0.0075$ in^2/in. (OK) $A_{l,min} = \dfrac{5\sqrt{4000} \times (18 \times 27)}{60{,}000}$ $- 0.023 \times 2 \times (18 + 27 - 7)\dfrac{60{,}000}{60{,}000}$ $= 0.81 in.^2 < A_l = 1.75\ in.^2$ (OK)	SHEAR 7.3.2
11.6.6.2	**Step 9**- Select longitudinal steel. These bars must be distributed around perimeter at a spacing not to exceed 12 in. The steel required for torsion is additive to that required for flexure. Bar shall have diameter at least 1/24 of the stirrup spacing but not less than #3 bar. There shall be a longitudinal bar in each corner of the stirrup.	Distributing the bars in three layers gives total of 8 bars. $A_{bar} = A_l/8 = 1.75/8 = 0.22$ in^2. Use 8#5 bars in 3 layers: 3#5, 2#5, and 3#5 $d_{b,l} = 0.625$ in. $> s/24 = 6/24 = 0.25$ in. (OK) > 0.375 in. (OK) $A_l = 8 \times 0.31 = 2.48$ in^2. And vertical spacing: s_{lv} = (h - 2 x cover - 2 x d_b(hoops) - $d_{b,l}$)/2 = (27 - 2 x 1.5 - 2 x 0.5 - 0.625)/2 = 11.2 in. < 12 in. (OK). Horizontal spacing: s_{lh} = (b - 2 x cover - 2 x d_b(hoops) - $d_{b,l}$)/2 = (18 - 2 x 1.5 - 2 x 0.5 - 0.625)/2 = 6.7 in. < 12 in. (OK)	REIN - FORCE - MENT 1

SHEAR EXAMPLE 17- Design for shear and compatibility torsion

Given:
f'_c = 4000 psi
f_y = 60,000 psi
b = 12 in.
h = 20 in.
Required V_u = 19.4 kips
Required T_u = 47.7 (k-ft)

ACI 318-95 Section	Procedure	Calculation	Design Aid
	Step 1- Determine whether shear reinforcement is required. Using SHEAR 7.1.2, read K_{vc}. If V_u > $0.5\phi K_{vc}$, then A_v is required.	For $\phi = 0.85$, ϕK_{vc} = 0.85 x 26.56 = 22.6 kips V_u = 19.4 kips > $0.5\phi K_{vc}$ = 0.5 x 22.6 = 11.3 kips, therefore A_v is required.	SHEAR 7.1.2
11.6.1	**Step 2**- Determine whether torsion reinforcement is required. Using SHEAR 7.2.2 read ϕK_{tcr}. If T_u > $0.25\phi K_{tcr}$, then A_t is required.	For $\phi = 0.85$, ϕK_{tcr} = 0.85 x 18.97 = 16.1 (k-ft). T_u= 47.7 (k - ft) > $0.25\phi K_{tcr}$ = 0.25 x 16.1 = 4 (k-ft), therefore A_t is required.	SHEAR 7.2.2
11.6.1 11.6.2	**Step 3**- Determine the design torsion : T_u is the lesser value of actual T_u and ϕK_{tcr}, where $K_{tcr} = 4\sqrt{f'_c}\left(\frac{A_{cp}^2}{P_{cp}}\right)$ *in (k-ft) units.*	T_u = 47.7 (k - ft) > ϕK_{tcr} = 16.1 (k-ft), therefore design for T_u = 16.1 (k-ft).	SHEAR 7.2.2
11.6.3.1 Eq. (11-18)	**Step 4**- Determine if section is adequate Using SHEAR 7.4.2, read K_t. If $\sqrt{\left(\frac{V_u}{5\phi K_{vc}}\right)^2 + \left(\frac{T_u}{\phi K_t}\right)^2} \leq 1$ then the section is adequate.	For $\phi = 0.85$, ϕK_t = 0.85 x 35.25 = 30 (k-ft) $\sqrt{\left(\frac{19.4}{5 \times 22.6}\right)^2 + \left(\frac{16.1}{30}\right)^2} = 0.56 < 1$ ∴ section is adequate.	SHEAR 7.4.2
11.6.3.6 Eq. (11-21)	**Step 5**- Determine transverse reinforcement for shear and torsion. Using SHEAR 7.1.2, and SHEAR 7.3.2, read K_{vs} and K_{ts}: $A_{vt}/s = K_{ct} = (V_u - \phi K_{vc})/\phi K_{vs} + T_u/\phi K_{ts}$, where $(V_u - \phi K_{vc}) \geq 0$	For $\phi = 0.85$, ϕK_{vc} = 22.6 kips, ϕK_{vs} = 0.85 x 1050 = 892.5 k/in. ϕK_{ts} = 0.85 x 596.03 = 507 (k-ft/in.) $(V_u - \phi K_{vc})$ = 19.4 - 22.6 = - 3.2 < 0; use $(V_u - \phi K_{vc})$ = 0 K_{ct} = 0 + (16.1 / 507) = 0.0318 in.^{2}/in.	SHEAR 7.1.2 SHEAR 7.3.2
11.6 .5 .2 Eq. (11-23)	**Step 6**- Check minimum transverse reinforcement: If $K_{ct} < 50b_w/f_{yv}$ then use $K_{ct} = 50b_w/f_{yv}$.	(50 x 12)/60,000 = 0.01 in.^{2}/in. < K_{ct} = 0.0318 in.^{2}/in. (OK)	

ACI 318-95 Section	Procedure	Calculation	Design Aid
11.6.6.1 11.5.4	**Step 7**- Compute closed stirrups spacing: $s = 2A_{bar}/K_{ct}$ in $s \le (b + h - 7)/4 \le 12$ in If $V_s = (V_u - \phi K_{vc})/\phi < 2K_{vc}$, $s \le d/2$ ≤ 24 in.	Using #3 hoops, $A_{bar} = 0.11$ in^2; $s = (2 \times 0.11)/0.0318 = 6.9$ in. $(b + h - 7)/4 = (12 + 20 - 7)/4$ $= 6.25$ in. < 6.9 in. $V_s = 0$ since $(V_u - \phi K_{vc}) = 0$ $< 2K_{vc}$ $d/2 = (20 - 2.5)/2 = 8.75$ in. > 6 in. (OK) Therefore use #3 @ 6 in.	REIN - FORCE - MENT 1
11.6.3.7 Eq. (11-22)	**Step 8**- Determine the additional longitudinal torsional reinforcement: $A_l = (b + h - 7)(T_u/\phi K_{ts}) f_{yv}/f_{yl}$ With $\theta = 45°$	$f_{yv} = f_{yl} = 60$ ksi, $\theta = 45°$ $A_l = (12+20-7)\text{x}(16.1/507)\text{x}(60/60)$ $= 0.794$ in^2.	SHEAR 7.3.2
11.6.5.3 Eq. (11-24)	**Step 9**- Determine the minimum total area of longitudinal torsional reinforcement $$A_{l,\min} = \frac{5\sqrt{f'_c}\,A_{cp}}{f_{yl}} - \left(\frac{A_t}{s}\right) p_h \frac{f_{yv}}{f_{yl}}$$ where $\frac{A_t}{s} \ge \frac{25\,b_w}{f_{yv}}$ and $A_t/s = T_u/2\phi K_{ts}$, $A_{cp} = bh$; $p_h = 2(b + h - 7)$	$A_t/s = T_u/2\phi K_{ts} = 16.1/(2\text{x}507)$ $= 0.016$ in^2/in. $> (25 \times 12)/60{,}000$ $= 0.005$ in^2/in. (OK) $$A_{l,\min} = \frac{5\sqrt{4000} \times (12 \times 20)}{60{,}000}$$ $$- 0.016 \times 2 \times (12+20-7)\frac{60{,}000}{60{,}000}$$ $= 0.465\,in.^2 < A_l = 0.794\ in.^2$ (OK)	SHEAR 7.3.2
11.6.6.2	**Step 10**- Select longitudinal steel. These bars must be distributed around perimeter at a spacing not to exceed 12 in. The steel required for torsion is additive to that required for flexure. Bar shall have diameter at least 1/24 of the stirrup spacing but not less than #3 bar. There shall be a longitudinal bar in each corner of the stirrup.	Distributing the bars in three layers gives total of 6 bars. $A_{bar} = A_l/6$ $= 0.794/6 = 0.13$ in^2. Use 6#4 bars $d_{b,l} = 0.5$ in. $> s/24 = 6/24 = 0.25$ in. > 0.375 in. (OK) $A_l = 6 \times 0.2 = 1.2$ in^2. Spacing $s_l = (h - 2 \times \text{cover} - 2 \times d_b$ (hoops) $- d_{b,l})/2$ $= (20 - 2 \times 1.5 - 2 \times 0.0.375 - 0.5)/2$ $= 7.9$ in. < 12 in. (OK).	REIN - FORCE - MENT 1

SHEAR EXAMPLE 18- Design for shear and compatibility torsion

Given:

f_c' = 4000 psi
f_y = 40,000 psi
b = 10 in.
h = 18 in.
Required V_u = 9 kips
Required T_u = 8 (k-ft)

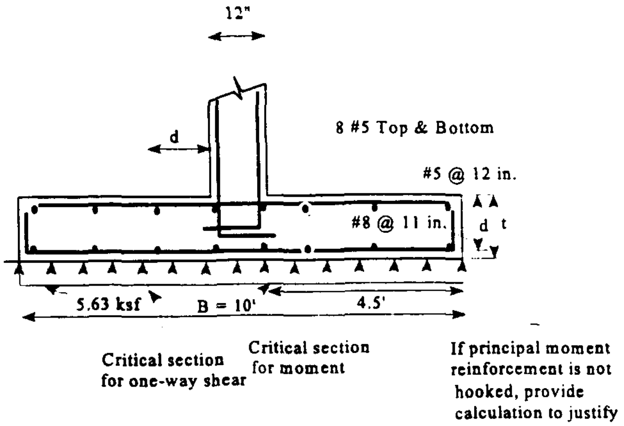
ACI 318-95 Section	Procedure	Calculation	Design Aid
	Step 1- Determine whether shear reinforcement is required. Using SHEAR 7.1.2, read K_{vc}. If $V_u > 0.5\phi K_{vc}$, then A_v is required.	For $\phi = 0.85$, ϕK_{vc} = 0.85 x 19.61 = 16. 7 kips. V_u = 9 kips > $0.5\phi K_{vc}$= 0.5 x 16.7 = 8.4 kips, therefore A_v is required.	SHEAR 7.1.2
11.6.1	**Step 2**- Determine whether torsion reinforcement is required. Using SHEAR 7.2.2 read ϕK_{tcr}. If $T_u > 0.25\ \phi K_{tcr}$, then A_t is required.	For $\phi = 0.85$, ϕK_{tcr} = 0.85 x 12.2 = 10.4 (k-ft). T_u= 8 (k - ft) > 0.25 ϕK_{tcr} = 0.25 x 10.4 = 2.6(k-ft), therefore A_t is required.	SHEAR 7.2.2
11.6.1 11.6.2	**Step 3**- Determine the design torsion : T_u is the lesser value of actual T_u and ϕK_{tcr}, where $K_{tcr} = 4\sqrt{f_c'}\left(\frac{A_{cp}^2}{P_{cp}}\right)$ *in (k-ft) units.*	T_u = 8 (k - ft) < ϕK_{tcr} = 10.4 (k-ft), therefore design for T_u = 8 (k-ft).	SHEAR 7.2.2
11.6.3.1 Eq. (11-18)	**Step 4**- Determine if section is adequate. Using SHEAR 7.4.2, read K_t. If $\sqrt{\left(\frac{V_u}{5\phi K_{vc}}\right)^2 + \left(\frac{T_u}{\phi K_t}\right)^2} \le 1$ then the section is adequate.	For $\phi = 0.85$, ϕK_t = 0.85 x 18. 95 = 16.1(k-ft). $\sqrt{\left(\frac{9}{5\times 16.7}\right)^2 + \left(\frac{8}{16.1}\right)^2} = 0.51 < 1$ ∴ section is adequate.	SHEAR 7.1.2 7.4.2
11.6.3.6 Eq. (11-21)	**Step 5**- Determine transverse reinforcement for shear and torsion. Using SHEAR 7.1.2, and SHEAR 7.3.1, read K_{vs} and K_{ts}: $A_{vt}/s = K_{ct} = (V_u - \phi K_{vc})/\phi K_{vs} + T_u/\phi K_{ts}$, where $(V_u - \phi K_{vc}) \ge 0$	For $\phi = 0.85$, ϕK_{vc} = 16.7 kips, ϕK_{vs} = 0.85 x 620 = 527 k/in. ϕK_{ts} = 0.85 x 267.03 = 227 (k-ft/in.) $(V_u - \phi K_{vc})$ = 9 - 16.7 = - 7.7 < 0 ∴ K_{ct} = 0 + 8/227 = 0.0352 in^2/in.	SHEAR 7.1.2 SHEAR 7.3.1
11.6 .5 .2 Eq. (11-23)	**Step 6**- Check minimum transverse reinforcement: If $K_{ct} < 50b_w/f_{yv}$ then use $K_{ct} = 50b_w/f_{yv}$.	(50 x 10)/40,000 = 0.0125 in.^{2}/in. < K_{ct} = 0.0352 in.^{2}/in. (OK)	

ACI 318-95 Section	Procedure	Calculation	Design Aid
11.6.6.1 11.5.4	**Step 7**- Compute closed stirrup spacing: $s = 2A_{bar}/K_{ct}$ in $s \le (b + h - 7)/4 \le 12$ in If $V_s = (V_u - \phi K_{vc})/\phi < 2K_{vc}$, $s \le d/2$ ≤ 24 in.	Using #3 hoops, $A_{bar} = 0.11$ in^2; $s = (2 \times 0.11)/0.0352 = 6.25$ in. $(b + h - 7)/4 = (10 + 18 - 7)/4$ $= 5.25 < 6.25$ in. $V_s = 0$ since $(V_u - \phi K_{vc}) = 0$ $< 2K_{vc}$ $d/2 = (18 - 2.5)/2 = 7.75$ in. > 5in.(OK) Therefore use #3 @ 5 in.	REIN - FORCE - MENT 1
11.6.3.7 Eq. (11-22)	**Step 8**- Determine the additional longitudinal torsional reinforcement: $A_l = (b + h - 7)(T_u/\phi K_{ts}) f_{yv}/f_{yl}$ With $\theta = 45°$	$f_{yv} = f_{yl} = 40$ ksi, $\theta = 45°$ $A_l = (10 + 18 - 7) \times (8/227) \times (40/40) = 0.74$ in^2.	SHEAR 7.3.1
11.6.5.3 Eq. (11-24)	**Step 9**- Determine the minimum total area of longitudinal torsional reinforcement $$A_{l,min} = \frac{5\sqrt{f'_c}\,A_{cp}}{f_{yl}} - \left(\frac{A_t}{s}\right) p_h \frac{f_{yv}}{f_{yl}}$$ $$where\ \frac{A_t}{s} \ge \frac{25\,b_w}{f_{yv}}$$ and $A_t/s = T_u/2\phi K_{ts}$, $A_{cp} = bh$; $p_h = 2(b + h - 7)$	$A_t/s = T_u/2\phi K_{ts} = 8/(2 \times 227)$ $= 0.018$ in^2/in. > $(25 \times 10)/40{,}000$ $= 0.00625$ in^2/in.(OK) $$A_{l,min} = \frac{5\sqrt{4000} \times (10 \times 18)}{40{,}000}$$ $$- 0.018 \times 2 \times (10 + 18 - 7)\ \frac{40{,}000}{40{,}000}$$ $= 0.683 in.^2 < A_l = 0.74\ in.^2\ (OK)$	SHEAR 7.3.1
11.6.6.2	**Step 10**- Select longitudinal steel. These bars must be distributed around perimeter at a spacing not to exceed 12 in. The steel required for torsion is additive to that required for flexure. Bar shall have diameter at least 1/24 of the stirrup spacing but not less than #3 bar. There shall be a longitudinal bar in each corner of the stirrup.	Distributing the bars in three layers gives total of 6 bars. $A_{bar} = A_l/6 = 0.74/6 = 0.13$ in^2. Use 6#4 bars $d_{b,l} = 0.5$ in. $> s/24 = 5/24 = 0.21$in. > 0.375 in. (OK) $A_l = 6 \times 0.2 = 1.2$ in^2. Spacing $s_l = (h - 2 \times cover - 2 \times d_b$ (hoops) $- d_{b,l})/2$ $= (18 - 2 \times 1.5 - 2 \times 0.375 - 0.5)/2$ $= 6.9$ in. < 12 in. (OK).	REIN - FORCE - MENT 1

FOOTINGS

FOOTINGS EXAMPLE 1 - Design of a continuous (wall) footing

Determine the size and reinforcing for the continuous footing under a 12 in. bearing wall of a 10 story building, founded on soil.



Given:
f'_c = 4 ksi
f_y = 60 ksi
Dead Load = D = 25 k/ft
Live Load = L = 12.5 k/ft
Wind O.T. = W = 4 k/ft
Seismic O.T. = E = 5 k/ft

Allowable due to D = 3 ksf = "a"
Allowable due to D + L = 4 ksf = "b"
Allowable due to D + L + (W or E) = 5 ksf = "c"

ACI 318-95 Section	Procedure	Calculation	Design Aid
	Step 1.- Sizing the footing.	D/a = 25/3 = 8.3 ft (D + L)/b = 37.5/4 = 9.4 ft ⇐ controls (D + L + W)/c = 41.5/5 = 8.3 ft (D + L + E)/c = 42.5/5 = 8.5 ft **Use B = 10 ft**	
9.2	**Step 2.** Required strength.	U = 1.4D + 1.7L = 1.4(25) + 1.7(12.5) = 56.3 k/ft or 5.63 ksf ⇐ controls design U = 0.75(1.4D + 1.7L + 1.7W) = 0.75(56.3 + 1.7(4)) = 47.3 k/ft or 4.73 ksf U = 0.9D + 1.3W = 0.9(25) + 1.3(4) = 27.7 k/ft or 2.77 ksf U = 0.75(1.4D + 1.7L + 1.87E) = 0.75(56.3 + 1.87(5)) = 49.2 k/ft or 4.92 ksf U = 0.9D + 1.43E = 0.9(25) + 1.43(5) = 29.7 k/ft or 2.97 ksf	

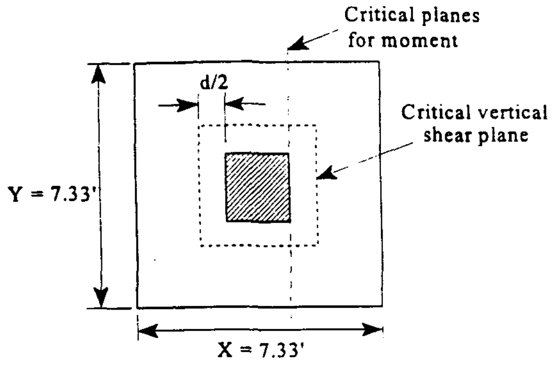
ACI 318-95 Section	Procedure	Calculation	Design Aid
9.3.2.3 11.1 11.3 7.7.1 11.1.1 11.3.1.1 11.1.3.1	**Step 3.** Shear.	$\phi_{shear} = 0.85$ Assume $V_s = 0$, (i.e. no shear reinforcement) $v_c \le 2\sqrt{f_c'} = 126\ psi$ Try **d = 17 in.** and **t = 21 in.** (3" min. cover) $V_u = (10/2 - 6/12 - 17/12)(5.63) = 17.4$ k/ft $V_n = V_u/\phi = 17.4/0.85 = 20.4$ k/ft $v_c = \frac{V_n}{12(d)} = \frac{20400}{12(17)} = 100\ psi < 126\ psi\ OK$ or $V_c = v_c bd = (.126)(12)(17) = 25.7\ k > V_n$ OK $d = \frac{17{,}400}{(0.85)(2\sqrt{4000})(12)} = 13.49\ in. < 17\ in.\ OK$	
9.3.2.1 10.5.4 10.3.3	**Step 4.** Moment.	$\phi_{flex} = 0.9$ $a_n = 4.45$ $\rho_{min} = .0018$ $\rho_{max} = .0214$ $M_u = \frac{wL^2}{2} = \frac{(5.63)(4.5)^2}{2} = 57\ ft\text{-}k/ft$ $Required\ A_s = \frac{M_u/\phi}{a_n d} = \frac{57/0.9}{(4.45)(17)} = 0.84\ sq.in./ft$ $\rho = A_s/bd = 0.84/(12)(17)$ $= .0041 > .0018$ OK **Use #8 @ 11" o.c.** $A_s = 0.86$ sq.in./ft > 0.84 OK	 Flexure 1 Flexure 1 Flexure 1 Reinf. 15

FOOTINGS EXAMPLE 2 - Design a square spread footing

Determine the size and reinforcing for a square spread footing that supports a 16 in. square column, founded on soil.

Given:
f'_c = 4 ksi
f_y = 60 ksi
16 in. x 16 in. column
Dead load D = 200 k
Live load L = 100 k

Allowable due to D = 4 ksf = "a"
Allowable due to D + L = 7 ksf = "b"

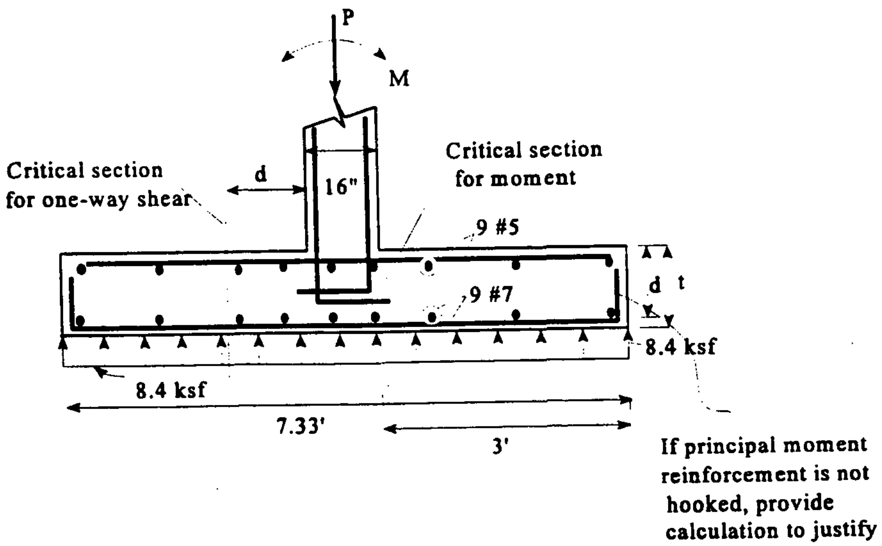


Design a square footing.

ACI 318-95 Section	Procedure	Calculation	Design Aid
	Step 1. Sizing the footing.	D/a = 200/4 = 50 sq. ft. ⇐ Controls (D + L)/b = 300/7 = 42.9 sq. ft. **Use 7.33 ft x 7.33 ft** A = 53.7 > 50 sq. ft. OK	
9.2	**Step 2.** Required strength.	U = 1.4D + 1.7L = 1.4(200) + 1.7(100) = 450 k or 450/53.7 = 8.4 ksf	

ACI 318-95 Section	Procedure	Calculation	Design Aid
9.3.2.3 11.1 11.12.2.1 7.7.1 11.1.1 11.12.2.1	**Step 3.** Shear.	$\phi_{shear} = 0.85$ Assume $V_s = 0$, (i.e. no shear reinforcement) $v_c \le 4\sqrt{f_c'} = 253\ psi$ (two-way action) Try **d = 16 in.** and **t = 20 in.** (3" min. cover) $b_0 = (4)(16+16) = 128$ in. $V_{u2} = [(7.33)^2 - ((16+16)/12)^2]\ (8.4) = 391.6$ k $V_{n2} = V_{u2}/\phi = 391.6/0.85$ k/ft $= 460.7$ k $d = \frac{391{,}600}{(0.85)(4\sqrt{4000})(128)} = 14.22\ in. < 16\ in.\ OK$ $d = \frac{391{,}600}{0.85\left(\frac{40x16}{128}+2\right)(\sqrt{4000})(128)} = 8.13\ in. < 16\ in.\ OK$ $v_c = \frac{V_n}{ShearArea} = \frac{460{,}700}{(4)(32)(16)} = 225 < 253\ psi\ OK$ (one-way action) $V_{u1} = (7.33)(1.67)(8.4) = 102.6$ k $d = \frac{102{,}600}{(0.85)(2\sqrt{4000}(88)} = 10.84\ in. < 16\ in.\ OK$	

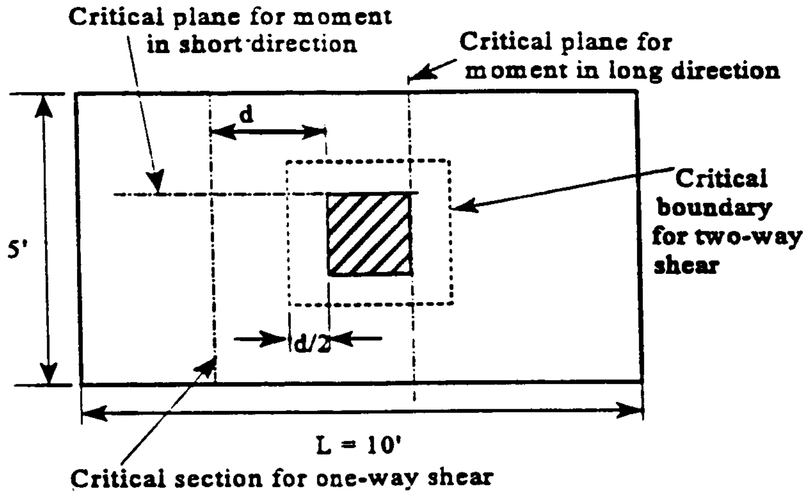
ACI 318-95 Section	Procedure	Calculation	Design Aid
9.3.2.1 10.5.4 10.3.3	**Step 4.** Moment.	$\phi_{flex} = 0.9$ $a_n = 4.45$ $\rho_{min} = .0018$ $\rho_{max} = .0214$ $M_u = (1/2)(8.4)(3)^2(7.33) = 277$ ft-k or $M_u = (1/2)[(8.9)(3)^2(7.33) + (10.5\text{-}8.9)(3)(7.33)(2)]$ $= 293.6 + 35.2 = 329$ ft-k **(Controls)**	Flexure 1 Flexure 1 Flexure 1
	Bottom reinforcement	Req'd $A_s = (M_u/\phi)/(a_n d) = (329/0.9)/[(4.45)(16)]$ $= 5.13$ sq.in. $\rho = A_s/bd = 5.13/[(7.33)(12)(16)]$ $= .0036 > .0018$ OK **Use 9 #7** $A_s = 5.41$ sq.in. OK	Reinf. 15
	Top reinforcement	Arbitrarily design to take 1/2 the seismic moment. $M_u = (1/2)[0.75(1.87)(200)] = 140$ ft-k $A_s = (M_u/\phi)/(a_n d) = [(140)/(0.9)]/[(4.45)(16)]$ $= 2.18$ sq.in. **Use 9 #5** $A_s = 2.76$ sq.in. > 2.2 sq. in. OK	Reinf.15



FOOTINGS EXAMPLE 3 - Design a rectangular spread footing

Determine the size and reinforcing for a rectangular spread footing that supports a 16 in. square column, founded on soil.

Given:
$f'_c = 4$ ksi
$f_y = 60$ ksi
16 in. x 16 in. column
Dead load = D = 180 k
Live load = L = 100 k
Wind O.T. = W = 120 k, axial load due to overturning under wind loading.



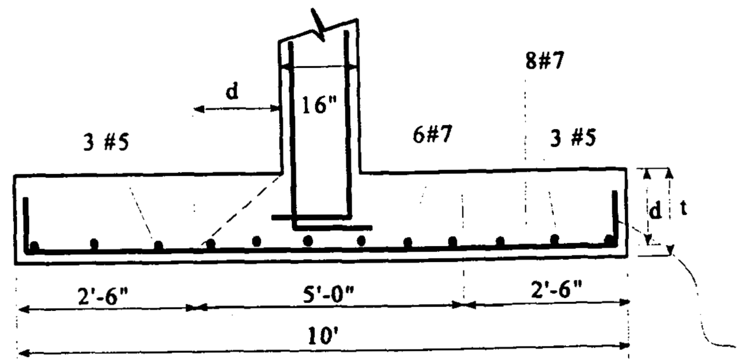
Allowable due to D = 4 ksf = "a"
Allowable due to D + L = 6 ksf = "b"
Allowable due to D + L + E = 8 ksf = "c"

Design a rectangular footing with B/L aspect ≤ 0.6.

ACI 318-95 Section	Procedure	Calculation	Design Aid
	Step 1. Sizing the footing.	D/a = 180/4 = 45 sq.ft. (D+L)/b = 280/6 = 46.7 sq.ft. (D+ L+ W)/c = 400/8 = 50 sq.ft. ⇐ Controls **Use 5 ft x 10 ft** A = 50 sq.ft. OK	
9.2	**Step 2.** Required strength.	U = 1.4D + 1.7L = 1.4(180) + 1.7(100) = 422 k or 422/50 = 8.4 ksf U = 0.75(1.4D + 1.7L + 1.7W) = 0.75[422 + 1.7(120)] = 469.5 k or 9.4 ksf ⇐ Controls U = 0.9D + 1.3W = 0.9(180) + 1.3(120) = 318 k or 6.4 ksf	

ACI 318-95 Section	Procedure	Calculation	Design Aid
9.3.2.3 11.1 11.12.2.1 7.7.1 11.1.1 11.12.2.1	**Step 3.** Shear.	$\phi_{shear} = 0.85$ Assume $V_s = 0$, (i.e. no shear reinforcement) $v_c \le 4\sqrt{f_c'} = 253\ psi$ (two-way action) Try **d = 23 in.** and **t = 27 in.** (3" min. cover) $b_0 = (4)(39) = 156$ in. $V_{u2} = [50 - ((16+23)/12)^2](9.4) = 370.7$ k $V_{n2} = V_{u2}/\phi = 370.7/0.85$ k/ft $= 436.1$ k $v_c = \frac{V_n}{ShearArea} = \frac{436{,}100}{(2)(39)(23)} = 243 << 253\ psi\ OK$ $d = \frac{370{,}700}{(0.85)(4\sqrt{4000})(156)} = 11.05\ in. < 23\ in.\ OK$ $d = \frac{370{,}700}{0.85(\frac{30x23}{156}+2)(\sqrt{4000})(156)} = 6.88\ in. < 23\ in. OK$ (one-way action) $b = (5)(12) = 60$ in. $V_{u1} = (5.0)(2.42)(9.4) = 113.74$ k $d = \frac{113{,}740}{(0.85)(2\sqrt{4000})(60)} = 17.63\ in. < 23\ in. OK$	

ACI 318-95 Section	Procedure	Calculation	Design Aid
9.3.2.1 10.5.4 10.3.3	**Step 4.** Moment.	$\phi_{flex} = 0.9$ $a_n = 4.45$ $\rho_{min} = .0018$ $\rho_{max} = .0214$	Flexure 1 Flexure 1 Flexure 1
	Long direction	$M_u = (1/2)(9.4)(5)(4.33)^2 = 440.6$ ft-k Req'd $A_s = (M_u/\phi)/(a_n d) = (440.6/0.9)/[(4.45)(23)]$ $= 4.78$ sq.in. $\rho = A_s/bd = 4.78/[(5)(12)(23)]$ $= .0035 > .0018$ OK **Use 8 #7** $A_s = 4.81 > 4.78$ sq.in. OK	Reinf. 15
  10.5.4	Short direction	$M_u = (1/2)(9.4)(10)(1.83)^2 = 157.4$ ft-k Req'd $A_s = (M_u/\phi)/(a_n d) = [(157.4)/(0.9)]/[(4.45)(23)]$ $= 1.71$ sq.in. $A_s = \rho_{min}bd = (0.0018)(10)(12)(23)$ $= 4.97$ sq.in.	
15.4.4.2		(Reinf. in Center 5' Band)/(Total Reinf.) $= 2/(\beta+1)$ $\beta = L/B = 2$ or $X/4.97 = 2/(2+1)$ $X = 3.31$ sq.in. **Use 6 #7** $A_s = 3.6$ sq.in. < 3.31 sq in. OK	Reinf.15
		Outside of Center Band: Req'd $A_s = (4.97 - 3.6) = 1.37$ sq.in. **Use 6 #5** (3 each side) $A_s = 1.86$ sq.in. > 1.37 OK	Reinf.15



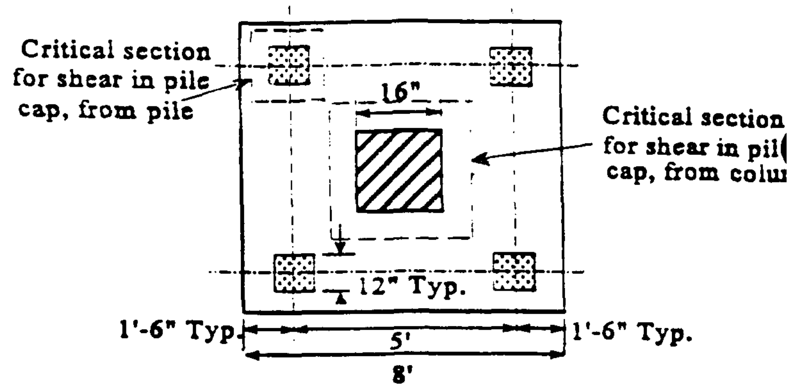
If principal moment reinforcement is not hooked, provide calculation to justify

FOOTINGS EXAMPLE 4 - Design a pile cap

Determine the size and reinforcing for a square pile cap that supports a 16 in. square column, on 4 piles.

Given:
f'_c = 5 ksi
f_y = 60 ksi
16 in. x 16 in. column
Dead load = D = 250 k
Live load = L = 150 k

16 x 16 in. reinforced concrete column
12 x 12 in. reinforced concrete piles,
4 each @ 5' o.c.



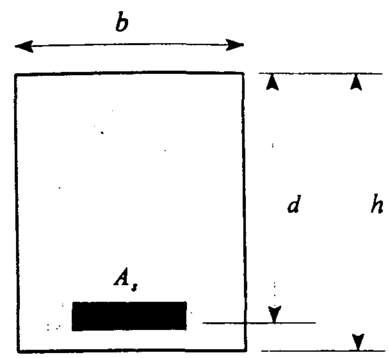
ACI 318-95 Section	Procedure	Calculation	Design Aid
9.2	**Step 1.**-Factored Loads.	Column: $P_u = 1.4D + 1.7L$ $= 1.4(250) + 1.7(150)$ $= 605 \text{ k} = V_u$ Piles: $P_u = 605/4 = 151 \text{ k} = V_u$	
9.3.2.3 11.1.1 11.1 11.12.2.1	**Step 2.** Shear	$\phi_{shear} = 0.85$ From Column: $V_n = V_u/\phi = 605/0.85 = 712 \text{ k} = P_{n\,col}$ From Piles: $V_n = P_{n\,col}/4 = 712/4 = 178 \text{ k} = P_{n\,pile}$ Assume no shear reinforcement $V_n = V_c$ $V_c = (v_c)(\text{shear area})$ $v_c \le 4\sqrt{f'_c} = 283\ psi$ (two-way action) Column load controls "d" **Try d = 29"** Shear area = 4(29+16)(29) = 5220 sq.in. v_c = 712,000/5,220 = 136 << 283 psi OK (This is conservative, but OK because of the overlapping of the shear cones of the column and the piles.) Check shear at piles: Shear area = 2(18+8+14.5)(29) = 2349 sq.in. v_c = 178,000/2349 = 76 < 283 psi OK	

ACI 318-95 Section	Procedure	Calculation	Design Aid
9.3.2.1	**Step 3.** Moment	$\phi_{flex} = 0.9$	
		$a_n = 4.45$	Flexure 1
10.5.1		$\rho_{min} = .0018$	Flexure 1
10.3.3		$\rho_{max} = .0214$	Flexure 1
		$M_u = 2(151)(1.83) = 553$ ft-k	
	Bottom reinforcement:	Req'd $A_s = (M_u/\phi)/(a_n d) = (553/0.9)/[(4.45)(29)]$ $= 4.76$ sq in.	
10.5.4		$A_{s\,min} = 0.0018(8)(12)(29) = 5.01$ sq.in.	Reinf. 15
		Use 10 #7 each way As = 6.0 in. > 5.01 OK	
	Top reinforcement:	Not required.	

DEFLECTION

DEFLECTION EXAMPLE 1-Effective moment of inertia for a rectangular section with tension reinforcement

Determine the effective moment of inertia I_e to be used for the rectangular section shown



Given:

b = 14 in.
d = 21.4 in.
h = 24 in.
A_s = 6.24 sq in.
f'_c = 4 ksi (normal weight concrete)
n = 8
M_a = 177 kip-ft under service load

ACI 318-95 Section	Procedure	Calculation	Design Aid
9.5.2.3 Eq. (9-8)	**Step 1**- Determine cracking moment. Find K_{cr} Compute $M_{cr} = K_{cr}\, b$	For h = 24 in. and f_c' = 4 ksi, read K_{cr} = 3.79 M_{cr} = 3.79(14) = 53.1 ft-kips	DEFLEC-TION 1.1
	Step 2-Determine cracked section moment of inertia. Compute $\rho = A_s / bd$ Find K_{i1} Compute $I_{cr} = K_{i1}\, bd^3$	ρ = 6.24 / (14 x 21.4) = 0.0208 For ρ = 0.0208 and n = 8 read K_{i1} = 0.080 I_{cr} = 0.080(14)(21.4)^3 = 11,000 in.^{4}	DEFLEC-TION 2
	Step 3-Determine moment of inertia of gross section. Compute $I_g = bh^3 / 12$	I_g = 12(24)^3 / 12 = 16,100 in.^{4}	
Eq. (9-7)	**Step 4**-Determine I_e. Compute I_{cr} / I_g Compute M_{cr} / M_a Find K_{i3} Compute $I_e = K_{i3}\, I_g$	I_{cr} / I_g = 11,000 / 16,100 = 0.68 M_{cr} / M_a = 53.1 / 177 = 0.30 For I_{cr} / I_g = 0.68 and M_{cr} / M_a = 0.30, find K_{i3} = 0.689 I_e = 0.689(16,100) = 11,100 in.^{4}	DEFLEC-TION 5.1*

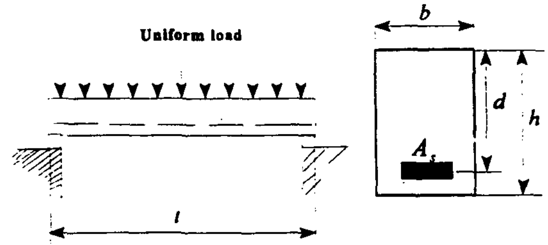
* From DEFLECTION 5, it can be deduced that there will be only small differences between I_e and I_{cr} unless M_{cr} / M_a is greater than I_{cr} / I_g.

DEFLECTION EXAMPLE 2-Deflection of a simple span, rectangular beam with tension reinforcement

Determine the live load deflection at midspan.

Given:

f'_c = 4000 psi
n = 8
A_s = 6.24 sq in.
M = 120 kip-ft
M_{d+l} = 177 kip-ft
b = 14 in.
d = 21.4 in.
h = 24.0 in.
l = 40 ft
(M_d and M_{d+l} under service loads)



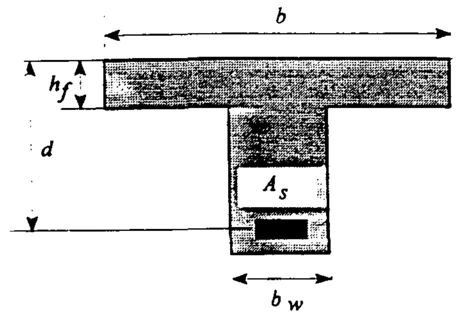
ACI 318-95 Section	Procedure	Calculation	Design Aid
9.5.2.3 Eq. (9-7)	Since concrete, section, and moment are the same, data from Deflection Example 1 can be used here. **Step 1**- Determine I_e for the dead-load moment. Compute M_{cr} / M_d; use M_c from Example 1. Find K_{i3}; use I_c/I_g from Deflection Example 1. Compute M_d, $I_{ed} = K_{i3} I_g$; use I_g from Example 1.	M_c/M_d = 53.1 / 120 = 0.44 For M_{cr} / M_d = 0.44 and I_{cr}/ I_g = 0.68, find K_{i3} = 0.707 I_{ed} = 0.707(16,100) = 11,400 in.^{4}	DEFLECTION 5.1
	Step 2-Determine initial dead- load deflection. Find K_{al} (*Note*: When f'_c = 4000 psi and $n = E_s / E_c$ = 8, concrete is normal weight, i.e., 145 pcf; see DEFLECTION 9.) *Find* K_{a3} *for uniform* load, simply supported Compute $a_d = (K_a/I_{ed})(K_{a3}M_d)$	For normal weight concrete f'_c = 4000 psi and l = 40 ft, read K_{al} = 15.81 For Case 2, read K_{a3} = 5.0 a_d= (15.81/11,400)(5)(120) = 0.83 in.	DEFLECTION 6. 2 DEFLECTION 6. 1
	Step 3-Determine total load deflection using I_e from Deflection Example 1. Compute $a_{d\text{-}l} = (K_a/I_e)(K_{a3} M_{d+l})$	a_{d+l} = (15.81/11,100)(5)(177) = 1.26 in.	
	Step 4-Determine live-load deflection. Compute $a_l = a_{d+l} - a_d$	a_l= 1.26 - 0.83 = 0.43 in.	Commen-tary on DEFLEC-TION 5.1

DEFLECTION EXAMPLE 3-Moment of inertia of a cracked T-section with tension reinforcement

Determine the cracked-section moment of inertia I_{cr} for the section shown.

Given:

$n = 9$
$b = 45$ in.
$b_w = 24$ in.
$h_f = 6.5$ in.
$d = 35.1$ in.
$A_s = 27.89$ sq in.



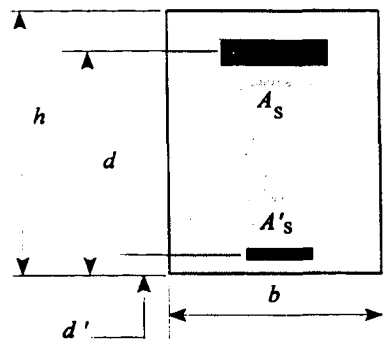
ACI 318-95 Section	Procedure	Calculation	Design Aid
	Step 1- Calculate constants for table. Compute $\rho_w = A_s / b_w d$ $\rho_w n$ $hf / 2d$ $\beta_c = \dfrac{\left(\dfrac{b}{b_w} - 1\right)\dfrac{h_f}{d}}{\rho_w n}$	$\rho_w = 27.98 / (24 \times 35.1) = 0.0331$ $\rho_w n = 0.0331(9) = 0.298$ $hf / 2d = 6.5 / (2 \times 35.1) = 0.0926$ $\beta_c = \dfrac{\left(\dfrac{45}{24} - 1\right)\dfrac{6.5}{35.1}}{0.298} = 0.54$	
9.5.2.3	**Step 2**-Find K_{i2}.	For $\beta_c = 0.50$, $\rho_w n = 0.298$, and $hf / 2d = 0.0926$, find $K_{i2} = 0.140$	DEFLECTION 4.1
		For $\beta_c = 0.60$, $\rho_w n = 0.298$, and $hf / 2d = 0.0926$, find $K_{i2} = 0.144$ Interpolating for $\beta_c = 0.54$, $K_{i2} = 0.142$	DEFLECTION 4.1
	Step 3-Determine cracked-section ment of inertia. Compute $I_{cr} = K_{i2} b_w d^3$	$I_{cr} = 0.142(24)(35.1)^3 = 147{,}000$ in.^{4}	

DEFLECTION EXAMPLE 4-Moment of inertia of a cracked section with tension and compression reinforcement

Determine the cracked-section moment of inertia I_{cr} for the section shown.

Given:

n = 9
b = 18 in.
h = 40 in.
d = 35.4 in.
d' = 35.4 in.
A_s = 13.62 sq in.
A_s' = 6.54 sq in.

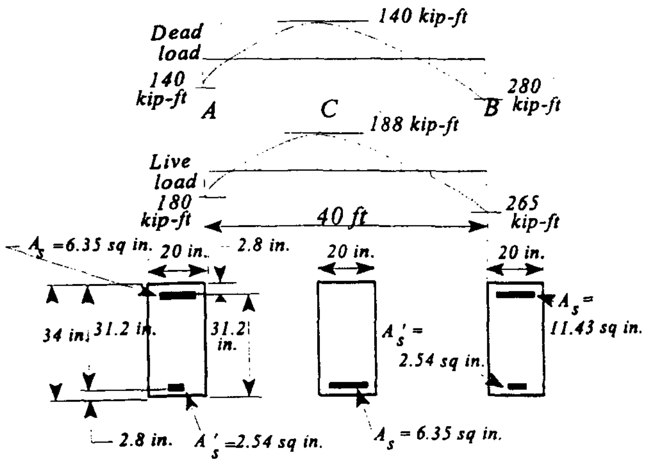


Negative moment section

ACI 318-95 Section	Procedure	Calculation	Design Aid
	Step 1- Calculate constants for table. Compute $\rho = A_s / bd$, $\rho = A_s' / bd$, ρn; $\beta_c = \frac{\rho'(n-1)}{\rho n}$; d' / d	ρ = 13.62 / (18 x 35.4) = 0.0214; ρ' = 6.54 / (18 x 35.4) = 0.0103; ρn = 0.0214(9) = 0.193; $\beta_c = \frac{0.0103(9-1)}{0.193} = 0.427$; d' / d = 2.6 / 35.4 = 0.0734	
9.5.2.3	**Step 2**- Find K_{i2}.	For β_c = 0.4, ρn = 0.193, and d' / d = 0.0734, find K_{i2} = 0.098	DEFLECTION 4.1
		For β_c = 0.5, ρ = 0.193, and d' / d = 0.0734, find K_{i2} = 0.101	DEFLECTION 4.1
		Interpolating for β_c = 0.427, K_{i2} = 0.099	
	Step 3-Determine cracked-section moment of inertia. Compute $I_{cr} = K_{i2} b_w d^3$	$I_{cr} = 0.099(18)(35.4)^3$ = 79,100 in.^{4}	

DEFLECTION EXAMPLE 5-Live load deflection of a continuous beam

Determine the deflection of the beam due to live load if the moment diagrams and cross sections are as shown. (All moments are due to service loads.)



Given:

f'_c = 3000 psi
f_y = 40,000 psi
n = 9
w_c = 145 pcf

Note that Section 9.5.2.4 of ACI 318-95 permits the use of Ie computed for the cross section at midspan (Section C of this example) rather than the average Ie. This simpler approach is illustrated in this ecample as Alternative 1 in Steps 4 through 8.

ACI 318-95 Section	Procedure	Calculation	Design Aid
	Step 1- Compute $I_g = bh^3 / 12$.	$I_g = 20(34)^3 / 12 = 65{,}500$ in.^{4}	
9.5.2.3	**Step 2**-Determine I_{cr} for each cross section. For Sections A and B, compute $\rho = A_s / bd$ $\rho' = A_s' / bd$ ρn $$\beta_c = \frac{(n-1)\rho'}{\rho n}$$ d'/d For β_c, ρn, and d'/d, read K_{i2} values. Compute $I_{cr} = K_{i2}bd^3 = K_{i2}(20)(31.2)^3$ For Section C, compute $\rho = A_s / bd$ Find K_{il} Compute $I_{cr} = K_{il}bd^3$	Section A / Section B ρ = 0.0102 / 0.0183 ρ' = 0.0041 / 0.0041 ρn = 0.092 / 0.165 β_c = 0.36 / 0.20 d'/d = 0.090 / 0.090 K_{i2} = 0.055 / 0.084 I_{cr} = 33,400 in.^{4} / 51,000 in.^{4} Section C $\rho = 6.35 / (20 \times 31.2) = 0.0102$ For n = 9 and ρ = 0.0102, read K_{il} = 0.053 $I_{cr} = 0.053(20)(31.2)^3 = 32{,}200$ in.^{4}	DEFLECTION 4.1 DEFLECTION 2
	Step 3-Determine cracking moment. Find Kcr Compute $M_{cr} = K_{cr}b$	For f'_c= 3000 psi and h = 34 in., read K_{cr} = 6.60 kip-ft per in. M_{cr} = 6.60(20) = 132 kip-ft	DEFLECTION 1.1
	Step 4-Determine I_{ed} for the dead load moments. Compute $M_{cr}/M_a = 132 / M_d$ Compute $I_{cr}/I_g = I_{cr} / 65{,}500$ Find K_{i3} for each section Compute $I_{ed} = K_{i3}I_g$	Section: A / B / C M_{cr}/M_d: 0.943 / 0.471 / 0.943 I_{cr}/I_g: 0.510 / 0.779 / 0.491 K_{i3}: 0.921 / 0.802 / 0.918 I_{ed}: 60,300 in.^{4} / 52,500 in.^{4} / 60,100in.^{4}	DEFLEC-TION 5.1

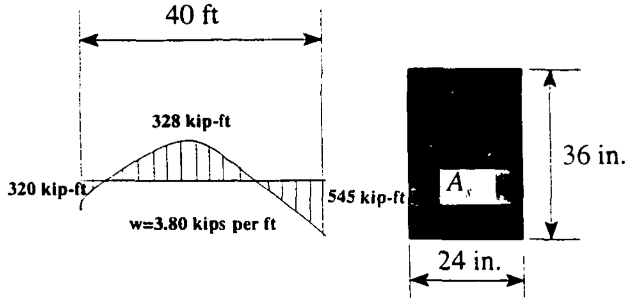
ACI 318-95 Section	Procedure	Calculation	Design Aid
9.5.2.4 ACI 318R-95 9.5.2.4	Compute average $I_{ed} = \sum I_{e}/3$ Alternative 1* Use I_{ed} @ C Alternative 2* Use weighted average	Average I_{ed} = (60,300 + 52,500 + 60,100)/3 = 57,600 in^4 I_{ed} = 60,100 I_{ed} = 0.70 (60,100) + 0.15(60,300 + 52,500) = 59,000	
ACI 318R-95	**Step 5**-Determine dead load deflection. Find K_{a1} Find K_{a3} for span with both positive and negative moments Compute ad = (K_{a3}/I_{ed})[Mc-0.1(M_A + M_B)]K_{a1} Alternative 1 Alternative 2	For 40-ft span, f'_c = 3000, and w_c = 145 pcf, read K_{a1} = 18.25 For Case 7, read K_{a3} = 5.0 a_d = (5/57,600)[140-0.1(140 + 280)] 18.25 = 0.155 in. a_d = (5/60,100)[140-0.1(140 + 280)] 18.25 = 0.149 in. a_d = (5/59,000)[140-0.1(140 + 280)] 18.25 = 0.125 in.	DEFLEC-TION 6.2 DEFLEC-TION 6.1 DEFLEC-TION 6.1
	Step 6-Determine I_e for live load plus dead load. Compute M_{cr} / M_a 132 / M_{d+l} Use I_{cr}/I_g computed above, and find K_{i3} Compute $I_e = K_{i3}I_g$ Compute average $I_e = \sum I_e / 3$ Alternative 1 Use I_e @ C Alternative 2 Use weighted average	Section: A / B / C M_{cr} / M_{d+l}: 0.412 / 0.242 / 0.402 K_{i3}: 0.544 / 0.782 / 0.523 I_e: 35,600 / 51,200 / 34,300 Average I_e = (35,600 + 51,200 + 34,300)/3 = 40,400 in^4 I_e = 34,300 I_e = 0.70 (34,300) + 0.15 (35,600 + 51,200) = 37,000	DEFLEC-TION 5.1
	Step 7-Determine total deflection for live load plus dead load. Compute $a_{d+l} = (K_{a3}/I_e)$[Mc-0.1(M_A + M_B)]K_{a1} Alternative 1 Alternative 2	a_{d+l} = (5/40,400)[328 - 0.1(320 + 545)] 18.25 = 0.545 in. a_{d+l} = (5/34,300)[328 - 0.1(320 + 545)] 18.25 = 0.642 in. a_{d+l} = (5/37,000)[328 - 0.1(320 + 545)] 18.25 = 0.596 in.	DEFLEC-TION 6.1
	Step 8-Determine live-load deflection. Compute $a_l = a_{d+l} - a_d$ Alternative 1 Alternative 2	a_l = 0.545 - 0.155 = 0.39 in. a_l = 0.642 - 0.149 = 0.49 in. a_l = 0.596 - 0.152 = 0.44 in.	Commen-tary on DEFLEC TION 5.1

*ACI 318-95 Section 9.5.2.4 allows Alternative 1. ACI 318R-83 Section 9.5.2.4 recommends Alternative 2.

DEFLECTION EXAMPLE 6 - Simplified method for approximate calculation of deflection

Determine the immediate deflection of the beam shown, using the simplified (approximate) method. (All moments are due to service loads.)

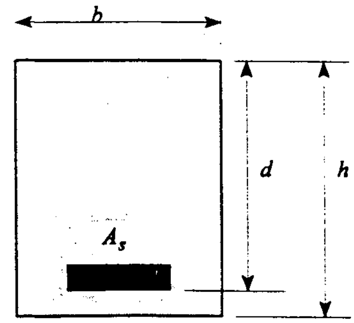
Given:
$\rho > 0.6\,\rho_{bal}$
$f'_c = 3$ ksi
$w_c = 145$ pcf
$w_u = 3.80$ k/ft



ACI 318-95 Section	Procedure	Calculation	Design Aid
9.5.2.2 9.5.2.3	**Step 1** Determine K_{a2} for simplified approximate method. Find K_{a2} in table. Correct for f'_c and w_c if necessary.	For h = 36 in. and l = 40 fti, read K_{a2} = 7.5 For f'_c = 3 ksi and w_c = 145 pcf, multiplier is 1.00 Therefore K_{a2} = 1.00 x 7.5 = 7.5	DEFLECTION 7 DEFLECTION 7
	Step 2 Determine δ_c for simplified approximate method.	For interior span with average reinforcement $\rho > 0.6\,\rho_{bal}$, read δ_c = 0.25	DEFLECTION 7 and Commentary
	Step 3 Compute deflection. $a_c = K_{a2}\,\delta_c\,w/b$	a_c = 7.5(0.25)(3.80) / 24 = 0.30 in.	DEFLECTION 7

DEFLECTION EXAMPLE 7-Effective moment of inertia of a rectangular beam with tension reinforcement

Determine the effective moment of inertia I_e for the rectangular section shown



Given:

A_s = 0.40 sq in. (2 #4)
h = 8 in.
d = 6.4 in.
b = 12 in.
f'_c = 5 ksi (normal weight concrete)
f_y = 60 ksi
M_a = 7.5 kip-ft

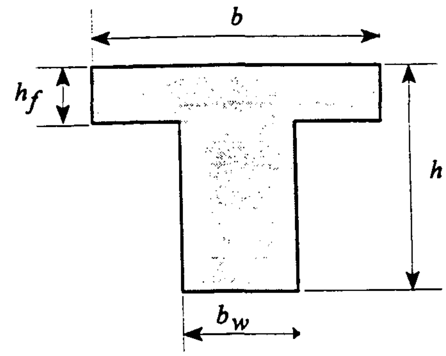
ACI 318-95 Section	Procedure	Calculation	Design Aid
8.5.1, 8.5.2	**Step 1**- Determine ρn. Compute $\rho = A_s / bd$ Find $n = E_s / E_c$ (For practical use, n may be taken as nearest whole number.) Compute ρn.	$\rho = 0.40 / (12 \times 6.4) = 0.0052$ For normal weight concrete (w_c = 145 pcf) and f'_c = 5 ksi, read n = 7 $\rho n = 0.0052(7) = 0.036$	DEFLECTION 9
	Step 2-Calculate d / h.	$d / h = 6.4 / 8 = 0.8$	
	Step 3-Calculate $I_g = bh^3 / 12$	$I_g = 12(8)^3 / 12 = 512$ in.^{4}	
	Step 4-Calculate the extreme fiber stress in tension on the gross section Fiber stress = $M_a y_t / I_g$ $y_t = h / 2$	Fiber stress = 7.5(12000)(4) / 512 = 703 psi	
	Step 5- Determine Ie. Compute $Ie = K_{i3} I_g$	Use graph to obtain K_{i3}: In left part of chart, on vertical line denoting f'_c = 5 ksi, locate fiber stress = 703 psi. Go horizontally to right to main chart and draw line slanting toward Ie / I_g = 1.0. Then, at upper left side of the chart, read ρn. = 0.036 and proceed horizontally to the right until meeting d / h = 0.8. Drop vertically to intersect the slanted line previously drawn. Then proceed horizontally to the right hand side of the graph and read Ie / I_g = 0.51. This procedure is shown on DEFLECTION 5.2 with heavy lines and arrows. $Ie / I_g = K_{i3} = 0.51$ $Ie = 0.51(512) = 261$ in.^{4}	DEFLEC-TION 5.2

DEFLECTION EXAMPLE 8-Cracking moment for T -section

Determine the cracking moment M_{cr} for the section shown, for use in ACI 318-95 Eq. (9-7).

Given:

f'_c = 3 ksi
w_c = 145 pcf
b_w = 18 in.
b = 90 in.
h = 40.5 in.
h_f = 4.5 in.

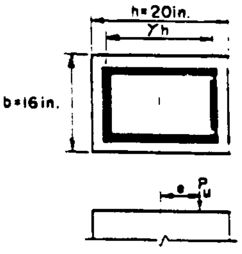


ACI 318-95 Section	Procedure	Calculation	Design Aid
9.5.2.3 Eq. (9-8) Eq. (9-9)	**Step 1**-Find K_{cr} for normal weight concrete.	For h = 40.5 in. and f'_c = 3 ksi, read K_{cr} = 9.36 kip-ft / in.	DEFLEC-TION 1.1
	Step 2-Determine value of K_{crt} Compute $\alpha_b = b / b_w$ $\beta_h = h_f / h$ Find K_{crt}	$\alpha_b = 90 / 18 = 5$ $\beta_h = 4.5 / 40.5 = 0.11$ For positive moment, read K_{crt} = 1.36 For negative moment, read K_{crt} = 2.3	DEFLEC-TION 1.2 DEFLEC-TION 1.3
	Step 3-Compute M_{cr} $M_{cr} = b_w K_{cr} K_{crt}$	For positive moment, M_{cr} = 18(9.36)(1.36) = 229 kip-ft For negative moment, M_{cr} = 18(9.36)(2.3) = 387 kip-ft	

COLUMNS

COLUMNS EXAMPLE 1-Required area of steel for a rectangular tied column with bars on four faces (slenderness ratio found to be below critical value)

For a rectangular tied column with bars equally distributed along four faces, find area of steel.



Given: Loading

Required nominal axial strength P_n = 800 kips
Required nominal moment strength M_n = 5600 kip-in.

Materials

Compressive strength of concrete f'_c = 4 ksi
Yield strength of reinforcement f_y = 60 ksi
Nominal maximum size of aggregate is 1 in.

Design conditions

Unsupported length of columns ℓ_u = 10 ft
Column is braced against sidesway

ACI 318-95 Section	Procedure	Calculation	Design Aid
	Step 1-Determine column section size.	Given: h = 20 in. b = 16 in.	
10.11.4.1 10.12.2 10.12.3	**Step 2**-Check whether slenderness ratio $k\ell_u/h$ is less than critical value. If so, slenderness effects may be neglected. If not, slenderness effects must be considered by magnifying moment M_n by factor δ_{ns}.		
10.11.3 10.11.4.1	**A)** Compute M_1/M_2, and read critical value of $k\ell_u/h$	In this case, it is given that M_1 = 5600 kip-in., but M_2 is not known. However, for rectangular columns, for all values of M_1/M_2, slenderness may be neglected where $k\ell_u/h < 6.6$	COLUMNS 1
 10.11.2.1 10.11.4.1	**B)** Compute $k\ell_u/h$ and compare with critical value; determine whether slenderness effects must be considered	For columns braced against sidesway: k = 1.0 Given: ℓ_u = 10 ft = 120 in. $k\ell_u/h = (1.0)(120)/20 = 6.0 < 6.6$ ∴ Slenderness effects may be neglected	

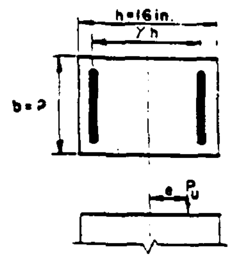
ACI 318-95 Section	Procedure	Calculation	Design Aid
9.3.2.2(b) 10.2.7	**Step 3**—Determine reinforcement ratio ρ_g using known values of variables on appropriate interaction diagram(s) and compute required cross section area A_{st} of longitudinal reinforcement.	Given: P_n = 800 kips M_n = 5600 kip-in. h = 20 in. b = 16 in. $\therefore A_g = b \times h = 20 \times 16 = 320$ in.^{2}	
	A) *Compute* $K_n = \dfrac{P_n}{f'_c A_g}$	$K_n = \dfrac{800}{4 \times 320} = 0.625$	
	B) *Compute* $R_n = \dfrac{M_n}{f'_c A_g h}$	$R_n = \dfrac{5600}{4 \times 320 \times 20} = 0.22$	
	C) Estimate $\gamma \approx \dfrac{h - 5}{h}$	$\gamma \approx \dfrac{20 - 5}{20} = 0.75$	
	D) Determine appropriate interaction diagram(s)	For a rectangular tied column with bars along four faces, f'_c = 4 ksi, f_y = 60 ksi, and an estimated γ of 0.75, use R4-60.7 and R4-60.8.	
9.3.2.2(b) 10.2 10.3	**E)** Read ρ_g for $P_n/f'_c A_g$ and $M_n/f'_c A_g h$	For $P_n/f'_c A_g = 0.625$ from Step 3A and $M_n/f'_c A_g h = 0.22$ from Step 3B: $\rho_g = 0.041$ for $\gamma = 0.7$ and $\rho_g = 0.039$ for $\gamma = 0.8 \Rightarrow \rho_g = 0.040$ for $\gamma = 0.75$	COLUMNS R4-60.7 and R4-60.8
	F) Compute required A_{st} from $A_{st} = \rho_g A_g$	Required $A_{st} = 0.040 \times 320$ in.^{2} = 12.8 in.^{2}	

COLUMNS EXAMPLE 2-For a specified reinforcement ratio, selection of a column section size for a rectangular tied column with bars on end faces only

For minimum longitudinal reinforcement ($\rho_g \approx 0.01$) and column section dimension h = 16 in., select column dimension b for a rectangular tied column with bars on end faces only.

Given: Loading

Required nominal axial strength P_n = 943 kips
Required nominal moment strength M_n = 3986 kip-in.



Materials

Compressive strength of concrete f'_c = 4 ksi
Yield strength of reinforcement f_y = 60 ksi
Nominal maximum size of aggregate is 1 in.

Design conditions

Slenderness effects may be neglected because $k\ell_u/h$ is known to be below critical value

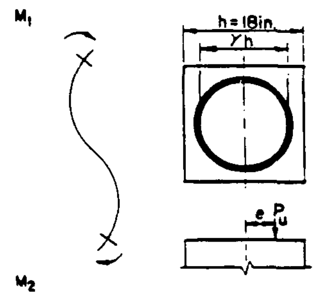
ACI 318-95 Section	Procedure	Calculation	Design Aid
	Step 1-Determine trial column dimension b corresponding to known values of variables on appropriate interaction diagram(s).	Given: P_n = 943 kips M_n = 3986 kip-in. $\rho_g \approx 0.01$ f'_c = 4 ksi f_y = 60 ksi h = 16 in.	
	A) Assume a series of trial column sizes b, in., and compute A_g = b x h, in.^{2}	24 26 28 384 416 448	
	B) *Compute* $K_n = \dfrac{P_n}{f'_c A_g}$	$\dfrac{943}{4 \times 384}$ = 0.61 $\dfrac{943}{4 \times 416}$ = 0.57 $\dfrac{943}{4 \times 448}$ = 0.53	
	C) *Compute* $R_n = \dfrac{M_n}{f'_c A_g h}$	$\dfrac{3986}{4 \times 384 \times 16}$ = 0.16 $\dfrac{3986}{4 \times 416 \times 16}$ = 0.15 $\dfrac{3986}{4 \times 448 \times 16}$ = 0.14	
	D) Estimate $\gamma \approx \dfrac{h-5}{h}$	0.7 0.7 0.7	
	E) Determine appropriate interaction diagram(s)	For a rectangular tied column with steel on end faces only, f'_c = 4 ksi, f_y = 60 ksi, and an estimated γ of 0.7, use L4-60.7	
9.3.2.2(b) 10.2 10.3	**F)** Read ρ_g for $P_n/f'_c A_g$ and $M_n/f'_c A_g h$ For γ = 0.7, select dimension corresponding to ρ_g nearest desired value of ρ_g = 0.01	0.018 0.014 0.011 ∴ Try a 16 x 28-in. column	COLUMNS L4-60.7

COLUMNS EXAMPLE 3-Selection of reinforcement for a square spiral column with reverse curvature (slenderness ratio found to be below critical value)

For the square spiral column section shown,
select the reinforcement

Given: Loading

Factored dead load P_d = 337 kips
Required nominal axial strength P_n = 943 kips
Required nominal moment strength at top of column M_1 = -943 kip-in.
Required nominal moment strength at bottom of column M_2 = +3771 kip-in.
$\beta_d = 1.4\ p_d/P_n = 0.5$
No transverse loading



Materials

Compressive strength of concrete f'_c = 4 ksi
Yield strength of reinforcement f_y = 60 ksi
Nominal maximum size of aggregate is 1 in.

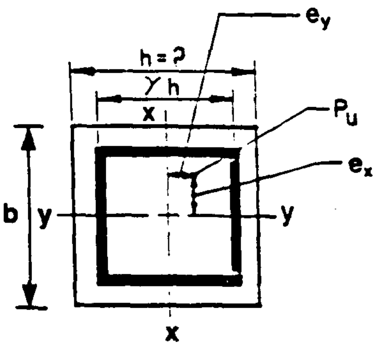
Design conditions

Column section size h = b = 18 in.
ℓ_u = 12 ft 10 in.
Column is braced against sidesway

ACI 318-95 Section	Procedure	Calculation	Design Aid
	Step 1-Determine column section size.	Given: h = b = 18 in.	
10.11.4.1 10.12.2 10.12.3	**Step 2**-Check whether slenderness ratio $k\ell_u/h$ is less than critical value. If so, slenderness effects may be neglected. If not, slenderness effects must be considered by magnifying moment M_n by factor δ_{ns}.		
10.11.4.1 10.11.3 10.11.4.1	**A)** Compute M_1/M_2, and read critical value of $k\ell_u/h$	M_1/M_2 = -943/3771 = -0.25 Critical value of $k\ell_u/h$ is 11.1	 COLUMNS 1
10.11.2.1 10.11.4.1	**B)** Compute $k\ell_u/h$ and compare with critical value to determine whether slenderness effects must be considered	Given columns braced against sidesway ∴ k = 1.0 Given: ℓ_u = 12 ft 10 in. = 154 in. $k\ell_u/h$ = (1.0)(154)/18 = 8.56 < 11.1 ∴ Slenderness effects may be neglected.	

ACI 318-95 Section	Procedure	Calculation	Design Aid
9.3.2.2(b) 10.2.7	**Step 3**—Determine reinforcement ratio ρ_g using known values of variables on appropriate interaction diagram(s) and compute required cross section area A_{st} of longitudinal reinforcement.	Given: $P_n = 943$ kips $M_n = 3771$ kip-in. h = 18 in. b = 18 in. $\therefore A_g = b \times h = 18 \times 18 = 324$ in.^{2}	
	A) Compute $K_n = \dfrac{P_n}{f'_c A_g}$	$K_n = \dfrac{943}{4 \times 324} \approx 0.73$	
	B) Compute $R_n = \dfrac{M_n}{f'_c A_g h}$	$R_n = \dfrac{3771}{4 \times 324 \times 18} = 0.16$	
	C) Estimate $\gamma \approx \dfrac{h - 5}{h}$	$\gamma \approx \dfrac{18 - 5}{18} = 0.72$	
	D) Determine appropriate interaction diagram(s)	For a square spiral column, $f'_c = 4$ ksi, $f_y = 60$ ksi, and an estimated $\gamma = 0.72$, use S4-60.7 and S4-60.8.	COLUMNS S4-60.7 and S4-60.8
9.3.2.2(b) 10.2 10.3	E) Read ρ_g for $P_n/f'_c A_g$ and $M_n/f'_c A_g h$	For $P_n/f'_c A_g = 0.73$, $M_n/f'_c A_g h = 0.16$ and, $\gamma = 0.70$: $\rho_g = 0.035$ $\gamma = 0.80$: $\rho_g = 0.031$ for $\gamma = 0.72$: $\rho_g = 0.0342$	
	F) Compute required A_{st} from $A_{st} = \rho_g A_g$	$A_{st} = 0.0342 \times 324$ in.^{2} $= 11.08$ in.^{2}	

COLUMNS EXAMPLE 4-Design of square column section subject to biaxial bending using resultant moment



Select column section size and reinforcement for a square column with $\rho_g \approx 0.04$ and bars equally distributed along four faces, subject to biaxial bending.

Given: Loading

Required nominal axial strength $P_n = 297$ kips
Required nominal moment strength about x-axis $M_{nx} = 2949$ kip-in.
Required nominal moment strength about y-axis $M_{ny} = 1183$ kip-in.

Materials

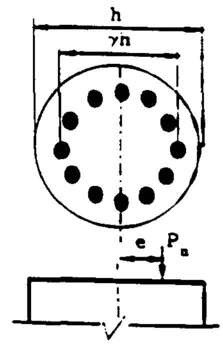
Compressive strength of concrete $f'_c = 5$ ksi
Yield strength of reinforcement $f_y = 60$ ksi

ACI 318-95 Section	Procedure	Calculation	Design Aid
	Step 1-Assume load contour curve at constant P_n is an ellipse, and determine resultant moment M_{nr} from	For a square column: h = b	
	$M_{nr} = \sqrt{M_{nx}^2 + \left(\frac{h}{b} M_{ny}\right)^2}$	$M_{nr} = \sqrt{(2949)^2+(1183)^2} = 3177$ *kip-in.*	
	Step 2-Determine trial column section size h corresponding to known and estimated values of variables on appropriate interaction diagram(s).	Given: $P_n = 297$ kips $\rho_g \approx 0.04$ $f'_c = 5$ ksi $f_y = 60$ ksi	
	A) Assume a series of trial values of h, in.	14 16 18	
	B) List $A_g = h^2$, in.^{2}	196 256 324	
	C) Compute $K_n = \frac{P_n}{f'_c A_g}$	$\frac{297}{5 \times 196} = 0.30$ $\frac{297}{5 \times 256} = 0.23$ $\frac{297}{5 \times 324} = 0.18$	
	D) Compute $R_n = \frac{M_{nr}}{f'_c A_g h}$	$\frac{3177}{5 \times 196 \times 14} = 0.23$ $\frac{3177}{5 \times 256 \times 16} = 0.16$ $\frac{3177}{5 \times 324 \times 18} = 0.11$	
	E) Estimate $\gamma \approx \frac{h - 5}{h}$	0.64 0.69 0.72	
	F) Determine appropriate interaction diagram(s)		COLUMNS R5-60.6, R5-60.7, and R5-60.8
	G) Read ρ_g for P_n/f'_cA_g and M_n/f'_cA_gh		
	For $\gamma = 0.60$	0.064 0.030	
	$\gamma = 0.70$	0.048 0.026 0.012	
	$\gamma = 0.80$	0.011	
		0.058 0.026 0.012	
		∴ Try h = 15 in.	

ACI 318-95 Section	Procedure	Calculation	Design Aid
	Step 3—Determine reinforcement ratio ρ_g using known values of variables on appropriate interaction diagram(s) and compute required cross section area A_{st} of longitudinal reinforcement.	Known: $A_g = h^2 = (15)^2 = 225$ in.^{2} $P_n = 297$ kips $M_{nr} = 3177$ kip-in.	
	A*) *Compute $K_n = \dfrac{P_n}{f'_c A_g}$	***A*)** $K_n = \dfrac{297}{(5)(225)} = 0.264$	
	B*) *Compute $R_n = \dfrac{M_{nr}}{f'_c A_g h}$	***B*)** $R_n = \dfrac{3177}{(5)(225)(15)} = 0.188$	
	C) Estimate $\gamma \approx \dfrac{h - 5}{h}$	$\gamma = \dfrac{15 - 5}{15} =$	
	D) Determine appropriate interaction diagram(s)	For a rectangular tied column, $f'_c = 5$ ksi, $f_y = 60$ ksi, and $\gamma = 0.67$, use R5-60.60 and R5-60.70	
9.3.2.2(b) 10.2 10.3	**E)** Read ρ_g for $P_n/f'_c A_g$ and $M_n/f'_c A_g h$	For $\gamma = 0.60$: $\rho_g = 0.043$ $\gamma = 0.70$: $\rho_g = 0.034$ and interpolating for $\gamma = 0.67$: $\rho_g = 0.0367$	COLUMNS R5-60.6 and R5-60.7
	F) Compute A_{st} form $A_{st} = \rho_g A_g$ and add about 15 percent for skew bending	$A_{st} = 0.0367 \times 225 = 8.26$ in.^{2} use $A_{st} \approx 9.50$ in.^{2}	

COLUMNS EXAMPLE 5-Design of circular spiral column section subject to very small design moment

For a circular spiral column, select column section diameter h and choose reinforcement. Use relatively high proportion of longitudinal steel (i.e., $\rho_g \approx 0.04$).



Given: Loading

P_u = 940 kips and M_u = 480 kip-in.
Note that the understrength factor (ϕ = 1.0)
Assume ϕ = 0.75
or, Required nominal axial strength P_n = 940/0.75 = 1253 kips
Required nominal moment strength M_n = 480/0.75 = 640 kip-in.

Materials

Compressive strength of concrete f'_c = 5 ksi
Yield strength of reinforcement f_y = 60 ksi
Nominal maximum size of aggregate is 1 in.

Design conditions

Effective column length $k\ell_u$ = 90 in.

ACI 318-95 Section	Procedure	Calculation			Design Aid
	Step 1-Determine trial column diameter h corresponding to known values of variables on appropriate interaction diagram(s).	Given: P_n = 1253 kips M_n = 640 kip-in. $\rho_g \approx 0.04$ f'_c = 5 ksi f_y = 60 ksi			
	A) Assume trial column sizes h, in.	12	16	20	
	B) Compute $\dfrac{M_n}{f'_c A_g h} = \dfrac{640}{\pi\left(\dfrac{h}{2}\right)^2 h\ x}$	0.094	0.040	0.020	
	C) Estimate $\gamma \approx \dfrac{h-5}{h}$	0.58	0.69	0.75	
	D) Determine appropriate interaction diagram(s)	C5-60.6	C5-60.7	C5-60.7 & C5-60.8	
9.3.2.2(b) 10.2 10.3	**E)** Read P_n/f'_cA_g for M_n/f'_cA_gh, ρ_g = 0.04, and γ γ after interpolation:	0.93 0.93	1.16 1.16	1.23 1.25 1.24	COLUMNS C5-60.6, C5-60.7, C5-60.8
	F) Compute $A_g = \dfrac{1253}{\phi P_n / f'_c A_g h}$	269	216	202	
	G) Compute $h = 2\sqrt{\dfrac{A_g}{\pi}}$ *in.*	18.5	16.6	16.0	
		∴ Try 17 in. diameter column			

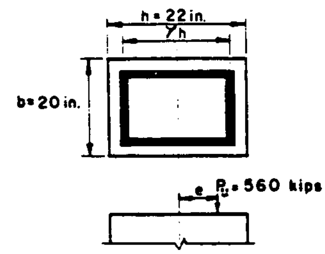
ACI 318-95 Section	Procedure	Calculation	Design Aid
10.11.4.1 10.12.2 10.12.3	**Step 2**-Check whether slenderness ration $k\ell_u/h$ is less than critical value. If so, slenderness effects may be neglected. If not, slenderness effects must be considered by magnifying moment by factor δ_{ns}.		
10.11.3 10.11.4.1	**A)** Compute M_1/M_2, and read critical value of $k\ell_u/h$.	In this case, M_1 and M_2 are not known, but for circular columns, for all values of M_1/M_2 slenderness may be neglected where $k\ell_u/h < 5.5$.	COLUMNS 1
	B) Compute $k\ell_u/h$ and compare with critical value; determine whether slenderness effects must be considered.	$k\ell_u/h = 90/17 = 5.3 < 5.5$ $\therefore$ Slenderness effects may be neglected.	
	Step 3-Determine reinforcement ratio ρ_g using known values of variables on appropriate interaction diagram(s), and compute required cross section area A_{st} of longitudinal reinforcement.	$A_g = \pi\left(\frac{17}{2}\right)^2 = 227\ in.^2$	
	A) Compute $P_n/A_g f'_c$	$P_n/A_g f'_c = 1253/227(5) = 1.10$	
	B) Compute $M_n/A_g h f'_c$	$M_n/A_g h f'_c = 640/227(17)(5) = 0.033$	
	C) Estimate $\gamma \approx \frac{h - 5}{h}$	$\gamma = \frac{17 - 5}{17} = 0.71$	
	D) Determine appropriate interaction diagram(s)	C5-60.7	
	E) Read ρ_g for $P_n/A_g f'_c$	For $P_n/A_g f'_c = 1.10$, $M_n/A_g h f'_c = 0.033$	
9.3.2.2(b) 10.2 10.3		$\therefore$ For $\gamma = 0.71$: $\rho_g = 0.036$	COLUMNS C5-60.7
	Required $A_{st} = P_g A_g$	$A_g = 0.036\pi\left(\frac{17}{2}\right)^2 = 8.17\ in.^2$	

COLUMNS EXAMPLE 6 - Selection of reinforcement for a rectangular tied column with bars on four faces (slenderness ratio found to be *above* critical value)

For a 22 x 20 in. rectangular tied column with bars equally distributed along four faces, select the reinforcement.

Given: Loading

Factored dead load P_d = 160 kips
Total factored axial load P_u = 560 kips
Factored end moment at top of the column M_2 = +3920 (k-in.)
Dead load moment unfactored at top of column M_d = 1120 (k-in.)
Factored end moment at bottom of column M_1 = +2940 (k-in.)
No transverse loading on member



Materials

Compressive strength of concrete f_c' = 4 ksi
Yield strength of reinforcement f_y = 60 ksi

Design conditions

Unsupported length of column l_u = 27.5 ft
Column braced against sidesway

ACI 318-95 Section	Procedure	Calculation	Design Aid
	Step 1-Determine column section size.	Given: h = 22 in. b = 20 in.	
10.12.3.2 Eq.(10-15)	Step 2-Check $M_{2,min} = P_u\,(0.6 + 0.03h)$	$M_{2,min}$ = 560(0.6 + 0.03 x 22) = 705.6 (k-in.) < M_2 = 3920 (k-in.) (OK)	
10.12.2 10.12.3	**Step 2**-Check if slenderness ratio is less than the critical value. If so slenderness effects may be neglected. If not slenderness effects must be considered by magnifying moment M_2 by factor δ_{ns}.		
10.11.3 10.11.4.1 10.12.2	A) Compute M_1/M_2 and read critical value of kl_u/h. (Where M_1/M_2 must not be taken less than -0.5)	M_1/M_2 = 2940/3920 = 0.7 > -0.5 (OK) Critical kl_u/h = 7.5	COLUMNS 1
10.12.1	B) determine k	For columns braced against sidesway: k = 1.0	
R10.12.1	C) compute kl_u/h and compare with critical value.	l_u = 27.5 x 12 = 330 in. kl_u/h = (1.0 x 330)/22 = 15 > 7.5	
10.12.3 Eq.(10-10) 9.2 10.0	D) determine moment magnification factor δ_{ns} -Compute $\beta_d = 1.4\,P_d/P_u$ -Compute $P_n(1 + \beta_d)/A_g$	∴ Slenderness effects must be considered β_d = (1.4 x 160)/560 = 0.4 For ϕ = 0.7 (tied column) P_n = 560/0.7 = 800 kips $P_n(1 + \beta_d)/A_g$ =(800 x 1.4)/(22 x 20) = 2.6 ksi	

ACI 318-95 Section	Procedure	Calculation			Design Aid
10.12.3.1 Eq.(1-14)	-Compute C_m $C_m = 0.6 + 0.4(M_1/M_2) \geq 0.4$ Estimate $\gamma = (h - 5)/h$	$C_m = 0.6 + 0.4(0.7) = 0.9 \geq 0.4$ $\gamma = (22 - 5)/22 \approx 0.75$			
	-Assume series of trial values of ρ_g	0.02	0.03	0.04	
	-Using COLUMNS 3.1 read K_c and K_s, (ksi)	593.24 537.75	593.24 806.62	593.24 1075.5	COLUMNS 3.1
9.3.2..2(b) 10.2 10.3	-Compute $\frac{h_e}{h} = \sqrt{0.5 + \frac{K_s}{2K_c}} \geq 1$	1	1.1	1.2	
	-Compute $\frac{kl_u}{h_e} = \frac{kl_u}{h} \div \frac{h_e}{h}$	15	13.6	12.5	
8.5.1 9.3.2 10.12.3	-Using COLUMNS 5.2, read δ_{ns}/C_m -For values of $P_n(1 + \beta_d)/A_g$ and kl_u/h_e determined above	1.8	1.6	1.5	COLUMNS 5.2
Eq.(10-10)	-Compute $\delta_{ns} = C_m \times \delta_{ns}/C_m$ from C_m and δ_{ns}/C_m determined above	0.9 x 1.8 = 1.62	0.9 x 1.6 = 1.44	0.9 x 1.5 = 1.35	
Eq.(10-9)	E) Compute $M_c = \delta_{ns}M_2$	6350.4	5644.8	5292	
9.3.2.2(b) 10.2.7	**Step 3**- Determine reinforcement ratio ρ_g using appropriate interaction diagram(s), and compute required cross-section area A_{st} of longitudinal reinforcement.	$A_g = 22 \times 20 = 440$ in.^{2}			
		For $\rho_g = 0.02$	For $\rho_g = 0.03$	For $\rho_g = 0.04$	
	B) Compute $K_n = P_n/f_c'A_g$	$K_n = 800/(4 \times 440) = 0.45$			
	$R_n = (\delta_{ns}M_2)/(\phi f_c'A_gh)$	0.243	0.21	0.193	
9.3.2.2(b) 10.2 10.3	C) Read ρ_g for K_n and R_n on interaction diagram for estimated γ of 0.75 and compare with assumed value of ρ_g	0.04 $\neq$ 0.02	0.031 $\approx$ 0.03	0.025 $\neq$ 0.04	COLUMNS 7.2.2, and 7.2.3 R4-60-7 R4-60-8

ACI 318-95 Section	Procedure	Calculation			Design Aid
	Using COLUMNS 3.1 read K_c and K_s for assumed ρ_g -Compute	∴ Repeat from Step 2D , assuming $\rho_g = 0.032$			COLUMNS 3.1
9.3.2.2(b) 10.2 10.3	$\frac{h_e}{h} = \sqrt{0.5 + \frac{K_s}{2K_c}} \geq 1$	$h_e/h = 1.11$			
	-Compute $\frac{kl_u}{h_e} = \frac{kl_u}{h} \div \frac{h_e}{h}$	$kl_u/h_e = 15/1.11 = 13.5$			
8.5.1 9.3.2.2 10.12.3 Eq.(10-10) Eq.(10-9)	Using COLUMNS 5.2, read δ_{ns}/C_m For values of $P_n(1 + \beta_d)/A_g$ and klu/h_e determined above Compute $\delta_{ns} = C_m \times \delta_{ns}/C_m$ from C_m and δ_{ns}/C_m determined above Compute $M_c = \delta_{ns}M_2$ Compute $R_n = (\delta_{ns}M_2)/(\phi f_c' A_g h)$	for $P_n(1 + \beta_d)/A_g = 2.6$ ksi (from Step 2D) and $kl_u/h_e = 13.5$: $\delta_{ns}/C_m = 1.55$ $C_m = 0.9$ (from Step 2D) $\delta_{ns} = 0.9 \times 1.55 = 1.4$ $M_c = 1.4 \times 3920 = 5488$ (k-in.) $R_n = 5488/(440 \times 22 \times 4 \times 0.7) = 0.2$			COLUMNS 5.2
9.3.2.2(b) 10.2 10.3	$K_n = P_n/f_c' A_g$ Read ρ_g for K_n and R_n on interaction diagram D) Compute required $A_{st} = \rho_g A_g$	$K_n = 0.45$ (from Step 3B) $\rho_g = 0.031 \approx 0.032$ assumed ∴ Use $\rho_g = 0.031$ $A_{st} = 0.031 \times 440 = 13.6$ in.^{2}			COLUMNS 7.2.2 , and 7.2.3 R4-60-7 R4-60-8
	Step 4-Select optimum reinforcement.				
10.9.1	A) Assume trial bar quantities	8	12	16	
	B) determine smallest bar size to provide A_{st},	#14	#10	#9	REIN-FORCE-MENT 2
	List resulting A_{st}, in.^{2},	18	15.2	16	
	Compute resulting $\rho_g = A_{st}/A_g$,	0.041	0.035	0.036	
10.9.2 7.10.5.1	And check that $0.01 \leq \rho_g \leq 0.08$	OK	OK	OK	COLUMNS 6.1
	C) List tie size	#4	#3	#3	
	D) refine γ and compare with estimated γ	0.74	0.77	0.78	
		γ is close enough to estimate of o.75 used in Steps 2D and 3C			

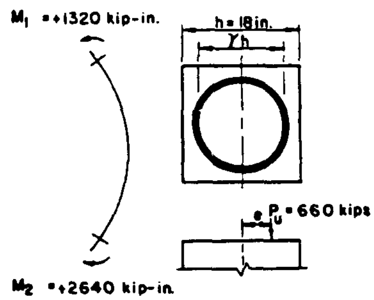
ACI 318-95 Section	Procedure	Calculation			Design Aid
3.3.3(c) 7.6.3 7.6.4	E) Check whether reinforcement can be accommodated along smaller face with	8#14	12#10	16#9	
7.7.1	-Bearing splices	OK	OK	OK	REIN-FORCE-MENT 22
7.10.5.1	-Normal lap splices	NO	OK	OK	
12.14.2.1 15.8.2	-Tangential lap splices	NO	OK	NO	
7.10.5.1	F) determine tie spacing as least of				
	-16 longitudinal bar diameter, in.	~~27~~	~~20~~	18	
	-48 tie bar diameter, in.	~~24~~	18	18	
	-least dimension of column, in.	20	~~20~~	~~20~~	
7.8 7.9 7.10	G) Select most cost-efficient reinforcement		Probable first choice		
	Solution	Use 12#10 bars with #3 ties spaced not more than 18 in. apart. (Choice is based on minimum steel requirement, use of #3 ties instead of #4 ties, ease of handling #10 bars instead of large bars, and suitability to all three types of splice.)			

COLUMNS EXAMPLE 7 - Selection of reinforcement for a square spiral column with single curvature (slenderness ratio found to be *above* critical value)

For an 18 x 18 in. square spiral column, select the reinforcement.

Given: Loading

Total factored axial load P_u = 660 kips
Factored end moment at top of the column M_1 = +1320 (k-in.)
Factored end moment at bottom of column M_2 = +2640 (k-in.)
$\beta_d = 1.4\ P_d/P_u = 0.5$
No transverse loading on member



Materials

Compressive strength of concrete f_c' = 4 ksi
Yield strength of reinforcement f_y = 60 ksi

Design conditions

Unsupported length of column l_u = 12 ft 10 in.
Column is braced against sidesway

ACI 318-95 Section	Procedure	Calculation	Design Aid
	Step 1-Determine column section size.	Given: h = 18 in. b = 18 in.	
10.12.3.2 Eq.(10-15)	Step 2-Check $M_{2,min} = P_u\ (0.6 + 0.03h)$	$M_{2,min}$ = 660(0.6 + 0.03 x 18) = 752.4 (k-in.) < M_2 = 2640 (k-in.) (OK)	
10.12.2 10.12.3	**Step 2**-Check if slenderness ratio is less than the critical value. If so slenderness effects may be neglected. If not slenderness effects must be considered by magnifying moment M_2 by factor δ_{ns}.		
10.11.3 10.11.4.1 10.12.2	A) Compute M_1/M_2 and read critical value of kl_u/h. (Where M_1/M_2 must not be taken less than -0.5)	M_1/M_2 = 1320/2640 = +0.5 > -0.5 (OK) Critical kl_u/h = 8.4	COLUMNS 1
10.12.1	B) determine k	For columns braced against sidesway: k = 1.0	
R10.12.1	C) compute kl_u/h and compare with critical value.	l_u = 12 ft 10 in. = 154 in. kl_u/h = (1.0 x 154)/18 = 8.56 > 8.4 ∴ Slenderness effects must be considered	
10.12.3 Eq.(10-10) 9.2 10.0	D) determine moment magnification factor δ_{ns} -Compute $\beta_d = 1.4\ P_d/P_u$ -Compute $P_n(1 + \beta_d)/A_g$	Given β_d = 0.5 For ϕ = 0.75 (Spiral column) P_n = 660/0.75 = 880 kips $P_n(1 + \beta_d)/A_g$ = (880 x 1.5)/(18 x 18) = 4.1 ksi	

ACI 318-95 Section	Procedure	Calculation			Design Aid
10.12.3.1 Eq.(1-14)	-Compute $C_m = 0.6 + 0.4(M_1/M_2) \geq 0.4$ Estimate $\gamma = (h - 5)/h$	$C_m = 0.6 + 0.4(0.5) = 0.8 \geq 0.4$ $\gamma = (18 - 5)/18 = 0.72$			
	-Assume series of trial values of ρ_g	0.03	0.035	0.04	
	-Using columns 3.4 read K_c and K_s, (ksi)	593.24 556.8	593.24 649.6	593.24 742.3	COLUMNS 3.4
9.3.2..2(b) 10.2 10.3	-Compute $\frac{h_e}{h} = \sqrt{0.5 + \frac{K_s}{2K_c}} \geq 1$	1	1.02	1.06	
	-Compute $\frac{kl_u}{h_e} = \frac{kl_u}{h} \div \frac{h_e}{h}$	8.56	8.4	8.1	
8.5.1 9.3.2 10.12.3	-Using COLUMNS 5.2, read δ_{ns}/C_m -For values of $P_n(1 + \beta_d)/A_g$ and klu/h_e determined above	1.32	1.32	1.27	COLUMNS 5.2
Eq.(10-10)	-Compute $\delta_{ns} = C_m \times \delta_{ns}/C_m$ from C_m and δ_{ns}/C_m determined above	1.06	1.06	1.02	
Eq.(10-9)	E) Compute $M_c = \delta_{ns}M_2$	2800	2800	2690	
9.3.2.2(b) 10.2.7	**Step 3**- Determine reinforcement ratio ρ_g using appropriate interaction diagram(s), and compute required cross-section area A_{st} of longitudinal reinforcement.	$A_g = 18 \times 18 = 324$ in.^{2}			
		For $\rho_g = 0.03$	For $\rho_g = 0.035$	For $\rho_g = 0.04$	
	B) Compute$K_n = P_n/f_c'A_g$	$K_n = 880/(4 \times 324) = 0.68$			
	$R_n = (\delta_{ns}M_2)/(\phi f_c'A_gh)$	0.16	0.16	0.15	
9.3.2.2(b) 10.2 10.3	C) Read ρ_g for K_n and R_n on interaction diagram for estimated γ of 0.72 and compare with assumed value of ρ_g	0.033 ≈ 0.03	0.033 ≈ 0.035	0.028 ≠ 0.04	COLUMNS 7.20.2 , and 7.20.3 S4-60-7 S4-60-8

ACI 318-95 Section	Procedure	Calculation						Design Aid
	D)Compute required $A_{st} = \rho_g A_g$	∴ Use $\rho_g = 0.033$ $A_{st} = 0.033 \times 324 = 10.7$ in.^{2}						
	Step 4-Select optimum reinforcement.							
10.9.2	A) Assume trial bar quantities	6	7	8	9	10	11	REIN - FORCE - MENT 2
	B) determine smallest bar size to provide A_{st},	#14	#11	#11	#10	#10	#9	
	List resulting A_{st}, in.^{2},	3.5	0.9	2.5	1.4	2.7	11	
7.10.4.2 7.10.4.3	C) Select spiral size and pitch	#4 2 in.	#4 2 in.	#4 2 in.	#4 2 in.	#4 2 in.	#4 2 in.	REIN - FORCE - MENT 20.2
Eq. (10-5)	D) Using COLUMNS 6.1 refine γ and interpolate for accurate ρ_g	0.68 0.036	0.7 0.034	0.7 0.034	0.71 0.034	0.71 0.034	0.72 0.033	COLUMNS 6.1
	E) Recompute required A_{st},in.^{2}, and compare with A_{st} provided (from B above)	11.7 OK	11.0 Not Ade-quate	11.0 OK	11.0 OK	11.0 OK	10.7 OK	
3.3.3 7.6.3 7.6.4 7.7.1 7.10.4.2 10.9.1 10.9.2 12.14.2.1	F) Check whether reinforcement can be accommodated along smaller face with -Bearing splices -Normal lap splices -Tangential lap splices	OK NO NO		OK OK NO	OK OK NO	OK NO NO	OK OK NO	REIN - FORCE - MENT 23
7.8 7.9 7.10.4	G) Determine recommended number of spacers. if spacers are used	2		2	2	2	2	
	H) Select most cost-efficient reinforcement				(Prob-able) first choice			
	Solution	**Use 9#10 bars with bearing or normal lap splices and #4spirals with 2in. Pitch. If spacers are used, recommended two spacers.**						

COLUMNS EXAMPLE 8—Determination of moment magnification factors δ_{ns} for each column and δ_s for each level and required reinforcement ratio ρ_g for columns in the first two stories of an unbraced frame

Note: This example illustrates the determination of moment magnification factors δ_b and $\boldsymbol{\delta}_s$, for columns in a structure that relies on column shears to resist horizontal force (unbraced frames). The illustration involves the total structure instead of one individual column in order to emphasize the fact that factors δ_s reflect the stability of the entire structure, whereas braced frame factors δ_b reflect the slenderness aspects of individual columns.

The structure contains 12 columns in each level. The four corner columns must resist the same sets of maximum forces, and the four columns A2, A3, C2, and C3 likewise resist identical maximum forces. Columns B1 and B4 are alike, and column B2 resists the same design forces as column B3. Thus, the structure contains four different types of columns. Reinforcement must be assigned for all five levels of the structure, but this example includes only the columns that support level 1 and level 2.

In this example, the required reinforcement ratio must be determined for the more severe of two loading conditions:
—gravity load only
—gravity load plus lateral forces on columns

Initially, the amount of reinforcement ρ_g is not known, and ACI 318-95 Eq. (10-13) can be used for column stiffness EI estimates. If resulting values of ρ_g are very large, ACI 318-95 Eq. (10-12) may produce larger EI values and permit somewhat less required ρ_g. If factors $2K_c$ in COLUMNS 3 are less than the amount of $K_c + K_s$ in COLUMNS 3, column stiffness was undervalued by Eq. (10-13).

With final ρ_g values determined from initial EI according to Eq. (10-13), the example is reworked with initial EI according to Eq. (10-12) in order to demonstrate the procedure with COLUMNS 3 and to illustrate typical reductions in ρ_g when Eq. (10-12) is employed for more heavily reinforced columns.

Given the structure shown on the sketch on the next page

Loading

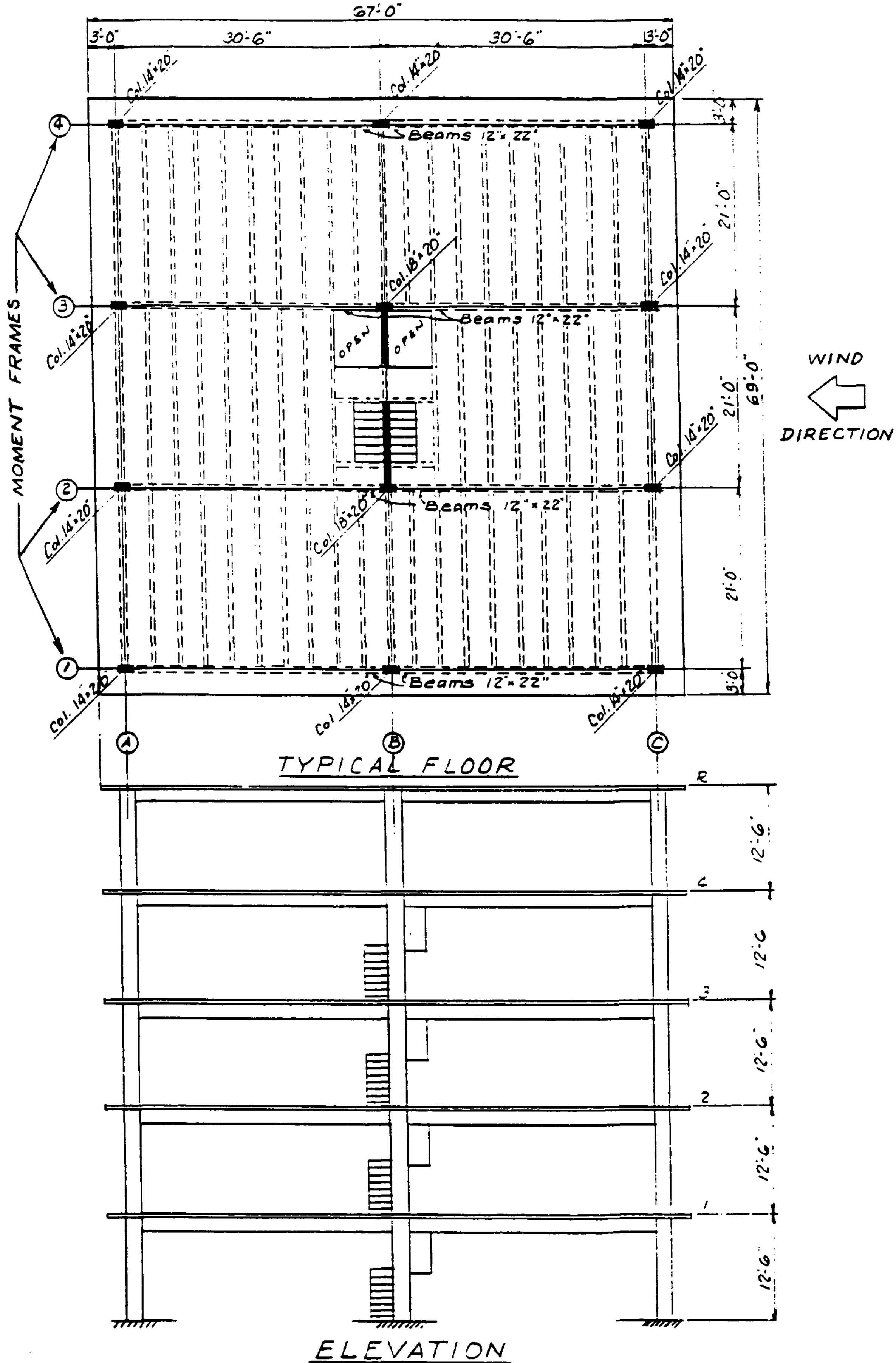
Loads and moments obtained by performing a first-order elastic frame analysis	Columns A1, A4, C1, C4		Columns A2, A3, C2, C3		Columns B1, B4		Columns B2, B3	
	supporting level		supporting level		supporting level		supporting level	
	2	1	2	1	2	1	2	1
Dead load thrust P_d, kips......................	112	136	160	193	178	215	251	303
Live load thrust P_l, kips.........................	128	158	198	244	208	257	327	404
x-axis bending moments applied to columns								
Dead load thrust M_d, ft-kips..................	30	30	34	34	0	0	0	0
Live load thrust M_ℓ, ft-kips.....................	61	61	75	75	59	59	84	84
Wind moment M_w, ft-kips on columns....	50	61	50	61	67	82	86	105
y-axis bending moments applied to columns....................	negligible							

Materials

Compressive strength of concrete $f'_c = 4$ ksi
Yield strength of reinforcement $f_y = 60$ ksi

Design Conditions

Rectangular tied column with reinforcement on four faces
Column cross-sectional dimensions
Columns A1, A2, A3, A4, B1, B4, C1, C2, C3, and C4: $b = 14$ in., $h = 20$ in.
Columns B2 and B3: $b = 18$ in., $h = 20$ in.
Unsupported length of column $l_u = 128$ in.
Vertical distance between floors $l_c = 150$ in.
Beam cross sectional dimensions
Beam width $b_w = 12$ in.
Beam thickness $h = 22$ in.
Span length of beam $l = 30.5$ ft


67'-0"
3'-0"
30'-6"
30'-6"
3'-0"
Col. 14"x20"
Col. 18"x20"
Beams 12"x22"
OPEN
MOMENT FRAMES
WIND
DIRECTION
21'-0"
69'-0"
3'-0"
TYPICAL FLOOR
R
4
3
2
1
12'-6"
12'-0
ELEVATION

ACI 318-95 Section	Procedure	Calculation								Design Aid
		Columns A1, A4, C1, C4		Columns A2, A4 C2, C4		Columns B1, B4		Columns B2, B3		
		support-ing level 2	1	support-ing level 2	1	support-ing level 2	1	support-ing level 2	1	
	Given: b	14	14	14	14	14	14	18	18	
	h	20	20	20	20	20	20	20	20	
	Step 1 -Calculate A_g	280	280	280	280	280	280	360	360	
10.13.2 10.13.1 10.11.5	**Step 2**-Check if slenderness ratio is less than the critical value. If so slenderness effects may be neglected. If not slenderness effects must be considered by magnifying moment M_2 by factor δ_{ns}. determine k, and compute kl_u/h and compare with critical value. *Note:* If $kl_u/h > 100$, second-order analysis as defined in section 10.10.1 must be made	Estimate k = 1.3 (unbraced frame) $kl_u/h = (1.3 \times 128)/(0.3 \times 20) = 27.7 > 22$ ∴ Slenderness effects must be considered								
	Step 3-For gravity load only, determine moment magnification factor δ_{ns}									
10.12.3.1	-Compute C_m	0.6	0.6	0.6	0.6	1	1	1	1	
9.2.1	-Calculate $P_u = 1.4P_d + 1.7P_l$	374	459	561	685	603	738	907	1111	
	and $M_u = 1.4M_d + 1.7M_l$	146	146	175	175	100	100	143	143	
10.0	-Compute $\beta_d = 1.4\, P_d/P_u$	0.42	0.41	0.4	0.39	0.41	0.41	0.39	0.38	
	-Compute $P_n(1 + \beta_d)/A_g$, where $P_n = P_u/\phi$	2.7	3.3	4.0	4.9	4.34	5.3	5.0	6.1	
	-Determine kl_u/h by assuming $kl_u/h_e = l_u/h$ where l_u and h are given: $kl_u/h_e = l_u/h$	6.4	6.4	6.4	6.4	6.4	6.4	6.4	6.4	

ACI 318-95 Section	Procedure	Calculation								Design Aid
		Columns A1, A4, C1, C4		Columns A2, A4 C2, C4		Columns B1, B4		Columns B2, B3		COLUMNS 5.2
		supporting level 2	1	supporting level 2	1	supporting level 2	1	supporting level 2	1	
Eq. (10-10)	-Using COLUMNS 5.2, read δ_{ns}/C_m -For values of $P_n(1+\beta_d)/A_g$ and kl_u/h_c determined above	1.1	1.13	1.17	1.2	1.18	1.23	1.2	1.27	
Eq. (10-12)	-Compute $\delta_{ns} = C_m \times \delta_{ns}/C_m \geq 1$ from C_m and δ_{ns}/C_m determined above	1	1	1	1	1.18	1.23	1.2	1.27	
	Step 4- Determine ρ_g or gravity load, using appropriate COLUMNS 7 graph									COLUMNS 5.2
7.7.1	A) $\gamma \approx (h - 5)/h = (20 - 5)/20 = 0.75$									
	B) Compute $K_n = P_u/\phi f_c' A_g$	0.48	0.6	0.7	0.88	0.8	0.94	0.9	1.1	
	C) Compute $R_n = (\delta_{ns} M_2)/(\phi f_c' A_g h)$	0.11	0.11	0.14	0.14	0.1	0.1	0.1	0.1	
9.3.2.2(b) 10.2 10.3	D) Read ρ_g from COLUMNS 7.2.2, and 7.2.3	0.01	0.01	0.02	0.032	0.01	0.024	0.022	0.039	COLUMNS 7.2.2, and 7.2.3
	Step 5-Determine moment magnification factor δ_s									
10.13.4.3 Eq. (10-19)	$\delta_s = \dfrac{1}{1 - \dfrac{\sum P_u}{0.75\sum P_c}}$									
Eq. (10-11)	where $P_u = 0.75(1.4P_d + 1.7P_l)$ and $P_c = \pi^2 EI/(kl_u)^2$									

ACI 318-95 Section	Procedure	Calculation				Design Aid
	A) Calculate gravity load part of the wind and gravity case $P_u = 0.75(1.4P_d + 1.7P_l)$ and ΣP_u, kips below level 2	281 344 4(281) +	420 514 4(420) +	452 553 2(452) +	680 833 2(680) = 5068	
	and Σp_u, kips below level 1	4(344) +	4(514) +	2(553) +	2(833) =6204	
	B) Calculate P_c and $0.75P_c$ below level 2 and below level 1					
8.5.1 8.5.2 10.12.3 Eq. (10-11) Eq. (10-13)	*Taking* $\pi^2 EI = 2K_c \dfrac{bh^3}{(1+\beta_d)}$ $0.75P_c = \dfrac{0.75(2K_c)bh^3}{(1+\beta_d)(kl_u)^2}$					
	where $2K_c$ is read from COLUMNS 3.1	1190	1190	1190	1190	COLUMNS 3.1
8.5.1 8.5.2 10.0	Assume no sustained lateral load, $\therefore \beta_d = 0$	0	0	0	0	
Fig. R10.12.1	To read k from COLUMNS 2 requires ψ_{top} and ψ_{bottom} where $\psi = \dfrac{\Sigma(EI/l_c)_{column}}{\Sigma(EI/l)_{beam}} = \dfrac{\Sigma(I/l_c)_{column}}{\Sigma(I/l)_{beam}}$					COLUMNS 2
	$\Sigma I_{column}/l_c = \Sigma bh^3/12l_c$, in.^{3}	146 146	146 146	146 146	188 188	
	$\Sigma I_{beam}/l = b_w h^3/12l_c$, in.^{3} where b_w, h, and l_c are given for Columns A and C:	31 31	31 31			
	for Columns B:			62 62	62 62	

ACI 318-95 Section	Procedure	Calculation				Design Aid
	ψ_{top} ψ_{bottom}	4.7 4.7 4.7 0.2	4.7 4.7 4.7 0.2	2.4 2.4 2.4 0.2	3.0 3.0 3.0 0.2	
R10.12.1	Read k from COLUMNS 2	2.15 1.51	2.151.51	1.691.35	1.82 1.4	COLUMNS 2
	$0.75P_c = \frac{0.75(2K_c)bh^3}{(1+\beta_d)(kl_u)^2}$	1232 2497	1232 2497	1994 3124	2210 3735	
	and $0.75\Sigma p_c$, kips below level 2 and $0.75\Sigma p_c$, kips below level 1 -Calculate	4(1232) + 4(2497) +	4(1232) + 4(2497) +	2(1994) + 2(3124) +	2(2210) = 18.264 2(3735) = 33.694	
10.13.4.1 Eq. (10-19)	$\delta_s = \frac{1}{1 - \frac{\sum P_u}{0.75\sum P_c}}$	Below level 2 δ_s =1.38 Below level 1 δ_s = 1.23				
9.9.2	**Step 6**-Determine ρ_g for gravity load plus lateral force, using COLUMNS 7. A)Calculate $K_n = 0.75\ P_u/\phi f_c'A_g$	0.36 0.44	0.54 0.66	0.58 0.35	0.68 0.83	
	B)Calculate $R_n = M_c/\phi f_c'A_g h$: -Check if any member has					
10.13.5 Eq. (10-20)	$\frac{l_u}{r} > \frac{35}{\sqrt{\frac{P_u}{f_c'A_g}}}$	NO	NO	NO	NO	
Eq. (10-17)	∴ M_{2ns} need not to be magnified. -Calculate $M_c = M_{2ns} + \delta_s M_{2s}$ (ft-kips), where: $M_{2ns} = 0.75(1.4M_d + 1.7M_l)$ and $M_{2s} = 0.75(1.7M_w)$	2370 2462	2631 2723	2477 2650	3373 3610	
	$R_n = M_c/\phi f_c'A_g h$	0.15 0.16	0.17 0.18	0.16 0.17	0.17 0.18	
9.2.2© 10.2 10.3	C)Read ρ_g from COLUMNS 7.2.2, and 7.2.3	0.0120.016	0.0220.031	0.02 0.029	0.02 0.038	COLUMNS 7.2.2, and 7.2.3

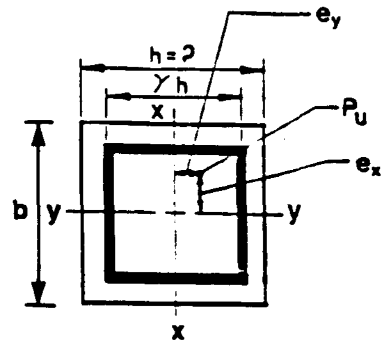
ACI 318-95 Section	Procedure	Calculation								Design aid
		Columns A1, A4, C1, C4		Columns A2, A4 C2, C4		Columns B1, B4		Columns B2, B3		
		supporting level 2	supporting level 1	supporting level 2	supporting level 1	supporting level 2	supporting level 1	supporting level 2	supporting level 1	
	Step 7—Compare ρ_g from Step 4 and ρ_g from Step 6 and list larger ρ_g, which governs. governing ρ_g	0.012	0.016	0.022	0.032	0.021	0.029	0.038	0.039	
	Step 8—Redetermine δ_{ns} using larger value of EI that is, using kl_n/h_e instead of in COLUMNS 5.2 A) Using COLUMNS 3.1 read K_c and K_s (ksi) B) Compute									COLUMNS 3.1
Eq. (10-12) Eq. (10-13)	$\frac{h_e}{h} = \sqrt{0.5 + \frac{K_s}{2K_c}} \geq 1$	1.0	1.0	1.02	1.12	1.01	1.09	1.08	1.19	
	C) Compute $\frac{kl_u}{h_e} = \frac{kl_u}{h} \div \frac{h_e}{h}$	6.4	6.4	6.3	5.7	6.3	5.9	5.9	5.4	
	D) List K_n from Step 4B	0.48	0.6	0.7	0.88	0.8	0.94	0.9	1.1	
10.12.2	E) Calculate $R_n = M_c/\phi f'_c A_s h$ —List $P_n(1 + \beta d)A_s$ from Step 3	2.7	3.3	4.0	4.9	4.34	5.3	5.0	6.1	
Eq. (10-10) Eq. (10-11)	Using COLUMNS 5.2, read δ_{ns}/C_m and since $C_m = 1$, $\delta_{ns} = \delta_{ns}/C_m$	1.0	1.0	1.0	1.0	1.18	1.19	1.17	1.17	COLUMNS 5.2
Eq. (10-9)	-Compute $M_c = \delta_{ns} M_2$ and then $R_n = M_c/g f'_c A_g h$	0.11	0.11	0.14	0.14	0.09	0.09	0.1	0.1	
	F) Read ρ_g from COLUMNS 7.2.2 and 7.2.3	0.01	0.01	0.021	0.031	0.01	0.024	0.022	0.038	COLUMNS 7.2.2 and 7.2.3

ACI 318-95 Section	Procedure	Calculation				Design Aid
	Step 9-Redetermine moment magnification factor δ_s using the larger of the values of *EI* calculated from Eqs. (10-12) and (10-13) in calculating P_c Therefore P_c is the larger of:	1.09 1.11	1.12 1.14	1.11 1.12	1.12 1.12	
8.5.1 8.5.2 10.12.3	$P_c = \dfrac{(2K_c)bh^3}{(1+\beta_d)(kl_u)^2}$ $or\ \ P_c = \dfrac{(K_c + K_s)bh^3}{(1+\beta_d)(kl_u)^2}$					
	A)List Σp_u from Step Σp_u, kips below level 2 Σp_u, kips below level 1	5068 6204				COLUMNS 3.1
	B)Read $2K_c$ from Step 5 and read $K_c + K_s$ from COLUMNS 3.1 and list larger of these values	1190 1190	1320 1530	1190 1240	1190 1610	
	C)Calculate $0.75P_c$, kips using $\beta_d = 0$ and k from Step 5	1232 2497	1366 3211	1994 3256	2210 5054	
	Compute $0.75\Sigma p_c$,kips below level 2	4(1232) +	4(1366) +	2(1994) +	2(2210) = 18.800	
	and $0.75\Sigma p_c$ below level 1	4(2497) +	4(3211) +	2(3256)	2(5054) = 39,452	
	D)Compute					
Eq. (10-19)	$\delta_s = \dfrac{1}{1 - \dfrac{\sum P_u}{0.75\sum P_c}}$					
	δ_s below level 2 and δ_s below level 1	$\delta_s = 1/(1 - 5068/18,800) = 1.37$ $\delta_s = 1/(1 - 6204/39,452) = 1.19$				

ACI 318-95 Section	Procedure	Calculation								Design aid
	Step 10—Redetermine ρ_s for gravity load plus lateral force, using δ_s from Step 8 and COLUMNS 7:									COLUMNS 7.2.2 and 7.2.3
9.2.2	A) List $K_n = 0.75P_u/\phi f'_c A_g$ from Step 6A	0.36	0.44	0.54	0.66	0.58	0.35	0.68	0.83	
	B) Calculate $R_n = M_c/\phi f'_c A_g h$	2362		2623		2466		3564		
Eq. (10-17)	Calculate $M_c = M_{2ns} + \delta_s M_{2s}$ (ft-kips) where $M_{2ns} = 0.75(1.4M_d + 1.7M_l)$ and $M_{2s} = 0.75(1.7M_w)$		2425		2686		2565		3418	
	$R_n = M_c/\phi f'_c A_g h$	0.15		0.168		0.157		0.179		
			0.153		0.171		0.164		0.168	
9.3.2.2©	C) read ρ_g from	0.012		0.022		0.02		0.033		
10.2 10.3	COLUMNS 7.2.2 and 7.2.3		0.014		0.029		0.028		0.036	
	Step 11—Compare ρ_s from Step 9 and ρ_g from Step 11 and list larger value, which governs.	0.012		0.022		0.02		0.033		
	Solution: required ρ_g		0.014		0.031		0.028		0.038	

COLUMNS EXAMPLE 9-Determination of adequacy of square tied column section subject to biaxial bending, using reciprocal load method with $1/P_{ni}$ equation

Determine adequacy of column shown; use reciprocal method and $1/P_{ni}$ equation.



Given: Loading

P_u = 140 kips, M_{ux} = 1404 kip-in., and M_{uy} = 636 kip-in.
Note that the understrength factor (ϕ = 1.0)
Assume ϕ = 0.7
or, Required nominal axial strength P_n = 140/0.7 = 200 kips
Required nominal moment strength M_{nx} = 1404/0.7 = 2006 kip-in.
Required nominal moment strength M_{ny} = 636/0.7 = 909 kip-in.

Materials

Compressive strength of concrete f'_c = 3 ksi
Yield strength of reinforcement f_y = 60 ksi
Eight #9 bars with #3 ties

Design conditions

Column section size h = b = 16 in.
Slenderness ratio is below critical value
so slenderness effects need not be considered
1½ in. concrete cover over reinforcement

ACI 318-95 Section	Procedure	Calculation	Design Aid
	Step 1-Determine K_n ratio and eccentricity about x and y axes.	Given: h = b = 16 in. P_n = 200 kips M_{nx} = 2006 kip-in. M_{ny} = 909 kip-in.	
	$K_n = \dfrac{P_n}{f'_c\, A_g}$	$K_n = \dfrac{200}{3(256)} = 0.26$	
	$e_x = \dfrac{M_{nx}}{P_n}$ & $e_y = \dfrac{M_{ny}}{P_n}$	$e_x = \dfrac{2006}{200} = 10.02\ in.$ $e_y = \dfrac{909}{200} = 4.543\ in.$	

ACI 318-95 Section	Procedure	Calculation	Design Aid
	Step 2-Find $P_{nx}/A_g f_c'$ associated with $M_{nx}/A_{st} f_c'$ and $P_{ny}/A_g f_c'$ associated with $M_{ny}/A_g f_c'$		
	A) Compute $\rho_g = A_{st}/A_g = nA_b/bh$	$\rho_g = 8(0.79)/16(16) = 0.0247$	
	B) Find γ	For #8 bars with #3 ties in a 16 x 16 in. column: $\gamma = 0.7$	COLUMNS 6.1
	C) Determine appropriate interaction diagram(s)	For square tied column, $f_c' = 3$ ksi, $f_y = 60$ ksi, and $\gamma = 0.7$: use R3-60.7	
9.3.2.2(b) 10.2 10.3	**D)** Find $P_{nx} = R_n A_g f_c' h/e_x$	For $K_n = 0.26$, $\rho_g = 0.0247$, and $\gamma = 0.7$: $R_n = 0.205$ Therefore $P_{nx} = 0.205(3)(256)(16)/10.02$ $= 251.4$ kips	COLUMNS R3-60.7
	E) Find $P_{ny} = R_n A_g f_c' h/e_y$	$P_{ny} = 0.205(3)(256)(16)/4.543$ $= 554.5$ kips	
9.3.2.2(b) 10.2 10.3	**Step 3**-Find strength at zero eccentricity P_{no} using COLUMNS 7.	For all values of γ, For $\rho_g = 0.020$: $P_{no} = 1.24(3)(256)$ $= 952.3$ kips For $\rho_g = 0.030$: $P_{no} = 1.43(3)(256)$ $= 1098.2$ kips $\therefore$ For $\rho_g = 0.0247$: $P_{no} = 1021$ kips	COLUMNS R3-60
	Step 4-Compute axial load strength P_n at e_x and e_y using $\frac{1}{P_{ni}} = \frac{1}{P_{nx}} + \frac{1}{P_{ny}} - \frac{1}{P_{nc}}$ and compare with nominal axial load P_{ni}	$\frac{1}{P_{ni}} = \frac{1}{251.4} + \frac{1}{554.5} - \frac{1}{1021}$ $= 0.0048$ $\therefore P_{ni} = 208$ kips	
	Solution	The calculated P_{ni} of 208 kips is within 4% of the given P_n of 200 kips; since this approximate method of analysis is known to be conservative, the section can be considered to be adequate.	

COLUMNS EXAMPLE 10—Determination of adequacy of square tied column section subject to biaxial bending, using load contour method and COLUMNS 10 and 11

Determine adequacy of a 16 x 16 in. column; use load contour method

Given: Loading

Axial load P_u = 140 kips

Moment about x-axis M_{ux} = 1404 kip-in.

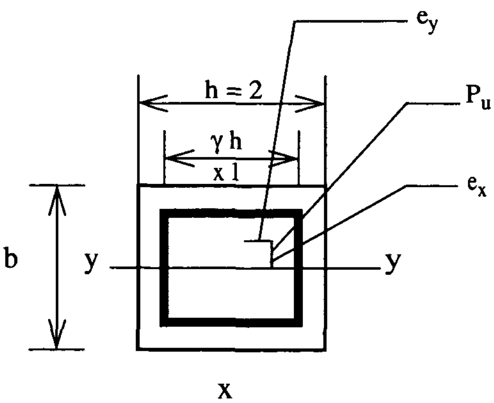
Moment about y-axis M_{uy} = 636 kip-in.

Assume ϕ =0.7

or, $P_n = P_u/0.7$ = 200 kips

$M_{nx} = M_{ux}/0.7$ = 2006 (k-in.)

$M_{ny} = M_{uy}/0.7$ = 909 (k-in.)



Materials

Compressive strength of concrete f_c' = 3 ksi

Yield strength of reinforcement f_y = 60 ksi

Eight #9 bars with #3 ties

Design Conditions

Slenderness ratio is *below* critical value, so slenderness effects need not to be considered

1- 1/2 in. Concrete cover over reinforcement

ACI 318-95 Section	Procedure	Calculation	Design Aid
	Step 1 - Determine required nominal axial strength at zero eccentricity P_o	A_g=b x h = 16 x 16 = 256 in^2.	
	A) Find γ	For #9 bars and #3 ties in 16 x 16 in. column: γ = 0.7	
	B) Compute ρ_g	ρ_g=(8 x 1.0)/256 = 0.031	
9.3.2.2(b) 10.2 10.3	C) Compute $P_o/f_c' A_g$	At zero eccentricity $R_n = M_n/f_c' A_g h = 0$ So, for γ = 0.7, R_n = 0 and ρ_g = 0.031 $P_o/f_c' A_g$ = 1.42	COLUMNS 7.1.2
	D) Compute $P_o = (P_o/f_c' A_g) \times f_c' A_g$	P_o = 1.42 x 3 x 256 = 1090.56 kips	
	Step 2 - Determine biaxial bending design constant β. A) Compute $\rho_g f_y/f_c'$ B) Compute P_n/P_o C) Using appropriate COLUMNS 10, read β for P_n/P_o	$\rho_g f_y/f_c'$ = 0.031(60/3) = 0.62 P_n/P_o = 500/1090.56 = 0.46 For the values above, read β from COLUMNS 10.2.2 β = 0.56	COLUMNS 10.2.2
	Step 3-Determine M_{nox}, the uniaxial moment capacity about the x-axis associated with P_{ny}. A) Compute $K_n = P_n/f_c' A_g$ B) Determine appropriate interaction diagram	K_n = 200/(3 x 256) = 0.26 For γ = 0.7, f_c' = 3 ksi and f_y = 60 ksi use COLUMNS 7.1.2	

ACI 318-95 Section	Procedure	Calculation	Design Aid
9.3.2.2(b) 10.2 10.3	C)Read $R_n = M_{nox}/f_c' A_g h$ D)Compute $M_{nox} = R_n \times f_c' A_g h$	For $K_n = 0.26$, and $\rho_g = 0.031$ $R_n = 0.236$ $M_{nox} = 0.236 \times 3 \times 256 \times 16$ $= 2900$ (k-in.)	COLUMNS 7.1.2
	Step 4-Determine M_{noy}, the uniaxial moment strength about the y-axis	Because of symmetry, $M_{noy} = M_{nox}$ $= 2900$ (k-in.)	
	Step 5-Determine M_{ny}, the nominal moment strength about the y-axis A)Compute (M_{nx}/M_{nox}) B)Using COLUMNS 11, read M_{ny}/M_{noy} for M_{nx}/M_{nox} and β.	 $M_{nx}/M_{nox} = 2006/2900 = 0.69$ For $M_{nx}/M_{nox} = 0.69$ and $\beta = 0.56$ $M_{ny}/M_{noy} = 0.41$	 COLUMNS 11
	Step 6-Determine nominal moment strength M_{ny} under biaxial bending and compare with required M_{ny}.	$M_{ny} = (M_{ny}/M_{noy}) \times M_{noy}$ $= 0.41 \times 2900 = 1189$ (k-in.) $M_{uy} = 0.7 \times 1189 = 832.3$ (k-in.) $>$ (required) $M_{uy} = 636$ (k-in.) Therefore section is adequate.	

TWO-WAY SLABS

SLABS EXAMPLE 1 - Two-way slab *without* beams, designed according to Direct Design Method

Design the two-way slab without beams shown in the sketch by the Direct Design Method.

Given: Loading

Live load, $w_l = 125 \cdot psf$

Mechanical load, $w_{mech} = 15 \cdot psf$

Exterior wall, $w_w = 400 \cdot plf$

Materials

Concrete strength, $f'_c = 3000$ psi for slabs and columns.

yield strength of reinforcement, $f_y = 40 \cdot ksi$

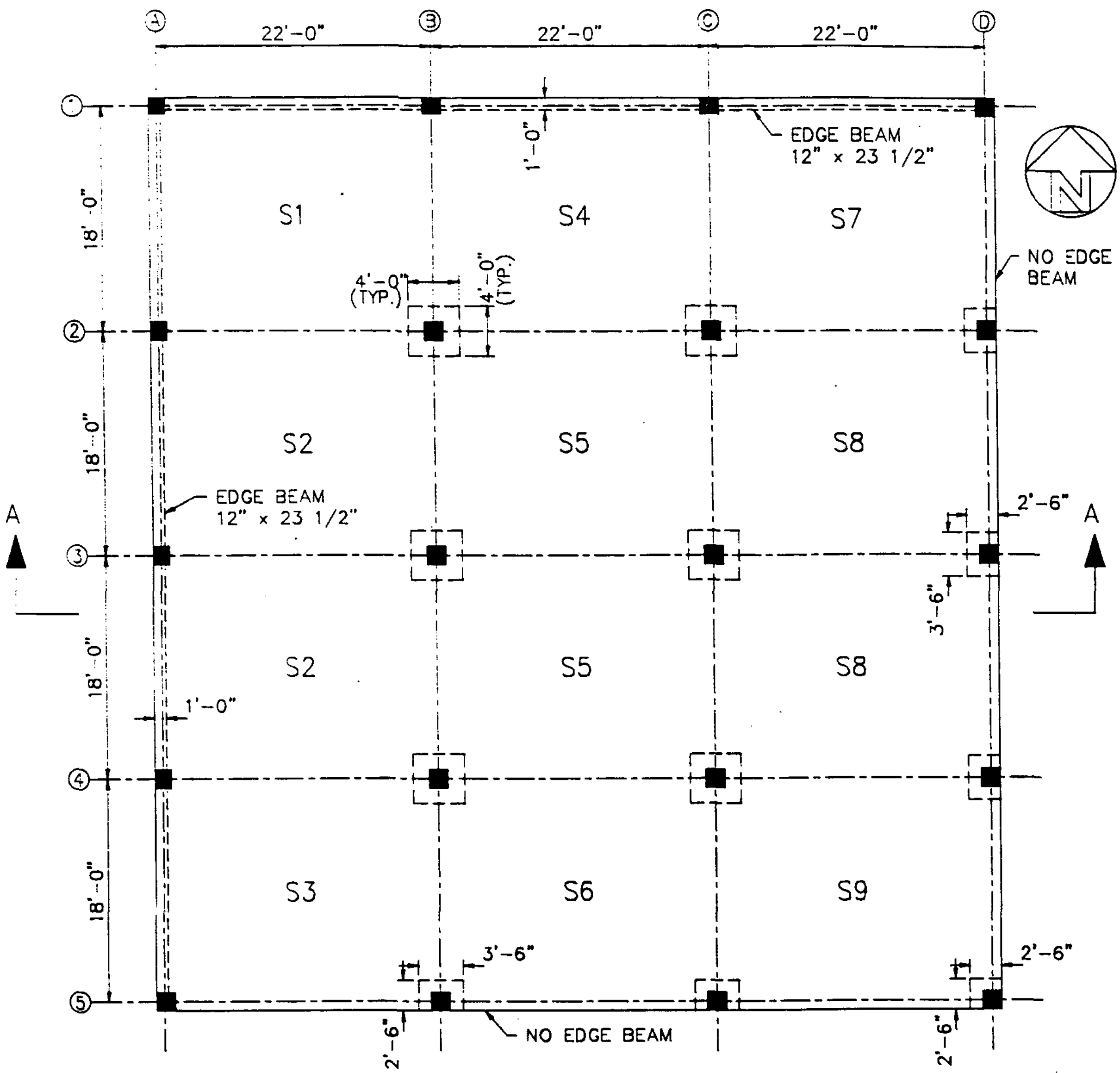
Weight of concrete, $w_c = 150 \cdot pcf$

Design conditions

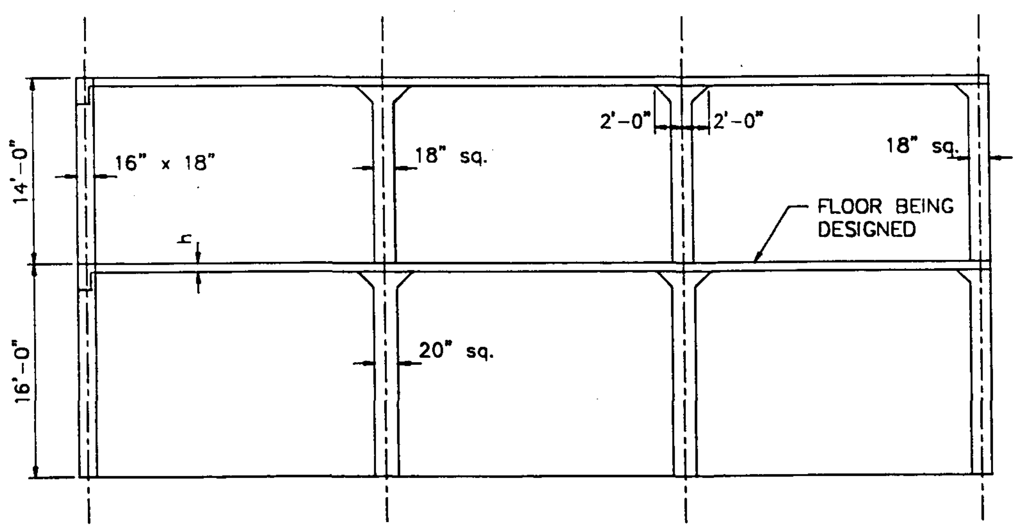
Interior columns below slab, 20 x 20 in.
Interior columns above slab, 18 x 18 in.
Corner column A1, 16 x 16 in.; other exterior columns where edge beams are used, 16 x 18 in., with longer dimension parallel to the edge of slab.
Exterior columns without beams, 18 x 18 in.
Column capitals may be used.
Edge beams extending not more than 16 in. below the soffit of the slab are permitted along building Lines (A) and (1). No beam is provided along Lines (5) and (D).
Floor to floor height below slab, 16 ft. 0 in.
Floor to floor height above slab, 14 ft. 0 in.

Note - To facilitate comparison of slabs designed by Procedures I and III, loading, material and design conditions are the same for Slabs Examples 1 and 3.

ACI 318-95 Section	Procedure	Calculation	Design Aid
	Step 1 — Determine whether slab geometry and loading satisfy conditions for use of Direct Design Method (DDM).		
13.6.1.1		**A)** At least three continuous spans in each Direction.	
13.6.1.2		**B)** Panels are rectangular and ratio of longer span to shorter span 22.0'/18.0' = 1.22 < 2.0.	
13.6.1.3		**C)** Successive span lengths in each direction do not differ.	
13.6.1.4		**D)** Columns are not offset.	
13.6.1.5	E) To verify that w_l does not exceed 3 w_d, estimate minimum likely w_d on basis of minimum allowable slab	**E)** $w_d = w_{mech} + \frac{h \cdot w_c}{12} = 15 + \frac{5}{12} \cdot 150$ $w_d = 77.5 \cdot psf$	
9.5.3.1	thickness of 5 in., and compare unfactored w_l and unfactored 3 w_d.	$3 \cdot w_d = 232.5 \cdot psf$	
13.6.1.5	Note — All loads considered are due to gravity only.	Given, $w_l = 125 \cdot psf < 3\ w_d$	
13.6.1.5		∴ DDM may be used.	



FLOOR PLAN



SECTION A-A

ACI 318-95 Section	Procedure	Calculation	Design Aid
	Step 2 — Select trial thickness on basis of deflection and shear requirements for critical panel..		
9.0 13.6.2.5	**A)** Consider Panel S9, compute trial slab thickness adequate for deflection requirements. Assume no column capitals.	**A)** For deflection (Panel S9): $l_n(S) = 22\cdot 12 - \left(\frac{20}{2}\right) - \left(\frac{18}{2}\right)$ $= 245$ in.	
9.5.3.1	*Note* — If no capitals are used and the successive spans in a given direction are equal, the critical panel for a constant-thickness slab is a corner panel with the least edge restraint.	$l_n(D) = 18\cdot 12 - \left(\frac{20}{2}\right) - \left(\frac{18}{2}\right)$ $= 197$ in. $h_s(\min) = \frac{245}{33} = 7.42$ in. Try $h_s = 7.5$ in.	SLABS 1.1
	B) Check trial slab thickness for shear capacity. Note — At this stage, since moments are not known, only a preliminary check for shear can be made. To provide some reserve shear capacity for transfer of unbalanced moments at column-slab connections, the depths required for shear alone must be increased. Suggested amounts for this increase are as follows: Interior columns 10 percent Exterior columns 40 percent Corner columns 70 percent Cover to centroid of steel layers assumed to be 1.25 in. (i.e., 3/4 in. cover with #4 bars). Perimeter shear load = V_u = total factored unit loading w_u x net slab area.	**B)** For $h_s = 7.5$ in. $w_d = \left(15 + \frac{7.5}{12}\cdot 150\right)\cdot 1.4$ $= 21 + 131 = 152$ psf $w_l = 125\cdot 1.7 = 213$ psf $w_u = 365\cdot\text{psf} = 0.365\cdot\text{ksf}$	
	B1) For perimeter shear load at interior column, net slab area = span length x design frame width less area of one column section. $V_u = (l_1\cdot l_2 - c_1\cdot c_2)\cdot w_u$	**B1)** For *interior* column: $V_u = \left[22\cdot 18 - \left(\frac{20}{12}\right)^2\right]\cdot 0.365$ $= 143.5$ kips $2\cdot(c_1 + c_2) = 2\cdot(20 + 20) = 80$ in.	

Entering chart at $f'_c = 3000$ psi, $V_u = 143.4$ kips and $2(c_1+c_2) = 80$ in., read $d = 7.2$ in. — SLABS 3.1

Allowing 10% for unbalanced moment:

$$d = 1.1 \cdot 7.2 = 7.9 \text{ in.}$$

Checking whether values from chart can be used without modification:

$$\beta_c = \frac{20}{20} = 1 < 2$$

and

$$b_o/d = \frac{2 \cdot (c_1 + c_2) + 4 \cdot d}{d}$$

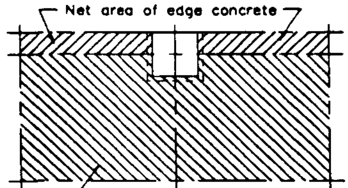
$$= \frac{80 + 4 \cdot 7.9}{7.9} = 14 < 20$$

∴ chart can be used without modification: use $d = 7.9$ in.

$$h_s(s) = 7.9 + 1.25 = 9.2 \text{ in.} > 7.5 \text{ in.}$$

indicating the trial slab thickness of 7.5 in. is not adequate in shear.

B2) Exterior column shear is calculated for load on net half-panel area plus weight of edge concrete and upper exterior wall with dead load factor.



B2) For *exterior* edge column:

$$V_u = \left(\frac{22 \cdot 18}{2} - \frac{9 \cdot 18}{12 \cdot 12}\right) \cdot 0.365 \ldots$$
$$+ \frac{9}{12} \cdot (22 - 1.5) \cdot 0.131 \ldots$$
$$+ (22 - 1.5) \cdot 0.400 \cdot 1.4$$
$$= 85.3 \text{ kips}$$

$$2 \cdot c_1 + c_2 = 2 \cdot 18 + 18 = 54 \text{ in.}$$

Entering the chart at $f'_c = 3000$ psi, $V_u = 85.3$ kips and $2c_1 + c_2 = 54$ in. read $d = 6.8$ in. — SLABS 3.4

Allowing 40% for unbalanced moment:

$$d = 1.4 \cdot 6.8 = 9.52 \text{ in.}$$

Checking whether values from chart can be used without modification:

$$\beta_c = \frac{18}{18} = 1 < 2$$

and

$$b_o/d = \frac{(2 \cdot c_1 + c_2) + 2 \cdot d}{d}$$

$$= \frac{54 + 2 \cdot 9.52}{9.5} = 7.7 < 15$$

ACI 318-95 Section	Procedure	Calculation	Design Aid

∴ chart can be used without modification: use d = 9.5 in.

$$h_s(s) = 9.5 + 1.25 = 10.8 \text{ in.} > 7.5 \text{ in.}$$

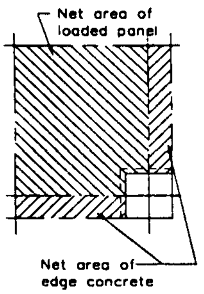
indicating the trial slab thickness of 7.5 in. is not adequate in shear.

B3) Calculation of shear at a corner column is included here as an example although this would not be necessary since it is already obvious, from the interior column and edge column calculations, that thickness required for shear greatly exceeds that required for deflection, therefore, column capitals should be used.

B3) For *corner* column:

$$V_u = \left[\frac{22\cdot 18}{4} - \left(\frac{9}{12}\right)^2\right]\cdot 0.365 \ldots + \left[\frac{9}{12}\cdot\left(\frac{22}{2} - \frac{9}{12}\right) + \frac{9}{12}\cdot\left(\frac{18}{2} - \frac{9}{12}\right)\right]\cdot 0.131 \ldots + \left(\frac{22+18}{2} - \frac{18}{12}\right)\cdot 0.400\cdot 1.4$$
$$= 48.7 \text{ kips}$$

Corner column shear is calculated for load on net quarter-panel area plus weight of edge concrete and exterior wall on two edges with dead load factor.



$$c_1 + c_2 = 18 + 18 = 36 \text{ in.}$$

Entering the chart at $f'_c = 3000$ psi, $V_u = 48.1$ kips and $c_1 + c_2 = 36$ in. read d = 6.3 in. — SLABS 3.8

Allowing 70% for unbalanced moment:

$$d = 1.7\cdot 6.3 = 10.7 \text{ in.}$$

Checking whether values from chart can be used without modification:

$$\beta_c = \frac{18}{18} = 1 < 2$$

and

$$bo/d = \frac{c_1 + c_2 + d}{d} = \frac{36 + 10.7}{10.7} = 4.4 < 10$$

∴ chart can be used without modification: use d = 10.7 in.

$$h_s(s) = 10.7 + 1.25 = 11.9 \text{ in.} > 7.5 \text{ in.}$$

indicating the trial slab thickness of 7.5 in. is not adequate in shear.

At all columns, thickness of the slab required for shear is greater than that required for deflection.

∴ Use column capitals.

ACI 318-95 Section	Procedure	Calculation	Design Aid

C) Select size of column capitals.

Design Procedure says reasonable assumption for interior capitals is $0.15l_a \leq c \leq 0.25l_a$ where l_a is average of longer and shorter spans.

Note 1 — Considerable freedom is given to the designer in selecting capital dimensions. Designer should consider dimensions easily formed with plywood or use of standard forms, as well as shear requirements.

Note 2 — No capitals are used for the exterior columns in Line (A) and Line (1) since there are beams along these lines.

C) Estimate capital size.
For interior columns, capital size may be:

$$0.15 \cdot l_a = 0.15 \cdot \left(\frac{18+22}{2}\right) = 3\,\text{ft}$$

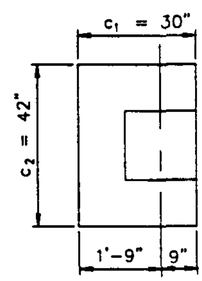
$$0.25 \cdot l_a = 0.25 \cdot \left(\frac{18+22}{2}\right) = 5\,\text{ft}$$

Choosing increments of 6 in., select square interior capitals:

$c_1 = c_2 = 4$ ft 0 in.

For edge columns, select capital size:
2 ft 6 in. perpendicular to edge.
3 ft 6 in. parallel to edge.

(In SLABS 3.4, dimension perpendicular to discontinuous edge is labeled c_1 and parallel dimension c_2. Therefore, c_1 = 2 ft 6 in. and c_2 = 3 ft 6 in.)



For corner column D5, select square capitals:

Repeat Steps 2A and 2B with capitals.

$c_1 = c_2 = 2$ ft 6 in.

A') Consider Panel S9 for deflection requirements after capitals are added.

A') For deflection (Panel S9) — SLABS 1.1

$$l_n(5) = 22 \cdot 12 - 21 - 21 = 222\,\text{in.}$$

$$l_n(D) = 18 \cdot 12 - 21 - 21 = 174\,\text{in.}$$

No edge beam present

$$\therefore h_{min} = \frac{l}{33}$$

$$h_{min} = \frac{222}{33} = 6.73\,\text{in.}$$

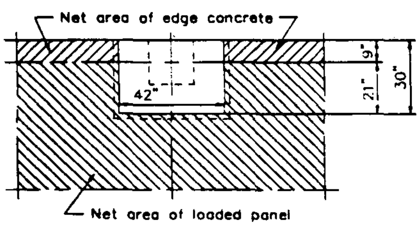
Try h = 7.5 in.

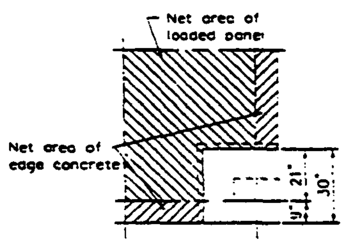
B1') Recheck slab thickness required by shear with column capitals.

B1') For *interior* column with a capital.

$$V_u = \left[22 \cdot 18 - \left(\frac{48}{12}\right)^2\right] \cdot 0.365$$

$$= 138.7\,\text{kips}$$

	Obtain value of d required to provide shear capacity and compare with d provided.	$2\cdot(c_1+c_2) = 2\cdot(48+48) = 192$ in.	
		For h = 7.5 in., d = 7.5 - 1.25 = 6.25 in.	
	Compute b_o/d based on trail dimensions. If $b_o/d > 20$, then modify k_1.	$b_o/d = \dfrac{2\cdot(c_1+c_2)+4\cdot d}{d} = \dfrac{2\cdot(48+48)+4\cdot 6.25}{6.25} = 34.7 > 20$	
		∴ Modification of k_1 required.	
		Entering chart at $f'_c = 3000$ psi, $V_u =$ 138.7 kips and read $k_1 = 1.60$.	SLABS 3.1
		mod. $k_1 = 1.60\cdot\left(\dfrac{10\cdot 6.25}{217}+0.5\right) = 1.26$	
		Entering chart with $k_1 = 1.26$ and $2(c_1+c_2) = 192$ in., read d = 4.5 in.	SLABS 3.1
	Increase by 10% for allowance of unbalanced moment and compare d required to d provided.	Allowing 10% for unbalanced moment: $d = 1.1\cdot 4.5 = 4.95$ in. < d = 6.25 in. O.K.	
	B2') Recheck for exterior edge column. 	**B2')** For *exterior* (edge) column with capital. $V_u = \left(\dfrac{22\cdot 18}{2}-\dfrac{21\cdot 42}{12\cdot 12}\right)\cdot 0.365\ ... + \dfrac{9}{12}\cdot(22-3.5)\cdot 0.131\ ... + (22-3.5)\cdot 0.400\cdot 1.4 = 82.2$ kips	
		$2\cdot c_1 + c_2 = 2\cdot 30 + 42 = 102$ in.	
	Compute b_o/d based on trial dimensions. If $b_o/d > 15$, then modify k_1.	$b_o/d = \dfrac{2\cdot c_1+c_2+2\cdot d}{d} = \dfrac{2\cdot 30+42+2\cdot 6.25}{6.25} = 18.32 = 18.32 > 15$	
		∴ Modification of k_1 required.	
		Entering chart at $f'_c = 3000$ psi, $V_u =$ 82.2 kips and read $k_1 = 2.70$.	SLABS 3.4
		mod. $k_1 = 2.70\cdot\left(\dfrac{7.5}{18.32}+0.5\right) = 2.46$	

ACI 318-95 Section	Procedure	Calculation	Design Aid
		Entering chart with $k_1 = 2.46$ and $2c_1+c_2 = 102$ in., read $d = 4.2$ in.	SLABS 3.4
	Increase by 40% for allowance of unbalanced moment and compare d required to d provided.	Allowing 40% for unbalanced moment: $d = 1.4 \cdot 4.2 = 5.9$ in. $<$ $d = 6.25$ in. <u>O.K.</u>	
	B3') Recheck for corner column with capital.  	**B3')** For corner column with capital. $V_u = \left[\frac{22 \cdot 18}{4} - \left(\frac{21}{12}\right)^2\right] \cdot 0.365 \ldots$ $+ \left[\frac{9}{12} \cdot \left[\left(\frac{22}{2} - \frac{21}{12}\right) + \left(\frac{18}{2} - \frac{21}{12}\right)\right]\right] \cdot 0.131 \ldots$ $+ \left(\frac{22 \cdot 18}{2} - \frac{2 \cdot 21}{12}\right) \cdot 0.400 \cdot 1.4$ $= 46$ kips $c_1 + c_2 = 30 + 30 = 60$ in.	
	Compute b_o/d based on trial dimensions. If $b_o/d > 10$, then modify k_1.	$b_o/d = \frac{c_1 + c_2 + d}{d}$ $= \frac{30 + 30 + 6.25}{6.25} = 10.6$ $= 10.6 > 10$ $\therefore$ Modification of k_1 required.	
		Entering chart at $f'_c = 3000$ psi, $V_u =$ 46 kips and read $k_1 = 4.80$. mod. $k_1 = 4.8 \cdot \left(\frac{5}{10.6} + 0.5\right) = 4.66$	SLABS 3.8
		Entering chart with $k_1 = 4.66$ and $c_1+c_2 = 60$ in., read $d = 4.$ in.	SLABS 3.8
	Increase by 40% for allowance of unbalanced moment and compare d required to d provided.	Allowing 70% for unbalanced moment: $d = 1.7 \cdot 4.0 = 6.8$ in. $\approx$ $d = 6.25$ in. <u>O.K.</u>	
	Since deflection thickness exceeds shear requirement with capitals, use thickness calculated for deflection requirements.	Use $h_{s(d)} = 7.5$ in. Confirm loading: $w_u = 213 + \left(15 + \frac{7.5}{12} \cdot 150\right) \cdot 1.4 = 365.25$ $= 365$ psf $= 0.365$ ksf	

ACI 318-95 Section	Procedure	Calculation	Design Aid

D) Check adequacy of slab thickness for other panels, as in Steps 2A and 2B.

D) Check Panel S2.

$$l_n(A) = 18 \cdot 12 - \frac{18}{2} - \frac{18}{2} = 198 \text{ in}$$

$$l_n(B) = 18 \cdot 12 - \frac{48}{2} - \frac{48}{2} = 168 \text{ in}$$

Note 1 — In slabs where not all spans are the same, it may be necessary to check several panels to determine which panel is critical for deflection. While Panel S2 is not critical by inspection for this slab, a check is made to illustrate the procedure when an edge beam is present.

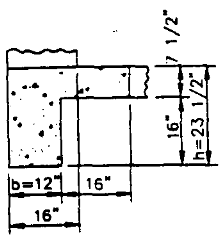
9.0

13.6.2.5 — *Note 2* — Clear span l_n is measured from face of capital where there is a capital and from face of column where there is no capital [Lines (A) and (1)] - except where there are beams, in which case l_n is measured to the face of beams for the slab thickness formulas. Minimum l_n is 0.65 l_1 — SLABS 1

First it is necessary to calculate α for the edge beam.

D1) To obtain α for the beam, compute from the cross section of the beam and slab the values h/h_s, u/h_s and l, and read α_f from SLABS 2; then find α from:

D1) Compute α: — SLABS 2



13.2.4

$$\alpha = \frac{E_{cb} \cdot b}{E_{cs} \cdot l} \cdot \left(\frac{h}{h_s}\right)^3 \cdot \alpha_f$$

$$h/h_s = \frac{23.5}{7.5} = 3.13$$

$$u/h_s = \frac{2 \cdot 12}{7.5} = 3.2$$

$$l = 0.5 \cdot l_2 + 0.5 \cdot \text{column dimension}$$

$$= 0.5 \cdot (22 \cdot 12) + 0.5 \cdot 16 = 140 \text{ in.}$$

Note 3 — In an edge beam, for the purpose of computing u/h_s, u=2b.

13.2.4 — If concrete is the same in beam and slab, E_{cb} and E_{cs} are the same so that:

Reading the graph for $h/h_s = 3.13$ and $u/h_s = 3.2$, find $\alpha_f = 1.46$ — SLABS 2

$$\alpha := \frac{b}{l} \cdot \left(\frac{h}{h_s}\right)^3 \cdot \alpha_f$$

$$\alpha = \frac{12}{140} \cdot 3.13^3 \cdot 1.46$$

$$3.84 > 0.80$$

$$h_{min} = \frac{l_n}{36} = \frac{198}{36} = 5.5 \text{ in.} < 7.5 \text{ in.}$$ — SLABS 1.1

ACI 318-95 Section	Procedure	Calculation	Design Aid
9.5.3.3	**D2)** Compute slab thickness for Panel S7 for deflection requirements. This panel contains an exterior edge [along line (D)] without edge beam, thus indicating a 10 percent thickness increase. The other exterior edge qualifying for a waiver of the 10 percent increase. It is recommended that the 10 percent increase be applied for this case. The large value of α along one edge does not seem a sufficient requirement to neglect the effect of an edge having no beam at all.	**D2)** Check Panel S7 $l_n(1) = 22 \cdot 12 - 18 = 246$ in. For α, u/hs = 3.2; h/hs = 3.13 $l = \frac{18 \cdot 12}{2} + 8 = 116$ in. Find $\alpha_f = 1.45$ $\alpha = \frac{12}{116} \cdot 3.13^3 \cdot 1.45 = 4.6 > 0.8$ $= 4.60 > 0.80$ $h_{min} = \frac{l_n}{33} = \frac{246}{33} = 7.45$ in. This is less than the 7.5 in. used.	SLABS 2 SLABS 1.1
	Step 3 — Divide the structure into design frames along the column lines.	**Step 3** — Interior frames along Lines (B) and (C) have a frame width $l_2 = 22$ ft; interior frames along Lines (2), (3) and (4) have a frame width $l_2 = 18$ ft; exterior frames along Lines (A) and (D) have a frame width $l_2 = 11$ ft 8 in. and 11 ft 9 in., respectively (from center line of panel to exterior face of slab); exterior frames along Lines (1) and (5) have a frame width $l_2 = 9$ ft 8 in. and 9 ft 9 in., respectively (from center line of panel to exterior face of slab).	
13.6.2.1 13.6.2.2 Eq. (13-3)	**Step 4** — Compute total static moment M_o for each span in a design frame centered on one column line (and for similar frames centered on similar column lines). where $M_o = \frac{w_u \cdot l_2 \cdot l_n^2}{8}$	**Step 4** — Considering the design frame along Line (3), compute moments in east-west direction: For End Span A-B on column Line (3), $l_2 = 18.0$ ft $l_n = 22 - 0.67 - 2.0 = 19.33$ ft $M_o = \frac{0.365 \cdot 18.0 \cdot 19.33^2}{8} = 307$ kip-ft	

ACI 318-95 Section	Procedure	Calculation	Design Aid
	Note — Steps 4 through 6 are carried out for column Line (3); then Steps 4 through 6 are repeated as Step 7 for each other column line in north-south and east-west directions.	For Interior Span B-C on column Line (3), $l_2 = 18.0$ ft $l_n = 22 - 2.0 - 2.0 = 18$ ft $M_o = \dfrac{0.365 \cdot 18.0 \cdot 18.0^2}{8} = 266$ kip-ft For Interior Span C-D on column Line (3), $l_2 = 18.0$ ft $l_n = 22 - 2.0 - \dfrac{21}{12} = 18.25$ ft $M_o = \dfrac{0.365 \cdot 18.0 \cdot 18.25^2}{8} = 274$ kip-ft	
	Step 5 — Compute distribution of factored moments for east-west design frame centered on Column Line (3). *Note* — Because of symmetry of the design, the results will be the same for adjacent design frames centered on Lines (2) and (4). A similar procedure is used to provide moment distribution for north-south design frames centered on Lines (B) and (C).		
13.6.3.3	Moments for an end span. Moment at exterior column $-M_e = 0.30M_o$ Positive moment $+M_e = 0.50M_o$ Moment at first interior column $-M_{ie} = 0.70M_o$	For design span centered on Line (3) dividing Panels S2, where M_o = 307 kip-ft (from Step 4): $-M_e = 0.3 \times 307$ $= 92$ kip-ft $+M_e = 0.5 \times 307$ $= 154$ kip-ft $-M_{ie} = 0.7 \times 307$ $= 215$ kip-ft	
13.6.3.2	Moments for an interior span. $-M = 0.65M_o$ $+M = 0.35M_o$	For design span centered on Line (3) dividing Panels S5, where M_o = 266 kip-ft (from Step 4): $-M = 0.65 \times 266$ $= 173$ kip-ft $+M = 0.35 \times 266$ $= 93$ kip-ft	

ACI 318-95 Section	Procedure	Calculation	Design Aid
13.6.3.3	Moments for the end span. *Note* — These calculation results will be the same for design frames centered on Lines (2) or (4).	For design span centered on Line (3) dividing Panels S8, where M_o = 274 kip-ft (from Step 4): $-M_e = 0.26 \times 274$ $= 71$ kip-ft $+M_e = 0.52 \times 274$ $= 142$ kip-ft $-M_{ie} = 0.7 \times 274$ $= 191$ kip-ft	
13.6.4	**Step 6** — Distribute frame factored moments to column and middle strips.		
13.6.4.2	A) For exterior frames, find proportion of *exterior negative* moment to be apportioned to column strip from table in Section 13.6.4.2. Consider east-west design Frames (2), (3), (4) framing into exterior columns in Lines (A) and (D). *Note* — Distribution of frame moment at discontinuous edge depends on relative torsional stiffness of edge beam transverse to direction in which moments are determined as measured by β_t, the ratio of torsional stiffness of edge beam to flexural stiffness of slab strip.	A) Design Frame (3) and Columns A3 and D3.	

Also depends on relative flexural stiffness of beam parallel to direction in which moments are determined — as measured by α_1, the ratio of beam flexural stiffness to slab flexural stiffness.

A1) Column D3

Where there is no torsional edge beam, as in Lines (5) and (D), $\beta_t = 0$. Where there is no flexural beam, as in design frames along Lines (2), (3), (4), (5), and (B), (C), (D), $\alpha_1 = 0$.

A1) Column Line (D) has no edge beam:

$$\beta_t = 0$$

If $\beta_t = 0$, distribute factored negative moment 100 percent to column strip.

If $\beta_t \geq 2.5$, and $\alpha_1 = 0$, distribute 75 percent of factored negative moment to column strip.

If $0 \leq \beta_t \leq 2.5$, interpolate linearly.

If $\alpha_1 > 0$, interpolate linearly.

100 percent of negative moment in column strip = 71 kip-ft.

A2) Column A3

A2) Column Line (A) Contains an edge beam. There is no flexural beam in the design strip in the direction in which moments are determined.

$\therefore\ \alpha_1 = 0$ and $\alpha_1 l_2 = 0$

13.7.5.3 — Compute C for the edge beam in Column Line (A).

(See sketch in Step 2D1)

for $x_1 = 12$ in. and $y_1 = 23.5$ in.

$$C_1 = 9181 \text{ in.}^4$$

for $x_2 = 7.5$ in. and $y_2 = 16$ in.

$$C_2 = 1586 \text{ in.}^4$$

$$C = C_1 + C_2 = 9181 + 1586 = 10{,}767 \text{ in.}^4$$

Compute I_s for slab strip cross section at Column A3.

$$I_s = \frac{l_2 \cdot h_s^3}{12}$$

For $h_s = 7.5$ in., $l_2 = 18$ ft., and $l_1 = 22$ ft.:

$$I_s = \frac{(7.5)^3 \cdot (18 \cdot 12)}{12} = 7594 \text{ in.}^4$$

13.0

$$\beta_t = \frac{E_{cb} \cdot C}{2 \cdot E_{cs} \cdot I_s}$$

$$\beta_t = \frac{C}{2 \cdot I_s} = \frac{10767}{2 \cdot (7594)} = 0.709$$

ACI 318-95 Section	Procedure	Calculation	Design Aid
		Interpolating in the table of Section 13.6.4.2 of ACI 318-95, between $\beta_t = 0$ and $\beta_t \geq 2.5$ for $\beta_t = 0.709$: Percent to column strip = $100\% - \frac{0.709}{2.5} \cdot (100\% - 75\%)$ = 93% of exterior negative moment $-M_{ie}$ at Line (A) to be assigned to column strip: $= 0.93 \cdot 92 = 86$ kip-ft Moments assigned to each half middle strip equal: $\frac{92 - 86}{2} = 3$ kip-ft	
13.6.4.1	**B)** For exterior and interior panels, find proportion of *interior negative* design moment to be apportioned to the column strip from table in Section 13.6.4.1.	**B)** Reading the table in Section 13.6.4.1 for $\alpha_1 l_2 / l_1 = 0$, find 75% of $-M_{ie}$ to be apportioned to the column strip. For Column B3 (east-west) Column Strip Negative Moment: $= 0.75 \cdot 215 = 161$ kip-ft For each half middle strip, Negative Moment: $= \frac{215 - 161}{2} = 27$ kip-ft For Column C3 (east-west) Column Strip Negative Moment: $= 0.75 \cdot 191 = 143$ kip-ft For each half middle strip, Negative Moment: $= \frac{191 - 143}{2} = 24$ kip-ft	
13.6.4.4	**C)** For exterior and interior panels, find proportion of *positive* design moment to be apportioned to the column strip from table in Section 13.6.4.4.	**C)** Reading the table in Section 13.6.4.4 for $\alpha_1 l_2 / l_1 = 0$, find 60% of $+M_e$ to be apportioned to the column strip. For Span A3-B3 (Panels S2), Column Strip Positive Moment: $= 0.60 \cdot 154 = 92$ kip-ft For half middle strip, Positive Moment: $= 0.20 \cdot 154 = 31$ kip-ft For Span B3-C3 (Panels S5), Column Strip Positive Moment: $= 0.60 \cdot 93 = 56$ kip-ft	

For half middle strip, Positive Moment:

$= 0.20 \cdot 93 = 19$ kip-ft

For Span C3-D3 (Panels S8), Column Strip Positive Moment:

$= 0.60 \cdot 142 = 85$ kip-ft

For half middle strip, Positive Moment:

$= 0.20 \cdot 142 = 28$ kip-ft

D) Summarize by span the distribution of factored moments to the column strip and the two half middle strips. For interior span, use the larger of the two negative moments at the column.

D) Factored Moment, kip-ft:

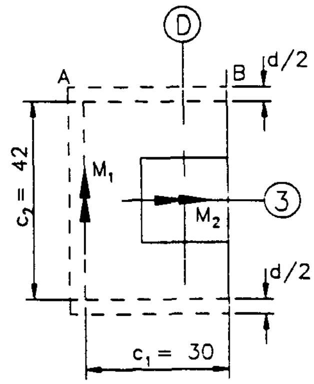
Panel and moment	Distribution to column strip	Total to two Half middle strips
Panel S2		
$-M_e$	93% (92) = 86	6
$+M_e$	60% (154) = 92	62
$-M_{ie}$	75% (215) = 161	54
Panel S5		
$-M$	75% (215) = 161	54
$+M$	60% (93) = 56	37
$-M$	75% (191) = 143	48
Panel S8		
$-M_e$	75% (191) = 143	48
$+M_e$	60% (142) = 85	57
$-M_{ie}$	100% (71) = 71	0

Note 1 — These calculation results will be the same for design frames centered on Lines (2) or (4). Distribution of north-south moments for columns in Line (5) will be 100 percent in columns strips, as already shown for distribution of east-west moments to columns in Line(D). Distribution for columns in Line (1) will be calculated in the same manner as Column A3. Note that frames along Lines (A) and (1) have flexural edge beams so that $\alpha_1 > 0$ when taking moments in direction along Line (A) or Line (1).

Note 2 — Minimum reinforcement steel is required in middle strips even when moment is distributed 100 percent to the column strip.

Step 7 — Repeat Steps 4 through 6 for all other column lines.

Step 7 — omitted in this example.

ACI 318-95 Section	Procedure	Calculation	Design Aid
	Step 8 — Determine whether trial slab thickness chosen is adequate for moment-shear transfer. Consider all possible critical columns.		
13.6.3 13.6.9.2	**A)** For interior columns compute moments.	**A)** Consideration of interior columns omitted in this example.	
11.12.6.2	**B)** For all columns, compute portion of moment that is to be considered transferred to the slab by eccentricity of the shear, and the properties of the critical shear section.	**B)** For Column D3: 	
	Compute dimensions for use in SLABS 3.4	$d = h_s$ - cover - d_b $= 7.5 - 0.75 - 0.5 = 6.25$ in. $2c_1 + c_2 = 2 \cdot 30 + 42 = 102$ in. $c_1 - c_2 = 30 - 42 = -12$ in.	
		To find k_1, enter the graph at d=6.25; proceed horizontally to $2c_1 + c_2$ = 102, drop to k_1 scale, and read k_1= 1.65.	SLABS 3.4
	SLABS 3.5 assumes $c_1 = c_2$; SLABS 3.6 corrects for $c_1 \neq c_2$.	To find k_2, first find k_2' and then correct for $c_1 \neq c_2$. Find k_2' by entering SLABS 3.5 at c_2 = 42 in., proceeding horizontally to d = 6.25 in., dropping to the k_2' scale, and reading k_2' = 0.42.	SLABS 3.5
		Then find k_2 by entering SLABS 3.6 at d = 6.25 in. and k_2' = 0.42, proceeding horizontally to $c_1 - c_2$ = -12 in., dropping vertically to d = 6.25 in., and reading k_2 = 0.56.	SLABS 3.6
13.6.3.6	**C)** For moment transfer between slab and an edge column, Section 13.6.3.6 requires that the fraction of unbalanced moment transferred by eccentricity of shear must be based on the full column strip nominal moment strength M_n provided.	**C)** For Column D3: $M_u = -M_e = 71$ kip-ft	

ACI 318-95 Section	Procedure	Calculation	Design Aid
	Determine reinforcement required for column strip negative moment.		
	Estimate j at 0.95 and compute $A_s = \frac{M_u}{\phi \cdot f_y \cdot j \cdot d}$	$A_s = \frac{71 \cdot \left(\frac{12 \cdot in}{ft}\right) \cdot \left(\frac{1000 \cdot lb}{kip}\right)}{0.9 \cdot 40000 \cdot 0.95 \cdot 6.25}$ $= 3.99\ in.^2$	
	Check validity of estimated value j by: $a = \frac{A_s \cdot f_y}{0.85 \cdot f_c \cdot b}$	$a = \frac{3.99 \cdot 40000}{0.85 \cdot 3000 \cdot 108} = 0.58\ in.$	FLEXURE 1
	and $j = 1 - \left(\frac{a}{2 \cdot d}\right)$	$j = 1 - \frac{0.58}{2 \cdot 6.25} = 0.95$ ∴ assumed j is OK.	FLEXURE 1
	Using #5 bars, for which $A_s = 0.31$, determine number of bars required.	$\frac{3.99}{0.31} = 12.9$ bars ∴ 13 bars required.	
	Determine bar spacing.	$\frac{108}{13} = 8.3$ in.	
13.4.2	Check whether this spacing exceeds twice the slab thickness.	$2h_s = 2 \cdot 7.5 = 15$ in. $8.3\ in. < 15\ in.$ ∴ spacing is OK, try 13 #5 bars.	
	Determine unbalanced moment $\gamma_f M_u$ transferred by flexure.		
13.3.3.2	$\gamma_f = \frac{1}{1 + \frac{2}{3} \cdot \sqrt{\frac{c_1 + \frac{d}{2}}{c_2 + d}}}$	$\gamma_f = \frac{1}{1 + \frac{2}{3} \cdot \sqrt{\frac{30 + \frac{6.25}{2}}{42 + 6.25}}} = 0.64$	
	Note — Where there is a capital, c_1 and c_2 refer to the capital dimensions as shown in the sketch in Step 8B, rather than to the column dimensions.	$M_u = 71$ kip-ft (from Step 9C) $\gamma_f \cdot M_u = 0.64 \cdot 71 = 45$ kip-ft	
	Calculate width of moment transfer section = $c_1 + 2(1.5h_s)$.	$42 + (2 \cdot (1.5 \cdot 7.5)) = 64.5$ in.	
	Determine reinforcement needed for unbalanced moment, again estimating j at 0.95. $A_s = \frac{\gamma_f \cdot M_u}{\phi \cdot f_y \cdot j \cdot d}$	$A_s = \frac{45 \cdot \left(\frac{12 \cdot in}{ft}\right)}{0.9 \cdot 40000 \cdot 0.95 \cdot 6.25}$ $= 2.51\ in.^2$	

Check validity of estimated value j by:

$$a = \frac{A_s \cdot f_y}{0.85 \cdot f'_c \cdot b}$$

$$a = \frac{2.51 \cdot 40000}{0.85 \cdot 3000 \cdot 64.5} = 0.61$$ FLEXURE 1

and

$$j = 1 - \left(\frac{a}{2 \cdot d}\right)$$

$$j = 1 - \frac{0.61}{2 \cdot 6.25} = 0.95$$ FLEXURE 1

∴ assumed j is OK.

Determine number of #5 bars needed in moment transfer section.

$$\frac{2.53}{0.31} = 8.2 \cdot \text{bars}$$

∴ 9 #5 bars are needed.

Use 9 #5 bars within 64.5 in. moment transfer section and two bars on each side for a total of 13 bars

Compute nominal moment strength.

$$M_n = \frac{A_s \cdot f_y \cdot j \cdot d}{\left(\frac{12 \cdot in}{ft}\right) \cdot \left(\frac{1000 \cdot lb}{kip}\right)}$$

$$M_n = \frac{3.99 \cdot 40000 \cdot 0.95 \cdot 6.25}{12 \cdot 1000}$$
$$= 79 \text{ kip-ft}$$

D) Compute total nominal shear stress v_n, and compare with maximum allowable value. As represented in SLABS 3

11.12 $$v_n = \frac{1}{\phi \cdot A} \cdot V_u + \frac{\gamma_{v1} \cdot c_{a1}}{\phi \cdot J_1} + \frac{\gamma_{v2} \cdot c_{a2}}{\phi \cdot J_2}$$

or

$$v_n = k_1 \cdot V_u + k_2 \cdot M_1 + k_3 \cdot M_2$$

where values of k_1, k_2, and k_3 depend only on values of c_1, c_2, and d, and are given in SLABS 3

D1) Initial check neglecting possible unbalanced north-south moment M_2 (which will be small for Column D3 for which spans on either side are of equal length and weight of wall).

D1) For column D3

$w_u = 0.365$ ksf (from Step 2)

$M_n = 79$ kip-ft (from Step 8C)

$-M_{ie} = 191$ kip-ft

$$V_u = \frac{0.365}{0.85} \cdot \left[(11 \cdot 18) - \frac{24.125 \cdot 48.25}{144}\right] \ldots$$
$$+ \frac{9}{12} \cdot \left(18 - \frac{48.25}{12}\right) \cdot \frac{0.131}{0.85} - \frac{\frac{191}{0.9} - 79}{22 - 3.75}$$
$$= 75.9 \text{ kips}$$

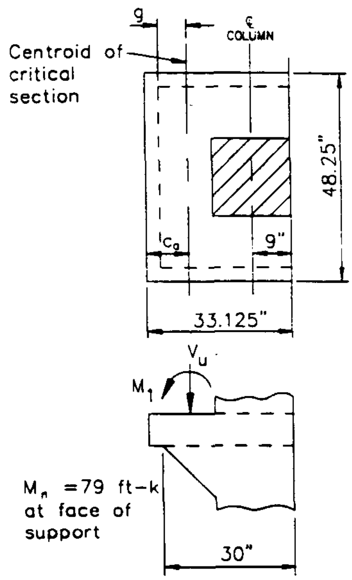
Obtain equivalent shear and moment at centroid of critical section.

$$c_u = \left(\frac{2\cdot 0.5\cdot 33.125^2}{2\cdot 33.125 + 48.25}\right) = 9.58 \text{ in.}$$

$$g = (30.0 - (33.125 - 9.58)) = 6.45 \text{ in.}$$

$$M_1 = M_n + V_u\cdot g$$

$$= 79 + 75.9\cdot\frac{6.45}{12} = 120 \text{ kip-ft}$$



With $k_1 = 1.65$ and $k_2 = 0.56$, the shear stress due to perimeter shear and effect of moment M_1 is:

$$\frac{v_u}{\phi} = (1.65\cdot 75.9 + 0.56\cdot 120) = 192 \text{ psi}$$

Note — The limiting shear stress v_u/ϕ can increase, necessitating a thicker slab if excessive reinforcement is used in the shear-moment transfer zone, raising the nominal moment strength M_n used in the calculation.

Shear stress caused by weight of upper wall = net half wall length on each side of capital x plf divided by area on each side corresponding to critical section width and slab depth.

$$v_n = \frac{V_w}{\phi\cdot 2\cdot\left(30 + \frac{d}{2}\right)\cdot d}$$

Shear stress due to exterior wall.

$$V_w = \left(18 - \frac{48.2}{12}\right)\cdot 0.4\cdot 1.4 = 7.83 \text{ kips}$$

$$v_n = \frac{7830}{0.85\cdot 2\cdot\left(30 + \frac{6.25}{2}\right)\cdot 6.25} = 22 \text{ psi}$$

Shear stress due to perimeter shear, effect of M_1, and weight of wall.

Total $v_n = 192 + 22 = 214$ psi

ACI 318-95 Section	Procedure	Calculation	Design Aid
11.12.2.1	Allowable v_n is the smallest of the values of v_c given by Eq. (11-36), Eq. (11-37), and Eq. (11-38):		
Eq. (11-36)	$v_c = \left(2 + \frac{4}{\beta_c}\right) \cdot \sqrt{f'_c}$ Where β_c is the ratio of long side to short side of the column, concentrated load or reaction area.	$\beta_c = \frac{l_n(\text{long\_side})}{l_n(\text{short\_side})}$ of reaction area $= \frac{48.25 \cdot in}{33.125 \cdot in} = 1.46$ $v_c = \left(2 + \frac{4}{1.46}\right) \cdot \sqrt{3000}$ $= 260$ psi	
Eq. (11-37)	$v_c = \left(\frac{\alpha_s \cdot d}{b_o} + 2\right) \cdot \sqrt{f'_c}$ Where α_s is 30 for edge columns.	b_o = perimeter of critical section at d/2 from capital edge. $= 2 \cdot \left(30 + \frac{6.25}{2}\right) + (42 + 6.25)$ $= 114.5$ in. $v_c = \left(\frac{30 \cdot 6.25}{114.5} + 2\right) \cdot \sqrt{3000}$ $= 199$ psi	
Eq. (11-37)	$v_c = 4 \cdot \sqrt{f'_c}$	$v_c = 4 \cdot \sqrt{3000}$ $= 219$ psi ∴ maximum allowable nominal shear strength $v_c = 199$ psi < 214 psi. Hence, either provide shearhead reinforcement allowing $v_c = 7 \cdot \sqrt{f'_c}$ or change slab thickness from 7.5 in. to 8 in.	
	Assume shearhead reinforcement is provided.		
	D2) Compute shear stress due to moment M_2 from possible pattern loadings and from weight of wall, add to previously computed shear stress, and again compare with maximum allowable shear stress.	**D2)** For column D3.	
Eq. (13-4)	$M' = 0.07 \cdot \left[\left(w_d + 0.5 \cdot w_l\right) \cdot l_2 \cdot l_n^2 \ldots + -w'_d \cdot l'_2 \cdot l'_n{}^2 \right]$		

ACI 318-95 Section	Procedure	Calculation	Design Aid
		For $w_d = w'_d = 1.4 \cdot \left(15 + \frac{7.5}{12} \cdot 150\right)$ $= 152$ psf $= 0.152$ ksf $w_l = 0.213$ ksf (from Step 2B). $l_2 = l'_2 = 22/2 = 11$ ft $l_n = l'_n = 18 - 3.5 = 14.5$ ft. : $M_2 = 0.07 \cdot \left[\begin{matrix}(0.152 + 0.5 \cdot 0.213) \cdot 11 \cdot 14.5^2 \ldots \\ + - 0.152 \cdot 11 \cdot 14.5^2\end{matrix}\right]$ $= 17.2$ kip-ft	
	Find k_3' corresponding to k_2' in SLABS 3.5. k_2' was previously determined to be 0.42 in Step 8B.	In the bottom scales of SLABS 3.5, where $k_2' = 0.42$, find $k_3' = 0.35$. Entering SLABS 3.7 at $d = 6.25$ in. and $k_3' = 0.35$, proceed horizontally to $c_1 - c_2 = -12$, drop to $d = 6.25$, and read $k_3 = 0.48$. $v_u = 214 + 0.48 \cdot 17.2$ $= 222$ psi $v_{u(total)} = 222 \approx v_{all} = 219$ psi	SLABS 3.5 SLABS 3.7
	Confirms previous determination of h_s in Steps 2A and 2C.	$h_s = 7.5$ in. is adequate	
	Step 9 — Select remaining slab reinforcement.		
	Step 10 — Design edge beams.	See examples of beam design in this Design Handbook (SP-17).	

SLABS REINFORCEMENT EXAMPLE 2 — Reinforcement spacing for crack control in panel of uniformly loaded two-way slab for severe environment

Select bar size and spacing necessary for crack control at the column reaction region of a 7-in.-thick slab which is uniformly loaded.

Select bar size for two conditions:

Condition A: Floor is subjected to severe exposure of humidity and moist air.

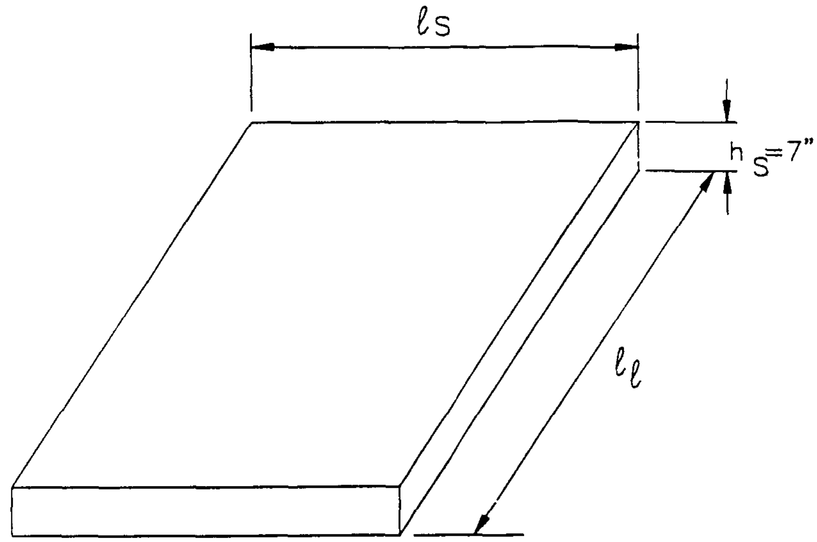
Condition B: Floor sustains an aggressive chemical environment where the design working stress level in the reinforcement is limited to 15 ksi.

Given:

$\beta = 1.20$

$l_s / l_l = 0.8$

$f_y = 60$ ksi



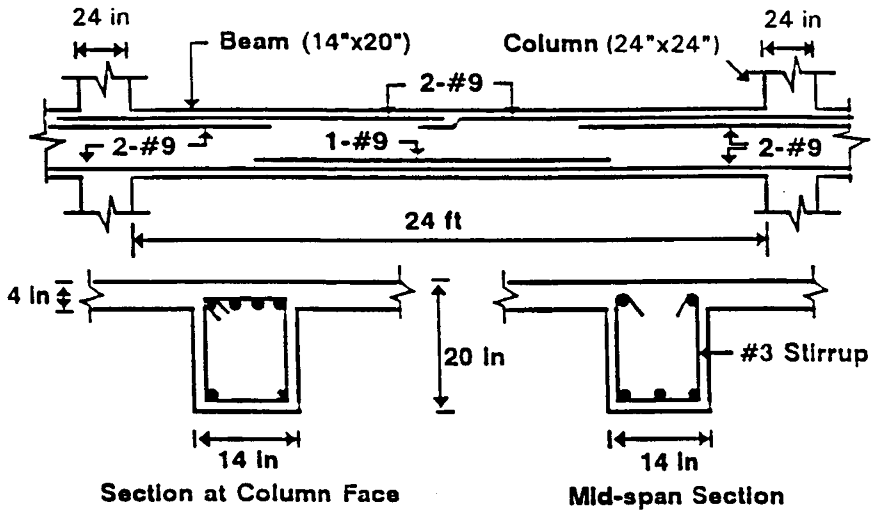
ACI 318-95 Section	Procedure	Calculation	Design Aid
	Step 1 — Select fracture coefficient K.	For concentrated reaction at column, $K = 2.1 \cdot 10^{-5}$	TWay Ac. RF. 1, Note 2
		For Condition A	
	Step 2(a) — Select applicable crack width.	For humidity and moist air, $w_{max} = 0.012$ in.	TWay Ac., RF. 2
	Step 3(a) — Find grid index M_l, assuming full reinforcement stress exists at working level.	$\frac{w_{max}}{\beta} = \frac{0.012}{1.2} = 0.01$ Reading the graph for $w_{max} / \beta = 0.01$ $K = 2.1 \times 10^{-5}$, find $M_l = 395$ in.^{2}	TWay Ac., RF. 3
	Step 4(a) — Calculate spacing for crack control.	Try #4 bars, for which $d_{bl} = 0.50$ in. $d_c = 0.75 + 0.25 = 1$ in.	 REINF. 1 REINF. 1
	Assume $s = s_1 = s_2$ for given panel aspect ratio of $l_s / l_l = 0.8$	$M_l = 395 = \frac{s^2 \cdot d_c}{d_{bl}} \cdot \frac{8}{\pi}$	

ACI 318-95 Section	Procedure	Calculation	Design Aid
		$= \frac{s^2 \cdot 1.0}{0.5} \cdot \frac{8}{\pi}$ $s = 8.80$ in.	
13.4.2	Also satisfies Section 13.4.2.	∴ Recommend #4 bars with maximum spacing of 8.75 in. center-to-center each way for crack control.	
	For Condition B		
	Step 2(b) — Select applicable crack width.	For chemical environment, $w_{max} = 0.007$ in.	TWay Ac., RF. 2
	Step 3(b) — Find grid index M_l, using prescribed design working stress level in crack control equation. * A low stress f_N is also used in designing sanitary or water-retaining structures.	Given: $f_N = 15$ ksi* $w_{max} = K \cdot \beta \cdot f_N \cdot \sqrt{M_l}$ $0.007 = 2.1 \cdot 10^{-5} \cdot 1.20 \cdot 15.0 \cdot \sqrt{M_l}$ $M_l = 343$ in.^{2}	Eq. on TWay Ac., RF. 4
	Step 4(b) — Calculate spacing for crack control.	Try #5 bars, for which $d_{bl} = 0.625$ in. $d_c = 0.75 + 0.312 = 1.06$ in. $M_l = 343 = \frac{s^2 \cdot d_c}{d_{bl}} \cdot \frac{8}{\pi}$ $= \frac{s^2 \cdot 1.06}{0.625} \cdot \frac{8}{\pi}$ $s = 8.9$ in.	REINF. 1 Eq. on TWay Ac., RF. 4
13.4.2	Also satisfies Section 13.4.2.	∴ Recommend #5 bars with maximum spacing of 9 in. center-to-center each way for crack control.	

SEISMIC

SEISMIC DESIGN EXAMPLE 1 - Adequacy of beam flexural design for seismic requirements

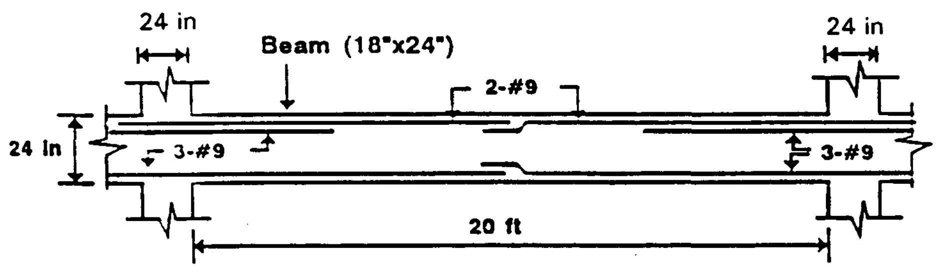
The beam shown below is designed for flexure, using factored loads.
Check if the beam meets seismic design requirements for flexure,
if the beam is to be considered as part of a frame that resists
earthquake induced inertia forces.
f'_c = 4,000 psi; f_y = 60,000 psi; clear cover: 1.5 in.

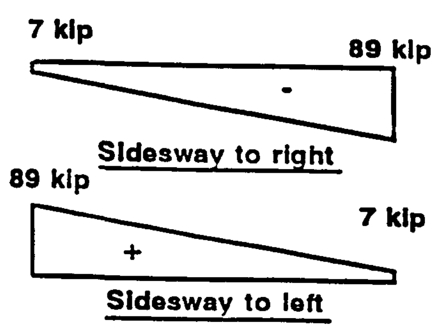


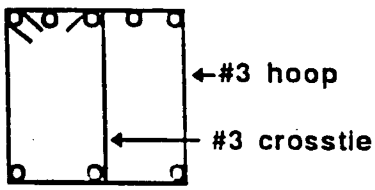
ACI 318-95 Section	Procedure	Calculation	Design Aid
21.3.1.2 21.3.1.3 21.3.1.4	**Step 1** - Check geometric constraints for the beam.	d = 20 - 1.5 - 0.375 - 1.128/2 = 17.6 in. **1** - Clear span ℓ_n = 24 ft ≥ 4d = 5.8 ft OK **2** - b_w/h = 14/20 = 0.7 > 0.3 OK **3** - b_w > 10 in. OK and $b_w = c_2$ OK	
21.3.2.1 10.5.1 Eq.(10-3)	**Step 2** - Check for minimum and maximum ratio of longitudinal reinforcement.	$(\rho_{min})_{top} = (\rho_{min})_{bott.} = 3\sqrt{f'_c}\ / f_y$ = 0.32 % $(\rho_{min})_{top} = (\rho_{min})_{bott.} = 200/f_y$ = 0.33% > 0.32% 2 # 9 bars result in ρ = 0.81% OK 2 # 9 top and bottom continuous bars ρ_{max} = 2.5 % OK	
21.3.2.2	**Step 3** - Check for minimum positive and negative moment capacity at each section.	**1** - $M_n^+ \geq 0.5M_n^-$ at column face; ρ^- = 4 (1.0) / [(14)(17.6)] = 1.62% $M_n^- = K_n F$ = (832)(0.361) = 300 ft-kips ρ^+ = 2 (1.0) / [(14)(17.6)] = 0.81% $M_n^+ = K_n F$ = (451)(0.361) = 163 ft-kips 163 > 0.5 (300) = 150 ft-kips OK **2** - $M_n^+ \geq 0.25\ (M_n^-)_{max}$ at any section; $(M_n^+)_{min}$ = 163 > 0.25 (300) = 75 ft-kip **3** - $M_n^- \geq 0.25\ (M_n^-)_{max}$ at any section; $(M_n^-)_{min}$ = 163 ft-k > 0.25(300) = 75 ft-k	FLEXURE 2.2, 5

SEISMIC DESIGN EXAMPLE 2 - Design of transverse reinforcement for potential hinge regions of an earthquake resistant beam

The beam shown below is part of a frame that resists seismic induced forces. Design the potential hinging region of the beam for transverse reinforcement. f'_c = 4,000 psi; f_y = 60,000 psi; clear cover: 1.5 in.; live load: 1.20 k/ft; dead load: 2.45 k/ft.



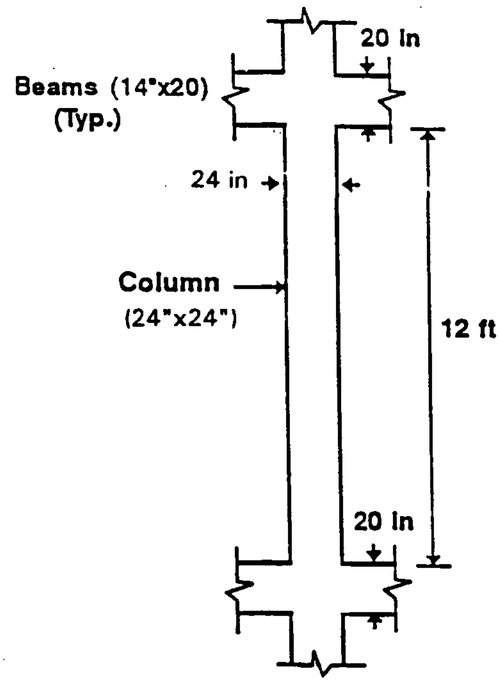
ACI 318-95 Section	Procedure	Calculation	Design Aid
21.3.4.1	**Step 1** - Determine design shear force V_e associated with formation of plastic hinges at beam ends. First compute probable moment strength (M_{pr}) for positive and negative bending.	Assuming #3 hoops, effective depth d: d = 24 - 1.5 - 0.375 - 1.128/2 = 21.6 in. ρ^- = 5 (1.0) / [18(21.6)] = 0.0129 K^-_{pr} = 830 psi; $M^-_{pr} = K^-_{pr}$ F M^-_{pr} = 830(18)(21.6)^2/12000 = 581 ft-k ρ^+ = 3 (1.0) / [18(21.6)] = 0.0077 K^+_{pr} = 528 psi; $M^+_{pr} = K^+_{pr}$ F M^+_{pr} = 528[(18)(21.6)^2]/12000 = 370 ft-k	 SEISMIC 1 SEISMIC 1
21.3.4.1	**Step 2** - Compute design shear force V_e associated with formation of M_{pr} at member ends while the member is loaded with factored gravity loads. Shear force diagrams; 	w_u = 0.75 (1.4 D + 1.7 L) w_u= 0.75[1.4(2.45) + (1.7)(1.20)] = 4.1 k/ft $V_e = \frac{w_u l}{2} \mp \left(\frac{M_{pr}^+ + M_{pr}^-}{l}\right)$ $V_e = \frac{4.1(20)}{2} \mp \left(\frac{370 + 581}{20}\right)$ V_e = 41 ± 48 kips $(V_e)_{max}$ = 89 kips	SEISMIC 2

ACI 318-95 Section	Procedure	Calculation	Design Aid
21.3.4.2	**Step 3** - Check the magnitude of seismic induced shear relative to total design shear and determine the contribution of concrete to shear strength, V_c.	$(V_e)_{max}/2 = 89/2 = 44.5$ kips $(M^+_{pr} + M^-_{pr})/ l = 48$ kips $> (V_e)_{max}/2$ Therefore, $V_c = 0$ (within hinging region, 2h)	
11.5.6.2	**Step 4** - Determine vertical shear reinforcement at the critical section. 	Use #3 perimeter hoops and cross ties as shown in the figure. $\phi V_s = V_e$; $V_s = 89/0.85 = 105$ kips $s = (A_v f_y d)/V_s$ $s = (3\text{x}0.11)(60)(21.6)/105 = 4.1$ in.	
21.3.3.1 21.3.3.2	**Step 5** - Provide hoop steel in the potential hinge region at member ends.	$s < d/4 = 21.6/4 = 5.4$ in. $< 8\,(d_b)_{long.} = 8(1.128) = 9$ in. $< 24\,(d_b)_{tr.} = 24(0.375) = 9$ in. < 12 in. spacing required for shear is 4.1 in. Therefore, use s = 4.0 in. within 2h = 2(24) = 48 in. (4 ft) distance from the column face at each end, with the first hoop located not more than 2 in. from the column face.	
21.3.3.3 7.10.5.3	**Step 6** - Check hoop detailing.	Perimeter hoops and cross ties provide lateral support to at least every other longitudinal reinforcement on the perimeter by the corner of a hoop or the hook of a cross tie. No longitudinal bar is farther than 6 in. from a laterally supported bar.	SEISMIC 3

SEISMIC DESIGN EXAMPLE 3 - Design of an earthquake resistant column

The column shown has a 24 in. square cross-section, and forms part of a reinforced concrete frame system that resists seismic induced forces. Design the column for longitudinal and confinement reinforcement. Assume that the slenderness effects are negligible, and the framing beams are the same as that given in SEISMIC DESIGN EXAMPLE 1.

f'_c = 4,000 psi; f_y = 60,000 psi; Clear cover : 1.5 in.; The column is bent in double curvature with required strength of ϕP_n = 1162 kip, $(\phi M_n)_{top}$ = 420 ft-kip, $(\phi M_n)_{bot.}$ = 380 ft-kip for sidesway to the right; ϕP_n = 980 kip, $(\phi M_n)_{top}$ = 395 ft-kip, $(\phi M_n)_{bot.}$ = 254 ft-kip for sidesway to the left.

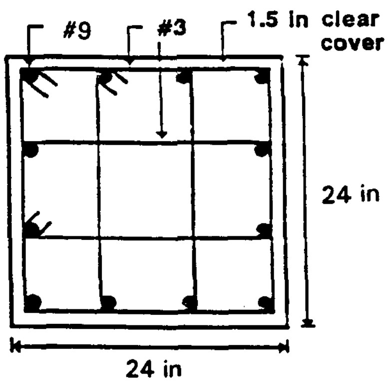


ACI 318-95 Section	Procedure	Calculation	Design Aid
	Step 1 - Determine column size.	Given: h = b = 24 in.	
10.11.4	**Step 2** - Check if slenderness effects may be neglected.	Given : Slenderness effects are negligible.	
21.4.1 9.3.2.2	**Step 3** - Check the level of axial compression.	$A_g f'_c/10$ = (576)(4000)/[(10)(1000)] = 230 kips ϕP_n = 1162 kips > 230 kips. Therefore, the requirements of section 21.4 apply. Also, ϕ = 0.7 for tied columns at this axial load level.	
21.4.1.1 21.4.1.2 21.5.1.4	**Step 4**- Check geometric constraints. Note that the largest longitudinal beam bar is #9 with d_b = 1.128 in.	h = b = 24 in. > 12 in. OK 24/24 = 1.0 > 0.4 OK h = 24 in. > $20d_b$ = 20(1.128) = 22.6 in. OK	
10.2 10.3	**Step 5** - Determine longitudinal reinforcement. First select the appropriate interaction diagram. Estimate γ for a column section of 20 in., cover of 1.5 in., and assumed bar sizes of #3 ties and #9 longitudinal bars.	γ = [24 - 2(1.5 + 0.375) - 1.128)] / 24 γ = 0.80 Square cross-section. If equal area of reinforcement is to be provided on four sides, select COLUMNS 7.2.3 interaction diagrams.	COLUMNS 7.2.3

ACI 318-95 Section	Procedure	Calculation	Design Aid
	Step 6 - Select the critical design loads and compute; $K_n = P_n / (f'_c A_g)$ and $R_n = M_n / (f'_c A_g h)$ Obtain reinforcement ratio ρ from interaction diagrams. Select longitudinal reinforcement	ϕP_n = 1162 kips; P_n = 1162/0.7 = 1660 kips ϕM_n = 420 ft-k; M_n = 420/0.7 = 600 ft-k A_g = (24)(24) = 576 in.^{2} $P_n / (f'_c A_g)$ = 1660 /[(4)(576)] = 0.72 $M_n / (f'_c A_g h)$ = (600 x 12) / [(4)(576)(24)] = 0.13 for γ = 0.80; ρ = 0.019 $A_s = A_g \rho$ A_s = 576 x 0.019 A_s = 10.94 in.^{2} (req'd); use 12#9 bars A_s = 12 x 1.00 = 12.0 in.^{2} (provided)	COLUMNS 7.2.3
21.4.3.1	**Step 7** - Check the limits of reinforcement ratio ρ.	$(\rho)_{prov.}$ = 12.0/576 = 0.021 0.01 < 0.021 < 0.06 OK	

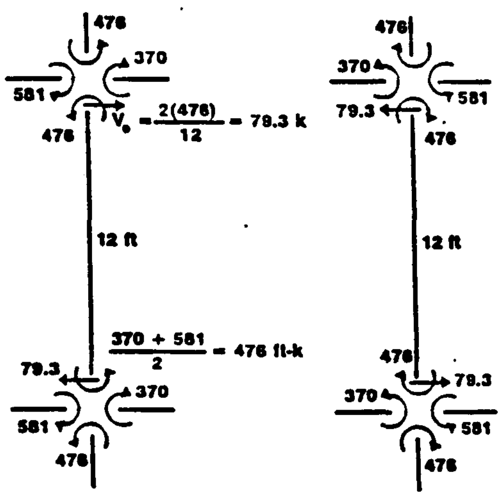
21.4.2
21.4.2.2
	Step 8 - Check flexural strengths of columns and beams at each joint.
	
	Determine column strength M_e from interaction diagram
	
	Determine strengths of the adjoining beams from SEISMIC DESIGN EXAMPLE 1
		$\sum M_e \geq (6/5) \sum M_g$
		
		<u>For sidesway to right:</u>
		ϕP_n = 1162 kips; P_n = 1660 kips
		for ρ = 0.021 and $P_n / f'_c A_g$ = 0.72;
		$M_n / (f'_c A_g h)$ = 0.14 when γ = 0.80
		
		M_n=(0.14)(4)(576)(24)/12= 645 ft-kips
		ϕM_n = 0.7 x 645 = 452 ft-kip
		$M_g^+ = \phi M_n^+$ = 147 ft-kip
		$M_g^- = \phi M_n^-$ = 270 ft-kip
		(2)(452) =904 >(6/5)(147+270) =500 OK
		
		<u>For sidesway to left:</u>
		ϕP_n = 980 kips; P_n = 1400 kips
		for ρ = 0.021 and $P_n / f'_c A_g$ = 0.60;
		$M_n / (f'_c A_g h)$ = 0.162 when γ = 0.80
		M_n = (0.162)(4)(576)(24)/12= 746 ft-kips
		ϕM_n = 0.7 x 746 = 522 ft-kip
		$M_g^+ = \phi M_n^+$ = 147 ft-kip
		$M_g^- = \phi M_n^-$ = 270 ft-kip
		(2)(522) = 1044 >(6/5)(147+270) = 500
			COLUMNS 7.2.3
			
			COLUMNS 7.2.3

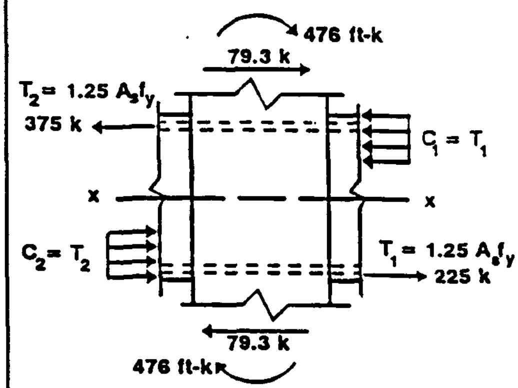
21.4.2.3 21.4.4.4 21.4.2.1		Therefore, confinement reinforcement is to be provided only within ℓ_0 from top and bottom ends of the column. Also, the contribution of the column to lateral strength and stiffness of the structure can be considered.	

ACI 318-95 Section	Procedure	Calculation	Design Aid
21.4.4.1 Eqs.(21-3) Eqs.(21-4)	**Step 9** - Design for confinement reinforcement.	$A_g = (24)^2 = 576$ in.^{2} $A_{ch} = [24 - 2(1.5)]^2 = 441$ in.^{2} $A_g/A_{ch} = 1.31$ $\rho_c = 0.0062 = A_{sh}/(sh_c)$; Try #3 overlapping hoops as shown, $A_{sh} = 4(0.11) = 0.44$ in.^{2}; $h_c = 20.6$ in. $s = 0.44/[(0.0062)(20.6)] = 3.45$ in.	SEISMIC 5
21.4.4.2	Check for maximum spacing of hoops.	$s < 24/4 = 6$ in. OK $s < 4$ in. OK	
21.4.4.3		spacing of hoop legs < 14 in. use #3 overlapping hoops @ 3.5 in. spacing. $\ell_0 \geq h = 24$ in. $\geq l/6 = 24$ in. ≥ 18 in.	
21.4.4.4	Provide hoops over potential hinging regions.	Provide hoops over 24 in. (2 ft) top and bottom, measured from the joint face.	

SEISMIC DESIGN EXAMPLE 4 - Shear strength of a monolithic beam-column joint

Check the shear strength of an interior beam-column joint.
The columns have 24 in. square cross-section, and 12 ft clear height.
The maximum probable moment strength of columns is $(M_{pr})_{col.}$ = 520 ft-kips.
The framing beams have the same geometry and reinforcement as those given in SEISMIC DESIGN EXAMPLE 2. f'_c = 4,000 psi; f_y = 60,000 psi.

ACI 318-95 Section	Procedure	Calculation	Design Aid
21.4.5.1 Fig. 21.3.4	**Step 1** - Compute column shear force V_e associated with formation of plastic hinges at the ends of columns, i.e. when probable moment strengths, $(M_{pr})_{col.}$ are developed.	$V_e = 2(M_{pr})_{col.} / 12$ $V_e = 2(520) / 12 = 86.7$ kips	SEISMIC 2
21.4.5.1 Fig. 21.3.4	**Step 2** - Note that column shear need not exceed that associated with formation of plastic hinges at the ends of the framing beams. Compute V_e when probable moment strengths are developed at the ends of the beams.	From SEISMIC DESIGN EXAMPLE 2; M^-_{pr} = 581 ft-k and M^+_{pr} = 370 ft-k  Sidesway to right Sidesway to left V_e = 79.3 kips < 86.7 kips Therefore, use V_e= 79.3 kips.	SEISMIC 2

ACI 318-95 Section	Procedure	Calculation	Design Aid
21.5.1.1	**Step 3** - Compute joint shear, when stress in flexural tension reinforcement of the framing beams is $1.25f_y$. Note that, in this case, because the framing beams have symmetrical reinforcement arrangements, the joint shear associated with sidesway to the left would be the same as that computed for sidesway to the right.	 Sidesway to right $V_{x\text{-}x} = T_2 + C_1 - V_e$ $V_{x\text{-}x} = 375 + 225 - 79.3 = 520.7$ kips	SEISMIC 6
21.5.3.1	**Step 4** - Compute shear strength of the joint. The joint is confined externally by four framing beams, each covering the entire face of the joint.	$V_c = 20\sqrt{f'_c}\ A_j$ $A_j = (24)(24) = 576$ in.^{2} $\phi\ V_c = (0.85)(20)\sqrt{4000}\ (576)/1000$ $= 619$ kips > 520.7 kips OK	
21.5.2.2	**Step 5** - Note that transverse reinforcement equal to at least one half the amount required for column confinement, has to be provided, with absolute maximum tie spacing relaxed to 6 in.		

COMMENTARY

COMMENTARY ON DESIGN AIDS FOR STRENGTH OF MEMBERS IN FLEXURE

In this handbook, strength design of members subject to flexure is based on Section 10.2 and Sections 10.3.1 through 10.3.4 of "Building Code Requirements for Structural Concrete (ACI 318-95)."

The design method employed uses equations of statics available in standard texts on reinforced concrete. These equations are summarized in Section 10.3 of "Commentary on Building Code Requirements for Reinforced Concrete (ACI 318-95)." The FLEXURE Design Aids are intended to facilitate use of these equations.

The following five types of flexural characteristics are covered by the FLEXURE Design Aids:

(a) Design moment strengths M_n, M_{n1}, or M_{nw} (from FLEXURE 2 and 4, or FLEXURE 5 and 6), and M_{n2} (from FLEXURE 3, when $f_s' = f_y$)

(b) Area A_s of tension steel (from FLEXURE 2 plus FLEXURE 3)

(c) Area A_s' of compression steel (from FLEXURE 2)

(d) Effective beam depth d (from FLEXURE 2, 5, or 6)

(e) Maximum reinforcement ratio ρ_{max} (from FLEXURE 2 or 6)

The design method is based the following assumptions in accordance with Section 10.2 of ACI 318-95.

1. Strain in reinforcement and in concrete is directly proportional to its distance from the neutral axis.

2. Maximum usable strain at extreme concrete compression fiber is 0.003.

3. Stress in reinforcement below specified yield strength f_y is the product of E_s (29,000,000 psi) times steel strain. For strains greater than that corresponding to f_y, stress in reinforcement is taken equal to f_y.

4. Tensile strength of concrete is neglected.

5. Stress distribution is represented by a rectangular compressive stress block in which an average stress of 0.85 f_c' is used with a rectangle of depth $a = \beta_1 c$; $\beta_1 =$ 0.85 for concrete with $f_c' \leq 4000$ psi, $\beta_1 = 0.80$ for $f_c' = 5000$ psi, and $\beta_1 = 0.75$ for $f_c' = 6000$ psi.

Note that the FLEXURE Design Aids include no strength reduction factor [Section 9.3.2 of ACI 318-95]. Design aids are based upon $\phi = 1$.

The designer is reminded to check the selection of beam depth against the deflection control provisions of Section 9.5 of ACI 318-95. The design must also meet the crack control requirements of Section 10.6.4 of ACI 318-95.

FLEXURE 1

FLEXURE 1 is intended for use in preliminary design of rectangular beams with no compression reinforcement. It enables the designer to assume cross-sectional dimensions and quickly estimate area of reinforcement needed. The uses of a_n and the preferred value of ρ are explained in the Commentary on FLEXURE 2. The design aid is calculated on the basis of the relations shown above the table.

FLEXURE 2

FLEXURE 2, together with FLEXURE 5, provide coefficients for use in solving the equation

$$M_n = \left[A_s f_y d\left(1 - 0.59\,\rho\,\frac{f_y}{f_c'}\right)\right] \quad (FL\text{-}1)$$

where

$$\rho = \frac{A_s}{bd}$$

For a rectangular beam with tension reinforcement only, these two design aids facilitate determining the following:

M_n: The nominal moment strength of a section of known concrete strength f_c', steel yield stress f_y, and dimensions b and d

A_s: The area of tension reinforcement meeting the strength design requirements of ACI 318-95 for the nominal moment M_n with known values of f_c', f_y, b, and d

b and d: The required beam width and required effective depth for the nominal moment M_n with known A_s, f_c', and f_y.

For rectangular beams with compression as well as tension reinforcement, FLEXURE 2 and FLEXURE 5 facilitate determining the following:

M_{nl}: The nominal moment strength of the section before additional top and bottom reinforcement is added to develop the required extra moment.

For flanged sections, FLEXURE 2 and FLEXURE 5 facilitate determining:

M_{nw}: The nominal moment strength of the web section before additional tension reinforcement is added to develop the tension force required to counterbalance any available additional compression forces of the flanges.

The equation for M_n (Eq. FL-1) serves for M_{nl} for beams with compression reinforcement (taking $\rho = A_{sl}/bd$) and for M_{nw} for flanged sections (taking $\rho = A_{sw}/bd$).

Eq. (FL-1) may be developed by using the rectangular stress block (see Fig. FL-1) permitted by Section 10.2.7 of ACI 318-95. The compressive force C and the tensile force T are

$$C = 0.85 f_c' \, ba$$
$$T = A_s f_y$$
$$C = T$$

$$a = \frac{A_s f_y}{0.85 f_c' \, b} + \frac{\rho b d f_y}{0.85 f_c' \, b} = \frac{\rho d f_y}{0.85 f_c'}$$

The nominal strength M_n is then

$$M_n = (C \text{ or } T) \times (\text{moment arm})$$

$$= A_s f_y \left(d - \frac{\rho d f_y}{1.70 f_c'} \right) = A_s f_y d \left(1 - 0.59 \, \rho \, \frac{f_y}{f_c'} \right)$$

Eq. (FL-1) may be rewritten, substituting $A_s = \rho bd$,

$$M_n = \rho f_y \left(1 - 0.59 \rho \frac{f_y}{f_c'} \right) \left(\frac{bd^2}{12{,}000} \right), \text{ ft-kips}$$

and letting $\omega = \rho(f_y/f_c')$

$$M_n = \left(f_c' \omega (1 - 0.59\omega) \right) \left(\frac{bd^2}{12{,}000} \right), \text{ ft-kips}$$

Then, letting $K_n = f_c' \omega \, (1 - 0.59\omega)$ and $F = bd^2/12{,}000$, Eq. (FL-1) can be written simply as

$$M_n = K_n F, \text{ ft-kips} \qquad (FL\text{-}2)$$

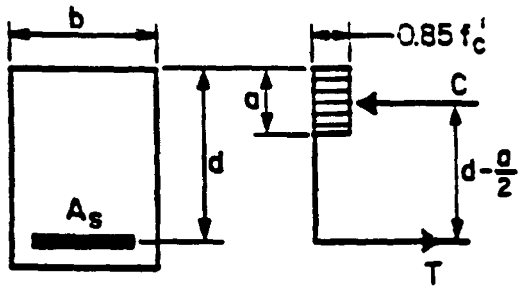


Fig. FL-1—Rectangular stress block for rectangular beam with tension reinforcement only

The coefficient K_n is tabulated in FLEXURE 2, and the coefficient F in FLEXURE 5.

Eq. (FL-1) may have its terms regrouped as

$$M_n = A_s d \left[f_y \left(1 - 0.59 \rho \frac{f_y}{f_c'} \right) \frac{1}{12{,}000} \right], \text{ ft-kips}$$

Then, again letting $\omega = \rho(f_y/f_c')$ and letting $a_n = f_y(1 - 0.59\,\omega)/12{,}000$, ft-kips/in., Eq. (FL-1) becomes

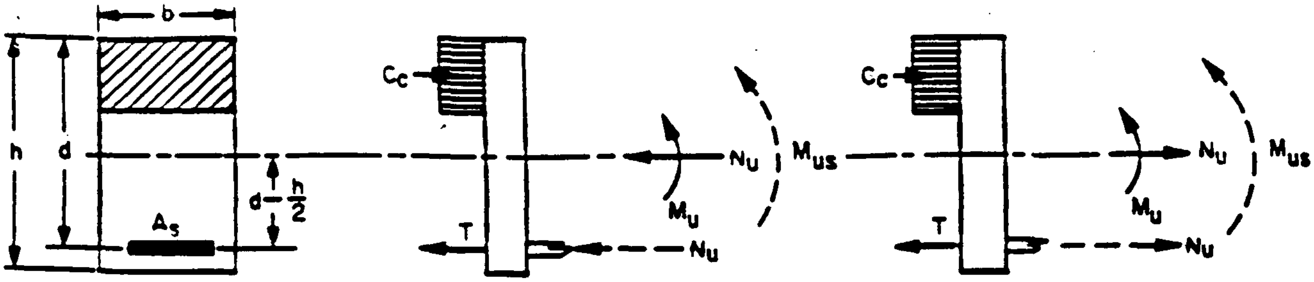
$$M_n = A_s d a_n, \text{ ft-kips} \qquad (FL\text{-}3)$$

The coefficient a_n is tabulated in FLEXURE 2.

Note that the coefficients F and a_n include divisors of 1,000 so the Eq. (FL-2) and (FL-3) give M_n in ft-kips when the units for b, d, and A_s are inch units and for f_y is psi.

In FLEXURE 2, values of ρ and corresponding values of a_n are tabulated only up to maximum permissible values (ρ_{max} as defined in Section 10.3.3 of ACI 318-95). Values of ρ_{max} are tabulated in FLEXURE 10. The value of ρ listed just below the upper heavy line in FLEXURE 2 is equal to (or just slightly greater) than the minimum permissible value of ρ (which is $\rho_{min} = 3\sqrt{f_c'}/f_y$ but not less than either $200 / f_y$ or minimum shrinkage reinforcement as provided in Sections 10.5.1 and 10.5.3 of ACI 318-95 respectively).

For rectangular beams with restricted construction depth, where compression as well as tension reinforcement is required, FLEXURE 2 and FLEXURE 5 are used in conjunction with FLEXURE 3.



Axial Compression

$$M_{us} = M_u + N_u\left(d - \frac{h}{2}\right)$$

$$A_s = \frac{M_{us}}{da_n} - \frac{N_u}{\phi f_y}$$

Axial Tension

$$M_{us} = M_u - N_u\left(d - \frac{h}{2}\right)$$

$$A_s = \frac{M_{us}}{da_n} + \frac{N_u}{\phi f_y}$$

For such beams, FLEXURE 2 and FLEXURE 5 are used to determine the area A_{s1} of tension reinforcement needed to counterbalance the compressive force of the concrete. FLEXURE 3 (depending on whether or not the compression reinforcement yields at design conditions) is used to determine the area of compression reinforcement A_s', while FLEXURE 3 is used to determine the additional area A_{s2} of tension reinforcement.

For use with flanged sections in which the flange thickness h_f is greater than the stress block depth a, FLEXURE 2 displays the dimensionless ratios a/d and j_n as a function of ω (which itself is a function of reinforcement ratio ρ and materials strength f_y and f_c'.

For flanged sections the value of the effective flange width b is determined by the provisions of Section 8.10 of ACI 318-95. Note that the minimum tension reinforcement ratio is based on the web width b_w rather than the flange width b.

Combined bending and axial load

Although FLEXURE 2 is intended primarily for pure flexure, it can be used for combined flexure and axial load. For flexural members subject to axial tension, the coefficients in FLEXURE 2 can be applied as tabulated, because the FLEXURE Design Aids are based on a strength reduction factor ϕ of 1.0 as required by Section 9.3.2 of ACI 318-95 applicable for flexure with or without axial tension. For flexural members subject to axial compression the strength reduction factor is less than 0.90 and may be as low as 0.70 for tied compression members, according to Section 9.3.2. The coefficients K_n and a_n tabulated in FLEXURE 2 must be revised accordingly.

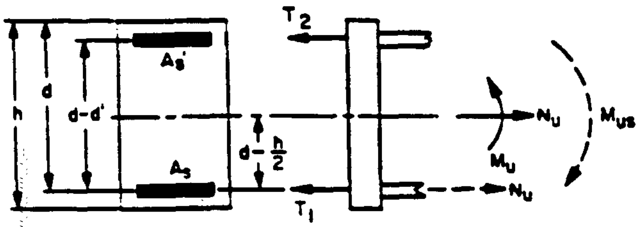
Design for combined bending and axial load can be simplified by transferring the force system on the section os that the axial load acts along the axis of the tension reinforcement where it may either increase or decrease the force in the reinforcement. The supplementary moment resulting from the transfer may be either larger or smaller than the original moment on the section as indicated in Fig. FL-2 and must be considered in the design for flexure.

A member loaded with M_u and with N_u in the line of action of the reinforcement and reinforced with

$$A_s = \frac{M_{us}}{da_n} \pm \frac{N_u}{\phi f_y}$$

(the sign depends on whether N_u is a tensile or compressive force) will have the same size compressive stress block and the same strains on the concrete and steel as a member loaded only with M_{us} and reinforced with $A_s = M_{ns}/(a_n d)$. The ϕ factor does not appear in the first term because it is included in the coefficient a_n. For analysis, the equivalent factored moment M_{us} may be interpreted as the equivalent design moment strength ϕM_{ns}.

In the case of axial tension in which N_u is large compared to M_u, Fig. FL-3, M_{us} may become negative, indicating tension rather than compression in the top fiber. In this case the entire section is in tension. Since the concrete tensile strength is neglected, both force systems must be carried by the steel, the moment as a couple and the axial load as a tension in what was considered the lower or tension reinforcement.



$$M_{us} = M_u - N_u\left(d - \frac{h}{2}\right)$$

$$A_s' = \frac{M_{us}}{\phi f_y (d - d')}$$

$$A_s = \frac{N_u}{\phi f_y} - A_s'$$

Fig. FL-3—Equivalent rectangular stress block and free body diagram for section in which axial tension N_u is large compared to the moment M_u

FLEXURE 3

For rectangular beams with compression as well as tension reinforcement, FLEXURE 3 contains coefficients for use in solving for M_{n2} or A_s' when the compression reinforcement yields and the compression strength of the displaced concrete is neglected. The following equation is used:

$$M_{n2} = [A_{s2} f_y (d - d')]$$

M_{n2}: that portion of total moment strength M assigned to compression reinforcement.

A_{s2}: area of additional tension reinforcement to provide M_{n2}; approximately equal to the area A_s' of compression reinforcement.

For flanged sections in which $h_f < a$, FLEXURE 3 tabulates coefficients for use in solving the corresponding equations

$$M_{n2} = [A_{sf} f_y (d - 0.5h_f)]$$

and

$$A_{sf} = \frac{0.85 f_c'(b - b_w)h_f}{f_y}$$

where

M_{n2}: that portion of total moment strength M assigned to the flanges

A_{sf}: the required area of additional tension reinforcement required to counterbalance the compression force in the overhanging portions of the flanges.

One may note that the total area of tension reinforcement A_s is obtained as follows:

For a rectangular section with compression reinforcement:

$$A_s = A_{s1} + A_{s2}$$

where

A_{s1}: area of tension reinforcement required for rectangular beam with tension reinforcement only (from FLEXURE 2)

A_{s2}: additional tension reinforcement to counterbalance additional compression force contributed by compression reinforcement (approximately compression reinforcement area A_s' if compression reinforcement yields and displaced concrete is neglected)

For a flanged section having $h_f < a$ with tension reinforcement only:

$$A_s = A_{sw} + A_{sf}$$

where

A_{sw}: area of tension reinforcement required for rectangular section of width b_w

A_{sf}: area of additional tension reinforcement to counterbalance additional compression force contributed by the flanges

Additional comments relating to stress f_s' for use in computing strength contributed by compression reinforcement

When using compression reinforcement in computation of strength, the stress f_s' used for such reinforcement must be compatible with the ultimate strain diagram. That is, for a strain ϵ_s' greater than f_y/E, the compression reinforcement yields, giving $f_s' = f_y$.

When ϵ_s' is less than f_y/E_s, compression reinforcement does not yield, making stress proportional to strain, and $f_s' = \epsilon_s' E_s$. The designer should note that on thin (shallow) members the strain (and therefore stress) computed at the location of the compression reinforcement based on location of neutral axis and compression reinforcement location may be subject to large error due to reinforcement placement tolerances.

Since the strength effectiveness of compression reinforcement is reduced if it does not yield when the strength M_n of the section is reached, Section 10.3.1 (A)(3) of the Commentary on ACI 318-95 suggests that for such cases where $f_s' < f_y$ the compression reinforcement may be neglected entirely. In such cases, the design moment strength M_n is computed as for a rectangular section with tension reinforcement only.

The case where the calculated strain in the compression reinforcement (when strength M_n is reached) is greater than or equal to the yield strain (i.e., $f_s' = f_y$) may be shown to be equivalent to the requirement that

$$\frac{(A_s - A_s')}{bd} \geq 0.85\beta_1 \frac{f_c' d'}{f_y d}\left(\frac{87{,}000}{87{,}000 - f_y}\right) \quad (FL\text{-}4)$$

in which case

$$M_n = \left[A_{s1} f_y \left(d - \frac{a}{2}\right) + A_{s2} f_y (d - d')\right] \quad (FL\text{-}5)$$

where

$$a = \frac{A_{s1} f_y}{0.85 f_c' b}$$

Eq. (FL-5) may be written

$$M_n = M_{n1} + M_{n2}$$

where

$$M_{n1} = \left[A_{s1} f_y \left(d - \frac{a}{2}\right)\right]$$

$$M_{n2} = A_{s2} f_y (d - d')$$

$$= A_s'(f_y - 0.85 f_c')(d - d')$$

Since

$$M_n = M_u/\phi \text{ , } \phi = 1.0$$

$$M_n = \left[A_s f_y d\left(1 - 0.59\,\rho\,\frac{f_y}{f_c'}\right)\right]$$

$$M_n = \left[A_s f_y \left(d - \frac{a}{2}\right)\right] \quad (FL\text{-}1)$$

[from Section 10.3.1 (A)(2) of Commentary on ACI 318-95], and

$$M_n = K_n F \qquad \text{(FL-2)}$$

it is apparent that $M_{n1} = M_n$ and therefore

$$M_{n1} = K_n F$$

Hence,

$$M_n = K_n F + [A_{s2} f_y (d - d')] \quad (FL\text{-}6)$$

Solving for A_{s2} gives

$$A_{s2} = \frac{(M_n - K_n F)12{,}000}{f_y (d - d')}$$

Letting

$$a_n' = \frac{f_y}{12{,}000}\left(1 - \frac{d'}{d}\right)$$

then

$$A_{s2} = \frac{M_n - K_n F}{d a_n'}$$

when displaced concrete is neglected, i.e., when $f_y - 0.85 f_c'$ is taken as approximately f_y, A_s' is approximately A_{s2}. Thus,

$$A_s' = \frac{M_n - K_n F}{d a_n'}$$

FLEXURE 3 contains $a_n^{\prime}$ for use in obtaining A_{s2} *or* $A_s^{\prime}$ from the above equation.

According to Section 10.3.3 of ACI 318-95: For flexural members the ratio of reinforcement provided shall not exceed 0.75 of the balanced reinforcement ratio that would produce balanced strain conditions for the section under flexure without axial load. For members with compression reinforcement, the portion of the balanced reinforcement ration equalized by compression reinforcement need not be reduced by the 0.75 factor.

For a beam with compression reinforcement, Section 10.3.3 provides that

$$\rho_{max} = 0.75\rho_b + \rho^{\prime}\frac{f_s^{\prime}}{fy} - \frac{0.85\ \rho^{\prime}f_c^{\prime}}{f_y}$$

where

ρ_b: balanced reinforcement ratio for a rectangular section with tension reinforcement only

$f_s^{\prime}$: stress in compression reinforcement corresponding to the strain in compression reinforcement at the moment strength of a beam containing ρ_{max} [Note that Table 10.3.2 of the Commentary on ACI 318-95 uses the simplifying assumption that the balanced condition should be used to determine $f_s^{\prime}$, in which case $f_s^{\prime} = f_{sb}^{\prime}$. FLEXURE 3 is calculated on the basis of this assumption.]

The third term in the ρ_{max} equation accounts for the concrete displaced by the compression steel.

Values of $(\rho - \rho^{\prime})$, which is equal to $(A_s - A_s^{\prime})/bd$ in Eq. (FL-4) are tabulated for d'/d up to that corresponding to 100 percent of ρ_b, beyond which the section would be overreinforced. ACI 318-89 does not specify a limit on ρ'.

For values of $d^{\prime}/d$ or of $(\rho - \rho^{\prime})$ less than those shown in FLEXURE 3, stress in compression reinforcement is less than yield strength f_y, and FLEXURE 4 must be used to determine $a_n^{\prime\prime}$ for use instead of $a_n^{\prime}$ for calculating the area of compression reinforcement $A_s^{\prime}$.

Flanged sections (with tension reinforcement only) for which $h_f < a$

When the depth a of the equivalent rectangular stress block is less than the flange thickness h_f, the compression zone is a rectangular one and the rectangular section strength [Eq. (FL-1)] is used. The effective flange width b rather than the web width b_w is then appropriate for the reinforcement ratio ρ.

When the depth a of the equivalent rectangular stress block is equal to or greater than the flange thickness, special equations are required. For this rare case,

$$M_n = M_{nw} + M_{n2} \qquad (FL-7)$$

where

$$M_{nw} = \left[A_{sw}\, f_y\left(d - \frac{a}{2}\right)\right]$$

$$M_{n2} = [A_{sf}\, f_y(d - 0.5h_f)]$$

A_{sw}: area of tension reinforcement for a rectangular section of width b_w containing tension reinforcement only (corresponds to A_s from Eq. 1)

$$A_{sf} = \frac{0.85f_c^{\prime}(b - b_w)h_f}{f_y}$$

$$a = \frac{A_{sw}f_y}{0.85f_c^{\prime}b_w}$$

One may note that M_{nw}, like M_{nl}, is equal to M_n. Thus,

$$M_{nw} = K_nF$$

and therefore Eq. (FL-7) becomes

$$M_n = K_nF + [A_{sf}\, f_y(d - 0.5h_f)]$$

Making the following substitutions in the expression above for A_{sf}

$$k_{nf} = \frac{0.85 f'_c}{12{,}000}\left(\frac{b}{b_w} - 1\right)$$

$$j_f = 1 - \frac{h_f}{2d}$$

$$a_{nf} = \frac{f_y}{12{,}000}\left(1 - \frac{h_f}{2d}\right)$$

then

$$A_{sf} = \frac{k_{nf}\, j_f b_w h_f}{a_{nf}}$$

FLEXURE 3 contains K_{nf}, j_f, and a_{nf} for use in the equation for A_{sf}.

FLEXURE 4

FLEXURE 4 facilitates determining the area of compression reinforcement A'_s in beams in which the compression reinforcement has yielded at design conditions. (Note that $A_s = A_{s1} + A_{s2}$, with A_{s2} approximately equal to A'_s to when compression reinforcement yields when strength of section is reached.)

For rectangular beams with compression reinforcement in which the strain ϵ'_s exceeds f_y/E_s (and therefore $f'_s = f_y$), the "extra" moment M_{n2} to be carried in the compression reinforcement is, from the development following Eq. (FL-5), approximately,

$$M_{n2} = A'_s f_y(d - d'),\ in.\text{-}lb$$

$$= \frac{A'_s f_y(d - d')}{12{,}000},\ ft\text{-}kips$$

Therefore,

$$A'_s = \frac{12{,}000 M_{n2}}{f_y(d - d')},\ sq\ in.$$

The area A'_s or A_{s2} is given in FLEXURE 4.

FLEXURE 4 is correct only for cases in which compression reinforcement yields before ϵ_c reaches 0.003. When compression reinforcement is to be used for ductility or for deflection control, the inaccuracy resulting from nonyielding compression reinforcement is negligibly small. However, in evaluating the moment strength of beams in which compression reinforcement is required for strength, the lesser stress of $E_s\epsilon_s$ or f_y must be used. Values of $\rho - \rho'$ tabulated in FLEXURE 3 provide a convenient check for yielding of compression reinforcement. If compression reinforcement does not yield, A'_s must be greater than the values tabulated in FLEXURE 4 and the quantity A_{cw} from FLEXURE 6 may be helpful for determining A'_s.

FLEXURE 5

The coefficient F (= $bd^2/12{,}000$) from FLEXURE 5 multiplied by K_n from FLEXURE 2 gives the following design moment strengths in ft-kips:

- M_n: The nominal moment strength of a rectangular beam with tension reinforcement only
- M_{nl}: The nominal moment strength of a section before compression reinforcement and extra tension reinforcement are added
- M_{nw}: The nominal moment strength of the web of a flanged beam

The equation $M_n = K_n F$ is Eq. (FL-2), which is derived above in the section on FLEXURE 2. It can be seen that M_{nl} and M_{nw} are equivalents of M_n.

FLEXURE 6

FLEXURE 6 contains solutions to the equation [given in Section 10.3, in graphical and tabular form, of the Commentary on ACI 318-95] for design moment strength of a rectangular section with tension reinforcement only in the form

$$M_n = A_s f_y d\left(1 - 0.59\rho\frac{f_y}{f'_c}\right) \quad (FL\text{-}1)$$

For 12 in. wide sections of slabs (or beams), FLEXURE 6 facilitates determining the following:

M_n (or M_{nl}) : Nominal moment strength for known values of area of reinforcement A and effective depth d

d: Effective depth required for known design moment M_n (or M_{nl}) in a slab of known effective depth d

While FLEXURE 6 contains moment strengths of 12 in. wide sections of slabs, the values can be used for other widths of slabs or beams by multiplying tabulated moment strength by a factor of section width divided by 12 in.

FLEXURE 6 provides the graphical solutions to the following equation for design moment strength of a rectangular section with tension reinforcement only (see Fig. FL-1):

$$M_n = \left[A_s f_y \left(d - \frac{a}{2}\right)\right]$$

from Section 10.3.1 (A)(1) of the Commentary on ACI 318-95, where

$$a = \frac{A_s f_y}{0.85 f'_c b}$$

FLEXURE 6 graphs consist of 6.X.1 graphs that facilitate determination of the nominal moment strength and tension reinforcement for 12 in. wide slab sections. These graphs include five selected ratios of known material strengths f'_c and f_y and variable effective depths d. Any one of the following three items may be read from the graphs when the other two are known:

M_n: Nominal moment strength for a slab (or beam) of known effective depth d, and area of tension reinforcement A_s

A_s: Required area of tension reinforcement for a slab (or beam) of known effective depth d, and nominal moment strength M_u

d: Required effective depth of a slab (or beam) of known area of tension reinforcement A_s to carry a nominal moment strength M_n

To aid the designer in making the appropriate selection, lines have been located on every graph indicating five useful reinforcement ratios:

$0.75\rho_b$: the maximum reinforcement ratio for a rectangular section with tension reinforcement only in accordance with Section 10.3.3

$0.5\rho_b$: the maximum reinforcement ratio which may be used when moments are redistributed in accordance with Section 8.4.3 of ACI 318-95

$0.25\rho_b$: the reinforcement ratio at which 15 percent of the moment may be redistributed in accordance with Section 8.4.1 of ACI 318-95

ρ for A_s: 0.002bh or 0.0018bh for (h-d) of 2½ and 1½ in.: the minimum reinforcement ratios for slabs of uniform thickness) specified in Section 10.5.3 of ACI 318-95

ρ_{min}: $3\sqrt{f'_c}/f_y$ but not less than $200/f_y$: the maximum reinforcement ratio for flexural members (other than slabs of uniform thickness) specified in Section 10.5.1 of ACI 318-95

For a rectangular beam with compression as well as tension reinforcement, the graphs of FLEXURE 6 also show

A_{cw}: The minimum area of tension reinforcement A_{sw} to keep the neutral axis low enough for compression reinforcement to reach yield strain at design conditions, i.e., if $A_{sl} < A_{cw}$, $f'_s = f_y$. A_{cw} is computed as

$$A_{cw} = \frac{0.85 b f'_c \beta_1}{\left(1 - \frac{f_y}{87{,}000}\right) f_y} d'$$

and for a flanged section, the graphs of FLEXURE 6 show

A_{s3}: The maximum area of tension reinforcement A_{sf} at which depth of the stress block a will be less than the flange thickness h_f, i.e., if $A_{sw} \le A_{s3}$, then $a \le h_f$... and the moment strength may be calculated as for a rectangular beam using Eq. (FL-1); if $A_{sw} > A_{s3}$, then $a > h_f$... and the nominal moment strength M_n must be computed using Eq. (FL-7). In such case,

$$A_{s3} = \frac{0.85 f'_c b}{f_y} h_f$$

It should be noted that while the FLEXURE 6.X.1 graphs are intended for design and analysis of slabs, they may be for beams by multiplying moments and reinforcement areas on the graphs by a factor of beam width b divided by 12 in.

COMMENTARY ON REINFORCEMENT DESIGN AIDS

REINFORCEMENT 1

REINFORCEMENT 1 shows nominal dimensions and weights of reinforcing bars in commercially available sizes. These bars can be obtained as deformed bars conforming with

ASTM A 615 Standard Specification for Deformed and Plain Billet-Steel Bars for Concrete Reinforcement (covers bar sizes #3 through #6 in Grade 40, sizes #3 through #18 in Grade 60, and sizes #11 through #18 in Grade 75),

ASTM A 616 Standard Specification for Rail-Steel Deformed and Plain Bars for Concrete Reinforcement including Supplementary Requirements (covers bar sizes #3 through #11 in Grade 60),

ASTM 617 Standard Specification for Axle-Steel Deformed and Plain Bars for Concrete Reinforcement (covers bar sizes #3 through #11 in Grades 40 and 60), and

ASTM A 706 Standard Specification for Low-Alloy Steel Deformed Bars for Concrete Reinforcement (covers bar sizes #3 through #18 in Grade 60).

Grades denote minimum yield strength of the material in thousands of pounds per square inch.

REINFORCEMENT 2

REINFORCEMENT 2 gives cross section areas for various combinations of bars. Instructions for use are given on the table; see also examples below table.

REINFORCEMENT 3

REINFORCEMENT 3 provides information concerning the properties of bars of various sizes bundled together in groups. Included are the diameter d_{be} of a single bar of equivalent cross section area and the distance of the centroid of each bundle from the bottom of the bundle. The latter is used in calculating the effective depth of the beam. The cross section of each bundle can be obtained using REINFORCEMENT 2. For minimum width-of-web requirements, see REINFORCEMENT 12. For maximum web width satisfying crack control provisions, see REINFORCEMENT 13.

REINFORCEMENT 4

REINFORCEMENT 4 gives the sectional properties for smooth and deformed wires in welded fabric and their cross section areas for various wire spacings.

REINFORCEMENT 5.1 and 5.2

REINFORCEMENT 5.1 lists ASTM standard specifications for wire and welded wire fabric. REINFORCEMENT 5.2 gives strength requirements of wire in welded wire fabric for reinforcing concrete. Both tables are taken from the *Manual of Standard Practice* published by the Wire Reinforcement Institute* with minor revisions according to ACI 318-95.

REINFORCEMENT 6

REINFORCEMENT 6 gives information on typical stock items of two-way wire fabric usually carried by various manufacturers in certain parts of the United States and Canada. Information for this table is taken from the WRI *Manual of Standard Practice** and the *Manual of Standard Practice* published by the Concrete Reinforcing Steel Institute.*

REINFORCEMENT 7.1 and 7.2

REINFORCEMENT 7.1 and 7.2 give the typical development lengths and splice lengths for welded smooth and deformed wire fabric. The information is excerpted from the WRI *Manual of Standard Practice** with minor revisions according to ACI 318-95

REINFORCEMENT 8.1 and 8.2

REINFORCEMENT 8.1 gives theoretical maximum effective tension area of concrete A in beams for various conditions. The value $A = b_w t/n$ computed for the selected beam size must be smaller than the corresponding table value to satisfy crack control provisions.

These provisions are based on the general expression given in ACI 318R-95 Section 10.6.4:

$$w = 0.076\beta f_s \sqrt[3]{d_c A}$$

where

w = crack width in units of 0.001 in.

β = the ratio of x_e to x_c shown in Fig. RE-1

f_s = the calculated flexural stress in the reinforcement at service loads, or 0.6 of f_y

d_c = the thickness of the concrete cover measured from the extreme tension fiber to the center of the bar located closest thereto (or the centroid of the bar bundle closest thereto)

A = the effective tension area of concrete surrounding the main tension reinforcing bars—and having the same centroid as that of the reinforcement—divided by the number of bars

Fig. RE-2 illustrates the Area A used in the equation for w. According to the definition of A in ACI 318-95 Section 10.0, A = effective tension area divided by number of bars when effective tension area has the same centroid as the flexural tension reinforcement. (Displacement of concrete by tension reinforcement

Manual of Standard Practice, 3rd Edition, Wire Reinforcement Institute, Inc., McLean, 1979, 32 pp.

Manual of Standard Practice, 24th Edition, Concrete Reinforcing Steel Institute, Schaumberg, IL., 1986, 75 pp.

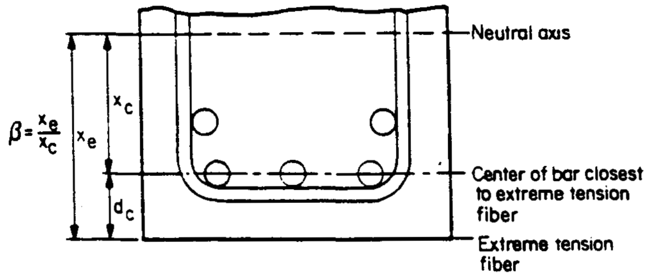


Fig. RE-1—Ratio β used in Gergely-Lutz equation for crack control ($w = 0.076\beta f_s \sqrt[3]{d_c A}$) discussed in ACI 318R-95 Section 10.6.4). β is usually taken as about 1.2 for beams and 1.35 for slabs

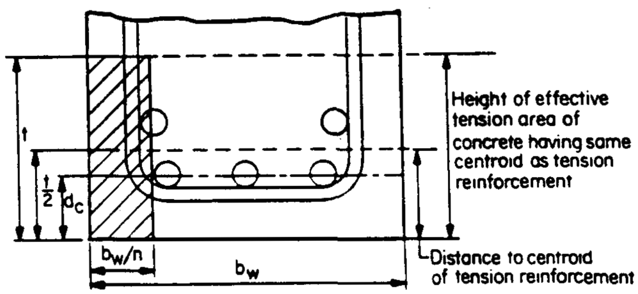


Fig. RE-2—Crosshatched area illustrates Area A [used in Gergely-Lutz equation mentioned in Commentary on ACI 318-95 Section 10.6.4 and in Eq. (10-4) of ACI 318-95] for case where five bars in two layers are used. *A* is defined as effective tension area of concrete surrounding the flexural tension reinforcement and having the same centroid as that reinforcement, divided by the number of bars. When the flexural reinforcement consists of different bar sizes the number of bars shall be computed as the total area of reinforcement divided by the area of the largest bar used

is neglected.) Therefore the height t of the effective tension area must be twice the distance from the centroid of the flexural tension reinforcement to the extreme tension fiber. The width of the effective tension area is b_w. Since A is effective tension area of concrete divided by number of bars,

$$A = \frac{tb_w}{n}$$

Note that when flexural reinforcement consists of different bar sizes, the number of bars shall be computed as the total area of reinforcement divided by the area of the largest bar used. Therefore n need not be a whole number.

Moving all constants to one side of the general equation for the crack width gives

$$\frac{w}{0.076\beta} = f_s \sqrt[3]{d_c A} = z$$

By introducing a maximum crack width w of 0.016 in. for interior exposure and 0.013 in. for exterior exposure and an average β of 1.2, ACI 318-95 Section 10.6.4 arrives at the following two equations which shall be satisfied to provide crack control.

For interior exposure: $f_s \sqrt[3]{d_c A} \leq 175$ kips/in.
For exterior exposure: $f_s \sqrt[3]{d_c A} \leq 145$ kips/in.

The above expressions for z are not particularly useful for the practicing engineer. The equations are easier to apply if solved for A:

$$\text{Interior exposure: } A \leq \frac{1}{d_c}\left(\frac{175}{f_s}\right)^3$$

$$\text{Exterior exposure: } A \leq \frac{1}{d_c}\left(\frac{145}{f_s}\right)^3$$

These two equations are the basis for REINFORCEMENT 8.1.

It is indicated by the Commentary on ACI 318-95, Section 10.6.4, and can easily be seen from Fig. RE-1, that the ratio β is not a constant and that values other than 1.2 may be more appropriate for different conditions. (Linear interpolation is sufficiently accurate.)

Tables and graphs have been evaluated for $f_s = 0.60 f_y$ as permitted by ACI 318-95, Section 10.6.4. Since this assumption ordinarily is high to be on the safe side, evaluation of the actual f_s may often be advantageous for critical conditions.

REINFORCEMENT 8.2 gives a graphical presentation of the basic relationships for crack control. If the chart values of REINFORCEMENT 8.2 are multiplied by the area of one bar, the A values of REINFORCEMENT 8.1 are obtained.

REINFORCEMENT 9

REINFORCEMENT 9 gives minimum web widths for two bars of a single size, along with the increment of width required for each additional bar.

The determination of what constitutes minimum width according to ACI 318-95, while apparently a simple task, is actually subject to controversy. The complicating factor is that the minimum stirrup bend permitted has an inside diameter of four stirrup bar diameters (ACI 318-95, Section 7.2.2). This means the bar nearest the side face of the beam usually is not the same diameter as the stirrup bend diameter. For example, using a #3 stirrup, the stirrup bend diameter is 4(0.375) = 1.5 in. All main bars smaller than 1.5 in. diameter will not be tightly cornered by the stirrup. Thus, the decision must be made regarding where the side bar is to be placed when computing a minimum web width table. Two of the possible assumptions are shown in Fig. RE-3. The 1973 edition of this design handbook used the assumption that the bar nearest the side face of the beam was located

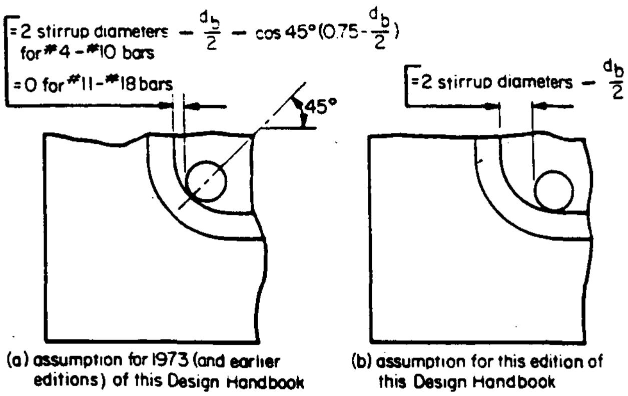


Fig. RE-3—Difference between location of outside bar assumed for 1973 (and earlier) editions of this design handbook and for this edition. Design Aids based on this latter assumption are REINFORCEMENT 9, 10, and 14

half way around the bend of the stirrup as shown in Section (a) of Fig. RE-3. This represented a compromise between (1) those who prior to the 1971 ACI Building Code treated the stirrup as being bent around a diameter equal to that of the bar around which the stirrup was to be placed, and (2) the conservative assumption of Section (b) of Fig. RE-3 that places the main bar so that it contributes the maximum effective depth.

Since around 1963 the ASTM bend requirements for stirrups have remained unchanged; however, the first ACI Building Code to reflect these bend requirements was the 1971 edition, and no change was made by the 1977, 1983 or 1989 ACI Building Code. The maximum difference in minimum width computed by these different procedures equals four stirrup bar diameters minus the main bar diameter. For #6 bars with #3 stirrups, the practical situation where the discrepancy between the 1973 (and pre-1973) and the 1983 (and 1981) design handbooks is the greatest, an additional web width of 0.75 in. will be indicated.

REINFORCEMENT 9 and 10 in this edition of the design handbook have been calculated using the more conservative assumption of Section (b) of Fig. RE-3. The minimum web width for inside layers of bars will be less than indicated in the tables because the bend diameter for the stirrup is not involved. When no stirrup bend is involved, a subtraction (4 stirrup diameters less the main bar diameter) may be made to the table value.

REINFORCEMENT 9 presents minimum web widths for ACI 318-95 requirements for both 3/4 and 1 in. maximum size aggregate, and for AASHTO requirements.

For bars of different sizes, determine from the table the beam web width needed for the given number of larger bars, and then add the indicated increment for each smaller bar.

REINFORCEMENT 10

REINFORCEMENT 10 gives the minimum web widths for various combinations of bars in one row for beam webs for which maximum aggregate size is 3/4 in. Outside bars are assumed to be located as shown in Section (b) of Fig. RE-3. (See discussion under commentary for REINFORCEMENT 9.) Enter table with largest bar size in group and read minimum beam width for entire combination.

REINFORCEMENT 11

REINFORCEMENT 11 gives the maximum bar spacing (concrete width per bar) for single bars in one to three layers, as required for crack control in beam webs or one-way slabs. Values in this table are based on ACI 318-95 Eq. (10-4):

$$z = f_s \sqrt[3]{d_c A}$$

Taking f_s as $0.6f_y$, d_c figured as clear cover of 2 in. to tension reinforcement (= clear cover of 1½ in. plus #4 stirrups), and $A = t(b_w/n)$, and solving for b_w/n gives

$$\frac{b_w}{n} = \frac{\left(\frac{z}{f_s}\right)^3}{d_c t}$$

For one layer of tension bars, $d_c = t/2$. However, since d_c is measured from the extreme tension fiber to the *centroid of the bar located closest thereto* and $t/2$ is measured from the extreme tension fiber to the *centroid of tension reinforcement*, d_c and $t/2$ are not equal when tension reinforcement bars are in more than one layer. Fig. RE-2 illustrates the difference between d_c and $t/2$.

For bars in one layer, $A = 2d_c b_w/n$ and

$$\frac{b_w}{n} = \frac{\left(\frac{z}{f_s}\right)^3}{2d_c t}$$

For bars in two layers with 1 in. spacing between layers, $A = 2(3 + d_b)b_w/n$ and

$$\frac{b_w}{n} = \frac{\left(\frac{z}{f_s}\right)^3}{d_c(5 + 2d_b)}$$

For bars in three layers with 1 in. spacing between layers, $A = 2(b_w/n)(3 + 1.5d_b)$ and

$$\frac{b_w}{n} = \frac{\left(\frac{z}{f_s}\right)^3}{d_c(3 + 1.5d_b)}$$

REINFORCEMENT 12

REINFORCEMENT 12 provides data on minimum beam web widths for various combinations of bundled bars when aggregate size is equal to or less than 3/4 in.

REINFORCEMENT 13

Bundled bars have less surface area in contact with surrounding concrete than the same bars used singly. Therefore, with bundled bars, bond stress level is higher, crack spacing is increased, and cracks are wider than with the same bars used singly.

To account for the reduced contact area of bundled bars, Nawy* has proposed that the value of n in the equation $A = (b_w/n)(t)$ be reduced by the ratio (called k in REINFORCEMENT 13) of the projected surface area of the bundle to the area of the individual bars: It can be shown that

For a bundle of two bars:	$k = 0.815$
For a bundle of three bars:	$k = 0.650$
For a bundle of four bars:	$k = 0.570$

Accordingly REINFORCEMENT 13 has been calculated on the basis that

$$\frac{b_w}{n} = \frac{\left(\frac{z}{f_s}\right)^3 (\text{number of bars per bundle})}{2d_c'}$$

where d_c' is distance from extreme tensile fiber to center of gravity of closest bundle, and units for b_w/n are in. per bundle.

REINFORCEMENT 14

REINFORCEMENT 14 helps to simplify the selection of main reinforcement for beams. It is arranged in increasing values of A_s rather than in the traditional arrangements by bar sizes.

Where feasible, for each A_s value are shown various possibilities for symmetrical arrangement of bars in one, two, or three layers or two or three bundles. With each listing are shown the minimum web widths as governed by placement requirements and the maximum web widths as governed by crack control requirements for $f_y = 60{,}000$ (see Section 10.6 of ACI 318-95).

The table shows from two to twelve bars usually of one size, and generally shows the least number of bars for a particular A_s. Combinations of bar sizes are listed where needed to bridge the large gaps that would exist in A_s values if only one bar size per beam were used. In most cases the change in A_s from one listing to the next does not exceed 6 percent, so that generally selection from this table alone is reasonable and economical.

REINFORCEMENT 15

REINFORCEMENT 15 gives areas of bars in a section 1 ft wide.

REINFORCEMENT 16

REINFORCEMENT 16 gives maximum permissible bar spacing in one-way slabs as governed by

(a) Crack control provisions of ACI 318-95, Section 10.6.4

(b) Requirement that primary flexural reinforcement shall not be spaced farther apart than three times slab thickness, nor 18 in., as given in ACI 318-95, Section 7.6.5 (this requirement governs in the majority of cases).

(c) Requirement that maximum spacing as governed by crack control provisions be not less than minimum spacing required by ACI 318-95, Section 7.6.1. (This consideration governs only in the case of #18 bars of grade 60 steel.)

The crack control provision is expressed in ACI 318-95, Eq. (10-4):

$$z = f_s \sqrt[3]{d_c A}$$

where $z = w/0.076\beta$ from the Gergely-Lutz equation discussed in the Commentary on Section 10.6.4. As the Commentary suggests, the values of z used for beams (that is, 175 for interior exposure and 145 for exterior exposure) are reduced for this table for one-way slabs by (1.2/1.25) for the 3/4 in. clear cover assumed for #3 through #11 bars, and by (1.2/1.35) for #14 and #18 bars.

The maximum spacing is calculated as

$$\text{spacing} = \frac{b_w}{n} = \frac{\left(\frac{z}{f_s}\right)^3}{2(\text{clear cover} + 0.5d_b)^2}$$

where

$$z = \frac{1.2}{1.25}(175) = 168 \text{ for \#3–\#11 bars and interior exposure}$$

$$= \frac{1.2}{1.35}(175) = 156 \text{ for \#14 and \#18 bars and interior exposure}$$

$$= \frac{1.2}{1.25}(145) = 139 \text{ for \#3–\#11 bars and exterior exposure}$$

$$= \frac{1.2}{1.35}(145) = 129 \text{ for \#14 and \#18 bars and exterior exposure}$$

$$f_s = 0.6f_y$$

and clear cover is 3/4 in. for #3–#11 bars and 1 1/2 in. for #14 and #18 bars, the minimum values required by ACI 318-95, Section 7.7.1.

*Edward G. Nawy, "Crack Control in Beams Reinforced with Bundled Bars Using ACI 318-71," ACI JOURNAL, *Proceedings* V. 69, No. 10, Oct. 1972, pp. 637–639.

REINFORCEMENT 17

REINFORCEMENT 17 gives development lengths for straight bars in tension.

REINFORCEMENT 17 presents a ratio of the basic development length and bar diameter of bars in tension calculated according to Section 12.2.2 as

Category I

For #6 and smaller bars $\frac{\ell_d}{d_b} = \frac{f_y \alpha \beta \lambda}{25\sqrt{f'_c}}$

For #7 and bigger bars $\frac{\ell_d}{d_b} = \frac{f_y \alpha \beta \lambda}{20\sqrt{f'_c}}$

Category II

For #6 and smaller bars $\frac{\ell_d}{d_b} = \frac{3 f_y \alpha \beta \lambda}{50\sqrt{f'_c}}$

For #7 and bigger bars $\frac{\ell_d}{d_b} = \frac{3 f_y \alpha \beta \lambda}{40\sqrt{f'_c}}$

Values for development lengths are rounded upward to nearest whole inch.

Category I presents bars with:

clear spacing $\geq d_b$,

clear cover $\geq d_b$,

and stirrups or ties throughout ℓ_d not less than the Code minimum or clear spacing not less than $2d_b$, clear cover not less than d_b.

Category II presents all other cases.

Note that coefficients α, β, and λ are assumed to be 1.0, therefore the values in the table should be multiplied by applicable coefficients according to the ACI 318-95 Section 12.2.4. However, the product $\alpha\beta$ need not be taken greater than 1.7.

Note also that ACI 318-95 Section 12.2.1 requires that ℓ_d for bars in tension shall not be less than 12 in.

The provision previously required by ACI 318-89 of $0.03 d_b f_y / \sqrt{f'_c}$, Section 12.2.3.6 in respect to the minimum development length limit is eliminated since the new equation compensates for it.

REINFORCEMENT 18.1 AND 18.2

REINFORCEMENT 18.1 tabulates basic development length l_{hb} for deformed bars in tension terminating in a standard hook (as described in Section 7.1) calculated according to Sections 12.5.2 and 12.5.3.1 as

$$l_{hb} = 1200\, d_b \frac{f_y}{60{,}000} \Big/ \sqrt{f'_c} \text{ in.}$$

Values of l_{hb} were calculated to three significant figures and rounded to the nearest tenth of an inch. It is expected that users will multiply l_{hb} by the applicable factors from Section 12.5.3.2 through 12.5.3.5 to obtain l_{dh} and round l_{dh} upward to the nearest whole inch. Note that Section 12.5.1 stipulates that l_{dh} shall not be less than 8 d_b (tabulated in the column at the far right of the table) nor less than 6 in.

REINFORCEMENT 18.2 gives minimum embedment lengths where embedment length is the sum of (a) hook extension required by Section 7.1.1, and (b) minimum bend radius required by Section 7.2.1 plus bar diameter, and (c) the customary 2 in. minimum cover to the end of the hook extension. These three dimensions are indicated in the sketch for this Design Aid. For the table, calculated values have been rounded up to the nearest whole inch.

REINFORCEMENT 19

REINFORCEMENT 19 tables aid in determining maximum bar size of positive moment tension reinforcement in beams meeting the requirement of Section 12.11.3 of ACI 318-95. That section provides that at simple supports and at points of inflection, positive movement tension reinforcement shall be limited to a diameter such that development length l_d satisfies

$$l_d \leq \frac{M_n}{V_u}\delta + l_a \qquad \text{from Eq. (12-2) of ACI 318-95}$$

where

- l_d = development length defined by Section 12.2 of ACI 318-95
- M_n = **nominal strength of the positive moment reinforcement embedded past the point of zero moment**
- V_u = **factored shear force at point of zero moment**
- δ = a factor introduced to apply the provision in Section 12.11.3 of ACI 318-95 that "value of M_n/V_u may be increased 30 percent when the ends of reinforcement are confined by a compressive reaction"
- l_a = **the embedment length from the point of zero moment (center of support or inflection point) to the end of the bar, but not greater than the larger of the effective depth *d* or 12 bar diameters.**

Fig. RE-4 illustrates the significance of this equation at a simple support; Fig. RE-5 illustrates its significance at a point of inflection.

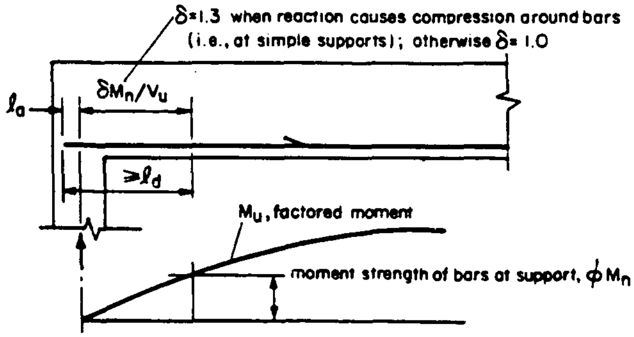


Fig. RE-4—Required development of positive moment tension reinforcement at simple support

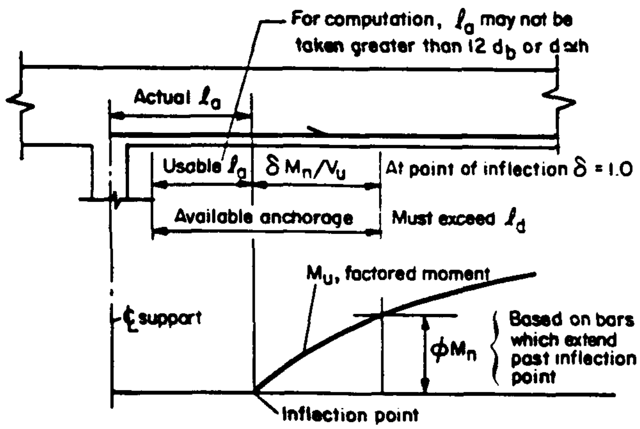


Fig. RE-5—Required development of positive moment tension reinforcement at point of inflection

For REINFORCEMENT 19:

(a) l_d is assumed to be basic development lengths (Section 12.2.2 of ACI 318-95) but not less than 12 in (Section 12.2.1 of ACI 318-95).

(b) l_a *at a support* is assumed to be zero; i.e., reinforcement terminates at the center of suppport. If bars terminate closer to the support face than the center of the support, then the span should be measured to the end of the bars rather than to the center of the support.

(c) l_a *at a point of inflection* is assumed to be the greater of $12d_b$ or the effective depth d calculated as equal to the thickness h as determined from Table 9.5(a) of ACI 318-95 [note that for f_y = 40,000 psi, Table 9.5(a) values must be multiplied by 0.8], but never greater than the distance from the point of inflection to the end of the span (assumed to be $0.15l$ for a continuous end, or $0.10l$ for a discontinuous end that is restrained). The assumptions for l_a mean that the bars extending past the point of inflection must be detailed so they actually extend the larger of $12d_b$ or h [as determined from Table 9.5(a)] beyond that point, but need not extend more than $0.15l$ at continuous ends and $0.10l$ at restrained discontinuous ends for the table values to be valid.

δ = 1.3 when the ends of the reinforcement *are confined* by a compressive reaction (as for a simple support provided by a column below).

δ = 1.0 when the ends of the reinforcement *are not confined* by a compressive reaction (as for a point of inflection).

For convenience in calculating and displaying the table values, each bar size has been assumed and the corresponding minimum permissible span length calculated. However, in practice, the designer knows the span lengths and will use the tables to determine maximum bar size complying with Section 12.11.3 of ACI 318-95. The appropriate situations must be selected from among the sketches on REINFORCEMENT 19. For those situations showing two l's (l_1 and l_5 in an interior bay, or l_2 and l_3 or l_2 and l_4 in an exterior bay) the maximum size reinforcement for each l should be obtained from the table, and bars no larger than the smaller allowable size used. For a discontinuous end bay, the bars at the exterior support are controlled by the span l_2 from the point of zero moment to the end of the span. The bars at the interior support are controlled by the span l_3. For a continuous end bay, similar dual criteria exist. Fig. RE-6 illustrates this point.

For the equations used for REINFORCEMENT 19, Eq. (12-2) of ACI 318-95 is rewritten as

$$\frac{M_n}{V_u} \geq \frac{l_d - l_a}{\delta} \qquad (1)$$

For the tables, the usable strength ϕM_n is assumed equal to the factored load M_u; thus

$$M_n = \frac{M_u}{\phi}$$

Further, it is assumed that

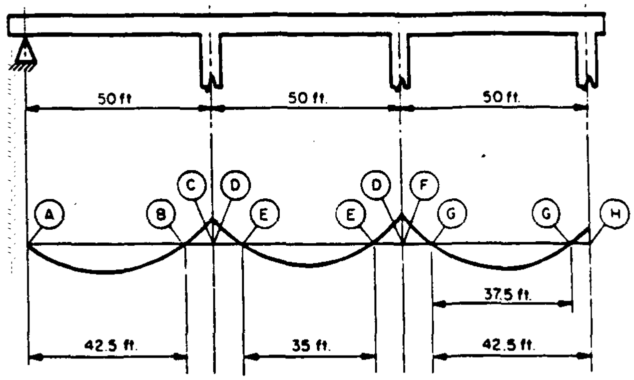
$$M_u = \frac{w_u l^2}{8}$$

where l is the span center-to-center of simple supports, from inflection point to inflection point, or from inflection point to center of a simple support. Also,

$$V_u = \frac{w_u l}{2}$$

Then

$$\frac{M_n}{V_u} = \frac{M_u/\phi}{V_u} = \frac{\dfrac{w_u l^2}{8(0.9)}}{\dfrac{w_u l}{2}} = \frac{l}{3.6}$$



Location	Controlling span	Maximum bar size
A	l_2 = 42.5 ft	#11
B	l_1 = 42.5 ft	#14
C	l_3 = 50 ft	#14
D	l_5 = 50 ft	#11
E	l_1 = 35 ft	#11
F	l_4 = 50 ft	#14
G	l_1 = 37.5 ft	#11
H	l_1 = 42.5 ft	#14

Fig. RE-6—Example illustrating spans l_1, l_2, l_3, l_4, and l_5 and giving maximum bar size (read from REINFORCEMENT 19.1) satisfying Section 12.11.3 of ACI 318-95 at various points of zero positive moment along column line. Maximum permitted bar size is #11. Note that at Location G, use of l_2 assumes inflection point is close enough to support that bars extending past are in fact compressed by the reaction at Location H

Substituting $l/3.6$ in Eq. (1) gives

$$\frac{l}{3.6} \geq \frac{l_d - l_a}{\delta}$$

or

$$l_{min} = \frac{3.6}{\delta}(l_d - l_a) \qquad (2)$$

For simple supports l_a is assumed to be zero. For points of inflection, it is assumed any one of three conditions may control the value to be used for l_a. The three conditions are: (1) $l_a = d$, effective beam depth, taken as approximately h, the minimum thickness from Table 9.5(a) of ACI 318-95. (2) $l_a = 12d_b$; and (3) l_a = the embedment length available between the point of inflection and the end of the span (assumed to be $0.15l$ for a continuous end or $0.10l$ for a discontinuous end restrained). The value of l_a used is the greater of (1) or (2) but not greater than (3).

Table values in REINFORCEMENT 19 for spans in which not all bars extend the full distance between supports are calculated assuming that M_n equals the required strength M_u/ϕ of the proportion of the reinforcement embedded past the point of zero moment.

Five cases, illustrated in Fig. RE-7, are considered in REINFORCEMENT 19.1 and 19.2.

Case 1

Minimum span length l_1 between points of zero moment where the points of zero moment are not confined by a compressive reaction (such as span length between points of inflection). For this case $\delta = 1.0$.

In computing l_1, l_a has been taken equal to $12d_b$. Thus l_1 values are applicable where bars extending to the point of zero moment actually extend at least $12d_b$ *past* that point. Thus, from Eq. (2),

$$l_{1\,min} = 3.6(l_d - 12d_b) \qquad (3)$$

Section 12.11.1 of ACI 318-95 provides that for simple

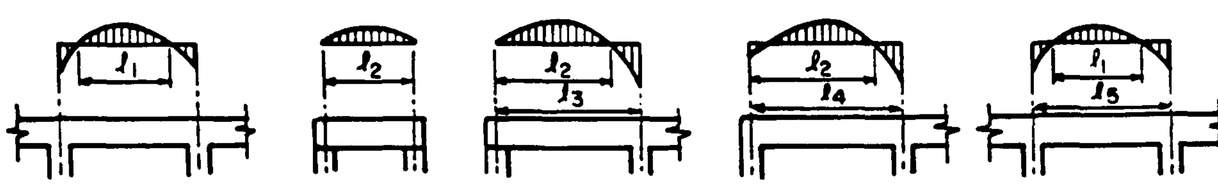


Fig. RE-7—Span lengths considered in REINFORCEMENT 19: l_1 is span between points of inflection in a beam in an interior bay of a continuous span; l_2 is span between points of zero moment in a span in which ends of positive moment reinforcement are confined by a compressive reaction—as for a simply supported span; l_3 is span length in an exterior bay of a continuous span in which the discontinuous end of the span is *unrestrained;* l_4 is span length in an exterior bay of a continuous span in which the discontinuous end of the span is *restrained;* l_5 is span length in an interior bay of a continuous span spans at least one-third of the bars shall extend through the span past the face of support at least 6 in. Eq. (3) assumes 100 percent of the positive moment reinforcement extends past the zero moment point at least $12d_b$.

When only one-third of the reinforcement extends through the zero moment point, it has been assumed that

$$M_n = \frac{1}{3}\frac{M_u}{\phi} = \frac{1}{3}\frac{w_u l_1^2}{8(0.9)} = \frac{w_u l_1^2}{21.6}$$

$$\frac{M_n}{V_u} = \frac{\dfrac{w_u l_1^2}{21.6}}{\dfrac{w_u l_1}{2}} = \frac{l_1}{10.8}$$

and

$$l_1 = 10.8\frac{M_n}{V_u} = \frac{10.8(l_d - 12d_b)}{\delta} = 10.8(l_d - 12d_b)$$

Similarly if one-half of the reinforcement extends through the span,

$$l_1 = 7.2(l_d - 12d_b)$$

Table values for one-half and one-third through bars are calculated for all the support conditions treated; values for one-quarter through bars are also calculated for Cases 3, 4, and 5.

Case 2

Minimum length l_2 between points of zero moment for situations where the ends of the reinforcement are confined by a compressive reaction (as for simply supported spans); thus $\delta = 1.3$.

Using Eq. (2) and conservatively assuming $l_a = 0$ gives for all bars through

$$l_{2\,min} = \frac{3.6}{1.3} l_d = 2.77 l_d$$

For one-third bars through

$$l_{2\,min} = 8.31 l_d$$

and for one-half bars through

$$l_{2\,min} = 5.54 l_d$$

Case 3

Minimum span length l_3 in an exterior bay of a continuous span in which the discontinuous end of the span is *unrestrained*, making $l_3 = l_{min}/0.85$. This case is based on the inflection point controlling and therefore is *without* the confinement by compressive reaction; $\delta = 1.0$.

For this case, using Eq. (2),

$$0.85 l_{3\,min} = 3.6(l_d - l_a)$$
$$l_{3\,min} = 4.24(l_d - l_a)$$

For this case l_a is the greater of (A) $l_{3\,min}/18.5$ for $f_y = 60{,}000$ psi (or $0.8 l_{3\,min}/18.5$ for $f_y = 40{,}000$ psi) from Table 9.5(a) of ACI 318-95 of (B) 12 d_b, but not greater than (C) the assumed $0.15 l_{3\,min}$ to the end of the span.

(A) For $f_y = 60{,}000$ psi, $l_a = d \approx h = \dfrac{l_{3\,min}}{18.5}$

(from Table 9.5(a) of ACI 318-95

$$l_{3\,min} = 4.24\left(l_d - \frac{l_{3\,min}}{18.5}\right) = 3.45 l_d$$

For $f_y = 40{,}000$ psi, $l_a = d \approx h = 0.8\dfrac{l_{3\,min}}{18.5}$

$$l_{3\,min} = 3.58 l_d$$

(B) For $l_a = 12d_b$

$$l_{3\,min} = 4.24(l_d - 12d_b)$$

(C) For $l_a = 0.15 l_{3\,min}$

$$l_{3\,min} = 4.24(l_d - 0.15 l_{3\,min}) = 2.59 l_d$$

The value of $l_{3\,min}$ tabulated is the lesser of $l_{3\,min}$ calculated by (A) or (B) but not less than that calculated by (C).

For one-half bars through, the equations are

(A) $l_{3\,min} = 5.81 l_d$ for $f_y = 60{,}000$ psi
$= 6.20 l_d$ for $f_y = 40{,}000$ psi
(B) $l_{3\,min} = 8.47(l_d - 12d_b)$
(C) $l_{3\,min} = 3.73 l_d$

For one-third bars through, the equations are

(A) $l_{3\,min} = 7.53 l_d$ for $f_y = 60{,}000$ psi
$= 8.20 l_d$ for $f_y = 40{,}000$ psi
(B) $l_{3\,min} = 12.7(l_d - 12d_b)$
(C) $l_{3\,min} = 4.37 l_d$

For one-quarter bars through, the equations are

(A) $l_{3\,min} = 8.83 l_d$ for $f_y = 60{,}000$ psi
$= 9.76 l_d$ for $f_y = 40{,}000$ psi
(B) $l_{3\,min} = 16.9(l_d - 12d_b)$
(C) $l_{3\,min} = 4.78 l_d$

Case 4

Minimum span length l_4 in an exterior bay of a continuous span in which the discontinuous end *is restrained*. The tables assume the inflection points are $0.10 l_4$ from the discontinuous end and $0.15 l_4$ from the continuous end, making $l_4 = l_{min}/0.75$. The case is based on the condition at the inflection point near the discontinuous end, a location when the bars are *not* confined by a compressive reaction: thus, $\delta = 1.0$.

For this case, using Eq. (2),

$$0.75l_{4min} = \frac{3.6}{1.0}(l_d - l_a)$$

giving

$$l_{4min} = 4.8(l_d - l_a)$$

where l_a is taken as the greater of (A) $l_{4\,min}/18.5$ for f_y = 60,000 psi ($0.8l_{4\,min}/18.5$ for f_y = 40,000 psi) from Table 9.5(a) of ACI 318.95 or (B) 12 d_b,but not more than (C) $0.10l_{4\,min}$.

For all bars through, the resulting equations are

(A) $l_{4\,min}$ = 3.81 l_d for f_y = 60,000 psi
= 3.97l_d for f_y = 40,000 psi
(B) $l_{4\,min}$ = 4.80 (l_d – 12d_b)
(C) $l_{4\,min}$ = 3.24l_d

and $l_{4\,min}$ is taken as the lesser of (A) or (B) but not less than (C).

For one-half bars through, the equations are

(A) $l_{4\,min}$ = 6.32l_d for f_y = 60,000 psi
= 6.78l_d for f_y = 40,000 psi
(B) $l_{4\,min}$ = 9.60 (l_d – 12d_b)
(C) $l_{4\,min}$ = 4.90l_d

For one-third bars through, the equations are

(A) $l_{4\,min}$ = 8.10l_d for f_y = 60,000 psi
= 8.87l_d for f_y = 40,000 psi
(B) $l_{4\,min}$ = 14.4(l_d – 12d_b)
(C) $l_{4\,min}$ = 5.90l_d

For one-quarter bars through, the equations are

(A) $l_{4\,min}$ = 9.42l_d for f_y = 60,000 psi
= 10.49l_d for f_y = 40,000 psi
(B) $l_{4\,min}$ = 19.2 (l_d – 12d_b)
(C) $l_{4\,min}$ = 6.58l_d

Case 5

Minimum span length l_5 in an interior bay of a continuous span. Here the inflection points are assumed to be located at $0.15l_5$ from each end of the span; thus $l_5 = l_{min}/0.70$. The inflection points do not have the bars confined by a compressive reaction; therefore, $\delta = 1.0$.

For this case, using Eq. (2),

$$0.70l_{5min} = \frac{3.6}{1.0}(l_d - l_a)$$

$$l_{5min} = 5.14(l_d - l_a)$$

where l_d is taken as the greater of (A) $l_{5\,min}/21$for f_y = 60,000 psi ($0.8l_{5\,min}/21$ for f_y = 40,000 psi) from Table 9.5(a) of ACI 318-95 or (B) 12 d_b, but not more than (C) $0.15l_{5\,min}$.

For all bars through, the resulting equations are

(A) $l_{5\,min}$ = 4.30l_d for f_y = 60,000 psi
= 4.13l_d for f_y = 40,000 psi
(B) $l_{5\,min}$ = 5.14(l_d - 12d_b)
(C) $l_{5\,min}$ = 2.90l_d

For one-half bars through, the equations are

(A) $l_{5\,min}$ = 6.90l_d for f_y = 60,000 psi
= 7.39l_d for f_y = 40,000 psi
(B) $l_{5\,min}$ = 10.28(l_d – 12d_b)
(C) $l_{5\,min}$ = 4.04l_d

For one-third bars through, the equations are

(A) $l_{5\,min}$ = 8.88l_d for f_y = 60,000 psi
9.71l_d for f_y = 40,000 psi
(B) $l_{5\,min}$ = 15.4(l_d – 12d_b)
(C) $l_{5\,min}$ = 4.65l_d

For one-quarter bars through, the equations are

(A) $l_{5\,min}$ = 10.39l_d for f_y = 60,000 psi
= 11.53l_d for f_y = 40,000 psi
(B) $l_{5\,min}$ = 20.56(l_d – 12d_b)
(C) $l_{5\,min}$ = 5.03l_d

For f_y = 40,000 psi, application of the equations given above for Cases 1-5 yields larger minimum span lengths for #4 bars than for #5 bars—or larger minimum span lengths for #4 and #5 bars than for #6 bars. Where this occurs in REINFORCEMENT 19.1, the calculated span lengths are enclosed in parentheses. These values result from an anomaly in ACI 318-95, Section 12.11.3; while the values may not be logical, they are conservative.

REINFORCEMENT 20

REINFORCEMENT 20.1 gives maximum allowable spiral pitch for circular spiral columns; REINFORCEMENT 20.2 gives maximum allowable spiral pitch for square spiral columns. Both tables are based on the definition of spiral reinforcement ratio ρ_s (Section 10.0 of ACI 318-95) and on Eq. (10-5). Equations used in calculating these tables are derived in this manner:

$$\rho_s = \frac{volume\ of\ reinforcement}{volume\ of\ core}$$

$$= \frac{A_{sp} \times length\ of\ one\ 360°\ loop\ of\ spiral}{\pi\left(\frac{1}{2}h_{core}\right)^2 \times s}$$

where

h_{core} = core diameter, in.
A_{sp} = area of bar from which spiral is formed, in^2.
s = spiral pitch, in.

Therefore,

$$\rho_s = \frac{A_s\sqrt{\left[\pi(h_{core} - d_{sp})\right]^2 + s^2}}{\pi\left(\frac{1}{2}h_{core}\right)^2 \times s}$$

where d_{sp} = diameter of bar from which spiral is formed, in.

Also,

$$\rho_s \geq 0.45\left(\frac{A_g}{A_c} - 1\right)\frac{f_c'}{f_y}$$

where A_c = area of core of spirally reinforced column, in.^{2}, or

$$\rho_s \geq 0.45\left(\frac{A_g}{\pi\left(\frac{1}{2}h_{core}\right)^2} - 1\right)\frac{f_c'}{f_y}$$

Therefore,

$$\frac{A_s\sqrt{\left[\pi(h_{core} - d_{sp})\right]^2 + s^2}}{\frac{\pi}{4}(h_{core})^2 \times s} \geq 0.45\left(\frac{A_g}{\frac{\pi}{4}(h_{core})^2} - 1\right)\frac{f_c'}{f_y}$$

and solving for s, s equals

$$\sqrt{\frac{\pi^2(h_{core} - dsp)^2}{\left(\frac{0.45\frac{\pi}{4}h_{core}^2}{A_{sp}}\right)^2\left(\frac{A_g}{\frac{\pi}{4}h_{core}^2} - 1\right)^2\left(\frac{f_c'}{f_y}\right)^2 - 1}}$$

For circular spiral columns,

$$s = \sqrt{\frac{\pi^2(h_{core} - d_{sp})^2}{\left(\frac{0.1125\ \pi}{A_{sp}}\right)^2(h^2 - h_{core}^2)^2\left(\frac{f_c'}{f_y}\right)^2 - 1}}$$

and for square spiral columns,

$$s = \sqrt{\frac{\pi^2(h_{core} - d_{sp})^2}{\left(\frac{0.45\ \pi}{A_{sp}}\right)^2\left(h^2 - \frac{\pi}{4}h_{core}^2\right)^2\left(\frac{f_c'}{f_y}\right)^2 - 1}}$$

However, the clear spacing between spirals-that is, the pitch s minus the spiral diameter d_{sp}-may not exceed 3 in. or be less than 1 in. (Section 7.10.4.3 of ACI 318-95). REINFORCEMENT 20.1 and 20.2 reflect this limitation and also the limitation that the nominal maximum size of the aggregate shall not be larger than three-fourths of the minimum clear spacing (Section 3.3.3(c) of ACI 318-95).

(REINFORCEMENT 20.2 does not tabulate values for S8-60 columns - and no interaction diagrams or basic limits tables are given for S80-60 columns in this volume - because for all column and spiral sizes listed, the maximum allowable pitch would result in clear spacing less than the required minimum of 1 in.)

Because it is customary to specify spiral pitch to the 1/4 in., values for spiral pitch in the tables have been rounded down to the nearest 1/4 in.

REINFORCEMENT 20.3 tabulates ACI 318R-95 recommendation for number of spacers for spirals, if spacers are to be used to hold spirals firmly in place, at proper pitch and alignment, to prevent displacement during concrete placement.

However, while ACI 318R-95 does require that spirals be held firmly in place and true to line, the 1995 code does not require that this be accomplished by installation of spacers, as earlier versions of the code did.

REINFORCEMENT 21, 22, AND 23

These tables are based on the requirements that

(a) Minimum concrete cover for reinforcement shall be 1-1/2 in. for columns [Section 7.7.1(c) of ACI 318-95]

(b) Clear distance between longitudinal bars shall not be less than 1-1/2 bar diameters of 1-1/2 in. (Section 7.6.3 of ACI 318-95)-that is, clear distance between bars shall be 1-1/2 in. for #3-#8 bars and 1-1/2d_b for #25-#55 bars.

(c) Clear distance limitation between bars applies also to clear distance between a contact lap splice and adjacent splices or bars (Section 7.6.4 of ACI 318-95)

(d) Minimum number of bars in a column shall be four for bars within rectangular or circular ties and six for bars enclosed by spirals (Section 10.9.2 of ACI 318-95)

(e) Reinforcement ratio ρ_g for columns shall not be less than 0.01 nor more than 0.08 (Section 10.9.1 of ACI 318-95)

The tables are calculated for #3 ties or spirals (d_b = 0.500 in.) A column which will accommodate a certain number of bars with #4 ties or spirals will, of course, accommodate at least that number with #3 ties (d_b = 0.375 in.).

REINFORCEMENT 21 gives minimum column size for various quantities of bars per face for rectangular columns having bars arranged along four sides or two sides of the rectangle with (1) bearing splices, (2) normal lap splices, and (3) tangential lap splices. These splices are illustrated in Fig. 10.

For bearing splices it can be seen from inspection of Fig. 10, if required bend diameters of ties and deformation of bars are neglected, that for #3-#8 bars:

$$b_1 = 2(cover + tie\ diameter) + nd_b + (n - 1)\left(1\frac{1}{2}\right),\ in.$$

and therefore for 1-1/2 in. cover and #3 ties (d_b = 0.500 in.).

$$b_1 = 2.5 + n\ (d_b + 1\text{-}1/2),\ in.$$

for #8-#18 bars:

$$b_1 = 2(cover + tie\ diameter) + nd_b + (n - 1)\left(1\frac{1}{2}d_b\right)\ in.$$

and therefore

$$b_1 = 4 + \left(2\frac{1}{2}n - 1\frac{1}{2}\right)d_b,\ in.$$

For normal lap splices, the width b_2 of the column section must be slightly larger than width b_1 for bearing splices because of the position of the splice bars nearest the corners. This is shown in Fig. 11, from which it can

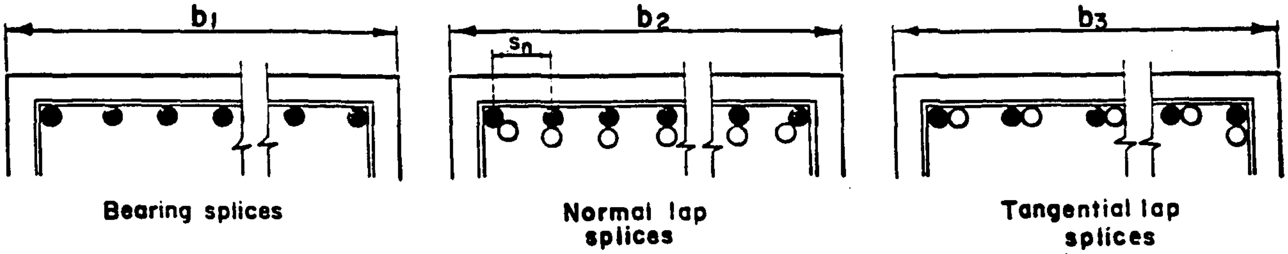


Fig. 10 - Three types of splices for which REINFORCEMENT 21, 22, and 23 give data

be determined that for #3-#8 bars:

$$b_2 = b_1 + 2\left[(s_n - d_b) - 1\frac{1}{2}\right]$$

$$s_n = \left(1\frac{1}{2} + d_b\right)\cos\theta + \sqrt{\frac{1}{2}}\, d_b$$

where s_n = spacing between the two bars nearest the corner of the section, in., as shown in Fig. 10 and 11. Subtracting d_b from both sides of the above equation gives

$$s_n - d_b = \left(1\frac{1}{2} + d_b\right)\cos\theta + \sqrt{\frac{1}{2}}d_b - d_b$$

$$= \left(1\frac{1}{2} + d_b\right)\cos\theta + \left(\sqrt{\frac{1}{2}} - 1\right)d_b$$

Substituting this latter expression for $(s_n - d_b)$ in the equation for b_2 gives

$$b_2 = b_1 + 2\left[\left(1\frac{1}{2} + d_b\right)\cos\theta + \left(\sqrt{\frac{1}{2}} - 1\right) \times d_b - 1\frac{1}{2}\right]$$

$$= b_1 + [(3 + 2d_b)\cos\theta - 0.586d_b - 3\ ]$$

where

$$\theta = \arcsin \frac{\left(1 - \sqrt{\frac{1}{2}}\right)d_b}{1\frac{1}{2} + d_b}$$

or #8-#18 bars:

$$b_2 = b_1 + 2\left[(s_n - d_b) - 1\frac{1}{2}d_b\right]\ in.$$

$$s_n = 2\frac{1}{2}\, d_b \cos\theta + \sqrt{\frac{1}{2}}\, d_b\ in.$$

$$s_n - d_b = 2\frac{1}{2}\, d_b \cos\theta \left(\sqrt{\frac{1}{2}} - 1\right)d_b\ in.$$

$$b_2 = b_1 + 2\left(2\frac{1}{2}d_b \cos\theta + \left(\sqrt{\frac{1}{2}} - 2\frac{1}{2}\right)d_b\right)\ in.$$

$$= b_1 + 5d_b \cos\theta - 3.586d_b\ in.$$

where

$$\theta = \arcsin \frac{\left(1 - \sqrt{\frac{1}{2}}\right)d_b}{2.5d_b} = \arcsin 0.1172$$

$$\theta = 6.73\ deg$$

$$\cos\theta = 0.9931$$

and therefore

$$b_2 = b_1 + 1.380d_b\ in.$$

For tangential lap splices it can be seen from inspection of Fig. 10 that for #3-#8 bars:

$$b_3 = 2(cover + tie\ diameter) + (2n - 1)d_b + (n - 1)\left(1\frac{1}{2}\right)\ in.$$

and therefore

$$b_3 = 2\frac{1}{2} + 1\frac{1}{2}\, n + (2n - 1)d_b\ in.$$

For #8-#18 bars:

$$b_3 = 2(cover + tie\ diameter) + (2n - 1)d_b + (n - 1)1\frac{1}{2}d_b\ in.$$

and therefore

$$b_3 = 4 + \left(3\frac{1}{2}n - 2\frac{1}{2}\right)d_b\ in.$$

$$b_3 = 4 + \left(3\frac{1}{2}n - 2\frac{1}{2}\right)d_b \ in.$$

For REINFORCEMENT 21,, all column sizes are rounded upward to the nearest ½ in.

For REINFORCEMENT 22, the equations used for REINFORCEMENT 21 are solved for the number of bars per face n (rounded downward to the nearest integer) that can be accommodated in a column of given size and the resulting number of bars per column computed. Then area A_{st} of the bars is computed as $A_{st} = n \times A_b$ (where n = number of bars per column), area of gross column cross section A_g is computed as $A_g = h^2$, and ρ_g is computed as $\rho_g = A_{st}/A_g$.

REINFORCEMENT 23 gives the number of bars that can be accommodated in a circular or a square column in which the bars are arranged in a circle. The calculations are based on these equations:

For bearing splices (Fig. 12):

$$\sin\frac{1}{2}\left(\frac{360}{n}\right) =$$

$$= \frac{\frac{1}{2}s}{\frac{1}{2}h - (cover + spiral\ diameter) - \frac{1}{2}d_b}$$

For #40-#8 bars:

$$s = 1\frac{1}{2} + d_b \ in.$$

and therefore for 1-1/2 in. cover and #3 spirals:

$$n = \frac{180}{\arcsin\left(\frac{1\frac{1}{2} + d_b}{h - 4 - d_b}\right)}$$

For #8-#18 bars:

$$s = 2\frac{1}{2}\ d_b \ in.$$

$$n = \frac{180}{\arcsin\left(\frac{2\frac{1}{2}d_b}{h - 4 - d_b}\right)}$$

For normal lap splices (Fig. 13)

$$\sin\frac{1}{2}\left(\frac{360}{n}\right) =$$

$$= \frac{1}{2}s \Bigg/ \left[\frac{1}{2}h - (cover + spiral\ diameter)\right] - 1\frac{1}{2}d_b$$

For #3-#8 bars:

$$s = 1\frac{1}{2} + d_b$$

$$\sin\frac{1}{2}\left(\frac{360}{n}\right) = \frac{(40 + d_b)/2}{\frac{1}{2}h - 51.3 - 1\frac{1}{2}d_b}$$

$$n = \frac{180}{\arcsin\left(\frac{40 + d_b}{h - 102.6 - 3d_b}\right)}$$

For #8-#18 bars:

$$s = 2\frac{1}{2}d_b$$

$$\sin\frac{1}{2}\left(\frac{360}{n}\right) = \frac{\frac{1}{2}\left(2\frac{1}{2}d_b\right)}{\frac{1}{2}h - 2 - 1\frac{1}{2}d_b}$$

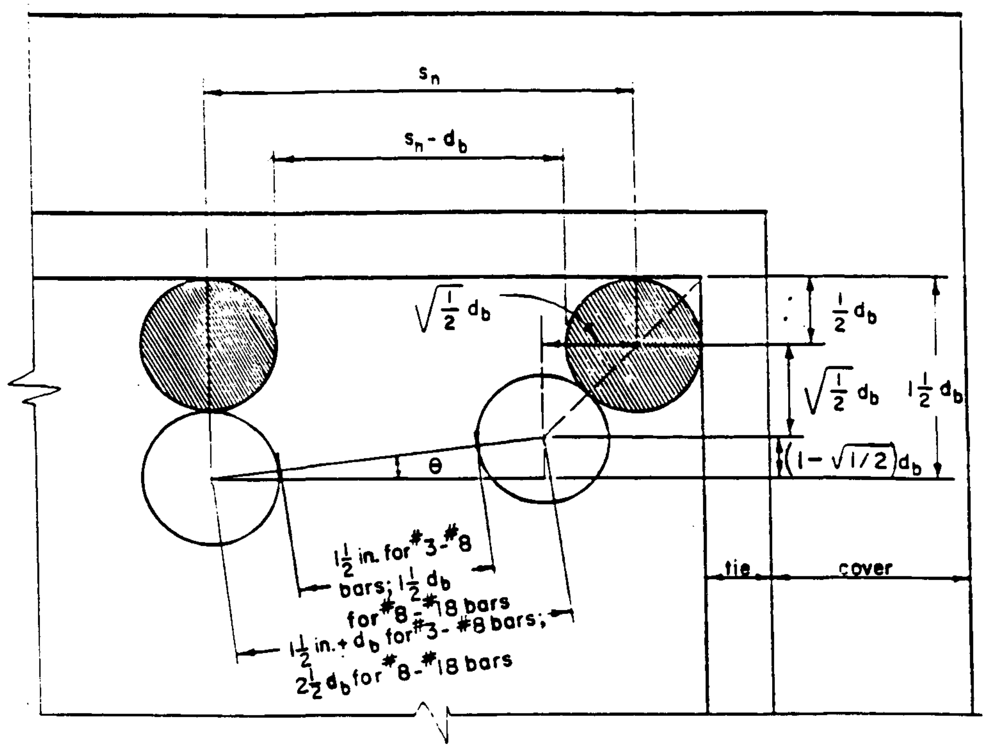


Fig. 11—Dimensions and angles used in deriving equations for normal lap case for REINFORCEMENT 21 and 22.2

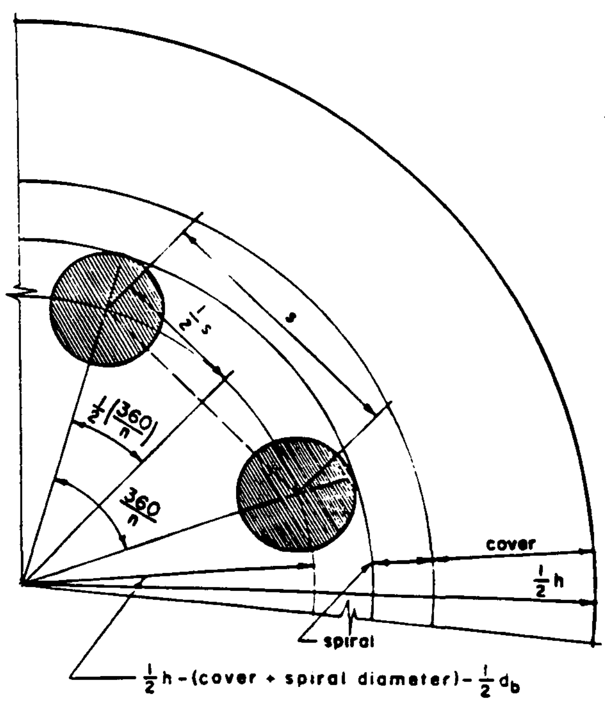


Fig. 12—Dimensions and angles used in deriving equations for bearing splice case for REINFORCEMENT 23.1

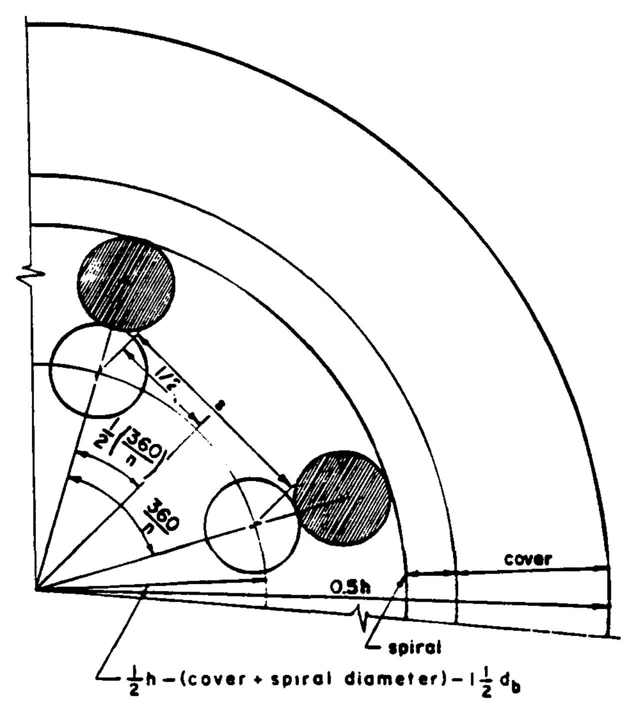


Fig. 13—Dimensions and angles used in deriving equations for normal lap splice case for REINFORCEMENT 23.2

$$n = \frac{180}{\arcsin\left(\dfrac{2\frac{1}{2}d_b}{h - 4 - 3d_b}\right)}$$

For tangential lap splices (Fig. 14)

$$\sin\frac{\theta}{2} = \frac{\frac{1}{2}d_b}{\frac{1}{2}h - (cover + spiral\ diameter) - \frac{1}{2}d_b}$$

$$\sin\frac{1}{2}\left(\frac{360}{n} - \theta\right) = \sin\left(\frac{180}{n} - \frac{\theta}{2}\right)$$

$$= \frac{\frac{1}{2}s}{\frac{1}{2}h - (cover + spiral\ diameter) - \frac{1}{2}d_b}$$

$$\frac{\theta}{2} = \arcsin\left(\frac{d_b}{h - 4 - d_b}\right)$$

$$\frac{180}{n} - \frac{\theta}{2} = \arcsin\left(\frac{s}{h - 4 - d_b}\right)$$

$$n = \frac{180}{\arcsin\left(\dfrac{s}{h - 4 - d_b}\right) + \arcsin\left(\dfrac{d_b}{h - 4 - d_b}\right)}$$

For #3-#8 bars:

$$s = 1\frac{1}{2} + d_b\ in.$$

and therefore

$$n = \frac{180}{\arcsin\left(\dfrac{1.5 + d_b}{h - 4 - d_b}\right) + \arcsin\left(\dfrac{d_b}{h - 4 - d_b}\right)}$$

For #8-#18 bars:

$$s = 2\frac{1}{2}d_b\ in.$$

and therefore

$$n = \frac{180}{\arcsin\left(\dfrac{2.5\,d_b}{h - 4 - d_b}\right) + \arcsin\left(\dfrac{d_b}{h - 4 - d_b}\right)}$$

For REINFORCEMENT 23, n is, of course, rounded downward to the nearest integer.

COMMENTARY ON DESIGN AIDS FOR SHEAR STRENGTH OF BEAMS AND SLABS

Design aids for shear are drawn for normal weight concrete. Some of these design aids can be adopted for use with lightweight concrete by multiplying f_c' by the factor:

$$\lambda = \frac{f_{ct}^2}{6.7^2 f_c'} \le 1.0$$

where f_{ct} is the average splitting tensile strength of the lightweight concrete.

SHEAR 1

In this design aid, values of $V_n / (b_w d)$ are plotted against f_c'. The design aid summarizes the stirrup design requirements of a beam with vertical stirrups and identifies five different regions for shear design:

(1) When $V_n / (b_w d)$ is less than $\sqrt{f_c'}$, stirrups are not required.

(2) Minimum stirrups are needed when $V_n / (b_w d)$ lies between $\sqrt{f_c'}$ and $\left(2\sqrt{f_c'}+50\right)$ psi. The spacing of these stirrups must satisfy following equations:

$$s \le \frac{A_v f_y}{50\ b_w}$$
$$s \le d/2$$
$$s \le 24\ in$$

(3) When $V_n / (b_w d)$ lies between $\left(2\sqrt{f_c'}+50\right)$ and $6\sqrt{f_c'}$, normal stirrups are needed and the spacing of such stirrups is calculated from:

$$s \le \frac{A_v f_y d}{V_s}$$
$$s \le d/2$$
$$s \le 24\ in$$

(4) Closely-spaced stirrups are required if $V_n / (b_w d)$ lies between $6\sqrt{f_c'}$ and $10\sqrt{f_c'}$. The spacing of these stirrups is calculated from:

$$s \le \frac{A_v f_y d}{V_s}$$
$$s \le d/4$$
$$s \le 12\ in$$

(5) The size of the cross section must be increased when $V_n / (b_w d)$ is greater than $10\sqrt{f_c'}$.

SHEAR 2

SHEAR 2 provides means of obtaining stirrup spacing for strength design, using triangular and trapezoidal shear diagram. The value

$$V_s = V_n - V_c$$

must be calculated at he critical section and at break points an the trapezoid. The distance where $V_s = 0$ must also be determined in feet. The value β_v and K_v $(= A_v f_y)$ can be obtained from the small tables at the top of SHEAR 2.

At the critical section and at the break points, the value

$$\frac{V_s}{\beta_v K_v d}$$

must be calculated.

A diagram of $V_s/(\beta_v K_v d)$ versus distance in feet may be plotted on the tracing paper using the same scale that is used for SHEAR 2. The plotted diagram is then placed on top of SHEAR 2 and the required stirrup spacing is read directly for every intersection of the $V_s/(\beta_v K_v d)$ diagram with a vertical line on SHEAR 2.

SHEAR 3

SHEAR 3 simplifies the determination of the minimum beam height needed to provide embedment required by ACI 318-95 Section 12.13.2.2 for #6, #7 and #8 stirrups with f_y equal to 60,000 psi. This section requires (1) a standard stirrup hook (defined in Section

7.1.3) bent around a longitudinal bar plus (2) an embedment between mid-height of the member and the outside end of the hook equal to or greater than

$$0.014\,d_b f_y / \sqrt{f_c'}\ .$$

For SHEAR 3, cover is assumed to be 1.5 in. and therefore required embedment is $h/2$ - 1.5 and

$$h/2 - 1.5 \geq \frac{0.014\,d_b f_y}{\sqrt{f_c'}}$$

or

$$\text{minimum}\quad h \geq \frac{0.028\,d_b f_y}{\sqrt{f_c'}} + 3.0\ ,\ \text{in.}$$

SHEAR 3 is calculated by this later equation.

For stirrups of #5 bar and D31 wire, and smaller, and of #6, #7, and #8 bars with f_y of 40,000 psi or less, full depth stirrups containing a standard hook are required, but are satisfactory without extra embedment between midheight and the outside end of the hook.

SHEAR 4

The shear strength of #3 and #4 U-stirrups is shown in SHEAR 4 for various combinations of values of d and s based on Eq. (11-16):

$$V_s = \frac{A_v f_y d}{s}$$

The shear strength V_s from stirrups can be added directly to the shear strength V_c from concrete when the total nominal strength V_n is desired.

The bottom line of each table shows maximum beam width above which minimum stirrup requirements of Section 11.5.5.3 are not met. Maximum values of b_w were calculated using Eq. (11-13) solved for b_w.

$$b_w = \frac{A_v f_y}{50\,s}$$

SHEAR 5

SHEAR 5 provides the effective depth d of a slab of uniform thickness required to resist a given shear load about an interior column or capital when no shear reinforcement is provided. SHEAR 5.1 is for rectangular supports of sides b and h. SHEAR 5.2 is for circular supports of diameter h.

If V_u is the factored shear force to be transmitted without shear reinforcement across a section located within a distance $d/2$ from the perimeter of the concentrated load or reaction area, then V_u/ϕ must be equal to or less than $V_n = V_c$, the nominal shear strength of concrete. Section 11.12.2.1 gives the nominal shear strength attributable to the concrete as

$$V_c = \left(2 + \frac{4}{\beta_c}\right)\sqrt{f_c'}\,b_o d \quad (11\text{-}36)$$

where b_o is the perimeter of the critical section and β_c is the ratio of long side to short side of concentrated load reaction area ($= h / b$).

Section 11.12.2.1 also indicates that V_c cannot be greater than $4\sqrt{f_c'}\,b_o d$. For maximum V_c the factor $(2 + 4 / \beta_c)$ in the code equation must be equal to 4. Then $2 + 4 / \beta_c = 4$, $2 + 4 / (h / b) = 4$, $4 / (h / b) = 2$, and the maximum value of h / b is 2. This is the reason that $h / b \leq 2$ for SHEAR 5.1.

Assuming the maximum permitted value of concrete shear strength, the equation becomes

$$V_c = 4\sqrt{f_c'}\,b_o d$$

and since $V_u / \phi = V_c$ when shear reinforcement is not used,

$$\frac{V_u}{\phi} = V_n = V_c = 4\sqrt{f_c'}\,b_o d$$

Substituting $b_o = 2(b + h + 2d)$ for rectangular support, we obtain (including the λ factor to make it also applicable for lightweight concrete)

$$V_c = (4)\sqrt{\lambda f_c'}(2)(b + h + 2d)d$$

$$= 8\sqrt{\lambda f_c'}(b + h + 2d)d$$

$$\frac{V_c}{8d\sqrt{\lambda f_c'}} = (b + h + 2d)$$

$$b + h = \frac{V_c}{8d\sqrt{\lambda f_c'}} - 2d$$

The final relationship forms the basis for the curves of SHEAR 5.1.

By a similar process for circular support, where $b_o = \pi (h + d)$, we obtain

$$V_c = 4\sqrt{\lambda f_c'}[\pi(h+d)]d$$

$$= 4\pi d\sqrt{\lambda f_c'}(h+d)$$

$$(h+d) = \frac{V_c}{4\pi d\sqrt{\lambda f_c'}}$$

$$h = \frac{V_c}{4\pi d\sqrt{\lambda f_c'}} - d$$

The final relationship forms the basis for the curves of SHEAR 5.2 for circular supports.

Section 11.12.2.1 (b) states that

$$V_c = \left(\frac{\alpha_s d}{b_o} + 2\right)\sqrt{f_c'}\,b_o d \quad \text{ACI 318-95 Eq. (11.37)}$$

in which $\alpha_s = 40$ for interior columns. Substituting this value of α_s in the above-noted equation yields

$$V_c = \left(\frac{40d}{b_o} + 2\right)\sqrt{f_c'}\,b_o d$$

Equating this equation to the limiting shear strength of concrete

$V_c = 4\sqrt{f_c'}\,b_o d$ gives $[(40d / b_o) + 2] = 4$, or $(40d / b_o) = 2$, or $40d = 2\,b_o$ or $20d = b_o$.

For rectangular columns, b_o is equal to $2(b + h) + 4d$. Therefore, for concrete stress reaching the limiting shear stress of $4\sqrt{f_c'}$, $2(b + h) + 4d$ equals $20d$, or $2(b + h) = 16\,d$, or $b + h = 8\,d$.

Similarly, for circular columns, b_o is equal to $\pi(h + d)$. Hence, $\pi(h + d) = 20\,d$, or $\pi h = d(20 - \pi)$, or $h = d(20 - \pi)/\pi$, $h = 5.37\,d$ when the concrete stress equals the limiting shearing stress of $4\sqrt{f_c'}$. The limiting curve representing $b + h = 8d$ for rectangular columns is plotted in SHEAR 5.1 and that representing $h = 5.37d$ for circular columns is shown in SHEAR 5.2. Note the values in SHEAR 5.1 and 5.2 above limiting curves are controlled by the concrete shear stress $= 4\sqrt{f_c'}$ and those below the limiting curves are controlled by a concrete shear stress $< 4\sqrt{f_c'}$.

SHEAR 6

SHEAR 6, in its lower group of curves (Lines A), shows values of nominal torsional moment

$$T_n = \sqrt{f_c'}\left(\frac{A_{cp}^2}{p_{cp}}\right)$$

for concrete strengths from 3000 to 6000 psi over range of values of A^2_{cp} / p_{cp}. According to Section 11.6.1, torsional moment, T_u / ϕ, lover than this value may be neglected.

The upper group of curves (Lines B) in SHEAR 6 relates to Section 11.6.2.2. This section permits design for a value of maximum nominal torsional moment T_n in cases of statically indeterminate torsion. The ACI 318-95 prescribed design strength T_n is intended to provide adequate torsional ductility so that redistribution of torsional moment in the member can occur. T_n may arbitrarily be taken as

$$T_n = 4\sqrt{f_c'}\left(\frac{A_{cp}^2}{p_{cp}}\right);$$

the upper curves show these T_n values for concrete strengths from 3000 to 6000 psi over range of values of A^2_{cp} / p_{cp}.

SHEAR 7

SHEAR 7 establishes a relationship between the width and the height of the section, providing numerical values K_{vc}, K_{vs}, K_{tcr}, K_{ts}, and K_t, for different f_c' and f_y.

In this design aid the strength reduction factor ϕ was assumed to be 1.0; therefore, in calculations, the table values should be multiplied by the proper ϕ.

The table value K_{vc} is used to determine whether shear reinforcement is required.

Shear reinforcement is required if:

$$V_u > \phi\sqrt{f_c'}\, b_w d$$

Setting $K_{vc} = \dfrac{\sqrt{f_c'}\, b_w d}{500}$ *(kips)* implies that,

if $V_u > 0.5\phi K_{vc}$, then stirrups are required.

To determine whether torsional reinforcement is required, the value K_{tcr} is used. ACI 318-95 requires providing torsional reinforcement if

$$T_u > \phi\sqrt{f_c'}\left(\frac{A_{cp}^2}{p_{cp}}\right)$$

Using $K_{tcr} = \dfrac{\sqrt{f_c'}}{3000}\left(\dfrac{A_{cp}^2}{p_{cp}}\right)$ $(k\text{-}ft)$

simplifies this process; where A_{cp} and P_{cp} are the gross area and perimeter of the section as shown in Figure 7.1.

If $T_u > 0.25\phi K_{tcr}$, then torsional reinforcement is required.

The values K_t and K_{vc} are used to determine if the section is adequate. ACI 318-95, Section 11.6.3.1, Eq. (11-18) requires an increase in the dimensions of the cross section if the condition below is not satisfied.

$$\sqrt{\left(\frac{V_u}{b_w d}\right)^2 + \left(\frac{T_u\, p_h}{1.7A_{oh}^2}\right)^2} \le \phi\left(\frac{V_c}{b_w d} + 8\sqrt{f_c'}\right)$$

Using K_t and K_{vc}, where:

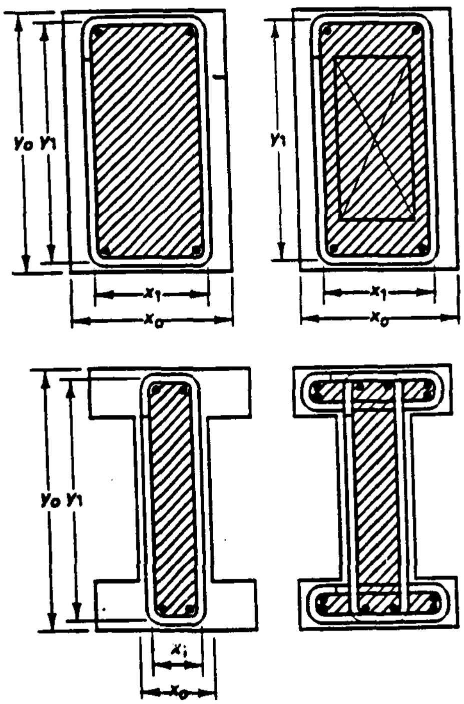
$$K_{vc} = \frac{\sqrt{f_c'}\, b_w d}{500} \quad (kips)$$

$$K_t = \frac{17A_{oh}^2\sqrt{f_c'}}{12000p_h} \quad (k\text{-}ft)$$

A_{oh} and P_h are the area and the perimeter of the section measured from center of the outermost transverse reinforcement:

$$\text{If} \quad \sqrt{\left(\frac{V_u}{5\phi K_{vc}}\right)^2 + \left(\frac{T_u}{\phi K_t}\right)^2} \le 1$$

then section is adequate.



$A_{cp} = x_o * y_o$ $\quad A_o = 0.85A_{oh}$
$A_{oh} = x_1 y_1$ = shaded area to center line of stirrups
$P_{cp} = 2(x_o+y_o)$ $\quad P_h = 2(x_1+y_1)$
Note: All stirrups should be closed.

Figure 7.1 Torsional geometric parameters.

SHEAR 7 also provides the means for calculating the area of transverse reinforcement for shear and torsion.

According to ACI 318-95, Sections 11.6.3.6 and 11.6.3.8, $A_{vt}/s = (A_v + 2A_t)/s$. Therefore, using the table values in SHEAR 7:

$$K_{vs} = f_{yv}d \text{ (kips/in.)}$$

$$K_{ts} = \frac{A_o f_{yv} \cot\theta}{12} \text{ (k-ft/in.)}$$

and the area of closed stirrups per spacing A_{vr}/s can be found as

$$\frac{V_u - \phi K_{vc}}{\phi K_{vs}} - \frac{T_u}{\phi K_{ts}}$$

where θ is taken as 45 degrees. $A_o = 0.85\ A_{oh}$ (see figure 7.1), and $(V_u - \phi K_{vc})$ must be greater than zero.

Also, to satisfy the minimum transverse reinforcement allowable by ACI 318-95, Section 11.6.5.2, Eq. (11-23) $A_v/s - 2A_t/s \geq 50b_w/f_{yv}$; the value K_{ct} is used. If $K_{ct} \leq 50b_w/f_{yv}$; then K_{ct} should be taken as $50b_w/f_{yv}$.

The same value K_{ct} is used for computing stirrup spacings according ACI 318-95, Section 11.6.6.1 $s = 2A_{bar}/K_{ct}$, where $s \leq P_h/8 = (b + h - 7)/4 \leq 12$ in. where $(b + h - 7)$ is equal to half the perimeter of the section measured from the center of the transverse reinforcement $(P_h/2)$, assuming concrete cover of 1.75 in. to the center of transverse reinforcement. Note that spacing limits of section 11.5.4 should also be satisfied.

The values for K_{ts} can also be used for computing the area of torsional longitudinal reinforcement. ACI 318-95, Section 11.6.3.7, Eq. (11-22), specifies:

$$A_\ell = \frac{A_t}{s} p_h \left(\frac{f_{yv}}{f_{yl}}\right) \cot^2\theta$$

Using the values for K_{ts} in SHEAR 7, simplifies this process as follows:

$$A_\ell = (b + h - 7)\left(\frac{T_u}{\phi K_{ts}}\right) \frac{f_{yv}}{f_{vl}} \text{ (in.}^2\text{)}$$

Design Aids COLUMNS 1-5 relate to column slenderness considerations. COLUMNS 6 tabulates information needed to select appropriate axial strength versus moment interaction data, which data are provided in graphical form in COLUMNS 7. These axial strength versus moment interaction data are for section of columns subjected to uniaxial bending. COLUMNS 9-12 make it possible to apply the axial strength versus moment data to design and analysis of cross sections of columns subject to biaxial bending.

COLUMNS 1

The design aid COLUMNS 1 simplifies checking the need for considering slenderness effects. The aid is based on ACI 318-95 Section 10.11.4.1, which provides that for columns braced against sidesway, slenderness effects may be neglected when

$$kl_u/r < 34 - 12(M_1/M_2)$$

where

- k: effective length factor
- l_u: unsupported length of column
- r: radius of gyration
- M_1: value of smaller nominal end moment on column due to the loads that result in no appreciable sidesway, calculated by conventional elastic frame analysis, positive if member is bent in single curvature, negative if bent in double curvature
- M_2: value of larger nominal end moment on column due to the loads that result in no appreciable sidesway, calculated by conventional elastic frame analysis, always positive

COLUMNS 1 gives critical column slenderness ratios in terms of overall column thickness h in the direction stability is being considered, as well as r; for rectangular columns r is taken as 0.30h and for circular columns r is taken as 0.25h, as permitted by Section 10.11.3 of ACI 318-95.

COLUMNS 2

The effective length of columns is expressed as a product of unsupported column height l_u and a coefficient k given in COLUMNS 2 such that the moment magnifier relationships (explained below under COLUMNS 5) for columns with hinged ends can be used for other conditions of column end restraint.

For columns braced against sidesway, the value of k can always be taken safely as unity (Section 10.11.2.1 of ACI 318-95). (A lower value may be used if analysis shows it is justified.)

For columns in frames that can sway, the value of k will always exceed unity. A value of k less than 1.2 for columns not braced against sidesway normally will not be realistic (ACI 318R-95 Section 10.11.2).

Frames that rely on the flexural stiffness of columns to resist lateral displacement must be considered subject to sway.

The value of k for columns depends on ψ, the ratio of column stiffness to stiffness of flexural members at a joint:

$$\psi = \frac{\Sigma EI/l_c}{\Sigma EI/l}$$

Values of ψ should be computed for every joint.

Values of moment of inertia for beams should be determined from the transformed areas of cracked beams. (Values of EI for columns should be based on gross column sections and may be determined from COLUMNS 3 or 4.)

In the nomograph of COLUMNS 2, which are taken from Fig. 10.11.2 of ACI 318R-95, values of the effective length coefficient k are displayed as functions of column-to-beam flexural stiffness ratio ψ at the two ends of the column. The accompanying table gives numerical values in accordance with the equations

$$\textit{for braced frames} \quad \psi = \frac{-2k}{\pi} \tan \frac{\pi}{2k}$$

$$\textit{for unbraced frames} \quad \psi = \frac{6k}{\pi} \cot \frac{\pi}{2k}$$

where $\pi/2k$ is in radians.

These equations are modifications of those derived in Steel Structures: Design and Behavior, by C.G. Salmon and J. E. Johnson, 2nd Ed., Harper & Row Publishers, New York, 1980, pp. 843-851.

COLUMNS 3 AND 4

For determination of column effective length factor k-and for some operations in frame analysis-the magnitude of an effective EI value is needed. The charts of COLUMNS 3 and the tables of COLUMNS 4 can be used for determining these EI values consistent with Eq. (10-10) and (10-11) of ACI 318-95.

Each COLUMNS 3 graph serves for a particular column configuration, and presents as its ordinates coefficients K_c and K_s useful for computing the EI components for concrete and for steel, respectively. The coefficient K_c is a function only of f'_c, whereas the value of K_s is a function of location and ratio of reinforcement.

The larger of the two values obtained from ACI 318-95's Eq. (10-10) and (10-11) is the value that should be used; it can be determined readily with the help of these graphs. If the K_s value is greater than the K_c is greater than K_s, Eq. (10-11) will give the larger value.

The constants K_c and K_s are derived from

$$EI_a = EI = \frac{\frac{E_c I_g}{5} + E_s I_{se}}{1 + \beta_d}$$

Multiplying both sides of the above equation by π^2 to obtain $\pi^2 EI_a$, a term useful in calculating the critical load P_c from

$$P_c = \frac{\pi^2 EI}{(kl_u)^2}$$

gives

$$\pi^2 EI_a = \frac{\frac{\pi^2 E_c I_g}{5} + \pi^2 E_s I_{se}}{1 + \beta_d}$$

The moment of inertia of the gross section about the axis of bending may be written

$$I_g = \xi_1 A_g h^2$$

where ξ_1 is a numerical constant depending on the shape of the column. The moment of inertia of the reinforcement about the axis of bending may be written

$$I_{se} = \xi_2 (\rho_g A_g)(\gamma h)^2$$

where ξ_2 is a numerical constant depending on the number and configuration of the bars.

Therefore for a rectangular column the equation for π^2 EI may be written

$$\pi^2 EI_a = \frac{\frac{\pi^2 E_c \xi_1}{5} bh^3 + \pi^2 E_s \xi_2 (\rho_g bh)(\gamma h)^2}{1 + \beta_d}$$

$$= \left(\frac{\pi^2 E_c \xi_1}{5} + \pi^2 E_s \xi_2 \rho_g \gamma^2 \right) \frac{bh^3}{1 + \beta_d}$$

Or, substituting $K_c = \pi^2 E_c \xi_1 / 5$ and $K_s = \pi^2 E_s \xi_2 \rho_g \gamma^2$ the equation for π^2 EI may be written

$$\pi^2 EI_a = (K_c + K_s) \frac{bh^3}{1 + \beta_d}$$

for a rectangular column

Similarly, for a circular column, the equation for π^2 EI may be written

$$\pi^2 EI_a = \left[\frac{\pi^2 E_c \xi_1}{5} h^4 + \pi^2 E_s \xi_2 \, x \, (\rho_g \frac{\pi}{4} h^2)(\gamma h)^2 \right] \div (1 + \beta_d)$$

$$= \left(\frac{\pi^2 E_c \xi_1}{5} + \frac{\pi^3}{4} E_s \xi_2 \rho_g \gamma^2 \right) \frac{h^4}{1 + \beta_d}$$

and substituting $K_c = \pi^2 E_c \xi_1 / 5$ and $K_s = \pi^3 / 4 \, E_s \xi_2 \rho_g \gamma^2$ the equation may be written

$$\pi^2 EI_a = (K_c + K_s) \frac{h^4}{1 + \beta_d}$$

Since EI may also be taken as

$$EI_b = EI = \frac{\frac{E_c I_g}{2.5}}{1 + \beta_d}$$

the corresponding expressions for π^2 EI may be written

$$\pi^2 EI_b = 2K_c \frac{bh^3}{1 + \beta_d} \textit{ for a rectangular column}$$

$$\pi^2 EI_b = 2K_c \frac{h^4}{1 + \beta_d} \textit{ for a circular column}$$

It should be remembered that K_c is dependent on ξ_1 and therefore, like ξ_1, K_c depends on the shape of the column cross section; K_s is dependent on ξ_2 and therefore, like ξ_2, depends on the number and location of the bars.

In calculating K_c, E_c is taken as $57000\sqrt{f_c'}$ psi (Section 8.5.1 of ACI 318-95) and ξ_1, the numerical constant for moment of inertia of the gross section, is 1/12 for a rectangular column and $\pi/64$ for a circular column. Therefore, for COLUMNS 3.1, 3.2, and 3.3 for rectangular columns and for COLUMNS 3.5 for square columns:

$$K_c = \frac{\pi^2(57000\sqrt{f_c'})\left(\frac{1}{12}\right)}{5}, psi$$

$$= 9.38 \times 10^3 \sqrt{f_c'}, psi$$

and for COLUMNS 3.4 for circular columns

$$K_c = \frac{\pi^2(57000)\frac{\pi}{64}\sqrt{f_c'}}{5}$$

$$= 5.52 \times 10^3 \sqrt{f_c'}, psi$$

In calculating K_s, E_s is taken as 29,000 ksi (Section 8.5.2 of ACI 318-95).

For rectangular columns with reinforcement considered as a thin rectangular tube,

$$\xi_2 = \frac{1}{6}$$

and

$$K_s = \pi^2(29,000,000)\left(\frac{1}{6}\right)\rho_g\gamma^2, psi$$

$$= 47.8 \times 10^6 \rho_g\gamma^2, psi$$

For rectangular columns with bars on end faces only,

$$\xi_2 = \frac{1}{12}$$

and

$$K_s = \pi^2(29,000,000)\left(\frac{1}{12}\right)\rho_g\gamma^2, psi$$

$$= 23.8 \times 10^6 \rho_g\gamma, psi$$

For circular columns with any number of bars arranged in a circle,

$$\xi_2 = \frac{1}{8}$$

and

$$K_s = \pi^2 (29,000,000)\left(\frac{1}{8}\right)\left(\frac{\pi}{4}\right)\rho_g\gamma^2, psi$$

$$= 28.1 \times 10^6 \rho_g\gamma^2, psi$$

For square columns with any number of bars arranged in a circle,

$$\xi_2 = \frac{1}{8}$$

and

$$K_s = \pi^2 (29,000,000)\left(\frac{1}{8}\right)\rho_g\gamma^2, psi$$

$$= 35.8 \times 10^6 \rho_g\gamma^2, psi$$

COLUMNS 4.1, 4.2, 4.3, 4.4, and 4.5 tables are for use with ACI 318-95 Eq. (10-11); they present values of

$$\frac{E_cI_g}{2.5} \times 10^{-5}$$

for concrete having compressive strengths f_c' of 3000, 4000, 5000, 6000, 9000, and 12000 psi, respectively. On each table, the extreme right-hand column is for circular columns; the rest of the applies to rectangular columns.

COLUMNS 5

Column cross sections must be checked for nominal axial load P_n obtained from conventional frame analysis and a nominal moment M_c that includes any possible magnification of moment due to column slenderness or frame displacement. The formula for the moment magnifier for braced frames given by ACI 318-95 is produces a value of EI in eq. (10-10) higher than that in Eq. (10-11). The formulation of the equations for δ_{ns}/C_m can be retained if an effective thickness h_e, reflecting the higher flexural stiffness, is used in place of the actual thickness h. The effective thickness h_e can be used most conveniently if it is inserted only in the slenderness portion of the magnifier equations while the stress ratio is left unmodified. Since moments of inertia I can be expressed as a product of area and the square of thickness, it is possible simply to use in the slenderness ratio an effective thickness h_e such that

$$h_e/h = \sqrt{(EI_a)/(EI_b)}$$

where EI_a is taken as greater than or equal to EI_b, and therefore h_e/h is greater than or equal to 1.00.

Values of h_e/h are listed for each amount of steel on the interaction diagrams (COLUMNS 7). The value of h_e should be used with the value of effective column height kl_u, but not in the expression for column cross section area. Therefore, the equations for moment magnifiers become

for rectangular columns

$$\frac{\delta_{ns}}{C_m} = \frac{1}{1 - \left(\dfrac{P_u(1+\beta_d)}{b \times h}\right)\left(\dfrac{kl_u}{h_e}\right)^2\left(\dfrac{3}{0.75E_c}\right)}$$

for circular columns

$$\frac{\delta_{ns}}{C_m} = \frac{1}{1 - \left(\dfrac{4P_u(1+\beta_d)}{\pi h^2}\right)\left(\dfrac{kl_u}{h_e}\right)^2\left(\dfrac{4}{0.75E_c}\right)}$$

The moment magnifier graphs, COLUMNS 5, contain values of δ_{ns}/C_m determined in accordance with these last two equations for various stress ratios and slenderness ratios. Values of the ratio δ_{ns}/C_m higher than 3 are not shown. When δ_{ns}/C_m higher than 3 are not shown. When δ_{ns}/C_m exceeds 3, larger cross sections should be selected. When values in the range 4 to 6 occur, frame buckling is imminent, and columns or beams should be stiffened.

COLUMNS 6

COLUMNS 6 gives, for rectangular and circular columns and most practical bar and tie or spiral sizes, values of the ratio γ, which is the ratio of the distance between centroids of longitudinal bars in opposite faces to the cross section thickness h in the direction of bending, as illustrated in the sketch at the top of the table.

This information on values of γ is helpful for determining the appropriate interaction diagram(s) or basic limits table(s) for use in determining load and moment capacities of a column of given size.

COLUMNS 7

Load/Moment Interaction Diagrams for Columns

The load/moment interaction diagrams contained in this Hand Book were plotted by Dr. Mohsen A. Issa and Alfred A. Yousif using a plotting computer program at the University of Illinois at Chicago. They used diskettes that contained data from computer solutions developed by Dr. Noel J. Everard. The equations used to develop the solutions data were derived by Dr. Everard in 1963, and were originally programmed in the FORTRAN II language for use on the IBM 1620 Computer at the University of Texas at Arlington. The solutions data obtained from that computer was hand plotted for 120 pages of column interaction diagrams, which were published in 1964 as ACI Special Publication SP-7, "Ultimate Strength Design of Reinforced Concrete Columns,"* by Noel J. Everard and Edward Cohen. Subsequently, the interaction diagrams were reproduced in volume II, Columns, ACI Special Publication SP - 17A, "Ultimate Strength Design Hand Book" in 1970.

Since 1963, students in classes in Reinforced Concrete Design at the University of Texas at Arlington were required to solve long hand problems in order to document the computer solutions. Students were assigned different neutral axis locations in order to cover the complete range of possible problems. Consequently, several thousand long hand solutions have documented the computer programs. Copies of many of the long hand solutions have been sent to ACI headquarters to be filed as documentation of the interaction curves.

The computer programs have been recompiled for an IBM Compatible 486 PC using Microsoft FORTRAN 5.1. The solutions were output on diskettes, and these were sent to Dr. Issa and Alfred Yousif for plotting using their plotting program on their computers. The resulting plots have been thoroughly checked for accuracy using the previously developed long hand and computer solutions. Those solutions covered the full range of points on all interaction charts for all types of cross-sections.

The equations that were programmed were based on statics and strain compatibility, and the equivalent rectangular stress block, as described in "Building Code Requirements for structural Concrete," developed by ACI Committee 318. The assumptions that were used were based entirely on the provisions of the ACI Code. The depth of the equivalent rectangular stress block was taken as: $a = \beta_1 c$, where c is the distance from the compression face to the neutral axis, and $\beta_1 = 0.85$ for $f_c' \leq 4.0$ ksi, and $\beta_1 = 0.85\text{-}0.05\ (f_c' - 4.0)$ for $f_c' \geq 4.0$ ksi , but not less than 0.65. The total depth of a cross-section was considered to be h.

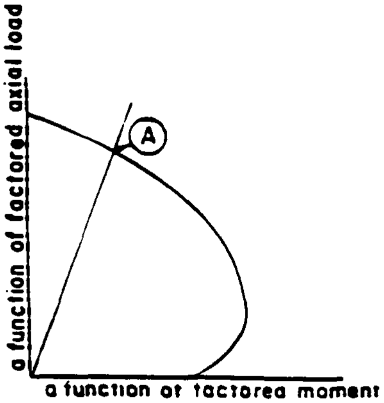


Fig.C-7-1 Interaction diagram

The ultimate strain on the compression face of the concrete cross-section was considered to be 0.003. The effects of the reinforcement area within the compression stress block was compensated for by subtracting $0.85f_c'$ from the compressive stress in the reinforcing bars.

The reinforcement was assumed to be represented by a thin rectangular tube in the case of rectangular cross-sections having longitudinal steel bars distributed along all four faces, and a thin circular tube for patterns of longitudinal steel bars arranged in a circle. For the rectangular steel patterns, five separate cases were developed, dependent on whether or not the tension steel or the compression steel had yielded. For the circular steel patterns, integration was used with the limits dependent on the yielding or non-yielding of the steel. For cross-sections having longitudinal steel on the end faces only, the side steel ratio, ρ_s was set equal to zero.

The compression forces in the concrete were calculated as the product of $0.85f_c'$ and the area under the equivalent rectangular stress block. The moments were obtained as the moments of the compressive forces in the concrete about the centroidal axis.

The forces in the steel were obtained as the product of the stress and the applicable cross-sectional areas of the assumed thin tubes, with the stress calculated as the product of the strain and the modulus of elasticity of the steel, E_s = 29000 ksi, but limited to f_y. The moment due to the steel was calculated as the forces multiplied by the distances of an increment of steel tube from the centroidal axis.

Neutral axis locations were located by starting with $c/h = 0$, and incrementing c/h by units of 0.01. The values of $K_n = P_n / (f_c' A_g)$ and $R_n = M_n / (f_c' A_g h)$ were calculated for each neutral axis location. Here, A_g is the gross area of the concrete cross-section. Approximately 100 coordinate points were developed to plot each individual interaction curve. The computer program that was used to perform the plotting utilized a spline fit routine to insure continuity of curvature of the diagrams.

It is important to note that the "strength reduction factor" (resistance factor) ϕ has been taken to be 1.0 in the interaction diagrams. This was done in order to make the interaction charts as universal and long lasting as possible, and considering that the ϕ factors described in Chapter 9 of the ACI Code might eventually change.

The American Society of Civil Engineers Standard, ASCE-7-95, "Minimum Design Loads for Buildings and Other Structures,"provides load factors different from those contained in Chapter 9 of the ACI Code. These arefor use when concrete is coupled with other materialsin design. An example is the design of a concrete lab that is composite with a structural steel beam.

Appendix A, Section A-6, of the 1995 ACI Code, "Alternate Design Method," calls for designing members subjected to axial load and flexure using 40 percent of the capacity according to Chapter 10 of the ACI Code, with $\phi = 1.0$. Hence, these interaction diagrams may be used directly for this purpose, using an overall load factor of 2.5 with service loads, axial loads and moments.

It can be shown that, when using the strength design factored loads as U = 1.4D + 1.7L, the *Composite Load Factor* will be close to 1.65 for ratios of L/D from 1.0 to 3.0. That is to say, very closely, 1.65/0.7 = 2.35. Similarly, for columns with closely spaced spirals designed according to the ACI Code, for which, in general, $\phi = 0.75$, the combined effects of the Strength Design load factors and the ϕ factor will be, closely, 1.65/0.75 = 2.2.

It follows then, that with the Alternate Design Method overall load factor of 2.5(L + D) with $\phi = 1.0$ is slightly more conservative than when axial loads and moments are factored as U = 1.4D + 1.7L with the corresponding ϕ factors 0.7 and 0.75 for columns.

However, the increase in ϕ factors to 0.9 for small axial loads does not apply to the Alternate Design Method.

Appendix B, Sections B.9.3 and RB.9.3, "Unified Design Provisions for Reinforced and Prestressed Concrete Flexural and Compression Members," contain new equations for determining ϕ in terms of the strain ($\in_t$) in the outermost bar at the tension face. Definitions for tension-controlled members ($\in_t$ = 0.005), compression-controlled members ($\in_t$ = 0.002 for f_y = 60.00 ksi and 0.0026 for f_y = 75.00 ksi), and transition members with strains $\in_y \leq \in_t \leq 0.0005$ are contained therein in terms of c/d_t. Here, d_t is the distance from the compression face to the center of the outermost bar at the tension face.

Furthermore, several U.S. government agencies and a number of foreign countries have used ϕ factors other than those that are contained in Chapter 9 of the ACI Code.

Table C-7-1 provides steel ratios ρ_g (for all values of f_c', f_y and γ for which the axial load-moment interaction diagrams were plotted) above which column strength is not *tension controlled* due to compressive axial loads. That is to say, the maximum tension strain ϵ_t is less than 0.005 and ϕ is less than 0.9. The table provides guidance for selecting column dimensions, cover over the steel and steel ratios ρ_g when ductility is a requirement; i.e., for structures designed for areas of seismic activity, high wind loads, for blast resistant structures, etc.

Where steel ratios are listed in the table as 0.08, the strain ratio ϵ_t can be obtained for all steel ratios permitted by the ACI Code, or for $0.01 \leq \rho_g \leq 0.08$.

For example, if one is using f_c' = 3.0 ksi, f_y = 60.0 ksi and γ = 0.6 for series "R" columns having steel equally distributed along all four faces, steel ratios ρ_g above 0.024 should not be used when ductility is required for the structure. The strain ratio ϵ_t = 0.005 can not be obtained for ρ_g greater than 0.024 in this specific case.

The strength reduction factors (ϕ) that are provided in Appendix "B" of ACI-318-95 are generally more liberal than those contained in Chapter 9 of the ACI Code. They may be easily obtained from the interaction diagrams using interpolation between appropriately plotted lines. Lines for ε_t = 0.0035 and 0.005 have been plotted on interaction diagrams for which f_y = 60 ksi, and lines for ε_t = 0.0038 and 0.005 have been plotted on interaction diagrams for which f_y = 75 ksi. All of the plotted lines for which $f_s/f_y \leq 1.0$ represent compression control conditions for which ϕ = 0.7 for columns with ties and ϕ = 0.75 for columns with closely spaced spirals conforming with the ACI Code. The lines for f_s/f_y = 1.0 represent the conditions for which the tensile strain ε_t in the reinforcing bars farthest from the compression face is equal to $\varepsilon_y = f_y/E_s$. For f_y = 60.0 ksi, ε_y = 0.00207, which was rounded to 0.002 in the 1999 ACI Code, Appendix B. For f_y = 75.0 ksi, ε_y = 0.002586, which can be rounded to 0.0026. For both f_y = 60.0 ksi and f_y = 75.0 ksi, the strength reduction factor φ is equal to 0.7 for members with properly designed ties, and 0.75 for members with properly designed spirals when $\varepsilon_t \leq \varepsilon_y$ (that is, $f_s/f_y \leq 1.0$). For columns with ties and f_y = 60 ksi, ϕ = 0.8 for ε_t = 0.0035 and ϕ = 0.9 for $\varepsilon_t \geq 0.005$. For columns with spirals and f_y = 60 ksi, ϕ = 0.825 for ε_t = 0.0035 and ϕ = 0.9 for $\varepsilon_t \geq 0.005$. For columns with ties and f_y = 75 ksi, ϕ = 0.8 for ε_t = 0.0038 and ϕ = 0.9 for $\varepsilon_t \geq 0.005$. For columns with spirals and f_y = 75 ksi, ϕ = 0.825 for ε_t = 0.0038 and ϕ = 0.9 for $\varepsilon_t \geq 0.005$. It is perfectly satisfactory to interpolate for ϕ between the lines for the various ε_t values.

In the vast majority of cases, ϕ will be 0.7 for tied columns and 0.75 for columns with spirals. Hence, those values may be used as first estimates in determining $M_n = M_u/\phi$ and $P_n = P_u/\phi$ for obtaining values of $K_n = P_n/(f_c'A_g)$ and $R_n = M_n/(f_c'A_g h)$ for use in entering the interaction diagrams. One may then find that ϕ may be increased. In such a case, another iteration is permitted.

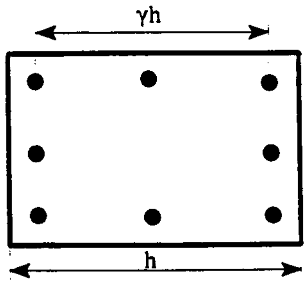
Hence, ϕ must be determined first, and then the nominal values, $M_n = M_u/\phi$ and $P_n = P_u/\phi$ are calculated using the values of M_u and P_u that have been obtained from the structural analysis. The dimensionless quantities $K_n = P_n/(f_c'A_g)$ and $R_n = M_n/(f_c'A_g h)$ are calculated, and used in the interaction diagrams to obtain the steel ratio ρ_g that is required.

Curves for f_s/f_y are provided on the interaction diagrams. Here, f_s is the stress in the outermost reinforcing bar on the tension side of the neutral axis. These curves may be used in reducing splice lengths when f_s/f_y is less than 1.0. The curve for f_s/f_y = 1.0 corresponds to the definition of "balanced conditions." That is, the outermost compression fiber of the concrete is strained exactly to 0.003 at the same time that the outermost reinforcing bar on the tension side is stressed to exactly f_y. The definition was developed in 1963 in discussions with Alfred Parme and Albert Gouwens, both members of ACI Committee 340, who were developing column design tables for the Portland Cement Association at that time. The curves for $f_s / f_y = 0$ designate the points at which the outermost bar at the tension face begins to experience compressive stress.

Curves for K_{max} are also provided on each interaction diagram. Here, K_{max} refers to the maximum permissible axial load on a column that is laterally reinforced with ties that conform with the definitions in the ACI Code. K_o refers to the theoretical axial load capacity of a tied column, $P_o = 0.85 f_c'(A_g - A_{st}) + A_{st} f_y$ and $K_{max} = 0.8\, K_o$. For columns with closely spaced spirals, $K_{max} = 0.85\, K_o$ Hence, in order to obtain K_{max} for a column with closely spaced spirals, the value of K_{max} from the charts is to be multiplied by 0.85/0.80.

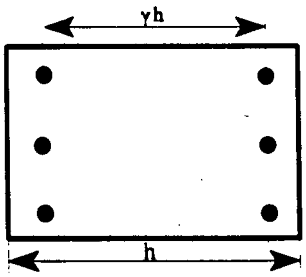
The number of longitudinal reinforcing bars that may be contained in any column cross-section is not limited to the number shown on the column cross-section illustrations on the interaction diagrams. However, for circular and square columns with steel bars arranged in a circle, and for rectangular columns with steel bars equally distributed along all four faces of the cross-section, at least eight bars (and preferably twelve bars) should be used. Although the side steel was assumed to be 50 percent of the total steel for

TABLE C-7-1. Steel Ratios ρ_g, above which strength is not controlled by tension due to compressive axial load ($\epsilon_t < 0.005$). Entries 0.08 indicate that section is controlled by tension for $0.01 \le \rho_g \le 0.08$.



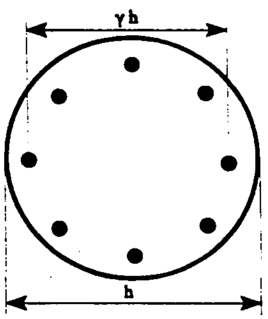
SERIES (R)

f_c' (ksi)	f_y (ksi)	γ = 0.6	γ = 0.7	γ = 0.8	γ = 0.9
3.0	60	0.024	0.0352	0.0557	0.0730
4.0	60	0.0317	0.0463	0.0726	0.0800
5.0	60	0.0370	0.0538	0.0800	0.0800
6.0	60	0.0414	0.0599	0.0800	0.0800
9.0	75	0.0422	0.0570	0.0800	0.0800
12.0	75	0.0553	0.0741	0.0800	0.0800



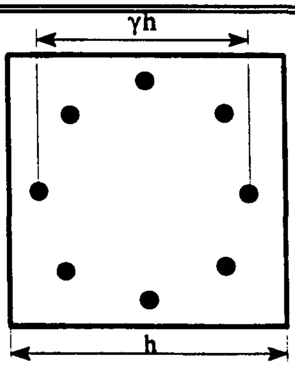
SERIES (L)

f_c' (ksi)	f_y (ksi)	γ = 0.6	γ = 0.7	γ = 0.8	γ = 0.9
3.0	60	0.0388	0.0800	0.0800	0.0800
4.0	60	0.0504	0.0800	0.0800	0.0800
5.0	60	0.0579	0.0800	0.0800	0.0800
6.0	60	0.0636	0.0800	0.0800	0.0800
9.0	75	0.0556	0.0800	0.0800	0.0800
12.0	75	0.0708	0.0800	0.0800	0.0800



SERIES (C)

f_c' (ksi)	f_y (ksi)	γ = 0.6	γ = 0.7	γ = 0.8	γ = 0.9
3.0	60	0.0161	0.0225	0.0327	0.0458
4.0	60	0.0214	0.0297	0.044.0	0.0579
5.0	60	0.0245	0.0337	0.0489	0.0676
6.0	60	0.0269	0.0370	0.0531	0.0731
9.0	75	0.0274	0.0356	0.0481	0.0671
12.0	75	0.0366	0.0468	0.0626	0.0800



SERIES (S)

f_c' (ksi)	f_y (ksi)	γ = 0.6	γ = 0.7	γ = 0.8	γ = 0.9
3.0	60	0.03.04	0.0278	0.0396	0.0543
4.0	60	0.0271	0.0367	0.053.0	0.0709
5.0	60	0.0319	0.0430	0.0605	0.0800
6.0	60	0.0359	0.0481	0.0650	0.0800
9.0	75	0.0390	0.0493	0.0649	0.012.0
12.0	75	0.053.0	0.0648	0.012.0	0.012.0

columns having longitudinal steel distributed along four faces, very accurate and conservative results are obtained when only 30 percent of the steel is contained along the side faces of a rectangular cross-section. For rectangular columns with steel bars along the two end faces only, at least four bars must be used. The maximum number of bars in any column cross-section is limited only by the available cover and clearance between the bars and the maximum permissible steel ratio, $\rho_g = 0.08$.

Many computerized studies concerning flexure with tension axial load have shown that the interaction diagram for tension axial load is very nearly linear between R_0 and K_{nt} as shown on Fig. C-7-2. Here, R_0 is the value of R_n for K_n equal to zero, and $K_{nt}=A_{st} f_y / (f_c' A_g)$, where A_{st} is the total cross-sectional area of longitudinal steel. Design values for flexure with tension axial load can be obtained using the equations:

$$K_n = K_{nt}\left(1.0 - \frac{R_n}{R_0}\right) \qquad \text{(C - 7 - A)}$$

and

$$R_n = R_0\left(1.0 - \frac{K_n}{K_{nt}}\right) \qquad \text{(C - 7 - B)}$$

Also, the tension side interaction diagram can be plotted as a straight line using R_0 and K_{nt}.

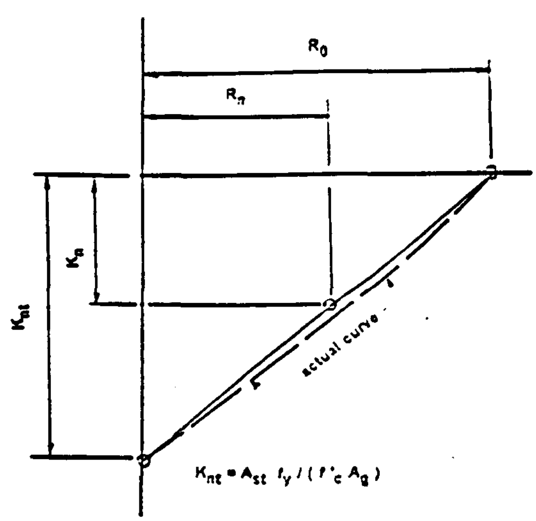


Fig. C-7-2 Flexure with axial tension

DESIGN AIDS COLUMNS 8-11 FOR COLUMNS SUBJECTED TO BIAXIAL BENDING AND AXIAL COMPRESSION

A *circular* column subjected to moments about two axes may be designed as a uniaxial column acted upon by the resultant moment $M_u = (M_{ux}^2 + M_{uy}^2)^{1/2} \geq \phi M_n = \phi(M_{nx}^2 + M_{ny}^2)^{1/2}$.

For the design of rectangular columns subjected to moments about two axes, this handbook provides design aids for two methods: (1) the reciprocal load ($1/P_i$) method suggested by Bresler[†], and (2) the load contour method developed by Parme[‡], Nieves, and Gouwens.

The reciprocal load method is the more convenient for making an analysis of a trial section; the load contour method is the more suitable design tool for selecting the cross section. Both of these methods use the concept of a failure surface to reflect the interaction of three variables, the nominal axial load P_n and the nominal biaxial bending moments M_{nx} and M_{ny}, which in combination will cause failure strain at the extreme compression fiber. In other words, the failure surface reflects the strength of short compression members subject to biaxial bending and compression. The notation used is defined in Fig. 2.

A failure surface S_1 may be represented by variables P_n, e_x, and e_y, as in Fig. 3, or it may be represented by surface S_2 represented by variables P_n, M_{nx}, and M_{ny} as shown in Fig. 4. Note that S_1 is a single curvature surface having no discontinuity at the balance point, whereas S_2 has such a discontinuity.

(When biaxial bending exists together with an nominal axial force smaller than the lesser of P_b or $0.1 f_c' A_g$, it is sufficiently accurate and conservative to ignore the axial force and design the section for bending only.)

COLUMNS 8 AND 9 (USED WITH RECIPROCAL LOAD METHOD)

In the reciprocal load method, the surface S_1 is inverted by plotting $1/P_n$ as the vertical axial, giving the surface S_3, as in Fig. 5. As Fig. 6 shows, a true point ($1/P_{nI}$, e_{xA}, e_{yB}) on this reciprocal failure surface may be approximated by a point ($1/P_{ni}$, e_{xA}, e_{yB}) on a plane S_3' passing through Points A, B, and C. Each point on the true surface is approximated by a different plane; that is, the entire failure surface is defined by an infinite number of planes.

Point A represents the nominal axial load strength P_{ny} when the load has an eccentricity of e_{xA} with $e_y = 0$. Point B represents the nominal axial load strength

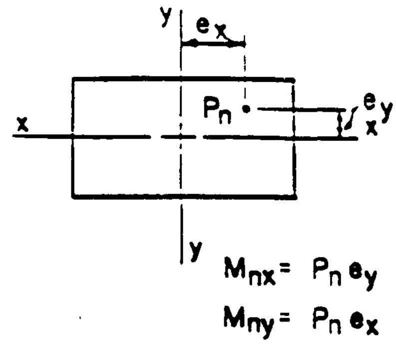


Fig. 2—Notation for column sections subjected to biaxial bending

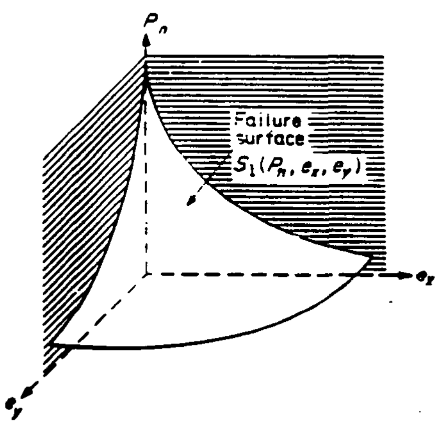


Fig. 3—Failure surface S_1 (P_n, e_x, e_y)

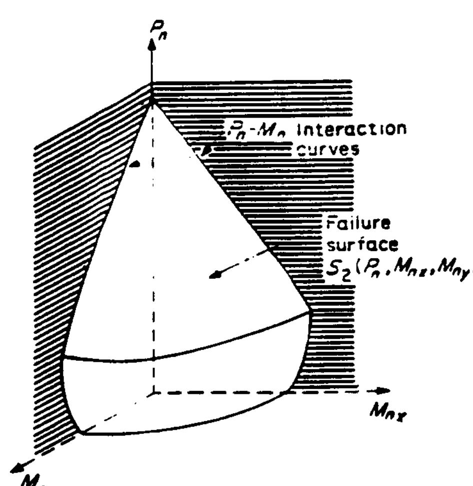


Fig. 4—Failure surface S_2 (P_n, M_{nx}, M_{ny}) for load contour method

P_{nx} when the load has an eccentricity of e_{yB} with $e_x = 0$. Point C is based on the axial capacity P_o with zero eccentricity.

The equation of the plane passing through the three points is

$$\frac{1}{P_{ni}} = \frac{1}{P_{nx}} + \frac{1}{P_{ny}} - \frac{1}{P_o}$$

where

- P_{ni}: approximation of nominal axial load strength at eccentricities e_x and e_y
- P_{nx}*: nominal axial load strength for eccentricity e_y along the y-axis only (x-axis is axis of bending)
- P_{ny}*: *nominal axial load strength for eccentricity* e_x along the x-axis only (y-axis is axis of bending)
- P_0: nominal axial load strength for zero eccentricity

*Definitions for P_{nx} and P_{ny} differ from those in ACI 318R-95 in Section 10.3.5 and 10.3.6.

For design purposes, when ϕ is constant, the $1/P_{ni}$ equation may be written:

$$\frac{1}{P_{ni}} = \frac{1}{P_{nx}} + \frac{1}{P_{ny}} - \frac{1}{P_o}$$

The 96 Uniaxial load-moment interaction charts (pages 146 - 241) were plotted in non-dimensional form so that they could be applied with any system of units (inch-pound, SI, etc.). The plots were made without including the strength reduction factor (ϕ), because several methods of determining ϕ are currently in use. The non-dimensional terms $K_n = P_n / (f_c' A_g)$ and $R_n = M_n/(f_c' A_g h)$ were used for plotting the curves. Note that the bending moment term may also be expressed as $M_n = P_n e$, where e is the eccentricity of the axial load from the axis about which bending occurs.

The design of cross-sections for bi-axial bending is usually one that involves several trials. A cross-section having trial dimensions b and h is selected, with an assumed trial reinforcement ratio (ρ_g). The values of R_{nx} and R_{ny} are calculated using M_{nx} and M_{ny} along with f_c' and A_g. Each is used separately with the steel ratio (ρ_g) to obtain K_{nx} and K_{ny} from the interaction curves. The value of K_0 is obtained as the value at which the steel ratio curve intersects the vertical axis on the chart, where $R = 0.0$

The reciprocal equation provided above can be used by calculating the three separate values of $P_n = K f_c' A_g$. However, if all of the values of P_n are divided by $f_c' A_g$, they become values of K_n, and the reciprocal equation can be expressed as:

$$K_{ni} = 1/K_{nx} + 1/K_{ny} - K_{n0}$$

Therefore, there is no need to calculate the separate values of P_n. The resulting value of P_{ni} is obtained as $P_{ni} = K_{ni} f_c' A_g$.

If the resulting value of P_{ni} is less than that of the actual axial load, the assumed cross-section size and/or reinforcement ratio (ρ_g) must be increased and another iteration must be performed. Similarly, if the calculated value of P_{ni} is sufficiently larger than the actual axial load, the section is over designed, and it should be revised.

Individuals who have used earlier versions of this Handbook may recall that radial lines representing eccentricity ratios e/h were included on the interaction diagrams. However, use of those e/h lines was extremely approximate when used with small values of axial load. The angular distance between $e/h = 6.0$ and $e / h = \infty$ was very small, and accurate interpolation was not possible. For this reason, the e/h lines were not included on the current interaction diagrams.

In COLUMNS 9, sometimes called the "skew bending chart," reciprocal values of stress are plotted along 45-degree diagonals; however, all ordinates are labeled with the values of stress instead of the value of the reciprocal. Points along the same diagonal line represent a constant same for component values of stress reciprocals from each axis. Reciprocal stress components can be added by determining the appropriate diagonal stress line; then a stress component can be subtracted by moving along the diagonal to one of the ordinate lines. Figure 7 illustrates how to use COLUMNS 9.

COLUMNS 10 AND 11 (USED WITH LOAD CONTOUR METHOD)

The load contour method uses the failure surface S_2 (Fig. 4) and works with a load contour defined by a plane at a constant value of P_n, as illustrated in Fig. 8. The load contour defining the relationship between M_{nx} and M_{ny} for a constant P_n may be expressed non-dimensionally as follows:

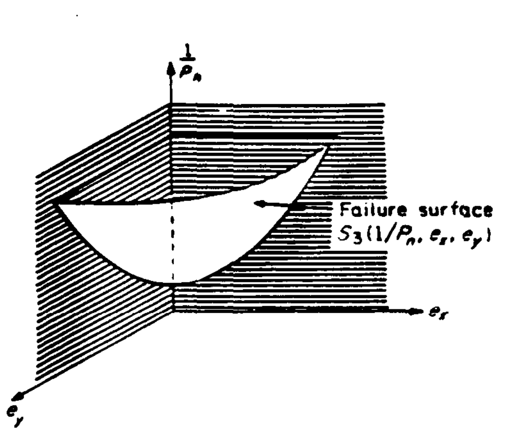


Fig. 5—Failure surface S_3 $(1/P_n, e_x, e_y)$ which is reciprocal of failure surface S_1. Failure surface S_3 is that for reciprocal load method

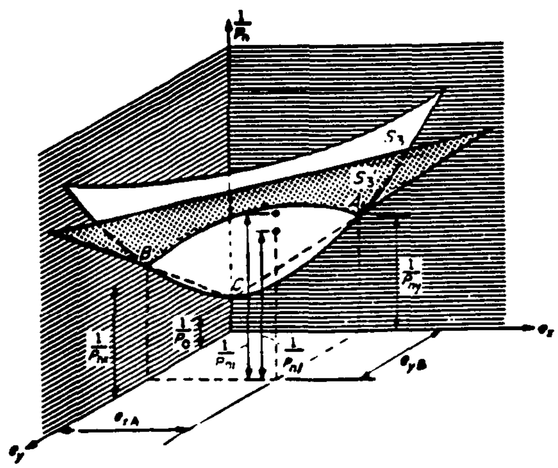


Fig. 6—Graphical representation of reciprocal load method

$$\left(\frac{M_{nx}}{M_{nox}}\right)^{\alpha} + \left(\frac{M_{ny}}{M_{noy}}\right)^{\alpha} = 1$$

For design, if each term is multiplied by ϕ, the equation will be unchanged. Thus M_{ux}, M_{uy}, M_{ox}, and M_{oy}, which should correspond to ϕM_{nx}, ϕM_{ny}, ϕM_{nox}, and ϕM_{noy}, respectively, may be used instead of the original expressions. This is done in the remainder of this section.

To simplify the equation (for application), a point on the nondimensional diagram Fig. 9 is defined such that the biaxial moment capacities M_{nx} and M_{ny} at this point are in the same ratio as the uniaxial moment capacities M_{ox} and M_{oy}; thus

$$\frac{M_{nx}}{M_{ny}} = \frac{M_{ox}}{M_{oy}}$$

or

$$M_{nx} = \beta M_{ox} \text{ and } M_{ny} = \beta M_{oy}$$

In the physical sense, the ratio β is that constant portion of the uniaxial moment capacities which may be permitted to act simultaneously on the column section. The actual value of β depends on the ration P_n/P_{og} as well as properties of the material and cross section; however, the usual range is between 0.55 and 0.70. An average value of $\beta = 0.65$ is suggested for design. Correct values of β are available from COLUMNS 10.

In terms of β, the load contour equation above may be written

$$\left(\frac{M_{nx}}{M_{ox}}\right)^{\log 0.5/\log \beta} + \left(\frac{M_{ny}}{M_{oy}}\right)^{\log 0.5/\log \beta} = 1$$

A plot of this appears as COLUMNS 11. This design aid is used for analysis; entering with M_{nx}/M_{ox} and the value of β from COLUMNS 10, one can find the permissible M_{ny}/M_{oy}.

The relationship using β may be better visualized by examining Fig. 9. The true relationship between Points A, B, and C is a curve; however, it may be approximated by straight lines for design purposes.

The load contour equations as straight line approximation are

$$M_{oy} = M_{ny} + M_{nx}\left(\frac{M_{oy}}{M_{ox}}\right)\left(\frac{1-\beta}{\beta}\right)$$

$$\text{for } \frac{M_{ny}}{M_{nx}} \geq \frac{M_{oy}}{M_{ox}}$$

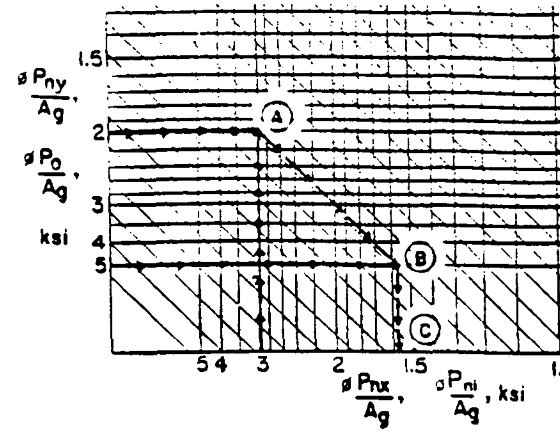


Fig. 7—Portion of COLUMNS 8 (skew bending chart). Example illustrating use: Enter from the left at $\phi P_{ny}/A_g$ = 2 ksi and from the bottom at $\phi P_{nx}/A_g$ = 3 ksi to locate intersection A. Enter from the left at $\phi P_o/A_g$ = 5 ksi; proceed from A in a direction parallel to the diagonal to intersection B. From B drop vertically to the horizontal axis, where at intersection C, P_{ni}/A_g is read as 1.57

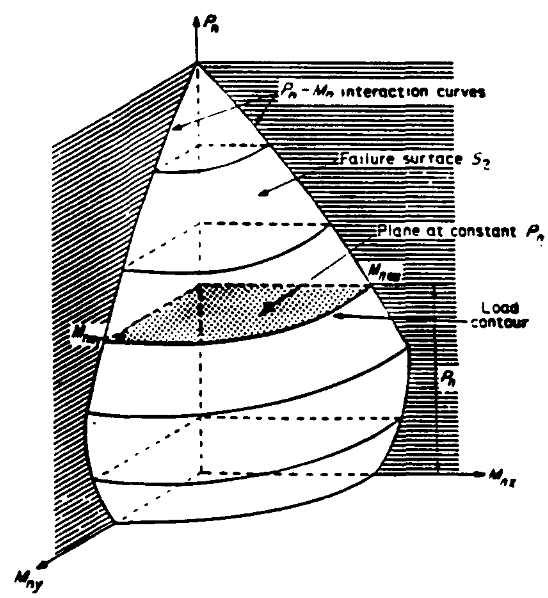


Fig. 8—Load contour for constant P_n on failure surface S_2

$$M_{ox} = M_{nx} - M_{ny}\left(\frac{M_{ox}}{M_{oy}}\right)\left(\frac{1-\beta}{\beta}\right)$$

$$for\ \frac{M_{ny}}{M_{nx}} \leq \frac{M_{oy}}{M_{ox}}$$

For rectangular sections with reinforcement equally distributed on all four faces, the above equations can be approximated by

$$M_{oy} \approx M_{ny} + M_{nx}\left(\frac{b}{h}\right)\left(\frac{1-\beta}{\beta}\right)$$

$$for\ \frac{M_{ny}}{M_{nx}} \leq \frac{M_{oy}}{M_{ox}}\ or\ \frac{M_{ny}}{M_{nx}} \leq \frac{b}{h}$$

where b and h are dimensions of the rectangular column section parallel to x and y axes, respectively.

Using the straight line approximation equations, the design problem can be attacked by converting the nominal moments into equivalent uniaxial moment capacities M_{ox} or M_{oy}. This is accomplished by

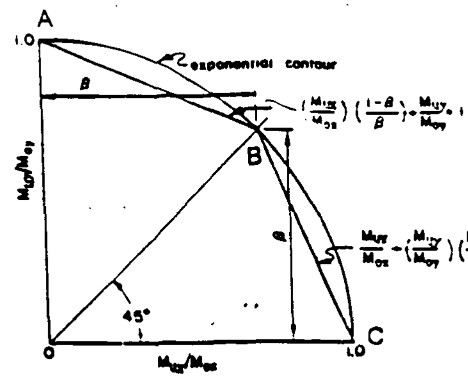


Fig. 9—Bilinear approximation of nondimensionalized load contour

(a) assuming a value of b/h
(b) estimating the value of β as 0.65
(c) calculating the approximate equivalent uniaxial bending moment using the appropriate one of the above two equations
(d) choosing the trial section and reinforcement using the methods for uniaxial bending and axial load

The section chosen should then be verified using either the load contour or the reciprocal load method.

All flexural members must meet the deflection control requirements of ACI 318-95, Section 9.5. Unless the thickness of a beam or one-way slab satisfies the minimum thickness-span ratios given in ACI 318-95 Table 9.5(a), long-time deflections must be computed to prove that they are smaller than or equal to the maximum limits given in ACI 318-95 Table 9.5(b). If a member supports or is attached to construction likely to be damaged by large deflections, deflections must be checked even if the requirements of Table 9.5(a) are satisfied. Immediate deflections are to be computed by elastic analysis, using for the moment of inertia the effective moment of inertia I_e of the cross section as determined from ACI 318-95 Eq. (9-7). The effective moment of inertia is not to be taken larger than the gross moment of inertia I_g.

DEFLECTION 1 through DEFLECTION 5.1 in these design aids have been provided to help in the algebraic evaluation of the effective moment of inertia as given in Eq. (9-7).

DEFLECTION 5.2 combines the several steps involved into one design aid, employing a graphical approach to evaluate the effective moment of inertia for rectangular beams with tension reinforcement only.

DEFLECTION 6.1 provides the moment coefficients of the elastic deflection formulas for the most common cases of loading for simple and continuous spans, and DEFLECTION 6.2 is intended to help in the computation of the immediate deflection by combining the span and the modulus of elasticity of the concrete into one factor.

The evaluation of the immediate deflection for a flexural member is, in spite of all helpful design aids, still a somewhat cumbersome procedure. For this reason, DEFLECTION 7 has been provided to help the designer in obtaining an approximate immediate deflection with relatively little effort. These tables should prove to be of great help, especially during the design stage.

All deflections evaluated with the help of DEFLECTION 1 to DEFLECTION 7 are immediate deflections occurring instantaneously upon each load application.

According to ACI 318-95, Sections 9.5.2.5 and 9.5.3.4, all immediate deflections due to sustained loads shall be multiplied by a factor as given in Section 9.5.2.5 to obtain the additional long-time deflection (creep and shrinkage part). When the creep and shrinkage part is added to the immediate live-load deflection, the total must be below the maximum limits given in Table 9.5(b). DEFLECTION 8 simplifies the evaluation of the factor given in Section 9.5.2.5 to obtain the additional long-time deflection of a flexural member.

DEFLECTION 9 furnishes the modulus of elasticity as computed for the strength and weight of the concrete used.

While tables giving several significant figures may suggest a high degree of accuracy is necessary in computation of deflections, such is not the case. The user is reminded that even under laboratory controlled conditions for simply supported beams, "there is approximately a 90 percent chance that the deflections of a particular beam will be within the range of 20 percent less than to 30 percent more than the calculated value," In[1] view of this degree of accuracy, interpolation in the use of deflection tables is generally unnecessary.

DEFLECTION 1.1

This chart gives the cracking moments in kip-feet for rectangular sections of various thicknesses h but of constant width b = 1 in. for f_c' values from 3000 to 7000 psi. According to ACI 318-95, Section 9.5.2.3, Eq. (9-8) and (9-9)

$$M_{cr} = \frac{f_r I_g}{y_t} \quad \text{and} \quad f_r = 7.5\sqrt{f_c'}$$

For rectangular sections,

$$M_{cr} = f_r \frac{bh^3}{6}$$

Expressed in kip-feet,

$$M_{cr} = b\,K_{cr}$$

where

$$K_{cr} = \frac{f_r}{12{,}000}\left(\frac{h^2}{6}\right)$$

$$= (h^2)\frac{\sqrt{f_c'}}{9600} \text{ kip - ft per inch of width}$$

ACI 318-95 Section 9.5.2.3(b) specifies that f_r shall be multiplied by 0.75 for "all-lightweight" concrete, and 0.85 for "sand-light weight" concrete (for the circumstance where f_{ct} is not specified). This means that K_{cr} from DEFLECTION 1.1 must be multiplied by these values when "all-lightweight" or "sand-lightweight" concrete is used unless f_{ct} is specified. If f_{ct} is specified, multiply K_{cr} by the ratio $f_{ct} / (6.7\sqrt{f_c'}) \leq 1$, in accordance with ACI 318-95 Section 9.5.2.3(a).

[1] ACI Committee 435, "Variability of Deflections of Simply Supported Reinforced Concrete Beams," ACI JOURNAL, *Proceedings* V. 69, No. 1, Jan. 1972, p.35.

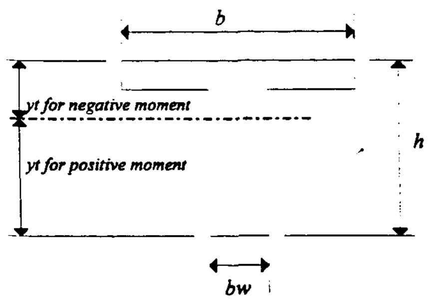


Fig. DE-1 - Illustration of y_t for positive and negative moments

DEFLECTION 1.2 AND 1.3

These charts provide the coefficient K_{crt}, which relates the cracking moment of a T-section to that of a rectangular section having the same width as the T-section web (b_w) and the same thickness as the overall thickness of the T-section (h). K_{crt} can be used in conjunction with coefficient K_{cr} from DEFLECTION 1.1, which is the cracking moment for a 1-in. width of rectangular beam. Multiplying K_{cr} by web width and K_{crt} computes the cracking moment of a T-beam.

DEFLECTION 1.2 provides values of K_{crt} for tension at the bottom (positive moment), and DEFLECTION 1.3 provides values for tension at the top (negative moment).

$$M_{cr} = \frac{f_r I_g}{y_t} \quad \text{ACI 318-95 Eq. (9-8)}$$

y_t = distance from centroid to tension face, as illustrated in Fig. DE-1.

For a rectangular section

$$\frac{I_g}{y_t} = \frac{b_w h^2}{6}$$

For a T- or L-section

$$\frac{I_g}{y_t} = \frac{b_w h^2}{6} K_{crt}$$

Dividing by 12,000 to change units from pound-inches to kip-feet,

$$M_{cr} = \frac{f_r}{12{,}000}\left(\frac{b_w h^2}{6}\right) K_{crt}$$
$$= b_w \left(\frac{f_r h^2}{72{,}000}\right) K_{crt}, \text{ Kip - ft}$$

or since K_{cr} (from DEFLECTION 1.1) $= \dfrac{f_r h^2}{72{,}000}$

$$M_{cr} = b_w K_{cr} K_{crt}, \text{ kip-ft}$$

The equations by which K_{crt} was evaluated are shown on the Design Aids.

DEFLECTION 2

This chart gives the moment of inertia I_{cr} of cracked rectangular sections with tension reinforcement only, for various modulus of elasticity ratios n and reinforcement ratios ρ. Since this condition represents a special case of the derivation for the moment of inertia of a cracked rectangular section with compression and tension reinforcement, refer to the commentary for DEFLECTION 4 where the general case is treated.

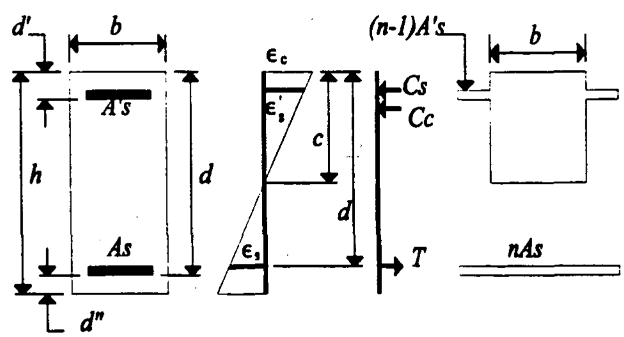


Fig. DE-2 - Internal forces and strains in cracked rectangular section with tension and compression reinforcement

Compression force in concrete $C_c = (bc / 2 - A'_s) f_c = (bc / 2 - \rho' bd) \epsilon_c E_c$

Compression force in compression reinforcement $C_s = A'_s f'_s = \rho' bd \epsilon'_s E_s$

Tension force in tension reinforcement $T = A_s f_s = \rho bd \epsilon_s E_s$

Modulus of elasticity of steel $E_s = 29 \times 10^6$ psi

DEFLECTION 3

This chart provides coefficient K_{i4} for obtaining the gross moment inertia for an uncracked T-section as

$$I_g = K_{i4}\,(b_w h^3 / 12)$$

This formula for K_{i4} is given on DEFLECTION 3.

DEFLECTION 4

This table furnishes the coefficient K_{i2} for calculating the moment of inertia of cracked rectangular sections with tension and compression reinforcement, and can also be used for the evaluation of the moment of inertia of cracked T-sections. Because deflection is the response of a structure at service loads, the derivations have been based on the linear relationship between stress and strain.

For a rectangular section with compression reinforcement as illustrated in Fig DE-2, we have from the equation of equilibrium,

$$T = C_s + C_c \quad \text{or} \quad T - C_s - C_c = 0$$

$$\rho bd\epsilon_s E_s - \rho' bd\epsilon_s' E_s - (bc/2 - \rho' bd)\epsilon_c E_c = 0$$

$$\rho bd\epsilon_s E_s - \rho' bd\epsilon_s' E_s - (bc/2)\epsilon_c E_c + \rho' bd\epsilon_c E_c = 0$$

Let $E_s = n\,E_c$. The term $\rho' bd\epsilon_c E_c$ represents the reduction in C_c caused by displacement of concrete by reinforcement. At the level of compression reinforcement, $\epsilon_c = \epsilon_s'$. Substituting ϵ_s' for ϵ_c in this term only,

$$\rho bd\epsilon_s nE_c - \rho' bd\epsilon_s' nE_c - (bc/2)\epsilon_c E_c - \rho' bd\epsilon_s' E_c = 0$$

$$d\epsilon_s - \frac{\rho'}{\rho} d\epsilon_s' \left(\frac{n-1}{n}\right) - \frac{c\epsilon_c}{2\rho n} = 0$$

Let

$$\beta_c = \rho'(n-1)/\rho n$$

Then

$$\epsilon_s - \beta_c \epsilon_s' - \frac{(c/d)\epsilon_c}{2\rho n} = 0$$

Since $\epsilon_s' = \epsilon_c(c/d - d'/d)/c/d$ and $\epsilon_s = \epsilon_c(1 - c/d)/c/d$,

$$\frac{1 - c/d}{c/d} - \beta_c\left(\frac{c/d - d'/d}{c/d}\right) - \frac{c/d}{2\rho n} = 0$$

From this c / d becomes:

$$c/d = \sqrt{[\rho n(1+\beta_c)]^2 + 2\rho n(1 + \beta_c d'/d)} - \rho n(1+\beta_c)$$

Assuming that a crack extends to the neutral axis of the section, the moment of inertia of the cracked section about the neutral axis is the sum of the moment of inertia of the uncracked portion of the concrete section (less the area occupied by compression reinforcement) plus the moments of inertia of the compression and tension reinforcement areas, with allowance for the relative moduli of elasticity of concrete and steel. Thus

$$I_{cr} = \frac{b(c/d)^3 d^3}{3} + n\rho bd(d-c)^2 + (n-1)\rho' bd(c-d)^2$$

$$\frac{I_{cr}}{bd^3} = \frac{(c/d)^3}{3} + \rho n(1 - 2c/d + (c/d)^2) + \rho n\beta_c\left\{(c/d)^2 - 2c/d\frac{d'}{d} + \left(\frac{d'}{d}\right)^2\right\} = K_{i2}$$

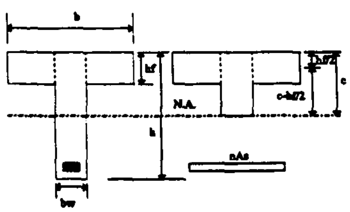


Fig. DE-3--Illustration showing how DEFLECTION 4 can be used to determine moment of inertia of cracked T-section by considering capacity of overhanging flange as compression reinforcement and redefining β_c as

$$\beta_c = \left(\frac{b}{b_w} - 1\right)\frac{h_f/d}{n\rho_w}$$

DEFLECTION 4 gives the values of K_{i2} for different β_c values. To find the moment of inertia of the cracked rectangular cross section, determine ρn, d' / d and β_c for the section under consideration to find K_{i2} which has to be multiplied by bd^3.

$$I_{cr} = K_{i2}\, bd^3$$

The most common case of a beam with tension reinforcement only is a special case of this derivation in which $\beta_c = 0$.

$$\frac{I_{cr}}{bd^3} = \frac{(c / d)^3}{3} + \rho n(1 - 2c/d + (c / d)^2) = K_{i1}$$

where; $c/d = \sqrt{\rho^2 n^2 + 2\rho n} - \rho n$

The moment of inertia of a cracked rectangular cross section with tension reinforcement only becomes then

$$I_{cr} = K_{i1}\, bd^3$$

The K_{i1} values can be read from DEFLECTION 2 for ratios of $n = E_s / E_c$ and $\rho = A_s / b\, d$.

DEFLECTION 4 can also be used to determine the moment of inertia of a cracked T-section by considering the capacity of the overhanging flange portion as a kind of compressive reinforcement and by consequently redefining β_c as shown in Fig DE-3. Approximating the stress in the flange's overhang by that at its middepth and finding an equivalent amount of reinforcement ρ'_e at $d' = h_f / 2$, we see from the diagram that

$$(b - b_w)h_f\epsilon_c E_c = \rho'_e\, b_w d\epsilon_s' E_s - A_s'\epsilon_c E_c$$

Since $\epsilon_s' = \epsilon_c$ and $E_s = n\, E_c$,

$$(b - b_w)h_f E_c = \rho'_e\, b_w d(nE_c - E_c)$$

or

$$(b / b_w - 1)(h_f / d) = (n - 1)\rho'_e$$

Since

$$\beta_c = \frac{(n - 1)\rho'_e}{n\rho_w}$$

we obtain

$$\beta_c = \left(\frac{b}{b_w} - 1\right)\frac{h_f / d}{n\rho_w}$$

This expression is used in DEFLECTION 4 to arrive at a substitute for compression reinforcement for the overhanging flanges.

DEFLECTION 5.1 AND 5.2

According to Section 9.5.2.3 of ACI 318-95, the immediate deflection of a flexural member is to be based on the "effective" moment of inertia of the cross section which is given by Eq. (9-7) as

$$I_e = \left(\frac{M_{cr}}{M_a}\right)^3 I_g + \left[1 - \left(\frac{M_{cr}}{M_a}\right)^3\right] I_{cr}$$

Dividing both sides by I_g

$$\frac{I_e}{I_g} = \left(\frac{M_{cr}}{M_a}\right)^3 + \left[1 - \left(\frac{M_{cr}}{M_a}\right)^3\right]\frac{I_{cr}}{I_g} = K_{i3}$$

The values of $K_{i3} = I_e / I_g$ can be evaluated therefore for various ratios of M_{cr} / M_a and I_{cr} / I_g, using DEFLECTION 5.1.

It is noted that the effective moment of inertia is, according to definition, not the same for all loading conditions. Since I_e based on M_{cr} / M_d is not the same as I_e based on M_{cr} / M_{d+l} (although in many practical cases they are approximately the same), live load deflections can be computed correctly only by an indirect process:

$$a_l = a_{d+l} - a_d$$

Here a_{d+l} is determined using I_e based on M_{cr} / M_{d+l} and a_d is determined using I_e based on M_{cr} / M_d. Attention is also drawn to the Commentary on ACI 318-95, Sections 9.5.2.2, 9.5.2.3, and 9.5.2.4.

DEFLECTION 5.2 may be used as an alternate method to determine the effective moment of inertia for cracked rectangular cross sections with tension reinforcement only. The guidelines on the graph (which refer to Deflection Example 7) explain its use.

DEFLECTION 6.1 AND 6.2

The diagram and chart were prepared considering the deflection at the center of a beam in terms of the variables. All deflections are stated in terms of the equation

$$a_c = \frac{\sum(K_{a3} M_c)\, l^2\, (1728)}{48 E_c\, I_e} = \frac{\sum(K_{a3} M_c)}{I_e} K_{al}, \text{ inch}$$

where

M_c = the moment at the center of the beam, or a moment value related to the deflection, kip-ft

l = span length, ft

E_c = modulus of elastcity of concrete

$= \dfrac{w^{1.5}\, 33\, \sqrt{f_c'}}{1000}$, ksi

I_e = effective gross moment of inertia, in.^{4}

K_{a3} = a coefficient relating the moment at midspan to the deflection at the center

$K_{al} = \dfrac{1728\, l^2}{48\, E_c}$

DEFLECTIONS 6.1 shows the values of K_{a3} and equations for M_c for use in the equation

$$a_c = K_{a1} \frac{\sum(K_{a3}\ M_c)}{I_e}\ ,\ \text{inch}$$

where $\sum K_{a3}\ M_c$ is the sum of all separate $K_{a3}\ M_c$ values for the individual loading conditions.

DEFLECTION 6.2 shows the coefficient K_{a1} for various spans and values of concrete weight and strength. The values of K_{a1} depend on E_c, but it is not necessary to know E_c which is related to the strength and unit weight of the concrete. K_{a1} can be read directly in terms of span, w_c and f_c'.

DEFLECTION 7

DEFLECTION 1 through 6.2 have been prepared to aid the designer in arriving at the immediate deflection of a reinforced concrete flexural member as prescribed by Sections 9.5.2.2 and 9.5.2.3 of ACI 318-95. Effective use of these tables can reduce considerable the designer's efforts in the evaluation of the to-be-expected immediate deflection; however, they are still time consuming.

Since deflection must be computed whenever members support or are attached to partitions or other construction likely to be damaged by large deflection, it is highly desirable to have reasonable assurance that, after the selection of the section and reinforcement, the check of the deflection will prove the design satisfactory.

DEFLECTION 7 provides an approximate immediate deflection applicable to uniform loading, based on the moment coefficients for an approximate frame analysis as given in Section 8.3 of ACI 318-95. A deflection computed with DEFLECTION 7 may prove particularly helpful during the design stage.

The approximate immediate deflection at the midspan can be expressed as

$$a_c = \delta_c\ K_{a2}\ \frac{w}{b}$$

As already shown in relation to DEFLECTION 6.1 and 6.2, the general equation for the deflection a of Point C at midspan can be written as:

$$a_c = \frac{K_{a1}}{I_e} \sum(K_{a3}\ M_c)$$

If all factors are evaluated for a unit load of 1 kip per ft and a unit width of 1 in., the equation for a_c would have to be multiplied by the actual load w and divided by the actual width b , as follows:

$$a_c = \frac{K_{a1}}{I_e} \sum(K_{a3}\ M_c)\ \frac{w}{b}$$

Multiplying the a_c equation by I_g / I_g gives

$$a_c = \frac{K_{a1}}{I_g}\ \frac{I_g}{I_e} \sum(K_{a3}\ M_c)\ \frac{w}{b}$$

The first part of the above equation, (K_{a1} / I_g), can be evaluated for various combinations of span and thickness. These values are called K_{a2} and are given in DEFLECTION 7. The second part of the above equation consists of two expressions:

1. The I_g / I_e can be evaluated for various reinforcement ratios. The use of only two categories can be considered to be enough for practical purposes:

$$\rho \geq 0.6\ \rho_{bal} \text{ and } \rho \leq 0.6\ \rho_{bal}$$

This division was arbitrarily selected to reflect the behavior of beams so heavily reinforced that the applied moment is considerably larger than the cracking moment. Therefore, $(M_{cr} / M_a)^3$ is negligibly small.

2. The $\sum(K_{a3}\ M_c)$ can be evaluated as follows:

for simple spans:

$$\sum(K_{a3}\ M_c) = 5\ (w\ l^2 / 8)$$

For continuous spans according to Case 7 of DEFLECTION 6.1:

$$\sum(K_{a3}\ M_c) = 5\ [\ M_c - 0.1(M_A + M_B)\ ]$$

If the computation of the deflection is based on uniformly distributed loadings and on the moment coefficients for an approximate frame analysis as given in Section 8.3 of ACI 318-95, then the above expression can be evaluated for each kind of bay with the appropriate moments.

Combining $\sum(K_{a3}\ M_c)$ with the I_g / I_e values as described under the first expression above gives the δ_c values in DEFLECTION 7. The final equation for the approximate deflection of a beam at midspan can be written as:

$$a_c = \delta_c\ K_{a2}\ \frac{w}{b}$$

where

a_c	=	deflection at midspan, in.
w	=	uniformly distributed load, kip/ft (Note that in all deflection calculations the *service loads*, not the factored loads, are to be used.)
b	=	width of the member, in.
K_{a2}	=	coefficient from DEFLECTION 7.1
δ_c	=	factor depending on type of span and degree of reinforcement

DEFLECTION 8

DEFLECTION 8 is similar to the graph in the Commentary on ACI 318-95, Section 9.5.2.5. It is used to obtain multipliers for use with computed immediate deflections to estimate the additional long-time (1 to 60 month) deflections due to the sustained part of the load, sometimes referred to as "creep deflection."

DEFLECTION 9

This is a plot of the equation $E_c = 33\ w_c^{1.5}\ \sqrt{f_c'}$ and is used to obtain modulus of elasticity as provided in Section 8.5.1 of ACI 318-95, when concrete strength and unit weight are known.

Values of E_c can be obtained from this design aid and used to calculate n and ρn for use in DEFLECTION 2 and DEFLECTION 4. The ratio n can also be read directly when $E_s = 29{,}000{,}000$ psi.

COMMENTARY ON GENERAL DESIGN AIDS

GENERAL 1.1 AND 1.2, 2.1 AND 2.2

These charts provide basic information on moments and properties of cross sections. Moment formulas used in developing the tables are shown in GENERAL 1.1 AND 1.2. GENERAL 2.1 AND 2.2 show the familiar formulas for area, moment of inertia, and radius of gyration for a number of different cross sections.

COMMENTARY ON SLAB DESIGN AIDS

SLABS 1

For two-way action slabs having ratio of long to short span not exceeding 2, minimum thickness is specified in ACI 318-95 Sections 9.5.3.1 trough 9.5.3.5 unless—as is permitted by 9.5.3.6—it is shown by calculation that with a slab thickness less than the minimum required by Section 9.5.3.1, 9.5.3.2, and 9.5.3.3, the deflection will not exceed the limits stipulated in ACI 318-95 Table 9.5(b).

ACI 318-95 deals with slabs without interior beams and provides that minimum slab thickness shall be in accordance with the provision of the section's Table 9.5(c), which is repeated in this volume as SLABS 1. (However, section 9.5.3.2 provides further that thickness shall not be less than 5 in. for slabs without drop panels and 4 in. for slabs with drop panels.) This design aid, ACI 318R-89 says, gives limits that have evolved trough the years and have not resulted in problems related to stiffness for short- and long-term load.

SLABS 2

SLABS 2 for design of slabs with beams is reproduced from Reference 1. The cross section to be used for the beam when computing the flexural stiffness is specified in Section 13.2.4 of ACI 318-95 which states "for monolithic or fully composite construction, the beam includes that portion of the slab on each side of the beam extending a distance equal to the projection of the beam above or below the slab, whichever is greater, but not greater than four times the slab thickness." Relative beam stiffness α is defined as

$$\lambda = \frac{E_{cb}\ I_b}{E_{cs}\ I_s}$$

The $4h_s$ restriction on the extent of slab to be considered acting with the beam stem accounts for the abrupt change in the slope of each curve at $h/h_s = 5$.

(The abrupt change at $h/h_s = 2$ is due to the scale change in the graph.) SLABS 2 can be used for both interior and exterior beams. For exterior beams, u is taken as twice the width of the beam stem.

To use SLABS 2, the ratios h/h_s and u/h_s are computed. Enter at the bottom of the chart with the value of h/h_s, proceed vertically to the value of u/h_s, then proceed to the left to obtain the value of α_f. Relative beam stiffness α is then computed from the equation

$$\alpha = \frac{E_{cb}}{E_{cs}}\left(\frac{b}{l_2}\right)\left(\frac{h}{h_s}\right)^3 \alpha_f$$

When there is no beam , $\alpha = 0$. The product bh^3 is proportional to I_b for rectangular beam cross section, and α_f converts it to a T-section. E_{cb} and E_{cs} cancel if concrete modulus is the same for slab and beam.

SLABS 3

These charts were derived to match the shear force expressions of ACI 318-95 Chapter 11. Note that charts are derived for $\phi = 1$.

In section 11.2.1.2, the critical section for perimeter shear is defined as a section located $d/2$ from the face of the column or column capital. The allowable shear load depends on the size of critical section, depth of slab, and strength of concrete in the slab.

The provisions of ACI 318-95 for the perimeter shear capacity of slab in the vicinity of column are based on the nominal shear strength of the concrete when no moment is transferred (Section 11.12.6). The charts given in as SLABS 3 can be used for both conditions and are taken from Reference 3.

The shear strength of the concrete V_c is the smallest by the following three equations in Section 11.12.2.1 of ACI 318-95:

$$V_c = \left(2 + \frac{4}{\beta_c}\right)\sqrt{f_c'}\, b_o\, d \quad \text{Eq. (11-35)}$$

$$V_c = \left(2 + \frac{\alpha_s d}{b_o}\right)\sqrt{f_c'}\, b_o\, d \quad \text{Eq. (11-36)}$$

$$V_c = 4\sqrt{f_c'}\, b_o\, d \quad \text{Eq. (11-37)}$$

The governing equation depends on β_c (the ration of the longer side to shorter side of the column or capital) and b_o / d (ratio of the perimeter of the critical section to the effective slab depth) as follows:

When

$\beta_c \le 2$ and $b_o/d \le \alpha_s/2$, $V_c = 4\sqrt{f_c'}\, b_o\, d$

Eq. (11-37)

$\beta_c > 2$, $V_c = \left(2 + \frac{4}{\beta_c}\right)\sqrt{f_c'}\, b_o d$

Eq. (11-36)

$$b_o / d > \alpha_s / 2, V_c = \left(2 + \frac{d_s d}{b_o}\right)\sqrt{f_c'}b_o d$$

Eq. (11-36)

In these expressions b_o is the perimeter of the critical section and can be evaluated from the following expressions:

Interior columns: $b_o = 2c_1 + 2c_2 + 4d$

Edge columns: $b_o = 2c_1 + c_2 + 2d$, where c_2 is support dimension parallel to the edge.

Corner columns: $b_o = c_1 + c_2 + 4d$

The required effective depth of the slab to resist perimeter shear at any column location can be obtained from SLABS 3.

When no moment is considered to be transferred between support and the slab, the required effective slab depth to resist perimeter shear at interior, edge and corner columns can be obtained from SLABS 9.1, 9.4, and 9.8 respectively. These charts were drawn for $\beta_c \leq 2$ and $b_o/d \leq \alpha_s/2$ but can be used for other cases as described below.

When $\beta_c \leq 2$ and $b_o/d \leq \alpha_s / 2$, enter the lower chart with the concrete strength (or adjusted strength if light-weight concrete); go across to the calculated value of the nominal shear force V_n, go vertically upward to the appropriate combination of c_1 and c_2, then left to find d.

When $\beta_c > 2$ or $b_o/d > \alpha_s/2$, the procedure is similar except that parameter k_1 (also used when moment is transferred) is introduced between upper and lower charts. The factor k_1 may be defined in terms of either geometry or in terms of permitted concrete shear stress and the shear strength of a section as follows:

When

$\beta_c \leq$ and $b_o/d \leq \alpha_s/2$,

$$k_1 = \frac{1}{b_o d} = 4\frac{\sqrt{f_c'}}{V_n}$$

$$\beta_c > 2, k_1 = \frac{1}{b_o d} = \left(2 + \frac{4}{\beta_c}\right)\frac{\sqrt{f_c'}}{V_n}$$

$$b_o/d > 2\alpha_s / 2, k_1 = \frac{1}{b_o d} = \left(2 + \frac{d_s d}{b_o}\right)\frac{\sqrt{f_c'}}{V_n}$$

When $\beta_c > 2.0$ the procedure for using SLABS 3.1, 3.4, and 3.8 is to enter the lower chart with the concrete strength (or adjusted strength if light-weight concrete); go across to the calculated value of the nominal shear force, ($V_n = V_n/\phi$), go vertically upward to read k_1. This value is than multiplied by the factor ($0.5 + 1/\beta_c$) to obtain the modified value of k_1 corresponding to $\beta_c > 2$. Enter upper chart with this modified value of k_1, go vertically upward to the appropriate combination of c_1 and c_2, then left to find d.

When $b_o/d > 2\alpha_s/2$, the procedure is same except that initial k_1 value is multiplied by the factor ($\alpha_s d/4b_o + 0.5$) to obtain the modified value of k_1 corresponding to $b_o/d > 2\alpha_s/2$.

When both $\beta_c > 2$ and $b_o/d > \alpha_s/2$, determine both modified values of k_1 and use the smaller modifier to find k_1 to find d.

When this method of correcting is used, it is unnecessary to apply the correction factor k_{va} given in SLABS 3.11.

When bending moment is to be transferred between slab and column, ACI 318-95 requires that a fraction of this moment be transferred by eccentricity of the shear about the centroid of the critical section described as above, this means that the total shear at a slab-column junction consists of a portion from the vertical shear force and a portion from the moment being transferred to the column in each direction by torsion. This later portion is small enough for interior columns when the loading and length of adjoining spans are nearly equal, but may be appreciable for edge and corner columns. For this reason when selecting the slab thickness originally for shear strength, allowance must be added for moment shear transfer. The shear stress resulting from the transfer of the nominal shear force V_n is:

$$v_n = = \frac{v_u}{\phi} = \frac{V_n}{b_o d} = k_1 V_n$$

Section 11.12.6.2 of the commentary on ACI 318-95 suggests the shear stress caused by moment be evaluated by the expression

$$v_{n(m)} = \frac{v_{u(m)}}{\phi} = \frac{\gamma_v M_u c_{AB}}{\phi J_c} = \frac{\gamma_v M_n c_{AB}}{J_c}$$

which can be written

$$v_{n(m)} = k_2 M_n$$

where

v_n = v_u / ϕ

γ_v = the portion of moment M_n transferred by eccentricity of shear

c_{AB} = the extreme distance of the critical section from the centroid of this section

M_n = the unbalanced moment transferred between slab and column = ϕM_n

ϕ = the strength reduction factor for shear

J_c = the torsional term comparable to the polar moment of inertia of the critical section about the centroid of that section

The column strip nominal strength M_n of the section must be used in evaluating the unbalanced transfer moment for gravity load and at the column edge (ACI 318-95 Section 13.6.3.6).

The terms γ_v, C_{AB}, and J_c are tedious to evaluate but are functions only of the geometry terms c_1, c_2, and d. For this reason the constant k_2 is also only a function of these geometric quantities.

A similar constant k_3 can be defined for moments in the perpendicular direction so that the total shear stress can be evaluated by the expression

$$v_n = \frac{v_u}{\phi} k_1 V_n + k_2 M_{nl} + k_3 M_{n2}$$

The charts in SLABS 3 may be used to evaluate the three factors k_1, k_2, and k_3 for any slab column geometry.

Factor k_1 is obtained from SLABS 3.1, 3.4, and 3.8, as mentioned above. To obtain k_1, enter the chart from the left with the value of effective slab depth d, proceed horizontally to the appropriate combination of column or capital dimensions, then vertically downward to scale marked k_1. Note that this value is correct for all values of β_c.

For interior columns and corner columns, values of both k_2 and k_3 may be found from the same charts by interchanging size of column dimensions c_1 and c_2. It should be noted that in SLABS 3.2 and 3.9 the dimension c_2 is parallel to the moment vector for the moment being transferred. For the edge columns (SLABS 3.5), c_2 is always measured parallel to the discontinuous edge.

For square columns or capitals, the value of k_2 and k_3 are obtained directly from SLABS 3.2, 3.5, and 3.9 and are designated as k_2' and k_3'. For rectangular columns or capitals, these values must be modified to obtain k_2 and k_3.

To obtain k_2 and k_3 from SLABS 3.2, 3.5, and 3.9, enter with the value of c_2 at the left of the chart, proceed horizontally to the effective slab depth d, and the n vertically downward to the scale marked either k_2' or k_3' as required. For square interior and corner columns, it is only necessary to read the chart once since $k_2' = k_3'$. For rectangular interior and corner columns, the charts are read twice to find k_2' and k_3', once for each value of c_2 as the side parallel to the moment vector.

For rectangular columns or capitals, the values k_2' and k_3' are modified in SLABS 3.3, 3.6, 3.7, and 3.10 to obtain values of k_2 and k_3. To use these charts, enter at the left with the value of effective slab depth d and either k_2' and k_3', proceed horizontally to the appropriate value of $c_1 - c_2$, and then vertically downward to the value of d and k_2' or k_3'. Note that the value of $c_1 - c_2$ may be either positive or negative.

It should be noted that evaluating the shear stress for moment-shear transfer is frequently a checking operation after most of the design has been completed and, unless the shear appears to be critical or c_1 and c_2 differ markedly, the correction of k_2' and k_3' for most columns is not required.

REFERENCES

1. *Notes on ACI 318-89 Building Code Requirements for Reinforced Concrete with Design Applications*, Portland Cement Association, Skokie IL, 1990, pp. 21-10.

2. Simmonds, S. H., and Misic. Janko, "Design Factors for Equivalent Frame Method," ACI JOURNAL, *Proceedings* V. 68, No. 11, Nov. 1971, pp. 825-831.

3. Simmonds, S. H., and Hrabchuk, L. C., "Shear Moment Transfer Between Slab and Column," Department of Civil Engineering, University of Alberta, Dec. 1976.

COMMENTARY ON TWO-WAY ACTION REINFORCEMENT

Proportioning of reinforcement in two-way slabs and plates is not covered by the crack control requirements of Section 10.6 of ACI 318-95; some guidelines, however, are given in this handbook based on the recommendations of ACI 224R. These guidelines do not relate to shrinkage and temperature cracking. While not mandatory, the guidelines are recommended for flexural crack control in structural floors where flexural cracking at service load and overload conditions can be serious such as in office buildings, schools, parking garages, industrial buildings, and other floors where the design service live load levels exceed those in normal size apartment building panels and also in all cases of adverse exposure conditions.

TWO-WAY ACTION REINFORCEMENT 1, 2, and 3, based on the recommendations of ACI 224R have been provided in the Design Aids on the assumption that designers might want to incorporate crack control in the design of such members.

Because flexural cracking behavior under two-way action is significantly different from that in one-way members, the maximum possible crack width (in.) at the tensile face of concrete is best predicted by:

$$w_{max} = K \beta f_s \sqrt{M_l}$$

where

M_l = the grid index = $(d_{bl}\, s_2 / \rho_{tl})$

$$= \frac{s_1 s_2 d_c 8}{d_{bl} \pi} \text{, in.}^2$$

K = fracture coefficient having the values given in Table 1

f_s = actual average stress in the reinforcement at service load level, or 40 percent of design yield strength f_y', ksi.

β = 1.25

ρ_{tl} = active steel ratio

$$= \frac{\textit{area of steel } A_{sl} \textit{ per 1 meter width}}{d_{bl} + 2c_{cl}} = \frac{A_{sl}}{d_c}$$

where c_{cl} is clear concrete cover to reinforcement in direction "1" (c_{cl} is taken as 0.75 in. for all bars smaller than #11, and as 1.5 in. for larger bars.)

The β values used in calculating REINFORCEMENT 2 differ from the values of β used in Section 10.6.4 of the Commentary on ACI 318-95 for the crack control criteria for beams because of the different distance of the neutral axis from the tensile fiber in slabs. The values used in the evaluation of the table are more appropriate for use with slabs.

In the evaluation of REINFORCEMENT 1,2, and 3, a clear concrete cover of 0.75 in. was used for interior as well as for exterior exposure for all bars smaller than #11, and 1.5 in. for larger bars. This assumption is based on the explanations contained in ACI 318R-95, Section 7.7, saying that under ordinary conditions even exterior slabs (underside) are not directly exposed to weather unless they are subject to alternate wetting and drying, condensation or similar effects.

REINFORCEMENT 1,2, and 3 are given for proportioning crack control reinforcement of two-way action slabs and plates. REINFORCEMENT 1 gives maximum spacing compatible with maximum allowable crack widths w_{max}, for the two standard exposure conditions of interior exposure ($w_{max} \le 0.016$ in.) and exterior exposure ($w_{max} \le 0.013$ in.). REINFORCEMENT 2 gives the values of maximum permissible crack widths in concrete structures for the various exposure conditions encountered in practice. REINFORCEMENT 3 is applicable to any exposure condition or crack width level. Both REINFORCEMENT 2 and 3 permit interpolation of the fracture coefficient K values for the various load and boundary conditions encountered in multipanel floor systems.

TABLE 1—VALUES FOR FRACTURE COEFFICIENT K*

K in.^{2}/kip	Fully restrained slabs and plates
2.8×10^{-5}	Uniformly loaded, square
2.1×10^{-5}	At concentrated loads and columns
2.1×10^{-5}	$0.5 < l_s/l_l < 0.75$
1.6×10^{-5}	$l_s/l_l < 0.75$

*For simply supported slabs multiply spacing values by 0.65. Interpolate multiplier values for intermediate span ratio l_s / l_l values or for partial restraint at boundaries such as cases of end and corner panels of multipanel floor systems.

Severe exposure conditions warrant limiting the permissible flexural crack width to values lower than the 0.013 in. permitted by ACI 318-95. Engineering judgment should be exercised in choice of the lower permissible crack widths in adverse exposure conditions.

Data are presented by Nawy and Blair (Reference 1) and in the summary ot Reference 2. See also ACI 224R. Requirements on the spacing of slab and plate reinforcement, other than those for crack control, are given in Section 13.4.2 of ACI 318-95.

REFERENCES

1. Nawy, Edward G., and Blair, Kenneth W., "Further Studies on Flexural Crack Control in Structural Slab Systems," *Cracking. Deflection, and Ultimate Load of Concrete Slab Systems*, SP-30, American Concrete Institute, Detroit, 1971, pp. 1-41.

2. Nawy, Edward G., "Crack Control Through Reinforcement Distribution in Two-Way Acting Slabs and Plates," ACI JOURNAL, *Proceedings* V. 69, No. 4, Apr. 1972, pp. 217-219.

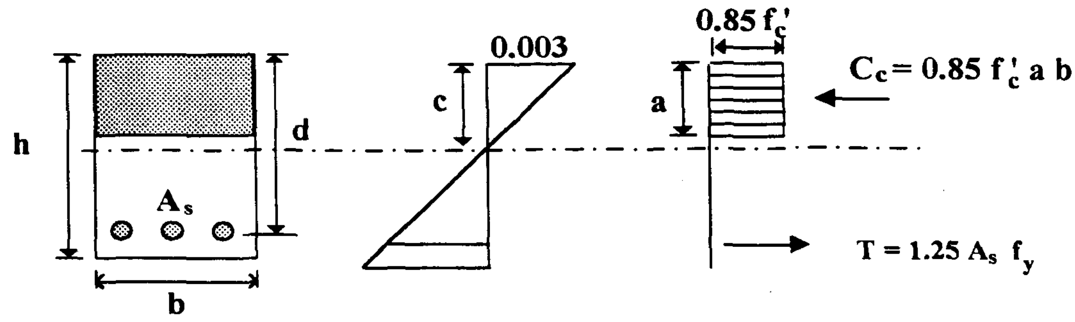


Fig. SE-1 Internal forces of a reinforced concrete section at probable moment resistance.

SEISMIC 1

SEISMIC 1 provides coefficients K_{pr} to determine the probable moment resistance M_{pr} of a rectangular section with tension reinforcement. The probable moment resistance of a section is computed when the tension reinforcement attains stress of 1.25 f_y during the formation of a plastic hinge. Figure SE-1 illustrates internal forces in a reinforced concrete section at this stage of loading. The coefficient K_{pr} is used in solving the following equation:

$$M_{pr} = 1.25\ A_s f_y (d - \frac{a}{2}) \qquad \textbf{(SE-1)}$$

where;

$$a = \frac{1.25 A_s f_y}{0.85 f_c' b} \qquad \textbf{(SE-2)}$$

$$A_s = \rho b d \qquad \textbf{(SE-3)}$$

Substituting Eqs. SE-2 and SE-3 into SE-1;

$$M_{pr} = K_{pr} b d^2 \qquad \textbf{(SE-4)}$$

where;

$$K_{pr} = 1.25\ \rho f_y (1 - 0.735 \rho \frac{f_y}{f_c'}) \qquad \textbf{(SE-5)}$$

SEISMIC 2

Seismic 2 gives free-body diagrams for the calculation of seismic design shears for beams and columns, associated with the formation of plastic hinges.

SEISMIC 3

Seismic 3 shows the details of transverse reinforcement for beams and columns.

SEISMIC 4

Seismic 4 provides the solutions of the following two equations, whichever gives a higher value of volumetric ratio for spiral reinforcement.

$$\rho_s \geq 0.45 \left(\frac{A_g}{A_c} - 1\right)\frac{f_c^{/}}{f_{yh}} \qquad \textbf{(SE-6)}$$

$$\rho_s \geq 0.12 \frac{f_c^{/}}{f_{yh}} \qquad \textbf{(SE-7)}$$

SEISMIC 5

Seismic 5 gives the confinement steel ratio in each cross-sectional dimension for columns confined by rectilinear reinforcement.

$$\rho_c = \frac{A_{sh}}{sh_c} \qquad \textbf{(SE-8)}$$

where, A_{sh} is the larger of the values obtained from the following two equations.

$$A_{sh} = 0.3 \frac{sh_c f_c^{/}}{f_{yh}}\left(\frac{A_g}{A_{ch}} - 1\right) \qquad \textbf{(SE-9)}$$

$$A_{sh} = 0.09 \frac{sh_c f_c^{/}}{f_{yh}} \qquad \textbf{(SE-10)}$$

SEISMIC 6

Seismic 6 illustrates forces acting on an interior beam-column joint for the computation of joint shear.

SEISMIC 7

Seismic 7 illustrates forces acting on an exterior beam-column joint for the computation of joint shear.