Physics Workbook FOR DUMMIES®

by Steven Holzner, PhD

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Steven Holzner is the award-winning author of more than 100 books, including *Physics For Dummies.* He did his undergraduate work in physics at Massachusetts Institute of Technology (MIT) and got his PhD from Cornell University. He's been on the faculty of Cornell for ten years, teaching Physics 101 and Physics 102, as well as on the faculty of MIT.

Dedication

To Nancy.

Author's Acknowledgments

The book you hold in your hands is the result of many people's work. I would especially like to thank Tracy Boggier, Kelly Ewing, Kathy Simpson, Elizabeth Rea, James Kovalcin, Heather Kolter, and Lynsey Osborn.

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Contents at a Glance

Table of Contents

Introduction

Physics is about the world and everything in it. Physics describes that world and the kinds of things that take place in it. Sometimes, however, physics seems like an imposition from outside — a requirement you have to get through.

That's a shame, because it's *your* world that physics describes. Under the burden of physics problems, though, things can get tough. That's where this book comes in, because it's designed to let you tackle those problems with ease.

Kirchhoff's laws? No problem. Carnot engines? No worries. Everything's here in this book. After you're done reading, you'll be a problem-solving pro.

About This Book

This book is crammed with physics problems, which is the idea; it's designed to show you solutions for the kinds of problems you may encounter in physics classes.

In this book, you can find solutions to problems similar to the ones you're having to deal with. And when you see how it's done, solving similar problems should be a breeze.

You can also read this book in any order you like instead of having to read it from beginning to end. Like other *For Dummies* books, this book is designed to let you move around as much as possible. You don't have to read the chapters in order if you don't want to; this book is yours, and physics is your oyster.

Conventions Used in This Book

Many books have endless conventions that you have to learn before you can start reading. Not this one. In fact, all you need to know is that new terms are given in italics, like *this,* when they're introduced. You should also know that vectors, which are those items that have both a magnitude and a direction, are given in bold, like this: **B**.

Those conventions are really all you have to know; no others are needed.

Foolish Assumptions

I'm assuming that you're using this book in conjunction with a physics class or textbook, because this book keeps the derivation of physical formulas to a minimum. The emphasis here is on solving problems, not deriving formulas. So some knowledge of the physics you're going to be using here is helpful. This book is designed to help you with the nitty-gritty, not to introduce the topics from scratch.

You should also know some algebra. You don't need to be an algebra pro, but you should know how to move items from one side of an equation to another and how to solve for values. Take a look at the discussion in Chapter 1 if you're unsure.

You also need a little knowledge of trigonometry, but not much. Again, take a look at the discussion in Chapter 1, where all the trig you need to know — a grasp of sines and cosines — is reviewed in full.

How This Book Is Organized

To make this book easier to handle, it's divided into parts. The following sections describe what's in each part to help you solve physics problems.

Part I: Applying Physics

This part gets the ball rolling by introducing the foundation you need for the rest of the book. The basics are all here: measuring systems, converting between units, and more.

Part II: May the Forces Be with You

This part covers a topic much prosed in physics: forces. If push comes to shove, you can find it in this part, which describes how to relate force to acceleration, change in momentum, and much more. You also see all about friction and how the force of friction opposes you when you're pushing things.

Part III: Being Energetic: Work

This part is all about energy and work, which are two topics near and dear to every physicist's heart. When you apply some force and move something, you're doing work, and this section gets quantitative on that. If you lift something high, you're giving it potential energy — and when you let it go and it's traveling fast, it's got kinetic energy. You get the lowdown on how to handle problems involving energy and work in this part.

Part IV: Obeying the Laws of Thermodynamics

How hot will that coffee be if you add a cube of ice? How much heat must you add to make that water boil? How much heat must you remove to make that water freeze? Those are the kinds of questions, which involve *thermodynamics* (the science of heat), in this part.

Part V: Zap: Electricity and Magnetism

This part is all about electrons in motion — that is, electrical current and how charges also give rise to magnetism. You discover how to use resistors and other elements in circuits and how to solve for the current in various branches of a circuit. You also discover how to find how much magnetic field a current is going to create — whether it's a straight wire of current or a loop.

Part VI: The Part of Tens

This part contains some good resources: ten great Web sites hosting physics tutorials, for example. And you also see the top ten mistakes people make when they try to solve physics problems — and how to avoid them.

Icons Used in This Book

You find a few icons in this book, and here's what they mean:

This icon points out helpful hints, ideas, or shortcuts that save you time, or that give you alternative ways to think about a particular concept.

This icon marks something to remember, such as a law of physics or a particularly juicy equation.

This icon means that what follows is technical, insider stuff. You don't have to read it if you don't want to, but if you want to become a physics pro (and who doesn't?), take a look.

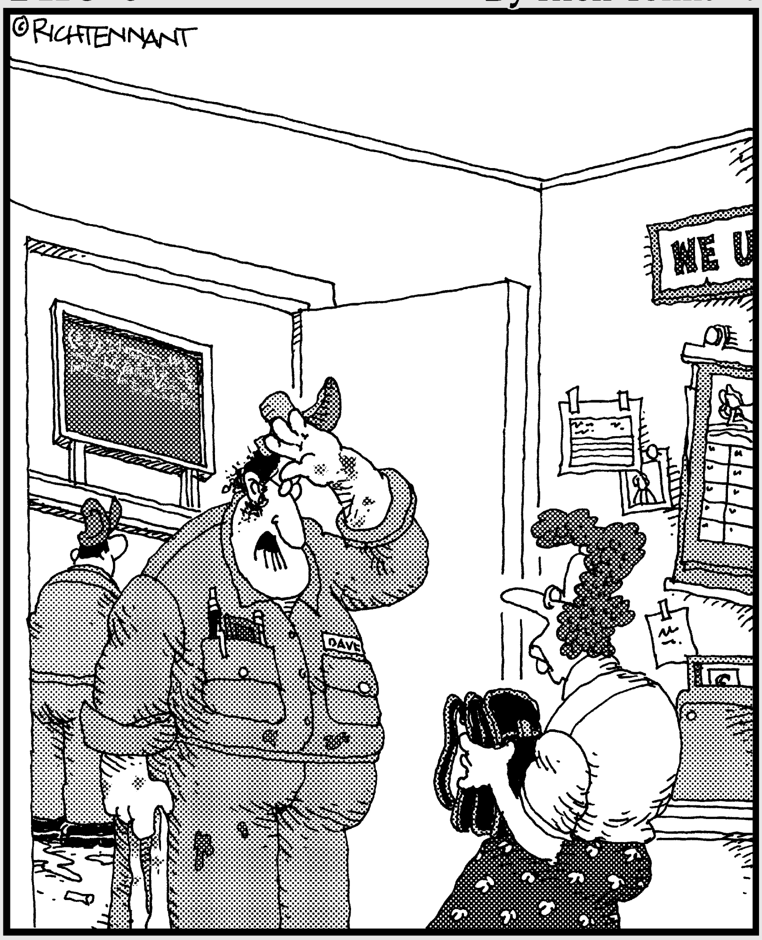
This icon highlights examples that show you how to work each type of problem.

Where to Go from Here

You're ready to jump into Chapter 1. You don't have to start there, of course. You can jump in anywhere you like; the book was written to allow you to do just that. But if you want some important general problem-solving background, take a look at Chapter 1 first.

Part I
Applying Physics

The 5th Wave By Rich Tennant



"I looked over your equations, Mrs. Dundt. Your concavity and inflection points are clean and there's nothing wrong with your velocity and acceleration. It might be your differentiation, but I won't be able to look at it until Thursday."

In this part . . .

This part gives you the story on physics in motion. Physics excels at measuring stuff and making predictions, and armed with just a few key equations, you can become a motion master. The chapters in this part offer up plenty of practice problems on velocity and acceleration, two physics favorites.

Chapter 1

Getting Started with Physics

In This Chapter

- Laying down measurements
- Simplifying with scientific notation
- Practicing conversions
- Drawing on algebra and trigonometry

This chapter gets the ball rolling by discussing some fundamental physics measurements. At its root, physics is all about making measurements (and using those measurements as the basis of predictions), so it's the perfect place to start! I also walk you through the process of converting measurements from one unit to another, and I show you how to apply math skills to physics problems.

Measuring the Universe

A great deal of physics has to do with making measurements — that's the way all physics gets started. For that reason, physics uses a number of measurement systems, such as the CGS (centiment-gram-second) system and the MKS (meter-kilogram-second) system. You also use the standard English system of inches and feet and so on — that's the FPI (foot-pound-inch) system.

REMEMBER

In physics, all measurements (except for some angles) have units, such as meters or seconds. For example, when you measure how far a hockey puck slid, you need to measure both the distance in centimeters and the time in seconds.

For reference, Table 1-1 shows the primary units of measurement (and their abbreviations) in the CGS system. (Don't bother memorizing the ones you're not familiar with now; you can come back to them later as needed.)

Table 1-1 CGS Units of Measurement

Measurement	*Unit*	*Abbreviation*
Length	centimeter	cm
Mass	gram	g
Time	second	s
Force	dyne	dyne

(continued)

Table 1-1 *(continued)*

Measurement	***Unit***	***Abbreviation***
Energy	erg	erg
Pressure	barye	ba
Electric current	biot	Bi
Magnetism	gauss	G
Electric charge	franklin	Fr

These are the measuring sticks that will become familiar to you as you solve problems and triumph over the math in this workbook. Also for reference, Table 1-2 gives you the primary units of measurement in the MKS system.

Table 1-2 MKS Units of Measurement

Measurement	***Unit***	***Abbreviation***
Length	meter	m
Mass	kilogram	kg
Time	second	s
Force	Newton	N
Energy	Joule	J
Pressure	Pascal	P
Electric current	Ampere	A
Magnetism	Tesla	T
Electric charge	Coulomb	C

EXAMPLE

Q. You're told to measure the length of a race-car track using the MKS system. What unit(s) will your measurement be in?

A. The correct answer is meters. The unit of length in the MKS system is the meter.

1. You're told to measure the mass of a marble using the CGS system. What unit(s) will your measurement be in?

Solve It

2. You're asked to measure the time it takes the moon to circle the Earth using the MKS system. What will your measurement's units be?

Solve It

3. You need to measure the force a tire exerts on the road as it's moving using the MKS system. What are the units of your answer?

Solve It

4. You're asked to measure the amount of energy released by a firecracker when it explodes using the CGS system. What are the units of your answer?

Solve It

Putting Scientific Notation to Work

Physics deals with some very large and very small numbers. To work with such numbers, you use *scientific notation.* Scientific notation is expressed as a number multiplied by a power of 10.

For example, suppose you're measuring the mass of an electron in the MKS system. You put an electron on a scale (in practice, electrons are too small to measure on a scale — you have to see how they react to the pull of magnetic or electrostatic forces in order to measure their mass) and you measure the following:

0.0000000000000000000000000000000091 kg

What the heck is that? That's a lot of zeros, and it makes this number very unwieldy to work with. Fortunately, you know all about scientific notation, so you can convert the number into the following:

9.1×10^{-31} kg

That is, 9.1 multiplied by a power of 10, 10^{-31}. Scientific notation works by extracting the power of 10 and putting it on the side, where it's handy. You convert a number to scientific notation by counting the number of places you have to move the decimal point to get the first digit in front of that decimal point. For example, 0.050 is 5.0×10^{-2} because you move the decimal point two places to the right to get 5.0. Similarly, 500 is 5.0×10^{2} because you move the decimal point two places to the left to get 5.0.

Check out this practice question about scientific notation:

Q. What is 0.000037 in scientific notation?

A. The correct answer is 3.7×10^{-5}. You have to move the decimal point five times to the right to get 3.7.

5. What is 0.0043 in scientific notation?

Solve It

6. What is 430000.0 in scientific notation?

Solve It

7. What is 0.00000056 in scientific notation?

Solve It

8. What is 6700.0 in scientific notation?

Solve It

Converting between Units

Physics problems frequently ask you to convert between different units of measurement. For example, you may measure the number of feet your toy car goes in three minutes and thus be able to calculate the speed of the car in feet per minute, but that's not a standard unit of measure, so you need to convert feet per minute to miles per hour, or meters per second, or whatever the physics problem asks for.

For another example, suppose you have 180 seconds — how much is that in minutes? You know that there are 60 seconds in a minute, so 180 seconds equals three minutes. Here are some common conversions between units:

- 1 m = 100 cm = 1000 mm (millimeters)
- 1 km (kilometer) = 1000 m
- 1 kg (kilogram) = 1000 g (grams)
- 1 N (Newton) = 10^5 dynes
- 1 J (Joule) = 10^7 ergs
- 1 P (Pascal) = 10 ba
- 1 A (Amp) = .1 Bi
- 1 T (Tesla) = 10^4 G (Gauss)
- 1 C (Coulomb) = 2.9979×10^9 Fr

The conversion between CGS and MKS is almost always just a factor of 10, so converting between the two is simple. But what about converting to and from the FPI system? Here are some handy conversions that you can come back to as needed:

- **Length:**
  - 1 m = 100 cm
  - 1 km = 1000 m
  - 1 in (inch) = 2.54 cm
  - 1 m = 39.37 in
  - 1 mile = 5280 ft = 1.609 km
  - 1 Å (angstrom) = 10^{-10} m
- **Mass:**
  - 1 kg = 1000 g
  - 1 slug = 14.59 kg
  - 1 u (atomic mass unit) = 1.6605×10^{-27} kg
- **Force:**
  - 1 lb (pound) = 4.448 N
  - 1 N = 10^5 dynes
  - 1 N = 0.2248 lb
- **Energy:**
  - 1 J = 10^7 ergs
  - 1 J = 0.7376 ft-lb

- 1 BTU (British Thermal Unit) = 1055 J
- 1 kWh (kilowatt hour)= 3.600×10^{6} J
- 1 eV (electron Volt) = 1.602×10^{-19} J

- **Power:**
  - 1 hp (horsepower) = 550 ft-lb/s
  - 1 W (Watt)= 0.7376 ft-lb/s

Because conversions are such an important part of physics problems, and because you have to keep track of them so carefully, there's a systematic way of handling conversions: You multiply by a conversion constant that equals one, and where the units you don't want cancel out.

Q. A ball drops 5 meters. How many centimeters did it drop?

A. The correct answer is 500 centimeters. To perform the conversion, you do the following calculation:

$$5.0\ \cancel{\text{meters}} \times \frac{100\ \text{centimeters}}{\cancel{\text{meters}}} = 500\ \text{centimeters}$$

Note that 100 centimeters divided by 1 meter equals 1 because there are 100 centimeters in a meter. In the calculation, the units you don't want — meters — cancel out.

9. How many centimeters are in 2.35 meters?

Solve It

10. How many seconds are in 1.25 minutes?

Solve It

11. How many inches are in 2.0 meters?

Solve It

12. How many grams are in 3.25 kg?

Solve It

Converting Distances

Sometimes you have to make multiple conversions to get what you want. That demands multiple conversion factors. For example, if you want to convert from inches to meters, you can use the conversion that 2.54 centimeters equals 1 inch — but then you have to convert from centimeters to meters, which means using another conversion factor.

Try your hand at this example question that involves multiple conversions:

EXAMPLE

Q. Convert 10 inches into meters.

A. The correct answer is 0.245 m.

1. You know that 1 inch = 2.54 centimeters, so start with that conversion factor and convert 10 inches into centimeters:

$$10\ \cancel{\text{in}} \times \frac{2.54 \times \text{cm}}{1\ \cancel{\text{in}}} = 25.4\ \text{cm}$$

2. Convert 25.4 cm into meters by using a second conversion factor:

$$10\ \cancel{\text{in}} \times \frac{2.54\ \text{cm}}{1\ \cancel{\text{in}}} \times \frac{1\ \text{m}}{100\ \text{cm}} = 0.254\ \text{m}$$

13. Given that there are 2.54 centimeters in 1 inch, how many centimeters are there in 1 yard?

Solve It

14. How many centimeters are in a kilometer?

Solve It

15. How many inches are in an angstrom, given that 1 angstrom (Å) = 10^{-8} cm?

Solve It

16. How many inches are in a meter, given that there are 2.54 cm in 1 inch?

Solve It

Converting Times

Physics problems frequently ask you to convert between different units of time: seconds, minutes, hours, and even years. These times involve all kinds of calculations because measurements in physics books are usually in seconds, but can frequently be in hours.

EXAMPLE

Q. An SUV is traveling 2.78×10^{-2} kilometers per second. What's that in kilometers per hour?

A. The correct answer is 100 km/hr.

1. You know that there are 60 minutes in an hour, so start by converting from kilometers per second to kilometers per minute:

$$2.78 \times 10^{-2}\ \frac{\text{km}}{\cancel{\text{sec}}} \times \frac{60\ \cancel{\text{sec}}}{1\ \text{minute}} = 1.66\ \text{km/minute}$$

2. Because there are 60 minutes in an hour, convert this to kilometers per hour using a second conversion factor:

$$2.78 \times 10^{-2}\ \frac{\text{km}}{\cancel{\text{sec}}} \times \frac{60\ \cancel{\text{sec}}}{1\ \cancel{\text{minute}}} \times \frac{60\ \cancel{\text{minutes}}}{1\ \text{hour}} = 100\ \text{km/hr}$$

17. How many hours are in 1 week?

Solve It

18. How many hours are in 1 year?

Solve It

Counting Significant Figures

You may plug numbers into your calculator and come up with an answer like 1.532984529045, but that number isn't likely to please your instructor. Why? Because in physics problems, you use significant digits to express your answers. *Significant digits* represent the accuracy with which you know your values.

For example, if you know only the values you're working with to two significant digits, your answer should be 1.5, which has two significant digits, not 1.532984529045, which has 13! Here's how it works: Suppose you're told that a skater traveled 10.0 meters in 7.0 seconds. Note the number of digits: The first value has three significant figures, the other only two. The rule is that when you multiply or divide numbers, the result has the number of significant digits that equals the smallest number of significant digits in any of the original numbers. So if you want to figure out how fast the skater was going, you divide 10.0 by 7.0, and the result should have only two significant digits — 1.4 meters per second.

Zeros used just to fill out values down to (or up to) the decimal point aren't considered significant. For example, the number 3600 has only two significant digits by default. That's not true if the value was actually measured to be 3600, of course, in which case it's usually expressed as 3600.; the final decimal indicates that all the digits are significant.

On the other hand, when you're adding or subtracting numbers, the rule is that the last significant digit in the result corresponds to the right-most column in the addition or subtraction. How does that work? Take a look at this addition example:

$$\begin{array}{r} 5.1 \\ +12 \\ +\ \ 7.73 \\ \hline 24.83 \end{array}$$

So is the result 24.83? No, it's not. The 12 has no significant digits to the right of the decimal point, so the answer shouldn't have any either. That means you should round the value of the result up to 25.

Rounding numbers in physics works as it usually does in math: When you want to round to three places, for example, and the number in the fourth place is a five or greater, you add one to the third place (and ignore or replace with zeros any following digits).

Q. You're multiplying 12.01 by 9.7. What should your answer be, keeping in mind that you should express it in significant digits?

A. The correct answer is **120.**

1. The calculator says that the product is 116.497.
2. Your result has to have the same number of significant digits as the least number of any two values you multiplied. That's two here (because of 9.7), so your answer rounds up to 120.

19. What is 19.3 multiplied by 26.12, taking into account significant digits?

Solve It

20. What is the sum of 7.9, 19, and 5.654, taking into account significant digits?

Solve It

Coming Prepared with Some Algebra

It's a fact of life: You need to be able to do algebra to handle physics problems. Take the following equation, for example, which relates the distance something has traveled (s) to its acceleration and the time it has been accelerated:

$$s = \frac{1}{2}at^2$$

Now suppose that the physics problem actually asks you for the acceleration, not the distance. You have to rearrange things a little here to solve for the acceleration. So when you multiply both sides by 2 and divide both sides by t^2, here's what you get:

$$\frac{2}{t^2} \cdot s = \frac{2}{t^2} \cdot \frac{1}{2} \cdot a \cdot t^2$$

Cancelling out and swapping sides, you solve for a like this:

$$a = \frac{2 \cdot s}{t^2}$$

So that's putting a little algebra to work. All you had to do was move variables around the equation to get what you want. The same approach works when solving physics problems (most of the time). On the other hand, what if you had to solve the same problem for the time, t? You would do that by rearranging the variables like so:

$$t = \sqrt{2s/a}$$

The lesson in this example is that you can extract all three variables — distance, acceleration, and time — from the original equation. Should you memorize all three versions of this equation? Of course not. You can just memorize the first version and use a little algebra to get the rest.

The following practice questions call on your algebra skills:

Q. The equation for final speed, v_f, where the initial speed was v_o, the acceleration was a, and the time was t is $v_f - v_o = at$. Solve for acceleration.

A. The correct answer is $a = \left(v_f - v_o\right)/t$

To solve for a, divide both sides of the equation by time, t.

21. The equation for potential energy, PE, of a mass m at height h, where the acceleration due to gravity is g, is $PE = m \cdot g \cdot h$. Solve for h.

Solve It

22. The equation relating final speed, v_f, to original speed, v_o, in terms of acceleration a and distance s is $v_f^2 - v_o^2 = 2as$. Solve for s.

Solve It

23. The equation relating distance s to acceleration a, time t, and speed v is $s = v_o \cdot t + \frac{1}{2} \cdot a \cdot t^2$. Solve for v_o.

Solve It

24. The equation for kinetic energy is $KE = \frac{1}{2} \cdot m \cdot v^2$. Solve for v, given KE and m.

Solve It

Being Prepared with Trigonometry

Physics problems also require you to have some trigonometry under your belt. To see what kind of trig you need, take a look at Figure 1-1, which shows a right triangle. The long side is called the *hypotenuse,* and the angle between x and y is 90°.

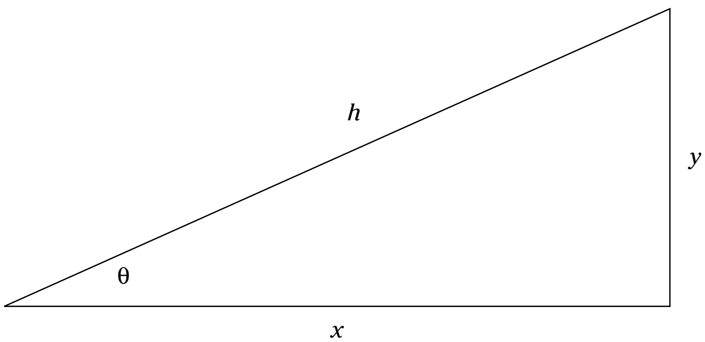


Figure 1-1: A triangle.

Physics problems require you to be able to work with sines, cosines, and tangents. Here's what they look like for Figure 1-1:

$\sin\theta = y/h$

$\cos\theta = x/h$

$\tan\theta = y/x$

You can find the length of one side of the triangle if you're given another side and an angle (not including the right angle). Here's how to relate the sides:

$x = h \cdot \cos\theta = y/\tan\theta$

$y = h \cdot \sin\theta = x \tan\theta$

$h = y/\sin\theta = h/\cos\theta$

And here's one more equation, the Pythagorean Theorem. It gives you the length of the hypotenuse when you plug in the other two sides:

$$h = \sqrt{x^2 + y^2}$$

25. Given the hypotenuse h and the angle θ, what is the length x equal to?

Solve It

26. If x = 3 and y = 4, what is the length of h?

Solve It

Answers to Problems about Getting Started with Physics

The following are the answers to the practice questions presented earlier in this chapter. You see how to work out each answer, step by step.

1 **grams**

The unit of mass in the CGS system is the gram.

2 **seconds**

The unit of time in the MKS system is the second.

3 **Newtons**

The unit of force in the MKS system is the Newton.

4 **ergs**

The unit of energy in the CGS system is the erg.

5 **4.3×10^{-3}**

You have to move the decimal point three places to the right.

6 **4.3×10^{5}**

You have to move the decimal point five places to the left.

7 **5.6×10^{-7}**

You have to move the decimal point seven places to the right.

8 **6.7×10^{3}**

You have to move the decimal point three places to the left.

9 **235 cm**

Convert 2.35 meters into centimeters:

$$2.35\ \cancel{\text{m}} \times \frac{100\ \text{cm}}{1\ \cancel{\text{m}}} = 235\ \text{cm}$$

10 **75 sec**

Convert 1.25 minutes into seconds:

$$1.25\ \cancel{\text{min}} \times \frac{60\ \text{sec}}{1\ \cancel{\text{min}}} = 75\ \text{sec}$$

11 **78.6 in**

Convert 2.0 meters into inches:

$$2.0\ \cancel{\text{m}} \times \frac{39.3\ \text{in}}{1\ \cancel{\text{m}}} = 78.6\ \text{in}$$

12 **3250 g**

Convert 3.25 kilograms into grams:

$$3.25\ \cancel{\text{kg}} \times \frac{1000\ \text{g}}{1\ \cancel{\text{kg}}} = 3250\ \text{g}$$

13 **91.4 cm**

1. 1 yard is 3 feet, so convert that to inches:

$$3\ \cancel{ft} \times \frac{12\ \text{in}}{1\ \cancel{ft}} = 36\ \text{in}$$

2. Use a second conversion factor to convert that into centimeters:

$$3\ \cancel{ft} \times \frac{12\ \cancel{in}}{1\ \cancel{ft}} \times \frac{2.54\ \text{cm}}{1\ \cancel{in}} = 91.4\ \text{cm}$$

14 **1.0 × 10^{-5} km**

1. Convert 1 centimeter to meters:

$$1\ \cancel{cm} \times \frac{1\ \text{m}}{100\ \cancel{cm}} = 1.0 \times 10^{-2}\ \text{m}$$

2. Use a second conversion factor to convert that into kilometers:

$$1\ \cancel{cm} \times \frac{1\ \cancel{m}}{100\ \cancel{cm}} \times \frac{1\ \text{km}}{1000\ \cancel{m}} = 1.0 \times 10^{-5}\ \text{km}$$

15 **4.0 × 10^{-9} in**

1. Convert 1 angstrom to centimeters:

$$1\ \cancel{\mathring{A}} \times \frac{10^{-8}\ \text{cm}}{1\ \cancel{\mathring{A}}} = 10^{-8}\ \text{cm}$$

2. Use a second conversion factor to convert that into inches:

$$1\ \cancel{\mathring{A}} \times \frac{10^{-8}\ \cancel{cm}}{1\ \cancel{\mathring{A}}} \times \frac{1.0\ \text{in}}{2.54\ \cancel{cm}} = 4.0 \times 10^{-9}\ \text{in}$$

16 **39.3 in**

1. Convert 1 meter into centimeters:

$$1\ \cancel{m} \times \frac{100\ \text{cm}}{1\ \cancel{m}} = 100\ \text{cm}$$

2. Use a second conversion factor to convert that into inches:

$$1\ \cancel{m} \times \frac{100\ \cancel{cm}}{1\ \cancel{m}} \times \frac{1\ \text{in}}{2.54\ \cancel{cm}} = 39.3\ \text{in}$$

17 **168 hours**

1. Convert 1 week into days:

$$1\ \cancel{\text{week}} \times \frac{7\ \text{days}}{1\ \cancel{\text{week}}} = 7\ \text{days}$$

2. Use a second conversion factor to convert that into hours:

$$1\ \cancel{\text{week}} \times \frac{7\ \cancel{\text{days}}}{1\ \cancel{\text{week}}} \times \frac{24\ \text{hours}}{1\ \cancel{\text{day}}} = 168\ \text{hours}$$

18 **8760 hours**

1. Convert 1 year into days:

$$1\ \cancel{\text{year}} \times \frac{365\ \text{days}}{1\ \cancel{\text{year}}} = 365\ \text{days}$$

2. Use a second conversion factor to convert that into hours:

$$1\ \cancel{\text{year}} \times \frac{365\ \cancel{\text{days}}}{1\ \cancel{\text{year}}} \times \frac{24\ \text{hours}}{1\ \cancel{\text{day}}} = 8760\ \text{hours}$$

19 **504**

1. The calculator says the product is 504.116.
2. 19.3 has three significant digits, and 26.12 has four, so you use three significant digits in your answer. That makes the answer 504.

20 **33**

1. Here's how you do the sum:

$$\begin{array}{r} 7.9 \\ +19 \\ +\ 5.654 \\ \hline 32.554 \end{array}$$

2. The value 19 has no significant digits after the decimal place, so the answer shouldn't either, making it 33 (32.554 rounded up).

21 **h = PE/mg**

Divide both sides by mg to get your answer.

22 $\mathbf{\dfrac{v_f^2 - v_o^2}{2a} = s}$

Divide both sides by 2a to get your answer.

23 $\mathbf{\dfrac{s}{t} - \dfrac{1}{2}at = v}$

1. Subtract $at^2/2$ from both sides:

$$s - \frac{1}{2}at^2 = vt$$

2. Divide both sides by t to get your answer.

24 $\mathbf{v = \sqrt{\dfrac{2KE}{m}}}$

1. Multiply both sides by 2/m:

$$\frac{2}{m}KE = v^2$$

2. Take the square root to get your answer.

25 **x = h cos θ**

Your answer comes from the definition of cosine.

26 **5**

1. Start with the Pythagorean theorem:

$$h = \sqrt{x^2 + y^2}$$

2. Plug in the numbers, and work out the answer:

$$h = \sqrt{3^2 + 4^2} = 5$$

Chapter 2

The Big Three: Acceleration, Distance, and Time

In This Chapter

- Thinking about displacement
- Checking out speed
- Remembering acceleration

Being able to connect displacement, speed, and acceleration is fundamental to working with physics. These things concern people every day, and physics has made an organized study of them.

Problems that connect displacement, speed, and acceleration are all about understanding movement, and that's the topic of this chapter — putting numbers into the discussion. You'll often find physics problems about cars starting and stopping, horses racing, and rocket ships zooming back and forth. And after you finish this chapter, you'll be a real pro at solving them.

From Point A to B: Displacement

Displacement occurs when something moves from here to there. For example, suppose that you have a ball at the zero position, as in Figure 2-1A.

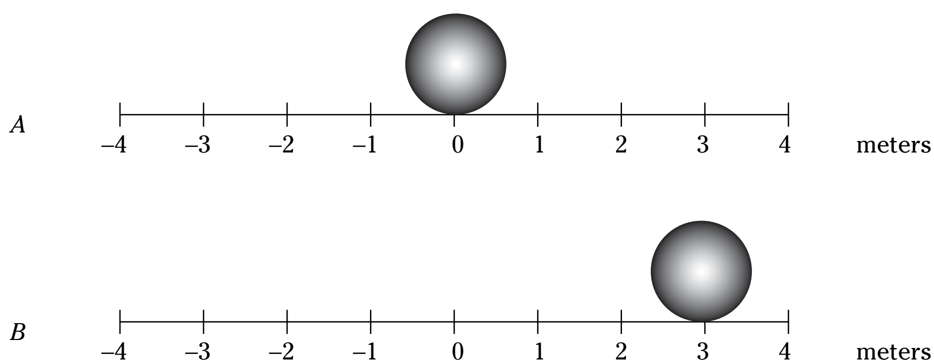


Figure 2-1: A moving ball.

Now suppose that the ball rolls over to a new point, 3 meters to the right, as you see in Figure 2-1B. The ball is at a new location, so there's been displacement. In this case, the displacement is just 3 meters to the right. In physics terms, you'll often see displacement referred to as the variable s. In this case, $s = +3$ meters.

Like any other measurement in physics, displacement is always expressed in units, usually centimeters or meters, as in this example. Of course, you also can use kilometers, inches, feet, miles, or even *light years* (the distance light travels in one year — 5,865,696,000,000 miles).

The following example question focuses on displacement.

EXAMPLE

Q. You've taken the pioneers' advice to "Go West." You started in New York City and went west 10 miles the first day, 14 miles the next day, and then back east 9 miles on the third day. What is your displacement from New York City after three days?

A. s = 15 miles west of New York City

1. You first went west 10 miles, so at the end of the first day, your displacement was 10 miles west.
2. Next, you went west 14 days, putting your displacement at 10 + 14 miles = 24 miles west of New York City.
3. Finally, you traveled 9 miles east, leaving you at 24 – 9 = 15 miles west of New York City. So s = 15 miles west of New York City.

1. Suppose that the ball in Figure 2-1 now moves 1 meter to the right. What is its new displacement from the origin, 0?

Solve It

2. Suppose that the ball in Figure 2-1, which started 4 meters to the right of the origin, moves 6 meters to the left. What is its new displacement from the origin — in inches?

Solve It

Reading That Speedometer

In physics terms, what is speed? It's the same as the conventional idea of speed: *Speed* is displacement divided by time.

For example, if you went a displacement *s* in a time *t*, then your speed, *v*, is determined as follows:

$$v = \frac{s}{t}$$

Technically speaking, speed is the change in position divided by the change in time, so you also can represent it like this if, for example, you're moving along the x axis:

$$v = \frac{\Delta x}{\Delta t} = \frac{x_f - x_o}{t_f - t_o}$$

Q. Suppose that you want to drive from New York City to Los Angeles to visit your uncle's family, a distance of about 2781 miles. The trip takes you four days. What was your speed in miles per hour?

A. $v = \frac{\Delta x}{\Delta t} = \frac{x_f - x_o}{t_f - t_o} = 28.97$ miles per hour

1. Start by figuring out your speed (the distance traveled divided by the time taken to travel that distance):

$$\frac{2781 \text{ miles}}{4 \text{ days}} = 695.25$$

2. Okay, the speed is 695.25, but 695.25 *what?* This solution divides miles by days, so it's 695.25 miles per day — not exactly a standard unit of measurement. So what is that in miles per hour? To determine that, you cancel "days" out of this equation and put in "hours." Because 24 hours are in a day, you can multiply as follows (note that "days" cancel out, leaving miles over hours, or miles per hour):

$$\frac{2781 \text{ miles}}{4 \cancel{\text{days}}} \times \frac{1 \cancel{\text{day}}}{24 \text{ hours}} = 28.97 \text{ miles per hour}$$

So your speed was 28.97 miles per hour. That's your average speed, averaged over both day and night.

3. Suppose that you used your new SpeedPass to get you through the tollbooths at both ends of your trip, which was 90 miles on the turnpike and took you 1 hour and 15 minutes. On your return home, you're surprised to find a traffic ticket for speeding in the mail. How fast did you go, on average, between the tollbooths? Was the turnpike authority justified in sending you a ticket, given that the speed limit was 65 mph?

Solve It

4. Suppose that you and a friend are determined to find out whose car is faster. You both start your trips in Chicago. Driving nonstop, you reach Los Angeles — a distance of 2018 miles — in 1.29 days, and your friend, also driving nonstop, reaches Miami — a distance of 1380 miles — in 0.89 days. Whose car was faster?

Solve It

Putting Pedal to Metal: Acceleration

In physics terms, *acceleration* is the amount by which your speed changes in a given amount of time. In terms of equations, it works like this:

$$a = \frac{\Delta v}{\Delta t}$$

Given initial and final velocities, v_o and v_f, and initial and final times over which your speed changed, t_o and t_f, you can also write the equation like this:

$$a = \frac{\Delta v}{\Delta t} = \frac{v_f - v_o}{t_f - t_o}$$

To get the units of acceleration, you divide speed by time as follows:

$$a = \frac{v_f - v_o}{t_f - t_o} = \frac{\text{distance/time}}{\text{time}} = \frac{\text{distance}}{\text{time}^2}$$

Distance over time squared? Don't let that throw you. You end up with time squared in the denominator just because it's velocity divided by time — that's something you get used to when solving physics problems. In other words, acceleration is the *rate* at which your speed changes because rates have time in the denominator.

So for acceleration, you can expect to see units of meters per second^2, or centimeters per second^2, or miles per second^2, or feet per second^2, or even kilometers per hour^2.

Q. Suppose that you're driving at 75 miles an hour and suddenly see red flashing lights in the rearview mirror. "Great," you think, and you pull over, taking 20 seconds to come to a stop. You could calculate how quickly you decelerated as you were pulled over (information about your law-abiding tendencies that, no doubt, would impress the officer). So just how fast did you decelerate, in cm/sec^2?

A. $a = \frac{\Delta v}{\Delta t} = \frac{3350 \text{ cm/sec}}{20 \text{ seconds}} = 168 \text{ cm/sec}^2$

1. First convert to miles per second:

$$\frac{75 \text{ miles}}{\cancel{\text{hour}}} \times \frac{1 \ \cancel{\text{hour}}}{60 \ \cancel{\text{minutes}}} \times \frac{1 \ \cancel{\text{minute}}}{60 \text{ seconds}} = .0208 = 2.08 \times 10^{-2} \text{ miles per second}$$

2. Convert from miles per second to inches per second:

$$\frac{2.08 \times 10^{-2} \ \cancel{\text{miles}}}{\text{second}} \times \frac{5280 \ \cancel{\text{feet}}}{1 \ \cancel{\text{mile}}} \times \frac{12 \text{ inches}}{1 \ \cancel{\text{foot}}} = \frac{1318 \text{ in}}{\text{second}}$$

3. Your speed was 1318 inches per second. What's that in centimeters per second?

$$\frac{2.08 \times 10^{-2} \ \cancel{\text{miles}}}{\text{second}} \times \frac{5280 \ \cancel{\text{feet}}}{1 \ \cancel{\text{mile}}} \times \frac{12 \ \cancel{\text{inches}}}{1 \ \cancel{\text{foot}}} \times \frac{2.54 \text{ centimeters}}{\cancel{\text{inch}}} = \frac{3350 \text{ cm}}{\text{second}}$$

4. What was your acceleration? That calculation looks like this:

$$a = \frac{\Delta v}{\Delta t} = \frac{v_f - v_o}{t_f - t_o} = \frac{0 - 3350 \text{ cm/second}}{20 \text{ seconds}} = -168 \text{ cm/sec}^2$$

In other words, –168 cm/sec^2, *not* +168 cm/sec^2. There's a big difference between positive and negative in terms of solving physics problems — and in terms of law enforcement. If you accelerated at +168 cm/sec^2 instead of accelerating at –168 cm/sec^2, you'd end up going 150 miles per hour at the end of 20 seconds, not 0 miles per hour.

In other words, the *sign* of the acceleration tells you *how* the speed is changing. A positive acceleration means that the speed is increasing in the positive direction, and a negative acceleration (also known as *deceleration*) tells you that the speed is increasing in the negative direction.

5. A rocket ship is going to land on the moon in exactly 2 hours. There's only one problem: It's going 17,000 miles an hour. What does its deceleration need to be, in miles per $second^2$, in order to land on the moon safely at 0 miles per hour?

Solve It

6. You're stopped at a red light when you see a monster SUV careening toward you. In a lightning calculation, you determine you have 0.8 seconds before it hits you and that you must be going at least 1.0 miles an hour forward at that time to avoid the SUV. What must your acceleration be, in miles per $hour^2$? Can you avoid the SUV?

Solve It

7. A bullet comes to rest in a block of wood in 1.0×10^{-2} seconds, with an acceleration of -8.0×10^4 meters per $second^2$. What was its original speed, in meters per second?

Solve It

8. The light turns red, and you come to a screeching halt. Checking your stopwatch, you see that you stopped in 4.5 seconds. Your deceleration was 1.23×10^{-3} miles per $second^2$. What was your original speed in miles per hour?

Solve It

Connecting Acceleration, Time, and Displacement

You know that you can relate speed with displacement and time. And you know that you can relate speed and time to get acceleration. You also can relate displacement with acceleration and time:

$$s = \frac{1}{2}a\left(t_f - t_o\right)^2$$

If you don't start off at zero speed, you use this equation:

$$s = v_o\left(t_f - t_o\right) + \frac{1}{2}a\left(t_f - t_o\right)^2$$

Q. You climb into your drag racer, waving nonchalantly at the cheering crowd. You look down the quarter-mile track, and suddenly the flag goes down. You're off, getting a tremendous kick from behind as the car accelerates quickly. A brief 5.5 seconds later, you pass the end of the course and pop the chute.

You know the distance you went: 0.25 miles, or about 402 meters. And you know the time it took: 5.5 seconds. So just how hard was the kick you got — the acceleration — when you blasted down the track?

A. 26.6 meters/second^2

1. You know that

$$s = \frac{1}{2}a\ t^2$$

You can rearrange this equation with a little algebra (just divide both sides by t^2 and multiply by 2) to get

$$a = \frac{2s}{t^2}$$

2. Plugging in the numbers, you get

$$a = \frac{2s}{t^2} = \frac{2\ (402\ \text{meters})}{(5.5\ \text{seconds})^2} = 26.6\ \text{meters/second}^2$$

What's 26.6 meters/second^2 in more understandable terms? The acceleration due to gravity, *g,* is 9.8 meters/second^2, so this is about 2.7 g. And that's quite a kick.

9. The light turns green, and you accelerate at 10 meters per second^2. After 5 seconds, how far have you traveled?

Solve It

10. A stone drops under the influence of gravity, 9.8 meters per second^2. How far does it drop in 12 seconds?

Solve It

11. A car is going 60 miles per hour and accelerating at 10 miles per hour^2. How far does it go in 1 hour?

Solve It

12. A motorcycle is going 60 miles per hour, and decelerating at 60 miles per hour^2. How far does it go in 1 hour?

Solve It

13. An eagle starts at a speed of 50 meters per second and, decelerating at 10 meters per second^2, comes to rest on a peak 5 seconds later. How far is the peak from the eagle's original position?

Solve It

14. A trailer breaks loose from its truck on a steep incline. If the truck was moving uphill at 20 meters per second when the trailer broke loose, and the trailer accelerates down the hill at 10.0 meters per second^2, how far downhill does the trailer go after 10 seconds?

Solve It

15. A block of wood is shooting down a track at 10 meters per second and is slowing down because of friction. If it comes to rest in 20 seconds and 100 meters, what is its deceleration, in meters per second^2?

Solve It

16. A minivan puts on the brakes and comes to a stop in 12 seconds. If it took 200 meters to stop, and decelerates at 10 meters per second^2, how fast was it originally going, in meters per second?

Solve It

Connecting Speed, Acceleration, and Displacement

Suppose you have a drag racer whose acceleration is 26.6 meters/second^2, and its final speed was 146.3 meters per second. What is the total distance traveled?

This scenario sets you up to use one of the important equations of motion:

$$v_f^2 - v_o^2 = 2as = 2a\,(x_f - x_o)$$

This is the equation you use to relate speed, acceleration, and distance.

EXAMPLE

Q. A drag racer's acceleration is 26.6 meters/second^2, and at the end of the race, its final speed is 146.3 meters per second. What is the total distance the drag racer traveled?

A. $s = \frac{1}{2a} v_f^2 = \frac{1}{2(26.6)} (146.3)^2 = 409 \text{ meters}$

1. To solve this problem, you need to relate speed, acceleration, and distance, so you start with this equation:

$$v_f^2 - v_o^2 = 2as = 2a\,(x_f - x_o)$$

2. In this scenario, v_o is 0, which makes this equation simpler:

$$v_f^2 = 2as$$

3. Solve for s:

$$s = \frac{1}{2a} v_f^2$$

4. Plug in the numbers:

$$s = \frac{1}{2a} v_f^2 = \frac{1}{2(26.6)} (146.3)^2 = 409 \text{ meters}$$

So the answer is 409 meters, about a quarter of a mile — standard for a drag racing track.

17. A bullet is accelerated over a meter-long rifle barrel at an acceleration of 400,000 meters per second^2. What is its final speed?

Solve It

18. A car starts from rest and is accelerated at 5.0 meters per second^2. What is its speed 500 meters later?

Solve It

19. A rocket is launched at an acceleration of 100 meters per second^2. After 100 kilometers, what is its speed in meters per second?

Solve It

20. A motorcycle is going 40 meters per second and is accelerated at 6 meters per second^2. What is its speed after 200 meters?

Solve It

Answers to Problems about Acceleration, Distance, and Time

The following are the answers to the practice questions presented earlier in this chapter. You see how to work out each answer, step by step.

1 **s = 4 meters**

The ball was originally at +3 meters and moved 1 meter to the right. +3 + 1 = 4 meters.

2 **s = –78.6 inches**

1. The ball started at 4 meters and moved 6 meters to the left, putting it at +4.0 – 6.0 = –2.0 meters with respect to the origin.
2. Convert –2.0 meters into inches:

$$-2.0 \ \cancel{\text{meters}} \times \frac{39.3 \text{ inches}}{1 \ \cancel{\text{meter}}} = -78.6 \text{ inches}$$

3 **v = 72 miles an hour. The ticket was justified.**

1. It took you one hour and fifteen minutes, or 1.25 hours, to travel 90 miles.
2. Divide 90 miles by 1.25 hours:

$$\frac{90 \text{ miles}}{1.25 \text{ hours}} = 72 \text{ miles/hour}$$

4 **Your speed = 1564 miles per day; your friend's speed = 1550 miles per day. You're faster.**

1. Note that to simply compare speeds, there's no need to convert to miles per hour — miles per day will do fine. First, calculate your speed:

$$\frac{2018 \text{ miles}}{1.29 \text{ days}} = 1564 \text{ miles/day}$$

2. Next, calculate your friend's speed:

$$\frac{1380 \text{ miles}}{0.89 \text{ days}} = 1550 \text{ miles/day}$$

So you were faster than your friend and probably more tired at the end of your trip.

5 **6.6 x 10^{-4} miles per second^2**

1. Start by converting 17,000 miles per hour into miles per second:

$$\frac{17{,}000 \text{ miles}}{\cancel{\text{hour}}} \times \frac{1 \ \cancel{\text{hour}}}{60 \text{ minutes}} \times \frac{1 \ \cancel{\text{minute}}}{60 \text{ seconds}} = \mathbf{4.72 \text{ miles per second}}$$

2. To land on the moon, v_f must be 0 miles per second, and $t_f - t_o$ = 2 hours, or 2 × 3600 seconds = 7200 seconds, so:

$$a = \frac{\Delta v}{\Delta t} = \frac{v_f - v_o}{t_f - t_o} = \frac{4.72 - 0}{7200 - 0}$$

3. Calculating this yields

$$a = \frac{\Delta v}{\Delta t} = \frac{v_f - v_o}{t_f - t_o} = \frac{4.72 - 0}{7200 - 0} = 0.00066 = 6.6 \times 10^{-4} \text{ miles per second}^2$$

So the rocket needs a constant deceleration of 6.6 × 10^{-4} miles per second^2 in order to land on the moon at a speed of 0 miles per second, touching down lightly.

6 **4.5×10^3 miles per hour^2**

You will avoid the collision.

1. Start by converting 0.8 seconds into hours in order to get all the quantities in units you want, miles and hours:

$$0.8 \text{ seconds} \times \frac{1 \text{ minute}}{60 \text{ seconds}} \times \frac{1 \text{ hour}}{60 \text{ minutes}} = 2.22 \times 10^{-4} \text{ hours}$$

2. Calculate the acceleration needed to get you to 1.0 miles per hour:

$$a = \frac{\Delta v}{\Delta t} = \frac{v_f - v_o}{t_f - t_o} = \frac{1.0 - 0}{2.22 \times 10^{-4}} = 4.5 \times 10^3 \text{ miles per hour}^2$$

That's about 0.05g, which is possible for any car. You'll avoid the SUV.

7 **$\Delta v = a(\Delta t) = 800$ meters per second**

1. You can calculate the change in speed, because it is acceleration multiplied by time:

$$\Delta v = a(\Delta t) = -8.0 \times 10^4 \times 1.0 \times 10^{-2} = -800 \text{ meters per second}$$

2. So if the bullet lost 800 meters per second of speed to come to a rest ($v = 0$), it must have been going 800 meters per second originally.

8 **$\Delta v = a(\Delta t) = 20$ miles per hour**

1. The change in speed is acceleration multiplied by time, so:

$$\Delta v = a(\Delta t) = 1.23 \times 10^{-3} \times 4.5 = 5.55\ 10^{-3} \text{ miles per second}$$

2. Convert this result to miles per hour:

$$\frac{5.55 \times 10^{-3} \text{ miles}}{\text{second}} \times \frac{60 \text{ seconds}}{1 \text{ minute}} \times \frac{60 \text{ minutes}}{1 \text{ hour}} = 20 \text{ miles/hour}$$

9 **125 meters**

1. You want to relate distance to acceleration and time, so use this equation:

$$s = \frac{1}{2} a\ t^2$$

2. Plug in the numbers:

$$s = \frac{1}{2} a\ t^2 = \frac{1}{2} 10.0\ 5^2 = 125 \text{ meters}$$

10 **–705 meters (that's 705 meters downward)**

1. To relate distance to acceleration and time, you use this equation:

$$s = \frac{1}{2} a\ t^2$$

2. Substitute the numbers:

$$s = \frac{1}{2} a\ t^2 = \frac{1}{2}(-9.8)\ 12^2 = -705 \text{ meters}$$

11 **65 miles**

1. To relate distance to speed, acceleration, and time, you use this equation:

$$s = v_o(t_f - t_o) + \frac{1}{2} a (t_f - t_o)^2$$

2. Plug in the numbers:

$$s = v_0(t_f - t_o) + \frac{1}{2} a (t_f - t_o)^2 = 60(1.0) + \frac{1}{2} 10\ 1.0^2 = 65 \text{ miles}$$

12 **30 miles**

1. You want to relate distance to speed, acceleration, and time, so you use this equation:

$$s = v_o(t_f - t_o) + \frac{1}{2}a(t_f - t_o)^2$$

2. Plug in the numbers:

$$s = v_0(t_f - t_o) + \frac{1}{2}a(t_f - t_o)^2 = 60(1.0) - \frac{1}{2}60\ 1.0^2 = 30 \text{ miles}$$

13 **125 meters**

1. To connect distance with speed, acceleration, and time, you use this equation:

$$s = v_o(t_f - t_o) + \frac{1}{2}a(t_f - t_o)^2$$

2. Plug in the numbers:

$$s = v_0(t_f - t_o) + \frac{1}{2}a(t_f - t_o)^2 = 50(5.0) - \frac{1}{2}10\ 5^2 = 125 \text{ meters}$$

14 **–300 meters**

1. To relate acceleration to speed and time, use this equation:

$$s = v_o(t_f - t_o) + \frac{1}{2}a(t_f - t_o)^2$$

2. Plug in the numbers:

$$s = v_0(t_f - t_o) + \frac{1}{2}a(t_f - t_o)^2 = 20\ 10 - \frac{1}{2}10.0\ 10^2 = -300 \text{ meters}$$

15 **–0.5 meters per second^2**

1. To relate acceleration to speed and time, use this equation:

$$s = \frac{1}{2}a\ t^2 + v_o t$$

2. Solve for a:

$$a = \frac{2(s - v_o t)}{t^2}$$

3. Plug in the numbers:

$$a = \frac{2(s - v_o t)}{t^2} = \frac{2(100 - 10 \times 20)}{20^2} = -0.5 \text{ meters per second}$$

16 **76.6 meters per second**

1. Start with this equation:

$$s = v_o(t_f - t_o) + \frac{1}{2}a(t_f - t_o)^2$$

2. Solve for v_0:

$$v_0 = \left[s - \frac{1}{2}a(t_f - t_o)^2\right]\ \frac{1}{t_f - t_o}$$

3. Plug in the numbers:

$$v_0 = \left[s - \frac{1}{2}a(t_f - t_o)^2\right]\ \frac{1}{t_f - t_o} = 76.6 \text{ meters per second}$$

17 **v_f = 894 meters per second**

1. Start with this equation:

$$v_f^2 - v_o^2 = 2as = 2a\,(x_f - x_o)$$

2. v_o is 0, so that makes things easier. Plug in the numbers:

$$v_f^2 = 2as = 2\ 400{,}000\ 1.0 = 800{,}000\ (\text{meters per second})^2$$

3. Take the square root:

$$v_f = 894 \text{ meters per second}$$

18 **v_f = 70.7 meters per second**

1. You want to find speed in terms of distance and acceleration, so use this equation:

$$v_f^2 - v_o^2 = 2as = 2a\,(x_f - x_o)$$

2. Plug in the numbers:

$$v_f^2 = 2as = 2\ 5\ 500 = 5{,}000\ (\text{meters per second})^2$$

3. Take the square root:

$$v_f = 70.7 \text{ meters per second}$$

19 **v_f = 4470 meters per second**

1. You want to find the speed of the rocket ship, having been given distance and acceleration, so use this equation:

$$v_f^2 - v_o^2 = 2as = 2a\,(x_f - x_o)$$

2. 100 kilometers is 100,000 meters, so plug in the numbers:

$$v_f^2 = 2as = 2\ 100\ 100{,}000 = 2.0 \times 10^7\ (\text{meters per second})^2$$

3. Take the square root to get the rocket's speed:

$$v_f = 4470 \text{ meters per second}$$

20 **v_f = 63.2 meters per second**

1. To determine the motorcycle's final speed, use this equation:

$$v_f^2 - v_o^2 = 2as$$

2. v_0 = 40 meters per second, so plug in the numbers:

$$v_f^2 - 40^2 = 2as = 2\ 6\ 200 = 2400\ (\text{meters per second})^2$$

3. That means that v_f^2 is

$$v_f^2 = 4000\ (\text{meters per second})^2$$

4. Take the square root to get v_f:

$$v_f = 63.2 \text{ meters per second}$$

Chapter 3

Vectors: Knowing Where You're Headed

In This Chapter

- Understanding what makes a vector
- Expressing vectors in different forms
- Converting vectors to different forms
- Adding vectors
- Expressing motion as vectors

Although an object in motion has a speed, it also has a direction. Together, the speed and the direction describe the object's motion. For example, you may be heading off to your grandmother's house at 60 miles/hour, but unless you're pointed in the right direction, you're not going to get there.

Physics takes note of the fact that objects in motion need two quantities to fully describe that motion — speed and direction — by saying that such motion is a *vector* quantity.

Creating a Vector

A *vector* is a combination of exactly two values: a magnitude (like the speed of an object in motion) and a direction (such as the direction of an object in motion). All kinds of things can be described with vectors, including constant motion, acceleration, displacement, magnetic fields, electric fields, and many more.

Vectors are defined by a *magnitude* (the length of the vector) and a direction. For example, take a look at the vector in Figure 3-1.

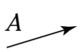


Figure 3-1: A vector.

In physics, vectors are written in bold type. The vector in Figure 3-1 — I'll call it **A** — represents the displacement of a golf ball from the tee. Its length is 100 yards, and its direction is 15° north of due east. That's all you need to have a vector — a magnitude and a direction.

Now take a look at the two vectors in Figure 3-2, **A** and **B.** These two vectors are considered equal, which is written as **A** = **B.**

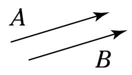


Figure 3-2: Two vectors.

Two vectors are considered equal if they have the same magnitude and direction. They do *not* need to start at the same point. The magnitude of a vector **A** — that is, its length — is written as A, not in bold type.

Figure 3-3 shows the standard coordinate system for vectors. Note the x and y axes, which vectors are measured against.

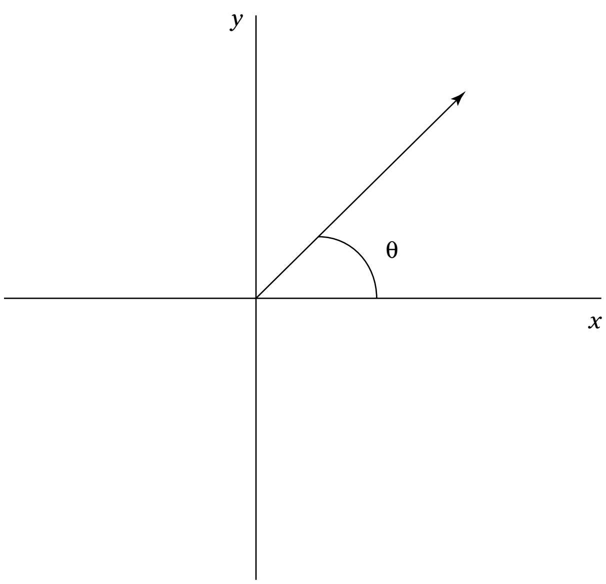


Figure 3-3: Vector coordinate system.

The x and y axes are measured using some standard physics units, such as centimeters. Positive x (also called *east*) is to the right, negative x is to the left; positive y (also called *north*) is up, negative y is down. The center of the graph, where the axes meet, is called the *origin.* A vector is commonly described by its length and its angle from the positive x axis (0° to 360°).

Q. Suppose the vector in Figure 3-3 is 3.0 centimeters long and at an angle of 45° with respect to the x axis. How would you exactly describe this vector?

A. The correct answer is 3.0 centimeters long and at an angle of 45° with respect to the x axis.

1. A marble starts at the origin and rolls 45 meters east. Describe where it ends up, in vector notation.

Solve It

2. A marble starts at the origin and rolls 45 meters east. Then it moves 90 meters west. Describe where it ends up, in vector notation.

Solve It

Understanding Vector Components

In addition to specifying a vector with a magnitude and a direction, you can specify it with a pair of coordinates as measured from the origin. For example, take a look at the vector in Figure 3-4, where the measurements are in centimeters.

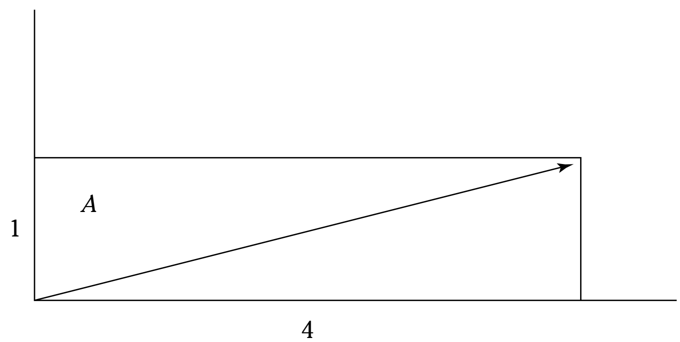


Figure 3-4: Resolving a vector.

You can describe the vector in Figure 3-4 with a length and an angle, of course, but you also can describe it by the coordinates of the tip of its arrow. In this case, that tip is at 4 centimeters to the right and 1 centimeter up from the origin. You notate that location as (4, 1), which is a valid way of expressing a vector.

So the two ways of expressing a vector are

- As a magnitude and a direction; for example, **A** is a vector along the x axis of length 5
- As a pair of coordinates corresponding to the tip of the vector (assuming the tail of the vector is at the origin); for example, **A** = (5, 0)

EXAMPLE

Q. Suppose a person walks 3.0 meters to the right of the origin. What is his displacement vector in terms of coordinates?

A. The correct answer is (3.0, 0).

The person's x coordinate is 3.0, and his y coordinate is 0, so his vector displacement is (3.0, 0).

3. A marble starts at the origin and moves to the right 5.0 centimeters. What is its new displacement, in vector coordinate terms?

Solve It

4. Suppose you move to the right of the origin by 3.5 meters and then up 5.6 meters. What is your final vector from the origin, in coordinate terms?

Solve It

Finding a Vector's Components

You can convert from the magnitude/angle way of specifying a vector to the coordinate way of expression. Doing so is essential for the kinds of operations you can expect to execute on vectors, such as when adding vectors together.

For example, you have one vector at 15° and one at 19°, and you want to add them together. How the heck do you do that? If you were to convert them into their coordinates, (a, b) and (c, d), the answer would be trivial because you only have to add the x and y coordinates to get the answer: (a + c, b + d).

To see how to convert between the two ways of looking at vectors, take a look at vector **v** in Figure 3-5. The vector can be described as having a magnitude v at an angle of θ.

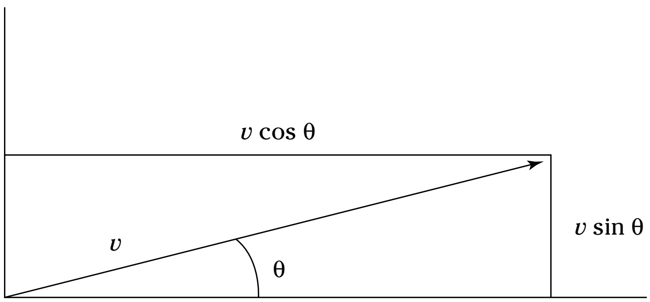


Figure 3-5: Finding a vector's components.

To convert this vector into the coordinate way of looking at vectors, you have to use the trigonometry shown in the figure. The x coordinate equals v cos θ, and the y coordinate equals v sin θ:

$$v_x = v \cos \theta$$

$$v_y = v \sin \theta$$

Keep this relationship in mind because you'll come across it often in physics questions.

EXAMPLE

Q. Suppose that you've walked away from the origin so that you're now at 5.0 kilometers from the origin, at an angle of 45°. Resolve that into vector coordinates.

A. The correct answer is (3.5, 3.5).

1. Apply the equation $v_x = v \cos \theta$ to find the x coordinate. That's 5.0·cos 45°, or 3.5.
2. Apply the equation $v_y = v \sin \theta$ to find the y coordinate. That's 5.0·sin 45°, or 3.5.

5. Resolve a vector 3.0 meters long at 15° into its components.

Solve It

6. Resolve a vector 9.0 meters long at 35° into its components.

Solve It

7. Resolve a vector 6.0 meters long at 125° into its components.

Solve It

8. Resolve a vector 4.0 meters long at 255° into its components.

Solve It

Finding a Vector's Magnitude and Direction

If you're given the coordinates of a vector, such as (3, 4), you can convert it easily to the magnitude/angle way of expressing vectors using trigonometry.

For example, take a look at the vector in Figure 3-5. Suppose that you're given the coordinates of the end of the vector and want to find its magnitude, v, and angle, θ. Because of your knowledge of trigonometry, you know that

$$x = v \cdot \cos\theta$$

$$y = v \cdot \sin\theta$$

In other words, you know that

$$\frac{x}{v} = \cos\theta$$

$$\frac{y}{v} = \sin\theta$$

Which means that

$$\theta = \sin^{-1}(y/v)$$

$$\theta = \cos^{-1}(x/v)$$

You can calculate the inverse sine ($\sin^{-1}$) or inverse cosine ($\cos^{-1}$) on your calculator. (Look for the $\sin^{-1}$ and $\cos^{-1}$ buttons.)

In Figure 3-5, you're given x and y, the coordinates, but not v, the magnitude. Dividing the expressions for y and x above gives you

$$\frac{y}{x} = \frac{\cancel{v}\sin\theta}{\cancel{v}\cos\theta} = \tan\theta$$

Where tan θ is the tangent of the angle. This means that

$$\theta = \tan^{-1}(y/x)$$

Suppose that the coordinates of the vector are (3, 4). You can find the angle θ as the $\tan^{-1}(4/3) = 53°$. And you can use the Pythagorean theorem to find the *hypotenuse* — the magnitude, v — of the triangle formed by x, y, and v:

$$v = \sqrt{x^2 + y^2}$$

Plug in the numbers for this example to get

$$v = \sqrt{3^2 + 4^2} = 5$$

So if you have a vector given by the coordinates (3, 4), its magnitude is 5, and its angle is 53°.

EXAMPLE

Q. Convert the vector given by the coordinates (1.0, 5.0) into magnitude/angle format.

A. The correct answer is magnitude 5.1, angle 78°.

1. Apply the equation $\theta = \tan^{-1}(y/x)$ to find the angle. Plug in the numbers to get $\tan^{-1}(5.0/1.0) = 78°$.
2. Apply the Pythagorean theorem $v = \sqrt{x^2 + y^2}$ to find the magnitude. Plug in the numbers to get 5.1.

9. Convert the vector (5.0, 7.0) into magnitude/angle form.

Solve It

10. Convert the vector (13.0, 13.0) into magnitude/angle form.

Solve It

11. Convert the vector (–1.0, 1.0) into magnitude/angle form.

Solve It

12. Convert the vector (–5.0, –7.0) into magnitude/angle form.

Solve It

Adding Vectors Together

You're frequently asked to add vectors together when solving physics problems. To add two vectors, you place them head to tail and then find the length and magnitude of the result. The order in which you add the two vectors doesn't matter. For example, suppose that you're headed to the big physics convention and have been told that you go 20 miles due north and then 20 miles due east to get there. At what angle is the convention center from your present location, and how far away is it?

You can write these two vectors like this (where east is along the x axis):

$$(0, 20)$$

$$(20, 0)$$

In this case, you need to add these two vectors together, and you can do that just by adding their x and y components separately:

$$\begin{array}{r} (0,20) \\ +(20,0) \\ \hline (20,20) \end{array}$$

Do the math, and your resultant vector is (20, 20). You've just completed a vector addition. But the question asks for the vector in magnitude/angle terms, not coordinate terms. So what is the magnitude of the vector from you to the physics convention? You can see the situation in Figure 3-6, where you have v_x and v_y and want to find v.

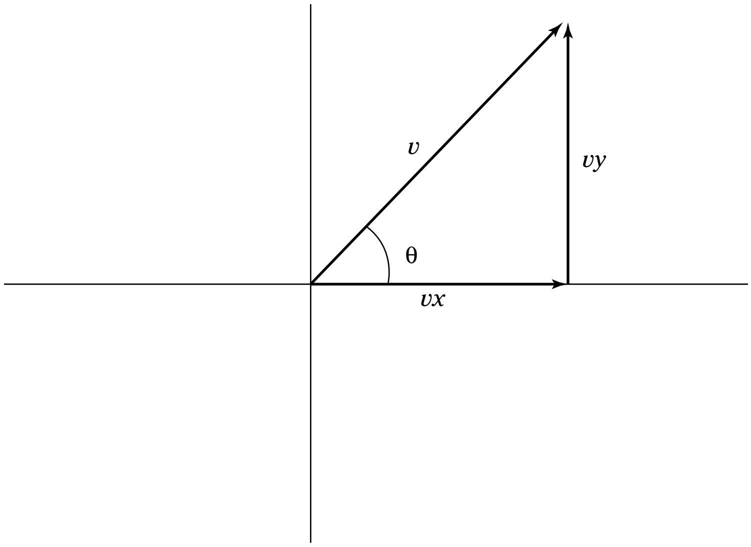


Figure 3-6: Adding two vectors.

Finding v isn't so hard because you can use the Pythagorean theorem:

$$R = \sqrt{X^2 + Y^2}$$

Plugging in the numbers to get

$$R = \sqrt{20^2 + 20^2} = 28.3 \text{ miles}$$

So the convention is 28.3 miles away. What about the angle θ? You know that

$$\theta = \sin^{-1}(y/h)$$
$$\theta = \cos^{-1}(x/h)$$

In this case, you can find the angle θ like so:

$$\theta = \sin^{-1}(y/h) = \sin^{-1}(20/28.3) = 45°$$

And that's it — you now know that the convention is 28.3 miles away at an angle of 45°.

Q. Add the two vectors in Figure 3-7. One has a magnitude 5.0 and angle 45°, and the other has a magnitude 7.0 and angle 35°.

A. The correct answer is magnitude 12.0, angle 39°.

1. **Resolve the two vectors into their components.** For the first vector, apply the equation $v_x = v \cos \theta$ to find the x coordinate. That's 5.0 cos 45° = 3.5.
2. Apply the equation $v_y = v \sin \theta$ to find the y coordinate of the first vector. That's 5.0 sin 45°, or 3.5. So the first vector is (3.5, 3.5) in coordinate form.
3. For the second vector, apply the equation $v_x = v \cos \theta$ to find the x coordinate. That's 7.0 cos 35° = 5.7.
4. Apply the equation $v_y = v \sin \theta$ to find the y coordinate of the second vector. That's 7.0 sin 35° = 4.0. So the second vector is (5.7, 4.0) in coordinate form.
5. To add the two vectors, add them in coordinate form: (3.5, 3.5) + (5.7, 4.0) = (9.2, 7.5).
6. Convert (9.2, 7.5) into magnitude/angle form. Apply the equation $\theta = \tan^{-1}(y/x)$ to find the angle, which is $\tan^{-1}(.82) = 39°$.
7. Apply the equation $v = \sqrt{x^2 + y^2}$ to find the magnitude, which is $v = \sqrt{9.2^2 + 7.5^2} = 11.9$. Converting to two significant digits gives you 12.

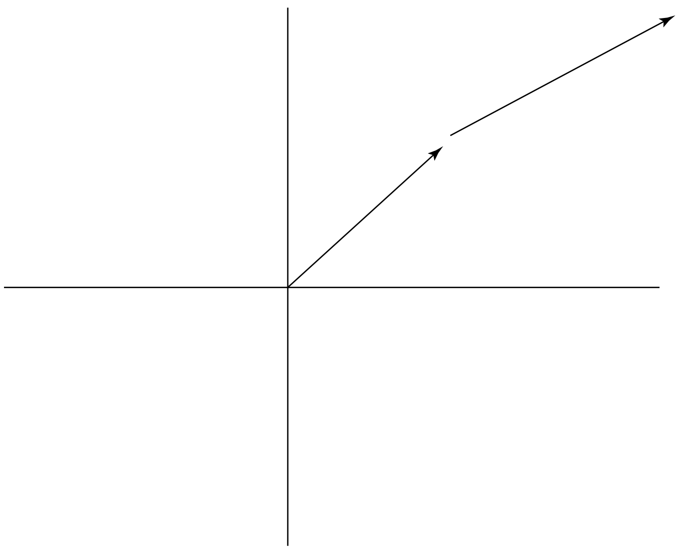


Figure 3-7: Two vectors being added.

13. Add a vector whose magnitude is 13.0 and angle is 27° to one whose magnitude is 11.0 and angle is 45°.

Solve It

14. Add a vector whose magnitude is 16.0 and angle is 56° to one whose magnitude is 10.0 and angle is 25°.

Solve It

15. Add two vectors: Vector one has a magnitude 22.0 and angle of 19°, and vector two has a magnitude 19.0 and an angle of 48°.

Solve It

16. Add a vector whose magnitude is 10.0 and angle is 257° to one whose magnitude is 11.0 and angle is 105°.

Solve It

Handling Motion As a Vector

Suppose you're in a car traveling east at 88 meters/second when you begin to accelerate north at 5.0 meters/second^2 for 10. seconds. What is your final speed?

You may think that you can use this equation to figure out the answer:

$$v_f = v_o + a \cdot t$$

But that's not a vector equation; the quantities here are called *scalars* (the magnitude of a vector is a scalar). This is a scalar equation, and it's not appropriate to use here because the acceleration and the initial speed aren't in the same direction. In fact, speed itself is a scalar, so you have to think in terms not of speed but of velocity.

Velocity is a vector, and as such, it has a magnitude and a direction associated with it. Here's the same equation as a vector equation:

$$\mathbf{v_f} = \mathbf{v_o} + \mathbf{a} \cdot t$$

Note that the speeds are now velocities (speed is the magnitude of a velocity vector) and that everything here is a vector except time (which is always a scalar). This change means that the addition you perform in this equation is vector addition, which is what you want because vectors can handle addition in multiple dimensions, not just in a straight line.

Here are the equations of motion, written as vector equations:

$$\mathbf{v_f} - \mathbf{v_o} = \mathbf{a} \cdot t$$
$$\mathbf{v} = \frac{\Delta \mathbf{x}}{\Delta t} = \frac{\mathbf{x_f} - \mathbf{x_o}}{t_f - t_o}$$
$$\mathbf{a} = \frac{\Delta \mathbf{v}}{\Delta t} = \frac{\mathbf{v_f} - \mathbf{v_o}}{t_f - t_o}$$
$$\mathbf{s} = \mathbf{v_o} \cdot \left(t_f - t_o\right) + \frac{1}{2} \cdot \mathbf{a} \left(t_f - t_o\right)^2$$

Q. You're in a car traveling east at 88 meters/second; then you accelerate north at 5 meters/second^2 for 10 seconds. What is your final speed?

A. The correct answer is 91 meters/second.

1. Start with this vector equation:

   $$\mathbf{v_f} = \mathbf{v_o} + \mathbf{a} \cdot t$$

2. This equation is simply vector addition, so treat the quantities involved as vectors. That is, v_o = (88, 0) meters/second and a = (0, 5) meters/second^2. Here's what the equation looks like when you plug in the numbers:

   $$\mathbf{v_f} = (88, 0) + (0, 5)(10)$$

3. Do the math:

   $$\mathbf{v_f} = (88, 0) + (0, 5)(10) = (88, 50)$$

4. You're asked to find the final speed, which is the magnitude of the velocity. Plug your numbers into the Pythagorean theorem.

5. You can also find the final direction. Apply the equation $\theta = \tan - 1\left(y/x\right)$ to find the angle, which is $\tan^{-1}(.57) = 29.6°$ in this case.

17. You're going 40 meters/second east, and then you accelerate 10 meters/second squared north for 10 seconds. What's your final velocity?

Solve It

18. You're going 44.0 meters/second at 35°, and then you accelerate due west at 4 meters/second^2 for 20 seconds. What's your final velocity?

Solve It

19. A hockey puck is going 100.0 meters/second at 250° when it's hit by a hockey stick, which accelerates it at 1000 meters/second^2 at 19° for 0.1 seconds. What's the final velocity?

Solve It

20. A car is driving along an icy road at 10 meters/second at 0° when it skids, accelerating at 15 meters/second^2 at 63° for 1.0 seconds. What's the final velocity?

Solve It

Answers to Problems about Vectors

The following are the answers to the practice questions presented in this chapter. You see how to work out each answer, step by step.

1 **45 meters at 0°**

The marble ends up 45 meters to the east of the origin, which is to say it ends up 45 meters from the origin, at 0°.

2 **45 meters at 180°**

1. The marble started at 45 meters at 0° and then moved left for 90 meters.
2. The marble ends up at 45 meters on the other side of the origin, at 45 meters at 180°.

3 **(5.0, 0) cm**

The marble moves to the right — in the positive x direction — by 5.0 meters, so its final location is (5.0, 0).

4 **(3.5, 5.6)·m**

1. You move to the right of the origin by 3.5 meters, leaving you at (3.5, 0).
2. Then you move up (in the positive y direction) by 5.6 meters, leaving you at (3.5, 5.6).

5 **(2.9, 0.8)·m**

1. Apply the equation $v_x = v \cdot \cos\theta$ to find the x coordinate: 3.0·cos 15°, or 2.9.
2. Apply the equation $v_y = v \cdot \sin\theta$ to find the y coordinate: 3.0·sin 15°, or 0.8.

6 **(7.4, 5.2)·m**

1. Apply the equation $v_x = v \cdot \cos\theta$ to find the x coordinate: 9.0·cos 35°, or 7.4.
2. Apply the equation $v_y = v \cdot \sin\theta$ to find the y coordinate: 9.0·sin 35°, or 5.2.

7 **(–3.4, 4.9)·m**

1. Apply the equation $v_x = v \cdot \cos\theta$ to find the x coordinate: 6.0·cos 125°, or –3.4.
2. Apply the equation $v_y = v \cdot \sin\theta$ to find the y coordinate: 6.0·sin 125°, or 4.9.

8 **(–1.0, –3.9)·m**

1. Apply the equation $v_x = v \cdot \cos\theta$ to find the x coordinate: 4.0·cos 255°, or –1.0.
2. Apply the equation $v_y = v \cdot \sin\theta$ to find the y coordinate: 4.0·sin 255°, or –3.9.

9 **Magnitude 8.6, angle 54°**

1. Apply the equation $\theta = \tan^{-1}(y/x)$ to find the angle: $\tan^{-1}(7.0/5.0) = 54°$.
2. Apply the equation $v = \sqrt{x^2 + y^2}$ to find the magnitude, which is 8.6.

10 **Magnitude 18.4, angle 45°**

1. Apply the equation $\theta = \tan^{-1}(y/x)$ to find the angle: $\tan^{-1}(13.0/13.0) = 45°$.
2. Apply the equation $v = \sqrt{x^2 + y^2}$ to find the magnitude, which is 18.4.

11 **Magnitude 1.4, angle –45°**

1. Apply the equation $\theta = \tan^{-1}(y/x)$ to find the angle: $\tan^{-1}(1.0/-1.0) = -45°$.
2. Apply the equation $v = \sqrt{x^2 + y^2}$ to find the magnitude, which is 1.4.

12 **Magnitude 8.6, angle 234°**

1. Apply the equation $\theta = \tan^{-1}(y/x)$ to find the angle: $\tan^{-1}(-7.0/-5.0) = 54°$. However, note that the angle must really be between 180° and 225° because both vector components are negative. That means you should add 180° to 54°, giving you 234° (the tangent of 234° is also $-7.0/-5.0 = 7.0/5.0$).
2. Apply the equation $v = \sqrt{x^2 + y^2}$ to find the magnitude, which is 8.6.

13 **Magnitude 23.7, angle 35°**

1. For the first vector, use the equation $v_x = v \cdot \cos\theta$ to find the x coordinate: $13.0 \cdot \cos 27° = 11.6$.
2. Use the equation $v_y = v \sin \cdot \theta$ to find the y coordinate of the first vector: $13.0 \cdot \sin 27°$, or 5.90. So the first vector is (11.6, 5.90) in coordinate form.
3. For the second vector, use the equation $v_x = v \cdot \cos\theta$ to find the x coordinate: $11.0 \cdot \cos 45° = 7.78$.
4. Use the equation $v_y = v \cdot \sin\theta$ to find the y coordinate of the second vector: $11.0 \cdot \sin 45° = 7.78$. So the second vector is (7.78, 7.78) in coordinate form.
5. Add the two vectors in coordinate form: (11.6, 5.90) + (7.78, 7.78) = (19.4, 13.7).
6. Convert (19.4, 13.7) into magnitude/angle form. Use the equation $\theta = \tan^{-1}(y/x)$ to find the angle: $\tan^{-1}(0.71) = 35°$.
7. Apply the equation $v = \sqrt{x^2 + y^2}$ to find the magnitude, which is $v = \sqrt{18.0^2 + 17.5^2} = 23.7$.

14 **Magnitude 25.1, angle 44°**

1. For the first vector, use the equation $v_x = v \cdot \cos\theta$ to find the x coordinate: $16.0 \cdot \cos 56° = 8.95$.
2. Use the equation $v_y = v \cdot \sin\theta$ to find the y coordinate of the first vector: $16.0 \cdot \sin 56°$, or 13.3. So the first vector is (8.95, 13.3) in coordinate form.
3. For the second vector, use the equation $v_x = v \cdot \cos\theta$ to find the x coordinate: $10.0 \cdot \cos 25° = 9.06$.
4. Use the equation $v_y = v \cdot \sin\theta$ to find the y coordinate of the second vector: $10.0 \cdot \sin 25° = 4.23$. So the second vector is (9.06, 4.23) in coordinate form.
5. Add the two vectors in coordinate form: (8.95, 13.3) + (9.06, 4.23) = (18.0, 17.5).
6. Convert the vector (18.0, 17.5) into magnitude/angle form. Use the equation $\theta = \tan^{-1}(y/x)$ to find the angle: $\tan^{-1}(0.97) = 44°$.
7. Apply the equation $v = \sqrt{x^2 + y^2}$ to find the magnitude, which is $v = \sqrt{19.4^2 + 13.7^2} = 25.1$.

15 **Magnitude 39.7, angle 32°**

1. For the first vector, use the equation $v_x = v \cdot \cos\theta$ to find the x coordinate: $22.0 \cdot \cos 19° = 20.8$.
2. Use the equation $v_y = v \cdot \sin\theta$ to find the y coordinate of the first vector: $22.0 \cdot \sin 19°$, or 7.16. So the first vector is (20.8, 7.16) in coordinate form.
3. For the second vector, use the equation $v_x = v \cdot \cos\theta$ to find the x coordinate: $19.0 \cdot \cos 48° = 12.7$.
4. Use the equation $v_y = v \cdot \sin\theta$ to find the y coordinate of the second vector: $19.0 \cdot \sin 48° = 14.1$. So the second vector is (12.7, 14.1) in coordinate form.
5. Add the two vectors in coordinate form: (20.8, 7.16) + (12.7, 14.1) = (33.5, 21.3).

6. Convert the vector (33.5, 21.3) into magnitude/angle form. Use the equation $\theta = \tan^{-1}(y/x)$ to find the angle: $\tan^{-1}(0.64) = 32°$.
7. Apply the equation $v = \sqrt{x^2 + y^2}$ to find the magnitude, which is $v = \sqrt{33.5^2 + 21.3^2} = 39.7$.

16 **Magnitude 5.2, angle 170°**

1. For the first vector, use the equation $v_x = v \cdot \cos\theta$ to find the x coordinate: $10.0 \cdot \cos 257° = -2.25$.
2. Use the equation $v_y = v \cdot \sin\theta$ to find the y coordinate of the first vector: $10.0 \cdot \sin 257°$, or –9.74. So the first vector is (–2.25, –9.74) in coordinate form.
3. For the second vector, use the equation $v_x = v \cdot \cos\theta$ to find the x coordinate: $11.0 \cdot \cos 105° = -2.85$.
4. Use the equation $v_y = v \cdot \sin\theta$ to find the y coordinate of the second vector: $11.0 \cdot \sin 105° = 10.6$. So the second vector is (–2.85, 10.6) in coordinate form.
5. Add the two vectors in coordinate form: (–2.25, –9.74) + (–2.85, 10.6) = (–5.10, 0.86).
6. Convert the vector (–5.10, 0.86) into magnitude/angle form. Use the equation $\theta = \tan^{-1}(y/x)$ to find the angle: $\tan^{-1}(-0.17) = 170°$. Because x is negative and y is positive, this vector must be in the second quadrant.
7. Apply the equation $v = \sqrt{x^2 + y^2}$ to find the magnitude, which is $v = \sqrt{5.10^2 + 0.86^2} = 5.2$.

17 **Magnitude 108 meters/second, angle 68°**

1. Start with this equation: $\mathbf{v_f} = \mathbf{v_o} + \mathbf{a} \cdot t$.
2. Plug in the numbers: $\mathbf{v_f} = (40, 0) + (0, 10)(10) = (40, 100)$.
3. Convert the vector (40, 100) into magnitude/angle form. Use the equation $\theta = \tan^{-1}(y/x)$ to find the angle: $\tan^{-1}(2.5) = 68°$.
4. Apply the equation $v = \sqrt{x^2 + y^2}$ to find the speed — the magnitude of the velocity, giving you 108 meters/second.

18 **Magnitude 50.7 meters/second, angle –29.8°**

1. Start with this equation: $\mathbf{v_f} = \mathbf{v_o} + \mathbf{a} \cdot t$.
2. Convert the original velocity into vector component notation. Use the equation $v_x = v \cdot \cos\theta$ to find the x coordinate of the original velocity vector: $44.0 \cdot \cos 35° = 36.0$.
3. Use the equation $v_y = v \cdot \sin\theta$ to find the y coordinate of the velocity: $44.0 \cdot \sin 35°$, or 25.2. So the velocity is (36.0, 25.2) in coordinate form.
4. Perform the vector addition: (36.0, 25.2) + (–4, 0)·(20) = (–44., 25.2).
5. Convert the vector (–44., 25.2) into magnitude/angle form. Use the equation $\theta = \tan^{-1}(y/x)$ to find the angle: $\tan^{-1}(.57) = -29.8°$.
6. Apply the equation $v = \sqrt{x^2 + y^2}$ to find the speed — the magnitude of the velocity, giving you 50.7 meters/second.

19 **Magnitude 86.1 meters/second, angle –45°**

1. Start with this equation: $\mathbf{v_f} = \mathbf{v_o} + \mathbf{a} \cdot t$.
2. Convert the original velocity into vector component notation. Use the equation $v_x = v \cdot \cos\theta$ to find the x coordinate of the original velocity vector: $100.0 \cdot \cos 250° = -34.2$.
3. Use the equation $v_y = v \cdot \sin\theta$ to find the y coordinate of the velocity: $100.0 \cdot \sin 250°$, or –93.9. So the original velocity is (–34.2, –93.9) in coordinate form.

4. Convert the acceleration into components. Use the equation $a_x = a \cdot \cos\theta$ to find the x coordinate of the acceleration: $1000 \cdot \cos 19° = 945$.

5. Use the equation $a_y = a \sin\theta$ to find the y coordinate of the acceleration: $1000 \cdot \sin 19°$, or 325. So the acceleration is (945, 325) in coordinate form.

6. Perform the vector addition: $(-34.2, -93.9) + (945, 325)(0.1) = (60.3, -61.4)$.

7. Convert the vector (60.3, –61.4) into magnitude/angle form. Use the equation $\theta = \tan^{-1}(y/x)$ to find the angle: $\tan^{-1}(-1.0) = -45°$.

8. Apply the equation $v = \sqrt{x^2 + y^2}$ to find the speed — the magnitude of the velocity, giving you 86.1 meters/second.

20 **Magnitude 21.4 meters/second, angle 38°**

1. Start with this equation: $\mathbf{v_f} = \mathbf{v_o} + \mathbf{a} \cdot t$.

2. Convert the original velocity into vector component notation: (10, 0) meters/second.

3. Convert the acceleration into components. Use the equation $a_x = a \cdot \cos\theta$ to find the x coordinate of the acceleration: $15 \cdot \cos 63° = 6.8$.

4. Use the equation $a_y = a \cdot \sin\theta$ to find the y coordinate of the acceleration: $15 \cdot \sin 63°$, or 13. So the acceleration is (6.8, 1.3) in coordinate form.

5. Perform the vector addition: $(10, 0) + (6.8, 1.3) \cdot (1.0) = (16.8, 13)$.

6. Convert the vector (16.8, 13) into magnitude/angle form. Use the equation $\theta = \tan^{-1}(y/x)$ to find the angle: $\tan^{-1}(.079) = 38°$.

7. Apply the equation $v = \sqrt{x^2 + y^2}$ to find the magnitude of the velocity, giving you 21.4 meters/second.

Part II

May the Forces Be with You

The 5th Wave By Rich Tennant

In this part . . .

This part is where you start solving problems that ultimately go back to the idea that "for every action, there is an equal and opposite reaction." Isaac Newton gets the spotlight here, helping you predict just what's going to happen when you apply a force to something. Force, mass, acceleration, friction — I cover it all in this part and include plenty of practice problems to keep you on your toes.

Chapter 4

Applying Force

In This Chapter

- Applying Newton's laws of motion
- Calculating force and acceleration
- Working with weight and mass

If all you have is objects in motion, things get pretty boring. Everything zipping around endlessly in straight lines, never changing direction or speed. So physics comes to the rescue by applying forces to those objects, and the application of force means changes in the objects' speed and direction. Now that's much more interesting.

This chapter covers Newton's Three Laws of Motion. More specifically, it examines force and the application of it on an object.

Newton's First Law of Motion

Whether you're asked to give it in physics class or not, you need to be familiar with Isaac Newton's First Law of Motion: "An object continues in a state of rest or in a state of motion at a constant speed along a straight line, unless compelled to change that state by a net force." What's the translation? The idea is that, if you don't apply a force to something in motion, it will stay in that same motion along a straight line. Forever!

The property of an object to stay in constant motion is called *inertia,* and mass is a measure of that inertia. In physics, it's very important to realize that there's a difference between mass and weight; objects have mass built in, but they don't acquire weight until you put them in a gravitational field.

Physics measures mass using the unit kilograms in the MKS system and grams in the CGS system. What's the unit of mass in the FPS (foot-pound-second) system? It's the oddly named *slug.* When you put a slug in the Earth's gravitational field, on the surface of the Earth a slug weighs approximately 32 pounds (you may not have heard of FPS slugs, but you've surely heard of pounds). (For a review of measurement systems, flip to Chapter 1.)

Q. Suppose that you have an SUV and want to measure its mass in the MKS system. What unit of measurement do you use?

A. The correct answer is kilograms because the MKS unit of mass is the kilogram.

1. You decide to measure the mass of an aircraft carrier in the CGS system. What unit does your measurement end up in?

Solve It

2. Your spouse asks you if she looks fat, so you decide to measure her mass in the FPS system. What unit is your measurement in?

Solve It

Newton's Second Law of Motion

Force saves you from the monotony of everything moving at the same speed and direction forever. Force can act on objects, changing their direction and/or speed. The relationship between force, mass, and acceleration is primary in physics classes, so this section (and even broader, this chapter) helps you become a pro at solving problems involving these quantities.

REMEMBER

To start, you need to know Newton's Second Law of Motion, which is a big one in physics: "When a net force ΣF acts on an object of mass m, the acceleration of that mass can be calculated by ΣF = ma." The translation version is that the force equals mass times acceleration, or ΣF = ma. The Σ stands for "sum," so ΣF = ma can be read as "the sum of all forces on an object (the net force) equals mass times acceleration." This equation is often just abbreviated as F = ma.

What is the unit of force? Table 4-1 gives you a rundown for the three measurement systems:

Table 4-1 Units of Force

System	*Unit*	*Name*	*Abbreviation*
MKS	kg-m/second^2	Newton	N
CGS	g-m/second^2	dyne	d
FPS	pound		lb

Here's how you relate the three different units of force:

1 lb = 4.448 N

1 N = 10^5 d

EXAMPLE

Q. You're at rest on an ice rink when you get hit from behind with a force of 50 N as someone bumps you. If your mass is 70 kg, what is your acceleration?

A. The correct answer is .71 m/sec^2.

1. Use the equation F = ma: Solving for a gives you a = F/m.
2. Plug in the numbers: a = F/m = 50 N/70 kg = .71 m/sec^2.

3. You come home to find a delivered package with a mass of 100 kg blocking the door. If you push it with a force of 100 N, what will its acceleration be if no friction is involved?

Solve It

4. You're gliding across a frictionless lake in a sailboat. If your mass is 70 kg and the boat's mass is 200 kg, with what force does the wind need to blow you to give you an acceleration of 0.30 m/sec^2?

Solve It

5. You have control of the space station, which has a mass of 4,000 kg. To give it an acceleration of 2.0 m/sec^2, what force do you need to apply with the rockets?

Solve It

6. You find a stone in the forest and give it a push of 50 N. It accelerates at 2.0 m/sec^2. What is its mass?

Solve It

7. You're applying a force of 10 N to a hockey puck with a mass of 0.1 kg. Starting from rest, how far has the puck gone in 1.0 seconds?

Solve It

8. You push a rowboat on a calm lake (no friction) with a force of 40 N. If the rowboat has a mass of 80 kg, how far has it gone in 10 seconds?

Solve It

9. A space station with a mass of 10,000 kg is moving toward a satellite at 5.0 m/sec. If you want to avoid crashing them together and have only 1,000 seconds in which to act, what force do you need to apply to stop the space station from colliding with the satellite?

Solve It

10. Your 1,000 kg car needs a push. Starting at rest, how hard do you have to push to get it up to 10 m/sec in 100 seconds?

Solve It

Force Is a Vector

Force, like displacement, velocity, and acceleration, is a vector quantity, which is why Newton's Second Law is written as $\Sigma\mathbf{F} = m \cdot \mathbf{a}$. Put into words, it says that the vector sum of the forces acting on an object is equal to its mass (a scalar) multiplied by its acceleration (a vector).

Because force is a vector quantity, you add forces together as vectors. That fits right into Newton's Second Law. (For more on the process of adding vectors, check out Chapter 3.)

EXAMPLE

Q. Suppose that you have two forces as shown in Figure 4-1: A = 5.0 N at 40°, and B = 7.0 N at 125°. What is the net force, $\Sigma\mathbf{F}$?

A. The correct answer is magnitude 8.9 N, angle 91°.

1. Convert force **A** into vector component notation. Use the equation $A_x = A \cos\theta$ to find the x coordinate of the force: 5.0 cos 40° = 3.8.
2. Use the equation $A_y = A \sin\theta$ to find the y coordinate of the force: 5.0 sin 40°, or 3.2. That makes the vector **A** (3.8, 3.2) in coordinate form.
3. Convert the vector **B** into components. Use the equation $B_x = B \cos\theta$ to find the x coordinate of the acceleration: 7.0 cos 125° = –4.0.
4. Use the equation $B_y = B \sin\theta$ to find the y coordinate of the second force: 7.0 sin 125°, or 5.7. That makes the force **B** (–4.0, 5.7) in coordinate form.
5. Perform the vector addition to find the net force: (3.8, 3.2) + (–4.0, 5.7) = (–0.2, 8.9).
6. Convert the vector (–0.18, 8.9) into magnitude/angle form. Use the equation $\theta = \tan^{-1}(y/x)$ to find the angle: $\tan^{-1}(-49) = 89°$.
7. Apply the equation $v = \sqrt{x^2 + y^2}$ to find the magnitude of the net force, giving you 8.9 N.

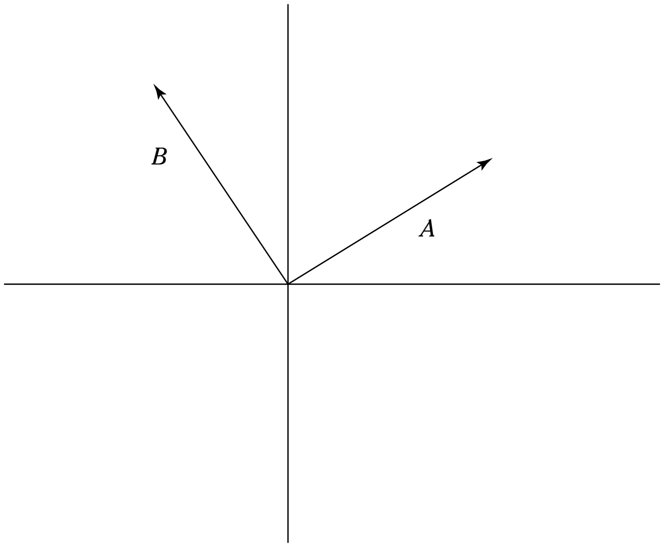


Figure 4-1: Two forces.

11. Add two forces: **A** is 8.0 N at 53°, and **B** is 9.0 N at 19°.

Solve It

12. Add two forces: **A** is 16.0 N at 39°, and **B** is 5.0 N at 125°.

Solve It

13. Add two forces: **A** is 22.0 N at 68°, and **B** is 6.0 N at 24°.

Solve It

14. Add two forces: **A** is 12.0 N at 129°, and **B** is 3.0 N at 225°.

Solve It

Calculating Net Force and Acceleration

Newton says Σ**F** = m**a**, which means that you add the force vectors **A** and **B** together to get the net force.

That's how it usually works when you have to figure out F = ma problems in physics. Most of the time, a number of force vectors are involved, and you have to solve for the net force in order to find the acceleration.

Take a look at the hockey puck in Figure 4-2. Two forces, **A** and **B**, are acting on the puck. What's going to happen to the puck?

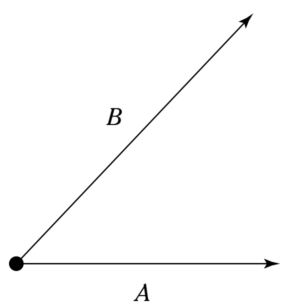


Figure 4-2: Two forces acting on a hockey puck.

You don't have to calculate the result of each force acting separately on the hockey puck because the net force is what's important. Calculate the net force first and then use that in **F** = m · **a**.

EXAMPLE

Q. Suppose that the forces acting on the hockey puck in Figure 4-2 are A = 9.0 N at 0°, and B = 14.0 N at 45°. What is the acceleration of the puck, given that its mass is 0.1 kg?

A. The correct answer is magnitude 213 m/sec^2, angle 27°.

1. Convert force **A** into vector component notation. Use the equation $A_x = A \cos \theta$ to find the x coordinate of the force: 9.0 cos 0° = 9.0.
2. Use the equation $A_y = A \sin \theta$ to find the y coordinate of the force: 9.0 sin 0°, or 0.0. That makes the vector **A** (9.0, 0.0) in coordinate form.
3. Convert the vector **B** into components. Use the equation $B_x = B \cos \theta$ to find the x coordinate of the acceleration: 14.0 cos 45° = 9.9.
4. Use the equation $B_y = B \sin \theta$ to find the y coordinate of the second force: 14.0 sin 45°, or 9.9. That makes the force **B** (9.9, 9.9) in coordinate form.
5. Perform the vector addition to find the net force: (9.0, 0.0) + (9.9, 9.9) = (18.9, 9.9).
6. Convert the vector (18.9, 9.9) into magnitude/angle form. Use the equation $\theta = \tan^{-1}(y/x)$ to find the angle of the net force: $\tan^{-1}(.52)$ = 27°.
7. Apply the equation $v = \sqrt{x^2 + y^2}$ to find the magnitude of the net force, giving you 21.3 N.
8. Convert 21.3 N into acceleration: a = F/m = 21.3/0.1 = 213 m/sec^2.

15. Assume that the two forces acting on a 0.10 kg hockey puck are as follows: **A** is 16.0 N at 53°, and **B** is 21.0 N at 19°. What is the acceleration of the hockey puck?

Solve It

16. Two forces act on a 1000 kg car. **A** is 220 N at 64°, and **B** is 90 N at 80°. Neglecting friction, what is the car's acceleration?

Solve It

17. Suppose that two forces act on a 100 kg boat. **A** is 100 N at 10°, and **B** is 190 N at 210°. What is the boat's acceleration?

Solve It

18. A marble with a mass of 1.0 g is hit by two other marbles that each apply a force for 0.3 seconds. If force **A** is 0.010 N at 63° and **B** is 0.050 N at 135°, what is the acceleration of the original marble?

Solve It

Sorting Out Weight and Mass

One of the forces you're asked to deal with frequently in physics is the force on an object due to gravity. In a gravitational field, all objects are accelerated due to gravity; on the surface of the Earth, the acceleration due to gravity is 9.8 m/sec^2, which is about 32 ft/sec^2.

Because the equation F = ma holds for all forces where the object being forced is free to accelerate, you can calculate the force an object feels due to gravity. Here are a few things to keep in mind when dealing with force and gravity:

- **The force on an object is proportional to the object's mass.** For example, twice the mass means twice the force.
- **Because a = F/m, twice the force still means the same acceleration if you have twice the mass.** It's the acceleration due to gravity that is constant in the Earth's gravitational field at the surface of the Earth, not the force.
- **The acceleration due to gravity points downward, toward the center of the Earth.**

The acceleration due to gravity at the surface of the Earth carries the symbol g. That means F = ma becomes F = mg; as a vector equation, that's F = m**g**. In practical terms, unless you're dealing with points so far apart that the curvature of the Earth matters, **g** is just considered downward.

Q. You're trying to lift a suitcase with a mass of 20 kg. How much force must you supply at a minimum to lift the suitcase?

A. The correct answer is 196 N.

To be able to lift the suitcase, you have to overcome the force due to gravity. Calculate it as follows: $F = mg = (20)(9.8) = 196$ N.

19. You're holding a basketball in your hands. If it has a mass of 0.8 kg, how much force must you provide to keep it where it is?

Solve It

20. A ball drops off a cliff. How fast is it going 1.0 seconds later (neglecting wind resistance)?

Solve It

21. A ball drops off a cliff. How far has it gone in 2.0 seconds (neglecting wind resistance)?

Solve It

22. A skydiver jumps out of a plane. How far has he gone 4.0 seconds later (neglecting wind resistance)?

Solve It

23. You throw a baseball straight up into the air at 60 m/sec. How fast is it going 1.0 seconds later?

Solve It

24. You throw a physics book straight up into the air at 30 m/sec. How fast is it going 4.0 seconds later?

Solve It

25. Holding a ball over the edge of a cliff, you throw it up at 10 m/sec. What is its position 5.0 seconds later?

Solve It

26. Holding a ball over the edge of a cliff, you throw it up at 20 m/sec. What is its position 7.0 seconds later?

Solve It

Newton's Third Law of Motion

Newton's Third Law of Motion is a famous one: "Whenever one body exerts a force on a second body, the second body exerts an oppositely directed force of equal magnitude on the first body." If that doesn't ring a bell, try this on for size: "For every action, there is an equal and opposite reaction."

Put simply, this law of motion says that, if your car pushes against the Earth, the Earth pushes back against your car with the same amount of force. This force causes your car to accelerate. What about the Earth? Doesn't it accelerate in response to the force your car exerts on it? Believe it or not, it does, but because the Earth has a mass about 6,000,000,000,000,000,000,000 times that of your car, the effect of your car on the Earth isn't very noticeable!

Newton's Third Law is particularly useful when it comes to problems of *equilibrium,* in which all the forces balance out. When you have equilibrium, you know that all forces sum to zero.

Q. In Figure 4-3, you see three connected ropes. If rope A has a tension of 10 N at 135°, and rope B has a tension of 10 N at 45°, what must the tension in rope C be to keep things in equilibrium?

A. The correct answer is 14 N downward.

1. Convert tension **A** into vector component notation. Use the equation $A_x = A \cos \theta$ to find the x coordinate of the tension: 10.0 cos 135° = –7.07.
2. Use the equation $A_y = A \sin \theta$ to find the y coordinate of the tension: 10.0 sin 135°, or 7.07. That makes the tension **A** (–7.07, 7.07) in coordinate form.
3. Convert the tension **B** into components. Use the equation $B_x = B \cos \theta$ to find the x coordinate of the tension: 10.0 cos 45° = 7.07.
4. Use the equation $B_y = B \sin \theta$ to find the y coordinate of the second tension: 10.0 sin 45°, or 7.07. That makes the tension B (7.07, 7.07) in coordinate form.
5. Perform vector addition to find the net tension: (–7.07, 7.07) + (7.07, 7.07) = (0, 14.1).
6. To counteract the net tension, the tension in rope C must be 14.1 N downward (that is, –14.1 N).

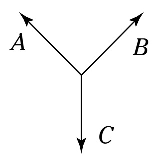


Figure 4-3: Three forces balancing out.

27. You have three ropes tied together in equilibrium. The tension in rope A is 15 N at 135°, and the tension in rope B is 15 N at 45°. What must the tension in rope C be?

Solve It

28. You have three ropes tied together in equilibrium. The tension in rope A is 17.0 N at 115°, and the tension in rope B is 18.0 N at 25°. What must the tension in rope C be?

Solve It

Answers to Problems about Force

The following are the answers to the practice questions presented in this chapter. You see how to work out each answer, step by step.

1 **grams**

The units of mass in the CGS system are grams.

2 **slugs**

The units of mass in the FPS system are grams.

3 **1.00 m/sec^2**

1. Solving F = ma for a gives you a = F/m.
2. Plug in the numbers: a = F/m = 100 N/100 kg = 1.00 m/sec^2.

4 **81 N**

1. Use the equation F = ma.
2. Plug in the numbers: F = ma = (70 + 200)(0.3) = 81 N.

5 **8000 N**

1. Use the equation F = ma.
2. Plug in the numbers: F = ma = (4000)(2.0) = 8000 N.

6 **25 kg**

1. Use the equation F = ma, and solve for the mass, giving you m = F/a.
2. Plug in the numbers: m = F/a = 50/2 = 25 kg.

7 **50 m**

1. Use the equation F = ma, and solve for the acceleration, giving you a = F/m.
2. Use the equation $s = \frac{1}{2}at^2$, and substitute F/m for a:

$$s = \frac{Ft^2}{2m}$$

3. Plug in the numbers:

$$s = \frac{Ft^2}{2m} = \frac{10\ 1.0^2}{2\ (0.1)} = 50\ \text{m}$$

8 **25 m**

1. Use the equation F = ma, and solve for the acceleration, giving you a = F/m.
2. Use the equation $s = \frac{1}{2}at^2$ and substitute F/m for a:

$$s = \frac{Ft^2}{2m}$$

3. Plug in the numbers: $s = \frac{Ft^2}{2m} = \frac{40\ 10^2}{2\ (80)} = 25\ \text{m}$

9 **–50 N**

1. Use the equation F = ma, and solve for the acceleration, giving you a = F/m.
2. Use the equation $v_f = v_o + at$. In this question, $v_f = 0$, so at = $-v_o$.
3. This becomes at $= \frac{Ft}{m} = -v_o$; solving for F gives you $F = \frac{-m\ v_o}{t}$.
4. Plug in the numbers, and you get F = –50 N (it's negative because it's in the opposite direction of travel).

10 **100 N**

1. Use the equation F = ma, and solve for the acceleration, giving you a = F/m.
2. Use the equation $v_f = v_o + at$. In this question, $v_f = 10$ and $v_o = 0$, so at = v_f.
3. This becomes at $= \frac{Ft}{m} = v_f$; solving for F gives you $F = \frac{m\ v_f}{t}$.
4. Plug in the numbers, and you get F = 100 N.

11 **Magnitude: 16 N; Angle: 36°**

1. Convert force **A** into vector component notation. Use the equation $A_x = A \cos \theta$ to find the x coordinate of the force: 8.0 cos 53° = 4.8.
2. Use the equation $A_y = A \sin \theta$ to find the y coordinate of the force: 8.0 sin 53°, or 6.4. That makes the vector **A** (4.8, 6.4) in coordinate form.
3. Convert the vector **B** into components. Use the equation $B_x = B \cos \theta$ to find the x coordinate of the acceleration: 9.0 cos 19° = 8.5.
4. Use the equation $B_y = B \sin \theta$ to find the y coordinate of the second force: 9.0 sin 19°, or 2.9. That makes the force **B** (8.5, 2.9) in coordinate form.
5. Perform vector addition to find the net force: (4.8, 6.4) + (8.5, 2.9) = (13, 9.3).
6. Convert the force vector (13, 9.3) into magnitude/angle form. Use the equation $\theta = \tan^{-1}(y/x)$ to find the angle: $\tan^{-1}(.72) = 36°$.
7. Apply the equation $v = \sqrt{x^2 + y^2}$ to find the magnitude of the net force, giving you 16 N.

12 **Magnitude: 17 N; Angle: 56°**

1. Convert force **A** into vector component notation. Use the equation $A_x = A \cos \theta$ to find the x coordinate of the force: 16.0 cos 39° = 12.4.
2. Use the equation $A_y = A \sin \theta$ to find the y coordinate of the force: 16.0 sin 39°, or 10.0. That makes the vector **A** (12.4, 10.0) in coordinate form.
3. Convert the vector **B** into components. Use the equation $B_x = B \cos \theta$ to find the x coordinate of the force: 5.0 cos 125° = –2.9.
4. Use the equation $B_y = B \sin \theta$ to find the y coordinate of the second force: 5.0 sin 125°, or 4.1. That makes the force **B** (–2.9, 4.1) in coordinate form.
5. Perform vector addition to find the net force: (12.4, 10.0) + (–2.9, 4.1) = (9.5, 14).
6. Convert the force vector (9.5, 14) into magnitude/angle form. Use the equation $\theta = \tan^{-1}(y/x)$ to find the angle: $\tan^{-1}(1.5) = 56°$.
7. Apply the equation $v = \sqrt{x^2 + y^2}$ to find the magnitude of the net force, giving you 17 N.

13 **Magnitude: 27 N; Angle: 59°**

1. Convert force **A** into vector component notation. Use the equation $A_x = A \cos \theta$ to find the x coordinate of the force: 22.0 cos 68° = 8.24.
2. Use the equation $A_y = A \sin \theta$ to find the y coordinate of the force: 22.0 sin 68°, or 20.3. That makes the vector **A** (8.24, 20.3) in coordinate form.
3. Convert the vector **B** into components. Use the equation $B_x = B \cos \theta$ to find the x coordinate of the force: 6.0 cos 24° = 5.5.
4. Use the equation $B_y = B \sin \theta$ to find the y coordinate of the second force: 6.0 sin 24°, or 2.4. That makes the force **B** (5.5, 2.4) in coordinate form.
5. Perform vector addition to find the net force: (8.24, 20.3) + (5.5, 2.4) = (13.7, 23).
6. Convert the force vector (13.7, 23) into magnitude/angle form. Use the equation $\theta = \tan^{-1}(y/x)$ to find the angle: $\tan^{-1}(1.7) = 59°$.
7. Apply the equation $v = \sqrt{x^2 + y^2}$ to find the magnitude of the net force, giving you 27 N.

14 **Magnitude: 12 N; Angle: 140°**

1. Convert force **A** into vector component notation. Use the equation $A_x = A \cos \theta$ to find the x coordinate of the force: 12.0 cos 129° = –7.6.
2. Use the equation $A_y = A \sin \theta$ to find the y coordinate of the force: 12.0 sin 129°, or 9.3. That makes the vector **A** (–7.6, 9.3) in coordinate form.
3. Convert the vector **B** into components. Use the equation $B_x = B \cos \theta$ to find the x coordinate of the force: 3.0 cos 225° = –2.1.
4. Use the equation $B_y = B \sin \theta$ to find the y coordinate of the second force: 3.0 sin 225°, or –2.1. That makes the force **B** (–2.1, –2.1) in coordinate form.
5. Perform vector addition to find the net force: (–7.6, 9.3) + (–2.1, –2.1) = (–9.7, 7.2).
6. Convert the force vector (–9.7, 7.2) into magnitude/angle form. Use the equation $\theta = \tan^{-1}(y/x)$ to find the angle: $\tan^{-1}(-.74) = 140°$.
7. Apply the equation $v = \sqrt{x^2 + y^2}$ to find the magnitude of the net force, giving you 12 N.

15 **Magnitude: 353 m/sec^2; Angle: 33°**

1. Convert force **A** into its components. Use the equation $A_x = A \cos \theta$ to find the x coordinate of the force: 16.0 cos 53° = 9.6.
2. Use the equation $A_y = A \sin \theta$ to find the y coordinate of the force: 16.0 sin 53°, or 12.8. That makes the vector **A** (9.6, 12.8) in coordinate form.
3. Convert force **B** into its components. Use the equation $B_x = B \cos \theta$ to find the x coordinate of the acceleration: 21.0 cos 19° = 19.8.
4. Use the equation $B_y = B \sin \theta$ to find the y coordinate of the second force: 21.0 sin 19°, or 6.8. That makes the force **B** (19.8, 6.8) in coordinate form.
5. Perform vector addition to find the net force: (9.6, 12.8) + (19.8, 6.8) = (29.4, 19.6).
6. Convert the vector (29.4, 19.6) into magnitude/angle form. Use the equation $\theta = \tan^{-1}(y/x)$ to find the angle of the net force: $\tan^{-1}(.66) = 33°$.
7. Apply the equation $v = \sqrt{x^2 + y^2}$ to find the magnitude of the net force, giving you 35.3 N.
8. Convert 35.3 N into acceleration: $a = F/m = 35.3/0.1 = 353$ m/sec^2.

16 **Magnitude: 308 m/sec^2; Angle: 69°**

1. Convert force **A** into its components. Use the equation $A_x = A \cos \theta$ to find the x coordinate of the force: 220 cos 64° = 96.
2. Use the equation $A_y = A \sin \theta$ to find the y coordinate of the force: 220 sin 64°, or 198. That makes the vector **A** (96, 198) in coordinate form.

3. Convert force **B** into its components. Use the equation $B_x = B \cos \theta$ to find the x coordinate of the acceleration: $90 \cos 80° = 16$.
4. Use the equation $B_y = B \sin \theta$ to find the y coordinate of the second force: $90 \sin 80°$, or 89. That makes the force **B** (16, 89) in coordinate form.
5. Perform vector addition to find the net force: (96, 198) + (16, 89) = (112, 287).
6. Convert the vector (112, 287) into magnitude/angle form. Use the equation $\theta = \tan^{-1}(y/x)$ to find the angle of the net force: $\tan^{-1}(2.5) = 69°$.
7. Apply the equation $v = \sqrt{x^2 + y^2}$ to find the magnitude of the net force, giving you 308 N.
8. Convert 308 N into acceleration: $a = F/m = 308/1000 = 0.31$ m/sec^2.

17 **Magnitude: 1.0 m/sec^2; Angle: 229°**

1. Convert force **A** into its components. Use the equation $A_x = A \cos \theta$ to find the x coordinate of the force: $100 \cos 10° = 98$.
2. Use the equation $A_y = A \sin \theta$ to find the y coordinate of the force: $100 \sin 10°$, or 17. That makes the vector **A** (98, 17) in coordinate form.
3. Convert force **B** into its components. Use the equation $B_x = B \cos \theta$ to find the x coordinate of the acceleration: $190 \cos 210° = -164$.
4. Use the equation $B_y = B \sin \theta$ to find the y coordinate of the second force: $190 \sin 210°$, or –95. That makes the force **B** (–164, –95) in coordinate form.
5. Perform vector addition to find the net force: (98, 17) + (–164, –95) = (–66, –78).
6. Convert the vector (–66, –78) into magnitude/angle form. Use the equation $\theta = \tan^{-1}(y/x)$ to find the angle of the net force: $\tan^{-1}(1.2) = 49°$. But that answer's not right because both components are negative, which means that the angle is actually between 180° and 270°. Add 180° to 49° to get 229°.
7. Apply the equation $v = \sqrt{x^2 + y^2}$ to find the magnitude of the net force, giving you 102 N.
8. Convert 102 N into acceleration: $a = F/m = 102/100 = 1.0$ m/sec^2.

18 **Magnitude: 54 m/sec^2; Angle: 125°**

1. Convert force **A** into its components. Use the equation $A_x = A \cos \theta$ to find the x coordinate of the force: $.01 \cos 63° = 4.5 \times 10^{-3}$.
2. Use the equation $A_y = A \sin \theta$ to find the y coordinate of the force: $.01 \sin 63°$, or 8.9×10^{-3}. That makes the vector **A** (4.5×10^{-3}, 8.9×10^{-3}) in coordinate form.
3. Convert force **B** into its components. Use the equation $B_x = B \cos \theta$ to find the x coordinate of the acceleration: $.05 \cos 135° = -3.5 \times 10^{-2}$.
4. Use the equation $B_y = B \sin \theta$ to find the y coordinate of the second force: $.05 \sin 135°$, or 3.5×10^{-2}. That makes the force **B** (-3.5×10^{-2}, 3.5×10^{-2}) in coordinate form.
5. Perform vector addition to find the net force: $(4.5 \times 10^{-3}, 8.9 \times 10^{-3}) + (-3.5 \times 10^{-2}, 3.5 \times 10^{-2}) = (-3.1 \times 10^{-2}, 4.4 \times 10^{-2})$.
6. Convert the vector (-3.1×10^{-2}, 4.4×10^{-2}) into magnitude/angle form. Use the equation $\theta = \tan^{-1}(y/x)$ to find the angle of the net force: $\tan^{-1}(1.2) = 125°$.
7. Apply the equation $v = \sqrt{x^2 + y^2}$ to find the magnitude of the net force, giving you 5.4×10^{-2} N.
8. Convert 5.4×10^{-2} N into acceleration: $a = F/m = 5.4 \times 10^{-2}/.001 = 54$ m/sec^2.

19 **7.8 N**

1. Use the equation $\mathbf{F} = m\mathbf{g}$.
2. The weight of the basketball is $\mathbf{F} = m\mathbf{g} = (.8)(9.8) = 7.8$ N.

20 **9.8 m/sec, downward**

1. The acceleration due to gravity is **g**.
2. The velocity of the ball is $v_f = \mathbf{a}t = \mathbf{g}t$.
3. Plug in the numbers: $v_f = (9.8)(1.0) = 9.8$ m/sec, downward.

21 **20 m**

1. The acceleration due to gravity is **g**.
2. The distance the ball has gone in a time t is $s = \frac{1}{2}gt^2$.
3. Plug in the numbers: $s = \frac{1}{2}gt^2 = \frac{1}{2}(9.8)(2.0)^2 = 20 \text{ m}$.

22 **78 m**

1. The acceleration due to gravity is **g**.
2. The distance the skydiver has gone in a time t is $s = \frac{1}{2}gt^2$.
3. Plug in the numbers: $s = \frac{1}{2}gt^2 = \frac{1}{2}(9.8)(4.02) = 78 \text{ m}$.

23 **50 m/sec**

1. The acceleration due to gravity is **g**.
2. Use the equation $v_f = v_o + at = v_o - gt$.
3. Plug in the numbers: $v_f = v_o + at = v_o - gt = 60 - 9.8(1.0) = 50$ m/sec.

24 **–9.2 m/sec**

1. The acceleration due to gravity is **g**.
2. Use the equation $v_f = v_o + at = v_o - gt$.
3. Plug in the numbers: $v_f = v_o + at = v_o - gt = 30 - 9.8(4.0) = -9.2$ m/sec.

25 **–73 m**

1. The acceleration due to gravity is **g**.
2. Use the equation $s = v_o t + \frac{1}{2}at^2$.
3. Plug in the numbers: $s = (10)(5.0) - \frac{1}{2}g(5.0)^2 = -73 \text{ m}$.

26 **–100. m**

1. The acceleration due to gravity is **g**.
2. Use the equation $s = v_o t + \frac{1}{2}at^2$.
3. Plug in the numbers: $s = (20)(7.0) - \frac{1}{2}g(7.0)^2 = -100. \text{ m}$.

27 **–21.2 N downward**

1. Convert tension **A** into vector component notation. Use the equation $A_x = A \cos\theta$ to find the x coordinate of the tension: 15.0 cos 135° = –10.6.
2. Use the equation $A_y = A \sin\theta$ to find the y coordinate of the tension: 15.0 sin 135°, or 10.6. That makes the tension **A** (–10.6, 10.6) in coordinate form.
3. Convert the tension **B** into components. Use the equation $B_x = B \cos\theta$ to find the x coordinate of the tension: 15.0 cos 45° = 10.6.
4. Use the equation $B_y = B \sin\theta$ to find the y coordinate of the second tension: 15.0 sin 45°, or 10.6. That makes the tension **B** (10.6, 10.6) in coordinate form.
5. Perform vector addition to find the net tension: (–10.6, 10.6) + (10.6, 10.6) = (0, 21.2).
6. To counteract this tension, the tension in rope C must be 21.2 N downward (that is, –21.2 N).

28 **24.7 N at 248°**

1. Convert tension **A** into vector component notation. Use the equation $A_x = A \cos\theta$ to find the x coordinate of the tension: 17.0 cos 115° = –7.18.
2. Use the equation $A_y = A \sin\theta$ to find the y coordinate of the tension: 17.0 sin 115°, or 15.4. That makes the tension **A** (–7.18, 15.4) in coordinate form.
3. Convert the tension **B** into components. Use the equation $B_x = B \cos\theta$ to find the x coordinate of the tension: 18.0 cos 25° = 16.3.
4. Use the equation $B_y = B \sin\theta$ to find the y coordinate of the second tension: 18.0 sin 25°, or 7.61. That makes the tension **B** (16.3, 7.61) in coordinate form.
5. Perform vector addition to find the net tension: (–7.18, 15.4) + (16.3, 7.61) = (9.12, 23.0).
6. Find the angle of the tension due to **A** and **B**: $\theta = \tan^{-1}(y/x) = \tan^{-1}(2.5) = 68°$.
7. Apply the equation $v = \sqrt{x^2 + y^2}$ to find the magnitude of tension due to **A** and **B**, giving you 24.7 N.
8. Find the angle of the tension in rope C. This must be opposite the tension due to **A** and **B**, which is 68° + 180° = 248°.

Chapter 5

Working with Inclined Planes

In This Chapter

- Calculating acceleration and speed on inclined planes
- Figuring out the coefficients of friction
- Factoring in friction

Many physics problems involve *inclined planes* — those ramps that you're always seeing balls and carts roll down in physics classroom labs. Gravitational force is what makes carts roll down ramps, of course, but there's more to it than that. In the classic beginning physics problem, you have to resolve the gravitational force along and perpendicular to the ramp in order to find the acceleration of the cart along the ramp.

In the real world, you also have friction. For example, if you're unloading a refrigerator from a truck using a ramp and the refrigerator slides down the ramp by itself, friction happens, and you have to take that into account.

In this chapter, I guide you through the wonderful world of inclined planes, providing you with plenty of practice questions to ensure that you come out a pro at handling this type of physics problem.

Breaking Ramps Up into Vectors

The first step in working with ramps of any kind is to resolve the forces that you're dealing with, and that means using vectors. (For more on vectors, check out Chapter 3.) For example, take a look at the cart in Figure 5-1; it's on an inclined plane, ready to roll.

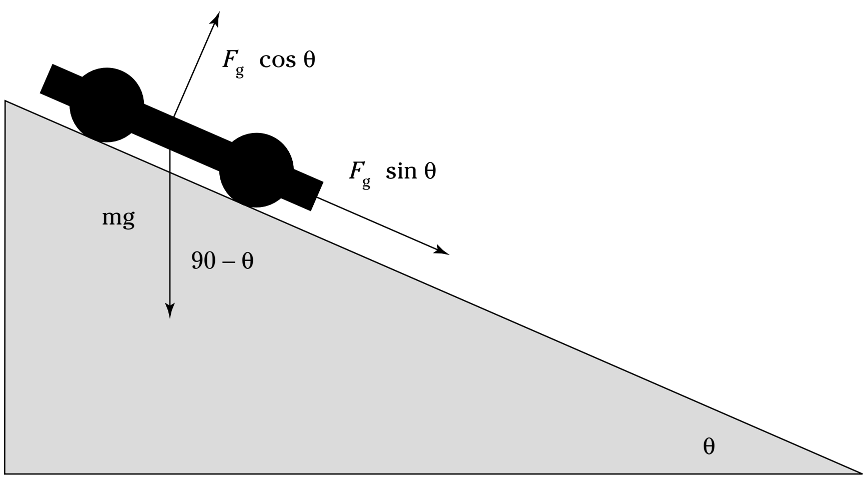


Figure 5-1: A cart on an inclined plane.

The force on the cart is the force due to gravity, $F_g = m \cdot g$. So how fast will the cart accelerate along the ramp? To get the answer, you should resolve the gravitational force — not in the x and y directions, however, but along the ramp's inclined plane and perpendicular to that plane.

REMEMBER

The reason you resolve the gravitational force in these directions is because the force along the plane provides the cart's acceleration while the force perpendicular to the ramp, called the *normal* force, does not. (When you start introducing friction into the picture, you'll see that the force of friction is proportional to the normal force — that is, it's proportional to the force with which the object going down the ramp presses against the ramp.)

Try out some questions involving force and ramps:

EXAMPLE

Q. In Figure 5-1, what are the forces along the ramp and normal to the ramp?

A. The correct answer is $F_g \sin \theta$ along the ramp, $F_g \cdot \cos \theta$ normal (perpendicular to the ramp).

1. To resolve the vector F_g along the ramp, you can start by figuring out the angle between F_g and the ramp. Here's where your knowledge of triangles comes into play. Because you know that a triangle's angles have to add up to 180°, the angle between F_g and the ground is 90°, and Figure 5-1 shows you that the ramp's angle to the ground is θ, you know that the angle between F_g and the ramp must be 90° – θ.
2. The angle between F_g and the ramp is 90° – θ. So what's the component of F_g along the ramp? Knowing the angle between F_g and the ramp, you can figure the component of F_g along the ramp as usual:

$$\mathbf{F_g} \text{ along the ramp} = \mathbf{F_g} \cdot \cos(90° - \theta)$$

3. Apply the following equation:

$$\sin\theta = \cos(90° - \theta)$$

$$\mathbf{F_g} \text{ along the ramp} = \mathbf{F_g} \cdot \cos(90° - \theta) = \mathbf{F_g} \cdot \sin\theta \text{ along the ramp}$$

Notice that this makes sense because as θ goes to 0°, the force along the ramp also goes to zero.

4. Solve for the normal force, F_n, perpendicular to the ramp: $F_g \cos(90° - \theta)$. Apply the following equation:

$$\cos\theta = \sin(90° - \theta)$$

$$\mathbf{F_n} \text{ perpendicular to the ramp} = \mathbf{F_g} \cdot \sin(90° - \theta) = \mathbf{F_g} \cdot \cos\theta$$

1. Suppose that the cart in Figure 5-1 has a mass of 1.0 kg and the angle $\theta = 30°$. What are the forces on the cart along and normal to the ramp?

Solve It

2. Suppose that the cart in Figure 5-1 has a mass of 3.0 kg and the angle $\theta = 45°$. What are the forces on the cart along and normal to the ramp?

Solve It

3. You have a block of ice with a mass of 10.0 kg on a ramp at an angle of 23°. What are the forces on the ice along and normal to the ramp?

Solve It

4. You have a refrigerator with a mass of 100 kg on a ramp at an angle of 19°. What are the forces on the refrigerator along and normal to the ramp?

Solve It

Acceleration and Inclined Planes

When you have a block of ice (read: frictionless) moving down a ramp, it's being acted on by forces, which means that it's accelerated. How fast is it being accelerated? When you know that F = ma, you can solve for the acceleration.

After you solve for the force along the ramp, you can get the acceleration (a = F / m) along the ramp. Your block of ice is going to slide down the ramp — and accelerate.

EXAMPLE

Q. Suppose that you have a block of ice on a ramp at 40°, and it slides down. What is its acceleration?

A. The correct answer is 6.3 m/sec^2.

1. What's important here is the force along the ramp: $F_g \sin\theta = m \cdot g \cdot \sin\theta$.
2. The acceleration of the ice is F/m = $m \cdot g \cdot \sin\theta / m = g \sin\theta$. In other words, the acceleration is the component of g acting along the ramp. Note that this result is independent of mass.
3. Plug in the numbers: $g \sin\theta = 6.3\ m/sec^2$.

5. Suppose that a block of ice is on a ramp with an angle of 60°. What is its acceleration?

Solve It

6. You're unloading a couch on a cart from a moving van. The couch gets away from you on the 27° ramp. Neglecting friction, what is its acceleration?

Solve It

7. You have a block of ice with a mass of 10.0 kg on a ramp with an angle of 23° when it slips away from you. What is its acceleration down the ramp?

Solve It

8. You're sliding down a toboggan run at 35°. What is your acceleration?

Solve It

Running Down Ramps: Speed

When objects slide (frictionlessly) down a ramp, they're acted on by a force, which means that they're accelerated and therefore their speed changes. The equation to use in physics problems like these is

$$v_f^2 - v_o^2 = 2 \cdot a \cdot s$$

REMEMBER

Finding the object's final speed under these circumstances is easy when you remember that $a = g \cdot \sin\theta$, s is the length of the ramp, and v_o is usually 0.

EXAMPLE

Q. Say you have a block of ice on a ramp at 20°, and it slides down a ramp of 5.0 meters. What is its final speed at the bottom of the ramp?

A. The correct answer is 5.8 m/sec^2.

1. The force along the ramp is $F_g \sin\theta = m \cdot g \cdot \sin\theta$.
2. The acceleration of the ice is $F/m = m \cdot g \sin\theta / m = g \sin\theta$.
3. Use the equation $v_f^2 = 2 \cdot a \cdot s = 2 \cdot g \cdot s \cdot \sin\theta$. Plug in the numbers: $v_f^2 = 34$, which means $v_f = 5.8$ m/sec^2.

9. Starting from rest, you go down a 100 m ski jump of 60°. What is your speed at takeoff?

Solve It

10. You're heading down a toboggan run of 1 km at an angle of 18°. What is your final speed?

Solve It

11. You have a block of ice on a ramp with an angle of 23° when it slips away from you. What is its speed at the bottom of the 6.0 m ramp?

Solve It

12. A cart starts at the top of a 50 m slope at an angle 38°. What is the cart's speed at the bottom?

Solve It

It's a Drag: The Coefficient of Friction

In the real world, when things slide down ramps, friction is involved, and the force of friction opposes the motion down the ramp. The force of friction is proportional to the force driving the two forces together; the stronger that force, the more friction is involved.

When something goes down a ramp, the force driving the two surfaces together is the normal force because it's the force perpendicular to the ramp's surface. The constant of proportionality is something you have to measure yourself; if the ramp is made of steel, there's a different amount of friction than if it's made of sandpaper. In the equation relating the normal force to the force of friction, F_f, the constant μ is called the *coefficient of friction* (a dimensionless number between 0.0 and 1.0):

$$F_f = \mu F_n$$

Following are a couple of questions to test your understanding of coefficient of friction:

EXAMPLE

Q. You're pushing a refrigerator along your kitchen floor and need to apply 100 N to get it moving. If your refrigerator has a mass of 100 kg, what is the coefficient of friction?

A. The correct answer is 0.1.

1. The force due to friction is $F_f = \mu F_n$, so $\mu = F_f / F_n$.
2. The force due to friction, F_f, is 100 N, and the normal force, F_n, is $(100) \cdot g = 980$ N.
3. Use the equation $\mu = F_f / F_n$ to get the coefficient of friction (note that μ has no units): $100 / 980 = 0.10$.

13. You're pushing a 15 kg box of books across the carpet and need to apply 100 N. What is the coefficient of friction?

Solve It

14. You're pushing a 70 kg easy chair from one room to the next. If you need to apply 200 N, what is the coefficient of friction?

Solve It

Starting from zero: Static friction

The two coefficients of friction correspond to two different physical processes. The first, called the *static coefficient of friction,* applies when you start pushing something at rest to get it moving. When you already have something moving and need to keep applying a force to keep it in motion, that's called the *kinetic coefficient of friction* (see the next section for more).

REMEMBER

The static coefficient of friction, μ_s, is usually larger than the kinetic coefficient of friction, μ_k, and both are between 0.0 and 1.0.

EXAMPLE

Q. Suppose that you need to move a 100 kg desk. If the static coefficient of friction between the floor and the desk is 0.2, how much force do you have to apply to get the desk to start to move?

A. The correct answer is 196 N.

1. The force of friction here is $F = \mu_s \cdot F_n = \mu_s \cdot m \cdot g$.
2. Plug in the numbers: $(0.2) \cdot 100 \cdot (9.8) = 196$ N.

15. You're standing at the top of a ski slope and need 15 N of force to get yourself moving. If your mass is 60 kg, what is the static coefficient of friction, μ_s?

Solve It

16. You've started to pull a garbage can out to the curb. If the can has a mass of 20 kg and you need to apply 70 N to get the can moving, what is the static coefficient of friction, μ_s?

Solve It

Already in motion: Kinetic friction

The second kind of friction is kinetic friction, which is usually less than the force of friction you need to overcome static friction. Kinetic friction occurs when you're pushing or dragging an object that's already in motion.

Static friction is the force needed to get something to move, and it's usually larger than kinetic friction, the force needed to keep it moving. The equation for kinetic friction is $F = \mu_k \cdot F_n$.

EXAMPLE

Q. Suppose that you have a 5.0 kg block of ice, and it takes 5 N to keep it moving across the floor. What is the kinetic coefficient of friction, μ_k?

A. The correct answer is 0.1.

1. The force of kinetic friction is $F_k = \mu_k \cdot F_g = \mu_k \cdot m \cdot g$.
2. Solve for μ_k: $\mu_k = F_k / m \cdot g$.
3. Plug in the numbers: $\mu_k = 5 / (5) \cdot (9.8) = 0.1$.

17. You're cross-country skiing and need 20 N of force to keep going. If you have a mass of 70 kg, what is the kinetic coefficient of friction, μ_k?

Solve It

18. You're skating and need 17 N to keep going. If you have a mass of 80 kg, what is the kinetic coefficient of friction, μ_k?

Solve It

Static Friction along Ramps

Figure 5-2 shows a box on a ramp. Suppose that the box contains a new widescreen TV that you're pushing up the ramp and into your house.

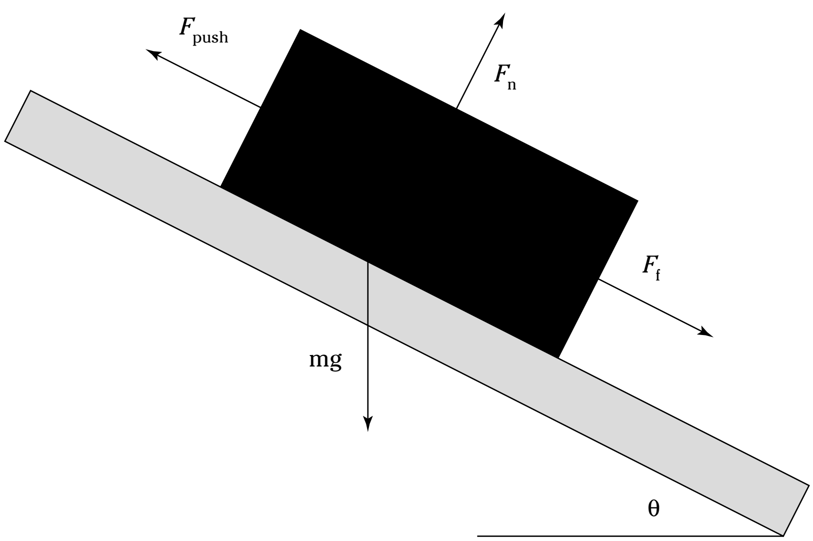


Figure 5-2: An object on a ramp.

A number of forces are acting on the box, in particular both gravity and friction, and you need to take both into account. There's also the force exerted upon the box as you push it up the ramp. So how do you balance all the forces? How much force is needed to get the box moving up the ramp into the house?

In order to figure that out, clearly you need to calculate the forces along the ramp. There's the force due to gravity, which is $m \cdot g \sin \theta$. But what is the force due to static friction along the ramp?

To find that, you use the equation $F = \mu_s F_n$. What is the normal force, F_n? You already know the answer — it's $F_n = mg \cos \theta$. That makes the force due to static friction $\mu_s mg \cos \theta$.

The force of static friction points along the ramp, and you've calculated the force due to gravity along the ramp. Both of these forces point down the ramp and need to be overcome by the force pushing up the ramp. So in other words, F_{push} is

$$F_{push} = F_g + F_s$$

Where F_g is the force due to gravity, and F_s is the force due to static friction. Plugging in these forces gives you:

$$F_{push} = F_g + F_s = m \cdot g \sin \theta + \mu_s \cdot m \cdot g \cos \theta$$

Q. Suppose that the widescreen TV's box has a mass of 100 kg, and the ramp has an angle of 23°. What is the force needed to get the box moving up the ramp if the coefficient of static friction is 0.20?

A. The correct answer is 562 N.

1. The equation to use is $F_{push} = m \cdot g \sin \theta + \mu_{s+} \cdot m \cdot g \cos \theta$.
2. Plug in the numbers: $F_{push} = 382 + 180 = 562$ N.

19. You're dragging your little brother up the 25° wheelchair ramp at the doctor's office. If he has a mass of 40 kg and the coefficient of static friction, μ_s, is 0.15, how much force will you need to apply to get him moving?

Solve It

20. Suppose that you're struggling to keep a 20.0 kg block of ice from sliding down a 40.0° ramp. If the coefficient of static friction, μ_s, is a low 0.050, how much force will you need to apply to overcome the weight pulling the block down the ramp and static friction?

Solve It

Kinetic Friction along Ramps

Two kinds of friction — static and kinetic — mean that you also have to handle ramp problems where kinetic friction is involved, as when an object on a ramp is sliding *down* that ramp. Because the object is moving, kinetic friction applies. That means you can solve problems with kinetic friction as easily as those that involve static friction.

Here's the equation for the force needed to get an object moving, and thus overcoming static friction:

$$F = F_g + F_k = m \cdot g \sin\theta + \mu_s \cdot m \cdot g \cos\theta$$

REMEMBER

To convert this equation to using kinetic friction, all you have to do is to change from using the static coefficient of friction, μ_s, to using the kinetic coefficient of friction, μ_k:

$$F = F_g + F_k = m \cdot g \sin\theta + \mu_k \cdot m \cdot g \cos\theta$$

That's all there is to it.

EXAMPLE

Q. Suppose that you're pushing a 40 kg block of ice up a 19° ramp. The kinetic coefficient of friction, μ_k, is 0.1, and you need to apply a force to keep the ice moving. What is the force you will need to apply?

A. The correct answer is 165 N.

1. The equation here is $F = F_g + F_k = m \cdot g \sin\theta + \mu_k \cdot m \cdot g \cos\theta$.
2. Putting in the numbers gives you $F = 128 + 37 = 165$ N.

21. You're dragging your little sister up the 25° wheelchair ramp at the doctor's office. If she has a mass of 30.0 kg and the coefficient of kinetic friction, μ_s, is 0.10, how much force will you need to apply to keep her going?

Solve It

22. You're pushing a box of books with a mass of 25 kg up a 40° ramp. If the coefficient of kinetic friction, μ_s, is 0.27, how much force will you need to apply to keep the box moving up the ramp?

Solve It

23. You're pushing a chest of drawers up an 18° ramp. If it has a mass of 50.0 kg and the coefficient of kinetic friction, μ_s, is 0.20, how much force will you need to keep it moving?

Solve It

24. You want to keep a 120 kg refrigerator moving up a 23° ramp. If the coefficient of kinetic friction, μ_k, is 0.20, how much force will you need to keep it moving?

Solve It

Acceleration along Ramps Including Friction

Suppose that you have a block of ice that has been mistakenly placed too near the top of a long ramp, and it starts sliding down that ramp. The preceding section helps you calculate how much force acts on that block of ice, so how about calculating its acceleration down the incline?

The object is sliding down the ramp — you're not pushing it — which means that the kinetic force of friction is opposing (not adding to) the component of gravity along the ramp. So the force on the block of ice is

$$F = F_g - F_k = m \cdot g \sin \theta - \mu_k \cdot m \cdot g \cos \theta$$

Because a = F / m, the acceleration of the block is

$$a = g \sin \theta - \mu_k \cdot g \cos \theta$$

EXAMPLE

Q. A block of ice slips down a 19° ramp with a kinetic coefficient of friction, μ_k, of 0.10. What is its acceleration as it slides?

A. The correct answer is 2.3 m/sec.

1. You can use the equation $a = g \sin \theta - \mu_k \cdot g \cos \theta$.
2. Plug in the numbers: a = 3.2 – .93 = 2.3 m/sec.

25. You're dragging a suitcase up a ramp into a luxury hotel when it gets away from you. If the angle of the ramp is 31° and the kinetic coefficient of friction is 0.1, what is the suitcase's acceleration down the ramp?

Solve It

26. You drop a 5.0 kg box on a ramp of 12°, and the kinetic coefficient of friction is 0.15. Will the box slide down the ramp?

Solve It

27. You drop a 1.0 kg book on a 15° ramp, and the kinetic coefficient of friction is 0.30. Will the book slide down the ramp?

Solve It

28. A refrigerator breaks away from the movers and slides down a 23° ramp that has a coefficient of kinetic friction of 0.25. What is its acceleration?

Solve It

Answers to Problems about Inclined Planes

The following are the answers to the practice questions presented earlier in this chapter. You see how to work out each answer, step by step.

1 **4.9 N along the ramp, 8.5 N normal to the ramp**

1. You know that the forces on the cart are $F_g \sin\theta$ along the ramp and $F_g \cos\theta$ normal to the ramp.
2. Plug the numbers into $F_g = m \cdot g$: $(1.0) \cdot (9.8) = 9.8$ N.
3. The force along the ramp is $F_g \cdot \sin\theta = (9.8) \sin 30° = 4.9$ N.
4. The force normal to the ramp is $F_g \cdot \cos\theta = (9.8) \cos 30° = 8.5$ N.

2 **21 N along the ramp, 21 N normal to the ramp**

1. The forces on the cart are $F_g \cdot \sin\theta$ along the ramp and $F_g \cdot \cos\theta$ normal to the ramp.
2. Plug the numbers into $F_g = m \cdot g$: $(3.0) \cdot (9.8) = 29$ N.
3. The force along the ramp is $F_g \cdot \sin\theta = (29) \cdot \sin 45° = 21$ N.
4. The force normal to the ramp is $F_g \cdot \cos\theta = (29) \cdot \cos 45° = 21$ N.

3 **38 N along the ramp, 90 N normal to the ramp**

1. The forces on the ice are $F_g \cdot \sin\theta$ along the ramp and $F_g \cdot \cos\theta$ normal to the ramp.
2. Plug the numbers into $F_g = m \cdot g$: $(10.0) \cdot (9.8) = 98$ N.
3. The force along the ramp is $F_g \cdot \sin\theta = (98) \cdot \sin 23° = 38$ N.
4. The force normal to the ramp is $F_g \cdot \cos\theta = (98) \cdot \cos 23° = 90$ N.

4 **320 N along the ramp, 930 N normal to the ramp**

1. The forces on the ice are $F_g \sin\theta$ along the ramp and $F_g \cos\theta$ normal to the ramp.
2. Plug the numbers into $F_g = m \cdot g$: $(100) \cdot (9.8) = 980$ N.
3. The force along the ramp is $F_g \cdot \sin\theta = (980) \cdot \sin 19° = 320$ N.
4. The force normal to the ramp is $F_g \cdot \cos\theta = (980) \cdot \cos 19° = 930$ N.

5 **8.5 m/sec^2**

1. The force along the ramp is $F_g \cdot \sin\theta = m \cdot g \cdot \sin\theta$.
2. The acceleration of the ice is $F/m = m \cdot g \cdot \sin\theta / m = g \cdot \sin\theta$.
3. Plugging in the numbers gives you $g \cdot \sin\theta = 8.5$ m/sec^2.

6 **4.4 m/sec^2**

1. The force along the ramp is $F_g \cdot \sin\theta = m \cdot g \cdot \sin\theta$.
2. The acceleration of the cart is $F/m = m \cdot g \cdot \sin\theta / m = g \cdot \sin\theta$.
3. Plugging in the numbers gives you $g \cdot \sin\theta = 4.4$ m/sec^2.

7 **3.8 m/sec^2**

1. The force along the ramp is $F_g \cdot \sin\theta = m \cdot g \sin\theta$.
2. The acceleration of the ice is $F/m = m \cdot g \cdot \sin\theta / m = g \cdot \sin\theta$.
3. Plugging in the numbers gives you $g \cdot \sin\theta = 3.8$ m/sec^2.

8 **5.6 m/sec^2**

1. The force along the ramp is $F_g \cdot \sin\theta = m \cdot g \cdot \sin\theta$.
2. Your acceleration is $F/m = m \cdot g \cdot \sin\theta / m = g \cdot \sin\theta$.
3. Plugging in the numbers gives you $g \cdot \sin\theta = 5.6$ m/sec^2.

9 **41 m/sec^2**

1. The force along the ramp is $F_g \cdot \sin\theta = m \cdot g \cdot \sin\theta$.
2. Your acceleration is $F/m = m \cdot g \cdot \sin\theta / m = g \cdot \sin\theta$.
3. Use the equation $v_f^2 = 2 \cdot a \cdot s = 2 \cdot g \cdot s \sin\theta$.
4. Solving for $v_f = 41$ m/sec^2.

10 **78 m/sec^2**

1. The force along the ramp is $F_g \cdot \sin\theta = m \cdot g \cdot \sin\theta$.
2. Your acceleration is $F/m = m \cdot g \cdot \sin\theta / m = g \cdot \sin\theta$.
3. Use the equation $v_f^2 = 2 \cdot a \cdot s = 2 \cdot g \cdot s \sin\theta$.
4. Solving for $v_f = 78$ m/sec^2.

11 **6.8 m/sec^2**

1. The force along the ramp is $F_g \cdot \sin\theta = m \cdot g \cdot \sin\theta$.
2. The block of ice's acceleration is $F/m = m \cdot g \cdot \sin\theta / m = g \cdot \sin\theta$.
3. Use the equation $v_f^2 = 2 \cdot a \cdot s = 2 \cdot g \cdot s \cdot \sin\theta$.
4. Solving for $v_f = 6.8$ m/sec^2.

12 **25 m/sec^2**

1. The force along the ramp is $F_g \cdot \sin\theta = m \cdot g \cdot \sin\theta$.
2. The cart's acceleration is $F/m = m \cdot g \cdot \sin\theta / m = g \cdot \sin\theta$.
3. Use the equation $v_f^2 = 2 \cdot a \cdot s = 2 \cdot g \cdot s \cdot \sin\theta$.
4. Solving for $v_f = 25$ m/sec^2.

13 **0.68**

1. Solve for μ. The force due to friction is $F_f = \mu F_n$, so $\mu = F_f / F_n$.
2. The force due to friction, F_f, is 100 N, and the normal force, F_n, is $(15) \cdot g = 148$ N.
3. Use the equation $\mu = F_f / F_n$ to get the coefficient of friction (note that μ has no units): $100 / 148 = 0.68$.

14 **0.29**

1. Solve for μ. The force due to friction is $F_f = \mu F_n$, so $\mu = F_f / F_n$.
2. The force due to friction, F_f, is 200 N, and the normal force, F_n, is $(70) \cdot g = 690$ N.
3. Use the equation $\mu = F_f / F_n$ to get the coefficient of friction: $200 / 690 = 0.29$.

15 **0.03**

1. Solve for μ_s. The force due to friction is $F_f = \mu_s \cdot F_n$, so $\mu_s = F_f / F_n$.
2. The force due to friction, F_f, is 15 N, and the normal force, F_n, is $(60) \cdot g = 590$ N.
3. Use the equation $\mu_s = F_f / F_n$ to get the static coefficient of friction: $15 / 590 = 0.03$.

16 **0.36**

1. Solve for μ_s. The force due to friction is $F_f = \mu_s \cdot F_n$, so $\mu_s = F_f / F_n$.
2. The force due to friction, F_f, is 70 N, and the normal force, F_n, is $(20) \cdot g = 196$ N.
3. Use the equation $\mu_s = F_f / F_n$ to get the static coefficient of friction: $70 / 196 = 0.36$.

17 **0.03**

1. Solve for μ_k. The force due to friction is $F_f = \mu_k \cdot F_n$, so $\mu_k = F_f / F_n$.
2. The force due to friction that you have to overcome, F_f, is 20 N, and the normal force, F_n, is $(70) \cdot g = 690$ N.
3. Use the equation $\mu_k = F_f / F_n$ to get the kinetic coefficient of friction: $20 / 690 = 0.03$.

18 **0.02**

1. Solve for μ_k. The force due to friction is $F_f = \mu_k F_n$, so $\mu_k = F_f / F_n$.
2. The force due to friction that you have to overcome, F_f, is 17 N, and the normal force, F_n, is $(80) \cdot g = 780$ N.
3. Use the equation $\mu_k = F_f / F_n$ to get the kinetic coefficient of friction: $17 / 780 = 0.02$.

19 **219 N**

1. Calculate the forces you need to overcome: The force due to gravity is $mg \sin\theta$; and the force due to friction, F_f, is $F = \mu_s F_n$. You need to find the normal force.
2. The equation for normal force is $F_n = mg \cos\theta$. Use the normal force to calculate the force due to friction: $\mu_s mg \cos\theta$.
3. The total force you have to overcome is $F = mg \sin\theta + \mu_s mg \cos\theta$.
4. Plug in the numbers: $F = mg \sin\theta + \mu_s mg \cos\theta = 166 + 53 = 219$ N.

20 **133 N**

1. Calculate the forces you need to overcome: The force due to gravity is $mg \sin\theta$; and the force due to friction, F_f, is $F = \mu_s F_n$. You need to find the normal force.
2. The equation for normal force is $F_n = m \cdot g \cdot \cos\theta$. Use the normal force to calculate the force due to friction: $\mu_s \cdot m \cdot g \cdot \cos\theta$.
3. The total force you have to overcome is $F = m \cdot g \cdot \sin\theta + \mu_s \cdot m \cdot g \cdot \cos\theta$.
4. Plug in the numbers: $F = m \cdot g \cdot \sin\theta + \mu_s \cdot m \cdot g \cdot \cos\theta = 126 + 7.5 = 133$ N. Note that most of this force is due to the component of the weight along the ramp.

21 **150 N**

1. Determine the forces you need to overcome: The force due to gravity is $m \cdot g \cdot \sin\theta$, and the force due to kinetic friction is $F = \mu_k \cdot F_n$. You need to find the normal force.
2. The equation for normal force is $F_n = m \cdot g \cdot \cos\theta$. Use the normal force to calculate the force due to kinetic friction: $\mu_k \cdot m \cdot g \cdot \cos\theta$.
3. The force you have to overcome is $F = m \cdot g \cdot \sin\theta + \mu_k \cdot m \cdot g \cdot \cos\theta$.
4. Plug in the numbers: $F = m \cdot g \cdot \sin\theta + \mu_k \cdot m \cdot g \cdot \cos\theta = 120 + 27 = 150$ N.

22 **210 N**

1. Calculate the forces you need to overcome: The force due to gravity is mg sin θ, and the force due to kinetic friction is $F = \mu_k \cdot F_n$. You need to find the normal force.
2. The equation for normal force is $F_n = m \cdot g \cdot \cos\theta$. Use the normal force to calculate the force due to kinetic friction: $\mu_k \cdot m \cdot g \cdot \cos\theta$.
3. The force you have to overcome to keep the box moving is $F = m \cdot g \sin\theta + \mu_k \cdot m \cdot g \cdot \cos\theta$.
4. Plug in the numbers: $F = m \cdot g \cdot \sin\theta + \mu_k \cdot m \cdot g \cdot \cos\theta = 160 + 50 = 210$ N.

23 **240 N**

1. Calculate the forces you need to overcome: The force due to gravity is mg sin θ, and the force due to kinetic friction is $F = \mu_k \cdot F_n$. You need to find the normal force.
2. The equation for normal force is $F_n = m \cdot g \cdot \cos\theta$. Use the normal force to calculate the force due to kinetic friction: $\mu_k \cdot m \cdot g \cdot \cos\theta$.
3. The force you have to overcome to keep the chest of drawers moving is $F = m \cdot g \cdot \sin\theta + \mu_k \cdot m \cdot g \cdot \cos\theta$.
4. Plug in the numbers: $F = m \cdot g \cdot \sin\theta + \mu_k \cdot m \cdot g \cdot \cos\theta = 150 + 93 = 240$ N.

24 **680 N**

1. Calculate the forces you need to overcome: The force due to gravity is mg sin θ, and the force due to kinetic friction is $F = \mu_k \cdot F_n$. You need to find the normal force.
2. The equation for normal force is $F_n = m \cdot g \cdot \cos\theta$. Use the normal force to calculate the force due to kinetic friction: $\mu_k \cdot m \cdot g \cdot \cos\theta$.
3. The force you have to overcome to keep the chest of drawers moving is $F = m \cdot g \cdot \sin\theta + \mu_k \cdot m \cdot g \cdot \cos\theta$.
4. Plug in the numbers: $F = m \cdot g \cdot \sin\theta + \mu_k \cdot m \cdot g \cdot \cos\theta = 460 + 220 = 680$ N.

25 **4.2 m/sec**

1. Calculate the forces on the suitcase: The force due to gravity is mg sin θ, and the force due to kinetic friction is $F = \mu_k \cdot F_n$. You need to find the normal force.
2. The equation for the normal force is $F_n = mg\cos\theta$. Use the normal force to calculate the force due to kinetic friction: $\mu_k mg\cos\theta$.
3. The net force on the suitcase along the ramp is $F = mg\sin\theta - \mu_k mg\cos\theta$.
4. Divide the net force along the ramp by m to get the acceleration: $a = g\sin\theta - \mu_k g\cos\theta$.
5. Plug in the numbers: $a = g\sin\theta - \mu_k g\cos\theta = 5.0 - 0.8 = 4.2$ m/sec.

26 **Yes, the box will slide.**

1. Calculate the forces on the box, and if the force down the ramp is larger than the force up the ramp, the box will slide. The force down the ramp is mg sin θ.
2. The force due to kinetic friction pointing up the ramp is $F = \mu_k \cdot F_n$, which means you need to find the normal force.
3. The equation for the normal force is $F_n = m \cdot g \cdot \cos\theta$. Use the normal force to calculate the force due to kinetic friction: $\mu_k \cdot m \cdot g \cdot \cos\theta$.
4. The net force on the suitcase along the ramp is $F = mg\sin\theta - \mu_k \cdot m \cdot g \cdot \cos\theta$.
5. Plug in the numbers: $F = m \cdot g \cdot \sin\theta - \mu_k \cdot m \cdot g \cdot \cos\theta = 10 - 7.1 = 2.9$ N down the ramp, so the box will slide.

27 **No, the book will not slide.**

1. Calculate the forces on the book, and if the force down the ramp is larger than the force up the ramp, the book will slide. The force down the ramp is $mg \sin \theta$.
2. The force due to kinetic friction pointing up the ramp is $F = \mu_k \cdot F_n$, which means you need to find the normal force.
3. The equation for the normal force is $F_n = m \cdot g \cdot \cos \theta$. Use the normal force to calculate the force due to kinetic friction: $\mu_k \cdot m \cdot g \cdot \cos \theta$.
4. The net force on the suitcase along the ramp is $F = mg \sin \theta - \mu_k \cdot m \cdot g \cdot \cos \theta$.
5. Plug in the numbers: $F = m \cdot g \cdot \sin \theta - \mu_k \cdot m \cdot g \cdot \cos \theta = 2.5 - 2.8 = -0.3$ N up the ramp, so the book will not slide.

28 **1.5 m/sec^2 down the ramp**

1. Calculate the forces on the refrigerator: The force down the ramp is $mg \sin \theta$, and the force due to kinetic friction pointing up the ramp is $F = \mu_k \cdot F_n$. You need to find the normal force.
2. The equation for the normal force is $F_n = m \cdot g \cdot \cos \theta$. Use the normal force to calculate the force due to kinetic friction: $\mu_k \cdot m \cdot g \cdot \cos \theta$.
3. The net force on the suitcase along the ramp is $F = m \cdot g \cdot \sin \theta - \mu_k \cdot m \cdot g \cdot \cos \theta$.
4. Divide by m to get the acceleration: $a = g \cdot \sin \theta - \mu_k \cdot g \cdot \cos \theta$.
5. Plug in the numbers: $a = g \cdot \sin \theta - \mu_k \cdot g \cdot \cos \theta = 3.8 - 2.3 = 1.5$ m/sec^2.

Chapter 6

Round and Round: Circular Motion

In This Chapter

- Converting angles
- Handling period and frequency
- Working with angular frequency
- Using angular acceleration

In physics, motion in circles is just as important as motion along lines, but there are all kinds of differences between the two. With circular motion, you no longer work with meters but instead work with radians, which are angular units. Velocity in circular motion is no longer in meters/sec but rather radians/sec.

The bottom line is that circular motion requires a whole new way of thinking. And the best way to get a handle on that switch is to break it down and work through some problems. That's what this chapter is about — solving problems involving angular motion, angular velocity, and angular acceleration.

Converting between Angles

The first step in working with angular motion is to know about the way of measuring that motion. You use radians, not meters, and you have to know what that means.

Take a look at Figure 6-1, where you can see an angle, θ, in a circle. Suppose that θ is 45°. What is that in radians?

There are 2π radians in a full, 360° circle. That means that in order to convert 45° from degrees to radians, you multiply by 2π / 360°, like so:

$$45^\circ \frac{2\pi}{360^\circ} = \frac{\pi}{8}$$

REMEMBER

Angles measured in radians don't have units! That's because they're expressed as a fraction of 2π. You'll sometimes see angles expressed as though radians were units, but technically, they're not.

The conversion factor, 2 π/360°, is usually written as π / 180°, which makes this conversion:

$$45^\circ \frac{\pi}{180^\circ} = \frac{\pi}{8}$$

Conversely, to convert from radians to degrees, you multiply by 180° / π. For example, the equation to convert π / 4 to an angle looks like this:

$$\frac{\pi}{4}\frac{180^\circ}{\pi} = 90^\circ$$

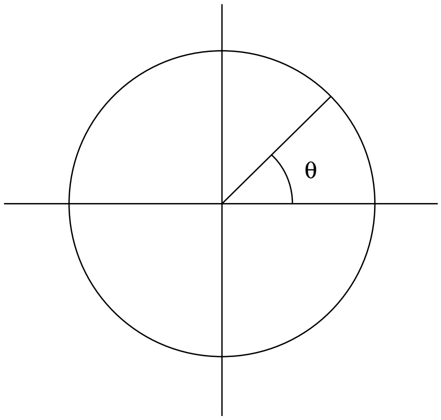


Figure 6-1: An angle in a circle.

EXAMPLE

Q. Convert 180° into radians.

A. The correct answer is π.

1. Use the conversion factor π / 180°.
2. Plug in the numbers:

$$180° \frac{\pi}{180°} = \pi$$

1. What is 23° in radians?

Solve It

2. What is π / 16 in degrees?

Solve It

Period and Frequency

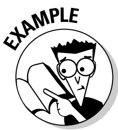
When describing the way things go in circles, you don't just use radians; you also can specify the time it takes. The time it takes for an object to complete an orbit is referred to as its *period.* For example, if the object is traveling at speed v, then the time it takes to go around the circle — the distance it travels in the circle's circumference, $2\pi r$ — will be

$$T = \frac{2\pi r}{v}$$

Note the symbol of the radius of a circle: r. That's half the circle's diameter, which is d. So $r = d / 2$. Note also the symbol for the period: T. With this equation, given an orbiting object's speed and the radius of the circle, you can calculate the object's period.

Another time measurement you'll see in physics problems is *frequency.* Whereas the period is the time an object takes to go around in a circle, the frequency is the number of circles the object makes per second. The frequency, f, is connected to the period like this:

$$f = \frac{1}{T}$$



Q. The moon's orbital radius is 3.85×10^8 m, and its period is about 27.3 days. What is its speed as it goes around the Earth?

A. The correct answer is 1024 m/sec.

1. Convert 27.3 days to seconds:

$$27.3 \text{ days} \frac{24 \cancel{\text{hours}}}{\cancel{\text{day}}} \frac{60 \cancel{\text{minutes}}}{\cancel{\text{hour}}} \frac{60 \text{ seconds}}{\cancel{\text{minute}}} = 2.36 \times 10^6 \text{ sec}$$

2. Use the equation for the period to solve for speed:

$$v = \frac{2\pi r}{T}$$

3. Plug in the numbers:

$$v = \frac{2\pi r}{T} = \frac{2 \cdot \pi \cdot 3.85 \times 10^8}{2.36 \times 10^6} = 1024 \text{ m/sec}$$

3. You have a ball on a string, and you're whipping it around in a circle. If the radius of its circle is 1.0 m and its period is 1.0 sec, what is its speed?

Solve It

4. You have a toy plane on a wire, and it's traveling around in a circle. If the radius of its circle is 10.0 m and its period is 0.75 sec, what is its speed?

Solve It

Getting into Angular Velocity

There are analogs of every linear motion quantity (such as distance, velocity, and acceleration) in angular motion, and that's one of the things that makes angular motion easier to work with. The velocity of an object in linear motion is shown in the following equation (this is actually a vector equation, of course, but I don't get into the vector nature of angular motion until Chapter 10, so I look at this equation in scalar terms):

$$v = \frac{\Delta s}{\Delta t}$$

What's the analog of this equation in angular terms? That's easy; you just substitute angle θ for the distance, so the angular velocity is θ / t. That means that angular velocity ω is the angle (in radians) that an object sweeps through per second.

$$\omega = \frac{\Delta\theta}{\Delta t}$$

Figure 6-2 shows a line sweeping around in a circle. At a particular moment, it's at angle θ, and if it took time t to get there, its angular velocity is θ / t.

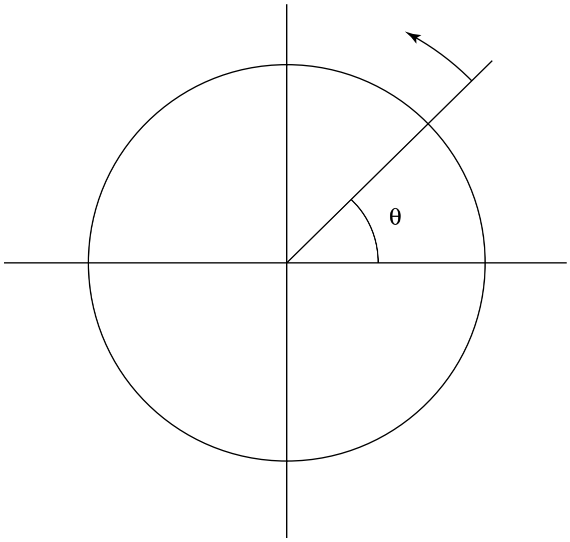


Figure 6-2: Angular velocity in a circle.

So if the line in Figure 6-2 completes a full circle in 1.0 sec, its angular velocity is $2\pi/1.0$ sec = 2π radians/sec (because there are 2π radians in a complete circle). Technically speaking, radian isn't a physical unit of measure (it's a ratio), so the angular velocity can also be written $2\pi\ \text{sec}^{-1}$.

The symbol for angular velocity is ω, so you can write the equation for angular velocity this way:

$$\omega = \frac{\Delta\theta}{\Delta t}$$

Given the angular velocity, you also can find the angle swept through in a number of seconds:

$$\Delta\theta = \omega \cdot \Delta t$$

Q. The moon goes around the Earth in about 27.3 days. What is its angular velocity?

A. The correct answer is 2.66×10^{-6} radians/sec.

1. **Convert 27.3 days to seconds:**

$$27.3 \ \frac{24\ \cancel{\text{hours}}}{\cancel{\text{day}}} \ \frac{60\ \cancel{\text{minutes}}}{\cancel{\text{hour}}} \ \frac{60\ \text{seconds}}{\cancel{\text{minute}}} = 2.36 \times 10^{6}\ \text{sec}$$

2. **Use the equation for angular velocity:**

$$\omega = \frac{\Delta\theta}{\Delta t}$$

3. **Plug in the numbers:**

$$\omega = \frac{\Delta\theta}{\Delta t} = \frac{2\ \pi}{2.36 \times 10^{6}\ \text{sec}} = 2.66 \times 10^{-6}\ \text{radians/sec}$$

5. You have a toy plane on a string that goes around three complete circles in 9.0 sec. What is its angular velocity?

Solve It

6. You're swinging a baseball bat around, getting ready for your shot at the ball. If you swing the bat in a half circle in 1.0 sec, what is its angular velocity?

Solve It

7. A satellite is orbiting the Earth at 8.7×10^{-4} radians/sec. How long will it take to circle the entire world?

Solve It

8. A merry-go-round is spinning around at 2.1 radians/sec. How long will it take to go in a complete circle?

Solve It

Whipping Around with Angular Acceleration

Just as with linear motion, you can have acceleration when you're dealing with angular motion. For example, the line in Figure 6-2 may be sweeping around the circle faster and faster, which means that it's accelerating.

In linear motion, the following is the equation for *acceleration,* the rate at which the object's velocity is changing:

$$a = \frac{\Delta v}{\Delta t}$$

As with all the equations of motion, you need only to substitute the correct angular quantities for the linear ones. In this case, v becomes ω. So the angular acceleration is $\Delta\omega / \Delta t$.

The symbol for linear acceleration is a, and the symbol for angular acceleration is α, which makes the equation for angular acceleration:

$$\alpha = \frac{\Delta \omega}{\Delta t}$$

The unit for angular acceleration is radians/sec^2 (or, technically, just sec^{-2}).

EXAMPLE

Q. Your toy plane on a string accelerates from ω = 2.1 radians/sec to 3.1 radians/sec in 1.0 sec. What is its angular acceleration?

A. The correct answer is 1.0 radians/sec^2.

1. Use the equation for angular acceleration:

$$\alpha = \frac{\Delta \omega}{\Delta t}$$

2. Plug in the numbers:

$$\alpha = \frac{\Delta \omega}{\Delta t} = \frac{(3.1 - 2.1)}{1.0} = 1.0 \text{ radians/sec}^2$$

9. Your model globe is turning at 2.0 radians/sec, which you decide isn't fast enough. So you give it a push, accelerating it in 10^{-1} sec to 5.0 radians/sec. What is its angular acceleration?

Solve It

10. You have a toy plane on a wire, and it's traveling around in a circle at 3.5 radians/sec. You speed it up to 5.4 radians/sec in 3.0 sec. What was its angular acceleration?

Solve It

11. You're square dancing, turning your partner around at 1.0 radians/sec. Then you speed up for 0.50 sec at an angular acceleration of 10.0 radians/sec^2. What is your partner's final angular speed?

Solve It

12. You're trying a new yoga move, and starting your arm at rest, you accelerate it at 15 radians/sec^2 over 1.0 sec. What's your arm's final angular velocity?

Solve It

Connecting Angular Velocity and Angular Acceleration to Angles

You can connect the distance traveled to the original velocity and linear acceleration like this:

$$s = v_0(t_f - t_o) + \frac{1}{2}a(t_f - t_o)^2$$

And you can make the substitution from linear to angular motion by putting in the appropriate symbols:

$$\theta = \omega_0(t_f - t_o) + \frac{1}{2}\alpha(t_f - t_o)^2$$

Using this equation, you can connect angular velocity, angular acceleration, and time to the angle.

EXAMPLE

Q. A marble is rolling around a circular track at 6.0 radians/sec and then accelerates at 1.0 radians/sec^2. How many radians has it gone through in 1 minute?

A. The correct answer is 2200 radians.

1. Use this equation:

$$\theta = \omega_0(t_f - t_o) + \frac{1}{2}\alpha(t_f - t_o)^2$$

2. Plug in the numbers:

$$\theta = \omega_0(t_f - t_o) + \frac{1}{2}\alpha(t_f - t_o)^2 = (6.0)(60) + \frac{1}{2}(1.0)(60^2) = 360 + 1800 = 2160$$

13. Your model globe is spinning at 1.0 radians/sec when you give it a push. If you accelerate it at 5.0 radians/sec^2, how many radians has it turned through in 5.0 sec?

Solve It

14. Your toy plane on a wire is traveling around in a circle at 8.0 radians/sec. If you accelerate it at 1.0 radians/sec^2 for 20 sec, how many radians has it gone through during that time?

Solve It

15. You're whipping a ball on a string around in a circle. If it's going 7.0 radians/sec and at the end of 6.0 sec has gone through 60.0 radians, what was its angular acceleration?

Solve It

16. A roulette wheel is slowing down, starting at from 12.0 radians/sec and going through 40.0 radians in 5.0 sec. What was its angular acceleration?

Solve It

Connecting Angular Acceleration and Angle to Angular Velocity

You can connect angle, angular velocity, and angular acceleration. The corresponding equation for linear motion is

$$v_f^2 - v_o^2 = 2 \cdot a \cdot s$$

Substituting ω for v, α for a, and θ for s gives you:

$$\omega_f^2 - \omega_o^2 = 2 \cdot \alpha \cdot \theta$$

This is the equation to use when you want to relate angle to angular velocity and angular acceleration.

EXAMPLE

Q. A merry-go-round slows down from 6.5 radians/sec to 2.5 radians/sec, undergoing an angular acceleration of 1.0 radians/sec^2. How many radians does the merry-go-round go through while this is happening?

A. The correct answer is 18 radians.

1. Start with the equation:

$$\omega_f^2 - \omega_o^2 = 2 \cdot \alpha \cdot \theta$$

2. Solve for θ:

$$\theta = \frac{\omega_f^2 - \omega_o^2}{2\,\alpha}$$

3. Plug in the numbers:

$$\theta = \frac{\omega_f^2 - \omega_o^2}{2\,\alpha} = \frac{(2.5)^2 - (6.5)^2}{2(-1.0)} = 18 \text{ radians}$$

17. A helicopter's blades are speeding up. In 1.0 sec, they go from 60 radians/sec to 80 radians/sec. If the angular acceleration is 10 radians/sec^2, what is the total angle the blades have gone through?

Solve It

18. Your ball on a string is traveling around in a circle. If it goes from 12 radians/sec to 24 radians/sec and the angular acceleration is 20 radians/sec^2, what is the total angle the ball has gone through during this acceleration?

Solve It

Handling Centripetal Acceleration

In order to keep an object going around in a circle, that object must be pulled toward the center of the circle. Take a look at the moon circling the Earth in Figure 6-3. The moon is accelerated toward the Earth along a radius from the Earth to the moon. The acceleration needed to keep an object (in Figure 6-3, it's the moon) going around in a circle is called the *centripetal acceleration,* and it's always perpendicular to the object's travel. The centripetal acceleration points toward the center of the circle.

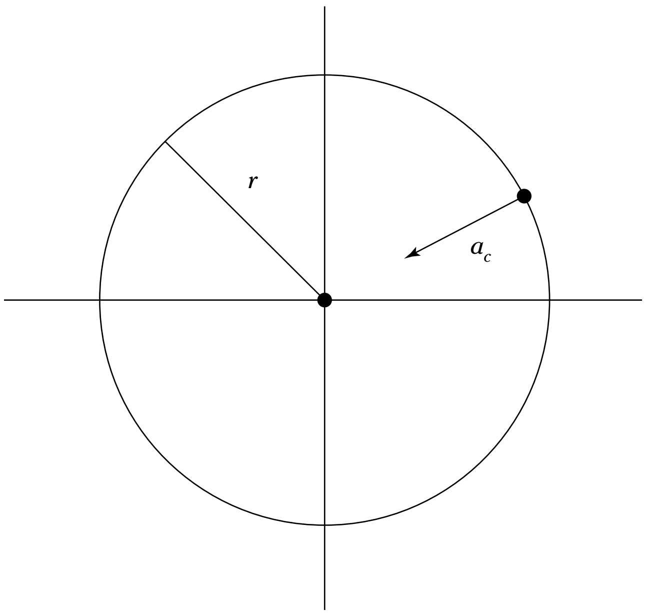


Figure 6-3: Centripetal acceleration.

If you're given the linear speed of the object going in a circle and the radius of the circle, you can calculate the centripetal acceleration, a_c, like this:

$$a_c = \frac{v^2}{r}$$

This equation gives you the acceleration needed to keep an object going around in a circle. You can think of this acceleration as continually bending the direction the object is going in so that it will keep going in a circle.

Q. Given that the moon goes around the Earth about every 27.3 days and that its distance from the center of the Earth is 3.85×10^8 m, what is the moon's centripetal acceleration?

A. The correct answer is 2.7×10^{-3} m/sec^2.

1. **Start with this equation:**

$$a_c = \frac{v^2}{r}$$

2. **Find the speed of the moon. It goes 2πr in 27.3 days, so convert 27.3 to seconds:**

$$27.3 \text{ days} \frac{24 \text{ hours}}{\text{day}} \frac{60 \text{ minutes}}{\text{hour}} \frac{60 \text{ seconds}}{\text{minute}} = 2.36 \times 10^6 \text{ sec}$$

3. **Therefore, the speed of the moon is**

$$\frac{2\pi r}{T} = \frac{2\pi\left(3.85 \times 10^8\right)}{2.36 \times 10^6} = 1024 \text{ m/sec}$$

4. **Plug in the numbers:**

$$a_c = \frac{v^2}{r} = \frac{1024^2}{3.85 \times 10^8} = 2.7 \times 10^{-3} \text{ m/sec}^2$$

19. The tips of a helicopter's blades are moving at 300 m/sec and have a radius of 7.0 m. What is the centripetal acceleration of those tips?

Solve It

20. Your ball on a string is rotating around in a circle. If it's going 60 mph at a radius of 2.0 m, what is its centripetal acceleration?

Solve It

Getting Forceful: Centripetal Force

To give an object moving in a circle the centripetal acceleration needed to keep moving, it needs a force applied to it. That force is called the *centripetal force.* Because $F = m \cdot a$, the centripetal force F_c is just $m \cdot a_c$. Here's the equation for centripetal force:

$$F_c = \frac{m \cdot v^2}{r}$$

Q. The moon goes around the Earth about every 27.3 days with a distance from the Earth of 3.85×10^8 m. If the moon's mass is 7.35×10^{22} kg, what is the centripetal force that the Earth's gravity exerts on it as it orbits the Earth?

A. The correct answer is 2.0×10^{20} N.

1. Start with this equation:

$$F_c = \frac{m \cdot v^2}{r}$$

2. Find the speed of the moon. It goes $2 \cdot \pi \cdot r$ in 27.3 days, so convert 27.3 to seconds:

$$27.3 \text{ days} \frac{24 \cancel{\text{hours}}}{\cancel{\text{day}}} \frac{60 \cancel{\text{minutes}}}{\cancel{\text{hour}}} \frac{60 \text{ seconds}}{\cancel{\text{minute}}} = 2.36 \times 10^6 \text{ sec}$$

3. Therefore, the speed of the moon is

$$\frac{2\pi r}{T} = \frac{2\pi \left(3.85 \times 10^8\right)}{2.36 \times 10^6} = 1024 \text{ m/sec}$$

4. Plug in the numbers:

$$F_c = \frac{m \cdot v^2}{r} = 2.0 \times 10^{20} \text{ N}$$

21. You're exerting a force on a string to keep a ball on a string going in a circle. If the ball has a mass of 0.1 kg and the angular velocity of the ball is 8.0 radians/sec at a distance of 2.0 m, what is the centripetal force you need to apply to keep the ball going in a circle?

Solve It

22. You have a 1.0 kg toy plane on the end of a 10 m wire, and it's going around at 6.0 radians/sec. What is the force you have to apply to the wire to keep the plane going in a circle?

Solve It

Answers to Problems about Circular Motion

The following are the answers to the practice questions presented earlier in this chapter. You see how to work out each answer, step by step.

1 **0.13 π**

1. Use the conversion factor π / 180°.
2. Plug in the numbers:

$$23^\circ \frac{\pi}{180^\circ} = 0.13\ \pi$$

2 **11.2°**

1. Use the conversion factor 180° / π.
2. Plug in the numbers:

$$\frac{\pi}{16}\frac{180^\circ}{\pi} = 11.2^\circ$$

3 **6.28 m/sec**

1. Use the equation for the period to solve for speed:

$$v = \frac{2\pi r}{T}$$

2. Plug in the numbers:

$$v = \frac{2 \cdot \pi \cdot r}{T} = \frac{2\ \pi\ 1.0}{1.0} = 6.28\ \text{m/sec}$$

4 **84 m/sec**

1. Use the equation for the period to solve for speed:

$$v = \frac{2\pi r}{T}$$

2. Plug in the numbers:

$$v = \frac{2 \cdot \pi \cdot r}{T} = \frac{2\ \pi\ 100}{0.75} = 84\ \text{m/sec}$$

5 **(2/3) π radians/sec (0.70 radians/sec)**

1. Use the equation for the angular velocity:

$$\omega = \frac{\Delta\theta}{\Delta t}$$

2. Plug in the numbers:

$$\omega = \frac{\Delta\theta}{\Delta t} = \frac{3 \cdot 2 \cdot \pi}{9.0} = (2/3)\ \pi\ \text{radians/sec}$$

6 **π radians/sec (3.14 radians/sec)**

1. Use the equation for the angular velocity:

$$\omega = \frac{\Delta\theta}{\Delta t}$$

2. Plug in the numbers:

$$\omega = \frac{\Delta\theta}{\Delta t} = \frac{0.5 \cdot 2 \cdot \pi}{1.0} = \pi\ \text{radians/sec}$$

7 **120 minutes**

1. Start with the equation for the angular velocity:

$$\omega = \frac{\Delta\theta}{\Delta t}$$

2. Solve for Δt:

$$\Delta t = \frac{\Delta\theta}{\omega}$$

3. Plug in the numbers:

$$\Delta t = \frac{\Delta\theta}{\omega} = \frac{2\pi}{8.7 \times 10^{-4}} = 7200 \text{ sec}$$

That's about 120 minutes.

8 **3.0 seconds**

1. Start with the equation for the angular velocity:

$$\omega = \frac{\Delta\theta}{\Delta t}$$

2. Solve for Δt:

$$\Delta t = \frac{\Delta\theta}{\omega}$$

3. Plug in the numbers:

$$\Delta t = \frac{\Delta\theta}{\omega} = \frac{2\pi}{2.1} = 3.0 \text{ sec}$$

9 **30. radians/sec^2**

1. Use the equation for angular acceleration:

$$\alpha = \frac{\Delta\omega}{\Delta t}$$

2. Plug in the numbers:

$$\alpha = \frac{\Delta\omega}{\Delta t} = \frac{(5.0 - 2.0)}{10^{-1}} = 30 \text{ radians/sec}^2$$

10 **0.63 radians/sec^2**

1. Use the equation for angular acceleration:

$$\alpha = \frac{\Delta\omega}{\Delta t}$$

2. Solve for $\Delta\omega$:

$$\alpha = \frac{\Delta\omega}{\Delta t} = \frac{(5.4 - 3.5)}{3.0} = 0.63 \text{ radians/sec}^2$$

11 **6.0 radians/sec**

1. Use the equation for angular acceleration:

$$\alpha = \frac{\Delta\omega}{\Delta t}$$

2. Solve for $\Delta\omega$:

$$\Delta\omega = \alpha \, \Delta t$$

3. Plug in the numbers:

$$\Delta\omega = \alpha\,\Delta t = (10)\cdot(0.5) = 5.0 \text{ radians/sec}$$

4. Add $\Delta\omega$ to the original angular velocity, ω_o:

$$1.0 + 5.0 = 6.0 \text{ radians/sec}$$

12 **15.0 radians/sec**

1. Use the equation for angular acceleration:

$$\alpha = \frac{\Delta\omega}{\Delta t}$$

2. Solve for $\Delta\omega$:

$$\Delta\omega = \alpha\,\Delta t$$

3. Plug in the numbers:

$$\Delta\omega = \alpha\,\Delta t = (15)\cdot(1.0) = 15.0 \text{ radians/sec}$$

4. Add $\Delta\omega$ to the original angular velocity, ω_o:

$$0 + 15.0 = 15.0 \text{ radians/sec}$$

13 **67.5 radians**

1. Use this equation:

$$\theta = \omega_o(t_f - t_o) + \frac{1}{2}\alpha(t_f - t_o)^2$$

2. Plugging in the numbers gives you:

$$\theta = \omega_o(t_f - t_o) + \frac{1}{2}\alpha(t_f - t_o)^2 = (1.0)(5.0) + \frac{1}{2}(\text{xx})(5.0^2) = 5.0 + 12.5 = \text{xx radians}$$

14 **360 radians**

1. Use this equation:

$$\theta = \omega_o\cdot(t_f - t_o) + \frac{1}{2}\alpha(t_f - t_o)^2$$

2. Plug in the numbers:

$$\theta = \omega_o\cdot(t_f - t_o) + \frac{1}{2}\alpha\cdot(t_f - t_o)^2 = (8.0)(20.0) + \frac{1}{2}\cdot(1.0)(20.0^2) = 160 + 200 = 360 \text{ radians}$$

15 **1.0 radian/sec^2**

1. Use this equation:

$$\theta = \omega_0\cdot(t_f - t_o) + \frac{1}{2}\alpha(t_f - t_o)^2$$

2. Solve for α, given that $t_o = 0$:

$$\alpha = \frac{2(\theta - \omega_o t_f)}{t_f^2}$$

3. Plug in the numbers:

$$\alpha = \frac{2(\theta - \omega_o t_f)}{t_f^2} = \frac{2(60 - (7.0)\cdot(6.0))}{6.0^2} = 1.0 \text{ radian/sec}^2$$

16 **–1.6 radian/sec²**

1. Use this equation:

$$\theta = \omega_0 \cdot \left(t_f - t_o\right) + \frac{1}{2}\alpha\left(t_f - t_o\right)^2$$

2. Solve for α, given that $t_o = 0$:

$$\alpha = \frac{2\left(\theta - \omega_o t_f\right)}{t_f^2}$$

3. Plug in the numbers:

$$\alpha = \frac{2\left(\theta - \omega_o t_f\right)}{t_f^2} = \frac{2\left(40 - (12.0)\cdot(5.0)\right)}{5.0^2} = -1.6 \text{ radian/sec}^2$$

17 **140 radians**

1. Use this equation:

$$\omega_f^2 - \omega_o^2 = 2 \cdot \alpha \cdot \theta$$

2. Solve for θ:

$$\theta = \frac{\omega_f^2 - \omega_o^2}{2\ \alpha}$$

3. Plug in the numbers:

$$\theta = \frac{\omega_f^2 - \omega_o^2}{2 \cdot \alpha} = \frac{80^2 - 60^2}{2(10)} = 140 \text{ radians}$$

18 **11 radians**

1. Use this equation:

$$\omega_f^2 - \omega_o^2 = 2 \cdot \alpha \cdot \theta$$

2. Solve for θ:

$$\theta = \frac{\omega_f^2 - \omega_o^2}{2\ \alpha}$$

3. Plug in the numbers:

$$\theta = \frac{\omega_f^2 - \omega_o^2}{2 \cdot \alpha} = \frac{24^2 - 12^2}{2(20)} = 11 \text{ radians}$$

19 **1.3×10^4 m/sec²**

1. Use this equation:

$$a_c = \frac{v^2}{r}$$

2. Plug in the numbers:

$$a_c = \frac{v^2}{r} = \frac{300^2}{7.0} = 1.3 \times 10^4 \text{ m/sec}$$

20 **360 m/sec²**

1. Use this equation:

$$a_c = \frac{v^2}{r}$$

2. Plug in the numbers:

$$a_c = \frac{v^2}{r} = \frac{26.8^2}{2.0} = 360 \text{ m/sec}^2$$

21 **13 N**

1. Use this equation:

$$F_c = \frac{m \cdot v^2}{r}$$

2. Calculate the velocity of the ball by relating v and ω like this:

$$v = \omega r$$

3. That makes the equation for centripetal acceleration this:

$$F_c = m \cdot \omega^2 \cdot r$$

4. Plug in the numbers:

$$F_c = m \cdot \omega^2 \cdot r = (0.1) \cdot (8.0^2) \cdot (2.0) = 13 \text{ N}$$

22 **360 N**

1. Use this equation:

$$F_c = \frac{m \cdot v^2}{r}$$

2. Calculate the velocity of the plane by relating v and ω like this:

$$v = \omega r$$

3. That makes the equation for centripetal acceleration this:

$$F_c = m \cdot \omega^2 \cdot r$$

4. Plug in the numbers:

$$F_c = m \cdot \omega^2 \cdot r = (1.0) \cdot (6.0^2) \cdot (10) = 360 \text{ N}$$

Part III

Being Energetic: Work

The 5th Wave By Rich Tennant

"You can take that old jar for your science project, I'm sure I have some baking soda you can borrow, and let's see, where's that old particle accelerator of mine... here it is in the pantry."

In this part . . .

A truck at the top of a hill has potential energy. Let the brake slip, and the truck rolls down to the bottom of the hill, where it has kinetic energy. Thinking in terms of work and energy lets you solve problems that Newton's laws couldn't attempt, putting you on track to be a physics problem-solving pro.

Chapter 7

Working the Physics Way

In This Chapter

- Understanding work
- Working with net force
- Calculating kinetic energy
- Handling potential energy
- Relating kinetic energy to work

This chapter covers all kinds of problems having to do with work in physics terms. Work has a specific definition in physics: It's force times the distance over which that force acts. For example, you may be holding up a refrigerator and thinking that you're doing a lot of work, but if that refrigerator is stationary, it's not considered work in physics. (On the other hand, lots of things are happening in your body at the molecular level to let you hold that refrigerator up, and those things can be considered physics — but that's another story, a biophysics one).

In addition to work, this chapter covers kinetic energy — that is, the ability to do work that comes from motion. Need an example? If you're at the bottom of a hill and trying to stop a speeding car, you get an instant introduction to kinetic energy. Also covered in this chapter is potential energy, which is the ability to do work stored up in an object. For example, a car at the bottom of a hill has less potential energy than one at the top of a hill because the one at the top of the hill can travel down, converting its potential energy into kinetic energy.

Finally, this chapter considers problems of power, or how much work you do in so much time. That's measured in Watts, which you're surely familiar with thanks to light bulbs.

A Different Kind of Work

In physics, *physical work* is defined as the applied force multiplied by the distance over which it was applied. So if you're pushing a refrigerator 2.0 m across the floor, and you need to apply 900 N, you've done this much work:

$$(900)\cdot(2.0) = 1800$$

1800 what? Work in the MKS system is measured in Joules, J, so that's 1800 J. You should almost always see work expressed in Joules in physics problems. The CGS unit for work is the (oddly named) erg, but it's rare that you'll see that used.

So applying a force of 1.0 N over a distance of 1.0 m means that you've done 1 Joule of work. 1.0 J = 1.0×10^7 ergs because ergs are dyne-centimeters, and there are 10^5 dynes per Newton and 10^2 centimeters in a meter. The FPS system of work uses the foot-pound as the unit for work.

But work isn't just the force applied multiplied by the distance traveled; it's also the force applied along the direction of travel. For example, look at the mass being pulled in Figure 7-1.

Figure 7-1: Dragging a mass.

The force applied along the direction of travel is what counts, so you want the component of F that is horizontal (assuming you're dragging the mass and not picking it up!). That horizontal component is F cos θ, so the work you do, W, is this:

$$W = F\,s\cos\theta$$

Here, F is the force applied, s is the displacement, and θ is the angle between them.

Q. Suppose that you're pulling the mass in Figure 7-1 by applying 200 N and that the angle θ is 40.0°. How much work do you do dragging the mass over 50.0 m?

A. The correct answer is 7660 J.

1. Use the equation $W = F\,s\cos\theta$
2. Plug in the numbers:

$$W = F\,s\cos\theta = (200)\cdot(50.0)\cdot(.766) = 7660 \text{ J}$$

1. You're pulling a chest of drawers, applying a force of 60.0 N at an angle of 60.0°. How much work do you do pulling it over 10.0 m?

Solve It

2. You're dragging a sled on a rope at 30°, applying a force of 20 N. How much work do you do over 1.0 km?

Solve It

3. You're pushing a box of dishes across the kitchen floor, using 100.0 J to move it 10.0 m. If you apply the force at 60.0°, what is the force you used?

Solve It

4. You're pushing an out-of-gas car down the road, applying a force of 800.0 N. How much work have you done in moving the car 10.0 m?

Solve It

Dealing with the Net Force

There's often more than one force involved when you're dragging a mass over a distance. Just think of the forces of friction and gravity.

For example, take a look at Figure 7-2, where a couch is being dragged up an incline. If you're applying force F, how much work is done as the couch is dragged up the incline?

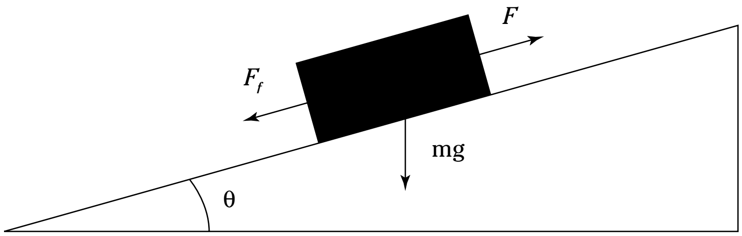


Figure 7-2: Dragging a mass up an incline.

Look at the forces involved in this scenario: There's the force you apply (your pull), the force due to friction that resists your pull, and the force due to gravity that also resists your pull. While all these forces do work on the couch as it's being pulled up the slope, the net work done on the couch is the product of the component of the *net* force multiplied by the distance traveled.

In other words, you may do 100 J of work on the couch; friction, which is opposing you, may do –50 J (the work is negative because the force here is opposite to the direction traveled); and the force of gravity may do –30 J. That means that a net work of 20 J was done on the couch by all the forces (100 – 50 – 30 = 20 J).

Q. Suppose that the couch you're dragging in Figure 7-2 has a mass of 75.0 kg, and the angle of the ramp is 24.0°. If the coefficient of kinetic friction is 0.170 and you're pulling the couch 2.00 m with a force of 800.0 N, how much work is being done by the net force on the couch?

A. The correct answer is 773 J.

1. Identify the forces on the couch. You're pulling with a force F in the direction of the ramp, so the component of your force in the direction of travel is exactly 800 N.

2. Look at the force due to gravity. The figure tells you that this force is $F_g = m \cdot g \sin\theta$. (You can verify this by noting that F_g goes to $m \cdot g$ as θ goes to 0°.)

3. Calculate the force due to friction. That's $F_f = \mu_k \cdot F_n$, where F_n is the normal force. The normal force in this scenario is mg sin θ, so that makes the force due to friction

$$F_f = \mu_k \cdot F_n = \mu_k \cdot m \cdot g \cos\theta$$

4. Calculate the net force:

$$\Sigma F = F - m \cdot g \sin\theta - \mu_k \cdot m \cdot g \cos\theta$$

Note that the forces due to friction and gravity are negative because they oppose the direction of travel.

5. The work done is F s cos θ. Because all forces act along the direction of travel, the net work, W, is equal to

$$W = \Sigma F \cdot s = (F - m \cdot g \sin\theta - \mu_k \cdot m \cdot g \sin\theta) \cdot s.$$

6. Plug in the numbers:

$$W = \Sigma F \cdot s = (F - m \cdot g \sin\theta - \mu_k \cdot m \cdot g \cos\theta) \cdot s = (800 - (75) \cdot (9.8) \cdot (0.41) - (0.17) \cdot (9.8) \cdot (75) \cdot (0.91)) \cdot (2.0) = (800 - 114 - 299) \cdot (2.0) = 773 \text{ J}$$

5. You're pulling a chest of drawers, applying a force of 600.0 N parallel to the slope. The angle of the slope is 40.0°. The chest has a mass of 40.0 kg, and there's a coefficient of kinetic friction of 0.12. How much net work do you do in pulling the chest of drawers 10.0 m up the inclined plane?

Solve It

6. You're applying a force of 800.0 N to yourself as you go up a ski slope on your new skis; this force is applied parallel to the slope. If the slope is 22.0°, the coefficient of kinetic friction is 0.050, and you have a mass of 80.0 kg, how much work do you do going up that slope 500.0 m? How much net work is done?

Solve It

Getting Energetic: Kinetic Energy

When you have objects in motion, you have kinetic energy. When, for example, you slide an object on a frictionless surface, the work you do goes into the object's kinetic energy.

If you have an object with mass m moving at speed v, its kinetic energy is

$$KE = \frac{1}{2} mv^2$$

That's the energy of motion.

EXAMPLE

Q. Say that you push a space ship, mass 1000.0kg, by applying a force of 1.00×10^4 N for 1.00 m. How fast does the space ship end up traveling?

A. The correct answer is 4.5 m/sec.

1. Find the total work done:

$$(1.0 \times 10^4\ N)\cdot(1.0\ m) = 1.0 \times 10^4\ J$$

2. The work done on the space station goes into its kinetic energy, so its final kinetic energy is 1.0×10^4 J.

3. Use the equation for kinetic energy and solve for v:

$$KE = \frac{1}{2} mv^2$$

$$1.0 \times 10^4\ J = \frac{1}{2} m \cdot v^2$$

$$\sqrt{20} = 4.5\ \frac{m}{s}$$

$$v^2 = \frac{2}{m}(1.0 \times 10^4\ J) = 20$$

4. The final speed is $20^{1/2}$ = 4.5 m/sec.

7. You're applying 600.0 N of force to a car with a mass of 1000.0 kg. You're traveling a distance of 100.0 m on a frictionless, icy road. What is the car's final speed?

Solve It

8. You're ice skating and traveling at 30.0 m/sec. If your mass is 65 kg, what is your kinetic energy?

Solve It

9. You're traveling in a car at 88 m/sec. If you have a mass of 80.0 kg and the car has a mass of 1200.0 kg, what is the total kinetic energy of you and the car combined?

Solve It

10. You're applying 6000.0 N of force to a hockey puck with a mass of 0.10 kg. It travels over a distance of 0.10 m on a frictionless ice rink. What is the puck's final speed?

Solve It

Getting Kinetic Energy from Work

Here's more on kinetic energy.

EXAMPLE

Q. In Figure 7-3, a 1000.0 kg safe full of gold bars is sliding down a 3.0 m ramp that meets the horizontal at an angle of 23°. The kinetic coefficient of friction is 0.15. What will the refrigerator's speed be when it reaches the bottom of the ramp?

A. The correct answer is 3.8 m/sec.

1. Determine the net force acting on the refrigerator. The component of the refrigerator's weight acting along the ramp is

$$F = mg \sin \theta$$

2. You know that the normal force is

$$F_N = mg \cos \theta$$

That means that the kinetic force of friction is

$$F_F = \mu_k F_N = \mu_k mg \cos \theta$$

3. The net force accelerating the safe down the ramp, F_{net}, is

$$F_{net} = mg \sin \theta - F_F = mg \sin \theta - \mu_k mg \cos \theta$$

4. Plug in the numbers:

$$F_{net} = (1000)(9.8)(\sin 23°) - (0.15)(1000)(9.8)(\cos 23°) = 3800 \text{ N} - 1350 \text{ N} = 2450 \text{ N}$$

5. The net force, 2450 N, acts over the entire 3.0 m ramp, so the work done by this force is

$$W = F_{net} = (2450)(3.0) = 7400 \text{ J}$$

6. 7400 J of work is done on the safe by the net force, and that work goes into the safe's kinetic energy. You find the safe's kinetic energy by using this equation:

$$W = F_{net} = 7400 \text{ J} = \frac{1}{2} mv^2$$

7. Solve for v^2:

$$v^2 = 14.8, \text{ so } v = 3.8 \text{ m/sec}$$

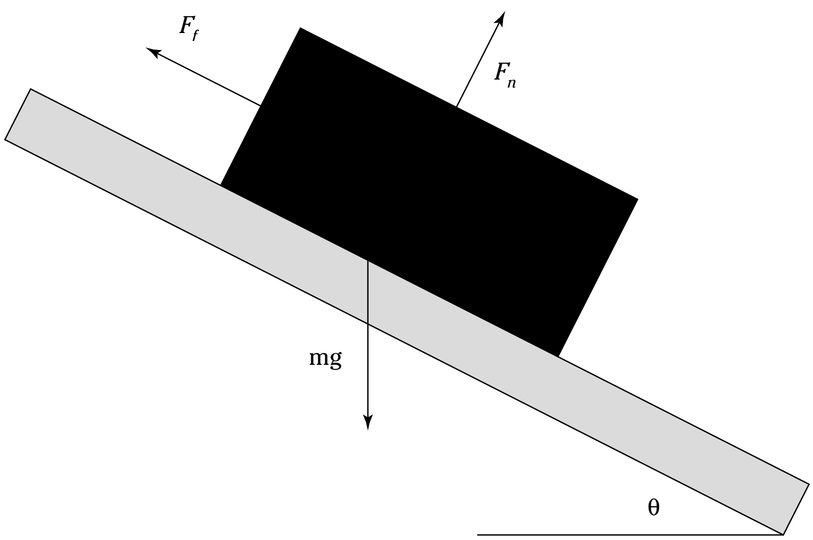


Figure 7-3: The safe slides down.

11. A 40.0 kg box of books is sliding down a 4.0 m ramp of 27°. If the coefficient of kinetic friction is 0.17, what is the box's speed at the bottom of the ramp?

Solve It

12. An 80.0 kg person trips and slides down a 20.0 m toboggan run of 27°. If the coefficient of kinetic friction is 0.10, what is the person's speed at the bottom of the run?

Solve It

13. A parked car with its wheels locked starts sliding down an icy street with an angle of 28°. If the kinetic coefficient of friction is 0.10, the street is 40.0 m to the bottom, and the car has a mass of 1000.0 kg, what is the car's speed at the bottom of the street?

Solve It

14. You're going down a 100.0 m ski jump at an angle of 40.0°. Your mass (including skis!) is 90.0 kg, and the coefficient of kinetic friction is 0.050. What is your speed at the bottom of the jump?

Solve It

Storing Your Energy: Potential Energy

Objects can have energy at rest simply by having a force act on them. For example, an object at the end of a stretched spring has energy because when you let the object go and it accelerates because of the spring, it can convert that stored energy into kinetic energy.

The energy that an object has by virtue of a force acting on it is called *potential energy.* For example, an object at height h in the gravitational field at the Earth's surface has this potential energy:

$$PE = m \cdot g \cdot h$$

For example, say that you have a basketball at height h. When it drops, its potential energy gets converted into kinetic energy. To figure out how fast the basketball (or any object) is going when it hits the ground, you use this equation:

$$PE = KE = mgh = \frac{1}{2} mv^2$$

Q. During a basketball game, a 1.0 kg ball gets thrown vertically in the air. It's momentarily stationary at a height of 5.0 m and then falls back down. What is the ball's speed when it hits the floor?

A. The correct answer is 9.9 m/sec.

1. Find out how much potential energy the ball has when it starts to fall. Use this equation for potential energy:

$$PE = mgh$$

2. This potential energy will be converted into kinetic energy, so you expand the equation like so:

$$PE = mgh = KE = \frac{1}{2} m \cdot v^2$$

3. Solve for v^2:

$$v^2 = 2 \cdot g \cdot h$$

4. Solve for v:

$$v = \sqrt{2gh}$$

5. Plug in the numbers:

$$v = 9.9 \text{ m/sec}$$

15. A 40 kg box of books falls off a shelf that's 4.0 m above the ground. How fast is the box traveling when it hits the ground?

Solve It

16. You jump out of an airplane at 2000 ft and fall 1000 ft before opening your parachute. What is your speed (neglecting air resistance) when you open your chute?

Solve It

17. The flagpole on top of a 300.0 m skyscraper falls off. How fast is it falling when it strikes the ground?

Solve It

18. If you're in an airplane at 30,000 ft, what is your potential energy if you have a mass of 70.0 kg?

Solve It

Powering It Up

When it comes to work in physics, you're sure to see problems involving *power,* which is the amount of work being done in a certain amount of time. Here's the equation for power, P:

$$P = \frac{W}{t}$$

W equals force times distance, so you could write the equation for power this way, assuming that the force was acting along the direction of travel:

$$P = \frac{W}{t} = \frac{F \cdot s}{t}$$

On the other hand, the object's speed, v, is just s / t (displacement over time), so the equation breaks down further to:

$$P = \frac{W}{t} = \frac{F \cdot s}{t} = F \cdot v$$

So power equals force times speed. You use this equation when you need to apply a force in order to keep something moving at constant speed.

EXAMPLE

Q. You're riding a toboggan down an icy run to a frozen lake, and you accelerate the 80.0 kg combination of you and the toboggan from 1.0 m/sec to 2.0 m/sec in 2.0 sec. How much power does that require?

A. The correct answer is 60 Watts.

1. Assuming that there's no friction on the ice, you use this equation for the total work:

$$W = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2$$

2. Plug in the numbers:

$$W = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2 =$$

$$\frac{1}{2}(80)\,2.0^2 - \frac{1}{2}(80)\,1.0^2 = 120\text{ J}$$

3. Because this work is done in 2 sec, the power involved is

$$P = \frac{120\text{ J}}{2.0\text{ sec}} = 60\text{ W}$$

19. A 1000 kg car accelerates from 88 m/sec to 100 m/sec in 30 sec. How much power does that require?

Solve It

20. A 60.0 kg person is running and accelerates from 5.0 m/sec to 7.0 m/sec in 2.0 sec. How much power does that require?

Solve It

21. A 120 kg linebacker accelerates from 5.0 m/sec to 10.0 m/sec in 1.0 sec. How much power does that require?

Solve It

22. You're driving a snowmobile that accelerates from 10 m/sec to 20 m/sec over a time interval of 10.0 sec. If you and the snowmobile together have a mass of 500 kg, how much power is used?

Solve It

Answers to Problems about Work

The following are the answers to the practice questions presented earlier in this chapter. You see how to work out each answer, step by step.

1 **300 J**

1. Use the equation $W = F\, s \cos\theta$.

2. Plug in the numbers:

$$W = F\, s \cos\theta = (60)\cdot(10)\cdot(0.5) = 300\text{ J}$$

2 **1.7×10^4 J**

1. Use the equation $W = F\, s \cos\theta$.

2. Plug in the numbers:

$$W = F\, s \cos\theta = (20)\cdot(1000)\cdot(.866) = 1.7 \times 10^4\text{ J}$$

3 **20 N**

1. Use the equation $W = F\, s \cos\theta$.

2. Solve for force:

$$F = \frac{W}{(s\cos\theta)}$$

3. Plug in the numbers:

$$F = \frac{W}{(s\cos\theta)} = \frac{100}{(10 \times 0.5)} = 20\text{ N}$$

4 **8000 J**

1. Use the equation $W = F\, s \cos\theta$.

2. Plug in the numbers:

$$W = F\, s \cos\theta = (800)\cdot(10)\cdot(1) = 8000\text{ J}$$

5 **3120 J**

1. Find the forces on the chest of drawers. You're pulling with a force F in the direction of the ramp, so the component of your force in the direction of travel is 600 N.

2. Find the force due to gravity. Keep in mind that this force will be going down the ramp.

$$F_g = m\cdot g \cos\theta$$

3. Find the force due to friction. That's $F_f = \mu_k\cdot F_n$, where F_n is the normal force. The normal force here is mg sin θ, so that makes the force due to friction

$$F_f = \mu_k\cdot F_n = \mu_k\cdot m\cdot g \sin\theta$$

4. Calculate the net force:

$$\Sigma F = F - m\cdot g \sin\theta - \mu_k\cdot m\cdot g \cos\theta$$

Note that the force due to friction and gravity are negative because they oppose the direction of travel.

5. The work done is F s cos θ. Because all forces act along the direction of travel, the net work, W, is

$$W = \Sigma F \cdot s = (F - m \cdot g \sin\theta - \mu_k \cdot m \cdot g \cos\theta) \cdot s$$

6. Plug in the numbers:

$$W = \Sigma F \cdot s = (F - m \cdot g \sin\theta - \mu_k \cdot m \cdot g \cos\theta)s =$$
$$(600 - (40) \cdot (9.8) \cdot (0.64) - (0.12) \cdot (9.8) \cdot (40) \cdot (0.77)) \cdot (10.0) =$$
$$(600 - 252 - 36) \cdot (10.0) = 3120 \text{ J}$$

6 You do 4.00×10^5 J. The net work is 2.35×10^5 J.

1. You're pushing yourself up the slope with a force of 800 N; this force goes along the direction of the slope.
2. The force due to gravity is $F_g = m \cdot g \sin\theta$. This force is going down the slope.
3. The force due to friction is $F_f = \mu_k \cdot F_n$, where F_n is the normal force. The normal force here is $m \cdot g \cos\theta$, so that makes the force due to friction:

$$F_f = \mu_k \cdot F_n = \mu_k \cdot m \cdot g \cos\theta$$

4. Calculate the net force:

$$\Sigma F = F - m \cdot g \sin\theta - \mu_k \cdot m \cdot g \cos\theta$$

5. The work done is given by F s cos θ. Because all forces act along the direction of travel, the net work, W_{net}, is

$$W_{net} = \Sigma F \cdot s = (F_a - m \cdot g \sin\theta - \mu_k \cdot m \cdot g \cos\theta) \cdot s$$

6. Plug in the numbers to find the net work:

$$W_{net} = \Sigma F \cdot s = (F - m \cdot g \sin\theta - \mu_k \cdot m \cdot g \cos\theta) \cdot s =$$
$$(800 - (80)\ (9.8)\ (0.37) - (0.05)\ (9.8)\ (80)\ (0.93))\ (500.0) =$$
$$(800 - 36 - 294) \cdot (500.0) = 2.35 \times 10^5 \text{ J}$$

7. The work you do is $(800)\ (500.0) = 4.00 \times 10^5$ J.

7 11 m/sec

1. Find the total work done:

$$(600 \text{ N})\ (100 \text{ m}) = 6.0 \times 10^4 \text{ J}$$

2. The work done on the car goes into its kinetic energy, so its final kinetic energy is 6.0×10^4 J.
3. Use the equation for kinetic energy and solve for v:

$$KE = \frac{1}{2} mv^2$$

4. Plug in the numbers:

$$F = \frac{W}{(s \cos\theta)}$$

$$F = \frac{W}{(s \cos\theta)} = \frac{100}{(10 \times 0.5)} = 20 \text{ N}$$

The final speed $= 120^{1/2} = 11$ m/sec.

8 **2.9×10^4 J**

1. The equation for kinetic energy is

$$KE = \frac{1}{2} mv^2$$

2. Plug in the numbers:

$$\frac{1}{2} mv^2 = 2.9 \times 10^4 \text{ J}$$

9 **4.9×10^6 J**

1. The equation for kinetic energy is

$$KE = \frac{1}{2} mv^2$$

2. Plug in the numbers:

$$\frac{1}{2}(m + M) v^2 = 4.9 \times 10^6 \text{ J}$$

10 **110 m/sec**

1. Find the total work done:

$$(6000 \text{ N}) (0.1 \text{ m}) = 600 \text{ J}$$

2. The work done on the puck goes into its kinetic energy, so its final kinetic energy is 600 J.
3. Use the equation for kinetic energy and solve for v:

$$KE = \frac{1}{2} mv^2$$

4. Plug in the numbers:

$$6000 \text{ J} = \frac{1}{2} mv^2$$

$$v^2 = \frac{2}{m}(600 \text{ J}) = 12{,}000$$

The final speed = $12{,}000^{1/2}$ = 110 m/sec.

11 **4.9 m/sec**

1. To find the net force on the box, start with the component of the box's weight acting along the ramp:

$$mg \sin \theta$$

2. The normal force on the box is

$$F_N = mg \cos \theta$$

which means that the kinetic force of friction is

$$F_F = \mu_k F_N = \mu_k mg \cos \theta$$

3. The net force accelerating the box down the ramp, F_{net}, is

$$F_{net} = mg \sin \theta - F_F = mg \sin \theta - \mu_k mg \cos \theta$$

4. Plug in the numbers:

$$F_{net} = (40) (9.8) (\sin 27^\circ) - (0.17)(40) (9.8) (\cos 27^\circ) = 180 \text{ N} - 60 \text{ N} = 120 \text{ N}$$

5. The net force is 120 N. This net force acts over the 3.0 m ramp, so the work done by this force is

$$W = F_{net} = (120)\ (4.0) = 480\ J$$

6. 480 J of work is done on the box, and that work goes into the box's kinetic energy. Find the box's kinetic energy:

$$W = F_{net} = 480\ J = \frac{1}{2}mv^2$$

7. Solve for v^2:

$$v^2 = 24, \text{ so } v = 4.9\ m/sec$$

12 **11.8 m/sec**

1. Find the net force on the sliding person. The component of the person's weight acting along the run is

$$F = mg \sin\theta$$

2. The normal force on the person is

$$F_N = mg \cos\theta$$

which means that the kinetic force of friction is

$$F_F = \mu_k F_N = \mu_k mg \cos\theta$$

3. The net force accelerating the person down the ramp, F_{net}, is

$$F_{net} = mg \sin\theta - F_F = mg \sin\theta - \mu_k mg \cos\theta$$

4. Plug in the numbers:

$$F_{net} = (80)\ (9.8)\ (\sin 27°) - (0.10)\ (80)\ (9.8)\ (\cos 27°) = 350\ N - 70\ N = 280\ N$$

5. The net force acts over the 20.0 m toboggan ramp, so the work done by this force is

$$W = F_{net} = (280)\ (20.0) = 5600\ J$$

6. 5600J of work is done on the person, and that work goes into the person's kinetic energy. Find the person's kinetic energy:

$$W = F_{net} = 5600\ J = \frac{1}{2}mv^2$$

7. Solve for v^2:

$$v^2 = 140, \text{ so } v = 11.8\ m/sec$$

13 **17 m/sec**

1. Find the net force on the sliding car. The component of the car's weight acting along the street is

$$F = mg \sin\theta$$

2. The normal force on the person is

$$F_N = mg \cos\theta$$

which means that the kinetic force of friction is

$$F_F = \mu_k F_N = \mu_k mg \cos\theta$$

3. The net force accelerating the car down the street, F_{net}, is

$$F_{net} = mg \sin\theta - F_F = mg \sin\theta - \mu_k mg \cos\theta$$

4. Plug in the numbers:

$$F_{net} = (1000)(9.8)(\sin 28^\circ) - (0.10)(1000)(9.8)(\cos 28^\circ) = 4600 \text{ N} - 865 \text{ N} = 3700 \text{ N}$$

5. This net force acts over the 40 m street, so the work done by this force is

$$W = F_{net} = (3700)(40) = 1.5 \times 10^5 \text{J}$$

6. That work goes into the car's kinetic energy:

$$W = F_{net} = 1.5 \times 10^5 \text{ J} = \frac{1}{2} mv^2$$

7. Solve for v^2:

$$v^2 = 300, \text{ so } v = 17 \text{ m/sec}$$

14 **35 m/sec**

1. Find the net force on you to start. The component of your weight acting along the ski jump is

$$F = mg \sin\theta$$

2. The normal force on you is

$$F_N = mg \cos\theta$$

which means that the kinetic force of friction is

$$F_F = \mu_k F_N = \mu_k mg \cos\theta$$

3. The net force accelerating you down the ski jump, F_{net}, is

$$F_{net} = mg \sin\theta - F_F = mg \sin\theta - \mu_k mg \cos\theta$$

4. Plug in the numbers:

$$F_{net} = (90)(9.8)(\sin 40^\circ) - (.05)(90)(9.8)(\cos 40^\circ) = 570 \text{ N} - 34 \text{ N} = 540 \text{ N}$$

5. This net force acts over the 100 m jump, so the work done by this force is

$$W = F_{net} = (540)(100) = 5.4 \times 10^4 \text{ J}$$

6. That work goes into your kinetic energy:

$$W = F_{net} = 5.4 \times 10^4 \text{ J} = \frac{1}{2} mv^2$$

7. Solve for v^2:

$$v^2 = 1200, \text{ so } v = 35 \text{ m/sec}$$

15 **8.8 m/sec**

1. Figure out how much potential energy the box has when it starts to fall. That potential energy is

$$PE = mg\,h$$

2. This potential energy will be converted into kinetic energy, so you have

$$PE = KE = mgh = \frac{1}{2}mv^2$$

3. Solve for v^2:

$$v^2 = 2gh$$

4. Solve for v:

$$v = \sqrt{2gh}$$

5. Plugging in the numbers gives you v = 8.8 m/sec.

16 **254 ft/sec**

1. Figure out how much potential energy you have when you start to fall. That potential energy is

$$PE = mg\,h_o$$

2. Find out how much potential energy is converted into kinetic energy when you open your parachute. The change in potential energy, $\triangle PE$, is

$$\triangle PE = mgh_o - mgh_f$$

3. This potential energy will be converted into kinetic energy, so you have

$$\Delta PE = mgh_o - mgh_f = KE = \frac{1}{2}mv^2$$

4. Solve for v^2:

$$v^2 = 2\,g\,(h_o - h_f) = 2 \cdot 32.2 \cdot (2000 - 1000) = 64{,}400$$

5. Plugging in the numbers gives you v = 254 ft/sec.

17 **77 m/sec**

1. Figure out how much potential energy the pole has when it starts to fall. That potential energy is

$$PE = mg\,h$$

2. This potential energy will be converted into kinetic energy, so you have

$$PE = KE = mgh = \frac{1}{2}mv^2$$

3. Solve for v^2:

$$v^2 = 2gh$$

4. Plug in the numbers:

$$v = 77 \text{ m/sec}$$

18 **6.3×10^6 J**

1. The equation for potential energy is

$$PE = mgh$$

2. Convert 30,000 ft into m using the fact that 1 ft = .305 m:

$$30{,}000\ \cancel{\text{feet}} \times \frac{.305\text{ meters}}{\cancel{\text{foot}}} = 9150\text{ m}$$

3. Plug in the numbers:

$$PE = mgh = (70)(9.8)(9150) = 6.3 \times 10^6 \text{ J}$$

19 **3.8×10^4 Watts**

1. The equation for power is

$$v = 9.9 \text{ m/sec}$$

2. The amount of work done is the difference in kinetic energy:

$$P = \frac{W}{t} = \frac{F \cdot s}{t} = F \cdot v$$

3. Therefore, the power is

$$P = \frac{1}{2t} m \left(v_2^2 - v_1^2\right)$$

4. Plug in the numbers:

$$P = 3.8 \times 10^4 \text{ Watts}$$

20 **360 Watts**

1. The equation for power is

$$v = 9.9 \text{ m/sec}$$

2. The amount of work done is the difference in kinetic energy:

$$P = \frac{W}{t} = \frac{F \cdot s}{t} = F \cdot v$$

3. Therefore, the power is

$$P = \frac{1}{2t} m \left(v_2^2 - v_1^2\right)$$

4. Plug in the numbers:

$$P = 360 \text{ Watts}$$

21 **4500 Watts**

1. The equation for power is

$$v = 9.9 \text{ m/sec}$$

2. The amount of work done is the difference in kinetic energy:

$$P = \frac{W}{t} = \frac{F \cdot s}{t} = F \cdot v$$

3. Therefore, the power is

$$P = \frac{1}{2t} m \left(v_2^2 - v_1^2\right)$$

4. Plug in the numbers:

$$P = 4500 \text{ Watts}$$

22 **7.5×10^3 Watts**

1. The equation for power is

$$v = 9.9 \text{ m/sec}$$

2. The amount of work done is the difference in kinetic energy:

$$P = \frac{W}{t} = \frac{F \cdot s}{t} = F \cdot v$$

3. Therefore, the power is

$$P = \frac{1}{2t} m \left(v_2^2 - v_1^2 \right)$$

4. Plug in the numbers:

$$P = 7.5 \times 10^3 \text{ Watts}$$

Chapter 8

Getting Things to Move: Momentum and Kinetic Energy

In This Chapter

- Working with impulse
- Gaining momentum
- Conserving momentum
- Conserving kinetic energy
- Handling two-dimensional collisions

Things collide all the time, and when they do, physics gets involved. Knowing about the conservation of momentum and kinetic energy lets you handle this kind of problem — when things hit each other, you can predict what's going to happen.

The conservation of momentum and kinetic energy uses two of the strongest tools that physics has in the physical world, and this chapter helps you become a pro at working with these tools.

Acting on Impulse

When you apply a force for a certain amount of time, you create an *impulse.* In fact, that's the definition of impulse — impulse equals the force applied multiplied by the time it was applied. Here's the equation:

$$\textbf{Impulse} = \mathbf{F}\,t$$

Note that this is a vector equation (for a review of vector equations, check out Chapter 3). Impulse can be an important quantity when you're solving physics problems because you can relate impulse to momentum (which I cover in the next section), and working with momentum is how you solve collision problems in physics.

Here's an example of impulse in action: You're playing pool, and you strike a pool ball with the cue. The cue may be in contact with the ball for only a millisecond, but there's an observable result — the ball flies off in the opposite direction. That result is the impulse.

What are the units of impulse? You have force multiplied by time, so the unit is the Newton-second.

EXAMPLE

Q. Suppose that you're playing pool and hit a pool ball for 10.0 milliseconds (a millisecond is 1⁄1000 of a second) with a force of 20.0 N. What impulse did you impart to the pool ball?

A. The correct answer is 0.2 N-sec, in the direction of the force.

1. Use the equation **Impulse** = **F** t.

2. Plug in the numbers:

Impulse = **F** t = $(20)\cdot(1.0 \times 10^{-2})$ = 0.20 N-sec, in the direction of the force.

1. You're disgusted with your computer and give it a whack. If your hand is in contact with the computer for 100.0 milliseconds with a force of 100.0 N, what impulse do you impart to the computer?

Solve It

2. You're standing under the eaves of your house when a huge icicle breaks off and hits you, imparting a force of 300.0 N for 0.10 seconds. What was the impulse?

Solve It

Getting Some Momentum

Momentum is the most important quantity when it comes to handling collisions in physics. Momentum is a physical quantity defined as the product of mass multiplied by velocity. Note that that's velocity, not speed, so momentum is a vector quantity. Its symbol is **p**; here's the equation for momentum:

$$\mathbf{p} = m\,\mathbf{v}$$

Note that **p** is always in the same direction as **v** because m is a *scalar value* (that is, a single value, not a value with multiple components like a vector). It turns out that momentum is conserved in collisions, which means that the momentum before a collision is the same as the momentum after a collision (assuming that minimal heat was generated by the collision — that little energy went into deforming the colliding objects). So if you know the original momentum in the collision, you can predict what the situation will be after the collision (and physicists are always delighted by such predictions).

What are the units of momentum? Momentum is mass times velocity, so its unit is kilogram·meters/second (kg-m/sec) in the MKS system.

EXAMPLE

Q. Suppose that you're in an 800.0 kg race car going 200.0 miles an hour due east. If you have a mass of 60.0 kg, what is the total momentum?

A. The correct answer is 2.5×10^5 kg-m/sec, due east.

1. Use the equation **p** = m **v**.

2. Plug in the numbers, assuming that 200 miles per hour is about 89.4 m/sec:

$\mathbf{p} = m\,\mathbf{v} = (800 + 60)\cdot(89.4) = 1.43 \times 10^5$ kg-m/sec, due east.

3. You're running north at 3.0 m/sec. If you have a mass of 80.0 kg, what is your momentum?

Solve It

4. You're falling out of an airplane, and before opening your parachute, you hit a speed of 100.0 m/sec. What is your momentum if you have a mass of 80.0 kg?

Solve It

5. You're pushing a 10.0 kg box of dishes across the kitchen floor at a rate of 4.0 m/sec. What is its momentum?

Solve It

6. You're pushing an 800.0 kg car down the road, and it's going at 6.0 m/sec west. How much momentum does it have?

Solve It

Relating Impulse and Momentum

It turns out that there's a direct connection between impulse and momentum. If you hit a pool ball with a cue, the cue imparts a certain impulse to the ball, causing the ball to end up with a particular momentum.

How can you relate impulse to momentum? Easy. The impulse you impart to an object gives it a change in momentum equal to that impulse, so:

$$\textbf{Impulse} = \mathbf{F}\cdot\Delta t = \Delta\mathbf{p} = m\cdot\Delta\mathbf{v}$$

EXAMPLE

Q. If you hit a stationary 400.0 g pool ball with a force of 100.0 N for 0.10 seconds, what is its final speed?

A. The correct answer is 25 m/sec.

1. Use the equation $\mathbf{F}\cdot\Delta t = \Delta\mathbf{p}$.

2. Find the impulse first:

$$\textbf{Impulse} = \mathbf{F}\cdot\Delta t = (100)\cdot(0.1) = 10 \text{ N-sec}$$

3. That impulse becomes the pool ball's new momentum:

$$p = (100)\cdot(0.1) = 10 \text{ kg-m/sec}$$

4. Momentum equals mass times velocity, so to find speed (the magnitude of velocity), you can solve for v, like so:

$$v = p / m = (10) / 0.4) = 25 \text{ m/sec}$$

7. You hit a hockey puck, mass 450 g, with a force of 150 N for 0.10 seconds. If it started at rest, what is its final speed?

Solve It

8. You're standing on an ice rink when another skater hits you, imparting a force of 200.0 N for 0.20 seconds. If you have a mass of 90.0 kg, what is your final speed?

Solve It

9. You kick a 1.0 kg soccer ball with a force of 400.0 N for 0.20 seconds. What is its final speed?

Solve It

10. You hit a 400 g baseball with a force of 400.0 N for 0.10 seconds. The baseball was traveling toward you at 40 m/sec. What is its final speed?

Solve It

Conserving Momentum

The major tool you have in calculating what's going to happen in collisions is the knowledge that momentum is conserved. You know that the total momentum before the collision is the same as the total momentum after the collision as long as there are no significant outside forces.

When you have two objects that collide (one is initially at rest and the other is moving), and you know the final velocity and mass of one object after the collision, you can calculate the final velocity of the other object. You can do this because the total momentum is conserved, so it's the same, before and after the collision, as shown by this equation:

$$\mathbf{p_o} = \mathbf{p_f}$$

EXAMPLE

Q. A pool ball with a mass of 400 g and a speed of 30 m/sec hits another pool ball that's at rest. If the first pool ball ends up going in the same direction with a speed of 10 m/sec, what is the new speed of the second pool ball?

A. The correct answer is 20 m/sec.

1. Use the equation $\mathbf{p_o} = \mathbf{p_f}$.
2. All travel is in the same direction in this question, so you can treat it as a scalar equation. Find the original total momentum:

   $$p_o = (0.4)\cdot(30) = 12 \text{ kg-m/sec}$$

3. The original total momentum equals the total final momentum, which is given by this expression, where p_{2f} is the final momentum of the second ball:

   $$p_o = 12 \text{ kg-m/sec} = p_f = (0.4)\cdot(10) + p_{2f}$$

4. Solve for p_{2f}:

   $$p_{2f} = 12 - 4.0 = 8.0 \text{ kg-m/sec}$$

5. Divide p_{2f} by the pool ball's mass to find its speed:

   $$p_{2f} / m = 8.0 / 0.4 = 20. \text{ m/sec}$$

11. A 450 g hockey puck traveling at 60 m/sec hits a stationary puck. If the first puck ends up going at 20 m/sec in the same direction as the second puck, how fast is the second puck moving?

Solve It

12. You're driving a bumper car at a circus at 18 m/sec, and you hit another car that's at rest. If you end up going at 6.0 m/sec, what is the final speed of the other car, given that both cars have 100.0 kg mass, you have 80.0 kg mass, and the other person has a mass of 70.0 kg?

Solve It

13. On the athletic field, a golf ball with a mass of 0.20 kg and speed of 100.0 m/sec hits a soccer ball at rest that has a mass of 1.0 kg. If the golf ball ends up at rest, what is the soccer ball's final speed, given that it travels in the same direction as the golf ball was originally traveling?

Solve It

14. You're stopped at a traffic light when a 1000.0 kg car (including driver) hits you from behind at 40.0 m/sec. Ouch. If the other car ends up moving at 12 m/sec, and if you and your car have a mass of 940 kg, what is your final speed?

Solve It

Conserving Kinetic Energy — or Not

In some types of collisions, called *elastic collisions,* kinetic energy and momentum are conserved. Here are the equations for the conservation of these factors:

$$KE_o = KE_f$$

$$p_o = p_f$$

Check out this idea in action: Suppose that you're in a car when you hit the car in front of you (elastically — no deformation of bumpers is involved), which started at rest. You know that momentum is always conserved, and you know that the car in front of you was stopped when you hit it, so if your car is car 1 and the other one is car 2, you get this equation:

$$m_1\mathbf{v}_{f1} + m_2\mathbf{v}_{f2} = m_1\mathbf{v}_{o1}$$

This equation can't tell you what $\mathbf{v}_{f1}$ and $\mathbf{v}_{f2}$ are, because there are two unknowns and only one equation. You can't solve for either $\mathbf{v}_{f1}$ or $\mathbf{v}_{f2}$ exactly in this case, even if you know the masses and $\mathbf{v}_{o1}$. So to solve for both final speeds, you need another equation to constrain what's going on here. That means using the conservation of kinetic energy. The collision was an elastic one, so kinetic energy was indeed conserved. That means that:

$$\frac{1}{2}m_1 v_{f1}^2 + \frac{1}{2}m_2 v_{f2}^2 = \frac{1}{2}m_1 v_{o1}^2$$

With two equations and two unknowns, v_{f1} and v_{f2}, you can solve for those unknowns in terms of the masses and v_{o1}.

You probably won't be asked to solve questions of this kind on physics tests because, in addition to being two simultaneous equations, the second equation has a lot of squared velocities in it. But it's one you may see in homework. When you do the math, you get

$$v_{f1} = \frac{(m_1 - m_2) \cdot v_{o1}}{(m_1 + m_2)}$$

and

$$v_{f2} = \frac{2 \cdot m_1 \cdot v_{o1}}{(m_1 + m_2)}$$

This is a more substantial result than you get from problems that just use the conservation of momentum; in such problems, you can solve for only one final speed. Here, using both the conservation of momentum and kinetic energy, you can solve for both objects' final speeds.

EXAMPLE

Q. You're in a car that hits the at-rest car in front of you. If you and your car's mass is 1000.0 kg, the mass of the car and driver ahead of you is 900.0 kg, and if you started at 44 m/sec, what are the final speeds of the two cars? Assume that all the action happens in the same line as your original direction of travel.

A. The correct answer is that your car moves 2.3 m/sec, and the other car moves 46 m/sec.

1. Use this equation to find the final speed of your car:

$$v_{f1} = \frac{(m_1 - m_2) \cdot v_{o1}}{(m_1 + m_2)}$$

2. Plug in the numbers:

$$v_{f1} = \frac{(m_1 - m_2) \cdot v_{o1}}{(m_1 + m_2)} = \frac{(100) \cdot (44)}{(1900)} = 2.3 \text{ m/sec}$$

3. Use this equation to find the final speed of the other car:

$$v_{f2} = \frac{2 \cdot m_1 \cdot v_{o1}}{(m_1 + m_2)}$$

4. Plug in the numbers:

$$v_{f2} = \frac{2 \cdot m_1 \cdot v_{o1}}{(m_1 + m_2)} = \frac{2 \cdot (1000) \cdot (44)}{(1900)} = 46 \text{ m/sec}$$

15. A 450 g hockey puck traveling at 60.0 m/sec hits a stationary puck with the same mass. What are the final speeds of the pucks, given that the collision is elastic and that all motion takes place along the same line?

Solve It

16. You're driving a bumper car at 23 m/sec, and you hit another bumper car that's at rest. If you and your car have a mass of 300 kg, and the mass of the other car and driver is 240 kg, what are the final speeds of the cars?

Solve It

Collisions in Two Dimensions

Collisions can take place in two dimensions. For example, soccer balls can move any which way on a soccer field, not just along a single line. Soccer balls can end up going north or south, east or west, or a combination of those — not just along the east-west axis. So you have to be prepared to handle collisions in two dimensions.

EXAMPLE

Q. In Figure 8-1, there's been an accident at an Italian restaurant, and two meatballs are colliding. Assuming v_{o1} = 10.0 m/sec, v_{o2} = 5.0 m/sec, v_{f2} = 6.0 m/sec, and the masses of the meatballs are equal, what are θ and v_{f1}?

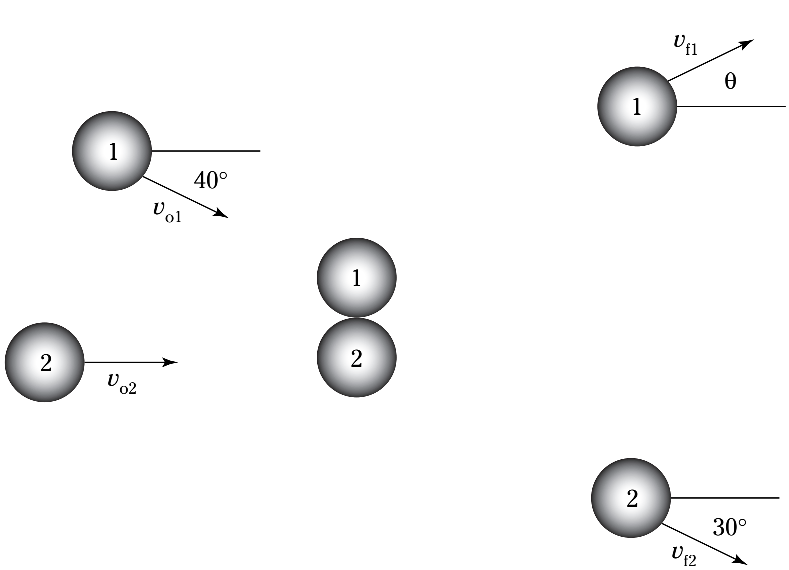


Figure 8-1: Colliding objects.

A. The correct answer is θ =24° and v_{f1} = 8.2 m/sec.

1. You can assume that collisions between meatballs don't conserve kinetic energy. However, momentum is conserved. In fact, momentum is conserved in both the x and y directions, which means that:

$$p_{fx} = p_{ox}$$

and:

$$p_{fy} = p_{oy}$$

2. Here's what the original momentum in the x direction was:

$$p_{fx} = p_{ox} = m_1 \cdot v_{o1} \cdot \cos 40° + m_2 v_{o1}$$

3. Momentum is conserved in the x direction, so:

$$p_{fx} = p_{ox} = m_1 \cdot v_{o1} \cdot \cos 40° + m_2 v_{o2} = m_1 \cdot v_{f1x} + m_2 v_{f2} \cos 30°$$

4. Which means that:

$$m_1 \cdot v_{f1x} = m_1 \cdot v_{o1} \cdot \cos 40° + m_2 v_{o2} - m_2 v_{f2} \cos 30°$$

5. Divide by m_1:

$$v_{f1x} = \frac{m_1 \cdot v_{o1} \cdot \cos 40° + m_2 v_{o2} - m_2 v_{f2} \cos 30°}{m_1}$$

And because $m_1 = m_2$, this becomes

$$v_{f1x} = v_{o1} \cdot \cos 40° + v_{o2} - v_{f2} \cos 30°$$

6. Plug in the numbers:

$$v_{f1x} = v_{o1} \cdot \cos 40° + v_{o2} - v_{f2} \cos 30° = (10)(.766) + 5.0 - (6.0)(.866) = 7.5 \text{ m/sec}$$

7. Now for the y direction. Here's what the original momentum in the y direction looks like:

$$p_{fy} = p_{oy} = m_1 \cdot v_{o1} \cdot \sin 40°$$

8. Set that equal to the final momentum in the y direction:

$$p_{fx} = p_{ox} = m_1 \cdot v_{o1} \cdot \sin 40° = m_1 \cdot v_{f1y} + m_2 v_{f2} \sin 30°$$

That equation turns into:

$$m_1 v_{f1y} = m_1 \cdot v_{o1} \cdot \sin 40° - m_2 v_{f2} \sin 30°$$

9. Solve for the final velocity component of meatball 1's y velocity:

$$v_{f1y} = \frac{m_1 \cdot v_{o1} \cdot \sin 40° - m_2 v_{f2} \sin 30°}{m_1}$$

10. Because the two masses are equal, this becomes

$$v_{f1y} = v_{o1} \cdot \sin 40° - v_{f2} \sin 30°$$

11. Plug in the numbers:

$$v_{f1y} = v_{o1} \cdot \sin 40° - v_{f2} \sin 30° = (10)(.642) - (6.0)(0.5) = 3.4 \text{ m/sec}$$

12. So:

$$v_{f1x} = 7.5 \text{ m/sec}$$
$$v_{f1y} = 3.4 \text{ m/sec}$$

That means that the angle θ is

$$\theta = \tan^{-1}(3.4/7.5) = \tan^{-1}(0.45) = 24°$$

And the magnitude of v_{f1} is

$$v_{f1} = \sqrt{v_{f1x}^2 + v_{f1y}^2} = \sqrt{7.5^2 + 3.4^2} = 8.2 \text{ m/s}$$

17. Assume that the two objects in Figure 8-1 are hockey pucks of equal mass. Assuming that v_{o1} = 15 m/sec, v_{o2} = 7.0 m/sec, and v_{f2} = 7.0 m/sec, what are θ and v_{f1}, assuming that momentum is conserved but kinetic energy is not?

Solve It

18. Assume that the two objects in Figure 8-2 are tennis balls of equal mass. Assuming that v_{o1} = 12 m/sec, v_{o2} = 8.0 m/sec, and v_{f2} = 6.0 m/sec, what are θ and v_{f1}, assuming that momentum is conserved but kinetic energy is not?

Solve It

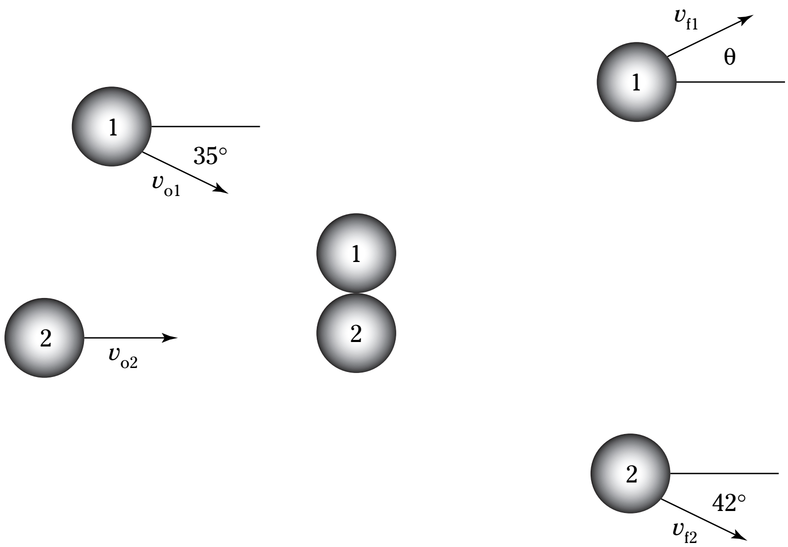


Figure 8-2: Dragging a mass.

Answers to Problems about Momentum and Kinetic Energy

The following are the answers to the practice questions presented in this chapter. You see how to work out each answer, step by step.

1 **10 N-sec**

1. Use the equation **Impulse** = **F** t.
2. Plug in the numbers:

$$\mathbf{Impulse} = \mathbf{F}\, t = (0.1)\,(100) = 10 \text{ N-sec}$$

2 **30 N-sec**

1. Use the equation **Impulse** = **F**·t.
2. Plug in the numbers:

$$\mathbf{Impulse} = \mathbf{F}\, t = (0.1)\cdot(300) = 30 \text{ N-sec}$$

3 **240 m-kg/sec north**

1. Use the equation **p =** m **v**.
2. Plugging in the numbers gives you:

$$\mathbf{p} = m\,\mathbf{v} = (80)(3.0) = 240 \text{ m-kg/second north}$$

4 **8000 kg-m/sec downward**

1. Use the equation **p** = m·**v.**
2. Plug in the numbers:

$$\mathbf{p} = m\,\mathbf{v} = (80)\cdot(100.0) = 8000 \text{ kg-m/sec downward}$$

5 **40 kg-m/sec**

1. Use the equation **p** = m·**v.**
2. Plug in the numbers:

$$\mathbf{p} = m\,\mathbf{v} = (10)\cdot(4.0) = 40 \text{ kg-m/sec}$$

6 **4800 kg-m/sec west**

1. Use the equation **p** = m·**v**.
2. Plug in the numbers:

$$\mathbf{p} = m\,\mathbf{v} = (800)\cdot(6.0) = 4800 \text{ kg-m/sec west}$$

7 **33 m/sec**

1. Use the equation **F**·△t = △**p.**
2. Find the impulse:

$$\text{Impulse} = F\cdot\triangle t = (150)\cdot(0.1) = 15 \text{ N-sec}$$

3. That impulse becomes the puck's new momentum:

$$p = (150)\cdot(0.1) = 15 \text{ kg-m/sec}$$

4. Momentum equals mass times velocity, so to find speed (the magnitude of velocity), you solve for v:

$$v = p / m = (15) / (0.45) = 33 \text{ m/sec}$$

8 **0.44 m/sec**

1. Use the equation $\mathbf{F}\cdot\triangle t = \triangle\mathbf{p}$.
2. Find the impulse:

$$\text{Impulse} = F\cdot\triangle t = (200)\cdot(0.2) = 40 \text{ N-sec}$$

3. That impulse becomes your new momentum:

$$p = (200)\cdot(0.2) = 40 \text{ kg-m/sec}$$

4. Momentum equals mass times velocity, so to find speed (the magnitude of velocity), you solve for v:

$$v = p / m = (40) / (90) = 0.44 \text{ m/sec}$$

9 **80 m/sec**

1. Use the equation $\mathbf{F}\cdot\triangle t = \triangle\mathbf{p}$.
2. Find the impulse:

$$\text{Impulse} = F\cdot\triangle t = (400)\cdot(0.2) = 80 \text{ N-sec}$$

3. That impulse becomes the soccer ball's new momentum:

$$p = (400)\cdot(0.2) = 80 \text{ kg-m/sec}$$

4. Momentum equals mass times velocity, so to find speed (the magnitude of velocity), you solve for v:

$$v = p / m = (80) / (1.0) = 80 \text{ m/sec}$$

10 **60 m/sec**

1. Use the equation $\mathbf{F}\cdot\triangle t = \triangle\mathbf{p}$.
2. Find the impulse first:

$$\text{Impulse} = F\cdot\triangle t = (400)\cdot(0.1) = 40 \text{ N-sec}$$

3. That impulse becomes the ball's change in momentum:

$$\triangle p = (400)\cdot(0.1) = 40.0 \text{ kg-m/sec}$$

4. The baseball started with a momentum of $(0.4)\cdot(40) = 16$ kg-m/sec, so its final momentum will be $16 - 40 = 24$ kg-m/sec.
5. Momentum equals mass times velocity, so to find speed (the magnitude of velocity), you solve for v:

$$v = p / m = (24) / (0.4) = 60 \text{ m/sec}$$

11 **40 m/sec**

1. Use the equation $\mathbf{p_o} = \mathbf{p_f}$.
2. The pucks travel in the same direction here, so you can treat this as a scalar equation. Find the original total momentum:

$$p_o = (0.45)\cdot(60) = 27 \text{ kg-m/sec}$$

3. This equals the total final momentum, which is given by this expression, where p_{2f} is the final momentum of the second puck:

$$p_o = 27 \text{ kg-m/sec} = p_f = (0.45)\cdot(20) + p_{2f}$$

4. Solve for p_{2f}:

$$p_{2f} = 27 - 9.0 = 18.0 \text{ kg-m/sec}$$

5. Divide p_{2f} by the puck's mass to find its speed:

$$p_{2f} / m = 18.0 / 0.45 = 40 \text{ m/sec}$$

12 **15.2 m/sec**

1. Use the equation $\mathbf{p_o} = \mathbf{p_f}$.
2. The bumper cars travel in the same direction here, so you can treat this as a scalar equation. Find the original total momentum:

$$p_o = (18)(100 + 80) = 3240 \text{ m-kg/sec}$$

3. This equals the total final momentum, which is given by this expression, where p_{2f} is the final momentum of the second bumper car:

$$p_o = 3240 \text{ m-kg/sec} = p_f = (100 + 80)(6.0) + p_{2f}$$

4. Solve for p_{2f}:

$$p_{2f} = 3240 - 1080 = 2160 \text{ m-kg/sec}$$

5. Divide p_{2f} by the bumper car and person's mass to find its speed:

$$p_{2f} /m = 2160 / (100 + 70) = 15.2 \text{ m/sec}$$

13 **20 m/sec**

1. Use the equation $\mathbf{p_o} = \mathbf{p_f}$.
2. Find the original total momentum:

$$p_o = (100)\cdot(0.2) = 20 \text{ kg-m/sec}$$

3. This equals the total final momentum, which is given by this expression, where p_{2f} is the final momentum of the soccer ball:

$$p_o = 20 \text{ kg-m/sec} = p_f = p_{2f}$$

4. Solve for p_{2f}:

$$p_{2f} = 20 \text{ kg-m/sec}$$

5. Divide p_{2f} by the soccer ball's mass to find its speed:

$$p_{2f} / m = 20 / (1.0) = 20 \text{ m/sec}$$

14 **30 m/sec**

1. Use the equation $\mathbf{p_o} = \mathbf{p_f}$.
2. Find the original total momentum:

$$p_o = (1000)\cdot(40) = 40{,}000 \text{ kg-m/sec}$$

3. This equals the total final momentum, which is given by the following expression, where p_{2f} is the final momentum of the car:

$$p_o = 40{,}000 \text{ kg-m/sec} = p_f = (1000)\cdot(12) + p_{2f}$$

4. Solve for p_{2f}:

$$p_{2f} = 28{,}000 \text{ kg-m/sec}$$

5. Divide p_{2f} by you and your car's mass to find its speed:

$$p_{2f} / m = 28{,}000 / (940) = 30 \text{ m/sec, about 67 mph}$$

15 **0, 60 m/sec**

1. Use this equation to find the final speed of the first puck:

$$v_{f1} = \frac{(m_1 - m_2)\cdot v_{o1}}{(m_1 + m_2)}$$

2. Substituting the numbers gives you:

$$v_{f1} = \frac{(m_1 - m_2)\cdot v_{o1}}{(m_1 + m_2)} = 0 \text{ m/sec}$$

3. Use this equation to find the final speed of the puck:

$$v_{f2} = \frac{2\cdot m_1 \cdot v_{o1}}{(m_1 + m_2)}$$

4. Putting in the numbers gives you:

$$v_{f2} = \frac{2\cdot m_1 \cdot v_{o1}}{(m_1 + m_2)} = \frac{2\cdot(450)\cdot(60)}{(900)} = 60 \text{ m/sec}$$

Note that when the masses are the same, the first puck stops, and the second puck takes off with the same speed as the first puck had.

16 **You: 2.6 m/sec, the other car: 26 m/sec**

1. Use this equation to find the final speed of your car:

$$v_{f1} = \frac{(m_1 - m_2)\cdot v_{o1}}{(m_1 + m_2)}$$

2. Plug in the numbers:

$$v_{f1} = \frac{(m_1 - m_2)\cdot v_{o1}}{(m_1 + m_2)} = 2.6 \text{ m/sec}$$

3. Use this equation to find the final speed of the second car:

$$v_{f1} = \frac{(m_1 - m_2)\cdot v_{o1}}{(m_1 + m_2)}$$

4. Plug in the numbers:

$$v_{f2} = \frac{2\cdot m_1 \cdot v_{o1}}{(m_1 + m_2)} = \frac{2\cdot(300)\cdot(23)}{(540)} = 26 \text{ m/sec}$$

17 **14 m/sec, 26°**

1. Momentum is conserved in this collision. In fact, momentum is conserved in both the x and y directions, which means that the following are true:

$$p_{fx} = p_{ox}$$
$$p_{fy} = p_{oy}$$

2. The original momentum in the x direction was

$$p_{fx} = p_{ox} = m_1 \cdot v_{o1} \cdot \cos 40° + m_2\, v_{o1}$$

3. Momentum is conserved in the x direction, so:

$$p_{fx} = p_{ox} = m_1 \cdot v_{o1} \cdot \cos 40° + m_2\, v_{o2} = m_1 \cdot v_{f1x} + m_2\, v_{f2} \cos 30°$$

4. Solving for $m_1 v_{f1x}$ gives you:

$$m_1 v_{f1x} = m_1 \cdot v_{o1} \cdot \cos 40° + m_2\, v_{o2} - m_2\, v_{f2} \cos 30°$$

5. Divide by m_1:

$$v_{f1x} = \frac{m_1 \cdot v_{o1} \cdot \cos 40° + m_2\, v_{o2} - m_2\, v_{f2} \cos 30°}{m_1}$$

   Because $m_1 = m_2$, that equation becomes

$$v_{f1x} = v_{o1} \cdot \cos 40° + v_{o2} - v_{f2} \cos 30°$$

6. Plug in the numbers:

$$v_{f1x} = v_{o1} \cdot \cos 40° + v_{o2} - v_{f2} \cos 30° = (15)\,(.766) + 7.0 - (7.0)\,(.866) = 12.5 \text{ m/sec}$$

7. Now for the y direction. The original momentum in the y direction was

$$p_{fy} = p_{oy} = m_1 \cdot v_{o1} \cdot \sin 40°$$

8. Set that equal to the final momentum in the y direction:

$$p_{fx} = p_{ox} = m_1 \cdot v_{o1} \cdot \sin 40° = m_1 v_{f1y} + m_2\, v_{f2} \sin 30°$$

   Which turns into:

$$m_1 v_{f1y} = m_1 \cdot v_{o1} \cdot \sin 40° - m_2\, v_{f2} \sin 30°$$

9. Solve for the final velocity component of puck 1's y velocity:

$$v_{f1y} = \frac{m_1 \cdot v_{o1} \cdot \sin 40° - m_2\, v_{f2} \sin 30°}{m_1}$$

10. Because the two masses are equal, the equation becomes

$$v_{f1y} = v_{o1} \sin 40° - v_{f2} \sin 30°$$

11. Plug in the numbers:

$$v_{f1y} = v_{o1} \sin 40° - v_{f2} \sin 30° = (15)\,(.642) - (7.0)\,(0.5) = 6.1 \text{ m/sec}$$

12. So:

$$v_{f1x} = 12.5 \text{ m/sec}$$
$$v_{f1y} = 6.1 \text{ m/sec}$$

That means that the angle θ is

$$\theta = \tan^{-1}(6.1/12.5) = \tan^{-1}(0.49) = 26^\circ$$

And the magnitude of v_{f1} is

$$v_{f1} = \sqrt{v_{f1x}^2 + v_{f1y}^2} = \sqrt{12.5^2 + 6.1^2} = 14 \text{ m/sec}$$

v_{f1} = ec

18 **15 m/sec, 15°**

1. In this situation, momentum is conserved in both the x and y directions, so the following are true:

$$p_{fx} = p_{ox}$$
$$p_{fy} = p_{oy}$$

2. The original momentum in the x direction was

$$p_{fx} = p_{ox} = m_1 \cdot v_{o1} \cdot \cos 35^\circ + m_2\, v_{o1}$$

3. Momentum is conserved in the x direction, so:

$$p_{fx} = p_{ox} = m_1 \cdot v_{o1} \cdot \cos 35^\circ + m_2\, v_{o2} = m_1 \cdot v_{f1x} + m_2\, v_{f2} \cos 42^\circ$$

4. Which means that:

$$m_1 \cdot v_{f1x} = m_1 \cdot v_{o1} \cdot \cos 35^\circ + m_2\, v_{o2} - m_2\, v_{f2} \cos 42^\circ$$

5. Divide by m_1:

$$v_{f1x} = \frac{m_1 \cdot v_{o1} \cdot \cos 35^\circ - m_2 v_{o2} - m_2 v_{f2} \cos 42^\circ}{m_1}$$

Because $m_1 = m_2$, this becomes

$$v_{f1x} = v_{o1} \cdot \cos 35^\circ + v_{o2} - v_{f2} \cos 42^\circ$$

6. Plug in the numbers:

$$v_{f1x} = v_{o1} \cdot \cos 35^\circ + v_{o2} - v_{f2} \cos 42^\circ = (15)(.82) + 7.0 - (7.0)(.74) = 14.1 \text{ m/sec}$$

7. Now for the y direction. The original momentum in the y direction was

$$p_{fy} = p_{oy} = m_1 \cdot v_{o1} \cdot \sin 35^\circ$$

8. Set that equal to the final momentum in the y direction:

$$p_{fx} = p_{ox} = m_1 \cdot v_{o1} \cdot \sin 35^\circ = m_1 \cdot v_{f1y} + m_2\, v_{f2} \sin 42^\circ$$

Solvng for $m_1 \cdot v_{f1y}$ gives you:

$$m_1 v_{f1y} = m_1 \cdot v_{o1} \cdot \sin 35^\circ - m_2\, v_{f2} \sin 42^\circ$$

9. Solve for the final velocity component of puck 1's y velocity:

$$v_{f1y} = \frac{m_1 \cdot v_{o1} \cdot \sin 35^\circ - m_2 v_{f2} \sin 42^\circ}{m_1}$$

10. Because the two masses are equal, the equation becomes

$$v_{f1y} = v_{o1} \cdot \sin 35^\circ - v_{f2} \sin 42^\circ$$

11. Plug in the numbers:

$$v_{f1y} = v_{o1} \cdot \sin 35° - v_{f2} \sin 42° = (15)\,(.57) - (7.0)\,(0.67) = 6.8 - 4.0 = 3.9 \text{ m/sec}$$

12. So:

$$v_{f1x} = 14.1 \text{ m/sec}$$
$$v_{f1y} = 3.9 \text{ m/sec}$$

Which means that the angle θ is

$$\theta = \tan^{-1}(3.9 / 14.1) = \tan^{-1}(4.8) = 15°$$

And the magnitude of v_{f1} is

$$v_{f1} = \sqrt{v_{f1x}^2 + v_{f1y}^2} = \sqrt{13.4^2 + 2.8^2} = 15 \text{ m/s}$$

v_{f1} =

Chapter 9

Winding It Up: Rotational Kinematics

In This Chapter

- Finding tangential speed and acceleration
- Getting angular velocity
- Calculating torque
- Working with rotational kinematics
- Handling rotational equilibrium

When things rotate, lots of physics happens. If you have a merry-go-round, for example, and apply a force to its edge, what happens next is explained by physics.

This chapter covers tangential speed, tangential acceleration, angular velocity and angular acceleration treated as a vector, and torque. I provide you with plenty of practice problems to help you become a master at handling these types of physics problems.

Finding Tangential Speed

Tangential speed is the magnitude of tangential velocity. Take a look at Figure 9-1, where you see a rotating ball going around the origin. As it sweeps around the origin with linear speed v (which keeps changing direction as the ball moves in a circle), the angle θ increases in time.

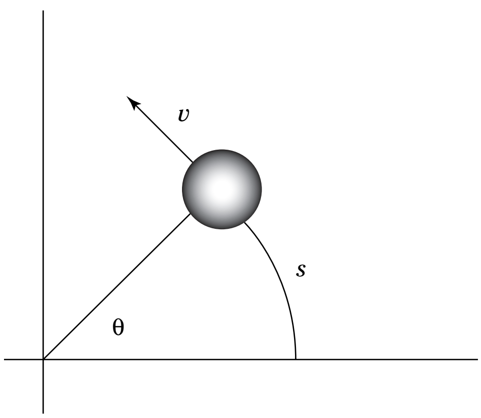


Figure 9-1: A rotating ball.

How do you relate the distance the ball has traveled, s, to the angle θ? If you measure θ in radians, that relationship is the following, where r is the radius of the circle:

$$s = r \cdot \theta$$

Also, you know that:

$$v = \frac{s}{t}$$

That means that you can substitute for s to get:

$$v = \frac{s}{t} = \frac{r \cdot \theta}{t}$$

And $\omega = \theta/t$, which means that:

$$v = \frac{s}{t} = \frac{r \cdot \theta}{t} = \frac{r \cdot \omega}{t}$$

So:

$$v = r \cdot \omega$$

REMEMBER

In this case, v is called tangential velocity — that is, it's the instantaneous linear velocity of the ball as it goes around in the circle. Tangential velocity is always perpendicular to the radius of the circle and is in the direction of travel of the object going around the circle.

EXAMPLE

Q. A ball on a string is going around in a circle at 6.0 radians/sec. What is its tangential velocity if the radius of the circle is 2.0 m?

A. The correct answer is 12 m/sec.

1. Use the equation $v = r \cdot \omega$.
2. Plug in the numbers:

   $v = r \cdot \omega = (2.0) \cdot (6.0) = 12$ m/sec

1. If a satellite is orbiting the Earth, which has an average radius of 3960 miles, at an altitude of 150 miles and an angular speed of 1.17×10^{-3} radians/sec, what is the satellite's tangential speed in mph?

Solve It

2. You're flying a toy plane on a string, and it's going around at 20.0 mph, 100.0 feet from you. What is its angular speed in radians/sec?

Solve It

3. A racing car is going around a circular track of 400.0-ft radius at 50.0 mph. What is its angular speed in radians/sec?

Solve It

4. The tip of an airplane propeller is going at 500.0 mph. If the propeller has a radius of 3.0 ft, what is its angular speed?

Solve It

Targeting Tangential Acceleration

Besides tangential velocity, you can have tangential acceleration. For instance, if you start a helicopter's rotors, the tip of any rotor starts with a tangential velocity of zero and increases with time. Because the velocity vector's magnitude increases and its direction changes, there's acceleration, which is expressed like so:

$$a = \frac{\Delta v}{\Delta t}$$

How can you relate this to angular quantities? Because tangential speed is $v = r \cdot \omega$, you can plug that into the acceleration equation:

$$a = \frac{\Delta v}{\Delta t} = \frac{\Delta(r \cdot \omega)}{\Delta t}$$

And because $\Delta\omega / \Delta t = \alpha$, which is the angular acceleration, this equation becomes

$$a = \frac{\Delta v}{\Delta t} = \frac{r \cdot \Delta\omega}{\Delta t} = r \cdot \alpha$$

Which breaks down to:

$$a = r \cdot \alpha$$

What this all means is that the tangential acceleration at radius $r = r \cdot \alpha$.

EXAMPLE

Q. A set of helicopter blades has a radius of 4.3 m. If a point on the tip of one blade starts at 0 m/sec and ends up 60 seconds later with a speed of 400 m/sec, what was the angular acceleration?

A. The correct answer is 1.6 radians/sec^2.

1. Use this equation:

$$a = \frac{\Delta v}{\Delta t} = r \cdot \alpha$$

2. Plug in the numbers:

$$a = \frac{\Delta v}{\Delta t} = \frac{(400 - 0)}{(60)} = r \cdot \alpha$$

3. Divide both sides by r:

$$\frac{(400 - 0)}{(4.3)(60)} = \alpha$$

4. Do the math:

$$\frac{(400 - 0)}{(4.3)(60)} = \alpha = 1.6 \text{ radians/sec}^2$$

5. If a point on the edge of a tire with a radius of 0.50 m starts at rest and ends up 3.5 minutes later at 88 m/sec (about 197 mph), what was the magnitude of its angular acceleration?

Solve It

6. You're flying a toy plane on a string, and it's going around at 20.0 m/sec, 10.0 m from you. If it accelerates to a final velocity of 30.0 m/sec in 80.0 seconds, what is its angular acceleration?

Solve It

Angular Velocity as a Vector

Angular velocity is really a vector, **ω.** The question is, which way does it point? Think of it this way: If you have a flying disk being tossed back and forth between two players, it's spinning. So which way can **ω** point so that it stays constant in magnitude and direction?

Take a look at Figure 9-2 for the answer. The **ω** vector points out of the plane of rotation.

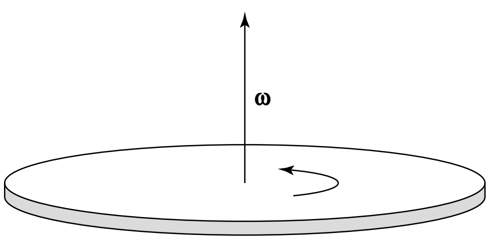


Figure 9-2: Angular velocity as a vector.

TIP: You find the direction of the **ω** vector by wrapping your right hand around in the direction of rotation. Your right thumb will point in the direction of the **ω** vector.

EXAMPLE

Q. A helicopter's blades are rotating in a horizontal plane, and they're going counterclockwise when viewed from above. Which way does **ω** point?

A. The correct answer is upward.

1. Curl your right hand in the direction of rotational motion — counterclockwise.
2. Your right thumb points upward, indicating the direction of the **ω** vector.

7. Suppose that you're flying a toy plane on a string, and it's going around clockwise as viewed from above. Which way does **ω** point?

Solve It

8. Suppose that you're driving forward. Which way does **ω** point for the left front tire?

Solve It

Angular Acceleration as a Vector

Like angular velocity, angular acceleration is a vector; it's represented by the symbol **α.** But unlike angular velocity, angular acceleration need not be perpendicular to the plane of rotation. The angular acceleration vector just points in the direction of change of the angular velocity vector.

Figure 9-3 shows angular acceleration in the same direction as the angular velocity vector. That means the angular velocity vector will grow in time.

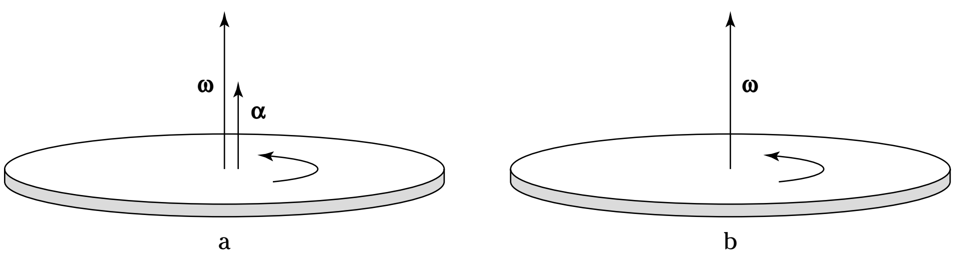


Figure 9-3: Angular acceleration as a vector.

Bear in mind that **α** need not be perpendicular to the rotation — it just points in the direction in which **ω** is changing. For example, if you're turning the wheels of a car, the vector **α**·t points in a direction so that when it's added to the original angular velocity, $\boldsymbol{\omega}_o$, you get the new angular velocity, $\boldsymbol{\omega}_f$.

Q. A helicopter's blades are rotating in a horizontal plane, and they're going counterclockwise when viewed from above. As they speed up, which way does $\boldsymbol{\alpha}$ point?

A. The correct answer is upward.

1. Curl your right hand in the direction of rotational motion — counterclockwise.
2. Your right thumb points upward, indicating the direction of the $\boldsymbol{\omega}$ vector.
3. The $\boldsymbol{\omega}$ vector is increasing in magnitude with time while staying in the same direction, which means that $\boldsymbol{\alpha} \cdot t$ points in that same direction — upward.

9. Suppose that you're flying a toy plane on a string, and it's going around clockwise as viewed from above. In time, the plane is slowing down. Which way does $\boldsymbol{\alpha}$ point?

Solve It

10. A flying disc tossed from one player to another spins as it flies and slows down. If it's spinning counterclockwise when viewed from above, which way does $\boldsymbol{\alpha}$ point?

Solve It

Doing the Twist: Torque

Torque, represented by the symbol τ, is the rotational analog of force in physics. Torque is much like force but differs in that it's the amount of twist, not push, that occurs. For example, take a look at the door, viewed from above, in Figure 9-4.

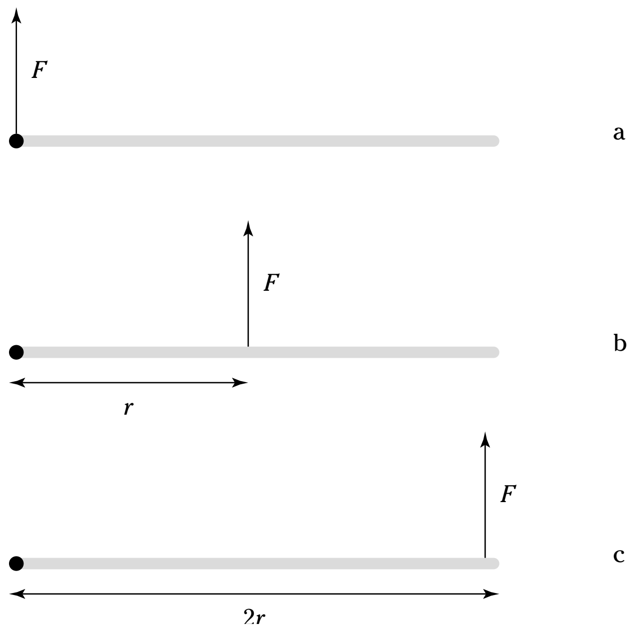


Figure 9-4: Torque at work.

If you push on the hinge, as shown in Figure 9-4a, the door doesn't open. That's because there's no torque on the door, turning it. On the other hand, if you push on the midpoint of the door, as shown in Figure 9-4b, the door turns. And if you push on the outer edge of the door, it turns even faster (that's Figure 9-4c).

The more force you apply, the more torque there is; the farther out from the turning point (the hinge) you push, the more turning force there is. Following is the equation for torque; **F** is the force you're applying, and l is the lever arm — the perpendicular distance from the axis of rotation to the point where you apply the external force:

$$\boldsymbol{\tau} = \mathbf{F} \cdot l$$

The units of torque are force multiplied by distance, so that's N-m in the MKS system, dyne-cm in the CGS system, and foot-pounds in the English system.

It's important to realize that the lever arm is the effective distance at which the force acts in a perpendicular direction. What does that mean? In Figure 9-4a, the torque is zero because the lever arm is zero. In Figure 9-4b, the torque is $\mathbf{F} \cdot \mathbf{r}$ because the lever arm is $\mathbf{r}$. In Figure 9-4c, the torque is $\mathbf{F} \cdot (2 \cdot \mathbf{r})$ because the lever arm is $2 \cdot \mathbf{r}$.

The lever arm is always perpendicular to the force applied, so how do you find the torque in a situation like the one shown in Figure 9-5, where the force isn't perpendicular to the door?

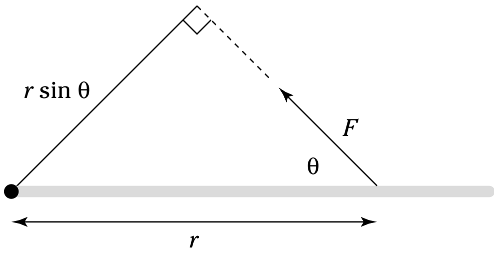


Figure 9-5: Lever arms at work.

For that reason, you have to find the effective perpendicular distance at which the force acts, as shown in Figure 9-5. (Alternatively, you can find the component of the force perpendicular to the door.) That means the torque is the following, where θ is the angle between the force and the door:

$$\boldsymbol{\tau} = \mathbf{F} \cdot \mathbf{r} \sin \theta$$

REMEMBER: Even if you didn't follow that business about lever arms and effective perpendicular distances, remember this equation, because it tells you what the torque is in general. If you apply a force **F** at a displacement **r** from a pivot point where the angle between that displacement and **F** is θ, the torque you produce is $\boldsymbol{\tau} = \mathbf{F} \cdot \mathbf{r} \sin \theta$.

Torque is a vector too. Like $\boldsymbol{\omega}$, $\boldsymbol{\tau}$ points out of the plane of motion; it's positive if it tends to increase the angle θ and negative if it tends to decrease the angle θ.

EXAMPLE

Q. You push a merry-go-round at its edge, perpendicular to the radius. If the merry-go-round has a diameter of 3.0 m, and you push with a force of 200 N, what torque are you applying?

A. The correct answer is 300 N-m.

1. Use the equation $\tau = F \cdot r \sin \theta$.
2. Plug in the numbers:

$$\tau = F \cdot r \sin \theta = (200) \cdot (1.5) \cdot \sin 90^\circ = 300 \text{ N-m}$$

11. You're opening a door by pushing on its outer edge with a force of 100.0 N. If the door is 1.3 m wide, what torque are you applying if you push perpendicular to the door?

Solve It

12. The hot water tap in your shower is stuck. In anger, you apply 200.0 N of force to the outer edge of the handle, which has a turning radius of 10.0 cm. What torque are you applying?

Solve It

13. You're shoveling snow, holding the shovel handle in your right hand. Your left hand is placed halfway down the shovel's shaft and is providing the lifting motion. If you apply 200.0 N of force at an angle of 78° to the shovel and the shovel is 1.5 m long, how much torque are you applying?

Solve It

14. You apply force on a wrench to loosen a pipe. If the wrench is 25 cm long and you apply 150 N at an angle of 67° with respect to the wrench, what torque are you applying?

Solve It

The Balancing Act: Rotational Equilibrium

Sometimes you encounter rotational equilibrium problems in physics. In such problems, the torques all balance out, and nothing rotates. For example, take a look at the situation in Figure 9-6, where a ladder with a person on it is balanced against a wall. Will the force of friction keep the ladder from moving, if θ is 45° and the static coefficient of friction with the floor is 0.70?

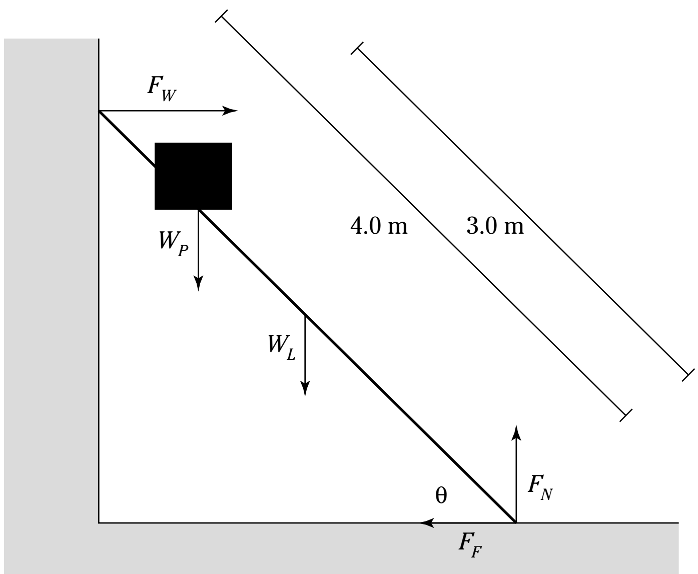


Figure 9-6: Checking a ladder.

This is a rotational equilibrium question: If the sum of the torques is zero ($\Sigma\tau = 0$), the ladder will not fall. But if the force of friction can't supply enough torque, the ladder will indeed rotate and therefore fall.

Q. Will the ladder in Figure 9-6 fall? (Assume that the frictional force between the wall and the ladder is insignificant.) Here are the forces involved:

- F_W = Force exerted by the wall on the ladder
- W_P = Weight of the person = 450 N
- W_L = Weight of the ladder = 200 N (you can assume it's concentrated at the ladder's center)
- F_F = Force of friction holding the ladder in place
- F_N = Normal force

A. The correct answer is that the ladder will not slip.

1. You want the ladder to be in both linear ($\Sigma\mathbf{F} = 0$) and rotational ($\Sigma\tau = 0$) equilibrium. To be in linear equilibrium, the force exerted by the wall on the ladder, F_W, must be the same as the force of friction in magnitude but opposite in direction because those are the only two horizontal forces. So if you can find F_W, you know what the force of friction, F_F, needs to be.

2. To find F_W, take a look at the torques around the bottom of the ladder, using that point as the pivot point. All the torques around that point have to add up to zero. The torque due to the force from the wall against the ladder is

   $$-F_W\,(4.0\text{ m})\sin 45° = -2.83\,F_W$$

   This torque is negative because it tends to produce a clockwise motion toward smaller angles.

3. The torque due to the person's weight is

   $$W_P\,(3.0\text{ m})\cos 45° = (450\text{ N})(3.0\text{ m})(.707) = 954\text{ N-m}$$

4. The torque due to the ladder's weight is

   $$W_L\,(4.0\text{ m})\cos 45° = (200\text{ N})(2.0\text{ m})(.707) = 283\text{ N-m}$$

5. Both torques are positive, so because $\Sigma\tau = 0$, you get this result when you add all the torques together:

   $$954\text{ N-m} + 283\text{ N-m} - 2.83\,F_W = 0$$

6. Solve for F_W:

   $$F_W = \frac{954\text{ N-m} + 283\text{ N-m}}{2.83\text{ m}} = 437\text{ N}$$

   That means the force the wall exerts on the ladder is 437 N. Note that that's also equal to the frictional force of the bottom of the ladder on the floor because F_W and the frictional force are the only two horizontal forces in the whole system. That means that the following is the force of friction needed:

   $$F_F = 437\text{ N}$$

7. What force of friction do you actually have? Use this equation:

   $$F_F\text{ actual} = \mu_s F_N$$

8. F_N is the normal force of the floor pushing up on the ladder, and it must balance all the downward pointing forces in this problem because of linear equilibrium. That means that you have the following:

   $$F_N = W_C + W_L = 650\text{ N}$$

9. Plug in the numbers, using the value of μ_s, 0.70:

   $$F_F\text{ actual} = \mu_s F_N = (0.70)(650) = 455\text{ N}$$

   That's your answer — you need 437 N, and you actually have 455 N available, so the ladder isn't going to slip.

15. You're opening a door by pushing on its outer edge with a force of 100 N at 90°, and someone is trying to keep the door shut by pulling on it one-third of the door's width from the hinge. What force does the other person need to supply perpendicular to the door to keep the door in rotational equilibrium?

Solve It

16. You're on a teeter-totter with a total length of 2 L with a person twice your weight. The other person's sitting only one-third of the distance from the pivot point, however. Where must you sit to balance the person out?

Solve It

17. A motor can provide 5000 N-m of torque. If you can provide 33 N of force, how far away from the motor's axle must you be to produce rotational equilibrium?

Solve It

18. The cap on top of the oil well is frozen. If it takes 450 N-m of torque to free it and you have a wrench that's 0.40 m long, how much force must you apply?

Solve It

Answers to Problems about Rotational Kinematics

The following are the answers to the practice questions presented earlier in this chapter. You see how to work out each answer, step by step.

1 **17,300 mph**

1. Use the equation v = r·ω.
2. Convert 1.17×10^{-3} radians/sec into radians/hr:

$$\frac{1.17 \times 10^{-4} \text{ radians}}{\cancel{\text{seconds}}} \frac{60 \ \cancel{\text{seconds}}}{\cancel{\text{minute}}} \frac{60 \ \cancel{\text{minutes}}}{\text{hour}} = 4.21 \text{ radians/sec}$$

3. The radius at which the satellite orbits is 150 miles added to the radius of the Earth, which is about 3960 miles, making that radius 4110 miles. That makes the satellite's tangential speed:

$$v = r \cdot \omega = (4110)\,(4.2) = 17{,}300 \text{ mph}$$

2 **0.30 radians/sec**

1. Use the equation v = r·ω.
2. Solve for ω:

$$\omega = v / r$$

3. Convert 20 mph to ft/hr:

$$\frac{20 \ \cancel{\text{miles}}}{\text{hour}} \frac{5280 \text{ feet}}{\cancel{\text{mile}}} = 1.1 \times 10^{5} \text{ ft/hr}$$

4. Convert from ft/hr to ft/sec:

$$\frac{1.1 \times 10^{5} \text{ feet}}{\cancel{\text{hour}}} \frac{1 \ \cancel{\text{hour}}}{60 \ \cancel{\text{minutes}}} \frac{1 \ \cancel{\text{minute}}}{60 \text{ seconds}} = 30. \text{ ft/sec}$$

5. Solve for ω:

$$\omega = v / r = 30. / 100 = 0.30 \text{ radians/sec}$$

3 **0.18 radians/sec**

1. Use the equation v = r·ω.
2. Solve for ω:

$$\omega = v / r$$

3. Convert 50 mph to ft/hr:

$$\frac{50 \ \cancel{\text{miles}}}{\text{hour}} \frac{5280 \text{ feet}}{\cancel{\text{mile}}} = 2.6 \times 10^{5} \text{ ft/hr}$$

4. Convert from ft/hr to ft/sec:

$$\frac{2.6 \times 10^{5} \text{ feet}}{\cancel{\text{hour}}} \frac{1 \ \cancel{\text{hour}}}{60 \ \cancel{\text{minutes}}} \frac{1 \ \cancel{\text{minute}}}{60 \text{ seconds}} = 73 \text{ ft/sec}$$

5. Solve for ω:

$$\omega = v / r = 73 / 400 = 0.18 \text{ radians/sec}$$

4 **240 radians/sec**

1. Use the equation v = r·ω.
2. Solve for ω:

$$\omega = v / r$$

3. Convert 500 mph to ft/hr:

$$\frac{500\ \cancel{\text{miles}}}{\text{hour}} \frac{5280\ \text{feet}}{\cancel{\text{mile}}} = 2.6 \times 10^6\ \text{ft/hr}$$

4. Convert from ft/hr to ft/sec:

$$\frac{2.6 \times 10^6\ \text{feet}}{\cancel{\text{hour}}} \frac{1\ \cancel{\text{hour}}}{60\ \cancel{\text{minutes}}} \frac{1\ \cancel{\text{minute}}}{60\ \text{seconds}} = 730\ \text{ft/sec}$$

5. Solve for ω:

$$\omega = v / r = 730 / 3.0 = 240\ \text{radians/sec}$$

5 **0.84 radians/sec²**

1. Use this equation:

$$a = \frac{\Delta v}{\Delta t} = r \cdot \alpha$$

2. Plug in the numbers:

$$a = \frac{\Delta v}{\Delta t} = \frac{88 - 0}{(3.5) \cdot (60)} = r \cdot \alpha$$

3. Divide both sides by r:

$$\frac{(88 - 0)}{(0.5)(3.5)(60)} = \alpha$$

4. Do the math:

$$\frac{(88 - 0)}{(0.5)(3.5)(60)} = \alpha = 0.84\ \text{radians/sec}^2$$

6 **1.3×10^{-2} radians/sec²**

1. Use this equation:

$$a = \frac{\Delta v}{\Delta t} = r \cdot \alpha$$

2. Plug in the numbers:

$$a = \frac{\Delta v}{\Delta t} = \frac{(30 - 20)}{(80)} = r \cdot \alpha$$

3. Divide both sides by r:

$$\frac{(30 - 20)}{(10)(80)} = \alpha$$

4. Do the math:

$$\frac{(30 - 20)}{(10)(80)} = \alpha = 1.3 \times 10^{-2}\ \text{radians/sec}^2$$

7 **Downward**

1. Curl your right hand in the direction of rotational motion — clockwise.
2. Your right thumb points downward, indicating the direction of the **ω** vector.

8 **To the left**

1. Curl your right hand in the direction of rotational motion.
2. Your right thumb points to the left, indicating the direction of the $\boldsymbol{\omega}$ vector.

9 **Upward**

1. Curl your right hand in the direction of rotational motion.
2. Your right thumb points downward, indicating the direction of the $\boldsymbol{\omega}$ vector.
3. In time, the rotation is slowing, so the $\boldsymbol{\omega}$ vector must be decreasing in magnitude. That means that $\boldsymbol{\alpha}\cdot t$ points in the opposite direction — upward.

10 **Downward**

1. Curl your right hand in the direction of rotational motion.
2. Your right thumb points upward, indicating the direction of the ω vector.
3. In time, the rotation is slowing, so the $\boldsymbol{\omega}$ vector must be decreasing in magnitude. That means that $\boldsymbol{\alpha}\cdot t$ points in the opposite direction — downward.

11 **130 N-m**

1. Use the equation $\tau = F\cdot r \sin\theta$.
2. Plug in the numbers:

$$\tau = F\cdot r \sin\theta = (100)\cdot(1.3)\cdot\sin 90^\circ = 130 \text{ N-m}$$

12 **20 N-m**

1. Use the equation $\tau = F\cdot r \sin\theta$.
2. Plug in the numbers:

$$\tau = F\cdot r \sin\theta = (200)\cdot(0.10)\cdot\sin 90^\circ = 20 \text{ N-m}$$

13 **148 N-m**

1. Use the equation $\tau = F\cdot r \sin\theta$.
2. Plug in the numbers:

$$\tau = F\cdot r \sin\theta = (200)\cdot(0.75)\cdot\sin 78^\circ = 148 \text{ N-m}$$

14 **35 N-m**

1. Use the equation $\tau = F\cdot r \sin\theta$.
2. Plug in the numbers to give you:

$$\tau = F\cdot r \sin\theta = (150)\cdot(0.25)\cdot\sin 67^\circ = 35 \text{ N-m}$$

15 **300 N**

1. Use the equation $\Sigma\tau = 0$.
2. Find the torque you apply using this equation, where L is the width of the door:

$$\tau = F\cdot r \sin\theta = (100)\cdot(L)\cdot\sin 90^\circ = 100\,L \text{ N-m}$$

3. Find the torque the other person applies using this equation, where F is the force that person applies:

$$\tau = F \cdot r \sin\theta = (F) \cdot (L/3) \cdot \sin 90° = F \cdot (L/3) \text{ N-m}$$

4. To have rotational equilibrium, set these two torques equal and solve for F:

$$100 \cdot L = F \cdot (L/3)$$
$$(3)\ 100 = F = 300 \text{ N}$$

16 **2 L/3**

1. Use the equation $\Sigma\tau = 0$.
2. Find the torque the other person applies using this formula, where F is the force that person applies and m is your mass:

$$\tau = F \cdot r \sin\theta = (2 \cdot m \cdot g) \cdot (L/3) \cdot \sin 90° = 2 \cdot m \cdot g \cdot (L/3) \text{ N-m}$$

3. Find the torque you apply using this equation, where x is your distance to the pivot:

$$\tau = F \cdot r \sin\theta = (m \cdot g) \cdot (x) \cdot \sin 90° = m \cdot g \cdot x \text{ N-m}$$

4. To have rotational equilibrium, set these two torques equal and solve for x:

$$\frac{2 \cdot m \cdot g \cdot L}{3} = m \cdot g \cdot x$$
$$x = 2\,L / 3$$

17 **152 m**

1. Use the equation $\Sigma\tau = 0$.
2. Find the torque you apply:

$$\tau = F \cdot r \sin\theta = (33) \cdot (x) \cdot \sin 90° = 33\ (x) \text{ N-m}$$

3. Set that equal to the torque of the motor and solve for x:

$$33 \cdot x = 5000$$
$$x = (5000/33) = 152 \text{ m}$$

18 **1130 N**

1. Use the equation $\Sigma\tau = 0$.
2. Find the torque you apply:

$$\tau = F \cdot r \sin\theta = (F) \cdot (0.4) \cdot \sin 90° = F\ (0.4) \text{ N-m}$$

3. Set that equal to the torque needed and solve for F:

$$F\ (0.4) = 450$$
$$F = 1130 \text{ N}$$

Chapter 10

Getting Dizzy with Rotational Dynamics

In This Chapter

- Calculating moments of inertia
- Doing rotational work
- Rolling with rotational work and ramps
- Handling angular momentum
- Working with rotational dynamics

Linear physics translates to rotational physics in a number of ways — in terms of angular speed and acceleration and in terms of angular momentum and rotational kinetic energy. What's the rotational analog of force? Torque!

How about F = ma? What the analog of that? It turns out to be $\tau = I \cdot \alpha$, where I is the moment of inertia. Just about everything in linear motion has an angular analog. This chapter covers moment of inertia, torque as it relates to angular acceleration, and rotational kinetic energy.

Putting Newton on Wheels

Figure 10-1 shows a rotating ball at the end of a stick. You should remember that the force needed to keep the ball moving in a circle is given by $F = m \cdot a$, so the torque, which equals $F \cdot r$, is

$$F \cdot r = \tau = m \cdot r \cdot a$$

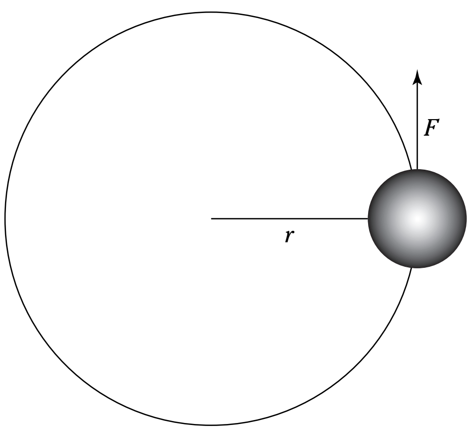


Figure 10-1: A rotating ball.

Look at this situation in terms of α, angular acceleration. Angular acceleration is one of those items you can multiply by the radius to get the linear equivalent, which in this case is equal to the tangential acceleration:

$$a = r \cdot \alpha$$

Substituting $a = r \cdot \alpha$ in the torque equation gives you:

$$\tau = m \cdot r^2 \cdot \alpha$$

This is an important result because it relates torque and angular acceleration. The quantity mr^2 is called the *moment of inertia,* I, and it represents the effort you need to get something to turn, so this equation is usually written as:

$$\tau = I \cdot \alpha$$

This equation is a general result, but the moment of inertia, I, differs depending on the situation. For example, I is different when you're spinning a solid cylinder versus a solid sphere. For a single small mass such as the ball on the end of a stick, $I = mr^2$.

The units of moment of inertia are kilogram meters^2 (kg-m^2) in the MKS system.

Q. You're whipping a cannonball around on the end of a 1.0 m iron rod. If the cannonball has a mass of 10.0 kg, what torque do you need to apply to get an angular acceleration of 0.50 radians/sec^2?

A. The correct answer is 5.0 N-m.

1. Use the equation $\tau = m \cdot r^2 \cdot \alpha$.
2. Plug in the numbers:

$$\tau = m \cdot r^2 \cdot \alpha = (10) \cdot (1.0^2) \cdot (0.5) = 5.0 \text{ N-m}$$

1. You're rotating a cannonball at the end of a 1.0 m rod in a circle and want an angular acceleration of 1.0 radians/sec^2. What torque do you need to supply?

Solve It

2. The cannonball on the end of a rod has an angular acceleration of 2.0 radians/sec^2. What torque are you applying?

Solve It

3. You're shoveling snow, holding the shovel handle in your right hand. Assuming that you can use the equation $I = mr^2$ to determine the moment of inertia of the shovel, if the 2.0 kg shovel has an angular acceleration of 10.0 radians/sec^2 and a length of 1.5 m, what torque are you applying?

Solve It

4. A 100.0 g clock pendulum on the end of a 1.0 m rod has an angular acceleration of 2.0 radians/sec^2. What torque is being applied?

Solve It

Moments of Inertia for Everyone

You know that the moment of inertia of a small mass on the end of a thin rod is $m \cdot r^2$. What are the moments of inertia for other configurations, such as a solid sphere? By treating each mass as a collection of small masses, the moment of inertia for a number of other shapes have been figured out; here are some of them:

- Disk rotating around its center: $I = \frac{1}{2} \cdot m \cdot r^2$
- Hollow cylinder rotating around its center (such as a tire): $I = m \cdot r^2$
- Hollow sphere: $I = \frac{2}{3} m \cdot r^2$
- Hoop rotating around its center (like a tire): $I = m \cdot r^2$
- Point mass rotating at radius r: $I = m \cdot r^2$
- Rectangle rotating around an axis along one edge: $I = \frac{1}{3} m \cdot r^2$
- Rectangle rotating around an axis parallel to one edge and passing through the center: $I = \frac{1}{12} \cdot m \cdot r^2$
- Rod rotating around an axis perpendicular to it and through its center: $I = \frac{1}{12} \cdot m \cdot r^2$
- Rod rotating around an axis perpendicular to it and through one end: $I = \frac{1}{3} m \cdot r^2$
- Solid cylinder: $I = \frac{1}{2} \cdot m \cdot r^2$
- Solid sphere: $I = \frac{2}{5} m \cdot r^2$

EXAMPLE

Q. A solid cylinder with a mass of 5.0 kg is rolling down a ramp. If it has a radius of 10 cm and an angular acceleration of 3.0 radians/sec^2, what torque is operating on it?

A. The correct answer is 0.075 N-m.

1. Use the equation $\tau = I \cdot \alpha$.
2. In this case, $I = \frac{1}{2} m \cdot r^2$.
3. Plug in the numbers:

$$\tau = \tfrac{1}{2} \cdot m \cdot r^2 \cdot \alpha = \tfrac{1}{2} \cdot (5.0) \cdot (0.1^2) \cdot (3.0) = 0.075 \text{ N-m}$$

5. You're spinning a 5.0 kg ball with a radius of 0.5 m. If it's accelerating at 4.0 radians/sec^2, what torque are you applying?

Solve It

6. A tire with a radius of 0.50 m and mass of 1.0 kg is rolling down a street. If it's accelerating with an angular acceleration of 10.0 radians/sec^2, what torque is operating on it?

Solve It

7. You're spinning a hollow sphere with a mass of 10.0 kg and radius of 1.0 m. If it has an angular acceleration of 15 radians/sec^2, what torque are you applying?

Solve It

8. You're throwing a 300.0 g flying disc with a radius of 10 cm, accelerating it with an angular acceleration of 20.0 radians/sec^2. What torque are you applying?

Solve It

9. If you're spinning a 2.0 kg solid ball with a radius of 0.5 m, starting from rest and applying a 6.0 N-m torque, what is its angular speed after 60.0 sec?

Solve It

10. If you're spinning a 2.0 kg hollow ball with a radius of 0.50 m, starting from rest and applying a 12.0 N-m torque, what is its angular speed after 10.0 sec?

Solve It

Doing Some Rotational Work

What if you apply some force to the edge of a tire to try to get a car moving, and you apply a force of 500 N, what work do you do over 1.0 m of travel? That work looks like this equation, where s is the distance the force was applied over:

$$W = F{\cdot}s$$

You can also apply force rotationally. In the case of you applying force to the edge of a tire to get a car moving, the distance s equals the radius multiplied by the angle through which the wheel turns, $s = r{\cdot}\theta$, so you get this equation:

$$W = Fs = F{\cdot}r{\cdot}\theta$$

But the torque, τ, equals $F{\cdot}r$ in this case. So you're left with this:

$$W = F{\cdot}s = F{\cdot}r{\cdot}\theta = \tau{\cdot}\theta$$

REMEMBER

Talk about a cool result — work equals torque multiplied by the angle through which that torque is applied.

EXAMPLE

Q. If you apply a torque of 500.0 N-m to a tire and turn it through an angle of 2π radians, what work have you done?

A. The correct answer is 3140 J.

1. Use the equation $W = \tau{\cdot}\theta$.
2. Plug in the numbers:

$$W = \tau{\cdot}\theta = (500){\cdot}(2\pi) = 3140\text{ J}$$

11. How much work do you do if you apply a torque of 6.0 N-m over an angle of 200 radians?

Solve It

12. You've done 20.0 J of work turning a steering wheel. If you're applying 10.0 N-m of torque, what angle have you turned the steering wheel through?

Solve It

13. How much work do you do if you apply a torque of 75 N-m through an angle of 6π radians?

Solve It

14. You've done 350 J of work turning a bicycle tire. If you're applying 150 N-m of torque, what angle have you turned the wheel through?

Solve It

Round and Round: Rotational Kinetic Energy

In Chapter 7, you review the equation for linear kinetic energy:

$$KE = \frac{1}{2}mv^2$$

Convert that equation to its angular analog:

$$KE = \frac{1}{2}I \cdot \omega^2$$

In other words, the first equation becomes the second when you're going rotational.

Q. You have a 100 kg solid sphere with a radius equal to 1.0 m. If it's rotating at ω = 10.0 radians/sec, what is its rotational kinetic energy?

A. The correct answer is 2000 J.

1. Use this equation:

$$KE = \frac{1}{2}I \cdot \omega^2$$

2. For a solid sphere, $I = \frac{2}{5} \cdot m \cdot r^2$.

3. Plug in the numbers:

$$KE = \frac{1}{2}I \cdot \omega^2 = \frac{1}{2}\frac{2}{5} \cdot m\ r^2\ \omega^2 =$$

$$(0.5) \cdot (0.4) \cdot (100) \cdot (1.0^2) \cdot (10^2) = 2000\ J$$

15. How much rotational kinetic energy does a spinning tire of mass 10.0 kg and radius 0.50 m have if it's spinning at 40.0 rotations/sec?

Solve It

16. How much rotational kinetic energy does a spinning tire of mass 12 kg and radius 0.80 m have if it's spinning at 200.0 radians/sec?

Solve It

17. How much work do you do to spin a tire, which has a mass of 5.0 kg and a radius of 0.40 m, from 0.0 radians/sec to 100.0 radians/sec?

Solve It

18. How much work do you do to spin a hollow sphere, which has a mass of 10.0 kg and a radius of 0.50 m, from 0.0 radians/sec to 200.0 radians/sec?

Solve It

Getting Working with Ramps Again

Figure 10-2 shows a hollow cylinder and a solid cylinder at the top of a ramp. What happens when they roll down the ramp? Which cylinder ends up with the greater speed?

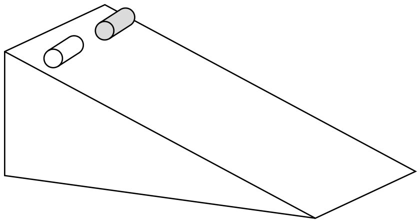


Figure 10-2: A hollow cylinder and a solid cylinder on a ramp.

You may think that you can simply set the original potential energy equal to the final kinetic energy and solve for the final speed that way:

$$KE = PE = \frac{1}{2} m \cdot v^2 = m \cdot g \cdot \Delta h$$

Unfortunately, you can't work out the answer that way because of what you know about rotational kinetic energy. Some of the potential energy of each cylinder goes into rotational kinetic energy as the cylinders roll to the bottom of the incline. Here's the correct equation to use:

$$m \cdot g \cdot \Delta h = \frac{1}{2} m \cdot v^2 + \frac{1}{2} I \cdot \omega^2$$

Further, you can relate v and ω with the equation $v = r \cdot \omega$, which means that $\omega = v / r$, so:

$$m \cdot g \cdot \Delta h = \frac{1}{2} m \cdot v^2 + \frac{1}{2} I \cdot \omega^2 = \frac{1}{2} m \cdot v^2 + \frac{1}{2} I \cdot \frac{v^2}{r^2}$$

Given that equation, this is the final equation for v:

$$v = \sqrt{\frac{2mgh}{m + I/r^2}}$$

So how do you evaluate this for the hollow cylinder and the solid cylinder? For a hollow cylinder, $I = m \cdot r^2$; for a solid cylinder, $I = \frac{1}{2} \cdot m \cdot r^2$. Substituting for I for the hollow cylinder gives you this:

$$v = \sqrt{gh}$$

But substituting for I for the solid cylinder gives you this:

$$v = \sqrt{\frac{4gh}{3}}$$

The solid cylinder will be going 1.15 times as fast as the hollow cylinder when they reach the bottom of the incline.

Q. If a solid sphere is at the top of a 3.0 m-high ramp, what is its speed when it reaches the bottom of the ramp?

A. The correct answer is 6.5 m/sec.

1. Use this equation:

$$v = \sqrt{\frac{2mgh}{m + I/r^2}}$$

2. For a solid sphere, $I = \frac{2}{5} \cdot m \cdot r^2$. That means that v is equal to this:

$$v = \sqrt{\frac{2mgh}{m + (2/5)m}}$$

3. That breaks down to:

$$v = \sqrt{\frac{2gh}{1 + (2/5)}}$$

4. Plug in the numbers:

$$v = 6.5 \text{ m/sec}$$

19. If a hollow cylinder is at the top of a 4.0 m-high ramp, what is its speed when it reaches the bottom of the ramp?

Solve It

20. If a solid cylinder is at the top of a 2.0 m-high ramp, what is its speed when it reaches the bottom of the ramp?

Solve It

21. A tire is rolling down a ramp, starting at a height of 3.5 m. What is its speed when it reaches the bottom of the ramp?

Solve It

22. A basketball (that is, a hollow sphere) is rolling down a ramp, starting at a height of 4.8 m. What is its speed when it reaches the bottom of the ramp?

Solve It

Can't Stop This: Angular Momentum

In linear motion, momentum looks like this:

$$\mathbf{p} = m\mathbf{v}$$

Momentum is conserved in collisions. In addition to linear momentum, you can have angular momentum, which is represented by the symbol L. Following is the equation for angular momentum; note that this is a vector equation and that **L** points in the same direction as **ω**, the object's angular velocity:

$$\mathbf{L} = I\cdot\boldsymbol{\omega}$$

In physics, angular momentum is conserved. For example, if you have a skater spinning around and then she spreads her arms (giving her a different moment of inertia), because angular momentum is conserved, you get this:

$$I_1\cdot\omega_1 = I_2\cdot\omega_2$$

With this equation, if you know the skater's original and final moments of inertia and her original angular speed, you can calculate her final angular speed like this:

$$\omega_2 = \frac{I_1}{I_2}\cdot\omega_1$$

Q. If a 500.0 kg merry-go-round with a radius of 2.0 m is spinning at 2.0 radians/sec and a boy with a mass of 40.0 kg jumps on the outer rim, what is the new angular speed of the merry-go-round?

A. The correct answer is 1.7 radians/sec.

1. Use this equation:

$$I_1 \cdot \omega_1 = I_2 \cdot \omega_{2w}$$

2. For a solid disc like the merry-go-round, $I = ½ m \cdot r^2$.

3. When the boy jumps on, he adds $m_b \cdot r^2$ to I, where m_b is the mass of the boy. This means that:

$$(½ \cdot m \cdot r^2) \cdot \omega_1 = (½ \cdot m \cdot r^2 + m_b \cdot r^2) \cdot \omega_2$$

$$(½ \cdot m \cdot r^2) \cdot \omega_1 = 2000 \text{ kg-m}^2$$

4. Solve for ω_2:

$$\omega_2 = \frac{\frac{1}{2} \cdot m \cdot r^2 \cdot \omega_1}{\left(\frac{1}{2} \cdot m \cdot r^2 + m_b \cdot r^2\right)}$$

where $½ \cdot m \cdot r^2 = 1000 \text{ kg} \cdot \text{m}^2$ and $m_b \cdot r^2 = 160 \text{ kg} \cdot \text{m}^2$.

5. Plug in the numbers:

$$\omega_2 = \frac{2000}{(1000 + 160)} = 1.7 \text{ radians/sec}$$

23. A merry-go-round with a mass of 500.0 kg and radius of 2.0 m is rotating at 3.0 radians/sec when two children with a combined mass of 70.0 kg jump on the outer rim. What is the new angular speed of the merry-go-round?

Solve It

24. A 2000.0 kg space station, which is a hollow cylinder with a radius of 2.0 m, is rotating at 1.0 radians/sec when an astronaut with a mass of 80.0 kg lands on the outside of the station. What is the station's new angular speed?

Solve It

Answers to Problems about Rotational Dynamics

The following are the answers to the practice questions presented earlier in this chapter. You see how to work out each answer, step by step.

1 **10 N-m**

1. Use the equation $\tau = m \cdot r^2 \cdot \alpha$.
2. Plug in the numbers:

$$\tau = m \cdot r^2 \cdot \alpha = (10) \cdot (1.0^2) \cdot (1.0) = 10 \text{ N-m}$$

2 **20 N-m**

1. Use the equation $\tau = m \cdot r^2 \cdot \alpha$.
2. Plug in the numbers:

$$\tau = m \cdot r^2 \cdot \alpha = (10) \cdot (1.0^2) \cdot (2.0) = 20 \text{ N-m}$$

3 **45 N-m**

1. Use the equation $\tau = m \cdot r^2 \cdot \alpha$.
2. Plug in the numbers:

$$\tau = m \cdot r^2 \cdot \alpha = (2.0) \cdot (1.5^2) \cdot (10) = 45 \text{ N-m}$$

4 **0.2 N-m**

1. Use the equation $\tau = m \cdot r^2 \cdot \alpha$.
2. Plug in the numbers:

$$\tau = m \cdot r^2 \cdot \alpha = (0.1) \cdot (1.0^2) \cdot (2.0) = 0.20 \text{ N-m}$$

5 **2.0 N-m**

1. Use the equation $\tau = I \cdot \alpha$.
2. In this case, $I = \frac{2}{5} m \cdot r^2$.
3. Plug in the numbers:

$$\tau = \frac{2}{5} m \cdot r^2 \cdot \alpha = \frac{2}{5} \cdot (5.0) \cdot (0.5^2) \cdot (4.0) = 2.0 \text{ N}-\text{m}$$

6 **2.5 N-m**

1. Use the equation $\tau = I \cdot \alpha$.
2. In this case, $I = m \cdot r^2$.
3. Plug in the numbers:

$$\tau = m \cdot r^2 \cdot \alpha = (1.0) \cdot (0.5^2) \cdot (10.0) = 2.5 \text{ N-m}$$

7 **100 N-m**

1. Use the equation $\tau = I \cdot \alpha$.
2. In this case, $I = \frac{2}{3} \cdot m \cdot r^2$.
3. Plug in the numbers:

$$\tau = \frac{2}{3} m \cdot r^2 \cdot \alpha = \frac{2}{3} \cdot (10) \cdot (1.0^2) \cdot (15) = 100 \text{ N}-\text{m}$$

8 **0.030 N-m**

1. Use the equation $\tau = I \cdot \alpha$.
2. In this case, $I = \frac{1}{2} \cdot m \cdot r^2$.
3. Plug in the numbers:

$$\tau = \frac{1}{2} m \cdot r^2 \cdot \alpha = \frac{1}{2} \cdot (0.3) \cdot (0.1^2) \cdot (20) = 0.030 \text{ N}-\text{m}$$

9 **1800 radians/sec**

1. Use the equation $\tau = I \cdot \alpha$.
2. In this case, $I = \frac{2}{5} m r^2$.
3. Solve for α and plug in the numbers:

$$\alpha = \frac{\tau}{I} = \frac{6.0}{\left(\frac{2}{5}\right) \cdot (2.0) \cdot (0.5^2)} = 30.0 \text{ radians/sec}^2$$

4. Use the equation $\omega = \alpha \cdot t$ and plug in the numbers:

$$\omega = \alpha \cdot t = (30) \cdot (60) = 1800 \text{ radians/sec}$$

10 **360 radians/sec**

1. Use the equation $\tau = I \cdot \alpha$.
2. In this case, $I = \frac{2}{3} \cdot m \cdot r^2$.
3. Solve for α and plug in the numbers:

$$\alpha = \frac{\tau}{I} = \frac{12.0}{\left(\frac{2}{3}\right) \cdot (2.0) \cdot (0.5)^2} = 36 \text{ radians/sec}^2$$

4. Use the equation $\omega = \alpha \cdot t$ and plug in the numbers:

$$\omega = \alpha \cdot t = (36) \cdot (10) = 360 \text{ radians/sec}$$

11 **1200 J**

1. Use the equation $W = \tau \cdot \theta$.
2. Plug in the numbers:

$$W = \tau \cdot \theta = (6.0) \cdot (200) = 1200 \text{ J}$$

12 **2.0 radians**

1. Use the equation $W = \tau \cdot \theta$.
2. Solve for θ and plug in the numbers:

$$\theta = \frac{W}{\tau} = \frac{20}{10} = 2.0 \text{ radians}$$

13 **1400 J**

1. Use the equation $W = \tau \cdot \theta$.
2. Plug in the numbers:

$$W = \tau \cdot \theta = (75) \cdot (6\pi) = 1400 \text{ J}$$

14 **2.33 radians**

1. Use the equation $W = \tau \cdot \theta$.
2. Solve for θ and plug in the numbers:

$$\theta = \frac{W}{\tau} = \frac{350}{150} = 2.33 \text{ radians}$$

15 **7.9 × 10⁴ J**

1. Use the equation for kinetic energy:

$$KE = \frac{1}{2} I \cdot \omega^2$$

2. For a spinning tire, $I = m \cdot r^2$.
3. Convert 40 rotations/sec to radians/sec. One rotation is 2π radians, so 40 rotations/sec is $(40) \cdot (2\pi) = 80\pi$ radians/sec.
4. Plug in the numbers:

$$KE = \frac{1}{2} \cdot I \cdot \omega^2 = \frac{1}{2} \cdot (10) \cdot (0.5)^2 \cdot (80\pi)^2 = 7.9 \times 10^4 \text{ J}$$

16 **1.5 × 10⁵ J**

1. Use the equation for kinetic energy:

$$KE = \frac{1}{2} I \cdot \omega^2$$

2. For a spinning tire, $I = m \cdot r^2 = (12) \cdot (.8^2) = 7.68$.
3. Plug in the numbers:

$$KE = \frac{1}{2} \cdot I \cdot \omega^2 = \frac{1}{2}(7.68)(200^2) = 1.5 \times 10^5 \text{ J}$$

17 **4000 J**

1. The work you do goes into the tire's final kinetic energy, so use the equation for kinetic energy:

$$KE = \frac{1}{2} I \cdot \omega^2$$

2. For a spinning tire, $I = m \cdot r^2$.
3. Plug in the numbers:

$$KE = \frac{1}{2} \cdot I \cdot \omega^2 = \frac{1}{2}(5.0)(0.4^2)(100^2) = 4000 \text{ J}$$

18 **33,300 J**

1. The work you do goes into the sphere's final kinetic energy, so use the equation for kinetic energy:

$$KE = \frac{1}{2} I \cdot \omega^2$$

2. For a hollow sphere, $I = \frac{2}{3} \cdot m \cdot r^2$.
3. Plug in the numbers:

$$KE = \frac{1}{2} \cdot I \cdot \omega^2 = (0.5)\left(\frac{2}{3}\right)(1.0)(0.5^2)(200^2) = 33{,}300 \text{ J}$$

19 **6.3 m/sec**

1. Use this equation:

$$v = \sqrt{\frac{2mgh}{m + I/r^2}}$$

2. For a hollow cylinder, $I = m \cdot r^2$.
3. That means that v is equal to this:

$$v = \sqrt{\frac{2mgh}{m + m}}$$

4. This equation breaks down to:

$$v = \sqrt{gh}$$

5. Plug in the numbers:

$$v = 6.3 \text{ m/sec}$$

20 **5.1 m/sec**

1. Use this equation:

$$v = \sqrt{\frac{2mgh}{m + I/r^2}}$$

2. For a solid cylinder, $I = \frac{1}{2} m r^2$.
3. That means that v is equal to this:

$$v = \sqrt{\frac{2mgh}{m + (1/2)m}}$$

4. This equation breaks down to:

$$v = \sqrt{\frac{2gh}{1 + 1/2}}$$

5. Plug in the numbers:

$$v = 5.1 \text{ m/sec}$$

21 **5.9 m/sec**

1. Use this equation:

$$v = \sqrt{\frac{2mgh}{m + I/r^2}}$$

2. For a tire, $I = m \cdot r^2$.
3. That means that v is equal to this:

$$v = \sqrt{\frac{2mgh}{m + m}}$$

4. This equation breaks down to:

$$v = \sqrt{gh}$$

5. Plug in the numbers:

$$v = 5.9 \text{ m/sec}$$

22 **7.5 m/sec**

1. Use this equation:

$$v = \sqrt{\frac{2mgh}{m + I/r^2}}$$

2. For a hollow sphere, $I = \frac{2}{3} m \cdot r^2$.

3. That means that v is equal to this:

$$v = \sqrt{\frac{2mgh}{m + (2/3)m}}$$

4. This equation breaks down to:

$$v = \sqrt{\frac{2gh}{1 + 2/3}}$$

5. Plug in the numbers:

$$v = 7.5 \text{ m/sec}$$

23 **2.3 radians/sec**

1. Use this equation:

$$I_1 \omega_1 = I_2 \omega_2$$

2. For a solid disc like the merry-go-round, $I = \frac{1}{2} \cdot m \cdot r^2$.

3. When the children jump on, that adds $m_c \cdot r^2$ to I, where m_c is the mass of the children. In other words:

$$\frac{1}{2} m \cdot r^2 \cdot \omega_1 = \left(\frac{1}{2} m \cdot r^2 + m_c \cdot r^2\right) \cdot \omega_2$$

4. Solve for ω_2 to get

$$\omega_2 = \frac{\frac{1}{2} \cdot m \cdot r^2 \cdot \omega_1}{\frac{1}{2} \cdot m \cdot r^2 + m_c \cdot r^2}$$

5. Plug in the numbers:

$$\omega_2 = \frac{3000}{(1000 + 280)} = 2.3 \text{ radians/sec}$$

24 **0.96 radians/sec**

1. Use this equation:

$$I_1 \omega_1 = I_2 \omega_2$$

2. For a hollow cylinder like the space station, $I = m r^2$.

3. When the astronaut lands, that adds $m_a \cdot r^2$ to I, where m_a is the mass of the astronaut. That change means that the following equation applies:

$$m \cdot r^2 \cdot \omega_1 = (m \cdot r^2 + m_a \cdot r^2) \cdot \omega_2$$

4. Solve for ω_2:

$$\omega_2 = \frac{m \cdot r^2 \cdot \omega_1}{(m \cdot r^2 + m_a \cdot r^2)}$$

5. Plug in the numbers:

$$\omega_2 = \frac{8000}{(8000 + 360)} = 0.96 \text{ radians/sec}$$

Chapter 11

Potential and Kinetic Energy Together: Simple Harmonic Motion

In This Chapter

- Using Hooke's law
- Working with simple harmonic motion
- Calculating simple harmonic motion velcoity
- Finding simple harmonic motion acceleration
- Working with springs

Simple harmonic motion — the motion of springs — is a very important topic in physics. This kind of motion is all based on *Hooke's law,* which says that the force on a spring in simple harmonic motion is proportional to the distance the object is away from its equilibrium position (the location where the oscillator will feel no unbalanced force). In other words, the farther a spring is stretched, the more it pulls back.

Hooking into Hooke's Law

Hooke's law is represented by this equation:

$$\mathbf{F} = -k \cdot \Delta\mathbf{x}$$

This deceptively simple equation is at the heart of explaining the motion of objects on springs. It says that the force on an object in simple harmonic motion is proportional to the displacement (that's $\Delta\mathbf{x}$) from rest.

The constant of proportionality is k (Hooke's constant, also called a spring constant), which must be measured for every situation (because no two springs are exactly identical, for example). The negative sign in the equation indicates that k is a restoring force — that is, that the force points toward the equilibrium position of the object.

What are the units of spring constants? Just check out the equation: Because $\mathbf{F} = -k \cdot \Delta\mathbf{x}$, k must have the unit N/m.

You'll commonly see Hooke's law applied to springs. Don't get confused by the minus sign in this equation; it's just there to indicate that the force opposes the displacement, which you know is true about springs.

TIP

If it's easier for you to understand, put in the minus sign after you've done the rest of the problem. You can easily figure out which way the force is going, and if it's in a positive direction as defined by the axes in the problem, the force is positive.

EXAMPLE

Q. You're stretching a spring with spring constant 5.0 N/m. If you stretch it 2.0 m, what pull do you feel from the spring?

A. The correct answer is –10.0 N.

1. Use the equation $F = -k \cdot \Delta x$.
2. Plug in the numbers:

$$F = -k \cdot \Delta x = -(5.0) \cdot (2.0) = -10.0 \text{ N}$$

1. You have a spring whose spring constant is 200 N/m, and you want to stretch it by 6.0 m. What force do you need to apply?

Solve It

2. You have a spring of spring constant 50.0 N/m. What force do you need to stretch it by 5.3 m?

Solve It

3. You have a spring of spring constant 73 N/m. What force do you need to stretch it by 15.0 m?

Solve It

4. It takes 200.0 N to stretch a spring 2.0 m. What is its spring constant?

Solve It

Simply Simple Harmonic Motion

Take a look at the spring in Figure 11-1. It starts at rest, drops down to distance A, and then moves back up to distance A.

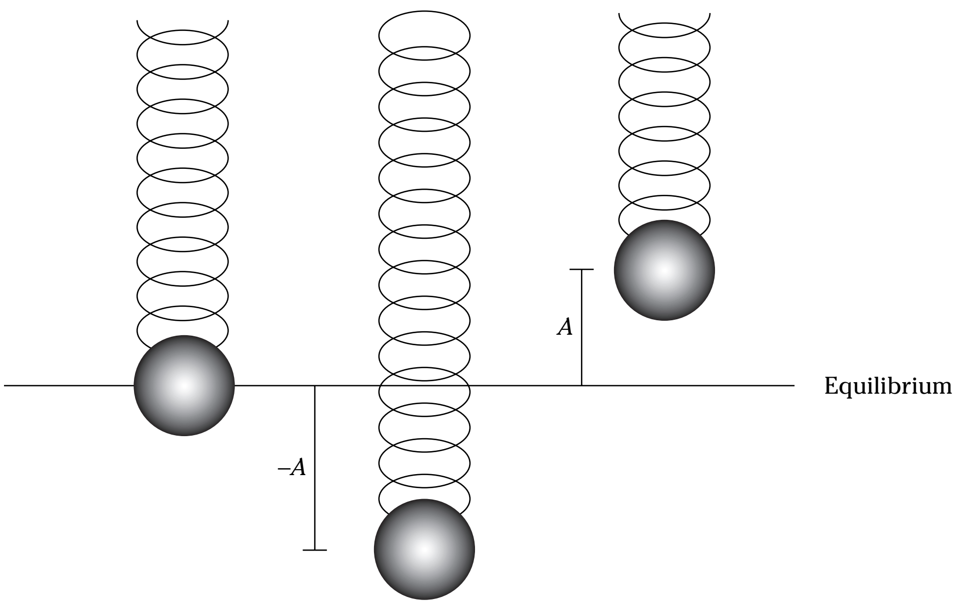


Figure 11-1: A weight on a spring.

How do you describe this motion? In terms of the distance A, called the *amplitude* of the spring's motion, of course:

$$y = A \cos(\omega \cdot t)$$

In this equation for displacement, t is time, and ω is the *angular speed.* (It's called angular speed here for a variety of reasons; it turns out that simple harmonic motion is only one component — hence the cosine — of full circular motion.)

That equation assumes that you start at full extension, t = 0. If you start at a different time, t_o, you can adjust the equation to match the spring's motion like this:

$$y = A \cos[\omega(t - t_o)]$$

EXAMPLE

Q. A weight on a spring is making that spring oscillate up and down. If the amplitude of the motion is 1.0 m and the angular speed is 1.0 radians/sec, where will the oscillating mass be after 10.0 sec?

A. The correct answer is –0.84 N.

1. Use the equation y = A cos (ω·t).
2. Plug in the numbers:

$$y = A \cos(\omega \cdot t) = (1.0) \cos[(1.0) \cdot (10)] = -0.84 \text{ m}$$

TIP

If you plug in the numbers in this equation into your calculator, either put it into radian mode because ω·t is in radians or convert to degrees by multiplying ω·t by 180 / π.

5. A spring with a weight on it has an amplitude of motion of 2.5 m and an angular speed of 2.0 radians/sec. Where will the weight be after 60.0 sec?

Solve It

6. A spring with a weight pulling it down has an amplitude of motion of 5.0 m and an angular speed of 16.0 radians/sec. Where will the weight be after 60.0 sec?

Solve It

7. A spring is at –3.0 m at time 60.0 sec and has an angular speed of 6.0 radians/sec. What is its amplitude?

Solve It

8. A spring is at –4.5 m at time 10.0 sec and has an angular speed of 16.0 radians/sec. What is its amplitude?

Solve It

Getting Periodic

A weight on the end of a spring bounces up and down periodically. The time it takes to bounce up and down, completing a full cycle and coming back to where it started, is called its *period,* represented by symbol T. Because $y = A\cos(\omega \cdot t)$, the object goes through 2π radians in period T, so you have this relation:

$$\omega = \frac{2\pi}{T}$$

In other words:

$$T = \frac{2\pi}{\omega}$$

The period, T, is measured in seconds.

Besides the period, oscillations are measured in *frequency,* which is the number of cycles per second. The frequency of a simple harmonic oscillator is equal to the inverse of the period. The equation for frequency is

$$f = \frac{1}{T}$$

Because $\omega = 2\pi / T$, you can change the frequency equation to get this equation:

$$\omega = \frac{2\pi}{T} = 2\pi \cdot f$$

Frequency is measured in Hertz, abbreviation Hz, which is one cycle per second.

It's worth noting that you can call ω angular speed, but when you're working with simple harmonic motion, it's usually called angular frequency.

Q. A weight on a spring is bouncing up and down with an angular frequency of 12.0 radians/sec. How long does it take to complete each cycle?

A. The correct answer is 0.52 sec.

1. Use this equation:

$$T = \frac{2\pi}{\omega}$$

2. Plug in the numbers:

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{12.0} = 0.52$$

9. A spring has a weight with an angular frequency of 4.5 radians/sec. What is the weight's period?

Solve It

10. A spring has a weight pulling it down, and the angular frequency of oscillation is 1.5 radians/sec. What is the weight's period?

Solve It

11. A spring with a weight on it has an angular frequency of 0.70 radians/sec. What is its frequency?

Solve It

12. A spring with a weight on it is bouncing up and down with an angular frequency of 1.3 radians/sec. What is its frequency?

Solve It

Considering Velocity

As I explain in the earlier section "Simply Simple Harmonic Motion," the displacement of an object on a spring looks like this:

$$y = A \cos(\omega \cdot t)$$

It turns out that you also can express the object's velocity; here's that equation:

$$v_y = -A \cdot \omega \sin\theta = -A \cdot \omega \sin(\omega \cdot t)$$

TIP

This equation assumes that you start at full extension, t = 0, which means that the initial velocity of the object is zero. If you want to adjust this equation to start at some other time, t_o, you alter the equation like so:

$$v_y = -A \cdot \omega \sin[\omega (t - t_o)]$$

When the object is at its fullest extension, the velocity is zero, and when it's swooping back through the equilibrium point, its velocity is at its maximum.

Q. A weight on a spring is bouncing up and down with an angular frequency of 3.4 radians/sec and an amplitude of 1.4 m. What is its speed at t = 5.0 sec?

A. The correct answer is 4.6 m/sec.

1. **Use this equation:**

$$v_y = -A \cdot \omega \sin(\omega \cdot t)$$

2. **Plug in the numbers:**

$$v_y = -A \cdot \omega \sin(\omega \cdot t) =$$
$$-(1.4) \cdot (3.4) \cdot \sin[(3.4) \cdot (5.0) \cdot (180 / \pi)] =$$
$$4.6 \text{ m/sec}$$

13. If you have a spring with a weight on it, and the weight's angular frequency is 1.7 radians/sec and amplitude is 5.6 m, what is the weight's velocity at 30.0 sec?

Solve It

14. A spring with a weight on it is bouncing up and down with an angular frequency of 2.7 radians/sec and amplitude of 1.7 m. What is its velocity at 10.0 sec?

Solve It

Figuring the Acceleration

In addition to calculating the displacement and velocity of an object on a spring, you can calculate its acceleration.

Displacement goes from –A to A, where A is the amplitude. And the velocity of an object on a spring goes from $-A\cdot\omega$ to $A\cdot\omega$; the velocity is at its minimum (zero) at either end of its maximum extension, and it's at a maximum at the equilibrium point (where it would be at rest). The acceleration varies from $-A\,\omega^2$ to $A\,\omega^2$, and it's at its maximum when the object is speeding up (or down!) the most, which is at the ends of its oscillations (when the velocity is zero). The acceleration of the object is zero when the object is passing through the equilibrium position because the net force on the object is zero at the equilibrium position.

Here's the equation for the acceleration of an object on a spring:

$$a = -A\,\omega^2\cdot\cos\theta = -A\,\omega^2\cdot\cos(\omega\cdot t)$$

Q. A weight on a spring is bouncing up and down with an angular frequency of 3.4 radians/sec and an amplitude of 1.4 m. What is its acceleration at 5.0 sec?

A. The correct answer is 4.6 m/sec.

1. Use this equation:

$$a = -A\,\omega^2\cdot\cos(\omega\cdot t)$$

2. Plug in the numbers:

$$a = -A\cdot\omega^2\cdot\cos(\omega\cdot t) = -(1.4)\cdot(3.4^2)\cdot\cos((3.4)\cdot(5.0)\cdot(180/\pi)) = 4.3\text{ m/sec}^2$$

15. A spring has a weight on it. If the weight's angular frequency is 1.3 radians/sec and its amplitude is 1.0 m, what is the weight's acceleration at t = 4.9 sec?

Solve It

16. You have a spring with a weight pulling it down; the angular frequency is 1.7 radians/sec and the amplitude is 6.0 m. What is the weight's acceleration at 60 sec?

Solve It

17. A spring with a weight on it has an angular frequency of 3.7 radians/sec and an amplitude of 1.4 m. What is the weight's acceleration at 15 sec?

Solve It

18. A spring with a weight on it is moving up and down with an angular frequency of 1.3 radians/sec and an amplitude of 2.9 m. What is its acceleration at 9.0 sec?

Solve It

Bouncing Around with Springs

What is the period of a spring in terms of its spring constant, k? You'll often come across that question in physics problems. You know that:

$$F = -k \cdot \Delta x$$

You also know that F = m a, so:

$$F = m \cdot a = -k \cdot \Delta x$$

For simple harmonic motion, you know that:

$$x = A \cos(\omega \cdot t)$$
$$a = -A\,\omega^2 \cdot \cos(\omega \cdot t)$$

Putting this all together gives you:

$$m \cdot a = -m \cdot A\,\omega^2 \cdot \cos(\omega \cdot t) = -k \cdot \Delta x = -k \cdot A \cos(\omega \cdot t)$$

That lengthy equation becomes

$$m\,\omega^2 = k$$

Which, if you solve for ω, gives you the angular frequency of an object on a spring:

$$\omega = \sqrt{\frac{k}{m}}$$

EXAMPLE

Q. A weight on a spring is bouncing up and down. The spring constant is 1.6 N/m, the mass is 1.0 kg, and the amplitude is 3.0 m. What equation describes the spring's motion?

A. The correct answer is $y = A \cos(\omega \cdot t) = 3.0 \cos(1.3 \cdot t)$.

1. Use this equation:

$$\omega = \sqrt{\frac{k}{m}}$$

2. So this equation describes the motion:

$$y = A \cos(\omega \cdot t) = 3.0 \cos(1.3 \cdot t)$$

19. If you have a spring with a mass of 1.0 kg on it and a spring constant of 12.0, what is the spring's period of oscillation?

Solve It

20. What is the period of oscillation of a spring with a mass of 300 g on it and a spring constant of 7.0?

Solve It

21. A weight on a spring is moving up and down. The spring constant is 1.9 N/m, the mass is 1.0 kg, and the amplitude is 2.4 m. What equation describes the spring's motion?

Solve It

22. A weight on a spring is oscillating up and down. The spring constant is 2.3 N/m, the mass is 1.3 kg, and the amplitude is 5.6 m. What equation describes the spring's motion?

Solve It

Talking about Energy

When you stretch a spring, you create potential energy (just like when you lift a weight against the force of gravity). For a spring of spring constant k stretched a distance x from equilibrium, the potential energy in the spring is

$$PE = \frac{1}{2}\,k\,x^2$$

EXAMPLE

Q. What is the potential energy in a spring of spring constant 40 N/m that's stretched 5.0 m from equilibrium?

A. The correct answer is 500 J.

1. **Use this equation:**

$$PE = \frac{1}{2}\,k\,x^2$$

2. **Plug in the numbers:**

$$PE = \frac{1}{2}\,k\,x^2 = 500\text{ J}$$

23. If you stretch a spring of spring constant 100 N/m by 5.0 m, what potential energy is in the spring?

Solve It

24. If you stretch a spring of spring constant 250 N/m by 6.5 m, what potential energy is in the spring?

Solve It

Following the Ticktock of Pendulums

In addition to springs, physics problems about simple harmonic motion may ask you to deal with pendulums like the one in Figure 11-2.

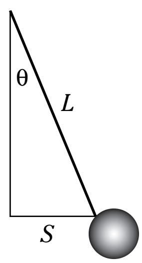


Figure 11-2: A pendulum.

You can calculate the angular frequency of a pendulum of length L using the following equation, where g is the acceleration due to gravity:

$$\omega = \sqrt{\frac{g}{L}}$$

EXAMPLE

Q. What's the angular frequency of a pendulum of length 1.0 m?

A. The correct answer is 4.6 m/sec.

Use this equation and plug in the numbers:

$$\omega = \sqrt{\frac{g}{L}} = 3.1 \text{ radians/sec}$$

25. What's the period of a pendulum with a length of 1.5 m?

Solve It

26. What's the period of a pendulum with a length of 3.0 m?

Solve It

27. If you have a pendulum with a length of 1.0 m with a weight on the end of it, what equation describes the x position of that weight if the amplitude of motion is 0.5 m?

Solve It

28. You have a pendulum with a length of 1.5 m with a weight on the end of it. What equation describes the x position of that weight if the amplitude of motion is 0.75 m?

Solve It

Answers to Problems about Simple Harmonic Motion

The following are the answers to the practice questions presented in this chapter. You see how to work out each answer, step by step.

1 **–1200 N**

1. Use the equation $F = -k \cdot \Delta x$.
2. Plug in the numbers:

$$F = -k \cdot \Delta x = -(200) \cdot (6.0) = -1200 \text{ N}$$

2 **265 N**

1. Use the equation $F = -k \cdot \Delta x$.
2. Plug in the numbers:

$$F = k \cdot \Delta x = -(50) \cdot (5.3) = 265 \text{ N}$$

Note that the sign is positive here because you're pulling on the spring; you're not interested in the force with which the spring pulls back.

3 **1100 N**

1. Use the equation $F = -k \cdot \Delta x$.
2. Plug in the numbers:

$$F = -k \cdot \Delta x = -(73) \cdot (15.0) = 1100 \text{ N}$$

Note that the sign is positive because it's the force with which you're pulling on the spring.

4 **100 N/m**

1. Use the equation $F = -k \cdot \Delta x$.
2. Solve for k:

$$k = -F / \Delta x$$

3. Plug in the numbers:

$$k = -F / \Delta x = (-200) / (2.0) = 100 \text{ N/m}$$

5 **2.04 m**

1. Use the equation $y = A \cos(\omega \cdot t)$.
2. Plug in the numbers:

$$y = A \cos(\omega \cdot t) = 2.5 \cdot \cos(2.0 \cdot 60) = 2.04 \text{ m}$$

6 **3.3 m**

1. Use the equation $y = A \cos(\omega \cdot t)$.
2. Plug in the numbers:

$$y = A \cos \omega t = 3.3 \text{ m}$$

7 **3.3 m**

1. Use the equation y = A cos (ω·t).
2. Solve for A and plug in the numbers:

$$A = y / \cos(\omega \cdot t) = -3.0 / \cos((6.0)(60)(180/\pi)) = 6.6\ \text{m}$$

8 **4.6 m**

1. Use the equation y = A cos (ω·t).
2. Solve for A and plug in the numbers:

$$A = y / \cos(\omega \cdot t) = -4.5 / \cos(16.0 \cdot 10) = 4.6\ \text{m}$$

9 **1.4 sec**

1. Use this equation:

$$T = \frac{2\pi}{\omega}$$

2. Plug in the numbers:

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{4.5} = 1.4\ \text{sec}$$

10 **4.2 sec**

1. Use this equation:

$$T = \frac{2\pi}{\omega}$$

2. Plug in the numbers:

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{1.5} = 4.2\ \text{sec}$$

11 **0.11 cycles/sec**

1. Use this equation:

$$\omega = \frac{2\pi}{T} = 2\pi \cdot f$$

2. Solve for f:

$$f = \frac{\omega}{2\pi}$$

3. Plug in the numbers:

$$f = \frac{\omega}{2\pi} = \frac{0.70}{2\pi} = 0.11\ \text{cycles/sec}$$

12 **0.21 cycles/sec**

1. Use this equation:

$$\omega = \frac{2\pi}{T} = 2\pi \cdot f$$

2. Solve for f:

$$f = \frac{\omega}{2\pi}$$

3. Plug in the numbers:

$$f = \frac{\omega}{2\pi} = \frac{1.3}{2\pi} = 0.21\ \text{cycles/sec}$$

13 **–6.38 m/sec**

1. Use this equation:

$$v_y = -A \cdot \omega \cdot \sin(\omega \cdot t)$$

2. Plug in the numbers:

$$v_y = -A \cdot \omega \cdot \sin(\omega \cdot t) = -(5.6)(1.7)\sin(1.7 \cdot 30) = -6.38 \text{ m/sec}$$

14 **–4.4 m/sec**

1. Use this equation:

$$v_y = -A \cdot \omega \sin(\omega \cdot t)$$

2. Plug in the numbers:

$$v_y = -A \cdot \omega \sin(\omega \cdot t) = -(1.7) \cdot (2.7) \cdot \sin(2.7 \cdot 10) = -4.4 \text{ m/sec}$$

15 **–1.7 m/sec**

1. Use this equation:

$$a = -A\ \omega^2 \cdot \cos(\omega \cdot t)$$

2. Plug in the numbers:

$$a = -A \cdot \omega^2 \cdot \cos(\omega \cdot t) = -(1.0) \cdot (1.3^2) \cdot \cos(1.3 \cdot 4.9) = -1.7 \text{ m/sec}^2$$

16 **-0.86 m/sec^2**

1. Use this equation:

$$a = -A \cdot \omega^2 \cdot \cos(\omega \cdot t)$$

2. Plug in the numbers:

$$a = -A \cdot \omega^2 \cdot \cos(\omega \cdot t) = -(6.0) \cdot (1.7^2) \cdot \cos(1.7 \cdot 6.0) = 0.86 \text{ m/sec}^2$$

17 **–10 m/sec^2**

1. Use this equation:

$$a = -A\ \omega^2 \cdot \cos(\omega \cdot t)$$

2. Plug in the numbers:

$$a = -A \cdot \omega^2 \cdot \cos(\omega \cdot t) A\ \omega^2 \cos \omega t = -(1.4)(3.7^2)\cos((3.7)(15)(180/\pi)) = -10 \text{ m/sec}^2$$

18 **–3.2 m/sec^2**

1. Use this equation:

$$a = -A \cdot \omega^2 \cdot \cos(\omega \cdot t)$$

2. Plug in the numbers:

$$a = -A \cdot \omega^2 \cdot \cos(\omega \cdot t) = -(2.9) \cdot (1.3^2) \cdot \cos(1.3 \cdot 9.0) = -3.2 \text{ m/sec}^2$$

19 **1.8 sec**

1. Use this equation and plug in the numbers:

$$\omega = \sqrt{\frac{k}{m}}$$

2. Now use this equation and plug in the numbers:

$$T = \frac{2\pi}{\omega}$$

20 **1.3 sec**

1. Use this equation:

$$\omega = \sqrt{\frac{k}{m}}$$

2. Now use this equation:

$$T = \frac{2\pi}{\omega}$$

21 **y = A cos (ω·t) = 2.4 cos (1.4·t)**

1. Use this equation:

$$\omega = \sqrt{\frac{k}{m}}$$

2. Now use this equation:

$$y = A \cos(\omega \cdot t) = 2.4 \cos(1.4 \cdot t)$$

22 **y = A cos (ω·t) = 5.6 cos (1.3·t)**

1. Use this equation:

$$\omega = \sqrt{\frac{k}{m}}$$

2. Now use this equation:

$$y = A \cos(\omega \cdot t) = 5.6 \cos(1.3 \cdot t)$$

23 **1250 J**

1. Use this equation:

$$PE = \frac{1}{2} k x^2$$

2. Plug in the numbers:

$$PE = \frac{1}{2} k x^2 = 1250 \text{ J}$$

24 **5280 J**

1. Use this equation:

$$PE = \frac{1}{2} k x^2$$

2. Plug in the numbers:

$$PE = \frac{1}{2} k x^2 = 5280 \text{ J}$$

25 **2.5 sec**

1. Use this equation:

$$\omega = \sqrt{\frac{g}{L}}$$

2. Now use this equation:

$$T = \frac{2\pi}{\omega}$$

26 **3.5 sec**

1. Use this equation:

$$\omega = \sqrt{\frac{g}{L}}$$

2. Now use this equation:

$$T = \frac{2\pi}{\omega}$$

27 **x = 0.50·cos (1.13·t)**

1. Use this equation:

$$\omega = \sqrt{\frac{g}{L}}$$

2. Now use this equation:

$$x = A\cos(\omega \cdot t) = 0.50\cos(3.13 \cdot t)$$

28 **x = 0.75·cos (2.56·t)**

1. Use this equation:

$$\omega = \sqrt{\frac{g}{L}} = 2.56 \text{ radians/sec}$$

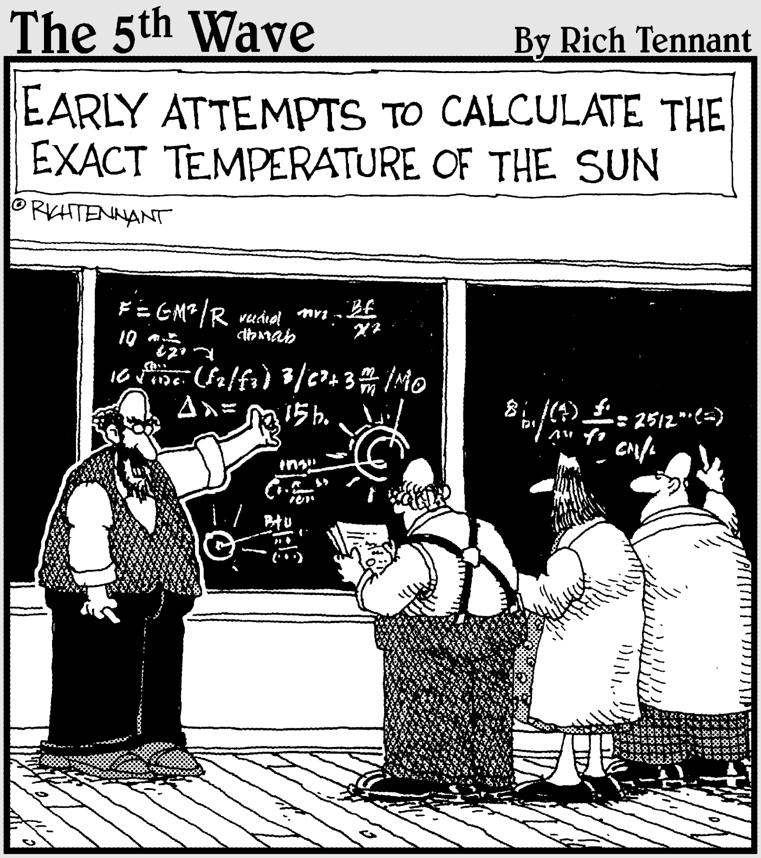
2. Use this equation and plug in the numbers:

$$x = A \cdot \cos(\omega \cdot t)$$

3. Plug in the numbers:

$$x = A \cdot \cos(\omega \cdot t) = 0.75 \cdot \cos(2.56 \cdot t)$$

Part IV
Obeying the Laws of Thermodynamics



"How's this? Hot enough for you?"

In this part . . .

Thermodynamics is the physics of heat and how heat flows. In this part, I cover problems that address how much temperature will change, how fast heat flows, and how to relate pressure to temperature and volume. I also include practice problems that illustrate how thermodynamics plays out in everyday situations — like how much ice you need to cool a cup of coffee or how much you need to cool water to make ice.

Chapter 12

You're Getting Warm: Thermodynamics

In This Chapter

- Converting between temperature scales
- Working with linear expansion
- Calculating volume expansion
- Using heat capacities
- Understanding latent heat

Thermodynamics is the study of heat. It's what comes into play when you drop an ice cube into a cup of hot tea and wait to see what happens — if the ice cube or the tea wins out.

In physics, you often run across questions that involve thermodynamics in all sorts of situations. This chapter refreshes your understanding of the topic and lets you put it to use with practice problems that address thermodynamics from all angles.

Converting Between Temperature Scales

REMEMBER

You start working with questions of heat by establishing a scale for measuring temperature. The temperature scales that you work with in physics are Fahrenheit, Celsius (formerly centigrade), and Kelvin.

Fahrenheit temperatures range from 32° for freezing water to 212° for boiling water. Celsius goes from 0° for freezing water to 100° for boiling water. Following are the equations you use to convert from Fahrenheit (F) temperatures to Celsius (C) and back again:

$$C = \frac{5}{9}(F - 32)$$

$$F = \frac{9}{5}C + 32$$

The Kelvin (K) scale is a little different: Its 0° corresponds to *absolute zero,* the temperature at which all molecular motion stops. Absolute zero is at a temperature of –273.15° Celsius, which means that you can convert between Celsius and Kelvin this way:

$$K = C + 273.15$$

$$C = K - 273.15$$

To convert from Kelvin to Fahrenheit degrees, use this formula:

$$F = \frac{9}{5}(K - 273.15) + 32 = \frac{9}{5}K - 459.67$$

Technically, you don't say "degrees Kelvin" but rather "Kelvins," as in 53 Kelvins. However, people persist in using "degrees Kelvin," so you may see that usage in this book as well.

EXAMPLE

Q. What is 54° Fahrenheit in Celsius?

A. The correct answer is 12° C.

1. Use this equation:

$$C = \frac{5}{9}(F - 32)$$

2. Plug in the numbers:

$$C = \frac{5}{9}(F - 32) = (0.55) \cdot (54 - 32) = 12°C$$

1. What is 23° Fahrenheit in Celsius?

Solve It

2. What is 89° Fahrenheit in Celsius?

Solve It

3. What is 18° Celsius in Fahrenheit?

Solve It

4. What is 18° Celsius in Kelvin?

Solve It

5. What is 18° Kelvin in Celsius?

Solve It

6. What is 57° Kelvin in Fahrenheit?

Solve It

Getting Bigger: Linear Expansion

Ever try to open a screw-top jar by running hot water over it? That hot water makes the lid of the jar expand, making it easier to turn. This simple solution is physics on the job — it's all about *thermal expansion.*

You can see an example of thermal expansion in Figure 12-1, where a bar is undergoing expansion in one direction, called *linear expansion.*

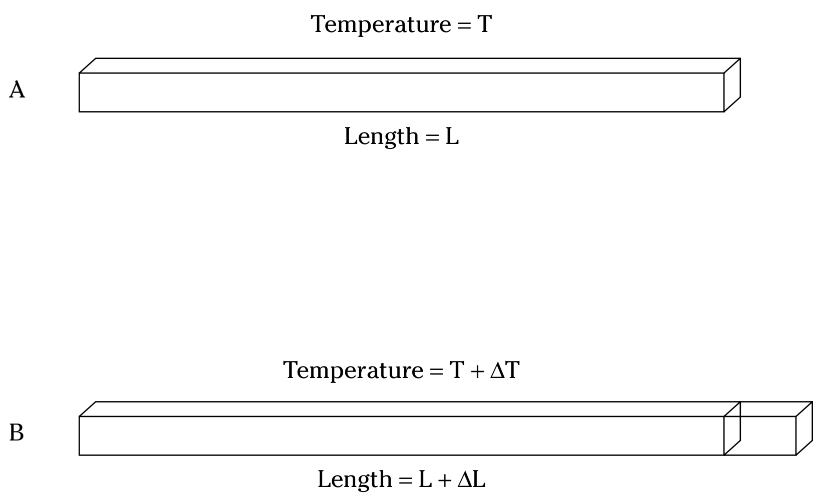


Figure 12-1: Linear expansion.

When you talk about the expansion of a solid in any one dimension under the influence of heat, you're talking about *linear expansion.* When you raise the temperature a small amount, this equation applies:

$$T_f = T_o + \Delta T$$

Linear expansion results in an expansion in any linear direction of the following:

$$L_f = L_o + \triangle L$$

If the temperature goes down a small amount, this equation applies:

$$T_f = T_o - \triangle T$$

You get a contraction instead of an expansion:

$$L_f = L_o - \triangle L$$

Like the coefficient of friction, a coefficient is in play here — the *coefficient of linear expansion,* which is given the symbol α. So you can write this:

$$\frac{\Delta L}{L_o} = \alpha \Delta T$$

This equation is usually written in this form:

$$\triangle L = \alpha L_o \triangle T$$

Here, α is usually measured in 1/°C, or $°C^{-1}$.

EXAMPLE

Q. You're heating a 1.0 m steel bar, coefficient of linear expansion 1.2×10^{-5} $°C^{-1}$, raising its temperature by 5°C. What is the final length of the bar?

A. The correct answer is $1.0 + 6.0 \times 10^{-5}$ m.

1. Use this equation:

$$\triangle L = \alpha L_o \triangle T$$

2. Plug in the numbers:

$$\triangle L = \alpha L_o \triangle T = (1.2 \times 10^{-5})\cdot(1.0)\cdot(5) = 6.0 \times 10^{-5} \text{ m}$$

3. The final length is

$$L_f = L_o + \triangle L = 1.0 + 6.0 \times 10^{-5} \text{ m}$$

7. You're heating a 1.0 m aluminum bar, coefficient of linear expansion 2.3×10^{-5} $°C^{-1}$, raising its temperature by 100°C. What is the final length of the bar?

Solve It

8. You're heating a 2.0 m gold bar, coefficient of linear expansion 1.4×10^{-5} $°C^{-1}$, raising its temperature by 200°C. What is the final length of the bar?

Solve It

9. You're heating a 1.5 m copper bar, coefficient of linear expansion $1.7 \times 10^{-5}\ {}^\circ C^{-1}$, raising its temperature by 300°C. What is the final length of the bar?

Solve It

10. You're heating a 2.5 m lead bar, coefficient of linear expansion $2.9 \times 10^{-5}\ {}^\circ C^{-1}$, raising its temperature by 40°C. What is the final length of the bar?

Solve It

Plumping It Up: Volume Expansion

In addition to linear expansion, physics problems can ask you to find volume expansion. Linear expansion takes place in only one dimension, but volume expansion happens in all three dimensions.

In other words, you have this:

$$\frac{\Delta V}{V_o}\ (\text{fraction the solid expands})\ \text{is proportional to}\ \Delta T\ (\text{change in temperature})$$

The constant involved in volume expansion is called the *coefficient of volume expansion.* This constant is given by the symbol β, and like α, it's measured in $^\circ C^{-1}$. Using β, here's how you can express the relationship shown in the preceding equation:

$$\frac{\Delta V}{V_o} = \beta \Delta T$$

When you multiply both sides by V_o, you're left with the following:

$$\Delta V = \beta\, V_o\, \Delta T$$

Which means that:

$$V_f = V_o + \beta\, V_o\, \Delta T$$

EXAMPLE

Q. You're heating a 1.0 m^3 steel block, coefficient of volume expansion 3.6×10^{-5} °C^{-1}, raising its temperature by 45°C. What is the final volume of the bar?

A. The correct answer is $1.0 + 1.6 \times 10^{-3}$ m^3.

1. Use this equation:

$$\triangle V = \beta V_o \triangle T$$

2. Plug in the numbers:

$$\triangle V = \beta V_o \triangle T = (3.6 \times 10^{-5}) \cdot (1.0) \cdot (45) = 1.6 \times 10^{-3} \text{ m}^3$$

3. The final volume is

$$V_f = V_o + \triangle V = 1.0 + 1.6 \times 10^{-3} \text{ m}^3$$

11. You're heating a 2.0 m^3 aluminum block, coefficient of volume expansion 6.9×10^{-5} °C^{-1}, raising its temperature by 30°C. What is the final volume of the block?

Solve It

12. You're heating a 2.0 m^3 copper block, coefficient of volume expansion 5.1×10^{-5} °C^{-1}, raising its temperature by 20°C. What is the final volume of the block?

Solve It

13. You're heating a 1.0 m^3 glass block, coefficient of volume expansion 1.0×10^{-5} °C^{-1}, raising its temperature by 27°C. What is the final volume of the block?

Solve It

14. You're heating a 3.0 m^3 gold block, coefficient of volume expansion 4.2×10^{-5} °C^{-1}, raising its temperature by 18°C. What is the final volume of the block?

Solve It

Getting Specific with Heat Capacity

It's a fact of physics that it takes 4186 J to raise the temperature of 1.0 kg of water by 1° C. But it takes only 840 J to raise the temperature of 1.0 kg of glass by 1° C.

You can relate the amount of heat, Q, it takes to raise the temperature of an object to the change in temperature and the amount of mass involved. Use this equation:

$$Q = m \cdot c \, \Delta T$$

In this equation, Q is the amount of heat energy involved (measured in Joules if you're using the MKS system), m is the amount of mass, ΔT is the change in temperature, and c is a constant called the *specific heat capacity,* which is measured in J/(kg-°C) in the MKS system.

REMEMBER

So it takes 4186 J of heat energy to warm up 1.0 kg of water 1.0°C. One calorie is defined as the amount of heat needed to heat 1.0 g of water 1.0°C, so 1 calorie equals 4.186 J. Nutritionists use the food energy term *Calorie* (capital C) to stand for 1000 calories, 1.0 kcal, so 1.0 Calorie equals 4186 J. And when you're speaking in terms of heat, you have another unit of measurement to deal with: the British Thermal Unit (Btu). 1.0 Btu is the amount of heat needed to raise one pound of water 1.0°F. To convert Btus to Joules, use the relation that 1 Btu equals 1055 J.

If you add heat to an object, raising its temperature from T_o to T_f, the amount of heat you need is expressed as:

$$\Delta Q = m \cdot c \cdot (T_f - T_o)$$

EXAMPLE

Q. You're heating a 1.0 kg copper block, specific heat capacity of 387 J/(kg-°C), raising its temperature by 45°C. What amount of heat do you have to apply?

A. The correct answer is 17,400 J.

1. Use this equation:

$$Q = m \cdot c \cdot \Delta T$$

2. Plug in the numbers:

$$Q = m \cdot c \cdot \Delta T = (387) \cdot (1.0) \cdot (45) = 17{,}400 \text{ J}$$

15. You're heating a 15.0 kg copper block, specific heat capacity of 387 J/(kg-°C), raising its temperature by 100°C. What heat do you have to apply?

Solve It

16. You're heating a 10.0 kg steel block, specific heat capacity of 562 J/(kg-°C), raising its temperature by 170°C. What heat do you have to apply?

Solve It

17. You're heating a 3.0 kg glass block, specific heat capacity of 840 J/(kg-°C), raising its temperature by 60°C. What heat do you have to apply?

Solve It

18. You're heating a 5.0 kg lead block, specific heat capacity of 128 J/(kg-°C), raising its temperature by 19°C. What heat do you have to apply?

Solve It

19. You're cooling a 10.0 kg lead block, specific heat capacity of 128 J/(kg-°C), lowering its temperature by 60°C. What heat do you have to extract?

Solve It

20. You're cooling a 80.0 kg glass block, specific heat capacity of 840 J/(kg-°C), lowering its temperature by 16°C. What heat do you have to extract?

Solve It

21. You put 7600 J into a 14 kg block of silver, specific heat capacity of 235 J/(kg-°C). How much have you raised its temperature?

Solve It

22. You add 10,000 J into a 8.0 kg block of copper, specific heat capacity of 387 J/(kg-°C). How much have you raised its temperature?

Solve It

Changes of Phase: Latent Heat

Heating blocks of lead is fine, but if you heat that lead enough, sooner or later it's going to melt. When it melts, its temperature stays the same until it liquefies, and then the temperature of the lead increases again as you add heat. So why does its temperature stay constant as it melts? Because the heat you applied went into melting the lead. There's a latent heat of melting that means that so many Joules must be applied per kilogram to make lead change phase from solid to liquid.

REMEMBER

The units of latent heat are J/kg.

There are three phase changes that matter can go through — solid, liquid, and gas — and each transition has a latent heat:

- **Solid to liquid:** The latent heat of melting (or heat of fusion), L_f, is the heat per kilogram needed to make the change between the solid and liquid phases (such as when water turns to ice).
- **Liquid to gas:** The latent heat of vaporization, L_v, is the heat per kilogram needed to make the change between the liquid and gas stages (such as when water boils).
- **Solid to gas:** The latent heat of sublimation, L_s, is the heat per kilogram needed to make the change between the solid and gas phases (such as the direct sublimation of dry ice (CO^2) to the vapor state).

The latent heat of fusion of water is about 3.35×10^5 J/kg. That means it takes 3.35×10^5 J of energy to melt 1 kg of ice.

Q. You have a glass of 50.0 g of water at room temperature, 25°C, but you'd prefer ice water at 0°C. How much ice at 0.0°C do you need to add?

A. The correct answer is 15.6 g.

1. The heat absorbed by the melting ice must equal the heat lost by the water you want to cool. Here's the heat lost by the water you're cooling:

$$\triangle Q_{water} = m \cdot c \; \triangle T = m \cdot c \cdot (T_f - T_o)$$

2. Plug in the numbers:

$$\triangle Q_{water} = m \cdot c \cdot \triangle T = m \cdot c \cdot (T_f - T_o) = (0.050)\,(4186)\,(0 - 25) = -5.23 \times 10^3 \text{ J}$$

3. So the water needs to lose 5.23×10^3 J. How much ice would that melt? That looks like this, where L_m is the latent heat of melting:

$$\triangle Q_{ice} = m_{ice} \cdot L_m$$

4. You know that for water, L_m is 3.35×10^5 J/kg, so you get this:

$$\triangle Q = m_{ice} \cdot L_m = m_{ice} \cdot 3.35 \triangle 10^5$$

5. You know that equation has to be equal to the heat lost by the water, so you can set it to:

$$\triangle Q_{ice} = \triangle Q_{water}$$

In other words:

$$m_{ice} = \frac{\Delta Q_{water}}{L_m} = \frac{5.23 \times 10^3 \text{ J}}{3.35 \times 10^5 \text{ J/kg}}$$

6. You know that the latent heat of melting for water is $L = 3.35 \text{ x } 10^5$ J/kg, which means that:

$$m_{ice} = \frac{5.23 \times 10^3}{3.35 \times 10^5} = 1.56 \times 10^{-2} \text{ kg}$$

So you need 1.56×10^{-2} kg, or 15.6 g of ice.

23. You have 100.0 g of coffee in your mug at 80°C. How much ice at 0.0°C would it take to cool 100.0 g of coffee at 80°C to 65°C?

Solve It

24. You have 200.0 g of cocoa at 90°C. How much ice at 0.0°C do you have to add to the cocoa (assuming that it has the specific heat capacity of water) to cool it down to 60°C?

Solve It

Answers to Problems about Thermodynamics

The following are the answers to the practice questions presented earlier in this chapter. You see how to work out each answer, step by step.

1 **–5° C**

1. Use this equation:

$$C = \frac{5}{9}(F - 32)$$

2. Plug in the numbers:

$$C = \frac{5}{9}(F - 32) = (0.55) \cdot (23 - 32) = -5°C$$

2 **31.7° C**

1. Use this equation:

$$C = \frac{5}{9}(F - 32)$$

2. Plug in the numbers:

$$C = (F - 32) \cdot \frac{5}{9} = (0.55) \cdot (89 - 32) = 31.7°C$$

3 **64° F**

1. Use this equation:

$$F = \frac{9}{5}C + 32$$

2. Plug in the numbers:

$$F = \frac{9}{5}C + 32 = (1.8) \cdot (18) + 32 = 64°F$$

4 **291° K**

1. Use this equation:

$$K = C + 273.15$$

2. Plug in the numbers:

$$K = C + 273.15 = 18 + 273.15 = 291°\ K$$

5 **–255° C**

1. Use this equation:

$$C = K - 273.15$$

2. Plug in the numbers:

$$C = K - 273.15 = 18 - 273.15 = -255°\ C$$

6 **–357° F**

1. Convert Kelvins to Celsius:

$$C = K - 273 = 57 - 273 = -216°C$$

2. Convert Celsius to Fahrenheit:

$$F = C \cdot \frac{9}{5} + 32 = 357°F$$

7 **$1.0 + 2.3 \times 10^{-3}$ m**

1. Use this equation:

$$\triangle L = \alpha \cdot L_o \cdot \triangle T$$

2. Plug in the numbers:

$$\triangle L = \alpha\ L_o\ \triangle T = (2.3 \times 10^{-5}) \cdot (1.0) \cdot (100) = 2.3 \times 10^{-3}\ \text{m}$$

3. The final length is

$$L_f = L_o + \triangle L = 1.0 + 2.3 \times 10^{-3}\ \text{m}$$

8 **$1.0 + 5.6 \times 10^{-3}$ m**

1. Use this equation:

$$\triangle L = \alpha \cdot L_o \cdot \triangle T$$

2. Plug in the numbers:

$$\triangle L = \alpha \cdot L_o \cdot \triangle T = (1.4 \times 10^{-5}) \cdot (2.0) \cdot (200) = 5.6 \times 10^{-3}\ \text{m}$$

3. The final length is

$$L_f = L_o + \triangle L = 1.0 + 5.6 - 10^{-3}\ \text{m}$$

9 **$1.0 + 7.6 \times 10^{-3}$ m**

1. Use this equation:

$$\triangle L = \alpha \cdot L_o \cdot \triangle T$$

2. Plug in the numbers:

$$\triangle L = \alpha \cdot L_o \cdot \triangle T = (1.7 \times 10^{-5}) \cdot (1.5) \cdot (300) = 7.6 \times 10^{-3}\ \text{m}$$

3. The final length is

$$L_f = L_o + \triangle L = 1.0 + 7.6 \times 10^{-3}\ \text{m}$$

10 **$1.0 + 2.9 \times 10^{-3}$ m**

1. Use this equation:

$$\triangle L = \alpha \cdot L_o \cdot \triangle T$$

2. Plug in the numbers:

$$\triangle L = \alpha \cdot L_o \cdot \triangle T = (2.9 \times 10^{-5}) \cdot (2.5) \cdot (40) = 2.9 \times 10^{-3}\ \text{m}$$

3. The final length is

$$L_f = L_o + \triangle L = 1.0 + 2.9 \times 10^{-3}\ \text{m}$$

11 **$2.0 + 4.1 \times 10^{-3}\ \text{m}^3$**

1. Use this equation:

$$\triangle V = \beta\ V_o\ \triangle T$$

2. Plug in the numbers:

$$\triangle V = \beta \cdot V_o \cdot \triangle T = (6.9 \times 10^{-5}) \cdot (2.0) \cdot (30) = 4.1 \times 10^{-3}\ \text{m}^3$$

3. The final length is

$$V_f = V_o + \triangle V = 2.0 + 4.1 \times 10^{-3}\ \text{m}^3$$

12 **$2.0 + 2.0 \times 10^{-3}$ m³**

1. Use this equation:

$$\Delta V = \beta \cdot V_o \cdot \Delta T$$

2. Plug in the numbers:

$$\Delta V = \beta \cdot V_o \cdot \Delta T = (5.1 \times 10^{-5}) \cdot (2.0) \cdot (20) = 2.0 \times 10^{-3}\ m^3$$

3. The final length is

$$V_f = V_o + \Delta V = 2.0 + 2.0 \times 10^{-3}\ m^3$$

13 **$1.0 + 2.7 \times 10^{-4}\ m^3$**

1. Use this equation:

$$\Delta V = \beta \cdot V_o \cdot \Delta T$$

2. Plug in the numbers:

$$\Delta V = \beta \cdot V_o \cdot \Delta T = (1.0 \times 10^{-5}) \cdot (1.0) \cdot (27) = 2.7 \times 10^{-4}\ m^3$$

3. The final length is

$$V_f = V_o + \Delta V = 1.0 + 2.7 \times 10^{-4}\ m^3$$

14 **$3.0 + 2.3 \times 10^{-3}\ m^3$**

1. Use this equation:

$$\Delta V = \beta \cdot V_o \cdot \Delta T$$

2. Plug in the numbers:

$$\Delta V = \beta \cdot V_o \cdot \Delta T = (4.2 \times 10^{-5}) \cdot (3.0) \cdot (18) = 2.3 \times 10^{-3}\ m^3$$

3. The final length is

$$V_f = V_o + \Delta V = 1.0 + 2.3 \times 10^{-3}\ m^3$$

15 **5.8×10^5 J**

1. Use this equation:

$$Q = m \cdot c\ \Delta T$$

2. Plug in the numbers:

$$Q = m \cdot c \cdot \Delta T = (387)\ (15.0)\ (100) = 5.8 \times 10^5\ J$$

16 **9.6×10^5 J**

1. Use this equation:

$$Q = m \cdot c \cdot \Delta T$$

2. Plug in the numbers:

$$Q = m \cdot c \cdot \Delta T = (562)\ (10.0)\ (170) = 9.6 \times 10^5\ J$$

17 **1.5×10^5 J**

1. Use this equation:

$$Q = m \cdot c \cdot \Delta T$$

2. Plug in the numbers:

$$Q = m \cdot c \cdot \Delta T = (840)\ (3.0)\ (60) = 1.5 \times 10^5\ J$$

18 **1.2×10^4 J**

1. Use this equation:

$$Q = m \cdot c \cdot \Delta T$$

2. Plug in the numbers:

$$Q = m\ c\ \Delta T = (128)\ (5.0)\ (19) = 1.2 \times 10^4\ J$$

19 **–7680 J**

1. Use this equation:

$$Q = m \cdot c \cdot \Delta T$$

2. Plug in the numbers:

$$Q = m \cdot c \cdot \Delta T = (128)\ (10.0)\ (-60) = -7680$$

20 **-1.1×10^6 J**

1. Use this equation:

$$Q = m \cdot c \cdot \Delta T$$

2. Plug in the numbers:

$$Q = m \cdot c \cdot \Delta T = (840)\ (80.0)\ (-16) = -1.1 \times 10^6\ J$$

21 **2.3° C**

1. Use this equation:

$$Q = m \cdot c \cdot \Delta T$$

2. Solve for ΔT:

$$\frac{Q}{m \cdot c} = \Delta T$$

3. Plug in the numbers:

$$\frac{Q}{m \cdot c} = \Delta T = 7600/\left[(235)(14)\right] = 2.3°C$$

22 **3.2° C**

1. Use this equation:

$$Q = m \cdot c\ \Delta T$$

2. Solve for ΔT:

$$\frac{Q}{m \cdot c} = \Delta T$$

3. Plug in the numbers:

$$\frac{Q}{m \cdot c} = \Delta T = 10{,}000/\left[(387) \cdot (8.0)\right] = 3.2°C$$

23 **10.4 g**

1. Calculate how much heat has to be lost by the coffee. Assuming it has the same specific heat capacity as water, that's:

$$\Delta Q_{cocoa} = m \cdot c \cdot \Delta T = m \cdot c \cdot (T_f - T_o)$$

2. Plug in the numbers:

$$\Delta Q_{cocoa} = m \cdot c \cdot \Delta T = m \cdot c \cdot (T_f - T_o) = (4186)\ (0.10)\ (80 - 65) = 6.28 \times 10^3\ J$$

3. How much ice do you need to remove 6.28×10^3 J? In this case, the heat supplied to the ice not only must melt the ice:

$$\triangle Q_{ice} = m_{ice} \cdot L_m$$

But this heat also needs to raise the temperature of the water that comes from melting the ice from 0°C to 65°C, so you have to add this:

$$\triangle Q_{ice} = m_{ice} \cdot L_m + m_{ice} \cdot c \cdot \triangle T = m_{ice} \cdot L_m + m_{ice} \cdot c \cdot (T_f - T_o)$$

4. This has to be equal to the heat lost by the coffee, so you get

$$6.28 \times 10^3 \text{ J} = m_{ice} \cdot [L_m + c \cdot (T_f - T_o)]$$

5. Solve for m_{ice}:

$$m_{ice} = \frac{6.28 \times 10^3 \text{ J}}{\left[L_m + c \cdot (T_f - T_o)\right]}$$

6. Plug in the numbers:

$$m_{ice} = \frac{6.28 \times 10^3 \text{ J}}{\left[L_m + c \cdot (T_f - T_o)\right]} = \frac{6.28 \times 10^3 \text{ J}}{\left[3.35 \times 10^5 + 4186 \cdot (65 - 0)\right]} = 0.0104 \text{ kg}$$

24 **42 g**

1. Find how much heat has to be lost by the cocoa. Assuming it has the same specific heat capacity as water, that's:

$$\triangle Q_{cocoa} = m \cdot c \cdot \triangle T = cm(T_f - T_o)$$

2. Plug in the numbers:

$$\triangle Q_{cocoa} = m \cdot c \cdot \triangle T = m \cdot c \cdot (T_f - T_o) = (0.20)\ (4186)\ (90 - 60) = 2.5 \times 10^4 \text{ J}$$

3. How much ice do you need to supply 2.5×10^4 J? In this case, the heat supplied to the ice, not only must melt the ice:

$$\triangle Q_{ice} = m_{ice} \cdot L_m$$

But this heat also needs to raise the temperature of the water that comes from melting the ice from 0°C to 60°C, so you have to add this:

$$\triangle Q_{ice} = m_{ice} \cdot L_m + m_{ice} \cdot c\ \triangle T = m_{ice} \cdot L_m + m_{ice} \cdot c \cdot (T_f - T_o)$$

4. This has to be equal to the heat lost by the cocoa, so you get

$$2.5 \times 10^4 \text{ J} = m_{ice} \cdot [L_m + c \cdot (T_f - T_o)]$$

5. Solve for m_{ice}:

$$m_{ice} = \frac{2.5 \times 10^4 \text{ J}}{\left[L_m + c \cdot (T_f - T_o)\right]}$$

6. Plug in the numbers:

$$m_{ice} = \frac{2.5 \times 10^4 \text{ J}}{\left[L_m + c \cdot (T_f - T_o)\right]} = \frac{2.5 \times 10^4 \text{ J}}{\left[3.35 \times 10^5 + 4186 \cdot (60 - 0)\right]} = 0.042 \text{ kg}$$

Chapter 13

Under Pressure: From Solid to Liquid to Gas

In This Chapter

- Handling heat convection and heat conduction
- Dealing with heat radiation
- Using Avogadro's Number
- Working with molecular motion

This chapter is concerned with all kinds of things heat related — heat transfer, for example, which is about how fast heat travels along iron bars, aluminum pot handles, and more. Want to know how long you can hold that pot over an open flame? This chapter is for you.

You also find here the famous ideal gas law, which connects the pressure, volume, and temperature of a gas. Want to know the pressure of a volume of gas at a particular temperature? Just turn to the ideal gas law, and it'll give you the answer.

Plenty of thermodynamics comes up in this chapter, from how fast heat travels to what pressure gas has at a certain temperature.

In this chapter, you discover the three primary means by which heat moves: convection, conduction, and radiation.

How Heat Travels: Convection

Take a look at Figure 13-1, where you see a pot of water being heated. The water in that pot moves in the pattern you see in the figure; as it circulates, the hot water moves from the bottom to the top.

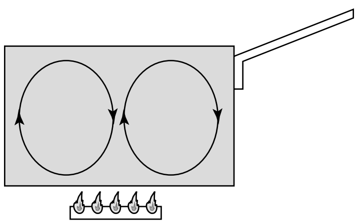


Figure 13-1: A convection example.

That's the first method of heat transfer: convection. *Convection* allows heat to move by the motion of heated matter.

EXAMPLE

Q. How does heat move from the bottom of the pot to the top in Figure 13-1?

A. The correct answer is convection.

The heat moves through convection. The water heats up near the flame and then rises in the pot.

1. How does heat move from the bottom inside a stove to the top?

Solve It

2. How does heat move from the Earth's surface to 2000 ft, where an eagle is coasting?

Solve It

How Heat Travels: Conduction

Take a look at Figure 13-2, where you see a metal pot being heated — and the handle's getting warm. The mechanism that transfers heat between points on the pot's handle is called *conduction.*

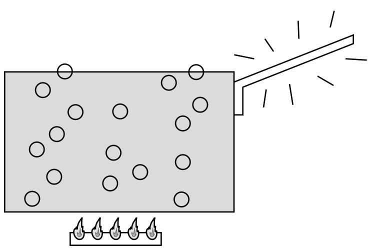


Figure 13-2: An example of heat conduction.

How much heat travels between two points via conduction? The amount of heat is proportional to the difference in temperature, so for heat conduction between the ends of a bar of metal, you'd have this:

$$Q \propto \Delta T$$

As you may expect, a bar twice as wide conducts twice the amount of heat. In general, the amount of heat conducted is proportional to the cross-sectional area:

$$Q \propto A$$

Also, the longer the bar, the less heat makes it all the way through. In fact, the conducted heat turns out to be inversely proportional to the length of the bar:

$$Q \propto \frac{1}{L}$$

Finally, the amount of heat transferred depends on the amount of time that passes — twice the time, twice the heat. That makes sense too. Here's how you express that mathematically:

$$Q \propto t$$

So here's what you get when you put it all together (k is some constant yet to be determined):

$$Q = \frac{k \cdot A \cdot \Delta T \cdot t}{L}$$

What is the constant k? This constant is called the thermal conductivity of a material, and its units are J/(s-m-°C).

EXAMPLE

Q. The thermal conductivity of copper is 390 J/(s-m-°C). How much heat is conducted per second between two points on a copper rod, cross section 0.1 m^2, 1.0 m apart, with a temperature difference of 69°C?

A. The correct answer is 2690 J/sec.

1. Use this equation:

$$Q = \frac{k \cdot A \cdot \Delta T \cdot t}{L}$$

2. Solve for Q/t:

$$\frac{Q}{t} = \frac{k \cdot A \cdot \Delta T \cdot t}{L}$$

3. Plug in the numbers:

$$\frac{Q}{t} = \frac{k \cdot A \cdot \Delta T \cdot t}{L} = \frac{(390) \cdot (0.1) \cdot (69)}{1.0} = 2690 \text{ J/sec}$$

3. The thermal conductivity of steel is 79 J/(s-m-°C). How much heat is conducted per second between two points on a steel rod, cross section 0.3 m^2, 2.0 m apart, with a temperature difference of 34°C?

Solve It

4. The thermal conductivity of silver is 420 J/(s-m-°C). How much heat is conducted per second between two points on a silver rod, cross section 0.40 m^2, 1.5 m apart, with a temperature difference of 91°C?

Solve It

5. If you have a steel rod, thermal conductivity 79 J/(s-m-°C), with a cross section 0.30 m^2 and a length of 50 cm, how long do you have to wait for 1000.0 J to be transferred from one end to the other if the temperature difference is 90°C?

Solve It

6. If you have a brass rod, thermal conductivity 110 J/(s-m-°C), with a cross section 0.50 m^2 and a length of 73 cm, how long do you have to wait for 3000.0 J to be transferred from one end to the other if the temperature difference is 180°C?

Solve It

7. You have a mystery substance, length 2.0 m, cross section 0.40 m^2, and apply a temperature difference of 69°C across its length. If the substance conducts 1000.0 J of heat in 0.50 sec, what is its thermal conductivity?

Solve It

8. You have an unknown material, length 1.0 m, cross section 0.30 m^2, and apply a temperature difference of 97°C across its length. If the substance conducts 1400.0 J of heat in 0.60 sec, what is its thermal conductivity?

Solve It

How Heat Travels: Radiation

The third way that heat travels, after convection and conduction, is *radiation.* Anything hot radiates heat in the form of electromagnetic radiation, which is the way the sun warms the Earth. The sun can't warm the Earth due to convection or conduction, because a vacuum exists between here and there. Instead, the sun beams light to the Earth in a wide range of frequencies, and that energy warms the Earth's surface.

The amount of heat transferred this way is proportional to the amount of time that the radiant object beams energy:

$$Q \propto t$$

As you may also expect, the amount of heat radiated is proportional to the total area doing the radiating — twice as much area doing the radiating, twice as much heat radiated. So you can write this equation, where A is the area doing the radiating:

$$Q \propto A \cdot t$$

You'd expect temperature, T, to be in here somewhere — the hotter an object, the more heat radiated. Hold on to your hat: It turns out that the amount of heat radiated is proportional to T in Kelvins to the fourth power, T^4. So now you have

$$Q \propto A \cdot t \cdot T^4$$

To make this relationship an equation, all you need to add is a constant, which is measured experimentally. This constant, called the Stefan-Boltzmann constant, σ, goes in like this:

$$Q = \sigma \cdot A \cdot t \cdot T^4$$

The value of σ is 5.67×10^{-8} J/(s-m^2-K^4).

This equation works only for objects that are so-called perfect emitters, however. Most objects are not perfect emitters, so you have to add another constant that depends on the substance you're working with. This constant is called its *emissivity,* e. So this equation, the Stefan-Boltzmann law of radiation, becomes

$$Q = e \cdot \sigma \cdot A \cdot t \cdot T^4$$

An object's emissivity depends on what it's made of, so you can expect emissivity to vary from material to material.

Q. A person has a surface area of 1.8 m^2, an emissivity of 0.65, and a temperature of 37°C. How much power does this person radiate?

A. The correct answer is 610 W.

1. Use this equation:

$$Q = e \cdot \sigma \cdot A \cdot t \cdot T^4$$

2. Solve for power, Q/t:

$$\frac{Q}{t} = e \cdot \sigma \cdot A \cdot T^4$$

3. Plug in the numbers, remembering to convert from Celsius to Kelvin:

$$\frac{Q}{t} = e \cdot \sigma \cdot A \cdot T^4 = (0.65) \cdot (5.67 \times 10^{-8}) \cdot (1.8) \cdot (37 + 273)^4 = 610 \text{ W}$$

9. A coffeepot has a temperature of 90°C and a surface area of 0.20 m^2. If its emissivity is 0.95, what power is it radiating?

Solve It

10. A toaster has a temperature of 120°C and a surface area of 0.15 m^2. If its emissivity is 0.90, what power is it radiating?

Solve It

11. If a 200 g coffeepot holds 300 g of coffee, its temperature is 80°C, and its surface area is 0.18 m^2, how long will it take for the coffee to cool to 75°C, given that the pot's emissivity is 0.90 (omitting the fact that the pot is heated by radiation coming from its surroundings)?

Solve It

12. If a 100 g teacup holds 200 g of tea, its temperature is 82°C, and its surface area is 0.07 m^2, how long will it take for the tea to cool to 76°C, given that the cup's emissivity is 0.60 (omitting the fact that the cup is heated by radiation coming from its surroundings)?

Solve It

A Biggie: Avogadro's Number

A certain amount of mass contains a specific number of atoms, and you can figure out just how many atoms a certain amount of mass has. In particular, a *mole* is defined as the number of atoms in 12.0 g of carbon isotope 12. (Carbon isotope 12, also called carbon-12 or just carbon 12, is the most common version of carbon, although some carbon atoms have a few more neutrons in them — carbon 13, actually — so the average works out to about 12.011.)

That number of atoms has been measured as 6.022×10^{23}, which is called Avogadro's Number, N_A. That number represents many atoms. Now you know how many atoms are in 12.0 g of carbon 12.

Does 12.0 g of sulfur have the same number of atoms? No. Just check a periodic table of the elements. You find that the atomic mass of sulfur is 32.06. But 32.06 what? It turns out to mean 32.06 atomic units, u, where each atomic unit is ¹⁄₁₂ of the mass of a carbon 12 atom. So if a mole of carbon 12 has a mass of 12.0 g, and the mass of an average sulfur atom is bigger than the mass of a carbon 12 atom by this ratio

$$\frac{\text{Sulfur mass}}{\text{Carbon 12 mass}} = \frac{32.06}{12\text{ u}}$$

a mole of sulfur atoms must have this mass:

$$\frac{32.06}{12\text{ u}}(12.0\text{ g}) = 32.06\text{ g}$$

Note that sulfur and carbon are composed of simple atoms; if you're dealing with a composite substance such as water, you have to think in terms of molecules instead. So instead of the atomic mass in cases like these, you look for the molecular mass (when atoms combine, you have molecules), which is also measured in atomic mass units. The molecular mass of water is 18.0153 u, so 1 mole of water molecules has a mass of 18.0153 g.

EXAMPLE

Q. How many molecules are in 10.0 g of water?

A. The correct answer is 3.3×10^{23} molecules.

1. You know that 1 mole of water has this mass:

   1 mole of water = 18.0 g

2. A mole has 6.022×10^{23} molecules, so 10.0 g has this many molecules:

$$6.022 \times 10^{23} \frac{(10.0)}{(18.0)} = 3.3 \times 10^{23}$$

13. You have 10.0 g of calcium, atomic mass 40.08 u. How many atoms do you have?

Solve It

14. You have 16.0 g of silicon, atomic mass 28.09 u. How many atoms do you have?

Solve It

15. You have 29.0 g of zinc, atomic mass 65.41 u. How many atoms do you have?

Solve It

16. You have 3.0 g of copper, atomic mass 63.546 u. How many atoms do you have?

Solve It

Ideally Speaking: The Ideal Gas Law

You can relate the pressure, volume, and temperature of an ideal gas with the ideal gas equation. An *ideal gas* is one whose molecules act like points; no interaction occurs among molecules except *elastic collisions* (that is, where kinetic energy is conserved). In practice, all gases act like ideal gases to some extent, so the ideal gas law holds fairly well. Here's that law:

$$P \cdot V = n \cdot R \cdot T$$

Here, P is pressure; n is the number of moles of gas you have; R is the *universal gas constant,* which has a value of 8.31 J(/mole-K); and T is measured in Kelvins. The volume, V, is measured in cubic meters, m^3. Sometimes, you'll see volume given in liters, where $1.0 \text{ L} = 10^{-3} \text{ m}^3$. Using this law, you can predict the pressure of an ideal gas, given how much you have of it, its temperature, and the volume you've enclosed it in.

One mole of ideal gas takes up 22.4 L of volume at 0°C and one atmosphere pressure, which is $1.013 \times 10^5 \text{ N/m}^2$, where N/m^2 (Newtons per square meter) is given its own units, Pascals, abbreviated Pa.

You can also write the ideal gas law a little differently by using Avogadro's Number, N_A, and the total number of molecules, N:

$$P \cdot V = n \cdot R \cdot T = (N/N_A)\, R \cdot T$$

The constant R/N_A is also called Boltzmann's constant, k, and it has a value of 1.38×10^{-23} J/K. Using this constant, the ideal gas law becomes

$$P \cdot V = N \cdot k \cdot T$$

EXAMPLE

Q. You have 1.0 moles of air in your tire at 0°C, volume 10.0 L. What is the gauge pressure within the tire?

A. The correct answer is 1.3×10^5 Pa.

1. Use this equation:

$$PV = nRT$$

2. Solve for P:

$$P = nRT / V$$

3. Plug in the numbers to get the pressure pushing out:

$$P = nRT / V = (1.0)\ (8.31)\ (273) / (10 \times 10^{-3}) = 2.3 \times 10^5 \text{ Pa}$$

4. Subtract the pressure from the surrounding air, which pushes in, assuming that the air is at 0°C too:

$$P = 2.3 \times 10^5 - 1.013 \times 10^5 = 1.3 \times 10^5 \text{ Pa}$$

17. You have 2.3 moles of air in your tire at 0°C, volume 12.0 L. What pressure is the tire inflated to?

Solve It

18. You have a bottle of 2.0 moles of gas, volume 1.0 L, temperature 100°C. What is the pressure inside the bottle?

Solve It

Molecules in Motion

The average kinetic energy of molecules in a gas is

$$KE_{avg} = \frac{3}{2} \cdot k \cdot T$$

Here, k is Boltzmann's constant, 1.38×10^{-23} J/K. Now you can determine the average kinetic energies of the molecules in a gas. And because you can determine the mass of each molecule if you know what gas you're dealing with, you can figure out their average speeds at various temperatures.

EXAMPLE

Q. What is the speed of air molecules at 28°C?

A. The correct answer is 517 m/s.

1. Use this equation:

$$KE_{avg} = \frac{3}{2} \cdot k \cdot T$$

2. Plug in the numbers:

$$KE_{avg} = \frac{3}{2} kT = \frac{3}{2}(1.38 \times 10^{-23})(301) = 6.23 \times 10^{-21} \text{ J}$$

3. You know that:

$$KE = \frac{1}{2} m \cdot v^2$$

4. Solve for v:

$$v = \sqrt{\frac{2KE}{m}}$$

5. Plug in the numbers. Air is mostly nitrogen, and each nitrogen atom has a mass of about 4.65×10^{-26} kg:

$$v = \sqrt{\frac{2KE}{m}} = 517 \text{ m/s} = 1150 \text{ mph}$$

19. What is the average speed of air molecules at 43°C?

Solve It

20. What is the average speed of air molecules at 900°C?

Solve It

Answers to Problems about Pressure

The following are the answers to the practice questions presented in this chapter. You see how to work out each answer, step by step.

1 **Convection**

The air is heated by the heating elements on the bottom, and the heated air rises, so the heat transfer takes place via convection.

2 **Convection**

The air is heated by being in proximity to the heated Earth, and the heated air rises.

3 **403 J/sec**

1. Use this equation:

$$Q = \frac{k \cdot A \cdot \Delta T \cdot t}{L}$$

2. Solve for Q/t:

$$\frac{Q}{t} = \frac{k \cdot A \cdot \Delta T}{L}$$

3. Plug in the numbers:

$$\frac{Q}{t} = \frac{k \cdot A \cdot \Delta T}{L} = \frac{(79) \cdot (.3) \cdot (34)}{2.0} = 403 \text{ J/sec}$$

4 **10,000 J/sec**

1. Use this equation:

$$Q = \frac{k \cdot A \cdot \Delta T \cdot t}{L}$$

2. Solve for Q/t:

$$\frac{Q}{t} = \frac{k \cdot A \cdot \Delta T}{L}$$

3. Plug in the numbers:

$$\frac{Q}{t} = \frac{k \cdot A \cdot \Delta T}{L} = \frac{(420) \cdot (.4) \cdot (91)}{1.5} = 10{,}000 \text{ J/sec}$$

5 **0.23 sec**

1. Use this equation:

$$Q = \frac{k \cdot A \cdot \Delta T \cdot t}{L}$$

2. Solve for t:

$$t = \frac{L \cdot Q}{k \cdot A \cdot \Delta T}$$

3. Plug in the numbers:

$$t = \frac{L \cdot Q}{k \cdot A \cdot \Delta T} = \frac{(0.5) \cdot (1000)}{(79) \cdot (0.3) \cdot (90)} = 0.23 \text{ sec}$$

6 **0.22 sec**

1. Use this equation:

$$Q = \frac{k \cdot A \cdot \Delta T \cdot t}{L}$$

2. Solve for t:

$$\frac{L \cdot Q}{k \cdot A \cdot \Delta T} = t$$

3. Plug in the numbers:

$$\frac{L \cdot Q}{k \cdot A \cdot \Delta T} = t = \frac{(0.73) \cdot (3000)}{(110) \cdot (0.5) \cdot (180)} = 0.22 \text{ sec}$$

7 **145 J/(s-m-°C)**

1. Use this equation:

$$Q = \frac{k \cdot A \cdot \Delta T \cdot t}{L}$$

2. Solve for k:

$$\frac{L \cdot Q}{t \cdot A \cdot \Delta T} = k$$

3. Plug in the numbers:

$$\frac{L \cdot Q}{t \cdot A \cdot \Delta T} = k = \frac{(2.0) \cdot (1000)}{(0.5) \cdot (0.4) \cdot (69)} = 145 \text{ J}/(\text{s}-\text{m}-^\circ\text{C})$$

8 **80 J/(s-m-°C)**

1. Use this equation:

$$Q = \frac{k \cdot A \cdot \Delta T \cdot t}{L}$$

2. Solve for k:

$$\frac{L \cdot Q}{t \cdot A \cdot \Delta T} = k$$

3. Plug in the numbers:

$$\frac{L \cdot Q}{t \cdot A \cdot \Delta T} = k = \frac{(1.0) \cdot (1400)}{(0.6) \cdot (0.3) \cdot (97)} = 80 \text{ J}/(\text{s}-\text{m}-^\circ\text{C})$$

9 **190 J/sec**

1. Use this equation:

$$Q = e \cdot \sigma \cdot A \cdot t \cdot T^4$$

2. Solve for power, which is Q/t:

$$\frac{Q}{t} = e \cdot \sigma \cdot A \cdot T^4$$

3. Plug in the numbers:

$$\frac{Q}{t} = e \cdot \sigma \cdot A \cdot T^4 = (0.95) \cdot (5.67 \times 10^{-8}) \cdot (0.2) \cdot (90 + 273)^4 = 190 \text{ W}$$

10 **180 J/(s-m-°C)**

1. Use this equation:

$$Q = e \cdot \sigma \cdot A \cdot t \cdot T^4$$

2. Solve for power, which is Q/t:

$$\frac{Q}{t} = e \cdot \sigma \cdot A \cdot T^4$$

3. Plug in the numbers:

$$\frac{Q}{t} = e \cdot \sigma \cdot A \cdot T^4 = (0.9) \cdot (5.67 \times 10^{-8}) \cdot (0.15) \cdot (120 + 273)^4 = 180 \text{ W}$$

11 **74 sec**

1. Use this equation (which works because the temperature difference involved, 80°C to 75°C, is small):

$$Q = e\sigma AtT^4$$

2. Solve for time:

$$\frac{Q}{e\sigma AT^4} = t$$

3. Solve for the amount of heat you need to lose:

$$\triangle Q = cm\triangle T$$

4. Plug in the numbers, assuming that coffee has the specific heat of water:

$$\triangle Q = cm\triangle T = (4186)(500\text{ g})(80 - 75) = 10{,}500 \text{ J}$$

5. Put in the numbers to find the time:

$$\frac{Q}{e\sigma AT^4} = \frac{10{,}500}{(0.9) \cdot (5.67 \times 10^{-8}) \cdot (0.18) \cdot (80 + 273)^4} = t = 74 \text{ sec}$$

12 **146 sec**

1. Use this equation (which works because the temperature difference involved, 80°C to 75°C, is small):

$$Q = e \cdot \sigma \cdot A \cdot t \cdot T^4$$

2. Solve for time:

$$\frac{Q}{e \cdot \sigma \cdot A \cdot T^4} = t$$

3. Solve for the amount of heat you need to lose:

$$\triangle Q = \text{heat lost by tea} + \text{heat lost by teapot}$$

4. Plug in the numbers, assuming that tea has the specific heat of water:

$$\triangle Q = m_1 \cdot c_1 \cdot \triangle T + m.\backslash_1 \cdot c_2 \cdot \triangle T = (4186) \cdot (200) \cdot (82 - 76) + (840) \cdot (100) \cdot (82 - 76) = 5530 \text{ J}$$

5. Put in the numbers to find the time:

$$T = \frac{Q}{e\sigma AT^4} = \frac{5530}{(0.6)(5.67 \times 10^{-8})(0.07)(82 + 273)^4} = t = 146 \text{ sec}$$

13 **1.5×10^{23} atoms**

1. You know that 1 mole of calcium has a mass of 40.08 g, so you have this many moles:

$$n = \frac{10.0}{40.08} = 0.25 \text{ mole}$$

2. With this many moles, you have this many molecules:

$$N = n \cdot (6.022 \times 10^{23}) = 1.5 \times 10^{23} \text{ atoms}$$

14 **3.4×10^{23} atoms**

1. You know that 1 mole of silicon has a mass of 28.09 g, so you have this many moles:

$$n = \frac{16.0}{28.09} = 0.57 \text{ mole}$$

2. With this many moles, you have this many molecules:

$$N = n \cdot (6.022 \times 10^{23}) = 3.4 \times 10^{23} \text{ atoms}$$

15 **2.6×10^{23} atoms**

1. You know that 1 mole of zinc has a mass of 65.41 g, so you have this many moles:

$$n = \frac{29.0}{65.41} = 0.44 \text{ mole}$$

2. With this many moles, you have this many molecules:

$$N = n \cdot (6.022 \times 10^{23}) = 2.6 \times 10^{23} \text{ atoms}$$

16 **2.8×10^{22} atoms**

1. You know that 1 mole of copper has a mass of 63.546 g, so you have this many moles:

$$n = \frac{3.0}{63.546} = 0.047 \text{ mole}$$

2. With this many moles, you have this many molecules:

$$N = n \cdot (6.022 \times 10^{23}) = 2.8 \times 10^{22} \text{ atoms}$$

17 **3.3×10^{22} Pa**

1. Use this equation:

$$P \cdot V = n \cdot R \cdot T$$

2. Solve for P:

$$P = n \cdot R \cdot T / V$$

3. Plug in the numbers:

$$P = n \cdot R \cdot T / V = (2.3) \cdot (8.31) \cdot (273) / (12.0 \times 10^{-3}) = 4.3 \times 10^5 \text{ Pa}$$

4. Subtract the pressure from the surrounding air, which pushes in, assuming that the air is at 0°C too:

$$P = 4.3 \times 10^5 - 1.013 \times 10^5 = 3.3 \times 10^5 \text{ Pa}$$

18 **6.2×10^6 Pa**

1. Use this equation:

$$P \cdot V = n \cdot R \cdot T$$

2. Solve for P:

$$P = n \cdot R \cdot T / V$$

3. Plug in the numbers:

$$P = n \cdot R \cdot T / V = (2.0) \cdot (8.31) \cdot (373) / (1.0 \times 10^{-3}) = 6.2 \times 10^6 \text{ Pa}$$

19 **530 m/sec**

1. Use this equation:

$$KE_{avg} = \frac{3}{2} \cdot k \cdot T$$

2. Plug in the numbers:

$$KE_{avg} = \frac{3}{2} \cdot k \cdot T = \frac{3}{2}(1.38 \times 10^{-23})(316) = 6.5 \times 10^{-21} \text{ J}$$

3. Here's the equation for kinetic energy:

$$KE = \frac{1}{2} m \cdot v^2$$

4. Solve for v:

$$v = \sqrt{\frac{2KE}{m}}$$

5. Plug in the numbers, assuming the molecular mass of nitrogen, 4.65×10^{-26} kg (which you can figure out by using Avogadro's Number and the mass of 1 mole of nitrogen):

$$v = \sqrt{\frac{2KE}{m}} = 530 \text{ m/sec}$$

20 **1020 m/sec**

1. Use this equation:

$$KE_{avg} = \frac{3}{2} \cdot k \cdot T$$

2. Plug in the numbers:

$$KE_{avg} = \frac{3}{2} \cdot k \cdot T = \frac{3}{2}(1.38 \times 10^{-23})(1173) = 2.4 \times 10^{-20} \text{ J}$$

3. Here's the equation for kinetic energy:

$$KE = \frac{1}{2} m \cdot v^2$$

4. Solve for v:

$$v = \sqrt{\frac{2KE}{m}}$$

5. Plug in the numbers, assuming the molecular mass of nitrogen, 4.65×10^{-26} kg (which you can figure out by using Avogadro's Number and the mass of 1 mole of nitrogen):

$$v = \sqrt{\frac{2KE}{m}} = 1020 \text{ m/sec}$$

Chapter 14

All about Heat and Work

In This Chapter

- Understanding the laws of thermodynamics
- Working with isobaric processes
- Handling isochoric processes
- Calculating isothermal processes

This chapter is all about the laws of thermodynamics. Those laws describe all kinds of heat processes. You also discover how to handle heat processes where pressure is constant, or volume is constant, or temperature is constant.

The First Law of Thermodynamics

The first law of thermodynamics says that energy is conserved. The internal energy in a system, U_o, changes to a final value U_f when heat Q is absorbed or released by the system and the system does work W on its surroundings, or the surroundings do work W on the system, such that:

$$U_f - U_o = \Delta U = Q - W$$

The value Q is positive when the system absorbs heat and negative when the system releases heat. The quantity W is positive when the system does work on its surroundings and negative when the surroundings do work on the system.

This law is useful because it says that total energy — work plus heat — is conserved.

Q. Say that a motor does 1000 J of work on its surroundings while releasing 3000 J of heat. By how much does its internal energy change?

A. The correct answer is –4000 J.

1. You know that the motor does 1000 J of work on its surroundings, so you know that its internal energy decreases by 1000 J.
2. The motor also releases 3000 J of heat while doing its work, so the internal energy of the system decreases by an additional 3000 J. Think of negative values of work and heat as flowing out of the system as negative, making the total change of internal energy this:

$$\Delta U = -1000 - 3000 = -4000 \text{ J}$$

1. You have a motor that absorbs 3000 J of heat while doing 2000 J of work. What is the change in the motor's internal energy?

Solve It

2. You have a motor that absorbs 2500 J of heat while doing 1700 J of work. What is the change in the motor's internal energy?

Solve It

Constant Pressure: Isobaric Processes

Take a look at Figure 14-1, in which a lid with a weight on it keeps constant pressure on a gas as that gas expands. Work that's done at constant pressure is called *isobaric* work.

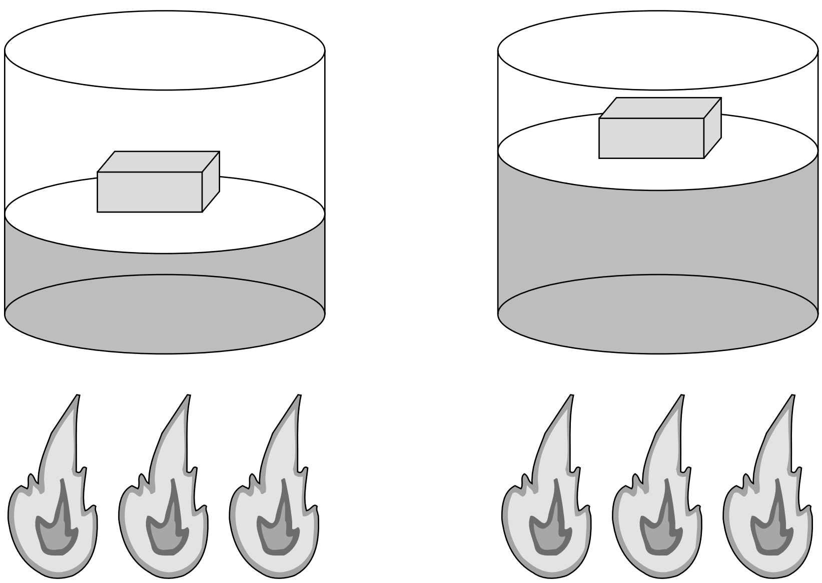


Figure 14-1: An isobaric system.

So the question is: What work is the system doing as it expands? Work = F·s, and F = P·A, where P is the pressure and A is the area. That means that:

$$W = F \cdot \Delta s = P \cdot A \cdot \Delta s$$

On the other hand, $A \cdot \Delta s = \Delta V$, the change in volume, so you have

$$W = P \cdot \Delta V$$

You can see what this looks like graphically for an isobaric process in Figure 14-2, in which the volume is changing while the pressure stays constant. Because $W = F\Delta V$, the work is the area beneath the graph as shown in the figure.

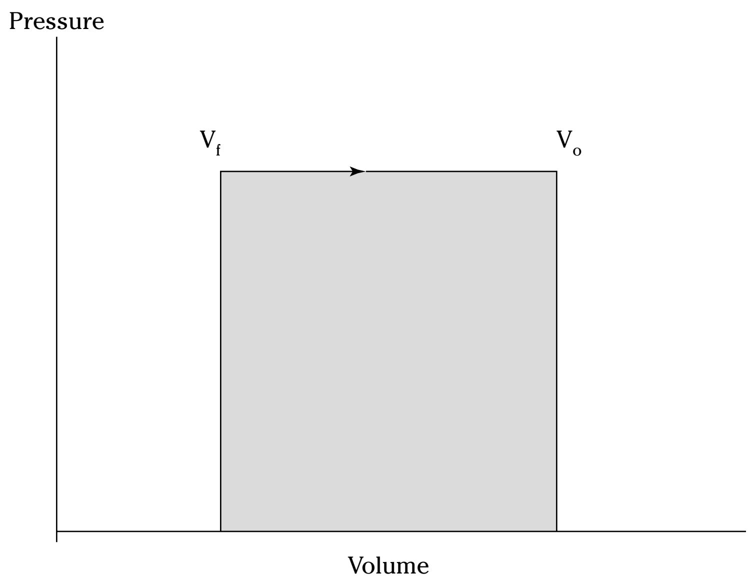


Figure 14-2: An isobaric graph.

All you've got to do is to plug in the numbers:

$$W = P \cdot \Delta V = (200) \cdot (120 - 60) = 12{,}000 \text{ J}$$

You get your answer: The gas did 12,000 J of work as it expanded under constant pressure.

EXAMPLE

Q. You have 60.0 m^3 of an ideal gas at 200.0 Pa and heat the gas until it expands to a volume of 120 m^3. How much work did the gas do?

A. The correct answer is 12,000 J.

1. Use this equation:

$$W = P \cdot \Delta V$$

2. Plug in the numbers:

$$W = P \cdot \Delta V = (200)\ (120 - 60) = 12{,}000 \text{ J}$$

3. You have 50.0 m^3 of an ideal gas at 1000.0 Pa and heat the gas until it expands to a volume of 300.0 m^3. How much work did the gas do?

Solve It

4. You have 300.0 m^3 of an ideal gas at 1500.0 Pa and heat the gas until it expands to a volume of 900.0 m^3. How much work did the gas do?

Solve It

5. You have 50.0 m^3 of an ideal gas at 1000.0 Pa, and the gas expands, doing 3000.0 J of work. What is the final volume of the gas?

Solve It

6. You have 100.0 m^3 of an ideal gas at 300.0 Pa, and it expands, doing 6000.0 J of work. What is the final volume of the gas?

Solve It

Constant Volume: Isochoric Processes

When an ideal gas's pressure increases at constant volume, how much work is done? Because

$$W = P \cdot \triangle V$$

the answer is simple: No work is being done. This is called an *isochoric* process, and you can see a graph of what's happening in Figure 14-3.

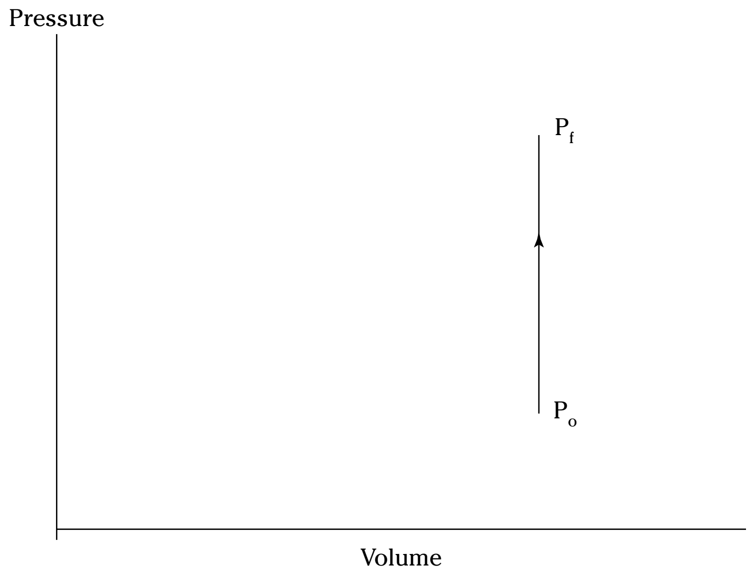


Figure 14-3: An isochoric graph.

As you see in the figure, the pressure rises while the volume stays the same, so no work is actually being done.

Q. You have 60.0 m^3 of an ideal gas at 200.0 Pa and heat the gas until its pressure is 300.0 Pa at the same volume. How much work did the gas do?

A. The correct answer is none.

1. Use this equation:

   $$W = P \cdot \triangle V$$

2. Because $\triangle V = 0$, no work was done. This process is an isochoric process.

7. You have 50.0 m^3 of an ideal gas at 1000.0 Pa and heat the gas until it has a pressure of 3000.0 Pa, still at the same volume. How much work did the gas do?

Solve It

8. You have 300.0 m^3 of an ideal gas at 1500.0 Pa and heat the gas until it has a pressure of 6000.0 Pa. How much work did the gas do?

Solve It

Constant Temperature: Isothermal Processes

Processes that take place at constant temperature are called *isothermal* processes. What's the work look like in this case as the volume changes? Because PV = nRT, the relation between P and V is

$$P = \frac{n \cdot R \cdot T}{V}$$

That relationship looks like the graph you see in Figure 14-4.

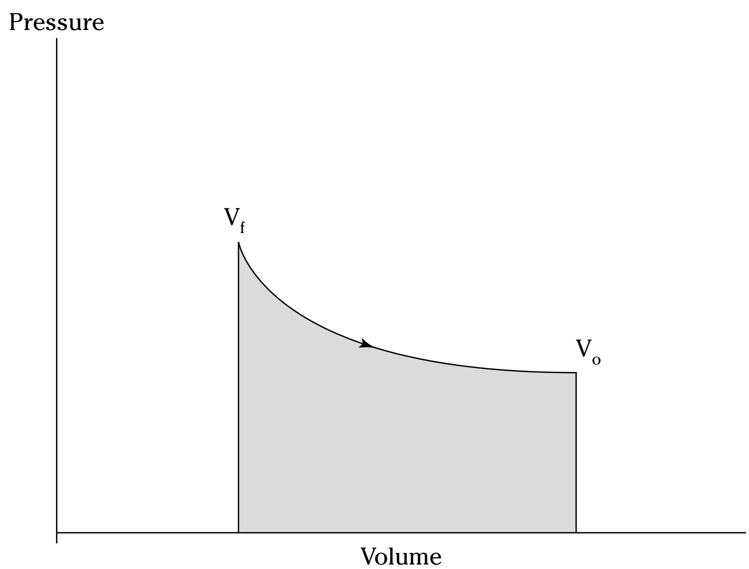


Figure 14-4: An isothermal graph.

As before, the work done is the area underneath the graph. What is that area? The work done is given by this equation in this case, where ln is the natural log:

$$W = n \cdot R \cdot T \ln(V_f / V_o)$$

Note that because in an isothermal process the temperature stays constant, and because for an ideal gas the internal energy = (3/2)·n·R·T, the internal energy doesn't change. So you have

$$\Delta U = 0 = Q - W$$

In other words:

$$Q = W$$

In this case, the work done is equal to the heat added to the gas.

EXAMPLE

Q. You have 1 mole of ideal gas at a constant temperature of 20°C, and you expand the gas from $V_o = 0.010\ m^3$ to $V_f = 0.020\ m^3$. What work did the gas do in expanding?

A. The correct answer is 1690 J.

1. Use this equation:

$$W = n \cdot R \cdot T \cdot \ln(V_f / V_o)$$

2. Plug in the numbers:

$$W = n \cdot R \cdot T \cdot \ln(V_f / V_o) = (1.0)\,(8.31)\,(273.15 + 20)\,\ln(0.020 / 0.010) = 1690\ J$$

3. The work done by the gas was 1690 J. The change in the internal energy of the gas is 0 J, as it must be in isothermal processes. Because Q = W, the heat added to the gas is equal to 1690 J.

9. You have 1 mole of ideal gas at a constant temperature of 30.0°C, and you expand the gas from $V_o = 2.0\ m^3$ to $V_f = 3.0\ m^3$. What work did the gas do in expanding?

Solve It

10. You have 0.60 mole of ideal gas at a constant temperature of 25°C, and you expand the gas from $V_o = 1.0\ m^3$ to $V_f = 3.0\ m^3$. What heat was supplied to the gas in expanding?

Solve It

At Constant Heat: Adiabatic

The last type of thermodynamic process is the *adiabatic* process, in which the total heat in the system is held constant. Take a look at the system in Figure 14-5; everything is surrounded by an insulating substance, so the heat from the system isn't going anywhere. When a change occurs, then, it's going to be an adiabatic change.

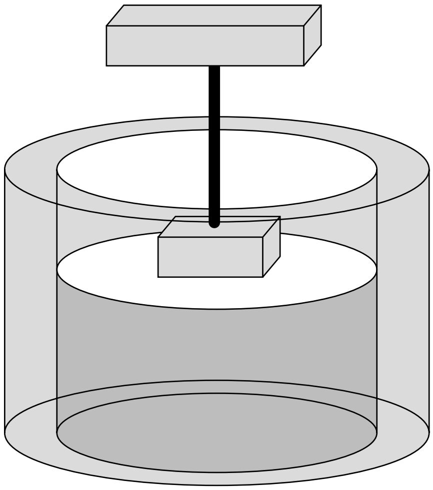


Figure 14-5: An adiabatic system.

Okay, what's the work done during an adiabatic process? Here, $\triangle Q = 0$, so $\triangle U = -W$. Because the internal energy of an ideal gas is $U = (3/2) \cdot n \cdot R \cdot T$, the work done is

$$W = \frac{3}{2} n \cdot R \cdot \left(T_o - T_f\right)$$

In other words, if the gas does work in an adiabatic process, the work comes from a change in temperature. If the final temperature is lower, the system does work on its surroundings.

You can see what P versus V looks like for an adiabatic process in Figure 14-6. The adiabatic curve here, called an *adiabat,* is not the same as the isothermal curves, called *isotherms.* The work done when the total heat in the system is constant is the area under the adiabat, as shown in Figure 14-6.

You can relate the initial pressure and volume to the final pressure and volume in an adiabatic process this way:

$$P_o \cdot V_o \gamma = P_f \cdot V_f \gamma$$

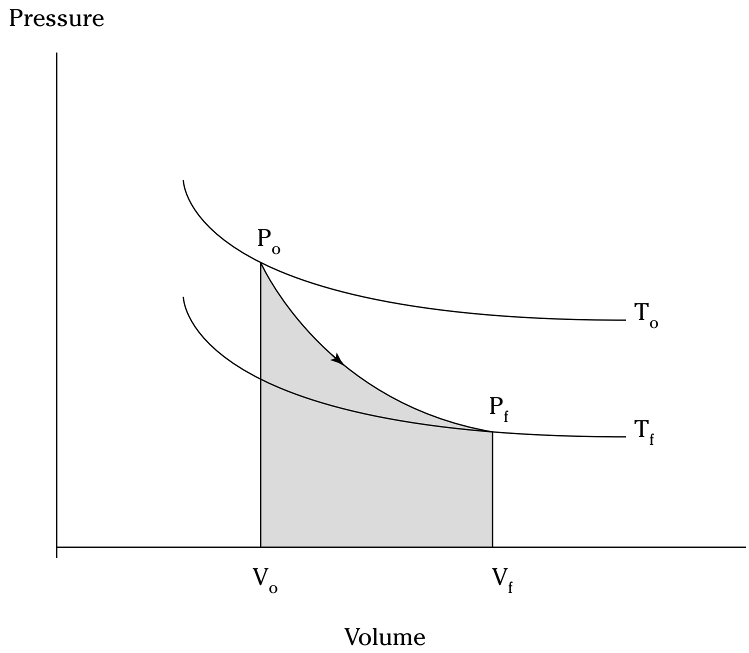


Figure 14-6: An adiabatic graph.

What's γ? It's the ratio of the specific heat capacity of an ideal gas at constant pressure divided by the specific heat capacity of an ideal gas at constant volume, c_p/c_v. What are c_p and c_v?

$$C_v = \frac{3}{2} \cdot R$$

$$C_p = \frac{5}{2} \cdot R$$

That makes the ratio γ this:

$$\gamma = \frac{C_p}{C_v} = \frac{5}{3}$$

So you can connect pressure and volume at any two points along an adiabat this way:

$$P_o \cdot V_o^{5/3} = P_f \cdot V_f^{5/3}$$

These processes are adiabatic; no heat is gained or lost.

Q. You start with 3.0 L of gas at 1.0 atmosphere and end up, after an adiabatic change, with 6.0 L of gas. What is the new pressure?

A. The correct answer is 0.31 atmosphere.

1. Use this equation:

$$P_f = \frac{P_o \cdot V_o^{5/3}}{V_f^{5/3}}$$

2. Plug in the numbers:

$$P_f = \frac{P_o \cdot V_o^{5/3}}{V_f^{5/3}} = \frac{(1.0)(3.0)^{5/3}}{(6.0)^{5/3}} = 0.31 \text{ atmosphere}$$

11. You start with 1.0 L of gas at 1.0 atmosphere and end up after an adiabatic change with 3.0 L of gas. What is the new pressure?

Solve It

12. You start with 1.5 L of gas at 1.7 atmosphere and end up after an adiabatic change with 2.9 L of gas. What is the new pressure?

Solve It

13. You have 1.0 mole of ideal gas that undergoes an adiabatic change, going from 30.0°C to 10.0°C. What work did the gas do?

Solve It

14. You have 3.0 moles of ideal gas that undergo an adiabatic change, going from 23°C to 69°C. What work was done on the gas?

Solve It

Heat Moves: The Second Law of Thermodynamics

You may see physics problems in the definition of the second law of thermodynamics, which says,

> Heat flows naturally from an object at higher temperature to an object at lower temperature, and does not flow of its own accord in the opposite direction.

EXAMPLE

Q. A red-hot horseshoe is placed on an anvil at room temperature. Which way does the heat flow?

A. The correct answer is into the anvil.

The second law of thermodynamics tells you that heat flows from hotter objects to cooler ones, so heat flows from the red-hot horseshoe to the room-temperature anvil.

Making Heat Work: Heat Engines

You know how a steam engine works: Heated steam does the work. Physics makes a study of this process, and you can see what's going on diagrammatically in Figure 14-7. Heat is supplied to a heat engine, which does work and sends its exhaust to a lower-temperature heat sink. (Often, the heat sink is just the surroundings of the heat engine, as is the case with steam engines.)

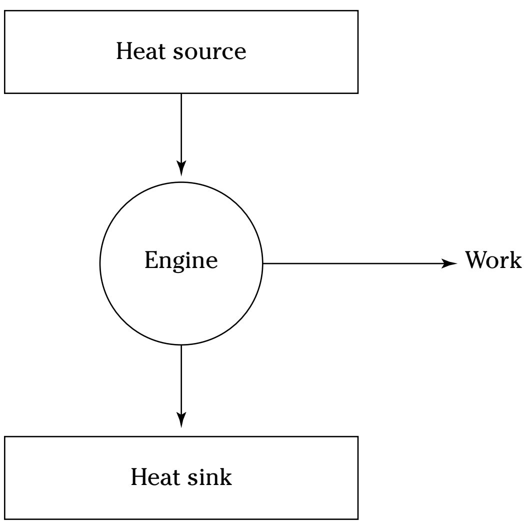


Figure 14-7: A heat engine.

Say that the heat supplied to the engine is Q_h, and the heat sent to the heat sink is Q_c (h and c stand for the hot and cold reservoirs of heat). In that case, you could say that the efficiency of the work engine in terms of turning heat into work is

$$\text{Efficiency} = \frac{\text{Work}}{\text{Heat input}} = \frac{W}{Q_h}$$

So if all the input heat is converted to work, the efficiency is 1.0. If none of the input heat is converted to work, the efficiency is 0.0.

Note that because total energy is conserved, the heat into the engine must equal the work done plus the heat sent to the heat sink, which means that:

$$Q_h = W + Q_c$$

That in turn means you can rewrite the efficiency in terms of just Q_h and Q_c this way:

$$\text{Efficiency} = \frac{W}{Q_h} = \frac{Q_h - Q_c}{Q_h} = 1 - \frac{Q_c}{Q_h}$$

EXAMPLE

Q. Your car is powered by a heat engine and does 3.0×10^7 J of work getting you up a small hill. If the heat engine is 80 percent efficient, how much heat did it use, and how much did it exhaust?

A. The correct answer is 3.75×10^7 J, 0.75×10^7 J.

1. Use this equation:

$$\text{Efficiency} = \frac{W}{Q_h} = \frac{3.0 \times 10^7 \text{ J}}{Q_h} = .80$$

2. Solve for Q_h:

$$\frac{3.0 \times 10^7 \text{ J}}{.80} = Q_h = 3.75 \times 10^7 \text{ J}$$

So the input heat was 3.75×10^7 J.

3. Use this equation:

$$Q_h = W + Q_c$$

4. Solve for Q_c:

$$Q_h - W = Q_c$$

5. Plug in the numbers:

$$Q_h - W = 3.75 \times 10^7 - 3.0 \times 10^7 = 0.75 \times 10^7 = Q_c$$

So the output heat was 0.75×10^7 J.

15. A car running on a heat engine does 7.0×10^7 J of work. If the heat engine is 76 percent efficient, how much heat did it use, and how much did it exhaust?

Solve It

16. A car running using a heat engine does 3.9×10^7 J of work. If the heat engine is 89 percent efficient, how much heat did it use, and how much did it exhaust?

Solve It

17. A 63 percent efficient heat engine does 3.8×10^{10}J of work. How much heat did it use, and how much did it exhaust?

Solve It

18. An 87 percent efficient heat engine does 4.5×10^{10}J of work. How much heat did it use, and how much did it exhaust?

Solve It

19. A heat engine does 4.6×10^{7} J of work when supplied 8.9×10^{7} J. What is its efficiency?

Solve It

20. A heat engine does 8.1×10^{7} J of work when supplied 10.9×10^{7} J. What is its efficiency?

Solve It

Maximum Efficiency: Carnot Heat Engines

An engineer named Sadi Carnot figured out that the maximum possible efficiency of a heat engine is this:

$$\text{Efficiency} = 1 - \frac{T_c}{T_h}$$

The temperatures T_h and T_c are the temperatures of the heat source and heat sink, respectively, measured in Kelvin.

That efficiency is the best a heat engine can get, assuming that no irreversible losses of energy occur due to friction or other causes. When you have a heat engine that does the best a heat engine can do, you have a Carnot engine, and the preceding equation is the expression for its efficiency.

EXAMPLE

Q. The heat source for a Carnot engine is at 100°C, and the heat sink is at 20°C. What is the engine's efficiency?

A. The correct answer is 21percent.

1. Use this equation:

$$\text{Efficiency} = 1 - \frac{T_c}{T_h}$$

2. Plug in the numbers:

$$\text{Efficiency} = 1 - \frac{T_c}{T_h} = 1 - \frac{(273.15 + 20)}{(273.15 + 100)} = 21\%$$

21. The heat source for a Carnot engine is at 87°C, and the heat sink is at 49°C. What is the engine's efficiency?

Solve It

22. The heat source for a Carnot engine is at 67°C, and the heat sink is at 29°C. What is the engine's efficiency?

Solve It

The Third Law of Thermodynamics

This chapter finishes with the third law of thermodynamics, which says

> You cannot reach absolute zero through any process which uses a finite number of steps.

In other words, you can get closer and closer to absolute zero step by step, but you can't actually reach it. This fact has been demonstrated in practice. Physicists have been able to approach absolute zero until they're just a fraction of a degree away, but no one has been able to reach it.

Answers to Problems about Heat and Work

The following are the answers to the practice questions presented earlier in this chapter. You see how to work out each answer, step by step.

1 **1000 J**

1. Use this equation:

$$U_f - U_o = \triangle U = Q - W$$

2. Plug in the numbers:

$$\triangle U = 3000\ [\text{heat coming in}] - 2000\ [\text{work going out}] = 1000\ \text{J}$$

2 **800 J**

1. Use this equation:

$$U_f - U_o = \triangle U = Q - W$$

2. Plug in the numbers:

$$\triangle U = 2500\ [\text{heat coming in}] - 1700\ [\text{work going out}] = 800\ \text{J}$$

3 **250,000 J**

1. Use this equation:

$$W = P{\cdot}\triangle V$$

2. Plug in the numbers:

$$W = P{\cdot}\triangle V = (1000)\ (300 - 50) = 250{,}000\ \text{J}$$

4 **9.0×10^5 J**

1. Use this equation:

$$W = P{\cdot}\triangle V$$

2. Plug in the numbers:

$$W = P{\cdot}\triangle V = (1500)\ (900 - 300) = 9.0 \times 10^5\ \text{J}$$

5 **52 m^3**

1. Use this equation:

$$W = P{\cdot}\triangle V = P{\cdot}(V_f - V_o)$$

2. Solve for V_f:

$$\frac{W}{P} + V_o = V_f$$

3. Plug in the numbers:

$$\frac{W}{P} + V_o = V_f = \frac{3000}{1500} + 50 = 52\ \text{m}^3$$

6 **120 m³**

1. Use this equation:

$$W = P\cdot\triangle V = P\cdot(V_f - V_o)$$

2. Solve for V_f:

$$\frac{W}{P} + V_o = V_f$$

3. Plug in the numbers:

$$\frac{W}{P} + V_o = V_f = \frac{6000}{300} + 100 = 120\ m^3$$

7 **0 J**

1. Use this equation:

$$W = P\cdot\triangle V = P\cdot(V_f - V_o)$$

2. Because no change in volume occurred, this process was isochoric, and no work was done.

8 **0 J**

1. Use this equation:

$$W = P\cdot\triangle V = P\cdot(V_f - V_o)$$

2. Because no change in volume occurred, this process was isochoric, and no work was done.

9 **1020 J**

1. Use this equation:

$$W = n\cdot R\cdot T\ \ln(V_f / V_o)$$

2. Plug in the numbers:

$$W = n\cdot R\cdot T\ \ln(V_f / V_o) = (1.0)\ (8.31)\ (273.15 + 30)\ \ln(3.0 / 2.0) = 1020\ J$$

3. The work done by the gas was 1020 J. The change in the internal energy of the gas is 0 J, as it must be in isothermal processes. Because Q = W, the heat added to the gas is equal to 1020 J.

10 **1630 J**

1. Use this equation:

$$W = n\ R\ T\ \ln(V_f / V_o)$$

2. Plug in the numbers:

$$W = n\cdot R\cdot T\ \ln(V_f / V_o) = (0.6)\ (8.31)\ (273.15 + 25)\ \ln(3.0 / 1.0) = 1630\ J$$

3. The work done by the gas was 1630 J. The change in the internal energy of the gas is 0 J, as it must be in isothermal processes. Because Q = W, the heat added to the gas is equal to 1630 J.

11 **0.16 atmosphere**

1. Use this equation:

$$P_f = \frac{P_o V_o^{5/3}}{V_f^{5/3}}$$

2. Plug in the numbers:

$$P_f = \frac{P_o V_o^{5/3}}{V_f^{5/3}} = \frac{(1.0)\cdot(1.0)^{5/3}}{(3.0)^{5/3}} = 0.16 \text{ atmosphere}$$

12 **0.57 atmosphere**

1. Use this equation:

$$P_f = \frac{P_o V_o^{5/3}}{V_f^{5/3}}$$

2. Plug in the numbers:

$$P_f = \frac{P_o V_o^{5/3}}{V_f^{5/3}} = \frac{(1.7)(1.5)^{5/3}}{(2.9)^{5/3}} = 0.57 \text{ atmosphere}$$

13 **250 J**

1. Use this equation:

$$W = \frac{3}{2}\cdot n\cdot R\cdot(T_o - T_f)$$

2. Plug in the numbers:

$$W = \frac{3}{2}\cdot n\cdot R\cdot(T_o - T_f) = (1.5)\cdot(1.0)\cdot(8.31)\cdot(31-10) = 250 \text{ J}$$

14 **1720 J**

1. Use this equation:

$$W = \frac{3}{2}\cdot n\cdot R\cdot(T_o - T_f)$$

2. Plug in the numbers:

$$W = \frac{3}{2}\cdot n\cdot R\cdot(T_o - T_f) = (1.5)\cdot(3.0)\cdot(8.31)\cdot(23-69) = -1720 \text{ J}$$

3. The work done by the gas is –1720 J, so the work done on the gas is 1720 J.

15 **9.2×10^7 J, 2.2×10^7 J**

1. Use this equation:

$$\text{Efficiency} = \frac{W}{Q_h} = \frac{7.0\times 10^7 \text{ J}}{Q_h} = .76$$

2. Solve for Q_h:

$$\frac{7.0\times 10^7 \text{ J}}{.76} = Q_h = 9.2\times 10^7 \text{ J}$$

So the input heat was 9.2×10^7 J.

3. Use this equation:

$$Q_h = W + Q_c$$

4. Solve for Q_c:

$$Q_h - W = Q_c$$

5. Plug in the numbers:

$$Q_h - W = 9.2\times 10^7 - 7.0\times 10^7 = 2.2\times 10^7 = Q_c$$

So the output heat was 2.2×10^7 J.

16 **4.4×10^7 J, 0.50×10^7 J**

1. Use this equation:

$$\text{Efficiency} = \frac{W}{Q_h} = \frac{3.9 \times 10^7 \text{ J}}{Q_h} = .89$$

2. Solve for Q_h:

$$\frac{3.9 \times 10^7 \text{ J}}{.89} = Q_h = 4.4 \times 10^7 \text{ J}$$

So the input heat was 4.4×10^7 J.

3. Use this equation:

$$Q_h = W + Q_c$$

4. Solve for Q_c:

$$Q_h - W = Q_c$$

5. Plug in the numbers:

$$Q_h - W = 4.4 \times 10^7 - 3.9 \times 10^7 = 0.5 \times 10^7 = Q_c$$

So the output heat was 2.2×10^7 J.

17 **6.2×10^{10} J, 2.4×10^{10} J**

1. Use this equation:

$$\text{Efficiency} = \frac{W}{Q_h} = \frac{3.8 \times 10^{10} \text{ J}}{Q_h} = 0.63$$

2. Solve for Q_h:

$$\frac{3.8 \times 10^{10} \text{ J}}{.63} = Q_h = 6.0 \times 10^{10} \text{ J}$$

So the input heat was 6.0×10^{10} J.

3. Use this equation:

$$Q_h = W + Q_c$$

4. Solve for Q_c:

$$Q_h - W = Q_c$$

5. Plug in the numbers:

$$Q_h - W = 6.0 \times 10^{10} - 3.8 \times 10^{10} = 2.2 \times 10^{10} = Q_c$$

So the output heat was 2.2×10^7 J.

18 **5.2×10^{10} J, 0.7×10^{10} J**

1. Use this equation:

$$\text{Efficiency} = \frac{W}{Q_h} = \frac{4.5 \times 10^{10} \text{ J}}{Q_h} = .87$$

2. Solve for Q_h:

$$\frac{4.5 \times 10^{10} \text{ J}}{.87} = Q_h = 5.2 \times 10^{10} \text{ J}$$

So the input heat was 5.2×10^{10} J.

3. Use this equation:

$$Q_h = W + Q_c$$

4. Solve for Q_c:

$$Q_h - W = Q_c$$

5. Plug in the numbers:

$$Q_h - W = 5.2 \times 10^{10} - 4.5 \times 10^{10} = 0.7 \times 10^{10} = Q_c$$

So the output heat was 0.7×10^{10} J.

19 **52 percent**

1. Use this equation:

$$\text{Efficiency} = \frac{W}{Q_h}$$

2. Plug in the numbers:

$$\text{Efficiency} = \frac{W}{Q_h} = \frac{4.6 \times 10^7}{8.9 \times 10^7} = 52\%$$

20 **74 percent**

1. Use this equation:

$$\text{Efficiency} = \frac{W}{Q_h}$$

2. Plug in the numbers:

$$\text{Efficiency} = \frac{W}{Q_h} = \frac{8.1 \times 10^7}{10.9 \times 10^7} = 74\%$$

21 **11 percent**

1. Use this equation:

$$\text{Efficiency} = 1 - \frac{T_c}{T_h}$$

2. Plug in the numbers:

$$\text{Efficiency} = 1 - \frac{T_c}{T_h} = 1 - \frac{(273 + 49)}{(273 + 87)} = 11\%$$

22 **11 percent**

1. Use this equation:

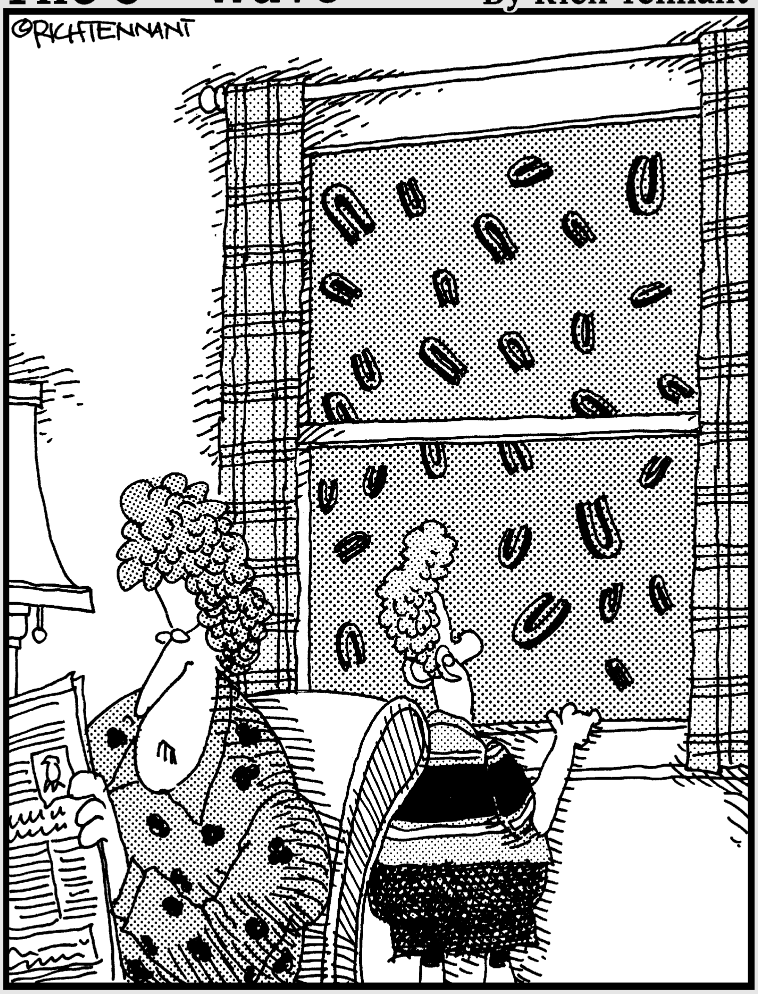
$$\text{Efficiency} = 1 - \frac{T_c}{T_h}$$

2. Plug in the numbers:

$$\text{Efficiency} = 1 - \frac{T_c}{T_h} = 1 - \frac{(273 + 29)}{(273 + 67)} = 11\%$$

Part V
Zap: Electricity and Magnetism

The 5th Wave By Rich Tennant



"I think what you mean, dear, is a 'magnetic' storm."

In this part . . .

People take electricity and magnetism for granted, and few know how to solve problems involving them. That's what this part is for — to clear up the mysteries surrounding both electricity and magnetism. You can try your hand at practice problems involving electricity and magnetism individually and together in the form of light.

Chapter 15

Static Electricity: Electrons at Rest

In This Chapter

- Working with electric charges
- Measuring forces from charges
- Handling electrical forces as vectors
- Understanding electric fields
- Calculating electric fields in parallel plate capacitors

This chapter is about electrical charges — the kind that go zap, not the kind that flow in wires. It's all about static electricity here. In this chapter, you see how to solve all kinds of problems, from forces between charges to electric field to voltage.

Talking about Electric Charges

The unit of charge in the MKS system is the *Coulomb,* symbol C. Electric charges are made up of subatomic charges such as electrons, and a Coulomb contains many, many electrons. Each electron has this charge, known as e:

$$e = 1.6 \times 10^{-19} \text{ C}$$

Charges come in two types — positive and negative, + and –. Electrons have negative charges, and protons have positive charges, for example.

Q. How many electrons are in 1 Coulomb?

A. The correct answer is 6.25×10^{18} electrons.

1. You know that the electric charge of an electron is

$$e = 1.6 \times 10^{-19} \text{ C}$$

2. To find the total number of electrons in 1.0 Coulomb, divide by the charge of a single electron:

$$\text{Number} = \frac{1.0}{1.6 \times 10^{-19} \text{ C}} = 6.25 \times 10^{18} \text{ electrons}$$

1. How many electrons are in 0.60 Coulomb?

Solve It

2. How many electrons are in 800.0 Coulombs?

Solve It

Getting Forceful with Charges

A force exists between electrical charges. Say you have two charges, q_1 and q_2, and they are a distance r apart. The force pulling them together (if the charges have opposite signs, + and –) or forcing them apart (if the charges have the same sign, + and + or – and –) is

$$F = \frac{k \cdot q_1 \cdot q_2}{r^2}$$

What is k? You can measure this constant, and its value is 8.99×10^9 N-m^2/C^2.

Sometimes, you also see this equation written using a constant ε_o, which is called the *permittivity of free space,* like this:

$$F = \frac{q_1 \cdot q_2}{4\pi\varepsilon_o \cdot r^2}$$

The constant ε_o has the value 8.854×10^{-12} C^2/(N-m^2).

EXAMPLE

Q. What is the force between 2 Coulombs of positive charge 1 km apart?

A. The correct answer is 8.99×10^3 N away from each other.

1. Use this equation:

$$F = \frac{k \cdot q_1 \cdot q_2}{r^2}$$

2. Plug in the numbers:

$$F = \frac{k \cdot q_1 \cdot q_2}{r^2} = \frac{(8.99 \times 10^9) \cdot (1.0) \cdot (1.0)}{(1000)^2} =$$

$$8.99 \times 10^3 \text{ N}$$

Because the charges have the same sign, the force pushes them apart.

3. What is the force between an electron and a proton 1.0×10^{-8} m apart?

Solve It

4. What is the force between two protons 1 cm apart?

Solve It

5. What is the force between a charge of 1.0×10^{-3} Coulombs and a charge of -3.0×10^{-3} Coulombs 1.0 m apart?

Solve It

6. What is the force between a charge of 1 Coulomb and a charge of 2 Coulombs, both positive, 10.0 m apart?

Solve It

Electrical Forces Are Vectors

Take a look at Figure 15-1, which shows three charges: two negative and one positive. The negative charges are at (–1.0, 0.0) and (1.0, 0.0), and the positive charge is at (0.0, 1.0), where all distances are in centimeters.

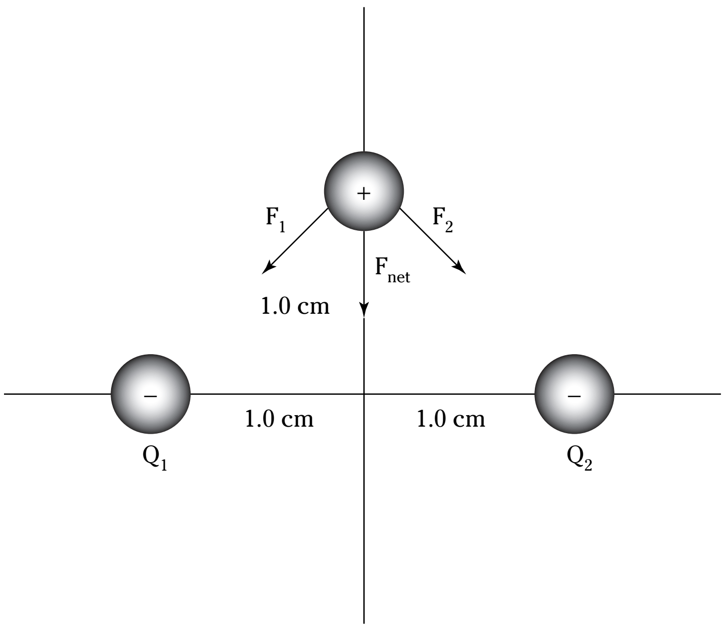


Figure 15-1: Three charges.

Because the force between electrical charges is a vector, those forces add as vectors. You might try to figure out the net force on the positive charge in the figure, for example.

EXAMPLE

Q. What is the net force on the positive charge if all charges have the magnitude 1.0×10^{-3} Coulombs?

A. The correct answer is 6.35×10^{7} N downward.

1. As you can see in the figure $F_1 = F_2$, so:

$$F_{net} = 2\ F_1 \cos(45°) = \sqrt{2}\ F_1$$

2. Solve for F_1:

$$F_1 = \frac{kQ_1Q}{r^2} = \frac{(8.99 \times 10^9)(1.0 \times 10^{-3})(1.0 \times 10^{-3})}{(0.01^2 + 0.01^2)} =$$
$$4.5 \times 10^7\ N$$

3. So that makes F_{net}:

$$F_{net} = 2\ F_1 \cos(45°) = \sqrt{2}\ F_1 =$$
$$\sqrt{2}\ 4.5 \times 10^7 = 6.35 \times 10^7\ N$$

As you can see in the figure, F_{net} points downward.

7. If the charges in Figure 15-1 all have the magnitude 1.0×10^{-8} C, what is the net force on the positive charge?

Solve It

8. If the charges in Figure 15-1 are two electrons and a proton, what is the net force on the proton?

Solve It

Force at a Distance: Electric Fields

The electric field at a particular location is defined as the force felt per Coulomb at that location. Here's the equation for electric field (note that it's a vector):

$$\mathbf{E} = \frac{\mathbf{F}}{q}$$

What is the electric field from a single item of charge (often called a *point charge* because its physical dimensions are negligible), such as with a single electron? You know that for a point charge Q, the force felt by a charge q distance r away is

$$F = \frac{k \cdot q \cdot Q}{r^2}$$

What's the electric field from this point charge? Just divide by the magnitude of your test charge, q, to get the force per Coulomb:

$$E = \frac{F}{q} = \frac{k \cdot Q}{r^2}$$

What are the units? You have force divided by Coulombs, so the units are N/C.

An electric field is drawn as a field of vectors, as you can see in Figure 15-2. It points inward — toward the point charge — for negative charges and outward for positive charges.

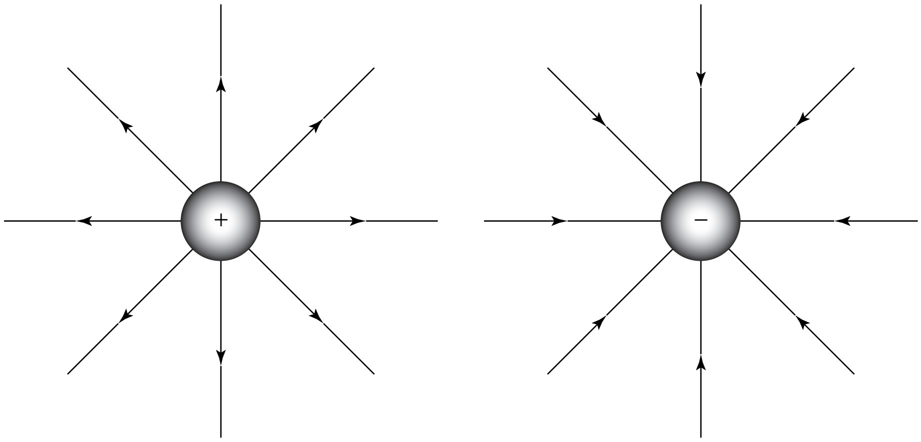


Figure 15-2: Electric fields from point charges.

EXAMPLE

Q. What is the electric field 1.0 cm away from a proton?

A. The correct answer is 1.4×10^{-5} N/C radially outward.

1. Use this equation:

$$E = \frac{kQ}{r^2}$$

2. Plug in the numbers:

$$E = \frac{k \cdot Q}{r^2} = \frac{(8.99 \times 10^{9}) \cdot (1.6 \times 10^{-19})}{(0.01)^2} =$$

1.4×10^{-5} N/C

The electric field of a positive charge extends radially outward.

9. What is the electric field from an electron 1.0 cm away from it?

Solve It

10. What is the electric field from a point charge of –1.0 C 1.0 m away from it?

Solve It

Easy Electric Field: Parallel Plate Capacitors

The electric field caused by point charges is a little difficult to work with because it's not constant in space. That's why you often see problems involving parallel plate capacitors, as shown in Figure 15-3, because between the plates of a parallel plate capacitor the electric field is indeed constant. Parallel plate capacitors are typically made of metal plates, each of which holds a charge — one plate positive and the other negative. Capacitors like this can store separated charge, and the electric field between them is constant.

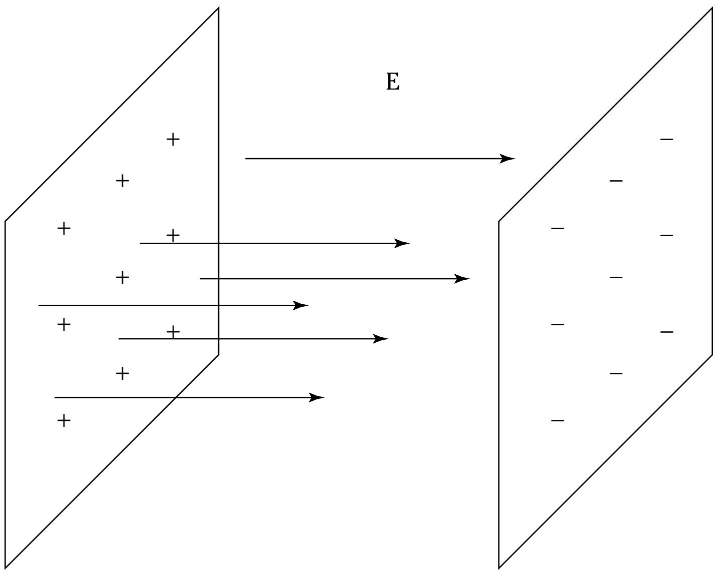


Figure 15-3: A parallel plate capacitor.

$$E = \frac{q}{\varepsilon_o A}$$

Where ε_o is the permittivity of free space, $8.854 \times 10^{-12}\ C^2/(n\text{-}m^2)$, q is the total charge on either of the plates (one plate has charge +q and the other –q), and A is the area of each plate. This is also often written in terms of the *charge density,* σ, on each plate, where $\sigma = q/A$ (the charge per square meter), and here's what charge density makes this equation look like:

$$E = \frac{q}{\varepsilon_o A} = \frac{\sigma}{\varepsilon_o}$$

This electric field is constant between the plates and points from the positive plate toward the negative one.

EXAMPLE

Q. You have a parallel plate capacitor with plates of $1.0\ m^2$, and the magnitude of charge on each plate is 1.0 C. What is the electric field between the plates?

A. The correct answer is 1.1×10^{11} N/C from the positive plate toward the negative one.

1. Use this equation:

$$E = \frac{q}{\varepsilon_o A} = \frac{\sigma}{\varepsilon_o}$$

2. Plug in the numbers:

$$E = \frac{q}{\varepsilon_o A} = \frac{1.0}{(8.854 \times 10^{-12})(1.0)} = 1.1 \times 10^{11}\ N/C$$

The electric field points from the positive plate toward the negative one.

11. You have a parallel plate capacitor with plates of 1.5 m^2, and the magnitude of charge on each plate is 2.0 C. What is the electric field between the plates?

Solve It

12. You have a parallel plate capacitor with plates of 2.3 m^2, and the magnitude of charge on each plate is 0.75 C. What is the electric field between the plates?

Solve It

13. You have a parallel plate capacitor with plates of 1.0 m^2, and the magnitude of charge on each plate is 2.0 C. What is the force on an electron between the plates?

Solve It

14. You have a parallel plate capacitor with plates of 1.3 m^2, and the magnitude of charge on each plate is 6.7 C. What is the force on an electron between the plates?

Solve It

Ramping Up Some Voltage

Electric potential, called *voltage* (and, as you already know, measured in Volts), is the work needed to move a charge from the negative side of an electric field to the positive side, divided by the amount of the charge you're moving. The electic potential is the potential energy gained per Coulomb gained by moving from the negative side to the positive side.

The work to move a charge q from the negative plate a distance s toward the positive plate in a parallel plate capacitor is

$$W = F \cdot s = q \cdot E \cdot s$$

And you know that $F = q \cdot E$, so:

$$W = F \cdot s = q \cdot E \cdot s$$

That work becomes the charge's potential energy, so the electric potential at that location is

$$V = \frac{PE}{q} = \frac{W}{q} = E \cdot s$$

For a parallel plate capacitor, you already know that:

$$E = \frac{q}{\varepsilon_o A}$$

And the voltage between the plates separated by a distance s is

$$V = E \cdot s$$

So:

$$V = \frac{q \cdot s}{\varepsilon_o A}$$

That's the voltage between two plates of area A and charge q, separation s.

EXAMPLE

Q. You have a parallel plate capacitor with plates of 1 m^2, 1.0 m apart, and the magnitude of charge on each plate is 1.0 C. What is the voltage between the plates?

A. The correct answer is 1.1×10^{11} Volts.

1. Use this equation:

$$V = \frac{q \cdot s}{\varepsilon_o A}$$

2. Plug in the numbers:

$$V = \frac{q \cdot s}{\varepsilon_o A} = \frac{(1.0)(1.0)}{(8.854 \times 10^{-12})(1.0)} = 1.1 \times 10^{11} \text{ Volts}$$

15. You have a parallel plate capacitor with plates of 1.0 m^2, 10.0 m apart, and the magnitude of charge on each plate is 0.6 C. What is the voltage between the plates?

Solve It

16. You have a parallel plate capacitor with plates of 2.0 m^2, 3.0 m apart, and the magnitude of charge on each plate is 0.10 C. What is the voltage between the plates?

Solve It

17. You have a parallel plate capacitor with plates of 2.0 m^2, 2.0 m apart, and the magnitude of charge on each plate is 0.20 C. How much work does it take to move 0.10 C of charge from the negative to the positive plate?

Solve It

18. You have a parallel plate capacitor with plates of 1.0 m^2, 1.3 m apart, and the magnitude of charge on each plate is 0.15 C. How much work does it take to move 0.30 C of charge from the negative to the positive plate?

Solve It

Electric Potential from Point Charges

What's the potential some distance away from a test charge? You know that in general:

$$V = E \cdot s$$

For a point charge Q, however, the electric field isn't constant as it is between the plates of a capacitor. You know that the force on a test charge q from point charge Q is equal to:

$$F = \frac{k \cdot Q \cdot q}{r^2}$$

And you know that the electric field around a point charge Q is equal to:

$$E = \frac{kQ}{r^2}$$

At an infinite distance away from the point charge, the potential will be zero. As you bring a test charge closer, to a point r away from the point charge, you have to add up all the work you do and then divide by the size of the test charge. That ends up looking like this:

$$V = \frac{k \cdot Q}{r}$$

That equation is the electric potential, measured in volts, at any point a distance r from a point charge of charge Q, where zero potential is at $r = \infty$. It's the work needed to bring a test charge from ∞ to the distance r from the point charge, divided by the magnitude of the test charge.

EXAMPLE

Q. What is the electric potential 1.0 m from a point charge of 1.0 C, assuming that the potential is zero at $r = \infty$?

A. The correct answer is 8.99×10^9 Volts.

1. Use this equation:

$$V = \frac{k \cdot Q}{r}$$

2. Plug in the numbers:

$$V = \frac{k \cdot Q}{r} = \frac{(8.99 \times 10^9) \cdot (1.0)}{(1.0)} = 8.99 \times 10^9 \text{ Volts}$$

19. What is the electric potential 100.0 m from a point charge of 0.10 C, assuming that the potential is zero at $r = \infty$?

Solve It

20. What is the electric potential 12.0 m from a point charge of 0.40 C, assuming that the potential is zero at $r = \infty$?

Solve It

21. How much work does it take to bring a 0.01 C charge from ∞ to 1.0 m from a 5.0 C charge?

Solve It

22. How much work does it take to bring a 0.020 C charge from ∞ to 4.0 m from a 10.0 C charge?

Solve It

Answers to Problems about Static Electricity

The following are the answers to the practice questions presented earlier in this chapter. You see how to work out each answer, step by step.

1 **3.75×10^{18} electrons**

1. Use this equation:

$$e = 1.6 \times 10^{-19}\ \text{C}$$

2. Plug in the numbers:

$$\text{Number} = \frac{0.6}{1.6 \times 10^{-19}\ \text{C}} = 3.75 \times 10^{18}$$

2 **5.0×10^{21} electrons**

1. Use this equation:

$$e = 1.6 \times 10^{-19}\ \text{C}$$

2. Plug in the numbers:

$$\text{Number} = \frac{800}{1.6 \times 10^{-19}\ \text{C}} = 5.0 \times 10^{21}$$

3 **2.3×10^{-12} N pulling them together**

1. Use this equation:

$$F = \frac{k \cdot q_1 \cdot q_2}{r^2}$$

2. Plug in the numbers:

$$F = \frac{k \cdot q_1 \cdot q_2}{r^2} = \frac{(8.99 \times 10^9) \cdot (1.6 \times 10^{-19}) \cdot (1.6 \times 10^{-19})}{(1.0 \times 10^{-8})^2} = 2.3 \times 10^{-12}\ \text{N}$$

Because the charges have opposite signs, the force pulls them together.

4 **2.3×10^{-24} N pushing them apart**

1. Use this equation:

$$F = \frac{k \cdot q_1 \cdot q_2}{r^2}$$

2. Plug in the numbers:

$$F = \frac{k \cdot q_1 \cdot q_2}{r^2} = \frac{(8.99 \times 10^9) \cdot (1.6 \times 10^{-19}) \cdot (1.6 \times 10^{-19})}{(1.0 \times 10^{-2})^2} = 2.3 \times 10^{-24}\ \text{N}$$

Because the charges have the same signs, the force pushes them apart.

5 **–27,000 N pulling them together**

1. Use this equation:

$$F = \frac{k \cdot q_1 \cdot q_2}{r^2}$$

2. Plug in the numbers:

$$F = \frac{k \cdot q_1}{r^2} = \frac{(8.99 \times 10^9) \cdot (1.0 \times 10^{-3}) \cdot (-3.0 \times 10^{-3})}{(1.0)^2} = -27{,}000\ \text{N}$$

Because the charges have opposite signs, the force pulls them together.

6 **1.8×10^8 N forcing the charges apart**

1. Use this equation:

$$F = \frac{k \cdot q_1 \cdot q_2}{r^2}$$

2. Plug in the numbers:

$$F = \frac{k \cdot q_1 \cdot q_2}{r^2} = \frac{(8.99 \times 10^9) \cdot (1.0) \cdot (2.0)}{(10.0)^2} = 1.8 \times 10^8 \text{ N}$$

Because the charges have the same signs, the force pushes them apart.

7 **6.36×10^{-3}N downward**

1. Add vectors:

$$F_{net} = 2\ F_1 \cos(45°) = \sqrt{2} \cdot F_1$$

2. Solve for F_1:

$$F_1 = \frac{k \cdot Q_1 \cdot Q_2}{r^2} = \frac{(8.99 \times 10^9)(1.0 \times 10^{-8})(3.0 \times 10^{-8})}{(0.01^2 + 0.01^2)} = 4.50 \times 10^{-3} \text{ N}$$

3. Solve for F_{net}:

$$F_{net} = 2\ F_1 \cos(45°) = \sqrt{2} \cdot F_1 = 6.36 \times 10^{-3} \text{ N}$$

The vector sum of the forces is downward.

8 **1.63×10^{-24} N downward**

1. Add vectors:

$$F_{net} = 2\ F_1 \cos(45°) = \sqrt{2} \cdot F_1$$

2. Solve for F_1:

$$F_1 = \frac{k \cdot Q_1 \cdot Q_2}{r^2} = \frac{(8.99 \times 10^9)(1.6 \times 10^{-19})(1.6 \times 10^{-19})}{(0.01^2 + 0.01^2)} = 1.15 \times 10^{-24} \text{ N}$$

3. Solve for F_{net}:

$$F_{net} = 2\ F_1 \cos(45°) = \sqrt{2} \cdot F_1 = 1.63 \times 10^{-24} \text{ N}$$

The vector sum of the forces is downward.

9 **Magnitude 1.4×10^{-5} N/C, pointing toward the charge**

1. Use this equation:

$$E = \frac{kQ}{r^2}$$

2. Solve for E:

$$E = \frac{k \cdot Q}{r^2} = \frac{(8.99 \times 10^9) \cdot (-1.6 \times 10^{-19})}{(0.01)^2} = -1.4 \times 10^{-5} \text{ N/C}$$

The electric field of a negative charge points radially inward, toward the charge.

10 **Magnitude 8.99 × 10^9 N/C, pointing toward the charge**

1. Use this equation:

$$E = \frac{kQ}{r^2}$$

2. Solve for E:

$$E = \frac{k \cdot Q}{r^2} = \frac{(8.99 \times 10^9) \cdot (-1.0)}{(1.0)^2} = -8.99 \times 10^9 \text{ N/C}$$

The electric field of a negative charge points radially inward, toward the charge.

11 **Magnitude 1.5 × 10^{11} N/C, from the positive plate toward the negative one**

1. Use this equation:

$$E = \frac{q}{\varepsilon_o A} = \frac{\sigma}{\varepsilon_o}$$

2. Plug in the numbers:

$$E = \frac{q}{\varepsilon_o A} = \frac{2.0}{(8.854 \times 10^{-12}) \cdot (1.5)} = 1.5 \times 10^{11} \text{ N/C}$$

The electric field points from the positive plate toward the negative one.

12 **Magnitude 3.7 × 10^{10} N/C, from the positive plate toward the negative one**

1. Use this equation:

$$E = \frac{q}{\varepsilon_o A} = \frac{\sigma}{\varepsilon_o}$$

2. Plug in the numbers:

$$E = \frac{q}{\varepsilon_o A} = \frac{0.75}{(8.854 \times 10^{-12}) \cdot (2.3)} = 3.7 \times 10^{10} \text{ N/C}$$

The electric field points from the positive plate toward the negative one.

13 **Magnitude 3.5 × 10^{-8} N, toward the positive plate**

1. Use this equation:

$$E = \frac{q}{\varepsilon_o A} = \frac{\sigma}{\varepsilon_o}$$

2. Plug in the numbers:

$$E = \frac{q}{\varepsilon_o A} = \frac{2.0}{(8.854 \times 10^{-12})(1.0)} = 2.2 \times 10^{11} \text{ N/C}$$

3. Use this equation to find the force on the electron:

$$F = qE$$

4. Plug in the numbers:

$$F = qE = (1.6 \times 10^{-19})\ (2.2 \times 10^{11}) = 3.5 \times 10^{-8} \text{ N}$$

The force on the negatively charged electron points toward the positive plate.

14 **Magnitude 9.32×10^{-8} N, toward the positive plate**

1. Use this equation:

$$E = \frac{q}{\varepsilon_o A} = \frac{\sigma}{\varepsilon_o}$$

2. Plug in the numbers:

$$E = \frac{q}{\varepsilon_o A} = \frac{6.7}{(8.854 \times 10^{-12})(1.3)} = 5.8 \times 10^{11} \text{ N/C}$$

3. Use this equation to find the force on the electron:

$$F = qE$$

4. Plug in the numbers:

$$F = qE = (1.6 \times 10^{-19})\,(5.8 \times 10^{11}) = 9.32 \times 10^{-8} \text{ N}$$

The force on the negatively charged electron points toward the positive plate.

15 **6.8×10^{11} Volts**

1. Use this equation:

$$V = \frac{q \cdot s}{\varepsilon_o A}$$

2. Plug in the numbers:

$$V = \frac{q \cdot s}{\varepsilon_o A} = \frac{(0.6) \cdot (10.0)}{(8.854 \times 10^{-12}) \cdot (1.0)} = 6.8 \times 10^{11} \text{ Volts}$$

16 **1.7×10^{10} Volts**

1. Use this equation:

$$V = \frac{q \cdot s}{\varepsilon_o A}$$

2. Plug in the numbers to find the voltage between the plates:

$$V = \frac{q \cdot s}{\varepsilon_o A} = \frac{(0.1) \cdot (3.0)}{(8.854 \times 10^{-12}) \cdot (2.0)} = 1.7 \times 10^{10} \text{ Volts}$$

17 **2.3×10^{9} J**

1. Use this equation:

$$V = \frac{q \cdot s}{\varepsilon_o A}$$

2. Plug in the numbers to find the voltage between the plates:

$$V = \frac{q \cdot s}{\varepsilon_o A} = \frac{(0.2) \cdot (2.0)}{(8.854 \times 10^{-12}) \cdot (2.0)} = 2.3 \times 10^{10} \text{ Volts}$$

3. Use this equation:

$$V = \frac{W}{q}$$

4. Solve for W:

$$W = q \cdot V$$

5. Plug in the numbers:

$$W = q \cdot V = (0.1) \cdot (2.3 \times 10^{10}) = 2.3 \times 10^{9} \text{ J}$$

18 **6.6×10^9 J**

1. Use this equation:

$$V = \frac{q \cdot s}{\varepsilon_o A}$$

2. Plug in the numbers to find the voltage between the plates:

$$V = \frac{q \cdot s}{\varepsilon_o A} = \frac{(0.15) \cdot (1.3)}{(8.854 \times 10^{-12}) \cdot (1.0)} = 2.2 \times 10^{10} \text{ Volts}$$

3. Use this equation:

$$V = \frac{W}{q}$$

4. Solve for W:

$$W = q \cdot V$$

5. Plug in the numbers:

$$W = q \cdot V = (0.3) \cdot (2.2 \times 10^{10}) = 6.6 \times 10^9 \text{ J}$$

19 **8.99×10^6 Volts**

1. Use this equation:

$$V = \frac{k \cdot Q}{r}$$

2. Plug in the numbers:

$$V = \frac{k \cdot Q}{r} = \frac{(8.99 \times 10^9) \cdot (0.1)}{(100)} = 8.99 \times 10^6 \text{ Volts}$$

20 **3.0×10^8 Volts**

1. Use this equation:

$$V = \frac{k \cdot Q}{r}$$

2. Plug in the numbers:

$$V = \frac{k \cdot Q}{r} = \frac{(8.99 \times 10^9) \cdot (0.4)}{(12)} = 3.0 \times 10^8 \text{ Volts}$$

21 **4.5×10^8 J**

1. Use this equation:

$$V = \frac{k \cdot Q}{r}$$

2. Plug in the numbers:

$$V = \frac{k \cdot Q}{r} = \frac{(8.99 \times 10^9) \cdot (5)}{(1.0)} = 4.5 \times 10^{10} \text{ Volts}$$

3. Use this equation:

$$V = \frac{W}{q}$$

4. Solve for W:

$$W = q \cdot V$$

5. Plug in the numbers:

$$W = q \cdot V = (0.01)\ (4.5 \times 10^{10}) = 4.5 \times 10^8 \text{ J}$$

22 **4.5×10^8 J**

1. Use this equation:

$$V = \frac{k \cdot Q}{r}$$

2. Plug in the numbers:

$$V = \frac{k \cdot Q}{r} = \frac{(8.99 \times 10^9) \cdot (10)}{(4.0)} = 2.25 \times 10^{10} \text{ Volts}$$

3. Use this equation:

$$V = \frac{W}{q}$$

4. Solve for W:

$$W = q \cdot V$$

5. Plug in the numbers:

$$W = q \cdot V = (0.02) \cdot (2.25 \times 10^{10}) = 4.5 \times 10^8 \text{ J}$$

Chapter 16

Electrons in Motion: Circuits

In This Chapter

- Getting to know current
- Handling resistance
- Working with power
- Developing series circuits
- Creating parallel circuits

This chapter is about what happens when you get electrons moving: You get electric current. Electric current runs through wires and has to contend with various items in those wires — resistors, capacitors, and the like. In this chapter, you see how to handle such items yourself.

Electrons in a Whirl: Current

If you have electrons moving through a wire, you have electrical current. *Current,* symbol I, is defined as the amount of charge that passes by in 1 second. One Coulomb per second is named 1 Ampere or just 1 Amp, symbol A. So this equation is how you find current:

$$I = \frac{q}{t}$$

Current is created when you have an electromotive force (EMF) providing a voltage across a conductor. EMF can be supplied by batteries, generators, and similar items in physics problems.

EXAMPLE

Q. How many electrons per second pass by a given point in a wire carrying 2.0 Amps?

A. **1.25×10^{19}**

1. Use this equation:

$$I = \frac{q}{t}$$

2. Solve for q:

$$q = I \cdot t$$

3. Plug in the numbers to find the total charge in 1 second:

$$q = (2.0) \cdot (1.0) = 2.0 \text{ C}$$

4. Divide by the charge per electron to find the number of electrons:

$$\text{Number} = \frac{2.0}{1.6 \times 10^{-19}} = 1.25 \times 10^{19}$$

1. How many electrons per second pass by a given point in a wire carrying 6.0 Amps?

Solve It

2. What total charge passes by a given point in a circuit carrying 12.0 Amps in 1 minute?

Solve It

Giving You Some Resistance: Ohm's Law

When you have an EMF providing voltage across a wire, how much current flows? The answer turns out to depend on how much resistance, symbol R, is in the wire. This equation shows the relationship among voltage, current, and resistance:

$$V = I \cdot R$$

Resistance is measured in ohms, symbol ω.

At the top of Figure 16-1, you see a battery with an EMF of 6.0 Volts. The current flows from the positive terminal of the battery — corresponding to the longer of the two lines in the battery symbol — to the negative terminal of the battery.

At the bottom of the figure is a resistor, R. How much current flows through the resistor? You know the answer from the equation $V = I \cdot R$.

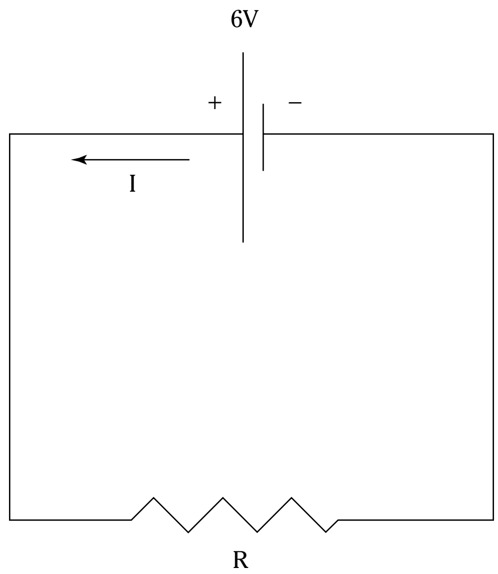


Figure 16-1: A resistor and a battery in a circuit.

EXAMPLE

Q. If the resistor in Figure 16-1 is 3.0 ω, what current flows through it, driven by the 6.0 V battery?

A. **2.0 Amps**

1. Use this equation:

$$V = I \cdot R$$

2. Solve for I:

$$I = \frac{V}{R}$$

3. Plug in the numbers:

$$I = \frac{V}{R} = \frac{6.0}{3.0} = 2.0 \text{ A}$$

---

3. If the resistor in Figure 16-1 is 1.5 ω, what current flows through it, driven by the 6.0 V battery?

Solve It

4. If the resistor in Figure 16-1 is 0.5 ω, what current flows through it, driven by the 6.0 V battery?

Solve It

5. What's the voltage across a 3.0 ω resistor with a current of 1.0 A going through it?

Solve It

6. What's the voltage across a 5.0 ω resistor with a current of 1.5 A going through it?

Solve It

Powering It Up

Notice how light bulbs get hot? That happens because the filament in the light bulb acts as a resistor — and resistors dissipate heat. How much heat does a resistor give off? This equation is how to calculate the answer in terms of current or voltage:

$$P = I \cdot V = \frac{V^2}{R} = I^2 \cdot R$$

EXAMPLE

Q. If a 300.0 ω resistor has a current of 1.0 Amps going through it, how much power does it turn into heat?

A. **300 Watts**

1. Use this equation:

$$P = I^2 \cdot R$$

2. Plug in the numbers:

$$P = I^2 \cdot R = (1.0^2) \cdot (300) = 300 \text{ Watts}$$

7. If a 500.0 ω resistor has a current of 2.0 Amps going through it, how much power does it turn into heat?

Solve It

8. If a 2000.0 ω resistor has a current of 1.5 Amps going through it, how much power does it turn into heat?

Solve It

One after the Other: Series Circuits

What if you have several resistors, one after another, in a circuit, as you see in Figure 16-2?

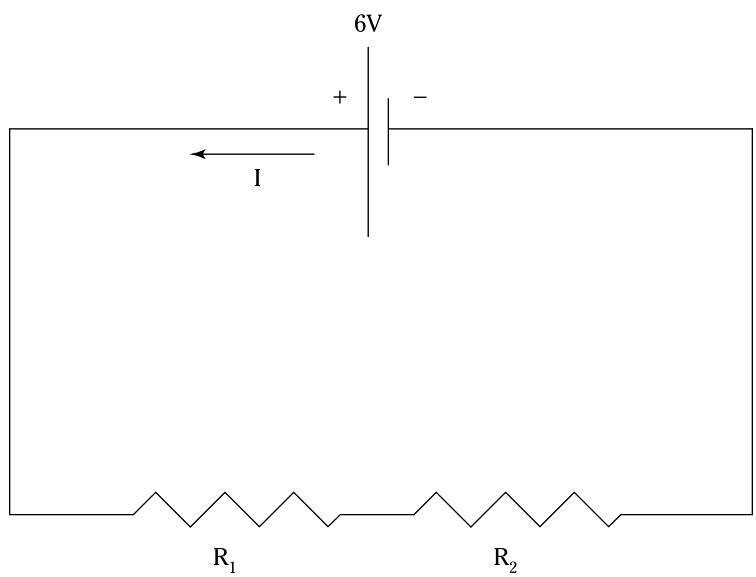


Figure 16-2: Resistors in series.

When you have resistors arranged this way, so that the current has to go through both of them, the arrangement is called *placing resistors in series.* To find the net resistance, you just add the two individual resistances:

$$R = R_1 + R_2$$

EXAMPLE

Q. If R_1 is 20 ω, and R_2 is 40 ω, what is the current flowing in Figure 16-2, in which the battery is 6.0 V?

A. **0.1 A**

1. Use this equation:

$$R = R_1 + R_2$$

2. Plug in the numbers:

$$R = R_1 + R_2 = 60\ \omega$$

3. Use this equation to find the current:

$$V = I \cdot R$$

4. Solve for I:

$$I = V/R$$

5. Plug in the numbers:

$$I = V/R = 6.0/60 = 0.1\ A$$

9. If R_1 is 60 ω, and R_2 is 100 ω, what is the current flowing in Figure 16-2, in which the battery is 6.0 V?

Solve It

10. If R_1 is 100 ω, and R_2 is 200 ω, what is the current flowing in Figure 16-2, in which the battery is 6.0 V?

Solve It

11. If R_1 is 100 ω, and R_2 is 200 ω, what is the voltage across R_1, in which the battery is 6.0 V?

Solve It

12. If R_1 is 100 ω, and R_2 is 200 ω, what is the voltage across R_2, in which the battery is 6.0 V?

Solve It

All for One: Parallel Circuits

Say you have resistors connected as you see in Figure 16-3, so that the current flowing can go through either resistor.

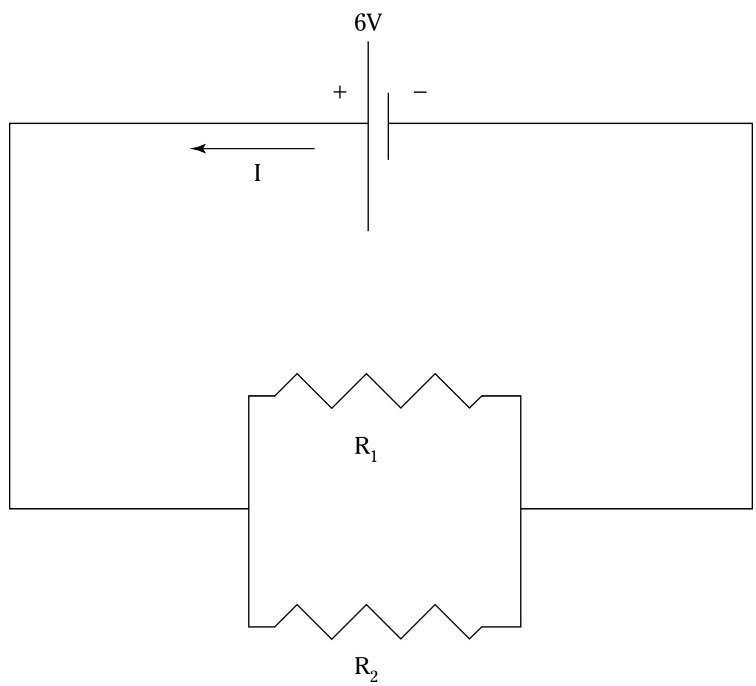


Figure 16-3: Resistors in parallel.

This arrangement is called *placing resistors in parallel.* Unlike resistors in series, in which the current has to go through the resistors one after another, in this arrangement part of the current goes through one resistor and part through the other.

What's the total resistance of two resistors in parallel? This equation shows you:

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

Q. If R_1 is 20 ω, and R_2 is 60 ω, what is the current flowing in Figure 16-3, in which the battery is 6.0 V?

A. **0.4 A**

1. Use this equation:

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

2. Plug in the numbers:

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{20} + \frac{1}{60} = 0.066$$

3. Find R, the total resistance:

$$R = \frac{1}{0.066} = 15\ \omega$$

4. Use this equation:

$$V = I \cdot R$$

5. Solve for I:

$$I = V/R$$

6. Plug in the numbers:

$$I = V/R = 6.0/15 = 0.4\ A$$

13. If R_1 is 30 ω, and R_2 is 90 ω, what is the current flowing in Figure 16-3, in which the battery is 6.0 V?

Solve It

14. If R_1 is 45 ω, and R_2 is 120 ω, what is the current flowing in Figure 16-3, where in which the battery is 6.0 V?

Solve It

15. If R_1 is 39 ω, and R_2 is 93 ω, what is the current flowing in Figure 16-3, in which the battery is 6.0 V?

Solve It

16. If R_1 is 42 ω, and R_2 is 56 ω, what is the current flowing in Figure 16-3, in which the battery is 6.0 V?

Solve It

The Whole Story: Kirchhoff's Rules

The circuit in Figure 16-4 is a whopper. What's I_1? What's I_2? Kirchhoff's laws make everything clear.

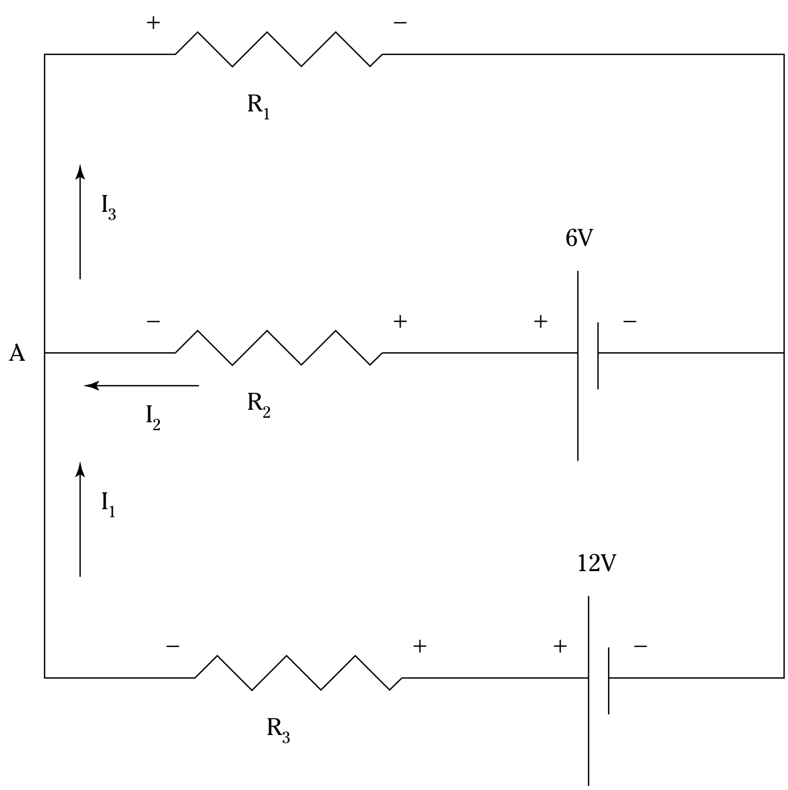


Figure 16-4: A circuit.

Following are Kirchhoff's laws for a circuit:

- **Junction rule.** The junction rule says that the total current going into any point in a circuit must equal the total current going out of that point. So $\Sigma I = 0$ at any point.
- **Loop rule.** The loop rule says that around any closed loop in a circuit, the sum of potential rises (as from a battery) must equal the sum of the potential drops (as from resistors). So $\Sigma V = 0$ around any loop.

An easy way of summing the voltages around a loop is to draw in the current arrows (it's no problem if you draw a current arrow backward; the current will just come out negative) and label each resistor with a + where the current goes in and a – where it comes out. Then go around the loop (either direction is fine), adding a +V if you encounter a battery's positive terminal first or a –V if you encounter its negative terminal first, and a +IR or –IR for each resistor, depending on whether you encounter the + or – signs you added to each resistor first. Then set that whole expression to zero, such as $-6V + IR_1 + 12V + IR_2 = 0$.

Q. If R_1 is 2.0 ω, R_2 is 4.0 ω, and R_2 is 6.0 ω, what are the three currents in Figure 16-4, in which the batteries are 6.0 V and 12.0 V?

A. **I_1 = 1.36 Amps, I_2 = 0.55 Amps, I_3 = 1.91 Amps**

1. The junction rule says that $\Sigma I = 0$ at any point, so use point A at left in the figure. I_1 and I_2 flow into it, and I_3 flows out of it, so:

$$I_1 + I_2 = I_3$$

2. The loop rule says that $\Sigma V = 0$. Figure 16-4 has three loops: the two internal loops and the external overall loop. Because you have three unknowns — I_1, I_2, and I_3 — all you need are three equations, and the $\Sigma I = 0$ rule has already given you one. So go around the two internal loops to get two more equations. From the top loop, you get

$$+6 - 2I_3 - 4I_2 = 0$$

3. From the bottom loop, you get

$$+12 - 6 + 4I_2 - 6I_1 = 0$$

4. Now you have three equations in three unknowns:

$$I_1 + I_2 = I_3$$

$$+6 - 2I_3 - 4I_2 = 0$$

$$+12 - 6 + 4I_2 - 6I_1 = 0$$

5. If you substitute the top equation for I_3 in the second equation, you get

$$+6 - 2\cdot(I_1 + I_2) - 4I_2 = 0$$

$$+12 - 6 + 4I_2 - 6I_1 = 0$$

So:

$$+6 - 2I_1 - 6I_2 = 0$$

$$+12 - 6 + 4I_2 - 6I_1 = 0$$

6. You can find I_1 in terms of I_2 by using the first equation:

$$I_1 = 3 - 3I_2$$

7. Then you can substitute this value of I_1 in the second equation to get

$$+12 - 6 + 4I_2 - 6\,(3 - 3I_2) = 0$$

So:

$$-12 + 22I_2 = 0$$

And:

$$I_2 = 12/22 = 6/11 = 0.55 \text{ Amp}$$

8. You now have one of the currents: $I_2 = 6/11$ Amp. Plug that into:

$$\frac{6}{11}$$

$$+6 - 2I_3 - 4I_2 = 0$$

Giving you:

$$+6 - 2I_3 - 4\,(6/11) = 0$$

Or, dividing by 2:

$$+3 - I_3 - 12/11 = 0$$

So:

$$I_3 = 21/11 = 1.91 \text{ Amps}$$

9. Now you can find I_1. Start with:

$$I_1 + I_2 = I_3$$

Which means that:

$$I_1 = I_3 - I_2$$

So:

$$I_1 = \frac{21}{11} - \frac{6}{11} = \frac{15}{11} = 1.36 \text{ Amps}$$

17. If R_1 is 3 ω, R_2 is 6 ω, and R_2 is 9 ω, what are the three currents in Figure 16-4, in which the batteries are 6.0 V and 12.0 V?

Solve It

18. If R_1 is 5 ω, R_2 is 10 ω, and R_2 is 15 ω, what are the three currents in Figure 16-4, in which the batteries are 6.0 V and 12.0 V?

Solve It

Answers to Problems about Circuits

The following are the answers to the practice questions presented earlier in this chapter. You'll see the answers worked out, step by step.

1 **3.75×10^{19}**

1. Use this equation:

$$I = \frac{q}{t}$$

2. Solve for q:

$$q = I \cdot t$$

3. Find the total charge passing in 1 second:

$$q = (6.0) \cdot (1.0) = 6.0 \text{ C}$$

4. Divide by the charge of one electron to find the total number:

$$\text{Number} = \frac{6.0}{1.6 \times 10^{-19}} = 3.75 \times 10^{19}$$

2 **720 Coulombs**

1. Use this equation:

$$I = \frac{q}{t}$$

2. Solve for q:

$$q = I \cdot t$$

3. Find the total charge passing in 1 minute:

$$q = (12.0) \cdot (60) = 720 \text{ C}$$

3 **4.0 A**

1. Use this equation:

$$V = I \cdot R$$

2. Solve for I:

$$I = \frac{V}{R}$$

3. Find the current:

$$I = \frac{V}{R} = \frac{6.0}{1.5} = 4.0 \text{ A}$$

4 **12.0 A**

1. Use this equation:

$$V = I \cdot R$$

2. Solve for I:

$$I = \frac{V}{R}$$

3. Find the current:

$$I = \frac{V}{R} = \frac{6.0}{0.5} = 12.0 \text{ A}$$

5 **3.0 V**

1. Use this equation:

$$V = I{\cdot}R$$

2. Plug in the numbers:

$$V = I{\cdot}R = (1.0){\cdot}(3.0) = 3.0\text{ V}$$

6 **7.5 V**

1. Use this equation:

$$V = I{\cdot}R$$

2. Plug in the numbers:

$$V = I{\cdot}R = (1.5){\cdot}(5.0) = 7.5\text{ V}$$

7 **2000 Watts**

1. Use this equation:

$$P = I^2{\cdot}R$$

2. Plug in the numbers:

$$P = I^2{\cdot}R = (2.0^2){\cdot}(500) = 2000\text{ Watts}$$

8 **4500 Watts**

1. Use this equation:

$$P = I^2R$$

2. Plug in the numbers:

$$P = I^2R = (1.5^2)(2000) = 4500\text{ Watts}$$

9 **0.0375 A**

1. Use this equation:

$$R = R_1 + R_2$$

2. Plug in the numbers:

$$R = R_1 + R_2 = 160\ \omega$$

3. Use this equation:

$$V = IR$$

4. Solve for I:

$$I = V/R$$

5. Plug in the numbers:

$$I = V/R = 6.0/160 = 0.0375\text{ A}$$

10 **0.020 A**

1. Use this equation:

$$R = R_1 + R_2$$

2. Plug in the numbers:

$$R = R_1 + R_2 = 300\ \omega$$

3. Use this equation:

$$V = I \cdot R$$

4. Solve for I:

$$I = V/R$$

5. Plug in the numbers:

$$I = V/R = 6.0/300 = 0.02\ A$$

11 **2.0 V**

1. Use this equation:

$$R = R_1 + R_2$$

2. Plug in the numbers:

$$R = R_1 + R_2 = 300\ \omega$$

3. Use this equation:

$$V = I \cdot R$$

4. Solve for I:

$$I = V/R$$

5. Plug in the numbers:

$$I = V/R = 6.0/300 = 0.02\ A$$

6. Use this equation:

$$V_1 = I \cdot R_1$$

7. Plug in the numbers:

$$V_1 = I \cdot R_1 = (0.02)(100) = 2.0\ V$$

12 **4.0 V**

1. Use this equation:

$$R = R_1 + R_2$$

2. Plug in the numbers:

$$R = R_1 + R_2 = 300\ \omega$$

3. Use this equation:

$$V = IR$$

4. Solve for I:

$$I = V/R$$

5. Plug in the numbers:

$$I = V/R = 6.0/300 = 0.02\ A$$

6. Use this equation:

$$V_2 = IR_2$$

7. Plug in the numbers:

$$V_2 = IR_2 = (0.02)(200) = 4.0\ V$$

13 **0.26 A**

1. Use this equation:

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

2. Plug in the numbers:

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} = 0.044$$

3. Find the total resistance:

$$R = \frac{1}{0.044} = 22.5\ \omega$$

4. Use this equation:

$$V = I \cdot R$$

5. Solve for I:

$$I = V/R$$

6. Plug in the numbers:

$$I = V/R = 6.0/22.5 = 0.26\ A$$

14 **0.18 A**

1. Use this equation:

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

2. Plug in the numbers:

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} = 0.03$$

3. Find the total resistance:

$$R = \frac{1}{0.03} = 33\ \omega$$

4. Use this equation:

$$V = IR$$

5. Solve for I:

$$I = V/R$$

6. Plug in the numbers:

$$I = V/R = 6.0/33 = 0.18\ A$$

15 **0.22 A**

1. Use this equation:

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

2. Plug in the numbers:

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} = 0.036$$

3. Find the total resistance:

$$R = \frac{1}{0.036} = 27\ \omega$$

4. Use this equation:

$$V = I{\cdot}R$$

5. Solve for I:

$$I = V/R$$

6. Plug in the numbers:

$$I = V/R = 6.0/27 = 0.22\ A$$

16 **0.25 A**

1. Use this equation:

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

2. Plug in the numbers:

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} = 0.042$$

3. Find the total resistance:

$$R = \frac{1}{0.042} = 24\ \omega$$

4. Use this equation:

$$V = I{\cdot}R$$

5. Solve for I:

$$I = V/R$$

6. Plug in the numbers:

$$I = V/R = 6.0/24 = 0.25\ A$$

17 **I_1 = 30/33 = 0.91 Amp, I_2 = 12/33 = 0.36 Amp, I_3 = 42/33 1.27 Amps**

1. $\Sigma I = 0$ at any point, so use point A at left in Figure 16-4. I_1 and I_2 flow into it, and I_3 flows out of it, so:

$$I_1 + I_2 = I_3$$

2. $\Sigma V = 0$ around a loop. Go around the two internal loops. From the top loop, you get

$$+6 - 3I_3 - 6I_2 = 0$$

3. From the bottom loop, you get

$$+12 - 6 + 6I_2 - 9I_1 = 0$$

4. You have three equations in three unknowns:

$$I_1 + I_2 = I_3$$
$$+6 - 3I_3 - 6I_2 = 0$$
$$+12 - 6 + 6I_2 - 9I_1 = 0$$

5. If you substitute the top equation for I_3 in the second equation, you get

$$+6 - 3(I_1 + I_2) - 6I_2 = 0$$
$$+12 - 6 + 6I_2 - 9I_1 = 0$$

So:

$$+6 - 3I_1 - 9I_2 = 0$$

$$+12 - 6 + 6I_2 - 9I_1 = 0$$

6. You can find I_1 in terms of I_2 by using the first equation:

$$I_1 = 2 - 3I_2$$

7. Then you can substitute this value of I_1 in the second equation, giving you:

$$+12 - 6 + 6I_2 - 9(2 - 3I_2) = 0$$

So:

$$-12 + 33I_2 = 0$$

And:

$$I_2 = 12/33 = 0.36 \text{ Amp}$$

8. Plug I_2 into:

$$+6 - 3I_3 - 6I_2 = 0$$

Giving you:

$$+6 - 3I_3 - 6\ (12/33) = 0$$

Or, dividing by 3:

$$+2 - I_3 - 24/33 = 0$$

So:

$$I_3 = 42/33 = 1.27 \text{ Amps}$$

9. Now you can find I_1. Start with:

$$I_1 + I_2 = I_3$$

Which means that:

$$I_1 = I_3 - I_2$$

So:

$$I_1 = \frac{42}{33} - \frac{12}{33} = \frac{30}{33} = 0.91 \text{ Amp}$$

18 **I_1 = 30/55 = 0.55 Amp, I_2 = 12/55 = 0.22 Amp, I_3 = 42/55 = 0.76 Amp**

1. $\Sigma I = 0$ at any point, so use point A at left in Figure 16-4. I_1 and I_2 flow into it, and I_3 flows out of it, so:

$$I_1 + I_2 = I_3$$

2. $\Sigma V = 0$ around a loop. Go around the two internal loops. From the top loop, you get

$$+6 - 5I_3 - 10I_2 = 0$$

3. From the bottom loop, you get

$$+12 - 6 + 10I_2 - 15I_1 = 0$$

4. You have three equations in three unknowns:

$$I_1 + I_2 = I_3$$

$$+6 - 5I_3 - 10I_2 = 0$$

$$+12 - 6 + 10I_2 - 15I_1 = 0$$

5. If you substitute the top equation for I_3 in the second equation, you get

$$+6 - 5(I_1 + I_2) - 10I_2 = 0$$

$$+12 - 6 + 10I_2 - 15I_1 = 0$$

So:

$$+6 - 5I_1 - 15I_2 = 0$$

$$+12 - 6 + 10I_2 - 15I_1 = 0$$

6. You can find I_1 in terms of I_2 by using the first equation:

$$I_1 = \frac{6}{5} - 3I_2$$

7. Then you can substitute this value of I_1 in the second equation, giving you:

$$+12 - 6 + 10I_2 - 15\left(\frac{6}{5} - 3I_2\right) = 0$$

So:

$$-12 + 55I_2 = 0$$

And:

$$I_2 = 12/55 = 0.22 \text{ Amp}$$

8. Plug I_2 into:

$$+6 - 5I_3 - 10I_2 = 0$$

Giving you:

$$+6 - 5I_3 - 10\ (12/55) = 0$$

Or, dividing by 5:

$$\frac{+6}{5} - I_3 - 24/55 = 0$$

So:

$$I_3 = 42/55 = 0.76 \text{ Amp}$$

9. Now you can find I_1. Start with:

$$I_1 + I_2 = I_3$$

Which means that:

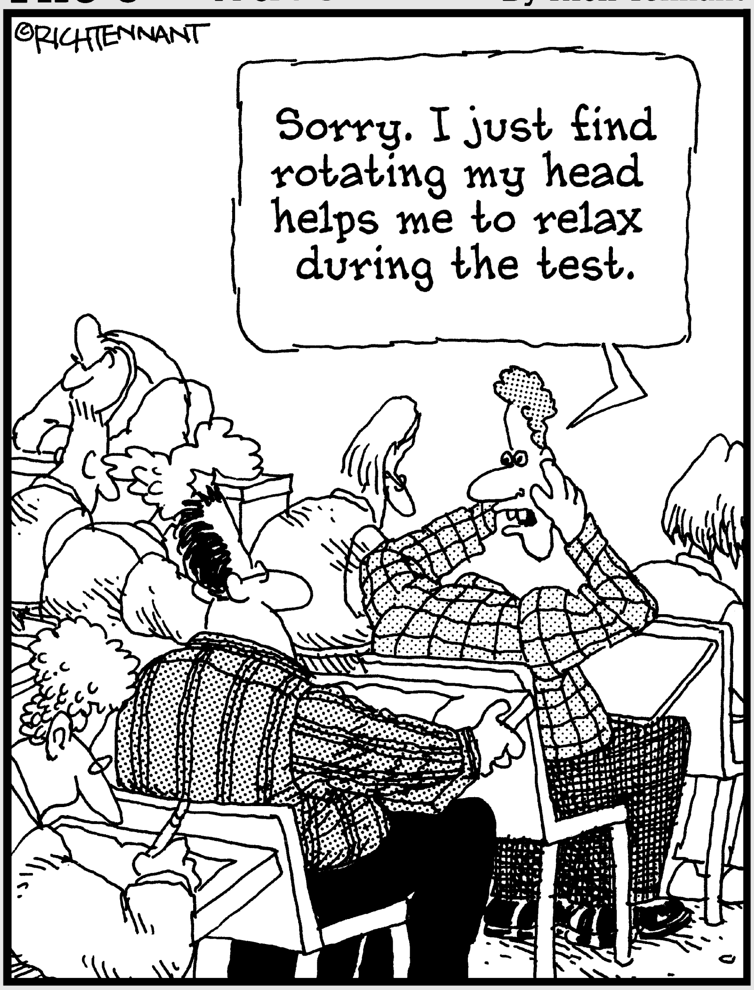
$$I_1 = I_3 - I_2$$

So:

$$I_1 = \frac{42}{55} - \frac{12}{55} = \frac{30}{55} = 0.55 \text{ Amp}$$

Part VI
The Part of Tens

The 5th Wave By Rich Tennant



In this part . . .

This part contains some handy information to give you an edge in your physics studies. I warn you about the ten most-common physics problem-solving mistakes — everything from mixing units to getting Kirchhoff's laws wrong. You also get my ten recommendations of online physics tutorials and study guides.

Chapter 17

Ten Common Mistakes People Make When Solving Problems

In This Chapter

- Handling the common mistakes people make
- Knowing how to spot potential problems

This section discusses the most common errors people make when working out physics problems. In my many years of teaching physics, certain types of problems stand out, and you see them here.

Mixing Units

The most common error made in solving physics problems involves mixing the units from one system with another system. If the problem is given to you in inches, kilograms, and seconds, convert it into a consistent system of units before proceeding to work out the answer. For example, if you want to use the MKS system, convert *everything* into MKS before working out the problem.

Expressing the Answer in the Wrong Units

If the problem asks for the answer in the MKS system, don't give it in CGS units. You'd be surprised at how common an error this is; people are so relieved that they've solved the problem that they goof it up in the last step.

Swapping Radians and Degrees

Degrees are commonly used in physics problems — except when it comes to angular velocity and acceleration. That's when you have to make sure you're working with radians. Use the $180° / \pi$ conversion factor to convert from radians to degrees when needed.

Getting Sines and Cosines Mixed Up

Physics students often make the mistake of interchanging sines and cosines. Take a look at Figure 17-1 and keep the following relationships in mind:

$\sin \theta = y/h$ = opposite/hypotenuse

$\cos \theta = x/h$ = adjacent/hypotenuse

$\tan \theta = y/x$ = opposite/adjacent

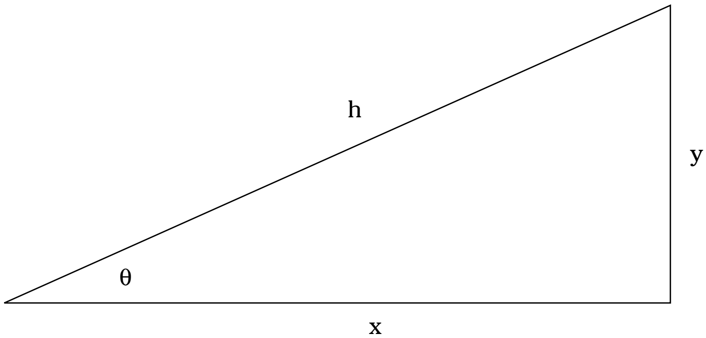


Figure 17-1: A triangle.

Not Treating Vectors as Vectors

When you add vectors, use vector addition. That means resolving vectors into components. Too many people just add the magnitudes of the vectors without realizing that they should be adding components instead.

Neglecting Latent Heat

When you're faced with a problem that involves a phase change, such as from ice to water, don't forget to take the latent heat into account. When ice changes into water, it absorbs latent heat that you have to account for in your solution.

Getting Refraction Angles Wrong

When you deal with refraction problems, make sure you get the angles right; they're measured with respect to a line perpendicular — called the *normal* — to the interface from one medium to the other. Many people incorrectly use the angle between the ray of light and the interface between the two mediums.

Getting the Signs Wrong in Kirchhoff Loops

You use Kirchhoff's laws to solve for the currents in a circuit, but many people run into trouble with Kirchhoff's-laws problems because they get the signs wrong.

To be sure you get the signs right, put in arrows for all the currents. Don't worry about getting the direction wrong for an arrow; if you do, the current will just come out negative. Then put a + sign where the current enters each resistor and a – sign where the current leaves each resistor, as shown in Figure 17-2.

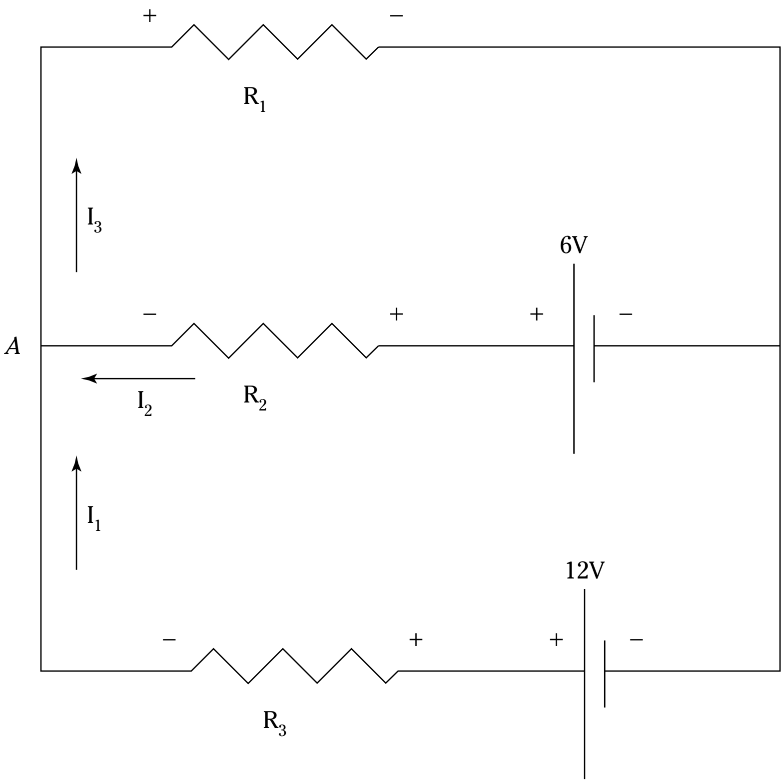


Figure 17-2: A circuit.

Adding Resistors Incorrectly

When you have resistors in series, the current has to pass through one after the other. Here's how you calculate the total resistance of two resistors in series:

$$R = R_1 + R_2$$

When you have two resistors in parallel, the current divides between the two of them, and you add the resistors like this:

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

Lots of people get these two confused — make sure you don't.

Using the Wrong Rays in Ray Diagrams

Ray diagrams are easy to get wrong, because you can easily use the wrong rays.

Bear in mind these rules for ray diagrams for mirrors (see an example in Figure 17-3):

- **Ray 1:** This ray goes from the object, bounces off the mirror, and goes through the center of curvature.
- **Ray 2:** This ray goes horizontally from the object to the mirror, bounces off, and goes through the focal point.
- **Ray 3:** This ray goes from the object through the focal point, bounces off the mirror, and ends up going parallel to the horizontal axis.

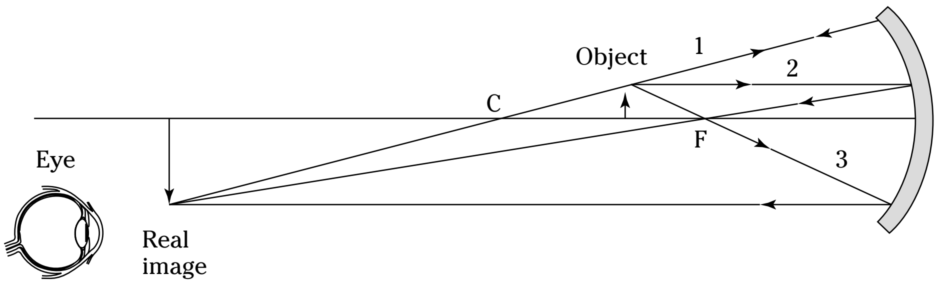


Figure 17-3: Ray diagram for a concave mirror.

And here are the rules for ray diagrams for lenses (see the example in Figure 17-4):

- **Ray 1:** This ray goes from the object through the center of the lens.
- **Ray 2:** This ray goes horizontally from the object to the lens and then goes through the focal point.
- **Ray 3:** This ray goes from the object through the focal point, through the lens, and ends up going parallel to the horizontal axis.

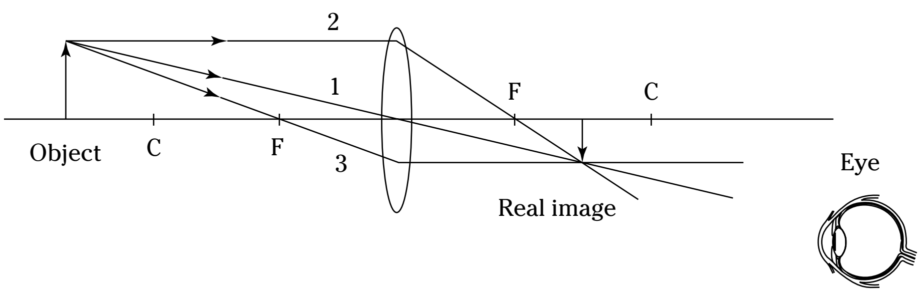


Figure 17-4: Ray diagram for a converging lens.

Chapter 18

Ten Top Online Physics Tutorials and Resources

In This Chapter

- Gearing tutorials for students of many levels
- Playing around with interactive sites

Tons of physics tutorials are online, and some are very useful, not to mention fun. Take a look at these resources in this chapter. There's a lot of physics just waiting for you out there.

The Physics Classroom

`www.physicsclassroom.com/Default2.html`

This famous physics tutorial and problem-solving site is touted as a high school physics tutorial, but it's a great resource for students at any level. It boasts good coverage on most topics (circuits are omitted).

ThinkQuest

`library.thinkquest.org/10796`

A good physics tutorial, this famous, well-designed site has built-in question-and-answer sections, where the answers appear when you click buttons.

HyperPhysics

`hyperphysics.phy-astr.gsu.edu/hbase/hframe.html`

An award-winning physics site. This site is very extensive. You find quick explanations on hundreds of topics, easily clickable to find the topic you want.

Roman Goc's Physics Tutorial

`www.staff.amu.edu.pl/~romangoc`

In addition to thorough explanations of a variety of physics topics and terms, this site includes problem-solving strategies.

Physics 24/7 Tutorial

`www.physics247.com/physics-homework-help/index.shtml`

An extensive site, with many different areas covered. This site also has quizzes, which are good for giving you practice for exams.

University of Guelph's Tutorial

`www.physics.uoguelph.ca/tutorials/tutorials.html`

Good coverage, but a limited selection of topics. The site does includes nice animations and plenty of well-illustrated graphs.

Tutor4Physics

`www.tutor4physics.com`

Eclectic site specializing in worked-out physics problems. A site worth checking out, it contains exceptionally clear explanations and very good step-by-step solutions.

Kenneth R. Koehler's Tutorial Page

`www.rwc.uc.edu/koehler/biophys/text.html`

Specially designed for biology or chemistry students who are taking physics. An extensive site that covers physics topics well, it includes some very nice hints that you won't find anywhere else.

Fear of Physics's Problem Solver

`www.fearofphysics.com/index2.html`

This Web site is unique because it's interactive. You can get practice by answering physics problems of the fill-in-the-blank variety — and you actually get to fill in the blanks!

Vector Resolver

`www.walter-fendt.de/ph14e/forceresol.htm`

This is an interactive Web page that helps you find the components of a vector. Your browser needs to support Java applets in order to work this one.

Index

Symbols

• A •

• B •

• C •

• F •

• G •

• H •

• I •

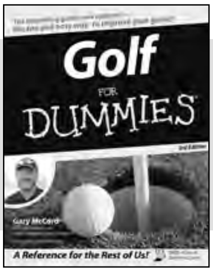
• T •

• U •

• V •

• W •

SPORTS, FITNESS, PARENTING, RELIGION & SPIRITUALITY



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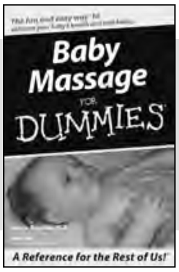

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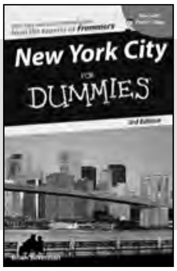

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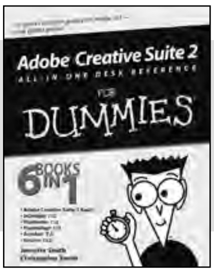

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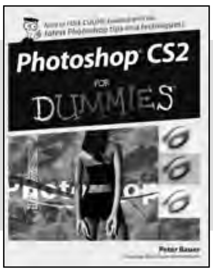

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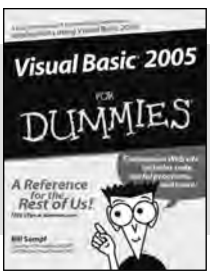

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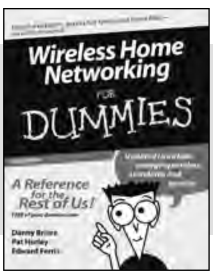

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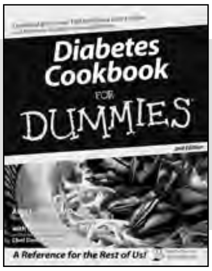

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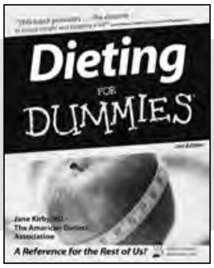

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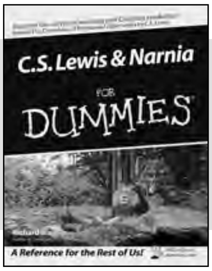

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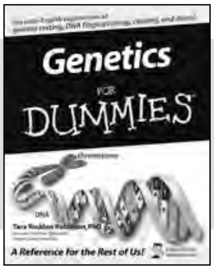

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