Book 2

Oxford Mathematics for the Caribbean

SIXTH EDITION

With online support

Nicholas Goldberg

with **Neva Cameron-Edwards**

OXFORD

BOOK 2

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Contents

About this book

This new edition of Oxford Mathematics for the Caribbean Student Book 2 has been revised to encompass recent changes to the mathematics curriculum throughout the region. Careful attention has been given to the major changes. As in the last editions, there is some content overlap across the three books in the series. This will allow schools to choose when certain topics are introduced but without any loss of continuity should they desire to begin a topic in a subsequent year.

In this edition, new activity boxes have been included that provide for more 'hands-on' experience and to offer a more student-centered approach. Some boxes provide links with other subject areas that teachers may wish to reinforce, while others include suggestions for group and project work.

To encourage greater use of technology in schools, the technology boxes and suggested websites have been revised. Many of these sites provide topic review and enhancement material – and many also showcase games to motivate learning. Support material is now available online at www.oxfordsecondary.com/9780198425748.

The major features of previous editions have been retained:

- Check-in boxes at the start of each unit to assess whether a student is ready to begin the unit.
- Worked examples are provided throughout the text.
- Graded exercises, generally easier questions are given in the earlier parts of an exercise with more challenging questions underlined:

  <u>1</u> a single underline will challenge the average student
  <u>2</u> a double underline will challenge the able student.

- Consolidation examples and exercises to provide further practice.
- Summary and checkout: these provide a quick review of the key points within the unit and enable the student to assess progress made.
- Review exercises with both extended response and multiple-choice format provided every four units.
- Units begin with a 'What's the point?' section to address links of the topic with the real world.

The aim of the book is to allow students to experience success and enjoyment in the learning of mathematics in the 21st century.

Nicholas Goldberg
Morne Jaune, Commonwealth of Dominica
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Number

- ✔ add and subtract fractions
- ✔ multiply and divide fractions
- ✔ use fractions to solve problems
- ✔ to solve problems involving ratios and proportions
- ✔ develop the concept of place value by using different number bases

How to share 1½chickens between the four of us?
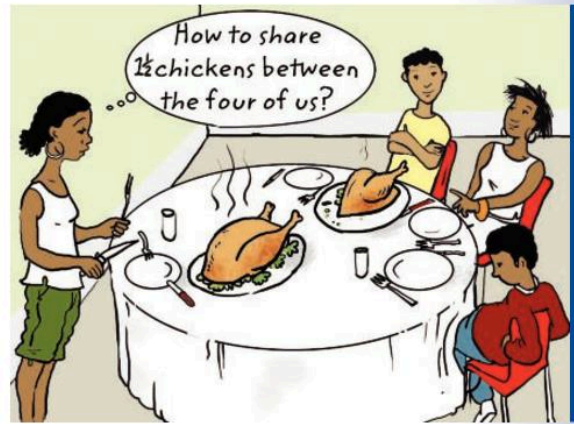
What's the point?

Carpenters and masons use whole numbers and fractions on a daily basis in their work.

Before you start

You should know ...

1 Fractions can be represented on diagrams:
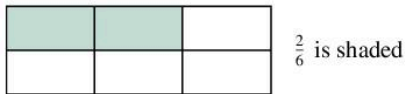
$\frac{2}{6}$ is shaded

2 Equivalent fractions show the same fraction using different numbers.

For example: $\frac{3}{4} = \frac{6}{8} = \frac{9}{12} = \frac{12}{16}$

3 Fractions can be simplified by dividing.
For example:

$$\frac{15}{27} = \frac{5}{9}$$

÷ 3 (numerator)
÷ 3 (denominator)

Check in

1 What fraction of each shape is shaded?

(a)   **(b)**
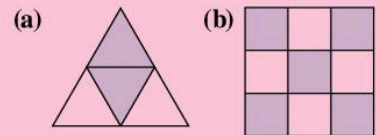
2 Copy and complete:
(a) $\frac{1}{4} = \frac{\square}{8}$ (b) $\frac{3}{8} = \frac{\square}{16}$
(c) $\frac{2}{3} = \frac{\square}{9}$ (d) $\frac{5}{8} = \frac{\square}{24}$

3 Simplify:
(a) $\frac{10}{15}$ (b) $\frac{12}{18}$ (c) $\frac{14}{21}$
(d) $\frac{32}{48}$ (e) $\frac{49}{77}$ (f) $\frac{72}{108}$

4   A mixed number is made up of a whole number and a fraction.

For example:

$$2\frac{2}{5} = \frac{2 \times 5 + 2}{5} = \frac{12}{5}$$

mixed number   improper fraction

4   Write as improper fractions:
(a) $2\frac{2}{3}$ (b) $3\frac{1}{4}$ (c) $4\frac{3}{4}$
(d) $1\frac{1}{6}$ (e) $4\frac{5}{6}$ (f) $6\frac{5}{8}$

5   A ratio compares the size of two quantities.

For example:

The ratio of boys to girls is 4 : 2
$4 : 2 = \frac{4}{2} = \frac{2}{1} = 2 : 1$

5   Write as ratios in their simplest form:
(a) $3 : 6$ (b) $2 : 8$ (c) $15 : 20$
(d) $21 : 14$ (e) $36 : 24$ (f) $51 : 34$

1.1   Addition and subtraction of fractions

You can always add or subtract amounts of the same object.

For example, 2 tables + 3 tables = 5 tables

In the same way:

2 eighths + 3 eighths = 5 eighths

or $\frac{2}{8}$ + $\frac{3}{8}$ = $\frac{5}{8}$

Exercise 1A

1   Work out the following:
(a) $\frac{1}{8} + \frac{3}{8}$ (b) $\frac{2}{6} + \frac{3}{6}$
(c) $\frac{2}{7} + \frac{4}{7}$ (d) $\frac{2}{3} + \frac{2}{3}$
(e) $\frac{5}{6} + \frac{3}{6}$ (f) $\frac{7}{8} + \frac{5}{8}$
(g) $\frac{6}{9} + \frac{7}{9}$ (h) $\frac{10}{12} + \frac{11}{12}$
(i) $2\frac{3}{4} + 1\frac{3}{4}$ (j) $4\frac{5}{8} + 3\frac{7}{8}$

2   Work out these subtractions:
(a) $\frac{3}{4} - \frac{1}{4}$ (b) $\frac{6}{8} - \frac{5}{8}$
(c) $\frac{4}{5} - \frac{2}{5}$ (d) $\frac{8}{12} - \frac{5}{12}$
(e) $1\frac{3}{4} - \frac{1}{4}$ (f) $3\frac{2}{3} - 1\frac{1}{3}$
(g) $2\frac{1}{3} - \frac{2}{3}$ (h) $1\frac{1}{4} - \frac{3}{4}$
(i) $4\frac{2}{5} - 2\frac{3}{5}$ (j) $3\frac{5}{12} - 1\frac{7}{12}$

3   In Mrs Bruno's class $\frac{5}{8}$ of the children walk to school, $\frac{2}{8}$ take a bus and the rest cycle to school.
(a) What fraction of the class walk or take a bus?
(b) What fraction cycle?

4   Two pieces of carpet $3\frac{1}{4}$ metres and $2\frac{3}{4}$ metres long are joined together. How long is the joined carpet?

5   Al saves $\frac{2}{7}$ of what he earns.
What fraction of his earnings does Al spend?

Adding fractions with different denominators

You **cannot** add or subtract numbers of **different** objects.

For example, you cannot work out

2 tables + 3 chairs

In the same way:

2 eighths + 3 fifths

or $\frac{2}{8}$ + $\frac{3}{5}$

cannot be added directly.

You have to use equivalent fractions to find a common denominator before you can add different kinds of fractions.

Example 1

Work out $\frac{3}{8} + \frac{2}{5}$

$\frac{3}{8} + \frac{2}{5}$ LCM of 8 and 5 is 40

$= \frac{15}{40} + \frac{16}{40}$

$= \frac{31}{40}$

$\times 5$ $\times 8$

$\frac{2}{8} = \frac{10}{40}$, $\frac{3}{5} = \frac{24}{40}$

$\times 5$ $\times 8$

Exercise 1B

1 Copy and complete:

(a) $\frac{3}{4} + \frac{3}{16} = \frac{\square}{16} + \frac{\square}{16} = \frac{\square}{16}$

(b) $\frac{3}{8} + \frac{1}{4} = \frac{\square}{8} + \frac{\square}{8} = \frac{\square}{8}$

(c) $\frac{1}{6} + \frac{2}{3} = \frac{\square}{6} + \frac{\square}{6} = \frac{\square}{6}$

(d) $\frac{2}{3} + \frac{1}{4} = \frac{\square}{12} + \frac{\square}{12} = \frac{\square}{12}$

(e) $\frac{2}{5} + \frac{2}{7} = \frac{\square}{35} + \frac{\square}{35} = \frac{\square}{35}$

2 Work out:

(a) $\frac{2}{7} + \frac{1}{3}$ (b) $\frac{3}{8} + \frac{1}{2}$ (c) $\frac{2}{5} + \frac{1}{4}$

(d) $\frac{3}{10} + \frac{2}{5}$ (e) $\frac{4}{9} + \frac{1}{3}$ (f) $\frac{3}{7} + \frac{1}{2}$

(g) $\frac{3}{4} + \frac{1}{8}$ (h) $\frac{2}{8} + \frac{2}{5}$ (i) $\frac{5}{7} + \frac{2}{9}$

(j) $\frac{4}{6} + \frac{2}{9}$

3 Cathy eats a quarter of an orange and her friend eats another $\frac{3}{8}$ of it. What fraction of the orange is eaten?

4 In a class, one third of the boys prefer football and another two fifths prefer cricket. What fraction like either game?

Adding mixed numbers

Mixed numbers can be added in a similar fashion.

Example 2

Work out $3\frac{2}{3} + 1\frac{3}{4}$

Add the whole numbers first, $3 + 1 = 4$

$3\frac{2}{3} + 1\frac{3}{4}$

$= 4 + \frac{2}{3} + \frac{3}{4}$

$= 4 + \frac{8}{12} + \frac{9}{12}$

$= 4 + \frac{17}{12}$

$= 4 + 1\frac{5}{12}$

$= 5\frac{5}{12}$

$\frac{2}{3} = \frac{8}{12}, \quad \frac{3}{4} = \frac{9}{12}$ (×4, ×3)

Exercise 1C

1 Use the method of Example 2 to work out:

(a) $3\frac{1}{2} + 2\frac{1}{4}$ (b) $3\frac{2}{3} + 2\frac{1}{4}$ (c) $4\frac{1}{2} + 2\frac{3}{4}$

(d) $2\frac{3}{8} + 3\frac{1}{3}$ (e) $4\frac{3}{4} + 3\frac{2}{5}$ (f) $5\frac{2}{3} + 2\frac{5}{8}$

(g) $4\frac{4}{5} + 3\frac{4}{7}$ (h) $6\frac{1}{2} + 7\frac{3}{4}$ (i) $4\frac{3}{7} + 3\frac{3}{5}$

(j) $2\frac{6}{7} + 3\frac{5}{6}$

2 Work out:

(a) $2\frac{1}{2} + 3\frac{1}{4} + 2\frac{1}{8}$

(b) $3\frac{2}{3} + 4\frac{5}{6} + 3\frac{7}{9}$

(c) $4\frac{7}{20} + 5\frac{2}{5} + 4\frac{9}{10}$

(d) $2\frac{1}{2} + 3\frac{1}{3} + 4\frac{1}{4}$

(e) $4\frac{3}{7} + 2\frac{2}{5} + 3\frac{2}{3}$

3 Mr Durant's car has $4\frac{1}{3}$ gallons of gas in its tank. He puts a further $2\frac{3}{4}$ gallons in it. How much gas is in the tank now?

4 Addison Thomas wishes to post three parcels with weights $1\frac{1}{2}$ kg, $2\frac{1}{3}$ kg and $4\frac{3}{4}$ kg. What is the total weight of his parcels?

5 Ambrose has $2\frac{1}{4}$ acres of land. His brother Anselm has $1\frac{2}{3}$ acres more than Ambrose. How much land do they have altogether?

Subtracting fractions with different denominators

You need to make sure that the denominators of the fractions are the same before subtracting, by using equivalent fractions.

Example 3

Work out $\frac{8}{9} - \frac{5}{6}$

$\frac{8}{9} - \frac{5}{6}$

$= \frac{16}{18} - \frac{15}{18}$

$= \frac{1}{18}$

LCM of 9 and 6 is 18

$\frac{8}{9} = \frac{16}{18}, \quad \frac{5}{6} = \frac{15}{18}$ (×2, ×3)

Exercise 1D

1 Copy and complete:

(a) $\frac{3}{8} - \frac{1}{4} = \frac{\square}{8} - \frac{\square}{8} = \frac{\square}{8}$

(b) $\frac{7}{9} - \frac{2}{3} = \frac{\square}{9} - \frac{\square}{9} = \frac{\square}{9}$

(c) $\frac{3}{4} - \frac{2}{3} = \frac{\square}{12} - \frac{\square}{12} = \frac{\square}{12}$

(d) $\frac{7}{8} - \frac{3}{5} = \frac{\square}{40} - \frac{\square}{40} = \frac{\square}{40}$

(e) $\frac{6}{7} - \frac{3}{4} = \frac{\square}{28} - \frac{\square}{28} = \frac{\square}{28}$

2 Work out:

(a) $\frac{4}{5} - \frac{2}{3}$ (b) $\frac{6}{7} - \frac{5}{8}$ (c) $\frac{2}{5} - \frac{1}{4}$

(d) $\frac{9}{10} - \frac{4}{5}$ (e) $\frac{8}{11} - \frac{3}{5}$ (f) $\frac{7}{10} - \frac{2}{15}$

(g) $\frac{4}{7} - \frac{2}{9}$ (h) $\frac{9}{14} - \frac{4}{21}$ (i) $\frac{8}{13} - \frac{5}{11}$

(j) $\frac{8}{12} - \frac{3}{8}$

3 Work out:

(a) $\frac{5}{8} - \frac{2}{9}$ (b) $\frac{11}{12} - \frac{3}{7}$ (c) $\frac{7}{9} - \frac{5}{12}$

(d) $\frac{11}{16} - \frac{4}{9}$ (e) $\frac{11}{12} - \frac{9}{10}$ (f) $\frac{7}{10} - \frac{4}{15}$

(g) $1 - \frac{1}{6}$ (h) $1 - \frac{3}{4}$ (i) $1 - \frac{10}{17}$

(j) $1 - \frac{23}{24}$

Subtracting mixed numbers

To subtract mixed numbers it is a good idea to turn them both into improper fractions.

Example 4

Work out:

(a) $2\frac{1}{5} - 1\frac{4}{5}$ **(b)** $3\frac{2}{3} - 1\frac{5}{9}$ **(c)** $2\frac{5}{6} - 1\frac{7}{8}$

(a) $2\frac{1}{5} - 1\frac{4}{5} = \frac{11}{5} - \frac{9}{5} = \frac{2}{5}$

(b) $3\frac{2}{3} - 1\frac{5}{9} = \frac{11}{3} - \frac{14}{9}$ $\left(\frac{11}{3} = \frac{33}{9}\right)$

Improper fractions

$\qquad\qquad = \frac{33}{9} - \frac{14}{9}$

$\qquad\qquad = \frac{19}{9}$

$\qquad\qquad = 2\frac{1}{9}$

(c) $2\frac{5}{6} - 1\frac{7}{8} = \frac{17}{6} - \frac{15}{8}$

$\qquad\qquad = \frac{68}{24} - \frac{45}{24}$

$\qquad\qquad = \frac{23}{24}$

Exercise 1E

1 Work out:

(a) $1\frac{1}{3} - \frac{2}{3}$ **(b)** $2\frac{3}{4} - 1\frac{1}{4}$ **(c)** $4\frac{5}{8} - 1\frac{7}{8}$

(d) $6\frac{1}{12} - 4\frac{7}{12}$ **(e)** $2\frac{1}{3} - 1\frac{5}{9}$ **(f)** $4\frac{11}{12} - 1\frac{5}{6}$

2 Work out:

(a) $6\frac{1}{3} - 2\frac{1}{6}$ **(b)** $2\frac{1}{6} - \frac{3}{4}$

(c) $3\frac{1}{2} - 1\frac{3}{5}$ **(d)** $4\frac{1}{7} - 1\frac{3}{4}$

(e) $6\frac{1}{9} - 5\frac{3}{4}$ **(f)** $5\frac{2}{5} - 3\frac{5}{8}$

(g) $3 - 2\frac{2}{3}$ **(h)** $4 - 2\frac{3}{5}$

(i) $3 - 2\frac{11}{12}$ **(j)** $5 - 4\frac{11}{17}$

3 A water tank holds 100 gallons. Mrs Shaw uses $6\frac{3}{4}$ gallons. How much water is left in the tank?

4 Kimani's home is $6\frac{1}{2}$ km from school. She walks $\frac{5}{8}$ km to the bus stop and takes a bus for the rest of the journey. How far is her school from the bus stop?

5 Olive has 3 parcels to post. The first parcel weighs $2\frac{1}{3}$ kg. The second parcel weighs $\frac{3}{4}$ kg. If the parcels weight 5 kg in total, how much does the third parcel weigh?

6 Copy and complete the diagram so that the sum of the numbers in any two circles equals the number in the square between them.
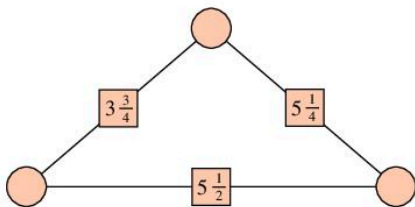
1.2 Multiplication and division of fractions

What is $4 \times \frac{2}{3}$?

$$4 \times \frac{2}{3} = \frac{2}{3} + \frac{2}{3} + \frac{2}{3} + \frac{2}{3} = \frac{8}{3} = 2\frac{2}{3}$$

You should see that

$$4 \times \frac{2}{3} = \frac{4}{1} \times \frac{2}{3} = \frac{8}{3} = 2\frac{2}{3}$$

You can use the same idea to multiply fractions by fractions.

Example 5

Work out:

(a) $\frac{2}{3} \times \frac{3}{4}$ **(b)** $4\frac{1}{2} \times 1\frac{1}{4}$

(a) $\frac{2}{3} \times \frac{3}{4} = \frac{6}{12} = \frac{1}{2}$

(b) $4\frac{1}{2} \times 1\frac{1}{4} = \frac{9}{2} \times \frac{5}{4}$

$\qquad\qquad = \frac{45}{8}$

$\qquad\qquad = 5\frac{5}{8}$

Exercise 1F

1 Copy the shape below.
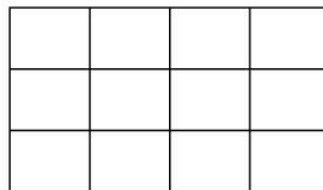
(a) Shade in $\frac{2}{3}$ of the shape.
(b) Shade in $\frac{1}{4}$ of the shaded part.
(c) What fraction of the shape has been shaded twice?
(d) Copy and complete:

$$\tfrac{1}{4} \text{ of } \tfrac{2}{3} = \tfrac{\square}{\square}$$

2 Copy the shape below.
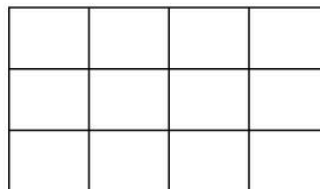
(a) Shade in $\frac{3}{4}$ of the shape.

(b) Shade in $\frac{2}{3}$ of the shaded part.

(c) What fraction of the shape has been shaded twice?

(d) Copy and complete:

$$\tfrac{2}{3} \times \tfrac{3}{4} = \tfrac{\square}{\square}$$

3 Draw shapes as in Questions 1 and 2 to show:

(a) $\frac{1}{2} \times \frac{1}{4} = \frac{1}{8}$ **(b)** $\frac{2}{3} \times \frac{1}{2} = \frac{1}{3}$

4 Work out:

(a) $4 \times \frac{1}{4}$ **(b)** $5 \times \frac{1}{2}$ **(c)** $8 \times \frac{3}{4}$

(d) $7 \times \frac{2}{5}$ **(e)** $6 \times \frac{3}{8}$ **(f)** $12 \times \frac{7}{8}$

5 Draw suitable pictures to check your answers in Question 4.

6 Calculate:

(a) $\frac{2}{3} \times \frac{1}{4}$ **(b)** $\frac{3}{4} \times \frac{1}{2}$

(c) $\frac{3}{5} \times \frac{3}{4}$ **(d)** $\frac{5}{8} \times \frac{3}{4}$

(e) $\frac{7}{8} \times \frac{2}{3}$ **(f)** $\frac{4}{5} \times \frac{1}{3}$

(g) $\frac{3}{16} \times \frac{4}{7}$ **(h)** $\frac{5}{8} \times \frac{5}{8}$

(i) $\frac{5}{12} \times \frac{6}{11}$

7 Work out:

(a) $2\frac{1}{2} \times 1\frac{1}{4}$ **(b)** $2\frac{2}{3} \times \frac{2}{3}$

(c) $4\frac{1}{4} \times \frac{3}{4}$ **(d)** $\frac{3}{8} \times 1\frac{2}{3}$

(e) $3\frac{3}{4} \times 1\frac{1}{2}$ **(f)** $4\frac{2}{3} \times 2\frac{2}{3}$

(g) $5\frac{1}{2} \times 2\frac{1}{4}$ **(h)** $6\frac{3}{4} \times 2\frac{4}{9}$

(i) $7\frac{1}{2} \times 2\frac{1}{2}$

8 Anisha has $\frac{2}{3}$ gallon of paint. She uses $\frac{3}{8}$ of it to paint a table.

(a) How much paint did she use?

(b) How much paint does she have left?

9 It takes a mechanic $2\frac{1}{4}$ hours to service a car. If the mechanic takes one third of this time to change the oil, how long does he take to perform the other tasks?

10 Fitzroy's school is $5\frac{1}{2}$ km from his home. Every day he walks $\frac{2}{5}$ of the way and takes a bus for the remaining distance.

He does the same on the return journey.

(a) How far does he walk altogether in one five-day week?

(b) How far does he travel by bus during the week?

Dividing fractions

What is $6 \times \frac{1}{2}$?

$$6 \times \tfrac{1}{2} = \tfrac{6}{2} = 3$$

Notice that

$$6 \div 2 = 6 \times \tfrac{1}{2} = 3$$

In other words, dividing by 2 is the same as multiplying by $\frac{1}{2}$.

Notice also that

$$2 \times \tfrac{1}{2} = 1$$

So $\frac{1}{2}$ is the **inverse** of 2 under multiplication. In the same way:

$\frac{5}{4}$ is the inverse of $\frac{4}{5}$
since $\frac{4}{5} \times \frac{5}{4} = 1$

$\frac{3}{7}$ is the inverse of $\frac{7}{3}$
since $\frac{7}{3} \times \frac{3}{7} = 1$

You can use this idea to divide fractions.

Example 6

Work out:

(a) $\frac{3}{4} \div \frac{2}{3}$ **(b)** $1\frac{1}{2} \div 2\frac{1}{4}$

**(a)** $\frac{3}{4} \div \frac{2}{3} = \frac{3}{4} \times \frac{3}{2}$

$$= \frac{\frac{3}{4} \times \frac{3}{2}}{\frac{2}{3} \times \frac{3}{2}}$$

$$= \frac{3}{4} \times \frac{3}{2}$$

$$= \frac{9}{8} = 1\frac{1}{8}$$

Dividing by $\frac{2}{3}$ is the same as multiplying by its inverse $\frac{3}{2}$.

(b) $1\frac{1}{2} \div 2\frac{1}{4} = \frac{3}{2} \div \frac{9}{4}$

$$= \frac{3}{2} \times \frac{4}{9}$$

$$= \frac{12}{18}$$

$$= \frac{2}{3}$$

Exercise 1G

1 Look at the figure below.
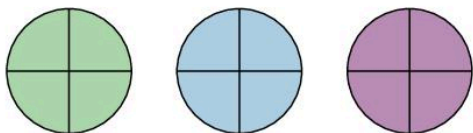
Use it to calculate $3 \div \frac{1}{4}$

2 Write down the inverses under multiplication of:
   (a) $\frac{1}{2}$ **(b)** 3 **(c)** 6 **(d)** $2\frac{1}{2}$
   (e) $\frac{3}{5}$ **(f)** $\frac{7}{8}$ **(g)** $\frac{6}{5}$ **(h)** $3\frac{3}{4}$

3 Work out:
   (a) $3 \div \frac{1}{2}$ **(b)** $6 \div \frac{1}{3}$ **(c)** $6 \div \frac{2}{3}$
   (d) $8 \div \frac{3}{4}$ **(e)** $4 \div \frac{2}{5}$ **(f)** $9 \div 1\frac{1}{2}$

4 Calculate:
   (a) $\frac{1}{2} \div \frac{1}{4}$ **(b)** $\frac{3}{4} \div \frac{1}{4}$
   (c) $\frac{5}{8} \div \frac{3}{4}$ **(d)** $\frac{3}{5} \div \frac{2}{3}$
   (e) $\frac{4}{5} \div \frac{3}{8}$ **(f)** $\frac{5}{12} \div \frac{5}{6}$
   (g) $\frac{7}{8} \div \frac{3}{16}$ **(h)** $\frac{7}{12} \div \frac{1}{6}$
   (i) $\frac{3}{11} \div \frac{2}{7}$

5 Calculate:
   (a) $3\frac{1}{2} \div 1\frac{1}{4}$ **(b)** $4\frac{1}{2} \div 1\frac{1}{2}$
   (c) $3\frac{2}{5} \div 1\frac{1}{5}$ **(d)** $4\frac{3}{4} \div 2\frac{1}{2}$
   (e) $6\frac{2}{3} \div 1\frac{2}{3}$ **(f)** $1\frac{3}{4} \div 2\frac{7}{8}$

6 How many thirds are there in:
   (a) 2 **(b)** $\frac{3}{5}$ **(c)** $2\frac{3}{4}$ **(d)** $\frac{2}{11}$?

7 How many one-third litre cups of fruit punch can be obtained from a bowl containing $4\frac{1}{2}$ litres of punch?

8 I have 6 grapefruits. How many people can I give $\frac{3}{4}$ of a grapefruit to?

9 Max has $4\frac{1}{2}$ gallons of paint. Each wall he paints requires $\frac{2}{3}$ gallon of paint. How many walls can he paint?

10 The area of a rectangular field is $106\frac{3}{4}$ m². What is the length of the field if its width is $9\frac{1}{3}$ m?

1.3   Ratio and proportion

In a school there are 100 boys and 150 girls. The ratio of boys to girls is

$$100 : 150 = 2 : 3$$

The last ratio indicates that for every 2 boys, there are 3 girls at the school.

You could also say that the

fraction of boys at school $= \frac{100}{250} = \frac{2}{5}$

fraction of girls at school $= \frac{150}{250} = \frac{3}{5}$

This shows how fractions are related to ratios.

You can use the idea of ratio to solve simple problems.

How would you share this cake between Jim and Leo, in the ratio $2 : 3$?

Jim will get 2 parts and Leo 3 parts. That is, they will have to share the cake into $2 + 3 = 5$ parts.

Example 7

Share $1 between Clara and Hope in the ratio 4 : 1.

For every 1 part Hope gets, Clara gets 4 parts.
4 + 1 = 5
You must divide $1 into 5 equal parts.

So 1 part is 20 cents.
Clara gets **80 cents**.
Hope gets **20 cents**.

In Example 7 you worked out the **proportion** of $1 that Clara and Hope each receive.

Sometimes you will be given one of the proportions and you will have to work out the other proportion or the total amount.

Example 8

The ratio of boys to girls in Rotterdam School is 3 : 2. There are 60 boys in the school. How many girls are there?

For every 3 boys there are 2 girls.
So for every 1 boy there are $\frac{2}{3}$ of a girl.
For 60 boys there are $60 \times \frac{2}{3}$ girls
$= 40$ girls.

Exercise 1H

1 A cake is shared between Tom and Judy, in the ratio 2 : 5.
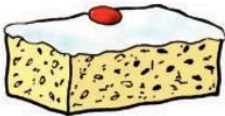
(a) If Tom gets 2 parts, how many parts should Judy get?
(b) How would you cut up the cake?

2 Bessie and Jason share some rice in the ratio 5 : 3.
(a) If Jason gets 3 parts, how many parts should Bessie get?
(b) Do you agree that the rice should be divided into 8 equal parts?
(c) How many kilograms does Bessie get?

3 (a) How would you share these apples between Ali and Sam, in the ratio 1 : 3?

(b) How many apples would Ali get?
(c) How many apples would Sam get?

4 Share $1 between Clara and Hope, in the ratio:
(a) 3 : 1 (b) 1 : 1 (c) 9 : 1
(d) 7 : 3 (e) 17 : 3 (f) 4 : 21

5 Share $5 between Clara and Hope, in the ratio:
(a) 3 : 1 (b) 9 : 1 (c) 17 : 3

6 Share 30 cows between Farmer X and Farmer Y, in the ratio:
(a) 2 : 1 (b) 1 : 1 (c) 4 : 1
(d) 7 : 3 (e) 4 : 11 (f) 3 : 27

7 Share the following between X and Y, in the given ratio.
(a) 20 apples, in the ratio 3 : 7
(b) 15 goats, in the ratio 2 : 3
(c) $60, in the ratio 1 : 4
(d) $5.50, in the ratio 4 : 1
(e) 3 metres of cloth, in the ratio 3 : 7
(f) 1 metre of rope, in the ratio 2 : 3
(g) 2 kilograms of honey, in the ratio 1 : 7
(h) 2 tonnes of sand, in the ratio 4 : 1
(i) 17.5 metres of cloth, in the ratio 3 : 7
(j) 15.6 kilograms of sugar, in the ratio 9 : 4

8 The ratio of boys to girls at the Valley School is 3 : 2. The school has 800 students.
(a) How many boys are at the school?
(b) How many girls?

9

The ratio of sand to cement to mix concrete is 4 : 1. How many wheelbarrows of sand does Sam need to make 25 barrows of concrete mix?

10 The ratio of men to women in the Acapulco Sports club is $4:3$. Use the method shown in Example 9 to find out how many women there are, if there are:

(a) 12 men (b) 40 men (c) 36 men
(d) 108 men (e) 400 men (f) 144 men

11 (a) Number of boys : number of girls $= 2:1$ in Tranquillity School. How many boys are there in the school, if the number of girls is:
(i) 25 (ii) 200 (iii) 75 (iv) 80?

(b) How many girls are there in Tranquillity School, if the number of boys is:
(i) 50 (ii) 200 (iii) 150 (iv) 80?

You can also use ratios to share quantities between more than 2 people.

Example 9

Share 50 elephants among Jo, Minny and Sam in the proportion $1:2:7$.

$1 + 2 + 7 = 10$ parts
There are 50 elephants. So there are 5 elephants in each part.

Jo gets **5 elephants**
Minny gets $5 \times 2 = 10$ elephants
Sam gets $5 \times 7 = 35$ elephants

Exercise 1I

1 A cake is shared between Annabel, Ria and Faith in the ratio $1:2:3$.

(a) If Annabel gets 1 part, how many parts should Ria get? How many parts should Faith get?
(b) Into how many equal slices should you cut the cake?

2 Share the following among A, B and C:
(a) 60 nuts, in the ratio $4:1:7$
(b) 100 colas, in the ratio $2:5:3$
(c) 75 metres, in the ratio $8:2:5$
(d) 1.8 metres, in the ratio $1:7:1$
(e) $25, in the ratio $2:13:5$
(f) $54, in the ratio $8:5:14$
(g) $100, in the ratio $2:4:19$
(h) 1000 kg, in the ratio $117:62:21$

3 Share:
(a) 725 camels, in the ratio $3:17:9$
(b) $24.50, in the ratio $1:1:12$
(c) 1.5 kg, in the ratio $13:2:5$

4 (a) Draw a line 12 cm long. Divide it in the ratio $2:1:3$.
(b) Draw a line 9 cm long. Divide it in the ratio $2:4:3$.

Technology

Need another look? Visit
www.onlinemathlearning.com
and follow the links to Arithmetic, Ratios.

Go over the examples and watch the video!

Activity

In Barbados the cost of 100 oz of liquid laundry detergent is $25.77

- A 150oz container costs $38.66.
- Is the relation between the amount of laundry detergent and cost proportionate?
- Is it more economical to buy the smaller size?
- Find the cost of different sizes of tooth paste in your country.
- Are they priced in proportion to the amount of toothpaste in each?
- Draw a graph to show the relationship.
- Repeat for other items.

1.4   Number bases

Our number system uses 10 as its base. So, for example,

$$396 = 3 \times 100 + 9 \times 10 + 6 \times 1$$
$$= 3 \times 10^2 + 9 \times 10^1 + 6 \times 1$$

The column headings in base 10 are all powers of ten.

Notice that only ten different digits (0, 1, 2, 3, 4, 5, 6, 7, 8, 9) are needed to represent any number.

The **binary** system uses 2 as a base.
The column headings are powers of 2:

1, 2, 2×2, $2 \times 2 \times 2$, $2 \times 2 \times 2 \times 2$, . . . etc
or 1, 2, 2^2, 2^3, 2^4, . . .

In the binary system there are only **two** different digits (0, 1). These two digits can be used to represent any number.

Example 10

Write the base 2 number 11010 as a base 10 number.

The column headings are

2^4	2^3	2^2	2	1
1	1	0	1	0

so 11010 (base 2)
$$= 1 \times 2^4 + 1 \times 2^3 + 0 \times 2^2 + 1 \times 2 + 0 \times 1$$
$$= 1 \times 16 + 1 \times 8 + 0 \times 4 + 1 \times 2 + 0 \times 1$$
$$= 16 + 8 + 2$$
$$= 26$$

To change a base 10 number to a binary number is more difficult. You need to find which powers of two will add up to the number.

Example 11

Write 45 as a base 2 number.

$$45 = 32 + 8 + 4 + 1$$
$$= 2^5 + 2^3 + 2^2 + 1$$
$$= 1 \times 2^5 + 0 \times 2^4 + 1 \times 2^3 + 1 \times 2^2 + 0 \times 2$$
$$+ 1 \times 1$$
$$= 101101 \text{ (base 2)}$$

Exercise 1J

1 Change these binary numbers to base 10.
   (a) 10   (b) 11   (c) 100
   (d) 101   (e) 1100   (f) 1010
   (g) 11010   (h) 10110

2 Change these base 10 numbers to base 2.
   (a) 2   (b) 3   (c) 5   (d) 8
   (e) 13 (f) 16 (g) 17 (h) 19
   (i) 23 (j) 29 (k) 41 (l) 63

3 Check your answers to Question 2 by converting them back to base 10.

Technology

The binary system is used in computing. Visit the site

www.computer.howstuffworks.com/bytes.html

to learn more about the binary system and its applications.

You can convert numbers into other number bases.

Base 5, for example, has column headings made up of powers of 5:

1, 5, 5^2, 5^3, ...

To convert a base 5 number to base 10, simply expand the number.

Example 12

Convert 3413 (base 5) into base 10.

The column headings are

5^3	5^2	5	1
3	4	1	3

So 3413 (base 5) $= 3 \times 5^3 + 4 \times 5^2 + 1 \times 5 + 3 \times 1$
$$= 3 \times 125 + 4 \times 25 + 5 + 3$$
$$= 375 + 100 + 5 + 3$$
$$= 483$$

To change a base 10 number to base 5, you can look at the powers of 5 that add up to that number.

Example 13

Change 83 to a base 5 number.

$83 = 75 + 5 + 3$
$= 3 \times 25 + 1 \times 5 + 3 \times 1$
$= 3 \times 5^2 + 1 \times 5^1 + 3 \times 1$
$= 313 \text{ (base 5)}$

A quicker way to change a base 10 number to another base is to repeatedly divide by the number representing the base and look at the remainders.

Example 14

Change 83 to base 5.

$5\lfloor 83$
$5\lfloor 16 \text{ r } 3$ Answer:
$5\lfloor 3 \text{ r } 1$ 313 (base 5)
$0 \text{ r } 3$

Exercise 1K

1 Write these base 5 numbers in base 10.
   (a) 41   (b) 33   (c) 132
   (d) 241   (e) 344   (f) 1403
   (g) 3333   (h) 1002   (i) 4141

2 Write down the first four powers of eight.

3 Write these base 8 numbers in base 10.
   (a) 11   (b) 37   (c) 71
   (d) 101   (e) 106   (f) 217
   (g) 476   (h) 1005   (i) 2165

4 Change these base 10 numbers to base 5.
   (a) 71   (b) 16   (c) 42
   (d) 80   (e) 121   (f) 314

5 Change these base 10 numbers to base 8.
   (a) 13   (b) 41   (c) 29
   (d) 63   (e) 131   (f) 493

Technology

Need more practice with number bases? Visit

www.purplemath.com/modules/numbbase.htm

Don't forget to try out the questions!

The operations in different number bases follow the same rules as with base 10.

Remember: counting in base 5 goes
1, 2, 3, 4, 10, 11, 12, 13, 14, 20, . . . etc.

Example 15

What is the sum of 304 (base 5) and 231 (base 5)?

Insert column headings and add.

5^2	5	1
3	0	4
2	3_1	1
1 0	4	0

4+1
=10 (base 5)

To multiply, again remember how to count in the base.
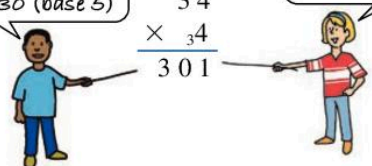
Example 16

Work out the multiplication 34×4 of base 5 numbers.

15 (base 10)
= 30 (base 5)

16 (base 10)
= 31 (base 5)

3 4
$\times \ _34$
3 0 1

Exercise 1L

1 Write down the first 20 counting numbers in:
   (a) base 5   (b) base 8

2 Write down the two times table in:
   (a) base 5   (b) base 8

3 Write down the three times table in:
   (a) base 5   (b) base 8

4 Work out these additions of base 5 numbers.
   (a) 1 2
      +1 4

   (b) 2 3
      +1 2

   (c) 3 2
      +2 3

   (d) 1 3 4
      +3 1 4

5 Work out these subtractions of base 5 numbers.
(a) $31 - 14$ (b) $43 - 24$
(c) $41 - 34$ (d) $123 - 44$

6 What is the sum of the first six base 8 counting numbers?

7 Work out these multiplications of base 5 numbers.
(a) 3×4 (b) 4×4 (c) 12×3
(d) 24×3 (e) 23×4 (f) 41×4

📦 **Activity**

- Find out why number bases are important.
- Why is the binary system (base 2) so useful in the modern world?

Exercise 1M – mixed questions

1 In a mixed school $\frac{3}{5}$ of the students are boys. What fraction are girls?

2 A bag of sugar has a mass of $\frac{3}{8}$ kg. A second bag has a mass of $\frac{9}{16}$ kg. What mass of sugar is there altogether?

3 A mouse eats $\frac{2}{15}$ kg of cheese each night.

How much cheese does it eat in:
(a) 30 nights (b) 16 nights?

4 A man walks $3\frac{1}{2}$ km each hour. How far does he walk in:
(a) 4 hours (b) $\frac{3}{4}$ hour?

5 Three oranges are divided equally between five people.

What fraction of an orange does each person get?

6 Change these base 5 numbers to base 10.
(a) 413 (b) 344 (c) 1312

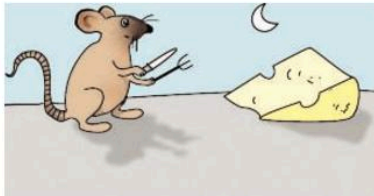**7** A length of string is 35 cm long. It is cut into six equal pieces. How long is each piece?

8 A barrel of rum holds 50 litres.

How many bottles each holding $\frac{3}{4}$ litre can be filled from the barrel?

9 A plank of wood is $1\frac{1}{2}$ cm thick. How thick is a pile of 25 planks?

10 Change these base 10 numbers to base 8.
(a) 80 (b) 131 (c) 467

11 For a certain type of concrete, the ratio of cement powder to sand is $3:5$.
How much cement powder is needed to make the concrete mix if the mass of sand is:
(a) 10 kg (b) 30 kg (c) 45 kg?

12
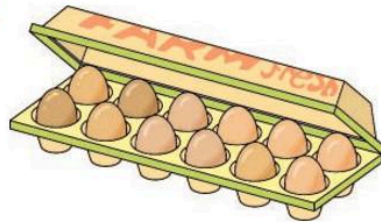
The cost of 12 eggs is $6.60.
(a) What is the cost of one egg?
(b) What is the cost of 9 eggs?

13 An orange squash is made of sugar, orange juice and water in the proportion $1:2:4$.
How many glasses of pure orange juice should be squeezed if 14 glasses of squash are required?

14 Mr Lloyd has no children so he decides to leave all his money to his three nephews, Bo, Bud and Billy. Bo get $\frac{3}{5}$ of the money. Bud gets $\frac{1}{4}$ of the money. What fraction does Billy get?

15 The table shows how Form 3 students in Radley High School come to school every day.

Means of transport	Number of students
Bus	28
Car	12
Bicycle	15
Walking	45

   (a) What fraction of the total number of students:
     (i)   come by bus
     (ii) come by car
     (iii) come by bicycle
     (iv) walk to school?
     Write each fraction in its simplest form.
   (b) What number do you get when you add the four fractions?

16

Benny's supermarket sells 25 tea bags of Tim's tea for $4.75 and 30 bags for $6.30.
(a) Work out the cost of one tea bag for each box.
(b) Which is the better buy?

17 30 m of electric cable is sold for $36.
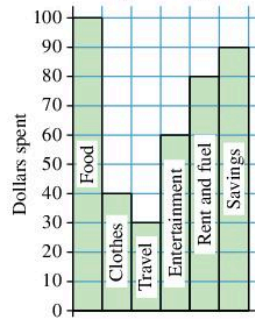How much does 25 m cost?

18 A loaf of bread for four people needs 440 g of flour.
How much flour will be needed to make a similar loaf for five people?

19 A 180 ml tube of 'Super fresh' toothpaste retails for $4.14.
What should a 270 ml tube retail for?
(assume the same price per ml)

20 The block graph shows how Mr Damon spends his monthly salary.

(a) How much does Mr Damon earn a month?
(b) Write down the fraction of his salary he uses for:
    (i)   food   (ii) clothes
    (iii) travel   (iv) entertainment
    (v) rent and fuel (vi) savings.
(c) When you add the six fractions, do you get 1 as your answer?

The money that Mr Damon spends on food is used in the following way:
$\frac{2}{5}$ on meat $\frac{1}{10}$ on fruit
$\frac{1}{4}$ on drinks $\frac{1}{10}$ on vegetables.
(d) What fraction is used for other foods?
(e) What fraction of his total salary is spent on:
    (i)   meat   (ii) drinks
    (iii) fruit   (iv) vegetables?

21 A book with 150 pages is $7\frac{1}{2}$ mm thick.
How thick is each page?

22 A bag of flour weighs $50\frac{2}{3}$ kg.
What is the weight of 10 bags?

23 Work out these multiplications of base 5 numbers,
(a) 43×4 (b) 3×13
(c) 12×13 (d) 24×14

24 Study this example.
$$6 \times 3\tfrac{2}{7} = 6 \times (3 + \tfrac{2}{7})$$
$$= 6 \times 3 + 6 \times \tfrac{2}{7}$$
$$= 18 + \tfrac{12}{7}$$
$$= 18 + 1\tfrac{5}{7}$$
$$= 19\tfrac{5}{7}$$

(a) Use the same method to work out:
    (i) $3 \times 2\tfrac{2}{7}$ (ii) $4 \times 6\tfrac{2}{5}$ (iii) $5 \times 4\tfrac{3}{4}$
    (iv) $6 \times 1\tfrac{3}{8}$ (v) $7 \times 2\tfrac{3}{5}$ (vi) $2 \times 1\tfrac{7}{8}$

(b) Do you prefer this method of multiplying fractions? Why?

25 Study this example.

$$\frac{3}{5} \div \frac{2}{3} = \frac{9}{15} \div \frac{10}{15} \qquad \left(\frac{3}{5} = \frac{9}{15}, \frac{2}{3} = \frac{10}{15}\right)$$
$$= \frac{9 \div 10}{15 \div 15} = \frac{9 \div 10}{1}$$
$$= 9 \div 10 = \frac{9}{10}$$

(a) Use the method to work out:

(i) $\frac{3}{4} \div \frac{4}{5}$ **(ii)** $\frac{2}{3} \div \frac{1}{6}$

(iii) $\frac{4}{7} \div \frac{5}{8}$ **(iv)** $1\frac{3}{4} \div \frac{4}{5}$

(v) $1\frac{3}{4} \div 1\frac{4}{5}$ **(vi)** $2\frac{3}{7} \div 4\frac{2}{7}$

(b) Do you prefer this method of dividing fractions? Why?

26 The area of a rectangular piece of card is $7\frac{1}{2}$ cm². What is the width of the card if its length is $3\frac{1}{4}$ cm?

27 John spent $\frac{1}{2}$ his money in the market and $\frac{1}{3}$ at the store.

(a) What fraction of his money does he have left?

(b) If he has $10 left, how much did he have to start with?

28 A $\frac{3}{4}$-full sack of flour weighs 40 kg.

How much does it weigh when it is:
(a) full   **(b)** $\frac{5}{8}$ full?

29 Write these fractions in order of size, smallest first:

$$\frac{1}{3}, \frac{3}{7}, \frac{2}{5}, \frac{7}{20}$$

30 It takes three men $1\frac{1}{2}$ days to dig a trench. How long would it take
(a) one man
(b) nine men
to do the same work?

31 Adam travels $\frac{2}{5}$ of his journey by car, $\frac{3}{10}$ by bus and the rest walking.

(a) What fraction of the journey does he walk?

(b) If he walks 15 km, how long was the journey?

Investigation

There are many ways of dividing fractions to give the answer $\frac{2}{3}$.

Here are two of the ways:

$$\frac{4}{9} \div \frac{2}{3} = \frac{2}{3}$$

$$\frac{3}{5} \div \frac{9}{10} = \frac{2}{3}$$

What other fraction pairs when divided give $\frac{2}{3}$ as an answer?

How many can you find?

Are there any rules?

Investigate further.

1 Consolidation

Example 1
Work out:

(a) $\dfrac{3}{5} + \dfrac{1}{3}$ (b) $2\dfrac{3}{4} - 1\dfrac{2}{3}$

(a) $\dfrac{3}{5} + \dfrac{1}{3} = \dfrac{9}{15} + \dfrac{5}{15}$ (b) $2\dfrac{3}{4} - 1\dfrac{2}{3} = \dfrac{11}{4} - \dfrac{5}{3}$

$\qquad\qquad = \dfrac{14}{15}$ $\qquad\qquad = \dfrac{33}{12} - \dfrac{20}{12}$

$\qquad\qquad\qquad\qquad\qquad = \dfrac{13}{12} = 1\dfrac{1}{12}$

Example 2
Calculate:

(a) $6 \times 3\dfrac{2}{5}$ (b) $4\dfrac{1}{2} \times 3\dfrac{2}{3}$

(a) $6 \times 3\dfrac{2}{5} = \dfrac{6}{1} \times \dfrac{17}{5}$ (b) $4\dfrac{1}{2} + 3\dfrac{2}{3} = \dfrac{9}{2} \times \dfrac{11}{3}$

$\qquad\qquad = \dfrac{102}{5}$ $\qquad\qquad = \dfrac{\overset{3}{\cancel{9}} \times 11}{2 \times \cancel{3}}$

$\qquad\qquad = 20\dfrac{2}{5}$ $\qquad\qquad = \dfrac{33}{2} = 16\dfrac{1}{2}$

Example 3
Calculate:

(a) $\dfrac{3}{4} \div \dfrac{1}{2}$ (b) $4\dfrac{3}{4} \div 2\dfrac{3}{5}$

(a) $\dfrac{3}{4} \div \dfrac{1}{2} = \dfrac{3}{4} \times \dfrac{2}{1}$

$\qquad\qquad = \dfrac{6}{4} = 1\dfrac{1}{2}$

> Dividing by $\frac{13}{5}$ is the same as multiplying by $\frac{5}{13}$

(b) $4\dfrac{3}{4} \div 2\dfrac{3}{5} = \dfrac{19}{4} \div \dfrac{13}{5}$

$\qquad\qquad = \dfrac{19}{4} \times \dfrac{5}{13}$

$\qquad\qquad = \dfrac{95}{52} = 1\dfrac{43}{52}$

Example 4
The ratio of men to women in a cricket crowd is 4 : 3.

(a) How many women are there if the crowd size is 7000?

(b) If there are 2800 men, how many women are there?

(a) There are $4 + 3 = 7$ parts
Each part is $7000 \div 7 = 1000$ people.
Number of women $= 3 \times 1000 = 3000$

(b) The 4 parts (men) represent 2800 men so one part is $2800 \div 4 = 700$
Number of women $= 700 \times 3 = 2100$.

Example 5
Change:
(a) 59 to base 5
(b) 263 (base 8) to base 10

(a)

$\begin{array}{l} 5\ \underline{|59} \\ 5\ \underline{|11}\ \text{r } 4 \\ 5\ \underline{|2}\ \text{r } 1 \\ \quad 0\ \text{r } 2 \end{array}$

so $59 = 214$ (base 5)

(b) 263 (base 8) $= 2 \times 8^2 + 6 \times 8 + 3 \times 1$
$= 2 \times 64 + 6 \times 8 + 3 \times 1$
$= 128 + 48 + 3$
$= 179$

Exercise 1

1 Work out:

(a) $\dfrac{1}{4} + \dfrac{1}{4}$ (b) $\dfrac{3}{5} + \dfrac{2}{5}$

(c) $2\dfrac{1}{4} + 1\dfrac{1}{4}$ (d) $3\dfrac{2}{5} + 1\dfrac{2}{5}$

(e) $4\dfrac{5}{9} + 2\dfrac{7}{9}$ (f) $\dfrac{1}{3} + \dfrac{1}{5}$

(g) $\dfrac{3}{4} + \dfrac{1}{3}$ (h) $\dfrac{3}{5} + \dfrac{3}{4}$

(i) $\dfrac{1}{6} + \dfrac{2}{3}$ (j) $\dfrac{3}{8} + \dfrac{5}{12}$

2 Calculate:

(a) $2\frac{3}{4}+1\frac{2}{3}$ (b) $3\frac{1}{2}+4\frac{3}{5}$

(c) $2\frac{1}{4}+3\frac{4}{5}$ (d) $6\frac{1}{8}+4\frac{2}{5}$

(e) $4\frac{5}{6}+2\frac{1}{2}+1\frac{3}{4}$

3 Work out:

(a) $2\frac{4}{7}-1\frac{3}{7}$ (b) $3\frac{2}{5}-1\frac{4}{5}$ (c) $\frac{3}{4}-\frac{1}{3}$

(d) $\frac{5}{8}-\frac{1}{2}$ (e) $\frac{4}{7}-\frac{2}{9}$ (f) $\frac{2}{3}-\frac{3}{5}$

(g) $3\frac{3}{4}-1\frac{3}{5}$ (h) $2\frac{1}{2}-1\frac{3}{4}$ (i) $3\frac{1}{3}-\frac{3}{4}$

(j) $2\frac{2}{5}-1\frac{3}{4}$ (k) $4\frac{2}{7}-1\frac{7}{8}$

4 Calculate:

(a) $\frac{1}{2}\times\frac{1}{4}$ (b) $\frac{2}{3}\times\frac{1}{4}$ (c) $\frac{3}{4}\times\frac{2}{5}$

(d) $\frac{4}{7}\times\frac{5}{6}$ (e) $\frac{3}{10}\times\frac{5}{6}$ (f) $1\frac{1}{2}\times1\frac{1}{2}$

(g) $2\frac{3}{4}\times\frac{3}{5}$ (h) $\frac{2}{3}\times4\frac{2}{5}$ (i) $3\frac{2}{3}\times1\frac{4}{7}$

(j) $4\frac{3}{10}\times3\frac{7}{10}$

5 Calculate:

(a) $\frac{1}{2}\div\frac{1}{4}$ (b) $\frac{1}{4}\div\frac{1}{2}$ (c) $\frac{3}{7}\div\frac{2}{7}$

(d) $\frac{4}{5}\div\frac{2}{3}$ (e) $\frac{3}{4}\div\frac{4}{5}$ (f) $3\frac{1}{2}\div\frac{1}{2}$

(g) $4\frac{3}{4}\div\frac{3}{4}$ (h) $2\frac{1}{2}\div1\frac{3}{4}$ (i) $2\frac{5}{6}\div1\frac{3}{10}$

(j) $4\frac{2}{3}\div2\frac{7}{9}$

6 Change:

(a) 71 (base 8) to base 10
(b) 314 (base 10) to base 5
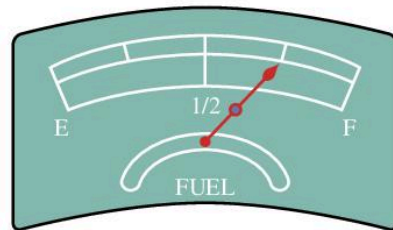(c) 213 (base 8) to base 10
(d) 7124 (base 10) to base 8
(e) 4311 (base 5) to base 10
(f) 6319 (base 10) to base 5

Application

7 Janice walked $5\frac{1}{2}$ km in $1\frac{1}{4}$ hours. What was her speed?

8 The drawing shows the fuel gauge in Ray's car. When full the fuel tank holds 120 litres.

(a) How much fuel does the tank hold now?
(b) After travelling a further 80 km, Ray noticed his tank was half full. How much fuel did he use?

9 There are 864 students at Greatfield High School. The roll is expected to increase by $\frac{1}{12}$ next year.
(a) How many students are expected at Greatfield next year?
(b) One third of the new intake will be boys. How many new girls are expected?

10 At Park Hill School there are 900 students and 50 staff.
a) What is the student teacher ratio?
b) How many more students would be needed for the student teacher ratio to be 20 : 1?
c) If 5 staff members left and were not replaced what would the student teacher ratio be?

Summary

You should know ...

1 To add or subtract fractions their denominators must be the same.

For example:

$$\frac{3}{4} + \frac{2}{5}$$
$$= \frac{15}{20} + \frac{8}{20}$$
$$= \frac{23}{20}$$
$$= 1\frac{3}{20}$$

You can't add 3 fourths to 2 fifths

but you can add 15 twentieths to 8 twentieths

2 How to multiply by fractions.

For example:

(a) $6 \times \frac{3}{4} = \frac{6 \times 3}{4} = \frac{18}{4} = 4\frac{2}{4} = 4\frac{1}{2}$

(b) $\frac{2}{7} \times \frac{3}{4} = \frac{2 \times 3}{7 \times 4} = \frac{6}{28} = \frac{3}{14}$

3 You should try to cancel fractions before multiplying to save working.

For example:

$\frac{6}{25} \times 3\frac{1}{3} = \frac{6}{25} \times \frac{10}{3} = \frac{{}^{2}6 \times \cancel{10}^{2}}{{}_{5}\cancel{25} \times \cancel{3}_{1}} = \frac{2 \times 2}{5 \times 1} = \frac{4}{5}$

4 To divide a fraction by a fraction, you turn over the second one and multiply.

For example:

$$\frac{3}{4} \div \frac{2}{5} = \frac{3}{4} \times \frac{5}{2}$$
$$= \frac{3 \times 5}{4 \times 2}$$
$$= \frac{15}{8}$$
$$= 1\frac{7}{8}$$

$\div \frac{2}{5}$ is the same as $\times \frac{5}{2}$

Check out

1 (a) Work out:

(i) $\frac{1}{2} + \frac{1}{4}$ (ii) $\frac{2}{3} + \frac{1}{4}$

(iii) $3\frac{1}{2} + \frac{2}{3}$ (iv) $1\frac{4}{7} + \frac{2}{9}$

(v) $\frac{2}{3} - \frac{1}{2}$ (vi) $3\frac{3}{4} - 1\frac{7}{8}$

(b) Beverly drinks $\frac{2}{3}$ litre of milk from a $1\frac{1}{2}$ litre bottle. How much milk is left in the bottle?

2 (a) Calculate:

(i) $3 \times \frac{1}{2}$ (ii) $6 \times \frac{2}{3}$

(iii) $\frac{3}{4} \times 8$ (iv) $\frac{2}{3} \times \frac{3}{4}$

(v) $\frac{3}{10} \times 1\frac{1}{2}$ (vi) $2\frac{1}{2} \times \frac{3}{5}$

(b) Kathy ate $\frac{3}{4}$ of a box of cornflakes. The box holds 152 g. What mass of cornflakes remains?

3 Cancel, then work out:

(a) $\frac{2}{3} \times 1\frac{1}{2}$

(b) $\frac{3}{5} \times \frac{5}{6}$

(c) $2\frac{3}{4} \times \frac{3}{22}$

(d) $3\frac{1}{2} \times 2\frac{3}{7}$

4 (a) Work out:

(i) $6 \div \frac{1}{2}$

(ii) $3\frac{1}{2} \div \frac{1}{2}$

(iii) $\frac{2}{3} \div \frac{3}{4}$

(iv) $3\frac{4}{5} \div 2\frac{2}{3}$

(b) How many $5\frac{1}{2}$ cm pieces of pipe can be cut from a pipe 66 cm long?

5 How to solve simple ratio problems.

For example:

Share $120 in the ratio 1 : 2 : 3

There are $1 + 2 + 3 = 6$ parts.

Each part is $120 \div 6 = 20

So the shares are

$20, $20 \times 2, 20×3 or

$20, $40 and $60

5 Share $90 in the ratio:

(a) 8 : 1   (b) 7 : 2

(c) 5 : 4   (d) 7 : 3

(e) 1 : 2 : 6   (f) 2 : 3 : 5

6 How to convert a number from one base into another.

6 Change:

(a) 312 (base 5) to base 10

(b) 417 (base 8) to base 10

(c) 72 to base 5

(d) 931 to base 8

Decimals

Objectives

- ✔ solve problems involving addition and subtraction of decimals
- ✔ solve problems involving multiplication and division of decimals
- ✔ calculate squares and square roots of decimals
- ✔ write a number to a given number of decimal places and significant figures
- ✔ write large numbers in standard form
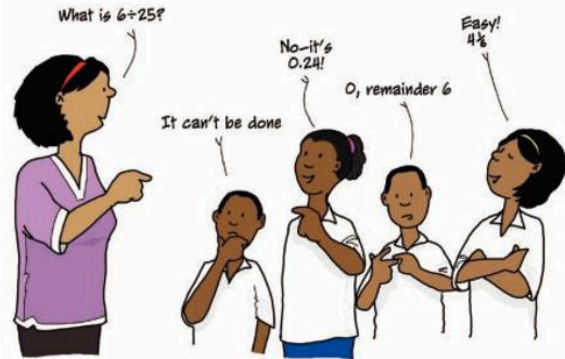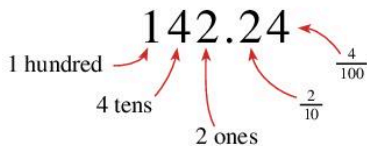

What's the point?

Medicine and nursing are just two professions that work with decimals on a daily basis. Have you gained weight? How tall are you? Do you have a temperature?

Before you start

You should know ...

1 A decimal is a way of writing a number using place values of tenths, hundredths etc.

For example:

$$142.24$$

1 hundred
4 tens
2 ones
$\frac{2}{10}$
$\frac{4}{100}$

2 A fraction with a denominator of 10 or 100 can be written as a decimal.

For example:

$\frac{78}{100} =$

U	$\frac{1}{10}$	$\frac{1}{100}$
0	7	8

$= 0.78$

Check in

1 Write down the value of each digit in these decimals.
- **(a)** 62
- **(b)** 6.2
- **(c)** 2.6
- **(d)** 25.6
- **(e)** 5.06
- **(f)** 621.4
- **(g)** 26.41
- **(h)** 2.416
- **(i)** 2.461

2 Write as decimals:
- **(a)** $\frac{3}{10}$
- **(b)** $\frac{13}{100}$
- **(c)** $\frac{3}{100}$
- **(d)** $1\frac{13}{100}$
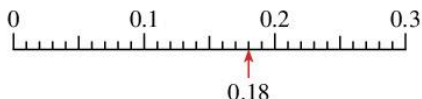- **(e)** $12\frac{2}{100}$

2.1   Ordering decimals

You can use a number line to help you order decimals.

Example 1

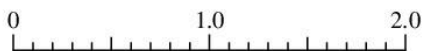Which is larger 0.2 or 0.18?

On a number line:


```
      0          0.1         0.2         0.3
      |..........|..........|..........|
                        ↑
                      0.18
```

So 0.2 is larger than 0.18

Exercise 2A

1   Draw number lines to help you find the larger of each pair of numbers.
    (a) 0.3, 0.26   **(b)** 0.53, 0.7
    (c) 0.04, 0.3   **(d)** 1.4, 0.86
    (e) 2.3, 2.04   **(f)** 5.07, 4.89
    (g) 16.1, 6.98   **(h)** 3.25, 3.3

2   **(a)** Copy this number line.

```
      0              1.0              2.0
      |..........|..........|..........|
```
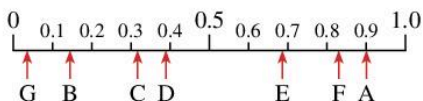
   (b) On your line mark the points

   A = 0.3,   B = 0.6,   C = 1.1
   D = 1.7,   E = 0.45,   F = 1.65

3   Estimate the value of the letters on this number line.

```
   0  0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0
   |   |   |   |   |   |   |   |   |   |   |
   ↑   ↑   ↑ ↑         ↑       ↑ ↑ ↑
   G   B   C D         E       F A
```

4   Write in order of size, smallest first:
    (a) 0.3, 0.21, 0.46, 0.18, 2.1
    (b) 4.8, 0.5, 0.46, 2.3, 0.41
    (c) 0.9, 0.81, 1, 1.01, 0.68
    (d) 2.4, 2.04, 4.2, 2, 2.13

5   Write down three numbers between 0.3 and 0.4.

6   This line is 4.2 cm long.

   ————————————————

What is the length of these lines in centimetres?

   (a) ——————————————

   (b) —————————————

   (c) ——————————

7   Which is longer 4 cm or 4.0 cm? Explain.

8   What is the value of each of the letters shown on this line?


```
   +---+---+---+---+---+---+---+
 6.34       ↑       ↑ ↑     ↑   6.35
            A       B C     D
```

2.2   Adding and subtracting decimals

To add and subtract decimals you must first line up the decimal points.

Example 2

Work out $2.40 + 0.61 + 9$

It helps to write in the column headings:

Remember
$9 = 9.00$

T	U		$\frac{1}{10}$	$\frac{1}{100}$
	2	.	4	0
	0	.	6	1
+	9	.	0	0
1	2	.	0	1

line up the decimal points

Example 3

Work out $6 - 0.23$

$6 = 6.00$

U		$\frac{1}{10}$	$\frac{1}{100}$
6	.	0	0
− 0	.	2	3
5	.	7	7

line up the decimal points

Exercise 2B

1 Work out:
(a) $2.3 + 1.4$ (b) $2.7 + 2.8$
(c) $4.2 + 5$ (d) $0.68 + 4.3$
(e) $1.98 + 2.1$ (f) $1.3 + 5 + 0.6$
(g) $16 + 2.5 + 0.9$ (h) $3 + 0.42 + 6.3$
(i) $102 + 0.3 + 6.25$ (j) $0.103 + 4.1 + 38$

2 Use your calculator to check your answers to Question 1.

3 Work out:
(a) $2.4 - 1.3$ (b) $2.4 - 1.6$
(c) $2 - 1.4$ (d) $5.3 - 2.24$
(e) $6.8 - 3.92$ (f) $9 - 0.41$
(g) $12 - 2.6$ (h) $103 - 7.9$
(i) $31.3 - 0.04$ (j) $11 - 0.94$

4 Use your calculator to check your answers to Question 3.

5 Teresa is 1.58 m tall. Her brother, Tony is 2 m tall. How much taller is Tony than Teresa?

6 For many years people thought that 100 m could not be run in less than 10 seconds.

In the 2008 Olympics Usain Bolt ran 100 m in 9.69 seconds. By how much did he break the 10-second barrier?

7 To install his new bath Ashton requires two pieces of pipe.

The first piece needs to be 3.45 m long and the second 4 m long.
(a) What is the total length of pipe that Ashton requires?
(b) If pipe is sold in 6 m lengths, how many lengths should he buy?
(c) How much pipe would be left over from the job?

8 In July 1999, Hicham El Guerrouj of Morroco set a world record for running the mile in 3 minutes 43.13 seconds. By how much did he beat the 4-minute mile barrier?

2.3 Multiplying decimals

Multiplying by 10, 100 and 1000

When you multiply a number by 10, 100 or 1000 you get a pattern:

$$36 \times 10 = 360$$
$$36 \times 100 = 3600$$
$$36 \times 1000 = 36\,000$$
$$36 \times 10\,000 = 360\,000$$

The same is true when you multiply a decimal by 10, 100 or 1000:

$$3.6 \times 10 = 36$$
$$3.6 \times 100 = 360$$
$$3.6 \times 1000 = 3600$$
$$3.6 \times 10\,000 = 36\,000$$

In Exercise 2C you will discover a rule for the pattern.

Exercise 2C

1 Use a calculator to work out:
(a) 10×3.65 (b) 10×4.3
(c) 10×9.1 (d) 10×2.06
(e) 10×0.63 (f) 10×0.124

2 Work these out in the same way:
(a) 100×3.65 (b) 100×4.3
(c) 100×9.1 (d) 100×2.06
(e) 100×0.63 (f) 100×0.124

3 Copy and complete the table for all your answers to Questions 1 and 2.

Number	Number × 10	Number × 100
3.65	36.5	365
4.3	43	
9.1		

4 Look at any number in the table in Question 3. What happens to the numbers in the decimal when you multiply by 10?
What happens to them when you multiply by 100?

5 Use the pattern you discovered above to work out without multiplying:
(a) 3.62×10 (b) 9.4×10
(c) 0.46×10 (d) 0.04×10
(e) 2.19×10 (f) 21.19×10

6 Repeat Question 5 but this time multiply by 100.

7 What do you think the rule would be for multiplication by 1000?

8 Write down the answers to:
(a) 6.253×1000 (b) 0.125×1000
(c) 6.48×1000 (d) 6.4×1000
(e) 0.4×1000 (f) 0.003×1000

9 Check your answers for Question 8 on a calculator. Did you get the decimal point in the right place?

10 Copy and complete these sentences.
(a) When a decimal is multiplied by 10, each number in the decimal moves … place to the …
(b) When a decimal is multiplied by 100, each number in the decimal moves… places to the …
(c) When a decimal is multiplied by 1000, each number in the decimal moves … places to the …

11 Another way to multiply a decimal by 10 or 100 is to change it back into a fraction. Look at these examples:

$$10 \times 2.68 = 10 \times (2 + \tfrac{6}{10} + \tfrac{8}{100})$$
$$= 10 \times 2 + 10 \times \tfrac{6}{10} + 10 \times \tfrac{8}{100}$$
$$= 20 + 6 + \tfrac{8}{10}$$
$$= 26.8$$

$$100 \times 0.17 = 100 \times (\tfrac{1}{10} + \tfrac{7}{100})$$
$$= 100 \times \tfrac{1}{10} + 100 \times \tfrac{7}{100}$$
$$= 10 + 7$$
$$= 17$$

Do these multiplications in the same way.
(a) $10 \times (20 + 4)$
(b) $10 \times (20 + 2 + \tfrac{3}{10})$
(c) $10 \times (1 + \tfrac{3}{10} + \tfrac{5}{100})$
(d) $10 \times (6 + \tfrac{7}{10} + \tfrac{4}{100} + \tfrac{9}{1000})$
(e) $100 \times (3 + \tfrac{2}{10})$
(f) $100 \times (5 + \tfrac{3}{10} + \tfrac{7}{100})$
(g) $100 \times (\tfrac{6}{10} + \tfrac{9}{100} + \tfrac{8}{1000})$
(h) 10×1.2
(i) 10×0.25
(j) 10×3.289
(k) 100×4.9
(l) 100×13.45
(m) 1000×7.163

Multiplying decimals by decimals

Multiplying decimals is like multiplying whole numbers. The only difference is you have to put the decimal point in the right place. You can do this from an estimate of the answer.

Example 4

Work out 7.4×8

The estimate is $7 \times 8 = 56$
$$\begin{array}{r} 7.4 \\ \times 8 \\ \hline 59_3.2 \end{array}$$
The digits in the answer are 592
The estimate is 56
Place the decimal point to give 59.2
That is, $7.4 \times 8 = 59.2$

The method is the same even if both numbers are decimals.

Example 5

Work out 2.3×0.6

The estimate is $2 \times 1 = 2$
$$\begin{array}{r} 2.3 \\ \times 0.6 \\ \hline 1.3_18 \end{array}$$
The digits are 138
The estimate is 2
Place the decimal point to give 1.38
That is, $2.3 \times 0.6 = 1.38$

Exercise 2D

1   Copy and complete:

Work out	Estimate	Answer
3.2×4	12	
6.9×9		
32.5×8		
1.68×3		
12.4×13		

2   Work out:
    (a) 0.4×8 **(b)** 0.9×6
    (c) 2.3×5 **(d)** 4.7×8
    (e) 6×1.4 **(f)** 12×0.6
    (g) 11×0.42 **(h)** 4×0.65
    (i) 3.24×8 **(j)** 4.85×12

3   Find the cost of 12 pencils each selling for $0.45.

4   If a packet of rice holds 1.4 kg, what is the mass of six packets?

5   Given that $45 \times 19 = 855$
    find
    (a) 45×1.9 **(b)** 4.5×19
    (c) 45×0.19 **(d)** 4.5×1.9
    (e) 0.45×19 **(f)** 0.45×1.9

6   What is the total length of 20 pieces of pipe each 3.25 m long?

7   A tyre costs $153.94.
    What would be the cost of six tyres?

8   Another way to multiply decimals is to change them to fractions. Look at the example below.

$$6 \times 4.32 = 6 \times 4\frac{32}{100}$$
$$= 6 \times \frac{432}{100}$$
$$= \frac{2592}{100}$$
$$= 25\frac{92}{100}$$
$$= 25.92$$

Use this method to work out:
    (a) 3×2.4 **(b)** 6.7×8
    (c) 4×0.3 **(d)** 12×0.7
    (e) 2.48×9 **(f)** 13×0.07
    (g) 1.3×1.6 **(h)** 4.6×0.7
    (i) 0.3×0.4 **(j)** 0.32×0.8

Activity

Estimation game (2 players)

15	0.3	2.46	8.4
0.07	1.4	5	2.1
12.7	18.9	68	0.12

- First player chooses two numbers from the set above and estimates their product (the answer when the numbers are multiplied).
- Second player also estimates the product.
- With a calculator, check who had the best estimate.
- The player to get ten best estimates wins.

2.4   Dividing decimals

Dividing by 10, 100 and 1000

As with multiplication there is a simple pattern when you divide a number by 10, 100 or 1000.

$42 \div 10$	$= 4.2$
$42 \div 100$	$= 0.42$
$42 \div 1000$	$= 0.042$
$42 \div 10\,000$	$= 0.0042$

Can you see the pattern?

Exercise 2E

1   Copy and complete these divisions:
    (a) $74.5 \div 10 = \square$ **(b)** $89 \div 10 = \square$
    (c) $3.4 \div 10 = \square$ **(d)** $30.4 \div 10 = \square$
    (e) $0.6 \div 10 = \square$ **(f)** $10.06 \div 10 = \square$
    (g) $184 \div 10 = \square$ **(h)** $216.3 \div 10 = \square$

2   Look at the divisions you completed in Question 1. What happens to each digit when you divide a decimal by 10? Write a rule for dividing decimals by 10.

3   Copy and complete these divisions:
    (a) $745 \div 100 = \square$
    (b) $890 \div 100 = \square$
    (c) $34 \div 100 = \square$
    (d) $304 \div 100 = \square$
    (e) $6 \div 100 = \square$

(f) $100.6 \div 100 = \square$
(g) $1840 \div 100 = \square$
(h) $2163 \div 100 = \square$

4 Copy and complete this table.

Number	Number ÷10	Number ÷100	Number ÷1000
74.5	7.45	0.745	0.0745
89	8.9	0.89	
3.4			
30.4			
0.6			
485			
1024			
102.4			
10.24			
6			

Use your calculator to check your answers.

5 Copy and complete these sentences.
   (a) When a decimal is divided by 10, each digit in the decimal moves … place to the … .
   (b) When a decimal is divided by 100, each digit in the decimal moves … places to the … .
   (c) When a decimal is divided by 1000, each digit in the decimal moves … places to the … .

Dividing decimals by whole numbers

Dividing decimals by whole numbers is similar to dividing whole numbers by whole numbers.

Example 6

Work out $3.8 \div 4$

$$\begin{array}{r} 0.95 \\ 4\overline{)3.80} \\ 3\ 6 \\ \hline 20 \\ 20 \\ \hline \end{array}$$ line up the decimal points

You can also work them out as 'short' divisions.

Example 7

Work out $0.175 \div 5$

$$\begin{array}{r} 0.035 \\ 5\overline{)0.1\overset{1}{7}\overset{2}{5}} \end{array}$$

line up decimal points

Exercise 2F

1 Work out:
  (a) $5.76 \div 4$ **(b)** $6.25 \div 5$
  (c) $9.66 \div 7$ **(d)** $2.75 \div 5$
  (e) $1.38 \div 6$ **(f)** $2.712 \div 8$
  (g) $0.64 \div 4$ **(h)** $0.23 \div 5$
  (i) $0.0784 \div 7$

2 Work out:
  (a) $15.1 \div 4$ **(b)** $19.32 \div 6$
  (c) $15.664 \div 11$ **(d)** $21.69 \div 9$
  (e) $28.5 \div 12$ **(f)** $31.36 \div 8$

What do you notice about some of the answers? Were some of them supposed to be recurring?

3 Four boys shared $63 among them equally. How much did each receive?

4 A book made from 400 sheets of paper is 2.5 cm thick. What is the thickness of each sheet of paper?

5 A 3 kg piece of cheese sells for $72.60. What is the price of cheese per kilogram?

6 The area of a rectangular field is $2136\ m^2$. What is the width of the field if its length is 60 m?

7 Nine tubes of toothpaste are sold for $88.65. How much do four tubes cost?

8 Half a lamb weighing 17 kg sells for $145.52. How much is the lamb per kilogram?

Dividing decimals by decimals

When you divide a decimal by a decimal, turn the divisor into a whole number.

> ### Example 8
>
> Work out $3.66 \div 0.6$
>
> *Multiply by 10 to turn 0.6 into a whole number.*
>
> $$3.66 \div 0.6 = \frac{3.66}{0.6}$$
>
> $$= \frac{3.66 \times 10}{0.6 \times 10}$$
>
> $$= \frac{36.6}{6}$$
>
> $$= 6.1$$

In Example 8 both the numerator and denominator were multiplied by the same number. Sometimes you will need to multiply by 100.

> ### Example 9
>
> Work out $0.028 \div 0.14$
>
> *Multiply by 100 to turn 0.14 into a whole number.*
>
> $$0.028 \div 0.14 = \frac{0.028}{0.14}$$
>
> $$= \frac{0.028 \times 100}{0.14 \times 100}$$
>
> $$= \frac{2.8}{14}$$
>
> $$= 0.2$$

Exercise 2G

1   Look at this division.

$$72 \div 0.9 = \frac{72}{0.9}$$

  (a) Is it easier to divide by 0.9 or by 9?
  (b) By what must you multiply 0.9 to give 9?
  (c) So by what must you multiply the 72?
  (d) Copy and complete:

$$\frac{72}{0.9} = \frac{72 \times 10}{0.9 \times 10} = \frac{\square}{\square} = \square$$

2   Look at this division.

$$6.182 \div 1.2 = \frac{6.182}{1.2}$$

  (a) Is it easier to divide by 1.2 or by 12?
  (b) By what must you multiply 1.2 to give 12?
  (c) So by what must you multiply the 6.182?
  (d) Copy and complete:

$$\frac{6.182}{1.2} = \frac{6.182 \times \square}{1.2 \times \square} = \frac{\square}{\square} = \square$$

3   Work out:
  (a) (i) $6.4 \div 0.8$ (ii) $64 \div 8$
  (b) (i) $3.5 \div 0.5$ (ii) $35 \div 5$
  (c) (i) $4 \div 0.5$ (ii) $40 \div 5$
  (d) (i) $4.8 \div 0.12$ (ii) $480 \div 12$
  (e) (i) $3.64 \div 0.4$ (ii) $36.4 \div 4$

Compare your answers to parts (i) and (ii) in each case. What do you notice?

4   Work out:
  (a) $5.7 \div 0.4$ (b) $5.7 \div 0.04$
  (c) $57 \div 0.04$ (d) $1.23 \div 1.5$
  (e) $61 \div 0.3$ (f) $34.6 \div 0.07$
  (g) $2.5 \div 3.6$ (h) $14.4 \div 2.8$
  (i) $1.34 \div 0.61$ (j) $1.34 \div 0.061$

5   Work out:
  (a) $5.28 \div 2.2$ (b) $19.52 \div 6.1$
  (c) $29.93 \div 4.1$ (d) $0.441 \div 1.4$
  (e) $2.52 \div 0.112$ (f) $0.336 \div 2.4$
  (g) $9.64 \div 0.16$ (h) $0.042 \div 0.14$
  (i) $6 \div 3.14$ (j) $1.75 \div 0.015$

6

Quick Wash SOAP POWDER — Extra large 1.3 kg

1.3 kg of Quick Wash soap powder costs \$10.66. How much does:
  (a) 1 kg of powder cost
  (b) 0.75 kg of powder cost?

7   There are 2.2 lb in 1 kg.

How many kilograms are there in packets of rice marked:
  (a) 1 lb   (b) 2 lb   (c) 7.5 lb?

8

A centipede is timed to travel 4.65 m in 7.5 s. What is its speed in metres per second?

9

(a) Measure the thickness of a 25 cent coin in millimetres.
(b) How many coins, placed on top of one another, would make a 10 cm tall pile?
(c) How many would make a 100 m pile?

Technology

That's quite a bit on decimals. You may need to review what you've done in your own time.
Visit

www.aaamath.com/dec.htm

or

www.coolmath.com/prealgebra

for a complete series of lessons on all decimal operations.

2.5 Squares and square roots of decimals

You will need a calculator.

● To **square** a number you multiply it by itself.

For example,

$$4^2 = 4 \times 4 = 16$$
$$12^2 = 12 \times 12 = 144$$
$$x^2 = x \times x = x^2$$
$$16^2 = 16 \times 16 = 256$$

3^2 is read as 'three squared'.
x^2 is read as 'x squared'.

Exercise 2H

1 Calculate:
(a) 6^2 (b) 10^2 (c) 15^2
(d) 25^2 (e) 0.4^2 (f) 0.1^2
(g) 0.02^2 (h) 1.5^2 (i) 1.41^2

2 Copy and complete the table of squares started below. Compare the three columns.

$1^2 = 1$	$10^2 = 100$	$0.1^2 = 0.01$
$2^2 = 4$	$20^2 = 400$	$0.2^2 = 0.04$
$3^2 =$	$30^2 =$	$0.3^2 =$
$4^2 =$	$40^2 =$	$0.4^2 =$
$5^2 =$	$50^2 =$	$0.5^2 =$
$6^2 =$	$60^2 =$	$0.6^2 =$
$7^2 =$	$70^2 =$	$0.7^2 =$
$8^2 =$	$80^2 =$	$0.8^2 =$
$9^2 =$	$90^2 =$	$0.9^2 =$

3 Copy and complete this list of square numbers.

$$1^2 = 1 \times 1 = 1$$
$$2^2 = 2 \times 2 = 4$$
$$3^2 = 3 \times 3 = 9$$
$$4^2 = 4 \times 4 = 16$$
$$5^2 = 5 \times 5 = 25$$
$$6^2 = 6 \times 6 = 36$$
$$7^2 = 7 \times 7 = 49$$
$$\vdots \quad \vdots \quad \vdots$$
$$20^2 = 20 \times 20 = 400$$

4 Copy and complete:
(a) $\square^2 = 36$ (b) $\square^2 = 100$
(c) $\square^2 = 225$ (d) $\square^2 = 625$
(e) $\square^2 = 0.16$ (f) $\square^2 = 0.01$

Square roots

The inverse of the square of a number is called its square root ($\sqrt{}$).

For example, as a flow chart:

$6 \longrightarrow \boxed{\text{square}} \longrightarrow 36$
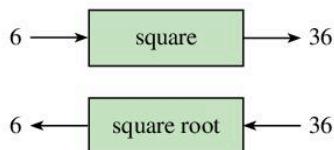
$6 \longleftarrow \boxed{\text{square root}} \longleftarrow 36$

That is $\sqrt{36} = 6$

Exercise 2I

1 Find the square root of each of these numbers.
   (a) 16   (b) 81   (c) 121   (d) 169
   (e) 289   (f) 225   (g) 361   (h) 441

2 In each list find the square number and work out its square root.
   (a) 35, 36, 39, 42, 48
   (b) 39, 49, 69, 89, 99
   (c) 63, 64, 65, 74, 75

3 Find all the numbers less than 400 which have whole number square roots.

4 Copy and complete:
   (a) $\sqrt{16} = \square$ (b) $\sqrt{64} = \square$
   (c) $\sqrt{100} = \square$ (d) $\sqrt{400} = \square$
   (e) $\sqrt{225} = \square$ (f) $\sqrt{900} = \square$

5 Find the answer, using the table in Question 2 of Exercise 2H:
   (a) $\sqrt{4}$ (b) $\sqrt{9}$ (c) $\sqrt{1600}$
   (d) $\sqrt{3600}$ (e) $\sqrt{8100}$ (f) $\sqrt{0.09}$
   (g) $\sqrt{0.25}$

Using a calculator

You can use the $\boxed{\sqrt{}}$ button on your calculator to find the square root of a number.

Example 10

Find the square root of 25.

Press

  $\boxed{2}$ $\boxed{5}$ $\boxed{\sqrt{}}$

to get $\sqrt{25} = 5$. On some calculators, the order is reversed, that is, press

  $\boxed{\sqrt{}}$ $\boxed{2}$ $\boxed{5}$ etc.

Exercise 2J

1 Use your calculator to find:
   (a) $\sqrt{81}$ (b) $\sqrt{144}$
   (c) $\sqrt{169}$ (d) $\sqrt{256}$

2 Use your calculator to find the square root of these numbers. Write down the full calculator display.
   (a) 0.419   (b) 3.146   (c) 10.15   (d) 0.695
   (e) 0.985   (f) 36.15   (g) 19.23   (h) 13.46
   (i) 12.64   (j) 20.16   (k) 0.905   (l) 0.425

3 Use your calculator to work out:
   (a) $\sqrt{45} + \sqrt{80}$ (b) $\sqrt{120} \times \sqrt{65}$
   (c) $\sqrt{10} \times \sqrt{19}$ (d) $\sqrt{148} - \sqrt{50}$
   (e) $\sqrt{98} + \sqrt{46}$ (f) $\sqrt{160} \div \sqrt{40}$
   (g) $\sqrt{96} \div \sqrt{28}$ (h) $\sqrt{105} - \sqrt{39}$

4 Find the edge length of a square with the area shown:
   (a)   (b)

   130 cm²   270 cm²

   (c)   (d)

   360 cm²   450 cm²

5 Copy and complete, using words and symbols from the list below.
   (a) 9^2 is a __ way of writing __ __ __.
   (b) The small raised __ is called the __ or the __.
   (c) The symbol __ stands for the words __ __ __.
   (d) The __ __ on a __ is pressed to find the __ of a number.

1. short	6. the square root
2. x^2 key	7. nine times nine
3. square	8. index
4. calculator	9. ^{2}
5. power	10. $\sqrt{}$

6 The product of two consecutive whole numbers is 2862. What are the numbers?

7 The product of two consecutive whole numbers is 61 256. What are the numbers?

Activity

You can use a trial and error method to find the square root of a number without using the $\sqrt{}$ button on your calculator.

To find $\sqrt{53}$

Guess	Check (square of guess)	Result
7	49	Too small
8	64	Too big
7.5	56.25	Too big
7.3	53.29	Too big
7.25	52.5625	Too small
7.28	52.9984	About right

So $\sqrt{53} = 7.28 \approx$

Use this method to find:

(a) $\sqrt{30}$ (b) $\sqrt{50}$ (c) $\sqrt{2}$ (d) $\sqrt{74}$

(e) $\sqrt{96}$ (f) $\sqrt{128}$ (g) $\sqrt{812}$ (h) $\sqrt{1121}$

Check your answers on a calculator.

2.6 Rounding numbers

Whole numbers are usually rounded to the nearest 10, 100 or 1000. To do this it helps to look at a number line.

For example, to round 67 to the nearest 10, show it on a number line:
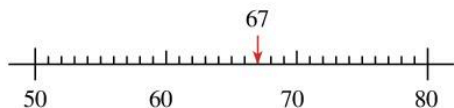
67 is closer to 70 than to 60 so
67 rounded to the nearest 10 is 70.
If the number is half way between, you round up.
For example, 85 is half way between 80 and 90, so rounded off to the nearest 10 it becomes 90.

Exercise 2K

1   Using a number line, round off to the nearest ten:
   (a) 43   (b) 68   (c) 92   (d) 104
   (e) 7   (f) 216   (g) 105   (h) 315

2   Round off to the nearest ten:
   (a) 33   (b) 34   (c) 35   (d) 36
   (e) 37   (f) 9   (g) 123   (h) 135
   (i) 200   (j) 1086   (k) 1545

3   (a) Is 639 nearer to 600 or to 700?
   (b) Round off 639 to the nearest hundred.

4   Write to the nearest hundred:
   (a) 98   (b) 103   (c) 149
   (d) 150   (e) 155   (f) 190
   (g) 235   (h) 550   (i) 601

5   Round off to the nearest hundred:
   (a) 3145   (b) 4799
   (c) 5310   (d) 1689
   (e) 3550   (f) 4480

6   The nearest whole number to 5.6 is 6. Give to the nearest whole number:
   (a) 4.1   (b) 4.4   (c) 4.5   (d) 4.6
   (e) 5.25   (f) 6.51   (g) 8.99   (h) 10.014

Rounding decimal values

To round a number to one decimal place you can also look at a number line.

Example 11

Round 3.14 to 1 decimal place.

First show 3.14 on a number line:
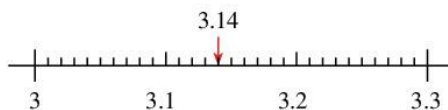
Notice 3.14 is closer to 3.1 than to 3.2, so 3.14 rounded to one decimal place is 3.1.

Notice that a number rounded to one decimal place has only **one** digit after the decimal point.

To round to one decimal place you need to look at the **second** digit after the decimal point.

● If the second digit is 0, 1, 2, 3 or 4, leave the first digit as it is.

For example,
0.83 = 0.8 (to 1 decimal place)
0.82 = 0.8 (to 1 decimal place)

● If the second digit is 5, 6, 7, 8 or 9, round up the first digit by one.

For example
0.68 = 0.7 (to 1 decimal place)
0.66 = 0.7 (to 1 decimal place)

Exercise 2L

1 The number line shows decimals from 1.5 to 1.8.
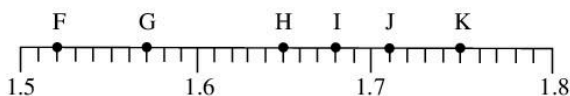
(a) Write down the decimal represented on the number line above by the point:
(i)   F   (ii) G   (iii) H
(iv) I   (v) J   (vi) K

(b) Now express each answer in part (a) to one decimal place.

2 Round to one decimal place:
(a) 1.49   (b) 0.52   (c) 0.55
(d) 2.83   (e) 3.98   (f) 3.02

3 Give the number to one decimal place:
(a) 4.614   (b) 5.002   (c) 1.911
(d) 0.471   (e) 2.599   (f) 0.894

4 When 3.858 is rounded off to two decimal places, it becomes 3.86.
Give to two decimal places:
(a) 1.285   (b) 0.036   (c) 0.476
(d) 0.832   (e) 1.509   (f) 2.006

5 Copy and complete this table.

Decimal value	1 d.p.	2 d.p.	3 d.p.
716.2581			
41.0215			
0.001 38			
1.764 31			
15.018 12			

6 A boy's height when rounded to one decimal place is 1.4 metres. What is the boy's maximum possible height?

Significant figures

Another way to round numbers is to round the number to a given number of *significant* figures.

Look at the number 682.

Which number is the most significant?

The answer is the 6 because it represents 6 hundreds (600).

It is more important than the 8 (80) and the 2 (2).

In the number 682, the first significant figure (sig. fig.) is the 6, the second is the 8 and the third is the 2.

To round 682 to 1 sig. fig. means rounding 682 to the nearest 100. That is,

682 = 700 (1 sig. fig.)

To round 682 to 2 sig. fig. means rounding 682 to the nearest 10. That is,

682 = 680 (2 sig. fig.)

The first significant figure in a number is always the first non-zero number.

For example, in the number 0.0703, the first significant figure is the 7, that is $\frac{7}{100}$ and

0.0703 = 0.07 (1 sig. fig.)

Example 12

Write:
(a) 6076 to
(i)   1 sig. fig   (ii)   3 sig. fig

(b) 0.00396 to
(i)   1 sig. fig   (ii)   2 sig. fig

(a) Inserting column headings
Th   H   T   O
6   0   7   6

(i) The first significant figure is thousands, so round to nearest thousand
6076 = 6000 (1 s.f.)

(ii) The third significant figure is tens, so round to nearest ten
6076 = 6080 (3 s.f.)

(b) Inserting column headings

1	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$	$\frac{1}{10000}$	$\frac{1}{100000}$
0	.0	0	3	9	6

  (i) The first significant figure is three thousandths, so round to nearest thousandth

$$0.00396 = 0.004 \text{ (1 s.f.)}$$

  (ii) The second significant figure is one ten thousandth, so round to nearest ten thousandth

$$0.00396 = 0.0040 \text{ (2 s.f.)}$$

Exercise 2M

1 Write correct to one significant figure:
  (a) 69   **(b)** 42   **(c)** 312
  (d) 418   **(e)** 569   **(f)** 608
  (g) 3416   **(h)** 43 204 **(i)** 12 002

2 Write to one significant figure:
  (a) 0.34   **(b)** 0.58   **(c)** 0.62
  (d) 0.315   **(e)** 0.042   **(f)** 0.806
  (g) 0.0026 **(h)** 0.0043 **(i)** 0.000923

3 Write to two significant figures:
  (a) 432   **(b)** 619   **(c)** 299
  (d) 3824   **(e)** 4706   **(f)** 1234
  (g) 14 968 **(h)** 39 804 **(i)** 688 426

4 Write to two significant figures:
  (a) 1.35   **(b)** 13.62   **(c)** 8.09
  (d) 0.0346 **(e)** 0.0172 **(f)** 0.00266
  (g) 31.004 **(h)** 206.34 **(i)** 293.6

5 Copy and complete:

Number	1s.f.	2s.f.	3s.f.
4139	4000		
6804			
2007			
138.6		140	
154.28			
0.03168			
2.0352			2.04
0.004169			

2.7 Standard form

Rounding off is one way of dealing with numbers that are very long because they have been written with many places of decimals.

Very large numbers can also be troublesome to write down. For example, the mass of the Earth is 5 980 000 000 000 000 000 000 000 kg!

To write these numbers in a simple way you can use *scientific notation* or *standard form*.

In standard form a number is written as:

Number including 1 but not including 10 × ten to a power

For example,

$$6\,000\,000 = 6 \times 1\,000\,000$$
$$= 6 \times 10^6$$
$$95\,000 = 9.5 \times 10\,000$$
$$= 9.5 \times 10^4$$

Using a power of ten makes a very long number short!

Mass of the Earth
= 5 980 000 000 000 000 000 000 000 kg
= 5.98×10^{24} kg

To get the idea, look at the next example.

Example 13

Write in standard form:
(a) 60 782   **(b)** 93 105 000

(a) $60\,782 = 6.0782 \times 10\,000$
            $= 6.0782 \times 10^4$

> This number must lie between 1 and 10!

(b) $93\,105\,000 = 9.3105 \times 10\,000\,000$
              $= 9.3105 \times 10^7$

Exercise 2N

1 Copy and complete:

(a) $3000 = 3 \times \underline{\quad} = 3 \times 10^3$

(b) $320 = \underline{\quad} \times 100 = \underline{\quad} \times 10^2$

(c) $6100 = \underline{\quad} \times 1000 = \underline{\quad} \times 10^-$

(d) $25\,000 = \underline{\quad} \times 10\,000 = \underline{\quad} \times \underline{\quad}$

(e) $251\,000 = \underline{\quad} \times 100\,000 = \underline{\quad} \times 10^-$

2 Write in standard form:

(a) 6000　　(b) 200　　(c) 90

(d) 40 000　　(e) 20　　(f) 13 000

(g) 200 000　　(h) 5 000 000　　(i) 170 000

3 Write in full:

(a) 2×10^1　　(b) 2×10^2　　(c) 2×10^3

(d) 2×10^4　　(e) 8×10^6　　(f) 9×10^7

(g) 6×10^5　　(h) 4×10^7　　(i) 3×10^9

4 Write in standard form:

(a) 450　　(b) 230　　(c) 910

(d) 4500　　(e) 2300　　(f) 91 000

(g) 325　　(h) 496　　(i) 7254

5 Write in full:

(a) 2.4×10^3　　(b) 1.2×10^4　　(c) 6.2×10^2

(d) 9.8×10^3　　(e) 4.25×10^4　　(f) 6.1×10^5

(g) 2.165×10^2　　(h) 8.14×10^3　　(i) 9.11×10^8

6 Research the distances of the planets from the Sun. Write each distance in standard form.

Activity

Find the mass in grams of 15 different animals from a mouse to an elephant. Make a display with pictures of the animals and their masses written in standard form.

Technology

Search the internet to find out more about scientific notation.
Why is it useful?
Visit

www.purplemath.com/modules/exponent4.htm

and

www.gcse.com/maths/standard_form.htm

to find out how to write a very large or very small number in scientific notation.

2.8 Writing fractions as decimals

Any fraction can be written as a decimal.
For example,

$$\frac{1}{2} = 1 \div 2 = 1.0 \div 2 = 0.5$$

$$\frac{3}{4} = 3 \div 4 = 3.00 \div 4 = 0.75$$

To convert fractions to decimals you will need to divide.

Example 14

Change the fractions to decimals:

(a) $\frac{1}{4}$　　　　(b) $\frac{3}{8}$

(a) $\frac{1}{4} = 1 \div 4 = 1.00 \div 4$

$$\begin{array}{r} 0.\,2\,5 \\ 4\overline{\smash{)}1.^10^20} \end{array}$$

So $\frac{1}{4} = 0.25$

(b) $\frac{3}{8} = 3 \div 8 = 3.000 \div 8$

$$\begin{array}{r} 0.3\,7\,5 \\ 8\overline{\smash{)}3.0^60^40} \end{array}$$

So $\frac{3}{8} = 0.375$

Exercise 2O

1 Change these fractions to decimals.

(a) $\frac{2}{5}$ (b) $\frac{3}{5}$ (c) $\frac{4}{5}$ (d) $\frac{1}{4}$

(e) $\frac{3}{4}$ (f) $\frac{3}{8}$ (g) $\frac{5}{8}$ (h) $\frac{7}{8}$

(i) $\frac{1}{16}$ (j) $\frac{5}{16}$

2 Change these fractions to decimals. Check each answer on your calculator.

(a) $\frac{9}{16}$ (b) $\frac{11}{16}$ (c) $\frac{3}{20}$ (d) $\frac{7}{20}$

(e) $\frac{11}{20}$ (f) $\frac{3}{200}$ (g) $\frac{7}{200}$ (h) $\frac{1}{250}$

3 Change these fractions to decimals.

(a) $\frac{1}{3}$ (b) $\frac{2}{3}$ (c) $\frac{1}{6}$

What do you notice?

These decimals are called **recurring decimals** because they go on for ever.

4 The fraction $\frac{1}{3} = 0.333\ldots$

The digits in the decimal repeat themselves. Decimals like this are called **recurring decimals**. Find four other recurring decimals, using your calculator to help you.

5 Look at this diagram. It shows ten identical strips of paper. A fraction of each is shaded.
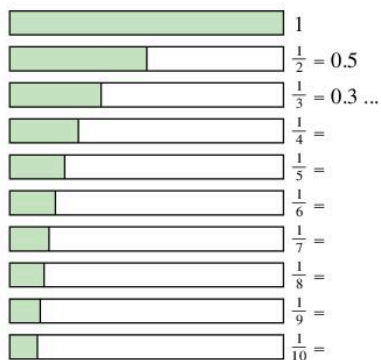
The fractions are written on the right-hand side. The first two have been converted to decimals.

(a) Why are there dots after the fraction for $\frac{1}{3}$?

(b) Copy the list of fractions and convert the rest of them to decimals.

(c) Which of these decimals are
   (i) terminating   (ii) recurring?

Use your calculator for these questions.

6 Convert the fraction to a decimal. If the decimal is non-terminating, find out whether it recurs:

(a) $\frac{5}{6}$ (b) $\frac{2}{7}$ (c) $\frac{4}{7}$ (d) $\frac{2}{9}$

(e) $\frac{5}{9}$ (f) $\frac{2}{11}$ (g) $\frac{1}{14}$ (h) $\frac{1}{12}$

7 In Question 6, did you find that all the decimals were recurring? When you change a fraction to a decimal, does the decimal always either terminate or recur?

8 The decimal

$$0.333\ldots = \tfrac{1}{3}$$

Find fractions that represent these decimals.

(a) $0.111\ldots$ (b) $0.555\ldots$ (c) $0.1818\ldots$

2.9   Using a calculator
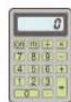
In real life, numbers are not always easy to work with. A calculator is often a great help.

Example 15

1 inch = 2.54 cm

What is, to one decimal place,
   (a) 13.4 inches in centimeters
   (b) 21.7 cm in inches?

(a) 13.4 inches = 13.4×2.54 cm

On the calculator enter

① ③ · ④ ✕ ② · ⑤ ④ =

The result is 34.036
So 13.4 inches = 34.0 cm (1 d.p.)

(b) 21.7 cm = $21.7 \div 2.54$ inches

On the calculator enter

② ① · ⑦ ÷ ② · ⑤ ④ =

The result is 8.543 307 087
So 21.7 cm = 8.5 cm (1 d.p.)

Notice that you press the keys on your calculator in the same order as you write down the calculation.

Exercise 2P

You will need your calculator to do some of these.

1 What is the cost of 93 books at $7.65 each?

2 36 plates cost $214.20. How much is each plate?

3 Given that

1 ounce (oz) = 28.35 g
1 pound (lb) = 0.4536 kg

Find:
(a) 3.8 oz in grams
(b) 76 g in ounces
(c) 18 lb in kilograms
(d) 9.6 kg in pounds

4 A plastic water pipe costs $168.75 for 25 m.
(a) Find the cost of 1 m of pipe.
(b) What will be the cost of 0.62 m?

5 (a) Change these fractions to decimals:
$$\frac{2}{3}, \frac{3}{8}, \frac{1}{4}, \frac{2}{5}, \frac{3}{7}$$
(b) Put them in order with the smallest first.

6 (a) Change the fractions:
$$\frac{1}{7}, \frac{2}{7}, \frac{3}{7}, \frac{4}{7}, \frac{5}{7}, \frac{6}{7}$$
to decimals. Write only the first six figures of your answers.
(b) What do you notice?

7 (a) What two identical numbers when multiplied together make 36? That is, find $\square$ if $\square \times \square = 36$.
(b) Did you find that $\square = 6$?

8 Use your calculator to find $\square$ if:
(a) $\square \times \square = 81$ (b) $\square \times \square = 289$
(c) $\square \times \square = 20$ (d) $\square \times \square = 30$
(e) $\square \times \square = 3$ (f) $\square \times \square = 2$
(g) $\square \times \square = 150$ (h) $\square \times \square = 942$

9 Check your answers to Question 8 with a friend. Which of you had the better answers? How can you tell?

10 (a) Look at these two brands of soap powder. Find the cost of 1 kg of each brand.
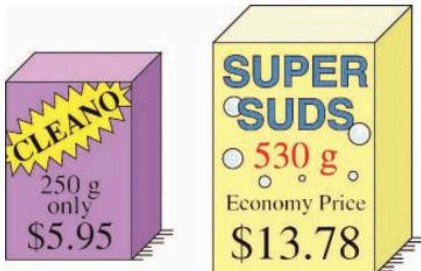
(b) Which powder is the better buy?

11 When performed on the calculator the division
$859 \div 31 = 27.709\ 677\ 42$
That is, $859 \div 31 = 27$ with a remainder.
Without doing the division, find out what the remainder is (as a whole number).

12 Use your calculator to find the exact value of:
(a) $19\ 836 \times 574\ 123$
(b) $712\ 384 \times 321\ 582$

Activity

Work with a partner. Choose a starting number and a target number. The aim is to reach the target number by finding the number which when multiplied by the starting number will give the target number. Players take turns, the person who is closest to the target number after four turns wins. For example, starting number 43, target 168.

	Starting number	Guess	Result
A	43	× 5	215
B	43	× 4	172
A	43	× 3	129
B	43	× 3.5	150.5
A	43	× 3.7	159.1
B	43	× 3.8	163.4
A	43	× 3.9	167.7
B	43	× 3.95	169.85

After four turns A wins because 167.7 is closer to the target, 168, than 169.85.
Try playing the game by finding the number which when divided into the starting number will give the target number.

Exercise 2Q – mixed questions

1 Chicken costs $6.49 per kg. How much does it cost to buy 10 kg?

2 The cost to hire a minibus for 15 people for one day is $138.75. How much does each person have to pay if the cost is shared equally?

3 A pay packet of $2980.11 is shared equally between seven men. How much does each receive?

4 Find the perimeter of each shape, with the measurements shown.

(a)   **(b)**

3.7 cm   3.5 cm   3.1 cm

2.8 cm   1.9 cm
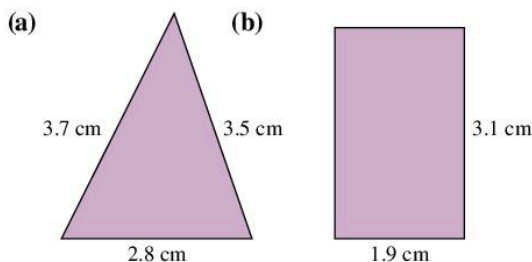
5 Find the perimeter of:
(a) a square with edges 3.7 cm
(b) an equilateral triangle with edges 4.6 cm
(c) a regular pentagon with edges 5.9 cm.

6 The perimeter of a square is 43.6 cm. Find the length of each edge.

7 The table below shows the winning times in the Olympic Games 400 metres.

Olympic Games 400 metres	
1896	54.2 s
1924	47.6 s
1968	43.8 s
1996	43.49 s
2008	43.53 s

Find the difference between the times for:
(a) 1896 and 2008
(b) 1924 and 1968
(c) 1968 and 1996

8 What is the area of this rectangle?

8.4 cm

3.1 cm
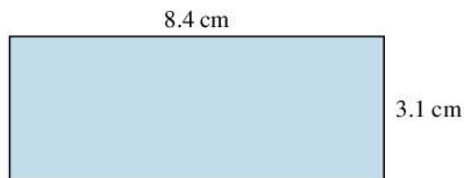
9 Copy and complete:

Number	1d.p.	2d.p.	3d.p.
6.7184			
0.0816			
0.9147			
1.0888			
12.1234			

10 Copy and complete:

Number	1s.f.	2s.f.	3s.f.
61400			
3175			
16.328			
0.01145			
0.9256			

11 **(a)** Find the number of seconds in:
   (i) 1 hour   **(ii)** 8 hours
   (iii) 1 day   **(iv)** 1 week
   (v) 52 weeks
(b) Write each answer in standard form:

12 Write in order of size, smallest first:
(a) $512, 4.9 \times 10^2, 8.1 \times 10^3, 1.72 \times 10^2$
(b) $8012, 8.1 \times 10^3, 8.01 \times 10^2, 8.01 \times 10^3$

13 A rectangular room has area 45 m^2. What is its length if its width is 8 m?

14 100 feet = 30.48 metres. What is the height of a 6-foot man in metres?

15 How would you use your calculator to find 6.7×0.13 if:
(a) the 6 button was broken
(b) the 3 button was broken?

16 How would you calculate 23×3.41 if:
(a) the × button was broken
(b) the 3 button was broken?

Σ→ Investigation

The fraction $\frac{1}{2}$ can be written as an exact (non-recurring) decimal

$$\frac{1}{2} = 0.5$$

The fraction $\frac{1}{3}$ gives a recurring decimal:

$$\frac{1}{3} = 0.33333 \ldots$$

(a) Which of these fractions give exact decimals?

$$\frac{1}{4}, \frac{1}{5}, \frac{1}{6}, \frac{1}{7}, \frac{1}{8}, \frac{1}{9}, \frac{1}{10}, \frac{1}{11}, \frac{1}{12}?$$

(b) Can you find a rule to help you tell whether a fraction is an exact decimal?

2 Consolidation

Example 1
Work out:
(a) $6.2 + 0.08 + 12$ (b) $17 - 0.13$

(a)

T	U	$\frac{1}{10}$	$\frac{1}{100}$
	6 .	2	
	0 .	0	8
+ 1	2 .	0	0
1	8 .	2	8

(b)

T	U	$\frac{1}{10}$	$\frac{1}{100}$
1	7 .	0	0
−	0 .	1	3
1	6 .	8	7

Example 2
What is:
(a) 0.63×10 (b) 17.4×100
(c) $0.63 \div 10$ (d) $17.4 \div 100$

(a) $0.63 \times 10 = 6.3$ (b) $17.4 \times 100 = 1740$
(c) $0.63 \div 10 = 0.063$ (d) $17.4 \div 100 = 0.174$

Example 3
What is 3.2×4.6?

An estimate of 3.2×4.6 is $3 \times 5 = 15$

$$\begin{array}{r} {}_1 32 \\ \times 46 \\ \hline 192 \\ 1280 \\ \hline 1472 \end{array}$$

The digits are 1472.
The estimate is 15.
Place the decimal point to give 14.72.

Hence $3.2 \times 4.6 = 14.72$

Example 4
Work out:
(a) $6.55 \div 5$ (b) $17.36 \div 0.31$

(a)
$$\begin{array}{r} 1 . 3 1 \\ 5\overline{)6 . 5 5} \\ 5\ 0 \\ \hline 1\ 5 \\ 1\ 5 \\ \hline 5 \\ 5 \end{array}$$

(b) $\dfrac{17.36}{0.31} = \dfrac{17.36 \times 100}{0.31 \times 100}$

$$= \dfrac{1736}{31}$$

Estimate : $\dfrac{2000}{30} \approx 66$

$$\begin{array}{r} 56 \\ 31\overline{)1736} \\ 155^{-} \quad 31 \times 5 \\ \hline 186 \\ 186 \quad 31 \times 6 \end{array}$$

So $17.36 \div 0.31 = 56$

Example 5
Round 13.218 to
(a) 2 decimal places
(b) 2 significant figures

(a) $13.218 = 13.22$ (2 d.p.)
(b) $13.218 = 13$ (2 s.f.)

Example 6
Write in standard form:
(a) $800\,000$ (b) $61\,508$

(a) $800\,000 = 8 \times 100\,000 = 8 \times 10^5$
(b) $61\,508 = 6.1508 \times 10\,000 = 6.1508 \times 10^4$

Example 7
Write as decimals:
(a) $\dfrac{3}{5}$ (b) $\dfrac{3}{7}$

(a) $\dfrac{3}{5} = 3 \div 5$

$$\begin{array}{r} 0.6 \\ 5\overline{)3.0} \end{array}$$

So $\dfrac{3}{5} = 0.6$

(b) $\dfrac{3}{7}$

$$\dfrac{3}{7} = 3 \div 7$$

$$\begin{array}{r} 0.428 \\ 7\overline{)3.000} \end{array}$$

So $\dfrac{3}{7} = 0.43$ (2 d.p.)

Exercise 2

1 Work out:
(a) (i) $0.7 + 0.5$ (ii) $5 + 0.6$
(iii) $12.3 + 0.14$ (iv) $6 + 0.02$
(v) $3.8 + 0.04 + 2$

(b) (i) $4 - 1.2$ (ii) $16 - 0.1$
(iii) $99 - 0.99$ (iv) $4.35 - 3.8$
(v) $101.1 - 0.011$

2 Calculate:
- **(a) (i)** 6.1×10 **(ii)** 6.15×10
  - **(iii)** 23.4×10 **(iv)** 0.13×100
  - **(v)** 2.784×100
- **(b) (i)** 1.3×2.4 **(ii)** 4.6×0.13
  - **(iii)** 0.38×0.72 **(iv)** 13.68×0.31
  - **(v)** 41.6×0.078

3 Calculate:
- **(a) (i)** $6.15 \div 10$ **(ii)** $0.615 \div 10$
  - **(iii)** $47 \div 10$ **(iv)** $0.3 \div 100$
  - **(v)** $36.2 \div 1000$
- **(b) (i)** $7.5 \div 5$ **(ii)** $0.75 \div 5$
  - **(iii)** $36.3 \div 3$ **(iv)** $15.3 \div 9$
  - **(v)** $0.688 \div 16$
- **(c) (i)** $4 \div 0.2$ **(ii)** $36 \div 0.09$
  - **(iii)** $4.78 \div 0.02$ **(iv)** $57.6 \div 0.24$
  - **(v)** $7.11 \div 0.9$

4 Round these numbers to:
- **(a)** 3 decimal places   **(b)** 2 decimal places
- **(c)** 1 significant figure   **(d)** 2 significant figures
  - **(i)** 4.611   **(ii)** 4.677
  - **(iii)** 0.0485   **(iv)** 61.7282
  - **(v)** 193.8888

5 Write in standard form:
- **(a)** 470   **(b)** 2190
- **(c)** 81 000   **(d)** 29 846
- **(e)** 93 000 000   **(f)** 151 284 000

6 Write as decimals:
- **(a)** $\dfrac{3}{4}$ **(b)** $\dfrac{2}{5}$ **(c)** $\dfrac{7}{8}$ **(d)** $\dfrac{3}{8}$
- **(e)** $\dfrac{5}{16}$ **(f)** $\dfrac{7}{24}$ **(g)** $\dfrac{2}{7}$ **(h)** $\dfrac{2}{9}$
- **(i)** $\dfrac{5}{6}$ **(j)** $\dfrac{6}{13}$ **(k)** $\dfrac{7}{15}$
- **(l)** $\dfrac{16}{17}$ **(m)** $\dfrac{1}{23}$ **(n)** $\dfrac{15}{26}$

Application

7 The cost of 9 kg of rice is \$111.24.
- **(a)** What is the cost of 10 kg?
- **(b)** What is the cost of 10.6 kg?

8 A scientist observes a snail move 3.1 cm in 26 seconds.
- **(a)** How far does the snail move, on average, in one second?
- **(b)** At this rate, how far could the snail go, in metres, in 100 seconds?

9 A square garden plot requires 63.4 m of fence to enclose it.
- **(a)** What is the area of the plot?
- **(b)** If the plot's area is increased by 10 m^2 but still remains square, how much fencing is required to enclose it?

Summary

You should know ...

1 When a decimal is multiplied by 10, 100 or 1000 each digit moves 1, 2 or 3 places to the left.
When a decimal is divided by 10, 100 or 1000 each digit moves 1, 2 or 3 place to the right.

For example:
$5.12 \times 10 = 51.2$
$5.12 \times 100 = 512$
$5.12 \times 1000 = 5120$

For example:
$72.1 \div 10 = 7.21$
$72.1 \div 100 = 0.721$
$72.1 \div 1000 = 0.0721$

Check out

1 Write down:
- **(a)** 2.5×10 **(b)** 0.4×10
- **(c)** 21.4×100 **(d)** 6.1×100
- **(e)** 6.8×1000 **(f)** $25 \div 10$
- **(g)** $6.4 \div 10$ **(h)** $14.5 \div 100$
- **(i)** $7.5 \div 1000$ **(j)** $0.4 \div 100$

2 You can multiply decimals by decimals.

For example:

$$2\,.\,8$$
An estimate is $3 \times 1 = 3$
$$\times\ 1_2.\,3$$

$$8\quad 4$$ The digits in the answer are 364

$$\underline{2_1\,8\quad 0}$$ The estimate is 3.

$$3\,.\,6\quad 4$$ Place the decimal point to give 3.64

3 To divide a decimal by a decimal you turn the divisor into a whole number.

For example: $3.8 \div 0.04 = \frac{3.8}{0.04} = \frac{3.8 \times 100}{0.04 \times 100} = \frac{380}{4} = 95$

4 How to round off a decimal.

For example:

4.62 is 4.6 to 1 decimal place

0.767 is 0.77 to 2 decimal places.

5 How to write a number to a given number of significant figures

For example:

$8160 = 8000$ to 1 s.f.

$0.0289 = 0.029$ to 2 s.f.

6 How to write a number in standard form

For example:

$291 = 2.91 \times 100 = 2.91 \times 10^2$

$81\,000 = 8.1 \times 10\,000 = 8.1 \times 10^4$

7 How to find the square root of a number using a calculator

For example:

$\sqrt{64} = 8$ because $8^2 = 64$

8 You can change fractions to decimals.

For example:

$\frac{1}{3} = 1 \div 3 = 1.00 \div 3$

This decimal goes on forever!

$$0.\,3\ 3\ \ldots$$
$$3\overline{)\,1.^10^10}$$

So $\frac{1}{3} = 0.33$ . .

---

2 Calculate:
(a) 3.2×0.5 (b) 0.4×7
(c) 12.2×6 (d) 1.4×3.5
(e) 9.42×6 (f) 1.36×0.42

3 Work out:
(a) $8 \div 0.5$ (b) $4 \div 0.2$
(c) $3.6 \div 0.3$ (d) $5.8 \div 0.02$
(e) $0.4 \div 0.02$

4 (a) Write to 2 decimal places:
(i) 4.715 (ii) 0.682
(iii) 17.991 (iv) 0.0453
(b) Write to 1 decimal place:
(i) 7.84 (ii) 0.683
(iii) 19.05 (iv) 0.0714

5 (a) Write to 1 significant figure:
(i) 892 (ii) 6030
(iii) 0.814 (iv) 0.000326
(b) Write to 2 significant figures:
(i) 18 920 (ii) 401 000
(iii) 0.0984 (iv) 0.0000396

6 Write in standard form:
(a) 60 (b) 609
(c) 2984 (d) 540 000
(e) 174 000 000 (f) 1 234 567

7 Find the square root of:
(a) 121 (b) 576
(c) 2 (d) 17.34

8 Write as decimals.
(a) $\frac{1}{4}$ (b) $\frac{2}{3}$
(c) $\frac{3}{8}$ (d) $\frac{5}{6}$
(e) $\frac{2}{9}$ (f) $\frac{3}{16}$

Angles

Objectives

✔ draw and measure angles
✔ construct angles using ruler and compass
✔ find angles in triangles, quadrilaterals and polygons
✔ discover properties of parallel lines
✔ use vertically opposite, corresponding and alternate angles to solve problems

This tree's angle is too great

You can't climb the coconut tree then?
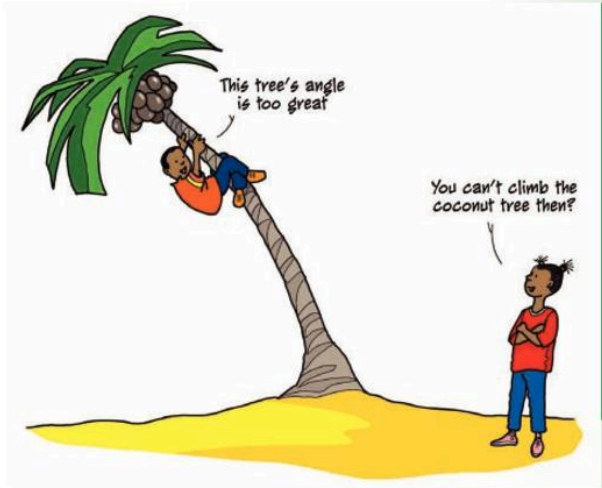
What's the point?

Angles are important in the design of many things. For example, reducing the angle made by the wing of a plane with its fuselage can reduce drag and enable the plane to fly at high speeds.

Before you start

You should know ...

1 How to use letters to name an angle.

angle AB̂C
or angle B̂
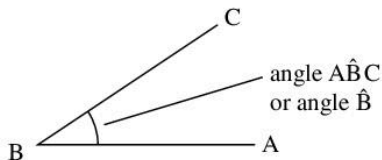
2 How to classify angles by the amount of turn.

acute angle   right angle   obtuse angle
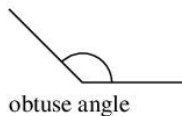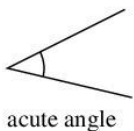
Check in

1 Name the angles marked.
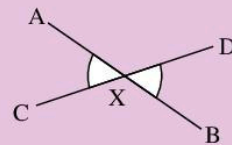
2 Classify these angles.
(a)   (b)
(c)   (d)
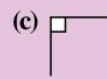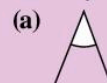
3   Angles that lie on a straight line are supplementary.
For example:
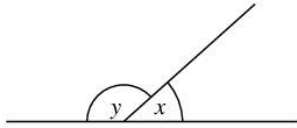
$x + y = 180°$
x and y are supplementary angles

3   Find the size of the angles marked a and b.
(a)   (b)
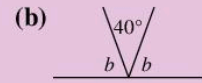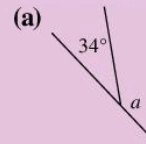
3.1   Measuring and drawing angles

You will need a protractor and ruler.

You use a protractor to measure and draw angles.

Example 1

Measure the angle ABC.
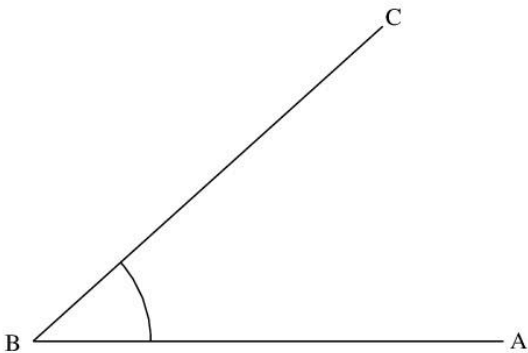
The first step is to see if the angle is more or less than a right angle.
$A\hat{B}C$ is less than 90°.
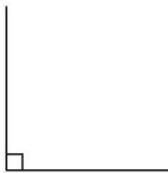
Next, place the centre of your protractor on B as shown below.
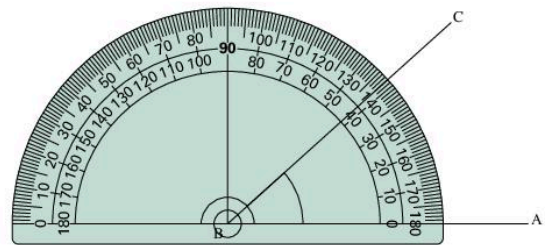
As $A\hat{B}C$ is less than 90°, you read the anti-clockwise scale.
$A\hat{B}C = 42°$

Exercise 3A

1   (a) Estimate the size of angle POQ.
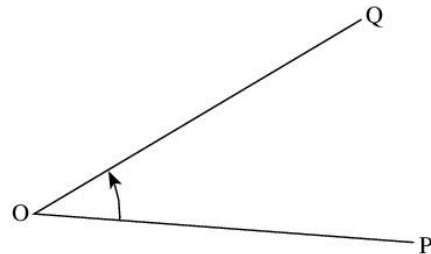
(b) Use your protractor to measure the angle POQ.
(c) How good was your estimate?

2 **(a)** Estimate the size of each of these angles.

(i)

(ii) E

(iii)
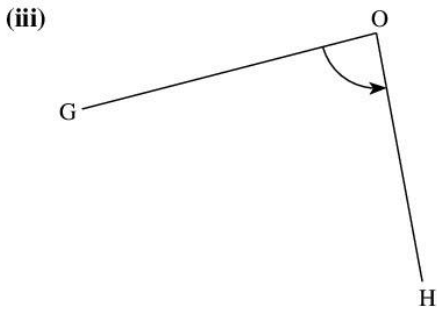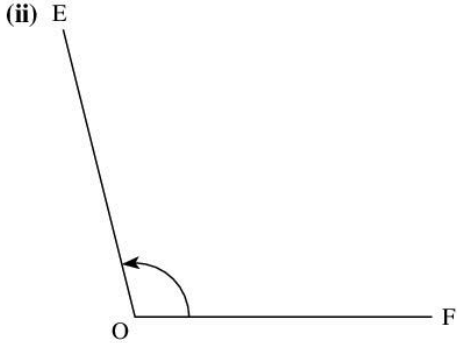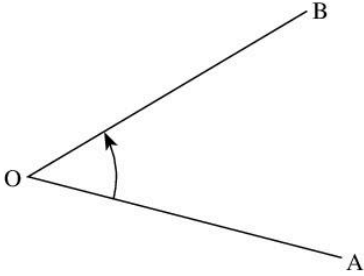
(b) Use your protractor to measure each of the angles.

(c) How good were your estimates?

3 Measure the angle a which shows the slope of this roof.
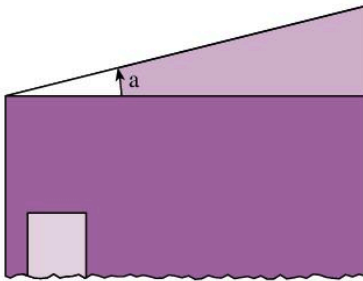
4 Measure the angle b between the cords holding up the picture.
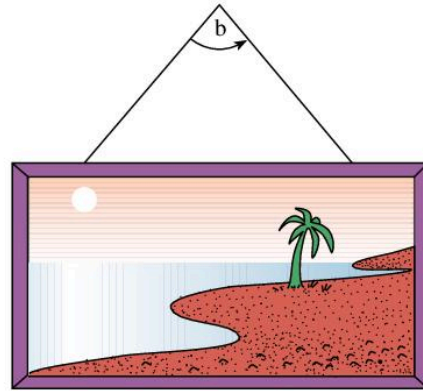
5 This picture shows a flag pole.
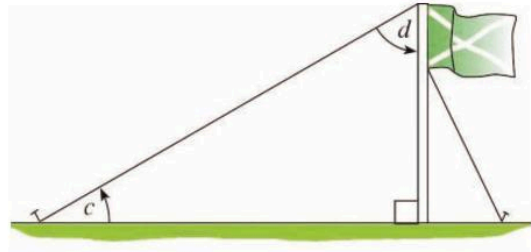
(a) Measure the angle c between the ground and the rope.

(b) Measure the angle d between the rope and the flag pole.

(c) What is the angle $(c + d)$?

6 **(a)** Measure $X\hat{O}Y$ and $Y\hat{O}Z$.

(b) What is $X\hat{O}Y + Y\hat{O}Z$?
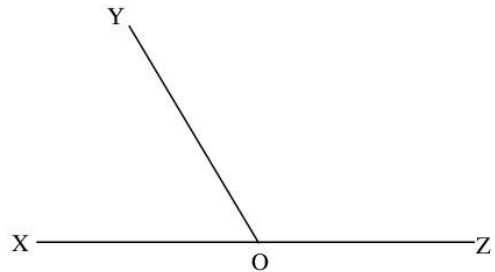
Technology

Try your hand at estimating angles by visiting

www.nrich.maths.org/1235

You've got to be fast to score well!

Drawing angles

You can use your protractor to draw an angle.

Example 2

Draw the angle $A\hat{B}C = 40°$

The angle ABC is made up of two lines AB and BC that meet at B. First draw the line AB.

B ————————— A

Place your protractor along the line with its centre at B.

Make a mark at 40°.

40° (Inner scale)
Put your mark here!
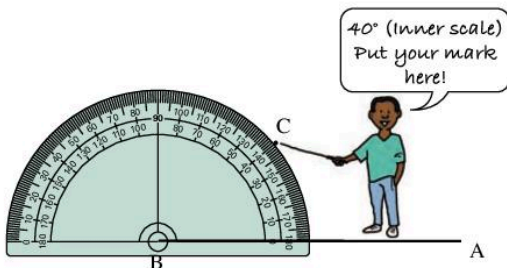
Remove your protractor and join your mark to the end of the line.
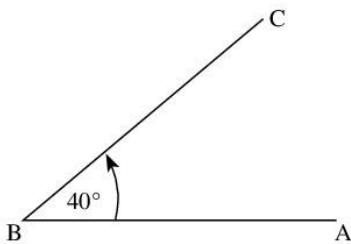
The lines AB and BC form the angle $A\hat{B}C = 40°$

Exercise 3B

1. Draw accurately angles of:
   (a) 15°   (b) 36°   (c) 98°
   (d) 109°   (e) 163°

2. (a) Draw an angle of 50°.
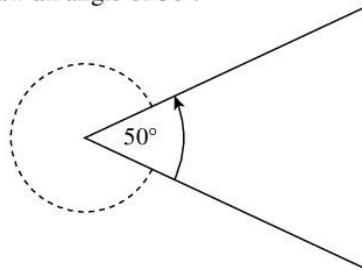
   (b) What is the exterior angle?

3. Use the idea in Question 2 to draw accurately angles of:
   (a) 190°   (b) 230°   (c) 270°
   (d) 330°   (e) 307°   (f) 354°

4. Draw accurately the triangle ABC, with :
   (a) AB = 8 cm, $\hat{A} = 40°, \hat{B} = 60°$
   (b) AB = 7 cm, $\hat{A} = 55°, \hat{B} = 38°$
   (c) AB = 6.7 cm, $\hat{A} = 100°, \hat{B} = 27°$
   (d) AB = 10.3 cm, $\hat{A} = 18°, \hat{B} = 121°$

5. Make an accurate drawing of the shape below.
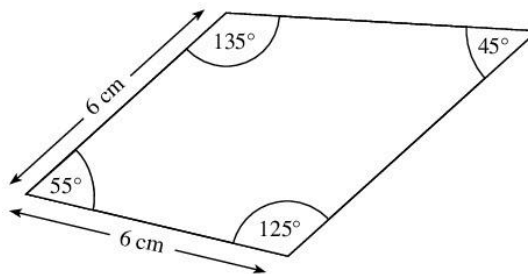
6. Make an accurate drawing of the pentagon below.
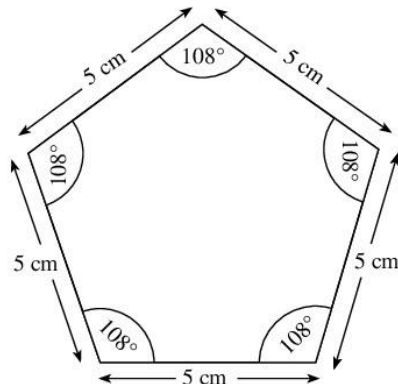
3.2 Constructing angles

You do not always need a protractor to draw an angle.
Angles such as 90°, 60° and 30° can be constructed
with a ruler and compasses only.

90° angle

You can construct an angle of 90° at a point X on a
line using a pair of compasses and a ruler. First, draw
two arcs centred at X to cut the line at A and B.
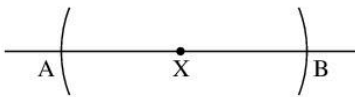
Increase the radius of your compasses. Draw arcs
centred at A and B to intersect each other at
P and Q. Join PQ.
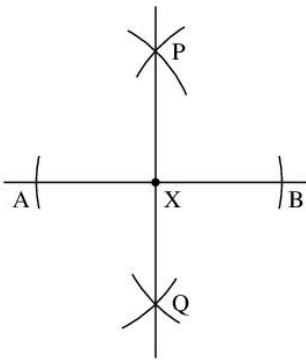
Angle AXP = 90°

Exercise 3C

1   Construct a 90° angle for yourself.

2   (a) Draw a line AB = 8 cm.
    (b) Construct a 90° angle at A.
    (c) Draw the line AC = 6 cm where BÂC = 90°
    (d) Join the points B and C.
    (e) Measure the line BC.

3   (a) Draw a line PQ = 7 cm.
    (b) Construct angles of 90° at both P and Q.
    (c) On the perpendicular through P mark the
         point S 5 cm from P.
    (d) Draw a perpendicular through S to meet the
         perpendicular through Q at R.
    (e) What is the shape PQRS?
    (f) Measure the length RQ.

60° angle

You can construct an angle of 60° at a point X on a
line in a similar way. Draw a large arc, with centre X,
to cut the line at P.
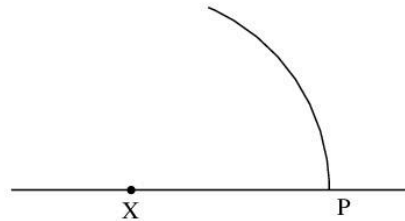
Using the **same** radius draw an arc, with centre P, to
cut the first arc at Q.
Join XQ.
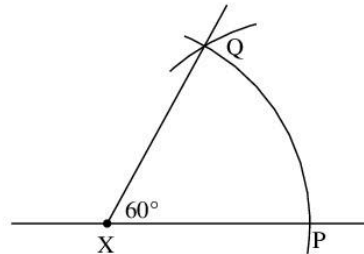
The angle QXP = 60°.

Exercise 3D

1   Construct a 60° angle for yourself.

2   Construct an equilateral triangle of sides 7 cm in
    length.

3   Construct the triangle ABC with
    AB = 10 cm, BÂC = 90° and AB̂C = 60°.

4   (a) Construct the angle BAC = 60°, with the
         length AC = 4 cm.
    (b) Draw the perpendicular from C to meet the
         line AB at D.
    (c) Measure the length AB.

Once you can construct angles of 90° and 60°, you can make angles of 45° and 30° by simply bisecting these angles.

Bisecting an angle

You can bisect the angle ABC with a pair of compasses and a ruler.
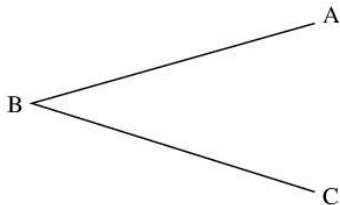
Draw an arc with centre B to cut AB at X and BC at Y.
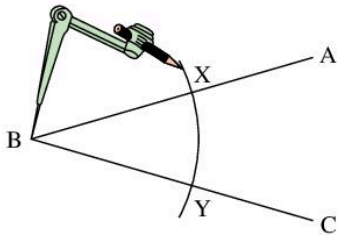
Draw two more arcs, with the same radius, centred at X and Y. Label the point where they meet Z.
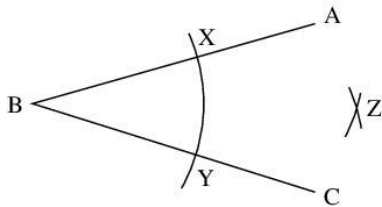
Join BZ. This line bisects AB̂C.
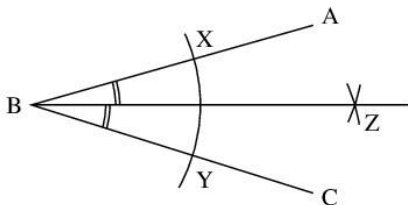
Exercise 3E

1 **(a)** Construct an angle of 90°.
   (b) Bisect it to make an angle of 45°.

2 **(a)** Construct an angle of 60°.
   (b) Bisect it to make an angle of 30°.

3 **(a)** Draw a straight line. This represents an angle of 180°.
   (b) Bisect the angle of 180° to get an angle of 90°.

<u>4</u> Construct an angle of $22\frac{1}{2}°$.

<u>5</u> Construct an angle of 15°.

<u>6</u> Construct an angle of 75°.

<u>7</u> Construct an angle of 135°.

<u>8</u> Construct the triangle ABC where AB = 8 cm, CÂB = 75° and AB̂C = $67\frac{1}{2}°$.

Technology

Need more practice?
Look at

www.onlinemathlearning.com/geometry-construction.html

or

www.mathopenref.com/constangle30.html

The videos show you how angles of 90° and 60° can be constructed using a ruler and compasses only. By bisecting these angles you can make angles of 45°, 30° and so on.

● Now construct these angles on your own.
   (i)   90°
   (ii) 60°
   (iii) 45°
   (iv) 30°
   (v)   120°
● Construct the triangle ABC where AB = 8 cm, BÂC = 30° and AB̂C = 45° using ruler and compasses only.

3.3 Angles in shapes

You can find the angle properties of shapes by working through Exercise 3D.

Exercise 3F

1 (a) Measure the angles at A, B and C.
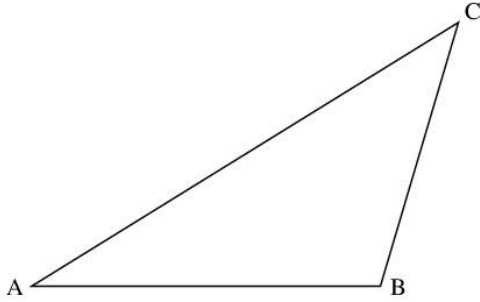
(b) What is the value of $\hat{A} + \hat{B} + \hat{C}$?

2 (a) Draw five different triangles ABC. In each case, measure the angles at A, B and C.
(b) Copy and complete the table.

Triangle	$\hat{A}$	$\hat{B}$	$\hat{C}$	$\hat{A} + \hat{B} + \hat{C}$
1				
2				
3				
4				
5				

(c) What do you notice about the sum of the angles in a triangle?

3 (a) With a ruler and pencil, draw a triangle. Cut it out very carefully.
(b) Now colour or shade each angle differently.
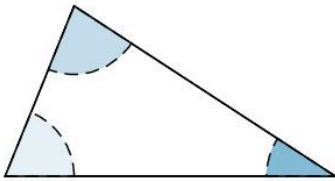
Cut off the angles along the dotted lines.
(c) Carefully fit the cut-out angles together. They should look like this:

(d) What angle is formed by the three angles?

4 (a) Repeat Question 3 for four other triangles.
(b) Do you get the same result?

5 (a) In the four-sided figure ABCD measure each of the angles.
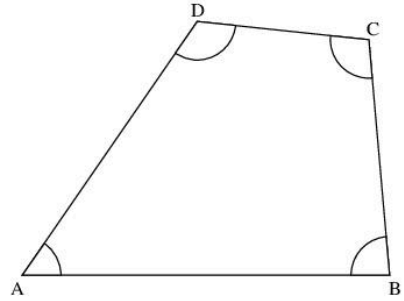
(b) What is the value of $\hat{A} + \hat{B} + \hat{C} + \hat{D}$?

6 (a) Draw five different quadrilaterals ABCD. In each case measure the angles at A, B, C and D.
(b) Copy and complete the table.

Quadrilateral	$\hat{A}$	$\hat{B}$	$\hat{C}$	$\hat{D}$	$\hat{A} + \hat{B} + \hat{C} + \hat{D}$
1					
2					
3					
4					
5					

(c) What do you notice about the sum of the angles in a quadrilateral?

7 (a) With a ruler and pencil, draw accurately a four-sided figure. Cut it out carefully.
(b) Now colour or shade each angle differently.
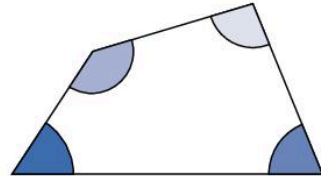
Cut off the angles along the dotted lines.
(c) Carefully fit the cut-out angles together. They should look like this:
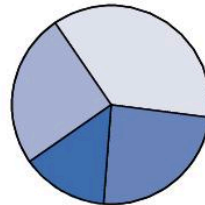
(d) What angle is formed by the four angles?

8 (a) Repeat Question 7 for four other quadrilaterals.
(b) Do you get the same result?

9 **(a)** In the triangle ABC, the side BC is extended to D. Measure the angles AB̂C, BÂC and AĈD.
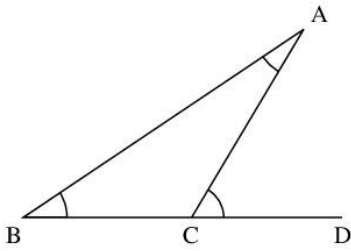
(b) What is the value of AB̂C + BÂC?

(c) Compare your answer to part (b) with AĈD What do you notice?

10 **(a)** Repeat Question 9 for four other triangles ABC extended to D.

(b) Do you get the same result?

In Exercise 3D you should have found the following results:

(1)   The angle sum in a triangle is 180°

$$a + b + c = 180°$$
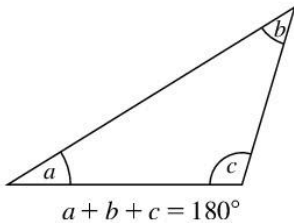
(2)   The angle sum in a quadrilateral is 360°

$$a + b + c + d = 360°$$
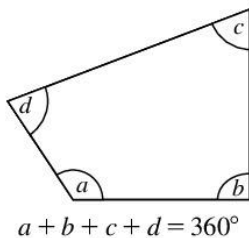
(3)   The exterior angle of a triangle is equal to the sum of the interior opposite angles

$$a + b = c$$
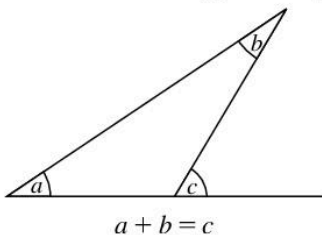
You can use these results to find missing angles in simple shapes.

Example 3

Find the angle x in the triangle XYZ.
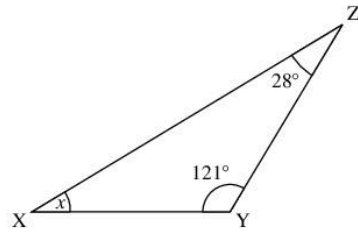
Since angle sum in a triangle is 180°
$$x + 121° + 28° = 180°$$
$$x + 149° = 180°$$
$$x = 31°$$

Exercise 3G

1   Find the missing angles in these triangles.

(a)

(b)

(c)

(d)
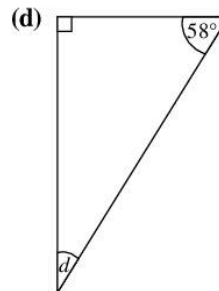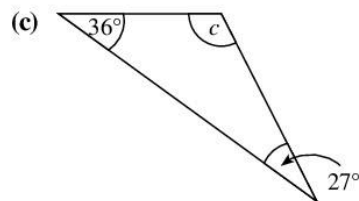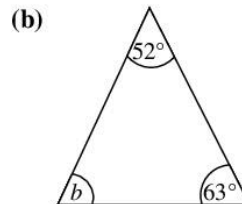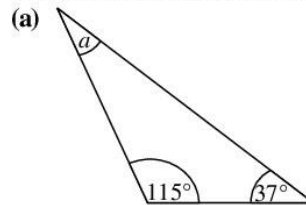
2   Find the missing angles in these quadrilaterals.

(a)
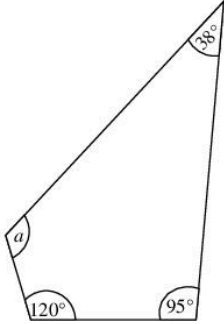
(b)
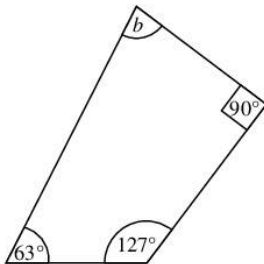
(c)
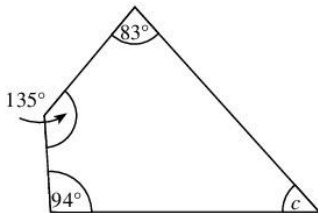
(d)
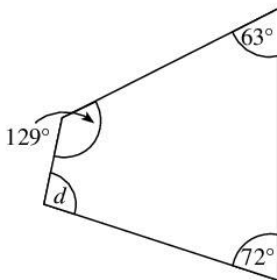
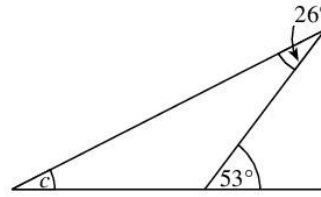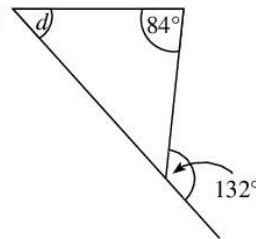3   Find the missing angles.

(a)
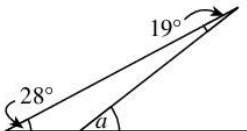
(b)
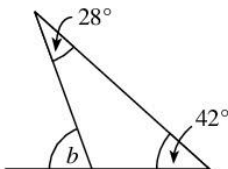
(c)

(d)

Angles in polygons

In a three-sided shape, a triangle, the sum of the interior angles is 180°. In a four-sided shape, a quadrilateral, the sum of the interior angles is 360°.
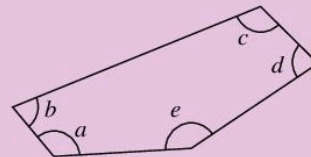
Shape	Number of sides	Sum of interior angles
Triangle	3	180°
Quadrilateral	4	360°
Pentagon	5	
Hexagon	6	
...	...	...

Can you complete this table for the interior angles of a pentagon, a hexagon, etc?

Activity

(a) Draw a five-sided shape or pentagon. With your protractor measure each of its five interior angles.

What is their angle sum?

$a + b + c + d + e = $ _____

(b) Repeat for a different pentagon. What is the angle sum?

(c) Repeat, this time for a six-sided shape, a hexagon. What is the angle sum?

In the activity, you should have found that the sum of the interior angles in a pentagon is 540° and 720° for a hexagon.

Exercise 3H

1   **(a)** Draw a seven-sided figure.
     (b) Measure each of its interior angles.
     (c) What is the angle sum?

2   **(a)** Draw an eight-sided figure.
     (b) Measure each of its interior angles.
     (c) What is the angle sum?

3   **(a)** Copy and complete the table:

Shape	Number of sides	Sum of interior angles
Triangle	3	180°
Quadrilateral	4	360°
Pentagon	5	
Hexagon	6	
Heptagon	7	
Octagon	8	

   (b) How does the angle sum for a shape increase with each extra side?

4   **(a)** What do you think is the angle sum for: a nine-sided a figure, a nonagon
     (b) a ten-sided figure, a decagon?

5   **(a)** What would be the sum of the internal angles of a twelve-sided figure?
     (b) What about an n-sided figure?

6   **(a)** How does a regular polygon differ from an irregular polygon?
     (b) If the sum of the interior angles of a regular polygon is 540°, what is the size of each interior angle?

7   A regular polygon has angle sum of 1440°.
     (a) How many sides does the polygon have?
     (b) What is the size of each interior angle?

8   A regular polygon has twelve sides. What is the interior angle of this polygon?

3.4   Angles and lines

Opposite angles

● When two straight lines intersect, two pairs of **vertically opposite** angles are formed. Vertically opposite angles are equal.
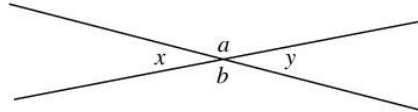
For example, in the drawing
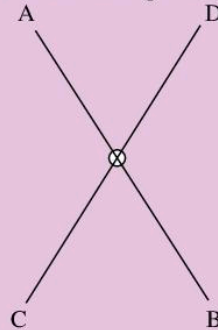a and b are vertically opposite angles.
$a = b$
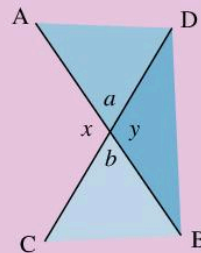x and y are also vertically opposite.
$x = y$

Activity

(a) Draw two intersecting straight lines.

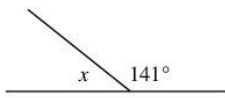**(b)** Shade them as shown and cut out the shaded parts.

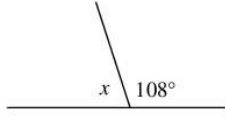**(c)** Place x over y and a over b. What do you discover about each pair of vertically opposite angles?

Exercise 3I

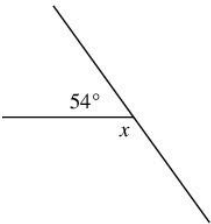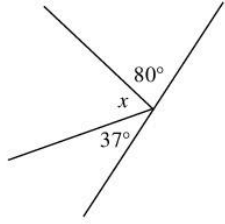**1** Without measuring, find the size of x.

(a)

x $\quad$ $141°$

(b)

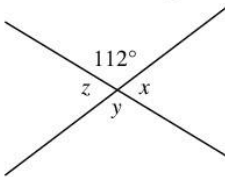
x $\quad$ $108°$

(c)

$54°$ $\quad$ x

(d)

$80°$ x $37°$

2 Without measuring, find the value of each letter.

(a)

$112°$ z x y

(b)

$42°$ f h g

3 Without measuring, find the value of each letter.

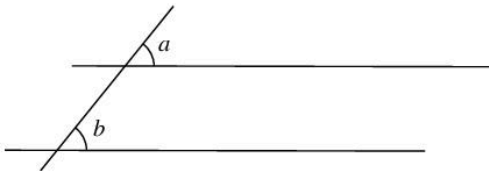a $90°$ b c $81°$ f $86°$ e d $95°$
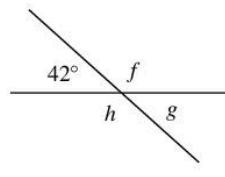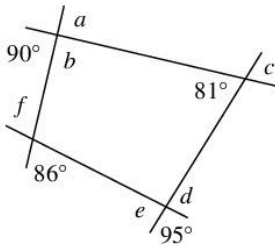
4 **(a)** Measure the angles a and b.

a b

(b) What do you notice about them?

5

C $\quad$ B
a
d $\quad$ b
c
A $\quad$ D
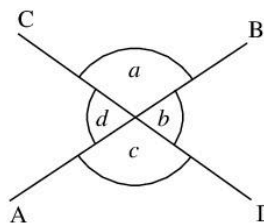
AB and CD are straight lines. If $a = 130°$
(a) find c
(b) find b and d

6

E $\quad$ B
C
s $\quad$ t
r $\quad$ O $\quad$ u
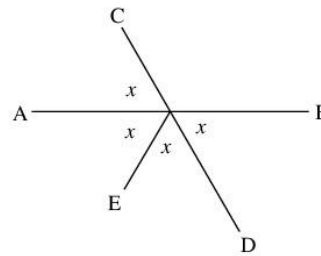w $\quad$ v
A $\quad$ D
F
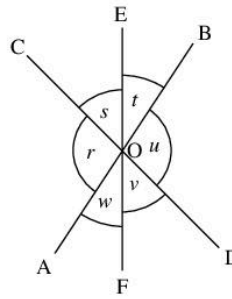
AB, CD and EF are straight lines that cross at O.
(a) If $s = 35°$ find v.
(b) If $r = 60°$ and $t = 65°$ find v.
(c) If $r + s = 120°$ and $v = 50°$ find u.

7

C
x
A $\quad$ x $\quad$ x $\quad$ B
x
E
D

Find the size of x.

8

P
$3x°$ $\quad$ $x°$
A $\quad$ O $\quad$ B
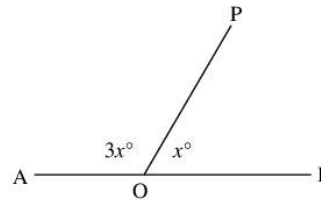
Find the value of x.

9

$5x°$
$2x°$
$5x°$
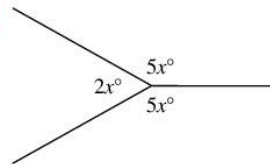
Find the value of x.

10

$4x°$
$x°$ $\quad$ $x°$
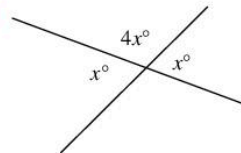
Find the value of x.

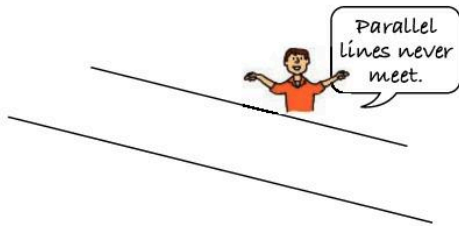# 3.5 Parallel lines

You will need tracing paper, plain paper, squared paper, a ruler and a protractor.

● Two straight lines that are always the same distance apart are **parallel**.

Parallel lines never meet.

Exercise 3J

1 **(a)** Look at the edges of your desk. Are they parallel?
(b) Are the edges of this book parallel?
(c) Are the edges of the classroom door parallel?
(d) Write down six examples of pairs of parallel lines that you can see around you.

2 Look at this diagram.
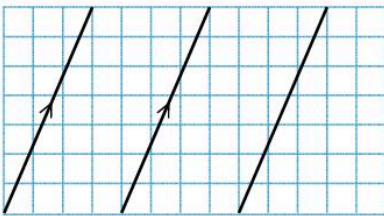
(a) The two lines marked with arrows are parallel. Is the third parallel to the other two? Can you tell by counting squares?
(b) Copy the diagram on to squared paper, and draw two more lines parallel to these.

3 Copy this diagram and draw a line parallel to each line.
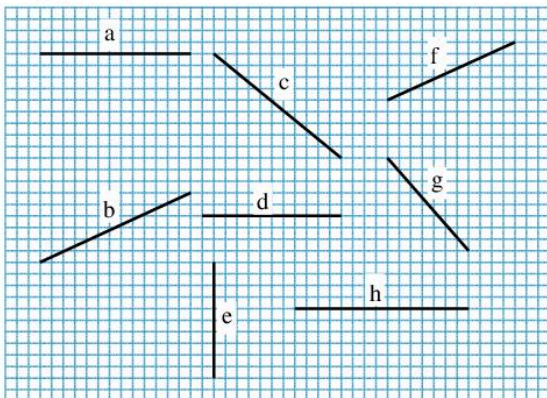
4 Pick out all the pairs of parallel lines from these diagrams.
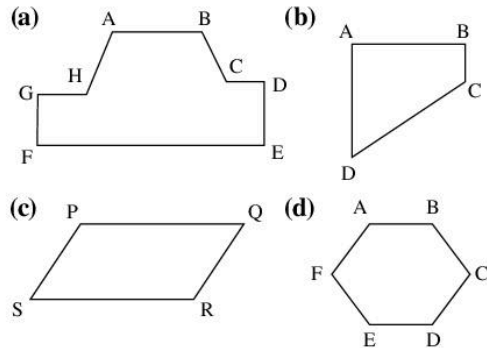
(a) **(b)** **(c)** **(d)**

5 The vertices, or corners, of this cube are A, B, C, D, E, F, G and H.
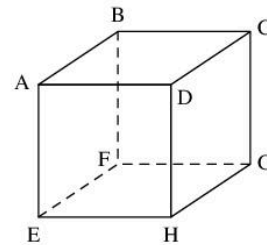
(a) Write down three edges that are parallel to AB.
(b) Find all the other sets of parallel edges.

6 A sketch of the box PQRSTUVW has been started. Copy and complete the sketch. Mark the parallel edges with the same number of arrows.
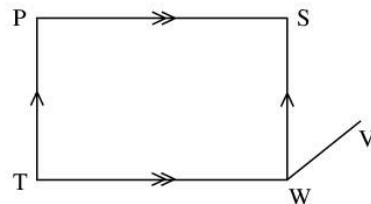
Copy and complete: A cuboid has . . . sets of parallel edges. There are . . . edges in each set.

Corresponding angles

In the diagram a line crosses two parallel lines. Measure with your protractor the angles x and y.
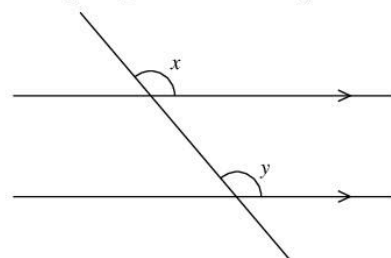
You should have found both angles were equal.
These angles are called **corresponding** angles.

- When a line crosses two parallel lines the
corresponding angles formed are equal.

$x = y$
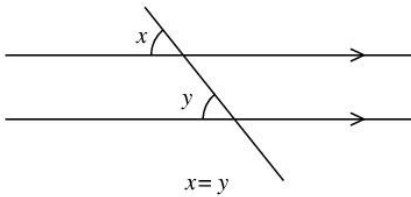
Example 4

Find $\hat{y}$ and $\hat{z}$ giving reasons.
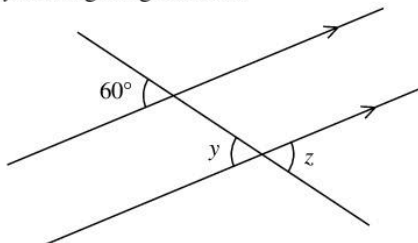
$y = 60°$ (corresponding angles)
$z = y$ (vertically opposite angles)
so $z = 60°$

Exercise 3K

1 In the diagram, which angle is:
   (a) the corresponding angle to x
   (b) the corresponding angle to y?
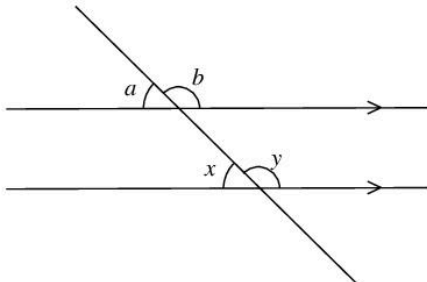
2 Which of these angles:
   (a) correspond to x
   (b) correspond to y?
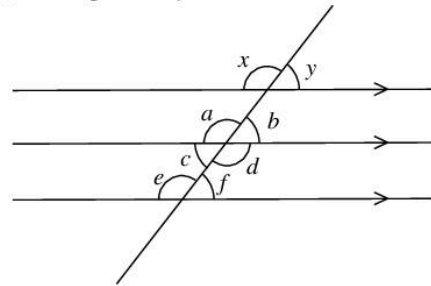
3 Find the angles marked by letters giving reasons
   for your answers.

(a)

(b)

(c)

(d)

(e)

55°
k
l
m
n
o
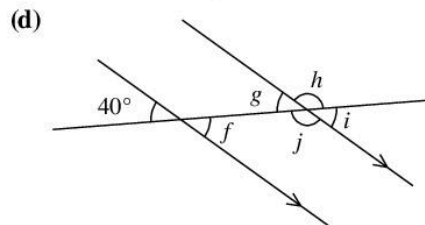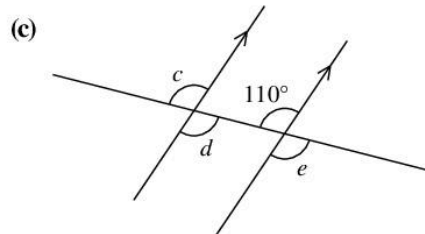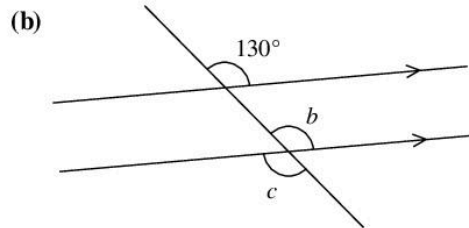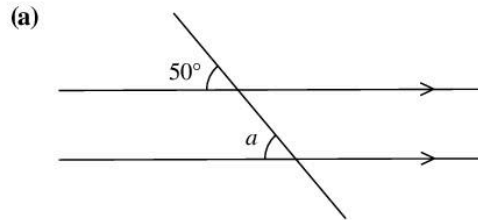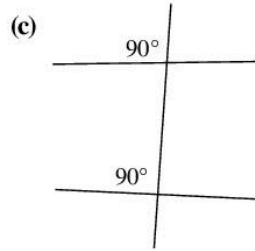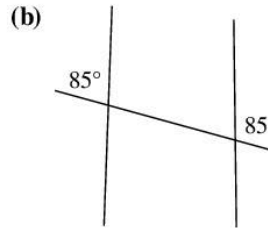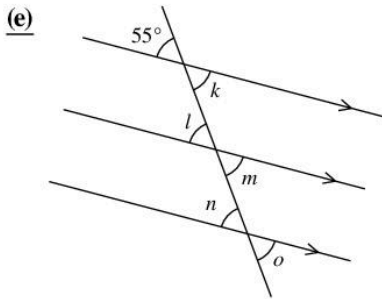
4 Without measuring, write down the value of each letter.

(a)

r
p q
125°

(b)

64°
x y
z

(c)

i h
g
90°
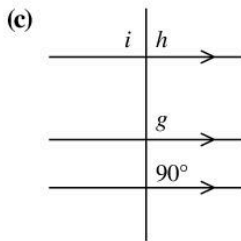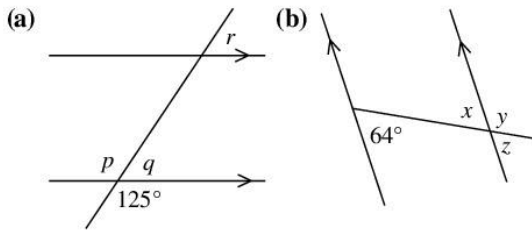
5 Calculate each of the unknown angles.

e
c
a
50° b d
f 110°
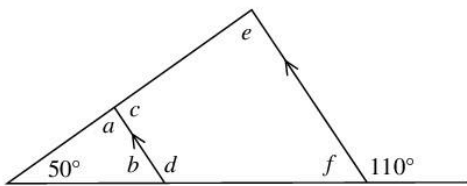
6 The diagrams in (a) to (c) have not been drawn properly. In which of them should the pair of lines be drawn parallel?

(a)

80°
80°

(b)

85°
85'

(c)

90°
90°
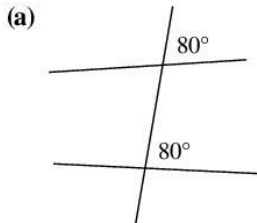
Copy and complete:

(d) When a line crosses a pair of parallel lines, corresponding angles are . . .

(e) When a line crosses a pair of lines, and corresponding angles are equal, the pair of lines must be . . .

Alternate angles

Measure with your protractor the angles p and q.

p
q
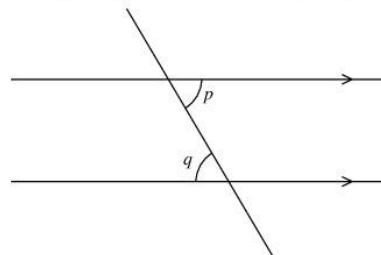
You should have found that the angles p and q were equal. These angles are called **alternate** angles.

• When a line crosses two parallel lines, the alternate angles formed are equal.

Exercise 3L

1 In the diagram which angle is:
   (a) the alternate angle to a
   (b) the alternate angle to b

a b
x y
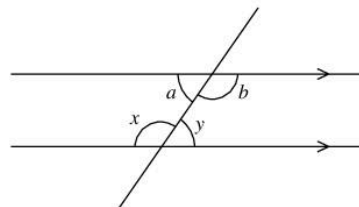
2 Look at this diagram.
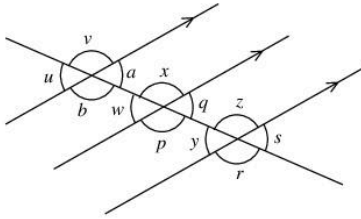
Which of the angles are:
(a) alternate to *a*
(b) corresponding to *a*
(c) vertically opposite to *a*?

3 Repeat Question 2 for angle *b*.

4 Write down the size of each angle marked by a letter.

(a)

54°
p

(b)

s
147°

(c)

37°
m

(d)

x   *y*
105° *z*
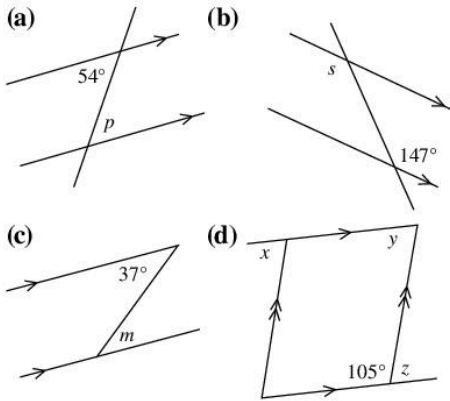
5 For Question 4(d), copy and complete:
(a) *x* and *y* are . . . angles.
(b) *y* and *z* are . . . angles.

6 Find the value of each letter.

(a)

56°
x
y

(b)

60°
m   *n*

(c)

g   63°
54°   *h*

(d)

48°
s
r   *t*
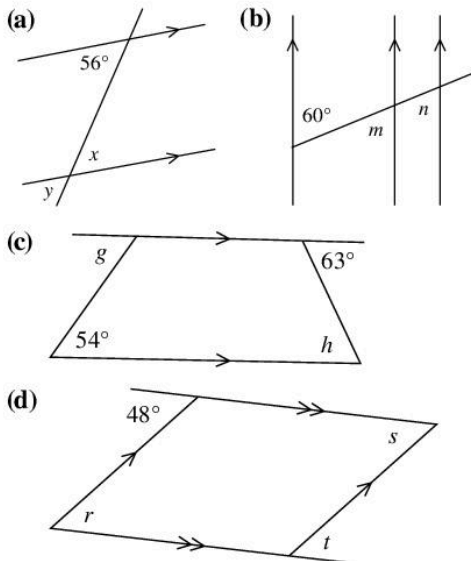
7 In the diagram, ABCD is a square. DF is a line cutting BC at E.
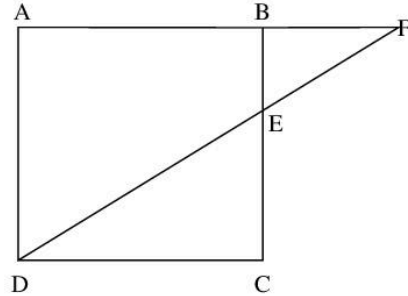
Copy and complete:
(a) AB is parallel to . . .
(b) AD is parallel to . . .
(c) AF̂D and FD̂C are equal because they are . . . angles.
(d) AD̂F and BÊF are equal because they are . . . angles.

8 Are the edges of your ruler parallel? Try to draw a large letter W, so that it is made up of two pairs of parallel lines. Use both edges of your ruler to help you. On your drawing, mark the equal angles.

9 These diagrams have not been drawn properly. In which of them should two lines be drawn parallel? Give reasons for your answers.

(a)

53°
53°

(b)

93°
93°

(c)

42°
42°

(d)

37°
27°
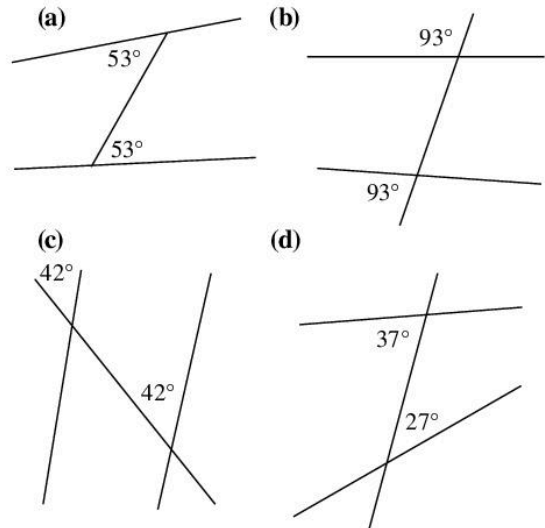
10 Copy the diagrams. Mark an x in all the angles that are equal to x.
Mark a y in all the angles that equal y.

(a)
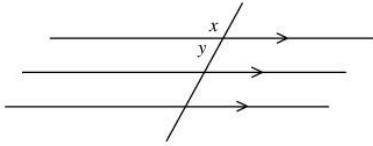
(b)
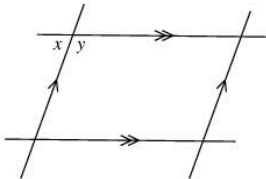
(c) Copy and complete:
$$x + y = \Box°$$

Drawing parallel lines

Exercise 3M

1 **(a)** Draw a pair of parallel lines on a piece of plain paper, by using both edges of your ruler, as shown below. Then move the ruler to draw a third line, parallel to the other two.
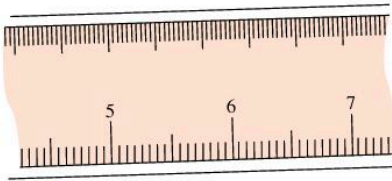
(b) Use the ruler to draw a pair of parallel lines that crosses these three.

(c) Use coloured pencils to mark all the equal angles.

2 **(a)** Use tracing paper to copy a pair of parallel lines from some ruled paper. Use your tracing to make up a tessellation of diamond shapes, as shown.
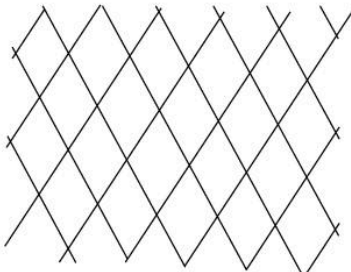
(b) Mark all equal angles with coloured pencils.

3 Copy this picture.
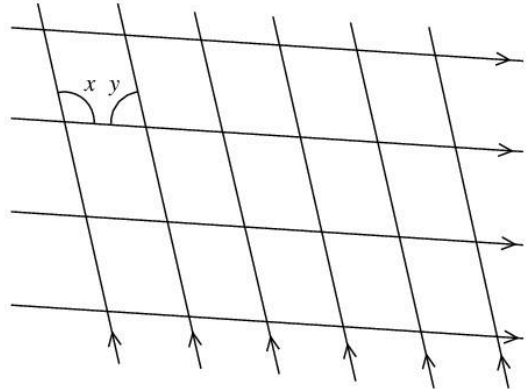
(a) Mark all the other angles that are equal to x and y.

(b) Explain why $x + y = 180°$

Technology

You can construct a line parallel to another using just a ruler and compasses. Look at

https://www.onlinemathlearning.com/pairs-of-lines.html#parallel

Watch the video to learn how!

Parallelograms

The four-sided shapes in Question 3 of Exercise 3M are called **parallelograms**.

The parallelogram ABCD is formed from two pairs of parallel lines:
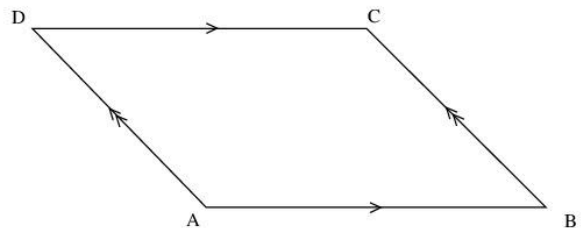
AD is parallel to BC
and AB is parallel to DC

The arrows show the lines are parallel.

Example 5

Find the angles x and y in the parallelogram WXYZ.
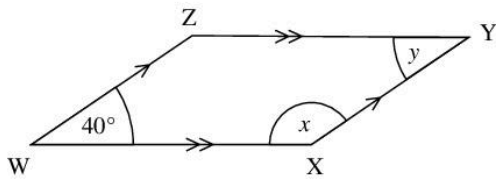
The angles W and X are supplementary.
$$40° + x = 180°$$
so $\qquad x = 140°$

The angles X and Y are also supplementary.
$$x + y = 180°$$
or $140° + y = 180°$
so $\qquad y = 40°$

Exercise 3N

1 Find the missing angles, marked by letters in these parallelograms.

(a)

(b)

(c)
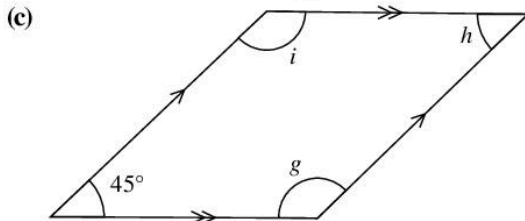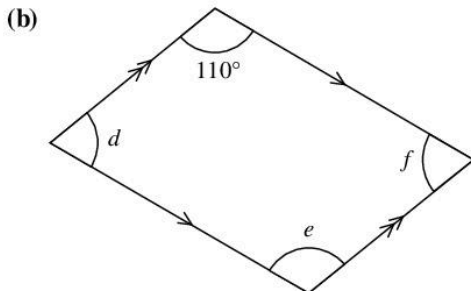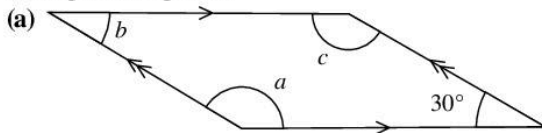
2 Write down the size of the angles marked by letters.

(a)

(b)

(c)

(d)
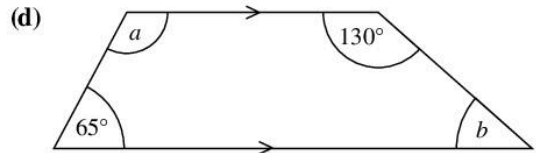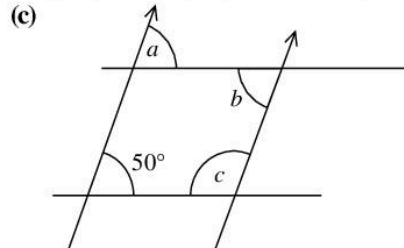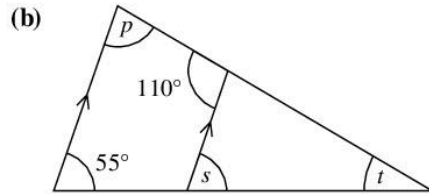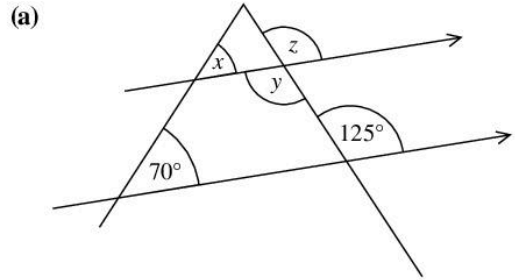
3 (a) Measure with your protractor the angles A, B, C and D.
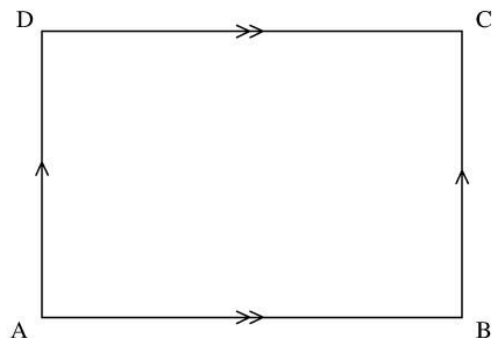
(b) What do you notice?

In Question 3 you should have found all the angles were right angles. This special parallelogram is a **rectangle**.

4   Draw, using the measurements given, each of these parallelograms.

(a)

3 cm

60°

5 cm

(b)

4 cm

115°

6 cm

(c)

4 cm

70°

4 cm

(d)

4 cm

5 cm
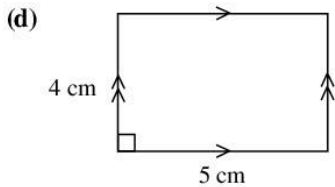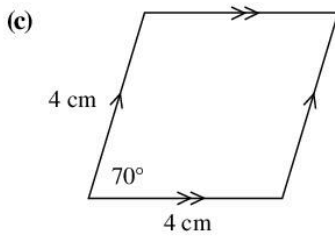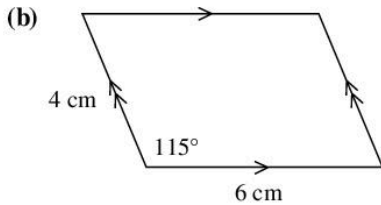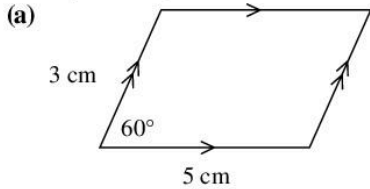
5   (a) Measure the sides WX, XY, YZ, and WZ of this parallelogram.

Z   Y

W   X
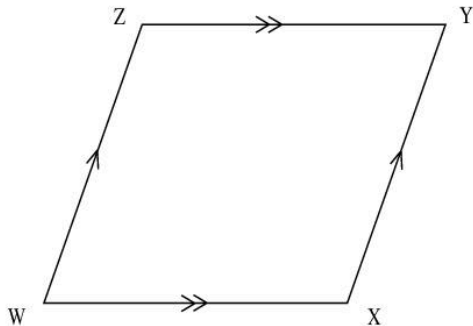
(b) What do you notice about them?

A parallelogram which has all its sides equal in length is called a **rhombus**.

6   (a) Draw accurately a square with each side 4 cm.
    (b) How is your square different from the rhombus in Question 5?

7   The Venn diagram shows a set of parallelograms and the subsets of rectangles and squares. Draw a copy of the diagram and complete the labels.

The set of parallelograms
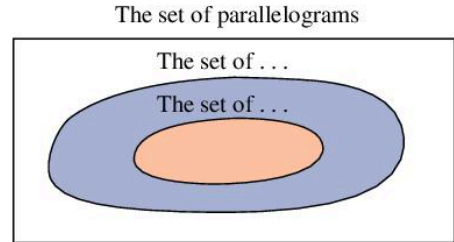
The set of . . .

The set of . . .

8   Draw a Venn diagram to show the set of parallelograms and the subsets of rhombuses and squares.

9   The Venn diagram below shows the set of parallelograms and the subsets of rectangles and rhombuses.

The set of parallelograms

the set of rhombuses

the set of rectangles
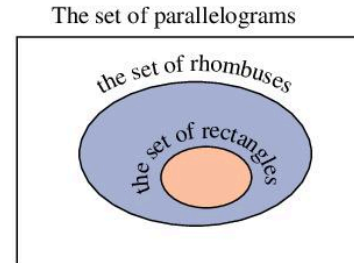
(a) Draw a large copy of the Venn diagram.
(b) Draw a shape in each region of your copy.
(c) Name the shape that is both a rhombus and a rectangle.

10 (a) Are all squares parallelograms?
    (b) Are all parallelograms rectangles?
    (c) Give reasons for your answers.

Investigation

Trace on card the two *identical* right-angled triangles below. Cut them out.
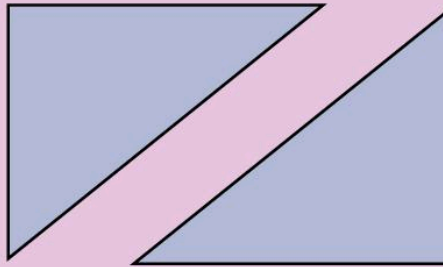
Which of the following shapes can you make from them:
(a) a parallelogram   **(b)** a square
(c) a rectangle   **(d)** a rhombus?

Repeat for two identical isosceles triangles of your own.

Repeat for two identical scalene triangles of your own.

③ Consolidation

Example 1
Measure these angles:
(a)

(b)

(a)
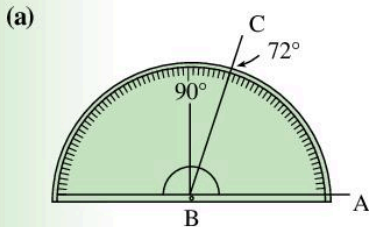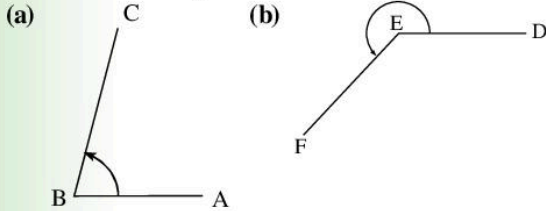
Place the centre of the protractor on B as shown.
Angle AB̂C = 72°

(b)
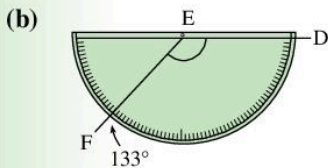
The angle DÊF is larger than 180°, so the smaller angle is measured. Place the centre of the protractor on E as shown.
The larger angle DÊF = 360° − 133°
= 227°

Example 2
Construct an angle of 30° using ruler and compass only.
First construct an angle of 60°.
Draw base line

Using a compass draw an arc centred at one end of the line and a second arc centred where the first arc cuts the line.

join end to where arcs meet
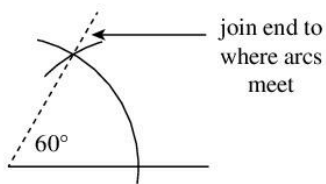
Then bisect the angle.
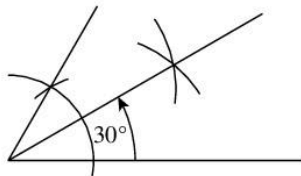
Example 3
Without measuring, calculate the angles indicated by the letters:
(a)

(b)
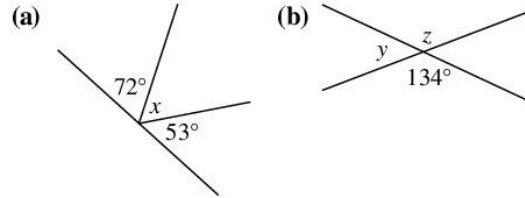
(a) 72° + x + 53° = 180°
x + 125° = 180°
x = 180° − 125°
= 55°
(b) z = 134° (vertically opposite angles)
y + 134° = 180° (angles on a straight line = 180°)
y = 180° − 134°
= 46°

Example 4
Find the angles marked by the letters:
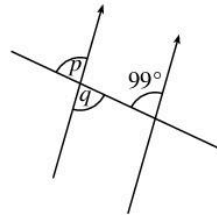
p = 99° (corresponding angles)
q = 99° (vertically opposite)

Exercise 3

1 Estimate these angles. Then measure with a protractor.
(a) (b)
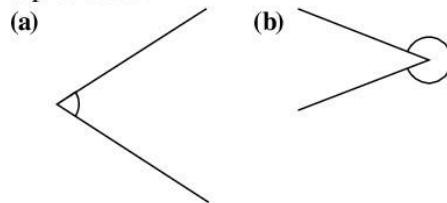
2 Accurately draw angles of:
(a) 25° (b) 66° (c) 125°
(d) 178° (e) 215° (f) 263°
(g) 279° (h) 310° (i) 325°
(j) 348°

3 Find the angles marked by the letters:

(a)

37°
a
68°

(b)

b c 32°

(c)

100°
d
d

(d)

e
27° 130°

(e)

75°
29° f g

(f)

k
j
34°

(g)

m
l n
103°

(h)

45° o
p

(i) 115° 120°
q r
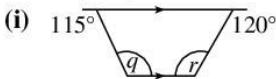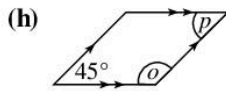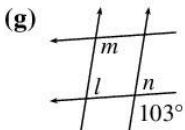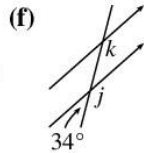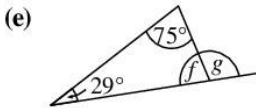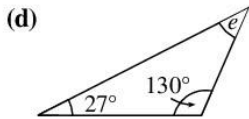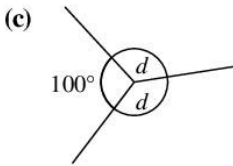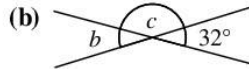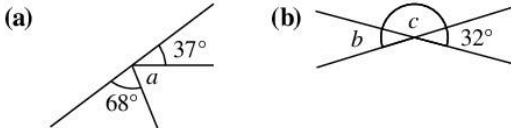
4 Construct the triangle ABC where AB = 8 cm, BÂC = 60° and AB̂C = 30° using ruler and compass only.

Application

5 **(a)** Draw six five-sided shapes (pentagons).
(b) Measure each of the internal angles with your protractor.
(c) What is the sum of the internal angles in each pentagon?
(d) What do you notice?
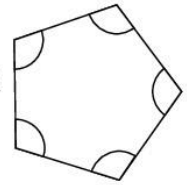
6 **(a)** Draw five triangles.
(b) Measure each exterior angle of your triangles with a protractor.
(c) What is the sum of the external angles in each triangle?
(d) What do you notice?
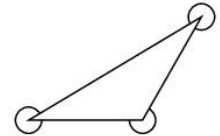
7 The design of an envelope ABCD is shown in the diagram. ABM is the flap and M is the mid-point of DC.
Assist in the construction of the envelope by calculating:
(a) angle ABM   **(b)** angle AMD
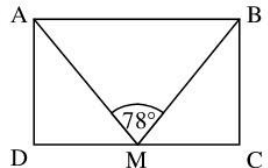
A B
D M C
78°

Summary

You should know ...

1 To measure the size of an angle larger than 180°, first measure the smaller angle.

First measure the smaller angle–It's 35°

The larger angle is 360° – 35° = 325°

A complete turn is 360°, so the larger angle is
360° − 35° = 325°

Check out

1 Measure this angle.

Q R
P

2 How to use ruler and compass to:
  (a) construct an angle of 90°
  (b) construct an angle of 60°
  (c) bisect an angle.

3 Angle sum in a triangle is 180°.
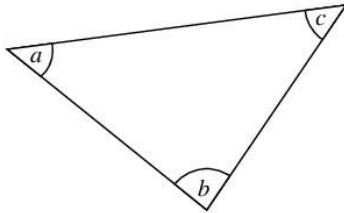
$$a + b + c = 180°$$

Angle sum in a quadrilateral is 360°.
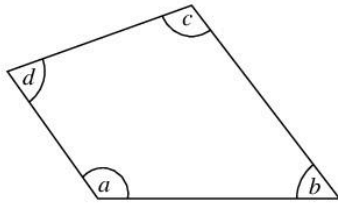
$$a + b + c + d = 360°$$

4 Vertically opposite angles are equal.
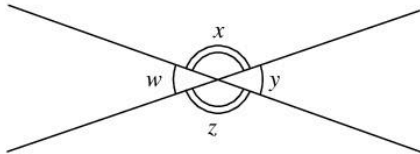
angle w = angle y
angle x = angle z

2 Construct an angle of:
  (a) 90°   **(b)** 45°
  (c) 60°   **(d)** 30°
  (e) 120°

3 Find the marked angles.
  (a)

  (b)

  (c)
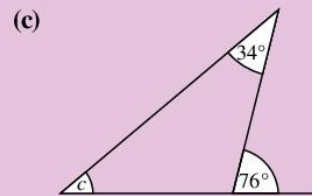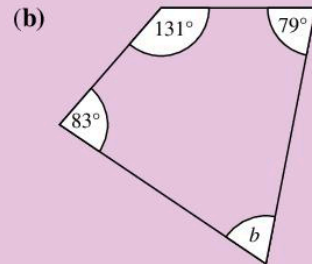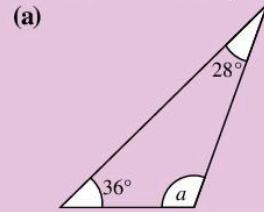
4 Give the value of the lettered angles.
  (a)

  (b)
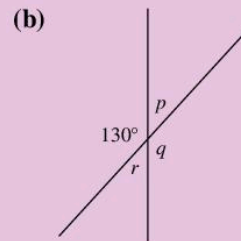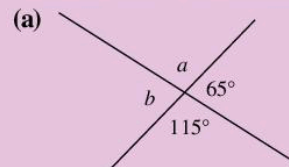
5 Parallel lines are always the same distance apart.

Parallel lines never meet
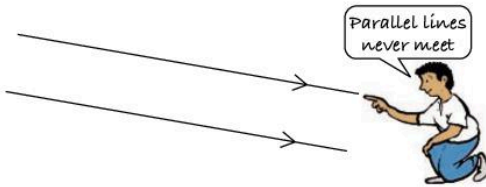
6 Corresponding angles on parallel lines are equal.

a

b

a = b
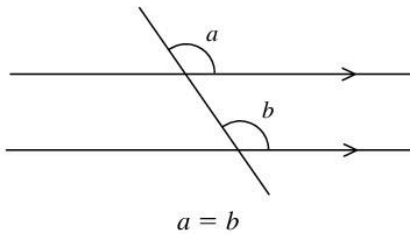
7 Alternate angles on parallel lines are equal.

y   *x*

x = y
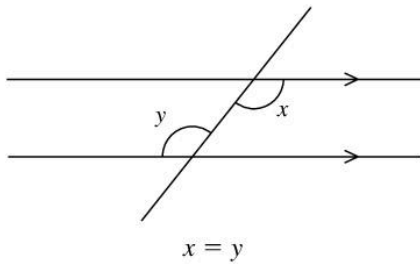
5 Pick out all the pairs of parallel lines from this diagram:

A   B

C   F

D   E

H   G
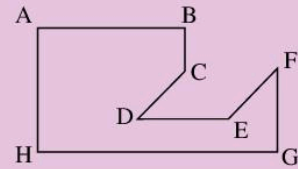
6 **(a)** Which angles in the diagram are corresponding?

75°   *a*

b   35°

c
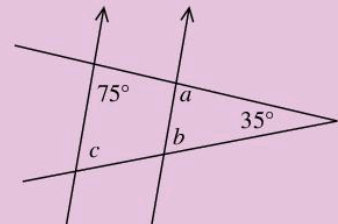
(b) What size are the angles *a*, *b* and *c*?

7 **(a)** Which angles in the diagram are alternate?

65°

f

d   60°   *e*
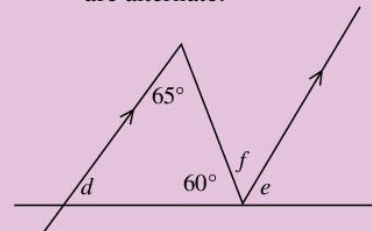
(b) What size are the angles *d*, *e* and *f*?

Sets

Objectives

✔ Find the number of subsets in a given set
✔ find the intersection and union of two sets
✔ find the complement of a set
✔ use the ideas of union, intersection and complement to solve simple problems

The set of pigs with wings is empty.
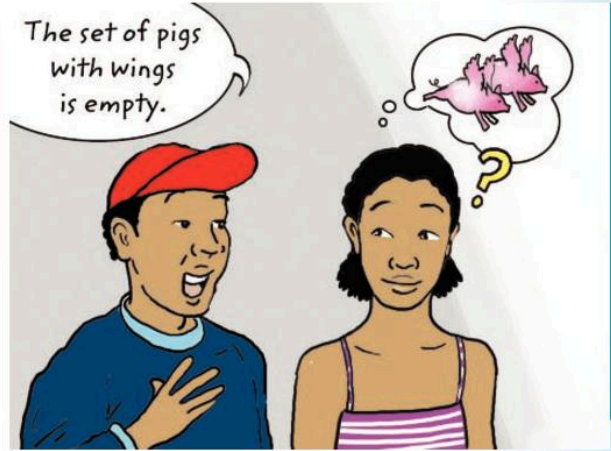
What's the point?

Scientists often study the characteristics of sets of things. For example, they have to answer questions such as, 'Can all birds fly?' or 'Which mammals are also sea creatures?'

Before you start

You should know ...

1 This is a Venn diagram.

Whole numbers from 1 to 10

The universal set   A subset
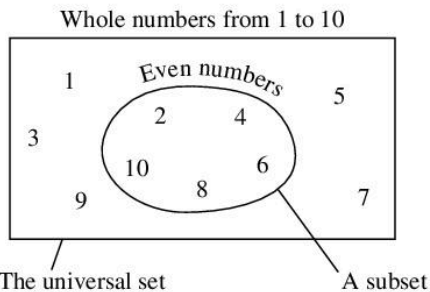
The elements of the set are 1, 2, 3, 4, 5, 6, 7, 8, 9, 10.

2 Sets can be written using curly brackets.
For example:
{even numbers} = {2, 4, 6, 8, 10, ... }

Check in

1 Copy and complete this Venn diagram.

Whole numbers from 1 to 10

Factors of 10
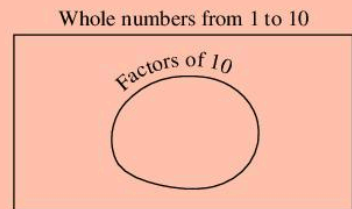
2 Using curly brackets write down:
   (a) the set of days of the week
   (b) the empty set.

3   The intersection of two sets consists of elements common to both sets.
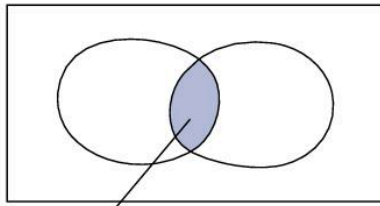
The intersection

3   Copy and complete this Venn diagram.

Whole numbers from 1 to 10

Even numbers   Numbers less than 5
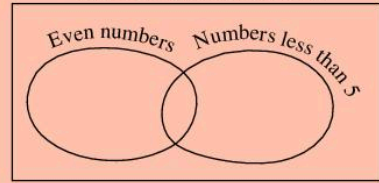
4.1   Sets and subsets
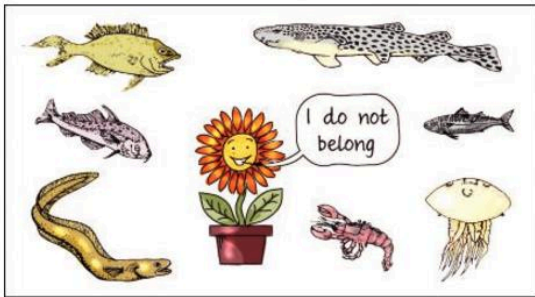
Exercise 4A

1   Using curly brackets, list these sets:
   (a) Even numbers greater than 8 and less than 12.
   (b) Vowels.
   (c) Whole numbers between 2 and 11.
   (d) Prime numbers less than 15.
   (e) Odd numbers between 11 and 13.

2   Using the people in your family as the universal set, draw a Venn diagram showing the subset of females.

3   Using {whole numbers from 1 to 20} as the universal set, draw separate Venn diagrams to show the subset of:
   (a) multiples of 5
   (b) prime numbers
   (c) factors of 16
   (d) square numbers
   (e) rectangle numbers.

4   Sylvia's friends

Vincent   Tony

In the science club

In the school play

                     Delia

        Betty

Bo   Roy   Judd

Sarah

        Reena

                     Samuel

Michael   Sally
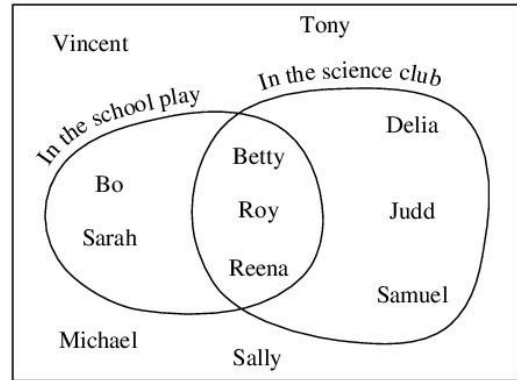
In the Venn diagram:
   (a) Which of Sylvia's friends are:
       (i)   in the school play *and* the science club
       (ii)   in the play *but not* in the science club
       (iii) in the science club *but not* the play
       (iv)   in *neither* the play *nor* the science club?
   (b) How many elements are shown in the Venn diagram?

5   Of the children in Content Village, John, Paul, Mark, Lottie and Kerry own bicycles. Sandra, Lottie, Jenny, Mark and Sue own skateboards.
   (a) Show this information on a Venn diagram.
   (b) Reverend Donaldson saw a boy cycling down the road with a skateboard under his arm. Which boy is it most likely to be?

Look at the set

{a, b, c}

What are the subsets of this set?

{a}, {b}, {c}, {a, b}, {a, c}, {b, c}

Are there any more? Yes.

{ } and {a, b, c}

Namely the empty set and the set itself.
Notice a set with three elements has 8 subsets.

Exercise 4B

1 List all the subsets of these sets:
   (a) {a, b}   (b) {1, 2, 3}
   (c) {5}   (d) {1, 2, 3, 4}

2 (a) Copy and complete

Set	Number of elements	Number of subsets
{5}	1	2
{a, b}	2	
{1, 2, 3}		8
{1, 2, 3, 4}		

   (b) What pattern do you see?

3 Use the result of Question 2 to find the number
   of subsets in a set with:
   (a) 5 elements
   (b) 8 elements
   (c) N elements.

4 Copy and complete the table for the set with
   three snakes.
   {adder, boa, cobra}

Subset	List	Number of subsets
0 members	{ }	1
1 member		
2 members		
3 members	{	1

5 Copy and complete the tables for the set with
   four snakes
   {adder, boa, cobra, daboia}

Subset	List	Number of subsets
0 members		
1 member		
2 members		6
3 members		
4 members		1

Technology

Visit

www.mathsisfun.com/activity/subsets.html

Learn more about subsets of sets.

See how your answers to Questions 4 and 5
of Exercise 4B are linked to Pascal's triangle.
Follow the link to learn more about Pascal's
triangle.

4.2   More about intersection

The Venn diagram shows the sets {whole numbers
1 to 5} and {even numbers 1 to 10}.

The subset {2, 4} is common to both sets.

{2, 4} is called the **intersection** of the two sets.

Whole numbers 1 to 10
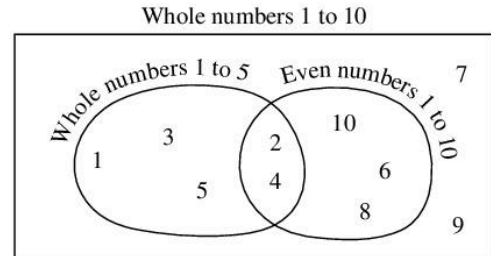
You can write:

$\{1, 2, 3, 4, 5\} \cap \{2, 4, 6, 8, 10\} = \{2, 4\}$

● $\cap$ is the mathematical symbol used for intersection.
   For example:
   $\{a, b, c, d, e\} \cap \{a, e, i, o, u\} = \{a, e\}$.

Exercise 4C

1 The picture above shows the boys in the Tambran steel band.
   (a) Write down the subset of names:
       (i) which begin with J
       (ii) which have four letters.
   (b) Which names are in the intersection of the two subsets?
   (c) Copy and complete:
       {names which begin with J} ∩ {names which have four letters} =

2 Use the picture above to copy and complete:
   (a) {names which have three letters} ∩ {names which begin with S} =
   (b) {names containing the letter g} ∩ {names containing the letter m} =
   (c) {names containing the letter s} ∩ {names containing the letter y} =

3 Copy and complete:
   (a) {blue, black, brown} ∩ {green, blue} =
   (b) {tennis, cricket, basketball} ∩ {swimming, surfing, diving} =
   (c) {c, a, k, e} ∩ {k, i, t, c, h, e, n} =
   (d) {2, 7, 8, 13, 24} ∩ {9, 10, 11, 12, 13, 14, 15} =
   (e) {1, 17, 21, 34, 41, 56} ∩ {13, 16, 17, 20, 21, 24} =
   (f) {3, 8, 13, 19, 24} ∩ {11, 15, 17, 21, 27} =
   (g) {0.1, 0.7, 0.8, 0.9} ∩ {0.1, 0.2, 0.9} =

4 Using {whole numbers from 1 to 30} as the universal set:
   (a) list the multiples of 3
   (b) list the odd numbers
   (c) copy and complete:
       {multiples of 3} ∩ {odd numbers} =

5 Using {whole numbers from 1 to 30} as the universal set, copy and complete:
   (a) {even numbers} ∩ {multiples of 5} =
   (b) {whole numbers from 10 to 20} ∩ {multiples of 4} =
   (c) {whole numbers from 1 to 10} ∩ {factors of 12} =
   (d) {odd numbers} ∩ {prime numbers} =
   (e) {prime numbers} ∩ {square numbers} =

6 Copy and fill in the missing elements:
   (a) {1, 3, , , 9} ∩ {2, 3, 4, 5, 6, 7} = {3, 5, 7}
   (b) {6, 15, 21, , 34, 41} ∩ {13, 15, 24, 28, , 41} = {15, 24, 34, 41}
   (c) $\{\frac{1}{4}, \frac{1}{8}, \frac{2}{3}, , , \frac{3}{4}, \frac{2}{7}\} \cap \{\frac{1}{16}, \frac{1}{3}, \frac{1}{8}, \frac{1}{9}, \frac{3}{11}\} = \{\frac{1}{8}, \frac{1}{16}, \frac{3}{11}\}$
   (d) {3, , 5, 6} ∩ {2, 4, 6} = {4, }
   (e) {Jill, Mary, Joan, Bob, Ricky} ∩ {Peter, Maxine, , } = {Joan, Ricky}
   (f) {n, p, a, f, } ∩ {s, , t, e} = {f, t}

Venn diagrams are a good way of showing intersections.

7 (a) How is a mammal different from a fish?
   (b) Draw a Venn diagram to show {mammals} and {fish}.
   (c) What is {mammals} ∩ {fish}?
   (d) Identify at least two members of each subset.

8 (a) Write down three members of the set {animals that can swim}.
   (b) Write down three members of the set {animals that breath air}.
   (c) Draw a Venn diagram to illustrate the two sets.
   (d) What is the intersection of the two sets.
   (e) Write down three animals that would lie in the intersection of the two sets.

9

   (a) Write down all the planets in the solar system.
   (b) List the set of planets that have masses larger than the Earth.

(c) List the set of planets that are further from the Sun than Jupiter.

(d) Draw a Venn diagram to show
{planets larger than the Earth}
{planets further from the Sun than Jupiter}

(e) Which planets lie in the intersection of the two sets? What can you say about them?

Activity

- Find out how scientists use sets to classify living things and non-living things.
- Draw some Venn diagrams to illustrate your findings.
- Present them to your class.
- Where else are sets useful for classification?

Example 1

Show on a Venn diagram:
{multiples of 3} ∩ {odd numbers}.
Universal set = {whole numbers from 1 to 10}.

Whole numbers from 1 to 10
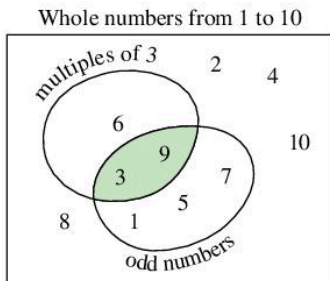
Example 2

Show on a Venn diagram:
{whole numbers less than 7} ∩ {factors of 4}.
Universal set = {whole numbers from 1 to 10}.

Whole numbers from 1 to 10
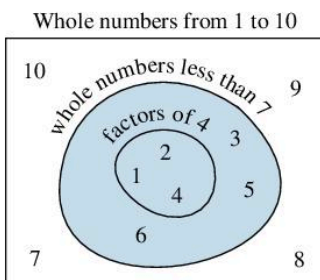
Exercise 4D

1 Show each answer to Question 6 of Exercise 4C on a Venn diagram.

2 Show on a Venn diagram:
(a) {multiples of 2} ∩ {multiples 4}
(b) {factors of 6} ∩ {factors of 12}
if the universal set is
{whole numbers from 1 to 12}.

3 State the intersection of the two indicated subsets in the following Venn diagrams.

(a)

(b)

(c)

(d)
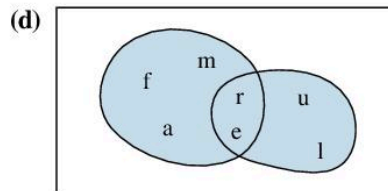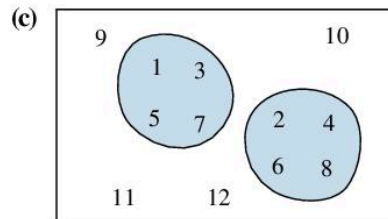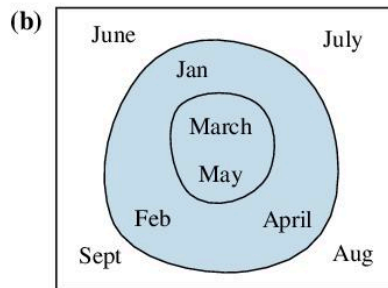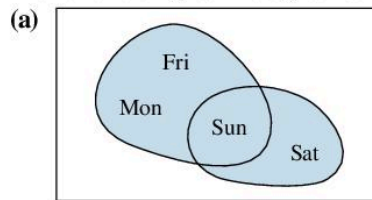
4 Last Tuesday night, a man broke into Mr Singh's shop, and stole $720. The thief left his coat behind him. In the pockets were a packet of cigarettes, a spectacles case, two door keys and a piece of string. Detective Starssky questioned several suspects. After talking to each of them, he

wrote their names on a Venn diagram. When he had finished, the Venn diagram looked like this:
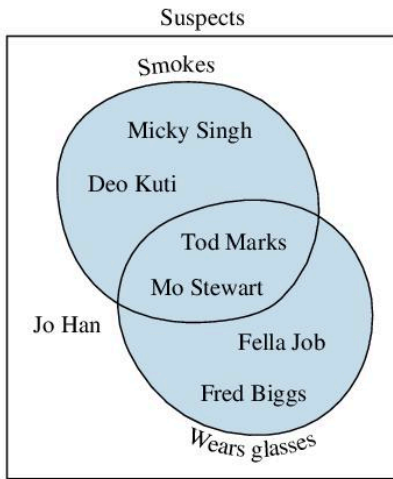
Suspects

(a) Why did Starssky let Jo Han go, after questioning him?
(b) Could Fella Job be the thief? Why?
(c) Which two suspects could be guilty? Why?
(d) What should Starssky do next?

4.3 Letters for sets

Often a letter is used instead of words, as a short name for a set.

For example,

T = {triangles}
S = {shoes}
V = {vowels}

could be the letters used for the sets in the following diagrams.
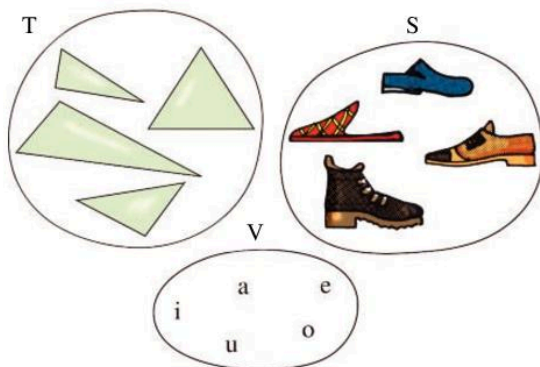
The letter U is usually chosen for the universal set.

Exercise 4E

1   In the diagram
    U = {whole numbers from 1 to 20}
    S = {factors of 16}
    T = {factors of 12}
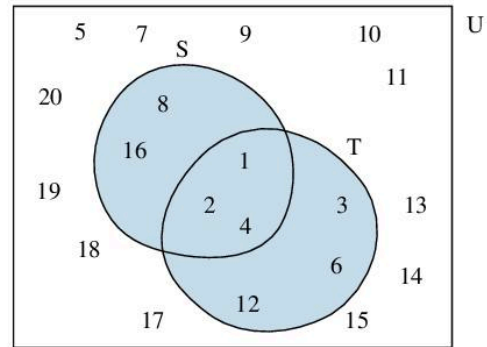
List the elements of:
(a) S   (b) T   (c) S ∩ T

2   If A = {2, 4, 6, 8}, B = {3, 6, 9, 12} and
    C = {6, 7, 8, 9}, write down:
    (a) A ∩ B   (b) B ∩ C   (c) A ∩ C

3   D = {days of the week} and
    T = {days of week beginning with T}.
    (a) Draw a Venn diagram to show the information.
    (b) List the members of the set D ∩ T.

4   (a) Draw a Venn diagram to show these sets:
        U = {months of the year}
        J = {months whose names begin with J}
        S = {months whose names have seven letters}
    (b) List the members of J ∩ S.

5   Draw Venn diagrams to show these sets:
    (a) U = {whole numbers from 1 to 10}
        A = {1, 3, 5, 7, 9}
        B = {2, 4, 5, 7}
    (b) U = {whole numbers from 1 to 15}
        P = {prime numbers less from 15}
        E = {even numbers less than 15}
    (c) U = {whole numbers from 1 to 20}
        T = {multiples of 3 less than 20}
        S = {multiples of 7 less than 20}
    (d) U = {mice}
        W = {white mice}
        B = {black mice}

6 **(a)** Using the Venn diagram below, write out a full name to describe:
   (i) the universal set
   (ii) the subset P
   (iii) the subset O
(b) List the elements of the intersection of P and Q.
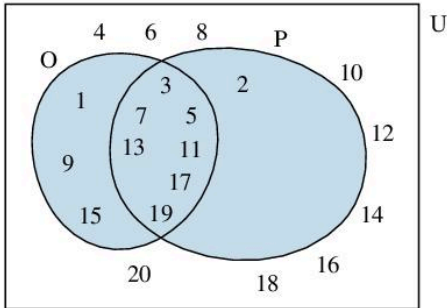
7 List the elements of
(a) F = {multiples of 4 up to 50}
(b) T = {multiples of 12 up to 60}
(c) F ∩ T
(d) Draw a Venn diagram to illustrate F and T.

8 For each Venn diagram describe the universal set U and the subsets A and B.
(a)

(b)

(c)

(d)
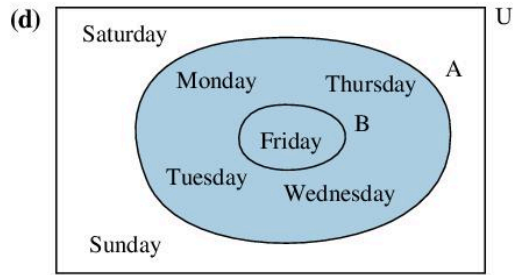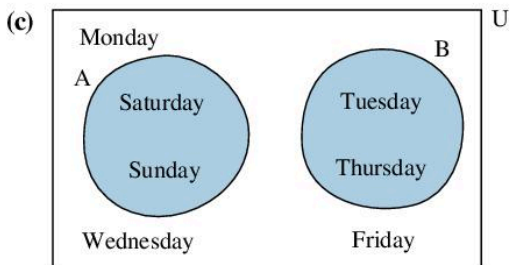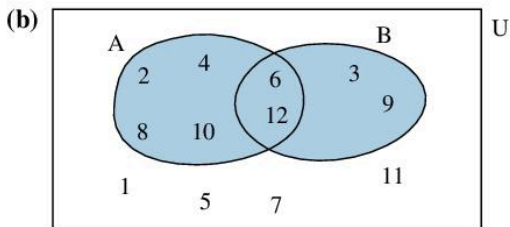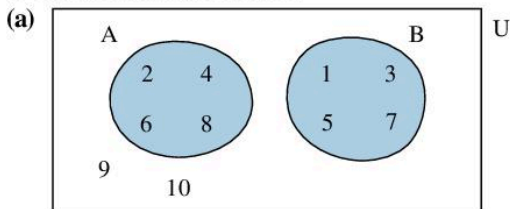
9 For each part of Question 8 write down the members of A ∩ B.

4.4 Union of sets

The **union** of two sets is a set containing all the elements of both sets.

● ∪ is the mathematical symbol used for union. For example,
{a, b, c, d} ∪ {a, e} = {a, b, c, d, e}

Example 3

Find A ∪ B where
   A = {1, 3, 5} and
   B = {2, 3, 5, 7}
A ∪ B = {1, 2, 3, 5, 7}
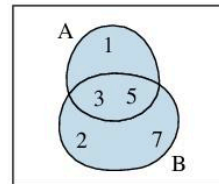
The shaded region shows A ∪ B.

Example 4

Find P ∪ Q where,
   P = {1, 3, 5} and
   Q = {1, 2, 3, 4, 5, 6}
P ∪ Q = {1, 2, 3, 4, 5, 6}
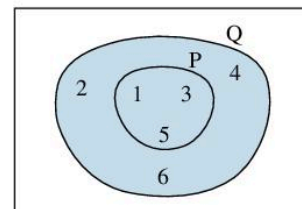
The shaded region represents P ∪ Q.

Exercise 4F

1 Use the diagram to list the elements in these sets:
(a) U **(b)** X **(c)** Y **(d)** X ∪ Y.
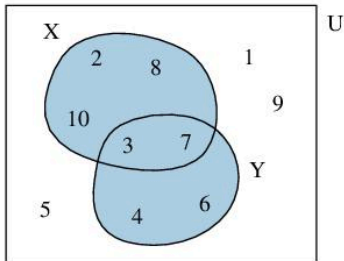
2 U = {1, 2, 3 . . . 10}
M = {even numbers}
N = {numbers less than 7}
State the members of the following sets:
(a) M ∪ N **(b)** M ∩ N
(c) N ∪ U

3

Don Lai Alice Lai Mona Lai Clare Lai Liz Hunt Tom Hunt Tina Sasaki Del Sasaki Paul Dean Clara Smith

In the picture above,
U = {tourists}
S = {tourists wearing sunglasses}
C = {tourists carrying a camera}
(a) Draw a Venn diagram to show the sets U, S and C.
(b) List the elements of
  (i) S   **(ii)** C   **(iii)** S ∪ C.
(c) List the names of the tourists who have
  (i) just sunglasses
  (ii) just a camera
  (iii) both sunglasses and a camera.

4 In the picture above
H = {tourists wearing a hat}
F = {females}
(a) List the names of those that are either in H or F or both.
(b) List the elements of H ∪ F.

(c) What do you notice about your answers to parts (a) and (b).
(d) Draw a Venn diagram and shade in H ∪ F.

5 **(a)** Copy and complete this Venn diagram:

Whole numbers from 1 to 20 (U)
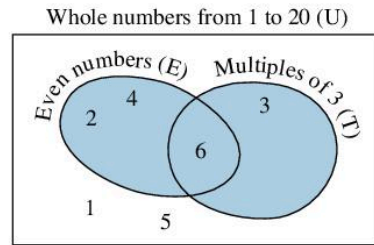
(b) From your completed Venn diagram, list the numbers in
  (i) E or T or both
  (ii) E ∪ T
  (iii) T ∪ E
  (iv) E but not in T
  (v) T but not in E
  (vi) E ∩ T.
  (vii) What do you notice about the answers to parts (ii) and (iii)?
  (viii) What do you notice about the answers to parts (i) and (ii)?
  (ix) What do you notice about the answers to parts (iv), (v) and (vi) combined?

6 Given A = {3, 6, 9, 12, 15},
  B = {2, 4, 6, 8, 10},
and C = {1, 3, 5, 7, 9}.

Write down the members of:
(a) A ∪ B **(b)** A ∪ C **(c)** B ∪ C
Draw a Venn diagram to help you if necessary.

7 {3, 4, 5, 6} ∪ {1, 2, 4, 5, 6}
  = {1, 2, 3, 4, 5, 6}

In the same way, copy and complete:
(a) {4, 7, 8} ∪ {2, 7, 8, 13} =
(b) {1, 5, 9, 15} ∪ {5, 10, 15, 20} =
(c) {2, 4, 9, 13, 17} ∪ {3, 4, 9, 12} =
(d) {9, 15, 18, 27} ∪ {7, 9, 18, 36} =
(e) {1, 2, 3, 4, 5} ∪ {2, 3, 4, 5} =
(f) {0.4, 0.8, 0.9} ∪ {0.5, 0.8, 0.9, 1.1} =

8 Copy, and fill in the missing numbers:
(a) $\{2, 6, 10\} \cup \{2, 6, \ , 14\} =$
  $\{2, 6, 10, 12, 14\}$
(b) $\{4, 5, 7, 11\} \cup \{1, 2, 4, 7, 11\} =$
  $\{1, \ , \ , 5, 7, \ ,\}$
(c) $\{6, \ , 9, 12\} \cup \{7, 12\} =$
  $\{6, \ , 8, \ , \ ,\}$
(d) $\{2, 4, 6, 8\} \cup \{2, 4, \ , \ , \ ,\} =$
  $\{2, 4, \ , \ , 10\}$

9 Given   U = {whole numbers from 1 to 24}
    and   A = {factors of 18}
        B = {factors of 12}
(a) Draw a Venn diagram to show this information.
(b) List the members of A $\cup$ B.

10 Make two larger copies of this Venn diagram.
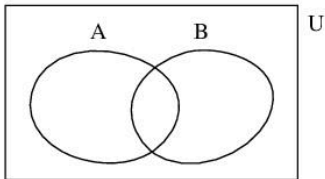
(a) On the first copy, shade the region that represents A $\cup$ B.
(b) On the second copy, shade the three regions that represent:
  (i) A $\cap$ B
  (ii) A *but not* B
  (iii) B *but not* A.
Compare your two copies.
What do you notice?

4.5 Complement of a set

This Venn diagram shows
U = {whole numbers from 1 to 10} and the subset
A = {even numbers from 1 to 10}.
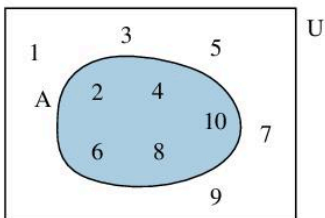
The subset outside the loop A is called the complement of A.

Complement of A = $\{1, 3, 5, 7, 9\}$

● The **complement** of A contains the numbers in the universal set which are not in A.

The symbol for the complement of A is A′.

Example 5

Describe the complement of A in this Venn diagram.
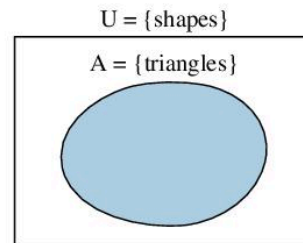
The set A contains triangles so the complement of A contains shapes that are not triangles.

Exercise 4G

1 List the members of the complement of A (A′) in these Venn diagrams:
(a)

(b)

(c)
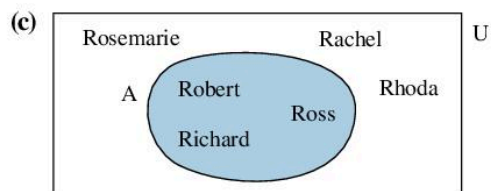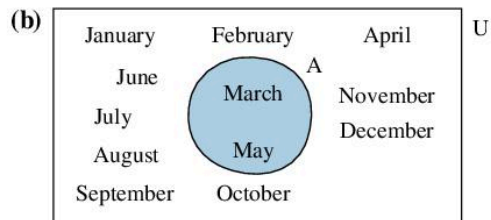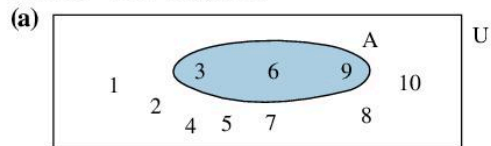
2 Sort this set of shapes into subsets.
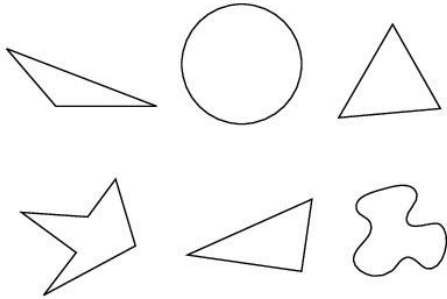
(a) Describe the subsets in words.
(b) Draw a Venn diagram to show the sets.
(c) Is one subset the complement of the other?

3 If $U = \{1, 2, 3, \ldots 10\}$ write down the complements of these subsets.
(a) $\{2, 3, 5, 7\}$
(b) $\{1, 2, 3, 4, 5\}$
(c) $\{5, 10\}$
(d) $\{7\}$
(e) $\{\}$

4 Describe the complement of the subset in each of these Venn diagrams.
(a)

(b)

(c)
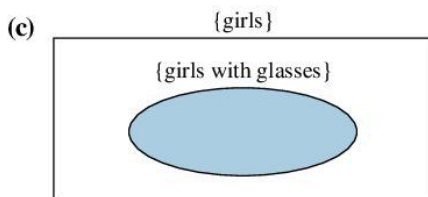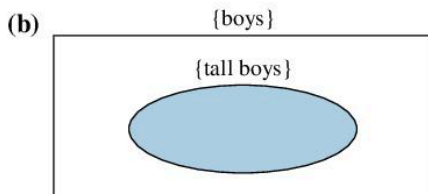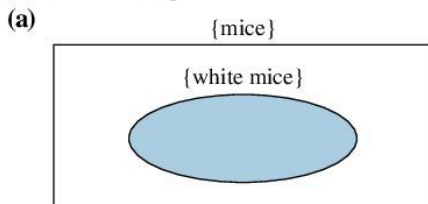
5 (a) The Venn diagram shows which games some boys in Form 2 like.
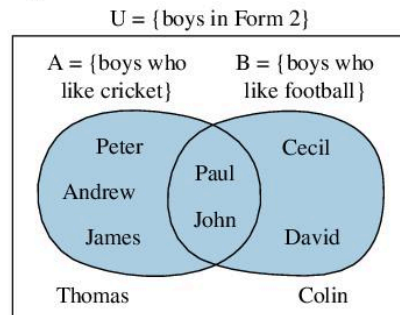
(i) Which boys like cricket only?
(ii) Which boys like cricket and football?
(iii) Which boys do not like cricket?
(iv) Which boys do not like cricket or football?
(v) Which boys like cricket but not football?

(b) List the members of
(i) $A \cup B$ (ii) $A \cap B$
(iii) A' (iv) B'

6 Let $U = \{$people$\}$, $A = \{$tall people$\}$ and $B = \{$teachers$\}$.
(a) Draw a Venn diagram to show this information.
(b) Describe the sets
(i) $A \cap B$ (ii) $A \cup B$
(iii) the complement of A

7 Let $U = \{$rats$\}$,
$S = \{$rats with sharp teeth$\}$
and $G = \{$greedy rats$\}$.
(a) Draw a Venn diagram to show this information.
(b) Describe the sets:
(i) $S \cap G$ (ii) $S \cup G$
(iii) the complement of G

8 Let $U = \{a, b, c, d, e\}$, $A = \{a, b, e\}$ and $B = \{d, e\}$. List the elements of
(a) A' (b) $A' \cap B$
(c) $A' \cap B'$ (d) $A \cup B'$
(e) $A \cup B$ (f) $(A \cup B)'$
(g) $A \cap B$ (h) $(A \cap B)'$

9 Copy and complete this table:

	Universal set	Set A	Set A′
(a)	{a, b, c, d, e, f, g}	{vowels}	
(b)	{whole numbers less than 10}	{factors of 8}	
(c)	{angle types}		{acute, right}
(d)		{p, a, r, t}	{l, y}

Exercise 4H – mixed questions

1 **(a)** Find the intersection and union of these sets.
   (i) P = {3, 6, 9, 12, 15} and
      Q = {6, 8, 10, 12, 14}
   (ii) X = {1, 3, 5, 7, 11} and Y = {1, 5, 11}
   (iii) R = {odd numbers less than 15}
      S = {prime numbers less than 12}
      We are assuming positive numbers.
   (iv) M = {multiples of 3 less than 15}
      N = {multiples of 2 less than 13}
      We are assuming positive numbers.
   (v) A = {multiples of 2 between 9 and 17}
      B = {multiples of 5 between 19 and 29}
   (b) Draw Venn diagrams to illustrate each pair
      of sets in part (a).

2 The table shows the games Jennifer's friends like playing.

	Rounders	Netball	Volleyball
Paula	✔	✔	
Mary		✔	
Ann	✔		✔
Theresa	✔	✔	
Susan			
Carol	✔		
Ruth			✔

R = {girls who like to play rounders}
N = {girls who like to play netball}
V = {girls who like to play volley ball}

(a) List the members of
   (i) R ∩ N   **(ii)** R ∩ V
   (iii) N ∪ V   **(iv)** R ∪ N
(b) Describe each of the sets listed in part (a).

3 If the universal set is your form, and
   G = {girls}, B = {boys},
   S = {students who wear spectacles},
   L = {students with long hair},
   list the elements of:
   (a) G ∩ S **(b)** G ∩ L **(c)** B ∩ S
   (d) B ∩ L

4 U = {names of the months}
   B = {names of months that contain the letter b}
   E = {names of months that contain the letter e}
   F = {names of months with more than five letters}
   (a) List the elements of:
      (i) B ∩ E **(ii)** E ∪ F **(iii)** F ∪ B
   (b) Describe in words the sets
      (i) E ∩ F **(ii)** B′

5 **(a)** Copy this Venn diagram.
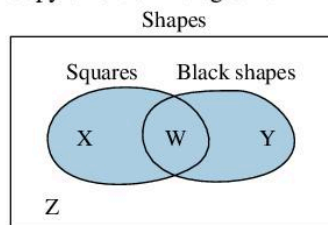
Shapes

(b) Draw any shape that belongs in the region
   marked:
   (i) W **(ii)** X **(iii)** Y **(iv)** Z

6 **(a)** If U = {triangles},
      R = {right-angled triangles}
   and I = {isosceles triangles},
   draw a Venn diagram to show this information.
   (b) Describe the sets:
      (i) R ∩ I **(ii)** R′

7 **(a)**

(b)
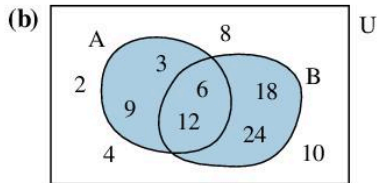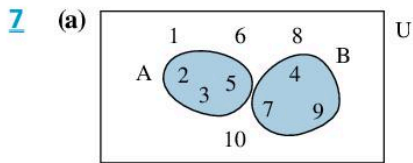
For each of the Venn diagrams above list the elements of the following sets.

(i)	U	**(ii)**	A
(iii)	B	**(iv)**	A ∩ B
(v)	A ∪ B	**(vi)**	(A ∩ B)′
(vii)	(A ∪ B)′	**(viii)**	A′
(ix)	B′	**(x)**	A′ ∩ B′
(xi)	A ∩ B′	**(xii)**	A′ ∩ B
(xiii)	A′ ∪ B′	**(xiv)**	A′ ∪ B

4 Consolidation

Example 1

(a) On a Venn diagram with U = {whole numbers 1 to 30}, show the multiples of 3 and the multiples of 5.

(b) What is {multiples of 3} ∩ {multiples of 5}?

(a)
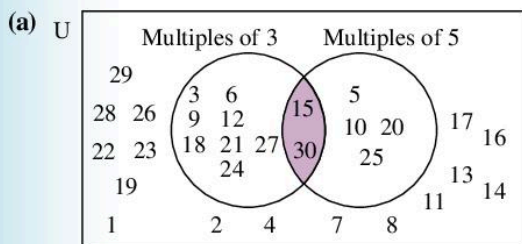
(b) Multiples of 3 = {3, 6, 9, 12, 15, 18, 21, 24, 27, 30}
Multiples of 5 = {5, 10, 15, 20, 25, 30}
{multiples of 3} ∩ {multiples of 5} = {15, 30}

Example 2

Given
A = {2, 3, 4, 5}, B = {2, 4, 6, 8}, C = {1, 3, 6, 9}, what is:

(a) A ∪ B
(b) B ∪ C

(a) A ∪ B = {2, 3, 4, 5} ∪ {2, 4, 6, 8}
= {2, 3, 4, 5, 6, 8}

(b) B ∪ C = {2, 4, 6, 8} ∪ {1, 3, 6, 9}
= {2, 3, 4, 6, 8, 9}

Example 3

If U = {1, 2, 3, 4, 5, 6, 7, 8} and
A = {2, 4, 6}, B = {1, 3, 5, 7}, what is:

(a) A′
(b) B′ ∪ A

(a) A′ = {1, 3, 5, 7, 8}
(b) B′ = {2, 4, 6, 8}
B′ ∪ A = {2, 4, 6, 8} ∪ {2, 4, 6}
= {2, 4, 6, 8}

Exercise 4

1 Draw Venn diagrams with
U = {whole numbers 1 to 20} to show the following sets:

(a) {even numbers}, {multiples of 5}
(b) {prime numbers}, {square numbers}
(c) {factors of 12}, {factors of 18}
(d) {odd numbers}, {multiples of 3}
(e) {factors of 30}, {multiples of 4}
(f) {multiples of 6}, {multiples of 3}

In each case write down the intersection of the two sets.

2 Given A = {2, 5, 8, 10}, B = {3, 5, 6, 12},
C = {1, 2, 3, 4} and D = {2, 5}, write down:

(a) A ∪ B   (b) A ∩ B   (c) A ∪ C
(d) B ∩ C   (e) A ∪ D   (f) B ∪ D
(g) D ∪ A   (h) A ∩ D   (i) B ∩ D
(j) C ∪ B

3 If U = {2, 4, 6, 8, 10}, A = {2, 4, 8,} and
B = {2, 8, 10}, find:

(a) A′   (b) B′   (c) A ∩ B
(d) A ∪ B   (e) A′ ∩ B   (f) A′ ∩ B′
(g) A′ ∪ B′   (h) A ∪ B′   (i) (A ∪ B)′
(j) (A ∩ B)′

4 The Venn diagram show two subsets X and Y of the universal set U. Find:
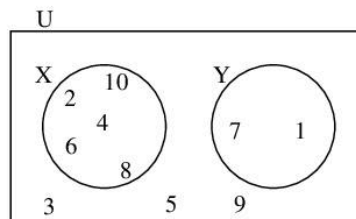
(a) U   (b) X ∩ Y
(c) X ∪ Y   (d) X′
(e) Y′   (f) (X ∪ Y)′
(g) X′ ∩ Y′   (h) X′ ∪ Y′
(i) X′ ∩ Y   (j) X ∩ Y′
(k) X ∪ Y′

Application

5 (a) Given the universal set U = {1 − 30}, draw a Venn diagram to show the subsets: {factors of 27} and {factors of 24}

(b) What is the intersection of the two subsets?

(c) What is the HCF of 24 and 27?

6 Given the universal set U = {1 − 36},
draw a Venn diagram to show the subsets:
{multiples of 3} and {multiples of 4}
  (a) What is the intersection of the two subsets?
  (b) What is the LCM of 3 and 4?

7 If U = {students in Form 2}, A = {students who
like football} B = {students who like cricket}:
  (a) Draw a Venn diagram to show these sets.
  (b) Describe these sets.
      (i) A ∩ B **(ii)** A ∪ B
      (iii) A′   **(iv)** (A ∪ B)′

(c) Write, in terms of A and B:
  (i) {students in Form 2 who do not
         like football}
  (ii) {students in Form 2 who like cricket but
         don't like football}
  (iii) {students in Form 2 who like football but
         not cricket}

Summary

You should know ...

1 The intersection of two sets can be shown on a Venn
diagram.
For example:
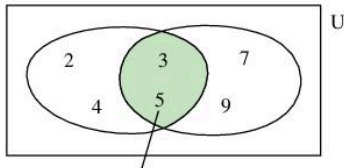
This is the intersection of the two sets.

Or using the symbol ∩
{2, 3, 4, 5} ∩ {3, 5, 7, 9} = {3, 5}

2 The union of two sets can be shown on a Venn diagram.
For example:
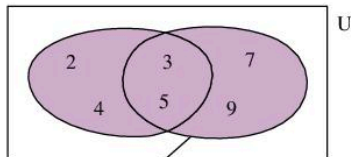
The shaded region represents the union
of both sets.

Or using the symbol ∪
{2, 3, 4, 5} ∪ {3, 5, 7, 9} = {2, 3, 4, 5, 7, 9}

Check out

1 **(a)** Copy and complete:
        {3, 6, 9, 12} ∩
                 {6, 8, 10, 12} =
    (b) Fill in the missing term:
        {2, 3, , 7} ∩
                 {2, 5, 11} = {2, 5}

2 **(a)** Copy and complete:
        {3, 5, 6, 8} ∪ {2, 3, 4, 5} =
    (b) Fill in the missing term:
        {2, 4, 6} ∪ {4, 5, }
             = {2, 4, 5, 6, 7}

3 Letters can be used to represent sets. The letter used for the Universal set is U.

3 These sets are represented by the letters U, P and Q. State which of the sets is U.
(a) {1, 3, 5, 7}
(b) {1, 2, 3, 4, 5, 6, 7, 8}
(c) {2, 3, 5, 8}

4 The complement of a set contains all the elements that are in the universal set, but are not in the given set.
For example:
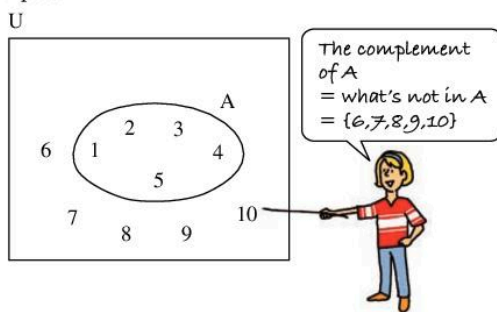
The complement of A = {6, 7, 8, 9, 10}
The complement of A can be written as A′

4 Given:
U = {5, 6 ... 15}
L = {6, 9, 12, 15}, and
M = {5, 7, 9, 11, 13, 15}
Find:
(a) L′
(b) L′ ∩ M′
(c) L ∪ M
(d) (L ∪ M)′

Revision exercise 1

Number

1 **(i)** **(ii)**
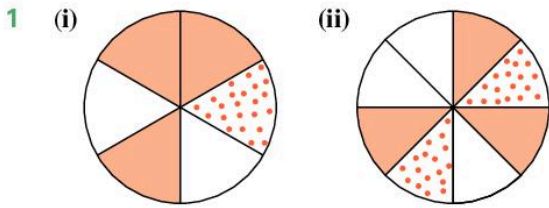
What fraction of each shape is:
(a) dotted **(b)** shaded
(c) non white **(d)** white?

2 Copy and complete:
(a) $\frac{2}{3}=\frac{\square}{6}$ **(b)** $\frac{7}{8}=\frac{21}{\square}$ **(c)** $\frac{8}{12}=\frac{\square}{3}$ **(d)** $\frac{15}{25}=\frac{3}{\square}$

3 Write the fractions in their simplest form:
(a) $\frac{6}{8}$ **(b)** $\frac{12}{16}$ **(c)** $\frac{16}{20}$ **(d)** $\frac{20}{24}$
(e) $\frac{15}{35}$ **(f)** $\frac{49}{84}$ **(g)** $\frac{48}{64}$ **(h)** $\frac{51}{85}$

4 Work out the following:
(a) $\frac{3}{8}+\frac{4}{8}$ **(b)** $\frac{7}{12}-\frac{5}{12}$ **(c)** $\frac{6}{7}+\frac{9}{7}$ **(d)** $\frac{17}{4}-\frac{9}{4}$
(e) $1\frac{3}{5}+3\frac{2}{5}$ **(f)** $7\frac{1}{4}-5\frac{3}{4}$ **(g)** $\frac{3}{8}+\frac{1}{4}$ **(h)** $\frac{5}{12}-\frac{1}{6}$
(i) $\frac{7}{9}+\frac{2}{3}$ **(j)** $\frac{11}{15}-\frac{3}{5}$ **(k)** $\frac{4}{5}+\frac{3}{7}$ **(l)** $\frac{9}{10}-\frac{6}{7}$

5 Find:
(a) $1\frac{3}{4}+2\frac{4}{5}$ **(b)** $3\frac{5}{8}-1\frac{2}{3}$
(c) $5\frac{1}{6}+3\frac{5}{9}$ **(d)** $4\frac{3}{8}-3\frac{5}{12}$

6 Work out the following:
(a) $7\times\frac{3}{5}$ **(b)** $\frac{4}{9}\times5$ **(c)** $12\times\frac{5}{6}$
(d) $8\times1\frac{1}{4}$ **(e)** $2\frac{2}{3}\times7$ **(f)** $3\frac{1}{3}\times4\frac{1}{4}$
(g) $\frac{1}{2}\div4$ **(h)** $\frac{2}{3}\div8$ **(i)** $5\div\frac{1}{3}$
(j) $8\div\frac{3}{4}$ **(k)** $9\div2\frac{1}{4}$ **(l)** $1\frac{3}{7}\div\frac{9}{10}$
(m) $\frac{5}{6}\div\frac{2}{3}$ **(n)** $\frac{7}{8}\div\frac{7}{9}$ **(o)** $4\frac{2}{3}\div\frac{7}{9}$

7 Anna, Brianna and Celia are to share a sum of money in the ratio 1 : 2 : 3.
(a) If Brianna gets $50 how much does Celia get?
(b) If Celia gets $120 how much does Anna receive?

8 **(a)** Eight tins of paint weigh $5\frac{1}{3}$ kg. What is the weight of each tin?
(b) Jan buys $2\frac{1}{2}$ kg of seed in one store and $1\frac{3}{5}$ kg in another. How much does he buy altogether?

(c) Sandra spent $\frac{2}{3}$ of her pocket money on sweets and $\frac{1}{7}$ on a comic. What fraction of her pocket money did she have left?
(d) Julia bought a piece of material $2\frac{1}{3}$ metres wide and $3\frac{1}{4}$ metres long. What was the area of this material?

9 How Clive spends his week.
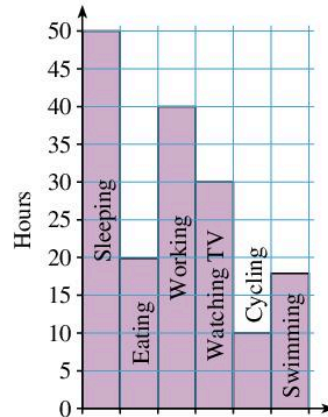
The block graph shows how Clive spends his week.
How many hours are there in a week?
Write down the fraction of each week Clive spends:
(a) cycling **(b)** working
(c) sleeping **(d)** swimming
(e) awake **(f)** taking exercise.

10 Change:
(a) 11011 (base 2) to base 10
(b) 247 (base 10) to base 5

Decimals

11 **(a)** Write these numbers to two decimal places.
(i) 14.321 **(ii)** 6.467 **(iii)** 0.325
(iv) 1.7241 **(v)** 7.016 **(vi)** 0.003 45

(b) Write the numbers in part **(a)** to one decimal place.

12 Write each of these fractions as a decimal:

(a) $\frac{7}{10}$ (b) $\frac{13}{100}$ (c) $\frac{429}{1000}$

(d) $\frac{4}{5}$ (e) $\frac{3}{4}$ (f) $\frac{7}{8}$

(g) $\frac{17}{20}$ (h) $\frac{23}{50}$ (i) $\frac{49}{50}$

(j) $\frac{5}{4}$ (k) $\frac{109}{25}$ (l) $\frac{413}{200}$

13 Work out:

(a) $8.72 \div 4$ (b) $0.35 \div 7$

(c) $6.4 \div 0.2$ (d) $2.5 \div 0.05$

(e) $25.6 \div 0.08$ (f) $4.3 \div 1.2$

(g) $0.072 \div 3.6$ (h) $0.038 \div 0.02$

14 Work out:

(a) $1.7 + 2.4$ (b) $5.46 + 4.38$

(c) $2.5 - 1.7$ (d) $8.71 - 3.29$

(e) $0.362 + 0.175$ (f) $0.917 - 0.431$

(g) 5.3×2 (h) $9.6 \div 4$

(i) 7.41×3 (j) $5.35 \div 5$

(k) 1.08×12 (l) $1.28 \div 16$

15 For each of the rectangles below find:

(a) the perimeter   (b) the area

Put in order of size, from the smallest to the largest the four perimeters and the four areas.

(i)

3.1 cm

1.7 cm

(ii)

2.3 cm

2.9 cm

(iii)

2.5 cm

2.5 cm

(iv)

2.8 cm

1.2 cm
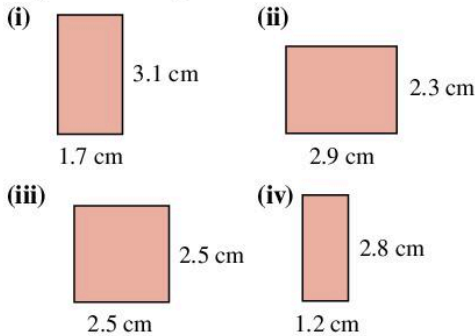
16 $123 \times 45 = 5535$ and $6.7 \times 8.9 = 59.63$. Without using your calculator, write down the answers for:

(a) 12.3×45 (b) 123×0.45

(c) 1.23×4.5 (d) 0.123×450

(e) 67×8.9 (f) 0.67×89

(g) 670×89 (h) 0.67×0.89

Now check your answers by using your calculator.

Use your calculator for the next questions.

17 (a) Books cost $5.65 each. How many can be bought for $163.85?

(b) A car averages 36.2 miles per gallon. How many gallons will be needed for 1267 miles?

(c) The area of a rectangular yard is 36.98 square metres. If its width is 4.3 metres, what is its length?

18 Copy and complete.

Number	1 decimal place	1 significant figure
16.32		20
205.692	205.7	
0.0014		
0.895		
1906.48		

19 Write in standard form:

(a) 2184   (b) 30 008

(c) 87.6   (d) 210 100

(e) 95 000 000   (f) 398 000 000

20 (a) Use your calculator to find:

$$\frac{1}{11}, \frac{2}{11}, \frac{3}{11}, \frac{4}{11}$$

Can you see a pattern in your answers? Without using your calculator, write down the decimals for:

$$\frac{5}{11}, \frac{6}{11}, \frac{7}{11}, \frac{8}{11} \text{ and } \frac{9}{11}$$

(b) Find a fraction for $0.13131313 \ldots$

Angles

21 Say whether each angle is: acute, right-angled, obtuse or a reflex angle.

(a)

(b)

(c)   **(d)**
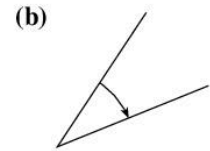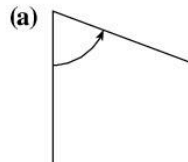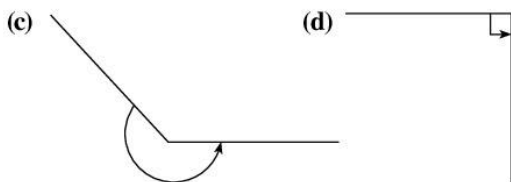
22 Estimate the size of each angle in Question 21. Use your protractor to check your estimates.

23 Write down, without measuring, the size of angles marked with letters. Give a reason for each answer.

(a)

k
d
$50°$

(b)

$137°$
m

(c)

n
$47°$

(d)

$123°$
p
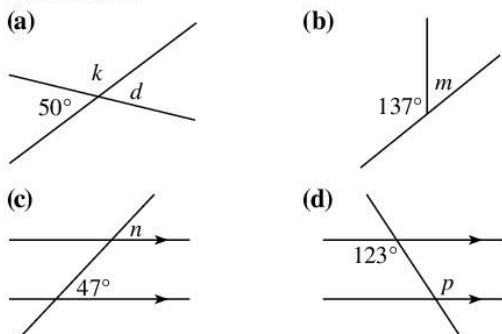
24 Using a ruler and compass only, construct:
(a) (i) an angle of 45°
  (ii) triangle ABC where AB = 10 cm, CÂB = 90° and CB̂A = 30°
(b) Measure the length AC in the triangle in **(a) (ii)**.

25 Calculate the angles marked with letters.

$64°$ r
s q $42°$
         $55°$
p t
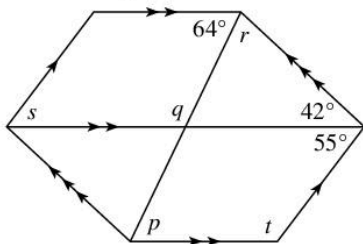
26 PQRS is a parallelogram.

S   R
$128°$

$32°$
P   Q
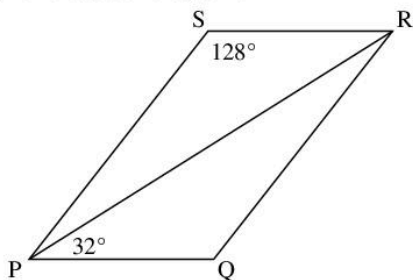
RP̂Q = 32° and PŜR = 128°

Find, giving reasons:
(a) PR̂S   **(b)** SP̂R   **(c)** PQ̂R

27 Calculate the interior angle of:
(a) a square
(b) a regular pentagon
(c) a regular octagon.

28 The shape KLMN below is made from two identical triangles KLM and KNM.

L
K   M
N
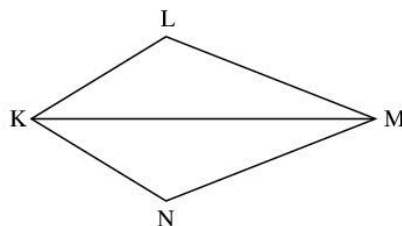
Which angle in triangle KLM are equal in size to:
(a) KL̂M   **(b)** LK̂M   **(c)** LM̂K?

29 Cut out from card a triangle like triangle KLM in Question 26. Make a shape like KLMN by tracing around your triangle. Now draw the diagonals KM and LN. Mark the point where they cross as X. What can you say about the angles KX̂L, KX̂N, MX̂L and MX̂N? What can you say about LX and XN?

30 Draw a parallelogram PQRS. Join the diagonals PR and QS and mark the point where they cross as X. What can you say about:
(a) PX and XR   **(b)** QX and XS
(c) the angles at X?

Sets

31 Write down a list of the elements in each set:
(a) {vowels in the alphabet}
(b) {the first five odd numbers}
(c) {the days of the week}
(d) {the last three months of the year}
(e) {multiples of four between 7 and 19}

32 (a) Write down all the subsets of the set {January, February, March}
(b) How many subsets are there?

33

Children in our village
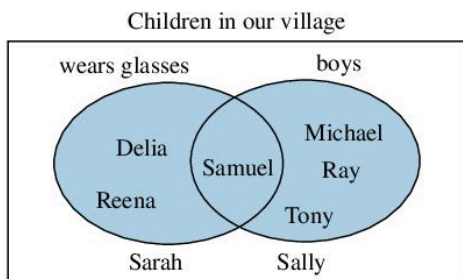
Look at the Venn diagram above. Which of the children:
(a) *do not* wear glasses
(b) are girls who wear glasses
(c) are *not* boys *and* do *not* wear glasses?

34 In Question 33, list the elements of the following sets.
(a) G = {children who wear glasses}
(b) B = {boys}
(c) G ∩ B
(d) the complement of B
(e) G ∪ B
(f) the complement of (G ∪ B)

35 If   U = {whole numbers from 1 to 20},
M = {multiples of three},
S = {square numbers},
write down the elements of:
(a) M   (b) S   (c) M ∩ S
(d) M ∪ S   (e) the complement of M.

36 Draw a Venn diagram to represent the universal set U, together with the subsets V and W where:
U = {the letters of the alphabet}
V = {vowels}
W = {the first 10 letters}

37 Using the sets in Question 36, write down the elements of:
(a) V ∩ W   (b) V ∪ W
(c) the complement of V
(d) the complement of W.

38 If U = {p, q, r, s} write down all the subsets of U with:
(a) one element   (b) two elements
(c) three elements.

39 Make a larger copy of the Venn diagram started below, and complete it by filling in all the whole numbers from 1 to 30.

Whole numbers from 1 to 30 (U)
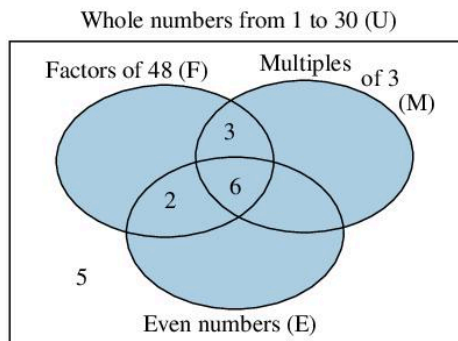
Write down the elements of:
(a) E   (b) F   (c) M
(d) E ∩ M   (e) F ∪ M

40 If   U = {names of the months}
A = {names of months that contain the letter a}
B = {names of months that contain the letter b}
E = {names of months that contain the letter e}

draw a Venn diagram like the one in Question 39 to show the universal set U and the subsets A, B and E.

Mixed questions 1

1 What is $\frac{1}{5} + \frac{2}{3}$?
A $\frac{3}{8}$ **B** $\frac{12}{53}$ **C** $\frac{13}{15}$ **D** $\frac{12}{15}$

2 Joan had $2\frac{1}{2}$ loaves of bread: David ate $\frac{3}{4}$ of a loaf. How many loaves did Joan have left?
A $3\frac{1}{4}$ **B** $2\frac{1}{4}$ **C** $1\frac{3}{4}$ **D** $\frac{3}{4}$

3 What is $\frac{2}{3}$ of $27?
A $11   **B** $18   **C** $27.66   **D** $40.50

4 How many quarter-litre bottles of juice can I get from a container holding 2 litres?
A 1   **B** 2   **C** 4   **D** 8

5 Alan lives $3\frac{1}{2}$ km from his school. He walks one-quarter of the way. How far does he walk?
A $\frac{7}{8}$ km   **B** $3\frac{1}{4}$ km   **C** $3\frac{3}{4}$ km   **D** 14 km

6 Janice has $2\frac{1}{2}$ kg of potatoes, Kimberly has $1\frac{1}{4}$ kg more potatoes than Janice. How many kilograms of potatoes do they have altogether?
A $1\frac{1}{4}$ kg   **B** $\frac{3}{4}$ kg   **C** $3\frac{3}{4}$ kg   **D** $6\frac{1}{4}$ kg

7 What is the area of a rectangular piece of card 4 cm long and $3\frac{3}{4}$ cm wide?

   A 15 cm²   **B** $7\frac{1}{3}$ cm²   **C** $12\frac{1}{3}$ cm²   **D** $13\frac{1}{3}$ cm²

8 What is the sum of the base 2 numbers 1101 and 1110?

   A 1111   **B** 11101   **C** 11011   **D** 111011

9 In a fruit squash, the ratio of orange to pineapple is 2 : 3. How many cups of pineapple juice are needed to make 75 cups of squash?

   A 5   **B** 15   **C** 30   **D** 45

10 Tom wants to share $\frac{1}{2}$ kg of sugar equally among his three brothers. How much sugar will each get?

   A $\frac{1}{6}$ kg   **B** $1\frac{1}{2}$ kg   **C** $2\frac{1}{2}$ kg   **D** $3\frac{1}{2}$ kg

11 The complement of 30° is added to the supplement of 150°. The result is:

   A 30°   **B** 60°   **C** 90°   **D** 120°

12 At 12.30 pm the smaller angle between the hands of a clock is:

   A 165°   **B** 155°   **C** 150°   **D** 145°

13 What type of angle is 290°?

   A acute   **B** obtuse   **C** straight   **D** reflex

14 Find angle x in the diagram.

   A 42°   **B** 38°   **C** 48°   **D** 28°
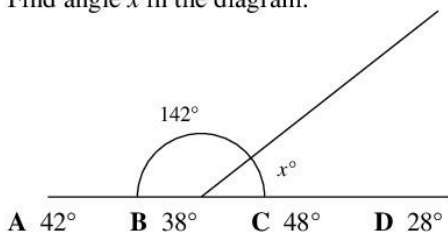
15 Find angle y in the diagram.

   A 55°   **B** 125°   **C** 35°   **D** 45°
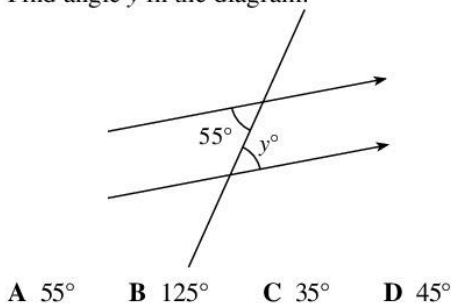
16 Find angle z in the diagram.

   A 144°   **B** 54°   **C** 36°   **D** 126°
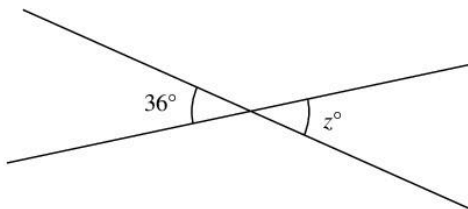
17 Two complete turns are made up of:

   A 4 right angles   **B** 6 right angles
   C 8 right angles   **D** 2 right angles

18 Find the value of x:

   A 120°   **B** 30°   **C** 90°   **D** 60°
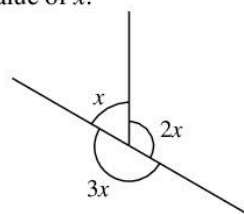
19 In the diagram, angles a and b are:

   A complementary   **B** supplementary
   C corresponding   **D** alternative
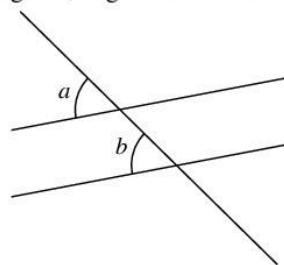
20 How many right angles are there in this diagram?

   A 16   **B** 20   **C** 9   **D** 8
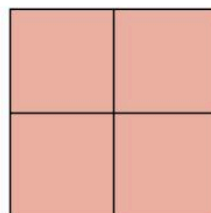
21 $\{3, 6, 9, 12\} \cup \{2, 3, 6, 12\} =$

   A $\{2, 3, 6, 9, 12\}$
   B $\{3, 6, 12\}$
   C $\{3, 6, 9, 12, 2, 3, 6, 12\}$
   D $\{3, 6, 9, 12\}$

22 The set A = {2, 3, 5}. How many subsets of A are there?

    A 3   **B** 4   **C** 8   **D** 16

23 U = {whole numbers from 1 to 10}
P = {prime numbers}
P′ =

    A {2, 3, 5, 7}   **B** {1, 2, 3, 5, 7}
    C {1, 4, 6, 8, 9, 10}   **D** {4, 6, 8, 9, 10}

24
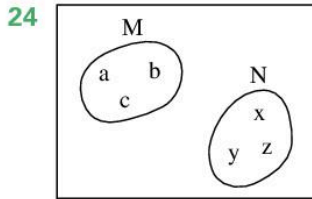
M ∩ N =

    A {a, b, c}   **B** {x, y, z}
    C {a, b, c, x, y, z}   **D** { }

25
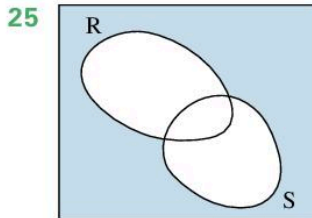
The shaded area represents

    A R ∩ S   **B** (R ∪ S)′
    C R′ ∩ S   **D** R ∩ S′

26 U = {students}
M = {students who like mathematics}
P = {students who like physics}
M ∩ P′ represents

    A students who like both mathematics and physics.
    B students who like physics but not mathematics
    C students who like neither mathematics nor physics.
    D students who like mathematics but not physics.

27
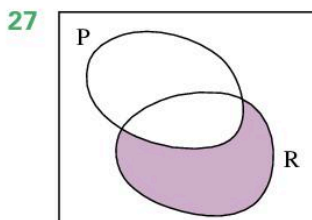
The shaded region represents

    A P′   **B** R   **C** P′ ∩ R   **D** R′ ∩ P

28 If A ⊂ B, and B ⊂ C, then A ∩ C is

    A {}   **B** A   **C** B   **D** C

29

The letter x represents the number

    A 0.4   **B** 0.45   **C** 0.5   **D** 0.65

30 John has 10 litres of juice. He sold 6.2 litres. How many litres of juice did he have left?

    A 16.2   **B** 5.2   **C** 3.8   **D** 4.2

31 What fraction is represented by the recurring decimal 0.666 . . . ?

    A $\frac{6}{10}$ **B** $\frac{2}{3}$ **C** $\frac{66}{100}$ **D** $\frac{666}{1000}$

32 Ten girls share $205 equally among them. How much does each receive?

    A $2050   **B** $205   **C** $20.50   **D** $2.05

33 1192.46 written to two significant figures is

    A 11   **B** 12   **C** 1100   **D** 1200

34 The fraction $\frac{5}{8}$ written as a decimal is

    A 8.5   **B** 5.8   **C** 0.625   **D** 0.58

35 What is the cost of 1.2 kg of fish at $17.25 per kilo.

    A $2.07   **B** $18.45   **C** $20.70   **D** $51.75

36 Which is the largest number?

    A 0.39   **B** 0.301   **C** 0.389   **D** 0.4

37 The product of two numbers is 248.4. If one number is 46, what is the other?

    A 202.4   **B** 46   **C** 5.4   **D** 0.54

38 Which of these is the best estimate of 1.39 × 17.26 is

    A 0.2   **B** 2   **C** 20   **D** 200

39 One part in 100 of a certain food additive is vinegar. How much vinegar is in 21.5 g of the additive?

    A 0.215 g   **B** 2.15 g   **C** 78.5 g   **D** 121.5 g

40 The Sun is 150 000 000 km from the Earth. What is this distance in metres, written in standard form?

    A 1.5×10^5 **B** 1.5×10^8
    C 1.5×10^{10} **D** 1.5×10^{11}

Measurement

Objectives

✔ compare imperial and metric units
✔ calculate the perimeter of shapes
✔ find the area of simple shapes
✔ read and use scale drawings

What's the point?

The ground staff who prepare a cricket pitch and set up the wickets measure lengths and areas regularly. Did you know that the height of the stumps should be 28 inches or 71.12 cm? Or that the distance between the stumps should be 9 inches or 22.86 cm?

Before you start

You should know ...

1 How to measure the length of objects using a ruler.
For example:
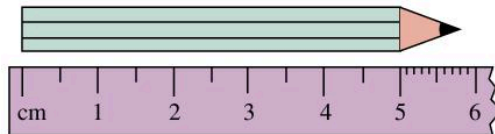
The pencil is 5 cm 8 mm or 5.8 cm long

2 Area can be measured in mm^2 and cm^2.
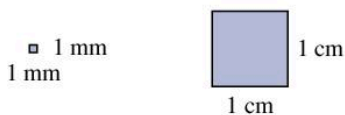
Check in

1 What is the length of each line?
(a) _____
(b) _____
(c) _____

2 Estimate the area of:
(a) a 25 cent coin
(b) a postage stamp
(c) your thumb nail
(d) a page of your exercise book.

3 You can find the area of a shape by counting squares.
For example:

Area A is 6 cm²   Area B is roughly 6 cm²
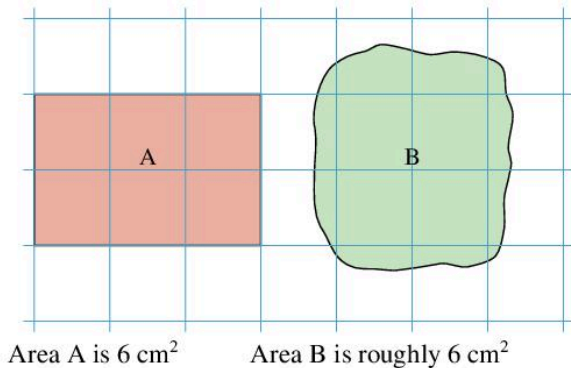
3 Find the areas of these shapes.
Each square is 1 cm².
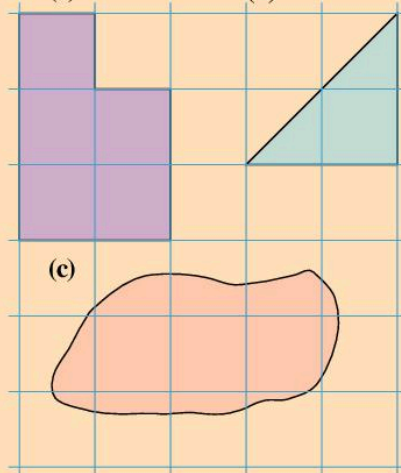
5.1   Units of measure

The main metric unit of length is the **metre (m)**. One metre is about one long stride.

Small lengths are measured in **centimetres (cm)** or **millimetres (mm)**.

1 m = 100 cm
1 cm = 10 mm

For mass, the **kilogram (kg)** is used.

A large pineapple weighs about 1 kilogram.

You probably weigh between 38 and 48 kilograms.

The **gram (g)** is used for small objects.
1 kg = 1000 g
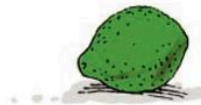
A lime weighs about 50 grams

A pencil weighs about 5 grams

The **tonne (t)** is used for heavy objects.
1 t = 1000 kg

A large car weighs about 1 tonne

Exercise 5A

1 Write down three things that are:
  (a) less than 1 cm long
  (b) less than 10 cm long
  (c) about 1 m long.

2 On a sheet of paper, mark two dots that you think are 5 cm apart.
  Now measure the distance between them, using a ruler. Were you nearly right?
  If not, try again.

3 Repeat Question 2 for dots that are:
(a) 10 cm apart (b) 15 cm apart
(c) 3 cm apart (d) 1 cm apart

4 Which metric unit would you choose, to measure:
(a) the length of this book
(b) the height of an electricity pole
(c) the length of your little finger
(d) the distance from the Earth to the Moon
(e) the height of the tallest building in your town or village
(f) the length of the hour hand on a watch?

5 For each part of Question 4, guess the measurement. Now check as many of the measurements as you can.

6 Write down three things that have a mass of:
(a) about half a kilogram
(b) about 1 kg (c) about 2 g

7 Which metric unit would you choose, to measure:
(a) the mass of a rhinoceros
(b) the mass of your maths teacher
(c) the mass of a pencil
(d) the mass of this book
(e) the mass of your desk
(f) the mass of Guyana's sugar crop?

8 Find, in metric units:
(a) your mass
(b) your height
(c) the distance around your waist.

Converting metric units

Sometimes you need to change metric units from one to another.

Example 1

(a) Alroy is 1 m 87 cm tall. What is his height in centimetres?
(b) A piece of wood is 263 mm long. What is its length in centimetres?

(a) 1 m 87 cm $= 1.87$ m $= 1.87 \times 100$ cm
$\qquad\qquad\qquad\qquad = 187$ cm
(b) 263 mm $= 263 \div 10$ cm $= 26.3$ cm

You can convert units of mass in the same way.

Example 2

An orange weighs 468 g. What is its mass in kilograms?

1 kg $= 1000$ g
so 468 g $= 468 \div 1000$ kg $= 0.468$ kg

Exercise 5B

1 Copy and complete:
(a) $\square$ g $= 1$ kg
(b) $\square$ kg $= 1$ t

2 Copy and complete:
(a) $\square$ cm $= 1$ m
(b) $\square$ m $= 1$ km
(c) $\square$ mm $= 1$ cm

3 Copy and complete this table.

Metres	Centimetres	Millimetres
6	$6 \times 100 =$	$6 \times 1000 =$
8		
	3400	
	500	
		4000

4 Copy and complete this table.

Kilograms	Grams
9	$9 \times 1000 =$
15	
3.1	
	4000
	800

5 This table shows the length of five objects. Copy and complete the table.

Object	Length (cm and mm)	Length (cm)	Length (mm)
Pencil	18 cm 5 mm	18.5	
Pen		13.3	
Eraser			50
Mat		95	
Fingernail			9

6 (a) Veda is 1.45 m tall. What is her height in centimetres?

(b) Veda weighs 52.3 kg. How much is this in grams?

7 Cherita Howard set Barbados national records in both the triple jump and long jump in 1996. She triple jumped 13.29 m and long jumped 6.43 m. What are these distances in centimetres?

Metric and imperial units

In some countries imperial units of length and mass are still used. You need to know that

1 inch is roughly 2.5 cm, 1 inch $\simeq$ 2.5 cm
1 mile is roughly 1.6 km, 1 mile $\simeq$ 1.6 km

and

1 pound is roughly 0.45 kg

The symbol $\simeq$ means "is roughly equal to".

Knowing these you can convert one unit to another. You may need your calculator.

Example 3

(a) My ruler is 12 inches long. How long is this in centimetres?
(b) It is 30 miles from Portsmouth to Roscau. How far is this in kilometres?

(a) 12 in $\simeq$ 12 $\times$ 2.5 cm = 30 cm
(b) 30 miles $\simeq$ 30 $\times$ 1.6 km = 48 km

Exercise 5C

1 How many centimetres are there in:
(1 foot = 12 inches, 1 yard = 3 feet)
(a) 1 foot **(b)** 1 yard?

2 Write these in centimetres:
(a) the height of a table 30 inches tall
(b) a 6-inch long pencil
(c) a 26-inch waist.

3 Convert these distances to kilometres:
(a) Dominica to Martinique, 60 miles
(b) St Lucia to Barbados, 100 miles
(c) St Kitts to Antigua, 50 miles
(d) Jamaica to Trinidad, 980 miles

4 What is the mass in kilograms of:
(a) a 200 lb man
(b) 2 lb of sugar
(c) a 100 lb bag of flour
(d) a six-pound baby?

5

Robert Wadlow was the tallest man ever measured. His height was roughly 9 feet. How tall was he:
(a) in centimetres
(b) in metres?

6 Pearl Washington once weighed about 880 pounds, a record. What was her mass in kilograms?

7

In Tokyo in 1991 Mike Powell set the current world record for the long jump when he jumped 29 ft $4\frac{1}{2}$ in. What is this distance in metres?

8 Mount Everest is 29 000 ft high. How many kilometres is this?

9 The long jumper James Beckford set a Jamaican national record when he jumped 27 ft $11\frac{1}{2}$ in in 1996. How far was he from the world record of 8.95 m?

10 The Cuban, Javier Sotomayer set a world high jump record of 2.45 m in 1993. By how many inches did he break the 8 ft barrier?

Technology

Look at the length conversions at

aaamath.com/mea69x10.htm

Do the practice and play the games. Can you beat the best score?

Need a quick metric converter? Have a look at the calculator at

http://www.worldwidemetric.com/ measurements.html

Check your answers to Exercise 5C with it.

5.2 Perimeter

- The **perimeter** is the distance around a shape.

Example 4

Find the perimeter of the shape below.
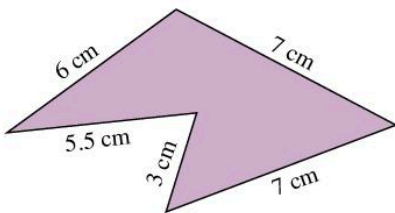
The perimeter is
6 cm + 7 cm + 7 cm + 3 cm + 5.5 cm
= 28.5 cm

Exercise 5D

1 Find the perimeter of these shapes:

(a) 6 cm, 7 cm, 9 cm

(b) 6 cm, 8.5 cm, 7 cm, 7.2 cm

(c) 10.5 cm, 9 cm, 8.7 cm, 12 cm, 9 cm

(d) 4 cm, 4 cm, 7 cm, 7 cm, 6 cm
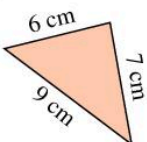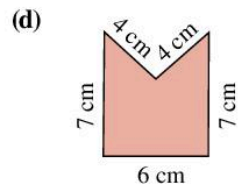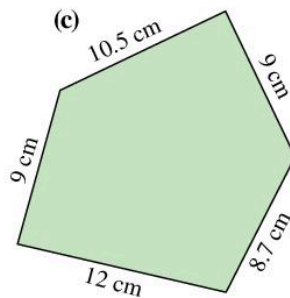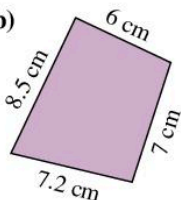
2 Find the lengths of the unknown sides in these shapes.

(a) 3 cm, 5 cm, 4 cm, 7 cm, a
Perimeter 21 cm

(b) 7 cm, 3 cm, b, 9 cm
Perimeter 26 cm

(c) 6.3 cm, 5 cm, c, 4 cm
Perimeter 22.5 cm

(d) d
Perimeter 25.8 cm
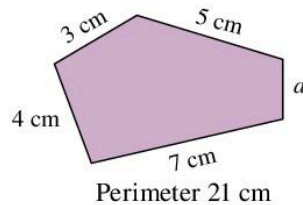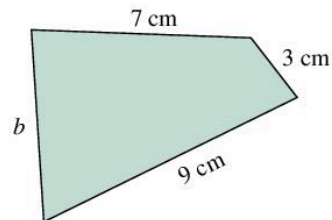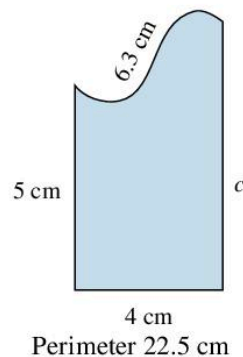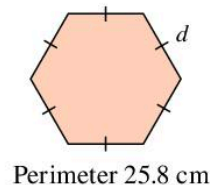
3 Find the perimeter of these rectangles:

(a)

4 cm
9 cm

(b)

3 cm
12 cm

(c)

18 cm
2 cm

(d)

6 cm
6 cm
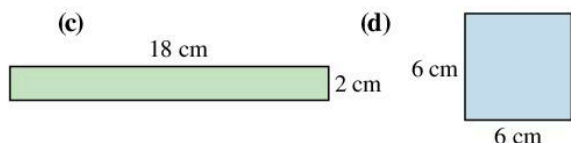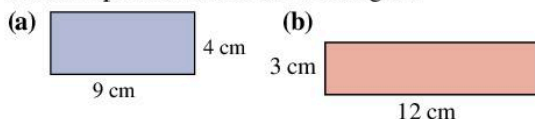
4 Find the unknown sides in these rectangles.

(a)

9 cm
x
Perimeter 48 cm

(b)

11 cm
y
Perimeter 36 cm

(c)

z
5.4 cm
Perimeter 14.6 cm

(d)

The marks show the lengths are equal

a cm
Perimeter 21.2 cm
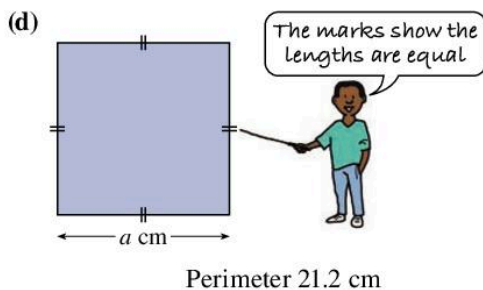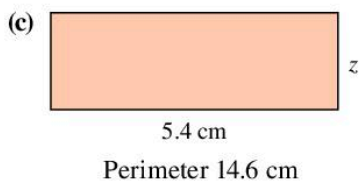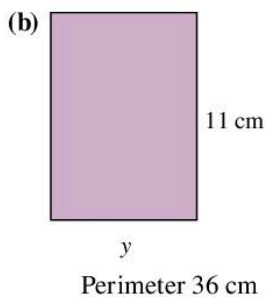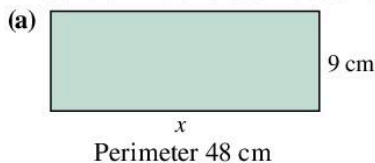
5 Copy and complete this table for rectangles.

Length	Width	Perimeter
(a) 6 cm	5 cm	
(b) 7 cm		22 cm
(c)	5.5 m	34 m
(d)	7.2 m	28.8 m
(e) 13.4 cm	8.2 cm	

6 **(a)** Find the perimeter of a square of side 7.3 m.
   (b) A square has a perimeter of 32 cm. What is the length of each side?

7 A rectangle is 17 cm long and 5 cm wide. What is the length of the side of a square whose perimeter is the same as that of the rectangle?

8 A swimming pool in the shape of a rectangle is fenced with material costing $15.75 per m. The sides of the pool are 11 m and 6 m. Find the cost of the fencing.

Perimeter of circles— circumference

This is a radius

The circumference is the distance around

This is a diameter
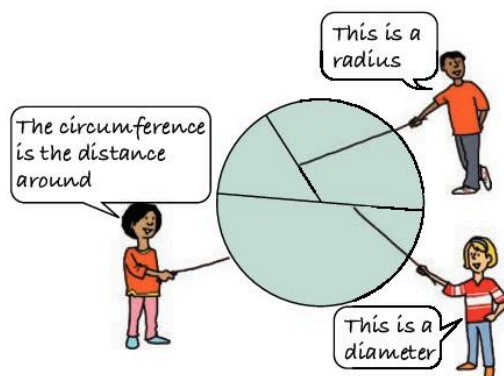
- The **radius** of a circle is the length of a line drawn from the centre to a point on the circle.

- The **diameter** of a circle is the length of a line passing through the centre from one point on the circle to another. (diameter $= 2 \times$ radius.)

- The **circumference** of a circle is the distance around it.

Activity

Here is a good way to find the circumference of a cylindrical tin.

(a) Wrap a thin strip of paper around it, make sure the ends overlap. Stick a pin through the overlap.
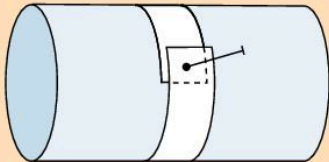
(b) Unwrap the paper and lay it flat on the table.
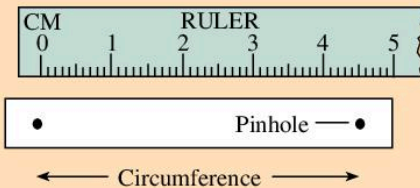
(c) Measure the distance between the two pin holes. This gives the circumference of the tin.

(d) Use this method to find the circumference, C of five cylindrical objects.

(e) With your ruler carefully measure the diameter, D of each.

(f) Copy and complete the table below.

Object	Circumference C cm	Diameter D cm	$C \div D$
tin	25.1 cm	8 cm	
plate			
cup			

What do you notice about the values of $C \div D$ in your table?

If you were careful you should have found that

$C \div D$ is roughly 3.1

That is:

the circumference of a circle is just over three times the diameter of the circle.

The circumference of a circle is given more accurately by the relation

$$C = \pi \times D$$

The Greek letter π, pronounced pi, cannot be found exactly. It is about 3.14 or $3\frac{1}{7}$.

Since

$$D = 2 \times \text{radius } (r)$$

You can also write

$$C = 2\pi r$$

● The circumference of a circle is approximately $3.14 \times$ diameter or $2 \times 3.14 \times$ radius.

Example 5
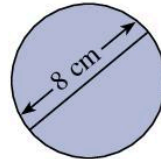
Find the circumference of a circle with diameter 8 cm. Take $\pi = 3.14$

Circumference $= \pi \times$ diameter
$$= 3.14 \times 8 \text{ cm}$$
$$= 25.12 \text{ cm}$$

Exercise 5E

1 Using 3.14 for π, calculate the circumference of a circle with a diameter of:
  (a) 2 cm   **(b)** 10 cm
  (c) 12 cm   **(d)** 21 cm

2 Using $3\frac{1}{7}$ for π, calculate the circumference of a circle with a diameter of:
  (a) 7 cm   **(b)** 14 cm
  (c) 42 cm   **(d)** 84 cm

3 Using $3\frac{1}{7}$ for π, calculate the circumference of a circle with a radius of:
  (a) 7 cm   **(b)** 35 cm
  (c) 49 cm   **(d)** 91 cm

4 Using 3.14 for π, calculate the circumference of a circle with a radius of:
  (a) 5 cm   **(b)** 8 cm
  (c) 13 cm   **(d)** 39 cm

5 The centre circle on a football pitch has a radius of 7.5 m. Find its circumference.

6 A bicycle wheel has a diameter of 70 cm. What is the circumference of the wheel?

70 cm

7 Each year the Earth goes around the Sun in a nearly circular path.

–SUN
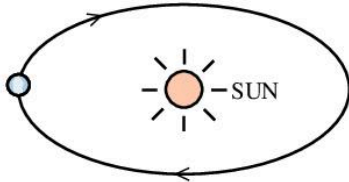
The Earth is about 150 000 000 km from the Sun. How far does the Earth travel each year?

8 The distance from the tip of the minute hand to the centre of a clock is 6 cm.
   (a) How far will the tip of the minute hand move in one hour?
   (b) How far will it move each minute?
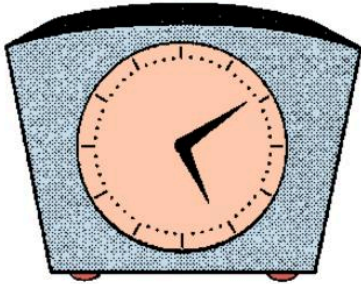
9 A circular toy railway has a radius of 1.4 m. Using $\pi = \frac{22}{7}$, calculate the time that a toy train will take to travel once round the track at a constant speed of 22 cm/s.

10 The diameter of the Earth is about 12 750 km. Find the distance around the equator. Take $\pi = 3.14$.
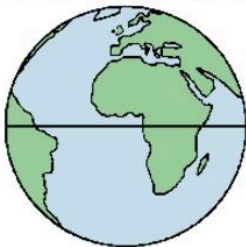
11 Find the perimeters of these shapes.
   (a)

   14 cm $(\pi = 3\frac{1}{7})$

   (b) 1 cm ⟶ 1 cm

   5 cm $(\pi = 3\frac{1}{7})$
   9 cm

   (c)

   20 cm $(\pi = 3\frac{1}{7})$
   21 cm

   (d)

   4 cm   5 cm $(\pi = 3.14)$
   3 cm

   (e)

   $(\pi = 3.14)$
   8 cm
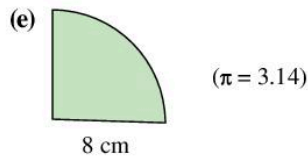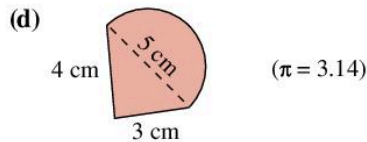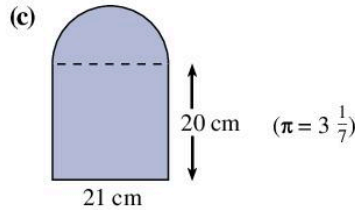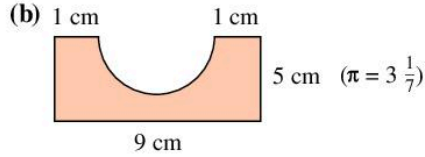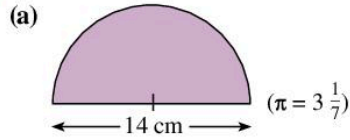
12 An athletics track consists of two equal semicircles joined by 100 m straights.

100 m
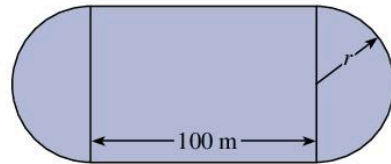
The radius of each semicircle is 35 m. Find the perimeter of the track. Take $\pi = 3\frac{1}{7}$.

13 The distance along the inner circle of a rubber tyre is 220 cm.
   (a) Find the inner radius.
   (b) If the thickness of the tyre is 14 cm, find the distance around the outside of the tyre.

5.3 Areas of rectangles, triangles and parallelograms
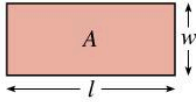
- The area of a rectangle, A is
$$A = l \times w$$
where l is the length and w the width of the rectangle.

Example 6
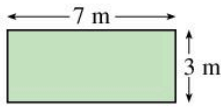
Find the area of a rectangular yard 7 m long and 3 m wide.

Area of yard = length × width
$$= 7 \text{ m} \times 3 \text{ m}$$
$$= 21 \text{ m}^2$$

Exercise 5F

1 Find the area of these rectangles.

(a) 2 cm, 3 cm

(b) 2 cm, 2 cm
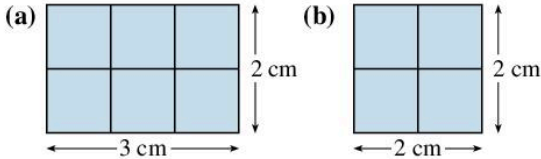
Check your answers by counting squares.

2 Calculate the area of these rectangles.

(a) 4 m, $6\frac{1}{2}$ m

(b) 3 cm, 8.4 cm
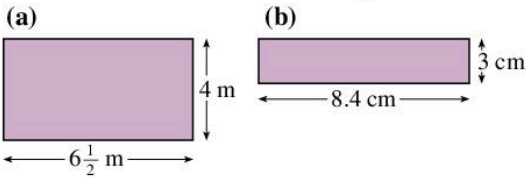
3 Copy and complete this table for rectangles.

	Length (cm)	Width (cm)	Area (cm²)
(a)	13	3	
(b)	25	5	
(c)		7	126
(d)	31		279
(e)	4.5	3.2	

4 George has a rectangular-shaped lawn with width 5.3 m and length 8.7 m.
Calculate the area of the lawn.

Areas of triangles

The area of a triangle is half the area of the rectangle that surrounds it.
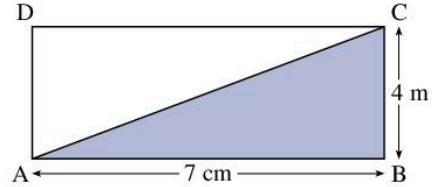
Area of shaded triangle $= \frac{1}{2}$ area of rectangle ABCD
$$= \frac{1}{2} (7 \text{ cm} \times 4 \text{ cm})$$
$$= 14 \text{ cm}^2$$

- Generally, the area of a triangle, A with base length b and vertical height h is
$$A = \frac{1}{2}b \times h$$
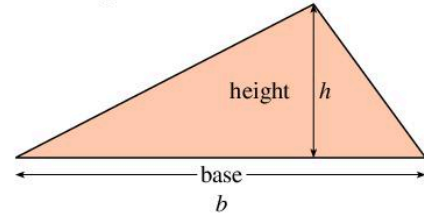
Example 7

Find the areas of these triangles.

(a) 3 cm, 4 cm

(b) 4 cm, 6 cm
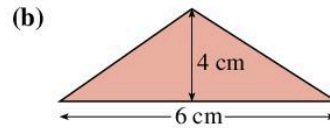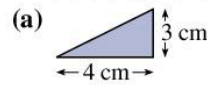
(a) Area of triangle $= \frac{1}{2}b \times h = \frac{1}{2} \times 4 \text{ cm} \times 3 \text{ cm}$
$$= 6 \text{ cm}^2$$

(b) Area of triangle $= \frac{1}{2}b \times h$
$$= \frac{1}{2} \times 6 \text{ cm} \times 4 \text{ cm}$$
$$= 12 \text{ cm}^2$$
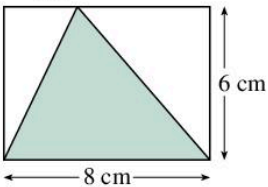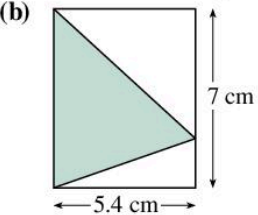
Exercise 5G

1 Measure the sides of the rectangle in centimetres, and find its area. Write down the area of the shaded triangle.
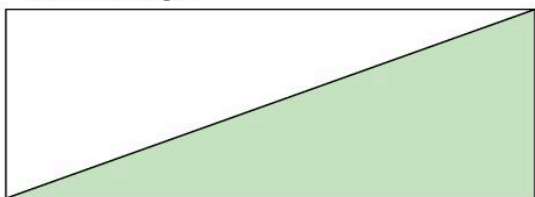
2 Find the area of:
  (a) the rectangle

  (b) each white triangle
  (c) the shaded triangle.
  (d) Compare your answers to parts (a) and (c).
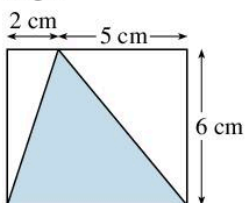
3 Find the area of the rectangle and shaded triangle.
  (a)   **(b)**

4 Work out the area of these triangles.
  (a)   **(b)**
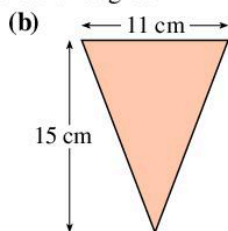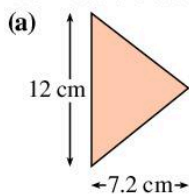
5 Measure the base of each triangle and its vertical height. Calculate their areas.
  (a)   **(b)**
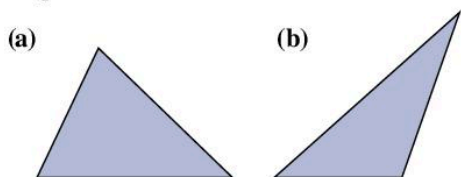
6 Copy and complete the table for triangles.

	Base (cm)	Vertical height (cm)	Area (cm²)
(a)	10	4	
(b)	16		96
(c)	4.5	12	
(d)		2.4	14.4
(e)	6.3	9.2	

Areas of parallelograms

Recall that a parallelogram is a four-sided shape with two pairs of parallel sides.
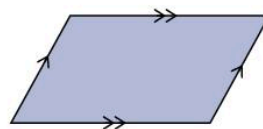
A parallelogram with base, b, and vertical height, h, is made up of two identical triangles.
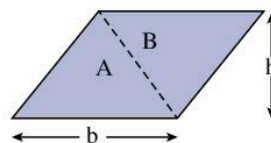
Area triangle $A = \frac{1}{2}b \times h$

Area triangle $B = \frac{1}{2}b \times h$ (why?)

Hence, area of parallelogram $= \frac{1}{2}b \times h + \frac{1}{2}bh$

$$= b \times h$$

The area A, of a parallelogram is A $= b \times h$

> ### Example 8
>
> Find the area of the parallelogram below.
>
>
>
> Area $= b \times h = 8$ cm $\times$ 6 cm $= 48$ cm²
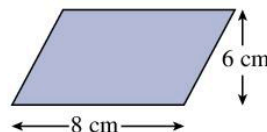
Exercise 5H

1 Find the area of each parallelogram.

(a)

5 cm

6 cm

(b)

7 cm

3 cm

(c)

4 cm

4 cm

8 cm

(d)

7.1 cm

3.8 cm
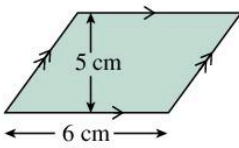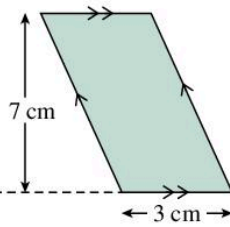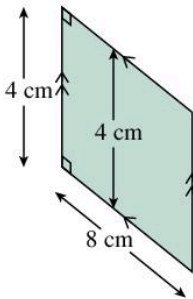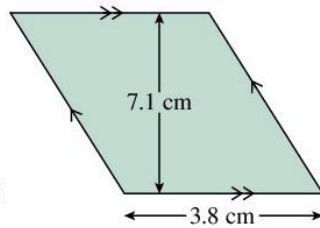
2 A parallelogram has area 48 cm². What is its vertical height, if its base length is

(a) 6 cm **(b)** 8 cm **(c)** 16 cm?

3 Copy and complete the table for parallelograms

	Base (cm)	Vertical height (cm)	Area cm²
(a)	4.8	5	
(b)		12	36
(c)		9	10.8
(d)	6.2	4.3	
(e)	13.84		110.72

4 A parallelogram has an area of 72 m².
Write down six possible dimensions for its base length and height.

5.4 Area of a circle

You will need a ruler, a pair of compasses, a protractor, scissors, cm squared paper and a calculator.

A square has been drawn around a circle of radius r. What is the area of the square?

Area of this square
$= r \times r$
$= r^2$
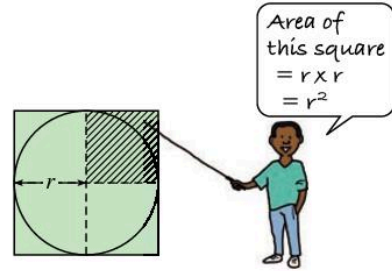
The area of the square is $4r^2$.

This time the shaded square has been drawn *inside* the circle of radius r. What is the area of the shaded square?

Area of this triangle
$= \frac{1}{2} r \times r$
$= \frac{1}{2} r^2$
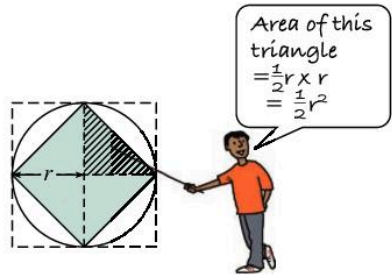
The area of the square is $2r^2$.

So, the area of the circle of radius r lies between the areas of the two squares; that is, between $4r^2$ and $2r^2$. The area of the circle is about $3r^2$.

Exercise 5I

1 Draw a circle of radius 4 cm on cm squared paper. Find the area of the circle by counting squares.

2 Repeat Question 1 for a circle of radius:
(a) 5 cm **(b)** 3 cm **(c)** 6 cm

3 **(a)** Use your answers to Questions 1 and 2 to help you copy and complete the table below.

Radius of circle r (cm)	3	4	5	6
r^2		16		
$3 \times r^2$		48		
Area of circle (cm²)				

(b) Do you agree that the areas of the circles you found in Questions 1 and 2 are slightly more than $3 \times r^2$?

4 Estimate the area of a circle of radius:
   (a) 8 cm   **(b)** 3 cm   **(c)** 6 cm

Activity

Draw a circle and divide it into 16 equal parts.

Circumference
= $2\pi r$

$\frac{1}{2}$ circumference
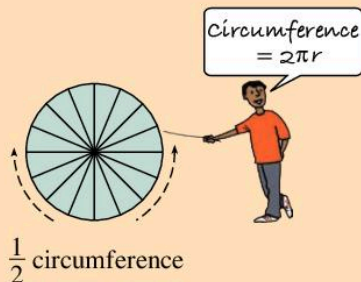
Cut out each part or wedge shape and fit them together.
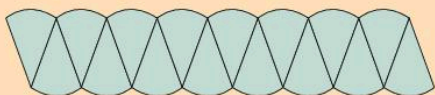
Your new shape is almost a parallelogram.
What is its height? (r)
What is its base length? (πr)
What is its area? ($\pi r \times r = \pi r^2$)

From Exercise 5H and the last activity you should see that:

● The area A, of a circle with radius r is given by the formula

$$A = \pi \times r^2$$

where π is about 3.14 or $3\frac{1}{7}$ as a fraction.

Example 9
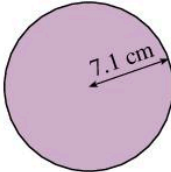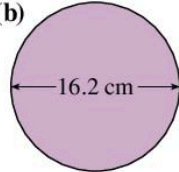
Find the area of a circle with radius 4 cm.

4 cm
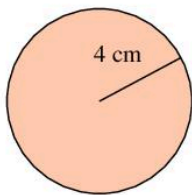
Take $\pi = 3\frac{1}{7}$

Area of circle $= \pi \times r^2$
$= 3\frac{1}{7} \times 4^2$
$= \frac{22}{7} \times 16$
$= \frac{352}{7}$
$= 50\frac{2}{7}$ cm^2

Exercise 5J

1 Using $3\frac{1}{7}$ for π, write down the area of a circle with a radius of:
   (a) 2 cm   **(b)** 7 cm
   (c) 10 cm   **(d)** 14 cm

2 Using 3.14 for π, write down the area of a circle with a radius of:
   (a) 1 cm   **(b)** 3 cm
   (c) 5 cm   **(d)** 10 cm

3 Using 3.14 for π, find the area of each circle.
   (a)   **(b)**

   7.1 cm   16.2 cm

4 Draw a flow chart that shows how to find the area of a circle, when you know its diameter.

5 Using 3.14 for π, find the area of a circle with a diameter of:
   (a) 20 cm   **(b)** 24.6 cm
   (c) 102 cm

6 Using $3\frac{1}{7}$ for π, find the area of the shaded part of each circle:
   (a)   **(b)**
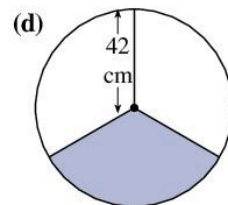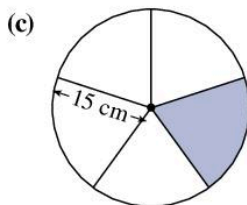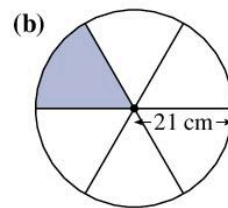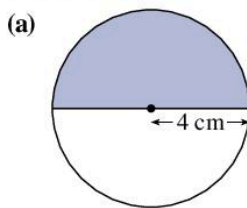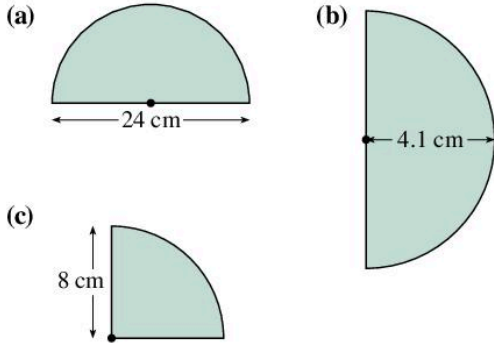
   4 cm   21 cm

   (c)   **(d)**

   15 cm   42 cm

7 Using 3.14 for π, find the area of these shapes.
(a)

24 cm

(b)

4.1 cm

(c)

8 cm

8 Using 3.14 for π, find the area of the shaded part of each shape.
(a)

4 cm

(b)

24 cm

9 Using 3.14 for π, find the area of the shaded part of each shape.
(a)

10 cm

(b)

4 cm
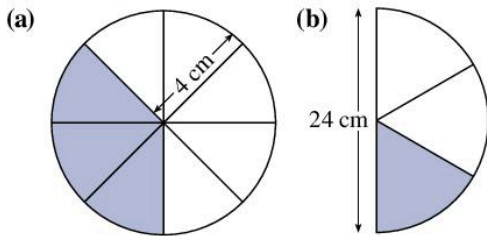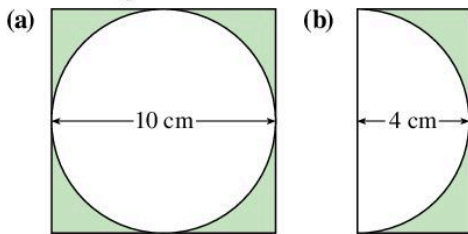
10 Using 3.14 for π, find the area of the shaded part of these shapes.
(a)

10 cm

(b)

6 cm
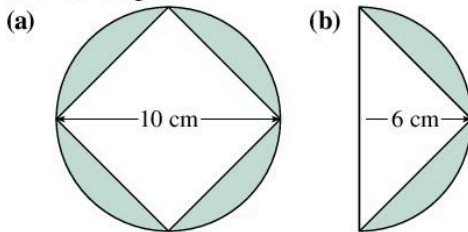
11 Using 3.14 for π, find the radius of a circle with an area of:
(a) 314 cm² **(b)** 12.56 cm² **(c)** 100 cm²

12 Find the area of the shaded parts of these shapes. Take π = 3.14.
(a)

7cm

21 cm

(b)

115 cm

70 cm

(c)

35 cm

35 cm

(d)

1.4 m

2.1 m
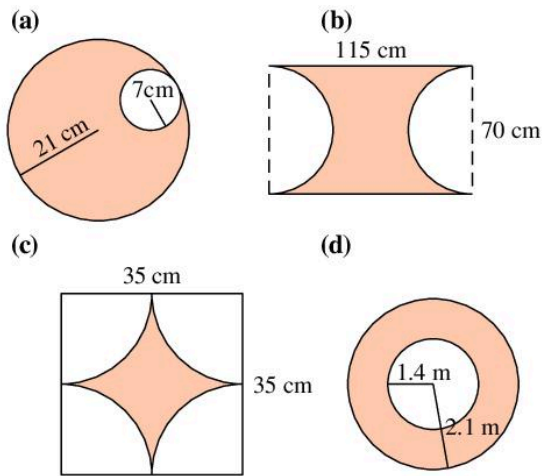
13 The largest possible circle is cut from a square sheet of paper of side 14 cm. What area of paper is left?

Technology

Review what you have learnt about the area of simple shapes by visiting

www.mathsisfun.com/area.html

Then do the review test at

www.bbc.co.uk/education/guides/z2mtyrd/test

5.5 Scale drawing

This is a scale drawing of a football field, which in real life is 80 m long and 55 m wide. It has a scale of 1 cm to 10 m.

Football field
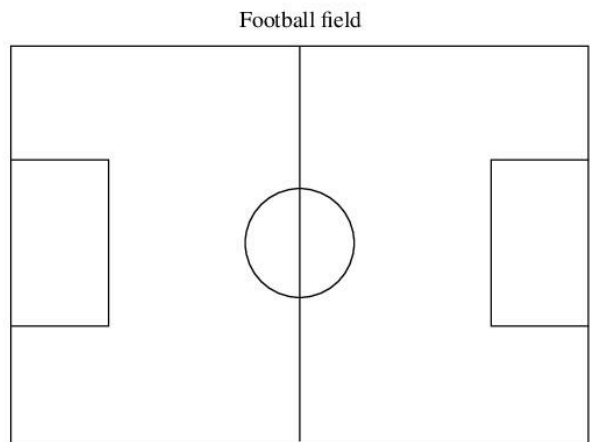
Scale: 1 cm represents 10 m

The scale drawing has a length of 8 cm and a width of 5.5 cm.

Example 10

A plan of a house is drawn with a scale of 1 cm to 2 m.
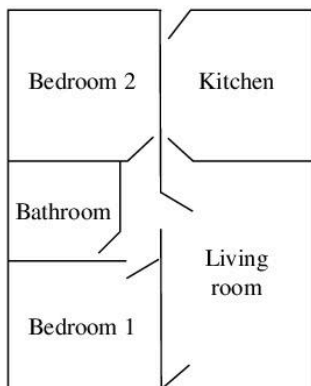
Scale: 1 cm represents 2 m

(a) What are the dimensions of the living room?
(b) A 20 m long wall is to be built at the back of the house. What would be the wall's length on the scale drawing?

(a) The living room on the plan is 3 cm long and 2 cm wide.

1 cm represents 2 m

so 3 cm represents 3×2 m = 6 m
and 2 cm represents 2×2 m = 4 m
Room is 6 m long and 4 m wide

(b)

2 m is represented by 1 cm

so 20 m is represented by $\frac{20}{2}$ cm = 10 cm
The wall would be shown by a 10 cm line on the plan.

Exercise 5K

1   These lines are scale drawings:
    (i) _____   (ii) _____
    (iii) _____   (iv) _____
    (a) Measure the lengths of the lines in cm.
    (b) What are the actual lengths of the lines if:
        A 1 cm represents 4 cm
        B 1 cm represents 10 cm
        C 1 cm represents 5 m?

2   Copy and complete this table.

Length on drawing	Scale	Actual length
(a) 7 cm	1 cm represents 5 m	
(b) 3.5 cm	1 cm represents 10 m	
(c) 8.2 cm		16.4 m
(d) 13 cm	1 cm represents 20 km	
(e)	1 cm represents 10 m	70 m
(f)	1 cm represents 25 km	170 km
(g) 12 cm		42 m

3   This map shows the island of St James.
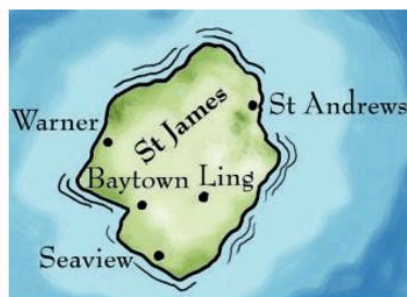
Scale: 1 cm represents 1 km

(a) On the map how far is it, in centimetres, from Warner to:
    (i)   St Andrews
    (ii) Ling
    (iii) Baytown
    (iv) Sea View?

(b) What are these distances in real life?

4   A rectangular room is 7 m long and 5 m wide.

Make a scale drawing of the room using a scale in which:
    (a) 1 cm represents 1 m
    (b) 1 cm represents 2 m

5   Use a scale of 1 cm to 1 m to make a scale drawing of a room measuring 12 m by 9 m. Find the distance between opposite corners of the room.

6 The plan shows the grounds of a school.
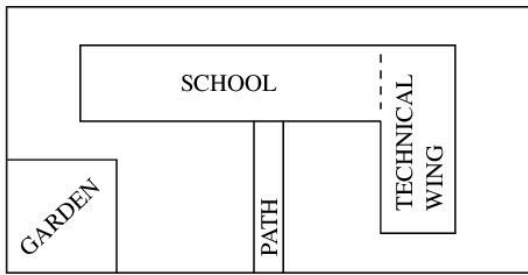
Scale: 1 cm represents 10 m

(a) On the plan measure with your ruler:
  (i) the length of the school
  (ii) the length of the technical wing
  (iii) the width of the school
  (iv) the width of the technical wing
  (v) the length of the garden
  (vi) the length of the path.
(b) What are these distances in real life?

7 Measure the dimensions of your classroom. Use a scale where 1 cm represents 1 m to make a plan of your classroom.

8 Make a scale drawing of your bedroom. What scale did you use?

Using ratios

Scales can be written as ratios.

Example 11

(a) Write the scale in which 1 cm represents 20 m in ratio form.
(b) A plan is drawn using a scale of 1 : 500. Find the actual measurement of a line of length 4 cm on the plan.

(a) 20 m = 100 × 20 cm
      = 2000 cm
So 1 cm represents 2000 cm.
The scale is 1 to 2000
which is written as 1 : 2000
(b) 1 : 500 means 1 cm represents 500 cm
So 4 cm represents 2000 cm = 20 m

Exercise 5L

1 Write these scales in ratio form:
  (a) 1 cm represents 1 m
  (b) 1 cm represents 5 m
  (c) 1 cm represents 10 m
  (d) 1 cm represents 1 km
  (e) 1 mm represents 10 m

2 Find the actual length and width of these rectangles if they have been drawn to a scale of
  (i) 1 : 100
  (ii) 1 : 50
  (iii) 1 : 2000
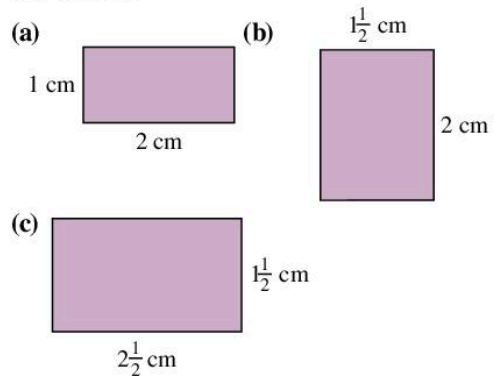
3 The map is a scale drawing of a part of St Kitts.
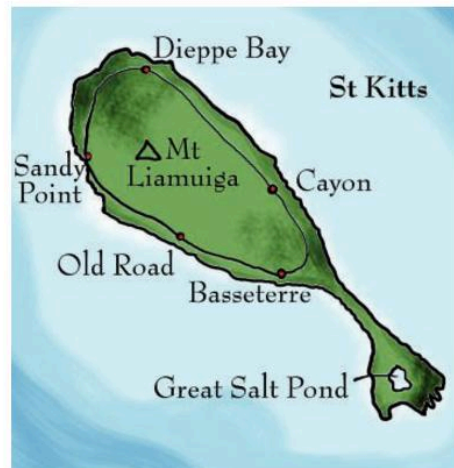
Scale 1 : 400 000

(a) What is the scale of the map?
(b) What distance does 1 cm on the map represent?
(c) What is the distance on the map of Basseterre from Sandy Point?
(d) What is the actual distance of Basseterre from Sandy Point?

(e) Find the actual distance of Basseterre from
  (i) Cayon
  (ii) Dieppe Bay
  (iii) Mt Liamuiga
  (iv) Great Salt Pond.

4 The diagram shows the plan of the town of Roaring Sands.
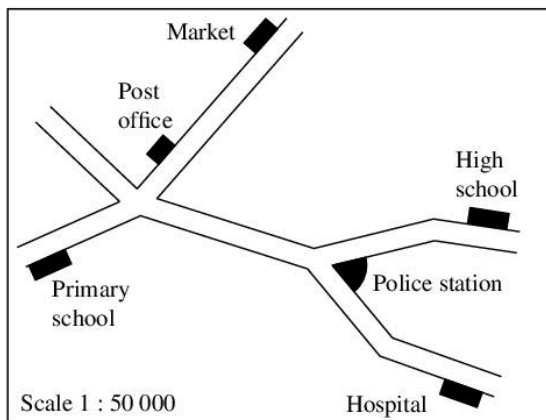
Scale 1 : 50 000

Using the scale of 1 : 50 000 find the shortest possible walking distance between
(a) the post office and the market
(b) the primary school and the high school
(c) the hospital and the police station
(d) the market and the hospital
(e) the post office and the primary school.

5 Copy and complete this table.

Length on drawing	Scale 1 : n	Actual length
0.85 cm	1 : 1000	
	1 : 50 000	2.1 km
5.2 cm		5.2 km
3.5 cm	1 : 250	
7.2 cm		360 m
	1 : 1 000 000	21 km

Activity

Using a scale of 1 cm to represent 1 m, make a scale drawing of your school.

Label your map of the school.

Is it good enough to direct visitors?

Exercise 5M – mixed questions

1 A square has sides of length 19 cm.
  (a) Find its perimeter.
  (b) Find the length of a rectangle with the same perimeter whose width is 7 cm.

2 A rectangular field is 750 m long and 380 m wide. Find its perimeter, in kilometres.

3 A map is drawn showing some buildings in the town of St Georges. The scale used is 1 : 50 000.
  (a) Find the actual distance between
    (i) the town hall and the jail when the distance on the map is 3.5 cm
    (ii) the hospital and the market when the distance on the map is 7.3 cm.
  (b) Find the distance on the map between
    (i) the high school and the primary school when the actual distance is 1.75 km
    (ii) the bakery and the supermarket when the actual distance is 0.35 km.

4 The length of the side of a square is 5 mm longer than the length of a rectangle. The perimeter of the square is 100 mm. What is the length of the rectangle?

5 (a) Find the lengths of the unknown sides in these figures.

(i)

(ii)

(iii)
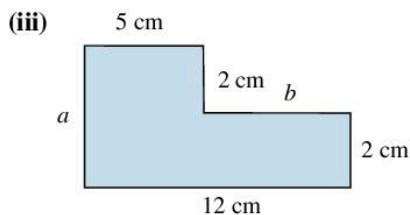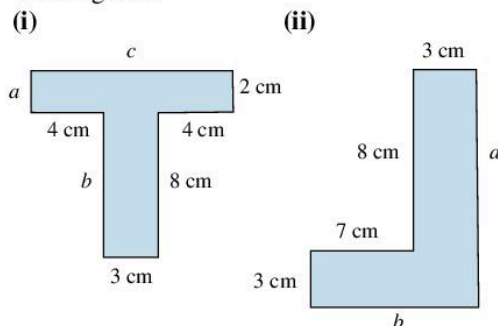
(b) Find the perimeter of each figure.
(c) Find the area of each figure.

6 Find the perimeter of these shapes ($\pi = 3\frac{1}{7}$).

(a)

32 m

28 m

(b)

40 cm

35 cm

(c)

3.5 cm

7 cm

3.5 cm

7 cm
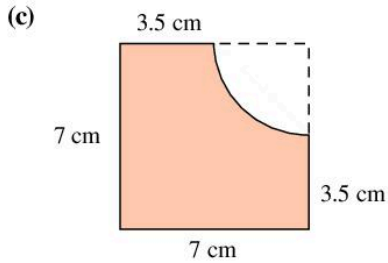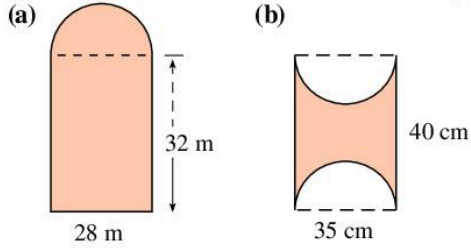
7 Find the area of the shapes in Question 6.

8 The diameter of a bicycle wheel is 42 cm.
  (a) Find the circumference of the wheel.
  (b) Find the number of revolutions that the wheel makes in travelling a distance of 1.98 km ($\pi = 3\frac{1}{7}$).

9 The circumference of a tyre is 54.95 cm. Find the diameter of the tyre ($\pi = 3.14$).

10 The minute hand of a clock is 49 mm long. How far does the tip of the hand travel in
  (a) 15 minutes
  (b) 45 minutes
  (c) 50 minutes?
  Use $\pi = \frac{22}{7}$.

11 A boy whirled a stone around which was attached to a piece of string 56 cm long. The string made 90 revolutions per minute. Find the speed of the stone in cm/s.

5 Consolidation

Example 1
What is:
(a) 6.3 m in centimetres
(b) 253 g in kilograms?

(a)   100 cm = 1 m
   So 6.3 m = 6.3 × 100 cm
         = 630 cm
(b)   1000 g = 1 kg
   So 253 g = 253 ÷ 1000 kg
         = 0.253 kg

Example 2
Find the perimeter of these shapes.
(a)

(b)
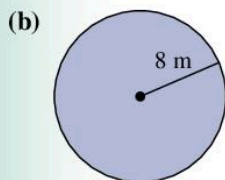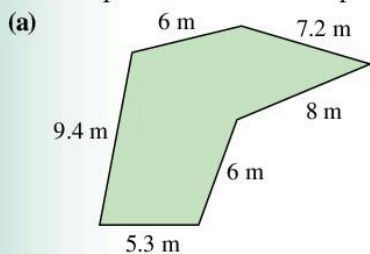
(a) Perimeter = distance around shape
   = 6 m + 7.2 m + 8 m + 6 m + 5.3 m + 9.4 m
   = 41.9 m
(b) Perimeter = circumference of circle
   = 2π × radius
   = 2 × 3.14 × 8 m
   = 50.24 m

Example 3
Find the area of these shapes.
(a)

(b)

(c)
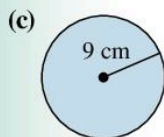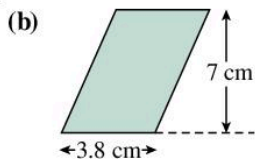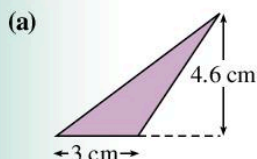
(a) Area of triangle = $\frac{1}{2}$ × base × height

$$= \frac{1}{2} \times 3 \text{ cm} \times 4.6 \text{ cm}$$

$$= 6.9 \text{ cm}^2$$

(b) Area of parallelogram = base × height
         = 3.8 cm × 7 cm
         = 26.6 cm²
(c) Area of circle = πr^2
         = 3.14 × 9² cm²
         = 3.14 × 81 cm²
         = 254.34 cm²

Example 4
A map has a scale of 1 : 50 000.
(a) What is the actual distance represented by 2.7 cm on the map?
(b) How is a distance of 6 km represented on the map?

(a) Actual distance = 2.7 cm × 50 000
            = 135 000 cm
            = 1350 m (or 1.35 km)
(b) Map distance = 6 km ÷ 50 000
            = 6000 m ÷ 50 000
            = 0.12 m
            = 12 cm

Exercise 5

1 (a) Write in centimetres:
      (i)   5 m   (ii) 28 m   (iii) 7.2 m
      (iv) 3 km   (v)   13.45 km
   (b) Write in metres:
      (i)   25 cm   (ii) 293 cm   (iii) 8.1 km
      (iv) 0.32 km (v)   2615 mm

   (c) Write in grams:
      (i) 4 kg   (ii) 0.4 kg   (iii) 3.2 kg
      (iv) 0.49 kg (v) 15.2 kg

   (d) Write in kilograms:
      (i)   500 g   (ii) 75 g   (iii) 5000 g
      (iv) 3168 g (v)   13 459 g

2 Find the perimeter of these shapes.

(a)

7 cm   6 cm   10 cm

(b)

5 cm 8.2 cm
5.1 cm   4.8 cm
12.3 cm

(c)

13.4 cm

3 Find the area of these shapes.

(a)

6.5 cm
6 cm

(b)

4 cm
3 cm
7 cm
10 cm

(c)

12 cm

(d)

7.4 cm

(e)

4.5 cm
6.4 cm
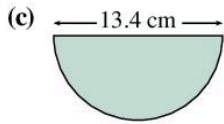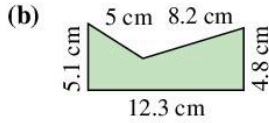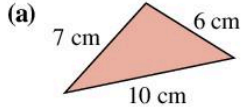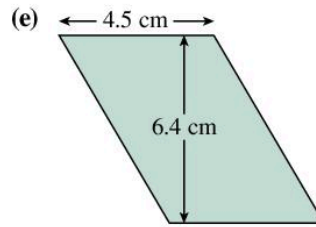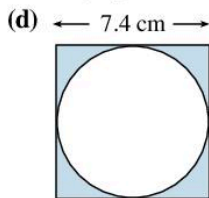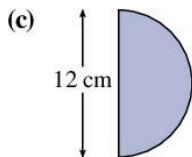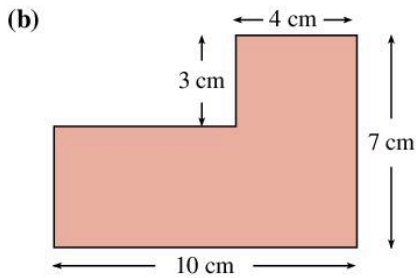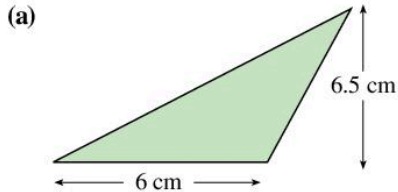
4 Copy and complete the table.

Length on drawing	Scale	Actual length
4 cm	1 : 50 000	
4.3 cm	1 : 250 000	
	1 : 50 000	4.5 km
	1 : 10 000	0.87 km
33 mm	1 : 25 000	
	1 : 25 000	267 m

Application

5 An athletics track is made up of two 100 m stretches of straight track and two semicircular ends each with diameter 120 m.
  (a) Make a sketch of the track.
  (b) How far is it around the track?
  (c) An athlete runs 15 times around the track. How far does he run in km?

6 Look in your atlas to find a map of your country.
  (a) What is the scale of the map?
  (b) Write down two towns named on the map.
    (i) What is the map distance between the towns?
    (ii) What is the actual distance between the towns?

Summary

You should now know ...

1   How to convert between metric and imperial units using:

$$1 \text{ inch} \simeq 2.5 \text{ cm}$$
$$1 \text{ mile} \simeq 1.6 \text{ km}$$

and $\qquad$ $1 \text{ pound} \simeq 0.45 \text{ kg}.$

2   Perimeter is the distance around the edge of a shape.
For example:
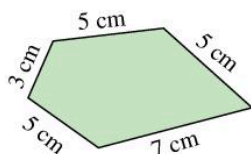
Perimeter $= (3 + 5 + 5 + 7 + 4)$ cm
$\qquad\qquad = 24$ cm

3   The circumference of a circle is πD or $2\pi r$ where $\pi = 3\frac{1}{7}$ or 3.14
For example:
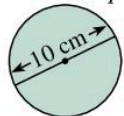
Circumference $= \pi D$
$\qquad\qquad\qquad = 3.14 \times 10$ cm
$\qquad\qquad\qquad = 31.4$ cm

4   Area of a triangle $= \frac{1}{2}$ base $\times$ height $= \frac{1}{2}bh$
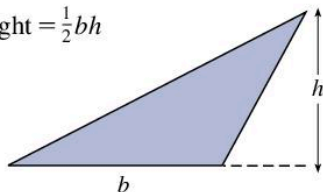
Area of a parallelogram $=$ base $\times$ height $= bh$
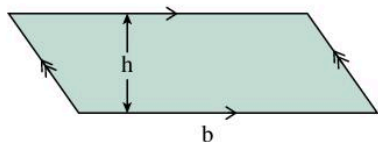
Check out

1   **(a)** Express 5 inches in centimetres.
      (b) Express 17 miles in kilometres.
      (c) Convert 22 pounds to kilograms.

2   The perimeter of this shape is 30 cm. Find the length of the unknown side.
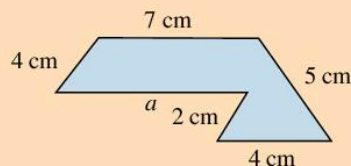
3   Find the circumference of these circles.

(a)   **(b)**
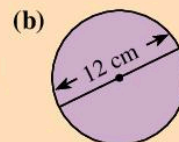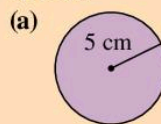
[Take $\pi = 3.14$]

4   Find the area of these shapes.

(a)

(b)
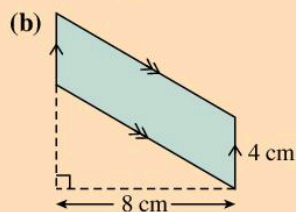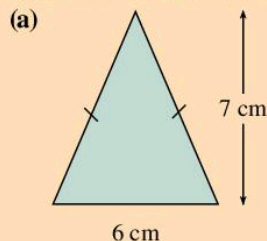
Area of a circle $= \pi r^2$

For example:
What is area of a circle
with radius 6 cm?
$$\text{Area} = \pi r^2 = 3.14 \times 6^2$$
$$= 3.14 \times 36$$
$$= 113.04 \text{ cm}^2$$

(c)

9 cm
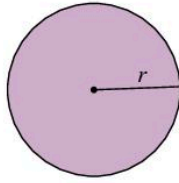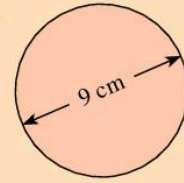
5 A scale drawing can be used to represent a larger object.
For example:
A distance of 8 m is represented by 4 cm on a plan. What is
the scale used?

4 cm represents 8 m (800 cm)

So　　　　　1 cm represents 200 cm

Scale used is 1 : 200.

5 A drawing has a scale of 1 : 500.
(a) Find the actual measurement
in metres of a length of 7.5 cm
on the drawing.
(b) Find the length on the
drawing for an actual
measurement of 135 m.

Integers

Objectives

- ✔ learn more about addition and subtraction of integers
- ✔ multiply integers
- ✔ divide integers
- ✔ use integers to solve problems

What's the point?

The alternator in an aircraft is used to generate electrical power. To check that the alternator is working properly the pilot looks at an instrument, called an ammeter, that measures the flow of current. When the ammeter shows a negative flow of current it means that the battery is discharging and the alternator is not supplying power to the system. The plane is in trouble!

Before you start

You should know ...

1 Negative numbers are smaller than zero.

| $^-5$ | $^-4$ | $^-3$ | $^-2$ | $^-1$ | 0 | 1 | 2 | 3 | 4 | 5 |

← —— Negative numbers —— →|← —— Positive numbers —— →
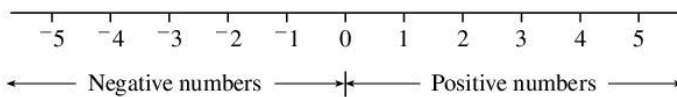
Numbers to the right of another number are always larger.
For example:
1 is larger than $^-3$
$^-2$ is larger than $^-4$

Check in

1 Write down the smaller of these number pairs
(a) $^-1, 5$ (b) $3, ^-2$
(c) $^-2, ^-3$ (d) $^-1, 0$
(e) $^-6, ^-4$ (f) $^-3, 1$

6.1 Adding integers

Numbers such as 1, 2, 3, . . . are positive whole numbers. Numbers such as $^-1$, $^-2$, $^-3$, . . . are negative whole numbers. The set of numbers that contains both positive and negative whole numbers and 0 is called the set of **integers**.

One way to see how to add and subtract integers is to work these operations using number lines.

Look at these number lines:

$6 + {}^-4 = 2$

is the same as

$6 - 4 = 2$
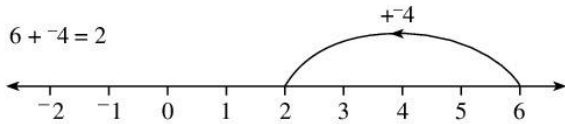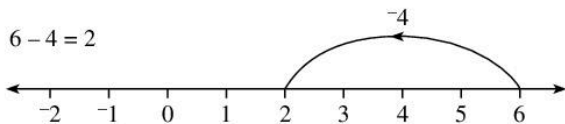
- The number lines show that **adding a negative number is the same as subtracting a positive number**.

Exercise 6A

1　(a) Draw diagrams to show $5 - 7 = {}^-2$ and $5 + {}^-7 = {}^-2$.

　(b) $+{}^-3$ is the same as -3. Use this idea to write down the answers to:
$5 + {}^-3 = \square, \quad 3 + {}^-3 = \square, \quad 1 + {}^-3 = \square.$

2　Write down the answers to:
　(a) $5 + {}^-2$　　　(b) $6 + {}^-3$
　(c) $8 + {}^-10$　　(d) $4 + {}^-4$
　(e) $1 + {}^-2$　　　(f) $2 + {}^-1$

3　Find the answer to:
　(a) $0 + {}^-3$　　　(b) $^-2 + {}^-3$
　(c) $^-1 + {}^-6$　　(d) $^-8 + {}^-2$
　(e) $^-11 + {}^-1$　(f) $^-1 + {}^-1$

4　Try these mixed questions:
　(a) $4 - 8$　　　　(b) $4 + {}^-2$
　(c) $3 + {}^-3$　　(d) $3 + {}^-6$
　(e) $^-8 + 6$　　　(f) $^-8 + 8$
　(g) $^-8 + 10$　　(h) $6 + {}^-3$

5　Work out:
　(a) $6 + {}^-6$　　　(b) $6 + {}^-7$
　(c) $^-2 - 5$　　　(d) $^-2 + {}^-5$
　(e) $^-3 + {}^-6$　　(f) $^-4 + {}^-3$

6　The commutative law can be used to add negative numbers:

$6 + {}^-4 = {}^-4 + 6 = 2$

Find the answer to:
　(a) $3 + {}^-5$　　　(b) $4 + {}^-2$
　(c) $8 + {}^-7$　　　(d) $8 + {}^-10$
　(e) $14 + {}^-6$　　(f) $5 + {}^-5$

7　Copy and complete this addition table. Use the pattern in the table to help you.

+	$^-5$	$^-4$	$^-3$	$^-2$	$^-1$	0	1	2
$^-5$								
$^-4$	$^-9$							
$^-3$			$^-6$					
$^-2$								
$^-1$								
0								
1								
2								

8　Work out:
　(a) $6 + {}^-2 + 4$　　　(b) $3 + {}^-4 + 2$
　(c) $8 + 2 + {}^-6$　　　(d) $4 + 10 + {}^-3$
　(e) $^-4 + {}^-3 + 7$　　(f) $^-4 + 6 + {}^-2$
　(g) $^-3 + {}^-2 + {}^-5$　(h) $5 + {}^-2 + 3$

9　Slim saves his money with the local bank. He keeps a record of his savings. If he puts in \$2 he writes down 2. When he takes out \$2 he writes down $^-2$. He starts with \$10. Here is a record of his entries for six weeks:

$3, {}^-2, 2, {}^-6, {}^-3, 7$

Find out how much he has in the bank at the end of the six weeks.

10　On Thursday afternoon the temperature fell by 3°C. On Friday it had risen by 5°C. If Thursday morning's temperature was $^-3$°C, what was the temperature on Friday?

Investigation

Copy and complete the magic square
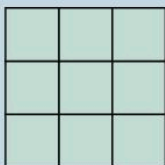
with the numbers ⁻10, ⁻8, ⁻6, ⁻4, ⁻2, 0, 2, 4, 6 so that all the rows, columns and diagonals add to the same number.

6.2 Subtracting integers

Look at these subtractions:

$4 - 4 = 0$
$4 - 3 = 1$
$4 - 2 = 2$
$4 - 1 = 3$
$4 - 0 = 4$
$4 - {}^-1 = \square$

These numbers increase by 1 each time.

What do you think $\square$ should be? The answers are increasing by one so $\square = 5$.

That is,

$4 - {}^-1 = 5$

● This shows that **subtracting a negative number** is the same as **adding a positive number**.

Example 1

Work out
(a) $6 - {}^-3$
(b) ${}^-4 - {}^-2$

(a) $6 - {}^-3 = 6 + 3$
$= 9$
(b) ${}^-4 - {}^-2 = {}^-4 + 2$
$= {}^-2$

You can use the same idea to work out harder calculations.

Example 2

Work out
(a) ${}^-6 - {}^-3 + 4$
(b) ${}^-7 + {}^-5 - {}^-4$

(a) ${}^-6 - {}^-3 + 4 = {}^-6 + 3 + 4$
$= {}^-6 + 7$
$= 1$
(b) ${}^-7 + {}^-5 - {}^-4 = {}^-7 - 5 + 4$
$= {}^-7 - 1$
$= {}^-8$

Remove the double signs first!

Notice that in part (b) ${}^-7 - 5 + 4$ can be worked out on the number line:
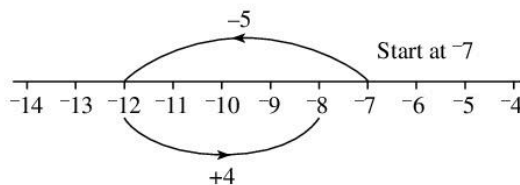
Exercise 6B

1 Copy and complete.
The first one has been done for you.
(a) $10 - {}^-7 = 10 + 7 = 17$
(b) $3 - {}^-6 = 3 + \square = \square$
(c) ${}^-2 - {}^-2 = {}^-2 + \square = \square$
(d) $0 - {}^-6 = 0 + \square = \square$
(e) ${}^-5 - {}^-3 = {}^-5 + \square = \square$

2 Work out:
(a) $4 - {}^-3$ (b) $10 - {}^-2$
(c) $5 - {}^-2$ (d) $11 - {}^-1$
(e) $3 - {}^-8$ (f) $2 - {}^-4$
(g) ${}^-3 - {}^-4$ (h) ${}^-5 - {}^-6$
(i) ${}^-6 - {}^-9$ (j) ${}^-13 - {}^-14$
(k) $17 - {}^-13$ (l) ${}^-5 - {}^-41$
(m) ${}^-34 - {}^-28$ (n) ${}^-16 - {}^-29$

3 Copy and complete this subtraction table. Use the pattern in the table to help you.

Second number

$-$	$^-4$	$^-3$	$^-2$	$^-1$	0	1	2	3
$^-4$								
$^-3$								$^-6$
$^-2$								
$^-1$	3							
0								
1							$^-1$	
2								
3								

First number

4 Find the answer:
  (a) $6 + {}^-2 + 3$ (b) $6 - {}^-2 + 3$
  (c) $6 + {}^-3 - 2$ (d) $6 - 4 - {}^-2$
  (e) $8 - {}^-3 + 1$ (f) $1 - 6 + {}^-7$
  (g) $8 - {}^-8 - 8$ (h) $3 + {}^-3 - 3$

5 Find the answer:
  (a) $4 + {}^-4 - {}^-7$ (b) $8 + {}^-5 - {}^-3$
  (c) $4 + {}^-1 - 5$ (d) $^-3 + {}^-3 + {}^-3$
  (e) $^-2 - {}^-2 + 2$ (f) $^-6 + {}^-5 - 3$
  (g) $^-9 + 7 + {}^-6$ (h) $^-12 - {}^-13 - 14$
  (i) $^-21 + {}^-6 + 19$ (j) $16 - {}^-17 - 23$

6 Find numbers that make these subtraction tables work.

(a)

$-$	Second number	
First number	1	4
	2	5

(b)

$-$	Second number	
First number	4	3
	5	4

Technology

Review what you have learnt about adding and subtracting integers. Visit

www.onlinemathlearning.com/integers.html

Study the examples and watch the videos!

Investigation

(a) Find two numbers such that when you subtract one from the other you get $^-4$.
(b) Can you find any other such pairs of numbers?
(c) Can you find a rule for finding other pairs of numbers?

6.3 Multiplying integers

To multiply by a negative number remember that multiplication is just repeated addition.

For example,
$$3 \times 4 = 3 + 3 + 3 + 3 = 12$$
so
$$^-3 \times 4 = {}^-3 + {}^-3 + {}^-3 + {}^-3$$
$$= {}^-3 - 3 - 3 - 3$$
$$= {}^-12$$

Multiplication is **commutative**, that is,
$$3 \times 4 = 4 \times 3$$
so
$$^-3 \times 4 = 4 \times {}^-3 = {}^-12$$

Exercise 6C

1 Copy and complete:
  (a) $^-2 \times 4 = {}^-2 + {}^-2 + {}^-2 + {}^-2 =$
  (b) $^-4 \times 5 =$
  (c) $^-6 \times 2 =$

2 Without writing down the addition, find the answer to:
  (a) $^-4 \times 3$ (b) $^-8 \times 2$ (c) $^-6 \times 3$
  (d) $^-5 \times 4$ (e) $^-1 \times 6$ (f) $^-7 \times 8$

3 Copy and complete.
  The first one has been done for you.
  (a) $2 \times {}^-3 = {}^-3 \times 2 = {}^-6$
  (b) $4 \times {}^-5 =$
  (c) $6 \times {}^-2 =$
  (d) $5 \times {}^-6 =$
  (e) $2 \times {}^-1 =$

4 Find the answer to:
  (a) $^-6 \times 4$ (b) $^-7 \times 3$ (c) $2 \times ^-2$
  (d) $^-1 \times 8$ (e) $8 \times ^-2$ (f) $10 \times ^-10$
  (g) $4 \times ^-7$ (h) $^-2 \times 9$ (i) $9 \times ^-3$

5 Find the answer to:
  (a) $^-9 \times 6$ (b) $^-8 \times 4$ (c) $^-10 \times 4$
  (d) $6 \times ^-4$ (e) $12 \times ^-2$ (f) $3 \times ^-11$
  (g) $3 \times ^-1$ (h) $^-1 \times 9$ (i) $4 \times ^-9$

Multiplication tables

You can show your times tables on a graph.

The diagram shows a graph of the 2-times table.

Graph of 2-times table

Number to be
multiplied by 2
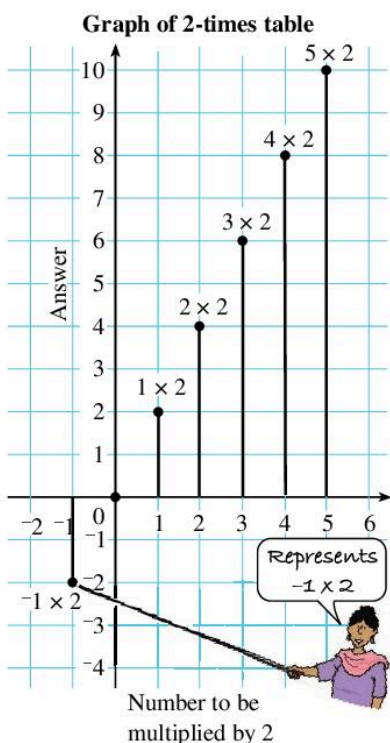
Notice all the points lie in a straight line.

Exercise 6D

1 (a) Draw a graph of the 3-times table for
    multiplying numbers from $^-3$ to 4. The
    answer line will need to show numbers from
    $^-9$ to 12.
  (b) Do all the answers lie on a straight line?
  (c) If you extended the line, what would be the
    answers to $^-4 \times 3$ and $^-5 \times 3$?

2 (a) Draw a graph of the 4-times table, for
    multiplying numbers from $^-4$ to 3.
  (b) If you extended the line, what answers would
    you find for $^-5 \times 4$ and $^-7 \times 4$?

3 (a) Write down the answers to:
    (i) $3 \times ^-2$ (ii) $4 \times ^-2$ (iii) $5 \times ^-2$
  (b) Copy the graph. Complete it with your
    answers to part (a).

Graph of $^-2$-times table
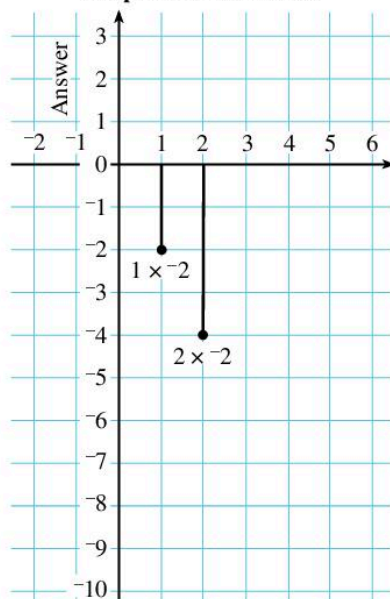
  (c) Lay a ruler along the points.
    What answer does the graph give
    for $^-1 \times ^-2$?
  (d) Extend the graph to find the answer
    to $^-2 \times ^-2$.

4 What does this graph show?
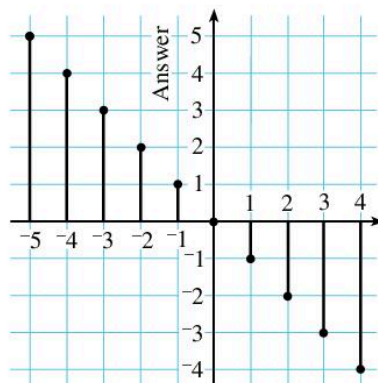
5 Use the graph in Question 4 to write down the answer to:

(a) $4 \times {}^-1$ (b) $^-4 \times {}^-1$ (c) $^-2 \times {}^-1$

6 Look at the graph of the $^-3$-times table,

(a) What is $0 \times {}^-3$?

(b) What answers does the graph suggest for $^-1 \times {}^-3$, $^-2 \times {}^-3$, $^-3 \times {}^-3$?

Graph of $^-3$-times table
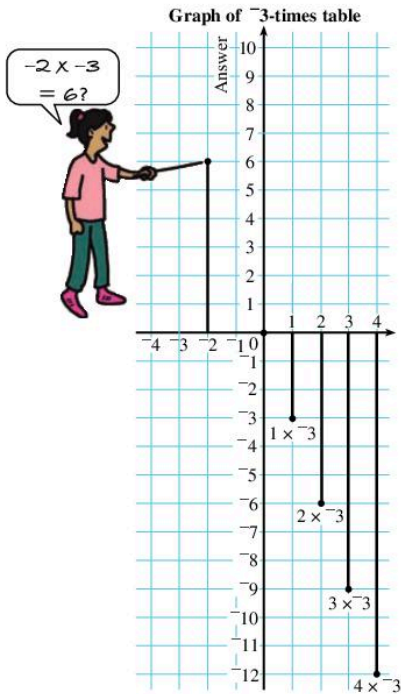
$\times$	$^-5$	$^-4$	$^-3$	$^-2$	$^-1$	0	1	2	3	4	5
$^-5$											
$^-4$											
$^-3$											
$^-2$											
$^-1$											
0											
1											
2					$^-2$					8	
3									6		
4											
5											

$3 \times 2 = 6$

Remember

- When you multiply a positive number by a negative number the answer is negative.
- When you multiply two negative numbers together the answer is positive.

7 Copy this table and match up the multiplications and answers, using arrows.

$^-3 \times {}^-2$	$\rightarrow$	$^-6$
$3 \times {}^-2$		6
$^-3 \times 2$		
3×2		

Example 3

Work out:

(a) $3 \times {}^-5$

(b) $^-3 \times {}^-5$

(a) positive $\times$ negative $=$ negative:
$3 \times {}^-5 = {}^-15$

(b) negative $\times$ negative $=$ positive:
$^-3 \times {}^-5 = 15$

8 Copy and complete the multiplication table. Use your graphs of $^-1$ times, $^-2$ times and $^-3$ times to help you. Follow the number patterns up the columns to complete the upper left-hand corner of the table.

The same rules hold when you multiply more than two numbers.

Example 4

Find $^-3 \times 4 \times ^-2$

$$^-3 \times 4 \times ^-2 = (^-3 \times 4) \times ^-2$$
$$= ^-12 \times ^-2$$
$$= 24$$

or $\quad ^-3 \times 4 \times ^-2 = ^-3 \times (4 \times ^-2)$
$$= ^-3 \times ^-8$$
$$= 24$$

Exercise 6E

1 Write down the answers:
(a) 4×5 (b) $^-4 \times 5$ (c) $4 \times ^-5$
(d) $^-4 \times ^-5$ (e) $2 \times ^-5$ (f) $^-4 \times ^-3$
(g) 2×4 (h) $^-2 \times ^-4$ (i) $^-2 \times 4$

2 Write down the answers:
(a) $^-6 \times ^-8$ (b) $6 \times ^-8$
(c) $^-2 \times 5$ (d) $^-7 \times ^-5$
(e) 14×5 (f) $^-5 \times ^-14$
(g) $100 \times ^-1$ (h) $37 \times ^-2$
(i) $^-1 \times ^-1$ (j) $^-3 \times ^-3$
(k) $^-20 \times ^-20$ (l) $20 \times ^-20$
(m) $2.7 \times ^-3.5$ (n) $^-1.75 \times ^-8$

3 Use the multiplication table in Question 8 of Exercise 2D to help find the missing numbers:
(a) $3 \times \square = 12$ (b) $3 \times \square = ^-12$
(c) $^-3 \times \square = ^-12$ (d) $^-3 \times \square = 12$
(e) $5 \times \square = ^-10$ (f) $\square \times ^-2 = 10$
(g) $\square \times ^-4 = 8$ (h) $\square \times ^-4 = ^-8$
(i) $^-5 \times \square = 15$ (j) $\square \times 4 = ^-16$

4 Find the answer to:
(a) $^-1 \times 3 \times ^-2$ (b) $^-6 \times ^-4 \times 2$
(c) $4 \times ^-1 \times 2$ (d) $^-8 \times ^-3 \times ^-2$
(e) $^-5 \times ^-2 \times 2$ (f) $8 \times 2 \times ^-1$
(g) $^-8 \times ^-1 \times ^-3$ (h) $4 \times ^-3 \times ^-9$

5 Find the answer to:
(a) $6 \times 2 \times ^-1 \times ^-3$ (b) $8 \times ^-3 \times ^-2 \times 2$
(c) $1 \times ^-2 \times ^-3 \times 4$ (d) $^-1 \times ^-2 \times ^-4 \times ^-8$
(e) $5 \times ^-4 \times ^-2 \times ^-3$ (f) $^-1 \times ^-4 \times 4 \times 5$

6 Find the missing number:
(a) $3 \times \square \times ^-2 = 12$ (b) $4 \times \square \times 4 = 64$
(c) $^-3 \times \square \times ^-3 = ^-27$ (d) $^-1 \times \square \times 8 = ^-16$
(e) $2 \times \square \times ^-6 = 72$ (f) $^-3 \times 4 \times \square = 72$
(g) $\square \times ^-3 \times ^-4 = 72$ (h) $^-9 \times \square \times ^-8 = ^-72$

7 Find the answer to:
(a) $^-1 \times ^-1$
(b) $^-1 \times ^-1 \times ^-1$
(c) $^-1 \times ^-1 \times ^-1 \times ^-1$
(d) $^-1 \times ^-1 \times ^-1 \times ^-1 \times ^-1$

8 Is the answer positive or negative, when you multiply together
(a) three negative numbers
(b) four negative numbers
(c) five negative numbers?

9 Find a rule that tells you whether the answer will be positive or negative when you multiply several negative numbers together.

6.4 Division of integers

Multiplication and division of positive numbers are connected like this:

$$3 \times 4 = 12 \qquad \text{so} \qquad 12 \div 4 = 3$$

For negative numbers:

$$^-3 \times 4 = ^-12 \qquad \text{so} \qquad ^-12 \div 4 = ^-3$$

Exercise 6F

1 Use the multiplication table in Question 8 of Exercise 2D to find the missing number in:
(a) $\square \times 3 = 6$ (b) $4 \times \square = 12$
(c) $^-4 \times \square = ^-20$ (d) $2 \times \square = ^-8$
(e) $3 \times \square = ^-12$ (f) $\square \times 5 = ^-15$

2 Use your answers for Question 1 to write down the answers to:
(a) $6 \div 3$ (b) $12 \div 4$
(c) $^-20 \div ^-4$ (d) $^-8 \div 2$
(e) $^-12 \div 3$ (f) $^-15 \div 5$

3 Rewrite as a multiplication, using $\square$.
(a) $18 \div 3 = \square$ (b) $^-8 \div 2 = \square$
(c) $12 \div ^-3 = \square$ (d) $9 \div ^-3 = \square$
(e) $\dfrac{16}{^-4} = \square$ (f) $\dfrac{^-4}{^-1} = \square$

4 Find the missing number in each part of Question 3.

5 How can you tell whether the answer to a division will be a positive or a negative number?

6 Copy this table. Match each division with its answer, using an arrow.

$\dfrac{15}{^-3}$	5
$\dfrac{15}{3}$	$^-5$
$\dfrac{^-15}{3}$	5
$\dfrac{^-15}{^-3}$	$^-5$

7 Multiplication and division of positive and negative numbers follow a pattern. Copy and complete these tables.

Second number

×	Positive	Negative
Positive	Positive	
Negative		

First number

Second number

÷	Positive	Negative
Positive		
Negative		

First number

8 Find the value of:

(a) $\dfrac{20}{^-2}$ (b) $\dfrac{8}{^-4}$ (c) $\dfrac{^-6}{3}$

(d) $\dfrac{^-100}{50}$ (e) $\dfrac{^-21}{3}$ (f) $\dfrac{^-21}{7}$

(g) $\dfrac{^-16}{^-4}$ (h) $\dfrac{^-4}{^-4}$ (i) $\dfrac{^-1}{1}$

9 Find the answer to:

(a) $\dfrac{^-4 \times ^-9}{^-3}$ (b) $\dfrac{^-21 \times ^-2}{^-7}$

(c) $\dfrac{^-70}{10} + \dfrac{^-35}{^-7}$ (d) $\dfrac{6}{^-2} - \dfrac{^-16}{4}$

10 (a) What is the square of $^-3$?

(b) Give two possible values of $\sqrt{9}$.

(c) Look at the multiplication table in Question 8 of Exercise 6D. Can you find a number whose square is $^-9$? Explain your answer.

6.5 Some ways we use integers
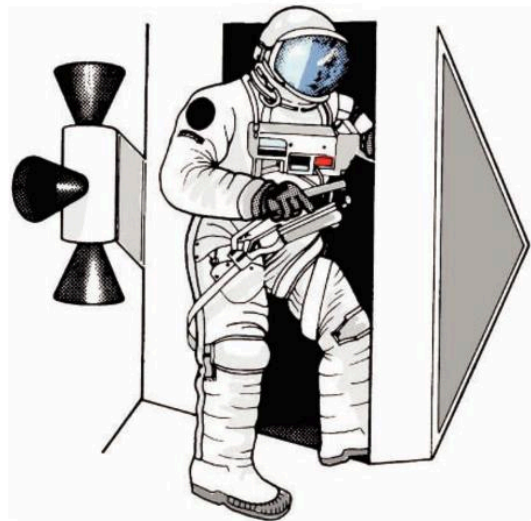
Time

The time when something is due to happen, like the launching of a space ship, is often called **time zero**. Times before zero are counted as negative, times after it are counted as positive.

Exercise 6G

1 This is a timetable for launching a space ship.

Time	Hour	Things to do
8.00 am	⁻4 h	Check weather report
9.00 am	⁻3 h	Fuelling
10.00 am	⁻2 h	Final check
11.00 am	⁻1 h	Count-down begins
12 noon	0 h	Launch
1.00 pm	1 h	Firing final rocket
2.00 pm	2 h	Into orbit
3.00 pm	3 h	Docking with space laboratory

  (a) The first check of the space ship is done at ⁻6h. What time is that?

  (b) On launch day, the astronaut sleeps until ⁻8h. How many hours are there between the astronaut waking up and docking with the space laboratory?

  (c) The astronaut must be dressed in a space suit $2\frac{1}{2}$ hours before launching. How would you record that on the timetable?

  (d) $1\frac{3}{4}$ hours after fuelling, the astronaut is locked into the space capsule. How would you show that on the timetable?

2 The Christian calendar begins counting years from the birth of Christ.
Years before the birth are called B.C.
Years after the birth are called A.D.
We could number B.C. years using negative numbers, if we say Christ was born at time zero.
Here are some events that happened in Asia.

Date	Event	Year
1250 B.C.	Jews leave Egypt	⁻1250
721 B.C.	Assyria destroys Israel	⁻721
63 B.C.	Pompey captures Jerusalem	⁻63
	Birth of Christ	0
70 A.D.	Romans destroy Jerusalem	70
638 A.D.	Muslims capture Jerusalem	638
1187 A.D.	Saladin captures Jerusalem	1187
1948 A.D.	New state of Israel founded	1948

How many years were there between:

  (a) the Jews leaving Egypt and the founding of the new state of Israel

  (b) the capture of Jerusalem by Pompey and by Saladin

  (c) the destruction of Israel by Assyria, and the capture of Jerusalem by Pompey

  (d) the Jews leaving Egypt and the birth of Christ?

3 Which is longer ago, 63 B.C. or 1250 B.C.?

4 Julius Caesar was murdered when he was 58 years old. He was born in 102 B.C. When did he die?

5 The first Olympic Games were held in Greece in 776 B.C.
The first modern Olympics were also held in Greece, 2672 years later.
In what year were the first modern Olympics held?

6 The city of Rome in Italy was founded in 753 B.C. To date how long has it been standing?

7
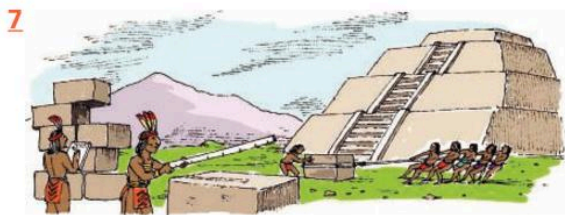
The pyramids of Egypt were built in 2600 B.C. How long will they have been standing in 2050 A.D.?

8 Imagine you are making up a new calendar, and that year 0 is the year your country became self-governing. Using your calendar, write down the new number for:

  (a) the year you were born

  (b) the year World War II broke out in Europe

  (c) the year Pompey captured Jerusalem

  (d) the year you started primary school.

Temperature

The **temperature** of something tells us how hot or cold it is.

It is measured using a thermometer. The temperature of melting ice is 0 °C.

Water
freezes

Thermometer
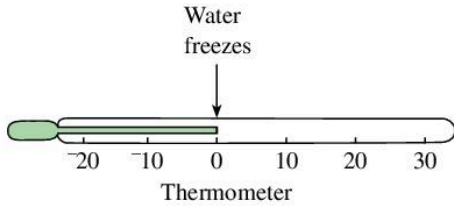
The C is short for Celsius (pronounced Sell-see-us) who was a famous scientist. 0 °C is read as *zero degrees Celsius*.

There are many things which are colder than ice. These things have negative temperatures. In many places, the temperature of the air is always colder than 0 °C.

Exercise 6H

1   Do you think that this picture shows a place that is colder than 0 °C?
    Why do you think so?

2   The temperature in Alaska on a certain day was 2 °C. Find the temperature the next day if:
    (a) it rose by 4 °C
    (b) it fell by 4 °C

3   Find the new temperature after:
    (a) a rise of 4 °C from ⁻1 °C
    (b) a rise of 6 °C from ⁻10 °C
    (c) a rise of 1 °C from ⁻9 °C
    (d) a fall of 2 °C from 3 °C
    (e) a fall of 4 °C from 3 °C
    (f) a fall of 7 °C from ⁻2 °C.

4   The temperature in Toronto was ⁻5 °C on Wednesday. It fell by 3 °C on Thursday. What was Thursday's temperature?

5   Liquid mercury freezes at ⁻39 °C and boils at 357 °C. What is the temperature difference between these states?

6   At a temperature of ⁻183 °C oxygen becomes a liquid. If its temperature is reduced by another 31 °C it freezes. What is the freezing point of liquid oxygen?

7   At a weather station in the Arctic the temperature was recorded as ⁻23 °C.
    Two hours later it had fallen by 8 °C.
    (a) What was the new temperature?
    (b) Four hours after the first readings the temperature was ⁻41 °C. By how much had the temperature fallen in four hours?

8   The table shows the average temperature of the air each month last year, in a North American city.

Month	Temperature
January	⁻4 °C
February	⁻11 °C
March	2 °C
April	15 °C
May	27 °C
June	31 °C
July	31 °C
August	33 °C
September	25 °C
October	17 °C
November	9 °C
December	⁻1 °C

(a) What was the temperature difference between the hottest and coldest months?
(b) Did the temperature rise between January and February? By how many degrees did it change?
(c) How many degrees did the temperature fall between November and December?
(d) How many degrees did the temperature rise between February and March?

Exercise 6I – mixed questions

1 Work out:
(a) $7 + {}^-8$ (b) $3 + {}^-9$
(c) $1 + {}^-3$ (d) $5 + {}^-11$
(e) $2 - 7$ (f) $6 - 13$
(g) ${}^-4 - 12$ (h) ${}^-9 - 13$

2 Work out:
(a) $3 - {}^-4$ (b) $7 - {}^-8$
(c) $6 - {}^-12$ (d) $4 - {}^-11$
(e) $11 - {}^-13$ (f) $12 - {}^-2$
(g) ${}^-9 - {}^-7$ (h) ${}^-18 - {}^-14$

3 Find the answer to:
(a) ${}^-1 \times 3$ (b) ${}^-4 \times 2$
(c) ${}^-6 \times 9$ (d) ${}^-5 \times 11$
(e) $7 \times {}^-9$ (f) $3 \times {}^-5$
(g) ${}^-6 \times {}^-7$ (h) ${}^-9 \times {}^-5$

4 Find the answer to:
(a) $9 \div {}^-3$ (b) $16 \div {}^-4$
(c) ${}^-12 \div 4$ (d) ${}^-15 \div {}^-3$
(e) $\dfrac{100}{{}^-10}$ (f) $\dfrac{25}{{}^-5}$
(g) $\dfrac{{}^-35}{7}$ (h) $\dfrac{{}^-42}{{}^-6}$

5 George noticed that the temperature was $3°C$ before he went to bed. During the night the temperature dropped by $17°C$.
What was the temperature in the morning?

6 The table shows the temperature in five cities on one day in December.

City	Temperature (°C)
Port of Spain	25
New York	${}^-3$
Castries	28
Toronto	${}^-11$
London	0

(a) How much hotter was it in Port of Spain than New York?
(b) Which city was the coldest on that day?
(c) How much colder was Toronto than London?
(d) Between which two cities was the temperature difference the greatest?
(e) Write down the cities in order of temperature beginning with the hottest.

7 The drawing shows three villages, X, Y and Z. The dotted line has been continued from the sea, to show sea level. X is above sea level. Y and Z are below sea level.
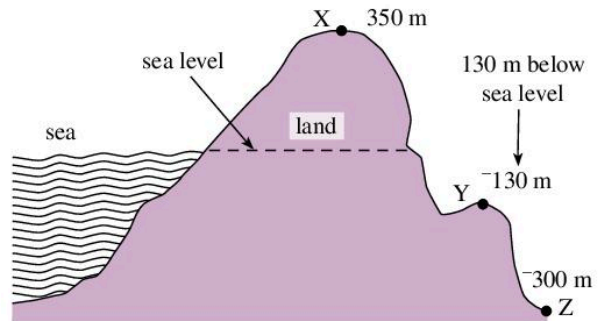
What is the difference in height between:
(a) Y and Z   (b) X and Y
(c) X and Z?

8 The highest place in the world where you could sit is on top of Mount Everest. It is 8840 m high. The lowest place where you could sit is on the shore of the Dead Sea. It is ${}^-393$ m high. What is the difference between these heights?

9 On Monday I had $23.50 in my bank account. On Tuesday I wrote cheques totalling $39. I paid $22 into my account on Wednesday. How much did I have in my account on Thursday?

10 Try to add negative numbers on your calculator. You will need to use the $\boxed{+/-}$ key to show a negative number. Use your calculator to work out:

(a) $6 + {}^-3$
(b) $10 + {}^-12$
(c) ${}^-3 + {}^-6$
(d) ${}^-12 + {}^-10$
(e) ${}^-3 + 5 + 8$
(f) ${}^-9 + {}^-3 + {}^-6$
(g) ${}^-114 + {}^-56$
(h) $63 + {}^-14 + {}^-29$

11 Copy and complete.
(a) $5 + \square = 3$ (b) ${}^-5 + \square = {}^-3$
(c) $\square + 2 = {}^-7$ (d) ${}^-3 - \square = {}^-9$
(e) $4 + \square - 3 = {}^-5$ (f) ${}^-6 + {}^-8 + \square = {}^-11$
(g) $\square + \square = {}^-7$ (h) ${}^-1.4 - \square = {}^-3.6$

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6 Consolidation

Example 1
Work out:

(a) $7 + {}^-11$

$$7 + {}^-11 = 7 - 11$$
$$= {}^-4$$

(b) ${}^-3 + {}^-8$

$${}^-3 + {}^-8 = {}^-3 - 8$$
$$= {}^-11$$
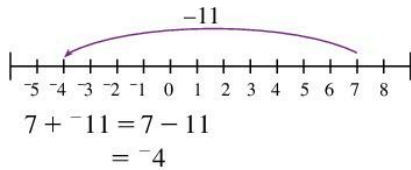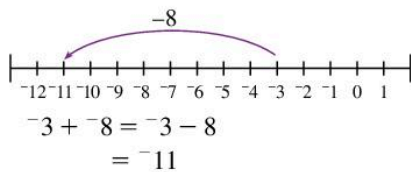
Example 2
Work out:

(a) $7 - {}^-4$

$$7 - {}^-4 = 7 + 4$$
$$= 11$$

(b) ${}^-7 - {}^-4$

$${}^-7 - {}^-4 = {}^-7 + 4$$
$$= {}^-3$$
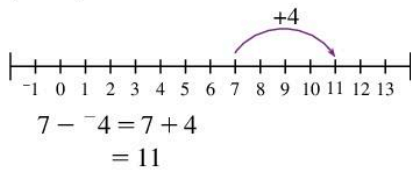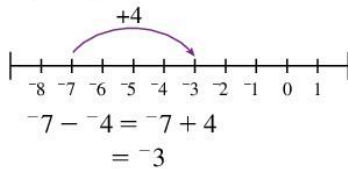
Example 3
Work out:

(a) ${}^-7 + 10 + {}^-3$ (b) ${}^-3 + {}^-6 + 5$

(a) ${}^-7 + 10 + {}^-3 = {}^-7 + 10 - 3$
$= {}^-7 + 7$
$= 0$

(b) ${}^-3 + {}^-6 + 5 = {}^-3 - 6 + 5$
$= {}^-9 + 5$
$= {}^-4$

Example 4
Work out:

(a) ${}^-3 \times {}^-5$ (b) $24 \div {}^-6$
${}^-3 \times {}^-5 = 15$ $24 \div {}^-6 = {}^-4$

Exercise 6

1 Work out:
(a) $3 + 4$ (b) $3 + {}^-4$
(c) ${}^-3 + 4$ (d) ${}^-3 + {}^-4$
(e) $6 + {}^-10$ (f) ${}^-6 + {}^-10$
(g) $7 + {}^-9$ (h) ${}^-3 + {}^-9$
(i) ${}^-13 + {}^-14$ (j) $2 + {}^-18$

2 Calculate:
(a) $3 - {}^-2$ (b) $5 - {}^-3$
(c) ${}^-5 - {}^-3$ (d) $3 - {}^-9$
(e) ${}^-7 - {}^-6$ (f) ${}^-6 - {}^-7$
(g) ${}^-13 - 8$ (h) ${}^-3 - 13$
(i) $14 - {}^-7$ (j) ${}^-7 - {}^-14$

3 Work out:
(a) $5 + {}^-3 - 2$ (b) ${}^-3 - 3 - 2$
(c) $3 - {}^-3 - 2$ (d) ${}^-6 - 4 - {}^-4$
(e) ${}^-4 + 6 - 8$ (f) $7 - {}^-6 + 4$
(g) ${}^-2 + 6 - {}^-4$ (h) ${}^-3 - {}^-4 - 9$
(i) $14 - 17 - 11$ (j) $8 + {}^-9 - {}^-4$
(k) ${}^-8 - 7 + {}^-6$ (l) ${}^-23 - 41 - {}^-19$

4 Work out:
(a) 3×5 (b) ${}^-3 \times 4$
(c) $3 \times {}^-4$ (d) ${}^-3 \times {}^-4$
(e) $5 \times {}^-4$ (f) ${}^-4 \times {}^-6$
(g) $7 \times {}^-2$ (h) ${}^-3 \times {}^-7$
(i) $11 \times {}^-12$ (j) ${}^-13 \times {}^-14$

5 Work out:
(a) $15 \div {}^-3$ (b) ${}^-15 \div 3$
(c) ${}^-15 \div {}^-3$ (d) $16 \div {}^-4$
(e) $4 \div {}^-16$ (f) ${}^-8 \div 12$
(g) ${}^-9 \div 15$ (h) ${}^-85 \div 17$
(i) ${}^-144 \div {}^-9$ (j) ${}^-182 \div 13$

6 Find the value of:
(a) $\dfrac{{}^-7}{7} - \dfrac{{}^-6}{2}$ (b) $\dfrac{{}^-3 \times {}^-9}{{}^-27}$

(c) $\dfrac{14}{{}^-7} + \dfrac{18}{6}$ (d) $\dfrac{{}^-25}{{}^-5} - \dfrac{24}{{}^-4}$

(e) ${}^-3 \times {}^-2 \times {}^-5$ (f) $7 \times {}^-4 \times {}^-2$

(g) $\dfrac{55}{{}^-11} + \dfrac{{}^-45}{5}$ (h) $\dfrac{{}^-1}{2} + \dfrac{{}^-1}{3}$

Application

7 The temperature in St Petersburg on 4th December was ⁻2°C. What was the temperature in:

    (a) Bratislava if it is 9°C hotter than St Petersburg

    (b) Murmansk if it is 4 deg C colder than St Petersburg

    (c) Volgograd if it is 3°C colder than St Petersburg?

8 The temperatures in four towns on 1st December were as follows:

Town	Temperature (°C)
Kingston	23
Moscow	⁻14
Calgary	⁻5
Bridgetown	31

    (a) What was the coldest town?

    (b) How much hotter is Bridgetown than Calgary?

    (c) How much colder is Moscow than Calgary?

    (d) Write the towns in temperature order, starting with the hottest.

9 The table shows the temperature in degrees Fahrenheit (°F) for given temperatures in degrees Celsius (°C).

°C	⁻20	⁻10	0	10	20
°F	⁻4	14	32	50	68

    (a) Plot these points on a graph.

    (b) Join the points with a straight line.

    (c) Use your graph to find:

        (i) the Fahrenheit temperature when it is 15°C

        (ii) the Celsius temperature when it is 0°F.

Summary

You should know ...

1 Adding a negative number is the same as subtracting a positive number.

For example:

$$^-2 + {}^-3 = {}^-2 - 3 = {}^-5$$

-3 so move 3 to the left
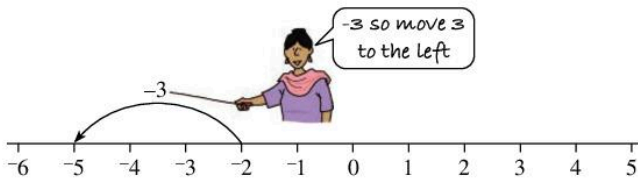
2 **(a)** Subtracting a negative number is the same as adding a positive number,

For example:

$$6 - {}^-2 = 6 + 2 \ = 8$$
$$^-5 - {}^-4 = {}^-5 + 4 = {}^-1$$

(b) More complicated additions and subtractions are done in the same way:

For example:

$$^-3 + {}^-5 - {}^-4 = {}^-3 - 5 + 4$$
$$= {}^-8 + 4$$
$$= {}^-4$$
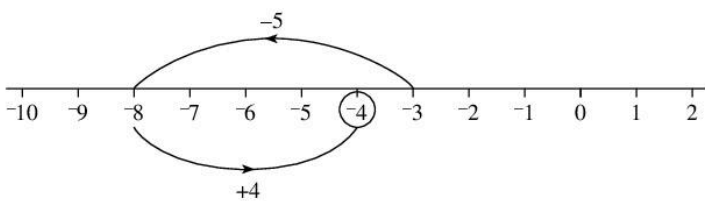
3 A number line can help you add and subtract numbers.

For example:

(a)

$$^-2 + 5 = 3$$

(b)

$$^-1 - 3 = {}^-4$$
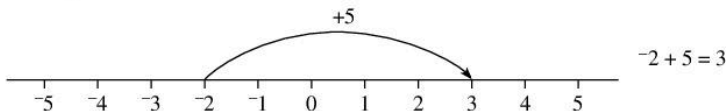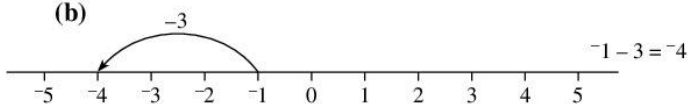
Check out

1 Work out:
(a) $2 + {}^-3$
(b) $3 + {}^-2$
(c) $^-3 + {}^-2$
(d) $^-5 + {}^-6$
(e) $3 + {}^-4 + {}^-5$
(f) $^-4 + 3 + {}^-5$

2 Calculate:
(a) $3 - {}^-2$
(b) $2 - {}^-3$
(c) $^-4 - {}^-5$
(d) $^-3 - {}^-7$
(e) $3 + {}^-2 - {}^-4$
(f) $^-4 + {}^-3 - {}^-5$

3 Use a number line to work out:
(a) $^-3 + 4$ **(b)** $^-4 - 2$
(c) $^-6 + 2$ **(d)** $3 - 5$
(e) $4 - 7$ **(f)** $^-2 + {}^-4$

4 **(a)** When you multiply two positive numbers together, the answer is positive.
For example: $5 \times 4 = 20$ and $12 \times 8 = 96$

(b) When you multiply two negative numbers together, the answer is positive.
For example: $^-5 \times {}^-3 = 15$ and $^-6 \times {}^-4 = 24$

(c) When you multiply a positive and a negative number together, the answer is negative.
For example: $^-3 \times 4 = {}^-12$ and $5 \times {}^-6 = {}^-30$

4 Calculate:
(a) $3 \times {}^-4$ **(b)** $^-3 \times 4$
(c) $^-3 \times {}^-4$ **(d)** $^-5 \times 3$
(e) $^-4 \times {}^-6$ **(f)** 4×5

5 **(a)** When you divide a positive number by a positive number, the answer is positive.
For example: $\dfrac{12}{3} = 4$ and $\dfrac{15}{5} = 3$

(b) When you divide a negative number by a negative number, the answer is positive.
For example: $\dfrac{^-6}{^-2} = 3$ and $\dfrac{^-30}{^-15} = 2$

(c) When you divide a positive number by a negative number, the answer is negative.
For example: $\dfrac{15}{^-3} = {}^-5$ and $\dfrac{21}{^-7} = {}^-3$

(d) When you divide a negative number by a positive number, the answer is negative.
For example: $\dfrac{^-14}{7} = {}^-2$ and $\dfrac{^-60}{20} = {}^-3$

5 Work out:
(a) $15 \div 3$ **(b)** $15 \div {}^-3$
(c) $^-15 \div 3$ **(d)** $^-15 \div {}^-3$
(e) $^-24 \div 4$ **(f)** $24 \div {}^-4$
(g) $^-36 \div {}^-9$ **(h)** $36 \div {}^-9$

Relations and graphs

(7)

Objectives

- ✔ draw and identify types of arrow diagrams
- ✔ identify mappings and represent them as ordered pairs
- ✔ read graphs to extract information
- ✔ draw straight line graphs
- ✔ find gradient and intercepts of straight line graphs
- ✔ show linear inequalities on graphs

Let me see your chart. Monday 99°, Tuesday 101° and my, my, Wednesday 104°. Now, what should we do about that?
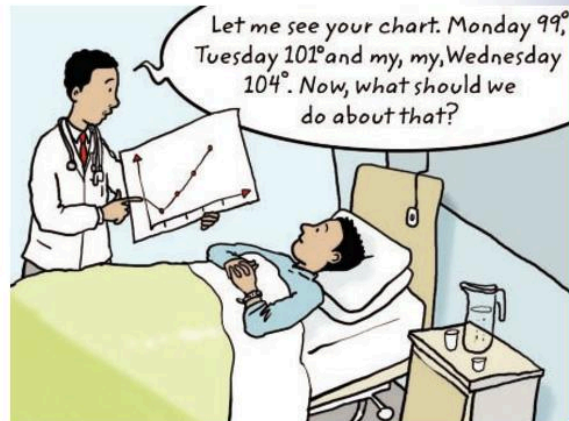
What's the point?

Graphs are powerful tools that show relationships between quantities. A quick look at a graph can show you what is happening and what may happen in the future. For example, a manager can look at a sales graph to see which employee is performing well, or not well enough.

Before you start

You should know ...

1 Mathematical relations.
For example:
(a) 3 'is less than' 7
(b) 8 'is double' 4
(c) 5 'is a factor of' 15

2 Number machines.

Input			Output
	Multiply by 3	Add 4	
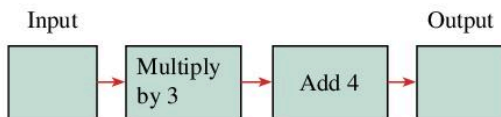
For example:
If 7 is input, then $7 \times 3 + 4 = 25$ is output.

Check in

1 Which of these statements are true?
(a) 7 is more than 10
(b) 3 is less than 4
(c) 7 is a factor of 21
(d) 12 is a multiple of 4

2 In the number machine shown, what is:
(a) the output when 6 is the input
(b) the output when 3 is the input
(c) the input if 7 is the output?

7.1   Arrow diagrams

This is an **arrow diagram**:

Plays against

Harry — Rita
Robert — Hazel
Paul — Pauline
Anthony — Andrea
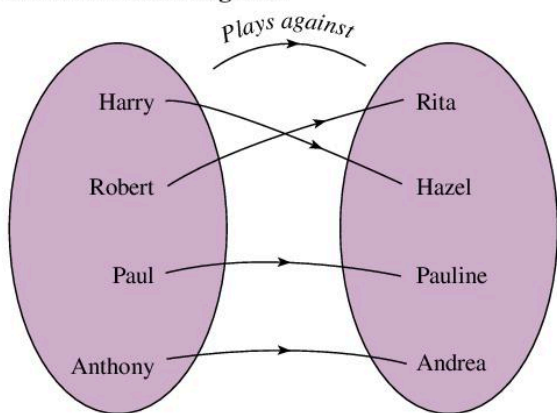
It shows how four boys and four girls from Tranquility School play each other in a tennis tournament. Each boy chooses one of the girls to play against.

● An **arrow diagram** links some or all of the elements in one set to the elements of a second set by means of a stated rule.

For example:

is 3 more than

5   6
7   2
9   4
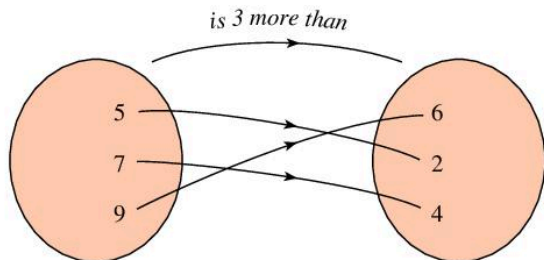
Exercise 7A

1   Copy and complete these arrow diagrams.

(a)

is 1 more than

1   0
3   1
5   2
    3
    4

(b)

is 2 less than

2   5
3   6
4   7
5   8
6   9

(c) *is double*

1   1
2   2
3   3
4   4
5   5
6   6
7   7
8   8

(d) *is a factor of*

1   1
2   2
3   3
4   4
5   5
6   6
7   7
8   8
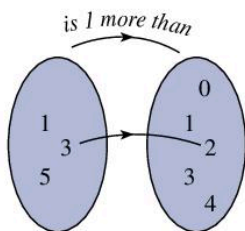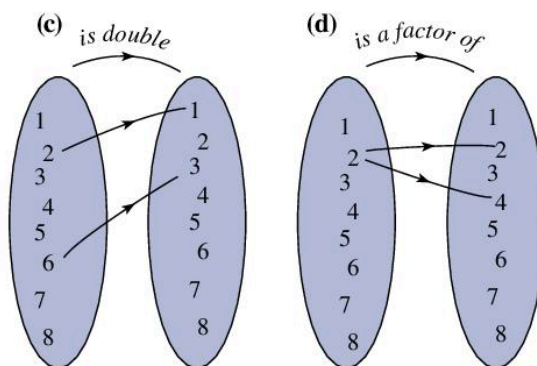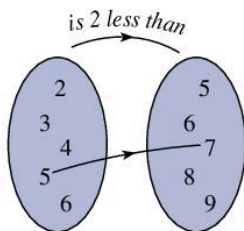
2   In the arrow diagram at the beginning of the chapter each boy could have picked a girl whose name has the same number of letters as his. Draw an arrow diagram to show this.

3   **(a)** Make a list of the subjects on your timetable today.
   (b) Now make a list of the books you use in lessons today.
   (c) Draw an arrow diagram to show how the set of books is related to the set of subjects. Think of a label for the arrow at the top of your arrow diagram.

4   Here is an arrow diagram for two sets A and B.

A   B

1
2   6
3   8
4   7
5   9
     10
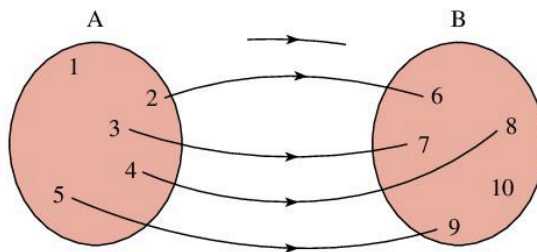
   (a) What do the arrows stand for?
   (b) Why is there no arrow going from 1 in set A?
   (c) Why is there no arrow going to 10 in set B?
   (d) Draw your own arrow diagram for the sets A and B to show *is three less than*.
   (e) Draw your own arrow diagram for the sets A and B to show *is half of*.

Arrow diagrams can represent more difficult relationships.

Example 1

$A = \{1, 2, 3, 6\}$
$B = \{2, 4, 6\}$

Draw an arrow diagram from A to B to show *is less than*.
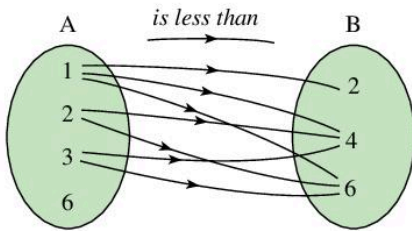
Exercise 7B

1 $X = \{$even numbers less than 12$\}$
$Y = \{1, 4, 9\}$
Using loops, show sets X and Y. On separate diagrams draw arrows from X to Y to show:
  (a) *is equal to*
  (b) *is greater than*
  (c) *subtracted from 11 gives*
  (d) *when divided by 2 gives.*

2 The members of two families are:
Mr Isaacs, Mrs Isaacs, Joey, Kara, Sam.
Mr Lam, Mrs Lam, Yen, Chang, Chung.
If P = {parents} and C = {children}
  (a) draw an arrow diagram from P to C to show *is a parent of.*
  (b) draw an arrow diagram from C to P to show *is a child of.*

3 $X = \{1, 2, 3, 4, 5\}$ $Y = \{2, 4, 6, 8, 10\}$
Draw arrow diagrams for X and Y to show:
  (a) *is half of*   (b) *is two times*
  (c) *is less than*   (d) *is greater than.*

4 $P = \{1, 2, 3, 4, 6\}$ $Q = \{2, 4, 6, 12\}$
Draw arrow diagrams for P and Q to show:
  (a) *is a factor of*   (b) *is a multiple of*
  (c) *is equal to*   (d) *is not equal to.*

5 G is a set of girls, B is a set of boys. The arrow diagram shows *is the sister of.*
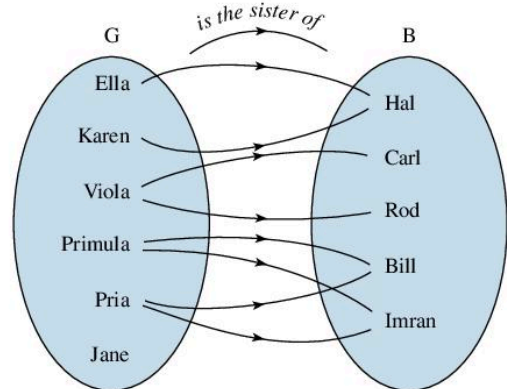
  (a) How many different families are the girls from?
  (b) What can you say about Carl and Rod?
  (c) What can you say about Ella and Karen?
  (d) What can you say about Jane?
  (e) What can you say about Bill and Imran?
  (f) Draw an arrow diagram to show *is the brother of*, using these sets B and G.

Another way of drawing an arrow diagram is to use a number line for each set.

Example 2

What rule is shown by the arrow diagram?
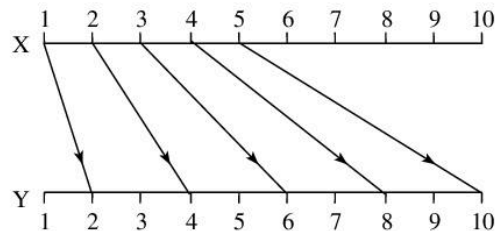
In the arrow diagram
  $1 \rightarrow 2$
  $2 \rightarrow 4$
  $3 \rightarrow 6$
So the rule is *multiply by 2.*

Exercise 7C

1 Find the rules shown by these arrow diagrams.

(a)
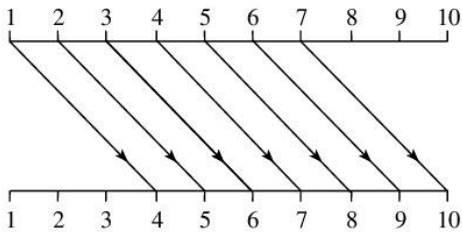
(b)
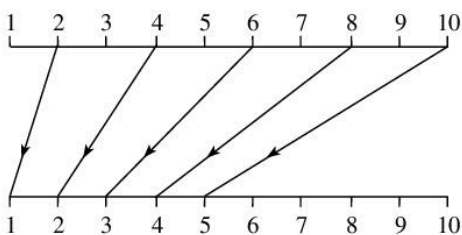
(c)
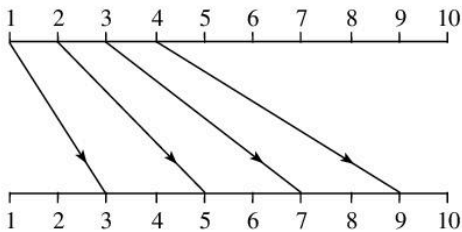
(d)
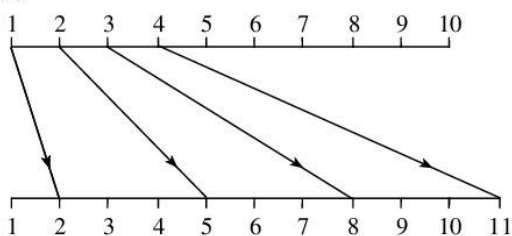
2 Draw arrow diagrams like the one in Example 2 to show:
(a) *add 3*
(b) *multiply by 2 then subtract 1*
(c) *multiply by 7 and write down the last digit of the answer.*
(d) *multiply by 5 and add the two digits of the answer.*

3 Find the rules shown by the arrows in these diagrams.

(a)
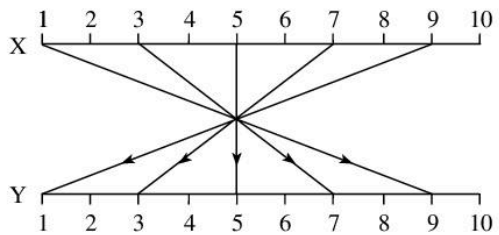
(b)
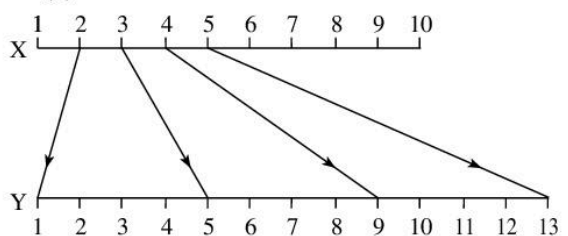
(c)
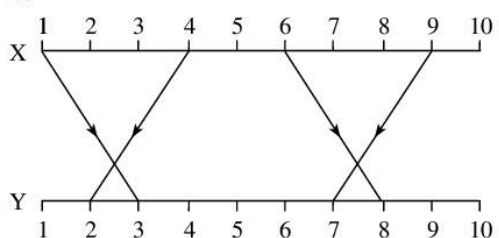
7.2 Mappings

This arrow diagram has the rule *add 3*.
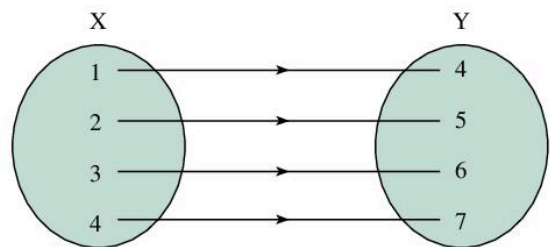
An arrow goes from each element of set X, into set Y. We say set X is **mapped** into set Y.
The relationship between the two sets is called a **mapping**.

- A mapping is a relationship in which an arrow must go from *every* element in the first set to an element in the second set.

Example 3

Which of these arrow diagrams show a mapping?

(a)

is the father of

(b)

is the brother of

(c)

is the mother of

(d)

is the sister of

(e)

number of sides

(f)

is the number of sides of a
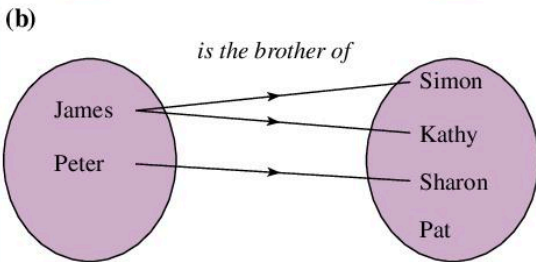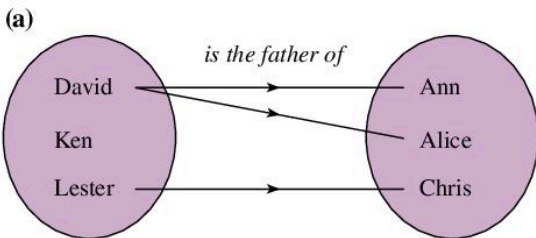
(a) Ken is not the father of Ann, Alice or Chris, so there is no arrow from Ken. So this is not a mapping.

(b) This is a mapping as an arrow goes from every element of the first set to the second set.
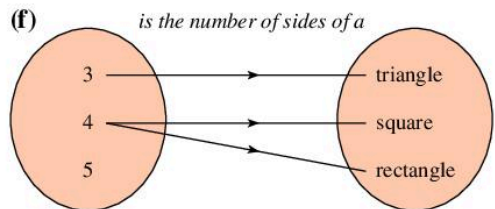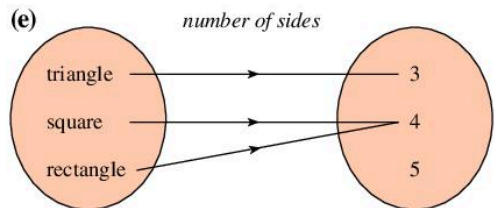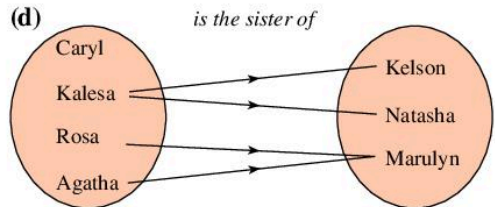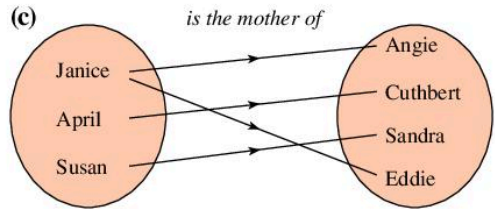
Exercise 7D

1 Which of these arrow diagrams show mappings?

(a)

multiply by 2

(b)

is smaller than
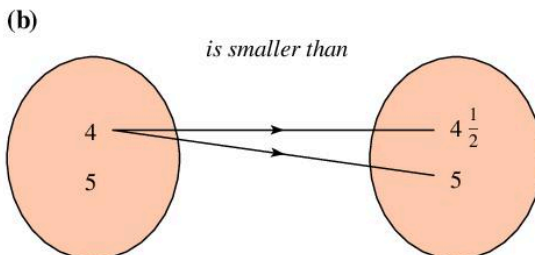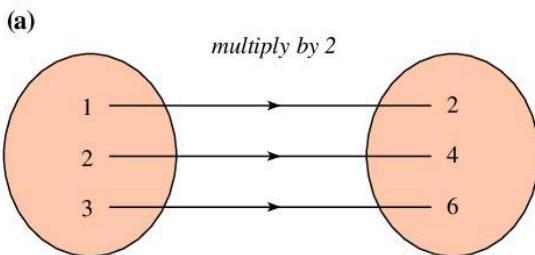
2 X = {1, 2, 3}
Y = {1, 2, 3, 4, 5, 6}
Draw arrow diagrams from set X to set Y for the rule
(a) *add 2*
(b) *multiply by 2*
(c) *multiply by 2, subtract 1*
(d) Which of these arrow diagrams are mappings?

3 A = {whole numbers from 1 to 5}
B = {whole numbers from 0 to 10}
Draw an arrow diagram to show how A maps to B for:
(a) *is equal to*
(b) *is half of*
(c) Why could you not show the mapping *is twice*?

4 The number lines show sets A and B.
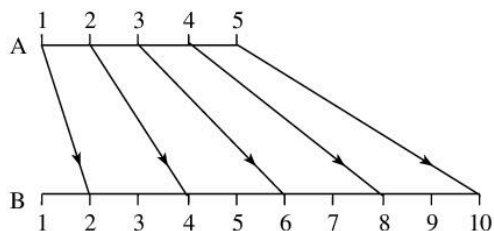
A = {1, 2, 3, 4, 5}
B = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}

(a) What is the rule shown by the mapping above?

(b) If 6 ∈ A where would 6 be mapped to under the rule?

Another way of showing the rule of a mapping is through a number machine, for example

| Input | → | Multiply by 2 | → | Output |
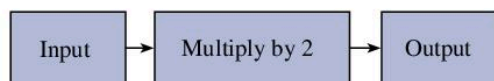
This machine takes any number and multiplies it by two.

Example 4

If X = {1, 3, 5, 7} and
Y = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10} draw an arrow diagram to show the mapping:

| Input | → | Subtract from 10 | → | Output |
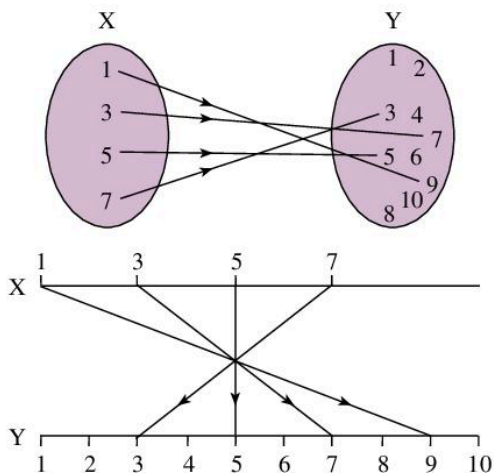
Exercise 7E

1 X = {1, 2, 3, 4, 5}
Y = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12}
Draw arrow diagrams to show the mappings on these number machines.

(a)
| Input | → | Multiply by 2 | → | Add 1 | → | Output |

(b)
| Input | → | Multiply by 2 | → | Subtract 1 | → | Output |

(c)
| Input | → | Add 1 | → | Multiply by 2 | → | Output |
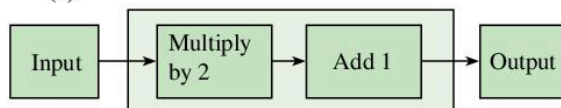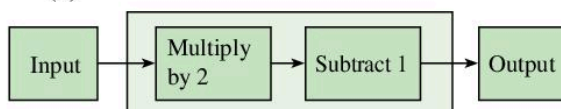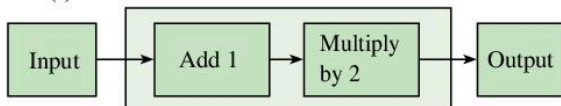
2 Draw the number machines used for the arrow diagrams below.

(a)

$12 \rightarrow 10$
$7 \rightarrow 5$
$3.2 \rightarrow 1.2$
$6\frac{1}{4} \rightarrow 4\frac{1}{4}$

(b)

$1 \rightarrow 4$
$2 \rightarrow 7$
$3 \rightarrow 10$
$4 \rightarrow 13$

(c)

$1 \rightarrow 1$
$2 \rightarrow 4$
$3 \rightarrow 9$
$4 \rightarrow 16$
$5 \rightarrow 25$
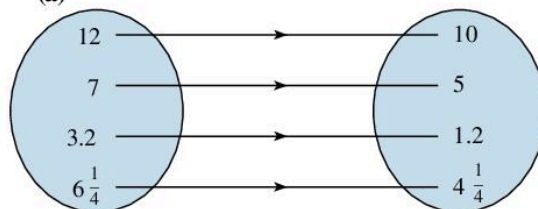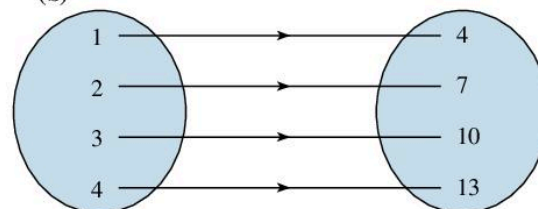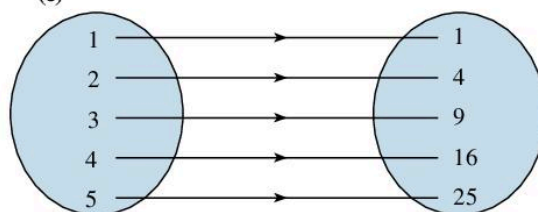
(d)
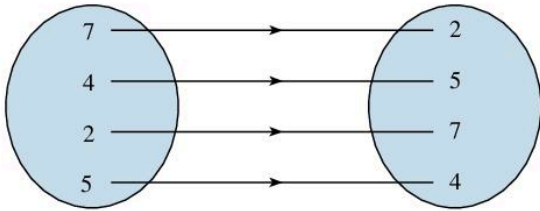
Generally, there are four different types of mappings.
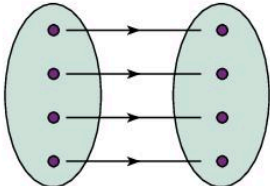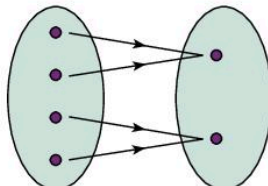
one-to-one (1–1)   many-to-one (M–1)
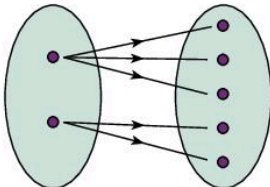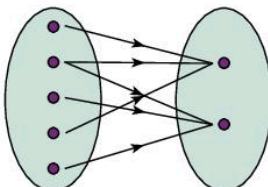
one-to-many (1–M)   many-to-many (M–M)

A mapping that is one-to-one or many-to-one is called a **function**.

Functions are really mappings that are uniquely defined. That is, each member in the domain that is, each member of the **domain** or initial set is sent to a single member of the **co-domain** or second output set is sent to a **single** number in the co-domain.

Example 5

Which of these maps are functions?

(a)

double

(b)
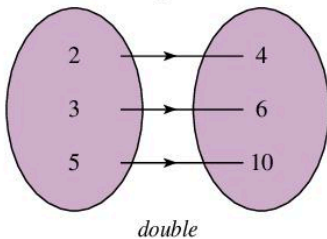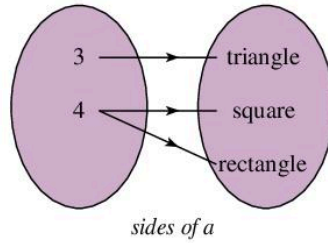
sides of a

(a) This is one-to-one and so is the number of sides of a function.

(b) This is one-to-many and so is not a function.

Exercise 7F

1 What type of mapping is shown in each diagram?

(a) *is a quarter of*

(b) *is a multiple of*

(c) *is the father of*

(d) *is the son of*
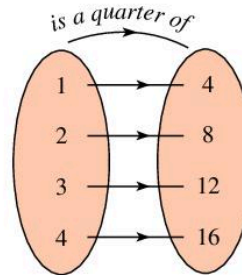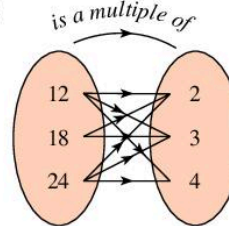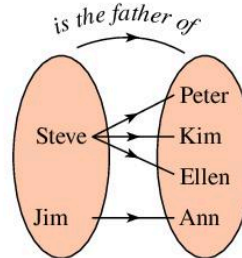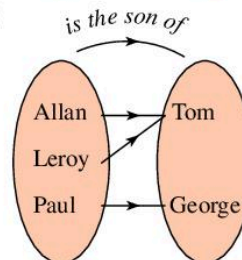
2 **(a)** Draw mapping diagrams to show the following.

           Domain   Rule
     (i) $\{1, 2, 3, 4\}$ *double*
     (ii) $\{1, 2, 3\}$ *square*
     (iii) $\{0, 3, 6, 9\}$ *double and add one*

    (b) Which of the above are functions?

3 $X = \{20°, 50°, 120°, 170°, 330°\}$
    $Y = \{acute, obtuse, reflex\}$
    (a) Draw an arrow diagram to show *is an angle that is.*
    (b) Is this mapping a function?

4 $X = \{J, K\}$
    $Y = \{Jack, Janice, Kay, Kim\}$
    (a) Draw an arrow diagram to show *name beginning with this letter.*
    (b) Explain why this map is not a function.

5 $X = \{1, 2, 3, 4, 5\}$
    $Y = \{4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16\}$
    (a) Draw an arrow diagram to show the mapping:

| Input | → | Multiply by 3 | → | Add 1 | → | Output |
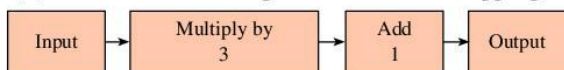
    (b) Explain why this map is a function.

If the number n is input into this number machine the output is given by $n - 2$.

| Input | → | Subtract 2 | → | Output |
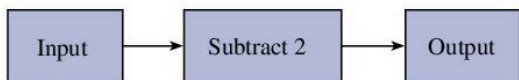
This mapping can be written as
    $n \rightarrow n - 2$

Example 6

If $X = \{1, 2, 3, 4, 5\}$
and $Y = \{whole\ numbers\ from\ 1\ to\ 15\}$,
draw an arrow graph of the mapping $x \rightarrow 3x$

The rule is multiply by 3
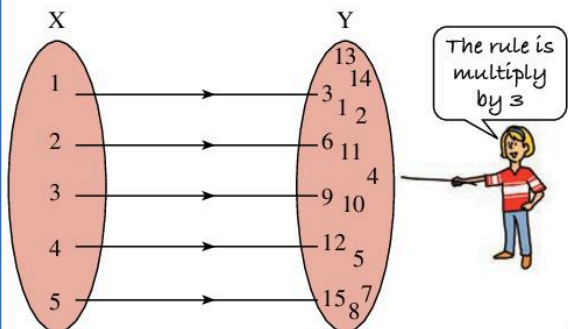
Exercise 7G

1 Describe the following mappings in the form $x \rightarrow \square$.

(a)

(b)

(c)

(d)
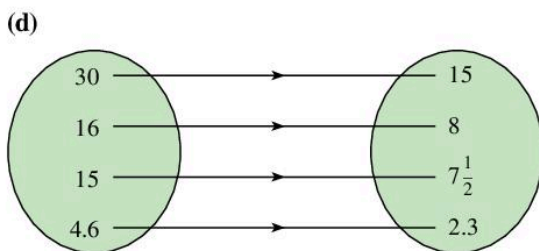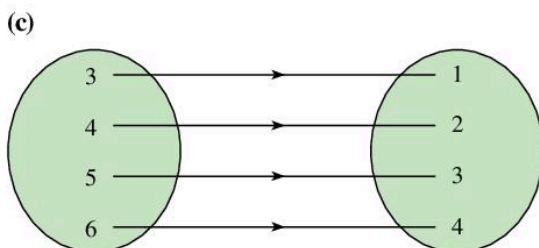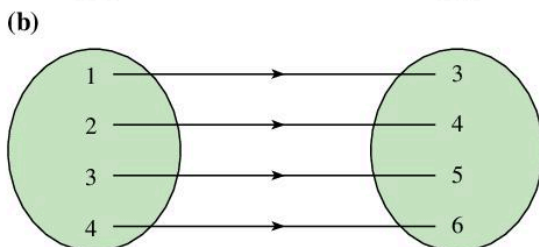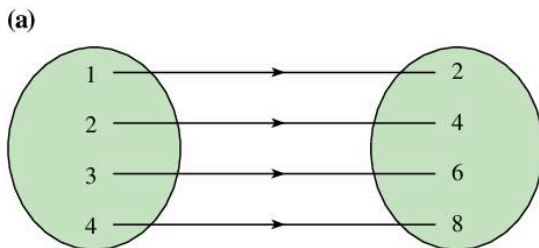
2 If $X = \{1, 2, 3, 4, 5\}$
and $Y = \{whole\ numbers\ from\ 0\ to\ 25\}$,
draw arrow graphs to show the mappings:
(a) $x \rightarrow 2x$
(b) $x \rightarrow x + 2$
(c) $x \rightarrow x - 1$
(d) $x \rightarrow 3x + 2$
(e) $x \rightarrow 5x - 3$

7.3 Ordered pairs

The mapping of the set J = {a, b, c} to the set
K = {1, 2, 3} is shown by the arrow diagram below.
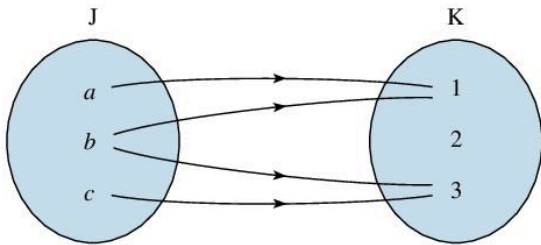
a maps to its image 1.

$$a \rightarrow 1$$

You can write the mapping

$$a \rightarrow 1$$

as the **ordered pair** (a, 1)

In the same way you can write (b, 1), (b, 3), (c, 3) to
show the rest of the mapping.

● An **ordered pair** is a pair of numbers written
in brackets, for example (m, n), in which m is
mapped to n.

Exercise 7H

1 Write down all the ordered pairs for each of
these mappings.

(a)
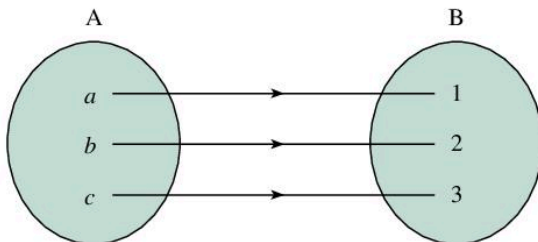
(b)
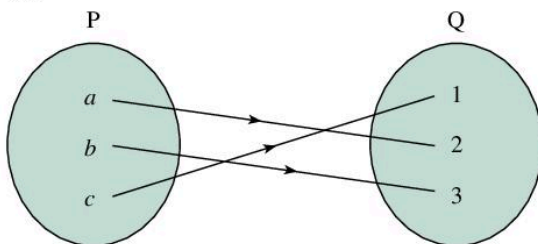
(c)
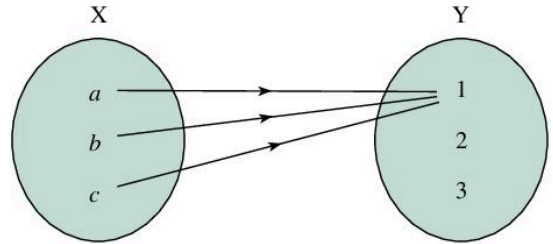
(d)
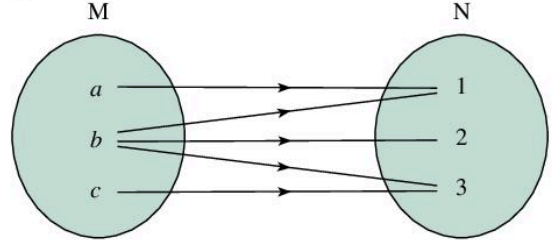
2 E = {a, b, c} and F = {1, 2}.
Draw an arrow diagram to show the mapping of
E to F described by each set of ordered pairs:
(a) {(a, 1), (b, 1), (c, 2)}
(b) {(a, 1), (a, 2), (b, 2), (c, 1)}
(c) {(a, 2), (b, 1), (b, 2), (c, 1), (c, 2)}

3 M = {March, April, May, June, July}
N = {28, 30, 31}
(a) Draw an arrow diagram to show the mapping
of M to N using *has the number of days*.
(b) Write down the ordered pairs of the mapping.

4 Draw an arrow diagram for each set of ordered
pairs. Say what the arrows represent.
(a) {(Apple, A), (Banana, B), (Pear, P),
(Pineapple, P)}
(b) {(31, January), (28, February), (31, May)}
(c) {(Square, 4), (Triangle, 3), (Pentagon, 5),
(Rectangle, 4), (Rhombus, 4)}

5 If 3 is put into this number machine, 9 will come
out the other end.
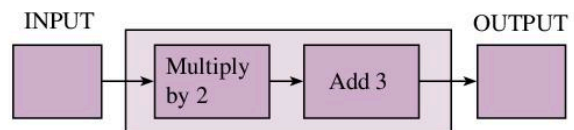
You can show the number put in and the number
coming out as an ordered pair. If 3 is put in, the
ordered pair is (3, 9).

(a) Copy and complete the following set of ordered pairs for the number machine above:
$\{(1, \ ), (2, \ ), (3, \ ), (\ , 15), (\ , 17), (\ , 23),$
$(12, \ ), (15, \ )\}$

(b) Draw an arrow diagram to show this set of ordered pairs.

6 $X = \{1, 2, 3, 4, 5\}$. Write down the set of ordered pairs (input, output) if each element of X is put into this number machine.

INPUT

| Subtract 1 | → | Multiply by 4 |

OUTPUT
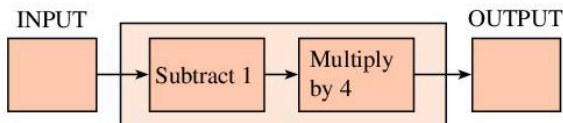
Investigation

Each person at a party shakes hands with every other person. In the following ordered pairs the first number represents the number of people attending the party, and the second number represents the number of handshakes. Find the values of a, b, c and d.

$(1, 0)$, $(2, 1)$, $(3, 3)$, $(4, a)$, $(5, b)$, $(6, c)$ and $(7, d)$

7.4 Plotting graphs

You will need centimetre squared paper.

Four boys are playing with dice.
Each of them throws one dice twice.
The arrow diagram shows how they score.

B   S

Deo
Jojo
Tej
Tony

1
2
3
4
5
6
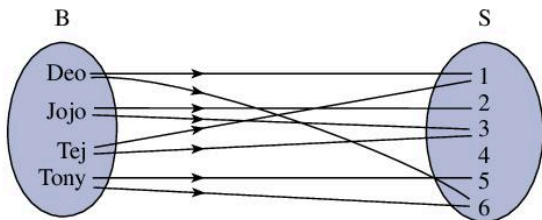
The information in the arrow diagram can also be written as ordered pairs:

(Deo, 1), (Deo, 6), (Jojo, 2), (Jojo, 3),
(Tej, 1), (Tej, 3), (Tony, 5), (Tony, 6)

These ordered pairs can be shown on a coordinate graph.

Dice scores

S (Scores)

Tony scored 5
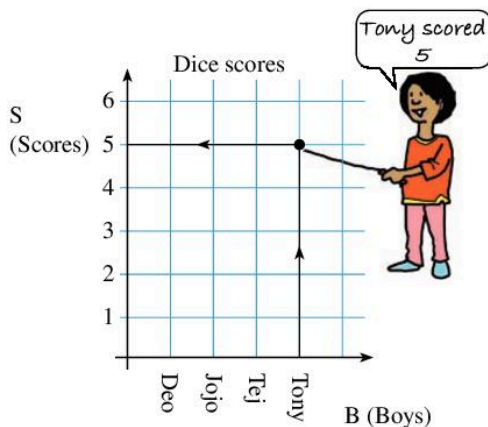
The graph shows one of Tony's throws.
The black dot shows Tony maps to 5.

Exercise 7I

1 Copy and complete the set of ordered pairs,
$\{(1, \ ), (4, \ ), (\ , 4), (\ , 2), (6, \ ), (3, \ )\}$ for the mapping of G to H shown below.

A mapping of G to H

This is the ordered pair (5, 4)
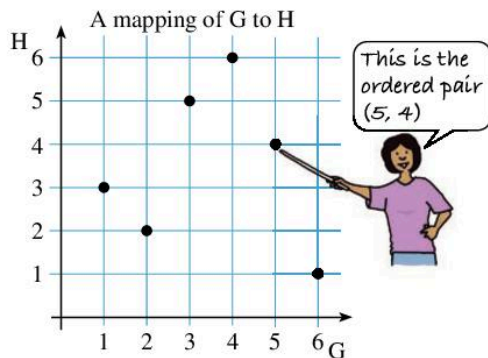
2 On a coordinate grid where X maps on to Y and
$X = \{0, 1, 2, 3, 4, 5\}$ and
$Y = \{0, 1, 2, 3, 4, 5\}$
plot the set of ordered pairs:
$(2, 0)$, $(4, 5)$, $(0, 3)$, $(2, 4)$,
$(5, 3)$, $(5, 5)$, $(1, 1)$, $(3, 3)$

3 Six girls each throw a dice twice. Each girl writes down her first and second score as an ordered pair.
Here is the set of their results:
$\{(6, 2), (6, 1), (1, 4), (3, 3), (2, 1), (5, 4)\}$

On squared paper, copy and complete this graph of the results.

Scores with two dice
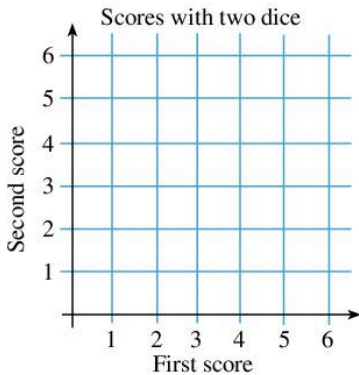
If the set of first scores is F and the set of second scores is S:

(a) List the elements of sets F and S.

(b) Draw an arrow graph to show the mapping of F to S.

4 The graph shows how a set of eight boys, B, maps to the set of months, M, using the rule *has his birthday in*.

Birthday months
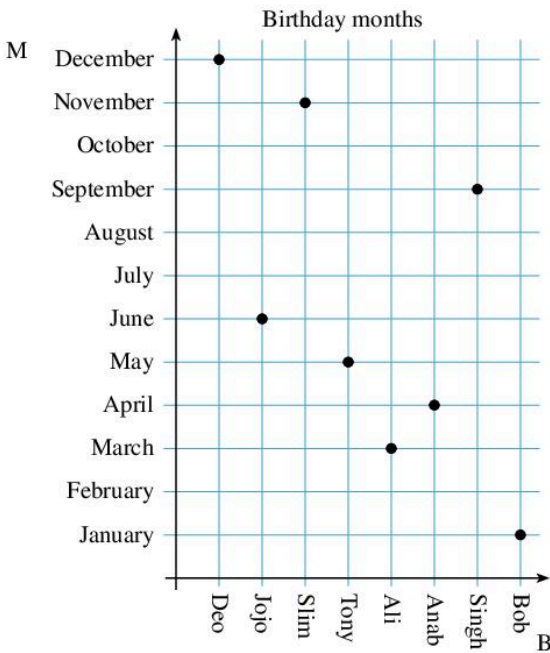
(a) List the ordered pairs of the mapping.

(b) Copy and complete: Tony → ☐.

5 A graph with rectangular axes has been started below. It shows set A on the horizontal axis, and set B on the vertical axis.
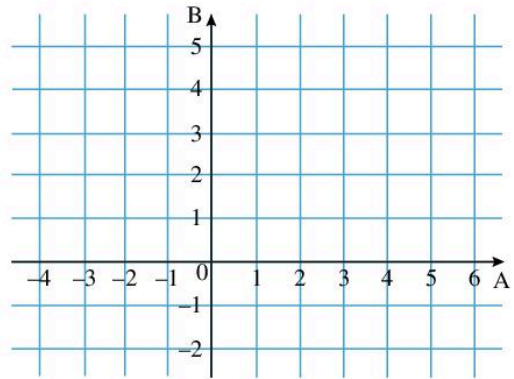
Make six copies of it on centimetre squared paper.

(a) On your copies show these sets of ordered pairs:

   (i) $\{(-2, -2), (-1, -1), (0, 0), (1, 1), (2, 2), (3, 3)\}$

   (ii) $\{(-2, 9), (-1, 8), (0, 7), (1, 6), (2, 5), (3, 4), (4, 3)\}$

   (iii) $\{(2, -2), (2, -1), (2, 0), (2, 1), (2, 2), (2, 3), (2, 4)\}$

   (iv) $\{(1, 4), (2, 4), (3, 4), (4, 4), (5, 4), (6, 4)\}$

   (v) $\{(0, -2), (1, 3), (2, 1), (3, 5), (4, 4), (5, 6)\}$

   (vi) $\{(1, -2), (2, 3), (3, 3), (3, 1), (-3, 4), (-3, -1)\}$

(b) Which of these mappings are functions? Explain.

6 Six boys each throw a dice twice.
The set of first scores, F, is $\{2, 3, 4, 6\}$.
The arrow diagram shows the mapping of F to the set of possible second scores P.
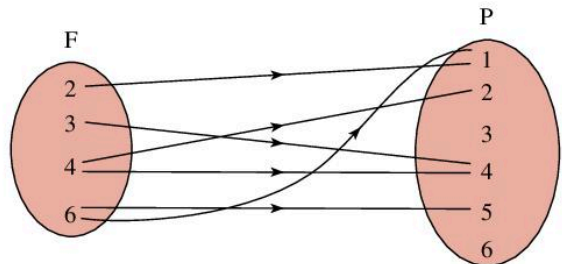
(a) List the ordered pairs for the mapping.

(b) Copy the grid and axes below. Draw a graph to show the mapping.

F (first score)
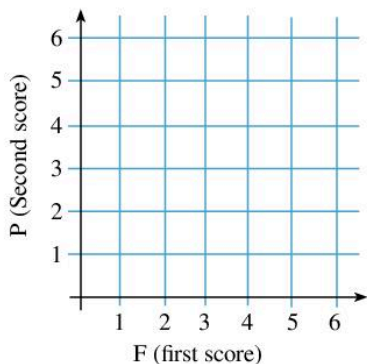
Coordinates

On a graph the horizontal axis is often called the *x*-axis and the vertical axis the *y*-axis. The ordered pairs that describe a mapping on rectangular axes are called coordinates.

In the graph below, $5 \in X$ and $8 \in Y$. The black point shows that 5 maps to 8. The coordinates of the point are (5, 8).

The mapping
$5 \longrightarrow 8$
is described by the coordinates
(5, 8)
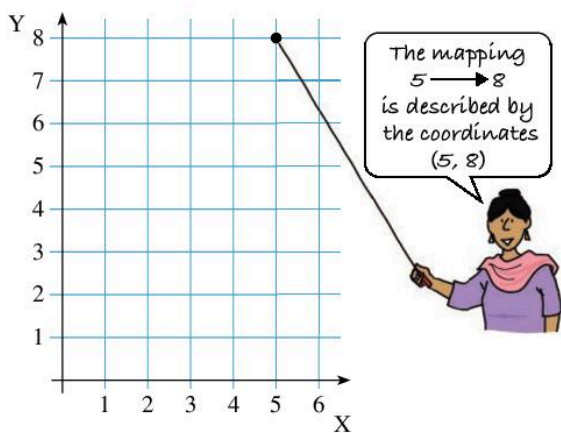
5 is the *x* coordinate.
8 is the *y* coordinate.
The *x* coordinate is *always* written first.
Can you suggest why?

- The coordinates (*a*, *b*) of a point on a graph represent the *x* and *y* coordinates respectively.

Exercise 7J

1   The graphs show the mapping of set A to set B on rectangular axes.

(i)   For each graph list the set of coordinates shown.

(ii) What rule has been used for the mapping?

(iii) Draw a number machine which will produce this mapping.

(a)

(b)

(c)

(d)
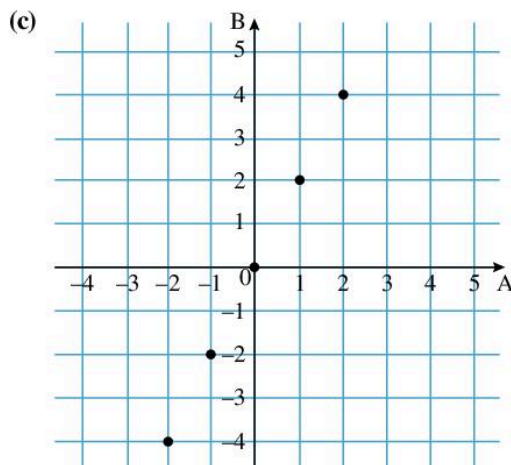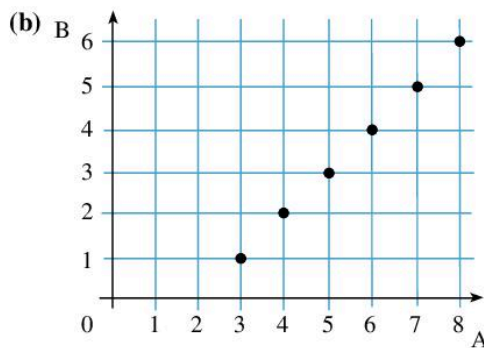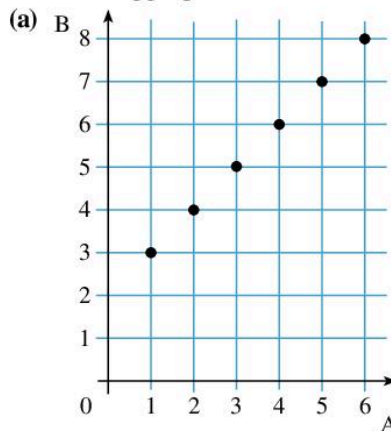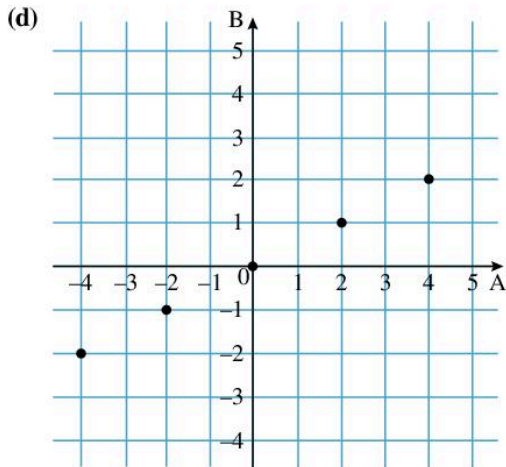
Example 7 illustrates how the rule for a mapping can be shown on a graph.

Example 7

$X = \{1, 2, 3, 4, 5\}$
$Y = \{\text{whole numbers}\}$

Draw a graph of the mapping

$$x \rightarrow x + 4$$

The rule is *add 4* so

$$1 \rightarrow 5, 2 \rightarrow 6, 3 \rightarrow 7, 4 \rightarrow 8, 5 \rightarrow 9$$

The point (5, 9) shows that $5 \longrightarrow 9$

Exercise 7K

1 $X = \{1, 2, 3, 4, 5\}$
$Y = \{\text{whole numbers}\}$
(a) Draw a graph of the mapping $x \rightarrow x + 2$.
(b) Draw an arrow diagram of the mapping $x \rightarrow x + 2$.

(c) List the ordered pairs of the mapping $x \rightarrow x + 2$.
(d) Copy and complete the diagram below for the mapping $x \rightarrow x + 2$.
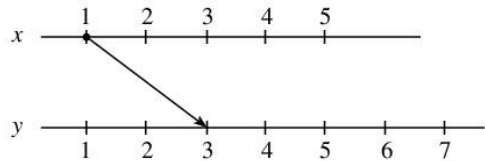
(e) Which of the four ways of showing the mapping $x \rightarrow x + 2$ do you prefer? Why?

2 $X = \{1, 2, 3, 4, 5\}$
$Y = \{\text{whole numbers from 1 to 20}\}$
$x \in X$.
X is mapped to Y by the rule $x \rightarrow 2x$, so
$1 \rightarrow 2, 2 \rightarrow 4, 3 \rightarrow 6$ and so on.
(a) List the set of coordinates of the mapping.
(b) Draw a graph of the mapping of X to Y.

3 $X = \{-3, -2, -1, 0, 1, 2, 3\}$
$Y = \{\text{integers}\}$;
Draw a graph of the mapping of X to Y using:
(a) $x \rightarrow 3x - 2$
(b) $x \rightarrow 2x + 3$
(c) $x \rightarrow x$

4   The number machine produces coordinates (input, output).
If 3 is the input, the output is 10. The coordinates are (3, 10).
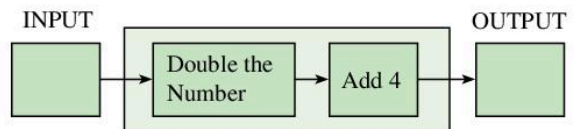
(a) List the set of coordinates produced when the set of numbers $\{1, 2, 3, 4, 5, 6\}$ is put in.
(b) Draw a graph to show the mapping. Label the axes.

7.5   Reading graphs

Graphs are often used to show numerical information. It is important to be able to read from graphs as well as plot them.

Example 8

The cost of mathematics textbooks is given by the graph below.

5 books cost $75 watch the scale.
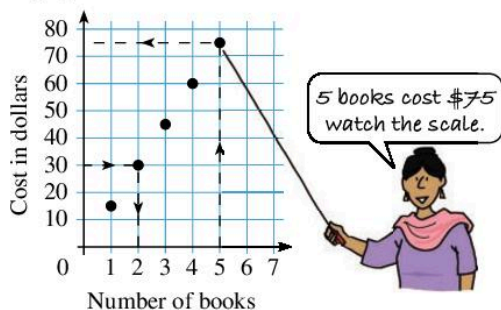
(a) How much do five books cost?
(b) How many books can be bought for $30?
(c) What is the cost of a single book?

(a) Five books cost $75 (notice the scale goes up by 10 each square on the y-axis, half a square represents $5).
(b) Two books can be bought for $30 (notice the ordered pair is (2, 30)).
(c) One book costs $15.

Exercise 7L

1 Six boys each throw a dice twice. The graph shows their scores.
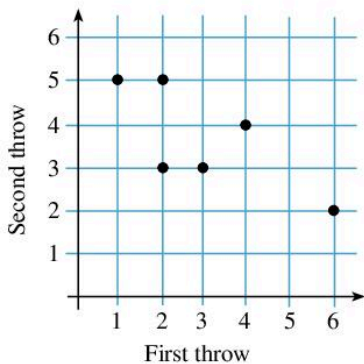
(a) Write down the ordered pairs of the graph.
(b) What is the highest total score?
(c) What is the lowest total score?
(d) How many boys scored 2 on the first throw?

2 The graph shows the maximum temperature each day during one week in August.
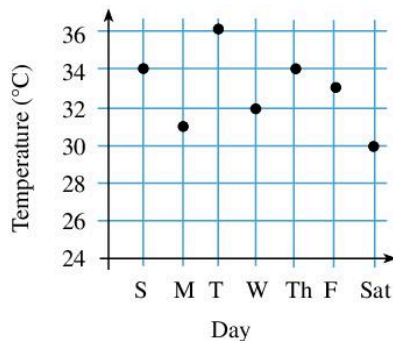
(a) What was the maximum temperature on Sunday?
(b) On which days was the temperature 34 °C?
(c) Which day was the hottest?
(d) What was the lowest maximum temperature recorded?

3 The cost for different sizes of Suds soap powder is given in the graph.
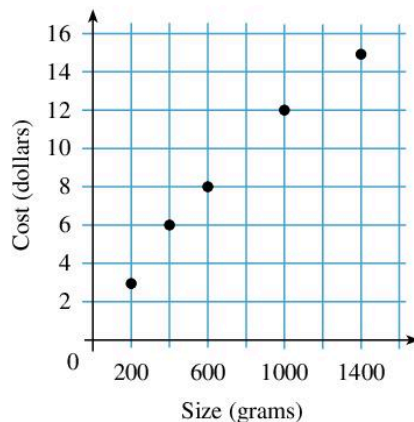
(a) What is the cost of 200 g of soap powder?
(b) How much soap powder can be bought for $12?
(c) Is it better to buy five 200 g boxes or one 1000 g box of soap powder?
(d) If an 800 g box was to be sold, how much should it cost?

4 Rani, Des, Bob, Tony and George took a spelling test. Two of them did badly and took the test again. Here are their scores:
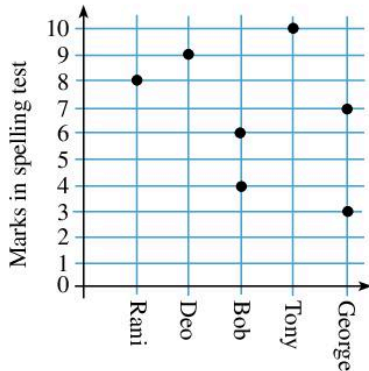
(a) Who got the best score?
(b) What are the names of the boys who had to repeat the test?
(c) Which boy scored six?

5 The graph shows John's height in centimetres at different birthdays.
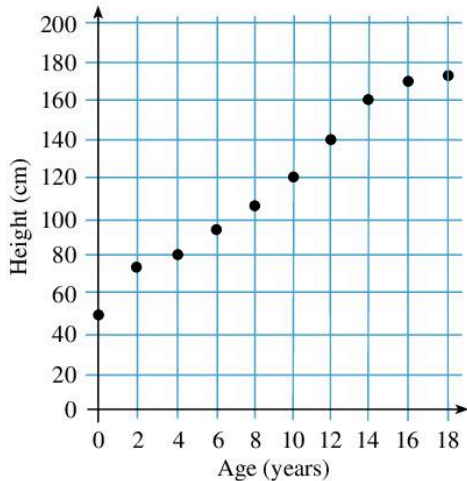
(a) What was his height on his fourth birthday?
(b) Estimate his height on his sixth birthday.
(c) On which birthday was he 170 cm tall?
(d) What do you think his height was on his thirteenth birthday?
(e) At what age do you think he was 100 cm tall?

The points on the graph in Question 5 could be joined to make a curve.

At $13\frac{1}{2}$ yrs John was 155 cm tall
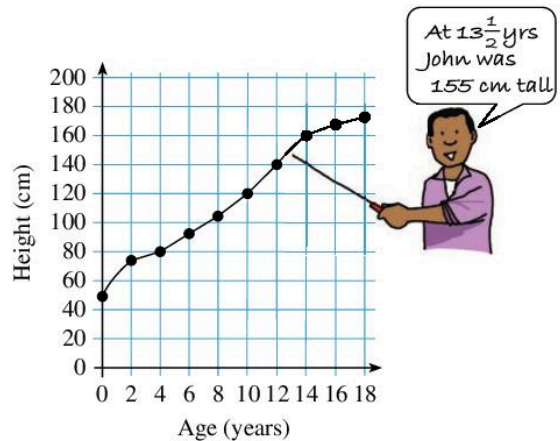
You should only join points on a graph if it makes sense.

For example, the points on the graph in Example 7 cannot be joined. The point (2.5, 37.5) would be meaningless—you could not buy $2\frac{1}{2}$ books.

Example 9

The temperature on 15th September is given by the graph:
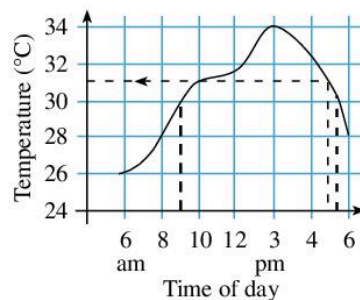
(a) What was the temperature at 8 a.m.?
(b) What was the temperature at 5 p.m.?
(c) When was the temperature 30°C?
(a) At 8 a.m. the temperature was 28°C.
(b) At 5 p.m. the temperature was 31°C, (see dotted lines on graph).
(c) The temperature was 30 °C at 9 am and again at 5.30 pm (see thick dotted lines).

Exercise 7M

1   The graph shows the cost of lengths of electric wire.
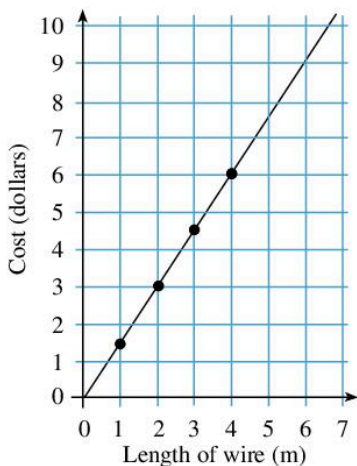
(a) How much does 4 m of wire cost?
(b) How much wire can be bought for $7?
(c) How much does 4.5 m of wire cost?
(d) Mr Lyn spent $9 on wire. How much did he buy?
(e) Copy and complete the statement:
      Wire costs $_____ per metre.

2   The graph shows the temperature on 3rd June.
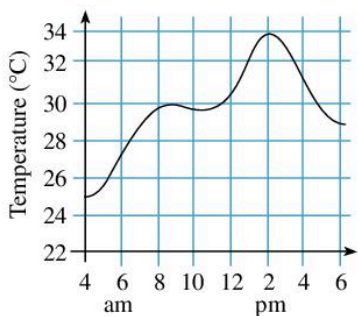
(a) What was the temperature at 6 am?
(b) What was the maximum temperature?
(c) When was the first time the temperature reached 30°C?
(d) What was the temperature at 6pm?
(e) What was the lowest temperature recorded?
(f) At what times was the temperature 29°C?

3   The graph shows the distance a man walks from his home one afternoon.
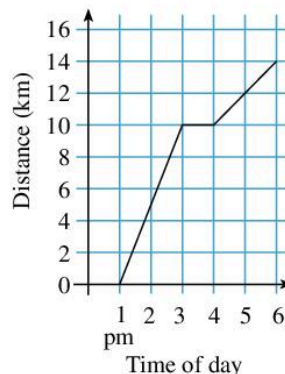
(a) What time did he leave home?
(b) How far had he travelled at 2 pm?
(c) What did he do between 3 pm and 4 pm?
(d) When was he 13 km from home?
(e) Between which times was he walking fastest?

7.6   Graphs of relations

A very simple relation is given by the rule *add 3*. This rule can be shown on an arrow diagram for $x = \{1, 2, 3, 4, 5\}$ and $y = \{$whole numbers$\}$.
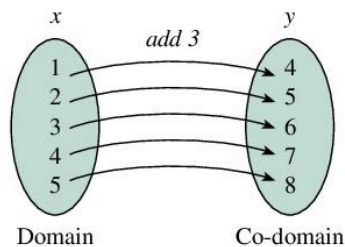
As a number machine this is:

Input $\rightarrow \boxed{+3} \rightarrow$ Output
  x y

or as a mapping:

$$x \rightarrow x + 3$$

Alternatively the mapping $x \rightarrow x + 3$ can be shown as ordered pairs on a graph:
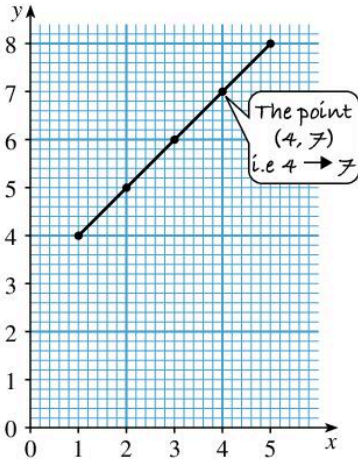
The point
$(4, 7)$
i.e $4 \rightarrow 7$

Instead of drawing an arrow diagram it is more usual to complete a table for the mapping:

x	1	2	3	4	5
y	4	5	6	7	8

The rule for the mapping $x \rightarrow x + 3$ is rewritten as the equation $y = x + 3$.

The points on the graph can be joined with a straight line.

We say the line is represented by the equation

$$y = x + 3$$

Example 10

(a) If $y = 2x + 1$, copy and complete the table, for the mapping.

x	$^-1$	0	1	2	3	4	5
y							

(b) Make a graph of this relation.

(a) The equation $y = 2x + 1$ can be written as the mapping:

$$x \rightarrow 2x + 1$$

or as the number machine

$$x \rightarrow \boxed{\times 2} \rightarrow \boxed{+1} \rightarrow y$$

so when $x = 5$, $y = 2 \times 5 + 1 = 11$

when $x = 4$, $y = 2 \times 4 + 1 = 9$

---

so the table becomes:

x	$^-1$	0	1	2	3	4	5
y	$^-1$	1	3	5	7	9	11

$(^-1, ^-1)$ $(1, 3)$ $(3, 7)$ $(5, 11)$
$(0, 1)$ $(2, 5)$ $(4, 9)$

(b) These ordered pairs can be plotted on a graph

The point $(3, 7)$ lies on the line $y = 2x+1$
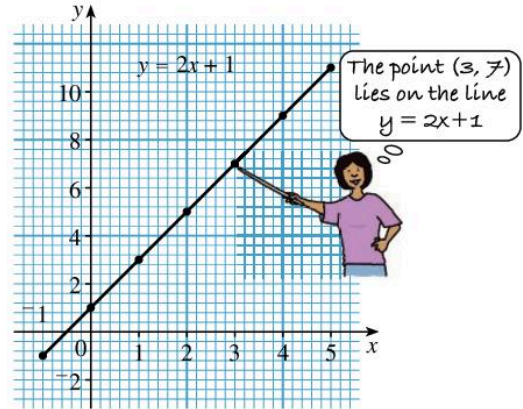
Exercise 7N

1 (a) Copy and complete the arrow diagram for the mapping $x \rightarrow 2x$.

$x \rightarrow 2x$
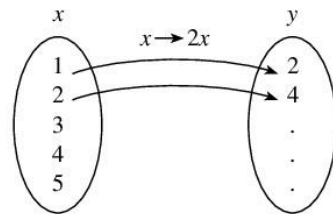
(b) Write down the ordered pairs of the mapping.
(c) Show the mapping on a coordinate graph.
(d) Join the points on the graph. What is the equation of the line?

2 Repeat Question 1 for each of these mappings.
(a)

$x \rightarrow 3x$

(b)

$x \rightarrow x + 2$

(c)

$x \rightarrow x - 3$

(d)

$x \rightarrow 2x + 2$
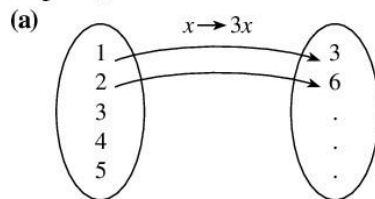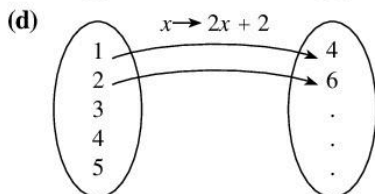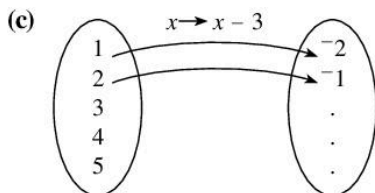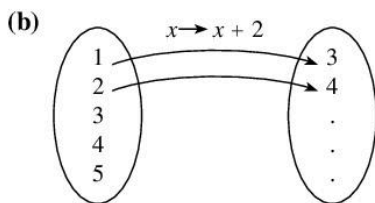
3 The number machine for the equation $y = 2x + 2$ is

$$x \rightarrow \boxed{\times 2} \xrightarrow{2x} \boxed{+2} \rightarrow 2x + 2 = y$$

Write down number machines for the following equations:

(a) $y = 3x$ (b) $y = x + 4$

(c) $y = x - 6$ (d) $y = 2x + 4$

(e) $y = 3x - 9$ (f) $y = \dfrac{x}{4} + 7$

(g) $y = \dfrac{x + 2}{5}$ (h) $y = \dfrac{3x + 2}{4}$

4 **(a)** If $y = 2x + 3$, copy and complete the table for the mapping:

x	0	1	2	3	4	5
y		5			11	

(b) Plot the points on a graph and join them with a straight line.

5 For each equation, copy and complete the table and then draw its graph.

x	0	1	2	3	4	5
y						

(a) $y = x + 4$ (b) $y = 3x + 4$

(c) $y = 3x - 2$ (d) $y = 2x - 5$

(e) $y = 5x - 6$

6 For each equation, copy and complete the table.

x	$^-5$	$^-4$	$^-3$	$^-2$	$^-1$	0	1	2	3	4	5
y											

(a) $y = 5x + 5$

(b) $y = 3x + 7$

(c) $y = 2x + 3$

7 **(a)** On centimetre square paper, draw a pair of rectangular axes. On the x-axis, show all the numbers from $^-5$ to 5. On the y-axis show all the numbers from $^-20$ to 30.

(b) Using these axes, show each set of ordered pairs (x, y) from your tables in Question 6. Use a different coloured pencil for each set of points.

(c) Join the points in each set with a straight line.

(d) Write down the coordinates of the point where each line crosses the y-axis (i.e. the y intercept).

(e) Compare your answer to part (d) with the equations. What do you notice?

(f) Which line is the steepest?

8 **(a)** For each equation, copy and complete the table.

x	$^-2$	$^-1$	0	1	2	3
y						

(i) $y = x$

(ii) $y = 2x$

(iii) $y = 3x$

(iv) $y = 4x$

(b) On suitable axes, plot each of these four graphs.

(c) Which graph is the steepest?

Technology

Review how to plot graphs of equations by visiting

www.purplemath.com/modules/graphlin.htm

and

www.mathisfun.com/algebra/linear-equations.html

Make sure you do the questions!

7.7 Straight-line graphs

The steepness of a straight-line graph is called its **gradient**.

Small gradient   Large gradient

The gradient of a line $= \dfrac{\text{vertical rise}}{\text{horizontal shift}}$.

Example 11

What is the gradient of the line $y = 2x - 1$?

First, complete the table

x	$^-1$	0	1	2	3
y	$^-3$	$^-1$	1	3	5

Then plot the points
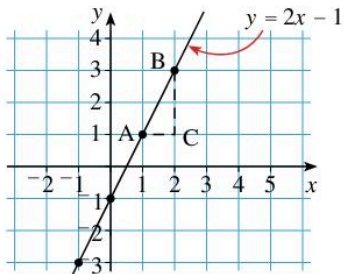
Look at two points on the graph:
A (1, 1) and B (2, 3).

$\text{Gradient} = \dfrac{\text{vertical rise}}{\text{horizontal shift}} = \dfrac{BC}{AC}$

$\qquad = \dfrac{3-1}{2-1} = \dfrac{2}{1} = 2$

The line $y = 2x - 1$ has gradient 2.

Exercise 7O

1 (a) Draw graphs of these equations
   (i) $y = x + 1$
   (ii) $y = 2x + 1$
   (iii) $y = 3x + 1$

by first copying and completing the table.

x	$^-1$	0	1	2	3
y					

(b) What is the gradient of each of these graphs?
(c) Where do each of these lines intercept the y-axis?

2 (a) Draw graphs of these equations
   (i) $y = 3x$
   (ii) $y = 3x + 1$
   (iii) $y = 3x + 2$

by first copying and completing the table.

x	$^-1$	0	1	2	3
y					

(b) What is the gradient of each of these lines?
(c) What do you notice about the gradient of each line?
(d) Where do each of these lines intercept the y-axis?
(e) What do you notice about the intercepts?

Gradients and intercepts

Look at the graphs you drew in Question 2 of Exercise 7O.

Same gradient, 3
Different intercepts
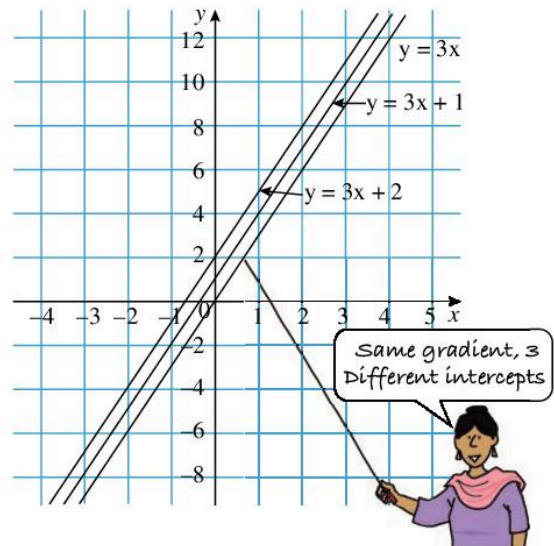
You should have noticed that all three lines have the same gradient, 3.

They **intercept** the y-axis at

$y = 0$, $y = 1$ and $y = 2$, respectively.

That is, the graph of the equation

$y = 3x$

has gradient 3 and intercept $y = 0$
The graph of the equation

$y = 3x + 1$

has gradient 3 and intercept $y = 1$
The equation

$y = 3x + 2$

has gradient 3 and intercept $y = 2$

In general, a linear graph of the form $y = mx + c$ has gradient m and intercept $y = c$.

Exercise 7P

1 (a) Draw graphs of the equations
  (i) $y = 2x - 3$
  (ii) $y = 2x - 1$
  (iii) $y = 2x + 3$
  by first copying and completing the table:

x	$^-2$	$^-1$	0	1	2	3
y						

  (b) What is the gradient of each line?
  (c) Where does each line intercept the y-axis?

2 Without drawing the graphs, find:
  (a) the gradient
  (b) the y-intercept of these lines:
  (i) $y = 4x - 1$ **(ii)** $y = 3x - 1$
  (iii) $y = 3x - 5$ **(iv)** $y = \frac{1}{2}x - 2$

3 Find the equation of a line with
  (a) gradient $= 2$,   intercept $y = 4$
  (b) gradient $= 3$,   intercept $y = \ ^-2$
  (c) gradient $= 1$,   intercept $y = 5$
  (d) gradient $= \frac{1}{4}$,   intercept $y = 6$

4 A line joins the points A(3, 1) to B(6, 4).
  (a) Show the points A and B on a graph.
  (b) Join the points A and B with a line.
  (c) Extend the line to find the y-intercept.
  (d) Calculate the gradient of the line.
  (e) Write down the equation of the line.

5 Repeat Question 4 for lines passing through the points:
  (a) A(0, 0), B(4, 4)
  (b) C(1, 1), D(3, 2)
  (c) E(0, 2), F(2, 0)
  (d) G(1, $^-1$), H($^-1$, 3)

6 A man walks 3 km in 1 hour and 6 km in 2 hours.
  (a) Draw a distance time graph to show this.
  (b) What is the gradient of the line drawn?
  (c) What does the gradient say about the speed of the man?
  (d) On the same graph draw the line of a man who walks at 5 km/h.
  (e) Is it true that $\dfrac{\text{change in distance}}{\text{change in time}} = $ gradient? Explain.

Activity

Research the gradient of ramps used for wheelchairs.

- What is the maximum gradient for such a ramp? What about gradients of roads?
- What is the steepest gradient for a road? Explain.

Make a presentation to your class to explain your findings.

What does the graph of an equation such as $y = 4$ look like?

Example 12

Draw a graph of the equation $y = 4$.

The equation $y = 4$ describes the mapping $x \rightarrow 4$.

An arrow diagram of this mapping is

$x \rightarrow 4$
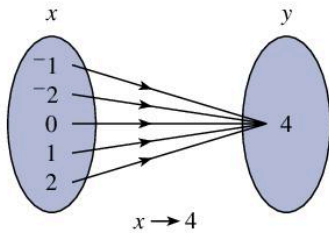
So a graph of the equation $y = 4$ is:
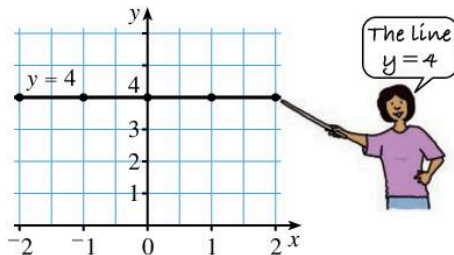
The line
$y = 4$

Exercise 7Q

1 (a) Using $x = \{^-2, 0, 2\}$ name three points that are on the line:
  (i) $y = 2$
  (ii) $y = \,^-1$
  (iii) $y = 6$
  (b) Draw the graphs of the equations in part (a).
  (c) What is the gradient of each of these lines?

2 Plot the coordinates $(3, \,^-2)$, $(3, \,^-1)$ $(3, 0)$, $(3, 1)$, $(3, 2)$ on a pair of axes. What is the equation of the line through these points?

3 For $y = \{^-2, \,^-1, 0, 1, 2\}$ draw graphs of:
  (a) $x = 4$ (b) $x = \,^-3$ (c) $x = 1\frac{1}{2}$

4
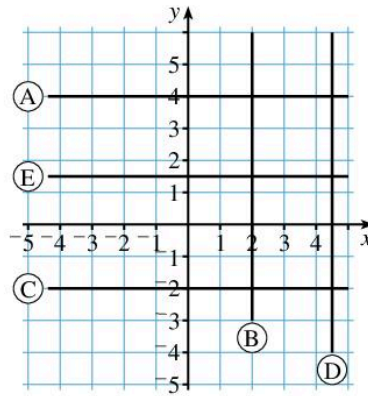
Write down the equations of the lines A, B, C, D and E.

5 Which line in the graph has the equation:
  (a) $y = 4$ (b) $x = 3$ (c) $y = x + 2$
  (d) $y = x - 1$ (e) $y = 4 - x$?
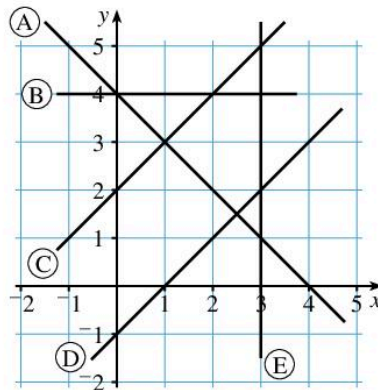
6 Look at the graphs in Question 5. Where do the following pairs of lines meet?
  (a) Ⓐ and Ⓑ
  (b) Ⓐ and Ⓒ
  (c) Ⓒ and Ⓔ
  (d) Ⓓ and Ⓔ
  (e) What can you say about the coordinates of these points of intersection?

7 **(a)** Copy and complete the table of values below for $y = 2x + 4$:

x	$^-2$	$^-1$	0	1	2	3	4	5
y							12	

and for $y = x + 1$:

x	$^-2$	$^-1$	0	1	2	3	4	5
y				2				

(b) Plot both graphs on the same axes.
(c) Where do the lines intersect?
(d) What does this point of intersection mean?

8 Draw graphs for each pair of equations and identify the point of intersection.
(a) $y = 2x - 3$ **(b)** $y = x + 2$
     $y = x - 2$ $y = 3x - 2$

9 The following four sets of points lie on straight lines:
(a) $\{(3, 2), (^-2, ^-3), (0, ^-1), (1, 0)\}$
(b) $\{(^-8, ^-10), (0, 6), (1, 8), (3, 12)\}$
(c) $\{(2, 0), (0, ^-1), (4, 1), (^-4, ^-3)\}$
(d) $\{(0, ^-8), (1, ^-4), (2, 0), (4, 8)\}$
Which of these equations represents which line?

$$y = 2x + 6 \qquad y = x - 1$$
$$y = 4x - 8 \qquad y = \tfrac{1}{2}x - 1$$

10 Write down an equation for each set of ordered pairs:
(a) $\{(0, 0), (1, 2), (2, 4), (3, 6), (4, 8)\}$
(b) $\{(0, 1), (1, 3), (2, 5), (3, 7), (4, 9)\}$
(c) $\{(0, ^-1), (1, 1), (2, 3), (3, 5), (4, 7)\}$
(d) $\{(0, ^-4), (1, ^-3), (2, ^-2), (3, ^-1), (4, 0)\}$
(e) $\{(0, ^-2), (1, 0), (2, 2), (3, 4), (4, 6)\}$

7.8 Inequalities

Recall that the sign $>$ means is "greater than" and $<$ means "is less than".

What values of x satisfy the inequality $x > 5$?

The answer is any number bigger than 5. You can show this on a number line as
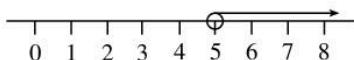
How about $x \leq 2$?

The shaded circle ● indicates that the value 2 is included in the set of numbers that satisfy the inequality $x \leq 2$.

In real life, inequalities are everywhere.

The safe age range for Inspiriton can be written

$$5 < A < 70$$

where A is the age of the person. On a number line, this is
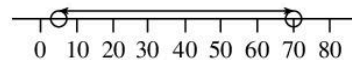
Exercise 7R

1 Draw number lines to show:
(a) $x < 2$ **(b)** $x \leq 3$ **(c)** $x > 4$
(d) $x \geq 70$ **(e)** $x \leq 38$

2 Draw number lines to show:
(a) $0 \leq x \leq 2$ **(b)** $0 < x < 3$
(c) $^-2 \leq x < 2$ **(d)** $^-30 < x \leq 30$

3 **(a)** Write inequalities to illustrate these situations.
(i) This product should be kept at a temperature between 15ºC and 20ºC.
(ii) Chapter 3 of a book is between pages 41 and 60.
(iii) Speed limit is 60 kph.
(iv) No admittance to children under 14 years of age.
(v) Police recruits should be at least 170 cm in height.
(vi) Only persons between the ages of 18 and 35 need apply.
(vii) Sale! All prices under $100.
(viii) Applicants should have a GPA of not less than 2.8.
(b) In each case, draw a number line to illustrate the answer.

You can also show inequalities on graphs.
Look at the line $y = 2$.
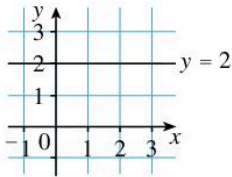
It divides the coordinate plane into two regions.
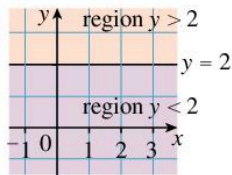
The shaded region above $y = 2$
shows the region where $y > 2$.
The shaded region below $y = 2$
shows the region where $y < 2$.

Example 13

Draw a graph to show the region $0 \leq x \leq 2$ and
$y > 3$.

First inequality:
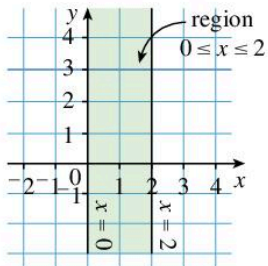
Second inequality:
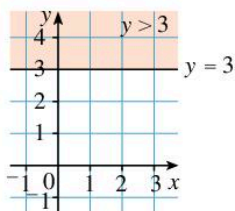
If you put the two graphs together, the shaded
regions overlap.

This is the region for which
$0 \leq x \leq 2$ and $y > 3$.
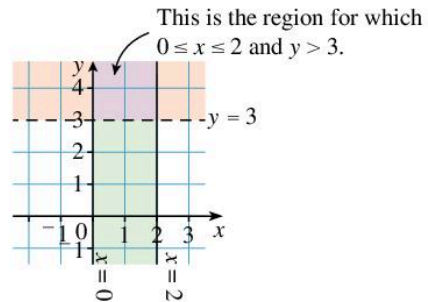
Exercise 7S

1 Sketch graphs to show:
   (a) $x > 2$ **(b)** $y < 3$
   (c) $x \leq 4$ **(d)** $y \geq {}^-2$
   (e) $0 \leq x \leq 3$ **(f)** $2 < y < 4$

2 Write an inequality which represents the shaded
region in these graphs.

   (a)   **(b)**

   (c)   **(d)**
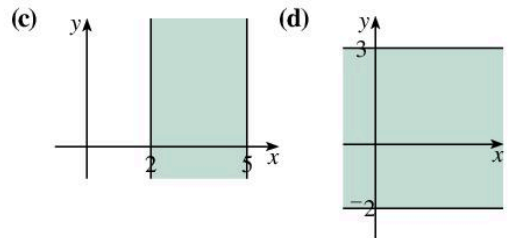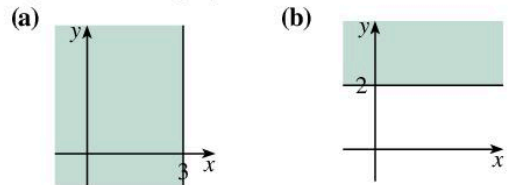
3 Draw graphs to show the regions defined by:
   (a) $x > 0, y > 0$
   (b) $x \leq 2, y \leq 2$
   (c) $0 \leq x \leq 3, y < 4$
   (d) ${}^-2 < x < 4, 3 < y \leq 5$

4 **(a)** Plot the graph $y = x + 2$ for $0 \leq x \leq 5$.
   (b) Shade the region where $y > x + 2$.
   (c) On the same axes, shade the regions $x > 1$
       and $y < 5$.
   (d) What shape is the region that satisfies all
       three inequalities?

5   **(a)** Plot the graphs of:
- **(i)** $y = 2x - 1$
- **(ii)** $y = 6 - x$
- **(iii)** $y = 1$
- **(iv)** $x = 1$

(b) On the same set of axes, shade the region defined by the inequalities:
$y \leq 2x - 1, y \leq 6 - x, y \geq 1, x \geq 1$.

Exercise 7T – mixed questions

1   **(a)** Copy and complete these arrow diagrams.

(i)

is a multiple of

(ii)

is 4 less than
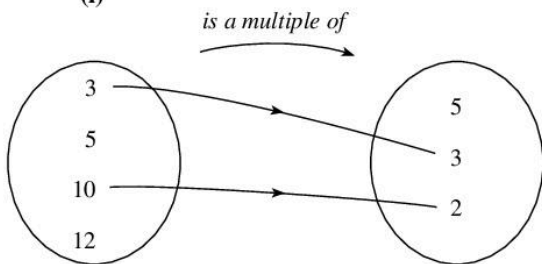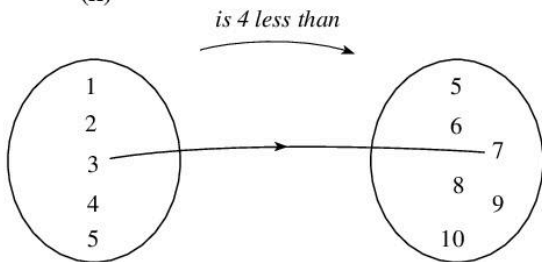
(b) X = {1, 2, 3, 4}
Y = {1, 2, 3, . . . , 12}
Draw an arrow diagram from X to Y to show *is one third of*.

2 Write down the rule for each of these mappings.

(a)
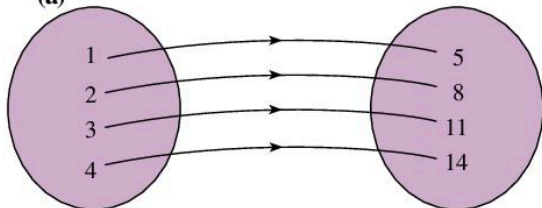
(b)
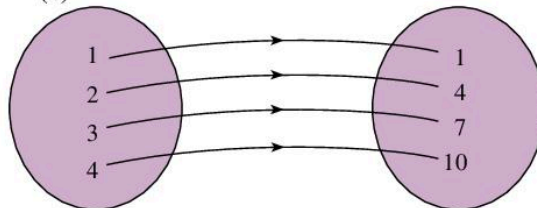
(c)
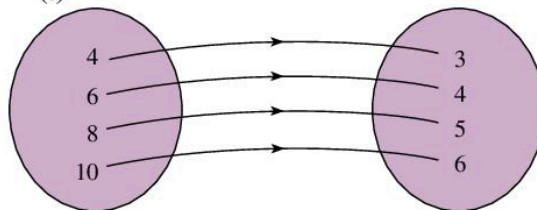
3 Describe these mappings in the form $x \rightarrow \square$.

(a)
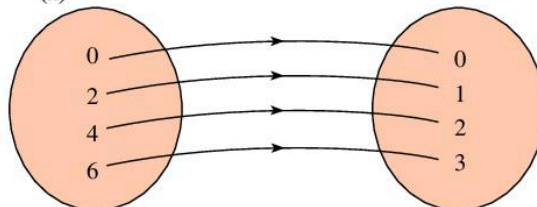
(b)
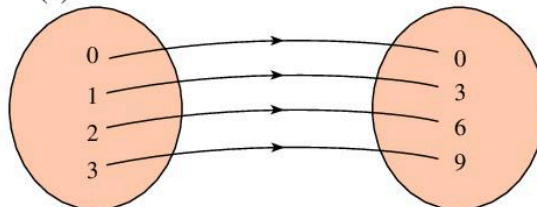
(c)
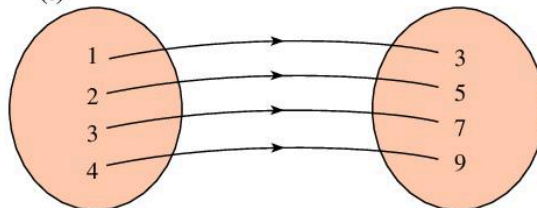
4   X = {0, 1, 2, 3, 4}
Y = {0, 1, 2, . . . , 10}
Draw arrow diagrams to show these mappings.
- **(a)** $x \rightarrow x + 3$ **(b)** $x \rightarrow 2x - 1$

(c)

| Input | → | Subtract from 9 | → | Output |
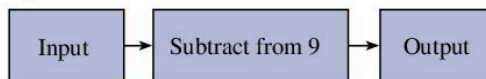
(d) Plot each map on a coordinate graph.

5 For each arrow diagram write down:
  (i) the rule
  (ii) the numbers marked *.
  (a)
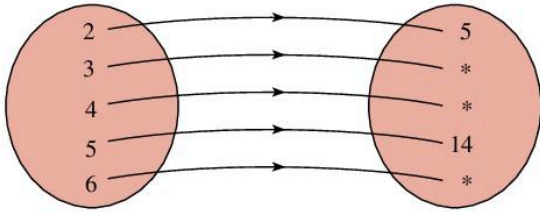
  (b)
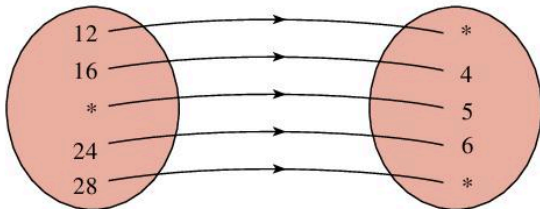
6 The graph shows a set of six children and the number of brothers and sisters they have.
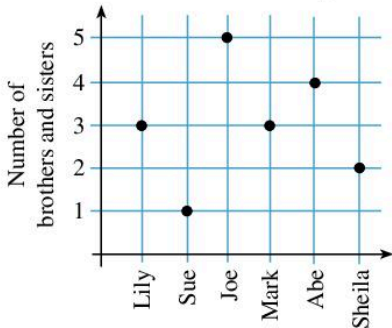
  (a) Complete the ordered pairs (Lily, 3) . . .
  (b) Two children have the same number of brothers and sisters. Name these children.
  (c) Name the child in the smallest family.
  (d) Name the child in the largest family.
  (e) Name the child whose family has two children.
  (f) Name the children whose family has three children.
  (g) Sheila's parents want a family of six. How many more children do they want?
  (h) If Joe has both parents at home, how many people are there in his family in total?

7 The diagram shows a map drawn on a coordinate graph.
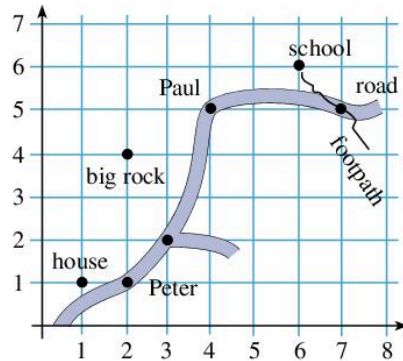
Peter and Paul leave home and walk along a road to school.
Write down the coordinates of:
  (a) the house
  (b) big rock
  (c) Peter's position
  (d) Paul's position
  (e) the school
  (f) the point at which the road branches to the right
  (g) the point where the footpath crosses the road.

8 Plot the following points on squared paper and join them in the order given.
  (4, 0) (4, 2) (5, 2) (5, 4) (2, 4)
  (5, 7) (3, 7) (5, 9) (4, 9) (6, 11)
  (8, 9) (7, 9) (9, 7) (7, 7) (10, 4)
  (7, 4) (7, 2) (8, 2) (8, 0) (4, 0)

9 The diagram shows an old map that Rory found in the attic.
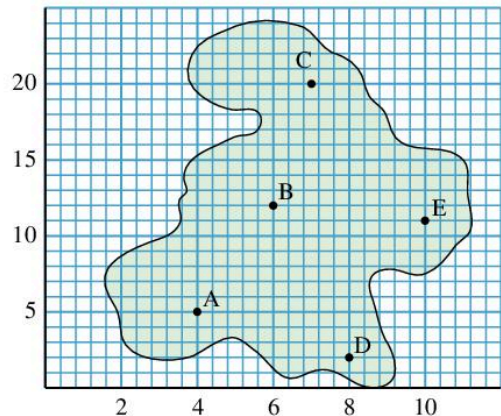
The points A, B, C, D and E show where the treasure might be.
Write down the coordinates of these points.

10 **(a)** The rule for an ordered pair is *multiply the first number by 3, then add 2.*
Copy and complete the following set of ordered pairs:
(1,), (3,), (4,), (, 23), (, 32)

(b) The rule for an ordered pair is *subtract from 11.*
Copy and complete these ordered pairs:
(2,), (3,), (, 6), (, 4), (, 0)

(c) P = {5, 4, 3, 2, 1}
Write down the set of ordered pairs for each element of P, if the rule is *subtract 1, then multiply by 2.*

11 **(a)** What are the rules for these sets of ordered pairs?
(i) (0, 0), (3, 1), (6, 2), (9, 3), (12, 4)
(ii) (0, 1), (1, 2), (2, 3), (3, 4), (4, 5)
(iii) (1, 4), (2, 7), (3, 10), (4, 13)

(b) Plot each set of ordered pairs on a coordinate graph.

(c) What is the equation that describes each graph?

12
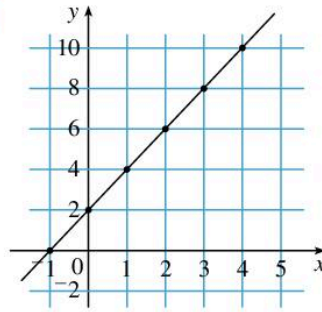
Look at the graph shown.
(a) Where does the line intercept the y-axis?
(b) What is the gradient of the line?

13 **(a)** Draw graphs of these linear relationships.
(i) $y = 3x + 2$ for $0 \leq x \leq 5$
(ii) $y = 3x - 1$ for $0 \leq x \leq 5$
(iii) $y = 2x + 3$ for $0 \leq x \leq 5$
(iv) $y = x + 4$ for $0 \leq x \leq 5$

(b) In each case, find the intercept with the y-axis.
(c) What is the gradient of each line?

14 Show the region defined by the inequalities.
$^-1 \leq x \leq 3$
$y \leq 4$
$y \leq 4x - 2$
$y \geq x$

7 Consolidation

Example 1
A = {2, 3, 4}, B = {2, 3, 4, 6, 8}
(a) Draw an arrow diagram from A to B to show *is a factor of*.
(b) What rule is shown by the arrow diagram?

(a)

is a factor of

(b)
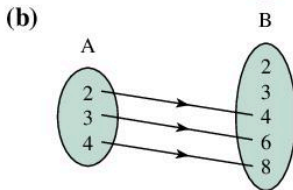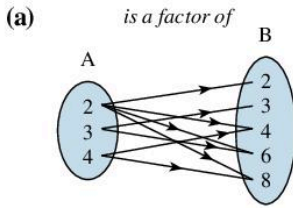
The rule is *is a half of*.

Example 2
Given A = {1, 2, 3, 4}, B = {0, 1, 2, 3, 4, 5, 6}, draw arrow diagrams to show the rules:
(a) *add 1*
(b) *multiply by 2 and subtract 2*

(a)

add 1

(b)

×2 and −2
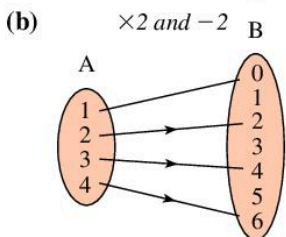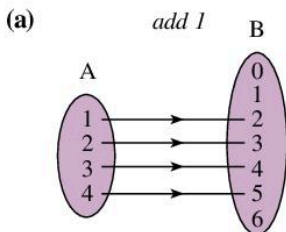
Example 3
The graph shows Ken's weight in kilograms at different birthdays.
(a) What was Ken's weight when he was 7?
(b) When did Ken weigh 30 kg?

(a) At 7 Ken weighed 25 kg.
(b) He weighed 30 kg when he was 9.
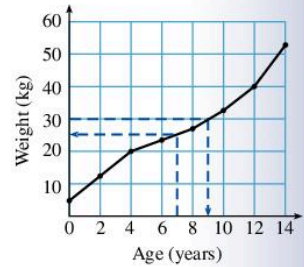
Example 4
Given X = {1, 2, 3, 4}, Y = {whole numbers}:
(a) Write down ordered pairs for the map $x \rightarrow 2x + 3$.
(b) Draw a graph of the mapping $x \rightarrow 2x + 3$.
(c) What is the gradient of the equation $y = 2x + 3$?
(d) Where does the line intercept the y-axis?

(a) The rule is *multiply by 2 then add 3* so
$1 \rightarrow 5, 2 \rightarrow 7, 3 \rightarrow 9, 4 \rightarrow 11$
The ordered pairs are:
(1, 5), (2, 7), (3, 9), (4, 11)
(b) The ordered pairs can be plotted on a graph.

The map $x \rightarrow 2x + 3$ can be written as the equation $y = 2x + 3$
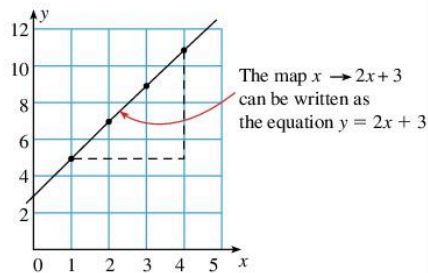
(c) Gradient $= \dfrac{\text{vertical height}}{\text{horizontal shift}} = \dfrac{11 - 5}{4 - 1} = \dfrac{6}{3} = 2$

(d) The line intercepts the y-axis at $y = 3$.

Exercise 7

1 A = {1, 2, 3, 4}, B = {2, 3, 4, 5, 6, 7, 8}
Draw arrow diagrams for A and B to show:
(a) *is a half of* (b) *is equal to*
(c) *is a factor of* (d) *subtracted from 6 gives*
(e) *is greater than* (f) *is less than*

2 Given P = {1, 2, 3, 4, 5},
Q = {0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10}, draw arrow diagrams to show the rules:
(a) *add 3*   (b) *subtract 1*
(c) *multiply by 2* (d) *multiply by 2 and subtract 1*

3 If X = {1, 2, 3, 4} and
Y = {whole number from 0 to 20}, draw arrow diagrams to show:

(a) $x \rightarrow x + 2$ (b) $x \rightarrow 2x$ (c) $x \rightarrow 2x - 1$

(d) $x \rightarrow 3x + 1$ (e) $x \rightarrow 5x$ (f) $x \rightarrow 4x - 3$

(g) $x \rightarrow 4x + 3$ (h) $x \rightarrow 3x - 2$

4 Given X = {1, 2, 3, 4, 5}, Y = {whole numbers}, write down ordered pairs and then draw the graph of these mappings:

(a) $x \rightarrow x + 2$ (b) $x \rightarrow 2x$ (c) $x \rightarrow 2x - 1$
(d) $x \rightarrow 3x + 1$ (e) $x \rightarrow 5x - 4$ (f) $x \rightarrow 3x + 4$
(g) $x \rightarrow 6 - x$ (h) $x \rightarrow 11 - 2x$

5 Given X = {1, 2, 3, 4, 5}, Y = {whole numbers}:
(a) Draw a graph of the mapping $x \rightarrow 3x - 2$.
(b) Join the points.
(c) Use your graph to find the value of *y* when $x = 2\frac{1}{2}$.

(d) What is the gradient of the line $y = 3x - 2$?
(e) What is the *y*-intercept of the line $y = 3x - 2$?

Application

6 The cost of different-sized tins of milk powder is shown in the graph.
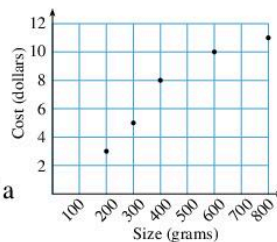
(a) What is the cost of a 300 g tin of milk?
(b) What sized tin can I buy with $3?
(c) Is it better to buy four 200 g tins or one 800 g tin? Explain.

7 The graph shows the height of a seedling in centimetres over a period of 24 days.
(a) What is the height of the seedling 12 days after planting?
(b) When was the seedling 12 cm tall?
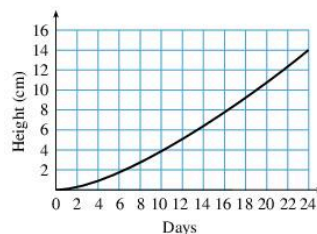
Summary

You should now know ...

1 An arrow diagram links some or all of the elements in one set to the elements of a second set by means of a stated rule.

For example:
If X = {1, 2, 3, 4, 5}, Y = {2, 3, 4, 5, 6} the rule *add 3* can be shown as
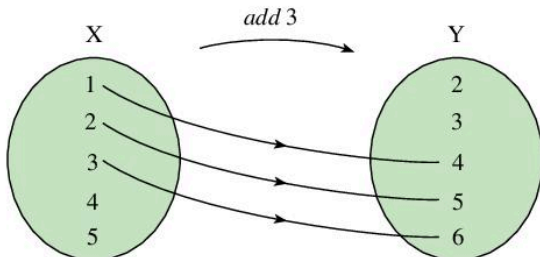
Check out

1 X = {1, 2, 3, 4}
Y = {2, 3, 4, 5, 6, 8}
Draw an arrow diagram from X to Y to show the rule *multiply by 2*.

2 Arrow diagrams can show maps that are:

double

one-to-one

is the brother of

one-to-many

is the son of

many-to-one

is a factor of

many-to-many
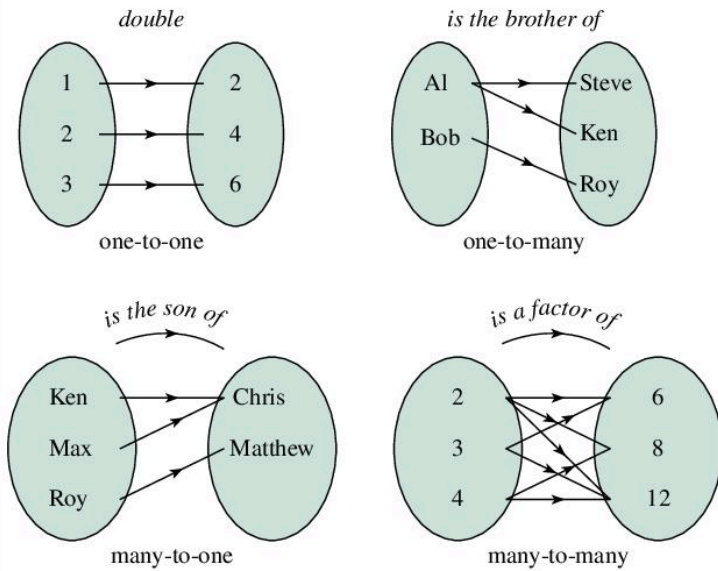
Mappings that are one-to-one or many-to-one are called functions.

3 Mappings can be represented as ordered pairs.

For example: In the arrow diagram
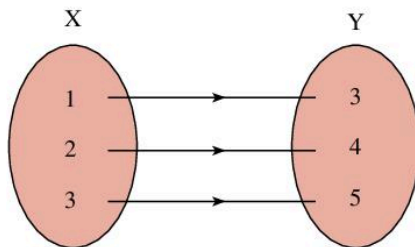
the pairs are (1, 3), (2, 4), (3, 5). These pairs can be shown on a graph.
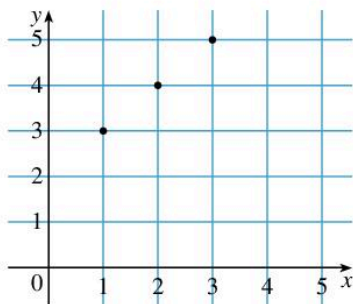
2 Draw an arrow diagram between the two sets using the relationship given.

(a) $\{8, 10, 12\}$, $\{2, 5, 6\}$
rule *is a multiple of*

(b) $\{2, 3, 4, 5\}$, $\{4, 6, 8, 10\}$
rule *is a half of*

(c) $\{$Brian, Bess, Kim, Kay$\}$, $\{$B, K$\}$

rule *has as its first letter*

In each case, state whether the mapping shows a function.

3 For each arrow diagram, write down the ordered pairs and plot them on a graph.

(a)

(b)
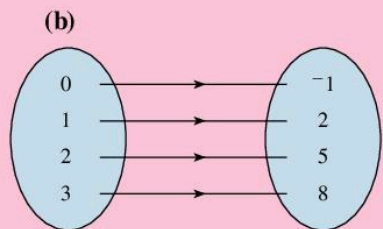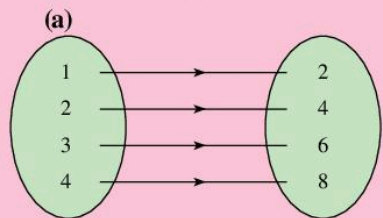
4 Information can be read from graphs.

For example:

The height of plant after 9 days is 4.5 cm

4

The graph shows the amount of peas that Mr Kelly reaped in one week.

(a) How many kilograms of peas were reaped on Tuesday?

(b) On which day were most peas reaped?

(c) How many kilograms of peas were reaped in the entire week?

5 How to draw the graph of an equation using a table of values.

For example:

$y = 3x + 1$

x	⁻1	0	1	2	3
y	⁻2	1	4	7	10

$y = 3x + 1$

5 Draw graphs of:

(a) $y = 2x + 1$

(b) $y = 3x - 2$

by copying and completing the table of values in each case.

x	⁻1	0	1	2	3	4
y						

6 How to find gradient and y-intercept of a straight line graph.

For example:

$y = 4x - 3$

The gradient is 4 and the y-intercept is $y = -3$.

6 **(a)** Find the gradient and intercept of the graphs of these equations:
- **(i)** $y = 2x + 5$
- **(ii)** $y = \dfrac{1}{2}x - 3$
- **(iii)** $y = 3x - 4$
- **(iv)** $y = 4x + 1$

(b) What is the equation of a line with:
- **(i)** gradient 2, intercept $y = -1$
- **(ii)** gradient 3, intercept $y = 2$

7 How to show an inequality on a graph.

For example:

7 Draw graphs to show these regions.
- **(a)** $0 \leq y \leq 6$
- **(b)** $x \geq 2$
- **(c)** $y \geq 3x - 1$

Objectives

✔ find the images of shapes after reflection
✔ find the images of shapes after translation
✔ describe translations as vectors
✔ rotate objects through 90°, 180° and 270°
✔ use rotation to identify rotational symmetry

What happened to that picture?

We've enlarged and rotated it

What's the point?

Symmetry occurs everywhere in nature from the radial symmetry of a starfish to the reflectional symmetry of a crab's shell. Studies even suggest that people are more attracted to symmetrical faces.

Before you start

You should know ...

1 A turn of 90° is a quarter turn:
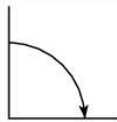
A turn of 180° is a half turn:

A turn of 360° is a full turn:
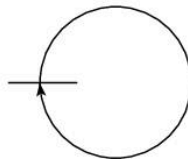
Check in

1 How many 90° turns are in
   (a) a quarter turn
   (b) a half turn
   (c) a full turn?

2 How to read coordinates.
For example:
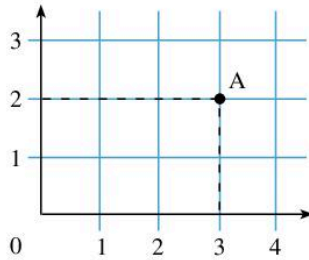
A has coordinates (3, 2).

2 Write down the coordinates of these points:
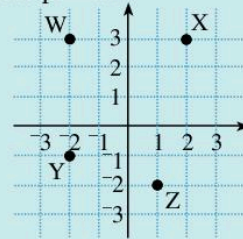
8.1 Reflection

When you look in a mirror you see your reflection.

Shapes can also be reflected in a mirror.
The shape is the **object** and its reflection is the **image**.

Example 1

Reflect the shape in the line X.
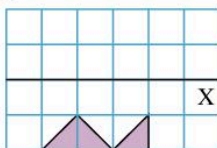
object

On reflection:
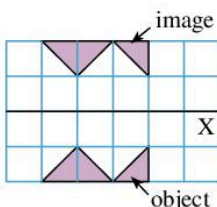
image

object

Notice that the image is the same distance from the mirror line as the object.

Exercise 8A

1 Copy these diagrams and reflect each shape in the mirror line.

(a) (b)

(c) (d)

(e) (f)
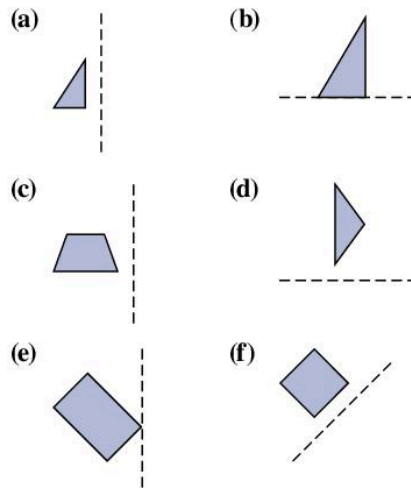
2 Copy these diagrams and reflect each shape in the mirror line, X.

(a) (b)

(c)
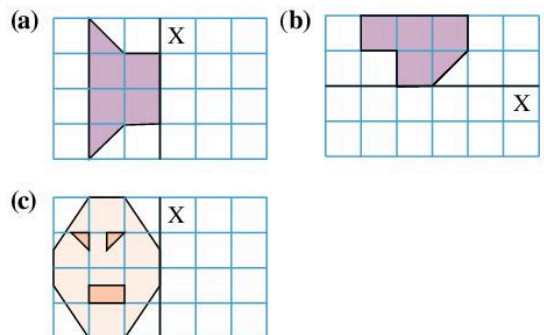
3 Copy these diagrams. Reflect each shape first in the line X and then in the line Y.

(a)

(b)

(c)
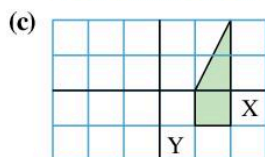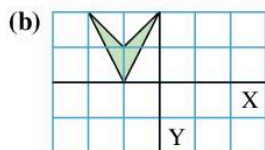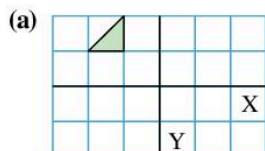
4 **(a)** Repeat Question 3 but this time reflect the shape first in the line Y and then in the line X.

(b) Does it matter which is done first?

You can use coordinates to show what happens when a shape is reflected.

Reflection on a grid

You will need squared paper and a coloured pencil.

In the diagram, the triangle on the left is reflected in the heavy line.
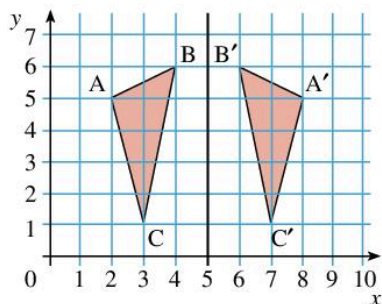
A′B′C′ is the mirror image of ABC.
We say that ABC **maps on to** A′B′C′, when it is reflected in the heavy line.

Vertex A maps to A′.
A has coordinates (2, 5).
A′ has coordinates (8, 5).
So we can say that (2, 5) maps to (8, 5).

The *x*-axis is sometimes used as a mirror line.
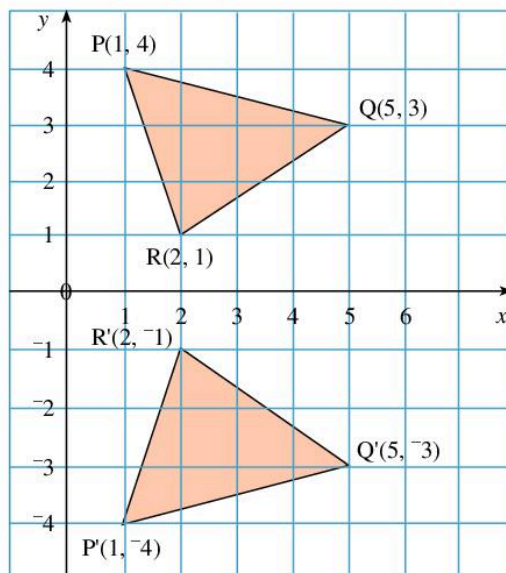
Triangle P′Q′R′ is the reflection of triangle PQR in the *x*-axis.
P(1, 4) is mapped on to P′(1, ⁻4)
Q(5, 3) is mapped on to Q′(5, ⁻3)
R(2, 1) is mapped on to R′(2, ⁻1)

Exercise 8B

1 In the diagram ABD is half of the shape ABCD. BD is a line of symmetry.
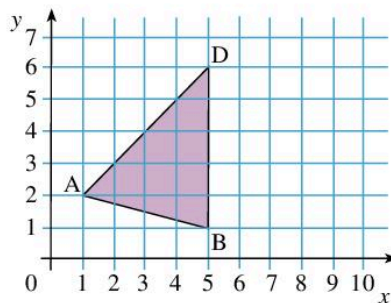
(a) Draw a copy of the diagram, and complete shape ABCD.

(b) What are the coordinates of point C?

2 Triangle PQR is reflected in the y-axis to give triangle $P_1Q_1R_1$ as its image.
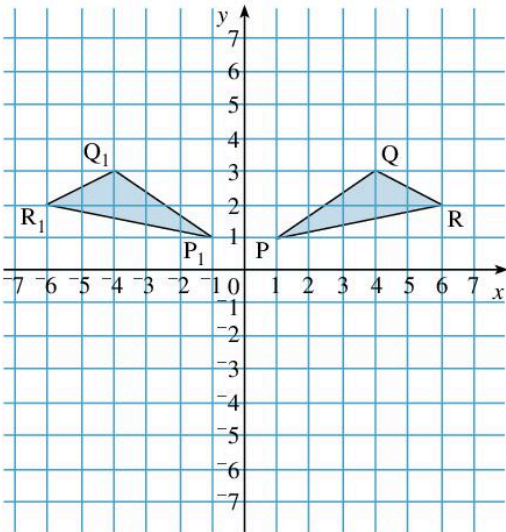
(a) Write down the coordinates of the vertices of the image.
(b) On squared paper, make a copy of the diagram.
(c) Using a coloured pencil, draw the mirror image of PQR when in is reflected in the x-axis. Call it $P_2Q_2R_2$.
(d) Write down the coordinates of the vertices P_2, Q_2 and R_2.
(e) On your copy, draw the mirror image of $P_1Q_1R_1$ when it is reflected in the x-axis. Call the image $P_3Q_3R_3$.
What are the coordinates of vertices P_3, Q_3 and R_3?
(f) Is $P_3Q_3R_3$ a mirror image of PQR? Why?
(g) Name the two triangles that have $P_3Q_3R_3$ as their mirror image.

3 In the diagram, the dotted lines are lines of symmetry. Copy and complete the shape, on centimetre squared paper.
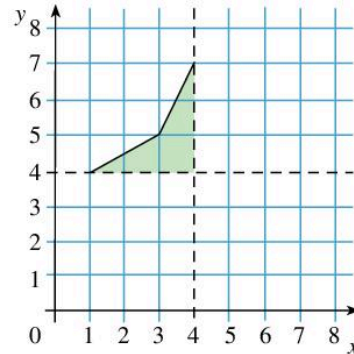
(a) What name would you give the completed shape?
(b) Write down the coordinates of all its vertices.

4 On squared paper, plot the points $(^-2, 4)$ and $(^-2, 0)$. Now, using a coloured pencil:

(a) Join $(^-2, 0)$ to $(0, 0)$ and join $(^-2, 4)$ to $(^-2, 0)$.
(b) Reflect the shape in the x-axis.
(c) Reflect this image in the y-axis.
(d) Reflect the original shape in the y-axis.
(e) What letter have you made?
(f) How many lines of symmetry does it have?

5 Copy this diagram. The kite on the left is a mirror image of the kite on the right.
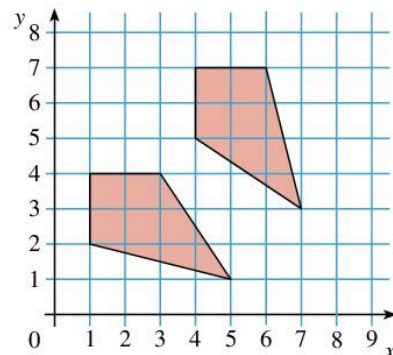
(a) On your diagram draw in the line of reflection.
(b) Write down the coordinates of three points on that line.

6 Repeat Question 5 for the shapes below.
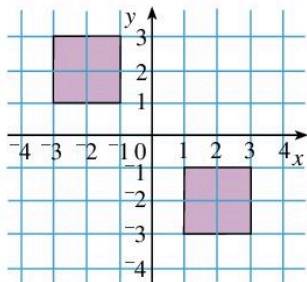
7 **(a)** On centimetre squared paper, plot the points A(4, 10), B(6, 7), and C(10, 7).
   (b) To complete the parallelogram ABCD, where would you put the fourth vertex?
   (c) Now reflect the parallelogram in the line which joins the points (1, 10) and (9, 2).
   (d) Write down the coordinates of the vertices of the mirror image.

8 Plot each set of points and connect them to form a polygon. Then draw the image after reflection in the given line.
   (a) A($^-$3, $^-$3), B($^-$1, $^-$5), C(1, 1) reflected in the y-axis.
   (b) P(0, 2), Q(3, 1), S(2, 9), D(0, 4) reflected in the x-axis.
   (c) M(3, 3), N(5, 2), O(5, 6) reflected in $y = x$.
   (d) P(3, 2), Q(6, 1), R(7, 2), D(2, 2) reflected in $y = -x$.

9 Plot the points and connect them to form a polygon. Draw their image after reflection in the given line.
   (a) A(3, $^-$2), B($^-$1, 1), C($^-$1, 3), D(2, 3) reflected in $y = 2$.
   (b) P($^-$3, 2), Q($^-$1, 0), R(1, 0), S(3, 2) reflected in $x = 3$.
   (c) Q(1, 5), R(3, 3), S(4, 5), T(3, 4) reflected in $x = 6$.

10 **(a)** The point (a, b) is reflected in the x-axis. What are the coordinates of its image?
   (b) The point (p, q) is reflected in the y-axis. What are the coordinates of its image?

8.2 Translating shapes

Another way of transforming a shape is by **translation**. A translation is simply a sliding movement in a particular direction.
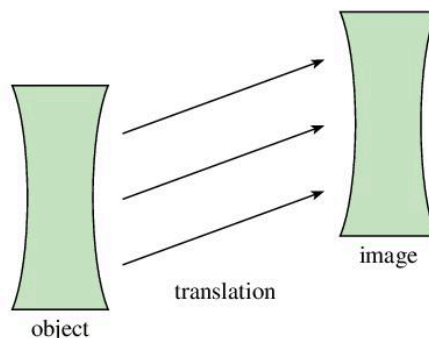
A translation can be described as a movement to the left or right followed by a movement up or down.

Example 2

Describe the translation that takes
(a) shape X → shape Y
(b) shape Y → shape Z
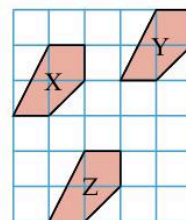
(a) X to Y: 3 right, 1 up
(b) Y to Z: 2 left, 4 down

Notice that a translation does not change the object's shape or size. The object and image are **congruent** to each other.

Exercise 8C

1 Describe these translations.
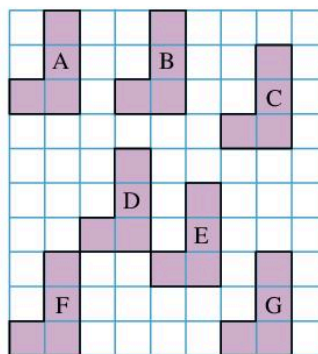
(a) A → B   **(b)** B → C
(c) A → C   **(d)** C → A
(e) D → E   **(f)** E → C
(g) F → E   **(h)** F → G
(i) G → A   **(j)** G → C

2
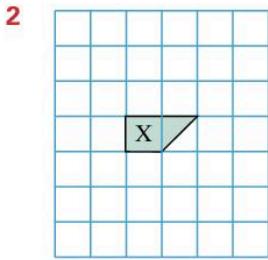
On squared paper, draw the image of the shape X after a translation:

(a) 2 right, 1 up

(b) 1 left, 1 up

(c) 3 right, 1 down

(d) 2 left, 3 down

(e) 3 left, 2 down.

Translations as vectors

Instead of writing a translation as, for example, "2 right, 1 up" you can write it as a column vector, $\begin{pmatrix} 2 \\ 1 \end{pmatrix}$.

In the diagram below,
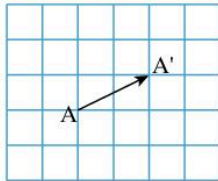
the vector $\vec{AA'} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$. It represents the translation or movement of the point A to A'.

Example 3

Write a column vector to describe the translations shown.

(a)

(b)

(c)

(a) $\vec{AA'}$ shows the point A translated 2 units to the right and 3 units upwards.

so $\vec{AA'} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$

(b) $\vec{BB'}$ shows the point B translated 3 units to the left (-3) and 2 units upwards

so $\vec{BB'} = \begin{pmatrix} -3 \\ 2 \end{pmatrix}$

(c) $\vec{CC'}$ shows the point C translated 3 units to the left (-2) and 4 units downwards (-4)

so $\vec{CC'} = \begin{pmatrix} -2 \\ -4 \end{pmatrix}$

In the same way you can draw the translation when given a column vector.

Example 4

Draw the column vectors.

(a) $\begin{pmatrix} 3 \\ -1 \end{pmatrix}$ **(b)** $\begin{pmatrix} -4 \\ -2 \end{pmatrix}$

(a) $\begin{pmatrix} 3 \\ -1 \end{pmatrix}$ represents a translation 3 units to the right and 1 unit down.
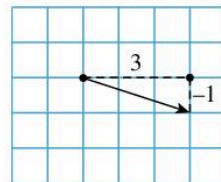
(b) $\begin{pmatrix} -4 \\ -2 \end{pmatrix}$ represents a translation 4 units to the left and 2 units down.
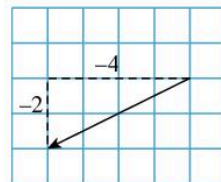
Exercise 8D

1   Write a column vector to describe each translation.

(a)

(b)

(c)

(d)

(e)

(f)

(g)

(h)

(i)

(j)
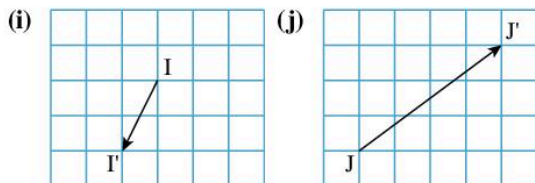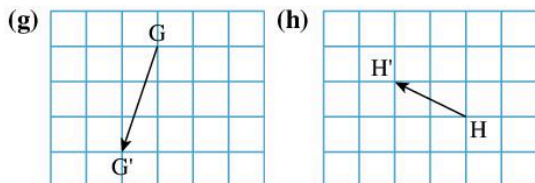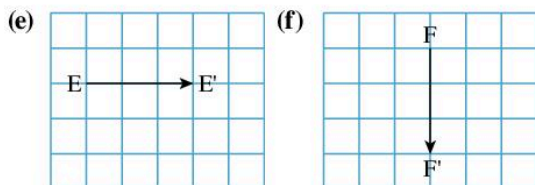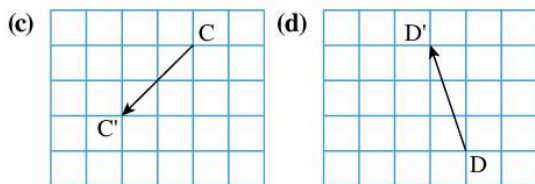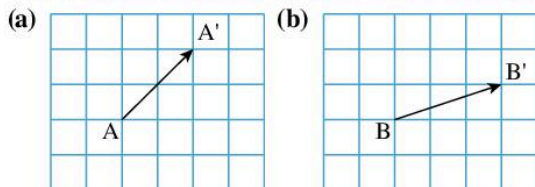
2   Draw the column vectors.

(a) $\begin{pmatrix} 2 \\ 2 \end{pmatrix}$ **(b)** $\begin{pmatrix} 3 \\ 1 \end{pmatrix}$ **(c)** $\begin{pmatrix} 3 \\ -1 \end{pmatrix}$

(d) $\begin{pmatrix} -3 \\ 1 \end{pmatrix}$ **(e)** $\begin{pmatrix} -3 \\ -1 \end{pmatrix}$ **(f)** $\begin{pmatrix} 2 \\ 0 \end{pmatrix}$

(g) $\begin{pmatrix} 0 \\ 2 \end{pmatrix}$ **(h)** $\begin{pmatrix} -2 \\ 0 \end{pmatrix}$ **(i)** $\begin{pmatrix} 0 \\ -2 \end{pmatrix}$

3   Describe, using column vectors, these translations:
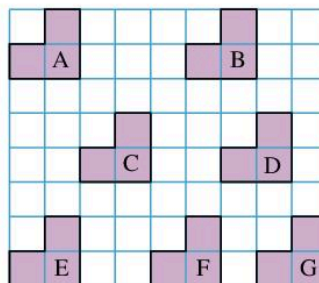
(a) A → B   **(b)** B → C
(c) A → C   **(d)** C → D
(e) D → A   **(f)** A → D
(g) C → E   **(h)** F → G
(i) G → B   **(j)** E → B

4   **(a)** Copy the diagram of the shape X.
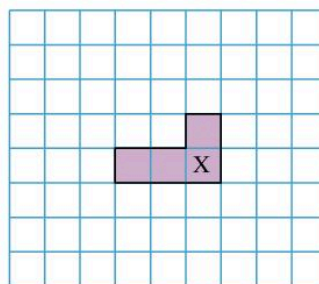
(b) On your diagram show its image after a translation represented by the vector:

(i) $\begin{pmatrix} 4 \\ 1 \end{pmatrix}$ **(ii)** $\begin{pmatrix} -2 \\ -3 \end{pmatrix}$ **(iii)** $\begin{pmatrix} 2 \\ 0 \end{pmatrix}$

You can use coordinates to show the image of a shape when it is translated.

Example 5

The triangle A(1, 0), B(2, 3), C(0, 2) is translated using the vector $\begin{pmatrix} 2 \\ -2 \end{pmatrix}$.

What are the coordinates of its image?

First plot the triangle *ABC* on the coordinates axes.
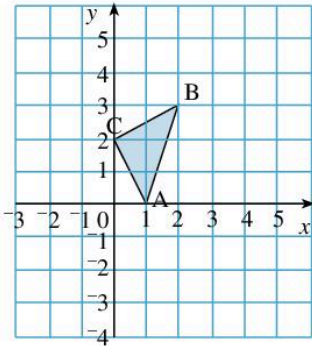
Each vertex of the triangle is translated through $\begin{pmatrix} 2 \\ -2 \end{pmatrix}$, that is, 2 units to the right and 2 units downwards.
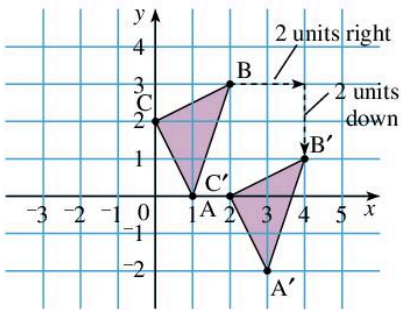
From the graph you can see that the coordinates of the image of the translation are A′(3, ¯2), B′(4, 1), C′(2, 0).

Exercise 8E

1 Write a column vector to describe the translation of shape *A* to its image shape *B*.

(a)

(b)

(c)

(d)
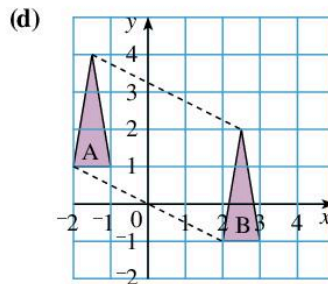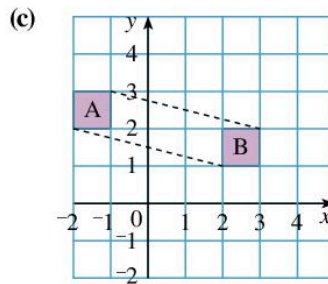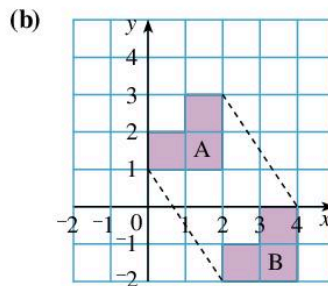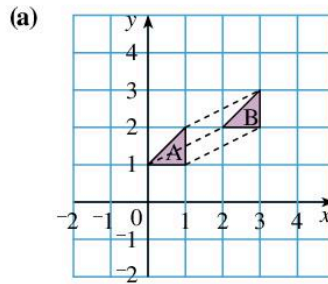
2 By drawing suitable axes and plotting these triangles
  (a) A(2, 0), B(0, 1), C(2, 2)
  (b) D(1, 3), E(1, 2), F(2, 0)
  (c) G($^-$1, 1), H(2, $^-$1), I(0, 0)
  (d) J(1, 3), K($^-$1, 2), L($^-$3, $^-$1)
  (e) M(1, $^-$3), N(2, $^-$1), O(1, 2)

find the coordinates of the images of the triangles under the translation $\begin{pmatrix} 1 \\ 2 \end{pmatrix}$.

3 Describe these translations as vectors.
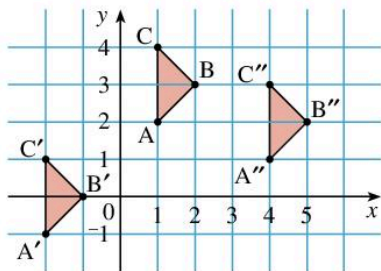
  (a) ABC → A$'$B$'$C$'$
  (b) A$'$B$'$C$'$ → A$''$B$''$C$''$
  (c) ABC → A$''$B$''$C$''$
  (d) A$'$B$'$C$'$ → ABC
  (e) A$''$B$''$C$''$ → A$'$B$'$C$'$
  (f) A$''$B$''$C$''$ → ABC

4 Describe the translations as vectors that map these triangles to their images.
  (a) A(2, 0), B(2, 2), C(0, 0)
       → A$'$(3, 1), B$'$(3, 3), C$'$(1, 1)
  (b) A($^-$1, 2), B(1, $^-$1), C($^-$2, $^-$2)
       → A$'$(1, 3), B$'$(3, 0), C$'$(0, $^-$1)
  (c) A(3, $^-$1), B($^-$2, 2), C($^-$1, 4)
       → A$'$(1, $^-$2), B$'$($^-$4, 1), C$'$($^-$3, 3)

Technology

Visit

www.geogebra.org

and download the GeoGebra Geometry app.

Use it to create lines and other shapes.

Show translations of these shapes using the transform icon.

Watch some videos on GeoGebra if you have difficulty!

8.3 Rotation

You will need some fine cardboard, a drawing pin, tracing paper, a pair of scissors and some squared paper.

Exercise 8F

1 **(a)** Copy this rectangle on to a piece of card.

Cut it out and put a black dot at the bottom left-hand corner.

Turn rectangle just a little.

drawing pin
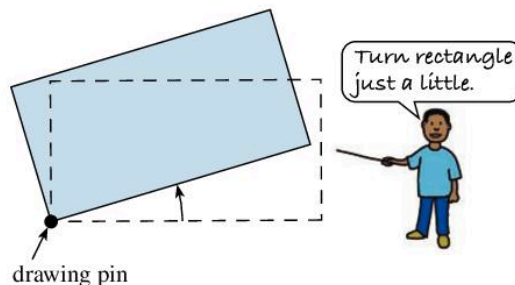
  (b) Place your rectangle on a page in your exercise book and draw round it. Stick a pin through the black dot. Rotate your rectangle just a little. Draw round it.
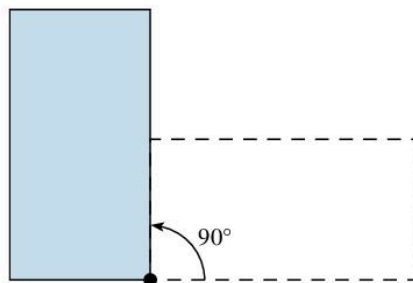
  (c) Now turn your rectangle through a $\frac{1}{4}$-turn (90°) about your black dot. Draw its new outline.

2 Turn the rectangle from Question 1 through:
  (a) $\frac{1}{2}$-turn (180°)
  (b) $\frac{3}{4}$-turn (270°)

Draw around the outline in each case.

3 Repeat Questions 1 and 2 for these shapes:

(a)

(b)

(c)
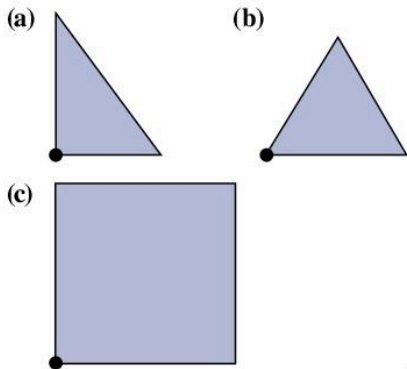
4 Draw the shape and its image after a $\frac{1}{2}$-turn rotation about the black dot.

(a)

(b)

(c)

(d)

Technology

Learn more about rotation by visiting

www.mathsisfun.com/geometry/rotation.html

Check carefully the interactive material on this site. You will be able to make rotations through any angle of your choice.

Make sure you attempt the questions!

Rotational symmetry

A shape which fits into the same position more than once when rotated through 360° has rotational symmetry.

Rectangle → Rotate → Rectangle after $\frac{1}{4}$-turn → Rotate → Rectangle after $\frac{1}{2}$-turn

The rectangle above fits exactly on to the rectangle rotated through a $\frac{1}{2}$-turn.

So the rectangle has rotational symmetry.

Example 6

Which of these letters has rotational symmetry?

(a) A

(b) H

(a) Trace the letter A on tracing paper.
Rotate it; these are some of its positions:

A → Start, $\frac{1}{4}$-turn, $\frac{1}{2}$-turn, $\frac{3}{4}$-turn, Full turn
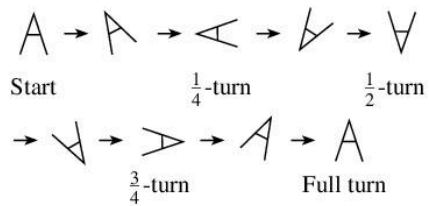
The letter A can fit exactly on to its start position only after a full turn. It has no rotational symmetry.

(b) H → Start, $\frac{1}{4}$-turn, $\frac{1}{2}$-turn, $\frac{3}{4}$-turn, Full turn
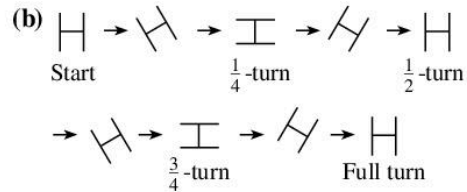
The letter H can fit exactly on to itself after a $\frac{1}{2}$-turn and after a full turn. So it has rotational symmetry.

The number of times a shape fits on to itself in a complete turn is the **order** of the rotational symmetry.

The letter H has rotational symmetry of order 2, so has the letter I.

$\frac{1}{2}$-turn $\frac{1}{2}$-turn

Full turn
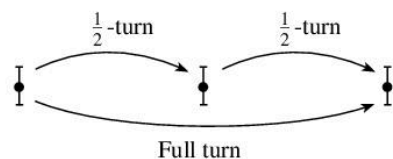
Exercise 8G

1 Trace these shapes and find their order of rotational symmetry.

(a)

(b)

(c)

(d)

(e)

(f)

(g)

(h)
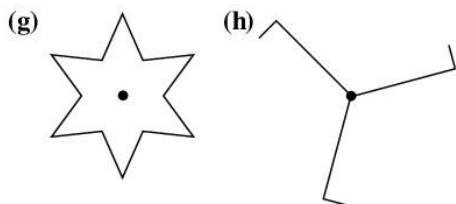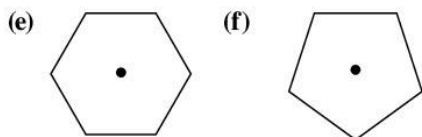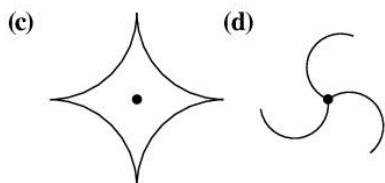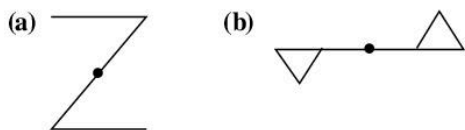
2 Find the order of rotational symmetry of each shape.

(a)

(b)

(c)

(d)

(e)
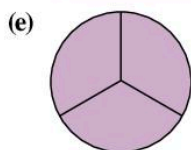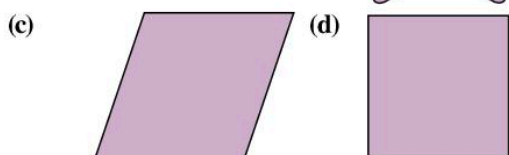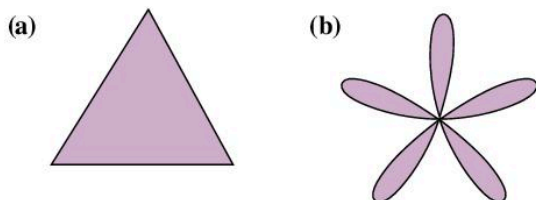
3 Which of these letters possess rotational symmetry?

A B C D E
F G H I J K
L M N O P
Q R S T U
V W X Y Z
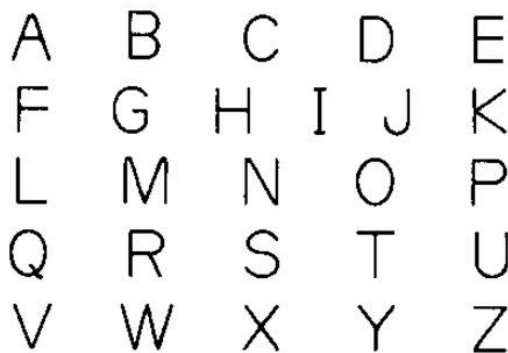
4 State the order of rotational symmetry of these shapes.

(a)

(b)

(c)

(d)

(e)

(f)

(g)

(h)
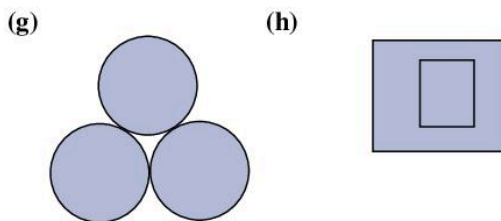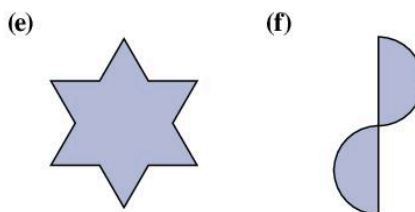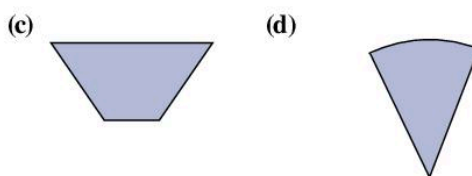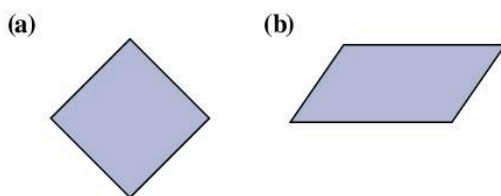
5 Which of these shapes have
  (a) rotational symmetry only
  (b) line symmetry only
  (c) rotational and line symmetry?

(i)
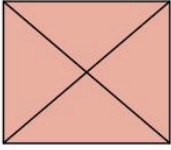
(ii)
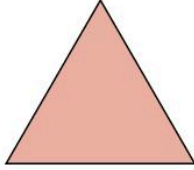
(iii)
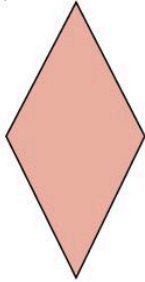
(iv)

(v)

(vi)

(vii)
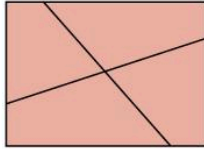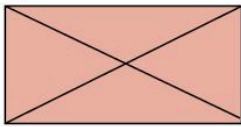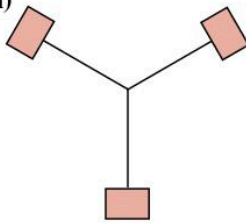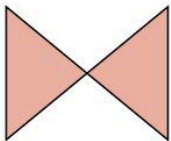
(viii)
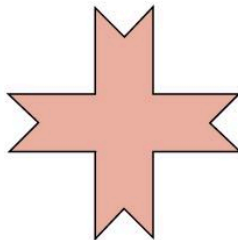
(ix)
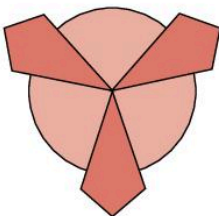
(x)
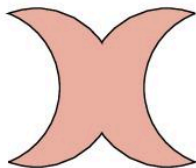
Investigation

(a) Can you draw a triangle with an order of rotational symmetry of
  (i) 2   **(ii)** 3?

(b) What quadrilaterals can you draw with rotational symmetry of order
  (i) 2   **(ii)** 3   **(iii)** 4?

(c) How about five-sided shapes?

Technology

- Conduct an internet search for pictures and photographs that illustrate line symmetry and rotational symmetry.
- Make a booklet on symmetry by pasting the photographs into a word processing program and giving brief descriptions of each picture.
- Visit Adrian Bruce's website on symmetry at

  www.adrianbruce.com

  for further ideas.

Exercise 8H – mixed questions

1
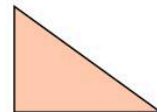
This triangle was made by folding a piece of paper in two.
  (a) What was the original shape of the piece of paper?
  (b) Could the piece of paper have been any other shape?

2 What original shapes of paper, when folded once, could make these shapes?

(a)

(b)

(c)

(d)
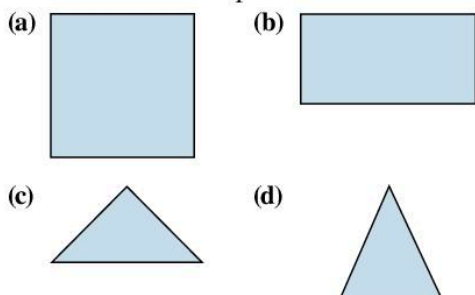
3 Take a rectangular piece of paper. Fold it in half. Fold it in half again.
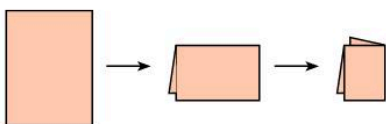
Draw a shape on the folded paper.

Shape drawn
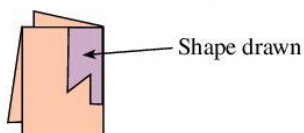
(a) Can you guess what shape you would get if you cut it out from the folded paper?

(b) Cut the shape out and check if your guess was correct.

(c) Repeat for other shapes.

4 Fold a rectangular piece of paper in two:

Cut out triangle shown
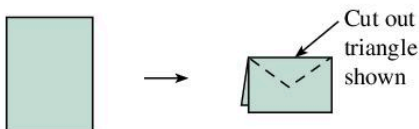
Cut a triangle from the folded edge. What shape do you get?

5 Repeat Question 4 for different triangles. What shapes can you make? Can you make a square? Can you make a rhombus?

6 (a) Fold a rectangular piece of paper twice.
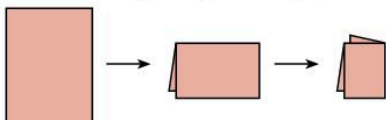
(b) Make a single straight cut with your scissors across the folded corner.

Cut here

Open out
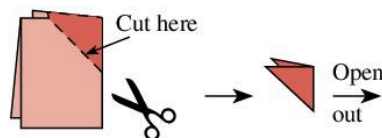
(c) Open up the folded triangle. What shape do you get?

(d) Experiment with different cuts. Can you make a square?

7 (a) Take a long strip of paper. Start at one end and fold it backwards and forwards like a concertina.

Paper strip

Fold repeatedly
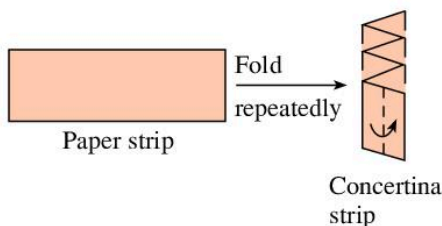
Concertina strip

(b) Fold your concertina strip again, this time down the middle:

Draw half the shape you want
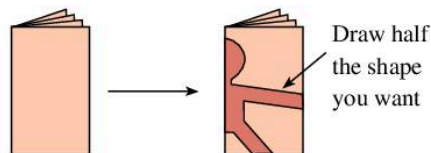
(c) Draw half the shape you want (half a man is shown above). Cut it out and unfold it.

8 (a) Cut a rectangle from inside a larger piece of card.

Card with rectangle removed
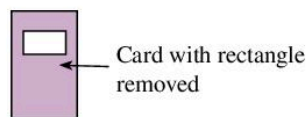
(b) Pin your card with a drawing pin to a piece of paper. Draw round the inside of the rectangle. Now turn the card through a small angle. Draw round the inside of the rectangle again.

(c) Repeat this many times.
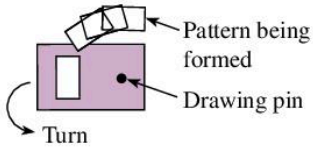
9 Repeat Question 8 but this time cut a different shape from your piece of card.

10 (a) Collect pictures, photographs or drawings of anything that shows symmetry.
(b) With a group of friends display in your classroom your pictures and some of the other work you have done in this section.

11 Draw a shape with
  (a) 4 lines of symmetry
  (b) 5 lines of symmetry
  (c) 6 lines of symmetry.

12 Draw a shape with rotational symmetry
  (a) of order 4
  (b) of order 5
  (c) of order 6.

13 Make the shapes you drew in Questions 11 and 12 by taking a piece of paper, folding it and making **one** cut with your scissors.

8 Consolidation

Example 1

What is the image of triangle A(1, 1), B(3, 2), C(2, 4) when reflected in the line $x = 4$?
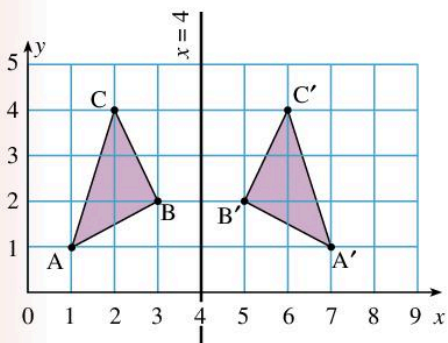
The image of triangle ABC is triangle A′B′C′ where:

A′ is (7, 1)
B′ is (5, 2)
C′ is (6, 4)

Example 2

What is the image of triangle A(1, 2), B(2, 4), C(0, 3) under the translation $\begin{pmatrix} -1 \\ -3 \end{pmatrix}$.
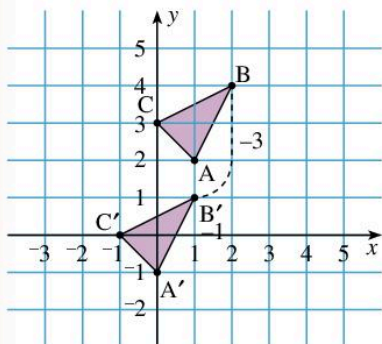
The image is A′(0, ⁻1), B′(1, 1), C′(⁻1, 0).

Example 3

What is the order of rotational symmetry of these shapes?

(a)   (b)
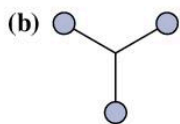
(a) Turn shape through 90° about centre.
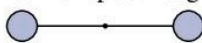
Turn through another 90° about centre and shape is same as original.
Shape has rotational symmetry of order 2.

(b) Rotate shape through 120° about centre and image fits exactly on original shape.
Rotate through another 120° and image fits again.
Shape has rotational symmetry of order 3.

Exercise 8

1 Find the coordinates of the images of the triangles shown after reflection in:
(a) x-axis   (b) y-axis.
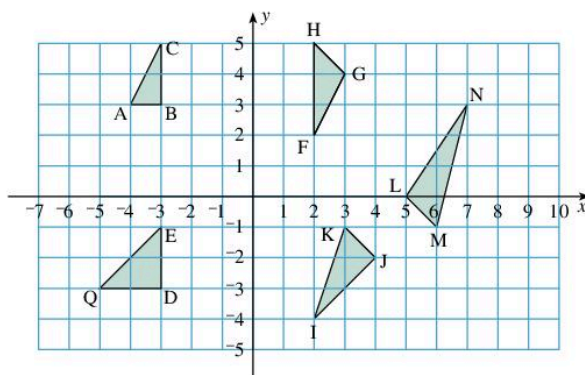
2 Find the coordinates of the image of the quadrilateral A(⁻3, 2), B(⁻5, 3), C(⁻4, 5), D(⁻2, 5) after:
(a) reflection in the x-axis
(b) reflection in the y-axis
(c) reflection in line $y = 1$
(d) reflection in line $x = ⁻2$
(e) reflection in line $x = 1$
(f) reflection in line $y = ⁻2$.

3 In the graph, describe the translations, as vectors, that send:
(a) ABC → A′B′C′
(b) A′B′C′ → ABC
(c) A″B″C″ → A′B′C′
(d) ABC → A″B″C″

(g) **(h)**
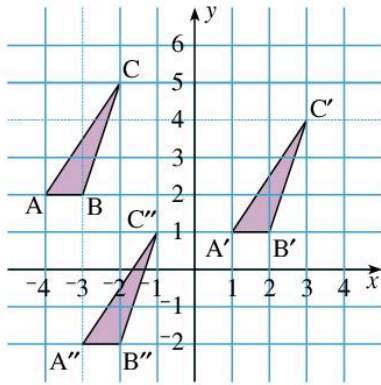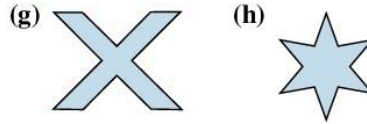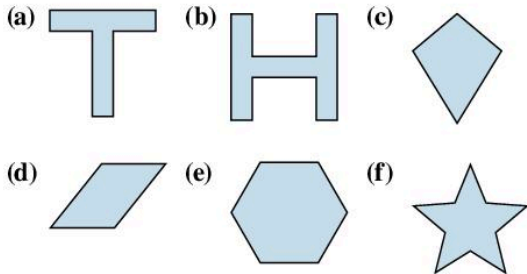
Application

5 Draw four different shapes with:
    (a) order of symmetry 2
    (b) order of symmetry 3
    (c) order of symmetry 4
    (d) order of symmetry 5.

6 **(a)** Identify three items/places or pictures that have reflectional symmetry. Make a sketch of each showing clearly the line(s) of symmetry.
    (b) Identify three items or pictures that have rotational symmetry. Make a sketch of each and state the order of rotational symmetry.

4 What is the order of rotational symmetry of these shapes?

(a) **(b)** **(c)**

(d) **(e)** **(f)**

Summary

You should know ...

1 A shape can be reflected in a line.
For example:

These distances are equal.

object

reflection line
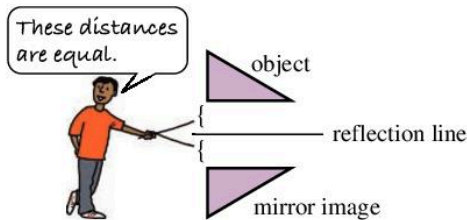
mirror image

Check out

1 Draw the image of the shape after reflection in the mirror line.
(a)

mirror line

(b)

mirror line
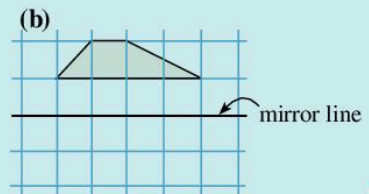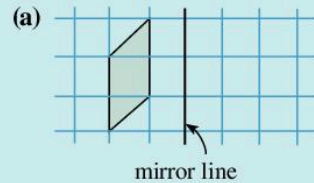
2   You can use coordinates to find image points after reflections. *For example:*
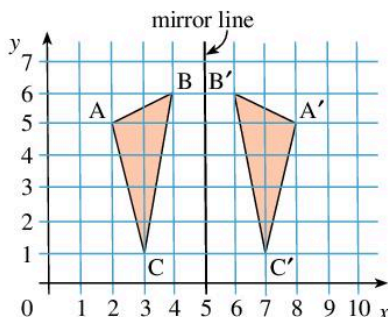
A′B′C′ is the mirror image of ABC.
A′(8, 5) is the mirror image of A(2, 5)

3   You can find the image of a shape after a translation. *For example:*
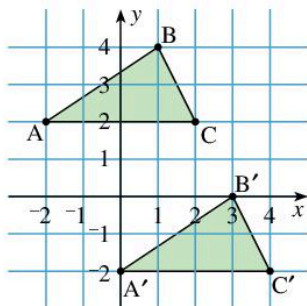
The image of A($^-$2, 2), B(1, 4), C(2, 2) under the translation $\begin{pmatrix} 2 \\ -4 \end{pmatrix}$ is A′(0, $^-$2), B′(3, 0), C′(4, $^-$2).

4   A shape has rotational symmetry, if when turned less than 360°, it fits on to itself. The order of rotational symmetry is the number of times the shape fits on to itself in a complete turn.
For example:

The order of rotational symmetry is 2.

---

2 (a) Plot the triangle A(3, 2), B(4, 5), C(6, 3).
   (b) Draw its image A′B′C′ after reflection in the *x*-axis.
   (c) Write down the coordinates of triangle A′B′C′.

3 (a) Plot the triangle A(3, $^-$1), B($^-$1, 2), C(0, 3).
   (b) Draw the image of this triangle under the translations:
      (i) $\begin{pmatrix} 2 \\ 1 \end{pmatrix}$ (label the image A′B′C′)
      (ii) $\begin{pmatrix} -1 \\ -2 \end{pmatrix}$ (label the image A″B″C″).
   (c) Write down the coordinates of triangles A′B′C′ and A″B″C″.

4 Find the order of rotational symmetry of these shapes.
   (a)   (b)

   (c)   (d)
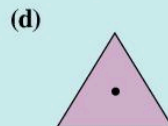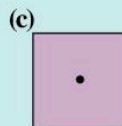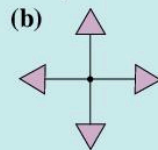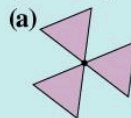
Revision exercise 2

Measurement

1 Dennis is 5 ft 8$\frac{1}{2}$ in tall. His sister Annette is 1.81 m in height.
If 1 inch = 2.54 cm, find who is the taller and by how much.

2 **(a)** Draw accurately five different rectangles each of which has an area of 36 cm².
(b) Write down the perimeter of each rectangle.
(c) If the area of a rectangle is 52 cm² and its length 8 cm, what is its width? What is its perimeter?

3 Find the area of these shapes.

(a)

(b)

(c)

(d)
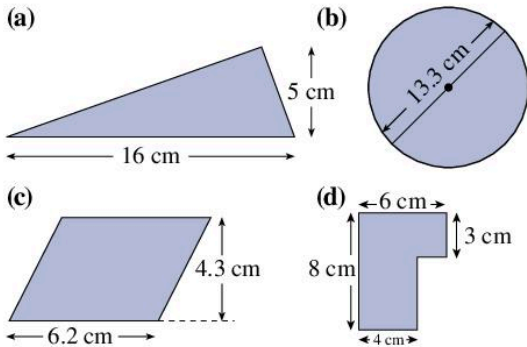
4 By using your calculator (take 3.14 for π), find the circumference of a circle whose radius is:
(a) 12.56 cm   **(b)** 37.68 cm
(c) 50 cm   **(d)** 157 cm

5 By using your calculator (take 3.14 for π), find the radius of a circle whose circumference is:
(a) 10 cm   **(b)** 42 cm
(c) 256 cm   **(d)** 589 cm
Give your answers correct to two decimal places.

6 A bicycle wheel has a radius of 35 cm.
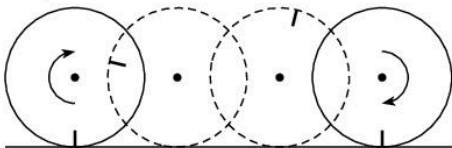
(a) Find how far it will move forward if it turns through 10 complete revolutions.
(b) Find how many revolutions it has made if it moves forward a distance of 5495 cm.

7 Each of the shapes below is made from part of a circle. Use 3.14 as π and find the area and perimeter of the shape.

(a)   **(b)**

(c)   **(d)**
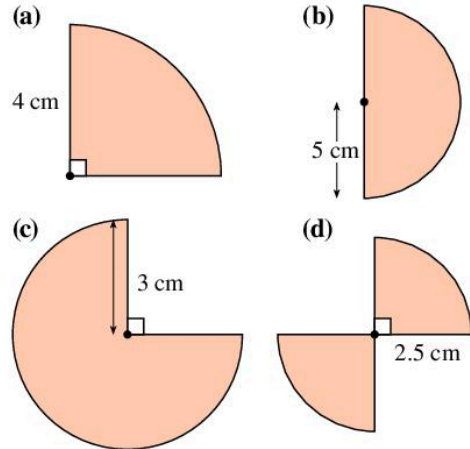
8 It is intended to make a 400 m running track from two 120 m straights and two semicircular ends as shown.
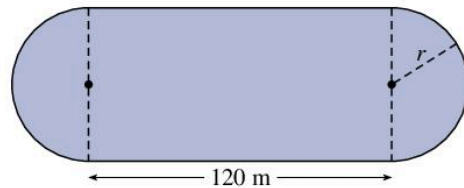
Using $\pi = 3.14$:
(a) What is the length of the curved part of the track?
(b) What is the radius of each of the semicircular ends?

9 **(a)** Make an accurate scale drawing of the running track in Question 8 using a scale in which 1 cm represents 10 m.
(b) On your scale drawing measure the length of the diagonal of the rectangle formed by the two straights and the two diameters.
(c) Use your answer for part (b) to write down the actual length of the diagonal on the running track.

10 A photograph of a painting involves a scale of 1 to 8. Write down:
   (a) the height of the painting if the photograph is 12.5 cm high
   (b) the width of the photograph if the painting is 68 cm wide
   (c) the area of the painting and the area of the photograph with dimension as given in (a) and (b).

Negative numbers

11 Work out:
   (a) $7 + {}^-8$ **(b)** $3 + {}^-9$
   (c) $1 + {}^-3$ **(d)** $5 + {}^-11$
   (e) $2 - 7$ **(f)** $6 - 13$
   (g) ${}^-4 - 12$ **(h)** ${}^-9 - 13$

12 Work out:
   (a) $3 - {}^-4$ **(b)** $7 - {}^-8$
   (c) $6 - {}^-12$ **(d)** $4 - {}^-11$
   (e) $11 - {}^-13$ **(f)** $12 - {}^-2$
   (g) ${}^-9 - {}^-7$ **(h)** ${}^-18 - {}^-14$

13 **(a)** $2 \times {}^-4$ **(b)** ${}^-3 \times 2$
   (c) ${}^-3 \times {}^-2$ **(d)** ${}^-6 \times {}^-4$
   (e) 5×4 **(f)** ${}^-9 \times 3$
   (g) $7 \times {}^-6$ **(h)** ${}^-3.5 \times {}^-2$

14 **(a)** $6 \div {}^-2$ **(b)** ${}^-14 \div 7$
   (c) ${}^-12 \div {}^-3$ **(d)** ${}^-18 \div 4$
   (e) $\frac{64}{-8}$ **(f)** $\frac{-72}{9}$
   (g) $\frac{-42}{-7}$ **(h)** $\frac{78}{-13}$

15 Copy and complete:
   (a) $3 + \square = 19$ **(b)** $19 + \square = 3$
   (c) ${}^-4 + \square = 6$ **(d)** ${}^-4 - \square = {}^-9$
   (e) $\square + {}^-3 = {}^-8$ **(f)** $\square - 9 = {}^-7$
   (g) $\square - 4 = {}^-5$ **(h)** $\square + {}^-15 = {}^-10$
   (i) $\square + \square + \square = {}^-12$ **(j)** $\square + \square = {}^-18$

16 The temperature of some ice-cream was $-2°C$. It was placed in a freezer where its temperature fell by $8°C$. What was its new temperature?

17 Curtly has $24 in his packet. He wishes to pay one bill for $9.73 and another for $17.40.
   (a) Does he have sufficient money to pay the bills?
   (b) His friend gives him $5. How much money does he have after bill payments?

18 The table below shows how the temperature varied during one night in January.

Time	Temperature
9.00 pm	5°C
10.00 pm	2°C
11.00 pm	1°C
12.00 midnight	${}^-1C$
1.00 am	${}^-5C$
2.00 am	${}^-5C$
3.00 am	${}^-6C$
4.00 am	${}^-1C$
5.00 am	3°C
6.00 am	5°C

   (a) At what time was the coldest temperature recorded?
   (b) Between which hours was the greatest fall in temperature?
   (c) Between which hours was the greatest rise in temperature?
   (d) By how much did the temperature change between 9.00 pm and 6.00 am?

19 Jan, Jim and Joy agreed to meet at the local cinema.
Jan arrived 20 minutes before the programme started, Jim arrived 5 minutes after the programme started and Joy was in her seat with 3 minutes to spare.
By how many minutes did:
   (a) Jan arrive before Jim
   (b) Jim arrive after Joy
   (c) Jan have to wait before Joy arrived?
If they all left the cinema 1 hour 47 minutes after the programme started how long did each spend in the cinema?

20 Look at the x, y graph below.
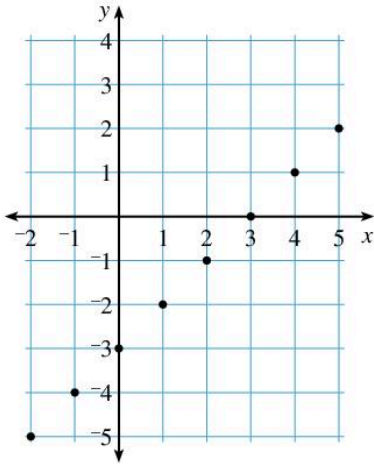
(a) Write down the coordinates of each of the eight marked points.

(b) Write down the rule for the point in the form $x \rightarrow$.

(c) Draw a number machine for these points.

(d) What number is output from your machine if $^-6$ is input?

(e) What number must be input to your machine if $^-6$ is output?

Relations and graphs

21 Write down for each arrow diagram:

(a) what the arrow stands for

(b) the missing numbers.

(i)

(ii)
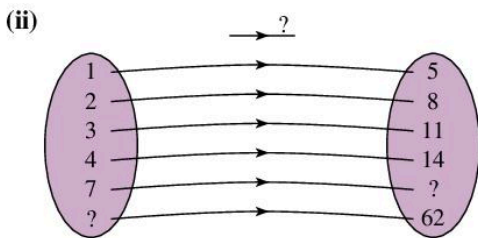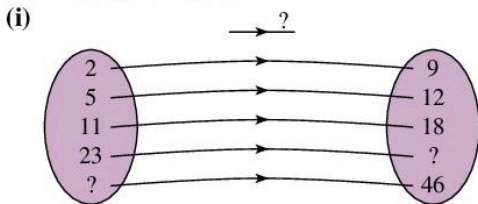
22 $P = \{1, 2, 3, 4\}$

$Q = \{$whole numbers 1 to 12$\}$

(a) Draw an arrow diagram from P to Q to show:

   (i) *is half of*   (ii) *is a factor of*

   (iii) $x \rightarrow x + 5$ (iv) $x \rightarrow 2x + 1$

(b) In each case, identify the type of mapping.

(c) Which of the maps show functions?

23 $X = \{1, 2, 3, 4, 5\}$

$Y = \{$whole numbers from 0 to 25$\}$

Draw arrow diagrams to show these number machines:

(a)

(b)

(c)
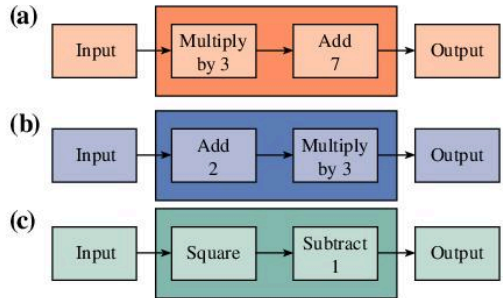
24 For the mapping below:

(a) draw the number machine

(b) write down the ordered pairs

(c) draw an X, Y graph on a grid
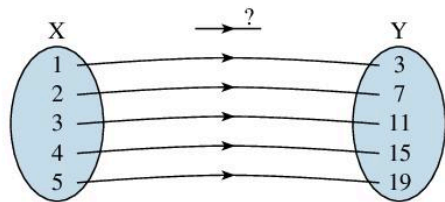
25 Look at the X, Y graph below.

(a) List the ordered pairs for the 7 points.

(b) Write down the rule as: $x \rightarrow$
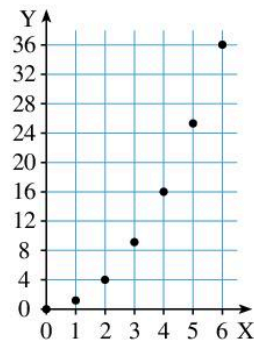
26 **(a)** Copy and complete the table of values

x	−2	−1	0	1	2	3
y						

for the equations:
(i) $y = 5x + 3$
(ii) $y = 7x - 6$
(iii) $y = 4 - x$
(b) Plot each equation on a suitable pair of axes.
(c) Find the gradient and y-intercept for each equation.

27 Look at the graph below for a walker.
(a) What time did he reach home?
(b) What was his furthest distance from home?
(c) When did he stop for a rest?
(d) When was he walking fastest? Why?

Graph of walker
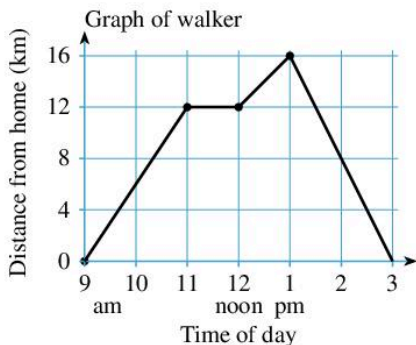
28 Make up a graph like the one above for a journey you might make on a bicycle. Describe your journey.

29 5 miles is approximately 8 km. Draw a graph to show how you would convert any distance from 0 to 100 miles into kilometres.

30 **(a)** Copy and complete the table of values for
   $y = 3x - 1$
   and $y = 2x + 3$

x	−2	−1	0	1	2	3	4
y							

(b) Draw the graphs of the two equations on the same axes.
(c) Write down the point of intersection of the two equations.

Transformations

31 Copy each shape, mark in any lines of symmetry and write down the order of rotational symmetry.

(a)

(b)

(c)
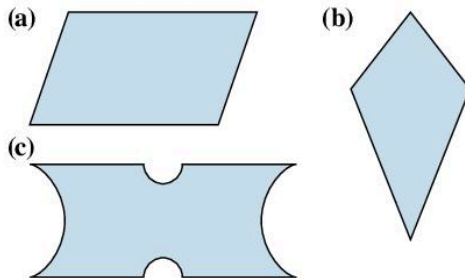
32 Using the grid on the right, by joining dots with straight lines, make up a shape which has:

(a) one line of symmetry but no rotational symmetry
(b) rotational symmetry of order 2 but no lines of symmetry
(c) rotational symmetry of order 2 and two lines of symmetry.

33 Show these points on a coordinate graph.
$(^-2, \,^-2), (0, \,^-3), (1, \,^-3), (3, \,^-2), (1, \,^-1), (0, \,^-1)$

Join them in order and then to $(^-2, \,^-2)$. What shape have you made? Mark in any lines of symmetry.

34 Reflect the shape in Question 33 in the x-axis. Write down the coordinates of the reflected shape.

35 **(a)** Copy the grid below.
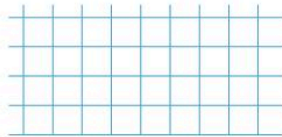
(b) On your grid draw the vectors
(i) $\vec{AB} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$ **(ii)** $\vec{CD} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

(iii) $\vec{EF} = \begin{pmatrix} -1 \\ 2 \end{pmatrix}$ **(iv)** $\vec{GH} = \begin{pmatrix} -3 \\ -2 \end{pmatrix}$

36 (a) On graph paper, draw triangles A(2, 0), B(0, 0), C(0, 2) and A′(4, 3), B′(2, 3), C′(2, 5).

(b) Find the vector that represents the translation of ABC → A′B′C′.

37 (a) Find the image X′Y′Z′ of triangle X (–2, 1), Y(0, –2), Z(1, 1) under the translation $\begin{pmatrix} 3 \\ 1 \end{pmatrix}$.

(b) Determine the vector that represents the translation X′Y′Z′ → XYZ.

38 The diagrams show a triangle XYZ and its image following a rotation. In each case write down the angle and centre of rotation.
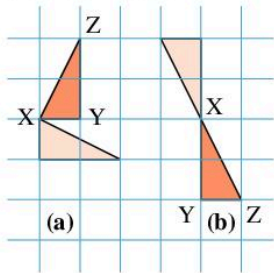
39 Copy the grid and triangle below.
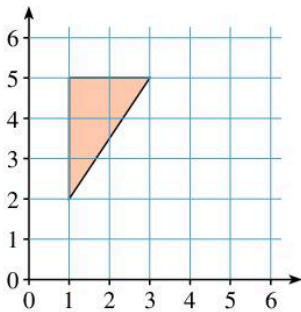
Draw the image of the triangle after a reflection
(a) in the line joining (4, 1) to (4, 6)
(b) in the line joining (0, 2) to (5, 2)
(c) in the line joining (1, 1) to (5, 5).

40 Repeat Question 39 for a shape formed by joining the coordinates (1, 1), (4, 4), (2, 5) and (1, 4).

41 Write down the coordinates of the vertices of the triangle in Question 39 if it is given a rotation of:
(a) a $\frac{1}{4}$-turn anticlockwise about (3, 5)
(b) a $\frac{1}{4}$-turn clockwise about (1, 2)
(c) a $\frac{1}{2}$-turn clockwise about (2, $3\frac{1}{2}$).

42 Repeat Question 41 for:
(a) a $\frac{1}{2}$-turn clockwise about (1, 5)
(b) a $\frac{1}{4}$-turn anticlockwise about (1, 5).

Mixed questions 2

1
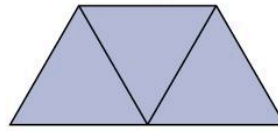
A trapezium is made from three equilateral triangles of side 8 cm.
The perimeter of the trapezium is:
A 40 cm   **B** 48 cm   **C** 56 cm   **D** 72 cm

2
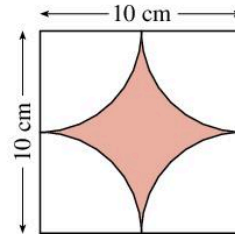
The perimeter of the shaded portion of the diagram is:
A 10 cm   **B** 90 cm
C 31.4 cm   **D** 68.6 cm

3
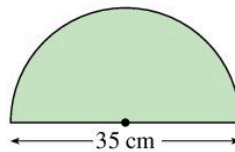
The perimeter of the semicircle is:
A 110 cm   **B** 90 cm
C 55 cm   **D** 52 cm

4 A circle has diameter 8 cm. What is its area? Take π = 3.14.
A 12.56 cm^2 **B** 25.12 cm^2
C 50.24 cm^2 **D** 200.96 cm^2

5 A bicycle wheel has a diameter of 70 cm. How many revolutions will the wheel make in travelling 1.1 km?
A 50   **B** 250
C 500   **D** 750

6 A map is drawn using a scale of 1 : 10 000. This can also be written as:
  A 1 cm represents 10 m
  B 1 cm represents 100 m
  C 1 cm represents 1000 m
  D 1 cm represents 1 km

7 A rectangular board 24 cm by 36 cm is cut into squares of side 3 cm. How many such squares are obtained?
  A 36   **B** 72   **C** 96   **D** 288

8 A map has a scale of 1 : 25 000. What is the real distance in metres represented by 4 cm?
  A 100 m   **B** 625 m   **C** 1000 m   **D** 6250 m

9 Which is the largest number?
  A ⁻5   **B** ⁻3   **C** ⁻9   **D** ⁻1

10 What is ⁻3 + 4?
  A ⁻7   **B** ⁻1   **C** 1   **D** 7

11
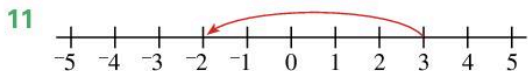
The number line shows:
  A 3 − 5 = ⁻2   **B** 3 + 5 = ⁻2
  C ⁻2 + 3 = 1   **D** 3 − 2 = 1

12 ⁻11, ⁻8, ⁻5, ⁻2, . . . What is the next number in this sequence?
  A ⁻5   **B** ⁻1   **C** 0   **D** 1

13 What is ⁻3 − ⁻5?
  A ⁻8   **B** ⁻2   **C** 2   **D** 8

14 One day in Washington the temperature was ⁻8°C. The next day it fell a further 2°C. What was the new temperature?
  A ⁻10°C   **B** ⁻6°C   **C** 6°C   **D** 10°C

15 What is ⁻3 + ⁻5 − 4?
  A ⁻12   **B** ⁻4   **C** ⁻2   **D** 6

16 6 + □ = ⁻5
  What is □?
  A ⁻11   **B** ⁻1   **C** 1   **D** 11

17 On Monday the temperature of a cold room was ⁻5°C. On Tuesday it was ⁻2°C. How did the temperature change from Monday to Tuesday?
  A It rose by 3°C   **B** It fell by 3°C
  C It rose by ⁻2°C   **D** It fell by 2°C

18 What is the value of ⁻2 + 3 × ⁻2?
  A ⁻8   **B** ⁻2   **C** 2   **D** 4

19 What is $(-3)^3$?
  A ⁻27   **B** ⁻9   **C** 9   **D** 27

20 Which arrow diagram shows a many-to-one map?

21
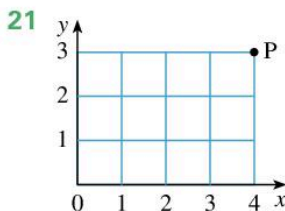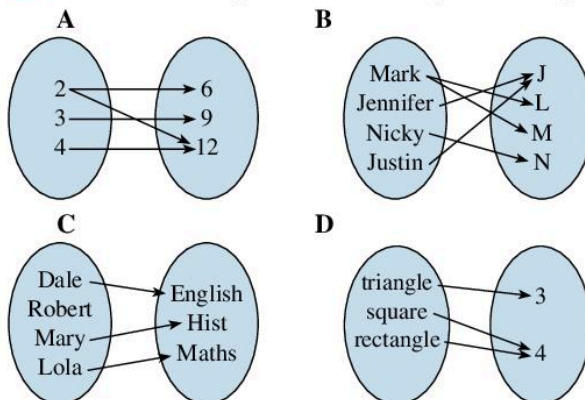
The coordinates of P are
  A (3, 4)   **B** (4, 3)   **C** (3, 3)   **D** (4, 4)

22
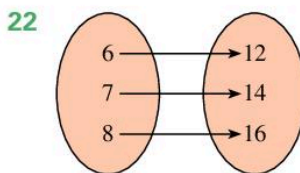
The arrow diagram shows the relation:
  A multiply by 2   **B** divide by 2
  C add 6   **D** add 8

23 Which point lies on the line $y = 3x + 2$?
  A (5, 1)   **B** (⁻1, 2)   **C** (0, 3)   **D** (1, 5)

24
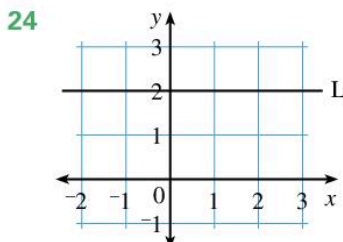
The line L on the graph has equation:
  A $x = 2$ **B** $y = 0$ **C** $y = 2$ **D** $x = 0$

25
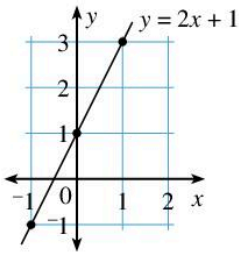
The graph shows the line $y = 2x + 1$. Where does it intercept the y-axis?

A $(^-1, \, ^-1)$ **B** $(1, 0)$
C $(0, 1)$ **D** $(2, 3)$

26 In the relation
$x \rightarrow 3x - 2$,
4 is mapped to:
A 2   **B** 5   **C** 6   **D** 10

27 The point that does not satisfy the mapping
$x \rightarrow 2x - 3$ is:
A $(6, 9)$ **B** $(4, 5)$ **C** $(2, 1)$ **D** $(0, \, ^-1)$

28
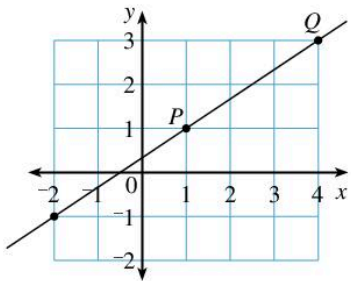
In the graph, P is the point $(1, 1)$ and Q is the point $(4, 3)$. What is the gradient of the line passing through P and Q?

A $\frac{-3}{2}$ **B** $\frac{-2}{3}$ **C** $\frac{2}{3}$ **D** $\frac{3}{2}$

29 Where does the line $y = 3x - 5$ intercept the y-axis?
A $^-5$ **B** $1\frac{2}{3}$ **C** 3   **D** 5

30 A line has gradient 3 and intercepts the y-axis at $y = -1$. What is the equation of the line?
A $y = -x + 3$ **B** $y = 3x + 1$
C $y = -3x + 1$ **D** $y = 3x - 1$

31
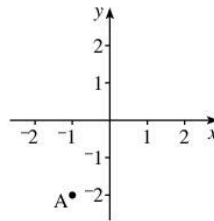
The coordinates of the image of the point A when reflected in the y-axis are:
A $(^-2, 1)$ **B** $(^-1, \, ^-2)$
C $(1, \, ^-2)$ **D** $(2, \, ^-1)$

32 What is the order of rotational symmetry of the letter I?
A 0   **B** 1   **C** 2   **D** 4

33 The diagram shows a regular hexagon. What is the order of rotational symmetry of this shape?
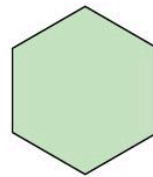
A 0   **B** 2   **C** 3   **D** 6

34
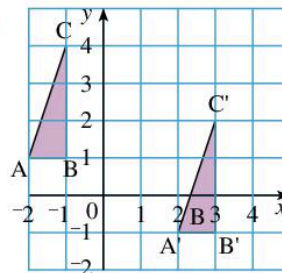
In the diagram triangle ABC is translated to A′B′C′. Which vector represents this translation?

A $\begin{pmatrix} 4 \\ -2 \end{pmatrix}$ **B** $\begin{pmatrix} -2 \\ 4 \end{pmatrix}$ **C** $\begin{pmatrix} 2 \\ -4 \end{pmatrix}$ **D** $\begin{pmatrix} -4 \\ 2 \end{pmatrix}$

35 Which diagram best represents the vector $\begin{pmatrix} 0 \\ -1 \end{pmatrix}$?

A

B

C

D
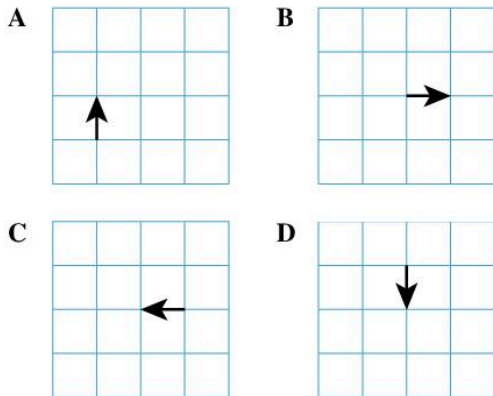
36
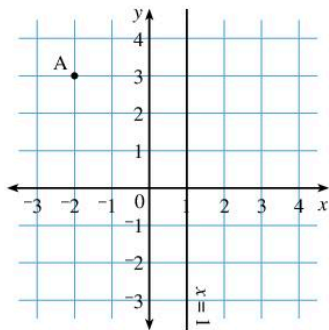
The point A is reflected in the line $x = 1$.
What are the coordinates of its image?
A (0, 3)   B (2, 3)   C (3, 3)   D (4, 3)

37 Which of these letters has rotational symmetry of order 2?
A S   B T   C U   D V

38
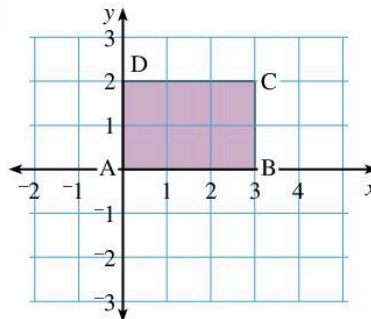
The rectangle A(0, 0), B(3, 0), C(3, 2), D(0, 2) is rotated anticlockwise about the origin through an angle of 90°.

What are the coordinates of image D′ of the point D?
A (0, 0)   B (0, 2)   C (0, –2)   D (–2, 0)

Statistics

Objectives

✔ read and interpret bar charts, pie charts and line graphs

✔ construct and use bar charts, pie charts and line graphs

✔ construct frequency tables

✔ Find the mean, mode and median from frequency tables

✔ How conduct a survey

Elect me and I will ensure you all will get above average wages
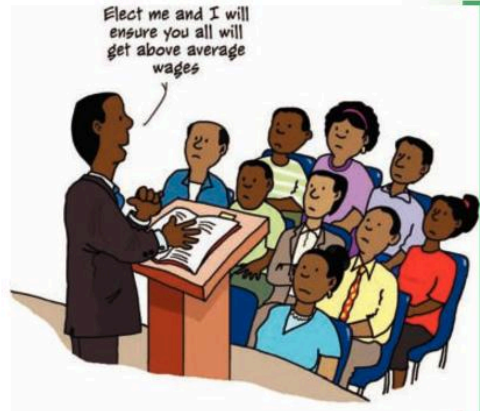
What's the point?

How can you tell who is the best West Indies batsman? You need to look at the data and the batting statistics. Only by reviewing the numbers of matches played and batting averages can you begin to answer the question.

Before you start

You should know ...

1 In a bar chart the frequencies are represented by bars.
For example:
In Form 2, 15 students like cricket best.
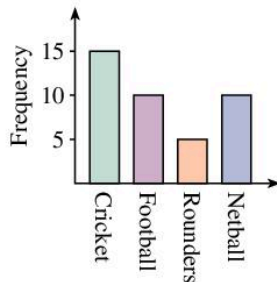
2 In a pie chart the frequencies are represented by sectors of a circle.
For example:
There are 40 students in Form 2 and $\frac{3}{8}$ of them like cricket best.
$\frac{3}{8} \times 40 = 15$
So 15 students like cricket best.
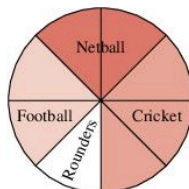
Check in

1 How many students in Form 2 like:
(a) rounders best
(b) cricket or rounders best?

2 How many students in Form 2 like:
(a) netball best
(b) football best
(c) football or rounders best?

9.1 Frequency tables

You will need a dice.

If you were asked to give information about your school you could include facts like:

the number of male students
the number of female students
the number of classrooms
the number of male teachers
the number of female teachers
the area of the playing fields
the total area of the school.

Each piece of numerical information collected is called a **statistic.**

● Statistics are numerical facts or data.

A frequency table is a good way of showing statistics. A tally can help you do this.

For example, a tally of the birthdays of the 36 students in Billy's class at Calder High School is:

Day	Tally	Frequency
Monday	||||	4
Tuesday	⊁⊁⊁	5
Wednesday	⊁⊁⊁ |	6
Thursday	|||	3
Friday	⊁⊁⊁ |||	8
Saturday	⊁⊁⊁ ||	7
Sunday	|||	3

← add the tally marks to give the frequency

each tally mark shows a birthday on a given day

Counting the tally marks gives the frequency.

Exercise 9A

1   (a) Copy the table below.

Score	Tally	Frequency
1		
2		
3		
4		
5		
6		

(b) Throw a dice 60 times. For each throw, put a tally mark beside the score you get.

(c) Now fill in the frequency column.

2   The 36 students in Billy's class did a spelling test. The marks they scored were:
9, 1, 7, 5, 4, 8, 6, 7, 3,
8, 6, 5, 7, 4, 10, 8, 4, 6,
8, 3, 9, 6, 7, 6, 5, 7, 8,
7, 6, 8, 7, 5, 9, 7, 6, 5

Copy and complete the table, for the set of marks.

Mark	Tally	Frequency
1	|	1
2		0
3	||	2

3   (a) Make a tally chart to show how often each vowel appears in this poem:

A funny old beast is the yeti,
Mostly he eats just spaghetti,
Tomatoes, ham
And iced mulberry jam,
Then he showers himself with confetti.

(b) Draw a frequency table to show this data.

9.2 Bar charts

A bar chart uses bars of different lengths to represent data.
Bar charts can be used to represent data collected in frequency tables.

Example 1

The number of children absent from Form 2A over a 20 day period were as follows:

5	2	2	0	4
1	1	3	1	0
2	1	2	2	5
3	1	0	0	4

(a) Complete a frequency table to show the data.
(b) Draw a bar chart to illustrate the absences.

(a)

Number of days absent	Tally	Frequency
0	IIII	4
1	ЖHT	5
2	ЖHT	5
3	II	2
4	II	2
5	II	2

(b)
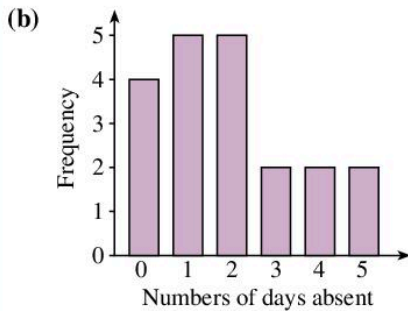
Exercise 9B

1 The favourite sports of a group of children are shown in the frequency table below:

Favourite sport	Frequency
Cricket	15
Football	20
Rounders	13
Netball	12
Athletics	16

(a) Draw a bar graph to show this data.
(b) What is the most popular sport?
(c) How many children were in the group?

2 The bar chart shows Form 2B's favourite subjects.
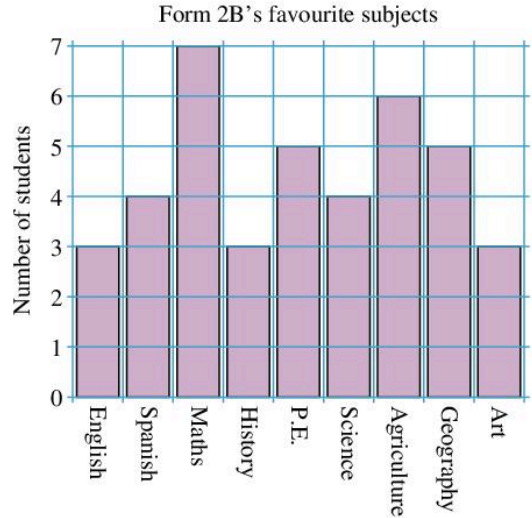
(a) How many students are in Form 2B?
(b) Draw a frequency table for the graph.
(c) What is 2B's favourite subject?

3 The bar chart shows the number of students at Faircross School.
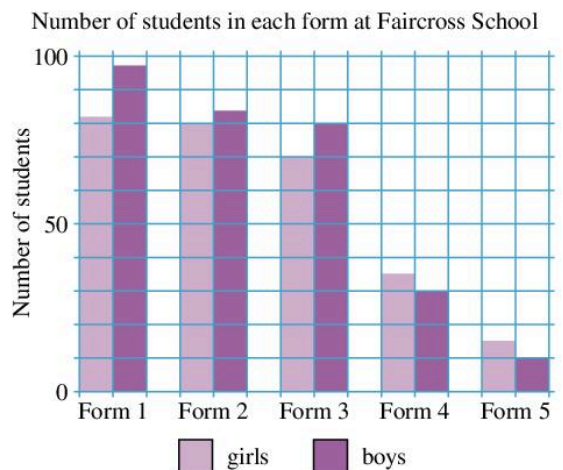
(a) How many girls are in Form 3?
(b) How many students are in Form 3?
(c) How many students are in Form 4?

4 The bar chart shows the rainfall statistics, for each month last year, for a certain South American country.
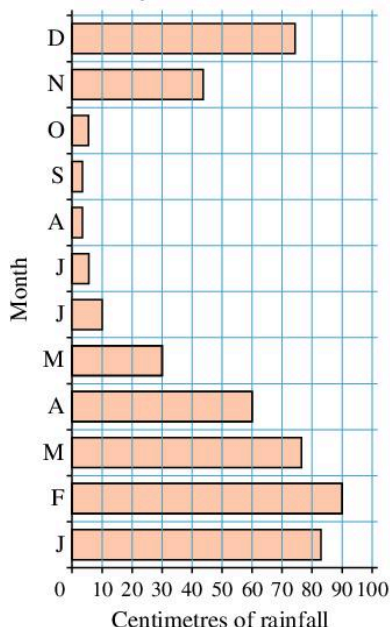
(a) What do, J, F, M, ... stand for?
(b) How much rain fell in July?
(c) In which month did most rain fall?
(d) When was the dry season?

5 (a) Find out the numbers of boys and girls in each year in your school.
(b) On squared paper draw a bar chart, to show your data.

Activity

- Find the shoe sizes of all the students in your class.
- Make a frequency table to show this data.
- Draw a bar chart to show the shoe sizes of your class.
- What is the largest size? What is the smallest size?
- How would you compare shoe sizes for boys and girls?

Grouped frequency charts

Sometimes it is impractical to construct a simple frequency table or bar chart to show certain data. Work through Exercise 9C to see why and how this problem is dealt with.

Exercise 9C

1 50 students took a geography test. Here are their marks.

$$35, 37, 38, 43, 45,$$
$$47, 47, 48, 49, 51,$$
$$53, 54, 54, 54, 55,$$
$$56, 57, 57, 57, 59,$$
$$59, 60, 60, 62, 63,$$
$$64, 64, 65, 66, 66,$$
$$67, 68, 68, 69, 69,$$
$$69, 70, 71, 72, 72,$$
$$72, 74, 76, 77, 78,$$
$$78, 81, 81, 84, 84$$

(a) What is the lowest mark?
(b) What is the highest mark?
(c) Copy and complete the frequency table below.

Mark	35	36	37	82	83	84
Number of students	1	0	1	0	0	2

(d) Why does the table begin at 35 and end at 84?
(e) Now draw a bar chart for the information in the table.
(f) Why is it difficult to draw this bar chart?

2 To avoid the difficulty encountered in Question 5 the marks can be put into groups, for example 35 to 39, 40 to 44, 45 to 49 and so on.
(a) Arrange the numbers from 35 to 84 in groups.
(b) Find out the number of students who got marks in each group.
(c) Copy and complete the frequency table.

Mark	35 to 39	40 to 44	45 to 49	50 to 54	55 to 59
Number of students	3	1	5	5	7

(d) Copy and complete the bar chart.

Marks in a geography test
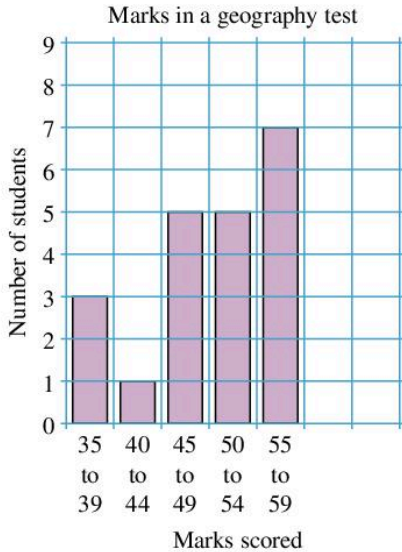
(e) Was this bar chart easier to draw than the one in Question 1?

3 Use the bar chart in Question 2 to answer the following questions (if possible).
  (a) How many students took the test?
  (b) How many students scored between 59 and 65?
  (c) How many students scored 70 or more?
  (d) How many students scored 50?
  (e) How many students failed, if the pass mark is 50?

4

How Form 2D counted

Number reached in 1 minute
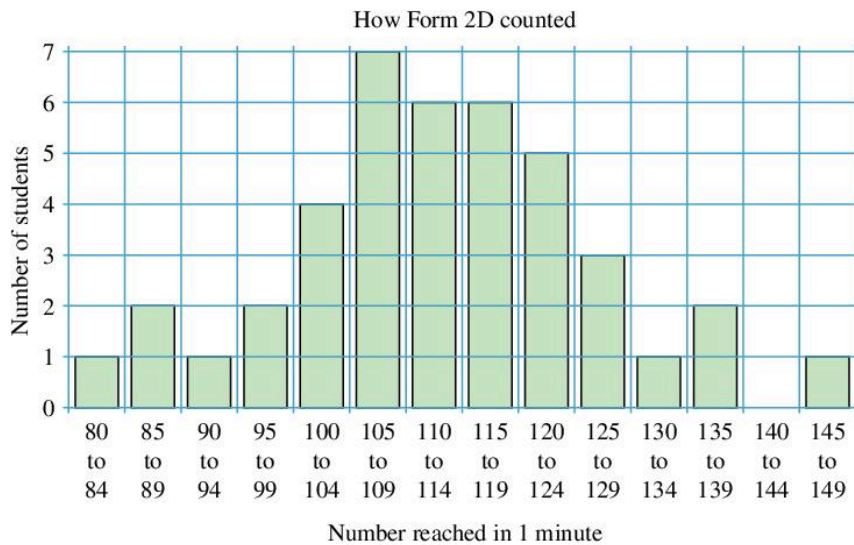
Mrs Singh asked Form 2D to count quietly *one*, *two*, *three* . . .
After exactly one minute, she asked them to stop. She wrote down the number that each student had reached.
The result is shown in the graph above.
Answer the following questions.
  (a) How many students are in Form 2D?
  (b) Why do the groups of numbers start at 80 instead of 0?
  (c) How many students counted to one of the numbers 105, 106, 107, 108 or 109?
  (d) How many students counted to between 124 and 130?
  (e) How many students counted to between 94 and 100?
  (f) Can you tell how many students counted to 131?
  (g) Can you tell whether anyone counted to 149?
  (h) How many students counted to 143?

5 Copy the graph from Question 4, but this time group the *Number reached* into tens, using groups 80–89, 90–99 and so on.

6 The heights of 25 children, measured to the nearest centimetre, are shown in the block graph.

Heights of 25 children
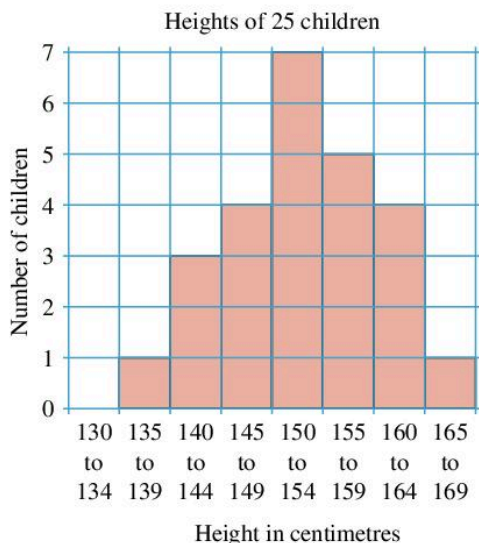
(a) How many of them are taller than 154 cm?
(b) How many are shorter than 145 cm?
(c) Redraw the graph, using groups 130–139 cm, 140–149 cm and so on.

Technology 1

Collect data in your class to find, for example:

- Favourite games of students
- Transportation used to reach school
- Latecomers over a 5-day period
- Favourite TV programme.

In each case, separate the results for boys and girls.

Insert the data collected as tables in an Excel spreadsheet.
For example:

	A	B	C
		Boys	Girls
1		Boys	Girls
2	Cricket	5	2
3	Football	8	3
4	Netball	0	10

Highlight your table and select the bar chart. Notice how the chart allows you to compare girls with boys.

Copy your charts into a word processing program and make brief notes to explain your results.

Bar charts are easy to draw using a spreadsheet.

Technology 2

Use the data from Question 1 of Exercise 9B. Type the data into separate cells of a spreadsheet. Highlight this table and select bar chart from the chart menu. The resulting bar chart should look like this. Bar chart of favourite sports
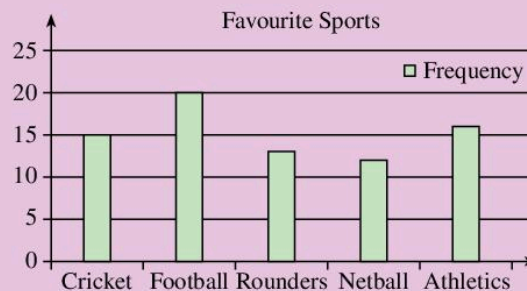
- Use a spreadsheet to draw bar charts for the other questions in Exercise 9B.

9.3 Pie charts

Pie charts are another way to show information. For example, this pie chart shows how Bob spends 24 hours.
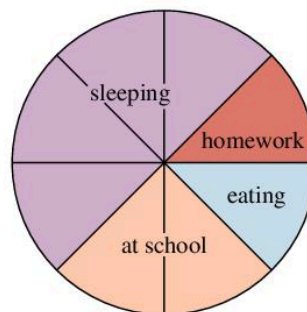
He spends $\frac{1}{8}$ of his time doing homework.

The angle of the sector is $\frac{1}{8} \times 360 = 45$

Example 2

Article bought	Shoes	Books	Shirt	Umbrella
Amount spent	$60	$40	$50	$30

Alan has $180. He uses it to buy the following: Show this data on a pie chart.

There are $360°$ in a circle.
Alan spends a total of $180. This is represented by $360°$.
That is, $1 is represented by $\frac{360}{180} = 2°$
Angle representing $60 on shoes $= 60 \times 2°$
$= 120°$

The other angles are:
Books: $40 \times 2 = 80°$
Shirt: $50 \times 2 = 100°$
Umbrella: $30 \times 2 = 60°$

Check the angle sum $=$
$120 + 80 + 100 + 60 = 360°$

Now draw a circle. With your protractor carefully mark in the angles.

Pie chart showing how Alan spends his $180
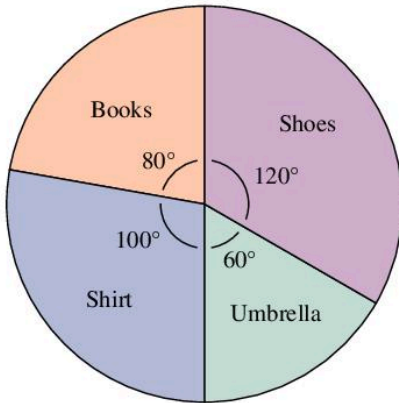
Exercise 9D

1 360 motorists together own four separate makes of cars. The numbers who own each make is listed.

Toyota	150
Suzuki	90
Ford	20
Mitsubishi	100

(a) What size of angle would represent one motorist on a pie chart?
(b) Copy and complete the table for each make of car.

Car make	Number of motorists	Size of angle on pie chart
Toyota	150	
Suzuki	90	
Ford	20	
Mitsubishi	100	

(c) Now draw a circle radius 6 cm. Using a protractor, draw an angle in your circle to represent each make of car. Label each sector.

2 The frequency table shows the favourite sports of Form 2C.

Favourite sport	Number of students
Cricket	14
Football	9
Badminton	5
Volleyball	4
Swimming	6
Tennis	2

(a) How many students are in Form 2C?
(b) What size of angle would represent one student, on a pie chart?
(c) Copy and complete the table, for each sport.

Favourite sport	Number of students	Size of angle on pie chart
Cricket	14	
Football	9	
Badminton		
Volleyball		

(d) Now draw a circle of radius 6 cm. Using a protractor, draw an angle in your circle to represent each sport. Label your chart.

(e) Do you think it is easier to read information from a pie chart than a bar chart? Why?

(f) Compare this pie chart for Form 2C with the bar chart for Form 2B, in Question 2 of Exercise 9B. What can you find out from the bar chart that you cannot find out from the pie chart?

3 The favourite singers of 30 students at Smalltown secondary school are:

Tarrus Riley	10	Mr Vegas	3
Busy Signal	2	Vybz Kartel	8
Machel Montano	7		

Show this information on a pie chart.

4 Kingstreet bookstore sold 60 magazines in one week. They were:

The Stars World	15	Cricket Reporter	12
News Review	9	Caribbean Focus	4
You and Me	20		

Show this information on a pie chart.

5 In a government survey, 240 men were asked their occupations. Their replies were:

Management	10	Services	30
Clerical	36	Skilled Labour	72
Teaching	12	Unemployed	50
Agricultural	30		

Show this information on a pie chart.

6 Here are the shoe sizes of 80 men:

Size 3	2	Size 7	12
Size 4	2	Size 8	28
Size 5	6	Size 9	16
Size 6	8	Size 10	6

Show this information on a pie chart.

7 Mr Pinder earns $400 a month. Here is what he did with his salary last month.

Food	$120	Entertainment	$40
Clothes	$50	Rent and fuel	$100
Travel	$30	Savings	$60

(a) In a pie chart to show this, $400 is represented by 360°.
What angle represents $1?

(b) Copy and complete the table.

	Amount	Angle
Food	$120	108°
Clothes	$50	
Travel	$30	

(c) Draw a pie chart to show the information in your table. Don't forget to label your pie chart.

8 Carry out a survey of 20 students. Find out their favourite:
(a) game **(b)** colour **(c)** singer.
Draw a pie chart to show the information in each case.

Technology

You can draw pie charts using Microsoft Excel (or other spreadsheet software). Use the data from Question 1 of Exercise 9D. First type the data into separate cells of a spreadsheet.

	A	B	C
1	Toyota	150	
2	Suzuki	90	
3	Ford	20	
4	Mitsubishi	100	
5			

Then, highlight your table. Click 'Chart' and select the pie chart.

Pie chart of car sales

□ Toyota   ▨ Suzuki   □ Ford   ▨ Mitsubishi

28%   41%   6%   25%
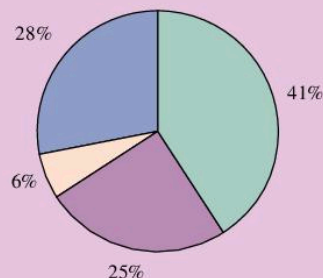
- Use a spreadsheet to draw pie charts for the other questions in Exercise 9D.
- Which do you prefer, pie charts or bar charts? Give reasons for your answer.
- Which type of chart is easier to interpret? Explain.

Reading pie charts

To read a pie chart you have to find the quantity or number that each given angle actually represents.

Example 3

There are 120 second-year students in St Mary's College. The pie chart shows their favourite colours.
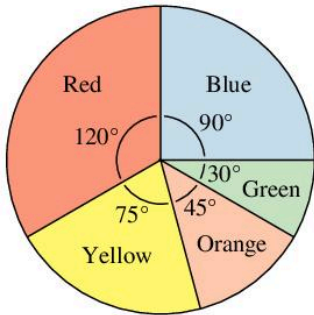
Find out how many students chose each colour.

120 students are represented by 360°
So 1 student is represented by $\frac{360}{120} = 3°$

The colour blue, 90°, represents $\frac{90}{3} = 30$ students

The other colours are chosen by
Red $= \frac{120}{3} = 40$ students
Yellow $= \frac{75}{3} = 25$ students
Orange $= \frac{45}{3} = 15$ students
Green $= \frac{30}{3} = 10$ students

Check:
total number of students
$= 30 + 40 + 25 + 15 + 10$
$= 120$

Exercise 9E

1   This pie chart represents the flavours of the 180 ice-creams sold by one man in one weekend.

Ice-cream flavours sold
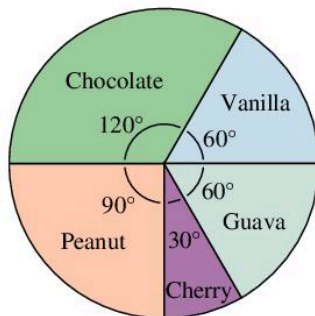
(a) What size of angle represents one ice-cream?
(b) How many chocolate ice-creams did he sell?
(c) Copy and complete the table.

Ice-cream flavour	Angle	Number sold
Chocolate	120°	
Vanilla	60°	
Guava	60°	
Cherry	30°	
Peanut	90°	

2   In an opinion poll 40 people were asked how they would vote in the next general election. The pie chart shows the results.
Political parties chosen by sample of voters
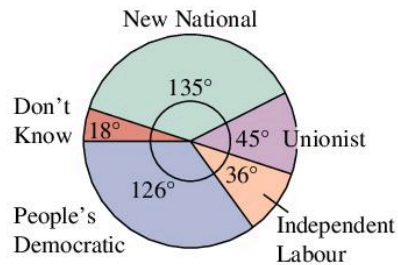
(a) What size of angle represents one person?
(b) How many of the people asked would vote Unionist?
(c) Find out how many would vote for each of the other parties.

3   The pie chart shows how the workers at the National Cement Works travel to work.

Method of travel used by cement workers
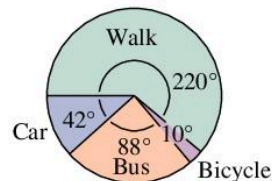
(a) If five people cycle to work, what angle represents one worker?
(b) Find out how many workers come by bus, by car, and by walking.
(c) How many workers are employed at the cement works?

4 The favourite subjects of Form 2A
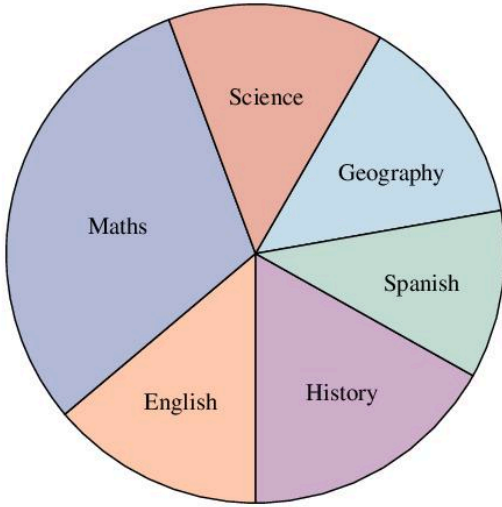
(a) From the pie chart, can you tell how many students are in Form 2A?
(b) Can you tell how many students prefer History?
(c) Measure the angle for History.
(d) If six students prefer History, what angle represents one student?
(e) Now measure the angle for each of the subjects.
(f) Calculate the number of students preferring each subject.
(g) How many students are in Form 2A?

5 The way John Boyd spent his wages last month is shown in this pie chart.
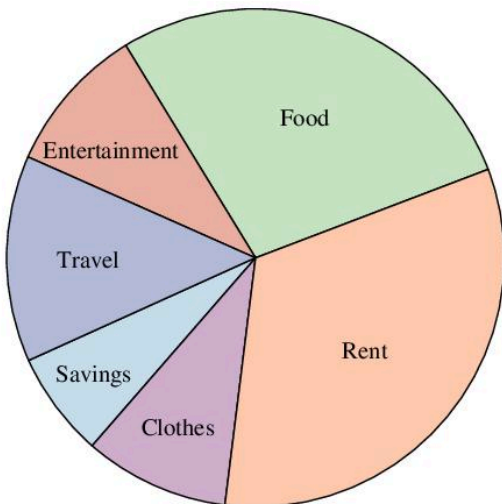
(a) About what fraction of his wages did John spend on rent?
(b) About what fraction did he save?
(c) Measure the angles at the centre. Write them down.
(d) Can you tell how much John earned last month?
(e) Last month John spent $200 on food. Work out how much money was used in each of the other ways.
(f) How much did John earn last month?
(g) If John's wages last month were $1080, calculate how much he spent on:
  (i) clothes   (ii) travel.

9.4 Line graphs and scales

A line graph is another way of showing data.

Example 4

Draw a line graph to show Bently's height from his birth in 2002.

Year	2002	2003	2004	2005	2006
Height (cm)	40	65	80	90	100
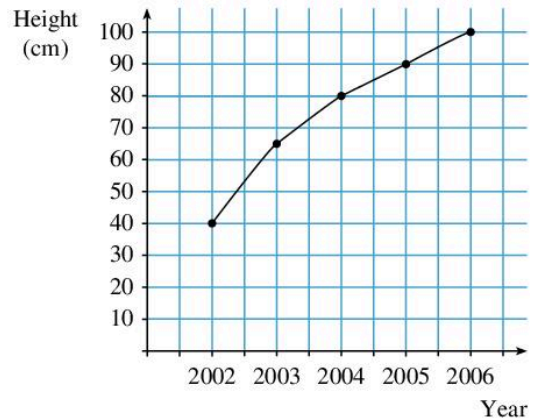
Why does it make sense to join the points?

Notice that in Example 4 the scale chosen is important. A change of scale can make the graph look different.
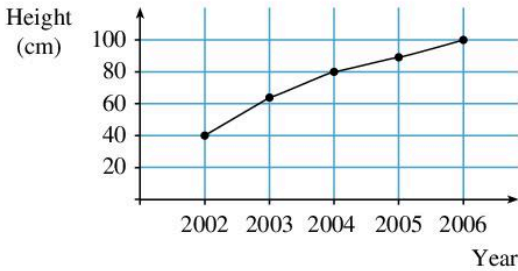
Which graph is a better representation of the data? Why?

Another change you could make is to begin the vertical axis above zero.
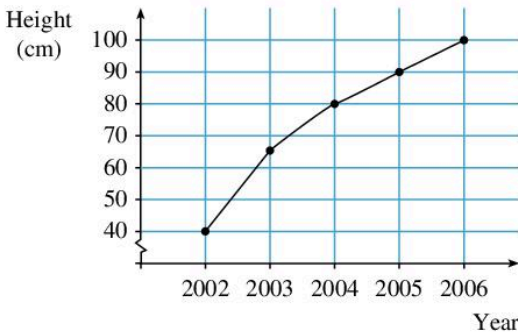
This graph looks very different from the second graph.

A person looking at the third graph may think Bently grew very rapidly in his first four years of life. A person looking at the second graph may think Bently did not grow very quickly!

You need to choose scales and axes carefully when drawing charts. If you do not, your graphs will mislead!

Exercise 9F

1 Draw a line graph to show sales at a shoe store over a six-month period.

Month	Jan	Feb	Mar	Apr	May	June
Sales ($)	5000	6000	5500	7000	6000	6500

2 (a) Redraw the graph you drew in Question 1. This time start the vertical axis (sales) from $5000.

(b) How does your graph differ from the one drawn in Question 1?

(c) Which shows the data better?

3 Look at these two graphs of sales at an electronics store over a six-month period.
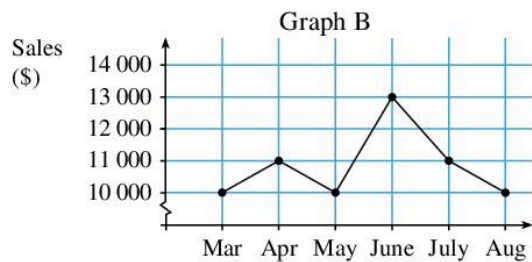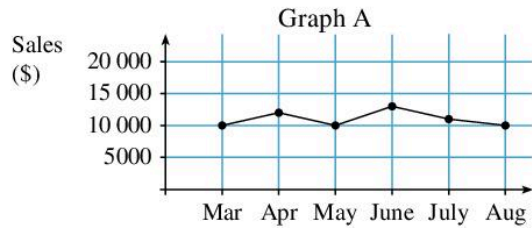
(a) Why do the graphs look different?

(b) Which graph is easier to read?

(c) Would it be fair to say that the sales in June were excellent? Why?

(d) Which graph would you show to the manager of the electronics store? Why?

4 (a) Draw a line graph to show the change of temperature over the course of a day. Start the temperature axis at 0°C.

Time	6 am	8 am	10 am	12 noon	2 pm	4 pm
Temperature (°C)	24	25	27	29	29	26

(b) Redraw the graph with the temperature axis beginning at 23°C.

(c) Which graph is easier to read?

(d) What do you think the temperature was at **(i)** 7 am **(ii)** 3 pm?

(e) At what times do you think the temperature was 28°C?

5   A group of 600 people were asked to choose their favourite fast food from pizza, hamburger or hot dogs. The results were:

Food	Votes
Hamburger	190
Hot dogs	210
Pizza	200

(a) Draw a bar chart with vertical (votes) axis starting at zero to show the data.

(b) Redraw your bar chart, this time with the votes axis beginning at 190.

(c) Which graph is a better representation of the data? Why?

6   Prime Minister Stevens of Rocky Island is running for re-election. He has been given two graphs on the number of robberies in the island.

Graph A
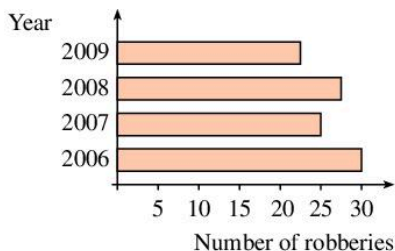
Graph B
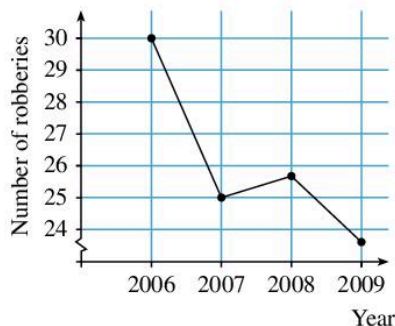
(a) Which graph is his press officer likely to publish? Why?

(b) Which graph gives a fairer picture of the level of robberies over the last four years? Why?

(c) In a speech, PM Stevens says that 'robberies have declined steeply' while he has been in charge. Is he correct? Explain.

7   Raymond Allison, Opposition Leader in Rocky Island, has been given a graph showing unemployment in the island.
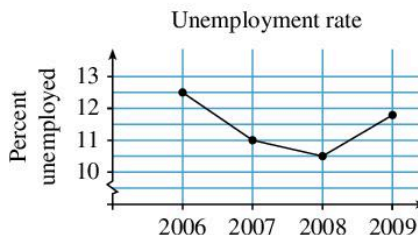
(a) Mr Allison says 'there has been a disastrous increase in unemployment' in the island. Is he correct? Explain.

(b) Redraw the graph to give a better representation of the data.

8   (a) Find some graphs from magazines or newspapers.

(b) For each graph, make a note of:
  (i)   the scale used
  (ii) whether the axes begin at zero
  (iii) the axes labels and graph title.

(c) Which graphs appear misleading? Why?

Technology

View the presentation on misleading graphs and charts at

https://venngage.com/blog/misleading-graphs/

or a similar site (do a search on misleading graphs).
Look for examples of misleading graphs and charts from magazines, newpapers or the internet.
Present some of these to your class.

9.5   Surveys

Bar and pie charts are pictorial ways of showing information that is easy for people to understand. They are especially useful when writing up the results of a **survey**. A survey is often conducted when you want to find the answer to a question or when you wish to know whether a particular statement is true.

For example,

- What are the television viewing habits of high school students?
- Are boys better than girls at mathematics at your school?
- Do taller people wear larger-sized shoes than shorter people?

There are four steps in carrying out a survey:

1 Decide on the question you want answered or the statement you wish to check.
2 Collect data to answer your question.
3 Analyse or study carefully the data you collected.
4 Interpret your results so as to answer your question or find out the truth of your statement.

Exercise 9G

In these questions, use data from the students in your class.

1 Check the accuracy of this statement.

> The pulse rate of students in your class is 72 beats per minute.

(a) Decide what data you need to collect.
(b) Decide how you will collect this data.
(c) Make a simple data collection sheet and use it to collect the data.
(d) Analyse the data and draw suitable charts to illustrate your results.
(e) Write up your results and explain whether the statement is true or false.

2 Try to answer the question:

> Do boys need more sleep than girls?

(a) Decide on the data needed to answer this question.
(b) Make a data collection sheet and use it to collect the data.
(c) Analyse the data and draw suitable charts to illustrate your results.
(d) Write up your results so as to answer the question.

3 Carry out the four steps of a survey for these questions/statements.
(a) What is the favourite television programme of girls/boys at your school?
(b) Girls spend more time doing homework than boys.

(c) Which teacher has the biggest classroom at your school?
(d) Left-handed people run faster than right-handed people.

Technology 1

The use of a computer is almost essential in carrying out a modern survey.
The computer can be used to:

- word process the data collection sheet
- enter results in table form in a spreadsheet
- use a spreadsheet to make charts
- word process the survey to include:

  (1) The question or statement to be investigated
  (2) How the data was collected and from whom
  (3) The results of the survey with suitable tables and charts
  (4) An explanation regarding the conclusions you reached.

Use a computer to do a complete write-up of one of the surveys you carried out in Exercise 9G or a survey of your choice.

This is a good introduction to CSEC's School Based Assessment.

The internet is a wonderful source of data on many areas of interest. You can use this information to answer questions such as 'Which Caribbean state has the lowest road fatality rate?' or 'Which island exports the most sugar?'

Technology 2

Visit the site

www.nationmaster.com

to find data on any country in the world.

Choose an area of interest, e.g. crime, and draw charts to show which island in the Caribbean has the lowest crime rate.
Word process and present your findings to your class.

9.6 Averages—mode, median and mean

An average is one number used to describe a set of numbers. There are three sorts of averages: the mode, the median and the mean.

The mode

● The **mode** is the most common item in any set of data.

Example 5

What is the modal height of these cricketers?

163 cm   166 cm   166 cm 166 cm 169 cm   172 cm   174 cm 175 cm   177 cm   177 cm   178 cm
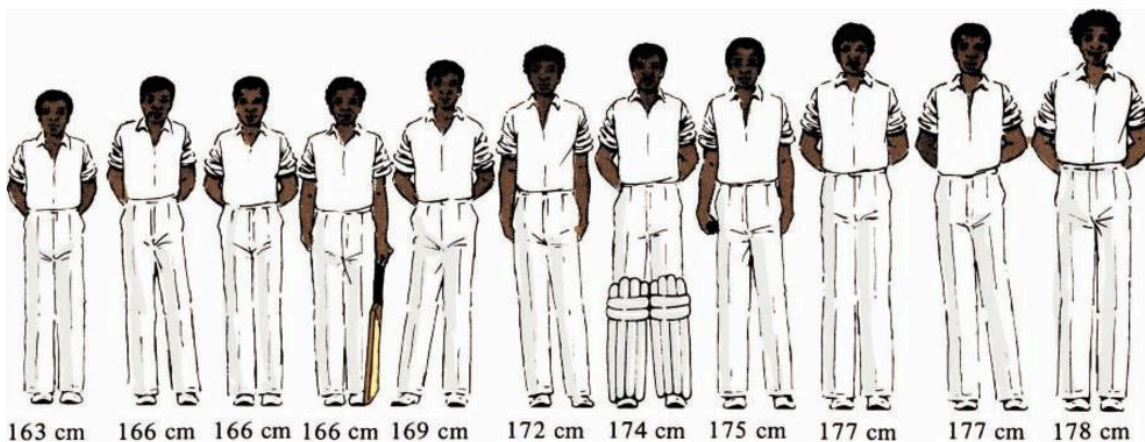
The cricket team has three players who are 166 cm tall, two who are 177 cm tall, and the rest are all of different heights. The mode of the crickets' heights is 166 cm.

Exercise 9H

1 The frequency table shows the size of a group of 30 families.

Family size	Number of families
1	1
2	3
3	4
4	6
5	5
6	4
7	3
8	4

(a) What is the smallest family size?
(b) What is the largest family size?
(c) What is the modal family size?
(d) Draw a bar chart to show this information.

2 30 students from Form 2 took part in a quiz. The bar chart shows their scores.

How 30 students scored in a quiz
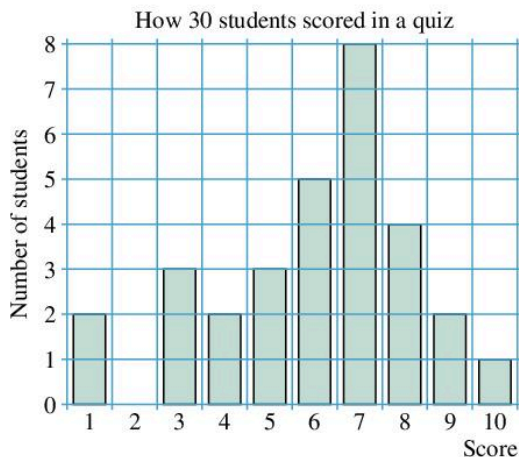
(a) How many students scored
   (i) the lowest mark
   (ii) the highest mark?

(b) What mark did most students score?
(c) What is the mode of the set of marks?
(d) Draw a frequency table to show this information.

3 The number of goals scored by Banfield Football team over 20 matches is as follows:

    1 0 2 0 1 3 5 0 0 1
    6 2 0 0 0 2 1 4 1 2

(a) Draw a frequency table to show the data.
(b) What was Banfield's best score?
(c) What was Banfield's worst score?
(d) What was Banfield's most frequent score?
(e) Draw a bar chart to show this information.

4 Dave sells soft drinks at his stall in the market. His sales for this week are shown in the bar chart.

Bottles of soft drink sold this week
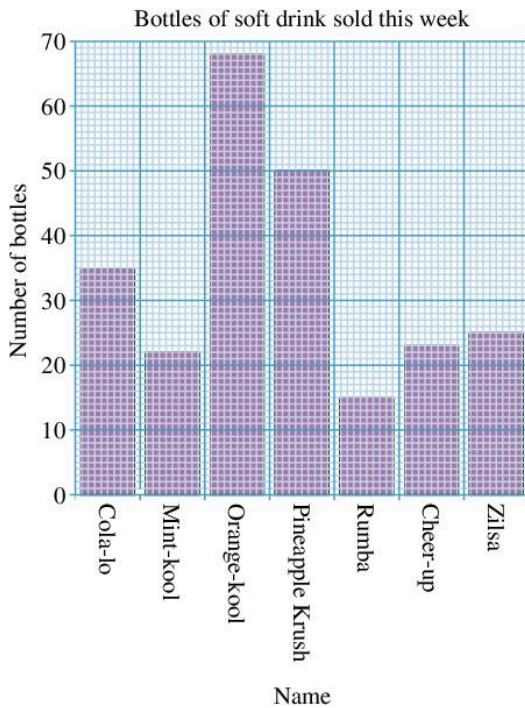
(a) How many bottles of soft drink has Dave sold altogether this week?
(b) Which soft drink was most popular?
(c) What is the mode?
(d) Which drink should Dave keep the largest stock of? Why?
(e) Which drink should Dave keep the smallest stock of? Why?
(f) Do you think the mode is a useful thing for Dave to know? Why?

5 Find out:
(a) the favourite record
(b) the favourite book
for each student in your class. Record your results on a tally chart. Which record is the mode? Which book is the mode?

6

The Fitu Shoe Company collected statistics for the shoe sizes of 1000 young women. The bar chart shows the results.

Shoe sizes of 1000 young women
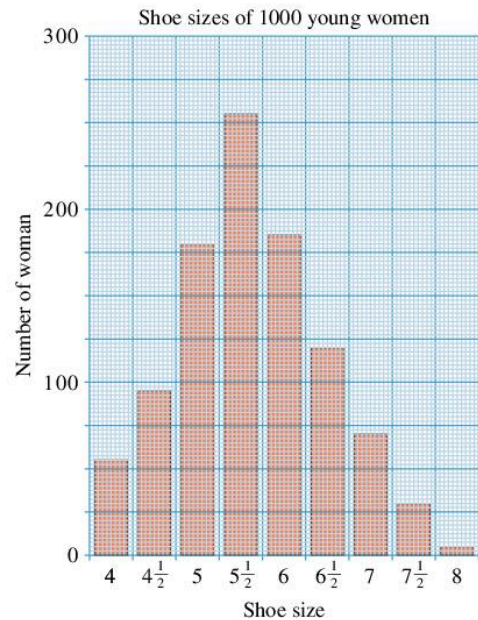
(a) Which shoe size is the mode?
(b) Use the bar chart to construct a frequency table to show the same information.
(c) The company is making a new style of shoe for young women. Should it make the same number of pairs for each size? Why?
(d) Of which shoe size should the company make most pairs?
(e) Of which shoe size should the company make least pairs?
(f) Do you think the shoe size bar chart would be useful to a shoe company?
(g) Write down three other products whose manufacturers could use a bar chart of this sort.

The median

● The **median** is the middle number when a set of numbers is arranged in order of size.

For example, the seven girls in the diagram are arranged in order of height.
The middle height of the set is Jill's height of 134 cm. The median height is 134 cm.

| Jane 98 cm | Julie 110 cm | June 122 cm | Jill 134 cm | Jankie 146 cm | Janice 158 cm | Jenny 170 cm |
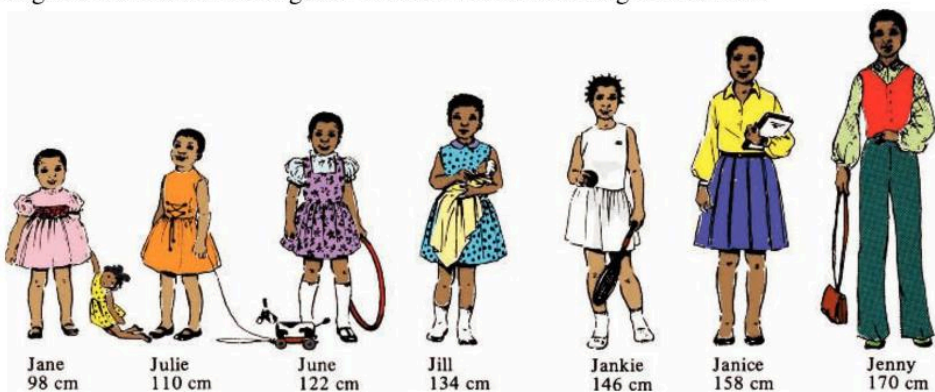
Example 6

Find the median of these numbers:

(a) 6, 3, 7, 2, 8
(b) 4, 1, 6, 6, 2, 5

(a) The numbers arranged in order are:
            2, 3, 6, 7, 8

            The middle number is 6,
So the median is 6.
(b) The numbers arranged in order are:
            1, 2, 4, 5, 6, 6
                    ↑
            median

There are two middle numbers 4 and 5.
The median is halfway between the two:

Median $= \dfrac{4+5}{2} = 4\frac{1}{2}$

Exercise 9I

1 The drawing below shows a set of rice bags of different masses. The mass has been written on each bag.

6 kg   8 kg   4 kg   12 kg   4 kg

(a) Draw the bags so that they are in order of mass, the lightest bag first.
(b) Put a circle round the middle bag.
(c) Which mass is the median?
(d) Which mass is the mode?

2 Look again at the cricketers on page 186.
(a) Are they arranged in order of height?
(b) Which height is the median?
(c) Is the median the same as the mode, for the cricketers?

3 Arrange each set of numbers in order, and find the median and the mode.
(a) 10, 9, 7, 8, 6, 8, 4
(b) 16, 16, 24, 17, 19
(c) 4, 6, 6, 5, 11, 3, 2, 7, 6
(d) 3, 9, 12, 14, 13, 14, 9, 9, 5
(e) 18, 13, 9, 21, 14, 13, 24, 22

4 Find the median for each set of scores.
(a) 6, 7, 13, 15, 17, 24
(b) 3, 4, 7, 8, 10, 17, 19, 27
(c) 4, 9, 8, 8, 11, 9, 2
(d) 8, 12, 7, 6, 13, 12, 4, 4
(e) 26, 2, 13, 41, 34, 9
(f) 103, 113, 117, 154, 109, 101

5 The frequency table shows the ages of the children in Form 3C at the Crown School.

Age	Frequency
13	2
14	12
15	13
16	2

(a) How many children are there in 3C?
(b) Write down their ages, in order, starting with the youngest.
(c) What is the median age of children in 3C?

The mean

● The **mean** is calculated by adding up all the data and dividing by the number of values.

$$\text{Mean} = \frac{\text{sum of the values}}{\text{number of values}}$$

It's mean because you have to work hard to find it!

Example 7

Find John Abrahm's mean mark.

LONG ACRE SCHOOL

ANNUAL REPORT

Name: John Abrahms
Form: 2B
No. in form: 36
Place in form: 10th

Subject	Mark	Comments
Mathematics	98	Top of class in this
English	62	Fair
General Science	80	A good student
Social Studies	39	Very lazy in class!
Agriculture	76	Satisfactory
Music	12	Should drop this subject
Spanish	57	More effort needed
Current Affairs	80	Satisfactory
Total Marks	504	

The total of all the marks is 504.
The number of different subjects is 8.
Mean $= \frac{504}{8} = 63$

The mean is the most commonly used average.

Exercise 9J

1 This sketch shows Robert's rice fields.
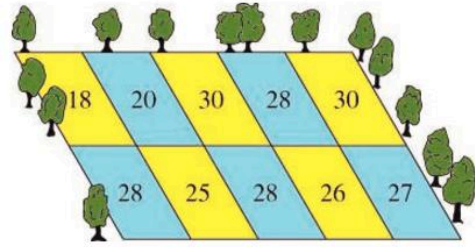
There are ten fields all the same size. The numbers show the bags of rice that Robert reaped in each field.

Robert told his friend 'I got 26 bags of rice per field.'
(a) What does he mean by *per field*?
(b) How did Robert work this out?
(c) Which sort of average is it?

2 John, Jack, Billy and Bud are playing cricket. Their runs for 10 overs are:

John   10, 8, 5, 8, 5, 9, 4, 5, 9, 7
Jack   6, 7, 1, 4, 5, 2, 2, 4, 4, 5
Billy   12, 8, 9, 11, 11, 14, 5, 3, 6, 1
Bud   15, 11, 14, 10, 9, 11, 8, 5, 5, 12

For each boy, find the mean score per over.

3 Calculate the mean score for each set of scores:
(a) 12, 18, 24
(b) 14, 26, 31, 15, 19
(c) 9, 4, 8, 8, 6, 7, 3, 11
(d) 84, 93, 89, 95, 98, 105
(e) 105, 110, 104, 108, 113, 120, 118, 118
(f) 1, 2, 3, 4, 5, 6, 7, 8, 9, 10
(g) 10, 15, 20, 25, 30, 35, 40, 45, 50, 55

4 Two groups of boys, A and B, are selling raffle tickets. The number sold by each boy is shown.

Group A		Group B	
John	12	Rani	10
Jo	15	Shea	16
Ali	18	Ben	18
		Bola	15
		Ed	11

(a) For each group, find the total number of tickets sold.

(b) Is is fair to compare the totals, to see which group did better? Why?

(c) Find the mean number of tickets sold per boy in Group A.

(d) Find the mean number of tickets sold per boy in Group B.

(e) In which group did the boys do better?

5 (a) If the mean of 8, 5, 17, *a*, 25 is 17.6, find *a*.

  (b) If the mean of 72, *b*, 87, 91, 95, 106 is 89, find *b*.

6 The table shows the scores of 30 students in a test.

Score	1	2	3	4	5	6	7	8	9	10
No. of students	2	0	3	2	3	5	8	4	2	1

Follow the flow chart below, to find the mean score for the 30 students.

> Multiply each score by its frequency

↓

> Add all the answers to give the total score.

↓

> Divide this total by the number of students.
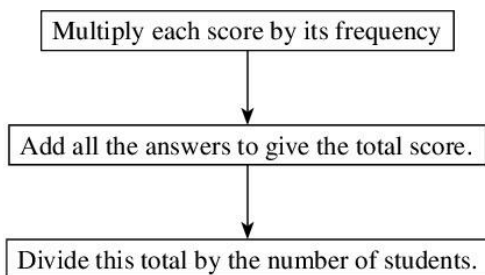
7 The table shows the ages of 20 students who entered a music competition.

Age in years	12	13	14	15	16	17
Number of students	4	4	5	3	3	1

(a) Find the total age of the 20 entrants.

(b) Find the mean age of the entrants.

8 Mr Bede sells 10 different sorts of comic. Here are the comics and their prices:

$1.50 each: Wanda, Metal Man, Super G
$2.50 each: Horror Stories, Spy Scope, Fan
$4 each: Kandy, Rag Mag, 3001, Terry

(a) If you buy one copy of each comic, what is the total cost?

(b) What is the mean price of a comic in Mr Bede's shop?

9

Mrs Akim sold 30 pairs of shoes today. She made $4.75 profit per pair on ten of the pairs. She made $3.55 profit per pair on fourteen pairs. She made $2.80 profit per pair on the remaining shoes.

(a) What was her total profit?

(b) Find the mean profit that Mrs Akim made on a pair of shoes today.

10 The average (mean) mass of Tom, Mick and Harry is 51 kg. Tom weighs 47.5 kg and Mick weighs 52 kg.

(a) What is the total mass of the three boys?

(b) How much does Harry weigh?

11 For each statement, say whether the average used is the mean, median or mode. Give a reason for your answer.

(a) In Trinidad, the average number of children per family is 4.3.

(b) In Guyana, the average age for a man to marry is twenty-four.

(c) The average number of days in a year in our calendar is 365.

(d) The average number of days in a month is 31.

Technology 1

You have been asked to choose a cricket World Eleven. The team should include the current world's best batsmen and bowlers. Don't forget a wicket keeper.

Visit

www.cricinfo.com

to find the recent form of top players. Decide your team based on batting and bowling averages. Write up your choice of team, giving reasons for your decisions.

Exercise 9K – mixed questions

1

Subject	Maths	Physics	Biology	Sociology	Accounting
No. of graduates	15	12	5	9	19

The table gives the number of graduates by subject from Seaview College in 2009.

(a) Calculate the number of students that graduated.

(b) What angle on a pie chart would represent one graduate?

(c) Draw a pie chart to show the information.

2
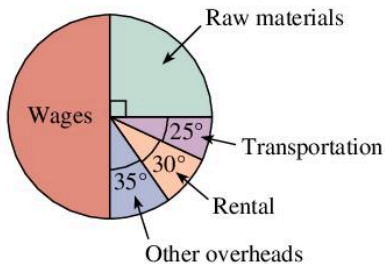
The pie chart illustrates how a firm spent its budget of $540 000 for a year on rental, raw materials, transportation, wages and other overheads.
Calculate the amount spent on each area.

3 The table shows the number of people employed at a building firm.

Types of personnel	Number employed
Unskilled workers	30
Masons	25
Carpenters	25
Draughtsmen	7
Tilers	3

Draw a bar chart to represent this information.

4 The bar graph shows the heights of some men.
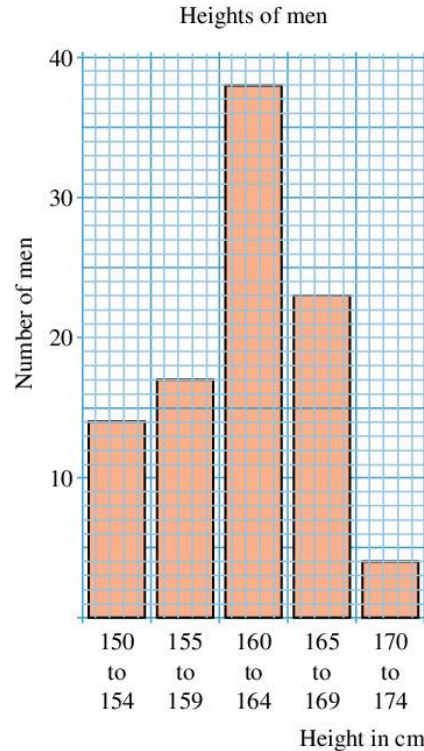
(a) How many men are represented in the bar chart?

(b) How many men are shorter than 160 cm?

(c) How many men are 165 cm tall or taller?

(d) How many men have a height of between 154 cm and 160 cm?

5 The number of tickets bought per person for a Reggae show are shown in the table.

No. of tickets bought	Frequency
1	13
2	42
3	37
4	25
5	3

Calculate the mean number of tickets bought per person for the Reggae show.

6 The pie chart shows how a local government agency allocates a budget to different areas.
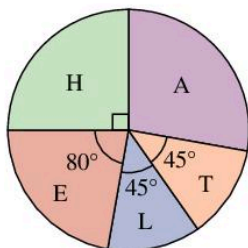
A: Agriculture
E: Education
H: Health
L: Labour
T: Transportation

The agency allocates $30 000 to Education.
Calculate:
(a) the total budget
(b) the amount allocated to Agriculture.

7

The mean length for four snakes is 51.4 cm.
The lengths for three of the snakes are 48.0 cm, 52.2 cm and 55.3 cm.
(a) What is the total length of the four snakes?
(b) How long is the fourth snake?
(c) A fifth snake of length 53.4 cm is added to the show case. What is the mean length of the five snakes?

8 The masses of 50 pupils in a school are shown in the table.

Mass (kg)	No. of pupils
53	8
54	7
55	12
56	10
57	13

Find the median mass for the group of children.

9 The mean height of nine choir members is 161 cm. Calculate the mean height if:
(a) a man of height 165 cm leaves the choir.
(b) a woman of 151 cm joins the **original** choir.

10 Five packets of spices have a mass of 21 g each. Three have a mass of 18.5 g each and two have a mass of 19.4 g each. What is the mean mass of the packets?

11 The table shows the number of children per family in 18 families.

No. of children	1	2	3	4	5
No. of families	3	5	6	3	1

Find
(a) the mode
(b) the median
(c) the mean.

12 Del did a maths test every week for nine weeks. Here are his scores out of 100:

5, 4, 86, 9, 4, 8, 8, 4, 7

(a) Find the mean, median and mode for the set of scores.
(b) Which of these averages do you think best describes how Del scored in his nine tests? Give reasons for your answer.

9 Consolidation

Example 1

The heights of 30 children in centimetres are:

128	143	162	152	147	143	137	129	145	152
132	137	141	146	149	153	151	148	147	161
126	133	142	146	138	139	156	151	149	143

Construct a frequency table using the groups
125–129, 130–134 etc.

Height	125–129	130–134	135–139	140–144
Tally	|||	||	||||	LHH
Frequency	3	2	4	5

145–149	150–154	155–159	160–164
LHH |||	LHH			||
8	5	1	2

Example 2

A class of 30 seven-year-olds were asked what sort
of job they would like when they grew up. Their
answers were as follows:

Occupation	Teacher	Policeman
Number of students	10	8

Doctor/Nurse	Farmer	Other
4	3	5

Draw a pie chart to show this information.
In the pie chart 360° represents the 30 children.
That is, one student is represented by $360 \div 30 = 12$

Angle representing teachers	$= 10 \times 12 = 120°$
Angle representing policemen	$= 8 \times 12 = 96°$
Angle representing doctors/nurses	$= 4 \times 12 = 48°$
Angle representing farmers	$= 3 \times 12 = 36°$
Angle representing others	$= 5 \times 12 = 60°$

Desired job
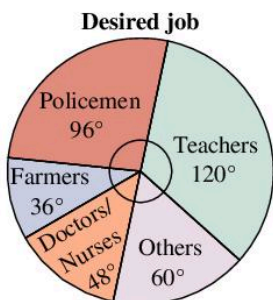
Example 3

The shoe sizes of ten boys are:
3, 7, 4, 6, 7, 6, 5, 3, 8, 6
Find the:
(a) mean
(b) mode
(c) median
shoe size.

(a) Mean $= \dfrac{3 + 7 + 4 + 6 + 7 + 6 + 5 + 3 + 8 + 6}{10}$

$= \dfrac{55}{10} = 5.5$

(b) Mode = most frequently occurring number
= 6 (occurs 3 times)

(c) To find the median write in size order:
3, 3, 4, 5, 6, 6, 6, 7, 7, 8

There are two middle numbers 6 and 6,
so median $= \dfrac{6 + 6}{2} = \dfrac{12}{2} = 6$.

Exercise 9

1 The weights of each of the students in Form 101
were recorded in kilograms as follows:

36, 34, 42, 53, 52, 45, 36, 47, 38, 50
47, 35, 39, 47, 44, 43, 51, 60, 46, 49
52, 38, 42, 43, 53, 41, 53, 61, 47, 50

(a) Construct a frequency table using the groups
30–34, 35–39 etc.
(b) Draw a bar chart to show this data.

2 The ages of the students in Form 101 are:

11, 12, 12, 11, 11, 13, 12, 12, 13, 10,
14, 12, 13, 13, 11, 12, 12, 13, 13, 14,
11, 11, 12, 14, 13, 12, 13, 10, 14, 13

(a) Construct a frequency table to show these
ages.
(b) Draw a bar chart to show this data.
(c) What is the most common age of this class?

3   Display each of these data sets on a pie chart.
  (a) 60 students select sports options as follows:
    20 choose football   10 choose cricket
    15 choose rounders   15 choose netball
  (b) 120 adults travel to work a follows:
    60 walk   20 car
    10 bicycle   30 bus
  (c) 72 doctors from the region attend a
    conference as follows:
    8 from St Lucia   12 from Dominica
    24 from Trinidad   8 from Grenada
    8 from St Vincent   12 from Barbados

4   Find the mean, mode and median of these sets of
  numbers.
  (a) 7, 3, 5, 4, 6, 2, 5
  (b) 54, 32, 56, 48, 32, 56
  (c) 12, 13, 11, 14, 13, 10, 8, 7, 13, 12

Application

5   Find the heights of all the students in your class.
  (a) Make a frequency table using groups
    130–134 cm, 135–139 cm etc.
  (b) Construct a bar graph to show this data.
  (c) How many students have heights over
    150 cm?

6   Using the data collected in Question 5, find the:
  (a) mean height   **(b)** median height
  (c) modal height of the students in your class.

7   Ask 30 students to name the sport they like best.
  (a) Draw a bar graph to show your results.
  (b) Draw a pie chart to show the same data.
  (c) Which representation do you prefer?
    Explain why.

8   **(a)** Conduct a survey in your class to find out
    whether girls use cell phones more than boys.
  (b) Write up or word process your findings.
  (c) Present your results to the class.

Summary

You should know ...

1 You can show data on a frequency table with the help of a tally.

For example:

The letters

a, e, i, i, a, e, i, u, i, o, a, i, i, i, u

can be shown on a frequency table.

Letter	Tally	Frequency
a	|||	3
e	||	2
i	⫲⫲ ||	7
o			1
u	||	2

2 A bar chart can be used to represent data from a frequency table.

For example:
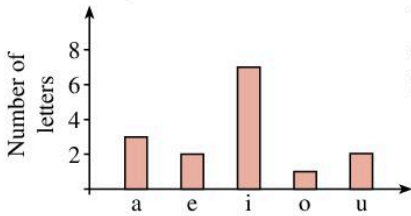
This bar chart shows the data from the frequency table above.

3 You can show information using a pie chart.

For example:

The favourite indoor games of 15 boys are:

| Dominoes | 7 | Draughts | 4 |
| Cards | 3 | Scrabble | 1 |

The 15 boys are represented by $360°$ so 1 boy is represented by $\frac{360}{15} = 24$

The angles representing games liked are:

Dominoes: $7 \times 24 = 168°$
Cards: $3 \times 24 = 72°$
Draughts: $4 \times 24 = 96°$
Scrabble: $1 \times 24 = 24°$
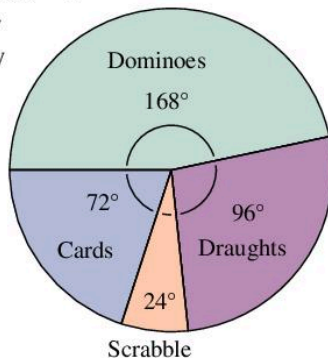
(Total angle = 168 + 72 + 96 + 24 = 360)

Check out

1 Draw a frequency table to show these marks obtained by 20 students in a test.

5, 5, 7, 6, 4, 3, 7, 5, 7, 8,
5, 6, 6, 4, 8, 4, 3, 6, 7, 7.

2 Draw a bar chart to show the data in Question 1.

3 The favourite colours of 20 girls are:

Yellow 3
Blue 6
Green 2
Red 5
Orange 4

Draw a pie chart to show this.

4   To conduct a survey you need to:
  - decide on the question you want answered
  - collect data to answer your question
  - analyse your data
  - interpret your results.

4   Conduct a survey in your class
     to find out whether girls watch
     more television than boys.
     (a) Decide on the data you need
         to collect.
     (b) Make a data collection sheet
         and collect the data.
     (c) Analyse your data and draw
         suitable charts to illustrate
         your results.
     (d) Word process your results so
         as to give a reasoned answer
         to the question.

5   There are three averages: mean, median and mode.
     (a) The mean of a set of numbers is

$$\frac{\text{sum of all the numbers}}{\text{total frequency}}$$

     (b) The median is the middle number in a set when the
         numbers are arranged in order.
     (c) The mode is the most common number in a set.

5   Find the:
     (a) mode
     (b) median of the following sets
         of numbers:
         (i) 6, 8, 3, 6, 4, 2, 6
         (ii) 5, 7, 4, 5, 10, 8
     (c) Find the mean of:
         (i) 3, 4, 5, 3, 6, 3, 4, 3, 5, 6.
         (ii)

Number	Frequency
4	6
5	5
6	4

Patterns and algebra

Objectives

- ✔ learn more about different number patterns
- ✔ use formulas to show relationships
- ✔ simplify simple algebraic expressions
- ✔ use linear equations to solve simple problems
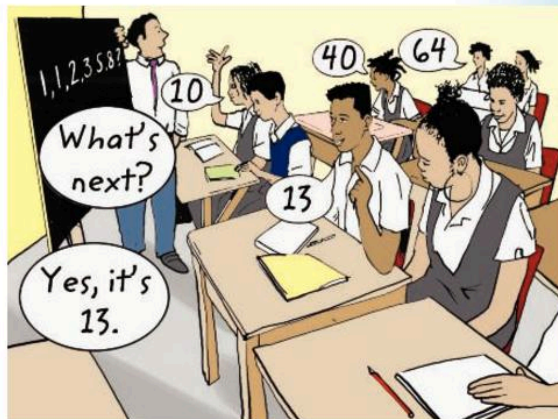
What's the point?

How many ways can three people line up to form a queue? 4 people? 5 people? 640 people? Searching for patterns and relationships that describe these patterns are at the heart of mathematics.

Before you start

You should know ...

1 A factor is a number that divides exactly into another number.
For example:
The factors of 12 are 1, 2, 3, 6 and 12.

2 A prime number is a number that has exactly two factors, itself and 1.
For example:
The only factors of 19 are 19 and 1
so 19 is a prime number.

3 Numbers with more than two factors are called composite numbers.
For example:
The factors of 9 are 1, 3 and 9 so 9
is a composite number.

Check in

1 Write down all the factors of:
(a) 4 (b) 10 (c) 24
(d) 35 (e) 72

2 Which of these numbers are prime?
(a) 3 (b) 15 (c) 51
(d) 53 (e) 87

3 Write down three composite numbers bigger than 20 but less than 30.

10.1 Rectangular, prime and square numbers

You can represent numbers by dots.
The dots of composite numbers can be lined up to form rectangles.
The length and width of the rectangle are factors of the composite number.
For example, 24 can be represented by:

12 dots, 2 dots
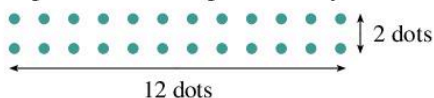
So, 2 and 12 are factors of 24.

- **Composite numbers** are also called **rectangular numbers.**

Exercise 10A

1   (a) What shape are these patterns?

(b) How many dots are in each pattern?
(c) Can you tell how many dots there are, without counting each dot?

2   Copy and complete, using the words from the list {prime, composite, factors, multiple}.
(a) 6 and 4 are ___ of 24.
(b) 24 is a ___ of both 6 and 4.
(c) 24 is a ___ number.

3   Using the dot patterns in Question 1 to help you, write down four factors of 24.

4   (a) Represent the following numbers with rectangular dot patterns:
(i) 16   (ii) 28   (iii) 36
(iv) 49   (v) 56   (vi) 96.
(b) Find all the factors you can from your rectangles.

Prime numbers

It is impossible to draw a rectangle of dots for the number 5:

not a rectangle   a line of dots (not a rectangle)
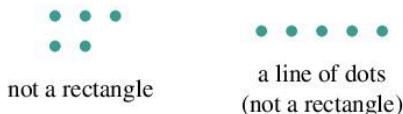
Numbers that cannot be shown as a rectangle of dots are **prime** numbers. A prime number has exactly *two* factors, itself and 1.

Exercise 10B

1   Is 1 a prime number? Explain.

2   Write down all the prime numbers less than 50.

3   Write down all the whole numbers from 1 to 60 in rows of 6:

1	2	3	4	5	6
7	8	9	10	11	12
13					

(a) Shade in all the prime numbers.
(b) What do you notice? Explain.
(c) What happens if you use rows of 4?

Investigation

In whatever order the digits 3 and 7 are arranged they make prime numbers:
37, 73
What other pairs of digits make prime numbers in whatever order they are arranged?
What if you used three-digit numbers?

Square numbers

You can show 36 as a *square* of dots:

So 36 is a square number.

36 can be written

as $36 = 6 \times 6$

or $36 = 6^2$ which is read as six squared.

- A **square number** can be made by multiplying a whole number by itself.

Exercise 10C

1 Find all the square numbers from 1 to 625.

2 Carry on this pattern by drawing similar squares to represent 16, 25, 36, 49, 64, 81.

$4 = 1 + 3$ $9 = 1 + 3 + 5$
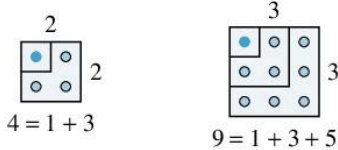
3 Study this pattern carefully.

$$1 + 3 = \left(\frac{1+3}{2}\right)^2 = 2^2$$

$$1 + 3 + 5 = \left(\frac{1+5}{2}\right)^2 = 3^2$$

$$1 + 3 + 5 + 7 = \left(\frac{1+7}{2}\right)^2 = 4^2$$

Work out:

(a) $1 + 3 + 5 + 7 + 9$
(b) $1 + 3 + 5 + 7 + 9 + 11$
(c) $1 + 3 + 5 + 7 + 9 + \ldots + 19$
(d) $1 + 3 + 5 + 7 + 9 + \ldots + 29$
(e) $1 + 3 + 5 + 7 + 9 + \ldots + 39$
(f) $1 + 3 + 5 + 7 + 9 + \ldots + 49$

4 Using the method of Question 3 write down the sum of:

(a) the first ten odd numbers
(b) the first twelve odd numbers
(c) the first twenty odd numbers
(d) the first twenty-five odd numbers
(e) the first n odd numbers.

Even and odd numbers

Dots can also be used to show odd and even numbers.

Even numbers can be shown by a rectangle with two rows of dots.

Even   Odd
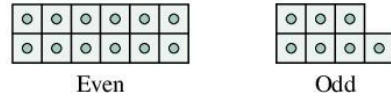
Odd numbers can be shown by a rectangle with two rows of dots, with one dot missing.

Exercise 10D

1 (a) Draw dot patterns to represent the first 10 even numbers.
  (b) Pick two numbers from part (a) and add them together. Is the answer even?
  (c) When you add two even numbers, is the answer always even? Explain.

2 (a) Draw dot patterns to represent the first 10 odd numbers.
  (b) Pick two numbers from part (a) and add them together. Is the answer odd?
  (c) When you add two odd numbers, is the answer ever odd? Explain.

3 The 1st even number is 2 (2×1)
  The 2nd even number is 4 (2×2)
  The 5th even number is 10 (2×5)

  (a) What is the 10th even number?
  (b) What is the 17th even number?
  (c) What is the nth even number?

4 The 1st odd number is 1 $(2 \times 1 - 1)$
  The 2nd odd number is 3 $(2 \times 2 - 1)$
  The 5th odd number is 9 $(2 \times 5 - 1)$

  (a) What is the 13th odd number?
  (b) What is the 20th odd number?
  (c) What is the nth odd number?

Σ+ Investigation

Look at the number 8.
Its square is $8^2 = 64$.
Both of the digits 6 and 4 are even.
What other numbers have squares with all their digits even?

Sum of whole numbers

> ### Example 1
>
> Find the sum of the first five whole numbers:
> First, show the numbers as bricks. Use one brick for each number.
>
>
>
> 1 2 3 4 5
>
> Now draw a second set of bricks like these.
> Draw it upside down on the first set.
>
>
>
> There are 6 bricks down the side.
> There are 5 bricks along the bottom.
> The total number of bricks in the *two* sets is
> $$6 \times 5 = 30$$
> So there are 15 bricks in *one* set.
> The sum of the first five whole numbers is 15.
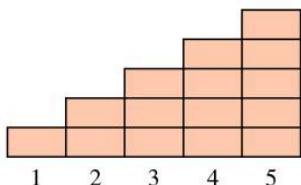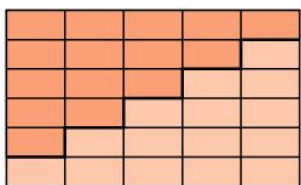
Exercise 10E

1   Using the method of Example 1, find the sum of:
    (a) the first seven whole numbers
    (b) the first nine whole numbers.

2   Without doing the addition or drawing bricks, find the sum of:
    (a) the first ten whole numbers
    (b) the first twelve whole numbers
    (c) the first n whole numbers.

3   By drawing bricks find the sum of:
    (a) the first four odd numbers
    (b) the first four even numbers
    (c) the whole numbers from 4 to 8.
    Does the method work for all of these?

4   Without drawing bricks or doing the addition, find the sum of:
    (a) the whole numbers from 7 to 13
    (b) the whole numbers from 10 to 20.

10.2   Triangle numbers

A number which can be shown as a triangle of dots, with the same number of dots on each edge is called a **triangle number**.

For example:
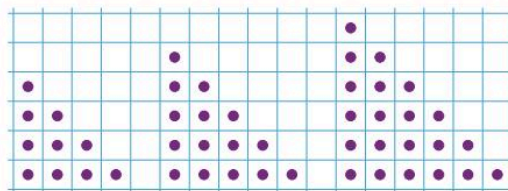
The first four triangle numbers are
1, 3, 6, 10.

Exercise 10F

1   Use dots to show the first eight triangle numbers.

2   (a) Copy and complete the additions.
    $$6 + 5 + 4 + 3 + 2 + 1 = \square$$
    $$5 + 4 + 3 + 2 + 1 = \square$$
    $$4 + 3 + 2 + 1 = \square$$
    $$3 + 2 + 1 = \square$$
    $$2 + 1 = \square$$
    $$1 = \square$$

    (b) Which of the answers to part (a) are triangle numbers?

3   The triangle numbers form the set
    $\{1, 3, 6, 10, 15, 21, 28, 36, \ldots\}$

    (a) What do you add to the 2nd triangle number, to get the 3rd triangle number?
    (b) What do you add to the 5th triangle number, to get the 6th?
    (c) Write down the 9th and the 10th triangle numbers.

4 **(a)** Look at the patterns of black and white dots. What shape is each pattern?

○ ○ ○ ○
○ ○ ○   ● ○ ○ ○
○ ○   ● ○ ○   ● ● ○ ○
● ○   ● ● ○   ● ● ● ○

○ ○ ○ ○ ○ ○
○ ○ ○ ○ ○   ● ○ ○ ○ ○ ○
● ○ ○ ○ ○   ● ● ○ ○ ○ ○
● ● ○ ○ ○   ● ● ● ○ ○ ○
● ● ● ○ ○   ● ● ● ● ○ ○
● ● ● ● ○   ● ● ● ● ● ○
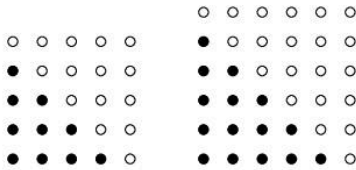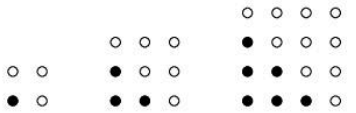
(b) Copy and complete:
$$2^2 = 1 + 3$$
$$3^2 = 3 + 6$$
$$4^2 = 6 + \square$$
$$5^2 = \square + \square$$
$$6^2 = \square + \square$$

5 What number do you get when you add:
(a) the 1st and 2nd triangle numbers
(b) the 2nd and 3rd triangle numbers
(c) the 5th and 6th triangle numbers
(d) the nth and $(n + 1)$th triangle numbers?
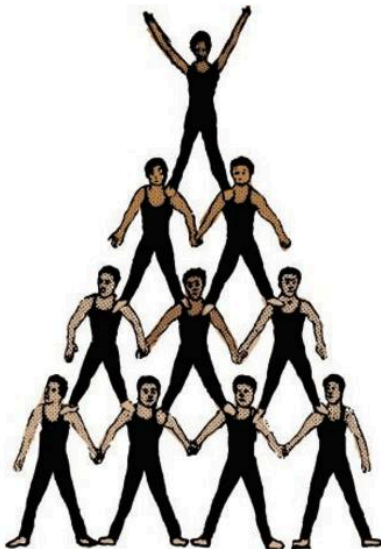
6 In the dot pattern, each triangle number is paired with itself, and makes a rectangle (except 1 which makes a line).
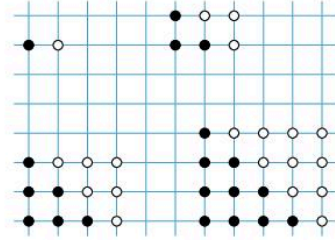
The first pattern has 1×2 dots.
The second pattern has 2×3 dots.
The third pattern has 3×4 dots.
The fourth pattern has 4×5 dots.
How many dots would be in:
(a) the 5th pattern
(b) the 10th pattern
(c) the 11th pattern
(d) the nth pattern?

7 Look again at the dot pattern in Question 6. The second triangle number is half the number of dots in the second pattern.

So it is $\dfrac{2 \times 3}{2}$.

The third triangle number is $\dfrac{3 \times 4}{2}$.

Use your answers to Question 6 to help you write down:
(a) the 5th triangle number
(b) the 10th triangle number
(c) the 11th triangle number
(d) the nth triangle number.

8 Write down:
(a) the 12th triangle number
(b) the sum of the whole numbers from 1 to 12.

9 Write down:
(a) the 99th triangle number
(b) the sum of the whole numbers from 1 to 99.

10 Work out these additions by first finding a triangle number.
(a) $1 + 2 + 3 + 4 + 5 + 6 + 7 + 8$
(b) $1 + 2 + 3 + \ldots + 13 + 14$
(c) $1 + 2 + 3 + \ldots + 19 + 20$
(d) $1 + 2 + 3 + \ldots + 24 + 25$
(e) $1 + 2 + 3 + \ldots + 91 + 92$
(f) $1 + 2 + 3 + \ldots + 199 + 200$
(g) $1 + 2 + 3 + \ldots + 2n$

11 Pick out the words in the box that describe the number 1.

composite, prime, square, rectangle, triangle

12 Copy and complete.
(a) When you add whole numbers in order, beginning at 1, the answer is always a . . . number.
(b) When you add a triangle number to the next triangle number, the answer is a . . . number.
(c) The nth triangle number is . . .
(d) When you add odd numbers in order, beginning at 1, the answer is a . . . number.
(e) The sum of the first n odd numbers is . . .

13 (a) Copy the Venn diagram and fill in the whole numbers from 1 to 30.

Whole numbers from 1 to 30
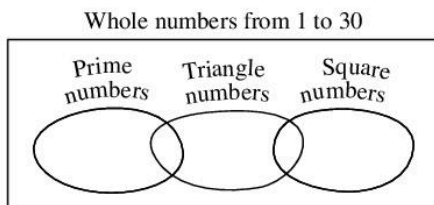
(b) Look at the numbers that are outside the small loops, in your Venn diagram. What are these numbers?

Technology

Watch the video TRIANGULAR NUMBERS (PART I) at

www.isallaboutmath.com

which summarises what you learnt about triangular numbers. Extend yourself by watching the second part of the video. Can you do the final problems?

10.3 Formulas

In Sections 9.1 and 9.2 you found expressions for the nth term of different number patterns.

For example,
the even numbers:
E_1 = first even number $= 2 \times 1 = 2$
E_2 = second even number $= 2 \times 2 = 4$
E_3 = third even number $= 2 \times 3 = 6$
E_4 = fourth even number $= 2 \times 4 = 8$
E_n = nth even number $= 2 \times n = 2n$
That is, the nth even number E_n is

$$E_n = 2n.$$

This equation is called a formula. This particular formula gives a simple way of finding even numbers.

Example 2

What is the 58th even number?
Use $E_n = 2n$ with $n = 58$
$E_{58} = 2 \times 58 = 116.$
So the 58th even number is 116.

Exercise 10G

1 Find the value of the following *even numbers* using the formula, $E_n = 2n$.
(a) 23rd   (b) 65th   (c) 75th
(d) 92nd   (e) 102nd

2 The formula for the nth odd number O_n is
$O_n = 2n - 1$.
What is:
(a) the 23rd odd number
(b) the 54th odd number
(c) the 95th odd number?

3 The formula for the nth square number S_n is
$S_n = n^2$.
What is:
(a) the 4th square number
(b) the 18th square number
(c) the 21st square number?

4 The formula for the nth triangle number, T_n, is $T_n = \frac{1}{2}n \times (n+1)$

What is:

(a) the 6th triangle number

(b) the 23rd triangle number

(c) the 25th triangle number?

Σ▸ **Investigation**

Take a piece of paper. Cut it in half. Put the pieces on top of each other and cut them in half again. Repeat the process.
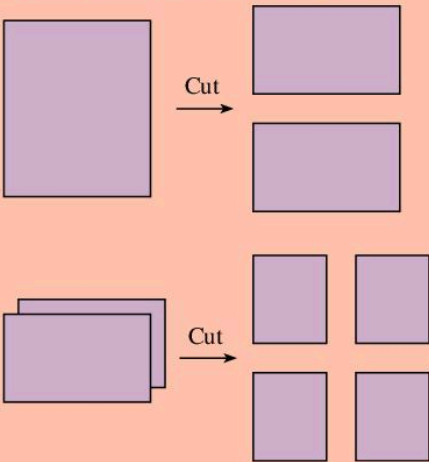
(a) Copy and complete the table.

Number of cuts	Number of pieces of paper
0	1
1	2
2	4
3	
4	
5	

(b) How many pieces of paper would there be after **(i)** 10 cuts **(ii)** 20 cuts?

(c) There are P pieces of paper after n cuts. Write down a formula for P.

Example 3

The area A of a rectangle is given by the formula
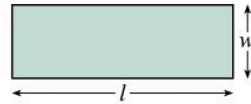
$$A = l \times w.$$

Find the area if $l = 5$ cm and $w = 3$ cm

$$A = 5 \text{ cm} \times 3 \text{ cm}$$
$$= 15 \text{ cm}^2$$

Exercise 10H

1 The perimeter P of a rectangle is given by the formula:

$$P = 2l + 2w$$

where l is the length and w is the width of the rectangle.

Find P if:

(a) $l = 3$ cm, $w = 4$ cm

(b) $l = 7$ cm, $w = 5$ cm.

2 The circumference C of a circle is given by the formula:

$$C = 2\pi r$$

where r is the radius of the circle and take $\pi = 3.14$.

Find C if:

(a) $r = 6$ cm   **(b)** $r = 13$ cm.

3 The volume of a cuboid V is given by the formula:

$$V = l \times w \times h$$

where l is the length, w is the width and h is the height of the cuboid.

Find the volume if:

(a) $l = 3$ cm, $w = 4$ cm, $h = 2$ cm

(b) $l = 5$ cm, $w = 6$ cm, $h = 2.5$ cm.

You have already met other formulas.

For example, formulas giving the area of a rectangle, $A = l \times w$, or the circumference of a circle, $C = 2\pi r$. These often involve more than one unknown quantity or variable.

4 The number of dollars, d, that can be made from c cents is given by:

$$d = \frac{c}{100}$$

How many dollars are there in:
(a) 1400 cents **(b)** 358 cents?

5 The distance d kilometres walked by a man is given by:
$d = s \times t$
where s is his speed in km/h and t is the time in hours he was walking.

Find the distance he walks if:
(a) $s = 3, t = 5$
(b) $s = 4\frac{1}{2}, t = 3$.

6 The density, D, of a liquid is given by the formula:

$$D = \frac{m}{V}$$

where m is the mass of the liquid in grams and V is the volume of the liquid in cm^3.

Find the density of a liquid if:
(a) $m = 30, V = 40$
(b) $m = 3.6, V = 4$.

7 If $P = 100r - t$, find the value of:
(a) P when $r = 0.5$ and $t = 15$
(b) P when $r = 0.15$ and $t = 10$.

8 Given that $N = 2(m - n)$, find the value of:
(a) N when $m = 10$ and $n = 4$
(b) N when $m = 25$ and $n = 12$.

9 Given that $S = \frac{1}{2}(b - a)$, find the value of:
(a) S when $a = 24$ and $b = 36$
(b) S when $a = 15$ and $b = ^-4$
(c) S when $a = ^-10$ and $b = ^-25$.

Making up formulas of your own

Formulas can represent all sorts of situations.

Example 4

A tin contains P peas. There are N tins of peas. Write down a formula giving the total number of peas, T, in the tins.

Number of peas in one tin $= P$
Number of peas in two tins $= 2 \times P$
so   Number of peas in N tins $= N \times P$
    So $T = N \times P$

Exercise 10I

1 Write down a formula for the perimeter P of each of these shapes:

(a)
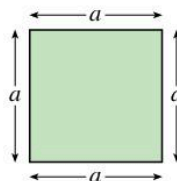
(b)
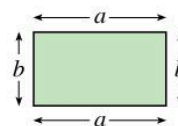
(c)
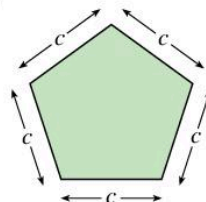
2 A class has B boys and G girls. The class has S students altogether. Write down a formula for the total number of students in the class, starting $S =$.

3 A glass of water has a total mass of M grams. The glass has mass G grams. Write down a formula giving the mass of the water, W.

4 Avril scored H marks in a History test and G marks in a Geography test. Her total score in both tests was T marks. Write down a formula, starting $T =$, and involving H and G.

5 M minibuses carry a total of T people. The number of people that one minibus holds is P. Write down a formula, starting $P =$, and involving M and T.

6 (a) Find the cost, C, of M mathematics books which cost \$7 each.
(b) Find the cost, K, of M mathematics books which cost \$$A$ each.

7 (a) Find the cost, C, of P pencils at 40 cents each and E erasers at 25 cents each.
(b) Find the cost, K, of P pencils at A cents each and E erasers at B cents each.

8 (a) B boys and G girls travel home by bus. The fare is \$2 each.
Find the total cost, C, of the fares.
(b) What is the total cost if the fare was D dollars?

9 A bucket holds W litres of water. John pours X litres of water away. Find how many litres, L, are left in the bucket.

10 A square tile has side length t cm.
(a) What is the area A of the tile?

(b) How many tiles, N, can be fitted into a room with floor area R cm^2?
(c) If each tile costs \$3, what is the cost, C, of tiling the room?

11 Tony sells A papers on Monday, B on Tuesday, C on Wednesday, D on Thursday, E on Friday and F on Saturday.

(a) How many papers, P, does he sell altogether?
(b) If each paper costs Y cents, how much money, M, does he bring in each week?

12 The cost of a knife is x dollars, a fork y dollars and a spoon z dollars. What is the cost, C, of a complete set of cutlery for:
(a) 4 persons   (b) P persons?

13 A man walks at n kilometres per hour for 6 hours.
(a) How far does he walk?
(b) If he walks at m kilometres per hour for another 3 hours, write down the total distance, D, that he travels.

14 Joanne buys M mangoes and G grapefruits in the market. Mangoes cost 45 cents, grapefruits 25 cents. Joanne paid a total of T dollars.
(a) How much did Joanne pay for her mangoes, in dollars?
(b) How much did Joanne pay for her grapefruits, in dollars?
(c) Write down a formula for T.

15 A cuboid has length a cm, width b cm and height c cm.

(a) How many faces does the cuboid have?
(b) What is the area of each face?
(c) The total surface area of the cuboid is S cm^2. Write down a formula for S.

Technology

Learn about another important number sequence—the Fibonacci sequence—by visiting

mathsisfun.com/numbers/fibonacci-sequence. html

Make a scrapbook or electronic file of pictures or photographs to illustrate this sequence.

10.4   Basics of algebra

In the previous section you saw how letters were used to generalise expressions or to write simple formulas.

Algebra is the study of generalised arithmetic that uses letters or symbols to represent numbers.

The rules of algebra are the same as those that apply for numbers.

$$2 + 2 + 2 = 3 \times 2$$
$$6 + 6 + 6 = 3 \times 6$$
$$a + a + a = 3 \times a$$
$$x + x + x = 3 \times x$$

$3 \times x$ is written $3x$ for short.
$3 \times a$ is written $3a$ for short.
3×2 is *not* written 32 for short!

Exercise 10J

1   Simplify:
    (a) $3 + 3 + 3 + 3$ **(b)** $b + b + b + b + b$
    (c) $f + f$ **(d)** $x + x + x + x$

2   Write out in full:
    (a) $6x$
    (b) $5y$
    (c) $4h$

This is an algebraic expression:

$$3a + 2b - 5c$$

These are called **terms**.

In an expression, you can collect **like terms**:

$$6x + 2y + x + 3y$$

These are   These are
like terms   like terms

$$= \quad 7x \quad + \quad 5y$$

so $6x + 2y + x + 3y = 7x + 5y$.

To **simplify** an algebraic expression you have to collect like terms.

Example 5

Simplify:
(a) $9t - 7t + 2t$
(b) $5a + 2b - 3a + 6b - a - b$

(a) $9t - 7t + 2t = 4t$
(b) $5a + 2b - 3a + 6b - a - b$
     $= 5a - 3a - a + 2b + 6b - b$
     $= a + 7b$

Exercise 10K

1   Simplify:
    (a) $3p + 2p + 5p$
    (b) $5s + 8s - 4s$
    (c) $15t - 3t + 8t$
    (d) $6l + 14l - 9l$

2   Rearrange these expressions putting like terms together.
    (a) $2a + b + a - 3b$
    (b) $x + 3y + 2x - 4y$
    (c) $f + 2g - 3f + g + f - 5$

3   Simplify:
    (a) $4a + 2b + 3a + 4b$
    (b) $a + b + 2a - b$
    (c) $c + 3d - 2c + 2d$
    (d) $2p - 3q - p + 4q$
    (e) $-3x + 5y + 5x - 3y$
    (f) $4f + 2g + h + 2f - g$
    (g) $r + 4s - 2t - r - 3s + t$
    (h) $m + 4n - 4p + 2n + p - 2m$
    (i) $5x + 2y + 3 - 2x - y - 2$
    (j) $14 - 4x + 3y - 2 + 6x - 2y$

4   You can write $a \times b$ in a shorter way as ab.
    Write in a shorter way:
    (a) $p \times q$
    (b) $m \times n$
    (c) $s \times t$
    (d) $g \times h$
    (e) $y \times z$
    (f) $u \times v$

5 $3 \times a \times b$ is usually written as $3ab$.
    Write in a shorter way:
    (a) $4 \times p \times q$
    (b) $5 \times 3 \times s$
    (c) $3 \times 4 \times r \times s$

Technology

Review basic ideas of simplifying an algebraic expression by watching some Youtube videos.

Then go over a complete course at

coolmath.com/prealgebra/15-intro-polynomials

Make sure you do the test questions!

Working with indices

You can write $3 \times 3 \times 3 \times 3$ in a short way.

$$3 \times 3 \times 3 \times 3 = 3^4$$

4 is the index or power

3 is the base

$q \times q$ can be written in a shorter way as q^2 (q squared) In the same way.

$$q \times q \times q \qquad = q^3 \text{ (q cubed)}$$
$$q \times q \times q \times q \qquad = q^4 \text{ (q to the fourth power)}$$
$$q \times q \times q \times q \times q = q^5 \text{ (q to the fifth power)}$$

Example 6

Simplify:
(a) $x \times x \times x \times x \times x \times x \times x$
(b) $x \times x \times y \times y \times y$
(a) $x \times x \times x \times x \times x \times x \times x = x^7$
(b) $x \times x \times y \times y \times y = x^2 \times y^3 = x^2 y^3$

You can add powers when multiplying as long as the letters are the same.
For example,
$$q^2 \times q^3 = q \times q \times q \times q \times q = q^5 \quad (2 + 3 = 5)$$
You can subtract powers when you divide as long as the letters are the same.
For example,
$$q^5 \div q^2 = \frac{\cancel{q} \times \cancel{q} \times q \times q \times q}{\cancel{q} \times \cancel{q}} = q^3 \quad (5 - 2 = 3)$$

Example 7

Simplify:
(a) $x^3 \times x^5$
(b) $y^{13} \div y^9$

(a) $x^3 \times x^5 = x^{3+5} = x^8$
(b) $y^{13} \div y^9 = y^{13-9} = y^4$

Exercise 10L

1   Simplify:
(a) $x \times x \times y \times y$
(b) $p \times p \times p \times p \times q$
(c) $w \times w \times w \times s \times s$
(d) $w \times w \times r \times r \times t \times t$

2   Simplify:
(a) $2 \times m \times n$
(b) $3 \times x \times y$
(c) $3 \times p \times q$
(d) $4 \times p^2 \times q$
(e) $3 \times g \times h^2$
(f) $2 \times a^2 \times b^2$
(g) $6 \times m^3 \times n^2$
(h) $4 \times c \times d^4$
(i) $3 \times m^2 \times n^3$

3   Simplify:
(a) $p^2 \times p^2$
(b) $p^2 \times p \times p$
(c) $p^2 \times p^4$
(d) $p^3 \times p^3$

4   Simplify:
(a) $m^3 \times m^2$
(b) $g^2 \times g^4$
(c) $d^2 \times d^3$
(d) $m^4 \times m^2 \times m$
(e) $q^2 \times q^2 \times q^2$
(f) $e^3 \times e^3 \times e^2$

5   Simplify:
(a) $p^5 \div p^2$
(b) $p^5 \div p^4$
(c) $p^6 \div p^2$

6 Simplify:
    (a) $b^3 \div b^2$
    (b) $g^4 \div g^2$
    (c) $m^6 \div m^3$
    (d) $n^{10} \div n^8$
    (e) $p^7 \div p^5$
    (f) $b^3 \div b$
    (g) $x^{10} \div x$
    (h) $g^{12} \div g^9$
    (i) $m^{15} \div m^{12}$

7 Simplify:
    (a) $g^2 \times g^6$
    (b) $3d^2 \times d^3$
    (c) $5m^4 \times m^2 \times 2n^3$
    (d) $2l^3 \times 3l^2 \times 5l$
    (e) $12b^3 \div 4b^2$
    (f) $7n^{10} \div n^8$
    (g) $\dfrac{10g^5 \times 9g^7}{15g^9}$

🖥 Technology

Need more practice?
Check out

www.coolmath.com/prealgebra

Have fun!

Substituting

When you know the value of the letters, you can find the value of an expression.
You substitute the value into the expression.

Example 8

If $x = 2$ and $y = 3$, find the value of
(a) $5x - 2y$ (b) $x^3 y^2$

(a) $5x - 2y = 5 \times x - 2 \times y$
             $= 5 \times 2 - 2 \times 3$
             $= 10 - 6$
             $= 4$

(b) $x^3 y^2 = x \times x \times x \times y \times y$
          $= 2 \times 2 \times 2 \times 3 \times 3$
          $= 8 \times 9$
          $= 72$

Exercise 10M

1 If $x = 4$, find the value of:
    (a) $2x + 3$ (b) $3x - 2$
    (c) $x^2 + 1$ (d) $2x^2$
    (e) $x^2 + 2x$ (f) $x^2 - 2x + 4$
    (g) $2x^2 + 3x - 5$ (h) $x^3 - 4x - 2$

2 If $p = 2$ and $q = 3$, find the value of:
    (a) $5p - 2q$ (b) $3p + q$
    (c) $7p - 2q$ (d) $4p \times q$
    (e) $p^2 q$ (f) $4p^2 q^2$
    (g) $5pq^3$ (h) $2p^4 - q^3$

3 (a) Write down the perimeter, P, of this shape.

    (b) Find the value of P if $a = 4$ cm, $b = 3$ cm and $c = 3\frac{1}{2}$ cm.

4 A stone is dropped from a height.
The distance, s metres, fallen by the stone after t seconds is given by the formula
$$s = 5t^2$$
Find the distance fallen when:
    (a) $t = 2$ (b) $t = 20$ (c) $t = 0.5$

5 (a) Find the area, A, of this shape in terms of x and y.

    (b) Find the value of A when:
       (i) $x = 4$ cm, $y = 6$ cm
      (ii) $x = 8$ m, $y = 10$ m

6 (a) Find, in terms of p, q and r, the volume, $V = $ length $\times$ width $\times$ height, of this cuboid.

(b) Find the value of V when:
  (i) $p = 4$ cm, $q = 5$ cm, $r = 6$ cm
  (ii) $p = 2$ cm, $q = 4$ cm, $r = 8$ cm.

The commutative and associative laws

An operation is **commutative** if you can combine two amounts in any order and still get the same result:

  $2 + 3$ is the same as $3 + 2$ so the operation $+$ is commutative.

Exercise 10N

1 Write True or False for each statement.
  (a) $3 + 7 = 7 + 3$ **(b)** $5 + 4 = 4 + 5$
  (c) $8 \times 5 = 5 \times 8$ **(d)** $3 \times 9 = 9 \times 3$
  (e) $5 - 4 = 4 - 5$ **(f)** $10 - 6 = 6 - 10$
  (g) $6 \div 2 = 2 \div 6$ **(h)** $18 \div 3 = 3 \div 18$

2 Which of the operations $+$, $\times$, $-$, $\div$ are commutative?

3 If $l * m = 3l + 2m$, find:
  (a) $5 * 4$ **(b)** $4 * 5$

4 If $a \circ b = a^2 + b^2$, find:
  (a) $3 \circ 2$ **(b)** $2 \circ 3$

5 If $x \triangle y = \dfrac{x \times y}{2}$, find:
  (a) $6 \triangle 2$ **(b)** $2 \triangle 6$

6 Which of the operations $*$, $\circ$, $\triangle$ in Questions 3 to 5 are commutative?

7 By working out what is inside the brackets first, write True or False for each statement.
  The first one is done for you.
  (a) $(3 + 4) + 5 = 3 + (4 + 5)$
    $7 + 5 = 3 + 9$
    $12 = 12$
         True.
  (b) $(5 + 6) + 4 = 5 + (6 + 4)$
  (c) $(3 \times 2) \times 5 = 3 \times (2 \times 5)$
  (d) $(6 \times 2) \times 3 = 6 \times (2 \times 3)$
  (e) $(15 - 3) - 2 = 15 - (3 - 2)$
  (f) $(18 - 8) - 4 = 18 - (8 - 4)$
  (g) $(18 \div 6) \div 3 = 18 \div (6 \div 3)$
  (h) $(20 \div 4) \div 2 = 20 \div (4 \div 2)$

In Question 7(a) the brackets make no difference to the answer so the operation, $+$, is **associative**.

8 Which of the operations $+$, $\times$, $-$, $\div$ are associative?

9 If $a * b = 2a + b$. find:
  (a) $3 * 5$
  (b) $2 * 3$
  (c) $2 * (3 * 5)$
  (d) $(2 * 3) * 5$.

Remember to do the brackets first

- Addition and multiplication are **commutative** because for any numbers a and b:
  $a + b = b + a$ and
  $a \times b = b \times a$.

- Addition and multiplication are **associative** because for any numbers a, b and c:
  $(a + b) + c = a + (b + c)$ and
  $(a \times b) \times c = a \times (b \times c)$.

10.5 Brackets

You can rewrite an expression containing brackets.
For example:
  $2(x + 3)$
You multiply each term inside the bracket by the term in front of the bracket:

$$2(x + 3) = 2 \times x + 2 \times 3$$

so $2(x + 3) = 2x + 6$
This is called expanding the brackets.
It illustrates the **distributive law**.
The distributive law says that if a, b and c are any three numbers then

  $a \times (b + c) = a \times b + a \times c$
            $= ab + ac$
  $a \times (b - c) = a \times b - a \times c$
            $= ab - ac$

You can use this idea to make multiplications easier.

Example 9

(a) $43 \times 2 = (40 + 3) \times 2$
          $= 40 \times 2 + 3 \times 2$
          $= 80 + 6$
          $= 86$
(b) $28 \times 3 = (30 - 2) \times 3$
          $= 30 \times 3 - 2 \times 3$
          $= 90 - 6$
          $= 84$

Exercise 10O

1 (a) Using the method in Example 9(a), find:
   (i) 34×2 (ii) 71×6
   (b) Using the method in Example 9(b), find:
   (i) 39×2 (ii) 58×3

2 Expand the brackets.
   (a) $3 \times (p + q)$ (b) $5 \times (p - q)$
   (c) $4 \times (l + m)$ (d) $3(x - y)$
   (e) $6 \times (r + s)$ (f) $5 \times (l - m)$
   (g) $8(p + 5q)$ (h) $9(2x + y)$

3 (a) Simplify, using brackets:
   (i) $a + a + a + a + b + b$
   (ii) $x - y + x - y - y - y$
   (b) If p and q stand for numbers, write down the perimeter of the rectangle. Express your answer in three different ways.

←— p cm —→

q cm

4 If jeans cost $\$j$, a shirt costs $\$s$ and a tie costs $\$t$, write down the cost of:
   (a) two shirts and three ties
   (b) one pair of jeans and four shirts
   (c) three pairs of jeans, three shirts and three ties.
   (d) Express your answer to part (c) in another way.

5

$19.50 each

$8.50 each

$46.50

   (i) Using the prices shown in the picture, find the cost of the items listed in Question 4.
   (ii) Do you find that Question 4 parts (c) and (d) give the same result?

6 Write without brackets:
   (a) $(j + s + t)3 =$
   (b) $4(2j + s + 3t) =$

7 Write $100k + 30l + 4m$ using brackets.

8 Write using brackets:
   (a) $2x + 2y$ (b) $3p + 3q$
   (c) $7k + 7l$ (d) $4x + 4y$
   (e) $5p - 5q$ (f) $6r - 6s$
   (g) $6a + 15b$ (h) $15m - 25n$

Simplifying expressions

Example 10

Expand the brackets.
(a) $5(6x - 3y)$ (b) $3m(4 - 6n)$

(a) $5(6x - 3y) = 5 \times 6x - 5 \times 3y$
$= 30x - 15y$

(b) $3m(4 - 6n) = 3m \times 4 - 3m \times 6n$
$= 12m - 18mn$

Example 11

Simplify:
(a) $5(x + 3y) + 6(x - 2y)$
(b) $a(b + 3) + b(2 + 3a)$

Multiply out brackets

(a) $5(x + 3y) + 6(x - 2y)$
$= 5x + 15y + 6x - 12y$
$= 5x + 6x + 15y - 12y$
$= 11x + 3y$

(b) $a(b + 3) + b(2 + 3a)$
$= ab + 3a + 2b + 3ab$
$= 4ab + 3a + 2b$

Collect like terms.

You write the number in front of the letters.

Exercise 10P

1 Expand the brackets.
   (a) $3(2x + 5)$ (b) $5(3y - 2)$
   (c) $7(p + 3q)$ (d) $4(2u - 5v)$
   (e) $k(3x + 2)$ (f) $l(3 - 5p)$
   (g) $m(2s + 7t)$ (h) $n(5r - 6s)$
   (i) $3n(6 - 2m)$ (j) $7p(q - 4p)$

2 Simplify:
   (a) $3(x + y) + 7(x + y)$
   (b) $6(p + q) + 5(p + q)$
   (c) $5(r + 2s) + 3(r + s)$

(d) $3(2r - 2s) + 4(r + 3s)$
(e) $7(q - 4p) + 6(2q + 5p)$
(f) $4(3x - 2y) + 6(3y - x)$
(g) $7(3 - 2z) + 3(z - 2)$
(h) $14 + 3(y + 4) - 2y$
(i) $7p + 6(4 - 2p) - 7$
(j) $3(x + 2) - 4 + 4(x - y)$

3 Simplify:
(a) $x(x + y) + y(x + y)$
(b) $a(a - 2b) + b(a - 2b)$
(c) $p(q - 2) + q(p - 2)$
(d) $2s(t + s) + t(2t + s)$
(e) $g(2h - g) + 2h(g - h)$
(f) $5(p + 2q) + 2p(q - 5)$
(g) $x^2 + x(y - x) + y(x - 2y)$
(h) $x(y - z) + y(z - x) + z(x - y)$
(i) $\frac{1}{7}(49x + 7) + \frac{1}{8}(64x + 8)$
(j) $\frac{1}{2}(6x + 2) + \frac{1}{5}(25x - 10)$

∑⊩ Investigation

(1) The large rectangle in the diagram has a length of $(x + 5)$ cm and a width of $(x + 2)$ cm. The area is $(x + 5)(x + 2)$ cm². Find the area of each of the four smaller rectangles then copy and complete:
$(x + 5)(x + 2) =$
(2) By using a suitable diagram show that:
$(x + 5)(x + 4) = x^2 + 5x + 4x + 20$
(3) Can you find out:
$(x + a)(x + b)$?

10.6 Solving linear equations

Using number machines

Example 12

Using the given number machine:
(a) find the result if you start with 5
(b) find the result if the starting number is x
(c) find the starting number if the result is 15.

(a)

The result is 13

(b)

The result is $2x + 3$

(c) To do this, use the machine backwards. Just subtract 3 and then divide by 2:
-3 is the inverse of $+3$
$\div 2$ is the inverse of $\times 2$

The original number was 6.

Exercise 10Q

1 For each machine, write down the result when the starting number is **(i)** 5 **(ii)** 12.

(a)

(b)

(c)

2 Draw a diagram to show the reverse machine for each part of Question 1.

3   For each machine in Question 1, find what starting number will give 26 as the result. Use your machines from Question 2 to help you.

4   For each machine in Question 1, write down the result when the starting number is x.

Simple equations can be solved using number machines.

Example 13

Solve the equation
$3x - 5 = 13$
First draw the machine which gives the result $3x - 5$, when you start with x.

Now draw reverse machine

Using the reverse machine you can see that $x = 6$.

Exercise 10R

1   Use the machines in Question 1 of Exercise 10Q and the method in Example 13 to solve the equations:
    (a) $3x + 2 = 26$ **(b)** $4x - 2 = 26$
    (c) $5x + 1 = 26$
    Do your results agree with your answers for Question 3 of Exercise 10Q?

2   Draw a number machine which gives this result when you start with x:
    (a) $5x + 2$ **(b)** $7x - 4$ **(c)** $4x + 9$

3   Use the machines you drew in Question 2 to help you solve:
    (a) $5x + 2 = 27$
    (b) $7x - 4 = 24$
    (c) $4x + 9 = 33$

4   Solve for x:
    (a) $3x - 5 = 19$ **(b)** $4x + 1 = 21$
    (c) $7x - 8 = 25$ **(d)** $2x + 12 = 13$
    (e) $\frac{1}{2}x + 3 = 5$ **(f)** $6x - 9 = {}^{-}5$

Balancing equations

Can you use a number machine to solve
$2x = x + 5$?

You cannot because there are xs on both sides of the equation.

The idea of a balance can be used to solve equations like these.

Look at the balance. On one side it has 2 tins of paint. On the other side it has 1 tin of paint and a 5 kg weight.

Each tin of paint weighs x kg.
The masses on each side of the balance are equal.
If you remove one tin of paint from each side of the balance

you will see that $x = 5$.
Algebraically you can write:
$2x = x + 5$
so, subtracting x from each side
$x = 5$

Example 14

Use the balance idea to solve:
    (a) $w + 5 = 9$ **(b)** $4w = w + 3$
    (c) $\frac{x}{3} + 4 = 8$

Subtract 5 from both sides!

    (a) $w + 5 = 9$
       Subtract 5 from both sides
       $w + 5 - 5 = 9 - 5$
            $w = 4$
    (b) $4w = w + 3$
       Subtract w from both sides
       $4w - w = w + 3 - w$
         $3w = 3$
       Divide both sides by 3
         $\dfrac{3w}{3} = \dfrac{3}{3}$
         $w = 1$
    (c) $\frac{x}{3} + 4 = 8$
       Subtract 4 from both sides

$$\frac{x}{3} + 4 - 4 = 8 - 4$$

$$\frac{x}{3} = 4$$

Multiply each side by 3

$$3\left(\frac{x}{3}\right) = 3(4)$$

$$x = 12$$

Exercise 10S

1 (a) Look at the diagram below.

Do you agree that is shows the equation
$5x + 1 = 2x + 10$?

(b) Can you explain how the diagram below was
obtained from the diagram in part (a)?
Do the two sides still balance?

(c) Write an equation to show the information in
the last diagram above.
(d) Draw a diagram like the one in part (b), but
this time remove 1 kg from each side. Do the
two sides still balance?
(e) Work out the mass of one tin.

2 Solve the following equations:
(a) $7x + 3 = 3x + 11$
    (*Hint*: Subtract $3x$ from both sides)
(b) $5x + 4 = 2x + 19$
    (*Hint*: Subtract $2x$ from both sides)
(c) $5x + 1 = 8 - 2x$
    (*Hint*: add $2x$ to both sides)
(d) $6x + 2 = 16 - x$
    (*Hint*: add x to both sides)

3 Use the balance idea to solve for x:
(a) $3x + 7 = 5x + 1$
(b) $9x + 3 = 4x + 5$
(c) $2x + 3 = 28 - 3x$
(d) $4x - 7 = 13 - x$
(e) $6x - 7 = 2x + 3$
(f) $5x - 12 = 3 + 2x$
(g) $9x - 11 = 3x + 13$
(h) $8x - 12 = 5x - 3$

4 Use the balance idea to solve the following
equations. Be sure to write down what you
are doing to both sides of the equation as in
Example 14.
(a) $x + 3 = 5$ (b) $x - 3 = 7$

(c) $2x = 6$ (d) $\frac{x}{3} = 4$

(e) $2x + 3 = 9$ (f) $2x - 3 = 9$

(g) $5x - 7 = 8$ (h) $\frac{x}{3} + 4 = 8$

(i) $7 + 6x = 7x$ (j) $4 - 2x = 2x$

(k) $\frac{x}{5} - 7 = 2$ (l) $7x = 20 - 3x$

(m) $3 - x = 2$ (n) $3x = x + 6$

5 Kevin tried to solve the equation
$5x - 4 = 3x + 6$
Here is his working.

$5x - 4 = 3x + 6$	
[−4]	$5x = 3x + 2$
[−3x]	$2x = 2$
[÷2]	$x = 1$

(a) What was Kevin's mistake?
(b) Solve the equation correctly for Kevin.

Sometimes you have to simplify equations by
first removing brackets.

Example 15

Solve
$$7(2p + 3) + 6(p - 2) = 10p + 9$$
First remove brackets:
$$14p + 21 + 6p - 12 = 10p + 29$$
Now simplify by collecting like terms:
$$20p + 9 = 10p + 29$$
$$(-10p) \quad 10p + 9 = 29$$
$$(-9) \qquad\quad 10p = 20$$
$$(\div 10) \qquad\quad p = 2$$

Exercise 10T

1   Solve the following equations:
   (a) $4x + 2(x + 1) = 16$
   (b) $3(x + 1) + 2(x + 2) = 17$
   (c) $2(2x - 7) + x = 2x + 6$
   (d) $4 + 3(x - 5) = 10$
   (e) $x + (x + 1) + (x + 2) = 63$
   (f) $2(x + 7) - 6 = x + 15$

2   Solve the equations:

   (a) $\dfrac{x}{3} + 4 = 8$ (b) $\dfrac{x}{5} - 6 = 1$

   (c) $\dfrac{x}{4} + 8 = 17$ (d) $\dfrac{(x + 2)}{3} = 10$

   (e) $\dfrac{(x + 3)}{2} + 3 = 7$ (f) $\dfrac{(x - 3)}{2} = 42$

Solving problems

Creating an equation and solving it is a powerful way of working many types of problem.

There are four steps to solving a problem.

1   Understand the problem

2   Devise a plan

3   Carry out your plan

4   Look back.

Example 16

w cm

10.5 cm

A rectangle with length 10.5 cm has perimeter 36 cm. What is its width?

Understand the problem
What is to be found?
The width in cm, call it w.
Devise a plan
Form an equation:
$$w + 10.5 + w + 10.5 = 36$$
Carry out the plan
Solve the equation.
First, collect like terms:
$$w + w + 10.5 + 10.5 = 36$$
Simplify:
$$2w + 21 = 36$$
Subtract 21 from both sides:
$$2w = 15$$
Divide both sides by 2:
$$w = 7.5$$
Look back
The width of the rectangle is 7.5 cm.
This is correct since:
$$10.5 + 7.5 + 10.5 + 7.5 = 36$$

Exercise 10U

In all of these questions, first write down an equation, then solve it.

1   The result when adding a number to 6 is 41. What is the number?

2   A certain number when multiplied by 2 and then added to 5 gives a result of 97. What is the number?

3   The sum of two consecutive whole numbers is 91. What are the numbers?

4   The sum of two consecutive even numbers is 214. What are the numbers?

5 Find the width of these rectangles.
   (a) length 5 cm, perimeter 24 cm
   (b) length 8.1 cm, perimeter 28.5 cm
   (c) length 17.3 m, perimeter 41.7 m

6 The perimeter of a triangle is 34 cm. What are the lengths of the sides if the first side is twice the length of the second side and the third side is 2 cm longer than the second side?

7 Janet is 6 years younger than Bernelle. How old is each girl if the sum of their ages is 26?

8 Anton has three times as many marbles as Brian. Brian has 4 more marbles than Cedric. How many marbles does each boy have if there are 96 marbles altogether?

9 In 16 years' time Jim will be three times his current age. How old will Jim be in 4 years' time?

10 The sum of three consecutive odd numbers is 243. What are the numbers?

Technology

Found this last section tough going?

First review some Youtube videos on solving linear equations then go over the complete course on equations at

coolmath.com/prealgebra/16-intro-to-solving-equations

Make sure you do the questions!

Activity

You will need a rubber/elastic band, some nails and a ruler.
- Hang an elastic band over a hook.
- Measure the length of the band.

elastic/rubber band
measure the length of the band
ruler

- Attach a nail to the band and remeasure its length.
- Repeat for up to ten nails.

measure length of band with nails attached

- Copy and complete the table.

Number of nails (N)	Length of band in cm (L)
0	
1	
2	
3	
10	

- Draw a graph of the number of nails against the length of the band, using the number of nails as the horizontal axis and the length of the band as the vertical axis

Length of band (cm) — L
Numbers of nails — N
0 1 2 3 4 5 6 7 8 9 10

- Draw a line of 'best fit' through your points plotted.
- Find the gradient of the line.
- What is the intercept value?
- Write down the equation of the line of best fit in terms N and L.
- Now redraw your graph, but this time use the length of the band, L, as the horizontal axis and the number of nails, N, as the vertical axis.

- Draw the line of 'best fit'
- What is the gradient of this line?
- What is the value of the intercept?
- Write down the equation of the line of best fit in terms of L and N.
- Look at the two equations for the line of fit plotted. What do you notice?

10.7 Changing the subject of a formula

In Section 10.3 you met some simple formula. For example, the circumference of a circle, C, is related to its radius, r by the formula

$$C = 2\pi r$$

You can use this formula to find the value of C when the radius is 6 cm, say, then

$$C = 2\pi r$$
$$= 2 \times 3.14 \times 6 \text{ cm}$$
$$= 37.68 \text{ cm}$$

Sometimes, however, you want to find the radius, r when given the circumference of the circle, C. This is called rearranging or changing the subject of the formula.

In the formula

$$C = 2\pi r$$

C is the subject of the formula. To make r the subject, you can use a number machine

So $r = \frac{C}{2\pi}$, where r is now the subject of the equation. Number machines are useful when changing the subject of simple formulae.

Example 17

Make x the subject of these formulae:

(a) $y = x + a$ (b) $y = \dfrac{x - x_1}{n}$

(a)

So $x = y - a$

(b)

So $x = yn + x_1$

Exercise 10V

1. Make x the subject of these formula
   (a) $y = x + c$
   (b) $y = mx$
   (c) $y = mx + c$

2. Make w the subject of the formula $A = l \times w$.

3. Make h the subject of the formula $P = mgh$.

4. Make l the subject of the formula $P = 2(l + w)$.

5. Make π the subject of the formula $A = \pi r^2$.

6. Make w the subject of the formula $V = lwh$.

7 Make x_2 the subject of the formula $\bar{X} = \dfrac{x_1 + x_2}{2}$.

8 Make m the subject of the formula $E = \dfrac{1}{2}mv^2$.

9 Make t the subject of the formula $v = u + at$.

10 Make V the subject of the formula $PV = RT$.

11 Make b the subject of the formula $A = \dfrac{1}{2}bh$.

12 Make a the subject of the formula $A = \dfrac{1}{2}(a + b)h$.

13 Make C the subject of the formula $F = \dfrac{9}{5}C + 32$.

14 Make r the subject of the formula $A = \pi r^2$.

15 Make l the subject of the formula $\dfrac{T}{2\pi} = \sqrt{\dfrac{l}{g}}$.

10.8 Linear inequalities

Look at the diagram.

The left-hand side is heavier than the right. The information can be shown by the inequality:

$$4x + 1 > 2x + 5$$

Make a copy of the diagram, in pencil. Use your copy and an eraser, to explain why:

(a) $2x + 1 > 5$ (b) $2x > 4$ (c) $x > 2$

Example 18 shows how the inequality can be set out and solved.

Example 18

Solve $\qquad 4x + 1 > 2x + 5$

Subtract $2x$ from each side
$(-2x) \qquad 2x + 1 > 5$
Subtract 1 from each side
$(-1) \qquad 2x > 4$
Divide each side by 2
$(\div 2) \qquad x > 2$
That is, x is greater than 2.

Exercise 10W

1 Use the method shown in Example 10 to solve each inequality.
(a) $5x + 2 > 17$
(b) $7x + 6 > 41$
(c) $4x + 3 < 15$
(d) $6x + 1 < 19$
(e) $4x + 3 > x + 9$
(f) $5x + 2 > x + 8$
(g) $3x + 4 < x + 7$
(h) $7x + 4 < 2x + 19$

2 Solve:
(a) $3x - 7 < 8$
(b) $2x - 3 > 15$
(c) $8x - 7 > 3x + 8$
(d) $7x - 11 < 4x + 10$
(e) $4x + 3 > 18 - x$
(f) $2x + 1 < 7 - x$
(g) $5x - 17 > 3x - 1$

3 Solve each inequality:
(a) $8(5x - 7) - 9 < 55$
(b) $5(3x + 2) + 6 > 91$
(c) $4(7x + 6) - 8 > 72$
(d) $\dfrac{2x - 3}{2} < 5$
(e) $\dfrac{3(x + 2)}{4} < 9$

Inequalities, number lines and solution sets

You can show the solution of an inequality on a number line.

In Example 10, the solution is $x > 2$.

It can be shown as:

The whole numbers from 1 to 9 which make $x + 1 > 7$ a true statement are 7, 8 and 9. This set of numbers $\{7, 8, 9\}$ is called the **solution set** of the inequality.

● The solution set of an inequality is the set containing the values that make the inequality true.

Exercise 10X

1 Use a number line to show the solution of the inequality in each part of Questions 1 and 2 of Exercise 10W.

2 Which *whole* numbers from 1 to 9 make the inequality a true statement?
  (a) $x + 3 > 7$ (b) $x + 5 < 8$
  (c) $2x > 10$ (d) $3x < 12$
  (e) $2x + 3 > 15$ (f) $5x - 1 < 24$

3 If x can be any *whole* number from 1 to 20, find the solution set for:
  (a) $x + 4 > 19$ (b) $x - 7 > 10$
  (c) $x + 9 < 13$ (d) $x - 1 < 5$
  (e) $2x + 5 < 13$ (f) $3x - 7 > 35$

4 If x can be any *whole* number from 21 to 25, find the solution set for:
  (a) $7x - 15 > 6x + 8$
  (b) $5x + 23 < 3x + 71$
  (c) $2x - 3 > 60 - x$

5 Which *whole* numbers from 1 to 9 make both inequalities true?
  (a) $x + 3 > 7$ *and* $x - 2 < 6$
  (b) $2x + 1 > 11$ *and* $3x - 1 < 23$

Solving problems involving inequalities

As you saw in Section 10.8, inequalities are often used to describe everyday situations. For example, a speed limit of 30 km/h, can be written as

$$v \le 30$$

where v is the speed of a vehicle. Inequalities also allow you to solve simple problems.

Example 19

The length of a rectangular room is 1 m longer than its width. What are the dimensions of the room if its perimeter is less than 20 m?
First, draw a diagram to show the information.

Let the width be w, then the length of the room is $w + 1$. Hence, the perimeter of the room

$$= w + (w + 1) + w + (w + 1)$$
$$= 4w + 2$$

Since the perimeter is less than 20 m

$$4w + 2 < 20$$
$$\Rightarrow 4w < 18$$
$$\Rightarrow w < 4.5\,\text{m}$$

Thus, the width of the room is less than 4.5 m and the length of the room is less than $4.5 + 1 = 5.5$ m.

Notice, again, that you first have to understand the problem. What is to be found? (The dimensions or the length and width of the room)? Next you need to decide a plan. In this case form an inequality. Carry out the plan by solving the inequality. Finally, look back to make sure your answers to the problem make sense!

Exercise 10Y

1 The area of a rectangular room is less than 50 m². Find the values of the width of the room if the length is 10 m.

2 The width of a rectangular room is 4 m. What values could the length of the room take if its area is greater than 20 m²?

3 A baker sells 8 loaves of bread for no more than $48. What is the maximum price of a loaf?

4 The area of a triangle is less than 60 cm². What would be the values of the height of the triangle if the base length was 5 cm?

5 A rectangular field has a perimeter of no more than 720 m. What is the maximum length of the field if its width is 8 m shorter than its length?

6 The perimeter of an isosceles triangle is at least 20 cm. Find the length of its two equal sides if its third side is 6 cm long.

7 Large ice-creams cost $5 and small ones are sold for $3. Khadija buys 15 ice-creams. What is the largest number of large ice-creams that she can buy if she cannot spend more than $60?

8 Twenty concert ticket are to be bought for a group. Concert tickets are priced at $10 for adults and $6 for children. Calculate the least number of children's tickets that can be bought if the total cost of the tickets must not be more than $160.

9 A rectangle has length less than 15 cm. What can you say about its width if the rectangle has a perimeter of 40 cm?

10 Johnson buys 36 soft drinks for his friends. Cola-aid costs $3, while Fanta-aid costs $4 a bottle. What is the least number of Cola-aid bottles he can buy if he cannot spend more than $120?

Exercise 10Z – mixed questions

1 Each of the following is a whole number less than 100. What are they?
  (a) An even, prime number.
  (b) An even, square number which is a multiple of 3.
  (c) A triangular, square number.
  (d) An odd, triangular number which is a multiple of 13.
  (e) An odd, triangular number which is also prime.

2 Make up similar descriptions for other numbers as in Question 1. Try them on a friend.

3 **(a)** Write down all the square numbers from 1 to 100.
  (b) Which prime numbers less than 100 can be written as the sum of two square numbers? (for example: $13 = 9 + 4$)

4 Look at this sequence:
  $$3, 6, 9, 12, \ldots$$

  (a) Write down the next three terms.
  (b) What is the **(i)** 10th term **(ii)** 20th term?
  (c) If S_n is the nth term, write down a formula for S_n.

5 Repeat Question 4 for these sequences.
  (i) 4, 8, 12, 16, . . .
  (ii) 5, 7, 9, 11, . . .
  (iii) 5, 6, 7, 8, . . .
  (iv) 56, 55, 54, 53, . . .
  (v) 5, 9, 13, 17, . . .

6 Check the solution for each of the equations shown in the table.

	Equation	Solution
(i)	$2x + 3 = 5$	$x = 1$
(ii)	$7x - 2 = 5$	$x = 2\frac{1}{3}$
(iii)	$3x + 4 = x + 2$	$x = 1$
(iv)	$5x - 2 = 3x + 2$	$x = 1$
(v)	$3(x + 2) = 2x - 6$	$x = 0$

  (a) Which solutions are wrong?
  (b) For each incorrect solution solve the equation correctly.

7 Look at the sequence
  $$1, 1, 2, 3, 5, 8, \ldots$$
  (a) What is the next number in this sequence?
  (b) What is the 10th term?
  (c) What is the 20th term?

8 The arrangement of numbers shown below is called Pascal's triangle.

```
              1
           1     1
        1     2     1
     1     3     3     1
  1     4     6     4     1
1     5    10    10     5     1
```

  (a) Continue the pattern for three more rows.
  (b) What patterns can you see in Pascal's triangle?
  (c) Where are the triangle numbers in Pascal's triangle?
  (d) What happens if you add the numbers in each row? Can you find a pattern?

9 **(a)** Adam, Brian and Curtis line up to buy cake at break time. In how many different ways can they line up?

(b) David joins the line. In how many different ways can the four boys line up?

10 A rectangle is 6 cm longer than it is wide. It has a perimeter of 90 cm.

(a) Write down an equation to show this information.

(b) Solve the equation to find the length and width of the rectangle.

11 Make x the subject of these equations:

(a) $\dfrac{x+3}{2} = y$

(b) $y = \dfrac{1}{3}(4x - 6)$

(c) $y = (x + a)^2$

(d) $x^2 + y^2 = r^2$

(e) $ax = bx + c$

12 Three times Fitzroy's age 4 years ago will be the same as twice his age in two years' time. What is his current age?

13 Two trucks each weighing 3 tonnes, when empty, pass on a bridge. If the first truck carries twice the load as the second, find the maximum load that can be carried by the second truck if the bridge cannot take a weight of more than 10 tonnes.

14 **(a)** Using the numbers 1, 2 and 3 once only, how many different three-digit numbers can you write down?

(b) Using the numbers 1, 2, 3 and 4 once only, how many different four-digit numbers can you write down?

(c) Copy and complete this table.

Numbers used	Number of different numbers made
1	1
1, 2	2
1, 2, 3	
1, 2, 3, 4	
1, 2, 3, 4, 5	

(d) How many different eight-digit numbers could you make using the numbers 1, 2, 3, 4, 5, 6, 7, 8?

15 **(a)** Choose two numbers from 1, 2, 3. How many different ways can you do this?

(b) Choose two numbers from 1, 2, 3, 4. How many different ways can you do this?

(c) How many ways can you choose two numbers from 1, 2, 3, 4, 5?

(d) Copy and complete the table.

Numbers used	Number of ways of choosing 2 numbers
1, 2	1
1, 2, 3	3
1, 2, 3, 4	
1, 2, 3, 4, 5	
1, 2, 3, 4, 5, 6	

(e) What is special about the numbers in the right-hand column of the table?

(f) How many ways could you choose two numbers from the first 20 numbers?

10 Consolidation

Example 1

Given that $\dfrac{P = 3(R - r)}{R}$, find the value of:

(a) P when $R = 8$ and $r = 5$.
(b) P when $R = 6.5$ and $r = 1.3$.

(a) $P = \dfrac{3(R - r)}{R} = \dfrac{3(8 - 5)}{8}$

$= \dfrac{3 \times 3}{8} = \dfrac{9}{8} = 1.125$

(b) $P = \dfrac{3(R - r)}{R} = \dfrac{3(6.5 - 1.3)}{6.5}$

$= \dfrac{3 \times 5.2}{6.5} = \dfrac{15.6}{6.5} = 2.4$

Example 2

A man walks at V km per hour for t hours.
(a) How far does he walk?
(b) If he walks at v km per hour for T more hours, what is the total distance, d, that he travels?

(a) Distance $=$ speed $\times$ time $= V \times t = Vt$ km
(b) Further distance
travelled $=$ speed $\times$ time $= v \times T = vT$
Total distance travelled $= d = Vt + vT$ km

Example 3

Simplify:
(a) $3x - 2y + 4y + 3(y + x)$
(b) $\dfrac{3x^2 \times 4y^3 \times x^3}{2y^2}$

(a) $3x - 2y + 4y + 3(y + x)$
$= 3x - 2y + 4y + 3y + 3x$
$= 3x + 3x - 2y + 4y + 3y$
$= 6x + 5y$

(b) $\dfrac{3x^2 \times 4y^3 \times x^3}{2y^2}$

$= \dfrac{3 \times x \times x \times 4 \times y \times y \times y \times x \times x \times x}{2 \times y \times y}$

$= \dfrac{3 \times x \times x \times x \times x \times x \times \overset{2}{\cancel{4}} \times y \times \cancel{y} \times \cancel{y}}{\cancel{2} \times \cancel{y} \times \cancel{y}}$

$= 6x^5 y$

Example 4

Solve:
(a) $3x - 2 = 7$
(b) $3(x - 2) = x - 4$

(a) $\qquad 3x - 2 = 7$
$[+2] \qquad 3x = 9$
$[\div 3] \qquad x = 3$

(b) $\qquad 3(x - 2) = x - 4$
$\qquad 3x - 6 = x - 4$
$[+6] \qquad 3x = x + 2$
$[-x] \qquad 2x = 2$
$[\div 2] \qquad x = 1$

Example 5

Make x the subject of the equation

$\qquad a(x - c) = y$

using a flow chart

Hence, $x = \dfrac{y}{a} + c$.

Exercise 10

1 The formula for pth triangle number, P is

$P = \dfrac{1}{2} p(p + 1)$.

(a) What is:
(i) the 8th triangle number
(ii) 19th triangle number?
(b) Which triangle number is:
(i) 10   **(ii)** 78?

2 Find the nth term of each of these sequences.
(a) $1, 3, 5, 7, 9, \ldots$ **(c)** $16, 18, 20, 22, 24, \ldots$
(b) $4, 7, 10, 13, 16, \ldots$ **(d)** $21, 19, 17, 15, 13, \ldots$

3 Simplify:
(a) $3x - 2y + 3x + 4y$
(b) $6(x - y) + 2(x + y)$
(c) $\dfrac{3x^3 \times 2y^2}{x^3}$
(d) $x^2(x^2 + 4y^2)$
(e) $4y - 2x + 3(x - 2y)$
(f) $7(x - 3y) + 3(4x - 2y)$

4 **(a)** Write down the formula for the perimeter P, and area A, of these shapes.

(i)

$x - 1$

x

(ii)

$x + 3$

(iii)

$x + 2$

x

$x + 1$

(b) Find the actual length of each of the sides of these shapes if the perimeter in each case is 25 cm.

5 Solve the inequalities, for x
   (a) $3x \leq 6$
   (b) $x + 8 > 5$
   (c) $3x - 4 < 6$
   (d) $2x + 5 \leq 6 - 3x$

6 Make y the subject of the equations
   (a) $ax + by = c$
   (b) $ay = bx$
   (c) $x = \dfrac{y + a}{b}$
   (d) $x = ay^2$

Application

7 A spring is 30 cm long and extends by 5 cm for each 100 g weight hung on it.
   (a) What is the spring's length when 300 g is hung from it?
   (b) What is the spring's length, z, when n 100 g weights are hung from it?
   (c) What weight has to be hung from the spring for its length to be 38 cm?

8 Ann walks for 2 hours at p km/hr.
   (a) How far does she walk?
   (b) If she walks at q km/hr for 3 more hours, what is the total distance she travels?
   (c) What is her average speed?

Summary

You should now know ...

1 Rectangle numbers can be shown as a rectangle of dots.
For example:

$\bullet\ \bullet\ \bullet$
$\bullet\ \bullet\ \bullet$ 6 is a rectangle number

Rectangle numbers are composite numbers.

2 A square number can be represented by a square of dots.
For example:

$\bullet\ \bullet\ \bullet$
$\bullet\ \bullet\ \bullet$ 9 is a square number
$\bullet\ \bullet\ \bullet$

$9 = 3 \times 3 = 3^2$

3 Prime numbers cannot be made into squares or rectangles.
For example:

$\bullet\ \bullet\ \bullet\ \bullet\ \bullet$ 5 is a prime number

4 Triangle numbers form a triangle of dots.
For example:

$\bullet$
$\bullet\ \bullet$ 6 is a triangle number
$\bullet\ \bullet\ \bullet$

The *n*th triangle number is the sum of the first *n* whole numbers $(1 + 2 + 3 + \ldots + n)$
For example:

> 10 is a triangle number

$\bullet$
$\bullet\ \bullet$
$\bullet\ \bullet\ \bullet$
$\bullet\ \bullet\ \bullet\ \bullet$

$1 + 2 + 3 + 4 = 10$

The *n*th triangle number is $\dfrac{n \times (n+1)}{2}$.

5 A formula is an equation relating different quantities.
For example:

$$A = l \times w$$

is the formula for the area A of a rectangle, with l = length and w = width.

Check out

1 **(a)** Write down the first five rectangle numbers.
(b) Which of these are rectangle numbers:
(i) 35 **(ii)** 36?

2 **(a)** Write down the first five square numbers.
(b) Which of these are square numbers:
(i) 169 **(ii)** 192 **(iii)** 576?

3 **(a)** Write down the first five prime numbers.
(b) Which of these are prime numbers:
(i) 41 **(ii)** 57 **(iii)** 83?

4 **(a)** Draw dots to show that 21 is a triangle number.
(b) Write down the 10th triangle number.
(c) What is the sum of the numbers from 1 to 200?

5 **(a)** What is the cost C of a bottles of juice at p dollars a bottle and b tins of milk at q dollars a tin?
(b) What is the surface area of a cube with side s?

6 How to multiply and divide terms with indices.
For example:
(a) $3m^3 \times 5m^2 = 15m^5$
(b) $12p^7 \div 4p^3 = 3p^4$

6 Simplify:
(a) $3a^4 \times a^5$
(b) $21x^3 \div 7x^2$
(c) $\dfrac{(3p^3)(12p^2)}{4p}$

7 You can simplify algebraic expressions involving brackets.
For example:
$2(5x - 4) + 3(2x + 3)$

> Multiply out brackets and collect like terms

$= 10x - 8 + 6x + 9$
$= 16x + 1$

7 Simplify:
(a) $3(7h - 1) + 4$
(b) $4(2w + t) + 6(w + 4t)$
(c) $2p + 3(2p - 4)$

8 You can use the idea of a balance to help you solve linear equations:

$3x - 5 = 19 - x$
$(+x)\quad 4x - 5 = 19$
$(+5)\quad\quad 4x = 24$
$(\div 4)\quad\quad x = 6$

> Bring all the x's to one side, the numbers to the other

8 Solve these equations.
(a) $3x + 7 = 19$
(b) $\dfrac{x}{2} - 2 = 5$
(c) $5(r - 4) = 3(8 - r)$

9 Inequalities can be solved in a similar way to equations.
For example:
$4x - 5 < 7$
$(+5)\quad 4x < 12$
$(\div 4)\quad x < 3$
You can show the solution on a number line.

9 Solve these inequalities and show the solutions on a number line.
(a) $4x \geq 2$
(b) $2x - 1 < 7$
(c) $\dfrac{x + 3}{4} \leq 6$

10 You can rearrange formula to change their subject using a flow chart.
For example:
Make x the subject of the formula
$y = \dfrac{x}{a} - b$

Hence $x = a(y + b)$

10 Make x the subject of the formula
(a) $ax = bcy$
(b) $y = \dfrac{1}{2}x + c$
(c) $y = ax^2 + b$

Volume

Objectives

✔ understand what volume is
✔ use the units of volume
✔ calculate the volume of simple solids
✔ calculate the volume of liquids
✔ find the surface area of simple solids

> There's 114 ml of toothpaste left in the tube!

> Yes, but I want to know how much toothpaste it really has!

What's the point?

What is the capacity of your freezer? Which is the bigger container? Which perfume bottle holds more? The wise consumer will always check before buying.

Before you start

You should know ...

1 How to multiply three numbers together.

2 How to find the areas of these shapes:
  (a) squares
  (b) rectangles
  (c) triangles.

Check in

1 Evaluate
  (a) $3 \times 12 \times 15$
  (b) $15 \times 10 \times 86$

2 Find the areas of these shapes.
  (a)

  15 cm

  15 cm

  (b)

  18 m

  12 m

  (c)

  15 m

  18 m

11.1   What is volume?

The cartoon shows how Supercrow got water from a jug.

(a) Was there *more* water in the jug after Supercrow had dropped in the stones?

(b) Why did the water rise to the top of the jug?

(c) If Supercrow kept on adding stones, what would happen to the water?

The water rose in the jug because the stones took up space in the bottom of the jug.

Anything that takes up space has volume

● The amount of space that something takes up is called its **volume**.

The volume of your body is much larger than the volume of this book. But it is much smaller than the volume of your classroom.

Exercise 11A

You will need two different jars, a cup, two medium-sized stones, a piece of chalk or crayon, some thread, some plasticine and some water.

1 (a) Take a jar. Half fill it with water. Mark the level of the water with chalk or crayon on the outside of the jar.

(b) Take two medium-sized stones. Tie a piece of thread round one of them.

(c) Lower this stone into the jar until it is completely covered. What happens to the water level? Why?

(d) Mark the jar at the new water level. Take out the stone. What happens to the water level? Why?

(e) Now tie some thread round the second stone. Carefully lower it into the jar. Put a mark at the new water level.

(f) Which stone has the larger volume? How can you tell?

2 (a) Take two jars of different shapes. Fill one with water up to the top.

(b) Now carefully pour this water into the second jar. Does it fill the second jar?

(c) Which jar can hold a larger volume of water?

3 Fill a cup with water. Now pour the water into a jar.

(a) Does the shape of the water change, when you pour it into the jar?

(b) Does the volume of the water change?

4 Take a piece of plasticine. Make it into a sphere. Now make it into a snake.

(a) Has the shape of the plasticine changed?
(b) Has the volume of the plasticine changed?

5 Ravi made cuboid A from eight wooden cubes. He then rearranged the cubes to make cuboid B.

(a) Are the two cuboids different shapes?
(b) Have both got the same volume of wood?

6 Here are the eight cubes from Question 5 arranged another way:

(a) Is this cuboid a different shape from the ones in Question 5?
(b) Has the volume of the wood changed?

7 Copy and complete:
A change in shape does not mean a change in
. . .

8 (a) Is the shape of your body different when you sit than when you stand?
(b) Is its volume different?

9 For each pair, pick out the one with the greater volume:
(a) a baby, a fully-grown adult
(b) a football, a table-tennis ball

(c) your exercise book, this maths book
(d) a rice grain, a grain of sugar
(e) a mango, a pineapple
(f) your left foot, your right foot.

10 How many different cuboids could you make using twenty-four cubes?
(a) Is the volume the same in each case?
(b) Is the shape different for each cuboid?

11.2 Units of volume

You can find the area of a surface by covering it with squares and counting them.

Area is the number of squares that can fit into the shape

In the same way you can find the volume of a space by filling it with cubes and counting them.

Volume is the number of cubes that can fit into the space.

Exercise 11B

1 Here is a box filled with cubes.

(a) How many cubes fill the box?
(b) Copy and complete:
The volume of the box is . . . cubes.

2 This is another box filled with the same sort of cubes as in Question 1.

(a) Copy and complete:
The volume of the box is . . . cubes.
(b) Is this box larger or smaller than the box in Question 1?

3 (a) How many cubes fill the box below?

(b) What is the volume of the box?
(c) Compare your answer with the answer you got for the box in Question 1.
What do you notice?
(d) Do both boxes look as if they have the same volume?
(e) Can you see any problem with this method of measuring volume?
(f) Can you think of a way to avoid the problem?

The cubic centimetre

To make sure everyone uses the same cubes for measuring volume, special sizes were chosen.
One of these is the cube with each edge 1 centimetre.

It has a volume of **1 cubic centimetre**.
This is usually written as 1 cm³.

This cuboid is made up of 10 cubes of edge 1 cm.

It has a volume of 10 cm³.

Exercise 11C

1 These shapes are made of centimetre cubes. Write down the volume of each shape.

(a)

(b)

(c)

2 These shapes are made of centimetre cubes. Write down the volume of each shape.

(a)

(b)

(c)

3 Make a guess at the volume in cm³ of:
(a) a chicken stock cube
(b) a matchbox
(c) a marble
(d) a pocket calculator
(e) an egg
(f) a pencil box

Compare your answers with those of your friends.

4 Jean found a small cardboard box 6 cm long, 4 cm wide and 3 cm high.
How many centimetre cubes could she pack into the box?

5

A

B

C

D

(a) Copy and complete the table.

	Number of cubes long L	Number of cubes wide W	Number of cubes high H	L × W × H	Total number of cubes
A	4	3	3	36	
B	3	3			
C	6				
D					

(b) What do you notice about the last two columns?

11.3 Volume of cuboids

Exercise 11D

1 These cuboids are made of centimetre cubes. Write down the volume of each cuboid.

(a)

(b)

(c)

(d)

(e)

2 Copy and complete the table for the shapes in Question 1.

Box	Volume (cm³)	Length (cm)	Width (cm)	Height (cm)	Length × Width × Height
(a)		2	2	2	8
(b)		1	1	2	
(c)					
(d)					
(e)					

3 Look at the table in Question 2. What do you notice?

● The volume of a cuboid, V, can be worked out using:

$$V = \text{length} \times \text{width} \times \text{height}$$

Example 1

Find the volume of the cuboid shown.

Volume of cuboid
= length × width × height
= 4 cm × 2.5 cm × 2 cm
= 10 cm² × 2 cm
= 20 cm³

Notice, for a cuboid, the volume, V, can be written as

$$V = A \times h$$

Where $A = l \times w$ is the area of the rectangular base and h is the height.

Example 2

Find the volume of the cuboid below.

The area of the rectangular base is 20 cm².

$$\text{Volume of cuboid} = A \times h$$
$$= 20 \text{ cm}^2 \times 5 \text{ cm}$$
$$= 100 \text{ cm}^3$$

Exercise 11E

1 Find the volume of these cuboids.

(a) Area = 18 cm², 4 cm

(b) Area = 35 cm², 9 cm

(c) 8 cm, 26 cm²

2 Find the volume of these cuboids.

(a) 3 cm, 2 cm, 3 cm

(b) 2 cm, 3 cm, 5 cm

(c) 2 cm, 1.5 cm, 6 cm

(d) 1.5 cm, 1.5 cm, 4.5 cm

3 **(a)** What is the volume of a cube of side
  (i) 5 cm
  (ii) 10 cm
 (b) How many cubes of side 5 cm are needed to make a cube of side 10 cm?

4 How many 2 cm sided cubes can fit into a box with base area 64 cm² and height 8 cm?

5 When a cuboid is measured in centimetres, its volume is given by:

$$l \text{ cm} \times w \text{ cm} \times h \text{ cm} = V \text{ cm}^3$$

Can you explain why the raised 3 is used in $V \text{ cm}^3$?

6 Copy and complete the table for cuboids.

Length (cm)	4	5	6.5	4.5	5
Width (cm)	2	2	3	2.5	1.5
Height (cm)	1	3	4	3	4.25
Volume (cm³)					

7 Copy and complete the table for cuboids.

V cm³	60	25	30	64
l cm	3	5	3	8
w cm	4	1	1	2
h cm				

8

A box of macaroni measures 12 cm by 20 cm by 4 cm.
What is the volume of the box?

9 (a) Think of two things with volumes too great to measure in cm³.
   (b) Think of two things with volumes too small to measure in cm³.

10 (a) The cubic centimetre is based on a unit of length. What unit is this?
   (b) Write down a metric unit of length that is larger than the centimetre.
   (c) Write down a metric unit of length that is smaller than the centimetre.
   (d) Do you think there could be units of volume based on these? Try to imagine them.

11 A matchbox has a volume of 24 cm³.
   (a) Draw and show the measurements of two boxes with this volume.
   (b) David made a matchbox that was 1 cm long, 1 cm wide and 24 cm high. Why is this not a good design?

Technology

Find out more about how to find the volume of common solids by visiting

www.mathguide.com/lessons

Try the tests (quizmasters) at the end of each lesson!

The cubic millimetre

Here is a cube with edge 1 cm.
Beside it is a jewel from Diane's ring.

The jewel has a volume less than 1 cm³.

The cm³ is too large a unit for measuring the volume of the jewel. For such small volumes, a smaller unit is used.

It is the **cubic millimetre**.

1 cubic millimetre is the volume of a cube of edge 1 mm.
1 cubic millimetre is usually written as 1 mm³.

Exercise 11F

1 (a) Write a list of five small objects whose volume you would measure in mm³.
   (b) Estimate the volume of each one.

2 (a) What is the name of the cuboid with all edges the same?
   (b) Copy and complete, for these cuboids:

Length	Width	Height	Volume
1 cm	1 cm	1 cm	
10 mm	10 mm	10 mm	

Remember: 1 cm = 10 mm

3 Study your table in Question 2. Now copy and complete:

$$1 \text{ cm}^3 = \square \text{ mm}^3$$

4

Sugar cubes have a volume of 1000 mm³.
How many such cubes can be packed in a box
measuring 15 cm by 10 cm by 5 cm?

5 How many sugar cubes of side 12 mm can be
made from a 100 cm³ container filled with
sugar?

The cubic metre

Suppose you wanted to measure the volume of the
school water-tank.
The cm³ is too small a unit—the answer might be
something like 10 000 000 cm³
So we need a larger unit, the **cubic metre**.
This is the volume of a cube of edge 1 metre, written
as 1 m³.

Exercise 11G

1 Write a list of five objects whose volume you
would measure in units of m³ then estimate the
volume of each one.

2 Find the volume in cubic metres of:

(a)

(b)

3 **(a)** Estimate the volume of your classroom in
cubic metres.
(b) Measure the length, width and height of your
classroom. What is its actual volume?
How good was your estimate?

4 Which unit of volume would you choose to
measure:
(a) a pencil box
(b) the inside of your classroom
(c) this book
(d) the inside of a gasoline tanker
(e) a sugar cube
(f) the top of your desk
(g) a shirt button
(h) a pea
(i) a toy building block
(j) a shoe-box?

5 **(a)** Copy and complete the table.

Length	Width	Height	Volume
1 m	1 m	1 m	
100 cm	100 cm	100 cm	

(b) Copy and complete:

$$1 \text{ m}^3 = \square \text{ cm}^3$$

6 How many cubic millimetres make 1 m³?
Remember: 1 m = 100 cm = 1000 mm.

Technology

Conversion from one metric unit of volume to another is usually straightforward. Converting metric to imperial units is more troublesome.
Did you know that
    1 litre = 2.11 pints
or that
    1 (US) gallon = 3.79 litres?
For fast conversions visit the site

www.convert-me.com

Use it to find the difference between US and British gallons, fluid ounces and pints. Make a table of important conversions.

11.4 Volumes of prisms and cylinders

In the last section, you saw that the volume, V, of a cuboid is given by the formula

$$V = A \times h$$

where A is the area of the base or cross-section and h is the height of the cuboid.

This formula can also be used to calculate the volume of solids with different cross-sectional areas, such as **prisms** and **cylinders**

| Triangular prism | Hexagonal prism | Cylinder |

A **prism** is a solid with two identical and parallel faces in the form of a polygon.

A **cylinder** is a solid with two identical and parallel circular faces.

To find the volume of a prism or a cylinder, you use the formula

$$V = A \times h$$

Example 3

Find the volume of these solids.

(a)

(b)

(a) The cross-sectional area, A, of the prism is a right-angled triangle

$A = \frac{1}{2}b \times h = \frac{1}{2}3 \text{ cm} \times 4 \text{ cm}$
$\quad = 6 \text{ cm}^2$

Volume of prism $= A \times h$
$\quad\quad\quad\quad\quad = 6 \text{ cm}^2 \times 10 \text{ cm}$
$\quad\quad\quad\quad\quad = 60 \text{ cm}^3$

(b) The cross-sectional area, A, of the cylinder is a circle

$A = \pi r^2$
$\quad = 3.14 \times 4^2$
$\quad = 3.14 \times 16$
$\quad = 50.24 \text{ cm}^2$

Volume of cylinder $= A \times h$
$\quad\quad\quad\quad\quad\quad = 50.24 \text{ cm}^2 \times 10 \text{ cm}$
$\quad\quad\quad\quad\quad\quad = 502.4 \text{ cm}^3$

Exercise 11H

1 Find the volume of these prisms, with marked cross-sectional areas and heights.

(a)

6 cm
12 cm²

(b)

5 cm
9 cm²

(c)

15 cm²   8 cm

2 Find the volume of these prisms.

(a)

4 cm
8 cm
5 cm

(b)

4 cm
6 cm
9 cm

(c)

5 cm
12 cm
5 cm

3 Find the volume of these cylinders

(a)

12 cm²
5 cm

(b)

14 cm
6 cm

(c)

8 cm
4 cm

4

1 cm
15 cm

A cylindrical flashlight has a radius of 1 cm and length 15 cm.
(a) Find the volume of the flashlight.
(b) How many such flashlights can be packed into a box with dimensions 6 cm × 8 cm × 15 cm?

5 **(a)** Find the volume of a cylindrical water tank with base area 0.5 m² and height 1m.
(b) What is its volume in cm³?

6

Vito cereal
15 cm
6 cm
12 cm

Vito Cereal is sold in a box with dimensions 6 cm by 12 cm by 15 cm.
(a) Find the volume of a box of Vito Cereal.
(b) How many such boxes can be packed into a box with dimensions 90 cm by 180 cm by 60 cm?

7 A tin of milk has a volume of 300 cm³.

Milk | 12 cm

(a) If the milk tin is 12 cm in height what is its base area?

(b) What would be the diameter of the tin?

11.5   Volume of liquids

You will need a sheet of stiff cardboard 45 cm × 25 cm, sticky tape, a bucket, a measuring cylinder and some water.

Liquids do not have a fixed shape, but you can still measure their volume.

Scientists often use a measuring cylinder to measure the volume of liquids.

This cylinder is holding 55 cc of water.

Measuring cylinder

cc is short for cubic centimetre that is 55 cc = 55 cm³.

Exercise 11I

1 Collect five bottles which usually contain liquid.
   (a) Estimate their volume.
   (b) Fill your containers with water. Use your measuring cylinder to find their actual volume.
   (c) Copy and complete the table:

Bottle	Estimated volume (cm³)	Actual volume (cm³)
1		
2		
3		
4		
5		

2 **(a)** Estimate the volume of a bucket of water.
   (b) Could you use a measuring cylinder to find its actual volume? Why might it be difficult?

3 **(a)** Estimate the volume of a thimble of water.
   (b) Could you use a measuring cylinder to find the actual volume of the thimble? Why might it be difficult?

● The amount of liquid a container can hold is the **capacity** of the container.

A litre is the volume of liquid that can be held in an open cube of edge 10 cm.

10 cm

This open cube holds 1 litre.

10 cm   10 cm

The volume of this cube = 10 cm × 10 cm × 10 cm
                        = 1000 cm³

That is   **1 litre = 1000 cm³**

In the same way, a cube with side 1 m

1 m

1 m   1 m

will hold a volume of

100 cm × 100 cm × 100 cm = 1 000 000 cm³

Since 1 litre = 1000 cm³, volume of liquid in an open cube side 1 m = $\frac{1\,000\,000}{1000}$ = 1000 litres

Example 4

What is the capacity, in litres, of a water tank built in the form of a cuboid, with base area 2 m² and height 1 m?

Volume of tank $= A \times h$
$= 2\text{ m}^2 \times 1\text{ m}$
$= 2\text{ m}^3$

Why?

$2\text{ m}^3 = 2\,000\,000\text{ cm}^3$

so tank holds $= \frac{2\,000\,000}{1000}$ litres
$= 2000$ litres

Another unit for measuring small volumes of liquid is the **millilitre**. It is written in short as **ml.** It is the amount of liquid that can be held in a cubic centimetre.

capacity 1 ml   1 cm   1 cm   1 cm

volume 1 cm³

$1\text{ cm}^3 = 1\text{ ml}$
$1000\text{ ml} = 1\text{l}$

Activity

Using thick card, make a net of a cube with side 10 cm. Fold it to make an open cube and fix it together.
Line your cube with a plastic bag and fill it with water. The volume of water in your cube is 1 litre.

Use your cube to find the capacity of a bucket.

Exercise 11J

1 (a) Guess the volume, in ml, of:
   (i) a teaspoonful of medicine
   (ii) a small bottle of ink
   (iii) a cupful of coffee

(iv) a full cola bottle
(v) a raindrop.
(b) Find the exact volumes of as many of these as you can.

2 Bottles and tins of liquid often have the volume of the contents written on the labels. Find five examples.

3 (a) What is the total volume of Judy's Punch in
   (i) litres
   (ii) millilitres?

(b) A glass holds 100 ml. How many glasses can Judy fill from her punch bowl?

Judy's Punch
5 litres lemonade
1 litre of pineapple juice
500 ml of water-melon juice

4

ml 300 / 200 / 150 / 100 — 2 to 8 weeks
ml 300 / 220 / 200 / 100 — 2 to 5 months
ml 280/300 / 200 / 100 — 6 to 12 months

These bottles show the volumes of feed for babies of different ages.
(a) Write the volume of feed in each bottle.
(b) Joan is 8 months old and has 4 feeds per day. Write in litres, the total volume of feed Joan has in one week.
(c) Ryan is 4 months old and has 6 feeds per day. Write in litres and millilitres the total volume of feed Ryan has in one week.

5 (a) Estimate how many cups of liquid you drink every day.
(b) How many litres is that?

6

(a) What is the volume in cubic centimetres of a cube of side 1 m?

(b) How many litres of water could such an open cube hold?

7 (a) Calculate the volume of a swimming pool with base area 400 m² and height 2 m.

(b) How many litres of water would be needed to fill it?

8 Find the capacities of the following rectangular tanks in cubic cm and litres.

(a) 40 cm by 30 cm by 20 cm

(b) 60 cm by 50 cm by 10 cm

(c) 30 cm by 15 cm by 8 cm.

9 For each person find how many days the medicine will last.

Andy	Ali	Anne
Take two 5ml spoonfuls per day.	Take one 5ml spoonful 3 times a day.	Take one 5ml spoonful twice daily.
300 ml	105 ml	150 ml

Activity

Can you think of a way to measure the volume of air you breathe out in one breath?

11.6 Surface area of solids

The area of the total surface of a solid is called its surface area.

You can think of the surface area of a solid as the amount or area of wrapping paper needed to just cover the solid as a gift.

If you checked the area of gift wrapping paper used, you would get the net of the solid.

To find the surface area of the cuboid, you would need to find the area of each of its six rectangular faces. That is, the area of the net.

Example 5

Find the surface area of a cuboid with dimensions 3 cm by 4 cm by 5 cm.

Area of front = 3 cm × 5 cm = 15 cm²
Area of side = 3 cm × 4 cm = 12 cm²
Area of top = 4 cm × 5 cm = 20 cm²

The total surface area = 2 × (15 + 12 + 20) cm²
= 94 cm²

Notice, in a cuboid, the faces are in pairs top and bottom, front and back, side and side.

Wedge shaped solids or triangular prisms have five faces, as you can see from its wrapping paper or net.

To find the surface area of a triangular prism, you have to find the area of its three rectangular faces and its two triangular faces.

Example 6

Find the surface area of the rectangular prism shown.

Area of back = 3 cm × 8 cm = 24 cm²

Area of bottom = 4 cm × 8 cm = 32 cm²

Area of top = 5 cm × 8 cm = 40 cm²

Area of triangular side = $\frac{1}{2}$ × 4 cm × 3 cm
                       = 6 cm²

Total surface area
= 24 + 32 + 40 + 6 + 6 cm²
= 108 cm²

There are two triangular sides

Exercise 11K

1 Find the total surface area of these cuboids.

(a)

(b)

(c)

2 Find the total surface area of a cuboid with dimensions:
   (a) 5 cm by 7 cm by 9 cm
   (b) 6.1 cm by 4.8 cm by 7.6 cm

3 Find the surface area of these triangular prisms.
   (a)

   (b)

   (c)

4

In the diagram of the solid shown, find the
(a) volume of the solid
(b) surface area of the solid.

Activity

Take a tin of soup or beans and carefully remove its label.

- What is the shape of the label?
- Measure its dimensions with a ruler.
- Measure the diameter of the tin.
- What is the circumference of the tin?
- How does the circumference relate to the dimensions of the label?
- What is the surface area of the label?
- Calculate the area of the circular base of the tin.
- Find the total surface area of the tin.

Surface area of cylinders

In the activity, you should have found that the label of a tin has the shape of a rectangle.

The width of the label is the same as the height of the tin.

The length of the label is simply the circumference of the tin.

A cylinder has two flat faces and one curved face.

Two flat, circular faces
area $= \pi r^2$

The total surface area of a cylinder
= curved surface area + area circular top + area circular bottom
$= 2\pi r \times h + \pi r^2 + \pi r^2$
$= 2\pi rh + 2\pi r^2$

Example 6

Find the
(a) curved surface area of the cylinder
(b) total surface area of the cylinder

(a) curved surface area
$= 2\pi r \times h$
$= 2 \times 3.14 \times 4 \text{ cm} \times 10 \text{ cm}^2$
$= 251.2 \text{ cm}^2$

(b) Total surface area
$= 2\pi r \times h + 2\pi r^2$
$= 251.2 \text{ cm}^2 + 2 \times 3.14 \times 4^2 \text{ cm}^2$
$= 251.2 + 100.48 \text{ cm}^2$
$= 351.68 \text{ cm}^2$

Exercise 11L

1 Find the curved surface area of these cylinders
(a) 6 cm, 8 cm
(b) 5 cm, 10 cm
(c) 6 cm, 6 cm
(d) 3 cm, 15 cm

2 Find the total surface area of the cylinders in Question 1.

3 A soup tin has diameter 8 cm and height 10 cm.
(a) What is the area of its circular base?
(b) What would be the area of a label for the tin?
(c) What is the total surface area of the tin?

4 Find the curved surface area of a cylindrical water tank with radius 0.6 m and height 1.2 m.

5 The label on a cylindrical milk can has an area of 300 cm².
(a) What is the radius of the can if it has a height of 12 cm?
(b) What is the area of the circular base of the can?
(c) What is the total surface area of the can?

Investigation

Investigate the cost of repainting the outside of your house.

Hints
(1) Find the cost of 1 litre or 1 gallon of paint.
(2) Find the size of the surface that 1 litre or 1 gallon of paint can cover.
(3) Make a sketch of the walls of the outside of your house and insert the dimensions of the each wall.
(4) Find the area of each wall.
(5) Find how much paint is needed to paint all the outside walls.
(6) Find the cost of painting the outside of the house.

You could use this investigation as part of a CSEC school based assessment.

Exercise 11M – mixed questions

1 Jayceline found the volume of a stone by placing it inside a measuring cylinder containing water. What is the volume of her stone?

Water   Water and stone

2 Find the volume of some solid objects using a measuring cylinder.

3 Could you use your measuring cylinder to find the volume of a cork?

4 The container marked A is full of water up to the mark. A pipe allows the water to flow from the top when it gets too full.

(a) What will happen when the stone is lowered into the water?
(b) If 15 ml of water flows into the measuring cylinder, what is the volume of the stone?

5 **(a)** Guess the volume of your largest text book. What unit should you use?
  (b) Measure to the nearest centimetre its length, width and height.
  (c) Now calculate the volume of the textbook, to the nearest cm³.

  Was your guess a good one?

6 **(a)** How much liquid does an ordinary teaspoon hold?

MEDICINE
Take two
teaspoons
twice a day
200 ml

  (b) For how many days must the medicine shown above be taken?

7 George is a normal twelve-year old. When he takes a deep breath he can breathe out about 2 litres of air. How many deep breaths does George need to blow a balloon up to a volume of 10 000 cm³?

8 A brick is 18 cm by 8 cm by 6 cm.
  (a) Find the volume of 1 brick.
  (b) Find the volume of a pile of 20 bricks.

9 A water tank at Dankwe Junior Secondary School is 3 m long, 2 m wide and 2 m high.
  (a) What is the volume of the tank?
  (b) How many litres of water does the tank hold when it is completely full?

10 How many small cubes, of edge 2 cm, could you fit into a cubic box of edge:
  (a) 10 cm   **(b)** 24 cm?

11 How many cubic wooden fruit crates, of edge 1.5 m, could you fit into a ship's hold that is 60 m long, 30 m wide and 10 m high?

12 A feeding trough for cattle has a right triangular cross-section and is 3 m in length. Given both the base and height of the triangular part are 40 cm in length, find the total surface area of the trough.

13 **(a)** A tank holds 140 m³ of water when it is full. If its heght is 4 m and its width is 5 m, what is its length?
  (b) If the tank is filled with water to a height of 2 m, how much water is there in the tank? Give your answer in   **(i)** m³ and
                            (ii) litres.

14 A cylindrical water container has diameter 10 cm and height 30 cm.
  (a) What is the volume of the water container?
  (b) How many litres of water does it hold?
  (c) If 1 litre of water is powered into the empty container, to what height will the water reach?

15

A new pencil is 18 cm long with a circular base of diameter 1 cm.
  (a) What is the curved surface area of the pencil?
  (b) The pencil is painted with yellow paint. How many such pencils can be painted with 1 litre of paint. Assume 1 litre of paint can cover area of 2 m².

16 A rectangular block of metal measuring 8 cm by 6 cm by 5 cm was melted down to make a block 10 cm long by 8 cm wide. How high was the block?

17 A tank contains 15 litres of water. When empty, the tank has mass of 0.5 kg. If 1 cm³ of water weighs 1 g find the mass of the tank when it is filled with water.

6 Consolidation

Example 1
Which unit of volume would you choose to measure:
(a) a ship's container
(b) medicine in an eye dropper
(c) toothpaste in a tube?

(a) Containers on ships hold very large objects, e.g. cars. Unit: m^3.
(b) An eye dropper is very small, it will hold very little medicine. Unit: mm^3.
(c) A tube of toothpaste. Unit: cm^3.

Example 2
Find the volume of these solids.

(a)

(b)

(a) Volume of cuboid = base area × height
$$= 8 \text{ cm} \times 6 \text{ cm} \times 3 \text{ cm}$$
$$= 144 \text{ cm}^3$$

(b) Volume of triangular prism
$$= \tfrac{1}{2} \times \text{base area} \times \text{height}$$
$$= \tfrac{1}{2} \, 5 \text{ cm} \times 7 \text{ cm} \times 8 \text{ cm}$$
$$= 140 \text{ cm}^3$$

Example 3
A rectangular tank has dimensions 50 cm by 30 cm by 40 cm.
How many litres of water can it hold?

$$\text{Volume of tank} = \text{length} \times \text{width} \times \text{height}$$
$$= 50 \text{ cm} \times 30 \text{ cm} \times 40 \text{ cm}$$
$$= 60\,000 \text{ cm}^3$$

1 litre = 1000 cm^3
So tank will hold $60\,000 \div 1000 = 60$ litres of water.

Example 4
Find the surface area of these solids.

(a)

(b)

(a) The cuboid has six rectangular faces.
Area front = 5 cm × 2 cm = 10 cm^2
Area side = 6 cm × 2 cm = 12 cm^2
Area top = 6 cm × 5 cm = 30 cm^2

Total surface area = $(10 + 12 + 30 \text{ cm}^2) \times 2$
$$= 104 \text{ cm}^2$$

(b) The triangular prism has three rectangular and two triangular faces.

Rectangular faces
Area back = 8 cm × 9 cm = 72 cm^2
Area bottom = 6 cm × 9 cm = 54 cm^2
Area front = 10 cm × 9 cm = 90 cm^2

Triangular faces
Area = $\tfrac{1}{2} \times 8 \text{ cm} \times 6 \text{ cm}$
$$= 24 \text{ cm}^2$$

Total surface area
$$= 72 + 54 + 90 + 24 + 24 \text{ cm}^2$$
$$= 264 \text{ cm}^2$$

Exercise 11

1 Which unit of volume would you use to measure the volume of:
(a) milk in a small tin
(b) a coca-cola bottle
(c) a grain of rice
(d) a grapefruit
(e) a refrigerator
(f) a water reservoir?

2   What is the volume of these solids?

(a)

1 cm
4 cm   7 cm

(b)

2 cm
4 cm
6 cm

(c)

5 cm
8 cm
← 4 cm →

(d)

3 cm
6 cm
10 cm

3   Find the total surface area of these solids.

(a)

8 cm
9 cm
3 cm

(b)

3 cm
5 cm   4 cm
12 cm

4   Write in litres:
(a) 3000 cm³   (b) 5000 cm³   (c) 6000 ml
(d) 7500 cm³   (e) 850 ml   (f) 1 m³
(g) 5 m³   (h) 925 cm³   (i) 3.4 m³
(j) 130 cm³   (k) 14 cm³   (l) 25 ml

Application

5   A water tank has a rectangular base with area
1800 cm². How many litres does the tank contain
if it has water to a depth of 40 cm?

6   A tin of polish has dimensions 12 cm
by 8.5 cm by 5 cm.
(a) What is the volume of the
tin in cm³?
(b) What is its capacity in ml?
(c) What is its capacity in litres?

7

A cylindrical water container has radius 25 cm
and height 40 cm.
(a) Find the volume of the container.
(b) How much water, in litres, can it hold?
(c) Sierra puts 20 litres of water into the
container when it is empty. How high does
the water rise?

Summary

You should know ...

1   Volume is the amount of space an object occupies.
A shape can change but its volume remains the same.
You can make

a ball
OR

...a snake

Different shape, but same volume!

out of a piece of plasticine.

Check out

1   Which, from each pair of
objects has the greatest volume?
(a) lime, grapefruit
(b) basketball, cricket ball
(c) a 10 cent coin, a 25 cent
coin.
(d) 1 kg of sand, 1 kg of
feathers.

2 Volume can be measured in cubic centimetres (cm^3).

1 cm 1 cm 1 cm 1 cm^3

For example:

This shape is made up of 8 cubes, so the volume is 8 cm^3

3 The volume, *V*, of a prism or a cylinder is given by the formula

$$V = A \times h$$

where *A* is the area of the base and *h* is the height of the solid.
For example:

(a) 2 cm 5 cm 8 cm

Volume of cuboid
$= A \times h$
$= 8 \text{ cm} \times 5 \text{ cm} \times 2 \text{ cm}$
$= 80 \text{ cm}^3$

(b) 5 cm 10 cm

Volume of cylinder
$= A \times h$
$= \pi r^2 \times h$
$= 3.14 \times (5 \text{ cm})^2 \times 10 \text{ cm}$
$= 785 \text{ cm}^3$

4 Volumes of small objects are measured in cubic millimetres (mm^3).
Volumes of large objects are measured in cubic metres (m^3).

2 The shapes below are made of centimetre cubes. What are their volumes?
(a)

(b)

3 **(a)** What is the volume of this matchbox?

Matches 3 cm 4 cm 1 cm

(b) Find the volume of a sugar cube that has 1.5 cm sides.
(c) Find the volume of a cylindrical can of tuna which has height 6 cm and base radius 4 cm.

4 cm Tuna 6 cm

4 Which unit of volume would you use to measure the volume of a:
(a) matchbox
(b) house
(c) pinhead
(d) lime?

5 Capacity is the space inside a container.
It is usually measured in litres (l).

This 10 cm x 10 cm x 10 cm cube will hold 1 litre of water

10 cm

1 litre = 1000 cm^3

10 cm

10 cm

1 cm^3 holds 1 millilitre (ml) of water.

5 **(a)** Estimate the capacity of:
    (i) a tablespoon of cough syrup
    (ii) a bucket of water.
  (b) How many litres of water can be held in a rectangular-based tank 2 m by 3 m by 2 m?

6 How to find the surface area of a prism or cylinder.
For example:
Find the total surface area of the triangular wedge.

6 cm   10 cm   12 cm   8 cm

The wedge has three rectangular faces and two, identical, triangular faces.

Area of base rectangle = $8 \times 12 = 96$ cm^2

Area of back rectangle = $6 \times 12 = 72$ cm^2

Area of top rectangle = $10 \times 12 = 120$ cm^2

Area of triangular face = $\frac{1}{2} \times 6 \times 8 = 24$ cm^2

Total surface area = $96 + 72 + 120 + 24 + 24$
                = 336 cm^2

6 Find the surface area of these solids

(a)

4 cm   8 cm   4 cm

(b)

3 cm   4 cm   5 cm   9 cm

(c)

8 cm

14 cm

Consumer arithmetic

Objectives

- ✔ distinguish between chequing and savings accounts
- ✔ calculate interest on savings and loans
- ✔ understand the purpose of taxation
- ✔ work out personal income tax
- ✔ calculate simple utility bills
- ✔ find out the hire purchase price of goods

What's the point?

When it comes to money, life is full of choices. Should you pay cash or write a cheque or use a credit card or a debit card? You can make good choices if you are informed about the alternatives.

Before you start

You should know ...

1 Fractions can be written as percentages.
For example:

$$\overset{\times 20}{\frac{3}{5}} = \frac{60}{100} = 60\%$$
$$\underset{\times 20}{}$$

2 How to multiply fractions by whole numbers.
For example:

$$\frac{6}{20} \times 5 = \frac{6}{20_4} \times \frac{5^1}{1} = \frac{6}{4} = 1\frac{2}{4} = 1\frac{1}{2}$$

Check in

1 Write these fractions as percentages.
  (a) $\frac{1}{10}$ **(b)** $\frac{17}{100}$ **(c)** $\frac{7}{20}$
  (d) $\frac{3}{25}$ **(e)** $\frac{16}{40}$

2 Work out:
  (a) $3 \times \frac{1}{2}$ **(b)** $\frac{3}{10} \times 5$
  (c) $\frac{11}{100} \times 20$ **(d)** $\frac{5}{100} \times 350$

12.1　Banking

Banks are places where you can deposit your money safely. When you put your money in a bank you are, essentially, lending the bank your money.

You deposit money into a bank **account**.

On any account, banks pay **interest**. Interest is the bank's payment to you for loaning them your money.

There are two common types of account:
- **chequing** or **current** accounts
- **savings** accounts.

In a chequing account you:
- can withdraw or deposit money at any time
- will receive a cheque book so you can make non-cash payments
- may receive an ATM card to make cash withdrawals
- will receive a small amount of interest on your balance.

In a savings account you:
- will not be able to withdraw your money without a period of notice
- will receive a larger amount of interest on your deposit.

Exercise 12A

1　(a) You wish to withdraw money from your account. What forms should you fill out?
　(b) What forms should you fill out if you wish to deposit money?

2　(a) Collect some forms for deposit and withdrawal from two different banks.

　(b) Fill out these forms for a:
　　(i)　deposit of $253.16
　　(ii)　withdrawal of $600.

3　

　(a) What are the advantages of using a cheque?
　(b) What are the disadvantages?
　(c) What should you always remember before you write a cheque?

4　(a) Get some sample cheques from a bank.
　(b) Use the samples to write the following cheques.
　　(i)　To: J. Williams for $216.17
　　(ii)　To: Dr Ramsaroy for $300
　　(iii) To: National Electricity Company for $529.06

5　(a) What is a crossed cheque?
　(b) How does it differ from one that is not crossed?

6　David has $503.16 in his chequing account. He makes transactions as follows:

Date	Reference	Withdrawal	Deposit
15/9	Cash	$250	
31/9			$650
5/10	Cheque	$217.36	
17/10			$315
28/10	Interest		$2.17

Balance David's account by copying and completing his pass book.

Date	Reference	Withdrawal	Deposit	Balance
				503.16
15/9	Cash	250		

7　(a) Find out what a credit union is.
　(b) How does a credit union differ from a bank?

8 Go to three banks/credit unions near your school and find out:
 (a) What types of account do they offer their customers?
 (b) What are their rates of interest on each type of account offered?
 (c) Copy and complete the table.

Bank	Account type	Interest rate

9 (a) What is an ATM card?
 (b) Why are such cards popular?
 (c) Which banks/credit unions in your area give ATM cards to their customers?

10 (a) Find out about direct debit cards.
 (b) Where can such cards be used?
 (c) Can these cards be used in other countries?

11

 (a) What is a credit card?
 (b) Which banks in your area issue credit cards to customers?
 (c) What are the advantages of having a credit card?
 (d) What are the disadvantages?

Activity

Find out as much as you can about 'on-line banking':

I do all my transactions on my cell

- What is on-line banking?
- How does it differ from going to a bank?
- How secure is on-line banking? What safeguards are there to make sure your money is secure?
- What are the advantages of on-line banking?
- Any disadvantages?
- What type of transactions are you able to do on-line?
- Make a powerpoint presentation to your class on on-line or **mobile** banking.

12.2 Interest on loans and deposits

When you borrow money as a loan from a bank or other financial institution you will always pay back more money than what you borrowed.

This additional payment is called **interest**.

Interest is the extra money paid in return for the use of someone's money.

You will also receive interest when you deposit money in a bank. Why?

The amount of interest paid is dependent on the bank's **rate of interest**.

The bank's rate on loans is higher than that paid for deposits. Why do you think this is the case?

For example,
Interest rate on deposits   2%
Interest rate on loans   8%

To calculate the amount of interest payable you need to be able to work with percentages.

Working with percentages

You often see percentages in the newspapers

5% Wage hike for teachers

For a teacher earning $2500 a month, his pay increase can be found.

$$\begin{aligned}
\text{Increase} &= 5\% \text{ of } \$2500 \\
&= \tfrac{5}{100} \times \$2500 \\
&= \tfrac{1}{20} \times £2500 \\
&= £2500 \div 20 \\
&= \$125
\end{aligned}$$

Here is another example:

Example 1

Find 10% of $6.30

First write the percentage as a fraction

$$10\% = \tfrac{10}{100}$$

$$\begin{aligned}
10\% \text{ of } \$6.30 &= \tfrac{10}{100} \times \$6.30 \\
&= \tfrac{1}{10} \times \$6.30 \\
&= \frac{\$6.30}{10} \\
&= \$0.63
\end{aligned}$$

Exercise 12B

1 Write these percentages as fractions in their simplest form.

  (a) 25%   (b) 50%   (c) 75%
  (d) 16%   (e) 85%   (f) 40%
  (g) 35%   (h) 3%   (i) 12%   (j) 8%

2 Write these fractions as percentages.

  (a) $\dfrac{38}{100}$ (b) $\dfrac{3}{100}$ (c) $\dfrac{4}{10}$

  (d) $\dfrac{7}{50}$ (e) $\dfrac{17}{25}$ (f) $\dfrac{9}{20}$

  (g) $\dfrac{3}{4}$ (h) $\dfrac{8}{40}$ (i) $\dfrac{3}{5}$

3 Copy and complete the table.

Fraction	Equivalent per cent fraction	Percentage
$\frac{3}{5}$	$\frac{60}{100}$	60%
$\frac{1}{4}$	$\frac{25}{100}$	
$\frac{1}{2}$		
$\frac{3}{4}$		
$\frac{7}{10}$		
$\frac{3}{20}$		
$\frac{4}{25}$		
$\frac{9}{50}$		
$\frac{1}{8}$		

4 Work out:

  (a) 5% of $100   (b) 12% of $100
  (c) 25% of $20   (d) 75% of $80
  (e) 37% of $200   (f) 20% of $7.50
  (g) 8% of $300   (h) 15% of $60
  (i) 3% of $1000   (j) 4% of $25

5 (a) What is wrong with this?

For shame!

$$\frac{\$6.30}{10} = \frac{\$6.3\cancel{0}}{1\cancel{0}} = \$6.3$$

  (b) Without making the mistake above, find:

    (i)   10% of $9.30   (ii)   10% of $11.40
    (iii) 20% of $6.50   (iv) 30% of $3.40
    (v)   15% of $25   (vi) 23% of $42
    (vii) 3% of $20   (viii) 5% of $10.50
    (ix) 7% of $95   (x)   4% of $7.50

6 A radio is priced at $120. What is the selling price if 15% value added tax is charged?

7 Factory workers in Newport want a 12% increase in pay. How much would a worker, who currently receives $230 a week, get after the increase?

8 Marlene's meal at the Top Class restaurant cost $95. How much did she pay altogether if there was a 10% service charge and an 8% government tax on all bills?

9 At St Joseph's Academy there are 800 students. 65% of the students are boys. How many students are girls?

10 Out of 160 boxes of bananas, 15% were rejected. How many boxes were good?

11 A sales tax of 3% is placed on goods. What is the selling price of a bicycle priced at $950 before tax.

Simple interest payments

It is easy to find the simple interest payable on an amount, once you know the interest rate.

Example 2

Nerissa puts $500 in her credit union for 4 years. The credit union gives 3% simple interest.
(a) How much interest does she earn?
(b) How much money does she have in her account after the 4 years?

(a) Amount = $500
    Interest after 1 year = 3% of $500

    $$= \frac{3}{100} \times \$500$$

    $$= \$15$$
(b) Interest after 4 years $= 4 \times \$15$
    $$= \$60$$
    Nerissa's account total after 4 years
    $$= \$500 + \$60$$
    $$= \$560$$

Exercise 12C

1 Vishan has $800 in his bank account. How much interest will he get after one year if the interest rate is:
(a) 2% (b) 3% (c) 5%?

2 In Question 1, how much money will Vishan have in his account altogether after one year?

3

$4590

Mrs Chan wishes to buy a refrigerator for $4590. She can take a loan from her credit union at rate of 8%.
(a) How much interest will she pay if she borrows this sum for one year?
(b) How much will Mrs Chan pay back to the credit union altogether?
(c) If she pays the money back in 12 equal monthly instalments, how much does she pay back each month?

4 Delroy is starting a small computer business. He applies to his bank for a loan of $12 000. If the interest rate is 11%, find the interest payable on his loan after:
(a) one year
(b) two years
(c) six years
(d) ten years.

5 Find the interest on these loans.
(a) $300 for 2 years at 10%
(b) $650 for 3 years at 8%
(c) $9000 for 5 years at 11%
(d) $25 000 for 4 years at $8\frac{1}{2}$%
(e) $58 000 for 3 years at $7\frac{1}{2}$%

6 In Question 5 find the total amount that is repaid to the bank.

7 The interest rate at the Nation's Bank is 2.5%. How much money will Delia have in the bank after 5 years, if she deposits:
(a) $50
(b) $500
(c) $6500
(d) $813
(e) $12 000
(f) $315.30?

8 Find the rate of interest at a bank if the interest on a deposit of $800 is:
(a) $40 after 5 years
(b) $200 after 10 years
(c) $120 after 4 years
(d) $300 after 4 years.

9 How long must Sanjay keep $1200 in his account for it to double in value if the interest rate at his bank is:
(a) 2%   (b) 4%   (c) 5%?

10 The interest rate on deposits at the St James' Credit Union is 3.5%. For how long will Anderson have to keep a deposit of $400 before it doubles in value?

Σ⊦ Investigation

Delia needs $2000 so she can visit her aunty in Curacao.
● Go to at least three different financial institutions and find out their interest rates.
● Find the interest charged if the loan is taken for a period of
(a) 2 years
(b) 5 years
● How much will she have to repay each month in each case?
● What conditions must be satisfied to receive the loan?
● Which financial institution would you advise her to use. Give reasons.

This investigation could be used for a CSEC school based assessment.

Compound interest

Suppose the interest rate at a bank is 3%, then the interest on a principal of $800 after one year is

$$\text{Interest} = 3\% \text{ of } \$800$$
$$= \frac{3}{100} \times \$800$$
$$= \$24$$

The **simple** interest after 2 years
$$= \$24 \times 2$$
$$= \$48$$

In practice, banks do not pay simple interest. Instead, the interest after each year is added to the principal and the interest is computed from these totals. This way of working out interest is called **compound interest**.

A calculator will help you to work it out easily.

Example 3

What is the compound interest on a principal of $800 kept in a bank for 2 years, if the rate is 3%?

$$\text{Interest for first year} = 3\% \text{ of } \$800$$
$$= \frac{3}{100} \times \$800$$
$$= \$24$$
$$\text{Principal after first year} = \$800 + \$24$$
$$= \$824$$

$$\text{Interest for second year} = 3\% \text{ of } \$824$$
$$= \frac{3}{100} \times \$824$$
$$= \$24.72$$
$$\text{Total compound interest} = \$24 + \$24.72$$
$$= \$48.72$$

Exercise 12D

You will need your calculator.

1 Find the compound interest on:
(a) $100 for 2 years at 5%
(b) $500 for 2 years at 4%
(c) $800 for 3 years at 5%
(d) $8000 for 3 years at 8%.

2 In Question 1 find the total amount at the end of each time period.

3 Explain which is the better investment:
(a) a deposit of $5000 in a bank for 2 years that has a simple interest rate of 4%
(b) a deposit of $5000 in a credit union for 2 years that gives compound interest at rate of 3%.

4 Daniel takes a loan of $10 000 from his bank.
(a) What is the compound interest payable if the interest rate is 10% and he takes the loan for 2 years.

(b) What is the total amount of money he has to repay?

(c) How much should he pay back each month?

5 Ashton takes a loan of $35 000 from his bank to buy a car. The bank has an interest rate of 11% on loans.
If he is to repay the loan in two years, find:
(a) the total compound interest he has to pay
(b) the total amount he must repay the bank
(c) how much he should repay the bank each month.

Technology

Find out how you can apply for a loan or a credit card.
For example, view the process at Jamaica's National Commercial Bank by visiting

www.jncb.com

What information do banks require? How easy is it to complete on-line applications for loans?

12.3 Taxes

Where does the money come from to pay for

- roads
- teachers' salaries
- the police
- hospitals
- airports?

The answer is from **taxes**.

Taxes are usually payments made to government to pay for the cost of services.

The main taxes used are:

- import duties
- value added tax (VAT) or general consumption tax (GCT)
- income tax.

The first two taxes are usually levied on goods and services. Income tax is levied on persons.
Taxes on goods and services are usually a charged as a percentage of the cost.

Example 4

In Jamaica the GCT is 16.5%. What is the sale price of a cell phone which costs $950 before GCT?

$$\text{GCT} = 16.5\% \text{ of } \$950$$
$$= \frac{16.5}{100} \times \$950$$
$$= \$156.75$$

Sale price of cell phone $= \$950 + \$156.75$
$$= \$1106.75$$

Exercise 12E

1 The prices of these items before VAT are shown below.

(a) $185   **(b)** $6950

(c) $3600   **(d)** $1250

What are the sale prices if VAT is charged at 15%?

2 Import duty on luxury items is charged at 40%. What is the duty on:
(a) a sports car valued at $145 000
(b) perfume valued at $203
(c) a DVD player valued at $460
(d) an electric toothbrush valued at $115?

3 General consumption tax in Jamaica is 16.5%. What will be the sale price of these items, whose values before tax are given?
(a) Mattress J$13 200
(b) Night table J$2900
(c) Chest of drawers J$21 500

4

A double room at the Hilton Hotel, Barbados costs $420 per night. James Jackson and his wife stay at the hotel for four nights. What will his bill be if a 10% hotel tax and a 7.5% service charge is added to their bill?

5 What is the rate of sales tax if a radio is priced at $130 before tax and $151.45 after tax?

6 What is the cost price of a chair which is priced at $330 after the imposition of VAT at 15%?

Income tax

Income tax is the tax paid to government on the money you earn.

Only part of your annual income is taxed. The part that is not taxed is called **tax-free income**.

The amount of tax-free income is decided by government, and is usually a fixed amount.

Taxable or **chargeable** income is the money you earn in excess of the tax threshold.

In Jamaica and Trinidad the income tax rate is 25%.

Example 5

Johnson earns $72 000 each year. The tax allowance and rate in his country are:

Tax-free allowance	Tax rate
$60 000	25%

(a) What is his taxable income?
(b) How much tax does he pay?

(a) Taxable income = annual income − taxfree allowance
$$= \$72\,000 - \$60\,000$$
$$= \$12\,000$$

(b) Tax payable $= 25\%$ of taxable income
$$= 25\% \text{ of } \$12\,000$$
$$= \frac{25}{100} \times \$12\,000$$
$$= \$3000$$

In Example 5, Johnson earns $72 000 (this is his **gross pay**) but pays $3000 in income tax each year. His take-home annual salary is $69 000 (this is his **net pay**).

Exercise 12F

1 (a) Find out about the tax-free allowances for earned income in your country.
(b) What are the income tax rates?

Use this table of tax rates and allowances for Questions 2–5.

Tax-free allowance	Tax rate
$60 000	25%

2 Albert Matthew earns $115 000 each year.
(a) What is his chargeable income?
(b) How much income tax does he pay?
(c) What is his net annual income?

3 How much income tax will Adrian Bolt pay if his annual salary is:
(a) $60 000   (b) $600 000   (c) $26 000?

4 In Question 3, what is Adrian's monthly net salary?

5 Melinda paid a total of $12 000 in income tax last year.
(a) What was her chargeable income?
(b) What was her net salary?
(c) What was her net monthly salary?
(d) What was her gross salary?

Use this table of tax rates and allowances for Questions 6 and 7.

Tax-free allowance	Tax rates	
$10 000	First $20 000	10%
	Next $20 000	20%
	Excess	40%

6 Anil Ramlogan has an annual salary of $48 500. Find:
(a) his chargeable income
(b) the amount of tax he pays
(c) his net income.

7 Find Anna Quash's net income if her annual salary is:
(a) $8000   (b) $80 000   (c) $115 000

12.4 Hire purchase

Have you ever wanted to buy an expensive item but did not have enough money for it? Years ago you would have had to do without it, but nowadays you can buy most things with a **hire purchase** agreement. This usually means that you can take the item home with you on a payment of a small deposit and an agreement to pay the store so much each month until the total hire purchase cost is met.

Cash price $870

OR

10% deposit &
12 monthly
instalments
of $85.75

Ramjit needs a bicycle to get to his new job. He buys a new bicycle on **hire purchase**.

This means he pays the store only a part of the price, called a **deposit**, and takes the bicycle away. He pays the rest of the price in small monthly amounts called **instalments**.

The cash price of the bicycle $= \$870$

$$
\begin{aligned}
\text{Deposit} &= 10\% \text{ of cash price} \\
&= 10\% \text{ of } \$870 \\
&= \frac{10}{100} \times \$870 \\
&= \$87
\end{aligned}
$$

Ramjit pays $87 and takes the bicycle home. Then he pays the store $85.75 a month for a year.

Exercise 12G

1 (a) In the example shown, what does *cash price* mean?
   (b) What instalment does Ramjit pay per month?
   (c) How much does he pay in instalments altogether, over 12 months?
   (d) How much does he pay altogether for the bicycle? (Don't forget his deposit!)

2 (a) Does the bicycle cost more on hire purchase? How much more?
   (b) Why do you think a store charges more, when you buy something on hire purchase?

3 (a) Think of two advantages of buying things on hire purchase.
   (b) Now see if you can think of two disadvantages of hire purchase.

4 What would happen if Ramjit did not pay his monthly instalments?

Activity

Find out about the conditions under which you can buy by hire purchase in your country.

- After you have paid the deposit, who owns the item?
- Can the store repossess the item if you miss one monthly payment?
- What happens if you damage the item during the purchase period?
- What happens if the item is defective?
- Can you terminate the hire purchase agreement?

When you buy something on hire purchase it costs more than if you paid cash. The extra money you pay is the interest charged.

Example 6

The cash price of a radio is $240. It can be bought on hire purchase with a deposit of 20% and 6 monthly payments of $35.
Find
(a) the hire purchase price
(b) the interest charged

(a) Deposit = 20% of $240

$$= \frac{2\cancel{0}}{1\cancel{0}\cancel{0}} \times \$24\cancel{0}$$

$$= \$48$$

    Total monthly payments
$$= 6 \times \$35$$
$$= \$210$$
    Total hire purchase price
$$= \text{deposit} + \text{total monthly payments}$$
$$= \$48 + \$210$$
$$= \$258$$

(b) Interest charged
$$= \text{hire purchase price} - \text{cash price}$$
$$= \$258 - \$240$$
$$= \$18$$

Some stores do not require a deposit but ask for weekly payments.

Example 7

The cash price of a bed is $895. It can be bought on hire purchase with no deposit and weekly payments of $10.64 over 2 years.
(a) What is the hire purchase price of the bed?
(b) What is the interest charged?

(a) Payments have to be made for two
    years $= 52 \times 2 = 104$ weeks
    Hire purchase price = total weekly payments
$$= \$10.64 \times 104$$
$$= \$1106.56$$

(b) Interest charged
$$= \text{hire purchase price} - \text{cash price}$$
$$= \$1106.56 - \$895$$
$$= \$211.56$$

Exercise 12H

1 Find the hire purchase prices of these goods. Assume there is no deposit and payments are for two years.

(a)

Cash $1195
Weekly
$14.20

(b)

Cash $695
Weekly
$8.26

(c)

Cash $395
Weekly
$4.69

(d)

Cash $1098
Weekly
$13.48

(e)

Cash $1395
Weekly
$17.13

2 What is the interest charged for each of the items in Question 1?

3 How much more is the hire purchase price than the cash price for these items?

(a)

Cash $800
10% deposit
12 monthly
payments
of $75.60

(b)

Cash $700
15% deposit
12 monthly
payments
of $63.40

(c)

Cash $850

20% deposit
24 monthly
payments
of $33.50

(d)

DVD

Cash $1100

25% deposit
30 monthly
payments
of $30.25

4 A child's bicycle is offered at $120 cash or a deposit of 10% and 18 monthly payments of $7.20. Find how much more it costs on hire purchase than by cash.

5 The same bicycle as in Question 4 is also offered at 10% deposit and 24 monthly payments of $5.50. Find the total interest paid, in dollars.

6 In the Walton Trading Stores, a certain bicycle is priced at $280. It can be bought on hire purchase with a deposit of 20% and 10 equal monthly payments.
(a) Find the deposit and the amount of each instalment, if *no* interest is charged.
(b) If the monthly instalments are $25.20, find the interest paid, in dollars.

7

FAN
Cash $270
or
40% deposit +
9 payments of $22·50

An electric fan, cash price $270 is offered at a 40% deposit and 9 monthly payments of $22.50. Find:
(a) the deposit
(b) the total price paid altogether
(c) the interest charged, in dollars
(d) the interest as a percentage of the cash price.

8

The sale of a child's bike, marked $380, is offered in three ways:
A 10% off for cash
B Easy payments: 20% deposit and $7.30 per week for 1 year
C Easy payments: $30 deposit and $7.20 per week for 1 year
Find:
(a) the total amount paid by each method
(b) the interest paid in methods B and C
(c) the best method if you want to pay out the least cash at the start
(d) the best method if you want to pay over one year.

9 Find:
(a) the total amount paid for the suit by methods A and B
(b) the percentage interest for method B.

A 15% discount for cash
B $500 deposit and 8 monthly instalments of $140.25

10 The selling price of a refrigerator is $3900. There are two ways you can buy it:
A Pay cash and get 15% discount
B Pay 20% deposit and 10 monthly instalments of $385

Find:
(a) the total amount paid for the refrigerator by each method
(b) the percentage interest for method B.

11 A television has a cash price of $1250. When sold on hire purchase the interest charged is 20%. What is the down payment if a customer makes 24 monthly payments of $53.50?

12 A furniture shop sells a single chair for $300 and a table for $800. Find the total amount paid for the table and **four** chairs, by each method.

A   Pay cash and get $12\frac{1}{2}\%$ discount
B   Pay 15% deposit and 8 monthly instalments of $245

13 Buying something on hire purchase is a bit like borrowing money from a bank. Can you see how? In Exercise 12G, is Ramjit taking the bicycle as a form of loan? If so, what is the rate of interest?

Activity

Find advertisements in newspapers or magazines, showing things being offered for sale by hire purchase. Cut them out.

(a) For each, find the total cost by hire purchase.
(b) Find the interest as a percentage of the cash price.
(c) Which stores give the best deals?

Investigation

Allana wishes to buy the latest smartphone.

● Go to two stores that sell this smartphone and find its cash price and hire purchase terms.
● Go to two lending institutions and find the terms of borrowing money to pay the cash price.
● Calculate
  (a) the hire purchase price from each store
  (b) the amount to be repaid to each lending institutions.

Show all your working.
● Draw a bar graph to show the relative costs of the cell phone.
● Discuss your results and state how you would advise Allana.

You could use this investigation as part of a CSEC school based assessment.

Exercise 12I – mixed questions

1

Complete this slip for a deposit of $3500.

2

Complete this cheque, making a payment of $1033.99 to J.R. Robinson.

3 Write down two advantages and two disadvantages of using a debit card as opposed to using a cheque.

4 Find:
(a) 6% of $75   (b) 9% of $220
(c) 7.5% of $150   (d) 2% of $5

5 Devon invested $3000 at 3% simple interest for 6 years.
Find:
(a) the simple interest
(b) the total amount in Devon's account after 6 years.

6 Find the total amount accrued on the sum of $250 when deposited in a credit union for 2 years which pays 3% compound interest.

7 The Nature Bank gives an interest rate of 5%. How long should Karen leave $5000 in the bank for it to grow to $6000?

8

Which is the cheaper hire purchase method for buying a refrigerator that costs $2500 cash?

A   Deposit of 20% and 24 monthly payments of $95.50
B   No deposit and 36 monthly payments of $87.95

9 Avril Flowers earns $65 000 a year. Allowances and tax rates are as follows:

Tax-free allowance	Tax rates
$30 000	20%

(a) What is Avril's chargeable income?
(b) How much income tax does she pay?
(c) What is her net income?

10 The tax rates and allowances in the island of St Michael are:

Tax-free allowance	Tax rate
$45 000	30%

(a) Ms Alport earns $93 200 each year.
  (i) What is her chargeable income?
  (ii) How much tax does she pay?
  (iii) What is her net salary?

(b) Mr Sampson pays $16 926.30 in income tax each year.
  (i) What was his chargeable income?
  (ii) What is his gross salary?
  (iii) What is his net salary?

11 What is the total cost of this bill?

4 Soups	$16.00
4 French fries	$13.00
2 Chicken legs	$15.50
+ 15% Service	
Total	

12 Andra bought a car for $60 000 in 2017. Each year it is worth 20% less. How much would it be worth in 2019?

13 Which earns the most simple interest?
(a) Deposit $5000 for 3 years at 5% simple interest rate.
(b) Deposit $4000 for 4 years at 4% simple interest rate.

14 The Nation Credit Union gives 4% simple interest on deposits. Marilyn puts $500 in the Nation. How long must she wait before her money doubles?

15 Kurt invests $7500 in a Trust Fund for 6 years at 8% simple interest.
(a) Calculate the total interest he receives in the six years.
(b) What sum would Kurt have to invest to receive $6000 interest after six years?

12 Consolidation

Example 1
Work out:
(a) 15% of $500
(b) 3% of $17.50

(a) $15\% \text{ of } \$500 = \dfrac{15}{100} \times \$500$

$= \dfrac{15}{100_1} \times \$500^{5}$

$= \$\dfrac{75}{1} = \$75$

(b) $3\% \text{ of } \$17.50 = \dfrac{3}{100} \times \$17.50$

$= \dfrac{52.50}{100}$

$= \$0.525$

$\simeq \$0.53$

Example 2
The People's Bank offers a 6% interest rate on deposits. What will be the interest given on a deposit of $5000 after:
(a) 1 year   (b) 7 years

(a) Interest = 6% of $5000

$= \dfrac{6}{100} \times \$5000$

$= 6 \times \$50$

$= \$300$

(b) Interest after 7 years = interest after 1 year × 7

$= \$300 \times 7$

$= \$2100$

Example 3
What is the sale price of a bed priced at $2340 if 15% VAT is added?

$\text{VAT} = 15\% \text{ of } \$2340 = \dfrac{15}{100} \times \$2340 = \$351$

Sale price $= \$2340 + \$351$

$= \$2691$

Example 4
A refrigerator has a cash price of $2400. What is its hire purchase price if it can be bought for a 15% deposit and 24 monthly payments of $95?

Deposit $= 15\% \text{ of } \$2400 = \dfrac{15}{100} \times \$2400$

$= \$15 \times 24 = \$360$

Total monthly payments $= \$95 \times 24$

$= \$2280$

Hire purchase price $= $ deposit + total monthly payments

$= \$360 + \$2280$

$= \$2640$

Exercise 12

1 Mr Ramsaroop earns $750 per week. He then gets a rise of 8%.
(a) What is his increase?
(b) What is his new weekly wage?

2 Find the total simple interest on a loan of $17 500 if the interest rate is 10% and the loan is taken for:
(a) 6 years
(b) $2\frac{1}{2}$ years
(c) 8 years.

3 VAT of 15% is imposed on all goods by a government. What is the selling price of:
(a) a radio priced at $80
(b) a television priced at $1250
(c) a book priced at $64
(d) a pen priced at $3.50
(e) a tin of milk priced at $2.30

(f) a box of soap powder priced at $13.50
(g) a bicycle priced at $975
(h) a wardrobe priced at $2180?

4   Find the hire purchase prices of these items.

(a)

CASH
$1299
10% deposit
24 monthly
payments
of $55.18

(b)

CASH $2229
5% deposit
Weekly
payments
of $24.50
for two years

(c)

CASH
$5499
$7\frac{1}{2}$% deposit
24 monthly
payments
of $217.95

5   The income tax rate in Trinidad is 25% on all earnings above $60 000 a year.
(a) How much tax is paid by a man earning
   (i) $47 375
   (ii) $93 140?
(b) What is the man's net pay in each case?

Application

6   (a) The simple interest rate at the New Credit Union is 4%. How much will Ashton have in the Credit Union after 1 year if he deposits:
   (i) $600
   (ii) $750
   (iii) $5000
   (iv) $14 250
   (v) $82?

   (b) How much money will he have on each of these amounts after:
   (i) 3 years
   (ii) 8 years?

7   The room rate at the Hastings Hotel is $210 per night. Justin spends two nights at the hotel. What is his bill if:
(a) a $7\frac{1}{2}$% service charge is levied?
(b) a 10% government tax is added?
(c) both a $7\frac{1}{2}$% service charge and 10% government tax are included?

8   The sale price of a reconditioned car is $45 000.
(a) How much will Carl pay for it if all sales are subject to 15% VAT?
(b) What will be Carl's payment if there is a 7% cash discount?
(c) If the car depreciates at the rate of 20% per year from the price including VAT, how much will it be worth after two years?

Summary

You should know ...

1 (a) The difference between a savings and chequing account.
  (b) How to complete deposit and withdrawal slips.
  (c) How to write a cheque.
  (d) The difference between a credit and a debit card.

2 How to find percentages of amounts.
For example:

$$6\% \text{ of } \$20 = \frac{6}{100} \times \$20$$
$$= \frac{\$12}{10} = \$1.20$$

3 How to calculate interest.
For example:
The interest rate at a bank is 3%.
  (a) How much simple interest is generated on a sum of $1300 kept in the bank for 2 years?
  (b) How much compound interest?

  (a) Interest after 1 year = 3% of $1300
$$= \frac{3}{100} \times \$1300$$
$$= \$39$$

  Simple interest after 2 years
$$= 2 \times \$39$$
$$= \$78$$

  (b) Total after one year = $1300 + $39
$$= £1339$$

  Interest in second year = 3% of $1339
$$= \$40.17$$

  Total compound interest = $39 + $40.17
  after 2 years $= \$79.17$

Check out

1 (a) Explain how a savings account differs from a cheque in account.
  (b) Complete a deposit slip for $638.15.
  (c) Complete a withdrawal slip for $925.
  (d) Make a cheque payable to The Gas Company' for $212.23.
  (e) Explain how a debit card differs from a credit card.

2 Work out:
  (a) 10% of $50 (b) 5% of $10
  (c) 3% of $6   (d) 12% of $25

3 The interest rate on loans at a bank is 9%. Ali wishes to borrow $15 000. What is his total repayment to the bank if he is to pay:
  (a) simple interest for 2 years
  (b) compound interest for 2 years?

4 The purpose of taxes and the types of taxes that are levied.

4 Explain in your own words the meaning of:
(a) income tax
(b) VAT
(c) consumption tax
(d) gross pay
(e) net pay.

5 Hire purchase is a way of buying something and paying for it by instalments over a period of time.
Often an initial down payment or deposit is required.
For example:

TV SET Cash price
$800
or DEPOSIT 15%
+ 12 EASY PAYMENTS of $60

Deposit = 15% of $800

$$= \frac{15}{100} \times \$800$$

$$= \$120$$

Total payments = 12 × $60 = $720
Hire purchase price = deposit + total payments
$$= \$120 + \$720$$
$$= \$840$$

5 A stereo has a cash price of $1250. It can be bought on hire purchase with a deposit of 20% and 18 monthly payments of $67.50.
(a) Find the hire purchase price of the stereo.
(b) What is the interest charged?

Revision exercise 3

Statistics

1 24 students in a maths test scored the following marks.

7, 9, 3, 8, 7, 2, 9, 7, 6, 2, 7, 4, 5, 1, 3, 7, 8, 5, 2, 7, 5, 3, 7, 8

(a) Copy and complete the frequency table.

Mark	Tally	Frequency
1	|	1
2	|||	
3		

(b) Which mark is the **mode**?

2 (a) Draw a block graph to represent the information in Question 1.

(b) Draw a second block graph using the groups 1–2, 3–4, 5–6, 7–8, 9–10 for the marks. Which group is the **mode**?

3 (a) Copy and complete the table below for calculating the angles in a pie chart to represent the information in Question 1.

Mark	Number of students	Angle
	24	360°
1	1	
2	3	

(b) Draw a **pie** chart to represent the information in part (a).

4 (a) Draw another pie chart to **represent** the information in Question 2(b).

(b) Which of your pie charts in Questions 3 and 34 is easier to read?

5 (a) Find the **mean** of the marks in Question 1.

(b) Find the **median** of the marks in Question 1.

6 (a) The mean of four marks is 73%. If three of the marks are 52%, 61% and 89%, what is the fourth mark?

(b) If the mean of four marks is 62% and when a fifth is added the mean becomes 68%, what is the fifth mark?

7 Look at the graph below.

Graph to show the ages of a group of athletes

(a) How many athletes are shown?

(b) Which age is the **mode**?

(c) Which is the **median** age?

(d) Find the **mean** age of the athletes.

8 (a) Copy and complete the table below for calculating the angles in a pie chart for Question 7.

Age	Number of atheletes	Angle
	20	360°
12	4	
13		

(b) Draw a **pie** chart to represent the information in part (a).

9 Look at the graph below.

Graph to show the ages of people in a block of flats

(a) Which age group is the **mode**?

(b) How many occupants are 20 or older?

(c) How many occupants are 29 or younger?

(d) What can you say about the **median** age of the group?

10 Draw a pie chart to represent the information shown in Question 9.

Patterns and algebra

11 (a) 1, 4, 9, 16 and 25 are the first five square numbers. Write down the next two square numbers and the sixteenth square number.

(b) Find the difference between each square number and the next one
What do you notice?

12 1, 1, 2, 3, 5, 8, 13 … are called Fibonacci numbers. Each new number is found by adding together the previous two (e.g. $5 + 8 = 13$).
(a) Write down the next two numbers in this sequence
(b) How many numbers are there in this sequence before you reach the next square number after 1?
(c) Which of the first twelve numbers in this sequence are also triangle numbers?

13 (a) Copy and complete this table:

1^3	2^3	3^3	4^3	5^3	6^3
$1 \times 1 \times 1$	$2 \times 2 \times 2$				
1	8				

(b) Copy and complete this table:
$1^3 = 1 = 1^2$
$1^3 + 2^3 = 9 = 3^2$
$1^3 + 2^3 + 3^3 =$
$1^3 + 2^3 + 3^3 + 4^3 =$
$1^3 + 2^3 + 3^3 + 4^3 + 5^3 =$
$1^3 + 2^3 + 3^3 + 4^3 + 5^3 + 6^3 =$

(c) What can you say about the results for each row in the table in part (b)?
(d) Can you write down the sum of the first 7 cubes? Check your answer with your calculator.

14 Look at a group of three numbers in the sequence in Question 12.

For example: 2, 3, 5 $3^2 = 9$, $2 \times 5 = 10$

          or   3, 5, 8 $5^2 = 25$, $3 \times 8 = 24$

Is it always true that the difference between the middle number squared and the product of the first and third is one?

15 For each sequence **(a)** to **(f)** below write down:
    (i)   the next three terms
    (ii) the 10th term
    (iii) the 20th term
    (iv) a formula for the nth term.
(a) 5, 10, 15, 20, 25, 30, . . .
(b) 6, 11, 16, 21, 26, 31, . . .
(c) 1, 4, 7, 10, 13, 16, . . .
(d) 2, 5, 8, 11, 14, 17, . . .
(e) 2, 5, 10, 17, 26, 37, . . .
(f) 0, 3, 8, 15, 24, 35, . . .

16 Look at the cube drawn below.

Write down a formula for:
(a) V, the volume of the cube
(b) S, the surface area of the six faces of the cube
(c) L, the total length of all the edges of the cube

17 (a) Find the mass, M, of n tins of paint each of mass m kg.
(b) Find the cost c of each book if the total cost of b books is T.
(c) Find A, the area of card left if a square of edge p cm is cut out from a square card of edge q cm.

18 Solve these equations.
(a) $2x + 3 = 9$ **(b)** $x + 4 = 2x - 2$
(c) $3(x + 1) = 6$ **(d)** $2x - 3 = x + 6$

19 If $\dfrac{1}{f} = \dfrac{1}{u} + \dfrac{1}{v}$ find f when:

(a) $u = 2$ and $v = 4$ **(b)** $u = 8$ and $v = 12$.

20 Make x the subject of these equations
(a) $y = ax - b$
(b) $y = ax^2 + b$

Volume

21 Each of these shapes is made of 1 cm cubes. Write down the volume of each shape.

(a)   **(b)**

22 Copy and complete the table for cuboids:

	Length	Width	Height	Volume
(a)	5 cm	4 cm	3 cm	
(b)	7 cm	6 cm	9 cm	
(c)	13 cm	8 cm	2.5 cm	
(d)	12 cm	9 cm	3.1 cm	
(e)	7 cm		5 cm	280 cm^3
(f)	12 cm	4 cm		360 cm^3

23 Use your calculator to find the volume of each cuboid below.

(a)   **(b)**

2.1 cm
4.7 cm
3.2 cm

7.9 cm
1.5 cm   1.3 cm

24 Find the surface area of each cuboid in Question 23.

25 Use your calculator to find the volume of each solid

(a)

2 cm
6.4 cm
2 cm
7.5 cm   3.9 cm

(b)

3.2 cm
2.4 cm
1.5 cm
1.3 cm
1.8 cm
2.1 cm
3.2 cm

26 Find the surface area of each shape in Question 25.

27 (a) Copy and complete the table for cubes.

Edge length (cm)	1	2	3	4	5	6
Volume (cm^3)						

  (b) Use your results for part (a) to write down the sum of:
    (i) the first two cubes in the sequence.
    (ii) the first three cubes in the sequence.
    (iii) the first four cubes in the sequence.
    (iv) the first five cubes in the sequence.
  (c) What do you notice about your answers for part (b)?
  (d) Is the sum of the first six volumes also a square number?

28 (a) Using you results from Question 27(a) as coordinates, draw a graph to show the volume of a cube for given edge lengths.
  (b) Use your graph to find an approximation for the edge of a cube of volume 50 cm^3.
  (c) Use your calculator to check your answer for part (b).

29 (a) Find the volume of these prisms.
  (i)

area = 12 cm^2
8 cm

  (ii)

5 cm
13 cm
12 cm
15 cm

(b) Calculate the surface area of the prism in (a) (ii).

30 (a) Find the volume of a cylindrical water tank with base radius 60 cm and height 2m.
(b) What is the capacity of the tank in litres?

31 Calculate the area of tin needed to make a can of milk with base radius 8 cm and height 15 cm.

Consumer arithmetic

32 Find:
(a) 20% of 35 cm   **(b)** 5% of $240
(c) 47% of 500 kg   **(d)** 13% of $29

33 What is the selling price of these items if 15% VAT is to be added?

(a)

$135

(b)

$248

(c)

$315

(d)

$925

34 Taxation rates in an island are:
Tax-free allowance $50 000
Tax rate 20%.
Brian earns $134 000 per year.
(a) What is his chargeable income?
(b) How much tax does he pay?
(c) What is his net income?

35 The cash price of a television is $3500. It can be bought on hire purchase for a deposit of 20% and 24 monthly payments of $130.
(a) What is the deposit?
(b) Calculate the total hire purchase price.
(c) Find the percentage interest rate.

36 (a) What is mobile banking?
(b) What are the advantages of mobile banking?
(c) How does it differ from 'internet banking'.

37 Copy and complete the table below for a sale in a clothes shop.

Item	Marked price	Discount	Sale price
Shirt	$8	5%	
Jacket	$42	12%	
Suit	$140	20%	
Dress	$32		$24
Jumper	$15		$13.50
Shoes		5%	$19
Hat		35%	$10.40

38 Sally wants to borrow $250 for a new cooker. The shop says she will have to pay 12% interest per year.
Find how much her cooker will cost if she borrows the money for:
(a) one year   **(b)** two years   **(c)** three years.
If Sally pays off the loan in equal monthly instalments, how much will she pay each month in part (a), in part (b) and in part (c)?

39 Jean is given $1000 by a rich uncle. She decides to invest it in the bank which offers her 8% interest per year.
Copy and complete the table to show how much interest she will receive if she leaves her money in the bank.

Number of years	Total interest received
1	
5	
6 months	
	$240
	$20
	$410

40 In Question 39 Jean decides to re-invest the interest she receives so the she will gain even more interest.
Copy and complete the table to show how much money she has in the bank altogether at the end of each year.
(You may use your calculator.)

Number of years	Total amount
0	$1000
1	$1080
2	
3	
4	
7	

Mixed questions 3

1 The number of children absent each day in a certain week is shown in the frequency table below.

Weekday	Tally	Frequency
Monday	|||	3
Tuesday	||||	4
Wednesday	||||	4
Thursday	|||	3
Friday	||||		6

What is the mean number of absences per day?
A 3   **B** 4   **C** 5   **D** 6

2 The favourite sport for a group of 30 students is as follows:

Sport	Football	Basketball	Cricket
Number of students	10	15	5

In order to show this information as a pie chart, the angle that represents the number of students who prefer football is:
A 10°   **B** 60°   **C** 120°   **D** 180°

3

The pie chart shows the results of an opinion poll taken of 120 persons on which political party they would vote for. How many persons indicated they would vote for the Working People's party?
A 30   **B** 60   **C** 90   **D** 120

4 The scores on a test were
6, 4, 9, 8, 6, 4, 10, 7
What was the median score?
A 4   **B** 6   **C** $6\frac{1}{2}$ **D** $7\frac{1}{2}$

5 In Question 4, what was the mean score?
A 4   **B** 6   **C** 6.5   **D** 6.75

6 The heights of 20 flowers in centimetres were:

16	15	20	6	12
15	6	15	12	14
11	10	9	17	13
16	14	19	7	13

What is the modal height?
A 6 cm   **B** 15 cm   **C** 16 cm   **D** 17 cm

7

The graph shows the temperature between 10 am and 3 pm on a certain day. What was the temperature at 11.30 am?
A 27°   **B** 28°   **C** 28.5°   **D** 29°

8 The manager of a shoe store records the mean, mode and median shoe sizes of shoes sold in a particular week. Which of these statistics would be of most use?
A Mean   **B** Median
C Mode   **D** None of these

9 What is the next number in this sequence? 5, 9, 13, 17, ___
A 21   **B** 22   **C** 23   **D** 27

10 Which of these is **not** a square number?
A 49   **B** 196   **C** 30   **D** 9

11 What is the 5th triangle number?
A 10   **B** 6   **C** 15   **D** 36

12 What value of x satisfies this equation?
$3x + 2 = x + 8$
  A 0   **B** 1   **C** 2   **D** 3

13 $\dfrac{3x^3y^2}{6y^3x^4}$ equals

  A $\dfrac{1}{2}$ **B** $\dfrac{1}{2xy}$ **C** $\dfrac{xy}{2}$ **D** $\dfrac{1}{2xy^2}$

14 What is the formula for the perimeter of this shape?

  A $x + y$ **B** xy **C** $x - y$ **D** $2x + 2y$

15

The number line shows the solution set that represents the inequality
  A $x > {}^-2$ **B** $x \le {}^-2$ **C** $x \ge {}^-2$ **D** $x < {}^-2$

16 The solution set of the inequality $3x - 2 < 4$ is
  A $x < 6$ **B** $x < 2$ **C** $x > 6$ **D** $x > 2$

17 If $3P - 2R = 4T$, then P equals

  A $3(4T + 2R)$ **B** $\dfrac{4T - 2R}{3}$
  C $\dfrac{4T + 2R}{3}$ **D** $3(4T - 2R)$

18 The expression $3(x - y) - 2(x + y)$ equals
  A $x - 5y$ **B** $x - y$ **C** $x + y$ **D** $x + 5y$

19 Coryl has three times as many oranges as Kim. Kim has two more oranges than Paula. If the three of them have 98 oranges, how many does Kim have?
  A 18   **B** 20   **C** 24   **D** 60

20 What is the cost of x tins of beans at y dollars a tin?
  A $\$xy$ **B** $\$x + y$ **C** $\$\dfrac{x}{y}$ **D** $\$x - y$

21 What units would you use to measure the volume of a raindrop?
  A mm^3 **B** cm^3 **C** m^3 **D** km^3

22 A cube has a volume of 64 cm^3. What is the length of an edge of the cube?
  A 16 cm   **B** 8 cm   **C** 4 cm   **D** 2 cm

23 What is the volume of this box filled with 1 cm cubes?

  A 24 cm^3 **B** 12 cm^3
  C 9 cm^3 **D** 8 cm^3

24 What is the volume of a crate measuring 1 m by 2 m by 1.5 m?
  A 6 m^3 **B** 4.5 m^3 **C** 3 m^3 **D** 1.5 m^3

25 What is the capacity of this box?

  A 0.4 ml   **B** 11 ml   **C** 20 ml   **D** 40 ml

26 The dimensions of the side of a box are 6 cm by 10 cm.
The dimensions of the front of the box are 6 cm by 8 cm.
What are the dimensions of the top of the box?
  A 6 cm by 6 cm   **B** 6 cm by 8 cm
  C 6 cm by 10 cm   **D** 8 cm by 10 cm

27 What is the surface area of a box with dimensions 3 cm by 4 cm by 5 cm?
  A 47 cm^2 **B** 60 cm^2 **C** 94 cm^2 **D** 120 cm^2

28

What is the volume of a cylinder that has a base area of 0.5 m^2 and a length of 50 cm?
  A 0.025 m^3 **B** 0.25 m^3
  C 2.5 m^3 **D** 25 m^3

29 How many boxes measuring 50 cm by 20 cm by 10 cm will fit into a cube with volume 1 m^3?
  A 10   **B** 100   **C** 1000   **D** 10 000

30 The volume of a crate is 120 m^3. The base of the crate measures 6 m by 5 m. What is the height of the crate?
  A 5 m   **B** 6 m   **C** 4 m   **D** 10 m

31 What is 8% of $50?

   A $1.60   **B** $4.00   **C** $16.00   **D** $40

32 An iron has a price of $120. What is its selling price if there is a 3% sales tax?

   A $3.60   **B** $36
   C $123.60   **D** $156

33 A gas stove bought for $2000 depreciates by 10% each year. What is its value after two years?

   A $200   **B** $400   **C** $1600   **D** $1620

34 A store gives a 5% cash discount on all purchases over $50. What is the cash price of a fan priced at $160?

   A $170   **B** $168   **C** $152   **D** $150

35 How much simple interest will Tony get after 3 years if he puts $500 in the New Bank which has a 7% interest rate?

   A $35   **B** $105   **C** $535   **D** $605

36 Celia buys 80 mangoes for $60 and sells them for $1.20 each. What is her profit?

   A $20   **B** $36   **C** $52   **D** $61.20

37

Tax allowance	Tax rate
First $35 000 tax-free	0%
Further amounts	30%

What is the tax paid by a man earning $40 000?

   A $1500   **B** $5000
   C $30 000   **D** $38 500

38 What is the hire purchase price of a refrigerator that can be paid for with a $150 deposit and 18 monthly payments of $120?

   A $150   **B** $2160   **C** $2310   **D** $2700

39 After one year $200 in the National Bank becomes $206. What is the rate of interest at the bank?

   A 3%   **B** 4%   **C** 5%   **D** 6%

40 What is the compound interest on $1000 invested in a bank for two years at an interest rate of 10%?

   A $100   **B** $110   **C** $200   **D** $210

Glossary

1 Number

Ratio Ratio is used to compare the size of two quantities

Base The base of a place value number system determines the relationship between the places

Binary Number system that uses just two digits: 0 and 1

2 Decimals

Significant figures Significant figures show the relative importance of the digits in a number. The most important digit is the first non-zero number.

For example:

	785 200	0.003 186
3 sig. fig.	785 000	0.003 19
2 sig. fig.	790 000	0.0032
1 sig. fig.	800 000	0.003

Standard form Standard form or scientific notation is a way of displaying a number as a product of a number lying between 1 and 10 and a power of ten.

For example: $785\ 200 = 7.852 \times 10^5$

$$0.003\ 186 = 3.186 \times 10^{-3}$$

3 Angles

Alternate angles The two equal angles formed on the opposite sides of a line cutting two parallel lines
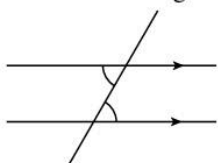
Corresponding angles The two equal angles formed on the same side of a line cutting two parallel lines
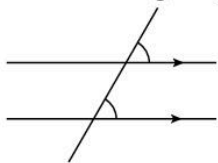
Exterior angle The exterior angle of a polygon is $180° -$ interior angle

Interior angle The interior angle of a polygon is the angle formed between two adjacent sides of the polygon

Parallelogram A quadrilateral with two pairs of parallel sides

Polygon A closed shape with three or more edges

Rhombus A quadrilateral with two pairs of parallel sides and all sides equal in length

Vertically opposite The two pairs of angles formed when two lines intersect
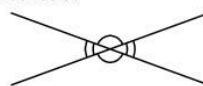
4 Sets

Complement The complement of a set contains all the elements that are in the universal set but are not in the given set

Intersection The intersection of two sets is the set that contains only elements common to both sets

Subset A set that contains part or all of another set

Union The union of two sets is the set that contains all the elements of both sets

5 Measurement

Circumference The distance around or perimeter of a circle

Diameter The distance across a circle through its centre

Perimeter The distance around a closed shape

Radius The distance from the centre of a circle to its circumference

Trapezium A quadrilateral with one pair of parallel sides

6 Negative numbers

Commutative An operation is commutative if you can combine two elements in any order and still get the same result

Number line A graduated line on which numbers can be marked

7 Relations and graphs

Arrow diagram   A diagram that links some or all elements in one set to the elements of a second set by means of a stated rule

Function   A one-to-one or many-to-one mapping

Gradient   The steepness or slope of a line

Intercept   An intercept of a straight-line graph is the point where it cuts an axis

Many-to-one map   A mapping in which one more than one element of the first set is sent to an element in the second set

Mapping   A mapping is a rule that sends every member of the first set to an element in the second set

One-to-one map   A mapping in which each element of the first set is sent to just one element of the second set

Ordered pair   Two numbers written so as to give the position of a point on a graph

8 Transformations

Congruent   Shapes are congruent to each other if they have the same size and shape

Order of rotational symmetry   The number of times a shape fits onto itself in a complete turn

Reflection   A transformation that gives the mirror image of a shape

Rotation   A transformation that rotates a shape through an angle about a point

Rotational symmetry   A shape has rotational symmetry if it fits onto the same position more than once when rotated through 360°

Translation   A transformation that moves a shape into a new position

9 Statistics

Mean   The mean value of a set of data is found by adding up all the data values and dividing by the number of values

Median   The median is the middle value after arranging the data set in order of size

Mode   The mode is the most common value in a set of data

Pie chart   A graph that displays data in the form of a pie or circle; each section of the pie chart represents a proportion or percentage of the total

Statistics   Numerical facts or data

Survey   A method of collecting or gathering information

10 Patterns and algebra

Algebra   A branch of mathematics dealing with generalised arithmetic where letters or symbols are used to represent numbers

Associative   An operation * is associative if $(a * b) * c = a * (b * c)$; for example, addition is associative since $(2 + 5) + 9 = 2 + (5 + 9)$

Commutative   An operation on two objects is commutative if the order in which it is carried out does not matter. For example: $a + b = b + a$

Composite number   A number that has three or more factors

Prime number   A number with only two factors

Square number   A square number can be made by multiplying a whole number by itself; the square numbers are 1, 4, 9, 16, 25, . . .

Triangle number   The numbers 1, 3, 6, 10, 15, 21, . . .

11 Volume

Cylinder   A solid with two identical and parallel circular faces
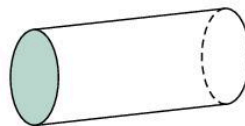
Prism   A solid with two identical and parallel faces in the form of a polygon
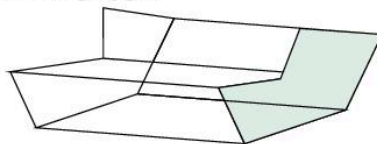
Capacity   The amount of liquid a container can hold

Surface area   The sum of the areas of each face of a solid

Volume   The amount of space taken up by a solid

12 Consumer arithmetic

Cheque A formal note promising to pay money to a named person or entity

Chequing account A bank account against which the depositor can draw cheques that are payable on demand

Compound interest Interest that is calculated on the principal and previously paid interest

Credit card A plastic card issued by a bank authorising payment for purchases; interest is charged on the outstanding balance

Debit card A plastic card which allows withdrawal of money or the cost of goods to be charged directly to the holder's bank account

Gross income The income before tax

Hire purchase A method of buying something by making regular payments over a period of time. A small down-payment or deposit is sometimes also required.

Income tax A tax on personal earnings and wages

Interest The extra money paid in return for the use of someone else's money

Net income The income that is left after payment of tax

Savings account A bank account where money is stored and accumulates interest

Simple interest Interest that is paid on the principal alone

Taxable income The portion of income that is subject to taxation

1 Number

Check in

1 (a) $\frac{2}{4}$ or $\frac{1}{2}$ **(b)** $\frac{5}{9}$

2 (a) $\frac{1}{4} = \frac{2}{8}$ **(b)** $\frac{3}{8} = \frac{6}{16}$ **(c)** $\frac{2}{3} = \frac{6}{9}$ **(d)** $\frac{5}{8} = \frac{15}{24}$

3 (a) $\frac{2}{3}$ **(b)** $\frac{2}{3}$ **(c)** $\frac{2}{3}$ **(d)** $\frac{2}{3}$ **(e)** $\frac{7}{11}$ **(f)** $\frac{2}{3}$

4 (a) $\frac{8}{3}$ **(b)** $\frac{13}{4}$ **(c)** $\frac{19}{4}$ **(d)** $\frac{7}{6}$ **(e)** $\frac{29}{6}$ **(f)** $\frac{53}{8}$

5 (a) $1:2$ **(b)** $1:4$ **(c)** $3:4$ **(d)** $3:2$ **(e)** $3:2$ **(f)** $3:2$

Exercise 1A

1 (a) $\frac{1}{2}$ **(b)** $\frac{5}{6}$ **(c)** $\frac{6}{7}$ **(d)** $1\frac{1}{3}$ **(e)** $1\frac{1}{3}$ **(f)** $1\frac{1}{2}$ **(g)** $1\frac{4}{9}$
   (h) $1\frac{3}{4}$ **(i)** $4\frac{1}{2}$ **(j)** $8\frac{1}{2}$

2 (a) $\frac{1}{2}$ **(b)** $\frac{1}{8}$ **(c)** $\frac{2}{5}$ **(d)** $\frac{1}{4}$ **(e)** $1\frac{1}{2}$ **(f)** $2\frac{1}{3}$ **(g)** $1\frac{2}{3}$
   (h) $\frac{1}{2}$ **(i)** $1\frac{4}{5}$ **(j)** $1\frac{5}{6}$

3 (a) $\frac{7}{8}$ **(b)** $\frac{1}{8}$ **4** $6\,\mathrm{m}$ **5** $\frac{5}{7}$

Exercise 1B

1 (a) $\frac{3}{4} + \frac{3}{16} = \frac{12}{16} + \frac{3}{16} = \frac{15}{16}$ **(b)** $\frac{3}{8} + \frac{1}{4} = \frac{3}{8} + \frac{2}{8} = \frac{5}{8}$
   (c) $\frac{1}{6} + \frac{2}{3} = \frac{1}{6} + \frac{4}{6} = \frac{5}{6}$ **(d)** $\frac{3}{8} + \frac{1}{4} = \frac{3}{8} + \frac{2}{8} = \frac{5}{8}$
   (e) $\frac{2}{5} + \frac{2}{7} = \frac{14}{35} + \frac{10}{35} = \frac{24}{35}$

2 (a) $\frac{13}{21}$ **(b)** $\frac{7}{8}$ **(c)** $\frac{13}{20}$ **(d)** $\frac{7}{10}$ **(e)** $\frac{7}{9}$ **(f)** $\frac{13}{14}$ **(g)** $\frac{7}{8}$
   (h) $\frac{13}{20}$ **(i)** $\frac{59}{63}$ **(j)** $\frac{8}{9}$

3 $\frac{5}{8}$ **4** $\frac{11}{15}$

Exercise 1C

1 (a) $5\frac{3}{4}$ **(b)** $5\frac{11}{12}$ **(c)** $6\frac{19}{20}$ **(d)** $5\frac{17}{24}$ **(e)** $8\frac{3}{20}$ **(f)** $8\frac{7}{24}$
   (g) $8\frac{13}{35}$ **(h)** $14\frac{1}{4}$ **(i)** $8\frac{1}{35}$ **(j)** $6\frac{29}{42}$

2 (a) $7\frac{7}{8}$ **(b)** $12\frac{5}{18}$ **(c)** $14\frac{13}{20}$ **(d)** $10\frac{1}{12}$ **(e)** $10\frac{52}{105}$

3 $7\frac{1}{2}$ gallons   **4** $8\frac{7}{12}$ kg   **5** $6\frac{1}{6}$ acres

Exercise 1D

1 (a) $\frac{3}{8} - \frac{1}{4} = \frac{3}{8} - \frac{2}{8} = \frac{1}{8}$ **(b)** $\frac{7}{9} - \frac{2}{3} = \frac{7}{9} - \frac{6}{9} = \frac{1}{9}$
   (c) $\frac{3}{4} - \frac{2}{3} = \frac{9}{12} - \frac{8}{12} = \frac{1}{12}$ **(d)** $\frac{7}{8} - \frac{3}{5} = \frac{35}{40} - \frac{24}{40} = \frac{11}{40}$
   (e) $\frac{6}{7} - \frac{3}{4} = \frac{24}{28} - \frac{21}{28} = \frac{3}{28}$

2 (a) $\frac{2}{15}$ **(b)** $\frac{13}{56}$ **(c)** $\frac{3}{20}$ **(d)** $\frac{1}{10}$ **(e)** $\frac{7}{55}$ **(f)** $\frac{17}{30}$ **(g)** $\frac{22}{63}$
   (h) $\frac{19}{42}$ **(i)** $\frac{23}{143}$ **(j)** $\frac{7}{24}$

3 (a) $\frac{29}{72}$ **(b)** $\frac{41}{84}$ **(c)** $\frac{13}{36}$ **(d)** $\frac{35}{144}$ **(e)** $\frac{1}{60}$ **(f)** $\frac{13}{30}$ **(g)** $\frac{5}{6}$
   (h) $\frac{1}{4}$ **(i)** $\frac{7}{17}$ **(j)** $\frac{1}{24}$

Exercise 1E

1 (a) $\frac{2}{3}$ **(b)** $1\frac{1}{2}$ **(c)** $2\frac{3}{4}$ **(d)** $1\frac{1}{2}$ **(e)** $\frac{7}{9}$ **(f)** $3\frac{1}{12}$

2 (a) $4\frac{1}{6}$ **(b)** $1\frac{1}{12}$ **(c)** $1\frac{9}{10}$ **(d)** $2\frac{11}{28}$ **(e)** $\frac{13}{36}$ **(f)** $1\frac{31}{40}$
   (g) $\frac{1}{3}$ **(h)** $1\frac{2}{5}$ **(i)** $\frac{1}{12}$ **(j)** $\frac{6}{17}$

3 $93\frac{1}{4}$ gallons

4 $5\frac{7}{8}$ km

5 $1\frac{11}{12}$ kg

6 From left to right: $2, 1\frac{3}{4}, 3\frac{1}{2}$

Exercise 1F

1 (d) $\frac{1}{4}$ of $\frac{2}{3} = \frac{1}{6}$

2 (d) $\frac{2}{3} \times \frac{3}{4} = \frac{1}{2}$

4 (a) 1 **(b)** $2\frac{1}{2}$ **(c)** 6 **(d)** $2\frac{4}{5}$ **(e)** $2\frac{1}{4}$ **(f)** $10\frac{1}{2}$

6 (a) $\frac{1}{6}$ **(b)** $\frac{3}{8}$ **(c)** $\frac{9}{20}$ **(d)** $\frac{15}{32}$ **(e)** $\frac{7}{12}$ **(f)** $\frac{4}{15}$
   (g) $\frac{3}{28}$ **(h)** $\frac{25}{64}$ **(i)** $\frac{5}{22}$

7 (a) $3\frac{1}{8}$ **(b)** $1\frac{7}{9}$ **(c)** $3\frac{3}{16}$ **(d)** $\frac{5}{8}$ **(e)** $5\frac{5}{8}$ **(f)** $12\frac{4}{9}$
   (g) $12\frac{3}{8}$ **(h)** $16\frac{1}{2}$ **(i)** $18\frac{3}{4}$

8 (a) $\frac{1}{4}$ gallon   **(b)** $\frac{5}{12}$ gallon

9 $1\frac{1}{2}$ hours   **10 (a)** $22\,\mathrm{km}$ **(b)** $33\,\mathrm{km}$

Exercise 1G

1 12

2 (a) 2 **(b)** $\frac{1}{3}$ **(c)** $\frac{1}{6}$ **(d)** $\frac{2}{5}$ **(e)** $\frac{5}{3}$ **(f)** $\frac{8}{7}$ **(g)** $\frac{5}{6}$
   (h) $\frac{4}{15}$

3 (a) 6 **(b)** 18 **(c)** 9 **(d)** $10\frac{2}{3}$ **(e)** 10 **(f)** 6

4 (a) 2 **(b)** 3 **(c)** $\frac{5}{6}$ **(d)** $\frac{9}{10}$ **(e)** $2\frac{2}{15}$ **(f)** $\frac{1}{2}$
   (g) $4\frac{2}{3}$ **(h)** $3\frac{1}{2}$ **(i)** $\frac{21}{22}$

5 (a) $2\frac{4}{5}$ **(b)** 3 **(c)** $2\frac{5}{6}$ **(d)** $1\frac{9}{10}$ **(e)** 4 **(f)** $\frac{14}{23}$

6 (a) 6 **(b)** $1\frac{4}{5}$ **(c)** $8\frac{1}{4}$ **(d)** $\frac{6}{11}$

7 $13\frac{1}{2}$ **8** 8 **9** $6\frac{3}{4}$ **10** $11\frac{7}{16}\,\mathrm{m}$

Exercise 1H

1 (a) 5 **(b)** into 7 equal pieces

2 (a) 5
   (b) Yes
   (c) $20\,\mathrm{kg}$

RICE
32 kg

3 (a) divide the apples into 4 equal groups   **(b)** 3 **(c)** 9

4 (a) $\$0.75$ and $\$0.25$ **(b)** $\$0.50$ and $\$0.50$
   (c) $\$0.90$ and $\$0.10$ **(d)** $\$0.70$ and $\$0.30$
   (e) $\$0.85$ and $\$0.15$ **(f)** $\$0.16$ and $\$0.84$

5 (a) $3.75 and $1.25   **(b)** $4.50 and $0.50
(c) $4.25 and $0.75

6 (a) 20 and 10 cows   **(b)** 15 and 15 cows
(c) 24 and 6 cows   **(d)** 21 and 9 cows
(e) 8 and 22 cows   **(f)** 3 and 27 cows

7 (a) 6 and 14 apples   **(b)** 6 and 9 goats
(c) $12 and $48   **(d)** $4.40 and $1.10
(e) 0.9 m and 2.1 m   **(f)** 0.4 m and 0.6 m
(g) 0.25 kg and 1.75 kg   **(h)** 1.6 t and 0.4 t
(i) 5.25 m and 12.25 m   **(j)** 10.8 kg and 4.8 kg

8 (a) 480   **(b)** 320   **9** 20

10 (a) 9   **(b)** 30   **(c)** 27   **(d)** 81   **(e)** 300   **(f)** 108

11 (a) (i) 50   **(ii)** 400   **(iii)** 150   **(iv)** 160
(b) (i) 25   **(ii)** 100   **(iii)** 75   **(iv)** 40

Exercise 1I

1 (a) Ria 2, Faith 3   **(b)** 6

2 (a) 20, 5 and 35 nuts   **(b)** 20, 50 and 30 colas
(c) 40 m, 10 m and 25 m   **(d)** 0.2 m, 1.4 m and 0.2 m
(e) $2.50, $16.25 and $6.25   **(f)** $16, $10 and $28
(g) $8, $16 and $76   **(h)** 585 kg, 310 kg and 105 kg

3 (a) 75, 425 and 225 camels   **(b)** $1.75, $1.75 and $21
(c) 0.975 kg, 0.15 kg and 0.375 kg

4 (a) segments: 4 cm, 2 cm, 6 cm
(b) segments: 2 cm, 4 cm, 3 cm

Exercise 1J

1 (a) 2   **(b)** 3   **(c)** 4   **(d)** 5   **(e)** 12   **(f)** 10   **(g)** 26
(h) 22

2 (a) 10   **(b)** 11   **(c)** 101   **(d)** 1000   **(e)** 1101
(f) 10000   **(g)** 10001   **(h)** 10011   **(i)** 10111   **(j)** 11101
(k) 101001   **(l)** 111111

Exercise 1K

1 (a) 21   **(b)** 18   **(c)** 42   **(d)** 71   **(e)** 99   **(f)** 228
(g) 468   **(h)** 127   **(i)** 546

2 8, 64, 512, 4096

3 (a) 9   **(b)** 31   **(c)** 57   **(d)** 65   **(e)** 70   **(f)** 143
(g) 318   **(h)** 517   **(i)** 1141

4 (a) 241   **(b)** 31   **(c)** 132   **(d)** 310   **(e)** 441
(f) 2224

5 (a) 15   **(b)** 51   **(c)** 35   **(d)** 77   **(e)** 203   **(f)** 755

Exercise 1L

1 (a) 1, 2, 3, 4, 10, 11, 12, 13, 14, 20, 21, 22, 23, 24, 30, 31, 32, 33, 34, 40
(b) 1, 2, 3, 4, 5, 6, 7, 10, 11, 12, 13, 14, 15, 16, 17, 20, 21, 22, 23, 24

2 (a) 2, 4, 11, 13, 20, 22, 24, 31, 33, 40
(b) 2, 4, 6, 10, 12, 14, 16, 20

3 (a) 3, 11, 14, 22, 30, 33, 41, 44, 102, 110, 113, 121
(b) 3, 6, 11, 14, 17, 22, 25, 30, 33, 36, 41, 44

4 (a) 31   **(b)** 40   **(c)** 110   **(d)** 1003

5 (a) 12   **(b)** 14   **(c)** 2   **(d)** 24   **6** 25

7 (a) 22   **(b)** 31   **(c)** 41   **(d)** 132   **(e)** 202   **(f)** 314

Exercise 1M – Mixed Questions

1 $\frac{2}{5}$ **2** $\frac{15}{16}$ kg   **3 (a)** 4 kg   **(b)** $2\frac{2}{15}$ kg

4 (a) 14 km   **(b)** $2\frac{5}{8}$ km   **5** $\frac{3}{5}$

6 (a) 108   **(b)** 99   **(c)** 207   **7** $5\frac{5}{6}$ cm

8 66 bottles   **9** $37\frac{1}{2}$ cm   **10 (a)** 120   **(b)** 203   **(c)** 723

11 (a) 6 kg   **(b)** 18 kg   **(c)** 27 kg

12 (a) $0.55   **(b)** $4.95

13 4 glasses   **14** $\frac{3}{20}$

15 (a) (i) $\frac{7}{25}$ **(ii)** $\frac{3}{25}$ **(iii)** $\frac{3}{20}$ **(iv)** $\frac{9}{20}$ **(b)** 1

16 (a) $0.19 and $0.21   **(b)** The 25 tea bag box

17 $30   **18** 550 g   **19** $6.36

20 (a) $400
(b) (i) $\frac{1}{4}$ **(ii)** $\frac{1}{10}$ **(iii)** $\frac{3}{40}$ **(iv)** $\frac{3}{20}$ **(v)** $\frac{1}{5}$ **(vi)** $\frac{9}{40}$
(c) Yes   **(d)** $\frac{3}{20}$ **(e) (i)** $\frac{1}{10}$ **(ii)** $\frac{1}{16}$ **(iii)** $\frac{1}{40}$ **(iv)** $\frac{1}{40}$

21 $\frac{1}{20}$ mm   **22** $506\frac{2}{3}$ kg

23 (a) 332   **(b)** 44   **(c)** 211   **(d)** 1001

24 (a) (i) $6\frac{6}{7}$ **(ii)** $25\frac{3}{5}$ **(iii)** $23\frac{3}{4}$ **(iv)** $8\frac{1}{4}$ **(v)** $18\frac{1}{5}$
(vi) $3\frac{3}{4}$

25 (a) (i) $\frac{15}{16}$ **(ii)** 4   **(iii)** $\frac{32}{35}$ **(iv)** $2\frac{3}{16}$ **(v)** $\frac{35}{36}$ **(vi)** $\frac{17}{30}$

26 $2\frac{4}{13}$ cm   **27 (a)** $\frac{1}{6}$ **(b)** $60

28 (a) $53\frac{1}{3}$ kg   **(b)** $33\frac{1}{3}$ kg   **29** $\frac{1}{3}, \frac{7}{20}, \frac{2}{5}, \frac{3}{7}$

30 (a) $4\frac{1}{2}$ days   **(b)** $\frac{1}{2}$ day   **31 (a)** $\frac{3}{10}$ **(b)** 50 km

Exercise 1

1 (a) $\frac{1}{2}$ **(b)** 1   **(c)** $3\frac{1}{2}$ **(d)** $4\frac{4}{5}$ **(e)** $7\frac{1}{3}$ **(f)** $\frac{8}{15}$
(g) $1\frac{1}{12}$ **(h)** $1\frac{7}{20}$ **(i)** $\frac{5}{6}$ **(j)** $\frac{19}{24}$

2 (a) $4\frac{5}{12}$ **(b)** $8\frac{1}{10}$ **(c)** $6\frac{1}{20}$ **(d)** $10\frac{21}{40}$ **(e)** $9\frac{1}{12}$

3 (a) $1\frac{1}{7}$ **(b)** $1\frac{3}{5}$ **(c)** $\frac{5}{12}$ **(d)** $\frac{1}{8}$ **(e)** $\frac{22}{63}$ **(f)** $\frac{1}{15}$ **(g)** $2\frac{3}{20}$
(h) $\frac{3}{4}$ **(i)** $2\frac{7}{12}$ **(j)** $\frac{13}{20}$ **(k)** $2\frac{23}{56}$

4 (a) $\frac{1}{8}$ **(b)** $\frac{1}{6}$ **(c)** $\frac{3}{10}$ **(d)** $\frac{10}{21}$ **(e)** $\frac{1}{4}$ **(f)** $2\frac{1}{4}$ **(g)** $1\frac{13}{20}$
(h) $2\frac{14}{15}$ **(i)** $5\frac{16}{21}$ **(j)** $15\frac{91}{100}$

5 (a) 2   **(b)** $\frac{1}{2}$ **(c)** $1\frac{1}{2}$ **(d)** $1\frac{1}{5}$ **(e)** $\frac{15}{16}$ **(f)** 7   **(g)** $6\frac{1}{3}$
(h) $1\frac{3}{7}$ **(i)** $2\frac{7}{39}$ **(j)** $1\frac{17}{25}$

6 (a) 57   **(b)** 2224   **(c)** 139   **(d)** 15 724   **(e)** 2581
(f) 200 234

7 $4\frac{2}{5}$ km/h   **8 (a)** 90 litres   **(b)** 30 litres

9 (a) 936   **(b)** 48

10 (a) 18 : 1   **(b)** 100   **(c)** 20 : 1

Check out

1 (a) (i) $\frac{3}{4}$ **(ii)** $\frac{11}{12}$ **(iii)** $4\frac{1}{6}$ **(iv)** $1\frac{50}{63}$ **(v)** $\frac{1}{6}$ **(vi)** $1\frac{7}{8}$
(b) $\frac{5}{6}$ litres

2 (a) (i) $1\frac{1}{2}$ **(ii)** 4 **(iii)** 6 **(iv)** $\frac{1}{2}$ **(v)** $\frac{9}{20}$ **(vi)** $1\frac{1}{2}$
(b) 38 g

3 (a) 1 **(b)** $\frac{1}{2}$ **(c)** $\frac{3}{8}$ **(d)** $8\frac{1}{2}$

4 (a) (i) 12 **(ii)** 7 **(iii)** $\frac{8}{9}$ **(iv)** $1\frac{17}{40}$
(b) 12

5 (a) \$80, \$10 **(b)** \$70, \$20 **(c)** \$50, \$40
(d) \$63, \$27 **(e)** \$10, \$20, \$60 **(f)** \$18, \$27, \$45

6 (a) 82 **(b)** 271 **(c)** 242 **(d)** 1643

2 Decimals

Check in

1 (a) 6 tens, 2 ones **(b)** 6 ones, 2 tenths
(c) 2 ones, 6 tenths **(d)** 2 tens, 5 ones, 6 tenths
(e) 5 ones, 0 tenths, 6 hundredths **(f)** 6 hundreds,
2 tens, 1 one, 4 tenths **(g)** 2 tens, 6 ones, 4 tenths,
1 hundredth **(h)** 2 ones, 4 tenths, 1 hundredth, 6 thousandths
(i) 2 ones, 4 tenths, 6 hundredths, 1 thousandth

2 (a) 0.3 **(b)** 0.13 **(c)** 0.03 **(d)** 1.13 **(e)** 12.02

Exercise 2A

1 (a) 0.3 **(b)** 0.7 **(c)** 0.3 **(d)** 1.4 **(e)** 2.3 **(f)** 5.07
(g) 16.1 **(h)** 3.3

3 A = 0.9, B = 0.14, C = 0.32, D = 0.39,
E = 0.69, F = 0.83, G = 0.03

4 (a) 0.18, 0.21, 0.3, 0.46, 2.1 **(b)** 0.41, 0.46, 0.5, 2.3, 4.8
(c) 0.68, 0.81, 0.9, 1, 1.01 **(d)** 2, 2.04, 2.13, 2.4, 4.2

6 (a) 5.1 cm **(b)** 3.4 cm **(c)** 2.8 cm

7 same length

8 A = 6.343, B = 6.346, C = 6.347, D = 6.349

Exercise 2B

1 (a) 3.7 **(b)** 5.5 **(c)** 9.2 **(d)** 4.08 **(e)** 3.99 **(f)** 6.9
(g) 19.4 **(h)** 9.72 **(i)** 108.55 **(j)** 42.203

3 (a) 1.1 **(b)** 0.8 **(c)** 0.6 **(d)** 3.06 **(e)** 2.88
(f) 8.59 **(g)** 9.4 **(h)** 95.1 **(i)** 31.26 **(j)** 10.06

5 0.42 m

6 0.31 seconds

7 (a) 7.45 m **(b)** 2 **(c)** 4.55m

8 16.87 seconds

Exercise 2C

1 (a) 36.5 **(b)** 43 **(c)** 91 **(d)** 20.6 **(e)** 6.3 **(f)** 1.24
2 (a) 365 **(b)** 430 **(c)** 910 **(d)** 206 **(e)** 63 **(f)** 12.4

3

Number	Number × 10	Number × 100
3.65	36.5	365
4.3	43	430
9.1	91	910
2.06	20.6	206
0.63	6.3	63
0.124	1.24	12.4

4 move one place to the left; move two places to the left

5 (a) 36.2 **(b)** 94 **(c)** 4.6 **(d)** 0.4 **(e)** 21.9 **(f)** 211.9

6 (a) 362 **(b)** 940 **(c)** 46 **(d)** 4 **(e)** 219 **(f)** 2119

7 digits move 3 places to the left

8 (a) 6253 **(b)** 125 **(c)** 6480 **(d)** 6400 **(e)** 400 **(f)** 3

10 (a) one place to the left **(b)** two places to the left
(c) three places to the left

11 (a) 240 **(b)** 223 **(c)** 13.5 **(d)** 67.49 **(e)** 320
(f) 537 **(g)** 69.8 **(h)** 12 **(i)** 2.5 **(j)** 32.89
(k) 490 **(l)** 1345 **(m)** 7163

Exercise 2D

1 $6.9 \times 9 = 62.1$; $32.5 \times 8 = 260$; $1.68 \times 3 = 5.04$;
$12.4 \times 13 = 161.2$

2 (a) 3.2 **(b)** 5.4 **(c)** 11.5 **(d)** 37.6 **(e)** 8.4 **(f)** 7.2
(g) 4.62 **(h)** 2.6 **(i)** 25.92 **(j)** 58.2

3 \$5.40

4 8.4 kg

5 (a) 85.5 **(b)** 85.5 **(c)** 8.55 **(d)** 8.55
(e) 8.55 **(f)** 0.855

6 65 m

7 \$923.64

8 (a) 7.2 **(b)** 53.6 **(c)** 1.2 **(d)** 8.4 **(e)** 22.32
(f) 0.91 **(g)** 2.08 **(h)** 3.22 **(i)** 0.12 **(j)** 0.256

Exercise 2E

1 (a) 7.45 **(b)** 8.9 **(c)** 0.34 **(d)** 3.04 **(e)** 0.06
(f) 1.006 **(g)** 18.4 **(h)** 21.63

2 move digits one place to the right.

3 (a) 7.45 **(b)** 8.9 **(c)** 0.34 **(d)** 3.04 **(e)** 0.06
(f) 1.006 **(g)** 18.4 **(h)** 21.63

4

Number	Number ÷ 10	Number ÷ 100	Number ÷ 1000
74.5	7.45	0.745	0.0745
89	8.9	0.89	0.089
3.4	0.34	0.034	0.0034
30.4	3.04	0.304	0.0304
0.6	0.06	0.006	0.0006
485	48.5	4.85	0.485
1024	102.4	10.24	1.024
102.4	10.24	1.024	0.1024
10.24	1.024	0.1024	0.01024
6	0.6	0.06	0.006

5 (a) one place to the right   **(b)** two places to the right
(c) three places to the right

Exercise 2F

1 (a) 1.44   **(b)** 1.25   **(c)** 1.38   **(d)** 0.55   **(e)** 0.23
(f) 0.339   **(g)** 0.16   **(h)** 0.046   **(i)** 0.0112

2 (a) 3.775   **(b)** 3.22   **(c)** 1.424   **(d)** 2.41   **(e)** 2.375
(f) 3.92

3 $15.75   **4** 0.00625 cm   **5** $24.2 per kg   **6** 35.6 m

7 $39.40   **8** $8.56 per kg

Exercise 2G

1 (a) by 9   **(b)** 10   **(c)** 10   **(d)** $720/9 = 80$

2 (a) by 12   **(b)** 10   **(c)** 10   **(d)** $61.82/12 = 5.15$

3 (a) (i) 8   **(ii)** 8   **(b) (i)** 7   **(ii)** 7   **(c) (i)** 8   **(ii)** 8
(d) (i) 40   **(ii)** 40   **(e) (i)** 9.1   **(ii)** 9.1
They are the same.

4 (a) 14.25   **(b)** 142.5   **(c)** 1425   **(d)** 0.82   **(e)** 203.3
(f) 494.3   **(g)** 0.694   **(h)** 5.14   **(i)** 2.20   **(j)** 21.97

5 (a) 2.4   **(b)** 3.2   **(c)** 7.3   **(d)** 0.315   **(e)** 22.5
(f) 0.14   **(g)** 60.25   **(h)** 0.3   **(i)** 1.91   **(j)** 116.67

6 (a) $8.20   **(b)** $6.15

7 (a) 0.45 kg   **(b)** 0.91 kg   **(c)** 3.41 kg

8 0.62 m/s

Exercise 2H

1 (a) 36   **(b)** 100   **(c)** 225   **(d)** 625   **(e)** 0.16   **(f)** 0.01
(g) 0.0004   **(h)** 2.25   **(i)** 1.9881

2

$1^2 = 1$	$10^2 = 100$	$0.1^2 = 0.01$
$2^2 = 4$	$20^2 = 400$	$0.2^2 = 0.04$
$3^2 = 9$	$30^2 = 900$	$0.3^2 = 0.09$
$4^2 = 16$	$40^2 = 1600$	$0.4^2 = 0.16$
$5^2 = 25$	$50^2 = 2500$	$0.5^2 = 0.25$
$6^2 = 36$	$60^2 = 3600$	$0.6^2 = 0.36$
$7^2 = 49$	$70^2 = 4900$	$0.7^2 = 0.49$
$8^2 = 64$	$80^2 = 6400$	$0.8^2 = 0.64$
$9^2 = 81$	$90^2 = 8100$	$0.9^2 = 0.81$

The figures in the second column are one hundred times those in the first. The figures in the third column are one hundredth of those in the first.

3 $8^2 = 8 \times 8 = 64$ $9^2 = 9 \times 9 = 81$
$10^2 = 10 \times 10 = 100$ $11^2 = 11 \times 11 = 121$
$12^2 = 12 \times 12 = 144$ $13^2 = 13 \times 13 = 169$
$14^2 = 14 \times 14 = 196$ $15^2 = 15 \times 15 = 225$
$16^2 = 16 \times 16 = 256$ $17^2 = 17 \times 17 = 289$
$18^2 = 18 \times 18 = 324$ $19^2 = 19 \times 19 = 361$

4 (a) $6^2 = 36$ **(b)** $10^2 = 100$ **(c)** $15^2 = 225$ **(d)** $25^2 = 625$
(e) $0.4^2 = 0.16$ **(f)** $0.1^2 = 0.01$

Exercise 2I

1 (a) 4   **(b)** 9   **(c)** 11   **(d)** 13   **(e)** 17   **(f)** 15
(g) 19   **(h)** 21

2 (a) 36, 6   **(b)** 49, 7   **(c)** 64, 8

3 0, 1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, 169, 196, 225, 256, 289, 324, 361

4 (a) 4   **(b)** 8   **(c)** 10   **(d)** 20   **(e)** 15   **(f)** 30

5 (a) 2   **(b)** 3   **(c)** 40   **(d)** 60   **(e)** 90   **(f)** 0.3   **(g)** 0.5

Exercise 2J

1 (a) 9   **(b)** 12   **(c)** 13   **(d)** 16

2 (a) 0.647302093   **(b)** 1.773696705   **(c)** 3.185906464
(d) 0.8336666   **(e)** 0.992471662   **(f)** 6.012487006
(g) 4.38520239   **(h)** 3.668787266   **(i)** 0.651920241
(j) 4.489988864   **(k)** 0.951314879   **(l)** 0.651920224

3 (a) 15.652   **(b)** 88.318   **(c)** 13.784   **(d)** 5.094
(e) 16.682   **(f)** 2   **(g)** 1.852   **(h)** 4.002

4 (a) 11.40 cm   **(b)** 16.43 cm   **(c)** 18.97 cm   **(d)** 21.21 cm

5 (a) 9^2 is a short way of writing nine times nine.
(b) The small raised ^{2} is called the power or index.
(c) The symbol $\sqrt{}$ stands for the words the square root.
(d) The x^2 key on a calculator is pressed to find the square of a number.

6 53 and 54

7 247 and 248

Exercise 2K

1 (a) 40   **(b)** 70   **(c)** 90   **(d)** 100   **(e)** 10   **(f)** 220
(g) 110   **(h)** 320

2 (a) 30   **(b)** 30   **(c)** 40   **(d)** 40   **(e)** 40   **(f)** 10
(g) 120   **(h)** 140   **(i)** 200   **(j)** 1090   **(k)** 1550

3 (a) nearer 600   **(b)** 600

4 (a) 100   **(b)** 100   **(c)** 100   **(d)** 200   **(e)** 200   **(f)** 200
(g) 200   **(h)** 600   **(i)** 600

5 (a) 3100   **(b)** 4800   **(c)** 5300   **(d)** 1700   **(e)** 3600
(f) 4500

6 (a) 4   **(b)** 4   **(c)** 5   **(d)** 5   **(e)** 5   **(f)** 7   **(g)** 9
(h) 10

Exercise 2L

1 (a) (i) 1.52   **(ii)** 1.57   **(iii)** 1.65   **(iv)** 1.68   **(v)** 1.71
(vi) 1.75   **(b) (i)** 1.5   **(ii)** 1.6   **(iii)** 1.7   **(iv)** 1.7
(v) 1.7   **(vi)** 1.8

2 (a) 1.5   **(b)** 0.5   **(c)** 0.6   **(d)** 2.8   **(e)** 4.0   **(f)** 3.0

3 (a) 4.6   **(b)** 5.0   **(c)** 1.9   **(d)** 0.5   **(e)** 2.6   **(f)** 0.9

4 (a) 1.29   **(b)** 0.04   **(c)** 0.48   **(d)** 0.83   **(e)** 1.51
(f) 2.01

5

Decimal value	Rounded to		
	1 d.p.	2 d.p.	3 d.p.
716.2581	716.3	716.26	716.258
41.0215	41.0	41.02	41.022
0.00138	0.0	0.00	0.001
1.76431	1.8	1.76	1.764
15.01812	15.0	15.02	15.018

6 1.45 m

Exercise 2M

1 (a) 70   **(b)** 40   **(c)** 300   **(d)** 400   **(e)** 600
(f) 600   **(g)** 3000   **(h)** 40 000   **(i)** 10 000

2 (a) 0.3   **(b)** 0.6   **(c)** 0.6   **(d)** 0.3   **(e)** 0.04
(f) 0.8   **(g)** 0.003   **(h)** 0.004   **(i)** 0.0009

3 (a) 430   **(b)** 620   **(c)** 300   **(d)** 3800   **(e)** 4700
(f) 1200   **(g)** 15 000   **(h)** 40 000   **(i)** 690 000

4 (a) 1.4   **(b)** 14   **(c)** 8.1   **(d)** 0.035   **(e)** 0.017
(f) 0.0027   **(g)** 31   **(h)** 210   **(i)** 290

5

Number	1s.f.	2s.f.	3s.f.
4139	4000	4100	4140
6804	7000	6800	6800
2007	2000	2000	2010
138.6	100	140	139
154.28	200	150	154
0.03168	0.03	0.032	0.0317
2.0352	2	2.0	2.04
0.004169	0.004	0.0042	0.00417

Exercise 2N

1 (a) 1000   **(b)** 3   **(c)** 6.1×10^3 **(d)** 2.5×10^4
(e) 2.51×10^5

2 (a) 6×10^3 **(b)** 3.2×10^2 **(c)** 9×10^1 **(d)** 4×10^4
(e) 2×10^1 **(f)** 1.3×10^4 **(g)** 2×10^5 **(h)** 5×10^6
(i) 1.7×10^5

3 (a) 20   **(b)** 200   **(c)** 2000   **(d)** 20 000   **(e)** 8 000 000
(f) 90 000 000   **(g)** 600 000   **(h)** 40 000 000
(i) 3 000 000 000

4 (a) 4.5×10^2 **(b)** 2.3×10^2 **(c)** 9.1×10^2 **(d)** 4.5×10^3
(e) 2.3×10^3 **(f)** 9.1×10^4 **(g)** 3.25×10^2 **(h)** 4.96×10^2
(i) 7.254×10^3

5 (a) 2400   **(b)** 12000   **(c)** 620   **(d)** 9800   **(e)** 42 500
(f) 610 000   **(g)** 216.5   **(h)** 8140   **(i)** 911 000 000

Exercise 2O

1 (a) 0.4   **(b)** 0.6   **(c)** 0.8   **(d)** 0.25   **(e)** 0.75
(f) 0.375   **(g)** 0.625   **(h)** 0.875   **(i)** 0.0625   **(j)** 0.3125

2 (a) 0.5625   **(b)** 0.6875   **(c)** 0.15   **(d)** 0.35   **(e)** 0.55
(f) 0.015   **(g)** 0.035   **(h)** 0.004

3 (a) 0.33...   **(b)** 0.66...   **(c)** 0.166...

5 (b) $\frac{1}{4} = 0.25, \frac{1}{5} = 0.2, \frac{1}{6} = 0.166..., \frac{1}{7} = 0.142..., \frac{1}{8} = 0.125,$
$\frac{1}{9} = 0.11..., \frac{1}{10} = 0.1$
(c) (i) $\frac{1}{2}, \frac{1}{4}, \frac{1}{5}, \frac{1}{8}, \frac{1}{10}$ **(ii)** $\frac{1}{3}, \frac{1}{6}, \frac{1}{7}, \frac{1}{9}$

6 (a) 0.833...   **(b)** 0.286...   **(c)** 0.571...   **(d)** 0.222...
(e) 0.555...   **(f)** 0.1818...   **(g)** 0.714...   **(h)** 0.0833...

7 Yes, yes

8 (a) $\frac{1}{9}$ **(b)** $\frac{5}{9}$ **(c)** $\frac{2}{11}$

Exercise 2P

1 $711.45

2 $5.95

3 (a) 107.73 g   **(b)** 2.68 oz   **(c)** 8.16 kg   **(d)** 21.16 lb

4 (a) $6.75   **(b)** $4.19

5 (a) 0.66 ..., 0.375, 0.25, 0.4, 0.42 ...   **(b)** $\frac{1}{4}, \frac{3}{8}, \frac{2}{5}, \frac{3}{7}, \frac{2}{3}$

6 (a) 0.142 857, 0.285 714, 0.428 571, 0.571 428,
0.714 285, 0.857 142
(b) The same six digits occur cyclically.

7 (a) 6   **(b)** Yes

8 (a) 9   **(b)** 17   **(c)** 4.47   **(d)** 5.48   **(e)** 1.73   **(f)** 1.41
(g) 12.25   **(h)** 30.69

10 (a) Cleano $23.80, Supersuds $26.00   **(b)** Cleano

11 22

12 (a) 11 388 303 828   **(b)** 229 089 871 488

Exercise 2Q – mixed questions

1 $64.90

2 $9.25

3 $425.73

4 (a) 10 cm   **(b)** 10 cm

5 (a) 14.8 cm   **(b)** 13.8 cm   **(c)** 29.5 cm

6 10.9 cm

7 (a) 10.67 s   **(b)** 3.8 s   **(c)** 0.31 s

8 26.04 cm^2

9

Number	1d.p.	2d.p.	3d.p.
6.7184	6.7	6.72	6.718
0.0816	0.1	0.08	0.082
0.9147	0.9	0.91	0.915
1.0888	1.1	1.09	1.089
12.1234	12.1	12.12	12.123

10

Number	1s.f.	2s.f.	3s.f.
61 400	60 000	61 000	61 400
3175	3000	3200	3180
16.328	20	16	16.3
0.01145	0.01	0.011	0.0115
0.9256	0.9	0.93	0.926

11 (a) (i) 3600   **(ii)** 28 800   **(iii)** 86 400
   (iv) 604 800   **(v)** 31 449 600
  (b) (i) 3.6×10^3 **(ii)** 2.88×10^4 **(iii)** 8.64×10^4
   (iv) 6.048×10^5 **(v)** 3.14496×10^7

12 (a) 1.72×10^2, 4.9×10^2, 512, 8.1×10^3
  (b) 8.01×10^2, 8.01×10^3, 8012×10^3, 8.1×10^3

13 5.625 m

14 1.8288 m

15 (a) Work out 5.7×0.13 then add 0.13
  (b) Work out 6.7×0.12 then add 6.7×0.01

16 (a) Add 3.41 to itself 23 times
  (b) Work out $(24 - 1) \times (2.41 + 1)$

Exercise 2

1 (a) (i) 1.2   **(ii)** 5.6   **(iii)** 12.44   **(iv)** 6.02   **(v)** 5.84
  (b) (i) 2.8   **(ii)** 15.9   **(iii)** 98.01   **(iv)** 0.55   **(v)** 101.089

2 (a) (i) 61   **(ii)** 61.5   **(iii)** 234   **(iv)** 13   **(v)** 278.4
  (b) (i) 3.12   **(ii)** 0.598   **(iii)** 0.2736   **(iv)** 4.2408
   (v) 3.2448

3 (a) (i) 0.615   **(ii)** 0.0615   **(iii)** 4.7   **(iv)** 0.003
   (v) 0.0362
  (b) (i) 1.5   **(ii)** 0.15   **(iii)** 12.1   **(iv)** 1.7   **(v)** 0.043
  (c) (i) 20   **(ii)** 400   **(iii)** 239   **(iv)** 240   **(v)** 7.9

4 (a) (i) 4.611   **(ii)** 4.677   **(iii)** 0.049   **(iv)** 61.728
   (v) 193.889
  (b) (i) 4.61   **(ii)** 4.68   **(iii)** 0.05   **(iv)** 61.73   **(v)** 193.89
  (c) (i) 5   **(ii)** 5   **(iii)** 0.05   **(iv)** 60   **(v)** 200
  (d) (i) 4.6   **(ii)** 4.7   **(iii)** 0.049   **(iv)** 62   **(v)** 190

5 (a) 4.7×10^2 **(b)** 2.19×10^3 **(c)** 8.1×10^4
  (d) 2.9846×10^4 **(e)** 9.3×10^7 **(f)** 1.51284×10^8

6 (a) 0.75   **(b)** 0.4   **(c)** 0.875   **(d)** 0.375   **(e)** 0.3125
  (f) 0.291...   **(g)** 0.285...   **(h)** 0.222...   **(i)** 0.833...
  (j) 0.461...   **(k)** 0.466...   **(l)** 0.941...
  (m) 0.043...   **(n)** 0.576...

7 (a) $123.60   **(b)** $131.02

8 (a) 0.119 cm   **(b)** 0.119 cm

9 (a) 251.2 m^2 **(b)** 64.6 m

Check out

1 (a) 25   **(b)** 4   **(c)** 2140   **(d)** 610   **(e)** 6800   **(f)** 2.5
  (g) 0.64   **(h)** 0.145   **(i)** 0.0075   **(j)** 0.004

2 (a) 1.6   **(b)** 2.8   **(c)** 73.2   **(d)** 4.9   **(e)** 56.52
  (f) 0.5712

3 (a) 16   **(b)** 20   **(c)** 12   **(d)** 290   **(e)** 20

4 (a) (i) 4.72   **(ii)** 0.68   **(iii)** 17.99   **(iv)** 0.05
  (b) (i) 7.8   **(ii)** 0.7   **(iii)** 19.1   **(iv)** 0.1

5 (a) (i) 900   **(ii)** 6000   **(iii)** 0.8   **(iv)** 0.0003
  (b) (i) 19 000   **(ii)** 400 000   **(iii)** 0.098   **(iv)** 0.000040

6 (a) 6×10^1 **(b)** 6.09×10^2 **(c)** 2.984×10^3
  (d) 5.4×10^5 **(e)** 1.74×10^8 **(f)** 1.234567×10^6

7 (a) 11   **(b)** 24   **(c)** 1.41   **(d)** 4.16

8 (a) 0.25   **(b)** 0.66 . . .   **(c)** 0.375   **(d)** 0.833 . . .
  (e) 0.222 . . .   **(f)** 0.1875

3 Angles

Check in

1 $A\hat{X}C$, $B\hat{X}D$

2 (a) acute   **(b)** obtuse   **(c)** right angle   **(d)** obtuse

3 (a) 146°   **(b)** 70°

Exercise 3A

1 (b) 35°

2 (b) (i) 45°   **(ii)** 104°   **(iii)** 86°

3 15°

4 80°

5 (a) 30°   **(b)** 60°   **(c)** 90°

6 (a) 59°   **(b)** 121°   **(c)** 180°

Exercise 3B

2 (b) 310°

Exercise 3C

2 (e) BC = 10 cm   **3 (e)** a rectangle   **(f)** RQ = 5 cm

Exercise 3D

14 (c) AB = 8 cm

Exercise 3F

1 (a) $\hat{A} = 32°$, $\hat{B} = 106°$, $\hat{C} = 42°$ **(b)** 180°

2 (c) They sum to 180°.

3 (d) 180°

4 (b) yes

5 (b) 360°

6 (c) They sum to 360°.

7 (d) 360°

8 (b) yes

9 (a) angle ABC = 35°, angle ACD = 60°, angle BAC = 25°
  (b) 60°
  (c) $A\hat{C}D = A\hat{B}C + B\hat{A}C$

10 (b) yes

Exercise 3G

1 (a) 28°   **(b)** 65°   **(c)** 117°   **(d)** 32°

2 (a) 107°   **(b)** 80°   **(c)** 48°   **(d)** 96°

3 (a) 47°   **(b)** 70°   **(c)** 27°   **(d)** 48°

Exercise 3H

1 (c) 900°   **2 (c)** 1080°

3 (a)

Shape	Number of sides	Sum of interior angles
Triangle	3	180°
Quadrilateral	4	360°
Pentagon	5	540°
Hexagon	6	720°
Heptagon	7	900°
Octagon	8	1080°

(b) It increases by 180°

4 (a) 1260°   **(b)** 1440°   **5 (a)** 1800°   **(b)** $(n-2)\,180°$

6 (a) In a regular polygon all sides are the same length and all angles the same size.

(b) 108°

7 (a) 10   **(b)** 144°   **8** 150°

Exercise 3I

1 (a) 39°   **(b)** 72°   **(c)** 126°   **(d)** 63°

2 (a) $x = z = 68°, y = 112°$ **(b)** $f = h = 138°, g = 42°$

3 $a = b = 90°, c = 81°, d = e = 85°, f = 86°$

4 (b) They are the same.

5 (a) 130°   **(b)** both 50°

6 (a) 35°   **(b)** 55°   **(c)** 70°

7 60°

8 45

9 30

10 36

Exercise 3J

1 (a) Yes   **(b)** Yes   **(c)** Yes

2 (a) Yes. Yes, because each line goes 3 squares across and 7 up.

4 (a) AB, GH, CD and FE are all parallel to each other; GF and DE are parallel   **(b)** AD and BC   **(c)** PQ and SR, PS and QR   **(d)** AB and ED, AF and CD, BC and FE

5 (a) DC, EF, HG   **(b)** BF, CG, AE and DH; BC, AD, EH and FG

6 A cuboid has three sets of parallel edges. There are four edges in each set.

Exercise 3K

1 (a) a **(b)** b

2 (a) a and e **(b)** b and f

3 (a) $a = 50°$ (corresponding angles)
(b) $b = 130°$ (corresponding), $c = 130°$ (vertically opposite)
(c) $c = 110°$ (corresp), $e = 110°$ (vert. opp), $d = 110°$ (corresp or vert. opp)
(d) $g = 40°$ (corresp), $f = 40°$ (vert. opp), $i = 40°$ (corresp), $j = h = 140°$ (supplementary)
(e) $l = n = 55°$ (corresp), $k = 55°$ (vert. opp), $m = o = 55°$ (corresp)

4 (a) $p = 125°, q = r = 55°$ **(b)** $x = 64°, y = 116°, z = 64°$
(c) all 90°

5 $a = 60°, b = 70°, c = 120°, d = 110°, e = 60°, f = 70°$

6 (a) and **(c)** should have a pair of parallel lines
(d) When a line crosses a pair of parallel lines, corresponding angles are equal
(e) When a line crosses a pair of lines, and corresponding angles are equal, the pair of lines must be parallel.

Exercise 3L

1 (a) y **(b)** x

2 (a) w, y **(b)** q, s **(c)** u

3 (a) x, z **(b)** p, r **(c)** v

4 (a) 54°   **(b)** 147°   **(c)** 37°   **(d)** all 75°

5 (a) corresponding   **(b)** alternate

6 (a) both 56°   **(b)** both 60°   **(c)** $g = 54°, h = 63°$
(d) all 48°

7 (a) AB is parallel to DC.   **(b)** AD is parallel to BC.
(c) alternate   **(d)** corresponding

8 Yes

9 (a) alternate angles   **(c)** corresponding angles

10 (c) 180°

Exercise 3N

1 (a) $a = 150°, b = 30°, c = 150°$
(b) $d = 70°, e = 110°, f = 70°$
(c) $h = 45°, i = 135°, g = 135°$

2 (a) $x = 70°, y = 125°, z = 125°$
(b) $p = 70°, s = 55°, t = 55°$
(c) $a = 50°, b = 50°, c = 130°$
(d) $a = 115°, b = 50°$

3 (b) they are all 90°

5 (a) all the sides are equal

6 (b) all angles are right angles

7 Middle region: The set of rectangles.
Inner region: The set of squares.

8 Outer region: The set of parallelograms.
Middle region: The set of rhombuses.
Inner region (contained within middle region): The set of squares.

9 (c) square   **10 (a)** yes   **(b)** no

Investigation page 55

You can make a rectangle and two different parallelograms with these two right-angled triangles. With two isosceles triangles you can make a parallelogram and a rhombus.

Exercise 3

1 (a) $67°$ (b) $317°$

2 Check students' work

3 (a) $a = 75°$ (b) $b = 32°, c = 148°$ (c) $d = 130°$
   (d) $e = 23°$ (e) $f = 76°, g = 104°$ (f) $j = k = 146°$
   (g) $l = n = 77°, m = 103°$ (h) $o = 135°, p = 45°$
   (i) $q = 115°, r = 120°$

5 (a) Check students' work   (b) Check students' work
   (c) $540°$ (d) They all add up to $540°$

6 (a) Check students' work   (b) Check students' work
   (c) $360°$ (d) They all add up to $360°$

7 (a) $51°$ (b) $51°$

Check out

1 $345°$

3 (a) $116°$ (b) $67°$ (c) $42°$

4 (a) $115°, 65°$ (b) $50°, 130°, 50°$

5 AB, DE and HG; AH, BC and FG; DC and EF

6 (a) a and $75°$; b and c (b) $75°, 70°, 70°$

7 (a) f and $65°$ (b) $55°, 55°, 65°$

4 Sets

Check in

1 Factors of $10 = \{1, 2, 5, 10\}$

2 (a) {Monday, Tuesday, Wednesday, Thursday, Friday, Saturday, Sunday}   (b) { }

3 Even numbers $= \{2, 4, 6, 8, 10\}$,
   Numbers less than $5 = \{1, 2, 3, 4\}$. Intersection $= \{2, 4\}$

Exercise 4A

1 (a) {10}   (b) {a, e, i, o, u}   (c) {3, 4, 5, 6, 7, 8, 9, 10}
   (d) {2, 3, 5, 7, 11, 13}   (e) { }

3 (a) inner region $= \{5, 10, 15, 20\}$
   (b) inner region $= \{2, 3, 5, 7, 11, 13, 17, 19\}$
   (c) inner region $= \{1, 2, 4, 8, 16\}$
   (d) inner region $= \{1, 4, 9, 16\}$
   (e) inner region $= \{4, 6, 8, 9, 10, 12, 14, 15, 16, 18, 20\}$

4 (a) (i) Betty, Roy and Reena   (ii) Bo and Sarah
   (iii) Delia, Judd and Samuel
   (iv) Vincent, Tony, Michael and Sally
   (b) 12

5 (a) intersection $= \{$Mark, Lottie$\}$
   (b) Mark, as he is the only boy in the intersection.

Exercise 4B

1 (a) {a, b}, {a}, {b}, { }
   (b) {1, 2, 3}, {1, 2}, {1, 3}, {2, 3}, {1}, {2}, {3}, { }
   (c) {5}, { }
   (d) {1, 2, 3, 4}, {1, 2, 3}, {1, 2, 4}, {1, 3, 4}
       {2, 3, 4}, {1, 2}, {1, 3}, {1, 4}, {2, 3}
       {2, 4}, {3, 4}, {1}, {2}, {3}, {4}, { }

2 (a)

Set	Number of elements	Number of subsets
{5}	1	2
{a, b}	2	4
{1, 2, 3}	3	8
{1, 2, 3, 4}	4	16

3 (a) 32   (b) 256   (c) 2^N

4

Subset	List	Number of subsets
0 members	{ }	1
1 members	{adder} {boa} {cobra}	3
2 members	{adder, boa}, {adder, cobra}, {boa, cobra}	3
3 members	{adder, boa, cobra}	1

5

Subset	List	Number of subsets
0 member	{ }	1
1 member	{adder}, {boa}, {cobra}, {daboia}	4
2 members	{adder, boa}, {adder, cobra}, {adder, daboia} {boa, cobra}, {boa, daboia} {cobra, daboia}	6
3 members	{adder, boa, cobra}, {adder, boa, daboia} {adder, cobra, daboia} {boa, cobra, daboia}	4
4 member	{adder, boa, cobra, daboia}	1

Exercise 4C

1 (a) (i) {Jigger, John, Jake}
       (ii) {Tony, John, Jake}   (b) John and Jake
   (c) {John, Jake}

2 (a) {Sam, Syd}   (b) {Midge}   (c) {Syd, Stanley}

3 (a) {blue}   (b) { }   (c) {c, k, e}   (d) {13}
   (e) {17, 21}   (f) { }   (g) {0.1, 0.9}

4 (a) 3, 6, 9, 12, 15, 18, 21, 24, 27, 30   (b) 1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25, 27, 29   (c) {3, 9, 15, 21, 27}

5 (a) {10, 20, 30}   (b) {12, 16, 20}   (c) {1, 2, 3, 4, 6}
   (d) {3, 5, 7, 11, 13, 17, 19, 23, 29}   (e) { }

6 The missing elements are: (a) 5, 7   (b) 24, 34   (c) $\frac{1}{16}, \frac{3}{11}$
   (d) 4, 6   (e) Joan, Ricky   (f) t

9 (a) {Mercury, Venus, Earth, Mars, Jupiter, Saturn, Neptune, Uranus}

(b) {Jupiter, Saturn, Neptune, Uranus}

(c) {Saturn, Neptune, Uranus}

(e) {Saturn, Neptune, Uranus}. They have masses larger than the Earth and they are further from the Sun than Jupiter.

Exercise 4D

2 (a) multiples of 4 = {4, 8, 12} inside
multiples of 2 = {2, 4, 6, 8, 10, 12}

(b) factors of 6 = {1, 2, 3, 6} inside
factors of 12 = {1, 2, 3, 4, 6, 12}

3 (a) {Sun}   **(b)** {March, May}   **(c)** { }   **(d)** {r, e}

4 (a) He doesn't smoke or wear glasses.   **(b)** No, he doesn't smoke.   **(c)** Tod Marks and Mo Stewart.   **(d)** Try the door keys at Tod Marks' and Mo Stewart's houses.

Exercise 4E

1 (a) {1, 2, 4, 8, 16}   **(b)** {1, 2, 3, 4, 6, 12}   **(c)** {1, 2, 4}

2 (a) {6}   **(b)** {6, 9}   **(c)** {6, 8}

3 (a) T = {Tuesday, Thursday} inside D = {Monday, Tuesday, Wednesday, Thursday, Friday, Saturday, Sunday}

(b) {Tuesday, Thursday}

4 (a) J = {January, June, July}, S = {January, October}

(b) {January}

6 (a) (i) {whole numbers from 1 to 20}
(ii) {prime numbers}   **(iii)** {odd numbers}

(b) {3, 5, 7, 11, 13, 17, 19}

7 (a) {4, 8, 12, 16, 20, 24, 28, 32, 36, 40, 44, 48}

(b) {12, 24, 36, 48, 60}   **(c)** {12, 24, 36, 48}

8 (a) U = {whole numbers from 1 to 10},
A = {even numbers less than 10},
B = {odd numbers less than 8}

(b) U = {whole numbers from 1 to 12}, A = {even numbers},
B = {multiples of 3}

(c) U = {days of the week},
A = {days of the week beginning with S},
B = {days of the week beginning with T}

(d) U = {days of the week},
A = {days of the week not beginning with S},
B = {days of the week beginning with F}

9 (a) { }   **(b)** {6, 12}   **(c)** { }   **(d)** {Friday}

Exercise 4F

1 (a) {1, 2, 3, ... 10}   **(b)** {2, 3, 7, 8, 10}

(c) {3, 4, 6, 7}   **(d)** {2, 3, 4, 6, 7, 8, 10}

2 (a) {1, 2, 3, 4, 5, 6, 8, 10}   **(b)** {3, 7}   **(c)** {1, 2, 3, ... 10}

3 (b) (i) {Liz Hunt, Tom Hunt}   **(ii)** {Don Lai, Liz Hunt, Tina Sasaki, Paul Dean} **(iii)** {Don Lai, Liz Hunt, Tom Hunt, Tina Sasaki, Paul Dean}

(c) (i) {Tom Hunt}
(ii) {Don Lai, Tina Sasaki, Paul Dean}   **(iii)** {Liz Hunt}

4 (a) {Alice Lai, Mona Lai, Clare Lai, Liz Hunt, Tom Hunt, Tina Sasaki, Clara Smith}   **(b)** {Alice Lai, Mona Lai, Clare Lai, Liz Hunt, Tom Hunt, Tina Sasaki, Clara Smith}

(c) The answers are the same

5 (b) (i) {2, 3, 4, 6, 8, 9, 10, 12, 14, 15, 16, 18, 20}
(ii) same as (i)   **(iii)** same as (i)
(iv) {2, 4, 8, 10, 14, 16, 20}   **(v)** {3, 9, 15}
(vi) {6, 12, 18}   **(vii)** They are the same
(viii) They are the same   **(ix)** They make E ∪ T.

6 (a) {2, 3, 4, 6, 8, 9, 10, 12, 15}

(b) {1, 3, 5, 6, 7, 9, 12, 15}   **(c)** {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}

7 (a) {2, 4, 7, 8, 13}   **(b)** {1, 5, 9, 10, 15, 20}

(c) {2, 3, 4, 9, 12, 13, 17}   **(d)** {7, 9, 15, 18, 27, 36}

(e) {1, 2, 3, 4, 5}   **(f)** {0.4, 0.5, 0.8, 0.9, 1.1}

8 (a) 12   **(b)** 2, 4, 11   **(c)** 8; 7, 9, 12   **(d)** 6, 8, 10; 6, 8

9 (b) {1, 2, 3, 4, 6, 9, 12, 18}

10 Both diagrams are the same.

Exercise 4G

1 (a) {1, 2, 4, 5, 7, 8, 10}   **(b)** {January, February, April, June, July, August, September, October, November, December}

(c) {Rosemarie, Rachel, Rhoda}

3 (a) {1, 4, 6, 8, 9, 10}   **(b)** {6, 7, 8, 9, 10}

(c) {1, 2, 3, 4, 6, 7, 8, 9} **(d)** {1, 2, 3, 4, 5, 6, 8, 9, 10}

(e) {1, 2, 3, ... 10}

4 (a) {mice that are not white}

(b) {boys that are not tall}   **(c)** {girls without glasses}

5 (a) (i) Peter, Andrew and James   **(ii)** Paul and John
(iii) Cecil, David, Thomas and Colin
(iv) Thomas and Colin   **(v)** Peter, Andrew and James

(b) (i) {Peter, Andrew, James, Paul, John, Cecil, David}
(ii) {Paul, John}   **(iii)** {Cecil, David, Thomas, Colin}
(iv) {Peter, Andrew, James, Thomas, Colin}

6 (b) (i) {tall teachers}
(ii) {people who are teachers or tall or both}
(iii) {people that are not tall}

7 (b) (i) {greedy rats with sharp teeth}
(ii) {rats with sharp teeth or greedy rats or both}
(iii) {rats that are not greedy}

8 (a) {c, d}   **(b)** {d}   **(c)** {c}   **(d)** {a, b, c, e}
(e) {a, b, d, e}   **(f)** {c}   **(g)** {e}   **(h)** {a, b, c, d}

9 (a) {b, c, d, f, g}   **(b)** {3, 5, 6, 7, 9}   **(c)** {obtuse}
(d) {letters in the word 'partly'}

Exercise 4H – mixed questions

1 (a) (i) {6, 12}, {3, 6, 8, 9, 10, 12, 14, 15}
(ii) {1, 5, 11}, {1, 3, 5, 7, 11}
(iii) {3, 5, 7, 11}, {1, 2, 3, 5, 7, 9, 11, 13}
(iv) {6, 12}, {2, 3, 4, 6, 8, 9, 10, 12}
(v) { }, {10, 12, 14, 16, 20, 25}

2 (a) (i) {Paula, Theresa}   **(ii)** {Ann}
(iii) {Paula, Mary, Ann, Theresa, Ruth}
(iv) {Paula, Mary, Ann, Theresa, Carol}

(b) (i) {girls who like to play rounders and netball}
(ii) {girls who like to play rounders and volleyball}
(iii) {girls who like to play netball or volleyball or both}
(iv) {girls who like to play rounders or netball or both}

4 (a) (i) {February, September, October, November, December}
  (ii) {January, February, June, August, September, October, November, December}
  (iii) {January, February, August, September, October, November, December}
  (b) (i) {names of months with more than five letters that contain the letter e}
  (ii) {names of months that do not contain the letter b}

5 (b) (i) a black square   **(ii)** a square that is not black
  (iii) a black shape that is not a square
  (iv) a shape that is not square or black

6 (b) (i) {right-angled isosceles triangles}
  (ii) {triangles that are not right-angled}

7 (a) (i) {1, 2, 3, ... 10}   **(ii)** {2, 3, 5}   **(iii)** {4, 7, 9}
  (iv) { }   **(v)** {2, 3, 4, 5, 7, 9}   **vi)** {1, 2, 3, ... 10}
  (vii) {1, 6, 8, 10}   **(viii)** {1, 4, 6, 7, 8, 9, 10}
  (ix) {1, 2, 3, 5, 6, 8, 10} **(x)** {1, 6, 8, 10}   **(xi)** {2, 3, 5}
  (xii) {4, 7, 9}   **(xiii)** {1, 2, 3, ... 10}
  (xiv) {1, 4, 6, 7, 8, 9, 10}

  (b) (i) {2, 3, 4, 6, 8, 9, 10, 12, 18, 24}   **(ii)** {3, 6, 9, 12}
  (iii) {6, 12, 18, 24}   **(iv)** {6, 12}
  (v) {3, 6, 9, 12, 18, 24}   **(vi)** {2, 3, 4, 8, 9, 10, 18, 24}
  (vii) {2, 4, 8, 10}   **(viii)** {2, 4, 8, 10, 18, 24}
  (ix) {2, 3, 4, 8, 9, 10}   **(x)** {2, 4, 8, 10}   **(xi)** {3, 9}
  (xii) {18, 24}   **(xiii)** {2, 3, 4, 8, 9, 10, 18, 24}
  (xiv) {2, 4, 6, 8, 10, 12, 18, 24}

Exercise 4

2 (a) {2, 3, 5, 6, 8, 10, 12}   **(b)** {5}   **(c)** {1, 2, 3, 4, 5, 8, 10}
  (d) {3}   **(e)** {2, 5, 8, 10}   **(f)** {2, 3, 5, 6, 12}
  (g) {2, 5, 8, 10}   **(h)** {2, 5}   **(i)** {5}
  (j) {1, 2, 3, 4, 5, 6, 12}

3 (a) {6, 10}   **(b)** {4, 6}   **(c)** {2, 8}   **(d)** {2, 4, 8, 10}
  (e) {10}   **(f)** {6}   **(g)** {4, 6, 10}   **(h)** {2, 4, 6, 8}
  (i) {6}   **(j)** {4, 6, 10}

4 (a) {whole numbers 1 to 10}   **(b)** { }
  (c) {1, 2, 4, 6, 7, 8, 10}   **(d)** {1, 3, 5, 7, 9}
  (e) {2, 3, 4, 5, 6, 8, 9, 10}   **(f)** {3, 5, 9}   **(g)** {3, 5, 9}
  (h) U   **(i)** {3, 5, 9}   **(j)** {1, 7}   **(k)** {2, 4, 6, 8, 10}
  (l) {2, 3, 4, 5, 6, 8, 9, 10}

5 (a)

  (b) {1, 3}   **(c)** 3

6 (a)

(b) {12, 24, 36}   **(c)** 12

7 (a)
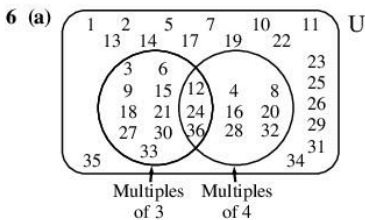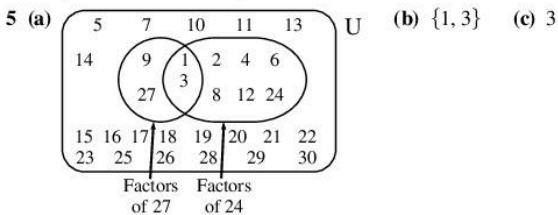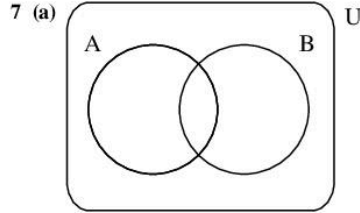
  (b) (i) {students in Form 2 who like football and cricket}
  (ii) {students who like football or cricket or both}
  (iii) {students who do not like football}
  (iv) {students who like neither football nor cricket}
  (c) (i) A′   **(ii)** B∩A′   **(iii)** A∩B′

Check out

1 (a) {6, 12}   **(b)** 5

2 (a) {2, 3, 4, 5, 6, 8}   **(b)** 7

3 (b)

4 (a) {5, 7, 8, 10, 11, 13, 14}   **(b)** {8, 10, 14}
  (c) {5, 6, 7, 9, 11, 12, 13, 15}   **(d)** {8, 10, 14}

Revision exercise 1

1 (i) (a) $\frac{1}{6}$ **(b)** $\frac{1}{2}$ **(c)** $\frac{2}{3}$ **(d)** $\frac{1}{3}$
  (ii) (a) $\frac{1}{4}$ **(b)** $\frac{3}{8}$ **(c)** $\frac{5}{8}$ **(d)** $\frac{3}{8}$

2 (a) $\frac{2}{3} = \frac{4}{6}$ **(b)** $\frac{7}{8} = \frac{21}{24}$ **(c)** $\frac{8}{12} = \frac{2}{3}$ **(d)** $\frac{15}{25} = \frac{3}{5}$

3 (a) $\frac{3}{4}$ **(b)** $\frac{3}{4}$ **(c)** $\frac{4}{5}$ **(d)** $\frac{5}{6}$ **(e)** $\frac{3}{7}$ **(f)** $\frac{7}{12}$
  (g) $\frac{3}{4}$ **(h)** $\frac{3}{5}$

4 (a) $\frac{7}{8}$ **(b)** $\frac{1}{6}$ **(c)** $2\frac{1}{7}$ **(d)** 2 **(e)** 5   **(f)** $1\frac{1}{2}$ **(g)** $\frac{5}{8}$
  (h) $\frac{1}{4}$ **(i)** $1\frac{4}{9}$ **(j)** $\frac{2}{15}$ **(k)** $1\frac{8}{35}$ **(l)** $\frac{3}{70}$

5 (a) $4\frac{11}{20}$ **(b)** $1\frac{23}{24}$ **(c)** $8\frac{13}{18}$ **(d)** $\frac{23}{24}$

6 (a) $4\frac{1}{5}$ **(b)** $2\frac{2}{9}$ **(c)** 10   **(d)** 10   **(e)** $18\frac{2}{3}$ **(f)** $14\frac{1}{6}$
  (g) $\frac{1}{8}$ **(h)** $\frac{1}{12}$ **(i)** 15   **(j)** $10\frac{2}{3}$ **(k)** 4   **(l)** $1\frac{37}{63}$
  (m) $1\frac{1}{4}$ **(n)** $1\frac{1}{8}$ **(o)** 6

7 (a) $75   **(b)** $40

8 (a) $\frac{2}{3}$ kg   **(b)** $4\frac{1}{10}$ kg   **(b)** $\frac{4}{21}$ **(d)** $7\frac{7}{12}$

9 168 hours **(a)** $\frac{5}{84}$ **(b)** $\frac{5}{21}$ **(c)** $\frac{25}{84}$ **(d)** $\frac{3}{28}$ **(e)** $\frac{59}{84}$ **(f)** $\frac{1}{6}$

10 (a) 27   **(b)** 1442

11 (a) (i) 14.32   **(ii)** 6.47   **(iii)** 0.33   **(iv)** 1.72   **(v)** 7.02
  (vi) 0.00   **(b) (i)** 14.3   **(ii)** 6.5   **(iii)** 0.3   **(iv)** 1.7
  (v) 7.0   **(vi)** 0.0

12 (a) 0.7   **(b)** 0.13   **(c)** 0.429   **(d)** 0.8   **(e)** 0.75
  (f) 0.875   **(g)** 0.85   **(h)** 0.46   **(i)** 0.98   **(j)** 1.25
  (k) 4.36   **(l)** 2.065

13 (a) 2.18   **(b)** 0.05   **(c)** 32   **(d)** 50   **(e)** 320
  (f) 3.583…   **(g)** 0.02   **(h)** 1.9

14 (a) 4.1   **(b)** 9.84   **(c)** 0.8   **(d)** 5.42   **(e)** 0.537
  (f) 0.486   **(g)** 10.6   **(h)** 2.4   **(i)** 22.23   **(j)** 1.07
  (k) 12.96   **(l)** 0.08

15 (a) (i) 9.6 cm   **(ii)** 10.4 cm   **(iii)** 10 cm
  (iv) 8 cm; 8, 9.6, 10, 10.4   **(b) (i)** 5.27 cm^2 **(ii)** 6.67 cm^2
  (iii) 6.25 cm^2 **(iv)** 3.36 cm^2; 3.36, 5.27, 6.25, 6.67

16 (a) 553.5   **(b)** 55.35   **(c)** 5.535   **(d)** 55.35   **(e)** 596.3
  (f) 59.63   **(g)** 59 630   **(h)** 0.5963

17 (a) 29   **(b)** 35 gallons   **(c)** 8.6 m

18

Number	1 decimal place	1 significant figure
16.32	16.3	20
205.692	205.7	200
0.0014	0.0	0.001
0.895	0.9	0.9
1906.48	1906.5	2000

19 (a) 2.184×10^3 **(b)** 3.0008×10^4 **(c)** 8.76×10^1
  (d) 2.101×10^5 **(e)** 9.5×10^7 **(f)** 3.98×10^8

20 (a) 0.090909 ... , 0.181818 ... , 0.272727 ... ,
    0.363636 ... , 0.454545 ... , 0.545454 ... ,
    0.636363 ... , 0.727272 ... , 0.818181 ...
  (b) $\frac{13}{99}$

21 (a) acute   **(b)** acute   **(c)** reflex   **(d)** right-angled

22 (a) 70°   **(b)** 35°   **(c)** 228°   **(d)** 90°

23 (a) $k = 130, d = 50$ **(b)** $m = 43$ **(c)** $n = 47$
  (d) $p = 123$

24 (b) 5.8 cm

25 $p = 64°, q = 116°, r = 74°, s = 55°, t = 125°$

26 (a) 32° (alt angles)   **(b)** 20° (angles in triangle)   **(c)** 128°

27 (a) 90°   **(b)** 108°   **(c)** 135°

28 (a) KN̂M   **(b)** NK̂M   **(c)** NM̂K

29 All angles 90°. Same length.

30 (a) same length   **(b)** same length   **(c)** opposite angles equal

31 (a) {a, e, i, o, u}   **(b)** {1, 3, 5, 7, 9}
  (c) {Sunday, Monday, Tuesday, Wednesday, Thursday, Friday, Saturday}
  (d) {October, November, December}
  (e) {8, 12, 16}

32 (a) {January, February, March}, {January, February} {January, March}, {February, March}, {January} {February}, {March}, { }
  (b) 8

33 (a) {Michael, Ray, Tony, Sarah, Sally}   **(b)** {Delia, Reena}
  (c) {Sarah, Sally}

34 (a) {Delia, Reena, Samuel}
  (b) {Michael, Ray, Tony, Samuel}   **(c)** {Samuel}
  (d) {Delia, Reena, Sarah, Sally}
  (e) {Delia, Reena, Samuel, Michael, Ray, Tony}
  (f) {Sarah, Sally}

35 (a) {3, 6, 9, 12, 15, 18}   **(b)** {1, 4, 9, 16}   **(c)** {9}
  (d) {1, 3, 4, 6, 9, 12, 15, 16, 18}
  (e) {1, 2, 4, 5, 7, 8, 10, 11, 13, 14, 16, 17, 19, 20}

37 (a) {a, e, i}   **(b)** {a, b, c, d, e, f, g, h, i, j, o, u}
  (c) {b, c, d, f, g, h, j, k, l, m, n, p, q, r, s, t, v, w, x, y, z}
  (d) {k, l, m, n, o, p, q, r, s, t, u, v, w, x, y, z}

38 (a) {p}, {q}, {r}, {s}   **(b)** {p, q}, {p, r}, {p, s}, {q, r}, {q, s}, {r, s}   **(c)** {p, q, r}, {p, q, s}, {p, r, s}, {q, r, s}

39 (a) {2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24, 26, 28, 30}
  (b) {1, 2, 3, 4, 6, 8, 12, 16, 24}
  (c) {3, 6, 9, 12, 15, 18, 21, 24, 27, 30}
  (d) {6, 12, 18, 24, 30}
  (e) {1, 2, 3, 4, 6, 8, 9, 12, 15, 16, 18, 21, 24, 27, 30}

Mixed questions 1

1 C   **2** C   **3** B   **4** D   **5** A   **6** D   **7** A   **8** C
9 D   **10** A   **11** C   **12** A   **13** D   **14** B   **15** A
16 C   **17** C   **18** D   **19** C   **20** A   **21** A   **22** C
23 C   **24** D   **25** B   **26** D   **27** C   **28** B   **29** B
30 C   **31** B   **32** C   **33** D   **34** C   **35** C   **36** D
37 C   **38** C   **39** A   **40** B

5 Measurement

Check in

1 (a) 2.3 cm   **(b)** 3.9 cm   **(c)** 0.9 cm or 9 mm

3 (a) 5 cm^2 **(b)** 2 cm^2 **(c)** Roughly 5 cm^2

Exercise 5A

4 (a) cm   **(b)** m   **(c)** mm   **(d)** km   **(e)** m   **(f)** mm
7 (a) kg or t   **(b)** kg   **(c)** g   **(d)** g   **(e)** kg   **(f)** t

Exercise 5B

1 (a) 1000 g = 1 kg   **(b)** 1000 kg = 1 t
2 (a) 100 cm = 1 m   **(b)** 1000 m = 1 km   **(c)** 10 mm = 1 cm
3

Metres	Centimetres	Millimetres
6	600	6000
8	800	8000
34	3400	34000
5	500	5000
4	400	4000

4

Kilograms	Grams
9	9000
15	15000
3.1	3100
4	4000
0.8	800

5

Object	Length (cm & mm)		Length (cm)	Length (mm)
Pencil	18 cm	5 mm	18.5	185
Pen	13 cm	3 mm	13.3	133
Eraser	5 cm		5	50
Mat	95 cm		95	950
Fingernail		9 mm	0.9	9

6 (a) 145 cm **(b)** 52 300 g

7 1329 cm, 643 cm

Exercise 5C

1 (a) 30 **(b)** 90

2 (a) 75 **(b)** 15 **(c)** 65

3 (a) 96 **(b)** 160 **(c)** 80 **(d)** 1568

4 (a) 90 **(b)** 0.9 **(c)** 45 **(d)** 2.7

5 (a) 270 **(b)** 2.7

6 396

7 8.8125 m

8 8.7

9 0.5625 m

10 2

Exercise 5D

1 (a) 22 cm **(b)** 28.7 cm **(c)** 49.2 cm **(d)** 28 cm

2 (a) 2 cm **(b)** 7 cm **(c)** 7.2 cm **(d)** 4.3 cm

3 (a) 26 cm **(b)** 30 cm **(c)** 40 cm **(d)** 24 cm

4 (a) 15 cm **(b)** 7 cm **(c)** 1.9 cm **(d)** 5.3 cm

5

	Length	Width	Perimeter
(a)	6 cm	5 cm	22 cm
(b)	7 cm	4 cm	22 cm
(c)	11.5 m	5.5 m	34 m
(d)	7.2 m	7.2 m	28.8 m
(e)	13.4 cm	8.2 cm	43.2 cm

6 (a) 29.2 m **(b)** 8 cm

7 11 cm

8 $535.50

Exercise 5E

1 (a) 6.28 cm **(b)** 31.4 cm **(c)** 37.68 cm **(d)** 65.94 cm

2 (a) 22 cm **(b)** 44 cm **(c)** 132 cm **(d)** 264 cm

3 (a) 44 cm **(b)** 220 cm **(c)** 308 cm **(d)** 572 cm

4 (a) 31.4 cm **(b)** 50.24 cm **(c)** 81.64 cm **(d)** 244.92 cm

5 47.1 cm **6** 220 cm

7 942 000 000 km (to 3 sig figs)

8 (a) 37.68 cm **(b)** 0.628 cm

9 40 seconds

10 40 035 km

11 (a) 36 cm **(b)** 32 cm **(c)** 94 cm **(d)** 14.85 cm **(e)** 28.56 cm

12 420 m

13 (a) 35 cm **(b)** 308 cm

Exercise 5F

1 (a) 6 cm^2 **(b)** 4 cm^2

2 (a) 26 m^2 **(b)** 25.2 cm^2

3 (a) 39 cm^2 **(b)** 125 cm^2 **(c)** 18 cm **(d)** 9 cm **(e)** 14.4 cm^2

4 46.11 m^2

Exercise 5G

1 7 cm by 2.5 cm; 17.5 cm^2; 8.75 cm^2

2 (a) 42 cm^2 **(b)** 6 cm^2, 15 cm^2 **(c)** 21 cm^2 **(d)** (c) is half (a)

3 (a) 48 cm^2, 24 cm^2 **(b)** 37.8 cm^2, 18.9 cm^2

4 (a) 43.2 cm^2 **(b)** 82.5 cm^2

5 (a) 2.6 cm, 1.7 cm; 2.21 cm^2 **(b)** 1.7 cm, 2.2 cm; 1.87 cm^2

6 (a) 20 cm^2 **(b)** 12 cm **(c)** 27 cm^2 **(d)** 12 cm **(e)** 28.98 cm^2

Exercise 5H

1 (a) 30 cm^2 **(b)** 21 cm^2 **(c)** 32 cm^2 **(d)** 26.98 cm^2

2 (a) 8 cm **(b)** 6 cm **(c)** 3 cm

3

	Base (cm)	Height (cm)	Area (cm^2)
(a)	4.8	5	24
(b)	3	12	36
(c)	1.2	9	10.8
(d)	6.2	4.3	26.66
(e)	13.84	8	110.72

4 e.g. 72 cm, 1 cm; 36 cm, 2 cm; 18 cm, 4 cm; 9 cm, 8 cm; 12 cm, 6 cm, 24 cm, 3 cm etc.

Exercise 5I

1 50 cm^2

2 (a) 79 cm^2 **(b)** 28 cm^2 **(c)** 113 cm^2

3 (a)

Radius of circle r (cm)	3	4	5	6
r^2	9	16	25	36
$3 \times r^2$	27	48	75	108
Area of circle (cm^2)	28	50	79	113

(b) Yes

4 (a) 192 cm^2 **(b)** 27 cm^2 **(c)** 108 cm^2

Exercise 5J

1 (a) $12\frac{4}{7}\,\text{cm}^2$ **(b)** $154\,\text{cm}^2$ **(c)** $314\frac{2}{7}\,\text{cm}^2$ **(d)** $616\,\text{cm}^2$

2 (a) $3.14\,\text{cm}^2$ **(b)** $28.26\,\text{cm}^2$ **(c)** $78.5\,\text{cm}^2$ **(d)** $314\,\text{cm}^2$

3 (a) $158\,\text{cm}^2$ **(b)** $206\,\text{cm}^2$

4 Divide the diameter by 2, square it then multiply by 3.14.

5 (a) $314\,\text{cm}^2$ **(b)** $475\,\text{cm}^2$ **(c)** $8167\,\text{cm}^2$

6 (a) $25\frac{1}{7}\,\text{cm}^2$ **(b)** $231\,\text{cm}^2$ **(c)** $141\frac{3}{7}\,\text{cm}^2$ **(d)** $1848\,\text{cm}^2$

7 (a) $226.08\,\text{cm}^2$ **(b)** $26.3917\,\text{cm}^2$ **(c)** $50.24\,\text{cm}^2$

8 (a) $18.84\,\text{cm}^2$ **(b)** $75.36\,\text{cm}^2$

9 (a) $21.5\,\text{cm}^2$ **(b)** $6.88\,\text{cm}^2$

10 (a) $28.5\,\text{cm}^2$ **(b)** $20.52\,\text{cm}^2$

11 (a) 10 cm **(b)** 2 cm **(c)** 5.6 cm

12 (a) $1230.88\,\text{cm}^2$ **(b)** $4203.5\,\text{cm}^2$ **(c)** $263.375\,\text{cm}^2$ **(d)** $7.693\,\text{cm}^2$

13 $42\,\text{cm}^2$

Exercise 5K

1 (a) (i) 1.9 cm **(ii)** 2.7 cm **(iii)** 1.5 cm **(iv)** 2.3 cm
(b) (i) A 7.6 cm, B 19 cm, C 9.5 m
(ii) A 10.8 cm, B 27 cm, C 13.5 m
(iii) A 6 cm, B 15 cm, C 7.5 m
(iv) A 9.2 cm, B 23 cm, C 11.5 m

2

Length on drawing	Scale	Actual length
(a) 7 cm	1 cm represents 5 m	35 m
(b) 3.5 cm	1 cm represents 10 m	35 m
(c) 8.2 cm	1 cm represents 2 m	16.4 m
(d) 13 cm	1 cm represents 20 km	260 km
(e) 7 cm	1 cm represents 10 m	70 m
(f) 6.8 cm	1 cm represents 25 km	170 km
(g) 12 cm	1 cm represents 3.5 m	42 m

3 (a) (i) 3 cm **(ii)** 2.2 cm **(iii)** 1.4 cm **(iv)** 2.5 cm
(b) (i) 3 km **(ii)** 2.2 km **(iii)** 1.4 km **(iv)** 2.5 km

5 15 m

6 (a) (i) 5 cm **(ii)** 2.5 cm **(iii)** 1 cm **(iv)** 1 cm
(v) 1.5 cm **(vi)** 2 cm **(b) (i)** 50 m **(ii)** 25 m
(iii) 10 m **(iv)** 10 m **(v)** 15 m **(vi)** 20 m

Exercise 5L

1 (a) $1:100$ **(b)** $1:500$ **(c)** $1:1000$ **(d)** $1:100\,000$
(e) $1:10\,000$

2 (a) (i) 2 m, 1 m **(ii)** 1 m, 0.5 m **(iii)** 40 m, 20 m
(b) (i) 1.5 m, 2 m **(ii)** 0.75 m, 1 m **(iii)** 30 m, 40 m
(c) (i) 2.5 m, 1.5 m **(ii)** 1.25 m, 0.75 m **(iii)** 50 m, 30 m

3 (a) $1:400\,000$ **(b)** 4 km **(c)** 3.3 cm **(d)** 13.2 km
(e) (i) 5.2 km **(ii)** 14.4 km **(iii)** 10.4 km **(iv)** 8.8 km

4 (a) 1 km **(b)** 3 km **(c)** 1.25 km **(d)** 3.85 km
(e) 1 km

5

Length on drawing	Scale 1 : n	Actual length
0.85 cm	1:1000	8.5 m
4.2 cm	1:50 000	2.1 km
5.2 cm	1:100 000	5.2 km
3.5 cm	1:250	8.75 m
7.2 cm	1:5000	360 m
2.1 cm	1:1 000 000	21 km

Exercise 5M – mixed questions

1 (a) 76 cm **(b)** 31 cm

2 2.26 km

3 (a) (i) 1.75 km **(ii)** 3.65 km
(b) (i) 3.5 cm **(ii)** 0.7 cm

4 20 mm

5 (a) (i) $a = 2$ cm, $b = 8$ cm, $c = 11$ cm
(ii) $a = 11$ cm, $b = 10$ cm **(iii)** $a = 4$ cm, $b = 7$ cm
(b) (i) 42 cm **(ii)** 42 cm **(iii)** 32 cm
(c) (i) $46\,\text{cm}^2$ **(ii)** $54\,\text{cm}^2$ **(iii)** $34\,\text{cm}^2$

6 (a) 136 m **(b)** 190 cm **(c)** 26.5 cm

7 (a) $1204\,\text{m}^2$ **(b)** $437.5\,\text{cm}^2$ **(c)** $39.375\,\text{cm}^2$

8 (a) 132 cm **(b)** 1500

9 17.5 cm

10 (a) 77 mm **(b)** 231 mm **(c)** 256.7 mm

11 528 cm/s

Exercise 5

1 (a) (i) 500 cm **(ii)** 2800 cm **(iii)** 720 cm
(iv) 300 000 cm **(v)** 1345 000 cm **(b) (i)** 0.25 m
(ii) 2.93 m **(iii)** 8100 m **(iv)** 320 m **(v)** 2.615 m
(c) (i) 4000 g **(ii)** 400 g **(iii)** 3200 g **(iv)** 490 g
(v) 15200 g **(d) (i)** 0.5 kg **(ii)** 0.075 kg **(iii)** 5 kg
(iv) 3.168 kg **(v)** 13.459 kg

2 (a) 23 cm **(b)** 35.4 cm **(c)** 34.438 cm

3 (a) $19.5\,\text{cm}^2$ **(b)** $52\,\text{cm}^2$ **(c)** $56.57\,\text{cm}^2$ **(d)** $11.73\,\text{cm}^2$
(e) $28.8\,\text{cm}^2$

4 4 cm, 1:50000, 2 km; 4.3 cm, 1:250000, 10.75 km;
9 cm, 1:50000, 4.5 km; 8.7 cm, 1:10000, 0.87 km; 33 mm,
1:25000, 825 m; 10.68 mm, 1:25000, 267 m

5 (a) Check students' work **(b)** 576.8 m **(c)** 8.652 km

6 Check students' work

Check out

1 (a) 12.5 cm **(b)** 27.2 km **(c)** 9.9 kg

2 8 cm

3 (a) 31.4 cm **(b)** 37.68 cm

4 (a) $21\,\text{cm}^2$ **(b)** $32\,\text{cm}^2$ **(c)** $63.64\,\text{cm}^2$

5 (a) 37.5 m **(b)** 27 cm

6 Integers

Check in

1 (a) $^-1$ **(b)** $^-2$ **(c)** $^-3$ **(d)** $^-1$ **(e)** $^-6$ **(f)** $^-3$

4 $2 \to 1, 4 \to 2, 6 \to 3, 8 \to 4$

Exercise 6A

1 (b) $2, 0, ^-2$

2 (a) 3 **(b)** 3 **(c)** $^-2$ **(d)** 0 **(e)** $^-1$ **(f)** 1

3 (a) $^-3$ **(b)** $^-5$ **(c)** $^-7$ **(d)** $^-10$ **(e)** $^-12$ **(f)** $^-2$

4 (a) $^-4$ **(b)** 2 **(c)** 0 **(d)** $^-3$ **(e)** $^-2$ **(f)** 0
(g) 2 **(h)** 3

5 (a) 0 **(b)** $^-1$ **(c)** $^-7$ **(d)** $^-7$ **(e)** $^-9$ **(f)** $^-7$

6 (a) $^-2$ **(b)** 2 **(c)** 1 **(d)** $^-2$ **(e)** 8 **(f)** 0

8 (a) 8 **(b)** 1 **(c)** 4 **(d)** 11 **(e)** 0 **(f)** 0
(g) $^-10$ **(h)** 6

9 \$11   **10** $^-1°C$

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4	$^-10$	0
$^-6$	$^-2$	2
$^-4$	6	$^-8$

Exercise 6B

1 (a) 17 **(b)** 9 **(c)** 0 **(d)** 6 **(e)** $^-2$

2 (a) 7 **(b)** 12 **(c)** 7 **(d)** 12 **(e)** 11 **(f)** 6
(g) 1 **(h)** 1 **(i)** 3 **(j)** 1 **(k)** 30 **(l)** 36
(m) $^-6$ **(n)** 13

4 (a) 7 **(b)** 11 **(c)** 1 **(d)** 4 **(e)** 12 **(f)** $^-12$
(g) 8 **(h)** $^-3$

5 (a) 7 **(b)** 6 **(c)** $^-2$ **(d)** $^-9$ **(e)** 2 **(f)** $^-14$
(g) $^-8$ **(h)** $^-13$ **(i)** $^-8$ **(j)** 10

6 (a)

$-$	2	$^-1$
3	1	4
4	2	5

(b)

$-$	$^-1$	0
3	4	3
4	5	4

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(c) The second number must be 4 greater than the first.

Exercise 6C

1 (a) $^-8$ **(b)** $^-20$ **(c)** $^-12$

2 (a) $^-12$ **(b)** $^-16$ **(c)** $^-18$ **(d)** $^-20$ **(e)** $^-6$ **(f)** $^-56$

3 (b) $^-20$ **(c)** $^-12$ **(d)** $^-30$ **(e)** $^-2$

4 (a) $^-24$ **(b)** $^-21$ **(c)** $^-4$ **(d)** $^-8$ **(e)** $^-16$
(f) $^-100$ **(g)** $^-28$ **(h)** $^-18$ **(i)** $^-27$

5 (a) $^-54$ **(b)** $^-32$ **(c)** $^-40$ **(d)** $^-24$ **(e)** $^-24$
(f) $^-33$ **(g)** $^-3$ **(h)** $^-9$ **(i)** $^-36$

Exercise 6D

1 (a)

(b) yes **(c)** $^-12, ^-15$

2 (a)

(b) $^-20, ^-28$

3 (a) (i) $^-6$ **(ii)** $^-8$ **(iii)** $^-10$ **(c)** 2 **(d)** 4

4 $^-1$ times table   **5 (a)** $^-4$ **(b)** 4 **(c)** 2

6 (a) 0 **(b)** $3, 6, 9$

7 $^-3 \times ^-2 = 6, ^-3 \times 2 = ^-6, 3 \times 2 = 6$

Exercise 6E

1 (a) 20 **(b)** $^-20$ **(c)** $^-20$ **(d)** 20 **(e)** $^-10$ **(f)** 12
(g) 8 **(h)** 8 **(i)** $^-8$

2 (a) 48 **(b)** $^-48$ **(c)** $^-10$ **(d)** 35 **(e)** 70
(f) 70 **(g)** $^-100$ **(h)** $^-74$ **(i)** 1 **(j)** 9 **(k)** 400
(l) $^-400$ **(m)** $^-9.45$ **(n)** 14

3 (a) 4 **(b)** $^-4$ **(c)** 4 **(d)** $^-4$ **(e)** $^-2$ **(f)** $^-5$
(g) $^-2$ **(h)** 2 **(i)** $^-3$ **(j)** $^-4$

4 (a) 6 **(b)** 48 **(c)** $^-8$ **(d)** $^-48$ **(e)** 20 **(f)** $^-16$
(g) $^-24$ **(h)** 108

5 (a) 36 **(b)** 96 **(c)** 24 **(d)** 64 **(e)** $^-120$ **(f)** 80

6 (a) $^-2$ **(b)** 4 **(c)** $^-3$ **(d)** 2 **(e)** $^-6$ **(f)** $^-6$
(g) 6 **(h)** 1

7 (a) 1 **(b)** $^-1$ **(c)** 1 **(d)** $^-1$

8 (a) negative **(b)** positive **(c)** negative

9 An even number of negative numbers multiplied together will give a positive answer, an odd number a negative answer.

Exercise 6F

1 (a) 2 **(b)** 3 **(c)** 5 **(d)** $^-4$ **(e)** $^-4$ **(f)** $^-3$

2 (a) 2 **(b)** 3 **(c)** 5 **(d)** $^-4$ **(e)** $^-4$ **(f)** $^-3$

3 (a) $3 \times \square = 18$ **(b)** $2 \times \square = ^-8$ **(c)** $^-3 \times \square = 12$
(d) $^-3 \times \square = 9$ **(e)** $^-4 \times \square = 16$ **(f)** $^-1 \times \square = ^-4$

4 (a) 6   **(b)** ⁻4   **(c)** ⁻4   **(d)** ⁻3   **(e)** ⁻4   **(f)** 4

6 $\frac{15}{-3} = -5$, $\frac{-15}{3} = -5$, $\frac{-15}{-3} = 5$

7

×	Positive	Negative
Positive	Positive	Negative
Negative	Negative	Positive

÷	Positive	Negative
Positive	Positive	Negative
Negative	Negative	Positive

8 (a) ⁻10   **(b)** ⁻2   **(c)** ⁻2   **(d)** ⁻2   **(e)** ⁻7   **(f)** ⁻3
    (g) 4   **(h)** 1   **(i)** ⁻1

9 (a) ⁻12   **(b)** ⁻6   **(c)** ⁻2   **(d)** 1

10 (a) 9   **(b)** 3, ⁻3
    (c) A number multiplied by itself can never be negative.

Exercise 6G

1 (a) 6.00 am   **(b)** 11 hours   **(c)** ⁻2.5 h   **(d)** ⁻1.25 h

2 (a) 3198 years   **(b)** 1250 years   **(c)** 658 years
    (d) 1250 years

3 1250 B.C.   **4** 44 B.C.   **5** 1896   **7** 4650 years

Exercise 6H

2 (a) 6 °C   **(b)** ⁻2 °C

3 (a) 3 °C   **(b)** ⁻4 °C   **(c)** ⁻8 °C   **(d)** 1 °C
    (e) ⁻1 °C   **(f)** ⁻9 °C

4 ⁻8 C   **5** 396 °C   **6** ⁻214 °C

7 (a) ⁻31 °C   **(b)** 10 °C

8 (a) 44 °C   **(b)** No. It fell by 7 °C   **(c)** 10 °C   **(d)** 13 °C

Exercise 6I – mixed questions

1 (a) ⁻1   **(b)** ⁻6   **(c)** ⁻2   **(d)** ⁻6
    (e) ⁻5   **(f)** ⁻7   **(g)** ⁻16   **(h)** ⁻2

2 (a) 7   **(b)** 15   **(c)** 18   **(d)** 15
    (e) 24   **(f)** 15   **(g)** 16   **(h)** 32

3 (a) ⁻3   **(b)** ⁻8   **(c)** ⁻54   **(d)** ⁻55
    (e) ⁻63   **(f)** ⁻15   **(g)** 42   **(h)** 45

4 (a) ⁻3   **(b)** ⁻4   **(c)** ⁻3   **(d)** 5
    (e) ⁻10   **(f)** ⁻5   **(g)** ⁻5   **(h)** 7

5 ⁻14 °C

6 (a) 28 °C   **(b)** Toronto   **(c)** 11 °C   **(d)** Castries, Toronto
    (e) Castries, Port of Spain, London, New York, Toronto

7 (a) 170 m   **(b)** 480 m   **(c)** 650 m

8 9233 m   **9** $6.50

10 (a) 3   **(b)** ⁻2   **(c)** ⁻9   **(d)** ⁻22   **(e)** 10   **(f)** ⁻18
     (g) ⁻170   **(h)** 20

11 (a) ⁻2   **(b)** 2   **(c)** ⁻9   **(d)** 6   **(e)** ⁻6   **(f)** 3
     (g) ⁻3.5 or any pair of numbers that sum to ⁻7   **(h)** ⁻2.2

Consolidation

Exercise 6

1 (a) 7   **(b)** ⁻1   **(c)** 1   **(d)** ⁻7   **(e)** ⁻4   **(f)** ⁻16
    (g) ⁻2   **(h)** ⁻12   **(i)** ⁻27   **(j)** ⁻16

2 (a) 5   **(b)** 8   **(c)** ⁻2   **(d)** 12   **(e)** ⁻1   **(f)** 1
    (g) ⁻21   **(h)** ⁻16   **(i)** 21   **(j)** 7

3 (a) 0   **(b)** ⁻8   **(c)** 4   **(d)** ⁻6   **(e)** ⁻6   **(f)** 17   **(g)** 8
    (h) ⁻8   **(i)** ⁻14   **(j)** 3   **(k)** ⁻21   **(l)** ⁻45

4 (a) 15   **(b)** ⁻12   **(c)** ⁻12   **(d)** 12   **(e)** ⁻20   **(f)** 24
    (g) ⁻14   **(h)** 21   **(i)** ⁻132   **(j)** 182

5 (a) ⁻5   **(b)** ⁻5   **(c)** 5   **(d)** ⁻4   **(e)** $\frac{-1}{4}$ **(f)** $\frac{-2}{3}$
    (g) $\frac{-3}{5}$ **(h)** ⁻5   **(i)** 16   **(j)** ⁻14

6 (a) 2   **(b)** ⁻1   **(c)** ⁻5   **(d)** 11   **(e)** ⁻30   **(f)** 56
    (g) ⁻14   **(h)** $\frac{-5}{6}$

7 (a) 7 °C   **(b)** −6 °C   **(c)** −5 °C

8 (a) Moscow   **(b)** 36 °C   **(c)** 9 °C
    (d) Bridgetown, Kingston, Calgary, Moscow

9 (c) (i) 59°F   **(ii)** 17.8°F

Check out

1 (a) ⁻1   **(b)** 1   **(c)** ⁻5   **(d)** ⁻11   **(e)** ⁻6   **(f)** ⁻6

2 (a) 5   **(b)** 5   **(c)** 1   **(d)** 4   **(e)** 5   **(f)** ⁻2

3 (a) 1   **(b)** ⁻6   **(c)** ⁻4   **(d)** ⁻2   **(e)** ⁻3   **(f)** ⁻6

4 (a) ⁻12   **(b)** ⁻12   **(c)** 12   **(d)** ⁻15   **(e)** 24   **(f)** 20

5 (a) 5   **(b)** ⁻5   **(c)** ⁻5   **(d)** 5   **(e)** ⁻6   **(f)** ⁻6
    (g) 4   **(h)** ⁻4

7 Relations and graphs

Check in

1 (b), (c) and **(d)**   **2 (a)** 22   **(b)** 13   **(c)** 1

Exercise 7A

1 (a) $1 \to 0; 3 \to 2; 5 \to 4$ **(b)** $3 \to 5, 4 \to 6, 5 \to 7, 6 \to 8$
    (c) $2 \to 1, 4 \to 2, 6 \to 3, 8 \to 4$
    (d) $1 \to 1, 2, 3, 4, 5, 6, 7, 8; 2 \to 2, 4, 6, 8; 3 \to 3, 6; 4 \to 4,$
     $8; 5 \to 5; 6 \to 6; 7 \to 7; 8 \to 8$

2 Harry $\to$ Hazel, Robert $\to$ Andrea,
    Anthony $\to$ Pauline, Paul $\to$ Rita

4 (a) is 4 less than   **(b)** There is no 5 in set B.
    (c) There is no 6 in set A.   **(d)** $3 \to 6, 4 \to 7, 5 \to 8$
    (e) $3 \to 6, 4 \to 8, 5 \to 10$

Exercise 7B

1 (a) $4 \to 4$ **(b)** $2 \to 1; 4 \to 1; 6 \to 1, 4; 8$
    $\to 1, 4; 10 \to 1, 4, 9$
    (c) $2 \to 9; 10 \to 1$ **(d)** $2 \to 1; 8 \to 4$

2 **(a)** Mr Isaacs → Joey, Kara, Sam;
Mrs Isaacs → Joey, Kara, Sam;
Mr Lam → Yen, Chang, Chung;
Mrs Lam → Yen, Chang, Chung
(b) Joey → Mr Isaacs, Mrs Isaacs;
Kara → Mr Isaacs, Mrs Isaacs;
Sam → Mr Isaacs, Mrs Isaacs;
Yen → Mr Lam, Mrs Lam;
Chang → Mr Lam, Mrs Lam;
Chung → Mr Lam, Mrs Lam

3 **(a)** 1 → 2, 2 → 4, 3 → 6, 4 → 8, 5 → 10   **(b)** 4 → 2
(c) 1 → all; 2 → 4, 6, 8, 10; 3 → 4, 6, 8, 10; 4 → 6, 8, 10;
5 → 6, 8, 10
(d) 3 → 2; 4 → 2; 5 → 2, 4

4 **(a)** 1 → all; 2 → all; 3 → 6, 12; 4 → 4, 12; 6 → 6, 12
(b) 2 → 2; 4 → 2, 4; 6 → 6   **(c)** 2 → 4, 4 → 4, 6 → 6
(d) 1 → all; 2 → 4, 6, 12; 3 → all; 4 → 2, 6, 12; 6 → 2, 4, 12

5 **(a)** 4
(b) They are brothers and their sister is Viola
(c) They are sisters and their brother is Hal
(d) She has no brothers or sisters
(e) They are brothers and their sisters are Primula and Pria.

Exercise 7C

1 **(a)** add 3   **(b)** divide by 2
(c) multiply by 2 then add 1
(d) multiply by 3 then subtract 1

2 **(a)** 1 → 4, 2 → 5, 3 → 6, 4 → 7, 5 → 8, 6 → 9, 7 → 10
(b) 1 → 1, 2 → 3, 3 → 5, 4 → 7, 5 → 9
(c) 1 → 7, 2 → 4, 3 → 1, 4 → 8, 5 → 5,
6 → 2, 7 → 9, 8 → 6, 9 → 3
(d) 1 → 5, 2 → 1, 3 → 6, 4 → 2, 5 → 7,
6 → 3, 7 → 8, 8 → 4, 9 → 9, 10 → 5

3 **(a)** subtract from 10
(b) multiply by 4 then subtract 7
(c) multiply by 3 and write down the last digit of the answer

Exercise 7D

1 **(a), (c)** and **(e)**

2 **(d)** They are all mappings.

3 **(c)** There would be no arrows from 1, 3 or 5.

4 **(a)** multiply by 2   **(b)** 12

Exercise 7E

1 **(a)** 1 → 3, 2 → 5, 3 → 7, 4 → 9, 5 → 11
(b) 1 → 1, 2 → 3, 3 → 5, 4 → 7, 5 → 9
(c) 1 → 4, 2 → 6, 3 → 8, 4 → 10, 5 → 12

2 **(a)** subtract 2 from input
(b) multiply input by 3 then add 1
(c) square input
(d) subtract input from 9

Exercise 7F

1 **(a)** one-to-one **(b)** many-to-many **(c)** one-to-many
(d) many-to-one

2 **(a) (i)**   **(ii)**

(iii)
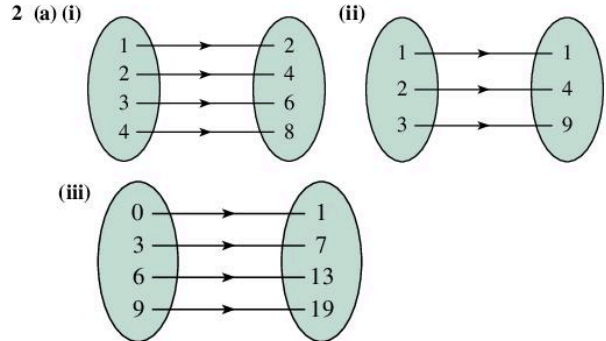
(b) all are functions

3 **(a)**

(b) yes

4 **(a)**
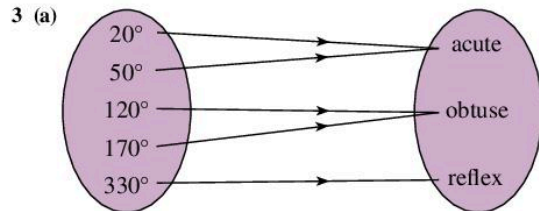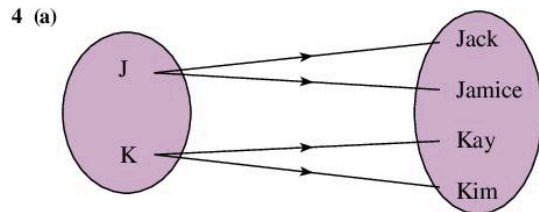
(b) It's a one-to-many map.

5 **(a)**
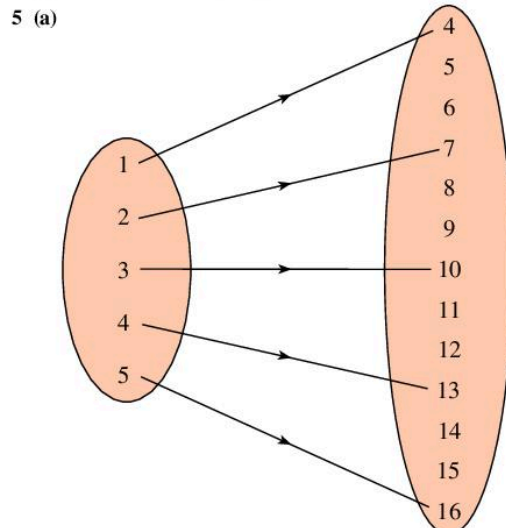
(b) Map is one-to-one and so is well defined.

Exercise 7G

1 (a) $x \to 2x$ (b) $x \to x+2$ (c) $x \to x-2$ (d) $x \to \frac{1}{2}x$

2 (a) $1 \to 2, 2 \to 4, 3 \to 6, 4 \to 8, 5 \to 10$
  (b) $1 \to 3, 2 \to 4, 3 \to 5, 4 \to 6, 5 \to 7$
  (c) $1 \to 0, 2 \to 1, 3 \to 2, 4 \to 3, 5 \to 4$
  (d) $1 \to 5, 2 \to 8, 3 \to 11, 4 \to 14, 5 \to 17$
  (e) $1 \to 2, 2 \to 7, 3 \to 12, 4 \to 17, 5 \to 22$

Exercise 7H

1 (a) $(a, 1), (b, 2), (c, 3)$ (b) $(a, 2), (b, 3), (c, 1)$
  (c) $(a, 1), (b, 1), (c, 1)$ (d) $(a, 1), (b, 1), (b, 2), (b, 3), (c, 3)$

2 (a) $a \to 1, b \to 1, c \to 2$ (b) $a \to 1, a \to 2, b \to 2, c \to 1$
  (c) $a \to 2, b \to 1, b \to 2, c \to 1, c \to 2$

3 (b) (March, 31), (April, 30), (May, 31), (June, 30), (July, 31)

4 (a) starts with the letter   (b) is the number of days in
  (c) has this number of sides

5 (a) $\{(1, 5), (2, 7), (3, 9), (6, 15), (7, 17), (10, 23),$
    $(12, 27), (15, 33)\}$

6 $\{(1, 0), (2, 4), (3, 8), (4, 12), (5, 16)\}$

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$a = 6, b = 10, c = 15, d = 21$

Exercise 7I

1 $\{(1, 3), (4, 6), (5, 4), (2, 2), (6, 1), (3, 5)\}$

3 (a) F = $\{1, 2, 3, 5, 6\}$, S = $\{1, 2, 3, 4\}$

4 (a) (Deo, December), (Jojo, June), (Slim, November),
    (Tony, May), (Ali, March), (Anab, April),
    (Singh, September), (Bob, January)
  (b) Tony $\to$ May

5 (b) (i) 1 to 1 map; (ii) 1-1 map; (iv) many to 1 map (v) 1 to 1 map;
    (iii) and (vi) are not functions 1 to many maps

6 (a) (2, 1), (3, 4), (4, 2), (4, 4), (6, 1), (6, 5)

Exercise 7J

1 (a) (i) $(1, 3), (2, 4), (3, 5), (4, 6), (5, 7), (6, 8)$ (ii) add 2
  (b) (i) $(3, 1), (4, 2), (5, 3), (6, 4), (7, 5), (8, 6)$ (ii) subtract 2
  (c) (i) $(-2, -4), (-1, -2), (0, 0), (1, 2), (2, 4)$ (ii) multiply by 2
  (d) (i) $(-4, -2), (-2, -1), (0, 0), (2, 1), (4, 2)$ (ii) half

Exercise 7K

1 (c) $(1, 3), (2, 4), (3, 5), (4, 6), (5, 7)$

2 (a) $(1, 2), (2, 4), (3, 6), (4, 8), (5, 10)$

3 Coordinates of points are:
  (a) $(^-3, ^-11), (^-2, -8), (^-1, ^-5), (0, ^-2), (1, 1), (2, 4), (3, 7)$
  (b) $(^-3, ^-3), (^-2, ^-1), (^-1, 1), (0, 3), (1, 5), (2, 7)$
  (c) $(^-3, ^-3), (^-2, ^-2), (^-1, ^-1), (0, 0), (1, 1), (2, 2)$

4 (a) $(1, 6), (2, 8), (3, 10), (4, 12), (5, 14), (6, 16)$

Exercise 7L

1 (a) (1, 5), (2, 3), (2, 5), (3, 3), (4, 4), (6, 2)   (b) 8
  (c) 5   (d) 2

2 (a) 34 °C   (b) Sunday and Thursday   (c) Tuesday
  (d) 30 °C

3 (a) $3   (b) 1000 g   (c) one 1000 g box   (d) $10

4 (a) Tony   (b) Bob and George   (c) Bob

5 (a) 80 cm   (b) 95 cm   (c) 16th   (d) 150 cm
  (e) 7 years old

Exercise 7M

1 (a) $6   (b) 4.7 m   (c) $6.75   (d) 6 m
  (e) Wire costs $1.50 per metre.

2 (a) 27 °C   (b) 34 °C   (c) 9 am   (d) 29°
  (e) 25 °C   (f) 7 am and 6 pm

3 (a) 1 pm   (b) 5 km   (c) he was stationary   (d) 5.30 pm
  (e) 1 pm and 3 pm

Exercise 7N

1 (b) (1, 2), (2, 4), (3, 6), (4, 8), (5, 10)   (d) $y = 2x$

2 (a) (1, 3), (2, 6), (3, 9), (4, 12), (5, 15); $y = 3x$
  (b) (1, 3), (2, 4), (3, 5), (4, 6), (5, 7); $y = x + 2$
  (c) (1, $^-2$), (2, $^-1$), (3, 0), (4, 1), (5, 2); $y = x - 3$
  (d) (1, 4), (2, 6), (3, 8), (4, 10), (5, 12); $y = 2x + 2$

3 (a) $\times 3$ (b) $+4$ (c) -6 (d) $\times 2, +4$ (e) $\times 3, -9$
  (f) $\div 4, +7$ (g) $+2, \div 5$ (h) $\times 3, +2, \div 4$

4 (a)

x	0	1	2	3	4	5
y	3	5	7	9	11	13

5 (a)

x	0	1	2	3	4	5
y	4	5	6	7	8	9

(b)

x	0	1	2	3	4	5
y	4	7	10	13	16	19

(c)

x	0	1	2	3	4	5
y	$^-2$	1	4	7	10	13

(d)

x	0	1	2	3	4	5
y	$^-5$	$^-3$	$^-1$	1	3	5

(e)

x	0	1	2	3	4	5
y	$^-6$	$^-1$	4	9	14	19

6 (a)

x	$^-5$	$^-4$	$^-3$	$^-2$	$^-1$	0	1	2	3	4	5
y	$^-20$	$^-15$	$^-10$	$^-5$	0	5	10	15	20	25	30

(b)

x	$^-5$	$^-4$	$^-3$	$^-2$	$^-1$	0	1	2	3	4	5
y	$^-8$	$^-5$	$^-2$	1	4	7	10	13	16	19	22

(c)

x	$^-5$	$^-4$	$^-3$	$^-2$	$^-1$	0	1	2	3	4	5
y	$^-7$	$^-5$	$^-3$	$^-1$	1	3	5	7	9	11	13

7 (d) $(0, 5)$; $(0, 7)$; $(0, 3)$
  (e) y-intercept has same value as constant term
  (f) $y = 5x + 5$

8 (c) $y = 4x$

Exercise 7O

1 (b) (i) 1   **(ii)** 2 **(iii)** 3
  (c) (i) $(0, 1)$ **(ii)** $(0, 1)$ **(iii)** $(0, 1)$

2 (b) (i) 3   **(ii)** 3   **(iii)** 3   **(c)** They are the same.
  (d) (i) $(0, 0)$ **(ii)** $(0, 1)$ **(iii)** $(0, 2)$
  (e) they are the same as the constant term in the equation

Exercise 7P

1 (b) 2, 2, 2   **(c) (i)** $(0, ^-3)$ **(ii)** $(0, ^-1)$ **(iii)** $(0, 3)$

2 (a) (i) 4   **(ii)** 3   **(iii)** 3   **(iv)** $\frac{1}{2}$
  (b) (i) $^-1$ **(ii)** $^-1$ **(iii)** $^-5$ **(iv)** -2

3 (a) $y = 2x + 4$ **(b)** $y = 3x - 2$ **(c)** $y = x + 5$
  (d) $y = \frac{1}{4}x + 6$

4 (c) $^-2$ **(d)** $y = 1$ **(e)** $y = x - 2$

5 (a) $1, y = x$ **(b)** $\frac{1}{2}, y = \frac{1}{2}x + \frac{1}{2}$ **(c)** $-1, y = -x + 2$
  (d) $-2, y = -2x + 1$

6 (b) 3   **(c)** 3 km per hour   **(e)** yes

Exercise 7Q

1 (a) (i) $(^-2, 2), (0, 2), (2, 2)$
      (ii) $(^-2, ^-1), (0, ^-1), (2, ^-1)$
      (iii) $(^-2, 6), (0, 6), (2, 6)$ **(c) (i)** 0   **(ii)** 0   **(iii)** 0

2 $x = 3$

4 A: $y = 4$, B: $x = 2$, C: $y = -2$, D: $x = 4.5$, E: $y = 1.5$

5 (a) B   **(b)** E   **(c)** C   **(d)** D   **(e)** A

6 (a) $(0, 4)$ **(b)** $(1, 3)$ **(c)** $(3, 5)$ **(d)** $(3, 2)$
  (e) They satisfy both equations.

7 (c) $(-3, -2)$

8 (a) $(1, -1)$ **(b)** $(2, 4)$

9 (a) $y = x - 1$ **(b)** $y = 2x + 6$ **(c)** $y = \frac{1}{2}x - 1$
  (d) $y = 4x - 8$

10 (a) $y = 2x$ **(b)** $y = 2x + 1$ **(c)** $y = 2x - 1$
  (d) $y = x - 4$ **(e)** $y = 2x - 2$

Exercise 7R

1 (a)

(b)

(c)

(d)

(e)

2 (a)

(b)

(c)

(d)
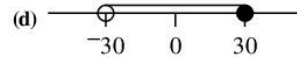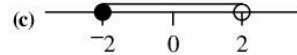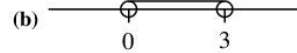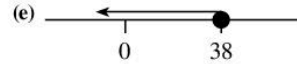
3 (a) (i) $15 < T < 20\,°C$ **(ii)** $41 \leqslant P \leqslant 60$
  (iii) $S < 60$ kph   **(iv)** $A \geqslant 14$
  (v) $H \geqslant 170$ cm   **(vi)** $18 \leqslant A < 35$
  (vii) $P < \$100$ **(viii)** GPA $\geqslant 2.8$

Exercise 7S

1 (a)   **(b)**

(c)   **(d)**

(e)   **(f)**

2 (a) $x \leqslant 3$ **(b)** $y \geqslant 2$ **(c)** $2 \leqslant x \leqslant 5$ **(d)** $^-2 \leqslant y \leqslant 3$

3 (a)   **(b)**

(c)   **(d)**
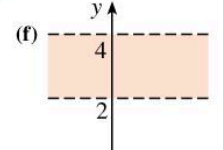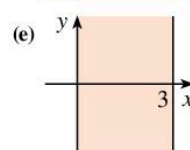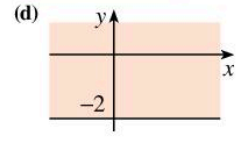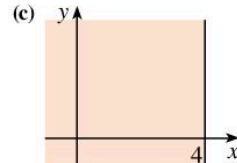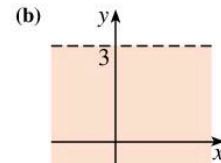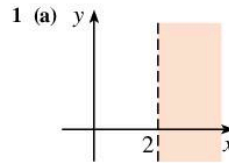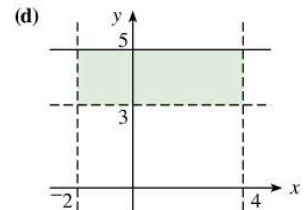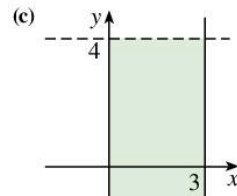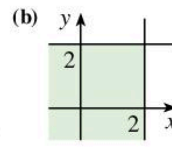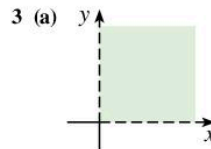
4 (a)–(c)
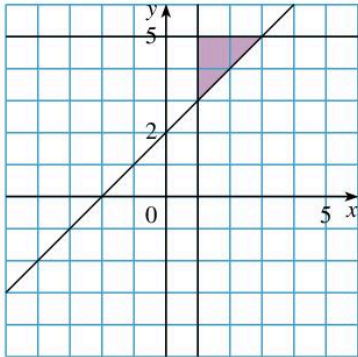
(d) a right-angled triangle.

5 (a)–(b)
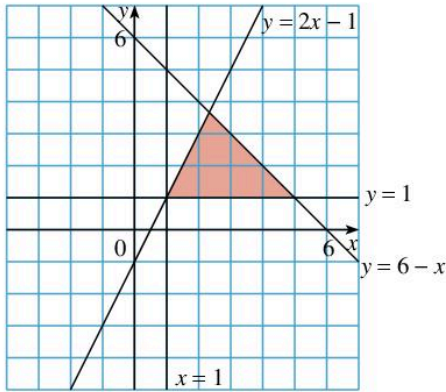
Exercise 7T – mixed questions

1 (a) (i) $3 \to 3; 5 \to 5; 10 \to 2; 10 \to 5; 12 \to 2; 12 \to 3$
(ii) $1 \to 5; 2 \to 6; 3 \to 7; 4 \to 8; 5 \to 9$
(b) $1 \to 3; 2 \to 6; 3 \to 9; 4 \to 12$

2 (a) multiply by 3 then add 2
(b) multiply by 3 then subtract 2   **(c)** divide by 2 then add 1

3 (a) $x \to \frac{1}{2}x$ **(b)** $x \to 3x$ **(c)** $x \to 2x + 1$

4 (a) $0 \to 3; 1 \to 4; 2 \to 5; 3 \to 6; 4 \to 7$
(b) $1 \to 1; 2 \to 3, 3 \to 5; 4 \to 7$
(c) $0 \to 9; 1 \to 8; 2 \to 7; 3 \to 6; 4 \to 5$

5 (a) (i) multiply by 3 then subtract 1
(ii) 8, 11, 17   **(b) (i)** divide by 4
(ii) Left: 20 Right: 3, 7

6 (a) (Lily, 3), (Sue, 1), (Joe, 5), (Mark, 3), (Abe, 4), (Sheila, 2)
(b) Lily and Mark
(c) Sue   **(d)** Joe   **(e)** Sue
(f) Sheila   **(g)** 3   **(h)** 8

7 (a) (1, 1)   **(b)** (2, 4)   **(c)** (2, 1)   **(d)** (4, 5)   **(e)** (6, 6)
(f) (3, 2)   **(g)** (7, 5)

8 Points shaped as a Christmas tree.

9 A (4, 5), B (6, 12), C (7, 20), D (8, 2), E (10, 11)

10 (a) (1, 5), (3, 11), (4, 14), (7, 23), (10, 32)
(b) (2, 9), (3, 8), (5, 6), (7, 4), (11, 0)
(c) (5, 8), (4, 6), (3, 4), (2, 2), (1, 0)

11 (a) (i) $x \to \frac{1}{3}x$ **(ii)** $x \to x + 1$ **(iii)** $x \to 3x + 1$
(b) (i) $y = \frac{1}{3}x$ **(ii)** $y = x + 1$ **(iii)** $y = 3x + 1$

12 (a) (0, 2)   **(b)** 2

13 (b) (i) (0, 2)   **(ii)** (0, ‾1)   **(iii)** (0, 3)   **(iv)** (0, 4)
(c) (i) 3   **(ii)** 3   **(iii)** 2   **(iv)** 1

14
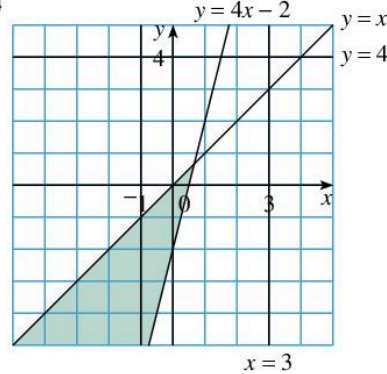
Exercise 7

1 (a) $1 \to 2, 2 \to 4, 3 \to 6, 4 \to 8$ **(b)** $2 \to 2, 3 \to 3, 4 \to 4$
(c) $1 \to 2, 3, 4, 5, 6, 7, 8; 2 \to 2, 4, 6, 8; 3 \to 3, 6; 4 \to 4, 8$
(d) $1 \to 5, 2 \to 4, 3 \to 3, 4 \to 2$ **(e)** $3 \to 2, 4 \to 2, 3$
(f) $1 \to 2, 3, 4, 5, 6, 7, 8; 2 \to 3, 4, 5, 6, 7, 8; 3 \to 4, 5, 6, 7, 8;$
$4 \to 5, 6, 7, 8$

2 (a) $1 \to 4, 2 \to 5, 3 \to 6, 4 \to 7, 5 \to 8$
(b) $1 \to 0, 2 \to 1, 3 \to 2, 4 \to 3, 5 \to 4$
(c) $1 \to 2, 2 \to 4, 3 \to 6, 4 \to 8, 5 \to 10$
(d) $1 \to 1, 2 \to 3, 3 \to 5, 4 \to 7, 5 \to 9$

3 (a) $1 \to 3, 2 \to 4, 3 \to 5, 4 \to 6$
(b) $1 \to 2, 2 \to 4, 3 \to 6, 4 \to 8$
(c) $1 \to 1, 2 \to 3, 3 \to 5, 4 \to 7$
(d) $1 \to 4, 2 \to 7, 3 \to 10, 4 \to 13$
(e) $1 \to 5, 2 \to 10, 3 \to 15, 4 \to 20$
(f) $1 \to 1, 2 \to 5, 3 \to 9, 4 \to 13$
(g) $1 \to 7, 2 \to 11, 3 \to 15, 4 \to 19$
(h) $1 \to 1, 2 \to 4, 3 \to 7, 4 \to 10$

4 (a) (1, 3), (2, 4), (3, 5), (4, 6), (5, 7)
(b) (1, 2), (2, 4), (3, 6), (4, 8), (5, 10)
(c) (1, 1), (2, 3), (3, 5), (4, 7), (5, 9)
(d) (1, 4), (2, 7), (3, 10), (4, 13), (5, 16)
(e) (1, 1), (2, 6), (3, 11), (4, 16), (5, 21)
(f) (1, 7), (2, 10), (3, 13), (4, 16), (5, 19)
(g) (1, 5), (2, 4), (3, 3), (4, 2), (5, 1)
(h) (1, 9), (2, 7), (3, 5), (4, 3), (5, 1)

Check points have been plotted correctly for graphs and ensure the points have not been joined by straight lines

5 (a), (b) Check the following points have been plotted:
(1, 1), (2, 4), (3, 7), (4, 10), (5, 13)   **(c)** $5\frac{1}{2}$ **(d)** 3   **(e)** –2

6 (a) \$5   **(b)** 200 g
(c) 800 g tin as this costs \$1 less than four 200 g tins

7 (a) 5 cm
(b) 21 days

Check out

1

2 (a)

(b)

(c)
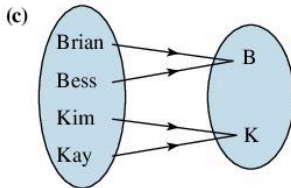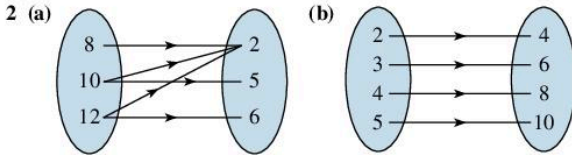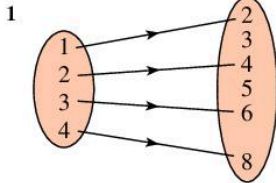
(b) and (c) are functions.

3 (a) (1, 2), (2, 4), (3, 6), (4, 8) **(b)** (0, ⁻1), (1, 2), (2, 5), (3, 8)

4 (a) 35 kg **(b)** Wednesday **(c)** 150 kg

5 (a)

x	⁻1	0	1	2	3	4
y	⁻1	1	3	5	7	9

(b)

x	⁻1	0	1	2	3	4
y	⁻5	⁻2	1	4	7	10

6 (a) (i) 2, 5 **(ii)** $\frac{1}{2}$, ⁻3 **(iii)** 3, ⁻4 **(iv)** 4, 1

(b) (i) $y = 2x - 1$ **(ii)** $y = 3x + 2$

7 (a)

(b)

(c)
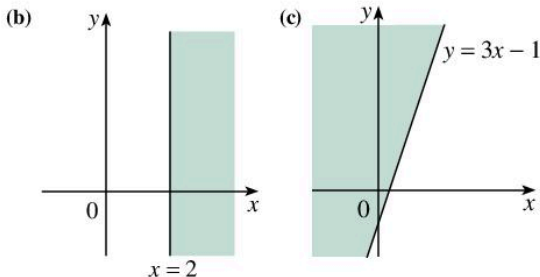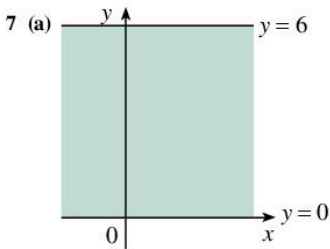
8 Transformations

Check in

1 (a) 1 **(b)** 2 **(c)** 4

2 W (⁻2, 3), X(2, 3), Y (⁻2, ⁻1), Z (1, ⁻2)

Exercise 8B

1 (b) (9, 2)

2 (a) P_1(⁻1, 1), Q_1(⁻4, 3), R_1(⁻6, 2)
   (d) P_2(1, ⁻1), Q_2(4, ⁻3), R_2(6, ⁻2)
   (e) P_3(⁻1, ⁻1), Q_3(⁻4, ⁻3), R_3(⁻6, ⁻2)
   (f) No. A mirror line cannot be found.
   (g) $P_1Q_1R_1$ and $P_2Q_2R_2$

3 (a) Octagonal star
   (b) (4, 7), (5, 5), (7, 4), (5, 3), (4, 1), (3, 3), (1, 4), (3, 5)

4 (e) H **(f)** 2

5 (b) (0, 8), (1, 7), (2, 6), (3, 5), (4, 4), (5, 3), (6, 2), (7, 1) or
   (0, 8) etc.

6 (b) (⁻4, ⁻4), (⁻3, ⁻3), (⁻2, ⁻2), (⁻1, ⁻1), (0, 0), (1, 1),
   (2, 2), (3, 3) or (4, 4)

7 (b) (8, 10) **(d)** A′(1, 7), B′(4, 5), C′(4, 1), D′(1, 3)

10 (a) $(a, ⁻b)$ **(b)** $(⁻p, q)$

Exercise 8C

1 (a) 3 right, 0 up **(b)** 3 right, 1 down **(c)** 6 right, 1 down
   (d) 6 left, 1 up **(e)** 2 right, 1 down **(f)** 2 right, 4 up
   (g) 4 right, 2 up **(h)** 6 right, 0 up **(i)** 6 left, 7 up
   (j) 0 right, 6 up

Exercise 8D

1 (a) $\begin{pmatrix} 2 \\ 2 \end{pmatrix}$ **(b)** $\begin{pmatrix} 3 \\ 1 \end{pmatrix}$ **(c)** $\begin{pmatrix} -2 \\ -2 \end{pmatrix}$ **(d)** $\begin{pmatrix} -1 \\ 3 \end{pmatrix}$ **(e)** $\begin{pmatrix} 3 \\ 0 \end{pmatrix}$

   (f) $\begin{pmatrix} 0 \\ -3 \end{pmatrix}$ **(g)** $\begin{pmatrix} -1 \\ -3 \end{pmatrix}$ **(h)** $\begin{pmatrix} -2 \\ 1 \end{pmatrix}$ **(i)** $\begin{pmatrix} -1 \\ -2 \end{pmatrix}$ **(j)** $\begin{pmatrix} 4 \\ 3 \end{pmatrix}$

3 (a) $\begin{pmatrix} 5 \\ 0 \end{pmatrix}$ **(b)** $\begin{pmatrix} -3 \\ -3 \end{pmatrix}$ **(c)** $\begin{pmatrix} 2 \\ -3 \end{pmatrix}$ **(d)** $\begin{pmatrix} 4 \\ 0 \end{pmatrix}$ **(e)** $\begin{pmatrix} -6 \\ 3 \end{pmatrix}$

   (f) $\begin{pmatrix} 6 \\ -3 \end{pmatrix}$ **(g)** $\begin{pmatrix} -2 \\ -3 \end{pmatrix}$ **(h)** $\begin{pmatrix} 3 \\ 0 \end{pmatrix}$ **(i)** $\begin{pmatrix} -2 \\ 6 \end{pmatrix}$ **(j)** $\begin{pmatrix} 5 \\ 6 \end{pmatrix}$

Exercise 8E

1 (a) $\begin{pmatrix} 2 \\ 1 \end{pmatrix}$ **(b)** $\begin{pmatrix} 1 \\ -3 \end{pmatrix}$ **(c)** $\begin{pmatrix} 4 \\ -1 \end{pmatrix}$ **(d)** $\begin{pmatrix} 4 \\ -2 \end{pmatrix}$

2 (a) (3, 2), (1, 3), (3, 4) **(b)** (2, 5), (2, 4), (3, 2)
   (c) (0, 3), (3, 1), (1, 2) **(d)** (2, 5), (0, 4), (⁻2, 1)
   (e) (2, ⁻1), (3, 1), (2, 4)

3 (a) 3 left, 3 down **(b)** 6 right, 2 up **(c)** 3 right, 1 down
   (d) 3 right, 3 up **(e)** 6 left, 2 down **(f)** 3 left, 1 up

4 (a) 1 right, 1 up **(b)** 2 right, 1 up **(c)** 2 left, 1 down

Exercise 8G

1 (a) 2 **(b)** 2 **(c)** 4 **(d)** 3 **(e)** 6 **(f)** 5
   (g) 6 **(h)** 3

2 (a) 3 **(b)** 5 **(c)** 2 **(d)** 4 **(e)** 3

3 H, I, N, O, S, X and Z

4 (a) 4   **(b)** 2   **(e)** 6   **(f)** 2   **(g)** 3

5 (a) (iv)   **(c)** (i), (ii), (iii), (v), (vi), (vii), (viii), (ix), (x)

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(a) (i) no   **(ii)** Yes, an equilateral triangle.

(b) (i) rectangle, parallelogram, rhombus   **(ii)** none
   (iii) square

Exercise 8H–mixed questions

1 (a) kite or triangle

2 (a) rectangle   **(b)** rectangle or square
   (c) isosceles triangle or square   **(d)** rhombus or kite

4 kite, square or rhombus

5 Yes. Yes.

6 (c) rhombus or square   **(d)** Yes

Exercise 8

1 (a) $A' = (^-4, ^-3)$, $B' = (^-3, 3)$, $C' = (^-3, ^-5)$
   $Q' = (^-5, 3)$, $D' = (^-3, 3)$, $E' = (^-3, 1)$
   $F' = (2, ^-2)$, $G' = (3, ^-4)$, $H' = (2, ^-5)$
   $I' = (2, 4)$, $J' = (4, 2)$, $K' = (3, 1)$
   $L' = (5, 0)$, $M' = (6, 1)$, $N' = (7, ^-3)$

   (b) $A' = (4, 3)$, $B' = (3, 3)$, $C' = (3, 5)$
   $Q' = (5, ^-3)$, $D' = (3, ^-3)$, $E' = (3, ^-1)$
   $F' = (^-2, 2)$, $G' = (^-3, 4)$, $H' = (^-2, 5)$
   $I' = (^-2, ^-4)$, $J' = (^-4, ^-2)$, $K' = (^-3, ^-1)$
   $L' = (^-5, 0)$, $M' = (^-6, ^-1)$, $N' = (^-7, 3)$

2 (a) $A' = (^-3, ^-2)$, $B' = (^-5, ^-3)$, $C' = (^-4, ^-5)$,
   $D' = (^-2, ^-5)$
   (b) $A' = (3, 2)$, $B' = (5, 3)$, $C' = (4, 5)$, $D' = (2, 5)$
   (c) $A' = (^-3, 0)$, $B' = (^-5, ^-1)$, $C' = (^-4, ^-3)$, $D' = (^-2, ^-3)$
   (d) $A' = (^-1, 2)$, $B' = (1, 3)$, $C' = (0, 5)$, $D' = (^-2, 5)$
   (e) $A' = (5, 2)$, $B' = (7, 3)$, $C' = (6, 5)$, $D' = (4, 5)$
   (f) $A' = (^-3, ^-6)$, $B' = (^-5, ^-7)$, $C' = (^-4, ^-9)$,
   $D' = (^-2, ^-9)$

3 (a) $\begin{pmatrix} 5 \\ -1 \end{pmatrix}$ **(b)** $\begin{pmatrix} -5 \\ 1 \end{pmatrix}$ **(c)** $\begin{pmatrix} 4 \\ 3 \end{pmatrix}$ **(d)** $\begin{pmatrix} 1 \\ -4 \end{pmatrix}$

4 (a) 1   **(b)** 2   **(c)** 1   **(d)** 2   **(e)** 6   **(f)** 5
   (g) 2   **(h)** 6

Check out

2 (c) $(3, ^-2)$, $(4, ^-5)$, $(6, ^-3)$

3 (c) $A''(4, ^-1)$, $B'(0, 1)$, $C''(1, 2)$

4 (a) 3   **(b)** 4   **(c)** 4   **(d)** 3

Revision exercise 2

1 Annette by 7.01 cm   **2 (c)** 6.5 cm, 29 cm

3 (a) 40 cm²   **(b)** 138.9 cm²   **(c)** 26.66 cm²   **(d)** 38 cm²

4 (a) 78.9 cm   **(b)** 236.6 cm   **(c)** 314 cm   **(d)** 986.0 cm

5 (a) 1.59 cm   **(b)** 6.69 cm   **(c)** 40.76 cm   **(d)** 93.79 cm

6 (a) 2198 cm   **(b)** 25

7 (a) 12.56 cm², 14.28 cm   **(b)** 39.25 cm², 25.7 cm
   (c) 21.2 cm², 20.13 cm,   **(d)** 9.81 cm², 17.85 cm

8 (a) 160 m   **(b)** 25.48 m   **9 (b)** 13 cm   **(c)** 130 m

10 (a) 100 cm   **(b)** 8.5 cm   **(c)** 6800 cm², 106.25 cm²

11 (a) $^-1$ **(b)** $^-6$ **(c)** $^-2$ **(d)** $^-6$
   (e) $^-5$ **(f)** $^-7$ **(g)** $^-16$ **(h)** $^-22$

12 (a) 7   **(b)** 15   **(c)** 18   **(d)** 15
   (e) 24   **(f)** 14   **(g)** $^-2$ **(h)** $^-4$

13 (a) $^-8$ **(b)** $^-6$ **(c)** 6   **(d)** 24
   (e) 20   **(f)** $^-27$ **(g)** $^-42$ **(h)** 7

14 (a) $^-3$ **(b)** $^-2$ **(c)** 4   **(d)** $^-4\frac{1}{2}$
   (e) $^-8$ **(f)** $^-8$ **(g)** 6   **(h)** $^-6$

15 (a) 16   **(b)** $^-16$ **(c)** 10   **(d)** 5   **(e)** $^-5$
   (f) 2   **(g)** $^-1$ **(h)** 5   **(i)** $^-4$ **(j)** $^-9$

16 $^-10°C$ **17 (a)** No   **(b)** \$1.87

18 (a) 3.00 am   **b)** 12.00 midnight and 1.00 am
   (c) 3.00 am and 4.00 am   **(d)** 0°C

19 (a) 25   **(b)** 8   **(c)** 17; 2 hours 7 minutes,
   1 hour 50 minutes, 1 hour 42 minutes

20 (a) from bottom left: $(^-2, ^-5)$, $(^-1, ^-4)$, $(0, ^-3)$,
   $(1, ^-2)$, $(2, ^-1)$, $(3, 0)$, $(4, 1)$, $(5, 2)$

   (b) $x \rightarrow x^-3$
   (c) $x \rightarrow [^-3] \rightarrow x^-3$
   (d) $^-9$
   (e) $^-3$

21 (a) (i) add 7   **(ii)** multiply by 3 then add 2
   (b) (i) $23 \rightarrow 30$, $39 \rightarrow 46$ **(ii)** $7 \rightarrow 23$, $20 \rightarrow 62$

22 (a) (i) $1 \rightarrow 2$, $2 \rightarrow 4$, $3 \rightarrow 6$, $4 \rightarrow 8$
   (ii) $1 \rightarrow 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12$; $2 \rightarrow 2, 4, 6, 8,$
      $10, 12$; $3 \rightarrow 3, 6, 9, 12$; $4 \rightarrow 4, 8, 12$
   (iii) $1 \rightarrow 6$, $2 \rightarrow 7$, $3 \rightarrow 8$, $4 \rightarrow 9$
   (iv) $1 \rightarrow 3$, $2 \rightarrow 5$, $3 \rightarrow 7$, $4 \rightarrow 9$
   (b) (i) one-to-one   **(ii)** many-to-many
      (iii) one-to-one   **(iv)** one-to-one
   (c) (i), (iii) and (iv)

23 (a) $1 \rightarrow 10$, $2 \rightarrow 13$, $3 \rightarrow 16$, $4 \rightarrow 19$, $5 \rightarrow 22$
   (b) $1 \rightarrow 9$, $2 \rightarrow 12$, $3 \rightarrow 15$, $4 \rightarrow 18$, $5 \rightarrow 21$
   (c) $1 \rightarrow 0$, $2 \rightarrow 3$, $3 \rightarrow 8$, $4 \rightarrow 15$, $5 \rightarrow 24$

24 (a)

Input	→	Multiply by 4	→	Subtract 1	→	Output
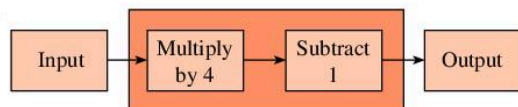
   (b) $(1, 3)$, $(2, 7)$, $(3, 11)$, $(4, 15)$, $(5, 19)$

25 (a) $(0, 0)$, $(1, 1)$, $(2, 4)$, $(3, 9)$, $(4, 16)$, $(5, 25)$, $(6, 36)$
   (b) $x \rightarrow x^2$

26 (c) (i) 5, 3   **(ii)** 7, $^-6$ **(iii)** $^-1, 7, (1, ^-4)$

27 (a) 3 pm   **(b)** 16 km   **(c)** between 11 am and 12 noon
   (d) between 1 pm and 3 pm, any good reason

30 (c) $(4, 11)$

31 (a) 2   **(b)** 1   **(c)** 2

33 hexagon, 2 lines of symmetry

34 $(^-2, 2)$, $(0, 3)$, $(1, 3)$, $(3, 2)$, $(1, 1)$, $(0, 1)$

36 (b) $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$

37 (a) X′(1, 2), Y′(3, −1), Z′(4, 2)　　**(b)** $\begin{pmatrix} -3 \\ -1 \end{pmatrix}$

38 (a) Rotation about X, 90° clockwise
   (b) Rotation about X, 180°

39 (a) vertices at: (5, 5), (7, 5), (7, 2)
   (b) vertices at: (1, 2), (3, ⁻1), (1, ⁻1)
   (c) vertices at: (5, 3), (5, 1), (2, 1)

40 (a) vertices at: (4, 4), (6, 5), (7, 4), (7, 1)
   (b) vertices at: (1, 3), (1, 0), (2, ⁻1), (4, 0)
   (c) vertices at: (1, 1), (4, 1), (5, 2), (4, 4)

41 (a) (3, 5), (6, 3), (3, 3)　　**(b)** (1, 2), (4, 2), (4, 0)
   (c) (3, 5), (3, 2), (1, 2)

42 (a) (1, 8), (1, 5), (⁻1, 5)　　**(b)** (1, 5), (4, 5), (1, 7)

Mixed questions 2

1 A	**2** C	**3** B	**4** C	**5** C	**6** B	**7** C	**8** C
9 D	**10** C	**11** A	**12** D	**13** C	**14** A	**15** A	
16 A	**17** A	**18** A	**19** A	**20** D	**21** B	**22** A	
23 D	**24** C	**25** C	**26** D	**27** D	**28** C	**29** A	
30 D	**31** C	**32** C	**33** D	**34** A	**35** D	**36** D	
37 A	**38** D						

9 Statistics

Check in

1 (a) 5　　**(b)** 20

2 (a) 10　　**(b)** 10　　**(c)** 15

Exercise 9A

2

Mark	Frequency
1	1
2	0
3	2
4	3
5	5
6	7
7	8
8	6
9	3
10	1

3 (b)

Vowel	Frequency
a	8
e	14
i	7
o	6
u	3

Exercise 9B

1 (b) Football　　**(c)** 76

2 (a) 40　　**(c)** Maths

3 (a) 70　　**(b)** 150　　**(c)** 65

4 (a) January, February, March, …
   (a) 5 cm　　**(b)** February　　**(c)** June to October

Exercise 9C

1 (a) 35　　**(b)** 84

2 (c)

60 to 64	65 to 69	70 to 74	75 to 79	80 to 84
6	9	6	4	4

2 (e) Yes

3 (a) 50　　**(b)** 6　　**(c)** 14　　**(d)** You can't tell　　**(e)** 9

4 (a) 41　　**(c)** 7　　**(d)** 3　　**(e)** 2　　**(f)** no
   (g) no　　**(h)** none

6 (a) 10　　**(b)** 4

Exercise 9D

1 (a) 1°　　**(b)** 150°, 90°, 20°, 100°

2 (a) 40　　**(b)** 9°　　**(c)** 126°, 81°, 45°, 36°, 54°, 18°

3 Tarrus Riley 120°, Mr Vegas 36°, Busy Signal 24°,
   Vybz Kartel 96°, Machel Montano 84°

4 The Stars World 90°, Cricket Reporter 72°, News Review
   54°, Caribbean Focus 24°, You and Me 120°

5 Management 15°, Services 45°, Clerical 54°, Skilled Labour
   108°, Teaching 18°, Unemployed 75°, Agricultural 45°

6 Sectors: 9°, 9°, 27°, 36°, 54°, 126°, 72°, 27°

7 (a) 0.9°　　**(b)** 108°, 45°, 27°, 36°, 90°, 54°

Exercise 9E

1 (a) 2°　　**(b)** 60　　**(c)** 60, 30, 30, 15, 45

2 (a) 9°　　**(b)** 5　　**(c)** 15 New National, 4 Independent
   Labour, 14 People's Democratic, 2 Don't Know

3 (a) 2°　　**(b)** bus 44, car 21, walk 110　　**(c)** 180

4 (a) no　　**(b)** no　　**(c)** 60°　　**(d)** 10°
   (e) clockwise from English: 50°, 110°, 50°, 50°, 40°, 60°
   (f) clockwise from English: 5, 11, 5, 5, 4, 6　　**(g)** 36

5 (a) $\frac{1}{3}$　　**(b)** $\frac{1}{14}$
   (c) clockwise from Food: 100°, 118°, 34°, 25°, 48°, 35°
   (d) no　　**(e)** clockwise from Rent: $236, $68, $50, $96, $70
   (f) $720　　**(g) (i)** $102　　**(ii)** $144

Exercise 9F

3 (a) Scales are different, upright axis in Graph B
   does not start at 0.　　**(b)** Graph B
   (c) No, they are little different from other months.

4 (d) (i) 24.5°C　　**(ii)** 27.5°C　　**(e)** 11 am and 2.40 pm

6 (a) Graph B, it appears to show a dramatic decrease in robberies.   **(b)** Graph A, bars begin from zero.
(c) Not really. Robberies have declined a little but not 'steeply'.

7 (a) Not really. Unemployment has risen over last year after declining in previous years.

Exercise 9H

1 (a) 1   **(b)** 8   **(c)** 4

2 (a) (i) 2   **(ii)** 1   **(b)** 7   **(c)** 7

3 (b) 6   **(c)** 0   **(d)** 0

4 (a) 238   **(b)** Orange-kool   **(c)** Orange-kool
(d) Orange-kool, it is more popular
(e) Rumba, it is least popular

6 (a) $5\frac{1}{2}$ **(c)** no   **(d)** $5\frac{1}{2}$ **(e)** 8   **(f)** yes

Exercise 9I

1 (a) 4 kg, 4 kg, 6 kg, 8 kg, 12 kg   **(b)** 6 kg   **(c)** 6 kg
(d) 4 kg

2 (a) yes   **(b)** 172 cm   **(c)** no

3 (a) 8, 8   **(b)** 17, 16   **(c)** 6, 6   **(d)** 9, 9   **(e)** 16, 13

4 (a) 14   **(b)** 9   **(c)** 8   **(d)** 7.5   **(e)** 19.5   **(f)** 111

5 (a) 29
(b) 2, 2, 14, 14, 14, 14, 14, 14, 14, 14, 14, 14, 14, 13, 13, 13, 13, 13, 13, 13, 13, 13, 13, 13, 13, 16, 16
(c) 15

Exercise 9J

1 (a) on average   **(b)** Total bags of rice ÷number of fields
(c) mean

2 7, 4, 8, 10

3 (a) 18   **(b)** 21   **(c)** 7   **(d)** 94   **(e)** 112
(f) 5.5   **(g)** 32.5

4 (a) A 45, B 70   **(b)** No, there are more boys in group B.
(c) 15   **(d)** 14   **(e)** A

5 (a) 33   **(b)** 83   **6** 6   **7 (a)** 280 years   **(b)** 14 years

8 (a) $28   **(b)** $2.80

9 (a) $114   **(b)** $3.80

10 (a) 153 kg   **(b)** 53.5 kg

11 (a) mean   **(b)** mode   **(c)** mode   **(d)** mode

Exercise 9K – mixed questions

1 (a) 60   **(b)** 6°   **(c)** sectors: 90°, 72°, 30°, 54°, 114°

2 clockwise from Raw Materials: $135 000, $37 500, $45 000, $52 500, $270 000

4 (a) 96   **(b)** 31   **(c)** 27   **(d)** 17   **5** 2.7

6 (a) $135 000 **(b)** $37 500

7 (a) 205.6 cm   **(b)** 50.1 cm   **(d)** 51.8 cm

8 55 kg

9 (a) 160.5 cm   **(b)** 160 cm

10 19.93 g

11 (a) 3 children   **(b)** 3 children   **(c)** 2.7 children

12 (a) mean 15, median 7, mode 4
(b) The median, as the mean is distorted by the highest value and the mode is the lowest value.

Exercise 9

1 (a)

Weight (kg)	30–34	35–39	40–44	45–49	50–54	55–59	60–64
Frequency	1	6	6	7	8	0	2

(b)
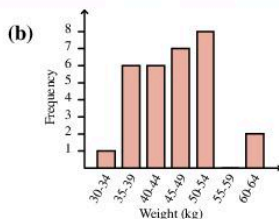
2 (a)

Age (years)	10	11	12	13	14
Frequency	2	6	9	9	4

(b)
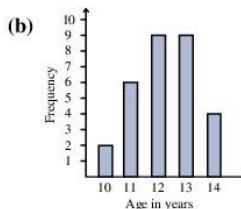
(c)   12 or 13 (there are two modes)

3 (a) Pie chart angles are football 120°, cricket 60°, rounders 90°, netball 90°   **(b)** Pie chart angles are walk 180°, car 60°, bicycle 30°, bus 90°   **(c)** Pie chart angles are St Lucia 40°, Dominica 60°, Trinidad 120°, Grenada 40°, St Vincent 40°, Barbados 60°

4 (a) mean = 4.6 (1dp); median = 5; mode = 5
(b) mean = 46.3 (1dp); mode = 32 or 56 (two modes); median = 51;
(c) mean = 11.3; median = 12; mode = 13

Check out

1

Mark	Frequency
3	2
4	3
5	4
6	4
7	5
8	2

3 sectors: 54°, 108°, 36°, 90°, 72°

5 (a) (i) 6   **(ii)** 5   **(b) (i)** 6   **(ii)** 6   **(c) (i)** 4.2   **(ii)** 4.9

10 Patterns and algebra

Check in

1 (a) 1, 2, 4   **(b)** 1, 2, 5, 10   **(c)** 1, 2, 3, 4, 6, 8, 12, 24
(d) 1, 5, 7, 35   **(e)** 1, 2, 3, 4, 6, 8, 9, 12, 18, 24, 36, 72

2 (a) and **(d)**

3 21, 22, 24, 26, 27, 28

Exercise 10A

1 (a) rectangular   **(b)** 24 in each   **(c)** multiply the number of rows by the number of columns

2 (a) factors   **(b)** multiple   **(c)** composite

3 Four of 1, 2, 3, 4, 6, 8, 12, 24

4 (b) (i) 1, 2, 4, 8, 16   **(ii)** 1, 2, 4, 7, 14, 28
(iii) 1, 2, 3, 4, 6, 9, 12, 18, 36   **(iv)** 1, 7, 49
(v) 1, 2, 4, 7, 8, 14, 28, 56
(vi) 1, 2, 3, 4, 6, 8, 12, 16, 24, 32, 48, 96

Exercise 10B

1 No, it only has one factor.

2 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47

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Number pairs are: 1 and 3 (13 and 31), 1 and 7 (17 and 71),
7 and 9 (79 and 97), 1 and 1 (11) and 7 and 7 (77).

Number triples: 1 and 1 and 3 (113, 131, 311) and 1 and 1 and 1 (111)

Exercise 10C

1 (a) 1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, 169, 196, 225, 256, 289, 324, 361, 400, 441, 484, 529, 576, 625

3 (a) 25   **(b)** 36   **(c)** 100   **(d)** 225   **(e)** 400   **(f)** 625

4 (a) $10^2 = 100$ **(b)** $12^2 = 144$ **(c)** $20^2 = 400$
(d) $25^2 = 625$ **(e)** n^2

Exercise 10D

1 (b) yes   **(c)** yes

2 (b) no   **(c)** no

3 (a) 20   **(b)** 34   **(c)** $2n$

4 (a) 25   **(b)** 39   **(c)** $2n - 1$

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$2^2 = 4, 20^2 = 400, 22^2 = 484, 68^2 = 4624$

Exercise 10E

1 (a) 28   **(b)** 45

2 (a) 55   **(b)** 78   **(c)** $\frac{1}{2}n(n+1)$

3 (a) 16   **(b)** 20   **(c)** 30

4 (a) 70   **(b)** 165

Exercise 10F

2 (a) 21, 15, 10, 6, 3, 1   **(b)** all of them

3 (a) 3   **(b)** 6   **(c)** 45, 55

4 (a) square made from two triangular patterns
(b) $4^2 = 6 + 10, 5^2 = 10 + 15, 6^2 = 15 + 21$

5 (a) 4   **(b)** 9   **(c)** 36   **(d)** $(n+1)^2$

6 (a) $5 \times 6 = 30$ **(b)** $10 \times 11 = 110$ **(c)** $11 \times 12 = 132$
(d) $n \times (n+1)$

7 (a) 15   **(b)** 55   **(c)** 66   **(d)** $\frac{1}{2}n(n+1)$

8 (a) 78   **(b)** 78

9 (a) 4950   **(b)** 4950

10 (a) 36   **(b)** 105   **(c)** 210   **(d)** 325   **(e)** 4278
(f) 20 100   **(g)** $n(2n+1)$

11 square, triangle

12 (a) triangle   **(b)** square   **(c)** $\frac{1}{2}n(n+1)$
(d) square   **(e)** n^2

13 (b) {8, 12, 14, 18, 20, 22, 24, 26, 27}

Exercise 10G

1 (a) 46   **(b)** 130   **(c)** 150   **(d)** 184   **(e)** 204

2 (a) 45   **(b)** 107   **(c)** 189

3 (a) 16   **(b)** 324   **(c)** 441

4 (a) 21   **(b)** 276   **(c)** 325

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(a)

Number of cuts	Number of pieces of paper
0	1
1	2
2	4
3	8
4	16
5	32

(b) (i) 1024   **(ii)** 1 048 576   **(c)** $P = 2^n$

Exercise 10H

1 (a) 14 cm   **(b)** 24 cm

2 (a) 37.68 cm   **(b)** 81.64 cm

3 (a) 24 cm^3 **(b)** 75 cm^3

4 (a) $14   **(b)** $3.58

5 (a) 15 km   **(b)** 13.5 km

6 (a) 0.75 g/cm^3 **(b)** 0.9 g/cm^3

7 (a) 35   **(b)** 5

8 (a) 12   **(b)** 26

9 (a) 6   **(b)** $^-$9.5   **(c)** $^-$7.5

Exercise 10I

1 (a) $P = 4a$ (b) $P = 2a + 2b$ (c) $P = 5c$

2 $S = B + G$

3 $W = M - G$

4 $T = H + G$

5 $P = T \div M$

6 (a) $C = \$7M$ (b) $K = \$MA$

7 (a) $C = (40P + 25E)$ cents   (b) $K = (PA + EB)$ cents

8 (a) $C = \$2(B + G)$ (b) $C = \$D(B + G)$

9 $L = W - X$

10 (a) $A = t^2$ (b) $N = R \div t^2$ (c) $C = 3N$ or $C = 3R \div t^2$

11 (a) $P = A + B + C + D + E + F$ (b) $M = YP$ or
     $M = (A + B + C + D + E + F)P$

12 (a) $C = 4(x + y + z)$ (b) $C = P(x + y + z)$

13 (a) $6n$ km   (b) $D = (6n + 3m)$ km

14 (a) $\$0.45M$ (b) $\$0.25G$ (c) $T = 0.45M + 0.25G$

15 (a) 6   (b) ac cm², bc cm² or ab cm²
     (c) $S = 2ac + 2bc + 2ab$

Exercise 10J

1 (a) 12   (b) $5b$ (c) $2f$ (d) $4x$

2 (a) $x + x + x + x + x + x$ (b) $y + y + y + y + y$
     (c) $h + h + h + h$

Exercise 10K

1 (a) $10p$ (b) $9s$ (c) $20t$ (d) $11l$

2 (a) $3a - 2b$ (b) $3x - y$ (c) $3g - f - 5$

3 (a) $7a + 6b$ (b) $3a$ (c) $5d - c$ (d) $p + q$ (e) $2x + 2y$
     (f) $6f + g + h$ (g) $s - t$ (h) $6n - m - 3p$ (i) $3x + y + 1$
     (j) $12 + 2x + y$

4 (a) pq (b) mn (c) st (d) gh (e) yz (f) uv

5 (a) $4pq$ (b) $15s$ (c) $12rs$

Exercise 10L

1 (a) x^2y^2 (b) p^4q (c) w^3s^2 (d) $w^2r^2t^2$

2 (a) $2mn$ (b) $3xy$ (c) $3pq$ (d) $4p^2q$ (e) $3gh^2$
     (f) $2a^2b^2$ (g) $6m^3n^2$ (h) $4cd^4$ (i) $3m^2n^3$

3 (a) p^4 (b) p^4 (c) p^6 (d) p^6

4 (a) m^5 b) g^6 (c) d^5 (d) m^7 (e) q^6 (f) e^8

5 (a) p^3 (b) p (c) p^4

6 (a) b (b) g^2 (c) m^3 (d) n^2 (e) p^2 (f) b^2
     (g) x^9 (h) g^3 (i) m^3

7 (a) g^8 (b) $3d^5$ (c) $10m^6n^3$ (d) $30t^6$ (e) $3b$
     (f) $7n^2$ (g) $6g^3$

Exercise 10M

1 (a) 11   (b) 10   (c) 17   (d) 32   (e) 24   (f) 12
     (g) 39   (h) 46

2 (a) 4   (b) 9   (c) 8   (d) 24   (e) 12   (f) 144
     (g) 270   (h) 5

3 (a) $P = 2a + 2b + c$ (b) $17\frac{1}{2}$ cm

4 (a) 20 m   (b) 2000 m   (c) 1.25 m

5 (a) $A = x^2 + y^2$ (b) (i) 52 cm²   (ii) 164 m²

6 (a) $V = pqr$ (b) (i) 120 cm³   (ii) 64 cm³

Exercise 10N

1 (a) true   (b) true   (c) true   (d) true   (e) false
     (f) false   (g) false   (h) false

2 $+$ and $\times$

3 (a) 23   (b) 22

4 (a) 13   (b) 13

5 (a) 6   (b) 6

6 $*$ is not commutative, the other two operations are

7 (b) true   (c) true   (d) true   (e) false   (f) false
     (g) false   (h) false

8 $+$ and $\times$

9 (a) 11   b) 7   (c) 15   (d) 19

Exercise 10O

1 (a) (i) 68   (ii) 426   (b) (i) 78   (ii) 174

2 (a) $3p + 3q$ (b) $5p - 5q$ (c) $4l + 4m$ (d) $3x - 3y$
     (e) $6r + 6s$ (f) $5l - 5m$ (g) $8p + 40q$ (h) $18x + 9y$

3 (a) (i) $2(2a + b)$ (ii) $2(x - 2y)$
     (b) $2(p + q)$ cm, $(2p + 2q)$ cm, $(p + p + q + q)$ cm

4 (a) $\$(2s + 3t)$ (b) $\$(j + 4s)$
     (c) $\$(3j + 3s + 3t)$ (d) $\$3(j + s + t)$

5 (a) $\$64.50, \$124.50, \$223.50$ (b) yes

6 (a) $3j + 3s + 3t$ (b) $8j + 4s + 12t$

7 $2(50k + 15l + 2m)$

8 (a) $2(x + y)$ (b) $3(p + q)$ (c) $7(k + l)$ (d) $4(x + y)$
     (e) $5(p - q)$ (f) $6(r - s)$ (g) $3(2a + 5b)$
     (h) $5(3m - 5n)$

Exercise 10P

1 (a) $6x + 15$ (b) $15y - 10$ (c) $7p + 21q$ (d) $8u - 20v$
     (e) $3kx + 2k$ (f) $3l - 5lp$ (g) $2ms + 7mt$ (h) $5nr - 6ns$
     (i) $18n - 6mn$ (j) $7pq - 28p^2$

2 (a) $10x + 10y$ (b) $11p + 11q$ (c) $8r + 13s$ (d) $10r + 6s$
     (e) $19q + 2p$ (f) $6x + 10y$ (g) $15 - 11z$ (h) $26 + y$
     (i) $17 - 5p$ (j) $7x - 4y + 2$

3 (a) $x^2 + 2xy + y^2$ **(b)** $a^2 - ab - 2b^2$ **c)** $2pq - 2p - 2q$
(d) $2s^2 + 2t^2 + 3st$ **(e)** $4gh - g^2 - 2h^2$ **(f)** $10q - 5p + 2pq$
(g) $2xy - 2y^2$ **(h)** 0   **(i)** $15x + 2$ **(j)** $8x - 1$

Investigation page 211

1 $x^2\,cm^2$, $2x\,cm^2$, $5x\,cm^2$, $10\,cm^2$; $(x + 5)(x + 2) = x^2 + 7x + 10$

3 $(x + a)(x + b) = x^2 + (a + b)x + ab$

Exercise 10Q

1 (a) (i) 17   **(ii)** 38   **(b) (i)** 18   **(ii)** 46
(c) (i) 26   **(ii)** 61

3 (a) 8   **(b)** 7   **(c)** 5

4 (a) $3x + 2$ **(b)** $4x - 2$ **(c)** $5x + 1$

Exercise 10R

1 (a) $x = 8$ **(b)** $x = 7$ **(c)** $x = 5$

2 Yes

3 (a) $x = 5$ **(b)** $x = 4$ **(c)** $x = 6$

4 (a) $x = 8$ **(b)** $x = 5$ **(c)** $x = 4\frac{5}{7}$ **(d)** $x = \frac{1}{2}$ **(e)** $x = 4$
(f) $x = \frac{2}{3}$

Exercise 10S

1 (b) $2x$ has been taken from both sides. The two sides still
  balance.   **(c)** $3x + 1 = 10$ **(d)** yes   **(e)** 3 kg

2 (a) $x = 2$ **(b)** $x = 5$ **(c)** $x = 1$ **(d)** $x = 2$

3 (a) $x = 3$ **(b)** $x = \frac{2}{5}$ **(c)** $x = 5$ **(d)** $x = 4$ **(e)** $x = 2\frac{1}{2}$
(f) $x = 5$ **(g)** $x = 4$ **(h)** $x = 3$

4 (a) $x = 2$ **(b)** $x = 10$ **(c)** $x = 3$ **(d)** $x = 12$ **(e)** $x = 3$
(f) $x = 6$ **(g)** $x = 3$ **(h)** $x = 12$ **(i)** $x = 7$ **(j)** $x = 1$
(k) $x = 45$ **(l)** $x = 2$ **(m)** $x = 1$ **(n)** $x = 3$

5 (a) He should have added 4 to both sides of the equation.
(b) $x = 5$

Exercise 10T

1 (a) $x = 2\frac{1}{3}$ **(b)** $x = 2$ **(c)** $x = 6\frac{2}{3}$ **(d)** $x = 7$ **(e)** $x = 20$
(f) $x = 7$

2 (a) $x = 12$ **(b)** $x = 35$ **(c)** $x = 36$ **(d)** $x = 28$
(e) $x = 5$ **(f)** $x = 87$

Exercise 10U

1 35

2 46

3 45, 46

4 106, 108

5 (a) 7 cm   **(b)** 6.15 cm   **(c)** 3.55 cm

6 8 cm, 10 cm, 16 cm

7 Bernelle, 16; Janet, 10

8 Anton, 60; Brian, 20; Cedric 16

9 12

10 79, 81, 83

Exercise 10V

1 (a) $x = y - c$ **(b)** $x = \dfrac{y}{m}$ **(c)** $\dfrac{y - c}{m}$

2 $w = \dfrac{A}{l}$ **3** $h = \dfrac{P}{mg}$ **4** $l = \dfrac{P}{2} - w$ **5** $\pi = \dfrac{A}{r^2}$ **6** $w = \dfrac{V}{lh}$

7 $2\bar{X} - x_1$ **8** $m = \dfrac{2E}{v^2}$ **9** $t = \dfrac{v - u}{a}$ **10** $v = \dfrac{RT}{P}$

11 $b = \dfrac{2A}{h}$ **12** $a = \dfrac{2A}{h} - b$ **13** $C = \dfrac{5}{9}(F - 32)$

14 $r = \pm\sqrt{\dfrac{A}{\pi}}$ **15** $l = \dfrac{gT^2}{4\pi^2}$

Exercise 10W

1 (a) $x > 3$ **(b)** $x > 5$ **(c)** $x < 3$ **(d)** $x < 3$
(e) $x > 2$ **(f)** $x > 1\frac{1}{2}$ **(g)** $x < 1\frac{1}{2}$ **(h)** $x < 3$

2 (a) $x < 5$ **(b)** $x > 9$ **(c)** $x > 3$ **(d)** $x < 7$
(e) $x > 3$ **(f)** $x < 2$ **(g)** $x > 8$

3 (a) $x < 3$ **(b)** $x > 5$ **(c)** $x > 2$ **(d)** $x < 6\frac{1}{2}$
(e) $x < 10$

Exercise 10X

2 (a) $\{5, 6, 7, 8, 9\}$ **(b)** $\{1, 2\}$ **(c)** $\{6, 7, 8, 9\}$
(d) $\{1, 2, 3\}$ **(e)** $\{7, 8, 9\}$ **(f)** $\{1, 2, 3, 4\}$

3 (a) $\{16, 17, 18, 19, 20\}$ **(b)** $\{18, 19, 20\}$ **(c)** $\{1, 2, 3\}$
(d) $\{1, 2, 3, 4, 5\}$ **(e)** $\{1, 2, 3\}$ **(f)** $\{15, 16, 17, 18, 19, 20\}$

4 (a) $\{24, 25\}$ **(b)** $\{21, 22, 23\}$ **(c)** $\{22, 23, 24, 25\}$

5 (a) $\{5, 6, 7\}$ **(b)** $\{6, 7\}$

Exercise 10Y

1 Width < 5 m   **2** Length > 5 m   **3** $6   **4** height < 24 cm
5 184 m   **6** length ≥ 7 cm   **7** 7   **8** 10   **9** width > 5 cm
10 24

Exercise 10Z – mixed questions

1 (a) 2   **(b)** 36   **(c)** 1 or 36   **(d)** 91   **(e)** 3

3 (a) 1, 4, 9, 16, 25, 36, 49, 64, 81, 100
(b) $5 = 4 + 1$, $13 = 9 + 4$, $17 = 16 + 1$, $29 = 25 + 4$, $37 = 36 + 1$,
$41 = 25 + 16$, $53 = 49 + 4$, $61 = 36 + 25$, $73 = 64 + 9$,
$89 = 64 + 25$, $97 = 81 + 16$

4 (a) 15, 18, 21   **(b) (i)** 30   **(ii)** 60   **(c)** $S_n = 3n$

5 (a) 20, 24, 28; 40, 80; $S_n = 4n$ **(b)** 13, 15, 17; 23, 43;
$S_n = 2n + 3$ **(c)** 9, 10, 11; 14, 24; $S_n = n + 4$
(d) 52, 51, 50; 47, 37; $S_n = 57 - n$ **(e)** 21, 25, 29; 41, 81;
$S_n = 4n + 1$

6 (a) (ii), (iii), (iv) and **(v)**   **(b) (ii)** $x = 1$ **(iii)** $x = {}^-1$
(iv) $x = 2$ **(v)** $x = {}^-12$

7 (a) 13   **(b)** 55   **(c)** 6765

8 (a) 1, 6, 15, 20, 15, 6, 1; 1, 7, 21, 35, 35, 21, 7, 1;
1, 8, 28, 56, 70, 56, 28, 8, 1
(c) along the third 'slope'   **(d)** successive powers of 2

9 (a) 6   **(b)** 24

10 (a) $4x + 12 = 90$ **(b)** 25.5 cm, 19.5 cm

11 (a) $x = 2y - 3$ **(b)** $x = \dfrac{3y + 6}{4}$ **(c)** $x = \pm\sqrt{y} - a$

   (d) $x = \pm\sqrt{r^2 - y^2}$ **(e)** $x = \dfrac{c}{a - b}$

12 16

13 $1\frac{1}{3}$ tonnes

14 (a) 6   **(b)** 24   **(d)** 40 320′

15 (a) 3   **(b)** 6   **(c)** 10   **(e)** triangle numbers   **(f)** 360

Exercise 10

1 (a) (i) 36   **(ii)** 190   **(b) (i)** 4th **(ii)** 12th

2 (a) $2n - 1$ **(b)** $3n + 1$ **(c)** $2n + 14$ **(d)** $23 - 2n$

3 (a) $6x + 2y$ **(b)** $8x - 4y$ **(c)** $6y^2$ **(d)** $x^4 + 4x^2y^2$
   (e) $x - 2y$ **(f)** $19x - 27y$

4 (a) (i) $4x - 2;\ x(x - 1)$ **(ii)** Either $(x + 3)^2$ or $x^2 + 6x + 9$
   (iii) $3x + 3;\ \frac{1}{2}x(x + 1)$ **(b) (i)** 5.75 cm, 6.75 cm
   (ii) 6.25 cm   **(iii)** 7.33 cm, 8.33 cm, 9.33 cm

5 (a) $x \le 2$ **(b)** $x \ge 3$ **(c)** $x < 3\frac{1}{3}$ **(d)** $x \le \frac{1}{5}$

6 (a) $y = \dfrac{c - ax}{b}$ **(b)** $y = \dfrac{bx}{a}$ **(c)** $y = bx - a$ **(d)** $y = \pm\sqrt{\dfrac{x}{a}}$

7 (a) 45 cm   **(b)** $z = 30 + 5n$ cm   **(c)** 160 g

8 (a) $2p$ km   **(b)** $2p + 3q$ km   **(c)** $\dfrac{2p + 3q}{5}$ km/h

Check out

1 (a) 4, 6, 8, 9, 10   **(b) (i)** and **(ii)**

2 (a) 1, 4, 9, 16, 25   **(b) (i)** and **(iii)**

3 (a) 2, 3, 5, 7, 11   **(b) (i)** and **(iii)**

4 (b) 55   **(c)** 20 100

5 (a) $C = ap + bq$ **(b)** $6s^2$

6 (a) $3a^9$ **(b)** $3x$ **(c)** $9p^4$

7 (a) $21h + 1$ **(b)** $14w + 28t$ **(c)** $8p - 12$

8 (a) $x = 4$ **(b)** $x = 14$ **(c)** $r = 5\frac{1}{2}$

9 (a) $x \ge \frac{1}{2}$ **(b)** $x < 4$ **(c)** $x \le 21$

10 (a) $x = \dfrac{bcy}{a}$ **(b)** $x = 2(y - c)$ **(c)** $x = \sqrt{\dfrac{y - b}{a}}$

11 Volume

Check in

1 (a) 540   **(b)** 12 900

2 (a) 225 cm^2 **(b)** 216 m^2 **(c)** 135 m^2

Exercise 11A

3 (a) yes   **(b)** no   **4 (a)** yes   **(b)** no

5 (a) yes   **(b)** yes   **6 (a)** yes   **(b)** no

7 A change in shape does not mean a change in volume

8 (a) yes   **(b)** no

9 (a) a fully-grown adult   **(b)** a football
   (c) this maths book   **(d)** a rice grain   **(e)** a pineapple
   (f) both the same

10 6 **(a)** yes   **(b)** yes

Exercise 11B

1 (a) 18   **(b)** The volume of the box is 18 cubes.

2 (a) The volume of the box is 24 cubes.   **(b)** larger

3 (a) 18   **(b)** 18 cubes
   (c) They have the same number of cubes   **(d)** no
   (e) The cubes are not the same size.
   (f) Use a standard-sized cube.

Exercise 11C

1 (a) 14 cm^3 **(b)** 25 cm^3 **(c)** 18 cm^3

2 (a) 27 cm^3 **(b)** 57 cm^3 **(c)** 80 cm^3

4 72

5 (a)

Number of cubes long L	Number of cubes wide W	Number of cubes high H	L × W × H	Total number of cubes
4	3	3	36	36
3	3	4	36	36
6	3	2	36	36
5	3	3	45	45

   (b) They are the same.

Exercise 11D

1 (a) 8 cm^3 **(b)** 2 cm^3 **(c)** 12 cm^3 **(d)** 16 cm^3
   (e) 36 cm^3

2

Box	Volume (cm^3)	Length (cm)	Width (cm)	Height (cm)	Length × Width × Height
(a)	8	2	2	2	8
(b)	2	1	1	2	2
(c)	12	3	2	2	12
(d)	16	2	2	4	16
(e)	36	4	3	3	36

3 Volume = length × width × height

Exercise 11E

1 (a) 72 cm^3 **(b)** 315 cm^3 **(c)** 208 cm^3

2 (a) 18 cm^3 **(b)** 30 cm^3 **(c)** 18 cm^3 **(d)** 10.125 cm^3

3 (a) (i) 125 cm^3 **(ii)** 1000 cm^3 **(b)** 8

4 64   **6** Volume (cm^3): 8, 30, 78, 33.75, 31.875

7 h cm: 5, 5, 10, 4   **8** 960 cm^3

10 (a) centimetre   **(b)** metre   **(c)** millimetre   **(d)** Yes

Exercise 11F

2 (a) cube   (b) Volume: 1 cm^3, 1000 mm^3
3 1 cm^3 = 1000 mm^3
4 750
5 57

Exercise 11G

2 (a) 30 m^3 (b) 150 m^3
4 (a) cm^3 (b) m^3 (c) cm^3 (d) m^3 (e) mm^3
   (f) cm^3 (g) mm^3 (h) mm^3 (i) cm^3 (j) cm^3
5 (a) Volume: 1 m^3, 1 000 000 cm^3 (b) 1 m^3 = 1 000 000 cm^3
6 1 000 000 000 mm^3

Exercise 11H

1 (a) 72 cm^3 (b) 45 cm^3 (c) 120 cm^3
2 (a) 80 cm^3 (b) 108 cm^3 (c) 300 cm^3
3 (a) 60 cm^3 (b) 1582.6 cm^3 (c) 803.8 cm^3
4 (a) 47.1 cm^3
   (b) 12 (a more difficult question, the answer 12 is the
       practical answer)
5 (a) 0.5 m^3 (b) 500 000 cm^3
6 (a) 1080 cm^3 (b) 900
7 (a) 25 cm^2 (b) 5.64 cm

Exercise 11J

3 (a) (i) 6.5 litres   (ii) 6500 ml   (b) 65
4 (a) 150 ml, 220 ml, 280 ml   (b) 7.84 litres
   (c) 9 litres 240 millilitres
6 (a) 1 000 000 cm^3 (b) 1000 litres
7 (a) 800 m^3 (b) 800 000
8 (a) 24 000 cm^3, 24 litres   (b) 30 000 cm^3, 30 litres
   (c) 3600 cm^3, 3.6 litres
9 30 days, 7 days, 15 days

Exercise 11K

1 (a) 208 cm^2 (b) 82 cm^2 (c) 192 cm^3
2 (a) 286 cm^2 (b) 224.24 cm^2
3 (a) 510 cm^2 (b) 108 cm^2 (c) 168 cm^2
4 (a) 960 cm^3 (b) 720 cm^2

Exercise 11L

1 (a) 301.44 cm^2 (b) 314 cm^2 (c) 226.08 cm^2
   (d) 282.6 cm^2
2 (a) 527.27 cm^2 (b) 471 cm^2 (c) 452.16 cm^2
   (d) 339.12 cm^2
3 (a) 50.24 cm^2 (b) 251.2 cm^2 (c) 351.68 cm^2
4 4.52 m^2
5 (a) 3.98 cm   (b) 49.76 cm^2 (c) 399.47 cm^2

Activity page 238

Breathe into a balloon.

Exercise 11M – mixed questions

1 35 ml
3 No, it would float.
4 (a) Water will pour into the measuring cylinder.   (b) 15 ml
5 (a) cm^3
6 (a) 5 ml   (b) 10 days
7 5
8 (a) 864 cm^3 (b) 17 280 cm^3
9 (a) 12 m^3 (b) 12 000 litres
10 (a) 125   (b) 1728
11 4800
12 2.56 m^2
13 (a) 7 m   (b) (i) 70 m^3 (ii) 70 000 litres
14 (a) 2355 cm^3 (b) 2.355 l   (c) 12.7 cm
15 (a) 56.52 cm^2 (b) 353
16 3 cm
17 15.5 kg

Exercise 11

1 (a) ml   (b) l   (c) mm^3 (d) cm^3 (e) l   (f) m^2
   These are suggestions, others are possible.
2 (a) 28 cm^3 (b) 48 cm^3 (c) 80 cm^3 (d) 90 cm^3
3 (a) 246 cm^2 (b) 156 cm^2
5 (a) 3 l   (b) 5 l   (c) 6 l   (d) 7.5 l   (e) 0.85 l
   (f) 1000 l   (g) 5000 l   (h) 0.925 l   (i) 3400 l
   (j) 0.13 l   (k) 0.014 l   (l) 0.025 l   4 72 l
6 (a) 510 cm^3 (b) 510 ml   (c) 0.51 l
7 (a) 78 500 cm^3 (b) 78.5 l   (c) 10.2 cm

Check out

1 (a) grapefruit   (b) basketball   (d) 1 kg of feathers
2 (a) 8 cm^3 (b) 12 cm^3
3 (a) 12 cm^3 (b) 3.375 cm^3 (c) 301.44 cm^2
4 (a) cm^3 (b) m^3 (c) mm^3 (d) cm^3
5 (b) 12 000 litres
6 (a) 160 cm^2 (b) 120 cm^2 (c) 1105.28 cm^2

12 Consumer arithmetic

Check in

1 (a) 10%   (b) 17%   (c) 35%   (d) 12%   (e) 40%
2 (a) $1\frac{1}{2}$ (b) $1\frac{1}{2}$ (c) $2\frac{1}{5}$ (d) $17\frac{1}{2}$

Exercise 12A

3 (a) You do not need to have cash.
(b) Cheques are not always accepted.
(c) To check that you have sufficient money in your account to cover the cheque.

6

Date	Reference	Withdrawal	Deposit	Balance
				503.16
15/9	Cash	250		253.16
31/9			650	903.16
5/10	Cheque	217.36		685.80
17/10		315		370.80
28/10	Interest		2.17	372.97

Exercise 12B

1 (a) $\frac{1}{4}$ (b) $\frac{1}{2}$ (c) $\frac{3}{4}$ (d) $\frac{4}{25}$ (e) $\frac{17}{20}$ (f) $\frac{2}{5}$ (g) $\frac{7}{20}$
(h) $\frac{3}{100}$ (i) $\frac{3}{25}$ (j) $\frac{2}{25}$

2 (a) 38% (b) 3% (c) 40% (d) 14% (e) 68%
(f) 45% (g) 75% (h) 20% (i) 60%

3

Fraction	Equivalent per cent fraction	Percentage
$\frac{3}{5}$	$\frac{60}{100}$	60%
$\frac{1}{4}$	$\frac{25}{100}$	25%
$\frac{1}{2}$	$\frac{50}{100}$	50%
$\frac{3}{4}$	$\frac{75}{100}$	75%
$\frac{7}{10}$	$\frac{70}{100}$	70%
$\frac{3}{20}$	$\frac{15}{100}$	15%
$\frac{4}{25}$	$\frac{16}{100}$	16%
$\frac{9}{50}$	$\frac{18}{100}$	18%
$\frac{1}{8}$	$\frac{12.5}{100}$	12.5%

4 (a) $5 (b) $12 (c) $5 (d) $60 (e) $74
(f) $1.50 (g) $24 (h) $9 (i) $30 (j) $1

5 (a) $6.30 = 6.3$ so no division by 10 has taken place
(b) (i) $0.93 (ii) $1.14 (iii) $1.30 (iv) $1.02
(v) $3.75 (vi) $9.66 (vii) $0.60 (viii) $0.525
(ix) $6.65 (x) $0.30

6 $138 7 $257.60 8 $112.10
9 280 10 136 11 $978.50

Exercise 12C

1 (a) $16 (b) $24 (c) $40
2 (a) $816 (b) $824 (c) $840
3 (a) $367.20 (b) $4957.20 (c) $413.10
4 (a) $1320 (b) $2640 (c) $7920 (d) $13 200
5 (a) $60 (b) $156 (c) $4950 (d) $8500
(e) $13 050
6 (a) $360 (b) $806 (c) $13 950 (d) $33 500
(e) $71 050

7 (a) $56.25 (b) $562.50 (c) $7312.50 (d) $914.63
(e) $13 500 (f) $354.71
8 (a) 1% (b) 2.5% (c) 3.75% (d) 9.375%
9 (a) 50 years (b) 25 years (c) 20 years
10 28.6 or 29 years

Exercise 12D

1 (a) $10.25 (b) $40.80 (c) $126.10 (d) $2077.70
2 (a) $110.25 (b) $540.80 (c) $926.10 (d) $10 077.70
3 (a) $400 (b) $304.50 (c) (a) is the better investment.
4 (a) $2100 (b) $12100 (c) $504.17
5 (a) $8123.50 (b) $43 123.50 (c) $1796.81

Exercise 12E

1 (a) $212.75 (b) $7992.50 (c) $4140 (d) $1437.50
2 (a) $58 000 (b) $81.20 (c) $184 (d) $46
3 (a) J$15 378 (b) J$3378.50 (c) J$25 047.50
4 $1974
5 16.5%
6 $286.96

Exercise 12F

1 (a) $55 000 (b) $13 750 (c) $101 250
3 (a) nothing (b) $135 000 (c) nothing
4 (a) $5000 (b) $38 750 (c) $2166.67
5 (a) $48 000 (b) $108 000 (c) $9000 (d) $120 000
6 (a) $38 500 (b) $5700 (c) $42 000
7 (a) $8 000 (b) $62 000 (c) $83 000

Exercise 12G

1 (a) The price you pay if you pay in full when you take the goods home. (b) $85.75 (c) $1029 (d) $1116
2 (a) Yes, $246 more.

Exercise 12H

1 (a) $1476.80 (b) $859.04 (c) $487.76 (d) $1401.92
(e) $1781.52
2 (a) $281.80 (b) $164.04 (c) $92.76 (d) $303.92
(e) $386.52
3 (a) $187.20 (b) $165.80 (c) $124 (d) $82.50
4 $21.60 5 $24 6 (a) $56, $22.40 (b) $28
7 (a) $108 (b) $310.50 (c) $40.50 (d) 15%
8 (a) A: $342, B: $455.60, C: $404.40 (b) $75.60, $24.40
(c) A (d) C
9 (a) A: $1275, B: $1622 (b) 8.13%
10 (a) A: $3315, B: $4630 (b) 18.7%
11 $216
12 A: $1750, B: $2260
13 Yes, 28.3%

Exercise 12I – mixed questions

4 (a) $4.50   **(b)** $19.80   **(c)** $11.25   **(d)** $0.10

5 (a) $540   **(b)** $3540

6 $15.23

7 4 years

8 A. $2792   B. $3166.20   Method A

9 (a) $35 000   **(b)** $7000   **(c)** $58 000

10 (a) (i) $48 200   **(ii)** $14 460   **(iii)** $78 740

(b) (i) $56 421   **(ii)** $101 421   **(iii)** $84 494.70

11 $51.18

12 $38 400

13 (a)

14 25 years

15 (a) $3600   **(b)** $12 500

Exercise 12

1 (a) $60   **(b)** $810

2 (a) $10 500   **(b)** $4375   **(c)** $14 000

3 (a) $92   **(b)** $1437.50   **(c)** $73.60   **(d)** $4.03
(e) $2.65   **(f)** $15.53   **(g)** $1121.25   **(h)** $2507

4 (a) $1454.22   **(b)** $2659.45   **(c)** $5643.23

5 (a) (i) nothing   **(ii)** $8285
(b) (i) $47 375   **(ii)** $84 855

6 (a) (i) $624   **(ii)** $780   **(iii)** $5200
(iv) $14820   **(v)** $85.28
(b) (i) $672, $840, $5600, $15960, $91.84
(ii) $792, $990, $6600, $18810, $108.24

7 (a) $451.50   **(b)** $462   **(c)** $493.50

8 (a) $51 750   **(b)** $48 127.50   **(c)** $31 120

Check out

2 (a) $5   **(b)** $0.50   **(c)** $0.18   **(d)** $3

3 (a) $17 700   **(b)** $17 821.50

5 (a) $146   **(b)** $215

Revision exercise 3

1 (a)

Mark	Frequency
1	1
2	3
3	3
4	1
5	3
6	1
7	7
8	3
9	2

(b) 7

2 (b) 7–8

3 (a) 15°, 45°, 45°, 15°, 45°, 15°, 105°, 45°, 30°

4 (a) sector angles: 60°, 60°, 60°, 150°, 30°

5 (a) 5.5   **(b)** 6.5

6 (a) 90%   **(b)** 92%

7 (a) 20   **(b)** 16   **(c)** 15   **(d)** 14.55

8 (a) 72°, 54°, 36°, 54°, 90°, 54°

9 (a) 0–9   **(b)** 16   **(c)** 19   **(d)** 20–29

10 sector angles: 96°, 72°, 60°, 84°, 48°

11 (a) 36, 49, 256   **(b)** They are consecutive odd numbers.

12 (a) 21, 34   **(b)** The twelfth Fibonacci number is $12^2 = 144$
(c) 1, 3, 21 and 55

13 (a)

3^3	4^3	5^3	6^3
$3 \times 3 \times 3$	$4 \times 4 \times 4$	$5 \times 5 \times 5$	$6 \times 6 \times 6$
27	64	125	216

(b) $6^2, 10^2, 15^2, 21^2$
(c) They are all squares of triangle numbers.
(d) $28^2 = 784$

14 yes

15 (a) (i) 35, 40, 45   **(ii)** 50   **(iii)** 100   **(iv)** $5n$
(b) (i) 36, 41, 46   **(ii)** 51   **(iii)** 101   **(iv)** $5n + 1$
(c) (i) 19, 22, 25   **(ii)** 28   **(iii)** 58   **(iv)** $3n - 2$
(d) (i) 20, 23, 26   **(ii)** 29   **(iii)** 59   **(iv)** $3n - 1$
(e) (i) 50, 65, 82   **(ii)** 101   **(iii)** 401   **(iv)** $n^2 + 1$
(f) (i) 48, 63, 80   **(ii)** 99   **(iii)** 399   **(iv)** $n^2 - 1$

16 (a) $V = x^3$ **(b)** $S = 6x^2$ **(c)** $L = 12x$

17 (a) $M = nm$ **(b)** $c = T \div b$ **(c)** $A = q^2 - p^2$

18 (a) $x = 3$ **(b)** $x = 6$ **(c)** $x = 1$ **(d)** $x = 9$

19 (a) $f = 1\frac{1}{3}$ **(b)** $f = 4\frac{4}{5}$

20 (a) $x = \dfrac{y + b}{a}$ **(b)** $x = \pm\sqrt{\dfrac{y - b}{a}}$

21 (a) 8 cm^3 **(b)** 11 cm^3

22 (a) 60 cm^3 **(b)** 378 cm^3 **(c)** 260 cm^3 **(d)** 334.8 cm^3
(e) 8 cm   **(f)** 7.5 cm

23 (a) 31.584 cm^3 **(b)** 15.405 cm^3

24 (a) 63.26 cm^2 **(b)** 48.14 cm^2

25 (a) 171.6 cm^3 **(b)** 34.209 cm^3

26 (a) 196.42 cm^2 **(b)** 75.42 cm^2

27 (a) 1, 8, 27, 64, 125, 216
(b) (i) 9 cm^3 **(ii)** 36 cm^3 **(iii)** 100 cm^3 **(iv)** 225 cm^3
(c) They are the squares of the triangle numbers
(d) yes, 21^2

28 (b) 3.7 cm

29 (a) (i) 96 cm^3 **(ii)** 450 cm^3 **(b)** 510 cm^2

30 (a) 2.26 m^3 **(b)** 2261 litres

31 1155.52 cm^2

32 **(a)** 7 cm **(b)** $12 **(c)** 235 kg **(d)** $3.77

33 **(a)** $155.25 **(b)** $285.20 **(c)** $362.25 **(d)** $1063.75

34 **(a)** $84 000 **(b)** $16 800 **(c)** $117 200

35 **(a)** $700 **(b)** $3820 **(c)** 9.14%

37

Item	Market price	Discount	Sale price
Shirt	$8	5%	$7.60
Jacket	$42	12%	$36.96
Suit	$140	20%	$112
Dress	$32	25%	$24
Jumper	$15	10%	$13.50
Shoes	$20	5%	$19
Mat	$16	35%	$10.40

38 **(a)** $280 **(b)** $310 **(c)** $340; $23.33, $12.92, $9.44

39

Number of years	Total interest received
1	$80
5	$400
6 months	$40
3	$240
3 months	$20
5 years $1\frac{1}{2}$ months	$10

40

Number of years	Total amount
0	$1000
1	$1080
2	$1166.40
3	$1259.71
4	$1360.49
7	$1713.82

Mixed questions 3

1 B **2** C **3** C **4** C **5** D **6** B **7** B **8** C

9 A **10** C **11** C **12** D **13** B **14** D **15** A

16 B **17** C **18** A **19** B **20** A **21** A **22** C

23 A **24** C **25** D **26** D **27** C **28** B **29** B

30 C **31** B **32** C **33** D **34** C **35** B **36** B

37 A **38** C **39** A **40** D

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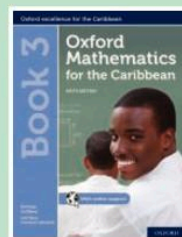- A workbook with each level for further practice
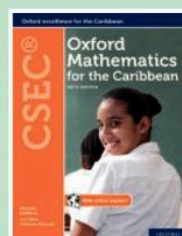
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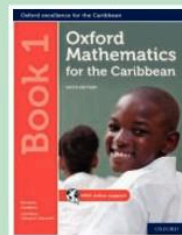
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