Attia, John Okyere. “Diodes.”
Electronics and Circuit Analysis using MATLAB.
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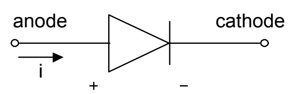
CHAPTER NINE

DIODES

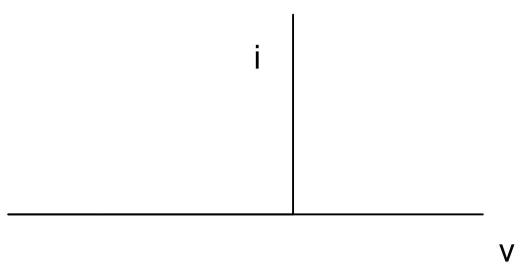
In this chapter, the characteristics of diodes are presented. Diode circuit analysis techniques will be discussed. Problems involving diode circuits are solved using MATLAB.

9.1 DIODE CHARACTERISTICS

Diode is a two-terminal device. The electronic symbol of a diode is shown in Figure 9.1(a). Ideally, the diode conducts current in one direction. The current versus voltage characteristics of an ideal diode are shown in Figure 9.1(b).



(a)



(b)

Figure 9.1 Ideal Diode (a) Electronic Symbol
(b) I-V Characteristics

The I-V characteristic of a semiconductor junction diode is shown in Figure 9.2. The characteristic is divided into three regions: forward-biased, reversed-biased, and the breakdown.

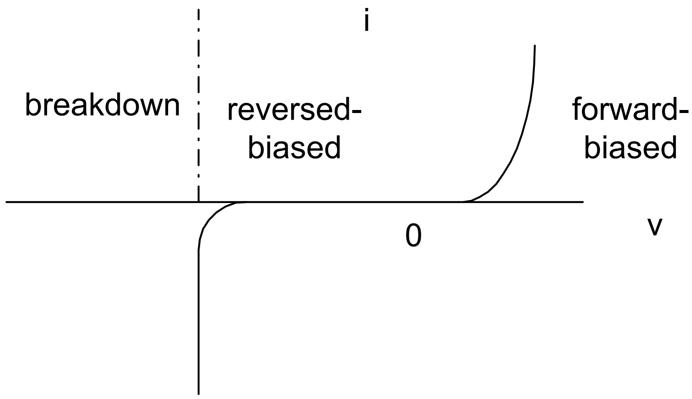


Figure 9.2 I-V Characteristics of a Semiconductor Junction Diode

In the forward-biased and reversed-biased regions, the current, i, and the voltage, v, of a semiconductor diode are related by the diode equation

$$i = I_S[e^{(v/nV_T)} - 1] \tag{9.1}$$

where

I_S is reverse saturation current or leakage current,
n is an empirical constant between 1 and 2,
V_T is thermal voltage, given by

$$V_T = \frac{kT}{q} \tag{9.2}$$

and

k is Boltzmann's constant $= 1.38x10^{-23}$ J / ^{o}K,
q is the electronic charge $= 1.6x10^{-19}$ Coulombs,
T is the absolute temperature in ^{o}K

At room temperature (25 ^{o}C), the thermal voltage is about 25.7 mV.

9.1.1 Forward-biased region

In the forward-biased region, the voltage across the diode is positive. If we assume that the voltage across the diode is greater than 0.1 V at room temperature, then Equation (9.1) simplifies to

$$i = I_S e^{(v/nV_T)} \tag{9.3}$$

For a particular operating point of the diode ($i = I_D$ and $v = V_D$), we have

$$i_D = I_S e^{(v_D/nV_T)} \tag{9.4}$$

To obtain the dynamic resistance of the diode at a specified operating point, we differentiate Equation (9.3) with respect to v, and we have

$$\frac{di}{dv} = \frac{I_s e^{(v/nV_T)}}{nV_T}$$

$$\left.\frac{di}{dv}\right|_{v=V_D} = \frac{I_s e^{(v_D/nV_T)}}{nV_T} = \frac{I_D}{nV_T}$$

and the dynamic resistance of the diode, r_d , is

$$r_d = \left.\frac{dv}{di}\right|_{v=V_D} = \frac{nV_T}{I_D} \tag{9.5}$$

From Equation (9.3), we have

$$\frac{i}{I_S} = e^{(v/nV_T)}$$

thus

$$\ln(i) = \frac{v}{nV_T} + \ln(I_S) \tag{9.6}$$

Equation (9.6) can be used to obtain the diode constants n and I_S, given the data that consists of the corresponding values of voltage and current. From

Equation (9.6), a curve of v versus $\ln(i)$ will have a slope given by $\frac{1}{nV_T}$ and y-intercept of $\ln(I_S)$. The following example illustrates how to find n and I_S from an experimental data. Since the example requires curve fitting, the MATLAB function **polyfit** will be covered before doing the example.

9.1.2 MATLAB function polyfit

The **polyfit** function is used to compute the best fit of a set of data points to a polynomial with a specified degree. The general form of the function is

$$coeff\_xy = polyfit(x, y, n) \qquad (9.7)$$

where

x and y are the data points.

n is the n^{th} degree polynomial that will fit the vectors x and y.

$coeff\_xy$ is a polynomial that fits the data in vector y to x in the least square sense. $coeff\_xy$ returns n+1 coefficients in descending powers of x.

Thus, if the polynomial fit to data in vectors x and y is given as

$$coeff\_xy(x) = c_1 x^n + c_2 x^{n-1} + \ldots + c_m$$

The degree of the polynomial is n and the number of coefficients $m = n + 1$ and the coefficients $(c_1, c_2, \ldots, c_m)$ are returned by the MATLAB polyfit function.

Example 9.1

A forward-biased diode has the following corresponding voltage and current. Use MATLAB to determine the reverse saturation current, I_S and diode parameter *n*.

Forward Voltage, V	Forward Current, A
0.1	0.133e-12
0.2	1.79e-12
0.3	24.02e-12
0.4	0.321e-9
0.5	4.31e-9
0.6	57.69e-9
0.7	7.726e-7

Solution

```
diary ex9_1.dat
% Diode  parameters

vt = 25.67e-3;
v = [0.1 0.2 0.3 0.4 0.5 0.6 0.7];
i  =  [0.133e-12  1.79e-12  24.02e-12  321.66e-12  4.31e-9  57.69e-9
772.58e-9];

%
lni = log(i); % Natural log of current

% Coefficients of Best fit linear model is obtained
p_fit = polyfit(v,lni,1);

% linear equation is   y = m*x + b
b = p_fit(2);
m = p_fit(1);
ifit = m*v + b;

% Calculate Is and n
Is = exp(b)
n = 1/(m*vt)

% Plot v versus ln(i), and best fit linear model
plot(v,ifit,'w', v, lni,'ow')
axis([0,0.8,-35,-10])
```

```
xlabel('Voltage (V)')
ylabel('ln(i)')
title('Best fit linear model')
diary
```

The results obtained from MATLAB are

```
Is  =      9.9525e-015

n  =       1.5009
```

Figure 9.3 shows the best fit linear model used to determine the reverse saturation current, I_S, and diode parameter, *n*.

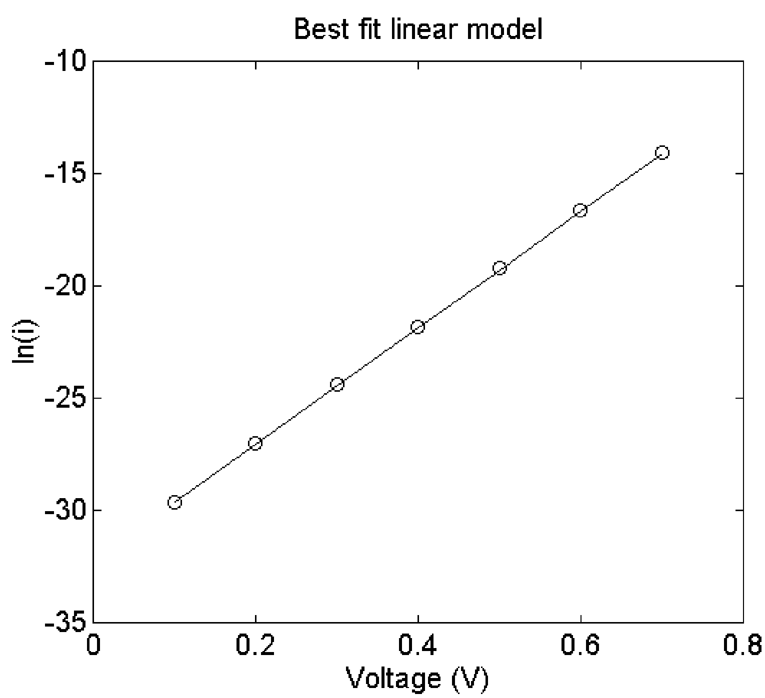


Figure 9.3 Best Fit Linear Model of Voltage versus Natural Logarithm of Current

9.1.3 Temperature effects

From the diode equation (9.1), the thermal voltage and the reverse saturation current are temperature dependent. The thermal voltage is directly proportional to temperature. This is expressed in Equation (9.2). The reverse saturation current I_S increases approximately 7.2% /°C for both silicon and germanium diodes. The expression for the reverse saturation current as a function of temperature is

$$I_S(T_2) = I_S(T_1)e^{[k_S(T_2 - T_1)]} \tag{9.8}$$

where

k_S = 0.072 / °C.

T_1 and T_2 are two different temperatures.

Since $e^{0.72}$ is approximately equal to 2, Equation (9.8) can be simplified and rewritten as

$$I_S(T_2) = I_S(T_1)2^{(T_2 - T_1)/10} \tag{9.9}$$

Example 9.2

The saturation current of a diode at 25 °C is 10^{-12} A. Assuming that the emission constant of the diode is 1.9, (a) Plot the i-v characteristic of the diode at the following temperatures: T_1 = 0 °C, T_2 = 100 °C.

Solution

MATLAB Script

```
% Temperature effects on diode characteristics
%
k = 1.38e-23; q = 1.6e-19;
t1 = 273 + 0;
t2 = 273 + 100;

ls1 = 1.0e-12;
ks = 0.072;
ls2 = ls1*exp(ks*(t2-t1));

v = 0.45:0.01:0.7;
```

```
I1 = Is1*exp(q*v/(k*t1));
I2 = Is2*exp(q*v/(k*t2));

plot(v,I1,'wo',v,I2,'w+')
axis([0.45,0.75,0,10])
title('Diode I-V Curve at two Temperatures')
xlabel('Voltage (V)')
ylabel('Current (A)')
text(0.5,8,'o is for 100 degrees C')
text(0.5,7, '+ is for 0 degree C')
```

Figure 9.4 shows the temperature effects of the diode forward characteristics.

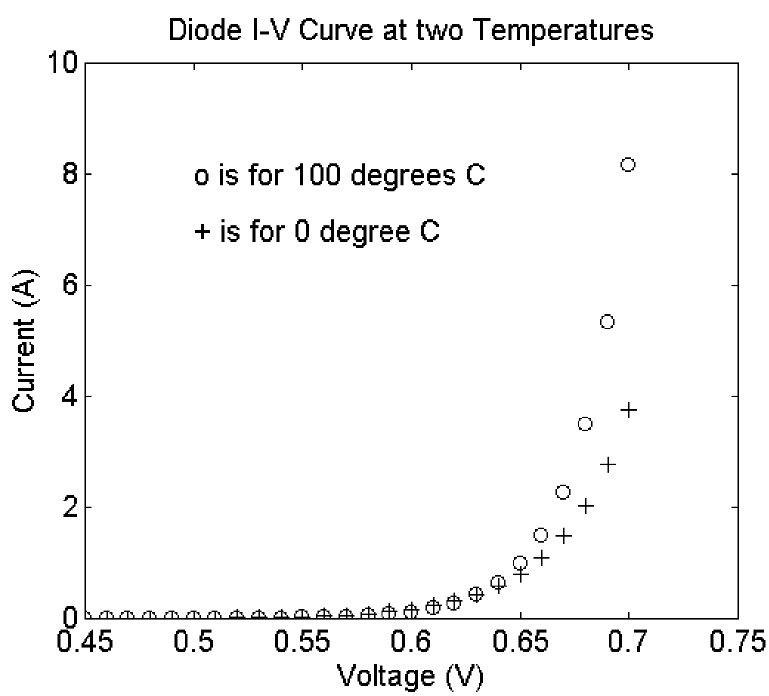


Figure 9.4 Temperature Effects on the Diode Forward Characteristics

9.2 ANALYSIS OF DIODE CIRCUITS

Figure 9.5 shows a diode circuit consisting of a dc source V_{DC}, resistance R, and a diode. We want to determine the diode current I_D and the diode voltage V_D.

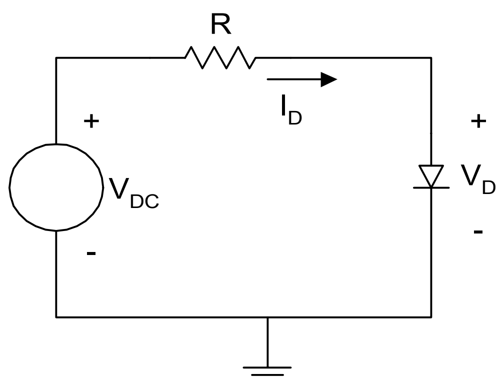


Figure 9.5 Basic Diode Circuit

Using Kirchoff Voltage Law, we can write the loadline equation

$$V_{DC} = RI_D + V_D \tag{9.10}$$

The diode current and voltage will be related by the diode equation

$$i_D = I_S e^{(v_D / nV_T)} \tag{9.11}$$

Equations (9.10) and (9.11) can be used to solve for the current I_D and voltage V_D.

There are several approaches for solving I_D and V_D. In one approach, Equations (9.10) and (9.11) are plotted and the intersection of the linear curve of Equation (9.10) and the nonlinear curve of Equation (9.11) will be the operating point of the diode. This is illustrated by the following example.

Example 9.3

For the circuit shown in Figure 9.5, if $R = 10\ \text{k}\Omega$, $V_{DC} = 10\text{V}$, and the reverse saturation current of the diode is 10^{-12} A and $n = 2.0$. (Assume a temperature of 25 ^{o}C.)

(a) Use MATLAB to plot the diode forward characteristic curve and the loadline.

(b) From the plot estimate the operating point of the diode.

Solution

MATLAB Script

```
% Determination of operating point using
% graphical technique
%
% diode equation
k = 1.38e-23;q = 1.6e-19;
t1 = 273 + 25; vt = k*t1/q;
v1 = 0.25:0.05:1.1;
i1 = 1.0e-12*exp(v1/(2.0*vt));

% load line 10=(1.0e4)i2 + v2
vdc = 10;
r = 1.0e4;
v2 = 0:2:10;
i2 = (vdc - v2)/r;

% plot
plot(v1,i1,'w', v2,i2,'w')
axis([0,2, 0, 0.0015])
title('Graphical method - operating point')
xlabel('Voltage (V)')
ylabel('Current (A)')
text(0.4,1.05e-3,'Loadline')
text(1.08,0.3e-3,'Diode curve')
```

Figure 9.6 shows the intersection of the diode forward characteristics and the loadline.

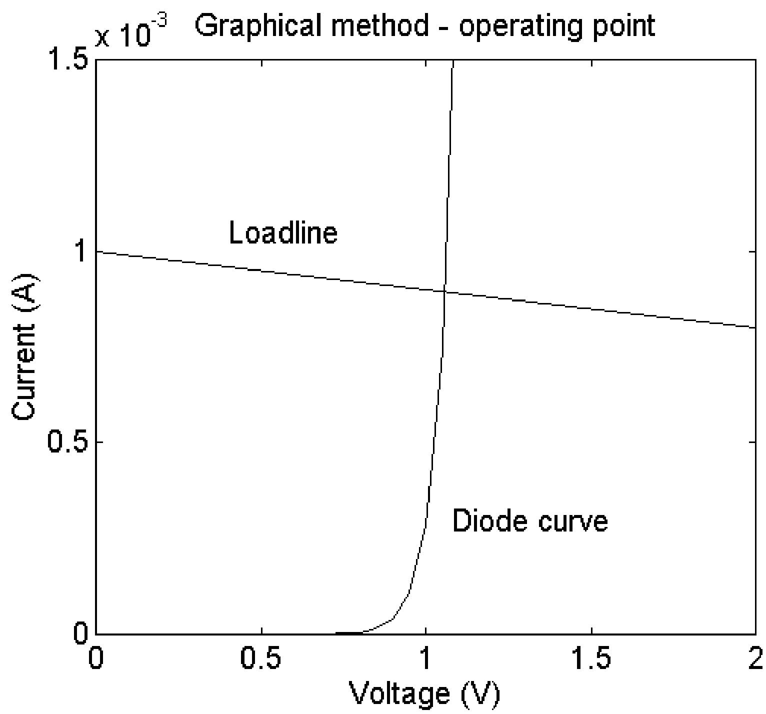


Figure 9.6 Loadline and Diode Forward Characteristics

From Figure 9.6, the operating point of the diode is the intersection of the loadline and the diode forward characteristic curve. The operating point is approximately

$$I_D = 0.9 \text{ mA}$$
$$V_D = 0.7 \text{ V}$$

The ***second approach*** for obtaining the diode current I_D and diode voltage V_D of Figure 9.5 is to use iteration. Assume that (I_{D1}, V_{D1}) and (I_{D2}, V_{D2}) are two corresponding points on the diode forward characteristics. Then, from Equation (9.3), we have

$$i_{D1} = I_S e^{(v_{D1}/nV_T)} \tag{9.12}$$

$$i_{D2} = I_S e^{(v_{D2}/nV_T)} \tag{9.13}$$

Dividing Equation (9.13) by (9.12), we have

$$\frac{I_{D2}}{I_{D1}} = e^{(V_{D2} - V_{D1}/nV_T)} \tag{9.14}$$

Simplifying Equation (9.14), we have

$$v_{D2} = v_{D1} + nV_T \ln\left(\frac{I_{D2}}{I_{D1}}\right) \tag{9.15}$$

Using iteration, Equation (9.15) and the loadline Equation (9.10) can be used to obtain the operating point of the diode.

To show how the iterative technique is used, we assume that I_{D1} = 1mA and V_{D1} = 0.7 V. Using Equation (9.10), I_{D2} is calculated by

$$I_{D2} = \frac{V_{DC} - V_{D1}}{R} \tag{9.16}$$

Using Equation (9.15), V_{D2} is calculated by

$$V_{D2} = V_{D1} + nV_T \ln\left(\frac{I_{D2}}{I_{D1}}\right) \tag{9.17}$$

Using Equation (9.10), I_{D3} is calculated by

$$I_{D3} = \frac{V_{DC} - V_{D2}}{R} \tag{9.18}$$

Using Equation (9.15) , V_{D3} is calculated by

$$V_{D3} = V_{D1} + nV_T \ln\left(\frac{I_{D3}}{I_{D1}}\right) \tag{9.19}$$

Similarly, I_{D4} and V_{D4} are calculated by

$$I_{D4} = \frac{V_{DC} - V_{D3}}{R} \tag{9.20}$$

$$V_{D4} = V_{D1} + nV_T \ln(\frac{I_{D4}}{I_{D1}}) \tag{9.21}$$

The iteration is stopped when V_{Dn} is approximately equal to V_{Dn-1} or I_{Dn} is approximately equal to I_{Dn-1} to the desired decimal points. The iteration technique is particularly facilitated by using computers. Example 9.4 illustrates the use of MATLAB for doing the iteration technique.

Example 9.4

Redo Example 9.3 using the iterative technique. The iteration can be stopped when the current and previous value of the diode voltage are different by 10^{-7} volts.

Solution

MATLAB Script

```
% Determination of diode operating point using
% iterative method
k = 1.38e-23;q = 1.6e-19;
t1 = 273 + 25; vt = k*t1/q;
vdc = 10;
r = 1.0e4;
n = 2;
id(1) = 1.0e-3; vd(1) = 0.7;
reltol = 1.0e-7;
i = 1;
vdiff = 1;
while vdiff > reltol
  id(i+1) = (vdc - vd(i))/r;
  vd(i+1) = vd(i) + n*vt*log(id(i+1)/id(i));
  vdiff = abs(vd(i+1) - vd(i));
  i = i+1;
end
k = 0:i-1;
% operating point of diode is (vdiode, idiode)
idiode = id(i)
```

```
vdiode = vd(i)
% Plot the voltages during iteration process
plot(k,vd,'wo')
axis([-1,5,0.6958,0.701])
title('Diode Voltage during Iteration')
xlabel('Iteration Number')
ylabel('Voltage, V')
```

From the MATLAB program, we have

```
idiode =
          9.3037e-004

vdiode =
          0.6963
```

Thus $I_D = 0.9304$ mA and $V_D = 0.6963$ V. Figure 9.7 shows the diode voltage during the iteration process.

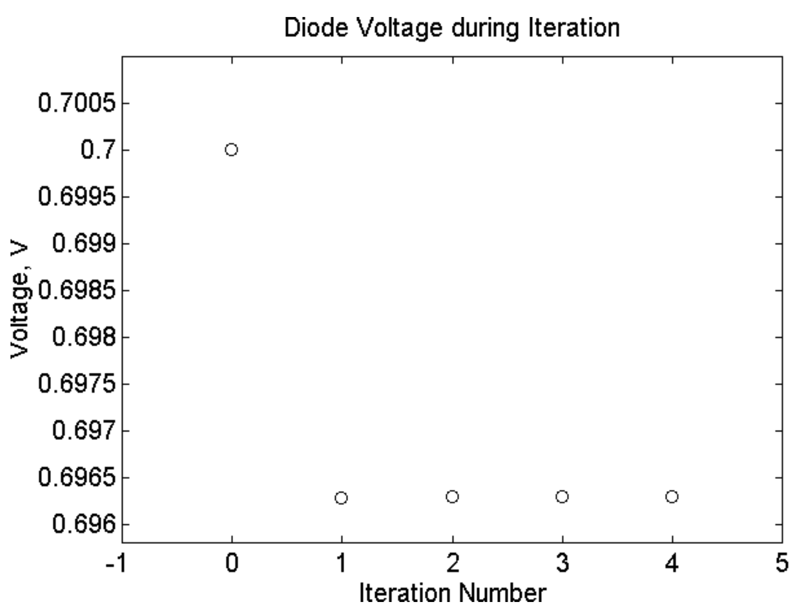


Figure 9.7 Diode Voltage during Iteration Process

9.3 HALF-WAVE RECTIFIER

A half-wave rectifier circuit is shown in Figure 9.8. It consists of an alternating current (ac) source, a diode and a resistor.

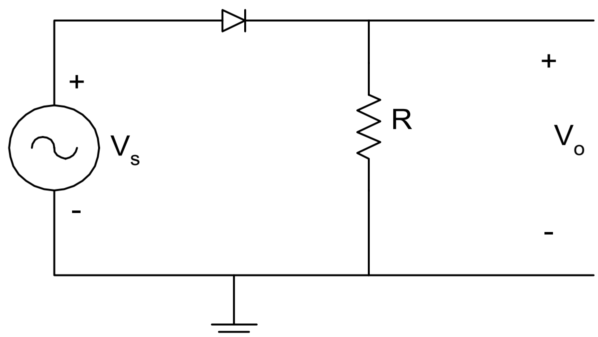


Figure 9.8 Half-wave Rectifier Circuit

Assuming that the diode is ideal, the diode conducts when source voltage is positive, making

$$v_0 = v_S \qquad \text{when } v_S \geq 0 \qquad (9.22)$$

When the source voltage is negative, the diode is cut-off, and the output voltage is

$$v_0 = 0 \qquad \text{when } v_S < 0 \qquad (9.23)$$

Figure 9.9 shows the input and output waveforms when the input signal is a sinusoidal signal.

The battery charging circuit, explored in the following example, consists of a source connected to a battery through a resistor and a diode.

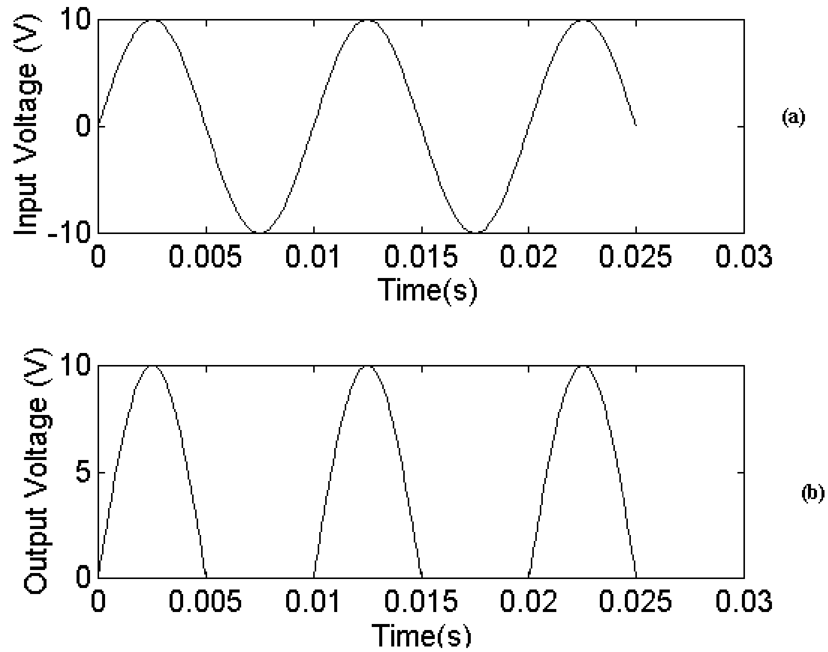


Figure 9.9 (a) Input and (b) Output Waveforms of a Half-wave Rectifier Circuit

Example 9.5

A battery charging circuit is shown in Figure 9.10. The battery voltage is $V_B = 11.8$ V. The source voltage is $v_S(t) = 18\sin(120\pi t)$ V and the resistance is R = 100 Ω. Use MATLAB (a) to sketch the input voltage, (b) to plot the current flowing through the diode, (c) to calculate the conduction angle of the diode, and (d) calculate the peak current. (Assume that the diode is ideal.)

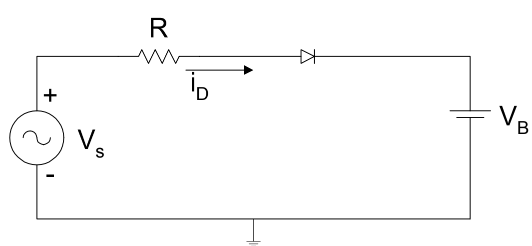


Figure 9.10 A Battery Charging Circuit

***Solution*:**

When the input voltage v_S is greater than V_B, the diode conducts and the diode current, i_d, is given as

$$i_d = \frac{V_S - V_B}{R} \tag{9.24}$$

The diode starts conducting at an angle θ, given by $v_S \geq V_B$, i.e.,

$$18\sin\theta_1 = 18\sin(120\pi t_1) = V_B = 11.8$$

The diode stops conducting current when $v_s \leq V_B$

$$18\sin\theta_2 = 18\sin(120\pi t_2) = V_B$$

due to the symmetry

$$\theta_2 = \pi - \theta_1$$

MATLAB Program:

```
diary ex9_5.dat
% Baltery charging circuit
period = 1/60;
 period2 = period*2;
inc =period/100;
npts = period2/inc;
vb = 11.8;

t = [];
for i = 1:npts
  t(i) = (i-1)*inc;
  vs(i) = 18*sin(120*pi*t(i));
    if vs(i) > vb
     idiode(i) = (vs(i) -vb)/r;
    else
     idiode(i) = 0;
    end
end
```

```
subplot(211), plot(t,vs)
%title('Input Voltage')
xlabel('Time (s)')
ylabel('Voltage (V)')
text(0.027,10, 'Input Voltage')
subplot(212), plot(t,idiode)
%title('Diode Current')
xlabel('Time (s)')
ylabel('Current(A)')
text(0.027, 0.7e-3, 'Diode Current')

% conduction angle
theta1 = asin(vb/18); theta2 = pi - theta1;
acond = (theta2 -theta1)/(2*pi)
% peak current
pcurrent = (18*sin(pi/2) - vb)/r
% pcurrent = max(idiode)
diary
```

The conduction angle, acond, and the peak current, pcurrent, are

```
acond =
       0.2724

pcurrent =
          0.0620
```

Figure 9.11 shows the input voltage and diode current.

The output of the half-wave rectifier circuit of Figure 9.8 can be smoothed by connecting a capacitor across the load resistor. The smoothing circuit is shown in Figure 9.12.

When the amplitude of the source voltage V_S is greater than the output voltage, the diode conducts and the capacitor is charged. When the source voltage becomes less than the output voltage, the diode is cut-off and the capacitor discharges with the time constant *CR*. The output voltage and the diode current waveforms are shown in Figure 9.13.

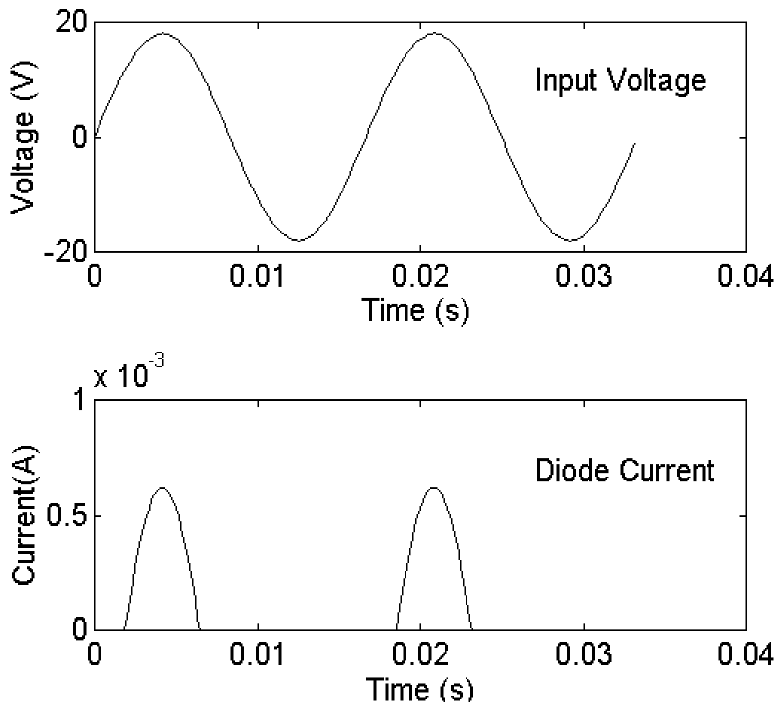


Figure 9.11 Input Voltage and Diode Current

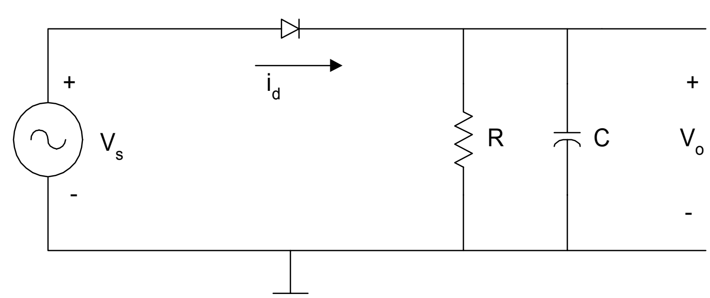


Figure 9.12 Capacitor Smoothing Circuit

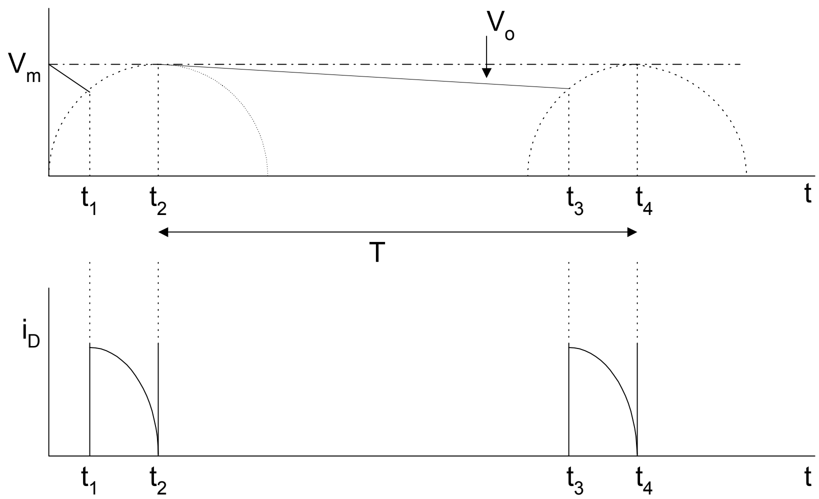


Figure 9.13 (a) Output Voltage and (b) Diode Current for Half-wave Rectifier with Smoothing Capacitor Filter

In Figure 9.12(a), the output voltage reaches the maximum voltage V_m, at time $t = t_2$ to $t = t_3$, the diode conduction ceases, and capacitor discharges through R. The output voltage between times t_2 and t_3 is given as

$$v_0(t) = V_m e^{-\left(\frac{t-t_2}{RC}\right)} \qquad t_2 < t < t_3 \qquad (9.25)$$

The peak to peak ripple voltage is defined as

$$\begin{aligned} V_r &= v_0(t_2) - v_0(t_3) = V_m - V_m e^{-\left(\frac{t_3-t_2}{RC}\right)} \\ &= V_m \left[1 - e^{-\left(\frac{t_3-t_2}{RC}\right)}\right] \end{aligned} \qquad (9.26)$$

For large values C such that $CR >> \left(t_3 - t_2\right)$, we can use the well-known exponential series approximation

$$e^{-x} \cong 1 - x \qquad \text{for } |x| << 1$$

Thus, Equation (9.26) approximates to

$$V_r = \frac{V_m(t_3 - t_2)}{RC} \tag{9.27}$$

The discharging time for the capacitor, $(t_3 - t_2)$, is approximately equal to the period of the input ac signal, provided the time constant is large. That is,

$$t_3 - t_2 \cong T = \frac{1}{f_0} \tag{9.28}$$

where

f_0 is the frequency of the input ac source voltage.

Using Equation (9.28), Equation (9.27) becomes

$$V_{r(peak-to-peak)} = \frac{V_m}{f_0 CR} \tag{9.29}$$

For rectifier circuits, because $RC >> T$, the output voltage decays for a small fraction of its fully charged voltage, and the output voltage may be regarded as linear. Therefore, the output waveform of Figure 9.12 is approximately triangular. The *rms* value of the triangular wave is given by

$$V_{rms} = \frac{V_{peak-to-peak}}{2\sqrt{3}} = \frac{V_m}{2\sqrt{3} f_o CR} \tag{9.30}$$

The approximately dc voltage of the output waveform is

$$V_{dc} = V_m - \frac{V_r}{2} = V_m - \frac{V_m}{2 f_o CR} \tag{9.31}$$

9.3.1 MATLAB function fzero

The MATLAB **fzero** is used to obtain the zero of a function of one variable. The general form of the **fzero** function is

$$fzero('function', x1)$$
$$fzero('function', x1, tol)$$

where

$fzero('funct', x1)$ finds the zero of the function $funct(x)$ that is near the point $x1$.

$fzero('funct', x1, tol)$ returns zero of the function $funct(x)$ accurate to within a relative error of tol.

The MATLAB function fzero is used in the following example.

Example 9.6

For a capacitor smoothing circuit of Figure 9.12, if R = 10KΩ, C = 100µF, and $v_S(t) = 120\sqrt{2}\sin(120\pi t)$,

(a) use MATLAB to calculate the times t_2, t_3, of Figure 9.12;

(b) compare the capacitor discharge time with period of the input signal.

Solution

The maximum value of $v_S(t)$ is $120\sqrt{2}$, and it occurs at $120\pi t_2 = \frac{\pi}{2}$, thus

$$t_2 = \frac{1}{240} = 0.00417 \text{ s}$$

The capacitor discharge waveform is given by

$$v_C(t) = 120\sqrt{2}\exp\left(-\frac{(t - t_2)}{RC}\right) \qquad t_2 < t < t_3$$

At $t = t_3$ $\qquad v_C(t) = v_S(t)$,

Defining $v(t)$ as

$$v(t) = 120\sqrt{2}\sin\left(120\pi(t - t_p)\right) - 120\sqrt{2}\exp\left(-\frac{(t - t_2)}{RC}\right)$$

Then,

$$v(t_3) = 0 = 120\sqrt{2}\sin\left(120\pi(t_3 - t_p)\right) - 120\sqrt{2}\exp\left(-\frac{(t_3 - t_2)}{RC}\right)$$

Thus,

$$v(t_3) = 0 = \sin\left(120\pi(t_3 - t_p)\right) - \exp\left(-\frac{(t_3 - t_2)}{RC}\right) \qquad (9.32)$$

MATLAB is used to solve Equation (9.32)

MATLAB Script

```
diary ex9_6.dat
% Capacitance discharge time for smoothing capacitor
% filter circuit
vm = 120*sqrt(2);
f0 = 60; r =10e3; c = 100e-6;
t2 = 1/(4*f0);
tp = 1/f0;

% use MATLAB function fzero to find the zero of a
% function of one variable
rc = r*c;
t3 = fzero('sinexpf1',4.5*t2);
tdis_cap = t3- t2;
fprintf('The value of t2 is %9.5f  s\n', t2)
fprintf('The value of t3 is %9.5f s\n', t3)
fprintf('The capacitor discharge time is %9.5f  s\n', tdis_cap)
fprintf('The period of input signal is %9.5f  s\n', tp)
diary

%
function y = sinexpf1(t)
t2 = 1/240; tp = 1/60;
rc = 10e3*100e-6;
y = sin(120*pi*(t-tp)) - exp(-(t-t2)/rc);
end
```

The results are

The value of t2 is 0.00417 s
The value of t3 is 0.02036 s
The capacitor discharge time is 0.01619 s
The period of input signal is 0.01667 s

9.4 FULL-WAVE RECTIFICATION

A full-wave rectifier that uses a center-tapped transformer is shown in Figure 9.14.

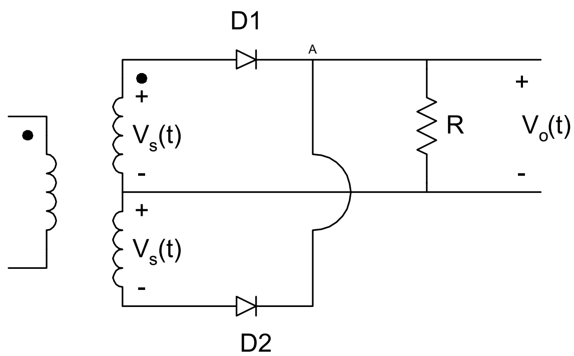


Figure 9.14 Full-wave Rectifier Circuit with Center-tapped Transformer

When $v_S(t)$ is positive, the diode D1 conducts but diode D2 is off, and the output voltage $v_0(t)$ is given as

$$v_0(t) = v_S(t) - V_D \tag{9.33}$$

where

V_D is a voltage drop across a diode.

When $v_S(t)$ is negative, diode D1 is cut-off but diode D2 conducts. The current flowing through the load R enters it through node A. The output voltage is

$$v(t) = \left|v_S(t)\right| - V_D \tag{9.34}$$

A full-wave rectifier that does not require a center-tapped transformer is the bridge rectifier of Figure 9.15.

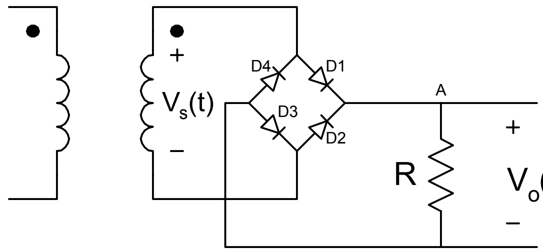


Figure 9.15 Bridge Rectifier

When $v_S(t)$ is negative, the diodes D2 and D4 conduct, but diodes D1 and D3 do not conduct. The current entering the load resistance R enters it through node A. The output voltage is

$$v(t) = \left|v_S(t)\right| - 2V_D \tag{9.35}$$

Figure 9.16 shows the input and output waveforms of a full-wave rectifier circuit assuming ideal diodes.

The output voltage of a full-wave rectifier circuit can be smoothed by connecting a capacitor across the load. The resulting circuit is shown in Figure 9.17.

The output voltage and the current waveforms for the full-wave rectifier with RC filter are shown in Figure 9.18.

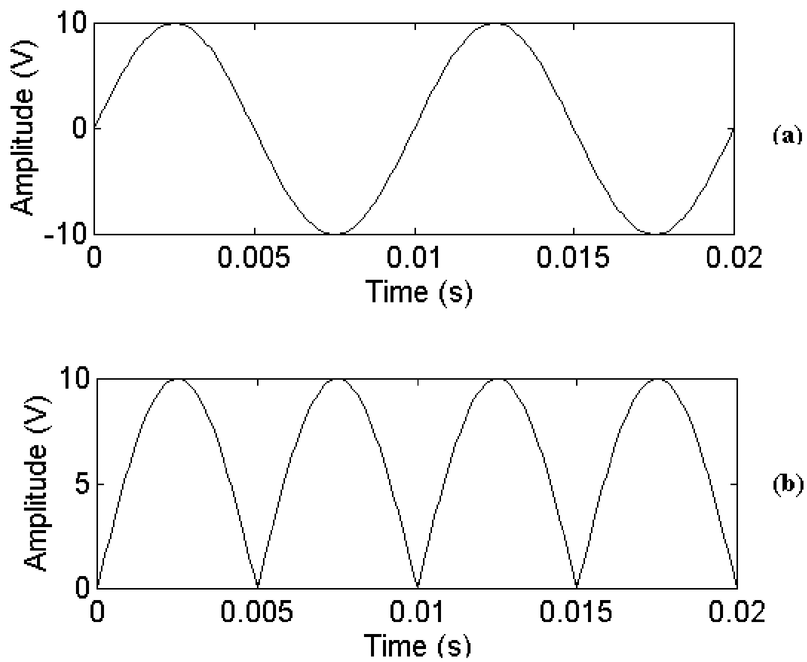


Figure 9.16 (a) Input and (b) Output Voltage Waveforms for Full-wave Rectifier Circuit

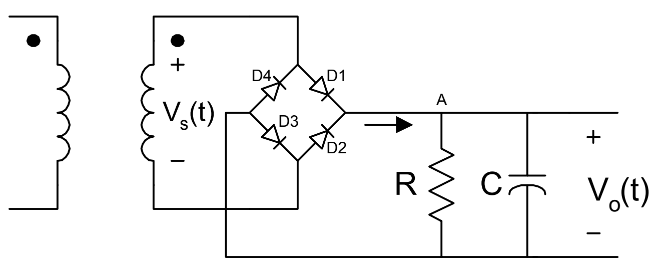


Figure 9.17 Full-wave Rectifier with Capacitor Smoothing Filter

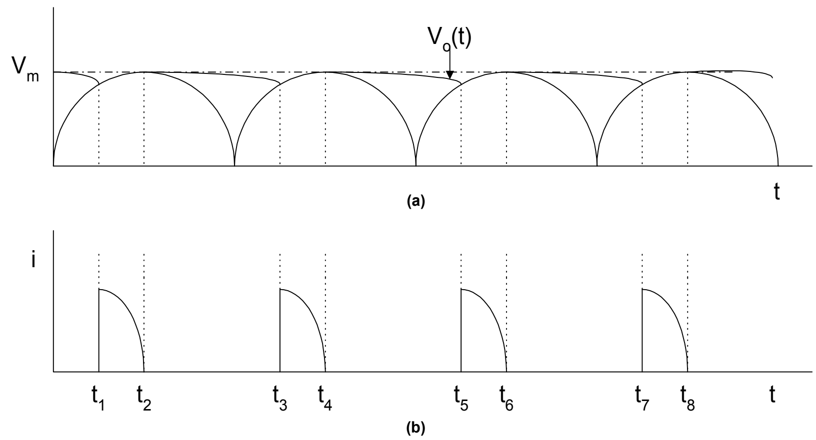


Figure 9.18 (a) Voltage and (b) Current Waveform of a Full-wave Rectifier with RC Filter

From Figures 9.13 and 9.18, it can be seen that the frequency of the ripple voltage is twice that of the input voltage. The capacitor in Figure 9.17 has only half the time to discharge. Therefore, for a given time constant, *CR*, the ripple voltage will be reduced, and it is given by

$$V_{r(peak-to-peak)} = \frac{V_m}{2 f_o CR} \tag{9.36}$$

where

V_m is peak value of the input sinusoidal waveform

f_0 frequency of the input sinusoidal waveform

The rms value of the ripple voltage is

$$V_{rms} = \frac{V_m}{4\sqrt{3} f_o CR} \tag{9.37}$$

and the output dc voltage is approximately

$$V_{dc} = V_m - \frac{V_r}{2} = V_m - \frac{V_m}{4 f_o CR} \tag{9.38}$$

Example 9.7

For the full-wave rectifier with RC filter shown in Figure 9.17, if $v_S(t) = 20\sin(120\pi t)$ and $R = 10\text{K}\Omega$, $C = 100\mu\text{F}$, use MATLAB to find the
(a) peak-to-peak value of ripple voltage,
(b) dc output voltage,
(c) discharge time of the capacitor,
(d) period of the ripple voltage.

Solution

Peak-to-peak ripple voltage and dc output voltage can be calculated using Equations (9.36) and (9.37), respectively. The discharge time of the capacitor is the time $(t_3 - t_1)$ of Figure 9.19.

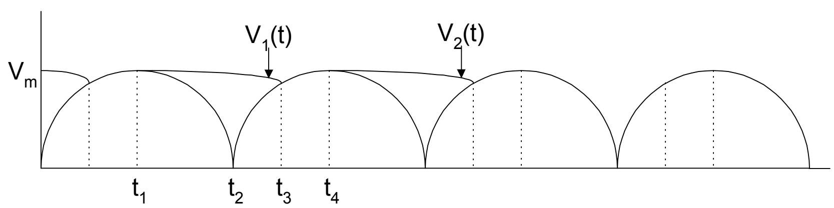


Figure 9.19 Diagram for Calculating Capacitor Discharge Time

$$v_1(t) = V_m \exp\left[-\frac{(t - t_1)}{RC}\right] \tag{9.39}$$

$$v_2(t) = \left|V_m \sin[2\pi(t - t_2)]\right| \tag{9.40}$$

$v_1(t)$ and $v_2(t)$ intersect at time t_3.

The period of input waveform, $v_S(t)$ is $T = \frac{1}{240}$ s

Thus,

$$t_1 = \frac{T}{4} = \frac{1}{240}\ \text{s}, \quad \text{and} \quad t_2 = \frac{T}{2} = \frac{1}{120}\ \text{s} \tag{9.41}$$

MATLAB Script

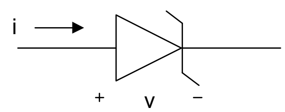
```
diary ex9_7.dat
% Full-wave rectifier
%
period = 1/60;
t1 = period/4;
vripple = 20/(2*60*10e3*100e-6);
vdc = 20 - vripple/2;
t3 = fzero('sinexpf2',0.7*period);
tdis_cap = t3 - t1;
fprintf('Ripple value (peak-peak) is %9.5f V\n', vripple)
fprintf('DC output voltage is %9.5f V\n', vdc)
fprintf('Capacitor discharge time is %9.5f s\n', tdis_cap)
fprintf('Period of ripple voltage is %9.5f s\n', 0.5*period)
diary

%
%
function y = sinexpf2(t)
t1 = 1/240; t2 = 2*t1; rc = 10e3*100e-6;
y = 20(sin(120*pi*(t - t2))) - exp(-(t-t1)/rc);
end
```

The results are

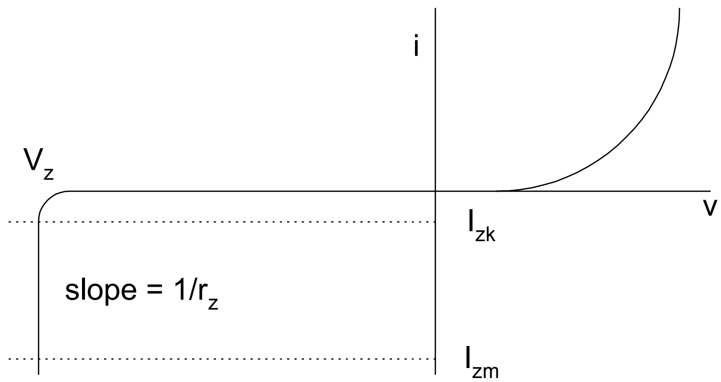
```
Ripple value (peak-peak) is   0.16667 V
DC output voltage is  19.91667 V
Capacitor discharge time is   0.00800 s
Period of ripple voltage is   0.00833 s
```

9.5 ZENER DIODE VOLTAGE REGULATOR CIRCUITS

The zener diode is a pn junction diode with controlled reverse-biased break-down voltage. Figure 9.20 shows the electronic symbol and the current-voltage characteristics of the zener diode.



(a)



(b)

Figure 9.20 Zener Diode (a) Electronic Symbol (b) I-V Characteristics

I_{ZK} is the minimum current needed for the zener to breakdown. I_{ZM} is the maximum current that can flow through the zener without being destroyed. It is obtained by

$$I_{ZM} = \frac{P_Z}{V_Z} \tag{9.42}$$

where P_Z is the zener power dissipation.

The incremental resistance of the zener diode at the operating point is specified by

$$r_Z = \frac{\Delta V_Z}{\Delta I_Z} \tag{9.43}$$

One of the applications of a zener diode is its use in the design of voltage reference circuits. A zener diode shunt voltage regulator circuit is shown in Figure 9.21

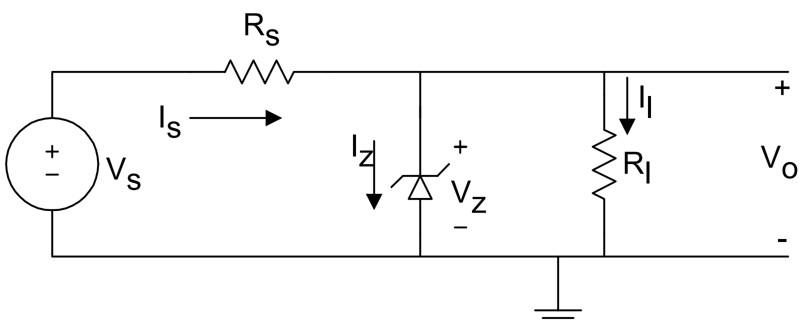


Figure 9.21 Zener Diode Shunt Voltage Regulator Circuit

The circuit is used to provide an output voltage, V_0, which is nearly constant. When the source voltage is greater than the zener breakdown voltage, the zener will break down ` and the output voltage will be equal to the zener breakdown voltage. Thus,

$$V_0 = V_Z \tag{9.44}$$

From Kirchoff current law, we have

$$I_S = I_Z + I_L \tag{9.45}$$

and from Ohm's Law, we have

$$I_S = \frac{V_S - V_Z}{R_S} \tag{9.46}$$

and

$$I_L = \frac{V_O}{R_L} \tag{9.47}$$

Assuming the load resistance R_L is held constant and V_S (which was originally greater than V_Z) is increased, the source current I_S will increase; and since I_L is constant, the current flowing through the zener will increase. Conversely, if R is constant and V_S decreases, the current flowing through the zener will decrease since the breakdown voltage is nearly constant; the output voltage will remain almost constant with changes in the source voltage V_S.

Now assuming the source voltage is held constant and the load resistance is decreased, then the current I_L will increase and I_Z will decrease. Conversely, if V_S is held constant and the load resistance increases, the current through the load resistance I_L will decrease and the zener current I_Z will increase.

In the design of zener voltage regulator circuits, it is important that the zener diode remains in the breakdown region irrespective of the changes in the load or the source voltage. There are two extreme input/output conditions that will be considered:

(1) The diode current I_Z is minimum when the load current I_L is maximum and the source voltage V_S is minimum.

(2) The diode current I_Z is maximum when the load current I_L is minimum and the source voltage V_S is maximum.

From condition (1) and Equation (9.46), we have

$$R_S = \frac{V_{S,\min} - V_Z}{I_{L,\max} + I_{Z,\min}} \tag{9.48}$$

Similarly, from condition (2), we get

$$R_S = \frac{V_{S,\max} - V_Z}{I_{L,\min} + I_{Z,\max}} \tag{9.49}$$

Equating Equations (9.48) and (9.49) , we get

$$(V_{S,\min} - V_Z)(I_{L,\min} + I_{Z,\max}) = (V_{S,\max} - V_Z)(I_{L,\max} + I_{Z,\min}) \tag{9.50}$$

We use the rule of thumb that the maximum zener current is about ten times the minimum value, that is

$$I_{Z,\min} = 0.1 I_{Z,\max} \tag{9.51}$$

Substituting Equation (9.49) into Equation (9.51), and solving for $I_{Z,\max}$, we obtain

$$I_{Z,\max} = \frac{I_{L,\min}(V_Z - V_{S,\min}) + I_{L,\max}(V_{S,\max} - V_Z)}{V_{S,\min} - 0.9V_Z - 0.1V_{S,\max}} \tag{9.52}$$

Knowing $I_{Z,\max}$, we can use Equation (9.49) to calculate R_S. The following example uses MATLAB to solve a zener voltage regulator problem.

Example 9.8

A zener diode voltage regulator circuit of Figure 9.21 has the following data:

$30 \le V_S \le 35$V; R_L = 10K, R_S = 2K

$$V_Z = -20 + 0.05I \qquad \text{for } -100 \text{ mA} \le I < 0 \tag{9.53}$$

Use MATLAB to

(a) plot the zener breakdown characteristics, (b) plot the loadline for V_S = 30V and V_S = 35 V, (c) determine the output voltage when V_S = 30V and V_S = 35V.

Solution

Using Thevenin Theorem, Figure 9.21 can be simplified into the form shown in Figure 9.22.

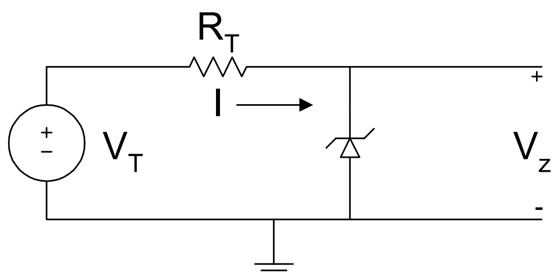


Figure 9.22 Equivalent Circuit of Voltage Regulator Circuit

$$V_T = \frac{V_S R_L}{R_L + R_S} \tag{9.54}$$

and

$$R_T = R_L \| R_S \tag{9.55}$$

Since R_L = 10K, R_S = 2K, R_T = (10)(2K) / 12 K = 1.67 KΩ

when V_S = 30V, V_T = (30)(10) / 12 = 25 V

when V_S = 35V, V_T = (35)(10) / 12 = 29.17 V

The loadline equation is

$$V_T = R_T I + V_Z \tag{9.56}$$

Equations (9.53) and (9.56) are two linear equations solving for I, so we get

$$V_Z = V_T - R_T I = -20 + 0.05I$$

$$\Rightarrow \qquad I = \frac{(V_T + 20)}{R_T + 0.05} \tag{9.57}$$

From Equations (9.56) and (9.57), the output voltage (which is also zener voltage) is

$$V_Z = V_T - R_T I = V_T - \frac{R_T (V_T + 20)}{R_T + 0.05} \tag{9.58}$$

MATLAB program

```
diary ex9_8.dat
% Zener diode voltage regulator
vs1 = -30; vs2 = -35; rl =10e3; rs = 2e3;
i = -50e-3: 5e-3 :0;
vz = -20 + 0.05*i;
m = length(i);
i(m+1) = 0; vz(m+1) = -10;
i(m+2) = 0; vz(m+2) = 0;
% loadlines
vt1 = vs1*rl/(rl+rs);
vt2 = vs2*rl/(rl+rs);
rt = rl*rs/(rl+rs);
l1 = vt1/20;
l2 = vt2/20;
v1 = vt1:abs(l1):0;
i1 = (vt1 - v1)/rt;
v2 = vt2:abs(l2):0;
i2 = (vt2 - v2)/rt;
% plots of Zener characteristics, loadlines
plot(vz,i,'w',v1,i1,'w',v2,i2,'w')
axis([-30,0,-0.03,0.005])
title('Zener Voltage Regulator Circuit')
xlabel('Voltage (V)')
ylabel('Current (A)')
text(-19.5,-0.025,'Zener Diode Curve')
text(-18.6,-0.016, 'Loadline (35 V Source)')
text(-14.7,-0.005,'Loadline (30 V Source)')
% output voltage when vs = -30v
ip1 = (vt1 + 20)/(rt + 0.05)
vp1 = vt1 - rt*(vt1+20)/(rt + 0.05)
% output voltage when vs = -35v
ip2 = (vt2 + 20)/(rt + 0.05)
vp2 = vt2 - rt*(vt2+20)/(rt + 0.05)
diary
```

The results obtained are

ip1 =
-0.0030

vp1 =
-20.0001

ip2 =
-0.0055

vp2 =
-20.0003

When the source voltage is 30 V, the output voltage is 20.0001 V.
In addition, when the source voltage is 35 V, the output voltage is 20.0003 V.

The zener breakdown characteristics and the loadlines are shown in Figure 9.23.

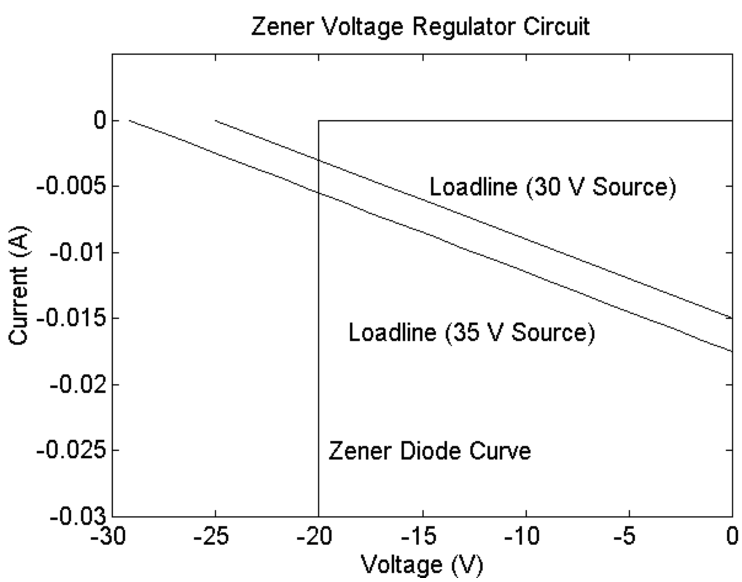


Figure 9.23 Zener Characteristics and Loadlines

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EXERCISES

9.1 Use the iteration technique to find the voltage V_D and the I_D of Figure P9.1. Assume that T = 25 ^{o}C, n = 1.5, I_S = 10^{-16} A. Stop current the iteration when $|V_n - V_{n-1}| < 10^{-9}$ V.

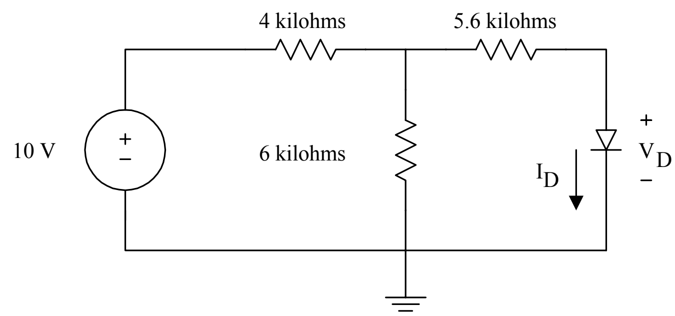

Figure P9.1 A Diode Circuit

9.2 A zener diode has the following I-V characteristics

Reverse Voltage (V)	Reverse Current (A)
-2	-1.0e-10
-4	-1.0e-10
-6	-1.0e-8
-8	-1.0e-5
-8.5	-2.0e-5
-8.7	-15.0e-3
-8.9	-43.5 e-3

(a) Plot the reverse characteristics of the diode. (b) What is the breakdown voltage of the diode? (c) Determine the dynamic resistance of the diode in its breakdown region.

9.3 A forward-biased diode has the following corresponding voltage and current.
(a) Plot the static I-V characteristics.

(b) Determine the diode parameters I_S and n.

(c) Calculate the dynamic resistance of the diode at V_S = 0.5 V.

Forward Voltage, V	**Forward Current, A**
0.2	7.54e-7
0.3	6.55e-6
0.4	5.69e-5
0.5	4.94e-4
0.6	4.29e-3
0.7	3.73e-2

.

9.4 For Figure P9.4,

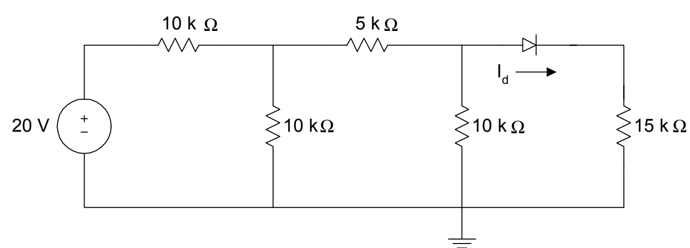

Figure P9.4 Diode Circuit

(a) Use iteration to find the current through the diode. The iteration can be stopped when $\left|I_{dn} - I_{dn-1}\right| < 10^{-12}$ A.

(b) How many iterations were performed before the required result was obtained? Assume a temperature of 25 °C, emission coefficient, n, of 1.5, and the reverse saturation current, I_S, is 10^{-16} A.

9.5 For a full-wave rectifier circuit with smoothing capacitor shown in Figure 9.17, if $v_S(t) = 100\sin(120\pi t)$ V, R = 50KΩ, C = 250μF, using MATLAB

(a) Plot the input and output voltages when the capacitor is disconnected from the load resistance R.

(b) When the capacitor is connected across load resistance R, determine the conduction time of the diode.

(c) What is the diode conduction time?

9.6 For the voltage regulator circuit shown in Figure 9.21, assume that 50 < V_S < 60 V, R_L = 50K, R_S = 5K, V_S = -40 + 0.01 I. Use MATLAB to

(a) Plot the zener diode breakdown characteristics.

(b) Plot the loadline for V_S = 50 V and V_S = 60V.

(c) Determine the output voltage and the current flowing through the source resistance R_S when V_S = 50V and V_S = 60V.

9.7 For the zener voltage regulator shown in Figure 9.21, If V_S = 35V, R_S = 1KΩ, $V_Z = -25 + 0.02I$ and 5K < R_L < 50K, use MATLAB to

(a) Plot the zener breakdown characteristics

(b) Plot the loadline when R_L = 5K and R_L = 50K.

(c) Determine the output voltage when R_L = 5KΩ and R_L = 50KΩ.

(d) What is the power dissipation of the diode when R_L = 50KΩ?