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Mathematics and the Divine:
A Historical Study

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MATHEMATICS AND THE DIVINE: A HISTORICAL STUDY

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This book is dedicated to our children

Quod mathematica nos iuvet plurimum
in diversorum divinorum apprehensione

(Mathematics helps us greatly
in understanding various divine truths)

Cusanus, *De Docta Ignorantia* I, XI

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Preface

The idea that is at the basis of the present book was born several years ago. A comprehensive historical study of the relations between mathematics on the one hand, and metaphysics, religion and mysticism on the other hand, did not exist. Because of the expanse of the subject we decided to combine the expertise of numerous experts. Although it proved to be impossible to give attention to everything in the present volume, a wide range of topics was covered, as is evident from the titles in the list of contents. We believe that at least the major aspects of the subject are covered. Wherever possible, sources of quotations are acknowledged. Every effort has been made to discover the original source of the illustrations used and to obtain permission to include them.

We are very grateful to the scholars who contributed the articles. Moreover, we gratefully acknowledge the following individuals who have assisted us in various ways in making this book a reality: Marc Bergmans, Henk Bos, Vera Brauns, Sébastien Busson, Georgia Gauley, Ineke Hilhorst, Jan Hogendijk, Jan Willem van Holten, Bas Jongeling, Jan van Mill, Mickaël Robert, Karel Schmidt Jr., Arjen Sevenster, Eric-Jan Tuininga.

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October 2004

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Introduction

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MATHEMATICS AND THE DIVINE: A HISTORICAL STUDY
Edited by T. Koetsier and L. Bergmans

1. The divine and mathematics

The history of mankind is a history of permanent struggle for survival. Human beings survive by means of cognitive systems. An essential part of a cognitive system is a ‘map’ of the world that helps us deal with reality. Animals possess cognitive systems as well, but human cognitive systems differ in that human beings have the capability of symbolic thought and symbolic language. Symbolic thought makes abstraction possible: with great precision human beings can describe situations and relations that they haven’t actually observed or experienced. Even in the Palaeolithic era the complexity of human cognitive systems was considerable as rock paintings and other archeological evidence show. The Palaeolithic cognitive systems can be called pre-scientific, but there is no essential difference between them and modern scientific cognitive systems other than that the latter are more systematic. The modern scientific cognitive systems developed out of the pre-scientific ones through a long process of trial and error.

It seems likely that from the moment human beings became aware of their own existence they started asking questions like: Who are we? Where are we from? Where are we heading? In this book religion will be interpreted as the response that people give to such questions.[1] The answers to these questions are part of the cognitive system of the civilization involved. The questions are all related to what the German theologian Paul Tillich called *ultimate concern*. Buddhists, Hindus, Sikhs, Taoists, Muslims, Jews, Christians, all of them display in their thinking this ultimate concern. Characteristics of an ultimate concern are: it has priority over other concerns, it is involved in all human experience and the emotions it involves are so unique that words like sacred or holy are used for it. Moreover this ultimate concern has its unique ritual and symbolic expressions.

In the course of history the religious aspect of human cognitive systems has undergone considerable development. In the 19th century E.B. Tylor proposed that religion began as animism and developed through polytheism to monotheism. And indeed the development of our cognitive systems shows a growing sophistication in all areas including religion. Features of a more sophisticated religious culture would be, for example, pantheism, mysticism and theology. In such a view, pantheism is the result of the development of reflective thought. Mysticism refers to the immediate experience of the divine. Theology is obviously also a product of higher religious culture; it is the endeavour to state in terms of general doctrines what is involved in religion. Tylor’s view of the general development of religion certainly has some value. Yet, because of the great variety of religious expression that has manifested itself in the course of history, it is too easy to view the development of religious thought as progressive. An extra complicating factor is that systems of allegiance like Nazism and Communism have much in common with religion.

In this book we will use the word divine to denote the whole of religious experience and its expression concerning the numinous or the transcendent. It has many aspects: knowledge, the methods used to acquire and represent this knowledge, the language used to speak about the divine, the institutions that support man’s relations with the divine, the experience of the divine of the individual, the applications of knowledge of the divine, etc.

Mathematical knowledge, in the widest sense of the word, has also always been a central part of human cognitive systems. It concerns structures, in particular the structure of space

[1] Cf. [21, p. 3].

and time, and as such, mathematical knowledge already played a role in Palaeolithic and Neolithic times in the form of counting and, for example, knowledge implicit in geometrical decorations. A great leap forward in mathematics was made when during the third and second millennia BCE more advanced forms of society evolved along the banks of some of the great rivers in Africa and Asia: the Nile, the Tigris and Euphrates, the Indus and Ganges, the Huang Ho and the Yang-tse. This oriental mathematics originated at least to a large extent as a practical science that facilitated calendar computations, tax collection, the administration of the harvest, the organization of public works, etc.

A major step forward in the development of mathematics was made in the context of Greek culture; both arithmetic and geometry developed into deductive theories and a wealth of new mathematical results was found. After the decline of Greek culture, mathematics continued to be practised, both in the Oriental and in the Greek traditions. Medieval mathematics in China and India, mathematics as developed in the world of Islam and mathematics in Medieval Europe all represent important chapters in the history of science. During and after the Renaissance the development towards modern mathematics really got underway. The history of mathematics after the Renaissance is one great success story. The invention of the calculus by Newton and Leibniz in the second half of the seventeenth century was so important that in the eighteenth century mathematical productivity concentrated almost exclusively on this particular invention and its applications. Until the nineteenth century mathematics could still be considered as the science of number and space. Especially the discovery of non-Euclidean geometries in the nineteenth century, but other developments as well, led to a different view of what mathematics is. For most mathematicians mathematics is now the science of mathematical structures. Euclidean space and the traditional number structures are very important, but only examples of mathematical structures amongst many others.

In this book we use the term mathematics in a broad sense. It refers to the objects of mathematical knowledge, to the knowledge itself, to the methods used to acquire and represent this knowledge, to the language of mathematics, to the role and nature of logic in its relation to mathematics, to the institutions that accompany mathematics, to the personal experience of the individual doing mathematics, to the applications of mathematics, etc.

The chapters of this book all concern the relation between mathematical and religious aspects of individual or collective cognitive systems. Mathematics in its relation with the divine has played a special role in the course of history. A particular complex of ideas that we owe among others to the Pythagoreans and Plato is responsible for this special role. Mathematics is abstract and it often seems absolute, universal, eternal and pure. More than other kinds of knowledge it possesses characteristics that we associate with the divine.

2. Three periods: The pre-Greek period, the Pythagorean–Platonic period and the period of the Scientific Revolution and its aftermath

We have not attempted to develop a general view of the relations between mathematics and the divine. The present book is primarily a collection of more or less chronologically ordered case studies. Yet it may be helpful to distinguish roughly three periods in the history of mathematics and its relation to the divine. In the first period mathematical knowledge

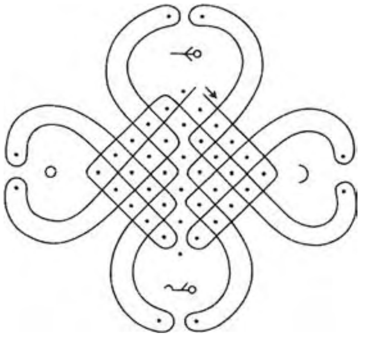


Fig. 1. The creation of the world according to the Tchokwe. Source: [47, p. 109].

was usually embedded in other forms of knowledge. This means that mathematical notions and mathematical truth were to a large extent not explicitly distinguished from other parts of reality. The Tchokwe people in Angola, for example, use geometrical drawings when they tell stories (Figure 1). They first set out orthogonal lattices of equidistant points with their fingertips. Then, while they tell the story, they draw a corresponding figure. The drawings should be made smoothly without lifting one finger. Figure 1 illustrates the creation of the world. The corresponding story runs as follows. First the Sun walked and walked until he found God. God gave him a cock. When the next morning the cock crowed, God said to the Sun: You may keep the cock, but you must return every morning. Since then the sun has appeared every morning. The Moon also went to see God. God gave him a cock as well. The next morning God ordered the Moon to come back every twenty-eight days. Since then the Moon has done exactly that. Finally man went to see God. God gave man a cock too. The next morning man had eaten the cock and God said: “The Sun and the Moon did not kill the cock; that is why they will never die. You ate the cock and that is why you must die too. But when you die, you will have to return here” [47, pp. 108–109]. The four characters represent God (top), man (bottom), Sun (left) and Moon (right).

This is an example of implicit mathematics. The feeling for patterns demonstrated by this drawing and the many other drawings the Tchokwe make when they tell stories, was not made explicit within their culture. Palaeolithic and Neolithic mathematics is implicit. The same holds for much of the mathematics in the irrigation societies that evolved on the banks of the great rivers in the last millennia BCE. Although it is probable that in the course of time an awareness developed in the irrigation societies with respect to the peculiar nature of mathematics, the Greeks most clearly discovered the possibility to study mathematics *in abstracto*.

With Greek mathematics the second period begins. The awareness of the abstract nature of mathematics implied the separation of mathematics from non-mathematical elements. It had an enormous effect on the way mathematics was done: the Greeks developed the possibility to do mathematics deductively. Until the twentieth century Euclid’s *Elements* (ca. 300 BCE) was considered a paradigmatic book. Book 1 (of the thirteen books) starts with Definitions, Postulates and Axioms that must be accepted without proof. Then follow Propositions that are all proved rigorously. There are no references in the *Elements* to applications, nor to any other non-mathematical aspects of life, such as religion. Yet the Pythagoreans and in particular Plato developed a very influential view of the world in which mathematics and the divine became closely associated. The divine origin of all things created the universe on the basis of mathematical principles and the believer wishing to get in touch with the divine had to study mathematics. Such ideas, especially in the form of Neo-Platonism, have exercised an enormous influence. During this period, which we will call the Pythagorean–Platonic period, magic, alchemy, astrology and other now discredited forms of knowledge were taken seriously by many philosophers and scientists.

The third period that we distinguish begins with the Scientific Revolution. The Scientific Revolution was followed by the Enlightenment. Until the Enlightenment there was a complex of religious values that was shared by everybody in the West. After the Enlightenment this was no longer the case. Mathematics played a major role in the Scientific Revolution. The universe came to be seen as a clockwork mechanism that could be understood in terms of mathematics. Although the Pythagorean–Platonic view that reality is structured mathematically was in fact confirmed, the effect was that for many the creator, God, eventually became a superfluous hypothesis. The reactions to the Scientific Revolution and its results were quite diverse. Some simply stuck to the idea that God created the world on a mathematical basis, some refused to accept the new developments in science, some divorced religion from science and finally there were those who abandoned religion and attempted to turn science, including mathematics, into a new faith. Another characteristic of this period is that it became the received view that positive science should be clearly distinguished from what came to be seen as pseudo-sciences: alchemy, magic, astrology, numerology, etc.

Our choice to distinguish three periods in the history of mathematics and its relation to the divine, the pre-Greek period, the Pythagorean–Platonic period and the period of the Scientific Revolution and its aftermath, is based on a Western perspective. Most of the contributions to this volume do indeed directly concern Western culture. In non-Western culture this periodisation makes no sense. In Chapter 1 of the present book, Ho Peng-Yoke discusses Chinese number mysticism. Chinese culture offers a nice example of mathematics being embedded in a wider context until long after Pythagoras. In fact, until the 19th century the Chinese equivalent of the word ‘mathematics’ encompassed philosophy, astrology, divination and aspects of mysticism. In Chapter 2, devoted to Indian culture, Kim Plofker, on the one hand, describes the picture of the cosmos presented in sacred texts such as the Puranas, consisting of divinely revealed truths. On the other hand, she describes the Siddhantas, astronomical treatises, written under the influence of Graeco-Babylonian and Hellenistic sources. Her theme is the delicate balance that existed between these two Indian cosmological traditions over the centuries and the way in which they have been viewed by historians.

3. The pre-Greek period and the ritual origin of mathematics

3.1. *Rock paintings and the Agnicayana*

In the early history of mankind, human cognitive systems were such that mathematical notions and mathematical truth were not clearly separated from truth concerning the divine. Let us consider two examples.

Fig. 2. Rock drawing in Puente Viesgo (Spain).

An aspect of the early cognitive systems was animism. The world was viewed as a Palaeolithic family, full of forces that were treated as individuals.[2] Shamanism was probably widespread. Characteristic of shamanism is that the shaman enters a trance in which he communicates with the spirits in order to heal the sick, foretell the future or control the behaviour of animals. Geometric rock paintings are found in many areas where shamanism traditionally occurred (Figure 2).

Clottes and Lewis-Williams have put forward an interesting hypothesis that connects shamanism and pre-historic rock paintings. If Clottes and Lewis-Williams are right, the geometric paintings correspond to geometrical figures that the shaman projected with open eyes on the wall in the first stage of his trance. The wall was experienced as a curtain separating the shaman from the world of the spirits. On the basis of modern studies about the effects of hallucinants, which describe similar phenomena, it is assumed that in the course of the trance the geometrical forms turned into animals with which the shaman could communicate. If this hypothesis is correct the geometrical rock paintings are among the earliest examples of mathematical patterns in a religious context.

Much more sophisticated is the mathematics in our second example. It concerns the role of geometry in an old Vedic ritual, called Agnicayana. The ritual is at least 2500 years old. In Vedic religion, fire, called *agni*, was worshipped and there was a cult of a plant called *soma*, probably a hallucinogen. The major Vedic rituals were dedicated to these two: Agni and Soma. We have a very good idea of what these rituals were like, because in 1955 Frits Staal became aware of the fact that this Vedic tradition was still alive in Kerala in southwest

[2] Cf. [9, p. 156].

Fig. 3. 17th century Siberian shaman. Source: [45]. Courtesy of the Library of the University of Amsterdam.

India, and in 1975 he documented Agnicayana, the “piling of Agni”, or, simply, Agni.[3] The Agnicayana is a complex ritual, which is the result of a long development. It takes twelve days of elaborate performances accompanied by recitations. Agni is a god and a divine messenger, who receives offerings during the ritual. Rituals like the Agnicayana are an essential element of Vedic culture and they have to be performed painstakingly according to strict rules.

One of the central elements of the Agnicayana ritual is the building of an altar consisting of five layers of bricks. The altar has the shape of a bird (Figures 4 and 5) and it is built in the course of the ritual in a precisely prescribed way out of bricks that have precisely prescribed shapes. Staal reports that the 1975 performance of the Agnicayana ritual was followed by a long series of other rites that were performed only to correct mistakes possibly committed in the course of the preceding twelve days [36, Vol. I, p. 15]. Figure 4 shows the order in which 57 of the bricks of the second layer are consecrated. These bricks have names. For example, the bricks 2 through 6 are called Skandhya or “Shoulder” and the bricks 22 through 26 are called Vrstisani or “Rain bringing”. The remaining bricks of this layer are called Space Fillers. Each layer consists of 200 bricks and each layer has an area of $\frac{71}{2}$ purusas. The word “purusa” means “man”. It denotes both the height of a man with his arms stretched upwards (approximately 2.2 m) and an area measure (approximately $2.2 \times 2.2 = 4.84$ m^2). Except for the vertical passage at the centre of the altar the interstices between bricks of any layer may never be above or under the interstices between bricks of an adjacent layer. This means that one needs two different patterns of bricks: one

[3]Cf. [36].

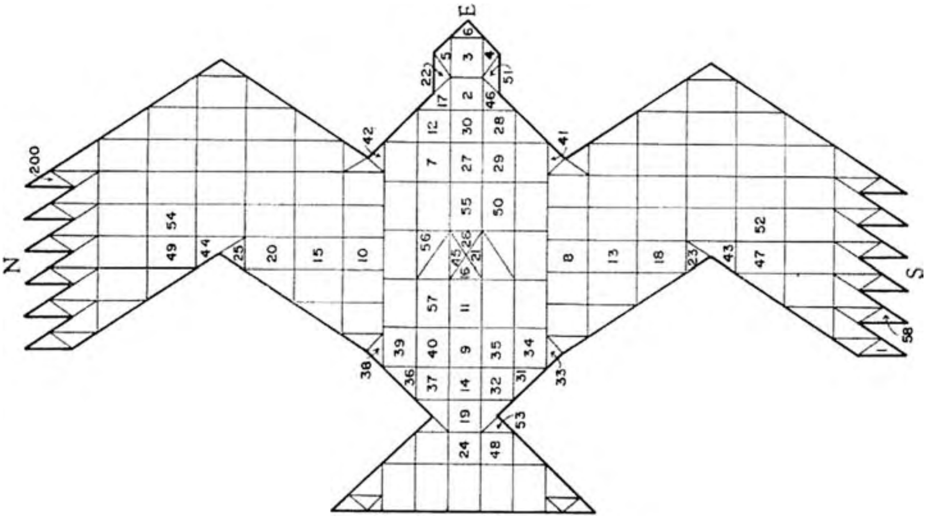


Fig. 4. Bricks in the second layer of the fire altar. Source: [36].

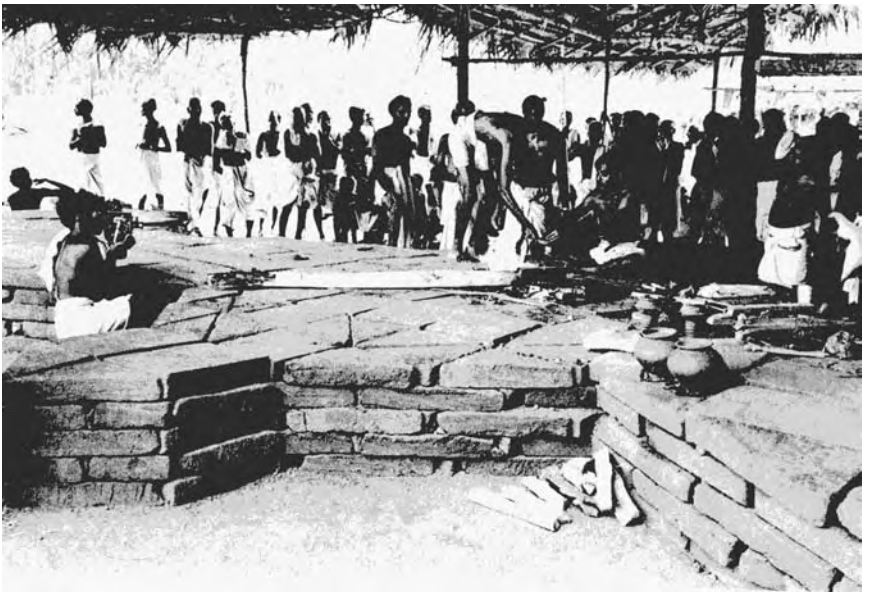


Fig. 5. A Vedic fire altar. Source: [36].

pattern for the 2nd and 4th layers and one for the 1st, 3rd and 5th layers. Rules like these concern the geometry of the altar and indeed there exists a class of geometrical sacred works, the Sulba Sutras (sulba = cord or rope), that have been called “manuals for altar construction”.

It is not easy to say what exactly the function of this ritual is, but it is clear that, just as in the case of the hypothesis of Clottes and Lewis-Williams, geometrical structures embedded in a context involving other elements are part of an approach to get in touch with the divine.

3.2. *Astronomy and the divine*

It has been argued that the more advanced forms of society that evolved in Africa and Asia during the fifth, fourth and third millennium BCE followed similar patterns in their development.[4] The increased productivity of these agricultural societies led to increased vulnerability. The growth in productivity led to an expansion of the population, which meant a greater dependence on agriculture, which in its turn led to greater vulnerability. The traditional dangers that man faced, natural disasters like plagues, droughts, inundations, locusts, etc., hostile groups of other people and the risk that the crop would fail or be spoilt by stupidity, negligence or greed, were increased. To cope with these threats a central role was played by a class of people who can be called priests. They possessed expertise in many areas and played a crucial role in the organization of society; they guarded and distributed the crops and by means of a strict system of rites, legitimized by their special relation with the supernatural, they compelled their people to behave in a disciplined way.[5]

In such societies the role of the "priests" will have automatically linked mathematics and religion. Nowadays we clearly distinguish between mathematics, astronomy, astrology and religion. In the mind of a Babylonian priest this was undoubtedly hardly the case at all.

The Babylonians described the earth as a flat disc floating on an ocean (Figure 6). On a Late Babylonian tablet in the British Museum we find a picture of the earth as conceived by the Babylonians. The earth is circular; the water surrounding it is called the 'Bitter River'. Beyond this river are several triangular regions for special creatures that are described in the text written above the drawing on the tablet. Near the upper edge of the disc there is a small region called 'mountain'. The two lines running down the map presumably represent the Tigris and the Euphrates. They run into the marshes near the Persian Gulf, represented by the two lower parallel lines. The small circles represent cities. The oblong above the centre seems to represent Babylon. The Babylonians may have thought of the sky as a dome or vault, although according to Lambert there are no indications of this.[6] The gods maintained the motions of the heavenly bodies. The powers of nature, the sun, the moon, the storm etc. were associated with gods and for the Babylonians celestial phenomena were astrologically significant. Both the mathematical regularity of the celestial phenomena and the occasional deviations from their normal regularity were believed to be related to sublunar events. The earliest known astrological omen texts are in fact Old Babylonian.[7] Lunar eclipses in particular attracted the attention of the Babylonian priests, but they were interested in other lunar phenomena and the sun and the weather as well. Presumably the first versions

[4] Cf. Johan Goudsblom, Eric Jones, Stephen Mennell, *The Course of Human History: Economic Growth, Social Process, and Civilization,* 2001.

[5] Cf. Op. cit.

[6] W.G. Lambert, The Cosmology of Sumer and Babylon, in [5, pp. 42–65].

[7] Although there was undoubtedly astronomical interest, according to [25, p. 33], there is no trace of any real astrology in the earlier Sumerian sources.

Fig. 6. The Babylonian Mappa Mundi: British Museum 92687.

of the important Babylonian astrological text *Enuma Anu Enlil* (EAE) were written during the Old Babylonian period. The EAE is an omen series consisting of about 70 tablets. The omens have the form of a description of a celestial phenomenon, followed by the repercussions it will have. For example: “An eclipse of the evening watch means plague, an eclipse of the middle watch means diminishing market, an eclipse of the morning watch means the sick will recover” [25, p. 106]. Babylonian astrology concerned the welfare of the state and the king. Only in Hellenistic astrology was the horoscope of the individual introduced.

In order to be able to describe the positions of the sun, the moon and the planets relative to the stars, a system of reference was needed. Initially the Babylonians used a system of reference consisting of seventeen stellar constellations. Omens dealt, for example, with particular planets entering or leaving certain constellations. In the seventh century BCE Babylonian astronomy and astrology underwent a change; the zodiac was introduced. The zodiac is the division of the apparent path of the sun in the sky, the ecliptic, in twelve equal parts, called signs.[8]

Between 1000 BCE and the 16th century CE a number of complex civilizations comparable to the irrigation cultures in Asia and Africa flourished in South- and Mesoamerica. We will make a few remarks about the Aztecs.[9] The dramatic fall of the Aztec empire is well known. In 1521 the capital Tenochtitlan, positioned in the middle of Lake Tetzcoco in the Valley of Mexico, surrendered. The Aztecs viewed the earth as flat. The outer perimeter

[8]Mathematical techniques have often played a role in divination. Cicero distinguished between natural and artificial divination. Natural divination concerns a direct message from the gods: e.g., in a dream or a vision. Artificial divination requires observation and calculation. Astrology is an example of artificial divination in which astronomical calculations play an important role. From a modern point of view the astronomical part of astrology should be sharply distinguished from the astrological interpretation.

[9]Much of what we will say applies to the Maya’s as well.

was conceived as a circle (or sometimes as a square), where the surrounding ocean met the dome of the sky. Both the heavens above and the underworld below the earth consisted of a series of layers. The Aztecs viewed everything in their world that represented a power as endowed with some sort of personality. They had a complicated calendar: they counted time in two different ways. The first was a 365-day solar year, called *xiuhpohualli*, the "counting of the years". It consisted of 18 months of 20 days plus a period of transition of 5 days. This is the Aztec ceremonial calendar that regulated the 18 "monthly" and various other celebrations directed at the earth, the sun, the maize, the mountains, the rain etc. The second system was *tonalpohualli*, the "counting of days", a 260 days cycle composed of 20 groups of 13 days, used for divination. The two systems operated together and a simple calculation shows that it takes 52 solar years for the 365-days calendar and the 260-days calendar to return to the same relative position.[10] In the second calendar, tonalpohalli, every 13-day period had its own associated deity and every day out of the 13 its Lord of the Day. In addition, there were 9 Lords of the Night. These Lords of the Night also recurred in a cycle in the *xiuhpohualli*. The 260-days calendar can be described by means of 3 simultaneously rotating gears and every day corresponded to a 13-day period Deity, a Lord of the Day and a Lord of the Night. Professional diviners interpreted the significance of the influence of these deities for newborn children or for the possible courses of action in a given situation.[11]

3.3. *Seidenberg's thesis*

The arithmetic of the two Aztec calendars is deeply embedded in a religious context. So is the geometry of the Agnicayana ritual and the astronomy of the Babylonians. Abraham Seidenberg has argued that mathematics as a whole has a ritual origin. He uses the contents of the above-mentioned Sulba Sutras as an argument to support his case for the ritual origin of geometry. For an understanding of the function of the altar Seidenberg refers to Hocart, who in 1927 in his book *Kingship* argued that the basic idea of the Indian sacrifice was to make an object equal to an altar and hence, by repetition of the ritual action, two things (distinct from the altar) equal to each other. By ritual action, we have: Falcon = Altar and Sacrificer = Altar, and therefore Sacrificer = Falcon, and hence the sacrificer can fly to heaven [28, p. 492]. The geometry went further than simple rules of thumb. In the successive augmentations of the falcon-shaped altar, the Theorem of Pythagoras is used, and according to Seidenberg, we have here the motive that led to its invention.

Seidenberg finds an analogous link between geometry and religion in the building of temples. The construction of a temple was a huge cooperative task. The labour of hundreds of participants had to be planned and coordinated in advance. In Sumeria the priests claimed that the plans were designed by the gods themselves and revealed to them in

[10] We must solve the equation $x \cdot 365 = y \cdot 260$. Division by 5 yields $x \cdot 73 = y \cdot 52$. Because 73 is prime we get: $y = 73$ and $x = 52$. The conclusion of the 52-years cycle was a reason for special celebrations.

[11] Townsend reported in 1992 that in Guatemala Maya Indian diviners still practice a form of calendrical divination. Starting from the day corresponding to the client's problem, they count out the days of the 260-day calendar by means of seeds and crystals en then they interpret the pattern [40, pp. 126–127].

dreams, and they may have believed this themselves [28, p. 521]. Temple building and the stretching of cords that was involved were rituals, according to Seidenberg.

In a second paper Seidenberg argues that counting also has a ritual origin. The argument runs as follows. What is needed for counting to be invented is "a definite sequence of words and a familiar activity in which they are employed" [29, p. 8]. According to Seidenberg the creation ritual offers exactly such a sequence and such an activity. Counting was a means of calling the participants in a ritual onto the ritual scene. More precisely, counting was born in the elaboration of a ritual procession re-enacting the Creation during which the participants appeared on the scene on being announced. The announcements took the form of numbers. This innovation took place once in human history and afterwards the invention diffused. In many different cultures all over the globe we find traces and further elaborations of this ritual counting and its significance.

Many of the examples that Seidenberg adduces are equally compatible with a practical origin of mathematics, and his thesis remains conjectural. Yet it is an interesting thesis, because it is undeniable that there are many phenomena in the history of human culture in which numbers have in one way or another a mythical or ritual significance. According to Seidenberg, for example, "in Babylonia, each of the numbers from 60 down to 1 came to be reserved for a special deity—there was a god Eight, a god Three, etc." and in the Satapatha Brahmana the numbers 1 to 101 are deities to whom offerings are made. Also "with the Maya the numbers 1 to 13 were (and still are) regarded as sacred beings and invoked as such" [29, p. 7].

4. The Pythagorean–Platonic period

4.1. *Pythagoras and Plato*

The Greek philosophers in Antiquity were the first to express the view that, compared with other forms of knowledge, mathematical knowledge is special. Pythagoras and his followers knew that mathematics is different from other kinds of knowledge and so did the Eleatic philosophers, Parmenides and Zeno of Elea. Although we do not know much with certainty about the precise doctrines these pre-Socratic philosophers held, it seems clear that for the Pythagoreans doing mathematics was a way to get in touch with the divine. Dodds has described Pythagoras as a great Greek shaman [14, p. 143]. It is possible that the opening of the Black Sea to Greek trade introduced the Greeks to shamanism. Tradition credits Pythagoras with the shamanistic powers of prophecy, bilocation and magical healing. He is also said to have visited the world of the spirits [14, p. 144]. In Chapter 3 of this book Reviel Netz discusses the Pythagorean views of mathematics. In his reconstruction he points out that in the mystery cult of the Pythagoreans active in the late fifth and early fourth centuries BCE, mathematics was an ideal means for the mortal individual to get in touch with the transcendent. Mathematics correlates the concrete and the abstract, the temporary and the eternal; this was most obvious for the Pythagoreans in mathematical musical theory. In Greek mathematics, proportions, as a means to correlate separate domains, played a major role. In Netz's view the Pythagoreans sensed that there is an incorporeal realm, but they did not fully reach beyond the corporeal. That is why for Plato the Pythagoreans were not dualistic enough, while for Aristotle they were too dualistic: the idea that mathematics calls for the assumption of a reality above the physical one, is simply wrong in Aristotle's view. Netz attempts to characterize the Pythagoreans on the basis of how they were perceived by Plato and Aristotle.

The ideas of the Pythagoreans were a major influence on Plato (427–347). In about 390 BCE Plato visited the Pythagoreans in Southern Italy. Dodds has described how, in his view, Plato transposed Pythagorean ideas from the level of revelation to the level of rational argument:

> The crucial step lay in the identification of the detachable 'occult' self which is [...] potentially divine with the Socratic *psyche* whose virtue is a kind of knowledge. That step involved a complete reinterpretation of the old shamanistic culture-pattern. Nevertheless the pattern kept its vitality, and its main features are still recognizable in Plato. Reincarnation survives unchanged. The shaman's trance, his deliberate detachment of the occult self from the body, has become the practice of mental withdrawal and concentration which purifies the rational soul [...]. The occult knowledge which the shaman acquires in trance has become a vision of metaphysical truth [...] [14, p. 210].

Because Plato wrote dialogues the interpretation of his work is not easy. Yet the dialogues and the passages that his pupil Aristotle[12] devoted to the views of his teacher make it possible to say some things with considerable certainty. First there is the doctrine of the two worlds: the eternal and intelligible world versus the changing world of the senses. Mathematical knowledge concerns the intelligible world. Secondly, there is the conception of a supreme divine intelligence ruling the world in accordance with the eternal ideas. Finally there is the notion of 'soul', both on the human and on the cosmic level; the soul operates in both worlds.[13]

For Plato true knowledge is knowledge of the forms. Mathematical knowledge is related to knowledge of the forms. In the *Meno* Plato has Socrates demonstrate that mathematical

[12] Aristotle (384–323 BCE), was very much influenced by Plato's ideas. Yet there are some fundamental differences. According to Plato true knowledge of the visible world is impossible. The reason is that the visible world is permanently in a flux and change cannot be a subject of rational knowledge. In Plato's view the mind's eye leads the philosopher to the Truth and in philosophy empirical evidence is in fact worthless. According to Plato a horse is a horse because the animal participates in some way in the idea Horse. However, the idea Horse is a subject of knowledge independently of all real, material horses. Aristotle rejected this view; for him empirical evidence was the only source of true knowledge. He believed, moreover, that a theory of change is possible and started to develop such a theory. He developed the theory of the four different types of interacting causes. The effective cause is the cause in the modern sense of the word, the material cause is the matter, the stuff a thing consists of, the formal cause is the form present in the matter, and the final cause is the purpose of the thing or process involved. The notion of final cause reflects Aristotle's view that nature has a purpose; everything in nature develops towards a goal. According to Aristotle Plato was the victim of an illusion: the illusion that next to the visible material world there is an independent realm of invisible things, the ideas. Aristotle treats the relation between ideas and visible objects the other way around. According to Aristotle our idea Horse is abstracted from the real horses that we run into, it is merely the form of the horse that we mentally separate from the matter. This theory of abstraction we find in the *Metaphysics*. It holds for mathematical notions like straight line, circle, etc. as well. How much Aristotle's views differ from Plato's is particularly clear in Aristotle's view of the human soul. For Plato the soul is eternal; for Aristotle the soul is merely the form of the human body and it does not survive the body.

[13] In Plato the cosmos is conceived, on the one hand, as a living organism. On the other hand, in the Timaeus, Plato describes the cosmos as an artifact, created by a superhuman craftsman. Not all Greeks shared this view of the universe. The fifth century atomists Leucippus and Democritus denied that the universe was created by an intelligent creator. They believed that the world is merely the result of the mechanical interaction of atoms. See G.E.R. Lloyd, Greek Cosmologies, in [5, pp. 198–224].

knowledge is different from other kinds of knowledge in the sense that it is acquired on the basis of insight and not on the basis of authority. Socrates compares a teacher of mathematics with a midwife; if the teacher asks the right questions, the pupil will by himself give birth to mathematical knowledge. Ian Mueller shows in Chapter 4 that Plato added a new element in the *Timaeus*, where he presented a mathematical creation myth. The creation of the cosmos was the result of the activity of a god, a representative of a higher level of reality, who imposed limits on unlimited matter, using geometrical forms. Plato's universe is geocentric and contained in the sphere of the fixed stars. The earth consists of four elements: earth, water, air and fire. Yet the universe is a single visible, living entity. In the *Republic* we find yet another view. Here mathematics is related to the divine because knowledge of it leads to knowledge of the forms. The future leaders of the Republic had to study arithmetic, plane and solid geometry, astronomy and harmonics in order to make their souls aware of a higher level of reality, and, in particular, of the form of the good.

Aristotle used to relate that those who came to hear Plato's lecture on the good, were disappointed because he spoke only of arithmetic and astronomy. We do not know what Plato said in that lecture, but mathematics and the theory of forms were obviously related in Plato's mind. According to Aristotle in his *Metaphysics* (A, 6, 9) Plato declared that

(i) forms are numbers,
(ii) things exist by participation in numbers,
(iii) numbers are composed of the One and the 'indeterminate duality'.

It is possible that Plato identified the forms with numbers so that he could find a principle of order in the mysterious world of the forms [10, p. 194]. The natural numbers can be generated from One and Two, if the Two is used as a principle of doubling the one. The sequence One, Two, Three, Four, etc., could then represent the first part of a logical generation of the universe. Similar tendencies existed in Pythagoreanism. Sextus Empiricus refers to the view that everything is derived from the monad or point. The point moves and generates a line. The line moves and generates a surface and three-dimensional bodies are generated by surfaces.

4.2. *Neo-Pythagoreanism and Neo-Platonism*

In Plato's time the Greeks were aware of the existence of Babylonian astrology, but only in the second century BCE astrology seems to have become fashionable among the Greeks. In Plato's dialogue the *Laws*, the stars, the sun and the moon are described as gods. In Plato's view divine minds were animating these heavenly bodies. Yet Plato did not take astrology seriously. Babylonian astronomy/astrology is based on the Babylonian cosmology: underneath the flat earth is the underworld, above it heaven. The Babylonians did not study the geometry of the orbits of the heavenly bodies; they studied the regularity of the celestial phenomena in arithmetical terms. With the Greeks astronomy became a geometrical science. The Greeks realized that the shape of the earth was spherical. It is a central element in the doctrine of the concentric spheres defended by Eudoxus. This doctrine was followed by the theory of epicycles and eccentrics, proposed by Apollonius in the third century BCE. Greek astronomy culminated in the work of Ptolomy (100 CE). In Ptolomy's model the stars are fixed on the inner side of a constantly rotating sphere with the earth at its centre.

Fig. 7. Petrus Apianus' 1553 view of the universe.

The sun, the moon and the five known planets execute precisely defined motions in the space between the earth and the sphere of the fixed stars. Ptolomy's geometrical model of the universe is highly sophisticated and very much in accordance with the observed celestial phenomena. Although heliocentric theories had been proposed, e.g. by Aristarchus of Samos (third century BCE), the geocentric model was the generally accepted theory of the universe until the Scientific Revolution in the seventeenth century (Figure 7).

When astrology became popular among the Greeks, Greek cosmology understandably superseded Babylonian cosmology in astrological theory. Present-day astrology is essentially Greek astrology.

In the second century BCE not only astrology became popular among the Greeks but other ideas that Plato might have considered with great scepsis as well. For example, the theory of occult forces immanent in certain animals, plants and precious stones gained popularity among the Greeks. Aristotle held that the properties of the material things in the world can be reduced to four elemental qualities: hot, cold, dry and wet. The four elements each have two elemental non-opposite qualities: for example, earth is cold and dry, fire is hot and dry, etc. There are however, many properties of things that cannot be accounted for by means of these *manifest* qualities. That is why non-manifest or *occult* qualities were introduced. The theory of occult qualities, or forces or virtues as they would also be called later, postulates, among other things, the existence of magical links between the celestial bodies and certain sublunary things. As Dobbs remarked:

> [. . .] if each planet had its representative in the animal, vegetable, and mineral kingdoms, linked to it by an occult 'sympathy', as was now asserted, one could get at them magically by manipulating these earthly counterparts [14, pp. 246–247].

In the first century BCE there was a revival of Pythagoreanism. In Chapter 5 Jean-François Mattéi describes the Pythagorean arithmetical-theological speculations of Nicomachus of Gerasa (fl. 100 CE). Mattei argues that the Greeks did not study man and the divine in terms of "persons", but, rather, in terms of "measure". And the supreme measure of the divine is Number, in so far as it repeats itself all over the universe with its unchangeable properties. Nicomachus describes the development, the manifestation of the divine in terms of ten steps, corresponding to the numbers one through ten, that each characterize an essential phase. One is the Monad, the supreme god, associated with Zeus, the ruler of the gods on Mount Olympus, and with Hestia, the goddess of the hearth. Two is the Dyad, the power of division and multiplication, associated with, among others, Isis, the Egyptian goddess of nature, Demeter, the giver of grain, and Rhea, that is mother earth, Gaia. Three is the Triad, representing the opposite of the Dyad, composition, and associated with a long list of gods. In this way the numbers one through ten, associated with the gods, are used to describe a ladder connecting heaven and earth, that descends from the Monad, the origin of all things, to the world we see around us.

The teaching in the Academy was influenced by Neo-Pythagoreanism. Some Neo-Platonist philosophers rejected the occult theories, but others were attracted by them. Plutarch of Chaeronea (born about 45 CE) was an eclectic Platonist who emphasized divine transcendance and aimed at a purer conception of God, with whom an immediate contact could be established. He denied that God was the author of evil and strongly opposed superstition. Yet he took prophecies seriously.

The most important Neo-Platonist philosopher, Plotinus (204–270), rejected astrology, while his successor Porphyry (234–ca. 305) incorporated it in his philosophy. Unlike Plotinus, Porphyry seems to have felt that knowledge of the influence of the heavenly bodies on the individual could help him to reach the divine mind.

Plotinus' views are a remarkable and sophisticated attempt to develop Plato's philosophy. He held that the source of everything is the *One*, which is identical with the 'Good' in Plato. The One is absolutely transcendent, ineffable, incomprehensible. The only predicates that Plotinus allows to be ascribed to the One are goodness and unity. Yet, even these can only be used on the basis of analogy. Plotinus must account for the multitude of finite things and he does this by applying the metaphor of 'emanation'. Emanation is not creation, because there is no will involved. It is a manifestation on the basis of the principle that every nature expresses itself in what is immediately subordinate to it. Plotinus also uses the metaphor of reflection in a mirror because in a mirror an object is duplicated without undergoing any change. The first emanation from the One is the *Intellect* or *Mind* (Nous): it encompasses the totality of all Platonic ideas, it is the divine mind characterized by intuition or immediate apprehension. The ideas arise from and are permanently maintained in the contemplation of One by itself. Nous is identified with the demiurg in Plato's *Timaeus*.

From the Intellect emanates the second principle in the hierarchy, the *Soul*, which corresponds to the World Soul of the Timaeus. The Soul arises through the contemplation of the Intellect by the One. The Soul consists of 'seminal reasons'. They are reflections of the ideas in the *Nous* and they are the productive power in the universe. In this way the forms have no direct connection with the sensible world. The Soul creates and animates the material universe. It is immanent in the world of becoming, but its higher part contemplates the intellect, while its lower part generates the world of the senses. The *Intellect* has an immediate grasp of reality. The Soul proceeds along the lines of discursive reasoning. Time is a product of the soul. The Soul has two aspects: the World Soul and the individual souls. The *Material World* is below the sphere of the *Soul*. Plotinus compares the emanations from the One to the radiation of light. The observed intensity of a light source diminishes the further one moves away from it until total darkness is reached. This darkness is matter in itself. Matter is the antithesis of the One, it is also the principle of evil. On this point Plotinus agreed with the Orphics and Neo-Pythagorean philosophers. It is possible for the individual soul to reach a mystical union with the One after a process of purification. Such ecstatic union, however, cannot last long in this life.

In Chapter 6 Dominic O'Meara discusses the ideas of the Neo-Platonist Proclus, a follower of Plotinus and one of the last representatives of philosophy in Antiquity. While Nicomachus used numbers to connect the world and the divine, Proclus used geometry in order to lead the pupils in his school in Athens to the divine. For the Neo-Platonists transcendence was characterized by an immediate and total unity of the thinking subject and its object. In Proclus' view the soul can attain such a self-discovery in geometry. In its geometrical projections the soul sees an image of itself and can thus attain a knowledge of truths concerning transcendent first principles, the gods. It is possible to experience some of the power of geometry as a means to connect man and god in the church of St. Sophia in Istanbul. If one stands in the centre of the church and looks upward, says O'Meara, one can see Proclus' geometry of the divine, translated into architecture. Religious buildings often symbolize, in various ways religious truths. Chapter 7, by Marie-Pierre Terrien, is devoted to the problem of religious architecture and mathematical symbolism during late Antiquity. Another aspect of religious architecture in relation with mathematics is treated in Chapter 8, in which David King discusses the sacred geography of Islam that developed out of the necessity to build mosques with the prayer-wall facing away from the direction of the Kaaba.

4.3. *The early Middle Ages*

Boethius (executed in 524 CE) and St. Augustine (354–430) transmitted the Neo-Platonic spirit to the Latin Middle Ages.[14] Boethius' work, for example, contributed to spreading the idea that music is number made audible. In *De Musica* he tells the story that Pythagoras once passed a forge and heard wonderful harmonies from four hammers beating on anvils. He weighed the hammers and discovered that the sound of the octave was produced by the weight ratio 2 to 1; the perfect fifth resulted from the ratio 3 to 2; and the perfect fourth from the ratio 4 to 3. The harmony corresponded to and was explained by simple numerical relations. Boethius divided music into three kinds, *musica instrumentalis*, *musica*

[14] From the 12th century onward Latin translations of Proclus and the great Arabic philosophers exerted influence as well.

humana, and *musica mundana*. *Musica instrumentalis* denotes both vocal and instrumental music. *Musica humana* refers to the harmony in the dimensions of the human body and the harmony between body and soul. *Musica mundana* is the (inaudible, at least for human beings) music that is made by the celestial bodies. This music is also called the music of the spheres or *musica caelestis*.

In the fourth century Christianity became the official religion of the Roman Empire. In the debates with opponents Christian theologians felt it was desirable to give profane proofs of Christian dogmas. Although early medieval theologians had only a limited knowledge of the original works of Plato and Aristotle, they made use of the Greek heritage. Until Aristotle's works became available in Latin translations in the 12th and 13th centuries, Platonic ideas dominated.[15] St. Augustine was the first major Christian theologian who tried to show along Platonic lines that our knowledge of God is as certain as our knowledge of geometry. In St. Augustine's view mathematical knowledge is knowledge of an eternal abstract realm to which we have access by means of an inner light, Divine illumination. The existence of eternal truth in mathematics implies the existence of the idea of Eternal Truth, which is an important ingredient of St. Augustine's proof of the existence of God and of the immortality of the soul.

The computation of the calendar was part of the task of the Greek astronomers–astrologers. It is remarkable that in the Latin West the science of time-reckoning and the construction of the calendar, the computus, was associated in a quite different non-astronomical way with the destiny of man. The computus was embedded in ideas about the religious significance of numbers. The number metaphysics represented by Nicomachus of Gerasa reached the Latin West via Boethius (ca. 480–ca. 525 CE). Abbo of Fleury, for example, wrote an exposition of the metaphysics of number based on Boethius' writings. In Chapter 9 Faith Wallis discusses a text by a pupil of Abbo, the English monk Byrhtferth of Ramsey, who around 1011 wrote a manual of computus. Wallis shows how in the early Middle Ages time-reckoning was identified with *numerus*. *Numerus* captures the core ideas of Christianized Pythagorean–Platonic numerology. Wisdom 11:21 says about the creation: "Thou hast made all things in measure, and number, and weight". In *numerus* this in fact meant: Thou hast ordered all things in time. In particular, before 1100 CE in the Latin West the calendar served as a natural vehicle for the survival of a complex of ideas relating mathematics and the divine.

[15] In Plato's hierarchy mathematics is inferior to dialectics, but on the whole mathematical knowledge has a very central position in Plato's philosophy as a sine qua non to gain true knowledge and at the same time the first and foremost example of what true knowledge amounts to. In Aristotle's hierarchy of the sciences mathematics is still inferior to metaphysics and above physics, but in fact the emphasis on the investigation of the changing visible world made mathematics lose its central role. This is clear from the work of Thomas Aquinas (1224/25–1274). Aquinas is the most famous of the scholastics who incorporated Aristotle's rediscovered ideas in their theology. Thomas rejected the Augustinian Divine illumination, because in his theory knowledge is separated from belief. Following Aristotle, Thomas argued that natural knowledge—including mathematical knowledge—only pertains to what can be abstracted from empirical experience, while belief assumes things that cannot be reached through natural knowledge. Although the existence and the unity of God can be proved, this only implies that belief in God is reasonable. Christian dogma, however, cannot be known on the basis of reasoning. The belief in these dogmas essentially depends upon the acceptance of Christian revelation.

4.4. *The later Middle Ages*

In the 12th and 13th centuries Aristotle was rediscovered and scholasticism developed. Christian theologians have always faced the question: How can reason be reconciled with revelation, science with faith? One of the core convictions of the Scholastics was that God is the author of all truths and that His creation cannot possibly show us the opposite of His revelation. In the thirteenth century Aristotle's rediscovered works opened new possibilities to deal with these problems. The Scholastics accepted Aristotle's doctrine of matter and form and his teleological view of nature. Yet they disagreed on, for example, the universality of matter. Thomas held that angels are immaterial forms; Franciscan theologians held that all created beings are material. Another point of dispute was whether there is only one form or whether there are several forms in each creature. In Chapter 13 Edith Sylla shows how such problems were in fact closely related to problems in the philosophy of mathematics.

At the same time other interesting developments occurred. In Babylonian astrology celestial phenomena were considered to be signs; the future could to a certain extent be read from the sky. The idea that the heavenly bodies could exert their influence by means of rays of light or heat occurs in the work of Robert Grosseteste (ca. 1168–1253), chancellor of the then newly founded Oxford University, after he had developed his light metaphysics (second decade of the thirteenth century).[16] This metaphysics is linked with his interest in optics. In the thirteenth and fourteenth centuries there was a great interest in optics among thinkers influenced by St. Augustine and Neo-Platonist thought. After all, in that tradition the light metaphor played an important role, for example in the idea of the illumination of the human intellect by divine truth. Alhazen's optics and Greek works on optics became available and exercised a powerful influence on the development of the theory. Although these works were probably not yet available to him, Grosseteste was one of the first to take up the study of optics. He argued that light was not only responsible for the dimensions of an object in space, but also the first principle of motion and efficient causation. Light was the first "corporeal form" of all material things. God had created light on the first day of the creation as the essential medium through which to bestow His divine grace. Crombie summarized Grosseteste's ideas as follows:

> Light emanated from a luminous body as a 'species' (the Latin word for 'likeness') which multiplied itself from point to point through the medium in a movement that went in straight lines. All forms of efficient causality, as for instance, heat, astrological influence and mechanical action, Grosseteste held to be due to this propagation of species, though the most convenient form in which to study it was through visible light [11, Vol. I, p. 112].

Although in retrospect astronomy and astrology can be distinguished clearly—there are the astronomical facts, observations and the mathematical theory to explain them, versus their supposed influence on the sublunar world, and criticism of different sorts of astrologers is actually at least as old as Antiquity—they were often studied and used concurrently until the seventeenth century. In the case of Grosseteste his belief in astrology was part of a serious intellectual enterprise.

[16] According to Shumaker Renaissance defenders of astrology very much emphasized the power of "rays" to exert influence by means of light or heat [33, p. 7].

Today one will not easily associate astrology, light, the theory of perspective and the defence of the Christian faith with each other. However, in the view of Samuel Edgerton Jr., the ideas of the Franciscan Roger Bacon, Grosseteste's pupil, represent a theoretical link between Grosseteste's metaphysics of light and the Renaissance of science and art after the fourteenth century. In his *Opus majus*, which Bacon sent to Pope Clement IV in 1267, he argued that in order to convince the Saracens, visual communication should be used:

> Oh, how the ineffable beauty of the divine wisdom would shine and infinite benefit would overflow, if these matters relating to geometry, which are contained in Scripture, should be placed before our eyes in corporeal figurations! For thus the evil of the world would be destroyed by a deluge of grace ... And with Ezekiel in the spirit of exultation we should sensibly behold what he perceived only spiritually, so that at length after the restoration of the New Jerusalem we should enter a larger house decorated with a fuller glory ... most beautiful since aroused by the visible instruments we should rejoice in contemplating the spiritual and literal meaning of Scripture because of our knowledge that all things are now complete in the church of God, which the bodies themselves sensible to our eyes would exhibit ... because to us nothing is fully intelligible unless it is presented before our eyes in figures, and therefore in the Scripture of God the whole knowledge of things to be made certain by geometric figuring is contained and far better than mere philosophy could express it [15, p. 45].

In the medieval West geometrical optics was called *perspectiva* (from the Latin 'perspicere' = looking through). Edgerton argues that Grosseteste's view that it is through *perspectiva* that God's grace spreads to the world, inspired Bacon and through Bacon others, and led to the idea that a geometrical theory of painting should be developed such that God's word could be spread more convincingly.

The Hebrew word 'kabbalah' means 'received tradition' and until the thirteenth century it covered the whole Jewish religious tradition. The thirteenth century is also the period when the mystical current that is now called 'Kabballah', gained momentum. One of the earliest representatives was Nahmanides. Nahmanides objected to Maimonides, who, in his *Commentary on the Scriptures*, had explained the biblical prophetic visions as a mere product of the prophets' imagination. Although influenced by it, the Kabbalists distanced themselves from Neo-Platonism and in the thirteenth century developed a sefirothic conception of the divine world. The sefiroth are the divine attributes or emanations, corresponding to the numbers one through ten. The sefiroth are "linked to the Unknowable, the *En-Sof* (Infinite), as the flame is joined to the coal; the *En-Sof* could exist without the flame, but it is the flame that manifests the Unknowable" [35, p. 248]. The names of the sefiroth, given by God himself, are: 1. *Kether Elyon* (the 'supreme crown' of God), 2. *Hokhmah* (the 'wisdom' of God), 3. *Binah* (the 'intelligence' of God), 4. *Hesed* (the 'love' or mercy of God), 5. *Gevurah* or Din (the 'power' of God), 6. *Rahamim* (the 'compassion' of God), 7. *Netsah* (the 'lasting endurance' of God), 8. *Hod* (the 'majesty' of God), 9. *Yesod* (the 'basis' or 'foundation' of all active forces in God), 10. *Malkhuth* (the 'kingdom' of God) [35, p. 248]. The sefiroth are the expression of God. There are important differences between philosophy and Kabbalah. For the medieval philosopher human actions like prayer only concern the destiny of the individual, because the individual intellects emanate from God, but are not part of it. For the Kabbalists the sefiroth are an expansion of God's manifestation. The Kabbalists are often experts in Jewish law as well, because they view human action as essential in the unfolding of the divine drama. Moreover, for the philosopher evil is the

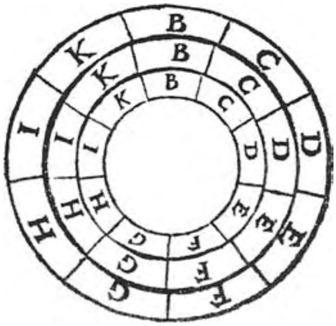


Fig. 8. The fourth figure of the Lullian *Ars Brevis*. Source: Opera, Strasburg, 1617.

absence of good, while for the Kabbalists evil is a positive force. In the *Zohar* (Book of Splendour) from the thirteenth century and the generally accepted central Kabbalistic text, it is indicated that evil can be associated with the sefiroth. Evil is said to have developed out of the separation of sefirah 4, Hesed, the love of God, from sefirah 5, Din, the power of God. The study of biblical and traditional texts was at the centre of the Kabbalah. Their content is seen as a key to the dynamics of the universe and indeed of God himself. In Chapter 10 Maurice-Ruben Hayoun discusses the sefiroth as they are treated in the *Zohar*.

Frances Yates has suggested that the ideas of the Spanish philosopher and mystic Ramon Lull (1232–ca. 1316) are best understood as a medieval form of Christian Kabbalah [46, p. 6]. When Lull was thirty years old he repeatedly had one and the same vision of Christ on the cross. The expression of great agony and sorrow had a tremendous effect upon Lull, who had until then lived a worldly life. From then on he devoted his life to Christ. Lull is the author of a vast number of works in Arabic, Latin, Catalan, French, Provençal and Italian. In everything that he wrote, his goal was to save souls, to convince others of the truth of the Christian faith. Lull had a system which he believed had been divinely revealed to him, commonly called the ‘Lullian Art’.

One aspect of the Lullian Art is that letters are used in combination with geometrical figures. The letter A represents a trinity: Essentia, Unitas and Perfectio. Then there are the attributes of God, the ‘dignities’ of the Art, represented by the letters B through K. In the *Ars Brevis* we find the following list: B = Bonitas (Goodness), C = Magnitudo (Magnitude), D = Eternitas (Eternity), E = Potestas (Power), F = Sapientia (Wisdom), G = Voluntas (Will), H = Virtus (Strength), I = Veritas (Truth), K = Gloria (Glory). Thorndike considered the Art of Lull as a logical machine [39, p. 865]. And indeed, in the Fourth Figure of the Art in the *Ars Brevis* (Figure 8) the two inner circles revolve, which makes possible an investigation of some sort of combinations of the dignities. Yet, although the combinatorial character of the Art is clear, the combinatorics are not simple. For example, although in Lull’s system the four elements, earth, water, air and fire, are a manifestation of the dignities, they are not represented in any simple sense as mathematical combinations of the dignities.

Another aspect of Lull’s Art is the symbolism of the tree. In Chapter 11 Charles Lohr describes Lull’s remarkable conception of nature. With the Kabbalists Lull shared a dynamic understanding of reality. Science was for Lull not the Scholastic ordering of pre-existing truths. Instead, in accordance with his view that God's activity of creation continues in the sense that everything in reality tends to its own perfection, he viewed science, including mathematics, as a productive art. Lull was very influential although his name was misused as well. Alchemical and kabbalistic works written by others have been associated with his name. Lullism proper influenced amongst others Nicholas of Cusa (Chapter 14), Giordano Bruno, Athanasius Kircher (Chapter 17) and Leibniz (Chapter 25). Although the combinatorics in the Lullian Art are not its essence, but merely a means of expression, Lull's combinatorial ideas exerted considerable influence as well. In *Gulliver's Travels* Swift describes a machine for rotating hundreds of cubes with words written on them. The machine is used by learned men on the island of Laputa to answer questions. Swift ridiculed the Lullian Art. On the other hand Leibniz's Lullian ideal of a *characteristica universalis*—a universal sign language in which all well defined problems could be solved through calculation—stimulated him to invent the calculus.

The term 'gnosticism' refers to a great diversity of sects that flourished at the beginning of the Christian era. The different sects have one element in common: they all assume that the human soul is a divine spark imprisoned inside the body as a result of an error; evil is due to the severance from the Godhead and the ultimate goal is salvation, i.e. the overcoming of the grossness of matter and the return of the soul to its divine origin. Gnosticism existed in Christian and non-Christian forms. Plotinus rejected gnosticism, but at the same time he was influenced by it. The Hermetic tradition is a non-Christian form of Hellenistic gnosticism. In Christian gnosticism, Christ is seen as the primary revealer, but the necessity of atonement is denied. Within the Christian tradition a wide range of apocryphal texts have survived outside of the New Testament canon. Some of them show clear gnostic influence. In Chapter 12 Hugue Garcia gives a gnostic interpretation of the twelfth century painting on the ceiling of the church Saint Martin de Zillis, in Zillis, Switzerland.

4.5. *The Renaissance*

The Renaissance introduced radical changes in all aspects of learning and culture in Europe in the 15th and 16th centuries, which constituted a break with the Middle Ages. In retrospect it is clear that the Renaissance prepared the way for the Scientific Revolution. In the later Middle Ages Platonism had lost popularity, but there was a revival of Neo-Platonism during the Renaissance. Possibly the first and no doubt the greatest Neo-Platonic philosopher of the early Renaissance was Nicholas of Cusa (Cusanus) (1401–1464). He developed a highly original view of how mathematics can deepen man's insight into his relationship with the divine. It is in the area of mathematics that man is able to come closest to an understanding of God's creative activity. Since the mathematician himself creates the objects which are considered and manipulated in this particular realm of thought, he becomes himself like a second creator, capable of reflecting on his creations as well as on the act of creating.

The two key concepts of Cusanus' philosophy, *docta ignorantia* (learned ignorance) and *coincidentia oppositorum* (coincidence of opposites) also have a strong bearing on mathematics. It is often by means of geometrical examples that Cusanus chooses to illustrate them. Considering a circle which he allows to grow indefinitely against a tangent, Cusanus

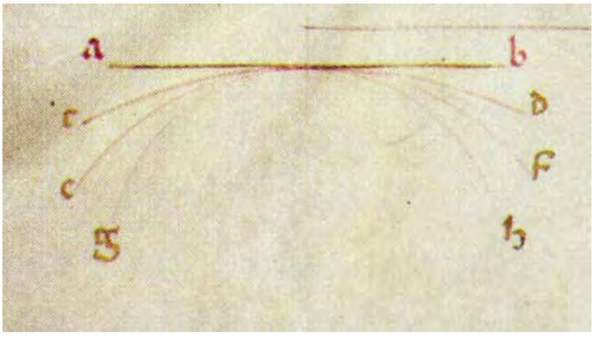


Fig. 9. Cusanus' coincidentia oppositorum. Drawing from the manuscript of *De docta ignorantia* in the Cusanus-Library, Kues, Germany.

demonstrates the limitations of rational thought, which fundamentally depends on distinctions, i.e. *opposita*, such as straight line vs. curved line (the tangent and the growing circle, respectively), and which, because of this dependence, fails to conceive of their ultimate fusion.

At the same time, Cusanus stresses the effect of contemplating such figures in motion, which is the intimation of something transcending rational understanding, the emergence of an inkling of what a mind superior to ours might see happening as the movement's ultimate stage, viz. the actual merging of the opposites. Significantly Cusanus makes his geometrical figures vanish. They can therefore be seen as special cases of what in the Netherlandic and German mystical traditions (Ruysbroeck, Meister Eckhart, Suso) was termed *ontbeelding/Entbildung*, i.e. the doing away with images, a necessary prerequisite to spiritual growth.[17]

The movement of the mind which allows one to recognize and hence go beyond the limitations of rational thought, thereby transforming relative ignorance into learned ignorance, is called *transsumptio* by Cusanus. This capacity, which one has to discover in oneself, is the same that allows one to see beyond the limitations of the Aristotelian principles of logic, in particular the principle of contradiction, which so strictly applies in rational thought and its most cherished domain, mathematics.

In Chapter 14, Jean-Michel Counet discusses the different ways in which Cusanus uses mathematics to approach the divine. Counet stresses that, according to Cusanus, any acquisition of human knowledge can be described as a form of measuring. The aim of all this measuring is to become aware of the limits of knowledge and, through this repeated experience of not-reaching, to attain the wisdom of unknowing.

The most important Neo-Platonist philosopher of the Italian Renaissance was Marcilio Ficino (1433–1499). Ficino adopted Plotinus' tripartite scheme of being: the One, the Intellect and the Soul. The Intellect contains the Platonic Ideas, the Soul contains the so-called seminal reasons that constitute the productive power of the soul. Ficino was a Neo-Platonic philosopher who combined a Platonic view of the universe with astrology and

[17] Cf. L. Bergmans "Nicholas of Cusa's vanishing geometrical figures and the mystical tradition of *Entbildung*" in the proceedings of the international conference "Nikolaus von Kues und die Mathematik", Irsee, 2003 (forthcoming).

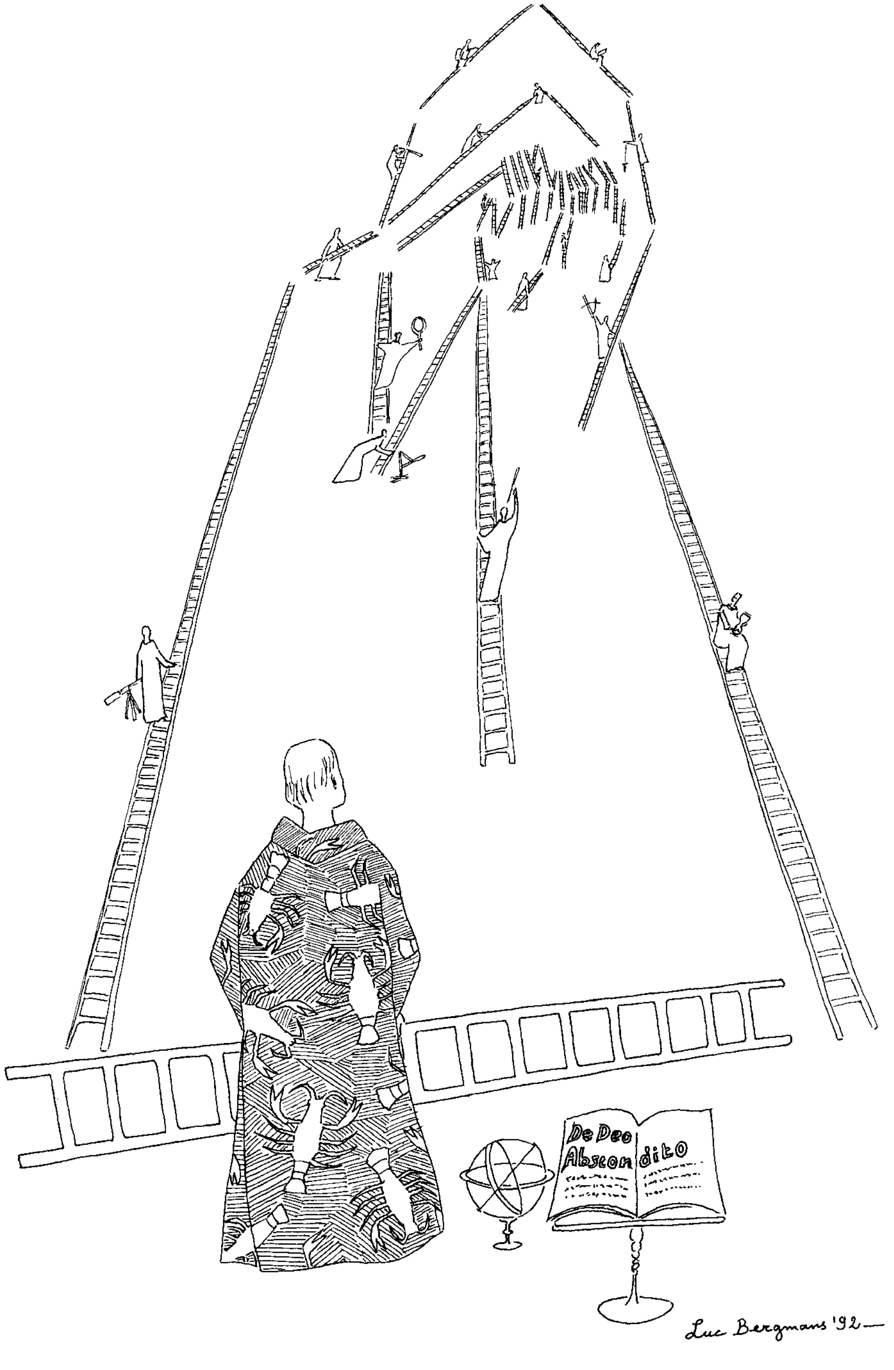


Fig. 10. The destiny of measuring human knowledge is to become aware of its limitations and to realise that the infinite remains unknown.

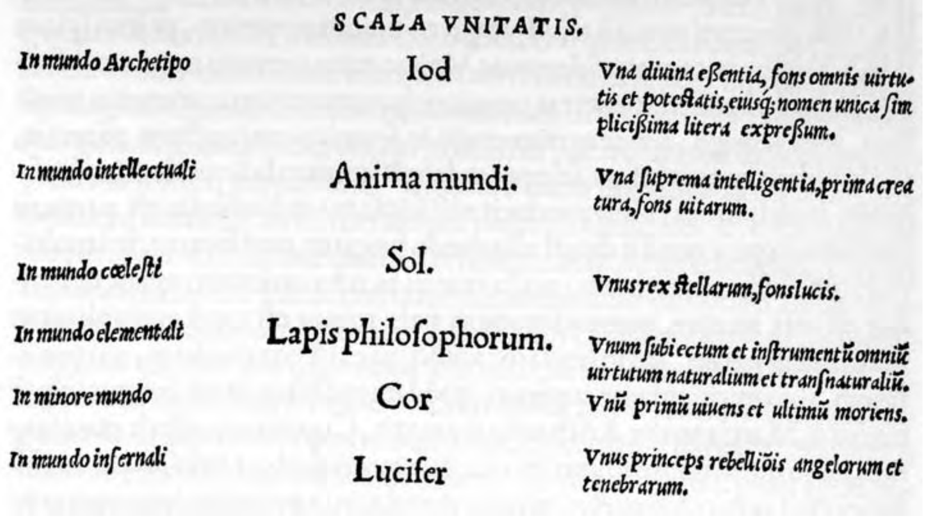
natural magic. In his explanation of magic, the concept of sympathy or correspondence played an important role. Not only natural substances like precious stones or herbs have such occult qualities, but also numbers and figures.

Ficino's views exerted considerable influence on Cornelius Agrippa (1486–1535). Agrippa was the author of a remarkable book: *De occulta philosophia libri tres* (first printed edition 1531). It is a complete compendium of the magic that steers clear of bad demons or evil, so-called white magic. In particular, considerable attention is paid to magic by means of numbers. Chapter I of the first book begins as follows:

> Seeing there is a Three-fold World—Elementary, Celestial and Intellectual—and every inferior is governed by its superior, and receive the influence of the virtues thereof, so that the very Original and Chief Worker of All doth by angels, the heavens, stars, elements, animals, plants, metals and stones convey from Himself the virtues of His Omnipotency upon us, for whose service He made and created all these things: Wise men conceive it no way irrational that it should be possible for us to ascend by the same degrees through each world, to the same very original World itself, the Maker of all things and First Cause, from whence all things are and proceed; and also to enjoy not only these virtues, which are already in the more excellent kinds of things, but also besides these, to draw new virtues from above. Hence it is that they seek after the virtues of the elementary World, through the help of physic, and natural philosophy in the various mixtions of natural things; thence of the Celestial World in the rays, and influences thereof, according to the rules of Astrologers, and the doctrines of mathematicians, joining the Celestial virtues to the former: Moreover, they ratify and confirm all these with the powers of the divers Intelligences, through the sacred ceremonies of religions.[18]

In accordance with this threefold division of the world, Agrippa distinguishes natural, celestial and ceremonial magic. In natural magic the magician tries to use the occult properties of things on earth. In celestial magic the goal is to influence the effects that the heavenly bodies have on events on earth. In ceremonial magic, finally, the magician attempts to influence the many spiritual powers that populate the universe. Book I is devoted to natural magic. Agrippa explains the distinction between the natural and the occult virtues of things. Natural virtues depend on the four elements: Fire, Water, Earth and Air. The occult virtues are "a sequel of the species and form of this or that thing". Their causes are hidden and not accessible to man's intellect. Herbs, stones, metals etc. have both elemental and occult properties. The occult properties do not derive from the nature of the elements, but are conveyed into them from above by the Spirit of the World. At the end of the first book Agrippa deals with the alphabet. In his view of the world, the order, the number and the shapes of letters are not accidental. They are not based on a convention that could easily have been different. Instead, the alphabet is related to the actual structure of the universe. This is in particular true of the Hebrew letters, which are the most sacred. The Hebrew alphabet consists of three parts: twelve letters are simple, seven letters are double and three letters are the 'mothers'. The simple letters correspond to the zodiacal signs, the double letters to the seven planets and the mothers to the three elements, earth, fire and water. The fourth element, the air, has a special position; it is the glue and spirit of the elements. In this way the Hebrew alphabet covers the entire universe and all kinds of relations between words and the world can be constructed. Wonderful mysteries concerning the past and the future can thus be drawn forth from the words that people use. Book II is interesting from a mathematical point of view. There are chapters on the numbers 1 through 10, a chapter

[18] Translation by Whitehead [44].



SCALA VNITATIS.

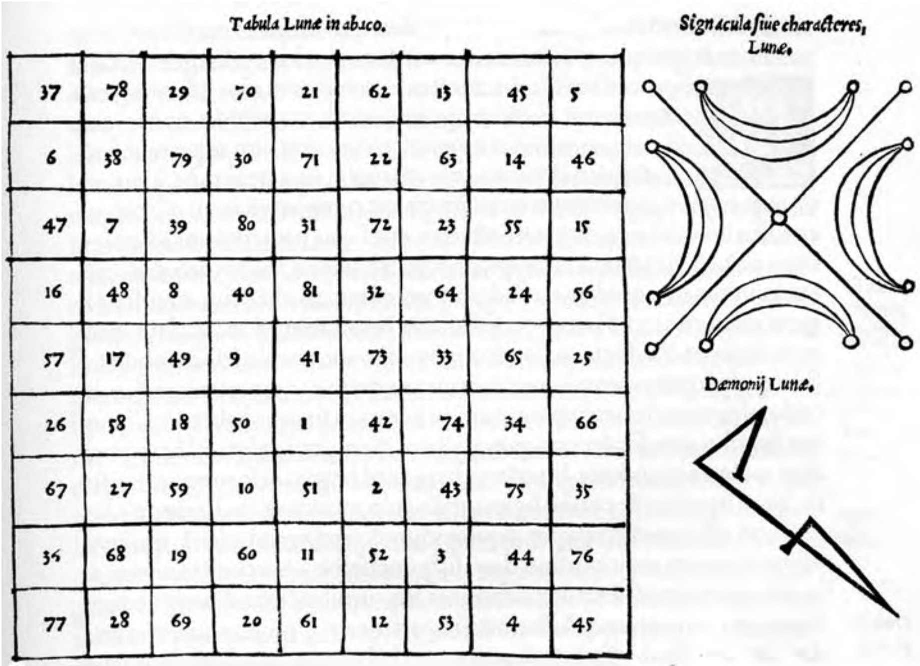
In mundo Archetipo	Iod	Vna diuina eßentia, fons omnis uirtutis et poteſtatis, eiusq; nomen unica ſimplicißima litera expreßum.
In mundo intellectuali	Anima mundi.	Vna ſuprema intelligentia, prima creatura, fons uitarum.
In mundo cœleſti	Sol.	Vnus rex ſtellarum, fons lucis.
In mundo elementali	Lapis philoſophorum.	Vnum ſubiectum et inſtrumentũ omniũ uirtutum naturalium et tranſnaturaliũ.
In minore mundo	Cor	Vnũ primũ uiuens et ultimũ moriens.
In mundo infernali	Lucifer	Vnus princeps rebelliõis angelorum et tenebrarum.

Fig. 11. The ladder of One in Agrippa's *De occulta philosophia*. Courtesy of the Akademische Druck- und Verlagsanstalt, Graz (cf. [1]).

on the numbers 11 and 12 and a chapter on numbers greater than 12. For Agrippa to each number there corresponds a ladder consisting of six levels ascending from the underworld, via the minor world (the human body), the elemental world, the heavenly world and the spiritual world to the world of ideas (see Figure 11 for the ladder of One). Numbers have powers and in order to be able to use these powers it is useful to know the ladders of the numbers. In Chapter 15 of Book II Agrippa explains that the power of large numbers is determined by the power of their divisors.

Some of the calculations relate individuals via their names to celestial bodies. Agrippa assigns the values 1 through 9 to the letters A through I of the Roman alphabet, to the letters K through S the values 10 through 90 and to the letters T, V (for U), X, Y, Z, J, V, HI (for JE, as in Hieronimus) and HU (for W as in Huilhelmus) the values 100 through 900. An example of a calculation would be the following. Let us consider someone whose name is Bogdan and whose parents are called Teunis and Mirjana. When we add the values of the letters in these names we get, respectively, 104, 444 and 170. We add these three numbers and get 1309. This total is divided by 9. In this case the remainder is 7. This means that the heavenly body under whose influence Bogdan functions is the Moon.[19] What implications does this have? Agrippa is not entirely clear about this. It may be that Bogdan would be well advised to have a silver amulet made for him with the Moon's magic square (see Figure 12) on it in order to apply the occult powers of the Moon to his advantage. It is very important that the amulet be made when the Moon is in a favorable position. Otherwise the charm can do harm (Book II, Chapter 23). The positive influence of the Moon will turn someone into a pleasant, respected person and it will provide protection during journeys.

[19] If the remainder is 1 or 4 the sun is involved. If it is 2 or 7 the moon is involved. If it is 3, the star is Jupiter. If it is 5, the star is Mercurius. If it is 6, Venus is involved. If it is 8, it is Saturnus. If it is 0, it is Mars.



37	78	29	70	21	62	13	45	5
6	38	79	30	71	22	63	14	46
47	7	39	80	31	72	23	55	15
16	48	8	40	81	32	64	24	56
57	17	49	9	41	73	33	65	25
26	58	18	50	1	42	74	34	66
67	27	59	10	51	2	43	75	35
36	68	19	60	11	52	3	44	76
77	28	69	20	61	12	53	4	45

Fig. 12. Magic square corresponding to the Moon in Agrippa's *De occulta philosophia*. Courtesy of the Akademische Druck- und Verlagsanstalt, Graz (cf. [1]).

If the square is etched on an amulet of lead while the Moon is in an unfavourable position, it will bring evil to the inhabitants of the area where it is buried.

In Chapter 15 Teun Koetsier and Karin Reich describe the numerology of Michael Stifel (1487–1567). Although Stifel's numerology is embedded in his Lutheran faith, and his interests merely concern the interpretation of the Bible and God's plans with the world, Stifel and Agrippa are kindred spirits. After Stifel the interest in numerology in Germany did not disappear. In Chapter 16 Ivo Schneider discusses the numerological theories of Johannes Faulhaber (1580–1635). Schneider also discusses the 'brotherhood' of Rosecrucians.

5. The Scientific Revolution and its aftermath

5.1. *The Scientific Revolution*

The Renaissance culminated in what is commonly called the Scientific Revolution. The new science that developed in the seventeenth century differed in several important respects from Greek and medieval science. A novel and very important characteristic is the great emphasis placed on observation and experiment. A second, no less important characteristic is the conviction that the structure of the material world is mathematical. This implied that the laws of nature had to be formulated in mathematical terms. The result was that mathematics invaded natural philosophy in an unprecedented way. In the Middle Ages philosophers and theologians had been the specialists on nature. The Scientific Revolution led in the course of time to the demise of their authority. In this process Galileo Galilei (1564–1642) played a pivotal role. In order to be taken seriously Galilei needed a metaphysical legitimation for his mathematical science. In Chapter 18 Volker Remmert shows how Galilei constructed this new legitimation. Galilei did not describe nature; he needed and wanted to read it. He presented mathematics as the only way to decode and understand the book of nature, a book written by God in the language of mathematics. Nature in this context included both terrestrial and celestial phenomena. The idea that the laws of nature are universal without a special status for the heavens, is a third characteristic of the Scientific Revolution. A fourth characteristic is a novel, very influential, mechanistic account of the material world. The result of this mechanistic view of nature was that in the seventeenth century all natural phenomena were explained in terms of moving particles. This led to mechanistic, non-theological accounts of the world and man. In this respect the French philosopher and mathematician René Descartes (1596–1650) played a crucial role. Descartes' fascination with mathematics is an essential clue to understanding his philosophical ideas. In the world view of Descartes and his followers, reason is given an absolute authority. His critics saw this as a threat to theology, and indeed, the Cartesians not only revolutionized science but Bible criticism as well. In Chapter 20 Jean-Marie Nicolle analyses the way in which mathematics functioned for Descartes as a model for his metaphysics.

Two of the greatest minds that brought about the Scientific Revolution were Johannes Kepler (1571–1630) and Isaac Newton (1642–1727). It is remarkable that both were very religious men for whom their scientific activities clearly had a religious dimension. Kepler was very much influenced by Neo-Platonism. Mysticism, mathematics and astronomy are all aspects of his work. In Kepler's view, God created the universe on the basis of mathematical principles. In the end Kepler's metaphysical speculations led to his three wonderfully simple and accurate laws for the motion of the planets around the sun. In Chapter 19 André Charrak discusses Kepler's views on how God had created the universe on the basis of mathematical principles. In Chapter 35, in which Albert van der Schoot traces the history of the divine proportion, he pays considerable attention to Kepler's views. Kepler's laws and the work of others—in particular that of Galileo Galilei (1564–1642), who derived the parabolic trajectory of a bullet—were the starting-point for Isaac Newton (1642–1727), who devised the theory that would be the core of mechanics and astronomy until the beginning of the twentieth century. Newton's work completed the transition from Renaissance science to modern science.

Until the 1650s the Copernican thesis that the earth moves around the sun had not really drawn much attention, in spite of Galilei's defence of it. In the middle of the seventeenth century it became a central element in the new mechanistic philosophy and after Newton's *Principia*, the universe was no longer seen as spherical but as an unbounded, infinite Euclidean space. Moreover, the motions of bodies relative to each other were in principle completely understood. The theory that Newton developed in his *Philosophiae naturalis principia mathematica* (1687) contained a unified axiomatic mathematical theory that implied both the parabolic trajectories that Galilei had derived and the elliptic planetary orbits that Kepler had discovered. Like Kepler, Newton was a religious man. His entire life can be said to have been devoted to a search for the truth concerning man, God and the universe. In Chapter 24 Kees de Pater discusses Newton's scientific work and the divine. A recent biography of Newton is entitled *Isaac Newton, the last sorcerer* [43]. And indeed, he studied theology, mathematics, alchemy, numerology and other subjects as part of an attempt to obtain answers to the ultimate questions. Maynard Keynes called him one of the last great Renaissance magi. As was the case in the pre-Greek period and in the Pythagorean–Platonic period, during the Scientific Revolution the human preoccupation with the divine had a considerable influence on the development of science and mathematics.[20]

5.2. *Secularization and the divine*

During the Renaissance there was still in Western civilization a shared core of faith. The Scientific Revolution sowed the seeds of secularization. Conflicts between believers were replaced by debates between believers and non-believers. The different views on mathematics and the divine that were expressed in this period reflect this radical development. In Chapter 21, Donald Adamson discusses the ideas of Blaise Pascal. To many the new mechanistic philosophy represented a threat to religion. Pascal tried to safeguard religion by strictly separating the scientific domain, with its geometric approach from the religious domain. In Chapter 23 Philip Beeley and Siegmund Probst treat the views of John Wallis (1616–1703). In the confrontation between theology and science Wallis protected religion by keeping his work in mathematics apart from his theological work. In this way he could handle infinity in mathematics in an unconstrained way and at the same time, on a theological level, attack Hobbes who had claimed that there was no argument to prove that the world has a beginning.

Descartes had not succeeded in solving the problem of the relation between body and mind. Baruch Spinoza (1632–1677) put forward a radical solution: mind and body are two different attributes of one and the same unique substance, God. Moreover, there is no free will. Spinoza was a determinist; he held that there is a strict parallellism between body and mind. Extension and thought are just two different ways to experience the same thing. If the development is completely determined on the level of matter, so it is on the level of thought. In Chapter 22 Ger Harmsen discusses Spinoza's views on the relation of mathematics and the divine and the way they have been interpreted. Spinoza's main work, the *Ethica*, is written 'more geometrico', in the manner of geometry. Opinions differ on whether the geometrical exposition is an essential element of Spinoza's philosophy. Harmsen briefly mentions an idea defended by Von Dunin-Borkowski at the beginning of the twentieth century.[21] Spinoza's strict parallellism is a solution to the problem of the relation between body and mind, left unsolved by Descartes. According to Von Dunin-Borkowski, Spinoza's solution was inspired by Descartes' invention of analytic geometry. Analytic

[20] Of course, Kepler and Newton were no shamans in the strict sense of the word, but, it is tempting to think that their motives were not unlike the motives of the shamans of the prehistory and the Pythagorean shamans. If this is the case our account of each of the three periods that we distinguish in the history of the relations between mathematics and the divine begins with shamans in a star role.

[21] Stanislaus von Dunin-Borkowski S.J., *Der junge De Spinoza, Leben und Werdegang im Lichte der Weltphilosophie*, Münster i. W., Aschendorffsche Buchhandlung, 1910.

geometry is based on the possibility to translate notions from the sphere of extension—line, point, etc.—into notions from the sphere of algebra, the sphere of manipulating formulae. In analytic geometry there is parallelism between geometry and algebra. Moreover, in the seventeenth century the manipulation of formulae is not infrequently associated with thought. It is therefore conceivable that Spinoza was inspired by analytic geometry. Se non è vero, è molto ben trovato.

Spinoza has been interpreted in different ways. Some have interpreted his pantheism and his emphasis on the *amor dei intellectualis* as a form of mysticism. Others view him as an atheist, because he abolished God as a creator. Jonathan Israel, for example, considers Spinoza as the crucial representative of what he calls the 'Radical Enlightenment'. Israel writes:

> Spinoza's prime contribution to the evolution of early modern Naturalism, fatalism and irreligion, as Bayle—and many who followed Bayle in this—stressed, was his ability to integrate within a single coherent or ostensibly coherent system, the chief elements of ancient, modern and oriental 'atheism' [22, p. 230].

Yet, if we take Spinoza's writings literally, pantheist seems a more appropriate term than atheist. Gottfried Wilhelm Leibniz (1646–1716) was a philosopher–mathematician who was a convinced Christian and at the same time had a high esteem for the power of human reason. In Chapter 25 Herbert Breger develops a coherent view of Leibniz's philosophical system, in particular with respect to mathematics and the divine.

Although the view that the world is based upon a divine mathematical concept exerted, as we have seen, great influence during the Scientific Revolution, in the eighteenth century a reversal took place. Before and during the Scientific Revolution nature *had* to be rational because God *is* rational. After the Scientific Revolution the rationality of nature became for many authors an observable phenomenon. It is no longer the divine concept of the world that delights man, but the mathematical laws of nature. In Hume's *Dialogues Concerning Natural Religion* the character Cleanthes compares nature to a machine and says:

> Since therefore the effects resemble each other, we are led to infer [...] that the causes also resemble; and that the Author of Nature is somewhat similar to the mind of man; though possessed of much larger faculties.[22]

The reversal is clear: "God *must* be an engineer, because nature *is* a machine" [3, p. 56]. If God is an engineer, He would have been a bad engineer if His involvement were still permanently required. First God became a distant creator, later He became for many superfluous altogether. In Chapter 26 Wolfgang Breidert shows how George Berkeley (1685–1753), Anglican theologian and Bishop of Clyne, attacked the foundations of the new mathematical calculus, invented by Newton and Leibniz, in order to defend theology. In Chapter 27, Rüdiger Thiele describes how the most productive mathematician of the eighteenth century, Leonhard Euler (1707–1783) maintained his religious convictions against the trend towards deism and freethinking.

Although the Scientific Revolution was in many different ways related to economic and social developments, it was primarily an affair of people that we would now call philosophers and scientists. Yet, everybody was soon to experience its effects. The Scientific Rev-

[22] Quoted in [3, p. 56].

olution directly led to the Enlightenment and the views of the Enlightenment philosophers formed the ideology of the French Revolution. The French Revolution took place when the (First) Industrial Revolution was in full swing. In the course of time, as a result of this revolution the world changed beyond recognition: steam took over from wind, water and muscle power. The Industrial Revolution—often taken to have covered the period 1780–1850—was to be followed by other industrial revolutions and the French Revolution was followed by other political revolutions. The result of all this was that in many countries the structure of society changed considerably. In traditional societies family ties and the church often interfere with economic processes. In modern bourgeois societies, religion is a private affair and economic activities are almost exclusively led by economic interests. During the First Industrial Revolution the role of science was still limited, but by the second half of the 19th century, science had begun to play an important role in industry. This role has only grown; today science, including mathematics, plays an important role in many sectors of society.

The problem of evil in the world is a problem no serious Christian philosophy can ignore. Leibniz had argued that the world as it is, is the best of all possible worlds. In Leibniz's view, before He created the world, God had solved a problem of optimization: which of all logically possible worlds is the best? Leibniz's view had been ridiculed by Voltaire in his satirical novel *Candide*. The deistic Enlightenment philosophers came up with a different solution. They had a highly optimistic view of the capabilities that man possesses to bring his own existence and the institutions that he lives by in harmony with the natural order. Equality, freedom and brotherhood, in combination with the liberating forces of reason and science, would bring about a better world. Some, Condorcet for example, advocated the view that the applications of the new differential and integral calculus would not remain confined to applications in the realm of nature, but would extend to the social sciences and to politics. Politics would be rationalised by means of the application of mathematics. This development might be called a *secularization of the divine*. The religious inclinations of secular humanism are particularly clear in the case of Auguste Comte (1798–1857), who dreamt about a positivist religion.

This Enlightenment optimism with its complete rejection of all established religion and its belief in man's capabilities to create a better world returned in the nineteenth and twentieth centuries in the workers' movement. Karl Marx, Friedrich Engels and Vladimir Ilyich Lenin created Marxism–Leninism, a view of the world that decisively influenced the course of history in the twentieth century. In Marxism–Leninism, science and mathematics were taken very seriously; they were an essential factor in the process that was supposed to bring about a workers' paradise on earth (See Figure 13). For most mathematicians and philosophers mathematics lost its connection with the divine. However, in most, the belief in the certainty of mathematical knowledge remained unshaken.Yet in this respect things were changing as well. In Chapter 32 François de Gandt discusses the sceptical views of the young Edmund Husserl (1859–1938) with respect to arithmetic.[23]

[23] In the second half of the 20th century Imre Lakatos developed a mitigated sceptical view of mathematics on the basis of Popper's philosophy of science. Cf. [24].

Fig. 13. Stamp of the USSR expressing the importance of mathematics in “living, working, studying in the communist way”.

5.3. *The 19th century. The freedom of mathematical thought. Neo-Thomism*

As for mathematics and the divine, the 19th and 20th centuries offer a very complex picture. The disappearance of a shared core of faith has led to a great plurality of views in Western culture. The secularisation of the divine represents one line of development. There are, however, others. In what follows we shall use the development of (pure) mathematics itself as a guiding principle.

The views of the Enlightenment philosophers remained influential in the following period. Yet, inevitably there had to be a reaction to the Enlightenment philosophers’ overemphasis on reason: this was the Romantic movement. The Romantic intellectuals praised imagination over reason, emotions over logic, and intuition over scientific rigour. One of the greatest mathematicians of this period was the French mathematician Augustin-Louis Cauchy (1789–1857). Cauchy did important work on the application of mathematics in science, but, in accordance with the Romantic reaction to the ideas of, for example, Condorcet, he totally rejected the idea that mathematics could be applied to matters of the heart. In the preface of his *Cours d’Analyse*, published in 1821, he wrote:

> Let us cultivate with passion the mathematical sciences, without wanting to extend them beyond their domain; and let us not imagine that one could attack history with formula’s, or give a foundation to morality by means of theorems from algebra or the integral calculus.[24]

Cauchy was a passionate royalist, totally opposed to the French Revolution. He was an orthodox catholic as well.[25] Cauchy was a stubborn man and his ultra-conservatism repeatedly landed him in difficulties. Yet, he is one of the mathematicians who heralded a new era in mathematics. In the eighteenth century the calculus was vexed by foundational problems. By a few radical innovations Cauchy put the development of analysis on a new track and brought about what is sometimes called the first revolution of rigour in analysis.

[24]“Cultivons avec ardeur les sciences mathématiques, sans vouloir les étendre au-delà de leur domaine; et n’allons pas nous imaginer qu’on puisse attaquer l’histoire avec des formules, ni donner pour sanction à la morale des theorems de l’algèbre ou de calcul integral”, *Cours d’analyse*, 1821, Introduction.

[25]Cf. [4].

He rejected the 'generality of algebra', meaning in particular the, from his point of view, careless eighteenth-century manipulation of infinite analytical expressions. In this way the ultra-conservative Cauchy played a crucial and progressive role in the fundamental changes that mathematics underwent in the nineteenth and twentieth centuries. Houraya Sinaceur has attempted to characterize these changes on a fundamental level [34]. One aspect of this transformation is that mathematicians became aware of their freedom in the creation of concepts. Great mathematicians like Karl Friedrich Gauss, Richard Dedekind, Georg Cantor, David Hilbert, Luitzen Egbertus Johannes Brouwer, and many others have emphasized the great importance of the freedom of mathematical thought. Galilei, Kepler, Newton, and others were decoding the book of nature written in the language of mathematics. For them the object of mathematics was given. At the beginning of the nineteenth century, probably also under the influence of Immanuel Kant's critical philosophy in which mathematics was described as dealing with the constructive activity of the human mind, mathematics and science became constructions that are brought about in a process of free creation. There are different ways in which this freedom can be more precisely defined. A mathematician who particularly emphasized the freedom of mathematical creation, was Georg Cantor, the creator of the theory of transfinite sets.

Cantor's discussions with Roman Catholic intellectuals are particularly interesting. On August 4, 1879 Pope Leo XIII issued the encyclical *Aeterni Patris*. The result of *Aeterni Patris* was that the interest in science among Roman Catholic intellectuals was greatly stimulated. The goal of the encyclical was a revival and modernization of Christian philosophy along Thomistic lines. According to the neo-Thomists the developments in science had led to false philosophies: materialism, atheism, liberalism. Leo XIII envisioned a reconciliation of modern science with Christian philosophy. In the encyclical he argued that modern science could greatly profit from Scholastic philosophy. It was the interest of Roman Catholic intellectuals in Scholastic philosophy and the infinite that brought Cantor into contact with them. Chapter 28, by Rüdiger Thiele is devoted to Cantor.

Neo-Thomism must have seemed very attractive to thinkers who were searching for the synthesis and unity offered by an all-encompassing philosophical and theological system. One of them was Jacques Maritain (1882–1973), who in his *Distinguer pour Unir ou Les Degrés du Savoir* (To distinguish in order to unite or the degrees of knowing) of 1946 tried to give a coherent and balanced survey of the different ways in which knowledge can be acquired. In doing so, Maritain assigned mathematics and mysticism their due places.

Within his own orthodox Russian tradition, Pavel Florensky related mathematics and the divine. In Chapter 31, Sergei Demidov and Charles Ford describe his life and views.

5.4. *The 20th century. Structuralist mathematics. Gödel. Process theology*

The German mathematician David Hilbert (1862–1943) was a key figure in the development towards the 'structuralist' view of mathematics that would dominate mathematics during most of the 20th century. In his *Grundlagen der Geometrie* (Foundations of Geometry) of 1899 Hilbert applied this approach to geometry. The book contains a rigorous, strictly axiomatic foundation of Euclidean geometry and the traditional non-Euclidean geometries. After 1900, when Hilbert gave his famous lecture on important unsolved problems in mathematics at the 2nd International Congress of Mathematicians in Paris, Hilbert's place of residence in Germany, Göttingen, became the centre of the mathematical world. Until 1933 Göttingen played a major role in the spread of the structuralist message and the application of the axiomatic method in all areas of mathematics and natural science. The astronomer and theoretical physicist Arthur Eddington, an active Quaker inclined to mysticism, was influenced by this view of mathematics. Physical knowledge is structural, of the kind investigated in the theory of groups, and the content of this structural knowledge is mind-stuff, he argued. Chapter 34 is devoted to him.

In structuralist mathematics the choice of a system of axioms is in principle completely free, the only restriction being that the system must be consistent, that is, it must be impossible to derive a contradiction from the axioms. A totally different philosophy of mathematics was developed by the Dutch mathematician and philosopher L.E.J. Brouwer (1881–1966). Brouwer completely rejected Hilbert's formalism and logicism. In Brouwer's view of mathematics the role of language and logic is absolutely secondary. Along Kantian lines Brouwer argued that mathematics consists in mental constructions which are executed on the basis of a fundamental mathematical intuition and that attempts to find a foundation for mathematics in language or in formal logic are essentially flawed. According to Brouwer the freedom of the mathematician does not lie in the choice of a system of axioms, but in the freedom to execute mental constructions. One of the consequences of Brouwer's views was a rejection of the principle of excluded third: 'p or non-p' is true for all statements p. In intuitionist mathematics a proof is by definition a mental construction. As a result a statement of the form 'p or non-p' is only considered proven if one can find either a construction corresponding to p, or a construction corresponding to non-p. In the 1920s Brouwers' intuitionistic papers began to appear in international journals. As a result the relationship between Brouwer and Hilbert, which had been cordial, rapidly deteriorated. Hilbert experienced intuitionism as a serious threat to classical mathematics. Hilbert decided to act and the result was an elaboration of his 1904 ideas on the way in which the consistency of mathematics could be guaranteed. Frege's work in logic and Russell and Whitehead's *Principia Mathematica* had suggested that a complete formalisation of mathematical theories was possible. The core of Hilbert's formalist foundational programme consisted of the following elements: (i) Define a formal system that captures the theory of the natural numbers, (ii) Prove that within the formal system no formula that expresses the inconsistency of the system can be formally derived, (iii) In order to avoid circularity this meta-mathematical proof of the consistency of Peano arithmetic should, of course, not implicitly assume this consistency. This is why the proof should only use what Hilbert called 'finitistic' means. This formalist programme was a brilliant answer to Brouwer's intuitionism, because 'finitistic means' essentially meant 'constructive means', i.e. means that should be acceptable to an intuitionist.

It cannot be denied that, compared to Brouwer's intuitionistic programme, Hilbert's formalist and finitist foundational programme has been extremely fertile. However, one of the most spectacular results produced within this programme, viz. Gödel's incompleteness theorem published in 1931, turned out to be a blow to Hilbert's position. What Gödel showed is that, if a formal system that contains Peano arithmetic is consistent, a proof of this consistency requires more than the methods of proof that are captured by the system. In a sense it turned out to be impossible to prove the consistency of Peano arithmetic.

It is remarkable that both Brouwer and Gödel were inclined towards mysticism. Brouwer's mysticism shows the influence of the Romantic movement (Chapter 30 by Teun Koetsier) while Gödel's mysticism is Platonic (Chapter 30, Section 7).

Let us, in this context, briefly turn to *process theology*. In process theology God has two natures, He is dipolar. Moreover, He is integrally involved in the cosmic process. God has a transcendent perfect nature and He has an immanent nature by which He is part of the process of the world itself. At the origin of this school of thought, which advocates the view that God himself is in process of development through His intercourse with the world, lies the work of A.N. Whitehead. Gödel's incompleteness theorem was as much a blow to Russell and Whitehead's *Principia Mathematica* as it was to Hilbert's views. The *Principia* were based on logicism and Peano arithmetic, and Gödel brought to light the limitations of both. Process theology as developed by Whitehead and others is therefore at least partly to be understood as an attempt at taking into account the new insights that resulted from Gödel's 1931 publication. As Granville C. Henry Jr. explains in *Logos: Mathematics and Christian theology*, process theology adopted and adopts an Augustinian point of view. In *De libero arbitrio* (On free will)[26] St. Augustine had established a close link between mathematics and a type of wisdom that is at the very heart of Christian faith. According to process theologians, this Augustinian view needed refinement in the light of the incompleteness theorem, for, as Henry puts it, proving the latter theorem made it clear that:

> There is no set of rules, no matter how large a (finite) number considered, that are appropriately formalized and axiomatized, that can give us *all* of the properties of number. The properties of arithmetic in their totality somehow transcend any other system. There is an intuitive presentation of the structures of number that cannot be mechanized by rules [19, p. 101].

In order to overcome the problem of the lack of a single all-encompassing formal system and to preserve the close relationship of mathematics and Logos (Christ as the Word) as well as the idea that God establishes and guarantees the unity of Logos, process theologians look for solutions in terms of plurality and increased potentiality. Radicalizing their views, Henry concludes

> It may be appropriate to understand the realm (however understood) of mathematical relationships, and hence of potential relationships, as *evolving*—a rather radical departure from modal Western philosophy or at least from its Christian adaptation [19, p. 112].

5.5. *Figurative mathematics*

Cusanus tried to understand the relation of the finiteness of human knowledge with the infinity of God by means of a sequence of circles converging towards a straight line. René Guénon's symbolism of the cross represents a similar form of relating mathematics and the divine. In Chapter 33 Bruno Pinchard discusses the ideas of this esoteric geometrician, who

[26] "Though we are unable to see clearly whether number is contained in wisdom, or derived from it or whether wisdom itself derives from number, and is contained in it, or whether it can be shown that both are names of the same thing, this much at least is clear, that both are true and are unchangeably true". *De libero arbitrio*, Chapter 11, 32, p. 142 quoted in [19, p. 95].

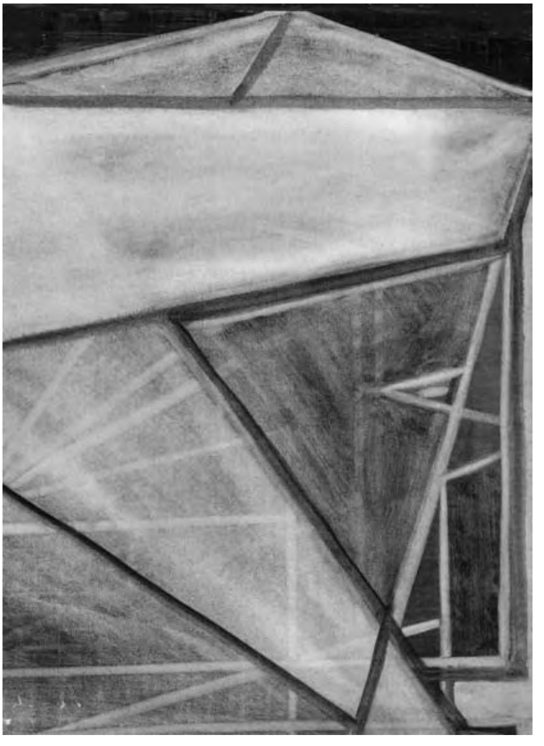


Fig. 14. Figurative mathematics (ca. 1916). Acquarel by Karel Schmidt. Courtesy of Karel Schmidt Jr.

did not adhere to his own tradition and displayed a remarkable openness towards other cultures in his *Symbolisme de la croix* (Symbolism of the cross). In this book Guénon presents space as the set of all possibilities characteristic of a particular being or of universal being. As Pinchard remarks, Guénon found the appropriate inspiration in the Eastern traditions of Taoism, Vedanta and Soufism, which view the centre of the universe as universal Potentiality rather than as the God of monotheism.

Cusanus and Guénon use mathematical examples as symbols to explain or illustrate non-mathematical divine truths. Another philosopher who used mathematical symbolism in this way was the Dutchman Mathieu H.J. Schoenmaekers (1875–1944). Schoenmaekers was a catholic priest, who left the church in 1903 because he had come to experience it as an oppressive institution that left no space for an authentic experience of faith. Schoenmaekers wrote several books in which he attempted to explain the mystery of the relation between the One and the Many, or between the Absolute and the Relative by means of mathematical metaphors. He called this use of mathematics *positive mysticism* or *figurative mathematics* (beeldende wiskunde). He used ellipses, sets of concentric circles and other mathematical figures to understand metaphysical truths.[27] It is remarkable that Schoenmaekers exerted considerable influence on a number of Dutch symbolist painters. One of them was Piet

[27] There is only one book on Schoenmaekers: [23].

Mondriaan. In a series of articles in the magazine *De Stijl* in 1919 Mondriaan used Schoenmaeker's terminology to explain that what he aimed at in his paintings was an expression of the universal harmony of the existing relations in nature. According to Mondriaan this harmony must be seen, it cannot be understood rationally. Carel Blotkamp and Mieke Rijnders describe Mondriaan as a representative of a romantic, mystical view of nature.[28] In Figure 14 we have reproduced a work by Karel Schmidt (1880–1920), another painter who was influenced by Schoenmaekers.

5.6. *The turn to the transcendental subject*

Gödel's incompleteness theorem represented another step in the *turn to the transcendental subject* characteristic of 19th and 20th century mathematics, represented among other things by the growing awareness of mathematicians of their freedom in the creation of concepts that we emphasized in Section 5.3 of this introduction. Gödel showed that whenever the human mind is capable of formulating some of its mathematical intuitions, this immediately yields new intuitive knowledge, for example intuitive knowledge of the consistency of the developed formalism. The mathematician's capacity for intuitive comprehension of truth therefore appears to extend beyond any formalistic reduction [13, p. 61, Footnote 4]. The mathematician seems indeed to have inexhaustible potential for departing from a given formalism and widening its scope. The technical results of Gödel's theorem shook the world of mathematics. The philosophical implications made their impact as well. However, the idea of a sovereign Subject forever beyond or ahead of formalization, which the incompleteness theorem naturally brings to the fore, was part of an existing undercurrent of thought that went back at least to the time of Kant. Kant is sometimes said to have brought about a Copernican revolution in philosophy. His ideas had far-reaching consequences in other fields of learning and scientific practice. The radical turn toward the *knowing subject* that Kant undertook, marked the beginning of a period in which man became the focus of attention in fields where his presence had not been felt before. Eddington's striking image of man studying the tracks of a being unknown to him, and then realizing that he was actually studying his own, fully applies here (cf. Chapter 34 by Teun Koetsier).

The plan to make this book was born from the editors' shared interest in aspects of the history of Dutch culture, particularly the history of the Dutch signific movement.[29] The term *significs* was coined at the end of the 19th century by Victoria Lady Welby (1837–1912), lady in waiting to Queen Victoria. She felt that a clarification of words and concepts would improve human communication and understanding. She expected that signific investigations would contribute to ending the mistrust and misunderstanding between social groups. Her ideas were taken seriously and she discussed them with Charles S. Peirce, Bertrand Russell, Bergson, Carnap, Lalande and others. Although the movement has sunk into oblivion it represented for some time an interesting pendant to the Unity of Science movement. In The Netherlands Lady Welby had considerable influence through the psychiatrist, poet and social reformer Frederik van Eeden (1860–1932). Van Eeden founded

[28] Cf. [7, p. 89].

[29] For the signific movement see [18].

the Signific Circle (Signifische Kring) of which the Dutch mathematicians L.E.J. Brouwer and Gerrit Mannoury were prominent members. In the works of the Dutch significians the turn towards the subject that we mentioned above is clearly present. Van Eeden wrote *Het Lied van Schijn en Wezen* (The song of appearance and essence), in which a host of original metaphors describe the workings of the Self. The turn towards the subject is also apparent in the search for the volitional element underlying every communicative act (cf. Chapter 29 on Gerrit Mannoury by Luc Bergmans). It was a dominant theme of signific literature that the most abstract and formalized expressions of the human mind, when studied properly, appear to be rooted in the deepest expressions of the will, in other words that no formalism stands on its own, but that it presupposes a past and continued research. It is by uncovering the volitional aspects of counting and measuring that Gerrit Mannoury linked mathematics and what he termed "mysticism", viz. every striving for the ultimate that manifests itself in individual and in shared convictions, be they of an explicitly religious nature or not. L.E.J. Brouwer, who, just like Mannoury, was both a significian and a mathematician, grounded his intuitionism on fundamental reflections regarding the will. Brouwer's preoccupation with the volitional has to be interpreted within its historical context. Above all, a comparison with the philosophy of Schopenhauer, author of the *The world as will and representation* suggests itself (cf. Chapter 30 by Teun Koetsier).[30]

5.7. *A modern creation myth?*

Traditional creation myths tell us about the beginning of the world and the origin of man as they are understood within a particular culture. A group of modern scientists, astrophysicists, geologists, chemists, biologists, paleontologists, historians, etc., faced with the question of how we, in our culture, think about the origin of the world and of ourselves, might give the following answer:

> Thirteen billion (13 000 000 000) years ago there was nothing. There wasn't even emptiness. Time did not exist. Nor did Space. In this Nothing, there occurred an explosion, and within a split second, something did exist [...] For a trillionth of a second it expanded faster than the speed of light, growing from the size of an atom to the size of a galaxy. Then the rate of expansion slowed [...] After about one billion years, huge clouds of hydrogen and helium began to gather and then collapse in on themselves under the pressure of gravity. As the centre of these clouds heated up, atoms fused together violently like vast hydrogen bombs, and the first stars lit up. Hundreds of

[30] It is important to realize that neither significs nor intuitionism can be reduced to psychological theory, even if the formulations used may have been affected by fashionable psychological vocabulary. It would be most interesting to compare Brouwer's views regarding the transcendental ego with those expressed by the later Husserl, who rejected psychologism in mathematics. In Chapter 32 a crucial phase in the development of Husserl is described by François de Gandt. He explains Husserl's turning away from a sceptical attitude, and heading towards the recognition of a transcendental ego, which could account for the objectivity of mathematics as well as for its being created. In Brouwer's writings the notion of transcendental ego is endowed with religious overtones reminiscent of eastern traditions referring to the Self, which is freely acting and beyond representation. The second-generation significian, Johan J. de Iongh, also held the relationship with the Subject to be an essential feature of mathematical truth. Some of his formulations suggest that for him the (obviously subject-related) platonic and Augustinian longing for truth was the very foundation of (mathematical) truth. In the paragraph on intuitionism of Luc Bergmans' chapter 29 on the significians, it is shown how Brouwer's treatment of infinity can be related to the notions of longing or searching for truth as opposed to possessing truth.

> billions of stars appeared, gathered in the huge communities we call "galaxies" [...] Our own sun was formed about four and half billion years ago [...] The planets of our solar system, were formed at the same time as the Sun, from the debris left over from the Sun's creation [9, pp. 165–166].

On the early Earth chemical reactions produced simple forms of life. Some of them developed the capability to extract energy from sunlight and eventually an interconnected web of Life was formed on Earth. From about 600 million years ago, there began to appear multicellular organisms. A mere 250 000 years ago, man appeared. The number of humans grew as they learnt how to live in more and more diverse environments, first in Africa, then in Eurasia, Australia, the Americas and finally on the islands of the Pacific. Agriculture was invented.

> As populations grew, the number and size of villages grew until there appeared the first large cities, from about 5000 years ago. These large, dense settlements required new and complex forms of regulation to prevent disputes and coordinate the activities of many people living at close quarters. In this way there appeared the first states, groups of powerful individuals capable of regulating the activities of the community as a whole [...] About 500 years ago, for the first time, these changes brought human communities in all parts of the world into contact with each other. For many communities this coming together was disastrous; it brought conquest, disease and exploitation, sometimes of the most brutal kind. But this merging of regional communities also helped trigger new technological breakthroughs that could now be shared throughout the world. In the last two centuries, new technologies, beginning with the harnessing of steam power, have given human societies access to the vast sources of energy locked up in fossil fuels such as coal and oil [9, pp. 165–166].

This modern creation story contains, with the exception of a few numbers, no mathematics. However, this is because it is a popular version of the full story. The complete story cannot be told without the formal language of mathematics. This is particularly clear, for example, in the case of the theory of the 'Big Bang', but in the other sciences involved the role of mathematics and computer simulations is ever growing. The modern creation story is to a considerable extent a mathematical story.

In modern Western culture science and religion have become separated. Religion has very much become a private affair. And indeed, one could argue that this modern creation story has got nothing to do with the divine and that it is essentially different from the traditional creation myths in the sense that it is merely a description, and not a normative text that helps us to make sense of our own existence. Such an argument is correct from a modern scientific point of view. God is no longer present in modern scientific theories. Yet the modern creation story can be interpreted in a religious way and, then, inevitably, its mathematical elements become related to the divine and there are many examples of individual scientists putting the story or aspects of the story in a religious perspective. At the end of this introduction we will briefly mention three examples.

The theory of the Big Bang is based on Einstein's equation in the theory of general relativity (See Figure 15). The entire content of general relativity can be summarized as follows: The curvature $g_{\mu\nu}$ of spacetime is related to the matter distribution in spacetime by Einstein's equation.[31] Given that the universe is expanding, Einstein's equation implies[32] that at a distant time in the past the universe was a singularity, in which the distance

[31] Cf. [42].

[32] There are some other reasonable assumptions involved.

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = - \frac{8\pi G}{c^4} T_{\mu\nu}$$

Fig. 15. Einstein's formula.

between all points of space was zero and the density and curvature of the universe were infinite. The explosive development of the universe from this singularity is called the "Big Bang". Two quotations may suffice to give an indication of Einstein's religious perspective. Einstein appreciated Spinoza's determinism and he recognized the divine in the harmony and beauty of nature. He said:

> I believe in Spinoza's God who reveals Himself in the orderly harmony of what exists, not in a God who concerns himself with fates and actions of human beings.[33]

His well-known remark, "I want to know God's thoughts [...] the rest are details", expresses the same idea. Elsewhere Einstein wrote:

> Science can only be created by those who are thoroughly imbued with the aspiration toward truth and understanding. This source of feeling, however, springs from the sphere of religion. To this there also belongs the faith in the possibility that the regulations valid for the world of existence are rational, that is, comprehensible to reason. I cannot conceive of a genuine scientist without that profound faith. Science without religion is lame, religion without science is blind.[34]

The group of scientists that we introduced at the beginning of this section might continue as follows:

> Human populations have grown faster than ever before [...] Today, human numbers are so great, and the impact of humans on the biosphere is so significant, that there is a real danger that we will do serious damage to the environment that is our home. And this could lead to a global collapse of human civilizations and have devastating impacts on other organisms as well. On the other hand, the ability of humans to share knowledge is now greater than ever before, and it may be that new technologies and new ways of organizing human societies will allow us to avoid the dangers created by our ecological virtuosity.[35]

The role of mathematics is just as important when we try to find answers to the ultimate question of where mankind is heading. Mathematics has become indispensable for the survival of the biosphere, the reduction of pollution and global warming, the preservation of biodiversity, etc. In this respect the views of Granville C. Henry Jr. give an interesting Christian perspective. He wrote in 1976:

> An adequate understanding of the relationship of science to the Christian religion depends upon an analysis of the relationship of mathematics to God. In the near future, decisions of vast political import will probably have to be made on the basis of mathematical models that relate population, pollution, industrialization, and other factors. A theology of mathematics concerned with God's

[33] Quoted in [31, pp. 659–660].
[34] Quoted in [27, pp. 121–122].
[35] [9, pp. 165–166].

> relationship to the discipline and its content ought to be available, especially to the Christian community, in order to aid it in such decisions.[36]

We will conclude with a completely different religious view of the evolution of mankind. Pierre Teilhard de Chardin (1881–1955), paleontologist and Jesuit priest, interpreted the evolution of the universe and man as a progression towards a higher state of consciousness. He claimed that the layer of human consciousness covering the globe would become a collective brain in the future and that the final goal of evolution is a union of humankind with the universal God-Omega. Teilhard wrote:

> The being who is the object of his own reflection, in consequence of that very doubling back upon himself, becomes in a flash able to raise himself into a new sphere. In reality, another world is born. Abstraction, logic, reasoned choice and inventions, mathematics, art, calculation of space and time, anxieties and dreams of love—all these activities of *inner life* are nothing else than the effervescence of the newly-formed centre as it explodes onto itself [38, p. 165].

The world wide web is connected by some followers of Teilhard with this mystical view of human evolution. Cyberspace would then be an important element of this collective brain that Teilhard called the 'noosphere'. Because computer programmes are abstract mathematical structures, cyberspace is on a certain level of interpretation a huge mathematical structure as well. We will not discuss the merits of Teilhard's views, but its recent transformations contain remarkable links between mathematics and the divine.

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CHAPTER 1

Chinese Number Mysticism

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Contents

MATHEMATICS AND THE DIVINE: A HISTORICAL STUDY
Edited by T. Koetsier and L. Bergmans

1. Introduction

The modern Chinese equivalent of the term "mathematics" is "*shuxue*" but before the middle of the nineteenth century this Chinese term had a much wider meaning, encompassing mathematics, philosophy, astrology and divination. We can find parallels in mediaeval Europe where the great Roman writer Boethius (ca. 480–524/525) divided mathematical disciplines into arithmetic, music, geometry and astronomy, which were together referred to as the *quadrivium* in early European universities. In a similar way traditional Chinese mathematics not only denoted mathematics in our sense of the word, but philosophy, astrology, divination and an element of mysticism as well. Numbers had other significance besides their numerical values.

2. The *Hetu* Diagram and the *Luoshu* Chart

In the Mingtang (Bright Hall) Chapter of Dai De's *Da-Dai Liji* (Record of Rites by the Elder Dai), a work dated about the year AD 80, there is an arrangement of the numbers 1 to 9 in sets of three as follows:

2	9	4
7	5	3
6	1	8

The purpose of these numbers is not explained in the book, and we do not clearly understand what their purpose is. We only know that the Bright Hall from the title of the chapter consisted of nine halls in which the Zhou emperors performed ceremonial rites. These nine numbers in three sets of three numbers also appeared in a probably earlier mathematical text, the *Shushu jiyi* (Memoir on Some Traditions of Mathematical Arts), said to have been written in the year 190 BC by Xu Yue. Here the set of numbers, which forms a magic square of the order 3, is referred to as the *jiugong* (nine-palaces) or the *jiugongtu* (nine-palaces diagram). The *Shushu jiyi* also mentions an arrangement of numbers for calculation known as *jiugongsuan* (nine-palaces calculation) but gives no details apart from mentioning numbers circulating round (within the nine-palaces). This might have been a system of divination because we know that this *jiugong* magic square was already employed for this purpose in Han China.

There were two important figures or diagrams that had dominated Chinese thought since ancient times. They were the *Hetu* (River Diagram) and the *Luoshu* (Luo River Chart). There are different descriptions of their origins, the most popular ones saying that the legendary emperor Fuxi obtained the *Hetu* Diagram from a dragon-horse at the Yellow River and that later during the 11th century BCE, Wenwang, the father of the first Zhou king, received inspiration from this diagram and produced the eight Trigrams used in his method of divination. As for the *Luoshu*, Yu, who later became the first king of the Xia dynasty (a dynasty that modern archaeologists in China are still searching for evidence to confirm), observed at the Luoshui River a tortoise carrying the *Luoshu* Chart on its back. Both the *Confucian Analects* and the *Yijing* (Book of Changes) mention the names of the

Hetu Diagram and the *Luoshu* Chart. However, we have no evidence in the form of actual figures of such Diagrams or Charts to tell us what they looked like at the time of Confucius.

Much discussion on the numbers in the *Hetu* and the *Luoshu* about their applications to astrology and divination took place in the succeeding generations. Then in the tenth century a famous Daoist by the name Chen Tuan interpreted the nine-palaces arrangement as the *Hetu* Diagram. But during the twelfth century Cai Yuanding (1145–1198), a disciple of Zhu Xi (1130–1220), identified the Bright Hall and the nine-palaces arrangement as the *Luoshu* Chart. Cai Yuanding himself was the greatest geomancer of his time. His view had the support of Zhu Xi, whose reputation as a Neo-Confucian scholar raised Cai's theory beyond challenge. Henceforth scholars talked about the *Luoshu* magic square as synonymous with the nine-palaces diagram. No matter what name it is known by, modern scholars regard it as the earliest magic square known to the ancient world.

The *Luoshu* Chart and the *Hetu* Diagram, as known since the twelfth century, are shown in Figures 1 and 2. Figure 1 clearly represents a magic square equivalent to the square of the nine-palaces diagram. Figure 2 does not qualify as a modern magic square, and has often been left aside by modern scholars studying Chinese magic squares. This is rightly so if one looks at magic squares pure and simple. However, to the traditional Chinese mathematician the two figures are of equal importance, both mystically and philosophically. In both Figure 1 and Figure 2 numbers are represented by little black and white circles, black circles for the odd or *Yang* numbers and white for the even or *Yin* numbers.

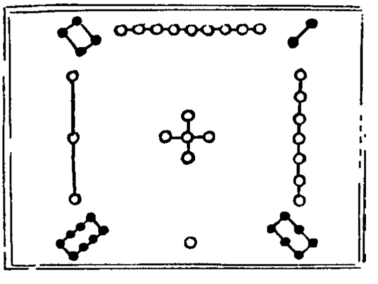

Fig. 1. The *Luoshu* Chart magic square.

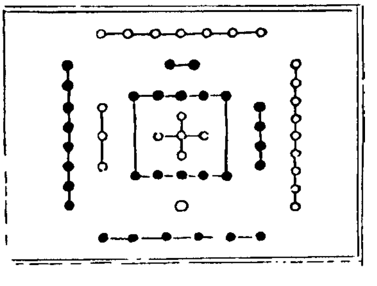

Fig. 2. The *Hetu* Diagram.

	Water	*Fire*	*Wood*	*Metal*	*Earth*
Heaven number	1	2	3	4	5
Earth number	6	7	8	9	10

Fig. 3. Heaven and earth numbers.

Numbers 1 to 5 form the heaven (*tian*) numbers, while 6 to 10 the earth (*di*) numbers. The numbers also represent the *wuxing*, a term that has been variously rendered as "Five Elements", "Five Agents", "Five Phases", etc. These are shown in Figure 3.

The so-called heaven numbers are defined in the *Book of Changes* as the numbers from 1 to 5 and the earth numbers as from 6 to 10. Odd numbers are *Yang* numbers, while even numbers are *Yin*. The *Book of Changes* also mentions 25, the sum of all odd numbers from 1 to 9, as being significant. Other significant numbers are the number 30, the sum of all even numbers from 2 to 10, and the number 55, which is the sum of all heaven and earth numbers, i.e. from 1 to 10. Neo-Confucian philosophers saw great significance in the number 5 being at the centre of the *Luoshu* Chart. To them 5 is the most important *Yang* heaven number, manifesting itself in the *wuxing*, in the five virtues in ethics, in the five colours, in the five tastes in human perceptions, in the five cereals that sustain human life, in the five human relationships that govern human behaviour, and so on. The number 9 is the highest *Yang* earth number. The number 9 together with the number 5 resulted in the office of the Chinese emperor often being referred to as having the exaltation of nine-and-five (*jiu wu zhi zun*). Let us take an example from Chinese astrology to illustrate the important role played by the number 9. Only seven stars were identified in the Plough by Chinese astronomers, but according to an old Star Manual, the *Xingjing* the Plough originally consisted of nine stars, two of which had already gone out of sight. Chinese astrology insists that there are nine stars, two of which have become invisible. One explanation is that the two stars concerned had gone beyond the circle of perpetual vision in the north sky, indicating that the antiquity of Chinese astronomy would be much beyond what many modern historians of astronomy think. Perhaps one may regard it instead as a reminiscence of the Pythagorean invention of a counter-earth to make up their perfect number 10.

Looking at the *Luoshu* magic square in Figure 1, let us begin with the two numbers 1 and 6 that represent *Yang Water* and *Yin Water*, respectively. Moving in the anticlockwise direction, we next find the two numbers 7 and 2 that represent *Yang Fire* and *Yin Fire*, respectively, which are conquered by the two previous *Waters*. Moving again in the same anticlockwise direction, we find *Yin Metal* 4 and *Yang Metal* 9 conquered by the two previous *Fires*. Proceeding likewise, we find the *Metals* 4 and 9 conquering *Yang Wood* 3 and *Yin Wood* 8, which in turn conquers *Yang Earth* 5 at the centre of the magic square. Finally *Earth* 5 conquers the *Yang Water* 1 and *Yin Water* 6. The process repeats itself in a cycle. Here the Chinese found a powerful symbolism that represents and "proves" the Principle of Mutual Conquest as applied to their *wuxing* theory.

Next look at the *Hetu* diagram in Figure 2 and begin again from the numbers *Yang Water* 1 and *Yin Water* 6, moving this time in the clockwise direction. We find the two *Waters* producing *Yang Wood* 3 and *Yin Wood* 8, which in turn produce the next *Yang Fire* 7 and *Yin Fire* 2. The two *Fire* numbers then produce *Yang Earth* 5 and *Yin Earth* 10 at the centre of the diagram. The two *Earth* numbers then produce *Yin Metal* 4 and *Yang Metal* 9, which

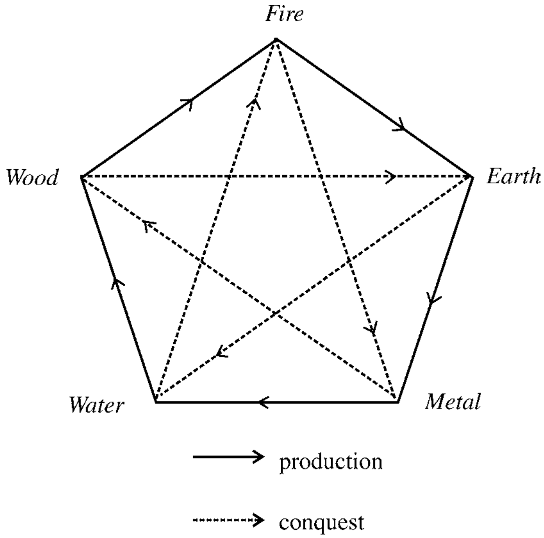

Fig. 4. The Orders of Mutual Conquest and Production.

produce *Yang Water* 1 and *Yin Water* 6. The process goes on and on. Here the Chinese thought that they had proven the Principle of Mutual Production of the *wuxing*. Figure 4 illustrates both the Principle of Mutual Conquest and that of Mutual Production.

Thus numbers were intricately linked with the very fundamental early Chinese thinking that went to explain everything from the natural world to the spiritual as well as human behaviour. Their connections were not restricted to the *Yin* and *Yang* principle and the *wuxing* theory. They played a big role in the divination system of the *Yijing* (Book of Changes), with which their mystic nature was further enhanced.

3. The system of the *Yijing* (Book of Changes)

The *Book of Changes* speaks about the *Dao*, as does the Daoist canon *Daodejing*, a text traditionally attributed to Laozi, an elder contemporary of Confucius in the fifth century BC. The *Daodejing* attaches the utmost importance to the number 1. A passage says:

> What have attained unity in the past are:
> Heaven has become bright and pure after attaining unity;
> Earth has become firm and sure after attaining unity;
> The spirits have become luminous on attaining unity;
> The valleys have been filled by their own void on attaining unity; and
> The myriad creatures have gained life on attaining unity.

Another passage in the text says:

> The *Dao* produces 1; 1 produces 2; 2 produces 3; and 3 produces the myriad creatures.

It is the latter that has struck a chord of resonance with the Book of Changes.

The great importance of the number unity here needs no further elaboration other than stating its parallel status in the Islamic world, where unity is regarded as a representation of Allah. The cell containing the number 1 in an Islamic magic square is often put either in the middle of the top row or left completely empty as a mark of respect. In the system of the *Book of Changes* unity denotes *Taiji* (Supreme Pole), which gives rise to the two cosmological forces (*liangyi*) of *Yin* and *Yang*. The two cosmological forces subdivide to form the four Symbols (*sixiang*), each of which in turn divides into two to form the eight Trigrams (*bagua*). Combinations of two Trigram produce sixty-four Hexagrams. These sixty-four Hexagrams can be represented by binary numbers from 000000 to 111111, i.e. from 0 to 63. They have been employed in a sophisticated system of divination in China for well over two thousand years. The *Book of Changes* says:

> The *Dayan* (Great Extension) number is 50, (but the system) uses (only up to number) 49. Divide it into two to symbolise the two cosmological forces of *Yin* and *Yang*. Suspend one (between the fingers) to symbolise the three (powers, *sancai*—heaven, earth and human beings). Divide into lots of four to symbolise the four seasons . . .

The imprecision of the above passage has given rise to many different interpretations as to how exactly one should put the system into practice. However, the general method has been to take 50 pieces of milfoils in one hand, put 1 piece aside to symbolise the *Taiji*, then with the other hand take some of the remaining 49 pieces at random to divide the milfoils into two parts, symbolising the formation of *Yin* and *Yang*. The next step is to remove 4 milfoils successively from those held in one of the two hands. The procedure beginning from this step varies according to different schools. For example, some schools referred to the bundle of milfoils held in the left hand and others the right. The differences, however, do not depart from the idea of a liturgy representing the process of the natural world in Chinese thinking. With the liturgy the diviner hoped to invoke the power of nature to his aid.

J.G. Frazer, the Cambridge anthropologist of *The Golden Bough* fame, had two “laws” on magic. The first was the law of similarity, where like produced like. The second was a law of contiguity or contagion, which said that things which had been in contact but were no longer so would still continue to act upon one another. The manipulation of milfoils by the diviner using the system of the *Book of Changes* is a case in point of the first law. The *Book of Changes* gives the Chinese another three significant numbers. They are 49, the number of milfoils used in divination; 50, the *Dayan* (Great Extention) number; and 64, the total number of Hexagrams.

To illustrate the mystical nature of the number 49 let us refer to the last problem in a sixth century Chinese mathematical text, the *Sunzi suanjing* (Mathematical Manual of the Master Sun). It tells how to calculate whether a pregnant mother will give birth to a male or a female child. The method is to take the number 49, then first subtract from it the month when the child is to be born (one version says the month when the child was conceived), followed by subtracting the age of the mother and the numbers 1, 2, 3 and 4 in succession. An odd remainder tells that the child will be male, and an even remainder that it will be female. Ruan Yuan, a well-known nineteenth century Chinese scholar, regarded this problem as too bizarre to have actually been written by Master Sun and concluded that it was a later unworthy addition to the original text. Perhaps the beginning of the acceptance of the modern meaning of the term mathematics in China had led him to overlook the fact

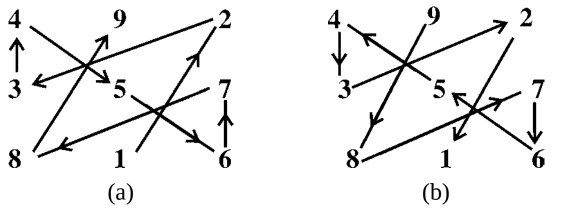

Fig. 5.

that the definition of the term mathematics in traditional China differed from that of his time. Divination, magic and mathematics were in traditional China covered by the same word. The numbers 1, 2, 3 and 4 must have referred, respectively, to (1) the *Taiji*, (2) the *Yin* and the *Yang*, (3) the "three powers" of heaven, earth and human beings, and (4) the four seasons, similar to the operation of the system of divination according to the *Book of Changes*. Hence the *Sunzi suanjing* provides us another example of Frazer's first law at play.

The 3 × 3 magic square finds a place in the traditional Chinese astrological calendar, where different colours are assigned to each number, and years, months and days are designated by a number together with its colour. The three numbers 1, 6 and 8 are white and are regarded as auspicious. The position of the numbers changes in the direction of the *yubu* steps, as in Figure 5(a) and (b), forming nine different 3 × 3 squares, of which only that with the number 5 at the centre is a true magic square. The positions of 1, 6 and 8 on a square indicate the lucky directions for the year, month and day concerned, subject to the combinations of the ten stems and twelve branches used to denote the year, the month, the day and the double-hour of the day.

4. Daoist liturgy

In Daoist ceremonies the priest moves his feet following the pattern of the nine stars of the Plough. This is called *Yubu* (Strides made by Yu the Great) or *bugang* (Stepping on the Stars of the Plough). By tracing the pattern of the Plough the performer of the rites tries to invoke the celestial power of the nine stars. This is Frazer's first law in operation. Now the Spirits of the nine stars of the Plough are also said to reside within the *jiugong* nine-palaces. This gave rise to another, and perhaps more powerful *Yubu* or *bugang* pattern by taking the *jiugong* magic square, taking the first step from 1 to 2, then to 3, etc., until reaching the number 9, or conversely starting from 9, and moving successively to 8, 7, 6, 5, 4, 3, 2 and 1. See Figure 5(a) and (b). By copying the pattern of the position of the Plough in the *jiugong* magic square, the performer hoped to invoke not only spiritual power from the Plough but also the force of nature embodied in the principles of the *Yin* and *Yang* and the *wuxing*, represented in the *Luoshu* Chart.

The magical *Yubu* or *gangbu* steps used in Daoist liturgy was also applied in the construction of Chinese magic squares. Take for example a 9 × 9 magic square found in the *Dayan suoyin*. This is a so-called composite magic square, consisting of nine 3 × 3 smaller

IV	**IX**	**II**
III	**V**	**VII**
VIII	**I**	**VI**

(a)

31	76	13	36	81	18	29	74	11
22	40	58	27	45	63	20	38	56
67	4	49	72	9	54	65	2	47
30	75	12	32	77	14	34	79	16
21	39	57	23	41	59	25	43	61
66	3	48	68	5	50	70	7	52
35	80	17	28	73	10	33	78	15
26	44	62	19	37	55	24	42	60
71	8	53	64	1	46	69	6	51

(b)

360	405	342
351	369	387
396	333	378

(c)

Fig. 6.

magic squares distributed in accordance with the order of the *jiugong* diagram. See Figure 6(a). The numbers 1 to 81 are divided into nine subgroups, namely 1 to 9, 10 to 18, 19 to 27, 28 to 36, 36 to 45, 46 to 54, 55 to 63, 64 to 72, and 73 to 81. Taking each subgroup in turn and distributing the numbers in the respective cells of the nine 3 × 3 small magic squares using the *bugang* steps results in Figure 6(b).

Let us look at the nine 3 × 3 magic squares I to IX in Figure 6(a). The sum of all nine cells in I to IX are 333, 342, 351, 360, 369, 378, 387, 396, and 405, respectively. These nine numbers also form another magic square as in Figure 6(c).

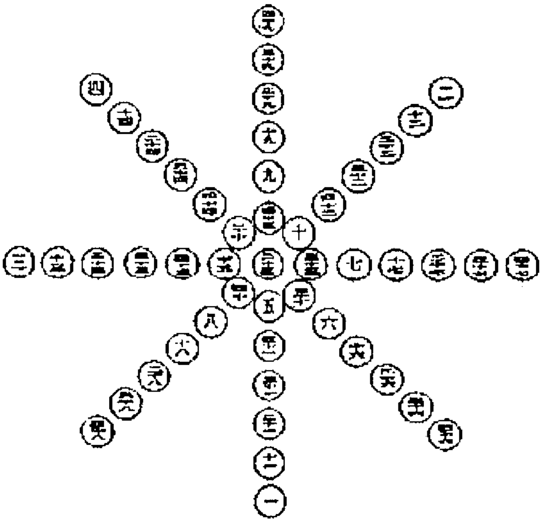

Fig. 7. Magic circle from the *Dayan suoyin*.

A number of other 9 × 9 magic squares can also be constructed by taking the *Yubu* steps backward or by a combination of forward and backward movement. The *Dayan suoyin* also contains some sort of a magic circle constructed using the same principle but in a more hidden form as shown in Figure 7. The vertical line from top to bottom reads 49, 39, 29, 19, 9, 45, 25, 5, 41, 31, 21, 11, and 1, with 25 at the centre. The horizontal line reads from left to right, 3, 13, 23, 33, 43, 15, 25, 35, 7, 17, 27, 37, and 47. The diagonal line in the NW and SE direction, starting from the number at the NW, reads 4, 14, 24, 34, 44, 20, 25, 30, 6, 16, 26, 36, and 46. The other diagonal line, starting from the number in the NE, reads 2, 12, 22, 32, 42, 10, 25, 40, 8, 18, 28, 38, and 48. Note that 25, the sum of the heaven and earth numbers, occupies the central position and that 49, the number of milfoils used in divination according to the system of the *Book of Changes*, occupies a prominent position right on top in the figure.

5. Mysticism in the Chinese magic square

In Figure 7 the numbers in every vertical, horizontal and diagonal line add up to the same number, 325, but not those in the circles. This is only an imperfect magic circle in the modern sense. Another example that modern historians of Chinese mathematics come across is the inclusion of the River Diagram (see Figure 2) among the magic squares given by the thirteenth century mathematician Yang Hui. As we can see, this diagram does not qualify as a magic square. It is important to note the difference in meaning and in purpose of a different culture and time. From the point of view of modern magic squares, some of the so-called Chinese magic squares and circles are imperfect, but the Chinese referred to them only as “traverse and transverse diagrams” (*zonghengtu*), while their main objective was mysticism, if not recreation, rather than number theory.

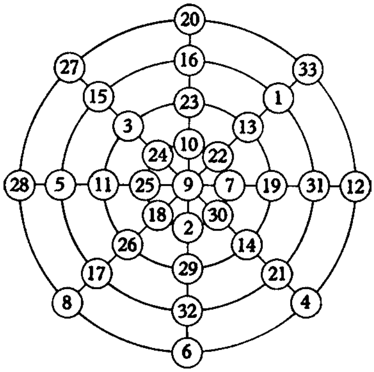

Fig. 8.

46	8	16	20	29	7	49
3	40	35	36	18	41	2
44	12	33	23	19	38	6
28	26	11	25	39	24	22
5	37	31	27	17	13	45
48	9	15	14	32	10	47
1	43	34	30	21	42	4

Fig. 9.

In the realm of mysticism Chinese magic squares were perhaps even more magical than modern magic squares.

The object of the exercise in Figure 7 was to demonstrate the important relation between the number 49 and the number 25. The main purpose of many of the earlier Chinese magic squares was to underline the importance of those numbers which were regarded as significant. Figure 8 is a true magic circle given by Yang Hui where all the cells taken in a circle, vertically, horizontally and diagonally, all add up to 147. Here the cell at the centre is 9, the highest *Yang* number.

Yang Hui also gave two 7×7 magic squares to show the significance of the *Yijing* number 49 together with the total number of heaven and earth, 25, as in Figures 9 and 10.

Note that in Figures 9 and 10 the two numbers 1 and 49 are symmetrically opposite, showing the relation between these two numbers. Furthermore, they add up to 50, the Great Extension number. In fact, magic squares provided one of the ways to "prove" the significance of the number 49. It is also interesting to note that one of the three *juan*s (section of a book occupying one complete scroll, roughly equivalent to a chapter) of Ding

4	43	40	49	16	21	2
44	8	33	0	36	15	30
38	19	26	11	27	22	32
3	13	5	25	45	37	47
18	28	23	39	24	31	12
20	35	14	41	17	42	5
48	29	34	1	10	7	46

Fig. 10.

1	62	7	60	13	55	54	8
63	15	48	23	46	43	20	2
6	18	25	39	38	28	47	59
61	49	36	30	31	38	16	4
9	24	32	34	35	29	41	56
51	44	37	27	26	40	21	14
12	45	17	42	19	22	50	53
57	3	58	5	52	10	11	64

Fig. 11.

Yidong's *Dayan suoyin* is devoted entirely to giving other reasons to explain the number 49. The significance of the number 64 is shown in an 8 × 8 magic square in Figure 11 by Yang Hui. The number 64 here is symmetrically opposite the number 1, indicating its importance.

The above discussion applies to Zhu Xi's (1130–1200) Non-Confucian school in general and to the *xiangshu* (Symbolic Numerology) branch in particular. From the ninth century some Chinese scholars introduced elements of Buddhist and Daoist philosophy into Confucian teaching. This was the beginning of Neo-Confucianism. Major development took place during the eleventh century. Cheng Hao (1031–1085) and his brother Cheng Yi (1032–1107) incorporated the *Book of Changes* into their philosophical thought and devised a metaphysics based on the Supreme Pole, *Yin* and *Yang*, and the *wuxing* principles to explain Confucian ethics. Zhang Zai (1020–1077) applied a similar metaphysics to natural science, while Shao Yong (1011–1077) used numerology to illustrate that past political events were all pre-determined. Some of the Neo-Confucian scholars were themselves exponents of the art of divination using the system of the Book of Changes. The final synthesiser of these teachings was Zhu Xi, the twelfth century Chinese thinker to whom comparison has been made with Thomas Aquinas. Neo-Confucianism dominated Chinese thinking until the nineteenth century.

One branch of Neo-Confucianism was the Symbolic Numerology (*xiangshu*) school, which placed emphasis on the Trigrams and Hexagrams, and on the *Hetu* Diagram and *Luoshu* Chart. Its diverse activities varied from divination, which was not confined exclusively to the system of the Book of Changes, to magic squares. In divination numerological calculations were performed not only to foretell future events but also to account for the past, while in magic squares numbers were used to "verify" or to "justify", if not to "explain" the fundamental concept of the school. In this respect number mysticism had played an important role in Chinese thinking. Zhu Xi's teaching had a great influence in pre-modern Japan and Korea. Hence one may say that this account on Chinese number mysticism also applies to East Asia in general.

6. Popular beliefs in number mysticism and conclusions

The last statement in the previous paragraph, however, is restricted not only in historical, but also in geographical, cultural and intellectual context. Neo-Confucian teaching was not general knowledge among the common people in East Asia, and not even among the Chinese themselves. In spite of the permeation of Neo-Confucian teaching into their societies over a long period, variation in belief among the people of East Asia is to be expected. The significance of the numbers 1, 6 and 8 in the astrological calendar, for example, is not general known among the Chinese and Japanese people still using that calendar today.

The number 1 is popular among East Asian societies also because of the hope of winning in a competition by coming first. In Japan *Daiichi* and *Ichiban* are familiar names used in the business world. *Hitotsubashi* is used as a name by a prestigious university. Denoting unity, the number 1 also has a political undertone in either national or a commercial sense, if not in both. In this context the number 2 means division or separation and does not enjoy the same popularity as the number 1. On the other hand, to the Chinese 2 is an auspicious number denoting a pair or couple. Here we have a divergence between the Chinese and Japanese preference for odd and even numbers. Odd numbers are more popular with the Japanese and even numbers with the Chinese. Japanese cookery sets, for example, are in five, while Chinese sets are in six. The number 3 is universally popular in East Asia, especially in the business world. In Japan we find names like *Mitsui*, *Mitsubishi*, *Mitsukoshi*, *Sanai*, *Sanwa*, *Sanyo*, *Mita* and *Mikasa*, in Korea there is *Samsung*, in Taiwan we come across *San Yang* and *San Shang*, to quote only a few names at random. The same word for "three" is used by the Chinese and the Japanese, and the Korean word also came from China. The word rhymes with or sounds like the Chinese word meaning life and growth. Frazer's first law of like producing like does explain the popularity of the number 3. Number 13 has no significance to the Chinese-educated, and to the Chinese business world in

Hong Kong it rhymes with two words, which together mean “certain to live” if not “definite to grow”, and is therefore a lucky number.

The Chinese word for the number 4 rhymes with the word meaning “death”. Again Frazer’s first law can explain its unpopularity among people in East Asia. The Japanese often avoid saying it *shi* in the Chinese way and read it as *yon* according to their native usage. Rhyming is dictated by the dialect of a language. In the middle of the twentieth century a well-established European car carrying the name “404” was on the market in Malaysia. It was quite popular with the connoisseur, but a large section of the Chinese community avoided it, because in their dialect the numbers rhymed with “four people die”, and if one of them had to take a ride in a 404 he would make certain that there were not exactly four people in the car.

Rhyming indeed decides the popularity of numbers among the Chinese. The number 8 rhymes with the word for prosperity, 18 with “definitely prosper”, 28 with “easily prosper”, 38 with “life and prosperity” and 48 with “prosperity to the extreme”. On the other hand, 58 sounds like “do not prosper” and has no appeal to the Chinese. A professor in Hong Kong once declined an offer to part with his three-year-old car in exchange for a new car of the same make, not that he remembered Anderson’s tale about the old street lamp that was going to be replaced by a new one, but because his car number plate read 1668, which rhymed with “prosperity all the way” in the Hong Kong Cantonese dialect. The number 138 may be read by some as “prosperity throughout one’s life”. The number 9 finds popularity among the Chinese because it rhymes with the word that conveys the sense of “long lasting” and “longevity”.

There is also special significance attached to some double-digit or even multi-digit numbers, some of which have already been referred to in the above paragraph. The number 10 is favoured by the Chinese because it conveys the sense of wholeness and completeness. Twenty is sometimes expressed as double-ten, where both the numbers 2 and 10 have an auspicious meaning. The number 60 is significant on account of the 60-year cycle in the Chinese calendar. The Japanese give a special meaning to the two numbers 77 and 88. Written in Japanese *kanji* characters, the former shows some resemblance to the character for “happiness”, while the latter forms a part of the character for “rice”, the traditional staple food for the Japanese. Hence the 77th and 88th birthdays of a person are both special occasions for celebration in Japan.

There is, however, no hard-and-fast rule, as there are always variations. One example is the number 6, which rhymes with the word meaning emolument, civil appointment or good job. It is also an auspicious astrological number like 1 and 8. However, some Hong Kong Cantonese-speaking people find it rhyming with the word meaning “falling down”. Room number 6 in a guest house in Edinburgh has not been popular with candidates from Hong Kong attempting professional examinations. Even with respect to the number 4 we can find some exceptions, especially in the case of established terms, such as four seasons and four seas. For example, *Shiki* is the name of a Japanese restaurant-chain, perhaps advertising that its food is good for all seasons, and *Sihai* is a popular Chinese name for the tourist trade, implying welcome to customers from all the four quarters of the Earth. *Yotsuya*, a place name in Tokyo meaning four valleys, is read in Japanese *kunyomi* and not in Chinese *onyomi*. However, the term “*Sirenbang*” (the Gang of Four) was never meant to be an exception to the rule. It is a derogatory term that sounds like “the Gang of Dead People”.

Coming back to the topic of motor car number plates, the Malaysian Chinese community has slightly different ideas from their Hong Kong counterpart. Malaysian Chinese motorists normally add up the four numbers in their car plates, removing tens from the total until the remainder is between 1 and 10. The higher the remaining number the better. Numbers 8, 9 and 10 are regarded as winning numbers. This I learned from personal experience. I went to a second-hand car dealer in Kuala Lumpur to sell my car. The dealer looked down at my car number plate and immediately offered me a price acceptable to me. Taken by surprise that the deal had been made within minutes, I asked him why he did not need to examine the engine and the body work. With a smile he said that he could do something about the engine and the car body if necessary but what was important was a good number that would attract a customer. He was proved absolutely right when I saw my old car on the road again the very next week. My official car in Kuala Lumpur carried the number 9399. I was approached several times to sell it at a premium, but that was not my own car. Second-hand cars with popular numbers fetch higher prices and sell better than those with less popular numbers, while those with unlucky numbers do not enjoy a good market. Commercial and residential properties are similarly affected by numbers. This provides a practical example of the application of Chinese number mysticism to modern economics, demonstrating that number mysticism is not all myth after all. The philosophical study of number mysticism helps towards the understanding of the construction of Chinese magic squares, while a knowledge of popular number mysticism would enhance the understanding of the people of East Asia.

7. Bibliographical comments

The subject on Chinese number mysticism is seldom touched upon specifically. The Bibliography gives general reference concerning Sections 1 to 5. Section 6 is written from the author's personal experience and observations and not academic research, which in this case falls within the province of an anthropologists or sociologist.

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CHAPTER 2

Derivation and Revelation: The Legitimacy of Mathematical Models in Indian Cosmology

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1. Indian science, Indian religion: "Orientalist" and "post-Orientalist" views

The importance in classical Indian culture of deities, religious devotion, and the general spiritual theme of transcending the human and mundane is a very familiar concept to Westerners. In fact, it has become commonplace to ascribe to India a uniquely pervasive preoccupation with the divine, a special status as a land of gods and mystics. Especially to the modern scientifically trained imagination, it may seem incongruous to combine such a preoccupation with a simultaneous interest in understanding the universe mathematically, that is, via self-consistent quantitative models; certainly, the details of mathematical and scriptural *Weltanschauungen* often sharply conflict. In the eighteenth and nineteenth centuries, it became a contentious question for many Indians and Anglo-Indians whether and how mathematics and the divine had coexisted in the "indigenous" Indian (Sanskrit) tradition. What roles had its science offered to divinely revealed truth (including the picture of the cosmos presented in sacred texts such as the Purāṇas) and mathematically deduced fact (which involved a very different picture of the physical universe)?

Though some colonial commentators on Sanskrit astronomy were frankly contemptuous of the entire tradition, others—especially Orientalists with some personal experience of and enthusiasm for Sanskrit literature—inclined to a more nuanced view.[1] This asserted that "Hindu" sciences in earlier times accepted mathematically rigorous models of the cosmos similar to those of the classical Greeks, but were subsequently adulterated by various fanciful superstitions derived from sectarian myths. Several Orientalists, particularly those with official responsibilities concerning native education, publicly deplored what they described as the consequent discrepancies between the classical Sanskrit astronomical tradition and the sacred cosmology of popular belief. Lancelot Wilkinson, a British Political Agent and educational reformer in Bhopal in the early nineteenth century, suggested using the former pedagogically to undermine the latter:

> The followers of the Puráns [Purāṇas] … maintain that the earth is a circular plane, having the golden mountain Merú in its centre … that the moon however is at a distance from the earth double of that of the sun; that the moon was churned out of the ocean; and is of nectar; that the sun and moon and constellations revolve horizontally over the plane of the earth, appearing to set when they go behind Merú, and to rise when they emerge from behind that mountain; that eclipses are formed by the monsters Ráhú and Ketú laying hold of the sun or moon … The jyotishís [astronomer/astrologers] or followers of the Siddhántas … teach the true shape and size of the earth … The authors of the Siddhántas … have spared no pains to ridicule the monstrous absurdities of … the Puráns … [W]e have only to revive that knowledge of the system therein [in the siddhāntas] taught, which notwithstanding its being by far the most rational, and formerly the best cultivated branch of science amongst the Hindus … has, from the superior address of the followers of the Puráns … been allowed to fall into a state of utter oblivion … Indeed, so general and entire is the ignorance of most of the joshís [or jyotishís] of India, that you will find many of them engaged conjointly with the Puránic bráhmans in expounding the Puráns, and insisting on the flatness of the earth, and its magnitude … as explained in them, with a virulence and boldness which shew their utter ignorance of their proper profession, which had its existence only on the refutation and abandonment of the Puránic system [25, pp. 504–509].

[1] Probably the most famous negative remark on the subject is that of Thomas Macaulay to the effect that Indian astral science "would move laughter in girls at an English boarding school" [8]. A survey of various other opinions held by European Indologists of the eighteenth and nineteenth centuries is given in [2].

Wilkinson felt that religious and scientific worldviews in Indian thought were more or less natural enemies, and that adherents of the former had caused, by their "superior address," the "oblivion" of the latter. Such evaluations have recently begun to receive some attention from colonial historians interested in the ways in which colonial powers used science to further imperial aims. Noting that these uses include not only the obvious roles of science in technology, education, public health, and so forth, but also colonial reactions to the indigenous scientific traditions of the colonized, several scholars have detected more than a little self-interest in colonial treatments of Indian cosmology. David Arnold, for example, identifies as "Orientalist" the notion that a Hindu "religious" worldview ousted in medieval times a more "scientific" one, and points out the political usefulness of this assessment to imperialists:

> Although the richness and diversity of India's ancient scientific traditions has long been recognised, over the past two centuries it has been the convention to see this as a history of precocious early achievement followed by subsequent decline and degeneration. The European Orientalist scholarship of the late eighteenth and early nineteenth centuries represented India as having had an ancient civilisation equalling, in some respects excelling or anticipating, those of classical Greece and Rome. . . . In astronomy, mathematics and medicine in particular, Hindu science was considered to have been remarkably advanced well before the dawn of the Christian era and to have been the source of discoveries and techniques that were only later taken up and incorporated into Western civilisation, such as "Arabic" numerals and the use of zero. However, according to this Orientalist interpretation, Indian civilisation was unable to sustain its early achievements and lapsed into decline. There followed an uncritical reliance upon earlier texts: tradition replaced observation as surely as religion supplanted science. . . . The history of Indian science thus served as a mere prologue to the eventual unfolding of Western science in South Asia as science was rescued from centuries of decline and obscurity by the advent of British rule and the introduction of the more developed scientific and technical knowledge of the West. This Orientalist triptych—contrasting the achievements of ancient Hindu civilisation with the destruction and stagnation of the Muslim Middle Ages and the enlightened rule and scientific progress of the colonial modern age—has had a remarkably tenacious hold over thinking about the science of the subcontinent. It was a schema deployed not only by British scholars, officials and polemicists but also by many Indians, for whom it formed the basis for their own understanding of the past and the place of science in Indian tradition and modernity [1, pp. 3–4].

In this analysis and in others like it,[2] what the colonial historian chiefly seeks to explain is why such claims about the relationship of science and religion in India were politically useful to those who made them. The focus is on the rhetorical uses to which these claims were put in serving or subverting colonial aims—how they were "deployed" in consolidating or "renegotiating" power. As Richard Young has suggested, however [26, p. 264; 27], analyses of political motives do not go very far in explaining what is meant by the "science" of the siddhāntic texts or the "religion" of the Purāṇas, nor how the one is supposed to have caused the deterioration of the other, nor how scientific and mathematical ideas actually interacted with religious ones in these texts. Labels like "Orientalist", "colonial", "nationalist", though they may be helpful in understanding historical actors' beliefs, tell us nothing about what those actors actually encountered in the traditions they were discussing. It is this background to such postcolonialist critiques that the present essay attempts to fill in. That is, was there in fact a shift in Indian cosmological views away from scientific

[2] See, for example, [7, pp. 20, 57–64] and [20, pp. 71–72, 83–85].

derivation towards religious revelation, and how were these Purāṇic and siddhāntic texts involved in it?

2. The Purāṇas

The Purāṇas form part of the Hindu sacred texts that are categorized as “smṛti”, literally “what is remembered” or mediated by human authorship as opposed to “śruti”, “what is heard”, e.g., the Vedic hymns themselves and other directly revealed texts; but especially in the later medieval period, they too were venerated as a source of divine truth. Within their innumerable legends of the exploits of the gods and other beings is a common picture, assembled in the first few centuries CE from various cosmological concepts up to several centuries older, of the structure of the universe. In this picture, as Wilkinson mentioned, the earth is a flat disk five hundred million yojanas in diameter with the sacred mountain Meru standing 84 000 yojanas tall in its center. (There is no single standard value for the length of the yojana, but it is more or less on the order of ten kilometers, which makes the diameter of the Purāṇic earth about five billion kilometers, or approximately equal to the modern value for the size of the orbit of Neptune; Mount Meru reaches more than twice as far as the distance to the moon by our reckoning.) Its surface is covered by the concentric rings of seven continents and seven oceans, and above it the sun, moon, constellations, and planets are carried in circles around the mass of Meru, which makes them appear to rise and set. The pole-star is above Meru’s summit, upon which is the city of the gods. The moon is higher above the earth than the sun in this system, so its phases as well as solar and lunar eclipses are explained by a demon who periodically devours the luminaries, and the five visible star-planets are higher than the constellations. These worlds will endure for one “day of Brahmā” or 4 320 000 000 years, called a kalpa, which is one thousand times as long as a “mahāyuga” or “great age”. Each mahāyuga in turn is divided into four unequal yugas of which the last, least, and worst is the Kaliyuga of 432 000 years. (See [16,19].)

This cosmology, unsurprisingly, is not adequate for mathematical prediction of the motions of the heavenly bodies as seen from the earth, and that was never its intended purpose. That function was performed in the last few centuries BCE by a collection of simple arithmetic rules for maintaining a relatively crude luni-solar calendar; it did not set up geometric models that challenged the Purāṇic cosmology.[3]

3. The siddhāntas

Shortly after the beginning of the current era, under the influence of Graeco-Babylonian and Hellenistic sources, more comprehensive astronomical treatises usually called siddhāntas—which in this context may be rendered by “astronomical systems”—began to appear. The remnants we still have of the earliest of these texts are devoted mostly to arithmetic schemes for predicting celestial events, comparable to those in older Babylonian texts. But as David Pingree has shown [16], by the fifth century at the latest a siddhāntic model

[3] Such texts are discussed in [15]. In fact, as noted in [16, p. 275], some details of the Purāṇic model are apparently adapted from the crude parameters of these schemes.

was established that assumed a spherical earth only about 5000 yojanas in circumference, suspended in the middle of a sphere of fixed stars, around whose center the planets including the sun and moon were considered to move in tilted circular orbits with other circles included to account for their orbital anomalies. The moon was now established as nearest to the earth and the constellations most distant from it. Where possible, compromises were made with the Purāṇic system: for example, Mount Meru was retained as the north terrestrial pole (though greatly reduced in size), and the unfamiliar southern terrestrial hemisphere served as a convenient receptacle for exotic geographical features such as the annular continents and oceans. The Purāṇic divisions of time were respected, and celestial rates of motion and distances were chosen so that all the bodies could complete integer numbers of revolutions about the earth from the same starting-point in one kalpa or lifetime of the worlds. But this siddhāntic model was now committed to certain mathematical constraints in return for its increased explanatory and predictive power. For instance, to explain the varying height of the pole star as seen at different localities, the earth must be more or less uniformly spherical; to account for the unchanging appearance of the stars' positions relative to one another, it must be tiny compared to the sphere of the heavens. Accounting quantitatively for eclipses and lunar phases by the configurations of three spherical bodies rather than by demonic agents requires that the moon's orbit be smaller than the sun's; and all the bodies' motions must be predictable and geometrically constrained so that their positions can be computed trigonometrically. These assumptions were retained by most of the siddhāntas of the medieval period, of which the last to have great influence was the *Siddhāntaśiromaṇi* composed by Bhāskara in the middle of the twelfth century.

4. Contradiction and concession

Apparently from their earliest stages, siddhāntic texts began making explicit their tensions with the existing Purāṇic model, although at first not systematically. For example, the astronomical treatise of Āryabhaṭa around 500 CE stressed the earth's sphericity:

> The globe of the earth [made of] earth, water, fire, and air, in the middle of the cage of the constellations [formed of] circles, surrounded by the orbits [of the planets], in the center of the heavens, is everywhere circular. In the same way that the [spherical] bulb of a kadamba-flower is entirely covered with blossoms, so is the globe of the earth [covered] by all the beings born of the water and the land.[4]

And it flatly contradicted the Purāṇic magnitude of Mt. Meru: "Meru is measured by one yojana ...".[5] In the same vein, Brahmagupta's siddhānta about 130 years later explicitly challenged the Purāṇic assumption that the moon is farther away than the sun: "If the moon [were] above the sun [as the Purāṇas indicate], how would [its] power of increase and decrease in brightness, etc., [be produced] from calculation [of the position of] the moon? The closer half would always be bright".[6] Brahmagupta's contemporary Bhāskara (not to

[4] vṛttabhapañjaramadhye kakṣyāpariveṣṭitaḥ khamadhyagataḥ | mṛjjalaśikhivāyumayo bhūgolaḥ sarvato vṛttaḥ ‖ yadvat kadambapuṣpagranthiḥ pracitaḥ samantataḥ kusumaiḥ | tadvaddhi sarvasattvair jalajaiḥ sthalajaiś ca bhūgolaḥ ‖ (Gola, 6–7 [22, pp. 258–259].)

[5] merur yojanamātraḥ ... (Gola, 11 [22, p. 261].)

[6] sitavṛddhihānivīryādi śaśāṅkāj jāyate kathaṃ gaṇitāt | upari raver induś ced arvāgardhaṃ sadā śuklam ‖ (*Brāhmasphuṭasiddhānta* 7, 1 [6, p. 100].)

be confused with the twelfth-century author of the same name), commenting on Āryabhaṭa's work, opposed his own geographic information to the Purāṇic values of terrestrial distances; he also (according to the summary of a later commentator) explained that the Purāṇic size of Meru was impossible because it would block northern stars from sight.[7]

In the middle of the eighth century, an astronomer named Lalla devoted an entire chapter (bluntly entitled "Errors") of his own siddhānta to refuting various assumptions such as the causation of eclipses by a demon and the flatness of the earth:

> If your opinion is that a demon invariably devours [the moon or sun] by means of magic, how is it [that the event can be] found by calculation? And how [is it that there is] no eclipse except [at] new or full moon?
> Eclipses, conjunctions of planets, risings, the appearance of the lunar crescent, the rule for [computing] the shadow at a given [time]—the solution of all five is accurate [when found] by means of the [siddhāntic] size of the earth. So how could it be [as] large [as the Purāṇas say]?
> Those who know calculation say [that] a hundredth part of the circumference [of the earth] is seen as flat. So the earth appears flat to this extent; it is just meant in that way.[8]

Here and in similar remarks by later authors,[9] the validity of the siddhāntic model was defended essentially on the grounds that it was mathematically effective. The fact that "calculation" or mathematical prediction agreed with observed result (or could be made to agree with it, in the case of the apparent flatness of the spherical earth) was an argument in favor of the predictive model's reality. So a more or less open breach was made between the Purāṇic model revealed by sacred texts and the siddhāntic model derived from calculation and observation, and remarks like these seem to make it clear which side an astronomer was supposed to be on.

But if we look at other remarks within the same texts, the picture becomes more complicated. It turns out that instead of simply defying revealed truth for the honor of mathematical consistency, siddhānta authors were often trying to have the best of both worlds—relying on their measurable and calculable universe while still availing themselves at need of Purāṇic authority. This was already indicated in the early inspiration of using features of the Purāṇic universe to fill in the gaps (literally and figuratively) in the geography or mechanics of the siddhāntic one. Contradiction continued to be softened by compromise: for example, Āryabhaṭa's above-mentioned rejection of the traditional height of Mount Meru did not imply any doubt as to whether the sacred mountain actually existed. In fact, his assertion that Meru was only one yojana tall was immediately followed by an orthodox Purāṇic description of its appearance, "shining" and "covered with jewels". Such minor concessions to scriptural knowledge are not particularly surprising, as many ancient cosmographers similarly incorporated tradition and legend to supply some of the vast lacunae in the available definite knowledge about terrestrial or celestial features. In addition, these concessions made no detectable difference to the siddhāntic universe: the authority of the

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[7] Gītikā 7, Gola 11 [22, pp. 28–29, 261–262].

[8] asuro yadi māyayā yuto niyato 'tigrasatīti te matam | gaṇitena kathaṃ sa labhyate grahakṛtparva vinā kathañca || grahaṇaṃ grahasaṅgamodayau śaśiśṛṅgonnatir iṣṭabhāvidhiḥ | syāt pratyayapañcakaṃ sphuṭaṃ kṣitimānena bhavet mahat katham || praguṇaḥ paridheḥ śatāṃśako gaṇitajñāḥ kathayanti dṛśyate | pratibhāti tadā samā mahī viṣaye yatra tathaiva gamyate || (*Śiṣyadhīvṛddhidatantra* 20, 22, 31, 35 [3, Vol. I, pp. 235–237]; other assertions criticized by Lalla in this chapter include non-scriptural speculations such as Āryabhaṭa's hypothesis of the earth's rotation.)

[9] See, for example, *Siddhāntaśiromaṇi* Gola 17, 11–16 [4, pp. 345–347].

Purāṇas might be deferred to in unverifiable details, but it was generally not allowed to contradict any point of real significance to the astronomers' model. Thus the Purāṇic oceans of wine, milk, etc., could be accepted by siddhānta authors as existing in the southern hemisphere, beyond the realm of direct knowledge; but the Purāṇic requirement that the earth be supported from below by an equally unobservable tortoise or serpent or elephants, which would imply a preferred "down" direction and thus jeopardize the credibility of uniformly stable positions everywhere on a spherical earth, came in for sharp criticism.[10]

What is more remarkable is the firmness with which siddhāntas often explicitly insisted on the need to conform to the models of sacred texts—though admittedly, this criterion was most often invoked by authors criticizing rival authors for having failed to meet it. For example, the same Brahmagupta who rejected the Purāṇic description of the moon's position made the following remark about Āryabhaṭa's views on eclipses:

> "How can the sun [illuminate] everything and [the demon] Rāhu [be] otherwise? Since there is variation in the [amount of] obscuration in a solar eclipse, an eclipse of the sun or moon is not caused by Rāhu": [what is] thus declared by Varāhamihira, Śrīṣeṇa, Āryabhaṭa, and others is opposed to popular [opinion] and is not borne out by the Vedas and smṛti ...
> "[The demon] Svarbhānu or Āsuri has afflicted the sun with darkness": this is the statement in the Veda. So what is said here is in agreement with śruti and smṛti ... The earth's shadow does *not* obscure the moon, nor the moon the sun, in an eclipse. Rāhu, standing there equal to them in size, obscures the moon and the sun.[11]

A good deal of this sort of disparagement was traded back and forth by siddhāntic authors, particularly with respect to one another's choices of values of the basic astronomical parameters such as the great divisions of time. Objections to the "heterodox" parameter choices of one pakṣa or major astronomical school by the adherents of another were most common, even though the different siddhāntic models all resembled one another far more closely than they did the Purāṇic system.[12] We cannot assume with certainty, though, that critics were always personally convinced of the unimpeachable authority of the Purāṇic positions that they criticized their rivals for contradicting: authors who indulged in disputes of this sort were generally lavish with their blame, and may have seized with equal willingness on any debatable point they could find. Likewise, their own innovations might be made for a variety of reasons—e.g., numerical elegance or ease of computation—and justified only ex post facto by appeals to smṛti.

[10] This generally took the form of pointing out that the idea led to an infinite regress of supporting and supported bodies (e.g., in *Śiṣyadhīvṛddhidatantra* 20, 41 [3, Vol. I, p. 238], *Vaṭeśvarasiddhānta* Gola 5, 5 [23, Vol. I, p. 326], *Siddhāntaśiromaṇi* Gola 17, 4 [4, p. 344]). But as [12] remarks, a more conciliatory note was struck by Jñānarāja in the sixteenth century when he suggested that the traditional serpent, etc., might support the earth from within its spherical shell. The development of such irenic solutions in astronomical works in the second half of the second millennium is further discussed below.

[11] kiṃ prativiṣayaṃ sūryā rāhuś cānyo yato ravigrahaṇe | grāsānyatvaṃ na tato rāhukṛtaṃ grahaṇam arkendvoḥ || evaṃ varāhamihiraśrīṣeṇāryabhaṭaviṣṇucandrādyaiḥ | lokaviruddham abhihitaṃ vedasmṛtisaṃhitāvāhyam || ... svarbhānurāsurir inaṃ tamasā vivyādha vedavākyam idam | śrutisaṃhitāsmṛtīnāṃ bhavati yathaikyaṃ taduktir itaḥ || ... bhūchāyendum ato hi grahaṇe chādayati nārkam indur vā | tatsthas tadvyāsasamo rāhuś chādayati śaśisūryau || (*Brāhmasphuṭasiddhānta* 21, 38–39, 43, 48 [6, pp. 372–374]. Here Brahmagupta has even taken the unusual step of discarding part of the mathematically predictive model in favor of the scriptural explanation.)

[12] E.g., Brahmagupta complained (*Brāhmasphuṭasiddhānta* 1, 9 [6, p. 3]) that Āryabhaṭa's divisions of kalpas and yugas deviated from the values defined in smṛti; Vaṭeśvara in the tenth century retorted (*Vaṭeśvarasiddhānta* 1, 10, 6 [23, Vol. I, p. 72]) that Brahmagupta's own presentation of these time-units was similarly defective.

Some authors went so far as to compose new texts that modified basic parameters to bring them more in line with the Purāṇic requirements. For instance, the author of the early ninth-century *Sūryasiddhānta*, wishing to conform to the Purāṇic time divisions preserved by Brahmagupta while still honoring Āryabhaṭa's requirement for a mean great conjunction of all planets at the start of the current Kaliyuga, got around the difficulty by hypothesizing a long period of quiescent creation at the beginning of the universe when the planets are motionless: they begin to revolve only at the right time to bring them to the desired conjunction (*Sūryasiddhānta* 1, 10–24 [5, pp. 5–11]). The entire siddhāntic school known as the Saurapakṣa was based on this hybridization, whose doctrinal authority was clinched by ascribing the authorship of the work to the Sun-god, Sūrya. However, not even inter-pakṣa disputes or "schisms" can be taken as evidence of unambiguous commitment to the reality of any one particular model, as the boundaries between pakṣas with different parameters were in fact highly permeable. Not only did some authors write texts in more than one pakṣa, changing their critical tendencies accordingly, but they also availed themselves of another form of compromise that took shape in the medieval period: namely, the so-called bījas or corrections for transforming computations starting with one set of parameters into their equivalents using the parameters of a different school [17]. Thus neither the siddhāntas' various parameter choices nor their theological ones seem to have been considered completely mutually unacceptable, in spite of their mutual criticisms.

To complete this kaleidoscope of perspectives on scientific truth, it should be noted that innovations contradicting the sacred tradition might even be justified by questioning the consistency of the sacred texts themselves. Āryabhaṭa's uncanonical division of the lifetime of the universe into 1008 rather than 1000 mahāyugas (which, as we have seen, was harshly criticized by Brahmagupta on account of its deviation from the Purāṇic value) was defended by his commentator Bhāskara as follows: since some religious texts agreed that the kalpa consisted of a thousand mahāyugas, while others maintained it was made up of fourteen 71-mahāyuga periods or 994 mahāyugas, then because of "the contradiction in their own words" implied by the unequal numbers, Āryabhaṭa's value could not be considered authoritatively contradicted (*Āryabhaṭīya* Kālakriyā, 8 [22, pp. 197–199]). Note that here, the charge that a mathematical model had abandoned the tenets of religious revelation was countered by the complaint that the revealed texts had fallen short of mathematical consistency.

In short, the siddhāntic model of the cosmos may be said to have substituted mathematical and physical truth criteria for the arguments from authority on which Purāṇic cosmology was based. Except that the authority of the Purāṇas was still active at some level in helping shape and validate the siddhāntic texts, and siddhāntic parameters might be modified on that account (even, allegedly, by direct divine intervention). Except that such modification of parameters did not necessarily imply the repudiation or obsolescence of previous, less canonical ones. Except that the validity of the canonical parameters themselves might be subject to testing by quantitative truth criteria. And so on, in an apparently endless cycle of caveats and qualifications.

The Muslim scholar and scientist al-Bīrūnī, who encountered this epistemological labyrinth in his study of Indian astronomy during his sojourn in India in the eleventh century, commented as follows:

> The religious books of the Hindus and their codes of tradition, the Purāṇas, contain sentences about the shape of the world which stand in direct opposition to scientific truth as known to their astronomers. By these books people are guided in fulfilling the rites of their religion, and by means of them the great mass of the nation have been wheedled into a predilection for astronomical calculation and astrological predictions and warnings. The consequence is, that they show much affection to their astronomers … For this the astronomers requite them by accepting their popular notions as truth, by conforming themselves to them, however far from truth most of them may be, and by presenting them with such spiritual stuff as they stand in need of. This is the reason why the two theories, the vulgar and the scientific, have become intermingled in the course of time, why the doctrines of the astronomers have been disturbed and confused, in particular the doctrines of those authors—and they are the majority—who simply copy their predecessors, who take the bases of their science from tradition and do not make them the objects of independent scientific research [21, Vol. I, pp. 264–265].

Al-Bīrūnī's interpretation of the "intermingling" of siddhāntic and Purāṇic models is itself somewhat reminiscent of the critiques of Orientalist scholarship mentioned above: it presents the "corruption" of mathematical astronomy by religious myth as a mere renegotiation of power on the part of astronomers seeking to enhance their professional authority. While his assertions do not do justice to the complexity of the interplay of various kinds of authority in siddhāntic texts as sketched above, they do show clearly that the idea that "tradition replaced observation" in Indian astronomy "as surely as religion supplanted science" is hardly a "European Orientalist" innovation.

5. The quest for non-contradiction

A fascinating new form of this "intermingling" of theories, developed apparently in the second half of the second millennium, has inspired growing scholarly interest in recent years;[13] it took shape as a separate subgenre of astronomical writings whose explicit goal was to reconcile the discrepancies between Purāṇic and siddhāntic truth claims. Beginning no later than the sixteenth century and extending at least well into the nineteenth, several authors composed entire works or sections of works discussing this topic of "virodhaparihāra", literally "removal of contradiction", or more simply "avirodha", "non-contradiction". Their basic approach was to defend the divine truth of the Purāṇic texts while acknowledging the usefulness of the siddhāntic model for practical computation, and/or to treat the siddhāntic universe as a small subset of the Purāṇic one (sometimes ingeniously reinterpreting siddhāntic texts for the purpose). As Richard Young has noted [27], some of the more extensive works of this type were composed in response to attempts by Westerners or Western-trained Indians to promote the Copernican model. One such text, the 1837 *Avirodhaprakāśa* or "Light on Non-contradiction" of Yajñeśvara, was in fact intended as a refutation of a heliocentrist treatise written by an Indian student of Wilkinson's. A sampling of its verses gives an idea of some of the arguments typically brought to bear on this issue:

> … The siddhāntas were produced by incarnations [of the gods] Sūrya and so forth. The circle of the earth, etc., stated there for the purpose of performing computations—which [is] much too small—having been taken for truth by men such as Ārya[bhaṭa] and Brahmagupta, controversy

[13] Cf. [16, p. 280], [9–13,18,26].

> arose. And by Westerners such as the red-faced ones [i.e., the British], once they had grasped the meanings set forth in the siddhāntas and observed some part of the earth by means of an instrument, a picture [of the cosmos] as it appears to the eye [was] constructed. And after consideration of the picture [according to] the siddhāntas [and that] of the Purāṇas, this non-contradictory knowledge is accurately stated by us.
>
> On the great earth as stated by the Purāṇas, between the salt sea and the Himālaya, the western portion of the land known as Bhārata is called the Bhārata region … It is raised in the middle like an overturned dundubhi drum, in the middle of which the pole-star should be seen on the top. … The pole-star, being visible in a place fully five thousand krośas [1250 yojanas] in extent [distant] from that, is seen [as] depressed [toward] the earth. …
>
> It is believed that a sphere of stars is on all sides of the earth [hanging] in space. Similarly, because of [false] confidence in the change in the height of the pole-star [in different places] on the earth, the resemblance of the earth to a ball, and by deduction [its] much-too-small dimensions, [are] assumed.[14]

Yajñeśvara here is rejecting Lalla's notion that mathematical predictive power gives a claim to reality; the siddhāntic picture, he avers, may be useful for computation but it was never intended by its divine architects to be mistaken for truth. At the same time, however, in order to maintain his claims for non-contradiction, Yajñeśvara must make at least a piece of the Purāṇic cosmos conform, albeit clumsily, to the geometric models set up by the siddhāntas. The earth's sphericity that changes the apparent height of the pole-star at different latitudes, for example, is more or less approximated by a raised central portion of the continent and by vertical parallax. These suggestions are geometrically quite feeble as equivalents for spherical astronomy (and in fact Yajñeśvara had to make some modifications to them in a later exchange with the same opponent [10]); but they illustrate the determination of avirodha authors not to relinquish their explicit adherence to the essential truth of the Purāṇic cosmology.

This brief example barely scratches the surface of its subject: unanswered questions abound concerning avirodha literature, its relation to the siddhāntas and to heliocentrism, its place in Indian epistemology, and its successors in later genres concerning "reconciliation" of scientific and religious texts. At present, however, this discussion must return to the siddhāntas themselves and to the ways in which they were vulnerable to the arguments of avirodha.

6. The status of siddhāntas in the nineteenth century

One reason that avirodha texts could consider spherical astronomy as merely a convenient computational fiction was that, paradoxically, it was treated in just this way by many of the

[14] … kṛtāvatāraiḥ sūryādyaiḥ siddhāntāḥ parikalpitāḥ ‖ gaṇitavyavahārāya tatra bhūmaṇḍalādikaṃ | uktam aty alpamānaṃ yat tadevādāyatattvataḥ ‖ vicāro vartitas tv āryabrahmaguptādibhir naraiḥ | yavanais tāmravaktrādyair api siddhāntasamsthitān ‖ arthān saṃgṛhya yantrena tathā pratyakṣadṛṣṭitaḥ | kañcid bhūbhāgamālakṣya parilekho vikalpitaḥ ‖ siddhāntānāṃ purāṇānāṃ parilekhasya cekṣaṇāt | asmābhir avirodho 'yaṃ vijñātaḥ sphuṭam ucyate ‖ purāṇoktamahābhūmau kṣārābdhihimaśailayoḥ | antargatasya varṣasya bhāratākhyasya paścimaḥ ‖ bhāgo bhāratakhaṇḍākhyaḥ … kṣāravārdhijalair vyāpto nyubjadundubhibhāṇḍavat | madhyonnato 'sti yanmadhye dṛśyeta śirasi dhruvaḥ ‖ … tasmāt samantataḥ pañcasahasrakrośasaṃmite | deśe vilokyamānas tu dhruvo bhūlagna īkṣyate ‖ … bhagolaś cābhito bhūmim ākāśe 'stīti kalpyate | tathā bhuvidhruvauccasya tāratamyapratītitaḥ ‖ bhūmeḥ kaṇḍukasādṛśyaṃ yuktyā cātyalpamānatā ‖ (*Avirodhaprakāśa*, 6–12, 13, 15, 18–19 [24, pp. 1–2].)

contemporary astronomers who used it. Lancelot Wilkinson noted that the "city joshí", an astronomical professional superior in status and training to the typical village astrologer,

> … can not only find the places of the sun, moon, and planets, but also work out eclipses. But the operation may be called purely mechanical, or an effort of memory. He can find the equatorial gnomonic shadow, from thence deduce the latitude … but is wholly ignorant as to what things in nature are expressed by these terms. The verses of [the sixteenth-century astronomical texts] the Graha Lághava and Tithí Chintámaní contain only abbreviated formulae for calculations; their wording is uncouth, and to the uninitiated, more unintelligible than an enigma. But though the ingenuity displayed in thus abbreviating calculations is considerable, it has had the effect above noticed of superinducing an utter neglect of the Siddhántas, in which the principles of the science are so fully, and in many respects so rationally, explained. I have met and cross-questioned many hundreds of joshís of late years; but in this large number, have found only two men who had a rational and full acquaintance with their own system [25, p. 508].

Wilkinson's knowledge of the contemporary state of Indian astronomy was of course imperfect. The copying of manuscripts of siddhāntas (which, at least in some cases, implied study and use of the texts) continued up to and through the nineteenth century; influential commentaries and even new siddhāntas continued to appear, although they did not equal in importance or popularity the major works of the earlier period. Furthermore, there continued to be at least some pandits who read and understood the texts well enough to discuss them with the European Sanskritists who translated them. On the other hand, there seems no reason to doubt Wilkinson's description of his own experience: it is not implausible that many professional astronomers were able to manipulate a set of rules whose geometric implications they did not fully understand (and according to al-Bīrūnī, such astronomers were not rare even back in the eleventh century).

The trend toward mechanical application would have been facilitated by the great influence of the two works mentioned by Wilkinson. The *Tithicintāmaṇi*, a set of astronomical tables, and the *Grahalāghava*, an astronomical handbook, were composed by the same author in sixteenth-century Gujarat, and were both extremely popular [14, Vol. 2, pp. 94–103]. Both table texts such as the *Tithicintāmaṇi*, which developed in imitation of Islamic models in the second millennium, and handbooks like the *Grahalāghava*, which are as old as siddhāntas, focus on the necessities of practical astronomical calculation and ignore underlying cosmological models. Astronomers who could be trained in the practices of their craft without ever needing to use directly the geometric models from which they were derived would indeed have less motive to maintain the reality of those models in defiance of the venerated Purāṇic cosmology, and might even, as Wilkinson found, remain entirely oblivious that any such contradiction existed.

7. Mathematical models in siddhāntas

Finally, there is the question of the integrity of the siddhantas' own mathematical models. Although, as previously noted, the basic siddhāntic cosmos is a quasi-Ptolemaic arrangement of spheres and circles, the computational practices of Indian astronomy usually held a greater share of the practitioners' interest than the geometric models underlying them or the physical consequences they implied. This almost always meant that when computational convenience or creativity came up against physical or geometric consistency, consistency took second place.

For example, a planet's mean position in its circular orbit was geometrically corrected by siddhāntas to its true position according to the position of its "apex", a point that could be considered to correspond to the apogee of an eccentric orbit. However, it was not clearly specified in the correction process whether the planet actually moved upon an eccentric circle or a mathematically equivalent epicycle. Even Ptolemy, of course, did not always make a confident physical assumption in favor of either of these mathematically indistinguishable structures. But the *Sūryasiddhānta*, for example, carried conventionalism much farther than Ptolemy would have dreamed of when it posited another mathematically indistinguishable alternative, in which the planet moves on its concentric orbit and the correction is caused by a demon that stands at the apogee and pulls on the planet with a cord of wind (*Sūryasiddhānta* 2, 1–3 [5, p. 31]). Similarly, the physical specification of circular planetary orbits was never explicitly reconciled with position-correcting procedures that modified the orbital radius depending on the planet's position, effectively turning the orbit into a quasi-ellipse. Moreover, various coefficients were used that changed the size of the epicycle radii, causing them to swell or shrink (sometimes discontinuously) depending on the planet's orbital anomaly or orientation with respect to the observer, but no physical reason was suggested for this phenomenon. Even the all-important sphericity of the earth and the heavens was not handled in an entirely self-consistent manner: a commonly used approximation for longitudinal difference effectively treated a slice of the earth as a cylinder, and celestial spherical triangles were routinely solved as if they were plane.[15] In addition, the early siddhāntic texts also took on, over time, a venerable status of their own, particularly when attributed to gods or seers. (Note the distinction drawn in Yajñeśvara's opening comment above between the original intent of divine siddhānta authors and its subsequent misinterpretations in the texts of merely human astronomers.) This meant that various approximate or erroneous formulas found in them could, and did, continue to coexist with more exact ones in later texts. Given the comparatively low priority allotted to strict mathematical consistency, and the correspondingly high tolerance for physical and geometric approximations, it seems less surprising that avirodha authors should venture to describe the whole siddhāntic cosmos as based on computational convenience and devoid of true physical reality.

8. Conclusion

What can we make, then, of the claims of Wilkinson and other British Orientalists that Indian astronomy had degenerated from an earlier state of comparative scientific purity under the influence of religious superstition? Certainly we can allow them a partial basis in fact. The sacred cosmology of the Purāṇas and the mathematical models of the siddhāntas did clash in many important respects, and several siddhāntic texts of the first and early second millennium issued open contradictions of Purāṇic assertions. For reasons not yet fully understood, after the middle of the second millennium it became increasingly necessary

[15] All these aspects of siddhāntic astronomy are illustrated in, for example, the editions of [3–6,22,23].

to tone down those contradictions. The arguments of avirodha were marshalled to explain them away, and the simultaneous growth in the influence of “cookbook” astronomical manuals made them irrelevant to many practicing astronomers. Mathematical arguments with which earlier authors had confidently countered scriptural ones now stood as examples of human limitation and error, in opposition to sacred knowledge. The recognition of these developments cannot be dismissed as merely the operation of imperialist self-interest in constructing an Orientalist historiography.

But Wilkinson’s assertion that the siddhāntas “had [their] existence only on the refutation and abandonment of the Purāṇic system” is also undeniably exaggerated. In fact, the idea of austere scholarly rigor heroically defending a “scientific method” of physical observation and mathematical deduction against superstitious priestcraft, however appealing it may have seemed to nineteenth-century Protestants thoroughly reconciled to Copernicus and Newton, does not fit the Indian model very well. Since Sanskrit astronomy was not delimited by the goal of testing a single self-consistent geometric and kinematic system against the results of observational experiment, it had greater liberty in its methods than did the Hellenistic/Islamic astronomy (and its modern European successor) that were so constrained. The mathematical models that Indian astronomy inherited and refined were legitimated by a wealth of different criteria that must have seemed startlingly incongruous to a nineteenth-century Englishman, as indeed they had also done to an eleventh-century Muslim. The validity of these models was variously claimed by the texts to be confirmed not only by internal consistency and agreement with observed phenomena, but by numerical elegance and symmetry, agreement with the cosmological premises of sacred texts, conformity to earlier siddhāntas which might be considered on a par with sacred texts themselves, and—the ultimate argument *ex auctoritate*—ascription to direct divine authorship. It is hardly to be wondered at that Wilkinson’s attempts to promote a heliocentric system (albeit a Sanskritized and siddhānticized one) that violated almost all of these criteria met with strong resistance. Wilkinson (who had himself studied and translated siddhāntas) and other Western scholars perhaps simply did not fully realize the complexity and variety of siddhāntic claims to truth; or perhaps they ignored or slurred over them in order to make a better case for modern Western theories, whose eventual dominance finally displaced Sanskrit astronomy’s delicate balance between the mathematical derivation of facts and the divine revelation of truth.

9. Bibliographical notes

Unfortunately, there is not a great deal of published research on this aspect of the relationship of mathematics and the divine in the Indian intellectual tradition. The influence of religious scripture does not usually receive close scrutiny from researchers studying Sanskrit astronomical texts. For one thing, it is difficult to draw a clear and consistent picture of the opinions of authors who reject some assumptions of sacred cosmology while espousing others; for another, to many scholars eager to validate the scientific achievements of medieval Indians according to modern criteria, the very notion of their deferring to scriptural authority at all is something of an embarrassment. Nonetheless, many interesting references to śruti and smṛti within important siddhāntic texts and commentaries are accessible in the published editions by Chatterjee, Chaturvedi, both Dvivedīs, and Shukla; for non-Sanskritists, the English introductions and/or translations and commentaries provided by Chatterjee and Shukla, though not very detailed on this topic, will have to suffice.

On the subject of avirodha literature and its attempts to reconcile sacred and secular cosmologies, the cited articles by Pingree, Young, and Minkowski, and the works therein referenced, contain virtually all the published research now available. The general study of science, science policy, and science scholarship in British India is a sizable and growing field, well mapped out in the monographs of Arnold, Kumar, and Prakash. The nineteenth-century writings of Wilkinson, Burgess, and Macaulay, and the many publications surveyed by Burgess, supply colonial Orientalists' views of these issues in their own words.

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CHAPTER 3

The Pythagoreans

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Contents

MATHEMATICS AND THE DIVINE: A HISTORICAL STUDY
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1. Introduction

The place of Pythagoras in the history of mathematics and the divine is, in a sense, well known. We may, for instance, follow Russell: "Pythagoras... was intellectually one of the most important man that ever lived... Mathematics... in him, is intimately connected with a peculiar form of mysticism. The influence of mathematics on philosophy, partly owing to him, has, ever since his time, been both profound and unfortunate... What appears as Platonism is, when analysed, found to be in essence Pythagoreanism. The whole conception of an eternal world, revealed to the intellect but not to the senses, is derived from him" [13, pp. 29, 37].

Much is told, in late sources, about a person from the sixth Century, called Pythagoras—in whom religion, mathematics and proto-Platonism come together. Of most of this, we are now less confident than Russell was in 1945. It appears that later antiquity had recreated the figure of Pythagoras and that later periods have contributed their own until the famous character of Russell's description was finally formed.[1] Little, then, will be said here on Pythagoras himself.

There are other people, 'Pythagoreans', active in the late fifth and early fourth centuries, whom we need to investigate. The most important of these were, apparently, Philolaus and Archytas. They did not suffer the fame of Pythagoras and thus the truth concerning them is somewhat easier to sift from the accretion of later legends. Still, what we know is little, and is always dependent on later sources. And so we are driven back—as is so often the case in the study of early Greek philosophy—to Plato and Aristotle.

What I mean is not just that the later tradition concerning Pythagoras and Pythagoreanism was formed, in antiquity, through the double prismatic effect of Plato and Aristotle (this is the case for all Pre-Socratic thought). I also mean that the historical significance of Pythagoreans lies precisely in this: that they were important for Plato and for Aristotle. It is clear that Plato and Aristotle opportunistically changed the meaning of the Pythagorean sources available to them; in my view, the important thing is that they saw there an *opportunity*. Plato and Aristotle perceived an affinity between a group of thinkers whom they associated with Pythagoras. They further saw in this group something useful for their own philosophy. To Plato, the usefulness was in the encouragement of a kindred spirit; to Aristotle, the usefulness was in the warning of a philosophical pitfall.

Having said that, let us be clear: Philolaus and Archytas—indeed, even Pythagoras himself—were real historical figures. Plato could hardly have *invented* them. Certainly, he did not invent Archytas as, in all probability, the two even met in the flesh. Ever since antiquity, this meeting led to speculation and that is how a certain account of Plato's growth is formulated:

Socrates dies when Plato is but twenty-nine. Plato, then, becomes the author of the early dialogues of Socratic refutation that prove the limits of everyday Athenian knowledge. Later, in his forties, he sails to the west; he stops at Tarentum and makes the acquaintance of Archytas—and soon thereafter the *Meno* is written. Now emerges a new, mathematical and Pythagorean Plato, with all that follows for western philosophy.[2]

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[1] On all of this, see [2].

[2] The best exposition of this account is in [16].

This is possible—though I find it difficult to believe that Plato, of all people, would have to be created from the outside, once by Socrates and then by Archytas. (He does not appear, from his dialogues, to have been easy to influence.) We just do not know who invented whom: did *Archytas invent Plato*—by making him a Pythagorean? Or did *Plato invent Archytas*—namely, did Platonist philosophy give, retrospectively, a meaning to the thought of so-called Pythagoreans?

Be that as it may: the truth is, to repeat, we do not know. From this basic ignorance, let us begin to look for Pythagoreanism and the divine. First, we look at the use made of the Pythagoreans by Plato and Aristotle (Section 2). Then we turn to the Pythagoreans themselves (Section 3). (Both Sections 2 and 3 are, of course, highly selective.) Section 4 offers a tentative explanation of why the Pythagoreans would initiate the history of the relation between mathematics and the divine.

2. Pythagoreanism in Plato and Aristotle

In a literary context, the mention of personal names would have had a certain jarring effect to the ears of ancient Greeks. Previous authors are typically merely alluded to, or their names are periphrastically suggested. Thus the name of Pythagoras is but infrequently mentioned in the writings of Plato and Aristotle. The most direct reference to Pythagoras in the writings of Plato is, at first sight, rather disappointing. Asking, in the last book of the Republic, whether Homer was indeed a great educator, Socrates asks if[3] "[Homer was] a *private* educational guide during his lifetime to individuals who cherished him for his company and passed on for posterity a Homeric way of life, just as Pythagoras was himself exceptionally cherished for this reason, and his successors even now call their way of life 'Pythagorean' and are somewhat distinctive among other men?"

No mathematics or religion here; but this in itself is instructive. We see that the essence of Pythagoreanism for Plato is in some personal quality, a 'way of life'. The basis of this in the historical Pythagoras—and in his memory in the 4th century B.C.—is clear: Plato thinks of the system of rules and taboos that governed the life of Pythagoras' followers. But it is not the practice itself that is important (and it would appear that Plato does not necessarily assume that the 'Pythagoreans' of his day live quite the same life of Pythagoras' immediate followers). What is important is the very distinctiveness of the practice, the willingness of Pythagoreans to see themselves as belonging to a 'Pythagorean' tradition, standing apart from their fellow-Greeks. The essence of Pythagoreanism, then, is in some self-imposed *difference*.

What about the philosophical views of Pythagoreans? Plato hints at them occasionally. Sometimes, indeed, they gain great significance. The 'Pythagoreans' are mentioned, explicitly, towards the end of the curriculum passage in the Republic:[4] "It appears… that these sciences [astronomy and harmony] are sisters, as the Pythagoreans say and we, Glaucon, agree… [having agreed, we do keep our own principle:] that our students shall not try to study those things short of perfection… as we have just said for astronomy. For don't

[3] Republic X 600a–b, translation from [9, p. 49].
[4] Republic VII 530d–531c, my abbreviation and translation.

you know that with harmony, too, they do this strange thing. . .—Yes, and absurd, too: they mention those so-called ‘quarter-tones’, and they stretch their ears [some claiming to hear, some not]. . . all of them put the ears before the soul. . .—You mean those worthy ones who approach the issue by the chords—torturing them on the rack. . . They do the same as in astronomy: They search the numbers in the audible harmonies, without passing onwards to abstract problems, studying just that—which numbers are harmonious and which are not”. This follows immediately upon the famous call to let the stars out of astronomy, and leads immediately to the culmination of Plato’s philosophy of the sciences in his description of supreme dialectics. Thus the curriculum appears, retrospectively, as a corrected version of Pythagoreanism. Since Archytas’ fr. 1 refers explicitly to the sciences of the curriculum as ‘siblings’, it appears that Plato had in mind a specific Pythagorean source (unless, of course, fr. 1 itself was written by a later Platonist, to construct the imputed source used by Plato. . .).

Another strategic reference to what may be a Pythagorean doctrine comes at the Phaedo, right before Socrates’ final great speech that introduces Platonic metaphysics at its fullness. Simmias, one of Socrates’ friends, has a worry about immortality:[5] “One could surely use the same argument [as about the soul] as about the attunement of a lyre and its strings, and say that the attunement is something unseen and incorporeal and very lovely and divine in the tuned lyre, while the lyre itself and its strings are corporeal bodies and composite and earthy and akin to the mortal. . . If then, the soul proves to be some kind of attunement, it’s clear that when our body is unduly relaxed or tautened by illnesses and other troubles, then the soul must perish. . .” While this is often taken to represent Pythagorean doctrine, there is some debate as to the source of the theory.[6] It is possible, indeed, that it was simply invented by Plato. But this debate may miss the point: there was no ‘doctrine of the Pythagoreans’ circulating in antiquity, and so, in the general context of what an ancient reader would expect of Plato and of his interlocutors, the mention of a theory having to do with music, which is held by friends of Socrates, but not quite by Socrates himself, would be taken in the same way as the ‘sibling sciences’ mentioned in the Republic. By the standards of ancient writing, this is a sufficiently explicit reference to ‘Pythagoreans’. So—following what, I admit, is a circular argument—we may put together these two key references to Pythagorean doctrine, from the Republic and from the Phaedo. The picture is consistent. The Pythagoreans have nearly reached the truth, and hence they can be mentioned at the very gates of Plato’s innermost philosophy. Their views are directly related to the Platonic interest in *duality*—the duality of the concrete and the abstract, the duality of body and soul. They sense somehow that there is an incorporeal realm, and connect it to the mathematical sciences; but they do not yet reach beyond the corporeal itself. The basic image is that of music. The Pythagoreans, alone of the Greeks, pursue it as mathematical; Plato goes further, and hears it as purely mathematical and abstract. We begin to see a connection between mathematics and the divine: a mathematical perception of music, serving as a stepping-stone leading from the perception of the world, to the perception of the otherworldly. The Pythagoreans lead in the right way (even if without reaching the goal).

[5] Phaedo 85e–86c, translated from [7, pp. 35–36].
[6] See, e.g., [8, pp. 306–319; 17, pp. 178–179].

Aristotle's attitude to the Pythagoreans is the mirror image of Plato's, and thus confirms it.

Of course, Aristotle was not the opposite of Plato. The distinction between them was much more fine-grained, and it appears that one of Aristotle's main questions was precisely that—"how am I different from Plato?". This is important to us, since one of Aristotle's strategies for distinguishing himself from Plato, was to distinguish himself from Pythagoreanism.

Let us consider the Metaphysics. In this work, Aristotle puts forward a world picture where reality ultimately depends on the perfection of divine stars, which are understood through an account put in the terms of mathematical astronomy. This is a bad start for anyone trying to distinguish himself from Plato or from the Pythagoreans. But then, Aristotle insists, it is a mistake to see reality in mathematical terms abstracted from their physical basis. This mistake he ascribes, specifically, to the Pythagoreans:[7] "The Pythagoreans, as they are called, devoted themselves to mathematics; they were the first to advance this study, and having been brought up in it they thought its principles were the principles of all things. Since in of these principles numbers are by nature the first, and in numbers they seemed to see many resemblances to the things that exist and come into being... since, again, they saw that the attributes and the ratios of the musical scales were expressible in numbers... they supposed the elements of numbers to be the elements of all things, and the whole heaven to be a musical scale and a number... E.g., as the number 10 is thought to be perfect and comprise the whole nature of numbers, they say that the bodies which move through the heavens are ten". Notice that, like Plato, Aristotle associates Pythagoreanism primarily with a certain way of life. The main characteristic of the Pythagoreans is their having devoted, or attached themselves (*hapsamanoi*), to mathematics, or 'mathematical learnings' (*mathemata*). The context of the description is well known: Aristotle goes through the philosophies preceding his own, classifying them according to their account of the first principles. These are all material in a sense and, for the philosophers mentioned prior to the Pythagoreans, we can easily see the origins of the views, as described by Aristotle, in the ordinary reality of daily life. (It is here, for instance, that we have the famous reference to Thales who thought that all was made of water—perhaps, Aristotle suggests, because of his realization that nourishment is moist![8]) In short, while ordinary people live a life of earth, air, fire and water, the Pythagoreans stand out, once again, in their very life. Living a strange life, 'attached to the mathematical learnings', an immediate consequence is that of a strange doctrine—'having been brought up in it they thought its principles were the principles of all things'.

The place of this comment is significant: in Aristotle's account, the Pythagoreans immediately precede Plato, whose philosophy is explained through a mixture of influences from previous thinkers, mainly that of the Pythagoreans.[9] Thus we see that the modern tendency to see Plato as an essentially *influenced* thinker begins with Aristotle. In this case, however, the motivation is clear. Aristotle asserts, effectively: "How do I differ from Plato? By not being influenced by the Pythagoreans".

[7] Metaphysics A 985b23–986a10, transl. Ross, in [1, p. 1559].
[8] Metaphysics A 983b22–23.
[9] Metaphysics A6.

Two books of the Metaphysics—M and N—effectively unpack this claim. Aristotle, once again, offers an account of Plato-under-influence. Plato (who is unnamed in this account) starts out from the Socratic search for the definitions of things; he then posits those definitions as possessing separate existence, apart from the things they define—following the example of Pythagoreans, who gave arithmetical definitions to things such as opportunity, justice, marriage and, apparently, saw those definitions as existing separately from any of the things they defined.[10] In short, the Pythagoreans provided Plato with a way of thinking about the otherworldly. Here, for instance, is 'justice'; and there is the number 4. The number 4 somehow underlies justice and is prior to it.[11] Thus the world is doubled, consisting of events of justice, on the one hand, and of the number 4, on the other hand. Aristotle's main claim in the books MN is that such a doubling is unnecessary, even in the cases where, for epistemological reasons, it has some prima facie credibility. Thus, even the study of the mathematical sciences themselves does not call for the assumption of any reality over and above the ordinary physical one. The Pythagorean extrapolation of their way of life into an ontology was, therefore, unfounded, even given the way of life itself. Properly conceived, mathematical learnings do not assume the existence of any separate reality. As mentioned above, Plato uses the Pythagoreans to suggest his dualities—a duality of the concrete and the abstract, a duality of the body and soul. For Plato, the Pythagoreans are not dualistic enough; for Aristotle, they are too dualistic.

In arguing against Pythagorean doubling, Aristotle uses two main strategies. One is serious and impersonal: the opponents are hardly alluded to, and the philosophical problem of the nature of mathematical abstractions is dealt with in a detailed philosophical analysis. The other strategy is much more ad hominem and rhetorical: Aristotle alludes to specific Pythagorean views, and makes fun of them. This is the climax of Metaphysics N:[12]

> They even say that Ξ, Ψ and Z are concords, and because there are three concords, the double consonants are also three. They quite neglect the fact that there might be a thousand such letters; for one sign might be attached to ΓP. These people are like the old Homeric scholars, who see small resemblances but neglect great ones. Some say that there are many such cases, e.g., that the middle strings are represented by nine and eight, and the epic verse has seventeen syllables, which is equal in number… no one could find difficulty either in stating such analogies or in finding them in eternal things, since they can be found even in perishable things.

Two points are interesting for us. First, Aristotle, in this critique, suggests that Pythagorean duality is a mere contrived analogy—they do not discover a real duality in the universe, but simply stitch up together two unrelated things. In other words, they engage in metaphorical language. Bear this in mind: this will be important for our discussion in Section 4. Second, notice the Aristotelian reference to the duality of the eternal and the perishable: this last point reminds us of the serious point of the joke. Aristotle, after all, subscribes to some dualist vision of the world. The shared assumption for everyone in the discussion—Pythagoreans, Plato and Aristotle—is that there is an eternal reality, to be distinguished from the reality of perishable things. Aristotle does not deny the duality: he merely grounds it in physical stars, and therefore can do away with intangible mathematical properties.

[10] Metaphysics M4.
[11] The identification is provided by Alexander's commentary to the Metaphysics, in Met. 38.10.
[12] Metaphysics N1093a20–b6, transl. Ross, in [1, pp. 1727–1728].

Let us see how Aristotle addresses the Pythagorean account of the stars themselves. Note the ad hominem—and comical note:[13]

> It is clear that the theory that the movement of the stars produces a harmony, i.e., that the sounds they make are concordant, in spite of the grace and originality with which it has been stated, is nevertheless untrue... Melodious and poetical as the theory is, it cannot be a true account of the facts. ... Indeed, the reason why we do not hear, and show in our bodies none of the effects of violent force [that follows from large noises] is easily given: it is that there is no noise... [and finally the authors of the view are identified] the very difficulty which made the Pythagoreans say that the motion of the stars produces a concord corroborates our view.

This is the famous harmony of the spheres, which Aristotle criticizes, typically, in a strictly physical way. Had the spheres produced a noise, Aristotle argues, we would expect certain physical consequences: the noise would be heard, and there would be other manifestations of that strong motion. Then what results is a 'poetical' theory that Aristotle would dismiss, once again, as mere metaphor. (Paradoxically, since the theory was taken literally, it is can now be read metaphorically only!)

Notice that Aristotle simply refuses to treat the theory as a more abstract metaphysical statement—e.g., that the true account of the motions of the stars is that they manifest the mathematical *structure* of musical harmony. According to that more abstract account, both music itself, as well as the stars, are explained through a more fundamental and abstract mathematical principle. This, probably, was Plato's intention when, in both the Timaeus and in the Republic's Myth of Er, he connected music and astronomy.[14] I make this comparison between Aristotle and a possible Platonic view, because we reach here the difficulty of disentangling this complex melee of Pythagoreanism, Platonism and Aristotelianism. Who was the author of the theory of the harmony of the spheres? Our first evidence, in fact, comes from Plato himself, who has a version of the theory as part of the imaginary after-world of the Myth of Er. There is nothing to prove that Plato relied on a previous Pythagorean account. Aristotle's criticism of the theory as Pythagorean might refer to authors who are contemporary or later than Plato himself; or it might be a characterization of the nature of the theory, rather than a description of its historical source. Let us assume for the sake of the argument that the theory is first put forward in Plato's Myth of Er, and is then first characterized as 'Pythagorean' in Aristotle's De Caelo. In this case, we see Pythagoreanism constructed by a tug-of-war between Plato and Aristotle. Plato, mathematicizing, has an astronomy anchored in the abstract properties of music; Aristotle, physicialising, gives Pythagoreanism its familiar shape, of the mathematical taken, concretely, to underlie the world. The music of the harmony of the spheres is taken literally, and so a strange, absurd world comes into being—an ever-present, never-heard soundtrack to accompany the universe. Aristotle's Pythagoreans listen to the inaudible.

In this we have come full circle to Plato's criticism of Archytas' studies in harmony. For Plato, the mistake of the Pythagoreans is that, while tuning themselves to the more abstract study of music, they still pay attention to the audible—the concrete reality of strings stretched on the rack. For Aristotle, the mistake of the Pythagoreans is that, while engaged in beautiful and important studies, they go beyond the concrete reality itself, into a realm of an inaudible, otherworldly layer of existence: a sound that is never heard.

[13] De Caelo II.9 290b12–291a9, transl. Stocks, in [1, p. 479].

[14] Timaeus 34b–36d, Republic 616b–617d.

For both Plato and Aristotle, then, Pythagoreanism is almost the closest philosophy to their own, and yet to be distinguished from their own. Pythagoreans look for a reality that is eternal and superior to the one immediately surrounding us, and they find it through the study of such objects as music and astronomy. So far, Plato and Aristotle follow. There they diverge from both the Pythagoreans as from each other. In both cases the Pythagoreans are guilty, as it were, in producing *too much music*. For Plato, the mistake is in the attention to concrete, heard sounds; for Aristotle, the mistake is the attention to abstract, unheard sounds. In both cases, music is suggestive of the more basic principle, of *duality*—one thing being simultaneously something else. This is the principle Plato wishes to exploit, and Aristotle to deny and reduce to mere metaphor.

For both Plato and Aristotle, the Pythagoreans are somewhat admired, they are somewhat sublime—but, even more, made fun of. They display a certain absurdity, paying attention to the tiny details, whether these are the quarter-tones that Plato's Pythagoreans stretch their ears to hear, or the three consonants that Aristotle's Pythagoreans describe in musical terms. This special position—sublime, and ridiculous—is perhaps what makes them most 'Pythagorean'. After all, this duality of the sublime and the ridiculous is central to the tradition of Pythagoras himself, the prophet of metempsychosis—and the hermit of beans. Once again: it is not so much the contents of the Pythagorean life itself that is important, as its very otherness. The Pythagoreans insist on living differently, and, in this way, they reach a different reality. Plato wishes to go beyond them into that reality, Aristotle wishes to stay nearer to ordinary reality, but both sense that the way to eternal, higher metaphysical realms is through a certain distance from ordinary reality; and that mathematics, in particular music, may lead the way.

So here is the formula we gain from Plato's and Aristotle's reception (or, perhaps, construction) of the Pythagoreans. *Otherness* (*based on a mathematical duality of concrete and abstract, in particular the duality of mathematical music*) *leads to the otherworldly.* Let us see if this formula is corroborated by the little we know of the Pythagoreans themselves.

3. Pythagoreanism: some evidence from the Pythagoreans

A number of people are mentioned in our sources as 'Pythagoreans', while others are identified as such by us, on the basis of what we know of their activity. For most of them, the little we know makes little sense. (Why, for instance, did Eurytus put pebbles together in the form of a human being?[15]) As a group, they belong to the late fifth to the early fourth century, but this again signifies little. The period begins where it does because only a few prominent thinkers are at all known from before the late fifth century (perhaps, not much intellectual activity, before that period, took the form of writing). The period ends where it does because, from the mid-fourth century onwards, the thinkers we would, coming from an earlier date, classify as 'Pythagorean', would instead be classified as 'Platonist'. (It is possible that Aristotle's 'Pythagoreans' included such later authors.) Thus the chronology and the very identity of the 'Pythagoreans' is a late construct made of the selection of the sources, and of our own nomenclature. There is a real identity of spirit between several

[15] [5], Vol. 1, pp. 419–420.

ancient authors, but this identity of spirit does not derive from anything like a 'Pythagorean school': the same is largely true for all such groups in early Greek thought. The ancient authors can be usefully classified, but they did not come together in clearly defined classes.

Working, then, with our own classification of 'Pythagoreans', we can see that two figures stand out. A little more is known about them than about their peers, and some sense of their intellectual personality can be formed. Philolaus lived in the late fifth century, Archytas in the early fourth century. Philolaus has been somewhat the more studied of the two (one can mention in particular the magisterial work, Philolaus of Croton, 1993, by Carl A. Huffman). This is in part because Philolaus' age makes him technically a 'Pre-Socratic' and thus of more interest to modern scholarship. There is another, more important reason for the relative neglect of Archytas by historians of philosophy. From their remaining fragments, Philolaus appears like a philosopher, while Archytas appears like a scientist. (That the term 'Pythagorean' can accommodate both is in itself significant.) To offer a general formula, Philolaus is the more suggestive and general author, while Archytas had produced accomplished and more detailed studies. Philolaus' metaphysics also appears, perhaps, to have been more speculative, while that of Archytas' might have been more grounded in the actual practice of science. To sum up the formula even more briefly, then, I suggest the following four-term proportion:

Philolaus : Archytas :: Plato : Aristotle

That is, the transition from Philolaus to Archytas may have been rather like the transition, a generation later, from Plato to Aristotle. It will also appear that Plato, when distancing himself from Pythagoreans, is thinking in particular of an Archytas-type interest in concrete reality; while Aristotle, when distancing himself from Pythagoreans, is thinking in particular of a Philolaus-type metaphysics.

Having said that, there are important continuities between the intellectual projects of Philolaus and of Archytas, which are then reflected later on in the main tradition of Greek philosophy preserved by Plato and Aristotle: in this lies their great historical significance. We shall concentrate here on those issues that have some bearing on the question of mathematics and the divine.

The best starting-point is Philolaus' fragments 6 and 6a, from which I quote following Huffman [11, pp. 123–124, 146–147]:

> Concerning nature and harmony the situation is this: the being of things, which is eternal, and nature in itself admit of divine and not human knowledge, except that it was impossible for any of the things that are and are known to us to have come to be, if the being of the things from which the world-order came together, both the limiting things and the unlimited things, did not preexist. But since these beginnings preexisted and were neither alike nor even related, it would have been impossible for them to be ordered, if a harmony had not come upon them, in whatever way it came to be. Like things and related things did not in addition require any harmony, but things that are unlike and not even related nor of the [the same speed(?)], it is necessary that such things be bonded together by harmony, if they are going to be held in an order. . . .
>
> The magnitude of harmonia (fitting together) is the fourth, and the fifth. The fifth is greater than the fourth by the ratio 9 : 8. For from [lowest tone] to the middle string is a fourth, and from the middle string to highest tone is a fifth. . . [A brief discussion of the elementary structure of the octave follows.]

The system seems to run as follows. The world is made to be known—in a sense, is made to exist—by having some order imposed upon it. This order is made primarily of two main components (Philolaus speaks about them elsewhere and they seem to have been the foundation of his system), *limited* and *unlimited* things. These may be, say, some chance length of string—*unlimited*—and its being compared in length to some other string, which comparison then makes it *limited*.

How can the measure apply to the measured? Philolaus mentions harmony, which we may consider by discussing further the example of the string. Suppose the lengths of the two strings have, to each other, the ratio of, say, the side of the square to its diagonal (what we call the square root of two). This is not a harmony, and no special relationship of sound would form when the two are plucked in sequence. No knowable, fixed object will come to be. However, if the two strings are to each other as, say, 3 to 4, the relationship would be that of the fourth. When plucked in sequence, they would form a musical unit: a precisely given individual, both knowable and real. By belonging to this real musical structure, the two lengths of string acquire a meaning—they are no longer mere unlimited lengths. The harmony—the fitting together of the strings—is thus, in a reasonable sense, ontologically prior to each of the components of the harmony.

This then is a sober, if bold, theory in epistemology and in metaphysics. Lest the wrong impression of Philolaus be formed, I hasten to quote testimony 16 [11, pp. 237–238]:

> Philolaus [says] that there is fire in the middle around the center which he calls the hearth of the whole and house of Zeus… And again another fire at the uppermost place, surrounding [the whole]. [He says] that the middle is first by nature, and around this ten divine bodies dance: heaven, planets, after them the sun, under it the moon, under it the earth, under it the counter-earth, after all of which the fire which has the position of a hearth about the center.

From such testimonia—and most testimonia are in this spirit—it appears that musical harmony is not, for Philolaus, a mere example of the more general principle of order. Rather, he develops a cosmology derived from numerical and harmonic principles: *ten* divine objects, moving in *dance*, surrounding a middle which is prior to them as *middle*—presumably, as being the middle term of some cosmic proportion. We can therefore see how both Plato and Aristotle could have used Pythagoreanism for their own purposes.

We also see that the sobriety of fragment 6 is misleading because—it appears—Philolaus deliberately aims at being *strange*. The cosmology is non-geocentric; a counter-earth is introduced. As usual, then, we see the Pythagorean emphasis on being different for difference's sake.

Now the following should be noted. It is likely that, as Philolaus brings forward the example of musical proportion, he uses a *recent* mathematical account of the nature of musical harmony. In other words, for Philolaus' audience there could have been a shock of surprise in the suggestion that the lower string to the middle string is as 3 to 4, rather like the shock of surprise in the suggestion that the heavenly bodies are to each other are as ten dancing figures. We, moderns, are now familiar with the mathematical account of music. We take it to be straightforward science. We have thus lost the shock of the strangeness of mathematical music and, as we read Philolaus' fragment 6a, we find there a straightforward scientific example for a straightforward idea. On the other hand, we do not think of the heavens as participating in a dance and therefore, as we read testimony 16, we still feel the intended shock, coming across the idea of balletic astronomy. What I suggest is this. To understand Philolaus, we should try not to rationalize balletic astronomy, so that we make it appear more natural to us; rather, we should somehow come to see the idea of mathematical music in its full strangeness. Fragment 6a should be as strange as testimony 16. An equation of strings, on the one hand, and numbers, on the other hand! The essence of Philolaus' system is in this, deliberate strangeness.

Note further the following. Philolaus' account of epistemology and metaphysics may appear sober but it is still, in itself, counter-intuitive. This is because Philolaus suggests, in fact, that the intangible precedes, epistemologically and ontologically, the tangible. The more abstract structure of harmonies is prior to the more concrete matter that it informs. This fundamental idea will of course flourish, in different ways, in the philosophies of both Plato and Aristotle, and in fact many parallels to this can be found in many other Pre-Socratic philosophers. This however does not make Philolaus' philosophy any more intuitive: it merely reminds us that the counter-intuitive was valued by people other than Philolaus himself.

We start, then, with a tangible world, which is meaningless and lacking in value; we distance ourselves from it and perceive musical harmonies, and through them we come to see a structured reality, which also gains in value. As we move away from the tangible to the abstract, we are also led gradually from the mundane to the divine—the world of Zeus set in the center of a cosmic ballet.

Moving on now to Archytas, it becomes difficult to speak of a philosophical system in the same sense. The most substantial fragments and testimonies do not provide us with a philosophy at all, but with four, apparently unrelated pieces of mathematical science.[16] I believe, however, that this mathematical science may form together a research program to complement Philolaus' metaphysics by offering a systematic, complete account of the science underlying it.

The first piece, which seems to come from a genuine fragment (number 1), is a physical theory of the origin of sounds.

The second piece, described (in somewhat different forms) in Ptolemy's *Harmony* and in Porphyry's commentary to the same work (testimonies 16–17, fragment 2), is a classification of ratios.

The third piece, reported only in Boethius (testimony 19), is a straightforward argument in number theory (probably genuine, for who would bother to forge an argument as straightforward as that?).

The fourth piece, very thoroughly and convincingly documented by the testimonies 14–15, is a brilliant solution to the problem of duplicating the cube.

The pieces fit together harmoniously, so to speak. They all belong to the study of proportions.

To begin with the first piece of the puzzle—fragment 1 and its acoustic theory—we should note that this study of the physical origin of sounds culminates, effectively, in a proportion. Archytas argues that pitch correlates to the speed of the motion of the air, and sums up fragment 1 in saying that 'it has been made clear to us by many [arguments, examples?] that the higher tones are moved faster, and the lower—slower'. While this does not yet have the mathematical form of proportion (i.e., Archytas does not offer the

[16] For the fragments and testimonies concerning Archytas, see [5, Vol. 1, pp. 421–439].

mathematical statement that the ratio of the speeds is the same as the ratio of the sounds) it would have been amazing, in light of his further views on musical ratios (of which see below) if he did not believe in some such theory. In other words, if the musical theory available to Philolaus had two separate domains connected by a single proportion—length of strings, say, and pitch of sounds—the musical theory available to Archytas adds to this another domain, covered by the same proportion—the speed of motion of air. This is an important advance since this domain of air motion is *universal*. This domain is not tied to any particular musical instrument, but is coextensive with any sound whatsoever. Thus Archytas had achieved a complete system uniting a mathematical structure with a physical structure.

This is corroborated by the second piece, where we see Archytas dealing in much greater detail with this system: this time, not with its physical–mathematical interface, but with the mathematical structure alone. Once again, Archytas' aim seems to have been the *completion* of the system. Thus he goes on to classify kinds of numerical ratios, corresponding to different musical relations. The details, especially as reported by Ptolemy, are quite complicated and reveal Archytas' computational fluency. The principle is simple: from the basic assumption that the octave is correlated to a 1 : 2 ratio, divided into, e.g., 2 : 3 and 3 : 4 (the fifth and the fourth) one can derive the entire system going as far as the quarter-tones (the debate as to their existence, we recall, was evoked, to comic effect, by Plato). One indeed reaches quickly pure numerical constructs whose auditory correlates are probably impossible to judge (how about the interval of 256 to 243?). A strange, purely numerical structure takes over the auditory world of music.

Even more abstract—but necessary for the sake of the completion of the system—is the third piece in the puzzle: the study in number theory reported by Boethius. Archytas proves that a ratio in integers of the form $(n + 1) : (n)$ cannot serve as the extreme terms in a continuous proportion in integers $(n + 1) : (k) :: (k) : (n)$. (This is not the trivial result that between two consecutive integers another integer cannot be found: note that the ratio $(n + 1) : (n)$, e.g., 9 : 8, is equivalent to many other ratios between non-consecutive integers, e.g., 18 : 16.) This proposition has immediate consequences on the possibility and impossibility of musical proportions, and in fact it appears as the third proposition of the musical treatise ascribed to Euclid, the *Sectio Canonis*. (Boethius, of course, quotes it in the course of his own musical treatise.) However, it does not feature any particular musical ratios. This study—no doubt, a fragment of a larger study in number-theory—moves beyond the actual numbers put forward by the facts of music, to a study of the possibility and impossibility of proportion in integers as such.

The same kind of extension, finally, may be true for the fourth and final piece of the puzzle—Archytas' solution of the problem of duplicating the cube. Both the problem, and Archytas' approach to its solution, are closely related to issues in proportion theory. We can see this as follows. How can one find, given two lengths, their geometrical mean? E.g., given the lengths 18 and 16, how can we find the length B that satisfies 18 : B :: B : 16? As we have seen, there is no purely numerical solution to the problem; however, it can be solved geometrically. In particular, we can fit together right-angled triangles in a circle so that, by triangle similarity, the triangles will display the geometrical progression A : B :: B : C (Figure 1). By producing A and C as 18 and 16, we find the desired B. Thus, by considering ratios in a more abstract way—moving beyond the ratios between integers to

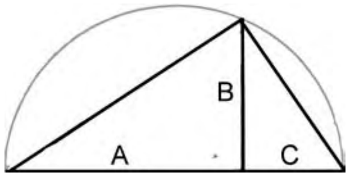

Fig. 1. A semi-circle with diameter A + C yields B with A : B = B : C.

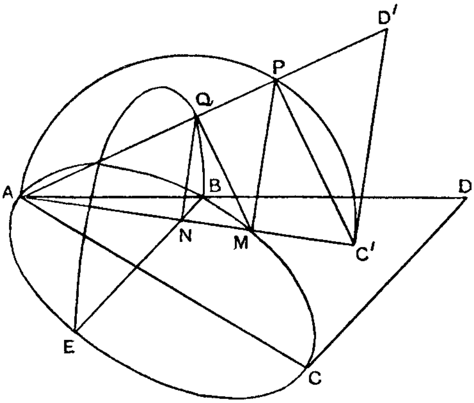

Fig. 2. Architas' duplication of the cube. Cf. [10, Vol. 1, p. 247].

the ratios between lengths as such—we have gained a new problem. To further complete this new abstract system, the following problem suggests itself:

Given two lengths, A and D, to find two means B, C that satisfy a geometrical progression, i.e., A : B :: B : C :: C : D.

This is the problem of finding two mean proportionals. We may note that it is equivalent to the problem of finding a cubic root (B is, effectively, the cubic root of the given D by the units of A) and therefore we should not be surprised with the name the problem has had attached to it 'the problem of duplicating the cube'. But let us remember that the problem, mathematically speaking, is simply an extension, inside the theory of geometrical proportions, of the problem of finding a proportion between two given lengths—as seen in Figure 1. Archytas' solution, interestingly, is an extension of Figure 1 in a rather direct sense. His approach seems to have been to look for ways of fitting right-angled triangles in space, so that their similarities would provide the proportion required by the terms of the problem. The reader may now consider Figure 2, of Archytas' solution, and compare it to Figure 1. It so happens that the problem of finding two mean proportionals calls for considerable spatial gymnastics, hence the charm and brilliance of Archytas' proof (probably, Archytas was the first to offer such a proof).

Archytas' research, then, fits together in a system—and, what is more, it fits together with Philolaus' metaphysics. The focus of research is proportions. These underly physical reality, but are also considered from an abstract perspective, as belonging to a purely mathematical system. It seems that the model of mathematical music is the key to Archytas' research—just as it was the key to Philolaus' metaphysics—and that, in both cases, the interest is not in the fact that mathematics has concrete manifestations but that, on the contrary, beneath the world of concrete sounds there is another world, richer and more abstract, of mathematical relations. Notice that when the theory of proportion reaches its mathematical culmination—we are likely to have Archytas' best work in his preserved solution of the problem of duplicating the cube—it becomes fantastically complicated and abstract, involving a cosmic dance no less remarkable than that of Philolaus' stars. The world of swirling semicircles, studied by Archytas, already belongs to a realm of pure mathematics that exists apart of any ordinary Greek experience.

Of course, it is impossible for us to tell what were Archytas' precise metaphysical views. Clearly, an admirer of Philolaus could have been excited by the science itself; while an admirer of Archytas could have enjoyed the science while criticizing the metaphysics. I have suggested above that these two routes were taken by Plato, and Aristotle, respectively. At any rate, we can now sum up the evidence provided by the four main protagonists—Plato, Aristotle, Philolaus and Archytas. We may recall the formula gained at the end of the previous section: *Otherness* (*based on a mathematical duality of concrete and abstract, in particular the duality of mathematical musical*) *leads to the otherworldly.* This is, indeed, corroborated by what we know of Philolaus' metaphysics and Archytas' science. In this section, though, we have noticed the predominant role of a special mathematical concept, that of *proportion*. Proportion, apparently, served the Pythagoreans to support their dualities of concrete and abstract. So let us reformulate this even more briefly:

Otherness, based on proportion, leads to the otherworldly.

In the next section, we shall try to explain how this formula came to be characterized as 'Pythagorean' and, finally, how the Greeks first formulated the question of mathematics and the divine.

4. Mathematics and the divine in the Pythagoreans: a suggestion

What made the Pythagoreans *Pythagorean*? That is, what made the ancients perceive a link between the thought of, say, Philolaus and Archytas, and the stories attached to person of Pythagoras? This question involves many unknowns. It does appear that, to have been called a 'Pythagorean', an ancient author had to come from the Greek West (where most of Pythagoras' activity took place). But does that mean that there was a historical continuity between the Pythagoreans of the sixth century, and those of the fourth? We simply do not know. It is better to concentrate, then, on the pattern of beliefs and practices that the name 'Pythagoras' evoked to the Greeks. Somehow, Philolaus and Archytas fitted that pattern. Why was that so?

On this question, too, one has to be speculative. Speaking somewhat dogmatically, then, we may offer the following account of 'Pythagoras' as present to the Greek imagination.

It appears that Pythagoras was primarily remembered as a charismatic religious leader of a special kind. He offered a way of life guaranteeing some form of release from pain and mortality. If this is indeed the case, then Pythagoras had embodied a Greek *type*—usually, however, fulfilled by mythical figures. Orpheus was another such savior-figure and so, in other ways, were other heroes and gods, such as Dionysius himself. Around such

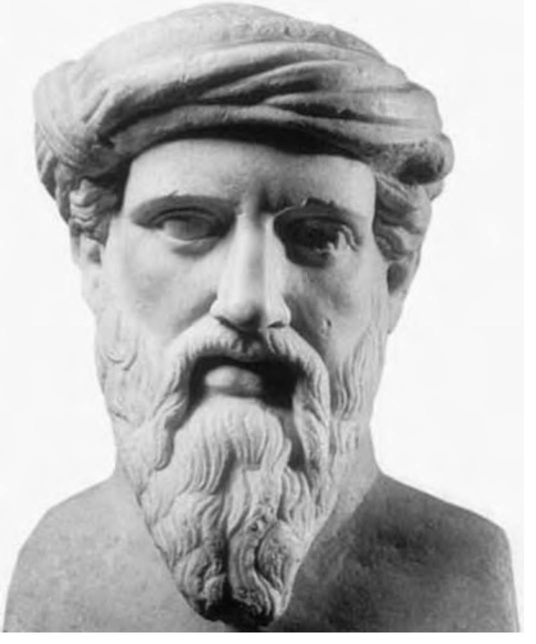

Fig. 3. Pythagoras. Courtesy of the Museo Capitolino, Rome.

savior-figures, special practices formed, the *mystery cults*.[17] These had a powerful role in the Greek imagination. It is therefore, probably, in relation to such mystery practices that we should understand what the term ‘Pythagorean’ evoked.

To follow mystery practices would be to stand out from the public Greek religion of the body of citizens in a city-state. There is something exclusive about being saved: as it were, the idea of one’s being saved implies that there are others, who are not. There is only so much room in the boat. A distinction is thus made between those who belong to a saved inner circle, and those who do not. The transition into the inner circle—the initiation—occurs through participation in an activity peculiar to the group, whether in the ascetic ritual of the Orphics or the Pythagoreans or in some special rite performed in a ritual center.

Briefly put, the core of such practices is the transition from the mundane to the divine through a process of becoming-other. This is precisely the formula we have gained for the Pythagoreans of the fifth and fourth century (minus the role given to mathematics and in particular music and its proportions as an agent in the initiation.) This then would be a reason for Greeks in the fourth century to conceive of Philolaus, for instance, as a Pythagorean. Authors such as Philolaus offered a way reaching from the *here* of the everyday to the *there* of the divine, passing through a strange form of life.

[17] See [3, pp. 296–304] and, in general for Greek mysteries, see [4].

How do Greeks bridge the gap between the mundane and the divine? We can follow a structure, whereby the bridge comes more and more to be, so to speak, under human occupation.

Religious ritual as such aims at bridging the gap between the mundane and the divine: but the traditional ritual of the City—by being ordained as ritual—belongs, in a sense, to the gods. Therefore a further, more personal ritual is called for, the mystery cult. Here, the individual makes a choice to perform the rite and thus assumes a control over his or her own destiny, beyond that provided by the ordained ritual of everyday religion. Pythagoras, by explicitly instituting a *new* mystery cult, brought the divine even nearer: the claim was that a practice, instituted by a mortal, could reach beyond mortality. Even so, the irrationality of the original Pythagorean practice implies that the practice is either worthless or divinely inspired. To make this claim plausible—of transformation to the transcendent, through human means—the transformative power should be given some rational basis. Thus the Pythagoreans of the fifth and fourth centuries went on, to *rationalize* mystery—to produce a systematic philosophical counterpart of the experience of mystery. There are important differences between the four stages—traditional ritual, mystery cult, Pythagoras and Pythagorean philosophy—but there is a basic continuity which explains, I suggest, why the Pythagoreans were called 'Pythagoreans'. In short, of all the intellectual variety on offer at the late fifth century, the Pythagoreans offered a life most closely resembling that of the mystery cult. Hence their special absurdity, as well as their special promise.

Why mathematics? That is, why did the Pythagoreans take *mathematics* as the agent for transformation to the transcendent? We can, I suggest, account for that as well. To outline this suggestion, let us look more closely at the mystery cult practice.

In a mystery cult, say the famous one of Eleusis, the initiate would follow a long and special rite: sacrifices, processions, fasts, all leading to a special ceremony conducted in an atmosphere of secrecy, terror and ecstasy; secrets lead on to secrets until the highest secrets are revealed. All of this, finally, is supposed to endow the initiate with a better after-life and to revoke the otherwise expected punishments of hell. The secrets, under close inspection (they were sometimes revealed in antiquity, despite the harsh rules against their being divulged) appear disappointing. What is the height of the Eleusis mystery? A priest announces "The mistress has given birth to a sacred boy, Brimo to Brimos", and then an ear of corn is displayed and cut in silence.

Any concrete representation of ecstasy is bound to appear disappointing and, with the appropriate preparation and staging, the rite would no doubt have been effective enough. But to concentrate on the staging effects is to miss a more important point, namely, that *any* rational statement would have been inappropriate at this context. Every text can be staged but some texts are more appropriate for ecstasy than others. The very irrationality of the Brimo-to-Brimos text endows it with a transformative power. The text cries out to be interpreted (Who is Brimo? Is she perhaps the goddess of Eleusis, namely Demeter? And who is her boy?). Such irrational texts provide us, then, with a verbal counterpart to the main theme of the ritual, which is that of becoming-other. The text itself is caught, so to speak, in the act of becoming-other: it is a metaphorical statement, whose non-metaphorical significance is left opaque. Precisely the same holds of the cut ear of corn: we immediately see the act as rich in metaphorical significance, but we are not given the indications which would specify a unique literal interpretation of the metaphor, so that the cut corn remains, as it were, a metaphor hanging in mid-air. (This, incidentally, is why the secrecy surrounding the mysteries would have been so important: with the ritual revealed, the assertions and acts would have acquired standard interpretations and would have thus turned into *dead* metaphors.)

The principle, then, is as follows. A certain 'mystery' statement—or act—is meaningless on its own, but it does suggest a meaning which however is different from its surface meaning. If we call the statement P, then the form of the expression is 'P is not P'. By immersing yourself in a system of expressions of this kind, you are made to become non-yourself, and this is the basis for the transformative power of mystery practice. If identity is put into question, then so is self-identity.

The transition accomplished in a mystery practice is mediated through strangeness. We can therefore begin to apply what we have seen from the mystery cults, to the Pythagoreans themselves. We have noted the aiming at difference for difference's sake, characteristic of both the early Pythagoras and the Pythagoreans themselves. We have also noted the sense of the sublime and the ridiculous, somehow combined, attaching to both. Pythagoras' abstaining from beans, or the Pythagoreans' interest in the three characters Ξ, Ψ and Z, are both as absurd and as meaningful as the ear of corn: they suggest ways of transformation, whether of the self into a reformed, ascetic person, or of the world into a musical structure.

We can be even more specific. Let us consider again the task as it faces the Pythagoreans. They are looking for a rational grounding for the mystery experience: that is, they can no longer rely on the Brimo-to-Brimos type of expression. They need some kind of literal statement that keeps the sense of the metaphorical, of a thing-being-something-else. We have offered above a formula to describe Pythagoreanism: *Otherness, based on proportion, leads to the otherworldly.* I now suggest that this can be derived from the task of Pythagoreanism: to rationalize mystery.

The best starting point for the rationalization of mystery would be a thing that *literally is something else*: an object that is simultaneously two radically separate things, so that for which, the paradoxical statement 'X is not X' could be literally valid.

Music, under its mathematical interpretation, offers just that, and in a peculiarly appropriate form: it is a concrete thing, the musical instrument, and it is simultaneously, under its mathematical interpretation, an abstract thing—a system of proportions. There is a single formula underlying both so that one can literally say 'the ratio gave birth to the harmony'—Brimo gave birth to Brimos—and so the world of strings and instruments (the mortal world in which we live) is simultaneously something else: another, intangible world. Here then is one reason why music would be an obvious mode for a systematic Pythagoreanism: it embodies the continuity of the worldly and the otherworldly.

This can be generalized. For what makes this continuity at all possible? How can it be possible to say, literally, that 'X is not X'? This is because we are dealing with proportion statements, 'this string is to that string as 3 is to 4'. This is proportion: the most basic way of saying, in a literal way, that two separate domains are, in some defined way, the same. They are different; and yet they embody the same relations. Thus proportion-statements are the most natural route to be taken by the Pythagoreans. Their project was to offer an intellectually systematic correlate of a mystery practice—as it were, to literalize metaphor without losing its metaphorical power. This is precisely what proportion statements are. We recall Aristotle complaining about the Pythagorean tendency to ground everything, in-appropriately, in a numerical and musical realm:[18] 'They even say that Ξ, Ψ and *Z* are concords, and because there are three concords, the double consonants are also three. . . . Some say that there are many such cases, e.g., that the middle strings are represented by nine and eight, and the epic verse has seventeen syllables, which is equal in number. . . no one could find difficulty either in stating such analogies or in finding them in eternal things, since they can be found even in perishable things'. From such criticisms, it is clear that the Pythagorean speculation was expressed by statements such as 'as X in domain A, so Y in domain B'—the general form of proportion or analogy statements or, indeed, the general form of metaphor. Aristotle criticizes Pythagoreanism for its metaphorical language, but metaphors are sometimes valid—when they are true analogy statements, that is proportions. Thus, for instance, if we take the harmony of the spheres as a proportion statement (as the motions of the stars to each other, so the motion of the strings on the octave) we obtain a statement which in principle could be simply *true*. At the same time, it is still—as Aristotle would put it—'poetical'. Indeed, the statement has the appropriate 'mystery' effect of alienating us from the mundane world around us and making it appear rather more 'divine', simply by virtue of its possessing a duality or metaphor inherent to its mechanism. If we are surrounded simultaneously by stars, and by musical harmonies, then the mundane world is not just mundane.

To sum up, then, there are two properties of Greek mathematics—and, in particular, of Greek musical theory—that would have made it appropriate for the Pythagorean project. First, the correlation of *the concrete and the abstract*. (This is most obvious in mathematical musical theory, but it is also a wider feature of Greek mathematics with its equation of a concrete diagram and a general, intangible theorem.) Second, Greek mathematics essentially relied on the tool of *proportion*, which is the general tool of correlating separate domains. Thus these two seemingly irreconcilable domains, too—Mathematics, and Mystery—could be brought together. Greek mathematics could have functioned as mystery, made literal.

The above account, of course, was merely a dogmatic statement of a speculative suggestion. This is perhaps inappropriate for a historical study and so I shall conclude by restating the suggestion made here as a philosophical claim concerning mathematics and the divine.

We might perhaps be surprised that there is any relationship between the two. This is because we often think, today, of mathematics as the domain of the *literal* par excellence. We feel that metaphor is more appropriate for discussing the divine—and that mathematics provides us with no metaphors. In mathematics—we tend to think—everything is just what it is and the only allowed relation is that of logical entailment, that is, identity. Thus mathematics is conceived as the domain of the statements of the form 'X is X'. How can it guide us, then, into the otherworldly—the domain, so to speak, of the not-X?

But, after all, this is not the only way in which to see mathematics and perhaps not even the most natural way to see it. After all, why did Russell so dislike Pythagoras? Not for his absurdity, but for his persuasiveness: Russell, himself, saw his own philosophical development as leading away from Pythagoreanism.[19] Russell, by his own account, had once been a Pythagorean.

[18] Metaphysics N1093a20–b6, transl. Ross, in [1, pp. 1727–1728].
[19] See [14].

By describing his philosophical growth in such terms, Russell meant the following. In his early, Logicist period, Russell believed that mathematics was informative, that is, it discovered non-obvious truths. Later on, based on what he understood to be the argument of Wittgenstein's Tractatus, Russell came to think of mathematics as a mere system of tautologies. That is, it was no more than a system of useful shorthand expressions of the form 'X is X'. Information could be gained through empirical investigation only, and, to such an investigation, mathematics gave no more than shape.

This later view was one of the standard accounts of the relationship between mathematics and science in the 20th century, and it may inform our own surprise when encountering the relationship between mathematics and the divine. It was based on such a view that, in his *History of western Philosophy*, Russell said that Pythagoreanism had an influence on philosophy 'both profound and unfortunate'. Pythagoreanism, to Russell, was the origin of his own youthful mistake—the view of mathematics as going beyond tautology, that is, asserting that 'X is not X'.

Now whether mathematics is informative or not is a question I shall not enter here—but the very fact that the youthful Russell Russell's believed it was, is of interest. This was the position held by Russell *while still engaged in mathematics* (if, that is, this is how we should call the writing of the Principia Mathematica). To the practitioner of mathematics, this intuition—that mathematics deals with 'X is not X'—is overwhelming. Quite simply, 'X is X' in never asserted in any mathematical text. That is: there are no statements in mathematics of the form 'The squares on the sides of the right-angled triangle are equal to the squares on the sides of the right-angled triangle'. Mathematical statements are always of the form 'The squares on the sides of the right-angled triangle are equal to the square on the hypotenuse'. Mathematics asserts the identity of the different, not of the same. This is what it most essentially is: the tool for making *valid* assertions of the apparent form 'X is not X'. Whether the deep form of such assertions may turn out, upon logical analysis, to be 'X is X', is a separate question: the fact remains that, at its surface, mathematics asserts the paradoxical identity of the different. At its surface, then, mathematics is a mystery, that is, an assertion of the identity of the different—an assertion which, however (unlike standard mystery assertions) also happens to be true.

This fact about mathematics—that it is, in a real sense, a mystery—is, I suggest, of historical significance. Mathematics deals with metaphor, in a rational way. It follows that those who care about both rationality and metaphor would, naturally, appeal to it. This, perhaps, may serve to explain the historical phenomenon of Pythagoreanism.

Notice for further reading

The fundamental study of Greek religion is Burkert [3].

The fundamental study of Pythagoras is Burkert [2]. Its deflation of Pythagoras-the-scientist has been followed here and is challenged in, e.g., Zhmud [18].

There are many accounts of the Pythagorean influence on Plato, of which I took Vlastos [16] as an example. The Pythagorean influence on Aristotle is less well known: a brilliant introduction is Sorabji [18].

As mentioned in the text, Huffman [11] is an essential study of Philolaus. There is nothing as useful on Archytas, whose achievement can be pieced together from such standard sources as, e.g., Heath [10].

An account of philosophical Pythagoreanism must be based on some picture of the history of Greek mathematics. In the view of this author, the scholarship on Greek mathematics underwent a sea change in the late twentieth century, rendering much of earlier scholarship obsolete (perhaps everyone thinks this way about his or her discipline). In particular, during the twentieth century, much of the literature was influenced by a belief in a late fifth century philosophical crisis, caused by the discovery of irrationality, leading to the formation of historical Pythagoreanism (as distinct from that of Pythagoras). Very few historians of Greek mathematics still believe in this story: see Knorr [12] for one of the first studies to doubt the traditional story, and Fowler [6] for a recent summary of the problem.

An account of philosophical Pythagoreanism that brings together the lessons of Burkert [2] with those of the recent research into the history of Greek mathematics, remains to be written.

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CHAPTER 4

Mathematics and the Divine in Plato

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Contents

MATHEMATICS AND THE DIVINE: A HISTORICAL STUDY
Edited by T. Koetsier and L. Bergmans

"*God is always doing geometry*"

1. Preliminary remarks

Plato (ca. 429–347 BCE) of Athens is known primarily as a philosopher.[1] It is clear that he had an intense admiration for mathematics and that his notion of philosophical method was based on reflection on mathematical reasoning.[2] His frequent references to and discussions of mathematics in his dialogues played a major role in establishing the idea that mathematics is of central human importance, and his making five branches of study (*mathêmata*), arithmetic, plane and solid geometry, astronomy, and harmonics, central to his notion of higher education in Book VII of the *Republic* is the source for the *quadrivium*, a fundamental component of later Western education. It is also very likely that Plato played a significant role in the rapid development of Greek mathematics in the fourth century BCE.[3] On the other hand, it is very unlikely that Plato made substantive contributions to mathematics;[4] indeed, many of the more specifically mathematical passages in his works have no clear and correct interpretation, and many of them can be read as the half-understandings of an enthusiastic spectator.

In the twentieth century much work has been done in trying to determine the chronological order in which Plato wrote his dialogues.[5] In this essay I am going to concentrate on two dialogues, which are a cornerstone of a traditional interpretation of Plato's philosophy and in which he gives his most influential treatments and uses of mathematics, namely the *Republic*, usually classed as a mature dialogue, and the *Timaeus*, usually classed as a late one.[6] We will see that there are some, at least apparent, tensions in the use of mathematics in the two dialogues. However, there is a much greater tension between the ways in which the two dialogues represent the divine,[7] even though they agree in their disregard for what might be called traditional Greek religion.

Explaining this tension requires me to bring in another aspect of Plato's philosophical outlook common to the *Timaeus* and the *Republic* and thought to be a linchpin of Plato's mature philosophy: the so-called theory of forms. The scholarly literature on this topic is both enormous and contentious, and I have no intention of pursuing interpretive issues here.[8] For purposes of this paper it suffices to think of forms as eternal, intelligible objects (like real numbers or functions as opposed to perceptible objects like tables or horses) corresponding to general terms such as 'just', 'large', 'triangle', 'fire', 'horse'. We might now refer to these objects as concepts, but it is important to realize that for Plato forms are real objects, indeed more real than things like tables or horses. For Plato the forms constitute

[1] On Plato in general I mention two collections of essays, [2] and [16].

[2] See [18].

[3] Cp. [17].

[4] On this topic see [7].

[5] See, for example, [4].

[6] There was much discussion of the place of *Timaeus* after the attempt by Owen [21] to group the *Timaeus* with the *Republic* as a mature work. But it seems that scholarly consensus now backs the response to Owen by Cherniss [8].

[7] On the concept of divinity in Plato see [25] and [11].

[8] Many of the articles in [2] discuss the forms.

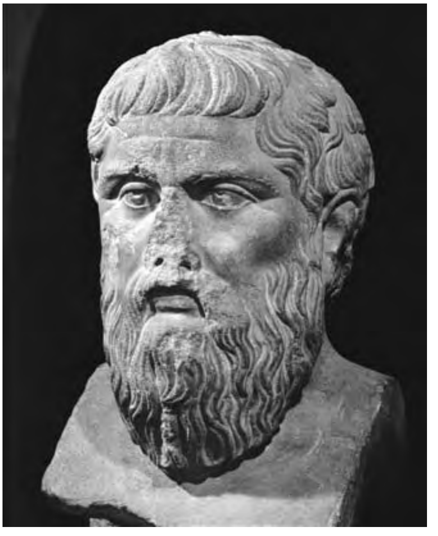

Fig. 1. Plato. Marble bust, Roman copy of an original dated 4th BCE. Height 43 cm. Louvre, Paris. Photo: Erich Lessing.

some kind of system, of which it is the task of the philosopher to gain an understanding. On the whole the knowledge of forms is conceptual knowledge. Philosophers who know the form corresponding to the term 'just' will have the answer to the question 'What is justice' and will be able to assess the justice of particular actions, programs, etc., things which are said to be just by participating or sharing in the form of justice.

In the *Timaeus* Plato describes the creation of our world or cosmos by a benevolent creator god (standardly referred to as a craftsman (*dêmiourgos*, frequently rendered as demiurge) or maker, but also called mind or reason (*nous*)), who tries to make the world as good as possible by making it resemble the forms as much as possible. This conception of a god as an intelligence making the world is found in other dialogues considered to be late, explicitly in the *Philebus* and with some complications in the *Laws*.[9] But it is not found in any explicit way[10] in the *Republic* or in other earlier dialogues. In the *Republic* Plato makes the distinction between forms and perceptible things the basis of a distinction between knowledge and mere belief or opinion. He makes knowledge of the forms the basis of the philosopher's superiority to other human beings, and he makes the form of the good, which he treats as the unhypothetical first principle of all things, the pinnacle of the system of the forms. The forms themselves are treated as objects of rational desire, understanding of which provides the fullest satisfaction. Plato is willing to speak of people with knowledge of the forms as divine, and it is quite clear that he thinks of the forms as divine as well. In this sense the "gods" of the *Republic* are the forms and the highest "god" is the form of the good. In the *Timaeus* mathematics is related to the divine because the

[9] See [15] or [11, pp. 186–191].

[10] At *Republic* 530a there is a reference to the demiurge of heaven, but that it presumably because a point is being made about heaven by reference to the products of craftsmen.

demiurge uses mathematics in fashioning the world; in the *Republic* mathematics is related to the divine because knowledge of it is an important step on the pathway to knowing forms.

Many attempts have been made to reconcile these apparently discrepant conceptions of divinity in the *Timaeus* and the *Republic*. The developmentalist alternative of saying that Plato came to believe in a creating god or mind in his later years is always available. The other most influential position, which has an ancient pedigree, involves the claim that the *Timaeus* account of creation is a narrative representation of eternal truths about an eternal world which reveals its composition, so that the demiurge can be treated as a fictional device. I confess that I am unhappy with either interpretation but unable to provide a satisfactory alternative. This unfortunate situation is, I think, ameliorated by the fact that the representations of the relationship of mathematics to the divine are quite parallel in the two dialogues. I shall then focus on that relationship and say little more about the intrinsic character of the divine in the two dialogues.

Before turning to the body of my paper I want to mention one set of texts other than Plato's dialogues which are sometimes brought to bear in discussions of the role of mathematics in his philosophy. Aristotle has many remarks, almost all critical, of positions taken by Plato and his associates. In a disconcerting number of cases the positions have no clear relation to anything we find in Plato's dialogues. Other later writers also mention these positions and discuss them in more detail than Aristotle does, although we usually do not know what their sources of information are. Many scholars react to this situation by referring to Plato's unwritten doctrines, and many of them think of the doctrines as ideas developed, probably very schematically, by Plato late in his life. There is, however, a group of philosophers who think of the unwritten doctrines or, perhaps now better, doctrine as a relatively worked out theory which in some sense is the core of Platonic philosophy and underlies in one way or another what is said in the dialogues. Since mathematical ideas play an important role in this interpretation, I shall make occasional references to the notion of unwritten doctrines, but I shall not go into this topic in any detail.[11]

2. Introduction

In one of the dialogues included in Plutarch's "Table-Talk"[12] the topic of conversation is what Plato meant when he made the statement I have used as an epigraph. Plutarch admits that there is no clear evidence that Plato ever did say this, but says that it is believable that he made the statement, which is in conformity with Plato's nature. The speakers offer four accounts of Plato's meaning which are more like accounts of the importance Plato attaches to mathematics. The first clearly reflects the role assigned to mathematics in the *Republic*:

1. Geometry turns us away from perception and towards the intelligible, eternal nature, contemplation of which is the goal of philosophy.

[11] On the unwritten doctrines see [12] and [13], and the collection of essays in the Argentinian journal *Methexis* 6 (1993). Ref. [22] gives a useful presentation of Aristotle's problematic descriptions of Plato's views.

[12] VIII.2 *Pôs Platôn elege ton theon aei geômetrein?*

The second, which is of less importance, has a relation to the *Republic*, but it picks up specifically on things said by Plato in the *Laws* and the *Gorgias*:

2. It is better to distribute goods on the basis of geometric than arithmetic proportion, since with the latter goods are distributed equally, whereas with the former they are distributed according to merit;[13]

The third and fourth relate to Plato's *Timaeus*, the dialogue I shall discuss first:

3. In creating the world god imposed limit on an unlimited matter, using geometrical shapes;
4. In the *Timaeus* Plato distinguished three principles which we call god, matter, and form; in making and maintaining the cosmos, god imposes form on the entire quantity of matter, and this is like solving that central geometric problem (Euclid, *Elements* VI, 25) of constructing a geometric figure equal to a given one and similar to another.

3. The *Timaeus*[14]

I have already mentioned that the *Timaeus*, which is named after the character who gives the discourse constituting most of the dialogue, is a description of the creation of the world by a benevolent god who strives to make the world as like the forms as possible. The most important form for god's creative activity is the form of living thing (*zôion*), a form including the forms of all living things and in imitation of which god creates the cosmos, "a single, visible living thing, containing within itself all living things ..." (30d–31a). The conception of the cosmos as in some way alive dominates ancient thinking about our world and separates it from modern physics, which purports to give a universal description of Plato's sensible world without making any specific reference to the fact of life, the domain of biology, or to intelligence, the domain of psychology. For Plato the cosmos is alive, and (by definition) that means it has a soul; for Plato, it also has a mind and intelligence (*nous*). It is difficult to spell out with any precision the exact consequences of this difference between the Platonic conception of the world as alive and the tendency of modern physics to abstract from the fact of life, but I think the most important related fact, if not consequence, is that Plato does not use mathematics for the formulation of laws on the basis of which experiments can be performed, measurements made, and outcomes predicted. For Plato the mathematical character of the world is most importantly a sign of its intelligent organization (and perhaps of the intelligence of its organizer) and—more or less indistinguishably—of its goodness and beauty.

[13]The conception involved here is difficult to formulate rigorously. But the idea is that in a geometric proportion v_1 is to m_1 as v_2 is to m_2, so that if v_1 and v_2 represent the values of goods g_1 and g_2 and m_1 and m_2 the merits of two people p_1 and p_2, then it will be just to assign g_1 to p_1 and g_2 to p_2; on the other hand if, g_1, g, and g_2 are in arithmetic proportion, then g is simply half of g_1 and g_2, and assigning goods on the basis of arithmetic proportion to p_1 and p_2 will allegedly be unjust because the two unequal people are given equal amounts.

[14]The standard translation and commentary for the *Timaeus* is [10]. I have used this translation with some revisions. Another very useful translation and commentary is [5].

3.1. *Why there are four "elements", earth, water, air, and fire*[15] *(Timaeus 31b–32b)*

Timaeus' first specific invocation of mathematics comes in his first description of the generation of the body of the world. I shall go into some detail on this passage because it is illustrative of significant difficulties in interpreting the way Plato uses mathematics. In outline Timaeus proceeds as follows:

(a) What has a body must be visible and tangible.
(b) Nothing can be visible without fire.
(c) Nothing can be tangible without something solid (*stereos*).
(d) Nothing is solid without earth.
(e_1) "But two things cannot be well united without a third; for there must be some bond between them drawing them together.
(e_2) And of all bonds the best is that which makes itself and the terms it connects one in the fullest sense; and it is of the nature of a proportion to effect this most perfectly.
(e_3) For whenever of three numbers $\langle a, b, c\rangle$ the middle one between any two that are either bulks (*ongkoi*) or powers (*dunameis*)[16] is such that "a is to b as b is to c and c is to b as b is to a, "then, since the middle becomes first and last and again the last and first become middle $\langle$i.e., b is to a as c is to $b\rangle$, in that way all will necessarily turn out to be the same thing and therefore they will all be one."
(f) If the body of the universe were a plane without depth, one mean would be sufficient, but it is a solid and solids are fit together by two, not one, mean;
(g) Therefore god made water and air means between fire and earth, and contrived it so that fire is to air as air is to water as water is to earth.

It is clear enough that some of the premisses, e.g., (b), (d), (e_1), are questionable and apparently *ad hoc*. But even if we grant all of the premisses which we understand, there remain premisses and a conclusion the meaning of which is not transparent. In what sense are 4, 6, and 9 one because 4 is to 6 as 6 is to 9? Timaeus' invocation of the possible positions of terms in a three-term proportion hardly seems an adequate answer, but the alleged unifying nature of proportion is clearly an important part of Plato's mathematical vision of the cosmos. And the way he invokes proportion suggests that he does not understand proportionality as mere proportionality in the way in which we might read an algebraic equation as merely an equation. Rather he reads a proportionality among entities as a kind of "force" for bringing things together, not simply as a way of describing relations among things.

A perhaps more important point concerns the whole structure of the reasoning. From our perspective its subject is physical reality, earth, water, air, and fire, out of which the universe is made. These are to be blended into a unified whole, but why should we think the simple proportionality of (g) will produce that unity? Moreover, what is the proportionality of (g) a proportionality of? The volumes of the "elements"? Their weights? Again Timaeus does not say, and it seems unlikely that Plato had specific quantities in mind. For him the picture

[15] I retain the standard appellation 'elements' for earth, water, air, and fire, but I put 'elements' in quotation marks because, as we shall see at the beginning of Section 3.3, Timaeus specifically denies that these 'elements' really are elements (48b–c).

[16] These words are not standard arithmetical or, more generally, mathematical terms, and their purpose is not understood. Dropping them from the assertion would not affect the truth—or falsity—of the claim that proportionality produces unity.

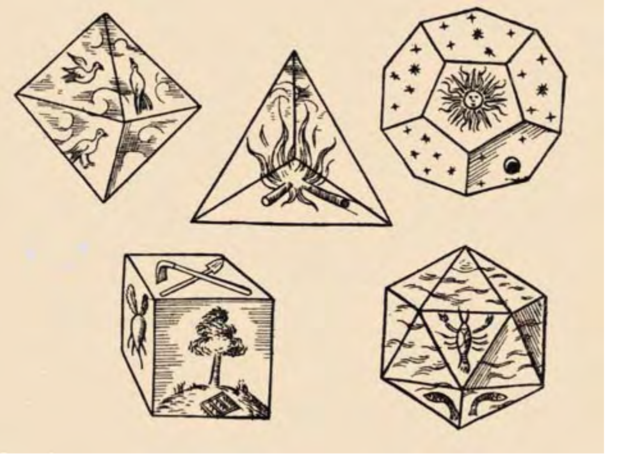

Fig. 2. The five (regular) Platonic solids as depicted in Johannes Kepler's *Harmonices Mundi*, 1619.

of there being something mathematical and relatively simple is an adequate support for the idea that the god's cosmos is good.

Finally there is the question of the apparently mathematical assertion (f). It seems unlikely that Plato was here thinking of some geometric truth.[17] He may have had in mind some version of what we know as propositions 18 to 21 of book VIII of Euclid's *Elements*:

- m and n are similar plane numbers if and only if there is one mean proportional number between m and n;
- m and n are similar solid numbers if and only if there are two mean proportional numbers between m and n.[18]

There is obviously a parallelism between these arithmetic truths and Timaeus' claims about the four "elements", but it is difficult to see any more specific relation between them. Plato moves from an abstract mathematical proposition about the existence of means to a claim about the make-up of the physical world without explaining how he passes from one to the other. He does not purport to look at the physical world and discover a mathematical

[17] If, for example, (f) presupposes that if there are two solids of volumes v_1 and v_3 there is no solid of volume v_2 where v_1 is to v_2 as v_2 is to v_3, then (f) presupposes something which is false from our perspective. We can refine the issue by introducing some notion of constructing a solid, and we can restrict the possible solids to convex ones contained by rectilinear planes, but it is not true either that a solid which is a mean between two such solids cannot sometimes be constructed or that between two such solids two mean solids always can be constructed. Of course, there are many other ways of trying to extract some geometric truth from (f), but I do not know of any satisfactory one.

[18] m and n are similar plane numbers if and only if, for some m_1, m_2, n_1, n_2, $m = m_1 \cdot m_2$ and $n = n_1 \cdot n_2$, and m_1 is to m_2 as n_1 is to n_2 (or equivalently, m_1 is to n_1 as m_2 is to n_2). Consequently, if m and n are similar plane numbers, $m_1 \cdot m_2$ is to $n_1 \cdot m_2$ as $n_1 \cdot m_2$ is to $n_1 \cdot n_2$, and $n_1 \cdot m_2$ is a mean proportional between m and n. On the other hand, suppose m is to l as l is to n; let m' and l' be the least numbers such that m' is to l' as m is to l. Then, for some j and j', $m = j \cdot m'$, $l = j \cdot l' = j' \cdot m'$, and $n = j' \cdot l'$, and j is to j' as $j \cdot m'$ is to $j' \cdot m'$ as $j' \cdot m'$ is to $j' \cdot l'$ as m' is to l', and m and n are similar plane numbers.
The treatment of similar solid numbers is analogous. However, it should be noted that the same numbers, e.g., 2^6 and 3^6, can be similar plane and similar solid numbers.

relationship among its elements. Rather he takes an alleged truth about proportionality and the fact that he is dealing with three-dimensional visible entities and infers (or at least makes a claim about) what the relationship of the “elements” of the cosmos is. Of course, I am not claiming that Plato actually came to the conclusion that earth, water, air, and fire are basic components of the world as a result of this kind of thinking. He undoubtedly took the idea over from other physicists, notably Empedocles. But he chooses to present the conclusion as a development of a mathematical proposition, and that kind of presentation proved to be extremely influential in later philosophy.

3.2. *The construction of the soul of the world (Timaeus 35a–36d)*

Plato’s presentation of the construction of the soul of the cosmos has the same *a priori* character. Timaeus imagines the demiurge producing a blend[19] of certain forms and then a division of this into portions. The ratios of the proportions, in fact, correspond to the standard diatonic scale of Greek mathematical harmonics, but Timaeus never mentions this fact; he simply gives an opaque series of arithmetic relations in terms of which the demiurge makes the division.[20] One might think that Plato was playing with mathematics and with the reader.

Once the division has been made, it drops from sight and the soul stuff is treated as a unified whole. This is divided into two equal strips which, in turn, are made into concentric circles set at an angle to one another and rotating in opposite directions, the “inner” circle itself being divided into seven unequal circles. Timaeus gives enough of a description of the motions of these circles to make clear that what underlies his account is a rough, geocentric, astronomical model in which the “outer” circle represents the sphere of the fixed stars, the inner circles sun, moon, and the five planets known at that time. But that this is so is only made clear some two pages later when Timaeus’ mentions some of the heavenly bodies explicitly.[21] So again mathematics comes into the *Timaeus* in the service of a kind of mathematical physics, in this case astronomy. But the mathematics is not presented as rising out of the physics, but is rather introduced in an apparently abstract way, the motivation of which is completely obscured.

3.3. *The geometry of the “elements” (Timaeus 48b–57c)*

Some ten pages after the discussion of the construction of the world soul Timaeus announces that, whereas he has been mainly discussing the creative activity of mind, he now has to bring in what he calls necessity and make in effect a new beginning:

[19] On a literal reading the whole description treats the soul as if it were a compound of extended magnitudes, a fact which Aristotle criticized as mistaken in the case of soul (*On the Soul*, I.3.406b26–407b11) because it is not an extended magnitude. Later Platonists insisted that Aristotle was reading a “mythological” passage too literally.

[20] For a more detailed discussion of this passage see the appendix.

[21] When the astronomical significance of the final arrangement of the soul stuff is put together with the musical ideas underlying its first division into portions, it is difficult to avoid the conclusion that some doctrine of heavenly music (the “harmony of the spheres”) lies behind what Timaeus is made to say. But it is not to be found explicitly in what Timaeus does say. (On the harmony of the spheres see [6, pp. 350–357].)

> We must, in fact, consider in itself the nature and properties of fire, water, air, and earth before heaven came to be. For to this day no one has explained their genesis, but we speak as if men knew what fire and each of the others is, positing them as elements or letters[22] of the universe. But one who has ever so little intelligence should not rank them in this analogy even so low as syllables. On this occasion, however, our contribution is to be limited as follows. We are not to speak of the first principle or principles—or whatever name men choose to employ—of all things, if only on account of the difficulty of explaining what we think by our present method of exposition. (48b–c)

After a few more words Timaeus launches into his new beginning and brings us into the third interpretation of the epigraph to this paper.

Timaeus begins by adding to the distinction between the eternal world of forms and its perceptible copy a third item which he calls (among other things) the receptacle and which later Platonists usually called matter. He compares the receptacle to a mass of soft gold which can be molded into a variety of shapes, but in itself has no particular shape; so too the receptacle receives the likeness of the eternal forms although in itself it has no particular character. "It is invisible and without form; it receives everything; and it participates in what is intelligible in a way which is puzzling and difficult to grasp" (51a–b). Timaeus goes on to describe a pre-cosmic situation in which the receptacle shakes and is shaken by rudimentary versions of the four "elements" in a wild way.

> Fire, water, earth, and air possessed indeed some vestiges of their own nature, but were altogether in such a condition as we should expect for anything when deity is absent from it. Such being their nature at the time when the ordering of the universe was taken in hand, the god began by shaping them by means of ⟨geometric⟩ forms[23] and numbers. That the god shaped them with the greatest possible perfection, which they had not before, must be taken, above all, as a principle we constantly assert; what I must now attempt to explain to you is the arrangement and genesis of each of them. The account will be unfamiliar, but you are schooled in those branches of learning ⟨i.e., mathematics⟩ which my explanations require, and so will follow me. (53b–c)

Timaeus now makes a quick series of mathematical moves when he asserts:

(a) The four "elements" are bodies, and bodies are three-dimensional.
(b) What is three-dimensional is contained by planes.[24]
(c) Rectilinear plane surfaces are divisible into triangles.
(d) All triangles are divisible into right triangles, whether isosceles or scalene.

Timaeus says he will hypothesize the right-angled triangles as the *archê* of the simple bodies, but, as he had done before at 48b–c, he announces that he is not going to talk about ultimate principles, ones known to god and anyone who might qualify as god's friend. Shortly after this Timaeus points out that, whereas all isosceles right triangles have the same "nature", presumably because they are all similar, there are infinitely many natures for the scalene right triangles of which he proposes to choose the most beautiful (*kallistos*), the half-equilateral. After saying that it would take too long to explain this choice, he opts for the triangle with sides in the ratio of 1, 2, and $\sqrt{3}$. I shall call these two kinds of triangles the rudimentary triangles.

[22] The Greek word for a letter of the alphabet is *stoikheion*, a word which came to be applied to elements or fundamental building blocks of the world.

[23] Timaeus speaks only of forms (*eidê*), which could mean Platonic forms, but the likelihood is that here he means geometric forms or shapes. In any case, so I shall assume. Cp. [5, p. 252, n. 386].

[24] This is obviously not true of all bodies, but only of those in which Timaeus is interested.

Just before selecting these two triangles as principles Timaeus says that his task is to find the four most perfect bodies "such that some can come to be from one another by dissolution" (53e2), a demand whose meaning becomes clear only when the task has been done, and the four simple bodies have been assigned to four of the five regular solids which Euclid constructs in the last book of the *Elements*:[25]

- the cube contained by 6 squares, assigned to earth;
- the triangular pyramid contained by 4 equilateral triangles, assigned to fire;
- the octahedron contained by 8 equilateral triangles, assigned to air;
- the icosahedron contained by 20 equilateral triangles, assigned to water;

Timaeus' job is to somehow reduce these four solids to his principles, the two rudimentary triangles. He does this by dividing the faces of the regular solids into rudimentary triangles and then indicating how the faces can be put together to form the solids.[26] The construction of the solids out of their faces is the basis of Timaeus' account of the way in which "elements" change into one another, which is a matter of these invisibly small solids breaking down into their faces and these faces recombining into different solids. This means that although water, air, and fire can be transformed into one another, earth is never involved in elemental change. When it is broken into its faces the faces can only reassemble into another cube (56d).[27]

There are a number of things to be said about this whole remarkable construction. Let us begin with an issue which Timaeus does not address: the connection between this treatment of the "elements" and the one discussed in Section 3.1, in which we were told that we had to deal with visible, tangible solid bodies and ended up with an unexplained "proportion":

- Fire is to air as air is to water as water is to earth.

There is no obvious way of correlating this proportion with a true one expressed in terms of triangular pyramid, octahedron, icosahedron, cube. And, although Timaeus does allude to the first proportionality (53e), his failure to make a connection suggests that Plato does not think there is one. Presumably, too, the first proportionality is not intended to apply to the pre-cosmic situation in which only traces of the "elements" exist and are shaken about in the receptacle.

Another issue of connection which is not addressed in detail by Timaeus brings us to a possible difference between the third and fourth interpretations of the epigraph. The third interpretation relates clearly and specifically to the passage I have been discussing. The fourth seems to be an artificial reading of the epigraph which draws an analogy between a particular geometric construction and god's imposition of form on matter, perhaps in order to avoid the specifically geometric interpretation of god's activity, which is now seen as the imposition of form on matter, say, in making a portion of that matter come to have the features of fire, i.e., to resemble the form of fire. It is possible to imagine

[25] Euclid also proves that these are the only five regular solids (XIII.17). In this essay I do not discuss Timaeus' curious remark at 55c about the dodecahedron or the curious discussion of the number of worlds which follows it.

[26] The division of the faces is curious because, Timaeus breaks the square into four isosceles right triangles and the equilateral triangle into six half-equilaterals, although he could have gotten away with two rudimentary triangles in each case. If there is some significance in Timaeus' choice, Plato has not chosen to tell us what it is. For discussion of this question see [19].

[27] Aristotle (*On the Heaven* III.7.306a1–17) ridicules Plato for adopting this position, which he sees as a case of promoting theory over what is observed.

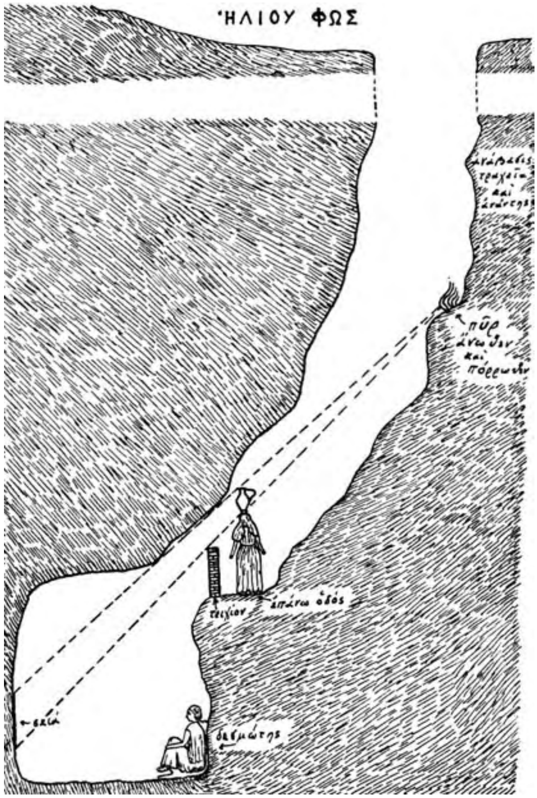

Fig. 3. Plato's cave.[28]

that god imposes the form of fire on matter without going through any geometrical steps. But Plato clearly thinks that God imposes such forms by mathematical means and that these mathematical means "explain" the characteristics of, say, fire. However, Timaeus' explanations of the correlations between the solids and the characteristics are, for us, mere analogies. He begins with earth:

> Let us next distribute the figures whose formation we have described among fire, earth, water, and air. To earth let us assign the cubical figure; for earth is the most immobile of the four kinds and the most moldable of bodies. The figure whose bases are the most stable must best answer that description; and as a base, if we take the ⟨rudimentary⟩ triangles we assumed at the outset, the face of the triangle with equal sides is by nature more stable than that of the triangle whose sides are unequal; and further of the two equilateral surfaces respectively composed of the two triangles, the square is necessarily a more stable base than the triangle, both in its parts and as a whole. Accordingly we shall preserve the plausibility of our account, if we assign this figure to earth. (55d–56a)

The remaining assignments are justified by assuming a correlation between a lower number of faces and greater mobility, sharpness, and lightness. It seems clear that Plato is much

[28]The drawing was made by Dr. B.Th. Koppers. Source: C.J. de Vogel, *Greek Philosophy, A Collection of Texts, Vol. 1, Thales to Plato,* 3rd edition, E.J. Brill, Leiden, 1963, p. 204.

more interested in indicating that there is a mathematical foundation for physics than in giving an account of physical mechanisms by which the foundation is realized.

The question of the relationship between mathematics and physics is made more difficult by the seemingly effortless way with which Timaeus moves between the purely mathematical and the physical. When he begins to describe the order imposed on the disordered receptacle by the demiurge, the four "elements" already exist, and apparently we know they are "bodies". The reduction of bodies to the rudimentary triangles appears to move us from the domain of physics to the domain of pure geometry, but Plato gives no indication of where he thinks the transition occurs. Bodies, he says, must have depth. This is, of course, true of physical bodies, but it is also true of geometrical solids. But as he proceeds to talk about what has depth being bounded by planes and planes being composed of triangles, and so on, he appears to be talking at the geometrical level rather than the physical one. But after he has reverted to the physical level by correlating the solids with the four "elements" and describing their transformations and non-transformations into one another, we are left with an apparently bizarre picture of two-dimensional surfaces floating in space and, where appropriate, combining to form a new solid. It is difficult to suppose that Plato wished to assign a physical reality to these triangles and squares, but equally difficult to see how he could treat them as purely mathematical objects and assign them physical effects.

The difficulties here do not appear to be difficulties for Plato. For him the physical world gains its intelligibility, and therefore, in some sense, its reality, from an ideal world, and, at least in the *Timaeus*, that ideal world is importantly mathematical. But, as I mentioned earlier, the mathematical intelligibility of the physical world is not a matter of what we would call mathematical laws, but rather of a mathematical structure, a system of ratios, a system of interacting spheres or circles, the system of regular solids, being present in the physical world.

3.4. *Mathematics and human happiness*

In the *Timaeus* these mathematical structures are said to be present in the world because a god put them there. And, at least in the case of the heavenly revolutions, our apprehension of them is a way of perfecting ourselves:

> The god invented and gave us vision in order that we might observe the circuits of mind in the heaven and profit by them for the revolution of our own thought, which are of the same kind as them, though ours be troubled and they are unperturbed; and that by learning to know them and acquiring the power to compute them rightly according to nature, we might reproduce the perfectly unerring revolutions of the god and reduce to settled order the wandering motions in ourselves. (47b–c)

This, roughly speaking, moral significance of mathematics is related to the second interpretation of the epigraph, in which geometric proportion is espoused over arithmetic proportion as a basis for social justice. The thought underlying the second interpretation is expressed clearly in Plato's *Laws* at VI.756e–758a, although in terms of two kinds of equality. But the moral significance of mathematics comes out much more clearly in a related passage in the *Gorgias*, where Socrates is in heated exchange with the opportunistic, self-seeking Callicles. Socrates insists that a person should control his appetites:

> He should not allow his appetites to be undisciplined or undertake to fill them up. Such a man, Callicles, could not be loved by another nor by a god, for he cannot participate in a community and without community there is no love. Wise men say, my friend, that community and love and order and moderation and justice hold together heaven and earth, gods and men, and that is why they call this universe a cosmos not a chaos or licentious revel. It seems to me that you do not put your mind to these things, even though you are versed in them; rather, failing to observe the great power of geometric equality among gods and among men, you think you should seek for more and more, because you neglect geometry. (507e–508a)

Putting this passage together with the one just quoted from the *Timaeus*, one can see that Plato brings together mathematical knowledge, morality, and a kind of deification in a way which is hard for a person who is accustomed to the modern compartmentalization of science, ethics, and religiosity to articulate. However we will see the same kind of assimilation in the *Republic*, where, as I have indicated, there is no analog of the god of the *Timaeus*.

3.5. *Higher principles (Timaeus 48b–c and 53d)*

Before turning to the *Republic* I want to signal the two passages in which Timaeus denies that in the second discussion of the “elements” he will present or has presented what we might call absolutely first principles, since these passages have played a role in the discussion of Plato’s unwritten doctrines. Timaeus has “reduced” the “elements” to geometric solids and the geometric solids to the rudimentary triangles, but he declines to proceed any further. For those who believe in the unwritten doctrines such further reductions would be included in them. Here I wish to mention only that the most obvious further reduction would be of the triangles to straight lines. Further reductions beyond that, e.g., of straight lines to points, would seem less straightforward although there are indications that Plato thought about some such thing. In addition there are indications that Plato thought about a reduction of geometry to arithmetic and a further reduction of mathematics to something less mathematical or even non-mathematical, namely the forms. All of these ideas are very nebulous and controversial, but I shall have a little more to say about them after discussing the *Republic*.

4. The *Republic*[29]

The central topic of the *Republic* is *dikaiosunê*, a term which applies to both social and political justice and to individual moral rectitude; it is standardly translated ‘justice’. However, the work is justifiably often thought of as the fullest representation we have of a

[29] For the *Republic* I have relied on the revised Grube translation [14]. The best annotated edition of the *Republic* remains [1]. Good commentaries include [28] and [3]. The classic essay on much of the material I discuss here is [9].

Platonic philosophy.[30] My concern here is not with Plato's account of justice, but with his depiction of (i) the ideally just person in a ideally just society, the philosopher who will be the ruler of this society, and (ii) his education. For Plato, the distinguishing feature of the philosopher–ruler is his knowledge of the forms and in particular of the form of the good, a knowledge which is displayed dialectically, that is, through successful defense of an account of goodness in the face of sharp and repeated questioning and successful refutation of alternative accounts through the same kind of questioning. The important passages bearing on mathematics occur in books VI and VII of the *Republic* in which the central speaker of the dialogue, Socrates, offers an indirect indication of what the form of the good is and a description of the special education of the philosopher–ruler.

4.1. *The education of the philosopher–ruler*

In books II and III of the *Republic* Socrates describes an elementary education focusing, as Greek education did, on music and literature and physical training. This education is aimed at the development of the moral and physical capacities of the students. It appears that these young people will also be exposed to mathematics by involving them in types of play which develop mathematical facility (VII.536d–e; the idea is developed more fully at VII.819b–d of the *Laws*). At the age of 20 the morally, physically, and intellectually best students will be separated out and begin a higher education with no precedent in fifth-century Greece. For ten years these people are to study mathematics in a serious way, bringing its branches together "into a unified view of their connection with one another and with the nature of reality" (VII.537c). After that the best of these students will spend five years in dialectical questioning and then a select few of them will spend 15 years in governmental and military affairs. Finally the best of these will be brought to an apprehension of the form of the good and spend the remainder of their life alternating between philosophical study and ruling.

Clearly Plato's educational scheme has a moral, a physical, a mathematical, a dialectical, and a practical component. Plato has nothing good to say for people who are highly developed in one of these components at the expense of the others, say, a brilliant mathematician who can't ride a horse or a skilled dialectician with no moral scruples. He sees these components as inextricably blended together in the education of his "divine" (VI.497c) philosopher–rulers. For this reason it is in an important sense misleading to isolate the mathematical component, but this must be done here, and doing it has the merit of bringing to the fore one of Plato's main contributions to western educational theory: the idea that a fully developed human being must be scientifically literate.

The mathematical curriculum

I mentioned at the very beginning of this essay that the five components of mathematical education for Plato are arithmetic, plane and solid geometry, astronomy, and harmonics. In his description of these sciences he does not give a further explanation of how one develops a "unified view of their connection with one another and with the nature of reality", although he does mention the need to do this at 531c–d. His emphasis is almost entirely on

[30] See, for example, [27, p. 115].

what is emphasized in the first interpretation of the epigraph, the way in which mathematics leads us away from the perceptible world to an intelligible reality. Arithmetic is praised because it requires us to deal with units or ones which, unlike single cows or horses, are absolutely indivisible.[31] Socrates wonders what arithmeticians would say if they were asked "What kind of numbers are you talking about, in which the one is as you assume it to be, each equal to every other, without the least difference and without parts", and imagines their answer:

> I think they would answer that they are talking about those which can only be thought about and can't be dealt with in any other way. (VII.526a)

Plane geometry is treated similarly, although Socrates regrets that geometers have to speak as if they were doing things such as constructing squares or moving a figure to a certain position, since the subject matter of geometry is really unchanging reality, not things that come into existence and go out of existence. Socrates does not say anything specific about solid geometry, but only urges that the study of it be promoted.

So, we might say that the first three sciences in Socrates' curriculum are there because they turn the mind away from human things toward divine ones. One might, of course, adopt a position of Aristotle and reject the claim that there are separately existing realities studied by mathematics.[32] But even if one adopts the Platonic view, it is difficult to see how it can be applied to the last two sciences of the curriculum, astronomy and harmonics, which appear to be about things we see and hear. We have seen in Section 3.4 that in the *Timaeus* Plato praises the study of the heavenly motions because by understanding them we see that they are more regular than they appear, and that understanding somehow affects our own emotional psychology. But Socrates goes much further in the *Republic*. The heavens are indeed beautiful, but their observed motions fall far short of their true ones, which must be grasped intellectually not by sight. The true ones are permanent, whereas the observed ones change and do not embody the truth about the "ratio of night to day, of days to a month, of a month to a year, or of the motions of the stars to these things or to each other". Socrates concludes:

> Then if, by really taking part in astronomy, we're to make the naturally intelligent part of the soul useful instead of useless, let us study astronomy by means of problems, as we do geometry, and leave the things in heaven alone. (VII.530b–c)

There are many obscurities in Socrates' discussion of astronomy, but it is clear that Socrates is envisaging a science which is quite different from anything we think of as astronomy, even mathematical astronomy, in which the observed heavenly motions are to some extent idealized to make them more susceptible to geometric representation. I will argue that the astronomical passages of the *Timaeus* are an indication of the sort of thing Plato had in mind for astronomy, but first I want to complete my account of Socrates' mathematical curriculum by mentioning his treatment of harmonics, which is quite parallel to his treatment of astronomy although formulated only negatively. In it Socrates complains about people who "look for numbers in heard concords and do not ascend to problems, investigating

[31] A somewhat different distinction is made at *Philebus* 56d–e between units of different sizes and units which do not differ from one another in any way.

[32] See, for example, Aristotle, *Metaphysics* M.2.1076b11–1077b14.

which numbers are concordant and which aren't and why". (531c) When his interlocutor says that such an investigation is superhuman (*daimonios*), Socrates responds that it is of use in the search for the good and beautiful.

In my description of Timaeus' construction of the world soul in Section 3.1 I pointed out that he partitioned the stuff of the world soul in accordance with the standard diatonic scale without ever mentioning the musical content of his description. And he arranged the stuff into moving circles constituting a rough model of the heavenly motions, but only pointed out that they are a model some two pages after doing the construction. In the *Republic* Plato is concerned with the task of getting the soul aware of a higher level of reality to which the soul is connected. Arithmetic and geometry provide him with examples of sciences which, he thinks, clearly deal with this level. Two other mathematical sciences, astronomy and harmonics, do not do so clearly, but Plato imagines that they could. On the other hand, in the *Timaeus* Plato is concerned to show how our world is derived from the higher level. He does this not only in the two cases I have just mentioned, but also in the explanation of the existence of the four "elements" based on a presumably arithmetic fact about proportion, and of their character on the basis of solid geometry. Timaeus' derivations are not anything like formal deductions, but they are based on the idea that mathematical science is a standard of rationality and that our world, being the best embodiment of a rational organization, is mathematically organized.

4.2. *The divided line (Republic VI.509d–511e)*

The sharp contrast between the perceptible and the intelligible world, which is emphasized in the *Republic* and many other dialogues, is usually treated as a central point of Platonic doctrine. Such a position creates difficulties for the interpretation of the *Timaeus*, which is devoted to an account of the perceptible world. I see this discrepancy between the two dialogues as a matter of emphasis rather than of doctrine, a position which I would like to buttress by looking at one further passage, part of Socrates' attempt to indicate in a rough way the nature of the form of the good.

The first part of that attempt is a comparison of the relationship of the sun to the perceptible world to that of the form of the good to the intelligible world. The second part, usually referred to as The Divided Line, compares the distinction between the intelligible and the sensible world to the division of a straight line AB into two unequal sections at C. However, Socrates then imagines a further division of AC at A_1 and CB at B_1 so that:

$$AC : CB :: AA_1 : A_1C :: CB_1 : B_1B.$$

The difference between CB_1 and B_1B is explained in terms of the distinction between perceptible things, animals, plants, artifacts (CB_1), and their images, shadows, reflections, etc. (B_1B). But the difference between AA_1 and A_1C is made in terms of a distinction between two intellectual methods. A_1C is explained by reference to geometers, arithmeticians, and the like, who hypothesize various things as known and give no account of them and then make inferences based on them; these people are also said to make use of visible

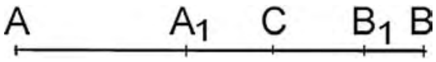

Fig. 4.

things in their reasoning even though they are not thinking about these things but about the things of which they are likenesses. The best model we have for this is probably elementary geometry done in the style of Euclid, which starts from definitions, postulates, and axioms, and reasons in terms of diagrams. AA_1 is associated with dialectic, which is said to use its hypotheses as underpinnings for rising to an "unhypothetical principle of everything" (certainly the form of the good) and then to move down to a conclusion, not using sensible things, but only dealing with forms.

In the context of the material we have been discussing it seems natural to think that A_1C is exhausted by mathematics, that is, presumably, the five subjects Socrates puts into his mathematical curriculum. At *Metaphysics* A.5.987b14–18 Aristotle asserts that Plato believed in mathematical objects which were like the forms and unlike perceptibles in being eternal and like sensibles and unlike forms in coming in multiple instances. Such a distinction might make some sense for arithmetic and geometry, but, as Aristotle himself points out (*Metaphysics* M.2.1076b39–1077a9 and B.2.997b12–24), it makes little sense for astronomy and harmonics. However, in the *Republic* nothing is said about a distinction between the objects in AC and A_1C, and, indeed, later (VII.534a) Socrates explicitly declines to enter into this issue. A striking feature of the way Socrates divides the line, but one which is not mentioned in the *Republic* or elsewhere in antiquity, is that it makes A_1C and CB_1 equal. I am not entirely convinced that Plato was aware of this fact, but the equality and Socrates' refusal to discuss ontological issues here might well be a way of expressing the nebulousness of the domain of mathematics, its transitional role between the sensible and the intelligible, both in the upward direction in the *Republic* and in the downward direction in the *Timaeus*.

5. Conclusion

To many modern readers Socrates' presentation of the divided line suggests the idea of a deductive system which in some mysterious way starts from the form of the good as an unhypothetical first principle and deduces an exhaustive description of the forms and the body of mathematical knowledge. My own sense is that the "downward movement" referred to in the connection with AA_1 and A_1C includes the kinds of applications of mathematics we see in the *Timaeus* and other kinds of reasoning which we would not class as deductive. Plato, of course, never provides anything appreciably more detailed than the kind of schematic presentation I have outlined here. For proponents of the unwritten doctrines, the absence of anything more detailed is only to be expected. The dispute over the unwritten doctrines is importantly a dispute over their significance for Plato. For me that question remains open, although I am inclined to think that some people have exaggerated their significance by making them the key to understanding all of Plato. Putting that issue aside, and assuming, what has also been disputed, that Plato did have the idea of some broad doctrine of the kind I have tried to sketch, there would still remain the question of how detailed and specific that doctrine was. I think that question is unanswerable, although it is important to say that we have no information about a detailed and specific doctrine.

However, even without that information, it seems clear to me that Plato did have a comprehensive picture of the cosmos divided into a higher divine, intelligible world and a lower human, sensible world, and saw the task of the individual, on which his well-being, his divinization, depends as a matter of somehow attaching himself to that higher world. A, and I suspect the, crucial link between these worlds is mathematics, in the broad sense determined by the mathematical curriculum of the *Republic*. In the *Republic* Plato emphasizes the role of mathematics in directing the attention of the potentially divine individual away from the lower world to the higher world and its apex, the form of the good. But in the *Timaeus* he emphasizes the way in which the higher world is expressed in the mathematical organization of the lower world. In both dialogues and elsewhere Plato insists on the moral benefit to be gained from the study of mathematics.

These fundamental beliefs of Plato found their expression in what has been called the world's first institution of higher learning, Plato's Academy. Having organized this essay around one, probably apocryphal, statement ascribed to Plato, I want to end with another, the inscription Plato is said to have put over the entrance to his Academy:

Let no one unversed in geometry enter here![33]

Appendix A. The division of the stuff of the world soul (*Timaeus* 35b–36b)

Here is Timaeus' description of the way in which the demiurge divided up the stuff of the world soul before reassembling it and forming it into circles corresponding to major heavenly motions:

> And he began the division in this way.
> (a) First, he took one portion from the whole, and next a portion double this; the third half as much again as the second, and three times the first; the fourth double the second; the fifth three times the third; the sixth eight times the first; and the seventh twenty-seven times the first.
> (b) Next he went on to fill up both the double and triple intervals, cutting off yet more parts from the original mixture and placing them between the terms, so that within each interval there were two means, (b1) the one exceeding and being exceeded by the extremes by the same part, (b2) the other exceeding and being exceeded by an equal number. These links gave rise to intervals of three to two and four to three and nine to eight within the original intervals.
> (c) And he went on to fill up all the intervals of four to three with the interval of nine to eight, leaving a part over in each of them. This leftover interval of the part had the ratio of the number two hundred and fifty six to the number two hundred and forty three.

The demiurge's procedure is the following.

(a) He first generates the series of powers of 2 and 3 in the order:

1 2 3 4 9 8 27

[33] "*Ageômetrêtos mêdeis eisetô*". On this inscription see [23].

(b) He then imagines these arranged as intervals (or ratios):

1	2	1	3
2	4	3	9
4	8	9	27

and fills these intervals $m \quad n$ with numbers k, l such that (b1) $k - m$ is to m as $n - k$ is to n and (b2) $l = \frac{m+n}{2}$, the so-called harmonic and arithmetic means between m and n. This yields:

1	4/3	3/2	2	1	3/2	2	3
2	8/3	3	4	3	9/2	6	9
4	16/3	6	8	9	27/2	18	27

Combining these two sequences we have:

1	4/3	4	3
4/3	3/2	9	8
3/2	2	4	3
2	8/3	4	3
8/3	3	4	3
3	4	4	3
4	9/2	9	8
9/2	16/3	32	27
16/3	6	9	8
6	8	4	3
8	9	9	8
9	27/2	3	2
27/2	18	4	3
18	27	3	2

where the right column expresses the ratio of the limiting terms in least numbers. Timaeus says that this process produces intervals representing the ratios 3/2, 4/3, and 9/8, which is true except for the case of 9/2   6/3.

(c) Timaeus then says the demiurge filled up the 4   3 intervals with 9   8 intervals leaving an interval of 256   243. These values correspond to the fact that $4/3 = 9/8 \cdot 9/8 \cdot 256/243$. Represented in terms of intervals one would have:

1   9/8   81/64   4/3

and these numbers would correspond to the first four notes of the diatonic scale, which I shall call do, re, mi, fa. Continuing we move up through two octaves as follows:

1			4/3	4	3
do	re	mi	fa		
4/3			3/2	9	8
fa			sol		
3/2			2	4	3
sol	la	ti	do′		
2			8/3	4	3
do′	re′	mi′	fa′		
8/3			3	9	8
fa′			sol′		
3			4	4	3
sol′	la′	ti′	do′′		

The continuation is a little less straightforward, since for the next octave we start from:

4			9/2	9	8
do′′			re′′		
9/2			16/3	32	27
re′′			fa′′		
16/3			6	9	8
fa′′			sol′′		
6			8	4	3
sol′′	la′′	ti′′	do′′′		

Although he does not say so, Timaeus presumably intends us to consider the combined interval:

4	16/3	4	3

and fill it up as in the case of the preceding two octaves. The remaining octave plus is represented by:

8	9	9	8
do′′′	re′′′		
9	27/2	3	2
re′′′	la′′′		

27/2	18	4	3
la'''	re''''		
18	27	3	2
re''''	la''''		

la''' and re''''' give us another 4 3 interval, which we can fill up in accordance with the diatonic scale if we use the order:

$$1 \quad 9/8 \quad 2304/1944 \quad 4/3$$

but there does not seem to be any way that the demiurge can construct quantities for mi''', fa''', sol''' or mi'''', fa'''', sol'''' following Timaeus' recipe.

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CHAPTER 5

Nicomachus of Gerasa and the Arithmetic Scale of the Divine

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MATHEMATICS AND THE DIVINE: A HISTORICAL STUDY
Edited by T. Koetsier and L. Bergmans

Introduction

Nicomachus of Gerasa, who lived in Palestine in the first half of the second century, was a major figure of the Neopythagorean movement, at the crossroads of mathematical traditions dating back to the oldest Pythagorism and ontological speculations about Ideas and Numbers coming from Academic Platonism. According to J.M. Dillon [1], Nicomachus is supposed to have lived between the era of Tiberius and that of the Antonines. Indeed Nicomachus mentions Thrasyllus, Tiberius' astrologer, in his *Harmonikon Enchiridion* (Jan, 260, 16). Cassiodorus refers to one Latin translation by Apuleius of *Arithmetic Introduction* by the same Nicomachus. If we know very little about this mathematician, we at least know the titles of his lost texts: *Ars Arithmetica*, mentioned by Jamblichus, *Life of Pythagoras*, quoted by Porphyrus, *Harmonic Introduction* [2], *Geometric Introduction* and *Platonic Reading* mentioned by Nicomachus himself in the writings which have been preserved. The three preserved writings are *Arithmetic Introduction* (*Arithmetiké Eisagogé*), *Manual of Harmonic* (*Harmonikon Egkeiridion*) and above all, *Arithmetic Discourses about the Gods* (*Theologoumena Arithmetika*). It is possible to reconstruct this latter piece of work through two late writings: the anonymous text from the *Theologoumena Arithmetica*, wrongly attributed to Jamblichus (ed. De Falco), and the summary of the Byzantine patriarch Photius in the 187 Codex of his *Library*.[1] According to Armand Delatte, *Theologoumena*, by Nicomachus and the anonymous compiler, might come from *Sacred Discourse* (*Hieros Logos*) or *Discourse about the Gods* (*Logos peri theôn*) that Jamblichus mentions in his *Life of Pythagoras* (§§146 and 152) [3], a text of the Roman Neopythagorean circles which would have, allegedly, for the first time, linked deities and mythological figures to numbers. Anyhow, Nicomachus' work, which has now been forgotten, had a great impact on Boetius, who was largely influenced by it in his *De institutione arithmetica libri II*, as well as on the Neoplatonists Jamblichus and Proclus.

Theologoumena Arithmetica

If we put aside the mathematical contents of *Arithmetic Introduction* [4] and the musical contents of *Manual of Harmony*, which both integrate Platonic developments into original Pythagorean theories, we find Nicomachus' speculations about the divine in *Theologoumena*. They obviously shocked Saint Photius, the patriarch of Constantinople, who, after mocking Nicomachus' "dear divine numbers" (τοὺς φίλους ἀριθμοὺς καὶ θεούς, 142 b 32–33) and rejecting a theology which deserved "neither secret nor respect", indicated that the book had become rare and had better disappear in spite of its fame. Nevertheless, Photius analysed it at length as if he were fascinated by the symbolic power arising from the mathematical properties attributed to deities.

In his *Arithmetic Theology*, in fact, Nicomachus no longer considers the natural properties of whole numbers (even and odd numbers, perfect numbers, linear and flat numbers, triangle numbers, tetragonal numbers and pentagonal numbers, arithmetic, geometric and harmonic medians). He reviews the first ten numbers, faithfully following the Pythagorean decade. The question is to relate "the nature of things" (τὴν τῶν ὄντων φύσιν) to "the hypostasis of numbers" (τὴν τῶν ἀριθμῶν ὑπόστασιν, 142 b 29–30) while letting us guess

[1] The original title was *Muriobiblion* (*Thousand Books*).

the sovereign presence of “gods and goddesses” who are likened to them (θεούς τε καὶ θεὰς, 142 b 35). What is the secret principle of this strange arithmetic theology? Photius sees it in the projection of the first quantity on the deities, who, in return, would deprive themselves of this same quantity so as to appear in all their splendour. In other words, the number would be a kind of springboard for reaching God at once without the mathematician caring to look back. Mathematics would not be worth an hour of effort for the initiate who is now one with his God.

However, this does not seem to be the initial intuition which allowed the Pythagoreans to identify Numbers with the various divinities of the Greek Pantheon. In the ninth century, Photius easily distinguished the field of the number from the field of the divine because it had been inscribed in the tradition of the Christian mystery of the Trinity since the baptismal formula of Matthew 28, 19: “Go (and) of all nations make disciples, baptise them in the name of the Father, the Son, and the Holy Spirit”. The Christian paradox of a God “one and yet multiple” after Hippolytus’ expression (*Contre Noët.*, 10) and, according to Tertullian, the very substance of the New Testament, for which “the Father, the Son and the Holy Spirit are believed to be three to make one God appear”? (*Contre Praxeas*, 31, 1–2) cannot support the Pythagorean paradox of deities whose sole manifestation is the Number. Still, in both cases, we are in an analogical perspective. Indeed, St. Augustine saw in the psychological analysis of the human triad: *esse*, *nosse*, *velle* (*Confessions*, XIII, 11, 12), *mens*, *notitia*, *amor* (*De Trinitate*, IX, 2–5) or *memoria*, *intelligentia*, *voluntas* (*De Trinitate*, X–XV) the three traces of the Father, the Son and the Holy Spirit because man is God’s image in every regard (Genesis 1, 26). Greek thinking does not apprehend man or divinity in terms of “person”, but, one might say, in terms of “measure”. And the supreme measure of the divine is the Number inasmuch as it is repeated in all the spheres of the universe with its immutable properties.

In this sense, it is not certain, as Photius puts it, that Nicomachus made “Gods out of numbers” (τοὺς ἀριθμοὺς θεοποιήσῃ, 143 a 9–10) nor that, conversely, he made numbers out of Gods by subtly introducing numerical principles in the divine. But rather, Nicomachus considers each god in the radiance of his cosmic measure, which inextricably combines arithmetic, geometric and astronomical properties. Because the divine is one, through the inexhaustible variety of its manifestations, the multiplicity of the number has to acknowledge its original unity, which is found, as we know, in the harmonic interval of the first ten numbers. From the Monad to the Decade, going through the Pythagorean Tetraktys which links the Whole to the first four numbers ($1 + 2 + 3 + 4 = 10$), Nicomachus sees the divinity deploying in ten steps, each one being an indispensable stage. Nicomachus’ scale is not comparable to Jacob’s ladder but it unites heaven and earth, god to matter, starting with the Monad, which is like the root of all things.

Photius’ account follows the natural scale of the first ten numbers, from the Monad to the Decade, as is found in the text by Pseudo-Jamblichus *Theologoumena arithmeticae* [5] as well as in the excerpts from Anatolius, a writer of the third century, partly preserved by Pseudo-Jamblichus.

(1) *The Monad*

The principle of all numbers is likened to the supreme God because it “contains all things” (πανδοχεὺς, 143 a 26). Nicomachus first calls it “chaos” (χάος) after Hesiod, then, "abyss" (χάσμα) in the Platonic tradition (*Republic*, *X*, 614 c–d), in order to evoke the primitive opening of which every being is born. Among the cosmic expressions qualifying it, the Monad is "the Axis of all beings" (ἄξων τέ ἐστιν αὐτοῖς, 143 a 31) and "Zeus' Tower" (Ζανὸς πύργος, 143 a 32), both images probably coming from Philolaos. The Pythagorean philosopher who, according to Theon of Smyrna, [7] gave the name of "Monad" to the "One", indeed identified the middle of the universe with a "guardroom" [8] and "the hearth of the universe" [9] which he called "Zeus' home". Zeus' home or tower is obviously associated with the figure of the goddess Hestia, in both Pythagorean and Platonic traditions. This is confirmed in an excerpt from Anatolius preserved by Ps. Jamblichus, which likens the intelligible unity to Hestia and the Earth. It seems obvious then, that the oldest Pythagorean teaching regarded the One, as Jamblichus puts it in his *Commentary on Nicomachus' Arithmetic Introduction*, as the "principle of all things", and that it located it in the middle of the celestial sphere: "The first harmonious compound, the One, which occupies the centre of the sphere is named Hestia" [11]. The Monad can thus rightly be called "hermaphrodite" (ἀρσενόθηλυς, 143 a 25) since it brings together in the centre of the world, the father of gods, Zeus, and his daughter Hestia who guards the divine home.

(2) *The Dyad*

With the Dyad, appears the other element of the number, which, by tearing itself away from the unity (Nicomachus, in a Plotinian way, defines the Dyad as "audacity" [τόλμα, 143 a 39]), allows the number to form itself and grow to infinity, according to the remark he makes in his *Arithmetic Introduction* (*II, 1*). The Dyad is thus associated with "matter" (ὕλη) since it generates the multiplicity of numbers in an endless process. If Photius recognises the natural arithmetic properties of the Dyad, he does not see more than a "teratological" development (ἅ δὲ τερατείας; 143 b 5) in the theological considerations that follow. He never notices the multiplication of feminine terms, which make of the Dyad, as "source" (πηγή), an endless depth of fecundity. Apart from the expression "equal Zeus" (ἴσον Δία), that reinforces the monadic image of the just God in the Dyad, the Dyad is "justice" (δίλην), "Isis" (Ἶσιν, Isin: a play on the words *ison*: equal or just, and *isin*: "the goddess Isis"), "Nature" (φύσιν), "Rhea" (Ῥέαν), "Zeus' Mother" (διομάτερα), and "Source of Repartition" (πηγὴν διανομῆς).

All the other divine terms are feminine: Demeter of Eleusis, Dictynna, Aeria, Artemis, Asteria (Artemis is likened to Hecate, whose mother was called Asteria), Disame, Substance, Aphrodite, Dione, Mychaia, Cytheraea (143 b 10–16). The names of these goddesses are associated with abstract terms which underline, in parallel, the dyadic power of otherness, division and endless multiplication, which is quite logically identified with the Lot assigned by Fate (μόρος), and therefore, with the tragedy of Death (θάνατος, 143 b 17–18).

(3) *The Triad*

As the first odd number in action—the One being neither even nor odd in the Pythagorean tradition—and as the first perfect number uniting the Monad and the Dyad, the Triad is going to be associated with everything that concerns "composition" in the world (σύστασις, 143 b 30) thus with "knowledge" (γνῶσις, 143 b 29). Photius finds such associations "gross" (φορτικά) in the field of science. Yet, they announce Hegel's idea of the Triad as the Absolute Spirit and Knowledge of the Whole. As for the myth, Nicomachus attributes a long series of deities to the Triad: Hecate, the three-headed magician (*trikephalos*) who presided at crossroads (*triodos*) and is often mistaken for Artemis; Leto, Apollo and Artemis' mother; Kronia (Kronos' mountain in Egidia); Amalthaea's horn (or Horn of Plenty); Ophion, who reigned over the Titans before Kronos; Thetis the Nereid; Harmonia; Polymnia, the Muse of geometry and history; Loxia or the Oblique (Apollo in his oblique solar walk); Arktos (Ursa Major), among various epithets evoking, encounter, marriage (Γάμον, 144 a 1), and unity associated with number Three (Triton and Tritogenia, goddesses of the sea). All these observations are proof that for Nicomachus the Triad is the divine expression of unity, reconciling the contraries and arranging them in unique harmony.

(4) *The Tetrad*

For Photius, the Number Four is the greatest wonder in the eyes of the Pythagoreans (θαῦμα μέγιστον, 144 a 4) because it is "the god in all things" (πάνθεος, 144 a 5). The text by this Byzantine author does not emphasise the general quadripartitions of the universe (four elements, four cardinal points, four seasons, etc.) which Ps. Jamblichus' *Theologoumena* discovered in *Peri Dekados* by Anatolius. Surprisingly Photius does not mention the Pythagorean Tetraktys, upon which Theon of Smyrna comments at length, distinguishing eleven tetrads (numerical by addition, numerical by multiplication, geometric, physic, stereometric, engendered, human, intellectual, parts of the animal, seasons, ages) [12]. But he underlines that, for Nicomachus, the Tetrad possesses the keys to all Nature. Thus it is associated with the male element and with power, of which the three gods Hermes, Hephaistos, and Dyonisus are the highest manifestations, as well as Herakles, the Hermes-Tetrad being also Maiadeus, Maia's son, here identified with the Dyad. It is with these first four founding numbers that the first book of Nicomachus' *Theology* ended as it should, thus being faithful to the primacy of the Tetraktys.

(5) *The Pentad*

At the beginning of the second book came the Pentad (ἡ πεντὰς) which holds the middle in all things in the same way as it forms the centre of the first ten numbers. Photius observes that it articulates the "natural number" in its ten-step deployment, with its two extremities, thus linking the unity, or Monad, the beginning of every being, to the totality, or Decade, which is their final term. Photius does not seek any justification for the cosmic properties of the Number Five and does not mention that, under the form of the star-shaped pentagram, it was the sign of recognition of the first Pythagoreans. As a matter of fact, the latter saw in the Pentad not only the cosmic number *par excellence,* attributed to the five regular solids inscribable in the sphere which Plato mentions at length in *Timaeus,* but also the nuptial number of procreation since it joined the first even female number or Dyad with the first odd male number, or Triad, hence its given name of *Gamelia*. Nicomachus, therefore, rightly insists on the first elements of the Whole which go by five (the four elements and ether, as quintessence) because the pentad is "division *par excellence*" (προσεχέστατον, 144 a 27–28). Photius does not seem to notice the particular appropriateness of the number Five to Justice, even though the divine epithets clearly point in that direction. In fact, the Pentad is "Justice" (Δικαισύνη), "Jurisdiction" (Δίκησις), "Nemesis" (Νέμεσις), while it is at the same time "the one which presides over cosmic regions" (Ζωναία), "the one which governs the cycles" (Κυκλιοῦχος), or else "The Middle of Middles" (μέσων μέσην, 144 a 36–41), and also, like the Monad, "Zeus' Tower" (Ζανὸς πύργος, 144 a 37). The same divine expressions are to be found in the anonymous *Theoulogoumena:* the Pentad is Gamelia, Zonaia, Kukliouchos, "the One which occupies Heaven", that is Aphrodite Ourania, the goddess of marriage and nature. Number Five is thus, at the same time, the number of fertile love (Aphrodite) and the number of Justice (*Dike* for Jamblichus, *Dikesis* and *Dikaiosune* for Nicomachus, in Photius).

In a famous study on Pythagorean politics, based on *Theologoumena arithmeticae* (p. 27 ss) and Jamblichus' *In Nicomachi arithmeticam introductionem* (p. 16 ss), Armand Delatte has clearly established that, for the Pythagoreans, Justice was the power to match what is equal and is located in the middle of an odd square number [13]. In the series of numbers from 1 to 9, their addition (45) divided by the number of digits (9) equals the quotient 5. Therefore, the Ennead balances small and large numbers by taking as set square in the centre, number 5, which is the arithmetic mean ($9-5=4$, attributed to $1=5$; $8-5=3$, attributed to $2=5$; $7-5=2$, attributed to $3=5$; $6-5=1$, attributed to $4=5$). I have myself developed the symbolism of justice linked to number 5 in Pythagorean philosophy as well as in Platonic myths in former writings to which I shall forward the reader [14].

(6) *The Hexad*

Photius is less loquacious about the last numbers, apart from the Decade. The Hexad, once more linked to Aphrodite because it unites the Dyad to the Triad by way of multiplication and not by addition like the Pentad, is called the "Articulation of the Whole" (Διάρθρωσις, 144 b 2) and the creator of "the state of life" (τῆς ζωτικῆς ἕξεως), from which it derives its name (ἑξάς). From the testimonies that have been left to us, the Pythagoreans seem to have hesitated about the symbolism of the numbers 5 and 6, just like the inhabitants of Atlantis mentioned by Plato in *Critias* (119 d) who mingled the product of the numbers 2 and 3 with their sum to establish the cycle of cosmic revolutions. This is probably the reason why Nicomachus defines the Hexad as well as the Pentad, as "the beginning", and "the middle of the Whole" (ἀρχὴν καί ἥμισυ παντός, 144 b 9), a cosmic property which, strictly speaking, is only appropriate to number 5. Anyhow, the divine attributes of the Hexad are more numerous in Nicomachus: Aphrodite, Lachesis, Trioditis or Trivia honoured at the crossroads, Amphitrite and Thalia. None of these names is truly revealing and only union, associated with Aphrodite, appears to be related to the Number Six, which reproduces in minor key what Nichomachus found in the Number Five, i.e., the articulation of the male and female elements and the ruling of cosmic regions.

(7) *The Hebdomad*

For Anatolius, quoted by Ps. Jamblichus, the Number Seven, the only number that is not generated by any of the first numbers except the unity, and the only one that does not generate any other number, is likened to Athena, the virgin motherless goddess. Nicomachus also comes to associate Athena with the Number Seven, in 144 b 2, without any comment from Photius, who lists the divine epithets. Nicomachus seems to identify Heptas with Septas without taking into account the Greek tradition, which had dropped the sigma. (*Theol. Arithm.*, 43; *Library*, 144 b 15). But like Anatolius, he does not insist on the realities which are seven in number: the ages of life, the openings of the head, the forms of the voice, the number of stars, etc. He simply identifies with the Number Seven, Kronos, often mistaken for Chronos and thus, for the favourable moment, *Kairos*, then Athena, Ares, Clio and Adrastaea (Nemesis, the "Inevitable" mentioned by Plato in the *Republic* (451 a 5)).

(8) *The Octad*

Photius found very little about the Number Eight in Nicomachus and he ironically observes that the octad does not even account for one eighth of the development of the other numbers. It is nevertheless associated with Rhea, the wife of Kronos, and the mother of Hestia, Demeter, Hera, Hades, Poseidon and Zeus. Considering that Rhea and Kronos had six children, it is difficult to find the link with the number eight. Then it is also associated with Cybele, herself later likened to Rhea by the Romans as Mother of the gods, and finally with Kadmeia, Neptolemus' sister. It is hard to understand why Cybele-Rhea's orgiastic cult, evoking Nature's indestructible power, should be represented by the Number Eight, or else by Themis, goddess of the Law, who is the ultimate attribute of the Octad, along with Euterpe, the Muse.

(9) *The Ennead*

The Ennead is given even less importance than the Octad. If Anatolius sees in it the first square of an odd number (3) and the duration of the gestation of a child, Nicomachus, according to Photius, does not resort to its arithmetic properties and likens it to the Ocean and the Horizon. Its divine attributes are *Halios* (= *Helios*, the Sun, identified with Apollo), Prometheus and Hecate (Perseia, Persaeus' daughter), Hera, Zeus' wife, and also Core (Persephone), and Terpsichore, the Muse of dance, but no explanation or analogy is given.

(10) *The Decade*

The Decade represents the largest development in Photius' account since it embodies "the Whole" for the Pythagoreans (τὸ Πᾶν, 145 a 5). The Decade is the supreme god who rules the categories as well as the parts of speech, though Photius' remarks are here tainted with Aristotelism. Ps. Jamblichus observes in *Theologoumena* that Speusippus' text *On Pythagorean Numbers* was half devoted to the Decade, "the number most able to generate beings and bring them to perfection" (De Falco 82, 10 = Philolaos A XIII).

An even number, Ten completes the series of numbers and contains an equal quantity of odd and even numbers, an equal quantity of integers (or prime numbers), as well as the totality of the ratios of equality, superiority, inferiority, superpartiality, linear, flat and cubic numbers (1 the point, 2 the line, 3 the triangle, 4 the pyramid). Nicomachus thus concludes that all geometric figures find their completion in Number Ten, which is consequently called "Faith" (Πίστις, 145 a 16; De Falco 81, 15) since it must be unflinchingly trusted. In Photius' summary the Decade is identified with all the figures of the Power covering the entire universe: Sky, Fate, Time, Force, Faith, Necessity, Atlas the indefatigable God, Phanes the orphic God, the Sun, Urania, Memory (Μνήμη), and Remembrance

(Μνημοσύνη) (145 a 15–17). Being "the God *par excellence*" (Ὑπέρθεός), the Decade contains "the power of all the divinity which lies in numbers" (τὸ ἄκρατος τῆς ἐν ἀριθμοῖς ἔχουσα θεότητος, 145 a 21–22). Therefore, Nicomachus need not repeat all the names of Gods that have been previously quoted, from the Monad to the Ennead. Inasmuch as the Decade contains all the numbers, it contains all the Gods, or rather it unifies them in the figure of the "Hyper God" who masters all numbers and all the regions of the universe. Similarly to the Number, which deploys like a flow of discrete unities, the Decade deploys the genesis of the Gods, which, at any moment, brings about the advent of the world.

How to interpret this long list of divine attributes that shocked the Patriarch of Constantinople as much by its apparent disorder as by the religious impiety which seems to enumerate the deities along with the numbers? It is not satisfactory to believe, like Photius, that Nicomachus or his Pythagorean predecessors planted divine figures, or, more simply, divine names on arithmetic beings, so as to deify the Number or to number the Deity. Nichomachus seems to have had the intimate conviction, beyond any surface analogy, not that the Number was divine and had to be honoured like the ancient Gods, but that ancient deities, obscured in the myths, were revealed more clearly by the infinite power of the Number. We may smile with Photius or ignore such wild imaginings. Yet, we cannot help thinking that these theological speculations were one of the ways that the rising rationalism opened up before establishing its identity in the neutrality of mathematic demonstration. Whether it be the trace of ancient mythologies or the sketch of a new rationality, arithmetic theology remains in man, not like "the dream of a shadow", to quote Pindarus, but rather, as Nicomachus wished it, like "the dream of a number".[2]

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[7] Theon of Smyrna, *Exposé des connaissances mathématiques utiles pour la lecture de Platon*, Hiller, 20, 19 = Philolaos A X; J. Dupuis (trans.), Paris, 1892.

[8] Athenagoras, *En faveur du christianisme*, 6; Schwartz, 6, 13 = Philolaos B XV.

[9] Aetius, *Placita, II, VII*, 7 = Philolaos A XVI.

[10] Jamblichus, *Commentaire sur l'introduction arithmétique de Nicomaque*, Pistelli (ed.), 77, 9 = Philolaos B VIII.

[11] Stobaeus, *Eclogœ, I*, XXI, 8 = Philolaos B VII.

[12] Theon of Smyrna, *Exposé des connaissances mathématiques utiles pour la lecture de Platon*, op. cit.

[13] A. Delatte, *Essai sur la politique pythagoricienne*, Université de Liège, fascicule XXIX, E. Champion (ed.), 1922, pp. 68–69. Cf. *La constitution des États-Unis et les Pythagoriciens*, Les Belles Lettres, Paris, 1948, pp. 15–17.

[2] A play on words in French: « rêve d'une ombre » and « le rêve d'un nombre ».

[14] J.-F. Mattéi, *L'Étranger et le Simulacre. Essai sur la fondation de l'ontologie platonicienne,* Presses Universitaires de France, Paris, 1983; *Pythagore et les Pythagoriciens*, Presses Universitaires de France, Paris, 1993; 3rd edition updated, 2001; *Platon et le miroir du mythe. De l'âge d'or à l'Atlantide*, Presses Universitaires de France, Paris, 1996; re-edition, Presses Universitaires de France, Quadrige, 2002.

CHAPTER 6

Geometry and the Divine in Proclus

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MATHEMATICS AND THE DIVINE: A HISTORICAL STUDY
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1. Philosophy as divinisation

Looking back in time, we tend to see Proclus as standing at the end of the history of ancient Greco-Roman philosophy and science, one of its last representatives, who did much to shape its transmission to the Byzantine and Islamic worlds and to the Latin West of the Middle Ages and the Renaissance. It is true that the resolutely pagan philosophical school in Athens of which he was head in 437–485 was soon to cease its teaching as a consequence of the anti-pagan measures taken by the most Christian Emperor Justinian in 529, the last members of the school (including Damascius and Simplicius) leaving for temporary exile in Persia. The sister school in Alexandria in Egypt, with which the Athenian philosophers had many close links, was not to survive much beyond the sixth century. And the works of these philosophical schools of Late Antiquity would only survive in Byzantine manuscripts, in Arabic and Latin translations, and in the appropriation and adaptation of their ideas by Christian and Islamic thinkers such as Boethius, Dionysius the Areopagite, Al-Farabi, Avicenna and Michael Psellos.

Proclus himself would have seen things quite differently. Although conscious of the danger, he was confident of his place in a long line of interpreters of a wisdom of whose eternal value and strength he had no doubt. These interpreters included philosophers identified today as ‘Neoplatonists’, in particular Plotinus (205–270), Porphyry (234–ca. 305) and Iamblichus (ca. 245–325), whose reading of Plato’s dialogues articulated the wisdom which Proclus wished to represent.[1] Plato himself, in Proclus’ view, and to a lesser extent his somewhat deviant pupil Aristotle, gave this wisdom scientific form, as compared to the somewhat enigmatic expressions it had found before in Pythagoras, in the Greek poets (in particular Homer) and indeed in ancient non-Greek religious revelations such as those of the Chaldaeans and Egyptians.

Wisdom thus included for Proclus a very wide range of knowledge, expressed in various ways in the works of Plato and of Aristotle, and in the (Neoplatonic) commentaries on these works, on earlier Greek philosophers and on ancient Greek and Barbarian religious traditions. This range of knowledge was unified and structured in terms of the attempt to reach a goal, the goal of philosophy as often defined by ancient Platonists, ‘assimilation to god as far as possible’ (Plato, *Theaetetus* 176b), what we might call the divinisation of human life. The training Proclus provided in his school in the various forms of knowledge, the use made in this connection of the work of Plato, Aristotle, Euclid, Ptolemy, Homer, Orphic poems, Chaldaean Oracles: all this was thoroughly organized so as to promote a transformation of the lives of pupils, leading them to higher, more divine levels. Lectures and texts were occasions, instruments, exercises for gaining access to a higher life. How this was to be achieved and how, in particular, mathematics had a function in this are questions which require that we consider first what ‘god’ or the ‘divine’ means for Proclus, what his conception of divinity means for the goal of divinisation, and how different forms of knowledge were taken by him to represent different degrees of divinisation.

[1] The link between Plato’s school in Athens (the Academy), which had long since disappeared, and Proclus’ Neoplatonic school is purely intellectual, not institutional or material.

2. Mathematics in the divinisation of human nature

In Plotinus, the realm of the divine extends from the life and order-giving force of the universe, immaterial soul, to transcendent principles presupposed by soul, Intellect (*Nous*) and the One.[2] The 'divine' thus has to do with what is causally prior and ontologically superior. The universe as a unified multiplicity requires a principle of unity other than it, causally prior to it, independent of it and superior to it. For Plotinus, this principle is soul. Soul, however, is also a unified multiplicity, albeit at a higher degree of unification. Soul therefore requires in turn a principle of its unity which is prior, independent and superior, Intellect. This divine Intellect reminds us of Aristotle's God (the 'unmoved mover' of *Metaphysics* XII, 6–10) whose object of thought Plotinus identifies with Plato's transcendent Forms. Divine Intellect, as subject and (multiple) object of thought, is itself a unified multiplicity, notwithstanding its maximal degree of unity, in which subject and object of thought are one. Plotinus therefore postulates as its principle of unity, as prior to it, independent and superior to it, an Ultimate which is absolutely non-multiple, in this sense 'one'. With the 'One' Plotinus identifies the Good 'beyond being' of Plato's *Republic* (509b). The One, as the ultimate principle of unity in all multiplicity, constitutes all things in a progressive order going from Intellect, through soul, to the natural world, each level deriving from and existing in its relation to the level above it. If then the 'divine' ranges from the One, through Intellect, to soul, the derivative levels of existence represented by Intellect and soul are 'divine' due to the perfection of existence or unity that they derive from the One.

Proclus essentially accepts Plotinus' metaphysics:[3] the 'divine', for him also, is found primarily in a transcendent ultimate principle and derives from it in a range going through Intellect to soul, a range surveyed in his manual of metaphysics, the *Elements of Theology*. However, due in large part to the influence of Iamblichus, Plotinus' metaphysical universe has become considerably more complicated and sophisticated in Proclus: mediating principles and levels intervene between the One and Intellect (the 'Henads'), between and within Intellect and soul. To this highly differentiated and gradated universe of transcendent principles Proclus fitted the populous pantheon of traditional Greek and Barbarian religions.

For humans, assimilation to god or divinisation can mean the attempt to live the life of any of these many degrees of divinity. However, the derivative character of lower degrees of divinity will imply the desire to reach higher degrees and ultimately that which is above all else and makes all else divine, the One. The goal of reaching higher degrees of divine life involves difficulties which might be sketched as follows. If human nature is primarily soul, then living the divine life of soul seems available to the extent that we disengage ourselves (as soul) from material preoccupations, discovering our true nature as a life and order-giving immaterial force: to (re)discover one's self, one's true nature and value, is already to live a degree of divine life. However, the transition to a higher mode of life, a life corresponding to divine Intellect, is made problematic to the extent that the intellectual activity characteristic of soul, discursive thinking, is inferior to the kind of thought proper to transcendent Intellect. Discursive thinking (*dianoia*) thinks objects exterior to it,

[2] Plotinus *Ennead* V, 1, 7. In *Plotinus: an Introduction to the Enneads*, Oxford 1993, I provide a sketch of Plotinus' metaphysics summarized here.

[3] A fundamental work on Proclus' metaphysics is Beierwaltes 1979 (see below Bibliographical note).

knowledge of which it seeks through the mediation of images (sense-images, for example) and through logical procedures whereby propositions are formulated and combined in arguments so as to yield conclusions. This mediated, indirect grasp of things (which constitutes also the fallibility and weakness of this type of knowledge) contrasts with the kind of knowledge which Neoplatonists supposed was characteristic of transcendent Intellect, in which the thinking subject is immediately and totally united with its object such that no imperfection, no error in this knowledge can occur. How is this higher form of thought, this higher mode of life lived by divine Intellect, to be attained, if the kind of thinking we live is of the discursive type? A further problem concerns the highest level of the divine: if the One transcends all forms of knowledge, if it transcends the highest level of knowledge represented by divine Intellect, how can it be known and approached? The greater degree of differentiation and gradation of Proclus' universe made these problems a good deal more acute and difficult of resolution than was the case as they posed themselves already for Plotinus.

Without developing these difficulties further, it is possible to give some indications already as to how the curriculum followed by Proclus in his school was designed so as to respond to such difficulties in the context of the overall aim of divinisation. This curriculum, first introduced by Iamblichus, followed a graduated approach in which lower stages or modes of life prepare for and lead to higher stages. These stages were constituted by a scale of philosophical instruction going from preliminary moral edification and training in logic, though the 'practical' sciences (ethics and politics), to the higher reaches of philosophy, the 'theoretical' sciences represented, in ascending order, by physics, mathematics and 'theology' (the science of the divine, or metaphysics). Each science in this scale was intended to develop a corresponding kind of moral excellence and life, the practical sciences promoting a divine life of soul as it orders materiality, the theoretical sciences bringing soul ever nearer to the life of divine Intellect. For each science corresponding texts were read, first as found in the works of Aristotle, and then in the dialogues of Plato.

In this scheme, mathematics clearly occupies a vital place: it comes very high in the scale of sciences and immediately precedes the highest science, theology or metaphysics. Plato had already given mathematics a pivotal role in his programme of an ideal philosopher's education in the *Republic* (510c–511d), where mathematics provides the passage from the material world to knowledge (called 'dialectic') of the transcendent Forms and of the Form of the Good. However, Proclus gives an interesting interpretation of why and how mathematics can assume this function in the account he supplies of the nature of mathematical thinking in his commentary on Euclid's *Elements*, one of the texts he used[4] as providing mathematical instruction.

3. The nature of mathematical science

What precisely are the objects of mathematical science? What is their status in the structure of reality? Arguing against Plato, Aristotle saw mathematical objects as conceptual abstractions which we produce by isolating the quantitative dimension of physical objects.

[4] With Nicomachus of Gerasa's arithmetic and Ptolemy's astronomy.

In contrast, Plato, according to Aristotle (*Metaphysics* I, 6), saw mathematical objects as realities existing independently of physical objects, occupying an intermediate realm between physical objects and the transcendent Forms. In Plato's dialogues, the position is less simple. In the *Republic*, for example, mathematics concerns objects that relate to the transcendent realm of Forms as images of the Forms. But what is not altogether clear is whether mathematical objects are images in the sense that they exist as image-like realities, or merely in the sense that they are the *way* in which the mathematician thinks immaterial objects. A further difficulty confronting the ancient interpreter of Plato was that concerning the relation between mathematical objects, as intermediates (in some sense) between Forms and physical objects, and soul, which also mediates in Plato between the Forms and the material world.

It is in connection with these difficulties that ancient Neoplatonists developed the interpretation of mathematical thinking that is presented by Proclus in commenting Euclid's geometry. According to this interpretation,[5] mathematical objects are concepts derived through logical procedures by soul from an innate knowledge common to all souls ('common notions', 'axioms'). This innate knowledge is projected (or constructed) by soul in quantity (arithmetic) and extension (geometry), with the purpose of developing (in the literal sense, 'unrolling') this innate knowledge so as to articulate and comprehend it more easily. The space in which geometrical development takes place is provided by imagination (*phantasia*): it is an imaginative space in which the contents of innate knowledge emerge as expressed by point, line, figure and solid, whereas arithmetic achieves a more unified, compact, purely numerical articulation of this knowledge. The innate knowledge in question, which mathematics seeks to unroll, might be described as metaphysical: it includes concepts which concern the transcendent principles of reality and the laws governing the progression of these principles from the One.

The following aspects of this theory of mathematical objects might be stressed. (1) Plato's intermediates, mathematical objects and soul, are related on this interpretation in the sense that mathematical objects are elaborations of concepts inherent in the nature of soul. (2) Mathematical objects are images of Forms in the sense that soul projects in a more accessible dimension its concepts of transcendent principles. (3) The theory involves both (Platonist) realism and constructivism in the sense that soul projects mathematical objects, but these objects are concepts expressive of transcendent realities; mathematical objects are not conceptual abstractions from physical objects. (4) Mathematical thinking is eminently discursive in its rigorous logical procedures. But this is also its weakness: it is because the soul has difficulty in grasping metaphysical truths that she has recourse to discursively elaborated expressions of them in mathematics. (5) Mathematics thus promotes perfection in the life of discursive reasoning, but it also prepares the soul for a higher level of reasoning, that of theology or metaphysics, the practice of which prepares the soul in turn for access to yet a higher level of divine life, that of non-discursive, perfect, complete knowledge, i.e. the life of divine Intellect. (6) There is a gradation within the mathematical sciences in the sense that arithmetic is a purely numerical articulation of metaphysical truths, whereas geometry extends these truths further into imaginative space. Among mathematical sciences, geometry is mediational and thus eminently suitable as representing the

[5]Cf. O'Meara 1989: 167–169, where references to further studies are to be found.

mediational role of mathematics in general: it is more accessible than arithmetic in its use of imaginative extension and of clearly applied demonstrative procedures. For Proclus, certainly, geometry is a privileged mediational science in the context of the divinisation of human life, a science exemplified perfectly for him in Euclid's *Elements*.

A passage from Proclus' commentary on Euclid beautifully summarizes his conception of the nature and philosophical importance of geometry:

> "So the soul, exercising her capacity to know, projects on the imagination, as on a mirror, the ideas of the figures; and the imagination, receiving in pictorial form these impressions of the ideas within the soul, by their means affords the soul an opportunity to turn inward from the pictures and attend to herself. It is as if a man looking at himself in a mirror and marvelling at the power of nature and at his own appearance should wish to look upon himself directly and possess such a power as would enable him to become at the same time the seer and the object seen. In the same way, when the soul is looking outside herself at the imagination, seeing the figures depicted there and being struck by their beauty and orderedness, she is admiring her own ideas from which they are derived; and though she adores their beauty, she dismisses it as something reflected and seeks her own beauty. She wants to penetrate within herself to see the circle and the triangle there, all things without parts and all in one another, to become one with what she sees and enfold their plurality, to behold the secret and ineffable figures in the inaccessible places and shrines of the gods, to uncover the unadorned divine beauty and see the circle more partless than any centre, the triangle without extension, and every other object of knowledge that has regained unity." [6]

For Proclus, the soul can reach self-discovery in geometry. In her geometrical projections, she sees an image of herself and, through this self-knowledge, reaches a knowledge of the presence in her of truths concerning transcendent first principles, the gods. In thinking these truths, soul already lives a life of knowledge that will bring her higher in the scale of divinity. This use of mathematics as an anticipatory imaging forth of the knowledge of the divine Proclus described as characteristic of the Pythagoreans.[7]

4. The metaphysics of geometry

In the passage quoted above, Proclus refers to the study of circles, triangles and other geometrical figures. In the following pages I will describe in a little more detail the use of geometrical thought in Proclus as expressive of metaphysical principles and laws as exemplified (i) in geometry in general and in specific geometrical figures, (ii) in the circle, (iii) semi-circle, (iv) triangle and (v) square.

[6] Proclus *Comm. in Eucl.* 141,4–142,2. Proclus' commentary is cited by page and line number in Friedlein's edition of the Greek text (the page numbers are given in the margins of Morrow's translation, which I quote here). It might seem that Proclus is providing in this passage a surprisingly positive interpretation of the Narcissus myth (compare Plotinus *Ennead* I, 6, 8, 8–16 and P. Hadot, 'Le mythe de Narcisse et son interprétation par Plotin', in: P. Hadot, *Plotin, Porphyre. Etudes Néoplatoniciennes*, Paris 1999, 225–266). However, Proclus speaks of looking in a *mirror*: the reference seems to be the mirror of self-knowledge (and knowledge of god) of Plato's *Alcibiades* (132d–133c), a dialogue Neoplatonists read as dealing with self-knowledge as the beginning of philosophy.

[7] *Comm. in Eucl.* 22, 9–14; cf. O'Meara 1989 for the revival of Pythagoreanism in the Neoplatonic schools of Late Antiquity.

(i) Coming back to Proclus' metaphysical system as traced in its main lines in his *Elements of Theology*, we might stress first that all reality, for him, relates in a differentiated range to an ultimate transcendent first principle, source of all existence, perfection, divinity, the One. From this Ultimate devolve, in a non-temporal, non-spatial successive order, levels of unified multiplicity, tending to ever greater multiplicity and lesser unity. Each stage in the devolution of reality (which we might summarize as the levels of Intellect, soul and the natural world) reflects a departure from unity, limitation, determination, in the direction of multiplicity, unlimitedness, indetermination, a departure limited at each stage, turned back, so as to result in a multiplicity that is unified, limited, determined. In this dynamic constitution of things we might note the priority of unity, limit, determinate identity, equality, perfection, which, combined with the secondary (but necessary) aspects of multiplicity, unlimitedness, indefiniteness, deficiency, inequality, generate the whole of being in ordered succession.

For Proclus, the very structure of the domain of mathematical objects manifests these metaphysical principles, both the flow of numbers (in arithmetic) from 1 (the monad, limit) and 2 (the dyad, unlimited) to 3 (the first determinate number) and the following members of the numerical series, and the flow (in geometry) from the point (limit) to line (unlimited in its tendency), plane figure (the line as limited) and solid. Euclid's *Elements* (Book I) follows this metaphysical order, in Proclus' view. Thus Euclid's 'Definitions' go from point (Def. 1) to line (Defs. 2–4), surface (Defs. 5–14) and the figures in correct order, circular and rectilinear (Defs. 15–34). So also do the 'Postulates' go from causes (I), through the progression of things from causes (II), to their constitution in a reversion to their causes which bounds them (187, 4–16). As for the Propositions of Book I, they concern the metaphysical properties of equality (primary) and inequality (greater/lesser) as regards lines and figures. The first group of Propositions (1–26) deals with triangles, the next with four-sided figures (354, 1–7).

There is a difficulty here, however, in matching the progression of Euclidean geometry with the principles of metaphysical progression. In Proclus' view, as will be seen shortly, the ordered series of geometrical figures begins with the circle (a line bounded by turning back on itself), semi-circle (two lines), triangle and quadrangle. One would therefore have expected Euclid to deal first with circles and semi-circles, and not with triangles, as he does. However, I think we can infer the solution Proclus would find to this difficulty. He notes that the construction of an equilateral triangle, the first of triangles, in Proposition 1 requires the use of circles (424, 3–6 with 209, 4–6). Circles are thus presupposed and prior to triangles in Euclid's demonstration.

(ii) Geometrical figures are composed of circular and rectilinear lines and by lines mixed from these. Circular lines represent limit and determination, as we have seen, whereas rectilinear lines tend to infinity, indefiniteness. Circular lines are therefore metaphysically prior to all other types of lines (107, 11–16; 164, 8–11). Proclus also claims that circular angles are prior to rectilinear angles (and prior to angles mixed from both types) as corresponding to unity, rectilinear angles corresponding to lower levels of being (129,20–130,6). In consequence, the geometrical figure constituted by circular lines and angles is the first of all such figures. In the progression point (monad), line (dyad), surface, the circle corresponds to the triad (centre, diameter, circumference), just as the first of the subordinate rectilinear figures, the triangle, is also triadic (115, 3–8).

Proclus often emphasizes the primary metaphysical properties embodied by the circle, its unity, self-identity, determinateness, equality, perfection.[8] It manifests therefore in imaginary space the properties of the primary transcendent principles constitutive of reality in general. In this it compares with secondary, rectilinear figures which correspond to secondary aspects of this constitution (146,24–153,9). The circle also manifests, in its internal structure, the dynamics of reality in general: from the centre, source of geometrical progression, source of unification, determination, stability, radiate lines, infinite in their multiplicity and tendency, reverting around the centre to constitute the circle (153,12–155,23).

(iii) The semi-circle comes after the circle in the metaphysical order and before the rectilinear figures, since it is bounded by two lines, the circle by one (the circumference) and rectilinear figures by three or more. In this sense the semi-circle mediates between the unity of the circle and the multiplicity represented by rectilinear figures (159, 4–7; 161, 7–12).

(iv) As constituted from straight lines, rectilinear figures express a tendency to progression, motion, change, production. This can be seen, according to Proclus (166, 14–18), in the use made of triangles in Plato's account of the constitution of the cosmos in the *Timaeus*. Coming after circles and semi-circles, which it presupposes (see above (i)), the triangle is, we have seen, the first of rectilinear figures, having a minimum of three sides. Of triangles the first, for metaphysical reasons, the most perfect, most unified and closest to the circle is the equilateral triangle, being composed of equal sides and angles (213,14–214,15).

(v) Following numerical progression, the first of rectilinear figures, coming after the triangle, is the quadrilateral. Of quadrilateral figures, the square is primary for the same reasons that give primacy in geometrical figures and in reality: the equality of sides and angles in the square represents, in quadrilateral figures, the maximum degree of equality, stability, limiting power. Hence its fundamental importance, for Proclus, after the triangle, in the constitutive structure of the world (172,15–174,21).

My brief summary might suffice to give an impression of the way in which Proclus, in commenting Euclid, finds in geometry an accessible, scientific articulation of metaphysical truths, of which the geometer's demonstrations are discursive 'images'. This use of geometry to image forth metaphysics Proclus regards as the main philosophical value of geometry. He is less interested in the technicalities of Euclidean geometry taken *per se* than in the role it can play in preparing the pupil for access to metaphysics or theology.[9] He does not fail to remind us that in going through geometrical exercises with Euclid, we are strengthening our discursive capacities and preparing our transition to a closer knowledge of the divine, a transition which also means sharing more closely in divine life (174, 17–21; 130, 22–23).

5. St. Sophia: a geometry of the divine?

Between 532 and 537 the emperor Justinian replaced an earlier church that had burnt down in Constantinople with the great church of St. Sophia that still stands in Istanbul. The

[8] A full discussion of this may be found in Beierwaltes 1979: 166–239.

[9] Cf. 84, 11–20; 130,23–131,2; 174, 17–21.

Fig. 1. Constantinople, Hagia Sophia interior view in the direction of the apse. Photo: Erich Lessing.

architects of the new church, a unique masterpiece, were Anthemius of Tralles and Isidore of Miletus. A contemporary, Procopius, describes the church as follows:

> "In the middle of the church there rise four man-made eminences which are called piers, two on the north and two on the south, opposite and equal to one another, each pair having between them exactly four columns... Upon these [piers] are placed four arches so as to form a square, their ends coming together in pairs and made fast at the summit of those piers, while the rest of them rises to an immense height... Above the arches the construction rises in a circle: it is through this that the first light of day always smiles... And since the arches are joined together on a square plan, the intervening construction assumes the form of four triangles. The bottom end of each triangle, being pressed together by the conjunction of the arches, causes the lower angle to be acute, but as it rises it becomes wider by the intervening space and terminates in the arc of a circle, which it supports, and forms the remaining [two] angles at that level. Rising above this circle is an enormous spherical dome which makes the building exceptionally beautiful. It seems not to be

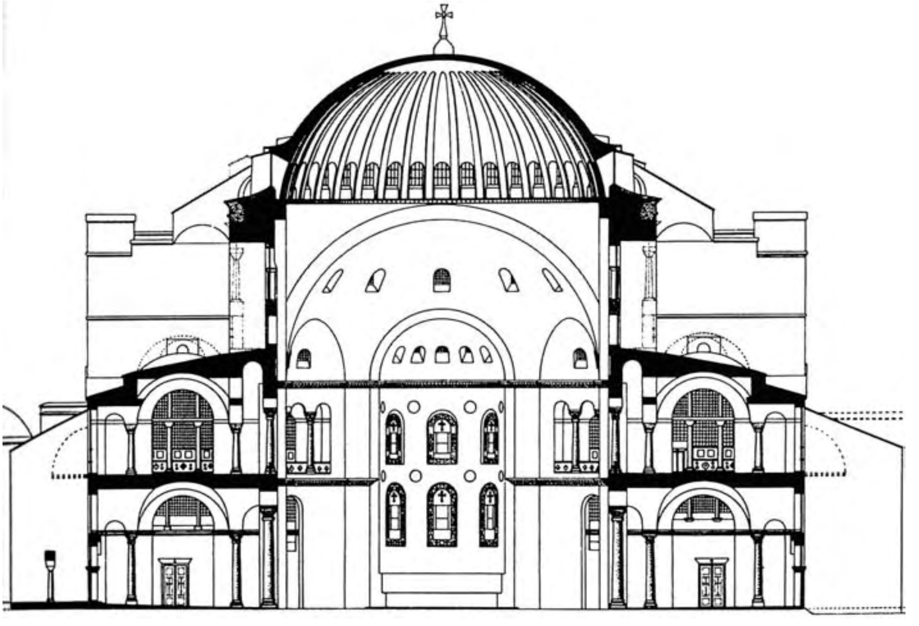

Fig. 2. Constantinople, Hagia Sophia transversal cut. Source: Andre Grabar, *L'Age d'Or de Justinien*, Gallimard, Paris, 1966. Courtesy of Gallimard, idem, p. 89.

> founded on solid masonry, but to be suspended from heaven by that golden chain and so cover the space. All of these elements, marvellously fitted together in mid-air, suspended from one another and reposing only on the parts adjacent to them, produce a unified and most remarkable harmony in the work... Whenever one goes to this church to pray, one understands immediately that this work has been fashioned not by human power or skill, but by the influence of God. And so the visitor's mind is lifted up to God and floats aloft, thinking that He cannot be far away, but must love to dwell in this place which He himself has chosen."[10]

In reading Procopius' description and in looking at the plan of St. Sophia (see illustration) it is almost as if we see, translated in stereometry, the geometry of the divine as interpreted by Proclus in his commentary on Euclid.[11] From the centre of the church, the lofty point from which radiates a dome, the church expands to the circular base of the dome, itself resting on four semi-circular arches. The circular base and semi-circular arches create four triangular spaces, the pendentives.[12] Arches and triangles lead down in turn to the

[10] Procopius *De aedif.* I, 1, 23ff. which I quote in the translation published by C. Mango, *The Art of the Byzantine Empire 312–1453*, Englewood Cliffs (1972), pp. 74–76. A description of the construction of St. Sophia can be read in C. Mango, *Byzantine Architecture*, Milan (1978), pp. 61–68.

[11] I limit myself here to the core of the building. A general and philosophically interesting description of the whole building can be found in R. Krautheimer, *Early Christian and Byzantine Architecture*, Harmondsworth (1975), pp. 215–230.

[12] These triangular spaces, bounded by the curved lines of the arches and dome (we remember that circular lines are prior to straight lines in Proclus' geometrical metaphysics), might be described, in geometrical terms, as 'spherical triangles', a conception first developed by Menelaus of Alexandria, a mathematician whose work was known to Proclus. In the course of the construction of St. Sophia, the structural deformation became such that

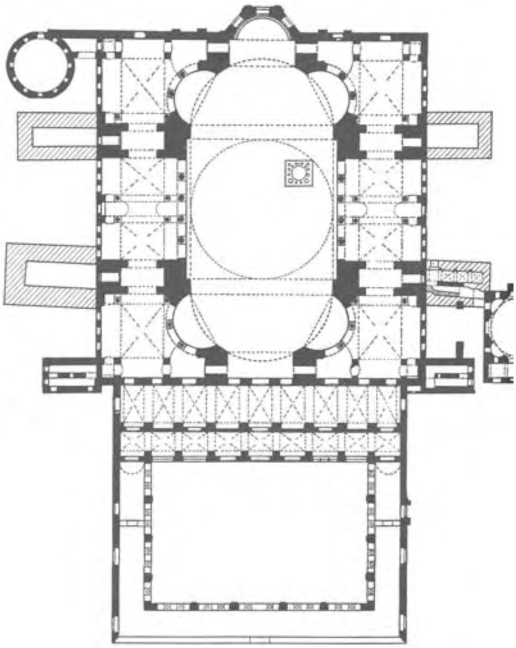

Fig. 3. Plan of the Hagia Sophia. Source: Andre Grabar, *L'Age d'Or de Justinien*, Gallimard, Paris, 1966. Courtesy of Gallimard, idem.

square composed by four massive piers. Expressed in solids, the sequence centre, circle, semi-circle, triangle and square manifests a perfectly controlled progression from unity to developing levels of perfectly unified multiplicity, ideal limitations of multiplicity which bring it back in stages to ever greater unity, back to the centre, transcendent source of all. The church thus corresponds, in visual space, to the metaphysical dynamics of unity and multiplicity, the progression of reality from, and reversion to, the Ultimate, as formulated by Proclus. Procopius does not omit to mention also the effect of the interior of the building on the human beholder, who feels drawn up by the power that radiates down, brought nearer to a life with God.

There are links between the architects of St. Sophia and Proclus' school in Athens. One of Proclus' pupils, Ammonius, became head of the Neoplatonic school in Alexandria. He too taught mathematics as part of the philosophical curriculum. In his school Eutocius the mathematician gave lectures on logic (and may have been Ammonius' direct successor as head of the school).[13] Eutocius addressed his commentary on Apollonius' *Conics* to Anthemius and an edition of Eutocius' mathematical work was prepared, it seems, by the other architect of St. Sophia, Isidore of Miletus, himself, like Anthemius, a mathematician in his own right and a teacher. A link between Proclus' philosophical teaching and architecture

the arches, leaning outwards, could no longer constitute a circular base for the dome, which became elliptical in form. The collapse of the dome in 558 led to an extension inwards from the arches which produced a circular base for the dome, but also a gap between the curves of the dome and of the arches bounding the pendentives.

[13]On Eutocius, see R. Goulet (ed.), *Dictionnaire des philosophes antiques*, Vol. III, Paris (2000), pp. 392–395.

may already be found in Proclus' works, one of which (the *De providentia*) is dedicated to Theodore, *mechanikos*. Proclus' authority in the philosophical circles of Athens and Alexandria, the use of his works in their curriculum and the personal links we have noted make it plausible that the architects of St. Sophia, themselves primarily mathematicians, were familiar with the ideas concerning the higher significance of geometry that Proclus conveyed in commenting on Euclid.

Can we go further? We can at least say that in visiting the new church of St. Sophia in the middle of the sixth century, those with an advanced education in the philosophy and mathematics of the time would with little difficulty have found there, expressed in three-dimensional space, the geometry of the divine that Proclus had found in Euclid.

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CHAPTER 7

Religious Architecture and Mathematics During the Late Antiquity

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MATHEMATICS AND THE DIVINE: A HISTORICAL STUDY
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Introduction

At the end of the Classical period and during the Early Middle Ages, the entire abstract, understandable, world appeared mystically represented in the perceptible world through symbolical forms. It was for this reason that the Fathers of the Church gave so much importance to symbols. According to Saint Augustine, the reason for symbols and their lesson was to light the fire of love and keep it burning, so that man could rise above himself, to reach a level he would not be able to reach alone.[1] The science of numbers had an enormous influence on the writers of this period, for numbers were essential, being signs corresponding to spiritual truths. We are going to see how numbers and geometrical figures played a fundamental role in the symbolism of religious building, a symbolic value which transcended any physical structure.

1. Religious architecture and heavenly measurements

In the Bible, the Temple was a copy of the heavenly Temple "prepared by Thee from the beginning".[2] The design of the Temple was not left to the architect's personal inspiration; it had been decided by God himself. The earthly Temple was built according to a heavenly plan which was communicated to man through a prophet who acted as intermediary: for the Ark, it was Noah who served as intermediary;[3] for the Tabernacle (the Ark of the Covenant), it was Moses;[4] for the Temple of Solomon, the intermediary was David, who had received the measurements from God.[5] This idea was taken up again in the New Testament: the New Jerusalem would come down from heaven.[6]

The four models given by Holy Scripture referred to by the founding fathers of the church were: Noah's Ark, the Tabernacle, the Temple of Solomon and New Jerusalem.

1.1. *The Ark*

Saint Irenaeus of Lyon (130–202) wrote about the Ark, which was a copy of the heavenly Temple.[7] Clement of Alexandria (150–216) states in *Stromata*[8] that the building was designed following divine intentions, using proportions full of meaning:

> The length of the Ark was 300 cubits, its width was 50 and its height 30; the Ark was completed up to a cubit from the roof, which became narrower from the base, in the shape of a pyramid, the

[1] Augustine, *Epistulae*, LV, ed. J.-P. Migne, *PL*, 33, 1845, col. 214.

[2] *Wisdom* 9, 8.

[3] *Genesis* 6, 14–16.

[4] *Exodus* 25, 8–22.

[5] *1 Chron.* 28, 11–12; *Wisdom* 9, 8.

[6] *Acts* 7, 44–50; *Apoc* 21, 2–3. Cf. [39].

[7] Irenaeus of Lyon, *Demonstration of the Apostolic Preaching*, IV, 10, ed. A. Rousseaux, Sources Chrétiennes n° 406, Paris, 1995, pp. 121 and 127.

[8] Clement of Alexandria, *Stromata*, VI, 11, ed. P. Descourtieux, Sources Chrétiennes n° 446, 1999, p. 235.

> figure of those who have been purified and tested by fire . . . . The proportions are in a ratio of 6 : 1 (300 to 50) or 10 : 1 (300 to 30) . . . . Some saw the symbol of the sign of the Lord in the 300 cubits, that of hope and redemption through Pentecost in the measure of 50; in the measure of 30 cubits, and according to some manuscripts, that of 12, scholars have tried to show the Gospel, as the Lord announced it when he was 30 and there were 12 apostles.

Tertullian (155–220) drew a parallel between the Ark and the Church,[9] as did Ambrose (339–397).[10] One can find this comparison in Jerome's writings (347–420).[11] As for John Chrysostom (340–407):

> The Ark is the Church, Noah is Christ, the dove is the Holy Spirit, the olive tree is the bounty of God.[12]

Going even further, for Augustine (354–430), the Ark was a forerunner of the Holy City.[13]

1.2. *The Tabernacle*

The cosmic symbolism of the Tabernacle was set out by Philo of Alexandria (20BC–50AD): the two-part division of the Tabernacle—the Holy for material things, the Holy of Holies for spiritual matters—was an image of the world.[14] According to Josephus (37–100), the three parts of the sanctuary corresponded to the three cosmic regions: the entrance represented "the sea", that is to say the lower regions, the Holy building the Earth, and the Holy of Holies represented Heaven; a three-part division between sky, earth and water, the natural elements of the universe.[15] Then the symbolism of the Tabernacle was taken up by Christian exegesis. So, for Origen (182–252), the Tabernacle was a representation of the whole world.[16] Theodore of Mopsuestia used the same symbolism; for him, the Tabernacle was a copy of creation.[17]

These comparisons reflect the concepts people had of the world at the end of the Classical period. In the 6th century, the link Cosmas Indicopleustes made between the Universe and the Tabernacle, one a model, the other a copy of the world with two regions one above the other, emphasises the resemblance between the world and a building.[18]

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[9] Tertullian, *De Baptismo, VIII*, eds. R.F. Refoulé & M. Drouzy, Sources Chrétiennes n° 35, Paris, 1952, p. 78.
[10] Ambrose, *De Noe et Arca*, VIII, 25 & XV, 52, ed. J.-P. Migne, *PL*, XIV, 1845, col. 371 & 386.
[11] Jerome, *Arca Noe ecclesiae typus fuit, Dialogus adversus Luciferianos*, 22, ed. J.-P. Migne, *PL*, 23, 1845, col. 176.
[12] John Chrysostom, *Homilia in terrae motum et in divitem et Lazarum*, VII, ed. J.-P. Migne, *PG*, 48, 1862, col. 1038.
[13] Augustine, *De civitate Dei*, XV, XVII, 1, ed. J.-P. Migne, *PL* 41, 1845, col. 472.
[14] *De Vita Mosis*, III, 6, quoted *in*: [9, p. 61].
[15] Josephus, *The Antiquities of the Jews*, III, 6, 4, ed. J. Weill, Paris, 1900, I, pp. 170–171.
[16] Origen, *In Exodum Homilia*, IX, 4, ed. M. Borret, Sources Chrétiennes n° 321, Paris, 1981, p. 295.
[17] Theodore of Mompsuestia, *Homilies*, XII, 3, eds. R. Tonneau & R. Devresse, Citta del Vaticano, 1949, p. 327.
[18] [43, p. 12ff].

1.3. *The Temple of Solomon*

The Temple seemed to be the model for all religious architecture, a model which every Christian building copied. For Augustine, it was the model of the Church.[19] For Theodoret, it was the model for all churches throughout the world.[20]

In the same way, the *tituli votivi* (monumental inscriptions placed in churches which help one interpret the sacred building, through references to examples found in the Scriptures) often refer to the Temple of Solomon: they draw a parallel between the building and the Temple of Solomon.[21] In this way, the person who founded the church is identified with Solomon, the king who had built the Temple in Jerusalem.[22]

1.4. *New Jerusalem*

The building had to be in the image of the eternal home, the New Jerusalem described by John.[23] The comparison with New Jerusalem spread during the 3rd century.[24] Eusebius of Caesarea (265–340) claimed that the Temple was an image of New Jerusalem.[25] According to him, the bishop, Saint Paulinus, built a temple in Tyre following divine inspiration, in order to make a physical reproduction of the heavenly plan, like a visible emblem of the invisible Temple.[26] The building had been designed "according to descriptions supplied by the holy oracle".[27] "Above all of these marvels are the meaningful and divine archetypes, prototypes and models".[28] The Word, who gave order to every thing, had himself made a copy of the heavenly plan on Earth, the Church of the "first-born, written in the heavenly book", the Jerusalem from above, Zion, the Mountain of God and the City of the Living God.[29]

Many texts compared the church to Heaven: in the 3rd century rite of the dedication of churches, the hymn *Urbs Beata Jerusalem* compared the church with New Jerusalem.[30]

In the East, the 5th century *Testamentum Domini Nostri* mentioned the existence of a heavenly prototype for the Christian building—the empire of God, the heavenly kingdom.[31]

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[19] Augustine, *Enarratio in Psalmum*, 126, ed. J.-P. Migne, *PL*, 37, 1845, col. 1668.

[20] Theodoret, *Libr. 1 Paralipomena q. 1*, ed. J.-P. Migne, *PG*, 80, 1860, col. 236.

[21] Paulinus of Nola, *Carmina* 27, 477 & 28, 311; ed. J.-P. Migne *PL*, 61,1847, col. 659 & 670; Apollinaris Sidonius, *Epistulae* VII, 9, 21 & IV, 18, 5, ed. A. Loyen, 1970, pp. 59 & 153; Fortunatus, *Carmina* II, 10 & III, 6, ed. M. Reydellet, I, Paris, 1998, pp. 66 & 92; Pietri, 1983, n° 16.

[22] Fortunatus, *Carmina* III, 20 (for Felix, Bishop of Bourges), *Carmina* VI, 2 (for King Charibert), ed. M. Reydellet, Paris, I, p. 118 & II, p. 56.

[23] *Apocalypse* 21, 2.

[24] [27, p. 14], [19, pp. 227–229].

[25] Eusebius of Caesarea, *Histoire Ecclésiastique*, X, 4, 3, ed. G. Bardy, Sources chrétiennes n° 55, Paris, 1967, p. 82.

[26] *Ibid*, X, 4, 26, p. 89.

[27] *Ibid*, X, 4, 43, p. 95.

[28] *Ibid*, X, 4, 55, p. 99.

[29] *Ibid*, X, 4, 65, 70, p. 103.

[30] [37, p. 103f].

[31] Cf. [40, p. 52], [42, p. 45].

One wonders how much influence these plans had on the architecture of this period.

According to Kitschelt, the Christian basilica represented the heavenly city of Jerusalem (the door = the gate into the town; the nave = the *via sacra*; aisles = arcades, the nave and transept = the *cardo* and the *decumanus*, the triumphal arch = the *arcus triomphalis*, the presbyterium = the throne room in the royal palace).[32] But this interpretation has been much criticised.[33] In fact, the building itself did not respect the archetype, but the idea of the archetype was transmitted through the building. A shape was imitated uniquely because of the value it bore. In this way, the building could refer to several archetypes at the same time.[34] Sometimes one even comes across a mixture of competing comparisons (e.g. the Temple of Solomon, the heavenly city, paradise). Thus, Eusebius[35] compared the church to Jerusalem, but Constantine is compared both to Bezalel and Solomon, builders, respectively, of the Ark of the Covenant and the Temple. It is important to realize that imitation and illusion were only partial. Many representations were "mixed" in the symbolism of the building.[36]

In fact, from the 2nd century, the church was above all a reflection of the Christian community.[37]

It must be said that the building was not designed according to the heavenly archetype, as, according to beliefs in the Middle Ages, the only way to pass beyond the perceptible world was through dissimilar designs. Giving something a shape did not mean that the object represented had that appearance. On the contrary, it was a question of giving it another appearance, changing the way one would look at it:

> this raw material itself, having received its physical aspect from Absolute Beauty, keeps, through all its physical alterations, some elements of intellectual beauty. It is possible, through the presence of this raw material, to raise oneself up to the spiritual archetypes. However, we must always be sure we understand the metaphors through their very lack of resemblance, that is to say, to consider them always in an identical way, taking into account the distance which separates the understandable from the perceptible and defining them in the way which suits each one.[38]

2. Religious architecture and geometrical measurements

Architecture, with its interplay of numbers and geometrical figures, was a way of creating the world.

Already in the Bible, numbers were very important. Everything was a question of measurement and proportion: "Thou hast ordered all things by measure and number and

[32] [27], [25, p. 20].

[33] See especially [28,42,4].

[34] Cf. [4].

[35] Eusebius of Caesarea, *Histoire Ecclésiastique*, X, 4, 3, ed. G. Bardy, Sources chrétiennes n° 55, Paris, 1967, p. 82.

[36] Cf. [40].

[37] Cf. [42, p. 36ff]. D. Iogna Prat has shown that this theory persisted until the 11th century: only in the 12th century did it become possible to represent the Biblical edifice. So, in *Gemma Animae*, Honorius Augustodunensis pointed out that the Tabernacle of Moses was divided in two—the first part corresponded in the Christian context to the *domus anterior* where the faithful congregated, while the clergy were in the second part (the *sancta sanctorum*). Example taken from [22].

[38] See Pseudo-Dionysius the Areopagite's theory on multiplicity in *On the Celestial Hierarchy*, XV, 114B–114C.

weight".[39] Earlier we saw that it was God who gave Noah the dimensions of the Ark, Moses those of the Tabernacle, Solomon those of the Temple. In a dream, Ezekiel received the dimensions of the new Temple.[40] Isidore of Seville (560–636) wrote a treatise on the "shadows which the Scriptures mention".[41]

Numbers were in fact necessary to pass from the terrestrial world to the celestial. To the Fathers of the Church, they really did have the status of magic keys.[42] It was Augustine who gave his backing to the acceptance of the symbolism of numbers as necessary to cross, by analogy, from the perceptible to the abstract: "We have praised God for ordering all things according to number and measure and weight".[43] He saw an image of the absolute in numbers. It was through visible shapes that divinity, the invisible, showed itself, made itself known.[44] Without perceptible form, man would not be able to reach spiritual realities. Visible analogies were necessary for men to approach the divine, the invisible, to establish a link between the outward visible sign and the invisible spiritual mystery. Geometrical shapes were applied as simulations comparable to divinity. Mathematics, the science of number, was the best way to gain access to the sphere of the abstract and the divine. So, numbers were the key which allowed access from the world of the senses to that of the intellect.

The numbers 4, 7 and 8 were particularly important.[45] Four, which evoked the four points of the compass and the four physical elements—earth, water, air and fire—was linked with the four evangelists, the four directions of the Cross, the four rivers of Paradise. The week had a place of honour for the Fathers of the Church. They all referred to the hexameron. The creation of the world in six days and God's rest on the seventh were seen to be spiritually significant: there was a link between the creation of the world and its development. The addition of 3 and 4, of the spiritual and the temporal, gave 7, which symbolised all numbers. Eight symbolised Eternity, the number of the resurrection of Christ and man in his perfect state. It was on the day following the Sabbath, and therefore the eighth day, that Christ came forth from the tomb. He embodied eternal blessedness. According to Augustine, the eighth day represented a return to the life before original sin, a life which had not disappeared for ever, but which had become eternal. And the eight people saved on the Ark from the waters of the flood were an allegory of the Resurrection.[46]

The circle and the square were the most common geometrical shapes found in Scripture and amongst the Fathers of the early Church. They were the first shapes. "When He established the heavens, I was there: When He set a circle on the face of the deep".[47] As for the Temple of Solomon: "This inner shrine was twenty cubits square and twenty cubits

[39] *Wisdom* 11, 20.

[40] *Ezekiel* 42.

[41] Isidore of Seville, *Liber numerorum qui in sanctis Scripturis occurunt*, ed. J.-P. Migne, *PL*, 83 , 1850, col. 179–200.

[42] See, for example, Augustine, *De ordine, XV*, 42–43 (*Geometria et Astronomia*), ed. J.-P. Migne, *PL*, 32, 1861, col. 1014–1015.

[43] Augustine, *De civitate Dei*, XI, 30, ed. J.-P. Migne, *PL*, 41, 1845, col. 343–344.

[44] For example, Augustine, *De Trinitate*, II, V, 10, ed. J.-P. Migne, *PL*, 42, 1841, col. 851.

[45] [37, pp. 62–65], [31], [13, pp. 60–65].

[46] Augustine, *Epistulae*, LV, 8–9 & 17–18, ed. J.-P. Migne, *PL*, 33, 1845, col. 208–209 & 212–213.

[47] *Proverbs* 8, 27.

high".[48] The Temple was often cited as a model;[49] in the same way, the Holy of Holies measured five cubits by five.[50] The idea of the square and the double square reappeared in Ezekiel.[51] New Jerusalem was also a square city.[52] Augustine chose the circle, after eliminating the triangle and the square, as the shape which would be most suitable to convey the unity of the soul, as its surface was limited only by a single, indivisible, line.[53]

In architecture we come across the Biblical squares and double squares. The plan of Hagia Sophia in Constantinople is made up of two double squares and a central square:[54] the circular central dome is inscribed in a 100 foot square, and the two low semi-domes which abut it also measure 100 feet in diametre. The secondary columns which surround it make a double square 100 feet by 200 feet, 15 feet apart. San Vitale in Ravenna is designed around a 50 foot square which defines the limits of the octagonal dome; this central square is flanked by four double squares measuring 25 feet by 50, separated from the central square by the thickness of the masonry of the arcade which supports the dome.[55] In the Baptistry of Nisibis, the three dimensions are identical, showing that the architect wanted a perfect cube.[56]

Ratios and proportions in the basilicas of the 5th and 6th centuries in Ravenna and on the northern coast of the Adriatic are based on the figures 4, 7 and 8:[57] for example, in the churches of San Giovanni Evangelista or Sant'Agata Maggiore in Ravenna, started in the 5th century and completed in the 6th, the naos is 4 squares by 7 in dimension and 4 squares by 8 with the apse. In plans dating from the 5th century, the ratio 4 : 7 always determined the proportions of the naos, while the apse was the part which gave the overall ratio between width and length as 4 : 8 (= 1 : 2); this meant that the design of the whole building was based on 2 squares. At Sant'Apollinare in Classe, the proportions of the whole complex are 4 : 7 : 11, terms of the mathematical series 1, 3, 4, 7, 11, 18, ..., where the sum of the last two terms is the next in the series. It was the idea of continuous division which made it possible to find harmony between the parts and the whole in a work of art. The narthex appeared in the 6th century; it was often added on to the naos and the apse to form a new ratio. Generally speaking, the centre of the apse was decided upon according to the line of the seventh square. The ratio between the nave and the aisle was always 1 : 2. The ratio 1 : 2, one square in width, two lengthways, defined the length and width of the whole building. These were the rules which had already been laid down in Vitruvius' books. The width must be at least a third, at most half, of the length, or there would be no Christian symbolism .... This ratio was very different from that of the Ark (6 : 10),[58] proof that the theoretical archetype did not bear decisive influence on the actual architecture.

[48] *1 Kings* 6, 20. Cf. [23, p. 88f].
[49] Cf. above.
[50] *Exodus* 27, 1.
[51] *Ezekiel Chs* 40 ff.
[52] *Apocalypse* 21, 16.
[53] Augustine, *De quantitae animae*, 10–16, ed. J.-P. Migne, *PL*, 32, 1861, col. 1042–1045.
[54] [23, pp. 201–207], [24, pp. 151–156].
[55] [24, pp. 161–163]. For other examples of square and double-square churches: [23, p. 94], [24].
[56] [26, pp. 409& 411].
[57] [34,1].
[58] Cf. above Section 1.

The cruciform plan was one of the liturgically-based demands, where its importance was underlined: the cross figured on the plans Eudoxia, wife of Theodosius II, sent to Saint Porphyry, bishop of Gaza, in the 4th century; she had asked him to have a church built on the ruins of a pagan temple: "*ecclesia in figuram crucis . . . factum ex divina revelatione*". The last words show the importance that was given to this symbol.[59] According to Gregory of Tours, Clermont cathedral was built at the end of the 5th century along the same lines,[60] taken from *Apostolic Tradition*. The Cross had become the Christian symbol par excellence, the symbol of redemption. Ambrose of Milan considered the Cross as the emblem of Christ's victory.[61] It was used by architects, who started by incorporating it in the martyria (cf. the Basilica Apostolorum in Milan). Then the shape was used for churches dedicated to the Holy Cross. In San Lorenzo Formoso in Ravenne, the building is in the shape of the Greek cross; a golden cross in the dome, to which Peter and Paul pay homage, indicates the East, the orientation of the apse of the palace church; on the path which took Lawrence to where he would be burnt to death, he carries Christ's cross; above the entrance, this same cross is set up by *Christus Victor*, peace incarnate enthroned in paradise. In the same way the sign of the cross, represented in the dome entirely dominates Empress Galla Placidia's mausoleum.

The octagon evoked the resurrection of Christ. Ambrose of Milan wrote of the Santa Tecla baptistry in Milan: "it was right that the room of holy baptism be built according to this number, which is the one by which the people obtained true salvation, in the light of the risen Christ".[62] This was the reflection of a conscious preoccupation with symbolism, intended from the start, and not a *post factum* explanation. In this case, it was religious thought which decided architectural form. But, when there is no written trace, which is the case most of the time, the question of meaning has to remain open: the choice of plan may have been dictated quite simply by architectural needs. In that case, Christian ideology was applied *a posteriori* to give a meaning to the architecture[63] or to tradition (with particular reference to Vitruvius).

3. The building in its Earth–Heaven dialectic, or the circle above the square

Not only was the design of the building given by God himself, but the whole sacred edifice itself was cosmic, made in imitation of the world. The circle standing above a square was enough to symbolise the universe in its Earth–Heaven dialectic, for the Temple was a miniature universe placed within reach of religious man, for use in his praise and worship of God, a microcosm reflecting the macrocosm. It was an image of heavenly, eternal truths.

Many writers established a correspondence between the earthly Temple and the heavenly one. The sanctuaries down here were a reflection of the home above, stone buildings were

[59] *Acta Sanctorum, februar*, quoted in [2, III, p. 105].

[60] Gregory of Tours, *Historia Francorum*, II, 16, ed. R. Latouche, Paris, 1995, pp. 105–106. Cf. the accounts of Gregory of Nazianzus, *Somm. de Anast. eccl. 2, 16. 60*; Procopius, *De aedif. Iustin., I, 4*; & Bede, *De locis sacris*, 15.

[61] *Inscriptiones latinae christianae veteres*, I, ed. E. Diehl, Berolini, 1925, n° 1800.

[62] Cf. [33].

[63] Cf. [39, pp. 160–161].

the reflection of heavenly truths.[64] For Fortunatus, the building ends up by looking like a reproduction of the heavenly universe; *carmen* III, 7 underlined the links between Earth and Heaven. "The roof of the church looks like another Heaven which has its own stars." The cover of the *ciborium* itself was compared to a heaven.[65]

The building was indestructible. It was eternal, like Christ: "*non peritura domus*".[66] Going even further, entering the church on Earth meant entering the heavenly sanctuary: "You start towards the altar; the angels watch you; they have seen you starting; they have seen your appearance".[67] Once in the sanctuary, they contemplated the hidden mysteries.[68]

According to the Fathers of the Oriental Church, the church was also a perfect image of the perceptible world. So, for Maximus the Confessor (580–662), the church "has the sky for sanctuary and the magnificent Earth for nave".[69] Maximus the Confessor's mystagogy could be adapted very easily to oriental churches, which were cubic structures surmounted by domes. The same idea appeared in the Syrian hymn of Edessa cathedral (6th century): "It's a wonderful thing, that in its smallness, this temple should resemble the wide world".[70] Philogathos, writing about the Cappella Palatino in Palermo, mentioned that its ceiling "decorated with fine sculptures, which form coffers and its shining gold, represents Heaven when, in still air, it shines through the choir of its stars".[71]

For Cosmas Indicopleustes, the form of the square surmounted by the circle was sufficient to symbolise the universe in its Earth–Heaven dialectic (a double journey—a horizontal one between nave and aisle—and a vertical one between cube and dome). These two spaces corresponded to the two catastases of man, that is to say, his two transcendental conditions—the first, made of death and change, of tests and conflict; the second, unchanging and eternal, where suffering cannot reach him and he is filled with perfect knowledge. Through a system of correspondence, God has set up a universe appropriate for these two states, and divided it into two distinct environments. The world down here, surrounded by the Earth and the firmament, and the world above, circumscribed by the firmament and the vault of the upper sky, the kingdom of Heaven, incorruptible and eternal. Once He had set up the universe in this way, God turned to the creation of man, "link of Creation", a physical and spiritual being, endowed with intelligence and perception, visible and invisible. This double nature alone set man apart for two roles. Starting off in this world, the things he learnt here allowed him to go through to the second space, the kingdom of Heaven. And it was Christ, man and God at the same time, who was sent to bring mankind from the first

[64] Paulinus of Nola, *The Mystery of the Trinity, Epistulae* 32, 5; 32, 10; 32, 13, ed. J.-P. Migne, *PL*, 61, 1847, col. 331, 335, 337; Fortunatus, *Carmina*, III, 7, ed. M. Reydellet, I, p. 96; The author of the titulus of Sts Peter and Damian in Rome, *Inscriptiones christanae Urbis Romae*, II, 71, 41.

[65] [3, p. 192], [41, p. 52ff].

[66] Apollinaris Sidonius, *Epistula*, IV, 18, 5, ed. A. Loyen, Paris, 1970, p. 153; Fortunatus, *Carmina*, I, 3, ed. M. Reydellet, I, p. 23; Paulinus of Nola, *Epistulae*, 32, 18 & 32, 23, ed. J.-P. Migne, *PL*, 61, 1847, col. 339 & 341–342; cf. [36, pp. 153f].

[67] Ambrose, *De Sacramentis*, IV, 5, ed. Dom Barbotte, Sources Chrétiennes n° 25 bis, Paris, 1961, p. 105.

[68] *Ibid*, III, 1 5, p. 93.

[69] Maximus the Confessor, *Mystagogy*, 3 & 6, *PG*, 91, ed. J.-P. Migne, 1860, col. 669 & 684.

[70] See [12] for the translation, [15] and [16] for the commentary. See also [32].

[71] Quoted *in*: [17, p. 675].

condition to the second, reestablishing the link of Creation which had been broken through Adam's transgression.[72]

It must be noted that the heavenly ladder as evoked by the Fathers of the Church from the 3rd century, and in Christian art from the 4th, reflected this belief: the ladder linked the physical world with Heaven and in this way evoked a hierarchical concept of the world, as is described notably by Pseudo-Dionysius the Areopagite.[73]

The building, as has been seen, was the image of the kingdom of God, of the divine order which encompassed the whole world (Earth and Heaven). Even the liturgy on Earth was the reflection of the heavenly liturgy. Mass was a sacramental representation of the sacrifice on the cross and a sacramental participation in the heavenly liturgy: every Christian initiation was a partaking in Christ's death and resurrection.[74] As all the ritual of the Temple was accomplished in Christ's incarnation, death and resurrection, the assembly united in communion and in the presence of the risen Christ, at the holy altar.[75]

The liturgy on Earth joined the heavenly liturgy which the angels celebrated; the community, during the eucharist, was "raised up into heaven", or rather, was in a risen space where there was no distinction between Heaven and Earth, between men and angels.

> Above, the angelic armies give praise. Down here, in church, men in choirs take up the same doxology. Up there, the angels of fire beat out the three-times holy hymn; here, man sends back the echo. The feast of those who live in Heaven joins that of the Earth-dwellers: one thanksgiving, one burst of happiness, one joy-filled choir.[76]

One can make an interesting comparison with the beliefs which were widespread in the East at the time. According to *The Interpretation of the Offices* a Chaldean text of the 7th–8th centuries, taking the cross to the *bema* symbolised Christ's going up to Jerusalem. Taking it away represented His arrest by the Jews who wanted to crucify Him and His carrying of the cross from Jerusalem to Calvary.[77] According to another source, *Explanation of the Offices of the Church* (a text which dates from the 9th century, but which accepted the liturgical prescriptions of the 7th and 8th centuries), the apse, which was the sanctuary or the Holy of Holies, symbolised Heaven. In front of the sanctuary there was a raised platform, higher than the level of the nave, the *qestroma*, and it represented paradise which rose to Heaven, but which belonged to the Earth. The nave represented the Earth. The *bema* in the middle of the nave symbolised Jerusalem, which was the centre of the world. In the middle of the *bema*, the altar symbolised Golgotha. Between the *bema* and the sanctuary there was a passage which should be relatively clear, symbolising the path of truth, which one took to reach Heaven. This was the place where any processions would take place, where the priests would walk two by two. At the end of the service, the veil of the sanctuary would be drawn shut, to indicate that heaven was closed again.[78]

This idea came up again in the writings of Narsai [consecrated bishop of Senna by Timothy I (780–823)]: the visible, physical church symbolised the church of Jerusalem, which

[72] [43, pp. 37–38 & 59].
[73] [20]. The reference is to Jacob's ladder, *Genesis* 28.
[74] See [8,39].
[75] [6, p. 25].
[76] John Chrysostom, *Homilia, 4, PG*, 56, ed. J.-P. Migne, 1860, col. 120.
[77] [10].
[78] Cf. [29], [30] and [40].

was on Earth. The *bema* represented Zion, the altar which was on the *ambon* represented the Ark of the old Covenant, the cross which was placed on it and the Gospel represented the New Testament and Christ's Earthly throne, as well as the link which existed between the Old Testament and the New. The assembly of priests on the *bema* symbolised the group of apostles and the passage which led from the *ambon* to the apse represented the narrow path to Heaven.[79]

Because the constantly renewed incarnation of the *Logos* was accomplished through the liturgy on the altar, it was necessary to show the Epiphany of the heavenly word on the triumphal arch. The iconography was linked to the liturgy as it was re-enacted at the altar. This depiction could be done in one of two ways: through the worship given to the heavenly master of the world (the *Parousia*), or through the *Logos* which came down to this world (the Incarnation). Both approaches were based on the law which linked Heaven and Earth, the divine throne and the altar, God and man.[80] For example, in the scene of the *Parousia* in the San Paolo basilica in Rome, (a Constantinian church added to in the 5th century by Galla Placidia, in the spirit of Old St Peter's), the bust of the *Pancreator* is depicted in front of an invincible sun which is emitting rays in a circle which floats above the zenith of the arch and among the four symbols of the evangelists: the bull, the angel, the lion and the eagle. The figures which surround Christ refer to Chapter 4 of the Apocalypse: on either side of the Saviour two prostrate winged angels represent the heavenly legions. The 24 elders worship Him also, giving Him their crowns, then in the arch drops one sees the princes of the apostles, Peter and Paul. On the triumphal arch of Santa Maria Maggiore in Rome (5th century), where scenes of the life of Christ are depicted, there are representations of Bethlehem and Jerusalem, the towns where the *Logos* was born and died in the Holy Land of this lower world. The circle of the zenith, around which the four evangelists float, with the apostles Peter and Paul on either side, surrounds an empty throne which is identified by the cross and the dove as the seat of the *Logos*.[81]

In the same way, in the Byzantine churches of the same period, the pictures were designed according to a precise plan: the central dome was decorated with a representation of Christ dominating all of creation. So, in San Vitale in Ravenna (6th century), the groin vault which dominates the altar has at its zenith, in the middle of a starry sky, the Lamb of God exactly over the centre of the altar, on which, at each celebration of the liturgy, His sacrifice is re-enacted (links between Earth and Heaven). A powerful triumphal arch goes up to the bases of the vault. It presents a frieze (much used after Galla Placidia): this frieze contains a continuous series of medallions of the apostles and saints and, at the apex, the bust of the Pancreator in the axis as the *Agnus Dei* which, through this represents the work of redemption of this Pantocrator, made flesh in the sacrament on the altar. "We are at the centre and in the exact spot where Heaven transforms Earth."[82]

This iconography was the expression of the union, re-enacted at each celebration of the Eucharist, between the heavenly church and the victorious and militant church on Earth.

During the Late Antiquity, the symbolism of Christian buildings developed at two levels: the church was at the same time a reflection of the heavenly church and an image of the

[79] [32, p. 95].
[80] [27, p. 58], [21], [14, p. 68f].
[81] [5, p. 55].
[82] [14, pp. 222 & 230].

universe. The use of numbers and geometrical figures revealed something deeper, more fundamental and more mysterious than visible reality, as the early Church fathers' writings, archeology, epigrams and pictures show us. It allows a link to be established between the two levels of cosmic reality, Earth and Heaven, and is a manifestation of God's presence. All the same, one cannot ignore the hiatus which, except in a few very special cases (e.g. the Baptistry of Milan), existed between the architectural plans common to all the architects of the time (the proportions of the basilicas, the cruciform plan, especially for mausoleums, wide-spread usage—by pagans as well as Christians—of the dome and the apse) and the Christian meanings which the Fathers of the Church attributed to them, using an intellectual approach which they often shared with pagan thinkers.[83]

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In the footnotes the following abbreviations are used:
PG: Patrologiae cursus completus, series graeca,
PL: Patrologiae cursus completus, Patres Ecclesiae Latinae.

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CHAPTER 8

The Sacred Geography of Islam

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MATHEMATICS AND THE DIVINE: A HISTORICAL STUDY
Edited by T. Koetsier and L. Bergmans

1. Introduction

In Islam, as in no other religion in human history, the performance of the most important ritual acts—prayer at specific times and in a specific direction, and fasting during a particular month—has been assisted by procedures involving astronomy and mathematics. This activity resulted from a distinctive conception of time and space in Islam. The organisation of the lunar calendar, the regulation of the astronomically-defined times of prayer, and the determination of the sacred direction towards the sacred Kaaba in Mecca—these are three topics of traditional Islamic science still of concern to Muslims today, and each has a history going back close to 1400 years. They form a small but highly significant part of a scientific tradition which knew no rival for several centuries. This paper deals with the third of these topics, the sacred direction in Islam and the corresponding conceptions of a sacred geography.

The Kaaba is a shrine of uncertain historical origin that served as a sanctuary and centre of pilgrimage for centuries before the advent of Islam in the early 7th century. In Muslim tradition the Kaaba was built by Abraham and Ishmael; otherwise practically nothing is known of its early history beyond what can be intimated from the nature and layout of the edifice. In any case, the *Qurân* advocates prayer towards the Kaaba, and the edifice was adopted as the focal point of the new religion, a physical pointer to the presence of God. Thus, since the early 7th century, Muslims have faced the Kaaba during their prayers. The sacred direction is called *qibla* in all languages of the Islamic commonwealth. Mosques are built with the prayer-wall facing the Kaaba, the direction being indicated by a *mihrâb* or prayer-niche. In addition, according to Islamic law, certain ritual acts—such as reciting the *Qurân*, announcing the call to prayer, and slaughtering animals for food—are to be performed facing the qibla. Also Muslim graves and tombs were laid out so that the body would lie on its side and face the qibla. Thus the qibla is of prime importance in numerous aspects of the life of every Muslim.

2. The dichotomy of science in Islamic civilisation

Islamic science functioned on two different levels throughout the period 750–1900. First, there was the folk scientific tradition, which had the backing of the religious authorities. Second, there was the mathematical tradition, which was cultivated by a small subset of society but whose prescriptions were generally too complicated to be put to into practice. Nowhere is this dichotomy more clear than in the situation regarding the obligation of Muslims to pray and to perform various other ritual acts facing the qibla.

The Muslims inherited the folk astronomical and folk cosmological notions of pre-Islamic Arabia, and when they expanded from the Arabian Peninsula they encountered the folk astronomy of the Hellenistic world. Arabic texts on folk astronomy, of which the most developed stem from the Yemen, deal with the changing seasons, with the stars and the lunar mansions, with the phases of the moon, with time-reckoning by shadow-lengths, as well as with the winds and the effects of the seasons on agriculture. In particular, directions were defined by astronomical horizon phenomena: the risings and settings either of the sun at the equinoxes and solstices or of the brightest fixed stars. As we shall see, the techniques of folk astronomy were employed by the legal scholars to determine the qibla, and a sacred geography defining the positions of various parts of the world in relation to the central shrine, the Kaaba, was developed (Section 3).

The Muslims were also heirs to the Hellenistic, Iranian and Indian traditions of mathematical astronomy and to the Hellenistic tradition of mathematical geography and cartography. They assimilated these with remarkable facility, and it was not long before they were in a position to criticise and improve upon them, and to select what they found most acceptable. Out of this amalgam of notions there developed a distinctly Islamic astronomy, which was without rival from the 9th to the 15th century. (The label "medieval" used for this period and the scientific tradition which it produced has no negative connotation since the tradition was unrivalled at the time; the term is here intended simply to distinguish between the scientific traditions of Antiquity and the modern age.) Muslim scientists

Fig. 1. A diagram of sacred geography with the qiblas of eight different regions of the Islamic commonwealth displayed around the four walls of the Kaaba. This is found in one of the copies of the cosmography of the early-15th-century scholar Pseudo-Ibn al-Wardî. [Taken from MS Istanbul Topkap AIII 3025, fol. 30v, courtesy of the Topkap Library.]

developed new trigonometric techniques and manifold tables for solving various astronomical problems. As we shall see (Section 4), they applied mathematical techniques to the available geographical data in order to calculate the qibla in various places. The scientists developed a sacred geography of their own, defining the directions of various cities to the centre, now Mecca, the city, rather than the Kaaba, the edifice. This distinction is significant and characterises the two different approaches.

3. The sacred geography of the legal scholars

The scholars of the sacred law of Islam created a sacred geography based on the notion that in order to face the Kaaba in any region of the world one should face the same direction in which one would be standing if one were directly in front of the appropriate segment

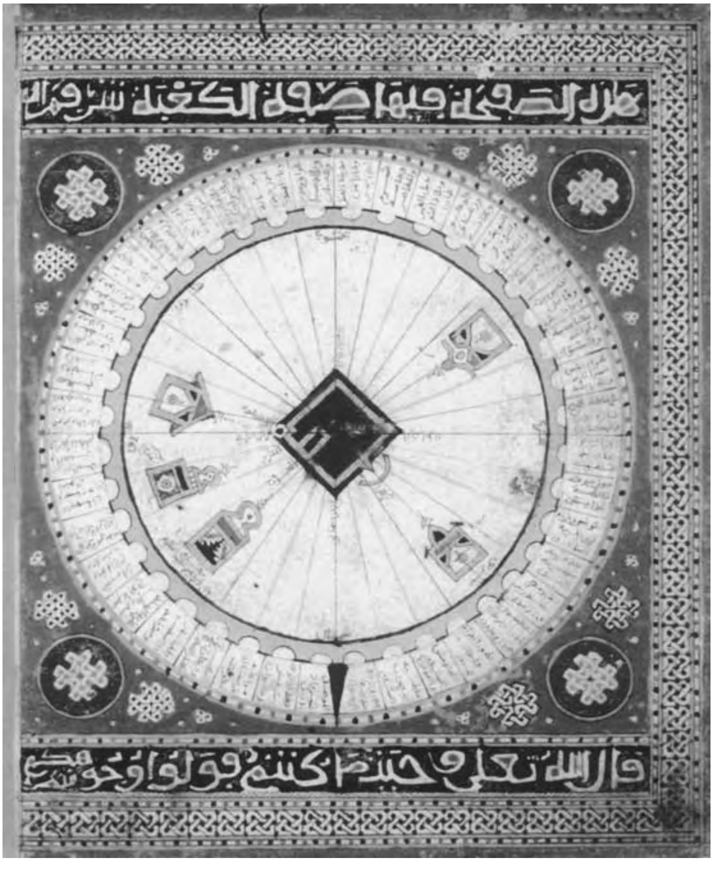

Fig. 2. A diagram of the world divided into sectors about the Kaaba, found in a copy of the navigational atlas of the 16th-century Tunisian scholar Ahmad al-Sharafî al-Safâqusî. [From MS Paris BNF ar. 2278, fol. 2v, courtesy of the Bibliothèque nationale de France.]

of the perimeter of the Kaaba. Since that sacred edifice is itself aligned in astronomically-significant directions, the directions adopted by the legal scholars for the qibla were towards the risings and settings of the sun at the equinoxes or the solstices or of various significant qibla-stars.

From the 9th to the 16th century Muslim scholars working exclusively in the folk astronomical tradition developed a tradition of sacred geography in which the qiblas of different regions of the world around the Kaaba were associated with particular astronomical horizon phenomena. Some 20 different schemes of this kind of sacred geography—sometimes illustrated in manuscripts, sometimes described in word—are known from some 30 different medieval sources, and there are surely more yet to be rediscovered. These directions adopted for the qibla obviously corresponded only roughly to the qiblas that were calculated by the Muslim astronomers. This tradition was particularly popular in the medieval Yemen, not least because it was a legal scholar of Yemeni origin named Ibn Surâqa who ca. 1000 first proposed various serious schemes with 8, 11 and 12 sectors around the Kaaba. In various later works, such as the geographical writings of Yâqût (Syria ca. 1200) and al-Qazwînî (Iraq ca. 1250), the information on the qibla is suppressed, and in yet later works such as the nautical atlas of Ahmad al-Sharafî al-Safâqusî (Tunis ca. 1575) and various other Ottoman compilations, the localities are uniformly distributed on a ring around the Kaaba with no specific qibla-values. Figures 1 and 2 show two examples of these schemes of sacred folk geography, involving no calculations whatsoever.

4. The sacred geography of the scientists

4.1. *Mathematical solutions to the qibla problem*

Muslim astronomers from the 8th century onwards concerned themselves with the determination of the qibla as a problem of mathematical geography. This activity required knowledge of geographical coordinates and involved the computation of the direction of one locality from another by procedures of geometry or trigonometry, such as the analemma (a sophisticated device for reducing problems on a sphere from three dimensions to two) or spherical trigonometry.

The qibla at any locality was defined as the direction to Mecca along the great-circle on the terrestrial sphere. The basic problem, illustrated in Figure 3, is to determine the direction to Mecca M from any locality X, given the latitudes of both localities, measured by MB $(=\phi_{\mathrm{M}})$ and XA $(=\phi)$, and the longitude difference AB $(=\Delta L = L - L_{\mathrm{M}})$. The qibla is measured by the angle AXM $(=q)$. We shall also have occasion to use the latitude difference XD $(=\Delta\phi = \phi - \phi_{\mathrm{M}})$ and the distance to Mecca XM $(=d)$.

Once the geographical data are available, a mathematical procedure is necessary to determine the qibla. The first Muslim astronomers who considered this problem developed a series of approximate solutions, all adequate for most practical purposes. The approximate solutions were all inspired by cartographic considerations. However, also in the early 9th century, accurate solutions by solid trigonometry and projection methods were formulated, and in the 10th century solutions by spherical trigonometry. Over the centuries one

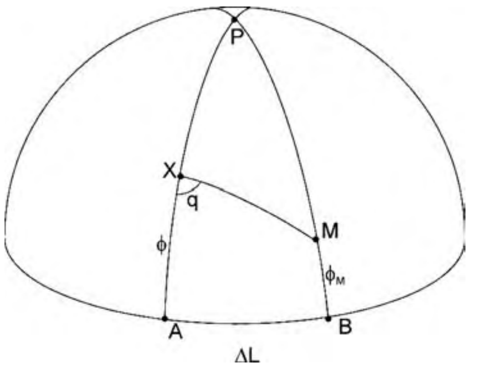

Fig. 3. The mathematical problem of determining the qibla.

"standard approximate method" was generally advocated in popular treatises on astronomy, and the most widely-known exact procedure amongst the experts was the "method of the *zîj*es". (A *zîj* is an astronomical handbook with tables, and some 225 works of this kind were compiled between the 8th and the 19th centuries.)

There is evidence that various scholars over the centuries favoured simple cartographic solutions to the qibla problem: in an anonymous treatise from 9th-century Baghdad three essentially cartographic methods are presented, together with an accurate solution by solid trigonometry and even a table displaying the qibla for a whole range of latitudes and longitudes. The author, as yet unidentified, was clearly capable of considering the qibla-problem in both cartographic and mathematical terms. The approximate procedures he presents are equivalent to:

$$q = \tan^{-1}\left\{\frac{\Delta L}{\Delta\phi}\right\}, \quad \tan^{-1}\left\{\frac{\Delta L\cos\phi_{\mathrm{M}}}{\Delta\phi}\right\}, \quad \tan^{-1}\left\{\frac{\sin\Delta L\cos\phi_{\mathrm{M}}}{\sin\Delta\phi}\right\}.$$

The corresponding basic triangles are shown in Figure 4(a)–(c):

Only the first of these had any following in later centuries, namely, on maps with square grids on which a given locality was simply joined to Mecca by a straight line.

Another approximate method first attested in the early 9th century but possibly earlier lends itself to the simple geometrical construction shown in Figure 4(d), and is equivalent to the formula:

$$q = \tan^{-1}\left\{\frac{\sin\Delta L}{\sin\Delta\phi}\right\}.$$

This, the "standard approximate method", was widely used until the 19th century but was generally not approved by the most serious astronomers.

The modern formula for the qibla is:

$$q = \cot^{-1}\left\{\frac{\cos\Delta L\sin\phi - \cos\phi\tan\phi_{\mathrm{M}}}{\sin\Delta L}\right\},$$

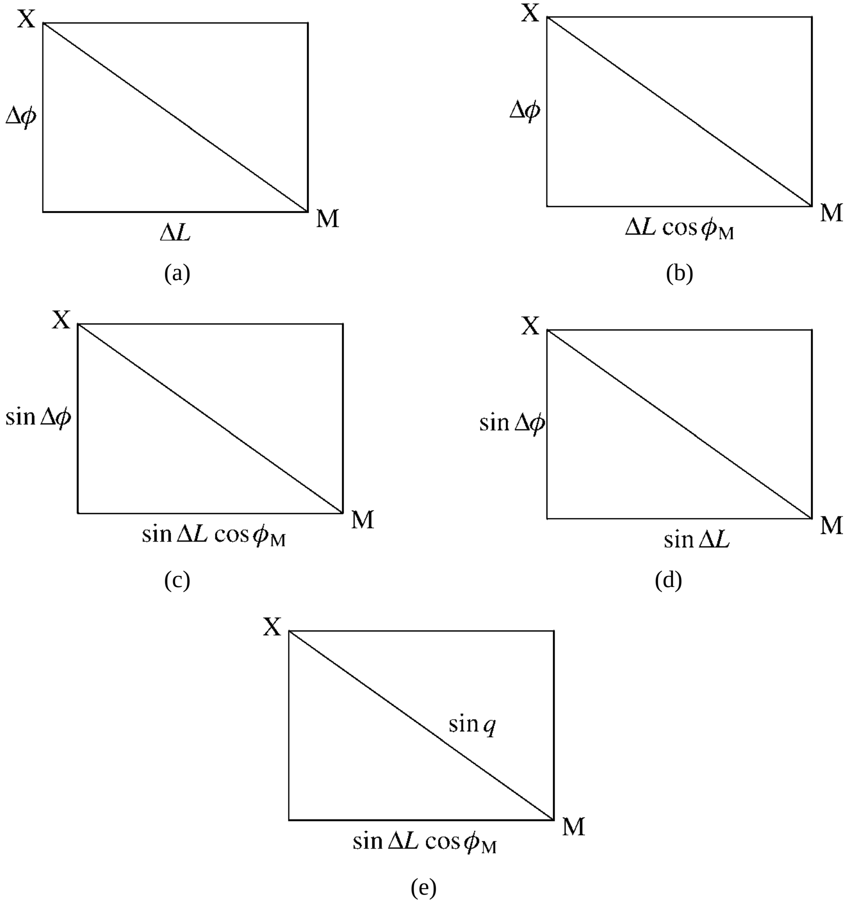

Fig. 4. (a)–(c) The underlying diagrams from which the 9th-century approximations can be derived. (d) The standard approximate construction for the qibla proposed by al-Battânî ca. 910. (e) The triangle from which the qibla can be derived from the distance to Mecca, according to the last procedure in the "method of the *zîj*es", first derived in the 9th century. This underlies the Mecca-centred maps discussed in 4(e).

and various accurate solutions were devised by Muslim astronomers from the 9th century onwards which were mathematically equivalent to this although they usually involved various steps. The distance to Mecca was also of interest. It may be derived using:

$$d = \sin^{-1}\left\{\frac{\sin\Delta L \cos\phi_{\mathrm{M}}}{\sin q}\right\}.$$

As noted above, an anonymous astronomer in 9th-century Baghdad derived an accurate formula for the qibla by solid trigonometry. From that century onwards, numerous Muslim scientists discussed the qibla problem, presenting other accurate solutions by spherical trigonometry, or reducing the three-dimensional problem to two dimensions and solving it by geometry or plane trigonometry. The celebrated early-11th-century scholar Ibn al-Haytham, who was born in Basra but later worked in Cairo, authored two brilliant studies of the determination of the qibla by means of spherical trigonometry and projection methods, respectively. The former, written first, is remarkable for the fact that all of the possible cases ($0 < \Delta L < 360°$ and $-90° < \phi < +90°$) are considered. The second, which uses an analemma construction, is spectacular for its conciseness, and from it the modern formula can be derived immediately. Also Ibn al-Haytham mentions in his autobiography that he wrote a treatise "on the determination of the azimuth of the qibla in all of the inhabited world by tables which I compiled..."; unfortunately, this treatise has not been preserved for us.

The so-called "method of the *zîj*es" for the determination of the qibla was developed in the mid-9th century, probably by Habash al-Hâsib, who is the first known author to have proposed it explicitly. It was also favoured by the major astronomers for centuries thereafter. The procedure was first derived by spherical trigonometry (anonymous 9th-century author, maybe Habash), then by projection methods (Habash), then by spherical trigonometry (various 10th-century scholars). It involves finding three auxiliary quantities "modified" from ΔL, ϕ_M, and $\Delta\phi$ and defined using:

$$\sin \Delta L' = \sin \Delta L \cos \phi_M; \qquad \sin \phi'_M = \frac{\sin \phi_M}{\cos \Delta L'}; \qquad \Delta\phi' = \phi - \phi'_M.$$

With these auxiliary quantities d and q are determined by:

$$\cos d = \cos \Delta L' \cos \Delta\phi', \qquad \sin q = \frac{\sin \Delta L'}{\sin d}.$$

What is involved here is essentially a conversion of equatorial/polar coordinates (longitude and latitude) to coordinates centred on a specific point on the sphere (qibla and distance).

The most comprehensive medieval discussion of mathematical qibla-procedures is by the celebrated al-Bîrûnî, who worked in Central Asia ca. 1025. In his treatise *Tahdîd nihâyât al-amâkin*, "The Determination of the Locations of Cities", the leading scientist of medieval Islam set out to establish the geographical coordinates of Ghazna (now in Afghanistan) in order to compute the qibla there: the result was an involved discussion of determinations of terrestrial latitude, the length of a degree along the terrestrial meridian, the differences in terrestrial longitudes by eclipse observations, the distances between cities by measuring caravan routes, and the derivations of the qibla for a given locality by mathematics, *inter alia*. Having first determined the longitude difference between Mecca and Ghazna with remarkable accuracy, al-Bîrûnî then applied complicated techniques of spherical trigonometry and projection methods to derive the qibla. His treatise is the most valuable work on mathematical geography from the medieval period.

4.2. *Tables displaying the qibla*

Muslim astronomers compiled a series of tables displaying the qibla q for each degree (within a certain range) of longitude different from Mecca ΔL and latitude difference $\Delta\phi$

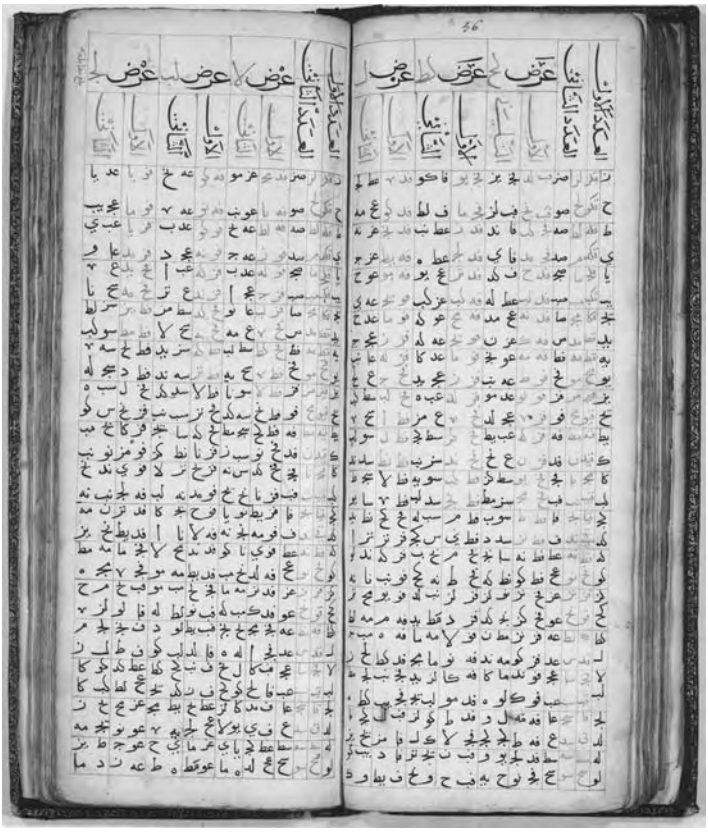

Fig. 5. An extract from the qibla-table of al-Khalîlî, showing qibla-values for the latitude range 28°–33°. The entries are written in standard Arabic alphanumerical notation. [From MS Paris BNF ar. 2558, fols. 55v–56r, courtesy of the Bibliothèque nationale de France.]

or latitude ϕ based on both approximate and exact formulae. The first of these was prepared in Baghdad, probably in the 9th century, and displays values of the qibla in degrees and minutes were given ostensibly for each 1° of ΔL and $\Delta\phi$ from 1° to 20°. Several other qibla-tables are known but the most impressive from the point of view of its scope and its accuracy is that of the 14th-century Damascus astronomer Shams al-Dîn al-Khalîlî, which displays $q(L, \phi)$ to degrees and minutes for each 1° of ΔL from 1° to 60° and each 1° of ϕ from 10° to 50°, with most of the 2 880 entries accurately computed: see Figure 5.

The extensive geographical tables that were a standard feature of Islamic *zîj*es rarely included qibla-values. The earliest table of this kind was by Abd al-Rahmân al-Khâzinî, who worked in Merw in Central Asia ca. 1125. It gives the qibla-values of about 250 localities along with their longitudes and latitudes. These basic coordinates appear to have been read from a map, for they are mainly those of his predecessor by one century, al-Bîrûnî, as found in his astronomical handbook, but generally rounded to the nearest 0; 10°, and occasionally there is a jump of 1° or 2° in one or other of the coordinates. al-Khâzinî's qibla-values are likewise given to the nearest 0; 10° but invariably they do not correspond to the longitudes and latitudes as well as one might expect; indeed, the errors can be as much as several degrees in places. Another table of this kind is found in a work by the Cairene astronomer Najm al-Dîn al-Misrî ca. 1325. This gives the longitudes, latitudes and qiblas (measured from the east or west) of some 150 localities, and whilst entries for both coordinates are given to the nearest degree, the qibla-values are computed to the nearest minute but do not correspond happily to the coordinates. Several other Egyptian tables of this kind present similar problems. A more successful table was that associated with al-Khalîlî (see above), serving 48 localities mostly in Syria and Palestine: here the qibla-values are remarkably accurate.

An enormous table very different from any of the others in its scope was compiled in 15th-century Timurid Central Asia. The author has not been identified, but he seems to have had a connection with the astronomical activity in Samarqand ca. 1425. In the available version of this table, values of the qibla and the distance to Mecca are given to the nearest minute, virtually without error, for 274 localities. But the original table had values of q and d to seconds, and these were accurate to within a few seconds. The information in this remarkable table, in its simplified form, was used in the gazetteers which were common on Iranian astrolabes and compasses from the 17th century onwards (see Section 4.3).

4.3. *Instruments for finding the qibla*

Several types of instruments were devised over the centuries for finding the qibla. One variety serves to actually determine the qibla of any locality: isolated examples of spherical devices for finding the qibla graphically are attested from the 12th century to the 14th. Another variety has markings based on computed qibla-values that provide a convenient practical means of displaying the qibla. From the 12th century onwards we find qibla directions for a specific locality marked on the back of an astrolabe or on a horizontal sundial. Iranian instrument-makers from the 16th century onwards were fond of representing graphically on the backs of astrolabes, for various localities, the solar altitude in the direction of the local qibla for all solar longitudes (that is, throughout the year).

Qibla-indicators in the form of magnetic compasses with information on the qiblas of various localities engraved around their rims are described in texts from the Yemen and from Egypt ca. 1300. A glazed ceramic bowl from Damascus ca. 1520 with qibla-values painted on it seems to attest to an earlier Iranian tradition. Brass qibla-indicators were particularly popular in Iran from the 17th century onwards, now using the qibla-values from the Timurid geographical table (see Section 4.2) in various degrees of corruption.

4.4. *Maps for finding the qibla*

Muslim scholars also prepared maps specifically for finding the qibla. One from early-13th-century Egypt is naive: on a standard coordinate grid fitted within a circular frame one simply joins one's locality to Mecca and measures the inclination to one's local meridian—see Figure 6. In spite of its simplicity, this approximate procedure works reasonably well

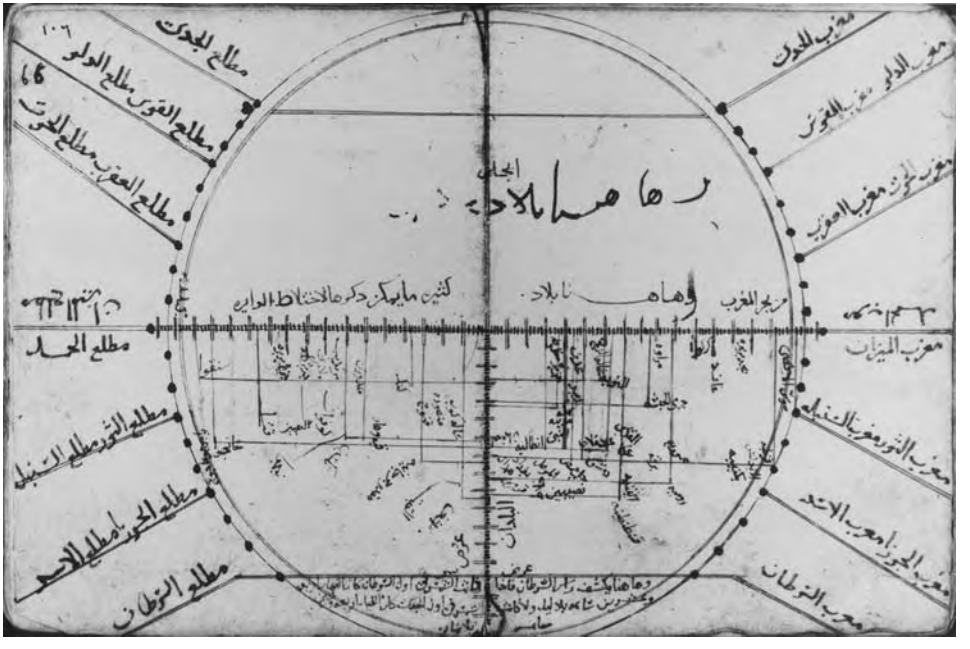

Fig. 6. A map of the world combining mathematical geography (localities plotted on a grid according to their coordinates) with folk geography (directions defined in terms of the rising and setting of the sun). The map occurs in a copy of an Egyptian treatise on folk astronomy compiled ca. 1210. The coordinates of the localities on the map are in the tradition of al-Bîrûnî (ca. 1025) and al-Khâzinî (ca. 1125), hence the map is of considerable historical interest. [From MS Princeton Yahuda 4657, fols. 65v–66r, courtesy of Princeton University Library.]

for localities in such regions as Egypt and Iran. Yet the map is not Mecca-centred and directions around the circumference are crudely associated with solar risings and settings. The map thus constitutes a unique example of the combination of mathematical cartography and folk astronomy. Another curious qibla-map with a grid based on an orthogonal projection is found on an astrolabe from Lahore dated 1666. A ‘qibla-map’ ‘invented’ by an Armenian for the Ottoman Grand Vizier in the late 1730s, is simply a European map of the landmass north of the equator between West Africa and Japan fitted with a magnetic compass and an additional pointer at Mecca, which, unhappily, is not at the centre. A later Ottoman world-map actually centred on Mecca is also known but has not been studied.

4.5. *Three Safavid world-maps centred on Mecca*

In 1989, 1995 and 2001, respectively, three remarkable world-maps from Safavid Iran came to light. Each is engraved on a circular brass plate some 22.5 cm in diameter and each is fitted with a highly sophisticated longitude and latitude grid centred on Mecca: see Figure 7. Some 150 localities are carefully marked on each map, the coordinates having being taken from the Timurid table mentioned above (Section 4.2). A non-uniform linear scale attached at the centre and a scale around the circumference enables the user to read off at a glance the direction of Mecca and the distance to Mecca. A grid achieving this purpose was previously unknown to the history of cartography, Islamic or European, before the early 20th century. The first map is unsigned and incomplete; its dating to the late 17th century seems secure on art-historical grounds. The second is signed by a certain Muhammad Husayn who appears to be one of the instrument-makers of 17th-century Isfahan, but who is known only from textual references and not by any other instruments: it looks as though it was made ca. 1800 but it is as genuine as the first and probably contemporaneous with it. The third is also complete and as elegantly finished as the first, and it is signed by an otherwise-unknown Hasan Husayn, who is, perhaps, however, to be identified with the maker of various unsigned sundials and qibla-indicators bearing a Shiite inscription mentioning both the imams Hasan and Husayn, which seem to date from the early 18th century. So even with the instruments themselves there are some unresolved problems.

Was the world-map grid designed by a 17th-century Safavid astronomer? I cannot accept that it was, since all of the contemporaneous writings from Safavid Iran that I have examined are devoid of scientific initiative. Now there were Europeans resident in Isfahan in the 17th century who were in contact with the Iranian instrument-makers. Also the instruments are fitted with a European-type magnetic compass (although the compass was

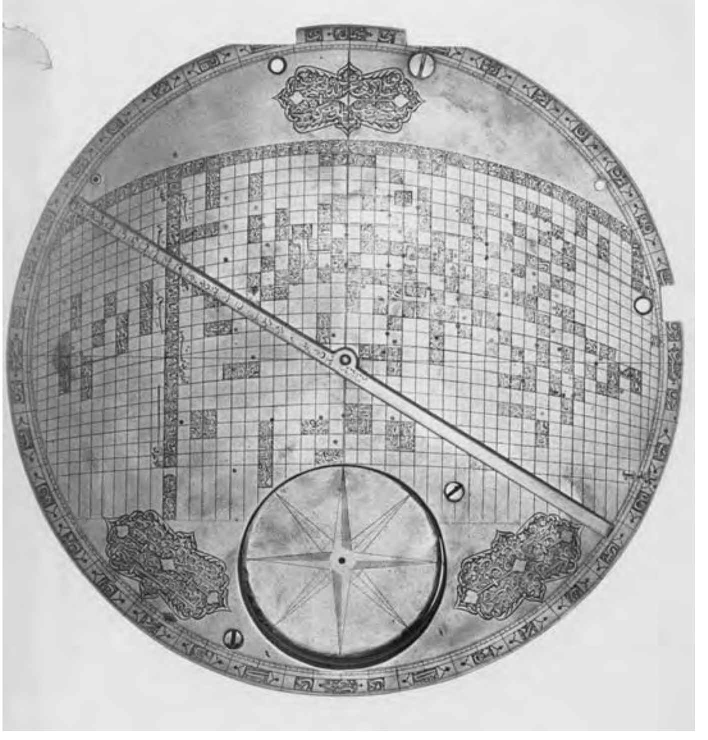

Fig. 7. The Safavid world-map rediscovered in 1989. [Dar al-Athar al-Islamiyyah, Kuwait, photo courtesy of Christie's of London.]

known in Islam at least since 1300) and a European-type inclining sundial (although such sundials were known in 14th-century Syria and 15th-century Egypt). If the grid was proposed to the Safavid instrument-makers by a European then that person would have had to have been exceptionally gifted. I could find no trace of European input on the grids, in spite of having considered what is known about the Europeans who were in Isfahan in the 17th century.

The problem with the maps is simply that they *look* very Islamic both from a geographical point of view and from a mathematical point of view: they feature essentially the world of medieval Islamic cartography and use somewhat outdated geographical data for regions of the world other than Greater Iran. Also, the underlying mathematics is in fact very simple when considered in terms of the well-known Islamic "method of the *zîj*es" for finding the qibla: see Figure 4(e). Furthermore, the latitude curves are approximated with arcs of circles (the centre of the arc with the smallest radius is still visible, the other centres would be off the map), and there is a well-attested tradition in Islamic instrument-making of using arcs of circles to approximate graphic representations of more complex curves (in this case, ellipses). Nevertheless, the idea of using such a method in this way to produce such a map-grid smacks of genius; also, even given the basic formula underlying the grid, the way to construct the grid is not self-evident. I have argued that the maps, all of which are without any doubt copies of an original, might have been inspired by a prototype developed alongside the monumental Timurid geographical table mentioned above. Concrete evidence of this is lacking. But I suspect that the original inspiration for such a grid might have come much earlier, and I have hypothesised that it was perhaps mentioned in some treatise now lost by al-Bîrûnî or his predecessor Habash. al-Bîrûnî wrote on mappings which preserve direction and distance to the centre, and Habash, some of whose works were available to al-Bîrûnî, devised an astrolabe based on such a projection (which is not the same as the one underlying the Safavid world-maps). I could show only that both of these early scholars and the later Timurid calculator had the necessary interest and ability to have devised a grid such as features on the Safavid world-maps, but not that they ever did.

The Safavid world-maps thus remain something of an enigma. Indeed, they have already become the object of some controversy, with some colleagues insisting, for example, that the possibility of European influence behind the grids should not be ruled out, or that one should not underestimate the Safavid astronomers. One colleague has proposed that the grids were added after the localities, which would be a first in the history of cartography, but he overlooked the fact that the deviations in the positions of the localities on the maps with respect to the longitudes and latitudes in the Timurid table clearly indicate that the grids were constructed first. Hence the inspiration for the Mecca-centred world-maps and their early development of remain clouded in obscurity.

Added in proof

In 2001, Professor Jan Hogendijk of Utrecht identified two Arabic treatises, one from 10th-century Baghdad and the other from 11th-century Isfahan, both dealing with the determination of the qibla by means of ellipse segments. The latitude curves on the maps are arcs of circles approximating segments of ellipses. We have thus come considerably closer to the origin of the Safavid world-maps, and the publication of Hogendijk's discoveries is awaited with anticipation.

5. The orientation of mosques and Islamic cities

The alignment of medieval mosques reflects the fact that the astronomers were not always consulted on their orientation. But since we now know from textual sources which directions were used as a qibla in each major locality we can better understand not only the mosque orientations but also can recognise numerous cities in the Islamic world that can be said to be qibla-oriented. In some, such as Taza in Morocco and Khiva in Central Asia, the orientation of the main mosque dominates the orientation of the entire city.

In the case of Cairo, various parts of the city and its suburbs are oriented in three different qibla directions: see Figure 8. The Fatimid city, founded in the 10th century, faces winter sunrise, which was the qibla of the Companions of the Prophet who erected the first mosque in nearby Fustat some three centuries previously. The Mamluk "City of the Dead" faces the qibla of the astronomers. The suburb of al-Qarâfa faces south, another popular qibla direction. The outer south-east walls of the splendid Mamluk mosques and madrasas on

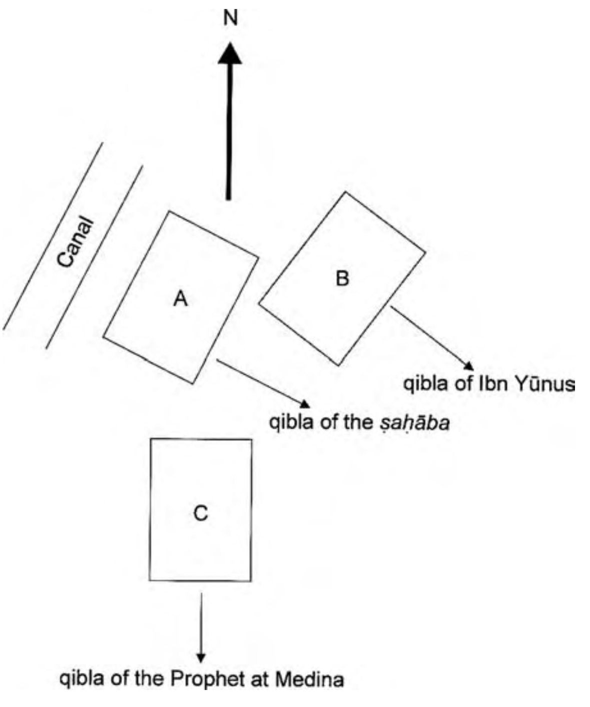

Fig. 8. The main orientations of three parts of medieval Cairo. Each part is qibla-oriented, the old city (A) in the qibla of the Companions of the Prophet (winter sunrise at ca. 27° S of E), the 'City of the Dead' (B) in the qibla of the astronomers (at 37° S of E), and the area around the tomb of al-Shâfi'î (C) in the qibla of the Shâfiite school, that is, the qibla of the Prophet when he was in Medina (due south).

the main thoroughfare of the city are aligned with the street plan, and the insides of the walls are aligned with the qibla of the astronomers; one can observe the varying thickness of the walls, which are at a 10° angle to each other, when standing in front of the windows inside the mosque overlooking the street outside. Figure 9 shows the qiblas used in Cairo and also Cordova and Samarqand.

Of all the regions of the Muslim world it is al-Andalus which has been best served by modern scholarship on orientations. A recent study by Mònica Rius on orientations in al-

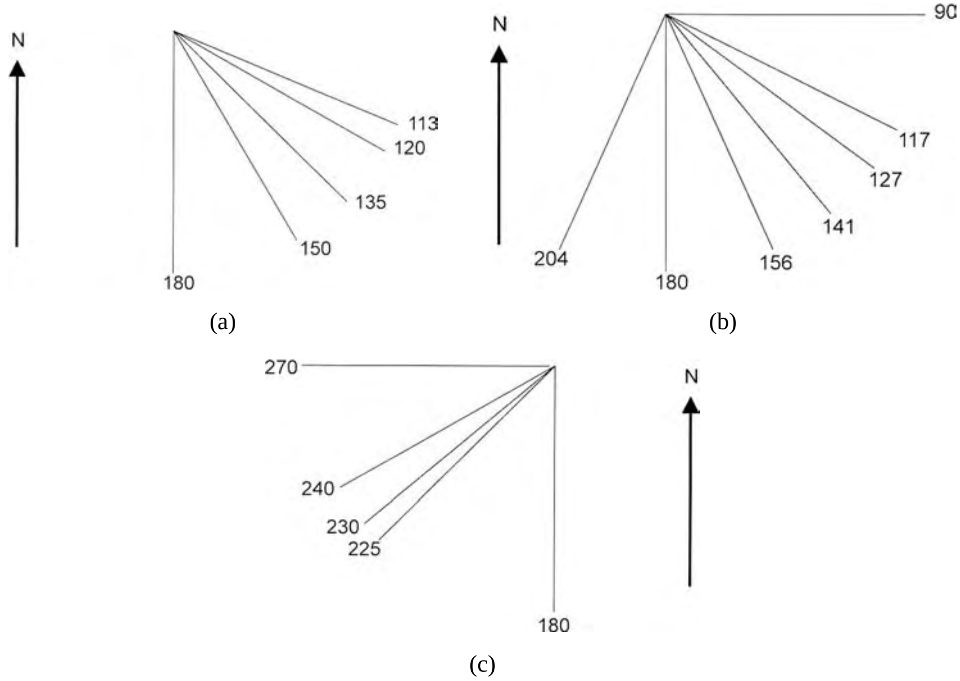

Fig. 9. Various qiblas accepted in Cordoba, Cairo-Fustat and Samarqand by different interest groups. (a) According to a 12th(?)-century Andalusi treatise on the astrolabe, mosques in Cordova were oriented in these different directions: 23° S of E, the qibla computed by the standard approximate formula (which works well for localities in the central regions of the Islamic world, but not for al-Andalus, where the error is more than 10°); 30° S of E, winter sunrise; 45° S of E, a compromise between due east and due south; 30° E of S, the direction of the Great Mosque (which is 'parallel' to the major axis of the Kaaba); and due south (not specifically mentioned in this text). (b) The Egyptian historian al-Maqrîzî (d. 1442) mentioned these qiblas as being used for mosque orientation in Cairo: due east (not specifically mentioned in this text); 27° S of E, winter sunrise, the qibla of the Companions of the Prophet; 37° S of E, the qibla of the astronomers, computed according to an exact procedure and first attested in the writings of the 10th-century Fatimid astronomer Ibn Yunus; 39° E of S, the qibla of the Mosque of Ibn Tûlûn, variously explained; and any direction between ca. 24° E of S and ca. 24° W of S, between the rising and setting of Canopus. (c) The legal scholar al-Bazdawî (d. 1089) reported these qiblas as being used for mosque orientation in Samarqand: due west, used by the Hanafite school of law and corresponding to the direction in which the road to Mecca left the city; 30° S of W, winter sunset, as used for the Great Mosque; 40° S of W, a value underlying a table for the altitude of the sun in the azimuth of the qibla, presented by al-Bazdawî but lifted from some earlier source; and due south, used by the Shâfiite legal school, corresponding to the qibla of the Prophet in Medina.

Andalus and the Maghrib compares the information in medieval folk astronomical texts with actual orientations. This is an area of the history of urban development in the Islamic world which has only recently been understood for the first time, not least because, prior to the discovery of the textual evidence, it was by no means clear which directions were used as qiblas, and even if a qibla direction at variance from the "true" qibla was clearly popular, it was not known why.

Only with the systematic scientific cartographic surveys of the 18th and 19th centuries did the first accurate measurements of geographical longitudes of localities in the Islamic world become available. Thus most of the accurately-computed qiblas of the Muslim astronomers were off by a few degrees anyway. Nowadays urban Muslims are content to use the qibla directions found by calculation from modern geographical coordinates. In rural areas where there is no mosque nearby, astronomical horizon phenomena are still used for the sacred direction. In recent years, various devices for finding the qibla have appeared on the market. The first were in the form of a magnetic compass with a list of directions for the qiblas in the major cities of the world. In some later electronic models one would feed in one's latitude and longitude, and the device would beep until it was turned to the appropriate direction for the qibla, upon which it would purr continuously; such devices were sometimes fitted to an electronic clock which would indicate the times of Muslim prayer. Most recently such devices are linked to a satellite which can locate the user and so does not need to be informed of his/her coordinates. If one were to take such a device into a mosque built in medieval times just before the call to prayer was to be announced, it is most unlikely that the device would show the same qibla as the *mihrâb* of the mosque, and also highly improbable that the thing would beep at the same time as the call to prayer.

6. Concluding remarks

The activities of both the legal scholars and the scientists are remarkable for the enthusiasm and ingenuity with which they were conducted, the sophistication and complexity they achieved, and not least for the occasional universality they attained, in both cases providing solutions to the qibla-problem for all localities. The legal scholars developed a sacred geography centred on the Kaaba, and the scientists a sacred geography centred on Mecca. In the long term it was irrelevant whether a particular qibla was "correct"; at the time it was important only that those who adopted it thought it was correct. Numerous historians of Islamic architecture have been misled into pronouncing the orientation of this or that mosque "incorrect"; it is they who stand to benefit most from our new insights about medieval notions of the qibla. Likewise, it is easy for us moderns with a mathematical bent to pour scorn on the legal scholars for not listening to the scientists and on some of the latter for not being able to compute the qibla properly. But one should not lose sight of the fact that the ultimate purpose of the adoption of a sacred direction in Islam was in obediance to a divine injunction that the devotion of the faithful should be focussed on a sacred shrine in the most sacred city at the heart of the Islamic world.

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[2] al-Bîrûnî, *Tahdîd*: J. Ali, *The Determination of the Coordinates of Cities ... by al-Bîrûnî*, American University of Beirut Press, Beirut, 1967; and E.S. Kennedy, *A Commentary upon Bîrûnî's Kitâb Tahdîd* [*Nihâyât*] *al-Amâkin, An 11th Century Treatise on Mathematical Geography*, American University of Beirut, Beirut, 1973. [Translation of and commentary on the most significant surviving medieval treatise on mathematical geography.]

[3] M. Chebel, *Symboles de l'Islam* and *Symbols of Islam*, with photographs by L. Hamani, both Éditions Assouline, Paris, 1997. [Beautifully illustrated.]

[4] *The Encyclopaedia of Islam*, new edition, 10 vols. to date, E.J. Brill, Leiden, 1960 to present. [Contains various survey articles including "Anwâ" [aspects of folk astronomy]; "Ilm al-hay'a" [astronomy]; "Kibla" [sacred direction]; "Layl and nahâr" [aspects of folk astronomy]; "Makka: As Centre of the World" [sacred geography]; "Matâli" [ascensions]; "Matla" [horizon directions]; "Mîkât" [astronomical time-keeping]; "Samt" [direction]; "Tâsa" [magnetic compass]; and "Zîdj" [astronomical handbooks with tables].]

[5] D.A. King, *Astronomy in the Service of Islam*, Variorum, Aldershot, 1993. [Reprints of various articles, including an analysis of the earliest mathematical methods for finding the qibla and a study of the orientation of the Kaaba and its influence on mosque orientations.]

[6] D.A. King, *Islamic astronomy*, Astronomy Before the Telescope, Christopher B.F. Walker, ed., British Museum Press, London, 1996, repr. 1999, pp. 143–174. [An overview.]

[7] D.A. King, *The Sacred Geography of Islam*, to appear. [An analysis of the numerous medieval schemes of folk geography in which the world was divided into sectors around the Kaaba; a summary is in the article "Makka, iv" in the *Encyclopaedia of Islam*.]

[8] D.A. King, *In Synchrony with the Heavens. Studies in Astronomical Timekeeping and Instrumentation in Medieval Islamic Civilization*, 2 vols., 1: *The Call of the Muezzin (Studies I–IX)*, and 2: *Instruments of Mass Calculation (Studies X–XVIII)*, E.J. Brill, Leiden, 2004. [See especially VIIa: "The Orientation of Medieval Islamic Religious Architecture and Cities", first published in *Journal for the History of Astronomy* 26 (1995), 253–274, and VIIb: "Architecture and Astronomy: The Ventilators of Medieval Cairo and Their Secrets", first published in *Journal of the American Oriental Society* 104 (1984), 97–133. See also [9].]

[9] D.A. King, *World-Maps for Finding the Direction and Distance of Mecca: Innovation and Tradition in Islamic Science*, E.J. Brill, Leiden, and Al-Furqan Islamic Heritage Foundation, London, 1999. [An overview of the treatment of the qibla-problem by Muslim scholars and a detailed study of the two 17th-century Mecca-centred world-maps with speculations on the source of their inspiration. The third world-map and the newly-discovered 10th- and 11th-century treatises are introduced in King, *In Synchrony with the Heavens*, VIIc.]

[10] D.A. King, J. Samsó, *Astronomical handbooks and tables from the Islamic world (750–1900): An interim report*, Suhayl—Journal for the History of the Exact and Natural Sciences in Islamic Civilisation (Barcelona) **2** (2001), 9–105. [A supplement to E.S. Kennedy's 1956 survey of Islamic *zîj*es, with some remarks on qibla-tables and geographical tables displaying the qibla for numerous localities.]

[11] M. Rius, *La Alquibla en al-Andalus y al-Magrib al-Aqsà*, Anuari de Filologia (Universitat de Barcelona) **XXI** (1998–1999) B-3, Institut "Millás Vallicrosa" de Història de la Ciència Àrab, Barcelona, 2000. [The first investigation of mosque orientations in the light of medieval folk astronomical and legal texts on the qibla.]

CHAPTER 9

"Number Mystique" in Early Medieval Computus Texts

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Introduction

Around A.D. 1011, the English monk Byrhtferth of Ramsey composed a manual of computus, the science of time-reckoning and calendar construction.[1] The first three books of his *Enchiridion* explain the basic structure of the Julian solar calendar, the lunar cycle, and the rules for coordinating the two to determine the date of Easter. The fourth and final book, however, is not about time or calendar problems; rather, it is "a handbook of the Christian exegesis of numbers", and is drawn almost entirely from theological sources.[2] Byrhtferth's own computus teacher, Abbo of Fleury, also wrote an exposition of the metaphysics of number, but based on Boethius' mathematical and logical writings, and cast as a commentary on a multiplication table designed by a fifth-century mathematician and computist, Victorius of Aquitaine.[3] Abbo's numerology stands in the neo-Pythagorean and neo-Platonic tradition of number metaphysics represented by the manuals of Nicomachus of Gerasa, and conveyed to the Latin West by Boethius' *De arithmetica*. This "philosophical numerology" derives the meaning of numbers from their mathematical properties: for example, Abbo characterizes seven as the "virgin number" symbolizing wisdom and the soul, because it is the only number less than ten which is not a product or a factor of any other number less than ten.[4] Byrhtferth, however, represents the Christian tradition of what might be called "allegorical numerology", where numbers derive their meanings from correspondence to Biblical events or theological concepts, alone or in combination (usually by addition or multiplication).[5] Byrthferth makes no mention of the "virginity" of seven,

[1] *Byrhtferth's Enchiridion*, ed. Peter S. Baker and Michael Lapidge, Early English Text Society S.S. 15, Oxford University Press, Oxford (1995). Hereafter, Byrhtferth's text will be cited as "*Enchiridion*", followed by book and section number, with the page reference in this edition in parentheses; references to Baker and Lapidge's introduction will be cited as "Baker and Lapidge".

[2] Baker and Lapidge, p. lxxiii.

[3] Ed. Alison Peden, *Abbo of Fleury and Ramsey: Commentary on the Calculus of Victorius of Aquitaine*, Auctores Britannici medii aevi 15, Oxford University Press for the British Academy, Oxford (2003). Portions of Abbo's commentary were printed by W. Christ, Über das *argumentum calculandi* des Victorius und dessen Commentar, *Sitzungsberichte der bayerischen Akademie der Wissenschaften, phil.-hist. Kl.* (1863), pp. 100–152. Victorius' *Calculus* was edited by G. Friedlein, Der Calculus des Victorius, *Zeitschrift für Mathematik und Physik* 7 (1871), 42–79. For analysis of Abbo's commentary, see G.R. Evans and A.M. Peden, Natural Science and the Liberal Arts in Abbo of Fleury's Commentary on the Calculus of Victorius of Aquitaine, *Viator* 16 (1985), 109–127, and A. van de Vyver, Les oeuvres inédits d'Abbon de Fleury, *Revue bénédicine* 47 (1935), 139–140; on Abbo's number metaphysic, see Alison Peden, Unity, Order and Ottonian Kingship in the Thought of Abbo of Fleury, *Belief and Culture in the Middle Ages: Studies Presented to Henry Mayr-Harting*, ed. Richard Gameson and Henrietta Leyser, Oxford University Press, Oxford (2001).

[4] Abbo, *Commentary* 3.17 (ed. Peden, 82–83); cf. Evans and Peden, p. 118.

[5] Classical number metaphysics: Walter Burkert, *Lore and Science in Ancient Pythagoreanism*, transl. Edwin J. Minar Jr., Harvard University Press, Cambridge, MA (1972); I. Mueller, Mathematical Method and Philosophical Truth, *Cambridge Companion to Plato*, ed. R. Kraut, Cambridge University Press, Cambridge (1992), pp. 170–199; Dominic J. O'Meara, *Pythagoras Revived: Mathematics and Philosophy in Late Antiquity*, Clarendon Press, Oxford (1990). Christian number symbolism: Heinrich Meyer, *Die Zahlenallegorese im Mittelalter: Methode und Gebrauch*, Münstersche Mittelalter-Schriften 25, Fink, Munich (1975) and *Lexikon der mittelalterlichen Zahlenbedeutungen*, Münstersche Mittelalter-Schriften 56, Fink, Munich (1987). These replace older syntheses such as V.F. Hopper, *Medieval Number Symbolism*, Columbia University Studies in English and Comparative Literature 131, Columbia University Press, New York (1938). For an accessible overview, see R.A. Peck, "Number as Cosmic Language", *By Things Seen: Reference and Recognition in Medieval Thought*, ed. David L. Jeffrey, University of Ottawa Press, Ottawa (1975), pp. 47–50.

save perhaps in his elliptical statement that "the number seven commences with unity (that is, from one) and thus it proceeds reverently, with complete holiness, to the triumph of its perfection".[6] Rather, seven is meaningful because it is the sum of three, the number of the Trinity, and four, the number of the Gospels, and because the gifts of the Holy Spirit are seven.

Christian numerology, as conveyed in handbooks for Bible study like Isidore of Seville's *Liber de numeris*, or in the commentaries of patristic and early medieval exegetes, is the fusion of this exegetical allegorical tradition with a Christianized version of the cosmological and contemplative philosophical tradition. Following the lead of Jewish exegetes like Philo, the Church Fathers took over the Pythagorean–Platonic concept of number as the structural principle of the cosmos, and re-framed it in the light of Wisdom 11.20: "But thou hast arranged all things by measure and number and weight." This text became the mantra of Christian philosophical numerology.[7] Number was the blue-print of a divinely-designed creation (not an eternal, divine cosmos), and was therefore the key to understanding the mind of Creation's God. In this way, Christian thinkers appropriated not only the cosmological framework of ancient number-lore, but also its contemplative and ascetic potential. For the pagan philosopher, meditating on number was a way to train oneself to think about abstract and immaterial entities, and to purify the soul through harmony and proportion;[8] for the Christian theologian, that immateriality and harmony was a revelation of God Himself, the source of all beatitude. That it was possible to fuse philosophical and allegorical numerology is exemplified by Augustine's assertion that the mathematical "perfection" of the number 6 (because it is the sum of its factors) explains why God chose to make the universe in six days as a sign of Creation's perfection.[9] But the fact that Byrhtferth and Abbo could opt to follow separate strands of the tradition should alert us to the fact that their fusion was neither total nor automatic, even in Augustine. In *De doctrina christiana*, his essay on the erudition appropriate for the Christian intellectual, Augustine discusses the science of numbers in connection with the role of the Liberal Arts in Christian intellectual life, and also in relation to the knowledge base needed by the exegete and preacher. First, he observes that though mathematics is classified with "useful knowledge of human origin", it is in fact divinely instituted, since its rules exist independently of human intervention: Virgil could change the pronunciation of "Italia", but no one can change the fact the three times three equal nine. The truths of number are not invented, but discovered, and for this reason the study of the science of number is an ascesis through which one learns to appreciate the immense difference between immutable divine truth and the mutable human mind. If one does not learn this, one may still be considered erudite, but one would

[6] *Enchiridion* 4.1 (208.174–176).

[7] Israel Peri, 'Omnia mensura et numero et pondere disposuisti': Die Auslegung von Weish. 11,20 in der lateinischen Patristik, *Mensura: Mass, Zahl, Zahlensymbolik im Mittelalter*, ed. Albert Zimmermann, Miscellanea mediaevalia 16, De Gruyter, Berlin (1983), 1.1–21.

[8] This idea would have been familiar to medieval readers through Macrobius, *In Somnium Scipionis* 1.5.4, ed. J. Willis, Teubner, Leipzig (1963), 15.9–17.

[9] Augustine, *De civitate Dei* 11.30, ed. B. Dombart and A. Kalb, Corpus Christianorum series latina (hereafter abbreviated CCSL) 48, Brepols, Turnhout (1955), pp. 1–12; *De Genesi ad litteram* 4.2, ed. I. Zycha, Corpus scriptorum ecclesiasticorum latinorum 28.1, Temsky, Vienna (1894), 97.19–98.22; cf. Boethius, *De institutione arithmetica* 1.19, ed. G. Friedlein, Teubner, Leipzig (1867), 41.6–15.

not be wise.[10] This is a lapidary statement of patristic “philosophical numerology”. But in the passage which immediately follows, Augustine takes a completely different tack. He envisions the production of a Christian encyclopaedia geared to the three kinds of signs found in Scripture: words, things, and numbers. Because ignorance of these matters leads to misunderstanding of figural expressions in Scripture, Augustine proposes that someone compile a reference work containing all the place names, facts of natural history, and numbers mentioned in the Bible—but *only* those numbers mentioned in the Bible—so that the Christian student would not have to waste time laboriously searching out information on these topics.[11] From being an open-ended process of meditation based on mathematical procedures, “number” here becomes a mode of explaining a corpus of literary allusions. It is “allegorical numerology”. Though Augustine included both, and could imagine their congruence, they were distinct in their scope and their epistemic objects.

Byrhtferth’s treatment of number symbolism is fairly conventional, and as I indicated, largely “allegorical” in its approach. However, in one critical respect it is rather unusual: it is found in a manual of computus. His editors have expressed puzzlement that this section of the *Enchiridion* seems not to have come to attention of either medieval or modern students of Christian number symbolism.[12] But why would anyone look for number symbolism in a computus textbook? Number symbolism is about the interpretation of Scripture, or if one follows the philosophic tradition represented by Abbo, about the *disciplina* of arithmetic as a propaedeutic to philosophy or theology. Computus was neither of these.

The aim of this essay is to explain the presence of book 4 in Byrhtferth’s *Enchiridion*. It will do this by showing how computus became a major vehicle for ideas about the metaphysical and religious dimensions of number in the early Middle Ages. It will also discuss how computus appropriated and shaped those ideas to create what I would term a “number mystique”—a rhetorical claim to profound significance, lofty philosophical pedigree, and even religious mystery, based on the identification of time-reckoning with *numerus*. Finally, it will consider the challenge to this mystique set out by Bede, and suggest some reasons why Byrhtferth might have chosen to ignore that challenge.

1. The shape and scope of computus

Computus—the science and technique of calendar construction—is an application of astronomy and arithmetic to an essentially religious problem.[13] Though pre-Christian mathematicians and astronomers addressed many issues related to calendar construction, it was

[10] *De doctrina christiana* 2.38.56–57, ed. Joseph Martin, CCSL 32 (1961), 71.1–72.24. Cf. Peck, p. 49.

[11] Ibid., 2.39.59 (72.24–73.47, esp. 36–38).

[12] Baker and Lapidge, pp. lxxiii–lxxiv, esp. p. lxxiii n. 3.

[13] For a bibliographical overview of general and technical scholarship on computus, see *Bede: The Reckoning of Time*, transl. Faith Wallis, Liverpool University Press, Liverpool (1999), pp. 430–431, and Faith Wallis, Chronology and Systems of Dating, in: *Medieval Latin Studies: an Introduction and Bibliographic Guide*, ed. George Rigg and Frank A.C. Mantello, Catholic University of America Press, Washington, DC (1996), pp. 383–387. Brief, reliable, and up-to-date introductions include: Stephen McCluskey, *Astronomies and Cultures in Early Medieval Europe*, Cambridge University Press, Cambridge (1998), Chs. 5 and 8; Georges Declercq, *Anno Domini: the Origins of the Christian Era*, Brepols, Turnhout (2000); and Baker and Lapidge’s introduction to *Byrhtferth’s Enchiridion*, pp. xxxiv–xl.

the specifically religious context of Christian theology, cultic practice, and ecclesiastical discipline which created computus as a distinctive domain of learning in Middle Ages. The medieval computist had to account for the precepts of divine law (the rules governing Passover in the Old Testament) and the data of sacred history (the account of Christ's passion and resurrection in the Gospels), as well as the motions of the heavenly bodies, and the conventions of the Julian calendar. He had to harmonize all this data and reduce it to a formula, and ideally, to a cycle, expressed in mathematical terms. Hence computus by its very nature embodies a profound concern for the relationship of mathematics and the divine.

However, it is not easy to define the precise character of that concern. The corpus of medieval *computistica* is vast, and largely still unpublished. Much of it is anonymous, derivative, and "uncanonized"—that is, it is not presented as authoritative texts to be faithfully transmitted and reverently expounded, but as an unstructured body of material which can freely be excerpted, interpolated, re-arranged, or adapted according to particular tastes and needs.[14] Moreover, its principal documents are not texts, or even mathematical formulae, but tables: the Julian solar calendar equipped with formulae (*argumenta*) or auxiliary tables with instructions (*canones*) to customize its information to any given year, and the Paschal listing the dates of Easter and related information for a sequence of years or a cycle of years.[15] Parallel to this graphic literature of computus is a body of textual literature which can broadly be divided into two genres: didactic and controversial. The premier textbook of computus, Bede's *De temporum ratione* (725) contains elements of both. Its didactic style and materials were widely imitated, condensed, glossed and updated in later centuries.[16] But it also aimed to prove the superiority of the Alexandrian computus of Dionysius Exiguus over its rivals, particularly an older Roman computus which continued to be used in the British and some Irish churches, and the system of Victorius of Aquitaine. It can be claimed that Bede's work settled these controversies forever, but by the 11th century, new subjects of debate had appeared, notably the problem of reconciling Dionysius' *annus domini* chronology to the historical and astronomical data of Christ's Passion. The new astronomy of the 12th and 13th centuries, based on Greco-Arabic documents and instrumentation, opened further problems: the progressive advance of the vernal equinox away from its official calendar date of 21 March, and the widening gap between the calculated and observed ages of the Moon. As equinox and lunar phase were the astronomical markers for Easter, this discrepancy between nature and convention was troubling. Various

[14] Canonized and uncanonized texts: Faith Wallis, The Experience of the Book: Manuscripts, Texts, and the Role of Epistemology in Early Medieval Medicine, in: *Knowledge and the Scholarly Medical Traditions*, ed. Don G. Bates, Cambridge University Press, Cambridge (1995), pp. 101–126. Computus as "uncanonized" knowledge: Faith Wallis, The Church, the World and the Time, in: *Normes et pouvoirs à la fin du moyen âge*, ed. Marie-Claude Desprez-Masson. Inedita et rara 7, Ceres, Montreal (1990), pp. 15–29; Baker and Lapidge, pp. xl–lx.

[15] There are variations on this formula: in the early medieval period, a computus MS might be structured around more or less stable dossiers of controversial and didactic material; the Carolingian period sees the emergence of the "classbook" or encyclopaedic anthology of quadrivial sciences, with computus at the core; in the Scholastic period, computus texts were often part of the *corpus astronomicum*. These will be discussed in my forthcoming volume for the series *Typologie des sources du moyen âge occidental,* entitled Computus: *Manuscripts, Texts and Tables*.

[16] See *The Reckoning of Time*, pp. lxxxv–xcvii.

solutions to these problems were debated from the 13th to the 16th centuries, culminating in the Gregorian reform of the calendar in 1582.

Computus therefore covers a broad spectrum of graphic and literary production on calendar construction spanning more than twelve centuries of Christian history. It was regarded throughout this period as a branch of clerical vocational learning, though after the 12th century a parallel secularized form of calendar science, allied to Scholastic astronomy and astrology, emerged. Nonetheless, most computus literature, including the most influential works such as *De temporum ratione*, takes a matter-of-fact approach to the mathematical dimensions of calendar reckoning. Computus demands some skill in calculation, but no mathematical theory. Moreover, it does not self-evidently lend itself to numerological treatment, because most of its data is furnished by astronomical fact, not Scripture or metaphysics. There is no inherent mystique to the 19 years of the Metonic lunar cycle, or the $29\frac{1}{2}$ days of the lunation, or the $365\frac{1}{4}$ days of the solar year, and for the most part, computists did not attempt to furnish any. In the case of Bede, the exception which proves the rule is 7, the number of the Christian week. The requirement to celebrate Easter only on Sunday introduced the week into the design of a formula for determining the date of Easter. However, the week is a time-unit like no other. Though the periods and cycles of the Sun and Moon served as "signs . . . for seasons and for days and years" (Genesis 1.14, cf. *De temporum ratione*, Ch. 6), the observance of the week was mandated by God directly, and is not governed by the heavens. In Ch. 2 of *De temporum ratione*, Bede gives the week as the classical example of "authoritative", as distinct from "natural" time-reckoning.[17] The week is the only division of time which inspires Bede to reflect on the symbolism of number, for it typifies the seven-fold structure of all sacred time. There are not only weeks of weeks—the seven-times-seven days between Easter and Pentecost—but weeks of years, both solar and lunar (Ch. 8). All of history is scanned into seven ages, which correspond to the seven ages of human life and the seven days of the historic weeks of Creation and Our Lord's Passion (Ch. 10, 66). Bede took this latter concept from his reading of the Fathers, especially Augustine, from whom he borrowed as well his understanding of the eschatological dimensions of the cosmic week—the sabbath rest of the Church Expectant in heaven, and the everlasting Lord's Day which will follow the end of time (Ch. 71).[18] What is exceptional about the week and the symbolism of 7 is that it is all but unique in *De temporum ratione*. For example, the 12 months and the 12 zodiac signs are discussed without the slightest mention of the rich Christian symbolism of this number, the number of the patriarchs and apostles, of the gems on the High Priest's breast-plate and the gates of the apocalyptic New Jerusalem. This is particularly surprising in that Bede revels in such contrapuntal allegorical numerology in his exegetical and hagiographical works.[19]

[17] C.W. Jones (ed.), *Bedae opera didascalica* 2, CCSL 123B (1977), 274.14–18; transl. F. Wallis, p. 13. Cf. Ch. 8, where Bede rejects as "the foolishness of the Gentiles" the idea that the seven-day week was derived from the seven planets (302.58; transl. F. Wallis, p. 34).

[18] *The Reckoning of Time*, pp. 353–375.

[19] Examples are cited by Claude Jenkins, Bede as Exegete and Theologian, in: *Bede, his Life, Times and Writings*, ed. A.H. Thompson, Clarendon Press, Oxford (1935), pp. 173–180; and Walter Berschin, 'Opus deliberatum ac perfectum': Why did the Venerable Bede Write a Second Prose Life of St. Cuthbert? in: *St. Cuthbert, His Cult and Community to AD 1200*, ed. Gerald Bonner, David Rollason and Clare Stancliffe, Boydell, Woodbridge (1989), pp. 95–102. On the relationship of the 12 stones of the High Priest's rationale to the 12 patriarchs, apostles, months

That being said, Byrhtferth's inclusion of a major exposition on number symbolism in his computus manual is far from eccentric or accidental. In fact, Byrhtferth is tapping into a tradition about computus which Bede actively rejected, a distinctively computistical "number mystique". This number mystique is the consequence of the cultural role of computus in the Middle Ages, and particularly in the period before 1100, when words like "philosophy" and "mathematics" were little more than labels on ancient boxes whose contents could no longer be located. For many computists, the calendar actually filled those boxes, and in doing so served as a vehicle for equally venerable notions of the metaphysical properties of number, and of the value of mathematics as spiritual ascesis.

2. Computus as *ratio numerorum*: the Irish computus of ca. 658

One of our oldest witnesses to this number mystique is a tract on computus in dialogue form, composed in Ireland about A.D. 658.[20] It opens with an exordium not ascribed to either participant in the dialogue (*magister* or *discipulus*), but which presumably represents the *doctrina* of the *magister*:

> Concerning the four divisions of Scripture, Augustine says: "Four things are necessary in the Church of God: the divine law (*canon*) in which the future life is described and foretold/preached (*praedicatur*); history, in which deeds are recounted; number, in which future events and divine celebrations are reckoned up (*facta futurorum et solemnitates diuinae dinumerantur*); grammar, in which the science of words is understood. Therefore there are four parts of Scripture: divine law, history, number, and grammar.[21]

The source of this statement of "Augustine" has not been traced, but the closest cognate appears to be *De Genesi ad litteram*: *In libris autem omnibus sanctis intueri oportet, quae ibi aeterna intimentur, quae facta narrentur, quae futura praenuntientur, quae agenda praecipiantur uel admoneantur* ("In every book of Scripture, it behooves one to take cognizance of what matters pertaining to eternity are intimated therein, what deeds are recounted, what future events are foretold, and what it commands or admonishes us to do.").[22] If this is what the author of the Irish computus thought he was quoting, then the changes introduced to the original are very significant indeed. Augustine is outlining four modes of Biblical exegesis: the literal (*quae facta narrentur*), the allegorical (*quae ibi aeterna intimentur*), the anagogical (*quae futura praenuntientur*) and the moral or tropological (*quae agenda praecipiantur uel admoneantur*). The Irish author has transformed these into topics or disciplines: sacred law replaces tropology; history corresponds to the literal meaning; grammar to allegory; and "number" to the anagogical mode. It is noteworthy that in common with most early

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and zodiac signs, see, for example, *De tabernaculo* 3, ed. D. Hurst, CCSL 119B (1969), 111.737–113.787; transl. Arthur G. Holder, Translated Texts for Historians 18, Liverpool University Press, Liverpool (1994), pp. 129–130.

[20] This tract is composed of the text entitled *De computo dialogus* in PL 90.647–652, and the piece which follows, *De divisionibus temporum* (653–664). The prologue and capitula were edited by C.W. Jones, *Bedae opera de temporibus*, Mediaeval Academy of America. Publications, 41, Mediaeval Academy of America, Cambridge, MA (1943), pp. 393–395, and by Alfred Cordoliani, Une encyclopédie carolingienne [*sic*] de comput: les *Sententiae in laude computi, Bibliothèque de l'École des Chartes* 104 (1943), 237–242. Dr Dáibhi Ó Cróinín of University College, Galway, is preparing a critical edition.

[21] PL 90. 647D. My translation.

[22] *De Genesi ad litteram* 1.1 (3.7–10).

medieval writers, on computus or other topics, the Irish teacher avoids the ambiguous term "mathematics", which could connote either mathematics in our modern sense, or astrology.[23] The preferred terms are *numerus* or *ratio numerorum*. The latter term incorporated its own ambiguity: "reckoning numbers" or "explaining the meaning of numbers". The first opened up the possibility of philosophical numerology; the second invited allegorical numerology. However, the Irish writer's equation of number specifically with the power to "reckon up future events and divine celebrations" indicates that "number" here is a synonym for computus. This identification is elaborated as the prologue continues:

> In praise of computus, Isidore says: "The *ratio* of numbers is not to be scorned, for it reveals the mystery contained in many passages of Holy Scripture. Not in vain is it said of God [in Wisdom 11.21]: 'Thou hast made all things in measure, number and weight'. The number six, perfect in its factors, proclaims the perfection of the cosmos by a certain numerical significance. Likewise the forty days wherein Moses, Elijah and our Lord himself fasted cannot be understood without a knowledge of number. There are other numbers in Holy Scripture whose symbolism (*figuras*) can only be unravelled by those knowledgeable in this science. Using the science of numbers, we have an ability to stand fast (*consistere*) to some degree, when through this science we discuss the course of the months or learn the span of the revolving year. Indeed through number we are taught so that we do not fall into confusion. Take number away, and everything lapses into ruin. Remove computus from the world, and blind ignorance will envelop everything, nor can men who are ignorant of how to calculate be distinguished from other animals".[24]

This passage comes from *Etymologiae* 3.4, where Isidore introduces arithmetic as one of the four branches of mathematics, and as the citation of Wisdom 11.21 would suggest, it leads into a discussion of the properties of different kinds of numbers based on philosophical numerology—though the allusion to Scriptural exegesis indicates Isidore's awareness of the parallel allegorical tradition. But Isidore's discussion of mathematics is not related directly to time-reckoning or the calendar, which he reserves for other sections of his encyclopaedia (the motions of the Sun and Moon in book 3, the units of time in book 5, and the Paschal reckoning in book 6, under the rubric of "ecclesiastical disciplines"). Moreover, Isidore's praise of computus—by which, it will be noted, he means simply "computation"—is a tissue of quotations from sources which have nothing to do with the calendar. The Augustinian borrowings are from exegetical and theological works,[25] and concern Biblical number symbolism. Cassiodorus' *Institutiones* 2.4.7 inspired the final section of this passage, beginning at "Using the science of numbers . . .". Here Cassiodorus is also discussing the mathematical properties of number in the "philosophical numerology" mode. His words echo those of Augustine in *De libero arbitrio*,[26] but he has altered Augustine's perspective in a significant way. For Augustine, "number" is the vehicle by which eternal Form is embodied in particular, material things. We can only know things insofar as they have "number", that is, insofar as they possess form. "Take away *these forms*",

[23]Contrast Isidore, *Etymologiae* 3, *praef.* (*mathematica* as science of abstract quantity) and 8.9.24–26 (*mathematici* as astrologers).

[24]PL 90. 647D–648D.

[25]"The *ratio* of numbers . . . measure, number and weight": cf. Augustine, *De civitate Dei* 11.30 (350.31–35). "The number six . . . a certain numerical significance": ibid. (350.1–9). "Likewise the forty days . . . without a knowledge of number": cf. Augustine, *De doctrina christiana* 2.16.25 (50.53–57).

[26]"Using the science of numbers . . . distinguished from other animals": cf. Cassiodorus, *Institutiones* 2.4.7, ed. R.A.B. Mynors, Clarendon Press, Oxford (1937), pp. 141.1–7; Augustine, *De libero arbitrio* 2.16.24, ed. W.M. Green, CCSL 29 (1970), 265.25–26.

he says, "and there will be nothing". Cassiodorus, however, is writing in praise of arithmetic and its manifold usefulness. For Cassiodorus, the comprehension of number is not the contemplation of forms, but the ability to *calculate*: "Take it away from the generation of those who now possess it, and blind ignorance will encompass all things; he who does not understand reckoning cannot be distinguished from the other animals". Calculation is useful for many things—financial accounting, for example, and also the reckoning of time, and it contributes to both intellectual and ethical discipline. The Irish author, however, singles out time-reckoning as the unique application of "the science of numbers", and ignores the others mentioned by Cassiodorus. "Computus" has acquired an exclusively calendrical connotation here,[27] without losing entirely its connection to the wider field encompassed by the Christian revision of classical philosophical numerology.

That connection is, in fact, celebrated by the Irish writer:

> DISCIPULUS: We would like to know where this calculation of numbers first came from.
> MAGISTER: From God, of course, because all wisdom and knowledge comes from the Lord God, by whom all things were made.
> DISCIPULUS: So tell me when this calculating was first discovered.
> MAGISTER: At the time when the creatures were made, that is, at the beginning of the universe. That is when number first began; as we read in Genesis, "And the evening and the morning were the first day" [Genesis 1.5]. He spoke of night and day when he said "evening" (that is, night) and "morning" (that is, day). But he spoke of number when he said "the first day", the second, third, fourth, fifth, sixth and seventh. God spoke of number again when he said about the sun and moon, "And they shall be signs for seasons and days and years" [Genesis 1.14]. For who can understand days and seasons and years except by number. Boethius says: Everything which is fashioned from the first nature of things (*a prima rerum natura*), is perceived to be given form by the *ratio* of numbers. For this was the principal exemplar in the Creator's mind. From it is derived the multiplicity of the four elements, and from it the changes of the seasons, and from it we understand the cycle (*conversio*) of the motion of the stars and of the heavens. Boethius also says: Properly speaking, every course of the stars and every astronomical *ratio* is constituted by the nature of numbers itself (*ipsa natura numerorum*). For thus we compute (*colligimus*) risings and settings, thus we track the slowing down and acceleration of the planets, thus we come to know the eclipses and manifold variations of the Moon.[28]

The Irish computist argues that time-reckoning is the primary manifestation of the role of number in generating and structuring God's creation. It is particularly significant that "number" is seen as prior to astronomy, a point which, as we shall see below, Bede challenged. But number is also a human science, and the foundation of "philosophy". Indeed, number is synonymous with philosophy.

> DISCIPULUS: This art—that is, number: what it is general name?
> MAGISTER: Philosophy, to be sure, because all wisdom is called philosophy.[29]

Not only are number and philosophy all but identical, but philosophy itself is defined in the most sweeping of terms:

> DISCIPULUS: How is philosophy defined?
> MAGISTER: Isidore defines it like this, saying: "Philosophy is the knowledge of things human and divine". This example shows that all wisdom, whether divine or human, is called philosophy.

[27] Changing meanings of *computus*: *The Reckoning of Time*, Appendix 4.
[28] PL 90.649A–B. The author is quoting from Boethius, *De institutione arithmetica* 1.2 (12.14–19), and 1.1 (12.6–10).
[29] PL 90.649B.

> Again, here is another definition of philosophy. "Philosophy is probable knowledge, insofar as this is possible, of divine and human things. And here is another definition of philosophy. "Philosophy is the art of arts and the discipline of disciplines".[30]

Then in a surprising turnabout, but one prepared by Isidore of Seville himself, our author proceeds to merge philosophy into Scripture, while retaining its secular origins:

> DISCIPULUS: How many divisions of philosophy are there?
> MAGISTER: Three: namely, physical, ethical, and logical—that is, natural, moral and rational. Hence Isidore says: "The form of philosophy is in three parts. One is natural, which in Greek is called φυσικὴ that is, physics, in which the investigation of nature is discussed. The second is moral, which amongst the Greeks is called ἠθικὴ, that is, ethics, and which deals with conduct. The third is rational, designated by the Greek word λογικὴ, that is, logic, in which, by disputation, truth itself is sought out, for instance, with respect to the causes of things or the conduct of life. Therefore the cause of inquiry is in physics, the order of discovery in ethics, and the law of understanding in logic [*Etym.* 2.24.3–4]." Hence, once again, there are three divisions of wisdom: physics, ethics and logic. The divine utterances consist of these three types of philosophy, for either they are wont to speak of nature, as in Genesis, Ecclesiastes, Proverbs, the Epistles of the Apostle Paul, or in the Song of Songs, and in the Gospels, and many other books, or else of conduct, or else of the higher celestial mysteries, that is, of logic [*Cf. Etym.* 2.24.8].
> DISCIPULUS: Who discovered these three parts of philosophy?
> MAGISTER: Thales of Miletus, one of the seven sages of Greece, discovered physics; Socrates discovered ethics; the philosopher Plato discovered logic.[31]

This delicate tight-rope walk between philosophy and the Bible has a very precise goal: to establish computus as the key to the relationship between number and the Divine, in line with the patristic revision of philosophical numerology. Computus, the Magister goes on to explain, is part of *physica*, but though *physica* is glossed "natural science", it is in fact the mathematical quadrivium. Of the four mathematical arts, arithmetic is the first in importance, for it is the exemplar of God's creative work.

> DISCIPULUS: Of these four divisions of *physica*, that is, of natural science, which should be learned first?
> MAGISTER: Arithmetic, without doubt. It is the beginning, and takes the role of mother to the others, so to speak. For arithmetic is prior to all, because God, the creator of all this mass of the cosmos, had [arithmetic] as the first model of his reasoning [*primum suae habuit ratiocinationis exemplar*]. From her he constituted everything, that is, by numbers he constituted and arranged all the elements.[32]

Arithmetic, synonymous with "number", which was earlier equated with computus, is both the foundation of philosophy and the governing principle of a creation ordered by "measure, number and weight". The climax of this intertwining of classical and Christian approaches to number is the endowing of number/arithmetic/computus with a double pedigree, divine (Abraham and Moses) and human (Pythagoras, Apuleius and Boethius).

> DISCIPULUS: Who first discovered number amongst the Hebrews and the Egyptians?
> MAGISTER: Amongst the Hebrews, Abraham first discovered number, and then Moses; and Abraham transmitted this science of number to the Egyptians, and taught it to them. So says Josephus.
> DISCIPULUS: Who first possessed this science of number amongst the Greeks and Latins?

[30] PL 90. 649B–C, quoting *Etymologiae* 2.24.1, 9, 3–4, and 8.
[31] PL 90. 649C–D.
[32] PL 90. 650A.

> MAGISTER: Pythagoras amongst the Greeks; Apuleius and Boethius amongst the Latins. Hence Isidore says: They say that Pythagoras as the first amongst the Greeks to write about the discipline of number, and that Nicomachus thereafter described it in a more extended manner, and that Apuleius and then Boethius translated this into Latin.[33]

The Irish computus of ca. 658 endows calendar science with "number mystique"—an essentially rhetorical claim to a distinguished pedigree, a close affiliation with philosophy, and a profound religious power, all based on the identification of time-reckoning with *numerus*. *Numerus* both captures and re-writes the core ideas of Christianized Pythagorean–Platonic numerology: the creation ordered by measure, number and weight becomes specifically the order of time, and the hierarchy of its divisions, which together form the theme of the second half of the Irish treatise. Though rhetorical in character, this mystique is not purely literary decoration, but appears to have inspired some distinctive practices in Irish computistical works. A notable feature of these texts is the interest in mathematical calculations which go well beyond what is necessary for calendrical purposes. For example, the author of the anonymous 7th century *De ratione conputandi* sets his students problems involving the calculation of extremely minute divisions of time which have no computistical application.[34] In Irish circles, the dry computistical formula or *argumentum* ("If you wish to know X, take Y, multiply by Z . . .") stimulated the production of numerous imitations and variations, many of which have little or no relevance for the calendar. It was arithmetic for its own sake.[35]

Computus' number mystique arises from a complex cultural process, in which the ancient concept of the arts as a preparation for philosophy mutates into the notion that the arts are a repository of erudition for the Christian intellectual. The handbook for Bible studies proposed by Augustine in *De doctrina christiana* never materialized; moreover, he appears to assume that someone in search of the first, or philosophical kind of number-lore would know where to turn. In practice, then, "number" as a branch of *doctrina christiana* had no canon of literature. Cassiodorus, acting on Augustine's assumption, collected and described the appropriate ancient mathematical treatises for his monks. But most of the works listed in the *Institutiones* were not readily available to western Europeans in ensuing centuries: even Boethius' *De arithmetica* was little known, outside some Irish circles, until the Carolingian period.[36] What was generally available, for example in Isidore's *Etymologies*, was relatively diluted. In consequence, much mathematics and *physica* was cut adrift without a secure "address" in the new Christian scheme of learning. Computus came, willy nilly, to offer a legitimate Christian shelter to these refugees from the ancient scientific encyclopaedia, and inherited its mystique. It was not the only applied science to do this: the technical literature of the *agrimensores* or surveyors sometimes offers reflections on

[33] PL 90. 650A–B. The quotation from Isidore is from *Etymologiae* 3.2.

[34] *De ratione conputandi* 108–109, ed. Dáibhí Ó Cróinín, in: Maura Walsh and Dáibhí Ó Cróinín (eds.), *Cummian's Letter 'De controversia paschali' and the 'De ratione conputandi'*. Studies and Texts 86, Pontifical Institute of Mediaeval Studies, Toronto (1988), pp. 209–210.

[35] Jones, *Bedae Opera de temporibus*, p. 71; Baker and Lapidge, p. xli.

[36] Gillian R. Evans, Introductions to Boethius' *Arithmetica* of the Tenth to the Fourteenth Century, *History of Science* 16 (1978), 22–41. Irish use of *De arithmetica*: Walsh and Ó Cróinín, *Cummian's Letter*, p. 122, and Ó Cróinín, The Irish as Mediators of Antique Culture on the Continent, *Science in Western and Eastern Civilization in Carolingian Times*, ed. P.L. Butzer and D. Lohrmann, Birkhäuser Verlag, Basel (1993), p. 44.

the spiritual and intellectual ascesis of studying geometry quite analogous to the "number mystique" of the Irish computus.[37] Computus, however, was distinctive in that it had an explicitly Christian religious mission, which allowed it to absorb allegorical numerology into its mystique. The potential scope of computus was also broader than that of the *agrimensores* texts: "time" was an elastic container which would accommodate, besides arithmetic and astronomy, materials on geography (the latitudes or climates defined by the varying lengths of the days), meteorology (the weather patterns of the seasons), music and prosody (acoustic phenomena in time), and even physiology and medicine (season regimina). One can see this in the shape of early computus manuscript anthologies, which incorporate an ever-expanding halo of extra-computistical scientific material.[38] The core of this material is arithmetic and astronomy, so when the Carolingian intellectuals sought to recover the ancient philosophical curriculum, they found the materials for two of the seven arts—arithmetic and astronomy—on the pages of *computi*.[39]

It would seem, then, Byrhtferth's justification for including a whole book dedicated to number symbolism in his *Enchiridion* was the tradition of computistical number mystique represented by the Irish computus text discussed above. That tradition was, as we shall see, taken up by the Carolingian computists who most influenced Byrhtferth, particularly Hrabanus Maurus. The story could end here, but it does not, for Byrhtferth, like Hrabanus before him, drew heavily on Bede's *De temporum ratione* for his treatise. And *De temporum ratione* promotes a different view of the relationship of computus to mathematics, one which is actually hostile to "number mystique".

3. Computus as *ratio temporum*: Bede's revision of computistical "number mystique"

It would be difficult to exaggerate the importance of Bede's *De temporum ratione* in the evolution of computus in the Middle Ages. Besides being a superbly lucid and well-organized text-book, and a persuasive argument in favour of the Alexandrian computus, this book is also a response to Augustine's proposal for a Christian encyclopaedia, in that it fuses computus with chronology (cf. *De doctrina christiana* 2.28.42–43) on the one hand, and natural history, especially cosmology, geography and physiology on the other.[40]

[37] Kurt Reindel, Vom Beginn des Quadriviums, *Deutsches Archiv zur Erforschung des Mittelalters* 16 (1959), 519–520.

[38] Probably the oldest extant computus MS, Vatican City BAV Reg. lat. 2077 (s. VII in., from Vivarium or a monastery in its orbit) contains, apart from Paschal tables, a treatise on the seven prophetic weeks, and a wind-rose: in short, a representation of allegorical numerology, and a token of the scientific "halo": F. Troncarelli, Il consolato dell'Anticristo, *Studi medievali* 3. ser. 30 (1989), 567–592, esp. p. 573, and ibid., Una pietà più profonda. Scienze et medicina nella cultura monastica medievale italiana, *Dall'eremo al cenobio. La civiltà monastica italiana dalle origini all'età di Dante*, Scheiwiller, Milan (1987), Fig. 536 (see also pp. 706–708).

[39] Dispersal of mathematics in the early Middle Ages into "functional" settings such as computus: Michael S. Mahoney, Mathematics, *Science in the Middle Ages*, ed. David Lindberg, University of Chicago Press, Chicago (1978), pp. 145–150; and Stephen McCluskey, *Astronomies and Cultures*, *passim*. Computus as vehicle for quadrivium in early Middle Ages: Brigitte Englisch, *Die Artes liberales im frühen Mittelalter (5.–9. Jh.): Das Quadrivium und der Komputus als Indikatoren für Kontinuität und Erneuerung der exacten Wissenschaften zwischen Antike und Mittelalter*. Sudhoffs Archiv, Beiheft 33, Franz Steiner, Stuttgart (1994).

[40] *The Reckoning of Time*, pp. lxiii–lxxi.

However, the “science of number” in either its philosophical or its allegorical mode, plays a surprisingly subdued role. This is all the more surprising in that Bede definitely knew the Irish computus of ca. 658.[41] But unlike the Irish computus, *De temporum ratione* contains no general disquisition on “number”, no encomium from Isidore or quotations from Boethius.[42] Bede never considers computus as a domain of knowledge that can be defined in its own right; indeed, he uses the term *computus* only twice, and without comment (once in the title of Ch. 1, and once in the chronicle in Ch. 66 to record Theophilus of Alexandria’s composition of a work on Paschal reckoning). He never alludes to computus as a branch of philosophy or an “art”. Furthermore, he neglects to praise number as the principle of cosmic created order, and does not quote Wisdom 11.20 on “measure, number and weight”—though he did not hesitate to do so in his own commentary on *Genesis*.[43] Finally, he never equates computus with *numerus* in *De temporum ratione*, even though when working in exegetical mode, he could regard “number” and “time” as interchangeable terms.[44]

For “computus”, Bede substitutes the phrase *ratio temporum*, a term which appears in the opening sentence of his textbook, and which gave it its conventional title.[45] His notion of *ratio temporum*, moreover, is grounded not in *ratio numerorum* but in cosmology. In fact, he goes out of his way to define time as a dimension of the created universe, and not as a numerical abstraction. Moreover, he reverses the hierarchy of the Irish computus by making arithmetic subordinate to astronomy. For example, he mentions the etymology of “month” from “measure”, only as a foil for his “more correct” explanation that “month” comes from the word for “Moon”.[46] Moreover, Bede’s case on behalf of the Alexandrian reckoning rests on the argument that the Alexandrian computus is based on the astronomical data which obtained at Creation. God must have created the Sun at the spring equinox (21 March) and the Moon must have been full. These phenomena are not recorded in Scripture, but they are justified by cosmological symbolism: first, God would not have created anything other than in a perfect state, and secondly, the positions of the planets at Creation are *figurae* of the relationship between Christ and the Church, whose redemption he achieved at Easter. Hence all the cycles of time which the computist measures must start from this point, and must end when the planets in question resume their status at the time of Creation.[47] This is what Bede calls *natura*, and the *naturalis ratio* of computus is calendar calculation based on this premise.[48] The broad array of extra-computistical material

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41 *The Reckoning of Time*, p. lxv.

42 Bede’s use of Irish *computistica*: Daíbhí Ó Cróinín, The Irish Provenance of Bede’s *Computus*, *Peritia* 2 (1983), 238–242. Bede’s apparent ignorance of Boethius’ *De arithmetica*: C.W. Jones, *Bedae pseudepigrapha: Scientific Works Falsely Attributed to Bede*, Cornell University Press, Ithaca and London (1939), p. 49.

43 *Libri quatuor in principium Genesis usque ad nativitatem Isaac et eiectionem Ismahelis adnotationum*, 1.1.31, ed. C.W. Jones. CCSL 118A (1967), 32.966–974.

44 See *praefatio* to his *Expositio Apocalypsis*, where Bede, transcribing Augustine’s summary of Tyconius’ rules for exegesis, inserts his own gloss on rule 5 “Of Times”: “It seems to me that it can also be called ‘of numbers’ ”: ed. Roger Gryson, CCSL 121A, Brepols, Turnhout (2001), pp. 277.75–76.

45 See *De temporum ratione* preface (263.2; transl. F. Wallis, p. 3).

46 *De temporum ratione* 11 (312.2–313.9; transl. F. Wallis, pp. 41–42).

47 *De temporum ratione* 6 (291–295; transl. F. Wallis, pp. 26–28) and 64 (456–459; transl. F. Wallis, pp. 151–155).

48 This point is discussed in detail in my essay “A New Approach to Bede’s Science”, in: *Bede: New Approaches*, ed. Scott De Gregorio (forthcoming).

which Bede folded into his treatise reflects this orientation to cosmology and astronomy: climates, tides, planetary phenomena like eclipses, the effects of seasonal changes on terrestrial life. There is, to be sure, a considerable amount of mathematics in *De temporum ratione*, some of it quite demanding, and Bede is relentless in his insistence that students learn to compute.[49] But mathematics *per se* is never mentioned. For Bede it is a technique, a means to the end of establishing a correct calendar. Unlike the Irish, he is not interested in making students perform complex calculations which serve no computistical purpose; indeed, he seems to find this activity suspicious. In Ch. 3, Bede discusses the divisions of time smaller than a day, but qualifies them as "not natural, but apparently agreed upon by [human] convention". By the time one reaches the smallest units of reckoning, like the *atomus*, one has left entirely the realm of practical calculation. Such minute division of time reminds Bede of astrologers (*mathematici*), who divide the zodiac into infinitesimal segments, and who base their claims to predict the future with precision on such feats. Bede rejects this claim with scorn: indeed, his reticence in using the term "computus" may be due to its implicit association with astrology.[50]

In Ch. 38, writing about the determination of the leap-year day, Bede waxes sarcastic about virtuoso calculators who lack substantial knowledge of the astronomical phenomenon they are measuring, or its computistical consequences:

> Some people are so clever at computation that they understand without difficulty how many fractions of the growing leap-year day are relentlessly added on each year, or month, or even week or day. Nonetheless, they would not be able to say how this same fraction increases, or what is its cause, or how the increase is calculated, or what troublesome error is engendered if the leap-year day is not intercalated in its sequence in the necessary way.[51]

Bede is certainly not averse to confronting the more extreme mathematical challenges of computus—in Ch. 41, for example, he thinks it worthwhile to examine "by what and how many particles of time" the Moon's leap-year day accrues, and in Ch. 46, he makes his students add up the total number of lunar and solar days in the 19-year cycle—but such feats are strictly subordinated to elucidating the problems of the calendar, and have no value beyond this.

Bede's reasons for steering away from "number mystique" are complex, and largely elusive, but one issue in particular seems to have made him especially sensitive to the dangers of over-enthusiastic computation: chiliasm. From the time he published his revised chronology of world-history in his early essay on computus, *De temporibus*, Bede had to contend with the objections of the literal-minded that the six ages of the world (a schema that Bede himself adopted[52]) had to unfold over 6000 years. The logic was that world-history replicated the week of creation (an analogy which Bede also accepted), and since Scripture states that a thousand years for God are as a day, the world was therefore destined

[49] E.g., *De temporum ratione*, Ch. 17–19, 21–23: see *The Reckoning of Time*, pp. 290–293, 296–299, re: Bede's preference for calculation over tables.

[50] *De temporum ratione* 3 (277.34–278.40; transl. F. Wallis, pp. 15–16); cf. *The Reckoning of Time*, Appendix 4: A Note on the Term Computus.

[51] Ch. 38 (399.8–14; transl. F. Wallis, pp. 106–107). The target seems to be the Irish: cf. the Irish-inspired Bobbio computus, Ch. 87 (PL 129.1315) which works out how much of the leap-year increment accrues in a single day.

[52] *De temporibus*, *Bedae opera didascalica* 16 (600–602); De *temporum ratione*, Chs. 10, 66. On Bede's treatment of the world ages, see *The Reckoning of Time*, pp. 353–366, and literature cited therein.

to end in *annus mundi* 6000. *De temporibus*, by adopting the chronology of the first two ages recorded in Jerome's Vulgate, defied this equation of an age with 1000 years much more radically than did the more conventional chronology of Eusebius, and brought the charge of heresy down on Bede. Bede's response in the *Epistula ad Plegwinum*, and in the preface and closing chapters of *De temporum ratione* itself, was to firmly deny that an "age" had any fixed measurement that could be calculated. Indeed, to do so would be heretical, for it would mean that we could know when the end would come by simply adding up the years since Creation, and Christ himself warned that no one, not even the Son, knew when this would be.

In the letter to Plegwin, Bede records his irritation not only at the "rustics" who pester him to tell them "how many years are left in the final millennium of the world", but at the "brothers" who dispute whether the end will come in *annus mundi* 6000 or *annus mundi* 7000.[53] Yet he himself was deeply committed both to the notion of the six world ages, and to a eschatological orientation. Indeed, contemplation of the elision of time and eternity replaces *numerus* as the focus of contemplation in Bede's computus. Rather than seek to identify computus with the mystique of mathematics as a way of understanding the divine mind, Bede chooses to imagine computus as a temporal crescendo, rising from the smallest unit of time to the world-ages, and from the world-ages to the world-to-come. Bede's closing chapters of *De temporum ratione* are a symphonic *tour de force* of overlapping temporal symbols, where the week of Creation, the Paschal week, and the cycles of calendrical time converge in a new "first day", an eternal Sunday and an everlasting Easter. In the light of his views on chiliasm, Bede was certainly running a risk in associating the reckoning of time with speculation on the future. But this association could hardly be avoided by the computist, whose mandate was to predict the future, if only the future date of Easter. In a sense, Bede's emphasis on the cyclical structure of computed time took the apocalyptic sting out of such reflections, for every Easter, every Sunday is a *figura* of eternity. But his awareness of latent danger may have made him especially cautious about computation and the "science of numbers", even though he was capable of exquisite subtleties of calculation himself, and by no means averse to applying numerology to time. We have already noted his interest in the symbolism of the seven-day week; to this we may add some more casual observations he draws from Augustine about the meaning of the 24 hours in the day, or of the quarter-day of the leap-year.[54] But this allegorical numerology is here used as discrete decoration. Bede sees no inherent religious meaning in number or in mathematical operations, no "number mystique".

Perhaps the most striking illustration of Bede's intention to de-mystify number is the opening chapter of *De temporum ratione*, whose title contains one of the rare appearances of the word *computus* in Bede's work: *De computo uel loquela digitorum*. Here Bede describes the Roman technique of finger-computation, justifying its study as "a very useful and easy skill" for students of the calendar. The exegete might also find that it is worth knowing something about finger-reckoning, because the positions of the hands which correspond to the numbers 30, 60 and 100 furnish a beautiful interpretation of the parable of

[53] *Ep. ad Plegwinum* 15, *Bedae opera didascalica*, pp. 624–625; transl. F. Wallis, *The Reckoning of Time*, pp. 413–414.

[54] 24-hour day: *De temporum ratione* 5 (quoting *De Genesi ad litteram*) (284.19–29; transl. F. Wallis, p. 19). Quarter-day: ibid., Ch. 39 (quoting *De Trinitate*) (402.26–404.59).

the Sower (Matthew 13.1–23). But curiously, after describing the manual positions representing the various numbers, Bede does not actually explain how the computist might perform calculations with the hands. Instead, he teaches his students how to use the manual numerals to convey secret messages, by representing each letter by a number corresponding to the numerical position of the letter within the alphabet. These secret messages could be deadly serious—outwitting enemies, for example—but Bede also describes the technique as a sort of party-trick. This is, of course, also the basis of *gamatria*, the computation of the numerical value of names. Bede was aware of *gamatria* as a technique of exegesis—his own commentary on the Apocalypse, for example, reproduced various solutions to the riddle of the name of the Beast coded into the number 666[55]—but in *De temporum ratione*, it is just a game.

4. Byrhtferth's choices

Carolingian educators and thinkers therefore inherited two distinctive traditions of calendar science. One tradition, represented by the Irish texts, called this science computus—"calculation"—and closely identified it with arithmetic, *ratio numerorum*. The tradition articulated by Bede, on the other hand, preferred to call calendar science *ratio temporum*—"the reckoning of time"—and stressed the roots of that reckoning in astronomy, the movements of the Sun and Moon. Arithmetic played a significant, but distinctly instrumental role. Byrthferth definitely espoused the "Irish" rather than the "Bedan" approach, preferring to imagine the world as number rather than as "natural history".[56] But he had the option of choosing the Bedan route, or even, like Hrabanus Maurus, of fusing the two. Hrabanus Maurus' *De computo* is essentially an adaptation of *De temporum ratione*, but prefaced with the encomium of *ratio numerorum* found in the Irish computus.[57] Moreover, he responded in a typically Carolingian way to one particular theme of "number mystique", namely, the alleged relationship of computus to the Liberal Arts, and to philosophy.

Nourished by the re-discovery of Martianus Capella's *De nuptiis* and other didactic texts from late Antiquity, Carolingian schoolmasters and intellectuals like Remigius of Auxerre meditated on the classification of the arts and their role as a preparation for philosophy. Philosophy itself recovered some of its ancient content through the diffusion of texts like Boethius' *De consolatione*, Calcidius' partial translation of and commentary on the *Timaeus* of Plato, and Macrobius' commentary on Cicero's *Dream of Scipio*. These works furnished at once a classificatory framework for the mathematical sciences, and a powerful *raison d'être* to study them: the quadrivium. What was missing, however, was a content for the category labelled "quadrivium". Apart from what was included in the texts of Calcidius, Macrobius or Martianus themselves, westerners had few resources for reconstituting

[55] *Expositio Apocalypsis*, c. 22 (415.79–417.94).

[56] Jennifer Neville, *Representations of the Natural World in Old English Poetry*, Cambridge Studies in Anglo-Saxon England 27, Cambridge University Press, Cambridge (1999), p. 155.

[57] *De computo* 1, ed. Wesley Stevens, Corpus christianorum continuatio mediaevalis 44, Brepols, Turnhout (1979), pp. 205–206. Hrabanus and number symbolism: B. Taeger, *Zahlensymbolik bei Hraban, bei Hincmar—und im 'Heliand'? Studien zur Zahlensymbolik im Frühmittelalter.* Münchner Texte und Untersuchungen zur deutschen Literatur des Mittelalters 30, Fink, Munich (1975).

the mathematical curriculum. An excellent illustration of how they improved this content through creative *bricolage* is the *Geometria Boethii I*, a composite work assembled probably at Corbie in the 7th century from parts of Boethius' translation of Euclid, sections of his *De arithmetica*, excerpts from Isidore and Cassiodorus, and some materials from the *agrimensores*.[58] By the mid-9th century, to be sure, there is evidence for the slowly expanding circulation of Boethius' *De arithmetica* and *De musica*, but their readership was, until the end of the 10th century, relatively small and specialized.[59] What filled the gap, was computus.

Computus already carried much of the quadrivium in its baggage, so to speak, and it had an additional advantage: an established position in the Carolingian program of *doctrina christiana*. This curriculum was designed in the first place to produce a literate clerical elite, and its contents were entirely vocational. As Charlemagne put it in his *Admonitio generalis*, students were to learn *psalmos* (i.e. how to read—the Psalter being the elementary textbook of clerical literacy), *notas* (writing), *cantus* (chant), *compotum*, *grammaticam*, and *libros catholicos* (the literature of the Christian tradition).[60] The Liberal Arts are not mentioned, with the exception of *grammatica*, which probably meant composition. *Compotus* meant calendar science.

That being said, Carolingian intellectuals persisted in developing the connection between computus and the quadrivium in a variety of ways. There seems to have been little question that computus belonged with astronomy, not arithmetic—in other words, that Bede's and not the Irish perspective on computus and number had prevailed. When Hrabanus Maurus re-cast Cassiodorus' *Institutiones* for his own day, and sought a place for computus (a branch of learning not mentioned by Cassiodorus), he chose to file it under astronomy.[61] Even more telling are the Carolingian computus manuscripts, some of *de luxe* quality, which set computus in the midst of an extensive penumbra of astronomical and cosmological materials,[62] or the Carolingian taste for dismembering the text of *De temporum ratione* in order to fit computus under *astronomia* in new compilations and anthologies.[63] Computus was on the way to being absorbed by the quadrivium.

This trend of separating mathematics from computus intensified in the 11th century, especially in the career of Gerbert of Aurillac (d. 1003), *scholasticus* of Reims and later Pope Sylvester II. Gerbert represents a significant turning point in the history of western mathematics, and his innovations are difficult to summarize.[64] We must content ourselves with

[58] Paul L. Butzer, Mathematics in West and East from the Fifth to Tenth Centuries: an Overview, *Science in Western and Eastern Civilization in Carolingian Times*, 451.

[59] Alison White, Boethius in the Medieval Quadrivium, in: *Boethius: His Life, Thought and Influence*, ed. Margaret Gibson, Basil Blackwell, Oxford (1981), pp. 164–169.

[60] Ed. A. Boretius, *Capitularia regum francorum*, Monumenta Germaniae Historica Leges (Quarto), 2, Berlin (1883), pp. 1.59–60.

[61] Hrabanus Maurus, *De clericorum institutione*, PL 108.403C–D; on *astrologia naturalis*, see Isidore, *Etymologiae* 3.27.1–2.

[62] Arno Borst, Alkuin und die Enzyklopädie von 809, *Science in Western and Eastern Civilization in Carolingian Times*, pp. 53–78; Bruce Eastwood, The Astronomies of Pliny, Martianus Capella and Isidore of Seville in the Carolingian World, ibid., pp. 161–180; Anton von Euw, Die künstlerische Gestaltung der astronomischen und komputistischen Handschriften des Westens, ibid., pp. 251–269.

[63] *The Reckoning of Time*, pp. xci–xcii.

[64] Essential literature on this topic: Uta Lindgren, *Gerbert von Aurillac und das Quadrivium: Untersuchungen zur Bildung im Zeitalter der Ottonen*, Sudhoffs Archiv Beiheft 18, Franz Steiner, Wiesbaden (1976); Joseph Navari,

three here. First, Gerbert raised the study of mathematical theory to a new level through his creative reading of Boethius' *De arithmetica*. Mathematics was for him a special form of logic, and he drew the attention of his students to the presence of challenging logical problems in the Boethian text. At the same time, he recovered something of Boethius' classical understanding of philosophical numerology, and the intellectual ascesis attendant on the study of mathematics. Responding to his patron Otto III's request for an explanation of *De arithmetica*, Gerbert wrote:

> If you were not firmly and immovably convinced that the power of numbers both contained the origins of all things in itself and brought forth all things from itself, you would not be hastening to a full and perfect knowledge of these numbers with such zeal.[65]

Secondly, Gerbert was the first teacher who is recorded as having specialized in the quadrivium as a "subject"; in other words, he was re-shaping the curriculum in a non-vocational direction. Gerbert's students were enthralled by his schemata showing the divisions of philosophy and the relationship of the *artes* to philosophy. One such taxonomy, circulating through the cathedral and monastic schools of the day, occasioned a famous public debate before Otto III himself between Gerbert and the *scholasticus* of Magdeburg, Otric—a debate which hinged on the relationship of *physica* and mathematics.[66]

Finally, Gerbert was responsible for introducing new content into the quadrivium which definitively pushed it beyond the boundaries defined by computus. In particular, his introduction of the classical abacus inspired a prolific output of treatises on calculation over the following century, and his promotion of the astrolabe from was the initial step in a process which would eventually result in the recovery of Ptolemy's *Almagest*.[67] In all these activities, he promoted the study of the mathematical sciences at a level well beyond what computus required, and without reference to its problems. Indeed, Gerbert seems never to have taught or written on computus.[68]

This is not to claim that a Gerbertian revolution replaced computus with the quadrivium overnight. To the contrary, the vocational style of education continued to be the norm for many decades after Gerbert. Indeed, those who followed in Gerbert's footsteps by composing treatises on the abacus or astrolabe, like Abbo of Fleury, Hermannus Contractus and

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The Leitmotif in the Mathematical Thought of Gerbert of Aurillac, *Journal of Medieval History* 1 (1975), 139–150; Carla Frova, Le opere matematiche di Gerberto d'Aurillac, *Studi sul medioevo cristiano offerti a Raffaelo Morghen*, Istituto Storico Italiano, Rome (1974), pp. 323–353; Guy Beaujouan, Les apocryphes mathématiques de Gerbert, in *Gerberto: scienza, storia e mito. Atti del Gerberti Symposiums (25–27 luglio 1983)*, Archivi Storici Bobiensi, Bobbio (1985), pp. 645–658.

[65] Ep. 187, ed. F. Weigle, *Die Briefsammlung Gerberts von Reims*, Monumenta Germaniae Historica, Briefe der deutschen Kaiserzeit, Berlin (1966), p. 224; transl. Harriet P. Lattin, *The Letters of Gerbert*, Columbia University Press, New York (1961).

[66] Richer, *Histoire de France (888–995)*, 3.56–65, ed. Robert Latouche, Classiques d'histoire de France au Moyen Age, 12, Les Belles Lettres, Paris (1967), pp. 66–80.

[67] W. Bergmann, *Innovationen im Quadrivium des 10. und 11. Jh.: Studien zur Einführung von Astrolab und Abakus im lateinischen Mittelalter*, Sudhoffs Archiv, Beiheft 26, Franz Steiner, Wiesbaden (1985); Guy Beaujouan, L'Enseignement du 'quadrivium', *La scuola nell' occidente latino dell' alto medioevo*, Settimane di studio del Centro italiano di studi sull' alto medioevo 19, Centro italiano di studi sull'alto medioevo, Spoleto (1972), pp. 639–667, 719–723; Arno Borst, *Astrolab und Klosterreform an der Jahrtausendwende*, Sitzungsberichte der Heidelberger Akademie der Wissenschaften, phil.-hist. Kl., Carl Winter, Heidelberg (1989).

[68] Lindgren, *Gerbert von Aurillac*, 44; Lindgren sees this as a point which distinguishes Gerbert from contemporary *scholastici* like Abbo of Fleury or Heriger of Lobbes: see pp. 52, 55, 57.

Gerlandus Compotista, also wrote on computus, and their works on the "new mathematics" are found side by side with computus materials in the manuscripts. Nonetheless, these new domains of mathematical activity appropriated the "number mystique" hitherto associated with computus by laying claim to a high philosophical vocation,[69] to *profunditas,*[70] and to number symbolism.[71]

Byrhtferth's teacher Abbo of Fleury was in the thick of this trend. His commentary on the *Calculus* of Victorius takes mathematics beyond computus, while capturing the mystique of number traditionally conveyed by the computus. Abbo's commentary is imbued with a Platonic philosophical numerology, yet he also stresses the role of number as the constituent principle of creation, and in so doing adopts a more conservative and Christian tone. Abbo titled his commentary *Tractatus de numero, mensura et pondere super calculum victorii*,[72] and he intended that all the theological resonance of that ubiquitous tag from the Book of Wisdom should be present in his work. Indeed, measure, number and weight held the key to understanding the meaning of the Trinity.[73] To be sure, the *Calculus* is defended as an indispensible adjunct to "all the arts which deal with the reckoning of numbers, that is arithmetic, geometry, music and astronomy"—the quadrivium, in other words—but what this essentially amounts to is its utility for those who must tackle problems of *ratio temporum.*[74] But in fact, computus is never explicitly discussed in the commentary. "Number mystique" has left its temporary shelter in calendar science, and resumed its ancient residence in the quadrivium.

Faced with an array of models—the Irish computus, still in circulation in the 11th century, *De temporum ratione* and Hrabanus' *De computo*, and the works of his master Abbo—Byrhtferth opted for a conservative, even archaic style of "number mystique" embedded in computus, and we can only conclude that he did so deliberately. This choice not only explains his inclusion of a tract on number-symbolism, but sheds light on his many poetic and strangely affecting invocations of the "mystery" of computus.

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[69] Gillian R. Evans, The Influence of Quadrivium Studies in the Eleventh and Twelfth Century Schools, *Journal of Medieval History* 1 (1975), 151–154; *Difficillima et ardua*: Theory and Practice in Treatises on the Abacus, 950–1150, *Journal of Medieval History* 3 (1977), 21–38.

[70] Gillian R. Evans, Schools and Scholars: the Study of the Abacus in English Schools ca. 980–ca. 1150, *English Historical Review* 94 (1979), 79–80.

[71] Gillian R. Evans, The Development of some Textbooks on the Useful Arts, *History of Education* 7 (1978), 90–91.

[72] *Commentary* II.1, ed. Peden, 65; see also Abbo of Fleury, *Quaestiones grammaticales* 50, ed. Anita Guerreau-Jalabert, Les Belles Lettres, Paris (1982), p. 275.

[73] Evans and Peden, pp. 112–114.

[74] *In presentiarum tamen intentio Victorii haec fuit, ut inerrato lector numerorum summas multiplicaret, diuideret; seu proponeretur aliquid de artibus, quae numerorum ratione constant, ut arithmetica, geometrica, musica, et astronomia, seu quaestio inesset de mensura et pondere, quae omnia calculatori sunt curae. Denique, et huius calculi quanta sit utilitas agnoscere potuit ut praecedentibus dictis, etiamsi ad quamlibet rationem temporum ob difficultatem multiplicandi nullomodo desudavit, qui talium rerum saltem parvam intelligentiam accepit. Verum, quoniam omnia creata sunt in numero mensura et pondere, in his singulis speculationem placet constituere, ut perspecta eorum natura, facilius pervideri possit singula quibus insunt in causa*: *Commentary* I.3, ed. Peden, 65.

> Yesterday, when the serene light of the golden sun had expelled the shadows from the innermost recesses of the mind, theology (that is, language concerning God) spoke concerning the enterprise of computus, and thereafter concerning the days of the solar year.[75]

> We have touched with our oars the waves of the deep water; we have seen as well the mountains by the shore of the salty sea, and with billowing sails and prosperous winds we have harboured on the coast of the fairest nation. The waves stand for this profound science [*deoper craeft*] and the mountains stand for the magnitude of this science. Therefore it says, ‘Where we saw the lily blossom’ (the beauty of the computus), ‘there we sensed the roses’ fragrance’ (we perceived the profundity of the computus).[76]

Bede had rejected computistical number mystique; Hrabanus paid it lip-service only; Abbo and his generation unyoked number mystique from computus. But for Byrhtferth, computus’ number mystique was still vital and valid. The secret of its persistent attraction may lie in the breadth of its appeal. Computus was a subject that all educated people, especially clergy, but laymen as well, were expected to know at least something about. The abacus, by contrast, was accessible only to a scholarly elite. In the broader stream of early medieval culture, numbering and calculating were overwhelmingly, almost automatically associated with the reckoning of time. Hrotswith of Gandersheim reveals how widely accepted this equation was. Her heroine Sapientia tells the emperor Hadrian how old her children are by means of a complex mathematical riddle. When the emperor dismisses the riddle as an “unprofitable dissertation”, she replies:

> It would be unprofitable if it did not lead us to appreciate the wisdom of our Creator, Who in the beginning created the world out of nothing, and set everything in number, measure and weight, and then, *in time and the age of man*, formulated fresh wonders the more we study it.[77]

The number mystique of computus was Pythagoreanism for Everyman. It had spoken on behalf of the nexus of mathematics and the divine for many centuries, and that voice still resonated at the turn of the millennium.

[75] *Enchiridion* 1.1 (16.164–166), and transl. p. 17.
[76] *Enchiridion* 1.1 (16.145–158), and transl. p. 17.
[77] *Sapientia* 3, transl. Christopher St. John, *The Plays of Roswitha*, Cooper Square, New York (1966), p. 140. My emphasis.

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CHAPTER 10

Is the Universe of the Divine Dividable?

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MATHEMATICS AND THE DIVINE: A HISTORICAL STUDY
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> "When the lights of the sefirot start to glow, to spread and to unite, the face of the community of Israel is radiant." (Zohar II, 232b)

1. The sefirotic concept of Divinity developed in response to philosophy and rational theology

As Tishby pointed out following Gershom Scholem, the great writings of the Spanish Kabbalah, one of them being the Zohar, developed ideas regarding God which were radically new and which were created in response to the philosophical conceptions of the times. However, this converting of philosophical concepts into mysticism-based theological ones brought about immeasurable separation between kabbalist representations on the one hand and concepts of rational theology on the other. Among the themes that were so dear to medieval philosophers, whether they were Jewish (Maïmonides), Christian (Thomas Aquinas) or Muslim (Ibn Badja, Ibn Tufayl and Averroes), the theme of divine immobility and unaffectedness was one of the more prominent ones. The Jews, who were very much attached to a non-conceptualised representation of God, and who identified with the naïve faith of their ancestors, as it was described in the Bible and greatly amplified by rabbinical literature, could not accept that a philosophical idea might take the place of the Divinity, who revealed himself to the Hebrews on Mount Sinai, who delivered them from Egyptian slavery, who satisfied their needs during the journey through the desert and who established them in the Holy Land. This is what Gershom Scholem wrote about the biblical God:

> "Before entering the melting pot of speculative theology which came about as the result of a long and difficult development, the biblical God was not an abstract construction, nor was he the end product of continued intellectual efforts. The biblical God must be seen as a historical experience of a community, an experience which the devout of the biblical times lived through individually at all levels of their existence. This God communicates with man, expresses himself and intervenes without mediation into the created world. His manifestations are too tangible to require proof. Whenever he exerts his power, this is felt in nature and especially in History. When he hides, then this is not due to his nature, but to the fact that we are not worthy of his revelation, because we ourselves have enveloped him in a veil, for which we are responsible. This God, whose ways and thoughts are incomparably superior to ours, is to a large and even very large extent assigned positive attributes by the biblical books. He is the creator, the king, the judge, the dispenser of blessings, the master of History, who leaves the mysteries which surround him in order to reveal himself." [1]

Finally the daily invocation of God during prayer would loose all its meaning if one were to follow the philosophers into their domain. If God is immobile and unaffected as well as gifted with eternal will, why should one direct one's prayers to him? In this context it is worth mentioning the importance which even pre-zoharic authors such as Azriel attached to the mysticism of prayer—an obvious reaction to the only marginal role which orison could play in the eyes of the medieval philosophers.

The sefirotic concept of Divinity, i.e. the concept which involves the ten primordial or ideal numbers (*sefirah*, plural: *sefirot*) was clearly meant as a response to a situation which the kabbalists felt had become most serious. The sefirot, they said, incarnated the dynamic unity of Divinity. Certainly, God transcends the categories of time and space and

[1] [5, p. 18]. All translations into English are the editors'.

his essence is not affected by passions as man's is, but all this does not detract from the fact that God is in some way present in the world.

This opposition between the dogma of the transcendence of God and the determination of the kabbalists to affirm his immanence reveals something of their philosophical culture. Even the most determined mystics could not leave the question of the immanence of divine essence to be a matter of naïve faith only. They decided to adopt the nomenclature of the sefirot in order to show that their God was a God of providence, that he cared for his creatures, that he could be implored, even if his name could not be pronounced.

2. *Eyn-sof*, the perfectly unknowable origin of the sefirot

The kabbalists insisted on attributing an occult origin to the world of the sefirot, an origin which was absolutely unknowable and the name of which rightfully called up the idea of the infinite, i.e. *Eyn-sof*. It is this polar tension between the eminent essences (the sefirot), which are barely accessible, and their absolutely occult source, which makes itself felt in zoharic literature, and even long before, i.e. in early mystical circles, such as those around Isaac the Blind. During revelation the sefirot serve somehow as a mirror to the hidden God. It would indeed be a *contradictio in adjecto* to expect of Eyn-sof to reveal itself. It is the sefirot which take care of this.

This occultation of Eyn-sof is very much present in the comments on the sefirot by Azriel of Gerona. Eyn-sof, so this author writes, escapes the grip of thought; no language can comprehend it and yet it is present everywhere. No letter, no speech, no name can contain it. Hence, even the biblical divine names do not apply to Eyn-sof. Jean Reuchlin, who had access to the works of Azriel, defined as follows the divine concepts of the kabbalists.

> "He is called Eyn-sof, i.e. infinity, which is the highest of things and which is itself incomprehensible and inexpressible; in a movement of retreat, which happens in the most secret part of his Divinity, he withdraws and hides into the inaccessible abyss of his light, which is the source. One has to understand that nothing proceeds from this. It is like the most absolute Divinity, immanent in its non-action, in its own retreat, naked without clothing, and without the surrounding things enveloping it. This Divinity does not diffuse itself. It has not extended itself by the generosity of its splendour. It is being and non-being without distinction, which, like a free and totally separated unity, envelops in a perfectly simple way all things which, to our intelligence, are contrary or contradictory." [2]

The Zohar literature, which no doubt seeks to respect scrupulously the absolutely occult nature of Eyn-sof, always insists on its inscrutable and inconceivable character. More than most other passages of the zoharic literature (e.g. I, 16b and I, 65a, where Rabbi Simeon reaches his arms to the sky) it is Zohar I, 21a, which defines the best that which is undefinable. The context of this last passage is indeed quite appropriate. It deals with the creation of lights. This allows the author to quote a passage from the vision of Ezekiel, and to stress its perfectly occult nature (1; 22: "Above the heads of the beings there was a kind of platform. It was like the impressive splendour of crystal. It spread itself over the upper part of their heads.").

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[2] J. Reuchlin, *De arte cabbalistica*, quoted in [5, p. 30, note 10].

Beyond this limit, there is no intellectual comprehension any more (*le-istakela u-le-minda'*), because this level is buried in divine Thought, which lies in the ultimate depths of superior mystery. It is not even possible to ask questions, let alone to draw any knowledge from them. Eyn-sof, so the Zohar continues, does not show any trace of something that human thought could have a grip on. Starting from occultation, the most occult Eyn-sof begins to descend, so to speak, to the realm of revelation. Eyn-sof then gives birth to an extremely thin light, which is unknowable and as difficult to see as the eye of a needle. This is, the Zohar concludes, the occult mystery of Thought.

3. The struggle between Plotinus' God and the God of the Bible

How did one ever arrive at this concept of a hidden God, which makes one think, whether one wants it or not, of a gnostic type of dualism? An opposition of this kind between a revealed God and a hidden God does not seem to have been known by rabbinical Judaism. It rather seems to have come from the just mentioned gnosticism and from Neoplatonism, which penetrated the whole of medieval philosophy. One should add in particular the influence of Plotinus.

Gershom Scholem devoted some shrewd reflections to this problem, which he presented as the struggle between Plotinus' God and the God of the Bible in the ancient Kabbalah.[3] Against the idea of a perfectly personal being, Plotinus sets, so Scholem says, the concept of the One, which is situated at the extreme opposite of the biblical God. Plotinus only very rarely gives this One (a neuter or masculine "the One", or "the thing which is one") the name "God". As a matter of fact, he writes whole chapters of the Enneads without ever using the word God. Just like Eyn-sof, this One is above and beyond everything: beyond life, pure being, the Good, thought, etc. No act of creation or revelation is attributed to this One. One easily measures the distance which separates this "occult Divinity" from the biblical God. By producing their theory of Eyn-sof, the kabbalists indeed touched upon regions which were completely foreign to their own. The terms which the Zohar applied to this "nameless entity", at least as to their spirit, resembled the ones which Plotinus used. Eyn-sof is the hidden thing, the light which occults itself, the mystery of occultation, the undivided unity, the absolute being, the root of all roots, etc.

Explaining the terminological origin of the expression *Eyn-sof,* Scholem quotes a brief but significant passage from a small work called *Book of the real Unity,* which was known by the circles of Isaac the Blind:

> "All spiritual forces, which reveal themselves, which shine and echo from the primordial wisdom come to unite themselves in Eyn-sof, and this is the place of unity of which they [the Neoplatonists] said: everything proceeds from the One and everything returns to the One."[4]

4. Thought and Will in zoharic and pre-zoharic literature

Some fifty years before the spreading of the Zohar, Azriel had, through certain of his commentaries on the *Aggadot* and through his definition of real faith, tried to bring together

[3] [5, pp. 18–53].

[4] [5, pp. 28–29, note 7].

the impersonal aspect of the Divinity and the creative will of the biblical God. The biblical God indeed wants creation, but one cannot say as much of Eyn-sof. Since it is thought or the act of thinking which is present at the beginning of God's manifestation, Azriel as well as later generations of kabbalists asked themselves questions regarding the links between divine thought and will.

In Azriel's commentary on the *Aggadot* we find a strange expression involving an antinomy. All by retaining a clear distinction of priorities regarding God's thinking and God's will, Azriel there speaks of the "will of thought" (*retson ha-mahashava*), as if dissociating thinking and willing in God were a problem to him. Thinking no doubt corresponds to the Plotinian, impersonal character of *Eyn-sof*, whereas the will better manages to do justice to the God of Bible. Bringing together the two in one and the same expression, could give the impression of a solution, but is it will which posits thinking and keeps it under its authority or is it actually the other way around?

Whatever may be the case here, the main characteristic of this will is infinity. Scholem remarks quite correctly that it is not infinite will which is referred to, but will in infinity or reaching right up to infinity (*ha-rom ad eyn-sof*). This aspect is very subtly brought to the fore by a kabbalist from Gerona, Jacob ben Shéshét, the author of the *Sefer ha-Emuna wé-ha-bittahon* (Book of belief and conviction). The latter takes literally a verse from the Psalms (19; 15), which was included in the liturgy and which contains the term *will*, but in an expression that actually signifies "to agree" or "to suit": *yhyu le-ratson imré fi* [may the words from my mouth be accepted]. In the spirit of ben Shéshét, who wrote around 1250, the verse of the Psalm becomes: "may the words from my mouth go to the Will! ", i.e. to that part of Eyn-sof which is hidden in the depths of the occult. In chapter V of the *Book of belief and conviction*, the author even writes that by wishing that the prayers reach the very Will, the one who prays "seeks to unite everything in the infinite".[5]

Does this mean that *Eyn-sof* and the Will are rigorously the same? By no means, but very tight ties unite them. The pre-zoharic kabbalists did not hesitate to speak of the "ravaging of plantations" in case one decided to dissociate the Will from Eyn-sof.

The writings of Moses of Leon, the author of the principal part of the Zohar, reflect the same difficulties, especially if one compares certain Aramaic versions to the parallel Hebrew texts published under his real name, in particular the *Sickle of sainthood*:

> "Before the Blessed-may-he-be created the world [the universe of the sefirot] he himself and his name—which was hidden in him—were one and the same and no thing could exist before he ascended into Thought's will to press the seal of existence onto Everything and to create the world. He traced the signs and constructed, but the things did not subsist until he enveloped himself in the cover of the supreme luminous splendour of thought; nothing subsisted before he created the world and produced the high mystical cedars from this supreme luminous splendour" (Zohar I, 29a).

5. The relations between Eyn-sof and the sefirot; the One and the multiple; otherness and immanence

Which relationship does there exist between Eyn-sof and the crown, *Kéter*, the first sefirah? Eyn-sof is the primordial point which gives birth to the ten lights or luminaries,

[5] See [2, p. 85].

but is *Kéter* really a sefirah just like the others? Zohar III, 269a, responds positively. Certainly *Kéter* is a sefirah situated at the very top of the sefirotic process, but it is a sefirah nonetheless! It is no doubt the master of the process of emanation, but at the same time, it is nothing more than an emanation. Some zoharic texts want to go further than this. They identify the Holy old man (*attiqa qadisha*) or *Arikh anpin* (the longanimous) with *Kéter* and have *Zé'ir anpin* correspond to the sefirot going from *Hokhma* (wisdom) to *Malkhut* (kingdom).

The *Ra'ya méhémna* (the True shepherd) and the *Tekkuné Zohar* tend to make use of a type of terminology which has a philosophical appearance and is adapted to mystical needs: the expression *sibbat ha-sibbot* or *illat ha-illot* occur very frequently. It seems that this impersonal designation covers at the same time *Eyn-sof* and *Kéter*, and that the ontological opposition between the *Deus absconditus* on the one hand and the sefirotic universe on the other is thus somewhat blurred. One might almost say that we are faced with one and the same Divinity with two aspects, an occult one and a revealed one.

It would be difficult to quote the quasi-totality of texts which support the preceding, but it does remain possible to pick out some of the most significant passages of the zoharic corpus. One passage from the *Zohar Hadash* (pericope Jethro, fol 55b–55d) speaks of the activity of Eyn-sof and, from the very beginning, insists on the paradox of the One and the multiple: Eyn-sof is the absolute unity which transcends the very idea of unity; nevertheless its actions reflect diversity: the sefirot which Eyn-sof installs are big, small or intermediary, and their acts reflect their own nature. Eyn-sof on the other hand, although being at the origin of everything, does not change. The text of the *Zohar Hadash* stipulates: he has created everything through *Bina*, but nobody created him. He is the one who shaped everything with the aid of *Malkhut*, but nothing shaped him. One is not allowed to separate him from the sefirot, which form an undividable whole, and which are called *Yod*, *Hé*, *Waw* and *Hé* (*YHWH*). It is Eyn-sof which constitutes a kind of architectural principle of the world by tying up and uniting the chariots of the angels and by linking up the inferior and superior universes.

Eyn-sof lets its existence be guessed thanks to the sefirot, which only subsist through him: his withdrawal would deprive them of science, existence and even life. This life-giving principle spreads itself all over the world. The Zohar quotes a verse from Isaï (40; 25): "Who would you assimilate me with so that I would become his equal?" says the saint in order to stress the absolute otherness and solitude of Eyn-sof. Only the Torah could offer a valid term of comparison. To that effect the text in question quotes a passage from Proverbs (3; 15): "She is more precious than pearls and nothing which is precious to you can match her".

In the same passage of the Zohar one further reads developments of ideas which strongly resemble philosophical considerations, such as: Eyn-sof is not knowledgeable through science nor omnipotent through power. What seems important to be stressed here is that in spite of otherness, so this text of the *Zohar Hadash* tells us, Eyn-sof is exterior to nothing. Nobody can escape his influence, although he himself is radically cut from it.

Another passage situated at the beginning of the Zohar (I, 22a), but which is actually part of the *Tikkunim*, develops interesting speculations regarding Eyn-sof, all by commenting on Deuteronomy (32; 39): "See now that it is me who is me, and that there is no God besides me; I am the one who puts to death and brings to life, who wounds and heals; no one can deliver out of my hand." In English the expression "it is me who is me" is already most intriguing, but it does not bring about the strange feeling that the Hebrew text leaves us with, which uses two different terms: *ani ani hu!* The Zohar says that what is referred to here is not the cause of causes but the Cause of all superior essences, i.e. Eyn-sof, which calls to being the sefirot.

In a study regarding the symbolism of colours, Gershom Scholem gives an excellent presentation of the relations which exist between Eyn-sof and the sefirot:

> "Considered in his pure transcendence and in the occultation of his being, which does not manifest itself, and which cannot be described by means of symbols and images, God is called *Eyn-sof* by the kabbalists, i.e. the infinite thing or being. This term was introduced by the kabbalists in order to refer to the nameless absolute as it is present in God. From him emanate the ten sefirot, which are not fundamental qualities of God in his relationship with creation, but active forces, or even rays of divine light. Through the sefirot the creative forces in God are represented, i.e. that which proceeds from him and which acts upon creation, or more precisely, provokes and determines creation; in other words, they represent the living God, who leaves his occultation and who manifests himself. The sefirot are no creations of God: they are the diversity which is contained in the dynamic unity of his life." [6]

Although we are mainly interested here in the Zohar and its conception of the sefirot, it may still be useful to take into account the later specifications suggested by a Jew of the beginning of the seventeenth century, Abraham Herrera, who around 1620 wrote in his *Sha'ar ha-Shamayim* (VII, 4 (*Porta Coelorum*)):

> "The sefirot are the mirrors of his truth and the analogies of his most sublime being, the ideas of his wisdom and the conceptions of his will, the reservoirs of his power and the instruments of his activity, the treasure chests of his felicity and the distributors of his grace, the judges of his empire, who reveal his verdict. They are also the designations, the attributes and the names of the one who is the highest and the cause of everything. The ten [sefirot] are inextinguishable; they are the ten attributes of his exalted majesty, the ten fingers of the hands, ten lights by which he reflects himself and ten clothes with which he covers himself. They are ten visions through which he appears, ten forms through which he shaped everything, ten sanctuaries where he is magnified, ten degrees of the prophecy through which he reveals himself, ten chains from the higher parts of which he dispenses his teaching, ten thrones from which he judges the peoples of the world, ten compartments in paradise destined to those who are worthy of them. They are ten steps which he goes down, and which one can go up toward him, ten zones producing all the influx and all the blessings, ten aims desired by all, but reached only by the Just, ten lights which illuminate every intelligence, ten types of fire which appease all desires, ten categories of glory which animate all reasonable souls, ten words through which the world was created, ten spirits which animate the world and keep it alive, ten numbers, weights and measures, which count, weigh and measure everything, ten touchstones, which watch over the accomplishment of everything." [7]

6. The origin of the term *sefirot*. A presentation of the nomenclature

Let us go back for a moment to trace the origin of the term sefirot. It is the Book of Creation, the *Sefer Yetsira*, which talks about *esser sefirot belima*. However, this book by the hand of a Pythagorean Jew (of the fifth century?) designates by this expression the ten primordial

[6] [5, p. 167].

[7] Knorr von Rosenroth, *Kabbala denudata, Apparatus in librum Zohar III & IV*, Sulzbach, 1678, quoted in [4, p. 60].

numbers which constitute the foundations of created order. It was only in the interpretation of the medieval kabbalists, that the sefirot became the acting forces of Divinity, or, using mystical terminology, that through which Divinity receives an appearance, a face. It is those sefirot, which in a sense, scan the rhythm of the intimate life of the Divinity, but also the rhythm of the pulsations of natural order into which they project themselves.

As the Divinity displays different aspects of itself according to the levels at which it manifests itself, the sefirot carry different names and correspond to a variety of symbols.

It is important to see this dynamic unity of the Divinity. It is totally present at every stage of its manifestation, but it preserves its absolute unity by a mystery (*raza de yhuda*).

We shall now present the nomenclature of the sefirot which is composed of three triads to which a final sefirah is added, which directly feeds our lower world, the so-called sublunary world.

The first triad consists of *Kéter elyon* (supreme crown), *Bina* (the discernment) and *Hokhma* (intelligence). Through this first group, *Eyn-sof*, the occult and perfectly impersonal, takes, in a manner of speaking, its first steps in the direction of the created. This first triad is rather neutral because it just emerges from the occult depth of *Eyn-sof*. One also tends to say, as is done in certain passages of the Zohar, that the very idea of creation becomes concrete (i.e. to the extent that one can speak of anything concrete at this level of the sefirot) in this passage from first sefirah to *Hokhma*.

The second triad contains the implacable rigour of judgment, called *Din*, then grace or *Hésed*, which dispenses its blessings, and finally the result of those two opposing forces, the sixth sefirah, *Tif'érét* or splendour, also called *Rahamin*, mercy. Tif'érét / Rahamin, which symbolises the community of Israël, is also called the superior *Shekina* as opposed to *Malkhut*, the inferior *Shekina* (which is the tenth and last sefirah). Situated in the very center of the sefirotic tree, *Tif'érét* exerts an appeasing influence in the sefirotic world by tempering rigour with mercy.

The third triad encompasses *Hod*, majesty, *Nétsah*, eternity and finally *Yesod*, i.e. foundation. This last sefirah, the second-last of the whole series, symbolises the Just, for a passage from the Proverbs (10; 25) speaks of the Just as of the foundation of the universe. In the configuration of primordial man, which constitutes an alternative to the sefirotic tree, the sefirah *Yesod* corresponds to a sign of alliance, i.e. the phallus. On the other hand, the very last sefirah, *Malkhut* (kingdom), is gifted with an intrinsically feminine symbol: This sefirah is indeed a real receptacle in which all the influx of higher entities is deposed. *Malkhut* revives the worlds by transmitting to them the *Shéfa'*, or directed flux.

7. Lights and colours

We saw how the theory of the ten primordial numbers of the Book of Creation gave rise to the nomenclature of the sefirot, which ended up imposing itself on all kabbalists starting with the authors of the zoharic literature. Let us now have a closer look at the theory of intellectual lights, which we know through the writings of the pre-zoharic *Iyyun*-circle (thirteenth century). In the *Sefer ha-'Iyyun* (book of theosophical speculation), God is described as "the One which is united in all its powers, just like the flame of fire, which unites

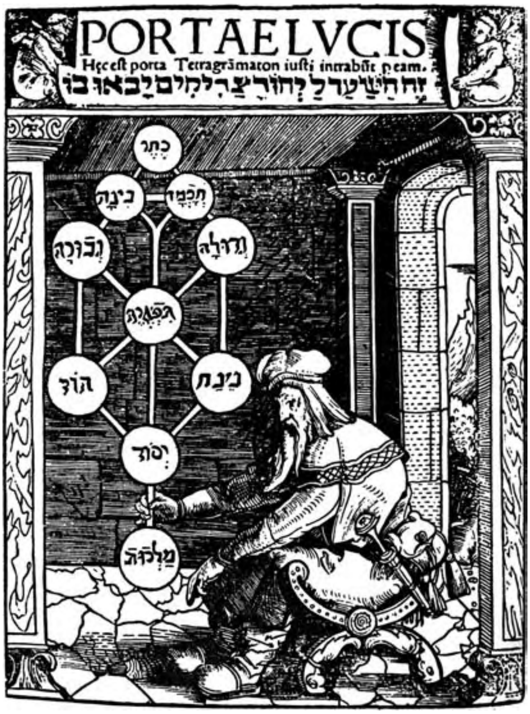

Fig. 1. The Sephirot. Frontispiece of Paulus Ricius, *Portae Lucis* (Gates of Light), Augsburg, 1516.

itself in all its colours, whereas its powers emanate from this unity like the light of the eyes comes from the black of the eye".[8] We may identify the ten sefirot with ten lights, because the mystical speculations of the beginning of the Zohar regarding *Beréshit*, the first term of the Torah, seem to come from the writings of the *Iyyun*-circle:

> "In the beginning, when the will of the king started to act, he engraved signs into the celestial aura. A dark flame emerged from the most intimate domain of the mystery of Eyn-sof, like a mist, which takes shape in something that does not have any, a dark flame enclosed in the ring of this aura, a flame which was neither white, nor black, nor red, nor green, and which was without any colour. It was only when this flame started to rise and to expand that it produced radiant colours. In the most intimate centre of this flame indeed a spring gushed forth, from which colours poured themselves onto everything that was beneath, a spring hidden in the mysterious secrets of Eyn-sof. The spring passed through the first ether without really travelling through it, and became completely unrecognisable until by the very power of this break through, a supreme and occult point started to glow. Beyond this point nothing can be known, and that is the reason why it is called *Réshit*, i.e. the beginning, i.e. the first word of creation." (Zohar I, 15a)

The author thus turns to the symbolism of light in order to explain how the emergence of the first superior entity from the occult depths of Eyn-sof takes place. This light sometimes carries the name of light that conceals itself and hides from sight (or *mit' allem*). It is the

[8] Quoted in [3, p. 330].

first sefirah, completely bereft of shape and identified with a mirror which does not have any colour, and hence renders them all.

The world of colours makes it possible for the kabbalists to show how the different powers of the Divinity, which are the sefirot, project themselves. The symbolism of colours plays a fundamental role in determining each of those superior entities.

> "As all the religions seem to agree on applying the concept of light to divine essence, colour, which is considered originally as the manifestation of light, cannot therefore have any other function than to refer to the Divinity as it manifests itself or as it appears. The different colours hence necessarily symbolise the different modes or ways in which the divine being manifests itself. They represent the divine being under different aspects, as well as in its relations with whatever is exterior to it. The symbolism of light therefore varies depending on the concept of divine essence and of its relationship with the world." [9]

8. The unbreakable dynamic unity governing the sefirot and respected in prayer

One may refer to the beautiful remark made by Tishby "the sefirot are the divine archetype of a universe which is not divine".[10] Fundamentally, the Zohar does not recognize the existence of barriers separating the world above from the world beneath, the world of the dead from the world of the living. The universe of the sefirot is precisely the obligatory passage between those different levels of being. Even the inferior sefirot are animated by the desire to come closer to their original source. They do not, however, turn away from our lower world, which they nourish with their blessings and which they, by doing so, keep alive. Different channels unite all the levels of the sefirot to one another. It is along those channels that people who pray in Israel orient their prayers, if they want the latter to be answered.

The Zohar (I, 12b) sometimes conceives of the last sefirah as being interwoven with "the other side", i.e. as being in close proximity with the forces of evil. The worst thing to do would be to give autonomous status to this last sefirah, *Malkhut*, and to cut if off from the others with which it forms an indivisible whole. *Malkhut* has to remain united to *Tif'érét*, because the harmony of the worlds depends on it.

If someone who is praying, isolates in his mind *Shekina* from the other sefirot, and addresses his prayers to *Shekina* only, he will be committing an act of idolatry and is bound to reinforce the grip of "the other side". The *Zohar hadash* (*Béréshit*, 18c) explicitly warns against this and quotes Exodus 22; 19: "Whoever sacrifices to the gods, and not to God only, exposes himself to anathema".

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CHAPTER 11

Mathematics and the Divine: Ramon Lull

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Edited by T. Koetsier and L. Bergmans

1. Introduction

Medieval Scholasticism sought, in accordance with its apologetic purpose, to present the various disciplines as Aristotelian "sciences", theology taking its principles from the articles of the Christian faith and philosophy from the axioms which Aristotle was thought to have found, to argue to "scientific" conclusions which would agree with one another. Our histories of human thought have consequently neglected the medieval development of another Aristotelian project, the description of methods or "arts" of production—medicine for example—so that many significant thinkers have been treated inadequately. The Catalan, Ramon Lull (1232–1316), belongs to this latter group.

Lull, who enjoyed a position in the court of the later king of Majorca (a realm which at that time extended as far as Montpellier), resolved at the age of about 30 years to dedicate his life to winning Muslims and Jews for the truth of Christianity. After studying both Latin and Arabic as well as Christian and Muslim theology, he composed a great number of works in the most diverse literary forms and travelled to the leading cities of the Mediterranean basin (including north Africa and Cyprus) in accord with this missionary purpose. During the period between 1270 and 1307, he drew up various forms of a productive "art" in the utopian conviction that through his "art" man would produce a world in which concord among the different religions of the world should reign.

2. The arbor elementalis

We all remember the dream of Nebuchadnezzar, recorded in the book of Daniel:

> I looked, and lo! there was a tree in the midst of the earth, and its height was great. The tree had grown great and strong, so that its height reached to the heavens, and its bound of vision to the end of the earth. Its leaves were fair, and its fruit was abundant, providing food for all (Daniel 4.10–12).

The symbolism of the tree played an important role in the Mediterranean context of Muslim Sufism and Jewish Kabbalism in which Lull's ideas developed. In all its forms the tree appears as a symbol of growth, of dynamic action, of striving for the infinite. It is "great", it "has grown great and strong", its "height reaches to the heavens".

The structure of the book on the *Arbor elementalis* (Elemental Tree, ROL XXIV 11–116) in Ramon Lull's *Arbor scientiae* (Tree of Science, Rome 1296) also resembles the description of the tree in Nebuchadnezzars's dream. Like the branches, leaves, and fruit of the tree described in the book of Daniel, Lull's *Arbor elementalis* grows great in its (Table 1).

Table 1

roots	principles of the art (*dignitates*)
trunk	chaos
branches	simple elements
boughs	composite elements
leaves	accidents
flowers	instruments
fruit	elementated things (*elementata*)

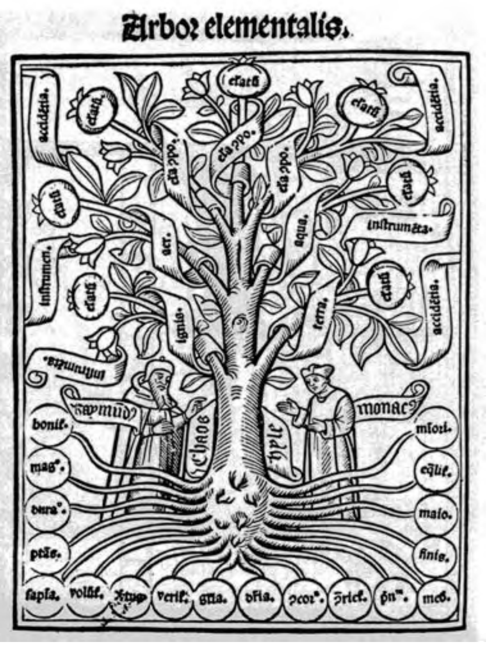

Fig. 1. Lull's Arbor Elementalis. Source: *Arbor Scientiae*, Lyon, 1515.

The *Arbor elementalis* grows from the Lullian principles, through substances and accidents, to the final *elementata*. This structure is basic, not only for all the sections of the *Arbor scientiae*, but also for all of Lull's works. I would like here to present Lull's idea of the emergence of all things from the vast realm of possibility which he called chaos, on which his mathematics is based.

The roots of the *Arbor elementalis* are the attributes of God, the "dignities" of the Lullian Art: *bonitas*, *magnitudo, duratio; potentia*, *sapientia*, *voluntas; virtus, veritas, gloria* (goodness, greatness, duration; power, wisdom, will; virtue, truth, glory). The inclusion of *greatness* among the principles shows that Lull was thinking of a tree which is "great and strong", the height of which "reaches to the heavens". In the dynamic spirit of Anselm of Canterbury's *id quo maius cogitari nequit* (that than which nothing greater can be thought), Lull maintained that all the dignities of the Art must be taken to stand for active powers striving for the infinite.

3. Lull's dynamic understanding of reality

Lull's analysis of what it means when we say that the powers of the principles are active is revolutionary. Lull spoke not only of principles of being, but also of principles of action. He proposed the thesis that God is not only productive contingently (*contingenter*) and **outwardly** (*ad extra*) in creation, but also necessarily (*necessarie*) and **inwardly** (*ad intra*) in the divine persons of the Christian Trinity. He maintained that all the principles of his

Art must be active *necessarily* and *inwardly*. True goodness must produce something good; true greatness must produce something great. We cannot truly call something good which does not produce a good.

In this way Lull introduced a completely new category in the history of thought. Because action presupposes a source, that which is produced, and a bond between them, he spoke not only of the "dignities", but also of their activity and the "correlatives" (*correlativa*) of their action. To designate these correlatives, he developed a new terminology:

> I call the acts of goodness bonifica**tive**, bonifica**ble**, to boni**fy**; also the acts of greatness are magnifica**tive**, magnifica**ble**, to magni**fy**; and so for the other divine dignities.
> Actus ... bonitatis dico bonifica**tivum**, bonifica**bile**, bonific**are**; actus etiam magnitudinis sunt magnifica**tivum**, magnifica**bile**, magnific**are**; et sic de aliis divinis dignitatibus (*Vita coaetanea* 26; ROL VIII 290).

Lull generalised this idea to the extent that he could speak even of the abstract moments of activity as *-tivum*, *-bile*, and *-are* (*-tive*, *-ble*, to *-fy*). The correlatives are the intrinsic and substantial moments of all agency (*agentia*). They are ontological principles, implicit in the striving of all things for infinity.

Applying his ideas to creation, Lull strongly emphasized not only the dynamic character of God's causal action, but also the dynamic activity of its effect which is the created world itself. For Lull creation is a likeness of God because of its dynamic character. Not only does God's activity of creation not cease with the bestowal of being on the world, the world which God has created is itself also active, tending to its own perfection. The divine activity is continued in the striving of all things for their own unlimited perfection.

Lull applied therefore the distinction between necessary **inward** (*ad intra*) activity and contingent **outward** (*ad extra*) activity to the activity of creatures. To explain how the distinction applies to creaturely activity, he employed the example of a property in a subject. Finite things like fire and whiteness have their own necessary, **inward** (*necessaria, intensa*) correlatives of action, but the outward things which they need to realize the perfection of their activity (earth or water, this or that body) are contingent and **extended** (*contingentia, extensa*). The forms of fire (*igneitas*) or of whiteness (*albedo*) are necessarily active within themselves, but when the form say of whiteness contingently whitens (*albificat*) this or that body, this form is present in the body as a perfection in a "contracted" (*contracte*) way.

Through this analysis Lull was able to apply a distinction which he had learned from Arabic authors to explain the origin of the predicamental accidents. This distinction he referred to as a distinction between "two intentions"—not in the Scholastic sense of first and second intentions in logic, but rather in the sense of a "first intention", which is the ultimate end of an action, and a "second intention" which includes the instruments and the operations necessary to attain the end.

Lull explains that the knowledge of the first and second intention and their correct order (*recta ordinatio*) are most necessary to men. He clarifies this distinction in one of his earliest works with the following example:

> Furthermore, we shall see that the apple tree or any other fruit-bearing tree brings forth leaves, so that it can better bear fruit. But it does not bear fruit for the sake of the leaves. Therefore, it bears fruit by a first intention, but it brings forth leaves by a second intention.
> Praeterea, iam videmus nos, quod pomerium vel quaelibet alia arbor fructifera, ut melius possit facere fructum, facit folia. Non facit autem fructum ad foliorum utilitatem. Facit ergo fructum

intentione prima; folia vero intentione secunda producit (*Compendium logicae Algazelis*, Montpellier 1271/72; ed. C. Lohr, §8.13).

Lull's use of the distinction reflects the much broader meaning employed by Arabic writers. The terms can be found in the philosophical encyclopaedia of the so-called "Brotherhood of Purity". This school explained that the first intention of the creator in the production of the world is the permanence and welfare of creatures, whereas corruption and pain are accidental and due to the imperfection of the matter which is necessary for creation. It was possibly through the tenth-century encyclopaedia that the distinction of two intentions came to Lull, either directly or perhaps through cultivated Majorcan Jews, of whose intellectual baggage the encyclopaedia formed an essential part.

Lull's emphasis on God's productive action in creating the world and on the consequent dynamic aspect of all reality led him to distinguish clearly between the end of an action and the means to the end. The first intention of an action is its proper end, the second intention is not the proper end, but that of the instrument which is for the sake of the proper end. In his earliest works Lull used this distinction primarily in the sphere of ethics: God should be loved for himself, while all other things are only a means to this end; sin reverses this order—it is contrary to the correct order (*recta intentio/recta ordinatio*). But he soon applied it in other areas as well: in the *Liber demonstrationum* (Book of Demonstrations, 1274–78) to the problem of the relationship between faith and reason, in the *Ars demonstrativa* (Demonstrative Art, ca. 1283) to the theory of knowledge. The *Liber de intentione* (Book concerning Intention, 1283) offers a generalized theory of the two intentions.

Lull applied this distinction to the action of the elements in his epoch-making *Liber chaos* (Book of Chaos), a work appended to the *Ars demonstrativa*. As indicated above, fire must by nature—in the first intention of its action—necessarily (*necessarie*) and actively burn **inwardly** (*ad intra*). But as finite, it requires a foreign matter—in second intention—to realize its activity. Whether fire warms air, water, or earth **outwardly** (*ad extra*) is not necessary, but contingent. In itself fire is active "substantially" (*substantialiter*); in water or in earth it is active "accidentally" (*accidentaliter*). Thus each created thing strives necessarily and "in first intention" toward its own *inward* perfection. But as finite, a created thing can not attain its goal—its perfection—without an *outward* activity. In order to realize its true self, a finite *-tivum* must go out of itself to an outward *-bile*. Extrinsic objects become thus the means or instruments which serve intrinsic perfection "in second intention". In this understanding of the properties of things action and passion are not, as in Aristotle, simply accidental. Because being and activity both belong to the substance of things, **inward** action and passion are necessary and substantial, whereas only **outward** action and passion are contingent and predicamental accidents.

Taken also as a whole, the world created by God is active and strives for its own perfection. Lull understood the individuals, the *species* and the *genera*, which we encounter in the world, as parts of a whole, as components of a unity reflecting the dynamic greatness of the creator. He presented his understanding of this whole—this universe—most clearly in the *Liber chaos*, making use of many ideas taken from the great Sufi master, Ibn al-'Arabî (d. 1240). For Lull, chaos is the complex of the intentional limits of the activity of all the things which were created by God in a unique act. Chaos embraces all the potentialities of created things. From chaos proceed the actualizations of all their potentialities.

Lull divided chaos into three degrees. In the first degree we find the causal seeds (*semina causalia*)—the *universal form* and the *universal matter*, the *genera* and *species*, the *substances* and *accidents*—which ground all things, as in the *Arbor elementalis* outlined above. In the second degree we find the first individuals of the particular species—Adam as the perfect, primordial man. In the third all the individuals deriving from these first ones.

But Lull's idea of the correlatives of all activity completely transformed the meaning of these ideas. Chaos is made up of the four abstract essences of the four elements: igneity, aereity, aqueity, terreity (*igneitas, aereitas, aqueitas, terreitas*). Each essence is active in its proper form (*ignificativum*), its proper matter (*ignificabile*) and its proper act (*ignificare*). The matter and the form of the various intentional realities in chaos should not be thought of as limited to the concrete individuals beneath them. Though abstract, they are real things, existing intentionally as real potentialities, as real goals for the striving of creation for its own perfection.

All the proper forms are joined together in a unique universal form—an soul of the world (*anima mundi*)—which is the sum of all the possible *-tiva*—and all the proper matters are joined together in a unique universal matter—a body of the world (*corpus mundi*)—which is the sum of all the possible *-bilia*. From the universal form and universal matter emerge the predicables and the predicaments—*genera* and *species*, *substances* and *accidents*—and from them in turn the individual things composed of particular forms and matters. Each particular form and each particular matter is grounded in the universal form and matter. Out of the unity of the universal form and the universal matter arises the one supposite (*unum suppositum*), which is the contract infinite Lull calls chaos. The creator has so created things that they tend to approach this contract infinite not only by numerical multiplication. The concrete individual things which emerge from the causal seeds (*semina causalia*) tend each of them also to their intrinsic, proper perfection. By nature all concrete things strive to fulfill the abstract essences from which they have emerged.

Lull's conception of nature as the source of the limitless striving in things sets his dynamic understanding of reality off from the ideas of his Scholastic contemporaries. Whereas the Latin Scholastics thought of the nature of a thing simply as its essence understood as a principle of activity, Lull distinguished between essence and nature, taking the latter to be that which is the source of its appetite for its proper act—its proper *-are*. The nature of man, for example, is that which gives him his appetite for the humanization of his animality (*appetitus homificandi animal*). By nature man seeks to transform his animality completely into that of a human being. That is, I think, the sense of the enigmatic definition of the *Logica nova* (New Logic, Genoa 1383) Man is a humanizing animal (*Homo est animal homificans*: I cap. 5, ROL XXIII 31).

In his *Liber novus de astronomia* (New Book of Astronomy, Paris 1297; ROL XVII 93–218) Lull explains the difference between nature and essence:

> There is a difference between essence and nature, because essence is related to constituting being (as humanity which constitutes human being, and lionness the being of a lion, and igneity which constitutes the being of fire). And nature is related to becoming a nature (as the nature of a man which gives the appetite of becoming a man, and as the nature of a lion which gives the appetite of becoming a lion, and as the nature of fire which has the appetite of becoming fire).
>
> Inter essentiam et naturam est differentia, quoniam essentia respectum habet ad constituendum esse, sicut humanitas, quae constituit esse humanum; et leonitas esse leonis; et igneitas, quae constituit esse ignis. Et natura respectum habet ad naturandum, sicut natura hominis, quae dat

> appetitum homificandi; et sicut natura leonis, quae dat appetitum leonificandi; et sicut natura ignis, quae habet appetitum calefaciendi (I 2 5 [1]; ROL XVII 163).

This chapter of the *Liber de astronomia* returns us to the tree of the *Arbor elementalis*. Lull's tree is made up of its matter and form as a material thing and the vegetative power which transforms the material thing into a living tree. Without the vegetative power, Lull tells us, the tree would dry up; the whole appetite of the tree would be inclosed within its own being and, in its essence, to being simply that which it is. The vegetative power completes the tree; it is its end and purpose. Through the vegetative power all the parts of the tree have the appetite to vegetate (*ad vegetandum*)—of going beyond themselves in branches, leaves and flowers. Through the vegetative power Lull's tree has the appetite to attain the end for which it exists and to multiply its nature and its similitude. In Lull's own words:

> The tree has an appetite through the vegetative power for the end for which it exists and for multiplying its nature and likeness.
> Arbor appetitum habet per vegetativam ad finem, propter quem est, et ad multiplicandum suam naturam et similitudinem (loc. cit.; ROL XVII 163).

This striving, this appetite is true both of the species and of the individuals which make them up. For Lull, all concrete things strive by nature to return to the *species* and *genera* from which they have emerged. In his *Liber de natura* (Book of Nature), Lull defined nature as the principle through which a concrete being—the individual man (*homo*), for example—approach ever more closely its abstract being—the essence of man (*humanitas*)—as to their perfection (Cyprus 1301; ed. Palma 1744, Int. IV p. 4).

Although the particular, individual things of which the world is made up are in constant process, tending toward their proper perfection, no individual thing can attain the full perfection of its species. The *species* is the limit to which the perfection of the individual approaches, the *genus* that of the *species*, the universe that of the *genera*. The abstract is already really present in the concrete, individual things as the limit of their striving toward infinity.

Chaos taken as a whole is understood as the created reflection of the divine infinity whose dynamism unfolds in a nature which is three-fold. At each degree of the return of created things to the original unity the respective *-tiva* form the *natura naturans*, the respective *-bilia* the *natura naturata*, and the respective *-are* the union of both—the *natura naturare*. Chaos is the trunk of the elemental tree because it is the limit of the striving of finite things for infinity—the limit of the striving of all created things for that than which nothing greater can be thought (*id quo maius cogitari nequit*).

4. The disciplines as productive arts

Lull's Art represents the attempt to understand and make use of this dynamic conception of reality. It is precisely because this conception was meant to include the striving of finite things for the infinite that Lull's thought is so foreign to one accustomed to Scholastic terminology. The Scholasticism of his contemporaries was built on the idea of science as ordering a truth which is already possessed, but hidden. Science was thought of as the ordered knowledge of the immutable and eternal essences of things. It was in accordance with this idea that Thomas Aquinas sought to arrange revealed Catholic truth in a "scientific" way on the model found in Aristotle and Euclid. But in Lull's works there appears the notion of truth not as something possessed, but as something to be gained. In Lull's work truth is not a beginning; it is rather an end to be sought.

Lull seems clearly to have known Aristotle's distinction of the disciplines concerned with knowing, doing, and making, particularly as it was developed in the Arabic tradition. Aristotle developed his conception of these disciplines in his treatment of the intellectual virtues in *Nicomachean Ethics* VI 3–5. Whereas for Aristotle—as for the Scholastics who took the idea from him—"science" is concerned with the knowledge of the immutable and universal essences of things, the practical disciplines he described as concerned with doing and making deal with the possible, with things capable of being other than they are. Practical reason has to do with things belonging to the vast realm Lull described as "chaos". Both doing and making seek something which does not exist, but is none the less real, because it can be. "Art" has to do with making; "prudence" is the virtue of correct conduct. The different ends of acting and making led Aristotle to make this further distinction. In making, something is produced—a house or a chair, for example. In acting, there is no product other than the prudent action itself.

"Art"—the technical knowledge which has to do with making—is defined as a virtue with respect to production alone. Although it aims at perfection, the level of technical skill is always limited; for instance, by the material being worked on. But here we must take account of an important distinction, which arises because of the difference between art and nature, as Aristotle observed in *Metaphysics* VII 9 (1034a8–30). Whereas the source of change in artistic production lies in the producer and his knowledge, nature brings forth change of itself.

Aristotle distinguished accordingly between two forms of art, according as the change involves natural change or not. The material which forms the basis of technical production is such that it either can or can not move itself. If the material (say, wood or stone in building a house) can not move itself, an extrinsic principle (an architect) is necessary for the production of the form (of a house). The house can not produce itself without an "artist". But if the material is capable of producing the change of itself, the active principle of the change is already present in the subject as its nature. The nature is an intrinsic principle which can develop and change of itself. The former type of change is that included in what was later called "fine art"; the latter is that with which the disciplines like medicine and rhetoric and the *Ars lulliana* are concerned. Health, for example, is a product of nature; the medicines which the art of the physician uses to heal simply help to restore the *natural* balance of the bodily humors.

In the medical tradition this latter type of art came to be known as a "conjectural art" (*ars coniecturalis*, Celsus, 1 praef.; 2, 6 fin.). The nature of those things having a principle of change in themselves lies in the form by which they are constituted. The term "nature" should be understood here in the sense of a dynamic process—a *natura naturans*. It does not mean simply the principle, but rather also the end, of the activity—an end which it seeks ever more closely to approach (*Physics* II 2 [193b12–18]). The nature which is the origin of the change is itself the goal of the change—the fullness and perfection of the essence of the thing. Rather than speaking of nature as a static principle of activity, one should speak of it as a "bridge"—a bridge rendering an estimate, a "conjecture", necessary, because the

Table 2

temperament	**hot quality**	**cold quality**	**wet quality**	**dry quality**
absolutely tempered	all primary qualities are in balance			
hot	hot dominates over cold		in balance	
cold	cold dominates over hot		in balance	
wet	in balance		wet dominates over dry	
dry	in balance		dry dominates over wet	
hot & wet	hot dominates over cold		wet dominates over dry	
hot & dry	hot dominates over cold		dry dominates over wet	
cold & wet	cold dominates over hot		wet dominates over dry	
cold & dry	cold dominates over hot		dry dominates over wet	

end of the activity is an ideal maximum, approached ever more closely, but never attained (Table 2).

Not only the individual elements seek a maximum; the activity of all the elements, taken together as a unit, also tends toward a highest value. Galen, the greatest physician of antiquity, used the "canon" (rule) of Polycleitos as an example of nature's way of working in healing (*Ars medica* cap. 14 [Kühn 341–343]). He compared the balance and symmetry produced by nature with the artist seeking the best possible organization of its subject through its victory over matter. He saw nature as a workman, a maker in a creative process. Nature is the immanent agent, bringing the form of health, which is adumbrated in man's abstract essence, to concrete realization. The ideal end of nature is the realization of the canon, the best possible organization of parts in balance, symmetry and proportion. In its course it follows a complex of rules, but because of the torpidity of matter, its end is a limit to which it can ever more closely approach, but never attain.

Galen stressed the due proportions which must be maintained among the primary qualities for the attainment of health. He presented the best possible temperament as the condition in which all primary qualities are in balance. But this temperament was an ideal, rarely if ever attained in the real world. Galen's doctrine is found in his *Medical Art* and a schematic presentation (see Joutsivuo 122) makes clear the balance which the action of the qualities of the elements seeks is an ideal value, sought, but rarely attained (see Table 2).

The balanced complexion (*aequa complexio*), the ideal value to which all qualitative dominion was to be referred in a system based upon the primary qualities, was the temperate or qualitatively balanced complexion. Galen's *De complexionibus* (On Complexions) describes the complexion as temperate when it is balanced between its possible extremes (I cap. 6 [Kühn I 547f.]; see McVaugh, "Introduction" 10). But the medical tradition, while accepting the possibility of an ideal mathematical equality, emphasized the relativity and variability of complexions in the real world in which the physician lives. For Avicenna, the human complexion "is something with a considerable range (*latitudo*) and cannot be precisely defined; it exists, not fixed, but between two extremities, which it may not surpass and still remain the human complexion" (*Canon* I fen 1 doct. 3 cap. 1 [Venice 1507, f. 2r]; see McVaugh 21).

These ideas—in both the Arabic and the Latin traditions—were also used to clarify the status of the discipline as an art. At the beginning of his famous *Colliget*, Averroes, the great Muslim Peripatetic of the twelfth century in Spain, defines the art of medicine as "an operative art, taking its departure from true principles, in which the preservation of the health of the human body is sought ... to the extent possible for each body" (I cap. 1 [Venice 1562, f. 3r–4r]). He states that in all the practical arts three things must be considered: the subject, the end sought, and the instruments necessary to attain the end in the subject in question (that is, first and second intention).

Averroes distinguished two types of knowledge in medicine. One type is theoretical and deals—in a "scientific" way—with the causes of health and sickness. The second type of knowledge is experiential and deals with the various types of medicine and the hidden powers of the medicinal compounds necessary to attain their ends. Averroes regarded the theoretical part of medicine as an Aristotelian "science" belonging to natural philosophy, because it deals with ultimate causes of sickness and health. The practical part of medicine takes only the "principles" of its art from the theoretical physician; for the "rules" of the art, it is limited to experience—to trial and error.

Averroes structured his *Colliget* in accordance with this distinction, as he stated at the beginning of the work. He explained that he would begin with the universal and then proceed to the particulars, employing rules formulated for the purpose. By means of his art, the practical physician tries to predict the working of nature, but can only have probable knowledge. He approaches the truth by way of estimate and approximation—*coniecturae*—and needs long experience. Averroes cited the aphorism of Hippocrates: *Vita brevis, ars longa* (Life is brief, art long).

In the medieval medical tradition a methodology was developed for these two processes, using the terminology of "**precepts**" and "**rules**". The universal principles applying to all cases were designated as "precepts" (*praecepta*) or "principles" (*principia*) and the means of arriving at the desired end in particular cases "rules" (*regulae*), as Arnau of Vilanova explained in his *Aphorismi de gradibus* (Aphorisms concerning Degrees), a treatise on the degrees of intensity in the operation of compound medicines. Rules in medicine, through which the intellect may attain truth to the extent possible, have, he tells us, to do with particular applications, but are based on theorems, that is, the conclusions deduced from the certain principles of medicine (ed. McVaugh, *Arnaldi Opera*, II prologus). The rules are meant to aid the intellect in its approach from merely probable, experiential knowledge on its way to production of some work. A "rule" in an art—understood in the medieval sense of "art"—is a statement which regulates the approximations and conjectures necessary for the artist as he approaches balance and symmetry in the end sought. The arts consist of a system of "precepts" and "rules" according to which a given end may be attained.

The idea of the disciplines as productive arts led to a radical revision of the theory of the traditional liberal arts. As early as the tenth century, Alfarabi, in his *Catalogue of the Sciences*, presented each of the liberal arts under two headings, first as "sciences", and then as "arts"—geometry, for example, not only as a "science" in the traditional Aristotelian sense, but also as an "art" concerned with production. The medieval Arabic tradition made great advances employing this conception of the arts. In Spain, under the influence of Alfarabi, a great effort was made clarify the notions of "scientific" and "artistic" practice, in a way which was somewhat similar to our distinction of theoretical and applied science. Theoretical geometry—the geometry of Euclid's *Elements*—was regarded as "scientific", considering by way of propositions, theorems and examples the universal essences of lines, surfaces and volumes and formulating demonstrations in a teaching context. But Aristotelian demonstrative "science" deals only with universal statements which are immutable because based on the immutable essences of things. In the Aristotelian theory there is no "scientific" knowledge of individuals or things subject to change. The **art** of geometry measures real dimensions toward making things, with the aid of instruments and fixed procedures. As an **art**, geometry must go beyond the universals—genus, species, difference, property, and accident—to the individual things which are to be produced.

Lull understood his Art in this way, but also as general, applying not only to geometry and medicine, but to theology as well, and to all the other disciplines, taken not as sciences, but rather as productive arts. The Art presupposes the reality of the vast realm of possible things described in the *Liber chaos*. Taking as his point of departure the existence of this realm called "chaos", Lull developed a new methodology to guide the "artist" in his production of the possible reality conjured up by man's imaginative faculty and his realization of its truth in its production.

It is as a part of this conception that the terms, "precepts" and "rules", used by Lull, must be understood. His Art is concerned with doing and making. To attain the end sought, his Art begins with principles or precepts and goes on to rules, as he found them especially in the medical tradition. In his *Ars inventiva veritatis* (Art of Discovering the Truth, about 1290) he attempted to formulate the rules by which the end sought—the work, its truth—could be achieved. The *Ars inventiva* seeks to incorporate his conception of the search for truth in the Art which he had first developed in his *Ars compendiosa inveniendi veritatem* (Compendious Art of Discovering the Truth, 1274) and his *Ars demonstrativa* (Demonstrative Art, 1283). From the time of the *Ars inventiva* the opening distinctions of Lull's Arts enumerate the **principles** and **rules** for this purpose.

Lull had begun to develop methods applicable not only to demonstrative science, but also to "artistic" production. He substituted in the *Tabula generalis* (General Table, Tunis 1293–Naples 1294) for the rules of the *Ars inventiva* nine new rules, meant to categorize the particular things which will be discovered in the phenomenal world. The *Tabula* begins with possibility and continues with rules based on the Aristotelian table of categories, as he had found them in the Arabic tradition, explicitly in the work of the Andalusian Sufi, Ibn Sab`în of Murcia (1217/18–1269/71 Mecca). The combinatoric which the *Tabula* introduces was meant to help in making the rules applicable to the phenomenal world. The *Ars generalis ultima* (The General Art in Its Final Form, Lyon 1305–Pisa 1308) represents the final stage of this development.

5. Geometry as an "art"

The idea of geometry as an "art", which followed from the vision of the creator as a maker, revolutionized mathematics. Lull related this vision to the medical theory of the intensity of qualities. The idea of a range—a "latitude"—in the intensity of qualities which had appeared in the medical tradition brought the discussion into the area of mathematics concerned with limits and continuous variability. These notions mark a break with the static, "scientific" geometry which the West had inherited from antiquity in favor of a geometry which takes account of change. In this new geometry, figures are *generated* by the movements of points and lines. The difference of approach may be illustrated by two possible ways of formulating definitions in geometry. The circumference of a circle can be defined statically as the locus of all points in a plane at a given distance from a center or dynamically as the curve generated by a point moving in a plane at a constant distance from a center.

Here Lull made some important distinctions. For Aristotle, qualities like heat were accidental forms. But whereas he had maintained that such qualities were capable of differences in degree only in the subjects in which they are found, not in themselves, Lull—in order to deal with such problems as the way in which a body grows hotter—suggested a quantitative treatment of increases or decreases in a quality (heat) by way of the addition or subtraction of degrees of intensity (temperature). Lull distinguished between an intense quantity (*quantitas intensa*), referring to the intensity of a perfection, and an extended quantity (*quantitas extensa*), characterized by three dimensions as it heats a foreign matter, and admitting only a potential infinity. In accordance with this distinction, the measure of the intensity of a quality was understood not as the extension of a quality in a subject (*quantitas extensa*, extended quantity, the quantity of heat), but rather in the sense of the inward quantity (*quantitas intensa*, temperature, for example).

Whereas Aristotle simply stated that, of the two qualities belonging to each of the four elements, one belongs to one element "more than" the other, Lull distinguished these two qualities, maintaining that heat is a quality "**proper**" to fire, while dryness is "**appropriated**" by it from another element, earth. With this distinction, Lull went beyond Aristotle's analysis of the activity of the elements. The distinction of proper and appropriated is based on Lull's conception of the two intentions—intense and extense—of finite activity. Lull understood the qualities of the elements as intimations of their dynamic activity, expressed in terms of his theory of the correlatives of action.

Heat (*caliditas*) is the quality, through which fire as a *calefactivum* is active (*calefacit*). In an intensive sense, the *caliditas* of fire has its object within itself, it is its own *calefactibile*. Lull called this species of *caliditas* a "proper" quality and an inseparable accident of fire; its action is a proper action and necessary. In an "extensive" sense, however, the *calefactibile* of fire is contingent, it is possible that it be an element like air or water (as when air or water is heated by fire) or a medicinal herb like pepper. In this case, the action of heat is an appropriated action and contingent. Because of the instruments necessary to gain its end, fire admits "appropriatedly" only a potential infinity. An appropriated quality can only approach the maximum; it is for this reason subordinate to its proper quality, according to the specific subject in which it is found.

These two types of quality—"proper" and "appropriated—are found in each of the elements, understood as dynamic natures. Lull maintained that because of the activity of the qualities of the elements one of the two assigned to each of the elements should be regarded as a "proper quality"—in the way the quality of heat is related to fire—while the other quality is merely "appropriated"—as is the dryness which is related to fire through earth. The distinction of first and second intention was important in this connection. The proper quality of each element is due in "first intention" to the necessary action of the element within itself, whereas the appropriated quality is due in "second intention" to its contingent action in another element. Lull distinguished therefore the qualities of heat and dryness, maintaining that heat is a quality "proper" to fire, while dryness is an "appropriated" quality, coming to fire from earth.

We may present Lull's conclusions as he himself did—as answers to the questions of the *Ars lulliana* which are meant to relate his abstract first principles to particularized phenomenal reality (*Logica nova* III 3 D–F; ROL XXIII 64). Since actions are—in accordance with the Aristotelian axiom—known through their objects, a distinction in respect of the two types of action discussed above is necessary. The distinction regards the question *Quando?* (When?) of the *Ars lulliana*. The proper action of things is—in *time*—instantaneous and continuous, because it is the ideal maximum which is the end of the action performed. But the phenomenal "diffusion" of the appropriated action of concrete things is in time successive and discrete, according to the concrete subject in which it is diffused.

A further question concerns the *Quantum*? (How much?) of quality. Lull distinguished two types of quantity according to the type of potency (*calefactibile*) which is reduced to act (*calefactum*). When the object is within the same genus (*ad intra*) as the quality which reduces it to act, ideal quantity is continuous. But when the object is of a different genus (*ad extra*), the phenomenal quality emerges as various discrete quantities.

A proper quality is continuous in quantity. The quantity of an appropriated quality is discrete, according to the specific subject in which it is found. Taking a standard example, Lull presents the quantity of the appropriated heat (*caliditas*) of fire—the *quantitas extensa*—as it is found in various herbs (Table 3).

Lull's theory of the continuous and discrete powers of things is a consequence of his vision of creaturely activity as striving for the infinite—for the Anselmian "that than which nothing greater can be thought" (*id quo maius cogitari nequit*). But because created things must turn to other created things to realize their own perfection, their "proper" action—which can be thought of as instantaneous and continuous—must remain an ideal to be reached only by "appropriated" effort which is successive and discrete. Continuous and instantaneous values are ideal *maxima*, quantities and times greater than which nothing can be thought (*quae maius cogitari nequeunt*), the limits to which created activity can only approach. Discrete and successive values are phenomenal, real-world values, values which increase and decrease in indivisible steps, in instants of time and atoms of matter, whose action can approach maximum values only as limits.

Lull did not make clear the manner in which transition from the continuous to the discrete is made. He asserted that although in thought the division of continuous magnitudes, such as space and time, may be continued indefinitely, nevertheless in actuality this process of subdivision is limited by the smallness of the parts obtained—that is, of atoms and instants.

Table 3

	Heat	dryness	humidity	coldness
pepper	4	3	2	1
cinnamon	3			
fennel	2			
anise	1			

With Lull's *Geometria nova* (New Geometry, Paris, 1299), constructed on these presuppositions, a radically new conception of geometry made its appearance. For Lull, natures are not static, immutable essences, but include an ontological intensity which can be measured as a distance between the relative nothingness of finite reality and a supreme terminus to which it tends as to its own unattainable perfection. The notion of continuous variability marks a break with the static geometry of pure form which the West had inherited from antiquity, in favor of a geometry of motion in which figures are generated by the movements of points and lines. The new geometry's concern with the problem of continuous variation led to the conclusion that knowledge consists in the determination of proportions or ratios and that perfect exactitude in measurement is unattainable.

To illustrate his idea that for the understanding of dynamic reality a "transcensus" from rational to "mathematical" knowledge is necessary, Lull took—in his *De quadratura et triangulatura circuli* (On the Quadrature and Triangulature of the Circle, Paris, 1299)—the example of a polygon as a variable magnitude inscribed in a circle. In his explanation of the measurement of the circle he used inscribed polygons. For rational knowledge the polygon and the circle are fixed essences which exclude one another, because according to the principle of contradiction an attribute cannot at the same time belong and not belong to a subject. But if we, by "conjecture", imagine a polygon the number of sides of which increases to infinity, we can see, by the vision of the intellect, that the polygon with a maximum number of sides will coincide with the circle. The triangle and the circle are the polygons with the smallest and the greatest number of sides. On this view, Lull proposed a characteristic quadrature of the circle. If the circle belongs among the polygons as the one with an infinite number of sides, its area may be found by the same means as that employed for any other polygon, by dividing it up into a number (in this case an infinite number) of triangles.

Lull regarded the validity of the propositions of mathematics as established by the intellect, the subject not being bound by the results of empirical investigation. Continuous motion belongs to the ideal world, not to the phenomenal world where real motion is composed of discrete, serially ordered states of rest. This view of mathematics as independent of the evidence of the senses encouraged speculation and allowed the indivisible and the infinite to enter, so long as no inconsistency in thought resulted. Such an attitude enriched the subject and eventuated in the methods of the calculus.

In a final step the mind extends these conclusions to the absolutely infinite God. Because he understood his "new geometry" as an application of his Art as productive, Lull saw it as a step beyond the finite mathematical figures of Euclid's *Elements* on the mind's ascent to the absolutely infinite. The mind ascends by transferring its considerations to infinite figures—the infinite which was the source and means, and at the same time the unattainable goal, of all knowledge.

6. Conclusion

Lull's Art was meant to bridge the gap between the necessary, universal conclusions of Aristotelian science and the contingent, particular facts which are produced by God. Lull saw God not as knower—not as "scientist", but as maker—he would have said, as "artist".

The absolutely infinite God is the architect who produces the things He wants to make manifest. Possible things are entities as the objects of His knowledge. Chaos, the trunk of the *Arbor elementalis*, is the reality which bridges the chasm separating Him and the cosmos.

These steps imply a different idea of truth than that of Aristotelian science. In his encounter with the divine exemplar—a real, transcendent exemplar which *is* his truth—man arrives at the end of his striving. The measure of his truth is the degree of intensity with which he reflects the divine exemplar. Man is a living image of God because the measure of the intensity of his perfection is the perfection of the exemplar. It was in this way that Lull learned the correct method of dealing with the infinite, not merely as a potentiality, but rather as an actuality, the upper limit of the activity of finite reality.

Lull's view of cosmic phenomena made him a peculiar player in the history of thought. Although he was certainly acquainted with Euclid's *Elements*, he was not a trained mathematician. Much that he wrote seems clumsy and baffling. But the things he wanted to address stemmed from the attempt to promote eternal truths in a shifting world.

The looseness of expression found in his works regarding the nature of the infinite was paralleled in Islamic Sufism and Jewish Cabala, which spoke above all in images. Man can know nothing of God's hidden, inner nature. Only through the divine attributes can man know something of the absolutely infinite. The attributes describe God's inner life and are not—like the Neoplatonic emanations—outside of him. They name the moments in which the divine life pulsates. Arabic and Hebrew writers as well as the *Ars lulliana* used especially the image of the living, growing tree to describe the way in which creation reflects this divine life. God's inner nature is the hidden root of the tree, beyond the limitless possibilities where the cosmos takes shape.

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CHAPTER 12

Odd Numbers and their Theological Potential. Exploring and Redescribing the Arithmetical Poetics of the Paintings on the Ceiling of St. Martin's Church in Zillis

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Contents

MATHEMATICS AND THE DIVINE: A HISTORICAL STUDY
Edited by T. Koetsier and L. Bergmans

> As the great increase of learning has made it necessary for most historians to limit their studies to one country, and one period of that one country, the importance of tradition as a factor in history has been somewhat obscured. Students of a particular area and age too often fail to realize that many of the actions they have to study were determined by patterns established *far away and long ago*.[1]

> By some strange illusion, it is often believed that history does not repeat itself; under the pretext that an historical event is isolated, it is imagined to be particular. Yet, numerous historical events are practically twins: they resemble each other like drops of water, and indeed, one can hardly make distinctions between the two. Only from a sociological viewpoint are they put into one single category. It is not that history consists in appreciating that which one will never see twice, it consists in appreciating twice as much what one sees occasionally.[2]

> Heavy eyelids that weigh on a fixed eye authenticate the seriousness of the national historian… The first shortcoming of the comparative historian is that he looks and critiques what first jumps out at him. No matter: his look must be alert and above all amused.[3]

> It is only the desire for a masterful author that makes one into a slavish reader.[4]

1. Introduction: displaying the method

This article will try to explain a certain mathematical logic found in the 153 paintings of the famous mediaeval ceiling in St. Martin's Church in Zillis.[5] In order maximise its intelligibility, this paper will attempt to redescribe such a remarkable (but mute about its origin[6]) work of art by piecing together multiple sources and references through analogical and not genealogical speculation and method.

This "excessive presumption" is an excellent heuristic tool, but let us define it right away: it is less the analysis of an object's intrinsic quality than the disposition of an on-looker or

[1] M. Smith, "On the Wine God in Palestine (Gn 18, Jn 2, and Achilles Tatius)", in *J.W. Baron Jubilee*, II, 1975, p. 815.

[2] "Par une étrange illusion, on croit que l'histoire ne se répète pas ; sous prétexte qu'un fait historique est individuel, on s'imagine qu'il est singulier. Pourtant, de nombreux faits historiques sont presque de vrais jumeaux ; ils se ressemblent comme deux gouttes d'eau ; ils n'en sont pas moins deux individus distincts et quand l'historien fait le recensement, ils comptent pour deux. Ce n'est qu'aux yeux d'un sociologue qu'ils tomberaient dans une seule et même catégorie. Il ne faut pas dire que l'histoire consiste à aimer ce que jamais on ne verra deux fois, elle consiste à aimer deux fois ce qu'on revoit à l'occasion", P. Veyne, *L'inventaire des différences. Leçon inaugurale au Collège de France*, Seuil, Paris, 1976, p. 38.

[3] "Les paupières lourdes sur oeil fixe authentifient le sérieux de l'historien national… L'oeil du comparatiste est primesautier, qualité dont on fera son premier défaut. Peu importe : son regard doit être vif et surtout amusé", Marcel Détienne, *Comparer l'incomparable*, Seuil, Paris, 2000, p. 17.

[4] Dennis A. Foster, *Confession and Complicity in Narrative*, Cambridge University Press, Cambridge, 1987, p. 19.

[5] For useful introductions, historical considerations and tentative interpretations, see E. Poeschel, *Kunstdenkmäler Graubündens IV*, Basel, 1943, pp. 223–246; R. Meier, *Suisse Romane*, Zodiaque, 1996, p. 85s.; E. Murbach and P. Heman, *Zillis. Images de l'univers roman*, Editions de Fontainemore/Bibliothèque des Arts, Paris (Atlantis, Zürich), 1967, p. 11ss.; D. Rudloff, *Zillis. Images de l'univers roman*, Zodiaque, 1989, p. 14ss.; H. Blanke, *Zillis. Evangelium in Bildern*, Theologischer Verlag, Zürich, 1994. Note also the more recent book of Ursi Tanner-Herter, *Zillis-Biblische Bilder*, Theologischer Verlag, Zürich, 2003.

[6] Very little is known about the Zillisian artist. One might imagine that he was surrounded by a team of students and apprentices, but died before the ceiling was finished, and that perhaps one of his students completed the project after his death (the last rows on the west side of the piece were apparently done by another's hand.

researcher, especially that of a comparatist researcher (here between text and painting[7]) who must choose, select and build his or her object in terms of "useful ends". The great historian of religions Jonathan Z. Smith recently defined this process as both scientific and humanist in a tellingly "discourse on method":

> Comparison does not necessarily tell us how things "are" (the far from latent presupposition that lies behind the notion of "genealogical" with its quest for real historical connections); like models and metaphors, *comparison tells us how things might be conceived*, how they might be "re-described", in Max Black's useful term.[8] *A comparison is a disciplined exaggeration in the service of knowledge*. It lifts out and strongly marks certain features within difference as being of possible intellectual significance, expressed in the rhetoric of their being "like" in some stipulated fashion. Comparison provides the means by which *we* "re-vision" phenomena as *our* data in order to solve *our* theoretical problems [...] The statement of comparison is never dyadic, but always triadic; there is always an implicit "more than", and there is always a "with respect to". In the case of the academic comparison, the "with respect to" is most frequently the scholar's interest, be this expressed in a question, a theory, or a model—recalling, in the case of the latter, that a model is useful precisely when it is different from that to which it is being applied.[9]

In light of such a resolutely heuristic perspective, ancient Gnosticism (which seems to have become my disciplinary expertise) can be (and will be) invoked as a legitimate and (for me) sufficiently intelligible resource for *translating* these images and bringing them forth from the unknown to the conceivable. To quote once again Jonathan Z. Smith:[10]

> The process [of comparison] by which this is accomplished ... is translation, that is to say, a proposal that the language that is appropriate to one domain (the known/the familiar) may translate the language characteristic of another domain (the unknown/the unfamiliar). The only effective grounds for rejecting such a proposal, is to attack the possibility of translation itself, most often expressed through arguments of incommensurability. Such arguments, if sustained, would render the human sciences sheerly impossible and are, therefore, always to be rejected ... I raise this issue in order to focus on one of its implications. Translation is never fully adequate: there is always discrepancy. To repeat the old tag: to translate is to traduce. This holds for natural language as well as for models. *Central to any proposal of translation is the double requirement of comparison and criticism* [...]
> To repeat: The implication of this is that comparison, *the negotiation of difference to some intellectual end*, is not an enterprise confined to odd surviving adherents of some *religionsgeschichtliche Schule*... The interesting arguments are over modes of comparison, their procedures and entail-

[7] Indeed, this is a relation not exempt from conflict and debate! See especially Léo Steinberg, *The Sexuality of Christ in Renaissance Art and in Modern Oblivion*, The University of Chicago Press, Chicago, 1996, p. 330ss.; see too the propositions of D.R. Cartlidge, "Which Path at the Crossroads? Early Christian Art as a Hermeneutical and Theological Challenge", in: Julian V. Hills (ed.), *Common Life in the Early Church. Essays Honoring Graydon F. Snyder*, Trinity Press International, Harrisburg, PA, 1998, pp. 357–372; and of R.M. Jensen, "Giving Texts Vision and Images Voice", ibid., pp. 344–356.

[8] Cp. this proposition to that, suggestively very Aristotelian, of Paul Veyne, op. cit., p. 62:

> It is more important to have ideas than to know truths ... this results in one ceasing to be naive and perceiving that which is could not have been. Reality is surrounded by an indefinite zone of non-realized potentialities. Truth is not the highest level in the values of knowledge.

[9] J.Z. Smith, *Drudgery Divine. On the Comparison of Early Christianities and the Religions of Late Antiquity*, Chicago University Press, Chicago, 1990, pp. 51–53.

[10] Cf. J.Z. Smith, "Social Formations of Early Christianities: A response to Ron Cameron and Burton Mack", *Method and Theory in the Study of Religion* 8, 1996, pp. 271–278; cf. p. 273s.

Fig. 1. The ceiling in St. Martin's Church in Zillis, the Johannine Gospel and the imaginary arithmetic of salvation.

ments, as well as questions as to the appropriate intellectual goals, and *not* over the question of whether or not to compare.

The ceiling's scene seems rich with literary references that apparently go beyond the framework of the New Testament.[11] The pictorial Zillisian imagery combines, in the style of a *diatessaron*,[12] multiple traditional scenes and constructs them in a particular "narra-

[11] On the importance of the apocryphal literature in Zillis, see H. Garcia, "Les diverses dimensions d'apocryphité dans le plafond de l'église Saint Martin de Zillis. Le cas du cycle zilliséen de la nativité de Jésus" (forthcoming in *Apocrypha. International Journal of Apocryphal Literatures*).

[12] This term refers to the major work of Tatian (second century) *Diatessaron* (dia tessarôn, "through four") that reunites, selects and recomposes in one coherent story the four (or more) "pre-canonical" gospels. See: W.L. Petersen, *Tatian's Diatessaron. Its Creation, Dissemination, Significance, and History in Scholarship*, Brill, Leiden, 1994); a substantial overview (less history of research) of this monumental study has already been rendered by the

tive" composition. This is like a silent cartoon in which the story and the commentary are in the hands of the on-lookers!

2. An anatomical description of the ceiling

The painting on the ceiling is rectangular and consists of 153 individual panels (17 rows with 9 panels and images each) that depict different scenes. In addition, there are 6 portraits that have been placed on the wall in the middle of a large, painted frieze that outlines the nave. The internal structure of the ceiling is easily discernible. Two areas can be clearly distinguished, through which a third area become visible after a relatively lengthy scrutiny. These areas are the following: 1 – the ceiling's evocatively colourful border that displays a water world filled with phantasmagorical and polymorphous or hybrid inhabitants and sea monsters (48 panels); 2 – an interior scene—by far the largest, consuming a total of 105 panels, only 7 of which depict the life of St. Martin—describing the life of Jesus and St. Martin, beginning with Jesus' childhood, at the east of the church, to his passion, and ending with a small row on St. Martin's life, at the west of the church; 3 – a prominent display of the glorious cross whose majesty dominates the ceiling and whose extremities include even the sea monsters at the painting's far borders; the cross is made visible or emphasised by an ornamental, double-friezed border of waves that consumes 25 panels.

Indeed, the entire painting is made up of a spectacular cosmography, an organised representation of the world that is both ordered and decorated in its entirety (a kosmos!) even to its maritime and monstrous extremities. Such a representation is as impressive as the environment in which it is framed; the church of Zillis and the valley of Schams (or the *Val Schons* in the local "réto-romanche" language) is a place that is geographically protected and enclosed by the *Via Mala Schlucht* in the north, *Ferrera* and *Rofla Schluchten* in the south.[13] Such a maritime scene, pregnant and luxurious in this alpine milieu[14] adds much to the artist's remarkable use of perspective, which creates a maximal widening effect and a veritable cosmological panoramic that zooms in on singular objects and situations. Indeed, with exquisite largesse, the created universe (and even the Almighty Christ) makes itself known in this tiny church in this hidden part of the world. As for Abraham[15] and other biblical or apocryphal personages[16] it is an artistic "heavenly tour" that is proposed to the visitor.

This scene, as a window into the "space of the infinite where I am not"[17] is consequently placed, put or even resting, and become like the natural centre of the painting: the fixed

same scholar Petersen in: H. Koester, *Ancient Christian Gospels. Their History and Development*, SCM, London, 1990, pp. 403–430.

[13] The presence of such a rich work in a church as small as this one seems to be another paradox (Cf. H.R. Meier, op. cit., p. 86).

[14] The mountainous environment abounds in traces of prehistoric times, when the marine universe dominated this region of Schams (see the fossiliferous site of *Piz Mellen*).

[15] See, for example, the *Testament of Abraham*, IX, 6ss.

[16] Cf. M. Himmelfarb, *Ascent to Heaven in Jewish and Christian Apocalypses*, Oxford University Press, New York/Oxford, 1993.

[17] Cp. the spatial–temporal anguish described by the French philosopher Blaise Pascal (*Les Pensées*, editées par F. Kaplan, Cerf, Paris, 1982; cf. §52, p. 114), whose style suggestively evokes that of the great horror and fantastic

space where one can come to contemplate the (imaginary) world. Such a dynamic seems to highlight the actual purpose of the Zillisian artist: to represent, from a determined point, a place that embraces all immensity; this privileged place is the valley of Schams, the village of Zillis and ultimately a church whose purpose is to elevate the faithful to the highest heights of God, as the Psalmist says (cf. Ps 113, 4–6; Jb 22, 12), and show the faithful how God sees all things.

The artist, however, is not content with only spiritual and mystic elevation since he is about to enshrine concretely his “point of view” in the very centre of this painted, symbolic geography of the world. At the very heart of this representation of the cosmos (panel J-V; n°101),[18] the Schams “physique”, seen from below (from the church’s main door, which faces west), is metonymically represented as the hillside facing the *Piz Vizan* and as the *Piz Vizan* itself, both situated in the southwest of the valley, the first on the right and the other on the left of the panel (as they are in physical reality). Suggestively, in the panel, the dark left wing of the devil completely takes the form of the *Piz Vizan*: This corresponds, perhaps, to the experienced fact that, in winter, around three o’clock in the afternoon (and later and later as winter gets underway), the sun always disappears precisely on the left side of the *Piz Vizan*. The symbolic correspondence is clear and evident: the devil is on the dark side of the mountain because he is the dark side *par excellence*! Through this detail, the only countryside that is geographically and physically accurate in the middle of an imaginary, mythic, symbolic, literary and undetermined painted universe,[19] that is, through this illusion that ingeniously manages and orders place and space,[20] the (local?) artist makes *Val Schons* (and the church of Zillis!) the centre of the world, even the centre of the gloriously displayed cross that covers and “crosses” the entire ceiling. But beware! Because, at the centre of the world, is the polymorphous figure of the devil who is shown struggling with the Savior himself![21]

writer Howard Phillips Lovecraft: “Je vois ces effroyables espaces de l’univers qui m’enferment, et je me trouve attaché à un coin de cette vaste étendue, sans que je sache pourquoi je suis plutôt placé en ce lieu qu’en un autre, ni pourquoi ce peu de temps qui m’est donné à vivre m’est assigné à ce point plutôt qu’en un autre de toute l’éternité qui m’a précédé et de toute celle qui me suit. Je ne vois que des infinités de toutes parts, qui m’enferment comme un atome et comme une ombre qui ne dure qu’un instant sans retour”; §§129, 130, p. 151s.: “Quand je considère la petite durée de ma vie, absorbée dans l’éternité précédant et suivant… le petit espace que je remplis et même que je vois, abîmé dans l’infinie immensité des espaces que j’ignore et qui m’ignorent, je m’effraie et m’étonne de me voir ici plutôt que là, car il n’y a point de raison pourquoi ici plutôt que là, pourquoi à présent plutôt que lors. Qui m’y a mis? Par l’ordre et la conduite de qui ce lieu et ce temps a-t-il été destiné à moi? Le silence éternel de ces espaces infinis m’effraie”. In the Zillisian context, the fact of quietly contemplating and mastering *de visu* all the created universe in its tiny spaces, corners, even those tremendous and far-out realities, constitutes a clever pedagogical aid.

[18] The ceiling is usually mapped out in two different ways: either as a naval battle (on one side, from the letters A to R, and on the other, from the numbers I to IX); or as a spiral, from 1 to 153. Here I am giving both these standard indications.

[19] The mountain represented in the panel O-VII (n°138), the prayer of Jesus at Gethsemane, is less a situated object from a specific point of view or a *panorama* than a “theatrical” support, a symbolic, decorative element signifying the “Mount of Olives”.

[20] On these concepts, see especially: J.Z. Smith, *To take Place. Toward Theory in Ritual*, University of Chicago Press, Chicago, 1987, p. 27ss.

[21] The very centre of the ceiling, panel J-V (n°101), represents the third temptation of Jesus, according to Matthew 4, 8–10.

When we take separately the ceiling's central and principal area, what initially seems striking are sensations of profusion, asymmetry and yet something lacking: profusion because we recognise a wide array of gospel and apocryphal scenes; asymmetry because certain scenes (or sets of scenes) are much more developed than others, probably because of the underlying stories that are more or less narrated in their original textual context; something lacking in that certain traditional stories are not at all mentioned (including the crucifixion and the resurrection of Jesus). Amazingly, as we have noticed, the entire pictorial story is organised on a rectangle consisting of 153 panels. The central space unfolds into four narrated parts: the infancy of Jesus, which occupies a very large third of the ceiling;[22] the public ministry of Jesus, which takes up another third;[23] Jesus' passion (perhaps somewhat shortened); and, to conclude, a depiction of the short but meaningful life of St. Martin. In fact, perhaps it is possible to reduce the ensemble to three parts: the passion being a part of Jesus' ministry; or even to two rather spatially asymmetrical parts narrating the lives of Jesus and St. Martin. Yet, in this *reductio ad unum*, the ensemble could be even more drastically reduced to one part: the narration of Jesus and Martin being co-ordinated in term of a Christological (and traditional) trajectory; as Burton L. Mack suggests: "It is as important for the disciples to be apostles as it is for Jesus to be the Christ".[24]

3. A theological imagination of odd numbers

Is it reckoning, chance or spatial constraint? There has been much speculation on the size of the ceiling's 153 panels, notably regarding issues of the corruption and disappearance of certain initial scenes or other problems relating to the ceiling's abrupt end. Several exegetical interpretations, suggestively with allegorical mathematical perspectives, have been proposed to explain this specifically Johannine number 153 (cf. Joh 21, 11).[25] Possibly, the accent of this Johannine passage is less on the precise number of the fishes than on the materially sufficient measure (1 cm $\rightarrow \times =$ big!) of the fishes and their total quantity *in only one net*. Moreover, not only does the net remain *untorn*, but Peter pulled it to shore *in one fell swoop*, according to the literary phenomenon of "miracles in the miracle". One can compare, in this context, Joh 11, 44, where the "resurrected" Lazarus walks all by himself with hands and feet tied without at all feeling disturbed or disconcerted!

Mathematical symbolism has possessed a long and rich religious and philosophical tradition since the mythical (and rather shamanistic) Pythagoras[26] and even Homer.[27] Its frequent and "natural" utilisation by Philo of Alexandria[28] and in Jewish and Christian

[22] Until panel H-IV (n° 93).

[23] Until panel O-III (n° 134).

[24] B.L. Mack, *Who Wrote the New Testament. The Making of Christian Myth*, HarperCollins, New York/San Francisco, 1995, p. 225.

[25] See, for instance, E.C. Hoskyns and F.N. Davey, *The Fourth Gospel*, Faber & Faber, London, 1947, p. 553ss.

[26] On the "philosophical" Pythagoras, see: J.P. Dumont et al. (eds.), *Les Présocratiques*, Gallimard, Paris, 1988, p. 53ss.; on philosophical posterity in late antiquity, cf. D.J. O'Meara, *Pythagoras Revived: Mathematics and Philosophy in Late Antiquity*, Clarendon Press, Oxford, 1989.

[27] See especially: F. Buffière, *Les Mythes d'Homère et la pensée grecque*, Les Belles Lettres, Paris, 1973 (1956), p. 559ss.

[28] See, for instance, *De Opificio Mundi*, 47ss.

apocalyptic literature[29] attest to its importance in the remarkable "bouillon de culture" of the first Christian century, from which the Johannine Gospel emerged, in all its arithmetic particularity. From then on, one cannot imagine that the mention of 153 fishes is meaningless or merely ornamental.

In fact, after scrutinising Johannine references, one finds an underscoring of certain numbers and, more precisely, a certain logic in the distribution of odd and even numbers. One can collect, for instance, the following data: Joh 1, 39 (the tenth hour); 2, 1 (the third day); 2, 6 (the six jars); 4, 16–18 (the five husbands); 4, 52–53 (the seventh hour); 5, 5 (the paralytic of 38 years); 6, 7 (200 denari); 6, 9 (five loaves and two fishes); 6, 10 (five thousand men); 6, 13 (12 baskets from five loaves left over); 6, 15 (Jesus alone); 6, 67.70–71 (the Twelve); 8, 57 (Jesus not yet 50 years old); 11, 1.3 (the two sisters—with one brother); 11, 39 (Lazarus having already been in the tomb for four days); 12, 1 (six days before Easter); 19, 4 (the sixth hour); 19, 23 (the clothes of Jesus separated in four parts *plus* one tunic); 19, 39 (100 pounds of myrrh and aloe brought by Nicodemus; 20, 4 (two disciples); 20, 12 (two angels); 21, 2 (two others disciples); 21, 11 (153 fishes!); 21, 14 (third appearance of the resurrected Christ); 21, 15–17 (Jesus' three questions to Peter); 21, 25 (the hyperbolic and infinite number of books about Jesus).

According to Pythagorean traditions,[30] widely popular (and modified) throughout the centuries, numbers have profound significance and reveal many things: they constitute ultimate reality and, accordingly, have become a privileged and ultimate explanation of the world. As such, they possess a divine character—indeed each Homeric god has his or her own essential or corresponding number[31]—and by extension, any person who could decipher the *arcana* of arithmetics was promised divine happiness.

For those living in ancient times, numbers are *like points which order themselves in forms and figures*; for example, numbers like four or nine form a square, whereas other numbers like ten or twenty-four make triangles.[32] Each number as such become a series of points or units (monads) that form figures and are fundamental elements of the number.[33]

Such a description of numbers is suggestively analogical to *military manœuvres* in which a *hoplite* is the principle and basic unity of the *Phalanx*, the army host's multiple forms or shapes. If numbers are transcribed into two columns, the significant difference between odd and even numbers becomes easily discernible: the even numbers make two equal columns and the odd numbers always create unevenness. In this ancient specific logic, even numbers are "hollow" (kenos) because one can cut them into two equal parts whereas odd numbers

[29]See especially: A. Yarbro Collins, "Numerical Symbolism in Jewish and Early Christian Apocalyptic Literature", *Aufstieg und Niedergang der römishen Welt,* II, 21.2, 1984, pp. 1221–1287.

[30]I am referring directly to the elements of F. Buffière's excellent explanation in op. cit., p. 560s. A good overview of mathematical traditions, notably from the ancient Near East (Mesopotamia, Egypt, etc.) is given by Jens Hoyrup, "Mathematics, Algebra and Geometry", *Anchor Bible Dictionary* 4, Doubleday, New York, 1992, pp. 602–612.

[31]Cf. F. Buffière, op. cit., p. 561ss. (Apollo, Proteus = 1; Eole = 4; Athena = 7; Hephaistos =, Hera = 9; Atlas = 10, etc.).

[32]Numerous examples are provided by Philo: *Quaestiones in Genesim*, II, 4–5; *De Opificio Mundi*, 47; *De Vita Mosis*, I, 96; *De Decalogo*, 26ss., etc.

[33]Cf. J. Hoyrup, op. cit., p. 610.

Fig. 2. Hybrid monster. Detail of the ceiling of St. Martin's Church in Zillis.

are "full" (perissos).[34] Odd numbers oppose this unity in surplus, like an irreductible bastion that resists every attempt to cut them. This surplus unity distinguishes the odd number from the even number and gives the odd its exceptional force and value.

In addition, this unity that stands out, "beyond the shape", is analogous to the male notion of fecundity or reproduction from which comes the principle that odd numbers are male and even numbers are female. Actually, combinations of numbers are considered to be *genuine marriages*: two numbers that come together beget a third number; yet in these unions, the odd number clearly plays the dominant role.

This is first evident in the ability of the odd number to make any sum with an even number, an odd number: odd + even = odd, always. The "stronger" number imposes itself, overtakes the weaker and stays on, *alone*. Moreover, the even number, added to itself, can never make anything but an even number (it can never get out of its "even sex"): even + even = even, always. On the contrary, the odd number is not so limited because an odd added to itself becomes an even, the "opposite sex": $5 + 5 = 10$. Indeed, an odd number is like a god capable of creating the companion with whom he can unite.

[34]The word perissos means "what goes beyond the limits or what goes past the limits"; it is composed from the preverb/preposition peri; that expresses the idea of completely encircling or surrounding, even from above—from which the uses of the term expressing the idea of superiority (cf., suggestively, Joh 10, 10).

Fig. 3. Fishermen. Detail of the ceiling of St. Martin's Church in Zillis.

The joining of Adam and Eve $(1+1)$ appears evident (cf. Gn 1, 26; 2, 18–24). In this "Genesiac" context, we can also pick out several *logia* of the *Gospel of Thomas* (11; 16; 23; 49; 75; 106) and its soteriological and ascetic "return to the odd": from two towards one or $1+1=1$![35]

All the power of the odd number, therefore, resides in its ability as unity that goes beyond; it is therefore the monad—the *one*, what is adding itself to the even—which constitutes the great principle of fecundity, union and order. Again, referring to the military metaphor, the monad is like the commander or general who leads his soldiers; without him walking ahead, *outside*, the military shape marching on would not be this beautiful organic group. It is the monad, the unity that distinguishes the odd number from the preceding even number, that seals a number and determines its role. The even number has no precise limits; its components can move about in any way. The monad, however, imposes a limit and enchains the even number. From the dynamic comes the notion that the monad is precision and determination and the dyad (the number 2, the pair or the double row of points figuring even numbers) is indetermination and imperfection. The monad is divine; the dyad, matter.

A *moral* interpretation naturally follows such a paradigm. With tautological obviousness, a monad signifies unity and a dyad, duality. Consequently, the monad is a source of

[35] See especially: M. Meyer, *The Gospel of Thomas. The Hidden Sayings of Jesus*, HarperCollins, New York/San Francisco, 1992, *in loco*.

Fig. 4. Jesus and the devil. Detail of the ceiling of St. Martin's Church in Zillis.

all good, the dyad is the root of all evil; unity is a symbol of peace and harmony, duality begets conflict and war; health is unity in the human body, illness duality. In the soul, odd and even numbers are, respectively, virtue and vice. In homes and cities, odd and even are translated into concord and discord. The number one is equated with coherence of *species*, whereas the number two connotes confusion and disorder. In the Zillisian context with its horde of polymorphous or hybrid monsters, it is possible to continue further in this vein and to record certain (fragmentary) developments concerning limitlessness and the undifferentiated from the pre-Socratic philosopher Anaximander.[36] Indeed, odd numbers are equal to peras, "limit, limitation, order, principle of organisation"; even numbers correspond to apeiron ou aoriston, "the limitless, the indiscernible, the disorganised".

What is the usefulness in exploring these allegorical and metaphysical dynamics to enlighten the Zillisian artwork? A few general descriptive remarks and suggestions can be made: 1 – As we have seen, the ceiling's entirety is composed of 17 rows of 9 panels, suggestively two odd numbers used by the artist to create (the dimensions of) his space; 2 – The maritime fresco of the sea monsters consumes 44 panels (with the exception of

[36]Cf. Dumont, op. cit., p. 24ss.

the four "cardinal" angels in each corner of the monsters' border). These four and "even" angels reveal the fundamental imperfection of even numbers: the perfectibility of creation itself manifested through the *subaltern* status of God's messengers who *yet* limit and control it! 3 – The glorious cross traverses all of the ceiling's regions and occupies 25 panels ($25 = 2 + 5 = 7$!). Given these *configurations*, what do the 153 Zillisian panels mean in light of the 153 Johannine fishes? Several possible combinations have been proposed by the Church Fathers, notably Origen, Cyril of Alexandria and Augustine.[37]

The sum of all the numbers from 1 to 17 equals 153 ($1 + 2 + 3 + 4 + \cdots + 17 = 153$), which together form a perfect equilateral triangle.[38] Moreover, 17 is the result of the "marriage" between $10 + 7$ (even + odd = odd),[39] two pregnant numbers in the Johannine narrative and symbolism. The ten refers to the "tenth hour" (Joh 1, 39), that marks the accomplishment—in fact, narratively, the *beginning* of the accomplishment of the revelation of the two disciples: $3 \times 3 = 9 + 1 = 10$ or $1 + 2 + 3 + 4 = 10$. The seven of the "seventh hour" (Joh 4, 52–53) likewise marks an analogous symbolic phenomenon (in the Johannine narration, the hour of *effective realisation* of the healing). On the other hand, the six ($7 - 1$) seems to mark the lack and the waiting (cf. Joh 2, 6; 12, 1; 19, 14).[40]

Other arithmetic combinations have been proposed with varied corresponding allegorical meanings such as: $153 = 3 \times 50 + 3$ ($50 = 7 \times 7 = 49 + 1$; $7 = 4 + 3$). All of these combinations always suggest the superiority and the domination of the odd number and the monad.

4. A zoomorphic and soteriological arithmetic

Other interpretations have been offered concerning these 153 Johannine fishes.[41] One suggests that this number reveals a zoological allusion to the supposed 153 species of fishes, each fish representing one species and symbolising one nation or human category. Jerome thus comments:

> Aiunt autem qui de animantium scripsere naturis et proprietate, qui halieutica tam latino, quam graeco didicere sermone, de quibus Oppianus Cilix est, poeta doctissimus, *centum quinquaginta tria esse genera piscium*.[42]

[37] See Hoskyns/Davey (eds.), op. cit., p. 556.

[38] Ibid., p. 553.

[39] Note again $17 = 5 + 12$ (cf. Joh 6, 9.13: five loaves and twelve baskets!).

[40] Cp. Philon, *De Opificio Mundi*, 89ss., on the sacred characteristic of the number seven. In Philo, the number six marks the completion, the end and the limit (cf. *De Opificio Mundi*, 13; *Legum Allegoriae*, I, 3-4; *Quaestiones in Genesim*, III, 38).

[41] Notably through the *gematria*, or the calculation according to the numerical value of the letters of the alphabet. On the notion of the *gematria* in relation to the 153 Johannine fishes, see J.A. Emerton, "The Hundred and Fifty-three Fishes in John XXI", 11", *Journal of Theological Studies* (JTS) 9, 1958, pp. 86–89; *id.*, "Some New Testament Notes. IV. Gematria in John 21, 11", *JTS* 11, 1960, pp. 335–336, which highlights the link between the number 153 and Hebrew words of place *En Guedi* and *En Eglaïm* in Ez 47, 10: "Guedi" signifying 17, "Eglaïm" signifying 153 by *gematria* , "En" being the word for "source"; note likewise the note by P.R. Ackroyd, "The 153 Fishes in John XXI, 11—a Further Note", *JTS* 10, 1959, p. 94, using the *gematria* on certain Greek transliterations of the words "En Guedi" et "En Eglaïm" (èggadi = 33; agalleim = 120); note to which the second article of Emerton responds.

[42] Jérôme, *Commentaria in Ezichielem*, *Lib.* XIV, *Cap*. 47 (PL 26. 474C).

This passing citation of Oppian's *Halieutica* is significant. In fact, Oppian does not propose a precise number;[43] he only says that he does not believe that the species of fishes in the sea are less numerous than the animals of the land; moreover, this important and undefined number measures up to the unexplored depths of this *other* world, the Ocean; to quote Oppian (I, 80–89):[44]

> Innumerable are the people and numberless are those who swim in the depths of the sea, and no one can name them with certitude. For no one has penetrated until the sea's limits; but at most men have been able to discover only three hundred fathoms of the sea's undulating depths. Yet much more—for indeed the sea's depths are measureless—remains hidden and such invisible things no mortal can ever make known, for the force and reason of mortal is small. Remarkably, the salty flood, I believe, does not nourish any less the human flock or tribes than does the fertile Mother Earth.[45]

The purpose of Jerome in thus linking these two heterogeneous but thematically analogous references (Oppian and the Johannine Gospel) is to subordinate the negative notions of the beyond and infinite (used by Oppian) to those of totality and universality (used by the Johannine Gospel): the catch of fishes, in the Johannine story, is miraculous in that it has effectively explored and controlled the depths of the sea and its innumerable creatures which symbolise the entirety of the human race *until the boundaries* of both land and sea. Presumably, Jerome seeks to comment on Ez 47, 10:

> People will stand fishing besides the sea from En-gedi to En-eglaim; it will be a place for the spreading of nets; its fish will be of great many kinds, like the fish of the Great Sea.[46]

This Messianic perspective seems to provide the scriptural backdrop for the Johannine story of the miraculous catch *via* the Lukan text (LK 5, 1–11) as a direct text source which highlights Peter's missionary role. Indeed, in the Johannine story, it is Peter *alone* who goes back into the water and *alone* pulls the net[47] from the sea (Joh 21, 11; cp. 21, 6; Lk 5, 6–7). Moreover, the Lukan passage implies a comparison and a movement of activity in which the expected end result is reversed:

> Do not be afraid; from now on you will be for their life catching people (esè/zôgrôn).[48]

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[43] In this regard, one can suspect that here the idea depends less on the citation of Oppian than on the reflexive imagination of Jerome interpreting Oppian (see R.M. Grant, "One Hundred Fifty-Three Large Fish (John 21, 11)", *Harvard Theological Review* XLII, 1949, p. 273).

[44] I refer to the recent edition of F. Fajen (Teubner, Stuttgart/Leipzig, 1999).

[45] Cp. Apuleius, *Apologia*, 39, about the "Gastronomy (*Hedyphagetica*) of Quintius Ennius: "innumerabilia genera piscium enumerat . . . alios etiam multis uorsibus decoravit".

[46] Translation of the *New Oxford Annotated Bible (New Revised Standard Version)*, Oxford University Press, New York, 1991. Again, the text from the *Septuagint* stresses less the number of diverse species of fishes and more their infinite number: plèthos polu sphodra. Moreover, one can imagine that the Johannine author made a sort of syntactical adjustment of the fishes from the "great" sea (hoi ichthues tès thalassès tès megalès) to the "great" fishes of the sea (tès thalassès tès Tiberiados . . . ichthuôn megalôn); this gentle manipulation logically places less emphasis on the (rather ridiculous) immensity of the sea of Tiberiades and more on the numerous "big" fishes one can occasionally catch!

[47] As we have seen, the net remains untorn (cp. inversely Lk 5, 6: dierrèsseto de ta diktua autôn; but it is perhaps a conative imperfect: the Lukan nets resisted and remained untorn!; see B.M. Fanning, *Verbal Aspect in New Testament Greek*, Clarendon Press, Oxford, 1990, p. 249ss.).

[48] Lc 5, 10.

Here the verb zôgreô,[49] being in the future periphrastic tense, has strong connotations and is rich with meaning; it signifies "to make alive, to make a prisoner alive, to spare the life of a prisoner of war". Homer already uses it to refer to a captive whose life has been saved.[50] A fisherman takes the life of the fishes he catches; the "apostles" as Christ's ministers capture people to give them life,[51] to revitalise, revive or reanimate them.

This missionary vocation is also found in these same terms of fishing technique and imagery in the parable of Matthew 13: 47–49:

> The Kingdom of heaven is like a net that was thrown into the sea and caught fish of every kind; when it was full, they dragged it ashore, sat down, and put the good into baskets but threw out the bad. So it will be at the end of the age. The angels will come out and separate the evil from the righteous.[52]

World is thus compared to sea and human beings to diverse species of fishes to be caught. By analogy with the archaic Greek image of the land as an *island* surrounded by the ocean[53] where people can easily throw their nets "from the centre", the Zillisian ceiling, formed by several distinct panels interlocked in an organised and ornamental pattern, convincingly creates the image of a net![54]

D. Rudloff[55] considers the fishing scene in panel B-IX (n° 10) to be representative of the miraculous catch described in Joh 21, 1–14 and of the specific role of the apostle Peter. In so doing, the "fish story" thematically contrasts with the neighbouring scene (C-IX; n° 11): Jonah fleeing from God and thus embarking "on the sea"[56]. These scenes illustrate two different responses to receiving a vocation from God.[57] This interesting interpretation, however, seems for me unlikely for several reasons: firstly, the traditional, stereotypical character of Simon Peter[58] is not represented on this fishing boat; secondly, from an exegetical point of view, Joh 21, 1–14, as all the Johannine story,[59] does not offer a positive image of the apostle Peter, and portrays him rather as resembling more a recalcitrant and

[49]Cf. 2 Tm 2, 26.

[50]Cf. *Iliad*, VI, 46; X, 378; V, 697. Note the expression ta zôagria, "the prize of salvation" that is given as recompense to the victor by the one whose life he spared (cf. *Iliade*, XVIII, 407; *Odyssée*, VIII, 462).

[51]Cf. E. Delebecque, *Evangile de Luc. Texte traduit et annoté*, Klincksieck, Paris, 1992 (1976), p. 28.

[52]Cf. Es 24, 17; Lc 21, 35 (the coming of the Kingdom of God as a net). The devil as monkey of God also benefits from this fishing technique: he too throws his nets and possesses his "miraculous catches" (cf. 2 Tm, 2, 26; 1 Tm 6, 9).

[53]See especially: J.M. Romm, *The Edges of the Earth in Ancient Thought*, Princeton University Press, Princeton, 1992.

[54]Cf. Murbach, op. cit., p. 18.

[55]Cf. Rudloff, op. cit., p. 53.

[56]Indeed, in the panel, Jonah embarking seems nearly to fall into the sea!

[57]Cp., in this regard, the proposal of R. Eisler (cf. Emerton, op. cit., 1958, p. 88): the number 153 is the sum of the numerical values of the word Simôn (= 76) and Ichthus (= 77); by *gematria*, many Johannine themes are thus joined together: Simon Peter, the fish and, indirectly, the testimony of Christ as saviour.

[58]See C.R. Matthews, "Nicephorus Callistus' Physical Description of Peter: An Original Component of the Acts of Peter?", *Apocrypha* 7, 1996, pp. 135–145. For the stereotypical description of the Zillisian Peter with white hair, see the following panels: J-VII/n° 103; L-III/n° 113; M-V/n° 122; M-VI/n° 123 or M-VIII/n° 125; O-IV/n° 135; O-VI/n° 137 (perhaps); O-VIII/n° 139; P-III/n° 141.

[59]See P. Perkins, *Peter. Apostle for the Whole Church*, University of South Carolina Press, Columbia, 1994, p. 95ss.

fleeing Jonah than the obedient follower of Christ![60] If there is a reference to the miraculous catch, it would probably refer to the one found in Luke, which is more favourable to Peter.

The eschatological Matthean parable that refers directly to the following passage in Habakkuk (1, 14–17), however, seems to be much more satisfying:

> *You have made men like fishes of the sea*, like crawling things that have no ruler. This one brings all of them up with the hook; he drags them out with his net, he gathers them in his seine; so he rejoices and exults. Therefore he sacrifices to his net and makes offerings to his seine; for by them his portion is lavish and his food is rich. Is he then to keep on emptying his net and destroying nations without mercy?

This Zillisian overall scene, redescribed and structured by these references and their common or "isotopic" theme, tends to give its "net trick" of 153 fishes an allegorical arithmetic meaning that connotes both the solidity and prolific effectiveness of Christian apostolic mission[61] which, guided by the Lord,[62] captures every person or nation without distinction.[63] It also connotes the Christian church that must remain *one*, despite the numerous apostles sent *into action* and the numerous faithful that are captured by these diverse fishers of men[64] (cf. again the image of the untorn net).[65]

In this large and coherent context of remarkable, though strange, mythological hermeneutics, the 153 Zillisian panels imply an enumerated or calculated expression of the entire world attained by Jesus' apostles and called to conversion, and the universal pertinence of Christ's message that must occupy all dimensions of human space and *kosmos*, as the Pauline letter to Ephesians records (3, 18s.):

> I pray that you may have the power to comprehend, with all the saints, *what is the breadth and length and height and depth*, and to know the love of Christ that surpasses knowledge, so that you may be filled with *all the fullness* of God.[66]

[60] See especially D. Gee, "Why did Peter spring into the Sea", *JTS* 40, 1989, pp. 481–490.

[61] That is, the work of the apostles of which the narratively immediate numbers make the sum of 17 (7 in Joh 21, 2 and 10 in Joh 20, 24) and, as we have already seen, 153 when multiplied to become a triangle. As Grant concludes (op. cit., p. 274), it seems probable that the importance of 17 comes directly from the fact that its components are ten and seven: two traditionally sacred numbers, as remarked with Philo; the value of 153 participates, therefore, in the same sort of background and thus symbolises the *demultiplication* of the works of Jesus' disciples effectively turned into *plenitude*.

[62] Cp. Mc 16, 20.

[63] One can note here that in Acts 2, 7ss., 17 geographic and political situations seem to be mentioned in a sort of "list of nations" probably borrowed from ancient geographers or even astrologers (see H. Conzelmann, *Acts of the Apostles*, Philadelphia: Fortress Press, 1987, p. 14s.).

[64] Hence the importance, in this context, of the final *reconciliation* between the character of Peter and the enigmatic "Johannine disciple/author", of the relative rehabilitation of the first and the non-competitive repartitioning of mission fields and functions between these two disciples (cf. Joh 21, 15ss.); cp. the polemical missionary context in 1 Co 1, 12–13; 3, 4–23; cp. likewise the apocryphal pacifying theme of this apostolic repartitioning of mission fields; see, for instance, *Acts of Andrew and Matthias*, 1 (cp. Acts 2, 1.47; Ph 2, 2); *Acts of Philip*, III, 1–2; VIII, 1–2 (cf. J.D. Kaestli, "Les scènes d'attribution des champs de mission et de départ de l'apôtre dans les Actes apocryphes", in: F. Bovon et al. (eds.), *Les Actes apocryphes des apôtres. Christianisme et monde païen*, Labor et Fides, Genève, pp. 249–264).

[65] Cf. M.J. Lagrange, *Evangile selon Saint Jean*, Gabalda, Paris, 1936, p. 527.

[66] In this passage, see the recent erudite commentary of E. Best, *Ephesians*, Clark, Edinburgh, 1998, p. 344s.

5. The cross as cosmic sign and structure

In the preceding descriptions, one saw how the ceiling in the Zillis church is constituted of a third space—in the shape of a cross—that traverses the other two spaces, penetrating both as if from the interior and the exterior and building a sort of *celestial framework* which keeps the sea monsters away from the centre. This cosmic and glorious cross[67] is all the more important, in the Zillisian context, because the pictorial narration of Jesus' life contains no crucifixion scene, *a fortiori* no resurrection scene. The illustrations of Jesus' life cease with the representation of his being crowned with thorns (panel P-VIII; n° 146), which constitutes a remarkable irony of situation by its clearly designated aspect of royal crowning and enthronement.[68]

A passage from Andrew's Passion (54; Armenian version, 16–19) offers some very interesting insights for entering into the logic of this celestial, though a little hidden, Zillisian cross and consequently for perceiving it with accurate intelligence, notably its autonomous or self- sufficient status as cross *sans* Christ:

> I study your image for which you stood. I saw mine in you as I etched yours upon me. If what I perceive is you existing, I like what I see, what I perceive, what I understand from you. What is your shape, O cross? What is your crossbeam? Where is the center? What is invisible in you? What is apparent? To what extent are you hidden? To what extent are you revealed through the cry of your companion (the gallows)? To what extent do you travail to find those who hear you? O name of the cross, entirely filled with deeds! Well done, O cross, who restrained the error of the world! Well done, vision of violence, that continually and violently treats violence with violence! Well done, shape of understanding, who shapes the shapeless! ... But until when shall I say this and not that which you showed, O cross? Standing before you I commend to you those who are listening, for there is not for us any other time to approach this vision, O cross.[69]

Clearly the Zillisian cross contains many "things to see", many actions, many painted panels that represent multiple scenes, especially the tempting devil facing Christ which is *at the centre*, at the intersection of the cross's "rays": the Zillisian cross, like the polyonymous cross in the *Acts of Andrew*, is swarming with many things!

In this regard, the Zillisian cross is not unlike certain Byzantine crosses decorated with medallion-shaped, in-laid portraits like the one situated in the "heaven" of the apse in the church of St. Apollinarus-in-Classe in Ravenna.[70] There is also the miniature of the Byzantine Bible entitled the "Bible of Leon" (tenth century)[71] which can be called a library-cross because it contains multiple circular-shaped pieces that together constitute the canonical list of the biblical books of the *Septuagint*: each circular piece carries the name of a book; the four evangelists, standing, surround the cross thus constructed; a medallion of Mary is

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[67] On this important theme in ancient Christianity, see, for instance, Mt 24, 26s.30; *Didachè*, 16, 6, *Acts de John*, 97ss.; *Gospel of Peter*, 39–42; *Epistle of the Apostles*, 16; *Apocalypse of Elijah*, III, 3; *Sibylline Oracles*, VI, 26–28; *Acts de Philip*, III, 14.

[68] This irony is probably present in the Johannine text and intention (see Joh 19, 13–16).

[69] Translation from R.D. MacDonald, *The Acts of Andrew and the Acts of Andrew and Matthias in the City of the Cannibals*, Scholars Press, Atlanta, GA, 1990.

[70] Cf. A. Kartsonis, "The Emancipation of the Crucifixion", in: A. Guillou, J. Durand (eds.), *Byzance et les images*, La Documentation Française, Paris, 1994, pp. 151–187; cf. pp. 158–160.

[71] See the beautiful photography in W. Ziehr, *Das Kreuz. Symbol und Wirklichkeit*, Silva Verlag, Zürich, 1997, p. 77.

just above the cross; a medallion of Jesus with its own caption or title, as have all the circular biblical pieces, is at the centre of the cross signifying not only that Jesus is/has a book, but that all the biblical holy books, announcing this Jesus at the centre, take artistically the prophetic form of the cross! This resemblance and analogy suggest a Byzantine influence in Zillis, notably with regard to how Jesus' life can be organised through and around the shape of a cross structuring inside and outside of itself multiple biblical or evangelical scenes.[72]

The lengthy speech on the cross in the *Acts of John* certainly constitutes another thematic and intertextual insight important for redescribing and understanding the structure of the Zillisian ceiling and the cross as the soteriological and cosmic ordering principle and ecclesiological symbol for *the arithmetic game of the one and the many*. This discourse is the very commentary of the Revealer, addressing his disciple John, about the Cross of light and what it signifies:

> This Cross of light is called by me now Word because of you, now Intellect, Jesus, Christ, Door, Way, Bread, Seed, resurrection, Son, Father, Spirit, Life, Truth, Faith, and Grace. The Cross is so called these names according to humanity. Yet what it is in reality, as conceived in itself and said in relation to us is the separation from all things, and the strong raising of that which is composed of unstable elements and in harmony with a wisdom that is harmoniously consistent. There are places right and left, Powers, Authorities, Principles and Demons, Energies, Threats, Furies, Devils, Satan and the roots of below, from which issues the nature of things that come to be.
>
> Such is the cross, that from a word fixed the All and separated that which is from the generation and the lower things and then combined everything into one unity. This cross is not the cross of wood that you must see, descending from here; and I am no longer the one who is on the cross that at this hour you do not see but only hear the voice. I have been taken for that which I am not, not being what is to the eyes of many, but something completely different. They called me something vile and unworthy of me. Just as the resting place is not seen or spoken of, indeed even more so the Lord this place will not be seen and no one will speak of him. As for the crowd of people who have different aspects and who surround around my cross, they are from the lower nature. And those who you see on the cross, they do not possess one form because each member of this one who is come down has not yet been understood. When human nature is assumed, together with the race that approaches me and obeys my voice, he who hears me will be united to him and will no longer be what he is now, but will be above others as I am now. For so long as you do not say that you belong to me, I am not what I am. But if you listen to me, in listening me you will become as I am and I shall be what I was, when you are like me beside me. For it is by me that you are what I am. Therefore, do not worry about the great crowd and despise those who are outside of the mystery.[73]

The cross of light is thus presented as the centre of the world, of which it is the pivotal structure, both on the inside and the outside and right and left, separating and limiting spaces and qualities of beings that spiritually (ontologically) differ.[74] Its polyonymous designation, through multiple evangelical names referring to many traditional stories,[75]

[72] See the reliquary *Sancta Sanctorum*, Vatican Museum (cf. W. Ziehr, op. cit., p. 157s.).

[73] Cf. *Acts of John*, 98–100; see the commentary of E. Junod and J.D. Kaestli, *Acta Johannis. Textus alii–commentarius–indices*, Brepols, Turnhout (CCSA 2), 1983, p. 617ss.

[74] This idea makes intelligible the function of the Zillisian cross in relation to the multiplicity of the many-aspected monsters that surround from right and the left.

[75] For example: the word "bread" refers to the stories of the multiplication of loaves and to its commentary by Christ in Joh 6; the word "door" alludes to the allegory of the shepherd (Joh 10), etc.; The designation of the cross as "Jesus" or "Christ" implies that the Logos that reveals himself is not limited to a form or a determined name. The Logos' apparition and determination as Jesus is just one manifestation out of many possibilities.

makes itself a reality where, effectively, many things are to be seen and named and told, whereas the cross itself remains unique. To quote again the *Acts of John* (98) as a summing up:

> He showed me a cross, dense with light, around which a large crowd had gathered, but not a single face could be discerned. But in the cross, there was only one face and one resemblance.

This dynamic of multiple names, forms and images reveals a metaphysical and epistemological insufficiency of human hearers (and seers!) and leads to a specific, notably liturgical, pedagogy.[76] In this human *Unterwelt*, nothing is *totum simul*; the Zillisean technique of *multiplying images* expresses at best this insufficiency and the *necessity* of this symbolic arithmetic game of the one (the structure, notably the "designed" cross) and the many (the painted panels). Such pedagogy is clearly exposed in the Valentinian collection called the *Gospel of Philip*:

> Names given to worldly things are very deceptive, since they turn the heart aside from the real to the unreal. And whoever hears the word "god" thinks not of the reality, but has been thinking of what is not real; so also, with the words "father", "son", "holy spirit", "life", "light", "resurrection", "church" [...] Only one name is not uttered in the world, the name that the father bestowed on the son; it is above every other—that is, the name of the father. For son would not become father had he not put on the name of the father. Those who possess this name think it but do not speak it. Those who do not possess it do not think it. Yet for our sakes truth engendered names in the world—truth, to which one cannot refer without names. Truth is unitary, ⟨...⟩ is multiple and it is for our sakes that ⟨it⟩ lovingly refers to this one thing by means of multiplicity. [...]
> Truth did not come to the world nakedly; rather, it came in prototypes and images.[77]

6. The celestial cross and its function as odd-number Eon

The description of the motifs and qualities of (or on) the cross in the *Acts of John* probably reflects the debates and mythological (theogonic and cosmogonic) speculations that occupied the thoughts of many second century theologians, especially in the so-called Gnostic[78]

[76] See the liturgical dance and litany in the *Acts of John*, 94-96!

[77] *Nag Hammadi Codices*, II, 3, p. 53, 23–54, 17; 67, 9–19 (translation from B. Layton, *The Gnostic Scriptures*, London: SCM, 1987, pp. 330 and 341). On the different forms of the Saviour that adapts himself to the various spiritual levels of his disciples, cf. ibid., pp. 57, 28–58, 9. On the *Gospel of Philip*, see especially M.L. Turner, *The Gospel according to Philip. The Source and Coherence of an Early Christian Collection*, Brill, Leiden, 1996.

[78] On this term and its particular endemic problems of definition, see the following studies: U. Bianchi (ed.), *The Origins of Gnosticism (Colloquium of Messina, 13–18 April 1966)*, Studies in the History of Religion 12, Brill, Leiden, 1967; M. Smith, "The History of the Term Gnostikos", in B. Layton (ed.), *The Rediscovery of Gnosticism. Proceedings of the International Conference on Gnosticism at Yale New Haven, Connecticut, March 28–31, 1978. Volume Two: Sethian Gnosticism*, Brill, Leiden, 1981, pp. 796–807; Kurt Rudolph, *Gnosis: The Nature and History of Gnosticism*, T&T Clark, Edinburgh, 1986; M. Tardieu, "Histoire du mot "gnostique" ", in: M. Tardieu and J.D. Dubois, *Introduction à la littérature gnostique* I, Cerf/CNRS, Paris, 1986, pp. 21–37; M.J. Edwards, "Neglected Texts in the Study of Gnosticism", *Journal of Theological Studies* 41/1 (1990), pp. 26–50; C. Markschies, *Valentinus gnosticus?, Untersuchungen zur valentinianischen Gnosis mit einem Kommentar zu der Fragmenten Valentins*, Tübingen: Mohr (WUNT I.65), 1992; *id.*, "Valentinian Gnosticism: Toward the Anatomy of a School", in: J.D. Turner, A. McGuire (eds.), *The Nag Hammadi Library after Fifty Years. Proceedings of the 1995 Society of Biblical Literature Commemoration*, Brill, Leiden, 1997, pp. 401–438.; A.H.B. Logan, *Gnostic Truth and Christian Heresy. A Study in the History of Gnosticism*, T&T Clark, Edinburgh, 1996; M.A. Williams, *Rethink-*

schools of thought. Indeed, in the first volume of his ambitious *Against Heresies*, Ireneus of Lyon dedicates a lengthy critique of Ptolemy's Gnostic speculations.

According to Ptolemy,[79] the double-named power called "Cross" and "Limit"[80] is a unique power, independent and separate from the thirty "Eons" of Plerome (the Valentinian divine world). Whereas the Eons all appeared as "zyzygies" or twin-numbered even numbers, the "Cross" is a sort of thirty-first Eon, a surplus Eon without peer (asuzugon) and is thus the very image (en eikoni idiā) of the First Principle or "Proto-Father" from which come all other Eons.

The Cross' double function determining its double name[81] is indicative of two things: firstly, its structural function, its internal consolidation of the unity of the Plerome (as Cross) as celestial framework; secondly, (as Limit) that of delimitation and of separation of Pleromatic realities from the reality that has become heterogeneous to the Plerome as a consequence of the "passion" of the thirtieth Eon called *Sophia* (Wisdom)[82] who has committed a sort of celestial original sin that resulted in the creation of the world below and the redemptive economy of Christ.[83]

This Plerome's heterogeneous reality issued from *Sophia*[84] as an *Ersatz,* rejected by the Cross/Limit and called *Enthumesis* or *Achamoth* ("weak and feminine fruit"), is declared to be "without form or figure" or an "abortion" (amorphos kai aneidos; ektrôma).[85] This heterogeneous entity is totally outside of the game, outside of the numbers game, outside of the structure; it does not have a number since it belongs, like the demiurge that comes from it,[86] to the world outside of Plerome, a world that has been created by its liquid humours (from its laughter, tears, sweat... and urine!);[87] it belongs to the undetermined multiplicity ...

ing "Gnosticism". An Argument for Dismantling a Dubious Category, Princeton University Press, Princeton, 1996; R. van den Brœk, "Gnosticism and Hermetism in Antiquity: Two Roads to Salvation", in: *id., Studies in Gnosticism & Alexandrian Christianity*, Brill, Leiden, 1996, pp. 3–21; R. Roukema, *Gnosis and Faith in Early Christianity. An Introduction to Gnosticism*, London: SCM, 1999 (Dutch original title: *Gnosis en geloof in het vroege christendom. Een inleiding tot de gnostiek*, Uitgeverij Meinema, Zoetermeer, 1998); M.G. Lancelloti, *The Naassenes. A Gnostic Identity among Judaism, Christianity, Classical and Ancient Near Eastern Traditions*, Ugarit Verlag, Münster (FARG—Forschungen zur Anthropologie und Religionsgeschichte 35), 2000. Finally, see especially K.L. King, *What is Gnosticism*, The Belknap Press of Harvard University Press, Cambridge, MA, 2003, which presents a remarkable, critical, update of the gnostic research and, historiographically more interesting, of the researchers and scholars treating these gnostic traditions!

[79] Cf. *Adversus Haereses*, I, 2, 2; I, 2, 4; I, 3, 5; I, 4, 1.

[80] This is a pun playing on the words stauros and horos.

[81] *Adversus Haereses*, I, 2, 4; I, 3, 5; as all of Plerome Eons (I, 2, 6), the Cross/Limit Eon is polyonymous and polymorphous as it measures and contains in itself all Eons.

[82] Cf. *Adversus Haereses*, I, 2, 2; this heterogeneous reality (indeed these heterogeneous realities) is called Enthymesis and, later, "Achamoth".

[83] This Christological economy happens first in the interior of the Plerome (*Adversus Haereses*, I, 2, 5).

[84] Sophia itself is finally reintegrated to the Plerome.

[85] Cf. *Adversus Haereses*, I, 2, 4; I, 4, 1.

[86] Cf. Irénée, *Adversus Haereses*, I, 5, 1ss.

[87] Cf. Irénée, *Adversus Haereses*, I, 4, 4.

CHAPTER 13

Swester Katrei and Gregory of Rimini: Angels, God, and Mathematics in the Fourteenth Century

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Edited by T. Koetsier and L. Bergmans

> "Tell me, daughter", he said, "doctors declare that in heaven a thousand angels can stand on the point of a needle (*tusent selen siczen jn dem himelrich uff einer nadel spicz*). Now rede me the meaning of this".
> She answered, "The doctors are right. You can see it in this way. The soul that enters into God owns neither time nor space nor anything nameable to be expressed in words. But it stands to reason, if you consider it, that the space occupied by any soul is vastly greater than heaven and earth and God's entire creation. I say more: God might make heavens and earths galore yet these, together with the multiplicity of creatures he has already made, would be of less extent than a single needle-tip compared with the standpoint of a soul atoned in God".

So spoke "Swester Katrei, Meister Eckhart's Strasburg Daughter" in a fourteenth-century German mystical work belonging to the tradition of the Brothers and Sisters of the Free Spirit.[1]

1. How many angels can dance on the point of a needle?

Nearly everyone has heard it said that scholastic theologians disputed how many angels can dance on the point of a needle (or, more frequently, on the head of a pin). In his 1791 *Curiosities of Literature*, under the heading "Quodlibets, or Scholastic Disquisitions", Isaac D'Israeli wrote:

> The reader desirous of being *merry* with Aquinas's angels may find them in Martinus Scriblerus, in Ch. VII who inquires if angels pass from one extreme to another without going through the *middle*? And if angels know things more clearly in a morning? How many angels can dance on the point of a very fine needle, without jostling one another?[2]

In their pseudonymous *Memoirs of the Extraordinary Life, Works, and Discoveries of Martinus Scriblerus*, written sometime before 1714, Pope, Arbuthnot, and Swift say nothing about angels dancing on the point of a needle, although they do include the first two questions D'Israeli lists, which are, in fact, very similar to questions found in Thomas Aquinas's *Summa Theologiae*, Part I, Questions 53 and 56.[3]

Moreover, despite extensive searches, no scholars up to the present have found any medieval quodlibet or other scholastic theological work containing a disputation asking how many angels can dance on the point of a needle. "Swester Katrei" is the only currently known medieval work in which even angels standing, or perhaps "perching" like birds (*siczen*), on points of needles is so much as mentioned. How did Swester Katrei's thousand angels standing on the point of a needle begin to dance and when did their numbers become uncertain?

[1] Swester Katrei, as in Franz Pfeiffer (ed.), *Deutsche Mystiker*, vol. 2, Leipzig, 1857, reprinted: Scientia, Aalen, 1962, p. 474; translated in Franz Pfeiffer, *Meister Eckhart*, trans. C. de B. Evans, John Watkins, London, 1924, Sister Katrei. Meister Eckhart's Strasburg Daughter, p. 333. George MacDonald Ross, Angels, *Philosophy* 60 (1985), 495, found a reference to this text in a letter to the editor of *The Times*, 26 November 1975. For the provenance of this text, which has sometimes been attributed to Meister Eckhart, see Franz-Josef Schweitzer, *Die Freiheitsbegriff der deutschen Mystik. Seine Beziehung zur Ketzerei der "Brüder und Schwestern vom Freien Geist", mit besonderer Rücksicht auf den pseudoeckartischen Traktat "Schwester Katrei"*, Peter D. Lang, Frankfurt am Main–Bern (1981), and, for an alternative perspective, Raoul Vaneigem, *The Movement of the Free Spirit*, trans. Randall Cherry and Ian Patterson, Zone Books, New York, 1994, pp. 149–152.

[2] Ross, Angels, p. 495.

[3] Ross, Angels, p. 496.

Fig. 1. White Angel at Christ's sepulchre. 13th century. Mileševa monastery, Serbia.

Fig. 2. Archangel Gabriel (marble relief), School of Benedetto Antelami, late twelfth–early thirteenth century, Princeton University Art Museum (Photo: Princeton University Art Museum).

In his 1678 *True Intellectual System of the Universe*, Ralph Cudworth wrote:

> And to conclude, though some who are far from Atheists, may make themselves merry, with that *Conceit*, of *Thousands of Spirits, dancing at once upon a Needles Point*, and though the *Atheists*, may endeavour, to *Rogue* and *Ridicule*, all *Incorporeal Substance* in that manner; yet does this run upon a clear Mistake of the Hypothesis, and make nothing at all against it; for as much as an *Unextended* Substance is neither any *Parvitude*, as is here supposed (because it hath no *Magnitude* at all) nor hath it any *Place*, or *Site*, or *Local Motion*, properly belonging to it; and therefore can neither Dance upon a Needles Point, nor any where else.[4]

Forty years before Cudworth, William Chillingworth, in his *Religion of the Protestants* of 1638, had defended Anglican clergy against a Jesuit critic, saying that they might be learned even though they "dispute not eternally, *Utrum chimera bombinans in vacuo possit comedere secundas intentiones*? 'Whether a million of angels may not sit upon a needle's point?' "[5] Chillingworth's sitting angels might have derived ultimately from a source like Swester Katrei.

[4]R. Cudworth, *The True Intellectual System of the Universe*: *The First Part*: *Wherein, All the Reason and Philosophy of Atheism is Confuted*; *and Its Impossibility Demonstrated*, Richard Royston, London, 1678, Book I, Ch. 5, pp. 776–777. Reprinted Stuttgart-Bad Cannstatt: Friedrich Frommann Verlag/Gunther Holzboog, 1964. See also: Meyer, Jena, 1733, pp. 1018–1019. In saying that incorporeal substances such as angels do not fall into the category of things in place, Cudworth drew upon Plotinus and other ancient authors (just before the text already quoted): "By this time we have made it unquestionably Evident, that this Opinion of *Incorporeal Substance* being *Unextended, Indistant, and Devoid of Magnitude*, is no Novel or Recent thing, nor first started in the *Scholastik Age*, but that it was the general Perswasion, of the most ancient and learned Asserters of *Incorporeal Substance*; especially, that the *Deity* was not Part of it *Here*, and Part of it There, nor the *Substance* thereof *Mensurable* by Yards and Poles, as if there were so much of it contained in one Room, and so much and not more in another, according to their several Dimensions; but that the whole *Undivided Deity*, was at once in Every *Part* of the world, and consequently No where *Locally* after the manner of *Bodies*. But because this opinion, seems so *Strange* and *Paradoxical*, and lies under so great *Prejudices*, we shall in the next place show, how these ancient *Incorporealists*, endeavoured to acquit themselves in repelling the several *Efforts* and *Plausibilities* made against it. The First whereof is this, That to suppose *Incorporeal Substances, Unextended* and *Indivisible*, is to make them *Absolute Parvitudes*, and by means of that, to render them all (even the *Deity* itself!) contemptible; since they must of necessity, be either Physical *Minimums*, that cannot *Actually* be *Divided* further by reason of their *Littleness* (if there be any such thing) or else meer *Mathematical Points*, which are not so much as *Mentally Divisible*: so that *Thousands* of these *Incorporeal Substances*, or *Spirits*, might *Dance together at once upon a Needles Point*. To which it was long since thus Replied by *Plotinus* . . . *God and all other Incorporeal Substances, are not so Indivisible, as if they were Parvitudes, or Little things, as Physical points*; *for so would they still be Mathematically Divisible*; *nor yet, as if they were Mathematical Points neither, which indeed are no Bodies nor Substances, but only the* Termini *of a Line. And neither of these ways, could the Deity* Congruere *with the world*; *nor Souls with their respective Bodies, so as to be all present with the whole of them*".

[5]William Chillingworth, *The Works of W. Chillingworth, M.S., containing his book, intituled* The Religion of the Protestants a Safe Way to Salvation; *together with his Sermons, Letters, Discourses, Controversies, &c. &c.*, 12th ed., London, 1940, p. 12. I owe this reference to Marty Helgesen at a web newsgroup found at http://geneva.rutgers.edu/src/faq/angels-dancing.txt, who quotes Arnold Lunn, *The Revolt Against Reason*, London: Eyre and Spottiswoode, 1950, p. 231, who quotes James Brodrick, in *The Tablet*, October 10, 1942, who was the person who noticed the passage in Chillingworth. The pseudo-scholastic question "Utrum chimera bombinans in vacuo possit comedere secundas intentiones?" comes from Rabelais' life of Gargantua, and is later repeated in Voltaire's *Philosophical Dictionary*, under "authority" and "atheist" (with reference to Vanini "a poor Neapolitan priest . . . a merciless argufier about quidities and universals, et utrum chimera bombinans in vacuo possit comedere secundas intentiones. But for the rest, there was not a drop of atheism in him"). According to Brodrick, Ebenezer Brewer, in *Dictionary of Phrase and Fable*, had said that Thomas Aquinas had "discussed the knotty point of how many angels can dance on the point of a needle", adding that what St. Thomas discussed in fact was "more strictly speaking, *Utrum angelus possit moveri de extremo ad extremum non transeundo per medium*," [i.e. can an angel

Cudworth's atheists, wanting to "rogue and ridicule all incorporeal substance" could have been someone like Thomas Hobbes. In his *Leviathan* (1651) Hobbes called the Papacy the "Kingdom of the Fairies".[6] The Catholic Church erred, Hobbes said, "by mixing with the Scripture divers reliques of the Religion, and much of the vain and erroneous Philosophy of the Greeks, especially of Aristotle".[7] Having decided that there can be "separated essences" or substances without matter, the scholastics were faced, he said, with the problem of the place of such entities:

> seeing that they will have these Forms to be reall, they are obliged to assign them some place. But because they hold them Incorporeall, without all dimension of Quantity, and all men know that Place is Dimension, and not to be filled, but by that which is Corporeall; they are driven to uphold their credit with a distinction, that they are not indeed any where *Circumscriptive*, but *Definitive*: Which Terms being meer Words, and in this occasion insignificant, passe only in Latine, that the vanity of them may bee concealed.
> Again, whereas Motion is change of Place, and Incorporeall Substances are not capable of Place, they are troubled to make it seem possible, how a Soule can goe hence, without the Body to Heaven, Hell, or Purgatory; and how the Ghosts of men (and I may adde of their clothes which they appear in) can walk by night in Churches, Churchyards, and other places of Sepulture. To which I know not what they can answer, unlesse they will say, they walke *definitive*, not *circumscriptive*, or *spiritually*, not *temporally*: for such egregious distinctions are equally applicable to any difficulty whatsoever.[8]

In so arguing, Hobbes may have spoken for intelligent lay people against the technical terminology of theologians, but in scholastic theology, the distinction between "circumscriptive" and "definitive" ways of being in place was well-established and not "insignificant" as Hobbes charged. In scholastic terms, corporeal substances or material bodies are in place "circumscriptively", in the sense that one part of them is in one part of the place and another part of them in another place. But the soul is in the body "definitively", i.e. wholly in every part: if a leg is amputated, one does not lose part of one's soul. Similarly, God is not in place circumscriptively, but omnipresent, wholly in every part of the universe.[9] Angels, which have no matter, are, like the soul, in place definitively: when they act on earth, they can be present to a larger or smaller, but always finite, volume, in such

move from one extreme to the other without passing through the middle] on which Brodrick remarked "now the 'more strictly speaking' of this emendation is a joyful thing, for it is as though one were to say: 'The man had red hair, or, more strictly speaking, his wife was wearing a new hat' " (Lunn, p. 231).

[6]Thomas Hobbes, *Leviathan*, ed. C.B. MacPherson, Penguin Books, London, 1968, reprinted 1985, p. 712.

[7]Cf. Hobbes, pp. 628–629.

[8]Cf. Hobbes, *Leviathan*, pp. 691–693.

[9]God is in space neither circumscriptively nor definitively, because God is not in any defined space, but everywhere. In the fourteenth century, theologian natural philosophers like Thomas Bradwardine and Nicole Oresme emphasized God's ubiquity outside the cosmos as well as inside, but said that, being wholly everywhere, God is present only in a non-extended sense. Isaac Newton would later come up with the parallel analogy that time is wholly the same in every part of space. See Edith Sylla, Imaginary Space: John Dumbleton and Isaac Newton, in: Jan Aertsen and Andreas Speer (eds.), *Raum und Raumvorstellungen im Mittelalter*, Miscellanea Mediaevalia 25, Walter de Gruyter, Berlin, 1998, p. 222. In Nicole Oresme's terms, the space outside the cosmos "is infinite and indivisible, and is the immensity of God and God Himself". Cf. E. Sylla, Imaginary Space: John Dumbleton and Isaac Newton, pp. 214–215. Just as God is eternal without temporal extension before the creation of the world and time, so God is ubiquitous without spatial extension outside the cosmos. If God were to annihilate everything inside the sphere of the moon, so John Buridan argued, what would be left inside would have no dimension, so that God could put huge new bodies into places that previously contained much less. See Edith Sylla, 'Ideo quasi mendicare oportet intellectum humanum': The Role of Theology in John Buridan's Natural Philosophy,

a way that, like the rational soul in the body, they are wholly everywhere within the given volume.[10]

When Swester Katrei said that the soul that enters God occupies a space vastly greater than God's entire creation, this was the sort of extravagant claim about the soul that might cause the Catholic Church to declare the Brothers and Sisters of the Free Spirit heretics, but it was a claim not unrelated to orthodox scholastic conceptions. Thus the Jesuit Franciscus Suarez, in a treatise on angels based on Thomas Aquinas's *Summa Theologiae* extending to more than 700 folios, discussed whether an angel may occupy a mathematical point or whether angels always occupy some volume which may be vanishingly small, but never zero.[11] In his discussion, Suarez used an additional concept of being in place, namely the concept of "*ubi*" or "where" derived from Aristotle's categories, which might be compared to location in a scheme of Cartesian coordinates, with the added connotation that the substance in question has a sort of intrinsic quality expressing its *ubi*.[12] If Thomas Hobbes

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in: J.M.M.H. Thijssen and J. Zupko (eds.), *The Metaphysics and Natural Philosophy of John Buridan*, E.J. Brill, Leiden–Boston–Köln, 2001, pp. 241–242. When Isaac Newton built upon such theological speculations to use God to "constitute" absolute space, he drew upon Platonic solids and something like Neoplatonic emanation from the One to convince his readers that empty space might have intrinsic dimension or extension without matter. E. Sylla, Imaginary Space, pp. 218–225.

[10] Perhaps the passion with which Hobbes and others attacked the idea of definitive place is to be explained by the fact that definitive place was sometimes used to explain the mode of being of Christ's body in the transubstantiated Eucharist. Christ's body is totally in every part of the Eucharist, so that when the bread is broken, Christ's body is not broken. One way of attacking the doctrine of transubstantiation was to attack the ways in which theologians had elaborated the doctrine.

[11] Franciscus Suarez, *De Angelis* in *Opera Omnia*, 20 vols., pp. 1613–1625. *Pars Secunda Summae Theologiae de Deo rerum omnium creatore. In tres praecipuos tractatus distributa, quorum primus De angelis*, Sumptibus Iacobi Cardan et Petri Cavellat, Lyon, 1620. Folio volume of 746 pages plus index. Later edition Vives, 1956. Suarez divides his work on angels, which follows the pattern established by Aquinas, into parts, the first half concerning what can be said about angels from a natural philosophical or metaphysical point of view (from which point of view the beings are usually called intelligences rather than angels), and the second half concerning what can be said about angels from the standpoint of grace. From the natural standpoint, Suarez's work is divided into four books considering in turn: Book 1, On the nature of angels, their production and attributes; Book 2, On the intellective potency of angels and their natural cognition; Book 3, On the will of angels with regard to their pure state of nature; Book 4, On the power of angels to effect transient actions. In Book 4, Chapters 9–11, Suarez concludes that the place of angels is not "omnino punctualis", but later he seems to say that an angel can be at a point and cites many authors including Thomas Aquinas in agreement. Suarez reports that the Thomists say there cannot be many angels in the same place (angels are in place by activity; there can only be one cause of one effect), while the Scotists say there can be many angels in one place. The maximum place of an angel is spherical. An angel cannot extend itself infinitely far by constricting its width—i.e. there is a maximum dimension in any direction as well as maximum volume it can have. Question 11 is Utrum angelus necessario sit in loco adequato, vel possit esse in quocunque minori, aut etiam in nullo. Suarez says angels are "extra genus quantitatis". God is indivisible, yet everywhere. Whether the minimum angelic size is intrinsic or extrinsic is, Suarez says, a philosophical (not a theological) question. The mode of presence of an angel is absolute. It depends on the will of the angel. An angel could be present in an imaginary space—it doesn't require a body. The many demons in a single man may interpenetrate—it would be absurd to suppose that they have to stay in separate places. The host of angels in the Christmas story need not be crowded because they can interpenetrate. No particular shape is necessary for angels "nihilominus satis probabiliter credo locum eius adequatum secundum circularem figuram designari, et limitari".

[12] Joseph Glanvill in talking about angels dancing on points of needles complains particularly about the concept of "*ubi*": "That the *Soul* and *Angels* are devoid of *quantitative dimensions*, hath the suffrage of the most; and that they have nothing to do with grosser *locality*, is as generally opinion'd: But who is it, that retains not a great part

had used the phrase "dispute how many angels can dance on the point of a needle", he might well have had his contemporary Suarez in mind: if any scholastic ever came close to disputing how many angels can dance on the point of a needle, it was Suarez, and Hobbes had many political as well as intellectual reasons for opposing Suarez.

Additional circumstantial evidence that Hobbes could have originated the satire derives from the fact that, after Chillingworth, the earliest mention of *disputations* about angels *dancing* on needles currently identified is in Henry More's 1659 *The Immortality of the Soul*, where he argues that the soul is extended and says that he thinks this is:

> worth taking notice of, that it may stop the mouths of them that, not without reason, laugh at those unconceivable and ridiculous fancies of the Schools: that first rashly take away all Extension from Spirits, whether Souls or Angels, and then dispute how many of them booted and spurr'd may dance on a needles point at once.[13]

Here More is writing about those who ridicule the Schools and it is only in the words of these satirists that the scholastics dispute how many angels "booted and spurr'd" (compare Hobbes' "their clothes in which they appear") may dance on a needle's point. In his defense of the reality of spirit, More argued against materialists like Hobbes. Thus I am persuaded that, if not Hobbes, it was some mid-seventeenth-century author very like him who first conflated for the sake of ridicule a mystical tradition which said that a thousand angels can stand or sit on the point of a needle and scholastic disputations involving the definitive place or *ubi* of angels. In sum, the most famous (supposedly) medieval connection between mathematics and the divine—the dispute how many angels can dance on the point of a needle—was almost certainly a seventeenth-century invention.

2. The place of angels: Gregory of Rimini

This said, it must be conceded that the scholastics did engage in disputations concerning the relation of angels to points, but I would like to argue here that what they had to say did not deserve the dismissal to which later satirists consigned it, not the least because, within their disputations, they had novel and progressive things to say about the mathematics of continuity and infinity. In the rest of this paper, I focus on the mid-fourteenth-century *Lectura* of Gregory of Rimini on Peter Lombard's *Book of Sentences* to show what the relation of mathematics and the divine in scholastic theology was really like.[14] I will begin with Gregory's questions concerning the place of angels and then show how Gregory's

of the imposture, by allowing them a *definitive Ubi*, which is still but *Imagination*? He that said, a *thousand* might dance on the *point of a Needle*, spake but grossly; and we may as well suppose them to have *wings*, as a proper *Ubi*". Joseph Glanvill, *The Vanity of Dogmatizing* (1661), p. 100. A revised version of this work was published in 1665 under the title *Scepsis Scientifica*, and with a dedication to the Royal Society. It was published again in 1676 under the title *Essays Against Confidence in Philosophy.* Reprint of all three works together with a critical introduction by Stephen Medcalf, The Harvester Press, Hove, Sussex, 1970. Note that, like Cudworth, Glanvill does not imply that there were scholastic disputations about angels dancing on the point of a needle.

[13] Henry More, *The Immortality of the Soul*, ed. Alexander Jacob, *Archives Internationales D'Histoire des Idées*, vol. 122, Martinus Nijhoff Publishers, Dordrecht/Boston/Lancaster, 1987, Book III, Chapter II, pp. 198–199.

[14] Gregory of Rimini, *Lectura super Primum et Secundum Sententiarum*, vol. 3, *Super Primum* (*Dist* 19–48), ed. A. Damasus Trapp and Venicio Marcolino, Walter de Gruyter, Berlin and New York, 1984; vol. 4, *Super Secundum* (*Dist* 1–5), ed. A. Damasus Trapp, Walter de Gruyter, Berlin and New York, 1979.

conception of God's knowledge of the world impacted what he had to say about infinity and continuity in progressive ways.

In Book II, distinction 2 of his *Lectura* on the *Sentences*, Gregory of Rimini had three questions on angels. First he asked whether the angels were created together with time, before time, or after time; second whether an angel may be in an indivisible or a divisible place; and third whether an angel can be in several places at the same time.[15] The second and third questions exemplify what the scholastics really did dispute concerning the place of angels. On the second question, having given a very brief principal argument, Gregory then devoted Article 1 to an extensive treatment of the continuum as a necessary background to the main question, considering whether a magnitude may be composed of indivisibles and whether there may be something indivisible within a continuum.[16] He concluded that no magnitude is composed of indivisibles; rather any magnitude is composed of magnitudes. Second any magnitude is composed of infinitely many magnitudes and, third, in no magnitude is there something indivisible intrinsic to it.[17] Gregory's arguments for these conclusions are a *tour de force*: he clears up several confusions about continuity in earlier authors, and he argues for the possibility of an actual infinity.[18] I will come back to Article 1 and its treatment of infinity and continuity below. Here I want simply to note that in the recent edition of Gregory's *Lectura*, Article 1 with its discussion of continuity and infinity takes up 53 pages (pp. 278–331), and only then does Gregory come to the question at issue concerning the place of angels:

> The second article, which, as is clear, concerns the main question, supposes that an angel is in a place. On this there is a common agreement among theologians, doctors, and saints. They all say in common that an angel is in place, not dimensively or circumscriptively, to use the modern terminology, but only definitively.[19]

There is, however, a difference of opinion about how an angel comes to be in place. Thomas Aquinas and Aegidius Romanus say that an angel is only in place by its operation.

[15]Gregory of Rimini, *Lectura* II, dist. 2, vol. 4, pp. 277–343, "Quaestio 1 additionalis: Utrum angeli fuerint creati simul cum tempore, an ante tempus, vel post". "Quaestio 2. Utrum angelus sit in loco indivisibili aut indivisibili". "Quaestio 3: Utrum angelus possit simul esse in pluribus locis".

[16]*Lectura* II, dist. 2, q. 2, art. 1, p. 278, "an magnitudo componatur ex indivisibilibus … et utrum in magnitudine sit aliquid indivisibile".

[17]Ibid., "nulla magnitudo componitur ex indivisibilibus … quaelibet magnitudo componitur ex magnitudinibus. Secunda quod quaelibet magnitudo componitur ex infinitis magnitudinibus. Tertia, quod in nulla magnitudine est aliquid indivisibile".

[18]See Anneliese Maier, *Die Vorläufer Galileis im 14. Jahrhundert: Studien zur Naturphilosophie der Spätscholastik*, 2nd ed., Edizioni di Storia e Letteratura, Rome, 1966, pp. 172–177; Eadem, Diskussionen über das aktuell Unendliche in der ersten Hälfte des 14. Jahrhunderts, in: Eadem, *Ausgehendes Mittelalter*, vol. 1, Edizioni di Storia e Letteratura, Rome, 1964, pp. 82–84; John Murdoch, "Infinity and Continuity", in: Norman Kretzmann, Anthony Kenny, and Jan Pinborg (eds.), *The Cambridge History of Later Medieval Philosophy*, Cambridge University Press, Cambridge, 1982, pp. 572–573; Richard Cross, Infinity, Continuity, and Composition: The Contribution of Gregory of Rimini, *Medieval Philosophy and Theology* 7 (1998), 89–110. It should be noted that Maier was mistaken in thinking that Gregory believed in infinitesimals. Cf. Cross, Infinity, Continuity, and Composition, p. 103, n. 58.

[19]*Lectura* II, dist. 2, q. 2, art. 2, p. 331, "Secundus articulus, qui est de principali quaesito, ut patet, supponit angelum esse in loco. De quo communis concordia est apud theologos, doctores et sanctos. Dicunt enim omnes communiter quod angelus est in loco, non quidem dimensive, seu circumscriptive iuxta moderniorem usum vocabuli, sed tantummodo definitive".

Article 203 of the Condemnations of 1277 opposes this view, however. Gregory concludes that angels are in place not only by operation, but through their substance.[20] Angels are not, he says, properly speaking in an indivisible place, but rather in a divisible place. This is so because there are no indivisibles in nature, as he had proved previously.[21]

Only after replying to the main question does Gregory of Rimini raise a doubt that might seem to be scholastic quibbling about angels and pins or needles. He asks whether an angel fills a determinate volume or whether angels may vary in size down to some minimum or up to some maximum.[22] There was general agreement among the scholastics that angels have a maximum possible size, because they are of a finite power.[23] Gregory agreed with this and added that an angel has a maximum extension in any dimension: it would not be possible for the given volume filled by an angel to become so extremely narrow that it could stretch, say, from heaven through the spheres of fire and air to earth. Richard Fitzralph (called Hibernicus) had argued that angels fill spheres of a certain size, so that if an angel were united to a very small body, the angel might extend into the air beyond the body. Gregory thought that this might be the case, but that it was more probable that an angel's volume could be constricted or shaped so that it did not overflow the body to which it was united; he thought it unreasonable to think that if an angel were united to a human form, that it would overflow into the surrounding air.[24] Moreover, in the case of humans we see that the volume occupied by the soul changes as the person changes size, even sometimes when a limb is amputated.[25]

Since angels are in place definitively, they are for Gregory in every part of any place they occupy, however small and of whatever shape. An angel could be in a whole house and, as such, in the stones, wood, iron, etc., that make it up.[26] The real question is whether the whole place of a given angel has a minimum or whether an angel could even be at a point. Some people, for instance John Duns Scotus, to avoid the possibility of the very long and very thin place previously mentioned, proposed that there was a minimum dimension

[20] Ibid., p. 335, "Dico igitur quod non solum per operationem angelus est in loco, sed etiam per substantiam suam, sic intelligendo quod eius etiam substantia praesens est loco et eo definitur et concluditur; quod etiam contingit, etiamsi non ibi operetur".

[21] Ibid., p. 335, "Quantum ad illud quod principaliter quaeritur, dico quod angelus non est in loco indivisibili proprie loquendo de loco scilicet corporali, sed est in loco divisibili. Ratio est, quia nullum tale indivisibile est in rerum natura, ut probatum est in primo articulo".

[22] Ibid., p. 335, "Sed tunc remanet dubium, an angelus in quantocumque loco possit naturaliter esse, vel sibi determinet magnitudinem loci, cui coexistere possit. Et si sic, est dubium, an sic determinet sibi magnitudinem aliquantam loci, quod nec in maiore possit esse naturaliter nec in minore, quamvis nullum determinatum locum secundum numerum sibi determinet ...".

[23] Ibid., p. 336.

[24] Ibid., p. 337, "Et determinat etiam sibi sphaericam figuram ita quod, si corpus aliquod sit minus illa quantitate, angelus non solum erit in eo, sed etiam erit in alio extra ipsum propinquo, cuius tantae parti coexistet, ut habeat integre sui loci quantitatem ... Quamvis autem istud dictum sit satis pulchrum et probabile, communius tamen tenetur oppositum, quod et mihi videtur etiam probabilius ... Confirmatur quia, cum angelus assumit sibi aliquod corpus in forma verbi gratia humana, non videtur rationabile quod tunc non solum sit in corpore illo, sed etiam in aere circumstante ...".

[25] Ibid., p. 337, "Ad hoc etiam aliqualiter persuadet quod videmus in anima humana, cuius unio et conformitas ad corpus est multo maior quam angeli ad locum, et tamen nunc minus nunc maius corpus informat secundum quod illud augetur vel minuitur naturaliter, et aliquando etiam per truncationem membri aut partis".

[26] Ibid., p. 337.

that an angel could occupy. Gregory, however, did not find Scotus' arguments for this position convincing. Moreover, Scotus also conceded that an angel could be at a point, which, Gregory suggested, was inconsistent with arguing for a minimum size in any given dimension.[27] Gregory himself concluded that neither position about minimum sizes of angels was impossible and neither was necessary. Each person might chose the position they preferred to defend, he said, so long as they were not too dogmatic, since this was a matter hidden from human comprehension, and the scriptures say nothing clear about it.[28]

In Question 3, Gregory then asked whether an angel could be in several places and, within this discussion, whether several angels can be in the same place. Thomas Aquinas had argued that two angels cannot be in the same place because an angel only comes to be in place by its operation. The net effect produced by such an operation must have a single cause, so that there cannot be two angels with different operations producing the same net effect. Since Gregory had argued that angels are in place by their substance and not only by their operation, this argument did not hold for him. Moreover, even if angels *were* in place by operation, he said, since they act freely, they could cooperate, each providing part of the cause of the net effect.[29] By God's power, even two bodies can be in the same place, so certainly two angels can be in the same place. Even if the question is whether two angels could be in the same place naturally, without a miracle, Gregory (agreeing with Duns Scotus) concluded that they probably can be. After all, matter and form are in the same place, so why not two angels, which are only in place definitively? Moreover, we see in obsessed people that they are simultaneously informed by their own souls and by the demon or spirit that possesses them.[30]

It is worth noting that the whole of Gregory's Question 3 takes up less than four pages in the recent edition, while the conclusions about angels at the end of Question 2 fill only a little more than three pages. This is not the sort of discussion that cries out for satire, but might be compared to modern discussions of the position of photons.[31] Gregory assumes the truths of natural philosophy and mathematics and has evidence about angels from scripture. Putting his information together, he gives reasonable answers to the ques-

[27] Ibid., p. 339, "Et mirum est de isto doctore, qui non reputat inconveniens, quod angelus possit esse in puncto, et videtur reputare inconveniens, quod in quantumcumque parvo possit esse . . .".

[28] Ibid., p. 339, "Dico igitur quod non video impossibilitatem alterius partis; nec tamen necessitatem. Eligat ergo quis quod velit, dummodo nihil temere affirmet. Nam haec materia abscondita est nobis, dum hic sumus. Et scriptura divina nihil tale nobis manifestat".

[29] Ibid., p. 342, "Ad primum . . . nego consequentiam. Cuius infirmitas patet: Tum quia supponit angelum non posse esse in loco nisi operetur circa illud, cuius oppositum in praecedenti quaestione probatum est; tum quia, dato quod operaretur, adhuc non sequitur quod essent duae causae totales, seu duo motores perfecti. Quoniam, cum ipsi libere moveant, posset quilibet movere citra ultimum suae potentiae, et citra illud quod requireretur ad movendum corpus illud, ita quod uterque esset partialis motor et ambo simul aequivalerent uni perfecto motori".

[30] Ibid., p. 342.

[31] See Matthew Fox and Rupert Sheldrake, *The Physics of Angels. Exploring the Realm Where Science and Spirit Meet*, Harper Collins, San Francisco, 1996, p. 21, "When Aquinas discusses how angels move from place to place, his reasoning has extraordinary parallels to both quantum and relativity theories. Angels are quantized; you get a whole angel or none at all . . . from the point of view of the angel this movement is instantaneous; no time elapses. This is just like Einstein's description of the movement of a photon of light . . . So in modern physics there are remarkable parallels to the traditional doctrines about angels, and I think the parallels arise because the same problems are being considered. How does something without mass, without body, but capable of action, move?"

tions about angels traditionally found in commentaries on the *Sentences*, in the course of which he cites previous authorities such as St. Augustine, Thomas Aquinas, Aegidius Romanus, John Duns Scotus, and others and agrees with or argues against them. Believing as other scholastics that angels really exist, Gregory tries to be as scientific as possible about them, admitting unavoidable uncertainty about some finer details.

3. God and the continuum: Gregory of Rimini

Now, however, I would like to discuss in more detail the 53 pages of mathematics and mathematical physics that formed Article 1 of Question 2. In this section, Gregory assumes an Aristotelian conception of the status of mathematics. For Aristotelians, quantities, or mathematical entities in general, have no existence separate from things or substances. They are features of things which mathematicians *consider* in abstraction from other attributes of things although they do not in reality *exist* separate from these other attributes.[32] If, in a Platonic conception, mathematics concerns mathematical forms that really exist in a realm separate from the physical world, in the Aristotelian conception of mathematics this is not the case. And not only do the objects of mathematics not exist separately, they do not in a strict sense even exist *in* objects.[33] Instead, they exist in the minds of mathematicians who talk about them using concepts or reasoning (*ex parte rationum sive conceptuum apud animam existentium*). Mathematics differs from metaphysics and physics not because it considers different entities, but because it considers the same entities under the aspect of quantity, whereas metaphysics considers them insofar as they are things, and physics considers

[32]Cf. Aristotle, *Metaphysics*, Book M (XIII), Ch. 1, 1076a33–37, "If the objects of mathematics exist, they must exist either in sensible objects, as some say, or separate from sensible objects (and this also is said by some); or if they exist in neither of these ways, either they do not exist, or they exist only in some special sense. So that the subject of our discussion will be not whether they exist but how they exist". Aristotle's conclusion (Ch. 3, 1077b11–17) is that the objects of mathematics exist neither *in* sensible objects, nor separately from sensible objects, but only in some special sense. Ch. 3, 1077b31–1078a30: "Thus since it is true to say without qualification that not only things which are separable but also things which are inseparable exist . . . It is true also to say without qualification that the objects of mathematics exist, and with the character ascribed to them by mathematicians. And as it is true to say of the other sciences too, without qualification, that they deal with such and such a subject . . . so too is it with geometry; if its subjects happen to be sensible, though it does not treat them *qua* sensible, the mathematical sciences will not for that reason be sciences of sensibles—nor, on the other hand, of other things separate from sensibles. Many properties attach to things in virtue of their own nature as possessed of each such character . . . so that there are also attributes which belong to things merely as lengths or as planes . . . Therefore, if we suppose attributes separated from their fellow-attributes and make any inquiry concerning them as such, we shall not for this reason be in error . . . Each question will be best investigated in this way—by setting up by an act of separation what is not separate, as the arithmetician and the geometer do. For a man *qua* man is one indivisible thing; and the arithmetician supposed one indivisible thing, and then considered whether any attribute belongs to a man *qua* indivisible. But the geometer treats him neither *qua* man nor *qua* indivisible, but as a solid. For evidently the properties which would have belonged to him even if perchance he had not been indivisible, can belong to him even apart from these attributes. Thus, then, geometers speak correctly; they talk about existing things, and their subjects do exist . . .".

[33]Gregory of Rimini, *Lectura*, vol. 3, In I *Sent*. dist. 24 q. 1, ad rationes principales, p. 33, "Ad secundam auctoritatem eiusdem dico quod opinio Commentatoris, ut videtur in verbis allegatis, et clarius adhuc in 10 *Metaphysicae* commento 8 fuit quod unum numerale significaret aliquod accidens inhaerens rei, quae dicitur una, sicut albus significat albedinem inhaerentem subiecto. Et in hoc ipse non fuit minus deceptus quam Avicenna, nec eius auctoritas in hac parte recipienda".

them insofar as they are mobile.[34] To fourteenth-century Aristotelians of a nominalistic bent, many of whom including Gregory of Rimini denied that mathematical indivisibles such as points, lines, planes, or instants really exist, mathematics might be considered a conditional science, saying that if such and such were the case, then this and that would follow.[35] When a theologian who denied the real existence of points or other indivisibles (a "non-entitist" in Rega Wood's terms[36]) engaged in debate with a theologian who thought

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[34]John Buridan, *Perutiles questiones in ultima eius lectura edite super duodecim libros Metaphysice*, Paris, 1518, ff. 33va–34ra, "Quaeritur secundo utrum philosophia speculativa bene dividatur in physicam, mathematicam, et metaphysicam ... Notandum est quod ista questio est satis difficilis, quia non est facile assignare unde proveniat sufficientia talis divisionis ... Et primo pono istam conclusionem: et ista divisio non sumitur ex distinctione rerum extra animam existentium, scilicet consideratorum in istis tribus scientiis, quoniam eedem res in istis tribus scientiis considerantur. De omnibus enim considerat metaphysicus. Et similiter etiam de omnibus considerat physicus, quia considerat terminos supponentes pro omnibus rebus ... Sed tunc sequitur alia conclusio: quod hec divisio provenit ex parte rationum sive conceptuum apud animam existentium, quia oportet eam provenire vel ex parte rerum extra vel ex parte rationum sive conceptuum. Tertia conclusio potest poni quod illa divisio non est sumenda ex dispositione conclusionum ... Alia conclusio ponitur quod etiam illa divisio non provenit ex distinctione principiorum complexorum ... Ideo finaliter concludo quod illa distinctio originaliter sumitur ex distinctione aliquorum principiorum incomplexorum. Cum enim debeat sumi hec distinctio ex parte rationum sive conceptuum, et tamen non ex parte complexorum, quia nec conclusionum nec principiorum, sequitur quod ex parte incomplexorum. Sed tunc est difficile assignare qui sunt illi termini et quare non ita bene sunt alii. Ad quod ego dico quod illi termini sunt 'ens', 'mobile' et 'quantum'. Omnia enim considerata in metaphysica considerantur secundum attributionem vel retributionem ad illum terminum 'ens' ... Deinde venio ad illum terminum 'quantum', et dico quod mathematicus non considerat que res est ipsum quantum nec que res est ipsa magnitudo vel numerus, scilicet utrum magnitudo sit substantia an non vel utrum numerus sit distinctus a rebus numeratis an non. De istis enim non curat mathematicus. Immo hec pertinent ad metaphysicum. Sed mathematicus considerat de rebus quante sunt, mensurando quantitates ignotas per quantitates notas, et sic habet considerare omnes proportiones quantorum adinvicem ... Unde mathematicus [*correxi ex:* metaphysicus] considerat de quanto abstrahendo secundum rationem non solum a motu, immo et ab ente, quia non curat de illis quid sunt nisi quantum ad quid nominis nec etiam a quibus causis sunt, de quo physica. Deinde consimiliter venio ad illum terminus 'mobile'...".

[35]William of Ockham, *Expositio in Libros Physicorum Aristotelis Bk. VI, Ch. I*, ed. R. Wood, R. Green, G. Gál, J. Giermek, F. Kelley, G. Leibold, and G. Etzkorn, St. Bonaventure University, St. Bonaventure, NY, 1985, *Opera Philosophica*, vol. V, pp. 461–462, "Et si dicas quod linea est longitudo sine latitudine, cuius extremitates sunt duo puncta, dicendum est quod mathematici imaginantur talia puncta ... Et iuxta illam imaginationem mathematicorum datur praedicta descriptio ... Sic ista definitio 'linea est longitudo' [etc.] debet sic intelligi: si esset aliqua longitudo et non latitudo et haberet puncta talia indivisibilia, tunc quaelibet linea haberet duas extremitates quae essent duo puncta ... Sic ergo dico quod ex principiis Aristotelis sequitur quod non sunt talia puncta indivisibilia copulantia et terminantia lineas, quamvis mathematici imaginentur talia ... ". Cf. John Murdoch, William of Ockham and the Logic of Infinity and Continuity, in: Normann Kretzmann (ed.), *Infinity and Continuity in Ancient and Medieval Thought*, Cornell University Press, Ithaca and London, 1982, p. 178. Gregory of Rimini, *Lectura*, vol. 3, p. 33, "Et, si esset aliquod omnino indivisibile habens situm, illud esset punctus. Utrum autem aliquod huiusmodi sit, alias videbitur. Sermo tamen Commentatoris in proposito condicionaliter debet intelligi. Secundo potest dici et forte melius quod Commentator in verbis illis et Philosophus etiam, qui ibidem dicit similia, non loquuntur secundum opinionem propriam, sed secundum opinionem tunc famosam ponentium mathematica separata et puncta et alia indivisibilia, quae, si esset vera, esset aliquid praecise de genere quantitatis carentis situ et indivisibile, et illud esset unitas principium numeri mathematici separati, esset etiam tale aliquid indivisibile habens situm, quod punctus ab illis vocabatur".

[36]See Rega Wood, *Adam Woodham, "Tractatus de Indivisibilibus": A Critical Edition with Introduction, Translation, and Textual Notes*, Kluwer, Dordrecht, Boston, and London, 1988, p. 11.

they did exist, the non-entitist might accept for the sake of argument that points exist.[37] Some "non-entitists" also argued that Aristotle did not believe in the real existence of indivisibles but only wrote of them following common or famous opinion. Gregory of Rimini took both of these positions in Article 1.[38] Thus Gregory argued about the points in a line while he denied that such points actually exist in the outside world.

To this Aristotelian conception of mathematics and physics, fourteenth-century theologians like Gregory of Rimini added ideas about God's sight or knowledge. Aristotle himself was of the opinion that a continuum is potentially infinitely divisible. No matter how many times a continuum has been cut into parts, the remaining pieces are still extended and may be cut further. At the same time, Aristotle thought that it was impossible to finish cutting a continuum into all the parts into which it could possibly be cut. One can cut as much as one likes and then more, but the result will always be a finite number of extended parts that could be cut further. One never has an actually infinite number of parts and, in fact, the concept of an infinite number is something of an oxymoron, since a number is supposed to have a certain quantity, while an infinite is immeasurable.

This conception worked for Aristotle, but it could not work in the same way for a scholastic Aristotelian like Gregory who asserted that God timelessly knows everything that has happened, is happening, or will happen.[39] If God timelessly knows all the places in which a continuum might be cut, does it not follow that God must know an actually infinite number of such places? If the places at which a continuum might be cut are points, does it not follow that God sees an actually infinite number of points in any line?

This introduction into discussions of continuity and infinity of God's knowledge or sight of infinitely many things had received a stimulus through notes that Robert Grosseteste wrote in the margins of his copy of Aristotle's *Physics*. Grosseteste wanted to explain how measurement would be possible if it were posited that only one line existed and nothing else by which it could be measured.[40] His suggested solution was to say that humans might not be able to make a measurement in such cases, but God would know. God might see, for

[37] Gregory of Rimini, *Lectura* II, dist. 2, q. 2, art. 1, vol. 4, p. 291, "Ad secundum, tertium et quartum possem simul respondere, negando id quod communiter supponunt, scilicet aliquod punctum esse in magnitudine. Hoc enim falsum est, sicut patebit in tertia conclusione. Quia tamen praeter hoc quodlibet etiam aliter deficit, respondeo ad secundum, supponendo pro nunc puncta esse in magnitudine secundum quod multi imaginantur, et false iudicio meo. Hoc tamen supposito dico . . .".

[38] Gregory of Rimini, *Lectura* II, dist. 2, q. 2, art. 1, vol. 4, p. 325, "Et pro duobus primis, sciendum quod, sicut patet ex diversis partibus philosophiae Aristotelis, quaedam opinio famosa fuit temporibus Aristotelis, quae huiusmodi indivisibilia posuit, non solum in anima, verum etiam extra animam. Et ex eis dicebant componi magnitudinem . . . Huius opinionis non fuerunt Augustinus, ut ex dictis patet, neque Philosophus; quinimmo secundum in pluribus locis impugnavit. Primum autem, videlicet puncta esse extra animam, aliquando sub dubio posuit . . . Aliquando vero expresse negavit . . . Nullibi tamen in propria forma exquisite discussit. Et ideo sicut moris sui fuit, antequam a proposito materiam tractasset et ubi etiam non principaliter de illa agebatur, loqui iuxta famosiorem opinionem, frequenter de punctis et lineis et superficiebus et contiguis et continuis locutus est iuxta opinionem illam famosam, et non secundum propriam".

[39] For a clear statement concerning various aspects of God's knowledge, cf. John Buridan, *In Metaphysicen Aristotelis*, ff. 75rb–76rb.

[40] Richard Dales (ed.), *Roberti Grosseteste Episcopi Lincolniensis Commentarius in VIII Libros Physicorum Aristotelis*, University of Colorado Press, Boulder, Colorado, 1963, pp. 90–91, "Sed cum linea numeretur, mirum est quo numeretur. Ponamus enim solam unicam lineam esse, et intelligamus eam abstractam ab omni materia. Hec iam metiri potest non per aliam lineam nec aliud longum, quia non sit alia dimensio longitudinis ab ipsa, nec per seipsam metiri potest. . .".

instance, all the infinitely many points in a one-foot magnitude and all the infinitely many points in a two-foot magnitude and, by a comparison of these two infinite multitudes, could know that one was twice as great as the other. The same might hold for periods of time before the creation of the cosmos.[41] To God, but not to humans, infinites are comprehensible, Grosseteste said.[42]

In the early fourteenth century, Henry of Harclay, drew non-Euclidean conclusions from this theological–mathematical–physical mix, concluding, for instance, that points are immediate to each other within lines or, in general, that indivisibles are immediate within continua. According to Harclay, if God sees all the infinitely many points of a line, then God must see the point immediate to the end point.[43] Imagine a line and that God removes the end point of the line. Then touch the end of the line. If you touch the line, you must touch something. What else could it be but the new end point of the line? Then before the original last point was removed, this new end point must have been immediate to it.

In the many fourteenth-century discussions of such cases, what we would consider mathematics and what we would consider physics are very frequently mixed: authors begin by discussing points and lines, but very frequently turn, for instance, to bodies of water being unified or separated. This happens naturally, because the authors assume that the mathematical entities such as lines really exist only as characterizations of bodies. In his *De continuo*, in which he tried to refute the views of Harclay and Grosseteste among others, Thomas Bradwardine explicitly stated that the properties of all continua, whether mathematical or physical, are isomorphic.[44]

In this mathematical–physical mix, many authors used God and angels to make possible such hypotheses as that a single point be removed from the end of a line, something that would be physically impossible. Or one assumed that a continuum had been divided into proportional parts and that an (indivisible) angel is located at the edge of each part.[45] To resolve such conundrums, the tools most frequently called upon were those of logic and, in particular, supposition theory.[46] The authors consider the truth or falsity of a proposition where this truth or falsity is assumed to depend upon the ways that the terms of the

[41] Cf. Edith Sylla, Thomas Bradwardine's *De Continuo* and the Structure of Fourteenth-Century Learning, in Edith Sylla and Michael McVaugh (eds.), *Texts and Contexts in Ancient and Medieval Science. Studies on the Occasion of John E. Murdoch's Seventieth Birthday*, E.J. Brill, Leiden, 1997, pp. 164–166.

[42] Augustine, *The City of God*, Book XII, Ch. 18 (cf. E. Sylla, Thomas Bradwardine's *De Continuo*, pp. 165–166 n. 36), had said the same thing, "Far be it, then, from us to doubt that all number is known to Him 'whose understanding', according to the Psalmist, 'is infinite'. The infinity of number, though of infinite numbers there is no number, is yet not incomprehensible to Him of whose understanding there is no number. And thus, if everything which is comprehended is defined or made finite by the comprehension of him who knows it, then all infinity is in some ineffable way made finite to God, for it is comprehensible by his knowledge".

[43] Cf. E. Sylla, Thomas Bradwardine's *De continuo*, pp. 171–177, and John Murdoch, Henry of Harclay and the Infinite, in A. Maieru and A. Paravicini-Bagliani (eds.), *Studi sul XIV secolo in memoria di Anneliese Maier*, Edizioni di Storia e Letteratura, Rome, 1982, pp. 228–229 n. 24.

[44] See E. Sylla, Thomas Bradwardine's *De continuo*, p. 157. It is worth noting that Bradwardine silently excludes the arguments of Grosseteste, Harclay, and Chatton that mention God. Cf. E. Sylla, ibid., pp. 162, 180.

[45] Cf. Gregory of Rimini's use of arguments from Richard Fitzralph in *Lectura*, II, dist. 2, q. 2, art. 1, vol. 4, p. 301, "Sexto, sit ita quod super huiusmodi infinitis partibus proportionalibus sint infiniti homines habentes corpora glorificata, 'ita quod corpus unum terminetur in puncto medio ipsius B, et corpus secundum terminetur in fine secundae partis, et sic de infinitis super partes infinitas procedendo versus A, ita quod non sint simul plures' ".

[46] Supposition theory is a theory of how terms in a proposition stand for things in the world. Cf. L.M. De Rijk, The origins of the theory of the properties of terms, in: Norman Kretzmann, Anthony Kenny, and Jan Pinborg

proposition supposit or stand for things in the world, now, in the past, or in the future.[47] It was in this context that nominalists frequently said that phrases such as "all the points of a line" are illegitimate, so that a proposition like "All the points of a line are infinite" cannot be true, because there are no things in the past, present, or future world for which "all the points of a line" can stand. But if many of the things that mathematicians talk about, such as points, lines, planes or even numbers, are not real, but imaginary or fictions, how do these words stand for things in the world? A tack that was used to solve such problems was to talk about possibilities. Thus Adam Wodeham replaced talk about points in a line with talk about possible cuts (*incisiones possibiles*) of the line and about the distinct parts that might result.[48] Aristotle had argued that it is impossible to divide a continuous magnitude into all the parts into which it could possibly be divided because those final parts could neither be divisible nor indivisible. Wodeham argued that when a line has been divided into parts then it no longer exists. Then one can ask concerning any collection of distinct parts, such as two halves or four fourths, whether they can be separated from each other, but it does not make sense to ask whether the original line can be divided into all its infinitely many potential parts.[49] Following this line of reasoning, Gregory of Rimini could conclude that it is impossible, even by God's power, that a line be actually divided into infinitely many distinct parts, such that the parts are actually separated from each other, but it is not impossible for God to distinguish conceptually infinitely many parts within a continuum, understanding that God sees at one time the two halves, the four fourths, the one thousand one-thousandths, *ad infinitum*, where all the parts he sees are themselves simultaneously divided in God's sight into infinitely many smaller parts, so that God knows the same magnitude as one, and as two, and as one thousand magnitudes, etc. In a pile of rocks, one may consider two rocks or three rocks or a hundred, and there need be no intrinsic quality in any given rock that makes it part of a pair or a triplet, etc. All the more is this the case if, instead of talking about a pile of rocks, we talk about a continuum that might possibly be divided in infinitely many different ways.[50]

(eds.), *The Cambridge History of Later Medieval Philosophy*, Cambridge University Press, Cambridge, 1982, pp. 166–167. Cf. Edith Sylla, God and the Continuum in the Later Middle Ages: The Relations of Philosophy to Theology, Logic, and Mathematics, in: *Was ist Philosophie im Mittelalter*?, Miscellanea Mediaevalia 26, Walter de Gruyter, Berlin and New York, 1998, pp. 791–797. Eadem, God, Indivisibles, and Logic in the Later Middle Ages: Adam Wodeham's Response to Henry of Harclay, *Medieval Philosophy and Theology* 7 (1998), 69–87.

[47] For some authors 'supposition' concerns the way in which terms stand for things in the present, but 'ampliation' resulting, for instance, from the tense of the verb, may cause a word to stand for things in the past or future. Cf. De Rijk, The origins of the theory of the properties of terms, p. 172.

[48] Cf. E. Sylla, God, Indivisibles, and Logic in the Later Middle Ages, pp. 81–86.

[49] Cf. E. Sylla, God, Indivisibles, and Logic, p. 84.

[50] For a discussion of related issues, cf. Buridan, *In Metaphysicen*, f. 60va–61rb, Queritur circa decimum *Metaphysice* primo utrum omne mensurabile mensuretur uno . . . Postea dico quod quidam binarius est ternarius, quia hec festuca est sue due medietates et sic est binarius et etiam est sue tres tertie, et sic est ternarius, et tunc per syllogismum expositorium sequitur hec festuca est ternarius et ipsa etiam est binarius, vel una si volueris, ergo binarius est ternarius. Et ille binarius que est ternarius vocetur B. Tunc ex dictis ego concludam quod omnis binarius est eidem, scilicet B, secundum multitudinem equalis et etiam inequalis, quia est omni binario equalis et omni ternario inequalis secundum prius dicta. Et causa huius est quia idem secundum aliam et aliam discretionem suarum partium est maior et minor multitudo . . . Et tunc ultra concludo quod non sequitur: hec sunt illis equalia secundum multitudinem, ergo hec sunt equalia, quia duo dolia vini sunt equalia secundum multitudinem duabus quartis, et tamen simpliciter falsum est dicere quod duo dolia sunt equalia duabus quartis . . . Et notan-

From this point of view, God knows not only all past, present, or future things and their parts and properties for which terms in a proposition could supposit, but God also knows all logically possible things or properties, whether or not they ever exist physically.[51] Then in cases in which, in the past, theorists had said that there was a potential, but not an actual infinite involved, William Heytesbury, Gregory of Rimini, and others concluded that there must be actually infinitely many possibilities in the pool from which any actual selection of a finite sub-set of supposita might be made with regard to a given proposition.[52] In this context it mattered whether one assumed that God knows in the same way that philosophers know, that is through knowing the truth of propositions about the world, or whether God knows individuals directly and not so-to-speak through the filter of propositions. In his *Metaphysics*, Buridan, for instance, argued that God knows individuals and not propositions, but he believed that God also knows relations between things.[53] Gregory of Rimini, on the other hand, thought that God knows propositions (*complexe significabile*) as well as individuals.[54] Whether God knows individuals outside himself and whether God knows propositions, most agreed that, as eternal and unchanging, God does not think discursively or in time. Whatever he understands, he must understand all at once. On the other hand, to be true a mathematical proposition needs to be formed, so there is a problem if the formation of a mathematical proposition is contingent, since it is not usually said that a mathematical proposition becomes true, just because it is first formed.[55] Perhaps it should

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dum est quod forte non sequitur: hec sunt illis secundum multitudinem inequalia, ergo hec sunt illis inequalia secundum multitudinem. Nam dicetur quod negatio implicita in iso termino 'inqualia' confundit distributive illum terminum 'multitudinem' quando ille terminus sequitur, sed non confundit si precedat. Et ideo si dico: hec sunt illis inequalia secundum multitudinem, sequitur quod secundum omnem discretionem sunt inequalia. Sed si dico: hec sunt illis secundum multitudinem inequalia, non est sensus nisi quod secundum aliquam multitudinem vel discretionem sunt inequalia, licet forte secundum aliam discretionem sint equalia".

[51] Cf. Buridan, *In Metaphysicen Aristotelis*, f. 76rb, "Ad aliam dico quod Deus similiter intelligit Sortem quando est et quando non est, et quando bene agit et quando male agit. Dico similiter quantum est ex identitate intellectionis, et tamen ab eterno per illam unicam et simplicissimam intellectionem ipse scit quandocumque Sortes est, erit, vel fuit, et quandocumque peccat, peccavit, vel peccabit, quia illa intellectio continet (ut dictum fuit) omnes partiales intellectiones quibus sic distincte intelligeremus Sortem . . . Ad aliam dicendum est quod Deus omnia intelligit intuitive: presentia, preterita, et futura, et possibilia, quia intuetur suam simplicem essentiam que est omnium sufficientissimum representativum, ita quod non intelligit ea per aliquam discursum nec per aliquem modum abstrahendi, et omnia satis sunt ipsi presentia, cum omnia sint in eius potestate". Such conclusions made use of new interpretations of logical possibility introduced in the early fourteenth century. See Simo Knuuttila, Medieval Theories of Modality, *Stanford Encyclopedia of Philosophy* [http://plato.stanford.edu/entries/modality-medieval/].

[52] Cf. Edith Sylla, William Heytesbury on the Sophism 'Infinita sunt finita', in: *Miscellanea Mediaevalia*, Band 13/2, edited by Albert Zimmermann, Walter de Gruyter, Berlin and New York, 1981, pp. 632–636.

[53] Buridan, *In Metaphysicen*, f. 75vb, "Ultima conclusio quantum ad presens est quod Deus nichil intelligit complexe, ita quod ibi sit complexio conceptuum simplicium sicut quando formamus propositionem, quia statim sequeretur in Deo multiplicitas et compositio. Et omnes istas conclusiones concesserunt tam fideles quam omnes philosophi. Positis conclusionibus in quibus fides et opinio omnium philosophorum concordant, ponenda conclusio in qua aliqui philosophi discordaverunt a fide quamvis etiam multi concordaverint. Dicimus ergo quod Deus omnia alia a se intelligit clarissime et sic distincte quod omne ens intelligit et omnem modum se habendi ipsum ad seipsum et ad quelibet alia et scit quod Sortes differt a Platone et scit quomodo differunt ab eo, et quando et quomodo unusquisque agit vel non agit".

[54] Gregory of Rimini, *Lectura*, vol. 3, 1984, pp. 276–277.

[55] Ibid., p. 257, "Et adverte quod proprie non dicitur propositio incipere esse vera ex hoc praecise, quod ipsa est vera et non fuit prius vera, quia, si numquam ipsa fuit prius et modo sit et sit vera, verum est quod ipsa est

be said that God knows all mathematical propositions that may possibly be formed. If mathematical propositions are conditionals, as Ockham and others argued, this introduces an additional layer of complexity.

In Article 1, then, Gregory decisively rejects Henry of Harclay's argument that, if God sees all the points of a line, God must see points immediate to each other.[56] He did this, in effect, by treating God's knowledge of the world as multilayered or involving many possibilities that are not compossible: God simultaneously sees the two halves of a continuum and the three thirds, and the thousand one-thousandths, and so forth.[57] These are all possible, and God sees them as such, but they are not all distinct from each other, since the quarters are parts of the halves, and so forth.

From this point of view it was possible to see why, even though God sees all the proportional parts of a continuum, there is no last or smallest part. Take the two halves of a line and divide one of those halves into proportional parts with the smaller parts on the side connected to the other half of the line. There will be no last proportional part touching the second half of a line, but infinitely many such parts. In God's eyes these are not potential, but actual parts. God sees them all and God sees that between any given part and the other half of the line there are other parts. God does not see a last proportional part, because there is no last proportional part—within every part that God sees there are smaller parts.[58] If one talked in terms of points rather than proportional parts, then one should say that God does not see any point next to the last point of a line because what is the case for points should be taken as analogous to what happens with proportional parts.

> To the fourth [objection] I would say that no point is between the first point and all the other points of the same line. And when it is inferred 'therefore the first point and all the other points are immediate and consequently some one of the other points is immediate to the first point', if in the conclusion the word 'all' is taken collectively, I concede the conclusion, whatever might be said about the inference. But I deny the further consequence by which it is said 'therefore some one of the other points is immediate to the first point'. A most certain counter example for me is because if there are two immediate bodies A and B dividing B in proportional parts such that the first proportional part is more distant from A and the second proportional part is half of the other half immediate to the first half of the whole, and the third is half of the rest, and so forth continuing toward A, then this is true: 'A and all the proportional parts of B are immediate', but this is false:

vera et numquam prius fuit vera. Nec tamen quaelibet talis proprie dicitur incipere esse vera, sed illa tantum quae fuit prius non vera, aut, si fuisset, fuisset non vera; alioquin quaelibet propositio vera noviter formata diceretur incipere esse vera, et sic una conclusio geometrica inciperet esse vera, quod tamen in communi usu loquendi absurdum diceretur". Of course, the real concern in most discussions of God's unchanging knowledge involved problems of predestination and free will, not mathematics. If human choices are contingent, how could God have determined timelessly who will or will not be saved?

[56] Gregory of Rimini, *Lectura*, vol. 4, p. 290, "Tertio ... suppono unum verum et a quolibet fideli concedendum, scilicet quod deus videt quodlibet punctum quod est in linea. Tunc quaero, an deus videat quod inter primum punctum lineae huius vel illius demonstratae et quodlibet aliud punctum ab eo visum in eadem linea intercipitur linea media, aut non. Si non, igitur videt punctum immediatum puncto. Si sic, igitur contra suppositum non videt quodlibet punctum illius lineae, quoniam in linea illa media sunt plurima puncta et nullum secundum hoc est visum a deo", p. 292, "Ad tertium dico, stante praedicta suppositione, quod deus videt quod inter primum punctum et quodlibet aliud eiusdem lineae intercipitur linea, et infinita etiam puncta, non tamen linea non visa, nec puncta non visa ab eo. Nullam tamen lineam deus videt intercipi inter primum punctum et quodlibet aliud punctum ab eo visum".

[57] Cf. Richard Cross, Infinity, Continuity, and Composition: The Contribution of Gregory of Rimini, p. 224.

[58] Cross, Infinity, Continuity, and Composition, p. 101. Gregory of Rimini, *Lectura*, vol. 4, pp. 304–305.

> ‘some proportional part of B is immediate to A’, since of the proportional parts none is the last towards A. Therefore, this [further consequence] does not follow from the first.[59]

In Book I, distinctions 35 and 36, Question 1 of his *Lectura*, Gregory had raised the issue of the composition of a continuum precisely in the context of God’s knowledge of things other than himself. Peter Aurioli had argued that God does not know individual things outside of himself on the grounds that God cannot know all the parts of a continuum.[60] Gregory replied that God can know all of the infinitely many parts of a continuum every one of which contains within itself infinitely many parts and such that there is no last and final division.

> To the seventh, when it is said ‘It is impossible for the division of a continuum to be totally completed’, speaking of division not as a real separation and breaking up into parts but of division as made by the understanding of the intellect distinguishing parts, which is compatible with the real continuity of the continuum so divided, in which sense the argument is made, I deny this assumption . . . To the proof, when it is asked whether this division stops at divisibles or indivisibles, I say that something may be called indivisible in two ways. In one way indivisible means lacking any sort of parts actually divided or possibly divided or distinguished in any way. In the

[59] Gregory of Rimini, *Lectura* II, dist. 2, q. 2, art. 1, vol. 4, p. 290 (against his first conclusion), “Quarto, et fortius. Aut aliquod punctum est medium inter primum punctum huius lineae verbi gratia A et omnia alia eiusdem, aut nullum punctum est medium inter punctum primum lineae A et omnia alia puncta eiusdem. Hoc patet, quia partes huius disiunctivae contradicunt mutuo. Si aliquod est medium, igitur illud est immediatum primo puncto; si nullum est medium, igitur primum punctum et omnia alia sunt immediata, et per consequens aliquod aliorum punctorum est primo immediatum”, p. 292, “Ad quartum dicerem quod nullum punctum est medium inter primum et omnia alia eiusdem lineae. Et cum infertur ‘igitur primum et omnia alia sunt immediata’, si in consequente ly ‘omnia’ teneatur collective, concedo consequens, quidquid sit de consequentia. Et nego ulteriorem consequentiam qua dicitur ‘igitur aliquod aliorum punctorum est primo immediatum’. Instantia certissima apud me est, quoniam si sint duo corpora immediata A et B, dividendo B in partes proportionales sic quod prima pars sit eius medietas remotior ab A, et secunda sit medietas alterius medietatis, immediata priori medietati totius, et tertia sit medietas residui, et sic semper procedendo versus A, tunc haec est vera ‘A et omnes partes proportionales B sunt immediata’, et tamen haec est falsa ‘aliqua pars proportionalis B est immediata ipsi A’, cum talium nulla sit ultima versus A. Igitur haec non sequitur ad primam”.

[60] Gregory of Rimini, *Lectura*, Book I, dist. 35 et 36, q. 1, vol. 3, pp. 211–214: “Utrum sequens praecise rationem naturalem habeat ponere quod deus intelligat alia a se vel non”. Seventh proof of Aristotle’s third conclusion: “Opinio Aristotelis et Commentatoris sui Averrois fuit quod deus nihil aliud a se intelligit. Unde 12 Metaphysicae ponit Philosophus circa intellectionem dei tres conclusiones: Quarum prima est quod deus est substantia intellectualis et semper actu intelligens. Secunda, quod sua substantia est sua intellectio. Tertio, quod deus nihil aliud intelligit praeter se ipsum”. “Septimo, ad eandem conclusionem arguitur sic per alios: ‘Impossibile est divisionem continui totaliter evacuari. Aut enim staret divisio in aliqua indivisibili’, et hoc non, quia tunc sequitur quod continuum componitur ex indivisibilibus, ‘aut staret divisio in aliqua divisibilia’, et hoc etiam dici non potest, quia sic non esset evacuata, sed possibilis esset ulterior divisio. Ex quibus patet quod impossibile est divisionem continui totaliter evacuari. Sed hoc sequeretur, si deus intelligeret alia a se, nam intelligeret continuum et eius divisio evacuata esset in conceptu eius. Quod probatur, quia aut in eius intuitu essent omnes partes continui divisae et distinctae, ita ut non relinquatur ulterior divisio, et per consequens ‘divisio continui erit evacuata’, aut aliquae partes tantum in certo numero, ‘quarum quaelibet esset divisibilis’, et hoc non potest esse, quoniam ‘tunc divinus intuitus esset in potentia et sibi aliquid posset accrescere, scilicet dividendo unamquamque illarum partium, quinimmo posset sic dividendo procedere in infinitum sicut et noster intellectus’; quod est penitus impossibile. Ex quo ultimate sequitur quod impossibile est deum intelligere alia a se”. Cf. a principal argument repeated in Buridan, *In Metaphysicen Aristotelis*, f. 75va, “Item vel Deus intelligit alia a se universaliter solum et hoc est intelligere confuse et imperfecte, quod non est dicendum, vel intelligit omnia singulariter, et hoc est impossibile, quia infinita sunt singularia, et infinitum secundum quod infinitum est ignotum, ut habetur primo *Physicorum*”.

> other way, something may be said to be indivisible because, even though it has parts, nevertheless each of them is divided in act from any other, such that it is not divisible in any way or in any part of itself unless it is actually divided in that way . . . I do not understand by 'stops' that there is some last division or any things which are finally divided that do not include within themselves parts divided in act, but I understand that the division 'stops' at the indivisible in the sense that it is totally divided in indivisibles in this second way, but that these parts are divisible in the first way. Thus I say that any part you take however small includes in itself parts actually divided in the divine mind (*conceptu*), indeed it includes infinitely many, nor can any be taken without taking infinitely many. And thus as it is a fact that any continuum has infinite parts potentially and any one of them no matter how small contains infinitely many, nor can any indivisible part be taken unless one takes infinitely many potential parts, so I say that in the mind of God a continuum is totally divided in act into parts of which any one is also totally divided in act and includes infinitely many actually divided parts.[61]

With this conclusion in place, in Book II, concerning the place of angels, Gregory could then make use of this earlier reply. God sees infinitely many parts of any continuum, but all of those parts also contain infinitely many parts.[62]

Thus adding God and angels into the picture of Aristotelian mathematics at first led fourteenth-century theologians into mathematical difficulties, but in the work of Gregory of Rimini it led to what we might consider progress in learning to deal with the mathematics of continuity and of actual infinites. The sort of intellectual labor that went into such mathematical progress does not deserve to be satirized by characterizing it as disputing how many angels can dance on the point of a needle.

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[61] Ibid., pp. 223–224, "Ad septimam, cum dicitur 'Impossibile est divisionem continui totaliter evacuari', loquendo de divisione non realis separationis vel discretionis, sed de divisione quae fit per significationem intellectus distinguentis, quae secum compatitur realem continuitatem continui sic divisi, in quo etiam sensu procedit ratio, nego istud assumptum, quamvis etiam loquendo de divisione actualis discretionis in effectu, si hoc intelligatur esse simpliciter impossibile etiam per divinam potentiam, aestimem ipsum esse falsum, sicut in quadam Responsione publica dixi, alias et forte alibi a proposito in speciali discutiam. Sed hoc non est necessarium ad propositum. Ad probationem, cum quaeritur, utrum ista divisio stet ad divisibilia vel indivisibilia, dico quod potest dupliciter dici aliquid indivisibile: Uno modo indivisibile, id est carens partibus quibuscumque actu divisis vel possibilibus quoquo modo dividi vel distingui, sicut multi imaginantur esse punctum et sicut in rei veritate quodlibet simplex est indivisibile. Alio modo potest dici aliquid indivisibile, quia, etsi habeat in se partes, quaelibet tamen est actu divisa a qualibet alia, ita quod ipsum non est aliquo modo vel secundum aliquid sui divisibile, quin sit illo modo actu divisum . . . Non intelligo autem per 'stare' quod sit aliqua ultima divisio vel aliqua ultimo divisa, quae in se non includat partes actu divisas, sed intelligo quod 'stat' ad indivisibilia, id est illud est totaliter divisum in indivisibilia isto secundo modo, quae tamen sunt divisibilia primo modo sumendo 'divisibile'. Unde dico quod, quamcumque partem quantumcumque parvam sumpseris, illa includit partes actu divisas in conceptu divino, immo infinitas includit, nec aliquam potest sumere quin sumas infinitas. Et, sicut de facto quodlibet continuum habet infinitas partes potentiales et quaelibet quantumlibet parva includit infinitas nec aliqua sumi potest indivisibilis et quin sumantur tales partes potentiales infinitae, sic dico quod in conceptu dei continuum est totaliter actu divisum in partes, quarum quaelibet etiam est totaliter actu divisa et includit infinitas actu divisas". Cf. Cross, Infinity, Continuity, and Composition, p. 103.

[62] Gregory of Rimini, *Lectura*, vol. 4, p. 293, "Dico quod nec in aliqua divisibilia nec in aliqua indivisibilia nec in aliqua omnino ultimate facta esset, quoniam nulla essent, quae non essent in alia resoluta. Unde sicut ante huiusmodi resolutionem nulla pars erat irresolubilis in alias, sic, posita totali resolutione, nulla esset non resoluta in alias, sicut satis propinque huic dictum est in primo libro distinctione 35 quaestione unica articulo 1 in responsione ad 7 argumentum opinionis ibi reprobatae de totali divisione continui in conceptu divino".

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CHAPTER 14

Mathematics and the Divine in Nicholas of Cusa

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1. Introduction

Nicholas of Cusa (1401–1464) is known above all as a philosopher and theologian. He was the author of numerous works on metaphysical and mystical topics, in most of which he tirelessly studied the various facets of the coincidence of opposites. The most famous of his books is *Learned Ignorance* (*De Docta Ignorantia*), published in 1440. Therein Nicholas tried for the first time to breathe life into his great intellectual discovery, the *coincidentia oppositorum*, which he had experienced while sailing home from Greece in 1438 with the emperor, the Patriarch of Constantinople and his entourage, whom Nicholas accompanied to Italy, where a Unification Council, gathering Latins and Greeks, was to take place.

Learned Ignorance and other subsequent works developing and prolonging his insights devote much attention to mathematics. The most important part of this study will concern the usefulness of mathematics in this search for truth. But it is also incumbent on us to look at Nicholas of Cusa's strictly mathematical works, where he tried to solve the celebrated problem of squaring the circle. Many books are devoted to this topic, among which we should mention the *Geometrical Transformations* (*De Geometricis Transmutationibus*) and *Arithmetical Complements* (*De Arithmeticis Complementis*) of 1445, *On Circle Squaring* (*De circula quadratura*) and *Squaring of the Circle* (*De Quadratura circuli*) of 1450, the *Mathematical Complements* (*De Mathematicis Complementis*) of 1454, etc. These are the more technical books, where the relation to the Divine is not particularly underlined, but which are nevertheless indispensable for a full idea of Nicholas's conception of mathematics. We shall see that some difficulties remain as to the exact bearing these mathematical treatises had in relation to the whole of his thought.

2. The place of mathematics in human knowledge and its symbolic value

Mathematics played a fundamental role in Nicholas's thought: it was the example par excellence, the model of all veritable human knowledge, to the point that Nicholas did not hesitate in extending to all human knowledge the gaps and the bounds he observed in mathematical approach of reality.

At the beginning of *Learned Ignorance*, we find very interesting developments as to the inherent limitations of human knowledge.

1. Human knowledge is compared to measuring size.
2. Measuring implies having a standard, something well known and in comparison with which the unknown becomes known.
3. Acquiring new knowledge is easy when few intermediary steps are needed to reduce the unknown to the already known; hard, should the contrary be the case.
4. It is impossible to make knowable that which has no proportion to it.
5. The infinite has no proportion with the finite. The infinite will never be known from the finite and will therefore always remain unknown.

Hence human knowledge cannot have total, exhaustive knowledge of reality as its objective goal because it is impossible. But its destiny is accomplished in becoming aware of its own limits and boundaries. This is the theme of Nicholas of Cusa's *Learned Ignorance*: this human knowledge conscious of its basic insufficiency in relation not only to God but also to other realities. The fact that there are limits by right to human knowledge of God, and not only limits of fact, has immediate repercussions on all other realities. In fact, to know the essence of a thing exactly would be to realise an infinite equality between the object and the knowledge man has of it. But only God is infinite in the strong sense of the term and hence only he can grasp the ultimate truth of anything. It is therefore impossible for man to know the essence of any reality whatever.

Given these circumstances, human knowledge is always conjectural, limited and symbolic: it strives to represent external realities within the mind but the precision is always a relative one and can always be improved.

Nicholas has recourse to two particularly striking images to help us understand the conjectural dimension of human knowledge:

1. The image of a series of polygons inscribed in a circle;[1] no more than the growing number of polygon sides can ever be identical with the circle in which they are inscribed, even if they get closer and closer, can human knowledge ever reach its object with absolute precision.
2. The image of a geographer[2] who seeks to map an unknown country; on the basis of observations and other information collected by his messengers, he traces the map which is only a likeness of the described reality, whose precision can always be improved by further attempts.

Both of these images, of a geometrical type, readily show the conjectural and symbolic dimension of any human knowledge and, particularly, knowledge of God.

2.1. *The symbolic bearing of mathematics*

In *Learned Ignorance*,[3] Nicholas strives to justify the great attention he has paid to mathematics, in the order of human knowledge, particularly in knowing invisible realities, for which we have no direct experience. It should be pointed out that, according to Nicholas, each part of reality has relations, proportions, to all others. It is therefore possible to represent and symbolise these invisible realities which escape direct knowledge by using others realities better known to us. In this respect, mathematical objects are quite interesting given their stability, the fact that they are exempt from any direct relation to change. This is what makes the reliability and strength of the disciplines dealing with them. Hence the Pythagoreans' and the Platonists' predilection for mathematics as a means of access to reality as a whole. Representatives of other philosophical trends were also involved: in a famous passage of his treatise *On the Soul* Aristotle compared forms to numbers.[4] This predilection, recognised and proclaimed by Greek philosophers, has been confirmed by numerous Christian thinkers, among whom Nicholas mentions St. Augustine and Boethius specifically.

The reliability and value of these mathematical objects justifies the fact of using them in aiming at the absolute maximum. This cannot be known in itself but has to be approached

[1] *Learned Ignorance*, I, 3, 10.
[2] *Compendium*, VIII.
[3] *Learned Ignorance*, I, 11–12.
[4] Aristotle, *On the Soul*, B 3 414b.

Fig. 1.

symbolically by other objects of knowledge which, as well as designating themselves, also refer to something other than themselves, that is, to God himself or to the absolute. The gap between the symbol and what is symbolised is already an (unavoidable) source of imperfection in human knowledge of God. One more reason to avoid a second shift factor as much as possible: which would come from variations and changes in the symbols themselves.

The merit of mathematical objects is precisely to hinder this second shift factor danger, due to their immutability.

Here Nicholas clearly takes up the classical tripartition of theoretical science, going back to the Platonic Academy: physics, mathematics and theology.

Given their characteristics as objects free of any movement while maintaining a link to matter, mathematical objects are intermediate between physical, material and changing realities and the reality theology treats, separated from all materiality and all change.

Nicholas's originality in this field consists in justifying the symbolic bearing of mathematical realities for the absolute maximum as he in fact conceives of it, i.e., as a coincidence of opposites. Geometrical figures can symbolise this absolute maximum because they are able to represent this coincidence in their own way. Nicholas shows this by means of what he calls theological figures.[5]

2.2. *The theological figures*

Different geometrical figures (circles, spheres, lines and triangles) can be identified with one another when they are increased to the infinite. Hence the infinite circle and the infinite line may be identified with one another, as present-day geometry readily admits.

In the same order of ideas, both equal sides of an isosceles triangle, of ever increasing altitude, become two parallel lines. Inversely, the same triangle reduces itself to a single line if its altitude is decreased until it vanishes.

Adding metaphysical considerations to this commonplace geometrical observation, Nicholas says that everything that is possible or potential in the domain of finite figures becomes actual in the case of corresponding infinite figures. For instance, it is possible to trace a circle from a single segment of a line by making this segment turn round its centre. This circle will itself generate a sphere by turning round any one of its diameters.

Therefore, the finite line contains the sphere potentially and what is potentially true for a finite segment becomes actual reality for an infinite line; so the infinite line is triangle,

[5] *Learned Ignorance*, I, 13–17.

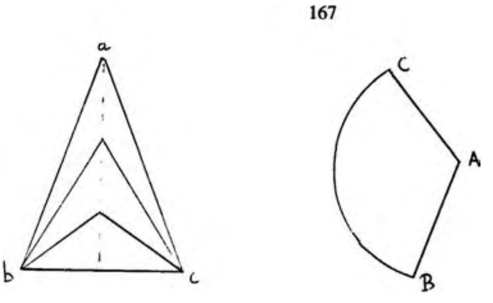

Fig. 2.

circle and sphere and is also identifiable with all other geometrical figures. In a general way, an infinite geometrical figure is identical with all of space because the infinite can lack nothing and hence all infinite figures coincide with one another. By going to the limit of theological figures, the human mind can get a feeling of the genuine coincidence of opposites as it exists in God. But it can only feel this because geometrical figures all lose their shape when they are increased to the infinite, and so disappear, as figures.

Yet what is only fiction on the level of geometrical figures and quantity is for Nicholas of Cusa an incontestable reality on the level of being: there is indeed an infinite form in which all differences and all opposites are identical.

2.3. *The great conjecture*

In his book *On Conjectures* (*De Coniecturis*), Nicholas sums up some themes he had treated in *Learned Ignorance*. He widens their development and lends them some new accents in the direction of the new metaphysics of the One and the new philosophy of mind he is working out here.

The main points of this new contribution to mathematics can be summarised as follows:

1. Every human cognition is conjectural and, on the grounds of sensible experience, tries to find its way back to ideas that, in the divine mind, are at the origin of that experience. So sensible reality is not, properly speaking, the object of human knowledge but simply its factual occasion. Human knowledge is therefore a sign or symbol of divine ideas and nothing more.
2. The human mind can claim to find in itself likenesses of the divine ideas because it is created in God's own image. This well-known biblical theme plays an important role in Nicholas of Cusa's view of man. Just as God is the creator of a real world through which he manifests his virtues and perfections, so the human mind creates an intentional world of concepts and conjectures in which it too reveals its internal aptitudes. Given this link of image to model between the mind and God, the concepts forged by the human mind by virtue of its internal dynamism naturally square with and are relevant to the real world created by God, who permits knowledge of it, properly speaking.

3. This process of knowledge is dynamic: the more the mind increases its knowledge, the more it exteriorises itself in a web of conjectures and concepts and knows itself, the more like God it becomes. The status of the image of God belongs to the intrinsic nature of the mind and can no more be lost than it can be increased, but the mind's likeness to God can be unceasingly increased. The level of similarity to God results from man's noetic activity.
4. These human conjectures in step with things are of a mathematical kind. Mathematical objects are therefore good symbols of the essences of things. In this context, on the grounds of a theory tracing back to the school of Chartres, Nicholas works out what he calls the Great Conjecture.

According to this theory, the different kinds of reality have to be symbolised by different kinds of numbers and unities. The divine unity, which transcends all created things while present to every finite reality it produces, should be symbolised by the first arithmetical unit (the one) which is the principle of all numbers. The unit of ten and its first multiples (20, 30, 40, 50, etc.) represent the order of pure intellects or intelligences, the unit of one hundred and its multiples (200, 300, 400, 500, etc.), the order of souls, and lastly the unit of one thousand should be linked to the world of bodies and materials.

God	1
Pure Spirits	10
Souls	100
Bodies	1000

The advantage of such conjecture is to give more precise content to the alleged correspondence between numbers and things. Nicholas also relies on the intrinsic symbolism of Arabic digits to make his conjecture plausible. Obviously God can be represented by pure unity, while creatures participate in this divine unity, all the while containing a certain measure of non-being, increasing with their distance from God; this is well symbolised by the series of units: 10, 100, 1000.

Moreover, the unit of ten, associated with the world of intelligences, is indeed the root of the two following worlds: the world of souls, symbolised by 10^2 and the world of matter or bodies, by 10^3. We find here the trace of a hierarchy of degrees of being in reality; according to the hierarchical scheme as it is worked out, for instance, by Dionysius; a lower degree of being receives its ontological consistency and perfection from the higher degrees it participates in.

It is also clear that $10 = 1 + 2 + 3 + 4$, as the Pythagoreans underlined long ago, in speaking of the Tetraktys. Thus intelligence appears to be precontained in the unity of God. In fact, $1 + 2 + 3$ represents the concrete divine nature in the coincidence of the opposites it incarnates (1 being oneness, 2 equality and 3 the union of oneness and equality). Subsequently, 4 is the number of the world, which the divine unity includes in itself as a possibility. What is most interesting is Nicholas of Cusa's statement that we actually have no knowledge of the different levels of reality in question other than that drawn from the analogy of the arithmetical units and their links with each other. Therefore, divine attributes like omnipotence, infinite fecundity, immanence and transcendence in relation to the world can be obtained by considering the links pure unity has with the numbers emanat-

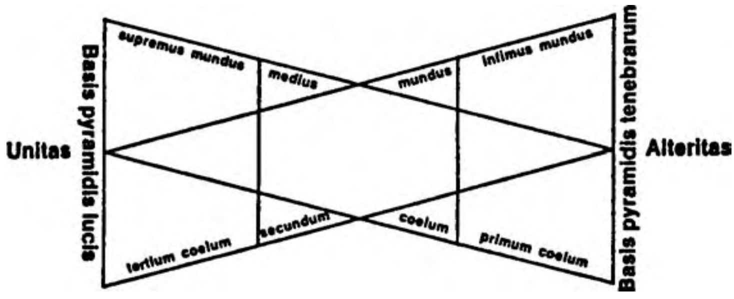

Fig. 3.

ing from it. For Nicholas, rational theology has no other positive content than that provided by the systematic use of this analogy of arithmetical unity in its relation to numbers.

We have here a particularly strong statement but which is merely the logical consequence of the identification of human knowledge and mathematics, already proclaimed in *Learned Ignorance*. When, at the end of the book, Nicholas describes his discovery during the return from Greece as consisting in "incomprehensibly grasping the incomprehensible in a learned ignorance transcending humanly knowable, incorruptible truths", incorruptible truths designate the level of mathematical knowledge. Becoming one with God in ignorance, transcending the order of immutable truths, means concretely transcending mathematical statements as well as those of Aristotelian metaphysics, which have the same epistemological status as mathematics, according to Nicholas.

Nicholas further develops this conjecture and strives to symbolise the various levels of reality through what he calls the paradigmatic figure (Figure 3).

It consists of two perfectly overlapping pyramids: the pyramid of unity or light and the pyramid of otherness or darkness. Together the pyramids delimit different levels determined by the more or less high degree of unity.

a. The base of the light pyramid coincides with the vertex of the other one. There otherness vanishes and unity is pure. It is the symbolic place of divine unity and the plenitude of being.
b. A second country is characterised by the prevailing of unity which includes otherness. It is the country of pure spirits or intelligences. This is the third heaven or supreme world.
c. A median zone where unity and otherness are externally opposed to each other and both have the same strength or approximately so, symbolises the domain of the souls.
d. A fourth country is marked by the victory of darkness and otherness over light and unity. It the matter domain of sensible bodies, corresponding to the first heaven and to the lowest world.
e. Finally we find a place where the vertex of light vanishes and is completely dominated by darkness. There is the not-being and nothingness.

As like numbers are looked upon as compounds of the One and the Other in the Pythagorean perspective, or of the One and the indefinite Dyad in the Platonic Academy, all things proceed from combinations of two ontological principles: Unity and Otherness.

It is important to point out that Aristotelian logic, with its principles of non-contradiction and of the excluded middle, is only valid in a limited part of reality. The two regions surrounding the middle one are governed by dialectical logics where one opposite is internal to the other. The role of unity in the intellectual heaven and of otherness in the corporal one is quite the same. Both regions are in fact the inverted reflection of each other, with the soul or rational level as a mirror. So many truths over the intellectual world could be extracted from a careful study of the corporal world.

But Nicholas goes still further and claims that each of the three worlds of creatures is actually a whole universe in which three sub-levels could be made out. And each of these sub-levels is also to be divided into three parts. He gives the example of angelic hierarchies and angelic choirs as the pseudo-Dionysius works it out in his *Celestial Hierarchy*.

Every reality can therefore be considered an image reflecting the whole universe and its ternary structure. He expresses this doctrine of everything is in everything through another typical figure that he call the figure U (U for universes). It develops the analogy with the number serial we have already seen in the Great Conjecture. Each world is a universe and so contains three heavens and therefore the three derived units: 10, 100, 1000, as the Figure 4 clearly shows.

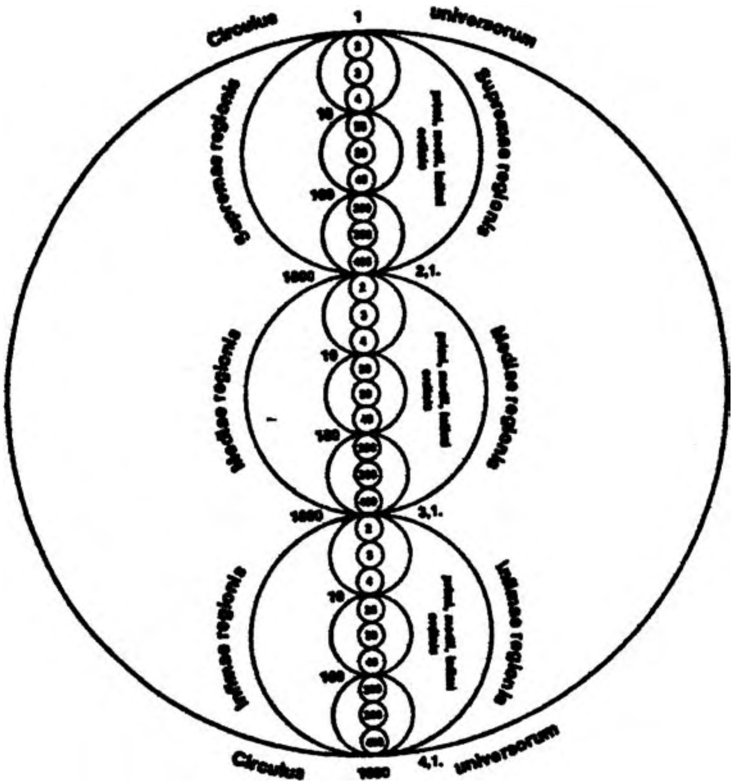

Fig. 4.

2.4. *Dianoetics and dialectics*

Although unfamiliar with Plato's famous allegory of the Line[6] and not using Plato's terminology, Nicholas of Cusa seems to sum up the core of Plato's distinction between two kinds of rationality:

1. Dianoetical (or mathematical) rationality, characterised by discursiveness, grounded in hypotheses accepted as self-evident, as well as in the sensible similarities upon which reasoning is based to facilitate demonstrations.
2. Dialectical rationality which starts with hypotheses to reach what is self-sufficient on the level of being and knowledge.

Dianoetical sciences, of which geometry is the clearest example, attain a certain grasp of their objects but the intelligibility obtained is not total because of the unjustified character of the starting hypotheses which condition the whole mental process. Dialectics, on the other hand, is not conditioned by the value of its starting point because that consists precisely in freeing itself from anything that starting point might have that is partial and limited to rise to the hypothetical. The mind's journey here is justified not by its starting hypothesis but by its term, beyond essence and determination, unconditioned, needing nothing else to prove its worth but attesting to itself and thereby attesting to the relative value of things other than itself.

The rationality suiting true theology is the type other than that which is useful for mathematics. That is the significance of the expression "learned ignorance" used by Nicholas. Theology is ignorance because it does not consist in a dianoetical knowledge of a mathematical type, identified here with human knowledge, as a whole. ("Learned ignorance" should not be confused with that trivial ignorance which knows nothing at all and, in particular, is not aware of its own ignorance.) Theology deals with an infinite reality which does not obey the same laws as mathematical realities: the coincidence of opposites escapes mathematical categories. It clearly belongs to what Plato called dialectical rationality, which Nicholas calls, for his part, ignorance conscious of itself. But the rapprochement with Plato shows that mathematics, even though destined to be transcended by dialectical procedure is, nonetheless, an indispensable preliminary support to dialectics. In this way, mathematics will continue to play a key role, even in theology, as proof that theology constantly needs to transcend itself in order to conquer its own epistemological identity.

The "ignorant" approach to the absolute maximum is actually going to take the form of a negation of the indicating characteristic of the finite. Now indicating the finite implies using numbers, as we have seen. Two different realities will be represented by distinct numbers, unequal to one another and distant from one another in the number series. These three aspects mutually imply one another but are not strictly identical to one another; for instance, unequal numbers may be more or less distant from one another. In this respect, proper to the finite, every creature that exceeds another is in turn transcended by another real or possible creature. Thus what exceeds is at the same time what is by that very fact exceeded. That is something fundamental for Nicholas of Cusa. We never find an absolute maximum or minimum among finite things but only factual relations of excedent and exceeded which can always be widened to other possibilities. The anterior–posterior is therefore a category

[6] *Republic* VI 509d–511e.

typical of the finite. That is a very important point in *Learned Ignorance*. For that matter, it is because the real is intrinsically characterised by relations of more or less (be that in perfection, origin or distance) that they can be so easily described in mathematical fashion.

On the contrary, God, as a coincidence of opposites, will be characterised by non-difference, that is to say by oneness (*unitas*), by non-inequality, meaning equality (*aequalitas*) and by that non-distance, meaning by union (*connexio*).

These three moments or aspects of divine unity, oneness, equality and union, are three ways of expressing the fact that God radically escapes the anteriority–posteriority typifying the finite.

The ternary structure of the absolute maximum also very clearly corresponds to the three persons of the Trinity: the Father corresponds to oneness, the Son to equality, and the Holy Spirit to union.

Here Nicholas reworks an approach to the Trinity already known to Thierry of Chartres[7] and his school, and whose roots go back to Saint Augustine, even if the concepts had no particular mathematical meaning for the great bishop of Hippo.

If Nicholas takes up the conceptual framework of the Line tentatively, he develops it by distinguishing three distinct dialectics more clearly than had been done before him:[8]

- the dialectics of matter and form;
- the dialectics of wholes and parts;
- the dialectics of the two natures in Christ.

2.5. *Matter–form dialectic*

The first dialectic aims at grasping the essences of things considered in and of themselves, independently of links they might have with their surroundings. The only relationship taken into account is that of creation which connects things to their creator and ontological principle, the absolute maximum. God himself is included within this dialectic inasmuch as he is the unique form free of every link with matter or movement, who gives existence to all particular forms embedded in matter–form compounds as actualising principles.

According to Nicholas of Cusa, God is, therefore, the form of being (*forma essendi* to follow Boethius's terminology) and the form of forms (the *forma formarum*, as Aristotelian and Plotinian traditions call him).

The mathematics that serve as the foundation for this first dialectic are mainly arithmetic and geometry. We have seen how a concept of absolute maximum can arise in terms of unity, equality and connection, from symbolically indicating creatures through numbers. In the same way, geometrical figures, by means of passage to the limit towards theological figures, turn out to be a good starting point in seeking the absolute maximum.

Nevertheless, we have here a transcending of ordinary mathematics, which symbolise things seemingly "existing on their own", that is to say, substances in the traditional sense of the term, but which are actually conditioned by and constitutionally dependent upon the

[7] Häring, Nicholas (1971), *Commentaries on Boethius by Thierry of Chartres and his school*, Pontifical Institute of Mediaeval Studies, Toronto; [12].

[8] Cf. Counet, [6, pp. 40–49].

absolute maximum and which are transcended to grasp the absolute maximum, the sole veritable substance that is reality on its own.

2.6. *The whole–parts dialectic*

The second is the whole–parts dialectic. It is the object of the second book of *Learned Ignorance* dedicated to the universe and plays a central role in other works like *On Conjectures* (*De Coniecturis*) and *On the Mind* (*De Mente*).

It actually splits into two branches:

a. A dialectic of the universe where Nicholas breaks with the ancient vision of the cosmos,[9] with the earth immobile at the centre of the world, the heavens composed of an essentially different matter than that of the sublunary world and a set of qualitatively differentiated places corresponding to the natural inclinations of the elements. This cosmos of the ancients, conceived as an aggregate of accidentally interconnected substances, is replaced by a boundless universe—not really infinite because God alone is, strictly speaking, infinite—a kind of sphere whose centre is everywhere and borders nowhere. This universe is a structure in the modern sense of the term, that is to say the elements composing it have essential links with one another, defining them and interconnecting them ontologically. One might even say that in every part of the universe the reality found there may be considered the result of all the interactions, of all the relations of the elements of the universe with one another, gathered in this place as in a focus. In other words, the universe may be considered to be a function, each particular thing being able to be seen as the value of the universe-function at the point in question. E. Cassirer[10] and H. Rombach[11] have insisted on the fact that by developing this vision of the material world, Nicholas of Cusa inaugurated the ontology and conceptuality which are the philosophical grounds of modern science.

   The mathematics involved here as the foundation of this second dialectic are principally astronomy and music. Traditional astronomy tries to describe the cosmos of the Ancients the help of some hypotheses admitted as obvious: there is an immobile centre of the universe, the paths of the planets can be reduced to circles, etc. With regards to this astronomy, which is clearly dianoetical learning in Plato's sense of the term, Nicholas of Cusa's cosmological vision must be seen as dialectical. By a transcending process, the mind comes to the sight of a reality that is not hypothetical, totally material reality, which is itself its own physical justification: the universe. Any explanation of a particular physical event will consist in reintegrating this event into the whole of the universe, where it has meaning.

   Nicholas's universe, which is actually neither infinite nor finite, whose centre is at once everywhere and nowhere, which is not exactly describable through mathematical notions, although mathematics can give an ever more precise account of it, corresponds to the so-called "astronomy-without-hypotheses" natural philosophers

[9] [13].

[10] [2–4].

[11] Rombach, Heinrich (1966). *Substanz, System, Struktur. Die Ontologie des Funktionalismus und der Philosophische Hintergrund der modernen Wissenschaften,* 2 vols., Alber Verlag, Freiburg–München.

had sought since the 14th century. In that respect, the revolution initiated by Nicholas of Cusa is far more radical than that of Copernicus, although Copernicus came after Cusa in time. No matter what is said, the Copernican system remains in the theoretical framework of traditional astronomy; it has simply changed the centre of the cosmos and assigned another role to the epicycles than had the Ptolemaic system.[12]

For his part, Nicholas introduced the modern view of the universe, accentuating the essential links between material realities within one and the same whole. In this respect, his conception is essentially musical. Proportions existing among all the parts of the world reflect the constitutive links among components of one and the same structure.

b. A dialectic of spirit. The universe is composed not only of material parts but also of spirits: the pure intelligence (the angels) and also embodied spirits like men and other intelligent animals that might exist on others planets. The fact of being the dwelling of numerous spirits and the fact of being a structure of interconnected material elements are correlative for Nicholas of Cusa. Indeed, minds can only exit in a multi-centred totality such as the "infinite" universe thought by Nicholas. In fact, perceiving and knowing reality in its totality, what every mind or spirit does in its rational activity, requires being at the centre of the world. Thus a manifold of finite minds necessitates as many centres as there are possible positions to be occupied.

An important work of Nicholas of Cusa, the *Vision of God* (*De Visione Dei*), illustrates this point very clearly. In order to introduce the monks of Tegernsee Abbey to mystical theology, Nicholas asked them to hang a picture he had sent to them on their dining hall wall. The picture represents a face with the peculiarity of always fixing its eyes on the person looking at it, from any position. Observers on the left get the impression the face is looking at their side, while people on the right believe it is staring at them. People moving around the room see the gaze following them and stopping if they stop. Thus everyone has the experience of being looked at and has the impression of being the only one looked at that way. In others words, everyone thinks he is the centre of the world and he really is, although he has to accept the fact that others are centres too. By sharing their experiences through language, everyone realises that others are living through exactly the same things.

Finite minds can, therefore, only live in a many-centred reality. Inversely, minds allow the unfolding of the universe as such and the unfurling of the manifold elements it contains. Further developing this view would take us beyond our goal; we shall content ourselves with simply remarking that, for Nicholas, the mind plays an active role in the manifestation of the world as such and of the mathematical proportions connecting the elements to one another. Briefly, number plays a role practically equivalent to that of an a priori category in Kant. Everything the mind grasps is seen as governed by number and proportion, even if this proportion eludes every attempt at exactly determining it.

This dialectic of mind is considered most intensely by Nicholas of Cusa in the mathematical context of the problem of squaring the circle. We have already seen that mathematics is a privileged means of studying the human mind because mathematics is its highest production. Ordinary mathematics illustrate reason's capacities but Nicholas wonders whether it might not be possible to forge new mathematics which, for its part, would express the

[12][15].

virtues of human intellect, which, unlike reason, is able to rise to the grasp of a conjunction of opposites. In these works, he talks about a new mathematics, also about what he calls an intellectual mathematics, in which such squaring would be possible. But this question will be considered in the last part of this paper.

2.7. *The third dialectic: the two natures of Christ*

The third part of *Learned Ignorance* relates to Christ and faith. It is not a simple appendix but the crowning achievement of the first two dialectics. One might speak here of a dialectic of man which would complete the matter–form and the whole–parts sections. Man is destined to partake in Christ's life and hence to undergo a metamorphosis and divinisation of his nature. For Nicholas, this accomplishment promised to the human mind was already perceptible in man's transcending capacities, capable of transcending finite geometrical figures towards theological figures and the divine infinite of which the latter are signs.

Christ, both man and God, whose two natures subsist in the divine person of the Word, who possesses in himself all the properties of human nature and hence identifies himself with every particular human nature, should be considered the equivalent in human reality of what the theological figures are with respect to ordinary geometrical figures.

$$\frac{\text{Christ}}{\text{Particular human nature}} = \frac{\text{Theological figures}}{\text{Geometrical figures}}.$$

This analogy alone strikingly summarises all the importance of mathematics in Nicholas of Cusa's thought and their propaedeutical role in acquiring true understanding of what man is capable of knowing and believing about the divine.

3. The problem of squaring the circle

This problem is one of the most famous in the whole history of mathematical science. It is above all renowned for having retained the attention of generations of mathematicians who tried to construct a square with a surface equal to that of a circle given at the outset, with the help of only a ruler and compass.

Archimedes[13] managed to demonstrate that the surface of a circle of radius R and of perimeter P was no greater or smaller than $RP/2$. Hence many concluded that the surface of the circle was therefore equal to that of a rectangle whose width is equal to the radius of the circle and the length of the half-diameter. The rectangle must then simply be transformed into a square, which is quite elementary, from a technical point of view.

This extrapolation of Archimedes' correct result is based on a postulate:[14] if there exists a square inferior in surface to a given circle, and if there exists a square superior to the same circle, there exists a square equal to it. Not everybody admitted that postulate and

[13] [5].
[14] [7, p. 37].

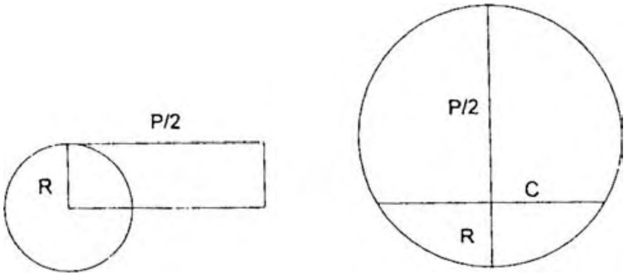

Fig. 5.

many maintained the incommensurability of rectilinear surfaces (generated by squares or, more generally, by polygons) and curvilinear surfaces (based on curves).

In a certain sense, this debate brought two views on equality to grips: either equality represents an actual quantitative identity, or one admits that a difference between two quantities is admitted as long as that difference is smaller than any rational fraction of the quantities concerned.

In squaring the circle, Nicholas saw a mathematical analogy to the coincidence of opposites as it exists in God or to the conjunction of opposites as the human intellect grasps it. He hoped to move the question along using principles from *Learned Ignorance*. In fact, his mathematical writings are those of a good amateur but do not really attain top level in rigour. Their interest is more philosophical than strictly mathematical.

Nicholas thinks the squaring is impossible on the level of ordinary mathematics.[15] His refusal of squaring plays an essential role here, comparable to his forbidding contradiction in formal logic. In others words, the older equality concept is the right one and there is no need to change it.

We nevertheless see Nicholas multiplying his attempts at practical approximations. There is a major paradox here: why spend so much time and energy on these many squaring attempts if the aim cannot theoretically be achieved? I suggest two paths for answering. Their value is quite conjectural:

1. According to the principles of learned ignorance, just because a truth may be inaccessible as such is not a reason not to try to approach it as closely as possible. That the essences of things, for instance, are unattainable does not stop human conjectures from being more and more exact, as we saw with the series of inscribed polygons or the geographer’s map drawing analogy. The problem with that explanation is that Nicholas’s practical approximations got worse and worse—rather than better—over time.
2. Practical approximations of squaring are valid only on the level of physical mathematics and not on the level of rational mathematics. But physical mathematics is the inverted reflection of intellectual mathematics, which is the principle, the ground of rational mathematics.

Many times in his works Nicholas mentioned the idea of intellectual mathematics, which is an extension of rational mathematics. The theological figures, dealing with the infinite

[15] [7, p. 40].

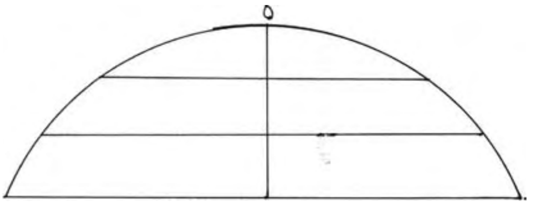

Fig. 6.

Fig. 7. Nikolaus von Kues (1401–1464) detail of the altarpiece in the chapel of the St.-Nikolaus-Hospital (Kues, Germany) founded by the philosopher.

figures we spoke of, belong to this intellectual mathematics, like the vanishing quantities obtained by going to the limit. For example, there is a difference between the length of an arc and the length of the corresponding chord. But as the height of the chord decreases, the difference between arc and chord decreases too. Finally both quantities vanish according to rational mathematics, but intellectual intuition grasps something that is not zero (we would speak today of the rapport, which is not equal to zero even if the terms are).

Clearly, intellectual mathematics deals with the infinitely great and the infinitely small. A coincidence or a conjunction of opposites exists there.

Where does this intellectual mathematics come from? It is distinct from ordinary mathematics in the sense that it is the light of the mind, thanks to which one can do ordinary mathematics. In Kantian terms, we might say that intellectual mathematics is the condition that makes possible ordinary mathematics.

Just as in the philosophy of nature, light is the source, the condition that makes possible different colours and, for that matter, contains all of them in itself in a state of unity, so intellectual light contains in its concrete unity all the figures and forms that are distinct for reason. Intellectual mathematics might be said to be nothing but new mathematics expressing the rules of these conditions of possibility of ordinary mathematics and allowing us to judge the truth or falsity of their propositions.

Nicholas insisted in saying that if squaring is impossible on the level of ordinary mathematics, it exists on the level of the light of the intellect and of the superior mathematics which is its expression. But it seems that this mathematics can only be studied indirectly, on the basis of physical mathematics.

As we saw in Figure 3, the intellectual world and the world of bodies are the inverse reflection of one another: unity includes in itself otherness within the intellect and otherness includes in itself unity in material bodies. Physical approximations may thus be signs pointing towards the true squaring, which is not to be found on the level of ordinary, rational mathematics but on that of the spiritual light making them possible.

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CHAPTER 15

Michael Stifel and his Numerology

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MATHEMATICS AND THE DIVINE: A HISTORICAL STUDY
Edited by T. Koetsier and L. Bergmans

1. Introduction

Michael Stifel (1486/87–1567) was born in Esslingen, Germany. As a young man he became a monk in the Augustine Order and in 1511 he became a priest. In the conflict between Luther and the Church he chose the side of the Reformation, left the monastery and became a minister. He spent two years in Tyrol but the persecution of the Lutheran movement forced him to move to Lochau (today called Annaberg), Germany not far from Wittenberg, Luther's residence. In 1532 Stifel anonymously published a booklet in Wittenberg under the title *Ein Rechenbüchlein vom Endchrist. Apocalypsis in Apocalypsim* (A Book of Arithmetic about the Antichrist. A Revelation in the Revelation). In the booklet Stifel used numerological arguments to show that the end of the world was very near and that the popes and the Catholic Church had to be identified with the Antichrist. After the publication Stifel continued his calculations and showed that the end of the world would take place on October 19, 1533, at 8 o'clock in the morning. Stifel gathered his faithful in the church and discovered that the world did not stop existing on this particular day. He was arrested by the authorities and spent several weeks in jail in Wittenberg. Luther, however, did not let down the man who had chosen his side at the very beginning of the Reformation and, although it took some time, in 1535 Stifel was again allowed to practise as a minister in Holzdorf, not far from Lochau. He never completely quit numerology but he clearly became more careful with respect to the interpretation of his calculations. Moreover, his interest in numbers led him to several original contributions to mathematics. In 1544 he published his main book, the *Arithmetica Integra*, printed by the man who had printed Copernicus's *Revolutiones* one year earlier. After the battle of Mühlberg in 1547, where the noblemen who had taken the side of the Reformation were defeated by Charles V, Stifel had to leave Holzberg. In 1559 he was teaching mathematics at the university in Jena.

In the history of sixteenth century German mathematics Stifel is a remarkable figure. His interest in numerology is sometimes depicted as a strange and irrational element in his work. We will show that this is a one-sided and superficial view. At the time, Stifel's numerological considerations were less strange than they appear to us now. In the sixteenth century there were many other men of learning engaged in similar calculations and it seems quite reasonable to view this work as a form of applied arithmetic that turned out to be sterile in the end.

2. The end of the world and the Antichrist

In the Bible the end of the world and the period preceding it are described on several occasions. At the very end Jesus Christ will return. Yet, preceding Christ's return, there will be signs announcing his coming. According to St. Luke, Jesus said:

> Nation shall rise against nation, and kingdom against kingdom:
> And great earthquakes shall be in divers places, and famines, and pestilences; and fearful sights and great signs shall there be from heaven. (Luke 21:10–11)

A remarkable figure associated with the nearing of the end of the world is the so-called Antichrist. The word “Antichrist” occurs only in Christian literature. The first mention is found in the Epistles of John. In 1 John 2 it says in verse 18:

> Little children, it is the last time: and as ye have heard that antichrist shall come, even now are there many antichrists; whereby we know that it is the last time.[1]

Understandably this text was associated with the following words that Paul the apostle wrote to the Thessalonians:

> Now we beseech you, brethren, by the coming of our Lord Jesus Christ, and by our gathering together unto him.
> That ye be not soon shaken in mind, or be troubled, neither by spirit, nor by word, nor by letter as from us, as that the day of Christ is at hand.
> Let no man deceive you by any means: for that day shall not come, except there come a falling away first, and that man of sin be revealed, the son of perdition.
> Who opposeth and exalteth himself above all that is called God, or that is worshipped; so that he as God sitteth in the temple of God, shewing himself that he is God. (2 Thessalonians 1–4)

The text is clear. Before the coming of Christ, a man of sin will be revealed, who will present himself as God. This “son of perdition” is often identified with the Antichrist announced by John.

In particular, the book Revelation contains many details about the events that will precede the end of the world. In Chapter 11 it says:

> And I will give power unto my two witnesses and they shall prophesy a thousand two hundred and threescore days, clothed in sack-cloth.
> These are the two olive trees, and the two candlesticks standing before the God of the earth.
> And if any man will hurt them, fire proceedeth out of their mouth, and devoureth their enemies: and if any man will hurt them, he must in this manner be killed.
> These have power to shut heaven, that it rain not in the days of their prophecy: and have power over waters to turn them to blood, and to smite the earth with all plagues, as often as they will.
> And when they shall have finished their testimony, the beast that ascendeth out of the bottomless pit shall make war against them, and shall overcome them, and kill them. (Revelation 11:3–7)

After three and a half days “the spirit of life from God” enters the two witnesses and they ascend to heaven. The witnesses represent the forces of good. The beast that kills them has often been interpreted as the Antichrist. Chapter 13 contains a similar prophecy:

> And I stood upon the sand of the sea, and saw a beast rise up out of the sea, having seven heads and ten horns, and upon his horns ten crowns, and upon his heads the name of blasphemy. [. . .]
> And there was given unto him a mouth speaking great things and blasphemies; and power was given unto him to continue forty and two months.
> And he opened his mouth in blasphemy against God, to blaspheme his name, and his tabernacle, and them that dwell in heaven.
> And it was given unto him to make war with the saints, and to overcome them: and power was given him over all kindreds, and tongues, and nations. (Revelation 13:1–7)

One notices in these texts the period of 1260 days equal to 42 months of 30 days. At the end of chapter 13 the number 666 occurs; the beast and its number are usually identified with the Antichrist:

[1] The Antichrist is also mentioned in 1 John 2, 22 and in 2 John 7.

Fig. 1. Dürer's depiction of St. Michael's battle with the dragon from Revelation 12:7–9.

> Here is wisdom. Let him that hath understanding count the number of the beast: for it is the number of a man; and his number is Six hundred threescore and six. (Revelation 13:18)

Although in the third century the Antichrist had already turned into the "one and only ultimate eschatological opponent of Jesus Christ" (Cf. [7, p. 1]), the medieval Antichrist tradition is very complex. It was developed from many sources and it varied from author to author. Nevertheless, according to Richard Kenneth Emmerson, it is possible to describe a widely accepted understanding of the Antichrist, which was from the tenth century on developed, modified and repeated throughout the later Middle Ages:

> Antichrist will be a single human, a man with devilish connections who will come near the end of the world to persecute Christians and to mislead them by claiming that he is Christ, he will be opposed by Enoch and Elias, whom he will kill, and will finally be destroyed by Christ or his agent [3, p. 7].

3. The autumn of the Middle Ages

In the *The Autumn of the Middle Ages* the Dutch historian Huizinga wrote about the fifteenth century:

> A human being who seriously contemplated the daily course of things and judged subsequently on life, only spoke of suffering and despair. This man saw how the time inclined towards the end [5, p. 35].

Such a sad opinion was strongly expressed not only by the religious, but by the chroniclers and the poets as well. In this context Huizinga quoted Deschamps:

> Times of sorrow and temptation,
> Ages of weeping, of desire and torment,
> Times of longing and damnation,
> Ages leading towards the end,
> Times full of horror that do everything wrongly
> I only see fools and folly,
> Times without honour and true judgement
> Age in sadness, that shortens life.[2]

The last centuries of the Middle Ages represented a fearful period, characterised by failing crops, epidemics and chronic misery caused by warfare. Wandering monks of the mendicant orders propagating the “memento mori” only reinforced the resulting feeling of despair. Moreover, at the end of the Middle Ages a new type of illustration entered the lives of many people: the woodcut, which spread the same message. Dürer’s woodcuts no longer depict the serene dignity of the figures of the Middle Ages. On the contrary, the fragility of human existence is omnipresent.

In Central Europe this state of mind reached a climax at the end of the fifteenth and at the beginning of the sixteenth century. It was a confusing period not understood by anyone: economic changes, usury, peasants revolting against the feudal order and citizens opposing the oligarchy in their cities. Moreover, the existing tradition of criticism of the Church and the clergy as for their attitude with respect to worldly goods and enjoyments reached an unknown level and intensity. In particular, the selling of indulgences by the Church represented to the critics a clear sign of degeneration. Last but not least there were the rumours about the Turks advancing in the southeast. What did it all mean? To many a mediaeval mind it seemed clear: at last the end of the world was really approaching.

The spirit of the times at the end of the fifteenth century is visible in the themes of many woodcuts. The Bible of Strasbourg of 1485 contains 17 illustrations, 8 of which concern the Apocalypse and in 1498 Dürer devoted a whole series of engravings to the Apocalypse. Dürer’s depiction of St. Michael’s battle with the dragon (Figure 1) still impresses the modern viewer. The dramatic impact on the mediaeval mind, not used to the multitude of images that we grow up with, must have been immense.

In the history of the Antichrist legend, images have always played an important role (Cf. [13]). This was not different at the end of the fifteenth century. The woodcut entitled *The Preaching and Fall of the Antichrist* from Schedel’s chronique of 1493 (Figure 2) gives us

[2] “Temps de doleur et de temptacion, Aages de plour, d’envie et de tourment, Temps de langour et de dampnacion, Aages meneur près du definement, Temps plains d’orreur qui tout fait faussement, Aages menteur, plein d’orgueil et d’envie, Temps sanz honeur et sans vray jugement, Aage en tristour qui abrege la vie” [5, p. 36].

Fig. 2. The preaching and the fall of the Antichrist (from Schedel's *Buch der Cronicken*, 1493).

in fact two scenes in one: a scene on earth and a scene in the sky. On earth, on the left, the Antichrist in the form of a man of learning is preaching to a crowd with a devil behind him whispering things into his ear; on the right the two witnesses (Revelation 11) are preaching to another crowd. In the sky the fall of the Antichrist is depicted. The Antichrist is depicted in a different way in the woodcut entitled *The Antichrist* from Brant's *The Boat of Fools* (Stultifera Navis) of 1498 (Figure 3). The woodcut was probably done by Dürer. The Antichrist is sitting on top of a capsized boot; following him leads to catastrophe. In the front St. Peter is pulling a boat full of the faithful to the safe shore with his key.

4. Stifel's numerology in 1532

Richard K. Emmerson wrote:

> "Early Protestants felt that they were living on the verge of Doomsday, for the millennium was in the past, the two witnesses—oppressed for 1260 years—had been resurrected, the beast had been wounded, and only the return of Christ remained in the future. This expectation of the imminent

Fig. 3. The Antichrist as master of the fools (From Brant's *Stultifera navis*, 1489).

> Second Advent led particularly to an increasing desire to determine the time of the end [. . .]" [3, p. 209].

Clearly, in his conviction that the end of the world was approaching Stifel was not alone. Moreover, before Luther the position of the papacy had never been called into question. Of course, late mediaeval opponents of the papacy had argued that its worldliness and degeneracy were proof of the last days and the work of the Antichrist. Yet, this criticism did not concern the church as an institution. However, when Luther broke with the church the identification of the institution of the papacy with the Antichrist seemed very natural, from the point of view of the Reformation.

In his booklet (Figure 4) Stifel constructed a correspondence between words on the one hand, and numbers on the other hand, by means of three methods. Stifel used Hindu-Arabic digits to denote numbers but he still calculated on the board (Figure 5).

Method I

One restricts oneself in a word or a phrase to all characters that have a numerical value in the Roman system. For example, consider the text

Iesvs Nazarenvs Rex Ivdeorvm

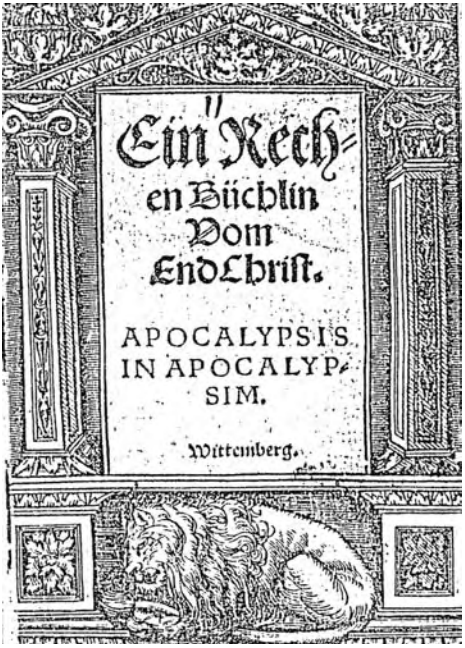

Fig. 4. Frontispiece of Stifel's *Endchrist*.

In the Roman system we have I = 1, V = 5, X = 10, L = 50, C = 100, D = 500 and M = 1000. If we put these values on the board we get 1532. According to the preface of his booklet Stifel at first did not consider relations such as the one constructed here between Jesus and the year 1532 too seriously. However, he changed his mind. The number of interesting discoveries must have made him change his mind. Another example: Take the name Leo Decimvs, pope from 1513 until 1522. The Roman characters corresponding with numbers are L, D, C, I, M, V. Adding them we get 1656, which in itself looks rather meaningless. On the board, however, 1656 is not much different from 666; all that is required is that we move one coin—the one that is on the thousands line—to the tens line. The fact that there are ten letters in Leo Decimvs and the fact that the M is the first letter of "mysterium" gave Stifel an argument to assume that in order to get the true number corresponding to Leo Decimvs the M had to be interpreted as 10. Obviously then, Leo Decimvs is the beast announced in the book of Revelation. It is interesting to read Stifel's account of this last discovery, written many years later. He describes how worried he was that the text in Revelation 21:8: "But the fearful, and unbelieving [. . .] shall have their part in the lake which burneth with fire and brimstone: which is the second death" would apply to him, fearful as he felt. Then, one day in the library he attempted to calculate the number of the beast in the described way. As for the "trick" with the M, he wrote: "I figured that M might mean "mysterium", went onto my cell, kneeled down and prayed to God about this matter. However, I did not pray for long; or I received such a consolation, that it consoles me even today, whenever I think about it. And after that I was no longer so fearful and despondent as I was before and from that time on I have always loved the Revelation of John" ([6, pp. 531–532]).

Fig. 5. Frontispiece of Jakob Köbel's textbook on calculating on the board, Augsburg, 1514.

Method II

The second numerological method that Stifel used is based on the order of the letters in the alphabet.

$$a \to 1,\quad b \to 2,\quad c \to 3,\quad d \to 4,\quad e \to 5,$$
$$f \to 6,\quad g \to 7,\quad h \to 8,\quad i \to 9,\quad k \to 10,$$
$$l \to 11,\quad m \to 12,\quad n \to 13,\quad o \to 14,\quad p \to 15,$$
$$q \to 16,\quad r \to 17,\quad s \to 18,\quad t \to 19,$$
$$v \to 20,\quad x \to 21,\quad y \to 22,\quad z \to 23.$$

Stifel illustrated the method using the name of the reformer Husz. He wrote

> "Nim nv zvm Exempel dieses Wort Hvsz/leg fvr das h 8, fvr das v 20,
> fvr das s 18 vnd fvr das z 23. So machet dir das gantz wort 69".

In English: "Take for example this word Husz/lay for the h 8, for the v 20, for the s 18, and for the z 23. So make yourself the whole word 69." It is clear from the language that Stifel was doing the addition on a calculating board. Stifel wrote that he entertained himself for some time with these calculations and had sometimes found things that amazed him. However, he did not succeed in calculating the number of the beast in this way and that is why he invented a third numerological method.

Method III

The first two methods were already very old in Stifel's time. The third numerological method could very well be an original contribution to numerology by Stifel. It is based on the sequence of so-called triangular numbers 1, 3, 6, 10, etc. This sequence is closely related to the sequence of the natural numbers and can be calculated from it as follows: the nth triangular number = the sum of the first n natural numbers. We get the following correspondence.

$$
\begin{aligned}
&a \to 1, \quad b \to 3, \quad c \to 6, \quad d \to 10, \quad e \to 15,\\
&f \to 21, \quad g \to 28, \quad h \to 36, \quad i \to 45, \quad k \to 55,\\
&l \to 66, \quad m \to 78, \quad n \to 91, \quad o \to 105, \quad p \to 120,\\
&q \to 136, \quad r \to 153, \quad s \to 171, \quad t \to 190,\\
&v \to 210, \quad x \to 231, \quad y \to 253, \quad z \to 276.
\end{aligned}
$$

Most of the calculations in Stifel's booklet are based on the triangular numbers. Stifel applies his third method to calculate the numerical value of words and phrases. Subsequently he shows that the resulting numbers, combined with the numbers denoting special years in the history of Christianity, and special numbers from biblical texts, like 1260, 666 and others, are related arithmetically in rather simple ways. The relations all take the form of an equality which, if the original words are again substituted, express, at least from Lutheran point of view, important truths. Before we will consider some examples it is necessary to know that in the history of the Church Stifel distinguished three periods.

(i) The period of the spiritual popes from Petrvs to Leo IV (847–855).

(ii) The period of the secular popes from Johannes VIII (856–???) to Leo X (1513–1522).

(iii) The period of the "furious" popes from Adrianvs VI (1522–1523) to Clemens VII (1523–1534).

N.B.: Clemens VII was still alive when Stifel published his booklet.

From the Lutheran point of view things went terribly wrong with Johannes VIII. According to Stifel he was a woman dressed like a man. Feminists might enjoy this possibility but Stifel did not. Moreover, one of the primary blasphemies of the secular and furious popes concerns the free will, the *libervm arbitrivm*. For both the reformers and the Catholic Church man is a sinner who cannot be saved without Divine Grace. However, while, according to Catholicism man can choose between accepting Divine Grace and refusing it, according to the reformers salvation depends exclusively on God. From the point of view of the Lutherans this Catholic belief in the role of the free will was a major blasphemy.

Stifel used neither characters for arithmetical operation nor the equality sign. We will sometimes use the equality sign. By means of the third numerological method we can now show that

Johannes Octavvs equals Libervm arbitrivm, because the numerical value on both sides of the equality sign is 1448. Because Petrvs = 859 and Leo Qvartvs = 1257, we have

Petrvs minus 856 plus Leo Qvartvs equals 1260.

Here 856 is the year in which Johannes VIII became pope. At the same time we have

1260 = Ira revelabitvr (the rage revealed) and Cessatio Papatvs = 1517.

The year 1517 is the year of the beginning of the Reformation.

In the Old Testament in the book of Daniel the numbers 1290 and 1335 occur.[3] Using Martinvs Lavter = 1574 and Romanvs Papa = 1051, Stifel deduced:

1290 plus 1335 equals Martinvs Lavter plus Romanvs Papa.

The result suggests a special relation among Luther, the pope and two special biblical numbers.

Stifel enlarged the scope of his investigation by allowing other arithmetical operations than addition and subtraction. He allowed the repetition of words as a sign of emphasis. Moreover, he uses an operation that he calls "duplatio". The duplatio of a number requires adding 3 to the number and subsequently doubling the result. So, because Leo Decimvs = 721 and Johannes Octavvs = 1448 and 2 times (721 + 3) yields 1448, Stifel gets

The duplatio of Leo Decimvs equals Johannes Octavvs.

Next to the duplatio Stifel uses the "mediatio": subtract 3 from a number and divide by 2. Stifel finds

The mediatio of 1335 equals 666.

At least once Stifel applies what we will call a "shift" on the calculating board. Such a shift is in fact a multiplication by 10. On page F6 he says: "Lay down Ira Dei and move all the coins one line forward and you will find Adrianvs Sextvs et Visio Danielis" And indeed, because Ira Dei = 269, Adrianvs = 1670, Danielis = 1020, and 10 times 269 equals 1670 + 1020, we have

Shift of Ira Dei equals Adrianvs plus Danielis.

Luther is reported to have told the story that Johannvs Husz, the Czech precursor of the Reformation, when they were about to burn him, said: "You can roast the goose, but in a hundred years the swan will sing". The name of Husz (Czech for *goose*) and the word Cygnus (Latin for *swan*) occur frequently in Stifel's calculations. For example, he derived

[3] "And from the time that the daily sacrifice shall be taken away, and the abomination that maketh desolate set up, there shall be a thousand two hundred and ninety days" (Daniel 12:11).
"Blessed is he that waiteth, and cometh to the thousand three hundred and five and thirty days" (Daniel 12:12).

among others the following numerological results (N.B. 1518 was the year in which Luther refused to withdraw his theses).

> 1518 plus Visio Danielis minus Johannes Husz equals 1290,
>
> Johannes Hvsz minus Visio Danielis + Johannes equals Cygnvs plus 1,
>
> Johannes Hvsz plus Cygnvs equals Leo Decimvs plus Antichristvs.

The reader who tries to check these calculations will find that Stifel made a mistake in the calculation of the value of Cygnvs ($= 759$). Stifel must have calculated by mistake the value of Czgnvs ($= 782$). Because he used this value repeatedly all results concerning Cygnvs are wrong.

At the end of the booklet there is a list of important words and combinations of words in alphabetical order with their numerical values on the basis of the third method. Stifel probably obtained many of his results on the basis of this list, simply trying combinations of numbers and operations.

After the publication of his book, Stifel continued his calculations and at a certain moment convinced himself of the fact that the end of the world would take place on October 19, 1533, at 8 o'clock in the morning. Unfortunately we do not know how he did this. It has been suggested that Stifel calculated the year 1533 using a phrase from John 19:37: "VIDebVnt In qVeM transfIXerVnt" ("They shall look on him whom they pierced").[4] The fact that Sunday, October 19, 1533 is the first day immediately after the 42nd week of the year 1533 may have played a role in Stifel's arguments as well.

5. Stifel's mathematics

Stifel was clearly a man who loved to calculate; he was also good at it. We do not know anything about Stifel's early mathematical development. There are indications that already in 1532 when he wrote his booklet his interest went beyond mere calculation. This is suggested by the application of triangular numbers but also by the occurrence of a magic square in the book. At the end of chapter XVI we find an example of a square consisting of the first 36 natural numbers arranged in such a way that the square is magic: the sum of the number in a row, in a column and on a diagonal equals 111 (Figure 6). Why is the square there? Probably only because the sum of all the numbers in the square is 666. It is, however, mathematically interesting as well.

Let us compare the square with the magic square on the well known etching of Dürer, *Melencolia*, done in 1514, the year in which Dürer's mother died (Figure 7).

If we take the four-by-four square in the centre of Stifel's square and substract 10 we get a slight variation on Dürer's square.[5] Stifel must have obtained his square the other way

[4]This calculation is given by Marcus Friedrich Wendelin in his *Cosmologia* (1648), Section 2, p. 349.

[5]The four by four square in the centre of Stifel's square is identical with Dürer's square if the second and third (horizontal) row are interchanged.

16	3	2	13
5	10	11	8
9	6	7	12
4	15	14	1

36	31	7	8	27	2
3	26	13	12	23	34
4	19	16	17	22	33
5	15	20	21	18	32
28	14	25	24	11	9
35	6	30	29	10	1

Fig. 6. The magic square in Dürer's *Melancholia* and the magic square from Stifel's *Enchrist*.

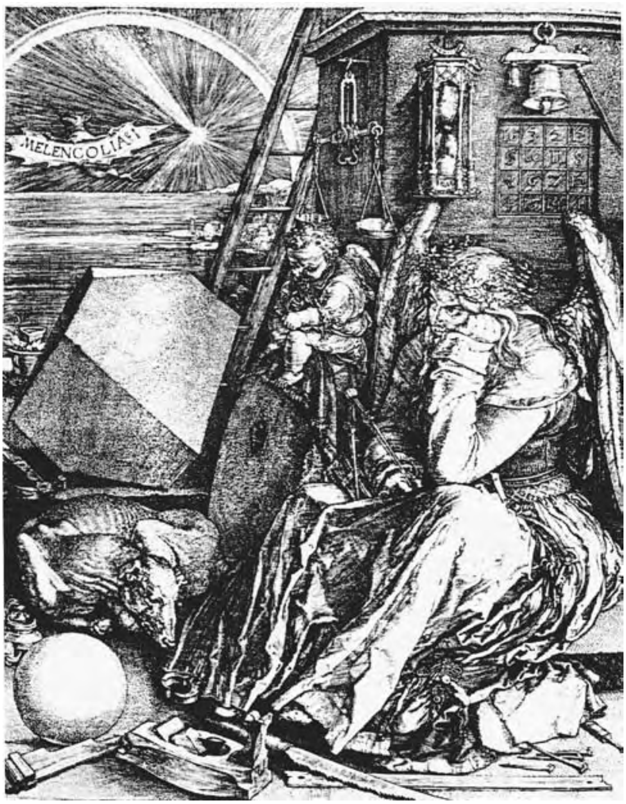

Fig. 7. Dürer's *Melancholia*.

around. He must have wondered whether the four-by-four square cannot be turned into a six-by six-square by adding a border. For the resulting square we then need the numbers

$$1, 2, 3, 4, \dots, 10\ [11, 12, 13, \dots, 25, 26]\ 27, \dots, 34, 35, 36.$$

The numbers 11 through 26 are the numbers 1 through 16 all raised by 10. With them we can make a magic square. Let us put them in the four-by-four centre square of an empty six-by-six square. Moreover, we have $1 + 36 = 37$, $2 + 35 = 37$, $3 + 34 = 37$, etc. This means that we can fill up the border by completing the rows columns and diagonals with the pairs $1, 36$ and $2, 35$ etc. in such a way that the sum of the numbers in the rows, columns and diagonals becomes 111. It turns out that this can be done in such a way that the resulting square is magic.

Stifel's major mathematical work is the *Arithmetica Integra* of 1544. It consists of three books. In the first book we find Pythagorean number theory like the theory of triangular numbers, etc. However, there is also a beautiful and original theory of certain magical squares, based on a generalisation of the idea that had led him from Dürer's magic square to the one in the *Endchrist*. The result is, for example, the 16-by-16 square of Figure 8. It is constructed from Stifel's six-by-six square by adding five more borders. For the details I refer to Joseph Hofmann [4].

256	9	247	246	12	13	243	242	16	17	239	238	20	21	235	2
3	226	213	45	46	210	209	49	50	206	205	53	54	201	32	254
4	33	200	63	193	192	66	67	198	188	70	71	185	58	224	253
252	34	59	178	169	89	90	166	165	93	94	161	80	198	223	5
251	222	60	81	160	101	155	154	104	105	151	98	176	197	35	6
7	221	196	82	99	146	141	117	118	137	112	158	175	61	36	250
8	37	62	174	100	113	136	123	122	133	144	157	83	195	220	209
23	38	73	173	107	114	129	126	127	132	143	150	84	184	219	234
24	218	183	85	108	115	125	130	131	128	142	149	172	74	39	233
232	217	75	86	148	138	124	135	134	121	119	109	171	182	40	25
131	41	76	87	147	145	116	148	139	120	111	110	170	181	216	26
27	42	180	162	259	156	102	103	153	152	106	97	95	77	215	230
28	43	179	177	88	168	167	91	92	164	163	96	79	78	214	229
228	202	199	194	64	65	191	190	68	69	187	186	72	57	55	29
227	225	44	212	211	47	48	208	207	51	52	204	203	56	31	30
255	248	10	11	245	244	14	15	241	240	18	19	237	236	22	1

Fig. 8.

The second book of the *Arithmetica Integra* is devoted to Euclid's theory of irrationalities. The third book of the *Arithmetica Integra* deals with the "Coss", i.e. algebra. At the time it was common practice among "cossists"—as algebrists were often called, because of the Italian word *cosa* (thing) denoting the unknown—to distinguish three different types of quadratic equations that were then solved by means of three analogous but different algorithms. According to Hofmann Stifel was (one of) the first mathematicians to realise that if the right rules for the addition, multiplication and division of negative numbers are used, the three different types turn out to be special cases of one general quadratic equation that can be solved by means of one algorithm [4, p. 31]. Because this is an important point we will illustrate it. Figure 9 shows part of a page from Stifel's last book, a revised and extended edition of an earlier book on algebra, *Die Coss* by Christoff Rudolff, 1525 [12].

Der andern vnderſchid. 266

Vom extrahiren der quadrat wurtzeln auſs ſollichen Coſſiſchen zalen.

240 — 8 𝔵
240 + 8 𝔵
20 𝔵 — 96

Erſtlich/ den halben teyl der zal die das zeychen 𝔵 hat multiplicir in ſich ſelbs/ vnd hab acht auff das zeychen — oder +. Denn du ſolt wiſſen das — vnd —. im multipliciren machet + (gleych ſo wol als + vnd +). Nu ſprich ich im erſten exemplo — 4. mal — 4. Facit + 16. Gleych wie ich im andern exemplo ſprich + 4 mal + 4. Facit + 16. vnd weyſet mich alſo das zeychen + in beyden exempeln auff das addiren. Drumb addir ich in beyden Exempeln die 16 zu der ledigen zal deſs geſetzten exempels/ nemlich zu 240. Facit 256. Darauſs extrahir ich radicem quadratam/ facit 16. Da von ſubtrahir ich im erſten Exemplo 4. vnd im andern Exemplo addir ich 4 zu 16. Drumb kompt im erſten Exemplo die geſuchte zal 12. aber im andern Exemplo komt 20.

Das ich aber zu letſt die 4 ſubtrahir von 16 iſt die vrſach das 4 im erſten exemplo ſteht bey dem zeychen — als der halbe teyl der zal ſo das zeychen 𝔵 hat.

Aber im andern Exemplo ſtehn die 4 bey dem zeychen + darumb addir ich hie.

Vom

Fig. 9.

The text says:

> About the extraction of square roots from such Cossic numbers as
>
> $240 - 8x$,
>
> $240 + 8x$,
>
> $20x - 96$.
>
> First, multiply half of the number that has the character x by itself/and watch the sign $-$ or $+$. Then you should know that $-$ and $-$ multiplied yield $+$ (just as $+$ and $+$). In the first example I take -4 times -4. Facit $+16$. Just as I get in the other example $+4$ times $+4$. Facit $+16$ and the sign $+$ points in both examples at an addition. That is why I add in both examples the 16 to the isolated number of the example, namely 240. Facit 256. From this number I extract the square root/facit 16. From this number I substract in the first example 4 and in the other example I add the 4 to 16. That is why in the first example the wanted number is 12, but in the other example 20.

Stifel did not yet admit negative roots and this means that his treatment of quadratic equations is not yet completely uniform. His treatment of the first two cases boils in modern notation down to the following rule: if $x^2 = p + qx$ and p and q have opposite signs, then $x = q/2 + \sqrt{(q/2)^2 + p}$.

6. Concluding remarks

The application of calculations to the prophetic interpretation continued to fascinate Stifel until his death. Some of his results are indeed remarkable. Stifel's 1532 booklet was not an isolated incident in his life. Twenty years later he came up with the following diagram in which the six numbers from the Book of Daniel and the Book of Revelations are related to the sentence "Christ, by faith alone, is for the elect the God of salvation eternal". It was found by Stifel [1, p. 191]. The calculations are based on the triangular numbers:

Christvs sola fide $(= 1416)$,

  $2300\ (= 1416 + 884)$,

Est Electis $(= 884)$,

  $1290\ (= 884 + 406)$,

Devs $(= 406)$,

  $1260\ (= 406 + 854)$,

Salvtis $(= 854)$,

  $1335\ (= 854 + 481)$,

Aeternae $(= 481)$,

  $666\ (= 481 + 185)$,

Amen $(= 185)$.

Neither was Stifel isolated in his interest in numerology. By the middle of the century it was common for Lutherans to offer "cabbalistic" support for the belief that with the Reformation had begun the Last Times. Barnes gives several examples of continental authors ([1], Ch. 5). Brady [2] is a good source for British authors. Some like Johannes Faulhaber of Ulm combined real mathematical talent with eschatological fervour. Faulhaber wrote many mathematical texts dealing to some extent with eschatological mysteries (see Chapter 16 of the present volume).

Another example of a mathematician involved in eschatological calculations was John Napier, Laird of Merchiston and inventor of logarithms. He published his *Plain Discovery of the Whole Revelation of Saint John* in 1593. Napier's book became quite popular. It was reprinted often and translated into Dutch, French and German.

It is interesting to compare briefly Stifel's 1532 booklet with Napier's book. Napier's book is more a theological treatise in which the Book of Revelations is interpreted. The numbers occurring in the book are interpreted as well but calculations play a much more limited role than in Stifel's work. Napier's book consists of two treatises; the first contains a "searching of the true meaning of the Revelation [...] after the manner of propositions". A few examples of propositions give a good idea of Napier's style.

1. Proposition. "In the propheticall dates of daies, weeks, moneths and years, euery common propheticall day is taken for a yeare".

3. Proposition. "The starre and locusts of the fift trumpet, are not the great Antichrist and his Cleargie, but the Dominator of the Turkes and his armie, who began their dominion, in anno Christi 1051".

4. Proposition. The Kings of the East, or foure Angels, specified in the sixt trumpet or sixt vial. cap. 9 &16 are the foure nations, Mohametanes beyond and about Euphrates, who began their empire by Ottoman, in the year of Christ, 1296 or there about".

The identification of the years 1051 and 1296 is based on a comparison of the events described in the Revelation with actual historical happenings. Because $1296 - 1051 = 245$, we have:

5. Proposition. "The space of the fift trumpet or vial, containeth 245 years, and so much also, euery one of the rest of the trumpets or vials doe containe."

12. Proposition. "The first of the seuen thunders, and the seuenth and last Trumpet or vial, begin both at once in Anno 1541."

The year 1541 is based on propositions 3 and 5. It is 1051 plus the spaces of the fifth and sixth trumpet, i.e. 1051 plus twice 245.

14. Proposition. "The day of Gods Iudgement appears to fall betuixt the yeares of Christ, 1688 and 1700."

The year 1700 is based on a simple addition. The "time that the daily sacrifice shall be taken away" (Daniel 12:11) is identified with the year 365. The 1335 days in Daniel 12:12 are taken as 1335 years and $365 + 1335$ yields 1700. According to Napier days should always be taken as years (see Proposition 1). The year 1688 is obtained by taking the year 1541 (see Proposition 12) and adding 3 times a period of thundering angels of 49 years [8, pp. 26–27].

23. Proposition. "The whore who in the Reuelation is stiled spiritual Babylon, is not reallie Babylon, but the very present citie of Rome."

24. Proposition. "The great ten-horned beast, is the whole bodie of the Latine Empire, whereof the Antichrist is a part."

26. Proposition. "The Pope is that only antichrist, prophecied of, in particular."

29. Proposition. "The name of the beast expressed by the number of 666 is the name λατεινοσ."

32. Proposition. "Gog is the Pope, and Magog is the Turks and Mahometans."

As for the interpretation of words as numbers Napier does not go beyond the possibilities offered by the Greek Ionic numeral system: the 24 letters of the alphabet extended with three obsolete symbols all had a numerical value and were used to denote numbers. For example, the 29th Proposition contains the following argument: 666 stands for Latinus or λατεινοσ, "for λ is 30. α is 1. τ is 300. ε is 5. ι is 10. ν is 50. ο is 70. And σ is 200 which altogether make sixe hundreth three score and sixe" [8, p. 69]. Moreover, in the Greek New Testament the number 666 is written in this way χξζ, for $\chi + \xi + \zeta = 600 + 60 + 6 = 666$ stood for the German Empire of his time. χ is the Greek cross, ξ is the same as the Latin X, again a cross, and the obsolete symbol stigma ζ or στ is also the beginning of the Greek word σταυροζ (cross). So 666 denoted the institution that was in every way abusing the cross in their "rings", the Catholic Church. Napier also declared that χξζ are not accidentally precisely the major initial letters in the phrase χαραμα του ξυλου ζαυρου ("character ligni crucis: the marke of the tree of the cross" [8, p. 69].

This brief comparison of Stifel's booklet with Napier's treatise shows that it was not at all uncommon for religious men with a talent for arithmetic to relate the Biblical numbers to the historical events of their time by means of calculations.

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CHAPTER 16

Between Rosicrucians and Cabbala—Johannes Faulhaber's Mathematics of Biblical Numbers

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MATHEMATICS AND THE DIVINE: A HISTORICAL STUDY
Edited by T. Koetsier and L. Bergmans

1. Introduction

Johannes Faulhaber was born the son of a weaver in Ulm in 1580. After an apprenticeship in Ulm with David Selzlin, a teacher of reading, writing and arithmetic, Faulhaber married and opened his own school as a *Rechenmeister* (teacher of arithmetic). In 1604 his first book appeared, a collection of cubic (third degree) problems entitled *Arithmetischer Cubicossischer Lustgarten* (Arithmetic Cubicossic Pleasure Garden), through which Faulhaber wanted to distinguish himself as a provider of methods to solve cubic equations, a hot topic after the appearance of Cardano's *Ars Magna* in 1545. So far nothing suggested the religious raptures and the heterodox views of the apparently very ambitious Faulhaber that would later involve him in numerous conflicts with the municipality and the representatives of the Protestant church in Ulm. Such a change became visible only in 1606 when he began to associate himself with the landlord of his court, the sanctimonious baker Noah Kolb. Kolb suggested to Faulhaber that he (Faulhaber) possessed an enlightenment deriving directly from God and he was strengthened in this conviction by his confessor, Johann Bartholome, a preacher at the minster, who, according to a contemporary report, "ordained and initiated [Faulhaber] by means of numerous ceremonies, prayers, incantations and the like, kneeling to one of the Latter-day prophets".[1] After this Faulhaber announced in Ulm, Memmingen, Augsburg and Hamburg the imminent (according to him) Day of Judgment. As a result of this he was imprisoned in the tower at the end of 1606 because of his fantastic ideas, together with Kolb. Because Faulhaber's wife was pregnant he was released quickly but for some time seriously limited in his freedom of movement and his contacts with others.[2] Officially no one was allowed to visit Faulhaber. In 1611 the new preacher at the minster, Peter Hueber, denied Faulhaber Holy Communion, because of, among other things, a suspicion of sorcery.[3] Because of his continuing contacts with Kolb, whom he also supported financially, Faulhaber was publicly reprimanded in 1613 and all contacts with Kolb were forever forbidden. Kolb was executed in 1615, in particular because of his confession—repeated in several interrogations, some under torture—of having committed fornication with several women and children; Faulhaber was one of the people interrogated in connection with Kolb's trial.

2. Pyramidal numbers and the Bible

The fantastic ideas (*Fantastereyen*) that Faulhaber was accused of were essentially connected with his mystic-cabbalistic number speculations about biblical numbers, which were in their turn based on the formulae that he had adopted and developed for the determination of polygonal and pyramidal numbers. Several of the 160 cubic problems in his *Arithmetischer Cubicossischer Lustgarten* concerned polygonal and pyramidal numbers, though no connection with biblical numbers was established. In this book Faulhaber had without further explanation introduced polygonal numbers—well known from the Greek

[1] Hermann Keefer, Johannes Faulhaber, der bedeutendste Ulmer Mathematiker und Festungsbaumeister des 17. Jahrhunderts, in: *Württembergische Schulwarte* 4, 1928, pp. 129–141, in particular p. 134.

[2] Report of the city council of Ulm for December 24, 1606.

[3] Report of the Kirchenbaupflegamt for July 4, 1611.

Fig. 1. Johannes Faulhaber in an engraving from 1630 (Courtesy of Birkhäuser Verlag).

tradition—and the pyramidal numbers based on them, in the manner of the German coss-tradition,[4] in order to get to the cubic problems via pyramidal numbers.[5]

Polygonal numbers are the sums of the first n terms of first order arithmetic sequences with first term 1 and difference d. The names of polygonal numbers depend on d. The Greek numeral that is used to denote the numbers corresponds exactly to $d + 2$. When $d = 1$ we get triangular (or triagonal) numbers; when $d = 2$ we get quadrilateral (or tetragonal) numbers, etc. When $d = 1$ the successive triagonal numbers, 1, 3, 6, 10, etc., can be read off the base of the sequence of nested isosceles triangles depicted on the left side in Figure 2, starting from the left vertex. If the sequence of nested isosceles triangles is extended with a similar sequence like that on the right side of Figure 2, again starting from the left vertex, we can now read off the quadrilateral numbers from the base: 1, 4, 9, 16, etc. In the terminology of the German *Rechenmeister* according to Faulhaber n had to be called the *square root* of the polygonal number and the nth term of the arithmetic progression, $1 + (n - 1)d$, was called the *polygonal root*.[6] The polygonal root equals the number of points on the nth gnomon in the sequence of polygons counted from the initial point.

[4]See Ivo Schneider, Textbooks of German Reckoningmasters in the Early 17th Century. In: *Journal of the Cultural History of Mathematics* 2, 1992, pp. 47–52, and Der Einfluß der griechischen Mathematik auf Inhalt und Entwicklung der mathematischen Produktion deutscher Rechenmeister im 16. und 17. Jahrhundert. In: *Berichte zur Wissenschaftsgeschichte* 23, Heft 2, 2000, pp. 203–217 (= *Nach oben und nach innen—Perspektiven der Wissenschaftsgeschichte, Festschrift für Fritz Krafft zum 65. Geburtstag* (ed. by Ulrich Stoll and Christoph J. Scriba).

[5]See Ivo Schneider, *Johannes Faulhaber (1580–1635)—Rechenmeister in einer Welt des Umbruchs*, Birkhäuser, Basel, 1993, Section 2.2.3.

[6]Faulhaber does not write n but uses the cossic character for the unknown, which is used here as a symbol for an arbitrary natural number.

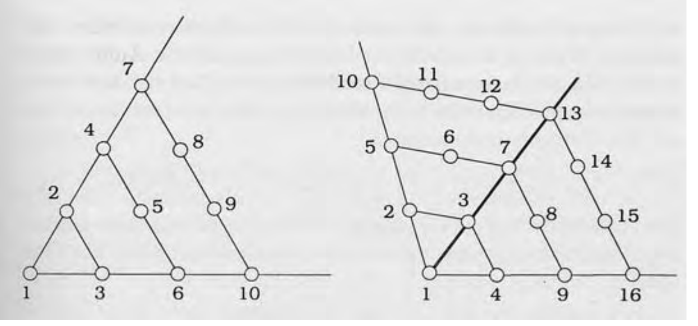

Fig. 2. Triangular and quadrilateral numbers.

The polygonal number itself is the totality of points in a polygon with a total length of the edges n and it equals

$$\sum_{i=1}^{n}\bigl[1+(i-1)d\bigr]=n+\binom{n}{2}d=\binom{n}{1}+\binom{n}{2}d.$$

When for a given d the polygonal numbers corresponding to all square roots from 1 to n are added, one gets the total number of points in the pyramid made by putting the associated polygons of points on top of each other in such a way that the result resembles a pyramid of cannon balls. That is why these numbers are called pyramidal numbers. Their value is

$$\sum_{j=1}^{n}\left[\binom{j}{1}+\binom{j}{2}d\right]=\binom{n+1}{2}+\binom{n+1}{3}d=\frac{dn^3+3n^2+(3-d)n}{6}.$$

In later works, in particular in his *Miracula Arithmetica* of 1622[7] and in his *Academia Algebrae* of 1631,[8] starting from figurate numbers like polygonal numbers and pyramidal numbers, Faulhaber introduced other solid numbers, like prismatic or dodecahedronal numbers. This led to arithmetic progressions of higher order and their sums, in particular sums of powers of natural numbers up to the exponent 17, for which he gave general formulae. He applied various methods, among others a calculus of differences to get his formulae.[9]

[7] Johannes Faulhaber, *Miracula Arithmetica. Zu der* Continuation *seines Arithmetischen Wegweisers gehörig.* Augsburg 1622 at David Franck.

[8] Johannes Faulhaber, Academia Algebrae. *Darinnen die miraculossiche Inventiones/ zu den höchsten Cossen weiters* continuiert *vnd* profitiert *werden. Dergleichen zwar vor 15. Jahren den Gelehrten auff allen* Vniversiteten *in gantzem Europa* proponiert, *darauff* continuiert, *auch allen* Mathematicis *inn der gantzen weiten Welt* dediciert, *aber bißhero/ noch nie so hoch/ biß auff die regulierte/* Zensicubiccubic *Coß/ durch offnen Truck* publiciert *worden. Welcher vorgesetzet ein kurtz Bedencken/ Was einer für Authores nach ordnung gebrauchen solle/ welcher die Coß fruchtbarlich/ bald/ auch fundamentaliter lehrnen vnd ergreiffen will.* Augsburg 1631 at Johann Ulrich Schönigk.

[9] See Ivo Schneider, *Johannes Faulhaber (1580–1635)—Rechenmeister in einer Welt des Umbruchs*, Birkhäuser Basel 1993, chapters 5 and 7.

Invariably the seven biblical numbers 2300, 1290, 1335, 666, 1260, 1600 and 1000, repeatedly interpreted figuratively, played a special role. The fact that the great mathematicians and philosophers of antiquity did not know anything about the biblical numbers, or at least did not express themselves on the interpretation of them, was understood by Faulhaber as an indication of God's intention to hide such information until his time.[10]

His claims to be able to reveal the intentions of God by means of the interpretation of the biblical numbers became explicit, at the latest, with his *Newer Mathematischer Kunstspiegel* (The New Artistic Mirror of Mathematics). In addition to the description of an instrument for surveying and a special pair of compasses, this book contains speculation about the biblical numbers. A Latin translation by Johannes Remmelin, a physician and pupil of Faulhaber's, appeared in the same year.[11] Both publications prompted the representatives of the Church authorities to object to the number speculations contained in them, as had happened on earlier occasions. The printer of both treatises in Ulm was subsequently urged to print no more work by Faulhaber or Remmelin in the future without permission of the council.[12] Apparently Faulhaber was only moderately impressed by the admonitions of the clerical and secular authorities in Ulm and in 1613 he published *Andeutung Einer vnerhörten newen Wunderkunst* (Indication of an Unheard New Miraculous Art), which was printed in Nuremberg and contained an interpretation of the biblical numbers as pyramidal numbers. In this book Faulhaber gave the most concise explanation of the sense of such number speculations. According to him, the biblical numbers are divine secrets and divine testimonies that occur continuously. Moreover, God Almighty has used them to fix all relationships and measures in nature.[13]

Specifically, Faulhaber wanted to represent the biblical numbers as pyramidal numbers, that is in the form

$$\binom{n+1}{3} d + \binom{n+1}{2},$$

where n and d are natural numbers $n > 1$. According to Faulhaber the biblical numbers are very special; they are specifically indicated by God. Excluding the trivial possibility $n = 2$ and $d = c - 3$ by means of which every natural number $c \geqslant 4$ can be represented as a pyramidal number, for a given biblical number b one should try stepwise whether for the numbers $n = 3, 4, 5, \ldots$ there is a corresponding natural number d by means of

[10] *Miracula Arithmetica*, p. 30.

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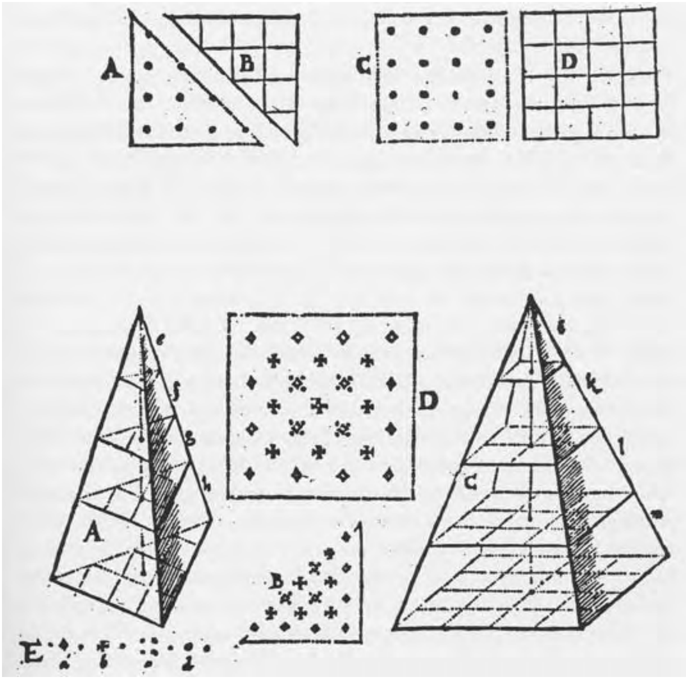

Fig. 3. Triangular, quadrilateral and pyramidal numbers as depicted in Johannes Remmelin's *Mysterium Arithmeticum* (Courtesy of Birkhäuser Verlag).

which b can be represented as a pyramidal number. Most of the biblical numbers treated by Faulhaber are divisible by 10. It is obvious that the choice of $n = 4$ yields the equation

$$10d + 10 = b \quad \text{or} \quad d = \frac{b - 10}{10}$$

which gives for each $b > 10$ divisible by 10 a natural number d. This representation does not hold for 666, which possesses (like all natural numbers $\geqslant 10$ that are not divisible by 4) for $n = 3$ a non-trivial representation as a pyramidal number, in this case with $d = 165$. Because there is for 666 only one non-trivial representation with $n = 3$ and $d = 165$, this representation of 666 as a pyramidal number is unique. A similar result holds for the other biblical numbers divisible by 10; they all permit a pyramidal representation of four levels with a unique d.

3. Gog and Magog

In the same year 1613 another book written by Faulhaber appeared, *Himlische gehaime Magia Oder Newe Cabalistische Kunst/ vnd Wunderrechnung/ Vom Gog vnd Magog* (Heavenly Secret Magic or New Cabbalistic Art and Miraculous Calculus about Gog and

Magog). It was published by Remmelin in Ulm and printed in Nuremberg. Faulhaber dedicated the book to the Emperor Matthias and sent him a copy as a well-timed present, before the beginning of the Reichstag. It is apparent from the dedication that Faulhaber expected protection from the Emperor against his opponents, in particular in his hometown. Three testimonies about technical inventions precede and hide the actual text. They were supposed to point out to the Emperor and other "potentates" that Faulhaber was an inventor and at the same time to identify Faulhaber as someone capable of interpreting the biblical numbers. They show that Faulhaber wanted to place his cabbalistic interpretations on the same level of usefulness and applicability as his technical inventions.

Faulhaber could have used the Emperor's protection that he expected for his book because the representatives of the clerical authorities in Ulm appeared rather angry about Faulhaber's new thrust into the realm of adventurous number speculations. They accused him of leaving the area of his actual competence as a mathematician and composing in an inadmissible way a prophecy "from letters, numbers and sealed words".[14] He was ordered to appear at the Dombauhütte in Ulm to be questioned and heard by the clergymen about the sense of his statements and the authorisation to use notions like "magic".[15]

In the preface and in the conclusion of his *Himlische gehaime Magia* (Heavenly Secret Magic) Faulhaber claims that, unlike those who believe they can decipher such divine secrets with their common sense, he himself had learned something about the secrets hidden in the "heavenly numbers" from God in person.

Here is a key to understanding the conflict between Faulhaber and his many critics. Until the end Faulhaber emphasised the intentionally esoteric character of the "biblical numbers". If God had wanted man to be able to discover the secrets by means of common sense, it would have been possible to decipher the numbers in the Book of Revelations much earlier. This however, at least in Faulhaber's view, was not the intention of God, who used particular chosen individuals to decipher the secret meaning of the biblical numbers. Such individuals had to be enlightened by God and, obviously, the result of the enlightenment lies outside the realm of human faculties. Faulhaber maintained that he belonged to those enlightened by God, and this raised him—in his own estimation—above all necessity to supply explanations for the knowledge to which he had gained access. To most of his contemporaries such a claim seemed unacceptable because they could not discover in Faulhaber much or indeed anything at all which would have justified the claim. They were moreover quite capable of giving everybody insightful explanations of the origin of Faulhaber's very vague utterances about the meaning of the biblical numbers. Faulhaber was thus depicted as a charlatan or a misguided religious fanatic.

In the text of *Himlische gehaime Magia* (Heavenly Secret Magic), a booklet of 10 pages, Faulhaber referred to the unsubstantiated necessity to assign by means of a "general key" to every "heavenly miraculous number its philosophical algebraic weight" and by doing so fix certain measures of time, from which predictions of the occurrence of important events can be deduced. He gave, moreover, a word calculus (*Wortrechnung*) relating not only to one but to four alphabets, which he used to decipher a biblical saying that he "observed through God's grace" in different texts of the Old and New Testaments.[16]

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[14] Report of the Kirchenbaupflegamt for 2.09.1613.
[15] Report of city council for 10.09.1613.
[16] "durch Göttliche Gnad observieret."

Only in 1619, the year in which the Emperor Matthias died, did Remmelin publish this saying together with the required word calculus in an apology for Faulhaber, *Sphyngis Victor*. The saying is: Gog and Magog, a high regent comes from the offspring of Japhet.[17] In the biblical book *Ezekiel* Gog from the land of Magog leads at the end of time an army of nations against the nation of Israel. After initial successes Gog is destroyed together with his followers by God himself. In many of his Cabbalistic works Faulhaber refers to Gog who, in the context of a very old Christian tradition, had assumed the traits of the Antichrist.

4. Word calculus and signs

Faulhaber and his contemporaries often used a word calculus in which the letters of an alphabet are given numerical values and vice versa; it was part of a long tradition going back at least to the secret Jewish teachings of the Cabbala. The goal of this word calculus, which had been revived in the 16th and 17th centuries, was, among other things, the interpretation of certain sayings by means of the assignment of numbers to the letters in them and, the other way around, the hiding of names and sayings by means of numbers. A prominent precursor of Faulhaber in Germany was Michael Stifel who, in 1532, in his anonymously published *Rechen Büchlein Vom End Christ*, by means of his form of word calculus identified the Beast of the Apocalypse[18] with Leo X, who was pope at the time that Luther nailed his 95 theses to the door of the Castle Church in Wittenberg. Stifel, moreover, used a suitable biblical text to predict the end of the world on October 18, 1533.

Stifel used the successive triangular numbers $\binom{n+1}{2}$ for $n = 1, 2, 3, \ldots, 23$ for the 23 letters of the alphabet in order to interpret the Latin sentence *id bestia leo* (this animal is [the pope] Leo), calculating the sum of the values of its letters, as the number 666, which in its turn stands for the Beast of the Apocalypse. Stifel's ideas were spread in Ulm by his pupil Conrad Marchtaler who founded, in 1545, a successful school for arithmetic in this town.[19]

The extent to which Faulhaber, who certainly knew very little, if anything at all, about the tradition of the Jewish Cabbala, was influenced in his interpretations by Stifel, Marchtaler and others, can no longer be established.

In the interplay of the very different mentalities that existed in the social microcosm of the city of Ulm in the year before the outbreak of the Thirty Years' War, Faulhaber was undoubtedly dependent on a group of supporters composed of representatives of all social ranks. This group was willing to interpret many observed events that were considered as particularly striking as tokens sent by God and to exempt them from all attempts to explain them rationally. Reports of such signs followed events that were experienced as particularly dramatic, like the death of a beloved sovereign or the outbreak of a war. This holds for the beginning of the Thirty Years' War in 1618. The beginning of the year 1618 was unpromising. The Emperor Matthias was very ill and hardly capable of taking care of government affairs. His death was expected soon. His cousin Ferdinand, who would succeed

[17] "Gog vnd Magog ein hoher Regent in Europa kompt auß Japheths Geschlecht."
[18] *Apocalypse* 13, 18.
[19] Keefer (Footnote 1), p. 129.

him and who had already been elected King of Bohemia in May 1617, was considered to be a stirrer in the battle between the religious denominations. When in the summer of 1618 lightning set fire to the tower of the castle of Preßburg, many contemporaries considered this to be a bad omen because the regalia of the Hungarian crown were kept in the tower. Although the regalia could be saved and on July 1, 1618, as planned, Ferdinand was crowned King of Hungary; on that same day the cardinal responsible for the imperial politics, Cardinal Klesl, was almost killed. A bullet from a gun salute just missed him. In spite of their lucky outcome, the events were interpreted as an unhappy omen, the more so because less than three weeks later Ferdinand deprived Klesl of his power forever. Klesl represented an obstacle to Ferdinand's plans and he was put in prison.

Such tokens, in combination with many other signs—for example, the finding of strange animals and human beings, rivers allegedly ensanguined, and the like—were ascribed by many to the immediate intervention of God. God wanted, through these signs, to deliver a message, the content of which, however, could only be understood by a chosen few. Three years before his death, Faulhaber, who claimed for himself immediate access to the interpretation of the signs sent by God, published the following text: *Vernünfftiger Creaturen Weissagungen/ Das ist: Beschreibung eines Wunder Hirschs/ auch etlicher Heringen vnd Fisch/ vngewohnlicher Signaturen vnd Characteren, so vnderschidlicher Orten gefangen/ worden. Auß den gehaimen Zahlen deß Propheten Danielis/ vnd der Offenbarung S. Johannis erklärt/ vnd was sie bedeuten möchten/ vermuthlich angezaigt.* (Prophecies of Reasonable Creatures That is: A Description of a Miraculous Deer and quite a few Herrings and Fish of unusual Signature and Character, caught on different Spots Explained from the secret Numbers of the Prophet Daniel and the Revelation of Saint John and what they may mean, presumably shown.)

Apparently he had published this treatise in connection with an expected meeting with the Swedish king Gustav Adolf. Such a meeting took place when Faulhaber, together with the burgomaster and some companions, visited Lauingen and was ordered to come to Donauwörth in order, as an engineer, to talk to the king about fortifications. Faulhaber used the opportunity to brief the king not only in this area but also to draw the king's attention to his interpretations of numbers, miracles and tokens. In this context he clearly pointed out God's intentions with respect to the decisive role of Gustav Adolf in the predetermined history of mankind. The treatise dedicated to Gustav Adolf was motivated by a miraculous deer that was shot in June 1630, the year in which the "Midnight Lion",[20] as Gustav Adolf was called, entered the war. The dimensions of this deer, for example, the length of the antler, the size of the head and the lengths of the legs, measured by means of a unit chosen by Faulhaber, yielded the biblical numbers, including 666. Faulhaber's interpretation of the miraculous deer implied that "a high regent" from the country of the reindeer, with an army, would "as fast as a deer" penetrate the area where the deer was shot. Faulhaber alluded to the extraordinary manoeuvrability and the impressive marching time of the Swedish army, which left at the time a lasting impression.

Among the signs that were given most attention by the followers of Faulhaber were the celestial phenomena because they belong to God's immediate sphere of influence. To a

[20] Carlos Gilly, The 'Midnight Lion', the 'Eagle' and the 'Antichrist': Political, religious and chiliastic propaganda in the pamphlets, illustrated broadsheets and ballads of the Thirty Years War in: *Nederlands Archief voor Kerkgeschiedenis* 80, 2000, pp. 46–77.

certain extent the events seen in the sky were viewed in direct relation with the events on earth in the sense of the macrocosm–microcosm relationship as it was defended by the followers of Paracelsus. Not only did many ordinary people see themselves as more or less helplessly dependent on the events in the sky, but quite a few of the political actors, like the imperial military commander Wallenstein, also made their decisions in accordance with the celestial phenomena.

5. The Rosicrucian movement

The opposition between the followers of such interpreters of tokens as Faulhaber and their opponents were intensified by the so-called Rosicrucian movement, triggered by the circulation of the two first Rosicrucian manifests, *Fama Fraternitatis* and *Confessio Fraternitatis*, first in the form of handwritten versions and, after 1614, in printed form.[21] It is to this day not known who wrote the *Fama* and the *Confessio*, although it is generally assumed that the protestant theologian Johann Valentin Andreae, who worked in Württenberg at the time, was party to it.

In both texts a brotherhood of the Rosicrucian cross is mentioned. In the *Fama* the history of the brotherhood is accompanied by references to the teachings of the brotherhood. The basic tenor is a criticism of the established authorities like Aristotle in philosophy and Galen in medicine. In order to read the only book that possessed authority, the book of nature written by God, the Rosicrucians instead based themselves on neo-Pythagorean, neo-Platonic and hermetic ideas, the microcosm–macrocosm correspondence in the harmony between man and nature, Paracelsus and the Cabbala. Numbers and their properties are in this view attributed with extraordinary explanatory power. With the aid of God's grace the members of the original brotherhood had acquired knowledge about the book of nature. The new generation of the brothers, who had rediscovered the grave of the founder of the order, foresaw a new reformation of mankind on the basis of the knowledge found in the book of nature and called upon the readers of the *Fama*, that they had written, to express themselves with respect to this first communication of the brotherhood.

The conglomerate of expectations expressed in the *Fama* was so extended that it led in the period 1614–1620 to more than 200 known Rosicrucian publications. The impact of the *Fama* and the next two Rosicrucian manifestos was not caused by the novelty of the ideas contained in them. Many of the ideas, like the Cabbalistic or neo-Platonic body of thought, the expectation of a more comprehensive reformation on the basis of a balance between theology and science, the critical discussion of the rigid forms of scholastic science and the idea of the creation of new forms of science that would lead to the actual solution of real practical problems, can all be found in texts that had been published before and independently of the Rosicrucian manifestos.

[21] *Fama Fraternitatis, Oder Brüderschafft/ des Hochlöblichen Ordens des R. C. An die Häupter/ Stände und Gelehrten Europae.* In: *Allgemeine vnd General Reformation der gantzen weiten Welt. Beneben der Fama Fraternitatis, Deß Löblichen Ordens des Rosenkreutzes/ an alle Gelehrte und Häupter Europae geschrieben: Auch einer kurtzen Responsion, von dem Herrn Haselmeyer gestellet/ welcher deßwegen von den Jesuitern ist gefänglich eingezogen/ und auff eine Galleren geschmiedet: Itzo öffentlich in Druck verfertiget/ und allen trewen Hertzen communiciret worden*. Kassel 1614, pp. 91–128 and *Confessio Fraternitatis, Oder Bekanntnuß der löblichen Bruderschafft deß hochgeehrten Rosen-Creutzes/ an die Gelehrten Europae geschrieben.* In: ibidem, pp. 54–82.

The success of the Fama was caused by the clever grouping of the expectations that were fostered by completely different circles. These were only united in their dissatisfaction with the after-effects of all forms of dogmatism. Also correspondingly multifaceted were the reactions to the Rosicrucian manifestos; not surprisingly the group of opponents consisted to a large extent of representatives of orthodox Protestantism. Some more or less sensational legal proceedings initiated by representatives of the Protestant church in Württemberg show that about 1620 the Protestant orthodoxy had succeeded in marginalising the Rosicrucian movement as heterodox and at the same time assessing the presuppositions for the use of the Cabbalistic, number-mystical and alchemistic elements in their construction as irrational.

Such appraisal hardly influenced the self-conception of those attacked by the orthodoxy, as two examples will show. The quite large number of mystics among the adherents of the Rosicrucian movement who followed a Cabbalistic tradition were in general mathematically trained and, as far as their mathematics went, beyond the accusation of irrationality. A statement about the goals of the Rosicrucians by the at times fervent follower of the Rosicrucian movement, Daniel Mögling, in his *Rosa Florescens*,[22] published in 1617 under the pseudonym Florentinus de Valentia, makes clear what the starting points were for the conflict with the orthodoxy. According to Daniel Mögling, the Rosicrucians abandon a literal understanding of the Holy Scripture in order to read the "true book of life" with the "eyes of the mind" and interpret it in harmony with the Bible. The realm of everything that is accessible to human reason can only be transcended with God's help. However, the means needed in order to assure oneself of God's help are no longer rational; moreover, in order to be able to effect God's help the special grace of God is required, which is not bestowed on everybody but only on a few.

Exactly on the question of the means to acquire human knowledge and the scope of such knowledge, the roads of the protestant orthodoxy and the Rosicrucian heterodoxy separated. While for the orthodoxy human knowledge was limited to the literal understanding of the Bible and a rational explanation of nature compatible with such understanding, the followers of the Rosicrucian movement were convinced of the fact that the revelations in the Bible are at least partially written in a symbolic language and that the rather limited sphere of rational knowledge of nature can only be transcended with God's help. If the symbolic language of the Holy Scripture is interpreted in the right way, it will not contradict what the eyes of reason see in the book of nature and what is on the pages that can only be interpreted by means of God's grace.

The followers of the Rosicrucian movement and their opponents can be viewed as representatives of two mentalities typical of this time. The orthodox Protestants on the side of the opponents represent the mentality of independence that is reached socially by means of personal responsibility and is reached in the area of knowledge by restricting the means of acquiring knowledge to the activity of the human mind. From their point of view God had created the world in accordance with a plan accessible to human understanding. It seemed agreeable to God that man should study the plan of the creation and explain as natural phenomena in the sense of the plan of the creation the many events seen by others as miracles brought about by God. This mentality corresponded to a high preparedness for competi-

[22] Florentinus de Valentia (Pseudonym for Daniel Mögling), *Rosa Florescens*, no place, 1617, f. 10 v.

tion and conflict. The mentality of the followers of the Rosicrucian movement was characterised by a longing for harmony in a world without conflicts, that would be guaranteed by unselfish labour for God and mankind in combination with access to new transcendent knowledge in possession of a group of chosen ones; these exceptionally gifted individuals would take over the responsibility that according to the orthodoxy everyone must bear individually.

From the beginning the texts of the Rosicrucians appealed to Faulhaber and they confirmed his ideas in many regards; like many other followers of the Rosicrucian movement he tried for years in vain to get in touch with the legendary brotherhood of Rosicrucians.[23] An anonymous Latin text published in 1615, of which Johannes Remmelin, Faulhaber's pupil and friend, later claimed to be the author, was one of such futile attempts.[24] This text was like a birdcall directed explicitly to the "above all enlightened and highly laudable men of the Fama of the Rosicrucian brotherhood". This did not protect Faulhaber, during the most intense disputes about the meaning of the comet of 1618, against being seen by the authorities as himself a member of the brotherhood. His booklet on the interpretation of the comet, *Fama siderea nova*, in which he again manipulated the number 666, and of which the title began with the word "Fama", like the first of the Rosicrucian texts, was connected in Ulm with the suspiciously watched meetings of the Rosicrucian movement in which Faulhaber, at least from the point of view of the church authorities in Ulm, played a significant part.[25] They had, for example, discovered that Faulhaber had secretly met up to 70 people, called in town *Rosenkreutz Brueder* (Rosicrucian brothers), that apart from such *Conventicula* communicated with each other in writing.

6. The comet of 1618

The special attention that was given to Faulhaber by the clerical and municipal authorities was sparked off by his interpretation of the comet of 1618, which had made big waves. In 1618 Johannes Kepler had observed three comets that were predominantly identified as one and the same heavenly body.[26] The first was from the end of August to the end of September only faintly visible and was apparently ignored by most people, as was the second, albeit not by Faulhaber. A third comet, easily visible to everybody, was only seen in November of 1618. From both the Catholic and Protestant pulpits sermons about its significance were delivered and in a flood of leaflets and treatises the astronomers and self-proclaimed experts on comets attempted to satisfy the curiosity of an intensely interested public.

The authors of the many texts on comets can be classified into two groups on the basis of their views. The first group, to which Faulhaber and most followers of the Rosicrucian movement belonged, viewed the comet as a "preacher of penance" (*Bußprediger*) put by

[23] Letter by Rudolf von Bunau for 21./31. January 1618 (Stadtarchiv Ulm).

[24] [Johannes Remmelin], *Mysterium Arithmeticum*, without place, 1615.

[25] Report of the Kirchenbaupflegamt in Ulm for 27. July, 1619, f. 647.

[26] Johannes Kepler, *De Cometis Libelli Tres*, Augsburg 1619; see Johannes Kepler, *Gesammelte Werke*, Bd. VIII, München 1963, pp. 129–262, in particular p. 177; cf. Werner Landgraf, Über die Bahn des zweiten Kometen von 1618, *Sterne* 61, 1985, pp. 351–353.

God on the "heavenly pulpit" (*Kanzel des Himmels*) in order to announce God's wrath and punishment if men did not abandon their sinful ways. The other group saw its task in particular as appeasing the population that was already afflicted with many problems, need and fears. Its representatives were concerned to explain the comet as a natural, for example atmospheric, phenomenon without further significance. The critics of Faulhaber from the second group first of all accused him of claiming without any justification the arrival of the comet, two months late, as a confirmation of his prediction. One of Faulhaber's friends objected to this, on the ground that in addition to Faulhaber other "credible learned folks" (*glaubwürdige gelehrte Leute*) had already seen a comet in September 1618, although it was not very visible, and because of that an irresistible "power was assigned" (*Kraft erteilt wurde*) by God to Faulhaber's prediction, made much in advance in a calendar for the year 1618, that a comet would appear on September 1.

The Protestant Free Imperial Town of Ulm (*Freie Reichsstadt Ulm*) had ordered this one-page calendar for the year 1618 with the intention of giving it to the civil servants of the town. Among the special references and predictions in this calendar was the entry "comet" on September 1. The small size of the entry on a page that had to refer to all the days of the year and the very much restricted distribution of the calendar were not suitable for making the public at large aware of Faulhaber's prediction. Faulhaber had informed his friend in Reutlingen, Matthäus Beger, of his observations of the comet he had seen in August, in a letter dated August 26, 1618, with the intention that these observations be sent to Professor Michael Maestlin in Tübingen. Beger had only learned from Tübingen[27] that in Tübingen and its environment such a comet could not be seen before November; this meant that Faulhaber had been the first to discover the comet in the sky.

Faulhaber saw no restriction on the validity of his prediction in the fact that the comet was observed later in Tübingen. In his *Fama Siderea Nova*, published in 1619, under the pseudonym Julius Gerhardinus Goldtbeeg from Jena, by Daniel Mögling, Faulhaber interpreted the observation without any restrictions as a confirmation of his special God-given faculties to interpret the comet as a token sent by God. The main opponents of Faulhaber in the discussions following the appearance of the *Fama* were the principal of the grammar school in Ulm, Johann Baptist Hebenstreit, and one of his teachers, the *Praeceptor* Zimbertus Wehe, who tried to hide behind a pseudonym in his two pamphlets written against Faulhaber. Hebenstreit and Wehe were acquaintances of Faulhaber, who had visited him at home up to the time of their criticism of the *Fama*. In particular the about-face of Hebenstreit came rather as a surprise. Faulhaber had given Hebenstreit lessons with respect to the observation of the comet(s) of 1618 and Hebenstreit had corrected the text of the *Fama* as well. With his treatise *Cometen Fragstuck* (The Question of the Comet) Hebenstreit wished to exploit the general interest in the appearances of the comet(s) as quickly as possible. A mistake he made in the booklet, confusing Mars and Arcturus, was apparently quickly discovered by one of his competitors in the market of texts on comets and used as a basis for a destructive criticism of Hebenstreit's text. Hebenstreit's view of the essence and the location of the comet provoked fairly lively disagreement.

Already in his *Cometen Fragstuck* Hebenstreit had asserted, without mentioning Faulhaber, that his eyes were "too foolish" (*zu blöd*) to see on September 1, 1618 a comet

[27] Matthäus Beger, *Problema Astronomicum: Die Situs Der Sternen Planetarum oder Cometarum zu observirn*, without place, 1619, f. D I v.

Fig. 4. Faulhaber's prediction of the 1618 comet depicted in his *Fama Siderea Nova* of 1619 (Courtesy of Birkhäuser Verlag).

that would only be visible much later.[28] In his second Latin booklet of 1619[29] Hebenstreit extensively attacked the possibility of predicting the appearance of comets on the basis of Cabbalistic number speculations. At the same time he criticised the contents of *Fama siderea nova*, again without mentioning Faulhaber. Hebenstreit and Wehe pointed out that Faulhaber had taken the prediction of the comet from a publication of the imperial mathematician and astronomer Johannes Kepler. Kepler had in his *Prognosticon* for the year 1618, in a section about diseases, granted the possibility of the appearance of a comet, be-

[28] Johann Baptist Hebenstreit, *Cometen Fragstuck/ auß der reinen Philosophia, Bey Anschawung, deß in diesem 1618. Jahr/ in dem Obern Lufft schwebenden Cometen, erläutert/ vnd auff etlicher Gelehrten vnd Vngelehrten Gegehren/ an Tag gegeben*. Ulm 1618.

[29] Johann Baptist Hebenstreit, *De Cabala Log-Arithmo-Geometro-Mantica*, Ulm 1619.

cause since 1607 no comet had been observed.[30] Although at the time it was not yet known that comets return after a period characteristic for their orbit, Kepler obviously assumed, on the basis of the astronomical observations he was familiar with, that comets would appear with more or less regularity. Kepler's argument was based on experience and was suitable, although he could not explain the observed regularity, and was written so as to make comets appear as something natural and not as something wonderfully supernatural.

Faulhaber was, moreover, familiar with Kepler's *Ephemeris* for 1618, in which for September 1618 (Julian calendar) the ecliptical longitude of Mars—calculated with reference to the meridian through Uraniborg, Tycho Brahe's observatory on the island of Hiven—and the ecliptical latitude of the moon as well, were fixed at $3°33'$.[31] Wehe followed Hebenstreit[32] and connected in his reconstruction of the background for Faulhaber's prediction of a comet in 1618 the possibility of the appearance of a comet that Kepler had mentioned without a more precise indication of the time, with the fact that the longitude of Mars and the latitude of the moon had the same value in the *Ephemeris* for 1618 on September 1 (old style).[33] Subsequently, said Wehe, Faulhaber had against all reason and every scientific rule interpreted the double occurrence of this value $3°33'$ as the double occurrence of the number 333, that is as the holy number 666 and construed this as a divine indication of the occurrence of a special event.

The testimonies of Hebenstreit and Wehe about the way in which Faulhaber proceeded to predict the comet are completely in accordance with the style of the Cabbalistic speculations Faulhaber engaged in elsewhere; neither Faulhaber, nor any of his defenders have contradicted this point of the two critics. This holds for both *Vorläufer einer Rechtfertigung Faulhabers* (Forerunners of a Justification of Faulhaber), a text written under the pseudonym of Justus Cornelius in defence of Faulhaber,[34] and *Fortsetzung der Rechtfertigung Faulhabers* (Continuation of the Justification of Faulhaber), which appeared subsequently, written by an author who used the pseudonym C. Euthymius de Brusca.[35] Anyway, both texts refer explicitly to the specifications of the positions of Mars and Kepler's *Ephemeris* for the year 1618.[36]

C. Euthymius de Brusca first asserted that Faulhaber saw Kepler's calendar for 1618, shown to him by Hebenstreit, in which a comet was mentioned, only in December 1618, long after the appearance of the comet.[37] A few pages later, after admitting the correctness

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[30] Quoted from Justus Cornelius, *Vindiciarvm Favlhaberianarvm Prodromus*, Ulm 1619, p. 17, from Johannes Kepler, *New vnnd Alter Schreib Calender sambt dem Lauff vnd Aspecten der Planeten auff das Jahr Christi M. DC. XVIII. Prognosticum Astrologicum auff das Jahr MDCXVIII. Von natürlicher Influentz der Sternen in diese Nidere Welt*. Linz 1618.

[31] Johannes Kepler, *Ephemeris nova Motuum Coelestium ad annum vulgaris aerae M D C XVIII. Ex obseruationibus potissimum Tychonis Brahei, Hypothesibus Physicis, & Tabulis Rvdolphinis; Nova etiam formâ disposita, ut Calendarii Scriptorii usum praebere possit. Ad Meridianum Vranopyrgicum in freto Cimbrico, quem proximè circumstant Pragensis, Lincensis, Venetus, Romanus*. Linz (no year), in: Johannes Kepler, *Gesammelte Werke*, Bd. XI, 1, München 1983, pp. 75–94, in particular p. 91.

[32] Johann Baptist Hebenstreit, *De Cabala*, Ulm 1619, p. 26.

[33] [Zimbertus Wehe] alias Hisaias sub cruce, *Expolitio famae sidereae novae Faulhaberianae.* Ulm 1619, p. 23.

[34] Justus Cornelius, *Vindiciarvm Favlhaberianarvm Prodromus*, Ulm 1619.

[35] C. Euthymius de Brusca, *Vindiciarvm Favlhaberianarvm. Continuatio.* Moltzheim 1620.

[36] Justus Cornelius, *Vindiciarvm Favlhaberianarvm Prodromus*, p. 13 and C. Euthymius de Brusca, *Vindiciarvm Favlhaberianarvm. Continuatio.* p. 24.

[37] C. Euthymius de Brusca, *Vindiciarvm Favlhaberianarvm. Continuatio.* p. 18.

of Wehe's description, C. Euthymius de Brusca attempts to paper over the cracks with the statement that Faulhaber owed the discovery, that 666 is a "tessaracondexagonal number" with the square root 6 and at the same time a prismatic number on the basis of a nonagon (polygon with 9 sides) with the same square root, to the "speculation and consideration" (*Speculation vnd Betrachtung*) of the longitude and latitude of, respectively, Mars and the Moon for September 1, 1618.[38] As a matter of fact, 666 can be represented as a polygonal number of square root 6 corresponding to a 46-gon, that is as sixth term of an arithmetic sequence of the second order with first term 1 and difference 44, and also as a prismatic number of square root 6 corresponding to a nonagon, that is as six times the sixth term of an arithmetic sequence of the second order with first term 1 and difference 7.

However, the two representations offer no connection with Kepler's value of 3°33′ both the longitude of Mars and the latitude of the Moon on September 1, 1618. After all, it is very probable that Hebenstreit and Wehe were right with their account of the way in which Faulhaber found his prediction of a comet; it even looks as if Hebenstreit and Wehe, who both, before the conflict about the comet of 1618, had a friendly relationship with Faulhaber, did not even have to speculate about Faulhaber's method, but had learned about it directly or via intermediaries.

When the clerical establishment felt that its authority could be eroded by "prophets" like Faulhaber, Hebenstreit, using his two texts on comets to attack Faulhaber, induced the Church to start an investigation of Faulhaber's theses. Thus in a colloquium in the autumn of 1619, not open to the public, an attempt was made to answer the main question: whether Faulhaber's prediction of a comet was the result of divine inspiration or of his own speculations. Faulhaber's testimony that he owed his knowledge about biblical numbers only to his zeal, in particular when studying arithmetic, and prayer, saved him from further sanctions.

When it became known that, despite his promise of the beginning of 1621, Faulhaber had talked again about Gog and Magog, absolution after confession was denied to him. He then obtained absolution from another confessor by misleading him. Moreover, he took Holy Communion in spite of repeated admonitions by Dr. Dieterich not to participate; the result was that he was excluded from the Holy Communion by the clerical authorities in Ulm.[39] Things escalated until the end of the year 1621; at this time the accusation of a conscious disesteem for and deception of the authorities played a decisive role. Moreover, in the same year, a text appeared[40] anonymously, in which the disciplinary actions against Faulhaber by the administration were vehemently attacked.[41] The authorities saw themselves prompted to proceed more strongly against Faulhaber, who denied all knowledge about the text and its author. Two intercepted letters from Faulhaber to his friend the physician Dr. Verbezius and the testimonies of the nobleman Hans Ludwig Schad, who had associated with Faulhaber and Verbezius,[42] fuelled the distrust and the suspicion against

[38] Ibidem, p. 24.

[39] Reports of the Kirchenbaupflegamt for 20. and 23.03.1621.

[40] *Gründliche Warhaffte Erzehlung Was in den Etlich Jahr wehrenden aber noch nit zu End gebrachten Stritten zwischen Johann Faulhaber und Gegentheil sich verloffen, von einer eifrigen Christlichen Persohn getreulich an Tag geben*, o. O. 1621; one suspects that David Verbez was the author of this text.

[41] Jakob Neubronner, manuscript of a biography of Faulhaber preserved in Stadtarchiv Ulm, p. 22 f.

[42] Report of the city council for 21.11.1621.

Faulhaber so strongly that it was considered justified to put Faulhaber in prison.[43] The intention was to question Faulhaber about the divine enlightenments that he had repeatedly claimed to have had and his participation in the brotherhood of the Rosicrucians.[44] However, Faulhaber absconded from imprisonment before Christmas Eve of the same year by fleeing to Augsburg,[45] where he was also excluded from Holy Communion by the authorities. After Johann Fugger the Elder and others from Augsburg had pleaded for him in Ulm, the authorities in Ulm promised not to imprison Faulhaber again.[46] Faulhaber returned to Ulm in March 1622, only in order to escape again to Tübingen for three months, having disobeyed another invitation by the authorities to exculpate himself at the *Dombauhütte*.[47]

Theologians of the University of Tübingen told Faulhaber that the Greek text of the New Testament, there where the biblical number 666 occurs, was corrupt.[48] This may have turned the balance, so that Faulhaber after long hesitations could return to Ulm at the beginning of 1624. After a discussion and reconciliation with representatives of the church he signed a profession of faith that was acceptable for the clerical authorities in Ulm.[49]

7. Pyrgoidal numbers

Although in Faulhaber's later texts the biblical numbers in general are taken as a starting point for mathematical developments and discoveries, they show that he never gave up his conviction that he possessed special abilities for the interpretation of the "heavenly" numbers in the Book of Revelation, though he had admitted he had erred. In the *Miracula Arithmetica* Faulhaber first defined so-called tower or pyrgoidal numbers,[50] that represent the number of lattice points of a tower built from a prism and a pyramid with the same base, by adding pyramidal and prismatic numbers or "columns" of which the bases are in each case equal but arbitrary polygonal numbers. This enabled him to form biblical numbers like 666, 1600 or 1000 as sums of pyrgoidal numbers by appropriately choosing d, that is, the number of sides of the fundamental polygon minus 2.

As soon as one knows that there exists for a biblical number a representation as a polygonal, pyramidal or pyrgoidal number, or as a sum of such numbers, corresponding to a certain d, one can determine the second variable n, the number of terms or, in Faulhaber's terminology, the square root of the polygonal number. This determination requires, for example, in the case of a polygonal number the solution of a quadratic equation and, in the case of a pyramidal or pyrgoidal number, the solution of a cubic equation.

In his next step Faulhaber attempts to represent the "holy" numbers as pyramidal numbers on the basis of generalised polygonal numbers. By the additional condition that the individual terms of the sequence that ends with the given "holy" number must correspond

43 Reports of the city council for 17., 19. and 20.12.1621.
44 Report of the city council for 6.04.1621.
45 Report of the city council for 24.12.1621.
46 Report of the city council for 20.03.1622.
47 Report of the Kirchenbaupflegamt for 15.05.1622.
48 Letter from Faulhaber to Sebastian Kurz of 20.02.1623 (BN Paris).
49 Report of the Kirchenbaupflegamt for 9.02.1624.
50 *Miracula Arithmetica*, pp. 41–43, Chapter 35.

with all the letters of a given alphabet, Faulhaber can assume n as given and d as to be determined. On the basis of, for example, the German alphabet, the first pyramidal number corresponds to the letter a, and the given "holy" number corresponds to the last letter. With the Latin, Hebrew and Arabic alphabets Faulhaber proceeds analogously.

Since a pyramidal number belonging to a polygonal number with the basis n can be expressed as follows:

$$\binom{n+1}{3}d+\binom{n+1}{2},$$

and n is given for a given alphabet, this yields

$$d=\left[b-\binom{n+1}{2}\right]:\binom{n+1}{3},$$

where b is a biblical or "holy" number. In contrast to the case of the proper polygonal numbers, where d is always necessarily a natural number, d can now be a positive rational number. Faulhaber gives four examples that are based on the German, Latin and Arabic alphabets, that is with $n = 24$, 23 and 29, and concern four different biblical numbers.

In a last generalisation Faulhaber also admits irrational differences d, by requiring that the nth term in the sequence of sums of correspondingly generalised pyrgoidal numbers equals a given biblical number. With it Faulhaber released himself by means of a formal arithmetic–algebraic generalisation from the concrete geometric notions that are the basis of the formation of the polygonal, pyramidal and pyrgoidal numbers. In particular, in his *Miracula Arithmetica* Faulhaber demonstrated his ability to formulate amazing generalisations. These generalisations not only concerned the application of the cossic notation to statements about binomial coefficients or power series for arbitrary natural numbers, but affected his entire mathematical work, as the 3-dimensional theorem of Pythagoras illustrates.

If one cuts off a corner from a cube and puts it down, the result is a triangular pyramid of which the three faces that meet at the top are mutually perpendicular. In this situation the theorem that was phrased by Faulhaber in his *Ingenieurs-Schul* (*Engineering School*) of 1630[51] holds: in all such pyramids the square of the base equals the sum of the squares of the three other faces. Faulhaber had already given the 3-dimensional theorem of Pythagoras in the *Miracula Arithmetica* of 1622. There,[52] restricting himself to pyramids with an equilateral base, Faulhaber had taken the number 666 as the length of the legs of the three mutually perpendicular isosceles triangular faces and shown that the square of the base equals the sum of the squares of the three other faces. Not satisfied with amazing his readers with the equality of the two calculated numbers, Faulhaber subsequently, without explanation or proof,[53] asserted that the same is true for all pyramids with three mutually perpendicular faces. Faulhaber pays no attention to the fact that the statement of the general validity of the relation that he had demonstrated, in the special case of isosceles triangular

[51] p. 153.
[52] pp. 73–76.
[53] p. 75.

faces with legs of length 666, annuls the allegedly exceptional position of this number, because he could have demonstrated the theorem in any particular case. This argument, however, would not have disturbed his followers. They interpreted the use of the very special biblical number 666 as an indication that the secret of the 3-dimensional theorem of Pythagoras, that, after antiquity, in principle could have been found by any mathematician, was intentionally revealed by God to Faulhaber, who found the theorem with 666 as test case.

A section in the second part of the *Ingenieurs-Schul*, which appeared together with the third and fourth part in Ulm in 1633 and in which problems of fortification are treated, shows that Faulhaber on various occasions extended his manipulations with the biblical numbers, so central to his self-imposed status as a prophet, into the area of technology. In chapter 13 of the second part under the title "About a wonderful fortress" (*Von einer wunderbahrlichen Fortressen*) Faulhaber deals with the construction of a non-regular fortification that results in the construction of a non-regular hexagon inscribed in a circle of which the edges sequentially are proportional to the following biblical numbers[54]

2300, 1600, 1290, 1000, 666, 1260, and 1335.

Until the plague of the year 1635 ended his life Faulhaber remained faithful to his convictions as an exponent of a mentality of which the representatives had already been pronounced outsiders by the Protestant church in the 1620s. Even more effective was the exclusion of Faulhaber and his Cabbalistic speculations on the profane level by the father of early rationalism, Réné Descartes, who, according to uncorroborated reports, stayed in Ulm in the winter of 1619–1620, at the climax of the quarrel about the prediction of the comet. With Descartes the representatives of a mentality that suited the protestant orthodoxy, could finally establish themselves as part of a development that led to the Enlightenment.

[54] Several authors treated the calculation of the radius of the circumference: Johann Melder in a letter to Faulhaber of 16.08.1629 (Stadtarchiv Ulm) and in the 19th century August Ferdinand Möbius (Ueber die Gleichungen, mittelst welcher aus den Seiten eines in einen Kreis zu beschreibenden Vielecks der Halbmesser des Kreises und die Fläche des Vielecks gefunden werden, in: *Crelle's Journal für die reine und angewandte Mathematik* 3, 1828, pp. 5–34) and Siegmund Günther (Über das irreguläre Siebeneck Faulhabers, in: *Sitzungs-Berichte der physikalisch-medizinischen Societät in Erlangen*, 1874, Heft 6). Faulhaber only gave the result of his calculation, which did not differ much from MELDER's, but not his method.

CHAPTER 17

Mathematics and the Divine: Athanasius Kircher

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Edited by T. Koetsier and L. Bergmans

1. Biographical introduction

Athanasius Kircher (1602–1680) was one of the most famous polyhistors of the Baroque period, one of the most important Lullists of his time, that is an adherent of Ramón Lull's (about 1235–1315) ideas. In 1618, Kircher was admitted as a novice to the German Jesuit College at Paderborn. After many travels and various appointments he became professor of mathematics at the College of Rome in 1638. After some eight years he resigned from this post and devoted himself to independent studies until his death.

Thirty-three monographs appeared during his lifetime dealing with nearly everything, including humanities, natural sciences and religion. Scientific research was no end in itself for him or for many of his contemporaries, but was the understanding of God's perfection. Kircher never envisioned any separation of science from religion.

2. God and the rational foundation of music

Using a quantitative-mathematical approach to being, the Baroque period saw a numerical foundation to all existence. This included music, whose own foundation was mathematical in those days. Solomon's apocryphal saying that God ordered all according to measure, number and weight (Wisdom 11: 20), is paraphrased again and again in musical text books of the Baroque period.

Order is the fundamental law of music, too. This applies especially to Kircher's tenth work which appeared in 1650, one of the most voluminous works he ever published: *Musurgia universalis*, *Universal musical art, or great art of consonance and dissonance, subdivided into ten books, by which the whole doctrine and philosophy of notes and the science of theoretical and practical music are treated with the greatest versatility. The wonderful forces and effects of the consonance and dissonance in the world and even in the whole of nature are revealed and proven by an equally new and unknown exhibition of the various examples. This is done with regard to the individual practical applications as well in almost every faculty, especially in philology, mathematics, physics, mechanics, medicine, politics, metaphysics, theology.*[1]

Already the Baroque title makes clear that consonance and dissonance were not only musical but also universal categories of philosophy. All natural or mental processes are dominated by agreement or disagreement ("consensus" or "dissensus"), by concord or discord ("concordia" or "discordia"). The musical equivalent of these pairs of notions were consonance and dissonance. In his *Magnet or on the magnetical art* (Rome 1641), Kircher had elaborated his understanding of a universal science for the first time, his magnetic philosophy. The force of magnetism became the key for his understanding of all processes in the world.

His *Universal musical art* shows how a rational conception of music, which makes number the determining principle of music and of its effects, could occur together with a re-

[1] Musurgia universalis sive ars magna consoni et dissoni in X libros digesta. Qua Universa Sonorum doctrina, et Philosophia, Musicaeque tam Theoricae, quam practicae scientia, summa varietate traditur; admirandae Consoni, et Dissoni, in mundo, adeoque Universa Natura vires effectusque, uti nova, ita peregrina variorum speciminum exhibitione ad singulares usus, tum in omni poene facultate, tum potissimum in Philologia, Mathematica, Physica, Mechanica, Medicina, Politica, Metaphysica, Theologia, aperiuntur et demonstrantur.

Fig. 1. Athanasius Kircher (1602–1680).

ligious conception of harmony. Though it taught the universal harmony of the world, it explicitly rejected Kepler's mathematics based harmony of the world.

In the frontispiece, the symbol of the holy Trinity is surrounded by the nine choirs of angels who sing a 36-part canon and are enlightened by the Trinity's rays. Musica is sitting on the world sphere holding Apollo's lyre and the panpipes of Marsyas. Pythagoras points with his right hand to his famous theorem, and with his left to the blacksmiths whose hammers, ringing on the anvil, allegedly led him to discover the relation of tone to weight. The muse is surrounded by musical instruments.

The outstanding importance of music for the Baroque period consisted in the possibility of transferring its laws to the harmonical structure of the world construction. God's creation became recognisable by analogical thinking where the fundamental notion was number. The world was a universal realisation of God's music. Or the other way around: number was God's instrument in order to create the world. Tones were—by Kircher's definition—sounding, harmonical numbers. The beauty of divine and human music consisted in the order, that is in the composition, combination, and arrangement of tones or sounding numbers ("numeri sonori"): to compose was identical with to combine. God was the first to practice this combinatorial art when he created the world. God was not only the first combinatorialist; his combinatorial activity is unlimited.

Fig. 2. Frontispiece of Kircher's *Musurgia Universalis*, Book I.

3. The rules of the divine, combinatorial art

Analogies and combinatorial art thus characterised Kircher's universal science. For that reason, he was deeply interested in the rules of that divine art and studied them in the eighth book of his *Universal musical art* in great detail.

The book is entitled *The eighth book of the great art of consonance and dissonance, that is the wonderful musical art, a new, recently invented musical–arithmetical skill, by which everyone is able to acquire in a short time a profound knowledge of composing, even if he knows nothing about music*.[2] It is divided into four parts:

(1) the combinatorial musical art,
(2) the rhythmic or poetical musical art,
(3) the practice of "song-building musical numbers" (*musarithmi melothetici*),
(4) the mechanical musical art or the various transpositions of certain "musical-arithmetical columns".

[2] Artis magnae consoni, et dissoni liber octavus de musurgia mirifica hoc est ars nova musarithmica recenter inventa, qua quivis etiam quamtumvis Musicae imperitus, ad perfectam componendi notitiam brevi tempore pertingere potest.

In detail greater than anywhere else Kircher explained some fundamental combinatorial rules and operations in the first part. The second part applies these rules to rhythms, that is notes and metrical sequences of syllables or metrical feet. The third part explains Kircher's new musical art, which consisted in an artistic composition of song-ordering columns. However this composition is done, it will result in a new harmony. The fourth part explains the use of a musical–arithmetical box. Kircher's message is clear: to compose means to combine, it is a mathematical activity. Not the idea matters but the disposition, elaboration of notes, which are based on rules.

As usual Kircher did not reveal his sources, though he completely depended on Marin Mersenne's two great monographs on music theory, the French *Universal Harmony* and the Latin *Books on Harmony*. Both works had appeared in 1636. He even used Mersenne's number-examples 9 and 22.

Thus he explained permutations, combinations or unordered selections, and arrangements or ordered selections. Though he did not discuss all of Mersenne's combinatorial problems, he still presented remarkable knowledge of combinatorics. It is worthwhile considering in more detail. Kircher wanted to demonstrate "the force of numbers" (*vis numerorum*), the huge variety of possible permutations and selections of relatively few, say n elements which constitutes the universal harmony of God's creation.

The number $P(n,n)$ of permutations without repetitions of n pairwise distinct elements is calculated by means of

$$\text{Rule 1:}\quad P(n,n) = 1\cdots n = n! \quad (\text{“}n\text{ factorial”})$$

and illustrated among others by the word-examples "ora" and "amen", which admit $3! = 6$ and $4! = 24$ permutations, respectively. Kircher presented the factorials from 1! to 24! (the latter is a number of 24 digits).

The number of permutations becomes smaller if one element of the n elements can be repeated k times, because it is

$$\text{Rule 2:}\quad \frac{n!}{k!} \quad \text{(division rule).}$$

Kircher's word-examples were "Maria", "ala", and "Amara", which admit $\frac{5!}{2!} = \frac{120}{2} = 60$, $\frac{6}{2} = 3$, and $\frac{120}{6} = 20$ permutations, respectively.

Mersenne had explained that there is one permutation of say nine equal notes. Kircher understood the notion of "mutatio", permutation, in the strict sense of the word, and asserted that there is no permutation at all of n equal elements. He did not notice that this assertion contradicted the division rule taken over from Mersenne.

Hence, he elaborated the following table:

1 Sequence of different things	2 Combinations of things which are all different	3 Combinations of things where 2 are equal	4 Combinations of things where 3 are equal
I	0		
II	2	0	
III	6	3	0
etc.			

It corresponds to the topic of his book that most of the following examples are taken from musical theory. Kircher discussed the more general problem that several of n notes are repeated. Mersenne had systematically studied all different types of repetitions regarding a certain number n like 9. This procedure recalls Lull who had selected nine fundamental notions for his language theory. In modern terms, he looked for the multinomial numbers

$$\text{Rule 3:}\quad \binom{n}{n_1 n_2 \cdots n_p} = \frac{n!}{n_1! n_2! \cdots n_p!},$$
$$\text{where } n_1 + n_2 + \cdots + n_p = n,\ n_i \geqslant 1,\ i = 1, \ldots, p.$$

The equation represents a partition of n. It means with regard to permutations with repetition that there are p different elements. The ith element is repeated n_i times. For example, there are thirty different partitions of 9; hence there are thirty different types of repetitions on the understanding that nine elements are combined. While Mersenne had actually considered all these types, Kircher enumerated only nineteen without referring to the number-theoretical context or the total number of different types. Here are three of his examples:

$$n = 9, \quad \text{type } 2\ 2\ 2\ 3,\ \text{number of permutations} = 7560,$$
$$\text{type } 3\ 3\ 2\ 1,\ \text{number of permutations} = 5040,$$
$$\text{type } 5\ 2\ 2,\ \text{number of permutations} = 756.$$

His “universal problem” deals with the number $P(n, k)$ of arrangements without repetitions or ordered selections of k elements out of n elements:

$$\text{Rule 4:}\quad P(n, k) = n(n-1)\cdots(n-k+1).$$

If $k = n$, we get $P(n, n) = n!$ as before, that is arrangements can be dealt with as generalizations of permutations. Kircher spoke of the “wonderful force of combination” (‘mira combinationis vis’) to describe the incredibly huge combinatorial numbers resulting from relatively small initial numbers.

The transition from arrangements without repetitions to arrangements with repetitions is especially simple if only two notes are arranged:

$$P(n, 2) + n$$

are all possible arrangements with or without repetitions. It is a special case of the “most universal problem” (see below) because

$$P(n, 2) + n = n(n-1) + n = n^2.$$

If the order of the elements or notes does not play any role, then the arrangements are replaced by combinations. Their number is calculated by means of

$$\text{Rule 5:}\quad C(n, k) = \frac{P(n, p)}{P(p, p)} = \binom{n}{k}.$$

Kircher's "most universal problem" dealt with arrangements with repetitions. The required number is

Rule 6: n^p

if we select p elements out of n. $n^p - P(n, p)$ is the number of those arrangements where at least one element is repeated.

Hitherto Kircher exclusively explained some of Mersenne's results, normally without giving a demonstration like his predecessor. Yet, he added an interesting detail. Let us suppose we permute n elements. The type of repetitions $n-k$, k leads to $P(n,n) = \frac{n!}{(n-k)!k!}$. This expression can be reduced to $\frac{n(n-1)\cdots(n-k+1}{k!} = C(n,k)$.

In other words, Kircher calculated $C(n,k)$ by means of the formula for permutations with repetitions. He knew, too, the principle of multiplication of choices:

If there are k successive choices to be made, and if the ith choice can be made in n_i ways, for $1 \leqslant i \leqslant k$, the total number of ways of making these choices is the product $n_1 n_2 \cdots n_k$.

Kircher applied the principle to four bars consisting of four, three, four, and five notes, respectively, and admitting certain numbers n_i of permutations in order to calculate the total number of possible permutations.

4. The world as God's organ

Not by chance the *Universal art of music* comprises ten books. Kircher entitled his tenth book *Decachord of Nature or organ with ten flutes where it is shown that the nature of things looked in all at musical and harmonic proportions and that the nature of the universe is consequently nothing else but the most perfect music*. God's all-penetrating force is nothing but a force, which joins together and unites all, he said, to a harmonic proportion. If one should separate harmonic ratios and nature, all will necessarily perish in a complete chaos and disaster, in nothing. The diligent reader will find, he said, the detected causes for the forces of stones, plants, animals, and their effects. This door is open to a certain new workshop of natural magic.

Kircher equated nature, which is ordered according to measure, number, and weight, with music. This identification led him to the theory that music or harmony itself is nothing else but measure, number, and weight. His illustration entitled *Harmony of the world coming into being* using a world organ relates the six organ stops to the six days of creation, which God needed to finish His work step by step.

After pulling the pertinent organ stops, the harmonics of the corresponding days of creation sound: The coming into being of light, the separation of water from land, the occurrence of plants, the creation of the stars, the occurrence of animals by water, land, and air, finally, the creation of man as God's image. All stops are pulled; the creation is finished. Beneath the keyboard we read: "In such a way God's eternal wisdom plays on the whole world". Kircher proves entirely to be a child of his time in this respect, because the ideal sound with Baroque period was produced by a richly coloured combinations of organ stops.

In the following explanations Kircher used the metaphorical expression of stop for the ten domains of being. Hence he dealt with the "symphonism" of the four elements of

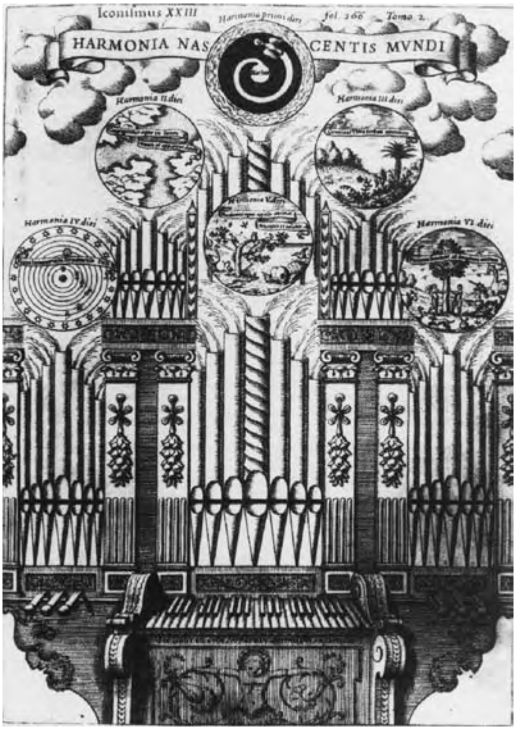

Fig. 3. Harmony of the world coming into being, *Musurgia Universalis*, Book II.

heavens, stones, plants, and animals; the microcosm and the megacosm; the harmony of the pulses and rhythms in the human body; the 'pathetic' symphonism or the music of the different affects of the spirit; the symphonism of the political world or the political music; metaphysical harmonics or music; the music of the angels; and finally the divine music or with the choir of the choirs', that is, with God's symphonism with the whole of nature.

His central statement was that neither intellect nor senses is able to detect the harmonic construction plan of the divine "Architect", as God was now called. Hence he rejected Kepler's world harmony, which was mathematically founded and based on scientific observations. He required the renunciation of the concept of harmony of the celestial spheres being quantifiable in the strict sense of the word. Instead, harmony consists in God's glorification when considering the perfection of the world. The harmony of the spheres consists in their admirable disposition and in a certain ineffable proportion working together in a unity. By this unity these world bodies correspond mutually to each other in such a way that after taking away just one world body the harmony of the whole world collapses.

5. Universal science as an imitation of God's art

Nineteen years later, Kircher took up again such mathematical studies related to combinatorics, in his *Great art of knowledge or combinatorial art, through which the broadest*

Fig. 4. Wisdom sitting on a throne. Frontispiece of Kircher's *Ars Magna Sciendi sive Combinatoria*.

door is opened for quickly acquiring knowledge in all arts and sciences. This invention is new, just as everybody will be able to speak about any proposed theme in an almost infinite amount of ways, if he is instructed with its help, and everybody will be able to obtain a certain summary knowledge of any doctrine.[3]

Wisdom sitting on a throne reveals the book of nature which has not seven but fifteen seals. They represent the disciplines theology, metaphysics, physics, logic, medicine, mathematics, moral ethics, ascetics, jurisprudence, politics, scriptural interpretation, controversy, moral theology, rhetoric, combinatorial art. The table shows twenty-seven (which is the number of books in the New Testament) fundamental notions of Kircher's alphabet of the art, that is nine notions forming the alphabet of absolute, of respective, and of universal principles, respectively, like goodness, difference, God, etc. Kircher promised that these notions contain the core of human knowledge, which comprehends the word sciences and the natural sciences as well. Hence he wrote on the base of the throne in Greek: "Noth-

[3] Ars magna sciendi sive combinatoria qua ad omnium artium scientiarumque cognitionem brevi adquirendam amplissima porta recluditur, quod uti inventum novum est, ita quoque eius subsidio usuque instructus, quilibet de quavis re proposita infinitis pene rationibus disputare, omniumque summariam quandam cuiuslibet doctrinae notitiam obtinere poterit.

ing is more beautiful than to know everything".[4] In his dedication to the emperor Leopold I he began by saying "Nothing is more divine than to know everything", attributing it to Plato. The human "great art" and "universal science" is an imitation of the divine art. Its significant structure is combinatorics for Kircher. Combinatorics as a specialty of the Lull school of the 16th and 17th centuries cannot be separated from the idea of a universal mathematization or quantification of being. Nature is the art of God.

The similarities are obvious between the declarations made in the title of the eighth book of the *Great art of consonance and dissonance* and in the title of the *Great art of knowledge*. Thanks to the *Combinatorial art* the special case of the musical art can be and is replaced by "all arts and sciences". This universal claim was the core and credo of Lullism. The fourth book is called the "own place of the combinatorial art" where its nature and method are explained. Before he discussed the various species of combination Kircher spoke about the marvelous and incomprehensible force of numbers, about the nature and variety of the combinatorial art. Number is, he said, a certain natural, exuberantly growing principle of rational construction ('rationalis fabrica'). Number is the principle of all that is attainable by reason. Unity is the principle of all numbers. This eternal unity is the essence of God, the beginning and end of all. All beings of created things are nothing else but signs of that supreme unity. Kircher explicitly referred to Augustin, Plato, Pythagoras, and the mythical author Hermes Trismegistus.

The combinatorial art consists mainly, he continued, in demonstrating how often and in how many ways arbitrarily given things might be combined with one another or permuted among one another. God's eternal wisdom practised this art for the first time when he created the world, when he formed by means of the confused bulk of chaotic mass such a great variety of things that it cannot be grasped by mind or sufficiently admired by men. Hence the combinatorial art is an "arithmological faculty" (*facultas arithmologica*). Its variety stems from the different things which are to be compared—for example letters, numbers, elements, or principles, terms, propositions, syllogisms of arts and sciences.

Regarding mathematical combinatorics, there is nothing new relative to the *Universal musical art*. On the contrary, Kircher repeated only some of his earlier rules, tables, results, namely the first two rules for permutations and the corresponding table for permutations with one repeated element. He extended the table for $n!$ up to 50! which has 67 digits. It is, however, by far still smaller than 64! which he found in Mersenne's two great monographs on musical theory mentioned earlier. Yet, perhaps he took it from Mersenne's book *La vérité des sciences*, which had appeared in 1625. There Mersenne not only calculated 1! up to 50! but also the number $2^n - 1 - n$ of possibilities altogether for selecting two or more (up to n) elements out of a set of n elements (combinations without repetitions in a body). This is exactly the result Kircher repeated in his *Great art of knowledge*.

6. Mystical arithmetic

Kircher had said in the *Universal art of music* that whoever knows mystical arithmetic, is able to penetrate into all secrets. No wonder that in 1665 he published a whole book

[4] Μηδὲν κάλλιον ἢ πάντα εἰδέναι.

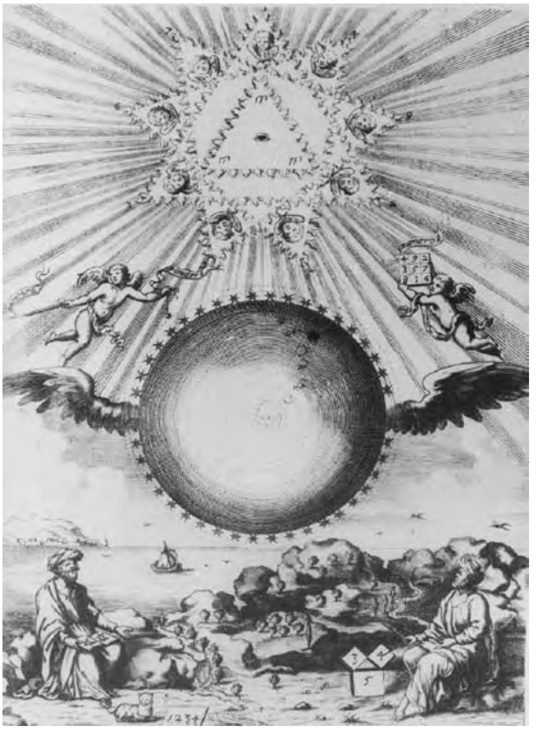

Fig. 5. The Divine Eye. Frontispiece of Kircher's *Arithmologia*.

on this subject, his *Arithmologia* (Science of numbers) *or on the hidden mysteries of numbers*. Therein he combined his theoretical interest in the mystical mathematics of the Pythagorean and Cabbalistic traditions with a more practical approach to the quantitative and computational matters of arithmetic [1, p. 15].

On the frontispiece the divine eye is surrounded by the nine orders of angels who occur again in his combined version of Cusanus's figures P and U (see below). The scrolls of the two cherubs allude to the biblical saying that God disposed all according to measure, weight, and number. The cherub on the right holds a so-called magical square consisting of the first nine natural numbers. Such squares are called "magical" if and only if the sum of the numbers of every row and column and of both main diagonals is always the same, in this case twelve. The winged sphere between the two angels is contained within the sphere of the forty fixed stars of the primum mobile and comprehends the Ptolemaic, geocentric world system consisting of the earth and the centre and the seven homocentric spheres of the "planets" Moon, Mercury, Venus, Sun, Mars, Jupiter, Saturn. This is worth mentioning because even Kircher no longer adhered to this out of date world system but to the Tychonian system. The Hebrew scholar on the left, whose book shows the stars of Solomon and David, symbolises the Cabbala. The figure on the right can be identified as Pythagoras, thanks to his theorem.

7. Mathematical theology

All important Lullists and adherents of a universal science of the 16th and 17th centuries were influenced by Nicolaus Cusanus (1401–1464). Kircher was no exception. The influence of the "mathematical theologian" Cusanus especially concerned two aspects of Kircher's thinking:

(1) The use of mathematical symbols to describe the structure of the universe conceived of as God's creation;
(2) the conception of the infinite.

(1) The use of mathematical symbols

In his treatise *On conjectures*, Cusanus introduced two figures, namely the paradigmatic figure P (chapter I, 9) and the circle of the universes U (chapter I, 13). Figure P consisted of two intersecting dark and light pyramids, figure U of forty circles of four different sciences vertically ordered in such a way that the greatest comprehended the remaining thirty-nine. Kircher enlarged their unification into one figure by two further closed curved lines.

It illustrates the overwhelming cosmological importance of the number three, four and ten for Cusanus as well as for Kircher, thus admittedly bearing witness to Pythagorean and Platonic tradition.

Kircher inserted it into the tenth book of his *Universal musical art* which itself was divided into ten registers. Its ninth register dealt with the music of the angels. Correspondingly, Kircher called the figure the "harmonical scheme" (*schema harmonicum*), the number forty the "most mystical number": Forty is the sum of 1, 3, 9, 27 that is of the unity one and of the first three powers of three. It is a product of ten multiplied by four. Ten itself is the sum of the first four numbers 1, 2, 3, 4, which is the Pythagorean "tetraktys". The simple unit one denotes God, the higher units ten, hundred, thousand denote intelligence, soul and body. Hence it is worthwhile considering the figure in more detail.

The two intersecting, horizontally arranged pyramids comprehend three worlds (the same applies to Cusanus's figure P): the supreme or angelic world, the middle or sidereal world, and the lowest or elemental world, each divided into nine. God as unity is the basis of the light; similarly, the basis of darkness is nothingness. Every creature is situated between God and nothingness. The nine levels of the angelic world are the choirs of the Seraphim, Cherubim, Throni, dominations, virtues, powers, principals, archangels, angels; that is the nine orders of angels. The nine levels of the sidereal world are the immobile sphere, the firmament of the fixed stars, the seven planets Saturn, Mars, Sun, Venus, Mercury, Moon. The nine levels of the elemental world are stones or metals (mixta), zoophytes, plants, animals, man and the four elements fire, air, water, soil.

The forty vertically arranged circles forming Cusanus's figure U represent the same world structure. Kircher's description almost literally repeats Cusanus's text but without mentioning his source. *On conjectures,* §67 corresponds to page 451 of Kircher's *Universal mathematical art,* volume II. This proves that Kircher completely depended on Cusanus, not on Robert Fludd who also used Cusanus's figures.

The simple unity of God touches four circles, Kircher explained: the greatest circle or that of the universe, the circles of the supreme world, of the supreme order, and of the supreme choir. Throughout, there is an analogical structure to the universe and the worlds

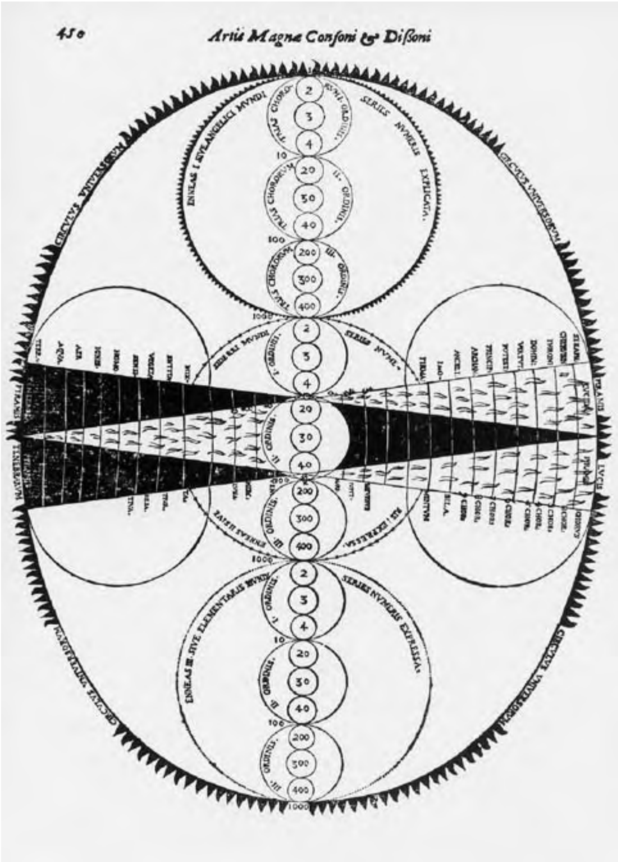

Fig. 6. The Harmonical Scheme. *Musurgia Universalis*, Book X.

contained therein: everything is to be found in everything (*omnia in omnibus*). Triads characterise the universe, the single worlds, the single choirs.

A related scheme was at least motivated by Cusanus. It occurs even twice; at the very beginning of the second volume of his *Great musical art*, and again in his tenth book.

Kircher's basic idea was that the world is God's organ. This idea is explained by three distichs saying that God's love is harmony by which the world is held together, that God is a "harmostes", "somebody who puts together". Already in the preface to the reader, Kircher had referred to Plato by defining music as being nothing else than τἄξιν than "to know the order of all things". Now he cited the mythical author Hermes Trismegistus from the third Christian century, who had said accordingly in his writing to the physician Asclepius: "ἡ μουσικὴ μηδέν ἐστιν ἕτερον, ἢ πάντων τάξιν εἰδέναι" (music is nothing other than to know the order of all things).

The circle represents God who binds all together (Kircher adopted this conception from Cusanus); the sevenfold divided scale representing an octave explains the different levels of

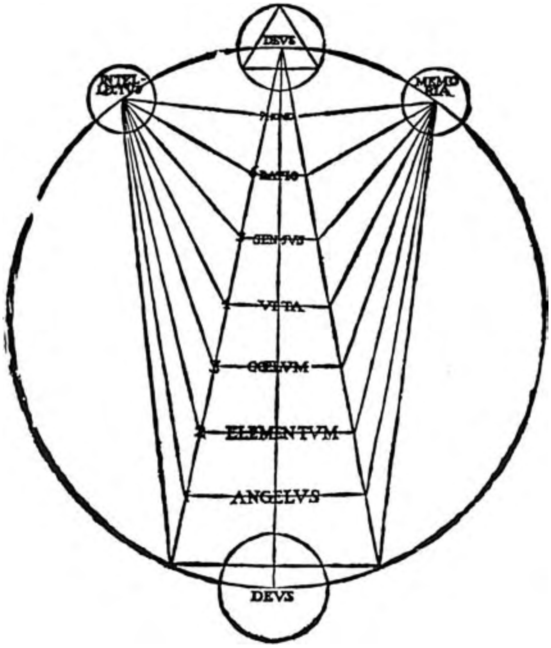

Fig. 7. The world as God's organ represented differently. *Musurgia Universalis*, Books II and X.

creation. The first chord refers to the angel, the last chord to human intellect, the intermediate chords to the perceivable levels of the world: element, heaven, life, sense, reason. The intellect, is so to speak, an eye, memory a mirror in which all pictures of the world reflect. Or the intellect might be an ear, and memory the reflexive mirror-like object. Kircher liked this metaphor. On the frontispiece of his *Great art of knowledge* the eye of reason (*ratio*) and the ear of use and experience (*usus et experientia*) occur again (see above Figure 3).

(2) The conception of the infinite

In 1656 Kircher published his only cosmological work, the *Ecstatic Journey, in which the construction of the world, that is the nature, forces, proprieties of the celestial expansion and of the planets and of the fixed stars as well, the composition and structure of the single stars from the lowest sphere of the earth up to the ultimate limits of the world explored under cover of fictive abduction, is truly explained by a new hypothesis in the form of a dialogue between Cosmiel and Theodidactus*.[5]

Its second dialogue especially deals with the magnitude of the world. Just as little as in the *Universal art of music* did Kircher reveal his source, that is Cusanus, though he repeated Cusanus's theory of the infinite. God is the absolute maximum, the only actual

[5]Itinerarium exstaticum quo mundi opificium id est Coelestis expansi, siderumque tam erantium, quam fixorum natura, vires, proprietates, singulorumque compositio et structura, ab infimo Telluris globo, usque ad ultima Mundi confinia, per ficti raptus integumentum explorata, nova hypothesi exponitur ad veritatem interlocutoribus Cosmiele et Theodidacto.

infinite. Hence, the world itself and all things in it cannot be infinite. They all are contracted to finite quantities. Yet, the world is the "contracted maximum".

This conception of the infinite, which is incomprehensible for the finite mind of men, implies a crucial consequence: the difference between God and the finite world remains infinite. Kircher called this difference the imaginary space, which had no equivalent in Cusanus. But its characterisation as being nothing and infinite at the same time was a reminiscence of Cusanus's famous coincidence of the opposites. Such a coincidence could not take place in mathematics, but only mathematical theology, in God who is the absolute coincidence.

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CHAPTER 18

Galileo, God and Mathematics

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Contents

MATHEMATICS AND THE DIVINE: A HISTORICAL STUDY
Edited by T. Koetsier and L. Bergmans

1. Introduction

It is not a particularly new insight that the mathematical sciences of the seventeenth century stood at the very core of what is commonly called the Scientific Revolution. True as this may be, it has to be realised that the actual message of this statement is twofold. It has long been seen that mathematical techniques and methods played a mayor role in fashioning the new sciences in the seventeenth and eighteenth centuries. As a rule a central place in this process is assigned to Galileo Galilei (1564–1642) who undertook the study of motion (free fall) in terms of mathematics and certainly gave a stimulus—the extent to which is highly debated—to the research on infinitesimal techniques by his disciples Buonaventura Cavalieri (ca. 1598–1647) and Evangelista Torricelli (1608–1647). On the other hand, the rapidly changing social and epistemological status of the mathematical sciences as a whole from the mid-sixteenth through to the seventeenth century is an important precondition of the Scientific Revolution.

In the early seventeenth century there was a growing consensus that the mathematical sciences should take up natural philosophy as an object, and the desire grew to legitimise this transgression of the existing boundaries. Therefore, one of the main aims of Galileo and of many of his contemporaries was to enhance the epistemological status of the mathematical sciences and to abolish the authority and monopoly of philosophers and theologians as keepers of the book of nature. This process not only required a legitimising strategy for the mathematical sciences, but also implied the disintegration and the restructuring of the ensemble of the mathematical sciences. This naturally led to the transformation of the whole hierarchy of scientific disciplines and, in the long run, to the formation of various specific scientific disciplines out of the whole of the mathematical sciences. In this process Galileo held a prominent position. The metaphysical legitimation he constructed for the mathematical sciences by eventually taking recourse to God and the divine is the topic of this paper.

2. The mathematical sciences in early modern Europe

For his contemporaries Galileo was a mathematician (*matematico*/*mathematicus*). I stress this as people today tend to consider him a natural philosopher. In early modern Europe the term “mathematical sciences” was used to describe those fields of knowledge that depended on measure, number and weight—reflecting the much quoted passage from the *Wisdom of Solomon* 11:20, “but thou hast ordered all things in measure and number and weight” (*sed omnia in mensura, et numero, et pondere disposuisti*). This included astrology and architecture as well as arithmetic and astronomy. The *scientiae mathematicae* were subdivided into *mathematicae purae*, dealing with quantity, continuous and discrete as in geometry and arithmetic, and *mathematicae mixtae* or *mediae*, which dealt not only with quantity but also with quality—for example, astronomy, geography, optics, music, cosmography and architecture. The frequent analogy between mixed mathematics and modern applied mathematics rather compares apples and pears, or, to be more precise, is quite anachronistic as pure and applied mathematics are subdivisions of a modern scientific discipline, mathematics, which did not even exist as a discipline in its own right around 1600.

The mathematical sciences, then, consisted of various fields of knowledge, often with a strong bent toward practical applications, which only became independent from each other and became scientific disciplines in the process of the formation of scientific disciplines from the late seventeenth to the nineteenth century.

Considering this context it is important to keep in mind the conceptual inaccuracy implied in the use of the terms (1) *mathematicus*, signifying either a (pure) mathematician or a practitioner of the mathematical sciences doing (mixed) mathematics, (2) *mathematica*, normally used as an adjective and only rarely but confusingly employed as a noun meaning pure mathematics and (3) *mathematicae* instead of *scientiae* or *disciplinae mathematicae*, denoting the whole ensemble of the mathematical sciences. This inaccuracy often renders it difficult to distinguish between the two branches of the mathematical sciences under discussion (*mathematicae purae* or *mixtae*).

Practitioners of the mathematical sciences were not expected to tackle physical problems like motion or to apply mathematical methods to such problems, since these belonged to the realm of natural philosophy. In the hierarchy of scientific disciplines of the Middle Ages and up to the sixteenth century, the mathematical sciences were subordinate to theology and philosophy, and to natural philosophy in particular. In the seventeenth century the picture began to change: Cinderella developed into *mathesis Regia*[1] and scientific modes of explanation increasingly dominated many branches of the sciences and segments of society. The mathematical sciences began to play a leading role in the hierarchy of scientific disciplines.

In the early modern era the mathematical sciences began to produce potential instruments of power and to supply technically and socially valuable knowledge—for use in engineering, administration, social control, etc. This ability to produce useful knowledge and potential instruments of power became the critical basis for the existence of the mathematical sciences; the publication and propagation of this capital was among the essential means through which their social and epistemological status was established.

3. Galileo, God and mathematics

Galileo boldly formulated the attempt to transgress the boundaries of the traditional hierarchy and the aspiration to participate in, if not to monopolise, the reading of the book of nature in his courtly pamphlet of 1623, *The Assayer*. Here his famous words on mathematics and the book of nature are to be found:

> Philosophy is written in this grand book, the universe, which stands continually open to our gaze. But the book cannot be understood unless one first learns to comprehend the language and read the letters in which it is composed. It is written in the language of mathematics, and its characters are triangles, circles, and other geometric figures without which it is humanly impossible to understand a single word of it; without these, one wanders about in a dark labyrinth.[2]

[1] As Claude François Milliet Dechales wrote in the letter of dedication of his *Cursus seu Mundus Mathematicus* (Lyon, 1676): “Plebeiae sunt ceterae disciplinae, mathesis Regia”.

[2] The translation is Stillman Drake’s (*Discoveries and Opinions of Galileo*, New York etc., 1957, p. 237f); cf. EN VI, p. 232: “La filosofia è scritta in questo grandissimo libro, che continuamente ci sta aperto innanzi a gli occhi (io dico l’universo), ma non si può intendere se prima non s’impara a intender la lingua, e conoscer i caratteri,

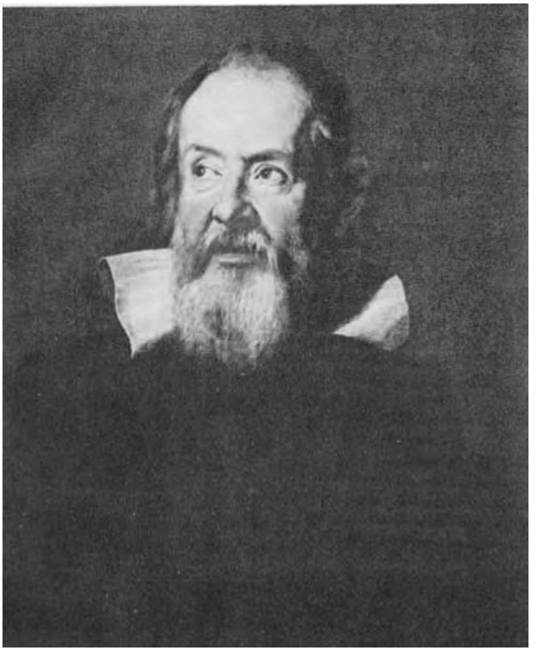

Fig. 1. Portret of Galilei by Giusto Sustermans, Uffizi, Florence.

The essence of these words seems at first sight to be clear: of all possible mathematical methods, the exploration of nature's secrets has to depend on geometry and the geometer alone is capable of reading the book of nature. But history is not as simple as rhetoric: Galileo, the working scientist, did not rely exclusively on classical Euclidean geometry, but rather on a mixture of a theory of proportions and early infinitesimal methods. On the other hand, Galileo as an "imperialistic" scientist sought to extend the bounds of his own competence and scenes of inquiry. Consequently, he could in his rhetoric not be content with a conception of geometry as being only one out of several possible languages for understanding and reading natural phenomena. He needed a cognitive tool guaranteeing exclusive and legitimate access to secure knowledge about the book of nature. He constructed (pure) mathematics as such a tool, thereby locking natural philosophers out of their own traditional domain.

This has to be explained. In discussing Galileo's thoughts about mathematics and the mathematical sciences, it is necessary to keep in mind what is frequently ignored: that he did not codify them and that he worked—as did many early modern scholars—with conflicting epistemologies. Furthermore, there is naturally a wide gap between his theoretical reflections on the nature of his scientific work and his actual procedures. In what follows I

ne'quali è scritto. Egli è scritto in lingua matematica, e i caratteri son triangoli, cerchi, ed altre figure geometriche, senza i quali mezi è impossibile a intendere umanamente parola; senza questi è un aggirarsi vanamente per un oscuro laberinto".

Fig. 2. Frontispiece of the original edition of *The Assayer*, Rome, 1623.

will concentrate on his theoretical reflections, as they are fundamental to the legitimation of his scientific claims.

Standard historiography—following modern scientific conceptions—would have Galileo using mathematics to *describe* nature. This cherished view is none the less mistaken. Galileo did not seek to *describe* nature. He wanted and needed to *read* it. The description of nature allows for a variety of possible languages whereas the reading of nature has to be done in the given and unequivocal language. It is not our conception of a (possibly) variable description that is the basis of Galileo's expressed world view, but rather the conviction that nature is to be understood and explained in the one and only God-given language of mathematics. And even the language of mathematics left room for interpretation as Galileo, the working scientist, knew quite well.

However, for the sake of his argument Galileo considered mathematics in a rather monolithic way and did not distinguish among its diverse branches like geometry, arithmetic, theory of proportions, theory of indivisibles, etc. To have an unequivocal conception of mathematics, he deliberately presented geometry alone as a model of mathematical rigor and exactness. This was, at the same time, to the convenience of the contemporary reader who might not have been at all familiar with more abstract mathematical concepts or with any kind of mathematics other than plane geometry. To confront, say a courtier, with such concepts would have been a fatal mistake because it is quite unlikely that he could have understood their meaning. But a minimum of knowledge or the illusion of such knowledge of plane geometry could be expected. Why was it so important for Galileo's scientific claims to legitimise mathematics as the definite language of the book of nature?

Galileo wanted his claims to be necessary. In particular, he wanted to escape the mere probability of his arguments for Copernicanism—for that was their status in the dominant scientific hierarchy—and give them the compelling character of mathematical proofs. He tried to achieve this by means of a double strategy. On the one hand, he sought to break the spell of theology by reducing the power of theological reasoning and formulas to an adequate domain, that is by excluding the explanation of natural phenomena from its competence. The book of revelations—the Bible—and the book of nature had both been written by God, but in different languages and for different purposes. Even though the languages of these books are not the same there can be no contradiction between their respective claims and truths. Galileo put this principle centre stage in his famous letter to the Grand Duchess Christina in 1615:

> Because of this, and because (as we said above), two truths cannot contradict one another, the task of a wise interpreter is to strive to fathom the true meaning of the sacred texts; this will undoubtedly agree with those physical conclusions of which we are already certain and sure enough through clear observations or necessary demonstrations.[3]

As a consequence, each of these books required its own exegetes. On the other hand—and this string of arguments is largely independent of the first one—Galileo tried to elevate the epistemological status of mathematics. Mathematics allowed the utmost epistemological certainty that could possibly be attained. This conviction was at the core of his mathematical cosmos. It was rooted in the sixteenth century debate *quaestio de certitudine mathematicarum* on the epistemological status of mathematical proofs. I do not want to give an account of this rather extensive debate, but just to state the main result accepted by most mathematical practitioners by the beginning of the seventeenth century: if mathematical proofs were not the most powerful within the ideal scientific hierarchy of early modern Aristotelians (*demonstrationes potissimae*) they still guaranteed the highest degree of certainty humanly attainable (*demonstrationes certissimae*). This perception was central to the revaluation of epistemological categories and deliberately ignored and undermined the Aristotelian hierarchy of the scientific disciplines.

Now Galileo not only considered mathematics absolutely certain, but he followed the sixteenth century *quaestio de certitudine mathematicarum* in attributing to pure mathematics (as exemplified by geometry and arithmetic) a logical structure superior to the logical systems of the early modern Aristotelians. Therefore, it was not only admissible to employ mathematical reasoning in the study of nature, it was compulsory to do so in order to guarantee creativity and epistemological certainty. In addition to the epistemological superiority of mathematical reasoning, Galileo declared in his *Dialogue Concerning the Two*

[3]The translation is Maurice A. Finocchiaro's (*The Galileo Affair: A Documentary History*, Berkeley/Los Angeles/London, 1989, p. 96); cf. EN V, p. 320: "Stante questo, ed essendo, come si è detto, che due verità non possono contrariarsi, è officio de'saggi espositori affaticarsi per penetrare i veri sensi de'luoghi sacri, che indubitabilmente saranno concordanti con quelle conclusioni naturali, delle quali il senso manifesto o le dimostrazioni necessarie ci avessero prima resi certi e sicuri".

Chief World Systems of 1632 that mathematical reasoning could produce knowledge that was equivalent in quality to divine knowledge. Galileo's spokesman Salviati explains:

> Salviati: ... But taking man's understanding intensively, in so far as this term denotes understanding some proposition perfectly, I say that the human intellect does understand some of them perfectly, and thus in these it has as much absolute certainty as nature itself has. Of such are pure mathematics; that is geometry and arithmetic, in which Divine intellect indeed knows infinitely more propositions, since it knows all. But with regard to those few which the human intellect does understand, I believe that its knowledge equals the Divine in objective certainty, for here it succeeds in understanding necessity, beyond which there can be no greater sureness.[4]

Galileo even insinuated that pure mathematics was the only way open to the human intellect to gain knowledge equivalent in quality to divine knowledge. In the *Dialogue* he spoke of "pure mathematics" (*le scienze matematiche pure*), namely geometry and arithmetic, as a way to certain knowledge. Although capturing Galileo's conception of mathematics as the exclusive way to knowledge equivalent in quality to divine knowledge, Stillman Drake's translation of *le scienze matematiche pure* as "the mathematical sciences alone" is still misleading, as this refers to the whole of the mathematical sciences—pure and mixed.[5] This interpretation neglects the direct connection of Galileo's statement to the *quaestio de certitudine mathematicarum*. It was not the possibly refutable mathematical knowledge of an individual that Galileo referred to in order to demolish the priority of theological knowledge but the whole branch of pure mathematics (*le scienze matematiche pure*) whose methods, merits and absolute certainty were accepted everywhere among mathematicians and were not open to debate.[6]

For Galileo the certainty of mathematics was not derived from God, but, on the contrary, it was a way to God. The certainty of mathematics was not only independent of God and consequently of theology, but was even the only way to epistemological perfection. Galileo's claims about the relation of pure mathematics and theology, though not officially mentioned in the trial of 1633, were among the points the inquisition held against him

[4]The translation is based on Stillman Drake's (*Dialogue Concerning the Two Chief World Systems. Translated with Revised Notes by Stillman Drake. Foreword by Albert Einstein*, Berkeley/Los Angeles, 1962, p. 103); cf. EN VII, p. 128f: "Salviati: ... ma pigliando l'intendere *intensive*, in quanto cotal termine importa intensivamente, cioè perfettamente, alcuna proposizione, dico che l'intelletto umano ne intende alcune così perfettamente, e ne ha così assoluta certezza, quanto se n'abbia l'istessa natura; e tali sono le scienze matematiche pure, cioè la geometria e l'aritmetica, delle quali l'intelletto divino ne sa bene infinite proposizioni di più, perchè le sa tutte, ma di quelle poche intese dall'intelletto umano credo che la cognizione agguali la divina nella certezza obiettiva, poichè arriva a comprenderne la necessità, sopra la quale non par che esser sicurezza maggiore".

[5]*Dialogue Concerning the Two Chief World Systems. Translated with Revised Notes by Stillman Drake. Foreword by Albert Einstein*, Berkeley/Los Angeles, 1962, p. 103. Emil Strauss' German translation as "purely mathematical knowledge" ("rein mathematische Erkenntnisse") is incorrect (*Dialog über die beiden hauptsächlichsten Weltsysteme, das ptolemäische und das kopernikanische. Aus dem Italienischen übersetzt und erläutert von Emil Strauss*, Leipzig, 1891, p. 108). Maurice A. Finocchiaro translates "the pure mathematical sciences" (*Galileo on the World Systems: A New Abridged Translation and Guide*, Berkeley/Los Angeles/London, 1997, p. 113).

[6]Galileo was well aware of the *quaestio de certitudine mathematicarum*. Not only did he own a copy of Francesco Barozzi's *Opusculum, in quo una Oratio, & duae Quaestiones: altera de certitudine, & altera de medietate Mathematicarum continentur* (Padua, 1560), but he also knew Giuseppe Biancani's *Aristotelis Loca Mathematica Ex universiis ipsius Operibus collecta, & explicata ... Accessere de Natura Mathematicarum Tractatio; atque Clarorum Mathematicorum Chronologia* (Bologna, 1615) as a note in his Discorsi proves (EN VIII, p. 165).

in 1632. After the complaints against Galileo's *Dialogue* Pope Urban VIII suspended further distribution of the book and appointed a special commission to investigate the affair. In the commission's report of September 1632, apart from Galileo's undisguised Copernican propaganda, mostly formal aspects of how he had dealt with Church censorship in the process of getting the imprimatur for the *Dialogue* were mentioned. However, the commission explicitly pointed to the passage under discussion:

> Moreover, there are in the book the following things to consider, as specific items of indictment:
> [...]
> vi. That he wrongly asserts and declares a certain equality between the human and the divine intellect in the understanding of geometrical matters.[7]

At the same time Galileo held mathematics to be the prescribed language of the book of nature and consequently mathematical reasoning was the exclusive way for the human intellect to decode the laws of divine creation. By this twofold construction Galileo wanted to secure an epistemological monopoly for mathematics in the explanation of natural things, rivalling that of theology and philosophy.

It does not come as a surprise that on this basis Galileo considered it legitimate to employ mathematics as the ideal tool in natural philosophy. Galileo combined five properties of mathematics in his argument: (1) God has written the book of nature—which is the object of natural philosophy—in the language of mathematics. (2) Man can learn this language and (3) apply it to the study of nature as it has a logical structure superior to that of early modern Aristotelians. (4) Handled with care this language cannot err or go astray because it is not only the most certain epistemological tool, but (5) even the most perfect one capable of elevating the human mind to divine knowledge. Actually Galileo did not just think it legitimate to use mathematics in natural philosophy but made it quite clear that it was the *only* legitimate way to proceed. Nature had to be read mathematically and was not to be described philosophically or theologically if any ambiguity was to be excluded and the (real) laws of nature were to be discovered.

Galileo automatically transferred the certainty of pure mathematics (*mathematicae purae*) to the mixed mathematical sciences (*mathematicae mixtae*) and particularly to the mathematical study of nature, the subject of the emerging *new science*. In this way the epistemological status of not only mathematics but of the mathematical sciences in their entirety and the applications of mathematics were enhanced and the hierarchy of scientific disciplines transformed. Of course, what one has to keep in mind is Galileo's desire to prove the Copernican system against scripture.

What Galileo needed and meticulously constructed was mathematics as a tool to refute counter-arguments from biblical exegesis and to fulfill the well-known demand of Cardinal Bellarmine and the Jesuit mathematicians that if the Copernican system was to be accepted and the Earth did in reality circle the sun, a necessary demonstration should be given instead of just probable arguments. The Jesuits' leading mathematician, Christopher Clavius (1538–1612), had left no doubt about this in one of his final statements in print, the preface to the fourth volume of his *Opera mathematica* published in 1612 and dealing

[7] The translation is Maurice A. Finocchiaro's (*The Galileo Affair: A Documentary History*, Berkeley/Los Angeles/London, 1989, p. 221f); cf. EN XIX, p. 326f: 'Nel libro poi ci sono da considerare, come per corpo di diletto, le cose seguenti: [...] 6. Asserirsi e dichiararsi male qualche ugualianza, nel comprendere le cose geometriche, tra l'intelletto umano e divino'.

with the theory of sun-dials.[8] And Cardinal Bellarmine put it bluntly in his 1615 letter to the Copernican Carmelite Antonio Foscarini: "But I will not believe that there is such a demonstration, until it is shown to me".[9] Regardless of the structure and truth of the proof Galileo eventually presented it had to be a mathematical proof or, to be more precise, a proof that definitely belonged in the sphere of the mathematical sciences. Only mathematical arguments, opening a highway to divine knowledge, could lead out of the impasse into which the Copernican debate had run. Mathematical arguments alone carried the power to counter biblical arguments as mathematics, too, was the language of God, the divine geometer.

4. God and mathematics around Galileo

An attempt to review thoroughly the divine element in seventeenth century mathematical sciences is beyond my scope but without doubt the divine quite naturally entered many a mathematician's theoretical reflections on the foundations of his discipline. Two of the standard arguments for a certain affinity between mathematics and divine creation have already been quoted, namely the saying attributed to Plato that in essence God himself was a geometer (*Deum semper Geometriam exercere*) and the famous passage from the *Wisdom of Solomon* that God had organised the world according to measure, number, and weight.

Nearness to God was explicitly drawn upon to legitimise the mathematical sciences by many of Galileo's contemporaries: by Johannes Kepler (1571–1630), who even considered the mathematical way as a kind of divine service; by Galileo's disciple Niccolò Aggiunti (1600–1635), who also served God mathematically; more cautiously by Evangelista Torricelli, who after the trial of 1633 knew about the pitfalls into which discourse about the divine and mathematics could lead; by Galileo's Dutch sympathiser Martin van den Hove (Martinus Hortensius, 1605–1639), who could freely confess to the close neighbourhood between mathematics and the divine; by the Jesuit mathematicians Giuseppe Biancani (1566–1624) and Charles Malapert (1580–1630), both of whom clearly saw that nearness to God and their epistemological perfection guaranteed the progress of the mathematical sciences; and many more.[10] It has to be taken into account that after the trial of 1633 the profession to close links between the mathematical and the divine may have been prevented by fear of ecclesiastical intervention, at least on the Italian scene.

A denunciation of October 1641 bears witness to this. The Piarist Mario Sozzi (1608–1643) had indicted some of his Florentine brothers before the Roman Inquisition. The

[8] Clavius, Christoph: *Operum mathematicorum tomus quartus complectens gnominices libros octo. fabricam et vsvm instrvmenti ad horologiorum descriptionem peropportuni. Horologiorum novam descriptionem. Compendivm brevissimvm describendorvm horologiorum horizontalium ac declinantium. Notas in novam Horologiorum descriptionem*, Mainz, 1612: "... vt nihil probabile admittant, sed illustribus omnia argumentis necessarijsque demonstrent ...".

[9] The translation is Maurice A. Finocchiaro's (*The Galileo Affair: A Documentary History*, Berkeley/Los Angeles/London, 1989, p. 68); cf. EN XII, p. 172: "Ma io non crederò che ci sia tel dimostratione, fin che non mi sia mostrata".

[10] For a detailed discussion of these topics cf. Volker R. Remmert, *Ariadnefäden im Wissenschaftslabyrinth. Studien zu Galilei: Historiographie–Mathematik–Wirkung*, Berne, 1998, Chapter 5.

Piarists (*Scolopi*) were an order devoted to teaching. In contrast to the Jesuits they did not teach advanced and elitist courses but founded elementary schools (*Scuole Pie*) to teach the children of the poor. The Florentine Piarists had been well known for their Galilean sympathies, which Sozzi deplored, but his main complaint about the Florentine brothers Clemente Settimi, Famiano Michelini and Ambrogio Ambrosi was another:

> All of them believe that there is no science truer and more certain than the one Galileo teaches following the mathematical way, calling it the new philosophy and the real way to do natural philosophy. The above mentioned, and Fathers Francesco Clemente and Ambrogio in particular, have often said that this was the true way to know God. And time and again Father Clemente tried to convince me to take up the study of their theories.
> . . . they said that this philosophy had been proved and that it was the true way to convert the heretics and to know God . . .[11]

This was sufficient for the Roman Inquisition to order Settimi to come all the way from Florence to Rome to justify himself. Even though the historical context and consequences of Sozzi's denunciation still lie in the dark, the affair nevertheless illustrates that to speak boldly about God and mathematics or the mathematical way as Galileo had done in his *Dialogue,* bore a certain risk of running into trouble with ecclesiastical authorities.

5. Concluding remarks

Obviously, there is a contradiction between Galileo's theory that mathematics is the only and unequivocal means to explain nature and his practice when he applied mathematical methods which called for the idealisation of nature and mathematical techniques adapted to the actual problems. But this contradiction breaks up into two fundamentally different concepts of mathematics. Galileo the working scientist employed mathematics as a research instrument, whereas Galileo the propagandist fashioned mathematics as a model of epistemological perfection.

Galileo followed two interwoven strategies to legitimise his position. He persisted with amazing success in advancing his own social status as a practitioner of the mathematical sciences. Mario Biagioli has shown how ingenious Galileo was in advancing his own career in the system of court patronage and how this contributed to the social legitimation of his scientific practice. On the other hand, Galileo outlined a complete legitimising strategy for the epistemological status of pure mathematics, its employment in other branches of knowledge and its emancipation from theology. Furthermore, on the foundation of his unshakeable belief in the epistemological excellence of the mathematical sciences in the hierarchy of scientific disciplines, Galileo devised and used many methods and mechanisms to enhance their status. By so doing, he could build on common procedures and patterns of argumentation.

[11] The translation is mine; cf. Picanyol, Leodegario: *Le Scuole Pie e Galileo Galilei*, Rome, 1942, pp. 141–143: "Tutti li sopradetti tengono che non ci sia né più vera, né più certa scienza di questa del Galileo che insegna per via di mattematica, chiamandola nova filosofia, e vero modo di filosofare; e più volte hanno detto i sopradetti e particolarmente il Padre Francesco Clemente et Ambrogio che questo è il vero modo di conoscere Dio, e più volte mi ha essortato il Padre Clemente a darmi a questo studio . . . hanno detto che questa filosofia è provata, che questo è il vero modo di convertire gli heretici e conoscere Dio; . . .".

Galileo's double strategy was extraordinary only insofar as it was exceptionally successful. To elevate the social status of the mathematical sciences and the epistemological status of pure mathematics by different means and mutually to transfer the epistemological status of pure mathematics to all the mathematical sciences and the social status of mixed mathematics to the realm of pure mathematics was totally conventional and accorded with the practice of many contemporary mathematicians. As did his contemporaries, Galileo referred to existing patterns of argumentation, employed the same sources and used the same structures and mechanisms to advance the common cause. His exceptional success rested on his high visibility as a client of the Medici, his outstanding scientific achievements and his rhetorical and intellectual brilliance.

But there is more to the story than a divine touch to Galileo's mathematical universe. Such developments around the mathematical sciences were closely related to the transformation of the hierarchy of scientific disciplines beginning in the early seventeenth century. Disciplinary boundaries were extended and blurred by the expansion of the mathematical sciences into hitherto forbidden scenes of inquiry. This expansion was particularly indebted to the construction of pure mathematics as a divine science and a model of perfection that had necessarily to be applied to the study of nature. The rising epistemological status of mathematics and the mathematical sciences automatically brought along epistemological weight for the originating new sciences depending on mathematical methods. This points directly, for instance, to the later role of physics as a leader in the hierarchy of scientific disciplines in the nineteenth and twentieth centuries (a *Leitwissenschaft* in the sense of Norbert Elias).

Without a doubt, the position of the new sciences originating in the changing hierarchy of scientific disciplines greatly profited from the changing position and imperialism of the mathematical sciences. The emancipation of the mathematical sciences, the elevation of their epistemological and social status, was a necessary precondition for the astonishing transformation of the hierarchy of scientific disciplines. And this transformation prepared the ground for the implementation of the modern organisation and structure of scientific disciplines. It was not merely the noble minds or backgrounds of its practitioners that gave the new fields of scientific inquiry credibility; it was, at the same time, the applicability of pure and mixed mathematics to these fields. This is not to say that the mathematical sciences were actually and efficiently applied to these fields but only that they could profit from their status—just think of the new branch of *physico-mathesis* or *fisicomatematica*, which not only incorporated mathematics into its name to prove that the mathematical sciences were to be used in physical inquiries but also to transfer the epistemological certainty of pure mathematics to the results of theses inquiries.

The importance the divine aspect had originally had in this process and in the thinking of Galileo and many of his contemporaries increasingly sank into oblivion as a more polished and rationalistic picture of the Scientific Revolution was painted in historiography. Indeed, taken at face value, Galileo's ideas on God and mathematics hardly seemed compatible with his position as a martyr of science (and not only science) or with the image of the founding father of modern physics. But seen in the context of Galileo's legitimising efforts and propaganda for the Copernican system, the divine fits perfectly into the picture.

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CHAPTER 19

The Mathematical Model of Creation According to Kepler

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Contents

MATHEMATICS AND THE DIVINE: A HISTORICAL STUDY
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Kepler not only embodies the birth of classical science with his model of the elliptical orbits of the planets and the formulation of the law of gases, but he is also often considered as one of the last representatives of a Renaissance scientific style, a style weighed down with questionable analogies and which is commonly presented, erroneously, as opposed to the modern project of a methodical mathematical expression of nature. Thus, the author of *Harmonices mundi* may have supposed that the movements which he observed in the sky were characterised by the same harmony as musical scales, and it is very tempting to think that Spinoza, when he criticised the cosmological application of traditional finalism, was talking about these attempts by Kepler: "[...] and finally, those [objects] which move the ears are said to produce noise, sound or harmony. Men have been so mad as to believe that God is pleased by harmony. Indeed there are Philosophers who have persuaded themselves that the motions of the heavens produce harmony".[1] This opposition between the rational core of his contributions to astronomy and the accompanying symbolism, explained away as a kind of intellectual "extravagance", is hardly satisfying. In fact it should be recognised that through the redevelopments that Kepler claims to have made in the interpretation of analogical correspondences, there exists an ambitious philosophical project to found the geometricisation of natural phenomena. To be more precise, on the one hand, it is clear that the mathematical research concerning the harmony of the world (that is to say, the orbits of the planets) has an apologetic purpose, "[...] seeing that we astronomers are, with the book of nature as our starting point, the priests of God the Almighty, we should seek above all not the praise of our own minds but the glory of the Creator".[2] Kepler interprets the Creation according to a mathematical model that allows him to maintain a similarity between the regions of which Boethius affirmed the correspondence, the music of the world (which governs natural phenomena), human music (which characterises the order of the parts of the soul and the purest pleasure) and instrumental music (which we can observe in music between different voices[3]). But, on the other hand, this similarity finds itself reinterpreted according to precise epistemological constraints: confronted with the first developments in algebra, the author of the *Harmonices Mundi* affirms the ontological primacy of geometrical objects which, alone, are truly real and constitute the archetypes of Creation. In so doing, Kepler abandons the Pythagorean tradition, which he explicitly challenges, even if this break is not sufficiently underlined by commentators. The critique of analogy, of which recent commentators have defined the metaphysical foundation,[4] is constructed with great attention to rigorous methodology, which will lead Kepler himself to reject a general appeal to finalist speculations. In short, Kepler's theory of harmony,

[1] *Ethics*, Ist Part, Appendix, *The Collected Works of Spinoza*, Edited and translated by Edwin Curley, Princeton University Press, Princeton, New Jersey, 1985, volume I, p. 445.

[2] Letter to H. von Hohenburg, 26 October 1598, *Gesammelte Werke* [*GW*], Band VI, Beck, München, 1937–1959: *GW* XIII, p. 193. All quotes from *Harmonices mundi* are taken from E.J. Aiton, A.M. Duncan, J.V. Fields, *The Harmony of the World* [*HW*], Translated into English with an Introduction and Notes, Memoirs of The American Philosophical Society, Philadelphia, vol. 209, 1997.

[3] See, for example, this precise version of the correspondence between instrumental music and music of the world which allows the order of Creation to be represented: "[...] for our polyphonists as they call them, no limit is set when they multiply the number of lines of melody to be sung together, as God the Creator also, in his tempering of the heavenly motions, preceded them, making a system of seven diapasons, and more" (HW, Book III, Chapter XI, p. 212).

[4] For example, J.-L. Marion, *Sur la théologie blanche de Descartes*, PUF, Paris, 1991, pp. 178–203.

Fig. 1. Johannes Kepler (etching by Jacob von Heyden).

which was to make explicit the true criteria of divine creation, will also have to set forth the conditions of a rigorous scientific explanation. In this respect, the problem is no longer that of the domination of Kepler's conception of science by theological preoccupations, but, on the contrary, of whether (and how) it is possible to avoid reducing his theological discourse to a discussion of the foundations of the mathematical sciences.[5]

1. Harmony and mathesis: the originality of Kepler

To illustrate his critique of finalism, Spinoza takes the example of the generalisation of the harmony of sounds to all the other parts of the natural order, and in particular to the order of the world or of nature; and we may, with certain reservations that we will examine later, consider that it is really according to this argument (from musical harmony to the harmony of the world, conceived and desired by God) that Kepler develops his conceptions. For it is firstly in music that man experiences the reality of harmony, and we should go back from this musical system (i.e., from the division of the octave and the classification of conso-

[5] In the present essay the relations between God and mathematics are examined and not Kepler's theological texts. The theological aspects of Kepler's work are presented in the studies (which are discussed by Marion in *Théologie blanche*) of E.W. Gerdes, "Keplers theologisches Selbstverständnis und dessen Herkunft", in: *Internationale Kepler Symposium, Weil-der-Stadt*, eds. F. Krafft, K. Meyer, B. Stricken, Hildesheim, 1973; "Johannes Kepler as Theologian", in: *Kepler. Four hundred years*, eds. A. Beer and P. Beer, Oxford-New York, 1975. The biographical chapters of Max Caspar, *Kepler*, translated and edited by C. Doris Hellman, Dover Publications, New York, 1993, deal with the religious context of Kepler's time.

nances) to the true foundation of relationships that it incarnates. This system which gives pleasure to the soul does not follow the whims of the musician, but finds its rational foundation in the same formal cause that God exploited in order to construct harmony in every region of creation: "Its construction is not arbitrary, as some may suppose, nor a human invention which may be modified, but entirely rational, and entirely natural, so much so that God Himself the Creator has given expression to it in adjusting the heavenly motions to each other".[6] This declaration gives us an essential indication: the intelligibility of the (rational) foundation guarantees that it is the (natural) cause of the diverse incarnations of harmony; and at the same time, Kepler must produce precise methodological criteria to define the domain of what is knowable. From this perspective, he takes great care to distinguish his research from traditional speculations, initiated by the Pythagoreans, about the correspondence between cosmological music and instrumental music (that which men sing or play) which gives us the idea of harmony.

The author of *Harmonices mundi* criticises the more or less suggestive comparisons between the different levels of Creation accepted before him, and he is motivated in this criticism by a truly demonstrative research based on the clear idea of the status of different mathematical objects which could (or could not) be called true causes. Thus, for example, he explicitly insists on "the difference [...] between the symbolism of Ptolemy and [his] own legitimate demonstrations ⟨*inter Ptolemaei symbolismos et meas demonstrationes legitimas*⟩".[7] Kepler's opposition to purely symbolic similarities is echoed in the criticisms he directed at certain of his contemporaries such as Robert Fludd. In the case of Fludd, as in that of Ptolemy or the Pythagoreans, the fundamental error consisted in first taking the numbers or, more generally, the discrete quantities (the relationships and proportions of which they are composed) for the causes of harmony which they try to explain: for example, Fludd "trusts the Ancients, who believed that the force of the harmonies comes from abstract numbers".[8] Thus the criticism of Renaissance symbolism is in reality connected to a strong conception of the relationships between discrete and continuous quantities, which leads Kepler to reject the (Pythagorean) notion that harmony comes from the beauty of numbers. The new doctrine of Harmony will therefore be based on a reformed *mathesis* that should be understood in a more precise context.

To begin with, this criticism of numerical causality seems to be aimed at prestigious but distant adversaries, above all the Ancients and Ptolemy in particular. For example, when, in 1621, he went back to the text of *Mysterium cosmographicum*, which, since 1597, had marked his adhesion to Copernicanism, Kepler argues, on the grounds of certain results established in *Harmonices mundi* in 1619, that Ptolemy is guilty of having looked for the causes of phenomena (in the same way as musical chords) in the properties of numbers: "Ptolemy does not look beyond the numbers as causes without considering the figures as counted numbers and so he, with the ancient writers, unjustly proscribes certain chords and accepts among the melodic intervals some which do not deserve that status".[9] Kepler knows, however, that the great astronomer endeavoured to avoid the errors of the Pythagoreans, but the criticism given here is situated at a much more general level.

[6] *HW*, Book III, Chapter II, p. 158.
[7] Ibid., Appendix to Book V, pp. 499–500; *GW* VI, pp. 369, 30–31.
[8] Ibid., p. 506.
[9] *Mysterium cosmographicum*, in caput duodecimum notae auctoris, n° 38, *GW* VIII, p. 77.

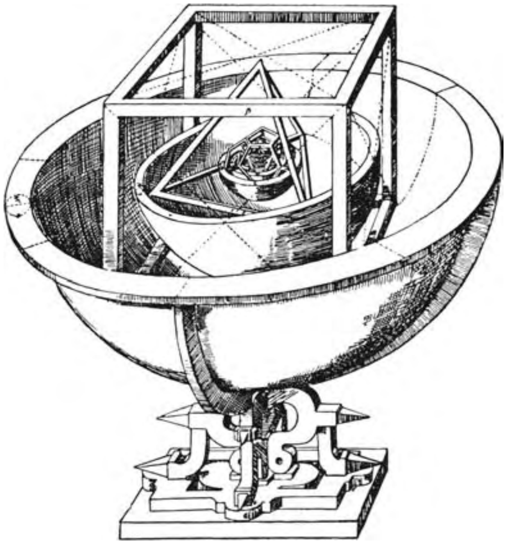

Fig. 2. The universe as depicted in Kepler's Mysterium Cosmographicum (1596).

Ptolemy's error stems from the fact that he does not see that numbers are nothing but the result of a measurement, and that only the objects counted possess true being: "[...] whereas Ptolemy looks for the basic principles of harmonies in abstract numbers, along with the Ancients, I, on the other hand, say that there is no force in the numbers as counting numbers ⟨*numeris ut numerantibus*⟩, and in their place establish as the basic principles of the harmonies the counted numbers ⟨*numeros numeratos*⟩, that is, the things themselves which are subject to the numbers, in other words the plane regular figures and the divisions of the circle which are to be controlled by them".[10] In itself, such a use of the distinction between counting numbers and counted numbers[11] has no originality. But in this criticism of apparently distant concerns, Kepler is actually arguing against the evolution of mathematical sciences of his time.

The refutation of all forms of numerical causality is allied to the criticism of other contemporary opponents that Kepler was engaged in from the first book of his *Harmonices mundi*. These opponents developed cossic methods and thus anticipated developments in algebra. Kepler targeted in particular Justus Burgius (Jost Bürgi), who wanted to provide an algebraic expression of the relation of a heptagon (which up to the 19th century was not a constructible polygon) to the radius of the circumscribed circle. We should note, in passing, that this object plays an important role in the theory of harmonics, because its examination engages the traditional exclusion of the number seven in defining the consonant intervals in music. Kepler maintains this exclusion with arguments taken from geometry. In general, the defect of algebra (or of its pre-figurations in cossics) lies in the fact that it has a tendency to create a belief in the existence of what is, in reality, a non-entity, because

[10] *HW*, Appendix to Book V, p. 500.

[11] Established, for example, by Aristotle, *Physics* IV, 11, 219b 6.

it allows the apparently rigorous definition of an object, such as the heptagon, which is not constructible: "[. . .] it is appropriate to add a word here addressed to Metaphysicians in connection with this algebraic treatment: let them consider if they can adapt this treatment to algebraic Axioms, since they say that which does not exist (a non-entity) has no characteristics and no properties. For here, indeed, we are concerning ourselves with entities which can be known; and we correctly maintain the side of the Heptagon to be among non-entities that cannot be known".[12] In conclusion, cossics, for Kepler, demonstrate that numbers lose their basis in being when their use is not strictly subordinate to the condition of measurement. We thus understand that the author of *Harmonices Mundi* is very careful concerning the redistribution of the relations between the two types of quantity, which modern developments in algebra define precisely as the problem of the objective correlate in mathematical writing. In this context, Kepler judged that it was becoming urgent to denounce explanations based on simple calculation (especially concerning proportions), in which the geometrical correlate in a constructible figure is not certain, in order to subordinate research into harmony to the preliminary elucidation of the status of objects of *mathesis*, of which the reality must be proven.

The remarkable point is that Kepler, starting with this discussion, reinterprets the whole of his etiology: the causes of the harmonies must be found in the *mathemata* which really possess being—that is to say, not in the geometrical figures whose numbers are subordinate, but in those that can be inscribed within a circle, to which Kepler attributes a special ontological status: it constitutes, for example, the nature of the soul. It is therefore the question of constructability that divides geometrical figures into knowable figures, which can be raised to the level of true causes, and their unknowable counterparts. In the framework of this article, the question is not really to know how the author of the *Harmonices mundi* can thus give a special primacy to the circle (which will be taken up by Descartes and Galileo) that means essentially that he maintains the schema of formal causality for certain mathematical objects; the real problem concerns the exclusive restriction of all causes in Creation to mathematical archetypes.

2. Mathematical ideas in God and creation

It can be considered, in a very general way, that Kepler conserves a model according to which certain mathematical objects are the formal causes of harmony, but we must be particularly attentive to the restrictions that he progressively assigns to this cosmological model and which lead him to produce an original interpretation of the passage of the book of Genesis in which God creates the world according to weight, measure and number. Firstly, we have seen how Kepler rejected the Pythagorean model of numerical causality: it was excluded after the first version of *Mysterium cosmographicum* and made way for a more technical interpretation in the criticism of Cossics given in 1619. However, with *Harmonices mundi,* Kepler also rejects the model of Platonic inspiration according to which the causes of phenomena must be looked for in the properties of regular solids. This second restriction turns out to be all the more serious in that it also targets Kepler's

[12] *HW*, Book I, Prop. XLV, p. 74.

own initial attempts, revealed in 1597, and of which he demonstrated the shortcomings first in the *Harmonices mundi* and then in the notes to the second edition of *Mysterium*. And it is this second question that gives him the space to discuss the passage in the book of Genesis, which interested all the founders of classical science. If it is acceptable to relate the harmony which reigns in phenomena to the properties of polyhedrons (as Kepler does again in *Harmonices mundi*), it is because such an analogy "is acceptable to me and to all Christians, since our Faith holds that the World, which had no previous existence, was created by God in weight, measure and number, that is, in accordance with ideas coeternal with Him".[13] In other words, the quote from Scripture legitimates the use of analogy; but analogies which focus on solid bodies are not sufficiently justified before recognition of the geometrical causes of such correspondences, and these are to be found only in the figures (it is this point that had not been understood in the *Mysterium cosmographicum*): "For their origin is not really in the properties of their solid angles but, rather, the properties of the solid angles are a consequence of the origin of the figures, being by their very nature something that comes later".[14] It is, therefore, obvious that the strict geometrical interpretation of formal causality serves to analyse philosophical discourse concerning the Creation of the World. The broader analogies that are suggested, for example, in Genesis, do not mean anything until they have been cross-referenced with the constructible figures which are the basis of the correspondence between different domains. The status of such comparisons was to be clarified in the second edition of the *Mysterium cosmographicum*. Kepler here distinguished two categories; an authentic relationship (*cognatio*), which truly associates the objects of the same mathematical type, and a simple affinity (*affinitas*), which may be revealed in objects of different species but which is also subordinate to the principles of all created things, the regular figures inscribed within a circle—the ancient comparisons between musical chords and regular solids will have only this status: "therefore, this perfection (or its opposite, mediocrity) belongs to the chords because of the plane figures themselves, and, furthermore, to the solid figures: consequently, and again, it is not a relationship (*cognatio*) but a simple affinity (*affinitas sola*) that exists between the double and imperfect harmonic sections and the Dodecahedron (primary figure) and the Icosahedron (secondary figure)".[15] This rarely quoted text seems to us to be absolutely necessary to an understanding of the profound change which marks the structure of comparisons proposed by Kepler between the diverse harmonies. It is no longer a question of envisaging the analogy between different levels which are more or less close to the formal archetype, but of recognising the existence of a unique formal causality in different sectors, which resemble (or not) each other only in this way.

Thus, only comparisons subservient to geometry are fully intelligible (and not simply pedagogic); but in return, geometrical knowledge guarantees us a full and complete intelligence of the formal criteria of Creation. If we can assimilate with complete certainty the unknowable (that is to say that which is not constructible) to non-being, it is because we share with God the principles of the mathematical sciences. Once again, Kepler's remarks concerning the heptagon prove particularly enlightening. On the one hand, the (non-) constructability establishes the limits of what is intelligible to the human mind, which does

[13] *HW*, Book II, Prop. XXV, p. 115.
[14] Ibid., p. 116.
[15] *Mysterium cosmographicum*, in caput duodecimum notae auctoris, n° 16, *GW* VIII, p. 74.

not know a figure until it can produce its formal description.—The heptagon must be considered as unknowable, in the strong sense of the word: "For a formal description of it is impossible; nor therefore can it be known to the human mind, since the possibility of being constructed is prior to the possibility of being known [...]". On the other hand, this restriction is not relative only to the capacity of our understanding: it concerns the Creator himself, whose science cannot reduce unrealised figures to the criteria of mathematical truth. God, who finds in himself the *mathemata*, does not know the heptagon any more than man: "[...] nor can it be known by the Omniscient Mind by a simple eternal act: because of its nature it is among unknowable things".[16] The impossibility of finding the geometrical procedure that would allow the inscription of a heptagon within a circle is not (and which, for example, would give the calculations of Burgius their objective correlate) is not due to an incapacity of the human mind, but, as G. Simon has written, "it is linked to the very nature of the mathematical object, and consequently, is just as valid for divine intelligence".[17] Above all, these declarations found in *Harmonices mundi* clearly indicate that the author supports the integral univocity between the mathematical knowledge of God and that of created intelligences.

Kepler affirms at the same time that the mathematical essences founded within being (that's to say the constructible figures) constitute all the formal material of Creation and that they are not known by God in a different way than by Man. In other words, geometry expresses God himself as the Creator and as intelligence. It is in the light of this double characterisation of the problem of the mathematical essence of things that the famous thesis "Geometry is coeternal with God" should be envisaged. In this affirmation, Kepler states more than just the eternity of Mathematical truths—as J.-L. Marion indicates, he says that these objects are "comparable (to God) on the basis of their relationship to eternity, and that (they) benefit from his eternity itself, as if one of the divine attributes spread out to that which does not come from God"[18]). Marion also observes that, at the same time, geometrical objects are placed on the same plan as *idea exemplares* which, according to Thomas, presided over the creation of the world—strongly attested to in the following text: "[...] geometry [...], is coeternal with God, and by shining forth in the divine mind supplied patterns to God [...], for the furnishing of the world, so that it could become best and most beautiful and above all most like to the Creator. Indeed all spirits, souls and minds are images of God the Creator if they have been put in command each of their own bodies, to govern, move, increase, preserve, and also particularly to propagate them. Then since they have embraced a certain pattern of the creation in their functions, they also observe the same laws along with the Creator in their operations, having derived them from geometry. Also they rejoice in the same proportions which God used, wherever they have found them".[19] In the light of such declarations we can follow Marion in concluding that, in Kepler, we witness a "a transcription of exemplarism in mathematical terms",[20] since it is obvious that in as much as they structure divine understanding, and thus the fields of possi-

[16] *HW*, Book I, Prop. XLV, p. 74.
[17] See concerning this point the fundamental work of G. Simon, *Kepler, astronome astrologue*, Gallimard, Paris, 1979, p. 156.
[18] Op. cit., p. 179.
[19] *HW*, Book III, Axiom VII, p. 146.
[20] Op. cit., p. 180.

bility, geometrical forms organise all forms of intelligibility (that is to say, also, that which created intelligence can attain) and function as the formal causes of all natural phenomena. It is within this remarkable perspective that we can add that the characteristics of knowable mathematics define a certain type of perfection of ideas that Kepler explicitly assimilates to their tendency to be present in nature. God finds in constructability a motive for realising the movement of the idea of constructible figures towards creation itself: "Since this property of Congruence, which shows itself in structure and bodily form, is such that it, as it were, of itself encourages the speculative mind to make something external to itself, to create, to fashion a solid body. Thus it has from eternity lain hidden in the supremely blessed divine mind, as one of the Ideas, and so far partook of the highest goodness that it might not be contained within its own abstraction but must break forth into the work of Creation, causing God the Creator to enclose bodies within particular figures".[21] The criteria of divine action and, finally, God himself seem thus to manifest themselves to the human mind which can attain, via mathematics, an apparently complete intelligence of the act of creation. The whole question is therefore to know if, as it seems to recent commentators, Kepler confines himself to this characterisation, which is at the same time exorbitant (from an epistemological perspective) and simplistic (from a theological perspective), of our relationship with God.

3. The limits of univocity

Here, it would be convenient to start with a precise question: is mathematical intelligence really sufficient to establish complete univocity between divine science and the typically human modalities of knowledge of harmonies? That is what the passages we have quoted lead us to understand, at least isolated from their systematic context. We must now reconstruct this context which shows that all human science is not established—at least not immediately—in the conditions of such certainty. The arguments that can be produced in this sense concern Kepler's remarks on the order of exposition of the works, his awareness of the novelty of his project and the limits that its radical position imposed on the writer to explain in truly remarkable terms. To catch the general feeling of research concerning the harmonies, we must pay attention to the explicit terms that Kepler himself uses to comment on his own work and also to the precautions that he often discusses in order to prevent a "naïve" interpretation of the mathematical model which, he argues, guarantees maximum intelligibility.

It is true that the human mind, when it has attained a clear consciousness of the mathematical causes of things, applies itself with the same certainty to the geometrical archetypes which define their nature as to God himself, with whom these objects are said to be coeternal—we have seen that this thesis implies, in particular, that the impossibilities encountered by Geometry with regard to certain objects apply equally to God, who, for example, cannot imagine, any more than we can, the constructability of a heptagon. But this strict identification *de jure* between human science and that of the creator is not, *in fact*, so easy to apply. And once it has been understood in principle, it does not lead, however,

[21] *HW*, Book II [Introduction], pp. 97–98.

to an effective explanation of the system of harmonies. This is the meaning of Keplers' remarks concerning the order of exposition that he follows in his works. In principle (*de jure*), the elucidation of the metaphysical status of harmony and its foundation in geometry should allow us to deduce its various aspects in God himself as creator, then in the world and finally in human music. However, if the inscribable polygons, as archetypes, constitute that which is most real, it remains that man *encounters* them only in abstraction based on sensory experience, which comes first in a chronological perspective. In other words, in the effective presentation of science, it is convenient to start with what is most derived, i.e. melody, and then progress up to the purest forms of harmony. Thus the beginning of the third book of *Harmonices mundi* makes it clear that once the mathematical premises of the research have been presented (especially the status of counted numbers which cossic methods tend to conceal), exposition of the harmonies must start by the examination of instrumental music, whose vocabulary is more familiar to us, and not the examination of the metaphysical theory of harmony, which is nevertheless the basis of the application of this concept to other areas of knowledge: "[. . .] it is indeed difficult to abstract mentally the distinctions, types and modes of the harmonic proportions from musical notes and sounds, since the only vocabulary which comes to our aid, as is necessary to expound matters, is the musical one".[22] In fact the situation is even more complicated than this. It does not consist only in tracing the order of exposition onto the order of invention, avoiding following the order of real generation of harmonies, which is "[. . .] what the nature of the subject requires".[23] A less quoted passage of the following book makes it clearer that in reality we must realise a double inversion, which consists in starting with the study of musical harmony and, furthermore, in including, the most abstract metaphysical considerations: "[. . .] the requirements of stating the arguments have persuaded us not only to reverse the order, starting from human song, passing from that to the works of Nature, and thus finally to the Work of Creation, which was the first and most perfect of all, but also to combine the end of abstract speculation with the beginning of actual harmonies in melody".[24] Such considerations are necessary in order to avoid reducing harmony to a simple relationship between sounds, which constitute the sensory starting-point of the investigation. We should note that Spinoza was to criticise such metaphysical extrapolation entrenched in the methodological composition of the work. Whichever direction we take, if the identification of human and divine logic leads Kepler, as Marion establishes, to push aside the inequality that is characteristic of relations of imitation and, in doing so, to reject *analogia entis*,[25] it is obvious that from the strict point of view of the order of exposition of harmonies, a difference remains, linked to the conditions of experience, which defines the conditions of its interpretation and affects the direction of our research.

But it is on the level of the very structure of science, and no longer on the conditions of its acquisition and exposition that the inordinate demands of complete univocity must be confronted. Therefore it is essential to turn careful attention to the passages of *Harmonices mundi* in which Kepler goes over his own reasoning in order to underline a novelty of which he is perfectly aware, even in its theological implications. The author does not hide the fact

[22] Ibid., Book III [Introduction], p. 129.
[23] Ibid.
[24] Ibid., Book IV, Preamble. Explanation of the Order, p. 283.
[25] See J.-L. Marion, op. cit., pp. 187–188.

that the reduction of archetypes to the ideas of geometry, accentuated in the mathematical transcription of the logic of the creator, may give rise to certain difficulties for theologians, even if he thinks that such difficulties do not of course undermine his ideas. To put it in another way, Kepler means to avoid (mistakenly, no doubt) the "theological loss" which sanctions the rewriting of exemplarism in a *mathesis* of continuous quantity. This attitude is illustrated once again in the very strong conclusions taken from the non-constructible nature of the heptagon: "[. . .] it is on account of this result that the Heptagon and other figures were not employed by God in ordering the structure of the World, as He did employ the knowable figures".[26] As we have seen concerning this figure, the central thesis of the univocity of mathematical reasoning in God and Man implies that the impossibilities encountered by the human mind in the order of mathematical knowledge constitute logical impossibilities for divine intelligence itself and, therefore, ontological impossibilities in the process of Creation. But it is obvious that the text of 1619 creates a remarkable shift (where a "theological loss" can be identified), following which the identical structuring of the *mathesis* for God and the human mind, illustrated in the fact that God used only the constructible figures in Creation, is generalised in the sense of complete univocity. The essential point is that, as Kepler himself recognises, such a project is scandalous and he tries to justify it by an identification between the definitions of mathematical knowledge and the formal rules of logic, which, for all writers (before Descartes), are common to all forms of intelligence. He argues that it is acceptable to say that we share the same logic with God and then affirm that this logic is fulfilled in mathematics. The coeternity which the geometrical objects contain means that they define the very principles of the working of intelligence, to which the creator must conform in order to remain consistent with his essence: "In case it should be supposed that these comments are blasphemous. One of my friends, a very practiced mathematician, thought they could be left out. But nothing is more habitual among Theologians than to claim that things are impossible if they involve a contradiction: and that God's knowledge does not extend to such impossible things, particularly since these formal ratios of Geometrical entities are nothing else but the Essence of God; because whatever in God is eternal, that thing is one inseparable divine essence: so it would be to know Himself as in some way other than He is if He knew things that are incommunicable as being communicable. And what kind of subservient respect would it be, on account of the inexpert who will not read the book, to defraud the rest".[27] From this point of view, knowing *more geometrico* boils down, not only to knowing like God, but to knowing God himself. But what does that mean exactly? It means that geometry, which gives us the material of divine understanding, reveals equally, concentrated in its *corpus*, the elementary logical criteria of its organisation, such that respect of the principle of contradiction is included as an internal demand of the *mathesis*. Such a reinterpretation allows Kepler to state, against the theologians, that he has not belittled divine science by discovering its true objects. Such a declaration should be judged in the light of the work accomplished in the first books of *Harmonices mundi* in order to realise that, far from being indifferent to the developments made in the domain of mathematical science, this radical conception of univocity assumes the novel conversion of a certain area of *mathesis pura* to architectonic science.

[26] *HW*, Book I, XLV Prop., p. 60.
[27] Ibid.

Kepler is sharply conscious of the radical break that he brings to the thesis of univocity between divine and human logic. This agreement not only concerns an indeterminate concept of *ens*,[28] but also the readable and perfectly attributable principles and also the detail of the classification of mathematical objects which, as far as they are constructible, constitute the archetypes of Creation. However, because of the problem of determining the complete univocity of the geometrical model which God himself has allowed us to know, Kepler increases the number of precautions to avoid fictional analogies that other authors, less aware of the criteria put into action within the *mathesis*, may have imagined between mathematics and God. This is the deeper meaning of numerous passages in which analogical arguments justifying the special dignity of certain numbers with more or less direct scriptural or theological references are refuted. Kepler takes care to denounce (in texts which commentators rarely take into account) what must be called a finalist prejudice, according to which man postulates a purely imaginary numerical causality which transforms the simple measurement of harmonies into their formal cause—thus he discusses the harmonic triad: "[. . .] this threefold number is not the efficient cause of the harmonies, but an effect of the cause, or a concomitant of the harmony which is realised. It does not give form to harmonies, but is a splendour of their form. It is not the matter of the harmonic notes, but is an offspring begotten by material necessity. It is not the end 'for the sake of which', but it is an eventual product of the work".[29] It is very remarkable, moreover, that the confusions that Kepler denounces here are put down to the ignorance of man who forges an imaginary relationship between the phenomena whose veritable causes he does not know: "[. . .] as this six fold does not come from the six days of Creation, so neither does that threefold depend on the Trinity of persons in the Deity. But since the threefold is common to divine and worldly things, whenever it occurs the human mind intervenes and knowing nothing of the causes marvels at this coincidence".[30] It goes without saying that such comments are aimed at marking out the domain pertinent to the methodical research of formal archetypes, whose perfection directed God in finalised Creation; but it remains that, here, Kepler's text describes the structure of a prejudice which associates an ignorance of the true causes and the presumption of a final causality which, taken from the simple consideration of numbers, is definitively invalidated by the effective mathematical presentation of phenomena. Thus, in the effort to distinguish his own attempt from the symbolism of the Pythagoreans and from Ptolemy, Kepler finally formulated the argument that Spinoza was to turn against him.

The analysis proposed here allows us to characterise the historical situation of the Keplerian doctrine of harmony by a double tension, which engages its metaphysical (its relation with the theological doctrines of analogy) and scientific (its place in the reflection on the principles of classical science) aspects. (1) As J.-L. Marion has established, Kepler's criticism of analogy sanctions a new conception of univocity, which, exclusively interpreted according to a mathematical model, leads Kepler to adopt *complete* univocity. And whatever the limits within which the author of *Harmonices mundi* affirms the pertinence of this project, it is impossible to contest the argument that the thesis of complete univocity between the logic of the Creator and the conditions of rigorous rational (mathematical)

[28] As Marion underlines, op. cit., p. 184.
[29] *HW*, Book III, Chapter III, pp. 169–170.
[30] Ibid., p. 170.

knowledge leads Kepler to neglect the transcendence of God, who finds himself measured "in function of our comprehension".[31] But this theological regression expresses at its foundation, in a tone which Kepler hopes to be apologetic, the demand for a methodical ordering of knowledge. One must, at the same time, be attentive to the fact that, in the striking passages that we have quoted, the explicit criticism of analogy entails a rejection of the finalist prejudice which weighed heavily on Renaissance theories of similarity. We are thus able to *read* God's designs in the world—a point that is essential to the doctrine of harmony in the world, up to Leibniz and even up to the 18th century. But this decoding can only be carried out on the basis of a rigorous mathematical epistemology which replaces the traditional power of numerical causality by a reformed *mathesis*. (2) In one sense, the criticisms of analogy developed later will apply to Kepler himself a line of argument which he directed against the Pythagorean tradition—without going as far as Spinoza, we can quote Mersenne: "[. . .] I'm surprised how Kepler dared to compare the figures with the consonances, in order to extract the reason for their number and their goodness". Such procedures lose all their scientific value, and can scarcely serve any pedagogical purpose in the work: Kepler's attitude "would be tolerable only if he contented himself with comparing the said figures with the consonances and dissonances by analogy, and for recreation".[32] But by the same movement, and if we admit the possibility of a natural theology, a whole field is freed for comparisons which will no longer be simply explanatory but essentially apologetic—which is why Mersenne will allow himself several purely symbolic comparisons, against which Kepler had warned his reader. The ideas of Mersenne concerning Harmony are characterised in this way by a sort of *chiasmus* in which harmony loses its status of scientific explanation but is reinstated as a general principle of order. And this situation is brought about by the (provisional) abandoning of the maximal demand, accepted by Kepler, for a conciliation between the geometrical science of phenomena and a metaphysic in which the formal causes are determining and harmonic.[33]

[31] Marion, op. cit., p. 186.

[32] *Harmonie Universelle, Traité des consonances, des dissonances, des genres, des modes et de la composition*, livre I, proposition XXXIII, Paris, Editions du CNRS, vol. II, p. 86.

[33] Qu'il nous soit permis de remercier ici Antony McKenna, directeur de l'UMR 5037, pour son aide précieuse dans l'établissement de ce texte.

Colour Figures

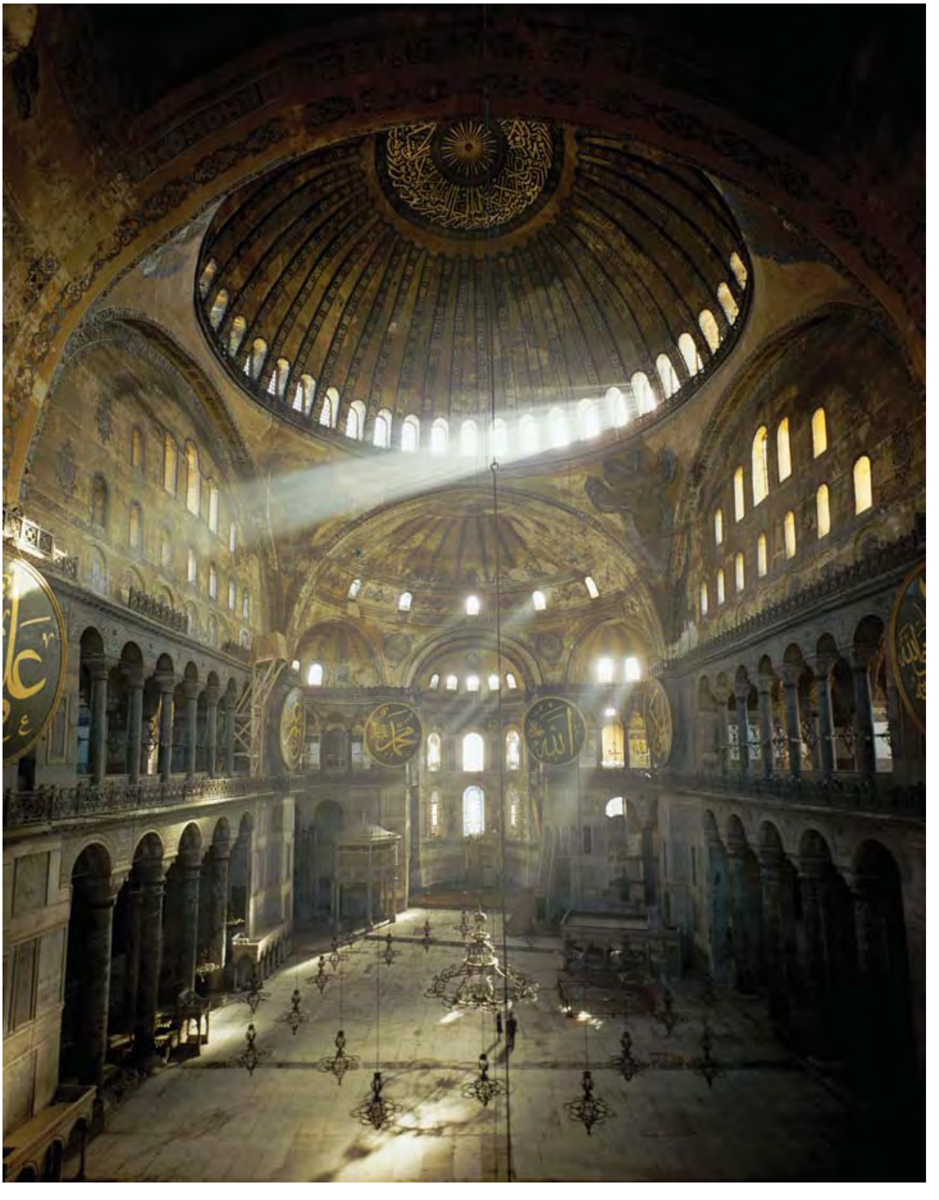

Fig. 1. Constantinople, Hagia Sophia interior view. Photo: Erich Lessing.

Fig. 2. Nikolaus von Kues (1401–1464) (left) and his brother; detail of the alterpiece in the chapel of the St.-Nikolaus-Hospital (Kues, Germany) founded by the philosopher. Courtesy of Helmut Gestrich and the Landesbildstelle Rheinland-Pfalz.


De docta ignorantia

Fig. 3. Titlepage of De docta ignorantia, codex 218 of the Cusanus-Library (Kues, Germany). Courtesy of the Bibliothek des St.-Nikolaus-Hospitals, Helmut Gestrich and the Landesbildstelle Rheinland-Pfalz.

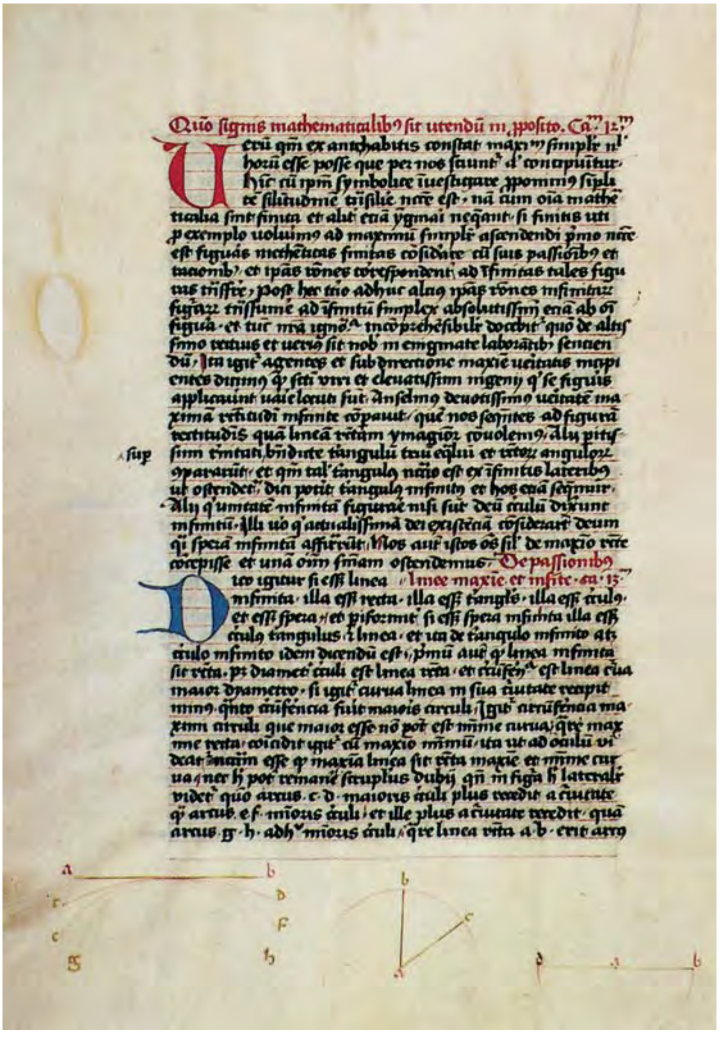

Fig. 4. Page from De docta ignorantia with geometrical figures (First book, end of chapter 13, Folio 6v), codex 218 of the Cusanus-Library (Kues, Germany). Courtesy of the Bibliothek des St.-Nikolaus-Hospitals, Helmut Gestrich and the Landesbildstelle Rheinland-Pfalz.

Fig. 5. Scene of the tapestry known as “la tenture de l’Apocalypse d’Angers” (14th century) based on drawings by John of Bruges. Illustrating Apocalypse (XI, 1): “And there was given me a reed like unto a rod: and the angel stood, saying, Rise, and measure the temple of God, and the altar, and them that worship therein.” Courtesy of the Château d’Angers, France.

Fig. 6. Trinity. Icon by Andrej Rublev (first quarter of the 15th century).

Fig. 7. Mathematical realization (ca. 1919). Acquarel by Karel Schmidt. Courtesy of Karel Schmidt Jr.

Fig. 8. Reason against Religion by E. Ščeglova. Anti Religious poster, USSR, 1970s.

Fig. 9. The Ancient of Days (1827?) by William Blake (1757–1827), relief etching finished in gold, body colour and watercolour; frontispiece (Plate I) from “Europe”. Courtesy of the Whitworth Art Gallery, The University of Manchester.

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CHAPTER 20

The Mathematical Analogy in the Proof of God's Existence by Descartes

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MATHEMATICS AND THE DIVINE: A HISTORICAL STUDY
Edited by T. Koetsier and L. Bergmans

1. Introduction: Descartes' plan

Disillusioned with the scholastic education at the College of La Flèche, where people confused science with erudition, thinking with quoting an author, and learning with knowing by heart, René Descartes undertook to base his knowledge on the sole practice of his reason. In this respect he was an heir of the Renaissance, when man became aware of the autonomy of his mind in relation to religion and politics: *My objective was never anything more than an attempt to reform my own thoughts and to build on a foundation that was entirely my own.*[1] (*Discourse on method*, A.T., VI, 15)[2]

1.1. *The reconstruction of our knowledge on the basis of reason alone*

To carry out his plan successfully, he has an instrument that everybody possesses, namely his reason (or common sense): *Common sense is the most equally distributed thing in the world*[3] (D.M., A.T., VI, 1). Reason is the faculty of producing reasoned arguments, that is putting ideas in order and judging about truth and falsehood. But why don't men who all have the same reason reach the same conclusions? Descartes answers: *The diversity of our views does not arise because some people are more reasonable than others, but simply from the fact that we guide our thoughts along different paths and do not think about the same things*[4] (D.M., A.T., VI, 2). Men don't use their reason in the same way; some of them use it correctly, others incorrectly; some of them proceed methodically, others don't: *It is much better never to try and seek the truth about anything, than to do so without method*[5] (*Rules for guiding One's Intelligence*, rule IV). But how should one find a method? Indeed, you will only know if a method is right once you have tried it and have been successful. There is no method to find a method. So, you have to begin a practice and, only afterwards, the results of the method become apparent.

1.2. *Mathematics as a model for metaphysics*

Descartes' metaphysical plan was clearly outlined in his letter to the theologians of the Sorbonne, in which he announced his Meditations. He wanted to prove the existence of God and the immortality of the soul, to convince the infidels by means of natural reason. If you want to convert them to faith and at the same time maintain that faith is a gift of God, you argue in a vicious circle. You therefore have to show that everything one can know

[1] Jamais mon dessein ne s'est étendu plus avant que de tâcher à réformer mes propres pensées, et de bâtir dans un fonds qui est tout à moi. (Discours de la méthode, in: *Oeuvres et Lettres*, Bibliothèque de la Pléiade, Paris, 1953, p. 135)

[2] References are to A.T.: *Oeuvres de Descartes*, published by Charles Adam and Paul Tannery, Léopold Cerf, Paris, 1897–1913.

[3] Le bon sens est la chose du monde la mieux partagée.

[4] La diversité de nos opinions ne vient pas de ce que les uns sont plus raisonnables que les autres, mais seulement de ce que nous conduisons nos pensées par diverses voies, et ne considérons pas les mêmes choses.

[5] Il est [. . .] bien préférable de ne jamais chercher la vérité sur aucune chose plutôt que de le faire sans méthode.

Fig. 1. René Descartes. Science Photo Library.

about God, can be discovered through the effort of one's own mind; you have to give them natural proofs: *I think that nothing more useful can be done in philosophy than to seek carefully, once and for all, the best of all those reasons and to expound them so accurately and so clearly that it is established for everybody once and for all that they are true proofs*[6] (A.T., IX-1, 6).

Descartes didn't claim that he provided new proofs, only that he put them in order, so that they will gain assent from everybody, like the proofs *from Archimedes, Apollonius, Pappus and others* ... (A.T., IX-1, 6). Descartes announced straightaway that in metaphysics he would take mathematics as a model and that he would make his metaphysical proofs at least as certain and obvious as geometrical proofs. We shall now examine this mathematical analogy in the proof of the existence of God. We'll see that it is consistently used by Descartes. Is this analogy an ordinary imitation of mathematical proofs, or is the proof of the existence of God really a mathematical proof? In the present paper we will investigate Descartes' use of mathematics as a model for metaphysics.

2. Mathematical truth as an example

First, Descartes turned to mathematics, which was flourishing at that time. According to him, mathematics is the most certain and easiest science; all you need to reach the truth,

[6] Je crois qu'on ne saurait rien faire de plus utile en la philosophie, que d'en rechercher une fois curieusement et avec soin les meilleures et plus solides, et les disposer en un ordre si clair et si exact, qu'il soit constant désormais à tout le monde, que ce sont de véritables démonstrations.

indeed, is to know how to deduce consequences. You need not confront experience to confirm its propositions. Moreover, mathematics applies to all the other sciences (physics, astronomy, mechanics) and is exemplary for all forms of research. Until 1630, mathematics was Descartes' main concern.

2.1. *Descartes' involvement with geometry*

In 1631, he solved a very difficult mathematical problem, Pappus' problem. Here is the beginning of the formulation of the problem: *with 3 or 4, or more line segments, given with their positions, a point is required, so that you may draw other lines to the given ones which determine given angles, and so that the products of the line segments, two by two, are equal*[7] (A.T., VI, 377–378). The elements of this problem are extremely general and awkward. Descartes chose two lines as axes for coordinates, AB and BC, which he postulates as given and which he calls x and y. He invented what we now call "Cartesian coordinates". It allowed him to represent each line by means of an equation; he "algebraised" geometry. To resolve Pappus' problem, he used a trick: he assumed he had already found the point he tried to find; he then deduced the consequences and inferred the causes of these consequences to discover the unknown term. This practice is called the "analysis". In his *Geometry*, in 1637, he defined it as follows: *You go over the difficulty according to the links between the lines. You put each unknown line into an equation, and, gradually, you reduce all the unknown lines to known ones*. He thought that he had rediscovered the geometrical analysis of the Greeks.[8]

Analytic geometry, the simplification of the algebraic notation, and the invention of the Cartesian coordinates are all great successes of Descartes' methodical mind.

2.2. *Towards a general method*

In the light of his experience, Descartes could make progress in his search for a new method. It was his investigation of logic, geometry and algebra that prompted Descartes to demand a new method.

Logic has two advantages: it imposes good practice and discipline on young minds, and it teaches one how to explain properly truths already known. However, in Descartes' time, logic was badly taught and had two disadvantages: it was useful to explain something already known, but it was useless to discover anything new, and it contained among many good rules, *so many others mixed up with them that are either harmful or superfluous*[9] (D.M., A.T., VI, 17), in other words rules that unsettle and deceive the mind.

[7] Ayant 3 ou 4 ou plus segments de droites, données par position, on demande un point tel qu'on puisse mener d'autres droites sur les droites données qui fassent avec elles des angles donnés et que les produits deux à deux des segments soient égaux.

[8] The Greeks wrote down their mathematical proofs in a synthetic way. They found these proofs, undoubtedly, often by means of analysis: starting from what one wants to prove, one works backwards towards postulates, definitions or propositions that have already been proved. Very little is known about the precise way in which the Greeks practised this analysis.

[9] tant d'autres mêlés, qui sont ou nuisibles, ou superflus.

In geometry, Descartes appreciated the analytical resolution of problems, but he recognised two disadvantages: *it concerns very abstract questions which seem to have no use*[10] (D.M., A.T., VI, 17)—Descartes wanted sciences to be useful—and, above all, geometry is bound to the observation of shapes. This contemplation of geometric figures he deemed dangerous for four reasons: 1 – it gives a solution of particular cases, but not one of absolute generality and as a result may lead to serious omissions; 2 – it concentrates the mind on certain lines chosen because of their classical properties (bisectors, medians, angles, and so on) and it prevents it from developing a global view of the problem; 3 – it tires the mind by being about pictures, not about ideas; 4 – it leads the mind to pseudo-proofs where the imagination sees the solution without understanding.

Finally, algebra makes it possible to economize on writing down the reasoning about abstracted quantities. Descartes had learned algebra from Clavius' textbook and he reproached Clavius for his difficult and confused algebra: *in the latter, one is so constrained by certain rules and symbols*[11] (D.M., A.T., VI, 18).

At the end of this critical investigation, Descartes could list the four characteristics of a good method: *By "method", however, I mean easy and reliable rules such that those who use them carefully will never accept what is false as true and will arrive at a true knowledge of everything within their capacity without wasting their efforts but constantly increasing their scientific knowledge*[12] (*Rules for* ***guiding One's Intelligence***, rule IV). Here we find: 1 – certainty by the elimination of risks of error, 2 – ease of thought by avoiding every useless effort, 3 – fertility by increasing knowledge, 4 – wisdom of the mind. Descartes began to write his ideas down—the *Rules for guiding One's Intelligence*—in 1628, but he had to stop at twenty one rules: there were too many and he had to simplify even more.

I thought that, instead of the multiplicity of rules that make up logic, I would have enough in the following four[13] (D.M., A.T., VI, 18). The famous four rules of the Cartesian method are enumerated in the *Discourse on Method* (1637). We mention only the first one, called the rule of obviousness. It consists of five precautions concerning obviousness. He does not mean a primary obviousness (that which is immediately obvious and seems true without any reflection), but a secondary obviousness, that is an obviousness worked out on the basis of critical thought. These precautions are: 1 – to carefully avoid haste, because otherwise you may think that you can skip stages and draw conclusions before you have obtained obviousness; 2 – to carefully avoid prejudices, that is false ideas picked up during one's childhood from parents, teachers and public opinion; 3 – to entertain only ideas that are clear—that is directly present in the mind, 4 – and distinct, that is specific, definite, and unmixed with others; 5 – to accept only knowledge that is certain and indubitable, so that nothing can call it into question. With such obviousness, you may begin a real work.

Descartes was proud of his discovery. He was certain of its universality: *All things that can fall within the scope of human knowledge* ***follow from each other*** *in the same way*[14]

[10] à des matières fort abstraites et qui ne semblent d'aucun usage.

[11] on s'est tellement assujetti, en la dernière, à certaines règles et à certains chiffres.

[12] Par méthode, j'entends des règles certaines et faciles, grâce auxquelles tous ceux qui les observent exactement ne supposeront jamais vrai ce qui est faux, et parviendront, sans se fatiguer en efforts inutiles mais en accroissant progressivement leur science, à la connaissance vraie de tout ce qu'ils peuvent atteindre.

[13] Au lieu de ce grand nombre de préceptes dont la logique est composée, je crus que j'aurais assez des quatre suivants.

[14] toutes les choses qui peuvent tomber sous la connaissance des hommes s'entresuivent en même façon.

(D.M., A.T., VI, 19). This universality can be understood in two ways: all problems about any matter are such that the method can be applied to them, and anybody will be able to solve these problems, if he uses these four rules carefully: *There cannot be anything so remote that it cannot eventually be reached nor anything so hidden that it cannot be uncovered*[15] (D.M., A.T., VI, 19).

2.3. *Certainty and simplicity*

Mathematics is a science which need not go out into nature and gather its truths. It differs from the natural sciences by two characteristics: the independence of experience and the direct knowledge of its objects based on reasoning. That is why Descartes had great confidence in mathematics: *Arithmetic and geometry are much more certain than other disciplines. The reason is that they alone are concerned with an object that is so pure and simple that they need not presuppose anything that experience might render uncertain, but consist exclusively* ***of*** *conclusions that are deduced by reason. They are therefore the easiest and clearest of all disciplines, and they have the kind of object that we require since it seems that, in their case, it is scarcely possible for someone to be mistaken except by not paying attention*[16] (*Rules for guiding One's Intelligence*, rule II, A.T., X, 365).

However, we have to admit that mathematics is not easy. Descartes refers to *the long chains of inferences, all of them simple and easy, that geometers normally use*[17] (D.M., A.T., VI, 19). Everybody is always surprised by this famous statement by Descartes. If the chains of inferences are long, how could they be easy? Why did Descartes write "simple and easy"? If they are simple, aren't they easy and vice versa? Unless simple is not so easy. To resolve a problem, I first have to *subdivide each of the problems that I was about to examine into as many parts as would be possible and necessary to resolve them better*[18] (D.M., A.T., VI, 18). In every investigation, you have to begin with the simple things. But the simplest is not necessarily the easiest.

Descartes begins his sixth rule of the *Rules for* ***guiding One's Intelligence*** as follows: *In order to distinguish the simplest things*.[19] If you need to distinguish them, it is because simplicity requires **work**. For example, Descartes calls God, soul, motion, extension, circle, "simple natures", not because of they are easy to know, but because they are not compound. As a result the mind is able to understand them directly. Here is his definition: *We apply the term "simple" only to those* [*entities*] *the knowledge of which is so clear and distinct that they cannot be divided by the mind into other things that are more clearly known. Shape,*

[15] Il n'y en peut avoir de si éloignées auxquelles enfin on ne parvienne, ni de si cachées qu'on ne découvre.

[16] L'arithmétique et la géométrie sont beaucoup plus certaines que toutes les autres sciences : c'est que leur objet, à elles seules, est si clair et si simple qu'elles n'ont besoin de rien supposer que l'expérience puisse révoquer en doute, et qu'elles ne consistent entièrement que dans des conséquences à déduire par la voie du raisonnement. Elles sont donc les plus faciles et les plus claires de toutes les sciences, et leur objet est tel que nous le désirons, puisque, à moins d'inadvertance, il semble à peine possible à un homme de s'y égarer.

[17] Ces longues chaînes de raison, toutes simples et faciles, dont les géomètres ont coutume de se servir.

[18] diviser chacune des difficultés que j'examinerais en autant de parcelles qu'il se pourrait et qu'il serait requis pour les mieux résoudre.

[19] Pour distinguer les choses les plus simples.

extension, motion, etc., are examples of this[20] (*Rules for guiding One's Intelligence*, rule XII). It is not their being easy to know that makes simple natures simple, but their simplicity, their not being compound. Simplicity is not given; it has to be sought patiently. Simplicity is difficult to get hold of.

Descartes applied this concept of simplicity in geometry, when he had to choose curves: *as the simplest curves should not just be considered those that can be described the most easily, nor the ones which make the construction or the demonstration of the problem easier, but mainly those that belong to the simplest kind that can be used to determine the required quantity*[21] (*Geometry*, L. III, §1). It would be a mistake to use a more complicated curve (of a higher degree) instead of a simpler one, if it is possible to find a solution with the simplest one. Simplicity is not necessarily easiness. Sometimes, simplicity requires a difficult approach.

3. The ontological proof of God's existence

At first sight, it seems as if Descartes repeats the famous proof by Saint Anselm in his *Proslogium*. Caterus, the author of the first objections against Descartes' Meditations, mentions Thomas of Aquinas' criticisms against Saint Anselm. However, Descartes subscribes to these criticisms because he thinks that his own proof is totally different: Saint Anselm relies on the definition of a name (the name of God denotes a thing that is such that nothing larger can be formed) from which he can only deduce truths in his mind, not in the things themselves, while Descartes **maintains** that he is investigating an essence: *What we understand clearly and distinctly belonging to the nature, or to the essence, or to the immutable and true form of something, may be truly asserted about this thing* (*First set of replies*, A.T., X, 91). That is why he denies that there is any **filiation** between him and Saint Anselm.

3.1. *The ontological proof*

We will examine the text in A.T., IX-1, 51–54, and follow its argument in detail. First, Descartes shows that essences are independent of his mind: *I find I have innumerable ideas of certain things* [...] *which are not, however, invented by me*[22] (*Meditation V*, A.T., IX-1, 51). The text begins as an internal exploration of the mind. I find in it, says Descartes, ideas which impose themselves on me and which I couldn't have invented myself. Such is the case for the ideas of infinity, perfection, and others. These ideas *have their own true and immutable natures*.[23] They not only impose themselves on the mind, but they have all the

[20] nous appelons simples celles-là seulement dont la connaissance est si nette et si distincte que l'intelligence ne peut les diviser en plusieurs autres connues plus distinctement : telles sont la figure, l'étendue, le mouvement, etc.

[21] par les plus simples on ne doit pas seulement entendre celles qui peuvent le plus aisément être décrites, ni celles qui rendent la construction ou la démonstration du problème proposé plus facile, mais principalement celles qui sont du plus simple genre qui puisse servir à déterminer la quantité qui est cherchée.

[22] je trouve en moi une infinité d'idées de certaines choses [...] qui ne sont pas feintes par moi.

[23] ont leurs natures vraies et immuables.

characteristics of eternal ideas. Descartes could have given the example of the idea of God, but he prefers to give that of the idea of a triangle: *For example, when I am imagining a triangle*.[24] The verb "to imagine" doesn't mean to create by means of one's fancy, but to present the triangle as a shape in space. Imagination is the faculty of **re**producing an object in the mind's eye, as a picture. Although an external object which exactly corresponds to the picture may not exist, *it clearly has some determined nature or essence or form, immutable and eternal, which has not been constructed by me and does not depend in any way on my mind*.[25] The entire scholastic vocabulary is apparent—nature, form, essence—from which Descartes cannot free himself. It does not matter for the moment whether a real object called a "triangle" exists outside of the mind, in the physical world. The main thing is that the essence of the triangle is not freely chosen or defined by myself. This essence imposes itself on my mind. I can examine it and deduce various properties, *for example, that its three angles are equal to two right angles, that the biggest angle is subtended by the longest side*.[26] This property (Euclid, *Elements*, I, 32) is not obvious at first sight. An examination of the triangle is necessary to discover these properties. *Even if I never thought of them previously when I imagined a triangle, I now know them clearly whether I want to or not, and therefore they were not invented by me*.[27] There is no invention, no construction. The independent object has imposed itself on the mind, and the mind has discovered the properties. Descartes rejects the hypothesis of the acquired origin for the idea of triangle, in other words, that the idea of a triangle could come from the observation of a triangular thing in nature, for he finds innumerable other shapes that have never been observed by senses and which are nevertheless clearly and distinctly understood. Then, in the following paragraph, Descartes makes the delicate link between truth and being. He makes this formidable assertion: *and therefore they are something and not simply nothing; for it is obvious that whatever is true is something*.[28] He joins two essential assertions: 1 – what I clearly and distinctly understand is true; 2 – what is true exists. How should we understand this connection?

An idea is not only an internal picture of external things, like a painting. An idea has its own reality. In an idea Descartes distinguishes its objective reality (*realitas objectiva*) and its formal reality (*realitas formalis*). The objective reality of an idea is its way of existence in the mind, that is to say some representing content which does not necessarily correspond to something outside. The formal reality of an idea is the actual reality, the real thing that the idea represents. For example, the objective reality of the sun is the sun in the mind, while its formal reality is the sun in the sky. The metaphysical question then is this: do ideas whose objective reality forces me to admit their formal reality, independent of my mind, exist? According to Descartes, only the idea of God, by its objective reality,

[24]par exemple, lorsque j'imagine un triangle.

[25]il ne laisse pas néanmoins d'y avoir une certaine nature, ou forme, ou essence déterminée de cette figure, laquelle est immuable et éternelle, que je n'ai point inventée, et qui ne dépend en aucune façon de mon esprit.

[26]à savoir que ses trois angles sont égaux à deux droits, que le plus grand angle est soutenu par le plus grand côté.

[27]lesquelles je reconnais très clairement et très évidemment être en lui, encore que je n'y aie pensé auparavant en aucune façon, lorsque je me suis imaginé la première fois un triangle.

[28]Et partant elles sont quelque chose, et non pas un pur néant ; car il est très évident que tout ce qui est vrai est quelque chose.

forces me to admit its formal reality outside of my mind. Descartes has now proved the two premises that he needs in order to turn to the ontological proof itself, that is to say that essences impose themselves on one's mind and that essences are true. He can now apply his reasoning to God.

From these premises he can now deduce *an argument to demonstrate God's existence*.[29] The verb "to demonstrate" shows that this is not a simple intuition or an immediate obviousness, but a deduction as in a mathematical demonstration. This demonstration proceeds with three analogies between God and a mathematical object: 1 – The idea of God imposes itself on me no less than the idea of the triangle: *Certainly I find within me an idea of God—that is, of a supremely perfect being—just as much as I find an idea of any shape or number*.[30] 2 – The deduction of his existence from his nature is known as clearly and distinctly as the deduction of a property of the triangle from its nature: *I understand that it belongs to God's nature that he always exists, as clearly and distinctly as I understand that whatever I demonstrate about any shape or number belongs to the nature of that shape or number*.[31] 3 – God's existence is a conclusion as certain as the conclusions about the properties of numbers and shapes: *Therefore* [...] *I should attribute to God's existence at least the same degree of certainty that I have attributed to mathematical truths until now*.[32]

When he finishes his demonstration, Descartes doesn't allow the reader to catch his breath, and he immediately adresses the possible suspicion that his argument is based on a sophism: *it seems to be some kind of logical trick*. And he explains this sophism: *Because I am used to distinguishing existence from essence in everything else, I easily believe that it is also possible to separate existence from the essence of God, and, in that way, that one could think of God as not existing*.[33] Descartes reminds the reader of the ordinary precaution that prevents one from deducing existence from essence about any idea. But is the idea of God an exception? Yes, it is: *But it is clear to whoever thinks about it more carefully that existence can no more be separated from God's essence than one can separate from the essence of a triangle the fact that the three angles are equal to two right angles, or than one could separate the idea of a valley from the idea of a mountain*.[34] The analogy is now threefold: God's existence is to his essence what the property that the sum of the three angles equals two right angles is to the triangle and what the valley is to the mountain. The common link is the necessary connection; existence necessarily belongs to God's essence, or negatively, it is a contradiction to think God without existence. So, God necessarily exists.

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[29] une preuve démonstrative de l'existence de Dieu.

[30] Il est certain que je ne trouve pas moins en moi son idée, c'est-à-dire l'idée d'un être souverainement parfait, que celle de quelque figure ou de quelque nombre que ce soit.

[31] Et je ne connais pas moins clairement et distinctement qu'une actuelle et éternelle existence appartient à sa nature, que je connais tout ce que je puis démontrer de quelque figure, ou de quelque nombre, appartient véritablement à la nature de cette figure ou de ce nombre.

[32] Et partant [...] l'existence de Dieu doit passer en mon esprit au moins pour aussi certaine, que j'ai estimé jusques ici toutes les vérités des mathématiques, qui ne regardent que les nombres et les figures.

[33] **Il semble y avoir quelque sophisme [...]** Ayant accoutumé dans toutes les autres choses de faire distinction entre l'existence et l'essence, je me persuade aisément que l'existence peut être séparée de l'essence de Dieu, et qu'ainsi on peut concevoir Dieu comme n'étant pas actuellement.

[34] Mais néanmoins, lorsque j'y pense avec plus d'attention, je trouve manifestement que l'existence ne peut non plus être séparée de l'essence de Dieu, que de l'essence d'un triangle rectiligne la grandeur de ses trois angles égaux à deux droits, ou bien de l'idée d'une montagne l'idée d'une vallée.

Descartes returns to the suspicion of sophistry in another way, although he had already answered it in the second paragraph. Indeed, somebody might object to Descartes that he considers his desires as reality; it is not because he imagines God as necessarily existing that he is really so: *My thought imposes no necessity on things*.[35] And now, about the analogy with the mountain, he concedes that if he cannot separate the mountain and the valley, it is a necessity of thought, but there is no necessity in the existence of the things. However, the necessity of God's existence is not only a necessity of thought; it is not just a connection freely established by thought: *It is not that my thought makes this happen or imposes any necessity on any thing; on the contrary, the necessity of the thing itself, namely the existence of God, makes me think this way. I am not free to think of God without existence*.[36] Descartes reverses the relation between thought and things. Concerning the mountain and the triangle, the inferences of the mind are necessary, but they don't involve the existence of things, while concerning God, it is the reality of the thing that imposes itself on the mind. I am not free to think of God as existing or not. The analogy between God and mathematical truths that Descartes had carefully maintained until now seems to break down suddenly with this radical difference about existence. Indeed, there is a difference between metaphysics, which studies existence, and mathematics, which doesn't care. This difference might destroy Descartes' systematic analogy. But before discussing it, let us consider the ontological proof in the context of the Meditations.

3.2. *The position of the ontological proof in the wider context of Descartes' meditations*

The fifth meditation, which contains the ontological proof of God's existence, is called "About the Essence of Material Things; and consequently about God, that he exists".[37] The question is no longer that of the truth of God's existence which has been discussed in the third meditation, "About God, that he exists".[38] In other words, the link is no longer that between God and truth, but that between God and essence. However, in spite of its title, the fifth meditation is not a demonstration of the existence of material things from their essences. It is an examination of the essences, in the first place mathematical essences. I know these essences through my clear and distinct ideas. For example, the idea of a particular triangle, even if such a figure exists nowhere outside of my mind, has some *determinate nature or form or essence, immutable and eternal*[39] (A.T., IX-1, 51), as is apparent from the fact that I can deduce various properties of this triangle. The truth of these essences has been guaranteed since the third meditation when Descartes proved that God exists and that he doesn't deceive.

In the third meditation Descartes proves the existence of God as follows: The only possible explanation of man possessing the clear and distinct idea of God is God's existence.

[35] ma pensée n'impose aucune nécessité aux choses.

[36] non pas que ma pensée puisse faire que cela soit de la sorte, et qu'elle impose aux choses aucune nécessité ; mais, au contraire, parce que la nécessité de la chose même, à savoir de l'existence de Dieu, détermine ma pensée à le concevoir de cette façon. Car il n'est pas en ma liberté de concevoir un Dieu sans existence.

[37] De l'essence des choses matérielles; et, derechef de Dieu, qu'il existe.

[38] De Dieu, qu'il existe.

[39] une certaine nature ou forme ou essence déterminée [. . .] immuable et éternelle.

The fact that we have such a clear and distinct idea of God implies therefore the existence of God. This argument is obviously related to the cosmological proof of God's existence or the proof of God's existence by his effects: God's existence is needed to explain the existence of something else. Moreover, according to Descartes everything that is conceived clearly and distinctly must have its origin in God. So, why did Descartes introduce a new proof of God's existence in the fifth meditation? Why didn't he treat it together with the proofs from effects in the third meditation? In order to understand this, we first have to consider the strong link between God and the triangle. The ontological proof is developed in analogy with a theorem on the triangle to show that the certainty of a theorem and the certainty of the ontological proof are of the same kind. The mathematical analogy is an analogy concerning certainty. Now, mathematical truths are clear and distinct ideas and God guarantees the objective reality of clear and distinct ideas. That is why Descartes writes "now" at the beginning of the fifth meditation: *now, having recognized what should be avoided and what should be done to reach the truth*[40] (A.T., IX-1, 50); now that the truth of the clear and distinct ideas has been proved, the ontological proof becomes possible. Moreover, the idea of God is, *at least as certain*[41] (A.T., IX-1, 54) as mathematical truths; it is the clearest and the most distinct of all essences. So, it is natural to begin with this idea when we begin to examine essences.

The order between the proofs of God's existence is justified as follows: the proofs from effects guarantee God's existence and this existence guarantees the objective reality of the essences. Since the ontological proof is based on the examination of the essences, it's natural to give it after having discussed the essences. Consequently, the proofs from effects in the third meditation are the main proofs and the ontological proof is **second**.

3.3. *The particular character of the ontological proof with respect to the proofs based on God's effects*

The ontological proof is a argument entirely different from the proofs from God's effects. The latter are investigations about the idea of infinity or about the idea of perfection which are in my mind; the idea is an effect and I have to seek its cause. From the ideas of an infinite and perfect being, I conclude that an infinite and perfect being necessarily exists, for he is the cause of the ideas of infinity and perfection. But the ontological proof doesn't consist in an investigation about a cause. It consists in an examination of the idea of God, considered as a mathematical essence, to find its necessary properties. God's necessary existence is deduced as a property included in his essence. Item 14 of *the Principles of Human Knowledge* asserts: *The existence of God is implied by the fact that necessary existence is included in our concept of God*[42] (A.T., IX-2, 31). God's existence is no longer the cause of an effect, but the necessary property of an essence. It is another reason why Descartes has separated this proof from the two proofs from the effects.

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[40] après avoir remarqué ce qu'il fallait faire ou éviter pour parvenir à la connaissance de la vérité.

[41] au moins aussi certaine.

[42] on peut démontrer qu'il y a un Dieu de cela seul que la nécessité d'être ou d'exister est comprise en la notion que nous avons de lui.

4. Arnauld's criticism: Descartes proof is circular

4.1. *Arnauld's criticism*

In the fourth objection to the Meditations, Arnauld thinks that there is a vicious circle in Descartes' ontological proof: God, with his veracity, guarantees the clear and distinct ideas, while Descartes uses clear and distinct ideas to prove God's existence: *I have one more difficulty. How does he avoid committing the fallacy of a vicious circle when he says that we are certain that what is perceived clearly and distinctly is true only because God exists? But we can be certain that God exists only because we perceive it clearly and distinctly. Therefore before we are certain that God exists, we have to be certain that whatever we perceive clearly and distinctly is true*[43] (A.T., IX-1, 166). Descartes seems to infringe his principle of the geometrical order, the same order as in Euclid's *Elements*: *The things which are set first must be known without the following ones, and the following things must be set so that they would be deduced from the first ones*[44] (A.T., IX-1, 121).

This criticism by Arnauld is linked with a criticism of the certain knowledge of mathematical truths, obtained by examining their essence. Arnauld gives the example of somebody who knows the theorem that says that a triangle inscribed in a semi-circle is a right-angled triangle (Euclid, *Elements*, III, 31), but who doesn't know the theorem attributed to Pythagoras (Euclid, *Elements*, I, 47). This man would know the essence of the right-angled triangle, but wouldn't know one of its main properties. *For he will say: I clearly and distinctly perceive that this triangle is right-angled, but I still doubt whether the square on the base is equal to the squares on the sides. Therefore it is not essential to the triangle that the square on the base is equal to the squares on the sides*[45] (A.T., IX-1, 157). This objection by Arnauld is directed at Descartes' famous argument to prove his own existence, I think, therefore I am, *cogito ergo sum*, and at the separation of body and soul. But it may be understood with reference to the knowledge of the essences. If we apply this objection against the ontological proof, it implies that we are able to know God's essence without his existence. Therefore, Descartes cannot convince an atheist.

4.2. *The analogy with the sphere*

In his fourth set of replies Descartes maintains the constant analogy between theological truths and mathematical truths. Concerning the inscribed triangle in a semi-circle, he

[43]il ne me reste plus qu'un scrupule, qui est de savoir comment il se peut défendre de ne pas commettre un cercle lorsqu'il dit que « nous ne sommes assurés que les choses que nous concevons clairement et distinctement sont vraies, qu'à cause que Dieu est ou existe ». Car nous ne pouvons être assurés que Dieu est, sinon parce que nous concevons cela très clairement et très distinctement; donc, auparavant que d'être assurés de l'existence de Dieu, nous devons être assurés que toutes les choses que nous concevons clairement et distinctement sont toutes vraies.

[44]qui consiste en cela seulement que les choses qui sont proposées les premières doivent être connues sans l'aide des suivantes et que les suivantes doivent après être disposées de telle façon qu'elles soient démontrées par les seules choses qui les précèdent.

[45]car, dira-t-il, je connais clairement et distinctement que ce triangle est rectangle ; je doute néanmoins que le carré de sa base soit égal aux carrés des côtés ; donc il n'est pas de l'essence de ce triangle que le carré de sa base soit égal aux carrés des côtés.

comes up with three additions: 1 – Pythagoras' theorem is not about a substance while the *cogito* is about a substance; 2 – if you admit Pythagoras' theorem, you must admit that this property belongs to a right-angled triangle, while you may clearly and distinctly understand soul without body, and vice versa; 3 – if you know the essence of the right-angled triangle without knowing Pythagoras' theorem, however, you cannot deny the existence of a proportion between the base and the sides, and you cannot deny the proportion expressed in Pythagoras' theorem. With these three points, Descartes insists on the difference between essence and property, and he insists on the necessary link between an essence and its properties.

Descartes introduces a new mathematical analogy inspired by Archimedes' work on the sphere. Indeed, Descartes defined God as "causa sui". Arnauld reproaches him for not distinguishing the cause and the effect. Nothing can be, at the same time, its own cause and its own effect. Descartes replies that on the basis of such an argument atheists would not admit God as a first cause for they would require another efficient cause different from God himself. Descartes compares Arnauld to an imaginary Archimedes who, after having proved the properties of the sphere on the basis of an analogy between sphere and rectilinear figures inscribed in it, would have refused to consider the sphere as a rectilinear figure with infinitely many sides (Cf. Archimedes, *About sphere and cylinder*, I, 24). In other words, if he would have maintained the specific difference between rectilinear and curvilinear figures, Archimedes wouldn't have been able to use his exhaustion method and wouldn't have succeeded. This reply means that the passage to the limit is permitted in mathematics and therefore it's permitted in metaphysics as well. Descartes is not anticipating the infinitesimal calculus before Leibniz here, but he shows that the same proof, when it doesn't concern the essence of the objects, is as good for the limited as for the unlimited. This analogy between theology and Archimedes' work on the sphere legitimises, once more, the mathematical analogy in the ontological proof of God's existence.

4.3. *The answer based on memory*

Descartes' reply concerning the vicious circle is already discreetly present in the fifth meditation: *But once I perceived that God exists and had also understood, at the same time, that everything else depends on him and that he is not a deceiver, I concluded that all those things that I clearly and distinctly perceive are necessarily true. And even if I no longer consider the reasons on account of which I made that judgement about its truth, no contrary reason can be found—as long as I remember having perceived it clearly and distinctly—that would make me doubt it*[46] (A.T., IX-1, 55–56). God guarantees the truth of my clear and distinct ideas; that doesn't mean that he guarantees the accuracy of my memory. Indeed, I may have false memories or I may not remember a truth accurately. But God guarantees an obviousness felt in the past. So, all I have to do is to remember I have well understood such a truth, while I have forgotten the reasons why, and this truth remains

[46]Mais après que j'ai reconnu qu'il y a un Dieu, parce qu'en même temps j'ai reconnu aussi que toutes choses dépendent de lui, et qu'il n'est point trompeur, et qu'en suite de cela j'ai jugé que tout ce que je conçois clairement et distinctement ne peut manquer d'être vrai : encore que je me ressouvienne de l'avoir clairement et distinctement compris, on ne me peut apporter aucune raison contraire, qui me le fasse jamais révoquer en doute.

certain through the vicissitudes of time. That's why, to break the vicious circle suspected by Amauld, you have to *distinguish between what we actually perceive clearly and what we remember having perceived clearly some time earlier*[47] (A.T., IX-1, 190). One has to distinguish obviousness in the present and obviousness in the past. So, when Descartes says that the idea of God, which is a clear and distinct idea, is a true idea, he refers to the third meditation. He doesn't use this obviousness to give a basis to the ontological proof. Descartes reverses the relation: he uses God's existence to guarantee the truth of clear and distinct ideas. His reply is short and accurate: *For we are certain, initially, that God exists because we consider the reasons that prove if. Subsequently, however, it is enough that we remember that we perceived something clearly in order to be certain that it is true. This would not be enough if we didn't know that God exists and that he does not deceive*[48] (A.T., IX-1, 190). Chronology and hierarchy are clear. Chronologically, we learn that God exists and, then, that he guarantees the truth of clear and distinct ideas all the time. Hierarchically, we learn that God exists and, then, we deduce the truth of clear and distinct ideas. The suspicion about a vicious circle comes from a careless mistake about the chronological difference: when I prove God's existence, when I understand God's essence in a clear and distinct idea, I remember that this idea is true, but I don't need to remember the reasons, namely the proof of God's existence from effects, to give a foundation for the ontological proof.

Descartes refuses to say more about the presence of a vicious circle in his argument, and he considers his reply to Arnauld as sufficient and definitive.[49] The divine veracity guarantees all knowledge based on the memory of an obviousness in the past.

5. Truth and existence

5.1. *Existence is not included in the mathematical perspective*

Unlike the idea of infinity, whose power forces my awareness of my own limited being to admit the existence of an unlimited being, the idea of God possesses a logical link with existence. Once more Descartes gives a mathematical demonstration: from the definition of an object, he rationally deduces all its properties. Descartes comes up with another mathematical analogy to reply to a "petitio principii". He might have defined God as having all kinds of perfections, in particular that of existence, and then pretended that he had discovered God's existence in the conclusion. He might also have believed *that all quadrilateral shapes can be inscribed in a circle*[50] (A.T., IX-1, 53), and then have deduced that the rhombus can be inscribed in a circle, which would have been a false conclusion. This would be

[47] faire distinction des choses que nous concevons en effet fort clairement, d'avec celles que nous ressouvenons d'avoir autrefois fort clairement conçues.

[48] premièrement, nous sommes assurés que Dieu existe, parce que nous prêtons notre attention aux raisons qui nous prouvent son existence; mais après cela, il suffit que nous ressouvenions d'avoir conçu une chose clairement, pour être assurés qu'elle est vraie ; ce qui ne suffirait pas si nous ne savions que Dieu existe et qu'il ne peut être trompeur.

[49] Quant à ce que vous ajoutez ensuite, j'y ai déjà suffisamment répondu.

[50] penser que toutes les figures de quatre côtés se peuvent inscrire dans le cercle.

a disgraceful manipulation. But, this is not the case: *Whenever I choose to think about the first and highest being and, as it were, to draw out the idea of God from the treasury of my mind, I must necessarily attribute all perfections to him*[51] (A.T., IX-1, 53). Descartes didn't place the property of existence in God's essence, like an artificial stone among diamonds, and didn't pretend to discover it at the end of his reasoning. He attentively examines the clear and distinct idea of God's essence and discovers his necessary existence, just as a diamond merchant examines the treasure he has received to extract each precious stone one by one. In the same way, if he had pretended to believe that all quadrilateral shapes can be inscribed in a circle, he would have invented a confused idea of quadrilateral shapes, while the clear and distinct idea of quadrilateral shapes rules out the deduction that it is possible to inscribe them all in a circle.

The mathematical analogy in the proof of God's existence is at the same time strong and feeble. It is strong because it shows very well that the way to truth is the same for divine truths and for mathematical truths. When he replies to Gassendi, Descartes points out that the way to truth is one: *you are wrong when you say that we don't prove God's existence such as we prove that every triangle has its angles equal to two right angles, for the reason is the same in the two proofs, except that the proof of God's existence is much simpler and more obvious than the other*[52] (*Fifth set of replies*, in: Descartes, *Oeuvres et lettres*, Paris, éd. Gallimard, 1953, p. 504). On the other hand, the analogy is feeble because it ignores an essential difference between metaphysics which deals with the question of existence and mathematics which rules out this question.

Such a difference has already been used; it is the difference between mathematical truths and the truth of the *cogito* argument. The latter truth is an essential certainty acquired when the subject coincides with himself, purely and simply, regardless of the content of his thoughts, while a mathematical truth is a certainty in relation to a content, a certainty acquired by a careful examination of an essence in the mind. The *cogito* argument gives a basis to the existence of the thinking subject. Mathematics, however, develops the content of an idea. That is why the *cogito* argument is more solid; it is immune to the Evil Genius argument. Even when an Evil Genius is distorting our thought, we are thinking. Mathematical truths are in principle vulnerable to the deceptions of such an imaginary Evil Genius. That is why Descartes needs divine veracity to save mathematics, while the *cogito* argument gives a foundation of its own truth without divine veracity.

5.2. *The limits of the analogy between mathematics and metaphysics*

Mathematics gives a foundation to the objective reality of its essences, while metaphysics proves that existence corresponds to my idea of God. Therefore, we have to consider again the breakdown in the analogy between mathematics and ontology. On the one hand, there

[51] Toutes les fois qu'il m'arrive de penser à un être premier et souverain, et de tirer, pour ainsi dire, son idée du trésor de mon esprit, il est nécessaire que je lui attribue toutes sortes de perfections.

[52] vous vous trompez grandement lorsque vous dites qu'on ne démontre pas l'existence de Dieu comme on démontre que tout triangle rectiligne a ses trois angles égaux à deux droits; car la raison est pareille en tous les deux, hormis que la démonstration qui prouve l'existence en Dieu est beaucoup plus simple et plus évidente que l'autre.

are many similarities, since God's existence is proved in the style of geometers, with certainty and obviousness. On the other hand, there is a manifest difference since mathematics doesn't prove the existence of its objects. Descartes also points out that the analogy between God and a triangle concerns the necessity of the link between an essence and a property (the property that the sum of three angles is equal to two right angles for the essence of the triangle, and the existence for God's essence), but the link is not between essence and existence about these two objects: *the existence of the triangle must not be compared with God's existence because, in God, existence has obviously a different link with the essence than in the triangle*[53] (*Fifth set of replies*, in: Descartes, *Oeuvres et lettres*, Paris, éd. Gallimard, 1953, p. 503). The similarities are consistently maintained in the analogy between God and a triangle. The difference is suddenly introduced: [...] ***rather than** I cannot think of God without existence*[54] (A.T. IX-1, 53).

At this moment, the reader may think that Descartes' entire enterprise is in ruins. What is the use of this mathematical analogy if it leads to this particular exception of the idea of God? Somebody may again suspect a sophism: Descartes tries to prove in the style of the geometers an idea which is not at all mathematical. To rediscover the Cartesian rationality, we have to remember that the idea of God and the idea of triangle are innate ideas. I can recall these ideas in my mind, as I want, but I cannot decide about their content; they have been given, ready-made, eternal and immutable: *the mathematical truths which you call eternal have been drawn up by God and depend entirely on him, like all other creatures* (*Letter to Mersenne, April the 15th 1630*, A.T. I, 145). I cannot modify the properties of my innate ideas. I cannot modify things in such a way that the three angles of the triangle are not equal to two right angles, and I cannot modify my ideas in such a way that God doesn't exist. In other words, the main similarity between these two ideas is the necessity of the link between the essence and its properties. To think of a triangle whose three angles don't equal two right angles is not to think of a triangle. To think of God without existence is not to think of God. I am not free to modify the content of the innate ideas. My mind imposes no necessity on things, while the innate ideas impose their own necessity on my mind. Necessity: this is the real link which is the basis, beyond the apparent difference, of the analogy between God and a triangle.

In addition, this analogy makes it possible to re-establish an essential religious truth. Indeed, if necessity were only a demand of my mind, it would impose restrictions on God's essence and deny his freedom. Now, since God has given me my innate ideas, of which he has freely chosen the content, I am not free to define God as I desire. It is he that gives me the knowledge that I have of him. This knowledge is a free and incomprehensible revelation, because God is beyond human understanding, and therefore we love him all the more.

Twenty-five years later, Molière makes Dom Juan say: *I believe that two and two are four, Sganarelle, and that four and four are eight*[55] (*Dom Juan*, Act III, scene 1). The atheist admits only mathematical truths. *I don't deny that an atheist can clearly know that the three angles of a triangle equal two right angles, but I only maintain that he doesn't*

[53] l'existence du triangle ne doit pas être comparée avec l'existence de Dieu parce qu'elle a manifestement en Dieu une autre relation à l'essence qu'elle n'a pas dans le triangle.

[54] au lieu que, de cela seul que je ne puis concevoir Dieu sans existence.

[55] Je crois que deux et deux font quatre, Sganarelle, et que quatre et quatre font huit.

know it with a true and certain knowledge[56] (A.T., IX, 111). According to Descartes, an atheist cannot be a geometer, for mathematical truths, however certain they may seem to natural reason, need to be guaranteed by a metaphysical truth, that of a God who has given us innate ideas and who doesn't deceive us.

6. Conclusion: Descartes' God and Pascal's God

Descartes is a rationalist and his theory about God is largely based on his philosophy. His God is the God of philosophers and scientists. His theology is, in spite of his precautions, a theology which serves his theory of knowledge. When we read his meditations, it is obvious that God is only an argument in his philosophy. God is the creator of the eternal truths, in particular of mathematical truths. Moreover, God guarantees truth, in particular the truth of the *cogito* argument. God is not a deceiver; God is distinguished from the Evil Genius. In this context it is remarkable that Descartes first proves his own existence as a thinking being, before he proves God's existence.

Pascal doesn't put up with Descartes' rationalism: *I cannot forgive Descartes; he would have liked to develop his entire philosophy without God, but he couldn't avoid letting him put the world in motion; afterwards he didn't need God anymore*[57] (*Thought* 77B). Pascal's God is the God of Abraham, Isaac and Jacob. He is the God of faith. His theology is revealed theology, separated from philosophy. According to Pascal, the proofs of God's existence don't lead to faith; reason has to admit that there are innumerable things that it doesn't understand. God is known by the heart, not by reason. The example of Jesus shows this: Jesus doesn't prove things, he talks in parables. He speaks to the heart.

Descartes' God is the creator of human nature; he has created me as a reasonable being, with innate ideas; he has left his marks in my mind, namely the idea of perfection. Descartes uses this mark as a proof of his existence. Descartes is sure that he is able to rationally prove God's existence. Descartes' God comes with human reason; he helps me to understand and to correct my mistakes, to control my will when I want to go beyond my understanding. God is the reason in the world; he is *the incomparable beauty of this immense light*[58] (A.T., IX-1, 41).

According to Pascal, man cannot ascend to God by the power of his reason, but God comes down to man and offers his love, his grace. Man has to respond to this proposal of God; in his famous debate against the Jesuits, *the Provincial letters*, Pascal defends Jansen's work, *Augustinus* (1640), which expounds the theory of double predestination. When somebody is born, God gives his grace or withholds it. Those who do not receive grace shall be damned. Those who have it may or may not cultivate it. Those who don't cultivate it lose this grace and shall be damned. Those who cultivate it shall be saved. Pascal's Christianity is pessimistic. There is not much space for human freedom. Man is predestined

[56] Or, qu'un athée puisse connaître clairement que les trois angles d'un triangle sont égaux à deux droits, je ne le nie pas; mais je maintiens seulement qu'il ne le connaît pas par une vraie et certaine science.

[57] Je ne puis pardonner à Descartes : il aurait bien voulu, dans toute sa philosophie, pouvoir se passer de Dieu ; mais il n'a pu s'empêcher de lui faire donner une chiquenaude, pour mettre le monde en mouvement ; après cela, il n'a plus que faire de Dieu.

[58] l'incomparable beauté de cette immense lumière.

by God's choice. The largest part of mankind is predestined to damnation because of original sin. Without God, alone, with only his reason, man is ruined. That is the theme of the misery of man without God. God is a mystery which infinitely exceeds us. While Descartes talks about light, according to Pascal *Jesus is a truly hidden God*[59] (*Thought* 751 B). God is obscure. Reason cannot save man. Reason cannot prevent him from sinning. The wager argument which we read in *Thought* 233B is based on the probability calculus to prove that it is better to bet that God exists than that he doesn't. This argument is not a proof of God's existence. It is only a means to disconcert a libertine and to lead him to prayer.

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CHAPTER 21

Pascal's Views on Mathematics and the Divine

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Contents

MATHEMATICS AND THE DIVINE: A HISTORICAL STUDY
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1. Introduction

Pascal's mathematical achievement is fourfold. Lacking Descartes's algebraic expertise, he chose to work in the traditional field of synthetic rather than analytic geometry, and thus contributed to the study of conic sections. Six years later, in *The Treatise on the Arithmetical Triangle* and in its first appendix *Multiple Numbers*, he published his findings on the theory of number (prime numbers and magic squares), propounding the method of combinatorial analysis known as 'Pascal's Triangle', and applying properties of the binomial theorem. In collaboration with Fermat, and at the prompting of his gambling acquaintance the Chevalier de Méré, he laid the foundations of modern probability theory. And towards the end of his life, in his *History of the Cycloid*, he resolved various problems in the geometry of indivisibles, thereby helping to create the infinitesimal calculus: this, though falling short of the generalized formulation which made Newton's integral calculus possible, nevertheless established the geometric laws applicable to a curve.

Such intermittent but highly concentrated scientific activity confirms him as one of the seventeenth century's greater mathematicians, yet in a famous letter he disparages such intellectual labours and pours scorn upon his arithmetical and geometric achievement. 'I would not go two steps out of my way for geometry's sake', he writes, adding that so different is the work on which he is currently engaged that he has almost forgotten about mathematics; 'for, to be frank with you about geometry, it is, in my opinion, the highest of mental exercises; but I also see that it is so useless that I draw hardly any distinction

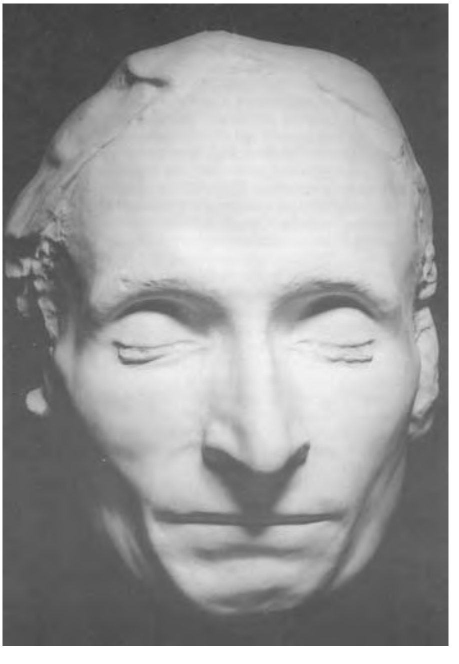

Fig. 1. Pascal's death mask. Courtesy of the Principal and Fellows of Newnham College, Cambridge.

between a mere geometrician and a skilled workman'; 'whilst geometry was a good means of testing one's mental powers, it was not a good means of employing them' (**522**).[1]

Nor is this disparagement of mathematics an isolated instance in Pascal's writings. The 'abstract sciences are not mankind's proper concern' (687). Descartes, he maintains (553), probed too deeply into science—by which he principally means mathematics—and should be censured accordingly. Moreover, 'I think it is right that one should not look closely into Copernicus's opinion; however, it affects our whole life that we should know whether the soul is mortal or immortal' (164).

Pascal reflects this situation in his celebrated Thought on the 'geometric' and the 'intuitive outlook' (for want of a better word to translate the admirably termed 'esprit de finesse'). In this antithesis the 'geometric outlook' is that of deductive, a priori reasoning, the reasoning of mathematics, whilst the 'intuitive outlook' not only embodies inductive, a posteriori reasoning—the thinking processes of natural philosophy—but also encompasses all the nuances and perceptions of human living, those very subtleties which the court-functionary Leibniz, the lawyer Fermat, the soldier turned Royal tutor Descartes (who in his *Treatise on Man* was also a natural scientist) and even Pascal's father, the tax commissioner Étienne Pascal, dealt with in the course of their professional careers. This Thought 512, in which almost the whole of Pascal's intellectual achievement is encapsulated, acknowledges that each of the two 'outlooks' and methods has its own particular validity—indeed its own necessity—within its sphere of influence. Pascal's so-called 'geometric' method foreshadowed the quasi-mathematical logic of the lemmas of Spinoza's *Ethics* as a method of reasoning geometrically from seemingly indubitable first principles.

To present the argument of this article: the characteristics of a God who is firmly embedded in human history suggest to Pascal a salvation-plan for the world that has been divinely created. God, it is suggested *to* and *by* Pascal, can be apprehended by both heart and mind. His existence, though not indubitable, can rightly be regarded as matter for informed mathematical speculation. A demonstration of the mathematical aspects of Pascal's religious dialectic concludes the article.

2. Characteristics of God

Pascal holds that, rare and admirable as the 'geometric outlook' may be, it does not have any bearing upon what is truly important in human life. What is essential from this standpoint is the 'intuitive outlook'; yet whereas the excogitations of the 'geometric outlook' do at least produce results that are universally acknowledged, those of the 'intuitive outlook' yield no similar certainty. Does God exist? Like Voltaire (somewhat disingenuously) a century later, Pascal maintains that belief in God should be the supremely important issue in the lives of his fellow men; yet not even the God of deism can be conclusively proved by mathematics; the God of revealed religion still less so! Mathematics is, he believes, 'useless in its profundity' (694); nor can the divine existence be proved by any of the traditional arguments of the schoolmen (Anselm's ontological argument or the Thomist arguments of

[1] The plain numbers within brackets denote some of Pascal's lesser known *Thoughts* as laid out in the second of L. Lafuma's editions (Seuil, 1962). The bold numbers within brackets refer to the pagination of Pascal's collected writings edited by J.L.A. Chevalier (Pléiade, 1954).

the Prime Mover, First Cause and Necessary Being), nor indeed by any moral argument nor by those from design or natural law.

In short, whether mankind has been created randomly or by a good God or a wicked demon cannot be decided by any individual's natural powers alone (131), though the natural order does indeed suggest that 'a necessary, eternal and infinite being' may well exist. Within nature there are, he says, powerful signs that God does exist and these in themselves are more cogent, if not more coherent, than the so-called metaphysical proofs, for the latter are interdependent and (in a manner of speaking) syllogistic so that not all of them can easily be comprehended at any one time, with the result that only an hour after a person has acknowledged their force they are all too easily confused, misunderstood and put to one side (190).

On the contrary, the God of Christianity is much more than the God of deism, natural religion being as distant from Christianity as is atheism itself—and both being equally abhorrent (449, 781). 'The Christian God', Pascal asserts, 'is not simply a God who is the author of geometric truths and of the natural order'; and therefore 'I shall not undertake here to prove by natural reasons either the existence of God or the Trinity, or the immortality of the soul, or anything of that kind, not only because I do not feel myself sufficiently able to find in nature proofs to convince hardened atheists, but also because this knowledge is useless and barren without Jesus Christ . . .' (449).

Pascal identifies three of the four principal attitudes towards divine belief as those of 'the Pyrrhonist, the deist and the Christian' (170): these are agnosticism, deism and Christianity, and 'doubt, assurance and submissiveness' are their distinctive features respectively. From this analysis he excludes atheism, refusing to make it central to his thesis. His reference in Thought 449 to 'hardened atheists' defines and circumscribes the main aim of his Apologia; this comprehensive defence of Christianity, as wide-ranging as those of St. Thomas Aquinas, Chateaubriand, Kierkegaard or Teilhard de Chardin, was a project upon which he had embarked at the urgent request of the Jansenist-inclined Duc de Roannez: intended to convert the agnostic rather than the atheist, it was a work which he contemplated for many years and for which his *Thoughts* were preparatory jottings. Doubt, which is the focus of the projected Apologia, characterizes the Pyrrhonist because, like Arcesilaus, Montaigne and Charron, he resists and refutes dogmatic attitudes. On the other hand, whereas the deist's attitude is one of complacent self-assurance, the Christian believer approaches his God reverently and submissively because he trusts in a personal Deity who has uniquely revealed himself in history and the Scriptures.

Never at any time does Pascal give us a palpable apprehension of the Godhead beyond affirming that he is the soul's 'path, object and final goal' (**551**), 'an infinite and immutable object', 'the sole principle . . . and sole end of all things': thus, in the purely abstract sense of the philosopher rather than the visionary, we know what is *meant* by him. Pascal resorts to arithmetic in order to explain the necessary haziness of man's perception of the Godhead: we do not know what God is like, he argues, just as in mathematics we are incapable of forming a mental picture of the infinite (418).

In mathematics Pascal sees the outlook of infinity; and the conviction which this gives him of the existence of some sort of infinite being adds vibrancy to his apologetic whilst also filling him with the same spacious and serene tranquillity expressed by Spinoza in his *Ethics* and by Plato in the *Timæus*. But whereas Spinoza is said by Balzac (in *Cousin*

Bette) to have ‘proved the existence of God by mathematics’, such was never Pascal’s intention. His God is no mathematical abstraction (449), nor is he the teleological Creator as seen by Aquinas and Bonaventura although the doctrine of divine grace undoubtedly has a teleological aspect. Nor is he the Prime Mover of the universe ‘who sets the world in motion with the flick of a finger’ (1001), which is how Pascal describes the God of Descartes’s *Discourse on Method*.

On the contrary, Pascal’s Godhead is voluntaristic, going much beyond the Deity in whom Newton believed, to the extent that through the Person of Jesus, through the Holy Spirit, through prophecy, miracles and Scriptural revelation, he actively intervenes in the unfolding of world-history, sustaining his Church throughout its tribulations, and putting each human soul to the test though by no means leading mankind into error. And therefore he is a personal God: not the Divinity ‘of the philosophers and scientists’ but the ‘God of Abraham, God of Isaac and God of Jacob’ whom he so vividly perceived during the two hours or so of a mystical vision and of whom, in his metaphor of tongues of fire, he has left a record in his *Memorial*. Pascal is indeed *the* ‘God-inebriated man’, to borrow the phrase invented for Spinoza by Novalis.

Pascal’s Godhead, in his interventions in the unfolding of world-history, is the *hidden God* of whom Isaiah prophesied; the God who, through the Holy Spirit, has inspired all prophetic utterance and who has shed some light but not too much, wishing to blind some people and enlighten others. He is the ‘God of Jesus Christ’ (913), without whom we cannot know the Father, for ‘Jesus Christ is . . . the true God of mankind’. And being through Jesus the epitome of self-sacrifice, he is also the God of love, ‘humbled’, ‘crucified’, and even actually ‘lost’ or ‘destroyed’: not quite abandoned upon the Cross, as Vigny was later to envisage his death, but ‘lost’ and ‘destroyed’ at the very heart and in the very fullness of the Godhead itself (471). Echoing in this respect all three Synoptic Gospels, Pascal emphasizes in *The Mystery of Jesus* that, though the Son may, in his humanity, have wished not to have to taste the bitterness of death, yet he was not abandoned by the Father: ‘he prays but once that the cup may pass from him, which he does submissively, and twice he prays that it may come to him if such be the will of God’. And it is this disquieting perception of both Father and Son being ‘lost’ or ‘destroyed’ which, through original sin, exactly mirrors the Fall of Man.

This God who has loved and redeemed us we must also love, and Pascal well realizes that there is a world of difference between knowing about God and actually loving him. ‘We must’, he says (618), ‘love only him, and not our transient fellow-creatures’—acknowledging him in our hearts as ‘saviour, father, priest, host, nourishment, king, sage, lawgiver, sufferer, pauper, and one who must produce a people whom he will lead, feed, and bring into the Promised Land’ (607).

Hence the fundamental Pascalian concept of a God of contradictions, or *contrariétés*, as he tends to call them: the Word who at his Nativity was unable to speak a word, the child who is God, the outsider who is King, the Messiah who is the spiritual rather than the secular saviour not merely of his own people but of all peoples, the Triune God in time yet out of time, who is of obscure birth and yet not so obscure that there are no signs of him. These are indeed the sorts of contradiction—reidentified and reiterated by Pascal—which, across most if not all denominations, have been the familiar substance of Christian worship and theology from the Fathers of the Church to the present day; and it is a tenet of

Pascal's faith that in the Second Person of the Trinity, 'Jesus Christ, all contradictions are reconciled'.

Within Pascal's view of God there are, however, two sorts of contradiction. There are the traditional contradictions previously mentioned, and these, perhaps better than anyone, he turns to convincing apologetic use: the fact, to which Isaiah bears witness, that in God's revelation of Jesus to mankind there is enough clarity for some to discern in him the God-Man but also enough darkness and obscurity for others to be mistaken. Likewise with the parables, of which Jesus himself says that he has told them to so many 'that seeing they might not see, and hearing they might not understand'. The shedding of light—and even the bestowal of faith—is 'a gift of God': it is not something of human origin.

Over and above the traditional contradictions, however, there are also those which, looking in at Pascal's conception of God from a standpoint which is not his, we equally perceive as being present. Predominantly, these contradictions are to be found in his attitude towards the doctrines of the Fall of Man, original sin, and predestination, matters which are confronted in his four *Writings On Grace* written in 1657–1658 although indirectly, in 1656, they had also been the subject of his earliest *Provincial Letters*.

Pascal, though not a theologian in the formal sense, assimilated a great deal about theology through the writing of these epistolary masterpieces, which, with their vivid and ironical dialogue, came brilliantly to the Jansenists' rescue in their struggle against the Jesuits. This dispute, which continued for well over a century, was between modernizers and traditionalists. Focusing on aspects of moral theology, upon the doctrines of sufficient and efficacious grace and upon the impact upon practical daily behaviour of any relaxation of moral teaching, it culminated in the Jansenists' defeat when, in 1713, their teaching was condemned in a Papal bull. In Pascal's particular phase of this prolonged acrimonious debate the intellectual ammunition concerning the Jesuits' doctrinal and ethical laxity was prepared for him by his friends at Port-Royal; but he, better than anyone, knew how to hit his target. And it is arguments about sufficient and efficacious grace that preoccupy him in his first three *Provincial Letters*.

In the Bible—and especially in Matthew's and Mark's Gospels and in the Pauline and supposedly Pauline Epistles—much is said about predestination, the Elect and the doctrine of divine grace. This central aspect of Christian teaching is nowadays understated to the extent that the Pelagianism or semi-Pelagianism once championed by the Jesuits has become the common currency of the Church's belief; but from any reading of the Epistle to the Romans it is clear why Augustine fervently denounced Pelagius and why, at the Synod of Carthage, Pelagianism was condemned as heretical as long ago as the fifth century.

Whereas Pelagius had denied original sin, teaching that the human will is equally free to do evil or good and thereby save one's soul, Augustine argued that God has an absolute will to save those who will be saved but, conversely, a conditional will to damn those who will be damned. Original sin, vehemently denied by Pelagius and of which (in Pascal's thought) the 'lost' or 'destroyed' God is so bold a metaphor, stems from the temptation by the serpent, the Fall of Man and the expulsion from the Garden of Eden—Adam, and through him all men, having thereby failed to fulfil God's plan for the salvation of the world.

Geometric and algebraic skills being close in nature to the logical syllogism and sorites, it is these which Pascal repeatedly uses in respect of the existence of the Christian God, the divine plan for the salvation of mankind, the Church's teachings on sufficient and efficacious grace, and, again in the *Provincial Letters*, the casuistical methods of probabilism and probabiliorism (e.g., **715–719**); to some extent Pascal also defends the doctrine of predestination in like manner. This apologetic activity, beginning with the *Provincial Letters*, was at its most coherent and singleminded in the *Writings on Grace* and continued until the end of his life, when he was preoccupied by his *Thoughts*.

3. God's Salvation-Plan

In the *Writings on Grace* these quasi-mathematical skills are applied to the consequences of Augustine's view of divine grace as Pascal tries to resolve some of the riddles and contradictions which that involves.

(1) *The Fall of Man*. In the beginning of things, he argues, the divine plan was that all people should be saved; all therefore received sufficient grace to enable them, with freewill, to work out their salvation (**965–967**). After the Fall of Man not only Adam but all his descendants became worthy of punishment. It is God's conditional will that some, but not all, shall be so punished (**952–954**).

(2) *Proximate Power*. Those whom God elects for salvation receive his efficacious grace, in addition to which a proximate power is needed to enable them to persist in prayer (**976–977**). A syllogism endeavours to establish that sometimes this proximate power is withheld even from the Elect:
   (a) if all the Elect have a proximate power to pray 'in the next instant', they must also have a proximate power to persist in prayer;
   (b) if the Council of Trent has laid down that not all the Elect are capable, at all times, of such persistence; *then*
   (c) it is contrary to the teaching of the Council of Trent to say that a proximate power to pray 'in the next instant' is always given to the Elect.

(3) *Efficacious Grace*. However, the salvation of the Elect is assured because they have that perseverance in faith which is God-given through Jesus Christ. But the non-Elect cannot be saved because they do not have the efficacious grace which is required for salvation.

(4) *Prayer*. Regular and devout prayer is sufficient to procure grace which will be efficacious in procuring salvation. Yet God can still abandon the prayerful man, perhaps because he can foresee the sins that he will commit of his own free will (**951**, **953**, **965**).

(5) *Moral Laxity*. Pascal denies that the belief that one belongs to the Elect may encourage in that person a lax attitude towards morality.

(6) *Scope for Human Effort*. He urges *all* men and women to do everything that could contribute to their salvation.

(7) *Ambivalence*. Pascal is reluctant to accept that 'God would have made the world in order to damn it' (864, cf. 725), or that he would damn an innocent child (131) or those people born before the Redemption. And if a man cannot achieve his salvation through faith and commitment, what is the point of an apologetic exercise?

The four *Writings on Grace* were the closest Pascal ever came to writing theology. Although, broadly speaking, they projected the Jansenists' theological point of view, it does not follow that Pascal himself entirely shared the beliefs he had analysed and defended in the *Provincial Letters*—great as was his admiration for Arnauld, Nicole etc. as human beings. In the aftermath of the Jesuit–Jansenist controversy, as also in many of his scientific discoveries, this for ever non-systematic man formulated his meditations in response to an adventitious stimulus. It was very far indeed from his intention to construct a theological system of his own. He was, on the whole, exceedingly content to abide by Scriptural revelation and the teachings of synods and ecumenical councils.

4. Heart and reason

In this all-important matter of the foundations of a belief in God Pascal's position should be distinguished from the positions of, say, Rousseau or Kant. To Jean-Jacques, unwavering in his religious belief, the two clearest evidences of God were 'the spectacle of nature' (Milton's 'harmony divine', Addison's 'spacious firmament on high') and that 'inner voice' of conscience and the moral law which also underlay Kant's conviction of the divine existence. But whereas Pascal agreed with Kant and Rousseau that religion could not strictly be founded on the a priori principles of mathematical reasoning, he held that the 'first principles' which are the basis of all deductive thinking are mediated to mankind by the heart, claiming that it was 'useless for reasoning, which has no part in the matter, to try to combat them . . . For the knowledge of first principles, of space, time, movement and numbers, is as solid as any that is given to us by reasoning, and it is upon such knowledge, conveyed by heart and instinct, that reason has to rely' (110). These first principles, the postulates of the laws of physics, are of an arithmetical or geometric kind and therefore—as was thought at that time about the common notions and postulates of Euclid—it might have been expected that for Pascal the truthfulness of the existence of these first principles was beyond question. Yet he plays down the part that reason might have to play in such processes, arguing that 'except for faith and revelation, we have no certainty of the truth of these principles other than the fact that within ourselves we have a natural feeling that they are true. However, this natural feeling is not a convincing proof of their truth, . . . there being no certainty outside faith' (131). The heart, says Pascal (418), knows that the number of finite numbers is infinite though reason cannot prove that this is so: he chooses to regard this infinitude of finite numbers not as a postulate but as evidence of the primordial importance of faith and revelation. It can, on the other hand, be demonstrated that 'there can be no square numbers one of which is double the other': that, for him, is the province of reason, not of the heart.

For Pascal, therefore, the foundation of divine belief is first and foremost a matter of the heart, but it is not a case of the heart inspiring the *first principles* from which reason can conclude *propositions* from which *belief* can stem. Rather, the God-given first principles that are confirmed to us by faith and revelation show the inadequacy and contingency of deductive reasoning—and, a fortiori, of inductive reasoning too. Faith, the ultimate guarantor of these *first principles*, stems from the heart, and 'those to whom God has given religious belief through feelings of the heart are blessed indeed and very properly persuaded, but those who have not that belief can only be given it by reasoning until such time as God will give it them through feelings of the heart, without which faith is merely human and quite useless for salvation' (110). This goes some way towards explaining the riddle that faith is bestowed by God only upon the Elect, for a 'merely human' faith is said to be 'quite useless for salvation' and reason is said to have little or no part to play in the conversion to religious belief. Other religions, argues Pascal, have 'suggested reasoning as the method of attaining faith, but it does not lead to faith, for all that'. 'Only two kinds of people can be called reasonable: either those who serve God with all their hearts because they know him, or those who seek him with all their hearts because they do not know him'. 'The heart has its reasons of which reason knows nothing': what is meant by this strange paradox? The reasoning mind proceeds, says Pascal, 'by principle and demonstration' whereas 'the heart's method is different': in the last analysis its apprehension of God is God-given. Faith is mediated by the grace of God—and Pascal considers that the two firmest foundations of religious belief are divine grace and miracles, 'both of which are supernatural' (861).

It is, then, a still stranger (though directly related) paradox that the avowed aim of the *Thoughts* is to bring about religious conversions. Pascal's God is not only the Deity who, for their original sin, has consigned to 'eternal damnation' (131) those who lived before the time of Jesus; he is also, and for the same reason, the God whose bestowal of mercy, although sometimes withheld, is always undeserved (149). He is the dispenser of grace which is not given to all but is necessary to all (110, 131, **984–991**), yet in whom there is *merit* in believing (**509**). For 'denying, believing and doubting come as naturally to man as does running to a horse' (505); 'religion . . . is uncertain'; hence, says Pascal, a man would not believe in God if there were no signs of him.

However, as is indicated by the analogy of the running horse, believing (*religious faith*) also comes as naturally to man as do denying (*atheism*) and doubting (*agnosticism*); and whether it be a case of joining battle, going to sea or walking along a plank, it is of the very nature of human endeavour to be for ever engaged in risk-taking (*travailler pour l'incertain*). Without being a mathematician, Augustine had also seen this element of risk and this fact of human nature (577), but how much more reasonable and meritorious it is to appreciate such things in the seventeenth rather than the fourth and fifth centuries, in the light of the probability calculus which he and Fermat happen to have pioneered! For the probability calculus demonstrates that people must take risks; and thus we are led to Pascal's famous—almost notorious—argument of the Wager, a leap of faith which is the most controversial aspect of the *Thoughts* and of his religious thinking generally.

5. The Wager argument (418)

Rather like the physical universe which he sees as being suspended between nothingness and infinity, Pascal portrays the individual human life as a sea-voyage on which man has embarked from an uncertain origin towards an uncertain goal. Yet the fact that he is uncertain of the goal does not mean that he should refrain from suppositions concerning it. Pascal, notwithstanding any reservations he may have about the Elect being foreordained to salvation, believes that all men should be mindful of the Last Things (death, judgment, Heaven, Hell), weighing in the balance—in the form of a probability calculus—whether it is better to enjoy present worldly delights to the possible detriment of one's eternal salvation, or whether it is better to forgo *concupiscence* with a view to gaining an infinitude of happiness which may or may not await one in a possible Hereafter. Man, he argues, should stake his present life for the sake of winning something infinitely better in the world to come. This is the broad sense of the Pascalian Wager, which, for all the apparent rigour of its mathematical formulation, has been rejected by Voltaire, Diderot and Laplace.

Objections to the Wager Argument:

- (i) *Existence of the Gain*. Contrary to the normal mathematical conventions of a wager, the interlocutor is urged to gamble on the *possible existence* of the prize.
- (ii) *Nature of the Gain*. How does the mathematics of Pascal's argument correlate to the specific concept of the Christian Heaven?
- (iii) *Compulsion*. The Pascalian Wager is not constructed on the normal model, where the risks are undertaken of the gambler's own free will. For reasons which are unclear Pascal says to his interlocutor that he *must* gamble because he has embarked. But is wagering for God actually believing in him or merely making the effort to do so?
- (iv) *Distaste for Immortality*. Unbelievers may serenely face the prospect of total extinction; and likewise the unhappy recluse. Both for the libertine and the unhappy recluse, but particularly for the former, the Wager argument should therefore have reposed on the concept of Hell.
- (v) *Crypto-materialism of the Notion of Gain*. Just as the Jesuits attacked the *Provincial Letters* for 'ridiculing holy things' (**779**, cf. **750**), so Pascal is also open to the criticism that holy things are quantified and reified in the Wager argument.
- (vi) *Differential Rewards*. Consequently, God might not reward infinitely those whose efforts to believe in him are prompted by mercenary considerations. And, rather similarly, he might differentiate belief that is based on faith from belief that is based on evidential reasons. Perhaps, therefore, different decision matrices are required for different persons.
- (vii) *Disproportion*. The lack of any recognizable relationship between the stake and the gain is an almost insuperable obstacle to those wagerers whose top priority is the maximization of gain. Conversely, the value that the stake still has for the unbeliever in terms of secular pleasure is no less a sticking-point for those who seek to minimize potential loss.

Both mathematically and otherwise the Wager argument perhaps therefore lacks the decisive force for which its creator hoped. But (α) it may be wondered whether this argument and the concept of *divertissement* were the specific expressions of a neurotic outlook; and, consequently, (β) the quasi-mathematical argument of the Wager might not have become supremely pivotal in opening the doorway from a pessimistic analysis of human nature into the effulgence of the Christian revelation. On the contrary, it would very probably have been one of the numerous convergent arguments used by Pascal in furtherance of his apologetic goal: in Newman's phrase, one of the 'powerful and concurrent' reasons—most of them non-mathematical—the sum of which may work towards religious conversion.

6. Mathematical aspects of presentation

What part is played by mathematics in Pascal's apologetic achievement and in his perception of the Godhead?

(a) *Proof*. The object of Euclidean geometry is proof. In the final part of the never-to-be-finished Apologia it seems that Pascal would likewise have sought to adduce proofs—and by a disproportionate process akin to that already noted in his Wager argument. There would perhaps have been five proofs from the prophecies (274), various proofs from Moses, i.e., from the Pentateuch (290–297), and there would certainly have been many proofs of and from Jesus Christ: all of these proofs are adumbrated in the *Thoughts*.

(b) *Probability Calculus*. Pascal's work with Fermat on probability had released the natural sciences from the confinements of absolute certainty, establishing instead the concept of a stochastic universe. These joint mathematical findings deeply influenced both Pascal's religious thought and its presentation: not all people will be saved, not all will be damned; impossible though it is to read the mind of God, we know through his revealed word that human salvation will depend upon the imparting of his grace, and human faith by itself will not be enough. The salvation-plan is therefore to a large extent deterministic, but not entirely so as human commitment is still required in order that that plan shall take effect. At a religious level the mind that had grappled with mathematical probability was universally aware of probability, including the casuistical methods of probabilism and probabiliorism. Furthermore, in the Wager argument his work with Fermat is at the core of his presentation of eschatological choice, although it can scarcely be said that the algebraic equations are proportionate. Pascal recognized that it is both natural and right to take calculated risks.

(c) *The Calculus of Indivisibles*. Cavalieri's work on *maxima and minima*, later perfected by Roberval, became the foundation of the infinitesimal and integral calculus. It would be incredible to suppose that Pascal was unfamiliar with their work on the summation of infinite series, especially as Cavalieri's *Six Geometrical Exercises* rapidly became a cornerstone of seventeenth-century mathematics; and he must have known of the earlier findings of Archimedes's method of exhaustion and Kepler's theory of infinitesimally small geometric quantities. Archimedes had calculated the area of a segment of a parabola by constructing an infinite sequence of triangles starting with one of area A and continuing to add further triangles between the existing ones and the parabola; by the same method of *maxima and minima*, or upper and lower limits, Cavalieri had calculated the quadrature of areas enclosed by certain curves. So too with Pascal's apologetic technique in the *Thoughts*, where a somewhat similar technique of movement between extremes, for ever approaching the resolution of the problem, was at the heart of his didactic purpose. 'There are two errors', he says (252), 'to take everything literally and to take everything spiritually'; 'if a man is boastful, I will humble him . . . And I shall go on contradicting him . . .' (130); 'I will not allow him to rest in either [view] so that, lacking both a stable base and peace of mind, . . .' (464). However, for Pascal there is an Absolute Truth compared with which all the contradictions of the phenomenal world (inconsistencies, for example, in the human view of justice) are stark evidence of human fallibility and its causation in original sin.

(d) *Infinity*. The mathematical concept of infinity, so familiar to Pascal from his knowledge of the summation of infinite series, naturally comes to his mind whenever he considers the infinite nature of God, and sometimes he thinks of it when he considers the cosmic situation of man: human beings are mysteriously suspended between two Infinites (199); 'unity added to infinity in no way increases it . . .' etc. (418). There is an infinite number of finite numbers: this is asserted in no less than three Thoughts (663, 418, 110), and to Pascal it is perhaps the clearest indication of the existence of an infinite being.

(e) *Syllogism*. The *Writings on Grace* can also be seen not as the work of a theologian but as speculation in the manner of a mathematician who, by means of syllogistic proof, seeks an elegant resolution of apparent contradictions; and the same criticism has been levelled at the *Provincial Letters*, that in some of them the mysterious relationships of human beings with God have been treated as if they were a geometric problem. The quasi-mathematical techniques of syllogism and sorites are primarily resorted to in the earlier letters' discussions of the rival doctrines of divine grace (e.g., Letters I–III) although they are also used with deadly effect in Pascal's mockery of the ethical implications of the Jesuits' casuistical devices (Letters VI, VII, XII). Even before embarking in earnest on the *Thoughts* Pascal had perfected his mastery of syllogism in the *Provincial Letters*. Self-evidently, his mastery of syllogism and near-syllogism must have contributed to a similar mastery of the apophthegmic discourse, as in the self-referential paradox that 'it is not certain that everything is uncertain' (521). Thought 192 is the best example of the near-syllogism: 'knowledge of God without knowing one's wretchedness leads to pride. Knowledge of one's wretchedness without knowing God leads to despair. Knowledge of J.C. is the happy medium because in him we find both God and our wretchedness'. This Thought has the dialectical form of the strict syllogism but is more in the nature of an elaborate antithesis.

(f) *Antitheses* are linked to syllogisms to the extent that they are the second term of the dialectical process. In Pascal's hands they become a sharply etched literary device which is also a *geometric* one. 'I should be much more fearful of being mistaken and finding that the Christian religion was true than of not being mistaken in believing that it was true' (387): such antitheses reflect the elegant mathematical harmony of Pascal's mind, a harmony that was so much at variance with the condition of the actual human world as he conceived it—damaged as that had been by original sin. Just as algebraic proofs could not, as we have seen, demonstrate the truth of the Word made Flesh, so likewise, in the defence of the Triune God, there is a danger that words themselves can be merely formulaic, their syllogistic neatness assuming the character of a priori demonstrations without particular reference to lived existence. This is particularly so when the antithesis almost amounts to syllogism, as in Thought 462: 'the prophets have predicted but were *not* predicted. In later times the saints were predicted but were not predictors. Jesus Christ was both predicted *and* a predictor'. But Pascal warns himself, just as much as his reader, against the danger of false antitheses (559). The stark contrasts of antitheses such as 'there is nothing on this earth . . .', in Thought 468, are not literary bravura. They are strictly logical in function in that they remind the reader of the categorical nature of the choice facing him, for Pascal writes in the spirit of Jesus's words (recorded in both Matthew and Luke) that 'he that is not with me is against me', and this saying of Our Lord is quoted in Thought 775. The best example of the syllogistic antithesis is the one in which Pascal suggests that there is greater certainty about religion than that we shall live to see tomorrow (577). Perhaps this Thought is a mere jotting and not a potential cornerstone, perhaps it would merely have helped to flesh out one of the 'powerful and concurrent' reasons; for the main philosophical objection to it is that such an argument could be applied to the defence of any religious, political or other belief.

(g) *Symbolism*. Pascal's training in geometry rather than algebra—but *awareness of* algebra nevertheless—would have stood him in good stead if he had ever come to the point of composing his Christian apologia; it has served him well in the *Thoughts*. For just as algebra is numerical symbolism, and as the geometry within which he worked can itself be made to perform exactly the same uses, so the *Thoughts* are permeated with the notion of *figures*, or prophetic metaphor: through the symbolism of the Messiah, the Suffering Servant, the Temple, the Babylonian Captivity and the Stone which the builders rejected Pascal would have sought to demonstrate that Old Testament prophecy had been fulfilled in Jesus's life and death. For example, the Gospel words 'I am the living bread', echoing Exodus XVI, are quoted in Thoughts 268 and 503, as Pascal thinks of Jesus as the manna, or divine bread, come down from Heaven to feed mankind. Algebra cannot prove the divinity of Jesus or that he is the living bread, but it is not surprising that Pascal, as a numerical symbolist, saw the whole world as a divine metaphor.

Indeed, in a certain sense mankind is a *figure* of the Godhead, made in God's image; and Pascal, like Kepler, believed that, because man was made to resemble God, so he was capable of understanding the universe which God had created. Kepler, following Pythagoras and Plato, also believed, however, that God had made the universe according to a mathematical plan and that consequently mathematics would provide a secure key to the understanding of the whole of that universe. Pascal had no such easy perception of the relationship of God and man. Original sin means that man's apprehension of the world and (in Kepler's phrase) of the *harmony of the world* is necessarily imperfect; the role of reason is limited. Infinity, the mathematician's concept to which Pascal always returns, terrifies him or his alter ego in the phenomenal world (198, 201). It is the perfect metaphor for eschatological uncertainty: 'the never-ending silence of those infinite spaces fills me with dread'. 'The last act is bloody' (165); man on earth is like a prisoner in a dungeon or death cell (163); human beings are like chained men watching their companions, one by one, being put to death (434). As for the eschatological decision, man is alone. Yet the relationship with God is not just a one-to-one relationship, for 'true religion teaches us our duties, our incapabilities, our pride and lusts; it also teaches us what the remedies are: humility and self-mortification'. The Christian religion is not only 'the object and centre towards which all things tend', it is also the vehicle of a moral message; and Pascal emphasizes what virtuous men and women there are in the Christianity community, he extols the good and happy lives they lead. This amounts to the important argument of Christian witness.

7. Conclusion

What shape would Pascal's apologia of the Christian religion ultimately have taken? Except for a few disconnected jottings in readiness for a *Provincial Letter* that was never written, practically nothing in the *Thoughts* seems irrelevant to the main purpose. What would he have added or omitted? Could the eventual work have had the beautiful simplicity of a theorem? 'The arrangement of the material is new', he explains (696). 'When playing tennis,

Fig. 2. French postage stamp commemorating the tercentenary of Pascal's death.

both players use the same ball, but one is better at handling it': all would have depended on the final order. 'The last thing you discover when you are writing a book is what you have to put first' (976). And yet, more than on logical orderliness, everything would have depended on Pascal's incomparable literary skill, which, contrary to the situation usual at that time, was modelled not on Latin authors but on Epictetus and Montaigne, the sceptical writers whom he had discussed in his *Conversation with M. de Saci*: 'I will write down my thoughts in a disorderly manner though not perhaps in purposeless confusion. That is the true order of presentation: in its very disorder it will always be indicative of the order I have in mind' (532). Thus the orderliness of the syllogism, the sorites, the dialectical antithesis would have been traversed by an alternative if not higher order of literary and linguistic presentation.

Pascal was a formal mathematician but not a formal philosopher, nor was he a formal theologian. In Archilochus's terms he was one of those who knew 'one really good thing'—and that thing did not belong to the realm of mathematics. Whether in religion, physics or mathematics he was not concerned to construct a unitary system of thought, tending instead to work brilliantly although sporadically in response to external stimuli (the vacuum experiments, the help he gave Méré, the Jansenists' controversy with the Jesuits, his involvement in the cycloid challenge). Given the a priori character of the 'geometric outlook', there was no way in which he could *not* have been a formal mathematician; but in philosophy and theology, where lay the 'one really good thing' which he recognized for what it was worth, his approach was quite the reverse. Pascal had undergone at least one unforgettable mystical experience, when for those two hours during his 'night of fire' (913) on 23–24 November 1654 he had a vision of the God of the Old Testament who is also the God of the New Covenant, lapped, as was the Holy Spirit (Acts II 2–4), in tongues of flame; and this experience had caused him to realize that insights which appear rational are essentially intuitive and that the phenomenal world can only ever be imperfectly understood. Thus miracles apparently contrary to the mathematical uniformity of the laws of physics would have had their all-important place within his apologetic discourse; and the presentational 'disorderliness within orderliness' would have served as a metaphor for miracles' disruption of the orderly sequences of the natural world. It is a voluntaristic approach in keeping with Jesus's conduct and assessment of his own ministry: 'except ye see signs and wonders, ye will not believe'.

'Contradiction', says Pascal (177), 'is not a sign of falsity, nor is the lack of contradiction a sign of truth', and this is as applicable to the *wonders* that are miracles as it is to the *signs* of the hidden God. And just as there are the twin aspects of contradiction, so also there are the twin aspects of truth, of which the deductive *and* inductive truths of mathematics/physics are but one aspect—and by no means the more important. Though he is deeply committed to Christian doctrine, Pascal dislikes the formulaic repetition of dogma, and in his *Writings on Grace* he explores the full implications of that doctrine, working them out in the manner of a mathematical problem. A facet of his seemingly random responsiveness to external stimuli is his devotion to truth at all costs: thus, he does not resile from the empirical finding that the fall of the column of mercury within the pipette or test tube is evidence of a vacuum, whatever the schoolmen may have said about nature abhorring the void and God not existing if the existence of a vacuum could be established. Pascal proves otherwise but still believes in God, and in Letters XII and XVII of the *Provincial Letters* he extols truth as something God-given and eternal. Likewise, the purpose of the *Thoughts* is to disseminate truth: 'wherever disputes occur, people enjoy seeing the clash of opinions—but as to the discovery and contemplation of truth? they have no interest in that' (773). Yet, he warns, 'truth itself becomes an idol, for truth without charity is not God: rather, it is his image and an idol which is neither to be loved nor worshipped, and still less must one love or worship its opposite, which is falsehood' (926).

This is a modern conception of the Christian religion, except for Pascal's seeming commitment to the Jansenists' cause, whose eventual defeat by the semi-Pelagianism of the Jesuits he did not foresee (662). The fact that his religious thought was unsystematic—in other words, that it was not, and did not purport to be, a philosophy or a theology—does not in any way subtract from its intrinsic value as an urgent eschatological entreaty, nor as the categorical outline of a way of life.

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CHAPTER 22

Spinoza and the Geometrical Way of Proof

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Contents

MATHEMATICS AND THE DIVINE: A HISTORICAL STUDY
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1. Spinoza's time

Spinoza (1632–1677), the greatest and most widely admired Dutch philosopher, was born in Amsterdam into a Spanish–Jewish family. Spinoza's major work, *Ethics*, represents an interesting link between metaphysics and mathematics because the book is written *more geometrico* (in the way of geometry). Spinoza applied to metaphysics the same method that Euclid had applied to mathematics when he wrote *The Elements*. However, before we can turn to the way in which mathematics and the divine are related in Spinoza's work, we must first devote some attention to the time and the society in which Spinoza lived.

In the Eighty Years' War (1568–1648) the Republic of the United Netherlands had fought itself free from the Spanish–Habsburg empire and the bourgeoisie had taken the place of the nobility. At the time of Spinoza the revolutionary spirit had disappeared but the social, economic and cultural gains remained. In the middle of the 17th century the provinces Zeeland and Holland were the centre of a powerful republic of merchants with vast colonial possessions, ruled by an oligarchy of regents and merchants led by a Grand Pensionary. The wars that the republic fought with England were about the hegemony on the seas and in the world.

The people lived under the spiritual discipline of Calvinist ministers to whom the broad-mindedness, the looser way of life and the greater wealth of the elite was a thorn in their flesh. The regents stimulated culture, art, philosophy and science, the character of which frequently did not harmonise with the views of the Christian orthodoxy. The freethinkers among them were on their guard against expressing their views.

At the time Amsterdam could rightly be called a multi-ethnic and multi-cultural society. Next to the Protestant state church the regents tolerated the Lutherans, the Baptists, the Catholics and other faithful as long as they avoided being conspicuous. Spinoza wrote in *Tractatus Theologico-Politicus*: "Taken the city of Amsterdam, whose enjoyment of this

Fig. 1. Benedict de Spinoza (1632–1677). Portrait from the 1677 edition of the *Opera Posthuma*.

freedom has made it great and greatly admired by the whole world. In this flourishing state, this city without a peer, men of every race and sect live in the greatest harmony [. . .] and no sect whatsoever is so detested that its members (provided that they harm no one, give every man his own, and live decent lives) are refused the protection of the civil authorities”.[1]

2. The Jews in the Netherlands

The Dutch Jewish community had grown very much in the 16th and 17th centuries. This was caused by the fact that in 1492 the Roman Catholic monarchy had driven the Moors out of their last bulwark, Granada, and consequently from the entire Iberian peninsula. This meant the end of centuries' long peaceful and fertile co-operation between Jews and Arabs, also in philosophy. Persecution and the burning of heretics took on considerable proportions. In 1580 Portugal came under Spanish rule and in this country also, which already had a moderate inquisition, the Jews were persecuted in a bloody way by the fanatically Roman Catholic monarchy. Even the new Christians (called *Marranos*, which means “swines”), who had converted to the Roman Catholic faith—at least for the outside world—did not escape persecution. Some of them emigrated and the wealthy businessmen among them, taking their networks of European customers with them, were seeking refuge in, amongst other cities, Hamburg and Amsterdam. The first Roman Catholic Jews arrived in Amsterdam in 1598. The Calvinist governors of the city requested them not to profess their religion in public. In 1602, however, Uri Ha Levi, who preached the undiluted Jewish faith came to Amsterdam from Emden in Germany. The regents permitted him to stay because in their eyes Roman Catholicism was actually worse than the Jewish faith. Synagogues were built and both orthodox and liberal persuasions of the Jewish faith developed. However, freethinking was not tolerated. For example, Uriel da Costa (1585–1640), the son of a Jewish mother and a Portuguese nobleman, was excommunicated twice with all the accompanying humiliations, and he finally committed suicide in 1640. Spinoza was seven years old when everybody around him talked about this event.

For the Calvinist part of the population as well, openly expressed doubt or denial of the truths of faith could lead to heavy punishments. This happened to Adriaan Koerbagh (1633–1669), who belonged to Spinoza's circle. After a severe sentence to imprisonment Koerbagh died in jail as a result of the barbaric regime there.

3. Descent and youth

Baruch de Spinoza came from a family of Spanish–Jewish merchants but he himself was born in Amsterdam. His father was a prominent merchant and furthermore played an im-

[1] “Urbs Amstelodamum exemplo sit, quae tanto cum suo incremento, et omnium nationum admiratione, hujus libertatis fructus experitur. In hac enim florentissima republica et urbe praestantissima omnes cujuscunque nationis et sectae homines summa cum concordia vivunt [. . .] et nulla omnino tam odiosa secta est cujus sectarii (modo neminem laedant, et suum unicuique tribuant, honesteque vivant) publica magistratuum authoritate et praesidio non protegantur”. Quoted from: Benedict de Spinoza, *The Political Works*, edited and translated by A.G. Wernham, Clarendon Press, Oxford, 1958, pp. 240–241.

portant role in the government of the synagogue. Baruch's increasingly free religious convictions were not in keeping with those of his family and the orthodoxy of the rabbis and, when attempts to convince him of the incorrectness of his views or to let him keep silent about them, failed, he was put under a ban. After he was banned from the Jewish community no one was allowed to associate with him any more. Because of this he could no longer practice his profession as a Jewish merchant.

After his expulsion Spinoza became a member of a group of Collegiants of which Mennonites were also a part. They were non-ecclesiastical Christians who met in living rooms. The Calvinist ministers followed the Collegiants with Argus eyes because they were not very dogmatic. Several of those who participated in the meetings became Spinoza's friends and pupils. However, he was always on the alert for people who sought contact with him with the hidden intention of interrogating him and eliciting blasphemous statements from him. Later, he had a rose and the word *caute* (careful) engraved in his signet ring. Yet, the idea that he led a quiet and withdrawn life is not in accordance with the facts. He not only received his Collegiant friends but philosophers and scientists as well. In 1660 the municipal regents considered it advisable to let him disappear from Amsterdam for a couple of months and accommodated him on a large country estate. Spinoza did not return to Amsterdam but established himself in Rijnsburg where the Collegiants met several times a year. Whenever he had the impression that he had got himself talked about, and for other reasons as well, he thought it safer to move. After Rijnsburg he subsequently lived in Voorburg and The Hague.

Numerous edicts were issued against his *Tractatus Theologico-Politicus* (1670) by ecclesiastical and lower governmental bodies. However, he had powerful protectors among the regents. One of them was the Grand Pensionary, Johan de Witt, who had published an authoritative book about mathematics: *Elementa curvarum linearum* (1659). The edicts were prevented or remained as long as possible a dead letter. After Johan de Witt and his brother had been tortured and lynched by fanatical followers of the house of Orange, others continued to protect Spinoza, for example, Johan Hudde, a burgomaster of Amsterdam and an important mathematician as well. None of the regents, however, would ever want to have been seen with Spinoza. After 1770 it seemed to Spinoza unadvisable to publish other texts and he came back on his plans to do so. Yet he apparently loved publishing and he had a need to propagate his own views, of which he had no low opinion. Spinoza was a fearless and self-conscious man; yet, like many freethinkers he attempted above all not to give offence. Reliable biographical data about Spinoza are rare.[2]

4. Influences

What were the spiritual influences that Spinoza underwent in his younger years? In particular around 1900 much work was done and published on this subject. *Der Junge Spinoza; Leben und Werdegang im Lichte der Weltphilosophie* (1910) by Stanislaus von Dunin Borkowski, S.J., is still a classic on this point. Since then substantial factual research has

[2]The well-wrought and balanced biography by Theun de Vries ([27], in Dutch) gives short shrift to several popular myths concerning Spinoza.

been done, but nevertheless much remains speculative. The list of books in Spinoza's library, however, offers a lead.[3] It shows in any case the books that he actually possessed and probably read as well. Spinoza underwent the influence of the Jewish–Arabic philosophy (Averroës, Maimonides) and via them he came into contact with Greek thought. This happened not only in this indirect manner but also directly. He had the works of Greek and Roman philosophers, historians and poets in his bookcase: Aristotle, Seneca, Cicero, Livius, Tacitus, Sallustius, Homerus, Ovidius and Vergilius. Of the great minds of the Renaissance he was familiar with Giordano Bruno, Tommaso Campanello and others. However, in the course of time, the rationalism of Descartes (1569–1650) became more and more important for him. Spinoza frequented the Flemish Roman–Catholic philosopher Franciscus van den Enden (1602–1674) at whose school he not only learnt Latin but became familiar with the philosophy of Descartes as well. At the same time he became here acquainted with the exact natural sciences. Moreover, Van den Enden would not have kept the radical-democratic views that appeared posthumously in his *Free Political Theses* (Vrije Politieke Stellingen) for himself.

For a good understanding of Spinoza it is necessary to make a few remarks about Descartes' views. Descartes did not want to base himself on mediaeval scholastical philosophy, while, as for the big philosophical questions, he did not want to base himself on sensory perceptions either. Where Spinoza straight out based himself upon God, Descartes' starting point was his own ego and radical doubt—*cogito ergo sum*—although in the end, according to Descartes, the existence of a perfect God could not be doubted because an imperfect human being would not be able to make up the notion of such a God. Everything that was "clear and distinct" had to be true. Descartes held next to God two completely separate substances, thought (mind) and extension (matter), that could not influence one another. In this way Descartes left philosophers after him, like Spinoza, who was 36 years younger, with the task of relating mind and matter in one way or another.

Descartes and Spinoza had completely different ideas about religion. Descartes spent 20 years in the Dutch Republic where he continued to profess the Roman Catholic faith with all the dogmas included. He denoted them as the *tri mirabilia* (three miracles): the creation from nothing, the free will and the incarnation of the Son of God, although in his philosophy there was no place for the first and the last. Descartes' rationalism caused commotion among the Calvinist ministers in the Netherlands because it implied the sovereignty of reason and the separation of philosophy and religion. His view that philosophy should not be the handmaiden of theology—*geem philosophia ancilla theologiae*—raised mistrust among, in particular, the theologians in Utrecht. They foresaw consequences of his philosophy that Descartes himself did not draw. That is why the theologians, among them the professor of divinity Gisbertus Voetius (1589–1667), feared his writings. They feared them less, however, than they would later fear the philosophy of Spinoza, so that several of Spinoza's followers pretended to be Cartesians.

Spinoza defended freethinking and rejected all religious coercion categorically. Moreover, according to Spinoza, philosophy had to be emancipated from theology and politics, from church and state. As a philosopher Spinoza made no concessions. On the basis of rational arguments he rejected every revealed religion and for him God, nature and substance

[3] J. Freudenthal, *Die Lebensgeschichte Spinoza's in Quellenschriften, Urkunden und nichtamtlichen Nachrichten*, 1899.

(in the sense of the fundamental principle of everything) were identical notions: *Deus sive natura*. He considered people who believed in miracles to be atheists because they denied the unshakeable causality of nature inherent to God.

Yet Spinoza saw the established church as a moral force that kept the masses under control, with the Bible in its hand. The masses ought to continue to go obediently to church. The state church, however, ought to avoid theological quarrels and deal only with "external acts of piety and worship; not of inward piety and the inward worship of God [. . .] inward worship and piety are part of the inalienable right of the individual".[4] The task of the church was only to preach justice and charity. According to Spinoza all revealed religions had justice and charity in common and this is what the morality of his philosophy, according to his own words, amounted to.

5. Interpretation

At the heart of Spinoza's philosophy is the idea that there are not, as Descartes held, three substances, God, mind and body, but there is only one unique infinite substance: God. The attributes of a substance are what the intellect perceives as constituting the essence of the substance. God has infinitely many attributes. Thought and Extension are two of them: God is a thinking thing and God is an extended thing. The implication of this view is that mind and body are one and the same thing conceived differently: conceived under the attribute of Thought it is mind, conceived under the attribute of Extension it is body.

Henricus Regius (1598–1679), physician and professor in Utrecht and a friend of Descartes, argued that thought and extension were not independent of each other and not equivalent either. As a physician he considered the mind as dependent on the body.[5] Spinoza criticised these ideas. For Spinoza there is strict parallellism between mind and body. For Spinoza this parallellism implies that there is no free will. In a letter to a young German who studied at the University of Leiden, Ehrenfried Walther von Tschirnhaus, Spinoza wrote about a stone set in motion by an external cause:

> Conceive now, if you will, that while the stone continues to move, it thinks, and knows that as far as it can, it continues to move. Of course since the stone is conscious only of its striving, and not at all indifferent, it will believe itself free, and to persevere in motion for no other cause that because it wills so. And this is that famous human freedom which everyone brags of having, and which consists only in this: that men are conscious of their appetite and ignorant of the causes by which they are determined. So the infant believes that he freely wants the milk; the angry boy that he wants vengeance; and the timid, fight.[6]

In spite of the geometrical argumentation that is the subject of this present essay, the philosophy of Spinoza can be interpreted in more than one way, so that for several centuries

[4]"[. . .] de pietatis exercitio et externo religionis cultu; non autem de ipsa pietate et Dei interno cultu [. . .] internus enim Dei cultus et ipsa pietas uniuscujusque juris est quod in alium transferri non potest", *Tractatus Theologico-Politicus,* Chapter XIX. Quoted from: Benedict de Spinoza, *The Political Works*, edited and translated by A.G. Wernham, Clarendon Press, Oxford, 1958, pp. 204–205.

[5]M.J.A. de Vrijer, *Henricus Regius, een 'cartesiaansch' hoogleraar aan de Utrechtsche hoogeschool,* 1917.

[6]Quoted from: Benedict de Spinoza, *A Spinoza Reader, The Ethics and other works*, edited and translated by Edwin Curley, Princeton University Press, 1994, pp. 267–268.

the discussion has been about the correct interpretation. In many of these interpretations mathematics plays no role whatsoever. First Spinozism got a place within the context of Descartes' rationalism. This rationalism mistrusted sensual perception (*empiricism*) in philosophy and based itself on reason (*ratio*). After Spinoza's death the theologians fought him for more than a century as an immoral atheist and his writings were officially forbidden for a long time. Although this was still the case in the second half of the eighteenth century, underground Spinozism had exerted considerable influence, in particular in Germany.[7] At the time the emphasis was on the pantheistic implications and the view that body and mind were expressions of the same thing. Remarkably, Christian faith does not exclude this. Although Spinoza had no affinity whatsoever with living nature, the experience of the unity of all forms of life was part of the Spinozism of the poet-thinker Goethe. It is expressed in the following ode on nature:

> Nature! It surrounds us and holds us—we cannot step outside her, and we cannot go deeper into her. Uninvited and without warning she draws us into the circulation of her dance [...] One obeys her laws, even when one resists them [...] Her crown is love. Only through love one can get closer to her.[8]

Goethe's poem Prometheus met with considerable resistance because in it man takes the place of God and points out God's shortcomings.

In the course of the nineteenth century Spinoza became popular among the freethinking workers and citizens outside the universities. In the same vein we see that around 1900 leading social-democrats viewed Spinozism as a philosophical foundation for Marxism. In the spirit of Plechanov a young Georgian bolshevist wrote in his book *Anarchism or socialism*: "The one and undividable nature, expressed in two forms, the material and the ideal; the unique and undividable social life, expressed in two forms: the material and the ideal: this is how we must view the development of nature and social life".[9] This young revolutionary would later become known as Stalin.

Around 1900 Spinozism was considered by many as a form of mysticism, although without much argumentation (Carl Gebhardt, Stanislaus von Dunin Borkowski, Ernst Cassirer, Heinz Pflaum, Anna Tumarkin, Johannes Diderik Bierens de Haan, Johan Herman Carp). It was an unreligious mysticism in which individuals break through their own boundaries and transcend themselves. Lalande gives the following definition of mysticism: "Strictly speaking the belief in the possibility of an intimate and direct union of the human mind with the fundamental principle of being".[10] Some students of Spinoza perceive a mystical touch only in the early works of Spinoza, but for some others this holds for the complete

[7] Cf. The publication of Jacobis *Spinoza Büchlein* in 1912. Moreover, not long before his death G.E. Lessing had expressed his agreement with Spinozism and, when this became known, it stimulated extensive discussions.

[8] "Natur! Wir sind von Ihr umgeben und umschlungen—unvermögend aus ihr herauszutreten, und unvermögend tiefer in sie hinein zu kommen. Ungebeten und ungewarnt nimmt sie uns in den Kreislauf ihres Tanzes auf [...] Man gehorcht ihren Gesetzen, auch wenn man ihnen widerstrebt [...] Ihre Krone ist die Liebe. Nur durch sie kommt man ihr nahe", J.W. Goethe, *Sämtliche Werke*, 1949 (Artemis-Edition), 16, p. 921. Although Goethe did not write this text, he agreed with it.

[9] Ger Harmsen, The attention for Spinoza in the socialist and communist workers' movement before 1970 (De aandacht voor Spinoza in de socialistische en communistische arbeidersbeweging van voor 1970). In Dutch. In *Bulletin Nederlandse Arbeidersbeweging*, 1995, nr. 39, p. 56 and further.

[10] "Proprement croyance à la possibilité d'une union intime et directe de l'esprit humaine au principe fondamentale de l'être", *Vocabulaire de la philosophie* (1902, 1968).

works and they attribute this to the influence of mystically inclined Jewish, Arabic Italian and Spanish thinkers. In particular the influence of Leone Ebreo (Juda Abravanes) (1460–1535) is supposed to have been important.[11] Spinoza possessed his book *Dialogos de amor*. Love is viewed as a uniting and transcending force. According to Pflaum, in Spinoza a passive knowledge of God (an experience of God that happens to him) is replaced by an active love (*amor dei intellectualis*). Man is no longer God's slave, but man finds in himself the power to find the way to God. The mind begins to play a more active role, for example by means of the mathematical way of reasoning. God mathematically proven, transcended from the *hic et nunc* (here and now). Yet in none of these interpretations does the geometrical deduction as such play a decisive role.

Although at the universities Spinoza's views were, for a long time, contested, in particular by the theologians, and he remained the philosopher of laymen outside the universities, admirers have appeared at the universities during recent decades, say since 1970. From Paris the Spinozistic structuralism of the Marxists–Leninists spread. This led to a considerable number of dissertations and other texts, mainly written by ex-communists who contributed their interpretations to the influence of Louis Althusser, Pierre Macherey, Martial Guéroult, Etienne Balibar and Antonio Negri. Furthermore, they meticulously explored the intellectual environment of Spinoza in search of friends, acquaintances and sympathisers. A unanimous reading is, however, further away than ever. After three centuries of diverging and even contradictory interpretation, because Spinoza offers everybody something and there is a case for every reading, I am inclined to consider all these explanations with scepticism. In any case interest in Spinoza is great and text editions and monographs of high quality are appearing regularly.

6. The mathematical way of reasoning

During the Renaissance the influence of Plato made itself felt again and because of the simultaneous need for independent observation of organic and inorganic nature, Aristotle temporarily retreated into the background. Descartes and Spinoza were not the first in European philosophy who, after antiquity, made use of mathematics in their philosophy. I will mention two great minds who used mathematics in their philosophy. Nicholas of Cusa (1401–1464), on the border of the Middle Ages and the Renaissance, wrote *De docta ignorantia*, still partially in the tradition of the scholastics. Cusanus wrote in Chapter XI of *De docta ignorantia*: "*mathematica nos iuvet plurimum in diversorum divinorum apprehensione*". Dijksterhuis described Cusanus' views as follows: "Infinity may be unknowable directly for our reason, but there are means" to get to know it "and it is mathematics that gives us these means. Mathematics is concerned with finite figures, but the consideration of their properties can open a way to the infinite".[12] Cusanus deeply influenced Spinoza. Giordano Bruno (1548–1600) is fully representative of the Renaissance. He died at the stake. In his work Bruno repeatedly refers to Cusanus. Bruno uses dry mathematical argumentation, but, as a reaction to this, also uses the more artistic form of the dialogue, as did Spinoza in his earliest work.

[11] H. Pflaum, *Die Idee der Liebe bei Leone Ebreo*, 1926, p. 105

[12] E.J. Dijksterhuis, *De mechanisering van het wereldbeeld*, 1950, p. 249.

Spinoza lived in a cultural climate in which mathematics and the exact sciences (astronomy and mechanics) were a tremendous success and brought about a scientific revolution. According to Galilei the book of nature is written in mathematical language, the letters are triangles, circles and other geometrical figures, and without these means it is humanly impossible to understand one word.[13] And indeed, the principles of the philosophy of nature as given by Newton in his *Principia* are mathematical principles and the book is structured in a mathematical way. The condemnation of Galilei's heliocentric views by the Roman Catholic Church authority shocked the scientific world and in particular Descartes, who had at the time a text ready for the printer, but withheld it.

I shall restrict myself to a concise comment on the significance and the position of the mathematical argumentation in the philosophy of Spinoza. I shall not go into the question of what he himself contributed or what he took from others: Descartes, Hobbes, Frans van Schooten.[14] Some experts (Lodewijk Meyer, Léon Brunschvicg, Guéroult, Herman de Dijn) view the geometrical argumentation as indispensable and essential; according to others (Harry Wolfson, Fritz Mauthner), it can easily be spared.[15] Among the Spinozists, Mauthner is the most extreme in his rejection of the significance of the geometrical style, but he adds: "It is irrelevant that Spinoza came in a scholastic way to his consistent great world view. Jesus came on a donkey too"[16] Guéroult screened and verified the argumentation in the first two books of *Ethica*. He found that philosophy and geometrical proof form in Spinozism an unbreakable unity. *Ethica* supplied its own interpretation, as long as we were prepared to judge the argumentation of this book on its own merits. Even if this were in accordance with Spinoza's intentions, it still says nothing about the tenability and truth content of this mode of thought. However, Spinoza himself wrote to a young critic: "I do not pretend to have found the best philosophy, no, I know that I understand the true philosophy" just as much as "the three angles of a triangle are equal to two right angles".[17]

In two of his works Spinoza carried out the geometrical argumentation. The first was: *Renati Descartes Principiorum philosophiae more geometrico demonstratae* (1663), to which he added *Cogitata Metaphysica*. Although at the time he himself already held the view that there is only one substance which he equated to God or nature, he gave a mathematical proof of the fact that there are, according to Descartes, two substances: mind and extension. In his main work, *Ethica*, he proceeds in an entirely geometrical way, while in his earlier *Short Treatise about God, Man and his Well-being* (*Korte verhandeling over God, de Mensch en deszelvs Welstand*) the mathematical method received only a modest place at the end. Yet this work deals to a large extent with the same metaphysical problems as *Ethica*.

Nowhere does Spinoza go into the question as to when and why he uses the mathematical argumentation. He left it to his friend Lodewijk Meyer to defend it briefly in his introduc-

[13] Dirk Struik, *Het land van Stevin en Huygens*, 1958, p. 81.

[14] Dirk Struik, *Het land van Stevin en Huygens*, 1958, p. 93.

[15] N.A. Brunt sees no good in the mathematical argumentation and as a physicist he rejects the philosophy of Spinoza completely [6].

[16] "Es tut nichts, daß Spinoza auf scholastischem Wege zu seiner einheitlich grossen Weltanschauung kommt. Auch Jesus kam auf einem Esel [. . .]" [16, p. 70].

[17] "Want ik heb niet de pretentie de beste filosofie te hebben gevonden, neen, ik weet dat ik de ware wijsbegeerte begrijp", in a letter to Albert Burgh. F. Akkerman, H.G. Hubbeling, A.G. Westerbrink, *Spinoza, briefwisseling* (Spinoza, correspondence in Dutch), Wereldbibliotheek, Amsterdam, 1977.

tion to the 1663 edition of Spinoza's *Descartes' "Principles of Philosophy"*. Meyer wrote in this introduction:

> All those who are striving for more insight than ordinary people possess, agree that the method of the mathematicians for investigation and exposition of scientific knowledge—the method in the course of which with cogency conclusions are drawn from definitions, postulates and axioms—is the best and safest way to seek and teach the truth.

Although Spinoza was occupied with mathematics and although he possessed a considerable number of books about mathematics, he himself did not accomplish anything special in this field. Johan de Witt and the above mentioned Johan Hudde were involved in mathematics as well, but Spinoza did not participate in their discussions about this subject.

It goes without saying that for Spinoza Descartes' mathematical work was of overriding importance. Descartes was not only the founder of modern philosophy but, at the same time, a pioneering mathematician following in the footsteps of François Viète (1540–1603) and others. In the 16th and 17th centuries it was no longer a matter of Euclid's *Elements*, of synthetic geometry in the Greek way. In his *Géométrie* (1637), which appeared as an appendix of the *Discours de la Méthode*, Descartes made the move that brought the entire classical geometry within the reach of algebra, such that from now on geometrical and algebraic methods could inseminate each other.[18] Spatial figures can be expressed in terms of numbers and numbers can be interpreted in terms of figures. Descartes himself made limited use of mathematical methods in philosophy, although Mersenne with whom he kept in close contact urged him to do this in his entire work. Descartes wrote about the question of which subjects lend themselves to mathematical argumentation and which do not:

> Already mathematicians in Antiquity in their writings made use of synthetic argumentation, not because they were entirely ignorant of analysis, but because they [...] reserved it for themselves alone, like an important secret. For me, I only followed the analytical road in my *Méditations*, because she seems to me to be the truest, and the most suitable way to teach, but as for the synthesis, she is not always so well applicable to matters belonging to metaphysics.[19]

According to Descartes this was caused by the fact that in metaphysics no unanimous and compelling axioms could be phrased, as one could in the exact sciences. All the same, in order to please Father Mersenne, Descartes cast some metaphysical thoughts in a synthetic form.

However, for the development of Spinoza's metaphysics analytical geometry was of a fundamental heuristic significance. It helped him to conquer the gap between thought and extension and in doing so develop his own metaphysics. According to Dunin Borkowski "like the formula is nothing different from the spiritualized curve, thought, the ideas are only the spiritual side of material reality".[20] And elsewhere Borkowski wrote: "The ideas

[18]Cf. [18, p. 102].

[19]"non qu'ils ignorassent entièrement l'analyse, mais parce qu'ils [...] la réservaient pour eux seuls, comme un secret d'importance. Pour moi, j'ai suivi seulement la voie analytique dans mes Méditations, parce qu'elle me semble être la plus vraie, et la plus propre pour enseigner; mais, quant à la synthèse [...] elle ne convient pas toutefois si bien aux matières qui appartiennent à la métaphysique". Descartes, Méditations, Objections et réponses, *Descartes, Oeuvres et lettres*, p. 388.

[20]"Wie die Formel nichts anders ist als die vergeistigte Kurve so ist das Denken, die Idee, nur die geistige Seite der stofflichen Realität" [8, p. 404].

follow from each other in exactly the same way as the things (ordo idearum idem est ac ordo rerum). [. . .] The body is the geometrical figure, the soul is the formula”:[21] Two totally different spheres and yet expression of one and the same and this cleared the way from the Aristotelian and later Scholastic notion of *substantia* to a complete transformation of this fundamental notion.[22] God is eternal, uncreated and as *causa sui* the cause of himself. I suppose that for Spinoza the power of the geometrical argumentation in philosophy was due to the fact that mathematics does not depend on empirical data from the outside world and concerns a construction of thoughts. The mathematical method gives Spinoza in his philosophical development the certainty and clarity that he needs.[23] Secondly, in mathematical argumentation no steps are omitted as happens easily in syllogistic argumentation. Thirdly, no teleological moment can sneak in and the argumentation remains strictly causal. Fourthly, anthropomorphic thoughts in thinking about God are avoided and mathematics eliminates the individual subjectivity. As was usual in his time, Spinoza tied in with the synthetic argumentation in Euclid’s *Elements*: definitions, axioms, propositions, proofs. In particular the definitions are of great importance. Dunin Borkowski pointed out that Spinoza was familiar with Hobbes *De Cive*, but also with the genetic definition in Hobbes’ book *De Corpore*. This definition is an elaboration of the Euclidean way of defining. The sphere is defined by constructing it and not by means of its description. A definition should give the cause of a thing, its becoming.[24] A century later, in the wake of Guéroult, Herman de Dijn would emphasise this aspect. *Ethica* starts with *Definitiones, Axiomata, Propositiones* and *Demonstrationes*, but, unlike Euclid, Spinoza goes beyond that. Sometimes there is, next to the proof of a proposition, an *Aliter* (another proof). There are extensive *Explicationes, Corrolaria, Scholia* (additions) and *Lemmata* (auxiliary propositions). Some of them take up more space than the proof in the strict sense does. Also in his choice of words Spinoza uses geometrical metaphors and clarifying mathematical examples. The geometrical argumentation is completely absent in numerous books on Spinoza’s philosophy. Apparently in the judgment of all these authors the geometrical argumentation can be missed without hampering understanding. For the reader of *Ethica* too, the geometrical argumentation remains rather accidental, somewhat external. *Korte Verhandeling* (Short treatise) shows that it can be done as well without the mathematical argumentation. So the question is whether it is more than a figure of speech. Mauthner’s judgement is severe:

> One should simply delete all proofs, moreover all definitions and theorems that are only meant to fill the holes in the system. Instead of this one orders the additions and explications, the prefaces and corollaries, one adds the necessary texts from his big treatises and from his wonderful letters, and at last the world will be able to read the unforgettable philosopher.[25]

Recently Henri Krop and Wiep van Bunge wrote about *Ethica*:

[21] “die Ideen folgen aus einander genau wie die Dinge (ordo idearum idem est ac ordo rerum). [. . .] Der Körper ist die geometrische Figur, die Seele die Formel” [8, p. 405].

[22] [24, p. 28] and further.

[23] Yet mathematics served many practical goals: military science, astronomy, mechanics, shipping, bookkeeping, etc.

[24] [8, p. 410].

[25] “Man lasse doch sämtliche Beweise einfach weg, dazu alle die Definitionen und Lehrsätze, die nur die Lücken des Systems auszufüllen bestimmt sind. Man ordne dafür die Zusätze und Erläuterungen, die Vorreden und Anhänge gut zusammen, man füge aus seinem grossen Traktate und aus seinem herrlichen Briefen das Nötige hinzu, und die Welt wird den unvergleichlichen Philosophen endlich lesen können” [16, p. 43].

> On the face of it, this book could not have been written differently from the way it was written. However, the fact that Spinoza appeared to be able to put his views into words in so many different ways, could very well indicate that also the famous 'geometrical' argumentation must be seen as a strategic choice.[26]

However, it may be that non-philosophical circumstances exerted influence on the radical application of the geometrical argumentation. One should remember that Spinoza had weak health. He suffered from tuberculosis and would die from it. His youth was not easy. As a young man he had to do without his mother and there was further mortality in his family. His expulsion from the Jewish community must surely have left traces. This all resulted in a stoic attitude to life and a flight into the restfulness of mathematical argumentation. His work radiates stillness. It is more related to Vermeer than to Rembrandt. The art historian Valentiner writes about this, after pointing out the mathematical constructions in Vermeer's work: "We have seen that Spinoza's rationalistic ideas were in many ways opposed to Rembrandt's. And while his spirituality was on the same high level, his philosophy had more in common with Vermeer",[27] someone of his own generation.

The Dutch theologian Antonius van der Linde compares Spinoza's work with the paintings of Ruysdael: "It resembles Ruysdael's melancholic landscapes that one hardly dares to approach without fearing to disturb the quiet loneliness".[28]

7. Social philosophy

In Spinoza's eyes, contrary to the inner life, political and social subjects do not lend themselves to the mathematical argumentation. His *Theological-Political Treatise* (Theologisch-Politiek Tractaat) and *Ethica* are methodologically separate: the one is—also because of Spinoza's knowledge of Hebrew—a pioneering and subtle historical–philological analysis of the Jewish and Christian faith of divine revelation in which no mathematical argumentation enters, and the other is a strictly rational exposition about God. Mathematical and historical analysis seem to be square with each other here. Yet Spinoza had intended to study human beings as if they were geometrical figures. In mathematics there was no good and evil, no sense and nonsense. He reprimands a child who screams of pain from a gallstone, because the screams would amount to a circle complaining not to be a sphere.

The philosophical views that Spinoza developed were certainly revolutionary, but his social conviction was moderately liberal. In *Het land van Rembrandt* (The Land of Rembrandt) Busken Huet writes: "Spinoza is a conservative with a revolutionary method".[29]

[26]"Het heeft er alle schijn van dat dit boek niet anders geschreven had kunnen worden dan het geschreven is. Dat Spinoza echter in staat bleek zijn denkbeelden op zoveel verschillende manieren onder woorden te brengen, zou er wel eens op kunnen wijzen dat ook die vermaarde "geometrische" betoogtrant als een strategische keuze moet worden gezien" [14, p. 21].

[27]W.R. Valentiner, *Rembrandt and Spinoza. A study of the spiritual conflicts in seventeenth-century Holland*, 1957, p. 79.

[28]"Es gleicht jenen wehmüthigen Landschaften von Ruysdael denen man kaum wagt nahe zu treten, ohne zu befürchten, dasz man die stille Einsamkeit derselbe stört". A. van der Linde, *Spinoza. Seine Lehre. Eine philosophisch-historische Monographie*, 1862, Diss.

[29]Conrad Busket Huet, *Het land van Rembrand*, 1987, p. 611.

Spinoza chooses a realistic point of departure that has nothing to do with geometry. Men are driven by their own interest and by ambition to persist in their own existence. This principle, often referred to as the *conatus*[30] doctrine is very fundamental in Spinoza's philosophy. He thought less negatively about the masses than did Thomas Hobbes, to whom, next to Descartes, Spinoza owes so much. Like Hobbes he embraces the positive effect of a fictitious social contract in which people for their own good successfully put power in the hands of a state authority. Spinoza differs from Hobbes in the sense that this need not be an absolute sovereign; he prefers a restricted form of democracy.

Spinoza was an austere human being but he did not find it necessary that people should deny themselves everything. They were allowed to enjoy modestly whatever they acquired or conquered as long as it did not become a goal in itself, like the interest of the masses in money: "However, money has supplied a token for all things, with the result that its image is wont to obsess the minds of the populace, because they can scarcely think of any kind of pleasure that is not accompanied by the idea of money as its cause".[31] He did not view the possession of money as wrong. Spinoza wrote: "But this vice is characteristic only of those who seek money not through poverty nor to meet their necessities, but because they have acquired the art of money-making, whereby they raise themselves to a splendid estate".[32] The greatest happiness, however, was in man himself. Spinoza felt that his metaphysics and his conception of life were only suitable for a philosophical elite. He adopted the Cicero's saying: "All excellence is as difficult as it is rare" (*Omnia praeclare tam difficilia quam rara sunt*).

The mathematical argumentation did not keep Spinoza from strong prejudices, such as with respect to the lower ranks of society that he called lecherous and indolent. Sometimes, however, he emphasised that it is shameful to restrict the failings that are characteristic of all people only to the rabble: "However, we let merely outer appearance and a certain refinement blind us and that is why we often, when two people do the same thing, say that one can do it without punishment and the other cannot, not because the act is different, but because the perpetrator is different".

He expressed himself positively on the small businessmen that he associated with and he considered them competent to judge in matters of state. The statement at the end of *Tractatus Theologico-Politicus* about women is severe:

> Yet if nature had made women equal to men, and had given them equal strength of mind and intellectual ability, in which human power and therefore human right mainly consists, surely among so many different nations some would be found where both sexes ruled on equal terms, and others where the men were ruled by the women and brought up in such a manner that they had less ability. But since this has nowhere happened, I am fully entitled to assert that women have not the same

[30] Spinoza expressed this conatus doctrine (from the Latin *conatus* = striving) as follows: "Each thing, as far as it can by its own power, strives to persevere in its being" (*Ethica*, Part III, Proposition 6).

[31] "Verum omnium rerum compendium pecunia attulit, unde factum, ut ejus imago Mentem vulgi maxime occupare soleat; quia vix ullam Laetitiae speciem imaginari possunt, nisi concomitante nummorum idea, tanguam causa", *Ethica*, IV, Appendix 28. Quoted from: Benedict de Spinoza, *Spinoza Complete works*, Translations by Samuel Shirley. Edited with Introduction and Notes by Michael L. Morgan, Hacket Publishing Company, Indianapolis/Cambridge, 2002, p. 361.

[32] "Sed hoc vitium eorum tantium est qui non ex indigentia, nec propter necessitates nummos quaerunt; sed quia lucri artes didicerunt, quibus se magnifice efferunt", *Ethica*, IV, Appendix 29 , Quoted from op. cit.

> right as men by nature, but are necessarily inferior to them; so that it is not possible for both sexes to rule on equal terms.[33]

This inferiority could not be deduced from reason so Spinoza had to make an appeal to the sensory perception that he as a rationalist had taught to mistrust. But Spinoza had another argument in store, pleading against the equal association of men and women, sex:

> If we also consider human passions, and reflect that men generally love women out of mere lust, judge their ability and wisdom by their beauty, are highly indignant if the women they love show the slightest favour to others, and so on, we shall easily see that it is impossible for men and women to govern on equal terms without great damage to peace.[34]

Spinoza's negative judgment of women cannot be deduced just as such from the spirit of the time, nor from the mathematical argumentation. Maybe this attitude can be explained by the lack of ties with educated women. With one exception, his circle of acquaintances consisted of married men. Descartes, on the other hand, had an extensive, affectionate and learned correspondence with two women (Princess Elisabeth of Bohemia who had fled to the Netherlands with her parents, and Kristina Wasa, Queen of Sweden). Moreover, in the Netherlands he had a love affair with a housemaid, Hélène Jansz. They had a daughter that Descartes called Francine in honour of his fatherland. When she died at the age of six, Descartes spoke of ”the greatest sorrow of his life”.

8. Living nature

Like the author of the book of Genesis in the Bible Spinoza possessed no respect for living nature:

> Except for mankind, we know of no individual thing in Nature in whose mind we can rejoice, and with which we can write in friendship or form some kind of close tie. So whatever there is in Nature external to man, regard for our own advantage does not require us to preserve it, but teaches us to preserve or destroy it according to its varying usefulness, or to adapt it to our own use in whatever way we please.[35]

[33]"Quod si ex natura foeminae viris aequales essent, et animi fortitudine et ingenio, in quo maxime humana potentia et consequenter jus consistit, aeque pollerent, sane inter tot tamque diversas nationes quaedam reperirentur ubi uterque sexus pariter regeret, et aliae ubi a foeminis viri regerentur, atque ita educarentur ut ingenio minus possent. Quod cum nullibi factum sit, affirmare omnino licet foeminas ex natura non aequale cum viris habere jus, sed eas viris necessario cedere; atque adeo fieri non posse ut uterque sexus pariter regat, et multo minus ut viri a foeminis regantur". Quoted from: Benedict de Spinoza, *The Political Works*, edited and translated by A.G. Wernham, Clarendon Press, Oxford, 1958, pp. 442–445.

[34]"Quod si praeterea humanos affectus consideremus, quod scilicet viri plerumque ex solo libidinis affectu foeminas ament, et earum ingenium et sapientiam tanti aestiment quantum ipsae pulchritudine pollent, et praeterae quod viri aeggerrime ferant ut foeminae quas amant aliis aliquo modo faveant, et id genus alia, levi negotio videbimus non posse absque magno pacis detrimento fieri ut viri foeminae pariter regant". Quoted from op. cit. pp. 444–445.

[35]"Praeter homines nihil singulare in natura novimus, cujus Mente gaudere, et quod nobis amicitia, aut aliquo consuetudinis genere jungere possumus; adeoque quicquid in rerum natura extra homines datur, id nostrae utilitatis ratio conservare non postulat; sed pro ejus vario usu conservare, destruere, vel quocunque modo ad nostrum usum adaptare nos docet". *Ethica*, IV, Appendix 23. Quoted from: Benedict de Spinoza, *Spinoza Complete works*, Translations by Samuel Shirley. Edited with Introduction and Notes by Michael L. Morgan, Hacket Publishing Company, Indianapolis/Cambridge, 2002, p. 361.

This was written in a period in which Leonardo da Vinci, Albrecht Dürer, Antoni van Leeuwenhoek, Jan Swammerdam and Dodonaeus teach us to see the beauty of nature. Spinoza, on the other hand, brings about a timeless mathematisation of nature. The *becoming* in the abstract form of mechanical movement is, in Spinoza's terminology, nothing more than an *infinite modus* and does not possess the *status* of an *attribute*. The *operari* is turned into the *sequi*. The mathematical aspect of reality is no longer the expression of reality but takes its place. A gap between God (*substantia*) and the empirical world, between eternity and finiteness, eliminates in fact the historical dimension and the uniqueness of the course of the world.

9. Summum bonum

When Spinoza talked about human happiness he thought primarily of God and not of happiness between people. Yet he was a proponent of a church based on the love between people. In everything, however, God is the point of departure. The love of God is man's highest happiness (*amor dei intellectualis*). The individual is not in the centre as in humanism. The beginning of the *Tractatus de intellectus emendatione* points this out. The pursuit of possessions, money, power, honour, pleasure can only give temporary satisfaction. It does not yield the true, permanent and highest happiness (*summum bonum*). His practical philosophy of life is thoroughly inspired by Stoicism, but without the radical "suffer deprivation so that you won't suffer deprivation".[36] He would not hear of the gloomy Calvinism that takes sinfulness as its starting point. Spinoza's maxim is: "A free man thinks of nothing less than of death, and his wisdom is a meditation on life, not on death".[37] Man is allowed to enjoy himself, although not too much.

In the fifth book of *Ethica* Spinoza indicates the road that leads to the knowledge of God and to spiritual freedom. Spinoza distinguished three kinds of knowledge.[38] The least reliable knowledge is based on reports from others or it comes to us from random experience, without interference of the intellect. The second kind of knowledge we have when the essence of a thing is inferred from another thing. The third kind of knowledge we have when a thing is perceived through its essence alone or through knowledge of its proximate cause. Spinoza illustrated this classification by means of a mathematical problem: find x when $a : b = c : x$ when $a, b,$ and c are given. If we use a rule that we once learned without understanding why it works, or if we apply a rule that we have found by trial and error, we acquire knowledge of the first kind (*imaginatio*). If we understand proposition 19 of book 7 of Euclid's *Elements*, which says that $a : b = c : d$ is equivalent to $a * d = b * c$, and apply it to the problem involved, we acquire deductive knowledge: the second kind of knowledge (*ratio*). The third kind of knowledge is intuitive knowledge and it is to be preferred. In the case of the equation $1 : 2 = 3 : x$ we can immediately, intuitively, without deduction see that $x = 6$ is the correct solution. This is the kind of knowledge that proceeds from an idea

[36] This is how the German poet Johann Heinrich Voss (1751–1826) characterised Cato's stoicism: "Damit du nichts entbehrst, war Catos weise Lehre, Entbehre".

[37] "Homo liber de nulla re minus, quam de morte cogitat, et eieus sapientia non mortis, sed vitae meditatio est". *Ethica*, IV, Proposition 67.

[38] *Ethica*, II, Proposition 40, Schol. 2.

of the attributes of God to knowledge of the essence of things. It is not a matter of feeling but of reason. Is this a rational mysticism?

The nature of God is strictly deterministic and freedom is the insight in the necessity of things. There is no free will. The fifth book concerns the spiritual freedom that results from the control of passion. Mankind finds himself to a certain extent at the mercy of the lower passions: lust, the desire for esteem, riches and wealth. An intellectual elite, however, craves a higher, spiritual happiness, for "the smell of higher honey".[39] In Spinoza the way to the eternal love for God leads in the end to such delight that in comparison all earthly enjoyments become insignificant. The *amor dei intellectualis* leads to a way of life that consists of *laetare et bene agere* (rejoice and act well, *Ethica*, IV, Proposition 73). W. Meijer called Spinoza the happy messenger of the mature mankind. The Christian idea that virtue would eventually be rewarded, in other words the idea that only the prospect of a reward in an afterlife brings people to live virtuously, was alien to him. The reward for a virtuous life was in the perpetration of virtue itself. According to Spinoza God loves only himself and not people; that is why he who loves God cannot desire that God should love him in return (*Ethica*, V, Proposition 19). This love for God cannot be defiled by envy or jealousy. The more people are joined to God by this bond of love, the stronger it gets (*Ethica*, V, Proposition 20). Spinoza's *opus magnum*, the *Ethica*, begins and ends with God. Proposition 21 in *Ethica*, V, seems to imply that the soul (mind) perishes together with the individual body. It says that the mind cannot imagine anything, nor remember past things, except as long as the body exists. Yet *Ethica*, V, Proposition 23 reads: "The human mind cannot be completely destroyed together with the body, but something of it remains which is eternal". Insofar as the human mind participates in the one and only substance, God, via the *amor intellectualis dei*, the mind is eternal. Clearly this does not concern the individual inner life (imagination, memories) but the general moment of the mind. Does not this thought remind us of the *intellectus agens* in the *Destruction destructionis* (1180) of Averroës?[40] In the end more mind than matter and no strict parallellism. This has not much to do with mathematics.

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[40] Ibn Roschd, or Averroës (1126–1198), as he was called by the Latins, was an Arabian philosopher and physician. He was very much influenced by Aristotle. His work *Tehafot al Tchafot* (Incoherence of the Incoherence), or in Latin *Destructio Destructiones* was a refutation of Al-Ghazali's *Destructio Philosophorum*. Al-Ghazali had argued: Islam is true, and where the philosophers contradict it they are wrong. Averroës reproduced the text of Al-Ghazali's book and commented on it. An important question concerned the individual mind. Al-Ghazali maintained that the individual human soul survives the death of the body. Averroes found this illogical. The visible world and also human bodies are moved by the *intellectus agens*. It is eternal, according to Averroës. The individual human intellect is not.

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CHAPTER 23

John Wallis (1616–1703): Mathematician and Divine*

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MATHEMATICS AND THE DIVINE: A HISTORICAL STUDY
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Against the backdrop of political and religious strife which characterised seventeenth-century England from the time of the Civil Wars (1642–1649) up to the Glorious Revolution of 1688, few scholars survived as well as the Savilian professor of geometry at Oxford, John Wallis. Appointed to this professorship on 14/24 June 1649, just a matter of weeks after England had been declared a republic, and largely in recognition of his services to Parliament as a decipherer during the military conflict, he retained it up to his death in October 1703. As is well known, during this time he succeeded in making major contributions to mathematics, most notably through his *Arithmetica infinitorum* (1656), to physics—his *Mechanica* (1670/71) was rightly acclaimed at the time—and also played a founding role[1] in the Royal Society, of which he subsequently became one of the most prolific members. But from his training Wallis was a theologian. Ordained by the Bishop of Winchester in 1640, he became one of the secretaries to the Westminster Assembly of Divines in 1644, and participated in numerous theological debates throughout his active life, on topics ranging from justification to the Trinity, and from the calculation of Easter to infant baptism. It was in this field in particular that dangers abounded, as is evidenced by the fate of many less compliant and more outspoken contemporaries after the Restoration. Wallis survived, however, not only on account of his continued importance as more or less official decipherer,[2] but also because of the course he chose to adopt. He describes this clearly in his autobiography, written in January 1696/7 for his friend Thomas Smith, at that time librarian at the Cottonian Library:

> "It hath been my endeavour all along, to act by moderate Principles, between the Extremities on either hand, in a moderate compliance with the Powers in being, in those places, where it hath been my Lot to live, without the fierce and violent animosities usual in such Cases, against all, that did not act just as I did, knowing that there were many worthy Persons engaged on either side. And willing whatever side was upmost, to promote (as I was able) any good design for the true Interest of Religion, of Learning, and the public good; and ready to do good Offices, as there was Opportunity".[3]

But if moderation is true of his political and theological stance, things are quite different when it comes to scientific affairs in general and to mathematics in particular. There was probably no other scholar in the Seventeenth Century who got embroiled[4] in quite as many disputes as the Savilian professor of geometry: disputes with Huygens (e.g. over Neile's priority in rectifying a curve of any kind), with Fermat and Frenicle (over questions in number theory), with Roberval (on priority in rectifying the Archimedean spiral), and so on. However, the most bitter and certainly the most long drawn-out battle was that with his fellow-countryman Thomas Hobbes, which ran from the publication of *De corpore* in 1655 until its author's death in 1679. And while it is true that the numerous disputes with Continental scholars are not without their religious and political overtones—due to

[1] See: John Wallis, *A Defence of the Royal Society. In Answer to the Cavils of Dr. William Holder*, London, 1678, especially pp. 7–10. Cf. Joseph Frederick Scott, *The Mathematical Work of John Wallis, D.D., F.R.S. (1616–1703)*, London 1938 (reprint: New York, 1981), pp. 8–14.

[2] Wallis held the official post of decipherer (together with his son-in-law William Blencowe) from April 1701 onwards. See *Officials of the Secretaries of State (1660–1782)*, compiled by J.C. Sainty, London, 1973, pp. 59, 66, 114.

[3] See Christoph J. Scriba, *The Autobiography of John Wallis, F.R.S.*, in: *Notes and Records of the Royal Society of London* 22 (1967), 17–46, 42–43.

[4] See: Scott, *Mathematical Work*, pp. 80–82.

Fig. 1. John Wallis. Science Photo Library.

increasingly strong religious and national rivalries—only in that with Hobbes is there an immanent relation between mathematical and theological considerations.

1. The controversy with Thomas Hobbes

After the publication of *Leviathan* (1651) Thomas Hobbes was attacked by a number of divines of the Anglican Church. Among them was the Savilian professor of astronomy at Oxford, Seth Ward, who published his *Philosophicall Essay* (1652) as an antidote against the theological impact of Hobbes' book. Ward tried to demonstrate the creation of the world by rejecting actual infinity in mathematics and thereby negating the basis of the concept of infinite duration.[5] Hobbes who maintained that it was impossible to argue for a finite age of the world from natural reason only, repeated his doctrine in *De Corpore* (1655). This time, Ward shared the task of answering Hobbes: while he prepared an answer[6] to the philosophical parts of *De Corpore*, Wallis reacted immediately to the shortcomings of the geometrical part of *De Corpore*, especially to the attempted solutions of the classical unsolved problems (the quadrature of the circle, the trisection of the angle, and the duplication of the cube), in his *Elenchus geometriae Hobbianae* (1655). During the following years, an avalanche of polemical treatises was published; the dispute involved a number of

[5] Seth Ward, *A Philosophicall Essay Towards an Eviction of the Being and Attributes of God. The Immortality of the Souls of Men. The Truth and Authority of Scripture*, Oxford, 1652. See Siegmund Probst, "Infinity and Creation: the Origin of the Controversy between Thomas Hobbes and the Savilian Professors Seth Ward and John Wallis", in: *British Journal for the History of Science* 26 (1993), 271–279.

[6] Seth Ward, in: *Thomae Hobbii philosophiam exercitatio epistolica*, Oxford, 1656.

British and continental mathematicians, among them Brouncker, Rooke, Huygens, Sluse, and Mylon.[7]

It is important to emphasise that Wallis (like Ward before him) vindicated his scientific critique of Hobbes with his religious motivation: In the dedicatory letter[8] of the *Elenchus*, addressed to John Owen, vice-chancellor of the University of Oxford, Wallis claimed that Hobbes' mathematical writings had no impact on the mathematical public. He would not have troubled his mind with refuting the pretended solutions of the classical problems had the author not been Hobbes—whose influence in philosophy was a danger to religion. The destruction of Hobbes' reputation in the field of mathematics aimed at undermining the epistemological basis of Hobbes' mechanical philosophy.

Wallis explicitly declares differences in the field of theology to be the main reason for the dispute; he mentions specifically the *Leviathan* and the theses put forward there to be stumbling blocks:[9] theses concerning God, sin, the Bible, non-corporeal substances, and the immortality of the soul ("tremenda plane & horrida, de Deo, de Peccato, de Scriptura sacra, de Substantiis incorporeis universim omnibus, de hominis Anima immortali, caeterisque gravioribus Religionis apicibus").[10]

The question of the beginning of the world was taken up by Wallis in *Hobbius Heautontimorumenos* (1662), a sarcastic polemic reaction to the attacks of Hobbes against him in the *Examinatio et emendatio mathematicae hodiernae* (1660). Wallis accused Hobbes of favouring atheism:

> "For, one while they find him affirming, That, beside the Creation of the World, there is no Argument to prove a Deity: Another while, That it cannot be evinced by any Argument, that the World had a beginning; and, That, whether it had or no, is to be decided not by Argument, but by the Magistrates Authority: And, *Jeering* upon every turn at *Immaterial Substances*: But, no where proving either the *Impossibility*, or the *Non-existence* of them".[11]

Hobbes answered the reproaches in a short biographical essay, written in the third person and entitled *Considerations upon the Reputation, Loyalty, Manners, and Religion of Thomas Hobbes* (1662), in no way confining himself to the defence of his person, but asserting that Wallis had committed treason against King Charles I by deciphering royalist letters immediately before the battle of Naseby.[12] Hobbes did not enter into a discussion of the contention of Wallis that his philosophy undermines the belief in the existence of

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[7] See Siegmund Probst, *Die mathematische Kontroverse zwischen Thomas Hobbes und John Wallis* (Diss. Univ. Regensburg), 1997, and Douglas M. Jesseph, *Squaring the Circle: The War between Hobbes and Wallis*, Chicago, 1999.

[8] The epistle dedicatory is dated 10/[20].X.1655, whereas the date for the completion of the manuscript is given as 27.VIII/[6.IX].1655.

[9] John Wallis, *Elenchus geometriae Hobbianae*, Oxford, 1655, sig. A3r: "Verum quidem est, hunc nostrum Philosophiae instauratorem de ipsius in Mathematicis peritia opinionem concepisse quam maximam, aliosque omnes sive Philosophos sive Mathematicos (quasi solus ipse haec intelligat) e longinquo despicere; idque adeo ut siquos ipsi in Theologicis aut Philosophicis contravenientes videat, aut dictata sua minus admissuros, supercilioso hoc putet responso amandandos, quod, cum Geometriae si[n]t imperiti, haec sua non intelligant".

[10] Wallis, *Elenchus*, sig. A2v.

[11] John Wallis, *Hobbius Heautontimorumenos*, Oxford, 1662, p. 6.

[12] Thomas Hobbes, *The English Works*, ed. W. Molesworth, 11 volumes, London, 1839–1845, IV, pp. 416–17; Hobbes already mentions the affair in the *Examinatio* and in the *Dialogus*. See Thomas Hobbes, *Opera philosophica quae latine scripsit omnia*, ed. W. Molesworth, 5 volumes, London, 1839–1845, IV, pp. 55, 290.

God but met the charge of atheism by referring to his obedience to the Bible. Nevertheless, he confirmed his objection to philosophical arguments about the beginning of the world:

> "For a third argument of atheism, you put, that he [sc. Hobbes] says: *besides the creation of the world, there is no argument to prove a Deity*: and, *that it cannot be evinced by any argument that the world had a beginning*: and, *that whether it had or no, is to be decided not by argument, but by the magistrate's authority*. That it may be decided by the Scriptures, he never denied; therefore in that also you slander him. And as for arguments from natural reason, neither you, nor any other, have hitherto brought any, except the creation, that has not made it more doubtful to many men than it was before".[13]

For a second time, the problem of the creation of the world was discussed by Wallis in his answer to the leaflets of Hobbes addressed to the Royal Society in 1671.[14] After the refutation of his *Rosetum geometricum* (geometrical rose garden) in the *Philosophical Transactions*[15]—Wallis qualified the work as "*fimetum*" (dung-hill)—Hobbes addressed the Royal Society in public. He had a leaflet printed in which he repeated the main arguments from the *Rosetum* against a theorem of Wallis and asked for an official expert judgement from the Royal Society on his dispute with Wallis. Moreover, Hobbes presented his paper (and two sequels) to King Charles II who demanded an expertise from the Society. Wallis was forced to react to the attack.[16] As soon as Wallis' *An Answer to three Papers of Mr. Hobbes, lately published in the months of August and this present September* had come out in the form of a leaflet, Hobbes replied in the *Considerations upon the Answer of Dr. Wallis*, leaving just enough time for Wallis to add a few comments for the reprint of his paper in the *Philosophical Transactions* for September 1671.

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[13] Hobbes, *English Works* IV, pp. 427–428.

[14] Since the founding of the Royal Society, Hobbes had sought to obtain public recognition from it as a leading natural philosopher. But his geometrical works had been repeatedly confuted by Wallis, Brouncker and Huygens, all three prominent members of the Society. Moreover, from 1666 on, his publications were critically reviewed in the *Philosophical Transactions*, mostly by his arch-enemy Wallis. See Review of Hobbes' *De principiis et ratiocinatione geometrarum*, in: *Philosophical Transactions* No. 14 (2 July 1666), pp. [253]–254; *Animadversions of Dr. Wallis, upon Mr. Hob's late Book, De Principiis & Ratiocinatione Geometrarum*, in: *Philosophical Transactions* No. 16 (6 August 1666), pp. 289–294; Review of Wallis' *Quadratura circuli, Cubatio sphaerae, Duplicatio cubi, confutata*, in: *Philosophical Transactions* No. 48 (21 June 1669), pp. 971–972; Review of Wallis' *Quadratura circuli, Cubatio sphaerae, Duplicatio cubi (secundo edita), Denuo refutata*, in: *Philosophical Transactions* No. 55 (17 January 1669/70), pp. 1121–1122; Review of Hobbes' *Rosetum geometricum*, in: *Philosophical Transactions* No. 72 (19 June 1671), pp. 2185–2186; Wallis, *An Answer of Dr. Wallis to Mr. Hobbes' Rosetum geometricum*, in: *Philosophical Transactions* No. 73 (17 July 1671), pp. 2202–2209; Wallis, *An Answer to Four Papers of Mr. Hobs*, in: *Philosophical Transactions* No. 75 (18 September 1671), pp. 2241–2250; Review of Hobbes' *Lux mathematica*, in: *Philosophical Transactions* No. 86 (19 August 1672), pp. 5047–5048; Wallis, *Dr. John Wallis' Answer … to the Book, Entituled Lux Mathematica*, in: *Philosophical Transactions* No. 87 (14 October 1672), pp. 5067–5073; Review of Hobbes' *Principia & Problemata aliquot Geometrica*, in: *Philosophical Transactions* No. 97 (6 October 1673), p. 6131; Review of *Decameron physiologicum*, in: *Philosophical Transactions* No. 138 (25 March 1678), pp. 965–967. Hobbes disappointment about this is clearly expressed in a letter to Aubrey a few years later. See Hobbes–Aubrey 24.II/[6.III].1675, in: *The Correspondence of Thomas Hobbes*, ed. N. Malcolm, 2 volumes, Oxford, 1994, II, pp. 751–752.

[15] Wallis, *An answer of Dr. Wallis to Mr. Hobbes' Rosetum geometricum*.

[16] Cf. Oldenburg–Wallis 5/[15].VIII.1671, in: *The Correspondence of Henry Oldenburg*, ed. A. Rupert Hall and Marie Boas Hall, 13 volumes, Madison, Milwaukee, London, 1965–1986,VIII, pp. 184–185.

Besides the rejection of the application of algebraic methods to geometry, the main concern of Hobbes was to discuss the use of infinite numbers by Wallis. He therefore started by quoting his own translation of part of the first proposition of chapter five of Wallis' *Mechanica*—a theorem on infinite rows which the Savilian professor employed in order to deduce formulas for calculating the areas of higher parabolas, hyperbolas, and similar problems:

> "If there be understood an infinite row of quantities beginning with 0 or 1/0, and increasing continually according to the natural order of numbers, 0, 1, 2, 3, &c. or according to the order of their squares, as 0, 1, 4, 9, &c. or according to the order of their cubes, as 0, 1, 8, 27, &c. whereof the last is given; the proportion of the whole, shall be to a row of as many, that are equal to the last, in the first case, as 1 to 2; in the second case, as 1 to 3; in the third case, as 1 to 4, &c".[17]

In order to provoke a dismissal of this theorem by the Royal Society, Hobbes asked several questions concerning the nature of *infinity*; (1) *whether there can be understood an infinite row of quantities, whereof the last can be given*; (2) *whether a finite quantity can be divided into an infinite number of lesser quantities, or a finite quantity can consist of an infinite number of parts*; (3) *whether there be any quantity greater than infinite*; (4) *whether there be* [...] *any finite magnitude of which there is no centre of gravity*; (5) *whether there be any number infinite*.[18]

Wallis first tried to conceal his use of the actual infinite by referring to the hypothetical character of the axioms of a mathematical theory, using Hobbes' definition of science as conditional knowledge against him, and asserted that his methods were in accordance with Euclid's procedures. Hobbes was not impressed and pointed to these weak points of Wallis' argumentation. In the end, Wallis defended the new infinitistic mathematics overtly with the progress achieved in quadratures of infinitely extended curves, etc.

On the other hand, the *Doctor of Divinity* Wallis was not at all a supporter of the emancipation of political and philosophical thinking from theology. Thus he tried to discuss the question of the infinite in a field more dangerous for Hobbes than for himself—again the cosmological problem of the beginning of the world:

> "Let him ask himself, therefore, if he be still of opinion, that *there is no argument in nature to prove, the World had a Beginning*; 1. Whether, in case it had not, there must not have passed an *Infinite number of years* before Mr. *Hobs* was born. (For, if but *Finite*, how many soever, it must have begun so many years before.) 2. Whether, now, there have not passed more: that is, *more than that infinite number*. 3. Whether, in that *Infinite* (or *more than infinite*), number of *Years*, there have not been a *Greater* number of *Days* and *Hours*: and, of which hitherto, *the last is given*. 4. Whether, if this be an Absurdity, we have not then (contrary to what Mr. *Hobs* would perswade us) an *Argument in nature* to prove, *the world had a beginning*".[19]

Wallis argued further that Hobbes could not choose the alternative of rejecting the infinite in the field of mathematics while conceding it in physics—a way out analogous to the one that Wallis claimed for the mathematician who considers the objects of his science

[17] Hobbes, *English Works* VII, pp. 431–432, col. 1. Apparently for reasons of symmetry, Wallis equates the value of 0 with that of its reciprocal 1/0; this identification, which is of course at odds with modern views, has however no effective significance in the calculation, nor does Wallis use it in the equivalent proposition 44 of his *Arithmetica infinitorum*. Cf. Scott, *Mathematical Work*, pp. 30, 34.

[18] Hobbes, *English Works* VII, p. 432, col. 1.

[19] Wallis, *An Answer to Four Papers*, p. 2243.

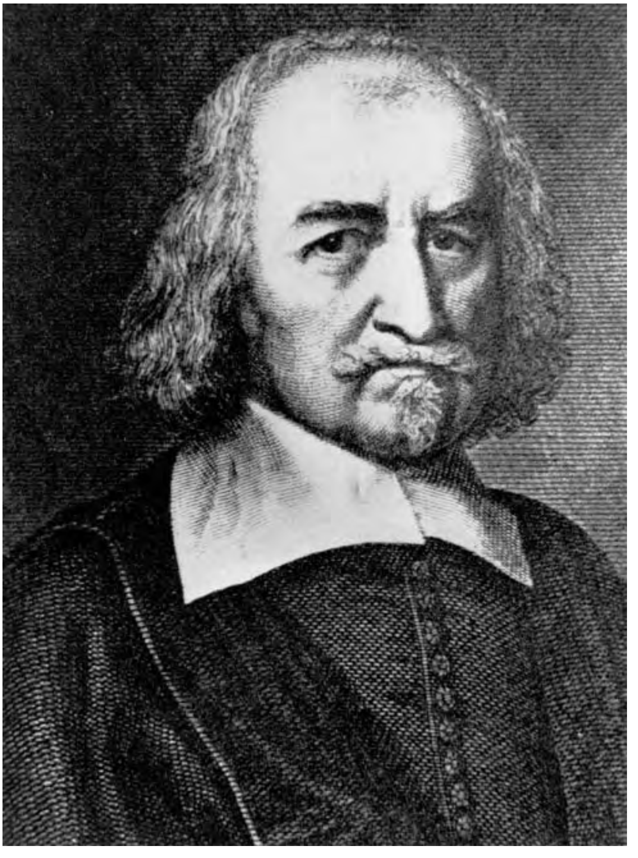

Fig. 2. Thomas Hobbes (1588–1679). Science Photo Library.

only hypothetically and is not subject to the finiteness of existing physical bodies. Hobbes was now confronted with the dilemma of having to object simultaneously to the incompatible standpoints of Ward and Wallis: the former's rejection and the latter's acceptance of actual infinity in mathematics. His answer consisted of two parts: first, he transferred the argument from the object *world* to the object *God* in order to show the danger of this argument for religion, as he did in the dispute with Ward. In a second step, he tried to point out the absurdity of infinite numbers by constructing an infinite number *smaller than infinite*:

> "To this I answer, not willingly, but in service to the Truth, that by the same Argument, he might as well prove that God had a beginning. Thus: in case he had not, there must have passed an Infinite length of time before Mr. *Hobbes* was born; but there hath passed at this day more than that Infinite length by eighty four years. And this day, which is the last, is given. If this be an Absurdity, have we not then an Argument in Nature to prove that God had a beginning? Thus 'tis when men intangle themselves in a Dispute of that which they cannot comprehend. But perhaps he looks for a Solution of his Argument to prove that there is somewhat greater than Infinite; which I shall do so far, as to shew it is not concluding. If from this day backwards to Eternity be more than

> Infinite, and from Mr. *Hobbes* his birth backwards to the same Eternity be Infinite, then take away from this day backwards to the time of *Adam*, which is more than from this day to Mr. *Hobbes* his birth, then that which remains backwards must be less than Infinite. All this arguing of Infinites is but the ambition of School-boyes".[20]

Hobbes' way out of the dilemma is the categorical rejection of the infinite, and even of an argumentation that draws on the paradoxical attributes of the infinite in order to conclude a negation of it. Of course, this standpoint is logically unassailable. The problem arises that not only have the questions of the beginning and of the extension of the world to be excluded from natural philosophy—as was done consequently by Hobbes in *De Corpore*[21]—but that, already by the middle of the seventeenth century a great deal of the achievements in mathematics was based on a more or less unrestrained handling of the infinite and had consequently to be regarded as unscientific according to Hobbes.

Not surprisingly, Wallis protested against Hobbes' transfer of the argument to the object *God*. As much as he favoured the application of algebraical concepts to different fields of mathematics, he could not accept an application of arguments from natural philosophy in the domain of theology:

> "What he would therein insinuate concerning *God* (that we may as well prove *Him* to have had a Beginning, as that the World had) smells too rank of Mr. *Hobs*. We are not to measure Gods *Permanent* Duration of Eternity, by our *successive* Duration of Time: Nor, his Intire *Ubiquity*, by Corporeal *Extension*".[22]

There is no further dispute about the creation of the world in the last publications of the controversy.[23] Hobbes' final statement on infinity is to be found in chapter XIII of his *Principia et problemata* of 1674. Hobbes denied that of two infinitely extended lines, starting from different points on a given finite line, one could be considered as longer than the other and that the same were true of eternal periods of time. Cautiously, Hobbes avoided asserting the equality of these infinites, and explicitly characterized the relation between them as *non-inequality*.[24] So Hobbes seems to have arrived at the conclusion that the law of the excluded middle is not valid in the case of these infinites; more than a century later, Immanuel Kant, confronted with the same problem, defined the relation between these infinites as *dialectical opposition*.[25]

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[20]Thomas Hobbes, *Considerations upon the Answer of Dr. Wallis*, London, 1671, p. 1; *English Works* VII, pp. 445–446.

[21]Hobbes, *Opera philosophica* I, pp. 334–336; *English Works* I, pp. 410–412.

[22]Wallis, *An Answer to Four Papers*, p. 2250.

[23]See Thomas Hobbes, *Lux mathematica*, London 1672; idem, *Principia et problemata aliquot geometrica*, London 1674; idem, *Decameron physiologicum: or Ten Dialogues of Natural Philosophy*, London, 1678.

[24]Hobbes, *Opera philosophica* V, p. 212: "Ergo de infinito secundum sensum hunc dici non potest aliud alio majus esse. [...] Itaque semidiametri sphæræinfinitæa quolibet centro [...] sunt inter se non-inæquales. Non est ergo una linea infinita major quam alia. [...] Per eandem ratiocinationem, si linea AB ponatur pro finito tempore, probari potest duo æterna inæqualia esse non posse".

[25]Immanuel Kant, *Kritik der reinen Vernunft*, A 504/B 533. Cf. also Dieter Wandschneider, *Il problema dell'inizio del mondo in Kant e Hegel*, in: *Kosmos. La cosmologia tra scienza e filosofia*, ed. Umberto Curi, Ferrara, 1989, pp. 55–65.

2. Wallis' defence of the Trinity

As the controversy with Hobbes documents, Wallis sought to maintain a clear distinction between his work on mathematics and his work on theological topics. One of the few areas where he departed from this, albeit only in the employment of an analogy, was in the course of the most extensive religious dispute in which he was involved, namely that on the Trinity. Here, the trading of positions by the various parties was already underway when Wallis entered the fray in 1690 with a published reply to the leading exponent of English Socinianism at the time, Stephen Nye (1648?–1719). It had been Nye's *Brief History of the Unitarians*[26] which had reopened the antitrinitarian discussion in 1687; since then the theologian William Sherlock (1641?–1707), who became Dean of St. Pauls in 1691, had defended[27] the Trinity, only to be rebuffed by authors of the opposing faction including Nye[28] and William Freke.[29]

Wallis' strategy in upholding the orthodox Anglican doctrine in the light of Socinian attacks was to retain the fundamental inexplicability of the Trinity as a matter of faith and at the same time to show that it was not inconsistent with reason—a strategy not unlike that adopted by Leibniz[30] on the same topic. In this respect Wallis distinguishes the question of the possibility of the Trinity from that of its truth, which, as he emphasises, are to be approached from different ways, the one from natural reason, the other from revelation: "There is nothing in natural reason (that we know of, or can know of) why it [sc. the Trinity] should be thought *Impossible*; but whether or not it be *so*, depends only upon Revelation".[31] In other words, while much in religion goes beyond reason, it is not therefore repugnant to it, but nevertheless cannot be discovered by rational means alone. Precisely herein lies for him the key to defeating the Socinian arguments against the Trinity: by showing that the concept of three divine persons in the one God is rationally sustainable, using the analogy of the mathematical cube, Wallis is in effect aiming to defeat the opponents with their very own tools.

The fundamental idea incorporated in the Athanasian Creed, in which the concept of the Trinity favoured by the Council of Nicaea in 325 found its fullest expression,[32] is that of worshipping one God in the Trinity and the Trinity in the Unity.[33] The three godly

[26] Stephen Nye, *A Brief History of the Unitarians, called also Socinians, in 4 letters*, London, 1687; second edition: London, 1691.

[27] William Sherlock, *The Acts of Great Athanasius, with Notes on his Creed; and Observations on the learned Vindication of the Trinity and Incarnation*, [London] 1690.

[28] See e.g. Stepehen Nye, *Some Thoughts upon Dr. Sherlock's Vindication of the Doctrin of the Holy Trinity, in a Letter*, London, 1690.

[29] See e.g. William Freke, *A Vindication of the Unitarians against a Late Reverend Author on the Trinity*, London, 1690.

[30] See *G.W. Leibniz: Sämtliche Schriften und Briefe*, VI, 1, Leipzig, 1930 (reprint: Berlin, 1990), p. 515.

[31] John Wallis, *The Doctrine of the Blessed Trinity Briefly Explained, in a Letter to a Friend*, London, 1690, p. 8. Reprinted (with other pieces) in: Wallis, *Theological Discourses containing VIII Letters and III Sermons Concerning the Blessed Trinity*, London, 1692. See also Wallis, *The Resurrection Asserted, in a Sermon... Preached on Easter Day, 1679*, Oxford, 1679, p. 24; Wallis, *An Explication and Vindication of the Athanasian Creed; In a Third Letter, persuant of two former, concerning the Sacred Trinity*, London, 1691, p. 23.

[32] See Wallis, *An Explication and Vindication of the Athanasian Creed*, p. 12.

[33] Wallis, *An Explication and Vindication of the Athanasian Creed*, p. 8. See also Wallis, *Three Sermons Concerning the Sacred Trinity*, London, 1691, p. 19.

persons are in some ways no more than distinguished, whereas the Arian view, rejected at Nicaea, emphasised the differences more. For the Socinians on the other hand it was inconceivable that three persons might somehow be in the one God. Inconceivable means however inconsistent with reason; Wallis' aim in using a mathematical analogy was to show that this is not the case.

Taking the example of a die, which as a material body belongs to "the most gross of finite beings",[34] he notes that the three dimensions length, breadth, and height can be easily distinguished, and not only in the imagination, but are nevertheless one cube. "There is no Inconsistence therefore, that what in one regard are Three (three Dimensions) may, in another regard, be so united as to be but One (one Cube). And if it may be so in Corporeals, much more in Spirituals".[35]

In fact, Wallis proceeds to develop this analogy further, supposing first that each of the dimensions be infinitely continued, "there being no limits in nature, greater than which a cube cannot be", so that an infinite cube encompassing all possible imaginary space results, and second, that this infinite cube and so also its dimensions be eternal. From this he is then able to conclude that the Trinity is not rationally inconsistent: "here is a fair resemblance (if we may parvis componere magna) of the Father, (as the Fountain or Original;) of the Son (as generated of him from all Eternity;) and of the Holy-Ghost (as eternally Proceeding from both:) And all this without Inconsistence".[36]

The analogy of the cube, which Wallis claims[37] to have discussed at Oxford already in 1649 with Seth Ward, featured prominently in the series of published letters and tracts putting forward the opposing views from 1690 to 1693. From the antitrinitarian point of view the fundamental flaw in the comparison was that while it is said of every person in the Trinity that he is God, it cannot be said of any one dimension that it be a cube.[38] Wallis however seeks to counter this by drawing on the philosophical distinction between the abstract and the concrete, claiming that "though we cannot say (in the Abstract) that *length* is a Cube (and so of the rest;) yet (in the Concrete) this *Long* thing (or this which is Long) is a Cube"[39] and that therefore the analogy can indeed be shown to hold true.

Such fine distinctions were of course not able to convince his opponents who saw in them almost the return to the worst side of scholasticism[40]. In their opinion, the plainest scriptures supported the Unitarian view, while Trinitarians like Wallis "have no scriptures

[34]Wallis, *The Doctrine of the Blessed Trinity*, p. 10.

[35]Wallis, *The Doctrine of the Blessed Trinity*, p. 12. See also Don Cupitt, *The Doctrine of analogy in the Age of Locke*, in: *Journal of theological Studies* NS 19 (1968), 186–202, 191.

[36]Wallis, *The Doctrine of the Blessed Trinity*, p. 13.

[37]See John Wallis, *An Explication and Vindication of the Athanasian Creed*, p. 42; idem, *A Fourth Letter, Concerning the Sacred Trinity; in Reply to what is Entituled, an Answer, to Dr. Wallis' Three Letters*, London, 1691, p. 18.

[38]W.J. Wallis 23.IX/[3.X].1690, Oxford, Bodleian Library MS Eng. Th. e. 22, f. 16r—printed in: John Wallis, *A Second Letter Concerning the Holy Trinity. Persuant to the former from the same hand; Occasioned by a letter (there inserted) from one unknown*, London, 1691, p. 5; [Anonymous], *Dr. Wallis' Letter Touching the doctrine of the Blessed Trinity, answer'd by his Friend*, 1691, pp. 8–9; [Anonymous], *Observations on the Four Letters of Dr. John Wallis, Concerning the Trinity and the Creed of Athanasius*, London, 1691, pp. 4–5.

[39]Wallis–? 27.IX/[7.X].1690, in: John Wallis, *Second Letter Concerning the Holy Trinity*, p. 9; [Anonymous], *Dr. Wallis' Letter*, pp. 14–15.

[40]See for example [Anonymous], *Observations on the Four Letters*, p. 5: "So it is; when they find themselves distrest, by clear either Argument or Answer, they fly to Metaphysicks, and Terms of the canting Schools. Then

left, but those that are obscure".[41] One author (who appropriately distinguishes himself from the Socinians), suggests that the problem of the approach chosen by Wallis reflects the limits of discursive reason when it comes to comprehending divine mysteries and finds that these limits are reflected in the sciences themselves:

> "Neither is it only in Divine Truths that we find our Consequences uncertain and fallacious, but also in Natural Truths, that lies more within the Compass of our Understanding; and even in that Order and Kind of natural Truths, that are thought of all others, to be most evident and demonstrable, I mean Geometry and Mechanicks. How many have found the *Perpetual Motion* and the *Quadrature of the Circle*, if their Consequences be true; for they generally begin from true Principles, and pursue them with all the Care and Exactness they are able, and conclude with a great Assurance that they have committed no Error, that their Deductions are clear and distinct, and consequently their Conclusion is as infallible as humane Wit can make it: Yet the World is satisfied, that neither the Circle is squared, nor the Perpetual Motion found out; and consequently that humane Ratiocinations, though from true Principles, are very subject to Error".[42]

In fact, Wallis himself was forced to accept that his analogy was far from perfect. But this was, he argued, something which rather reflected the limits of the definitions which we employ—just as the definitions of mathematical concepts in different authors often deviated from one another:

> " 'Tis well known, that a Cone in Euclide doth not signifie just the same as in Apollonius nor a Triangle in Euclide, just the same as in Theodosius, and others, who write of Sphericks: But when we meet with these words in Euclide, we must there understand them as they are defin'd by Euclide; and when in others so as they are defin'd by those others, And so when we speak of Persons in the Deity, we must be so understood as we there define: that is, for somewhat Analogous, but not just the same, with what is meant by, when applyed to Men; and, particularly, not so distinct as to be *three Gods*".[43]

Precisely Wallis' cautious approach using the method of analogy avoided the implication of tritheism[44] which could be found in the writings of Sherlock and which thus played into the hands of the Socinians.[45] Wallis, who nevertheless defended[46] Sherlock, confined his discourse to a single point: that there is no impossibility in the Trinity.[47] And this he saw as being sufficient for his purpose, namely to defeat the antitrinitarian arguments while at the same time retaining the essential mystery of the Christian doctrine.

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come in Abstract, Concrete, Paternity, Personality, and an Infinity of other barbarous and insignificant Words: only to hide clear Truth from Persons, who can be shifted off with obscure and sensless Words; Words which denote nothing that is really existent in Nature, but only the Chimera's of the Metaphysician". The author later suggests that "Tis somewhat surprizing, that a Mathematician should not be more considerate; in giving an Instance belonging to his Profession".

[41] [Anonymous], *Dr. Wallis' Letter*, p. 4.

[42] W.J., *The Third Letter from W.J. to the Reverend Doctor Wallis, Professor of Geometry in Oxford, upon the Subject of two former Letters to him, concerning the Sacred Trinity*, London, 1693. pp. 7–8.

[43] John Wallis, *An Eighth Letter Concerning the Sacred Trinity; Occasioned by some Letters to him on that Subject*, London, 1692, p. 13.

[44] See W.J.–Wallis 32.IX/[3.X].1690, Oxford, Bodleian Library MS Eng. Th. e. 22, f. 15r–15v.

[45] See Cupitt, *Doctrine of analogy*, p. 191.

[46] See Wallis, *An Explication and Vindication of the Athanasian Creed*, p. 42. Cf. Wallis–Elys 7/[17].III.1690/1, Oxford, Bodleian Library MS Eng. Th. e. 22, f. 130r–130v.

[47] See John Wallis, *A Fifth Letter, Concerning the Sacred Trinity; in Answer to what is Entituled, the Arians Vindication of himself against Dr. Wallis' Fourth Letter on the Trinity*, London, 1691, pp. 7–8.

3. Mathematics and calendar reform

That England in the second half of the seventeenth century began to play an increasingly important role in the advances made in mathematics in Europe generally was to a large part due to the efforts of Wallis, who from early on was an active participant, both through publications[48] and correspondence,[49] in the contemporary scientific discussion. In this sense the Savilian professor can be truly seen as a modernizer, but at the same time—and here he reflects the fundamentally historical orientation of many Protestant scholars[50]—he held a deep respect for tradition, as is particularly evident in his critical editions of classical texts such as Ptolemy's *Harmonica*[51] and Archimedes' *Arenarius*,[52] as well as that of the text *De pascha computus*,[53] (incorrectly) ascribed to the church father Cyprian. As he explains in his autobiography, not only in mathematics, but also in other studies "I made it my business to examine things to the bottom; and reduce effects to their first principles and original causes. Thereby the better to understand the true ground of what hath been delivered to us from the Antients, and to make further improvements of it".[54]

As both theologian and mathematician, Wallis demonstrated this historical orientation in no context more consistently than in the late seventeenth-century debates in England on calendar reform, where his opinion precisely on account of his competence on both fields was seen to carry a great deal of weight. In this respect, already in 1684 Wallis had made clear where his priorities lay, when an Oxford almanac appeared in which St. Matthias' day was incorrectly set—no adjustment had been made for that year being a leap year. Soon after its appearance, Wallis wrote[55] to John Fell, who as Bishop of Oxford was responsible for printing the almanac, and while he was cautious not to offend him, he spoke of it being a reasonable presumption[56] that the church should not depart from ancient practice and therefore also from the received rules of Ecclesiastical computation.

The question of introducing the Gregorian calendar into England naturally represented a far greater challenge.[57] While repeated moves to bring about reform had in the past been

[48] Especially his monumental *Opera mathematica*, 3 volumes, Oxford, 1693–1699 (reprint Hildesheim, New York, 1972).

[49] See *The Correspondence of John Wallis (1616–1703)*, ed. Philip Beeley and Christoph J. Scriba, vol. 1, Oxford, 2003.

[50] See Charles Webster, *The Great Instauration. Science, Medicine and Reform 1626–1660*, London, 1975, p. 15.

[51] *Claudii Ptolemaei Harmonicorum libri tres. Ex. Cod. mss. Undecim, nun primum Graece editus. Johannes Wallis*, Oxford, 1682; reprinted in: Wallis, *Opera mathematica* III, Oxford, 1699.

[52] *Archimedis Syracusani arenarius et dimensio circuli. Eutocii Ascalonitae in hanc commentarius. Cum versione et notis Joh. Wallis*, Oxford, 1676; Reprinted in: Wallis, *Opera mathematica* III, Oxford, 1699.

[53] *Cypriani de Pascha computus, (cum notis,) nunc primum ex mss. editus. Johannes Wallis, in: Sancti Caecilii Cypriani Opera recognita et illustrata per Johannem Oxoniensem Episcopum* [John Fell], Oxford, 1682.

[54] Scriba, *Autobiography*, pp. 40–41.

[55] Wallis–Fell ?.1684, Oxford, Bodleian Library MS Tanner 107, f. 50r–61v; MS Savile 56, f. 54r–65v. See also Wallis' *Advertisement concerning St Matthias Day*, MS Savile 56, f. 53v. and Wallis–Blencowe 8/[18].I.1694/5, MS Savile 56, f. 1r–1v.

[56] Wallis–Fell ?.1684, Oxford, Bodleian Library MS Tanner 107, f. 50v; MS Savile 56, f. 54v.

[57] See Wallis–Sloane 27.VII/[6.VIII].1699, London, British Library MS Sloane 4025, f. 320r: "I hear, it hath been under consideration, whether (at this time) to change our Julian Year, for the Gregorian. Of which (upon occasion of a Letter from the Lord ArchBishop of Canterbury) I have signified my thoughts to his Grace; who (if it be desired) will, I presume, impart it if there be occasion". See further Michael Hoskin, *The Reception of*

fought off, the pressure at the end of the century was considerable: not only was the difference between the Gregorian and the Julian calendar about to extend in 1700 from ten to eleven days, but also and more importantly the Protestant German states were themselves about to change.[58] And after the Imperial Diet had unsuccessfully sought to persuade William III to follow its decision, Leibniz had written to the Royal Society to the same end.[59] Added to which the Archbishop of Canterbury, Thomas Tenison, appeared to sympathise with moves for reform.

In part, the debate focussed again on the tradition established by the Council of Nicaea, at which the whole Christian church had adopted the Julian calendar as part of a project to fix a common date for Easter—an issue which had long caused contention and even schism.[60] According to Nicaea, Easter was to be celebrated on the Sunday following the first full moon after the Vernal equinox. From an astronomical point of view this required two things: the determination of the Spring equinox and thus of the exact length of the tropic year, and, secondly, the calculation of the length of time required for the synodic circulation of the moon and the establishment of the days in the year on which the full moon appears. In both these calculations errors had crept in during the course of the centuries which meant that certain demands of the celebration could no longer be fulfilled. Put simply, the Julian calendar was about eleven minutes a year too long, so that, even with the insertion of a leap year every four years, the beginning of Spring, which at the introduction of the calendar under Julius Caesar in 46 BCE had occurred on 25 March, had gradually moved forward: at Nicaea it fell on 21 March and by the seventeenth century it fell a full ten days earlier than this on 11 March.[61] In addition, the employment of the cyclical calculation of the occurrence of the full moons using the 19 year metonic or lunar cycle had in the context of the Julian calendar led to a significant deviation from the actual appearance of the heavens.[62] The Gregorian reform of 1582, drawn up under Christoph Clavius,[63] removed the excessive ten days to bring the calendar back to the same relation to the heavens which it had in 325 and introduced a modified pattern of leap years to keep it there.[64] Additionally, in place of the 19 year lunar cycle a system of so-called epacts was introduced, enabling a more exact calculation of the full moons.

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the Calendar by Other Churches, in: *Gregorian Reform of the Calendar*, ed. G.V. Coyne, S.J., M.A. Hoskin, and O. Pedersen, Vatican City, 1983, pp. 255–264.

[58] See Jürgen Hamel, *Erhard Weigel und die Kalenderreform des Jahres 1700*, in: *Erhard Weigel: 1625 bis 1699. Barocker Erzvater der deutschen Aufklärung*, ed. R.E. Schielicke, K.-D. Herbst, and S. Kratochwil, Thun and Frankfurt am Main, 1999, pp. 135–156, especially pp. 147–149.

[59] See Leibniz–Sloane 20/30.I.1700, London, Royal Society, Letter Book XII, pp. 314–315; *The Conclusion of the Protestant States of the Empire, of the 23rd of Sept. 1699. Concerning the Calendar, Communicated by Mr Houghton F.R.S.*, in: *Philosophical Transactions* No. 260 (January 1700), pp. 459–463; Robert Poole, *"Give us our eleven days!": Calendar Reform in Eighteenth-Century England*, in: *Past & Present* 149 (1995), 95–139, 108.

[60] Poole, *"Give us our eleven days!"*, pp. 105–106.

[61] See Wallis–Barlow 21/[31].XII.1677, Oxford, Bodleian Library, MS Savile 56, f. 6r–52v; f. 7r and f. 9r; Wallis–Blencowe 14/[24].V. 1698, in: *Philosophical Transactions* 240 (May 1698), 185–189, 188.

[62] See Wallis–Blencowe 14/[24].V. 1698, p. 189.

[63] Christoph Clavius, *Romani calendarii a Gregorio XIII pontifice maximo restituti explicatio*, in: *Opera mathematica, V Tomis distributa*, Mainz 1611–1612; volume 5.

[64] Wallis–Barlow 21/[31].XII.1677, f. 7r.

The proposal for introducing the Gregorian reform in England found no favour with Wallis, who suspected "a latent Popish interest"[65] behind it, since it would mean in practice "a kind of tacit submission to the Pope's supremacy, or owning his Authority". Even tacit agreement to papal authority was of course anathema to Anglicans, particularly those with strong Presbyterian leanings like Wallis. The question of calendar reform therefore had at once both political and religious dimensions, as already the case of John Greaves, Seth Ward's predecessor as Savilian professor of astronomy, had made clear. In a letter to William Lloyd, the Bishop of Worcester, Wallis points out that Greaves' proposal[66] to introduce the calendar during the Civil Wars had served to prejudice the King's cause in the eyes of those on the parliamentary side who supposed him "to be too much influenced by Popish Councils".[67] The implication was thereby that this was still very much a relevant issue in 1699.

In putting the case against reform Wallis was able to bring forth numerous arguments. He referred for instance to the "more simple composition"[68] of the Julian calendar in comparison to its Gregorian rival and to the fact that the question of Easter—which was apparently Gregory XIII's primary motivation—concerned only the Ecclesiastical year and could therefore be rectified without affecting the civil year at all.[69] The computation of Easter, as he made clear, did not require that the relation of the civil year to the heavens be as it was at the time of the Nicene Council, for the Vernal equinox was simply reputed to be on 21 March of the Julian calendar, irrespective of the astronomical calendar. The fundamental rule of Nicaea for computing Easter day was thus understood as being that Sunday which falls upon,[70] or next after, the first full moon which happens next after 21 March on the civil calendar: "For we are not to judge, either the Equinox, or the Full moon, according as they happen in the Heavens, or in the Almanacks; but according to the Paschal Tables, fitted to the time of the Nicene Council".[71]

In fact, it was precisely here where Wallis saw the possibility of rectifying the errors in the calculation of Easter. It could, as he explained to the Secretary of the Royal Society, Hans Sloane, be left to the care of the astronomers to calculate the equinoxes and full

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[65] Wallis–Tenison 13/[23].VI.1699, in: *Philosophical Transactions* No. 257 (October 1699), pp. 343–348; p. 345.

[66] See John Greaves, *Reflexions made on the foregoing paper* [sc. Report by the Lord Treasurer Burleigh], in: *Philosophical Transactions* No. 257 (October 1699), pp. 356–359; Poole, *"Give us our eleven days!"*, p. 107.

[67] Wallis–Lloyd 30.VI/[10.VII].1699, in: *Philosophical Transactions* No. 257 (October 1699), pp. 350–354; p. 354. See also Wallis postscript to Wallis–Tenison 13/[23].VI.1699 (dated 31.VIII/[10.IX].1699), in: *Philosophical Transactions* No. 257 (October 1699), pp. 348–349; p. 349.

[68] Wallis–Sloane 11/[22].V.1700, London, Royal Society Early Letters W2, No. 66.

[69] Wallis–Tenison 13/[23].VI.1699, p. 346.

[70] Wallis always maintained that according to the correct understanding of the Easter rule, if the first full moon after the Vernal equinox falls on a Sunday that Sunday is Easter day. He thus corrects what he sees to be a fault in Leibniz's letter to Sloane of 20/30 January 1700—see Wallis–Sloane 11/[22].V.1700: "But, when Mr Leibniz (speaking of the Day of the Ful-moon next after the Vernal Aequinox) says quam sequens proxime dominica dies debet esse Paschalis; he should rather have sayd in quam Incidens, vel proxime sequens, &c. For, if that Full-moon be Sunday; Easter is to be That Sunday; not, the Sunday next After. (There is, I confess, such a mistake in the Rule for Easter, in our Common Prayer Book; but, in the Tables, it is rectified.)". See also Wallis–Fell ?.1684, MS Tanner 107, f. 60v; MS Savile 56, f. 64v; Wallis–Barlow 21/[31].XII.1677, f. 9r.

[71] Wallis–Blencowe 14/[24].V.1698, p. 186.

moons, as they do for the rest of the almanac, thus allowing that the Easter rule be employed as it had been originally formulated at Nicaea.[72]

> "For if, in the Rule for Easter, instead of saying Next after the One and Twentieth of March, it be sayd Next after the Vernal Equinox; the work is done. (And we may be excused the trouble of the Paschal Tables, and the Intricate Perplexities of the Gregorian Epacts.)"[73]

Nevertheless, he also points out that there are no compulsive theological grounds for rectifying the Julian calendar at all, for "the celebration of Easter a Week or Month sooner or later, doth nor influence at all our solemn commemoration of Christ's resurrection".[74] The exact computation of the date is of secondary importance to the religious component of the feast itself.

In allowing that moveable feasts such as Easter be determined astronomically, Wallis took up a standpoint very similar to that of the Protestant states in Germany, which in order to redress the inconvenience to commerce with their Roman Catholic neighbours adopted the Gregorian calendar as their civil year in 1700, but continued to maintain an Ecclesiastical calendar independent from Rome, by having their Easter day calculated by astronomers rather than according to the Gregorian formula.[75] Indeed, Wallis sympathised with the situation of the Protestant states and applauded their solution.[76] He found himself however unable to extend the argument to England, not least on account of the need for a common calendar with that employed in Scotland.[77]

Aside from such pragmatic considerations Wallis found little if anything good in the Gregorian reform. Not only had its introduction "caused a Confusion in Stile throughout Christendome",[78] but also the goal which it sought to achieve, namely the alignment of the civil and the celestial year back to the time of the Nicene CouncilCouncil of Nicaea|) was both arbitrary and flawed.[79] In particular, he writes, "it was never pretended that the Civil year must needs agree (exactly to a minute) with the Celestial"; such is moreover impossible to be had "for the solar year and the sidereal year, differ more from each other, than the Julian from either, which is a middle betwixt them".[80] He notes also the dangers of removing temporal landmarks for our understanding of historical events: "such changes may have a further prospect than Men at first sight are aware of", leaving us at a loss "in History to judge distinctly of dates".[81]

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[72] Wallis–Sloane 11/[22].V.1700.

[73] Wallis–Sloane 6/[16].IX.1699, London, Royal Society, Early Letters W2, No. 74, p. 4. See also Wallis–Lloyd 30.VI/[10.VII].1699, pp. 352–353.

[74] Wallis–Tenison 13/[23].VI.1699, p. 346.

[75] *The Conclusion of the Protestant States*, especially pp. 459–460. This suggestion had originally been made by Kepler and was later taken up by Erhard Weigel. See Max Caspar, *Johannes Kepler*, fourth edition, Stuttgart 1995, pp. 270–271; H.M. Nobis, *The Reaction of Astronomers to the Gregorian Calendar*, in: *Gregorian Reform of the Calendar*, ed. G.V. Coyne, S.J., M.A. Hoskin, and O. Pedersen, Vatican City, 1983, pp. 243–251; Hamel, *Erhard Weigel und die Kalenderreform*, pp. 142–145.

[76] See Wallis–Sloane 11/[22].V.1700.

[77] Wallis–Tenison 13/[23].VI.1699, p. 346; Wallis–Lloyd 30.VI/[10.VII].1699, pp. 350–351.

[78] Wallis–Sloane 11/[22].V.1700. See also Wallis–Tenison 13/[23].VI.1699, p. 344.

[79] See Wallis–Lloyd 30.VI/[10.VII].1699, p. 353; Wallis–Sloane 6/[16].IX.1699, p. 4.

[80] Wallis–Tenison 13/[23].VI.1699, p. 346. See also Wallis–Sloane 6/[16].IX.1699, p. 2.

[81] Wallis–Lloyd 30.VI/[10.VII].1699, p. 351. See also Wallis–Tenison 13/[23].VI.1699, p. 344. Evidence of this aspect is provided Wallis' work on finding the Julian period given the number of the solar and lunar cycles, as

Mathematics and religion coincide in Wallis in his understanding of the importance of historical tradition. Just as he saw the need in mathematics to return to the ancients, to comprehend what results they had achieved, and to make improvements on these, so too the established practice of the church over the centuries was to be upheld, while at the same time allowing for minor adjustments and improvements as the need arose. Both these aspects merged in the question of the calendar and its relation to astronomical calculations. Although he did not acknowledge or simply disregarded the fact that the Julian calendar differs to a greater extent from the astronomical year than the Gregorian, Wallis was nevertheless correct to point out that astronomers necessarily base their calculations employing historical data on the former. And thus even those who subscribe to the Gregorian reform "are fain (or find it advisable) first to Adjust their Calculations to the Julian Year, and thence transfer them to the Gregorian".[82] At a time when the criterium of usefulness provided an important guiding line for scientific achievement, this was an argument which was hard to refute.

well as that of the Roman indictions for any year. See John Wallis, *De periodo Juliana*, in: idem, *Exercitationes tres*, London, 1678; Wallis–Oldenburg 16/[26].XI.1667, *Correspondence of Henry Oldenburg* III, pp. 598–601.

[82] Wallis–Lloyd 30.VI/[10.VII].1699, p. 352. See also Wallis–Tenison 13/[23].VI.1699, p. 347; Wallis–Sloane 6/[16].IX.1699, p. 3.

CHAPTER 24

An Ocean of Truth

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1. In search of ultimate truth

In his old age, shortly before his death, Newton looked back on his life, which had been long and fruitful. He asked himself who he had been and what the significance of his achievements was. He answered in the form of a self-portrait as an (infinitesimally) small boy on the wide beach of a vast ocean. He expressed it in one sentence:

> "I don't know what I may seem to the world, but, as to myself, I seem to have been only like a boy playing on the sea shore, and diverting myself in now and then finding a smoother pebble or a prettier shell than ordinary, whilst the great ocean of truth lay all undiscovered before me".[1]

The modern reader, for whom Newton is the eminent scientist and mathematician, may wonder whether there isn't an element of false modesty in this 'undiscovered ocean'. The harvest hadn't been that lean. Was the *Principia* with its famous theory of gravitation not a pearl of great beauty? And were the optical 'pebbles' and the mathematical 'shells' not fine specimens in Newton's collection? The impression that the 'collector' had made with these beautiful finds had been so great that he had received adulation from all sides in his lifetime.

This veneration for the great man was linked with a certain view of his achievements. For the generations after him in the enlightened eighteenth century he was the founder of the clockwork universe that had been set in motion by God in the beginning, and that had kept itself going ever since. The Enlightenment thinkers were aware that Newton had also dabbled in, for them, objectionable subjects like theology and chronology, but they relegated these activities to the post-*Principia* era of the older Newton, and they never doubted that Newton had kept science and religion strictly separate.

They considered him as the man who not only had disclosed the fundamental laws of the cosmos, but who in doing so had demonstrated what can be achieved by the human intellect. Many members of the intellectual elite had become convinced that the Newtonian method had opened the way to a splendid future for mankind. From this perspective he became the hero of the Enlightenment. Modern research by historians (of science) like Westfall, Dobbs, McGuire, Force, Snobelen and many others who have made the Newton manuscripts accessible, has shown this view to be totally wrong. The picture of the two Newtons that had been created by the Enlightenment—the young physicist genius and the old, declining theologian—has been relegated to the realm of fantasy.[2] Newton's conception of God was very different from the deistic view, and he was not in the least interested in isolated knowledge of the natural world as an aim in itself. His concern was the all-embracing truth about God, history and nature; or rather, he was seeking knowledge of the one true God and the one truth of His self-revelation and His acting in nature and history. It is precisely this religious aspect that links the seemingly disparate areas in which he was active.

The historical truth of many stories with a large dose of heroism has been equally doubted. This includes Newton's self-portrait as the playing boy, which for many historians is no more than an amusing anecdote that was meant to show how modest the great

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[1] Fara, *Newton*, pp. 206, 294 n.28; cf. Brewster, *Memoirs*, II, pp. 407–408; I, p. 462; Westfall, *Never at rest*, p. 863; Dobbs, *Janus faces*, p. 12.

[2] Castillejo, *Exp. force*, pp. 15, 78; Force/Popkin, *Newton and religion*, p.x; Ramati, *Hidden truth*, p. 418ff; cf. Gjertsen, *Newton's success*, pp. 23–41.

Fig. 1. Isaac Newton. Science Photo Library.

man was. There are actually many indications that modesty was not Newton's forte. And yet it seems to me that his deepest feelings are appositely expressed in this simile. How could someone who seeks to see God through nature and history, who sees the biblical prophecies are decipherable codes of God's works, who is convinced that both morality and religion, and science are time and again perverted and who therefore attempts to find the true original doctrine, how could somebody like that not be aware of his limitations, even though he was convinced that he was one of the happy few, chosen by God to rediscover the original teaching. It is not surprising that somebody with such a heavy burden on his shoulders should have to recognize at the end of his life that the great ocean of truth still lies undiscovered before him.[3]

2. Natural philosophy and mathematical principles

More than anyone else, Dobbs has emphasised that "the unity and consistency of Newton's thinking lay in his overwhelming religious concern to establish the manner of God's acting in the world".[4] His investigation of the natural world also served this purpose. When the *Principia* and the *Opticks* are considered only as 'science', the religious–metaphysical

[3]Fara, "Tales of genius", *Newton*, pp. 202–213; Force, *Sleeping arg.*, pp. 118–119; Kochavi, *One prophet*, pp. 116–117; Ramati, *Hidden truth*, p. 417 and 417 n.2; Hutton, *Seven trumpets*, p. 167; cf. Dobbs, *Janus faces*, p. 12; Westfall, *Never at rest*, pp. 862–863.

[4]Dobbs, *Janus faces*, p. 12; cf. Dobbs, *Unity*, p. 108.

background and therefore the historical context is lost sight of. Newton and his contemporaries were not engaged in doing 'science' in the modern sense. The domain of natural philosophy was the investigation of natural bodies, their forces, natures, actions and interactions, the religious aspect of which was of fundamental importance. In the words of Schuster:

> "Every system of natural philosophy (. . .) purported to describe and explain the entire universe and the relation of that universe to God, however conceived. The enterprise also involved, explicitly, a concern with the place of human beings and society in that universe".[5]

The essential difference with modern science is that natural philosophy considers God and His creation. This orientation towards God was taken for granted and there was no need for an author to state it explicitly. However, if a natural philosopher had an unorthodox view of God and His relation with the creation, he would have to define his position clearly and defend it against attacks. There was strong opposition to supposed atheist tendencies in prevailing systems. The rise of the mechanist natural philosophy in the seventeenth century occasioned fierce polemics between representatives of the old Aristotelianism and those of the new mechanist philosophies (Descartes amongst others), while Newton and his eighteenth-century followers opposed Cartesianism. Newton in his turn was attacked by Leibniz and others, who rejected his philosophy to a large extent on religious grounds. They felt that Newton had endowed absolute space with the properties of God, and that a God who had to interfere with His creation had created an imperfect world, which implied that Newton's God was Himself imperfect.[6]

Newton defended his cosmology against these accusations in the Scholium Generale included in the second edition of the *Principia* (1713), in which he gave an account of God's properties and the way He governs the cosmos. In this context he remarked that "to treat of God from phenomena is certainly a part of natural philosophy".[7] The debate was continued in 1715 by Leibniz and Clarke, who propounded the Newtonian standpoint, and ended only with Leibniz's death in 1716. The controversy shows how strongly cosmological problems and questions about God's relation to the cosmos were interconnected in the natural philosophies of Newton and Leibniz.[8]

That Newton thought that the *Principia* was suitable to be used for apologetic purposes is apparent from his correspondence with Richard Bentley (1692), who wanted to use the theory of gravitation as a weapon in the struggle against atheists and deists, and who asked the author of the *Principia* for advice. Newton's reply begins with the well-known sentence:

> "When I wrote my Treatise about our Systeme, I had an eye upon such Principles as might work w[i]th considering men for the beleife of a Deity & nothing can rejoyce me more than to find it usefull for that purpose".[9]

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[5]Cunningham, *How the Principia*, pp. 380–381; McGuire/Rattansi, *Pipes*, 1966, p. 138; Gabbey, *Active powers*, p. 329ff; Schuster, *Scientific revolution*, pp. 224–225; cf. Reif, *Textbook trad.*, pp. 17–32; Dobbs, *Janus faces*, p. 89; cf. Snobelen, *God of gods*, p. 202ff.

[6]Cunningham, *How the Principia*, pp. 382, 384; Force, *Nature*, p. 270.

[7]Newton, *Principia*, third ed., transl. Cohen, p. 943.

[8]Alexander, *Leibniz–Clarke*; Meli, *Leibniz–Clarke*, pp. 460–463.

[9]Turnbull, *Corr.* III, pp. 233–256, in part. p. 233; Cohen, *Papers*, pp. 271–394, in part. p. 280; cf. "Therefore Newton's excellent treatise will stand as a mighty fortress against the attacks of atheists; nowhere else will you

Although the foundations of Newton's main work had been laid in 1665–1666, the general law of gravitation was still far away at that time. Many problems had to be overcome before the *Principia* could be published in 1687, a process in which an inspiring visit by Edmond Halley in 1684 and his involvement in the following years had been most helpful. After the example of Euclid's *Elements* the work is made up of definitions, axioms and propositions.[10] The definitions concern matter, motion and forces. The axioms comprise three laws of motion, from which propositions can be derived on all kinds of motions under the influence of forces, both in vacuum (book I) and in resisting media (book II). Together they form the actual mathematical principles of the mathematical physics that Newton develops in books I and II. In book III he applies these principles to celestial phenomena, and he derives the general law of gravitation which explains planetary motion, the fall, the tides and various other phenomena. It seems therefore as if there is a certain dichotomy in the *Principia*, which is more or less confirmed by Newton's critics, who highly esteemed the mathematical pyrotechnics of book I and II, but who refused to accept the physical reality of gravitation presented in book III, because it lacked a mechanist explanation.

The American historian of science I.B. Cohen is of the opinion that Newton's method is characterized by a particular revolutionary 'Newtonian style', which is claimed to consist in a sharp separation between the domains of mathematical fiction and physical reality. In the first area (books I and II), within the mathematical systems, or 'mathematical constructs' as Cohen calls them, propositions are derived that apply to points and bodies under the influence of mathematical forces. The proofs are furnished with the help of lines, tangents and curves.[11] Cohen recognizes that Newton's constructs are not entirely arbitrary, as the choice of his problems of motion and the way they are worked out show that in the mathematical domain Newton always had the real world in view as a physical horizon. To forestall his critics Newton emphasises again and again that he treats forces only mathematically, without making any claims about their physical character. In the Preface of the *Principia* he points out that the gravitation of Book III is meant as an illustration of how the propositions of Books I and II can be applied. In a critical reaction to Cohen's 'Newtonian style' Dobbs points out that gravitation is a cornerstone of the *Principia* rather than just an example of how the principles of the first two books can be applied.[12] Newton's contemporaries recognised this as well. John Locke, for example, writes:

> "There are fundamental truths that lie at the bottom, the basis upon which a great many others rest, and in which they have their consistency. These (...) give light and evidence to other things, that without them could not be seen or known. Such is that admirable discovery of Mr. Newton, that all bodies gravitate to one another which may be counted as the basis of natural philosophy".[13]

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find more effective ammunition against that impious crowd. This was understood long ago (...) by (...) Richard Bentley (...)", Newton, *Principia*, Cotes, preface second ed., transl. Cohen, p. 398.

[10] Westfall, *Never at rest*, pp. 402–468; Roche, *Principia*, pp. 43–61; Christianson, *Presence*, pp. 281–331; Gjertsen, *Handbook*, pp. 455–502; Dobbs/Jacob, *Newton*, pp. 38, 45; Newton, *Principia*, transl. Cohen, pp. 416–417 (axioms).

[11] Cohen, *Newt. Revolution*, pp. 52–154; Newton, *Principia*, transl. Cohen, par. 3.5, pp. 61–64; cf. Whiteside, *Math. Papers*, VIII, p. 455, 455n.40.

[12] Newton, *Principia*, transl. Cohen, p. 382; Dobbs/Jacob, *Newton*, pp. 38, 45.

[13] Aarslef, *State*, p. 104; comparison with Locke's second example shows the importance of gravity for him: "Our Saviour's great rule, that 'we should love our neighbour as ourselves', is such a fundamental truth for the regulating human society", Aarslef, *State*, p. 104.

Bechler rejects Cohen's dichotomy, as it is based on the idea that science has an autonomous core that is free of hypotheses, within a surrounding physical interpretation that can only be hypothetical and therefore does not belong to the essence of science. On Cohen's assumption Newton would have freely used the concept of centripetal force, without actually believing in a physical action at a distance. Bechler claims that Newton actually had an entirely different philosophy of science. He was concerned about truth and certainty. His famous statement "Hypotheses non fingo" has nothing to do with a retreat into a hypotheses-free zone of mathematical constructions, but it criticises strongly the hypothetico-deductive method of Descartes and others, because their hypotheses were not deduced from the phenomena and therefore uncertain and perhaps untrue.

Another point concerns Cohen's 'physical horizon'. Newton supposedly adjusted and refined his models into more and more complex mathematical models, while keeping an eye on the physical world. According to Bechler he rather transformed his overly simplistic mathematical models into a physical theory. Newton starts with the simple mathematical model of a mass point that moves under the influence of a central force, and then he makes the transition to a physical model. The centre of force becomes a material source and the mass point becomes an extended mass. It follows from the laws of motion that the central mass of the system cannot be stationary if a direct and immediate action of the central body is assumed. This is exactly what Newton does. Strictly speaking his planetary system is therefore not Keplerian, and this is not an observable fact, but something that is necessitated by the physical theory. Because the direct action on the planets of an attractive force residing in the sun is an hypothesis, the law of gravitation derived from it is also hypothetical in character, as was noticed by Roger Cotes, the editor of the second edition of the *Principia*.[14]

This raises the question of why Newton thought his theory of gravitation superior to that of his opponents. This point is closely linked with the place in Newton's system of God, who acts on His creation through gravitation. Hughes has pointed out that we encounter God far less in Newton's *Principia* than in Descartes' *Principia Philosophiae.* Newton apparently separates starting-points from final goals, i.e. physics, which takes its starting point in the world of phenomena, from metaphysics, which is the final aim, but this separation should not mislead us. For Newton the two form one single whole. Physics deals with bodies in space and time, but space and time exist because God exists and has His abode in them and is in this way permanently linked with His creation. This infinite God and Creator dwells in an infinite world that is forced by His infinite power to obey strict and exact laws. In his extensive discussion of space and time Newton implicitly provides the logical and philosophical foundation of his mathematics.[15] Just as for Galileo, the book of nature is for Newton written in mathematical language. His natural philosophy is therefore founded on mathematical principles, which are the tools to discover the laws infixed by God in the creation and in this way to refute anybody who excludes God from the cosmos. The theory of gravitation could explain a large number of phenomena on the basis of just a

[14]Cohen, *Newt. Revolution*, p. 113; Turnbull, *Corr.* II, p. 440 and V, p. 392; Bechler, *Introduction*, pp. 2–12; Bechler, *Newton's physics*, pp. 283–286; Newton, *Principia*, transl. Cohen, p. 943 ("I do not 'feign' hypotheses"); cf. Koyré, *Newt. Studies*, pp. 273–282.

[15]Hughes, *Phil. persp.*, pp. 3–4; cf. Markley, *Fallen Languages*, p. 139; Bechler, *Introduction*, pp. 12–14; McGuire, *Tradition*, pp. 15–16.

few principles. It seems that for Newton this was a decisive principle of harmony to claim the truth of the theory, the more so as a mechanist explanation seemed impossible without divine intervention.[16]

3. Anticartesianism: passive matter and active principles

In the second edition of the *Principia* Newton defended himself against both physical and metaphysical accusations by Leibniz and others, but a quarter of a century earlier he had confronted Descartes in his book. Even in the title he seems to oppose the (mathematical) *principles* of his (natural) *philosophy* to those of Descartes.[17]

As a student in the early 1660s Newton had become acquainted with the works of the French philosopher and he had been converted permanently to the idea that natural phenomena have to be explained in terms of moving particles. The first traces of this view can be found in the *Questiones*.[18] A growing number of critical questions in the following years shows that Newton was influenced by the well-known Cambridge Platonists Ralph Cudworth and Henri More. Just like them, Newton in the end rejected the separation of mind and matter advocated by Descartes, Hobbes and others, who accepted only material causes as explanations of phenomena in the physical world, a view that seemed to have all kinds of atheist and materialistic implications.

Before the publication of the *Principia*, Newton already attacked Descartes in *De Gravitatione*, in particular his conceptions of space, place and body. He rejected the strictly mechanistic type of explanation according to which phenomena of motion are purely the effect of contact between passive particles of matter. By denying the existence of a mechanically active medium that fills all space, he rejected a cornerstone of Cartesianism, the identification of space and extension.[19] This had a deeper religious background:

> "If we say with Descartes that extension is body, do we not manifestly offer a path to Atheism, both because extension is not created but has existed eternally, and because we have an absolute idea of it without any relationship to God, and so in some circumstances it would be possible for us to conceive of extension while imagining the non-existence of God?"[20]

At first sight this accusation directed at Descartes seems undeserved. In his *Principia Philosophiae* he had explicitly proved the existence of God. However, the French philosopher wanted to derive natural phenomena only from his principles on matter and motion—principles that he had actually partially adopted for religious reasons. From these principles

[16] Cunningham, *How the Principia*, p. 380; Mandelbrote, *Duty*, p. 301; Leshem, *Newton on math.*, pp. 20, 31.

[17] Newton, *Principia*, transl. Cohen, pp. 43–49; cf. Whiteside, *Prehistory*, p. 34.

[18] Newton, ed. McGuire, *Certain phil.*, pp. 317–320.

[19] Westfall, *Never at rest*, pp. 301–302; Dobbs, *Janus faces*, pp. 144–146; in Dobbs' opinion (p. 144) *De gravitatione* was written as late as 1684, but others think that Newton completed the work already in the late 1660s; Newton, ed. Hall/Hall, *Sc. papers*, pp. 90–114, 122–150, cf. 76–85; Turnbull, *Corr.* III, p. 244; Cohen, *Papers*, p. 310.

[20] Newton, ed. Hall/Hall, *Sc. papers*, pp. 142–143, 144; in Newton's opinion extension was only one of the properties of matter, Newton, *Principia*, transl. Cohen, pp. 795–796; Hall/Hall, *Sc. papers*, pp. 138, 141–142; Newton, *Opticks*, ed. 1952, pp. 400–403; Dobbs, *Janus faces*, p. 114.

he deduced the present state of the entire cosmos, so that God was actually made superfluous in the Cartesian system. This consequence was unacceptable for Newton, as is clear from the Scholium Generale of the *Principia*:

> "This most elegant system of the sun, planets, and comets could not have arisen without the design and dominion of an intelligent and powerful being. And if the fixed stars are the centers of similar systems, they will all be constructed according to a similar design and subject to the dominion of *One* ..."[21]

In his active involvement with the creation God is sovereign and free. When the claim that the total quantity of motion—the sum total of all products of the amount of matter and velocity of all particles—in the universe is constant is derived from God's immutability, as was done by Descartes, God's freedom and power become limited. Therefore feigning these kinds of hypotheses has to be strongly rejected. It is the natural philosopher's task to discover what God has actually accomplished in the creation, and not what He should have or might have accomplished because our thinking prescribes this to Him. In other words, Newton is not satisfied with anything but the truth.[22]

In the *Principia*, mathematics appears to be a welcome ally of religion when Newton confronts Descartes' natural philosophy with the phenomena. Descartes assumed that the planets are dragged around the sun in a vortex of subtle fluid. In book II Newton calculated the ratio between the period of the matter in such a vortex and the distance to the centre. The result is inconsistent with the ratio described in Kepler's third law. It seems that the second book of the *Principia* plays a key role in Newton's efforts to deliver the final blow to the Cartesian cosmology.[23] The successful overthrow of Cartesianism must have given Newton a great deal of satisfaction, as it seemed to have made the identification of matter and extension impossible.

Newton was concerned about other questions, in particular about the phenomenon of life. Already in the late 1660s he became convinced that a mechanist explanation of vital processes was doomed to fail. The great richness of forms was a problem in itself: for him it was inconceivable how the mathematical properties of particles all moving in the same way could produce the great richness of forms in the living world. His mature conclusion was clearly formulated in the third edition of the *Principia* (1726):

> "No variation in things arises from blind metaphysical necessity, which must be the same always and everywhere. All the diversity of created things, each in its place and time, could only have arisen from the ideas and the will of a necessarily existing being".[24]

This leaves the question of how God's interaction with the created world comes about. For an Arian theologian as Newton was, this was an immense difficulty. While according to the orthodox doctrine of the Trinity Father, Son and Holy Spirit are coequal, in Arianism God the Father is a totally transcendent deity, who is entirely outside the material world. The Son is a lower being, created by God, the first and most important of all creatures.

[21] Newton, *Principia*, transl. Cohen, p. 940; cf. "(...) I do not think explicable by mere natural causes but am forced to ascribe it to ye counsel and contrivance of a voluntary Agent (...)", Newton to Bentley, ed. Turnbull, *Corr.* III, p. 234; Cohen, *Papers*, p. 282.

[22] Newton, *Principia*, Scholium generale, transl. Cohen, pp. 393, 943.

[23] Newton, *Principia*, transl. Cohen, pp. 187–188, 779–790; McGuire, *Tradition*, p. 190.

[24] Newton, *Principia*, third ed., transl. Cohen, p. 942.

He is not only an intermediary between God and His creatures in soteriology, but also in cosmology. God the Father communicates with his creatures through this Arian Christ. As a result, in the 1670s Newton became more and more convinced that God as the primary cause of everything does not do directly what He can do through the intermediary of secondary causes, as this agrees better with his essence.[25] If passive particles of matter cannot explain vital processes and the richness of forms in nature, there should exist, by analogy with Christ as mediator, some intermediary divine agent that produces the visible order and organisation of life.

The spiritual dimension of this problem led him to alchemy, which was the area where natural and divine principles met. At the end of the 1660s Newton started an thorough investigation to find an answer to the insistent problems with which the mechanist philosophy of nature presented him. From the start he was keen to find evidence for the existence of a vegetative principle active in the cosmos. Early on, probably in 1669, he advocated in a number of 'Propositions' the existence of such a vital agent, universal in its operation and diffused through all things. First he called this agent mercurial spirit, then fermental virtue or vegetable spirit and finally (in the *Opticks*) force of fermentation. This vital agent is basically anti-mechanistic, and it does not work by pressure or impact of particles, but in a way that suggests conscious acting. Newton was thinking of an uninterrupted purposive process of disorganisation to formless matter and of reorganisation to new forms of organisation. The vital agent was apparently the cause of seemingly spontaneous processes like fermentation, putrefaction, generation and vegetation, the last term denoting all processes having to do with life and growth. God used this agent in order to carry out his will concerning the organisation of matter in the cosmos.

This early conviction of the existence of a universal animating spirit was soon reinforced by the views of the Stoics, who postulated the existence of a world ether, a cosmic pneuma or spirit, an all-pervasive warm breath of life, consisting of air and fire. At the top of the ladder of being there is the deity, the most creative and subtle form of pneuma, entirely creative fire, from whom all things receive their form and essence. In an alchemist treatise entitled *Of natures obvious laws & processes in vegetation* (ca. 1672) Newton describes a vitalistic ether, that replaces the mechanistic ether from the *Questiones*. The earth is a living being that breathes in and out. The etherial breath regulates vegetation, the vital processes that are the same for plant, animal and mineral. The mechanical (the mechanic action of particles of matter) and the vegetable (vegetation as divinely guided activity) are sharply contrasted, a pair of concepts that is later denoted as passive vs. active.[26]

Newton's own optical investigations, started in 1666, may have played a role here. For him, as a seeker after the truth about God and the cosmos, light did not just belong to physics but also to metaphysics. Both in Christianity and in Neoplatonism, light bore the hallmark of the divine. In addition there is a strong parallelism between the fire of the Stoics and the light of the Neoplatonists as cosmic vegetative principles.

In various writings it is apparent that Newton's view of natural processes was strongly influenced by these traditions. In *An hypothesis explaining the properties of light* (1675), for example, he suggests that all bodies are composed of certain etherial spirits or vapours,

[25] McGuire, *Tradition*, p. 227n.156; Dobbs/Jacob, *Newton*, p. 53; Henry, *Pray*, p. 133.
[26] Dobbs, *Janus faces*, pp. 29–33; Dobbs/Jacob, *Newton*, pp. 25–30.

ether and light, that are condensed to a different extent and in different forms. The force to produce this formation and coagulation was initiated by God when he ordered his creatures to increase and multiply. One of these spirits is the ether. A second, that is dispersed through the first one, is light. These two spirits, ether and light, interact continually, so that 'nature is a perpetual worker'. In Newton's later work, when his ideas have matured, we find similar statements. In the 'Queries' of the *Opticks* there are many remarks about interactions between bodies, light and ether.[27]

Especially in the alchemist tradition Newton found a close relation between light and God's activity in the cosmos, and there is a strong emphasis there on active principles that act on passive matter. As Newton was convinced of the 'analogy of nature', he concluded that not only in alchemy, but elsewhere as well, active principles were operative. Newton's abhorrence of materialism, atheism and deism explains his unremitting efforts in the area of alchemy, in search of a proof of providential divine acting.[28]

Especially Dobbs has shown in her publications how important alchemy was for Newton's views of the interaction of God and the material world through active principles:

> "Alchemy offered Newton access to spiritualistic guiding principles in the natural world and provided hints for the experimental examination of them and of non-mechanical causation in general. An effective demonstration of such principles at work in the organization of matter would have been a demonstration of divine activity in nature. Through the argument of design, a proof of divine interaction with the material world would not only have been an experimental proof of the existence of God (and thus a resounding blow struck against both skepticism and atheism) but also a decisive refutation of the materialistic and deistic [and even atheistic, d.P.] implications of Cartesian mechanism".[29]

This conception of active forces operative in nature was first applied by Newton to celestial phenomena as described in the theory of gravitation of the *Principia*. Although in the *Principia* Newton seems to have little interest in the cause of gravitation, he was actually intensively engaged with the problem. Far from being a positivist, here as elsewhere he was a seeker after truth.[30]

As is apparent from the *Questiones*, in 1664 Newton still accepted the Cartesian idea of an etherial medium that caused gravity on earth through contact. The introduction of the vitalist ether in 1672, mentioned above, was a radical correction of Descartes' mechanist system in general terms, but Newton continued to explain gravity mechanistically. In 1675 the mechanist and the vitalist action of the ether were even more closely linked and the action was extended to the entire solar system. After having proposed a few more variants on this mechanist theme, he rejected, according to Dobbs around 1684/85, a mechanist explanation, as it was—as we saw earlier on—inconsistent with the observed celestial motions, which obey Kepler's laws. In the *Principia* Newton did have a mathematical law to calculate the effects of gravitation, but he had no physical explanation.

In 1690 Newton seems to have expressed his sympathy with the new mechanist explanation proposed by Nicolas Fatio, a young Swiss mathematician. As appears from his later

[27] Turnbull, *Corr.* I, p. 366; Dobbs, *Janus faces*, pp. 29–33; Newton, *Opticks*, 4th ed., pp. 348ff, 374 (Qu 18ff, 30).

[28] Dobbs, *Janus faces*, pp. 4–5, 52; Dobbs/Jacob, *Newton*, p. 51; Newton, *Principia*, transl. Cohen, p. 794; Newton, *Opticks*, 4th ed., pp. 376, 397; Hall/Hall, *Sc. papers*, p. 307; McGuire, *Force*, pp. 193, 198.

[29] Dobbs, *Unity*, p. 110; cf. Bono, *Paracelsus*, p. 75.

[30] Dobbs, *Janus faces*, pp. 92, 146, 149–150, 169, 185ff, in part. p. 188.

report, Fatio was proud of Newton's interest, but it is remarkable that he added the following observation: "…he would often seem to incline to think that gravity had its foundation only in the arbitrary will of God".[31] As Newton was convinced of the 'analogy of nature' he favoured the notion of *active principles*, actually divine principles that mediated between God and the world. Not only in fermentation, heat, electricity, stimulus conduction and magnetism, but also in gravitation, active principles are operative, spiritual agents through which God interacts with the material world. It is precisely to the fact that Newton supplemented the narrow mechanist categories with other principles derived from alchemy, Stoicism and Neoplatonism, which were linked with his theological views, that Newton owes his lasting fame as scientist.[32]

4. Prisca theologia and prisca sapientia

God's involvement with His creation not only concerns the material world, according to Newton, but also the moral world. Alchemy recounts the story of God's lasting involvement with the material world through active principles, and history shows us God's providential acting in the moral world. Just like many Renaissance authors, Newton was convinced that mankind had lapsed from the pristine Noachian religion, so that not only religion, but also science was corrupted, as without true religion no right conception of the Creator and therefore of His creation is possible. From the religious perspective that we know God only from His works, Newton made a thorough study not only of alchemy, but also of (church) history, biblical prophesy and chronology in order to gain a better understanding of His providential care.

These prisca theologia and prisca sapientia are treated extensively by Newton in his manuscript *Theologiae gentilis origines philosophicae*.[33] Originally all nations knew the same religion, which consisted in the precepts of the sons of Noah. The two pillars of this religion were the love of God and the love of one's neighbour. This original religion was handed down to Abraham, Isaac and Jacob. Later Moses became acquainted with it, and he passed it on to the people of Israel. The evangelists added on to it the doctrine that Jesus was the Christ, whose coming had been foretold by the prophets of Israel. After His resurrection He instructs His disciples to preach conversion and forgiveness of sins. For Newton sin, as he writes in the *Irenicum*, is the transgression of the fundamental laws that together, for Jesus, constitute the great commandment of the law, the love of God

[31] van Lunteren, *Fatio*, p. 56; Dobbs, *Janus faces*, pp. 188–189.

[32] Newton, *Principia*, Rule III, transl. Cohen, p. 795 ("…nor should we depart from the analogy of nature, since nature is always simple and ever consonant with itself"); cf. Newton, *Opticks*, 4th ed., pp. 376, 379; McGuire, *Tradition*, pp. 190–191; Dobbs, *Newton's alch… act. princ. of grav*, pp. 53–74; Dobbs, *Janus faces* pp. 92–121, 130–150, 185–212, 218–230; Dobbs/Jacob, *Newton*, pp. 30, 34–37, 45–56; Leshem, *Newton on math.*, pp. 88–92,99ff; Turnbull, *Corr.* III, pp. 253–254; Cohen, *Papers*, pp. 302–303; Henry, *Pray*, p. 141 (contra Cohen, Hall and Koyré): "[Newton] believed gravity to be a superadded inherent property of body which was capable of acting at distance", cf. pp. 126, 145n.28); Koyré, *Newt. Studies*, p. 149 (" It is well known that Newton did not believe gravity to be an 'innate, essential and inherent property of matter' ").

[33] See a.o. Castillejo, *Exp. force*, p. 57ff; Manuel, *Religion*, pp. 53–79; Westfall, *Is. Newton's Theol. Gent.*, pp. 23–30; Ramati, *Hidden truth*, pp. 418–422; McGuire/Rattansi, *Pipes*; Goldish, *Judaism*, pp. 39–55; Goldish, *Of the Church*, pp. 145–154; Knoespel, *Interpr. strateg.*, pp. 179–202.

and the love of one's neighbour. The true religion was not limited to Israel. The Greek Pythagoras, for example, became acquainted with it during his travels in eastern countries, and he passed it on to his disciples. As a result other peoples learned about it. Therefore this pristine Noachian religion can be called the Moral Law of all nations. In the course of time this religion became involved in a cycle of corruption and restoration, even in the Christian era. After Jesus had purified it again, a new force of decay crept into the true religion when in the fourth century the reprehensible dogma of the trinity entered the church thanks to Athanasius. As we saw, Newton sided with his adversary Arius. The decay of the true religion became visible in the rise of tyrant governments. Tyranny was for Newton a pernicious characteristic of the Church of Rome with its papal hierarchy, a church that had been corrupted by the doctrine of the trinity anyway, just like the Church of England. Not surprisingly, Newton was fiercely anti-Roman-Catholic.[34]

De prisca theologia had also been corrupted by the Cabbala, by Platonism and by Gnosticism, because these schools had introduced metaphysics into theology, which had perverted the original Christianity, as Holy Scripture has not been given to man to teach it within metaphysics, but in order to teach him the morality of the double commandment of love. In addition these metaphysical schools assume creation by emanation instead of creatio ex nihilo, so that the supreme power of the omnipotent Creator is undermined. Emanation myths degraded God by creating a throng of co-substantial gods, so that the original monotheism was changed into polytheism. God, however, is one and indivisible. Divisibility is pre-eminently a property of matter. Only monotheism is the true fountainhead of good science, which after all aims at finding the unifying cause of all natural phenomena. The law of simplicity applies both to the interpretation of Holy Scripture and to the book of nature; we saw already that 'nature is very consonant and conformable to her self'.[35] An incorrect view of the Creator and the creation not only corrupts religion in its foundations, but it also corrupts science. Products of this corruption, according to Newton, are the geocentric worldview of antiquity, and the contemporary Cartesian philosophy that tries to explain all processes mechanistically. A consequence of this approach is that not only true religion, but also true science is to be found in a distant past. In concrete terms this means that Newton was convinced that the true nature of the creation, the structure of the cosmos, was fully known to Pythagoras and other thinkers in antiquity. In his view, the Pythagorean philosophy was the first and the most authentic representative of the original prisca theologia and sapientia, as this philosophy knew about atoms and gravitation, at least according to Newton. This means that both heliocentrism and the law of gravitation, assumed to be explicable in non-mechanistic terms, are to be found in Pythagoras and other ancient philosophers. Implicitly this shows of course that Newton viewed himself as the one who had exposed the (Cartesian) corruption of science and who had revealed the original true science.[36]

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[34] McLachlan, *Theol. manuscr.*, pp. 28, 31, 48, 52–53, 129–141; Iliffe, *Anti-cathol.*, pp. 97–119.

[35] Coudert, *Rosicr. Enl.*, pp. 31–32, 35–36; Goldish, *Kabbalah*, pp. 89–90, 92–93, 95–96; McLachlan, *Theol. manuscr.*, pp. 49–50; Castillejo, *Exp. force*, pp. 65ff; Manuel, *Religion*, pp. 48–49; Newton, *Opticks*, 4th ed., p. 476 (cf. note 32).

[36] Coudert, *Rosicr. Enl.*, p. 36; McGuire/Rattansi, *Pipes*, pp. 198–199; Ramati, *Hidden truth*, p. 432; Leshem, *Newton on math.*, p. 153.

In his published works Newton refers only sparingly to ancient writers, but in his manuscripts such references are common. A well-known example is a treatise from 1686 that describes the planetary system in a popular way. It was originally part of the *Principia*, but Newton had replaced it by a more mathematical treatment in book III. It was published shortly after his death in 1728 under the title *De systemate mundi*. This work begins as follows:

> "The most ancient opinion of the Philosophers was that the fixed stars stood motionless in the highest parts of the world, and that the planets revolved about the Sun beneath these stars; that the Earth likewise is moved in an annual course, as well as with a daily motion about its own axis, and that the Sun or heart of the Universe rests quietly at the centre of all things. For this was the belief of Philolaus, of Aristarchus of Samos, of Plato in his riper years, of the sect of the Pythagoreans, and (more ancient than these) of Anaximander and of that most sage king of the Romans, Numa Pompilius (...) Afterwards Anaxagoras, Democritus and several others taught that the Earth stands unmoved in the middle of the world".[37]

After the publication of the Principia Newton wrote a number of notes in which he extensively discussed the knowledge the ancients had of gravitation and questions linked with it. They were probably meant to be included in the second edition as scholia with the propositions IV-IX of book III. They were never published, perhaps for fear of criticism. Newton, however, did give them to the mathematician David Gregory, who used them in the preface of his *Astronomiae physicae & geometricae elementa*.[38]

The manuscript is known under the name 'Classical Scholia' and contains a detailed collection of physico-astronomical conjectures derived from a great variety of ancient sources. In one of the scholia there is a passage in which Newton 'proves' that the law of gravitation can be derived from the well-known relation between proportions of string lengths and harmonic sounds. He relates the story of the experiments Pythagoras performed in the blacksmith's shop and of the musical scale, and he concludes:

> "Pythagoras afterwards applied the proportion he had thus found by experiments, to the heavens, and from thence he learn'd the harmony of the spheres. And by comparing these weights of the planets, and the intervals of the tones produced by the weights, with the interval of the spheres; and lastly, the lengths of strings with the distances of the planets from the center of the orbs; he understood, as it were by the harmony of the heavens, that the gravity of the planets (according to whose measures the planets move) were reciprocally as the squares of their distance from the sun".[39]

Not only did the ancient writers mentioned know the correct cosmological system, but according to Newton they were also convinced that for the existence of matter and its motion a deity, however interpreted, was necessary. The question that Newton asks in a draft variant to query 23 of the *Opticks* (Latin ed. 1706) he answers as follows:

> "The ancient Philosophers who held Atoms & Vacu[u]m attributed gravity to Atoms without telling us the means unless perhaps in figures: as by calling God Harmony & representing him and matter by the God Pan & his Pipe, or by calling the Sun the prison of Jupiter because he keeps

[37] Casini, *Scholia*, pp. 1, 20n.1; Newton, *The System of the World*, in Newton, *Principia*, transl. Motte/Cajori, p. 549.

[38] Casini, *Scholia*, pp. 2, 47–58.

[39] Casini, *Pyth. myth*, pp. 196–198; Casini, *Scholia*, pp. 9, 32–33, 57–58; Gouk, *Harm. Roots*, pp. 120–124; McGuire/Rattansi, *Pipes*, pp. 115–117; Leshem, *Newton on math.*, pp. 102, 108, 153, 186.

Fig. 2.

> the Planets in their Orbs. Whence it seems to have been an ancient opinion that matter depends upon a Deity for its laws of motion as well as for its existence".[40]

In the Scholium Generale of the *Principia* Newton speaks of the ubiquity and omnipotence of God, the Pantocrator, and in a note he adds that this was also the view of the ancients, Pythagoras, Thales, Anaxagoras and others.[41]

Different lines of argument come together here. Newton's conception of God requires a permanent involvement with the creation. His criticism of Descartes is connected with this point and makes him look for non-mechanistic agents in nature. Alchemy showed him the existence of active principles. Gravitation, one of these principles and prominently present in the *Principia*, he traces back to the ancients, in whom he also finds indications of the true religion and the notion of God as the cause of gravitation.

5. Prophecy and comets

In Newton's quest for traces of God's providential acting in history the study of the Biblical prophecies plays a key role. Especially the books of Daniel and Revelations were studied extensively by Newton. He points out that a book written in a foreign language is only accessible if one has a thorough command of the language. This is also the case with the Biblical prophecies. They contain coded messages, which Newton tried to decipher. According to Newton all prophets used "one and the same mystical language". Once the "necessary significations" of this language has been found, all other interpretations have to be rejected as the "offspring of luxuriant fancy", because "no more significations are to be admitted for true ones than can be proved".[42]

This is reminiscent of the first rule of reasoning in book III of the *Principia*: "No more causes of natural things should be admitted than are both true and sufficient to explain their

[40] Casini, *Scholia*, pp. 2, 20n.2; McGuire/Rattansi, *Pipes*, p. 118.
[41] Newton, *Principia*, third ed., transl. Cohen, pp. 940–942; Casini, *Scholia*, pp. 2, 20 n.4.
[42] Castillejo, *Exp. force*, p. 32; Mandelbrote, *Duty*, p. 296; McLachlan, *Theol. manuscr.*, p. 120; cf. Manuel, *Religion*, p. 117; Newton, *Observations*, pp. 16ff.

phenomena".[43] In his explanation of this rule Newton states that nature delights in simplicity. We find this same notion of simplicity in his *Rules for methodizing the Apocalyps*. In one of the relevant passages he himself makes the link between nature and prophecy:

> "Truth is ever to be found in simplicity, and not in the multiplicity and confusion of things. As the world, which to the naked eye exhibits the greatest variety of objects, appears very simple in its internall constitution when surveyed by a philosophic understanding, and so much the simpler by how much the better it is understood, so it is in these visions. It is the perfection of ⟨all⟩ God's works that they are all done with the greatest simplicity. He is the God of order and not of confusion. And therefore as they that would understand the frame of the world must indeavour to reduce there knowledg [sic] to all possible simplicity, so it must be in seeking to understand these visions".[44]

When such rules reduce things to the greatest possible simplicity, as Newton of course assumes of his own 'constructions', this is sufficient. In that case no attention need be paid to other keys to the Apocalypse, except if there are evident flaws in the rules that have been used. The methodological parallel of this rule is to be found in the Principia, where Newton states in his fourth rule of reasoning that inductive generalisations have to be considered true or nearly true, except if there are empirical grounds showing that they need correction. Clearly, the method in science and that in theology are parallel, or rather there is only one method. God's simplicity justifies inductive reasoning both in reading the book of nature and reading the books of the prophets. The inductive method is justified by our theological understanding of the nature of God. Whoever rejects this method, actually makes Him as irrelevant.[45]

It was not Newton's intention to predict future events with the help of the Bible, but rather to find a code to interpret the prophecies and to confirm that Holy Scripture was right. This would make God's providence visible to the world. If events take place that have been foretold ages ago, this will be convincing proof that the world is governed by providence. In a number of cases the reader can derive a scheme of the course of future events from the facts and keys provided by Newton. Only in this sense it can be said that Newton makes predictions, for strictly speaking his calculations never go further than his own time.[46] Because of the complicated calculations there is a direct link between mathematics and the explanation of the prophetic books, and in addition Newton calls in the assistance of astronomy to date events. Then there is a more indirect relation through temples and other sanctuaries. Castillejo speaks of a "triple alliance between the Temple, the Universe, and the political-mystical events of an ecclesiastical community". The constituent parts of the universe are recognized by Newton in the parts of the temple:

[43] Newton, *Principia*, transl. Cohen, p. 794.

[44] Manuel, *Religion*, p. 120 (rule 9); Rogers, *Guarant. God*, p. 233; Force, *Ancients*, p. 244; as to the "necessary significations" of a "prophetic phrase" Newton remarks: "I received also much light in this search by the analogy between the world natural and the world politique. For the mystical language was founded in this analogy (. . .)", Castillejo, *Exp. force*, p. 32; The 'Rules for methodizing the Apocalyps' form part of the fifteen 'Rules for interpreting the words and language in Scripture', Manuel, *Religion*, pp. 116ff; Kochavi, *One prophet*, pp. 108–109; Force, *Ancients*, pp. 244–247; cf. quotation in note 32.

[45] Rogers, *Guarant. God*, p. 234; Manuel, *Religion*, p. 12; Castillejo, *Exp. force*, pp. 32–33; Newton, *Principia*, transl. Cohen, p. 796.

[46] Newton, *Observations*, p. 251; Castillejo, *Exp. force*, p. 34; Murrin, *Apocalypse*, pp. 205 n.9, 217; Snobelen, *Newton, heretic*, pp. 391–392; cf. for some calculations based on Newon's data: Castillejo, *Exp. force*, pp. 53–55.

> “[They] have the same signification with the analogous parts of the world. For Temples were anciently contrived to represent the frame of the Universe as the true Temple of the great God. Heaven is represented by the Holy place or main body of the edifice, the highest heaven by the most Holy or Adytum, the throne of God by the Ark, the Sun by the bright flame of the fire of the Altar or by the face of the Son of man shining through this flame like the Sun in his strength, the moon by the burning coals upon the Altar convex above and flat below like an half moon, the stars by the lamps (. . .) And all these parts of the Temple have the same signification with the parts of the world which they represent”.[47]

Solomon’s temple seems to be the model here for Newton’s interpretation of temples and sanctuaries in general. His temple studies lead to two important conclusions. On the one hand in its physical shape the temple is a representation of the cosmos, including the heliocentric system. Sanctuaries therefore represent the scientific knowledge of the ancients. On the other hand the temple is the place where the Son of man is present and the scene of the events in the book of Revelation. The temple service with all its ceremonies is a model for Christian worship and the government of the church. Every object and every ceremony has a prophetic meaning. This interpretation is fundamental for Newton’s entire theology.

We also come across the relation between cosmos and sanctuary in Newton’s *Origenes*. He focuses there on the prytaneum—the central hearth of a city in ancient Greece, representing the unity and the vitality of the community—or rather on the sacred fire itself contained in such a building. In its most developed form this fire is a manifestation of the original religion:

> “The religion most ancient & most generally received by the nations in the first ages was that of the Prytanea or Vestal temples. (. . .) The placing of ye fire in the common center of the Priests court & of ye outward court or court of ye people in the Tabernacle & in Solomons Temple [& the framing ye Tabernacle & Temple so as to make it a symbol of the world] is a part also of ye religion wch ye nations received from Noach. For they placed ye fire in ye middle of ye Prytanaea. And as the Tabernacle was a symbol of the heavens, so were the Prytanaea amongs ye nations. The whole heavens they recconed to be ye true & real Temple of God & therefore that a Prytanaeum might deserve ye name of his Temple they framed it so as in the fittest manner to represent the whole systeme of the heavens. (. . .)”[48]

Newton assumes that the circumambulatory movements of the priests symbolised the motions of the planets. Striking again is the close relation between theology and natural philosophy in Newton’s view. To some extent they can even be said to coincide:

> “So then was one design of ye first institution of ye true religion to propose to mankind by ye frame of ye ancient temples, the study of the frame of the world as the true temple of ye great God they worshipped. And thence it was yt ye Priests anciently were above other men well skilled in ye knowledge of ye true frame of Nature & accounted it a great part of their Theology”.[49]

This intimate relation between natural philosophy and theology provides us with a key to combine Newton’s view of the conflagration of all things as described in II Peter with his treatment of comets in the *Principia*. Already in the *Questiones* we find the following remarks on the end of the world under the heading ‘Of Earth’:

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[47] Castillejo, *Exp. force*, p. 33.
[48] Goldish, *Judaism*, p. 95; Knoespel, *Interpr. strateg.*, pp. 194–195.
[49] Knoespel, *Interpr. strateg.*, p. 196.

> "Its conflagration testified Peter 2, Chapter 3, verses 6, 7, 10, 11, 12. The wicked probably to be punished thereby, Peter 2, Chapter 3, verse 7. The succession of worlds is probable from Peter 2, Chapter 3, verse 13, in which text an emphasis upon the word "we" is not countenanced by the original. Revelations, Chapter 21, verse 1; Isaiah, Chapter 65, verse 17, Chapter 66, verse 22. Days and nights after the judgment, Revelations, Chapter 20, verse 10".[50]

The expression 'succession of worlds' shows that early on Newton reckoned with the possibility of cyclical return of events in the universe. In his later view comets play a key role so that prophecy, astronomy and conic sections are closely interwoven. The extensive analysis of the comet of 1680 in the *Principia* is closely linked with the conflagration at the end of time, but also with the expectation of a new world as formulated in Peter's Letters. Comets provide both the fuel for a conflagration and the seeds for a new world. In the *Principia* Newton compares the indispensability of vapours rising from the sea for the growth of vegetation on earth and the significance of comets:

> "…comets seem to be required, so that from the condensation of their exhalations and vapors, there can be a continual supply and renewal of whatever liquid is consumed by vegetation and putrefaction and converted into dry earth (…). I suspect that that spirit which is the smallest but most subtle and excellent part of our air, and which is required for the life of all things, comes chiefly from comets".[51]

Dobbs points out that in the post-*Principia* era Newton had a predilection for regularity in his interpretation of both natural phenomena and historical processes. He therefore needed a physical agent that acted with 'majestic regularity', but that at the same time could explain unusual happenings, including those of sacred history. According to a memorandum of Gregory (May 1694) Newton held the opinion that the great differences between planets and comets, particularly their eccentricities, constitute evidence of a divine hand, which has apparently destined the comets for a different purpose than the planets. A few years before his death he revealed his thoughts on this point to John Conduitt. His idea was that the coagulating vapours and light emanating from the sun would in successive phases solidify into a body, then into a primary planet and finally into a comet. After a number of revolutions this comet would approach the sun more and more closely, and finally by coagulation be able 'to recruit and replenish the sun', just as new fuel stimulates a fire, with serious consequences for the earth. The comet of 1680, which according to Newton and Halley had a period of more than 500 years would probably bring this about.[52]

Newton's ideas about comets are closely linked with his tour de force as found in the Principia to tame the until then elusive comets into members of the solar system, moving around the sun in narrow ellipses and like the other celestial bodies subject to the law of gravitation. For Newton these celestial bodies with their now regular motions were no longer harbingers of doom, but themselves agents of destruction:

> "With comets, then, Newton had found a precise mechanism, described by his mathematical principles, by which the dissolution of this world could be effected. The dissolution would take place through natural law, but this law operated under divine guidance. Newton's ideas had roots in Judeo-Christian texts (…) and were congruent with the Stoic doctrine of *ekpyrosis;* yet one must

[50] McGuire/Tamny, *Certain phil.*, pp. 374–376, 375–377; Dobbs, *Janus faces*, p. 234.
[51] Newton, *Principia*, transl. Cohen, p. 926.
[52] Dobbs, *Janus faces*, pp. 233–234; Turnbull, *Corr.* III, pp. 334, 336; Markley, *Fallen Languages*, p. 159; cf. Newton, *Principia*, transl. Cohen, pp. 937, 938.

> in addition recognize that within the rigorous mathematical system of the world for which he is so justly famous, Newton had firmly established a providential God—at least as he himself understood what he had done".[53]

The unity of Newton's thought, strongly emphasised by Dobbs, here assumes concrete shape: mathematics, astronomy and gravitation are linked through the comets to prophecy and the providence of a God who is Lord over nature and history.

6. Prisca geometria and fluxions

In the quotation above Dobbs emphasises God's providence, but at the same time she indicates the fundamental role of Newton's mathematical principles. The great importance she attaches to the unity of Newton's thought also concerns the "mathematical principles of God's design", and the question arises how these are connected with Newton's concept of God and his ideas about prisca scientia and with his thought as a whole.

Newton's interest in mathematics was aroused in 1664 when he studied the works of Viète, Descartes, Wallis and others and became acqainted with the 'new analysis' of the seventeenth century. It took him only a few months to make himself familiar with the new ideas and techniques, and already during the winter of 1664–1665 he produced a first important contribution of his own to the new area with the binomial theorem named after him. He also occupied himself with series expansion. In 1666 he laid the foundations for his most important contribution to analysis, the calculus of fluxions, which he applied to a wide range of problems in 1670–1671. This was a new method of doing calculations with finite and infinitely or indefinitely small quantities, with the extra rule that $A + a = A$, if A is a finite quantity and a an infinitely small quantity.[54]

From the start the kinematic orientation dominated in Newton's calculus of fluxions. In his *Methods of series and fluxions* (1670) he considered quantities "as though they were generated by continuous increase in the manner of a space which a moving object describes in his course." A continually moving point generates a line and a continually moving line generates a plane. Such quantities generated by 'flow' are called 'fluents' by Newton, and their instantaneous velocities 'fluxions', while the "infinitely small additions by which those quantities increase during each infinitely small interval of time" are named 'moments'. These moments were considered by Newton at that time as proportional to their 'speeds of flow', that is, their fluxions. In the notation he used after 1690: if x is the fluent at time t and $\dot{x}$ the fluxion, then the moment is $\dot{x}o$, if o is the infinitely small period of time, so that the quantity in question equals $x + \dot{x}o$ at time $t + o$. Newton warns the reader not to identify the time t with the absolute real time, as any fluent with constant fluxion can play the role of fluxional time.[55]

With his new method Newton could successfully tackle all kinds of problems, such as determining maxima and minima, quadratures, tangents, arc lengths, surface areas and centres of gravity. Newton's great achievement was to reduce all these problems to two

[53] Dobbs, *Newton as alch.*, p. 137; Newton, *Principia*, transl. Cohen, pp. 888–938; cf. Hughes, *Princ. and comets*, pp. 53–74; Dobbs, *Janus face*s, pp. 235–237.

[54] Leshem, *Newton on math.*, pp. 19ff; Guicciardini, *Reading*, pp. 17–18, 20–22.

[55] Whiteside, *Math. papers*, III, pp. 72–73, 78–81; Guicciardini, *Reading*, pp. 21ff; Arthur, *Fluxions*, pp. 334ff.

fundamental questions, viz., to find the fluxion with a given fluent, and vice versa, and in addition to prove the fundamental theorem that these two operations are each other's inverse.[56]

In his early works Newton prided himself as a promoter of the 'new analysis', but in the course of the 1670s this changed. Two factors seem to play a role here. In the first place there is his anticartesianism. His growing opposition to Descartes' mechanism, which in his view leaned towards atheism, seems to have made him critical of everything produced by the French philosopher, including the new analytical method in mathematics. Secondly, there was his preoccupation with the prisca scientia. In his quest for lost knowledge he encountered again the geometry of the Greeks. He considered their synthetic method superior to the analytic method of the moderns: it was simple and elegant, and it could be interpreted in terms of actually existing objects, so that it could be converted more easily into ordinary language than a algebraic calculation, as he argues in *Three books of geometry* (1693). Of the modern mathematicians Descartes was singled out for criticism, while of the ancients, in addition to Euclid and Apollonius, Pappus was often mentioned with approval. Pappus was also adduced as an authority in the Preface of the *Principia*.[57] Newton pointed out that, although the ancients made their discoveries with the help of analysis, they furnished the final proof by way of synthesis. That was the way of truth and certainty, "for the force of geometry and its every merit lay in the utter certainty of its matters, and that certainty in its splendidly composed demonstrations".[58]

Newton never abandoned this new position of criticism of the moderns and praise of the ancients. Although this was the main component of his attitude, on the other hand it is certainly true that he continued to use and develop the new analysis. His ideal remained the restoration and improvement of the old geometry, although actually his mathematical methods transcend the boundaries of classical geometry. There are two elements in Newton's geometry that are completely absent in Greek mathematics, viz., motion and the infinitely small. This is clear in the *Principia*, where Newton's argumentation is strictly geometrical. An argument is linked to a figure, from which ratios and proportions are read off that are then reduced until the desired result has been obtained, again in terms of proportions, as is for example the case with the law of gravitation. The great innovation compared to the ancients is that Newton always checks what form the relations that have been obtained take when some elements in the figure reach certain boundary values or become infinitesimally small. Newton speaks of 'first and ultimate ratios', and he discusses them in the *Principia* in the very first section, apparently aware of the novel character of this geometrical approach. A second important point in which Newton differs from Euclid and Apollonius is that his geometry is full of kinematic notions. This means that here there is actually a rift with the ancients, as they only studied constant quantities and figures, while Newton's quantities and figures often change over time and approach boundary values.[59]

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[56] De Gandt, *Force*, pp. 209–210, 225; Guicciardini, *Reading*, pp. 26, 212–213.

[57] Guicciardini, *Reading*, pp. 20, 28–29, 222–223; Whiteside, *Math. papers*, III, p. 33; cf. IV, pp. 221–222, 222–223, 274n4, 276–277: "their method is more elegant by far than the Cartesian one", cf. p. 423: "(. . .) more simply effected by synthesis", p. 450; VII, pp. 199, 199n67 (Newtons esteem for Huygens), pp. 251, 342; Newton, *Principia*, Preface, transl. Cohen, p. 381.

[58] Whiteside, *Math. papers*, VIII, pp. 451, 453, 455 (relation between motion and geometry).

[59] Newton, *Principia*, transl. Cohen, pp. 433–443; Guicciardini, *Reading*, pp. 34–35, 99–117; De Gandt, *Force*, pp. 224–226.

Newton uses his earlier discoveries in the calculus of fluxions, but he replaces the calculus of fluxions by a geometry of fluxions. He denotes the new approach as the synthetic method of fluxions, which he distinguishes from his earlier analytic method of fluxions. Later in his polemics with Leibniz there is a third element. In his *Quadrature of curves* (1691ff) he writes:

> "Mathematical quantities I here consider not as consisting of least possible parts, but as described by a continuous motion. Lines are described and by describing generated not through the apposition of parts, but through the continuous motion of points (. . .) times through continuous flux, and the like in other cases. These geneses take place in the reality of physical nature and are daily witnessed in the motion of bodies. And in much this manner the ancients by 'drawing' mobile straight lines into the length of stationary ones, taught the genesis of rectangles".[60]

In addition to the appeal to the ancients we find here the express rejection of Leibniz's infinitesimals, in which the ontological content of the method of fluxions plays an explicit role. Fluents and fluxions are realities, or to use the words of his anonymous review of the *Commercium epistolicum* (1713): "whilst indivisibles upon which the Differential method is founded have no being either in Geometry or in nature".[61]

Hall has pointed out in his *Philosophers at war*, in which he describes the controversy between Newton and Leibniz about the discovery of the calculus, that Leibniz's calculus and Newton's method of fluxions "are products of very different minds". Newton's calculus of fluxions is based on the continuity in time of a permanently present God, and Leibniz's infinitesimal calculus on the discontinuity between monads in a world in which God does not interfere. This difference raises the question, asked by Force in 1990, of what is the relation between mathematics and metaphysics, particularly in the *Principia*, if an 'integrated system of thought' is ascribed to Newton.[62] Force emphasises that behind the mechanical framework of the *Principia* there is the image of Newton's God. He is convinced that this is a God of continuous dominion, who is not only the Creator, but also the one who maintains and governs His creation either via the normal way of secondary causes or via the unusual, but always possible way of direct mediation on the basis of the free exercise of His will. Newton owes his fame as a scientist to the link between mathematics and the empirical world, but both facets of his method are inextricably linked with his voluntarist conception of God. Even the ordinary concourse of a law of nature together with its mathematical description is inconceivable without a God who maintains the regular operations of nature that He has created.

According to Force, Newton's method of fluxions is also closely linked with his conception of God because it is based on the continuity of flow as supervised by the God of dominion, who maintains the current state of natural law. Several scholars have discussed Force's question. In a paper on fluxions (1990) Arthur points out that the method of fluxions enabled Newton to treat mathematics and physics within the one framework of his conception of space and time, which is inextricably connected with his conception of God.

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[60] Whiteside, *Math. papers*, VIII, pp. 122, 123; Guicciardini, *Reading*, p. 35.

[61] Guicciardini, *Reading*, p. 35; Whiteside, *Math. papers* VIII, pp. 597–598; Hall, *Phil. at war*, p. 295 (Appendix, p. 205); Ramati, *Hidden truth*, p. 343.

[62] Hall, *Phil. at war*, p. 258; Force, *Newton's God*, pp. 88, 100n.38.

The mathematics of continually changing quantities and the mathematics of geometrical physics form an indivisible whole.[63]

In a number of publications in which he builds on the work of Dobbs, Snobelen, Force, McGuire, Guicciardini, De Gandt, Arthur and others, Ayval Ramati (aka Ayval Leshem)[64] has again made a link between the calculus of fluxions and Newton's God. He extensively discusses Newton's quest for the truth about nature and history, which finds its unity in the one God who as Lord of dominion, as Pantocrator, is in touch with the material world through space as his sensorium, and who exercises his dominion over both human history and the natural world.[65] Ramati says that there are reasons to assume that calculating fluxions should help the wise man to obtain the objective perspective of God and achieve the simplicity of true worship. Our perspective is distorted, because we are part of the physical world in which we perceive objects. Bodily perceptions are subject to the law that every action evokes an equal and opposite reaction. When impressions reach our sensorium, the centre of perception in the brain, they arrive there through the mediation of the senses between the objects without and our sensorium within, which leads to confusion, error and corruption. But because God is omnipresent and everything has its existence in Him, everything is present in Him and He observes all objects directly, without distortion. This means that our perception of things differs from God's objective uniformly flowing perception.

From this Ramati concludes that Newton may have viewed his mathematical method of fluxions as a divine path through which people can purify their bodily senses. With the help of the method of fluxions it is possible to calculate the distortions that curves representing continually flowing quantities suffer. In the context of the world of phenomena this enables human beings to purify their senses and eventually to converge with God's infinite objective sensorium. Ramati points out how important tangents are in Newton's method of fluxions. In every curved path of a physical body the tangents express at every moment the inertial flow of the body, provided that the vis insita at the time in question has not been disturbed. Tangents are actually fluxions flowing along the equable flow of God's endurance. They are informative devices, as they recover the equable flow of God's absolute, mathematical, true and pure time. Ultimately, Ramati argues,

> "that we have good reasons to see Newton's association of his fluxions with God's absolutes, time and space, as well as his *Principia* project, as a modern form of the old religious worship of Noah and his sons around vestal fires, incorporating the mathematical concepts inherent in the ceremony reflecting the motion of the planets around the sun".[66]

In this way prisca scientia, prisca theologia, physics, history and even morals are linked with mathematics, "the fittest of all languages to describe God's governance over creation".[67] This is in perfect agreement with Dobbs who has emphasised again and again the unity of Newton's thought:

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[63] Arthur, *Fluxions*, pp. 325–326.

[64] See Leshem, *Newton on math.*, p. 75n.1.

[65] Newton, *Opticks*, 4th ed., p. 403 ("to move bodies in his boundless uniform sensorium"); Ramati, *Hidden truth*, p. 418; cf. Snobelen, *God of gods*, pp. 175–180, 202ff.

[66] Ramati, *Hidden truth*, pp. 427, 433, 435; Leshem, *Newton on math.*, p. 75, cf. pp. 26, 34, 62–63, 66, 80–81, 118; Dobbs, *Alchemy*, pp. 73–74; Westfall, *Origines*, pp. 22–24.

[67] Leshem, *Newton on math.*, p. 85, cf. pp. 78–79, 84; cf. Markley, *Corruption*, p. 140: "(. . .) Mathematics becomes akin to revelation, both analytical method and moral shield against the corruption of the physical world"; Newton, *Opticks*, 4th ed., pp. 405–406.

"For Newton, true knowledge was all in some sense a knowledge of God. Truth was one, its unity guaranteed by the unity of God. Reason and revelation were not in conflict but were supplementary. God's attributes were recorded in the written Word, but were also directly reflected in the nature of nature. Natural philosophy thus has immediate theological meaning for Newton, and he deemed it capable of revealing to him those aspects of the divine never recorded in the Bible or the record of which had been corrupted by time and human error. By whatever route one approached truth, the goal was the same. Experimental discovery and revelation; the productions of reason, speculation, or mathematics; the cryptic, coded messages of the ancients in myth, prophecy, or alchemical tract; the philosophies of Platonists, atomists, or Stoics—all, if correctly interpreted, found their reconciliation in the ultimate unity and majesty of the Deity".[68]

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CHAPTER 25

God and Mathematics in Leibniz's Thought

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Edited by T. Koetsier and L. Bergmans

1. Introduction

Unlike many other philosophers, Leibniz published no major work in which he expounded on his philosophical system. His thought has to be reconstructed from numerous remarks scattered throughout several books and a number of published articles as well as from a huge number of letters and drafts in his Nachlass. Nevertheless, the result of such a reconstruction is fairly coherent.

Leibniz believed in the God of Christianity and he also had an extraordinarily high esteem for reason and its capabilities. One might try to develop part of his thought on the topic from these two suppositions. Firstly, it follows that there is a concept of God, not just a vague imagination of him. This concept must evidently be that of "the most perfect being" [16, ser. VI, vol. 4B, p. 1531]. The most perfect being is necessarily the most rational being. The principle of sufficient reason ensures us that for all decisions and acts of God there is a rational reason. Since knowledge is a perfection,[1] the most perfect being must be omniscient. As being active is more perfect than being passive, God has to be all-powerful. To be more precise, the most perfect being has to be almighty insofar as it is logically possible to be almighty. In other words, the definition of "almighty" should not be contradictory. Intellectual conundrums such as discussing the question whether God can create a stone which is so heavy that he himself cannot lift it, do not fit into Leibniz's style of thought. As for the moral dimension, Leibniz is firmly convinced that there is a close connection between rationality and goodness: the most perfect being will always act according to the maximum of goodness which is characteristic of him. This necessarily implies that God will create the best of all possible worlds [16, ser. VI, vol. 4B, pp. 1533–1534] because otherwise he would either not be almighty, or not be absolutely good, or not be omniscient. Leibniz seems to be aware of the imperfections of this world we live in, but the principle of the best possible world is a direct consequence of his high esteem for rationality. Incidentally, Leibniz anticipates how a mathematician might possibly object: a mathematician faced with an increasing infinite sequence knows that there need not be a maximum. So Leibniz states: if it were true that for every good possible world there exists a better possible world, then God would not have created any world at all, because in this case there would not have been sufficient reason for selecting and creating this particular world which exists [15, VI, pp. 107, 364]. The absolutely rational being will never act arbitrarily.

Before we continue with this consideration of God, two remarks might be useful. Firstly, Leibniz's project of a *characteristica universalis* should be discussed briefly. Leibniz was well aware of the fact that our philosophical reasoning often lacks the cogency of mathematical reasoning (cf. [15, IV, p. 468]). There are no Euclidists and Archimedians in mathematics, as there are Aristotelians and Platonists in philosophy [16, ser. VI, vol. 4A, p. 695]. Whereas mathematicians have their own means of discovering possible mistakes, philosophers, who do not have such means at their disposal, should adopt rigorous reasoning all the more [16, ser. VI, vol. 4A, p. 705; ser. II, vol. 1, pp. 475, 478; 15, IV, p. 469]. Rationality should be such as to allow for a mathematisation of our thought; just as mathematicians have introduced letters and other symbols to designate mathematical objects

[1] Qualities which do not have a maximal degree—like velocity or number—cannot be considered to be perfections [15, IV, p. 294; 16, ser. VI, vol. 4B, p. 1531].

and rules for operating with them, Leibniz proposed finding symbols and rules, in order to formalize a considerable part of our thought [20,10]. Then two philosophers with different opinions on a philosophical topic would no longer need to quarrel; they could say to each other "calculemus" (let's calculate) [16, ser. VI, vol. 4A, p. 493]. Therefore, Leibniz's invention of a calculating machine had a strong philosophical relevance. And besides, Leibniz tells us, this characteristica universalis will be an efficient means of converting pagans, because the true religion is the most rational religion, and it is impossible to resist rational arguments. In addition, apostasy will no longer occur, just as a mathematical truth, once understood, is never rejected.[2]

Leibniz made several efforts to find suitable symbols for the representation of our thinking. A very simple and nonetheless very interesting one was his idea that there are primary or irreducible notions and composite notions. If this is true, a map from notions to natural numbers can be defined, mapping primary notions to prime numbers and the relation "implies" between notions to the relation "is divisible by" between numbers. For illustration's sake, Leibniz gives the example of the traditional definition of the human being as the rational living being. If the notion "rational" is mapped to the number 2 and the notion "being alive" to the number 3, the notion "being a human being" has to be mapped to the number 6 [16, ser. VI, vol. 4A, pp. 182, 201–202]. As there is an infinite number of prime numbers, the model is more powerful than it might seem at first glance. In other drafts, Leibniz maps every notion to an ordered pair of a positive and a negative integer [16, ser. VI, vol. 4A, pp. 224–256]. As Leibniz is aware, even with such a characteristica universalis the deduction of an individual statement like "Caesar was murdered on the ides of March" would be impossible, because such a statement involves an infinity of causes and an individual notion like Caesar is composed of an infinity of elements.

The use of numbers for a characteristica universalis even has a metaphysical foundation. Leibniz quotes [16, ser. VI, vol. 4A, p. 263; cf. also ser. I, vol. 12, p. 72 and 15, VI, p. 604] the well-known statement [Plato, Philebos 55e; Sapientia Salomonis 11, 21] that God made everything according to measure, number and weight. Admittedly, Leibniz continues, some entities do not have weight, and some entities do not have parts and therefore lack measure. But there is nothing which does not allow for a number. So number is "quasi figura quaedam metaphysica" [16, ser. VI, vol. 4A, p. 264], and arithmetic is therefore a doctrine for the exploration of the powers of things, and thereby the perfection of God's creation.

The second remark concerns the difference between human and divine reason. According to Descartes, if God had wanted, he could have decided that the radii of a circle have different lengths, that the sum of 1 and 2 is different from 3, or that the sum of the angles of a rectangular triangle is different from two right angles[3]. Leibniz argues that mathematical truths are not fictions; on the contrary, they do exist in the region of ideas [15, VII, p. 305], which is nothing other than God's reason [15, VI, p. 614; II, pp. 304–305; 16, ser. VI, vol. 6, p. 447]. They do not depend upon his will [16, ser. VI, vol. 4A, pp. 1532–1533; 15, VI, p. 226, p. 614]; God would not be able to change the necessary truths without abolishing himself [4, p. 310]. One might interpret this as mathematics being autonomous. In finding mathematical truths, human beings discover part of God's reason. This is not only

[2][16, ser. VI, vol. 4A, p. 269; ser. II, vol. 1, p. 491]. Nearly twenty years later Leibniz was more sceptical: there are people who even reject indisputable arguments [16, ser. I, vol. 13, pp. 553–554].

[3]R. Descartes, *Œuvres*, ed. by Adam/Tannéry, vol. 1, Paris 1969, pp. 145, 152; vol. 2, Paris 1972, p. 118.

valid for mathematics and logic, but also for a number of truths in metaphysics, including some statements on goodness, justice and perfection. Leibniz calls these truths necessary or eternal truths as opposed to contingent truths. Necessary truths are valid in every possible world, whereas contingent truths depend on the particular structure of this world which God created and which he could have created otherwise. So Kepler's laws, Galilei's law for bodies falling in a vacuum and the proposition "Caesar was murdered" are all contingent truths.

That part of reason which human beings possess is compatible with the whole; it differs from the whole only as a drop of water differs from the ocean, or, more precisely, as the finite differs from the infinite [15, VI, p. 84]. Thus, reason is a property common to God and man; it is not only what links all human society and is the basis of friendship, but is also the link between God and man [17, pp. 10–11]. "The true quietness which can be found in the Holy Scriptures, in the writings of the Church Fathers and in reason is achieved through liberation from the external pleasures of the senses, in order to listen to God's voice, that is, the inner light of eternal truths".[4] In this respect, doing mathematics seems to be a way of listening to God's voice and might even be comparable to divine service, although of course the active endeavour for the welfare of others would be indispensable for a Christian anyway [9, pp. 194–195, 202–203].

2. The best of all possible worlds

Now imagine God considering all possible worlds in order to decide which one is to be created (this formulation is not meant to imply a temporal order, because God does not exist in time). There must be a rational basis for God's decision. Given that, for Leibniz, thought has a fundamentally mathematical structure, God's reasoning about possible worlds must be precise and, in principle, understandable for a mind of sufficiently high capabilities. God must be a perfect mathematician [16, ser. VI, vol. 4B, p. 1616; 15, II, p. 105; 15, III, p. 52; 15, IV, p. 571] calculating all possible worlds and selecting the best one. The ancient idea of God as mathematician [19] is given an emphatically new meaning by Leibniz. "As God calculates and executes thought, the world comes into being".[5] The act of creation is done by "divine mathematics",[6] and is nothing other than the solution of an extreme value problem. Among all the possibilities, God selects the maximum of essences compatible with each other. This selection is much more complicated than it might seem. It does not simply mean a maximum of material objects, but rather the essences of all living creatures having all possible courses of life. Moreover, all kind of perfections, such as moral goodness, justice, ontological variety and so on, have to be taken into account. For example, God decided to create lions although they are dangerous to human beings, but without any lions the world would have been less perfect [15, VI, p. 169]. In addition, there is the temporal dimension. There might have been a possible world which would have been better up

[4][16, ser. I, vol. 5, p. 600]: "La veritable Quietude qu'on trouve dans la Sainte Écriture, dans les Peres, et dans la raison est de se detourner des plaisirs exterieurs des sens, afin de mieux écouter la voix de Dieu, c'est à dire la Lumiere interieure des Verités eternelles".

[5]"Cum Deus calculat et cogitationem exercet, fit mundus" [16, ser. VI, vol. 4A, p. 22].

[6]"Mathesis quaedam Divina" [13, VII, p. 304].

to the present time, but which would have contained less possibilities for future progress and was, therefore, not selected [15, VI, p. 237]. Obviously we cannot understand how God calculated all this. But in all things, Leibniz argues, there is a principle of determination, which is the maximum of effect achieved with a minimum of expenditure [15, VII, p. 303]. Time and space, or generally the capacity of the world, are to be considered as expenditure or terrain on which to erect the building of the world, whereas the variety of its forms and the number and elegance of its rooms constitute the effect. To elucidate the principle of determination Leibniz poses that a triangle has to be made. If no constraints are given, an equilateral triangle will be the wise man's choice [16, ser. VI, vol. 4B, p. 1617]. If two points are to be connected, the shortest line will be the best solution. Liquids in heterogeneous matter tend to form spheres, which are the most voluminous shapes. These are just a few examples of the principle of determination [15, VII, p. 304]. To create a maximum of essences requires using very simple laws for their interaction. Otherwise God would have been like an architect using round bricks, which cost more space than they take up [16, ser. II, vol. I, p. 478].

As God created this world according to this construction principle, Leibniz can state the fundamental rule of his philosophy, or in other words, the fundamental rule for our knowledge of this world: the reason of things is the same everywhere, but the forms and degrees of perfection vary infinitely [16, ser. VI, vol. 6, pp. 71, 72, 472, 490]. The philosopher should think of the unknown or only vaguely known objects according to the same pattern as the distinctly known objects; thus philosophy becomes easy with respect to the reason of things, and it will be very rich with respect to the different manners of execution. Nature varies these manners infinitely with as much order and decoration as can ever be imagined.

Three hundred years later we tend to have questions such as this: Is it really possible that mathematics, even divine mathematics, can solve the huge problem of calculating all possible worlds? As every individuality implies an infinity of aspects [16, ser. VI, vol. 6, pp. 289–290], it must be very difficult to calculate the life of one single person alone. We know that there are systems of differential equations which cannot be solved formally, and thus God, being bound to proven mathematical results, could not solve them either. In fact, Leibniz himself indicates a kind of limitation to God's capacity of calculation by exposing an analogy between real numbers on the one side and all truths on the other [16, ser. VI, vol. 4B, pp. 1616, 1657–1658]. The necessary truths correspond to the rational numbers; the necessary truths can be proved in a finite number of steps just as the rational numbers can be calculated in a finite number of steps producing a finite or a periodical decimal representation. The contingent truths correspond to the irrational numbers; the contingent truths could be proved only in an infinite number of steps, just as the irrational numbers can be calculated only in an infinite number of steps giving an infinite non-periodical decimal representation. For Leibniz there is only potential infinity in mathematics [6, pp. 320–323]; therefore even God cannot finish an infinite calculation: the impossibility of ever being finished is the very essence of infinity. So Leibniz makes it unmistakably clear that God cannot calculate in a strict sense the infinity of contingent truths for each possible world. Admittedly, in some cases a finite calculation might be sufficient. Although we are not able to execute an infinity of additions in order to find the sum of the geometrical series, we are nevertheless able to calculate the sum. But here Leibniz is not referring to this fact; evidently he accepts the impossibility of the calculation of contingent truth in general.

(Incidentally, this impossibility has some advantage for Leibniz's theory of freedom). This does not imply that God does not know what he is doing, because he possesses the ability of vision: God sees at once the result of an infinity of calculations, although even he would not be able to finish all the steps one after the other. So our questions are not really answered, but at least it is difficult to raise further objections in this respect.

The discussions about the paradoxes of set theory suggest further objections. The totality of all possible worlds does not seem to be well defined; it seems possible to give analogies of Russell's paradox and Richard's antinomy for possible worlds. If this is true, Leibniz's God cannot overcome these difficulties, and Leibniz would have to reformulate his theory of possible worlds, using neither Cantor's nor Zermelo–Fraenkel set theory [11, pp. 103–105].

3. The binary system and creation

There is another relation between mathematics and the act of creation. In a letter to Duke Rudolf August of Wolfenbüttel in 1697, Leibniz suggested stamping a medal to demonstrate the analogy between the creation of the world out of nothing and the binary number system [16, ser. I, vol. 13, pp. 116–125; cf. also ser. I, vol. 12, pp. 66–72]. The medal was to carry the words "image of creation", "To develop everything out of nothing, unity is sufficient" and "One is necessary".[7] Furthermore some numbers in binary notation would be shown as well as an example of addition and of multiplication. In the 17th century, the Latin word "nullum" meant "zero" as well as "nothing". The words "One is necessary" may be considered as an allusion to Jesus' words to Martha [Luke 10, 42] [16, ser. I, vol. 13, p. 119, line 24–25; p. 120, line 6–7; 22, 1973, p. 49], but they probably also refer to Leibniz's notion of God as the necessary being; we will return to this later in the discussion of Leibniz's proofs for the existence of God.

In his letter to Rudolf August, Leibniz explains that void and darkness (Genesis 1, 1) correspond to zero and nothingness, whereas the spirit of God hovering above the waters (Genesis 1, 1) correlates with the Almighty One.[8] The analogy goes further than might at first be expected; Leibniz repeatedly states that all creatures have their perfection from God and their imperfection from themselves ([15, VI, pp. 603, 613; IV, p. 476]; cf. also [13, VII, p. 239; 16, ser. I, vol. 15, p. 560]). In a slight change of terminology, Leibniz claims in a tract on mystical theology that all creatures derive from God and nothing; their "essence" is from God, whereas their "nothingness" or "bad condition" is from themselves.[9] This is demonstrated in a wonderful way by numbers, as Leibniz continues, and the essence of all things is equal to numbers.[10] Every creature has some nothingness, otherwise it would be

[7] "Imago creationis", "Omnibus ex nihilo ducendis sufficit unum", "Unum est necessarium".

[8] Leibniz was not the only one in the 17th century to compare God with the number 1 [K. Radbruch, *Mathematische Spuren in der Literatur*, Darmstadt, 1997, pp. 30–31].

[9] "Selbstwesen" or "Unwesen" [21, p. 130]; the latter word means the negation of existence as well as evil essence. Leibniz was an adherent of the traditional theory authored by Plotin according to which evil is a privation, a lack of perfection, instead of anything positive.

[10] In other passages Leibniz refers to this famous saying from the Pythagorean tradition in a weaker sense, namely: the essence of all things is as if it were numbers ([16, ser. I, vol. 12, pp. 66, 71]; cf. also [22, pp. 43–48]).

Fig. 1. Medal designed by Leibniz on the binary systems.

God. The lack of perfection in the creatures is darkness in knowledge, vacillation in will, passion, limitation of their essence. On the other hand, in our essence there is infinity, a footprint, even an image of God's omniscience and omnipotence [21, pp. 128, 130], and sometimes Leibniz even compares human beings to small gods [16, ser. VI, vol. 6, p. 389; 15, II, p. 125; IV, p. 479]. In other words, the creatures might by analogy be considered as certain mixtures of 1 and 0.

The analogy with the binary system illustrates not only the creation out of nothing, but also the beauty and the perfections of the world [16, ser. I, vol. 13, p. 117; vol. 12, pp. 69–70]. In the decadic number systems regularities are not so easily seen; in the binary system every column has its own periodicity, if the numbers are written one beneath the other. Similarly, the disorder which seems to be present in God's creation disappears if the right perspective is found; then beauty and harmony prevail (another example of this would be the Copernican system [15, VII, p. 120]). In the same way, periodicities can be found if, for example, the square numbers or the cubes or the multiples of any natural number are written beneath each other [22, pp. 245, 255–256]. In another letter Leibniz takes the analogy even further: 0 denotes the void which preceded the creation. At the beginning of the first day of creation, there was 1, that is God. At the beginning of the second day there was two, that is heaven and earth, which had been created before. Finally, at the beginning of the seventh day, everything had been created, and that is why this day is the most perfect day or the sabbath; the seven is 111 in the binary system and thus immediately shows the perfection of the seventh day, which is the sacred day. Moreover, the sign 111 refers to the trinity [22, p. 285].

Evidently, Leibniz is entering here, if he had not earlier, the region of traditional number mysticism. Some other examples of relations to traditional mysticism may be mentioned. In some passages Leibniz seems to agree with a traditional mystical saying, according to which God is like a circle the centre of which is everywhere whereas the periphery is nowhere [16, ser. VI, vol. 1, p. 531; 12, pp. 31, 49; 15, VI, p. 604, cf. also IV, p. 562; 3, pp. 24–25]. Leibniz could have read this in the writings of Pascal, More and others [17, pp. 19–34].

Leibniz's remark "God enjoys odd numbers"[11] (in his publication of the later so-called Leibniz series for pi divided by four) might seem to be another example of number mysticism. But this is a quotation from Virgil [Eclogae 8, 75] [22, p. 41], and it seems rather natural for an educated scholar of the 17th century to refer to this classical statement on such an occasion.

The Italian mathematician Grandi had also tried to illustrate the creation of the world out of nothing by mathematical means. He discussed the diverging infinite series $1-1+1-1+1\cdots$, remarking that $1+(-1+1)+(-1+1)\cdots=1$ and $(1-1)+(1-1)+\cdots=0$. In a similar way, God might have created something out of nothing. Leibniz considered this not an inelegant illustration, but he objects that the mere repetition of an infinity of zeros is not comparable to God's creation, which really brings about a new reality [13, V, p. 382]. Evidently, Leibniz did not agree with every possible analogy in this field. Mahnke [18, p. 21] claimed Pythagoreanism to be a fundamental trait of Leibniz's metaphysics. It is true that Leibniz's quest for a characteristica universalis has its ultimate origin in ideas of Rosicrucians or other religious mystics [17, p. 19; 9, p. 191]. On the other hand, Leibniz generally liked to give mathematical analogies for philosophical statements [15, II, pp. 112, 258; III, p. 635; VI, pp. 261–262, 508]. These analogies were not necessarily related to (Neo-)Pythagoreanism. And Leibniz liked to combine as many thoughts of other people as possible in his own thinking; he repeatedly made statements like "I have the general maxim to despise few things and to benefit from the good everywhere".[12] When Leibniz was asked by one of his correspondents whether it would be possible to compose a tract on theology using mathematical methods, he confirmed this, but on the condition that at least the relevant part of philosophy would have been formulated with mathematical method before [16, ser. I, vol. 13, p. 551]. For Leibniz, mathematics, physics, philosophy and theology are like the steps of a staircase leading to God. Let us have a look at these interconnected steps of human knowledge.

4. A staircase leading to God

Knowledge of God is the foundation or origin of all knowledge and wisdom [15, III, p. 54; 21, p. 128]. Every newly found truth, every experiment or theorem is a new mirror of the

[11] [13, V, pp. 118–122, drawing 23 in the appendix: "Numero Deus impare gaudet"; cf. also 16, series I, vol. 12, p. 66].

[12] [15, III, 384: "J'ay cette Maxime generale de mepriser bien peu de choses et de profiter de ce qu'il y a de bon par tout." Cf. also III, p. 620; II, p. 539; VI, p. 19; 13, VI, p. 236.]

beauty of God [16, ser. IV, vol. 1, p. 535]. Perfection of the mind, which results from the progress of our knowledge, unites man with God [16, ser. II, vol. 1, p. 270]. Mathematics is particularly suitable for entering the realm of ideas [15, IV, p. 571]; it pleases us by giving us a glimpse of God's ideas [15, VI, p. 262]. It is a divine science, a test for our thought, and Leibniz confesses that he has an immoderate love for it [16, ser. II, vol. 1, pp. 475, 433]. An important mathematical achievement is the surest sign of a sound mind [16, ser. II, vol. 1, p. 492]. Leibniz hoped that his mathematical achievements would help to draw attention to his philosophical and theological statements [16, ser. I, vol. 13, pp. 556–557; ser. II, vol. 1, pp. 433–434, 491–493; cf. also ser. I, vol. 13, p. 516, line 24–25]. Here there is evidently a parallel to Pascal; and whatever Descartes' intentions may have been, there is little doubt that his mathematical achievements supported him as a philosopher. On the other hand, in a letter to the mathematician L'Hôpital, Leibniz complained about philosophical and theological discussions, which were taking up his time [13, II, p. 219]. He certainly had a genuine interest in mathematics and science, as, for example, was shown by his attitude towards the Jesuit missionaries in China. They tried to use the superiority of European mathematics and astronomy in order to impress the Chinese and to persuade them of the superiority of Christian religion. Leibniz did not disapprove of this—according to him [12] the ideas of the Chinese were fairly close to a reasonable Christianity anyway—, but he was concerned that the Jesuits might thereby fail to learn from the Chinese as much as they would otherwise do [16, ser. I, vol. 13, p. 516; ser. III, vol. 4, pp. 409–416; 14, vol. IV, part 1, p. 82].

Cartesian mathematics dealt only with algebraic problems, but most scientific problems are transcendental and therefore need infinitesimal calculus. Leibniz gave a philosophical explanation for this fact: In nature everything bears the signature of an author with an infinite nature; therefore infinitesimal calculus, which is a science of the infinite, is essential for any application of mathematics in natural science [13, V, p. 308]. The difference between nature and art (that is, engineering) is not a quantitative, it is a qualitative, one: Whereas every machine made by man only has a finite number of organs, a living being has an infinite number, thus exhibiting the infinite power and wisdom of its creator [15, IV, pp. 396, 482, 505].

Moreover, as Leibniz explains, the principle of continuity derives from infinity. Being the foundation of infinitesimal calculus [7], it is absolutely necessary in mathematics, but it is also successfully applied in physics, because the author of all things acts as a perfect mathematician. In particular, the principle implies that Descartes' laws of impact must be wrong [15, II, p. 105; III, p. 52] because the prerequisites of one of the laws can be transformed into the prerequisites of another of the laws by continual change, but the consequences would not be transformed into each other by this change.

The demon of Laplace has a precursor in Leibniz's writings: Everything in nature takes place mathematically, which means, as Leibniz explains, inevitably. Someone with sufficient insight and memory and reason to calculate would be a prophet and would be able to see the future in the present [15, VII, p. 118; IV, p. 557].

Not only is the world as a whole the solution of an extreme value problem, but the same is true for several of its parts. A mathematical example may be given for this: If the line of quickest descent (the brachystochrone) between two points has been determined, then the same line is the line of quickest descent for any other two points situated on the original curve. In fact, the world as a whole is most perfect only because it is most perfect in all its parts [15, VII, pp. 272–273]. Leibniz seems to have changed his opinion later; he states elsewhere that a certain amount of disorder in parts is possible [15, III, p. 636], and the parts of a beautiful thing need not be beautiful [15, VI, pp. 245–246]. There are not only natural laws which state a relation between cause and effect, but also principles of finality. A mechanical system is in a stable state if the centre of gravity is as low as possible [15, VII, p. 304]. In optics, catoptrics and dioptrics, there is the principle of the easiest way: light will always take the path with the least resistance [15, VII, pp. 273–278; IV, p. 506]. Leibniz's thoughts on this topic are superior to those of a later mathematician, Maupertuis, insofar as Leibniz claims only the existence of a most determinate case (a maximum or a minimum) [15, VII, p. 270], whereas Maupertuis postulated a principle of God's economy, implying that in all processes of nature, action always takes a minimum (which is wrong).

Objections might be raised here from the point of view of today's philosophy of science. Whatever the causal laws of nature may be, it is probably always possible to express them as principles of finality, with this or that physical expression becoming an extreme value [11, pp. 106–109]. So the existence of finality principles in physics is probably due rather to the efforts of physicists than to God's wise creation.

After mathematics and physics, philosophy is the next step on the staircase to God. Mathematicians ought to be philosophers, just as philosophers ought to be mathematicians [15, I, p. 356]; in this quotation, "mathematician" (as opposed to geometer) probably includes physicists. "My fundamental considerations revolve round u n i t y and i n f i n i t y",[13] Leibniz declares, the individual souls being the unities and material bodies being infinities, for the investigation of which infinitesimal calculus would be necessary. In the draft of a letter to L'Hôpital, Leibniz even contends "My metaphysics is completely mathematical so to speak or could become so".[14] Evidently he was thinking of his project of a characteristica universalis when making this statement. Leibniz confessed that he would prefer to have metaphysics proven than to find a treasure [16, ser. II, vol. 1, p. 475]. In another letter Leibniz uses only slightly different terms: Mathematics is closely related to the art of making new inventions, which is true logic, and metaphysics is hardly different from this art; metaphysics is natural theology, and God is the origin of all our knowledge [16, ser. II, vol. 1, p. 434; cf. also ser. I, vol. 13, p. 554].

The next step is the step from philosophy to theology. "I start as a philosopher, and I end up as a theologian".[15] Knowledge of God can be gained if philosophy has a firm foundation in mathematics. This epistemological claim is defended against Descartes, who had maintained that finite beings cannot know anything about the infinite [Descartes: *Œuvres*, ed. by Adam/Tannéry, vol. 8, Paris 1905, pp. 14–15]. Yet mathematics proves the contrary. Not only do we know about asymptotes and the sum of infinite series, but also about infinite geometrical figures (like Torricelli's infinite solid) having a finite volume [15, IV, pp. 360, 570]. The philosopher F.M. van Helmont had argued that the Messiah was a being between God and man. Had he known more about mathematics, Leibniz argued, he would not have said that: Anything between the finite and the infinite must itself be either finite or infinite.

[13][16, ser. I, vol. 13, p. 90]: "Mes Meditations fondamentales roulent sur deux choses, sçavoir sur l'*u n i t é* et sur l'*i n f i n i*".

[14][16, ser. III, vol. 6, p. 253]: "Ma metaphysique est toute Mathematique pour dire ainsi, ou la pourroit devenir".

[15][5, p. 58]: "Je commence en philosophe, mais je finis en theologien".

Admittedly, there are different degrees of infinity (as is indicated by second differentials, etc.), but there is an infinity of these degrees, so there should be an infinity of Messiahs, which is absurd [16, ser. I, vol. 11, pp. 18–19].

5. The existence of God

It will not come as a surprise that Leibniz trusted in being able to demonstrate the existence of God. I will mention only two of these demonstrations, both related to my topic, without trying to go into difficult details. The first one is the so-called ontological proof, originating from Anselm of Canterbury and agreed to by Descartes. Its main idea is roughly this: God is the most perfect being; lack of existence would imply lack of perfection, therefore the most perfect being must exist. Leibniz's criticism of this reasoning (and his efforts to improve upon it) may remind mathematicians of the discussion about the paradoxes of set theory at the beginning of the 20th century. According to Leibniz the ontological proof is correct, if, in addition, proof is given that the notion of most perfect being is consistent [15, VI, p. 614; VII, p. 310, IV, pp. 405–406; 16, ser. VI, vol. 4B, p. 1531]. The second proof starts with the statement that there are eternal or necessary (in the sense of being valid in every possible world) truths, the huge majority of them being truths of mathematics and logic. The reality of these truths must be founded in something actually existing. That is, it must be founded in the existence of a necessary being whose essence implies existence; in other words, for which existence is the consequence of being possible [15, VI, p. 614; VII, p. 305].

A particular problem of theology was of course that of the Trinity. Leibniz neither denied the Trinity, as Newton did, nor could he accept that the doctrine of Trinity implied a contradiction. The Athanasian creed states that there are three persons but only one God. With reference to this creed, Leibniz comments: If B is A, and C is A, and B is not C, and C is not B, then there are two As. The analogue of this statement for three entities B, C, D, all of them being A but being different from each other, is also valid. He continues: "And this is the origin of numbers".[16] Leibniz's solution is that the notion "A" (that is, "God") is used in two different senses: God in an essential sense denotes the divine essence, which is one and of which the three persons partake, whereas God in a personal sense denotes any one of the Trinitarian persons.

In other passages, he compares the problem with a mind reflecting upon itself. The reflecting entity and the entity it is reflecting upon are active or passive, respectively, and therefore different, but they are nonetheless the same. In a similar way the difference within the Trinity might be derived from the three fundamental roots of all rational action, namely power, knowledge and will, all of them belonging to the same essence [4, p. 313; cf. also 16, ser. VI, vol. 4B, p. 1461; vol. 4C, p. 2292]. So for Leibniz the doctrine of the Trinity is not contrary to reason, although it is above reason, because we do not completely understand it. Leibniz does not try to prove the Trinity—as he claims to have proven monotheism—but rather to refute the charge of inherent contradiction made against the Trinity. Not everything in religion can be explored by reason,

[16] "Atque haec origo est Numerorum" [2, pp. 2, 10 (footnote 7)]; cf. also [16, series VI, vol. 4A, pp. 552, 2291–2292, and 15, VI, pp. 63–64].

Fig. 2. German stamp in commemoration of Leibniz.

but religion must be founded on reason, otherwise it would be superstition [4, p. 309; cf. also 16, ser. VI, vol. 6, p. 494]. Leibniz is careful not to provoke criticism from orthodox theologians, but the general impression is that he always stressed the common ground between Catholicism and Protestantism, and that he also stressed the common ground with the Mosaic religion, Islam, and what he considered to be Chinese religion.

6. Concluding remark

The topic of this article has led us to concentrate on Leibniz's rationalism. But there are some tendencies in his thought which should at least be mentioned in order to avoid a biased view. In particular the idea of individuality is certainly as important in his thought as rationality. There is no uniformity in the world; not only is every living being different from every other, but any two leaves, any two eggs, and in general any two bodies are different from each other [15, IV, p. 554]. So the power of rational abstraction and similarity everywhere is supplemented by the infinite variety of individual entities. The importance of "small perceptions" in his philosophical thought may be another example. Finally, Leibniz's idea of an infinity of living creatures in every particle of matter might even shock today's rationalists. And one does not expect a typical rationalist to argue for mathematical investigation of games by saying: "The human mind shows itself more clearly in games than in serious matters".[17]

[17] [16, ser. VI, vol. 6, p. 466]: "l'esprit humain paroissant mieux dans les jeux que dans les matieres les plus serieuses". Cf. also [15, IV, p. 570; VI, p. 639].

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CHAPTER 26

Berkeley's Defence of the Infinite God in Contrast to the Infinite in Mathematics

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Edited by T. Koetsier and L. Bergmans

1. Berkeley's intent

George Berkeley, Anglican theologian and Bishop of Cloyne from 1734–1753, fostered an ambiguous attitude towards mathematics and mathematicians. His first publications (Arithmetica/Miscellanea mathematica) strove to promote mathematical enthusiasm in students, since he highly valued mathematics for its educational merits. It would therefore be mistaken to surmise a broad aversion regarding mathematics in Berkeley's thinking. But in spite of those early writings on mathematics and science, Berkeley was convinced that he must safeguard the Christian revelation against hostile assault. Foremost, he considered the deists and freethinkers to be opponents of religion, viewing some of them as disguised atheists. In reality, these spiritual trends did not constitute one uniform movement. Some authors dealt with the liberty of religion, desiring emancipation from regimentation by ecclesiastical authority; others denied various dogmas (e.g. the Trinity, miracles), and yet others rejected Christian revelation in favour of rational or "natural" religion. Berkeley's mathematical and scientific attainments can be appropriately appreciated when considered in light of his motivation to defend the Christian faith, although they should not be judged solely from that perspective. Though it is perhaps true that the impetus to Berkeley's conceptions and reasonings is founded deeply in his religious convictions, it demands examination from an epistemological viewpoint. The logic of discovery differs from the logic of justification. Berkeley himself claimed to proceed scientifically rather than dogmatically.

As we know from Berkeley's early private notes (*Philosophical Commentaries*, hereafter cited as *PC*), at the time he wrote them he had already planned to impair the great reputation of mathematicians–referring to them invectively as "nihilarians"; the denotation "freethinker" was not yet common. Berkeley's essays published in the *Guardian* (1713) attacked the freethinkers as "minute philosophers", unable to see the beauty of the whole universe as would flies on a cathedral pillar, and recommended that they expand their narrow minds by studying astronomy. "Astronomy opens the mind, and alters our judgment, with regard to the magnitude of extended beings; but Christianity produceth an universal greatness of soul". Berkeley answers the presumptiveness of the scientists with an arrogance of the Christian religion, which "ennobleth and enlargeth the mind beyond any other profession or science whatsoever" (Works, VII, p. 208). Around 1730, when Berkeley thought that the failure of his Bermuda project—the foundation of a college for Christianising America—could be due to resistance by the freethinkers, his opposition to them grew. The unfortunate disappointment Berkeley sustained at Newport was the immediate occasion for writing his great works concerning freethinkers (*Alciphron*, *Analyst*), but the essential thrust for publishing these books is readily recognisable in his earliest writings. The underlying dichotomy, i.e. the relation between religion and science, disquieted Berkeley himself, who was a scientist as well as a pious man. Therefore, he asked explicitly: "Shall we not admit the same method of arguing, the same rules of logic, reason, and good sense, to obtain in things spiritual and things corporeal, in faith and science?" (*Alciphron*, VII, Sect. 8). He replies to this rhetorical question by calling his opponents to the common "maxim, that the same logic which obtains in other matters must be admitted in religion" (ibid.). "There is no need to depart from the received rules of reasoning to justify the belief of Christians" (*Alciphron*, VII, p. 15). Berkeley's disregard for mathematicians is not based on a vague appeal to the irrationality of religion, as might be found in obscure mysticism.

Fig. 1. George Berkeley. Science Photo Library.

On the contrary, he requests that mathematicians observe the rationality which they themselves claim, as he would use it in theology. Religion must not admit any point contrary to reason, but must be above reason. Moreover, mysteries might "with better right be allowed of in Divine Faith than in Human Science" (*Analyst*, Sect. 50, Qu. 61 f.).

In Berkeley's view, the scope of divinity was threatened by modern science on points such as these:

(1) Modern mathematics increasingly occupied the domain of infinity, which formerly had been reserved for theology.
(2) Regarding the paradigm of exactness (*akribeia*), mathematics claimed more authority than was conceded to theology.
(3) The marvellous results reached by the mathematical sciences in terms of theoretical explanation of the world as well as in mathematically influenced technical practice diminished esteem for the Christian religion of revelation, being based on mysteries.
(4) Materialistic and physicalistic philosophies were supported by such successes, and therewith undermined the prestige of the Christian religion, since its God is spirit (Joh. IV, 24).

Altogether, Berkeley considered most of the mathematicians as potential foes of the Christian religion, but faced with the results of modern science, he was compelled to specify his opposition to "the" mathematicians.

In his *Philosophical Commentaries* we find entry No. 375: "Mathematicians have some of them good parts, the more is the pity". On several occasions Berkeley refers to Newton as the most famous exception to contemptible mathematicians (No. 372). He explicitly mentions his high esteem for Newton's "original and free genius", characterising him as "an extraordinary mathematician, a profound naturalist, a person of the greatest abilities and erudition" (*Defence*, Sect. 13 and Appendix, Sect. 2), and particularly advocates Newton's religious attitude, which had been derided by mathematicians (*Analyst*, Sect. 50, Qu. 58). Whereas many of the most famous mathematicians—e.g. Pascal, Leibniz, Euler—combined their mathematical pursuits with active engagement in faith, others more or less kept their distance from the church. Edmond Halley, former professor of geometry at Oxford, was among them. Generally he was not assumed to be a Christian, but he was considered an extraordinary astronomer with solid knowledge of the world and splendid skills in methods of mathematical proof. Since in matters of faith personal reputation is not unimportant, Berkeley was not unreasonable regarding that suspicion. Once Addison told him about a learned mathematician who supported his infidel attitude concerning another infidel mathematician. As Berkeley's first biographer, Stock, reported, Halley could be identified as this infidel mathematician influencing Samuel Garth, who was both a doctor and a poet and acquainted with both Berkeley and Addison. Before he died in 1719, Garth met Addison but refused all religious discussions, being convinced by Halley that the Christian doctrine would be incomprehensible and that religion was nothing more than deception (Works IV, 56 f.). The Halley episode had long passed when the *Analyst* was published; thus it was not the immediate occasion for writing that book, but in any case Berkeley did consider Halley an important representative of the freethinkers.

In Berkeley's view, most mathematicians—with few exceptions—constitute a conspired scientific community, structurally no different than a communion, an idea similar to the slightly modified notion of scientific community as applied to scientists by Thomas S. Kuhn.

In his controversy opposing the mathematicians Berkeley pursues several aims:

(1) He tries to undermine the significant authority exercised by the mathematicians.
(2) Mathematics and theology should be distinctly separated with the former subordinate to the latter.
(3) It must be shown that theology can be justified as thoroughly as mathematics or physics. In support of this point Berkeley refers to the struggle between the Cartesians, the Leibnizians, and the Newtonians over the physical notion of force. Their instances of "force" were as opaque as the theological notion of "grace" (*Alciphron* VII, Sect. 6 f.).
(4) On the levels of ontology and theory of science Berkeley tries to overcome atheistic materialism with his philosophy of immaterialism. "Existence is *percipi* or *percipere* or *velle* i.e. *agere*" (*PC*, No. 429, 429a).

2. God and infinity

In European philosophy the question of infinity gained increasing significance beginning with the patristical thinkers of the church, who made infinity to be the most important attribute of God. Enthusiastic study of infinity reached a preliminary culmination during the age of scholasticism, then it entered mathematics and science in the seventeenth centuries, when not only the concept of Newtonian space was developed, but also infinity of science itself and that of scientific evolution was welcomed, whereas Nicolaus Cusanus had written as a man of his time: mathematical things are finite. Due to that development the groups of theologically-orientated scholars split into two different parties: some authors recognised the danger that the outstanding attribute of God, i.e. infinity, could be claimed by the other sciences; the rest of the scholars considered infinity in mathematics or sciences to be a mundane trace of God. Berkeley demanded that the infinite be reserved for God alone. Therefore he rejected any acceptance of infinity in mathematics (infinite space, infinite extension, infinite division etc.). He also refused entirely every idea intending to attribute any extension to God, because an extended God would be in danger of becoming a mundane thing, which, in the end, could be eliminated by materialistic science.

In *De Motu* (Sect. 54) it is evident that Berkeley considered Newtonian space to be a threat to theology because this space was considered to be something "which cannot be annihilated", and that "it exists necessarily of its own nature, and that it is eternal, uncreated, and even shares the divine attributes". In a letter to Samuel Johnson (1730) Berkeley wrote: "I know some late philosophers have attributed extension to God, particularly mathematicians, one of whom, in a treatise De Spatio Reali, pretends to find out fifteen of the incommunicable attributes of God in Space. But it seems to me that they all being negative, he might as well have found them in Nothing". The philosopher in question is Joseph Raphson, who in a treatise characterised paradigmatically the absolute space with divine attributes, namely as something *actu infinitum* and *aeternum*. From Berkeley's early paper *Of Infinites* it is obvious that he had knowledge of Raphson's book at that time. The infinity of space and the Spinocistic tendency to an extended god repeatedly irritated Berkeley. For the same reason one of the rhetorical questions at the end of the *Analyst* is: "Whether extension can be supposed an attribute of a Being immutable and eternal?" Berkeley rejected any endeavour to make plausible the religious mysteries by mathematical analogies, such as was undertaken by John Wallis, the famous mathematician and theologian, who illustrated the Holy Trinity in terms of a three-dimensional cube, which is one cube, but has three dimensions (length, breadth, height). In Berkeley's opinion mathematics should not be misused for religious analogies, and, he argues, someone who admits such absurdities as an abstract general triangle into his demonstrations should at least also admit the Holy Trinity itself (*Alciphron* VII, Sect. 8). Berkeley held the notion of an abstract general triangle to be empty because no triangle exists which is neither acute-angled nor right-angled nor obtuse-angled (*Defence*, Sect. 45). To reserve infinity for God alone it is necessary that all of the mundane sciences, including mathematics, either renounce the infinite or limit themselves to the application of potentially infinite considerations, only, i.e. that they admit quantities of arbitrary, but finite, magnitude. For that purpose Berkeley was compelled to restrict each mathematical reasoning to finite quantities. Therefore, he criticised the doctrines of calculus for both the Newtonian theory of fluxions and the Leibnizian infinitesimals. In his opinion, which was founded on his theory of perception, geometry should employ only perceptible quantities of finite extensions. Since in the perception of extended things there are minimal quantities, Berkeleyan geometry would be restricted to discrete quantities. Nevertheless, Berkeley admits the divisibility of arbitrarily assignable quantities. To divide a minimal quantity it is necessary to replace it by a more extended one. This representative is further divisible. If needed, these proceedings may be iterated. Because of these sequences of representatives Berkeley applies the usual expression used for divisibility of geometrical extension, but in his view that way of speaking did not force one to accept quantities which are actually infinite. Accordingly, Berkeley asked rhetorically at the end of the *Analyst*, "Whether there be any need of considering quantities either infinitely great or infinitely small?"

With his perspicuous view of the difficulties in which eighteenth century mathematics was involved, Berkeley was concerned with the problem of the angle of contiguity, i.e. the angle between a circle and its tangent. It seemed to be evident that these lines constitute an angle but it also seemed to be evident that this angle is smaller than any assignable angle. Whereas John Wallis concluded that the angle of contiguity should be a *non-quantum*, Berkeley held that problem to be a further argument for his assertion that the notions in mathematics were not sufficiently clear (*Alciphron* VII, Sect. 15). To substantiate that imputation he referred to the definition of an angle delivered by Pardies: "When two lines meet in a point, the aperture, distance, or inclination between them is called an angle" (*PC*, No. 432). According to Berkeley's opinion, the notion of an angle of contiguity must be rejected generally, because it contradicts the Aristotelian principle of homogeneity by putting in relation a right line and a curve and that relation is said to be logically unacceptable (*PC*, No. 309). Berkeley attacks the use of infinites not only in mathematics but also in physics. Therefore, he dismisses speaking of the "infinity of impact", which was under discussion according to the conviction that the force of impact or the kinetic energy is infinitely greater than the static force (*De Motu*, Sect. 14; *Alciphron* VII, Sect. 6). To avoid the notion of limit involved in the integrations corresponding thereto Berkeley insisted on the Aristotelian doctrine of heterogeneity of "forces".

3. Mathematical exactitude

Berkeley recognises clearly that the mathematicians' endeavour for exactitude shares common roots with the subtlety exercised in scholastics. Thus mathematicians behaved similar to scholastics (*PC*, No. 327). Also in the doctrine of motion Berkeley tries to argue that modern scientists approach Aristotelian scholastics (*De Motu*, Sect. 19). As they follow Aristotle, the mathematicians similarly blindly follow Newton with "ipse dixit" (*Defence*, Sect. 50; *Alciphron* VII, Sect. 9).

Even if one were to object that, contrary to the mathematicians the scholastics were dominated by dogmatic thinking, Berkeley would reply that even mathematical books could be read subject to dogmatism, namely following the author with no comprehension (*PC*, No. 335). And he imputes such dogmatic narrow-mindedness to most of the mathematicians in their blind admiration for Newton (*Analyst*, Sect. 50, Qu. 15; *Defence*, Sect. 3 and 7). Mathematicians of this type, Berkeley thought, should be designated as "believers of mysteries" (*Defence*, Sect. 50). In any case, reverence for mathematicians is unwarranted, since one could achieve the same results by common sense (*PC*, No. 368).

Previously, in his *Philosophical Commentaries*, Berkeley had censured the lack of exactitude in mathematical works of his time. He especially reproved the use of infinitesimals in mathematics. In his paper *Of Infinites* he attacks with disdain the mathematics of infinitesimals with respect to Leibniz, who had rejected scrupulosity in mathematics. Indeed, Leibniz intended thereby to justify formulas such as

$$x = x + \mathrm{d}x.$$

Clearly, Berkeley recognised that it seems to be an inconspicuous postulate used in calculus but nevertheless it was employed by the mathematicians as “a corner-stone or ground-work of their speculations” (*Analyst*, Sect. 6). On the contrary, he insisted on the principle of exactitude (*akribeia*) as a scientific pattern prevailing since antiquity (Cf. Aristotle, *Metaphysics* 982a13; *De Anima* 402a2). In Berkeley’s view the mathematicians using calculus abandoned the endeavour for exactness by neglecting some quantities (i.e. infinitesimals) used in their calculations. In any case the infidelity of freethinking is not a necessary consequence or result of mathematical exactitude. That mathematics by itself would imply disbelief is an opinion Berkeley explicitly rejected (*Defence*, Sect. 5).

He accepted not in the least for himself the part of a zealous inquisitor, which was imputed to him by his immediate opponents, namely Jurin and Walton (*Defence*, Sect. 8 f.; Appendix, Sect. 4).

4. The astounding results of modern mathematics

Even Berkeley was not able to disavow the great results reached by the new calculus. Therefore, he was compelled to deliver a justification for the supposed fact that some methods, which he held to be unfounded—and which were unfounded in his time!—nevertheless produced many useful results. By his schooling in logic he knew that true conclusions may be drawn from false premises by logically correct inferences. Using that knowledge he asserted that the mathematicians would obtain their great results with the help of a “compensation of error”.

Berkeley considered the computation of the subtangent of a parabola as a plain example (for the case of derivation).

By similar triangles

$$\frac{\mathrm{TP}}{y} = \frac{\mathrm{d}x}{\mathrm{d}y + z}$$

Berkeley’s argumentation is as follows. On the one hand, the mathematicians take $\mathrm{d}y$ instead of $\mathrm{d}y + z$, i.e. a quantity less than the true quantity, putting

$$\frac{\mathrm{TP}}{y} = \frac{\mathrm{d}x}{\mathrm{d}y}. \tag{1}$$

On the other hand, $\mathrm{d}y$ may be calculated by their rules of derivation from the original equation $y^2 = px$, namely,

$$2y\frac{\mathrm{d}y}{\mathrm{d}x} = p,$$

therefore

$$\mathrm{d}y = \frac{p\,\mathrm{d}x}{2y}. \tag{2}$$

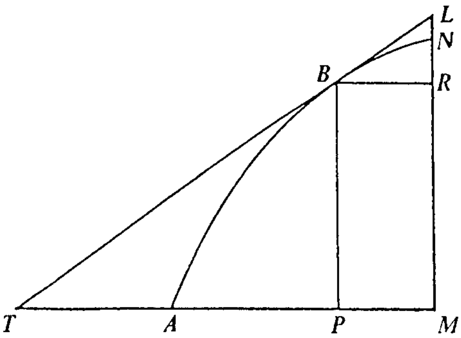

Fig. 2. $y^2 = px$, TP = subtangent, AP = x, PB = y, PM = BR = dx, RN = dy, $NL = z$.

But this value—Berkeley argues—is bigger than the true value, as may be seen by substitution of y by $y + \mathrm{d}y$ and y by $x + \mathrm{d}x$, because then we have

$$y^2 + 2y\,\mathrm{d}y + (\mathrm{d}y)^2 = p\,\mathrm{d}x + p\,\mathrm{d}x,$$

therefore

$$\mathrm{d}y = \frac{p\,\mathrm{d}x}{2y} - \frac{(\mathrm{d}y)^2}{2y}.$$

If y designates a finite value, in accordance with the figure, then the values of dy both in (1) and (2) are wrong, and the errors do compensate each other, as Berkeley emphasised. Some modern interpreters met with difficulties in understanding Berkeley's argumentation because they erroneously took dy and dx to be differentials resp. infinitely small quantities, but Berkeley insists strictly on the suppositions made at the beginning, where the quantities dx, dy, and z were introduced as quantities differing from zero.

Berkeley's intent was to use only finite quantities and in fact there is neither any infinitesimal nor any limiting value included in his explanations. Indeed, some errors may be found in his considerations but it is strange to see that they were not clearly recognised in any case by either Berkeley's contemporaries or by his modern interpreters. For example, Berkeley never discussed the question of whether the supposed method of compensation of error, which he demonstrated in the case of the parabola, would also work in a more general case. He was never asked such questions. Strangely enough, his opponents did not attack him where he was mathematically most vulnerable. One of the most famous of these weak points is his consideration in Sections 28 and 29 of the *Analyst*, where his aim is to demonstrate the "derivation" of x^n without the application of infinitesimals. There he is comparing some analytical terms with some geometrical figures (surfaces) and his opinion is that it would be possible to adjoin homological terms corresponding to each other in a natural way. He does not drop any hint of a definition of "homology" between an algebraical term and a geometrical quantity, much less how to find it out. Perhaps Berkeley was seduced on this point by the antique principle of homogeneity allowing the comparison of solely homogeneous quantities, i.e. lines with lines, surfaces with surfaces, and bodies with bodies. It seems that Berkeley's opponents did not notice the weakness of this argumentation or that they recognised that the evidence of an error in one of Berkeley's demonstrations would not in the least confirm the foundations and methods of their calculus.

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CHAPTER 27

Leonhard Euler (1707–1783)

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MATHEMATICS AND THE DIVINE: A HISTORICAL STUDY
Edited by T. Koetsier and L. Bergmans

> Every man may rest assured that, from all eternity, he entered into the plan of the universe. [...] How ought this consideration to increase our confidence and our joy in the providence of God, on which all religion is founded!
>
> Leonhard Euler, *Letters to a German Princess*, January 3, 1761.[1]

1. Historical background

The Enlightenment changed the established ways of living. This movement of thought and belief was concerned with concepts of God and reason. It was convinced that every problem could be settled by the proper use of reason and therefore everything was "dragged" before the court of pure reason. The established ways of thinking and believing were also under attack because of the rise of modern science; and as a by-product of science anti-Christian rationalism arose. It is often assumed that the essential developments in the Age of Reason took place in philosophy (and it is then named after Voltaire, Locke or some other philosopher). It was, however, also a great time for the natural sciences, a time in which a new framework of scientific thinking was established. The new mechanics of Isaac Newton (1643–1727) are a beautiful example. Newton's theory offered an impressive explanation of the way the universe functions. It was the progress of the natural sciences that influenced the rise of industry and, as a consequence, increasingly determined the conditions of human life up to this day.

What about Christian faith under the rule of the sharp sword of pure reason? Did the spirit of the time, which contrasted sharply with religion, allow for any Christian belief or any "enlightened religion"? The life of Leonhard Euler is an interesting example in this respect. He was without doubt the leading natural scientist of the 18th century. A century after Euler's death the German historian Hermann Hankel (1839–1873) correctly remarked that Euler was the best representative of the scientific consciousness in the middle of the 18th century.[2] In our day, Clifford Truesdell (1919–2000) has estimated that in the 18th century Euler alone wrote about one third of all the mathematical works (including mathematical physics) that appeared in that period. It seems interesting to investigate Euler's religious belief, the belief of an outstanding natural scientist of the day. Indeed, the relationship of the Christian religion and the natural sciences during the Enlightenment determined the entire life of Leonhard Euler.

2. Life

Euler's father, Paul Euler (1670–1745) was a protestant minister. His mother, Margaretha Brucker (1677–1761), was the daughter of a minister. Euler was born in Basel in 1707, but in 1708 the family moved to Riehen. In this little village, a place two or three miles

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[1] Chaque homme peut être assuré, que de toute éternité il est entré dans le plan du monde. [...] Combien cette considération doit-elle augmenter notre confiance et notre amour pour la providence Divine, sur laquelle est fondée toute la religion.

[2] H. Hankel, *Untersuchung über die unendlich oft oszillierenden und unstetigen Funktionen*, Mathematische Annalen 20 (1882), 63–112, quotation p. 64: "Euler, der das wissenschaftliche Bewußtsein in der Mitte des vorigen Jahrhunderts am vollständigsten vertritt [...]".

Fig. 1. Leonhard Euler (1707–1783).

from Basel with about one thousand inhabitants, Leonhard Euler grew up with his parents and later with his two younger sisters in the two rooms of the parsonage. He was surrounded by educated people, most whom were both ministers of religion and mathematicians. His learned father, who had been a pupil of the famous mathematician Jakob Bernoulli (1645–1705), gave Leonhard his first instruction, including mathematics. Later, when the young Leonhard moved to Basel to attend a grammar school in which mathematics was not taught, he received private instruction in mathematics from a Calvinist minister, Johann Burckhardt (1691–1743).

Among the mathematicians and theologians Euler encountered in his youth, it seems that, in addition to his father, it was above all Burckhardt who played an important role in Euler's education and in forming Euler's beliefs. The young Leonhard had intensive, liberal discussions on the modern views of the planetary system (Copernican theory) and of the biblical account of creation, and more generally of the relationship of scientific knowledge and religious belief. On the other hand, the Calvinist clergy—even its most pious members—strongly advocated "law and order", mainly for political reasons. This created an ambiguous religious situation, and it is this spirit of submission to religious discipline in which Leonhard Euler grew up and that he upheld later in Berlin and St. Petersburg. He remained a devout Calvinist all his life. The doctrines he held throughout his life were those of Calvinism: he was pious and full of devotion. Every evening Euler conducted family prayers for his whole household, usually finishing with a sermon. He handled parish affairs in the same spirit to the very end. In Berlin he was a member of the parish council (Gemeindeältester) and it was in the spirit of his father that he reformed certain affairs like the instruction given to candidates for confirmation and that he promoted the printing and distribution of sermons. When Euler gave the example of "a man who, on hearing a beautiful sermon, is affected by it, repents, and is converted [...]"[3] in the Letters to a German Princess, January 6, 1761, he stressed the value of a good sermon. He was convinced that man can do nothing by himself; everything depends on divine grace. Therefore Euler advocated prayer (letter of January 3, 1761). Although the human will is free, by supplying motives God can influence human decisions.

Paul Euler wanted his son to follow him into the church. In October 1720, at the age of 13, as was quite usual then, Leonhard Euler enrolled in the University of Basel, which had been founded in 1460 as the first university in Switzerland. The famous Dutch scholar Erasmus of Rotterdam (1469?–1536) had taught at this university, making the city a centre of humanism in the 16th century. A central purpose of humanism was to serve religion. At this traditional university Euler started to study theology. Although the glorious days of Erasmus were long past, under Johann Bernoulli (1667–1748), a younger brother of Jakob Bernoulli, the University of Basel again became a centre of learning in Europe, this time with the main focus on mathematics. At this small university, then one of the smallest in Europe with 19 professors and only about 100 students, it was inevitable that the student Leonhard Euler and the leading mathematician, Johann Bernoulli, would meet each other. Indeed, Euler's mathematical abilities soon earned him the esteem of Bernoulli who advised him to study mathematics. Leonhard Euler's passion for mathematics grew stronger and stronger, so that his father finally gave in and allowed him to follow the bent of his genius. In 1723 Euler, now a Master of Arts, left the Faculty of Theology and began the study of mathematics under the supervision of Bernoulli. In 1727 Euler moved to St. Petersburg. First he became an associate of the St. Petersburg Academy of Sciences, but in 1733 he succeeded Daniel Bernoulli (1700–1782) to the chair of mathematics.

Theology remained one of Euler's favorite interests, and it would be wrong to regard Euler's transition from theology to mathematics as implying that he lost interest in theology. On the contrary, theology remained very important for him. It was Galileo Galilei (1564–1642) who had spoken of the two books that God had given mankind: Holy Writ and Nature. The language of the book of nature, the language of a world well-ordered by the Creator, was mathematics, the science of order. The study of mathematics would lead necessarily to an insight into the Creator's construction programme. However, the rise of natural science had actually led to a serious problem: the success of the scientific method pressed in the direction of the autonomy of science, free from religious influence, and in the direction of Deism or even Atheism. Later, above all in the environment of the liberal deïst, King Frederick II (1712–1786), in his Berlin period (1741–1766), Leonhard Euler defended the Christian faith against freethinkers and atheists. He also strongly opposed the rationalism (monadology) of Leibniz and Wolff,[4] and he took part in several

[3]"Si, par exemple, un pécheur en entendant un beau sermon, en est frappé, rentre en soi même et se convertit [...]".

[4]The monadology is Leibniz's metaphysics which is expressed in its mature form in the correspondence with Burcher de Volder (professor of philosophy at the University of Leyden). Monads are the basic individuals, they are immaterial entities without spatial parts. Each monad is unique and indestructible, distinguished from other monads by its degree of consciousness. God created the universe in such a way that the monads are perfectly

fierce philosophical–theological debates, the most sensational of which was the ill-famed controversy on Maupertuis' celebrated Principle of Least Action. The debate became intense and turned into an outright conflict. Voltaire (1694–1778) and even King Frederick II were involved. On this occasion Leonhard Euler supported the President of the Prussian Academy, Pierre-Louis Moreau de Maupertuis (1698–1759).[5] Euler's physico-teleological attitude was rather ambiguous: because he interpreted the principle as a theological one, he was compelled to defend religion against the hated ideology of free-thought; on the other hand, he correctly formulated the principle for some cases in dynamics, and he strongly believed that nature generally operates in such a teleological way. This belief is expressed early on in his book "Methodus inveniendi" (A method for discovering curved lines, 1744), a book on the calculus of variations:

> "For, since the fabric of the Universe is most perfect and the work of a most wise Creator, nothing at all takes place in the Universe in which some rule of maximum or minimum does not appear".[6]

Although the King of Prussia often employed Euler to perform calculations with respect to money, the fountains of Sans Souci (the summer residence of the king), the Finow canal, salt mines, calendars, maps, and other practical problems, Euler was unpopular at Frederick's court, mainly because Frederick preferred French culture and its representatives: the deist Voltaire, and the Roman Catholic Maupertuis. King Frederick, but also many others, in Berlin were put off by Euler's penetrating attacks against free-thought ("the rabble of freethinkers (die Rotte der Freygeister), these wretched people (diese elenden Leute)").[7]

The situation in Berlin finally led to Euler's return to Russia, where he had started his career and where he was to end it. Blind, but active to the end, Leonhard Euler died in St. Petersburg on September 18, 1783, at the age of 76 years, 5 months, and 3 days. $22\frac{3}{4}$ years earlier, on January 13, 1761, in his "Letters on different subjects in natural philosophy addressed to a German Princess", he had written on the subject of death:

> "It is also the influence of the soul upon the body which constitutes its life, which continues as long as this union subsists. [...] Death, then, is nothing but the dissolution of this union, and the soul has no need to be transported elsewhere; for as it resides in no place, all places must be indifferent to it. [...] This supplies us with a clear elucidation of the omnipresence of God: his power extends to all bodies contained in the universe. [...] God is everywhere present. [...] It is only bodies that cannot be in two places at the same time; but there is nothing to prevent spirit, which has no relation to place. [...] We can form some idea of the state of the soul after death. As the soul, during life, acquires all its knowledge by means of the senses, being deprived by death of the information communicated through the senses, it no longer knows what is happening in the material world; this state is more or less similar to that of a man who suddenly becomes blind, deaf, dumb, and deprived of the use of all the other senses. [...] Sleep likewise furnishes

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synchronised in preestablished harmony (illustrated by two well-synchronised clocks). In the 18th century harmony is often understood in the sense of action (effort). Euler rejected the monadology because in the system of preestablished harmony no freedom of the will would be possible (no spirit can act on bodies). Freedom, however, is a condition of spirits, just as extension is one of bodies (see Letters December 9 & 13, 1760; January 3, 1761).

[5] The principle of the least action is based on the idea that whenever something moves in nature, this happens in the most efficient way, i.e. some quantity assumes a minimum value. Maupertuis was in fact accused of having taken this principle from Leibniz, which he had not.

[6] Cum enim Mundi universi fabrica sit perfectissima atque a Creatore sapientissimo absoluta, nihil omnino in mundo contingit, in quo non maximi minimive ratio quaepiam eluceat. (Methodus inveniendi. Additamentum I. Opera omnia I/24, p. 231)

[7] Rettung der göttlichen Offenbarung, §53; also in: Opera omnia III/12.

> us with something like an example of this state. [. . .] Thus, after death, we will find ourselves in a more perfect state of dreaming, which nothing shall be able to discompose. [. . .] And this, in my opinion, is just about all we can say of it, at least with any appearance of reason".[8]

3. Euler on religion

During Euler's lifetime the role of Christianity became problematic: atheism and unorthodox religious attitudes such as natural theology[9] and deism appeared, and conflicts arose between them and traditional Christianity. Euler took part in these religious discussions and conflicts. His religious belief is expressed chiefly in two publications: the celebrated "Letters to a German Princess" (1760–1762) and the "Rettung der göttlichen Offenbarung gegen die Einwürfe der Freygeister" (The Deliverance of Divine Revelation from the Censures of the Freethinkers, 1747), and it is touched on in some philosophical controversies, especially in the debate on Maupertuis' famous principle of least action (1744–1759). (Incidentally, the flagship publication of the Age of Enlightenment, the French "Encyclopédie" in 17 volumes, was published between 1751 and 1772.) While atheists and deists attacked the literal interpretation of the Scriptures as divine revelation, Euler defended the trustworthiness of the Bible by comparing it with that of science. There are contradictions and paradoxes in science too, even in mathematics, and yet no sane person will ever doubt the trustworthiness of science. The same approach is found in the critique of the calculus by George Berkeley (1685–1753): "Whether mysteries may not with better right be allowed of in Divine faith than in Human Science?" (The Analyst, Question 62).

For Euler there are three fundamentally different classes of knowledge: there is the truth of the senses (experience), that of understanding (reasoning), and that of belief (history), and the true foundation of human knowledge is different for each of them. Each class of knowledge requires its own type of proof of its trustworthiness:

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[8]C'est aussi l'influence de l'ame sur le corps qui en constitue la vie, qui dure aussi longtems que cette liaison subsiste. [. . .] La mort n'est donc autre chose que la destruction de cette liaison: ensuite l'ame n'a pas besoin d'être transportée autre part; car puisqu'elle n'est nulle part, elle est indifférente à tous les lieux. [. . .] Cela nous fournit un bel éclaircissement pour concevoir comment Dieu est partout; c'est que son pouvoir s'étend à tout l'univers et à tous les corps qui s'y trouvent. [. . .] Dieu est présent par tout. [. . .] Ce ne sont que les corps qui ne peuvent être en même tems en deux endroits, mais pour les esprits qui n'ont aucun rapport aux lieux en vertu de leur nature. [. . .] On peut se former quelque idée de l'etat de l'ame après la mort. Comme l'ame pendant la vie tire toutes ses connoissances par le moyen des sens, étant dépoulliée par la mort de ce rapport des sens, elle n'apprend plus rien de ce qui se passe dans le monde matériel; elle parvient à peu près dans le même état, où se trouveroit un homme, qui seroit devenu tout d'un coup aveugle, sourd, muet, et privé de l'usage de tous les autres sens. [. . .] Le Sommeil nous fournit aussi un bel échantillon de cet état. [. . .] Ainsi après la mort nous nous trouverons dans un état des songes les plus parfaits, que rien ne sera plus capable de troubler. [. . .] Et c'est à mon avis à peu près tout ce que nous saurions en dire de positif.

[9]Natural theology in the 18th century bases itself on knowledge of God drawn from nature and can be characterised as establishing religious truth (existence of God, immortality of the soul, God's providential control of the world, etc.) by rational arguments and without referring to God's revelations. Natural theology is distinguished from revealed theology as well as is contrasted with it (no access to the existence of Jesus Christ).

> "*I have seen or felt*, is the proof of the first class; *I can demonstrate it*, is that of the second: we likewise say, *I know it is so*. Finally, *I receive it on the testimony of persons worthy of credit*, or *I believe it on solid grounds*, is the proof of the third class". (April 4, 1761)[10]

These three different species of knowledge, which comprehend all human knowledge, must be considered equally certain. They correspond to the only sources of human knowledge, derivation from our own experience, reasoning, and reports of others.[11] Religious belief belongs to the third kind of knowledge. But, how to handle the different kinds of knowledge? Euler remarked:

> "The three sources from which our knowledge is derived all require certain precautions, which must be carefully observed, in order to acquire assurance of the truth; but it is possible, in each, to carry matters too far, and one should always steer a middle course. The third source clearly proves this". (April 18, 1761)
> "Therefore, as logic prescribes rules for correct reasoning, where intellectual truth is concerned, there are equally certain rules for the first source, that of our senses, and for the third, that of belief". (April 11, 1761)[12]

The theology of revelation explains the articles of faith in that it rests upon the Bible as the only source of religious truth (*sacra doctrina*); on the other hand, natural theology approaches God and his creation with the help of pure reason. It is exactly this difference that Euler had discussed earlier with his father and the professors of theology in Basel: Did God in the Bible reveal the physical structure of our world? Euler rejected any literal interpretation of biblical texts as scientific explanations, and in his "Rettung" he gave reasons why God has not supplied us with a revelation of the fabric of the universe. The opening line reads: "Our mental powers manifest themselves in two capacities, one of which is called the understanding (Verstand), the other the will (Wille). Because all supreme happiness consists in perfection, the supreme happiness of the soul can only be promoted by the perfection of the understanding and the perfection of the will".[13] The understanding serves to find truth, and by the will our duties are then deduced from truth. The aim of life is bliss or complete happiness and both powers serve to complete our bliss. This means that to increase our bliss we have to perfect both powers. But God's revelation is concerned

[10] *Je l'ai vu ou senti*, est la preuve de la premiere classe: *Je puis le démontrer,* est la preuve de la seconde classe, de laquelle on dit aussi, qu'on sait les choses: Enfin, *Je le tiens par le témoignage de personnes dignes de foi: ou je le crois par des raisons solides*; c'est la preuve de la troisieme classe.

[11] With a similar intention to distinguish knowledge and faith Immanuel Kant (1742–1804) noted in his "Critique of Pure Reason" (second edition 1787): "I have therefore found it necessary to abolish knowledge in order to make room for faith". (Ich mußte also das Wissen aufheben, um zum Glauben Platz zu bekommen. In: Kritik der reinen Vernunft, 1787. Vorrede zur 2. Auflage, B XXX.)

[12] Toutes les trois sources d'où nous tirons nos connoissances, exigent chacune certaines précautions, qu'on doit bien observer pour être assuré de la vérité, mais dans chacune on peut pousser la chose trop loin, et il faut toujours tenir un certain milieu. La troisieme source ne prouve cela que trop ouvertement.—Donc comme la Logique prescrit les regles des raisonnemens justes qui nous mettent à l'abri de l'erreur à l'égard des vérités intellectuelles, il y a aussi des regles également certaines, tant pour la premiere source, de nos sens, que pour la troisieme, de la foi.—Euler used rigorous demonstrations in his philosophical (or theological) expositions. Because Kant's work is so well known Euler's role and importance are overshadowed.

[13] §1. Die Kräfte der Seele äussern sich in einem gedoppelten Vermögen, davon man eines den Verstand, das andere den Willen nennet. Da nun alle Glückseeligkeit in der Vollkommenheit bestehet, so kan die Glückseligkeit einer Seele nicht anders als durch die Vollkommenheit des Verstandes, und durch die Vollkommenheit des Willens befördert werden. (Rettung, §1; Opera omnia, III/12, p. 268.)

with our will only, not with our understanding. Why is this? Our complete happiness rests upon the complete submission of our will to God's will. However, the more we extend our knowledge, the more we extend our duties which depend on the state of our knowledge. A revealed knowledge, i.e. an absolute knowledge, would involve an infinite set of duties which would make complete acquiescence impossible. To prevent this dilemma, in his infinite goodness God has taken our limited mental capacities into account. In addition, the biblical revelation not only proscribes duties but also supplies advice.

Euler's rejection of metaphysics rests on this insight: our ability to acquire knowledge is too limited to supply a sufficient theory of cognition. However, we can admire the eternal works of God.[14]

While the Prussian Kant said that the majesty of duty has nothing to do with enjoyment of life (Critique of Practical Reason), according to the Swiss Euler we will taste enjoyment, but not before there is a complete correspondence of human and divine will. In Euler we find Descartes' dualism of body and soul, but in an extended form. In a letter concerning the nature of spirit we read:

> "To think, to judge, to reason, to feel, to reflect and to will, are qualities incompatible with the nature of bodies; and beings invested with them must be of a different nature. Such are souls and spirits; and He who possesses these qualities in the highest degree is God. There is, then, an infinite difference between body and spirit. Extension, inertia, and impenetrability—qualities which exclude all thought—are the properties of body; but spirits are endowed with the faculty of thinking, of judging, of reasoning, of feeling, of reflecting, of willing, or of determining in favour of one object over another. There is here neither extension, nor inertia, nor impenetrability; these material qualities are infinitely remote from spirit. [...] It is asked, What is spirit? I acknowledge my ignorance in this respect; and I reply, that we cannot tell what it is, as we know nothing of the nature of spirit. [...] This union of the soul with the body undoubtedly is, and ever will be, the greatest mystery of the Divine Omnipotence—a mystery which we shall never be able to unfold. [...] These two species of beings are nevertheless most intimately united; and upon their union principally depend all the wonders of the world, which are the delight of intelligent beings, and lead them to glorify their Creator. It is certain that spirits constitute the principal part of the world, and that bodies are introduced into it merely to serve them". (November 29, 1760)[15]

[14]It is precisely this attitude that is expressed by Kant (Critique of Practical Reason, 1787): "Two things fill the mind with ever new and increasing admiration and awe: the starry heaven above me and the moral law within me". (Zwei Dinge erfüllen das Gemüt mit immer neuer und zunehmender Bewunderung und Ehrfurcht, je öfter und anhaltender sich das Nachdenken damit beschäftigt: Der bestirnte Himmel über mir, und das moralische Gesetz in mir; Kritik der Urteilskraft, Beschluß, S. 191.)

[15]Penser, juger, raisonner, sentir, réfléchir et vouloir sont des qualités incompatibles avec la nature des corps; et les êtres, qui en sont revêtus, doivent avoir une nature tout-à-fait différente. Ce sont des ames et des esprits, dont celui qui possede ces qualités au plus haut degré, est Dieu. Il y a donc une différence infinie entre les corps et les esprits. Aux corps il ne convient que l'étendue, l'inertie, et l'impénétrabilité, qui sont des qualités, qui excluent tout sentiment: pendant que les esprits sont doués de la faculté de penser, de juger, de raisonner, de sentir, de réfléchir, de vouloir ou de se décider pour un objet plutôt que pour un autre. Ici il n'y a ni étendue, ni inertie, ni impénétrabilité; ces qualités corporelles sont infiniment éloignées des esprits. [...] Mais on demande ce que c'est qu'un esprit? sur cela j'aime mieux avouer mon ignorance et répondre que nous ne saurions dire ce que c'est qu'un esprit, puisque nous ne connoissons rien du tout de la nature des esprits. [...] Or cette même union de chaque ame avec son corps est sans doute et restera toujours le plus grand mystere de la Toutepuissance Divine, que nous ne saurions jamais pénétrer. [...] Cependant ces deux especes d'êtres sont liées ensemble de la maniere la plus étroite, et c'est principalement de ce *lien* que dépendent toutes les merveilles du monde, qui ravissent les êtres intelligens et les portent à glorifier la Créateur. Il n'y a aucun doute que les esprits ne constituent la principale partie du monde et que les corps n'y soient introduits que pour leur service.

Elucidating the nature of spirits Euler continued:

> "To ask, In what place does a spirit reside? would be for the same reason an absurd question, for to connect spirit with place is to ascribe extension to it. No more can I say in what place an *hour* is; though assuredly an hour is something; something, therefore, may exist without being attached to a certain place. [...] Just as it may be with truth affirmed of the hour now passing, that it exists neither in my head nor outside of my head. A spirit exists, then, though not in a certain place; but if our reflection turns on the power which a spirit has of acting upon a body, the action is most undoubtedly performed in a certain place". (January 10, 1761)[16]

Euler dealt with the liberty of spirits, "a stumbling-block in philosophy" (Euler):[17]

> "But [in comparison with bodies] spirits are of a very different nature, and their actions depend on principles directly opposite. Liberty, entirely excluded from the nature of body, is the essential portion of spirit to such a degree that without liberty a spirit could not exist; and this it is which renders it responsible for its actions. This property is as essential to spirit as extension or impenetrability is to body; and as it would be impossible for the Divine Omnipotence itself to divest body of these qualities, it would be equally impossible for it to divest spirits of liberty. A spirit without liberty would no longer be a spirit, as a body without extension would no longer be a body". (December 16, 1760)
>
> "It is accordingly of importance to remark, that God acts in a manner totally different towards bodies and spirits. God has established for bodies laws of rest and motion, conformably to which all changes necessarily take place; as bodies are merely passive beings, whereas spirits are susceptible of no force or constraint, but are governed of by God through precepts and prohibitions. [...] When it is said to be the will of God that men should love one another, we mean by that expression a commandment which men ought to obey; but this is very far from being the case. God does not force men to it, this would be contrary to the liberty which is essential to them. [...] It always depends on the will of man whether he is to obey or not. In this sense we are to understand the will of God, when it refers to free actions of spiritual beings". (December 27, 1760)[18]

[16] Ce sera donc aussi une question absurde de demander, en quel lieu un esprit existe? car dès qu'on attache un esprit à un lieu, on lui suppose une étandue. Je ne saurois dire non plus en quel lieu se trouve une *heure*, quoiqu'une heure soit sans doute quelque chose: ainsi quelque chose peut être, sans qu'elle soit attachée à un certain lieu. De la même maniere je puis dire, [...] aussi peu que l'heure d'à-présent, dont je puis dire véritablement, qu'elle n'existe ni dans ma tête ni hors de ma tête. Un esprit existe donc sans qu'il existe dans un certain lieu; mais si nous faisons réflexion au pouvoir, qu'un esprit peut avoir d'agir sur un certain corps, cette action se fait sans doute dans un certain lieu.

[17] This point was later (independently) picked up by the German philosopher Johann Gotthelf Fichte (1762–1814) in his philosophical system Wissenschaftslehre (theory of science), outlined above all in two books "Grundlage der gesamten Wissenschaftslehre" (Foundations of the entire Wissenschaftslehre, 1794) and "Grundriß der Eigentümlichkeiten der Wissenschaftslehre" (Outline of the Distinctive Character of the Wissenschaftslehre, 1795). Fichte arrived at the contradiction: "If the self [Ich] posits itself as determined, it is not determined by the not-self [Nicht-Ich]; if it is determined by the not-self, it does not posit itself as determined (Science of Knowledge). So he deduced: "What sort of philosophy one chooses depends, therefore, on what sort of person one is".

[18] Mais les esprits sont d'une nature entierement différente, et leurs actions dépendent de principes directement opposés. Comme la liberté est entierement exclue de la nature des corps, elle est la partage essentiel des esprits; de sorte qu'un esprit ne sauroit être sans la liberté; et c'est la liberté qui le rend responsable de ses actions. Cette propriété est aussi essentielle aux esprits, que l'éntendue ou l'impénétrabilité l'est aux corps; et comme il seroit impossible, même à la Toutepuissance Divine, de dépouiller les corps de ces qualités, il lui est également impossible de dépouiller les esprits de la liberté. Car un esprit sans liberté ne seroit plus un esprit, tout de même qu'un corps sans étendue, ne seroit plus un corps. [...] Aussi est-il fort important de remarquer que Dieu agit d'une maniere tout-à-fait différente envers les corps et les esprits. Pour les corps, Dieu a établi les loix du repos et du mouvement, conformément auxquelles tous les changemens arrivent nécessairement, les corps n'étant que

From this point of view Euler deduced the existence of evil and sin:

> "The origin and permission of evil in the world is an article which has in all ages greatly perplexed theologians and philosophers. To believe that God, a supremely good Being, should have created this world, and to see it overwhelmed with such a variety of evil appears so contradictory, that some found themselves reduced to the necessity of admitting two principles, the one supremely good, the other supremely evil. This was the opinion entertained by the ancient heretics known in history by the name of Manicheans. [...] Though the question be extremely complicated, this single remark, that liberty is a quality essential to spirits, dispels at once a great part of the difficulties which would otherwise be insurmountable. [...] In this respect, therefore, the government of God over spirits, or rational beings, is infinitely different from that which men exercise over men like themselves, and we greatly err if we imagine that a government which appears the best in the eyes of men is really so in the judgment of God. This is a reflection of which we ought never to lose sight". (March 14, 1761)[19]

For Euler it is "an established truth that Christ has risen from the dead" and that "the Divinity of Christ's mission in this world cannot possibly be called into question". From this he argues that "we can absolutely trust in all the promises given in the Gospel".[20]

4. Physico-theological arguments in Euler

Euler, one of the most distinguished scientists of the 18th century, deduced his philosophical and theological statements with the help of then modern arguments, i.e. he evaluated the astronomical, physical, chemical, and biological progress in view of (physico-)theological deductions. An example: because of the nature of the ether which produced friction, Euler regarded it as a matter of fact that the present state of the planetary system could not be in existence from eternity to eternity and in this way found confirmation of the biblical statement of the finiteness of our world, which lasts from the creation to doomsday (Rettung, §50). Euler's physico-theological view was manifest in his defense of Maupertuis' principle which stated that in all the changes which occur in nature, the cause will be the smallest that can produce the effect. Maupertuis proclaimed his economy principle to be a

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des êtres passif [...] au lieu que les esprits ne sont susceptibles d'aucune force ou contrainte, et que c'est par des commandemens ou des défenses que Dieu les gouverne. [...] Quand on dit que Dieu veut que les hommes s'aiment mutuellement, c'est une toute autre volonté de Dieu: c'est un commandement, auquel les hommes devroient obéir; mais il s'en faut beaucoup qu'il soit exécuté. Dieu n'y force pas les hommes, ce qui seroit une chose contraire à la liberté qui leur est essentielle. [...] C'est sur ce pied qu'on doit juger de la volonté de Dieu, quand elle se rapporte aux actions libres des êtres spirituels.

[19] L'Origine et le permission du mal dans le monde est un article, qui a de tout tems fort embarrassé les Théologiens et les Philosophes. Croire que Dieu, cet être souverainement bon, ait créé ce monde et y voir fourmiller tant de maux, paroit si contradictoire, que plusieurs d'entr'eux ont cru être forcés d'admettre deux principes, l'un souverainement bon et l'autre souverainement méchant: c'étoit le sentiment des anciens hérétiques connus sous le nom des Manichéens. [...] C'est donc à cet égard que le Gouvernement de Dieu sur les esprits ou êtres raisonnables est infiniment différent de celui que les hommes exercent sur leurs pareils; et on se trompe beaucoup, quand on s'imagine, qu'un gouvernement qui paroit meilleur aux yeux des hommes, le soit réellement au jugement de Dieu. C'est une réflexion que nous ne devons jamais perdre de vue.

[20] It is "eine ausgemachte Wahrheit, daß Christus von den Todten auferstanden" and it is clear that "die Göttlichkeit der Sendung Christi auf diese[r] Welt unmöglich in Zweifel gezogen werden kann". Moreover, "Wir können auch alle Verheissungen, welche uns in dem Evangelio [...] gethan werden, mit der festesten Zuversicht glauben". (Rettung, §36)

general law of nature, most worthy of the creator, whereas critics who mocked Maupertuis, claimed that the principle turned the almighty God into a stingy, or at least a parsimonious, creator.

Euler expounded the Principle as perfectly founded in the very nature of body, and he had the strong belief that basic physical principles can always be expressed by certain minimal or maximal properties. But mathematically, in his paper "On the motion of bodies in a non-resisting medium, determined by the method of maxima and minima" (Appendix II in the "Methodus inveniendi"), he limited himself to a few special applications in dynamics. He said explicitly that he would leave the general case to the Philosophers,[21] and he did not use the principle at all to prove the existence of God mathematically. Here he differs from other celebrated physico-theological writers like William Derham (1657–1735), the author of "Physico-theology or a demonstration of the Being and Attribute of God from his Works of Creation" (1713), John Ray (1628–1705, "Three physico-theological discourses", 1692, and "The Wisdom of God Manifested in the Works of Creation", 1691), Johan Swammerdam (1637–1680, "Bybel der natuur", The bible of nature or the history of insects, ed. by Boerhave 1737), Bernard Nieuwentyt (1654–1718, "Het regt gebruik der werelt beschouwingen, ter overtuiginge van ongodisten en ongelovigen aangetoont", The correct use of the world views, demonstrated in order to convince atheists and unbelievers, 1714). Most of them showed more industry in collecting examples for anthropomorphic justification of Divine Providence than in providing a sound theoretical foundation.

It is the self-confidence of the rising bourgeoisie that interpreted the Creator like an ideal human being (where there is order, there is mind). The optimistic physico-theological view can also be regarded as a reaction to baroque nihilism. It failed to see any misfortune and sorrow in the world because almost everything served to demonstrate the glory of God. Although Euler was guided throughout by general teleological principles (by a priori conviction) his realistic view (the a posteriori corroboration of the principles by true and sound dynamic methods) appears in the following lines taken from his "Methodus inveniendi":

> "Although this conclusion does not seem sufficiently confirmed, nevertheless, if I show that it agrees with a truth known a priori, so much weight will result that all doubts which could originate on this subject will completely vanish. Even better, when its truth will have been shown, it will be very easy to undertake studies in the profound laws of Nature and their final causes, and to corroborate this with the firmest arguments".[22]

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[21] Quod negotium aliis, qui Metaphysicam profitentur, relinquo. (Methodus inveniendi, Additamentum I, Opera omnia I/24, p. 308.)

[22] Quae conclusio etsi non satis confirmata videatur, tamen, si eam cum veritate iam a priori nota consentire ostendero, tantum consequetur pondus, ut omnia dubia, quae circa eam suboriri queant, penitus evanescant. Quinetiam, facilius erit in intimas Naturae leges atque causas finales inquirere hocque assertum firmissimis rationibus corroborare. (Methodus inveniendi, Additamentum II, Opera omnia I/24, p. 298).

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CHAPTER 28

Georg Cantor (1845–1918)

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Credo, ut intellegam.
(I believe in order to understand.)
Anselmus[1]

1. The mathematician Cantor

Any reader of a modern mathematical textbook will be confronted with notions and concepts that were introduced by Georg Cantor, the founder of *set theory*. He founded this highly original theory in a series of only a few papers from 1874 to 1897. However, this unique work in which Cantor extended mathematics into philosophy and in which he violated the principles of religion was the subject of much criticism as well. Laurence Young (1905–2000) wrote in his book "Mathematicians and Their Times":[2]

> "In times of rule of thumb authoritarianism, regardless of truth, even the best mathematicians can be quarrelsome and intolerant, especially to someone who breaks not only with longstanding mathematical traditions but [also with] established beliefs outside mathematics, such as the authorship of Shakespeare's plays, and the Immaculate Conception".

However, among Cantor's supporters, les Cantoriens, as Henri Poincaré (1854–1912) called them, were the leading mathematicians of Göttingen. At the beginning of the 20th century when antinomies began to appear in set theory, it was David Hilbert (1862–1943) who said:

> "No one shall expel us from the paradise that Cantor has created for us".[3]

It is rather remarkable that Hilbert uses here a biblical metaphor like the mathematicians of the bygone 18th century used to do. The reason to take up biblical images such as this might root in the estimation of Cantor by Hilbert who regarded him as the profoundest mathematician of our age. And it was Stanislaw Ulam (1909–1984) who put Cantor into a line with Jesus (0?–33?), Karl Marx (1818–1883), and Sigmund Freud (1856–1939). David Hilbert praised Georg Cantor as a scholar who is unrivalled by all mathematicians from Euler to Einstein.[4] On the other hand, it was no other than Henri Poincaré who regarded Cantor's set theory as an illness, "un beau cas pathologique".[5]

Who was this mathematician, Georg Cantor, who created the revolutionary set theory and the concepts of infinity, but who never lectured on these topics at a university.[6] What were the beliefs of that outstanding mathematician whose theological writings destroyed

[1] The paradox "Credo quia absurdum" (I believe because it is absurd) is ascribed to Tertullian (155–220). Augustine opposed this idea with the words "Crede ut intellegas". Anselmus of Canterbury (1033–1109) echoed Augustine's words.

[2] North Holland, Amsterdam, 1981, p. 232.

[3] "Aus dem Paradies, das Cantor uns geschaffen, soll uns niemand vertreiben können". Quoted from *Über das Unendliche*. Lecture delivered in Münster 1925. Mathematische Annalen **95** (1925), also in the 7th edition of *Grundlagen der Geometrie*, Teubner, Leipzig, 1930.

[4] "kein Mathematiker aller Zeiten von Euler bis Einstein" übertrifft ihn. Draft of a letter, Niedersächsische Staats- und Universitätsbibliothek Göttingen, Handschriftenabteilung, Cod. Ms. D. Hilbert 457.

[5] *L'avenir des mathématiques*, in: Atti del IV congresso internazionale dei matematici, vol. 1, G. Castelnuovo, ed., Academia dei Lincei, Roma, 1909, p. 182.

[6] Yet, in the summer of 1885 Cantor called his class on number theory an "Introduction to the theory of order types", with an oblique reference to the revolutionary theory that he was developing. Moreover, he lectured on set theory in seminars.

his friendship with the good-natured Hermann Amandus Schwarz (1843–1921), a fellow-student of his college days in Berlin. It perturbed Schwarz that his old friend quoted the Fathers of Church to elucidate irrational numbers:

> "Now that I have had the opportunity to study the text at leisure I cannot hide the fact that I regard it as pathological aberration. What in the world have the early fathers to do with irrational numbers? I deeply trust it will not prove that our patient has arrived at that crooked course from which the unhappy Zöllner[7] never found his way back to occupation with concrete tasks".[8]

On the occasion of the first International Congress of Mathematicians in Zurich 1897, Adolf Hurwitz (1859–1919) openly expressed his great admiration of Cantor, we read in a history of set theory.[9] On the other hand, at the same time Hermann Minkowski (1864–1909), a close friend of Hurwitz, wrote to David Hilbert:

> "Hurwitz's brother writes that Cantor of Halle has been offered a chair in Munich. Seems rather peculiar. The chair of Shakespearology?"[10]

And Georg Cantor himself? He was unhappy with both his unsatisfying position in Halle and the lacking appreciation of his set-theoretic work. Nevertheless, guided by his philosophical mind he declared (here in the words of Hilbert): "There are many objections which are of interest only for whom who makes them".[11]

For Cantor, however, scientific knowledge and religious belief were untearably connected, both were rolled into one, united in the end. Understanding the (earthly) theory of infinite sets was for Cantor a necessary condition to begin to understand the infinity of God:

> "If I have told you earthly things, and ye believe not, how shall ye believe, if I tell you of heavenly things"? (John 3:12)

[7] Johann Karl Friedrich Zöllner (1834–1882), physicist, astrophysicist, founder of astrophotometry, belonged to the staff of the University of Leipzig which is not far from Cantor's hometown Halle. In the end, Zöllner turned towards "spiritism".

[8] "Nachdem ich Gelegenheit erhalten habe, den Aufsatz [*Mitteilungen zur Lehre vom Transfiniten, I*, Zeitschrift für Philosophie und philosophische Kritik **91** (1887), 81–125, 251–270; also *Gesammelte Abhandlungen*, pp. 240–265] mit Muße anzusehen, kann ich nicht verhehlen, daß mir derselbe als eine krankhafte Verirrung erscheint. Was haben denn in aller Welt die Kirchenväter mit den Irrationalzahlen zu tun? [...] Möchte sich doch die Befürchtung nicht bewahrheiten, daß unser Patient auf derselben schiefen Linie angelangt sei, von der unglückliche Zöllner den Rückgang zur Beschäftigung mit concreten Aufgaben nicht mehr gefunden hat". Letter to Weierstraß of October 17, 1887.

[9] P.E. Johnson, *A History of Set Theory*, Boston, MA, 1972.

[10] "Hurwitz' Bruder schreibt, dass Cantor aus Halle nach München berufen sei. Die Nachricht klingt sehr seltsam. Etwa auf einen Lehrstuhl für Shakespearology"? H. Minkowski, *Briefe an David Hilbert*, L. Rüdenberg, ed., Springer, Berlin, 1973, p. 97. The 105 letters are preserved in the Niedersächsische Staats- und Universitätsbibliothek Göttingen, Handschriftenabteilung. Nachlass Hilbert, Cod. Ms. D. Hilbert 258.—Minkowski hints at Cantor's habit to leave mathematics which often took place after Cantor suffered a period of depression and then turned towards religion, philosophy and literature. For example, Cantor was deeply convinced and defended strongly his belief that Francis Bacon (1561–1626) wrote the plays of William Shakespeare (1564–1616), see Section "Religious denomination". He published some pamphlets on this subject, the first of them was "Resurrectio Divi Quirini Baconi", Halle 1896. Invited to the 500th anniversary of the foundation of the University of St. Andrews in Scotland in 1911, Cantor (mis)used the opportunity and talked at length on that Bacon–Shakespeare question.

[11] "Es gibt viele Einwendungen, die nur für den Interesse haben, der sie macht—pflegte Cantor zu sagen". Niedersächsische Staats- und Universitätsbibliothek Göttingen, Handschriftenabteilung, Cod. Ms. D. Hilbert 681, Bl. 19.

2. Short vita of Cantor

Georg Cantor was born in St. Petersburg, Russia, in 1854 where his father Woldemar Georg Cantor (1813?–1863) was a successful merchant. As Herbert Meschkowski (1909–1990) noted

> "the letters to his son [Georg] attest to a cheerfulness of spirit and deep appreciation of art and religions".[12]

Among the ancestors of his mother Marie Böhme (1819–1896) were some well-known musicians, and Georg Cantor described himself also as "rather artistically inclined". His father was Protestant, his mother was Roman Catholic. Georg Cantor married Vally Guttmann (1849–1923) in 1874. They were married by a Protestant minister and their six children received their first communion at the Protestant St. Bartholomew church in Halle. So Cantor was a Protestant, but the fact that his mother was a Roman Catholic may have had some influence on him. As a matter of fact, because of Cantor's philosophical ideas concerning the infinite he developed links with Catholicism, as we will see. In the end Georg Cantor was buried by a Protestant priest in Halle at the Giebichenstein cemetery (i.e. he was, at least formally, a parishioner of a Protestant community). He was later interred in a new grave when this cemetery was transformed into a park in the 60s.

In 1856, Cantor's father became ill, and the family moved from St. Petersburg to Germany. After a short period at the University of Zürich, Cantor went to the University of Berlin where he attended the lectures of Karl Weierstraß (1815–1897), Leopold Kronecker (1823–1891), and Ernst Eduard Kummer (1810–1893). In 1867 he wrote his dissertation "De aequationibus secundi gradus indeterminandi" (On indeterminate equations of second order), and was habilitated two years later in Halle in 1869. His friend Schwarz, then professor at the University of Halle, managed to install Cantor in Halle when he left for Zürich in 1869. Although Cantor started his Halle career without a salary he remained for the rest of his life in the small Prussian town. In a letter to Charles Hermite (1822–1901) he thanked God for his being in Halle:

> "I am only grateful to God, the all-wise and infinitely kind that he denied me the fulfillment of these desires [a call from Berlin or Göttingen],[13] for in doing so he compelled me, through penetration into the theology, better to serve him and his Holy Roman Catholic Church then I had done with my poor mathematical talent, through extension exclusively to mathematics".[14]

From 1884 Cantor suffered sporadically from mental illness (manic depression) and in all he spent more than four years in hospitals. Nevertheless, he remained active in mathematics and in organizing mathematical congresses, the foundation of the German Association of Mathematicians (Deutsche Mathematiker-Vereinigung), etc. At the turn of the century, his work was finally accepted as fundamental to mathematics, moreover his set theory was

[12] *Biographical Dictionary of Mathematicians*, vol. 1, Scribners, New York, 1991, p. 400.

[13] It was Cantor's dream to receive a call from Berlin (or, at least, from Göttingen).

[14] "Allein nun danke ich Gott, dem Allweisen und Allgütigen, daß er mir die Erfüllung dieser Wünsche [Berufung nach Göttingen oder Berlin] versagt hat, denn so hat er mich gezwungen, durch ein tieferes Eindringen in die Theologie Ihm und seiner heiligen römischen-katholischen Kirche beßer zu dienen, als ich es, nach meinem wahrscheinlich schwachen mathematischen Talent, durch *ausschließliche* Beschäftigung mit der Mathematik hätte tun können". Letter to Hermite of January 24, 1894.

Fig. 1. Georg Cantor. Courtesy of the University of Hamburg.

regarded as a landmark in human thought. Hilbert's first problem in the famous collection delivered at the Paris Congress in 1900 concerned the celebrated Continuum Hypothesis (*CH*), a conjecture that without Cantor's set theory could not even be formulated. The problem was only solved half a century later by Kurt Gödel (1906–1978), Paul Cohen (born 1924), and Petr Vopenka (born 1935).[15]

3. Mathematics and metaphysics

From Antiquity until the end of the nineteenth century, in mathematics the "potential infinite" was rigorously distinguished from the "actual infinite". Aristotle (384–322 BC), René Descartes (1596–1650), Blaise Pascal (1623–1662), and Carl Friedrich Gauß (1777–1855), just to mention a few names, had rejected the actual or complete infinite as unknowable and avoided its application like the devil avoids holy water. At the time of Cantor's life Leopold Kronecker was probably the most zealous defender of such views. However, not all mathematicians, opposed the actual infinite. Here we can mention Gottfried Wilhelm Leibniz (1646–1716) and Bernhard Bolzano (1781–1848).

In this tradition of discussions on infinity Cantor was a major participant. A very brief description of his mathematical results would run as follows. He turned arbitrary collections of things into mathematical objects and in a very general sense of the word he counted

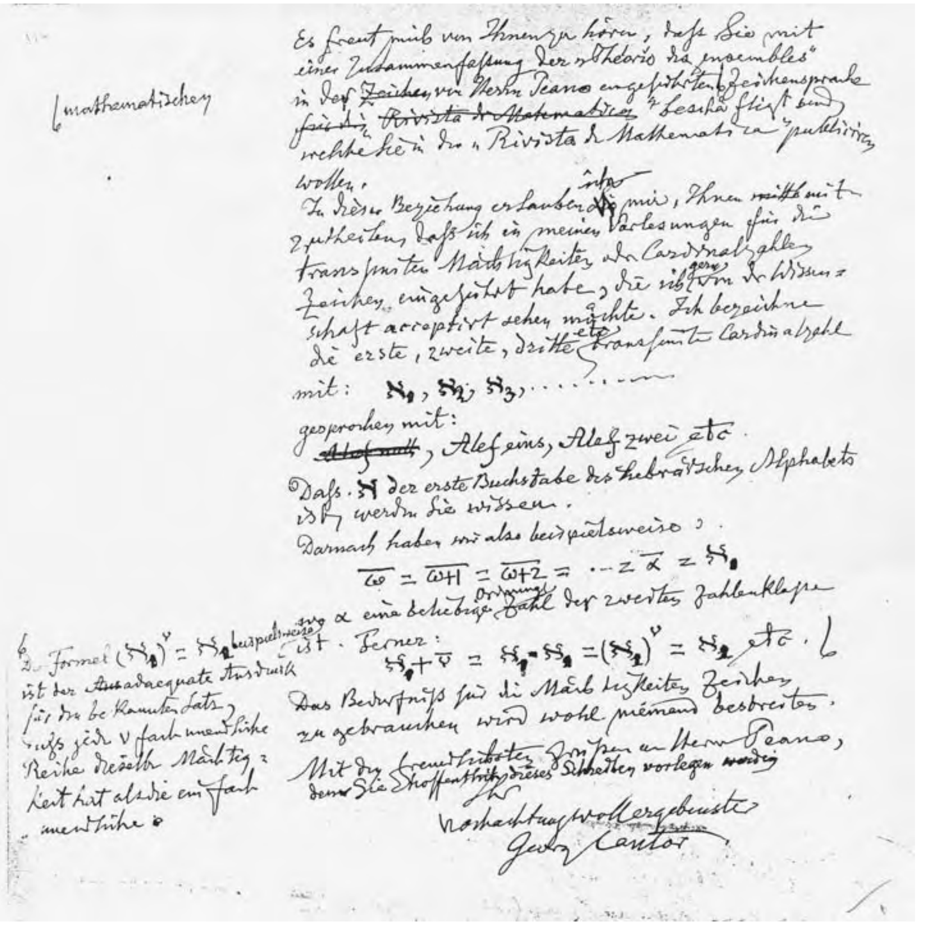
[15] It was solved negatively, in the sense that it became clear that *CH* is independent of the other axioms of set theory.

Fig. 2. Georg Cantor. Science Photo Library.

these collections and calculated with the resulting numbers (the finite and transfinite numbers). A possible consequence of this approach could be that mathematics is nothing more than a generalized set theory, which would imply that all foundational problems of mathematics would be reduced to the problems of the foundation of set theory.

A decisive circumstance in Cantor's consideration was the fact that not all infinite sets have the same power or mathematical size (Mächtigkeit). In Weierstraß's seminar Cantor had learned that the set of rational numbers can be counted in the sense that with every rational number corresponds a unique natural number. In 1873 Cantor wrote to Richard Dedekind (1831–1916) that the set of real numbers cannot be counted (letter of December 7, 1873; in: *Briefe*, 1991, pp. 35–36). In 1874 the result was published in Cantor's paper "Über eine Eigenschaft des Inbegriffs" (About a property of the totality)[16] and this publication can be legitimately seen as the birth of set theory. Cantor expresses the power of an infinite set by means of so-called *transfinite cardinal numbers*. The development of Cantor's conception of the cardinal numbers can briefly described as follows. First, from 1877 until 1885 Cantor viewed a cardinal number as a "determined set, consisting of only ones [units]".[17] In a letter to Dedekind of August, 1899, we find a cardinal number defined as an equivalence class[18] and finally after 1899 Cantor introduced a cardinal number as the first number of a number class.

[16]Crelle Journal für die reine und angewandte Mathematik **77** (1874), 258–262.
[17]*Beiträge zur Begründung der transfiniten Mengenlehre*, Mathematische Annalen **46** (1895), 481–512. Also in *Gesammelte Abhandlungen*, p. 283.
[18]In: *Briefe*, 1991, pp. 405–413.


mit: $\aleph_1, \aleph_2, \aleph_3, \ldots$

gesprochen mit: Alef eins, Alef zwei etc.

Dass ℵ der erste Buchstabe des hebräischen Alphabets ist, werden Sie wissen.

Darnach haben wir also beispielsweise:

Das Bedürfniss für die Mächtigkeiten Zeichen zu gebrauchen wird wohl niemand bestreiten.

Georg Cantor

Fig. 3. Concept of a letter written on Sept. 14, 1891 by Georg Cantor to Prof. Lossen in Heidelberg. Courtesy of the Niedersächsische Staats- und Universitätsbibliothek Göttingen.

We will now proceed with a brief introduction to the theory of transfinite numbers. In the "Grundlagen einer allgemeinen Mannigfaltigkeitslehre" (Foundations of a general theory of manifolds) of 1883 Cantor's research reached a milestone. The transfinite ordinal numbers were presented as a coherent extension of the natural numbers. The *transfinite ordinal numbers* can be introduced as follows. The sequence of natural numbers 0, 1, 2, 3, 4, ... can be obtained by the repeated addition of a unit. There is no largest element. Yet, we can imagine a first transfinite ordinal number ω that follows the entire sequence of natural numbers. Once we have ω we can go on adding units and we get

$$0, 1, 2, 3, 4, \ldots, \omega, \omega+1, \omega+2, \omega+3, \ldots$$

Again, we can imagine a first number $\omega + \omega$ that follows this entire sequence. In this way Cantor goes on, precisely defining the way in which this sequence of transfinite ordinal numbers continues:

$$0, 1, 2, 3, 4, \ldots, \omega, \omega+1, \omega+2, \omega+3, \ldots, \omega+\omega, \omega+\omega+1, \omega+\omega+2, \ldots,$$
$$\omega+\omega+\omega, \omega+\omega+\omega+1, \ldots$$

A remarkable property of the sequence of ordinal numbers is that each number corresponds to the well-ordered set of numbers that proceeds in the sequence: if we count $\{0, 1, 2\}$ we get 3, if we count $\{0, 1, 2, 3, \ldots, \omega, \omega+1, \omega+3\}$ we get $\omega+3$, etc.[19]

The addition of finite ordinal numbers can be defined as follows. $7+3$ means: count till seven and add three more units; the result is 10. For the transfinite ordinal numbers Cantor defined the addition in the analogous way. $\alpha+\beta$ means: count till α and add β more units. In other words: Take well-ordered sets A and B so that α and β correspond to A and B respectively; $\alpha+\beta$ is then the numbering of the well-ordered set obtained by considering the ordered elements of A followed by the ordered elements of B. Clearly, addition is associative; however, it is not commutative, because $3+\omega$ is by definition of addition the numbering of the well-ordered set $\{0, 1, 2, 0, 1, 2, 3, 4, \ldots\}$. This set has the ordinal number ω, which is unequal to $\omega+3$. In the "Grundlagen" Cantor convincingly showed that such transfinite ordinal numbers possess all properties that are required of a regular mathematically interesting number system.

An important notion in Cantor's set theory is the notion of power. Two sets are of the same power, if there is a one-to-one correspondence between their elements. For example, the set of natural numbers and the set of even natural numbers are of the same power. If we map by means of the mapping $n \to 2n$ the natural numbers $0, 1, 2, 3, \ldots, n, \ldots$ on the even numbers $0, 2, 4, 6, \ldots, 2n, \ldots$ respectively, we have a one-to-one correspondence, which proves it. One of Cantor's first results was the discovery (1873) that the set of natural numbers $\mathbb{N}$ and the set of real numbers do not have the same power: $\mathbb{R}$ is as for power essentially larger than $\mathbb{N}$. This was a very remarkable result. It implies that $\mathbb{R}$ is non-denumerable; it is not possible to find a well-ordering of $\mathbb{R}$ such that the ordinal number of the set is ω.

In the "Grundlagen" Cantor does not yet treat these powers of sets as transfinite numbers. Yet, in 1883, the year of publication of the "Grundlagen", he was already aware of the fact that powers of sets can be considered as numbers. For example, all sets of two elements obviously have the same power, by definition this is the finite cardinal number 2. The smallest transfinite cardinal number is the number of the set of natural numbers $\mathbb{N}$. Cantor represented it as $\aleph_0$ (aleph-zero) using the first letter of the Hebrew alphabet. $\aleph_0$ is the cardinal number of many sets: the even natural numbers, the integers, the prime numbers, the rational numbers, etc. The sequence of cardinal numbers starts as follows: $\aleph_0, \aleph_1, \aleph_2, \aleph_3, \aleph_4, \ldots$. Cantor defined an arithmetic for cardinal numbers as well. Exponentiations is defined as follows: $2^{\aleph_0}$ is by definition the power of the set of all functions from a set of two elements to $\mathbb{N}$. The repeated application of the operation of exponentation quickly leads to huge cardinal numbers. $2^{\aleph_0}$ is the cardinal number of the set of all subsets of $\mathbb{N}$; $\aleph_1$ the cardinal number of the set of $\mathbb{R}$, the numbers of points of a line (and astonishingly in any Euclidean space, etc.). The equality $2^{\aleph_0} = \aleph_1$ is in fact the above mentioned

[19]Systematically: $\emptyset = 0$, $\{0\} = 1$, $\{0, 1\} = 2$, $\{0, 1, 2, \} = 3$, etc.

Fig. 4. Balancing on an aleph, Cantor, who felt he derived the foundation of his theory of the infinite from "the first cause of all things created", feels secure facing the attacks of his critic Kronecker (drawing by Barbe).

Continuum Hypothesis (*CH*). It expresses that there are no sets with a cardinality between the cardinalities of $\mathbb{N}$ and $\mathbb{R}$. Cantor thought *CH* was true, but he did not succeed in proving it. He was troubled by the fear he could not prove the "continuum hypothesis", and he was right. The question was whether the order of infinity of the real numbers was the next after that of the natural numbers. In Cantor's picture of set theory as a staircase of alephs leading to God's throne (the absolute and complete infinity) the first step of the stairs is already missed. Today we know there is a bifurcation at the beginning of the staircase, and at present most mathematicians use that part of the stairs which is formed in a series of stages, the iterative hierarchy delivered by the Zermelo–Fraenkel axioms (*ZFC*).

Let us shortly look back. Cantor started with well-ordered sets which consisted of denumerably elements. The totality of such well-ordered sets represents a new number class which is called second number class. The sets contained in this second number class are non-denumerable. Proceeding in the same way we arrive at a third number class, etc. In this way different orders of infinity are built up. The procedure coincidences with our usual understanding for finite sets: Every transfinite multiplicity, that is, every consistent transfinite set, must have a definite aleph as its cardinal number. However, all was not going well. David Hilbert was the first, yet before Cesare Burali-Forti (1861–1931) in 1897 and even before Cantor himself in 1895, who publicly referred to contradictions in totalities of set theory (we will in detail deal with later).[20]

[20] O. Hölder, *Die mathematische Methode*, Springer, Berlin, 1924, p. 547; cf. also B. Rang and W. Thom, *Zermelo's discovery of the 'Russel Paradox'*, Historia Mathematica **8** (1981), 15–22.

The system of transfinite ordinal numbers enabled Cantor to study questions concerning incredibly large sets. Cantor's arithmetic of infinite sets touches on the old question of the limits of human ability to handle the infinite. The human mind is finite, limited. To what extent is it possible for such a limited human brain to grasp the infinite, to transmit and express it in finite terms? Can we conceive of the infinite or represent it in finite terms? Such structures constructed by Cantorian concepts appear to come from a different world, from a world beyond that of our senses, i.e. are we able at least to get a glimpse of it as in Plato's famous allegory of the Cave? Cantor's paradise of set theory does not seem to be part of our (physical) world. The philosophical question is: Does the Infinite ("Transfinitum") exist and in what sense? The Greeks were the first to study such questions. In Platonic thought eternal truth cannot come from the world of changing and temporal things; man is not able to encounter the world of Ideas and eternal Truth through the senses. Getting in touch with the Ideas is a spiritual affair. Augustine (354–430) picked up Plato's views and in his religious interpretation the Platonic world of Ideas becomes a Divine world of truth. In this view mathematical truth exists before human being knows it, such truth is a creation of God.

Cantor was greatly attracted by such mathematical-philosphical-theological considerations, and that is why he was strongly influenced by the philosophical works of such scholastic Catholics as Augustine and Nicholas of Cusa (1401–1464). Felix Klein (1849–1925) pointed out in his lectures on the development of Mathematics in the 19th century:

> "If we strip away this mantel from the scholastic sophistries which superficially appear purely theological sophistries, it turns out that they are frequently the most concrete beginnings of what we now call the theory of sets. So even Georg Cantor, the creator of set theory, learnt his craft by the scholastics".[21]

Laurence Young wrote that concepts of infinity introduced by Bradwardine (about 1295–1349) and other contemporaries had to wait 600 years to be developed by Georg Cantor. Moreover, Cantor extended his mathematical research to the metaphysical foundation of mathematics and, furthermore, to theological doctrines. Hilbert described Cantor's work as one of the most beautiful realization of human activity in the domain of the purely intelligible. All in all, regarding such new questions how to distinguish powers in the realm of the infinite and such new extensions to the metaphysical foundation of mathematics using a biblical metaphor we can say with Cantor the dawn of the eighth day of Creation had come.

4. Mathematics and religion

Gioacchino Pecci (1810–1903) became Pope Leo XIII in 1878. For Leo XIII Thomas Aquinas (1225?–1274) was the authoritative theologian and philosopher, the *Dr. ecclesiae*. On August 4, 1879 Leo XIII issued the encyclical "Aeterni Patris". The result of "Aeterni

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[21] "Entkleidet man die scholastischen Spitzfindigkeiten dieses Gewandes, das sie dem oberflächlichen Blick als reine theologische Spitzfindigkeit erscheinen läßt, so erweisen sie sich häufig als die konkretesten Ansätze dessen, was wir heute als "Mengenlehre" bezeichnen. [...] Tatsächlich ist denn auch Georg Cantor, der Schöpfer der Mengenlehre, bei den Scholastikern in die Schule gegangen". In: *Vorlesungen über die Entwicklung der Mathematik im 19. Jahrhundert*, Springer, Berlin, 1925, vol. 1, p. 52.

Patris" was that the interest in science among Roman Catholic intellectuals grew considerably. Constantin Gutberlet (1837–1928), in 1888 founder of the "Philosphisches Jahrbuch der Görres-Gesellschaft",[22] a review journal of clear neo-Thomistic signature, was one of them. Others were Father Esser (1850–1920), and Cardinal Johannes Baptist Franzelin (1816–1886). The goal of the encyclical was a revival and modernization of Christian philosophy along Thomistic lines. According to the neo-Thomists the developments in science had led to false philosophies: materialism, atheism, liberalism. Leo XIII envisioned a reconciliation of modern science with Christian philosophy. In the encyclical he argued that modern science could greatly profit from Scholastic philosophy. It was the interest of these Roman Catholic intellectuals in scholastic philosophy and the infinite that was brought Cantor into contact with them. In February 1896 Georg Cantor expressed himself on the encyclical to father Thomas Esser, SJ, as follows:

> "In this sense, I believe, in the encyclical Aeterni Patris and on many other occasions the holy Father Leo XIII urges the philosopher to study the natural sciences (and consequently mathematics as well, because without it science cannot exist) in order that the Christian philosophy as revitalized by the great pope, will not look like an Egyptian mummy, but rather represent an image of Adam, the Adam as he emerged from the hand of the Creator and was breathed upon by Him before the moment of the tragic Fall".[23]

In 1891 Cantor wrote to the Jesuit Tilman Pesch (1836–1899) that he is very pleased to see that the order (Society of Jesu) is taking more and more interest in higher mathematics (letter of September 8, 1891). As we see in a letter to Father Thomas Esser, SJ, Cantor also wished to contribute:

> "It would please me best if my work would to be of benefit to the Christian philosophers dearest to my heart, to the 'philosophia perennis'[lasting philosophy]".[24]

A few month later he wrote that there is

> "an inseverable bond which links metaphysics with theology" (unzerreiß bares Band, das Metaphysic mit Theologie verbindet) and "the foundation of the principles of mathematics and natural sciences devolves upon metaphysics" (die Begründung der Mathematik und Naturwissenschaften fällt der Metaphysik zu).[25]

[22] Joseph von [since 1839] Görres (1776–1848) was an influential catholic author and from 1827 to 1848 professor of literary science in München. After the foundation of the German Reich in 1871 from 1872 a confrontation—called "Kulturkampf" (culture war)—occurred between in particular the Prussian state and the catholic church. In 1876 at the one hundredth anniversary of Von Görres as an answer to the Kulturkampf catholic intellectuals founded the Görres-Gesellschaft (Görres Society) for the advancement of the sciences (excluding theology). Under Leo XIII (pope since 1878) the culture battle was arranged.

[23] "In diesem Sinn wird, glaube ich, in der Encyclica "Aeterna Patris" und auch sonst bei vielen Gelegenheiten vom heil.[igen] Vater Leo XIII das Studium der Naturwissenschaften (und folglich auch der Math.[ematik], ohne welche jene nicht durchführbar sind) dem Philosophen an's Herz gelegt, damit die vom großen Papste repristinirte [wieder belebte] christliche Philosophie nicht einer ägyptischen Mumie gleiche, sondern vielmehr ein Bildnis Adams sei, und zwar Adams, so wie [er] aus der Hand des Schöpfers hervorgegangen [ist] und von ihm angehaucht vor dem Zeitpunkt des traurigen Sündefalls beschaffen war". Quotation from a letter to father Esser of February 1, 1896, in: H. Meschkowski, *Aus den Briefbüchern Georg Cantors*, Archive for History of Exact Sciences **22** (1962–1966), 503–519, quotation is on p. 512.

[24] "Am Meisten würde es mich aber freuen, wenn meine Arbeiten der meinen Herzen am nächsten stehenden christlichen Philosophen, der 'philosohia perennis' zu Gute käme". Letter of December 19, 1895.

[25] Letter to Father Esser February 1, 1896.

Mathematical research corresponds to considerations on Creation, and its results are therefore steps towards God. In different ways Cantor expressed repeatedly the intimate involvement of mathematics with the Divine via metaphysics. According to him mathematical research corresponds to considerations on creation, and its results are therefore steps towards God. For Cantor mathematics serves metaphysics and religion, and set theory is even integrated into metaphysics (as science of the existing). In a letter to Father Thomas Esser he wrote:

> "Every extension of our insight into the origin of the creatively-possible therefore must lead to an extension of our knowledge of God".[26]

The statement has a physico-theological ring: the extension of our scientific knowledge leads us to a theological truth. Moreover, on the other hand metaphysics is essentially involved in science:

> "In my opinion, without a modicum of metaphysics no foundation of any exact science is possible".[27]

And a few lines later Cantor explained his Platonic thinking:

> "As I view it, Metaphysics is the science of the existing, the world as it is, and not as it appears to us to be. Everything that we perceive with the senses and all that we conceive with our abstract thinking is non-existing, and thus at most a clue to that which exists of itself".[28]

From this Cantor deduced that in any metaphysical discussion theology is necessarily profoundly involved and that each actual progress in metaphysics enlarges the precepts of theology.

Cantor distinguished sharply between in concreto and in abstracto (concrete and abstract).[29] Those oriented toward Platonic philosophy accepted actual infinity in abstracto but not in concreto whereas Cantor believed that the actual infinitum (Transfinitum) occurs as well concretely and abstractly (in concreto and in abstracto). Cantor was convinced that in concreto the atoms of our physical world form an actually infinite set with cardinality $\aleph_0$[30] and he believed that the set of the atoms of the universal ether is essentially bigger and possesses the power $\aleph_1$ of the continuum, i.e. has the cardinality $2^{\aleph_0}$.

With Cardinal Johannes Baptist Franzelin Cantor discussed whether infinite sets occur in abstracto or in concreto in God's Creation and whether Creation is a necessary consequence of God's completeness. Cantor believed in the transfinitum in concreto in the

[26]"Jede Erweiterung unserer Einsicht in das Gebiet des Creatürlich-möglichen muß daher zu einer erweiterten Gotteserkenntnis führen". Quoted from the Archive for History of Exact Sciences **2** (1965), 511.

[27]"Ohne ein Quentchen Metaphysik läßt sich meiner Überzeugung nach, keine exacte Wissenschaft begründen". Page of the Nachlass, about 1913.

[28]"Metaphysik ist, wie ich sie auffasse, die Lehre vom Seienden, also von der Welt wie sie an sich ist, nicht wie sie uns erscheint. Alles was wir mit den Sinnen wahrnehmen und mit unserm abstracten Denken uns vorstellen, ist Nichtseiendes und damit höchstens eine Spur des an sich Seienden". Page of the Nachlass, about 1913.

[29]By means of the ontological question that the theory of sets brought up and the contradiction (paradoxes) associated with the theory in combination with the attempts to neutralize them, Cantor shook the platonic way of thinking in mathematics. For David Hilbert the main foundational problem concerning formal mathematical systems did not concern truth or meaning, but certainty (consistency). Apparently, Cantor himself did not notice this shift in focus concerning the foundations of mathematics, that he had caused.

[30]In 1931 Sir A. Eddington computed the total number of electrons and protons in the universe as 1.3×10^{79} each.

created world (in natura naturata or in the universe) and in the transfinitum in abstracto in thought (transfinite numbers in human knowledge). He wrote:

> "This groundwork, which I consider to be the sole correct one, is held only by a few. While possibly I am the very first in history to take this position so positively, [...] I know for sure that I shall not be the last".[31]

The Cardinal agreed that Cantor deduced the *possibility* of a Transfinitum from the concept of the all-mighty of God, however, he disagreed when Cantor deduced the necessity of the Creation of a Transfinitum from the infinite Goodness and Glory of God. Cardinal Franzelin regarded the *necessity* of Creation by God as a contradiction in itself and he argued that on the contrary God's *freedom* to create is essential. Franzelin pointed out that Cantor's view that the transfinite exists in nature is a dangerous view, because any attempt to identify God's infinity with the natural world seemed to be closed to pantheism. Politely in his answer in 1886, Cantor weakened the arguments into that of our subjective feeling for a necessity of Creation. He wrote that one ought not forget the distinction between the absolute infinite and the actual infinite. The actual infinite we can grasp in our set-theoretic considerations. The absolute infinite, however, we cannot know. With that distinction the cardinal could live and saw no longer danger to religious concepts in Cantors transfinitum.

Elsewhere Gottfried Wilhelm Leibniz maintained: God is pleased with odd numbers. In a speech Cantor's teacher Ernst Eduard Kummer explained this statement. Numbers are God's not man's Creation, they are therefore objects like those in natural science. In Cantor (like Hilbert) all that is needed for mathematical objects to exist is that there is no logical contradiction in their concept. In mathematics the Almighty Creator is (only) bond to this logical basic; in nature, however, the Creator has freedom to formulate laws (i.e. to determine gravitation as depending on mass and distance in this or that manner). This physico-theological view is found in Baron Christian Wolff (1679–1754), a century before Cantor professor in Halle, for instance in his "Theologia naturalis",[32] and accepted by Cantor. However, Cantor insisted that the nature of mathematics consists in its freedom (within the bonds, of course, with logic). The theologian Friedrich Schleiermacher (1768–1834) who was in Halle for some time (1804–1809), would have sided rather with Cantor. A modern theologian Hans-Georg Fritzsche (1926–1986) says in this connection:

> "The world is as it is, however only founded on fact, but not on any absolute necessity. God is more than the totality of the laws of nature. Despite he created these laws, he could have had in mind quite another laws and could have created those".[33]

One of the leading 19th century German neo-Thomists, Constantin Gutberlet, a disciple of Cardinal Franzelin, wrote the book "The infinite regarded metaphysically and mathe-

[31] "Auf diesem Boden, den ich für den einzig richtigen halte, stehen nur wenige, vielleicht bin ich zeitlich der erste, der diesen Standpunkt mit voller Bestimmtheit [...] vertritt, doch das weiß ich sicher, daß ich nicht der letzte sein werde, der ihn verteidigt!". In: *Gesammelte Abhandlungen*, p. 372.

[32] Halle, 1739, Reprint 1978, esp. §799.

[33] "Die Welt ist, wie sie ist, doch nur faktisch so, aber nicht aus einer ihr zukommenden absoluten Notwendigkeit. Gott ist mehr als der Inbegriff der Naturgesetze, die Gott zwar geschaffen, die er aber auch ganz anders hätte wollen und schaffen können". In: *Lehrbuch der Dogmatik, Teil II: Lehre von Gott und der Schöpfung*, Evangelische Verlagsanstalt, Berlin, 1967, p. 124.

matically".[34] Gutberlet used Cantor's transfinite numbers to support his own ideas on the absolute infinity of God's existence. With this author Cantor discussed the way in which the arguments were presented in the book (and which Cantor found rather scholastic). Nevertheless, in these discussions Cantor acquired a great deal of his knowledge of the scholastic philosophy. To the mathematician Ivar Bendixson (1861–1935) Cantor wrote that the author is a Catholic theologian and this would explain much of his style (letter of November 11, 1886).

In his textbook "Allgemeine Metaphysik" (General Metaphysics) which appeared in 1879 and saw a third edition in 1897, Gutberlet dealt in Chapter 8, which is called "Die Vollkommenheit" (The Perfection), with infinite sets. In the Preface to the first edition Gutberlet mentions "an unassailable principle of the holy Thomas [of Aquino] about relative infinity". Thomas had corrected Aristotle's conception of infinity. Aristotle only accepted potential infinity. Thomas had argued that although a "constructive" notion of the actually infinite is inconsistent, it is perfectly acceptable to study the "relative actually infinite" (infinitum actu secundum quid). We cannot actually construct the set of natural numbers, because it has no maximum element. Yet, we can consistently consider the totality of all natural numbers as an actually infinite set by, as it were, stepping out of the process of construction and look at it from the outside. In the second edition of his book Gutberlet mentions Cantor, whose "brilliant works about the theory of manifolds and the transfinite numbers" have strengthened the opinion of the author:

> "With much perspicacity the brilliant mathematician G. Cantor defends the actually infinite set and he has even created a mathematics of the transfinite number which he explained and founded in numerous writings".[35]

Gutberlet also quotes Louis Couturat (1868–1914) which considered Cantor's works as fitting answers to the objections of G.W. Leibniz and Immanuel Kant (1724–1804) and for whom "the giddy piling of infinities on top of each other"[36] was in existence.

Gutberlet demonstrated that there is no contradiction in the concept of an actually infinite set; the headline of §4 in Chapter 4 in his book "Metaphysik" reads: In the concept of an actually infinite set we cannot point out contradictions.[37] From this point of view he found Cantor's set theory as a support of his concepts. We read: "By G. Cantor's stroke of genius on set theory the opinion of the author concerning the infinite entities has got a very strong mainstay". (Metaphysik, Preface of the second edition 1890). However, there were differences between Cantor and Gutberlet too. Cantor wrote in the mentioned letter to Bendixson: "You will note that he [Gutberlet] is not everywhere successful in defending of the actual infinite because among others he tries to deduce the actual infinite from the differentials and so misses his aim completely because differentials represent a potential

[34] *Das Unendliche, metaphysisch und mathematisch betrachtet,* Mainz, 1878.

[35] "Mit vielem Scharfsinn vertheidigt der geniale Mathematiker G. Cantor die actual unendliche Menge und[, er] hat sogar eine Mathematik der transfiniten Zahl geschaffen und in zahlreichen Schriften erklärt und begründet". *Allgemeine Metaphysik,* 3rd edition, p. 183.

[36] Gutberlet in his Metaphysik (3rd edition, p. 184) quotes in German and gives as source: Couturat, *De l'infini mathématique,* Paris, 1896. "Cet échafaudage vertigineux d'infinis superposés, qu'ils croyaient inconcevable, existe aujourd'hui, construit par un subtil et profond mathématicien [Cantor]", p. 456.

[37] Kapitel 8, §4: "In dem Begriff einer actual unendlichen Menge [. . .] läßt sich kein Widerspruch nachweisen", *Allgemeine Metaphysik,* Theissing'scher Verlag, Münster, 1879, 3rd edition 1897.

infinite only".[38] In a letter to Franz Goldscheider (1852–1926) from September 24, 1886 Cantor regarded the "actual infinitely small quantities" (actual unendlich kleinen Größen) as "paper entities" (papierne Größen) because that "trifling objects does exist only on paper of their inventors and partisans".[39]

A central idea in Thomism (called the fourth way) is that all objects of everyday world are finite but they point beyond themselves to the infinite and, moreover, they demand for completion. Entities participating in such a perfection do not belong to everyday world (a world without infinitum actu in natura), they must have received that perfection from something else, not from the surrounding world. In the field of mathematics for example, S. Thomas Aquinas (1225–1274) said there are no kinds of infinite numbers ("nulla autem species numeri est infinita"),[40] i.e. there does not exist a largest natural number. However, if one wishes to have such an upper bound for all natural numbers it cannot exist within the set of natural numbers but is to be considered as an exterior element (ideal element) belonging to another kind. Cantor coined for that kind of numbers the word "transfinite number". Again, for the kind of transfinite numbers limiting natural numbers there does not exist a major element, and the process begins again, etc. From this point of view we can read Thomas in a new sight: "The existence of an actual infinite multitude is impossible. [...] Sets of things are specified by the number of things in them".[41]

The difference between Thomas and Cantor is: Thomas did not accept actual infinities in the real world, and he attributed the absolute (actual) infinite to God. However, he regarded certain relatively infinite sets as admissible. Cantor, however, was convinced that he had found a principle to insert such (relatively) actual infinities of different kinds, the transfinite numbers (actu infinite), into logic and mathematics and into the real world as admissible objects. For this purpose Cantor extended the successor function as used in finite aggregates to infinite multiplicities just by means of transfinite numbers. Cantor's biographer Herbert Meschkowski once remarked: "For Cantor the theory of sets was not only a mathematical discipline. He also integrated it into metaphysics, which he respected as a science. He sought, too, to tie it in with theology, which used metaphysics as its 'scientific tool'. Cantor was convinced that the actually infinite really existed 'both concretely and abstractly' ".[42]

At any rate, we should take into account that the concept of infinity is predominant philosophical (logical and mathematical, respectively) and that is why although Thomas and Cantor travelled the same route in the end they went into different directions. In the Sacred Scripture infinity is found in such concepts as God's omnipotence and almighty, but also in God's incomparable love. Let us quote Herbert Meschkowski once more:

[38] See Cantor's letter to Aloys von Schmid (1825–1910) of March 26, 1887, and the given arguments for the infinitely small quantities (differentials) as only a potentially infinite; in: *Briefe*, 1991, p. 287.

[39] "weil diese Dingerchen nur auf dem Papiere ihrer Entdecker und Anhänger existieren". Letter from October 20, 1895. Niedersächsische Staats- und Universitätsbibliothek Göttingen, Handschriftenabteilung, Math.-Archiv 47.

[40] *Summa theologica I*, q. 7, a. 4.

[41] Unde impossibile est esse multitudinem infinatam actu [...] Unde necesse est quod sub certo numero omnia creat comprehendatur, *Summa theologica I*, q. 7, a. 4. See also on Thomas' views Gianfranco Basti's paper on infinity in the *Dizionario Interdisciplinare di Scienza e Fede*, Giuseppe Tanzella-Nitti and Alberto Strumia, eds., Rome, 2002. Available in English on the web at http://www.disf.org/en/Voci/13.asp.

[42] *Biographical Dictionary of Mathematicians*, vol. 1, C. Gillespie, ed., Scribner's Sons, New York, 1991, p. 404.

> "To the end Cantor believed that the basis of mathematics was metaphysical, even in those years when Hilbert's formalism was beginning to take hold. Found among his papers after his death was a shakily written penciled note (probably from 1913) in which he reaffirms his view that "without some grain of metaphysics" [43] mathematics is unexplainable. By metaphysics he meant 'the theory of being' ".[44]

This echos Kant's view: "There will always metaphysics in the world, and what is more in everyone, especially in every thinking man".[45]

All numbers comply with rules that are given by the Creator and that is also true for Cantor's transfinite numbers and their arithmetic developed by Cantor. Transfinite numbers therefore can be subsumed under the words of the Scripture (Wisdom 12: 11):

> "You have ordered all by means of number, measure, and weight". (Omnia in pondere, numero et mensura disposuisti.)

It is remarkable that in the number system Cantor excluded non-archimedian systems, i.e. he emphatically rejected infinitesimally small magnitudes. In a letter to Franz Goldscheider[46] he argued that the Archimedian axiom (axiom of measurability)[47] is only a consequence of the concept of linear magnitudes (which is indeed true if we presuppose the continuity of the magnitudes in the system).[48] Later Cantor toned down his rejection of such quantities and regarded them as quantities that are brought into being only by writing on paper (letter to Goldscheider, 24.9.1886) only whereas earlier he had even condemned them as a dangerous "infinitarian bacillus" that threatens Italy (infinitäre Bacillus, der Italien bedroht; letter of December 13, 1893 to Guilio Vivanti (1859–1949)).[49]

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[43] "Ohne ein Quentchen [old German measure in pharmacy, Lat. Quentinu)] Metaphysik läßt sich, meiner Überzeugung nach, keine exacte Wissensachft begründen" (In my opinion, without some grain of metaphysics it is impossible to found any exact science), in: H. Meschkowski, *Mathematik und Realität*, Bibliographisches Institut, Mannheim, 1979, p. 96.

[44] *Biographical Dictionary of Mathematicians*, vol. 1, C. Gillespie, ed., Scribner's Sons, New York, 1991, p. 405.

[45] I. Kant, *Prolegomena zu einer jeden künftigen Metaphysik*, Hartknoch, Riga, 1783, p. 367. *Prolegomena to Any Future Metaphysics* (transl. by p. Lucas). Liberal Art Press, New York, 1951, p. 163. "Es wird also in der Welt jederzeit und, was noch mehr, bei jedem vornehmlich nachdenkenden Menschen Metaphysik sein".

[46] After Cantor had difficulties in March 1885 to publish his theory of types in the Acta mathematica (Letter to G. Mittag-Leffler of February, 21 and March 15, 1885, in: Meschkowski and Nilson, 1991, pp. 240–242), he found in the Berlin teacher (Gymnasiallehrer) Franz Goldscheider, his former student, an interested partner to whom he explained his theory in some quite detailed letters. The concepts of these letters can be found in a book of letters (Briefbuch) in the Cantor Nachlass in the Niedersächsische Staats- und Universitätsbibliothek Göttingen, Handschriftenabteilung, Cod. Ms. G. Cantor 16–18.

[47] Euclid, *The Elements*, Book V, Definition. 4. "Magnitudes are said to have a ratio to one another which are capable, when multiplied, of exceeding one another". T. Heath's edition, vol. 2 (reprint), Dover, New York, 1956, p. 114.

[48] In *Gesammelte Abhandlungen*, p. 409.

[49] Let us recall the desperation of Blaise Pascal (1632–1662) that we are incapable of knowing both the infinitely large as well as the infinitely small, we are wedged in between both extrema and neither bears any relation to us. "Car enfin qu'est-ce que l'homme dans la nature? Un néant à l'égard de l'infini, un tout à l'égard du néant, un milieu entre rien et tout. Infiniment éloigné de comprendre les extrêmes, la fin des choses et leur principe sont pour lui invinciblement cachés dans un secret impénétrable, également incapable de voir le néant d'ou il est tiré, et l'infini où il englouti. [...] Qui suivra des étonnantes démarches? Le auteur de ces merveilles les comprend. Tout autre ne le peut faire". B. Pascal, *Pensées*, d'après l'édition de M. Brunschvigg [sic]. Preface E. Boutroux, G. Crès, Paris, 1913, pp. 26–27; in Brunschvicg's edition section II, no. 72.

5. Paradoxes and truth

The belief that infinity is paradoxical persisted until the 19th century, and a majority of mathematicians had therefore avoided the introduction of the actual infinite. Then Cantor tried to grasp and, moreover, to manipulate infinity using rigorous logical demonstration in a finite rational thought. Do his considerations open the way to a complete understanding of the (at least mathematical) infinite or even of God's Creation?

Cantor's theory faced opposition, but the theory generated its own internal problems as well. Surprisingly contradictions appeared, first that of Cesare Burali-Forti in 1897, later the more influential great paradox of Bertrand Russell (1872–1970) in 1902.[50] Burali-Forti considered the entire succession of all ordinal numbers and argued that it had to have a corresponding ordinal number, bigger than all ordinal numbers, i.e. bigger than itself. Russell considered the set of all sets that are not an element of itself and showed that such a set cannot exist. In these counterexamples we again encounter the limits to rational thought, in view of self-referential reflections clearly pointed out by Kurt Gödel in 1931.

Strictly speaking Cantor's theory did not exclude such arguments and the counter examples shocked many mathematicians. Actually Cantor himself discovered the inconsistent sets (like, for example, the superset of all sets) in 1895 or even earlier. Some attempted to rescue set theory by excluding inconsistent sets (axiomatic systems of Ernst Zermelo (1871–1953) or Abraham Fraenkel (1891–1965)). Such axiomatic systems are meant to save set theory by erecting walls against enormous quantities, against non-sets. Henri Poincaré, who rejected the actual infinite, asked whether after the flock of sheep is fenced in, the wolf might have already been among the flock.

By Karl Friedrich Heman (1839–1919) Cantor was asked about the relation of two sentences I and II, and in a in a letter June 21, 1888 we read:

> "I. The world and its time begun before a finite space of time, or, in other words, the bygone time of the world (e.g. measured by hours) is finite, which is a true sentence and a Christian dogma (article of faith).
> II. There are no actual infinite numbers, which is false and haethen and therefore no Christian dogma."

Cantor distinguished the different levels of the statements and his response reads as follows: Sentence I refers to "concrete world creatures" (concrete creatürliche Welt) and sentence II to the "ideal realm of numbers" (ideale Gebiet der Zahlen) and the truth of sentence I is no consequence of the truth of sentence II. Finally he remarked: "I do not know which Roman Catholic misguided you to consider sentence II as a secret of faith".[51] Moreover, Cantor was absolutely sure that the late Cardinal Franzelin who was versed in scholastic philosophy would tolerate this opinion.

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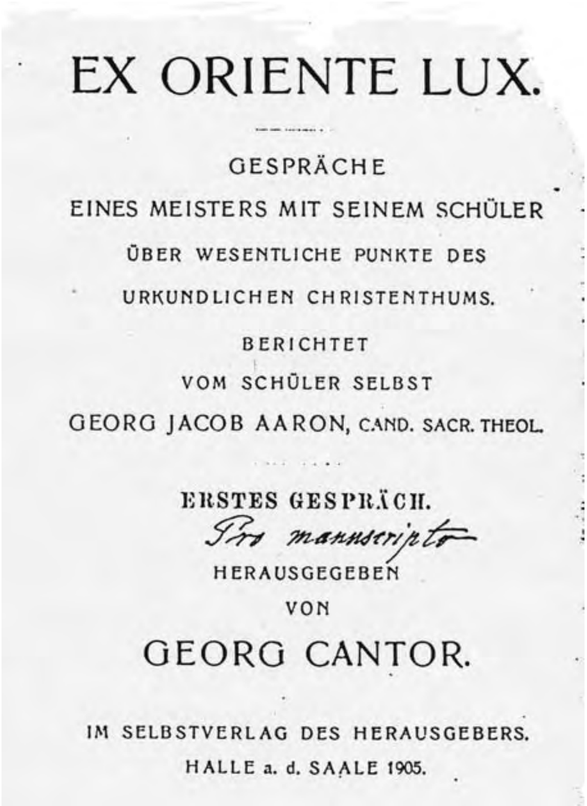
[50] Cf. R.M. Sainsbury, *Paradoxes*, 2nd edition, Cambridge University Press, Cambridge, 1995; cf. footnote 20.

[51] "Es wird von Ihnen das Verhältnis der beiden Sätze: I. Die Welt hat sammet der Zeit vor einem endlichen Zeitabschnitt angefangen oder was dasselbe sagt, die bisher verflossenen Zeitdauer der Welt ist (mit dem Maaß etwa einer Stunde gemessen) eine endliche, welcher wahr und christlicher Glaubenssatz ist, und II. Es giebt keine actual unendlichen Zahlen, welcher falsch und heidnisch ist und daher kein christlicher Glaubenssatz sein kann. [...] Ich weiß nicht, welcher Katholik Sie zu dem gefährlichen Irrtum verleitet hat, den Satz II gewissermaßen für einen Glaubenssatz zu halten". Niedersächsische Staats- und Universitätsbibliothek Göttingen, Handschriftenabteilung, Cod. Ms. G. Cantor, Briefbücher 16.



EX ORIENTE LUX.

GESPRÄCHE
EINES MEISTERS MIT SEINEM SCHÜLER
ÜBER WESENTLICHE PUNKTE DES
URKUNDLICHEN CHRISTENTHUMS.
BERICHTET
VOM SCHÜLER SELBST
GEORG JACOB AARON, CAND. SACR. THEOL.

ERSTES GESPRÄCH.
Pro manuscripto
HERAUSGEGEBEN
VON
GEORG CANTOR.

IM SELBSTVERLAG DES HERAUSGEBERS.
HALLE a. d. SAALE 1905.

Fig. 5. Frontispiece of *Ex Oriente Lux*. Courtesy of the Universitäts- und Landesbibliothek Halle.

Much of mathematics is or can be founded on the concept of the infinite, and therefore the discovery of inconsistent sets shocked the axiomatic wing in mathematics first of all. Such inconsistencies did not really bother Cantor. He was aware of the complexity and reach of infinity, only because of the long Western tradition in which God is identified with the infinite. In a hierarchical system of infinity God is its completion, he is the Absolute. That is why no ultimate answer can be given by a finite human mind. Reason alone cannot decide the foundation of science, we need some grain of metaphysics which in Cantor's belief is supplied by God's existence. Cantor revolutionized the metaphysical foundation of mathematics by boldly insisting on the existence of actually infinite sets as mathematical objects. However, for Cantor the existence of inconsistent sets was in a certain way quiet necessary because our limited mind cannot completely grasp the whole infinite God and his creations in their total existence; inconsistent sets simply belong to God's Creation too. Cantor solved the mathematical problem created by inconsistencies by considering *consistent sets* only ("a set considered as finished"; eine als fertig gedachte Menge, letter to D. Hilbert of October 2, 1897) and pointed that fact out to be his *most distinguished and most minded Theorem in set theory* ("zum wichtigsten und vornehmsten Satz der Mengenlehre", same letter to D. Hilbert).

From Emmy Noether (1882–1935) we have Cantor's impressive description of his concept of a set:

> "As to the concept of a set, Dedekind said that he regards a set as a pursed sack, a sack that contains certain specific things, but you do not see them and know nothing of them other than that they are specific and present. Some time later, Cantor gave his own idea of a set. He draw himself up to full height, described with his arm a grandiose gesture, and, with inscrutable gaze, breathed: 'I visualize a set as an abyss' ".[52]

Noether's comparison of Cantor and Dedekind is not accidental, because Cantor and Dedekind had mathematical interests in common, which included the merging theory of sets; moreover, they were close friends. In his booklet "Was sind und was sollen die Zahlen"? (What are and what should the numbers?) of 1887, in the Sections 1 through 5 Dedekind considered systems of things or totalities (called "Systeme", "Mannigfaltigkeiten", "Inbegriffe", and "Gesamtheiten" by Dedekind), that are very similar to Cantor's notion of set. Dedekind characterized the infinity of a system by means of the property that it is similar to one of its real part (Section 5). Already in 1882 Dedekind had written Cantor about this characterization. Cantor wrote about the difference of opinion with Dedekind that was impressively described by Noether as follows:

> "My other opposition to Dedekind consists, as you know, in the fact that he considers every determined multiplicity as consistent, that means he does not admit the difference between consistent and inconsistent multiplicities".[53]

Inconsistencies did not shock Cantor. Moreover, his theory had generated other paradoxical results. In a letter to Dedekind of June 20, 1877 he had demonstrated the equal cardinality of certain sets in Geometry with *different* dimensions. Cantor did not dare to express his astonishment on that paradoxical result in his native language but wrote in French: "Je le vois, mais je ne le croix pas!" (I see it but I don't believe it).[54]

A demonstration of the existence of God is not intended to convince any Atheist but presupposes the Christian belief as such and then unfolds or elucidates certain religious properties. It is exactly this background against which Cantor deduced from God's infinite goodness and glory the necessity of the actual Creation of a Transfinitum (aus Gottes "Allgüte und Herrlichkeit auf die Notwendigkeit der tatsächlichen erfolgten Schöpfung eines Transfinitums" schloß).[55] Cantor was quite sure of God's existence. Our knowledge will be extended to the Absolute step by step, i.e. to God. Set theory is integrated in this procedure.

[52] "Dedekind äußerte hinsichtlich des Begriffs der Menge: er stelle sich eine Menge vor wie einen geschlossenen Sack, der ganz bestimmte Dinge enthalte, die man aber nicht sehe, und von denen man nichts wisse, außer daß sie bestimmt und vorhanden seien. Einige Zeit später gab Cantor seine Vorstellung einer Menge zu erkennen: Er richtete seine kolossale Figur auf, beschrieb mit erhobenem Arm eine großartige Geste und sagte mit einem ins Unbestimmte gerichteten Blick: 'Eine Menge stelle ich mir vor wie einen Abgrund' ". In: R. Dedekind, *Werke*, Vieweg, Braunschweig, 1932, vol. 2, p. 449.

[53] "Mein anderer Gegensatz zu Dedekind besteht, wie Sie ja wissen, darin, daß er jede bestimmte Vielheit für consistent [widerspruchsfrei] hält, also den Unterschied von consistenten und inconsistenten Vielheiten nicht zugiebt". Letter to D. Hilbert of January 27, 1900 in: Briefe, 1991, p. 427. The other opposition to Dedekind that Cantor mentions concerns the character of the axioms. There is a difference of opinion with Gauß as well; the reason here is that Gauß considers all infinite multiplicities for inconsistent.

[54] Quoted by Fraenkel in Cantor's Gesammelte Abhandlungen; also confirmed by Wilhelm Stahl, a grandchild of Cantor.

[55] Letter to Cardinal Franzelin, January 22, 1886.

Gerhard Kowalewski (1876–1950) who during his Leipzig period about the turn of the century at the joint meetings of the Mathematicians of Leipzig and Halle (in a mathematical circle ["Kränzchen"] every two weeks) regularly met Cantor reported in his autobiography "Bestand und Wandel" (Duration and Change):

> "The cardinal numbers, Cantor's alephs, were for Cantor something holy, steps so to say leading upwards to the thrown of Infinity, to the thrown of God".[56]

Both the paradoxes of set theory and the concept of the Almighty God are rather similar. Already in Pascal the Almightiness of God is questioned: Can God create a stone so heavy that he cannot pick up it? Because God is infinitely beyond our comprehension we are incapable of determining his infinite properties from our finite viewpoint.[57] Reason cannot solve this question completely; if we try to do so, we get concepts without content (in the sense of Kant). In the 18th century some minds distinguished between mathematical necessity and physical possibility in a physico-theological manner in their reflections, i.e. they separated form and content in their considerations.[58] The mathematical necessity rests on the logical Theorem of contradiction and once the logical rules were given by God, the Creator himself was bound to follow them. However, he is free in his choice in the sense of that physical possibility that bears no relation to logic (Theorem of sufficient reason, Satz vom zureichenden Grund).

6. Religious denomination

Like his father Georg Cantor belonged to the Protestant faith, and he remained a Protestant throughout his life. Although for metaphysical reasons he was rather interested in Catholic dogmas he never did change his denomination. However,

> "I am not a Catholic, yet judge the unholy confusion within Protestantism quite similar as Adeodatus[59] and as a positive Christian I am inwardly, and if need be outwardly, on good terms with Catholicism and its spiritual leader [...] and I by no means share the fundamental enmity of the majority of Protestants toward all that Catholicism holds to",[60]

he wrote to Heman (letter of July 28, 1887). It seems quite clear that he was not strictly denominational and formed a belief of its own between Catholic and Protestant doctrines. In a letter to Constance Pott (1833–1915) he confessed a decade later:

> "As to religious questions and relations my point of view is not denominational as I do not belong to any of the existing denominations [false]. My religion is that of a Trinitarian and my theology is

[56]"Die Mächtigkeit, die Cantorschen Alephs, waren für Cantor etwas Heiliges, gewissermaßen die Stufen, die zum Thron der Unendlichkeit, zum Thron Gottes emporführen". Oldenburg, München, 1950, p. 201.

[57]For a more detailed discussion cf. R. Thiele, *Hilbert's 24th problem*, American Mathematical Monthly **110** (2003), 1–24.

[58]An important German philosopher in this respect is Johann Nicolaus Tetens (1736–1807), cf. J.N. Tetens, *Die philosophischen Werke*, vols. III–IV, J. Engfer, ed., Olms, Hildesheim, 2005.

[59]Aurelius Adeodatus (pseudonym), author of a book on Thomas's philosophy (Colon, 1887).

[60]"Ich bin kein Katholik, beurtheile aber die heillose Zerfahrenheit innerhalb des Protestantismus ganz ähnlich wie Adeodatus und stehe als positiver Christ innerlich und, wenn nöthig, auch nach Außen hin freundschaftlich zum Katholicismus und dessen geistigem Oberhaupt [...] und ich theile sicher nicht mit der Mehrzahl der Protestanten die grundsätzliche Feindschaft gegen alles zum Catholicismus gehörige".

> founded in the God's word and work revealed by himself whereas above all I revere as my masters also the Apostolic Fathers, the Early Fathers, and the most respected Masters of the Church during the first 15th centuries of our era (i.e. the time before the Reformation of Church in the 16th century)".[61]

Cantor believed in a personal God not in abstract metaphysical supreme Being. When Cantor grew older his interests shifted. He considered obscure historical affairs far removed from mathematical research. To his correspondent Charles Hermite, a Catholic colleague, Cantor confessed:

> "Metaphysics and theology, I can but confess, have laid siege to my soul that I find too little time for my first love, mathematics".[62]

In a writing he even considered the question who was the actual Father of Jesus Christ. In a letter to Philip Jourdain (1879–1919), the English translator of the mentioned pamphlet, Cantor gave an answer to Jourdain's question, what he thinks of Christ's resurrection:

> "As to Christ's resurrection I believe in it as a matter of fact and do not muse on the 'how' of it, it is attested best and to the fullest in the Scriptures of the New Testament".[63]

During the last decades of his life such odd interests of Cantor were probably effected by his mental illness; in that period Cantor somewhat lost the right track. A course on Leibniz' philosophy delivered by Cantor at the University of Halle ended in disaster: the initial number of about 20 students rapidly decreased to zero. Cantor regarded Francis Bacon (1561–1626) as the true author of the works of William Shakespeare (1564–1616), and he held some public lectures on the topic. He tried to prove his opinion in public lectures, papers, and pamphlets.[64] He was, moreover, completely convinced that Bacon was the greatest Englishman who had ever lived and also in the end Bacon had come very close to the Roman Catholic faith (letter to Hermite, 30.11.1895). Cantor asked his Catholic colleague to inform the Rector Maurice Lesage d'Hautecour d'Hulst (1841–1896) of the Catholic University of Paris on this fact. Cantor hoped to bring d'Hulst to a revised and enlarged edition of the old book "Christianisme de Bacon" in two volumes (1799) by Jacques André Emery (1732–1811). And in 1896 Cantor even felt the necessity to set Pope Leo XIII (1810–1903) right concerning the true confession of Bacon ("confessionem fidei Baconi [. . .] in memoriam revocare opportunum existimavi"). Cantor was invited by d'Hulst to attend a planned Congress of learned Catholics at Freiburg, in the Black Forest, however, as a member of a Protestant university, though. Cantor hesitated, considered

[61] "In religiösen Fragen und Beziehungen ist mein Standpunkt kein confessioneller, da ich keiner der bestehenden organisierten Kirchen angehöre. Meine Religion ist die vom dreieinigen Gott, einem und einzigen Gott selbst geoffenbarte und meine Theologie gründet sich auf Gottes Wort und Werk, wobei ich außerdem als meine Lehrer hauptsächlich die apostolischen Väter, die Kirchenväter und die angesehensten Kirchenlehrer der ersten 15 Jahrhunderte unserer Zeitrechnung verehre (d.h. der Zeit, welche der Kirchenrevolution des 16. Jahrhunderts vorangegangen ist)". Letter of March 7, 1896.

[62] "Metaphysik und Theologie haben, ich will es offen bekennen, meine Seele in einem solchen Grade ergriffen, daß ich verhältnismäßig wenig Zeit für meine erste Flamme [Mathematik] übrig habe". Letter of January 24, 1894.

[63] "Was die Auferstehung Christi betrifft, so ist sie durch die Schrift des Neuen Testaments auf's Beste und Umfassendste bezeugt; ich glaube fest daran, als eine Tatsache und grüble nicht über das 'Wie' derselben".

[64] *Confessi fidei Baconi*, Halle, 1896; *Resurrectio divi Quirium Baconi*, Halle, 1896; *Die Rawleysche Sammlung von 32 Trauergedichten auf Bacon*, ed. by Cantor with a Preface, Halle, 1897; *Shakespeareology and Baconism*, Magazin für Literatur **69** (1900), col. 196–203.

attending anonymously, and in the end did not take part. On paper Cantor was rather reserved, in real life, however, people were impressed by "his unconventional friendliness" (Grace Chisholm-Young (1868–1944)).

Another example of Cantor's own sight is to be found in a letter to Bertrand Russell (1872–1970):

> "I am a Baconion in the Bacon–Shakespeare question and I am quite an adversary of Old Kant, who, in my eyes[,] has done much harm and mischief to philosophy, even to mankind; as you [can] easily see by the most perverted development of metaphysics in Germany in all that [sic; who] followed him, as in [sic] Fichte, Schelling, Hegel, Herbart, Schopenhauer, Hartmann, Nietzsche, etc. etc., on to this very day". (Letter of September 19, 1911; in Cantorian English)

We finally mention Cantor's pamphlet "Ex oriente lux" written during a stay in a neuropathic hospital (Universitätsnervenklinik) in Halle and published in 1905. In a letter to Grace Chisholm-Young, who together with her husband William Young (1863–1942) wrote a book "Theory of sets of points" (1906), Cantor announced his booklet:

> "The muse[65] afforded to [granted] me I employed to a renewed study of our Bible with opened eyes and postponing all prejudices. The result has been highly remarkable, as you will see by a little pamphlet (anonymous) of half a sheet, that I will send you [in] perhaps in [sic] a week; it is now in the printing office. The title is: 'Ex Oriente lux. Gespräche eines Meisters mit seinem Schüler über wesentliche Puncte des urkundlichen Chistenthums. Bericht vom Schüler selbst'. Erstes Gespräch". (Cantorian English)[66]

In the pamphlet Cantor rejects the Immaculate Conception and tries to demonstrate Joseph of Arimathaea (town in Judaea) to be the father of Jesus and not Joseph of Nazareth (town in Galilee), who is the foster-father of Jesus only. This Joseph of Arimathaea is not mentioned in Bible with the exception of his request to bury Jesus Christ (Mark 15:43; John 19:38). Cantor identified this patrician of Arimathaea as a former Roman officer named Josephus Pandera who was for a short time married to Mary and begat Jesus then. Cantor's demonstration is founded in such quotations from John:

> "They asked, 'Where is your [human] father'? Jesus replied 'You know neither me nor my Father' ".[67]

We quote the concluding remark of the Master:

> "By our comprehension [...] we deal a mighty blow at all present-day theological tendencies, and shake profoundly at the existing, mutually hostile, ecclesiastical organizations. We deprive orthodox Jewry, too, of its grounds for denying Christ the Messiah. But there remains for all time, reposing unshakably upon the Rock of Christ, the Church Invisible, that he has founded. He is The Supreme, and has need of no vicar on Earth".[68]

[65] Cantor meant *leisure* (in German Musse or Muße) not *muse* (in German Muse).

[66] Letter of April 5, 1904; "Ex oriente lux. Conversation of a Master with his Pupil on essential points of documented Christianity. Reported by the pupil. First Conversation".

[67] John 8:19. See also Cantor's letter to Jourdain, May 3, 1905.

[68] "Mit unserer Auffassung [Joseph von Arimatheae ist der leibliche Vater von Jesus] treffen wir, wie mit einem wuchtigen Hiebe, alle theologischen Richtungen der Gegenwart und erschüttern auf's Tiefste die bestehenden, sich gegenseitig anfeindenden kirchlichen Organisationen. Auch entziehen wir der jüdischen Orthodoxie den Grund, warum sie Christo die Anerkennung der Messianität bisher versagt hat. Es bleibt aber bis zum Ende der Tage auf einem unerschütterlichen Fels, Christo selbst ruhend, die unsichtbare Kirche, welche Er gegründet hat, bestehen. Er ist ihr Oberhaupt, das keinen Statthalter auf Erden braucht (pp. 11–12).

Fig. 6. Cantor's grave in Halle. Photo: Rüdiger Thiele.

Indeed, this is a very strong critique of churches, above all the foundation of the Roman Pope as the Vicar of Christ is clearly neglected. Moreover, "I am only answerable to God" (Ich bin nur vor Gott verantwortlich), he wrote to Constance Pott (letter of March 13, 1896).

In his lifetime Cantor always had to fight for his revolutionary views and this militant character revealed itself also elsewhere. He did not recoil from the defense of any radical viewpoint when he was convinced of its correctness. He did not take such matters personally but for its own sake. In 1883 he had written to his friend Gösta Mittag-Leffler (1846–1927):

> "Far be it from me to take credit personally for my discoveries. I am merely the tool of a higher power, that will pursue its course when I am gone, even as it revealed itself thousands of years ago in Euclid and Archimedes".[69]

In conclusion we might express this statement in Biblical language: "The wind bloweth where it listeth; and thou hearest the sound thereof, but canst not tell whence it cometh, and whither it goeth: so is every one that is born of the spirit" (John 3:8).

[69] "Ich bin weit davon entfernt, mir meine Entdeckungen zum persönlichen Verdienst anzurechnen, denn ich bin nur ein Werkzeug einer höheren Macht, die nach mir weiter wirken wird, ebenso wie sie vor Jahrtausenden in Euclid und Archimedes sich offenbart hat". Letter of December 23, 1883.

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CHAPTER 29

Gerrit Mannoury and his Fellow Significians on Mathematics and Mysticism

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Edited by T. Koetsier and L. Bergmans

1. The search for unifying explanations

This chapter will deal with the way in which the Dutch philosopher and mathematician Gerrit Mannoury (1867–1956), as well as his pupils and friends, who were active in the so-called Dutch signific movement, established a link between mathematics and the higher concerns of the human mind.

In the last decades of the nineteenth century and the first decades of the twentieth the Netherlands saw the rise of currents of ideas which, dealing with subjects like society, science and the arts, sought to give all-encompassing explanations of human faculties and behaviour. Among these currents we must count Neo-Hegelianism, represented by G.J.P.J. Bolland, and psychic monism, advocated by G. Heymans.[1] Bolland and Heymans both influenced Gerrit Mannoury who was to become the main representative of a third current of ideas known as *Nederlandse significa* (Dutch significs).

As an unconventional and independent thinker, Mannoury, partly through a formal curriculum but largely through autonomous study, developed into one of the most many-sided scholars of Dutch intellectual life in the twentieth century. Mannoury was first of all a mathematician and, about the results of his purely mathematical research, his pupil David van Dantzig remarked: "[They are] viewed in the light of their time and environment,

Fig. 1. Gerrit Mannoury.

[1]There were also those individualistic thinkers who developed all-encompassing systems of thought that were too personal to lead to the founding of a school. The most fascinating among those scholars was no doubt Johan Andreas Dèr Mouw, a poet, mathematician, Sanskritist and philosopher. In a considerable number of poems he expressed the joy he felt at realising that his deepest Self was Brahman. In this so-called Brahman-poetry, he did not hesitate to use mathematical imagery. Dèr Mouw characteristically chose the *nom de plume* Adwaita (Sanskrit for "He who is without two-ity").

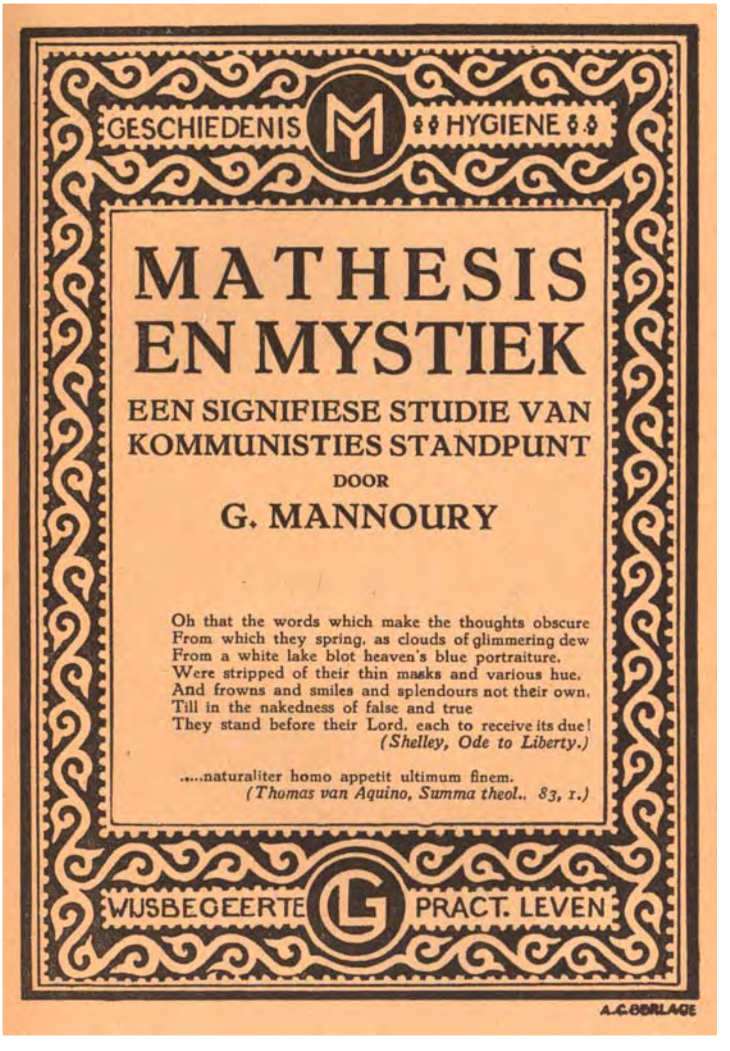
[...] quite outstanding and possess an uncommon and quite individual beauty" [26, p. 1]. Through his friendship with Frederik van Eeden, Mannoury became acquainted with significs. It was Lady Victoria Welby, lady in waiting to Queen Victoria, who coined the word significs and launched this new discipline which espoused to purify and enhance human communication and understanding through the clarification of words and concepts used in different realms of intellectual activity and through the criticism of metaphors in literary and non-literary contexts.

Frederik van Eeden had befriended Lady Welby and corresponded with her over many years. He developed his own views on language and communication, and summarised them in a series of theorems, published as *Redekunstige Grondslag van Verstandhouding* (Logical foundations of understanding), which is considered to be the first signific treatise in the Netherlands. What Mannoury held to be the main contribution of *Redekunstige Grondslag* was its leading idea of the "ultimate connection of all concepts". One of the main objectives of Van Eeden's treatise was indeed to show the basic unity of everything that is experienced or expressed, and to shed light upon the gradual transitions, which link up the clear-cut and unambiguous formulae of mathematics and formalised science to the fluid and evocative expressions of poetry and religious language. The interrelation of the most divergent phenomena of human life remained one of the dominant themes in signific literature as well as in the discussions among the members of the different study groups that were formed in the course of the history of the signific movement.

2. Convictions and ideologies

Strikingly divergent convictions and ideologies were to be confronted in the lively debates among the significians,who believed as much in the art of distinguishing as in the art of connecting. The evolving network of friends included Frederik Van Eeden, an unorthodox religious mind, who, in later years, converted to Catholicism; Luitzen Egbert Jan Brouwer, whose religious views were influenced by Eastern and Western mystical literature; Jacob Israël de Haan, who remained very attached to his Jewish origins; Henri Borel, an orientalist, who studied the religions of India, China and Japan; and Jacques Van Ginneken, who was a Catholic priest. Gerrit Mannoury himself had embraced communism, which he interpreted in his own personal way. He recognised the scientific importance of the Marxist analyses of society and human conduct, but more strongly believed in the spreading of communism as a heartfelt conviction in which volition and emotion had to play the dominant part. In this respect Mannoury's opinions must be viewed as akin to those of the communist Dutch poets Herman Gorter and Henriëtte Roland Holst. Mannoury's *Mathesis en Mystiek. Een SignifieseStudie van Kommunisties Standpunt* (Mathematics and mysticism. A signific study from a communist point of view), which is no doubt one of his wittiest and most personal writings, must be interpreted as an attempt to convince Catholic workers of the importance of *belief* in communist ideology.

There was room for faith in communism, but not for the old faith. In order to reach his end, Mannoury first analyses the simplest acts of counting and measuring, detecting in each of them an element that escapes denotation. One cannot count, nor can one measure


GESCHIEDENIS HYGIENE

MATHESIS EN MYSTIEK

EEN SIGNIFIESE STUDIE VAN KOMMUNISTIES STANDPUNT

DOOR

G. MANNOURY

Oh that the words which make the thoughts obscure
From which they spring, as clouds of glimmering dew
From a white lake blot heaven's blue portraiture,
Were stripped of their thin masks and various hue,
And frowns and smiles and splendours not their own,
Till in the nakedness of false and true
They stand before their Lord, each to receive its due!
(Shelley, Ode to Liberty.)

.....naturaliter homo appetit ultimum finem.
(Thomas van Aquino, Summa theol., 83, 1.)

WIJSBEGEERTE PRACT. LEVEN

A.C. BERLAGE

Fig. 2. Frontispiece of *Mathematics and Mysticism*.

without including something which goes beyond that which is counted or measured. Elaborating this point in the second section, Mannoury shows how any cognitive act involves the workings of the will. The third and last section of *Mathematics and Mysticism* deals with further reaching goals as expressed in religious and unreligious ideologies. Mannoury there fiercely opposes the old faith of Catholicism, which in his eyes fosters an asceticism hostile to human happiness and which uses symbols and imagery hopelessly out of tune with the aspirations of modern humanity. As the last pages of *Mathesis en Mystiek* show, to Mannoury the new belief needed to free man materially as well as spiritually, is the burning conviction of the struggling communist worker.

Through discussions and publications the significians aimed to improve understanding among people with diverging opinions. They did not generally eliminate this divergence, nor was it in most cases their aim to do so. They did feel, however, that disentangling the denotative content from the emotional or volitional value of expressions clarified the discussion and revealed the nature of the divergence. Clearly Jacques van Ginneken, the traditionalistic, Catholic member of the Signific Circle could only see Mannoury as a radical opponent in matters ideological and, moreover, as a zealous opponent trying to tear away the Catholic worker from the bosom of the church. There is no more convincing example of the emotional and volitional basis of ideologies and of the signific claim, worded by Wim Scheffer as “wanneer Uw gevoel spreekt, dàn spreekt U waarheid” [10, p. 17] [When your emotion speaks, then you speak the truth], than this lively exchange of ideas and troubled friendship between Gerrit Mannoury and Jacques van Ginneken. In the archives of Mannoury’s son Jan, there is a handwritten copy of a letter which reveals itself to be most informative as to the pain caused by the ideological opposition of the two friends and in particular by the way in which Mannoury used significs as a weapon against the church. In this letter, Mannoury asks for forgiveness, using the very religious language of his opponent and friend.

The heartfelt honesty about his own pain shows that there is reason to believe that Mannoury at one point felt very close to Catholicism and that his communist convictions were the result of inner struggle. If this is so, Mannoury provides us with a perfect illustration of his own views on ideological convictions as based on a strong emotional negation (see Section 7).

In Mannoury’s mind, communism and relativism were inseparable. In the article “Lenin en Trotski” published in *De Tribune* of 25 August 1925, he refers to “the dialectical or relativistic way of thinking, which I [Mannoury] feel ought to be the real foundation of our communist conviction” [15, p. 45]. Mannoury was perceived as unorthodox by many fellow party members. He was particularly fiercely attacked A. de Vries, who reproached him for his “systematic urge to doubt” [15, p. 77]. Mannoury responded that the so-called facts, which in De Vries’ eyes were solely capable of convincing any opponent of communism of his/her error, belong to “science-in-the-narrow-sense”, e.g. to the outer shape of communist ideology. Conviction, however, grows from within and develops through shared strength and enthusiasm. By refusing founding status to facts and attributing founding status to belief, Mannoury no doubt shocked many. It was, however, his concern with preserving what should be living and in continuous progress from being narrowed down and pin-pointed that inspired these views. This same concern led him to encourage the leadership of his party to abandon all idolatry of words and persons. As a significian, Mannoury indeed abhorred slogans that were bereft of their living content. As an admirer of Lenin, Mannoury believed that the communist cause, e.g. Lenin’s cause, was better served by cremating the latter’s remains than by embalming them. The frankness with which Mannoury voiced these ideas caused his exclusion from the Dutch communist party in 1929.

3. Does $2 + 2 = 4$ and is there a God to an all-levelling relativist?

Gerrit Mannoury quite naturally extended his relativism to every domain of human action and thought. In particular, he believed it possible to be a relativist as a mathematician.

In this domain too, he met with fierce criticism, coming in particular from the protestant Th. Van Vollenhoven, who in his *De Wijsbegeerte van de Wiskunde van Theïstisch Standpunt* (The philosophy of mathematics from a theistic point of view) criticised Mannoury's "alles nivelleerend relativisme" [35, p. 238] [all-levelling relativism]. As Mrs. Vuysje-Mannoury, Gerrit Mannoury's daughter, told the author in a personal communication in 1985, her father gladly accepted the honorary sobriquet "all-levelling relativist".

Mannoury's writings contain several formulations of the problem of the certainty attributed to expressions like $2 + 2 = 4$. In one case he states that $2 + 2 = 4$ is uncertain because it is not clear which uncertainty is suppressed by it. Mannoury indeed perceives certainty and uncertainty as fundamentally related to one another. Certainty can neither exist nor be understood by itself, but presupposes the uncertainty from which it springs.

A more elaborate explanation of mathematical certainty in general is given where Mannoury focuses on the principle of identity and where he remarks that the psychic factors which underlie expressions like $1 = 1$ do not allow for rigorous observation or clear-cut definition. On many occasions Mannoury denounces the seemingly self-evident definiteness of mathematical objects. Definiteness is seen by him as proceeding from an *effort* to define. In *Methodologisches und Philosophisches zur Elementar-Mathematik* (Methodological and philosophical comments on elementary mathematics) Mannoury asks his reader to consider what seems to be the simplest and at the same time least indispensable mathematical concept: the concept of unit (*Einheit*), i.e. the concept of definiteness (*Bestimmtheit*). Regarding the question of the origin of this concept, Mannoury remarks that nothing in nature, nothing in the exterior world, presents us with clear-cut units, so that we do not have any other choice but to look for the foundational conditions of the concept in the inner world (*Innenwelt*), i.e. in our own mind. But, however hard we try to isolate one single sensation, which would then seem indivisible to us, the numerous and manifold relations which link up this sensation to other similar or dissimilar ones, always impose themselves upon us. There is no escape from this tight network of connections. When we, for example, try to imagine a single spot of light, we can only do so by mentally situating this spot in our field of vision, i.e. by relating it to other spots, which we either represent or remember. To all this Mannoury proposes the following conclusion. Since it is the case that even within ourselves we are not able to observe anything that would satisfy the notion of a single unit, then the *mathematical* unit, which can only be a product of our mind, must inevitably display this fundamental characteristic of indefiniteness and relativity. It is only insofar as we are able to forget or disregard the imperfection of the concept of unit, that we shall be able to attribute an appearance of exactness to mathematics.

Mannoury's relativistic remarks regarding definiteness and certainty in mathematics reveal the polar nature of the human mind. Mathematics seeks definiteness but cannot free itself from the indefiniteness that it is trying to repel. Mysticism, which, according to Mannoury's use of the word, includes any strife for the all-encompassing or ultimate reality, turns away from definiteness, but still uses definite landmarks as points of departure in order to perform this expanding movement of the mind.

The notion of polarity, which is crucial in Mannoury's thinking, involves two aspects: contrariety and connectedness. Both of these features determine the efforts as well as the limitations of the human mind. Contrariety is that which urges one to ask the most radical questions, connectedness is that which makes these questions impossible to answer and even to ask. This is the conclusion which follows from Mannoury's train of thought in *Mathesis en Mystiek*:

> Is er een God [...] Is er iets, dat waarlijk *is*? Onzeggelik, en toch beleefbaar, onomvattelijk en toch vermoedbaar, ontelbaar-en-onmeetbaar-en-onweetbaar en toch werkelik, levend, *zijnd*? [...]
> De vraag, die ik heb gesteld, of liever: die ik vergeefs getracht heb te stellen, is niet voor beantwoording vatbaar. Niet voor beantwoording door ja of neen, maar ook niet voor beantwoording door "misschien" of "ik weet het niet". Want de vraag naar het algemene is een, menselikerwijs gesproken, natuurlike vraag, waar ons rekenen en meten, ons weten en willen, redelikerwijs toe leidt, maar toch is het geen redelike vraag, omdat, wie haar uiteindelijk stelt, daarmede het biezondere stelt.
> En evenzeer natuurlik en evenmin redelik is die àndere vraag, waar de tegengestelde gedachtengang onverbiddelijk op uitloopt: *is* er 'n waarheid buiten het leven, *is* tweemaal-twee vier? [14, pp. 92, 93, 99].
> [Is there a God? [...] Is there something which really *is*? inexpressible, but still liable to experience, undefinable, but still presumable, uncountable-and-unmeasurable-and-unknowable but still real, living and existent? [...]
> The question, which I asked, or rather: which I vainly tried to ask, is not liable to response; it cannot be answered by yes or no, neither can it be by "maybe" or by "I do not know". Because the question regarding the general is—speaking from a human point of view—a natural question, to which our counting and measuring, our knowing and our striving naturally lead, but yet it is not by itself a reasonable question, because, whoever ultimately asks it, thereby posits the particular.
> And equally natural and equally unreasonable is that other question, to which the opposite train of thought inescapably leads: *is* there truth outside life, *does* two times two equal four?]

4. Frederik van Eeden on the foundational status of uncertainty

As we observed before, Gerrit Mannoury admired Frederik van Eeden (1860–1932) for his ability to bring alive in *Redekunstige grondslag van verstandhouding* the ultimate connection of all concepts. The entanglement of the divergent tendencies of the human mind had been sharply observed by Frederik van Eeden. The same idea became a major theme of Mannoury's writings. His conclusion, "voert de mathesis tot mystiek en de mystiek tot mathesis" [14, p. 100] [Mathematics leads to mysticism and mysticism to mathematics], formulated in the last pages of *Mathesis en Mystiek*, expresses the very core of his thinking.

Besides the establishment of a connection between opposite tendencies, there is another feature of Van Eeden's signific views which is equally important to the understanding Mannoury's trains of thought regarding the foundations of mathematics. This is the idea that the opposite tendencies do not have equal status. Van Eeden links the definite propositions of mathematics and formalised science with the fluctuating evocations of poetry, but there is no doubt in his mind as to which of the two types of expression more directly, more eloquently and more deeply render the truth.

As a member of the Dutch literary movement of the 1880s, the so-called "*tachtigers*", Van Eeden had assimilated Percy Bysshe Shelley's romantic view on the status of poetry. In this view, it is the privilege of the real poet to have at the same time the best insight into truth and the best means of describing this insight. Only music, which is unhindered by the misleading symbolic character of words, can surpass poetry in nearing truth. When poetry is at its best, it can hardly be distinguished from the emotion from which it springs and, as we have seen, from a signific point of view, emotion is truth. Formulae of mathematical and

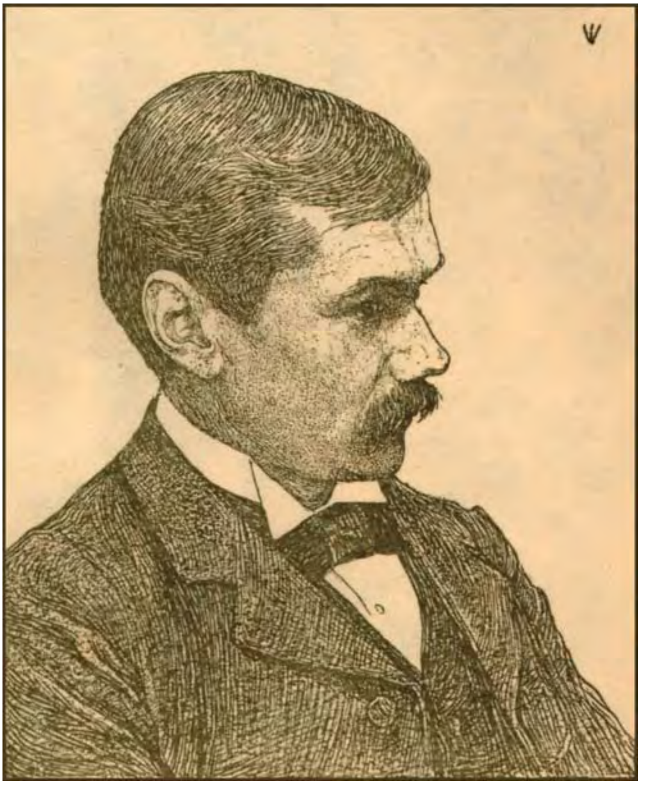


Fig. 3. Frederik van Eeden by Jan Veth.

scientific language do not have this close connection with truth. The rules which govern formal language are unable to contain that which essentially resists fixation and clear-cut distinction. Mathematical and scientific language are removed from truth and are derived, whereas poetical language is fundamental.

It is easy to see then that the title of Van Eeden's treatise, *Redekunstige Grondslag van Verstandhouding* (Logical foundation of understanding), is misleading, for ultimately no foundation can be expected from logic. As we learn from the preface, it is the author's aim to make concessions to the reader who expects rigorous reasoning and believes "in de vlottende aandoeningen van poëtische beelding en fantasie [. . .] geen genoegzame bevrediging te kunnen vinden" [29, p. 7] [not to be able to find sufficient satisfaction in the fluctuating sensations of poetical imagery and phantasy]. Van Eeden, therefore, strictly applies a logical method. *Redekunstige Grondslag van Verstandhouding* presents itself as a well-ordered series of theorems, not unlike the ones that can be found in Spinoza's *Ethica* or Wittgenstein's *Tractatus*. The ideas conveyed through this clear and rigorous reasoning, however, are largely meant to relativise the importance of logic and systematic philosophising as means of expressing the deepest truth. It seems that Van Eeden temporarily gives in to the reader's need for fixedness in order to subsequently deliver him of the idea that fixed concepts, expressions and notations could ever be reliable in the pursuit of truth. It is the language of the poet which is capable of the most intimate contact with truth and which, therefore, deserves the deepest trust. In other words, Van Eeden *himself* finds "satisfaction in the fluctuating sensations of poetical imagery and phantasy" and he hopes to bring his reader closer to finding the same satisfaction.

Van Eeden's philosophy of language is in line with his more general views on the status of certainty and uncertainty. When in his later years he developed a project for an ideal city in co-operation with the architect Jaap London [30], the maps that accompanied the description of this *Lichtstad* (City of Light) showed a concentric pattern, the inner circles of which were entirely designed for religious activity. In Van Eeden's project every world religion was indeed to be represented by a temple situated in a *heilige ring* (holy circle). However, in the middle of this holy ring, i.e. at the very centre of the City of Light, was to be constructed *het Godshuis* (the House of God), which, lacking any reference to particular religions, was to become the temple of the searching human soul. Hence, as was reflected by the very structure of the City of Light, any guidance which this holy place could offer the world would be coming, not from certainty, but from search.

Similarly, Frederik van Eeden describes the structure of a person's inner life in a way that associates uncertainty with depth. People, so Van Eeden says, are like kingdoms that border each other. Successful communication between the kingdoms is possible as long as matters regarding the peripheries are the subject of conversation. In these outward zones the well-established language of daily social intercourse guarantees the certainty of superficial understanding. As soon as more central needs and strivings are concerned, however, this guarantee disappears. At the very heart of a person's inner kingdom dwells "een Farao, die van de goden afstamt en sterrenwichelt" [28, p. 74] [a Pharaoh, who descends from the gods and divines]. In the most secret, but also most fundamental, part of his inner self, a human being is at the same time the closest to the divine and the furthest removed from a certainty expressible through words.

Van Eeden voiced his idea on certainty in the most pregnant and dramatic way in the play *De Heks van Haarlem. Treurspel der Onzeekerheid* (The witch of Haarlem. Tragedy of the uncertainty). The title character comments on the events of the play in a way that is not unlike Macbeth's witches and sings the following cryptical song:

> Wat eindt dat schijnt.
> Wat schijnt dat eindt.
> Dood is zeker.
> Zeker is dood.
> Maar wie weet leeft nog [31, pp. 33–34].
>
> [What ends [only] appears to be.
> What [only] appears to be [necessarily] ends.
> Death is certain.
> Certain[ty] is death.
> But Who knows [?] is still alive.]

It becomes clear from the context of the scene that the certainty referred to is the certainty of the dead letter of the Scripture and of the unalterable moral rule or law of the land. The tragedy of the play is directly related to one of the main characters' unconditional clinging to such exterior standards of conduct, thereby causing the death of his child. The young

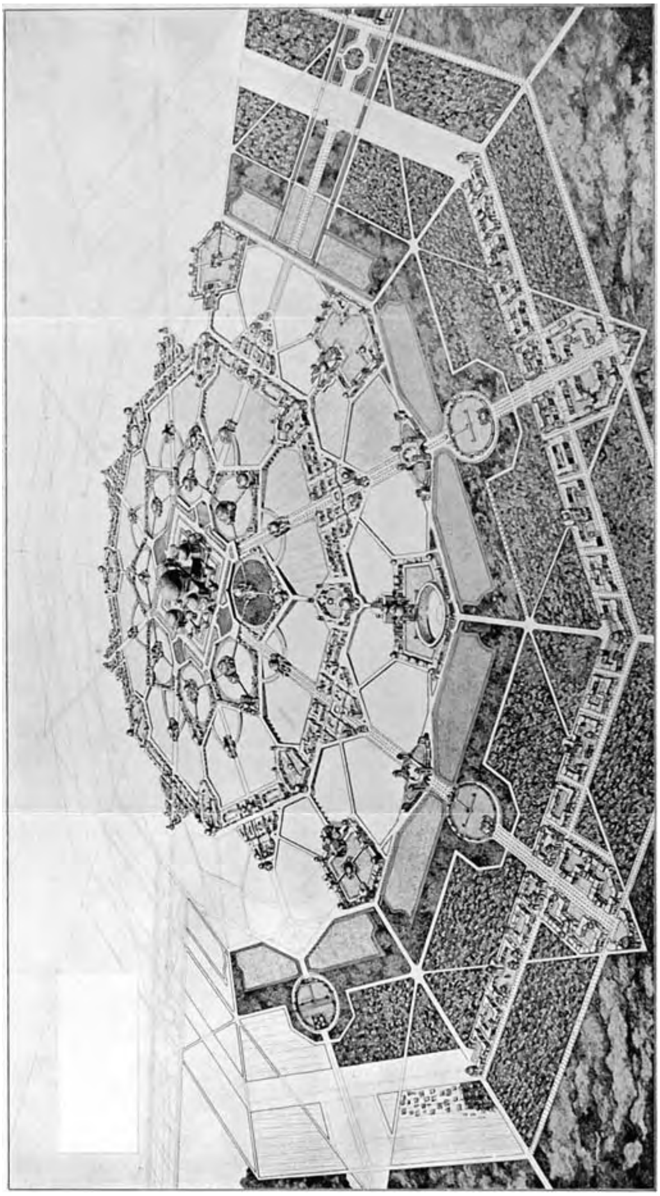


Fig. 4. The City of Light seen from the air.

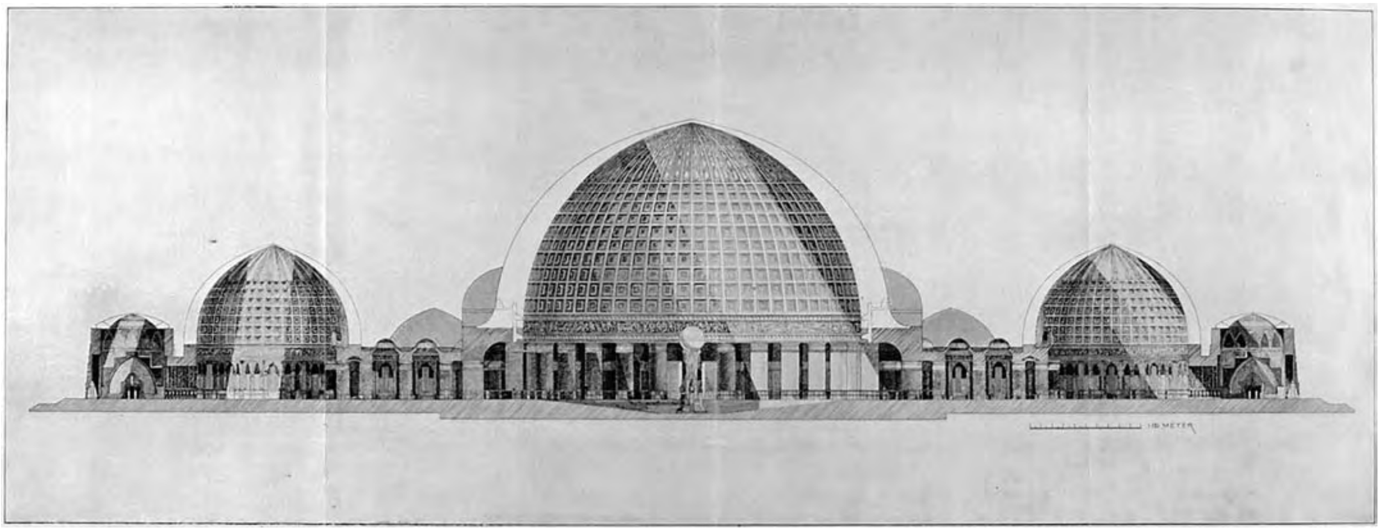


Fig. 5. The House of God (outside).

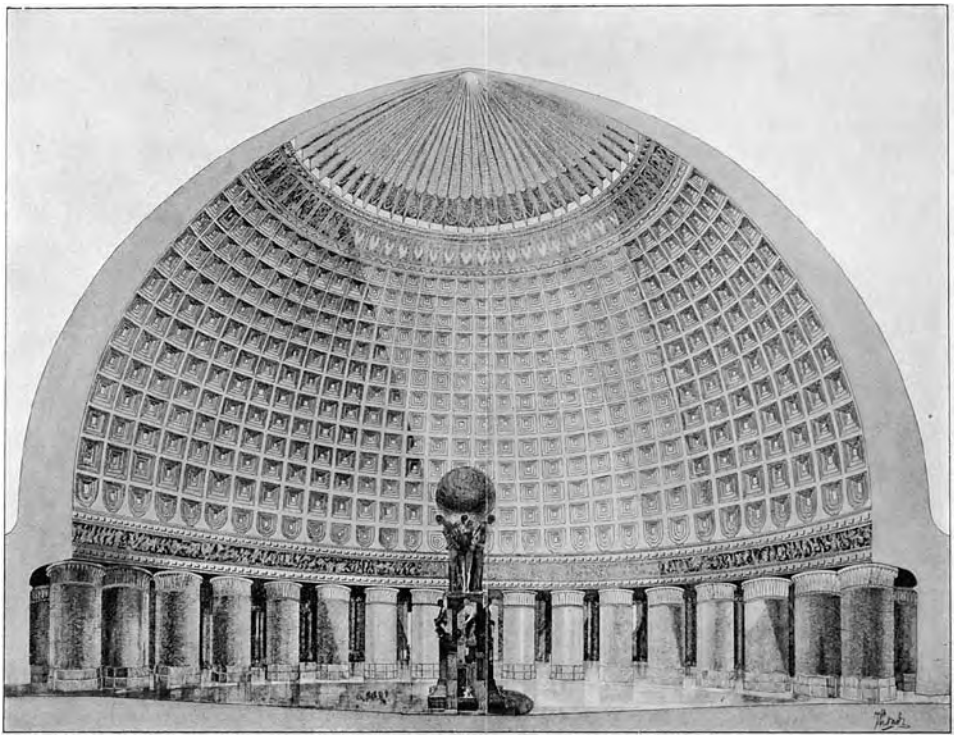


Fig. 6. The House of God (inside).

woman who dies is called Sterre (Star). Her name suggests the eternally unattainable. What provokes Sterre's death is her father's unquestioned obedience to the strict terms of the law. Terms terminate, if people do not question them. What Van Eeden seems to be saying is that, above all, one should be unwavering in asking "Who knows?". It is by rejecting the formalism of words and rules that we respect the unattainable character of truth. It is by allowing for uncertainty that we keep alive the search for truth.

5. Gerrit Mannoury on the foundational status of uncertainty

All of these ideas have to be kept in his mind when trying to understand Gerrit Mannoury, whose manner of thinking in the fields of philosophy and mathematics was so closely akin to that of his signific colleague and friend Frederik Van Eeden. In the article *Breekt* (Break!) published in *De Tribune*, Mannoury stated: "Een weten dat het vragen verleerd heeft is het weten niet waard" [15, p. 10] [A type of knowing which has lost the ability to ask is not worth knowing], a principle which he applied as strictly to mathematics as to any other field of knowledge. Mathematics is a type of learning which no doubt better than others covers up its asking. That is why to the layman, mathematics simply knows and does not ask. In order to bring this idea alive, Mannoury introduces the distinction between *spreekwiskunst* (speaking/spoken or active mathematics) and *hoorwiskunst* (hearing/heard or passive mathematics). The latter has to be interpreted as a catalogue of eternal truths and unalterable formulae. The epitome of *hoorwiskunst* is the logarithmic tables. Calculated and printed a long time ago, they do not seem liable to change. However, as Mannoury continues to show with shrewdness and perseverance, *spreekwiskunst* remains a prerequisite to any accomplishment in the field of mathematics. Certainty is born of the seeking mind, and any future result which can be expected from mathematics will require a new commitment of the seeking mind. The history of this discipline changes as a result of continuous tension between active and passive mathematics, the former of which inevitably brings about a turn to uncertainty. To use a distinction made by Henri Poincaré, Mannoury uncovers the often forgotten *science qui se fait* (science in the process of being practiced or produced), which hides under the misleading stability of the *science faite* (science as accomplished results). In doing so, Mannoury points to the subjective nature of all mathematical knowledge. This link between the outer shape of mathematics and the subject of the mathematician may be unconsciously repressed or consciously ignored for reasons of convenience, but there is no way denying its existence, for that would mean denying progress, renewal and creativity in mathematics. In *Mathesis en Mystiek*, Mannoury expresses this as follows:

> ’t Is het oude liedje: de spreek-wiskunst en de hoor-wiskunst liggen overhoop! [...] Spreek-wiskunst zoekt, vermoedt, gist, raadt of raadt mis, geniet en lijdt, duizelt en slaat spijkertjes, maar hoor-wiskunst blijft er kalm bij en verschanst zich in kant-en-klare definities en laat logarithmetafels drukken, met stereotypplaten. En wil zijn moeder niet meer kennen! Ja, als niet komt tot iet ... [14, p. 31].
> [It is the old song. Speaking mathematics and hearing mathematics are at loggerheads! [...] Speaking mathematics searches, supposes, wagers, guesses right or wrong, enjoys or suffers, feels dizziness and hits nails, but hearing mathematics stays calm and entrenches itself in ready-made definitions and has logarithmic tables printed, with stereotypes. And does not want to know its mother any more! Oh well, when one rejects one’s origins ...]

Mannoury insists that however far mathematics develops in the direction of pure formalisation, and however radically it reduces its number of basic distinctions, there is no way such a reductionist program could set aside the subject of the mathematician.

6. Beyond psychological foundations of mathematics: the unobjectifiable subject and the idea of the unreligious separation of subject and object

Gerrit Mannoury and his fellow significians lived at a time when the psychological way of expressing ideas was fashionable. Mannoury, moreover, developed a general psychological framework destined to harmonise subjective and objective types of expression [19,20]. One might, therefore, be easily led to think that he perceived psychology as the discipline that could ultimately provide the foundations of mathematics. The title *Fondements Psycholinguistiques des Mathématiques* (The psycholinguistic foundations of mathematics) is downright misleading in this respect. A thorough reading of Mannoury’s entire work shows that the situation is considerably more complex. What is mathematically true cannot possibly be derived from an objectifying description of what goes on in the mathematician’s mind. This insight, which was sharply formulated by Husserl, became very prominent in Mannoury's address on leaving the University of Amsterdam. This lecture—no doubt one of the most profound ever delivered by Mannoury—bore the title *De schoonheid der wiskunde als signifisch probleem* (The beauty of mathematics as a signific problem) and ended as follows:

> De volmaakte zekerheid der wiskunde mag dan slechts schijn en menselijk bedenksel zijn, het is dan toch in elk geval een stralend schone schijn en een bewonderingwekkend bedenksel. En deze schoonheid en deze bewonderende ontroering zijn *geen* schijn, want zij zijn mijn beleving zelf, zij *zijn* mijzelf, en dus voor nadere ontleding of helderder belichting niet vatbaar. De vlam verlicht zichzelve niet! [17, p. 201].
> [The perfect certainty of mathematics may be no more than illusion and human fabrication, it is at any rate an illusion of radiant beauty and a fabrication that inspires admiration. And this beauty and this emotion of admiration are *no* illusion, because they are what I live through, they *are* myself, and hence not liable to any further analysis or further illumination. The flame does not shed light upon itself.]

What these words mean is that there is an aspect to mathematics which is essential to its meaning but which cannot be defined by the mind or captured by any formula. The subject of the mathematician in its active pursuit of truth does not allow for objectification. It does not receive the light it sheds.

It is important for the history of the Dutch philosophy of mathematics to take into account the close relationship between Mannoury's thinking and intuitionism. L.E.J. Brouwer, the father of Dutch intuitionism and an active member of the signific movement, was a pupil and close friend of Mannoury. As Brouwer made clear in an address to Mannoury, it was the latter who made his student Brouwer aware of the fact that mathematics was something more than a sterile catalogue of unalterable true statements. The young Brouwer learned from his master that there was room for enthusiasm and for the experience of beauty in mathematics. In other words, he was made to see the importance of *spreekwiskunst*, of active and lively mathematics.

Just as in the case of Mannoury's signific views on mathematics, there has been a tendency to interpret intuitionism as an attempt to find psychological foundations for mathematics. Brouwer, however, clearly rejects the idea of objectified mathematical truth. The subject of the mathematician is the sovereign guard of truth. This self freely creates the mathematical objects and freely contemplates its creations. Inaccessible to psychological reduction, it moreover keeps intact the link with its creations. Any objectifying would imply a limitation of the subject's freedom, and Brouwer insists in his doctoral dissertation [5] that mathematics is a *free* creation of the mind. There is no other foundation of mathematical truth than the unhindered workings of the self.

Brouwer considers any attempt at separating the subject from something which is not this subject as a fatal and unreligious act. An essential feature of his intuitionistic mathematics is, therefore, its explicit safeguarding of an intimate link with the subject.[2] This can be illustrated by means of a passage from the notes of Brouwer's Cambridge lectures on intuitionism:

[2] Reflections regarding truth in its relationship with the subject in mathematics, mysticism and philosophy were also at the heart of G.F.C. Griss' realm of ideas (see [11, p. 30]). Griss developed an idealistic world-view which he combined with a radical type of intuitionism. In recent years, Jan Bauwens has taken up the idea of the necessarily subject-related nature of mathematical truth in his *Mathematica Christiana* [1].

> Only after mathematics had been recognized as an autonomous interior constructional activity [...] the criterion of truth and falsehood of a mathematical assertion was confined to mathematical activity itself, without appeal to logic or to hypothetical omniscient beings. An immediate consequence was that for a mathematical assertion a the two cases of truth and falsehood, formerly exclusively admitted, were replaced by the following three:
> (1) a has been proved to be true;
> (2) a has been proved to be absurd;
> (3) a has neither been proved to be true nor to be absurd, nor do we know a finite algorithm leading to the statement either that a is true or that a is absurd [27, p. 92].

Brouwer opens up the binary system of classical mathematics in a typically signific attempt at rendering the language of the field under consideration, i.e. the language of mathematics, more expressive and more precise. The accuracy of the intuitionistic refinements of mathematical language lies above all in the preservation of the indefinite character of the infinite as well as in the rejection of bold statements about the infinite. Brouwer achieves this refinement and precision by explicitly recognising the active role of the subject of the mathematician in the assignment of truth. In cases (1) and (2) above, the subject is capable of constructing a proof for a or for the absurdity of a, respectively. In (3) the subject is not capable of such construction. It is especially through the introduction of this third option that Brouwer manages to overstep the limitations of classical mathematics. Cases (1) and (2) roughly correspond to the traditional binary system, which strictly respects the principle of the excluded third (a is either true or false/absurd). By complementing (1) and (2) with (3), the intuitionist relativises this logical principle and expresses himself in a more careful way on the infinite, to which the subject of the mathematician does not have direct access.

7. Uncertainty and *docta ignorantia*. The role of negation in significs, intuitionism and theology

It is not surprising that Brouwer, the significian and admirer of Van Eeden, makes reference to a type of uncertainty in his tripartite truth-assignment. Option (3) in the passage from the Cambridge lectures quoted above allows for openness, and it is this openness which makes the intuitionist language more precise. It is no more surprising that Brouwer breaks the rigidity of a well-established logical principle by using new distinctions which bring to the fore the workings of the subject. Frederik van Eeden had already shown in *Logical Foundations of Understanding* that ... no foundations can ever be expected from logic! Logic is no more than an outward scheme. Foundations have to be looked for elsewhere, namely in the more central emotions and sensations, to which the poet, who remains unhindered by logical constraints, has direct access. There can be no doubt then that, according to Brouwer, the author of *De onbetrouwbaarheid der logische principes* (The untrustworthiness of the logical principles) [6], the mathematician and the poet display great affinity in their search for truth.

Referring again to the witch's song in *De Heks van Haarlem*, one might say that the dead formulae of the logical principles, which yield no more than a deceptive certainty, should be broken through in order to free the searching subject, whose uncertainty allows for unlimited openness.

Brouwer does indeed introduce into the language of mathematics (which belongs to the abstract, derived and formalised language-types) particular elements that allow him to pierce right through to the other pole of the mind, thereby re-establishing the sacred link with the subject and its emotions and volitions. Among these elements, *negation* is no doubt the most important.

Mannoury, too, attributes special status to negation in significs and draws the attention of his reader to the existence of a type of negation which is purely emotional or volitional. Such radical negation repels vigorously without bringing to the mind a well-defined alternative to what is being repelled. This is typical of expressions of conviction (religious, political or other) and underlies words such as *infinity*, which belong to what Mannoury terms the language of the general.

Although Mannoury's and Brouwer's views on negation are different in a number of respects, one can safely say that a radical and volitional negation is at work in Brouwer's option (3): a has NEITHER been proved to be true, NOR to be absurd, NOR do we know a finite algorithm leading to the statement either that a is true or that a is absurd.

As an expression of the will this negation is naturally associated with the subject of the mathematician. In addition, it significantly expresses the idea that the searching subject is in a state of relative ignorance (*nor do we know . . .*). To Brouwer, the explicit admission of ignorance regarding the (as yet) unconstructed entities is to be considered an advance on classical mathematics, which expresses itself too boldly in this respect, namely taking for available that which can only be suspected to be.

Regarding the infinite, Brouwer's thinking may be said to display the wisdom of the well-known unknowing. There are, therefore, very serious reasons for establishing the analogies between intuitionism and the views on mathematics expressed in the works of Nicholas of Cusa, the father of the *docta ignorantia*. This was attempted by A.F. Heijerman [12].

The role of negation in expressions such as option (3) in Brouwer's assignment of truth-values may be compared to the role which negation plays in negative or apophatic theology. Just like intuitionistic negation, which expresses relative ignorance and allows one to hint at mathematical entities that are beyond the reach of the constructing subject, negation in negative theology allows one to allude to that which is inaccessible to any attempt at pinpointing it through names, predicates or concepts.

8. Johan J. de Iongh. Shared longing as the religious meaning of mathematical dialogue

Johan J. de Iongh (1915–1999), professor of mathematics at the University of Nijmegen, belonged to the second generation of significians. As a pupil of Mannoury, an associate of Brouwer and a good friend of both, he developed a view on the philosophy of mathematics which was termed by his own pupil, Wim Veldman, "a platonically tinged intuitionistic one" [34, p. 17].

Very much aware of the need for precise formulation as well as for restraint in the discussion of matters that verge on human incomprehensibility, he advocated the refinements of intuitionistic language in his courses on the foundations of mathematics.

Fig. 7. Johan Jilles de Iongh.

De Iongh's view on human limitations, which, when recognised, allow for openness and advance, was not confined to these refinements but concerned mathematics as a whole. In one of his very few publications—De Iongh took Plato's critique of the written word very seriously—he stated the following:

> Deze worsteling van de mensheid om op dit beperkte gebied grotere duidelijkheid, grotere zekerheid, omvattende waarheid te bereiken [is] voor mij een van de diepste ervaringen, zowel door de grootsheid van de bereikte partiële resultaten als door de principiële noodzaak van een blijvend falen, een blijvende spanning tussen het bereikte en de niet positief vast te leggen, maar wel analoog en in dit falen ook negatief ervaren, dit bereikte transcenderende, waarheidsidee [8, p. 256].
>
> [This struggle, which humanity has to take up in order to achieve greater clarity, greater certainty and all-encompassing truth in this limited field [viz. that of mathematics], is to me one of the deepest experiences, and this is so as much because of the impressiveness of the achieved partial results, as because of the fundamental necessity of continuous failure, of a continuous tension between that which has been achieved, and the idea of truth, which transcends the achieved, and which cannot be pinned down positively, but which can be experienced in an analogical and, through failure, also in a negative, way.]

The awareness of imperfection and non-achievement intensifies the search of the mathematician, and it is this enthusiasm, this longing for truth, which two equally gifted friends feel, when they set their minds on the solution of a problem, that, in De Iongh's eyes, becomes fundamental for mathematics. Veldman remarks in this respect:

> [A]s Plato explains in his Seventh Letter, truth may show itself, unexpectedly and suddenly, as a spark of fire, by something like divine grace, where friends have been patiently seeking after it together, possibly for a very long time. Johan shared Plato's hankering after such events [34, p. 17].

Just like Brouwer, De Iongh feels that the truth of mathematics is ultimately founded in the subject of the mathematician, but more than Brouwer, De Iongh insists that the fundamental intention of the subject is a *shared* longing.

In a handout for a lecture "Signifiese beschouwingen over de grondslagen van de wiskunde" (Signific reflections on the foundations of mathematics) held in 1948, De Iongh concluded:

> Wordt deze scheiding tussen wiskunde en wiskundige opgeheven, dan krijgt de vraag naar de waarheid en de dan daarmee vrijwel samenvallende schoonheid van de wiskunde een nieuwe, niet meer matematiese, maar nog wel redelijke, intersubjectieve, nu wijsgerig-religieuze zin [7, p. 2].
> [If this separation between mathematics and the mathematician is removed, the question of the truth of mathematics as well as the question of its beauty, which then almost coincides with its truth, receives a new meaning, which is no longer mathematical, but still reasonable and intersubjective and which reveals itself as philosophical and religious.]

In particular, the mathematical dialogue between friends and the growing enthusiasm that accompanies their limited search for truth call up something much deeper, namely the question of the unlimited truth and the hoped for unlimited dialogue with the Divine Subject.

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CHAPTER 30

Arthur Schopenhauer and L.E.J. Brouwer: A Comparison

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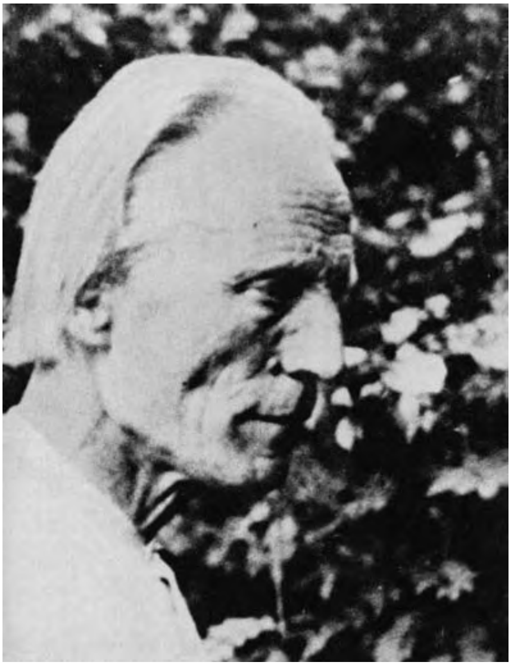

Fig. 1. L.E.J. Brouwer. Courtesy of the Brouwer Archive.

1. Introduction

Luitzen Egbertus Jan Brouwer[1] (1881–1966) is one of the few truly great Dutch mathematicians.[2] Brouwer's intellectual development is full of surprises. In 1905 he published a controversial booklet, *Leven, Kunst en Mystiek* (*Life, Art and Mysticism,* [2,26]), in which science and technology are attacked; the human intellect and everything it brought about are depicted as evil. Yet this did not stop Brouwer from exploiting his own intellectual capacities and pursuing an academic career. In 1907 he defended his doctoral dissertation, *Over de grondslagen van de wiskunde* (*On the Foundation of Mathematics,* [3]). Moreover, after proving in the period 1908–1912 a number of topological theorems that made him famous, Brouwer became in 1912 professor of mathematics at the University of Amsterdam. At the end of World War I, Brouwer's work once more radically changed course. In 1917 he started developing mathematics without use of the principle of the excluded third, the unreliability of which he had already shown in 1908. The consequences were dramatic: in so-called intuitionistic mathematics considerable parts of classical mathematics—including Brouwer's topological results—do not in fact occur.

[1] His friends called Brouwer Bertus, but in the literature he is almost exclusively called L.E.J. Brouwer or Brouwer.

[2] Next to Christiaan Huygens, Simon Stevin and Thomas Jan Stieltjes. I add Stevin, whom the Dutch share with the Belgians because he was born in Flanders, but spent more than half his life in the Netherlands. Stieltjes was born in Holland, but moved to France, so I guess we share him with the French.

Elsewhere [13] it is shown that there is no real opposition between Brouwer's topological work and his intuitionistic work. If one considers Brouwer's intellectual development, it is clear that his pre-1917 intuitionism was immature in the sense that he believed that most of the hard core of classical mathematics was sound and could be saved intuitionistically. Brouwer's topological work was closely related to his intuitionistic views at the time. In the foundation of mathematics that he presented in 1907 topological notions played a central role and his later work in topology was a very natural continuation of parts of the dissertation. Moreover, although Brouwer rejected the use of the principle of the excluded third from 1908 on, before 1917 in his view the principle continued to possess important heuristic value. When he used the principle while proving his spectacular topological results in the period 1908–1912, he assumed that in the course of time the classical proofs would be eventually replaced by intuitionistic ones. Only after 1917 did Brouwer realise that this would be very difficult.

In this paper I wish to concentrate on the other seeming opposition in Brouwer's intellectual development: the opposition that appears to exist between *Life, Art and Mysticism*, on the one hand, and Brouwer's career as a mathematician, on the other hand. Van Stigt has discussed this matter [25] and he has ended the intolerable inaccessibility of the text of *Life, Art and Mysticism* (*LAM*) by publishing an English translation [26].[3] Yet *LAM* remains controversial. In the present paper I wish to emphasise that it is wrong to consider it as a weird mix of ideas from different sources, written down by a frustrated young man. In the book the young Brouwer philosophically chooses a position and he does it very seriously. I will show that his position has much in common with the views of the German idealist Schopenhauer. I will also show that there is no real opposition between Brouwer's criticism of the role of the intellect, on the one hand, and his academic career, on the other hand.

Inspired by a paper by Van Atten and Tragesser, I will at the end of the paper put Brouwer's views on mathematics and the divine into a different perspective by comparing them with the views of the great twentieth century logician and mathematician Kurt Gödel.

2. From Kant to Schopenhauer

The German philosopher Immanuel Kant (1724–1804) brought about what he himself called a "Copernican revolution" in metaphysics. Kant argued that it had always been assumed that knowledge must conform to objects outside of our mind and he compared this view to the geocentric hypothesis. The revolution that he brought about consisted in the introduction of the view that the objects of our knowledge are to a large extent the product of certain a priori conditions on the part of the knowing subject; he compared this view to the heliocentric hypothesis. This new hypothesis enabled Kant to explain the existence of a

[3] Unless stated otherwise the translations into English from Dutch or German are mine. It is not easy to translate a literary text like *LAM*, written in a Dutch that, on the one hand, reflects the way in which Dutch was written a century ago, but on the other hand shows Brouwerian peculiarities. Van Stigt published a complete translation of *LAM* [26]. Yet I decided to use my own translations which are less elegant, but stay somewhat closer to the word order of the original.

Fig. 2. Brouwer talking to F.E. Hacket at the Royal Society in 1960. Courtesy of the Brouwer Archive.

priori knowledge like mathematics and logic. If such knowledge were empirical knowledge of an abstract kind, its necessity would be inexplicable.

Let us take a simple illustration. Take the experience of watching an apple lying on a table. This experience is the result of, on the one hand, some ultimate material—the apple and the table in and of themselves ("die Dinge an sich", the things in themselves)—and, on the other hand, an activity of the mind. This mental activity consists of two elements: *sensibility* (the receptivity of the mind for the impressions generated by the apple and the table) and *understanding* (which deals with concepts and their relations). Sensibility positions the apple and the table in space and time. Space and time are in this view a product of the mind; they are the a priori forms of sensibility. The apple and the table are finally recognised by the observer as an apple and a table only because of understanding: without the concepts of apple and table, the apple and the table would not be recognised as such. In this view mathematics deals with the a priori forms of sense intuition, space and time. Logic essentially differs from mathematics: it deals exclusively with the laws of understanding, i.e. the relations between concepts. The mental activity that leads to things being positioned in space and time is described by Kant as a mental construction. One could probably say that in Kant's view doing mathematics boils down to doing mental constructions without empirical input. The idea of *causality* is for Kant also an a priori concept. If we remove the table, the apple falls. Whatever happens in space and time, it must have a cause; we cannot think reality differently.

Kant was not a romantic philosopher. The emphasis in his philosophy is on the isolation of the a priori elements in our knowledge. The post-Kantian German idealist philosophers were, on the contrary, all influenced by Romanticism, defined here as the cult of subjectivism, of feeling and passion. A beautiful example is Arthur Schopenhauer (1788–1860), the philosopher of pessimism. Schopenhauer's major work, *Die Welt als Wille und Vorstellung* (*The world as will and idea*) dates from 1818. His other work is at heart a further elaboration of themes already present in this book. Schopenhauer follows Kant with respect to the spatio-temporal world, that is subject to the law of causality. However, where the things-in-themselves are for Kant unknowable, Schopenhauer maintained that they actually constitute one unitary principle that explains the whole of nature, including individual human nature. This principle he termed the “Will”. It is an energy that manifests itself everywhere, in physical forces, but also in individual desires and in unconscious instincts. While for Kant the multiplicity of phenomena constitutes reality, Schopenhauer, who was influenced by the Indian Upanishads, views the phenomena negatively; they are merely appearance. For him, mathematics and physics deal with an illusion brought about by our mental structure. The Will, on the other hand, is, in Schopenhauer’s view, the only true reality. However, Schopenhauer’s Will is not the result of a rational weighing of possibilities; it is the basic urge, the basic passion that controls us. The Will that manifests itself in nature is a blind force that is never satisfied and never comes to rest. In human life the Will manifests itself in a perpetual desire for happiness and pleasure that are never reached, or, if reached, quickly vanish. The Will also brings about consciousness and the idea that it is necessary to chase worldly things, like wealth, women, etc. Split up into numerous objectivations in space and time, the Will is hopelessly divided: everybody and everything wills. Everything that becomes an individual in space and time is at war with everything else. In the end, there is always only suffering and pain. From Schopenhauer’s point of view, the superficial optimist goes on living with the illusion that the world can be improved. The philosopher, however, understands the treacherous game of the Will. He should say “no” to life and try to release himself from the spell of the Will. For the metaphysical pessimist Schopenhauer, there are two ways of escaping from the sad world. There is the temporary escape in aesthetic contemplation and there is the lasting escape consisting in the denial of the will to live. Christian, Hindu and Buddhist saints exemplify this lasting solution.

In the following two sections I shall present the way in which the young Brouwer, as a student in Amsterdam, experienced and viewed himself and the world. I shall quote rather generously from his correspondence and from *LAM*. In Section 5, I shall compare the views of Brouwer and Schopenhauer.

3. Brouwer, the prophet

On August 9, 1903, Bertus Brouwer wrote to his friend, Carel Adama van Scheltema, the following:

> “It is unfortunate that a king has a body, like his subordinates, which is only a remnant from an earlier period of development, but which he has nevertheless and must lead properly to the grave. This does not make the task of the king easier; the subordinates are not allowed to know his corporeal troubles.
> And this corporeal trouble is for us much heavier, yes almost impossible. This body with its life of passions in the troubled brain, needs warmth of soul, and human warmth of soul flows downwards; who gives something to the King?” [4]

[4] “ ’t Is jammer dat een koning ook een lichaam heeft, zooals zijn onderdanen, een overblijfsel weliswaar uit een vroegere ontwikkelingsperiode, maar hij heeft het, en moet het behoorlijk naar ’t graf geleiden. Dat maakt de koningstaak niet lichter; de onderdanen mogen zijn lichamelijke zorgen niet weten. En die lichamelijke zorg

On November 15, 1903, Brouwer wrote:

> "The final harmonisation of our life seems to proceed in the heaviest, the slowest, the most laborious way for people of our kind. It seems, that in the succession of the generations of each parental pair the eldest child may not be sent away innocently in the large stream [...] but must be made as opening sacrifice to the Gods of consciousness, the in the earthly flux sterile consciousness—by way of compensation these Gods give up their rights with respect to the other children.
> So let these holy sacrificial animals be conscious of their role and not be jealous of the rough ryebread of the common members of the herd.
> Oh yes, my dear fellow-sufferer and brother-in-arms, the purest of ourselves, the resignation and the dedication to our task lives best in solitude. Association with people is necessary, but so different from our heart's jewels, as ryebread, which is also necessary, from ambrosia".[5]

These quotations express that Brouwer saw himself as a prophet, as a messenger of God, blessed with insight, but at the same time suffering from his lonely position far above the crowd. In the August 9, 1903, letter he put it quite succinctly:

> "chosen ones are we—not for our pleasure in the world—we are the prophets, who, messengers between God and mankind, guide and inspire its development, working, growth and blooming".[6]

On June 16, 1904 Brouwer finished his studies at the University of Amsterdam with honours and became 'doctorandus'. On July 4, 1904 he wrote to his friend Adama van Scheltema:

> "During the coming winter I will be either in Blaricum [...] working on a philosophical profession, that will be the prologue of my work—or in London [...] for my dissertation 'The Value of Mathematics'. If I were searching for a kingship on earth, it would be good maybe, to immure myself in mathematics, and let me crown as a pope in the Vatican, captive on his throne. But I desire a kingship in better regions, whereby not the goal but the driving force of the heart is primary. We are not on earth for our pleasure, but with a mission, for which we have to render account".[7]

The philosophical profession, *Life, Art and Mysticism* [2], and the dissertation, *On the Foundation of Mathematics* [3], were both written and published. The two publications belong together. They are the clue to the understanding of Brouwer's entire work.

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is óns veel zwaarder, ja haast onmogelijk. Dat lichaam met zijn passieleven in 't troebel brein, heeft zielewarmte noodig, en 's menschen zielewarmte vloeit naar beneden; wie geeft den Koning wat?" [17, p. 40].

[5] "Die eindelijke harmonizeering van ons leven schijnt bij mensen van ons slag het zwaarst, het langzaamst, het moeizaamst te gaan. Het schijnt, dat in den voortgang der generaties van ieder ouderpaar het oudste kind niet onnoozel weg mag worden meegestuurd in den grooten stroom [...] maar als openingsoffer [...] moet worden gebracht aan de Goden der bewustheid, der in 't aardsch beweeg onvruchtbare bewustheid—ter compensatie doen die goden dan afstand van hun rechten op de andere kinderen. Laten die gewijde offerdieren zich dus bewust zijn van hun rol, en niet jaloersch worden op 't grove roggebrood der kuddedieren. Ach ja, mijn beste lotgenoot en medestrijder, het zuiverste van onszelf, de resignatie en overgave aan onze taak, dat leeft het best in eenzaamheid. Omgang met mensen is nodig, maar van onze hartskleinodieën zo diep verschillend, als roggebrood, dat ook nodig is, van ambrosia" [17, p. 44].

[6] "[...] uitverkorenen zijn wij—niet voor ons pleizier in de wereld—wij zijn de profeten, die, boden tusschen God en het menschdom, de ontwikkeling, het werken, de groei, de ontbloeiing daarvan leiden en bezielen [...]" [17, p. 40].

[7] "Den komenden winter zal ik zitten òf in Blaricum [...] werkend aan een filosofische belijdenis, die de proloog van mijn werk zal zijn—òf in Londen [...] voor mijn dissertatie 'De waarde der Wiskunde' [...] Zocht ik een koningschap op aarde, dan was het misschien goed, mij in de wiskunde te ommuren, en mij te laten kronen als een paus in 't Vaticaan, gevangen op zijn troon. Maar ik begeer een koningschap in betere gewesten, waarbij niet het doel, maar de drijfveer in het hart primair is. Wij zijn op aarde niet voor ons plezier, maar met een zending, waarvan wij rekenschap hebben af te leggen" [17, p. 52].

4. Life, art and mysticism

At school Brouwer was a brilliant pupil who excelled in all subjects. He registered at the University of Amsterdam at the age of 16, two years earlier than usual, after having passed with full honours the entire scale of secondary school examinations that existed in the Netherlands at that time. Highly remarkable is the profession of faith that he read at the age of 17, in 1898, in the Haarlem Remonstrant Church. With respect to God Brouwer said:

> "That I believe in God originates by no means in an intellectual consideration in the sense that I should conclude from various phenomena that I observe around me the revelation of a higher 'power', but precisely in the utter powerlessness of my intellect. For to me the only truth is my own ego of this moment, surrounded by a wealth of representations in which the ego *believes*, and that makes it *live*. [. . .] My *life* at the moment is my *conviction of my ego*, and *my belief in my representations*, and the belief in That, which is the origin of my ego and which gives me my representations, independent of me, is directly linked to that. Hence something that, like me, lives and that transcends me, and that is my *God*. One should by no means read in the many words that I have used above, an intellectual *deduction* of the existence of God, for this belief in God is the bedrock, from which can be deduced, but that itself is not deduced. The belief in God is a *direct spontaneous emotion* in me". [18, p. 18] (translated by van Dalen).

About other people Brouwer said:

> "[. . .] hardly anywhere do I recognise my own thoughts and spiritual life: the shadows of men around me are the ugliest part of my conceptual world". (Ibidem, p. 20)

Fig. 3. Arthur Schopenhauer in 1815. Courtesy of the Stadt- und Universitätsbibliothek, Frankfurt am Main, Schopenhauer-Archiv.

One notices that the fragments from Brouwer's letters to Adama van Scheltema show that there is great continuity in the way Brouwer experienced life. This would never change. Even when Brouwer was relatively happy, he would write (to Carel Adama van Scheltema, August 7, 1906) something like:

> "Life is a magic garden. With miraculously soft shining flowers, but amidst the flowers the little people walk, that I am so afraid of, they stand on their heads, and the worst is, that they cry out to me that I must also stand on my head, now and then I try it, and I burn with shame; but sometimes the little people then shout that I do it very well, and that I am after all a real gnome too. But under no circumstances are they going to make me believe that".[8]

In *Life, Art and Mysticism* of 1905, which is the extended text of a series of lectures that he gave at Delft University, Brouwer gave the "philosophical profession, the prologue of his work". It is also a controversial text, difficult to discuss.[9] In 1916, in the middle of World War I, the Dutch author Frederik van Eeden wrote a review under the title "Een krachtig brouwsel" ("A Potent Brew"—'brouwer' is Dutch for 'brewer') in which he said, ten years after the publication of the book:

> "It is unbelievable! for more than ten years this piece of prose can be read by the public. [...] And nobody knows it, nobody talks about it, it lies there silently in its sweet, white cover, with its threatening, terribly black words—and round about the gang laughs and blethers and swindles, and philosophises and dances and shoots each other to death—while the gloomy Brouwer of that infernal-heavenly brew walks around freely, now in a neat jacket, now in a stately robe, and is considered by everybody as a more or less interesting and not only harmless, but even very respectable and useful individual that fits in our society" [21, p. 75].[10]

Van Eeden wrote the review after Brouwer had become professor and he expressed his astonishment. He assumed that the excellencies and highly learned who appointed Brouwer, who even made him a member of the Royal Academy, had read it and that they must have thought that it needed not be taken seriously.[11] They must have thought, in Van Eeden's opinion, that the idea that logic is a false doctrine and the intellect a gift of the devil that we must get rid of, were merely a juvenile sin of a 23 year old student. However, this is, according to Van Eeden, a serious mistake. And, indeed, it was not a juvenile sin. Brouwer always remained proud of *LAM*.

[8]"Het leven is een tovertuin. Met wonderzacht blinkende bloemen, maar tusschen de bloemen loopen de kaboutermannetjes, daar ben ik zo bang voor, die staan op hun kop, en het ergste is, dat ze mij toeroepen, dat ik ook op mijn kop moet gaan staan, een enkele maal probeer ik het, en schaam me dood; maar soms roepen dan de kabouters dat ik het erg goed doe, en toch ook een echte kabouter ben. Maar dat laat ik me in geen geval wijsmaken" [17, p. 68].

[9]Fearing misunderstanding or worse, only a small part of it was included by Heyting in Volume 1 of Brouwer's collected works. Van Stigt wrote to me that he originally intended to publish his translation as a book, but could not find a serious publisher. People feel uncomfortable with the text.

[10]"Het is ongeloofelijk! meer dan tien jaren ligt dat stuk proza oopenlijk ter leezing. Eenige honderden hebben het den schrijver hooren voordragen, en vermoedelijk hebben nog eenige honderden het geleezen. En niemand kent het, niemand praat er oover, het ligt daar maar stil, in zijn lieven, witten omslag, met zijn dreigende, verschrikkelijke zwarte woorden—en rondom lacht en zwatelt en zwendelt de bende, en filosofeert en danst en schiet elkander dood—terwijl de sombere Brouwer van dat helsch-heemelsche brouwsel vrij rondloopt, nu eens in een net colbertje, dan weer in een deftige toga, en door iedereen wordt beschouwd als een in onze samenleving passend, min of meer interessant en niet alleen onschadelijk, maar zelfs zeer achtenswaardig en nuttig individu" [21, p. 75].

[11]Van Dalen, Brouwer's biographer, believes that probably no one had bothered to have a look at the booklet.

The book consists of 9 chapters, 98 pages altogether. The structure of the treatise is rather simple. The first four chapters (I. The Sad World, II. Introspection, III. The Fall through the Intellect, and IV. The Reconciliation) seem to contain the message that Brouwer wants to convey. In the remaining five chapters (V. Language, VI. Immanent Truth, VII. Transcendent Truth, VIII. The Liberated Life, and IX. Economics) Brouwer elaborates on several aspects of the situation in which man finds himself. I will concentrate on the first four chapters and afterwards only briefly discuss the rest of the treatise.

Brouwer's message is indeed a powerful brew, a mixture of philosophical considerations and the strong feelings with respect to existence that we met earlier.

In the first chapter Brouwer writes down what is, from his point of view, wrong with the world. Man does not live in harmony with nature. On the contrary, the world is sad and characterised by heavy battles against nature and between men:

> "The life of mankind as a whole is an arrogant eating away at its nests over the sound earth, a messing about with her mothering crops, gnawing, violating, making sterile its rich creative power, until it has devoured all life, and around the barren earth the human cancer withers away".[12]

Man believes that he can change the world for the best. However, his theories are merely castles in the air that are continuously replaced by others.

> "The foolishness in their head that accompanies this, and makes them crazy themselves, they call: 'Understanding the world' ".[13]

In this first chapter Brouwer also indicates an explanation of this situation. In chapter III on the fall through the intellect he would return to this problem. The cause of the sad world is dissatisfaction; man wants always more. By means of the intellect man attempts to realise his wishes. The intellect, however, is essentially limited in its possibilities:

> "the mind never sees the world as a whole, and the means that it dictates in the direction of the limited goal one has one's eye on, will only do damage to the whole in unfathomable ways".[14]

In this context Brouwer quotes the German mediaeval mystic, Meister Eckhart, with approval:

> "If it were the case in this life that we would have all the time a mirror in front of us; in which we would see and understand in one moment all things in one picture, then our actions and our knowledge would not be a hindrance. However, because we must turn from the one thing to the other we cannot stay with one without hindering the other".[15]

[12] "Het leven van de mensheid als geheel is een arrogant uitvreten van haar nesten over de gave aarde, een knoeien aan haar moederend gewas, knagend, schendend, een steriel maken van haar rijke scheppingskracht, totdat ze alle leven heeft vervreten, en om de dorre aarde dort de mensenkanker weg" [2, p. 10]. At the end of his career Brouwer repeated these views in a slightly altered formulation [5, p. 1242].

[13] "De dwaasheid in hun hoofd, die dát begeleidt, en hen zelf gek maakt, noemen ze: 'De wereld begrijpen' " [2, p. 10].

[14] "het verstand ziet nooit de wereld als geheel, en de middelen die het dicteert in de richting van het begrensde in 't oog gevatte doel, zullen langs ondoorgrondelijke wegen aan het geheel slechts schade doen" [2, p. 9]. Van Stigt treats this text, which is given by Brouwer with quotation marks, as a quotation from Eckhart's work. If this is correct, it is remarkable that the original text is in Dutch, while in the rest of *LAM* all quotations from Eckhart's work are in German. The context, moreover, is such that the quotation marks could be meant rhetorically. That is my assumption.

[15] "Wäre es so bestellt in diesem Leben, dass wir allezeit einen Spiegel vor uns hätten, in dem wir in einem Augenblicke alle Dinge in einem Bilde sähen und erkennten, so wäre uns wirken und Wissen kein Hindernis. Da

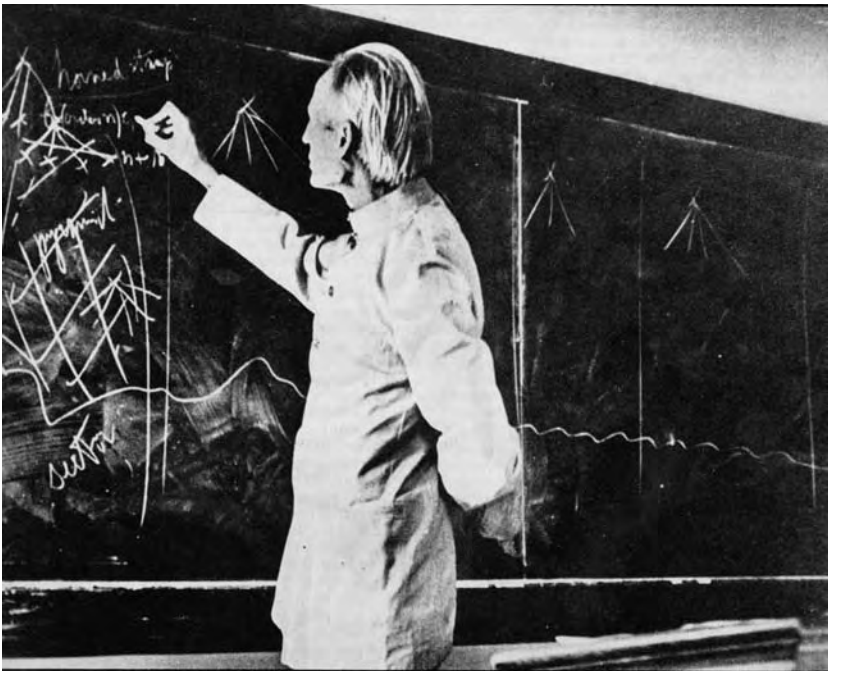

Fig. 4. Brouwer lecturing in Cambridge, about 1950. Courtesy of the Brouwer Archive.

In the second chapter, which is about God, Brouwer points to a solution and he urges the reader to engage in introspection (Brouwer uses here the Dutch word 'inkeer', which means introspection, but at the same time 'repentance'; Brouwer means introspection but there is a moral undertone). To a certain extent man controls the contents of his consciousness. Moreover,

> "you know within yourself, now and then, the religious feeling, which is to you, as if you are withdrawing from your passions, your fear and desire, from time and space, from your entire world of observation".[16]

This introspection is not easy.

> "However, if it is given to you, to conquer all inertia and go further, then passions will keep silent, you will feel the dying off of the old world of observation, of time and space and all other plurality, and the no longer bandaged eyes of a glad silence will open".[17]

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wir uns aber von einem zum anderen wenden müssen, darum können wir uns nicht bei dem einen aufhalten ohne Hinderung des andern" [2, p. 9].

[16]"kent ge bij uzelf, nu en dan, het religieuze gevoel, waarbij het u is, alsof ge uit uw hartstochten, uit vrees en begeerte, uit tijd en ruimte, uit de gehele aanschouwingswereld retireert" [2, p. 13].

[17]"Wordt het u niettemin gegeven, alle traagheid te overwinnen, en voort te gaan, dan gaan de hartstochten zwijgen, ge voelt u afsterven van de oude aanschouwingswereld, van tijd en ruimte en alle andere veelheid, en de niet langer gebonden oogen eener blijde stilte gaan open" [2, pp. 13–14].

Here Brouwer quotes the German mystics Eckhart and Jakob Böhme, who expressed the same thought in a similar way. Böhme wrote:

> "It is inside of you; and if all your willing and sensing allows you to be silent for an hour, then you will hear unspeakable words of God".[18]

Then Brouwer devotes two pages to the situation that results if the introspection is successful:

> "You feel free, to return at your own discretion into the shackles of plurality, separation, time, space and bodily feeling, but you do not do it; or rather do it and do it not at the same time; while you remain free outside, you live your tied bodily life in the world of men in clarity, you live in your bonds, seeing, how you accept them yourselves in freedom, and how they only exist, as long as you shroud yourself freely in mist".[19]

This quotation points to the reconciliation, which is the subject of chapter IV. However, Brouwer first studies in chapter III the mechanism of the intellect:

> "This Intellect does in the Life of Desire men the diabolic service of the goal-means bond between two fantasies. The intellect points out to them, caught in the desire for the one thing, the pursuit of the other thing as a means for that; for example, in order to move the bed of a river: the making of a dam; in order to allow one's jealousy of someone else to run its course: setting fire to his house; in order to be safe from beasts of prey: building one's house on piles; in order to let the sun shine on one's house: the cutting of trees".[20]

In fact the act always somewhat misses the goal. If this repeats itself several times, if the jump from goal to means is repeated several times, it can happen that in the end one counteracts the direction of the original goal. In this context Brouwer describes the coming into existence of industry, a means to produce products in order to simplify the living conditions, followed by the building of instruments to facilitate production, followed by the development of science in the service of industry, etc.

Sometimes man regrets what he does. There is

> "the melancholy about the lost Paradise, the vague conviction of the quiet happiness with life that was originally marked out for man, the desire for bliss, for religious certainty, for the free, surrendered life, this desire, stepping in the forms of the sad world, becomes hunger for higher, elevating, for transcendent things".[21]

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[18] "Es ist in dir; und so du magst eine Stunde schweigen von allem deinen Wollen und Sinnen, so wirst du unaussprechliche Worte Gottes hören" [2, p. 14].

[19] "Ge voelt u vrij, om naar willekeur terug te keren in de boeien van veelheid, scheiding, tijd, ruimte en lichaamsgevoel, doch doet het niet; of liever doet het tegelijk wel en niet; vrij buiten blijvend, leeft ge uw gebonden lichamelijke leven mee in helderheid in de menschenwereld, leeft in uw banden, ziende, hoe ge ze zelf in vrijheid aanvaardt, en hoe de er slechts zijn, zoolang ge u in vrijheid omnevelt" [2, p. 15].

[20] "Dat Intellect doet in het Leven van Begeerte den menschen den duivelsdienst van den band doel-middel tusschen twee fantazieën. Hun geeft, vastgeslingerd in 't begeeren van het eene ding, het intellect het streven naar een ander ding als middel daartoe aan; zoo om de bedding eener rivier te verleggen: het maken van een dam; om op een ander zijn jaloezie uit te vieren: diens huis in brand te steken; om voor roofdieren beveiligd te zijn: zijn huis op palen bouwen; om de zon op zijn huis te laten schijnen: boomen omhakken" [2, pp. 19–20].

[21] "de weemoed over het verloren Paradijs, de vage overtuiging van het stille levensgeluk, dat oorspronkelijk den menschen was bestemd, de hang naar zaligheid, naar religieuze vastheid, naar het vrije leven in overgave, die hang, die tredend in de vormen der droeve wereld, wordt honger naar hoogere, verheffende, naar transcendente dingen" [2, p. 24].

But conscience is being lulled. If it speaks in the bounded world, the attention for it is diverted by the luxurious satisfaction of other needs or it is recognised as a need within the system, susceptible to satisfaction. In this context Brouwer writes:

> "Art and religion in the world are merely morphine-industry on a large scale; the desire for a better life is being lulled, anaesthetised".[22]

The result is that introspection is made difficult. Yet it is possible and in chapter IV called "Reconciliation" Brouwer describes the possibility of appeasement. He writes:

> "In this way you are reconciled with the erring world, and you find her disconsolateness self-evident; but, moreover: you feel the fact that you are driven away from the Self, placed in Life, where pain and work, desire and fear are your share, and all Truth is veiled for you, as your accomplished Karma, that you are reconciled with and that you must fulfill; you view this life as the direction of your duty and you live it directed from the Self, that is to say that you recognise all bonds of the earth, that are your accomplished Karma until God releases you from them".[23]

With approval Brouwer quotes Böhme, who argued that the fall is necessary to know the light in its fullness:

> "From the Contrarium one understands what is Love or Suffering".[24]

Chapter IV is about language:

> "The intellect is directly accompanied by language. With life in the intellect comes the impossibility of relating to each other in a direct way—instinctively by gesture or glance of the eyes, or even more immaterially, through all spatial separations—and they start to train themselves and their offspring through harsh sounds, hard and—rather powerless, because so far never anybody has communicated his soul to someone else".[25]

This is followed by a further analysis of the role of language. For example:

> "[...] people with language loose primary desires, that, although sinful, were much closer to the Self; afraid of loneliness, their only fatherland, they become automatons in the service of the monster machine: public interaction".[26]

Moreover, according to Brouwer, if other forms of understanding show up, we immediately subject them to scientific studies, instead of simply opening ourselves to them in an open-minded way.

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[22] "Kunst en religie in de wereld zijn slechts morfine-industrie op groote schaal; de hang naar beter leven wordt er gesust, verdoofd" [2, p. 25].

[23] "Zoo zijt ge verzoend met de dwalende wereld, en vindt haar troosteloosheid vanzelfsprekend; maar meer nog: ge voelt als uw voldongen karma, waarmee ge zijt verzoend en dat ge te vervullen hebt, u, weggedreven uit het Zelf, geplaatst te zien in 't Leven, waar pijn en werk, begeerte en angst, uw deel zijn, en alle Waarheid voor u is omsluierd, ge ziet dat Leven als de richting van uw plicht en leeft het uit het Zelf gedirigeerd, dat wil zeggen erkent alle banden der aarde, die uw voldongen karma zijn, en die zo lang, tot God er u van losmaakt" [2, pp. 31–32].

[24] "An dem Contrarium wird erkannt was Liebe oder Leid sei" [2, pp. 33].

[25] "Het intellect gaat direct vergezeld van de taal. Met het leven in het intellect komt de onmogelijkheid, om zich op directe wijze—door gebaar en blik van oogen instinctief, of nog materieloozer, door alle afstandsscheiding heen—met elkaar in betrekking te stellen, en gaan ze zich en hun nakroost dresseeren op een teekenverstandhouding door grove klanken, moeitevol en—vrij machteloos, want nooit nog heeft door de taal iemand zijn ziel aan een ander meegedeeld [...]" [2, p. 37].

[26] "[...] menschen met taal verliezen primaire begeerten, die, hoe zondig ook, het Zelf veel nader waren; bang voor de eenzaamheid, hun enig vaderland, worden ze automaten in dienst van de monstermachine: het publiek verkeer" [2, p. 41].

In chapter V, "Immanent Truth", Brouwer argues that the Self manifests itself everywhere within the context of our bounded life in the form of Truth. But in order to find the Self more than the Truth is needed; necessary is something that is

> "above the forms of this world, and that can only be interpreted mystically with the word: Divine grace".[27]

Immanent Truth is Truth in the forms of the bounded world in so far as it points to the collision of opposing and irreconcilable interests. This seems to mean that wherever in the world there are essential oppositions that cannot be reconciled, there is Immanent Truth. Transcendent Truth is different; it is Truth in the forms of the bounded world in so far as it points to the possibility of Introspection to the Self.

About 15 pages are devoted to the concept of Immanent Truth. Often art shows us the Immanent Truth. For example, according to Brouwer:

> "Art, that is real truth, will everywhere belie common sense, causality and science: it will trample down the optimism that keeps the foolish earthly doings going; it will see in every human life the avenging Fate, it will see that the illusion of hopeful trust in worldly certainty turns into misery high above the delusion of causality"[28]

and

> "In times when there is no belief in other knowledge than in the intellect and in other forces of nature than the ones that are placed in practical life, the immanent truth in art will imperturbably continue to speak of magic, presentiments, murder by thought, resurrection of the dead, healings through love, apparitions, celestial interventions; she will not see people die of tuberculosis and gout and blood-poisoning, but, because their time has come".[29]

From Brouwer's point of view time is an illusion. Most clearly the tragedies show us this Immanent Truth. When Hamlet dies he has lost all illusions, happiness, faith, love. It requires only a slight extrapolation to see that life itself is an illusion. A tragedy can reveal to us that time is an illusion; the plastic arts, on the contrary, can reveal to us the illusion of space, the illusion of plurality.

At this point Brouwer proceeds with a characterisation of the woman as temptation:

> "lust of power, cupidity, ambition; and [. . .] the illusion of the woman. Also the latter is karma aggravation; because for no man is there room for the woman in the accomplished karma: she is a siren that lures him from his path".[30]

[27] "boven de vormen van deze wereld, en is alleen mystiek te duiden met het woord: 'Goddelijke genade' " [2, p. 47].

[28] "Kunst, die echte waarheid is, zal overal gezond verstand, causaliteit en wetenschap logenstraffen: het optimisme, dat het dwaze aardsch gedoe in gang houdt, vertrappen; in ieder menschenleven het wrekend Noodlot zien, dat illusie van hoopvol vertrouwen op wereldsche vastheid verkeert in ellende hoog boven de causaliteitswaan" [2, p. 49].

[29] "In tijden, dat niet wordt geloofd aan ander weten, dan in 't intellect, en aan andere natuurkrachten, dan die in het practische leven geplaatst zijn, zal de immanente waarheid in de kunst onverstoorbaar blijven spreken van magie, voorgevoelens, moord door gedachte, doodenopstanding, genezingen door liefde, geestverschijningen, zendingen des hemels, zij zal geen menschen zien sterven aan tuberculose en jicht en bloedvergiftiging, doch, omdat hun tijd gekomen is [. . .]" [2, p. 49].

[30] "heerschzucht, geldzucht, eerzucht; en [. . .] illusie van de vrouw. Ook de laatste is karmaverzwaring; want voor geen man heeft in 't voldongen karma de vrouw een plaats: zij is een van zijn weg aflokkende sirene" [2, p. 52].

Actually, for a modern reader one of the striking elements of Brouwer's thought is his idea that women are essentially different from men. To this theme Brouwer devotes many pages. For a man a woman represents a temptation that he must avoid. A woman, however, is nothing without a man. It is her nature and her fate to serve and do the ignoble work for man. She possesses no individuality. The situation in the sad world of a man was described above. For a woman, the situation in the sad world is different. A woman who does not wish to sin against her karma should totally sacrifice herself to the one she loves. She should not try to possess him and she should not engage in male activities.

This part of the book contains several references to Shakespeare, Goethe and others. Faust and Gretchen, Antonius and Cleopatra, Hamlet and Ophelia and other examples show us the immanent truth: a woman is for a man merely a temptation that he should avoid while it is a woman's fate to love and serve man.

Brouwer closes this chapter with the statement:

> "And in the most apparent way the immanent truth breaks through in the world in the eternal occurrence of unhappiness within the forms of the pursuit of happiness. Unhappiness denies happiness, by occurring within its forms as its frustration; the small houses of cards in which people lock themselves up so fearfully, once will all collapse, to everyone who is dying it becomes visible that another life was hollow, and a warning speaks that in spite of all labour and all bungling, Fate keeps the world within its paths".[31]

Those who know the immanent truth, that is the insight that whatever one tries in the world is going to fail, are directed by that insight towards Transcendent Truth, that is introspection to the Self. In chapter VII, Brouwer writes about the manifestations of Transcendent Truth in the world; for example in Bach's music, but also, of course, in mystical literature. chapter VIII returns to the way in which the life of a liberated man develops. In chapter IX Brouwer argues that as long as a liberated man is still in some ways tied to society, he should stay away from economics. A liberated man leaves the attempts to change society to "fools with ambition" [2, p. 95].

5. Schopenhauer and Brouwer: A comparison

5.1. *Brouwer's philosophical mysticism*

Let us briefly compare the ideas of Brouwer with those of Eckehart (1260–1327) and Böhme (1575–1624), from which they differ in several respects. In Eckehart's sermons, the central problem and the highest goal for a human being is the mystical union of the human and the divine. In order to achieve this a process of purification is necessary. Eckehart spoke much about what this mystical union of the "spark of the soul" and the Divine amounts to.

[31] "En het voelbaarst breekt de immanente waarheid in de wereld door in het eeuwig optreden van het ongeluk binnen de vormen van het streven naar geluk. Het ongeluk loochent geluk, door binnen diens vormen als verijdeling er van op te treden; de kaartenhuisjes waarbinnen de menschen zich zoo angstig opsluiten, storten éénmaal álle in, aan ieder stervende wordt zichtbaar, dat weer een leven hol is geweest, en spreekt een waarschuwing, dat trots alle arbeid en geknoei, het Noodlot de wereld binnen haar banen houdt" [2, p. 62].

In Böhme's work there is a theoretical aspect, absent in Eckehart's teachings. Böhme's central speculative problem was to show how good and evil, light and dark, etc. all emanate from God. For Böhme the Creation was a process of giving birth, resulting in repeated oppositions. In God, however, all these oppositions coincide. Böhme had considerable influence on Hegel and others.

Characteristic of most mystical literature is that the author had a mystical experience, sometimes of such intensity and of such a strange nature that only after a long time could something coherent be written about it. A difference between Brouwer, on the one hand, and Eckehart and Böhme, on the other, is that, as far as I know, Brouwer never had such an experience. The following quotation from chapter VIII of *LAM* also seems to suggest that Brouwer's mysticism is based more on philosophical considerations than on intense mystical experiences.

> "In this way at the start he will have shared the habits and ideals of his environment humbly and diligently in his intellect, carefully listening and patiently waiting for the relevation of the inner contradiction of that intellect; and *because he did not force this revelation, he will have received it in the ultimate consequences of philosophy*, in which he got jammed like in the top of a cone; but then for him the delusion of the world had flown away".[32] (italics added)

Another difference between Brouwer and other mystics is the great attention that Brouwer pays to the role of culture: science, language, art, etc. In every respect Brouwer is much closer to Schopenhauer than to the mystics.

5.2. *A further comparison of Brouwer's and Schopenhauer's views*

Schopenhauer's works encompass a few thousand pages. *LAM* counts 99 pages. This implies that a comparison is not unproblematic. Moreover, the early Brouwer never refers to Schopenhauer. It cannot be excluded that Brouwer actually read at the time Schopenhauer's work or about Schopenhauer, but that he considered his own views to be rather different. Quoting Schopenhauer would have meant agreement and there are subtle differences with respect to almost every aspect of Schopenhauer's philosophy. Yet, as I will show, Brouwer and Schopenhauer are in many respects two of a kind.

Both Schopenhauer and Brouwer are obviously followers of Kant in the sense that for them the objects of our knowledge are to a large extent the product of certain a priori conditions on the part of the knowing subject.

Both modified the idealistic Kantian views. At heart Schopenhauer follows Kant with respect to the spatio-temporal world and the law of causality. Yet for Schopenhauer the principle of causality, interpreted in a very general sense as the principle of sufficient reason, plays a central role. It represents the essence of time: succession. He wrote:

[32] "Zoo zal hij in den aanvang gewoonten en idealen zijner omgeving gedwee en ijverig hebben meegeleefd in zijn intellect, bij nauwlettend luisteren en geduldig afwachten naar de openbaring der innerlijke tegenstrijdigheid van dat intellect; en daar hij die openbaring niet heeft geforceerd, zal ze hem eerst zijn geworden in de uiterste consequenties der filosofie, waarin hij als in den top van een kegel vastliep; maar toen was hem de waan der wereld weggevlogen" [2, p. 84].

> "Succession is the form of the principle of sufficient reason; succession is the whole essence of time".[33]

The principle of sufficient reason also represents the essence of space:

> "this [space] is the possibility of the mutual determination of its parts with respect to each other, which is called position".[34]

Compare this to Brouwer in *LAM*:

> "The phenomena succeed each other in time, bound by causality, because you yourself, wrapped up in haze, want the phenomena in that regularity".[35]

Brouwer expressed himself much more clearly in his dissertation. There he compares his own views with those of Kant. For Kant Euclidean three-dimensional space and measureless time cannot be separated from external experience; moreover they occur necessarily on the basis of the organisation of the human intellect in the mathematical "receptaculum" of experience. For Brouwer a priori knowledge is knowledge that follows *immediately* from the primordial intuition of mathematics or the intuition of time. *That means that only the possibilities of mathematical construction that are implied by the primordial intuition of time are a priori* [3, Chapter 2].

It is quite possible to emphasise the differences between Brouwer and Schopenhauer, but there remains much common ground. Clearly Schopenhauer at heart followed Kant with respect to time and space, while Brouwer much more radically developed his own view. With respect to the "Ding-an-sich" there are also differences. Brouwer seems to have been more of a Kantian in this respect than Schopenhauer. Yet also in Schopenhauer's work is there the inclination to reduce time, space and causality to a simple primordial principle.

In the same way Brouwer's notion of will is not exactly Schopenhauer's Will. Brouwer's will is most of the time the individual will, but it plays the same negative role as Schopenhauer's Will. This is clear from the passages from Böhme that Brouwer quoted in *LAM*. Böhme wrote, for example:

> "The first property of nature is *desire*, like a magnet, as the possessiveness of the will, that wants to be something and at the same time has nothing to make something from; that is why it moves to an attractive object, impresses and attaches itself to a something, and yet it is nothing but an acute, magnetic *hunger*, a bitterness, hardness and a coldness".[36]

According to Böhme, in the end this possessiveness of the will leads to fear. Brouwer agrees.

I have written that Schopenhauer was a metaphysical pessimist. What does this mean? This pessimism consists, in the words of Copleston, in the conviction that

[33] "Succession ist die Gestalt des Satzes vom Grunde in der Zeit; Succession ist das ganze Wesen der Zeit" [24, p. 35].

[34] "dieser [. . .] ist [. . .] die Möglichkeit der wechselseitigen Bestimmungen seiner Teile durcheinander, welche Lage heißt" [24, p. 35].

[35] "De verschijnselen volgen elkaar in den tijd, gebonden door de causaliteit, omdat ge zelf in omneveling de verschijnselen in dien regelmaat wilt" [2, p. 15].

[36] "Die erste eigenschaft der Natur ist die *Begierde*, gleich einem Magnet, als die Einfaszlichkeit des Willens, der etwas sein will und doch nichts hat, woraus er etwas mache; und so führt er sich in eine Annehmlichkeit seiner selbst, impreszt und faszt sich selber zu einem Etwas, und ist doch nichts als ein scharfer, magnetischer *Hunger*, eine Herbigkeit, Härte und Kälte" [2, p. 77].

> "the passing years, the succeeding phases of history, could only mean a progressive increase in suffering and misery".[37]

Logically this pessimism is based on:

> "his conception of the Ding-an-sich, which he views as Will, an irrational, blind, self-conflicting impulse to existence".[38]

Brouwer shares the pessimism with respect to the course of history. He wrote:

> "People lived originally separately, and each for himself attempted to keep this balance in the supporting nature among sinful temptations; this filled their life, no interest in each other, no worries for the morrow. So no work either, and no sorrow; no hatred, no fear; no pleasure either".[39]

Dissatisfaction led to the world in which we live now:

> "ever more painful labour for the suppressed, ever more infernal conspiracy for the rulers, and all are suppressed and rulers at the same time; and the old instinct of separation lives on as pale envy and jealousy".[40]

According to Brouwer, an example of this development is the way in which the Dutch disturbed the original subtle equilibrium which existed between them and the water by building dykes [2, p. 7].

What is this pessimism based on? We saw above that the cause of the sad world is dissatisfaction; man always wants more. Moreover, the intellect by means of which man tries to realise his wishes is essentially limited in its possibilities.

Let us also compare Schopenhauer and Brouwer with respect to the solution that they offer man. Above we saw that for Schopenhauer there are two ways of escaping the sad world: the temporary escape in aesthetic contemplation and the lasting escape consisting in the denial of the will to live. Suicide is for Schopenhauer no real solution because it represents no permanent solution from the shackles of the will. Death implies that our intellect and our ego cease to exist, but as timeless will we continue to exist and we may reincarnate. The real escape lies in the denial of the will to live. If that denial succeeds, nothing remains. The atmosphere of Schopenhauer's writings on this point is one of atheistic religiosity.

What does Brouwer say? On the one hand, unlike Schopenhauer, Brouwer often mentions God. However, Brouwer's God is certainly not like the Christian God. In the profession of faith that he made at the age of 17 Brouwer described God as "that which is the origin of my ego and which gives me my images; which is therefore independent of me, something that like myself is alive and which is higher than me; that something is my God". This statement, but also the long quotations from the *Bhagavad Gita*, suggests that Brouwer's notion of God was close to Schopenhauer's.

It seems fair to assume that Brouwer uses the notion of "karma" in the sense of the *Bhagavad Gita*. The doctrine of action (karman) teaches that the actions of man, whether

[37] [6, pp. 74–75].

[38] [6, p. 75].

[39] "De menschen leefden uit oorsprong gescheiden, en ieder voor zich zocht te houden zijn evenwicht in de dragende natuur tusschen zondige verleidingen; dát vulde hun leven, geen belangstelling in elkander, geen zorg om den dag van morgen. Dus ook geen werk, en geen verdriet; geen haat, geen vrees; ook geen genot" [2, p. 7].

[40] "steeds pijnlijker arbeid voor de onderdrukten, steeds helscher samenspanning voor de machthebbers, en allen zijn onderdrukten en machthebbers tegelijk; en het oude instinct van scheiding leeft voort als bleeke nijd en jaloezie" [2, pp. 7–8].

good or bad, produce a binding effect in the sense that they necessitate rebirth. The Gita teaches that the binding quality of an act lies in the motive and in the desire that prompt it. Gradually the desire should be abandoned.

Let us consider some other aspects of Schopenhauer's and Brouwer's work. Schopenhauer distinguished between the intuition and the abstractions of understanding. The clear sunlight of the intuitive is, in Schopenhauer's view, superior to the moonlight of the abstract:

> "As long as we follow our intuition everything is clear, fixed and certain [...] But with abstract knowledge, with reason, doubt and mistake entered theory and worry and pity entered practice".[41]

The employment of abstract reasoning (peculiar to man) is only a temporary expedient. Compare this to Brouwer in *LAM* when he writes that only in

> "very narrowly circumscribed fantasies [...] understanding can be maintained rather long and well; about 'equal', about 'triangle', little misunderstanding will be possible; yet while doing so two people will never feel precisely the same, and even in the most restricted sciences, logic and mathematics, that cannot really be separated sharply,[42] no two individuals will think the same on the fundamental notions that they are constructed from".[43]

On women Schopenhauer and Brouwer also agreed. Schopenhauer described them in his essay "On Women" in *Parega and Paralipomena* (1851) as the number Two of the human race, incapable of undertaking an objective interest in anything, without a sense of justice and with little sense of truth. However, in both Schopenhauer's and in Brouwer's work, one can leave out all the passages on women without changing the core of their views.[44]

6. The turn to mathematics

When Brouwer wrote *LAM* he was already engaged in mathematical research. The following quotation from a letter in 1908 shows his attitude towards mathematics:

> "Korteweg and De Vries want to appoint me as 'privaatdocent' after the holidays, I want to escape, resist, lay down conditions [...] and will perhaps yield in the end; I love that subject, and why then not serve it in society as well? And if I should be more philosopher than mathematician, then it will also break through that straightjacket yet".[45]

[41]"Solange wir uns rein anschauend verhalten, ist alles klar, fest und gewiß. [...] Aber mit der abstrakten Erkenntnis, mit der Vernunft, ist im Theoretischen der Zweifel und der Irrtum, im Praktischen die Sorge und die Reue eingetreten" [24, pp. 66–67].

[42]N.B. In his dissertation Brouwer had changed his mind on this point.

[43]"zeer eng afgegrensde fantazieën [...] daar is het elkander verstaan vrij lang en goed vol te houden; over 'gelijk', over 'driehoek', zal weinig misverstaan mogelijk zijn; toch zullen daarbij nooit twee personen precies hetzelfde voelen, en zelfs bij de meest beperkte wetenschappen, logica en wiskunde, die eigenlijk niet scherp te scheiden zijn, zullen bij de grondbegrippen, waaruit ze zijn opgebouwd, geen twee hetzelfde denken" [2, p. 34].

[44]By the way, it seems to me that this negative attitude towards women is psychologically interesting. It is also remarkable that both Brouwer and Schopenhauer were obsessively occupied with their physical health and both repeatedly felt unreasonably treated by others.

[45]Korteweg en De Vries willen mij na de vacantie privaat-docent maken, ik wil ontvluchten, stribbel tegen, stel voorwaarden [...] en zal in 't eind misschien toegeven; ik heb dat vak lief, en waarom het dan niet dienen ook in de samenleving; wat is een God zonder altaren op aarde? En als ik meer philosoof dan mathematicus mocht zijn, dan zal het ook door die dwangbuis nog wel heenbreken" [17, p. 85].

In 1907, two years after the publication of *Life, Art and Mysticism*, Brouwer defended his doctoral dissertation, *On the Foundation of Mathematics* (*Over de Grondslagen der Wiskunde*) [3]. Then followed his topological work and his career started.

How serious is the opposition between the ideas that Brouwer expressed in *LAM* and the fact that he pursued a career in mathematics? As far as I know he himself never explicitly answered that question.

I shall try to give an answer. First of all the views expressed in *LAM* do not at all exclude work and an active participation in social life. The *Bhagavad Gita*, quoted by Brouwer, says on this point:

> "Therefore without attachment ever perform the work that thou must do; for if without attachment a man works, he gains the Highest". (*Bhagavad Gita*, III, 19; [11, p. 98])

In another translation:

> "Therefore perform Thou that which Thou hast to do, at all times unmindful of the event; for the man who doeth that which he hath to do, without attachment to the result, obtaineth the Supreme". (*Bhagavad Gita*, III, 19; [12, p. 19])

An Indian commentator, Rangacarya, explained that if a man works without attachment "there is among all the things existing in this world not one which is related to him as an object to be desired by him" [11, p. 98].

Well, is not a career in pure mathematics for someone who possesses the right talents the ideal way to work without attachment to the material world? Brouwer must have thought so. A passage by Brouwer, that was meant for but in the end not included in the dissertation, seems to confirm this:

> "And that is why science is only meaningful as a factor in the struggle of men against nature for their fellowmen by counting and measuring calculation, in other words, natural science has value as a *weapon*, but otherwise does not touch life, yes, it is there as disturbing as *everything* that is related to struggle. While mathematics, undertaken for its own sake, can acquire all the harmony [...] of music and architecture and can give all the illicit enjoyments that lie in the free unfolding of faculties without compulsion from outside".[46]

Mathematics undertaken for its own sake is not attached to the material world. It does not hinder the liberated human whose life is characterised by Brouwer in *LAM* as follows:

> "Your journey through the sad world is a steady journey in a light colourful cloud, in love for all in it that goes without saying, also for your wandering and desiring fellowmen, for you do no longer see it as a reality separated from the Self, but directed out of the Self, and along with the Self".[47]

Not only in this way is the dissertation very much in accordance with the ideas expressed in *LAM*. In the dissertation Brouwer describes true mathematics as a free creation that consists of mental constructions that are executed within the isolation of the individual human mind.

[46] "En daarom heeft de wetenschap ook alleen zin als factor in den strijd der mensen tegen de natuur om hun medemensen door tellende en metende berekening, m.a.w. de natuurwetenschap heeft waarde als **wapen**, maar raakt verder het leven niet, ja is er even storend, als **alles** wat aan strijd annex is. Terwijl wiskunde, om zichzelf bedreven, alle harmonie [...] van muziek en architectuur kan krijgen en al de ongeoorloofde genietingen kan geven, die liggen in de vrije ontplooiing van faculteiten, zonder dwang van buiten" [9, p. 30].

[47] "Uw gang door de droeve wereld is een gestadige gang in een lichte kleurenrijke wolk, in liefde voor al het van zelf sprekende daarin, ook voor uw dwalende en begeerende medemenschen, want ge ziet haar niet meer als een van het Zelf gescheiden werkelijkheid, maar gericht van uit het Zelf, en met het Zelf mee" [2, p. 16].

Language is exterior to mathematics and merely an imperfect means to communicate one's mental constructions to others. Brouwer rejects logicism and formalism as solutions in the foundations of mathematics because they are based on an attempt to found mathematics in language.

It is not possible to discuss in this paper Brouwer's work on the foundations of mathematics and his topological work. A good introduction with considerable biographical detail and many references can be found in [18]. For an elaboration on the point that there is in Brouwer's work a basic unity and that both his topological work and his intuitionistic work are in perfect harmony with the idea that mathematics is a free mental construction I refer to [13].

7. Brouwer compared to Gödel

Schopenhauer was very influential. Several of his doctrines reappeared with later philosophers. He influenced Eduard von Hartman (1842–1906), author of the *Philosophy of the Unconscious*. In the opinion of Richard Wagner (1813–1883), for example, Schopenhauer's philosophy must be the definitive foundation of all spiritual and moral culture. Wilhelm Busch was a follower of Schopenhauer and Nietsche was considerably influenced by Schopenhauer. These ideas reached the Netherlands and although we do not know exactly how this happened, it seems to me that I have convincingly shown that Schopenhauer's ideas must have reached and influenced Brouwer, directly or indirectly.[48]

Although Brouwer's work in philosophy and in mathematics is not easily accessible, it is a consistent whole. At the end of his book on Schopenhauer, Copleston writes that, although the reader of Schopenhauer is rewarded by many passages of striking beauty, Copleston did not succeed in discovering under the rhetoric a consistent systematic philosophy, a coherent psychology, ethic or theory of value. This makes a very precise comparison of the ideas of Schopenhauer and Brouwer difficult.

As for Brouwer's ideas on mathematics and the divine, it is clarifying to make another comparison. I shall compare Brouwer's views with the views of Kurt Gödel. It is remarkable that although these two great minds were working in completely opposed mathematical research programmes, they shared an interest in mysticism.

First I must devote a few words to Gödel's mathematical work. In the 1920s David Hilbert (1862–1943), who was at the time one of the world's leading mathematicians, felt that Brouwer's intuitionist mathematics represented a threat to classical mathematics. Hilbert's defence was brilliant. He came up with the so-called formalist programme to prove the consistency of classical mathematics. The central problem in the formalist programme turned out to be the following: to prove the consistency of the classical theory of natural numbers, the Peano arithmetic. In order to do so Hilbert represented the Peano arithmetic by means of a formal system, i.e., a formal representation in such a way that correct proofs of theorems are represented by sequences of formulae satisfying criteria so simple that a machine could check them. Hilbert assumed that the intuitionist threat to classical mathematics would be neutralised if one could give a intuitionistically acceptable

[48] After I finished this paper I discovered that in 1976 Eggenberger had already described Brouwer as a philosophical continuator of Schopenhauer [22, pp. 54].

Fig. 5. Schopenhauer in 1859 (Photo by Johannes Schäfer) Courtesy of the Stadt- und Universitätsbibliothek, Frankfurt am Main, Schopenhauer-Archiv.

proof of the formal consistency of this formal system.[49] However, in 1931 Kurt Gödel (1906–1978), became instantly famous when he published his *incompleteness theorem*: For every formal system encompassing the Peano arithmetic one can show that if it is formally consistent, the proof of this formal consistency requires more than the possibilities of deduction that are captured in the formal system. Gödel's result showed that Hilbert's programme could not be fully executed.

Let us now consider Gödel's views on mathematics and the divine. According to Gödel mathematics is not a mental construction but the description of a part of objective reality. Although the abstract objects that mathematicians study differ from concrete things, in Gödel's view mathematical entities exist independently of the activities of the mathematicians that study them. Gödel was an arch-rationalist who always sought rational order behind apparent chaos [20, p. 2]. Gödel had a vivid theological interest. His rationalism led him, for example, to an attempt to formalise Anselm's ontological proof of the existence of God: God's existence follows from the assumption of his perfection [1]. Gödel argued in private discussions that a system of postulates could be phrased for notions such as "God" and the "soul". He seems to have felt that in a purely rational way a theological Weltanschauung could be reconciled with all the known scientific facts [20, p. 210] or even that the existence of God would in the end turn out to be logically necessary [20, p. 165].

[49] Such a proof would, for example, show the impossibility of deriving formally within the formal system a formula of the form A&non-A.

Gödel had an interest in mysticism. Rudy Rucker, who talked to Gödel in 1977, wrote:

> "The central teaching of mysticism is this: *Reality is One*. The practice of mysticism consists in finding ways to experience this higher unity directly. This One has variously been called the Good, God, the Cosmos, the Mind, the Void, or (perhaps most neutrally) the Absolute. No door in the labyrinthine castle of science opens directly onto the Absolute. But if one understands the maze well enough, it is possible to jump out of the system and experience the Absolute for oneself. [. . .] I asked Gödel if he believed there is a single Mind behind all the various appearances and activities of the world. He replied that, yes, the Mind is the thing that is structured, but that the Mind exists independently of its individual properties. I then asked if he believed that the Mind is everywhere, as opposed to being localized in the brains of people. Gödel replied, 'Of course. This is the basic mystic teaching'. [. . .] then I asked him my last question: 'what causes the illusion of the passage of time'? [. . .] Finally he said this: 'The illusion of the passage of time arises from the confusing of the *given* with the *real*. Passage of time arises because we think of occupying different realities. In fact we occupy only different givens. There is only one reality' " [23, pp. 170–171].

About another conversation with Gödel in 1972 Rucker wrote:

> "I asked him how best to perceive pure abstract possibility. He said three things. (i) First one must close off the other senses, for instance, by lying down in a quiet place. It is not enough, however, to perform this negative action, one must actively seek with the mind. (ii) It is a mistake to let everyday reality condition possibility, and only to imagine the combinings and permutations of physical objects—the mind is capable of directly perceiving infinite sets. (iii) The ultimate goal of such thought, and of all philosophy, is the perception of the Absolute. Gödel rounded off these comments with a remark on Plato: 'When Plato could fully perceive God, his philosophy ended' " [23, p. 169].

Van Atten and Tragesser have argued that a comparison of Brouwer's views on mysticism and mathematics with the views of Gödel [15] suggests a refutation of what they call the common core thesis in mysticism (CCT). This thesis consists of two propositions:

(a) Mysticism holds that Reality is Good. Mystical practice aims to perceive this Good.
(b) This Good is objective, i.e. the same for all varieties of mysticism.

Put succinctly, Van Atten and Tragesser argue as follows:

(i) For Brouwer doing mathematics prohibits access to the Good or the Absolute,
(ii) For Gödel doing mathematics is the way of accessing the Good,
(iii) The conclusion from (i) and (ii) seems to be that Gödel and Brouwer are not talking about the same Good. Ergo: we have an argument against the common core thesis.

Van Atten and Tragesser essentially make a logical point: The statements "X can never be reached by means of method A" (Brouwer) and "Y can only be reached by means of method B" (Gödel) can only both be true if at least one of the statements "$X = Y$" or "$A = B$" is false. So if the two great men were both right and $A = B$ then necessarily $X \neq Y$.

This point clarifies the situation. Yet it is not conclusive, as Van Atten and Tragesser indeed also point out. It is obvious that Gödel and Brouwer had very different ideas about how to reach the Absolute. Gödel felt that abstract notions lead us towards the Absolute, while Brouwer thought that mathematics and even pure mathematics lead us away from the Absolute. There are different ways of avoiding the conclusion that the CCT is wrong. Indeed Gödel felt, like Plato, that ideas would have to lead the way, but like Plato he may have envisaged the full perception of God as being more than philosophy, as in some way transcending abstract thought. The opposition between the two great men, Gödel and Brouwer, may primarily concern the way to get to the Absolute, not so much the experience of the Absolute. So it is possible to maintain the CCT and assume that one of the two, Gödel or Brouwer, was simply wrong, or to assume that they were both right, in the sense that there are very different roads to reach the same goal.

8. Final remarks

In this paper I have tried to describe the ideas in *LAM*. I have not attempted to deal with the question of whether Brouwer was in any way right. In a 1982 series of five articles on Brouwer the Dutch author Rudy Kousbroek [14] discussed *LAM*. After praising the literary qualities of the text, Kousbroek made a number of very critical remarks. Kousbroek confronts Brouwer's mystical views with a "rational", "scientific" view of the world. He first establishes that Brouwer's description of the course of human history as a fall from an ideal life without diseases and without war, in harmony with nature, to a situation of misery and distress, is wrong if taken literally. It must obviously be read as a symbolic representation of our individual mental development: from a pure, unconscious, naked Self, on the one hand, to a less pure conscious mind on the other hand. Kousbroek argues that from a rational point of view there is nothing wrong here. There is only something wrong from Brouwer's mystical point of view: the symbolic opposition is taken literally and Brouwer feels compelled to view the fall as sinful; the pure Self is depicted as divine and consciousness as evil. Kousbroek finds this dangerous. If reason is rejected it becomes impossible to distinguish between correct or wrong mystical insights and "intuition" becomes like Pandora's box, a potential source of totalitarian ideologies. At this point Kousbroek emphasises the solipsism in Brouwer's ideas and its unattractive ethical consequences: other people and their suffering are considered to be annoying. Real communication is impossible and the limited communication that is possible is judged negatively. Compassion, tolerance, democracy, the principle of equal rights for everybody all go down the drain. Of course, Brouwer's views on women are also in this respect used against him by Kousbroek. I do no want to go into the question of whether or not Kousbroek is right. For our purposes this is not important. I have mentioned his papers because Kousbroek is one of the very few authors who took Brouwer's mystical ideas seriously enough to discuss them.

There is a considerable literature on Brouwer and on Gödel as well. The literature on their mystical interest is, however, limited. For the non-specialist the best places for further reading are, undoubtedly, the biographies: Van Dalen's biography of Brouwer [18] and Dawson's biography of Gödel [20]. As for Brouwer's mysticism the reader should turn to the original *Life, Art and Mysticism* ([2] or the English translation [26]). As for Gödel's mysticism the sources are very limited: beyond what Dawson tells us there are brief reports of conversations by Rudy Rucker [23] and Hao Wang [28].

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CHAPTER 31

On the Road to a Unified World View: Priest Pavel Florensky—Theologian, Philosopher and Scientist

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MATHEMATICS AND THE DIVINE: A HISTORICAL STUDY
Edited by T. Koetsier and L. Bergmans

Fig. 1. Pavel Florensky in the uniform of a student at the Moscow Theological Academy.

1. Early years

The theologian, philosopher and scholarly encyclopedist Father Pavel Aleksandrovich Florensky was born 9 January (old style) 1882 in the township of Yevlakh, Yelizavetpol district (present day Azerbaijan), where his father, a railway construction engineer, had worked on the construction of the trans-Caucasian railway. His father was Russian, born in Kostroma, his mother, maiden name Saparova (Saparian), was from a cultured Armenian family of Karabakh nobility, which moved to Georgia. His childhood was spent in Tiflis (Tbilisi) and Batumi, where his father worked on the construction of the military Batumo-Akhaltsykhsky railway. His contact with the natural surroundings of the Caucasas left a profound impression on his creative senses [1]. In 1892 he entered the Second Classical Gymnasium in Tiflis where his classmates included the future religious figure A.V. Yelchaninov (1881–1934) and the philosopher V.F. Ern (1881–1917). (See [2, vol. 1, pp. 3–6]. This article makes much use of the introduction to Florensky's collected works [2, vol. 1, pp. 3–36], written by Igumen Andronik (A.S. Trubachev), a grandson of Florensky.)

The family of Florensky, as with many intellectual Russian families of this period, was not religious. This family shared the ethos of the broad circle of the Russian democratic intelligentsia. The central component of this ethos was faith in science, the conviction that science could be the panacea for the problems of society, enabling the construction of a beneficent and just society. The family shared this enthusiasm for science and its technical applications. This ethos—including indifference to religion—was shared by the youthful Florensky. "Educated in complete isolation from any religious notions and even from simple fairy tales", he later wrote [2, vol. 1, p. 6]. "I regarded religion as something completely alien to me and any religious lessons in the Gymnasium drew from me only hostility and mockery".

In the summer of 1899 he had a spiritual conversion. He described it in his *Reminiscences* [1, pp. 211–212]. He fell into a deep sleep, as if he had fainted, accompanied by "a mystical experience of darkness, non-existence, confinement". He "experienced himself in a penal colony, maybe in the mines". He "was in the grip of inescapable despair" when "a thin, tiny ray, a light which could not be seen, a sound which could not be heard, pronounced the name of God, ... I was confronted face to face with a new fact, as incomprehensible as it was unquestionable: there is darkness and death, but there is also salvation from it ... For me this was a revelation, a discovery, a shock, a blow. Out of the darkness of this blow I suddenly awoke ... and ... shouted out loud: No, it is impossible to live without God!" At another time [1, pp. 214–215] he suddenly awoke from a spiritual jolt, which "was so strong and decisive", that he ran out to the yard which was awash in moonlight. "At just this moment the event occurred for the sake of which I was drawn outside. In the night air I heard a voice, distinct and loud, which called my name two times: 'Pavel! Pavel!' ... I did not know and do not know to whom this voice belonged, but there is no doubt that it came from a higher world".

The sensation was so powerful that it led to a profound spiritual crisis, which ended in his conversion. The limitedness and relativeness of scientific knowledge became apparent to him and he took the first deliberate step toward a religious point of view. During this period he wanted to work among the common people, but his parents insisted that he continue his education. The nineteenth century had seen in Russia the rise of revolutionary ideology and the rejection of religion, especially among the intelligentsia. This movement accelerated in the 1890s with the spread of marxism and eventually lead to the Bolshevik Revolution of 1917. The conversion of Florensky was part of a countervailing movement that began around 1900 and has become known as the Russian religious philosophical renaissance. It saw some leading intellectuals turn, or return, to the Russian Orthodox Church. Pavel Florensky was to become one of the most important figures in this movement.

2. Moscow University

In 1900 after graduating from the Gymnasium he entered the Mathematics Department of the Physico-Mathematical Faculty of Moscow University. Beside his strong interest in mathematics this choice is explained by the necessity of having a solid mathematical foundation for constructing a world view for a philosophical and theological understanding of the world, which the young Florensky had already chosen as his primary task. The choice of Moscow University was very fortunate. In contrast to Petersburg mathematical circles, which were dominated by positivism, the atmosphere of Moscow mathematics was strongly influenced by the ideology of the so-called Moscow philosophical mathematical school and its leader, the well known Moscow mathematician Professor N.V. Bugayev (1837–1903). Bugayev was in fact a very interesting philosopher, the author of his own philosophical system, evolutionary monodology. He was critical of the analytical world view which dominated XIXth century science and promoted another world view, the mathematical foundation of which was the theory of discontinuous functions. He put forth his own version, arithmology, which had many supporters and followers [3]. Among the topics proposed by Bugayev for his students for independent work was the question of discontinuity, not regarded simply from a narrow mathematical perspective but in broad philosophical context.

Florensky was both greatly surprised and extremely pleased to discover that the interests which he had developed in the Gymnasium coincided with the interests of a well known mathematician. Thus it happened that the topic chosen by Florensky already in his first year, discontinuity as a component of a world view, became the subject of his research for the entire period of his University education. This work was entitled “On singularities of plane curves as places disturbing its continuity”. The first part, “On singularities of algebraic curves”, was presented as his dissertation for his Candidate Degree in 1904 [4]. The second part was never completed. Florensky was remarkably active during his years at the University. He inherited from Bugayev a strong interest in the theory of discontinuous functions. He studied the works of Georg Cantor (1845–1918) on set theory. He was attracted especially by its theological connections, the very thing which provoked such strong negative reaction from the Petersburg mathematicians, with their strong positivist orientation. Florensky published the first Russian language article about Cantor’s set theory [5].

This article was published not in any scientific journal, but in the short-lived journal of the Religious–Philosophical Society of Writers and Symbolists, in which Florensky had earlier published two theological treatises. This Society held meetings from 1901 to 1903 in which church leaders and lay intellectuals came together to discuss the importance of a revival of religious life in Russia [6, pp. 58–59]. It was an important episode in the course of the Russian religious philosophical renaissance. In the meetings of the Society various speakers stressed the importance of freeing the Church from control of the state, to which it had been subordinated by Peter the Great. Such outspoken advocacy of Church independence lead to the suppression of the Society in April 1903.

Florensky organized the Student Circle of the Moscow Mathematical Society [7] and served as its first secretary, a position which he later offered to his friend, the future mathematician N.N. Luzin. In the sessions of this Circle, Florensky himself, other students, and sometimes even the professors, gave lectures on topics including set theory and the theory of functions of a real variable. These activities of Florensky helped to create the atmosphere in which one of the most important mathematical schools of the XXth century was born, the Moscow School of the theory of functions [8]. Its birth is connected above all with the names of Florensky’s teacher D.F. Egorov (1869–1931) and his friend N.N. Luzin (1883–1950). The relationship between Egorov and Florensky is discussed in [9,10]. Correspondence spanning eighteen years between Luzin and Florensky has been published [11–14].

In addition to the study of mathematics, Florensky attended the lectures of the well known philosopher L.M. Lopatin from the Faculty of History and Philology and visited the seminar of the important philosopher S.N. Trubetskoy. He was intensely interested in art history and poetry. In these years he was very close to the symbolists and became friends with the son of N.V. Bugayev, the famous Russian poet Andrei Belyi (B.N. Bugayev, 1880–1934). In 1907 he published his own collection of poems *In Eternal Azure*. In 1904 he graduated from the University with the Candidate Degree. On the recommendation of the famous applied mathematician N.Ye. Zhukovsky (1847–1921), who loved Florensky and was very interested in his work, he was offered a fellowship for post-graduate study. This, however, he did not accept. To become a mathematician was not his goal. He explained his real goal to his mother in a letter written in the spring of 1904 [2, vol. 1, pp. 8–9]. "I do not think I will continue at the University . . . To create a synthesis between religious and secular culture . . . to take all the positive teachings of the Church and a scientific, philosophical world view, together with art and so on—this is my real goal".

3. Theological Academy

His years at the University were also years of intense spiritual searching. In this period he seriously considered entering monastic life. However, his spiritual father, starets Bishop Antony (Florensov, 1847–1920), who lived in retirement in the Donskoi Monastery, recommended that he not rush into this decision but rather try other possibilities, first of all to receive a theological education at the Moscow Theological Academy. Florensky followed this advice. In 1904 he entered the Moscow Theological Academy, located 60 kilometers north of Moscow in Sergiev Posad.

The revolution of 1905 posed challenges to Florensky. One of his friends, V.F. Ern, like Florensky a future priest, observed that Florensky was strongly opposed to any sort of social activism, insisting instead that we should influence individual persons. Thus Ern was quite surprised to learn from one of the seminarians that Florensky had been arrested on 23 March and imprisoned in the Tagansky prison in Moscow. The Moscow Theological Academy had the practice of allowing seminarians to preach in the Cathedral. On 12 March

Fig. 2. The Philosophers Pavel Florensky and Sergei Bulgakov. Painting by Mikhail Nesterov, 1917.

1906 Florensky had preached sermon entitled “Appeal of Blood”. It was delivered during Holy Week just when the news arrived of the execution of Lt. Pyotr Shmidt, one of the activists of the revolution. Florensky spoke with great passion against the executions and shootings then being carried out. His sermon was greeted with elation by the students, who decided to print it on their own, avoiding censorship [12, note 2, p. 134]. The authorities drew up orders for his arrest.

He was in prison for almost a week. Influential churchmen intervened for his release. While in prison he wrote a treatise *On the elements of the α number system*. On the inside cover he wrote “Responsibility for this work goes to F.V. Dubasov—dedicated by the grateful author”. Admiral Dubasov was the governor of Moscow in the years 1905–1906 and the organizer of the defeat of the armed uprising in December 1905 in Moscow. Shortly after his release, Florensky wrote to Luzin, apologizing for not having written for nearly a year [12, pp. 132–133; 14, p. 239]. “You, of course, know the troubled condition of our country, even worse, it seems, the condition of mother (for me at least)—Church, and the muddle and turmoil in the minds and hearts now ruling in Russia”. In 1908 Florensky graduated from the Academy [13, pp. 336–338] and, after presenting two trial lectures “The cosmological antinomies of Kant” and “The general human roots of idealism”, was appointed Docent in the Chair of the History of Philosophy.

Florensky’s interest in monastic life persisted and he came to know several monks personally. In 1908, for example he authored a treatise *Salt of the earth* about Hieromonk Isidore whom he knew and loved [15]. He had known Father Serapion and would later come to know Father Anatole, among others. The tension of finally turning away from monastic life caused a crisis in Florensky’s life around the year 1910. In a letter to a friend dated 14 March 1910 [16, note 17, p. 78], S.N. Bulgakov (1871–1944), a close friend of Florensky wrote “P.Al. Florensky came somehow, stayed a long time, was infinitely dear, simple and sincere. I was not troubled because the crisis is not religious, but personal, human, partly on the grounds of overwork. He is upset with the Academy and again dreaming of the priesthood and family life, but, alas, it can not be created on command”. Indeed, on 25 August 1910 Florensky was married to A.M. Giatsintova (1889–1973). Later this year his friend Luzin wrote that Florensky “makes a very good impression now. He dreams about a ‘son’, whom he is sure that he will have”. “He still dreams of fleeing the Academy for the ‘village’, or at least for the priesthood” [12, note 2, p. 162]. In fact the marriage produced three sons and two daughters. In 1911 Florensky was ordained a priest.

4. The pillar and ground of the truth

In 1914 Florensky defended his magister dissertation *On Spiritual Truth*: *An Essay of Orthodox Theodicy*. In the same year he published a more complete version of this dissertation: *The Pillar and Ground of the Truth. An Essay in Orthodox Theodicy in Twelve Letters* [17] which became one of the greatest works in Orthodox theology in the XXth century (French translation [18], English translation [19]). According to Florensky the world of religious thought was divided, roughly speaking, into two parts, described by Igumen Andronik as follows [17, p. vii]. The first “on the basis of faith and Church life, to secure a foundation, to obtain The Pillar and Ground of the Truth (as the Church was designated by the Apostle Paul in 1 Timothy 3:15)". The second "from this foundation to develop the teaching about the world and the person" in communion with God. The first he named theodicy (using this term in a little different sense than usual) and the second anthropodicy. *The Pillar,* as indicated in its title, is dedicated to theodicy.

The book is structured as a story about the path which leads the author to the world of the Orthodox Church. The initial spiritual situation is an experience of the fallen world. The goal is to obtain an absolute foundation, to achieve the Truth. The first stage is to obtain this through rationalistic philosophy. This leads to total failure—the Truth does not exist on this path. In the second stage the author sets aside the intention to obtain the Truth but tries to describe its main qualities on the assumption that the Truth exists. As a result he learns many of these qualities, among them that the Truth exists in antinomies and (central for Christianity) that the Truth is trinitarian. "The Truth is one essence with three hypostases . . . the Trinity is of one substance and indivisible" [19, pp. 37, 39]. In this case there is only one way to reach the Truth "to leave the domain of concepts and enter the sphere of living experience" [19, p. 47]. And because the Truth is already revealed as a Trinity, for the third stage there is only one path, to enter the Church. The central topic becomes the Church—"The Pillar and Ground of the Truth". The fundamental conceptions on which the story constructs the basic philosophical themes of *Pillar*—existence before the fall, the

Fig. 3. Trinity. Icon by Andrej Rublev.

connection between our existence and that before the fall—is the conception of Sophia, Divine Wisdom. Because of this the philosophy of *Pillar* is included in the Sophiological tradition, a branch of the metaphysics of all-unity. A large part of the book is devoted to the problem of knowledge, in the interpretation of which ("knowledge expressed in love") Florensky followed the Russian philosophical tradition of "ontological gnoseology"—the patristic fathers, A.S. Khomyakov, V.S. Solovyev. (About Russian philosophy and in particular about the conception of Sophia and the philosophy of all-unity see [20–22]. For understanding the dialectic of Florensky we underscore the importance of his teachings about antinomies.)

In this period it was totally unusual for a work on religious philosophy to introduce material from mathematics and the natural sciences, questions from art history and linguistics. The appearance of this book had an explosive effect on both religious and secular culture. It became a major event in Russian theological and philosophical circles and gave its author wide exposure. The defense of his dissertation was a real triumph for Florensky. In that same year, 1914, he was appointed as an Extraordinary Professor in the Chair of the History of Philosophy at the Academy. From 1912 to 1917 he was editor-in-chief of the journal of the Academy *Theological Messenger*.

5. Concrete metaphysics

The dramatic events of 1914–1922—the First World War, revolution, the Bolshevik seizure of power, civil war and, finally, the establishment of the Soviet state—ruptured the life of the country and the life of Florensky as well. Persecution of the Church began immediately. In 1918 the work of the Academy was transferred to Moscow, where it was soon forced to close. In 1921 the church in Sergiev Posad where Fr. Pavel served as priest was closed. This difficult time became a period of exceptionally creative activity. He applied all of his energy to working out his anthropodicy, having earlier devoted himself to theodicy.

Florensky named his philosophy "concrete metaphysics" which he intended to present in his monumental work *On the Watersheds of Thought* [23]. The connection between phenomenon and thought, between phenomenon and nomenon is, for Florensky, mutual [23, pp. 3–12]. Both together precisely explain each other and form a perfect diunity, the symbol. The symbol is the central object of his philosophy. The central understandings and conceptions with which he worked are translated, "concretized", in the language of symbols. Thus his "concrete metaphysics" is a form of philosophical symbolism.

Florensky considered nomena to exist in a special space which is in an "inverted", "reversed", or "imaginary" relationship with physical space. Both worlds, the physical and the spiritual, are inseparable. Florensky interpreted them as lying on the two sides of a single surface. Thus the two sides of a symbol, the phenomenal and the nomenal, are obtained from each other by literally turning inside out. He worked this out in an interpretation of complex numbers in his 1922 book *Imaginary Values in Geometry*, as is explained in [24]. The phenomenon expresses the nomenon perfectly only in the perfect being. In our fallen being we do not have a symbolic expression of reality, we must receive it. This receiving is anthropodicy. It is realized in the Church cult—the sanctification of reality. The philosophy of cult is the most important part of concrete metaphysics.

The task of concrete metaphysics is the recognition and study of symbols. In this sense it can be regarded as an exercise in practical symbolism which must describe the symbols which exist in all areas of reality, classify them, and make a complete alphabet of symbols. In *Watersheds* the classification of symbols is made by means of perception—visual perceptions, auditory perceptions, sense perceptions and so on. The basis for such an approach is the correlation between man and the world, between man and the cosmic order—in modern terminology, the anthropomorphic principle. The first chapter of *Watersheds* is organized according to this principle, covering visual symbols (icons, pictorial images, graphic images) and auditory symbols (the Name of God, proper names, words).

The kernel of the philosophical symbolism of Florensky consists of energy principles. The connection between nomenon and phenomenon is realized by joining their energies. This energy gives life to the symbol. On the basis of these energies, the origins of which Florensky saw in the theology of energies of Saint Gregory of Palamas, he treats the word, the name, language, the connection between thought and language, and the meaning of its auditory aspects. The entirety of his symbolic picture of the world must be subject to "energization". Florensky began the resolution of this problem but was not able to complete it.

In his philosophical and theological constructions Florensky touched on questions from many different sciences. He studied seriously several contemporary problems: linguistics, art history, and others. He can be regarded as one of the founders of modern semiotics. He came surprisingly close to the general theory of information flow. His concept of "pneumatosphere", which has points of contact with the concept of "noosphere" of V.I. Vernadsky (1863–1945), is discussed today by philosophers and ecologists.

Florensky gave a special place to mathematics. Above all he regarded mathematics as a means for understanding the structure of complicated phenomena in different fields, including philosophy and theology. Many of his mathematical constructions served this purpose. For example the hierarchy of transfinite sets facilitated an understanding of the divine hierarchies [5]. The theory of Paul Dubois-Reymond about types of increasing functions was used as a tool for understanding the possibilities for the perfection of persons [25]. Imaginary complex values enabled him to construct a model of a mutually inverted pair of worlds, the spiritual and the physical [26,27]. These constructions, as Florensky himself underlined, "are not analogies or comparisons, but in fact indications of similarity in essence, not something to be accepted or rejected depending on your goal but something the legitimacy of which is determined by properly formulated premises: briefly—necessary mental schemes" [2, vol. 1, p. 284]. Mathematical structures play a fundamental role in his explanation of how the phenomena of the physical world and the nomena are united in a single entity by energies, the key to the understanding of which can be found in his interpretation of imaginary values. In his approach mathematics and philosophy are identical. Both have infinity as their subject. For Florensky [28, p. 12] "infinity is not an ideal or material conception but something living, which can be perceived by the senses".

Florensky wanted to present his "concrete metaphysics" in his monumental work *On the Watersheds of Thought*. The first outline of this plan of which we are aware is dated 23 October 1917 (old style) on the eve of the October Revolution which was fatal to the realization of this work. In the period from 1918 to 1922 he wrote a whole series of segments of the first part. The tragic events of the Soviet period did not allow him to continue working with the same intensity. During his lifetime only some fragments of the work were published. The entire work, only partly completed, has been published only in our own day ([23] and also as the two parts of volume 3 of [2]).

6. Church and revolution

During his years as a student at the Moscow Theological Academy Florensky became involved with a group of Orthodox intellectuals centered around M.A. Novosyolov (1864–1938). Correspondence between Florensky and Novosyolov had been published in a book that contains an informative introduction by I.V. Nikitina and S.M. Polovinkin [29]. In 1907 this group officially organized itself as the “Circle of Seekers of Christian Enlightenment” [29, p. 19]. Novosyolov was a strong critic of state control of the Church. In 1910, for example, he came out in print against Rasputin and his harmful influence on the Church, for which he was vilified by partisans of Rasputin [29, pp. 26–29].

On 6 October 1912 Novosyolov wrote Florensky to enlist his help in writing important work related to the movement “Praisers of the Name” [29, pp. 31, 73]. Florensky became one of the most convinced adherents of this movement. Arising in the Orthodox tradition, it promoted a specific teaching about the respect, esteem, and honor due the Name of God. The ideas of Florensky about the Name of God (and the nature of names in general), especially as he would later formulate in *Watersheds*, correspond closely with the thought of this movement [10; 19, pp. 303–311]. This teaching was propagated at the beginning of the XXth century in the 1907 book *On the Caucasian Mountains* by the hermit monk Ilarion, who lived in monastic isolation in the Caucasian Mountains. It provoked a serious

Fig. 4. The liturgy of hierarchs. Serbian Orthodox Fresco, Studenica, 1208/1209.

conflict in Russian religious society. In 1913 open conflict erupted in the Church over this movement [10, pp. 123–128].

As a result of the abdication of Emperor Nikolas II in March 1917 the Church was finally freed from its subjugation to government control that had been established under Peter the Great. Reform spread rapidly [30, pp. 25–30]. In August 1917 a Council of the Russian Orthodox Church was convened that met until September 1918 [31, pp. 193–198]. Both Florensky and Novosyolov were enlisted in the work of the Council [29, p. 32].

As a result of the Bolshevik seizure of power in October 1917 the Moscow Theological Academy was closed in September 1918. However, the Academy continued to function informally, first at the Danilov monastery in Moscow and later at the Petrov monastery [2, vol. 1, p. 24]. Already by the spring of 1918 members of the "Circle of Seekers of Christian Enlightenment" began presenting a series of theological lectures in the apartment of its leader Novosyolov. In November 1919, Novosyolov moved into the Danilov Monastery to continue this work [29, p. 33]. As the marxist onslaught against the Church intensified, the Danilov Monastery became a center of resistance to compromise with the Bolsheviks. Around 1921 Novosyolov secretly took monastic vows and in 1923 was secretly consecrated a bishop. About this time he began to undertake "catacomb" activities within the Church that continued through much of the 1920s [10, p. 143; 29, p. 40].

Those interested in the "Praisers of the Name" movement included Florensky's teacher Egorov and the philosopher A.F. Losev (1893–1988), who was much influenced by Florensky. A discussion group formed around them that took a special interest in this movement [32, pp. 108–112]. Egorov was able to meet with Patriarch Tikhon concerning the movement [32, p. 111]. On 29 June 1921, Egorov sent a letter to Florensky indicating the Patriarch's personal interest and inviting Florensky to join in a conversation on the subject [10, p. 153; 29, pp. 235–236]. As suppression of the Church continued through the 1920s, people engaged in activities like those of Novosyolov, Egorov, and Losev became, in effect, a "catacomb" church which became known as the "True Orthodox Church" [10, pp. 141–144; 30, pp. 150–158]. The arrest of Novosyolov late in 1928 [29, p. 41] was a major step in an attempt to suppress it. That arrest was followed by many more, including Losev in April 1930 [32, p. 129] and Egorov in October 1930 [10, p. 140].

7. Religious and scientific work

Florensky continued to reside in Sergiev Posad until early 1921. In 1918, Professor N.A. Berdyayev (1874–1948) and others established a "Free Academy of Spiritual Culture" in Moscow, which continued until Berdyayev and other intellectuals were expelled from the Soviet Union in 1922 [31, pp. 206–207]. Florensky presented lectures under the auspices of this Academy and other organizations well into the 1920s [27, note 67, p. 39]. He attended the church of Fr. Aleksei Mechev (1860–1923), which, at the beginning of the 1920s, was a meeting point for Russian religious intellectuals [27, note 61, p. 38]. Florensky was one of the speakers in a series of debates between religious believers and atheists that were sponsored by the Bolsheviks in the first years after the revolution. They began to draw huge crowds and, though stacked with party activists, often ended in applause for the religious speakers. This led to their discontinuation by the regime [33, pp. 38–39].

During this time Florensky worked very hard as the Scientific Secretary of the Commission for the Preservation of Monuments of Art and Antiquity at the Troitse-Sergieva Lavra, the great religious complex at Sergiev Posad [2, vol. 1, p. 25; 27, p. 8]. In addition to the study of museums and the arts, his goal here was to preserve these sacred national treasures from extermination. In particular, he was one of those who hid the relics of the patron Saint of Russia, Sergei of Radonezh, from the desecration that became normal during these years of militant atheism. His work on this Commission earned him his first public denunciation—for trying to set up an "Orthodox Vatican". Later when the Soviets closed the famous Optina monastery Florensky launched a campaign to "Save Optina".

During this time Florensky worked on a variety of questions in religious philosophy, art history, the organization of museums, and on his *Reminiscences* [1, pp. 24–266]. His famous work *Iconostasis* was published in 1922 [34] (German translation [35], French translation [36], English translation [37]). This work overlaps into both art history and religious philosophy. His ideas on the role of space in the arts, in particular his original theory of inverse perspective, provoked great interest from painters and specialists in art history and he was invited to join the faculty of The Higher Artistic Technical Studio (VKhUTEMAS). He gave lectures there in 1921–1924 [38]. In these lectures he emphasized reverse perspective and its use in icons and other religious art. The orthodox view of icons as a window to the divine corresponds well with Florensky's views in *Watersheds*. He was later denounced for supposedly creating an 'idealistic coalition' at this institution.

The year 1924 was a time of deep spiritual crisis, engendered by the oppressive antireligious atmosphere. The state increased its persecution of the Church and attacked every appearance of free philosophical thought that deviated in any way from official marxism. In these conditions he turned his creative abilities to scientific and technological questions as his theological and philosophical studies gradually went underground. After 1925 we can find no traces of such studies in any of his papers.

During these years he returned to his studies in mathematics and physics and worked on technical problems in material science. In 1921 he began to work in different departments of Glav-Electro of the All-Union National Economy (VSNKh) of the RSFSR, the newly created electrification commission of the Russian Federation. He worked in the Carbolite Commission, in the Departments of Mechanics and Chemistry, in the Section of High-voltage Technology, in the State Experimental Electrotechnical Institute, and from 1925 served as the Chief of the Department of Material Science, a Department which he himself had developed. In 1924 his monograph *Dielectrics and its Technical Applications* appeared. His research was directed toward a search for new materials for the long-distance transmission of high-voltage electricity.

In 1925 the future physicist I.Ye. Tamm worked at Glav-Electro with Florensky as a young engineer and had the opportunity to observe him [39]. Some of Tamm's impressions of Florensky have been passed down verbally and were told to us by S.S. Khoruzhy. Tamm thought that Florensky had the intellectual power to have discovered quantum theory, but that his intellectual perspective prevented him from doing so, except in the realm of electricity, where Tamm believed that Florensky had probably discovered, or at least anticipated, the zone theory of semiconductors [40, p. 40].

In 1927–1933 Florensky was an editor of the *Technical Encyclopedia* and author of more than 100 of its articles. The article on dielectrics, for example, begins with a page written by the well known physicist A.F. Ioffe and a page and a half by Florensky. In the second edition of the Encyclopedia, after his arrest, all references to him were removed. In 1928 his book *Carbolite, its Production and Qualities* was published, describing an inexpensive substitute for bakelite. In 1930 he became Assistant Director for Science of the All-Union Electrotechnical Institute. Florensky continued to publish in scientific journals. As late as 1932 several of his articles appeared in the journal *Socialist Reconstruction and Science* (SORENA). One of these [41] describes an analog calculator for approximate solutions to algebraic equations of high degree. It gives a specific example of Florensky's general thesis that the entire structure of mathematical knowledge rests upon physical intuition.

8. Enemy of the people

The work of Florensky and others in preserving religious artistic treasures at Troitse-Sergieva Lavra had met with some success. Several book were published about the religious treasures stored there, including one co-authored by Florensky entitled *Amvrosy, Troitsky Engraver from the XV Century*, which appeared in 1927 [27, note 65, pp. 38–39]. Excursions of workers and others would visit the site [42, p. 24].

In May 1928 a major media campaign was launched against the Troitse-Sergieva Lavra [2, vol. 1, pp. 29–30]. Newspaper and magazine articles and pamphlets thundered against the "counter-revolutionary scum" gathered in this "nest of black-hundreds" just outside of Moscow. One author, whose article appeared on 17 May in a major Moscow publication, specifically cited the book on Amvrosy. He was particularly incensed that "scholars" working for a state scientific institution could publish religious books for mass distribution. "These must be really devious, insolent, persons to give, under the guise of a 'scientific book' in the tenth year of the revolution, such trash to the Soviet reader when every Pioneer knows that the legend of the existence of Christ is nothing but a piece of priestly charlatanism" [42, p. 24].

Soon after, the museum's archive was closed and a large number of people were arrested and sent to Butyrki, a prison in Moscow. Florensky was sentenced to exile and on 14 July was sent to Nizhny Novgorod (renamed Gorky during the Soviet period). Influential people intervened on his behalf, after which he was granted early release and by September had returned to Moscow. Photographs of the time show that he still continued to attend meetings of various scientific committees dressed in the priestly cassock and apparently even wearing a pectoral cross.

As he continued his scientific work, in 1930 Florensky began to came under attack in the Soviet science community. His most militant opponent was Arnost Kolman (1892–1979), who was born in Czechoslovakia and became a prominent ideologue with influence in Soviet science [43]. The first attack came in an address by Kolman to the Society of Mathematical-Materialist Dialecticians on 29 November 1930 [44]. Kolman began by citing the religious philosophical tradition of Moscow mathematicians and mentioned such figures as Bugayev, Florensky, and Egorov (who had just been arrested the previous month).

Making specific reference to *Imaginary Values in Geometry*, Kolman railed against the use of "mathematics in the service of religion", "mathematics in the service of priestcraft", “mathematics in the service of obscurantism”. He pointedly remarked that Florensky was still working in institutions of Soviet science [44, p. 30]. The continued publication of articles by Florensky in Soviet journals prompted a major article by Kolman in 1933 [45]. Referring to the 1932 article in SORENA mentioned above, he launched a vituperative attack on Florensky. “Diplomaed lackeys of priestcraft right under our noses are using mathematics for a highly masked form of religious propaganda” [45, p. 91]. By the time this article appeared, however, Florensky was already in Siberia. He had been arrested on 25 February 1933 and on 26 July was sentenced to ten years in “corrective labor camps” [2, vol. 1, p. 31; 42, pp. 24, 26; 46, pp. 142, 144].

In the labor camps Florensky continue to do important research. The first year he spent in BAMLag in eastern Siberia, where he studied the phenomenon of permafrost at the scientific station in Skovorodino. He wrote on 27 November 1933 [2, vol. 4, pp. 48–49] “I envision large tasks for the economy of the local region from studying and maybe using permafrost etc. and I hope that in the future my special knowledge will be applicable and profitable for the state”. On 6 December he wrote [2, vol. 4, p. 51] “I am beginning a major undertaking in the study of the physics of permafrost. . . . In approximately two months I will leave for the permafrost station, where it will be possible to perform experiments. This work is, in the main, connected with part of the work I did in Moscow. I hope to do something useful for the economic development of the regions where there is permafrost, including, among others, the far east. Permafrost is connected to very many characteristics and peculiar phenomena of the local nature”. In the spring of 1934 his group produced two major manuscripts which were sent to the Academy of Sciences for its permafrost congress [2, vol. 4, p. 109]. The results of this pioneering work on permafrost and the nature of ice were later included in the book *Permafrost and Construction on it*, published in 1940 [2, vol. 1, p. 31], which was used as a guide to construction in areas with permafrost. Florensky’s name is absent from the list of authors.

Upon receiving in the spring of 1934 the news that his extensive library collection had been taken away by the OGPU [KGB] he wrote as follows to the head of construction of the camp system [2, vol. 1, p. 33]. “My whole life has been devoted to scientific and philosophical work, so much so that I have never allowed myself rest, recreation, or pleasure. To this service to mankind I gave not only all my time and all my strength, but also most of my modest earnings, which I spent on the purchase of books, photographs, correspondence, etc. . . . But now the labor of my life has been lost . . . [it is] a heavy blow to me . . . the annihilation of the results of my life’s work is a far more cruel punishment to me than physical death”. In the fall of 1934 he was transferred to the camp at Solovetsky, formerly a monastery on an island in the White Sea, where he developed and constructed technology for the extraction of agar and iodine from seaweed. In 1937 Florensky saw the destruction of the huge makeshift arrangement which he had organized for the production of agar, just as had happened earlier with the production of iodine. He also learned that the credit for his research into seaweed and permafrost had gone to others. “So everything is taken away from me at which I worked, in which I attained results and in preparation for which I expended great effort” [2, vol. 4, p. 702].

He then undertook to list the various fields in which he had worked, as a legacy for his family. In a letter to his son Kirill dated 21 February 1937 Florensky summed up his life’s work. “What have I been doing all my life? I have been studying the world as a single whole, as a coherent picture and reality, but at every moment, or rather at every stage of my life I looked at it from a definite angle. I examined its correlations in a cross-section of the world made in a definite direction and on a definite plane, and I tried to understand the structure of the world according to the aspect of current interest to me. The planes of the cross-section changed, but one did not cancel out another; on the contrary, one enriched the other. Hence—the continuous dialectical quality of thinking (the change of planes of observation), with an unchanging perception of the world as an integral whole" [2, vol. 1, p. 35], [2, vol. 4, p. 672].

In one of his last letters, dated 4 June 1937, he wrote [2, vol. 4, p. 705] "Everything has abandoned me (everything and everybody) . . . above all I think about you, but with worry. Life is dead and at the present time we, more than ever, feel ourselves isolated from the mainland. It is June and there is no sign of summer, it is more like November". During a 'cleansing' of the camp in November 1937 he was sentenced for the second time on 25 November by the special Troika of the UNKVD of the Leningrad district and condemned to death. He was shot on 8 December 1937 [2, vol. 1, p. 35].

His name was placed on the list of 'enemies of the people'. His works were excluded from libraries. Even to mention his name in the press was strictly forbidden. Only at the end of the 1960s was a procedure begun, very gradually at first, to allow his name to return to civil and religious culture. His works began to be published, in the *Journal of the Moscow Patriarchate*, in the series *Theological Works*, and in *Works on Symbolic Systems* published in Tartu Estonia. Eventually the first studies of his ideas and conferences devoted to his creative legacy were organized. Presently the creative legacy of P.A. Florensky is among the most intensively studied subjects in both Russia and abroad.

9. Conclusion

Florensky has frequently been called the Russian Leonardo Da Vinci. This comparison is appropriate if we consider the breath of his creative scope. In another sense, however, this evaluation is profoundly mistaken. Florensky was not consumed by the thirst for knowledge so characteristic of the Renaissance. His activity had a special goal—to construct a unified world view which included both science and religion. Contemporary science does not see itself bound by any limits in its expansion, rejecting in its methodology the ethical norms of humanity. Pretexts for this include the claim that science is indifferent to ethics, that only the utilization of science can be ethical or unethical. In the opinion of Florensky, such a science, the paradigmatic figure of which was Leonardo, was destined for the same dead end as the magical ideas of primitive societies, only "more severe, more merciless because this science was itself merciless to man" [23, p. 350]. Florensky saw an escape from this in the construction of a unitary religious world view in which science is included as an organic part (these ideas of Florensky are described in [47]).

In his study of cultures, Florensky noted two which periodically alternated with each other—medieval culture and renaissance culture. The first type he characterized as "organic, objective, concrete, and having the ability to concentrate itself, while the second is characterized by disunity, subjectivity, abstractness and superficially" [2, vol. 1, p. 39]. He was convinced that "the Renaissance culture of Europe . . . had come to an end at the beginning of the XXth century and from the first years of the new century one could see in all strands of culture the first seeds of the culture of the other type” [2, vol. 1, p. 39], that is of the high culture of the medieval type. All his creative activity was applied to the task of its reconstruction. This was the life’s work of Father Pavel Florensky—theologian, philosopher, scholar, and martyr for the faith.

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CHAPTER 32

Husserl and Impossible Numbers: A Sceptical Experience

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Contents

MATHEMATICS AND THE DIVINE: A HISTORICAL STUDY
Edited by T. Koetsier and L. Bergmans

Introduction

This is an unusual story. Mathematicians believe in the certainty of mathematics. Husserl did not. Philosophers consider mathematics as the paradigm of science. Not Husserl. For him mathematics was the central motif in an experience of deep scepticism, which is documented by a few published texts and several manuscripts and letters dating from the years 1887–1900, between the completion of his Habilitation on the concept of number and the publication of the *Logical Investigations.* Much later, in 1928, Husserl told Shestov that when he wrote the *Logical Investigations,* he was completely desperate about the possibility of any real knowledge.[1] This sceptical stage was a crucial event in the birth and subsequent orientation of phenomenology, and gives a new and striking meaning to the philosopher's thesis about the "crisis of the sciences" and the role of philosophy as a science of science.

1. Beyond Hume

Traditional scepticism à la Hume destroys our knowledge of regularities in matters of fact, the ability of science to predict natural events. Very early on Husserl was a convinced Humean in this respect, as we can infer from the first thesis appended to his Habilitation of 1887:

> "Every natural law is a hypothesis".[2]

Husserl's scepticism went further. His doubt extended to relations of ideas, it concerned all a priori sciences, especially mathematics and in particular the most robust core of mathematics, arithmetic.

Reasoning itself was to be considered suspicious, insofar as it involved not ideas, but signs. The problem lies with the signs. We do not think thoughts or concepts, we play with signs, we operate on words and symbols. Blind thought (the *cogitatio caeca* of Leibniz) is our principal and usual mode of thought. Intellectual life is based on signs, on what Husserl calls "representation":

> "I have just called representation an extremely remarkable function. [...] In the flux of conceptual thought, in most cases the optical and acoustical flow of words solely, or almost solely, represent. [...]
> [...] great and unresolved riddles are here; we are touching on the most obscure parts of epistemology. What I have in mind is not the psychological clarification of this oversight and of the entire state of affairs, although this would be basic for all which is to follow. Rather it is the possibility of knowledge at all.
> Scientific knowledge—the kind of knowledge which first come to one's mind here—rests entirely on the possibility of abandoning oneself to the greatest extent to a merely symbolic or otherwise extremely inadequate thinking or in being permitted (with certain precautions) to prefer it deliberately to a more adequate thinking. But then how is understanding possible, how does one arrive in such a fashion even at only empirically adequate results?

[1] K. Schuhmann, *Husserl Chronik,* Den Haag, 1977, pp. 330–331.

[2] "Jedes Naturgesetz ist eine Hypothese", Husserliana **XII**, p. 339.

> Mathematics is considered to be a model of exact science. But the arguments of its representatives, which have dragged on through the centuries and have not even yet been resolved about the meaning of its elementary concepts and the foundation of the soundness of its method, are in a peculiar conflict with the supposedly thorough evidence of its procedures. In various periods of his life the great mathematician Cauchy defended totally different theories of imaginary numbers. Which of them actually guided his evident procedures? [. . .] Theories change, but the procedure remains the same. The evidence produced by such a procedure—there is no doubt about this at all—is mere illusion.
>
> But how then can results be arrived at which agree to such an amazing degree with subsequent "tests" or with experience and thus condition a corresponding measure of practical confidence? Since analogous difficulties hold for all the sciences (and also for ordinary thought), should we return to the viewpoint of Humean scepticism and extend it further than its great originator did, to mathematics and to all "a priori" sciences as well? We turn in vain to the old and to the new logic for resolution of such doubts; logic here leaves us entirely in the lurch. Logic, the "theory of science", must concede if it is to be honest that all science is a mystery to it".[3]

This text was published at the end of a study on intuition and representation in 1894, but Husserl left many other texts unpublished, he remained almost silent until the publication of the *Logical Investigations* in 1900, and the arguments which justify his position must be sought in manuscripts and letters.

2. Prelogical use of signs

In principle signs should stand for something. But there are cases where signs do not represent and cannot represent. In his unpublished "Semiotic" of 1890, Husserl draws up a list and a classification of the uses and inadequacies of signs, of the possible distance between signs and objects.[4] It may happen that signs fail to give access to the thing, because things are too remote, or very difficult to reach. It may even be the case that any reference is excluded. The sign then stands for nothing, it cannot represent a thing, since the thing is an impossible one. This extreme case of completely void symbols may seem rare at first glance, but Husserl, as we shall see, met important examples in "higher arithmetic" (or "universal arithmetic", the name given to algebra by Newton and others).

In general the use of signs can be a rather go-as-you-please practice, governed by the sole interests of life and of comfort. It would be much too optimistic to believe that signs obey rational rules, that they follow a respectable grammar. The most common case is what Husserl calls a "prelogical use" of signs, or a "prelogical device". Even when signs are devised for the purpose of knowledge, they are most probably the fruit of chance and their correct use is only a result of innumerable trials. The philosopher (or logician) may then ask for a logical justification of each symbolic procedure:

> "Every artificial operation with signs serves in a certain way goals of knowledge; but not every one leads to knowledge, in the true and authentic sense of logical insights. Only when the device itself is a logical one, when we have the logical insight that, as it is and because it is so, it must lead to truth, then the result is not only a de facto truth, but a knowledge of truth. Only then are we

[3] Husserl, *Psychologische Studien zur Elementarlogik* II (1894), transl. R. Hudson and P. McCormick, in: Husserl, *Shorter Works*, ed. P. McCormick and F. Elliston, Univ. of Notre Dame Press, Notre Dame, 1981, pp. 140–141.

[4] "Zur Logik der Zeichen (Semiotik)", Husserliana **XII**, pp. 340–373.

> fully sure to be protected from error, and do we not judge from blind impulse (*Drang*), not out of a more or less vivid conviction, but from a clear insight. In this sense, we distinguish (1) prelogical sign operations, which aim at truth and perhaps attain it, although the application of these methods (as already its invention) is not based on logical comprehension; (2) logical sign operations, whose use is based on rigorous grounds of knowledge and for that reason produce not merely truth, but guaranteed truth (*gesicherte Wahrheit*). [...]
> It is useful to point out here that a systematic use of signs for the purpose of gaining knowledge, is not necessarily a logical one. A systematic investigation and use of signs can already take place at the prelogical level. One can easily notice that signs improve our knowledge, without being at all clear about the grounds of this improvement. This will in particular be possible when the propositions obtained in a symbolic way (symbolic judgments) by the transition from sign to thought, lead to genuine judgments that legitimate themselves because they can, in each case, be verified. That is the situation in mathematics".[5]

Truth is not enough. Manipulating signs blindly may have led us to a truth, the final result being a statement endowed with meaning, which we can see to be true. We know from somewhere else (not from the interplay of signs, but after the event and independently) that it is true, we can check its adequacy. But how do we know that the procedure will necessarily produce correct results? We have used it thousands of times, and generations of men before us have, but where is the guarantee that it must be so? Each time we reach a de facto truth, we hit on truth by chance, but we have not arrived at "guaranteed truth", we do not possess knowledge in the authentic sense.

3. Husserl's philosophical quest

Are the procedures of mathematics logical or only prelogical devices? This question marks the point that Husserl arrived at in 1890, in a long quest for certainty.

He had been trained as a professional mathematician by the best teachers, following the courses of Weierstrass and becoming his assistent, after a doctoral dissertation devoted to the foundations of the calculus of variations.[6] There was no place in his dissertation for philosophical considerations, but the subject is interesting for our purpose. Husserl tried to give necessary and (if possible) sufficient conditions for the existence of a function which minimizes the value of an integral between two fixed points. The problem was to avert the danger of non-existence. Mathematicians run the risk of operating with objects which evanesce under close scrutiny. Thus, as a mathematician, Husserl was perfectly aware of the fact that a sequence of signs, seemingly meaningful and rich, may reveal itself as a mere phantom. A similar and famous accident had occurred some years before, with an important and fruitful theorem of Riemann.

While preparing his dissertation on the calculus of variations, Husserl became actively interested in philosophy, and under the influence of Brentano, decided to become a philosopher:

[5]Husserl, "Zur Logik der Zeichen (Semiotik)" (1890) in *Philosophie der Arithmetik mit ergänzenden Texten*, Husserliana **XII**, Den Haag, 1970, pp. 368–369 (personal transl.).

[6]*Beiträge zur Theorie der Variationsrechnung*, 1882.

> "Brentano's lectures gave me for the first time the conviction that encouraged me to choose philosophy as my life's work, the conviction that philosophy too was a serious discipline which also could be and must be dealt with in the spirit of the strictest science".[7]

Philosophy is a serious and noble task, and Husserl embarked on a vast project, the philosophical analysis of the mathematical sciences. The themes current at that time, such as the status of the new geometries and the euclidean character of space, seemed too ambitious, and he felt the need to proceed from the simplest questions to more elaborate ones. He worked on a philosophical clarification of arithmetic, starting with the definition of number. He employed the array of tools at his disposal: a rather crude theory of abstraction, a classification of relations in several groups and subgroups, and an already refined analysis of consciousness.

The difference between philosophical foundation and descriptive psychology was not clear to him. But above all, he proceeded only one step in the large enterprise, and the result could not be used for the next task, a clarification of arithmetic. He had given a kind of genetic account of plurality and number: number is the fruit of a reflection upon acts of "colligation" (or collective unification, which is the act of consciousness corresponding to the logical particle "and"). But this accounted only for very elementary numbers, 2, 3, 4. How to explain and to justify the handling of large numbers? Even 0 and 1 were excluded from the description, they had to be treated as artificial extensions of the directly given numbers.

Arithmetic deals with arbitrarily large numbers, expressing them with the help of various symbolic tools, operating on them in computations governed by mechanical rules. Mathematicians—and engineers or merchants—have invented new kinds of numbers, negative, fractional, decimal. These realms of numbers cannot be founded on the elementary concept of number.[8]

4. Impossible concepts

The worst case was the use of imaginary numbers. They represented the greatest stumbling-block in Husserl's path. He explained his failure to Stumpf:

> "Since I originally conceived signs only as related to signified concepts, $\sqrt{2}$, $\sqrt{-1}$ had to be held to be representatives of "impossible" concepts. Therefore, I first tried to clarify how thought operations with contradictory concepts could lead to correct propositions".[9]

A key element lies in the form of the result:

> "I finally noticed that through the computation itself and its rules, as they are defined for those fictitious numbers, the impossible disappears (*wegfällt*) and there remains a correct equation".[10]

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[7] Husserl, "Recollections of Franz Brentano" (from Kraus, *Franz Brentano*, Munich, 1919), transl. by R. Hudson and P. McCormick, in: Husserl, *Shorter Works*, ed. P. McCormick and F. Elliston, Univ. of Notre Dame Press, Notre Dame, 1981, p. 343.

[8] See a letter to Stumpf (1890), *Studien zur Arithmetik und Geometrie*, Husserliana **XXI**, pp. 244–251.

[9] Ibid., p. 247.

[10] Ibid.

Once the result is obtained, verification is possible, directly and without recourse to the signs representing the impossible:

> "The computation process unfolds itself once more with the same signs, but related to valid concepts, and the result is correct again".[11]

Logical or metaphysical justifications, any discussion of the nature of the underlying reality, are totally irrelevant:

> "Therefore "possibility" or "impossibility" are irrelevant here. Even if I figured that the contradictory has existence, even if I sustained the most absurd theories about the content of such numerical concepts, as several great mathematicians have done, the computation will remain correct if it conforms to the rules. Therefore the signs themselves must do the job, with their rules (*Also müssen es die Zeichen machen, und ihre Regeln*)".[12]

It is a defeat for the philosophers: whatever you think or imagine about the content of those symbols, the computation works. There is an impassable gulf between the computational practice and the supposed logical or philosophical justifications.

The central and fundamental example, which Husserl mentions several times in the manuscripts and lectures of that period,[13] is the "casus irreductibilis" of the equation of the third degree.[14] An equation is given, and we have a general formula (found by Cardano) to find its three roots. In cases where the three roots are real, the formula which furnishes them involves imaginary numbers (square roots of minus one), which miraculously disappear during the computation. The meaningless symbols are only auxiliary elements and their meaninglessness has no importance.

Bombelli, who first developed this treatment of third degree equation, studied for instance the equation $x^3 = 15x + 4$ which has the three real roots 4, $-2 + \sqrt{3}$, $-2 - \sqrt{3}$. The formula which mechanically produces these three roots must contain imaginary numbers, which mutually cancel when operating according to the rules. Mathematicians tried in vain to find other expressions free from imaginaries. The detour through imaginaries seemed unavoidable. This was a scandal which stimulated the thinking of many great minds, d'Alembert and others.[15]

Husserl considered this case not as an isolated oddity, but as an example of a more general pattern in mathematical procedures. Infinitesimals or negative numbers have the same status: they may be perfectly void of any content, they serve only as intermediates in the chain of operations that leads to a meaningful and correct result.[16] The result is meaningful, that is it contains no infinitesimals or empty symbols, and it is true, as can be checked directly (this presupposes that a result which contains infinitesimals or imaginaries is not a result properly speaking, and needs further elaboration).

Husserl insists on the importance of the form of the result: the final stage of the procedure contains no meaningless symbols, the impossible (more precisely: the sign representing the

[11] Ibid.

[12] Ibid.

[13] See, for instance, the *Vorlesungen* for the winter semester 1889–1890, *Studien zur Arithmetik und Geometrie,* Husserliana **XXI**, p. 236 ff.

[14] "Irreducible" not in the sense of Galois theory and XX century algebra.

[15] See the entry "cas irréductible" in the *Encyclopédie*.

[16] *Studien zur Arithmetik und Geometrie*, pp. 234–237.

impossible) has disappeared after it has played its computational role. The rules are devised precisely in order to allow a vanishing or mutual cancellation of the empty signs.

Another aspect is equally important: it is possible, once the result has been obtained, to check it via another route, one that involves no meaningless symbols. Once the roots of the equation have been obtained, for instance, it is very simple to verify that they satisfy the equation. We then have exactly what Husserl calls a de facto truth.

5. Natural selection

The lesson is devastating: higher arithmetic has no logical dignity, it is just an "intellectual machine",[17] forged by generations of calculators in a long sequence of trials. In a supreme blasphemy against the queen of the sciences, Husserl even treats arithmetic as a by-product of natural selection:

> "It can be claimed: universal arithmetic, with its negative, irrational and imaginary ("impossible") numbers, was invented and applied for centuries, before it was understood. The most contradictory and incredible theories were held about the meaning of these numbers, but this did not impede their use. Each time a proposition was derived with their help, one could become convinced of its correctness through an easy verification, and after innumerable such experiences a natural confidence developed in the unrestricted applicability of these procedures, which were then increasingly extended and refined—all without the slightest insight into the logic of the affair, which has made no real progress in spite of many efforts since the time of men like Leibniz, D'Alembert and Carnot untill now. [...]
> Indeed, arithmetic in its fully developed form is independent of a logical understanding of its artificial procedures. Arithmetic has not emerged fully-fledged from the head of a single individual; it is the product of a development over many centuries. It originated through a kind of natural selection (*natürliche Auslese*). In the struggle for existence, truth overcame errors that had become untenable, and arithmetical methods were developed accordingly, as gradually modifications were introduced that ruled out the remaining errors. The correctness of the result could serve as a touchstone of the method, since it was possible to convince oneself of the correctness of the result without using the method (by way of the above mentioned verification)".[18]

The motives for doubt are now explicit: the success of the methods owes nothing to logical clarity, it derives from long practice. Even more: the search for logical justifications is useless, it is a priori "blocked" so to speak, since the symbols do not represent anything and the various pseudo-justifications may well proliferate freely, they have nothing to do with the blind mechanism which furnishes the results.

6. Structural mathematics and categorial connections

To this dark picture of science, objections can be raised. Why doesn't Husserl pay attention to the efforts to gain insight among mathematicians themselves? Mathematics is not only the production of "results", it is also a vast endeavour to understand itself. In the case of

[17] "Zur Logik der Zeichen (Semiotik)", in *Philosophie der Arithmetik und ergänzende Texte*, Husserliana **XII**, p. 368.

[18] Husserl, "Zur Logik der Zeichen (Semiotik)" (1890) in *Philosophie der Arithmetik mit ergänzenden Texten*, Husserliana **XII**, Den Haag, 1970, pp. 369–371 (personal transl.).

equations and roots for instance, the discoveries of Galois, the elaborate constructions of algebraic number theory, are at least remarkable steps towards answering the question of "why it must be so".

The texts I have quoted or paraphrased are of course not the end of the long Husserlian story. They offer a striking contrast with the image of mathematics presented at the end of the *Prolegomena to Pure Logic* of 1900. Whereas in 1890–1894 arithmetic and the rest of mathematics are in the end nothing but a collection of recipes, a bunch of tricks sanctified by long use, inherited from traditional know-how and vaguely safeguarded by an array of criteria and precautions, in 1900 Husserl sings the eulogy of the great formal constructions of audacious nineteenth-century theory builders like Hamilton, Grassmann, Lie and Cantor.[19]

In short, Husserl discovered "structures"—he called them "Mannigfaltigkeiten". The texts on that theme are better known and well commented upon, for instance in Suzanne Bachelard's book on Husserl's logic. What happened in the intermediate period? How did Husserl proceed from the "intellectual machine" to the "Mannigfaltigkeiten"? We do not as yet possess enough documents, *Vorlesungen*, etc. to be able to describe and understand precisely the process of his conversion. We know that his reading of Bolzano played a decisive role, and that one leading thread through his new reflections concerns the idea of the unity of a theory: under which conditions is a science one?[20]

It also seems that the study of space in 1894–1896 opened new issues. Husserl began to distinguish sharply between the study of lived space, through an analysis of the constitution of our spatial intuition, and the formal study of possible systems of geometries. Observing how euclidean geometry underwent formal re-elaborations, he arrived at the idea that a mathematical theory possesses a strong unity, governed by the connecting laws that bind the elements together. Such connections are not different in their essence from logical or linguistic connections. Husserl calls them "categorial".

We can very briefly summarize his later position on mathematics by quoting a passage from *First Philosophy*:

> Geometry needs spatial intuition, its concepts must come back to a factual sphere, to spatiality. In arithmetic on the contrary, there are concepts which express the modalities of something in general, like set, plurality, and the evidence which is required here is in principle of the same kind as the evidence gained with the logico-apophantic concepts of the consequence between judgments. More precisely, the whole of arithmetic and indeed the whole of analytical mathematics are just analytics, but oriented otherwise, just logic of consequence, but oriented otherwise; instead of predicative position, of judgment, it is related to positions of "thought-objects".[21]

The same ideal connections are at the basis of logical articulations and mathematical structures. A number and a judgment seem to be entities of a very different nature, but the act of counting and the act of judging are not different in principle. Only the orientations differ: in the first case the posited object, number, seems to be endowed with a kind of autonomy, whereas in the study of judgments the act of thought comes to the fore. The

[19] *Prolegomena zur reinen Logik, Logische Untersuchungen* I, §70.

[20] A technical solution of the problem of the imaginaries is even possible within the study of Mannigfaltigkeiten, when these are "closed" in a certain sense (*definit*). See the 1901 conference "on the imaginary in mathematics" (Husserliana **XII**).

[21] *Erste Philosophie*, Husserliana **VII**, p. 29 (a course from 1923).

progressive unification of classical logic and mathematics, guided by the Leibnizian ideal of a mathesis universalis, is the result of the fundamental kinship between several sorts of ideal connections.[22]

7. De facto truth and the need of a science of science

The sceptical and silent years of Husserl in the period 1890–1900, his doubts about the certainty of mathematics, are not a passing episode in a long quest. They give a richer meaning, a weight, to central theses of Husserl that may appear as philosophical rhetoric to a superficial reader.

The constant attitude of defiance, the constant warning by Husserl against "technicization" of science, from the *Prolegomena* (§4 etc.) in 1900 to the *Krisis* (esp. §9g) in 1935–1937, must be viewed in this light. Science becomes a pure technique, uncertain and blind, if

(1) it rests purely on success, on the attainment of de facto truth,

(2) it relies on the mechanical use of symbols.

The two conditions are linked: blind use of symbols is justified only by success, by verification *après coup*, not by logical clarity. And vice versa, the exclusive attention to final success, to de facto truth, allows an unlimited tolerance in the intellectual tools, until errors arise. Certainty is the goal, logical clarity is the instrument. Uncertainty derives from blindness.

The scandal of imaginary numbers shows that a pragmatic view of truth is possible even in mathematics. Husserl was not satisfied with it. He was silent for a long time, but not resigned. Philosophy, for him, starts with the attempt to go beyond de facto truth, beyond a pragmatic conception of knowledge.

I have gained knowledge, but do I really possess it? Will it not fly away on the next occasion? The tools I have used have proved efficient in all the cases I have had to deal with. But how can I be certain that these tools will again lead to the truth in other situations? In the *Meno* (98 a) Plato compares knowledge by chance to the statues of Daedalus. They are so alive and free that they may walk away in the next instant if we do not bind their legs with a chain. My knowledge is in danger of disappearing if I do not attach it firmly with the chain of a cause, explaining why (dioti) it must be so. Then I become *ametapeiston*, not susceptible to being convinced by the other side, firm in belief. The adjective describes science in Aristotle's treatise on demonstration.[23]

Husserl places himself in this tradition of Greek dialectic, which is for him the first step in the development of philosophy.[24] Truth is not enough, science requires a second level, it must give an account (*Rechenschaft geben*) of its own method. The idea of a science of science is not a new slogan, it is for Husserl philosophy itself, from its beginning among mathematical disciplines and various scientific practices.

[22] See *Formale und Transzendentale Logik*, esp. §§24–26 and 34–37.

[23] Aristotle, *Post. Anal.*, I, 2, 72 b 4; *ametapeiston* occurs also in *Top.* V, 2 to characterize science.

[24] See *Erste Philosophie*, chapters I–III and *Formale und Tr. Logik*, Introduction.

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CHAPTER 33

Symbol and Space According to René Guénon

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Contents

MATHEMATICS AND THE DIVINE: A HISTORICAL STUDY
Edited by T. Koetsier and L. Bergmans

1. Life and work of René Guénon (Blois 1886 – Cairo 1951)

René Guénon's timeless work is endowed with a strangely modern quality. His icy style, his unshakable aloofness, ironic detachment and muted anger arguably make him singularly adept at formulating a lucid synthesis of the past century. If we still remember Guénon today, it is hardly because of his engagement with the problems of the century or because of his contribution to the development of the Humanities. If he cannot be ignored, it is first of all because of his unique capacity for dialogue with what passes humanity by, what he calls "the non-human", as well as the unrivalled profundity of his judgement.

Yet it would be difficult to imagine more modest beginnings than the birth of René Guénon at Blois, on November 15, 1886, into a family of winegrowers of the Loire region, where his father is *architect expert* for an insurance company. Nothing more banal, again, than the curriculum of this exemplary pupil who is influenced by his philosophy teacher and, following the latter's example, will strive with utmost rigour to distinguish appearance from reality.

But that would be to ignore Blois and its region, that peerless land of Celtic legends and royal symbols, which throughout the centuries continued to foster exceptional individuals invariably imbued with deep spirituality, whether a Guillaume Postel meditating on Chambord or a Madame Guyon exiled at Blois.

Yet the course of Guénon's life resembles that of none of his known predecessors. To be sure, he marries in 1912 and pursues—in Blois, in the neighbourhood of Paris and briefly in Algeria—a career as a teacher of philosophy that was never to be crowned by the *agrégation*, yet the essential part of his life remains hidden and belongs, so to speak, to the realm of the invisible. Whether one views him as the culmination point of the entire French esoteric tradition or as the dedicated spokesman of oriental masters, and regardless of whether one considers him to have been expressly chosen to introduce Europe to the viewpoint of Islamic esoterism, which he embraces as of 1912, the fact remains that René Guénon in his early twenties already demonstrates a unique mastery of both eastern and western theological and initiative traditions. But what strikes one most in this polyglot and unrivalled theoretician is not so much his scholarship as his unshakable sense of mission, the true nature of which remains to be explored.

It must of course be remembered that mathematics was not alien to this all-round education, as Guénon had initially intended to enroll at the *École polytechnique*. Having been forced to renounce this course for reasons of health, he would nevertheless remain in contact with mathematics all his life, and his entire work can, as we shall see, be considered as a complete geometrisation of Tradition.

The general orientation which his work is to take becomes clear as early as 1924, through a book that will come to be seen as a milestone: *Orient et Occident* (*East and West*). Guénon here shows himself at once a ruthless critic of Western civilisation in the aftermath of the first World War and a man capable of transcending the aesthetical temptations that would plague a Malraux (who owes him a great debt: *La Tentation de l'Occident* dates from 1926), a man who could forge a new and reinvigorating link between the two tendencies that vie for the destiny of the planet: that of the West, enthralled by the concept of quantity ever since Descartes, and that of the East, which through appropriate traditions was able to

Fig. 1. René Guénon in Paris, ca. 1925 [10].

preserve a regular access to the invisible centre of the universe—not so much the God of monotheism as the Universal Possibility illustrated by Tao, Vedanta and Sufism alike.

These were the foundations upon which Guénon began to build his considerable production, consisting of numerous books and countless essays in various esoteric and catholic periodicals, invariably resplendent with an imperturbable lucidity and a tireless indictment—even in his *prima facie* most generous overtures—of the spiritualistic, syncretistic and humanitarian values of modernity.

Guénon was beginning to gain a reputation, in spite of the distance which he deliberately created between the various manifestations of humanism and the continually emphasised necessity of steering a fundamental course aimed at what is *higher than human*. Then, in 1928, he suffered the twofold loss of his wife, leaving him childless, and his aunt, Mme Duru, who had been sharing the family's life both in Paris and in Blois.

Shaken by this emotional crisis, convinced that in Europe the worst was to come about as a result of the confluence—which he had denounced from the very start—of nationalism, anti-Semitism and pan-Germanism, he left France on a study trip to Egypt in March 1930, settling for good in Cairo where, having converted to the Muslim religion, he eventually married and founded a family.

Guénon's departure for Cairo, by its suddenness and its far-flung implications, made the author's life the stuff of legends. Yet the choice of Cairo never meant that he had given in to the temptations of orientalism. If Guénon can, in one sense, be considered the spiritual heir of the great French travellers in the East, in reality he imparted a radically new meaning to the Egyptian adventure. The latter was no longer the fruit of some egyptomania born of a fascination with pyramids and occult sciences. Such motives would have been considered by Guénon as a sign of the most obscure worrying affects. Rather, in a fundamentally geopolitical and international perspective, he saw Cairo as the meeting point between East and West, as well as the means to found, at a time when the world was about to collapse, a kind of intellectual realm that would preserve the best parts of human knowledge till the beginning of the next cycle of time.

Thus Guénon was to establish, through his books (on Dante, on the multiple states of being, on symbolism, on initiation, on the modern world) and through his letters to correspondents all over the world, a unique *magisterium* aimed at the broad acceptance of the idea that, over and above the diversity of particular religions, there exists a primal Tradition which would allow the most enlightened part of humanity to gain an understanding of the course of history and to overcome the darkest phases of the cycle into which the West was about to drag along the entire world.

It would be wrong to portray as incorporeal and a-historic a doctrine which only takes interest in Eternity because it holds the key to Time. Guénon similarly refers to what is "non-human" only because the infra-human dominates the world, whether it is Industry taking hold of the body or Suggestion claiming possession of the mind. Thus Guénon attempts to keep alive a spiritual vigil, rooted in the very reality of civilisations and whose central focus invariably corresponds to the highest degrees of being. The force of his thought ultimately derives from the fact that it is based on none of the values which could be seen, in his own times, to be gradually foundering: Nation, Man, History, Art, the Churches, Ethics, the Unconscious—even Being itself... Hence it addresses itself to the peoples and individuals who are condemned to outlive them.

Guénon's work, no doubt, has the icy tone of the coming age. Its coldness rivals that of modern terror. It lives up to its times. It is in this certainty that the author dies in Cairo, on January 7, 1951, invoking the name of Allah.

2. The symbolism of space

In spite of Guénon's rejection of systematisation of any kind, his doctrine of the symbolic world is so rigorous and so closely articulated that it is difficult to give a simple overview, even if he himself relished the idea of an "aperçu", a word that recurs in the title of several of his books. It is true that the *aperçu* provides a vista on what Guénon calls "the multiple states of being". Yet one should not allow oneself to be fooled by this apparent modesty of the argument. As soon as it broaches the fundamental dimension of space and mathematical entities, Guénon's symbolic knowledge gives rise to genuine axiomatisation, a fact which (the message bears repeating) radically distinguishes the author's enterprise from all sorts of vague Pythagoreanism and precarious numerology. In the same way as Guénon tried to reduce esoterism to a unified order, so his meditation on space hearkens back to fundamental principles corresponding to the requirements of a metaphysic which is first of all a process of thinking *according to principles.* That such an enterprise should have been applied to mathematics, in a century wholly absorbed by the quarrel over the latter's "foundations", is not the least paradoxical aspect of a philosophical approach which was not conventional enough to remain indifferent to the mathematics of its time. That, at least, is what—within the limits of my competence—I intend to demonstrate.

I shall take as my starting point two remarks of René Guénon. The first is a purely metaphysical one which, nevertheless, immediately implies a link with geometrical ideas. Freely paraphrasing Mircéa Eliade in a review published in the *Revue de l'Histoire des Religions* in 1946, René Guénon writes that "only at the "Centre" it is possible to "break through the levels", to pass from one "cosmic region", i.e. from one state of being, to another" [11, XXV, p. 188]. A resolutely indefinite boundless ontology like the one claimed by the author, which allows all states of being, including the possibilities of manifestation and non-manifestation, is no merely virtual ontology. It amounts to a hierarchical ordering of states, admittedly undefined as to their several dimensions but organised around a *focal principle*, the "center", that has both a metaphysical and a spatial meaning.

In *Le symbolisme de la croix* (Symbolism of the cross), which describes this imbrication of states and spaces, we read this other fundamental proposition:

> Space here symbolises the set of all possibilities, either of a particular being or of universal Existence [5, IV, p. 47].[1]

It is not, however, space in general which becomes a symbol with Guénon, but its construction starting from a cruciform three-dimensional system, viewed as a coordinate system whose function it is to centre that space [5, IV, p. 47]. This latter statement of Guénon is extremely important. This three-dimensional cross is not first and foremost a religious sign. It is a system of coordinates to which the entire space, *and not merely Euclidean space*, can be related:

> It is obvious that the so-called "Euclidean" expanse, which is the object of ordinary geometry, is but a special case of a three-dimensional expanse, as it is not the only conceivable modality [5, XXX, p. 223].

Guénon does not fail to emphasise the strong "logical coherence" of the various non-Euclidean geometries. Does this mean that these geometries exclude one another in the construction of a rigorous spatial symbolism based on three dimensions?

> The various Euclidean and non-Euclidean geometries obviously cannot apply to one and the same space; but this does not prevent the diverse corresponding spatial modalities from coexisting in the totality of spatial possibilities, where each of them must be realised in its own way, according to what we shall have to say on the effective identity of the possible and the actual [6, II, p. 20, note 3].

The cross, then, here signifies the introduction of a metric in a space which—the unicity of the principle notwithstanding—comprises an infinite number of dimensions:

> The principle of universal manifestation, while being *one* and even unity itself, necessarily contains multiplicity; and the latter, in all its indefinite developments, and actualizing itself indefinitely in an indefinite number of directions, proceeds entirely from that primordial unity [6, V, p. 39].

[1] The references are to the French editions of Guénon's work. For some of the translations I have used: http://www.cis-ca.org/voices/g/guenon-mn.htm.

If René Guénon has therefore rightfully been considered the most ruthless of critics of the modern ways of life [and the modern world] and of the historicism of the various forms of knowledge, on the other hand he is entirely in tune with the contemporary geometry of his time, as long as the innovations which characterize it, in the realm of topology in particular, do not serve to confirm conventionalism or a mere axiomatic structuralism lacking all ontological purport. While it is true that he refuses the facile solutions offered by the "fourth dimension" much beloved by theosophists, he nevertheless posits that only our temporal state anchors us within classical three-dimensional space [5, p. 135]. As soon as the relationship with time changes, the fecundity of space increases. This is exactly what could happen in a world bearing the imprint of Einsteinian relativity:

> All that can be said in this regard is that, with the expansive tendency of space no longer opposed and restricted by the compressive action due to the compressive tendency of time, space must hence naturally experience, in one way or another, a dilatation which in a sense raises its indefiniteness to a higher power [7, p. 159].

While this power can still be symbolically represented, it is nevertheless clear that it transcends the classical physical and quantitative framework.

In any case the Guénonean symbolism, according to this cruciform pattern, unfolds naturally through a succession of three-dimensional spaces resulting from a process of subdivision, articulation and analogy (through a system of correspondences which "obviates the need to go outside the three-dimensional expanse", [5, XIII, p. 114], and which could, as pointed out by Enrico Barazetti, be assimilated to a true fractal [16, p. 78, note 45]). This choice is actually more pragmatic than ontological "the establishment of this correspondence between the two representations, *which allows an easy transition from one to the other,* obviates the need to go outside the three-dimensional expanse." [5, XIII, p. 114; my emphasis]. But this is easily understood to the extent that the use of spatial representations has no other meaning than a symbolic one, and should never constrain thought by tying it to a specific ontology (e.g., a neo-Pythagorean, a Cartesian or a Malebranchean one).

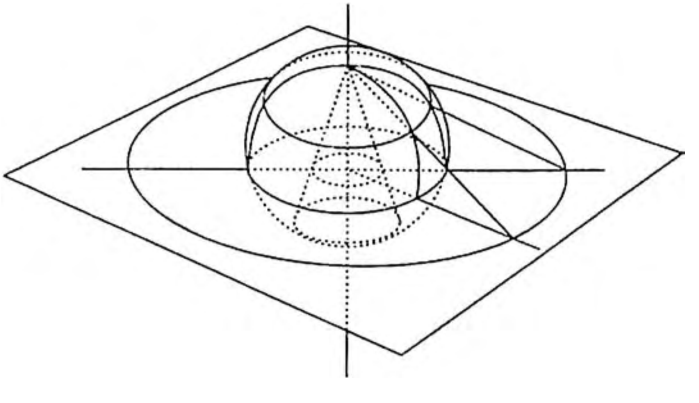

Fig. 2. The sphere, representing all possibilities, is projected on planes that symbolize receptive substrates of these possibilities [16, p. 200]. The projections on different parallel planes represent the indefinite number of realizations of the same degree of existence.

It is indeed necessary to carefully distinguish between the calculability of space and its symbolic function, which chiefly proceeds from the possibility of moving from a *finite* starting point to an indefinite dimension. As to ontology itself, it can be summarized in a single proposition: “Being is Being” [5, XVII, p. 42]; any other consideration would cause reflection to fall prey to dualism, which is certainly still within the domain of symbolism (by polarisation), but no longer within that of pure thought. Therefore no choice of space should be allowable that impinges directly on ontology, because no form of Quantity can express the truth of a being which is essentially limitless. All geometric representations are to some extent intrinsically imperfect; their value is determined by the degree of indefiniteness which they allow:

> We are naturally forced to position them in a particular space, within a determined expanse, and space—even envisioned in its full possible extent—is no more than a special condition contained in one of the degrees of universal Existence [5, XVIII, p. 148].

True metaphysical reflection attempts to enfranchise itself from such conditions. If in philosophy this enfranchisement comes about through knowledge mediated by concepts, for Guénon it can only be achieved by way of a symbol deployed in some appropriate space.

> The so-called “Hypergeometry”, as Guénon underscores, is particularly apt at appealing to the imagination of those whose mathematical knowledge is insufficient to allow them to understand the true nature of an algebraic construct expressed in geometrical terms [7, XVIII, p. 125]; cf. [8, XVI, p. 103].

Guénon’s mathematical competence guards him against such fetishism and enables him to exploit mathematical analogy on a purely operational level, making it subservient to the representation of the multiple states of being. His mathematical knowledge is never mere knowledge of quantity *per se*, but of quantity *to the extent that it opens up infinite dimensions* in space. Nor is space valued for itself, but (only) to the extent that it is capable of surpassing its own limitations through its own operations. This, by the way, confers to continuous quantity its superiority over discontinuous quantity: that the latter can approach the former through continuous variation only. Infinitesimal calculus is the result of this conversion of a discrete universe into a continuous one, where space has primacy:

> The most perfect representation of continuous quantity is achieved by the consideration of quanta which are no longer fixed and determinate [. . .], but variable, because in that way their variation can itself be regarded as arising in a continuous fashion [8, IV, p. 37].

By way of space, even the world of numbers achieves true indefiniteness. One could nevertheless ask why a thoroughly metaphysical way of thought should explicitly be framed in terms of indefiniteness rather than infinity. On this point, Guénon is unequivocal, and it constitutes the essence of numerous criticisms which he directs at the ambiguities of mathematicians from Leibniz onwards:

> There can be no such thing as mathematical or quantitative infinity—the expression itself is meaningless, because quantity itself is a form of determination; numbers, space, time, to which some would apply this notion of so-called infinity, are determinate conditions which, as such, are necessarily finite; they are particular possibilities, or particular groups of possibilities, along and outside which there are others, which obviously implies their limited nature [8, 1, pp. 15–16].

This explicature is of fundamental importance, because it firmly positions the universe of symbols in the realm of the *indefinite*, while at the same time justifying its mathematical representation, which is itself indefinite. Guénon thus challenges both the simple conventionalism of mathematicians, which would erect indefinite operations into infinity, and the pretensions of an ontological interpretation of symbols that would view mere representations as full-blown manifestations of infinity:

> As far as the totality of being is concerned, an indefinite entity is here being used as a symbol of the Infinite, to the extent that it is allowable to say that the Infinite can be symbolically represented; but it should be quite clear that this in no way means that the two are to be confounded, as is habitually done by Western mathematicians and philosophers [5, XXVI, p. 198].

It is now necessary to come back to the symbolic function and to recognize that it could not exist without reference to a centre in space. While it is important to refrain from reducing Guénon's thought to alleged "sources"—his approach being solely based on intellectual intuition and its capacity to proceed *a priori*—it must be recognised that this primacy of the centre, apart from constituting a manifest reference to all traditional religions, retraces very closely the teachings of Plotinus. Plotinus indeed deals with such a centripetal movement, but he does so in order to distinguish geometry and the analogic use of geometry:

> The soul is not a circle in the sense of the geometric figure but in that it at once contains the Primal Nature as centre and is contained by it as circumference [... We] hold through our own centre to the centre of all the centres, just as the centres of the great circles of a sphere coincide with that of the sphere to which all belong. Thus we are secure [13, VI, 9, pp. 183, 14–22].[2]

And Plotinus goes on to show that the link between the soul and its centre is not local, and that the souls do not lie around this centre, but that the relationship is one of intelligible similitude and kindred. For Guénon, however, what constitutes the centre is not so much a Nature as a more abstract form of centrality. The latter is not determined by the knowledge which the soul has thereof, but specifically by its power of symbolic representation. If Neoplatonism ultimately assigns everything to the soul, Guénonean speculation strives to restrict itself to a resolutely symbolic and therefore metapsychic dimension.

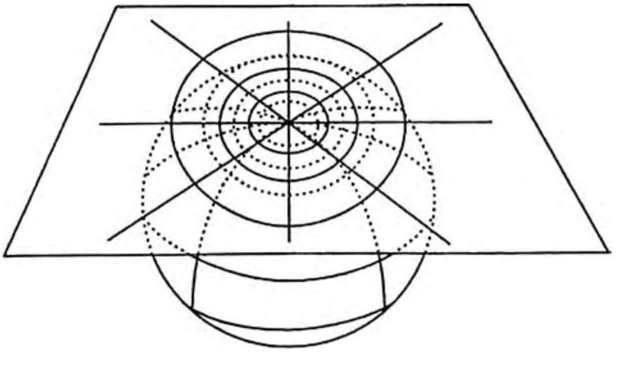

Fig. 3. When the sphere is projected on a tangent plane this corresponds to "the all seeing eye" or "God looking down" [16, p. 201].

[2]The translation used is the one given on http://www.essene.com/MysterySchool/Plotinus/Ennead.

Fig. 4. René Guénon in Cairo, ca. 1945 [10].

But the specific novelty of the Guénonean representation remains the relationship which he establishes between the cruciform coordinate system linked to the three directions of space, and its embedding within a sphere of indefinite radius:

> we view the three-dimensional cross as being traced from the centre of a sphere [5, V, p. 52].

Each and every state of being, whether singular or universal, must trace its meaning to it, and this provides the occasion to explore a profound analogy of the world, because the cosmoses, even those which modern science has ultimately invalidated from a physical viewpoint, regain a dimension of truth through the spherical dimension which they embody. Such a proposition, however, is only admissible insofar as reflection transcends the enclosed teleological aspect of these cosmologies and their latent geocentrism to contemplate Universal Possibility, which, alone, comprehends all dimensions of the metaphysical object.

From this absolute perspective, the mind is able to conceive reality as pure vibration:

> Thus, in order to convey the idea of totality, the sphere must [. . .] be indefinite, like the axes which constitute the cross and which are three orthogonal diameters of the sphere; in other words the

> sphere, which arises through the very radiation of its centre, never closes upon itself, the radiation being indefinite and filling the entire space with a series of concentrical waves, each of which reproduces the two phases of concentration and expansion of the initial vibration [5, VI, pp. 58–59].

One must read these texts in order to fully comprehend both the fertility and the limitations of symbolism (*any* symbolism), which as a matter of course presumes that the signified inevitably overbears the signifier. This is nowhere more evident than where discourse reaches its ultimate term: the universal Vortex.

> The unfolding of this spheroid is, in the end, nothing but the indefinite propagation of a vibratory (or undulatory, which is essentially the same thing) movement, no longer merely in a horizontal plane but throughout the whole three-dimensional expanse, from a starting point which can in fact be considered its centre. If one regards this expanse as a geometrical, that is to say spatial, symbol of the totality of the Possible (a necessarily imperfect symbol, being constrained by its very nature), we have arrived at a figuration which represents—to the extent that this is possible—the universal spherical vortex along which the realisation of all things is accomplished, and which the metaphysical tradition of the Far East calls *Tao*, that is to say “the Way” [5, XX, p. 159-59].

This Vortex is not to be understood merely on its own, as an integrative force with respect to the world of the possible. It provides, first of all, the matrix of interintelligibility of all the particular symbols transmitted by tradition. From such a Vortex spring the—invariably centered—figures of the spiral, the staircase, the enclosure, the hemispheres, the swastika, the pole, the Grail, the Sacred Heart, the Holy City, the Tree of Life..., which, along with many others, fill the pages of such an encyclopedic work as *Symboles de la Science sacrée* (*Fundamental Symbols of Sacred Science*). But it could be said that the undeniable success of that book would be deeply misleading if it were to be used as a kind of dictionary, without being constantly related to the metaphysics of the multiple states of being. In the middle of his symbolic constructions, Guénon never fails to remind us: “This multiplicity of states of being, which constitutes a fundamental metaphysical truth...” [5, I, p. 21]. For it alone enables one to focus not only on the whole set of degrees of Existence, but even more on the boundless domain of universal Possibility. And while the latter allows of a specifically geometrical expression, it can equally be arrived at by other means, as was brilliantly demonstrated in the work that would follow immediately after *Symbolisme de la croix*, which announces it in its last lines: *Les États multiples de l’être* (*The Multiple States of Being*), published in 1932. It, too, is concerned with Universal Possibility, but it leaves out geometry, and the essential limitlessness of Universal Possibility is established by a purely discursive argument.

In such a context, Man cannot provide the ground for any simple “humanist” assertions. Man only finds his true symbolic meaning if he positions himself according to such a dimensional arrangement and recovers his place as a cosmic one. Man is no subject, nor an absolute; he is a state of being endowed with limitless extensions, provided that this particular state recognizes its centre:

> The integration of the human condition, represented by the horizontal cross, is part of the order of existence, like an image of the very totalisation of being, represented by the vertical cross [5, VI, p. 60].

But this viewpoint, which subsumes the action of the transcendental upon the immanent, only makes sense if reference is made to the reflection of the centre upon itself:

> If one only considers the horizontal cross, the vertical axis is represented in it by the central point itself, which is the point where it meets the horizontal plane; thus every horizontal plane, symbolising some particular state or degree of Being, has in that very point which can be called its centre (as it is the origin of the coordinate system to which every point in the plane can be related) that same image of immutability [4, VII, p. 66].

Everything is therefore traced back to the point, in other words to a reality which exists outside any context of symbolisation and spatialisation. Just like space becomes symbolic space only through representing a reality which eludes it, so every measure of the states of being, even one devised by Man, is but the explosive dilatation of a point which eludes all representation, both because of its fundamental incorporality and its primordial nature. In this context, Guénon likes to recall an excerpt from Dante, who recognized two names of God, *I* and *El*, declaring *I*, the point, to be absolutely the first name of the Divinity [14, Par. XXVI, v. 134–136]. Guénon also cites this commentary of Moses of Leon, which summarises the teachings of the Kabbalah on this topic:

> The mystery of forthbringing Thought corresponds to the hidden "point". It is within the interior Palace that the mystery linked to the hidden "point" can be understood, because the pure and elusive aether always remains mysterious. The "point" is the aether rendered tangible (by concentration, which is the starting point of differentiation of whatever kind) in the mystery of the Interior Palace or Holy of Holies [5, IV, p. 46].

Thus we see that Rabelais—who only makes us laugh all the more surely to guide us to the "Manor of Truth" [15, *Quart Livre*, chapter LV, p. 668]—does not gratuitously define himself as an abstractor of quintessence, the latter being the alchemistic representation of an energy that existed prior to space and which holds primacy over all ulterior constructions derived from it. Incidentally, the Jubilee of 1550 which concludes the work is nothing but such a return to the principle of all forms before their ultimate recapitulation in a wine which is a source of mystery [15, *Cinquiesme livre*, Prologue, pp. 723–724]; cf. [5, IV, p. 42 and VII, p. 66; IX, p. 93]. In this way, by attending the interplay of point and space, one is drawn into a rhythm of recapitulation and expansion which sums up what is best in esoterical culture, which is first and foremost a law of transition from chaos to order, without any loss or exclusion.

> From his position at the centre of the "cosmic wheel", the perfect sage moves it imperceptibly, by his mere presence, without partaking in its movement and without having to concern himself with exerting any kind of action [5, pp. 70–71].

This ideal transcends that of ataraxy. It is truly speaking the deployment, as applied to Man, of the symbolism of the centre and its various spatial projections. That symbolism here is more than just a way of knowledge: it is also a particular conduct of the will which signifies for Man the return to the Primordial State, where the human state merges into the figure of universal Man and ascends, along the vertical dimension, through the successive states of being:

> The "pivot of the norm" is what in almost all traditions is termed the "Pole", that is to say [. . .] the fixed point around which all revolutions of the universe take place, according to the norm or law which rules all manifestations and which itself is but the direct emanation of the centre, in other words the expression of the "Will of the Heavens" in the cosmic order [5, VIII, p. 76].

Here one finds again the centre of all geometries and cosmologies to which Plato referred in the X book of the *Republic*:

> [A place] where they could see from above a line of light, straight as a column, extending right through the whole heaven and through the earth, in colour resembling the rainbow, only brighter and purer. [...] For this light is the belt of heaven, and holds together the circle of the universe, like the under-girders of a trireme [12, 615 b-c, transl. Benjamin Jowett].[3]

It could be said that the entire oeuvre of Guénon was written in this at once central and all-encompassing light, and that like Er, the hero mentioned by Plato, he is "the messenger who would carry to men the report of the other world", i.e. of the world of the invisible [12, 614d].

Life and death, too, obey the order thus reconstituted, and to these must be added the characteristic modalities of the human state as well as our various subtle corporeal and psychical extensions which together constitute our "complete individuality". The geometry of the centre then allows a representation of the overall equilibrium of multiplicities which traverse the human state and which only he who stands at that centre can comprehend by way of an intuition which is at once a sense of eternity and a sense of unity. These real multiplicities are such that they cannot be numbered: they are the very qualities of universal existence and escape the laws of numeration and quantification, thus evading the grip of the false idea of "infinite number", which Guénon, after Leibniz and perhaps more rigorously than he, refutes [8, II and III].

The geometry of a single being can thus become so complex that the only way to relate it to all the other beings in the Universe is by stepping outside the three-dimensional expanse. This proliferation of dimensions leads to the adage which is at the root of the work of a Giordano Bruno: "Every point could be a centre, and it may be said that it is one *in potentia*" [5, XII, p. 112]. Such a distinction between the potential and the actual nevertheless makes clear that Guénon does not yield to the temptations of an infinitism of the immanent, and his thought, in Leibnizean fashion, retains a focus on the singular, for he is quick to add: "but in fact, a particular point needs to be determined". Yet it remains true, in the case of space as in that of the human body, that all individuality "in reality contains an indefinite multitude of coexisting modalities", such as the indefinite multitude of cells each of which has its own existence [5, XIII, p. 116].

One of the most suggestive metaphors used by Guénon to explain his conception of the construction of space is the process of *weaving*. Not only does it link the symbolism of the cross to the imagery of the cloth, of writing, of books and nets, of the hairs of Shiva or the thread of the Fates, but it also creates the possibility to unify two kinds of representation, viz. the system of rectilinear coordinates and the system of polar coordinates: the cross in the latter case is no longer part of a system of straight lines, but of a system of concentric circumferences. Referring to the "spider's web" in Hindu thought, for example, he points to the reciprocal transformation which links these two cartographies of symbolic space:

> The warp is here represented by the threads radiating from the centre, and the woof by the threads laid out as concentric circumferences. To relate this to the customary model of weaving, it suffices to picture the centre as being at an indefinite distance, such that the rays become parallel lines [5, XIV, p. 123].

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[3] I used Benjamin Jowett's translation on http://www.worldwideschool.org/library/books/phil/ancientmedievalorientalphilosophy/TheRepublic/chap10.html.

One is struck by the extreme flexibility of these models and the necessity, for the symbolic mind, to accustom itself to a plurality of representations, lest it become petrified by the constraints of a single vocabulary.

This is obviously a far cry from the prevailing reception of Guénon's thought, of which all too often only the results are remembered by those unable to reconnect the latter to the truly indefinite substrate which confers them their aura and which chiefly derives from his laws of transformation. The Guénonean discourse, as we have said, is always but an "*aperçu*", caught in a never-ending rotation where only the centre remains fixed. That is why it is necessary to apply a *principle of continuity* which, alone, sheds light on the symbolisms employed as well as any other form of ontological statement that accompanies them. It must be stressed here that the universe envisioned by Guénon ultimately isn't that of the circle but that of the spiral, the infinitesimal spiral, i.e. an *open* concentric circumference. The sphere itself, for him, is neither closed nor planar [5, XXI, p. 163], just as Universal Possibility can never be limited:

> Its limitation would be inconceivable, as, having to comprise all Possibility, it could never be confined within it [5, XIV, p. 128].

In these conditions, any structuring system would, strictly speaking, be impossible, whether it be a system of categories, an eternal Return or some principle of reincarnation. This result is but the outcome of a rigorous application of the principle of indiscernibles, an eminently metaphysical principle which, like the principle of continuity, Guénon shares with Leibniz. In this universe nothing coincides—there is mere universal correspondence which continues into infinity. Here, once more, the notion recurs of a vibratory movement propagating indefinitely along some appropriate plane, like the surface of a liquid.

> All the systems that we have mentioned are in reality but different positions of one and the same system [5, XX, p. 157].

The very essence of the symbolism of the cross is that it suggests this kind of progressive universalisation of the symbol by introducing an ever-increasing indeterminacy, i.e. the ever higher powers to which infinity is raised. Transcendence and immanence in the resulting Vortex, considered in its plurality of states, are never absolute. A state, by the very fact that it is centered, is subject to transcendental influence only as an object of simultaneous contemplation of a plurality of states. This means that the force of attraction of transcendence in any case makes itself felt only through the centre of each state, "i.e. the point where equilibrium is reached and wherein this manifestation resides" [5, XXIII, p. 172]. In other words, there is transcendence only in those environments which are harmonious in the sense that they have attained the balanced conditions of their manifestation. Rather than enforcing their identity, transcendence presupposes their capacity for centration. In this sense it can be said above all to be *non acting.*

For a finite being, it is perceptible first of all by the establishment of a specific direction within the expanse, the straightness or "rectitude" corresponding to the vertical axis. Thus the indeterminate Absolute carries within itself a principle of determination, which assuredly leaves it unaffected but which allows the individual to have access to a way of perfection, if he manages to relate to this initial axis a *horizontal* plane whence he can manifest itself according to his particular wholeness. Thus is obtained a comprehensive construction of the individual according to the sundry directions of space, which could be conceived as a true "incarnation". But let us never forget that the individual plane upon which the Ray of Heaven will operate can be *any* state, "as all states are perfectly equivalent when viewed from Infinity" [5, XXVII, p. 203], the former being but the infinitesimal variations of the latter. Every road is a road of reintegration towards the centre, and Paradise and Hell, to the extent that they are linked to the universal vertical axis, are the symmetric ways of the individual realization [5, XXV, p. 192].

This nevertheless requires an initial vibration to illuminate the chaos, after the fashion of the *fiat lux* which sparks off cosmic creation. The individual unfolds in it like a lotus flower upon the waters, moving from the circumference of manifestation to the non-manifested centre. The notions of holiness, grace, salvation, which are characteristic of the religious outlook, are not thereby rejected but given their proper infinite background, detached from moralism and unfettered by naturalistic limitations of whatever kind. The conflicts of theology are far removed from such a perspective based above all on a principle of continuity applied to the full, which, alone, is worthy of the divine and allows a reconciliation of the cross-currents which pervade orthodoxy and heresy alike. Compared with the liberty offered by the symbolic world, theology, ethics and philosophy are held by Guénon to be discriminating practices which correspond to certain aspects of the cosmic cycle, but cannot without a thorough symbolic transformation lead to the perfection which is man's own and which is always achieved by synthesis rather than analysis.

Clearly, then, it is absolutely impossible to even begin approaching the meaning of the great currents of René Guénon's vast oeuvre without a thorough understanding of the mathematical symbolism that governs it from beginning to end. Whether he discourses upon primordial Tradition, the King of the World, or regular initiation, or whether he criticises modernity for being a space of solidification, always the reader has to keep in mind that his point of departure is an infinitesimal and variational conception of states which, alone, results in a correct understanding. While Guénon may borrow entire parts of his system from theosophy, from anthropology, from his casual readings even, he always reinterprets them within these holistic dynamics which result in a truly unique power of synthesis, thus turning the tracing of his sources into a sterile endeavour. Every Guénonean postulate must be engaged not through what it decrees, but through the perspectives it opens up. His reflection is the least confining one there is. Its value, once again, lies in the indefiniteness which it conjures up; and it surely constitutes one of the most irrefutable proofs of the correctness of Guénon's verdict on modernity, that the vibratile universe which he envisioned should have been rigidified, by his enemies and by some of his disciples, into a universe at once oppressive and reactionary.

Leaving aside these incomprehensions, it is important to say that these doctrines must additionally be viewed in the light of two other stances. Firstly, these truths are not merely the outcome of the modern scientific view of space. Rather, according to Guénon, modern science itself is the product of traditional doctrine. Modern science has, above all, a residual character, and this puts into a proper perspective the technical notation systems put to use in the course of Guénon's demonstrations. Secondly, and in the same line of thought, there is not an ancient sacred text, no temple, shibboleth or symbol which has come down to us which does not completely verify this representation of the universe or fails to confirm it by an anthropological truth corresponding to the theses first expounded at the metaphysical level.

Hence Guénon's oeuvre, in an era of radical breakdown of knowledge, succeeded in proposing the paradoxical notion of a universe of symbolic boundlessness, while at the same time creating precise correspondences which unite history and geometry, the theory of dreams and archaeology, the interpretation of sacred texts and cosmic life, in such a way that it is just as impossible to fault it with respect to the accuracy of its factual data as with respect to the rigour of its *a priori* argumentation. Against this backdrop of absolute openness, it finally becomes possible to acknowledge the true qualities of this unique body of work, which certain hurried readers have disqualified before acquainting themselves with it; it is in fact these unfair judgements, which erect their intrinsic limitations into self-evident criteria, which reveal themselves as the certain wellsprings of the most implacably oppressive systems.

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CHAPTER 34

Eddington, Science and the Unseen World*

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*The only recent paper that I know of about the relation between science and religion in Eddington's thinking is [1]. It contains many useful references. For a technical reconstruction of Eddington's work on the fundamental theory, I refer to [12].

MATHEMATICS AND THE DIVINE: A HISTORICAL STUDY
Edited by T. Koetsier and L. Bergmans

1. Introduction

Arthur Stanley Eddington (1882–1944) is remembered as a brilliant astronomer and physicist with a great gift for popular exposition. However, Eddington was a philosopher of science as well.[1] He was, moreover, an active Quaker inclined towards mysticism. In this paper I shall identify how the ideas of the physicist Eddington, the philosopher Eddington and the Quaker Eddington are related. My goal in this paper is to demonstrate the coherence of the three different aspects of Eddington's intellectual life.

2. Astronomer and physicist

Eddington received his scientific training at Trinity College, Cambridge. From 1906 to 1913 he was chief assistant at the Royal Observatory in Greenwich. In 1913 he became

Fig. 1. Arthur Eddington. Science Photo Library.

[1] Some professional philosophers considered Eddington to be a philosophical amateur who did not really know what he was talking about (cf. [15]). Others loved his writings; Merleau-Ponty considered him to be one of the greatest thinkers of the twentieth century ([14], 1965).

Plumian professor of astronomy at Cambridge and, in 1914, director of the university observatory. He is famous for his studies of stellar motion and the internal constitution of stars. In 1917 he became fascinated by Einstein's theory of relativity and he played a central role in one of the two expeditions to observe the total eclipse of the sun on May 29, 1919. Eddington participated in the expedition to Principe, an island off the west coast of Africa in the Gulf of Guinea. The photographs that Eddington obtained there verified one of the predictions of the theory of general relativity: stars observed close to the eclipsed disk of the sun were slightly displaced away from the centre. This deflection of light by a gravitational field had not been suspected before the introduction of general relativity.

Eddington was extraordinarily gifted in popular exposition and his books, *Space, Time and Gravitation* [5], written for the intelligent layman, and *The Mathematical Theory of Relativity* [6], written for the specialist, did much to make the theory of relativity well known to the English-speaking world. Soon after the publication of *The Mathematical Theory of Relativity* in 1923, Dirac succeeded in showing how the special theory of relativity is completely consistent with the equation that describes the electron in quantum mechanics, if it is written in the right form. This result had an enormous influence on Eddington. During the last fifteen years of his life he worked on a controversial and rather speculative theory that should bring about the unification of general relativity and quantum mechanics. A guiding idea in this work, which is already present in his first books, is that the fundamental physical theories can be derived a priori and the experiment is merely used to identify the constants that occur in the theory. On the basis of Dirac's work Eddington calculated a priori several numbers which physicists normally considered as experimental results. He identified what he called the seven primitive constants of nature: (1) the mass of an electron, (2) the mass of a proton, (3) the charge of an electron, (4) Planck's constant, (5) the velocity of light, (6) the constant of gravitation, (7) the cosmical constant. Eddington developed a theory of their relationships:

> "We may thus look on the universe as a symphony played on seven primitive constants as music is played on the seven notes of a scale." [10, p. 231][2]

Eddington's a priori calculations led to the number 1.848 for the ratio of the masses of a proton to an electron, while the experimentally measured result was 1.836. Another constant that Eddington calculated on the basis of purely theoretical considerations was the fine structure constant which occurs in the fine structure of the lines in the hydrogen spectrum. Eddington calculated the value 137; the observed value was 137.036.[3] The closeness between the calculated values of these dimensionless constants and the measured values only reinforced Eddington's conviction that he was right. Most physicists, however, did not share his ideas. As a result Eddington spent the last ten years of his life in isolation. He insisted that the many constants that he had calculated were correct and that the experimental results only had to be interpreted in the proper way.

[2]This has a somewhat Pythagorean ring, but it would be wrong to see Eddington as a twentieth century Pythagorean. Eddington wrote a few years later: "if it were necessary to choose a leader from among the older philosophers, there can be no doubt that our choice would be Kant. We do not accept the Kantian label; but, as a matter of acknowledgement, it is right to say that Kant anticipated to a remarkable extent the ideas to which we are now being impelled by the modern developments of physics" [11, pp. 188–189].

[3]The number 137 became a central number in his theory. Allegedly, he once told a colleague that he preferred to put his hat on peg number 137 in the cloakroom of a conference hall [16, p. 146].

3. Quakerism

One of the keys to understanding Eddington's philosophy of physics is, as we shall see below, his interest in relativity theory. Another key to a full understanding of Eddington's ideas is his Quakerism. Founded by George Fox, who emphasised the immediacy of Christ's guidance, Quakerism arose in the mid-17th century within the Puritan movement. The early Quakers, the Society of Friends, worshipped without liturgy or appointed ministers. The Meeting gathered in silence, although in this silence some of the worshippers might be led to speak. In the Quaker way of life the silent communion of the Meeting is the centre. First hand experience has been characteristic of Quakerism throughout the centuries. Although Quakers view their beliefs as consistent with the Bible and Christian tradition, they criticise infallible doctrines and creeds. They believe that creeds tend to crystallise thought and turn matters that cannot be put in words into a caricature of what they are supposed to express. Friends have no form of observance of the sacraments. In politics Quakers are inclined towards pacifism. In such an atmosphere Eddington grew up. The silent communion of the Meeting creates an atmosphere quite open to mystical experiences. Eddington wrote in 1928:

> "If I were to try to put into words the essential truth revealed in the mystic experience, it would be that our minds are not apart from the world; and the feelings that we have of gladness and melancholy and our yet deeper feelings are not of ourselves alone, but are glimpses of a reality transcending the narrow limits of our particular consciousness—that the harmony and beauty of the face of nature is at root one with the gladness that transfigures the face of man." [8, p. 321]

4. The problem

If mystical experience reveals to us that our minds are part of a reality that transcends us, a reality that expresses itself in nature as well, it seems natural to ask what the position of science is with respect to this reality. Eddington discusses this problem, the "perplexing dualism of spirit and matter" in *The Domain of Physical Science* [7, p. 202] as follows. On the one hand,

> "The spiritual phenomenon of consciousness is the one thing of which our knowledge is immediate and unchallengeable." [7, p. 202]

On the other hand, common sense and physics teach us of the existence of a material world around us, in which the "first reality", consciousness, seems to be a very late arrival. The physicist has no use for the notion of consciousness in his scheme. The religious mind is puzzled here.

> "It would welcome an admission from the physicist that his material world is not self-sufficient and would dissolve if it were not sustained by a spiritual reality, which, it is felt, must be deeper than all material reality." [7, p. 203]

The problem of the dualism of spirit and matter is related to other problems that concern the seemingly contradictory ways in which the same part of reality can be experienced and described. Eddington uses very suggestive examples to explain his point. For example, in *The Nature of the Physical World* Eddington describes how Lamb in his classic *Hydrodynamics* [13] deals with the generation of waves by wind. After two pages filled with impressive equations Lamb concludes: “Our theoretical investigations give considerable insight into the incipient stages of wave-formation” [8, p. 316]. Eddington now puts this description next to quite another description of the generation of waves by wind contained in Rupert Brooke’s words:

> “There are waters blown by changing winds to laughter
> And lit by the rich skies, all day. And after,
> Frost, with a gesture, stays the waves that dance
> And wandering loveliness. He leaves a white
> Unbroken glory, a gathered radiance,
> A width, a shining peace, under the night.”
> [2, p. 22][4]

Eddington writes:

> “The magic words bring back the scene. Again we feel Nature drawing close to us, uniting with us, till we are filled with the gladness of the waves dancing in the sunshine, with the awe of the moonlight on the frozen lake. These were not moments when we fell below ourselves. We do not look back on them and say, ‘It was disgraceful for a man with his six sober senses and a scientific understanding to let himself be deluded in that way. I will take Lamb’s Hydrodynamics with me next time.’ It is good that there should be such moments for us. Life would be stunted and narrow if we could feel no significance in the world around us beyond that which can be weighed and measured with the tools of the physicist or described by the metrical symbols of the mathematician.” [8, p. 317]

Eddington continues as follows on the experience described in Brooke’s poem: “Of course it was an illusion” (Ibidem) and he describes how vibrations of various wavelengths set the mind to work and how, on the basis of meagre incoming material, the mind wove an impression.

> “It was an illusion. Then why toy with it longer? These airy fancies [. . .] should be of no concern to the earnest seeker after truth. Get back to the solid substance of things, to the material of the water moving under the pressure of the wind and the force of gravitation in obedience to the laws of hydrodynamics.” [8, p. 318]

And then he makes his point:

> “But the solid substance of things is another illusion. It too is fancy projected by the mind into the external world.” [8, p. 318]

This last thought, which indicates the idealistic nature of Eddington’s philosophy, is illustrated elsewhere by means of the problem of the two tables. In the introduction to *The Nature of the Physical World* [8] Eddington tells that he wrote the book while sitting at a table that had been familiar to him from his earliest years: a table with an extension, comparatively permanent, coloured and above all substantial. This is Table No. 1, a solid thing, the opposite of empty space. At the same time, however, he wrote his book at Table No. 2, a table less familiar to him. Actually the work of Rutherford in 1911 had revealed the existence of Table No. 2 to Eddington. In nineteenth century physics the space inside a solid was still well filled. However, Rutherford showed that the solid objects consist

[4] It is remarkable in Brooke’s poem that the description of the waves “blown to laughter”, that when frozen leave a “white and unbroken glory”, is a metaphor for the young men killed in the Great War. Eddington seems to read it as merely a description of nature.

mainly of empty space. Only a one-billionth part of Table No. 2 is substantial and it is not at all clear what kind of substance this is. In physics, elementary particles are described by mathematical equations and by means of these equations the physicist can predict the results of experiments. Eddington concludes that the world as it is revealed by physics has become a world of shadows. Eddington:

> "In the world of physics we watch a shadowgraph performance of the drama of familiar life. The shadow of my elbow rests on the shadow table as the shadow ink flows over the shadow paper. It is all symbolic, and as a symbol the physicist leaves it. Then comes the alchemist Mind who transmutes the symbols. The sparsely spread nuclei of electric force become a tangible solid; their restless agitation becomes the warmth of summer; the octave of aethereal vibrations becomes a gorgeous rainbow [. . .]." [8, p. xvii]

5. The four-dimensional world

Apparently Rutherford's work made Eddington wonder about the status of the material world around us. However, relativity theory had an even greater impact on his views. In relativity theory time and space have a different interdependency. This can be illustrated by means of the strange and paradoxical implications of the so-called Lorentz-contraction. Eddington, who was very good at finding entertaining examples, illustrated the contraction of time as follows. Imagine an aviator travelling at 161,000 miles a second and imagine that we have an agreement that says that at the moment he passes us, we will both light a cigar and at the end of 30 minutes, when our cigars are finished, we will both give a signal. Then will both discover that the other smoker's cigar lasted twice as long as our own cigar [5, pp. 24–25].

Relativity theory implies that space and time are no longer intrinsic qualities of the world; they are brought about by the particular perspective that the mind has with respect to the world. The universe should no longer be seen as a three-dimensional space in which, in the course of time, things happen. No, the "World" in relativity theory is a four-dimensional aggregate that encompasses past, present and future. The World consists of point-events. Whenever a point-event or a relation of point-events is observed by an observer, this happens at a certain moment in time in three-dimensional space, but the way in which the World is "split" into a space-part and a time-part is different for different observers. In "The Domain of Physical Science" of 1925, Eddington expressed his agreement with Hermann Weyl:

> "It is a four-dimensional continuum which is neither time nor space. Only the consciousness that passes on in one portion of this world experiences the detached piece which comes to meet it and passes behind it as history, that is as a process that goes forward in time and takes place in space." [7, p. 211]

In relativity theory length and duration lost their status of things inherent in the external world; they turned out to be merely relations of things in the external word to a specified observer. This idea was only strengthened by general relativity theory, which also deals with accelerated motion. In general relativity theory the existence of gravitation and the laws of mechanics of matter can all be deduced from the geometry of the curved four-dimensional continuum, the "World".

6. Eddington's idealism

According to relativity theory, our experience of the World depends on the particular observer. However, Eddington went further. Relativity theory convinced him that the world of physics as a whole is a product of the human mind. The mind is also responsible for our experience of matter in space and time. While Eddington's argument is rather technical, it is also crucial: it turned Eddington into an idealist. In 1929 he wrote:

> "I would like to recall that the idealistic tinge in my conception of the physical world arose out of mathematical researches on the relativity theory. In so far as I had any earlier philosophical views, they were of an entirely different complexion." [8, p. viii]

The earliest published exposition of these idealistic views is two papers in *Mind* in 1920 [3,4]. The argument concerning matter is central to these publications. According to Eddington [3, p. 146], experiences take place in space and time. Nevertheless, the space and time of an experience are derived concepts and they are not intrinsic qualities of the World. However, by means of mathematical notions like tensors we can get to intrinsic qualities of the world. For example, the vanishing of a tensor denotes an intrinsic condition independent of space and time. Relativity theory implies, for example,

$$G_{\mu\nu} - \frac{1}{2} g_{\mu\nu} G = 0, \quad \text{for empty space}$$

and

$$G_{\mu\nu} - \frac{1}{2} g_{\mu\nu} G = -8\pi T_{\mu\nu}, \quad \text{for mass present.}$$

At this point Eddington objects to the usual view that the first equation is a law inherent in the continuum, while the second equation indicates how the continuum is disturbed when matter intrudes. Eddington writes:

> "I think there is something incongruous in introducing an object of experience (matter) as a foreign body disturbing the domestic arrangements of the analytical concepts from which we have been building a theory of nature. It leads to a kind of dualism." [3, p. 151]

Eddington asks himself a rhetorical question: What impression do the two equations make on our senses? He answers that the first equation corresponds to the perception of emptiness and the second equation to the perception of matter. Thus the two equations are not inherent laws of the external world, but they describe how the quantity measured by the expression $G_{\mu\nu} - \frac{1}{2} g_{\mu\nu} G$ (nowadays called the Einstein-tensor) is appreciated by the human mind. The Einstein-tensor satisfies a condition of continuity which implies that if space and time are measured in a specified number of ways matter will be permanent: if a particle disappears, at a neighbouring point a corresponding mass will appear [3, p. 153]. In this way Eddington arrives at a definition of matter in terms of the geometrical concepts of relativity theory. There are many ways in which the mind could resolve a four-dimensional continuum, he points out, but the mind singled out the familiar space and time. Why, asks Eddington? The reason is the fact that the mind has a predilection for living in a permanent universe. Eddington wrote:

> "The intervention of mind in the laws of nature is, I believe, more far-reaching than is usually supposed by physicists. I am almost inclined to attribute the whole responsibility for the laws of mechanics and gravitation to the mind, and deny the external world any share in them." [3, p. 155]

In the second paper in *Mind* [4] Eddington repeats the same message:

> "The mind is dealing with a real objective substratum; but the distinction of substance and emptiness is the mind's own contribution, depending on the kind of pattern it is interested in recognising. It seems probable that the reason for selecting the particular type of pattern is that this pattern has (from its own geometrical character, and independently of the material in which it is traced) a property known as Conservation. Reverting from the four-dimensional world to ordinary space and time, this property appears as permanence." [4, p. 420]

The same ideas return in *Space, Time and Gravitation* [5]:

> "Mind filters out matter from the meaningless jumble of qualities, as the prism filters out the colours of the rainbow from the chaotic pulsations of white light. Mind exalts the permanent and ignores the transitory; and it appears from the mathematical study of relations that the only way in which mind can achieve her object is by picking out one particular quality as the permanent substance of the perceptual world, partitioning a perceptual time and space for it to be permanent, and as a necessary consequence of this Hobson's choice the law of gravitation and mechanics and geometry have to be obeyed. Is it too much to say that the mind's search for permanence has created the world of physics?" [5, p. 198]

And somewhat further he adds:

> "the conclusion is that the whole of those laws of nature which have been woven into a unified scheme—mechanics, gravitation, electrodynamics and optics—have their origin, not in any special mechanism of nature, but in the workings of the mind." [5, p. 198][5]

As a Quaker Eddington experienced the dualism of mind and matter as a problem. His argument concerning the Einstein tensor changed the problem. It did not solve the problem entirely because the problem of the dualism of the mind and the "meaningless jumble of qualities" from which the mind "filters out matter" remained. Yet, on a heuristic level, this could be felt as a step towards a final solution.

7. Mind stuff

The idealistic views that Eddington expressed in *Mind* in 1920 didn't undergo any significant change over the rest of his life. He elaborated on them in several publications. He also related them to his solution to the perplexing dualism of matter and mind (in particular in [7–9]. In this section I shall try to give a sketch of this solution. It consists of three elements.

The notion of the primacy of mind

First of all, the following view is like a postulate for Eddington: the one thing of which our knowledge is immediate and unchallengeable is the content of our own consciousness. He

[5] On p. 199 Eddington points out that there is one law that bothers him, the law of atomicity: "Why does that quality of the world which distinguishes matter from emptiness exist only in certain lumps called atoms or electrons, all of comparable mass? The mind seems to prefer continuous perception".

finds this self-evident. Eddington: "Mind is the first and most direct thing in our experience; all else is remote inference" [9, p. 24].

The principle of permanence

There exists an undeniable difference between a priori knowledge and a posteriori knowledge; we need to distinguish between the two. A posteriori knowledge concerns the "external world". According to Eddington the motive for the conception of an external world lies in the existence of other conscious beings. If I would be the only conscious being, there would be no need for the construction of an external world, according to Eddington. However, because there are more conscious beings, we compare notes and place the elements common to all conscious beings "in neutral ground—an *external* world" [7, p. 192]. In this way the external world is in the final analysis the mental synthesis of appearances from all possible points of view. In daily life this implies that the external world is the synthesis of the elements common to the consciousness of all mentally healthy human beings. What happened in relativity theory is merely an extension of this attitude with respect to certain aspects of reality. In relativity theory the traditional point of view is modified to include the points of view of observers travelling at high velocities.

However, there is more. According to Eddington, there is a principle of selection playing "the all-important part in determining whether a law such as the conservation of energy shall take rank as a law of nature" [7, p. 213]. This principle is partly determined by the fundamental idea that the external world shall embody the elements common to the experiences of individuals, but on top of that

> "The principle of selection followed by the mind appears to be primarily a search for the things which are permanent." [7, p. 213]

So, our conception of the external world, in the form it has in our daily life or in the form it has in the fundamental theories of physics, is the result of a principle of selection which includes the intersubjective and permanent and excludes the subjective and temporary.

The notion of mind-stuff

Our knowledge of the external world is structural knowledge. This is particularly clear in the case of the physical theories. Talking about the theory of relativity Eddington writes:

> "An yet, in regard to the nature of things, this knowledge is only an empty shell—a form of symbols. It is knowledge of structural form, and not knowledge of content. *All through the physical world runs that unknown content, which must surely be the stuff of our consciousness. Here is a hint of aspects deep within the world of physics, and yet unattainable by the method of physics.* And, moreover, we have found that where science has progressed the farthest, the mind has but regained from nature that which the mind has put into nature.
> We have found a strange foot-print on the shores of the unknown. We have devised profound theories, one after another, to account for its origin. At last, we have succeeded in reconstructing the creature that made the foot-print. And Lo! it is our own." ([5, pp. 200–201], italics added.)

The thought contained in the italicised sentences returns again and again in Eddington's work. Physical knowledge, say, of atoms is structural. However, there must be something that possesses this structure, there must be content:

> "Because we see that our precise knowledge of certain aspects of the behaviour of atoms leaves their intrinsic nature just as transcendental and inscrutable as the nature of mind, so the difficulty of interaction of matter and mind is lessened. We create unnecessary difficulty for ourselves by postulating two inscrutabilities instead of one." [7, p. 208]
>
> "The mind-stuff of the world is, of course, something more general than our individual conscious minds; but we may think of its nature as not altogether foreign to the feelings in our consciousness." [8, p. 277]
>
> "The mind-stuff is not spread in space and time [...] But we must presume that in some other way or aspect it can be differentiated into parts. Only here and there does it rise to the level of consciousness, but from some islands proceeds all knowledge. Besides the direct knowledge contained in each self-knowing unit, there is inferential knowledge. The latter includes our knowledge of the physical world." [8, p. 277]

In the Tarner Lectures of 1938 [11], Eddington again returns to the same questions. In the first eight chapters he elaborates the doctrine that the laws of nature that are usually classified as fundamental correspond to a priori knowledge and are wholly subjective. In the remaining chapters he argues that the knowledge of the physicist is entirely structural. The mind has primacy:

> "Let us now consider the common root from which scientific and all other knowledge must arise. The only subject presented to me for study is the content of my consciousness." [11, p. 195]

The principle of selection is there. Eddington even writes that if it were necessary to give a short name to his philosophy, he would hesitate between "selective subjectivism" and "structuralism" [11, p. viii]. In 1919 Eddington had known that physical knowledge was structural. Twenty years later, it became clearer. In 1939 he could write:

> "it is the structure of the kind defined and investigated in the mathematical theory of groups" [11, p. 147]

and with even more emphasis he could give his solution to the dualism of mind and matter:

> "The recognition that physical knowledge is structural knowledge abolishes all dualism of consciousness and matter." [11, p. 150]

In 1939 he did not repeat his ideas about mind-stuff; he restricted himself to the comment that he had discussed mysticism elsewhere.

8. Concluding remarks

Many see their own consciousness as a strange, insignificant spark of spirit in a huge material universe. Eddington totally rejected this idea. For him mind is the first and most direct thing in our experience; all else is remote inference. In *Science and the Unseen World* [9] Eddington imagined how a passage in an obituary notice, which mentioned that the deceased had loved to watch the sunsets from his country home, led to a long series of letters on the teaching of Copernicus. Everybody forgot the only thing that really counted: the fact that the deceased man had looked out for an experience each evening and not a creed [9, pp. 51–53]. Scientific knowledge is only preliminary knowledge of a very restricted kind. Questions concerning good and evil, beautiful and ugly, meaningful and meaningless are not touched upon by science. Such questions can only be answered by "intimate knowledge" [8, p. 321]. Again and again Eddington emphasised that the sphere of personal experience is primary and that the experiences of beauty or melancholy are important. Within this personal experience Eddington emphasised the seeking. A mystic is for Eddington someone who takes his or her intimate experiences seriously. He wrote:

> "We have no creed in science, but we are not lukewarm in our beliefs. The belief is not that all the knowledge of the universe that we hold so enthusiastically will survive in the letter; but a sureness that we are on the road. If our so-called facts are changing shadows, they are shadows cast by the light of constant truth. So too in religion we are repelled by that confident theological doctrine which has settled for all generations just how the spiritual world is worked; but we need not turn aside from the measure of light that comes into our experience showing us a Way through the unseen world." [9, pp. 55–56)]

Eddington's philosophy is very much a search for an all-encompassing view of the world and ourselves. It is a Quaker search. Summarising, we can say that the core of Eddington's solution to the problem of the dualism between mind and matter consists of the idea of the primacy of mind, the principle of selection and the notion of mind-stuff. Because the principle of selection concerns a particular activity of the mind as well, the solution in fact boils down to a reduction of everything to "mind".

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CHAPTER 35

The Divined Proportion

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Contents

MATHEMATICS AND THE DIVINE: A HISTORICAL STUDY
Edited by T. Koetsier and L. Bergmans

Introduction

There is no doubt about the fact that around 300 BC the ancient Greeks were aware of the ratio now generally referred to as the *golden section*. Euclid has us divide a line according to this ratio and he also writes extensively about the pentagon and the dodecahedron. He explains how to construct these and how to inscribe the one in a circle and the other in a sphere.

But what Euclid coined the *division in extreme and mean ratio* is by far not yet a *golden* or *divine* proportion. In this article, I shall trace how and when the deification of an originally mathematical notion came about.

1. No logo

Roger Herz-Fischler, who has taken the trouble to track down the record of this division from antiquity through the eighteenth century,[1] lists a number of examples of pentagrams (the figure typically used to show the golden section ratio) found in sources preceding the Christian era. Inscriptions of pentagrams in clay that were found in Egypt and Mesopotamia have been archeologically determined as dating back from before 3000 BC. The oldest Sumerian cuneiform pentagrams stem from the same period and a flint scraper from Palestine showing an incised pentagram may even be older:[2] Yet, whatever the denotation or symbolic meaning of these shapes may have been, there is nothing to suggest that the golden section as such should have been of any importance to the oldest civilisations around the Mediterranean. Documents like the famous Rhind papyrus from the seventeenth century BC, an important source of our knowledge on Egyptian mathematics, do not make mention of it.

Nor do the Pythagoreans, although known for their interest in proportions, add anything as far as the golden section is concerned. Of course there is hardly any first hand information from the earliest generations of Pythagoreans. Our oldest source is Philolaus (late fifth century BC), whose writings we know through scattered fragments transmitted by later writers. Some of these fragments enter into specific issues of mathematics and of music, but none of the remarks on proportions has any bearing on the divine proportion.

The same goes for the later Pythagorean texts.[3] The most informative sources on Pythagorean mathematics date from the second to the fifth centuries AD. Nicomachus of Gerasa (beginning of the second century AD) is very explicit on the Pythagorean theory of proportions and especially on their idea of *mediation*. Mediation is the intervention of a third number to bring about a balance between two other numbers. Thus, the numbers 2 and 8 can be mediated by 4, since $2 : 4 = 4 : 8$. The same numbers can also be mediated by 5, since $8 - 5 = 5 - 2$. And even 6 would qualify, since $2 : 6 = (8 - 6) : (8 - 2)$. In this way, Nicomachus treats ten different types of mediation. Now, if the Pythagoreans had shown any interest at all in the golden section, Nicomachus' text would have been the

[1] *A Mathematical History of Division in Extreme and Mean Ratio*, Waterloo (Canada), 1987.

[2] *A Mathematical History...*, Ch. III.

[3] See K.S. Guthrie (ed.), *The Pythagorean Sourcebook and Library* (1920), Grand Rapids, 1987, for the classical documentation of the Pythagorean school.

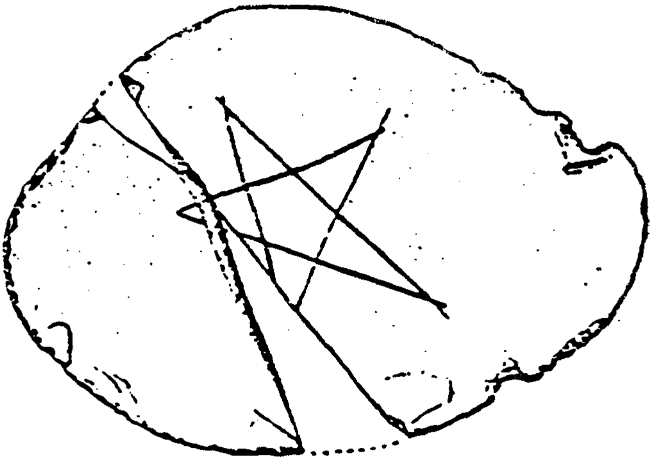

Fig. 1. Flint scraper from Palestine, Herz-Fischler, 1987, p. 58.

place to find the traces of such an interest. But again, there is nothing of the sort. Yet, Nicomachus' extensive dealing with mediation helps us understand why there was not, indeed could not have been, any cultivation of the divine proportion among the Pythagoreans: all his examples consist of numbers in the proper sense of the word; that is, of integers (*arithmoi*). The divine proportion calls for a further level of abstraction, one that would involve irrational quantities. Nicomachus could have taken this level into consideration had he been dealing with the state of the art of mathematics in his own time, which he was not; the mathematical simplicity of his account adds to the chance that we are dealing here with authentic Pythagorean notions of proportionality. From the sources available to us it becomes clear that the breakthrough in mastering the mathematical concept of irrationality must have taken place around the fourth century BC. The most obvious confrontation with an irrational ratio for a people thinking in geometrical forms rather than in abstract quantities must have been the relationship between elements of one and the same figure; how could these not be understandable from the same unit? The long history of reflections on the problem of squaring the circle, as we find with Cusanus and many others, goes to show that the irrational relationship between the circle's circumference and its diameter remained counterintuitive for a long period of time. This question was even more pressing in the light of the primordial status of circle and square in architecture; in treatises on Roman, Greek-Orthodox and Islamic architecture we find the importance of these two basic forms confirmed.

For Aristotle, writing in the middle of the fourth century BC, the ratio of the side and diagonal of the same square is a frequently returning issue.[4] At one point he uses it as an indication of the distinction between laymen and those who have received a proper education in mathematics. This passage also shows how closely mathematics is intertwined with the divine. A mathematical (and, more generally, philosophical) education is the type of instruction that helps to set man free; it is knowledge (ἐπιστημη) for its own sake, not instrumental to something else, and man gains mental sovereignty when he surpasses the

[4]See the listing in David Fowler, *The Mathematics of Plato's Academy*, Oxford, 1987, p. 295.

stage of acquiring knowledge for production (ποιησις) or action (πραξις) and is ready for pure contemplation (θεωρεια). In a sense, this is also what makes him superhuman:

> For human nature is in many ways servile: so that (. . .) God alone may have this prerogative; and it is fitting that a man should seek only such knowledge as becomes him. (. . .) This science alone may be divine, and in a double sense: for a science which God would most appropriately have is divine among the sciences; and one whose object is divine, if such there be, is likewise divine.

Then, Aristotle goes on to show how the initially counterintuitive turns into the expected:

> For all men begin, as we have said, by being amazed that things are as they are, as puppets are amazing to those who have not yet understood how they work, or the solstices, or the incommensurability of a square's diagonal with the side, for it seems curious that there is something which cannot be measured even with the smallest unit. Finally, however, in the progress of our science, the directly contrary and, as the proverb has it, the better state, is reached, the state reached by those who, as in the cases mentioned, have accepted instruction; for there is nothing which would surprise a geometer more than if the diagonal of a square became commensurable with the side.[5]

But if the irrationality of the divine ratio was a problem preventing the Pythagoreans from recommending it as a proportion with special qualifications, could it then be that they cherished it in its geometrical shape? Siegfried Heller, investigating the discovery of the divine proportion by the Pythagoreans, speaks about the pentagram as being "das Bundeszeichen der Pythagoreer"[6] and this has been a widely accepted idea. But what is this idea based upon? Much has been made of an isolated remark by an author from the generation after Nicomachus. In fact, this is a strange remark, in a strange text, by a very strange author. The notorious cynic Lucian of Samosata (±115–±200) found his inspiration for writing the short text *Pro lapsu inter salutandum* (*In defence of a slip of the tongue in greeting*) by accidentally using the wrong word when greeting a friend. Reflecting on ways of greeting and on the words and symbols used for that purpose, he speaks about "the pentagram, which they [i.e. the Pythagoreans] used as a token serving to recognise the members of their sect".[7] The few other texts from antiquity mentioning the pentagram with this function are merely repetitions of Lucian's remark.

So much for the foundation of the belief that the pentagram is a Pythagorean sign: a very weak basis for such an established conviction. Extensive studies of the Pythagorean tradition, such as De Vogel's, do not come up with any evidence that the pentagram was recognised by the Pythagoreans as their *logo*. On the contrary: De Vogel showed that the many southern Italian coins on which the image of the pentagram can be found date from periods and places not dominated by the Pythagoreans.[8]

2. Elemental truths

Thus, irrationality is a key term in understanding why the divine proportion did not, in reality, receive all the attention which later historiography ascribed to it. We would have a

[5] *Metaphysics* 982b29–983a20. See Aristotle, *Metaphysics*, transl. Richard Hope, Ann Arbor, 1960, pp. 8/9.

[6] S. Heller, *Die Entdeckung der stetigen Teilung durch die Pythagoreer*, Abhandlungen der deutschen Akademie der Wissenschaften zu Berlin, Klasse für Mathematik, Physik und Technik, Nr. 6, 1958, p. 5.

[7] Quoted in De Vogel, *Pythagoras and Early Pythagoreanism*, Assen, 1966, p. 47.

[8] De Vogel, *Pythagoras and Early Pythagoreanism*, ch. III.

better understanding of its impact on the Pythagorean mind and on its relation to the whole of Pythagorean teaching if more contemporary accounts were available to us. It is very difficult to evaluate the credibility of the many anonymous scholia and alleged emendations that were added to the antique documents in later centuries. Could these monks and other scholars really have known something that we cannot read in the surviving texts ourselves? Conjecture is the name of the game; yet the answer to that question cannot be a categorical negation. Sometimes their interpretations could indeed have been based on copies of manuscripts which they could still have disposed of, but which got lost afterwards. So much is certain that, in the sources available to us, there is no unequivocal reference to the divine proportion before it turns up, at the end of the fourth century BC, in Euclid's *Elements*. In Book VI, def. 3, Euclid defines: "A straight line is said to have been cut in extreme and mean ratio when, as the whole line is to the greater segment, so is the greater to the less".[9] This is not the first time, though, that this proportion occurs: in Book II, prop. 11, Euclid instructs us to "cut a given straight line so that the rectangle contained by the whole and one of the segments is equal to the square on the remaining segment".[10] This implies that the division of AB in H must be made according to extreme and mean ratio (AE = EC, EF = EB). (See Fig. 2.)

The same division is also involved in the construction of several polygons in Book IV. Book VI, prop. 30 finally tells us straightforwardly "to cut a given finite straight line in extreme and mean ratio".[11]

Euclid's Xth Book is explicit about the conceptions of incommensurability and irrationality which the Pythagoreans still shunned. Understanding of these conceptions must have grown during the fourth century in order for Euclid to be able to incorporate them into his compilation; Van der Waerden[12] credits Plato's friend Theaetetus with the intellectual ownership of the material dealt with in Book X as well as Book XIII. This latter book is especially interesting to us because here Euclid treats a number of propositions concerning

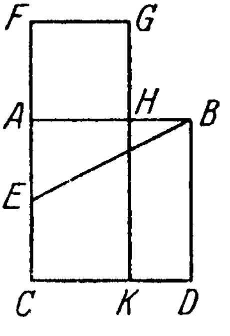

Fig. 2. Euclid's division in extreme and mean ratio, *Elements* II, 11.

[9]Euclid, *Elements*, Book VI, def. 3; see *The thirteen books of Euclid's Elements*, transl. Th. Heath (1908), Cambridge, 1926, Vol. II, pp. 188/9.
[10]Euclid, *Elements*, Book II, prop. 11, in *The thirteen books* . . . , Vol. I, p. 402.
[11]Euclid, *Elements*, Book VI, prop. 30, in *The thirteen books* . . . , Vol. II, p. 267.
[12]B.L. van der Waerden, *Science Awakening* I, Dordrecht, 1988 (1954), chs. V & VI.

the division in extreme and mean ratio before proceeding to the inscription of the five regular solids in a sphere. Two later supplements to this book were considered for a long time to be authentic Euclidean and referred to as Books XIV and XV. These contain additional theorems concerning the pentagon and the regular solids.

It is, therefore, beyond any doubt that Euclid was thoroughly familiar with the divine proportion and some of its peculiar properties. But it also becomes clear that terms like "divine proportion" or "golden section", with their typically evaluative connotations, are, in fact, anachronistic when related to Euclid. Euclid deals with arithmetic and with geometry, not with physical objects showing particular ratios. Moreover, the scattered distribution of propositions involving the division in extreme and mean ratio shows that Euclid found no reason to emphasise it. There is nothing in Euclid which makes this particular division stand out among the other mathematical insights presented and used for further proofs, and it is simply hilarious to call Euclid "above all, the man of the golden section ...", as I have read recently on the Internet in a description of Euclid's appearance in Raphael's *School of Athens*.

3. Potentia mirabilis

Can we find anything golden in Cusanus' writings? We can, but not the golden section. In 1459, Cusanus wrote down a theorem which he referred to as the *aurea propositio in mathematicis*. The theorem claims that three straight lines, originating in the same point and forming two equal angles of 45° or less, are always in the same ratio to the boundary line, whether this boundary line is a chord or an arc. In Fig. 3: $ab + ah + ac$ is to bhc as $ab + ad + ac$ is to the arc bdc, etc.

Cusanus reflects on this theorem not only as a mathematician, but also as a theologian. Mathematical contemplation opens the gate to loftier speculations beyond the sphere of empirical perception. In his reflections on the squaring of the circle (as well as in other texts, such as the *Idiota de mente*), Cusanus explains his belief in the importance of mathematics for theology: mathematical thinking resides in the highest regions of the mind, as it regards the figures in their true Forms, that is, not contaminated by the tricks of ever

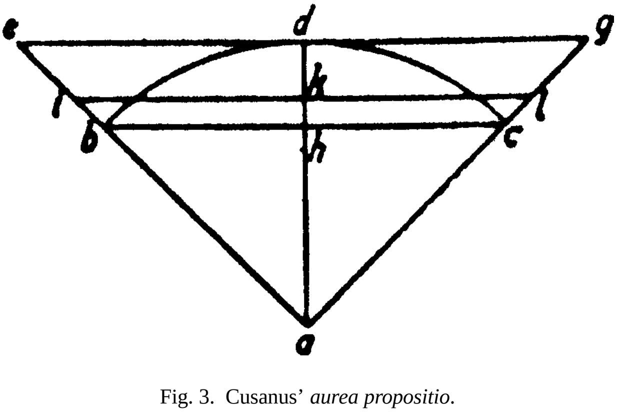

Fig. 3. Cusanus' *aurea propositio*.

changeable matter. Mathematics, therefore, assists the theologian in coming closer to the understanding of the Form of Forms, the primary Form in which all the other forms coincide.

The (in)commensurability of the linear and the curved is a recurring theme in Cusanus' mathematical writings, by which he illustrates the incomparability between the objects of the sublunary world and the austere power which has neither beginning nor end. This infinite power of eternity is as incommensurable to any finite power as the surface of the circle is to any other surface.[13] Yet, the golden proposition leads him to think of an infinitely large circle, with the implication that the difference between chord and arc would then be dissolved. Referring to the common origin of the three lines, he ends his explanation of the golden proposition by the statement that the highest speculation of the wise man will be directed to the Trinitarian origin from which all things emanate.

Although it is clear that this golden proposition bears no relation to the golden proportion, Cusanus' comment is still helpful in understanding the vein of the exalted exclamation which we find added to one of the theorems concerning the divine proportion in another manuscript: the thirteenth century translation of Euclid's *Elements* by Johannes Campanus, another mathematically gifted clergyman. This translation was later, in 1482, to become the first Euclid in print.

> Admirable therefore is the power of a line divided according to the ratio with a mean and two extremes; since very many things worthy of the admiration of philosophers are in harmony with it, this principle or maxim proceeds from the invariable nature of superior principles, so that it can rationally unite solids that are so diverse, first in magnitude, then in the number of bases, then too in shape, in a certain irrational symphony.[14]

This is the comment as we find it added to book XIV, prop. 10—a theorem concerning the proportional relationship between the volumes and the sides of a dodecahedron and an icosahedron inscribed in the same sphere. As previously mentioned, the XIVth book is not originally Euclidean, but it was considered as such during Campanus' days (and long after). And Campanus might just as well have written these words as a comment on the authentic book XIII, with its many propositions concerning the properties of the division in extreme and mean ratio.

One may consider Campanus' statement a sign of aesthetic admiration for the golden section but it certainly has nothing to do with art or with human creativity. It is aesthetic in the sense that Campanus is impressed by the formal properties or, if you wish, the beauty of this proportion, in much the same way as one may be aesthetically impressed by the Pythagorean proposition or by one of its many proofs. Yet, it is a metaphysical rather than an aesthetic statement, in the same vein as Cusanus' reflections on the *aurea propositio*.

[13] Nikolaus von Cues, *Die mathematischen Schriften*, ed. J.E. Hofmann, Hamburg, 1952, p. 51.

[14] "Mirabilis itaque est potentia lineae secundum proportionem habentem medium duoque extrema divisae; cui cum plurima ph[i]losophantium admiratione digna conveniant hoc principium vel praecipuum ex superiorum principiorum invariabili procedit natura ut tam diversa solida tum magnitudine tum basium numero tum etiam figura irrationali quadam simphonia rationabiliter conciliet". Quoted after Pacioli's edition of Campanus' translation (Venice, 1509), p. 137v.

4. Stupendous effects

We now turn to the author of the first book on the golden section, the only book on the subject to appear before the middle of the nineteenth century. Like Campanus and Cusanus, Fra Luca Pacioli (±1445–1517) combined his ecclesiastical duties (being a monk in the order of the Minorites) with a professional interest in mathematics. He was the author of an influential compilation of much of the mathematical knowledge of his time, the *Summa de Arithmetica, Geometria, Proportioni et Proportionalità* (1494), and he taught mathematics in many Italian cities. During the last few years of the fifteenth century, before the city was conquered by the French in the winter of 1499–1500, Pacioli was the humble servant of Ludovico il Moro, at the Sforza court in Milan. So had been, since 1482, Leonardo da Vinci—a happy coincidence, since Leonardo was eager to understand more about mathematics and to study Euclid, with the help of Fra Luca. In return, Leonardo took care of the sixty illustrations in Pacioli's peculiar study of 1498, which he named *Divina Proportione*—thus coining a new name for the proportion hitherto known by its Euclidean name of division in extreme and mean ratio.[15] This study, dedicated to the duke, contains a lot of expositions on the proportion in question, as well as on the five regular and a number of semiregular bodies and on their stellated and truncated derivatives. The printed edition of the *Divina Proportione*, which appeared in Venice in 1509, also included a treaty on architecture (without any reference to the golden section, let alone to the idea of using the golden section as a proportion in architecture), a geometrically well-considered alphabet, to be used for engraving texts on buildings or monuments, and the translation of Piero della Francesca's treaty on the five regular bodies. The painter Piero della Francesca, an older fellow-villager of Pacioli's, had been his teacher of mathematics in earlier days.

In his approach to the divine proportion, Pacioli must have taken his cue from the single, isolated statement in which Campanus praised this unique division. Pacioli also provided for a new edition of Campanus' translation of Euclid's *Elements*, so there is no doubt that he was well acquainted with the work of his thirteenth century predecessor.

Campanus' awe as expressed in that short comment on an allegedly Euclidean proposition has now turned into full-fledged cultivation in the *Divina Proportione.* The tone of the book proceeds with Campanus' admiration for the peculiar properties of the division according to extreme and mean ratio. Pacioli goes a number of steps further in almost rewriting those parts of the *Elements* which deal in particular with the golden proportion, mainly from Book XIII and the so-called Book XIV. It is instructive to compare Euclid's own phrasing of the peculiarities of this proportion with the elaborate descriptions that Pacioli gives in the *Divina Proportione*, now and then addressing the duke to whom the script is dedicated, and referring to contemporary persons and events. Each of the 13 *stupendi effecti* of the divine proportion which Pacioli singles out is dealt with in a separate chapter (the first one, with some supplementary explanations to the duke, even takes four chapters). Of course space does not allow a full rendering of this treatment but the mere qualifications of the properties will suffice to see how Euclid's down-to-earth mathematical statements have, under the baton of Maestro Luca, turned into movements of Campanus'

[15] After 500 years, the first English translation is expected to appear soon: *The divine proportion*, New York, 2005.

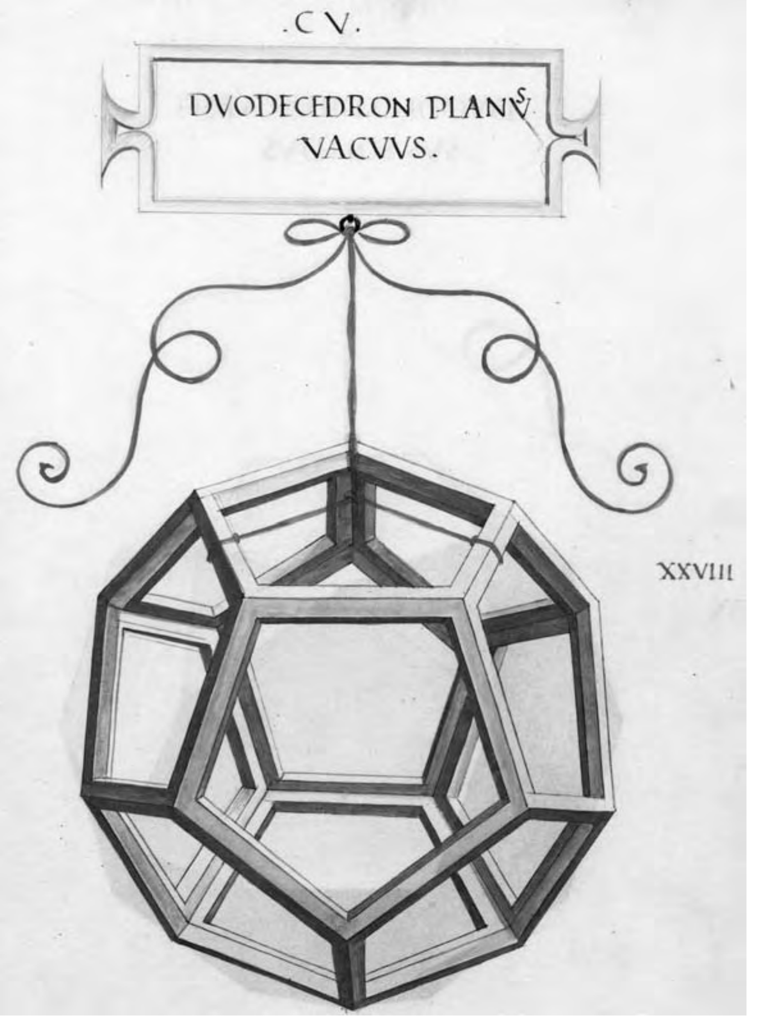

Fig. 4. Leonardo da Vinci's illustration of the open dodecahedron.

simphonia irrationalis. These are the qualifications in Pacioli's own *volgare*, each of which is the heading for the exposition of one of the properties of the divine proportion:

- *primo effecto de una linea divisa secondo la nostra proportione*,
- *suo secondo essentiale effecto*,
- *terzo suo singulare effecto,*
- *quarto suo ineffabile effecto*,
- *quinto suo mirabile effecto,*
- *suo* 6° *innominabile effecto,*
- *septimo suo inextimabile effecto*,
- *effecto converso del precedente,*

– *suo* 9. *effecto sopra glialtri excessivo,*
– *suo supremo effecto,*
– *suo* 11. *excellentissimo effecto,*
– *suo* 12. *quasi incomprehensibile effecto,*
– *suo dignissimo effecto*.

Yet, in spite of these high-sounding qualifications, Pacioli does not add any new mathematical insights to what can be found in Euclid's Book XIII and the supplement known as Book XIV. What he says in the sixteen chapters dealing with the stupendous effects shows that he knows his Euclid, but it does not justify his qualification of the admired proportion as being *divine*. At first sight, the choice of the term *divine* seems to fit in with adjectives like admirable, excessive and supreme, but the clergyman Pacioli takes it in a more literal sense: he submits that the character of the division according to extreme and mean ratio is inherently connected to the character of the Almighty. Pacioli illustrates this by explaining exactly which properties the venerated ratio has in common with God—and this is where his own ideas come to the fore, since these are not properties which he could have derived from Euclid.

The first property which God and the divine proportion have in common is their *singularity*. Neither belongs to a group that has more members; both are truly unique. The second is the Holy *Trinity*. The divine power is made up of Father, Son and Holy Spirit, and in much the same way, Pacioli believes, the divine proportion consists of three terms—that is, of one middle term and two extremes. The third property is their *irrationality*, a term taken here in the double meaning of "being beyond reason", and "not being expressible as the ratio of two integers". There are no words that equal His name; there are no numbers that equal this proportion. Both will always remain occult and secret. The fourth property they share is their invariable *continuity*. Both connect the parts with the whole in a way that does not allow for change. God is omnipresent; he is all in all as well as all in every part. The proportion shows the same type of omnipresence, says Pacioli: whether the parts are large or small, their ratio is always the same, both between the whole and the larger part and between the parts themselves. Finally, there is a common fifth property: the instauration of the *fifth essence*. Pacioli's theo-mathematical association here is that the dodecahedron, showing the divine proportion in its pentagonal faces and also in some of its other properties, is the "fifth body" that symbolises heaven.

To understand this last argument, the reader has to be familiar with Plato's *Timaeus*, in which the only five conceivable regular bodies are introduced (whence their alternative name: Platonic bodies) and given their function in the creation of the cosmos. In the cosmogony as narrated by Timaeus, four of the regular bodies take care of the ordering of the elements, which until then had not yet received a fixed form. The sharp angles of the tetrahedron make this the preferred form for fire, the octahedron informs air, the swirling of the waves calls for the twenty faces of the icosahedron to give a form to water, and the solid cube is the obvious shape of earth. The fifth shape, the dodecahedron, is called in to sustain heaven. Although Plato is not quite clear about what function the dodecahedron fulfils, this is the basis of the notion of a fifth essence (or quintessence), which the tradition has often related to a "fifth element"; that is, to ether.

Pacioli's explanations illustrate that he is dealing here with non-Euclidean mathematics in the tradition in which Cusanus also conceived of mathematics as a gateway to the understanding of divine power. Pacioli's dual interpretation of the concept of irrationality is the key to his understanding of the golden proportion as being divine, that is, supernatural. More than anything else, it is the miscomprehension of this theological conception of irrationality which has impeded later authors on the golden section from understanding what is meant by Pacioli's considering this proportion as *divine*, and why this divine section will not be recommended in this text to be used as a proportional guideline for a human craft like architecture. Incidentally, Pacioli refers to a biblical text to affirm his point, for instance when he compares the position of the divine proportion among the other proportions with the position of Christ among the people. In the Sermon on the Mount, Christ summons his followers: "Think not that I am come to destroy the law, or the prophets; I am not come to destroy, but to fulfil" (Matthew 5:17). Well then, says Pacioli, such is also the case of our proportion: sent from Heaven, it shares the conditions of all the run-of-the-mill proportions, not to disqualify these, but to glorify them by invariably maintaining the reign of unity, independent of the different quantities.[16]

Perhaps the text invites us to look for more associations. What are we to make of the *numbers* of the properties in the two lists—13 stupendous effects, and 5 properties in common between God and the divine proportion? We know that 5 and 13 occur in the Fibonacci Series, the series that evolved from the question "how many pairs of rabbits are born during one year if each pair gives birth to one new pair every month, beginning in the second month after its own birth?". This was one of the riddles from the *Liber Abaci* (1202) by Leonardo of Pisa, later nicknamed Fibonacci. Given the relationship between the golden section and this series, could it be accidental that Pacioli mentions 13 effects and 5 properties, whereas he could easily have made these lists longer or shorter?

The answer, in all likelihood, must be: yes, that is accidental. Pacioli did not mean to build in a hidden reference to the rabbit series. Nor did he choose these numbers to celebrate the existence of 5 regular and 13 semi-regular bodies. Attractive as these associations are, they are ours, not Pacioli's. Since only 6 semi-regular bodies are included in the *Divina Proportione*, Pacioli probably was not even aware of their total number. It is true that he was well acquainted with Fibonacci's work. Yet, he never related the divine proportion to the Fibonacci series. This is less surprising than it may seem at first sight. Pacioli's world of mysticism and ineffability is totally opposed to the world of everyday experience in which Leonardo of Pisa stages his riddles. It would have been impossible for him to understand his invariable celestial proportion as being related to the ever expanding additive series of promulgating rabbits. The rabbit riddle in the *Liber Abaci* was just one out of many, and there was no reason for Pacioli to start thinking about the connection between this particular riddle and the ratio to which he had devoted his *Divina Proportione*.

5. The blueprint of the universe

So who, then, was the first to realise the relationship between Pacioli's divine proportion and Fibonacci's series? Here is a surprise from history: this was Johannes Kepler, who

[16] "Cosi questa nostra proportione dal ciel mandata con laltre sacompagna in diffinitione e conditioni e non le degrada anci le magnifica piu amplamente tenendo el principato de lunita fra tutte le quantita indifferentemente e mai mutandose ..." *Divina Proportione*, ch. VII.

in the transitional period from the Renaissance to the mechanisation of the world picture was better equipped than anyone to unite religious understanding with empirical observation. In his research, as laid down in monumental works like *Mysterium Cosmographicum* (1596) and *Harmonices Mundi* (1619), Kepler united his two fields of study: theology and astronomy. Both should lead him to a better understanding of the lock, stock and barrel of Heaven.[17]

The problem in our understanding of Kepler today is that we are familiar with the answers he found but have become alienated from the questions he asked. We have learned to read Kepler's work in the way Newton spelled it out for us. In that way, Kepler is the discoverer of the three kinetic laws which describe the movements of the planets. But Kepler himself was unaware of having discovered these three kinetic laws, for the simple reason that he never asked a question to which mechanical laws could be the answer. For Newton, a planet's orbit results from the totality of forces exerted on it; for Kepler, on the other hand, the orbit is the expression of how God had meant the universe to be. God devised his Creation according to a plan, and Kepler saw it as his task to reveal God's blueprint. He received his inspiration from the Pythagorean tradition, which taught him that everything is ordered according to number and form. The way in which number and form are accessible to our senses is through music, and Kepler was sure that there must be a strong similarity between the formal cause of the harmonic character of musical intervals and the formal cause of the orbits of the planets. In fact, the *Harmonices Mundi* is the gospel of the music of the spheres. It was meant as such, and only after Newton read it, it turned out to contain the coping-stone of Kepler's laws of planetary motion.

From the passages in the *Harmonices Mundi* and in the other writings in which Kepler refers specifically to the division in extreme and mean ratio, it does not show that he himself had read Pacioli's book. He knows that it is a proportion "which today's geometers call *divine*",[18] yet he does not follow Pacioli's theological reasons for doing so. But although he believes that the term "divine" was chosen only "because of its wonderful nature and its many peculiar properties",[19] he does develop a new connection between the divine proportion and the divine Creation: Kepler believes that the divine proportion is the *archetype of procreation*. This was not a subject that Fra Luca had paid any attention to, and it shows the difference between the two Renaissance scholars in their approach to form. Pacioli sees the analogy between God and the divine proportion more *in abstracto*, in their common properties which elevate them above everyday knowledge. For Kepler, on the other hand, the ineffable divine proportion is the preformation of the ongoing biological process by which the similar is generated by the similar.[20] We may say that in his search for a

[17] For a more extensive discussion of Kepler's quest for the formal cause of the universe, see A. van der Schoot, "Kepler's search for form and proportion", in *Renaissance Studies*, Vol. 15, No. 1, 2001, pp. 59–78.

[18] "(P)roportione illa, quam hodierni Geometrae Divinam appellant"; *Strena Seu de Nive Sexangula*, in *Gesammelte Werke*, Band IV, eds. M. Caspar and F. Hammer, München, 1941, p. 270.

[19] "Hodierni et sectionem et proportionem ejus cognominant Divinam, propter admirabile ejus ingenium, et multiplicia privilegia". *Harmonices Mundi* (1619), Book I, def. xxvi, in *Gesammelte Werke*, Band VI, ed. M. Caspar, München, 1940, p. 29.

[20] "Hanc ego geometricam proportionem puto Ideam fuisse Creatori ad introducendam generationem similis ex simili: quae est etiam perennis (I believe that this geometrical proportion led the Creator to the Idea of introducing the generation of the similar from the similar: for this, too, is perennial)". Kepler wrote this on May 12, 1608, in a letter to Joachim Tanck. *Gesammelte Werke*, Band XVI, ed. M. Caspar, München, 1955, p. 156.

form that serves as a *causa formalis* for something yet to come, Kepler is really in the middle between the static metaphysical conception of Pacioli, who accepts an irrational and ineffable form as an eternal symbol for God's hidden omnipresence, and the dynamic mechanical conception of Newton, who took leave of formal causality in favour of *effective* causality. Kepler is different from both in that he thinks Platonically: he tries to establish a relationship between a supernatural *Urbild* (primordial image) and a natural *Abbild* (representation), that is, his search is one for a primordial image of which the phenomena known in our experience of nature partake.

So why does Kepler believe the divine proportion to be such an *Urbild* regarding natural procreation?

Several indications point in the same direction—if one follows Kepler's thinking, that is. The reader is warned in advance that none of his arguments will sound more convincing in our ears than did Pacioli's criteria for the divinity of our proportion, and, moreover, that their combination leads up to a very strange hotchpotch.

First of all, there is the argument from the regular bodies. Kepler has extensively studied Plato's *Timaeus*, whose importance we have already seen in Pacioli. Kepler adopts the notion of the dodecahedron as somehow being the shape of the heaven above the earth. He expands that picture in his *Mysterium Cosmographicum* by separating the orbits of the planets by means of the regular bodies: each time a different body between each pair of planets (five bodies sufficing for separating the six planets known in his time). The dodecahedron is then situated between Earth and Mars, whereas Earth and Venus are separated by the icosahedron. This was not just Kepler's way of adorning the universe; he was convinced that this was the model which God had materialised when creating heaven and earth. This would imply that God had situated the earth, the only planet where procreation takes place, in between the two bodies whose construction depends on the divine proportion.

Next to this argument from stereometry, there is also a planimetric argument. Like many others, Kepler has noted that flowers often show a pentagonal form. The pentagon is the face of the dodecahedron and it is characterised by the divinely proportional relationships between its sides and its diagonals. The fact that this very shape is visible in the flowers must be a symbol of their seminal faculty: "I believe that the ability to reproduce has come about by analogy with this self-generating proportion: thus, in blooming, the authentic pentagonal banner of reproductive ability leads the way".[21]

This leads to the third argument, the argument from continuity. Here, Kepler comes somewhat closer to Pacioli's line of thinking. The continuity of the division according to extreme and mean ratio implies that every new division will produce the same result, in terms of proportion, output being equal to input. So this is really the birth of the similar from the similar, in an endless continuation; Kepler understands that this property is unique for this particular proportion.

And then, finally, there is the argument which Kepler derives from his insight into the relationship between the golden section and the Fibonacci series. Kepler realises that in every consecutive triad of Fibonacci numbers, the product of the first and the third term is

[21] "Ad huius proportionis seipsam propagantis similitudinem, puto effictam esse facultatem seminariam: itaque in flore praefertur seminariae facultatis γνησιον vexillum Quinquangulum". *Strena Seu de Nive Sexangula,* p. 270.

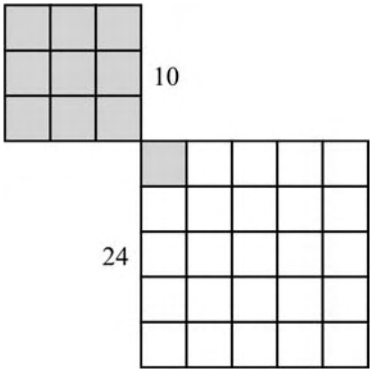

Fig. 5. Kepler's geometrical sexology.

always (alternately) one unit larger or smaller than the squared middle term:

$$2, 3, 5: \quad 2 \times 5 = 3^2 + 1,$$
$$3, 5, 8: \quad 3 \times 8 = 5^2 - 1,$$
etc.

Kepler gets entangled in surprising diagrams to show the morphological analogy between this mathematical insight and the practice of animal procreation: are the two partners, who produce the similar from the similar, not characterised by the fact that the male body shows a protrusion (the "unit" of Kepler's diagrams) which neatly fits into the opening of the female body? We are indeed dealing here with an author who tries to find the archetypes of God's creation, and believes these can be found in mathematical constellations.

What I have to emphasise here is that Kepler's juggling with the divine proportion stands in complete isolation, both from the earlier, likewise theologically inspired speculations of Pacioli, one hundred years before Kepler, and from the playful exposition of the procreation of rabbits, as presented in the form of a riddle by Leonardo of Pisa, at the beginning of the thirteenth century. Kepler is also completely isolated from the later developments which link the divine proportion in particular to architecture and pictorial art. For centuries after his death, Kepler's writings on the divine proportion were either not accessible to the educated readership (the *Strena* and the aforementioned letter to Tanck), or too complicated and far-fetched to be met with much enthusiasm (the *Harmonices Mundi*). And since Pacioli's book was not very widespread either, it took more than two centuries after Kepler's death before the divine proportion appeared again on the scene, this time for a larger audience.

6. Unity in variety

Adolf Zeising, the German philosopher responsible for this revival, knew nothing of either Pacioli's or Kepler's involvement with the proportion to which he devoted his 1854 monograph,[22] referred to here as *Neue Lehre*; this in spite of his effort to find out what had happened earlier in the field of preference for certain proportional relationships over others, especially with regard to the appreciation of the shape of the human body. In the first part of his book, he discusses as many as eighty predecessors before finally presenting his universal solution to all problems of proportional nature.

Zeising tries to mention every philosopher who had anything at all to say about proportions, from Pythagoras, Plato, Aristotle and Plotinus up until Burke, Kant, Hegel and Schelling, as well as practising artists who left their ideas on proportions on record, with an understandable emphasis on the artists from the Renaissance. He even does research into the works of anthropobiologists and physicists who established norms for the proportions of the human body: Quetelet, Hay, Camper, Carus and a host of others. And yet, among those whom he fails to mention are the only two people who played a significant role in the previous history of his favourite proportion: Luca Pacioli and Johannes Kepler. Leonardo da Vinci is considered, but only as the author of the *Trattato della Pittura*, not as the illustrator of the *Divina Proportione*.

It is, therefore, without any appeal to his predecessors that Zeising reveals to the world that for proportion to meet the demands of beauty, unequal parts must yet appear as being parts of one totality. Therefore, their inequality in size must be balanced by an equality of the ratio of the parts to one another and the ratio of part and whole.[23] This is just another way of phrasing the divine proportion, and it comes as no surprise that this is the "morphological foundation" which the title of his book announces and which, in Zeising's view, is constitutive for all but every shape in nature, particularly the human body, as well as for the proportional build-up of works of art, especially in architecture. Zeising now refers to this proportion by the new term which had come into vogue among mathematicians during the decades preceding the publication of his book: *der goldene Schnitt*, the golden section.

Once he has established the golden section as the aesthetically most rewarding proportion (and again: there is no evidence of any earlier author linking this particular proportion to aesthetic attractiveness), there is no doubt in his mind that it must be traceable in the shape which, in Zeising's view, has always been the ideal of perfect proportional articulation: the human figure. The structure of the human figure must, therefore, be understood from nature's applying the golden section in each and every detail of the body. (See Fig. 6.)

With Adolf Zeising, we have reached a framework of thinking completely different from the theologically inspired speculations of Pacioli and Kepler. Yet, after having considered the omnipresence of the golden section in nature, architecture, music and poetry, Zeising ends his book with some reflections on the importance of the proportional law in the field of ethics and religion.

These reflections are based on the insight that life always presents us with a tension between unequal forces; an ethical balance must be found in an equal validation of these.

[22] *Neue Lehre von den Proportionen des menschlichen Körpers, aus einem bisher unerkannt gebliebenen, die ganze Natur und Kunst durchdringenden morphologischen Grundgesetze entwickelt und mit einer vollständigen historischen Uebersicht der bisherigen Systeme begleitet*, Leipzig, 1854.

[23] "Wenn die Eintheilung oder Gliederung eines Ganzen in ungleiche Theile als proportional erscheinen soll: so muss das Verhältniss der ungleichen Theile zu einander dasselbe sein, wie das Verhältnis der Theile zum Ganzen". *Neue Lehre*, p. 158.

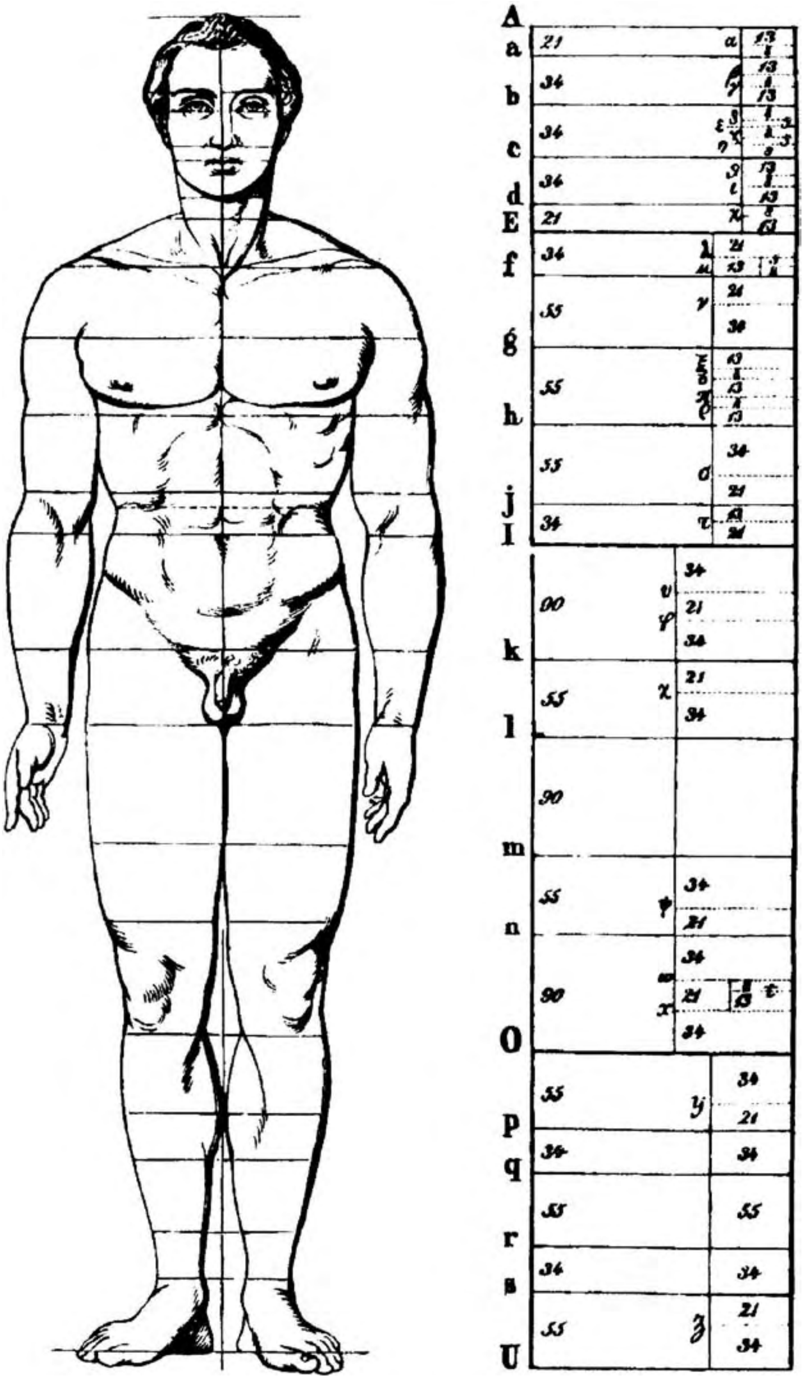

Fig. 6. Zeising's golden dissection.

Therefore, Zeising claims, neither eternal peace and communist equalising, nor continuous war and social suppression are acceptable solutions for a humane society. A similar balance has to be found in our conception of God: neither the strict, monotheistic religion of the Jews, in which there is an unbridgeable gap between God and man, nor a polytheistic arbitrariness that blurs the difference between the one Creator and the many creatures, could satisfy our religious need. Fortunately, Christianity provides for a sensible synthesis in the Divine Trinity, the perfect unification of unity and multitude. Christ is the personification of this synthesis, being both the son of God and the son of man.

In this last argument, Zeising seems to get closer to the mental atmosphere of Pacioli than in the earlier parts of his book. Yet, he is merely applying an idea shared by many German thinkers of the first half of the nineteenth century: the idea that, both in nature and in culture, development results from a mediated confrontation between two polar opposites. This principle of *Polarität*, advocated especially by Goethe and Schelling, was widely adopted in those days, in physics even more than in the study of culture. The exact value of the divine proportion has no meaning in this conception, but it does add the lustre of scientific exactness to general convictions about the "truth being somewhere in the middle". The "golden section" has now left its mathematical past behind, and has become a synonym of the "golden mean".

In his later, equally voluminous book on *Religion und Wissenschaft* (1873), in which he defends the Old-Catholic opposition against the dogma of papal infallibility, as proclaimed by the First Vatican Council in 1870, Zeising does not return to the omnipresent proportion. But the idea of the *Neue Lehre* struck root; and before too long, many art historians were convinced that the golden section had been a leading device for artists in antiquity and the Renaissance.

A more extensive account of the history of the golden section is to be found in A. van der Schoot, *Die Geschichte des goldenen Schnitts*, Stuttgart, 2004.

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