CHAPTER XXXVII

ON THE NOTION OF 'SIMULTANEITY'

> So we see that we cannot attach any *absolute* signification to the concept of simultaneity, but that two events which, viewed from a system of co-ordinates, are simultaneous, can no longer be looked upon as simultaneous events when envisaged from a system which is in motion relatively to that system. (155)
>
> A. EINSTEIN

In the older days we accepted as self-evident the structural assumption that there is sense in such a statement as that an event A on the sun was 'simultaneous' with an event B on the earth. We assumed also that the 'moments of our consciousness' had a universal 'meaning'. We tacitly assumed, for instance, that when we saw or photographed an event on the sun, that it happened just the moment we saw it. Such structural assumptions were rudely disturbed by the discovery of the *finite* velocity of light. Today we know that when we see or photograph an event on the sun, that event happened approximately eight minutes earlier, as it takes about eight minutes for the light from the sun to reach our earth. We begin to realize that the moments of our perceptions have no universal significance.

We inquire first what we mean structurally by simultaneity. We do not need to go into details. The application of functional and contact methods, even in the rough, will assist us. We can speak in terms of instruments. For instance, we can build a special, very fast moving picture camera, C, with two lenses D and E, at two opposite sides, and a calibrated film, F. running rapidly through the middle of the camera as shown in Fig. 1. If we focus our double camera on two flashes, A and B. occurring at 'equal distances', *L*, from the film, we say that the flashes occur simultaneously by definition if the pictures, a and b of the flashes A and B. appear exactly opposite each other on the film, or if we have *one* picture. If, under the conditions of the experiment, where the distances between the origins of the flashes and the film are equal, and our film is moving very rapidly, the pictures of the flashes do not occur exactly opposite each other, but one picture is separated from the other, then we have two pictures, and conclude, by *definition*, that the flashes are *not* simultaneous.

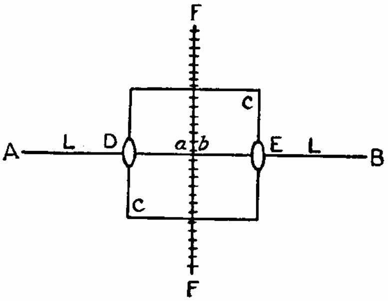

FIG. 1

We introduce this hypothetical instrument to show that, in discussing physics, and the theory of Einstein in physics, we do not speak of 'psychology' or personal 'subjectivity', but that we do deal with the inherent physical subjectivity of the instruments and the finite velocity of propagation. When we discuss the psycho-logical or methodological or semantic significance of science and scientific method, we deal with different subjects.

When we use the term ‘observer’ we mean an observer so equipped that he can do whatever is demanded of him.

What was said about the definition of ‘simultaneity’ by the aid of the camera, applies also to ourselves.

The problem of prime importance before us is to find out if ‘simultaneity’, as defined, has an ‘absolute’ and universal significance, or if it is perhaps a private and relative notion.

We will carry out the analysis in two ways, the first by example, which will be instructive, though perhaps not completely conclusive; the other, by the use of the Lorentz-Einstein transformation.

Let us perform our last experiment, which, with modern methods seems to be feasible, in a slightly more complicated form.

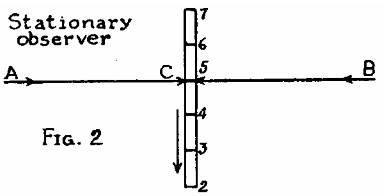

FIG. 2

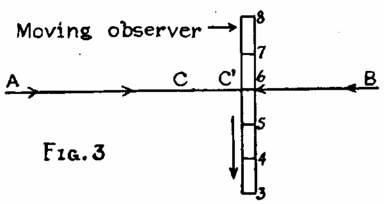

FIG. 3

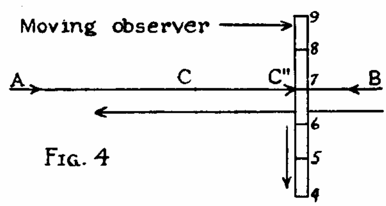

FIG. 4

We can select a dark night in which flashes will photograph well even at considerable distances. We can place powerful projectors at A and B. and we place our camera so that the film will come exactly at C, midway between A and B. We can start the mechanism of the rapidly moving film and, by an electrical contact made at C, we can produce a short flash from each of the two projectors. Because of the assumptions, AC=CB, and equal velocity of the propagation of electrical currents and light-waves in all directions we shall have by the structural definitions which condition the experiment, *one* picture in Fig. 2, say at the spot of our moving film marked by 5. The rays of light from A and B would arrive ‘simultaneously’—that is, ‘at the same time’—and would affect our moving film in *one spot.* Our definition was for a stationary observer, and under the conditions, the experiment was fairly definite—all the underlying structural assumptions, of course, being taken for granted.

Now consider an observer, as shown in Fig. 3, moving uniformly in the direction from A to B.

Let us assume that he is also equipped with a similar sort of moving picture camera as the stationary observer, and that just before he passes the point C the electrical impulse to the projectors is sent. Let us assume further that the mark 5 on his moving film is exactly at the focal point of the camera as C is passed. The electrical impulses travelling from C to A and B would travel the distance AC = BC, produce the flashes A and B which again would travel with finite velocity in all directions. During the interval these impulses and light-waves are travelling, our observer is moving from A toward B. and spot 5 on his moving film is no more at the focus of the camera. Obviously he will meet the light-wave from B first, at C', let us say, when mark 6 on his film is at the focus (Fig. 3). After another short interval when he reaches C" and mark 7 on his film is at the focus, the light-wave from A overtakes him (Fig. 4).

So we see that what was ‘simultaneous’ (by definition) and produced *one* impression on the moving film of the stationary observer, was not ‘simultaneous’, (again by definition), for the moving observer, as *his* film registers *two* pictures.

As both observers use similar instruments and one set of definitions, obviously both are entitled to claim that their records on the film are conclusive. So the first can claim that the flashes were ‘simultaneous’, the second can claim that they were not ‘simultaneous’. The reverse is equally true. If the moving observer had *one* picture, and claimed ‘simultaneity’, the stationary observer would have two pictures, and deny ‘simultaneity’.

But when two observers are equally *justified* in making *two* opposing claims where, by their very meanings, there is only one possible, we must conclude that the claim itself is meaningless. We see that ‘absolute simultaneity’ is a fiction and impossible to ascertain, as it would depend on some impossible ‘absolute motion’, or ‘infinite velocity’ of propagation of signals.

The analytical form of showing the impossibility of ‘absolute simultaneity’ is very simple, and follows directly from the Lorentz-Einstein transformation.

Let us imagine two observers, one in an S system of co-ordinates (x, y, z, t) and another in an S' system of co-ordinates (x', y', z', t') moving relatively with the velocity v.

Let us assume two events happening in the unprimed system at the point (x_1, y_1, z_1) at the ‘time’ t_1, and the other at the point (x_2, y_2, z_2) at the ‘time’ t_2. According to the Lorentz-Einstein transformation the ‘times’ at which the two events occur relatively to the primed system are given by the formulae:

$$t'_1 = \beta\,(t_1 - x_1 v/c^2),\ t'_2 = \beta\,(t_2 - x_2 v/c^2),\ \text{where as usual } \beta = 1/\sqrt{1 - v^2/c^2}\ .$$

If we assume that in our unprimed system S the two events were ‘simultaneous’, which means that they ‘occurred at the same time’, t_1 would be equal to t_2, that is, $t_1=t_2$, or $t_1-t_2=0$. Let us find the difference between the two primed ‘times’ in the moving system S', and see if this difference is zero, which would mean that the primed ‘times’ are equal.

Returning to our formulae which give us the values for the primed system ‘times’, we express their difference as

$$t_1'-t_2' = \beta\,(t_1-x_1v/c^2) - \beta\,(t_2-x_2v/c^2) = \beta\,(t_1-t_2+x_2v/c^2-x_1v/c^2).$$

But we assumed $t_1-t_2 = 0$; therefore $t'_1-t'_2= \beta\,(x_2v/c^2-x_1v/c^2)$.

This last formula shows clearly that $t'_1-t'_2$ cannot be zero; or in other words, t'_1 cannot be equal to t'_2 unless $x_1 = x_2$.

The two events which, for an observer in the unprimed system, happen 'simultaneously', ($t_1=t_2$, or $t_1-t_2=0$) at different places, (or x_1 not equal to x_2, $x_1 \neq x_2$), cannot be 'simultaneous' for the moving observer in the primed system S', but will happen at different 'times' (t'_1 is not equal to t'_2, or $t'_1-t'_2 \neq 0$).

It is extremely instructive to consider further what happens in measuring 'times' and 'lengths' in systems which are moving relatively to each other.

If, in the equation (1) above, we assume $x_1=x_2$, this means that both events occur at one place in the stationary unprimed system S.

By changing the signs and cancelling the terms with x_1 and x_2, which are equal and of opposite signs, we have $t'_2-t'_1=\beta(t_2-t_1)$ whence, substituting for β its value $1/\sqrt{1-v^2/c^2}$, we obtain

$$t'_2-t'_1=\frac{t_2-t_1}{\sqrt{1-v^2/c^2}}$$

This last formula brings out a few remarkable issues. In terrestrial velocities the square of the velocity of the motion v^2 of the observer in the primed system S' is very small as compared with the square of the velocity of light c^2, so the fraction v^2/c^2 is small, $\sqrt{1-v^2/c^2}$ differs very little from unity but the whole denominator is *less* than unity, and so $t'_2-t'_1$ is not equal to t_2-t_1, but greater.

In other words, the interval of 'time' between the two events appears *larger* to the observer in the primed moving system than to the observer in the stationary unprimed system. In general, among all systems in a state of uniform relative motion, that one in which two events occur at *one place*, is characterized by the fact that the 'time' interval between the two events appears *shortest* to an observer in this system. The shortest interval means that to an observer in the system, the events run their course most rapidly. A process which, with reference to a given system, occurs in *one* place, appears to run its course most rapidly to an observer in that system, but more slowly to a moving observer in any other system.

The more rapid the relative motion, the slower the process will appear, and, in the limit, if an observer could move with the velocity of light, $v^2=c^2$, the denominator of our equation would become 1-1=0 and $t'_2-t'_1$ would become 'infinite' and all events would be at a standstill.

As the formulae for length, x and x', involve the 'times' and, as we see, the intervals of 'time' are dependent on the relative velocities, by a similar process of reasoning we find that the standards of length are also relative, and that the length L' in the primed system is represented by $L'=L\sqrt{1-v^2/c^2}$. In other words, to an observer who sees the rod in motion, it will appear 'shortened', and among all systems in a state of uniform relative motion, the one in which the rod is at rest is distinguished from all others by the fact that in it the rod appears longer than in any other system. For instance, a metre rod lying on the earth in the direction of its motion would appear to an observer on the sun to be shortened by 5×10^{-7} cm. In the limit, when $v=c$, the fraction $v^2/c^2=1$, 1-1=0 and $L'=0$, which means that to an observer moving with the velocity of light, a three-dimensional body would appear as two-dimensional, or a two-dimensional figure as one-dimensional. The co-ordinates *y* and *z*, as we have seen, do not enter into consideration as they are equal in both systems moving relatively in the *X* direction, and the ‘time’ co-ordinates are independent of them.

If a body at rest appears to the observer in the unprimed system as a sphere, it will appear as an *oblate spheroid* to an observer in the primed system.

We see that structurally not only ‘simultaneity’ and ‘time’ are not absolute but also that length, and therefore *shape*, is relative.

We have seen that the ‘shortest’ and ‘longest’ values are important characteristics of the motion. This suggests why in the general theory of Einstein we are interested in, and introduce, geodetics.

It should be mentioned here that the Lorentz transformation has been reached by difficult considerations involving Maxwell’s electromagnetic field equations, unrelated to the Einstein theory. Einstein found the Lorentz transformation by the *simplest consideration* connected with his theory. The finding of such important equations by two methods, entirely different structurally, must be considered as a convincing proof of the fundamental importance of such formulae, the more so since they follow from very simple and fundamental structural principles which in themselves cannot be denied because they are negative in character. Negative statements are on a different footing in the new systems; they follow structurally from a $\overline{A}$ orientation, just as the older positive dogmas were the structural results of aristotelianism and the delusional results of identification.

The facts mentioned concerning the measures of length and the behaviour of clocks do not present any paradoxes. They simply say that these discrepancies are mutual and inevitable, as any measurement is only a measurement when it can be registered by an instrument, or seen, or recorded in some way. If the measuring rods and clocks are moving relatively to us, what we see or what our instruments record is *not* what is happening on the moving system, which no one can see or record from outside the system. What reaches us is simply what the light-waves or other signals moving with finite velocity (and therefore retarded by a motion away from us) bring to us. As all existing methods of communication and all known signals have *finite velocities*, these structural differences which are conditioned by the inherent characteristics of the world should be taken into consideration in modern science.

If we draw a square *ABCD* (Fig. 5) and an aviator E were to pass this square sign with a velocity of 161,000 miles a second* in the direction *AB*,

* I deliberately select such a velocity so as to make the contraction given by the formula $L' = L\sqrt{(1 - v^2/c^2)} = L/2$. With this aim we must make the fraction represented by $v^2/c^2 = 3/4$, then $1 - 3/4 = 1/4$ and $\sqrt{1/4} = 1/2$. We find the square of our velocity v by taking 3/4 of the square of the velocity of light $v^2 = 3/4\ c^2$ and find $v = c/2\ \sqrt{3}$ = 161,000 miles a second.

he would see—and any instrument carried by him would register it—the sides of *our square (AB=BC=CD=DA)* in the direction of his flight, namely *AB* and *CD,* as 'contracted' to half their length. If he turned at right angles, the sides *AB* and *CD* would 'expand' and the other sides, which are at right angles, *BC* and *DA,* would 'contract'. For us the sides *AB* and *BC* are *equal,* for him one appears twice the other. To him our square appears oblong.

Under such *natural structural* conditions it is a fundamental fallacy to ascribe to 'lengths' or 'shapes' or 'times' any 'absolute' significance. If we grasp the structural fact that 'length' and 'duration' are not *things* inherent in the external world, nor are 'matter', 'space', and 'time', but that they appear as relations between events and some specified observer, and forms of representations, then all paradoxes would disappear.

A suggestion which concerns visualization may be helpful. If we realize the structural fact that words *are not* the objects they represent, we shall always discriminate automatically between what we *see, feel.* , on the level of lower order abstractions, and what we *say* on the level of higher order abstractions. When we have conquered that single difficulty we could never then identify the two different orders of abstractions. We would evaluate the *terms* 'matter', 'space', and 'time' as forms of representation, and non-objects, and we would describe events in a functional, operational, behaviouristic language of order. If we realize and feel the *finite* velocity of propagation of all processes, we may visualize all that has been explained here. Diagrammatizing and even following with one's hand, the *visualized order* of occurrences, helps enormously. Try to visualize how the aviator in the last example is flying away and how much more slowly the light impressions from the earth are reaching him or his instruments, and the difficulties will soon vanish.

We shall also be greatly helped in our power of visualization when we become acquainted with the structure of the Minkowski four-dimensional world. An explanation of this appears in the next chapter.