PART IX

ON THE SIMILARITY OF EMPIRICAL AND VERBAL STRUCTURES

The theory of relativity has resulted from a combination of the three elements which were called for in a reconstruction of physics: first, delicate experiment; secondly, logical analysis; and thirdly, epistemological considerations.
(457) BERTRAND RUSSELL

The essence of Einstein's generalization is its final disentanglement of that part of any physical event which is contributed by the observer from that which is inherent in the nature of things and independent of all observers. (21) E. T. BELL

Even Leibniz formulated the postulate of continuity, of infinitely near action, as a general principle, and could not, for this reason, become reconciled to Newton's Law of Gravitation, which entails action at a distance and which corresponds fully to that of Coulomb. (547) HERMANN WEYL

This limitation to what is directly observable is ultimately based on Mach's philosophy and, directly inspired by Mach, led three decades ago to the propagation of the so-called theory of "Energetics," which sought to recognise only quantities of energy as physically given and observable quantities.
(481) A. SOMMERFELD

CHAPTER XXXV

ACTION BY CONTACT

> The difficulty involved is that the proper and adequate means of describing changes in continuous deformable bodies is the method of *differential equations.* . . . They express mathematically the physical conception of contiguous action. (45)MAX BORN

The analysis of 'matter', 'space', and 'time' from the point of view of structure and of orders of abstractions has led us to far-reaching conclusions. Let us summarize the semantic results, and consider some of the immediate consequences.

We may begin by recalling the difference between the lower order and higher order abstractions. The lower order abstractions are given to us by the lower nerve centres. They are 'dynamic', 'continuous', non-permanent, shifting, unreliable, and above all *un-speakable.*

They have a character of immediacy, because, structurally in terms of order, they are closest to outside events. They come first in order in the functioning of the nervous system. We always associate with them some 'objectivity' as, by necessity, the eventual definition of an 'object' starts at this level.

It should be emphasized over and over again that, speaking correctly, *on this level* we cannot define anything, since abstractions on this level are fundamentally *un-speakable.* We may look, listen, handle, feel. , but *cannot speak* and therefore cannot define. The moment we *define* our objects, we are no longer on the level of lower order abstractions. By neurological structural necessity we have passed to the higher nerve centres (speech), and higher order abstractions. This is what is meant when we say that this lower level is *un-speakable.*

Because these lower order abstractions are closer to the outside events, and because they come *first in order,* they have a special character of immediacy, with which we *must start.* The struggle begins when, through some primitive-made doctrines or structural assumptions (metaphysics), we try to avoid going any further than these lower order abstractions. As a matter of fact, this is an impossibility, because of the very structure of our nervous system. However intensely we believe that it is possible to do so, and however 'emotionally' we attempt to do it, we are cherishing delusions, which easily become *morbid* identifications, delusions, illusions, and often hallucinations.

This level being *un-speakable,* the only way to function on this level is to look, listen. , but to be *silent* outwardly as well as *inwardly.* This last condition represents a most beneficial semantic state, really-difficult, perhaps impossible, to acquire without training.

The higher order abstractions appear to be products of the activity of the higher nerve centres, further removed from the external events and lacking, therefore, in immediacy. But these higher abstractions are static and so may be analysed. They have a separable unit ‘quantum’ structure, which can be treated individually. It should be carefully noticed that the static character of these higher order abstractions is the origin of their separable quantum character, conditioned by the human nervous structure. They are, if properly treated, reliable and are uniquely responsible for our being time-binders.

Again, by the structural necessity of our nervous system, we deal first with lower order abstractions, and next with higher order abstractions. It must be noticed that *no one,* unless he is (pathologically) entirely deprived of the higher nerve centres, is, or can be, an exception. We all deal with lower abstractions first, and with the higher next, no matter how perfect or imperfect these abstractions may be.

The general confusion of orders of abstractions, the lack of theories, and therefore of structural understanding of the entirely different characters of these distinct orders of abstractions, leads to, and must result in, identification or confusion of orders of abstractions. As the different processes are going on, whether we will it or not, in every single one of us, they may result in the *delusional* ascribing of the characteristics of the higher order abstractions to the lower order abstractions, as for instance, permanence, immutability. , somewhere involving ‘infinities’. When objectified, we have such semantic disturbances as fanaticism, absolutism, dogmatism, finalism, which often become morbid semantic states.

A similar confusion may lead to the delusional ascribing of the characteristics of the lower order abstractions to the higher ones. Under such delusions we ascribe to the higher abstractions fluidity, shiftiness, non-permanence, ‘non-knowability’. , which results in pessimism, cynicism, disregard for science, bitterness, fright, hopelessness and other equally vicious semantic disturbances. These in turn affect by structural necessity the proper working of the entire organism, which always works as-a-whole.

The *A*-system and other older systems were not only built before these facts became structurally known but were actually based on such confusion. Hence their viciousness. By building a language and a method of this nature they perpetuated and made effective *mechanically* through the structure of language, a harmful confusion. This language being not in accordance with the structure and functioning of the nervous system and the world, must produce pathological results somewhere.

We have already seen that the use of the ‘is’ of identity is unconditionally delusional. Naturally, attitudes (affective, lower order abstractions) which can assert, (higher order abstractions) that so and so on objective levels ‘is’ so and so, must lead to pathological results. In science this is a profoundly unsatisfactory state of affairs and needs structural revision.

Mathematicians, though in the main unconscious or innocent of the structural, semantic, and neurological issues involved, nevertheless have solved this problem by producing methods of passing from one order of abstractions to another, from dynamic to static, and vice versa. The influence of these discoveries has also affected the other sciences *unconsciously*. Without consciously recognizing it, the modern trend of science is to banish from its habits and methods the application of the 'is' of identity.

So in science we have to use an actional, 'behaviouristic', 'functional', 'operational' language, in which we do *not* say that this and this 'is' so and so, but where we describe *extensionally* what happens in certain *order.* We describe how something *behaves,* what something *does,* what we *do* in our research work, . If one asks, for instance, what *is* 'length', what *is* 'space', what *is* 'time', what *is* 'matter'. , the only correct answer would be, 'As you asked the question verbally, and I answer it *verbally,* the above *terms remain terms,* which beside structure, have no connection whatsoever with the external world'. Yet undoubtedly we are interested in this external world and we should like to use a language which would help us in understanding this world better. What shall we do ? It seems that if we produce a language which is *similar in structure,* to the external world, somehow, as a map or picture is similar in structure to the region it pictures, we should have a uniquely appropriate language. How can we do it ? It is quite simple the moment we discover the principle. First of all, abandon completely the *A* 'is' of identity, and, instead, describe ordered happenings in an actional and functional language. Such a language shares with the external world at least the multi-dimensional order of happenings, and it gives us a solution.

It is easy to see that arguments (verbal) about 'matter', 'space', 'time'. , will never become anything else than verbal. All uses of the 'is' of identity, must lead to delusional evaluation. The situation is radically changed when we use an actional or functional language, when we describe what a physicist does when he finds his 'length' or 'second' or any other entity he is interested in.

We should notice here that the above procedure involves extremely far-reaching structural and semantic consequences. First of all, we abandon the vicious use of the 'is' of identity, and eliminate the semantic disturbance called identification. We introduce automatically the full psycho-logical working mechanism of *order, extensional* methods and discrimination between the orders of abstractions. We introduce the four-dimensional and differential methods, we build up static units, 'quanta', and so introduce *measurement* and its language called mathematics, which leads to structure and so to knowledge at each date.

It will be useful to recall why mathematics and measurements are somehow so important in our lives. Our nervous system, as we have seen, exhibits different activities on different levels. On one level the abstractions are shifting, non-permanent; on the other static and permanent in principle. This is expressed in our lives in a longing for some permanency, some security, some 'absolutes'. Mathematics formulated this tendency first and with *full success.* Mathematics has not only formulated full and successful theories of 'change', as, for instance, the theory of functions and the different calculi, but also full-fledged and remarkable theories of *invariance* under transformations. These new theories of invariance are actually *absolute* formulations in the only sense in which the term ‘absolute’ has a meaning; namely, relative, no matter to what; all of which leads to the only content of knowledge—structure.

The whole Einstein theory should, in this sense, be called the ‘theory of the absolute’, and can be expressed as the simple demand that ‘universal laws’ should be formulated in an invariant form, a most revolutionary demand and yet so *structurally natural* that no one can deny it.

When we mathematize or speak about potential or actual measurements, we are dealing with *ordered, extensional,* actional, behaviouristic, functional and operational entities, and so we build up a language which at least has a similar *structure* to the external events. Numbers imply units, quanta, but also order. It seems that number is the only abstraction upon which we all must agree. We never doubt that a statement, such as that ‘I have in my pocket five pennies’, may be perfectly definite and ascertainable for all. The specific and unique relations called numbers seems to have absolute significance. It must be added that the existence of non-quantitative branches of mathematics does not alter what is said here. In these branches, the asymmetrical relation of order remains paramount and we may treat numbers from either of their two aspects, the cardinal or the ordinal.

The epoch-making significance of the Einstein-Minkowski work consists precisely in the fact that they were the first to *apply* the above, though without, it is true, formulating the general principle. The lack of such a general, $\overline{A}$, epistemological formulation retards considerably the understanding of their work, and so laymen miss the enormous structural, and semantic beneficial effect upon the proper working of our nervous system and our sanity.

Before giving a short methodological account of the Einstein theory it will be well to recall some structural and semantic conclusions which the differential calculus suggests.

When we were dealing with the notion of a variable, we saw that the variable might be *any* element selected out of an ordered aggregate of elements. We can select elements relatively widely separated from each other, as, for instance, the numbers 1 and 2, or points, let us say, an inch apart. It is obvious that if we choose, we can make the gaps smaller, and postulate an infinity of intermediate steps. When we make our gaps smaller, the elements are ordered more densely and closer together. In the limit, if we choose indefinitely many elements between any two elements, our series become compact, if we still have a possibility of gaps; or they eventually become what we call *continuous,* when there are no more gaps.

Without legislating as to whether the entities we use in physics are ‘continuous’, ‘compact’, or ‘discontinuous’, we may grant that the maximum elucidation of the above terms in mathematics is very useful. We can easily see that in terms of *action* a continuous series gives us *action by contact,* since consecutive elements are indefinitely near each other. As the differential and integral calculus were built on the structural assumption of *continuity,* the use of the calculus brings us in touch not only with our x but also with its indefinitely close neighbour $x+dx$. We see that the calculus introduces a most important structural and semantic innovation; namely, that it is a language for describing *action* by *contact,* in sharp contradistinction to the structural assumption of action at a distance.

Let us illustrate the above by a structural example. Consider a series of equal small material spheres connected with each other by small spiral springs as shown on Fig. 1.

FIG. 1

These little spheres all have inertia, because of which, and because of the little springs, they resist displacement. If we displace the first of our spheres either in the transverse or longitudinal direction, it acts upon the second sphere, which in turn acts upon the third, . We see that the disturbance of equilibrium of the first little sphere is transmitted like a wave to the next sphere and so along the whole series. The most significant point in the analysis of such a wave of excitation is that it is not transmitted with some 'infinite velocity', or 'infinitely quickly' or in 'no time'. The action of each sphere is slightly delayed owing to its inertia, that is, it does not respond 'instantaneously' to an impulse. It must be noticed that the displacement is not due to a velocity, but to an acceleration, which is a change of velocity and requires a short interval of 'time'. The change in velocity again requires an interval of 'time' to overcome inertia and produce displacement. Similar reasoning applies to a long train just being started by the engine. The cars being coupled together by more or less elastic means, the engine may be moving uniformly and some of the last cars still be stationary. The pull of the engine is *not* transmitted instantaneously but with a *finite velocity,* due again to the inertia of the cars.

We see that the only structurally adequate means of describing changes in continuous, deformable materials is to be found in differential equations which express a method of dealing with *action* by *contact.*

We have already seen that this action by contact involves also the *finite velocity* of propagation, a fact of crucial structural and semantic importance. In the history of science we can distinguish three periods. The first was naturally the period of action at a distance, the best exemplified by the work of two great men, Euclid and Newton. In it we find of course, a superabundance of 'infinities'. With the advent of the differential calculus, and the introduction of differential equations in the study of nature, the notion of action at a distance became more and more untenable. We had a period of pseudo-contiguous action, which indeed involved differential equations; but the *velocity of propagation* was not introduced explicitly, and so there remained an implicit structural assumption of 'infinite velocity' of propagation. As an example of such pseudo-contiguous action we can cite the older theories of potential, which give differential equations for the change in the intensity of the field from place to place, but which do not contain members that express a change in 'time', and hence do not take into account the transmission of electricity with finite velocity.[1]

The modern theories, as for instance, the Maxwell theory of electromagnetism, and the Einstein theory, are based on *action by contact.* These theories not only use the differential method, but they also introduce explicitly the *finite velocity* of propagation.

The invention of differential geometry with the recent contribution of Weyl, which we have already mentioned, transforms the geometry of Euclid from a language of action at a distance into contact geometry, or a language of indefinitely near action.

It should be mentioned perhaps that the riemannian differential geometry is more general than all the $\overline{E}$ geometries which preceded it, and includes them, as well as the E geometry, as special cases. Perhaps, as Weyl points out,[2] the investigation of the famous fifth postulate, which was the beginning of $\overline{E}$ geometry, was accidental in importance and the main structural value of the $\overline{E}$ geometries lies precisely in the application of the differential methods to geometry which was originated by the great work of Riemann. This work, we see, has carried us from metaphysical action at a distance to a physical action by contact. In passing from the older mechanics to electromagnetic events a very striking analogy appears, which explains the finite velocity of propagation.

In mechanics, when we have waves in an elastic medium, the finite velocity of propagation is due to the delay which occurs due to the inertia of materials. Now inertia is determined by acceleration (d^2s/dt^2), which represents the rate of change of the velocity ($v=ds/dt$), velocity itself being a rate of change of displacement. We see that this retardation, or negative acceleration, is represented by a double differentiation.

Something analogous occurs in electromagnetic events. The rate of change of the electric field (de/dt) determines the magnetic field; and then the rate of change (dh/dt) of the latter determines the electric field at a neighbouring point. The advance of the electric field from point to point is thus conditioned by two differentiations with respect to 'time', which is quite analogous to acceleration.

It is due to this double differentiation with respect to 'time' that the formulation of electromagnetic waves are structurally possible. If the partial effects were to occur without loss of 'time', no propagation of the electric waves would occur. The maxwellian 'field equations' not only express the above-mentioned relations, but introduce structurally the finite velocity of propagation which makes the Maxwell's electromagnetic theory structurally a contact theory.

The Einstein theory is also structurally a contact theory, and it may be said that it was originated by this contact tendency, and has carried it to the limit, as we shall see later. The gaussian theory of surfaces, whose extension to any number of dimensions was made by Riemann, also represents action by contact. This theory does not state the laws of surfaces on a large scale, but only their differential properties, the coefficients of the measure determination, the invariants which we can form, and the curvature and its measure. The form of a surface and its characteristics can then be calculated by a process similar to the solution of differential equations in physics.

We are now in a position to understand why the newer physics and the $\overline{N}$-systems, which are built entirely on the foundations of action by contact, found the *E*-system unsatisfactory. The *E*-system was built on the structural assumption of action at a distance, and we had to select the $\overline{E}$ geometries as originated by Gauss, Lobatchevski, Riemann, and others, which gave to physics the necessary geometry of action by contact.

But the question of action at a distance versus action by contact has also an experimental aspect which makes the latter theory more satisfactory.

Faraday (1791-1867) was not a learned academician, and he was much freer from scientific prejudices than any of his contemporaries. From a book-binder's apprentice he became through his genius one of the founders of modern physics. His method of experimenting was to try every possible experiment and note what happened.

In 1838 Faraday made an important structural discovery; namely, that the mutual action between two electrically charged bodies depends upon the character of the intervening medium. Faraday established by this experiment that the capacity of a spherical condenser changes when another material is used as the separating medium, rather than air. He found that the capacity became twice as large when the medium was paraffin, three times as large for shellac, six times as large for glass, and about eighty times as large for water.

This experiment became the foundation of the new theory. The old 'action at a distance' theory postulated that the electrostatic field was merely a geometrical structure without physical significance, while this new experiment showed that the field had physical significance. Every charge acts first upon its immediate surroundings, and it is only through the medium of these that the action is propagated. The discovery of displacement currents necessitated an extension of his point of view to all distances.[3]

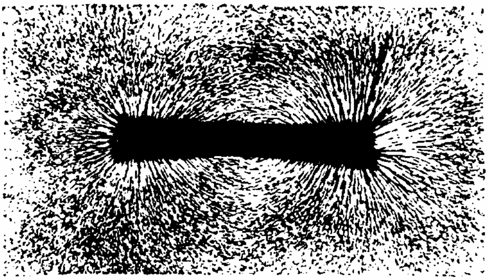

FIG. 2

Faraday was so impressed by this discovery that he abandoned the older theories of action at a distance and formulated a structurally new theory of contiguous action for electric and magnetic events. Any one can convince himself of the fact that the fields represent very actual physical conditions by taking a sheet of paper, sprinkling some iron dust upon it and putting a magnet under the paper. He will find that the particles of iron dust arrange themselves in a very definite structure as shown in Fig. 2.

Faraday also discovered that the forces between two magnetic poles likewise depend on the medium that happens to be between them. He concluded correctly that the electric as well as the magnetic forces are produced by a state of tension in the intervening medium.

These two examples will suffice as illustrations, but it can be said in general that all modern physics gives ample proofs of the correctness of Faraday's structural point of view. Some physicists, for instance Helmholtz, built special devices to test the correctness of this theory. As a matter of fact the success of the whole electromagnetic theory of Maxwell, which is structurally built as a contact theory, in which the velocity of propagation is considered finite, is in itself one of the best proofs of the correctness of the theory.

The finite velocity of light was discovered by Olaf Romer in 1676 and has since been repeatedly verified. This velocity is usually denoted by c and is known to be approximately 300,000 kilometres per second, that is, $c=3\times10^{10}$ cm./sec., or 186,000 miles per second.
In 1856 Weber and Kohlraush calculated a certain constant which appears in the electromagnetic theory, and discovered that the constant had the dimension of velocity, $[c] = [L/T]$, and that its numerical value was 3×10^{10}cm./sec., which is the exact value for the velocity of light. This fact led Maxwell to associate light with electromagnetic waves, a view justified by experiments. In 1888 Hertz not only established once more the interrelation of optics and electrodynamics but found that the velocity of propagation of electromagnetic waves is finite and exactly equal to the velocity of light.[4]

Outside the exact sciences the principle of action by contact is making but slow progress, perhaps because of *A s.r* and the lack of structural formulations of the general issues at hand. We are happy to find a notable exception in the biological work of Professor C. M. Child, who has laid. down a foundation for $\bar{A}$ biology and his system is structurally based on action by contact. This $\bar{A}$ biology has been applied to neurology by Professor C. J. Herrick. This present work, being a $\bar{A}$-system, must follow the methodological and structural advances explained here, and the $\bar{A}$ biology and neurology founded by Child and Herrick.

It is interesting to follow up the structural merging of geometry and physics. There are certain Smiths and Browns who call themselves physicists. There are some rooms with various instruments, which are called physical laboratories. The activities of the physicists which interest us are twofold. First, these scientists come to their laboratories, manipulate their instruments, note the positions of some pointers, manipulate the instruments again, note positions again, . This represents the *un-speakable* level of activity. Whatever happens happens, but there is no speaking to be done on that level.

Later the scientist describes his experiments in words. Obviously there are two entirely different stages in building physics, which usually we do not distinguish.

Quite obviously the *un-speakable* level cannot be called ‘physics’, and so we must apply the term to the higher order abstractions on the *verbal level;* namely, to the reasoned verbal account of what the experimenter saw, or felt, or experienced, in general abstracted on the lower levels; summarized, generalized. , in higher orders.

Physics represents then a verbal discipline. Being verbal, it needs a language. What language shall we select ? As we want to have a science called physics, we shall naturally try to use the most structurally correct language in existence, so by necessity we must look in the direction of mathematics.

In mathematics we find originally two entirely different disciplines. One we may call arithmetic; the other, geometry. Becoming acquainted with these two originally separated languages, we find that the actual experiments and the stimuli for many experiences of importance to us, are outside our skins; so we try to choose the one of these two languages which is the more closely related in structure to the lower abstractions—that is, to what we see, feel, . Naturally we have an inclination toward the geometrical languages, dealing with ‘lines’, ‘surfaces’, ‘volumes’. , terms for which we find immediate and quite obvious applications. By further investigation we find that of late both languages have become so developed in structure, that either can be translated perfectly into the other. This fact makes geometry the link between the higher order abstractions and the lower order abstractions. We have seen that *physics*, as well as geometry, must be considered *verbal* disciplines and their fusion becomes a very natural fact.

It is true that, as yet, ‘time’ appears as the bothersome factor, but ‘time’ may very well be represented geometrically, except that our diagrams and figures look a little different. For instance, a flat circular orbit in two-dimensional ‘space’ becomes a helix in three-dimensional space-time, a vibrational motion in one-dimensional ‘space’ becomes a wave-line in two-dimensional space-time, . ‘Time’, when properly represented, becomes simply another geometrical dimension.

It should not be forgotten that mathematicians obtain most of their structural inspirations from physics and build up mathematical theories to supply the structural needs of the physicist. We see an excellent example of this in the $\bar{E}$ geometries. In the days of Euclid, when physics hardly existed, we had ‘emptiness’, ‘action at a distance’, and such notions as were quite satisfactory for the needs of surveyors and builders. With the development of astronomy and physics, curved lines became more and more structurally important, and the haziness of the definition of ‘straight line’ also became apparent. The notion of ‘emptiness’ also became slowly structurally untenable. Such geometers as Gauss, Lobatchevski, and others, began to demand that the axioms of geometry be tested by experiment. With the introduction of ‘curvature’, the ‘straight line’ became only a special case of a curve with zero curvature.

The invention of the differential calculus also had a tremendous structural influence. It introduced continuity as a basic assumption in the vast structure of science, and cleared the way for psycho-logically trained scientific workers in structural continuity, and therefore in *action by contact.*

The discovery that light appears as electromagnetic waves and the finite velocity of both, made the notion of 'absolute emptiness' structurally untenable; and so *E* geometry with its action at a distance, 'emptiness' and neglect of gravitation and electricity, became very unsatisfactory. Indeed if our universe were *E*, light could not reach us.

Leibnitz, who invented the differential calculus independently of Newton, formulated a postulate of action by contact, and therefore could not become reconciled to Newton's Law of Gravitation which was structurally a law of action at a distance, corresponding fully to Coulomb's law in electricity. The latter law states that the force exerted by two electrically charged bodies upon each other is inversely proportional to the square of the distance between them, and acts in the direction of the line joining them.[5]

The introduction by Faraday of the structural notion of a 'field', instead of the notion of electrical charges acting at a distance, introduced the notion of a strain of the electrical field, which appears structurally as 'lines of forces'. Here we already have a 'fulness' of 'lines' and a big step toward the structural fusion of physics with geometry has been taken.

The transition from *E* to riemannian geometry corresponds structurally to the transition from physics based on action at a distance to physics based on action by contact. The fundamental metrical theorem of *E* geometry is the pythagorean rule, which expresses the fact that the square of the distance between two points is a quadratic form of the co-ordinates of the points. If we regard this theorem as strictly valid only in the case of points which are very near each other, we pass at once from *E* geometry to differential geometry. By doing so we gain a notable structural advantage, as we dispense with the necessity of defining our co-ordinates more precisely; because the pythagorean law, when expressed in differential form, is invariant for arbitrary transformations.[6]

Semantically, Riemann was the immediate predecessor of Einstein, although Einstein was not directly influenced by him. In differential geometry we ought to start with indefinitesimally near points, and depend for the analysis of greater distances, areas and volumes, on integration. The difficult notion of 'straight line' has to be replaced by the notion of the shortest line (geodetic), which is easily defined by differential methods and found empirically. In the older method, the length of a curve was to be found in general, by the process of integration. The length of a 'straight line' between two points was supposed to be defined as a whole, and not as the limit of a sum of indefinitely little bits. Riemann considered that a 'straight line' does not differ in this respect from a curve. Measurements which are always performed by means of some instrument are *physical* operations, and their results depend for their interpretation upon the theories of physics. Dealing with geodetics is therefore preferable to dealing with 'straight lines'.[7]

We see that the problem was ripe for a final stroke of genius. Einstein's structural discovery of the dependence of 'space' and 'time', and Minkowski's success in giving a geometrical interpretation to the Einstein theory accomplished the probably irreversible fusion.

Three-dimensional kinematics becomes four-dimensional geometry, *three-dimensional dynamics* can be considered as *four-dimensional statics.*

We see immediately the human, psycho-logical, semantic and neurological importance of this fact. Our nervous system by its structure produces abstractions of different orders, dynamic on some levels, static on others. The problems of sanity and adjustment become problems of translation from one level to another, for which the structural advances in science supply us with methods of solution.

It should be noticed that the semantic gain due to the above facts is considerable, and that being structural, it is practical as well as theoretical. The fact that geometry has lost its old restricted status, which formerly applied principally to what could be 'intuitively visualized' and has been further abstracted to apply to what can be 'conceived', has merged geometry with the rest of mathematics. This merging represents a great structural and semantic step forward, and makes possible the treatment of geometrical problems by purely analytical means. It liberates geometry from the restrictions of lower order abstractions. By using 'geometrical intuition' (lower order abstractions) we find again a great help in analysis.

In the cyclic nerve currents, our so-called 'intuitions' (lower order abstractions) are not structurally isolated from our 'conceptions' (higher order abstractions), but both are intimately connected and influence each other. Modern advances are not only in perfect accord with the 'organism-as-a-whole' principle, but indeed give us excellent proofs that this principle is sound. 'Psychologists' miss a great deal by disregarding this important and unique form of human behaviour which we call mathematizing.