Constructing The Universe Activity Books
Volume 2

A Voyage From 6 to 12

Explore Harmony in Mathematics, Nature and Art

Michael S. Schneider

Author of

A Beginner's Guide To Constructing The Universe:
The Mathematical Archetypes Of Nature, Art And Science
(HarperPerennial)

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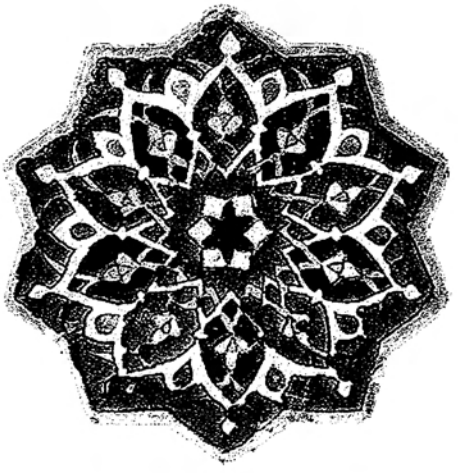
Activity Book

Volume 2

Create and Explore
Geometric Patterns
of Nature and Art

A Voyage From 6 To 12



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Cover illustration: "Woman Teaching Geometry" is the Frontispiece of Adelard of Bath's Latin translation of Euclid's *Elements* c 1310.

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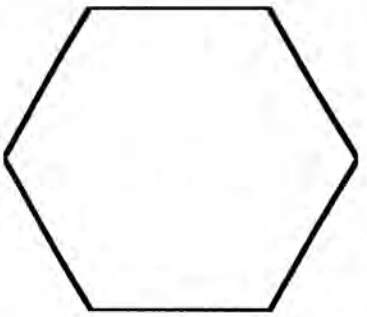
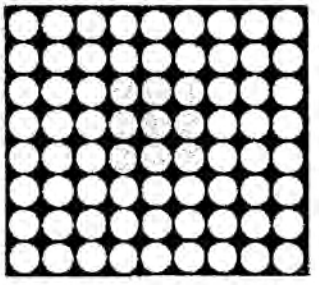
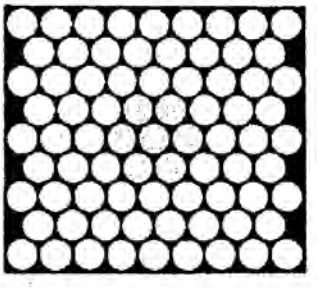
6 The Hexagon

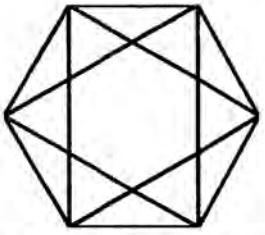


A regular Hexagon has six equal corner angles, six equal sides and three equal diameters. It is a simple shape, made out of the even simpler Triangle.

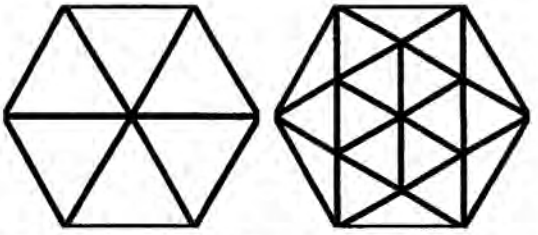
Hexagons are famous for packing alot into a small space. Look at these two black boxes. They are the same size, and the white Circles are the same size in each. In one, the Circles are packed like Squares, with eight around one in the center. In the other box, they're packed like Triangles and Hexagons, with six around one in the center. Count the Circles in each. Even though there are some larger unused spaces in the six-around-one box, it still contains more Circles than the other box, or any other packing scheme.



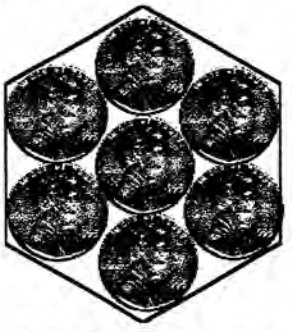




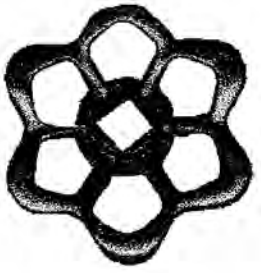
As with all polygons having an even number of corners, we have to lift our pencil to make the Hexagram star.



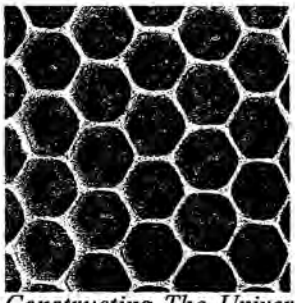
Triangles inside a Hexagon are the secret to it's close packing, great strength, and balance.

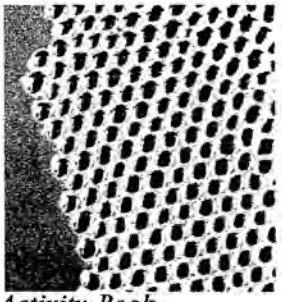


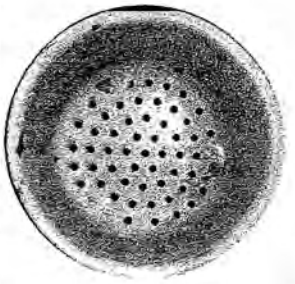
Many forms of nature and technology make use of the Hexagon's six-around-one packaging properties.



Activity: arrange six pennies around one in the center to see how round objects pack neatly as a Hexagon.





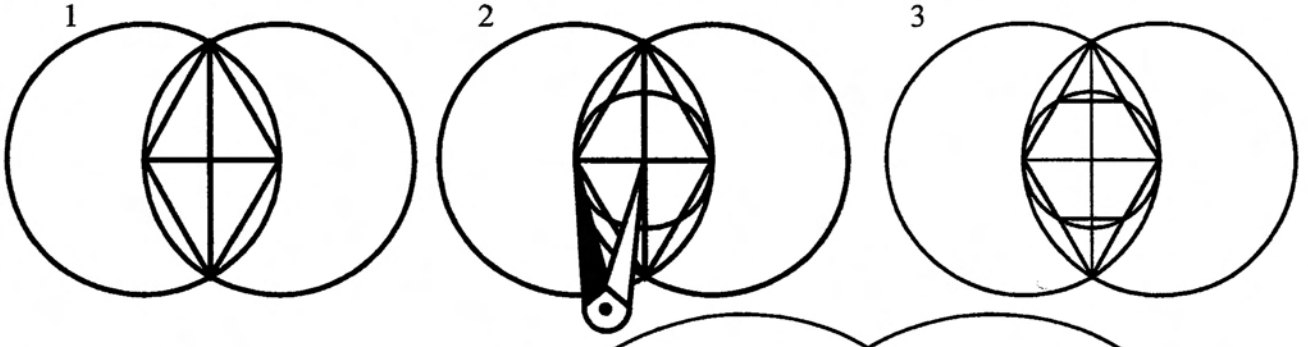


Snowflake, flower, faucet handle

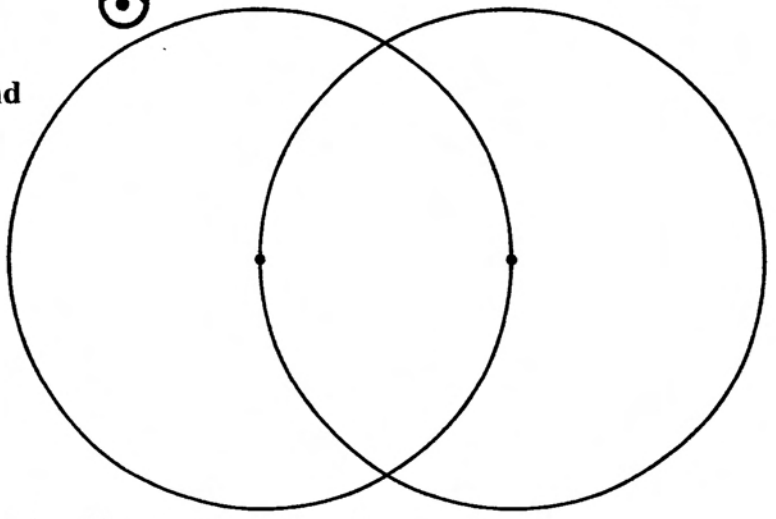
Beehive, froth of bubbles in the sink, colander.

Construct A Regular Hexagon In The Almond

(1) Construct an Almond. (To save space you might just turn large arcs.) Then connect the Circles' centers and crossings to make regular Triangles.
(2) Place your compass point at the Almond's center and open the scribe to one Circle's center. Turn a Circle within the Almond.
(3) Connect the points of the Triangle which cross the Circle. They form a regular Hexagon.

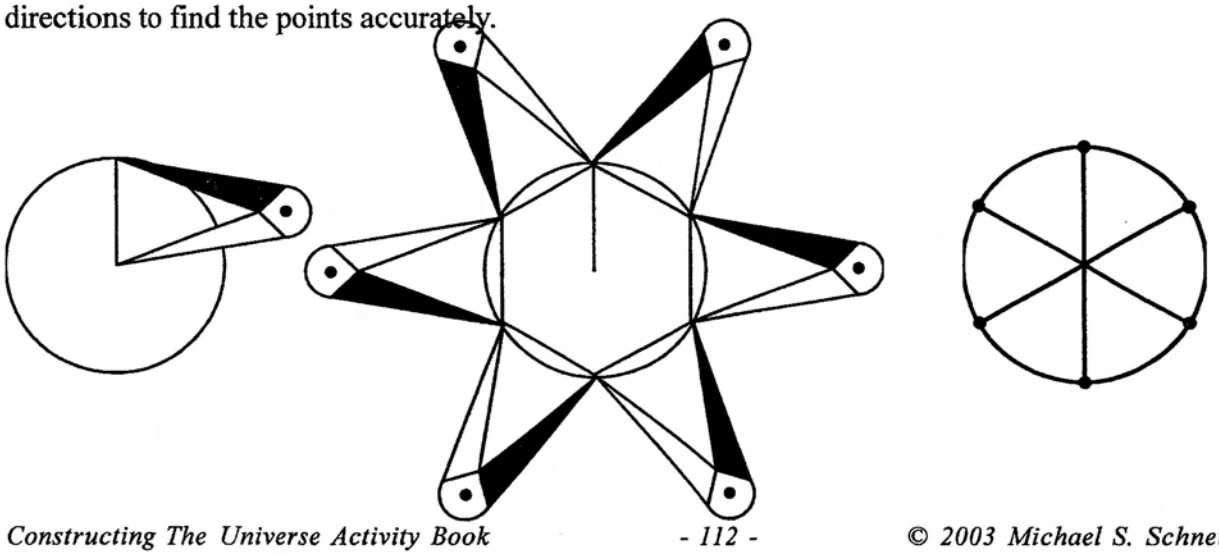


Construct a Hexagon in this Almond



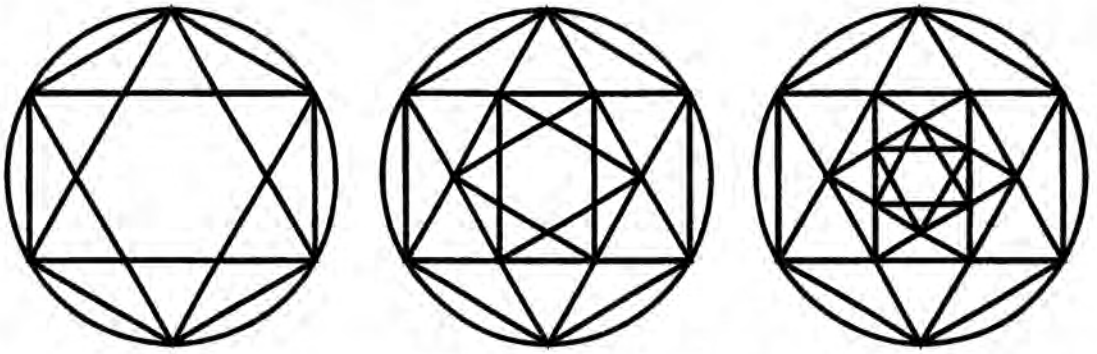
Construct A Hexagon By "Walking Six-Around-One"

We already saw in Chapter 3 (page 31) how to inscribe a Hexagon within a Circle by taking advantage of the curious Truth that each side of a Hexagon is equal to the radius of its Circle. Practice constructing Hexagons on blank paper. Remember to "walk" the compass around in both directions to find the points accurately.

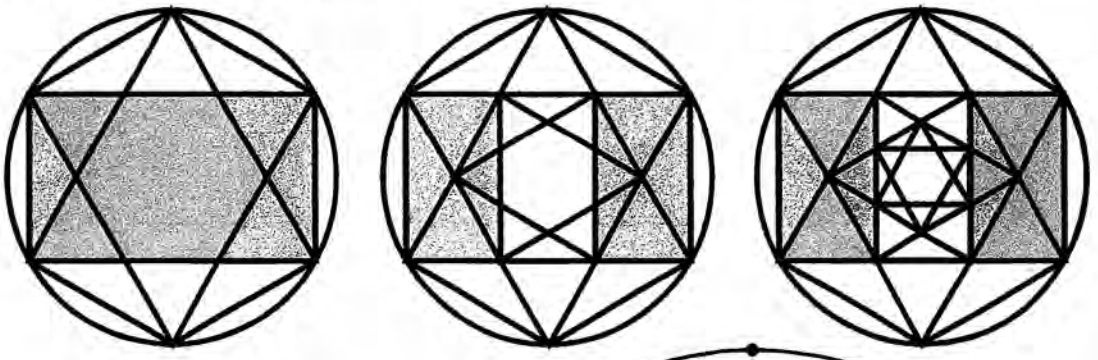


Subdivide A Hexagon

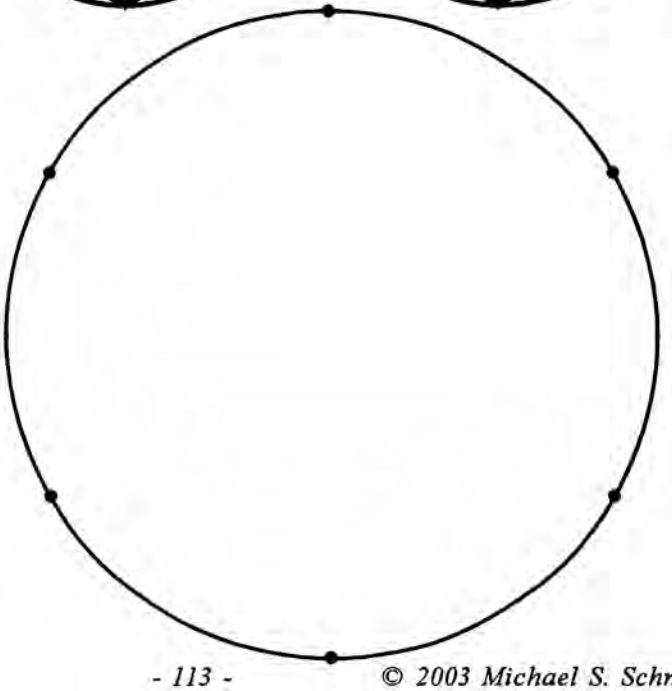
It's very useful to know how a Hexagon subdivides naturally into smaller versions of itself by constructing stars within it. At the center of each Hexagram star is a smaller, turned Hexagon. Simply connect the corners of the inner Hexagon to make another, but turned, Hexagram star with a Hexagon at *its* center. This process of making smaller Hexagons and Hexagrams keeps repeating endlessly smaller.



Notice how they make rectangles (shaded areas below) which keep dividing into three smaller rectangles, each a turned version of the whole rectangle! Changing its size but not its proportion is part of the Hexagonal's design appeal.

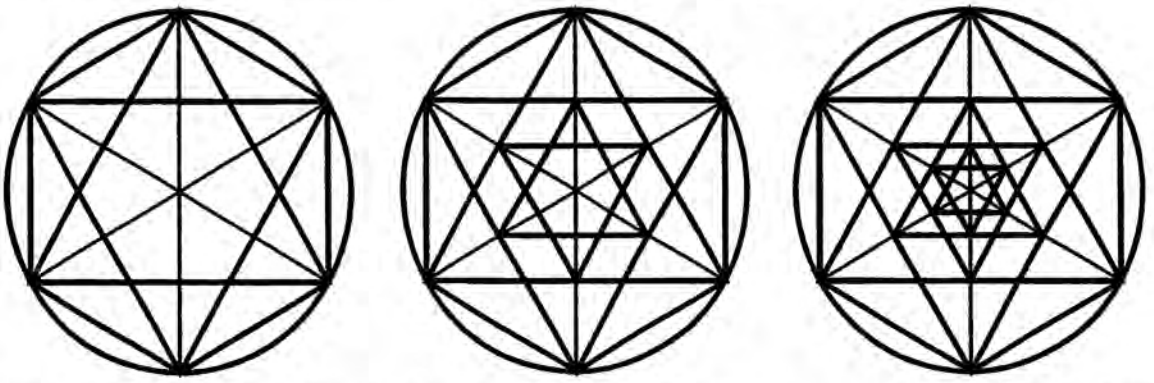


Use a straightedge and pencil to draw and subdivide a Hexagon in this Circle, or on a blank sheet of paper.

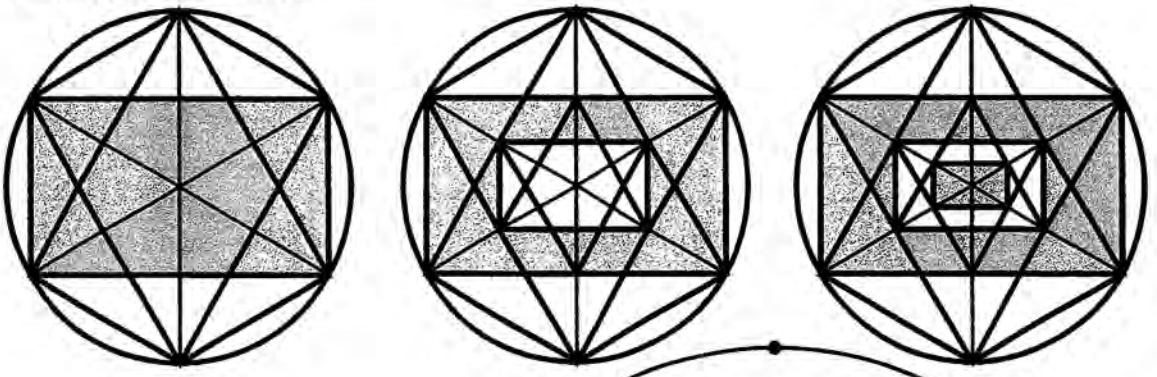


Another Subdivision Of The Hexagon

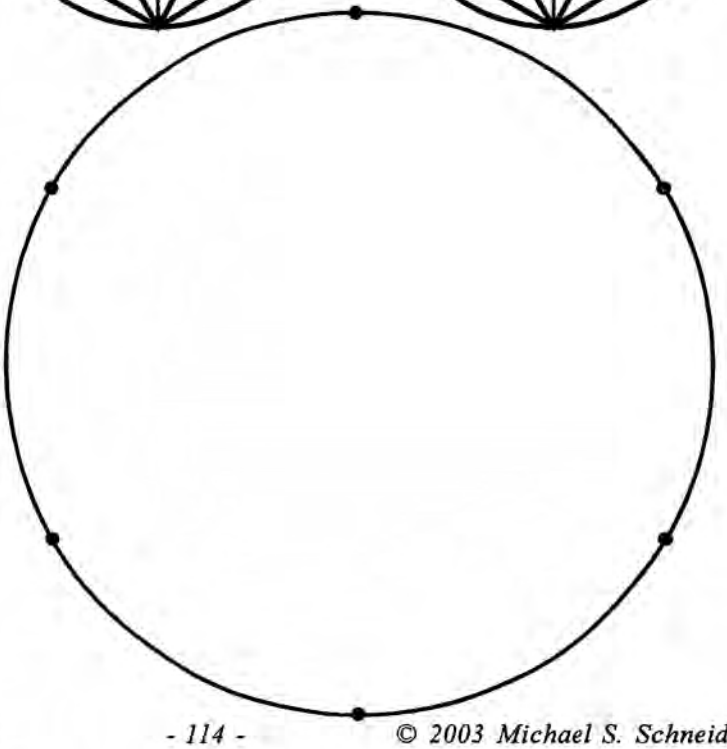
This begins the same way as the previous subdivision: with a Hexagram star. Draw the three diagonals across the corners of the Hexagon. Then, connect the points where these diameters cross the Hexagram star. This makes a smaller Hexagram star in the center Hexagon, but it is not turned. This process can be repeated infinitely smaller to make an interesting frame.



Like the first method of subdividing the Hexagon, this also creates rectangles (shaded below) which are models of each other in different sizes. But notice how they are arranged differently than the previous method of subdivision.

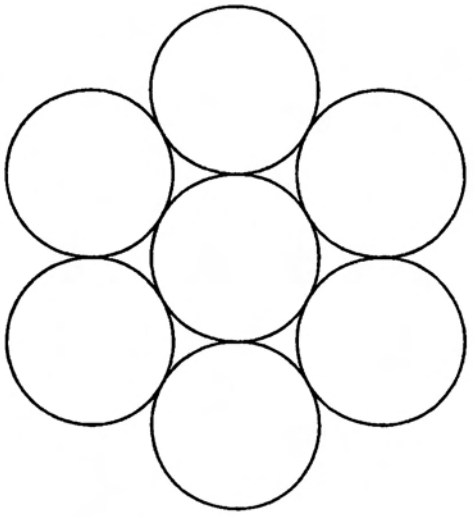


Use a straightedge and pencil to draw and subdivide a Hexagon by this method, or on a blank sheet of paper.

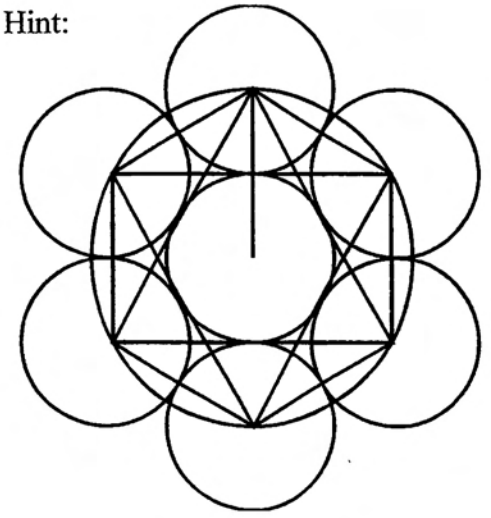


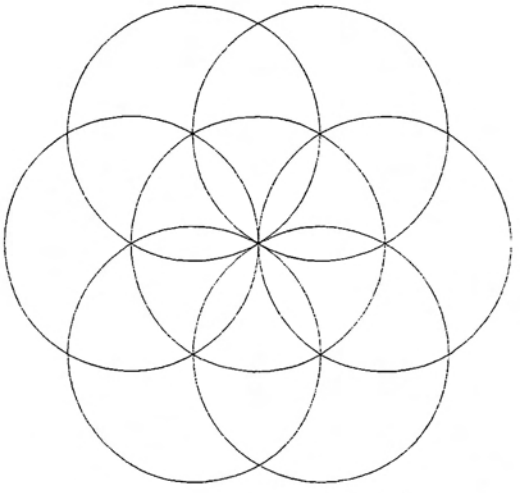
Replicate These Hexagonal Patterns

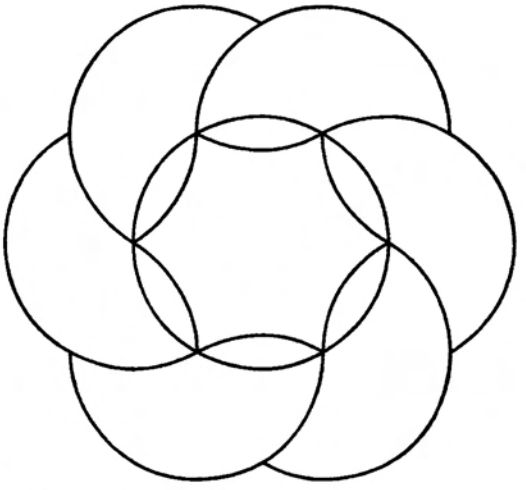
Do them on a blank sheet of paper. Each design usually starts by constructing the six pointed Circle. Remember: Only open your compass between *existing* points. Don't invent points but *find* them where Circles and line segments cross each other. Explore and develop each construction further.

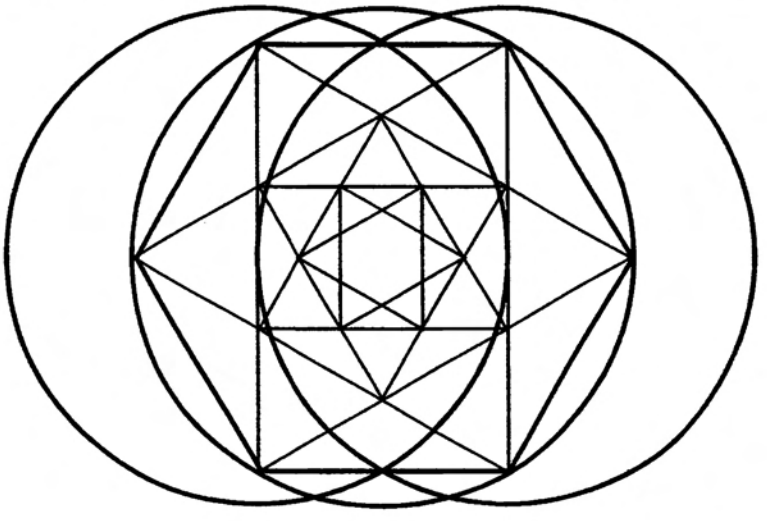


Hint:



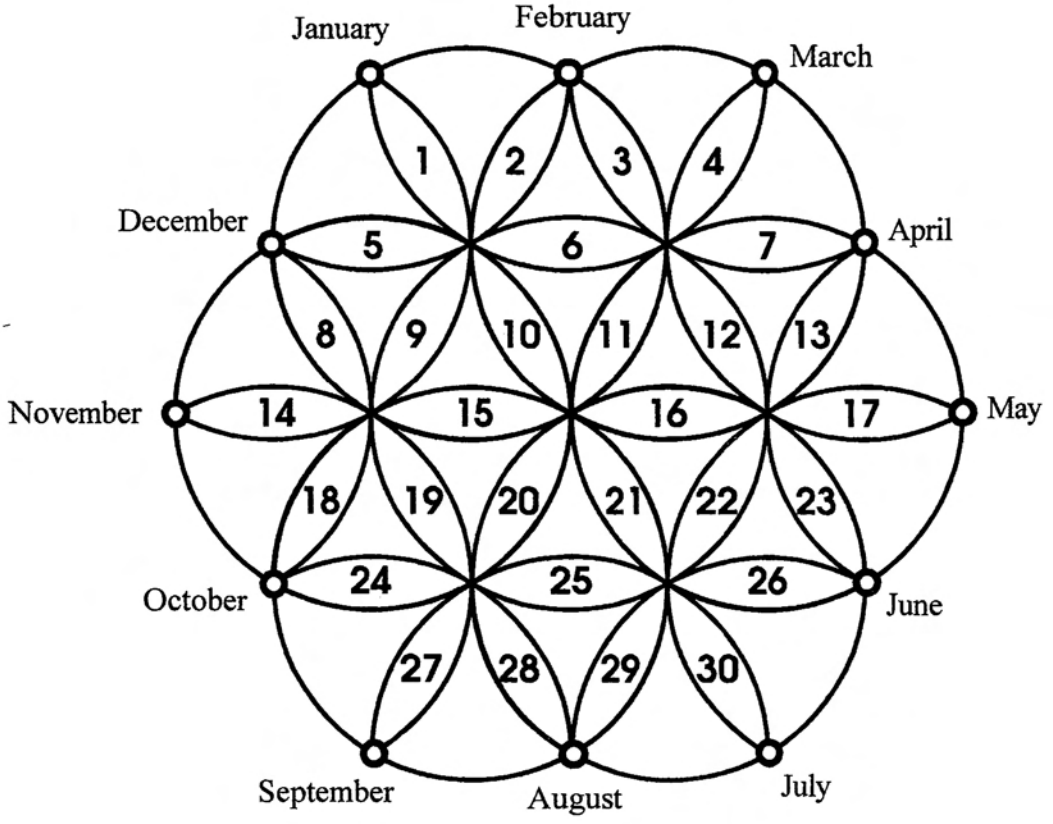






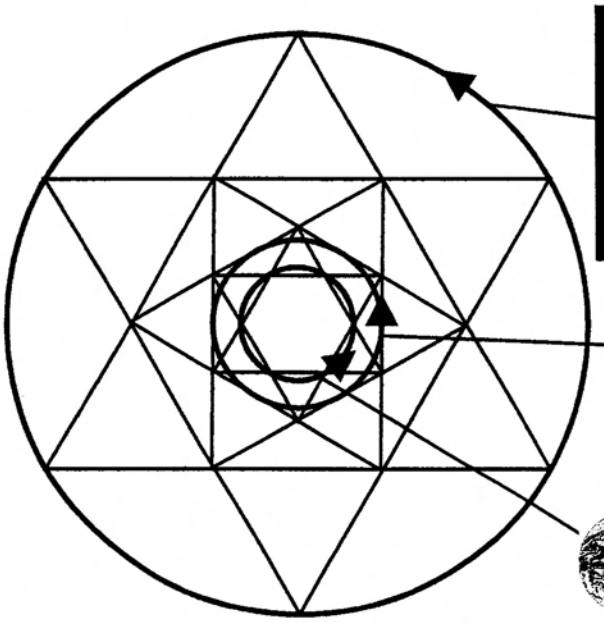
Roman Calendar

This design can be found made of tiles and carved as graffiti wherever the Roman Legion conquered. It was a yearly twelve-month, thirty day calendar. A hole in each date held a peg pointing to the day. Another peg in holes at each of the twelve points around the outside indicated the month. Modern calendars are different since not all months have thirty days. Can you see how to replicate this design? Construct it on a blank sheet of paper or on wood.



The Mean Orbits of Earth, Mars and Jupiter

The elliptical orbits of planets can be converted to Circles show their average or mean orbits. The great distance between Mars and Jupiter (about 350 million miles) is seven times the distance from Earth to Mars. Most asteroids orbit between Mars and Jupiter. This Hexagonal construction shows it with 99.9% accuracy.

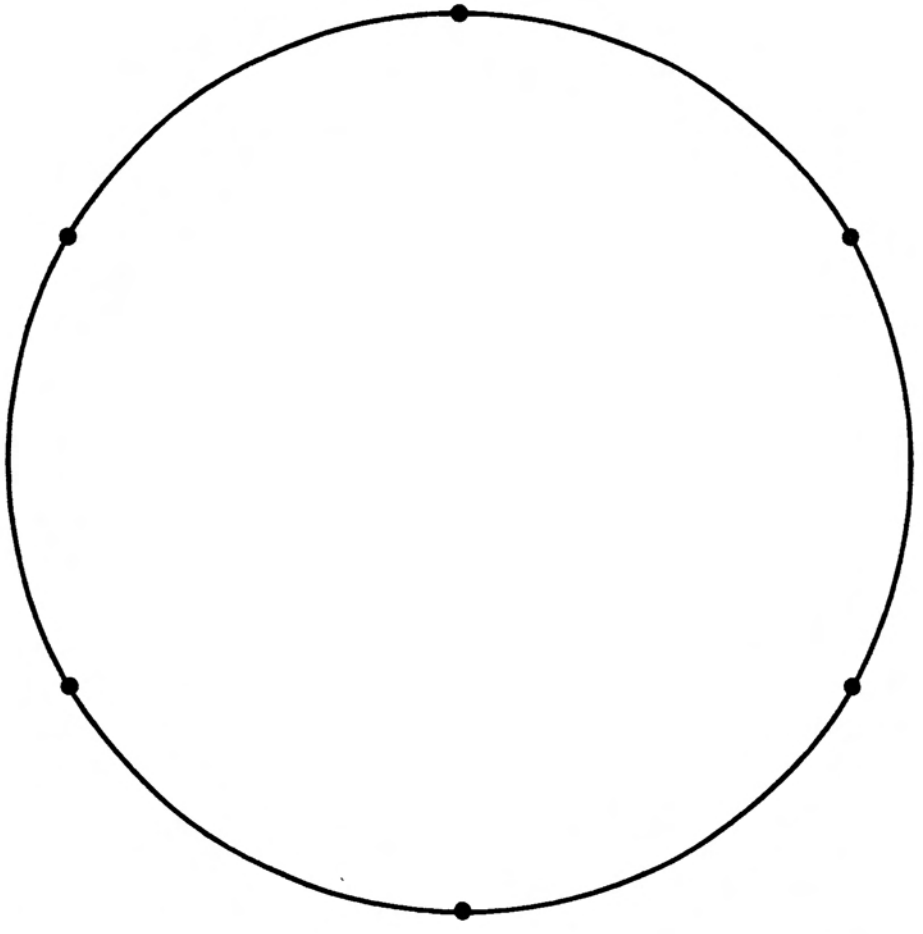


Jupiter

Mars

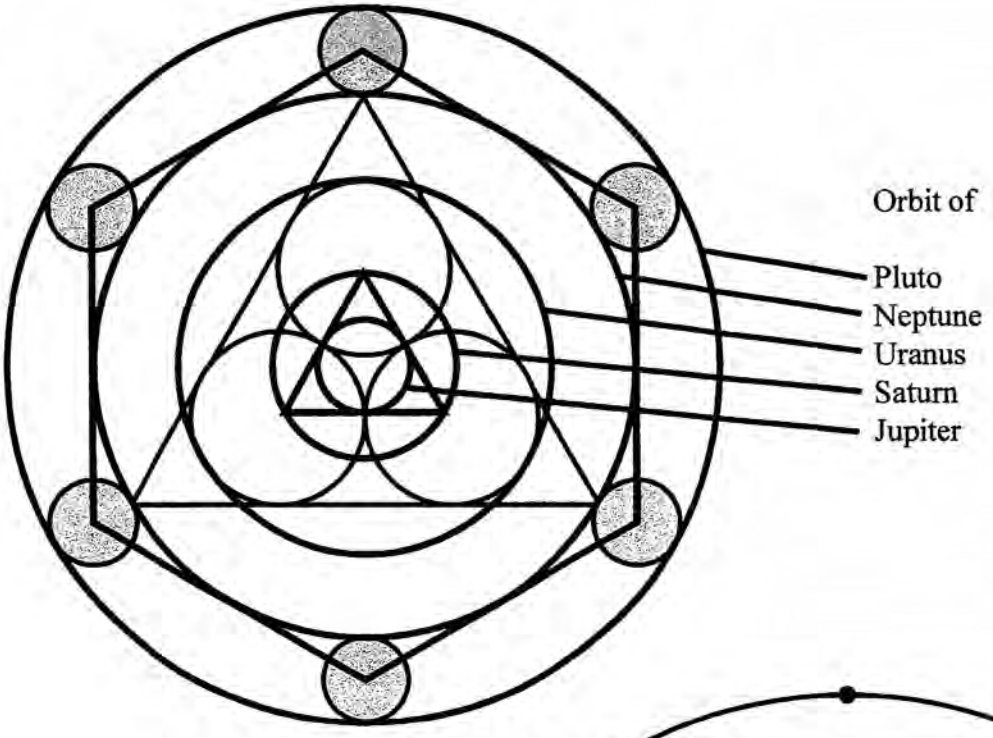
(Planet sizes are not drawn to scale.)

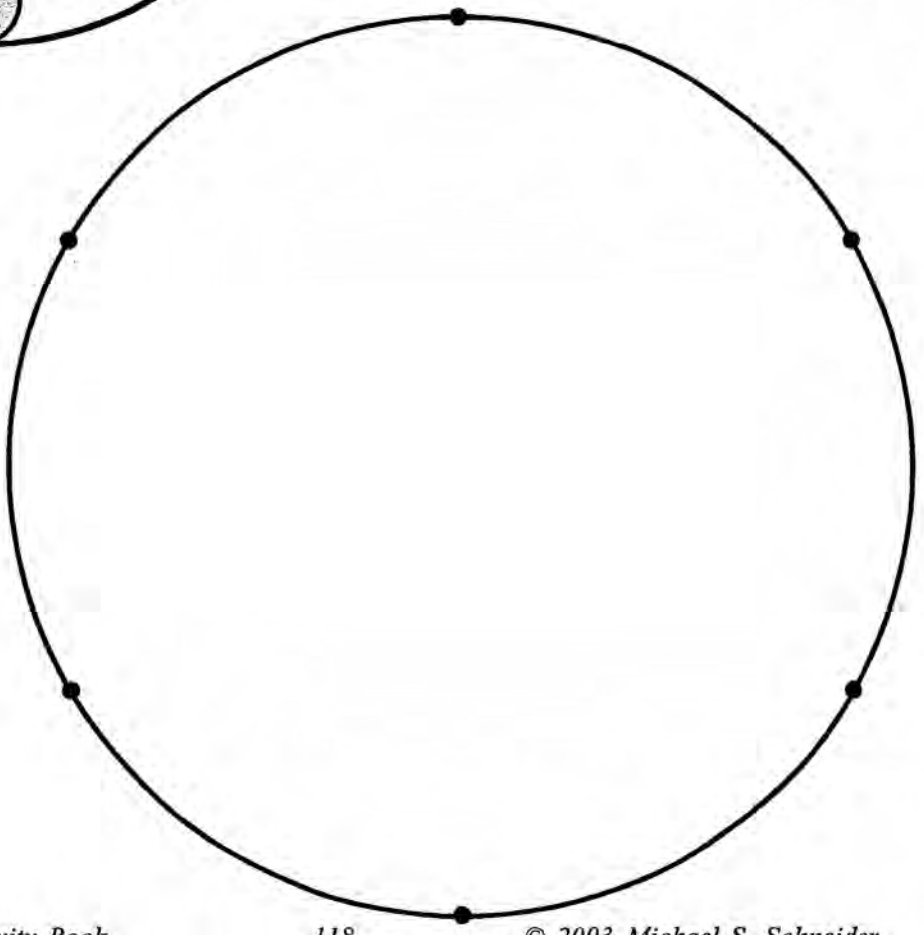
Earth



The Mean Orbits Of The Outermost Planets

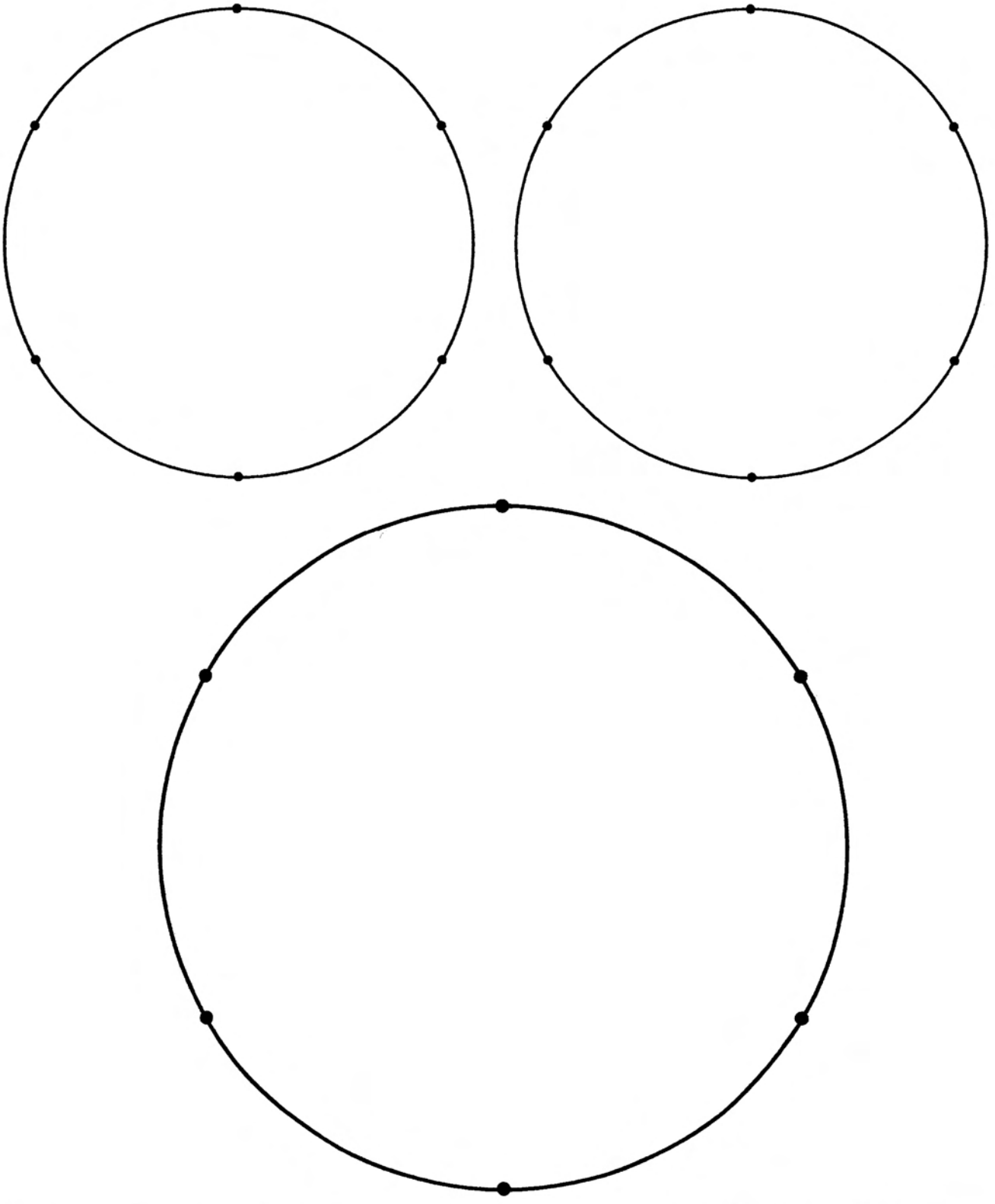
This construction shows the mean (circular) orbits of the planets furthest from the Sun: Jupiter, Saturn, Uranus, Neptune and Pluto. The entire construction on the previous page fits inside the smallest (Jupiter orbit) Circle.





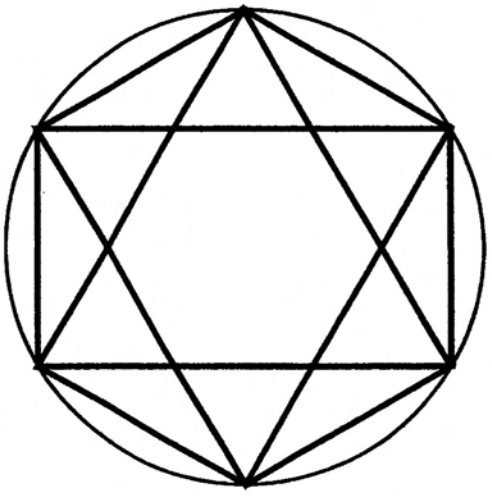
Create Your Own Hexagonal Patterns

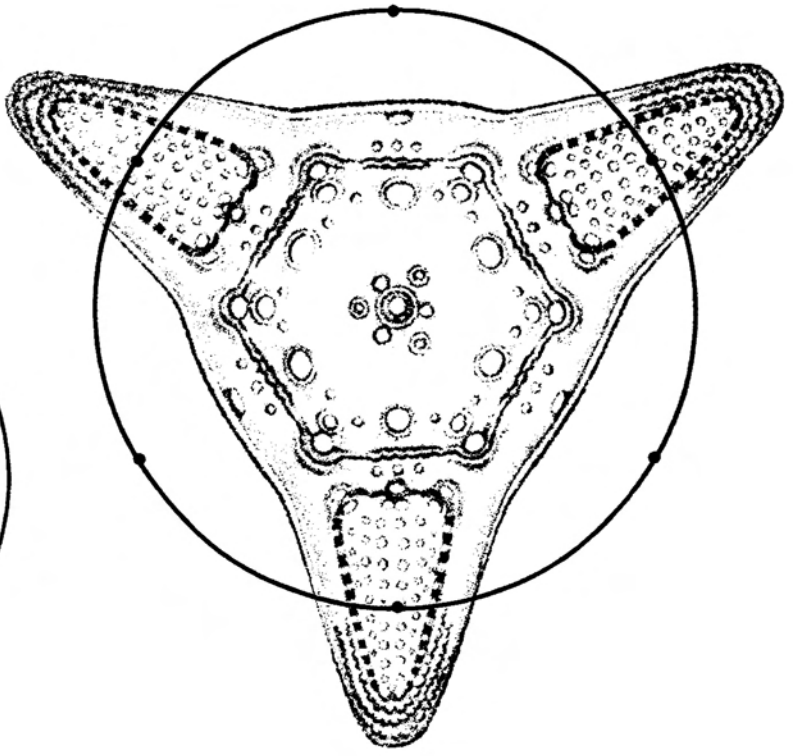
Use these Circles and your imagination to create and explore your own constructions.
Make more patterns on blank paper.
Color them.



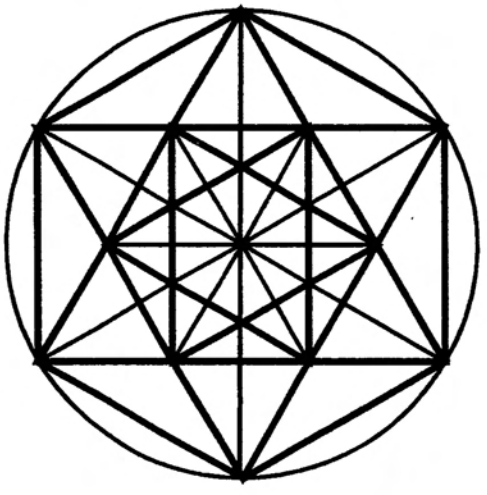
Hexagonal Design In Nature

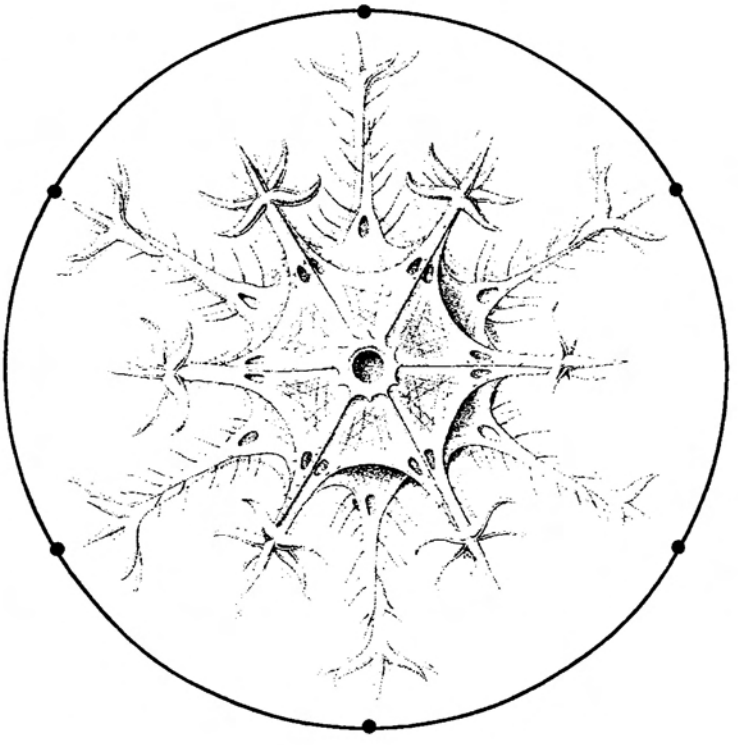
Microscopic Diatom (plant)



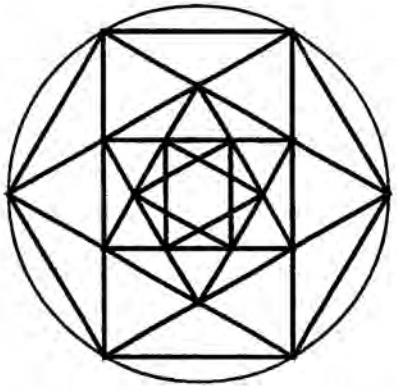


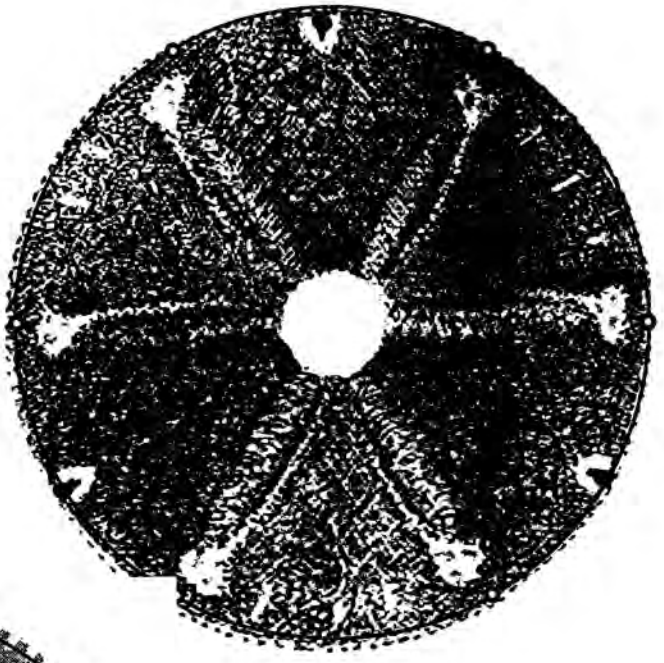
Microscopic Radiolarian (animal)



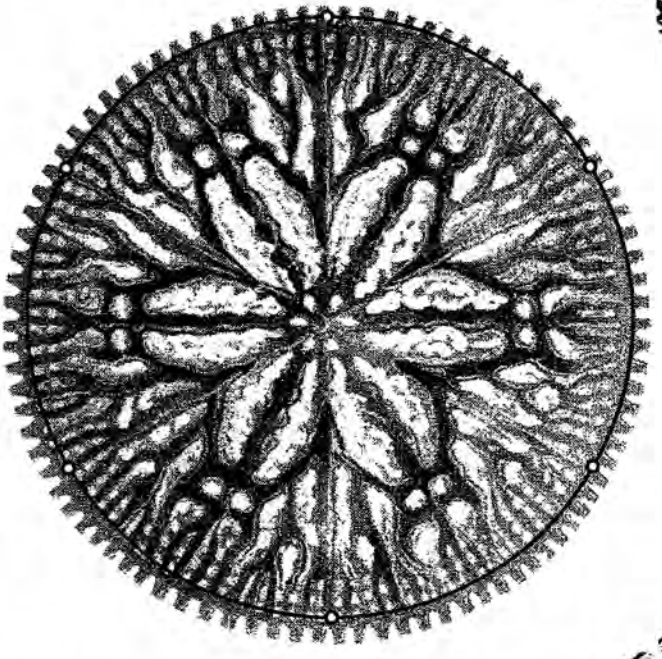


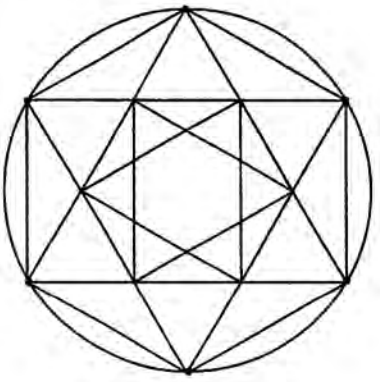
Diatom



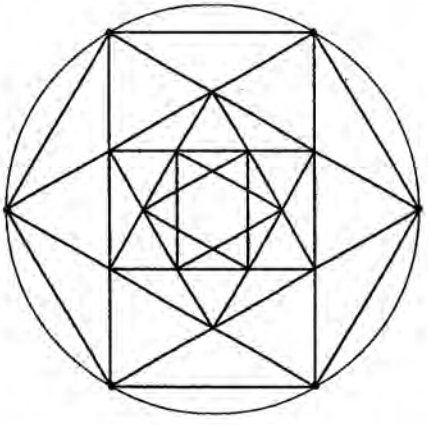


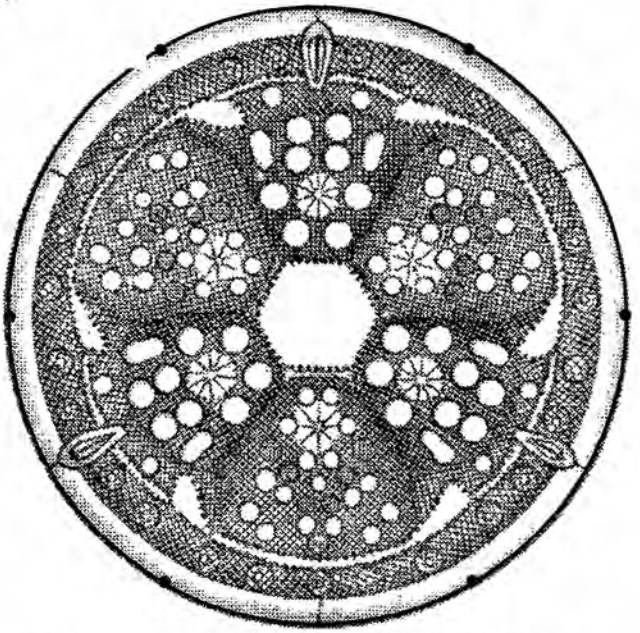
Coral





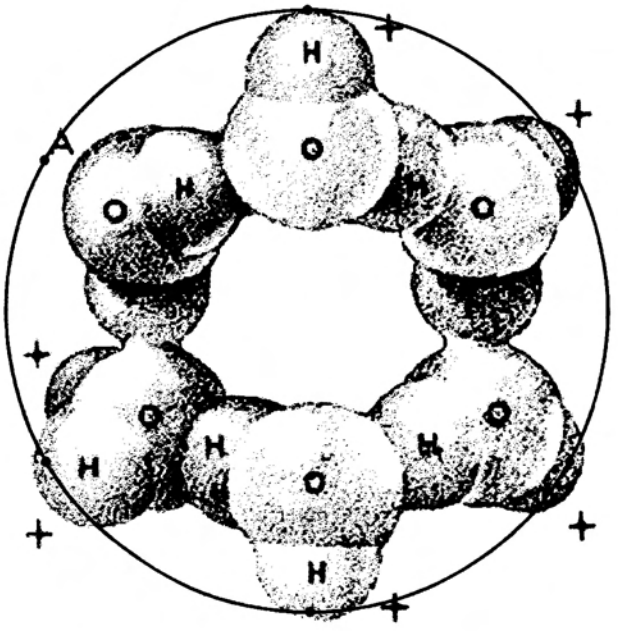
Diatom

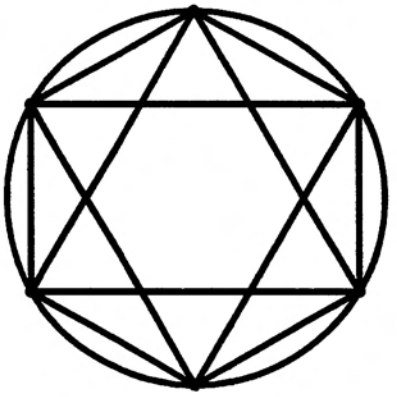


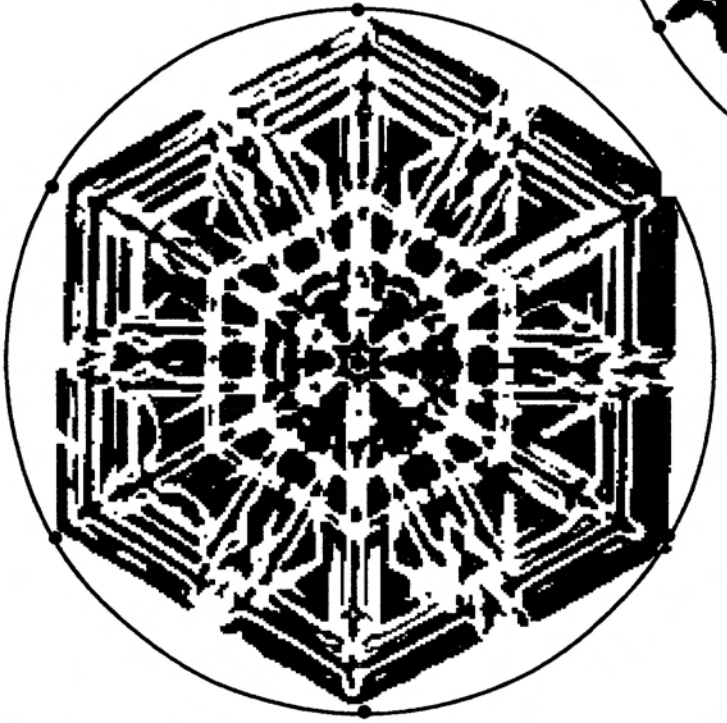


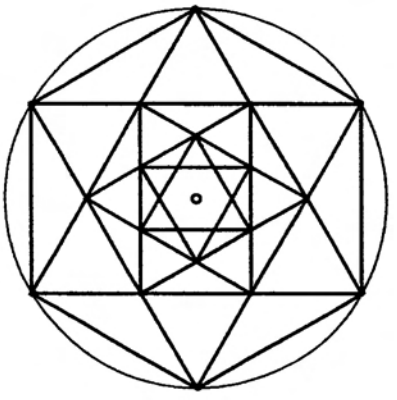
Water and Ice

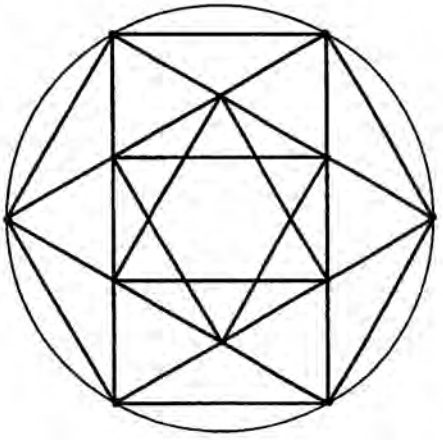
Six water (H_2O) molecules pack together as a Hexagon at the center of every snowflake.

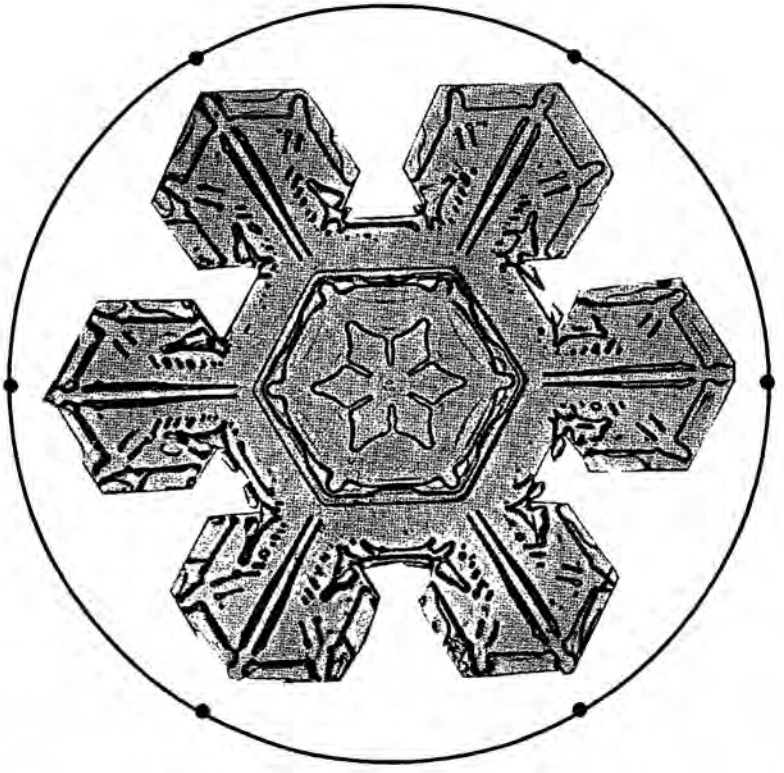


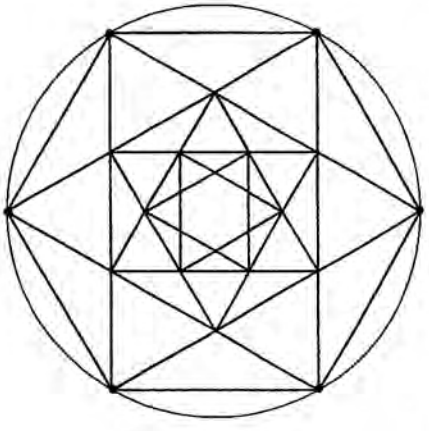


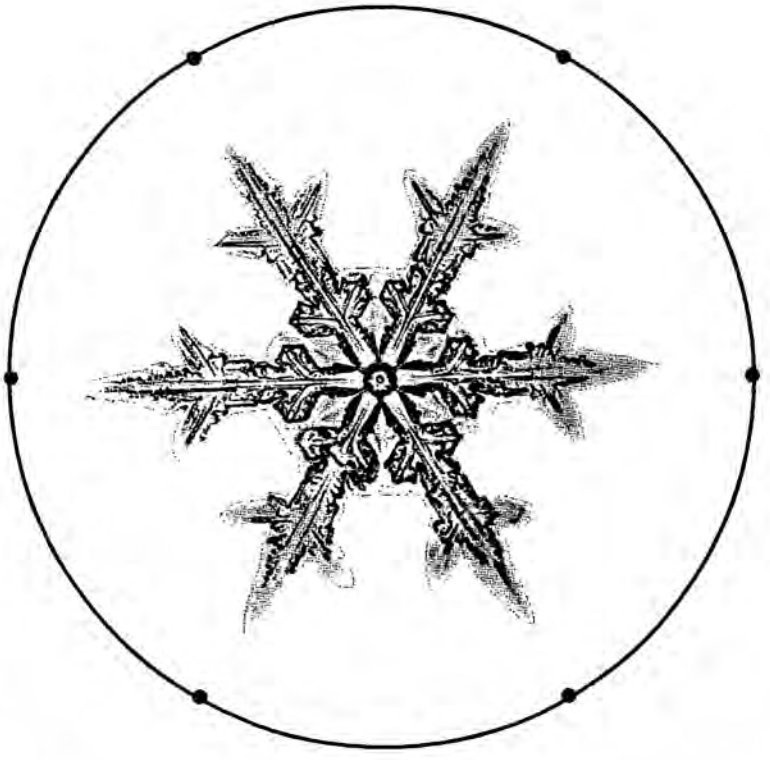






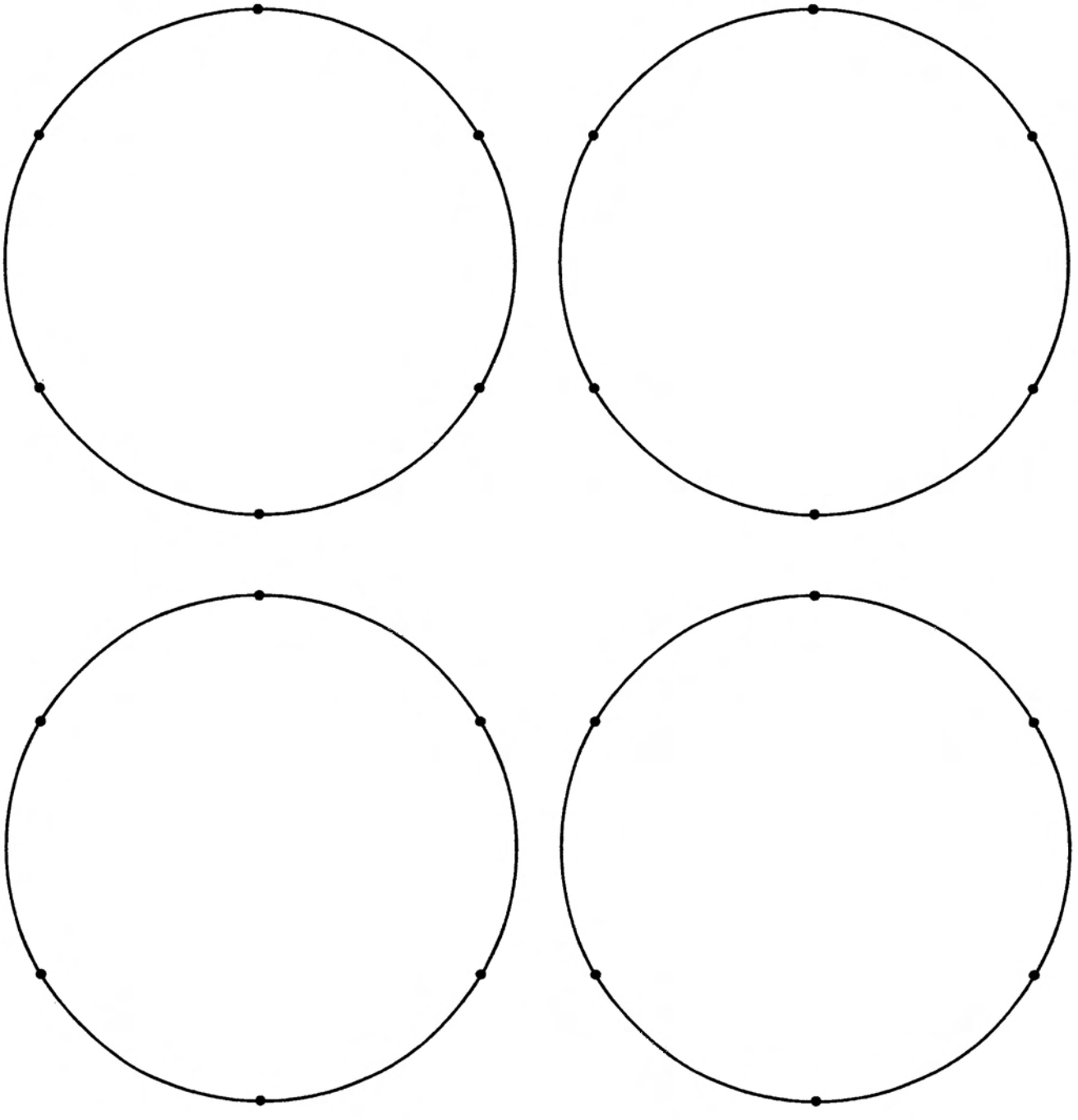






Draw Microscopic Life And Snowflakes

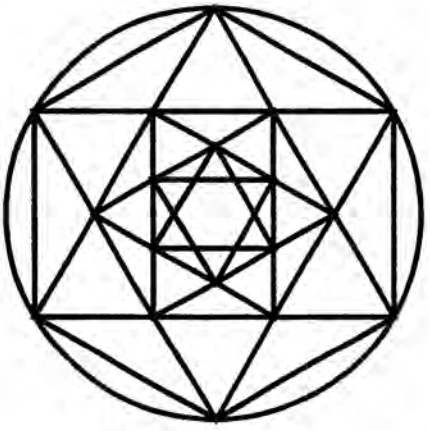
Do some geometric constructions and see what microscopic designs they might suggest.
Use your imaginature and colored pencils to draw them.

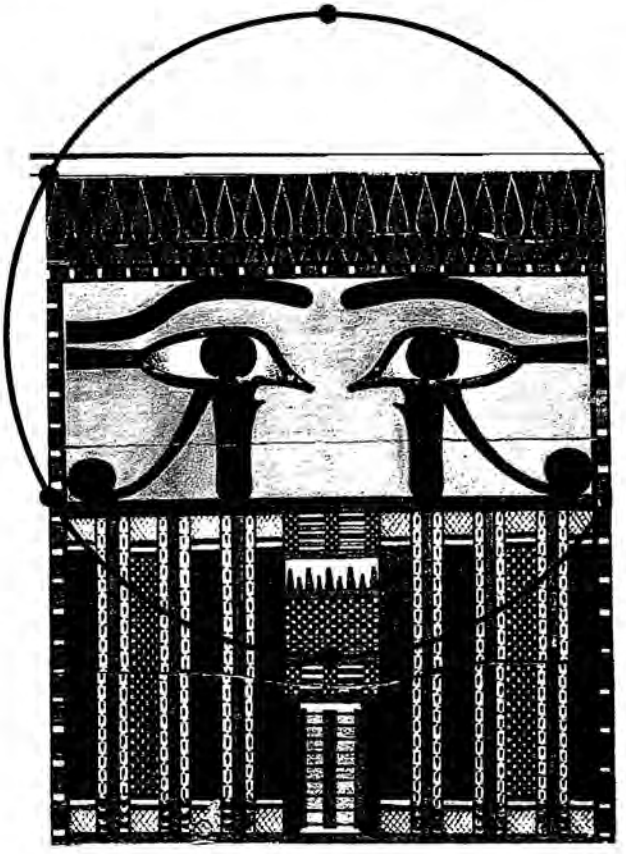


Hexagonal Designs In Art

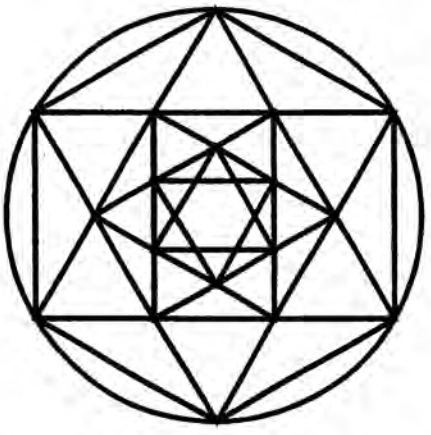
Each of these images from different cultures has a Circle with six equally spaced points around it. Replicate the construction shown next to it to see how geometry guides the scene. Artists can lea great deal by observing how geometry was used by past masters to align elements of the composti You can learn even more by developing each geometric construction further.

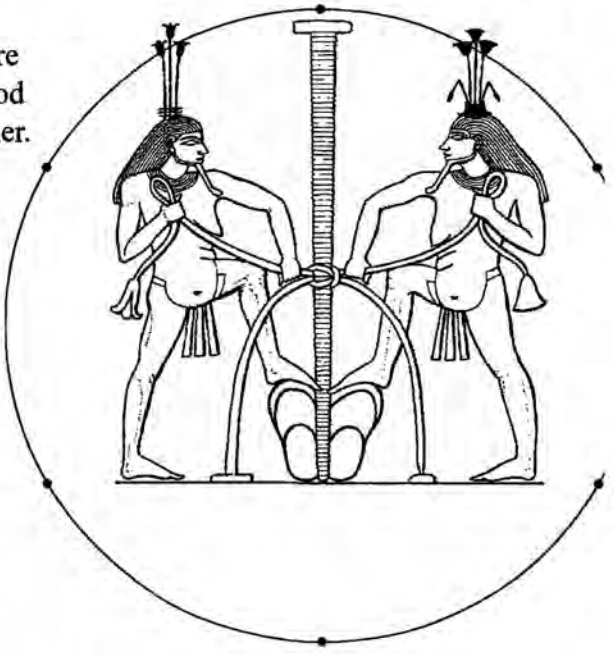
The side of an Egyptian mummy's coffin has eyes painted to look outward. Notice how the tears dripping under the eyes align with the geometry, dividing the scene into three parts. What other lines and crossings reveal the painting's layout?





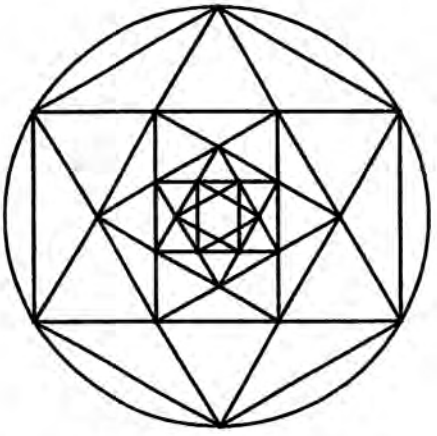
The two ends of the Nile river, south and north, are represented by (nearly) mirror-images of the Nile god *Hapi*. They tie a magical knot holding Egypt together.

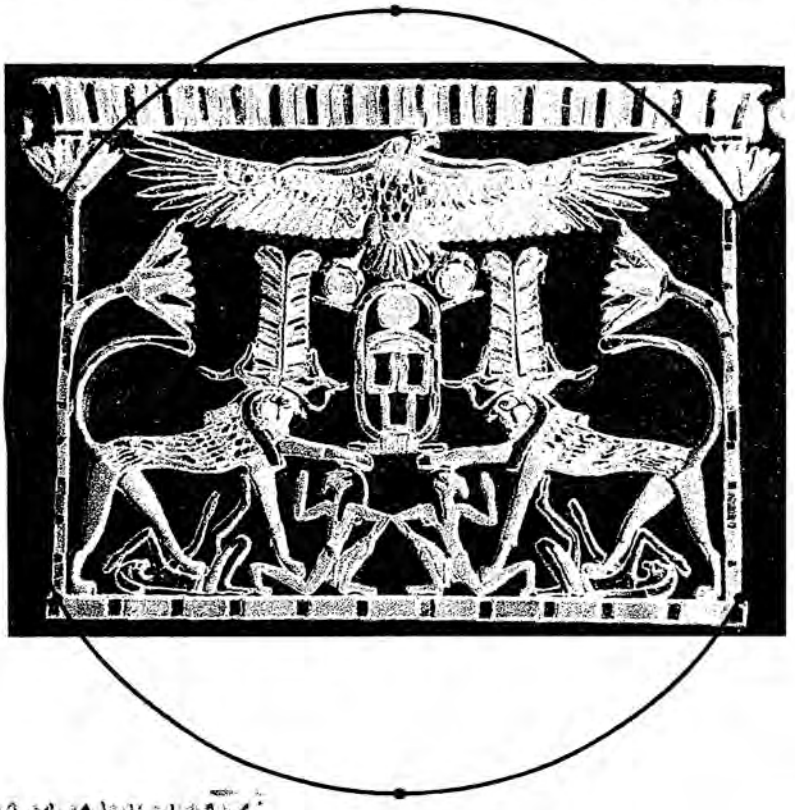




Pectorals Of Queen Mereret

These two Egyptian pectorals hung upon necklaces belonging to Queen Mereret of the 12th Dynasty (around 1810 BCE). They are inlaid with carnelian, lapis lazuli, turquoise and polychrome glaze. Notice how geometric they obviously appear. That's because a geometric scheme guides each scene, and the Egyptians were rather exact in their alignments. Also notice how each scene is divided into three parts: between each feather in the headdress of the griffin-pharaoh (upper pectoral), and in front of each pharaoh's face (lower). In both, the innermost Hexagram frames the central *cartouche* oval containing one of the pharaoh's sacred names. The height of each pectoral can be found from points within the scene.





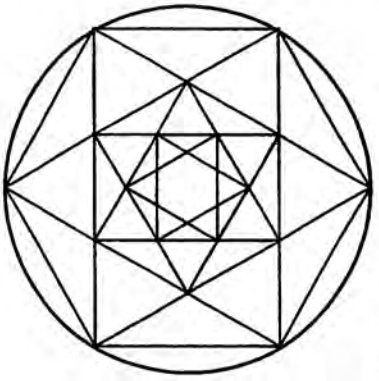
For the experience of doing more of the construction, two points are given to for you to turn the Circle and create a Hexagon and Hexagram stars as above.

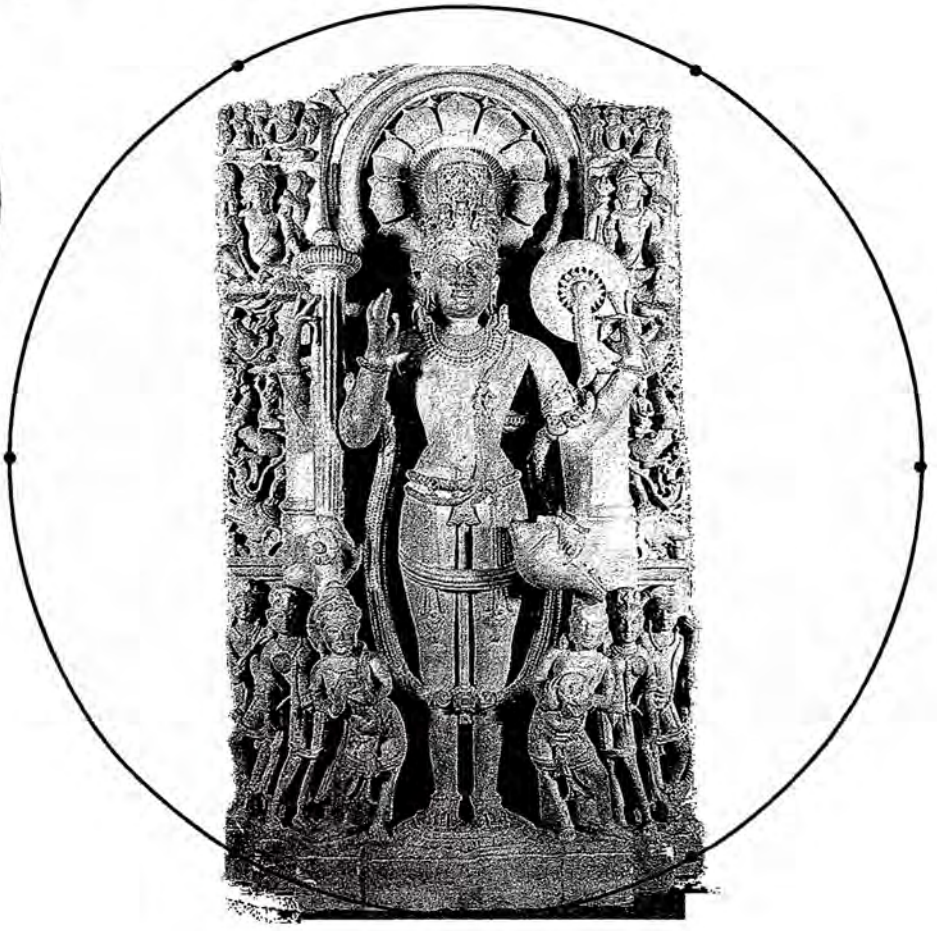
Start by placing the compass point on the point between the pharaoh's lower hands. Open the scribe to the point at the pectoral's lower left corner.

Turn a Circle and "walk" the radius around six times (in each direction).

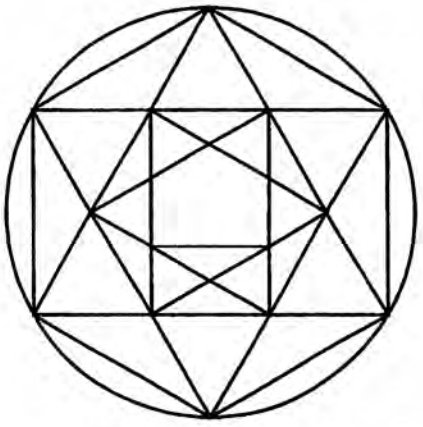
The Hindu Deity Vishnu

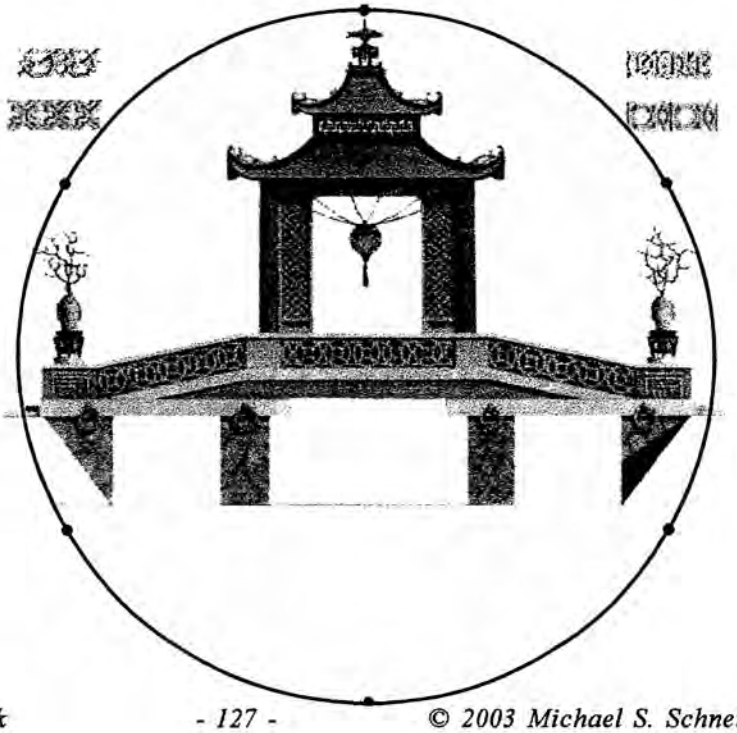
Many examples of art depicting Vishnu start with a Circle centered at his navel.



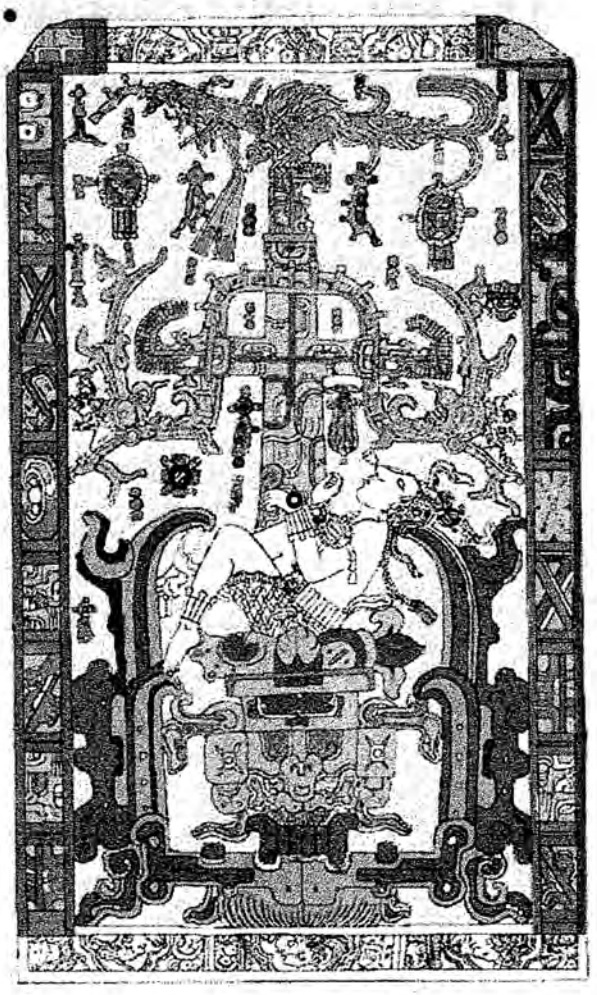


A Chinese Pagoda Design

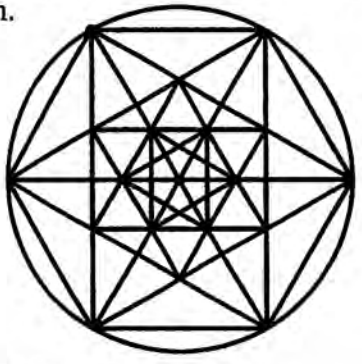




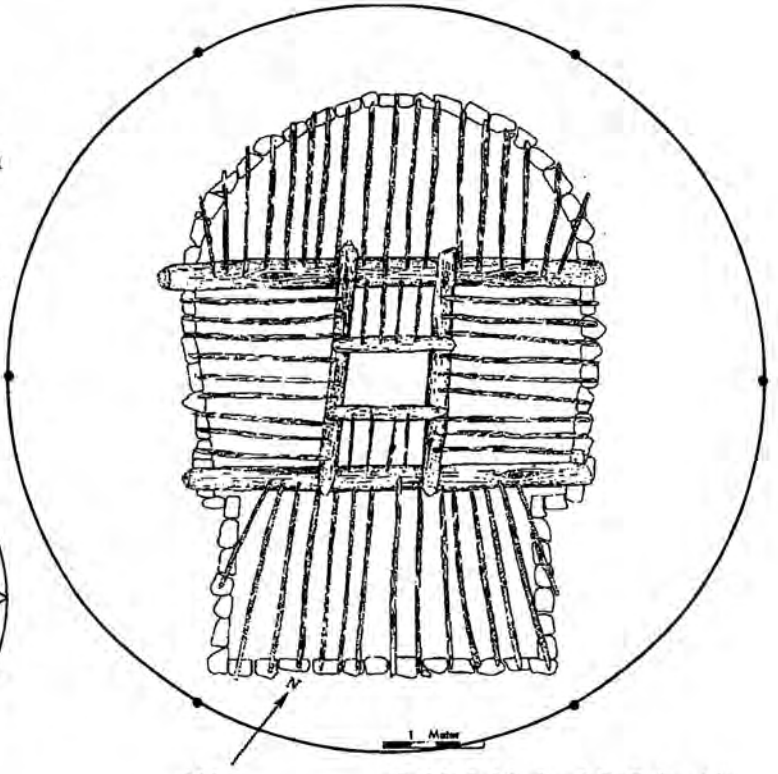
Lid Of Pacal's Sarcophagus



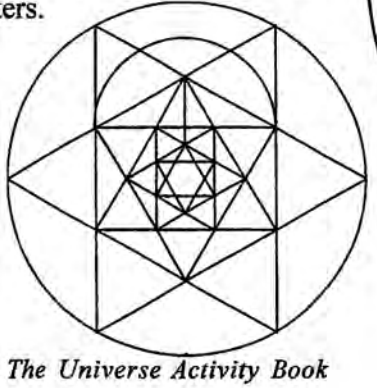
Pacal, also known as *Hanab-Pakal* (Pacal the Great) and Lord Shield Pacal, was one of the greatest Mayan rulers. At the age of 12 years and 125 days (the Mayans were very precise in calendar matters), in the year 615, he ascended the throne. He ruled from 615 to 683. Pacal died at age 80 years 158 days and is entombed in the sarcophagus of the Temple of the Inscriptions in Palenque. On this image of the lid of his stone sarcophagus is a center point at his wrist for your compass point, and a point in the upper left corner for the scribe. Turn a Circle and explore it, starting with this construction.



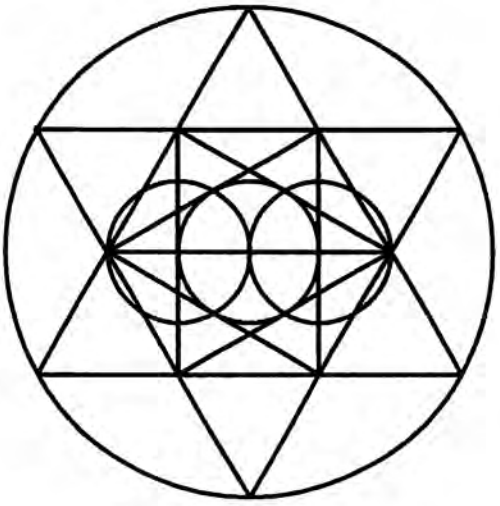
Native American Kiva Roof

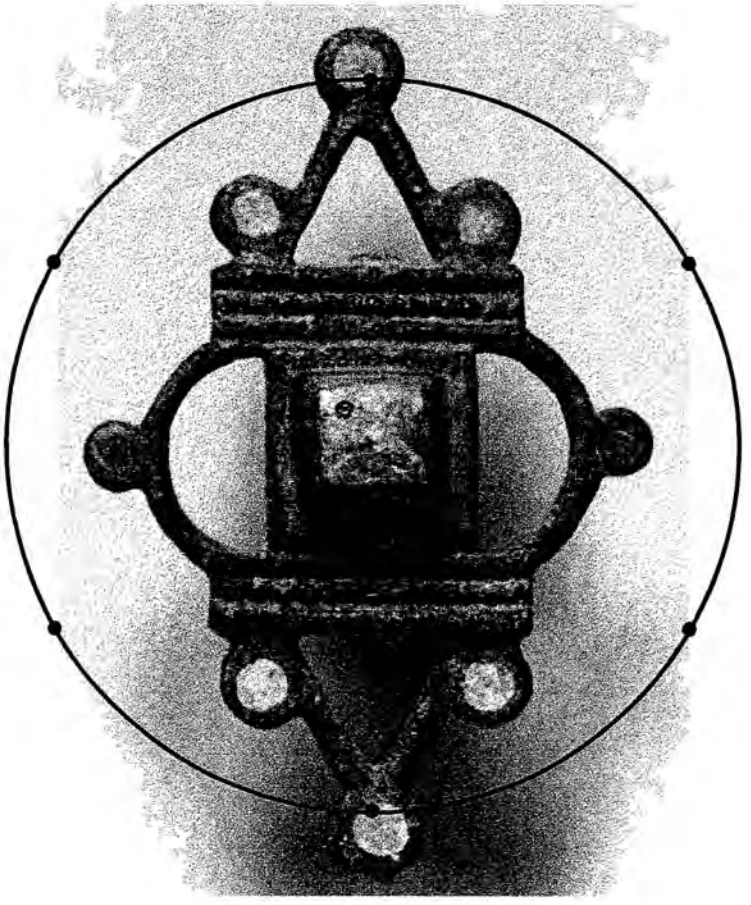


A *Kiva* is a sacred structure partly (or completely) buried in the Earth. Its entrance is usually through an opening in the roof. This roof was reconstructed after having been burnt centuries earlier. Yet notice how the geometry still guides the beams all the way down to the size of the entrance hole. The geometry symbolically sanctifies the passage of whoever enters.

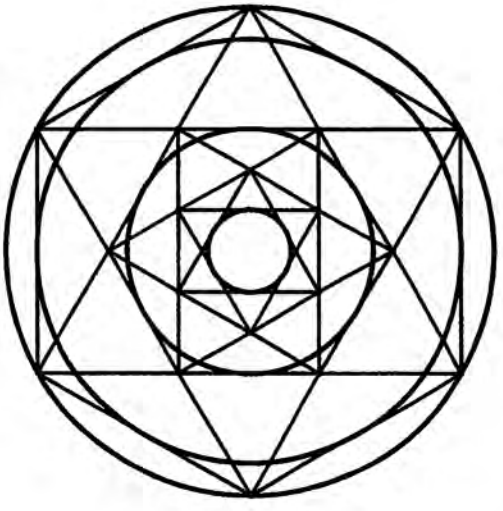


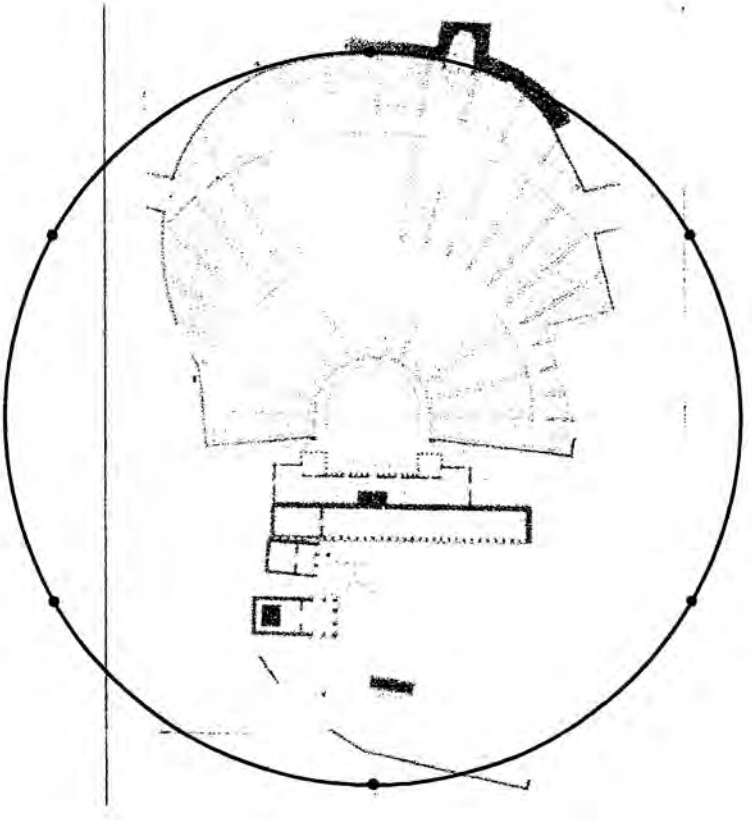
Celtic brooch from Roman Gaul,
Second century.



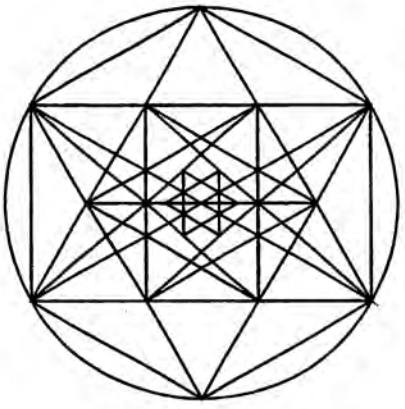


Greek Amphitheater

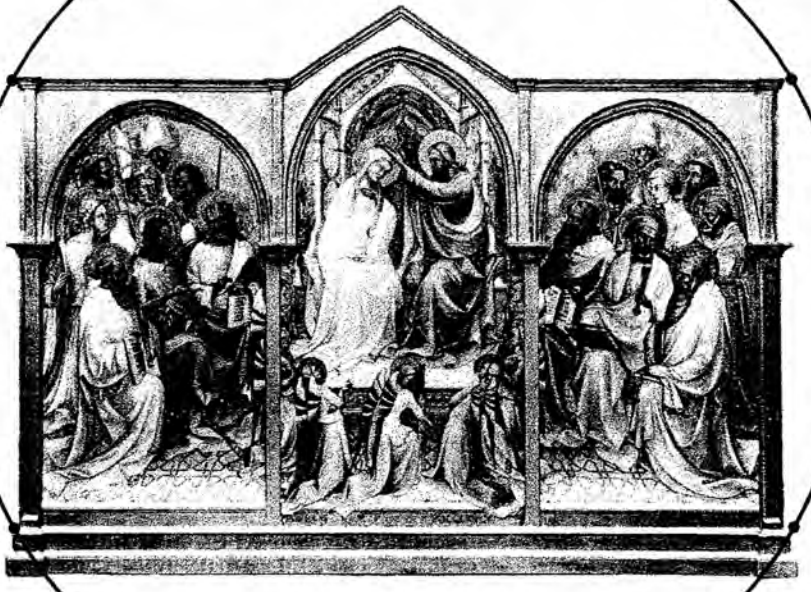


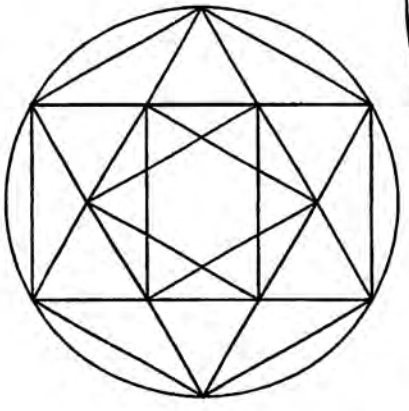


Ethiopian painting depicting
Jesus and his disciples



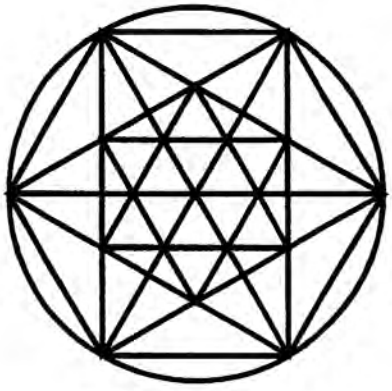
Italian triptych of
Jesus Crowning Mary





Baptism of Jesus

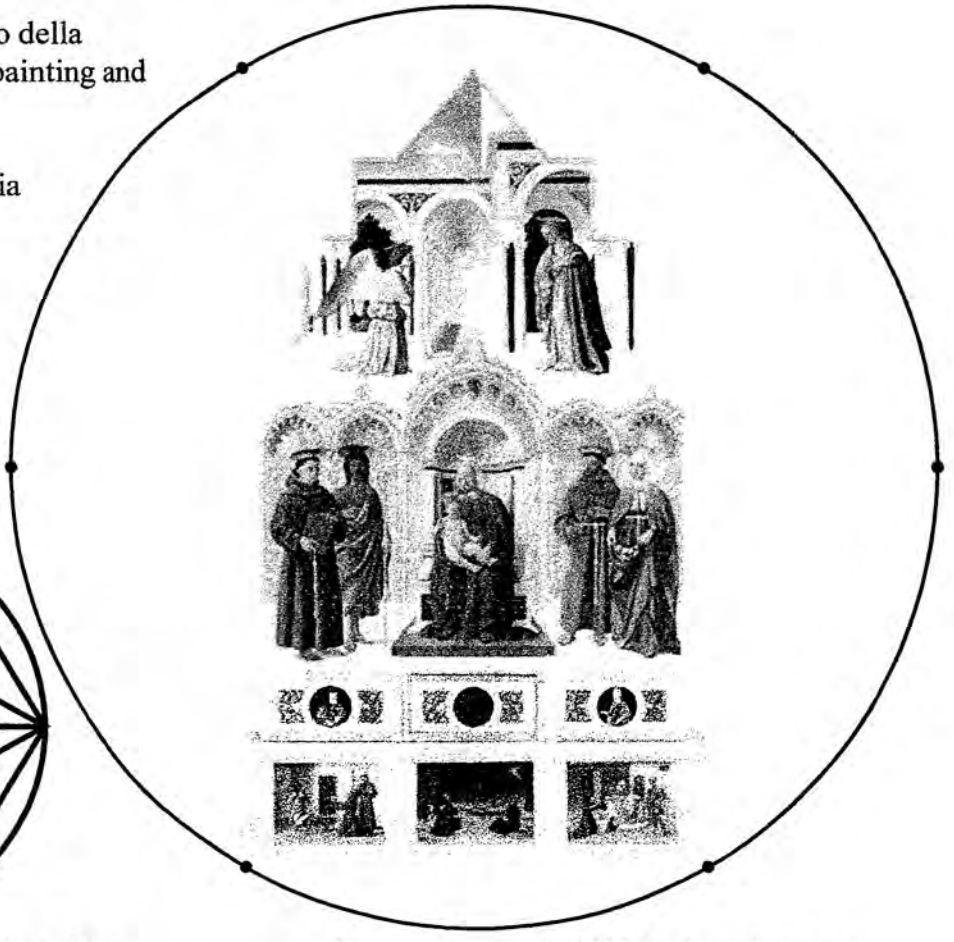
Note that his finger in this Byzantine painting points to the exact center of the Circle.

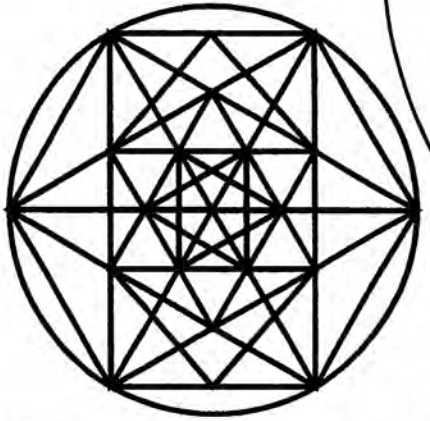


Polittico di Perugia

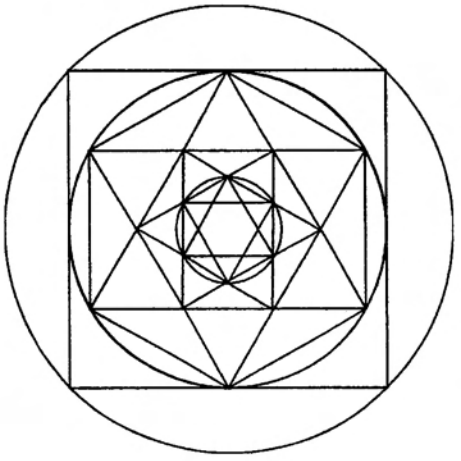
Painted around 1470 by Piero della Francesca, a master of both painting and geometry.

Galleria nazionale dell'Umbria

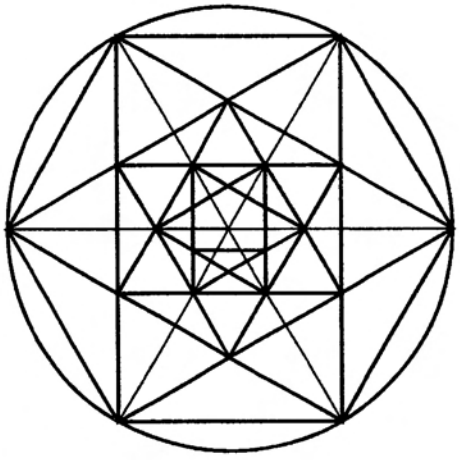




Jewish *Seder* plate for the Passover holiday meal. Notice that a large Square makes the area of the rim equal to the area of the center (page 51). The center is based on Hexagonal geometry. Points for the Square and Hexagon have been provided.

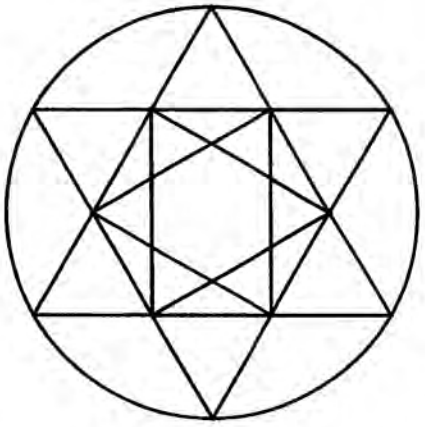


An Islamic painting depicts the **Angel Gabriel** blowing his trumpet, making an annunciation.



The designs of official seals of all nations are based upon the ancient geometric rules of heraldry.

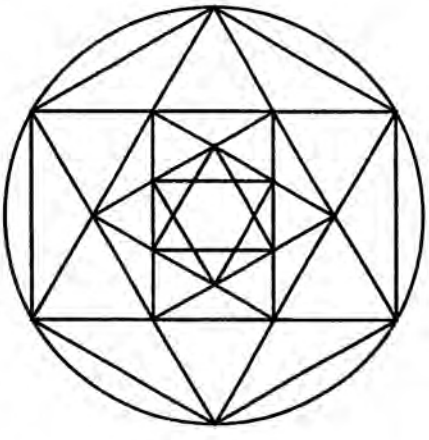
A British Coat Of Arms



Great Seal Of The United States (Eagle Side)

The Hexagram arrangement of the stars in the glory of clouds above the eagle's head shows us the geometric design of the whole seal. You've done the geometry accurately if the eagle's eye is in a small Triangle.

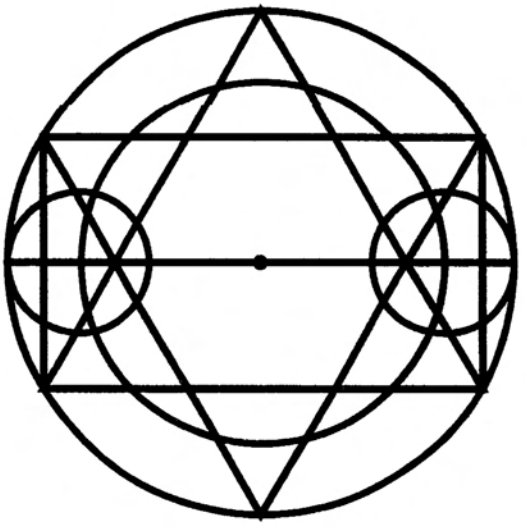
It's on every dollar bill.

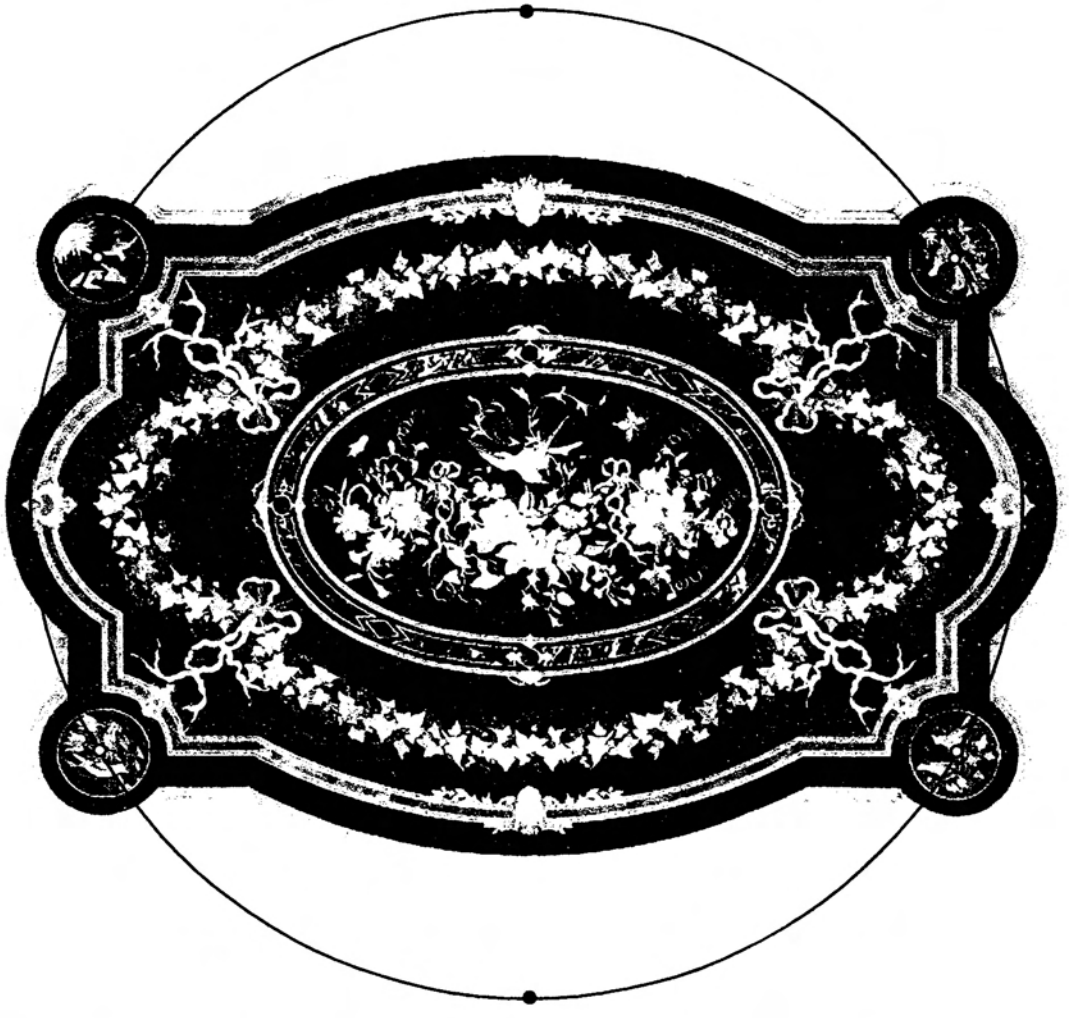


Herter Table Top

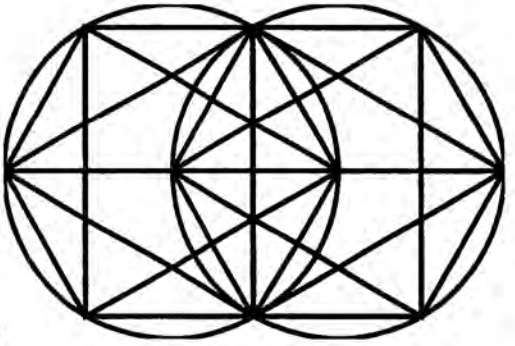
Designed by Herter Brothers of 19th century New York, who were known for applying classic and graceful geometry to their creations of interior design.

Metropolitan Museum Of Art, New York





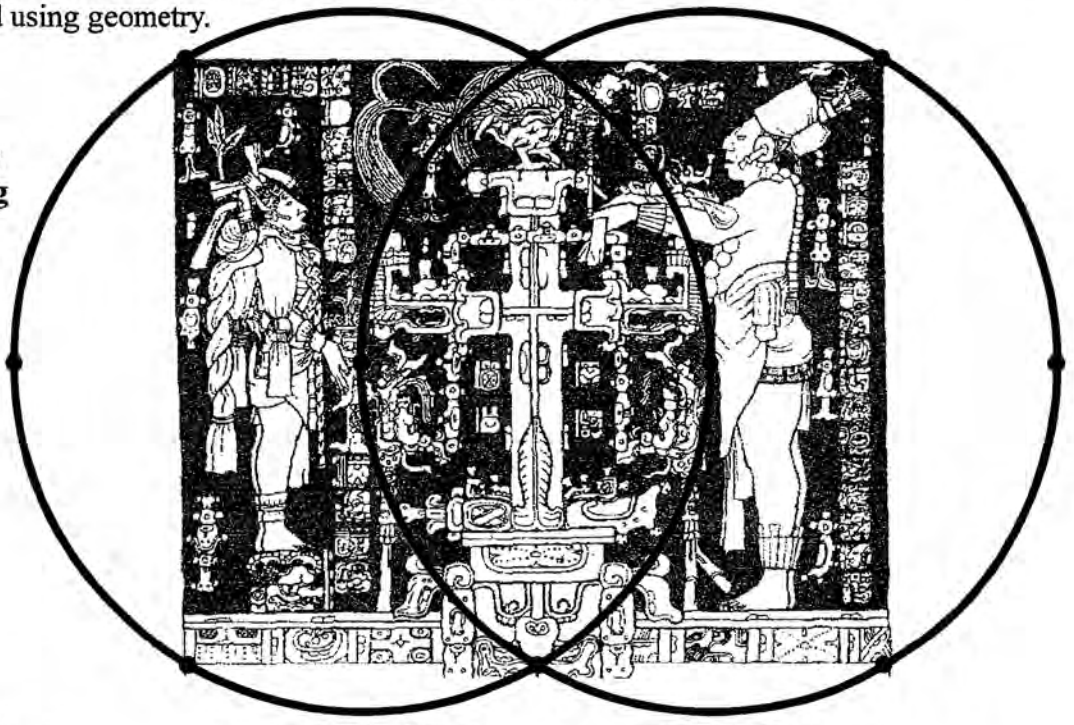
Overlapping Six-Pointed Circles



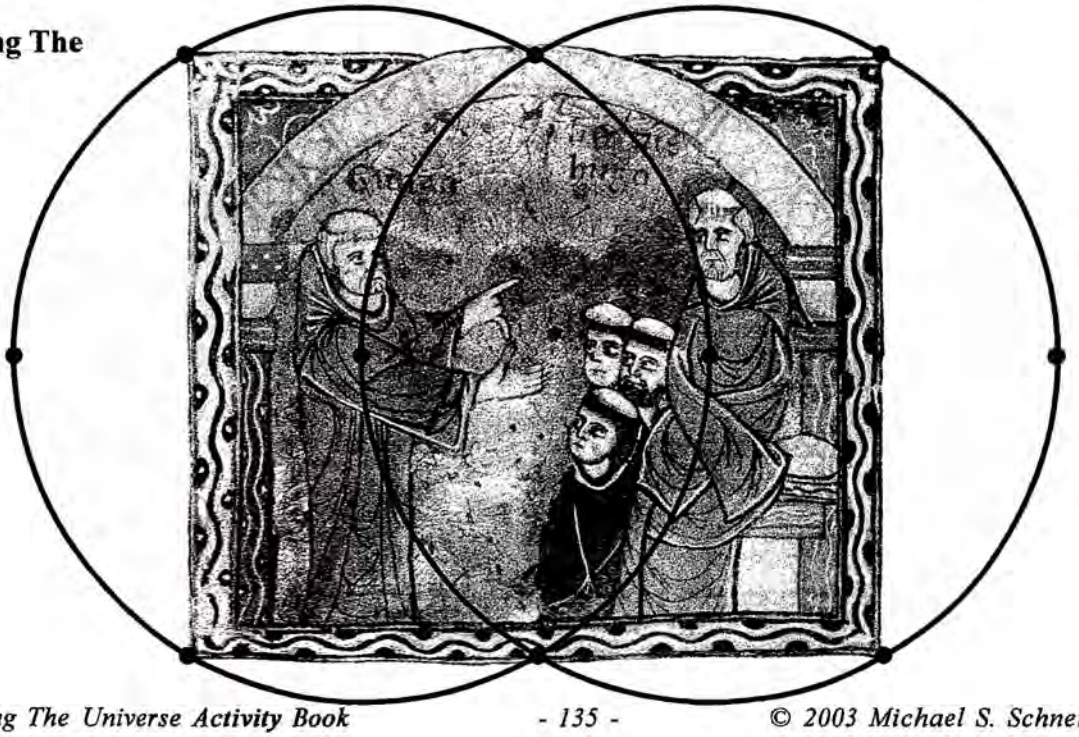
If we "walk" the compass around each Circle of the Almond construction we have two overlapping six-pointed Circles. And more art to examine.

Explore these works from the Mayan and Christian cultures by doing geometric constructions using the points on the Circles. You can draw and subdivide Hexagrams, or draw any other lines, arcs or Circles to discover how the scenes were composed using geometry.

Priest Making Offering

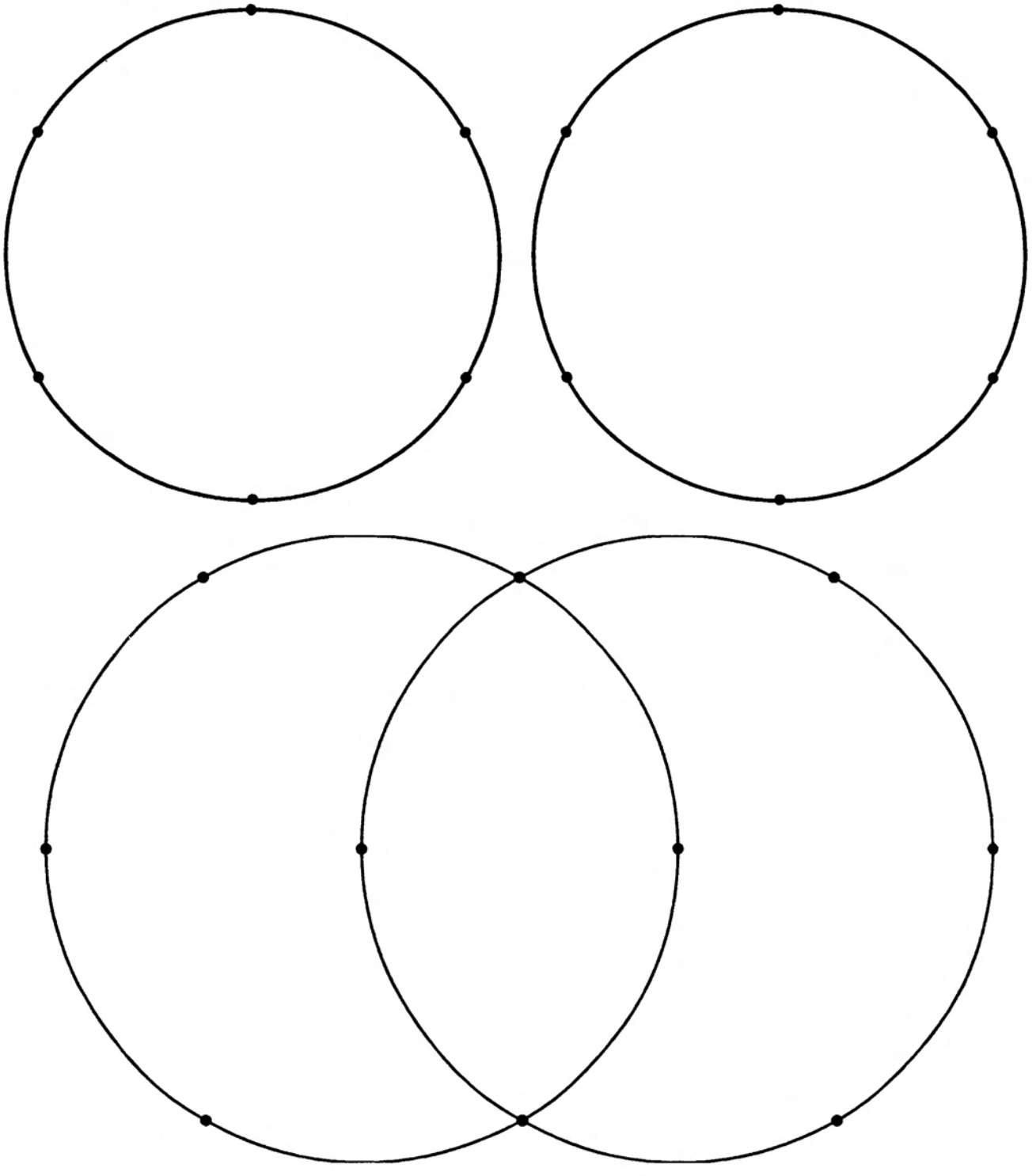


Gunzo Directing The Monks

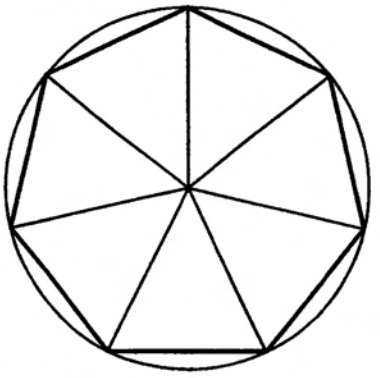


Design Your Own Hexagonal Art

Use your imagination and what you have seen to design your own art. Do a Hexagonal construction below, and on blank paper. Let your design be guided by its points, lines, arcs and areas.



7 The Heptagon



Every Heptagon (Greek *hept-* = seven) has seven corners and seven sides. But no regular Heptagon with equal corner angles and equal sides can be made with a compass and straightedge, which produce only approximations. But we'll learn constructions which will create Heptagons close enough for us to enjoy its properties.

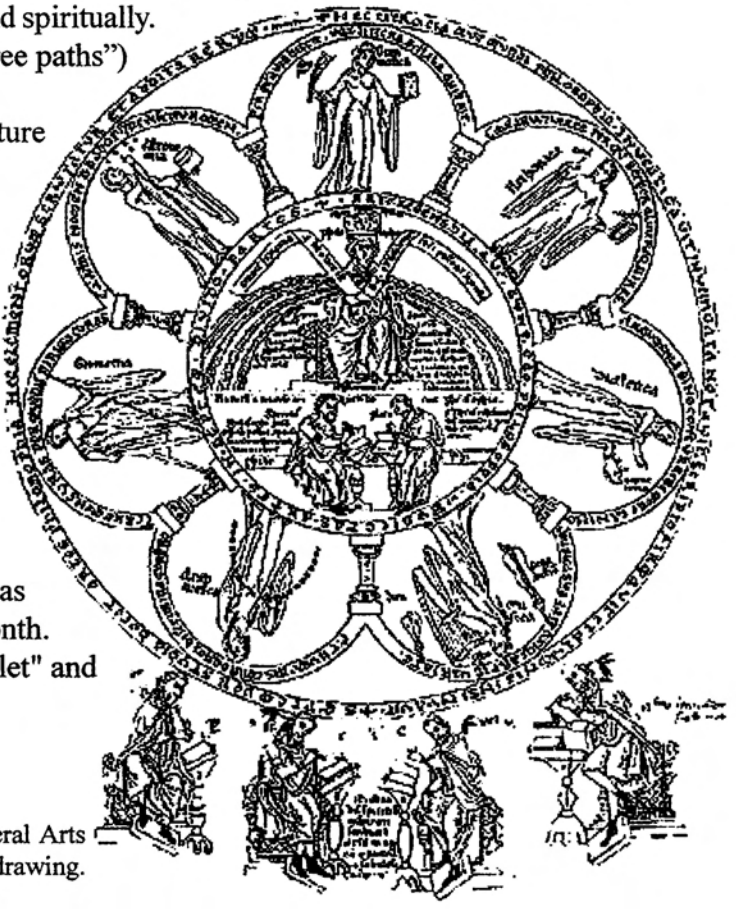
Around the world the number seven holds a special place in religion, philosophy and mystery. There are many stories with seven parts, seven creatures, cities or objects. A course of education known in the west as The Seven Liberal Arts was similar to that found in other cultures including India, China, Greece and Rome. Four basic topics, known as the Quadrivium ("four paths") were considered to be the foundation of all knowledge and learning:

Arithmetic—	Number
Geometry —	Number in space
Music —	Number in time
Astronomy —	Number in space and time

Numbers were considered to be the foundation of everything, since nothing can exist without quantity, shape, size or weight. Arithmetic and geometry were most important because everything else came from them. The lessons of arithmetic and geometry were applied to language, law and everyday affairs. Philosophy was once considered to be quite practical and the Liberal Arts were intended to help "liberate" people physically and spiritually.
Three more topics, known as the Trivium ("three paths") consisted of:

* Grammar — an introduction to literature and the mathematical structure of language;
* Rhetoric — an introduction to law, history and how to speak in a way which follows the numerical patterns of reality;
* Logic — or dialectic, the art of understanding "both sides", the "pro and con", a path to philosophical reason and discovering the truth of things.

Word Origins: Many Greek words beginning with "H" were changed to "S" in Rome. The Greek prefix *Hepta* became the Roman *Septa,* as when "September" was the seventh Roman month. And that's why we say "Hexagon" but "Sextuplet" and "Sextile" for different uses of "six".



Students learning from the Seven Liberal Arts are depicted in this Medieval drawing.

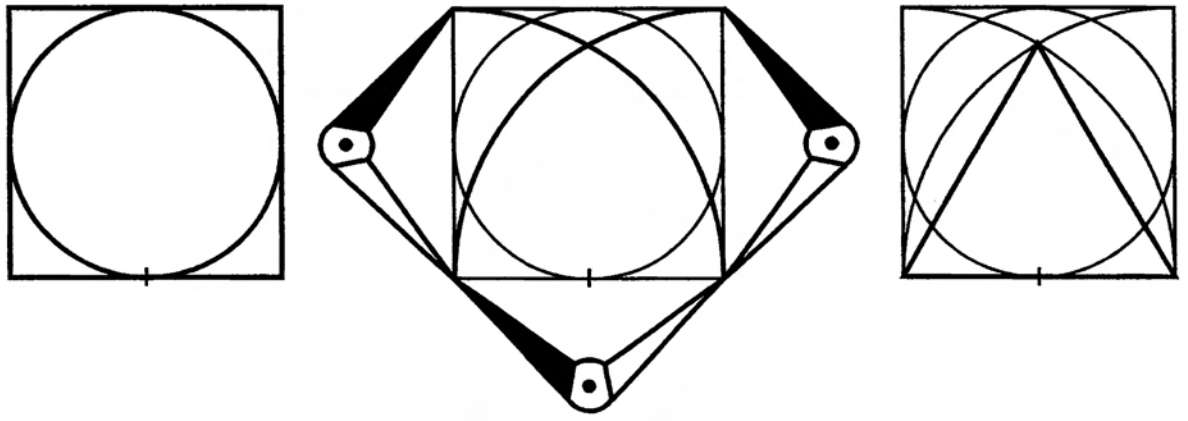
A Heptagon From Circle, Triangle And Square

(1) Begin with the construction of a Square around a Circle (page 47).

(2) Open the compass along the bottom of the Square. Swing an arc from each bottom corner to the top corners.

(3) Connect the crossing point of these arcs with the bottom corners of the Square. Since the three line segments are equal, this is an equilateral (regular) Triangle.

1 2 3

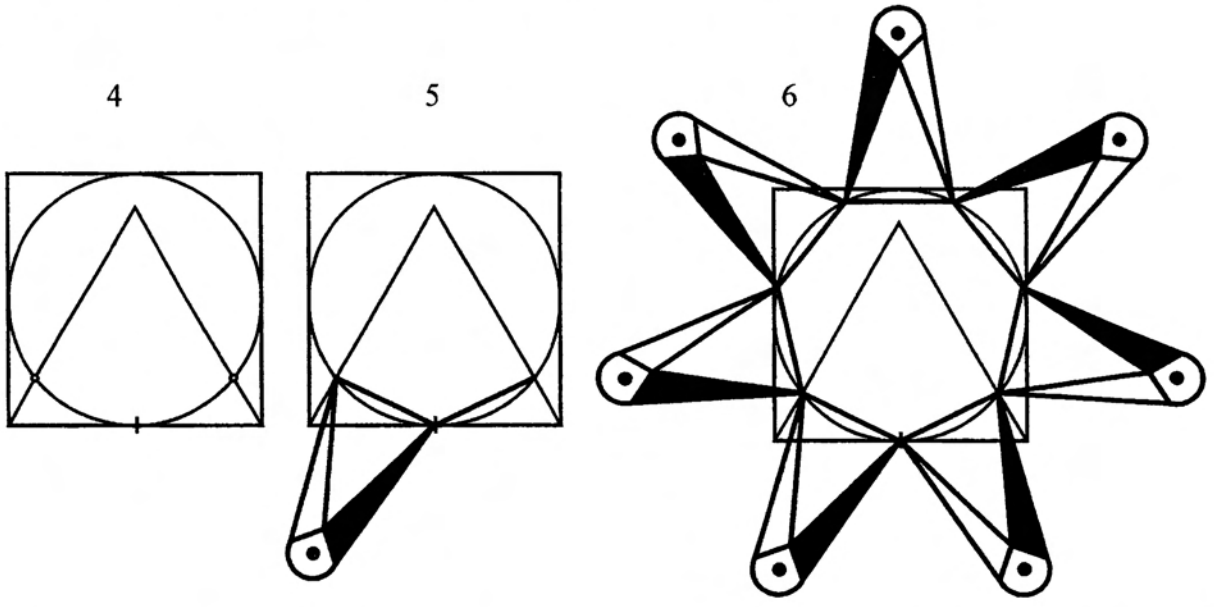


(4) Note the two points where this Triangle crosses the Circle.

(5) Connect the bottom midpoint of the Square with these crossing points. These are two sides of the Heptagon we're building. (This creates a Heptagon pointing downward. If you want it to point upward, make the starting Triangle (steps 2 and 3) point downward.)

(6) "Walk" this compass opening around the Circle *in both directions* from the same starting point. The seven best points will be those *between* the pairs of marks from each direction. Connect these points with your straightedge to inscribe a Heptagon in the Circle.

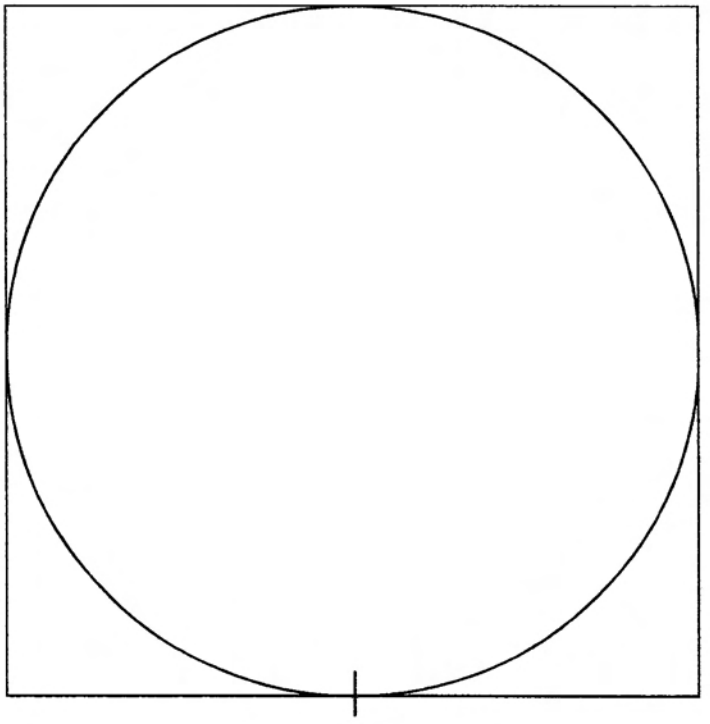
4 5 6



Construct A Heptagon

Use the method which combines Circle, Square and Triangle.

"A 7-sided figure is impossible to draw
With perfect mathematical precision;
And if you try to do it
    you are absolutely sure
To find your efforts treated with derision.
And yet there are philosophers
    who readily declare
That nothing in this world is really true,
And so I've drawn a Heptagon
    by Triangle and Square.
For any human purpose it will do."
-- John Michell



Heptagram Stars

The Heptagon contains two Heptagram stars.

(1) Connecting every other point produces one star.

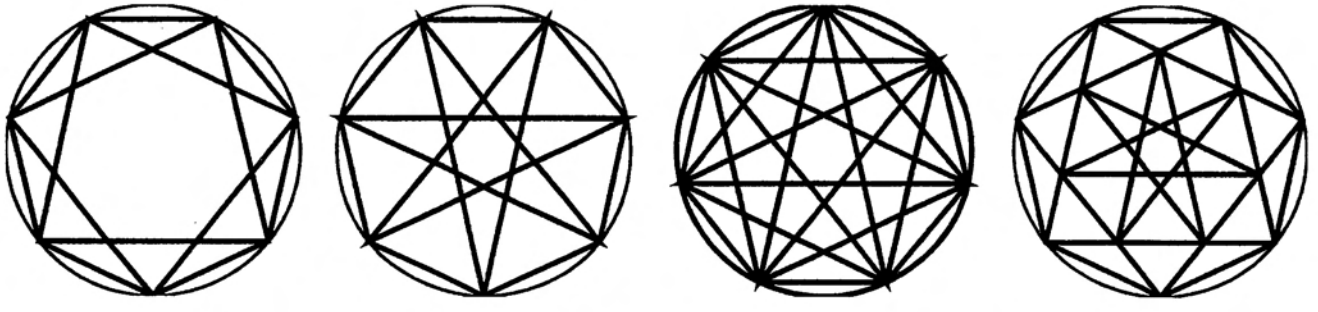
(2) Connecting every third point produces the other.

(3) A Heptagon and both Heptagram stars together.

(4) One Heptagram star within the other.

Add Heptagram stars to your construction above.

1 2 3 4



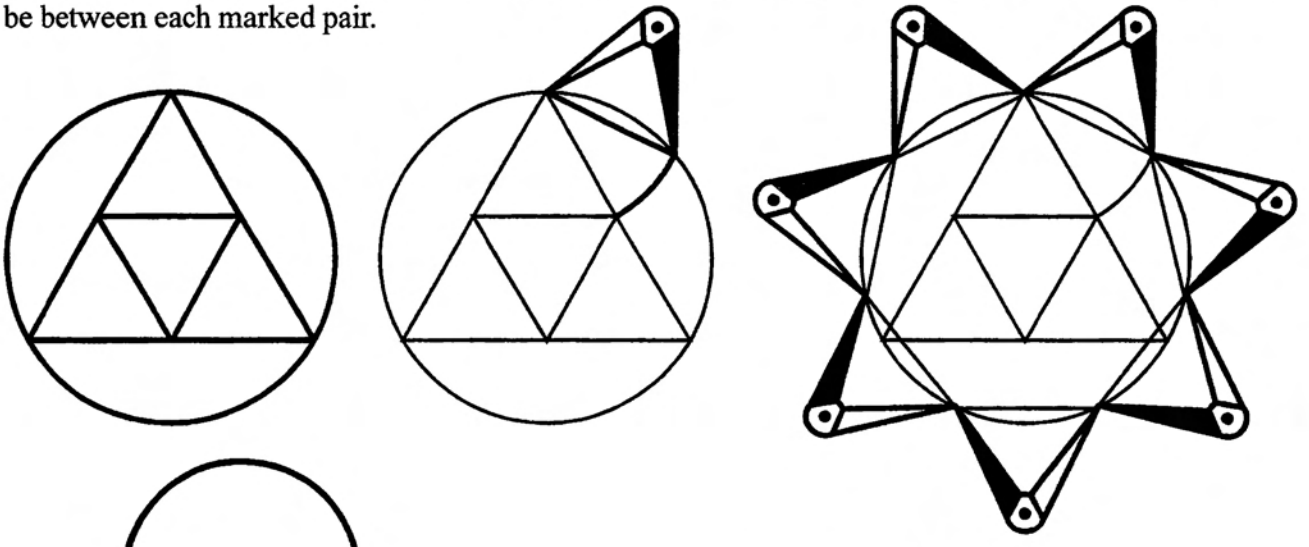
5

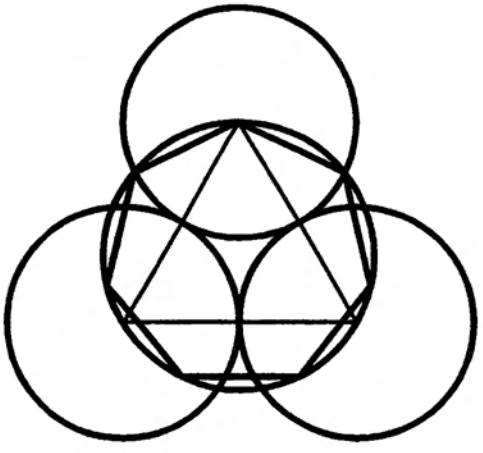
A Heptagon From A Triangle

(1) This begins by constructing a Triangle in a Circle, and then subdividing a Triangle within it. We saw this "six-around-one" construction in Chapter 3 (page 31) and Chapter 6 (page 112).

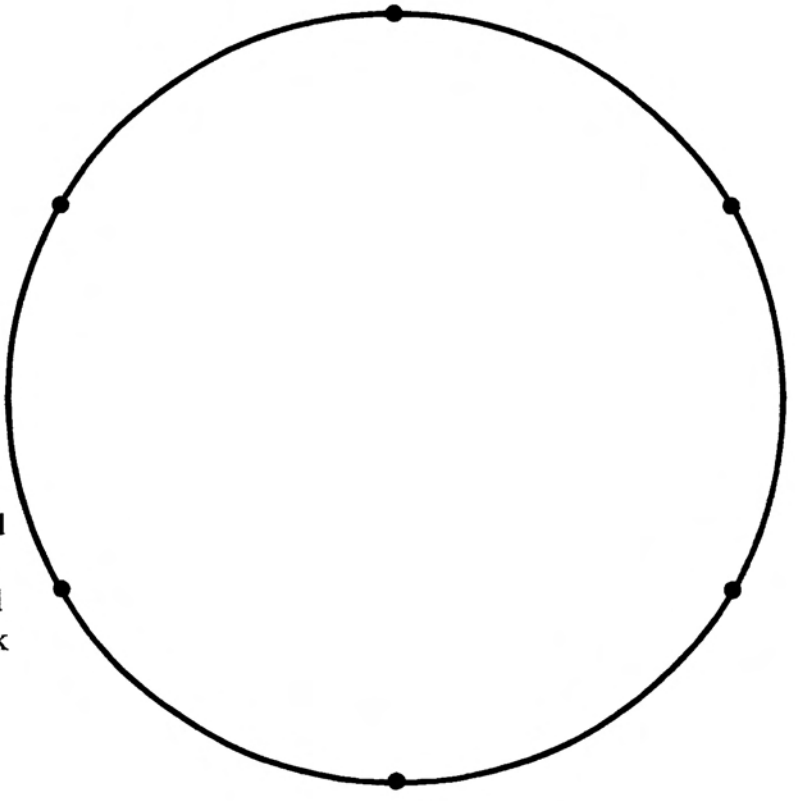
(2) Place the compass point at the top of the large Triangle and open the scribe to the corner of the small Triangle, the midpoint of the side of the large Triangle. Turn an arc until it crosses the Circle. (If you have room, turn the full Circle from each corner of the large Triangle to see the full construction.)

(3) Walk this opening of the compass around the Circle in both directions. The best seven points will be between each marked pair.





The construction with full circles



Construct a Heptagon by this method

Start with this circle's *six* equally spaced points, or do the construction on a blank sheet of paper.

Construct A Heptagon With Knotted String Or Rope

(after Keith Critchlow)

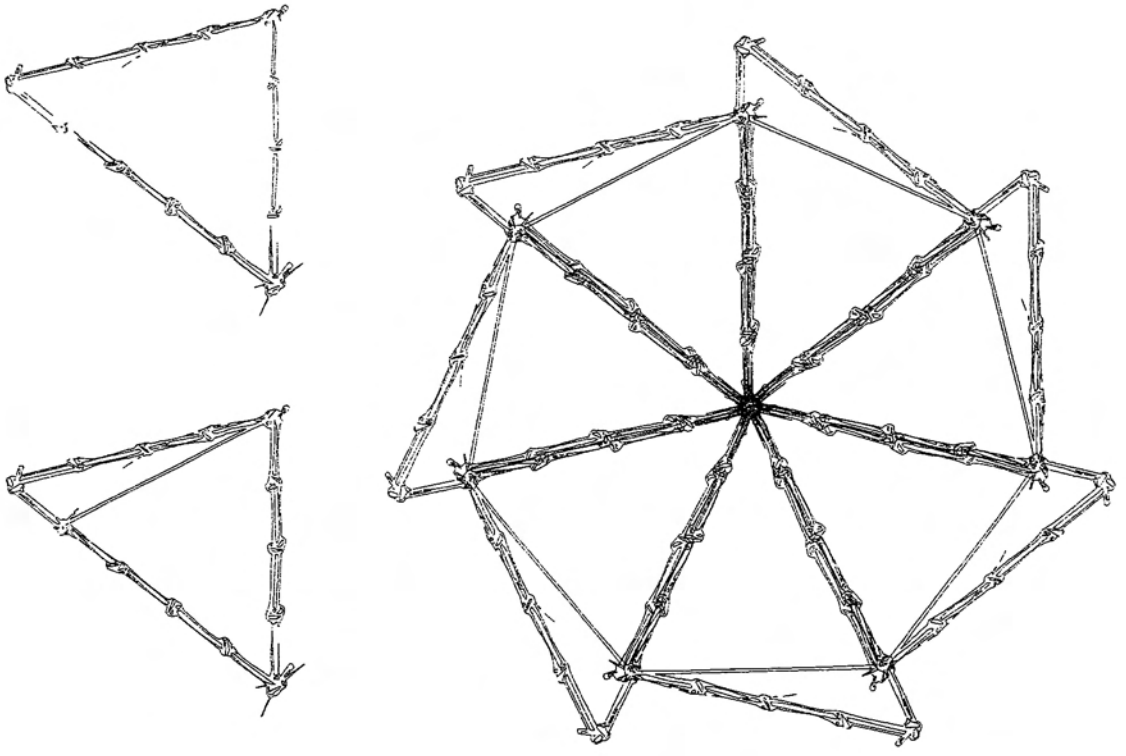
To do this construction on paper you'll need a length of string and push-pins.
To do it outdoors you'll need a rope, wooden or metal stakes, or paint.

(1) Create a loop with thirteen *equally-spaced* knots in it. If that's not easy to make, you can fold or measure equal distances along the string or rope and insert stakes, or paint it instead.

(2) Use push-pins or stakes to make the string or rope into a Triangle whose sides are four, four and five spaces between knots.

(3) Place a push-pin in the paper one knot inward on the longest side (as shown). With rope, stretch a cord from one corner to a stake at one knot in on the long side.

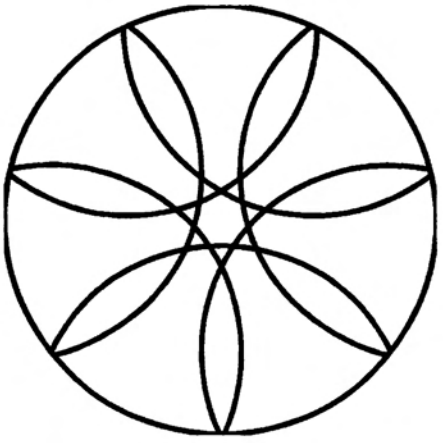
(4) Rotate the triangular loop around the center push-pin or stake. At each turn, place a stake at the point one knot in on the long side (see illustration). This process should take you around the Circle seven times. Connect the stakes to form a Heptagon.

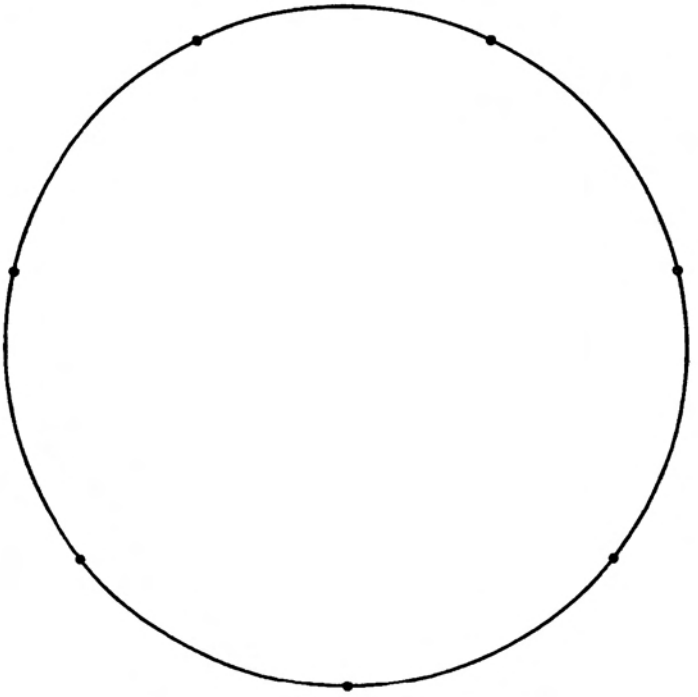


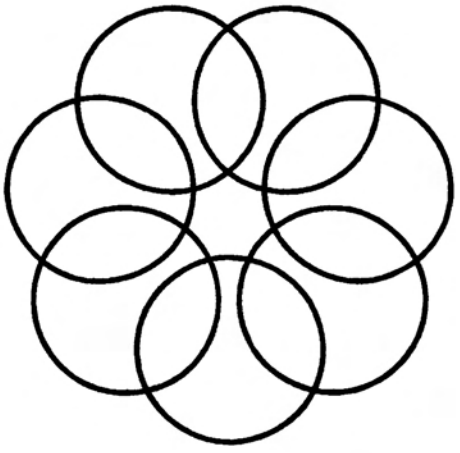
Replicate Heptagonal Patterns

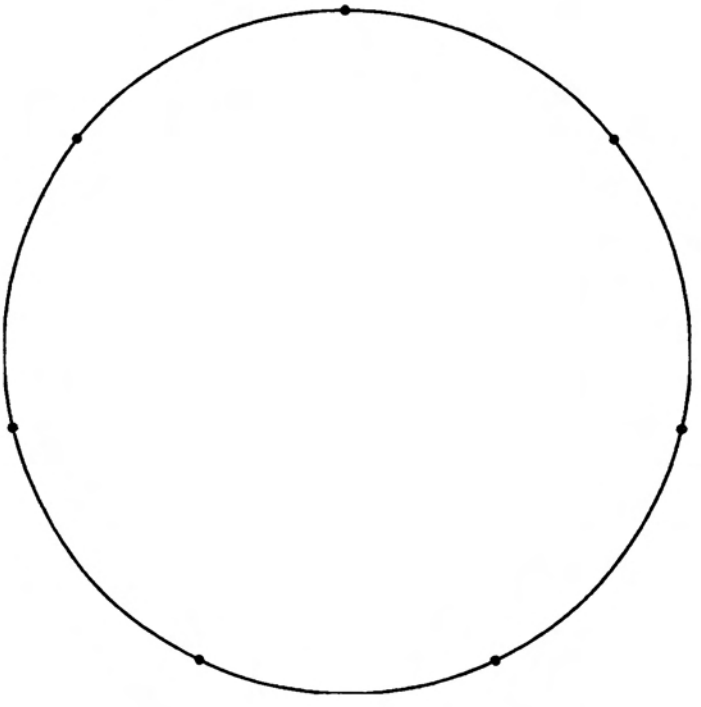
Use these circles or do the constructions on blank paper. Use colored pencils.

At each step, think: * Where is the *center* of each Circle (or arc)?
* Where is the scribe open to?

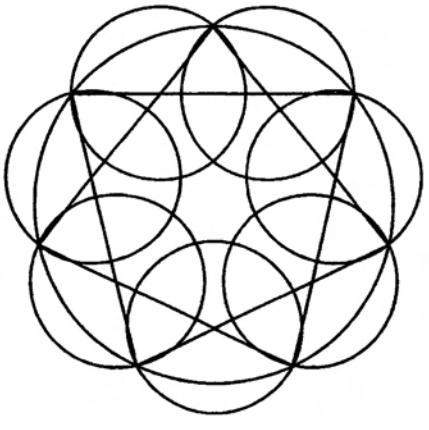


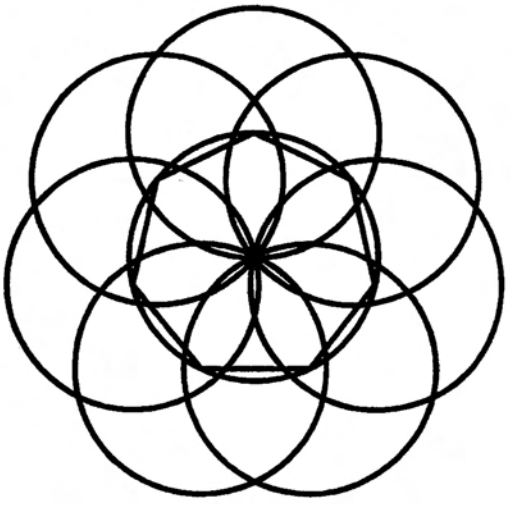


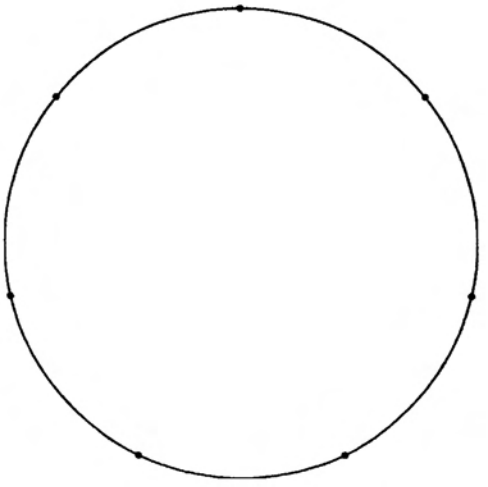


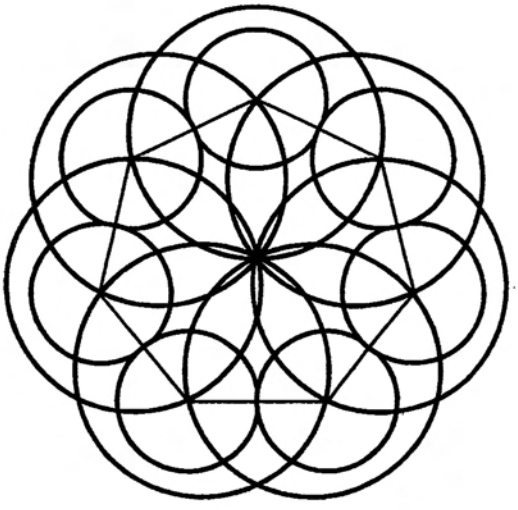


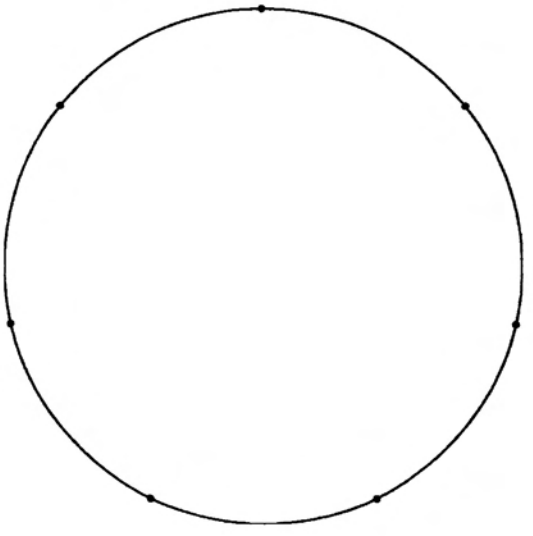
Hint:





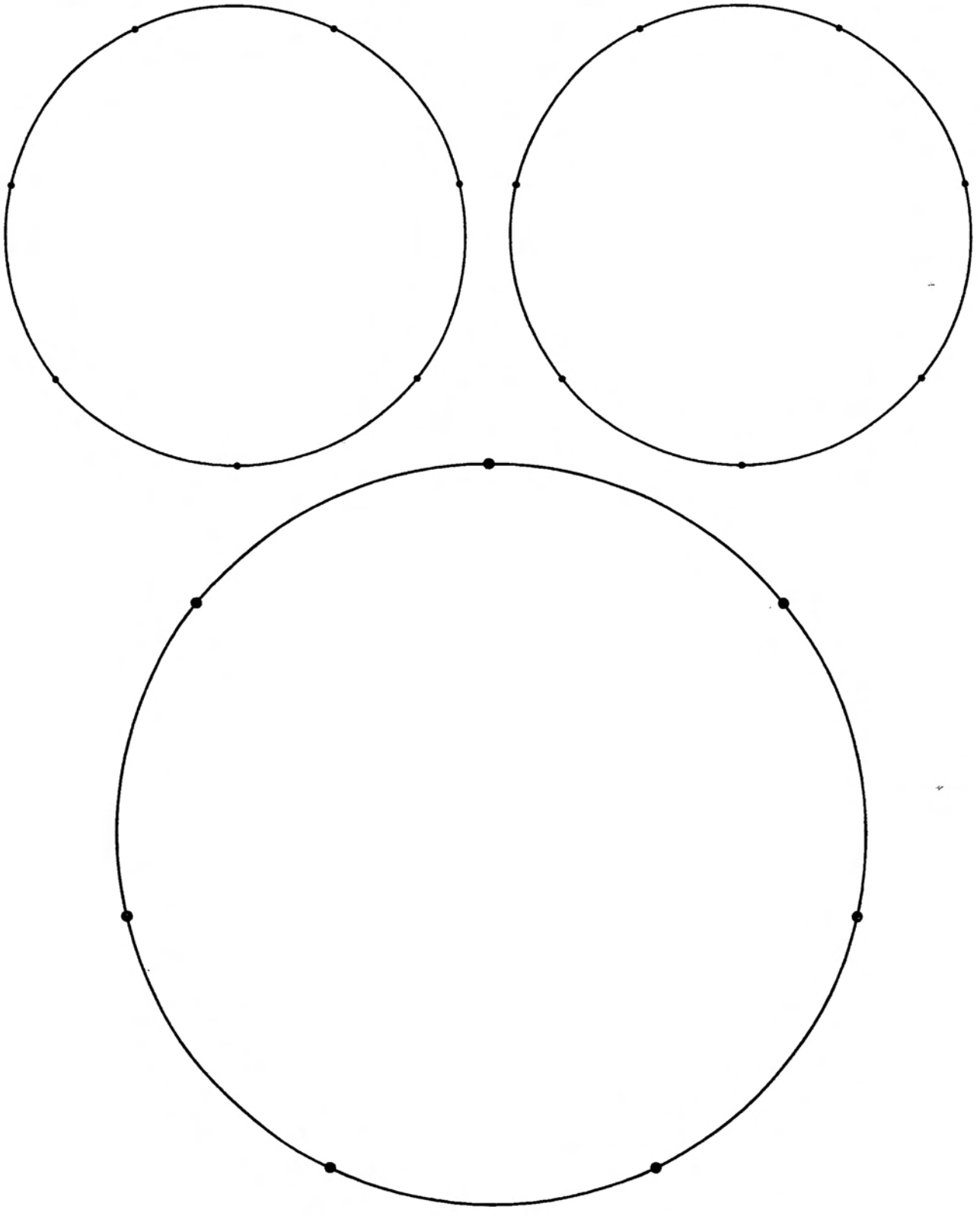




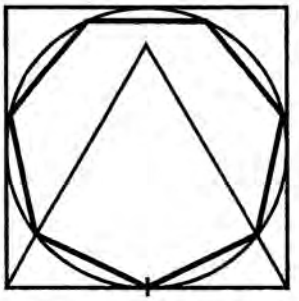


Create Your Own Heptagonal Designs

Create your own constructions starting with these seven pointed Circles.

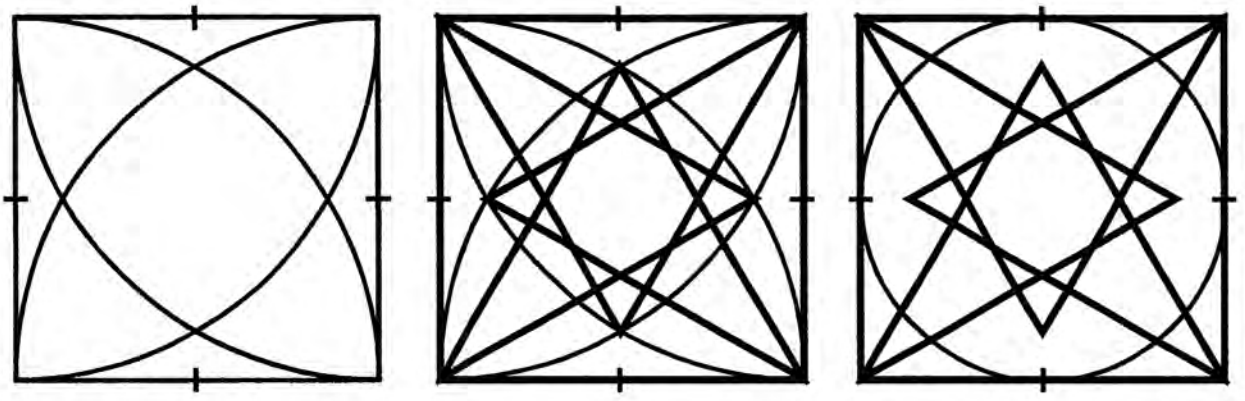


Twenty Eight Points Around A Circle



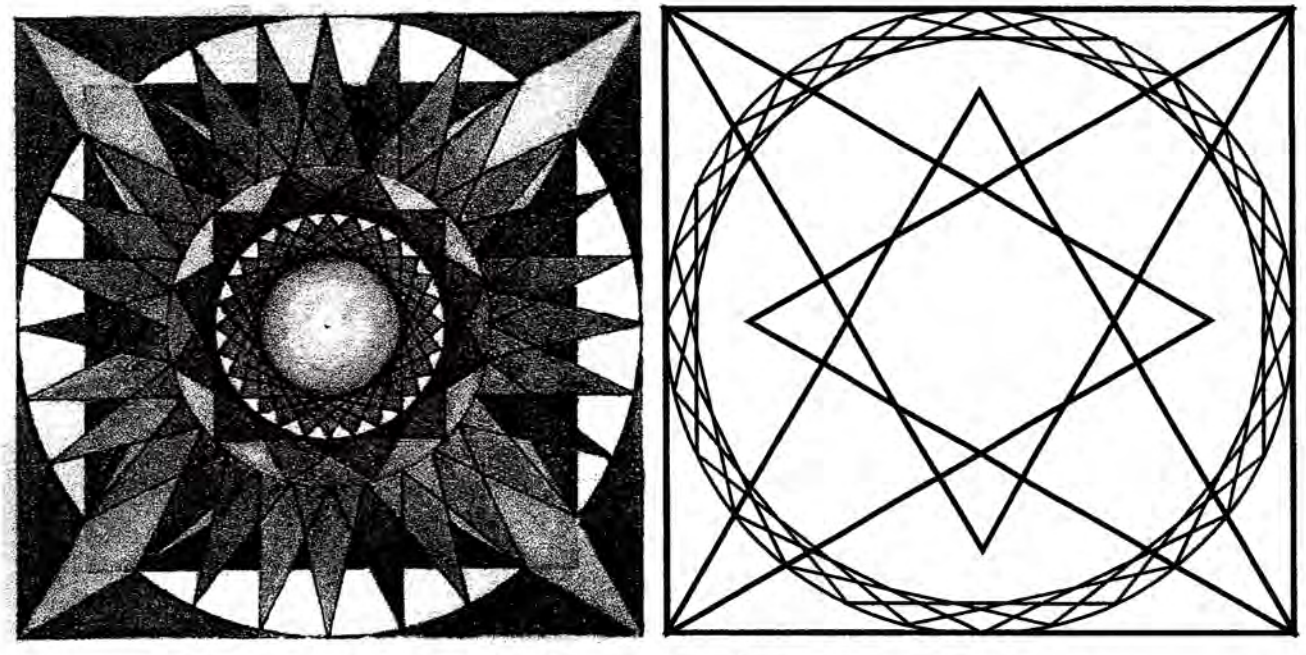
By constructing a Heptagon by the Square, Circle and Triangle method (page 138), you can make a Triangle *on each of the four inner sides of the Square*. This produces four Heptagons and twenty eight (28 = 4x7) equally spaced points around the Circle (see below).

If you have patience and can be accurate, you can do this construction on a sheet of blank paper. Construct a Triangle on each inner side of the Square and find the seven points from the triangle/Circle crossings.



Use your compass, straightedge and colored pencils to investigate and develop this construction of twenty eight points further.

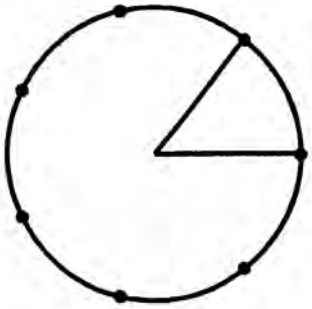
We'll see more of it in Chapter 12 (page 210).

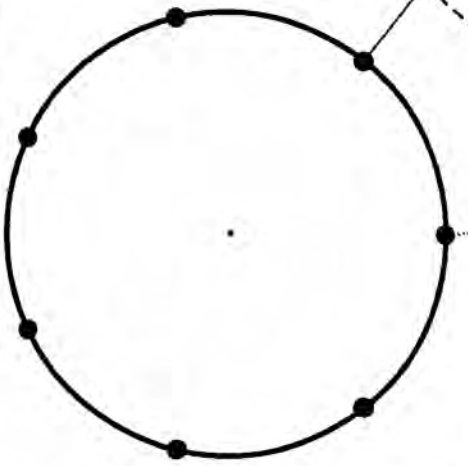


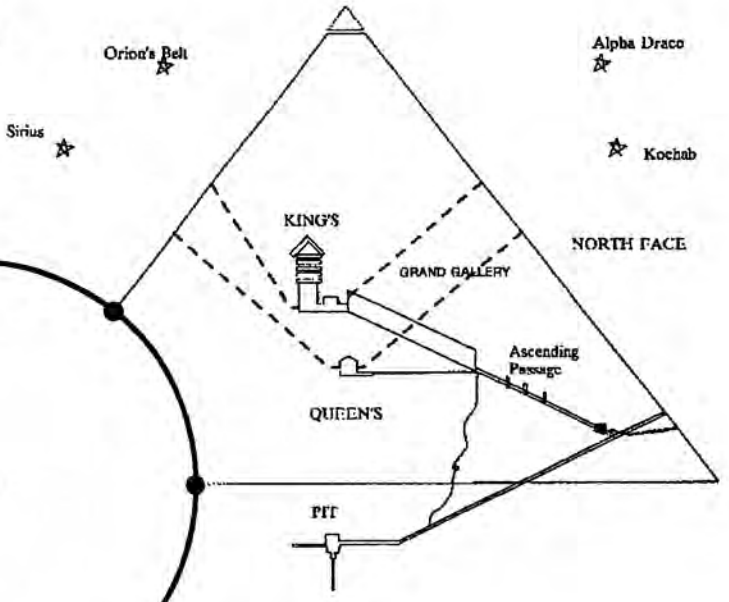
This watercolor by John Michell is based on the above geometric construction.
Can you discover how?

Heptagonal Designs In Art

Complete the corner of the **Great Pyramid.**



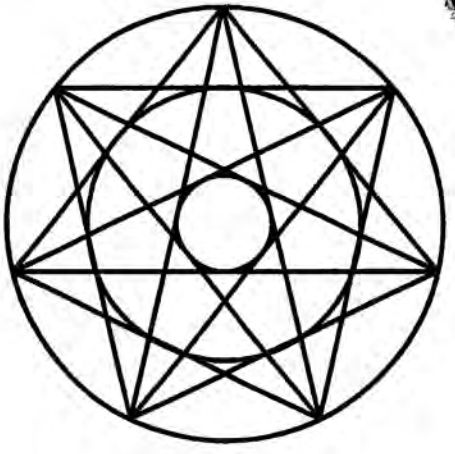




Chinese *Pi* Disc

The *Pi* disc was the highest emblem of Chinese noble status. Among other ritual appearances, it was used to guide a deceased spirit to heaven through the Pole Star, symbolized by the hole at the disc's center.

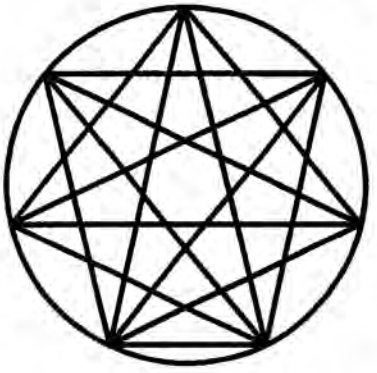
This *Pi* disc was excavated in 1916 from a tomb of the Han Dynasty at Lo-lang in Korea (National Museum of Seoul).



In worldwide circular art, the ground line upon which people, animals, deities and mythical creatures stand provides a clue about the polygon guiding the scene. Another clue may be found in patterns around the rim. Here are three of countless examples from Greek art, circular pictures from the bottoms of their *kylix* drinking bowls. Look for seven white points added to each and do the nearby construction to see how the scenes appear to have been guided by Heptagonal geometry.

Greek Angel
Notice the ground line, and the seven crosses spaced around the rim.

Greek Sphinx

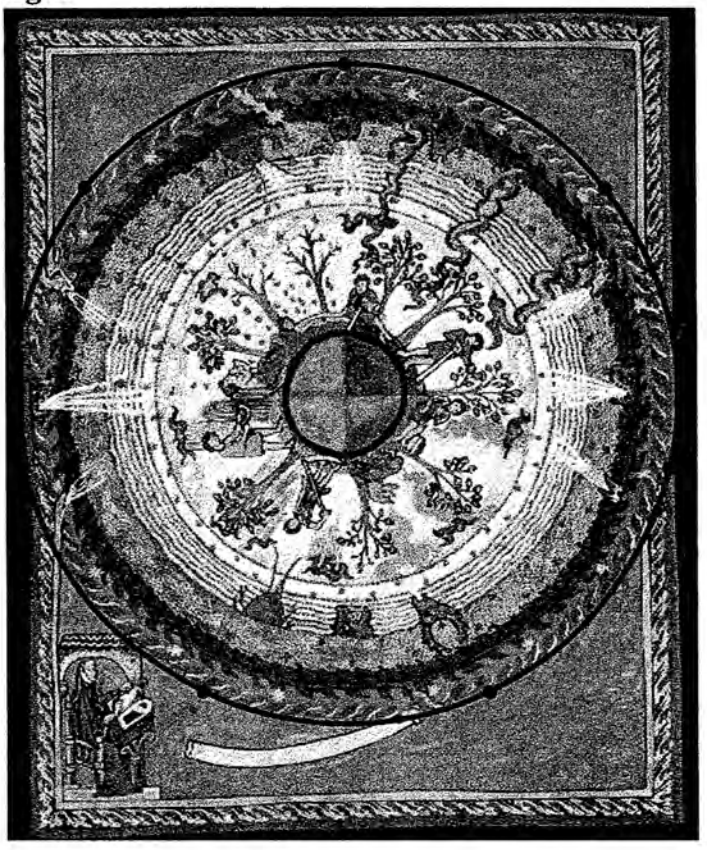


Stag in a Circle at the bottom of a **drinking bowl**.

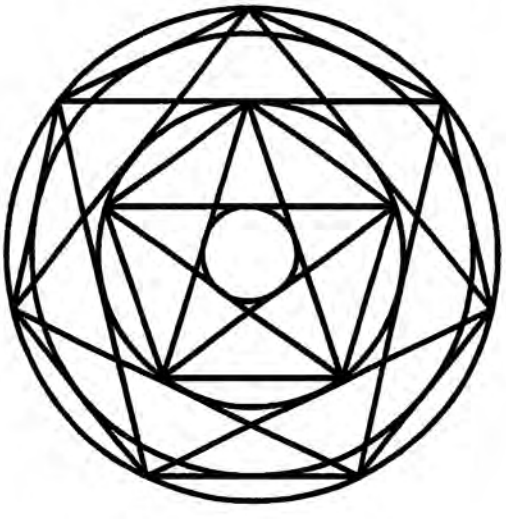
Paintings Of Visions Of Hildegard Of Bingen

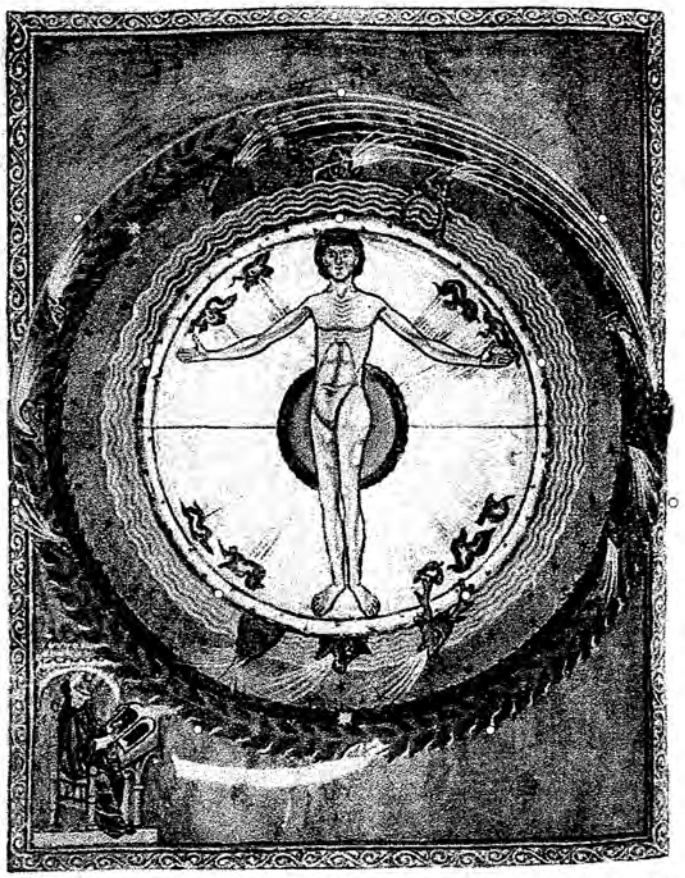
Hildegard was a nun (seen bottom left in each painting) who described her visions to a monk who painted them according to traditional rules of geometry, art and symbolism. From the center of each we see expanding circles of earth, water, air and fire in the outermost ring. Mythical creatures blow winds upon the earth.

The Four Seasons



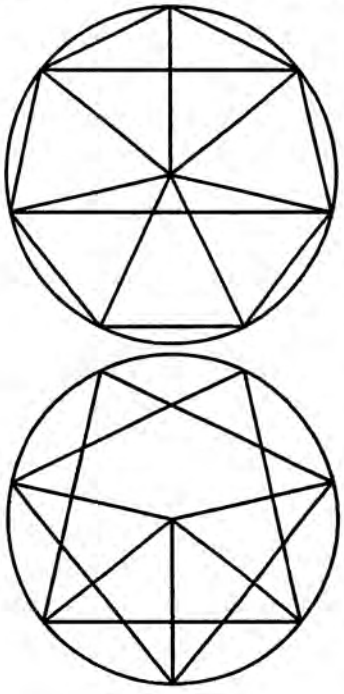
Here we see a man at the center of earth, water, air and fire. A Pentagonal geometry guides the human Circle. A Heptagonal construction surrounds it.

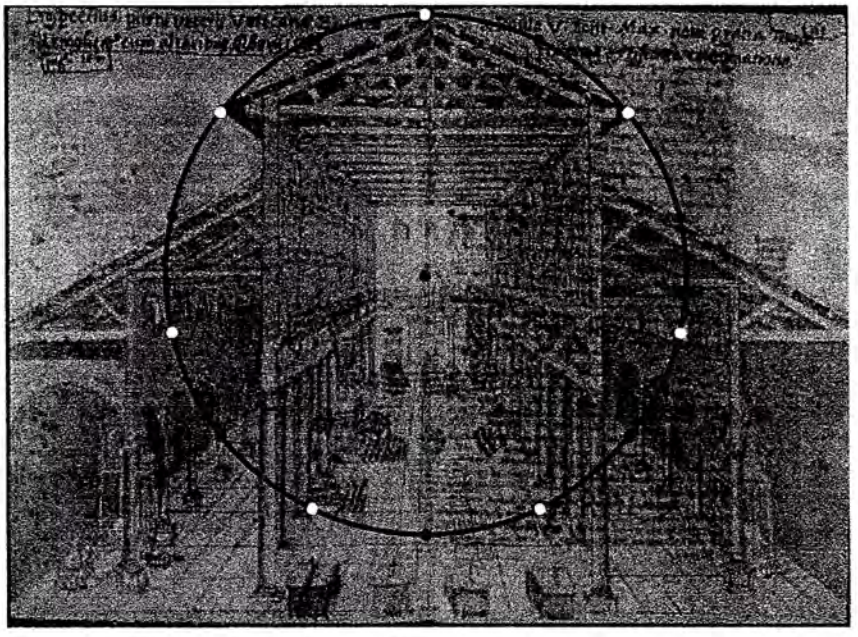




Basilica Design

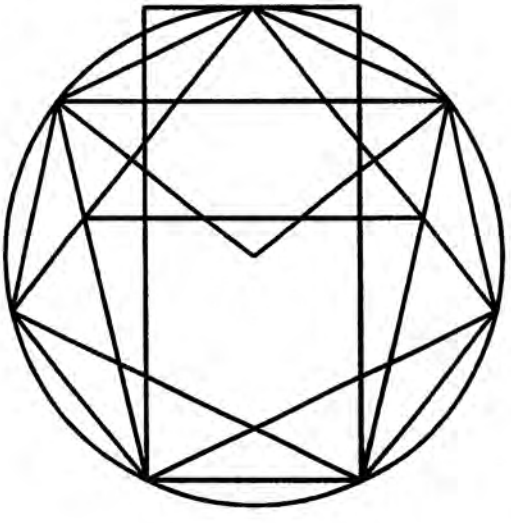
To understand this architectural drawing of a Basilica (Greek *basil* = "king"), fourteen (= 2x7) equally spaced points (alternating white and black) have been placed around one Circle. Its center is the drawing's "vanishing point". Different parts of the plan may be understood by constructions from each of the two Heptagons and the Heptagram stars they make.

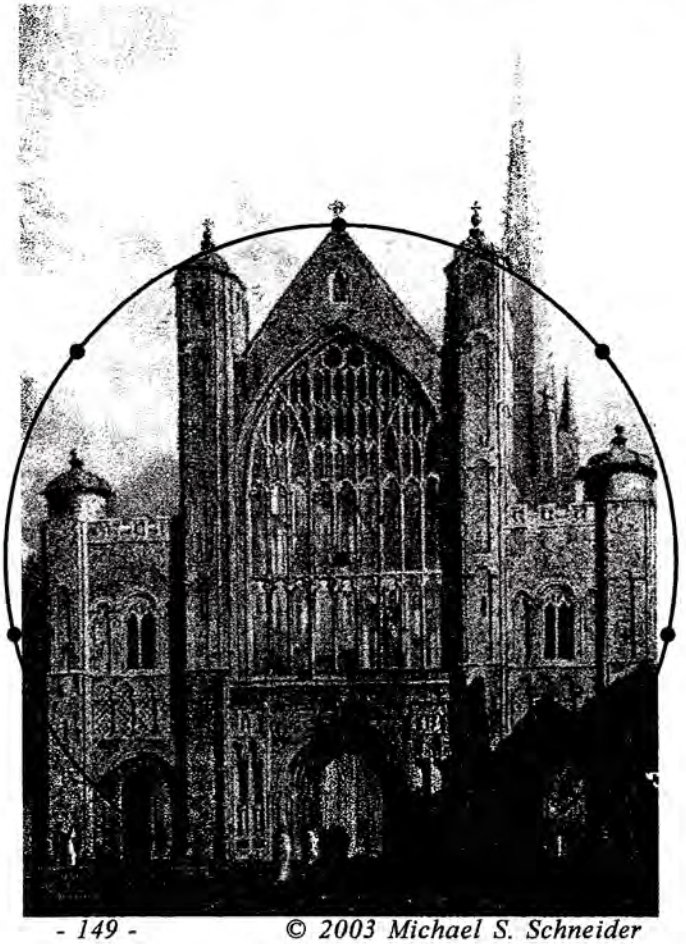




Norwich Cathedral

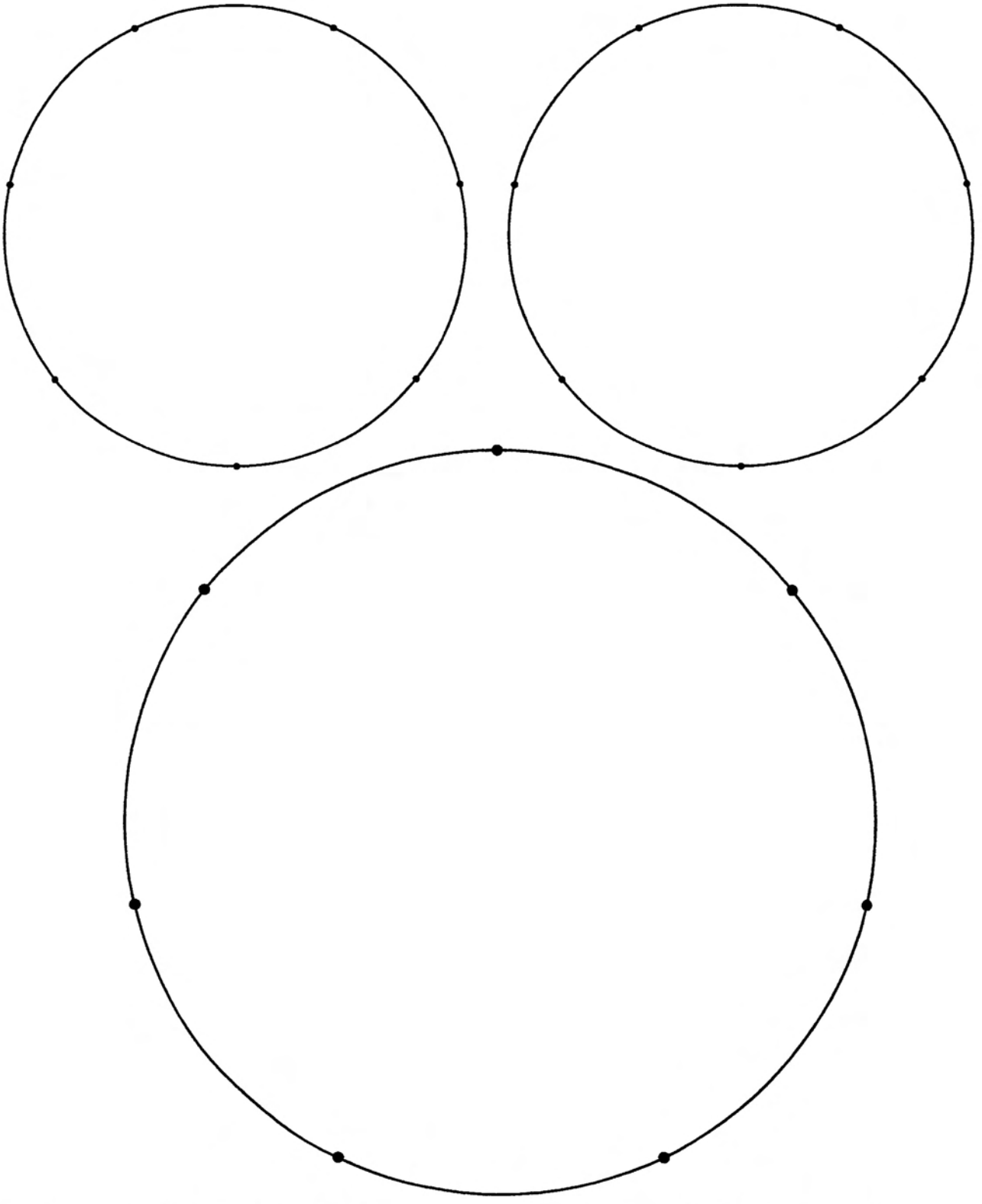
The geometry of Basilicas developed into the geometry of the Cathedral (*-hedra* = "seat (of the Bishop")). Different cathedrals display different polygons, but often something of the Heptagon appears.





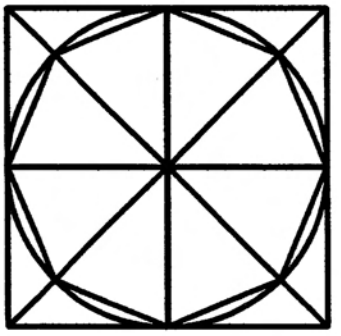
Design Your Own Heptagonal Art, Crafts And Architecture

Use this Heptagon, or construct one on blank paper, and create your own construction. Then use it as a plan to guide the elements of your art.



8 The Octagon

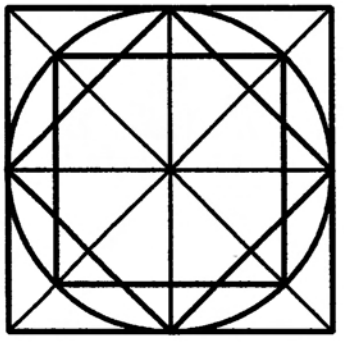
1



A regular Octagon has eight equal angles, eight equal sides, and four equal diagonals. It creates two Octagram stars. Octagonal geometry appears in the designs of nature and art, crafts and architecture of all cultures.

Construct A Regular Octagon

2

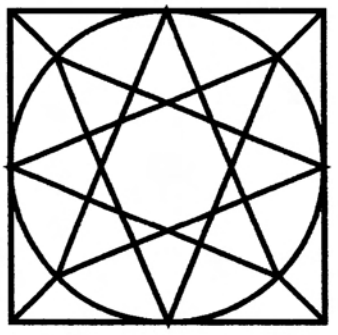


To construct a regular Octagon, just do the construction of a Square around a Circle (page 47). Everything you need will already be there.

(1) Notice that the square's two diagonals and the lines connecting the midpoints of the sides cross the Circle at eight equally spaced points. To construct an Octagon, just connect every consecutive point.

(2) To construct the first Octagram star, connect every other point.

3

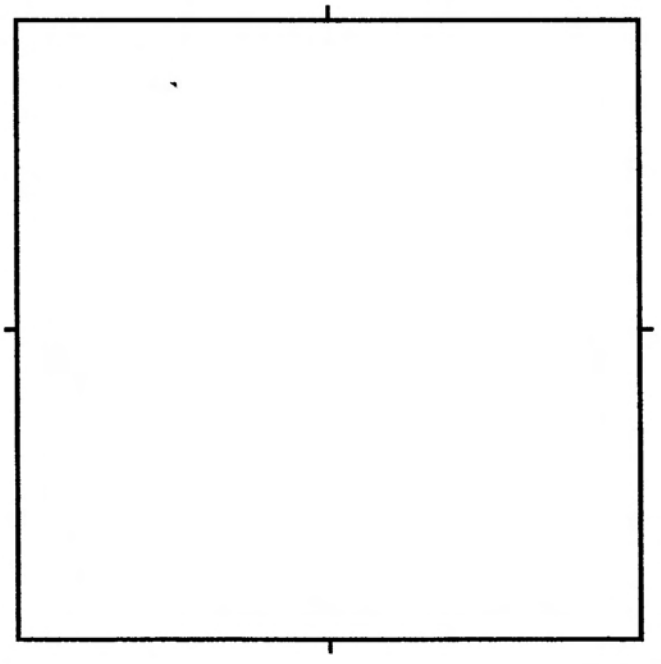


(3) To construct the second Octagram star, connect every third point.

Notice that each Octagram star contains an Octagon at its center.

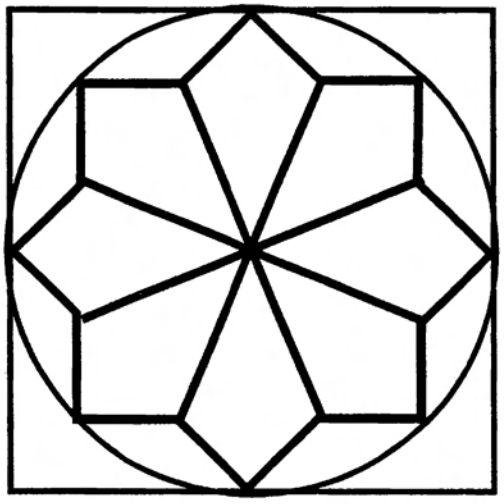
Construct The Octagram Stars

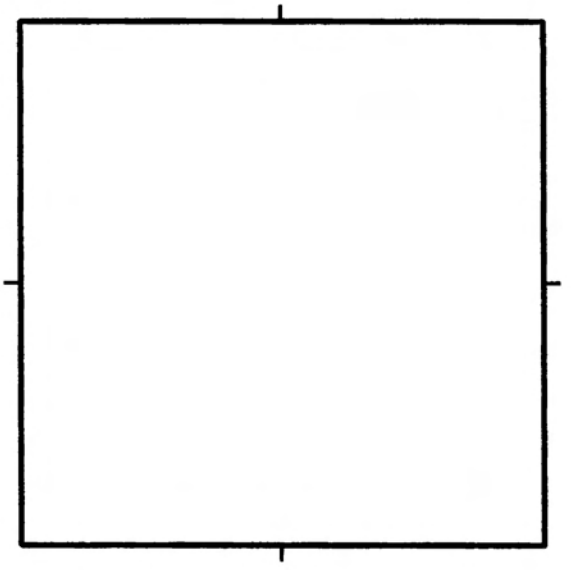
The center of each side of this Square has been marked.
Draw its diagonals and connect the midpoints to find its center.
Then turn the Circle and construct an Octagon with its Octagram Stars.

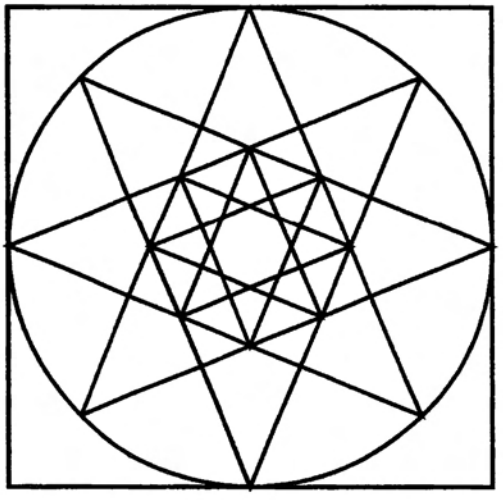


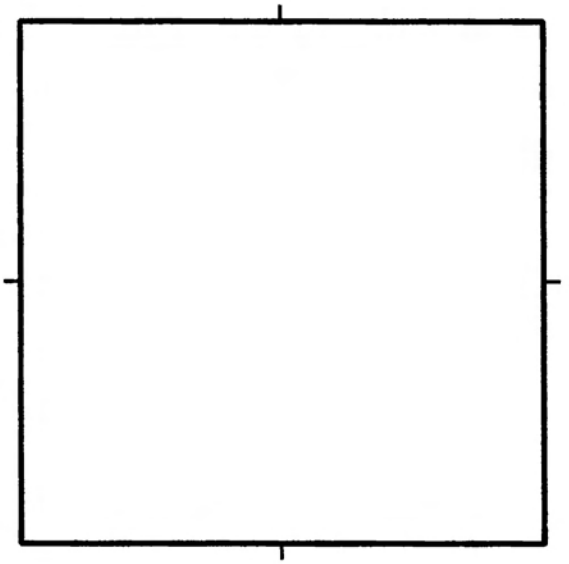
Replicate These Octagonal Patterns

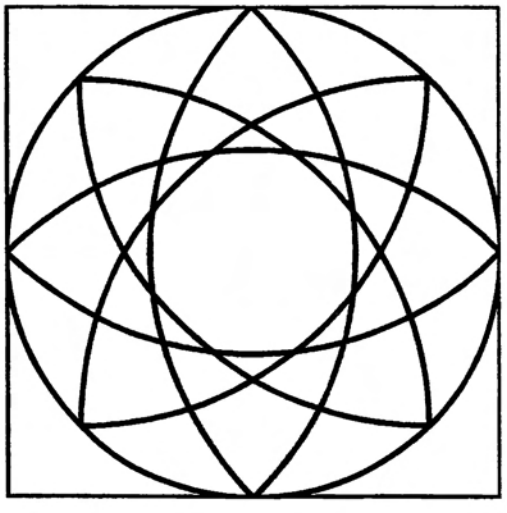
Use colored pencils.

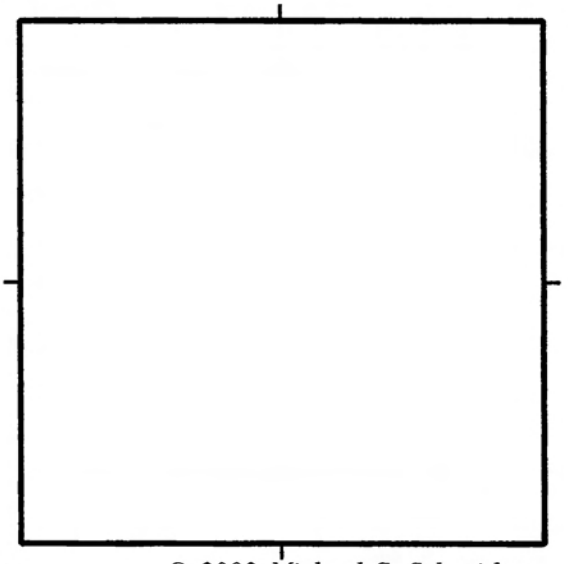


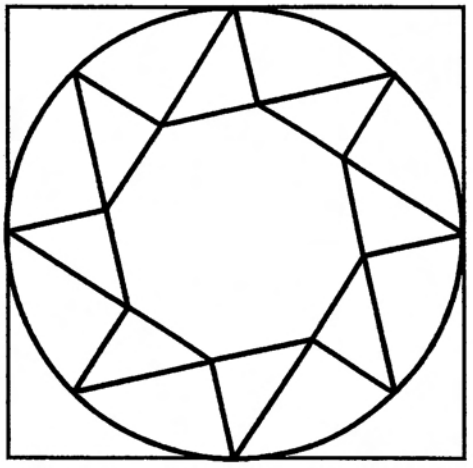


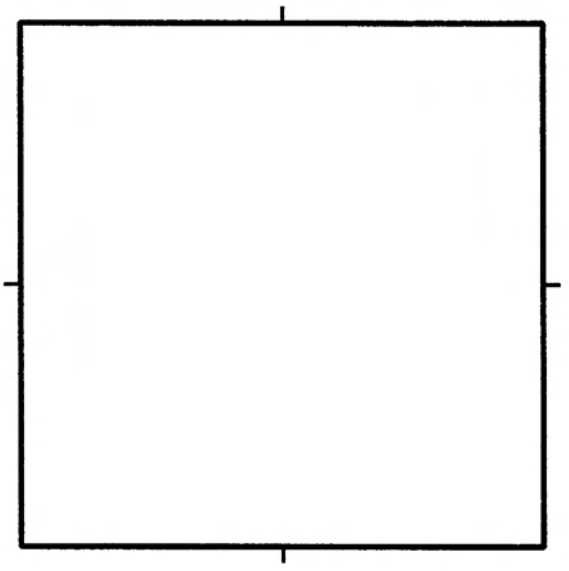


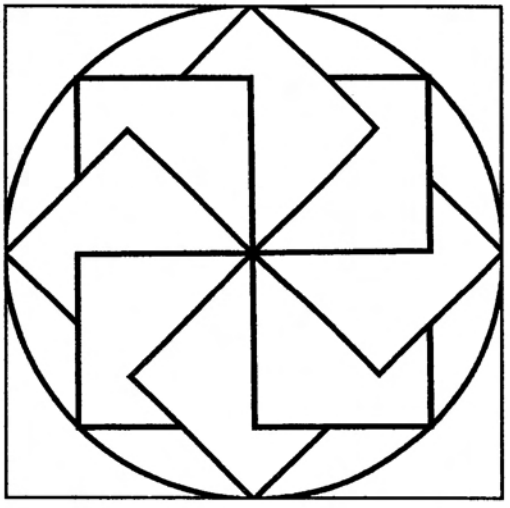


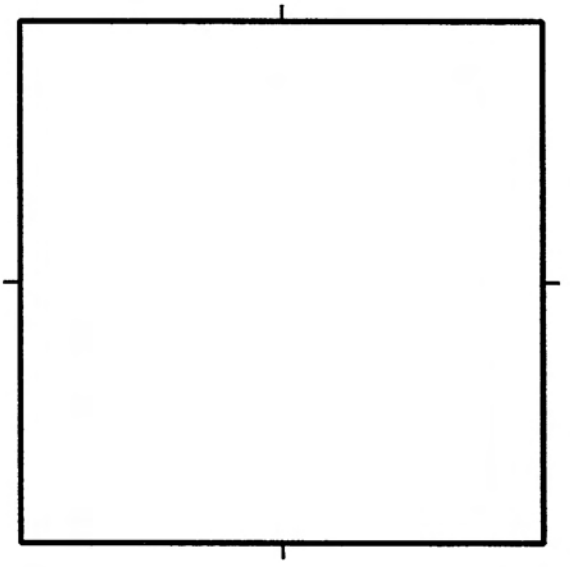


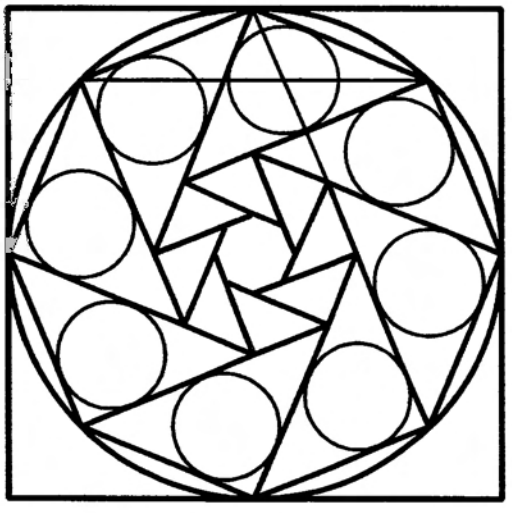


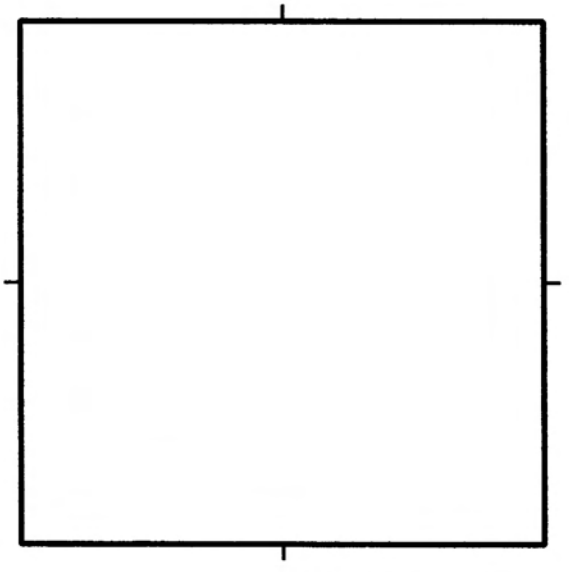


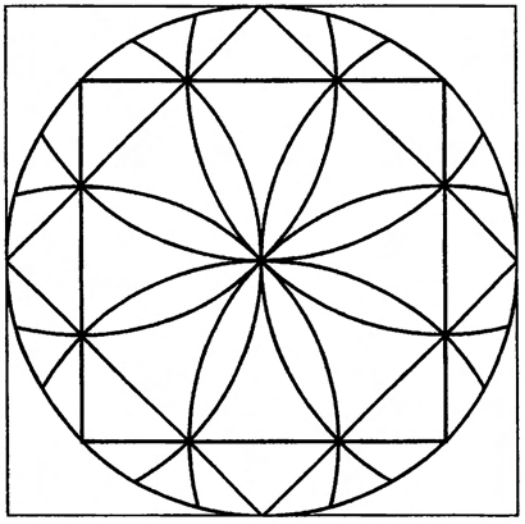


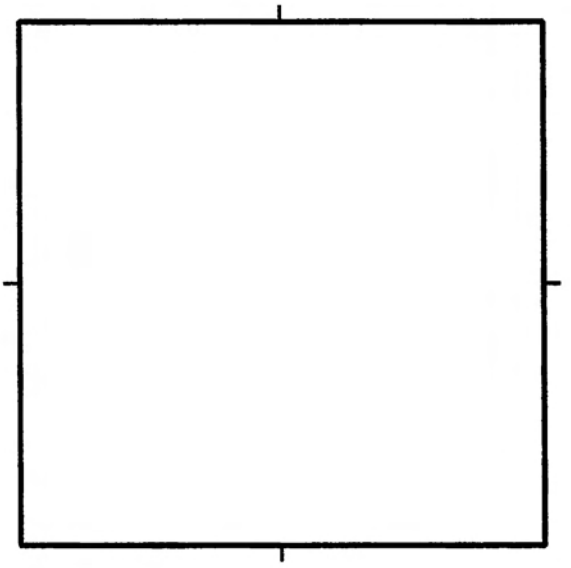


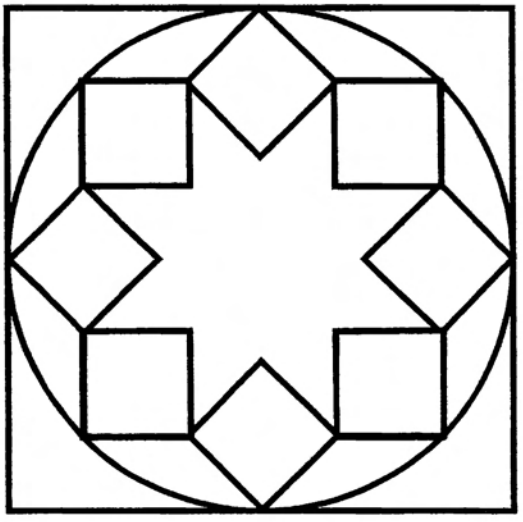


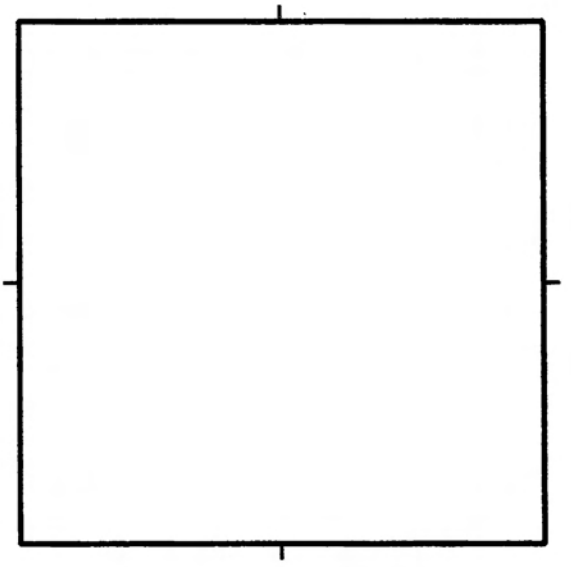


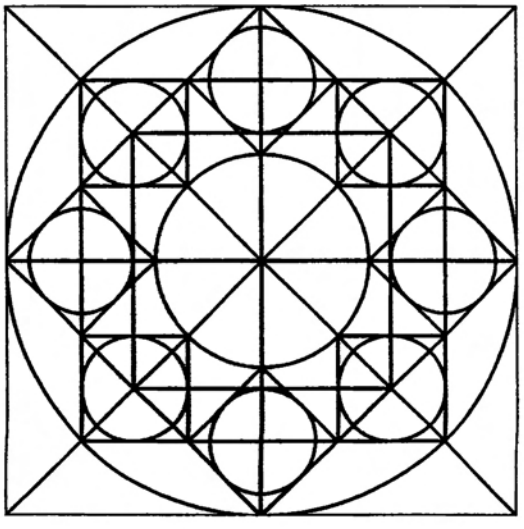


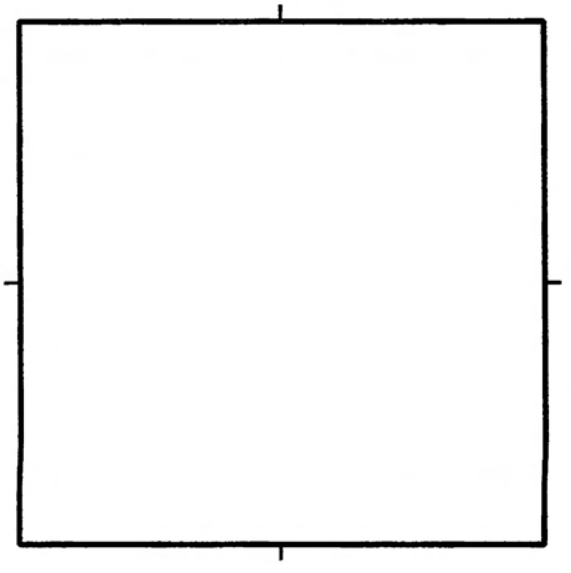


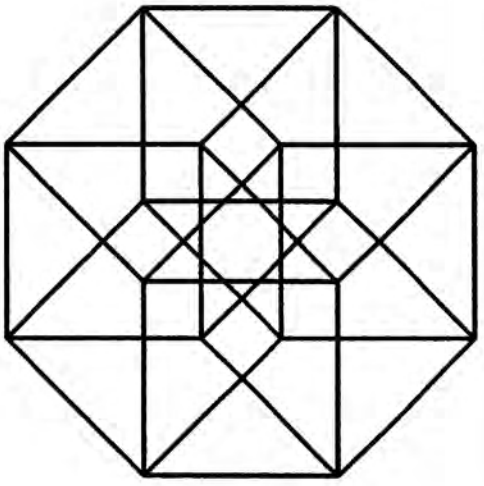


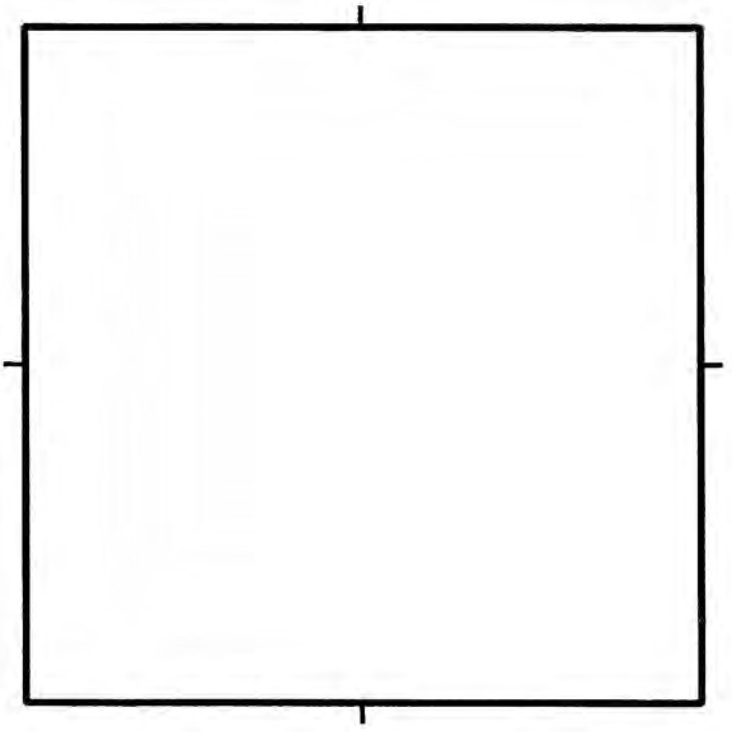


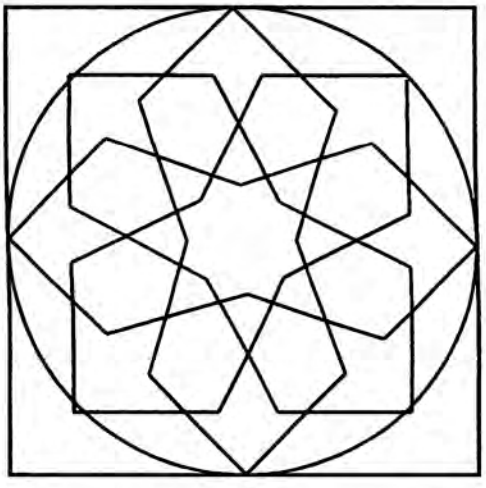


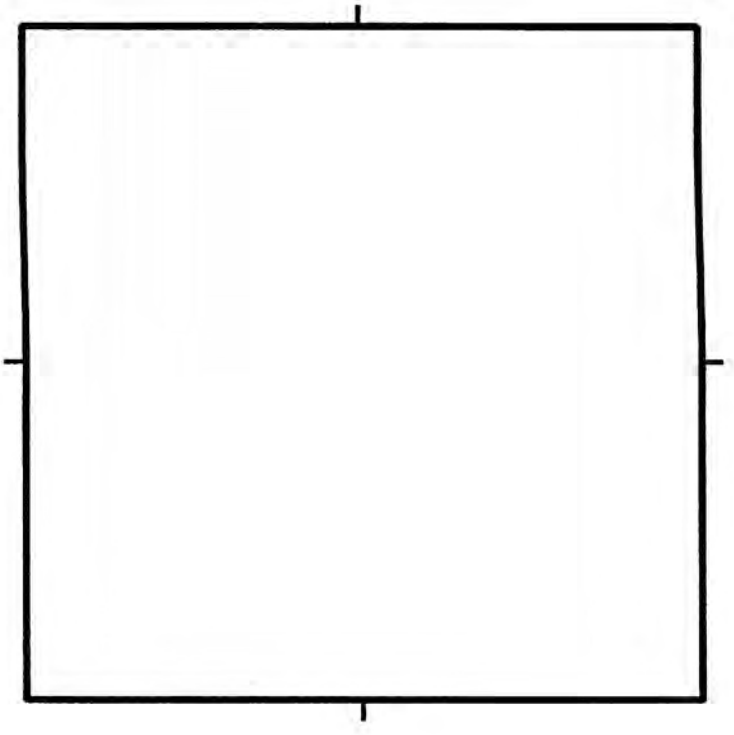






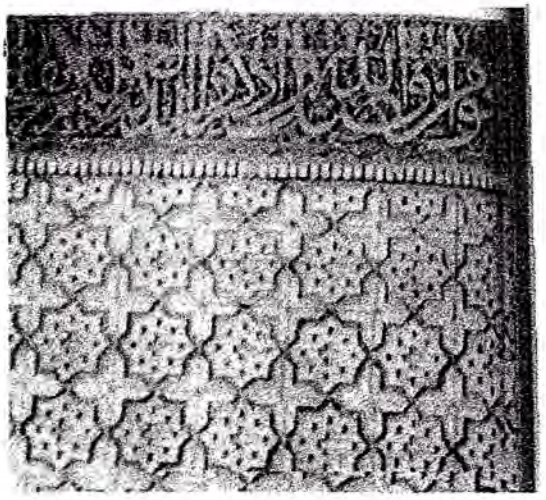






Islamic Tile

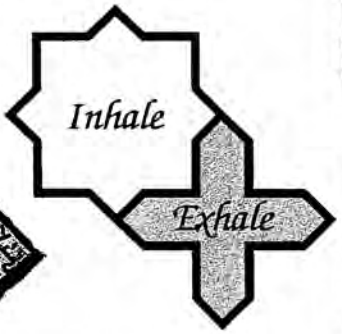
The "Breath Of The Compassionate" Pattern

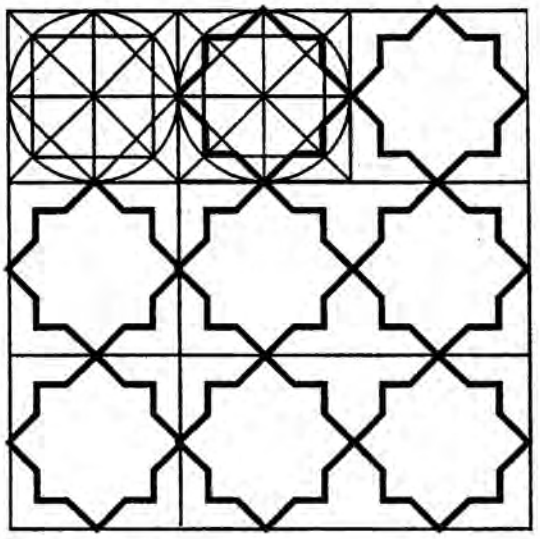


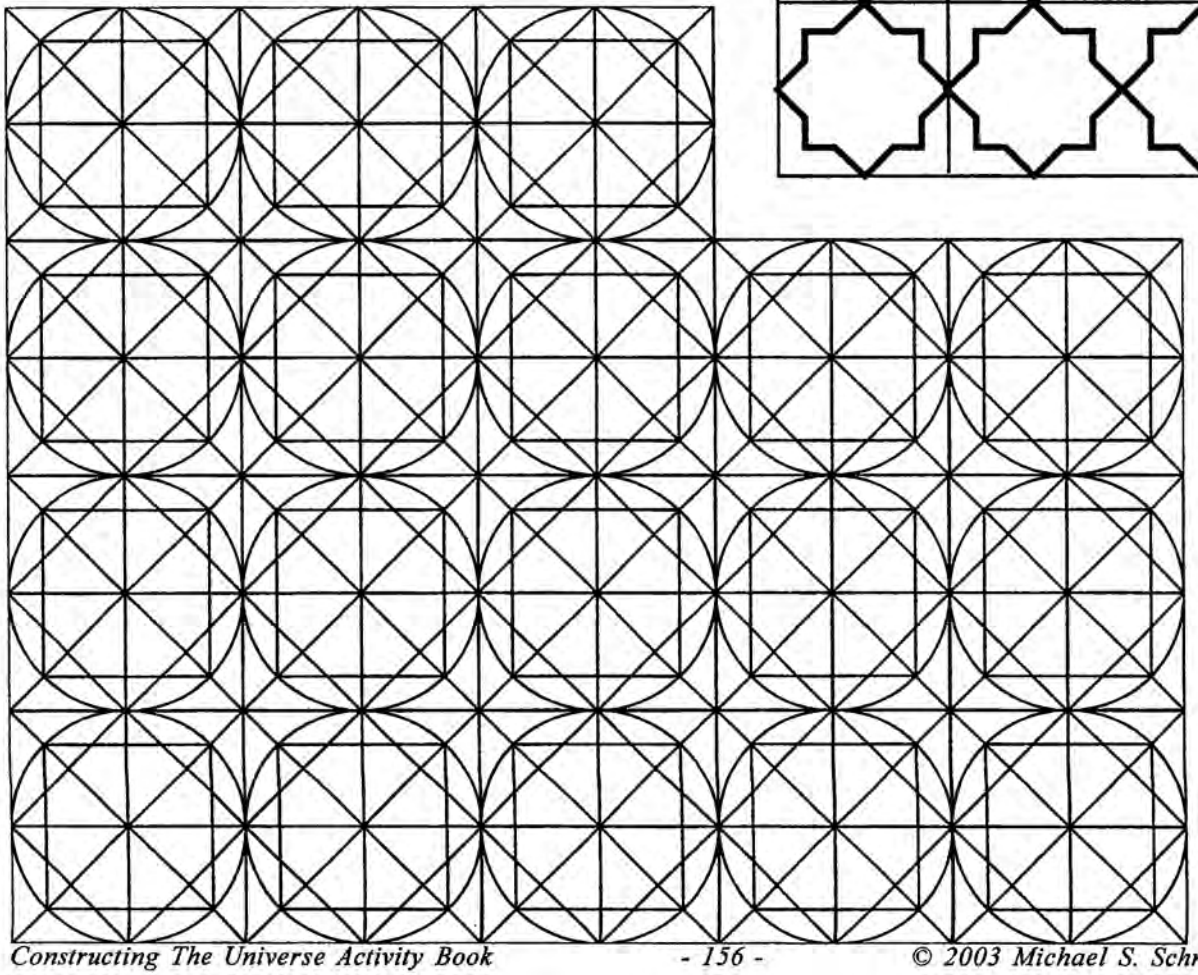
In Islamic tradition, the breath of Deity maintains the universe. Inhaling and exhaling are represented by Octagram stars and the crosses between them. You can see that they really occur together in one grid of Octagonal geometry.

Use colored pencils to shade the grid below to show this "Breath" pattern.

Tile from Iran showing the mythological Phoenix Bird arising



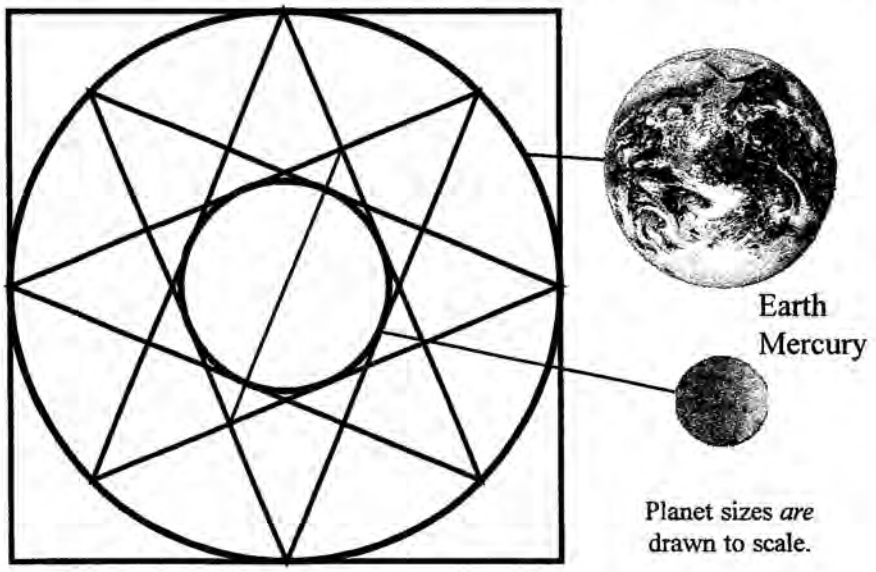




The Mean Orbits *And* Relative Sizes Of Mercury And Earth

When the elliptical orbits of Mercury and Earth are averaged and rounded to Circles, this construction shows their orbits with 99.9% accuracy.

What is curious about this diagram is that it also accurately shows the relative *sizes* of Mercury and Earth, which is the same ratio as their orbits! This also occurs only with the orbits and sizes of Earth and Saturn. Mercury and Saturn are the innermost and outermost planets visible from Earth with our eyes alone.

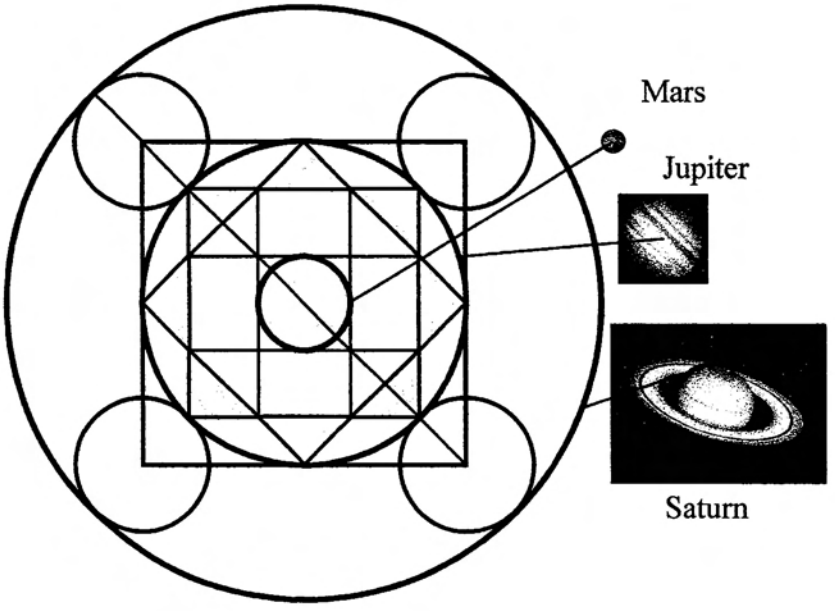


Planet sizes *are* drawn to scale.

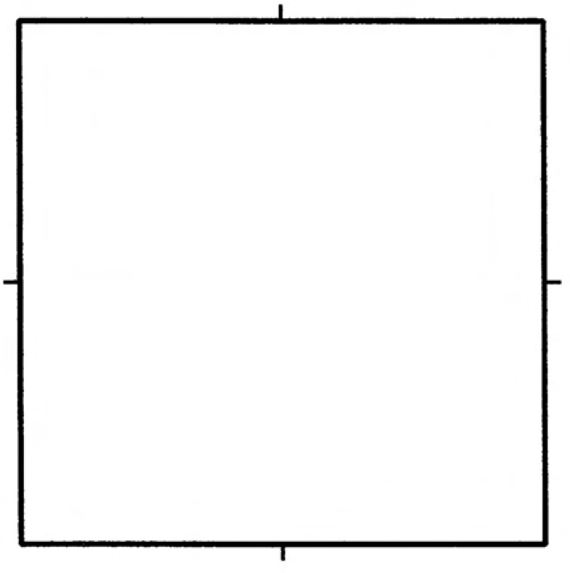
Do the construction of the orbits and sizes of Mercury and Earth here:

The Mean Orbits Of The Middle Planets

When the ovals of the planetary orbits are averaged to become circles, this geometric construction shows the relative orbits of Mars, Jupiter and Saturn with 99.9% accuracy. Replicate it below. Start with the Square in the middle of the construction.



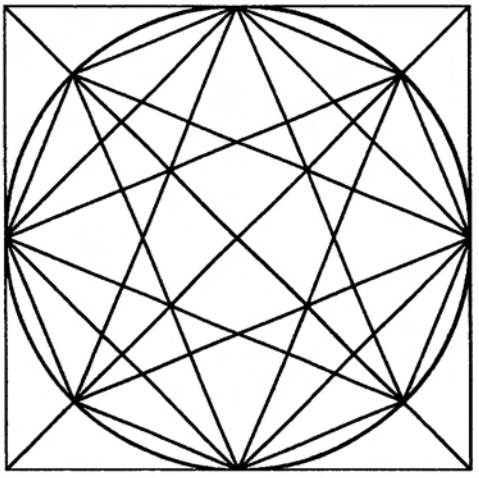
Planet sizes are not drawn to scale.

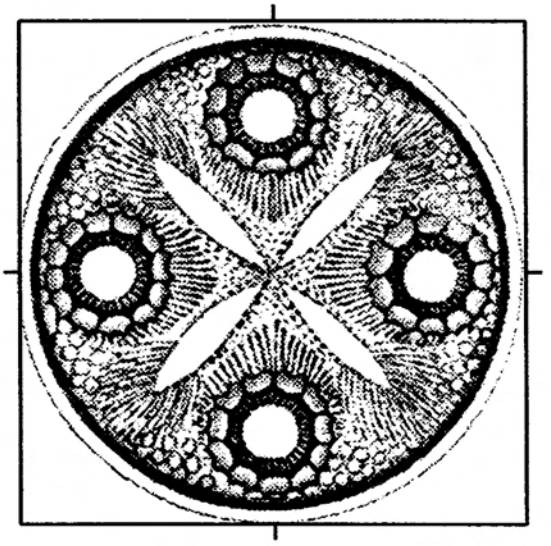


Octagonal Patterns In Nature

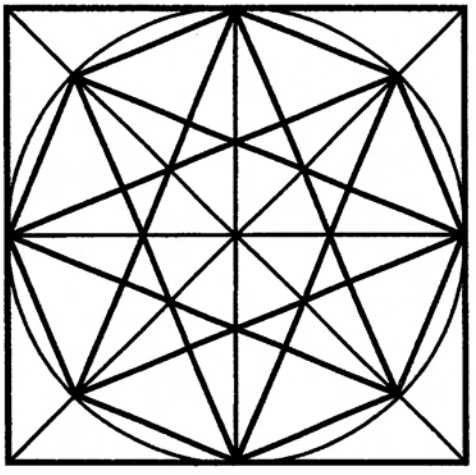
Use colored pencils to do the constructions on these natural forms.

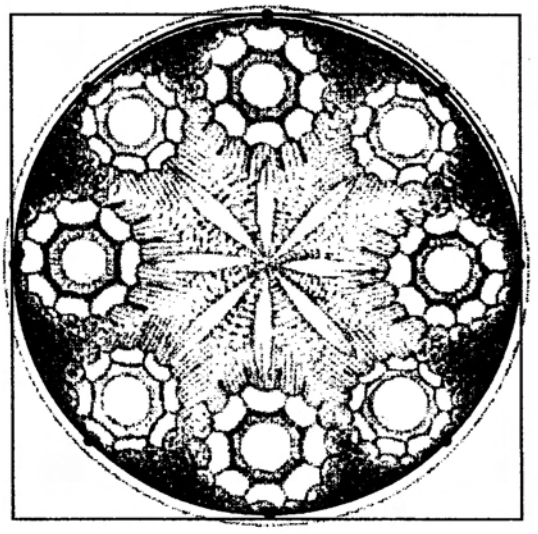
Diatom



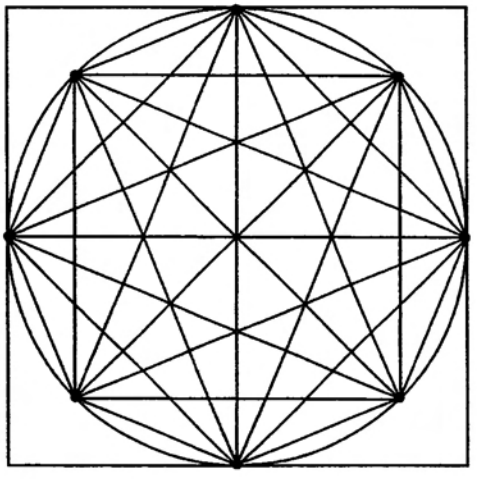


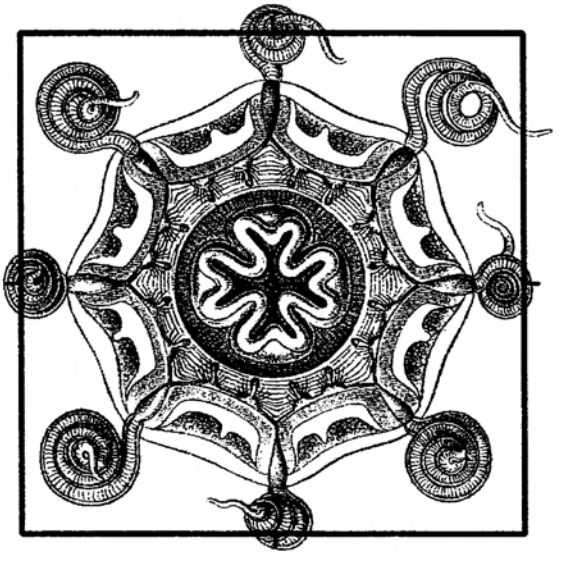
Diatom



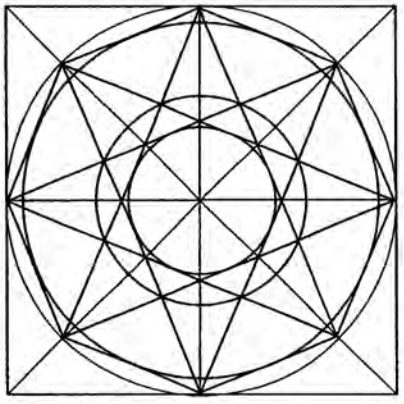


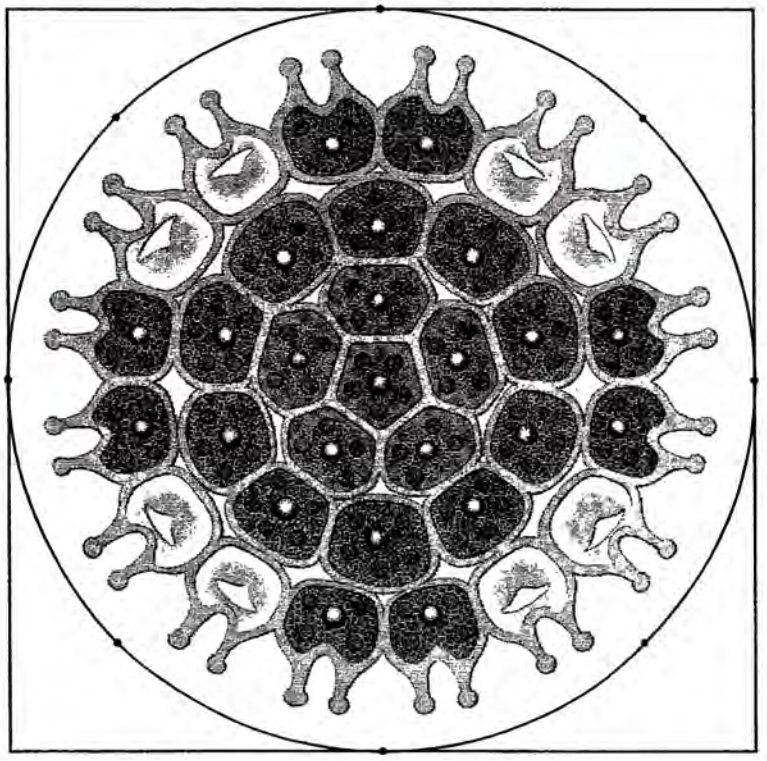
Hydra



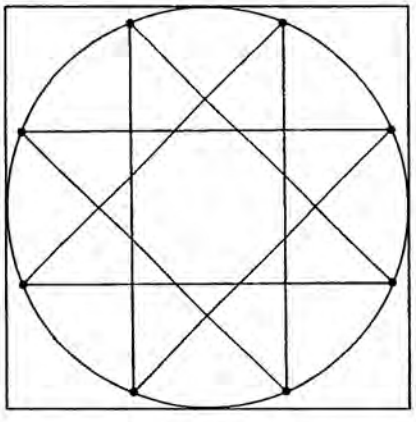


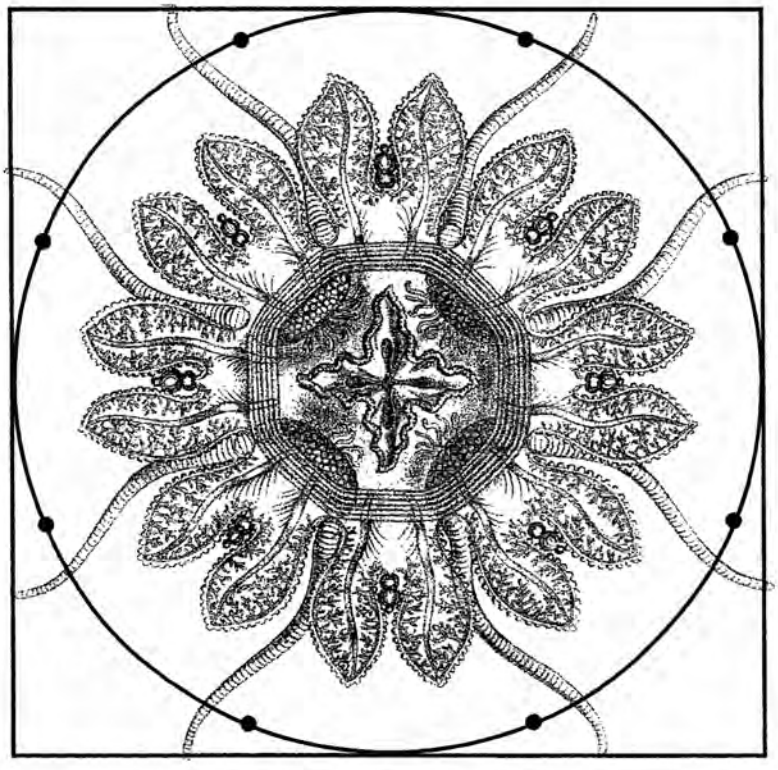
Algae





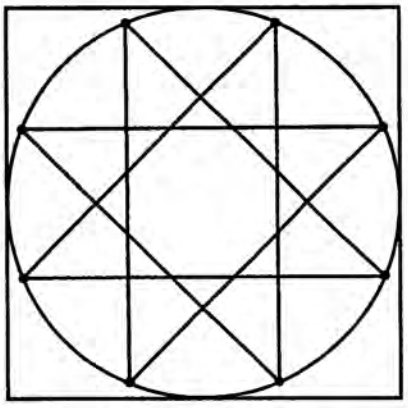
Jellyfish

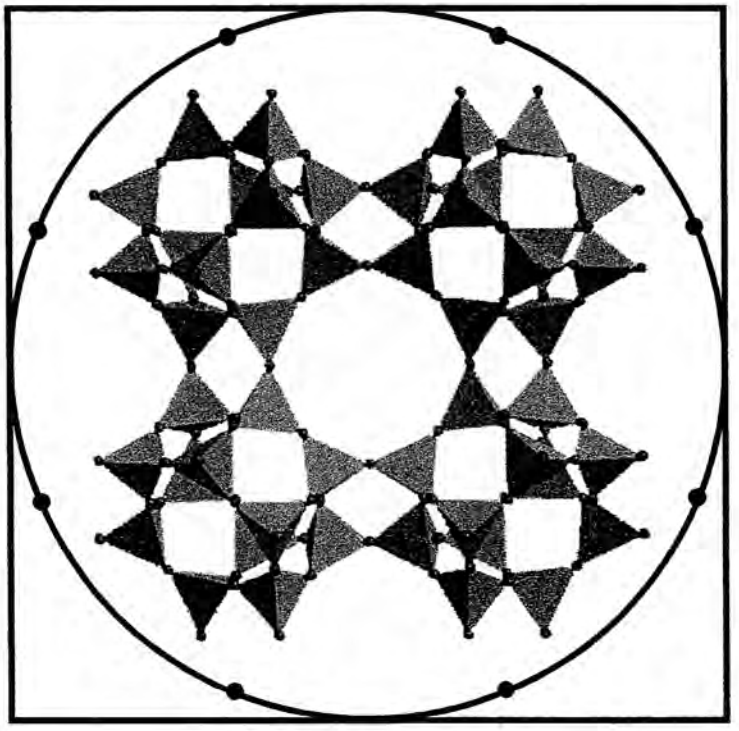




Zeolite Crystal

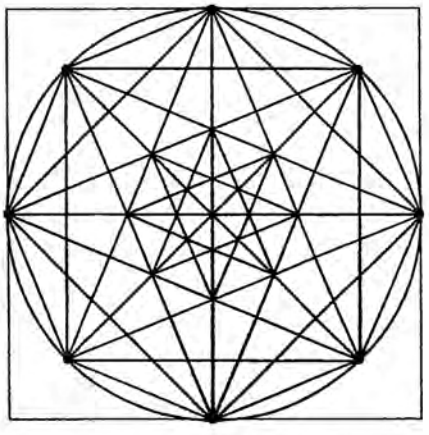
Zeolite is a crystal made of three types of atoms: silicon, aluminum and oxygen. Its name means "boiling stones" since it comes from volcanoes. As you can see, a Zeolite crystal has many holes. When it's heated, Zeolite puffs up to make the holes even larger. With plenty of space inside to trap tiny particles of dust and minerals, Zeolite is used to purify water, absorb the odor from kitty litter and absorb the radioactive elements from nuclear fuel.

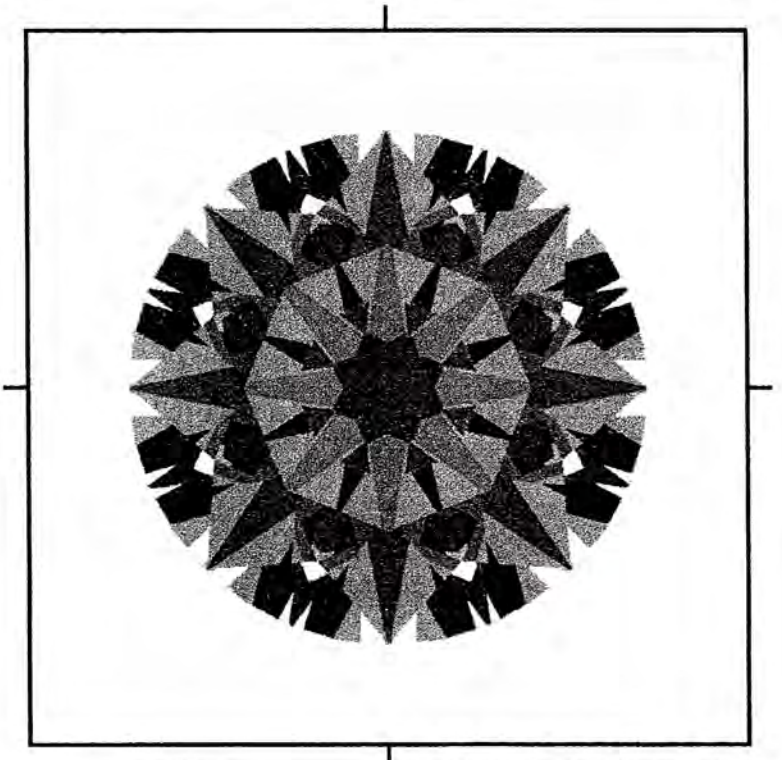




Cut Diamond Pattern

Rough diamonds are carefully cut into geometric patterns so that light enters, bounces around and sparkles back outward.

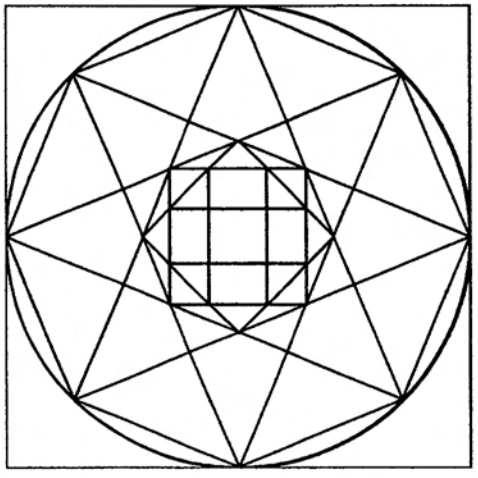




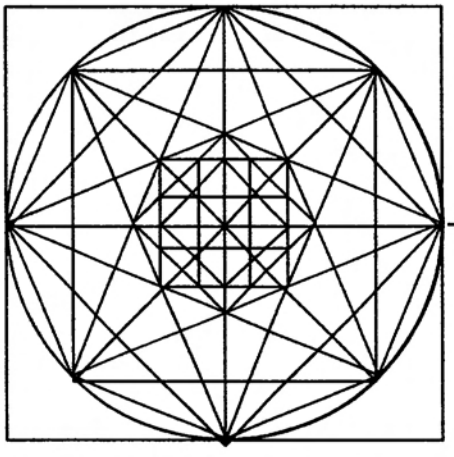
Octagonal Geometry In Arts, Crafts And Architecture

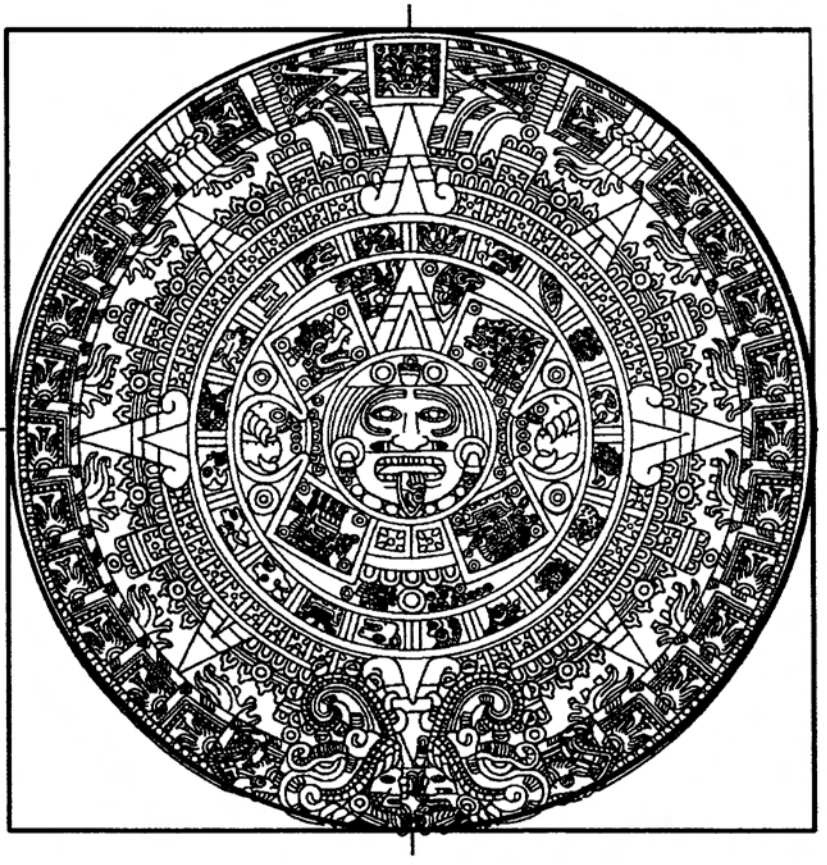
Replicate each geometric plan on the image to see how geometry was used to guide the design of tl object and its pattern.

Bowl From Panama



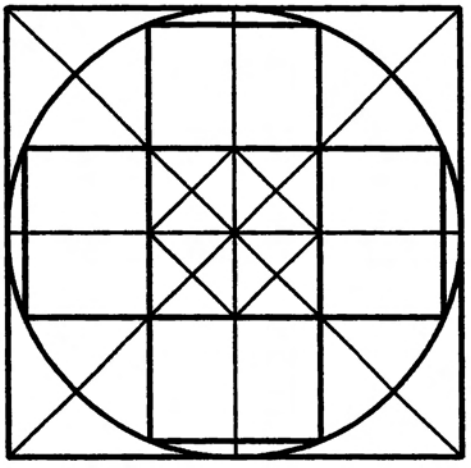
Aztec Sun Stone

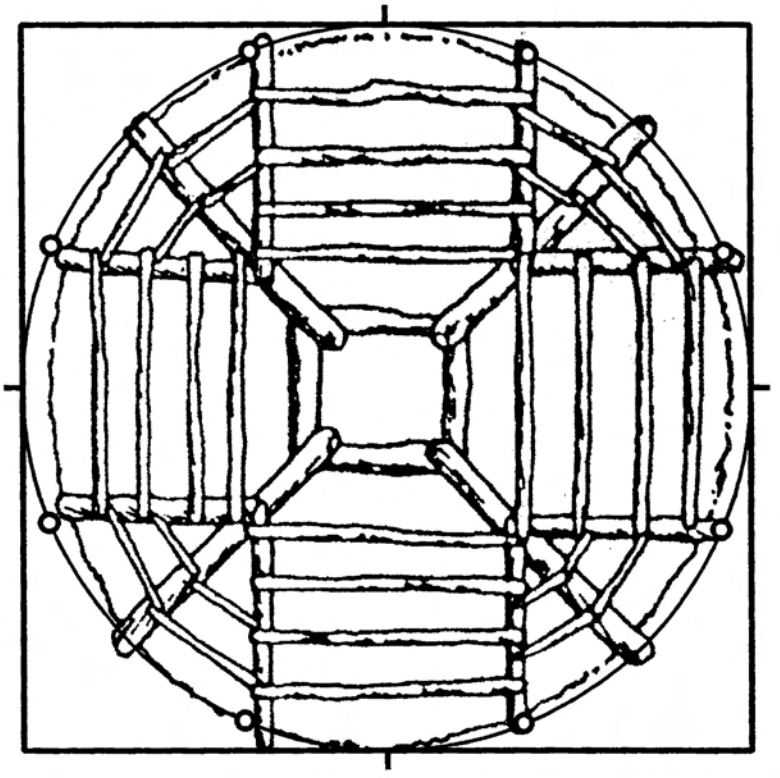




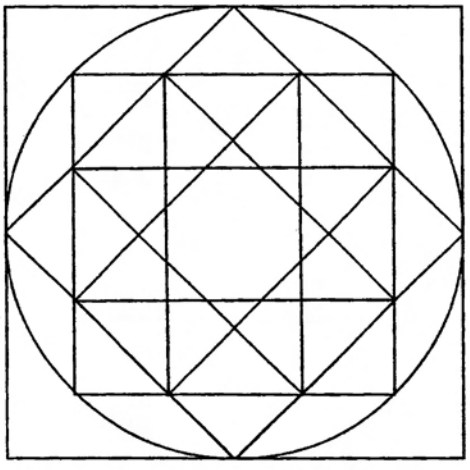
Native American *Kiva* Roof

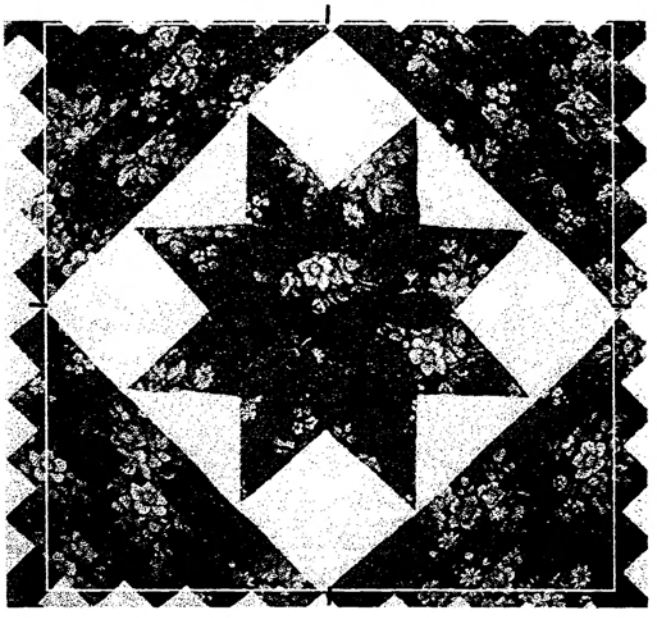
Start with the eight points on the Circle drawn around the roof.



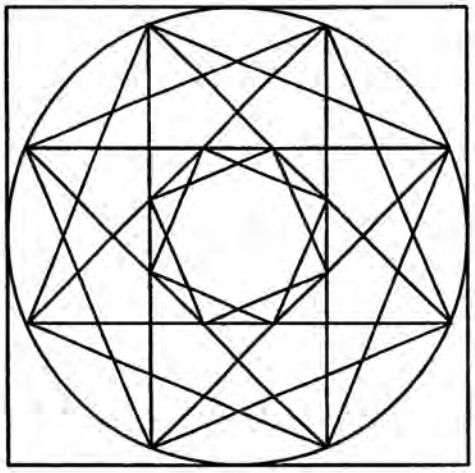


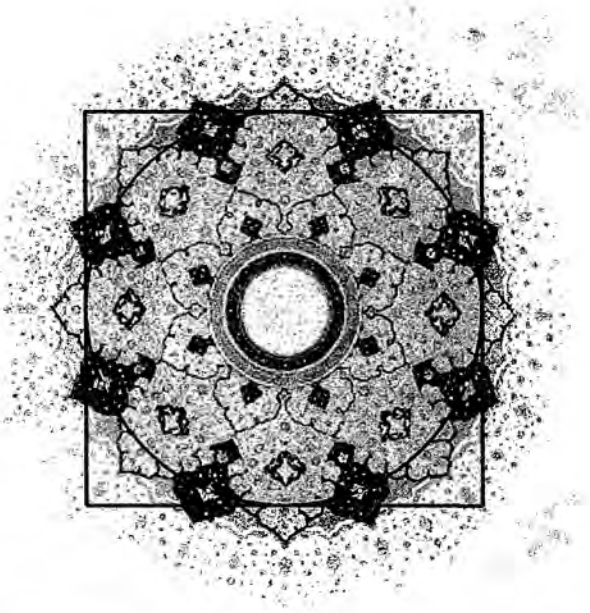
Quilt Pattern

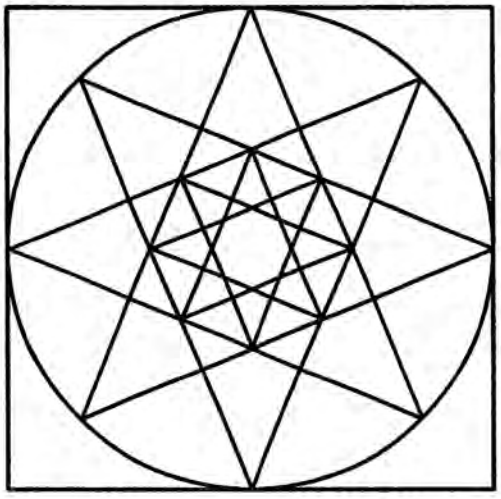


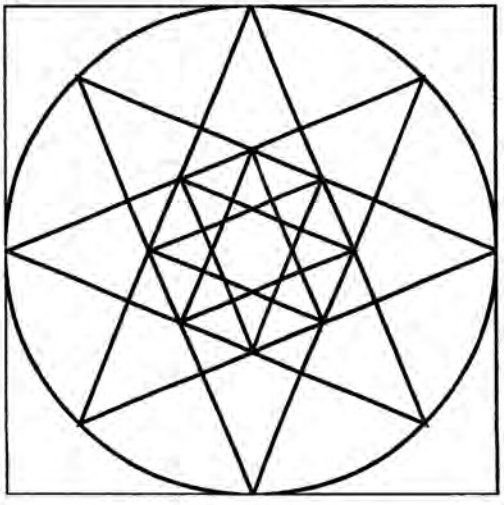


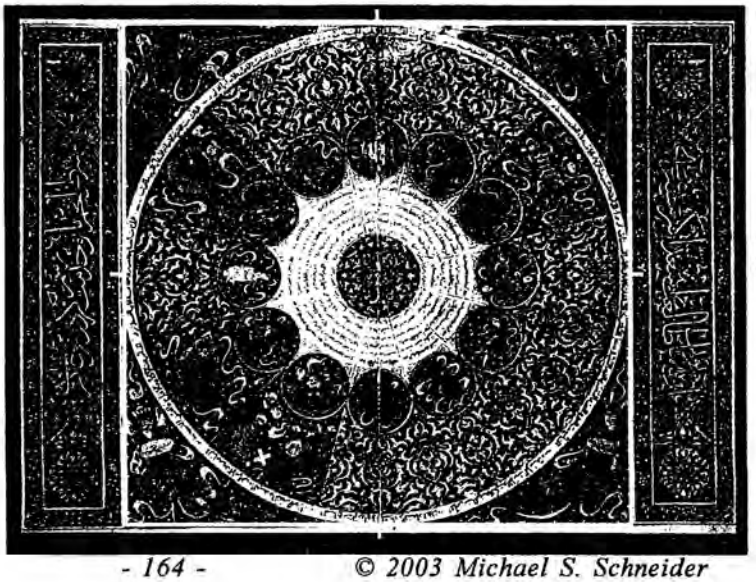
Islamic Designs



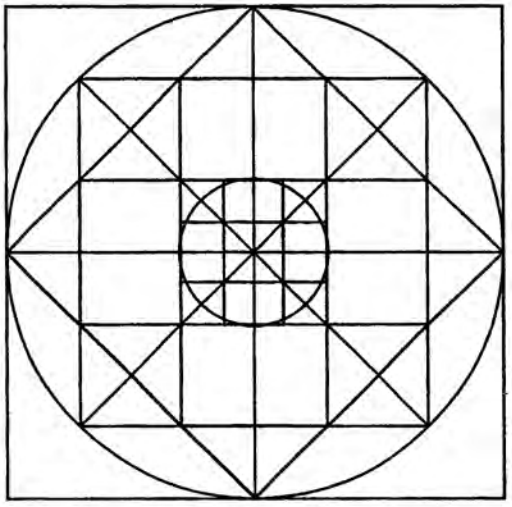


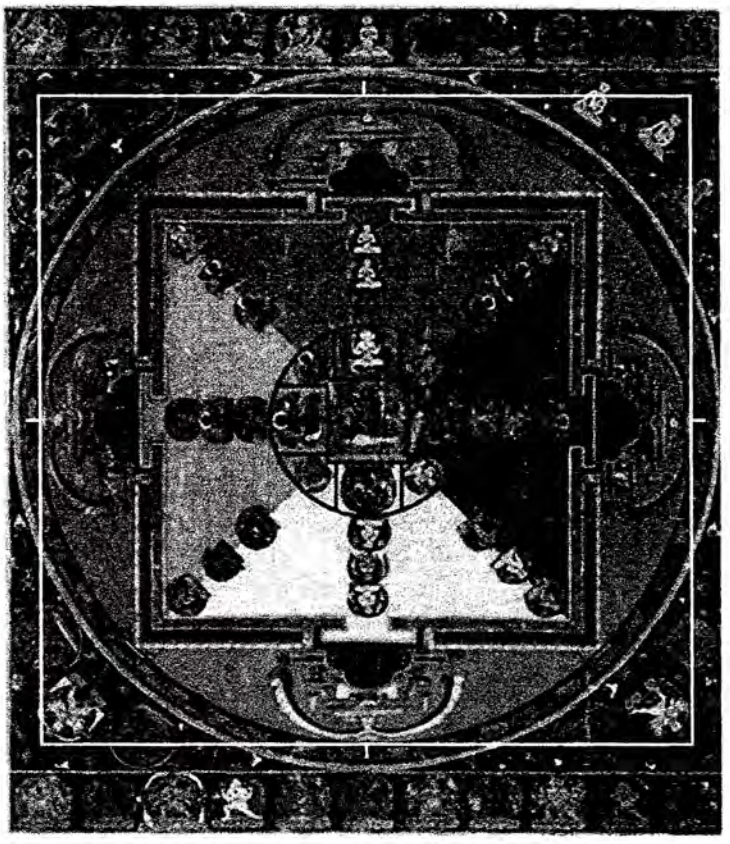




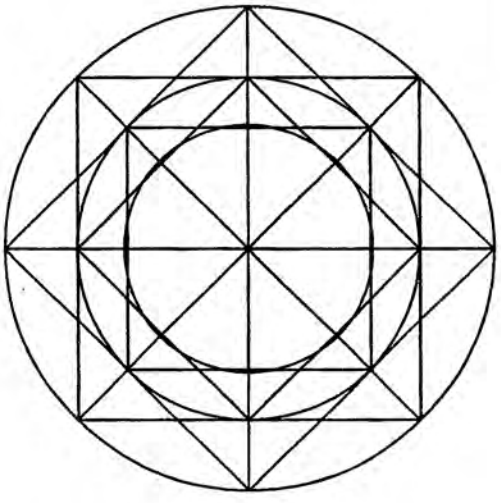


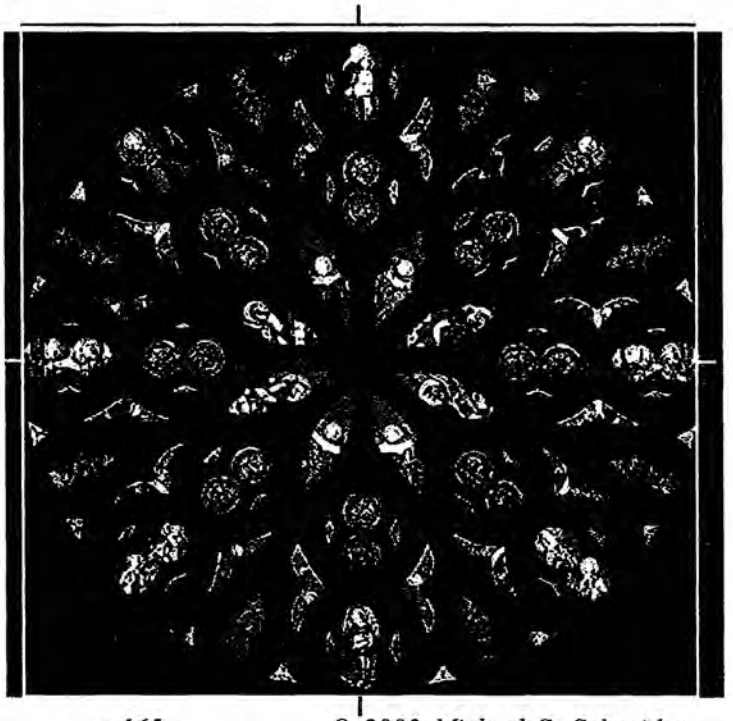
Tibetan Buddhist Mandala, 13th Century





Foucauld Rose Window

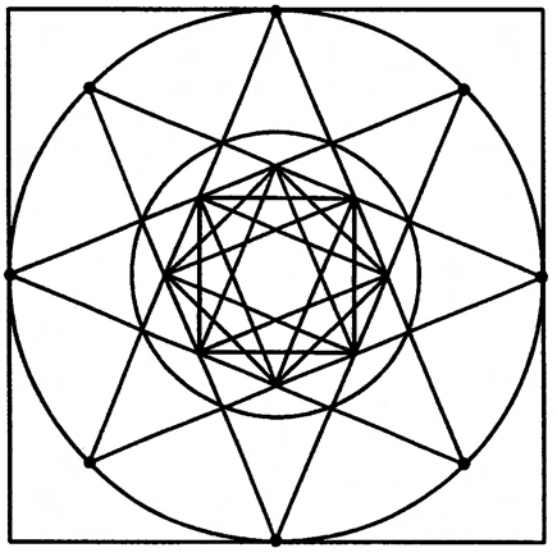


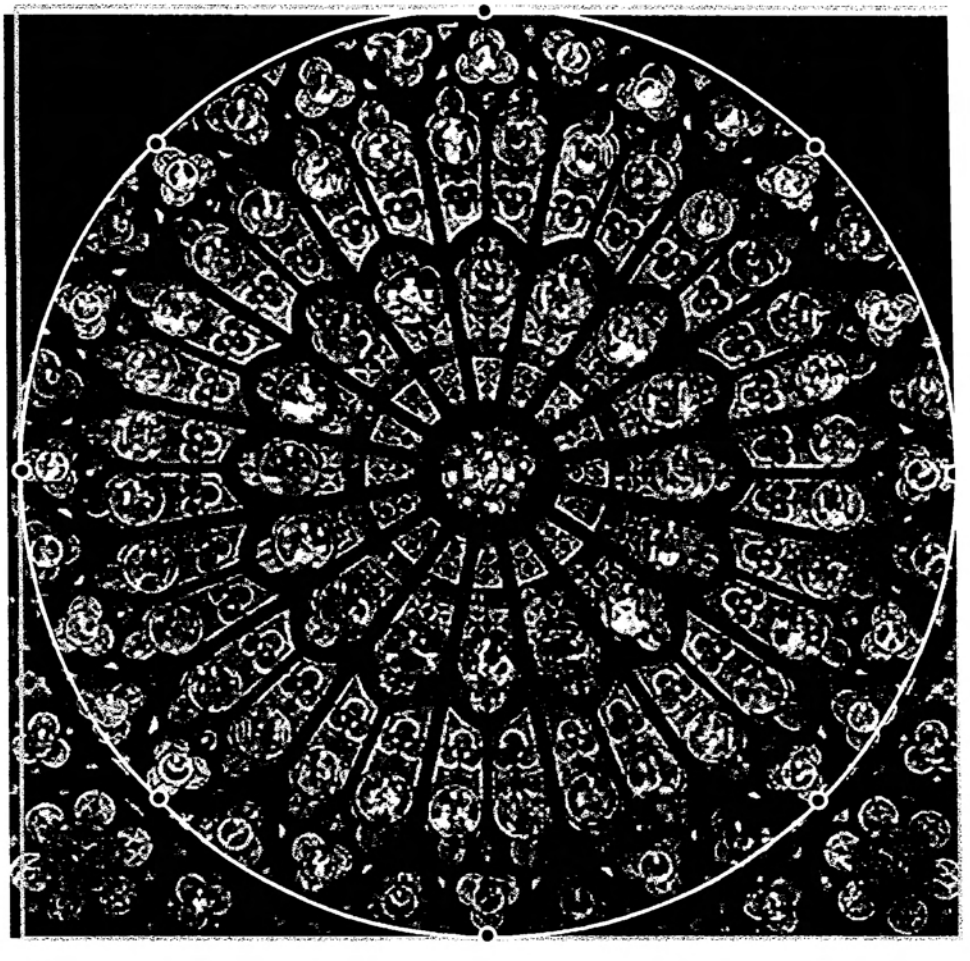


Notre Dame Cathedral, Paris

Rose Window, South Transept

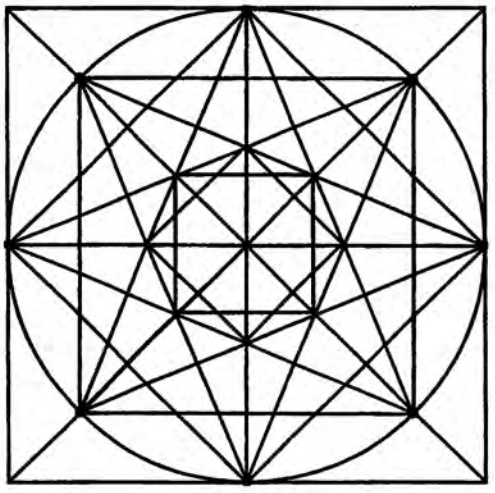
Made around 1260



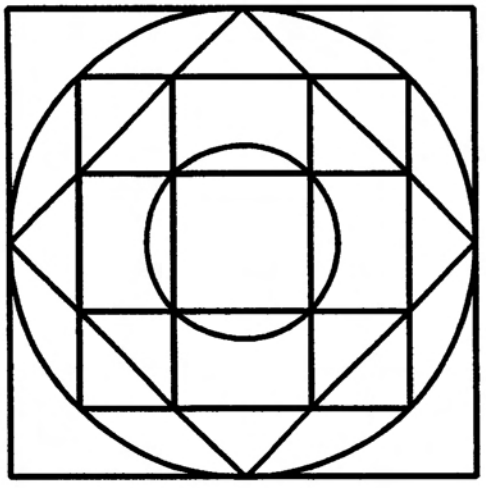


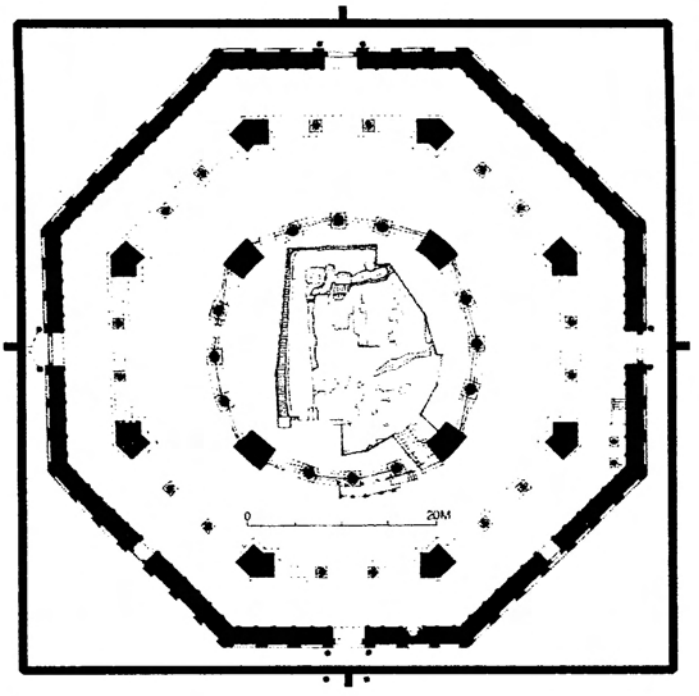
"Angels Direct Saint John to Write the Book of Revelation"

Spain, 1180



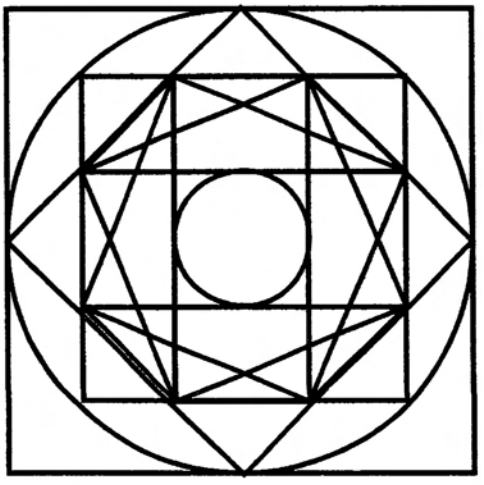
Floorplan of the Dome Of The Rock, Jerusalem

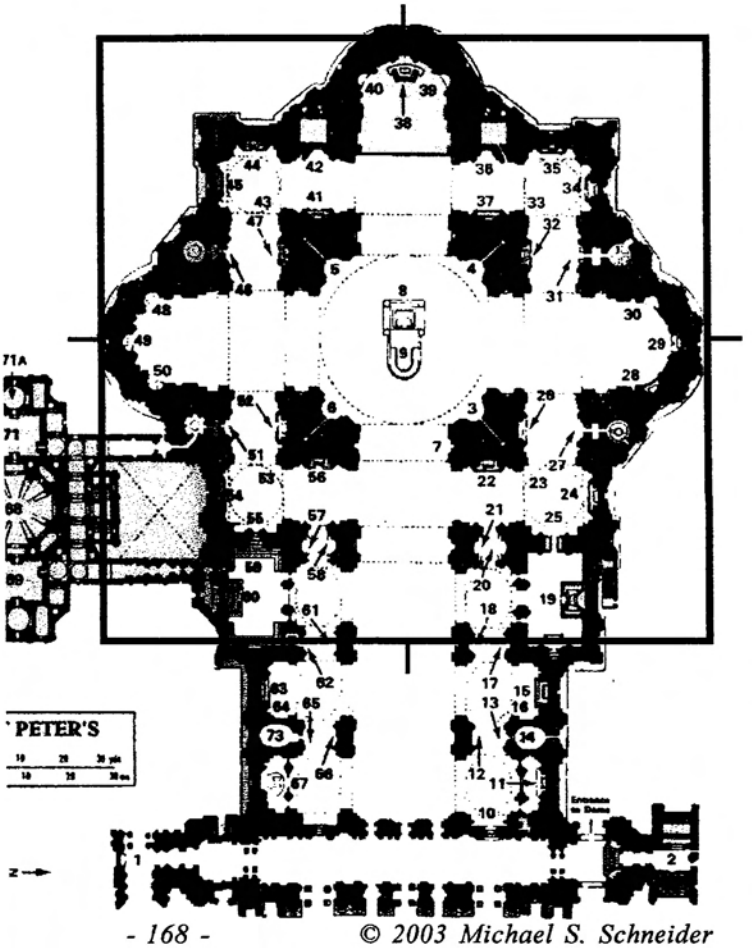




Octagonal geometry often symbolizes "between Heaven (Circle) and Earth (Square)"

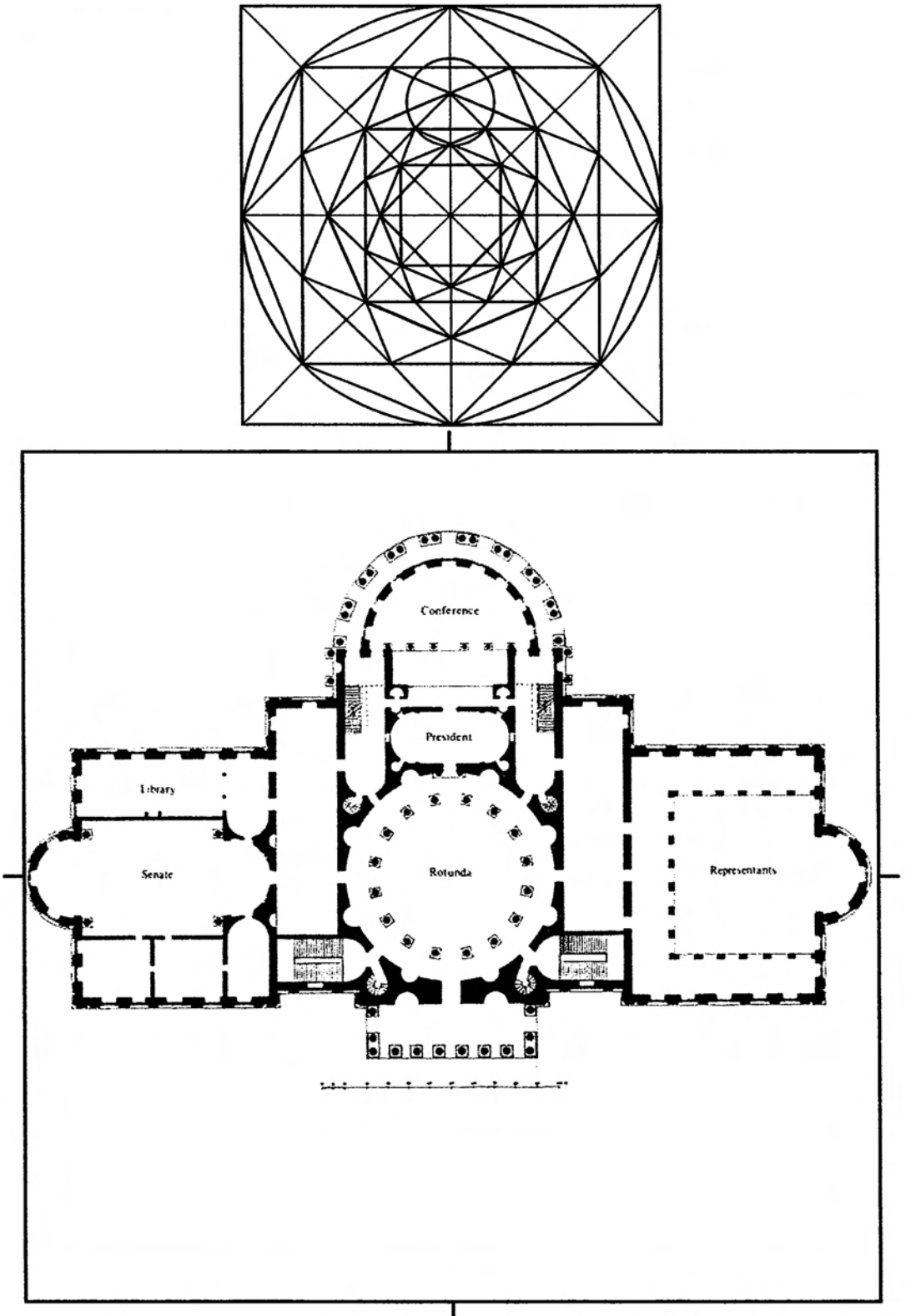
St. Peter's, Rome





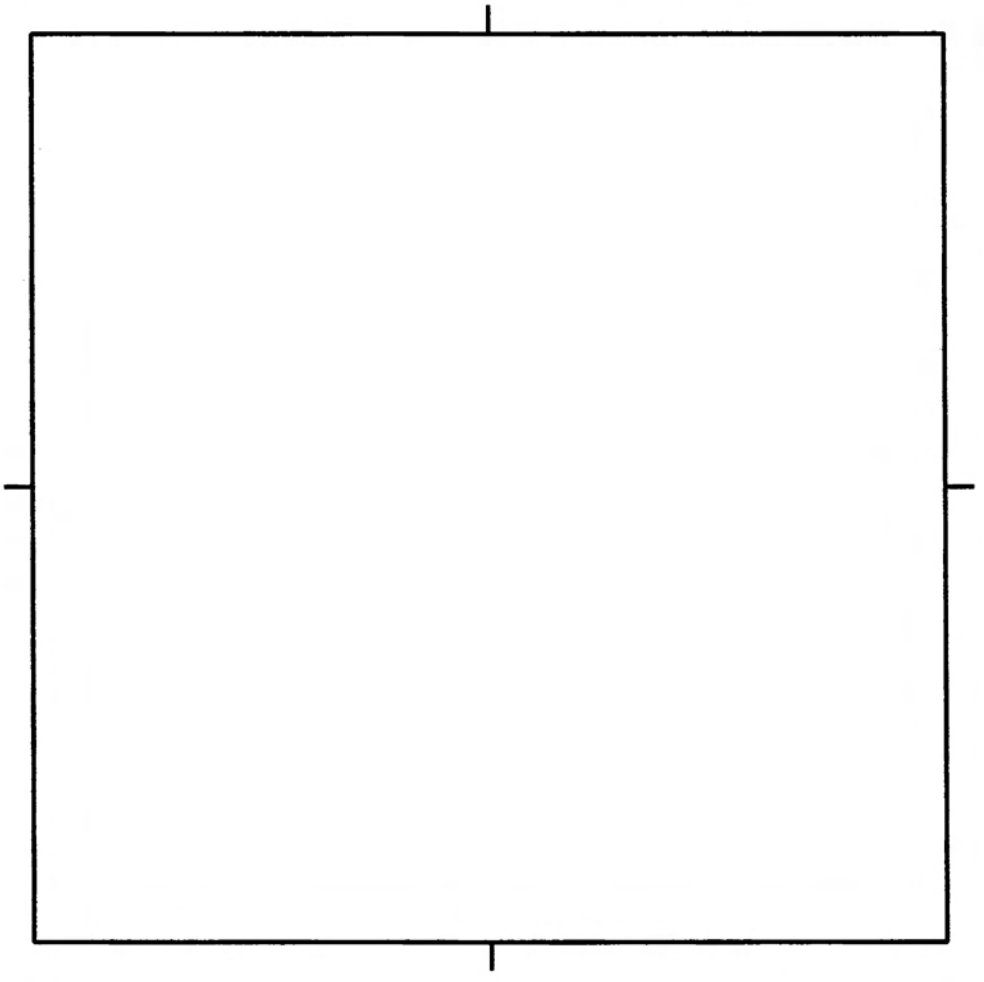
U.S. Capitol Building, Washington, D.C.

The midpoints of a Square around the Capitol are given.
Connect the midpoints and draw the diagonals to find the center.
Turn a Circle and identify eight points around it.
Notice how each Octagram star identifies more parts of the building.



Design Your Own Octagonal Art

The midpoints of the Square are marked. Create and explore your own Octagonal geometric construction. Then let the points, lines and areas guide your design for art, crafts or architecture. Use colored pencils. Design more on blank sheets of paper.



9 The Enneagon

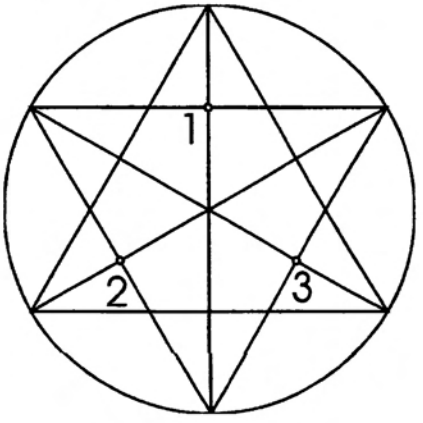
Every Enneagon (Greek *Ennea* = nine) has nine corners and nine sides. But as with the Heptagon (Chapter 7), a regular Enneagon cannot be constructed precisely with a compass and straightedge. Thus, there are not many appearances of nine-sidedness in nature or art. Here's one approximation.

Construct An Enneagon

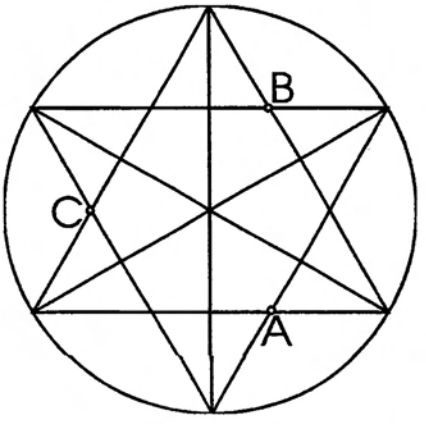
(1) Start by inscribing a regular Hexagram star in a Circle (page 112). Draw its three diagonals. Notice where they cross the downward pointing Triangle at three places, labeled points 1, 2 and 3.

(2) Also notice the three points where the two triangles of the Hexagram star cross, labeled points A, B and C.

1



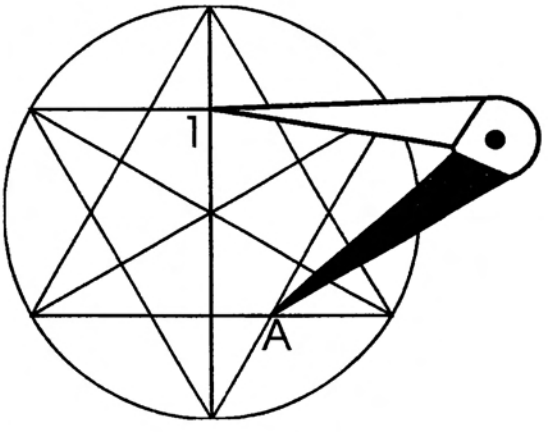
2



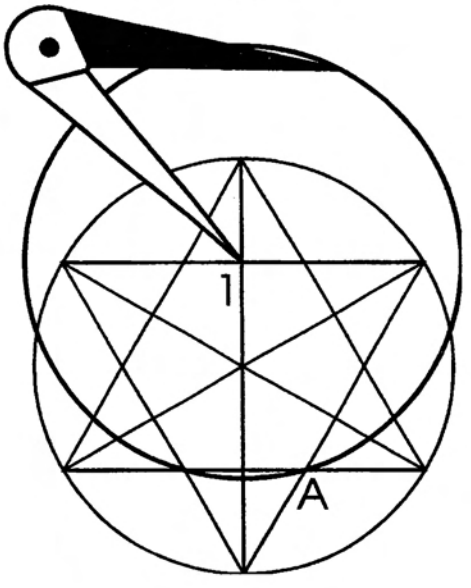
(3) Place the compass point at crossing point 1 and open the scribe to crossing point A.

(4) Turn a Circle.

3



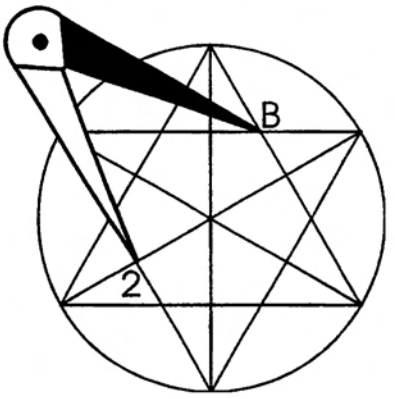
4



(5) With the same opening, place the compass point on crossing point 2 and the scribe on crossing point B.

(6) Turn a Circle.

5



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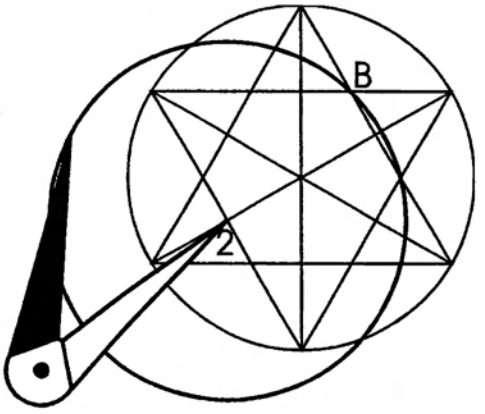
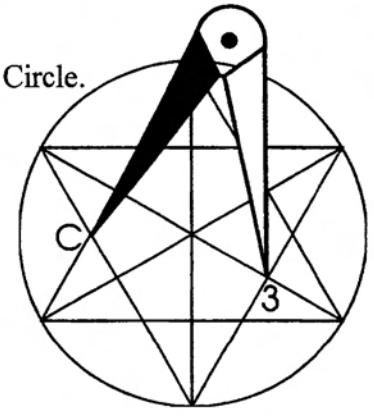


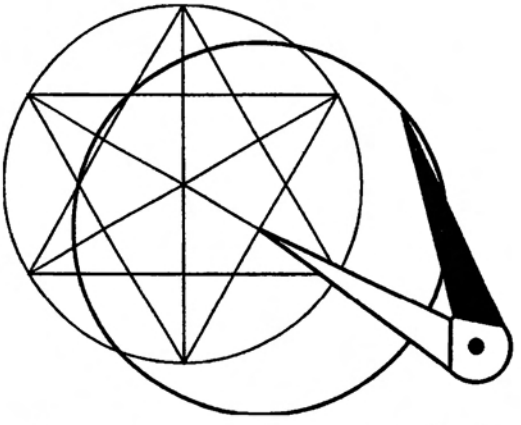
(7) With the same opening, place the compass point on crossing point 3 and the scribe on crossing point C.

(8) Turn a third large Circle.

7



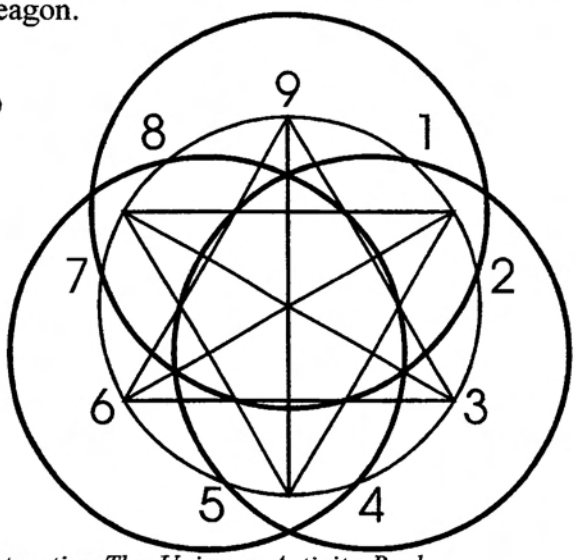
8



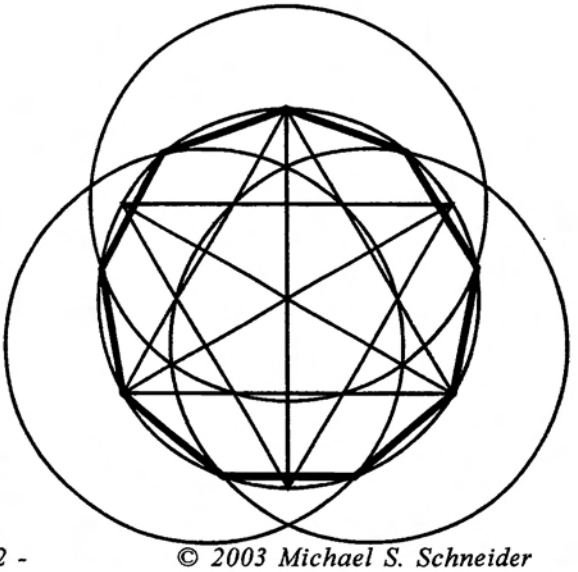
(9) The construction should look like this. Notice that the three large circles cross the original Circle at 6 points. These six, plus the three points of the upward pointing triangle will give us nine *nearly* equally spaced points.

(10) Connect these nine points with the straightedge to finish the construction of the near-regular Enneagon.

9

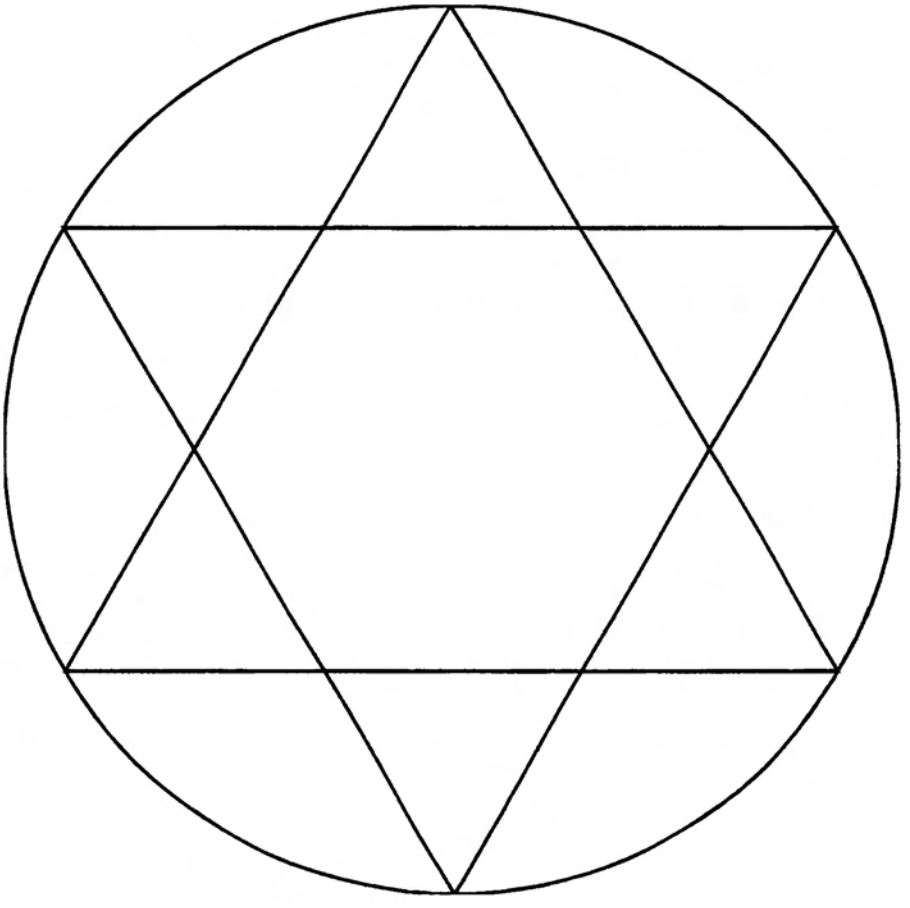


10



Construct An Enneagon

Start with this Hexagram star, or construct one larger on blank paper.
Follow the previous steps to inscribe an Enneagon in this Circle. Learn how to do it without looking at the instructions.



Draw Enneagram Stars

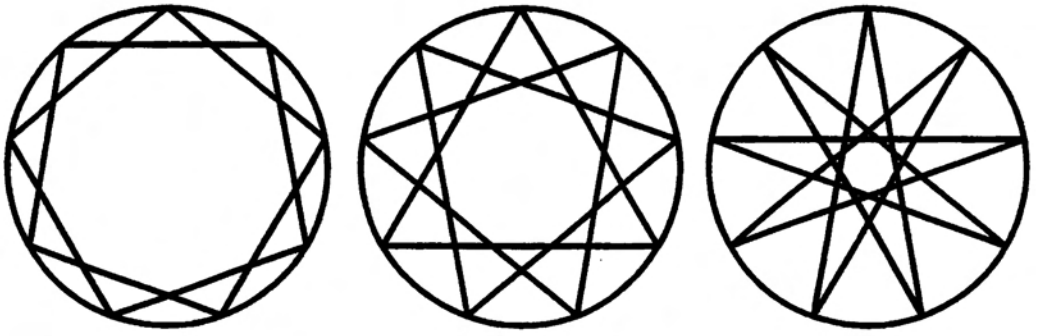
The Enneagon has three natural Enneagram stars.

(1) The first is made by skipping one point and connecting every other point. This can be done without lifting your pencil.

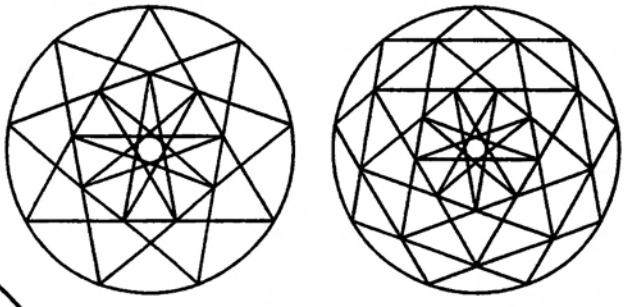
(2) The second is made by skipping two points, connecting every third point. You must lift your pencil to construct the three non-regular Triangles.

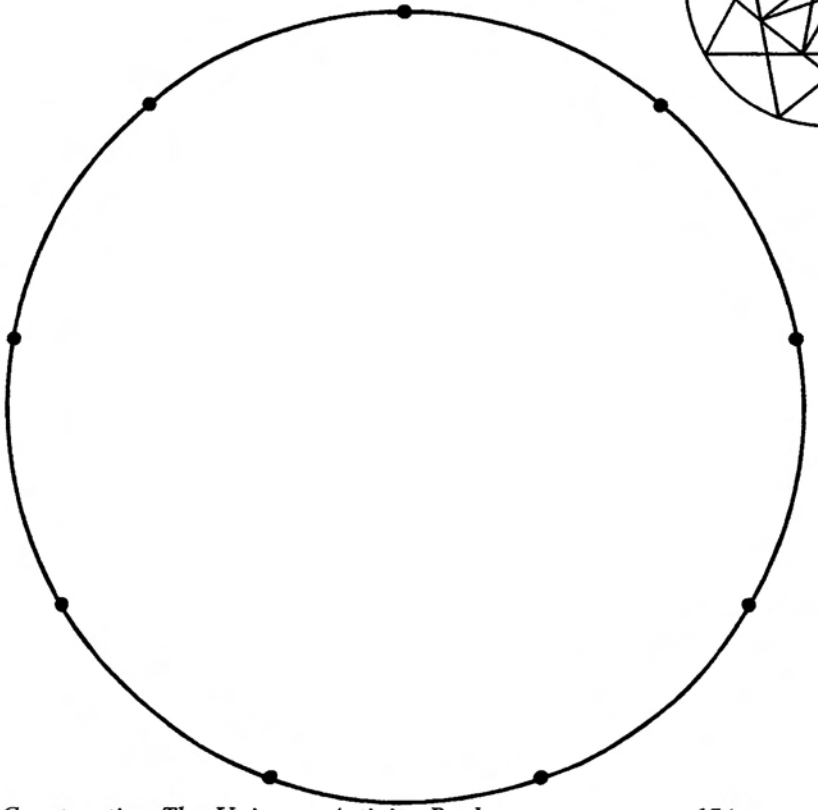
(3) The third is made by skipping three points, connecting every fourth point. You need not lift your pencil for this.

1 2 3



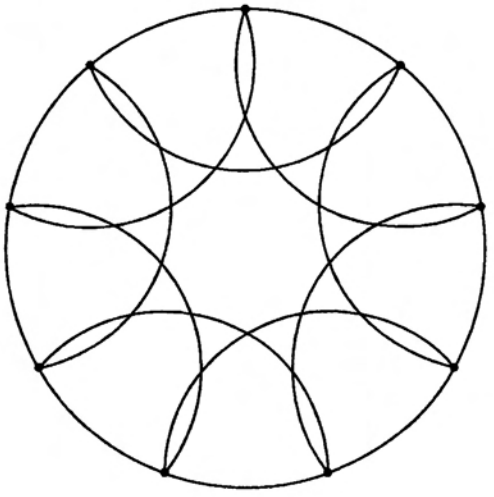
At the center of each Enneagram star is an Enneagon. Of course, they may be combined many ways.

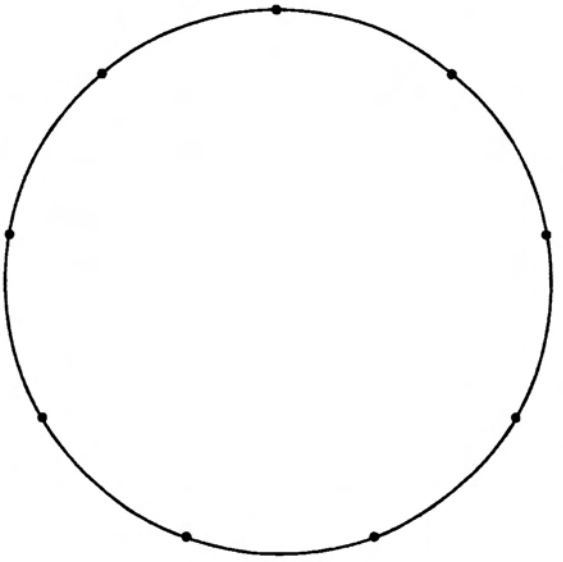


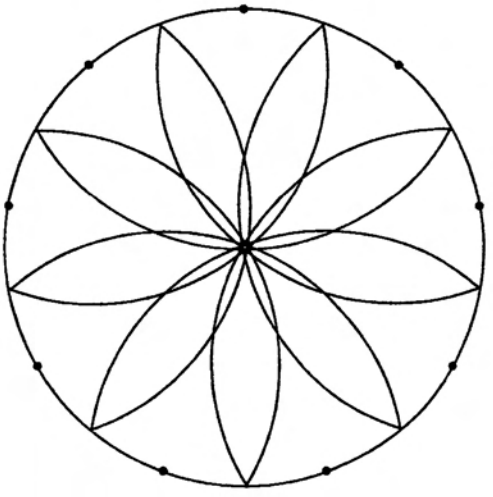


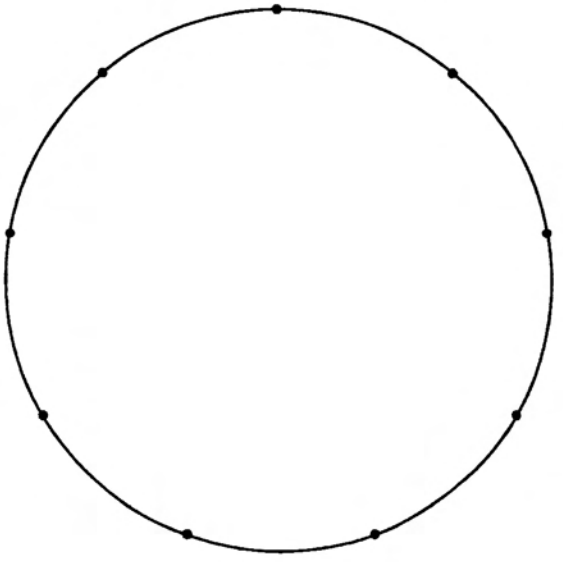
Start with these nine points around the Circle and use your straightedge and colored pencils to construct your own Enneagram star design.

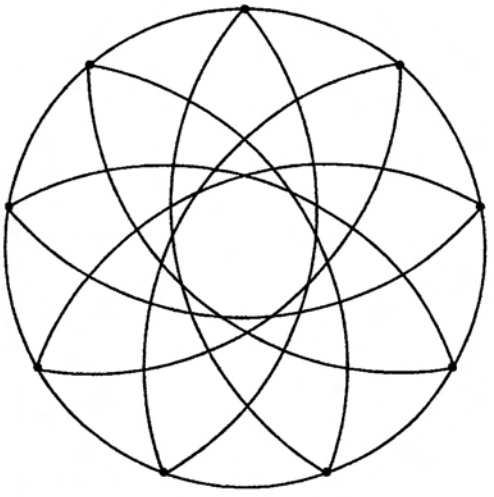
Replicate Enneagonal Patterns

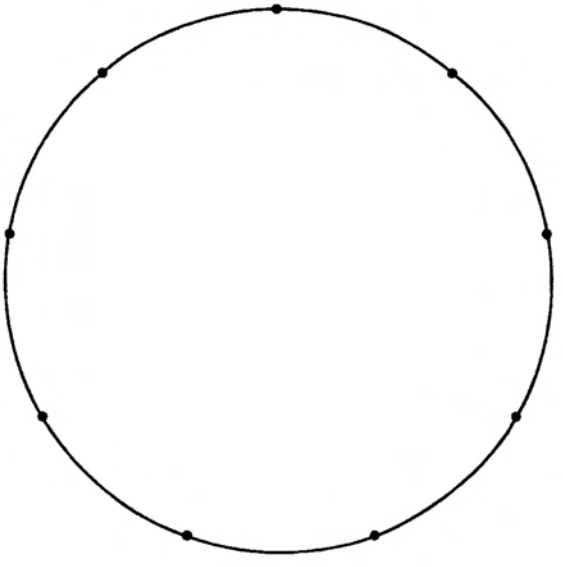


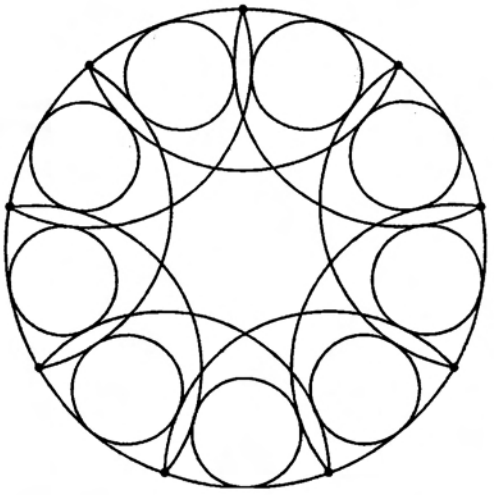


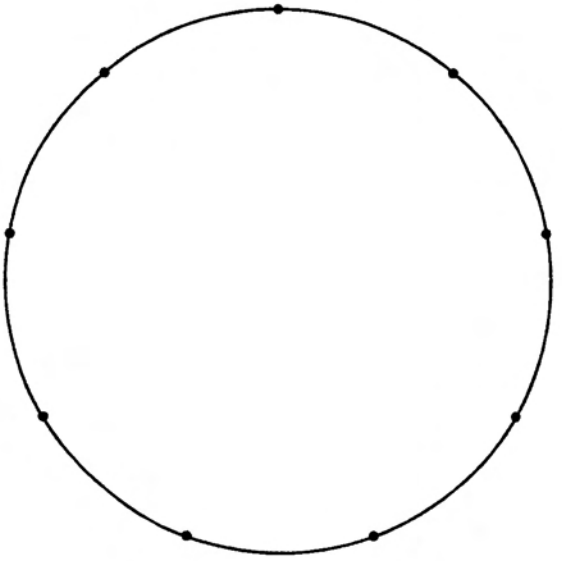




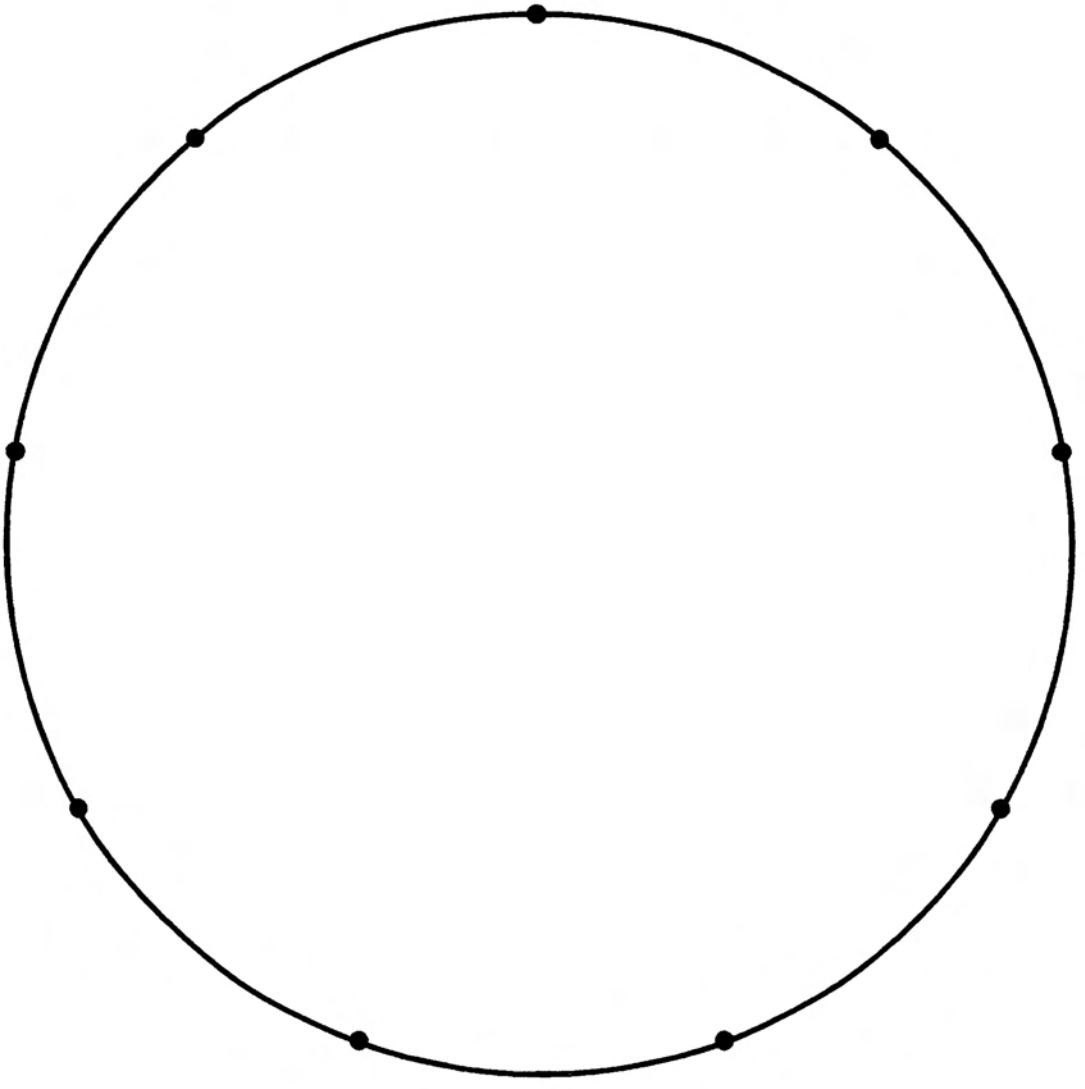








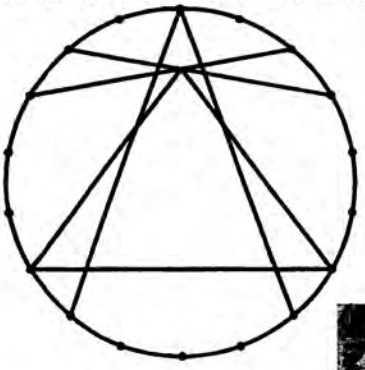
Create Your Own Enneagonal Patterns

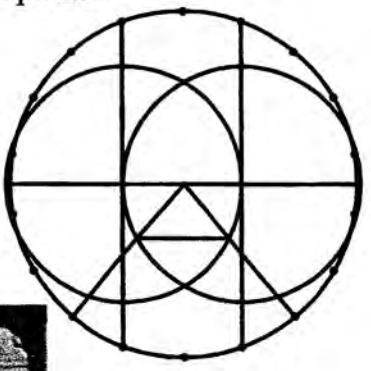


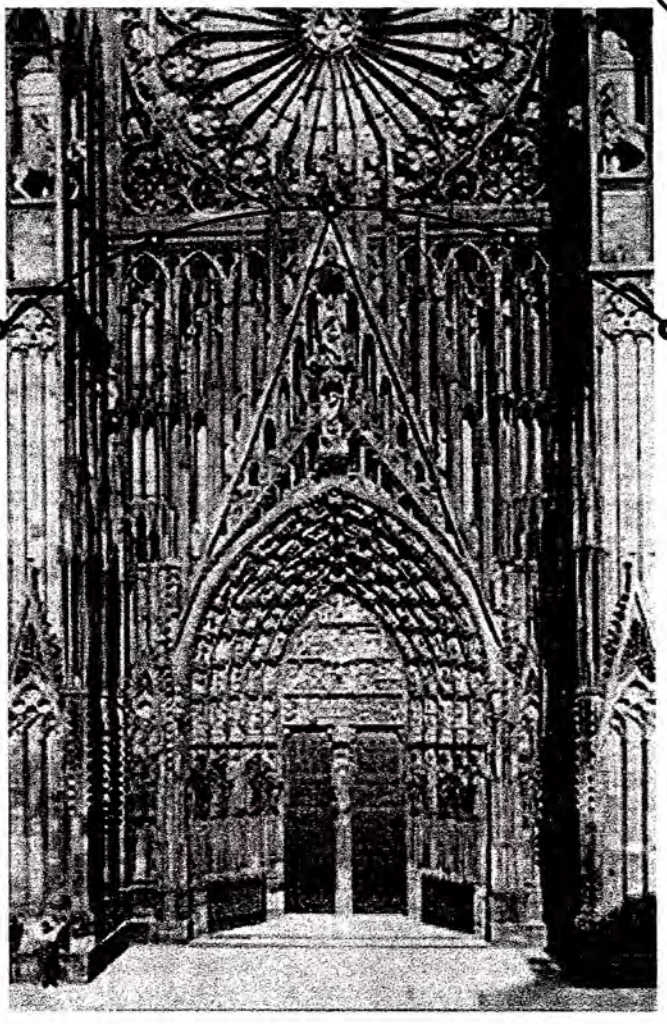
Strasbourg Cathedral Main Portal

France

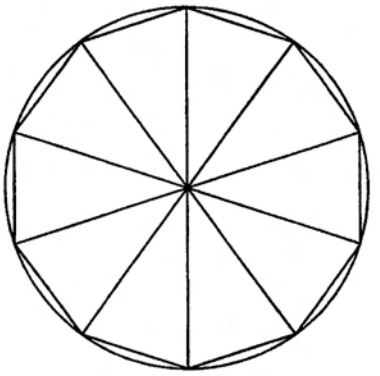
Understanding some of its design requires a circle with 18 (=2x9) equally spaced points around it. These points can also be understood as six regular Triangles, three regular Hexagons or two Enneagons. Draw these constructions in the circle and you'll see how the lines guide the Cathedral's architecture. There's more for you to discover about the design from these points.







10 The Decagon

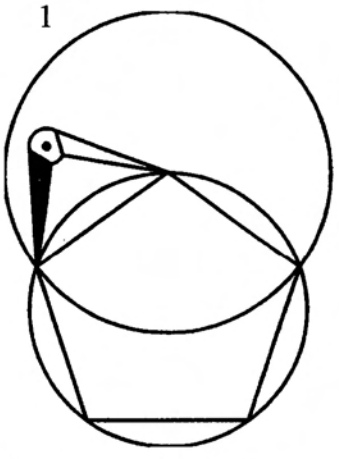


A regular Decagon (Greek *deka* = 10, as in "decade" of ten years) has ten equal corner angles, ten equal sides and five equal diameters. It may be constructed from a regular Pentagon inscribed in a Circle (page 69). Our approach will be to divide one side of the Pentagon in half using the Almond construction (page 9).

Construct A Decagon

(1) Open the compass between the ends of one side of the Pentagon and turn a Circle (or just the Almond arc if you don't have space).
(2) Reverse the compass and turn another Circle (or arc).
(3) Connect the crossing points of the Almond. This divides the side of the Pentagon into halves. But it also shows us the midpoint of the arc on the Circle between the two corners of the Pentagon. We want to find the midpoints of all five arcs which, added to the Pentagon's five corners, gives us the Decagon's ten points.

1



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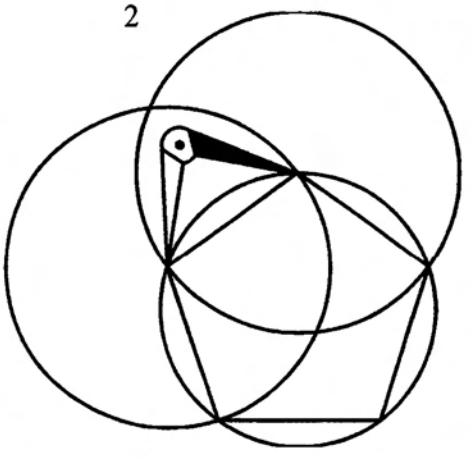
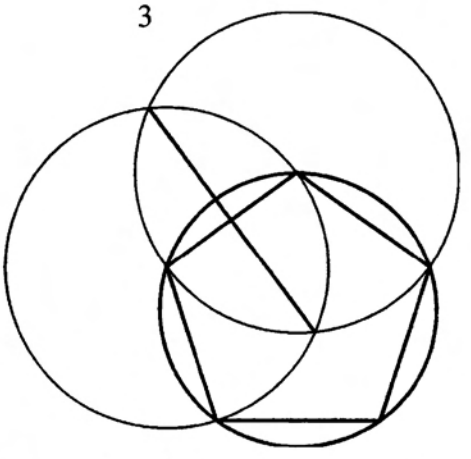
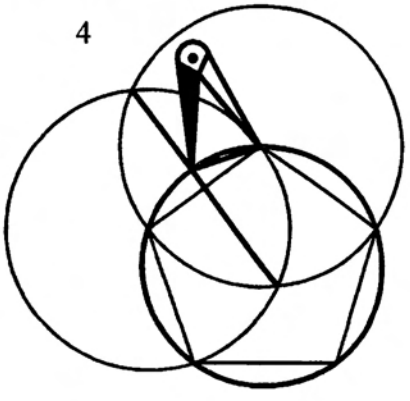


3



(4) Draw a line between one corner of the Pentagon and the point where the Almond line crosses the Circle. This is one side of the Dodecagon. Open the compass between these two points.
(5) Place the compass point at each corner of the Pentagon and turn an arc until it crosses the Circle. The five new points will be where the arcs meet (or halfway between them if they don't).
(6) A regular Decagon can then be inscribed in the Circle.

4



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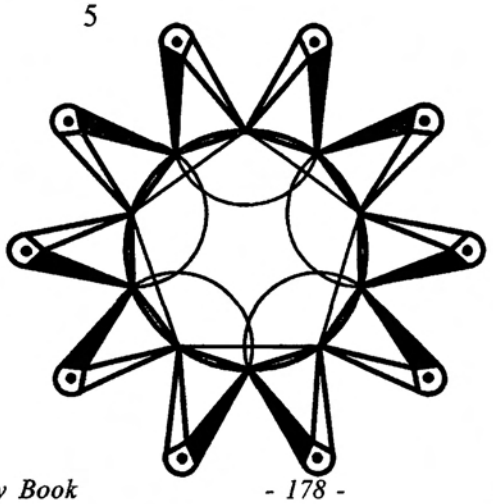

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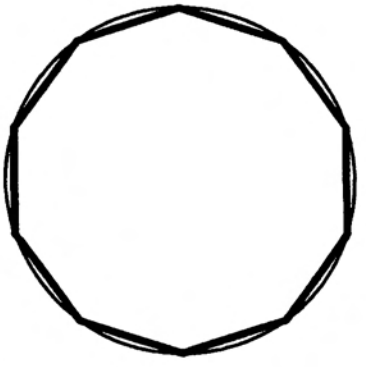
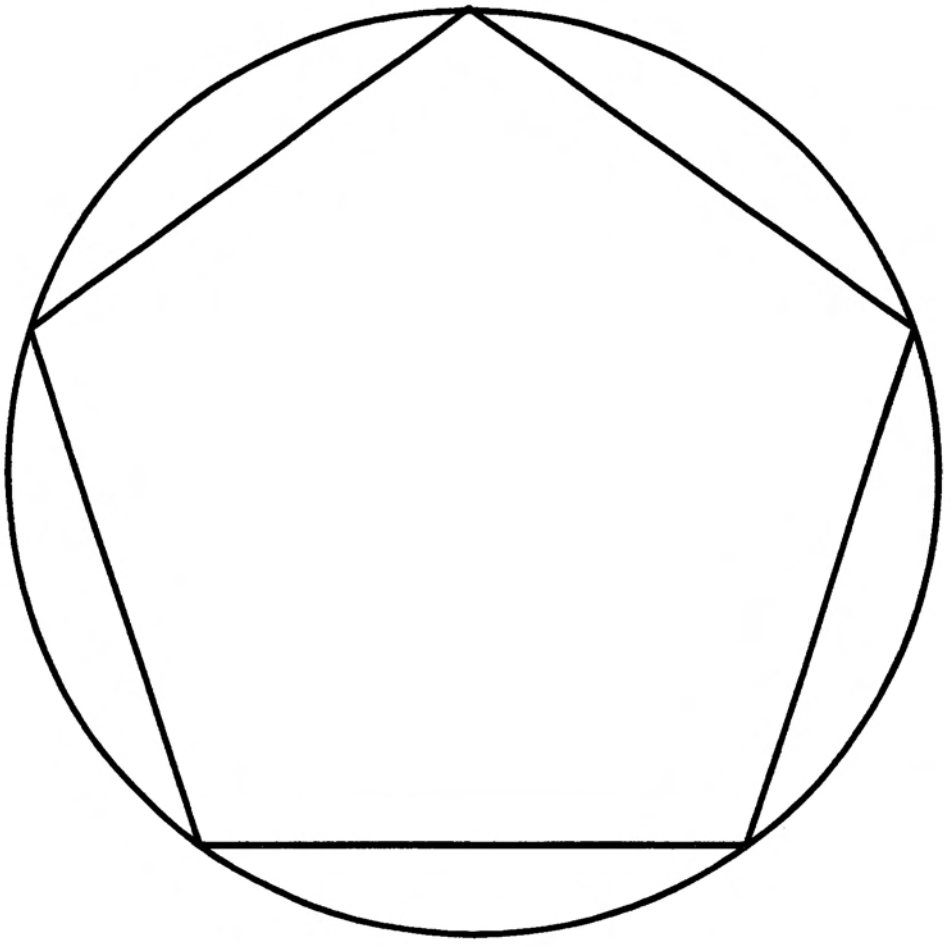


Construct A Decagon

Start with this Pentagon inscribed in a Circle.



A Decagon From A Square

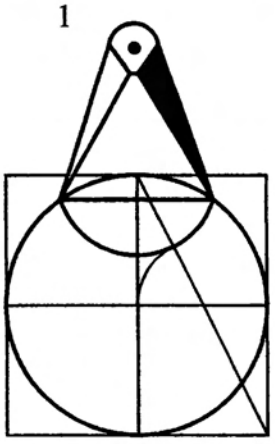
When we constructed a regular Pentagon in a Square (page 71) we developed everything we need to construct a regular Decagon too.

(1) Through a series of steps we found one side of the Pentagon.

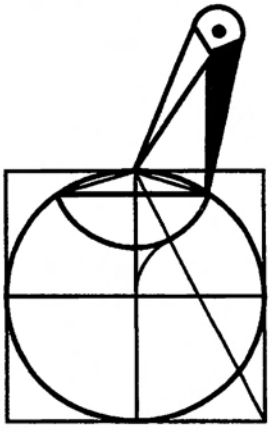
(2) But it also gives us two sides of a Decagon within the Circle.

(3) "Walk" the compass around the Circle *in both directions* to find the ten points of the Decagon.

1



2



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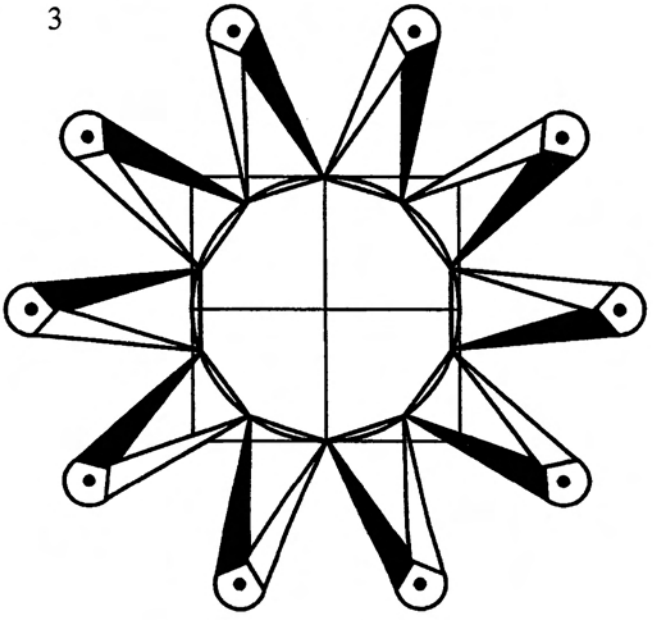
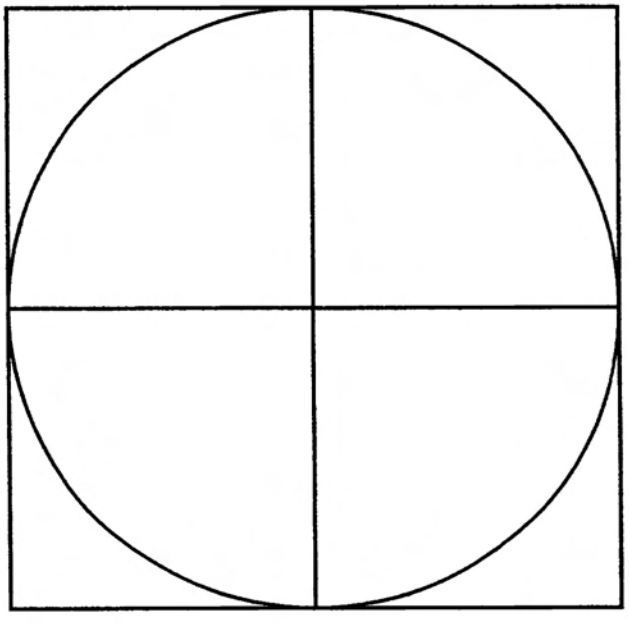


Construct A Decagon In This Square

Begin with the steps for the construction of the Pentagon in a Square (page 71).



Construct The Three Decagram Stars

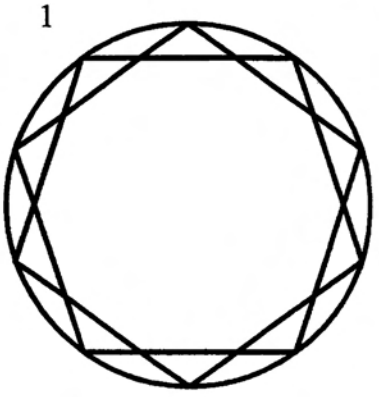
There are three Decagram stars in a regular Decagram.

(1) The first is made by skipping one point (connect every other point) of the Decagon. You must lift your pencil to draw these two interlaced Pentagons.

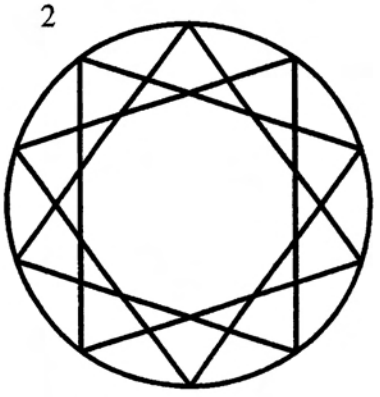
(2) The next is made by skipping two points (connect every third point) of the Decagon.

(3) The last is made by skipping three points (connect every fourth point) of the Decagon. You must lift your pencil to draw these two opposing Pentagram stars.

1



2



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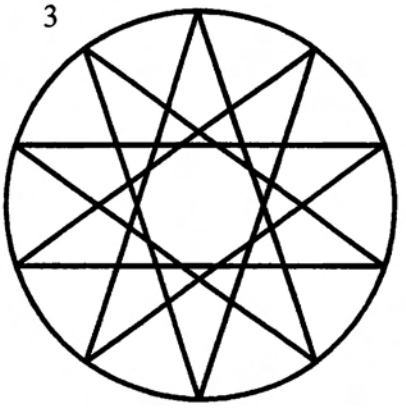
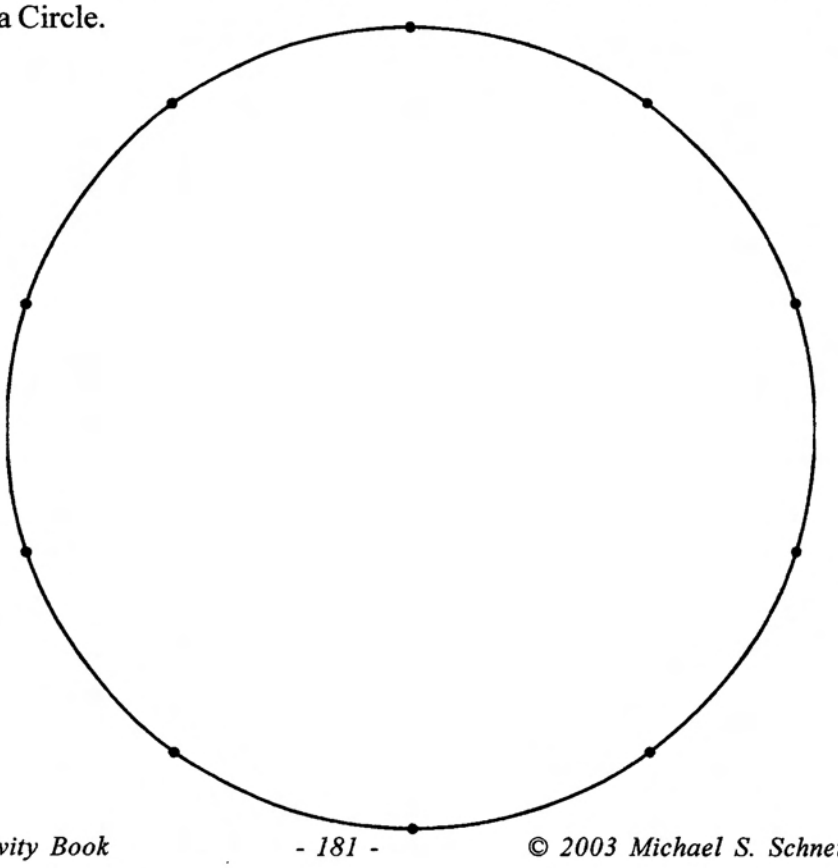
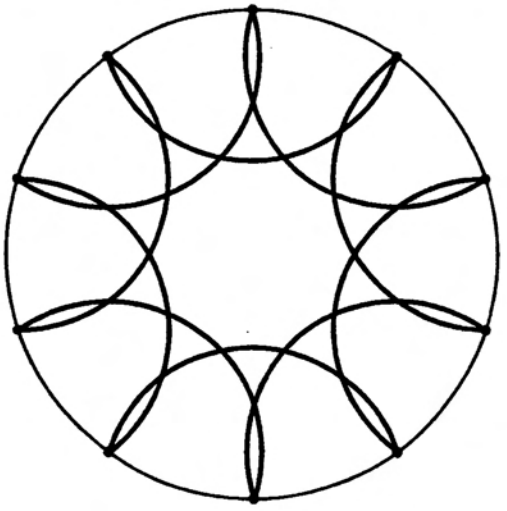


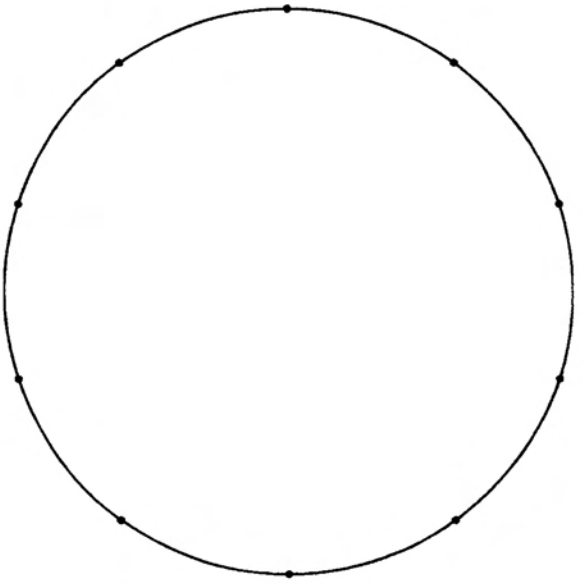
Of course, they may be combined with each other as well.

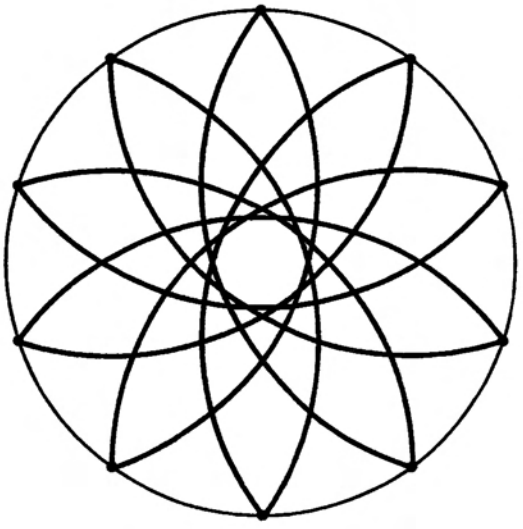
Use a straightedge and colored pencils to draw any combination of Decagram stars with these ten equally spaced points around a Circle.

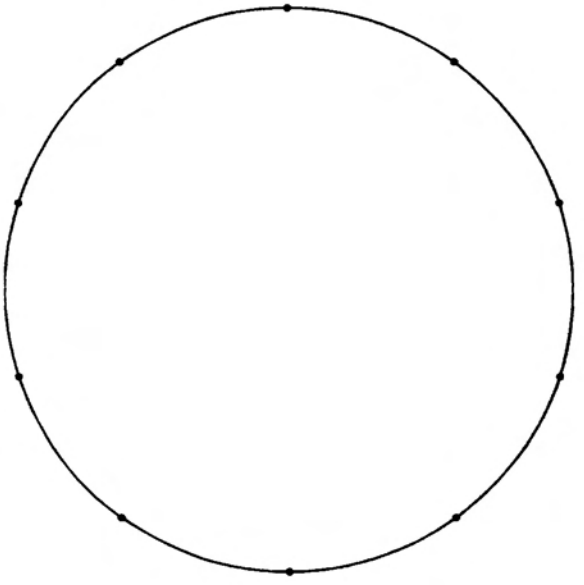


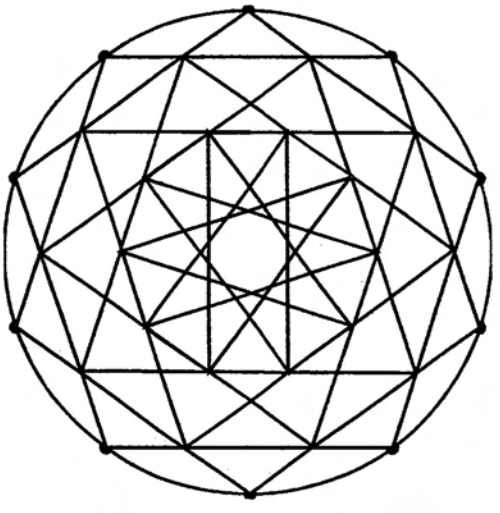
Replicate Decagonal Patterns

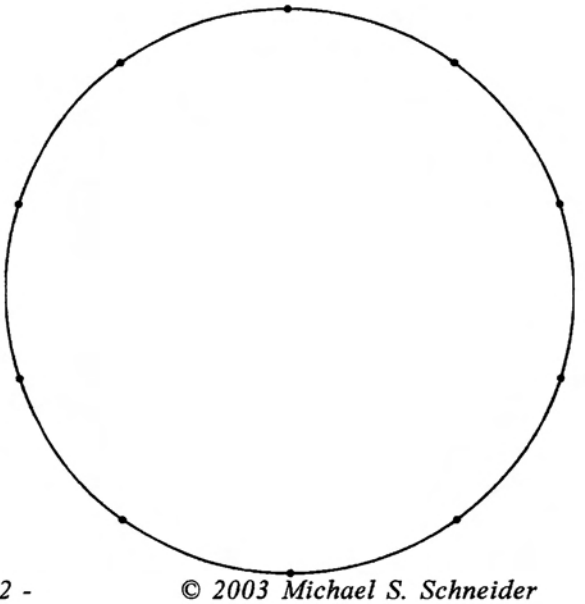


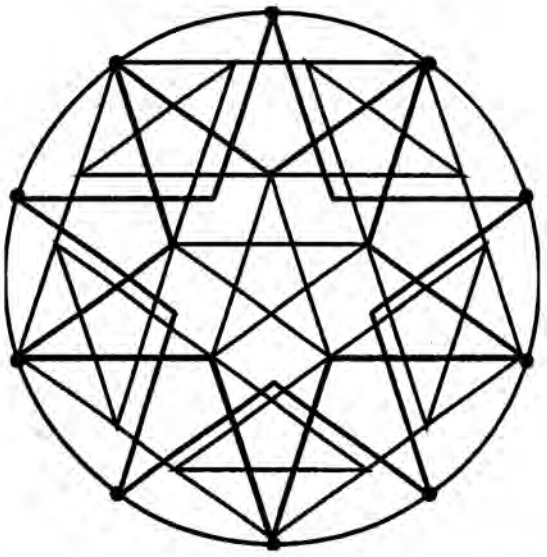


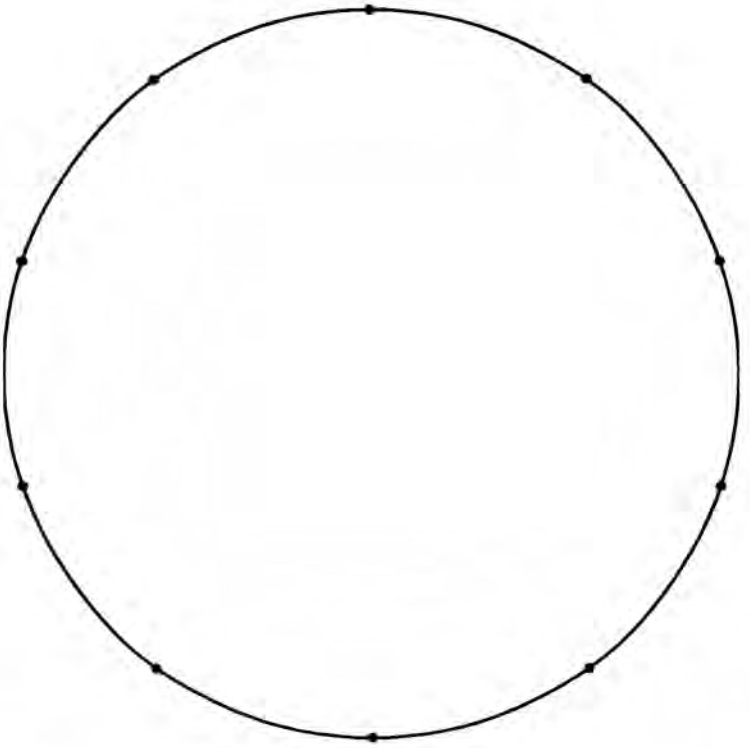


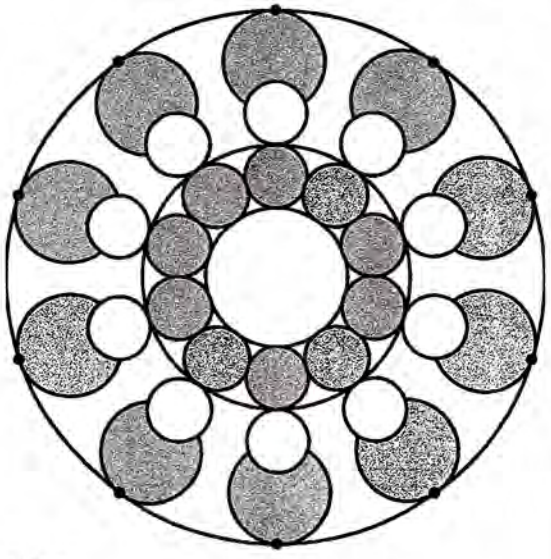


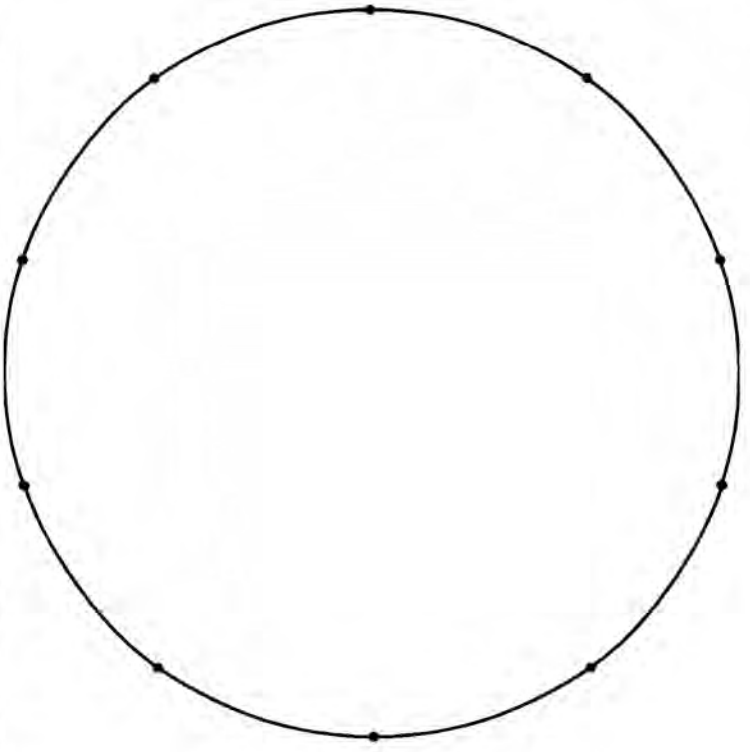




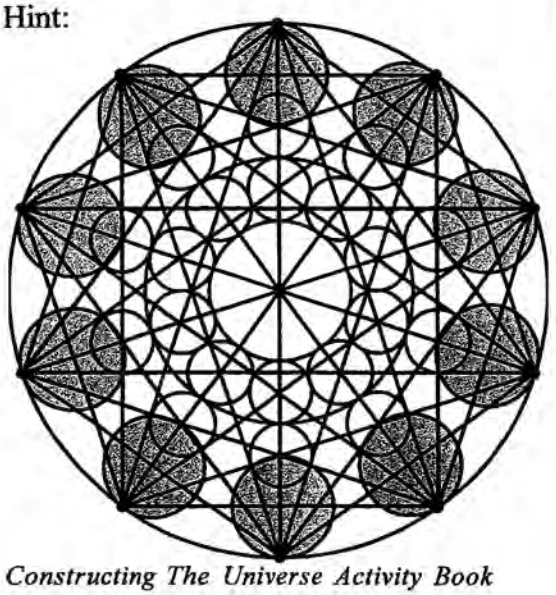




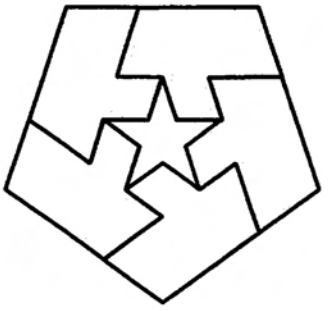




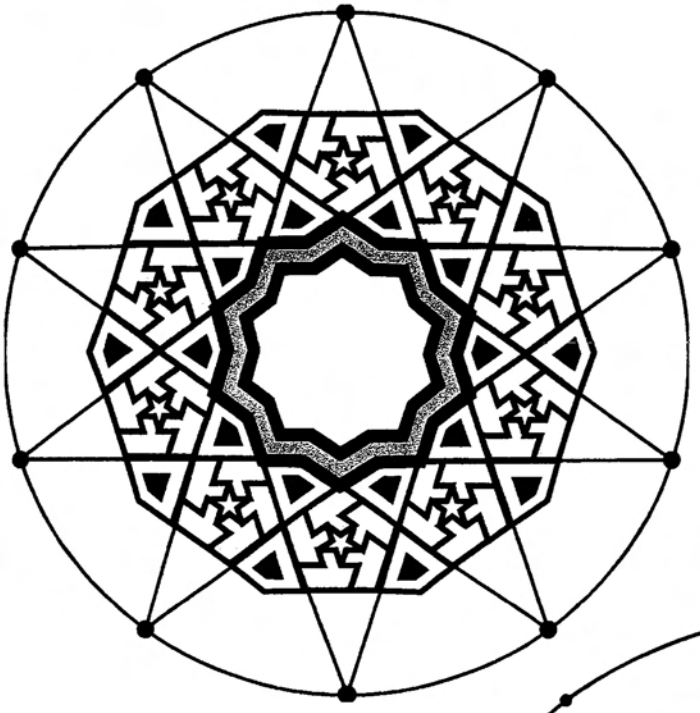
Hint:



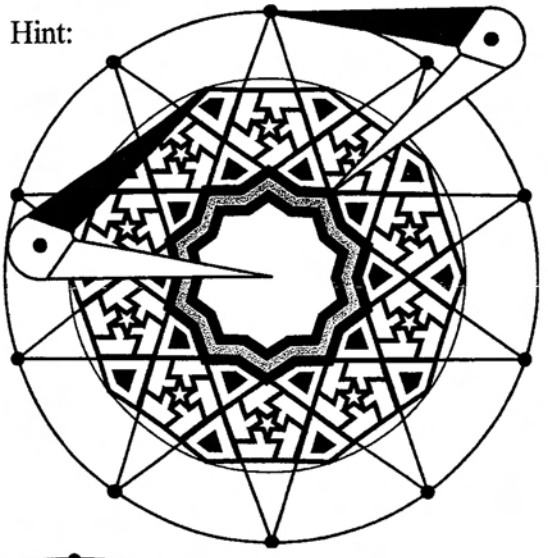
An Islamic Tiling Pattern

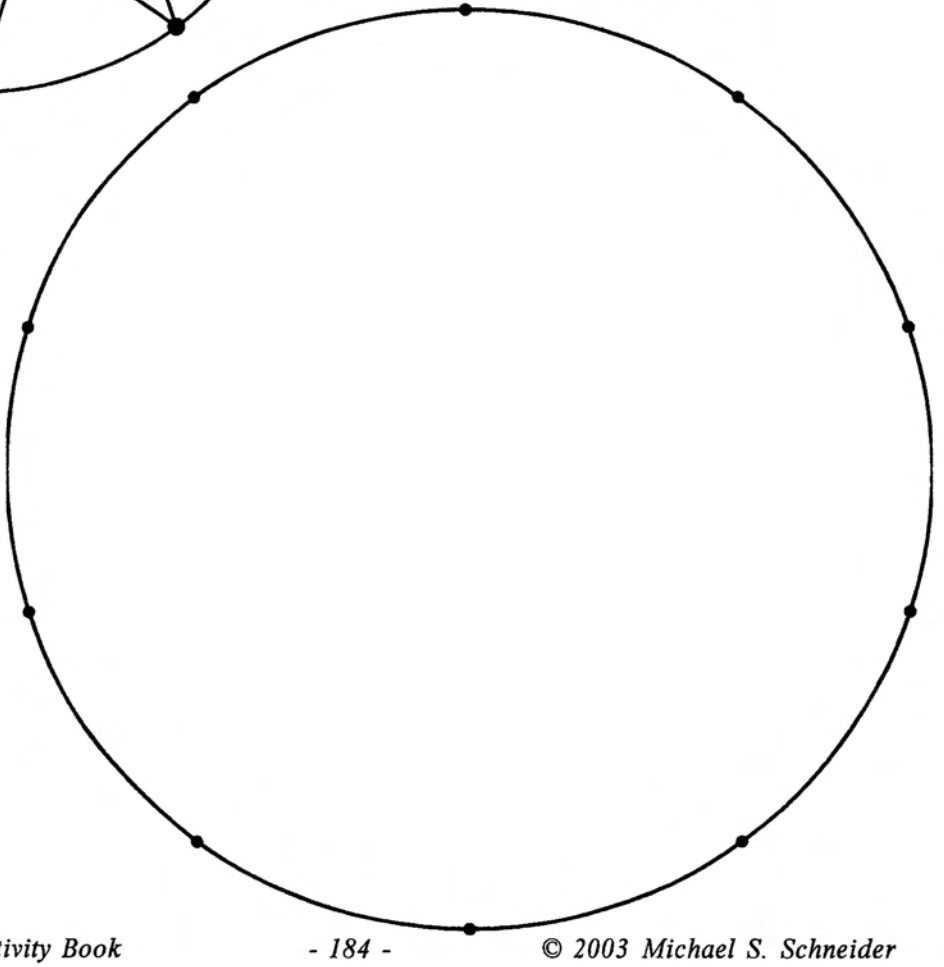


If you constructed the Five Piece Pentagon Puzzle in Chapter Five (page 95) then you know nearly enough to construct this Islamic tile design. You can construct it in the ten pointed Circle below, or actually construct and cut out the pieces and arrange them in this pattern.



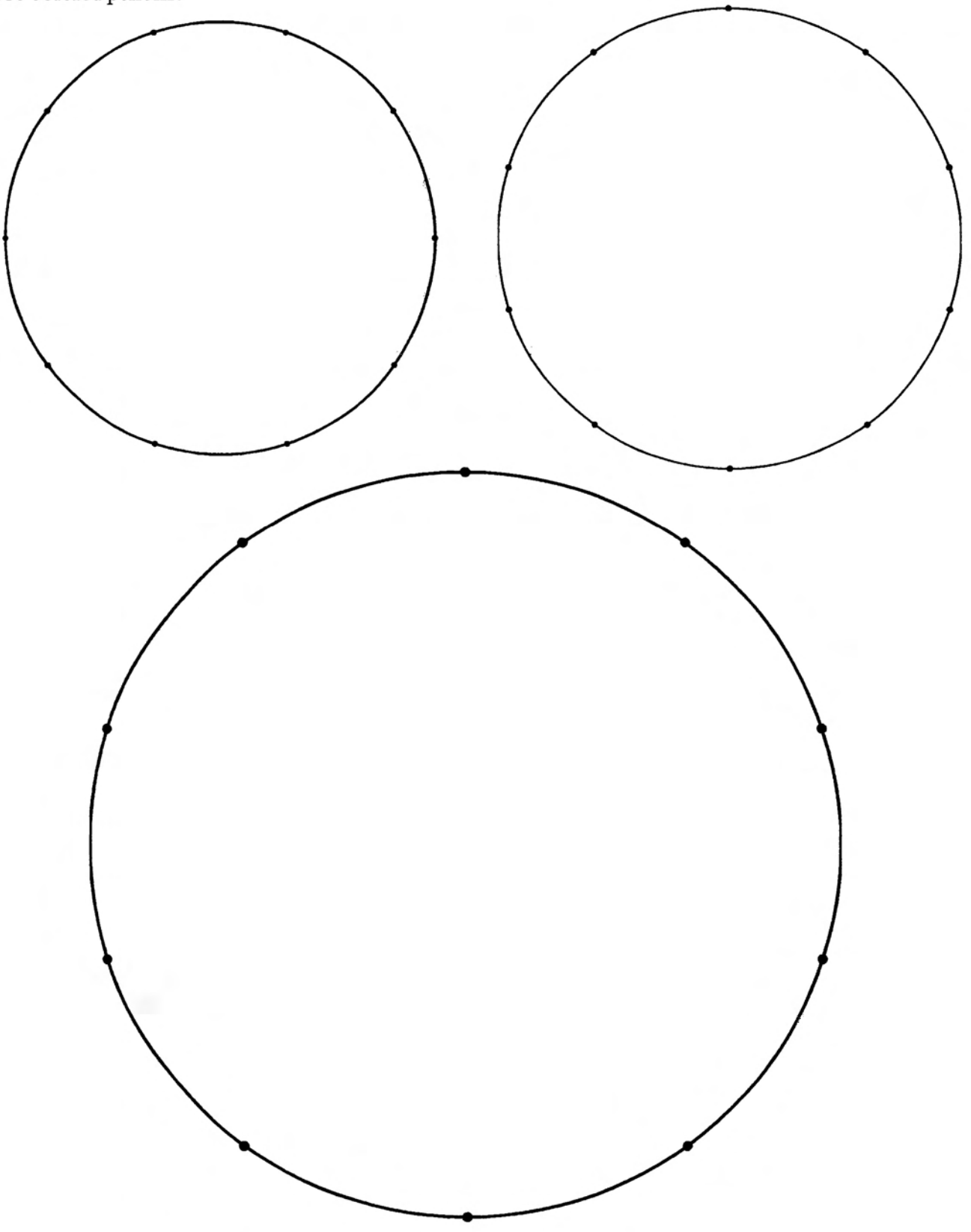
Hint:





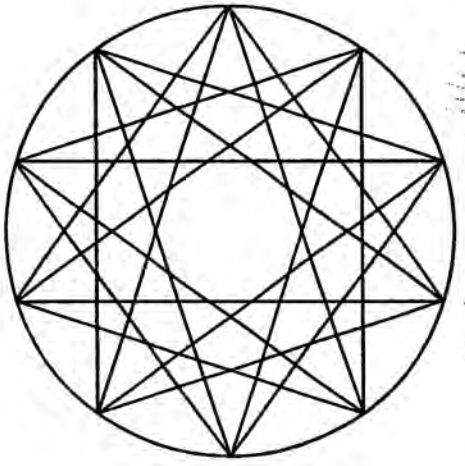
Create Your Own Decagonal Designs

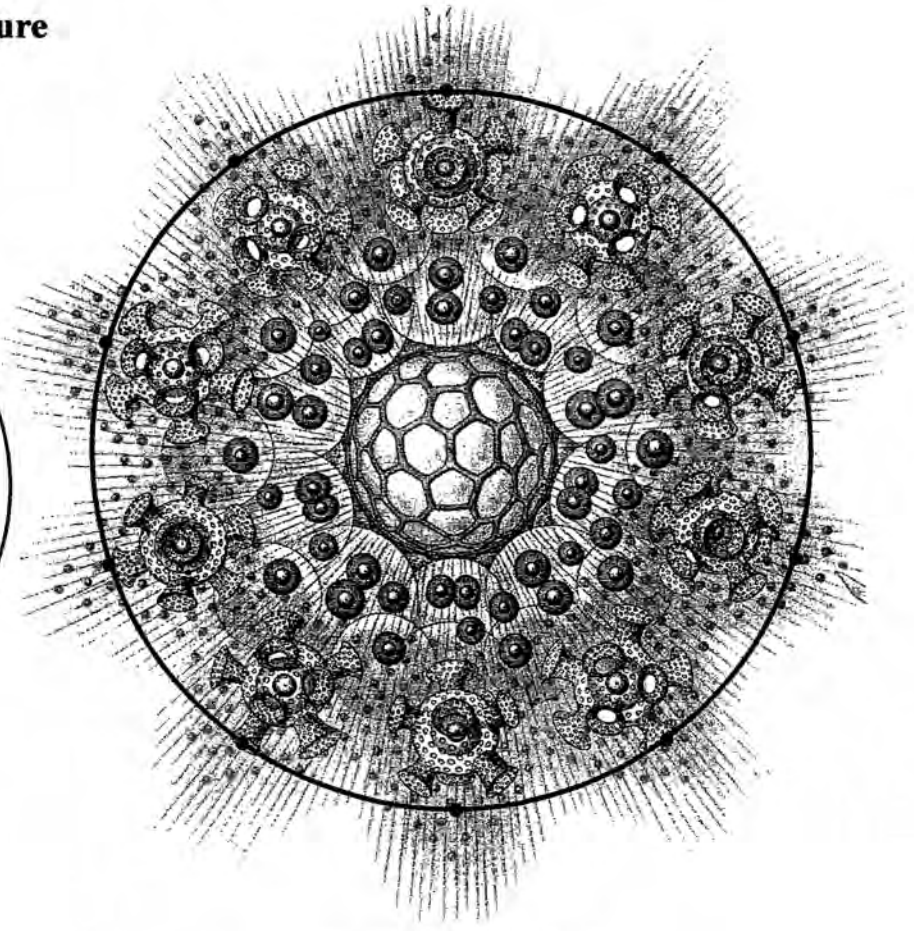
Construct them in the ten pointed circles below or on blank sheets of paper.
Use colored pencils.



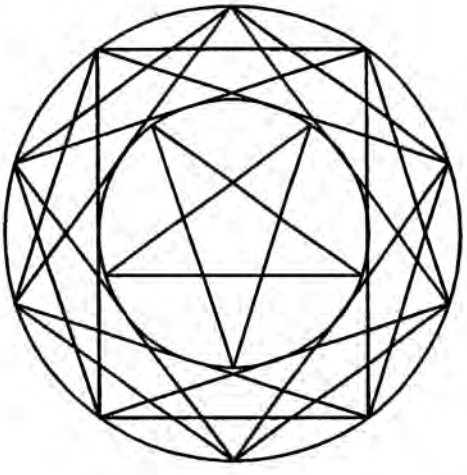
Decagonal Geometry In Nature

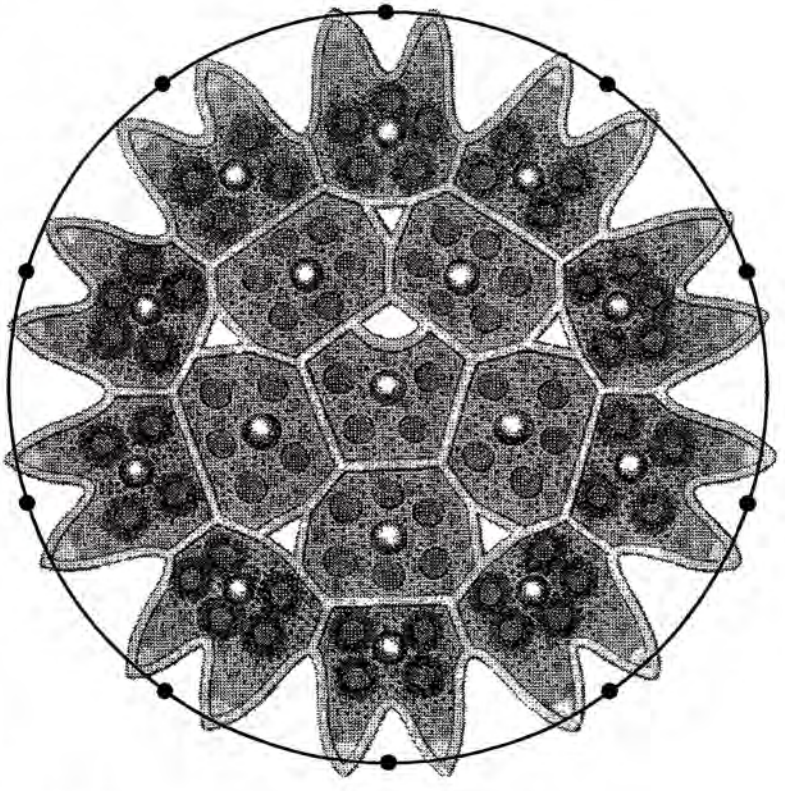
Microscopic Radiolarian



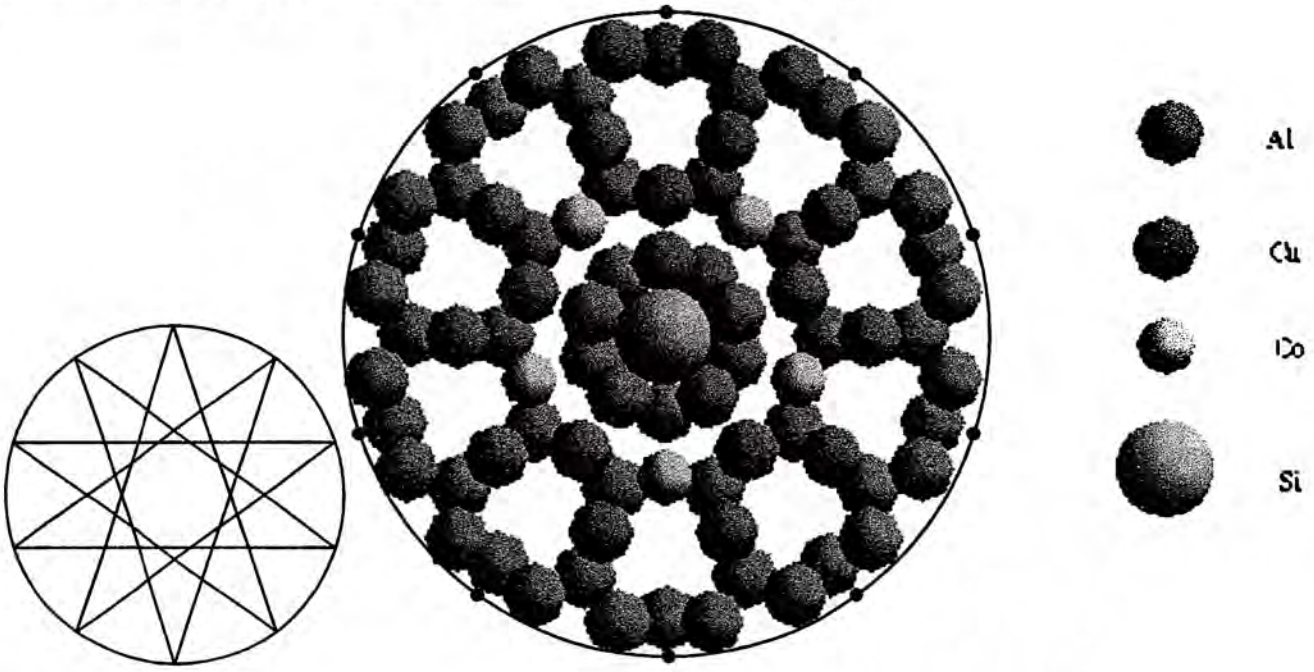


Algae Colony



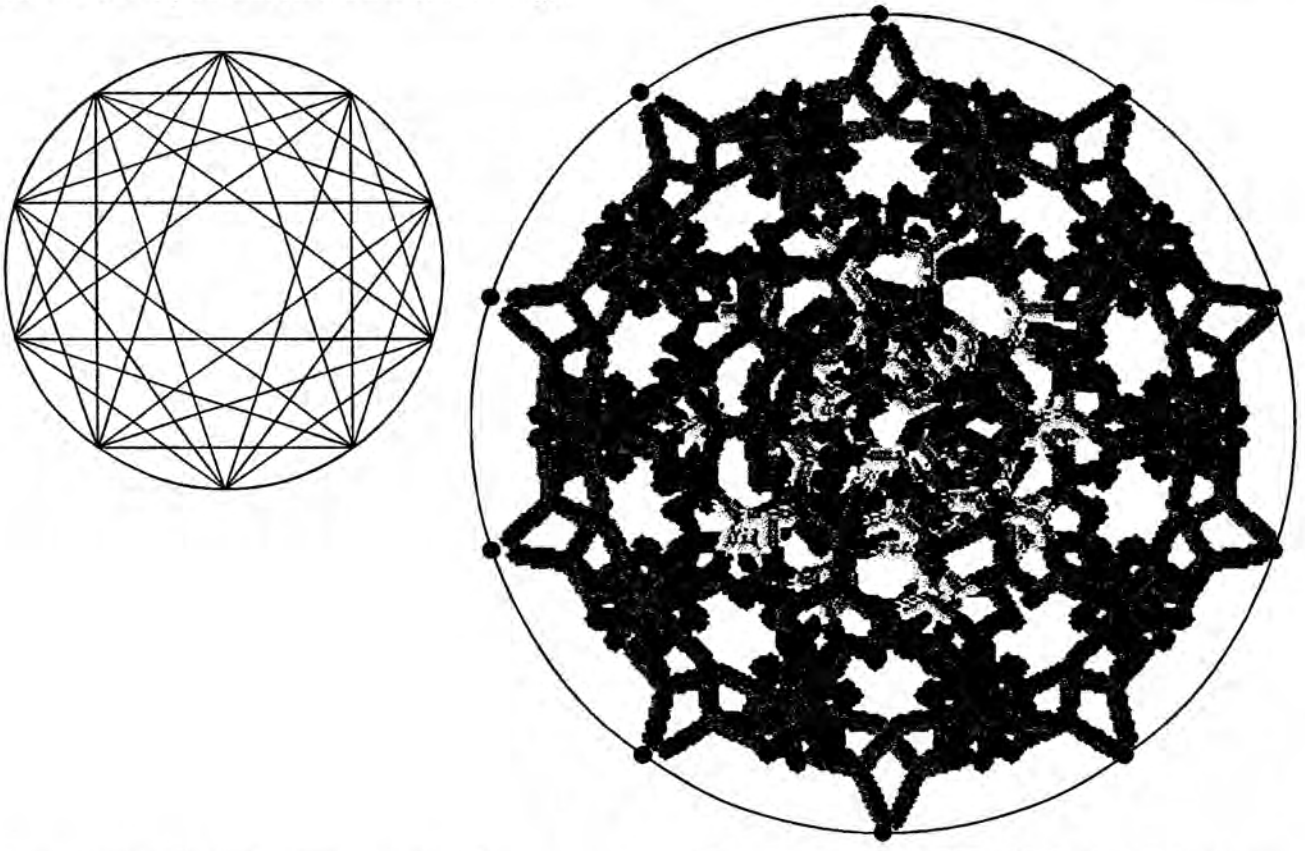


Aluminum, Copper, Cobalt and Silicon Molecule ($SiCo_5Cu_{15}Al_{60}$)

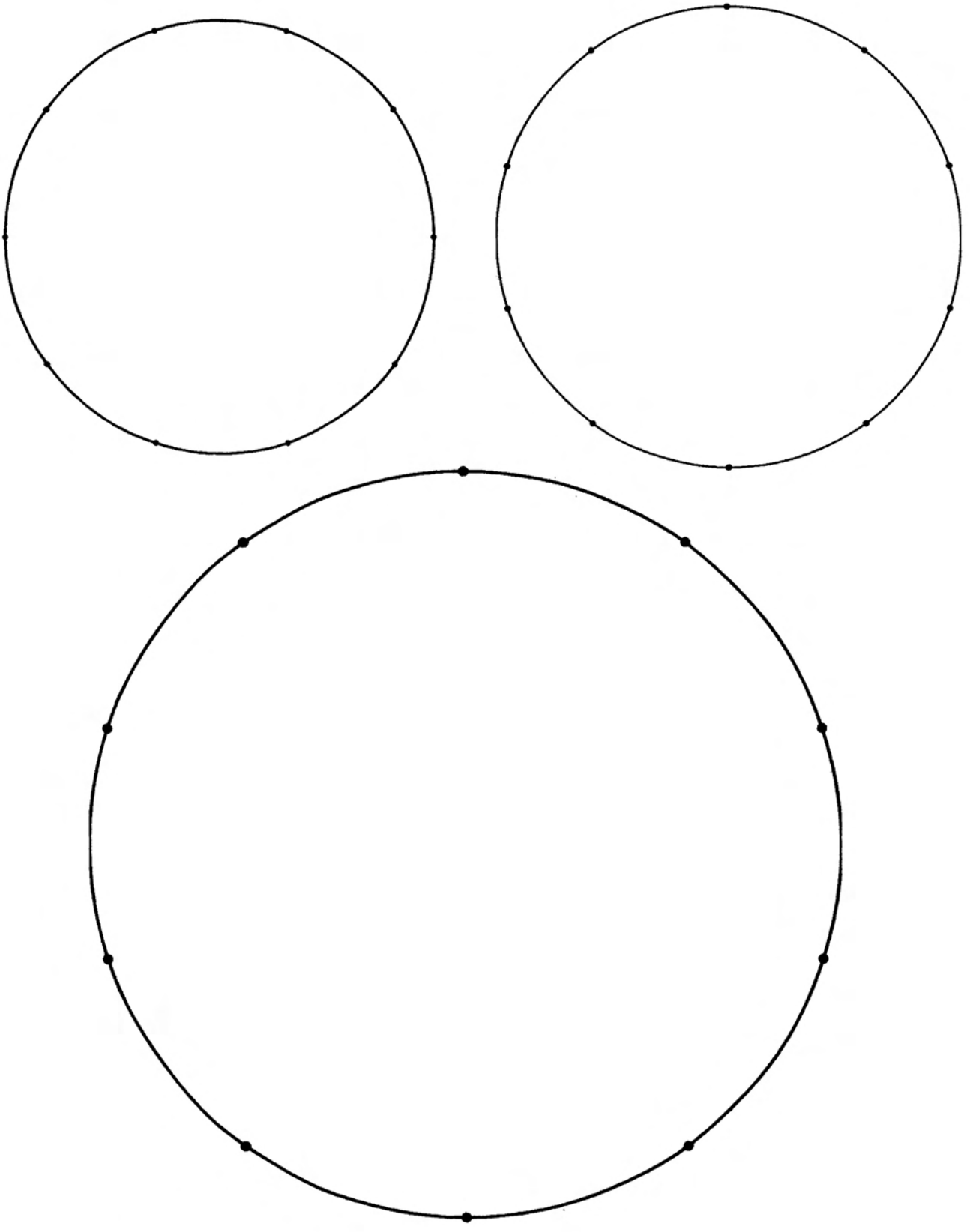


Human DNA

Seen from *above* the end of the double-helix, DNA shows ten points around a Circle. But its twist makes it not quite a regular Decagon. The geometry of DNA is more complex than this construction shows. See if you can find more of its design.



Design Your Own Natural Decagonal Forms

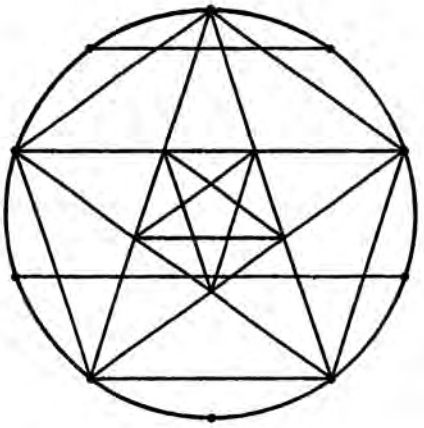


Decagonal Geometry In Art

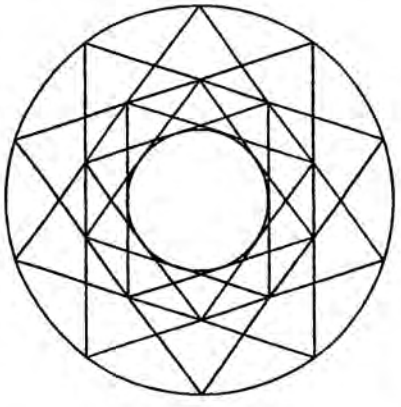
Replicate the geometry on these images.

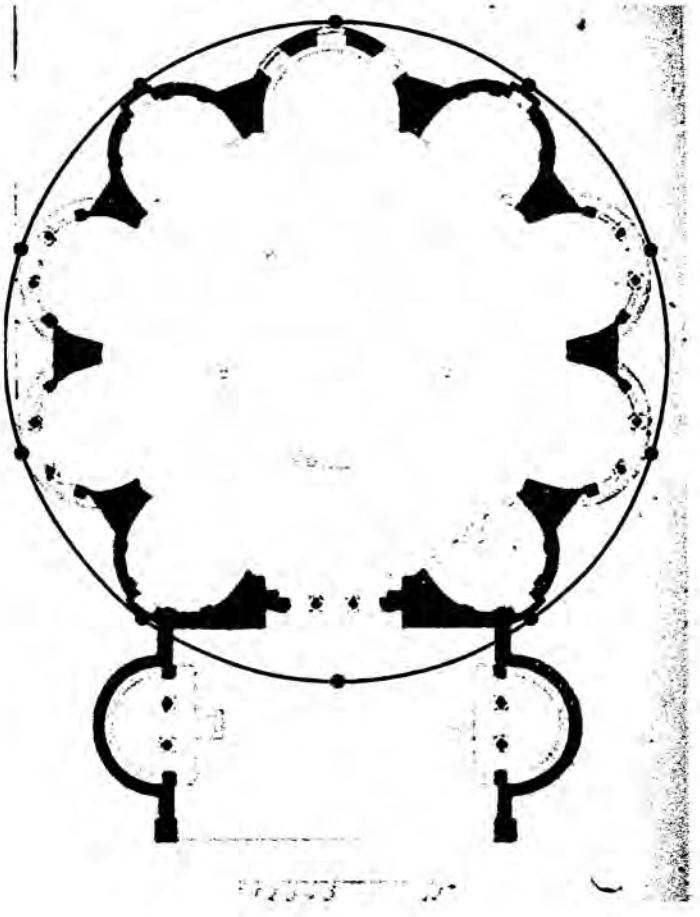
Painting In A Greek Drinking Bowl

Find out why the boy is sitting and pointing where he does.



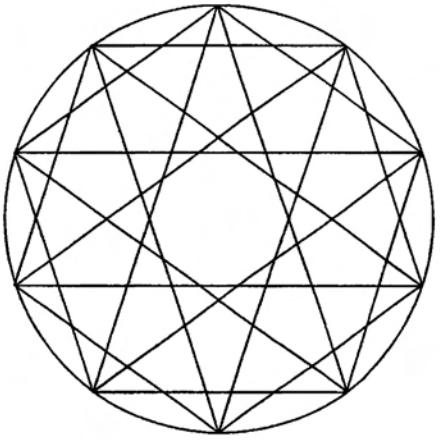
Plan of the Temple of Minerva in the Licinian Gardens, Rome

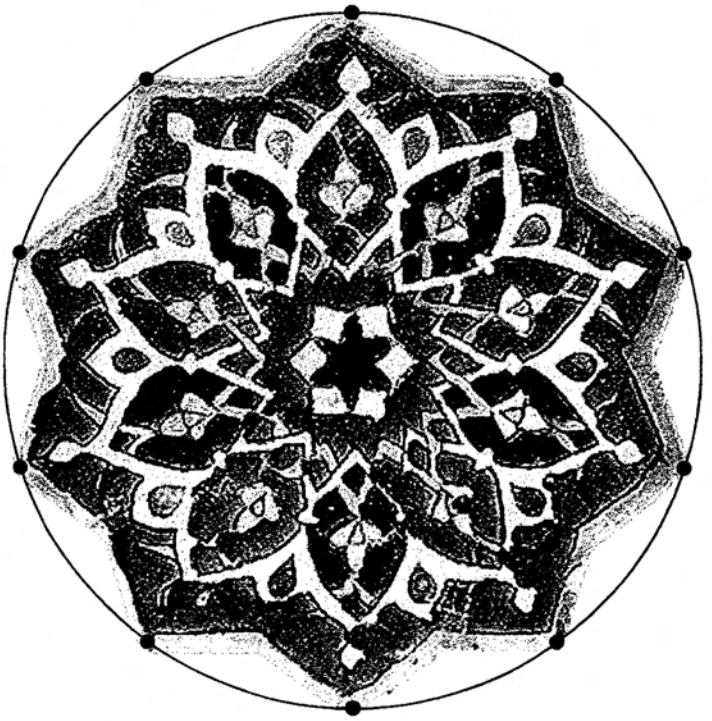




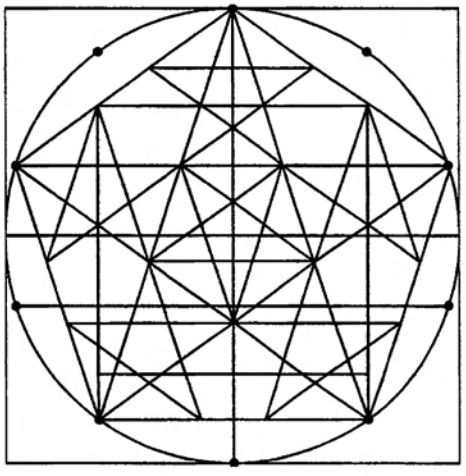
Persian Tile

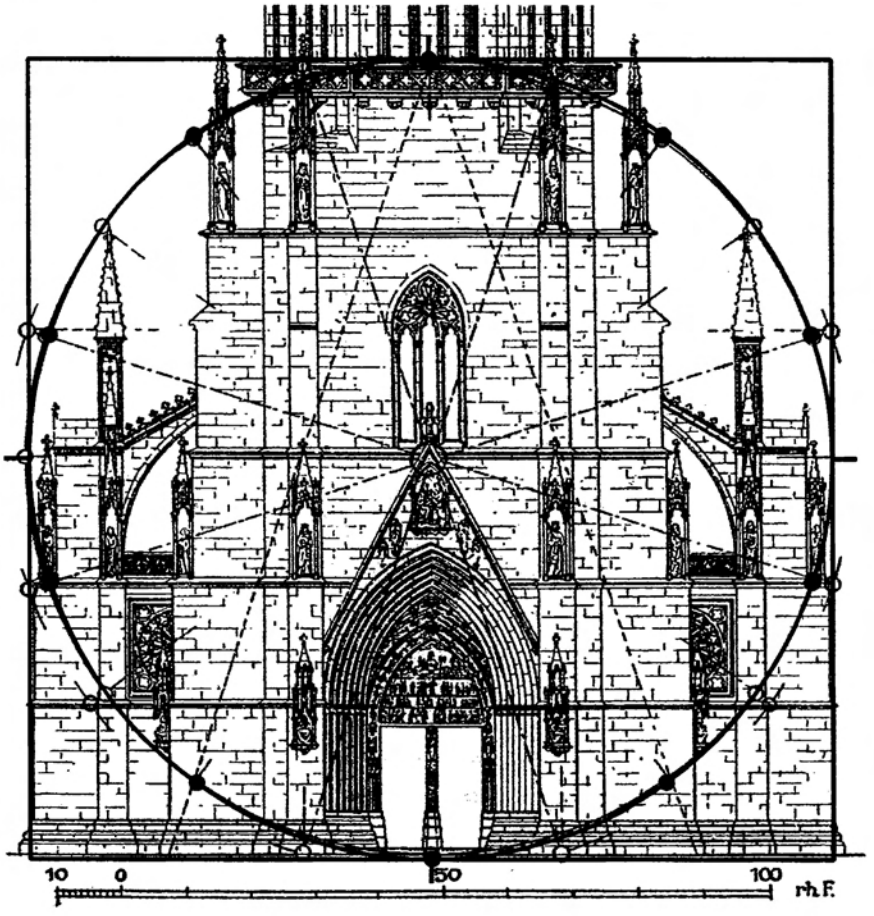
15th century, Timurid Cuereda Seca



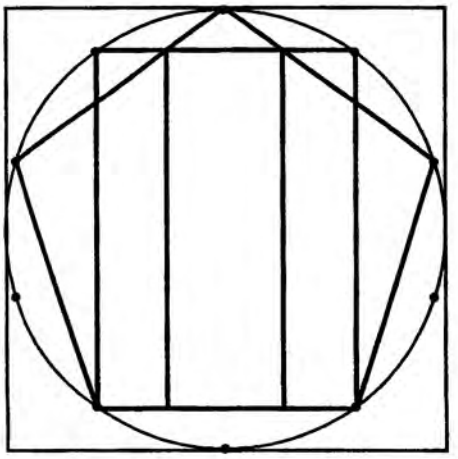


Munster Cathedral in Freiburg, Germany



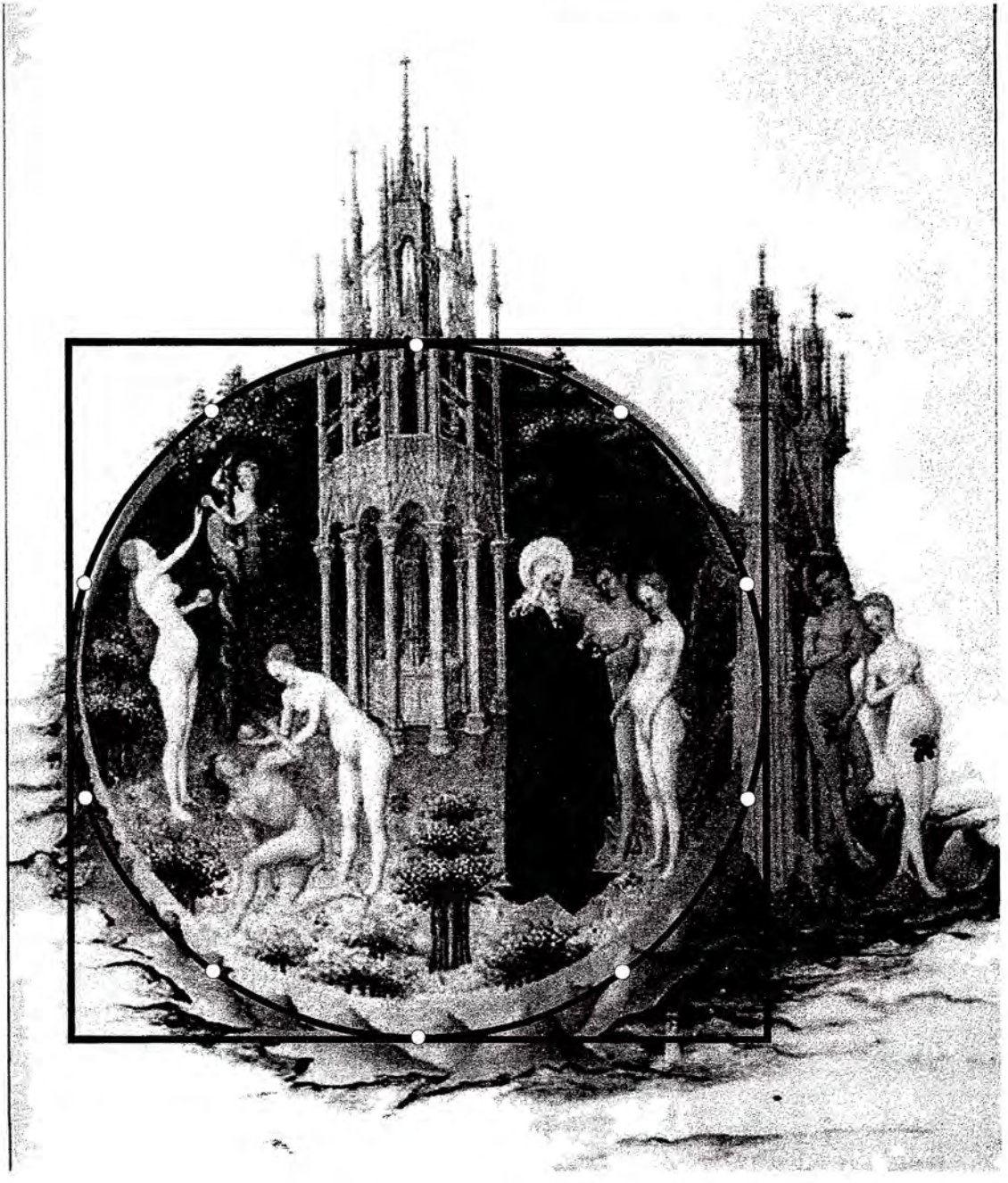


"The Fall and the Expulsion from Paradise"

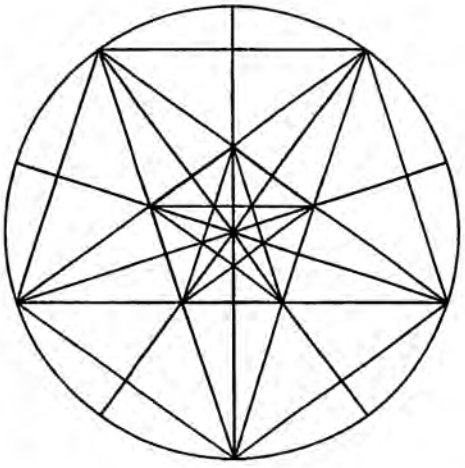


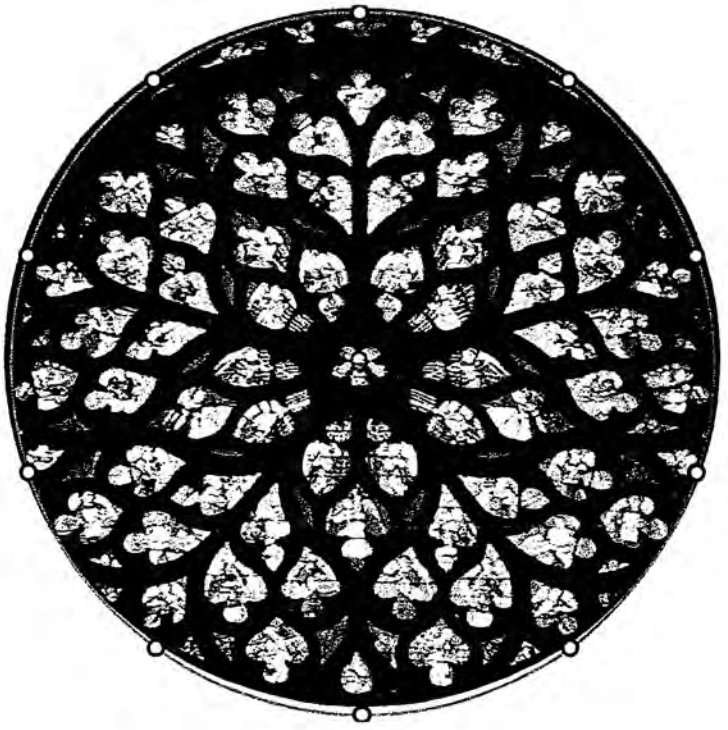
Painted 1415-16 by the famous Limbourg brothers (c.1370-1416), the geometry divides the scene into different parts of the story. This analysis may be found in *The Painter's Secret Geometry* by Charles Bouleau.

Musée Condé, Chantilly, France.

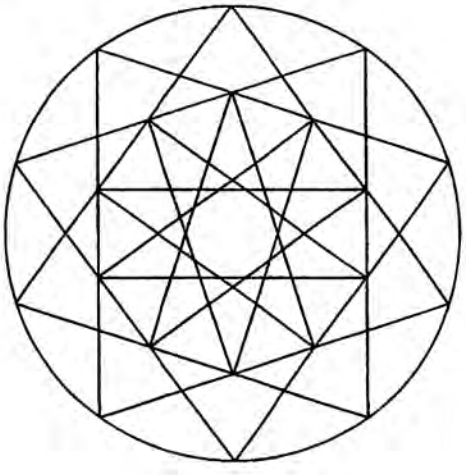


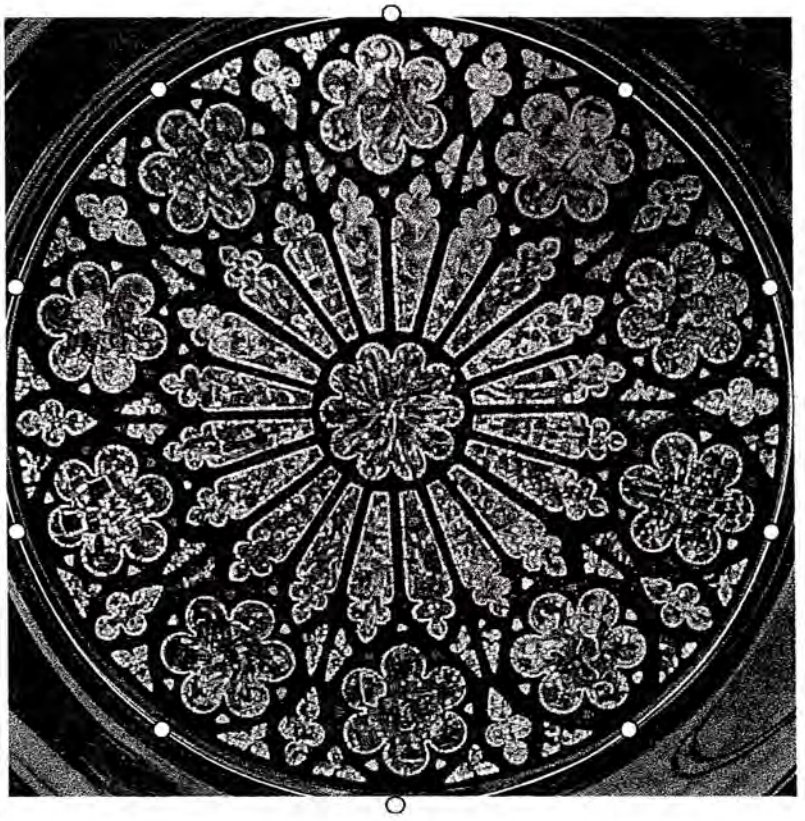
Rose Window Sens Cathedral
France
1140 - 1164





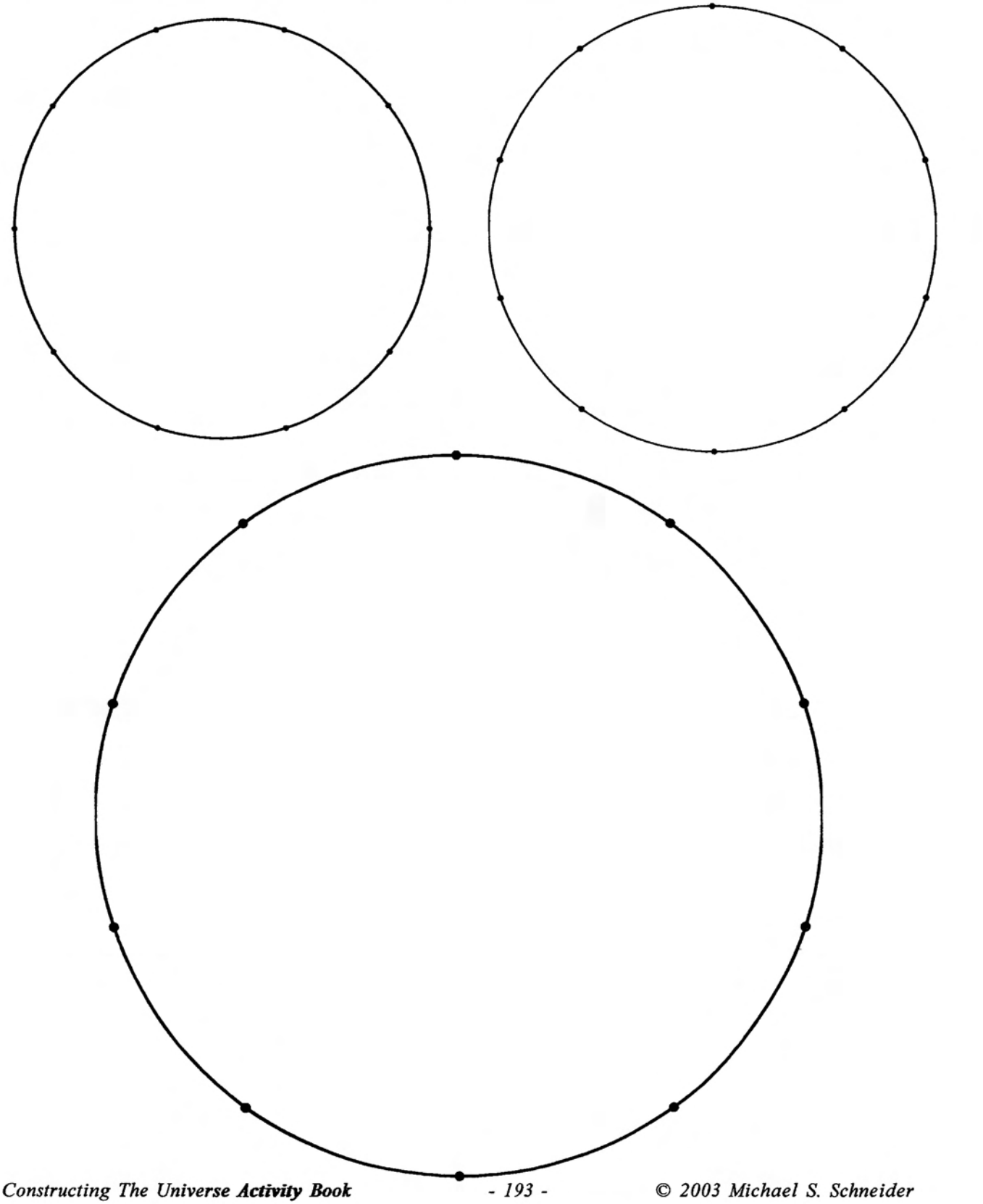
The Creation Rose Window
at the National Cathedral in
Washington, D.C.





Design Your Own Decagonal Art

Create constructions in the ten pointed circles below, or on blank sheets of paper. Use their lines as a frame guiding your designs for art, crafts and architecture.



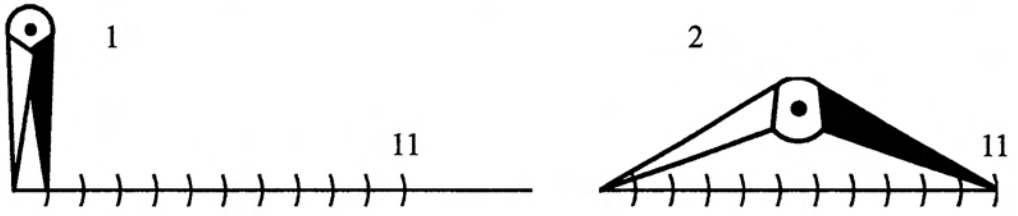
11 The Hendecagon

A Hendecagon (Greek *Hen* = 1 plus *Deka* = 10) has eleven corners and eleven sides. Like the Heptagon and Enneagon, a regular Hendecagon cannot be constructed with mathematical precision using a compass and straightedge. But here is a pretty good approximation by a technique which may be used to construct polygons of *any* number of sides.

Construct A Hendecagon

(1) We need a line segment eleven units long. (Note: the number of segments marked will result in a polygon of that many sides.) Use your straightedge to draw a line segment. Place your compass point on one end and open it to a small size (depending on the size of your line). "Walk" the compass along the line marking eleven segments.

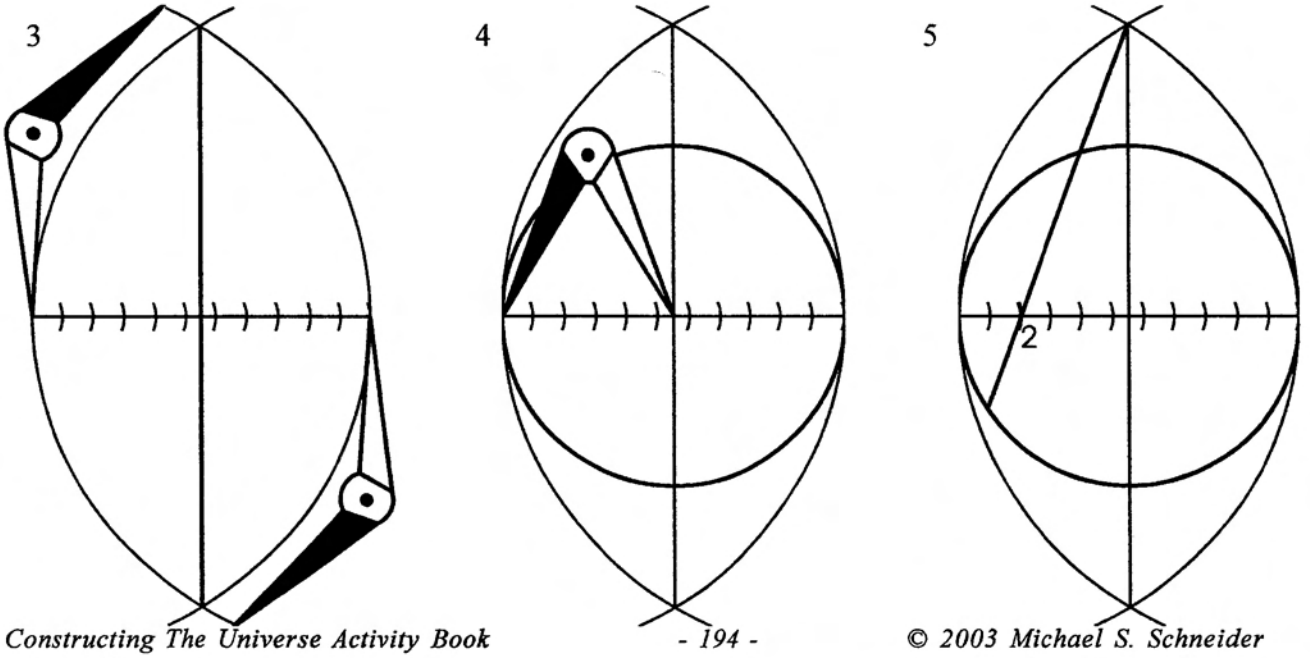
(2) Find the middle of that line by dividing it in half by an Almond construction (page 9). Open your compass between the two ends of the eleven-section line segment. (Ignore the remainder beyond the eleventh mark.)



(3) Swing the compass from each end to make the Almond. Connect its crossings to find the center of the line.

(4) Place the compass point at the center, open it to the end of the line, and turn a Circle within the Almond.

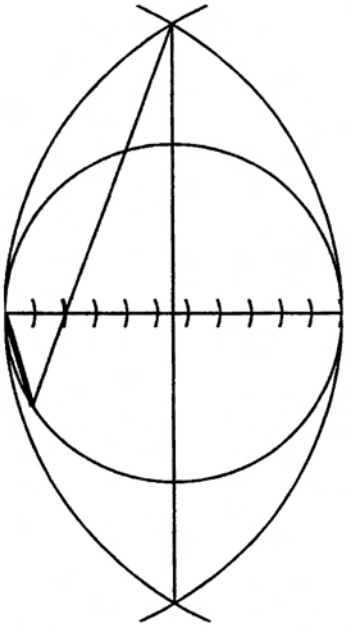
(5) Draw a straight line from the top crossing of the Almond to point number 2 along the diameter. But keep drawing the line *through* the point, extending it until it crosses the Circle.



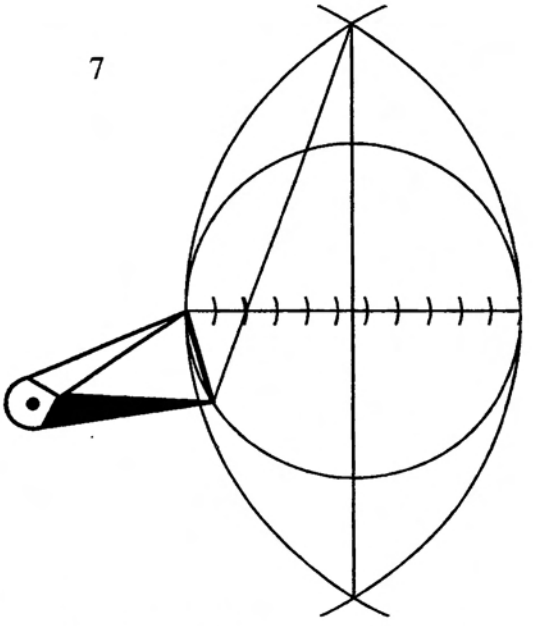
(6) Draw a straight line from this point on the Circle to the end of the diameter. This is one side of the Hendecagon.

(7) Open your compass between the ends of this line segment.

6



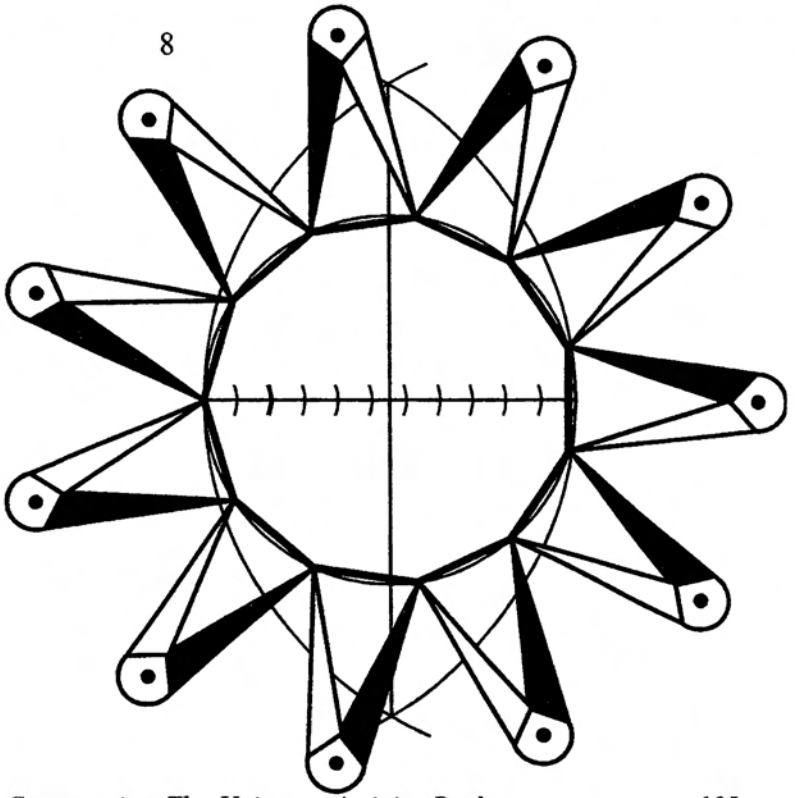
7



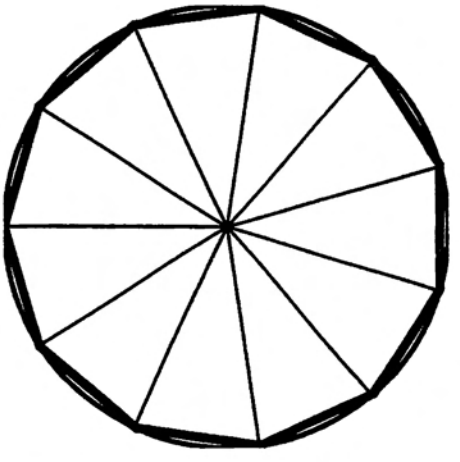
(8) "Walk" the compass around the Circle. If the eleventh step doesn't match the beginning point, "walk" the compass around again but in the opposite direction. The eleven best points will be halfway between each pair of markings.

(9) A non-regular Hendecagon with its eleven radii. (Note: if you want the Hendecagon to point upwards, begin the construction by marking the line segment in step 1 downward.)

8

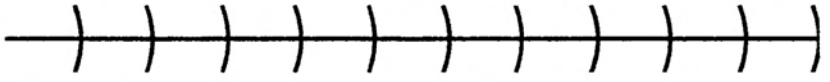


9



Construct A Hendecagon

A line segment is already divided into eleven segments for you.



Draw The Hendecagram Stars

Start with a Hendecagon. Each of its four Hendecagram stars may be drawn without lifting the pencil from the paper since no number less than 11 (except 1) divides evenly into 11.

(1) Connect every other (second) point.

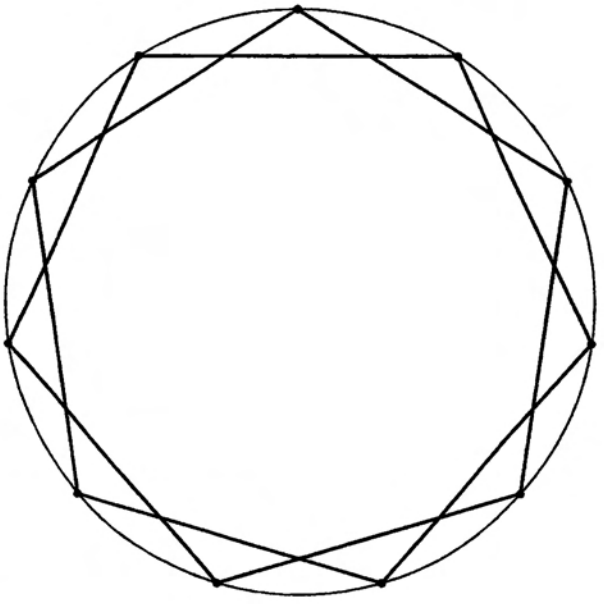
(2) Connect every third point.

(3) Connect every fourth point.

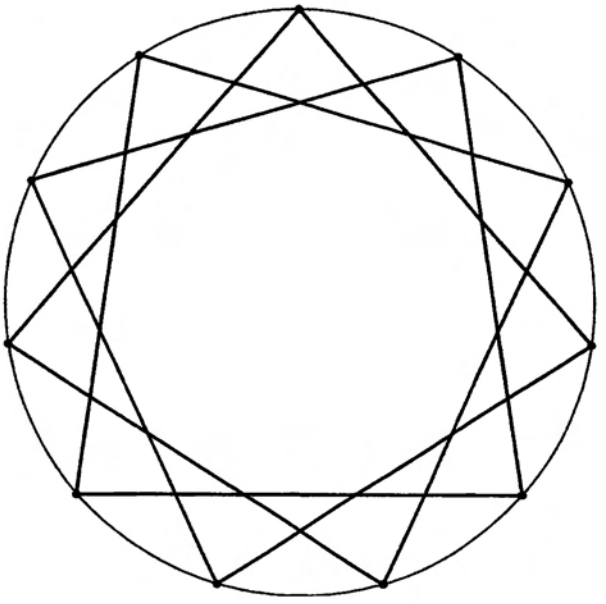
(4) Connect every fifth point.

Add to the Hendecagram stars below with your compass and straightedge.

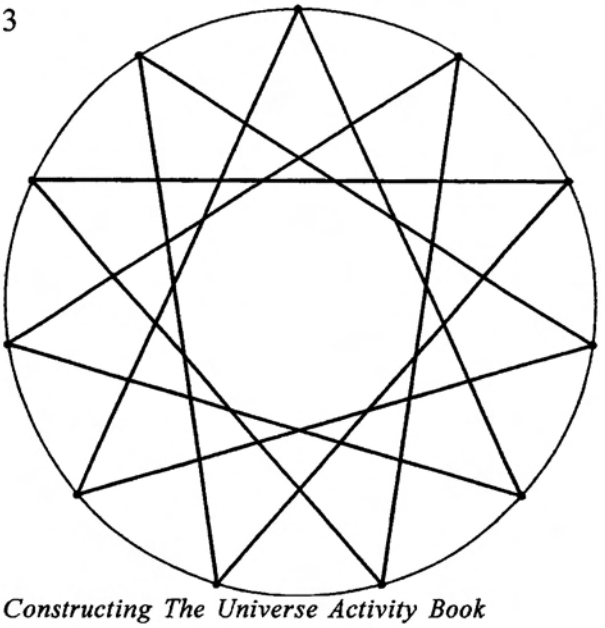
1



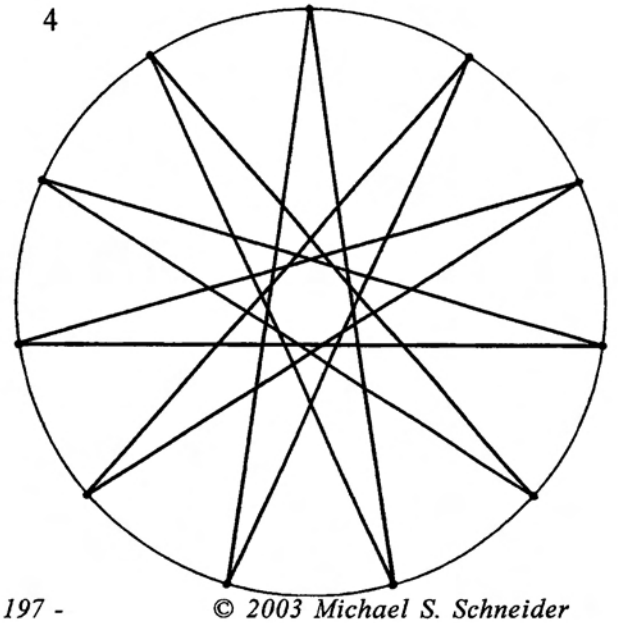
2



3



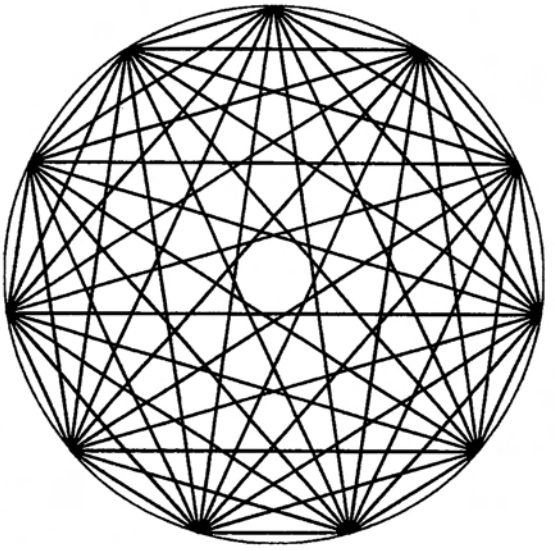
4



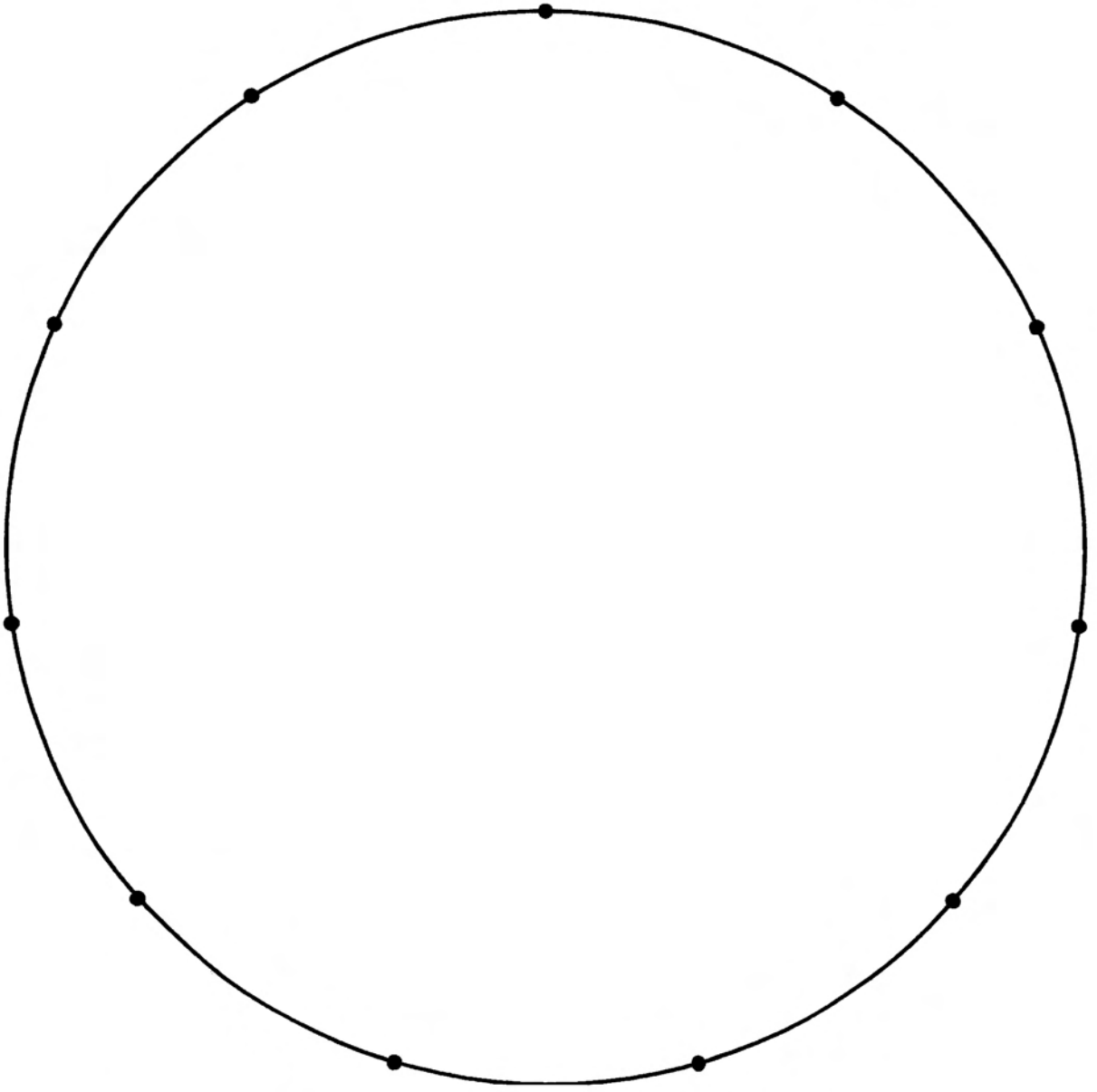
Draw The Hendecagram Stars

Draw one or more of the four Hendecagram stars along or within each other.

And, of course, they can be combined in different ways alongside or within each other.



The Hendecagon and all four of its Hendecagram stars.



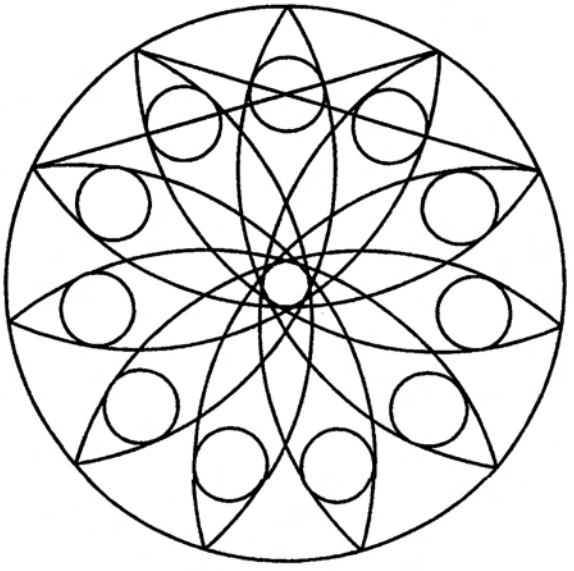
Replicate These Hendecagonal Designs

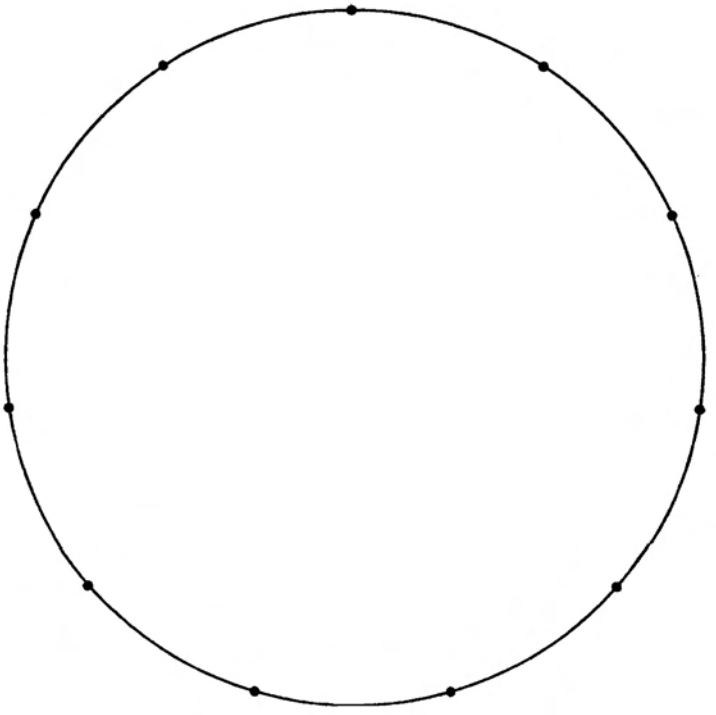
What must be done first?

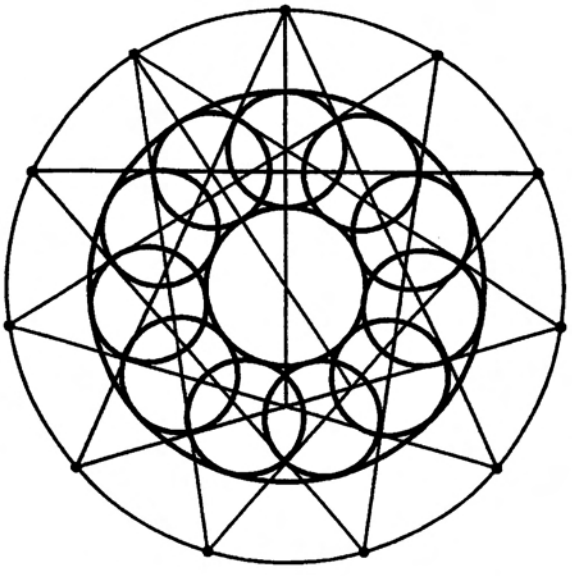
At each step, ask:

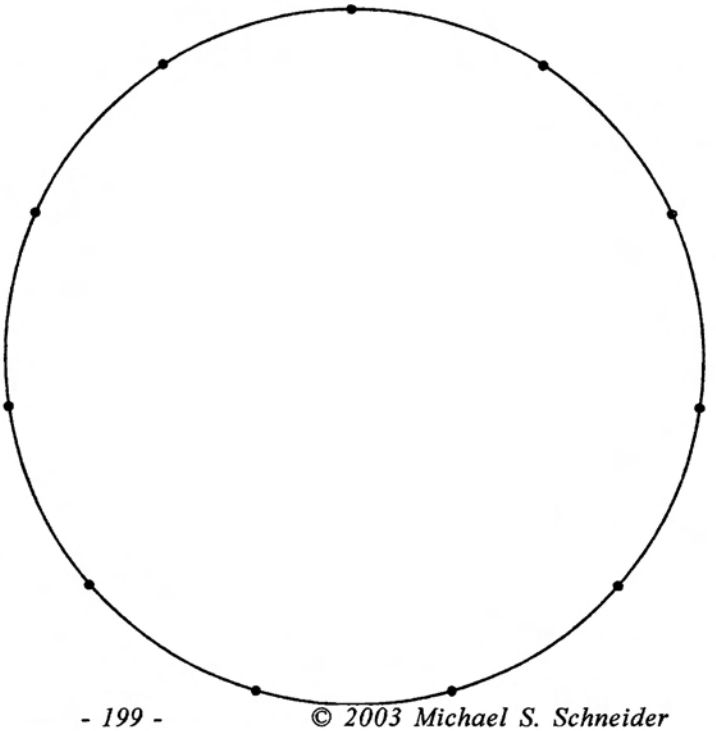
Where is the compass point? Watch for crossing points to be the centers of circles.

Where is the compass open to?



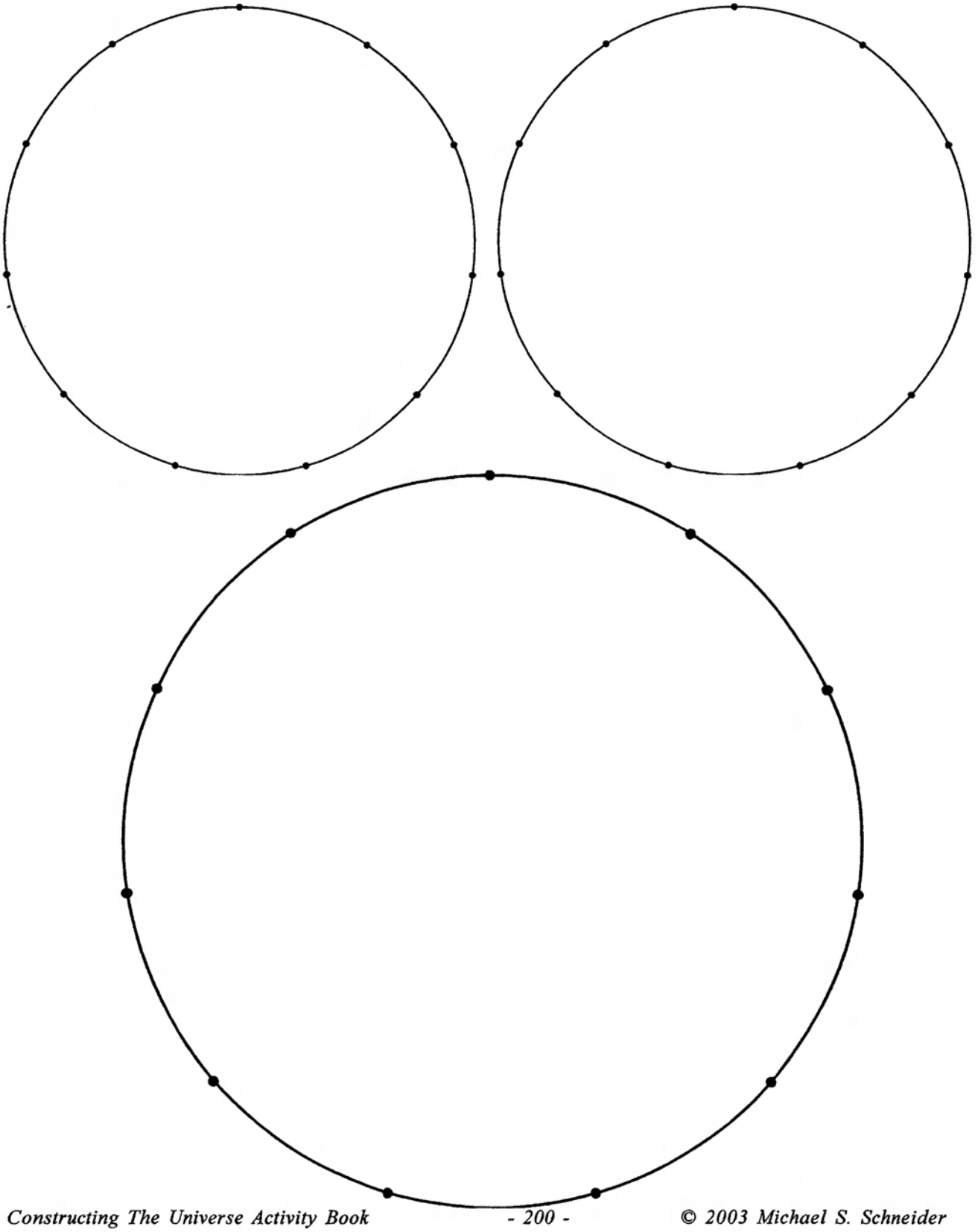






Create Your Own Hendecagonal Designs

Use colored pencils.

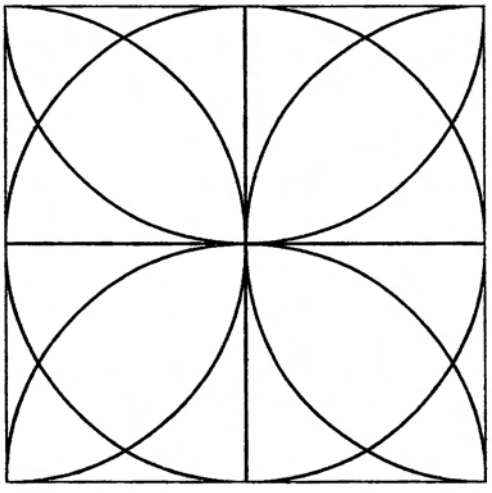


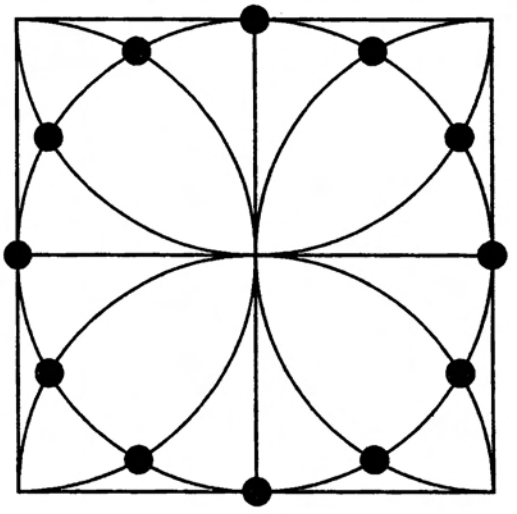
12 The Dodecagon

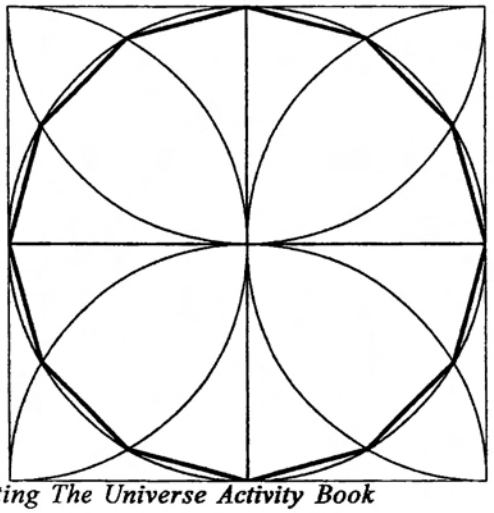
A regular Dodecagon (Greek: *Do* = 2 plus *Deka* = 10) has twelve corners with equal angles, twelve equal sides and six equal diameters. This twelve sided figure provides our ultimate geometric construction due to the friendly way that twelve relates to other numbers. It has many divisors for a small number, and relates nicely to the numbers less than and greater than it. One, two, three, four, six and twelve itself divide evenly into twelve. Eight, nine and ten join with it as simple fractions. Even seven has its special relationship with twelve (as we will see on page 210). A Dodecagon holds two regular Hexagons, three Squares and four equilateral Triangles. The next number with any more divisors is twenty four, a multiple of twelve.

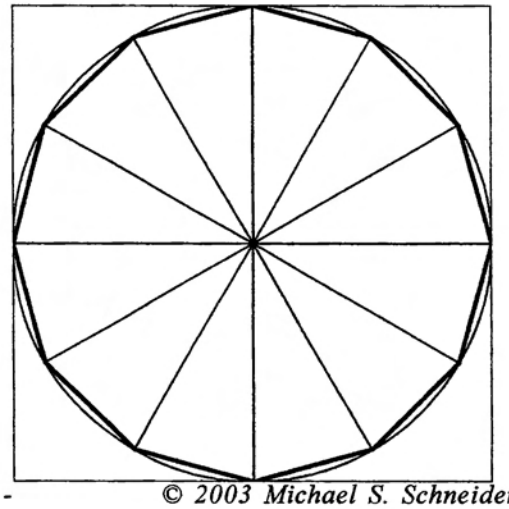
There are many ways to construct a Dodecagon.

Some ways begin by constructing a Square around a Circle (page 47). You found the corners of the Square by turning arcs with the compass point at the midpoint of each side, with the scribe open to the center of the Circle. These arcs made four leaves or petals. The curious Truth is this: they happen to cross the original Circle at eight points. These eight, plus the midpoints of the four sides of the Square, show us twelve equally spaced points around the Circle. These twelve points are the corners of a regular Dodecagon.









Construct A Dodecagon By The Four-Leaf Method

Start with this Square. The midpoints of its sides are marked.

Place your compass point on one midpoint, and open the scribe to the corner.

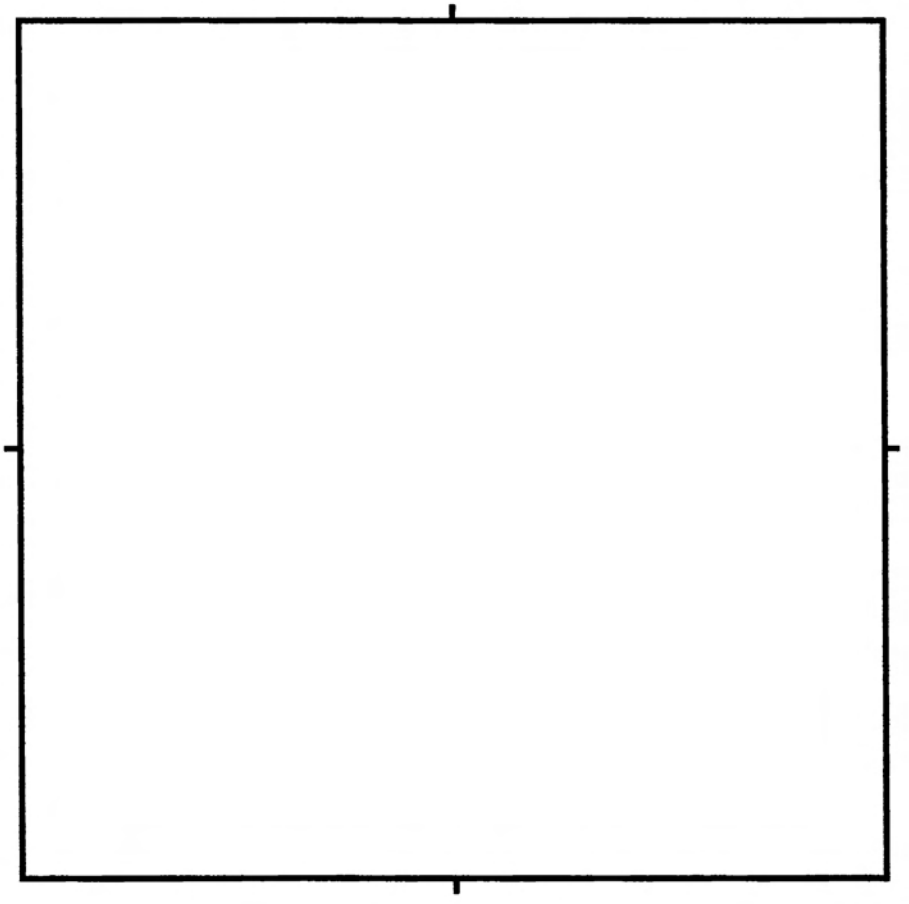
Turn an arc.

Repeat this process with the compass point at the midpoint of each side.

Now find the center of the Square by connecting midpoints or drawing diagonals.

Place the compass point at the center and open the scribe to the midpoint of the side of the Square. Inscribe a Circle in the Square.

Mark the twelve equally spaced points around the Circle where it crosses the arcs.



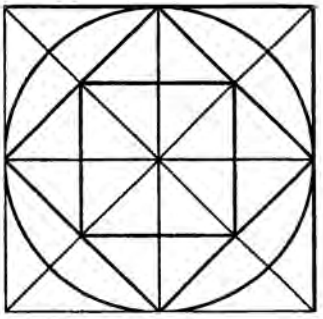
Another Dodecagon From The Square

(1) Again, construct a Square around a Circle and subdivide it as shown.

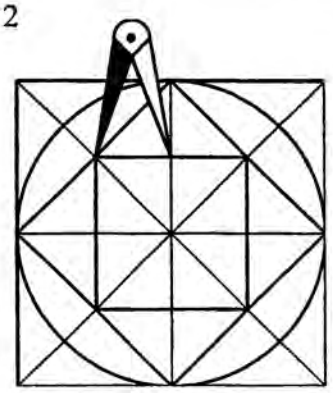
(2) Open your compass between the points as shown along the side of the small Square.

(3) Keep this opening and place the compass point on the midpoint of a side. Mark a point on each side of it along the Square. Do this at the midpoint of each side. Connect the points to make a four by four grid. (Another way to make the 4x4 Square was seen on page 61.)

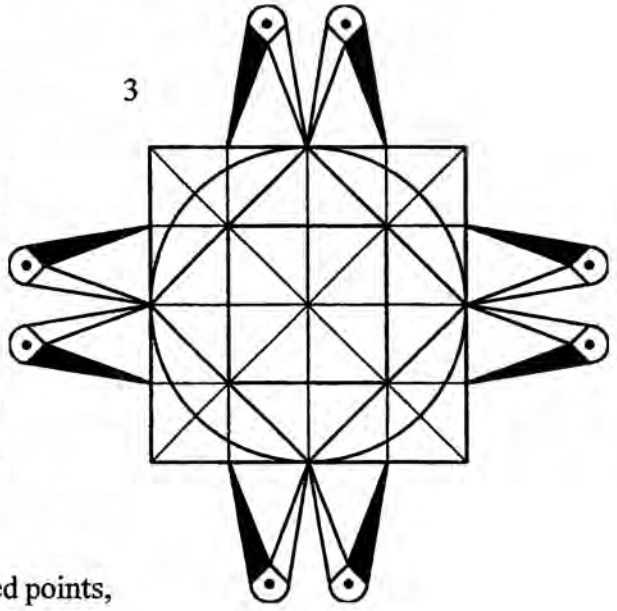
1



2

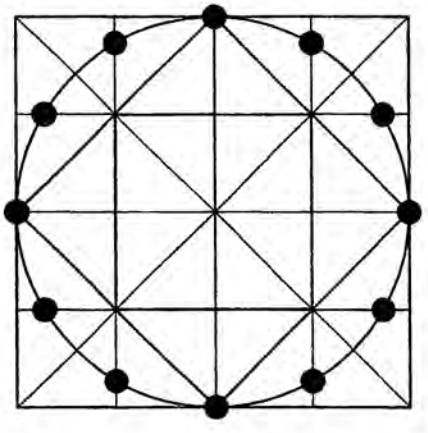


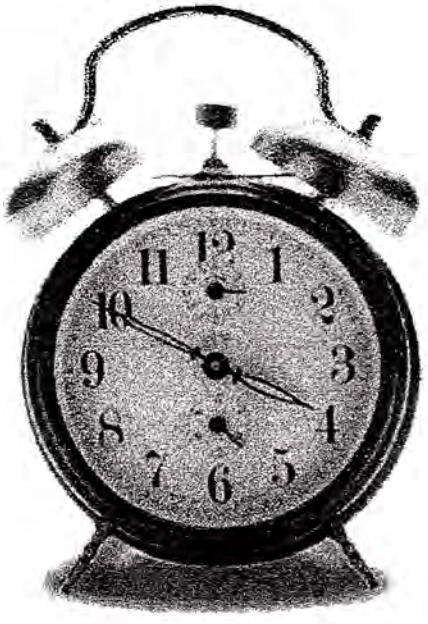
3



(4) The grid crosses the Circle at twelve equally spaced points, like the numbers on a clock.

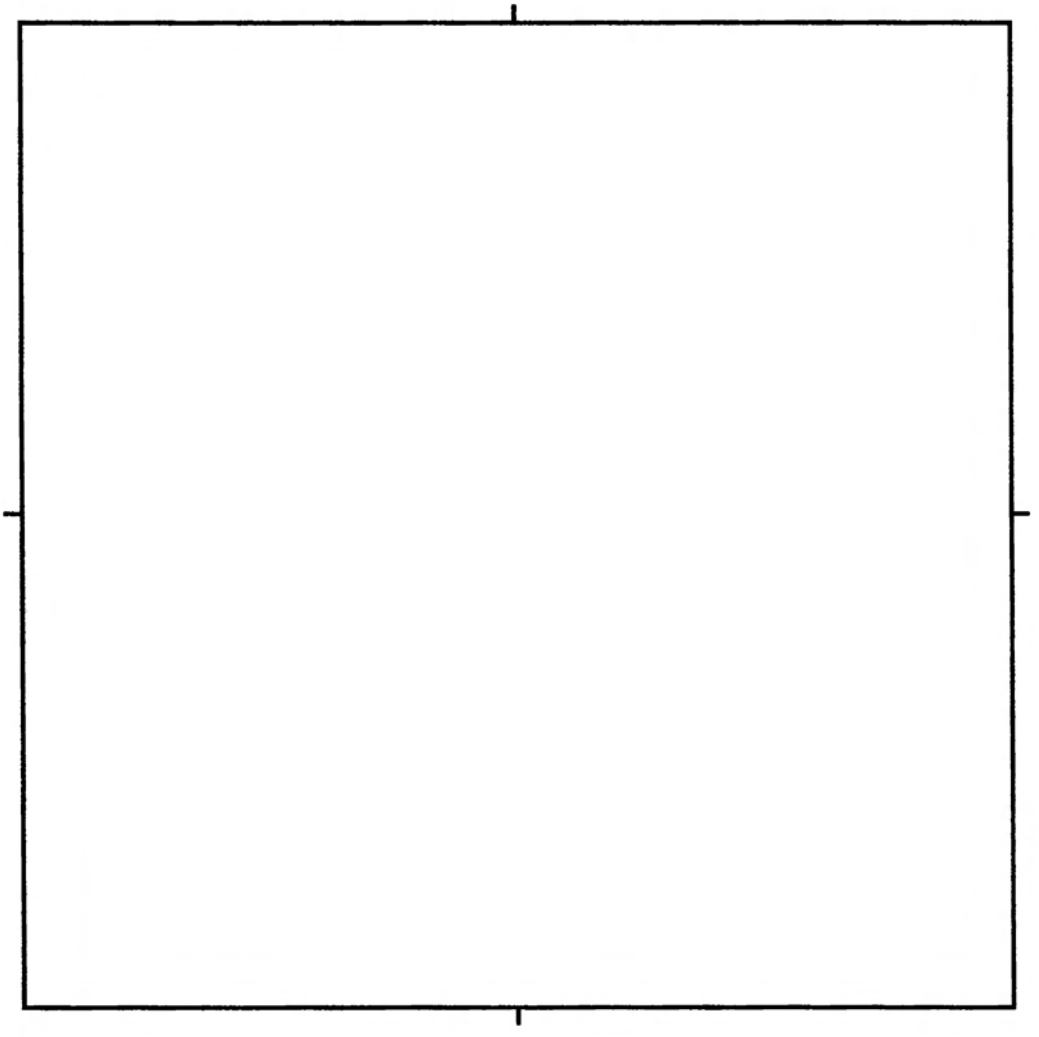
4





Construct A Dodecagon By This Method

First subdivide this Square (page 52) using the marks at the midpoints of its sides. Then follow the directions on the previous page.



Construct The Dodecagon From A Hexagon

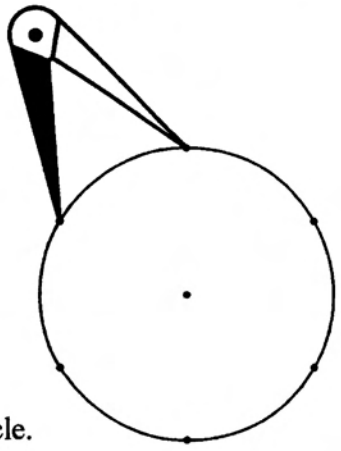
Since 12 = 2x6, then another way to construct a Dodecagon is from a Hexagon. Bisect the arcs around its sides in half using the Almond (page 9), as we did this to transform a Pentagon into a Decagon (page 178).

(1) First, start with a Circle which has six equally spaced points around it. Open the compass between two adjacent points.

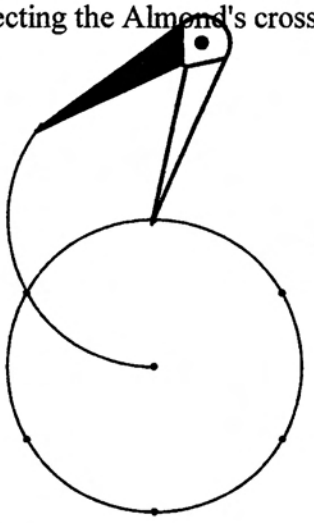
(2) Swing an arc. If it passes through the Circle's center, you're accurate.

(3) Turn the compass around between the points and swing another arc to complete an Almond. Draw a straight line segment connecting the Almond's crossings. Note the point where it crosses the Circle.

1



2



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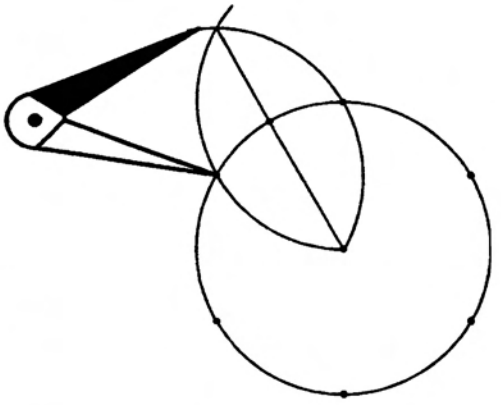
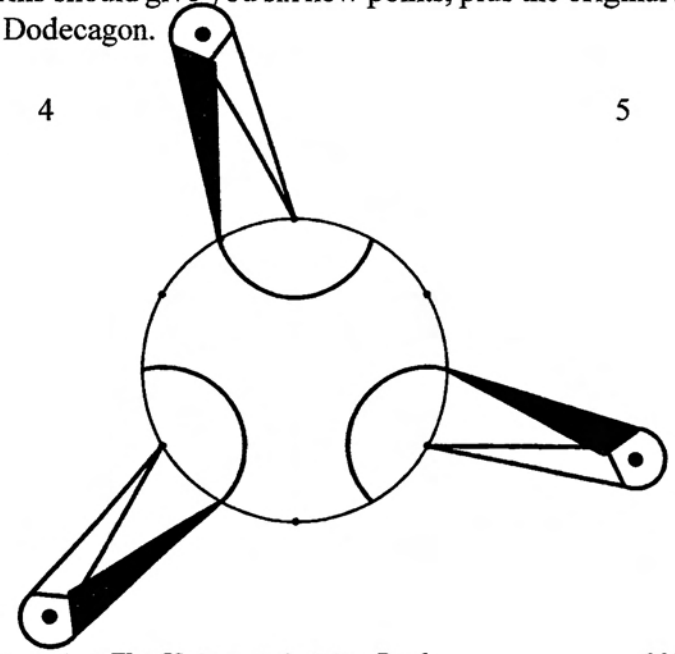


(4) Put the compass point on an original point and open the scribe to this new point. Swing an arc so it crosses the Circle in two places. Do this from every other point around the Circle (three times).

(5) This should give you six new points, plus the original six, making the twelve equally spaced points of a Dodecagon.

4



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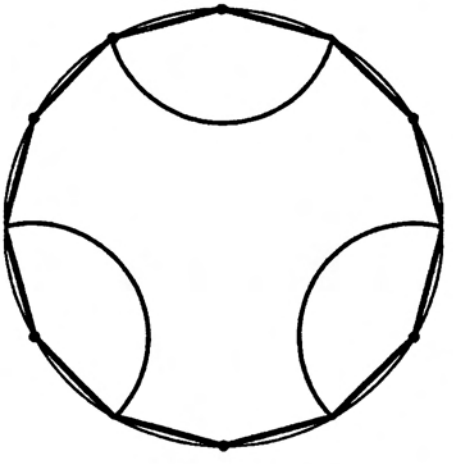
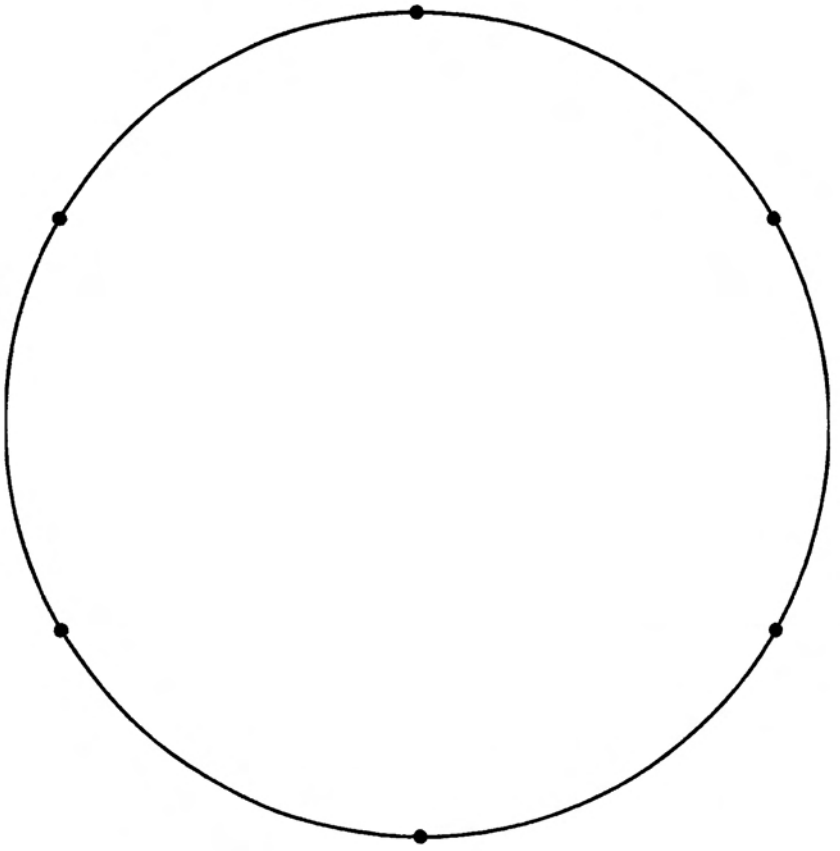


Construct The Dodecagon From A Six Pointed Circle

Construct a Dodecagon by this method on a blank sheet of paper or in this Circle.

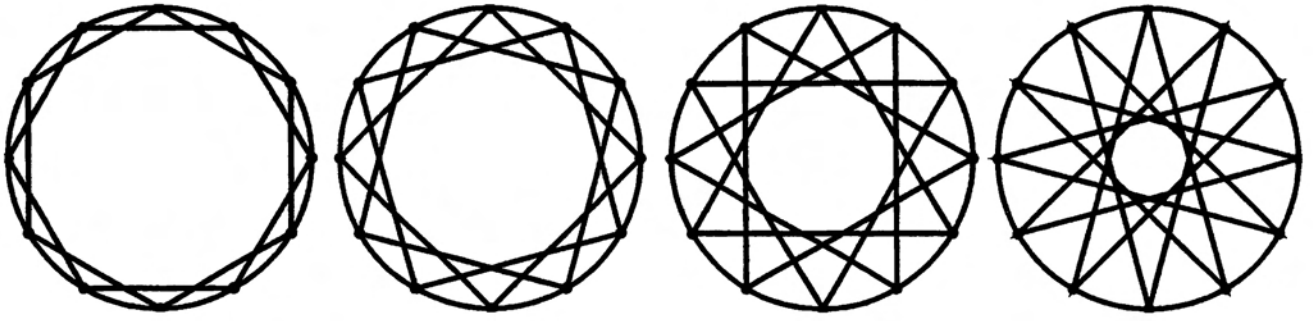


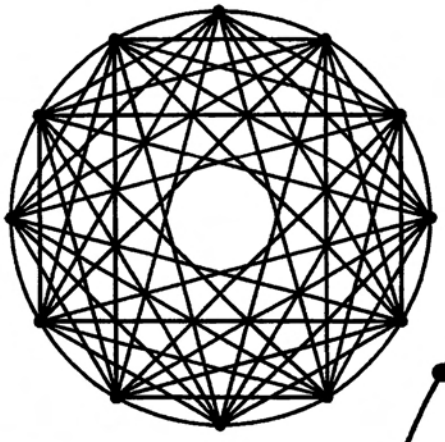
Construct The Dodecagram Stars

Start with twelve equally spaced points around a Circle. For three of its four Dodecagram stars you must lift your pencil.

(1) Connect every other (second) point to make two separate regular Hexagons.
(2) Connect every third point to make three Squares.
(3) Connect every fourth point to make four equilateral Triangles.
(4) Connect every fifth point without lifting your pencil.

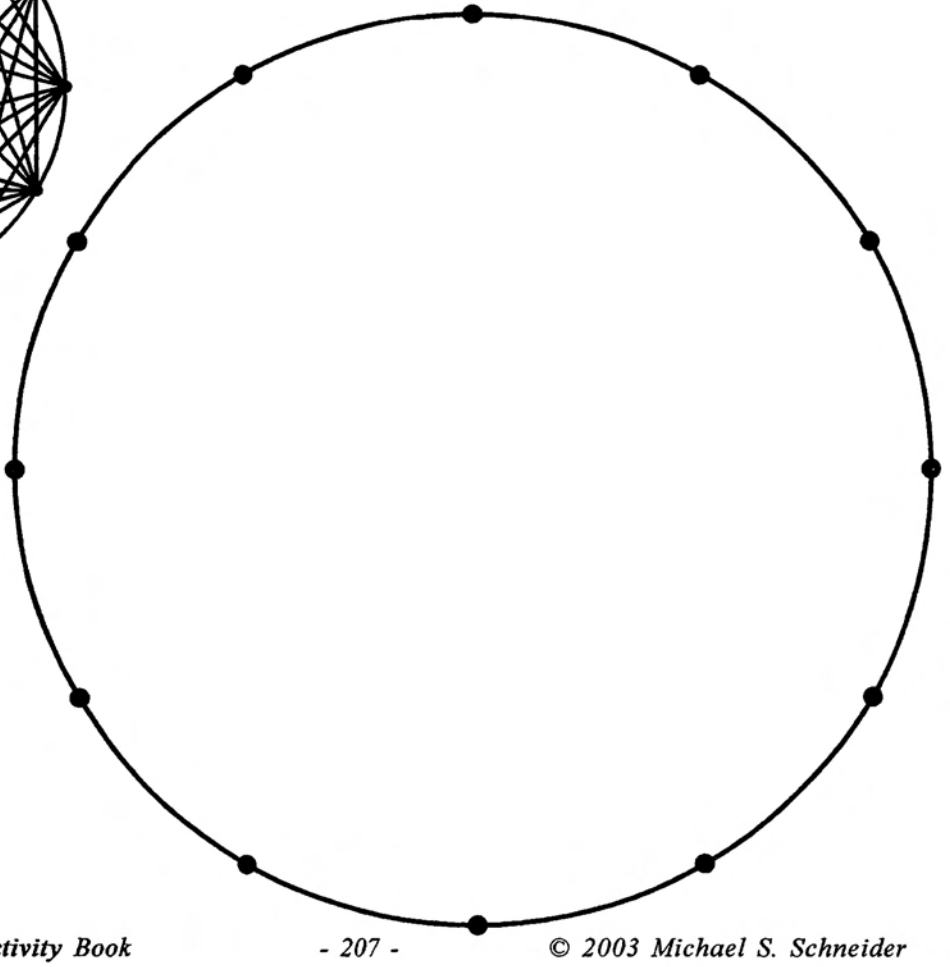
1 2 3 4





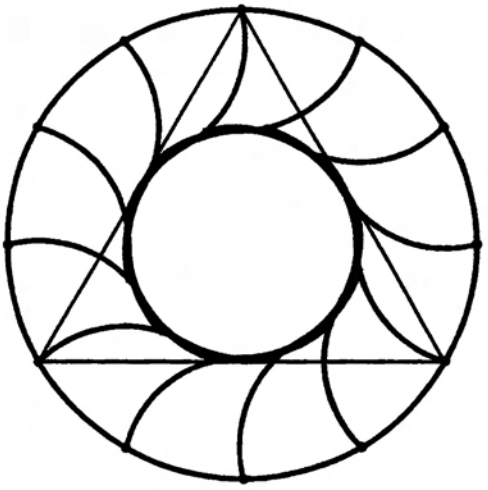
All the Dodecagram stars together

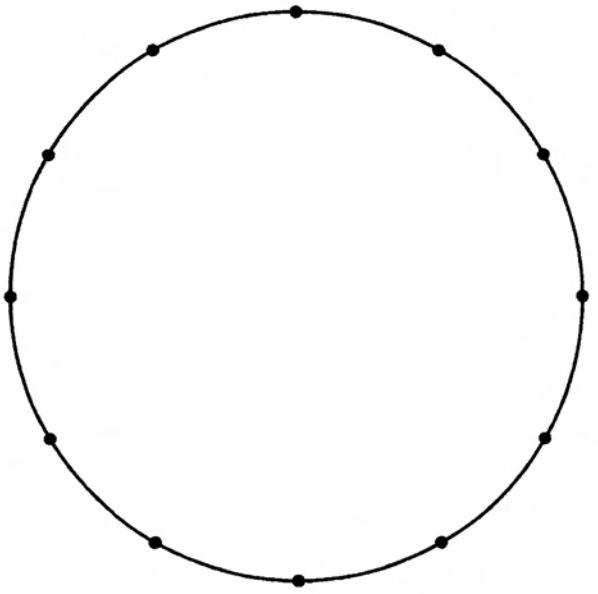
Draw Dodecagram stars here

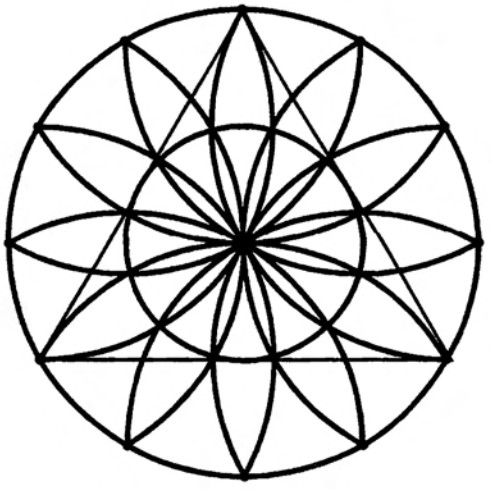


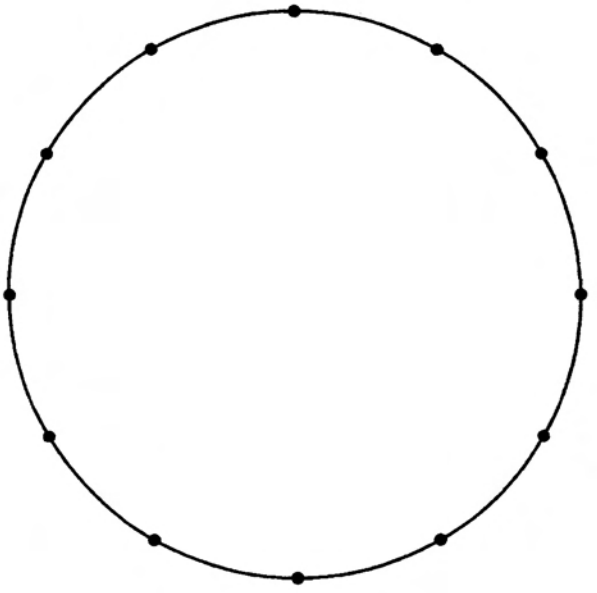
Replicate These Dodecagonal Designs

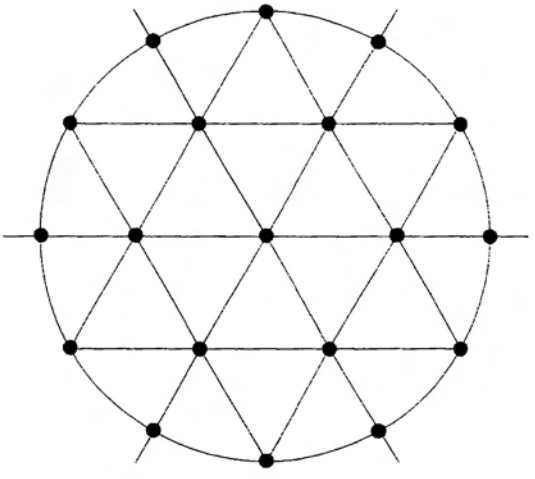
Then develop them further.

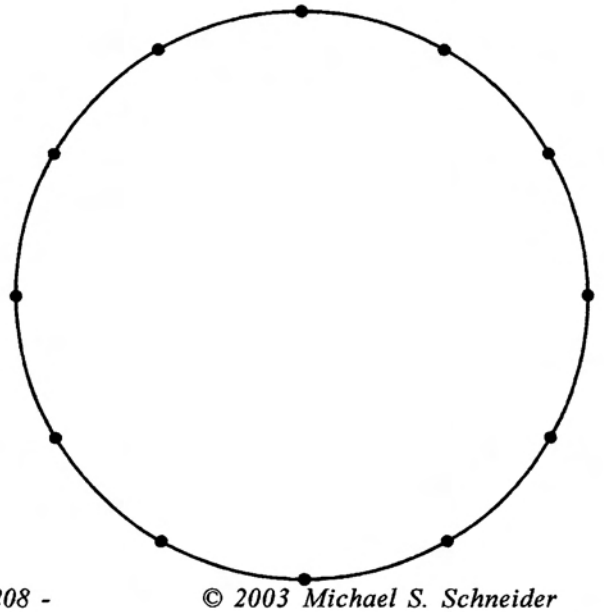






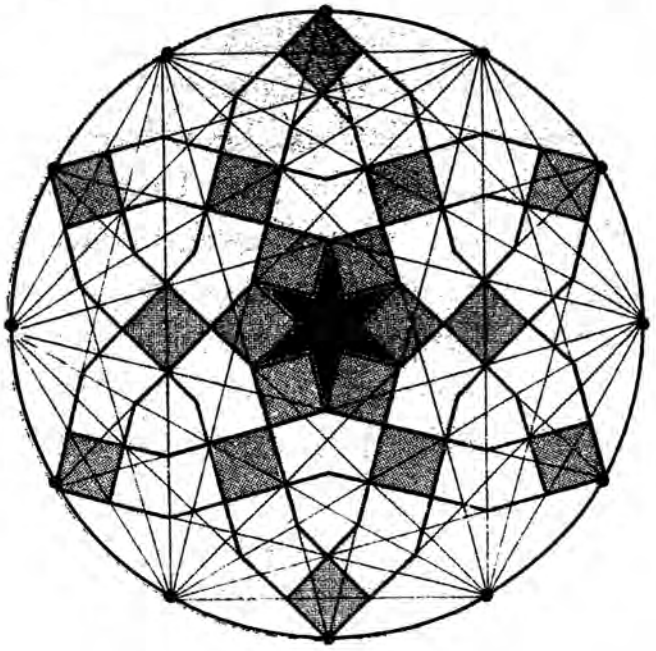


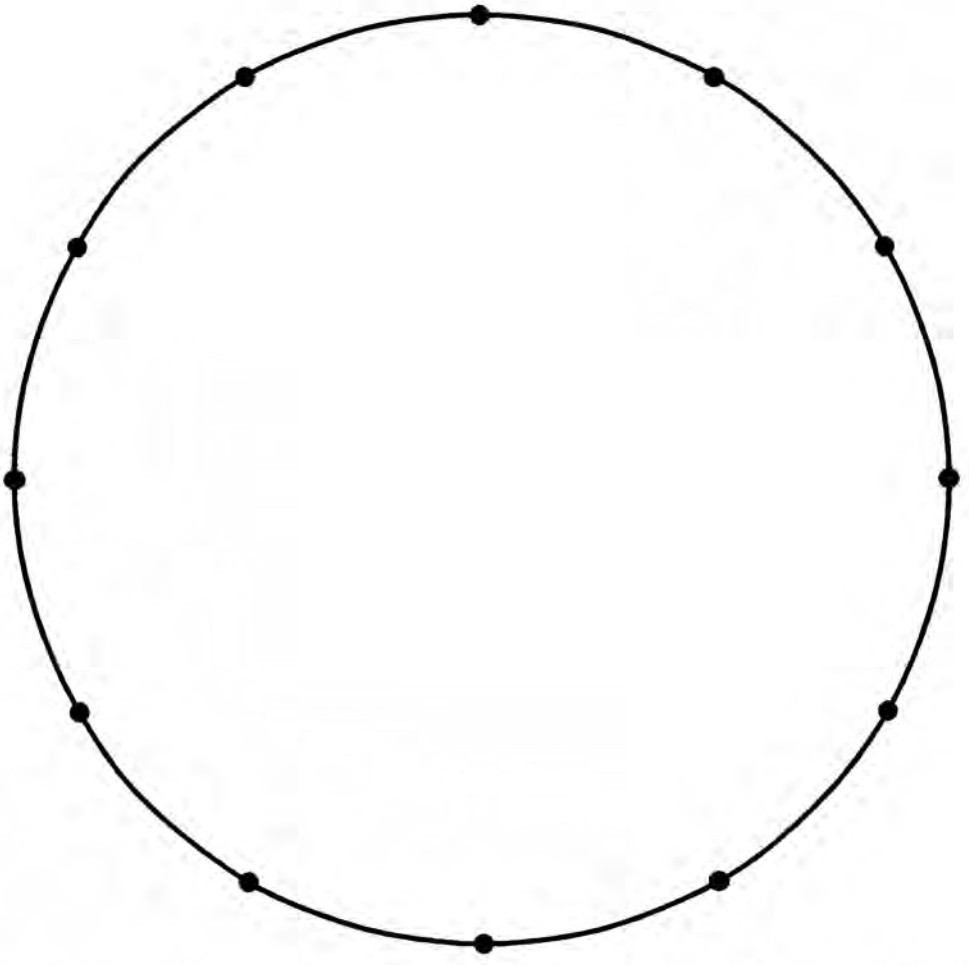




The Lover's Knot

This famous tiling pattern emerges from a Dodecagon and Dodecagram stars. The dark lines show the knot. Use colored pencils to shade your construction.



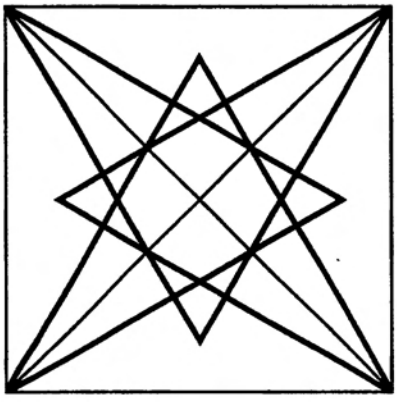


Twelve And Seven

The construction which gave us the Heptagon from the Circle, Triangle and Square (page 138) also holds within it a Dodecagon and Dodecagram star. This shows to us one of many interesting relationships between seven and twelve. We should have suspected as much, knowing that 3 + 4 = 7 and 3 x 4 = 12.

(1) Construct the Square with four equilateral Triangles inside it. Draw the diagonals of the Square.
(2) Place the compass point at the center of the Square and open the scribe to the point of one Triangle. Turn a Circle. It should touch the tips of all four Triangles.
(3) Draw straight lines between the crossing points made by the Triangles and this Circle as shown.
(4) Look carefully: you have identified twelve points around this Circle, and smaller rings of twelves around the center. It's a construction well worth investigating further on your own.

1



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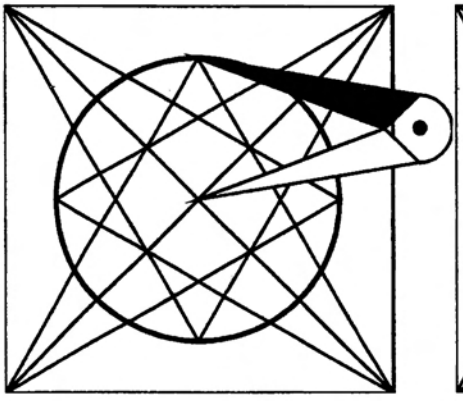

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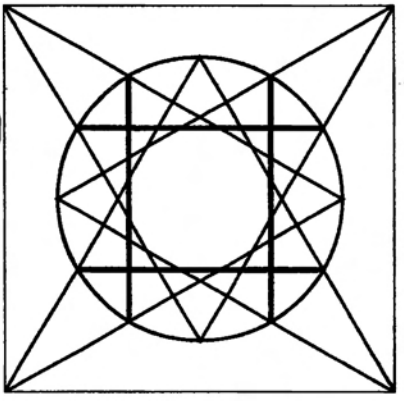

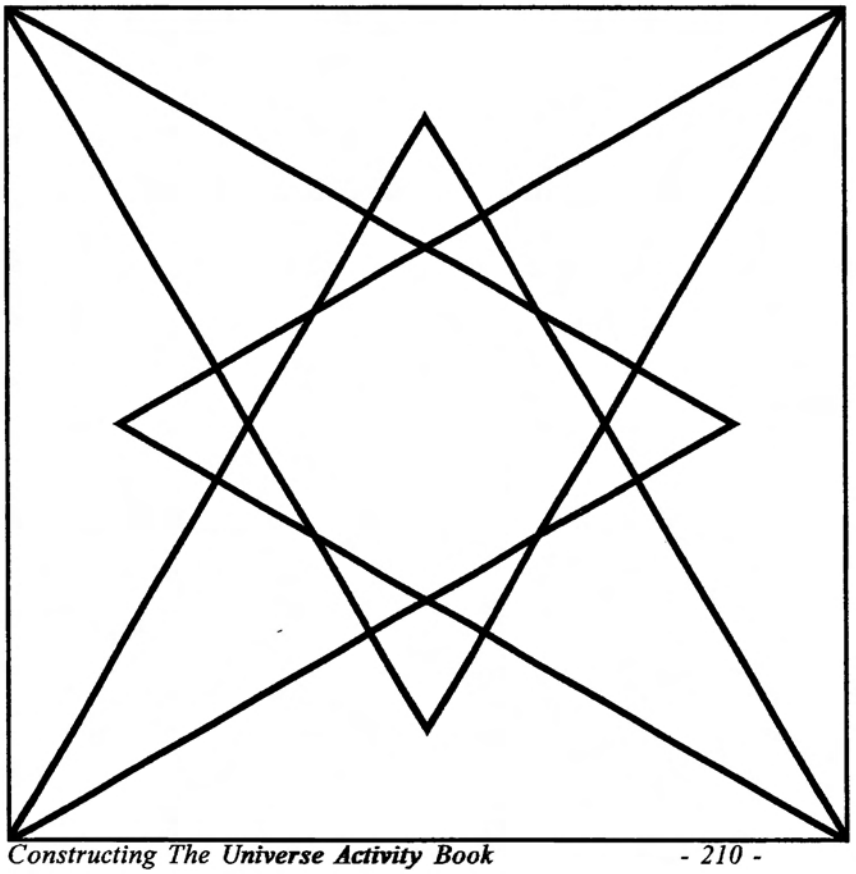

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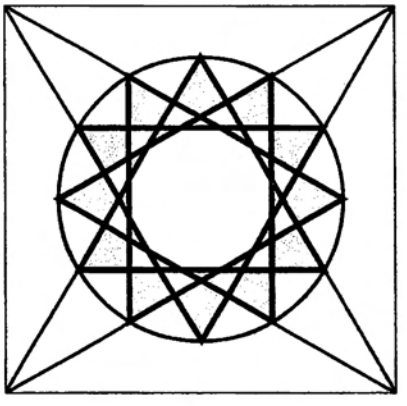
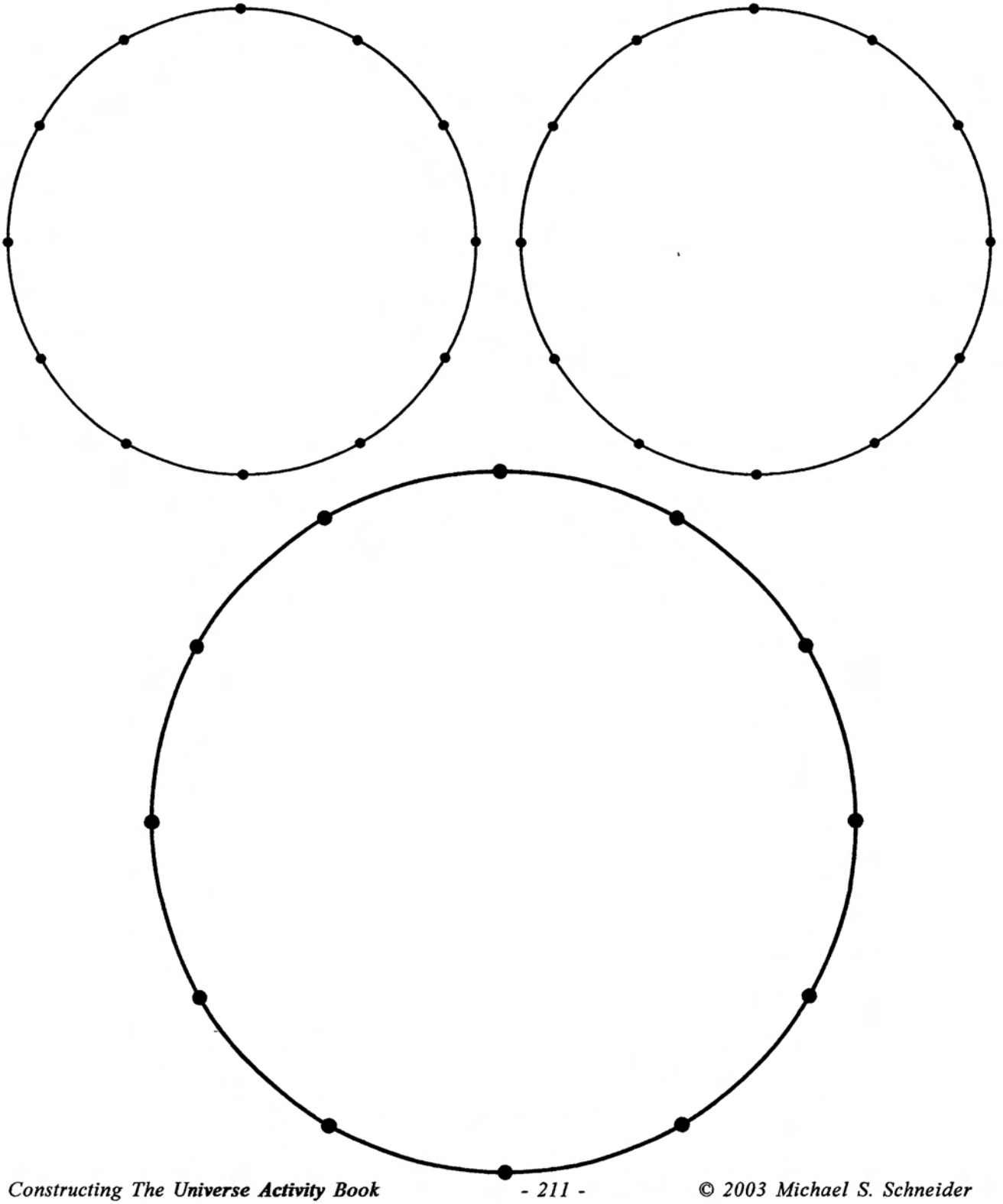


Do this construction in this Square with four Triangles.

Use your compass and straightedge to explore and develop it further.

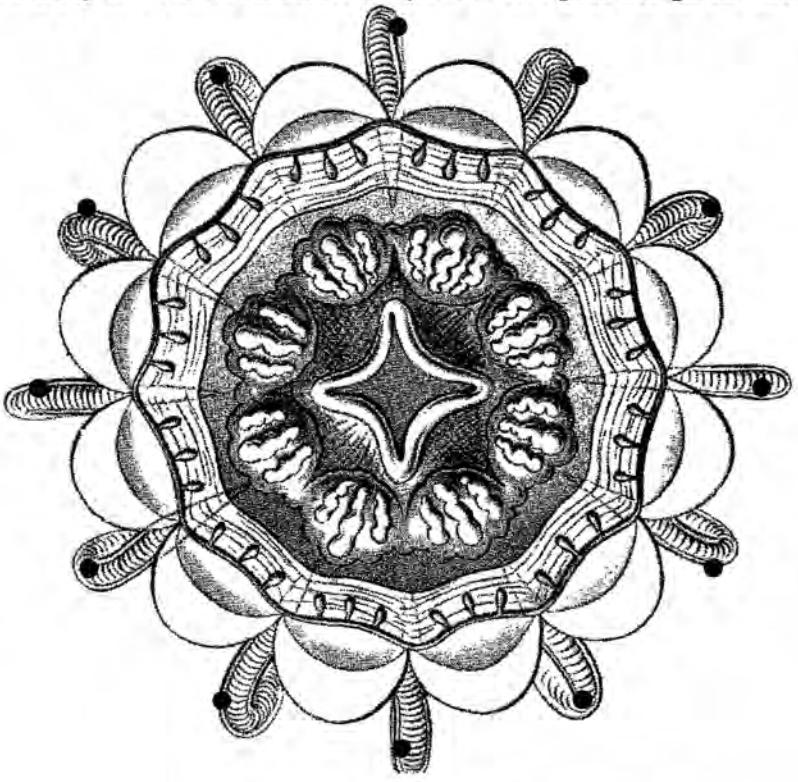
Create Your Own Dodecagonal Designs

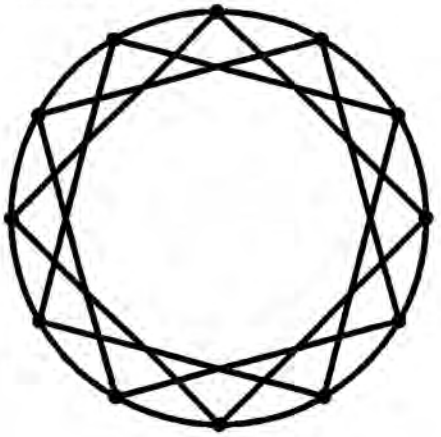
Now that you've seen a variety of ways that the Dodecagon can be constructed, start with twelve points around a Circle and follow your own explorations.

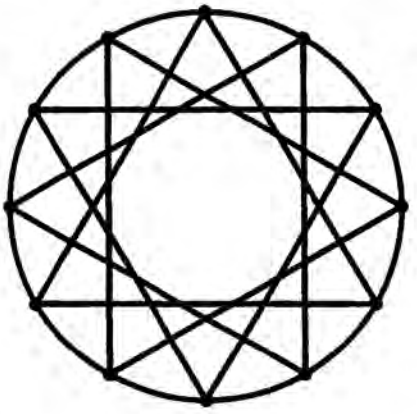


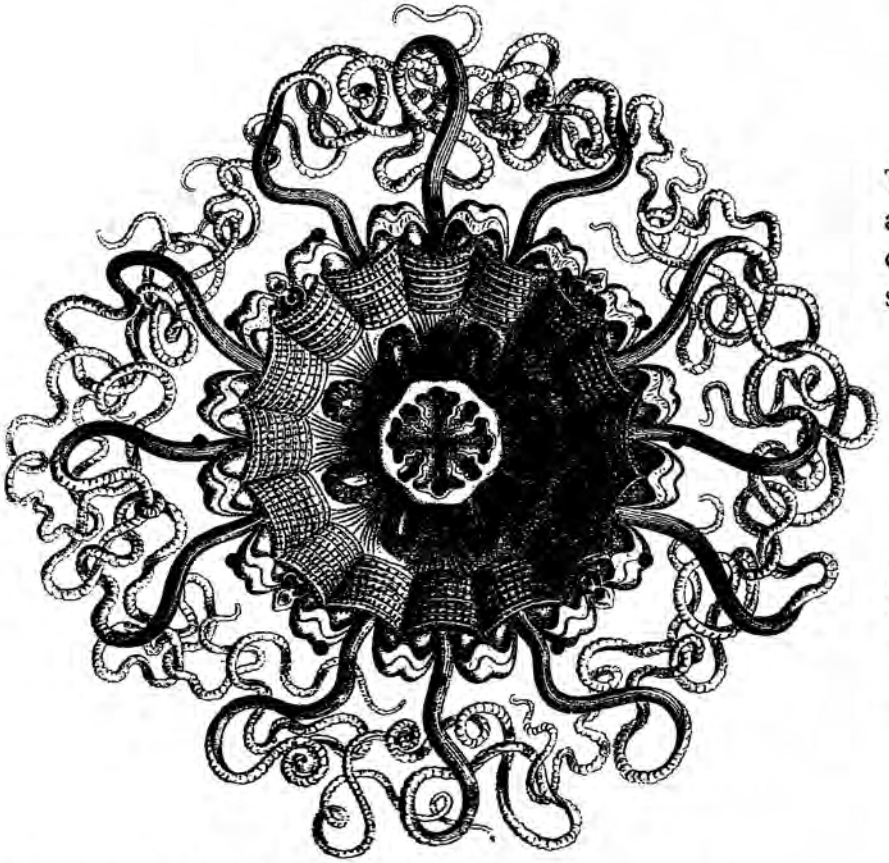
Dodecagons In Nature

This **Hydra** can be understood by its twelve part design as three Squares or four Triangles.

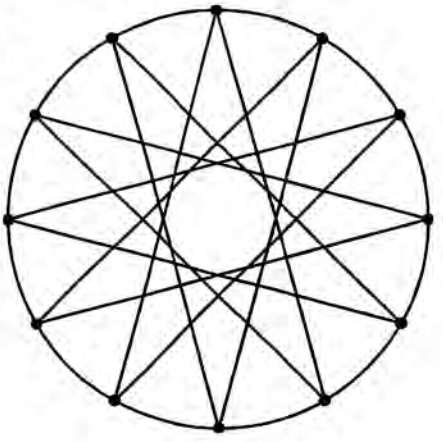




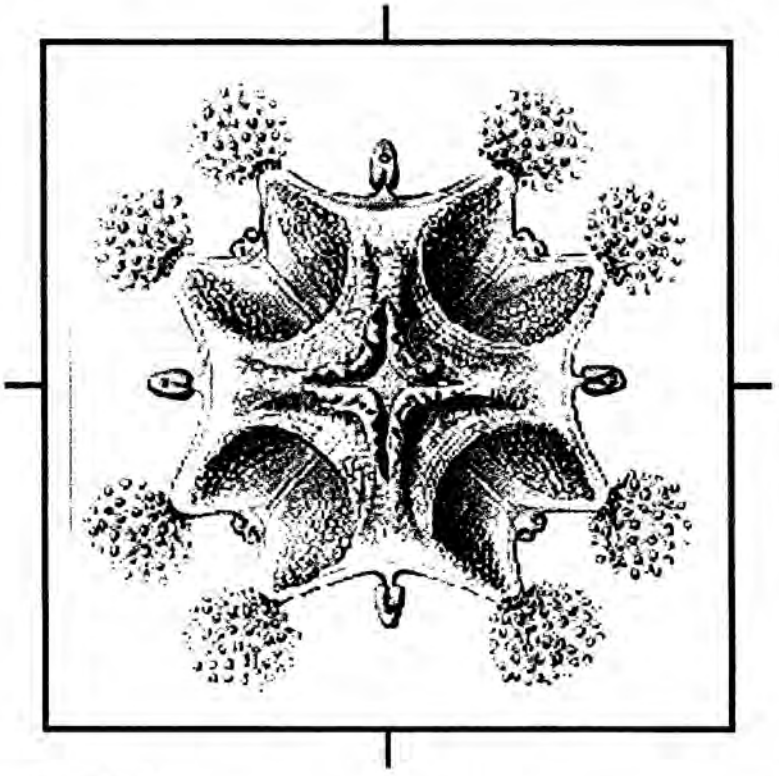


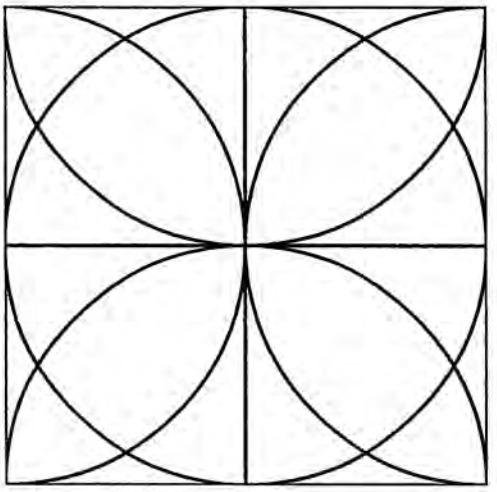


The structure of this **Jellyfish** can also be understood by applying one or more of the Dodecagram stars to the twelve points given.

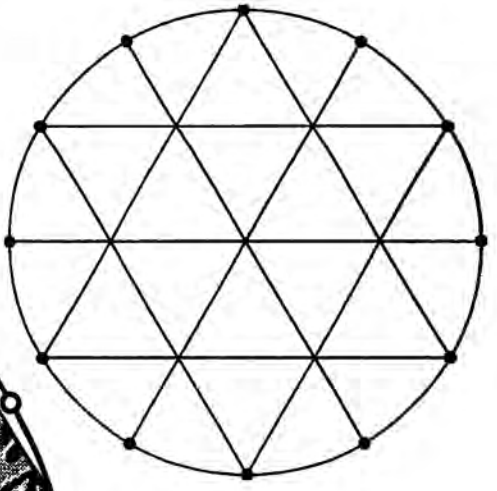


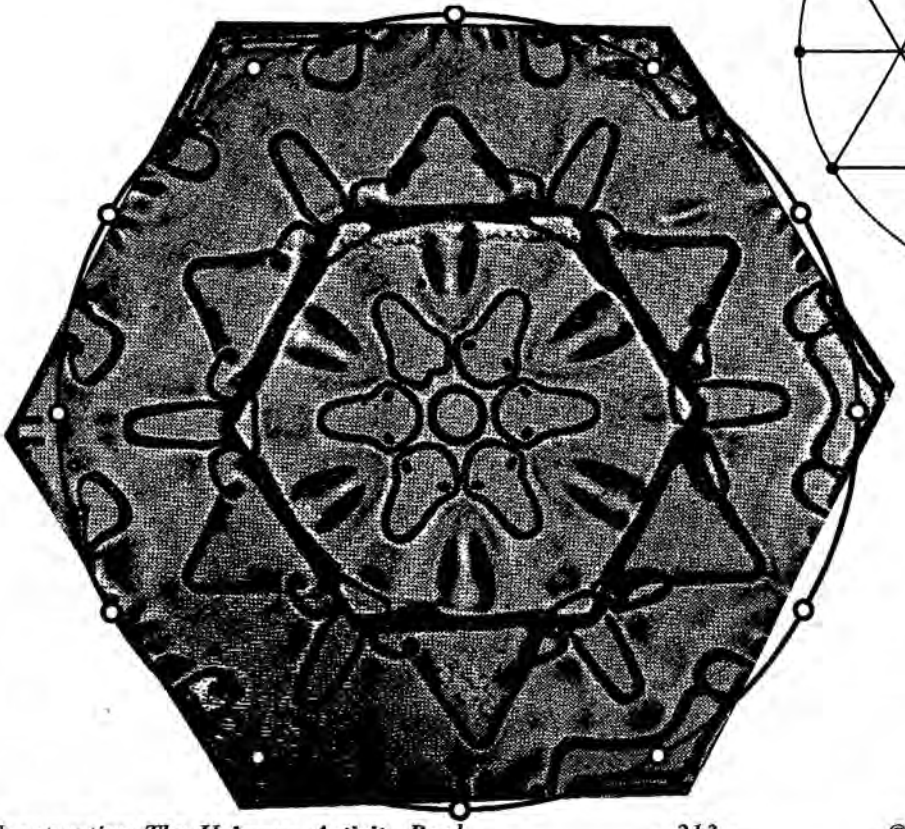
Hydra





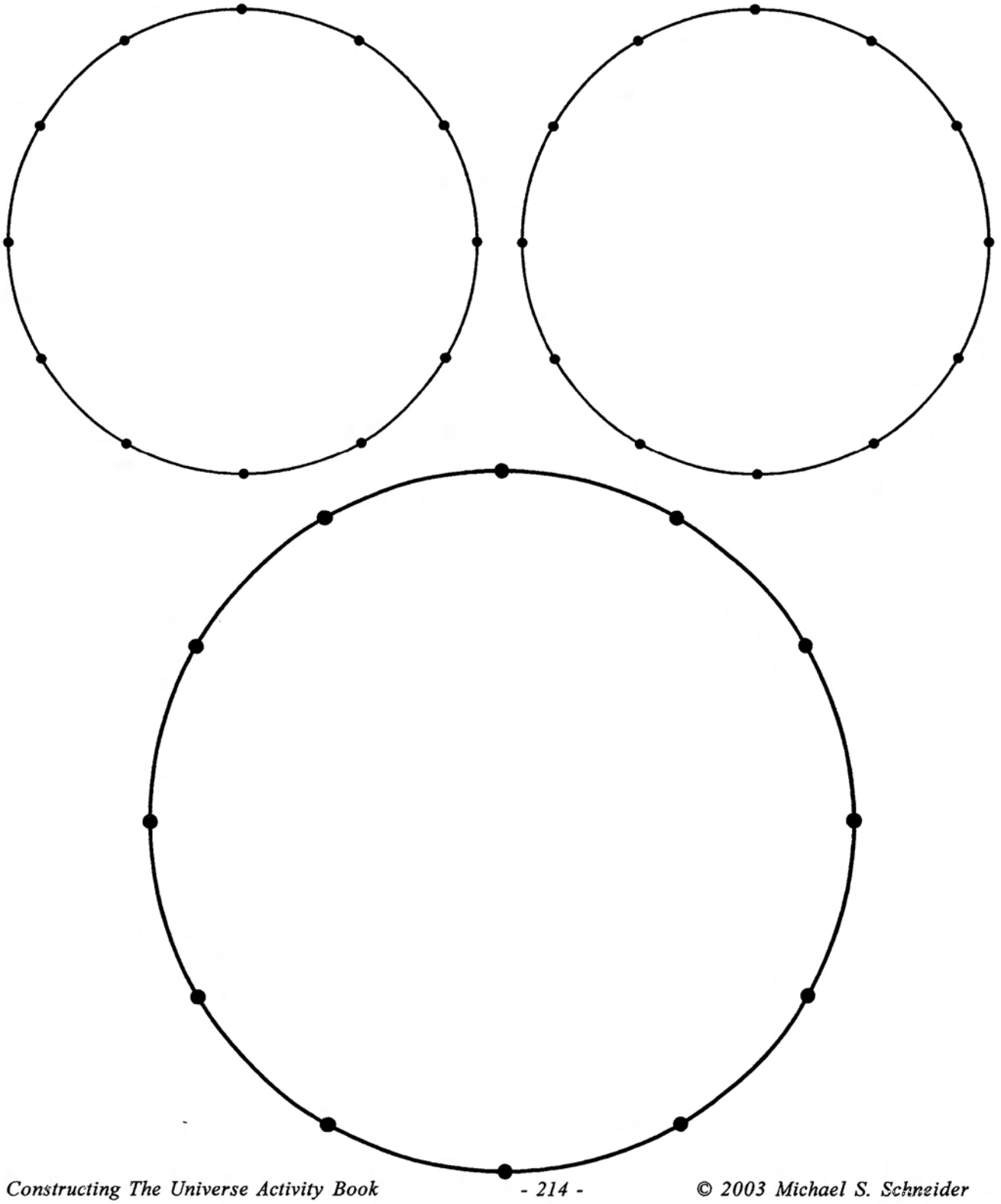
Snowflake





Design Natural Dodecagons

Now that you've seen some ways that the Dodecagon appears in nature, start with twelve points around a Circle, do a construction, and use colored pencils to design something guided by it.

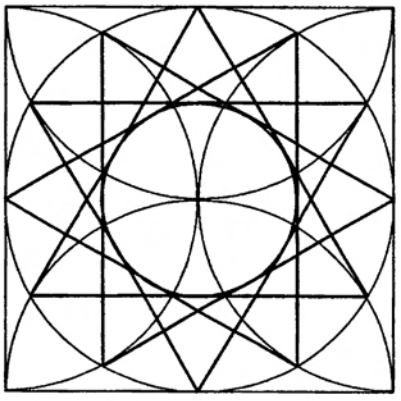


Dodecagonal Design In Arts, Crafts And Architecture

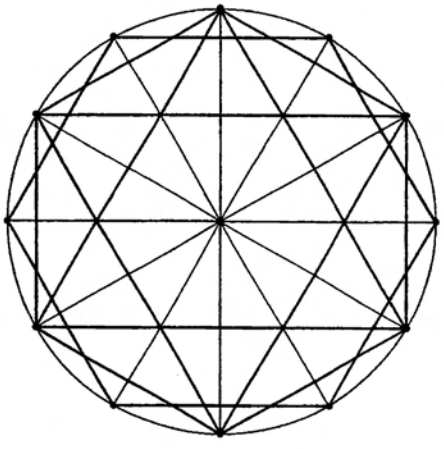
Bowl Of Twelve Greek Gods

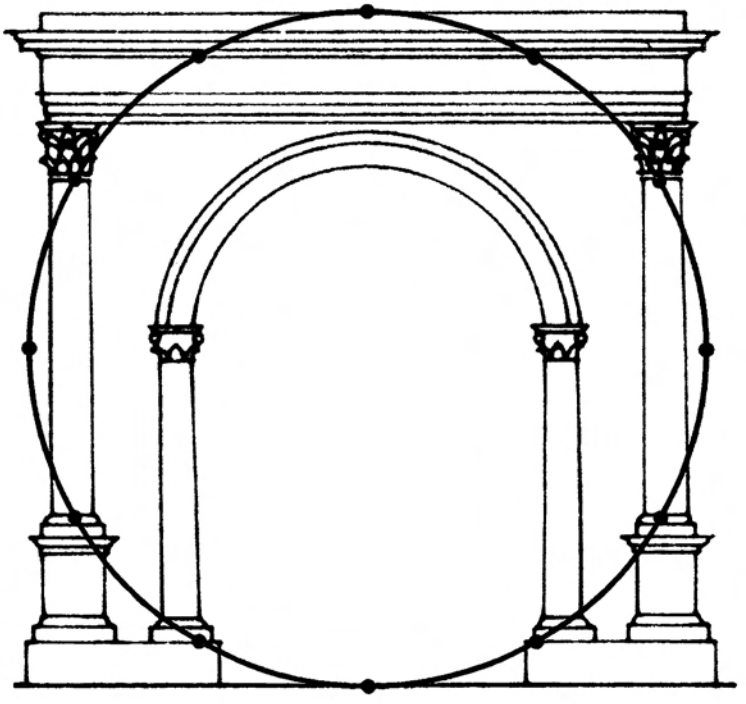
The mythological gods of Olympus are shown around the rim of this bowl.

To find out why the rim around the hole is the size it is, find twelve points around the bowl's Circle and connect the points as a Dodecagram star of four triangles.



Roman Arch





Traditional Navajo Sand Painting

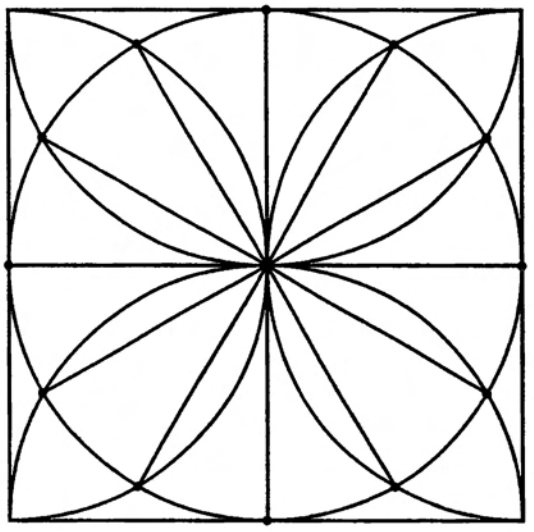
"Whirling Logs"
(See also page 56).

Turn arcs from the midpoints of the sides of the Square to its corners to find twelve points around the outermost Circle.

Draw the six diameters.

Do you see how the painting is guided by the lines?

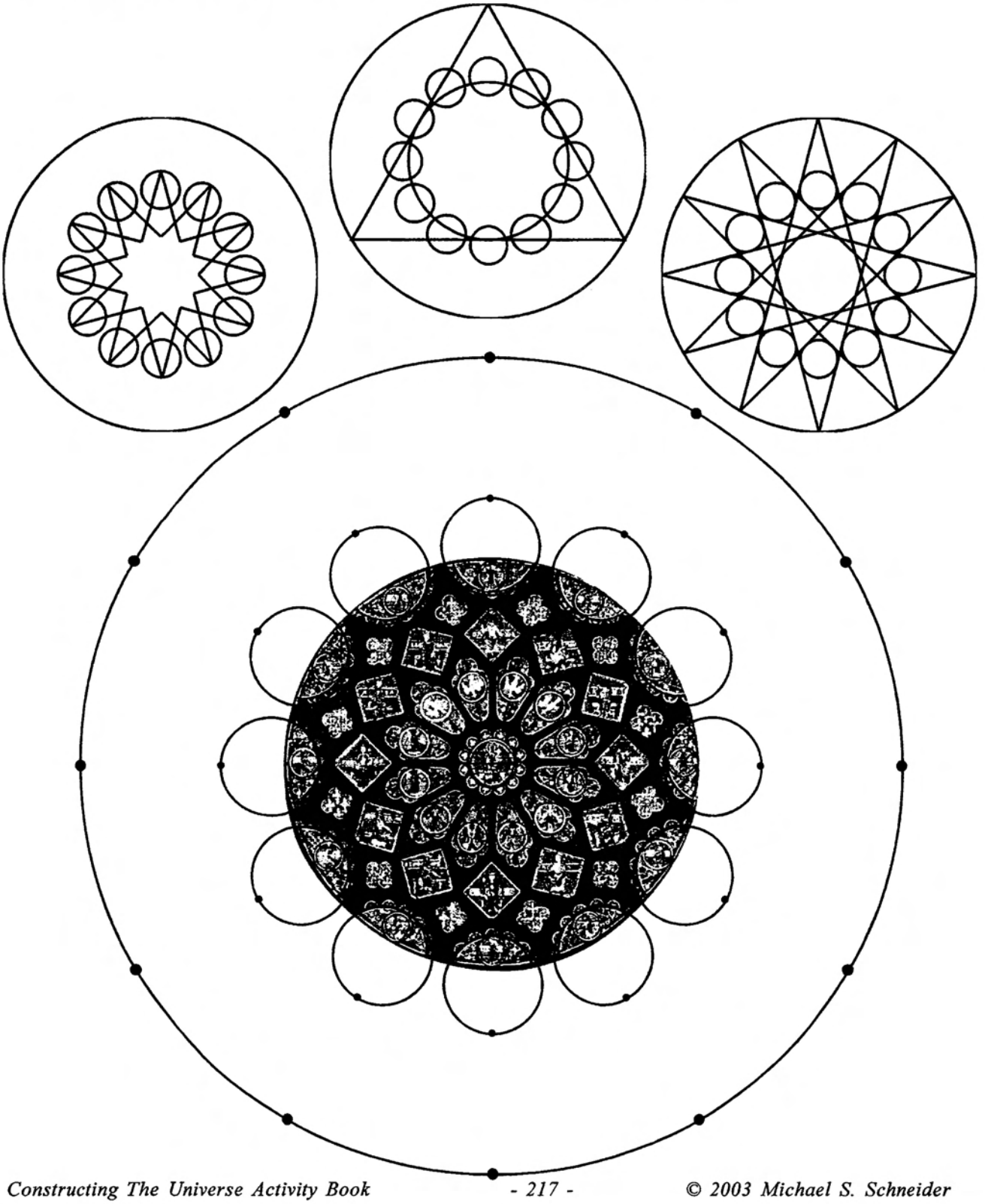
Explore it further and you will surely find more alignments.



Chartres Cathedral North Rose Window

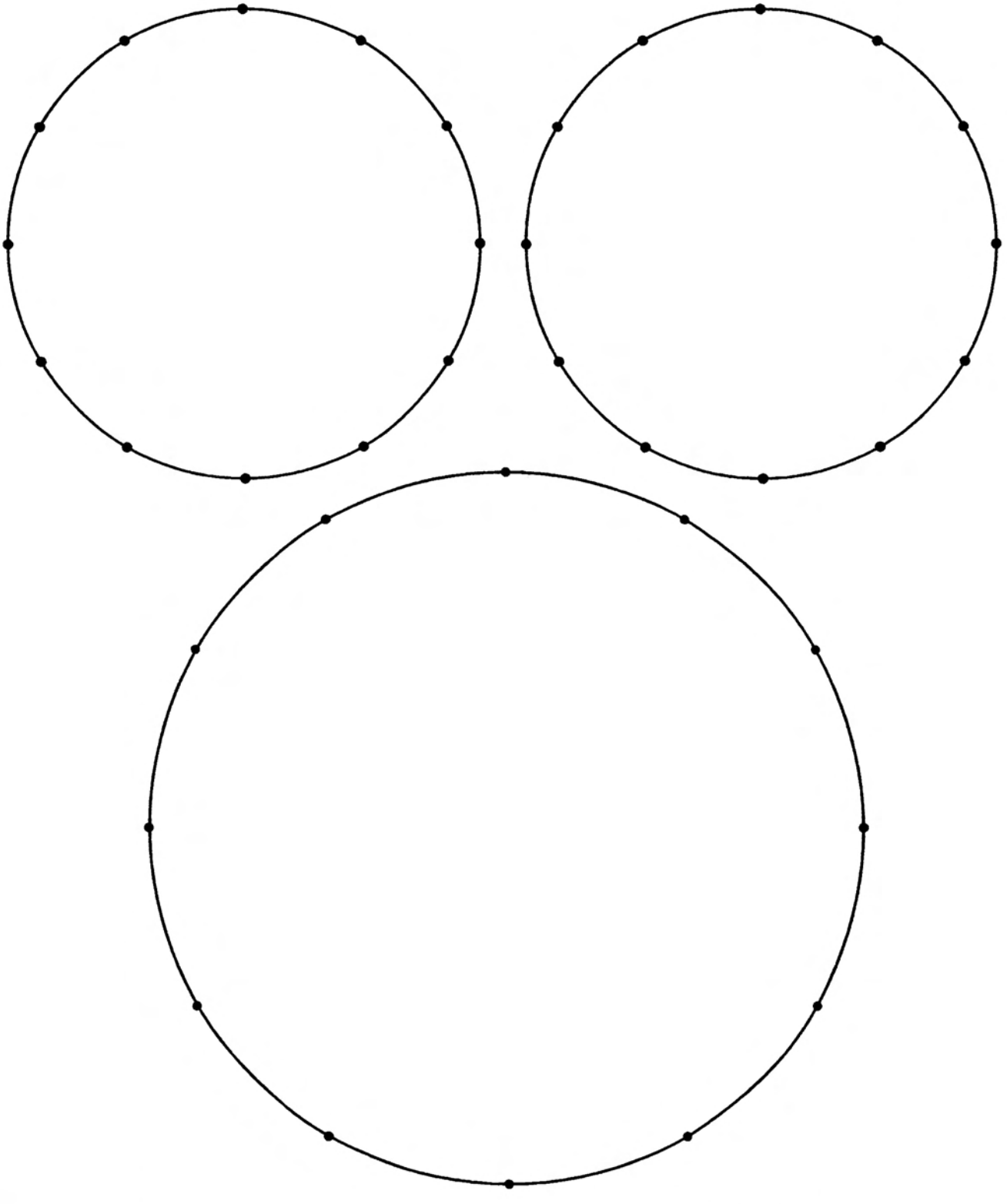
Each of these constructions will show you more of the window's design.
Use colored pencils.

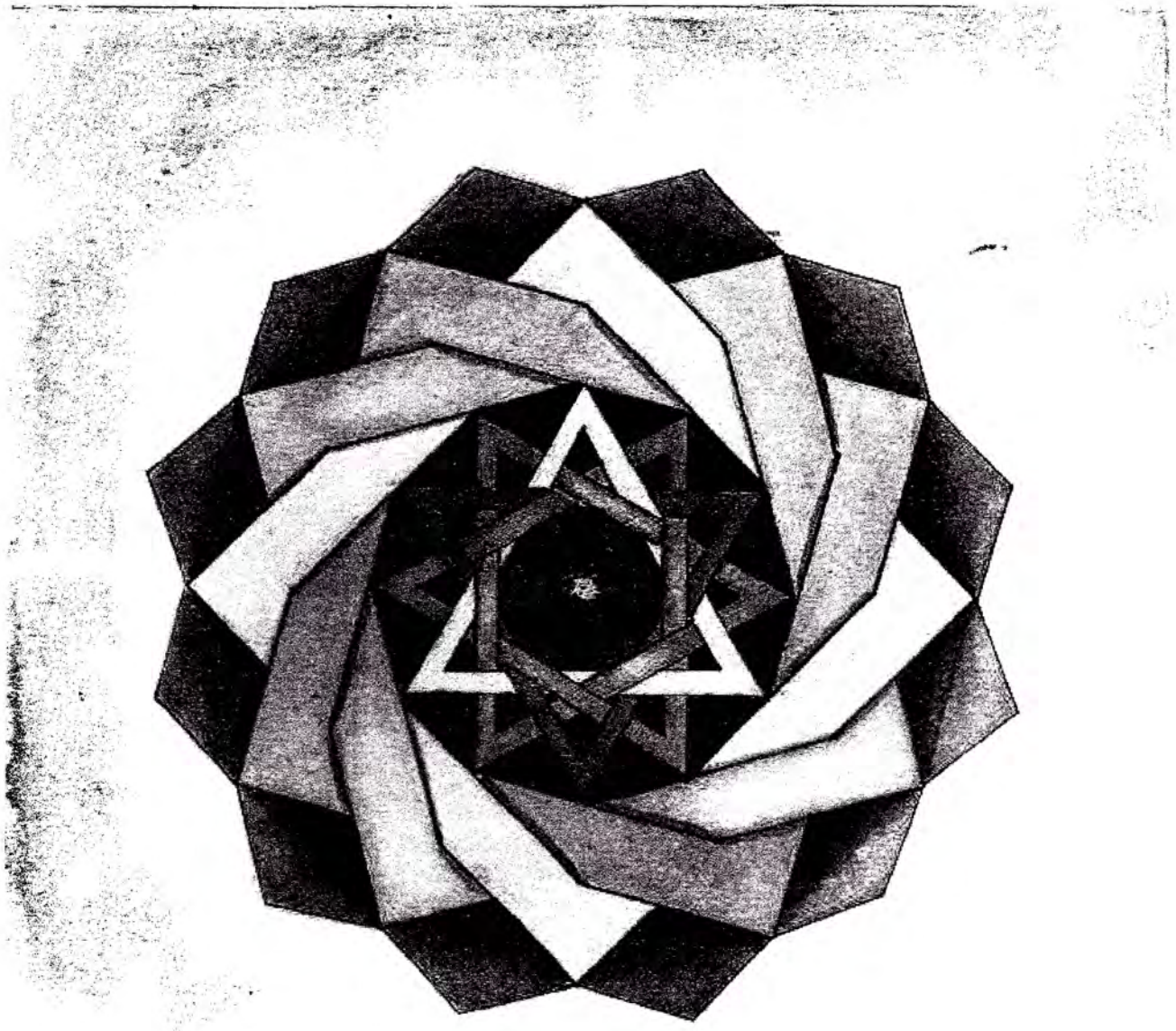
Can you find the centers of the small circles from elsewhere in the construction?



Design Your Own Dodecagonal Art

Now that you've seen some ways the Dodecagon has structured artistic designs, start with twelve points around a Circle and do a construction which guides your art, craft or architectural design.





The universe is simple, if you use imagination.
Its pattern is the number Twelve which structures all Creation.
You need not study physics nor be versed in numerology
To draw from Threes & Fours the duodecimal cosmology.

John Michell

