Constructing The Universe Activity Books
Volume 4

Dynamic Rectangles

Explore Harmony in Mathematics and Art

Michael S. Schneider

Author of

A Beginner's Guide To Constructing The Universe:
The Mathematical Archetypes Of Nature, Art And Science
(HarperPerennial)

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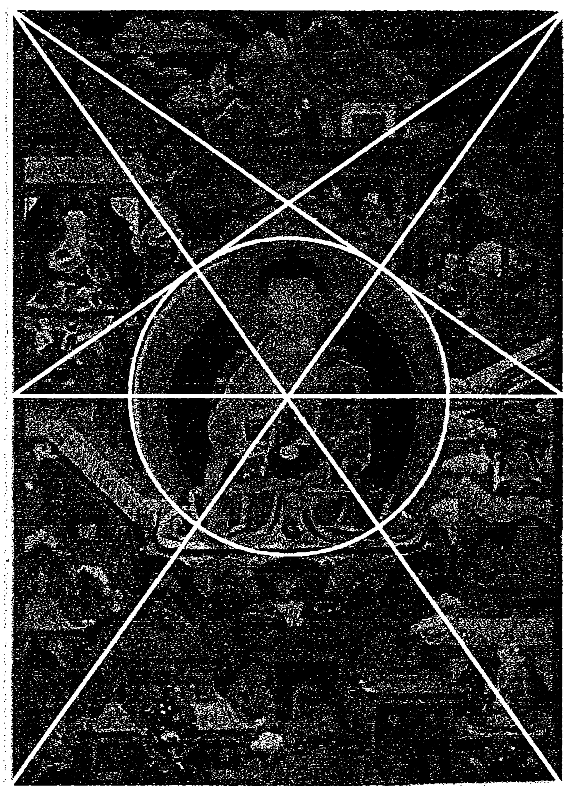

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"And since geometry is the right foundation of all painting,
I have decided to teach its rudiments and principles to all youngsters eager for art..."
Albrecht Dürer (1471-1528) "Course in the Art of Measurement" 1525

Cover illustration: Geometric analysis of Tutankhamun's jewelry box (see page 63).

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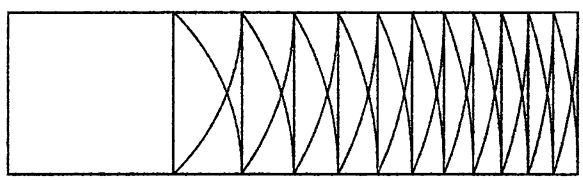
Introduction

Welcome to a book about Root Rectangles, a topic long overdue for hands-on exploration. I didn't mention it much in "A Beginner's Guide To Constructing The Universe" but it is so important that it certainly deserves its own workbook.

Why a book about rectangles? Well, look around you! Rectangles are omnipresent in countless varieties in architecture, technology and art, that we hardly notice the shape of boxes, books, doors, tables, windows, pictures, placemats and too many objects to list. Obviously, rectangles aren't all the same. But some have special properties, so interesting that artists, craftspeople, architects and designers from the deep past through today have focused upon them. What did they see and value in this set of shapes called Root Rectangles?

Since all rectangles have four right-angles, the difference must be in the ratio of their sides. Anyone who spends time with rectangles realizes that they each have a different "feel" to them. Tall or wide, they seem to have different "pressure" on their sides squashing them into that particular shape. But sides in certain ratios relate internally to themselves in different ways than others will. These are the "Root Rectangles" whose sides relate as square roots. No complex mathematics is necessary to appreciate their properties or to recognize how they've been used in art, or even to use them in your own creative efforts. They're not difficult to understand: they grow out of a square by continually extending their corners and diagonals, each leading to the next.



Their beauty is that they're interrelated and, as we shall see, they divide and subdivide internally into replicas of themselves and each other. Thus they convey a sense of unity underlying diversity, an idea artists have seized upon to design and create powerful works.

The name "Dynamic Rectangles" was used by the writer, architect, artist and influential teacher of artists Jay Hambidge (1867-1924) as part of his system of Dynamic Symmetry to describe the proportioning systems found in nature and worldwide art. Reading Euclid in Greek he came upon the term *dunamei summetros*, dynamic symmetry, meaning "commensurate in square". That's another secret of these Dynamic Root Rectangles. Their sides may be irrational, never-ending decimals, but when made into rectangles their *areas* become whole numbers and rational fractions, as do their subdivisions. Hambidge described these proportions as "active" since the parts relate to each other and the whole in harmonious relationships. This contrasts with "passive" or "static symmetry" which builds with simple multiples or modules. Static symmetry yields a limited way to incorporate disparate parts and thus easily feels lifeless and disjoint when applied to art. Hambidge found dynamic symmetry in nature and in Greek pottery, sculpture and architecture. Beyond lengths, also included in this harmony are areas and volumes. We examined these ideas in Volume 3 regarding the dynamic, self-similar Golden Rectangle.

In this book the term "Square-Root Rectangle" will be shortened to "Root Rectangle", as in the term "Root Two Rectangle" for a rectangle whose sides relate as the square-root-of-two to one.

What do Root Rectangles allow an artist to do with space that's so useful? They're used as a design instrument for organizing visual forces. They allow artists to control space by offering a variety of proportional suggestions which correlate and unify the elements of a design. They achieve a just balance among parts and whole. In effect, they can banish incoherence in a design.

While certain ratios and proportions have a powerful effect upon a viewer, it should be remembered that nothing outside us effects us unless we agree to it. Nevertheless, something deep in us recognizes these archetypal proportions and we agree to resonate with them, their ratios, proportions and areas. They're archetypal, not merely cultural, and so penetrate deeply into whoever sees them. Thus, there's also been a symbolic aspect to their use, a pan-cultural language associated with each of these rectangles, as there are for the regular shapes like circle, triangle and square, worldwide symbols of wholeness, sacredness and earth. Please see "A Beginner's Guide To Constructing The Universe" for much more about this symbolic aspect.

A word about the book's organization and what you'll do. First there's a general introduction to the Root Rectangles, the first five in particular. Activities with a compass and straightedge (and colored pencils) will teach you the basic geometric skills and understanding of them. The remainder of the book is focused on the attributes of two particular Root Rectangles: Root Two and Root Three. (Root One, the square and Root Four the double-square were seen in Volume 1 Chapter Four, and Root Five in Volume 3.) In each you'll apply these constructions to images of art so you can learn directly from the masters how to use them effectively. The reader is always encouraged to use this knowledge and skills to create your own art based on this powerful structuring system.

So please enjoy the beauty and pervasiveness of these Dynamic Rectangles.

Michael Schneider
San Anselmo, California
May 22, 2006

PS -- I didn't plan to make this book 141 pages, it just turned out to be *100 times the square root of two!*

The Mathematics of Root Rectangles

All geometric constructions begin with a point and circle. From this simple start we can produce a square, and from that foundation build the infinite progression of dynamic, root rectangles.

The square is a rational shape since the ratio of its sides is always unity, one-to-one-to-one-to-one. Like other regular polygons, the square is a mask of Unity. It offers a stable, calm space having the feel of equal pressure on all sides. Thus the square is the shape for a natural foundation from which the Dynamic Root Rectangles grow in a natural expansion.

So let's begin our geometric journey by constructing a square.

A Square Foundation

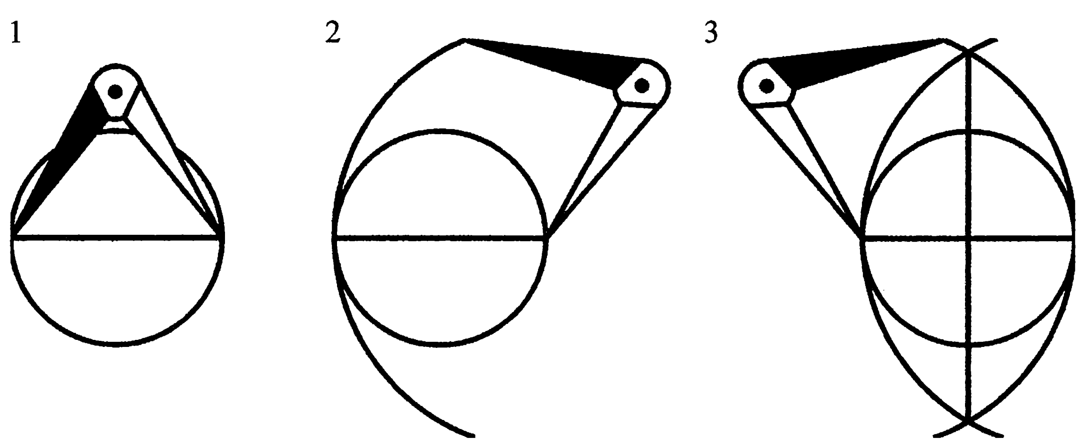
There are many ways to construct a Square. The method you choose depends on the situation and your purpose for creating it. (For more about the square see Volume 1 Chapter 4).

Here is perhaps the oldest method for constructing a Square, dating back to Babylon, India and elsewhere around the world. It is useful for constructing a Square when you know where its center is. Do this on a blank piece of paper, or on the blank page opposite this one, or start with the circle given on page 9.

(1) First, turn a Circle and draw its diameter. Open the compass to the ends of the diameter.

(2) Turn a Circle (if you have the space) or just an arc (as shown).

(3) Reverse the compass on the diameter and turn another Circle (or arc) to make an Almond. Draw a vertical line between the crossings which cuts the Circle at two points, top and bottom.



(4) Place the point of the compass on one of the four points (top, bottom, left, right) around the Circle and open the scribe to the center.

(5) Turn a full (or half) Circle.

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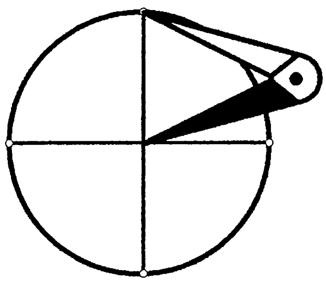

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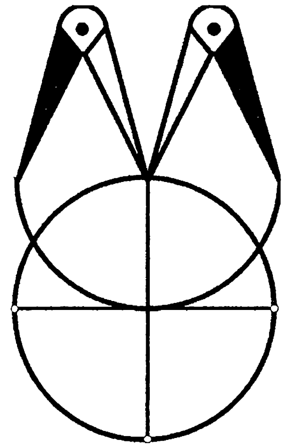


(6) Repeat this by placing the compass point on each of the four directions of the Circle and turn a Circle (or arc).

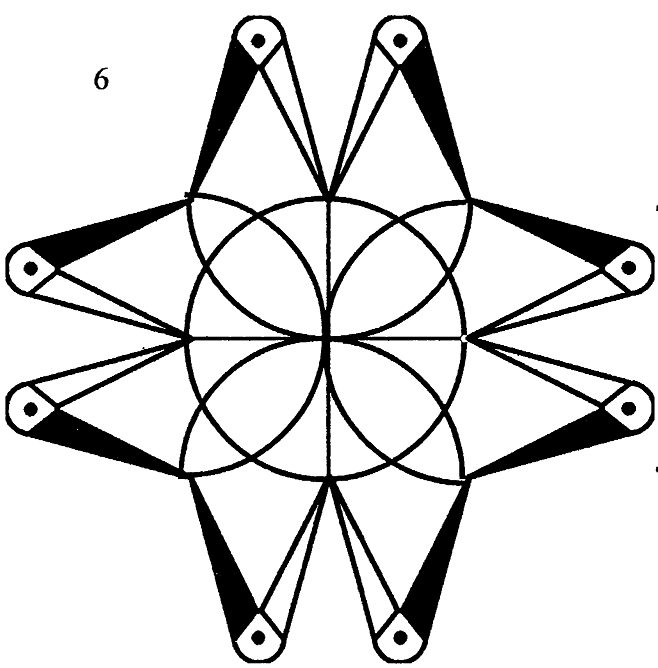
(7) We're interested in the four points made where the arcs cross.

(8) Connect these four points to construct a Square. If you've done it accurately, the sides of the Square will be tangent to (just touching) the sides of the Circle.

This construction is also useful for constructing the Octagon (Volume 2, Chapter 8) and the Dodecagon (Chapter 12).

Practice this construction in the Circle on the next page. How accurately do the sides of your square skim the sides of the circle? Learn to construct a Square without looking at these pages. Practice!

6



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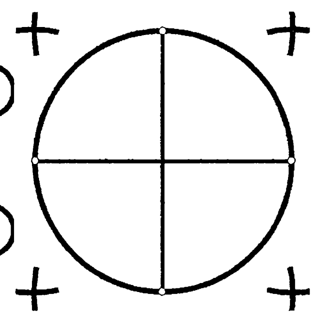

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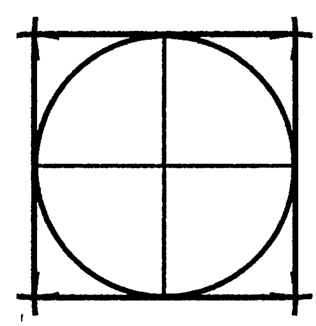
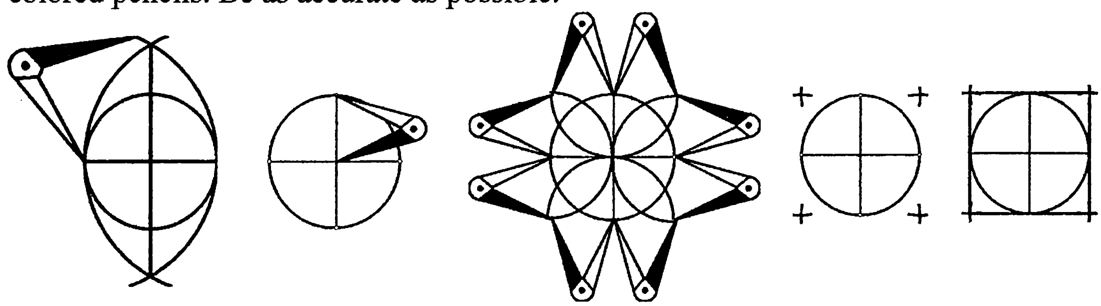
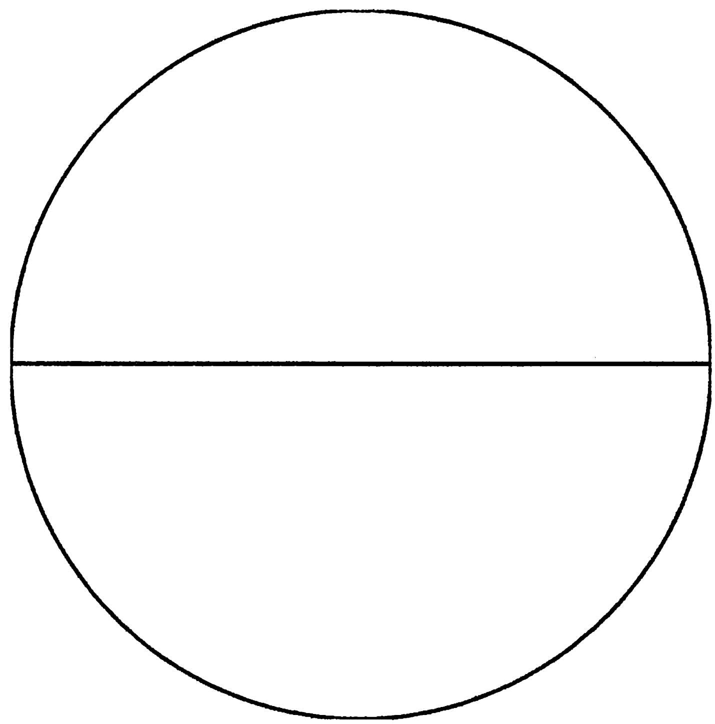


Construct a Square

Use colored pencils. Be as accurate as possible.





The Square's Diagonal

What do we know about squares? No matter a square's size, it has four equal sides and four right-angle corners which sum to 360 (=4x90) degrees, like the 360 degrees around a circle's center. (In contrast, a triangle's corners sum to 180 degrees and a pentagon to 540 degrees.)

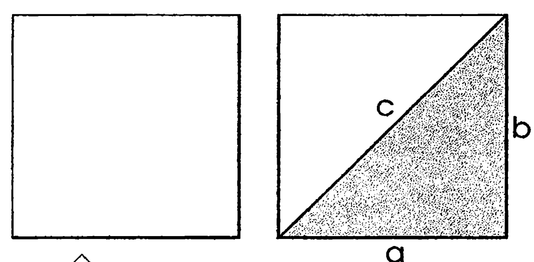
A square's area equals the length of its side times itself. That's why multiplying any number by itself is said to be "squared". Multiplying any number by itself is like multiplying the sides of a square to find its area.

And a square, like every rectangle, has two equal diagonals stretching between opposite corners. The Greek word we translate as "corners" they wrote as "knees" (*gonia*) since angles, corners and knees are each bent. The word "diagonal" comes from the Greek *diagonios* or *dia* + *gonios* "across the knees" or "from corner to corner".

The diagonal is always longer than a square's sides. But by how much? It's always a fixed ratio. And notice that the diagonal cuts the square into two isosceles right triangles, each with two 45 degree angles and one 90 degree angle. We can calculate the eternally fixed relationship between the diagonal of a square and its side by using the famous Pythagorean Theorem (actually known to the Chinese a thousand years earlier around 1500 BCE):

Simply put, in a right triangle having sides called a, b and c, then

$$a^2 + b^2 = c^2.$$



So it's about squares!

The area of a square made on the side "a"

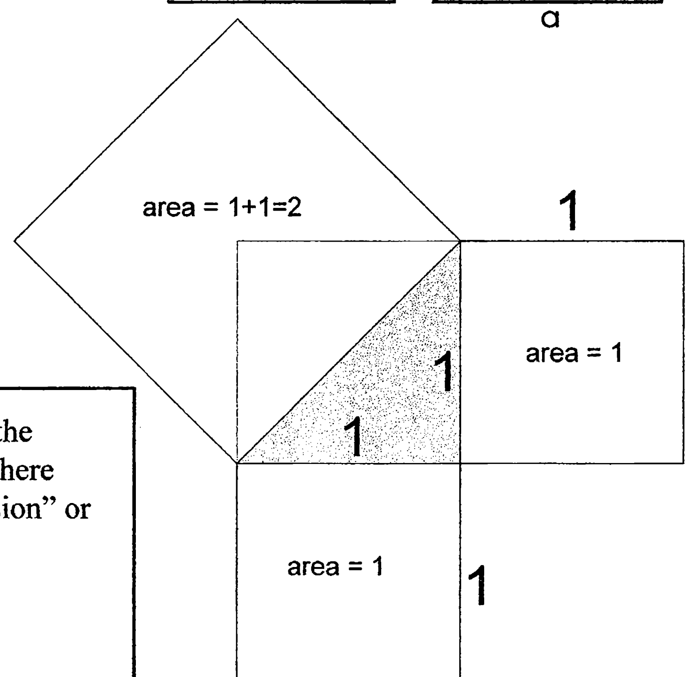
plus

The area of a square made on the side "b"

equals

The area of the square on the longest side "c".

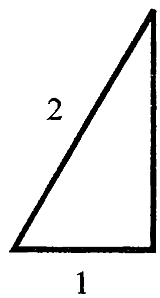
That's what the formula $a^2 + b^2 = c^2$ means.

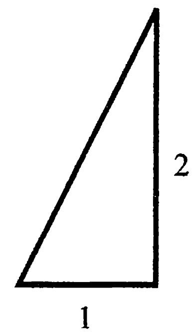


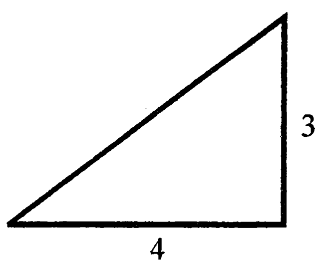
> The longest side of a right triangle is called the "hypotenuse" from the Greek *hypotenusa*, where *hypo* means "under" and *tenusa* means "tension" or "stretch".
>
> So side "c" is a hypotenuse, the line which "stretches under" the right angle.

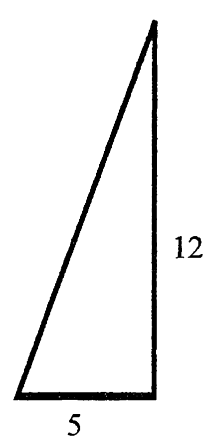
Find The Missing Sides

Use the Pythagorean Theorem to find the missing sides of these right triangles.
Draw squares on their sides and show their areas.









The Scarecrow's Misstatement

Upon receiving his "brain" (which is only a paper diploma in the health care system of Oz) the scarecrow demonstrates his mental boost by proclaiming:

"The sum of the square roots of any two sides of an isosceles triangle is equal to the square root of the remaining side."

... but he's wrong!

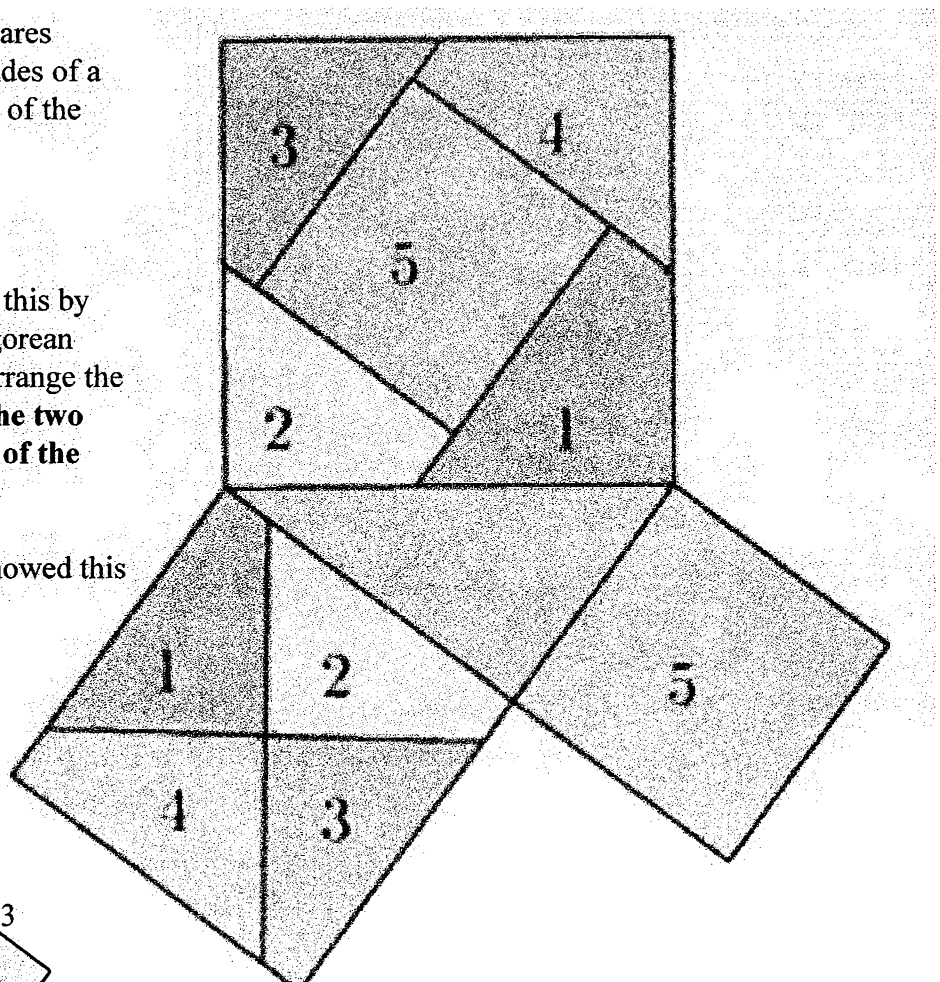
He's *trying* to recite the **Pythagorean Theorem** which actually states:

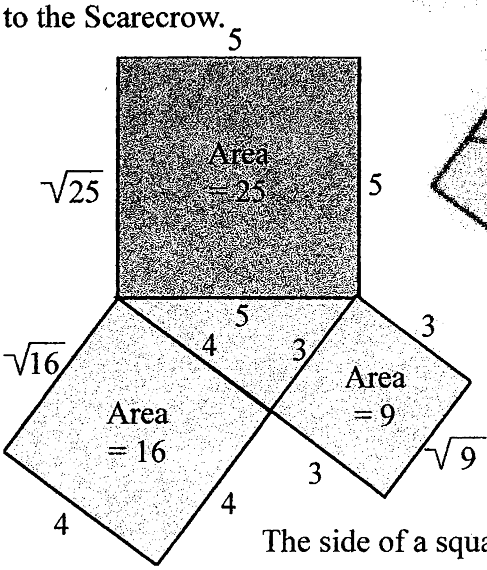
"The sum of the areas of the squares constructed on the two shorter sides of a right triangle is equal to the area of the square on the longest side".

That's what $a^2 + b^2 = c^2$ means.

It also means that you can prove this by tracing or cutting out this Pythagorean Theorem construction. Then rearrange the parts to show that **the areas of the two smaller squares equal the area of the large square.**

Perhaps someone should have showed this to the Scarecrow.





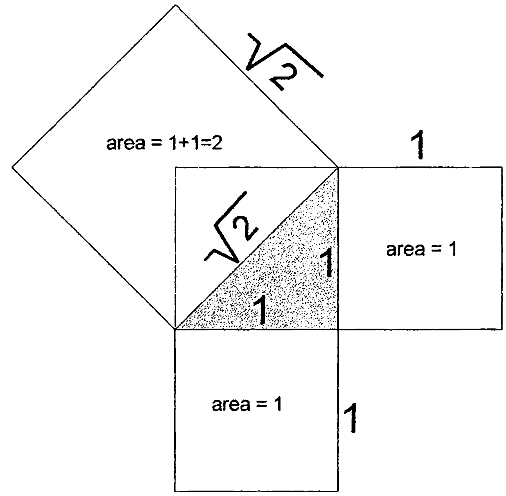
The side of a square is the "root" of its area.

The Square Root

So what actually is the length of the diagonal of a square, if the square's sides are each equal to one?

Let's start with what we know: Since all the sides of a square are the same length, we can symbolize this by

$$a = b = 1$$



Applying the Pythagorean Theorem $a^2 + b^2 = c^2$ we have

$$1^2 + 1^2 = c^2$$

$$1 + 1 = c^2$$

$$2 = c^2$$

taking the square root of each side, we find that

c = the square root of 2 = $\sqrt{2}$ which has a value of 1.414... , just a little under one-and-a-half.

So the diagonal of a square is always "the square root of 2" or 1.414... times the length of its side.

Since $\sqrt{2}$ has an irrational value (that is, it cannot be described by a simple ratio of whole numbers) it produces a neverending decimal. But when irrational values are multiplied together they can produce rational *areas*. This is what Jay Hambidge meant by saying that irrational lengths are "commensurate in area".

Any limited number of the digits of the square root of two multiplied times itself will always fall short of two. Try multiplying these approximations by themselves to see how it gets closer and closer to two:

Approximation	Squared value
1.4	
1.41	
1.414	
1.4142	
1.41421,	
1.414213	
1.4142135	

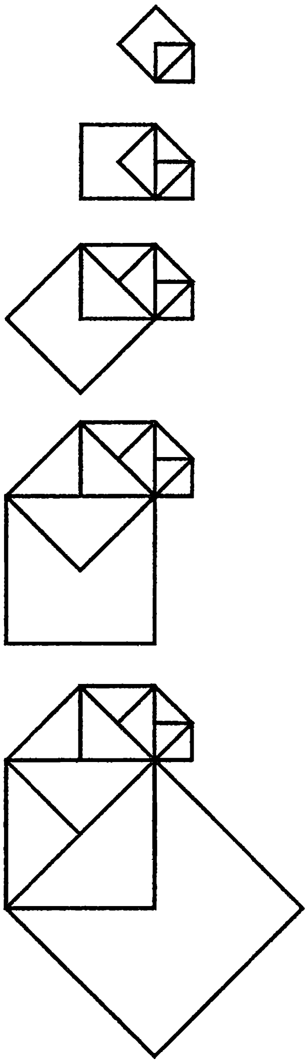
The Square Root Symbol $\sqrt{}$

is called a "radical" from the Latin word *radix* or "root". (The side of a square is considered the "root" from which its area grows.)

Ancient mathematicians actually wrote out their word for root, but it was eventually simplified to the letter R. In the Seventeenth century it morphed into the familiar radical sign which resembles a small letter "r".

Spiral Of Doubling Areas

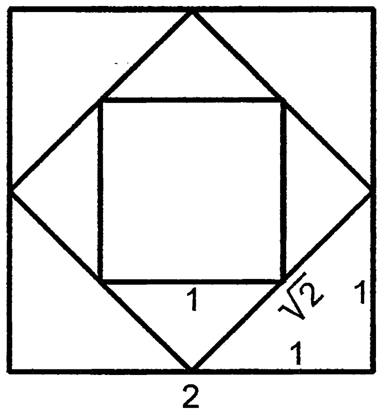
Since a square's diagonal creates the side for a new square, we can continue the process indefinitely and create a spiral of squares. And since the sides keep increasing by the square root of two, the area of each new square is *double* the one it came from. This produces a chain of doubling areas in the famous "binary" sequence 1, 2, 4, 8, 16, 32, 64, 128, 256, 512, ... at the heart of modern computers.

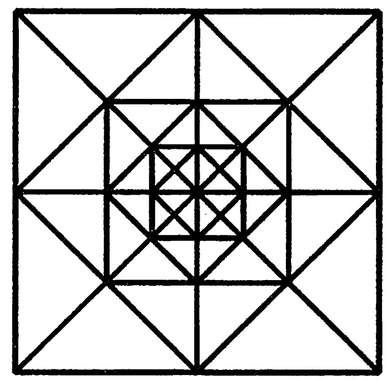


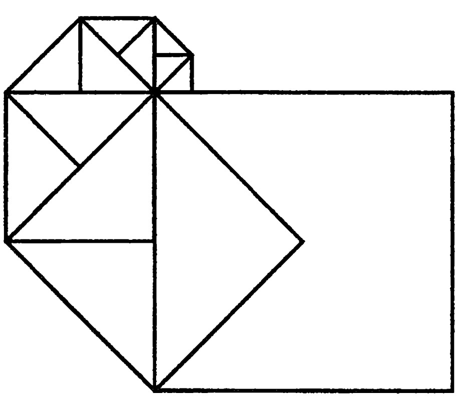
Use colored pencils to shade areas.

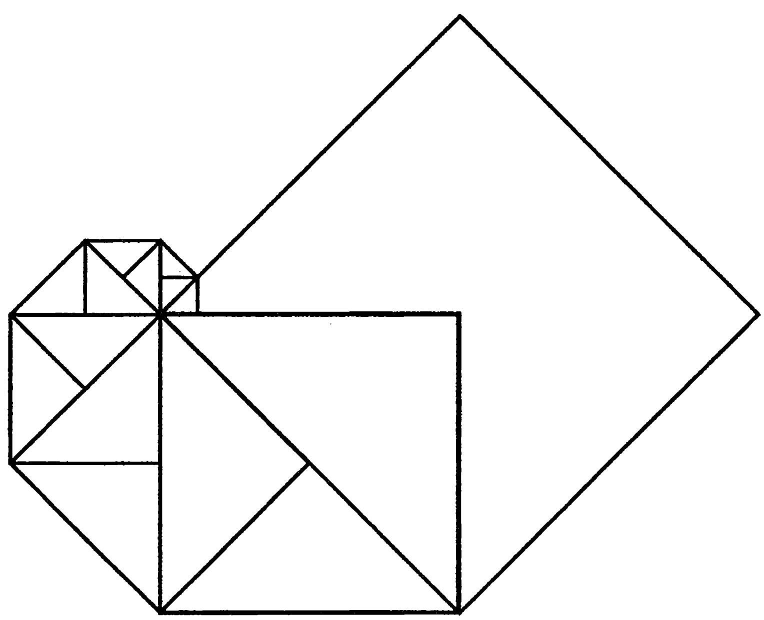
Draw the next square in the series and continue the spiral.

The binary sequence can diminish as well as grow larger. Connect the midpoints and diagonals of the square below and replicate this pattern to create squares whose areas continually diminish by half.





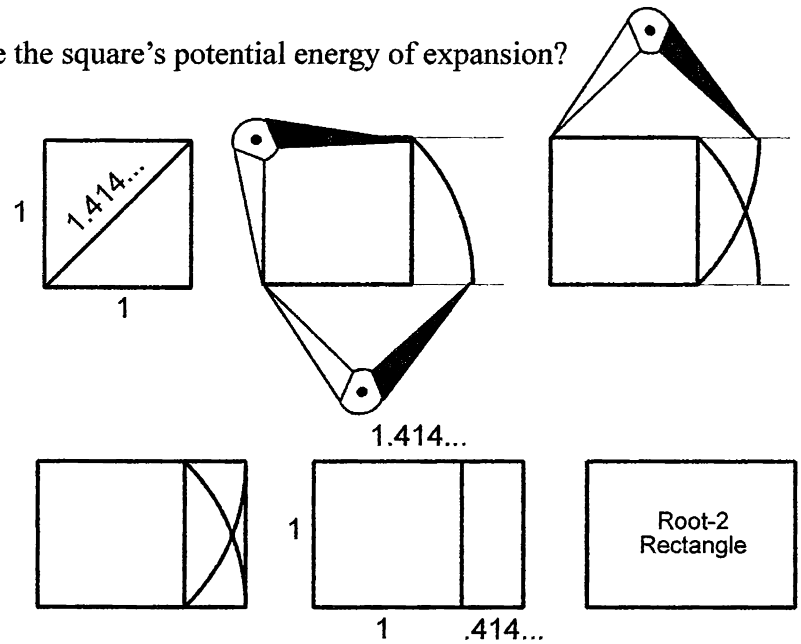




The Expansive Power Of The Diagonal

If you look at squares and different rectangles notice that they each have a different "visual feel" as if different pressures are squeezing their sides. Wider rectangles seem to have more pressure on their top and bottom than on their short sides. But a square feels visually as it if has equal pressure on all sides. Thus the square is static, a space at rest. But its diagonal is like a spring within waiting to expand the square beyond itself.

How can we use the diagonal to release the square's potential energy of expansion? The answer is to use our compass.



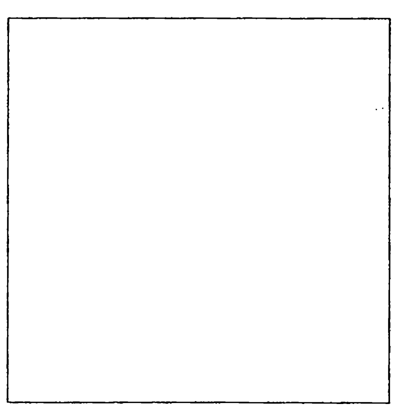
Since the latent energy within the square is held in its diagonal, open your compass to the two ends of a square's diagonal. Then swing the compass downward until it crosses the bottom level of the square. Repeat this with the other diagonal, swinging upward. Extend the square's sides to cross the arcs. Then draw a vertical line segment connecting the new corners to create a new rectangle, the only one which can emerge from the square this way.

Since the length of the diagonal became the new rectangle's long side, it equals the square root of two, or about 1.414, so the rectangle is called a "Root Two" rectangle.

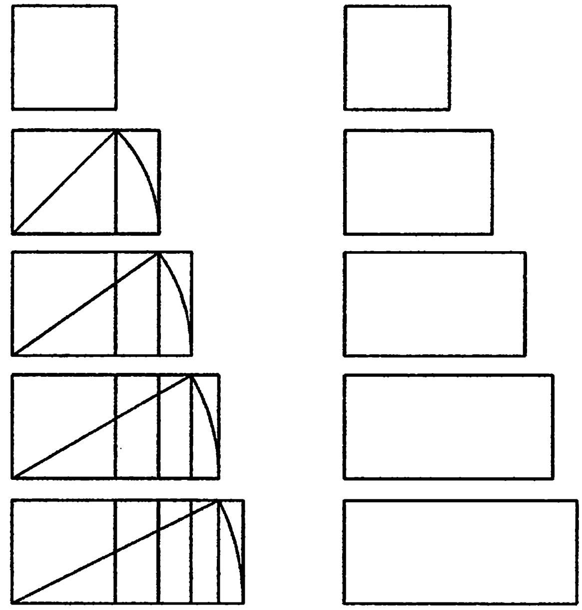
This foundation Square is also known as the "Square Root of 1" Rectangle, or just a Root One Rectangle.

Construct a Root Two Rectangle

Start with this square and extend its top and bottom sides to the right. Open your compass across the diagonal and follow the steps shown above. You don't have to show the arcs but just make marks where they would cross the lines of the extended square.



The Root Rectangle Ruler



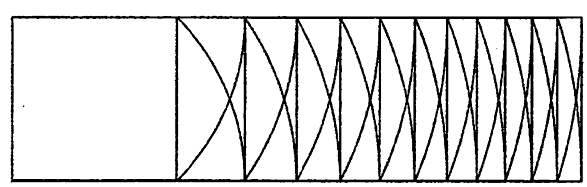
The Root Two rectangle is made by a harmonious extension grown from a square's diagonal. And notice that it too has a diagonal longer than its own side. Thus it can produce an extension onto the Root Two Rectangle to create another rectangle.

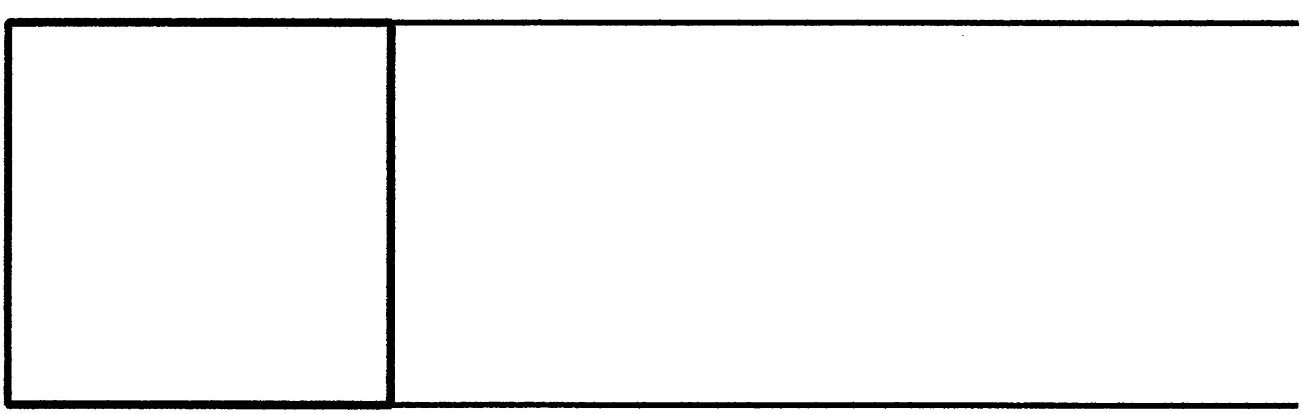
What potential energy does the diagonal of a Root Two rectangle hold? The Pythagorean Theorem tells us that the length of the diagonal of a Root Two rectangle has a value equal to the square root of three (= 1.732...). Thus we can use the diagonal within the Root Two Rectangle to construct the next rectangle in the series, a Root Three Rectangle. Astonishingly, the length of the diagonal of a Root Three Rectangle is equal to the square root of four, whose own rectangle has a diagonal equal to the square root of five, and so on. This is the beginning of an infinite journey along a chain of Root Rectangles built from the expansive energy of each new rectangle's diagonal. We can think of their progression as a Root Rectangle Ruler.

Construct a Root Rectangle Ruler

Use the square (with extended sides) below to construct a chain of Root Rectangles, the Root Rectangle Ruler. To start it, simply place your compass point to the bottom left corner to the square's diagonal and swing an arc downward to cross the extended line segment. Then move the compass point to the upper corner and swing upward to cross the upper line. Connect the crossings with a vertical line to complete the Root Two Rectangle.

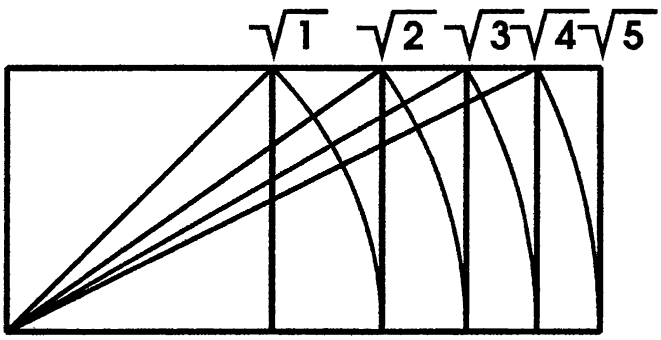
Continue this process using the diagonals of each new rectangle to produce an ongoing series of root rectangles. They're limited only by the size of your compass. These are the Dynamic Rectangles, the Root Rectangles which are each unique but harmoniously linked by their diagonals to every other Root Rectangle. Together, they form a Root Rectangle Ruler. Create as many rectangles as the reach of your compass will allow. Use colored pencils to shade different areas.





Calculate Square Roots

You can use the Pythagorean Theorem to prove that each new rectangle is a Square Root Rectangle. That is, their long and short sides form the ratios (beginning with the Square), Root 1, Root 2, Root 3, Root 4, Root 5 and so on. Each of these "roots" have numerical values. Here are approximations, but use your calculator to find more decimal places for each, and extend their values below.



$\sqrt{5} = 2.236$

$\sqrt{4} = 2.000$ = Double Square

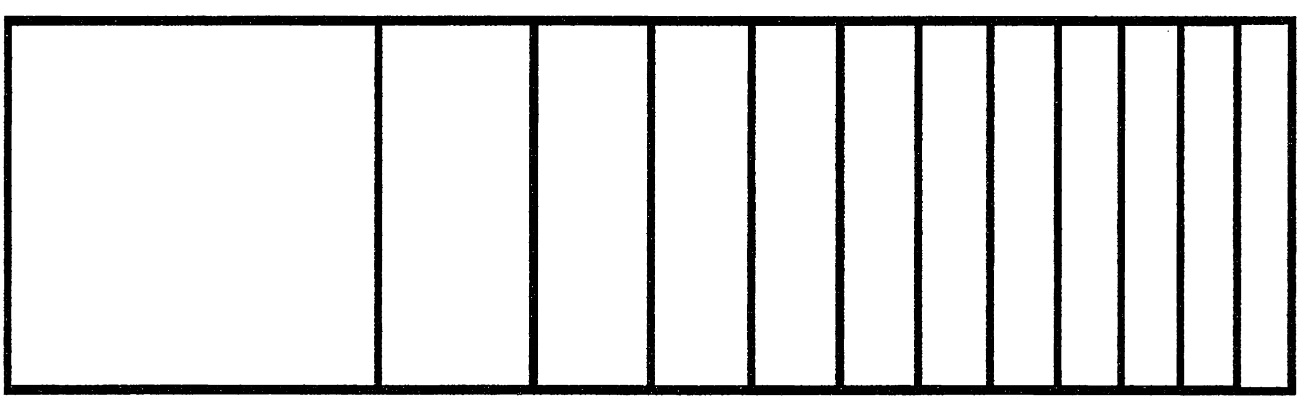
$\sqrt{3} = 1.732$

$\sqrt{2} = 1.414$

$\sqrt{1} = 1.000$ = Square

Write the decimal value of each square root above (or in) its rectangle's extension below.

The square is a Root One Rectangle, with a side length = 1.



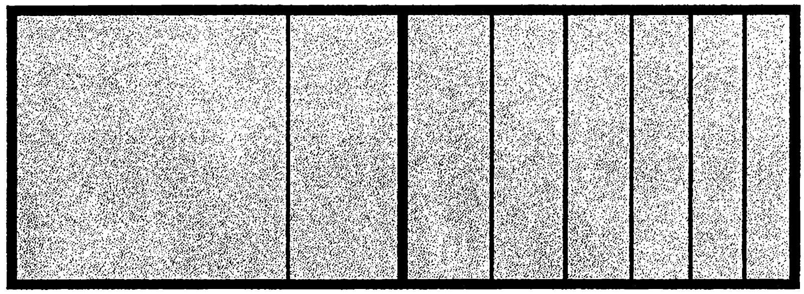
Multiples Of Root Rectangles

Many larger Root Rectangles can be understood as sums or multiples of smaller Root Rectangles.

Here are some examples. Use your compass to confirm them.

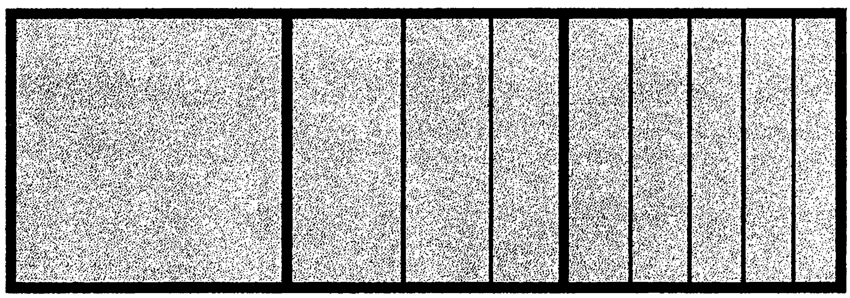
$\sqrt{8}$ Rectangle

= 2 Root Two Rectangles



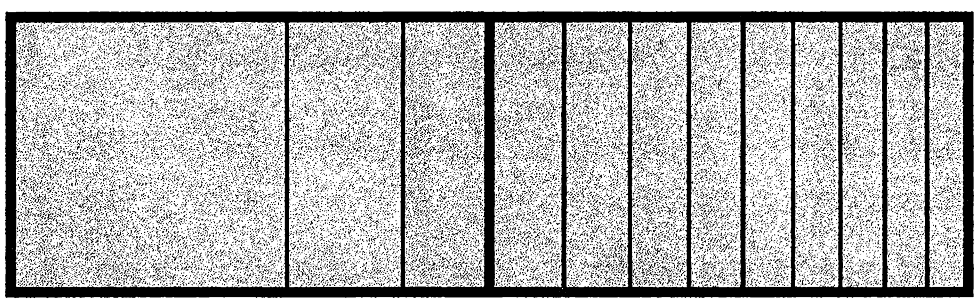
$\sqrt{9}$ Rectangle

= 3 Squares

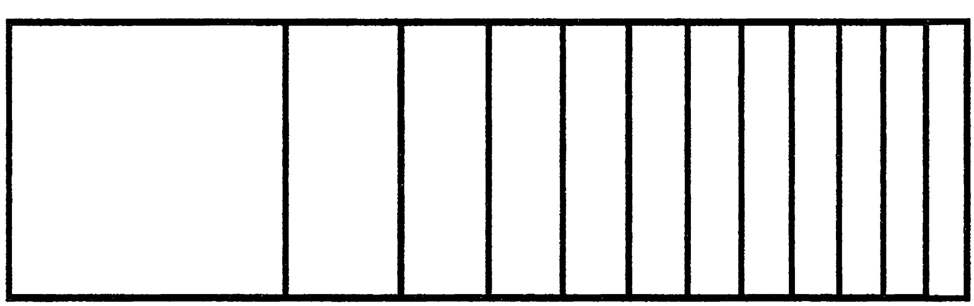


$\sqrt{12}$ Rectangle

= 2 Root ___?___ Rectangles



See if you can divide this Root Rectangle (or part of it) into identical multiples (or combinations) of smaller of Root Rectangles.

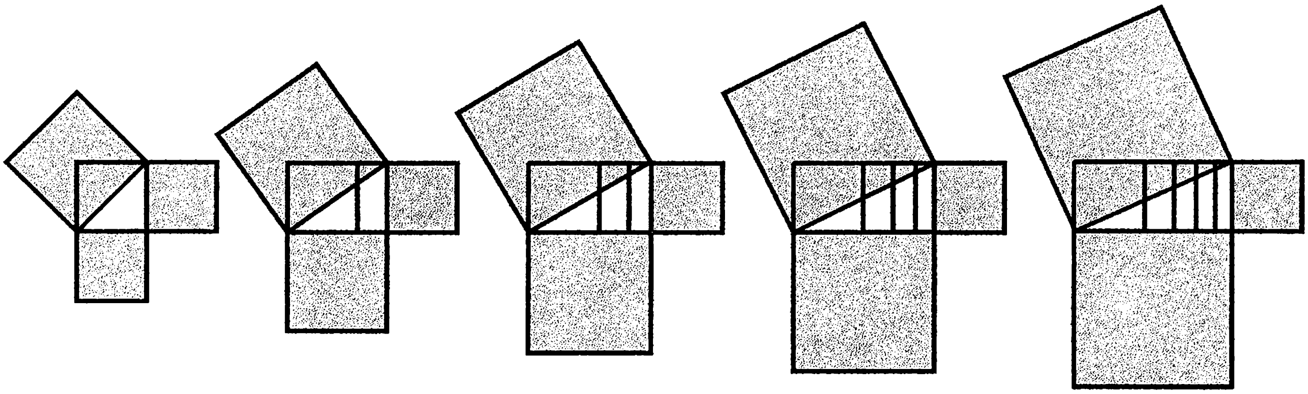


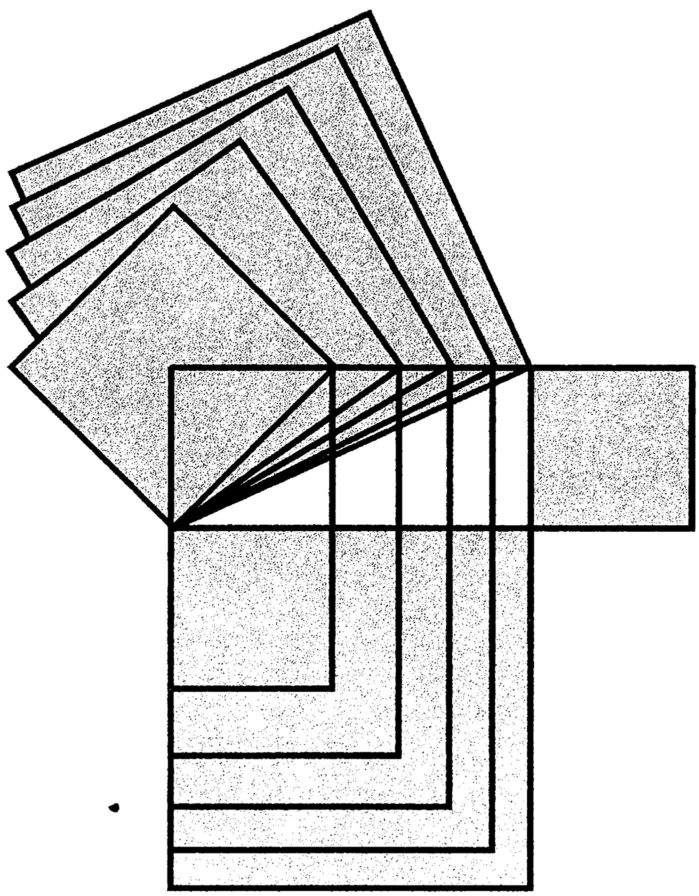
A Sequence of Squares

These irrational lengths of Root Rectangle diagonals produce squares with whole number, rational areas!

First, use the Pythagorean Theorem to find the areas of this sequence of squares. The smallest square has a side of one. Write their areas in the squares.

Do you see the pattern?





Use your results from above (and simple subtraction) to find the areas of these backwards-L shaped sections below the root rectangles.

Write their areas in their sections.

Do you see a pattern?

This L-shaped section, which always creates a larger square, was called by the ancient Greeks a "gnomon", their name for a carpenter's square-drawing tool.

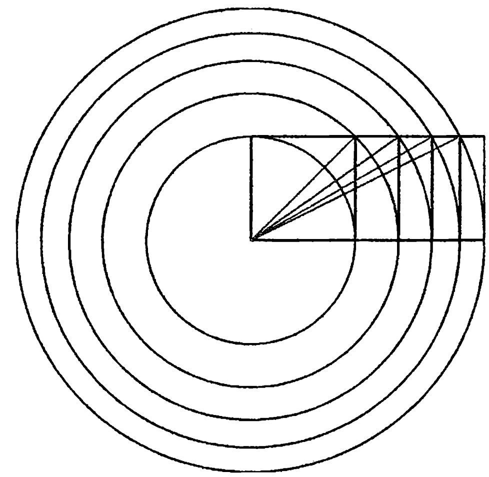
Don't confuse this type of gnomon with the shadow maker of a sundial, which has the same name.

Root Rectangle Ripples

The proportions of a Root Rectangle Ruler aren't only linear, but can also be expressed as concentric circles, like ripples of water expanding in a pond (but gradually slowing down).

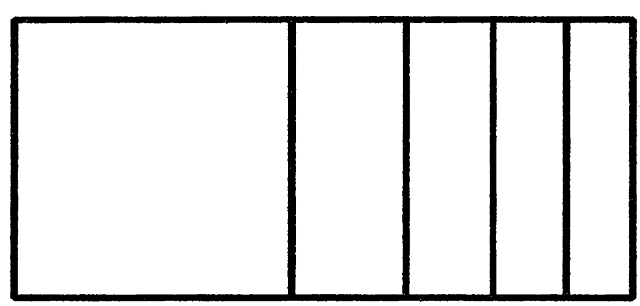
Try this: Place your compass point at the rectangle's lower left corner, where all the diagonals meet. Open the pencil along the bottom to the first vertical line you encounter, the corner of the square. Then turn a circle.

Open the pencil point further to the corner of the next rect-angle, and turn another circle.

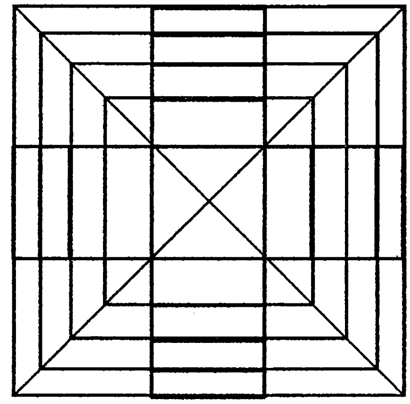


Repeat this with all the Root Rectangle corners.

Do this with a larger Root Rectangle Ruler you construct. Do these circles have any kind of "feel" to them?



Draw a Root Rectangle Platform



Here are four Root Rectangle Rulers overlapping on their square in four directions.

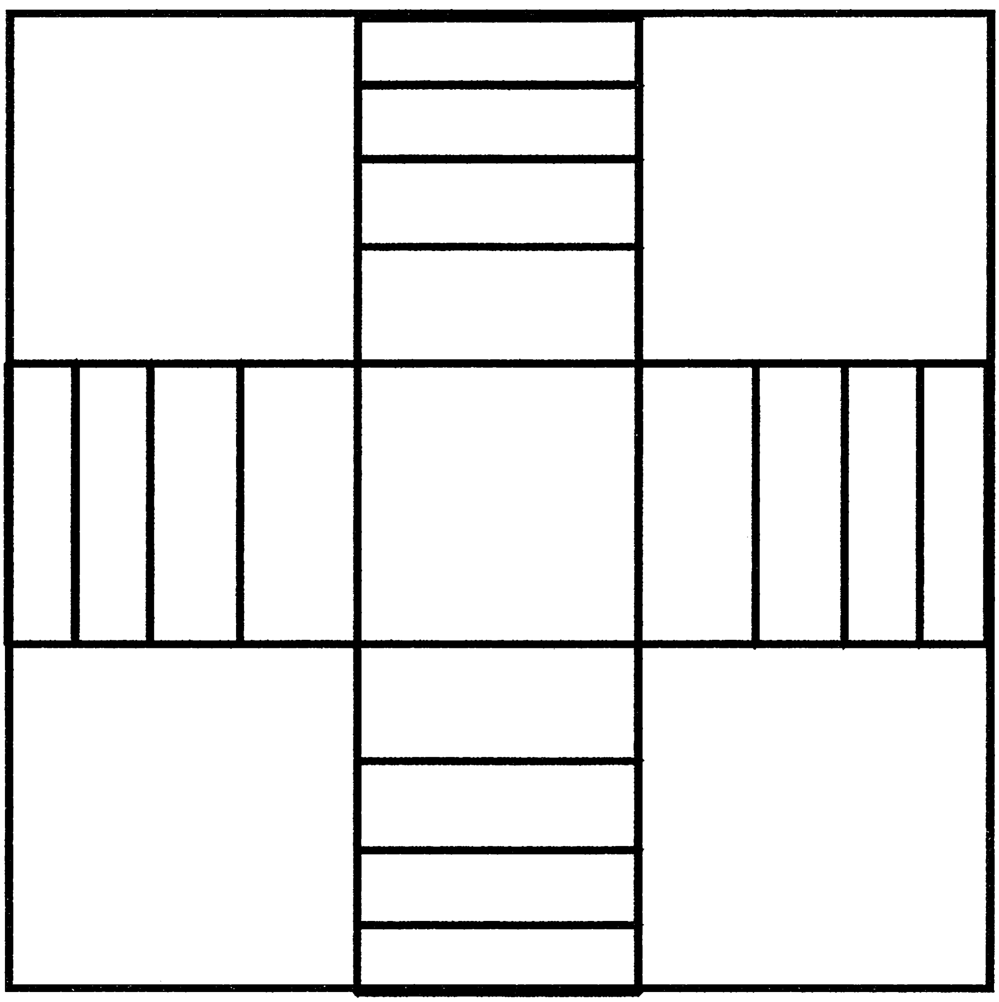
Use your compass to "move" the measurements of the Root Rectangles onto the sides of the square.

Then connect opposite points to create concentric squares and a "Root Rectangle Platform" as seen from above.

The sides of the squares expand in square root proportions.

Extend the square in all directions by continuing the Root Rectangle Rulers.

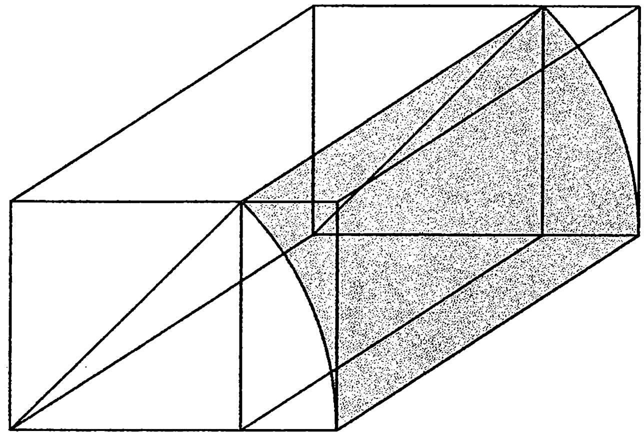
Use colored pencils to shade different areas.



Square Roots In Three Dimensions

Any two dimensional geometric constructed on flat paper can be extruded into three dimensions.

Construction of a Root Two Rectangle from a square as it would appear in three dimensions.



The continuous ladder of square roots also occurs in other geometric situations.

The sequence of square roots also unfolds naturally in three dimensions.

Look at the sides and diagonals found in two stacked cubes to see the square roots from one to five.

Can you find the square root of *six*? Draw it.

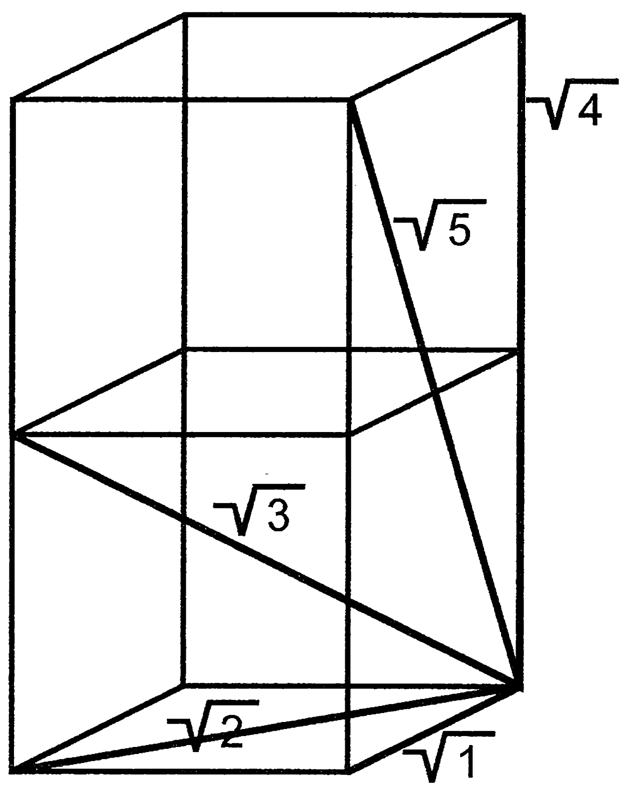
Then prove it using the Pythagorean Theorem.

(Hint: Shade a right triangle.)

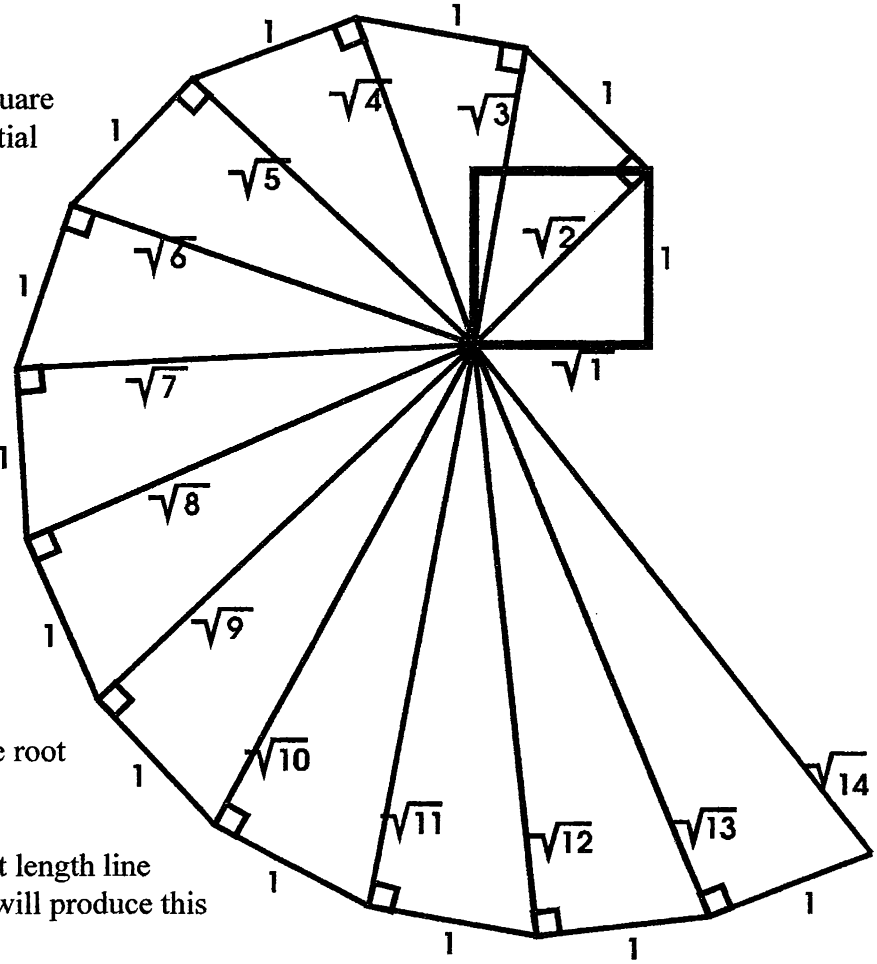
What square roots would *three stacked cubes* hold?

Draw it.

Prove your findings using the Pythagorean Theorem.



A Spiral Of Square Roots



We can construct the series of square roots outward and around the initial square as an expanding spiral which (theoretically) turns forever.

Observe this illustration carefully.

Notice how a right angle appears at the end of the square's diagonal, one unit length long.

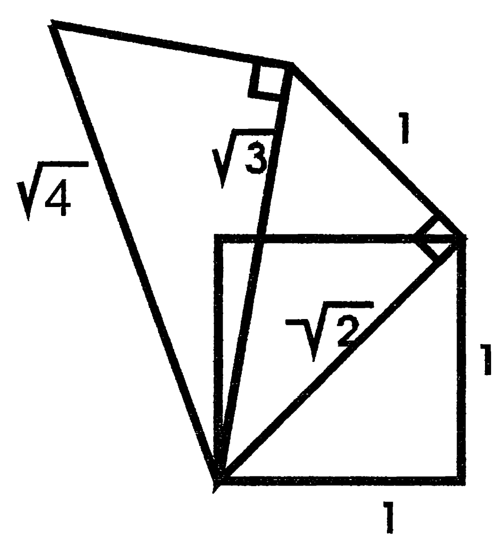
A line drawn from its end to the origin equals the square root of two. A right angle of unit length from this point reveals the square root of three.

By constructing or drawing a unit length line at the end of each new diagonal will produce this spiral series of square roots.

Try it yourself below.

Start with this, done to Root Four. Construct perpendicular right angles by using your compass to make the Almond (see Volume 1 Chapter 2).

Or, you might use a plastic right-angle triangle to draw them. (Be careful -- quick but less accurate!)

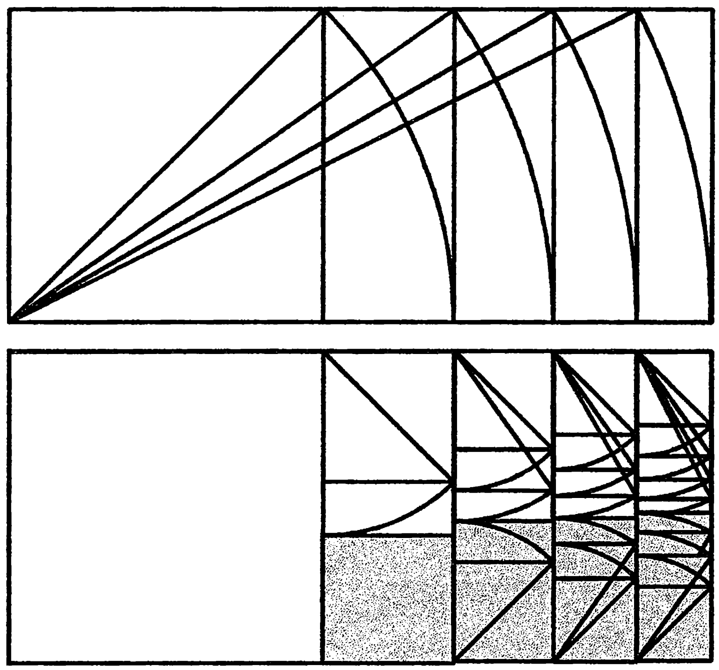


Explore The Root Ruler's Extensions

The Root Rectangles grew by constructing smaller, turned rectangles at the end of each Root Rectangle. Is there anything special about these additional extensions?

The natural question to ask is "are they themselves root rectangles?" The answer is no. But they're always *combinations* of two root rectangles. Specifically, **each section is made of a smaller, turned version of the whole root rectangle, plus the smaller root rectangle which came before it.**

For example, the extension onto the square which makes a Root Two Rectangle can be subdivided into a small, turned Root Two Rectangle, plus a small square.



Likewise, the rectangle which extended the Root Two into a Root Three Rectangle is made of a small Root Three Rectangle plus a Root Two Rectangle.

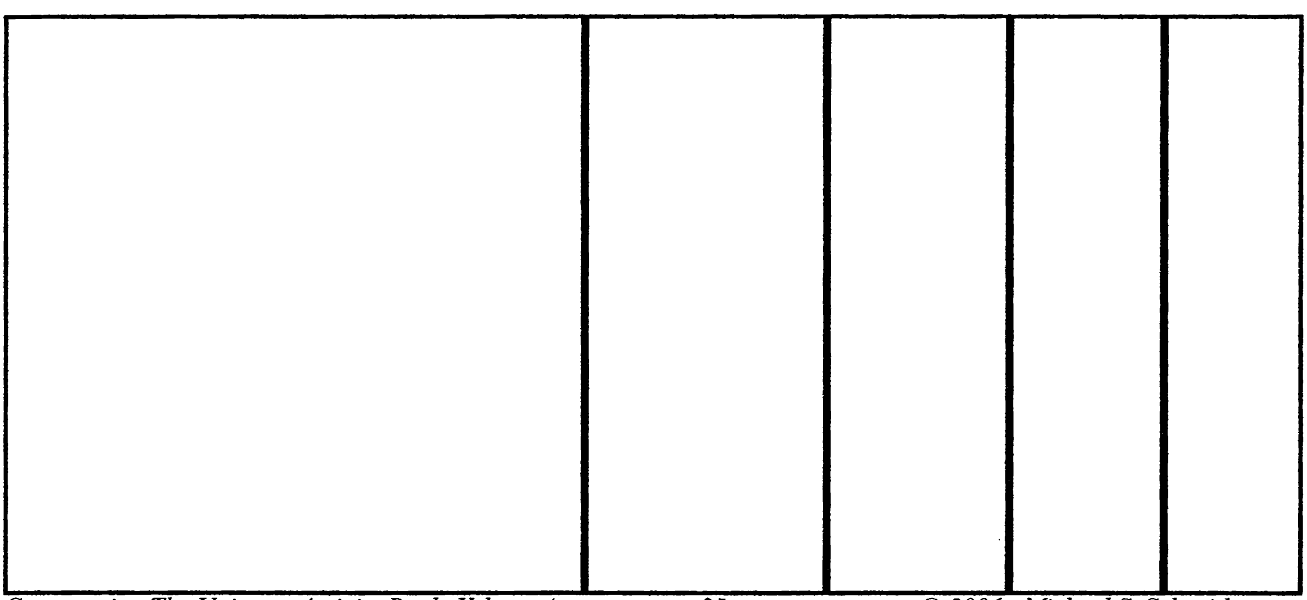
The extension which makes a Root Four rectangle is made of a smaller Root Four Rectangle plus a Root Three Rectangle.

Each rectangle extension may be subdivided in the same way.

Try it yourself below!

Use your compass to subdivide each extension rectangle below into smaller root rectangles.

This ongoing interconnectedness is why they're called harmonious, *dynamic* rectangles.



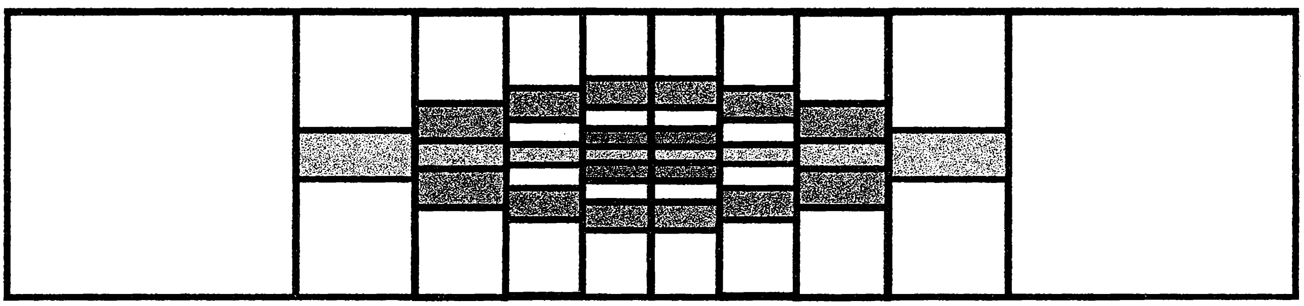
Root Rectangle Extension Patterns

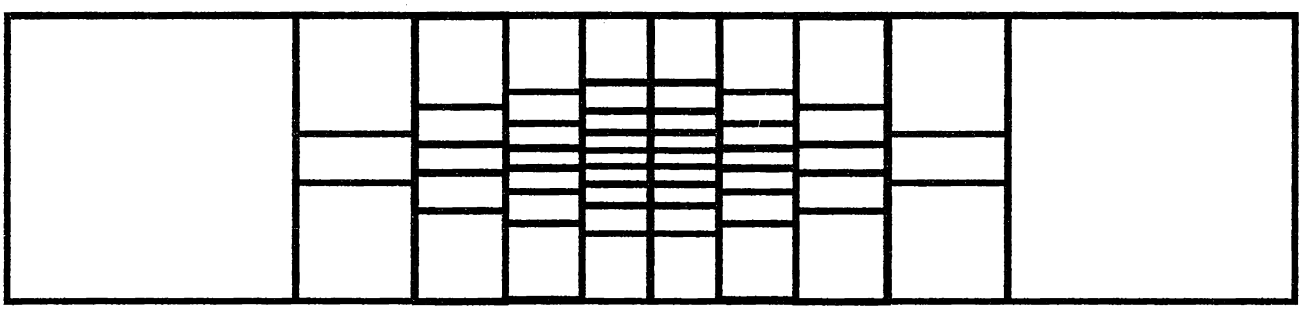
When the Root Rectangle extensions are subdivided into smaller Root Rectangles, their mathematical patterns can be seen visually. One example can be seen below (top) made with left-right mirror images of two Root Five Rectangles.

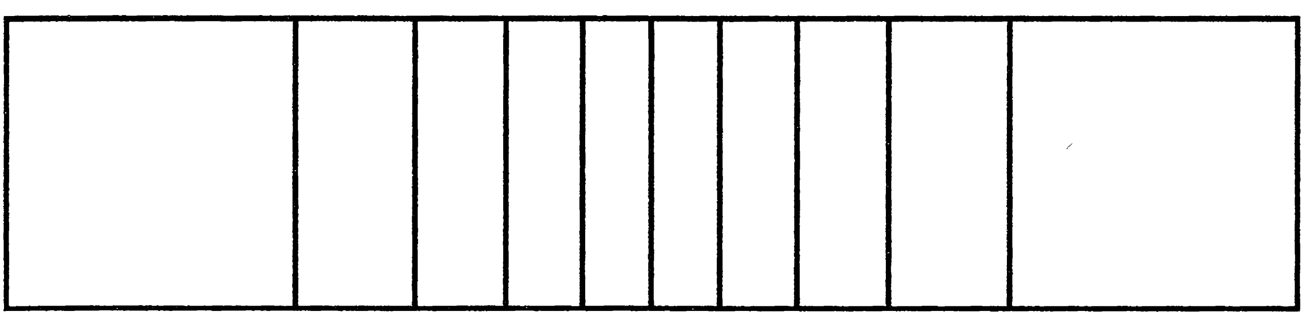
Use colored pencils to do your own shading in the middle illustration.

The bottom Root Ruler pattern is left for you to subdivide into smaller Root Rectangles with your compass into any pattern you can discover.

Other activities with these patterns include using beads, weaving fabrics, baskets and other crafts.







Constructing Root Rectangles *Within* A Square

We've constructed root rectangles starting with a square and expanding diagonals out beyond them. But there are also geometric forces pressing *within* the rigid bonds of a square which can shape a sequence of root rectangles within it. This is yet another way to make a Root Ruler.

(1) Begin with a square (see next page). Open your compass between the bottom corners and swing an arc upward to the opposite corner. Then reverse the compass and swing another arc upward.

(2) Draw the square's two diagonals.

(3) Notice the points where the arcs cross the diagonals. A line connecting them and extended to the square's sides show us the top of a Root Two Rectangle.

(4) Now draw two diagonals in that Root Two Rectangle. Notice where they cross the original arcs. The line segment joining them is a the top of a Root Three Rectangle.

(5) The diagonals in the Root Three Rectangle cross the arcs to show us the top of a Root Four Rectangle, and its diagonals cross the arcs to show us the top of a Root Five Rectangle. This process can continue as far as the pencil will allow to produce more and more Root Rectangles within a square. Always notice where the new diagonals cross the arcs.

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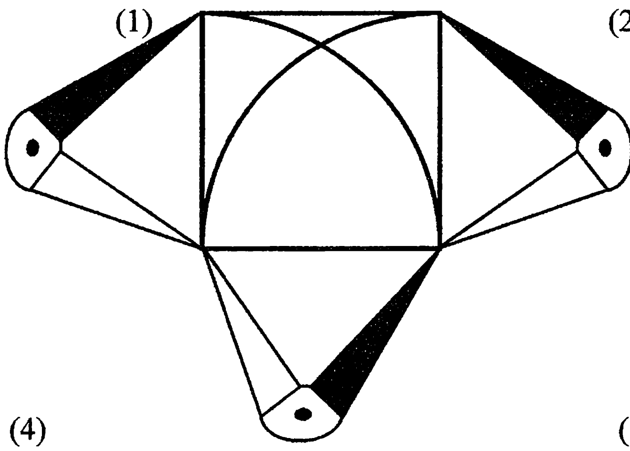

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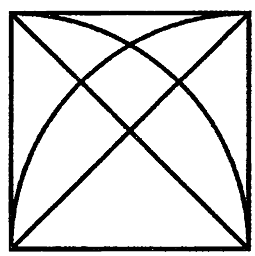

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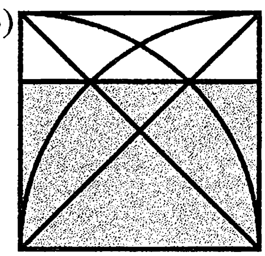

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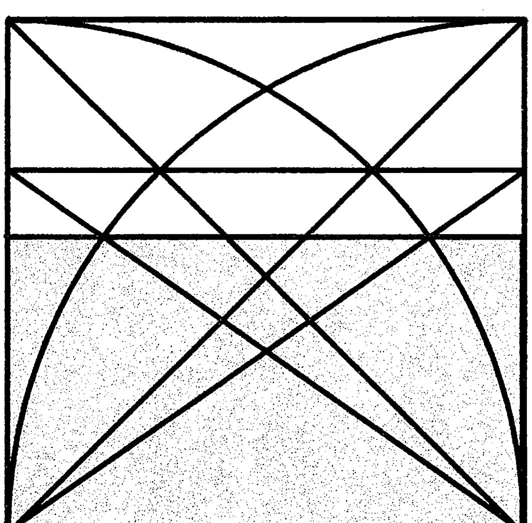

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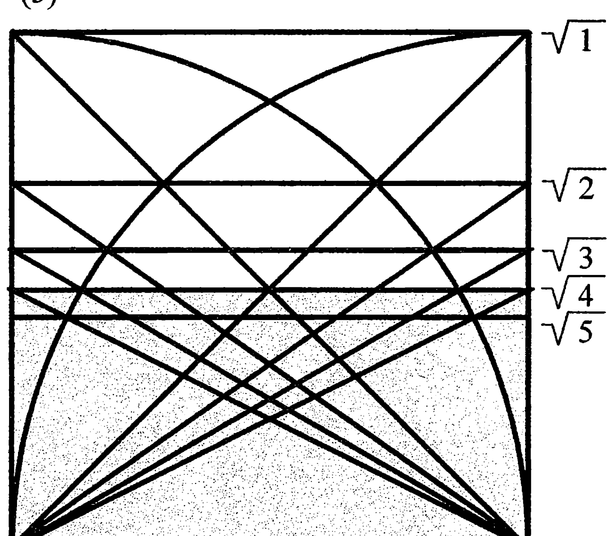


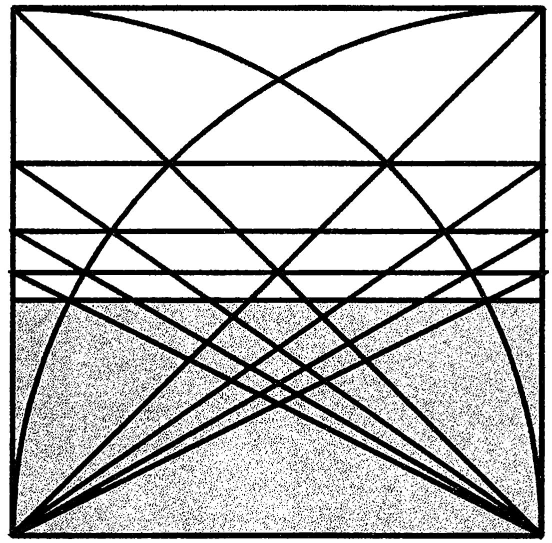
Construct A Root Ruler Within A Square

Use your compass and straightedge to construct the series of Root Rectangles within the square below or in another one.

Continue as far as your pencil will allow.

Shade different sections with colored pencils.

Start by drawing two arcs and two diagonals.

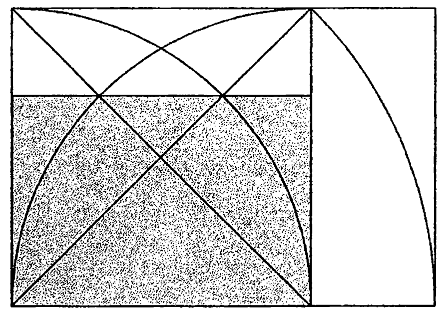


Relative Areas In A Root Rectangle

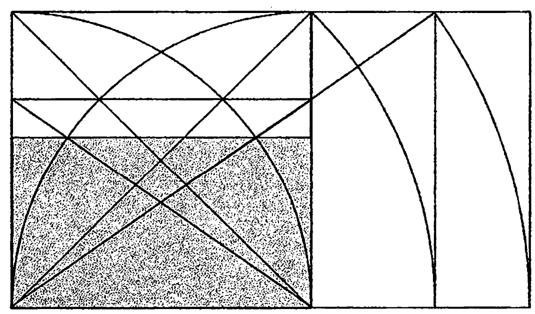
There's a clear relationship between the large root rectangles extending *beyond* the square and the root rectangles *within* the square.

Each small root rectangle is that number's fraction of the whole.

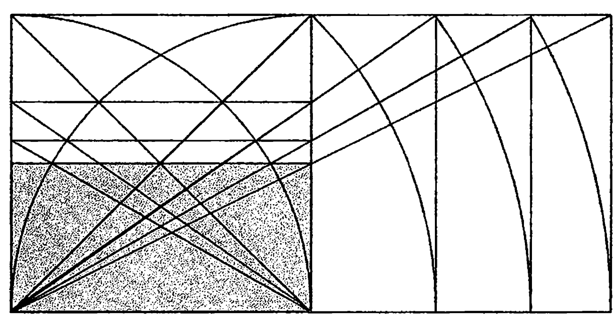
The Root Two Rectangle within the square is *one-half* the area of the full Root Two Rectangle constructed beyond the square.



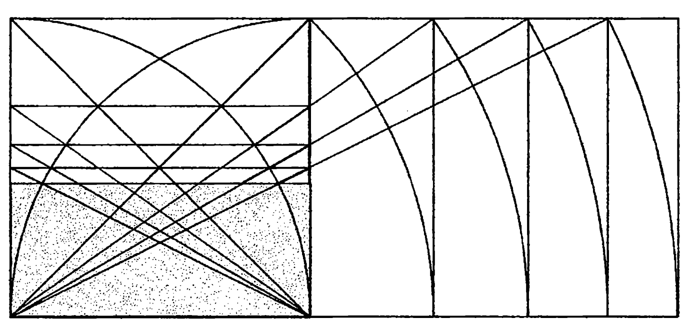
The Root Three Rectangle within the square is *one-third* the area of the large Root Three Rectangle.



The Root Four Rectangle within the square is *one-fourth* the area of the large Root Four Rectangle.

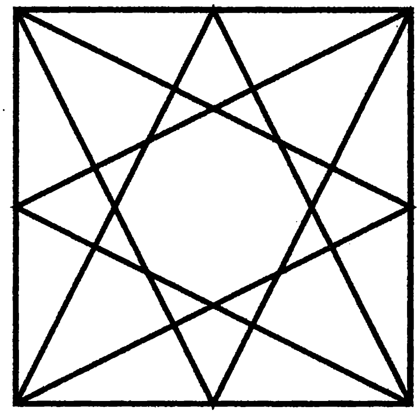


The Root Five Rectangle within the square is *one-fifth* the area of the large Root Five Rectangle.



Can you prove this mathematically?

Divide Any Rectangle Into Equal Parts

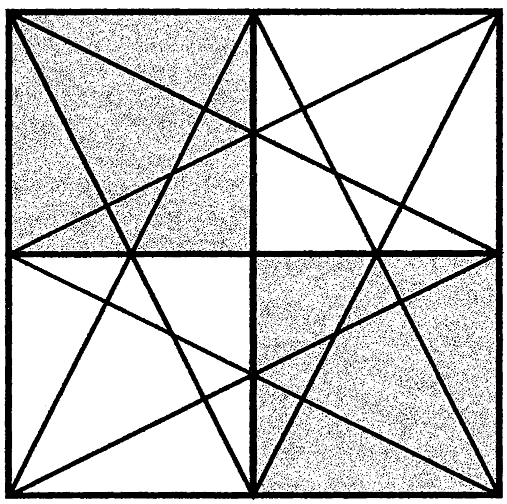


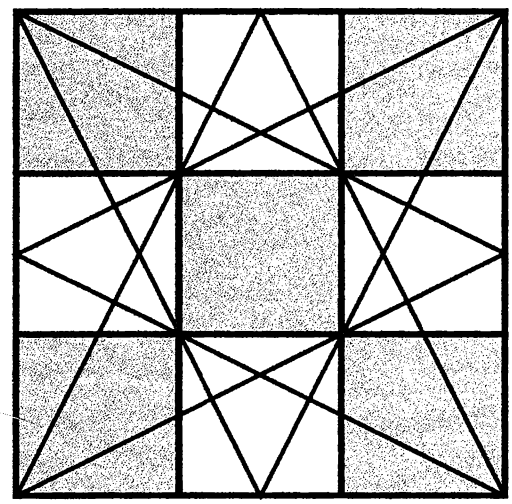
If you can find the midpoints of the sides of any rectangle you can divide its sides into equal sections, and area into equal areas. (The centers of sides appeared during the square's construction.)

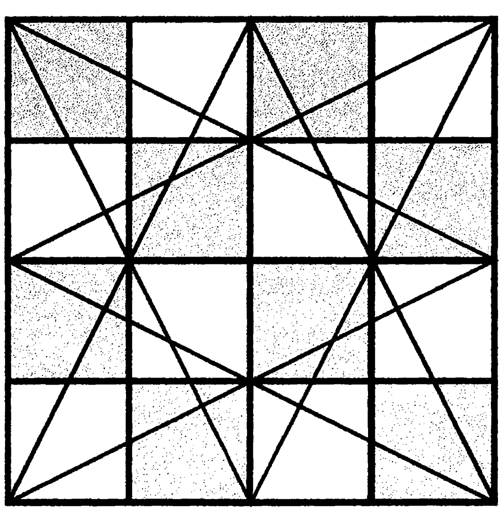
This is a useful geometric tool for exploring geometry and art.

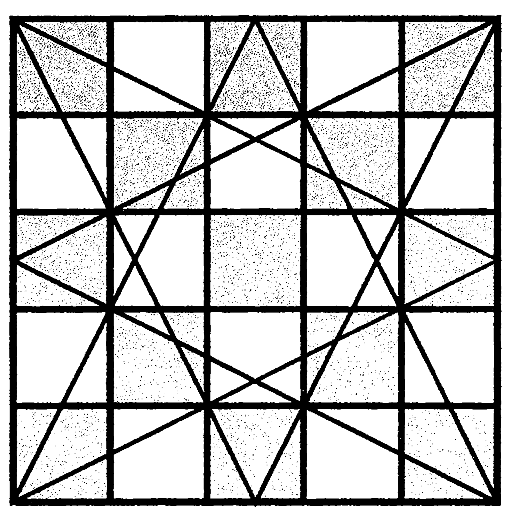
Connecting any rectangle's corners and midpoints (shown here) creates crossing points which may be connected to reveal equal divisions of the rectangle.

Look very carefully at these divided squares to see *which sets of crossing points* will divide the whole into 2x2, 3x3, 4x4, or 5x5 parts.

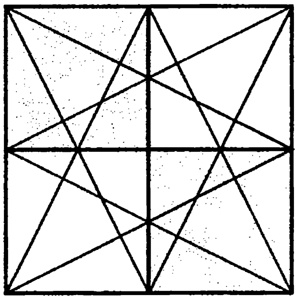
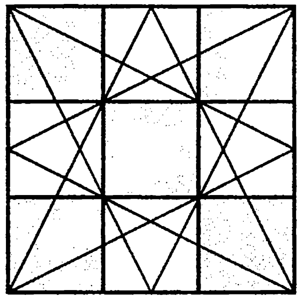
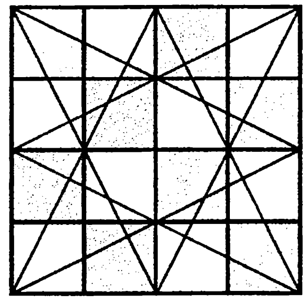
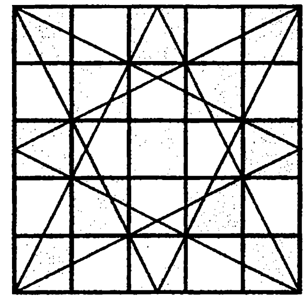


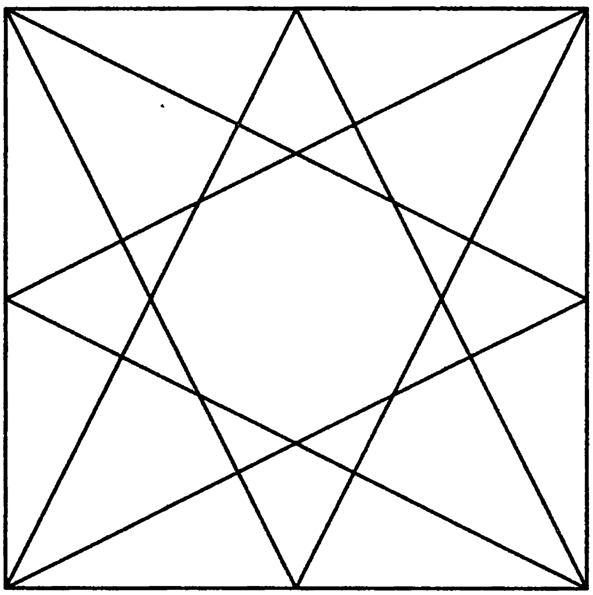
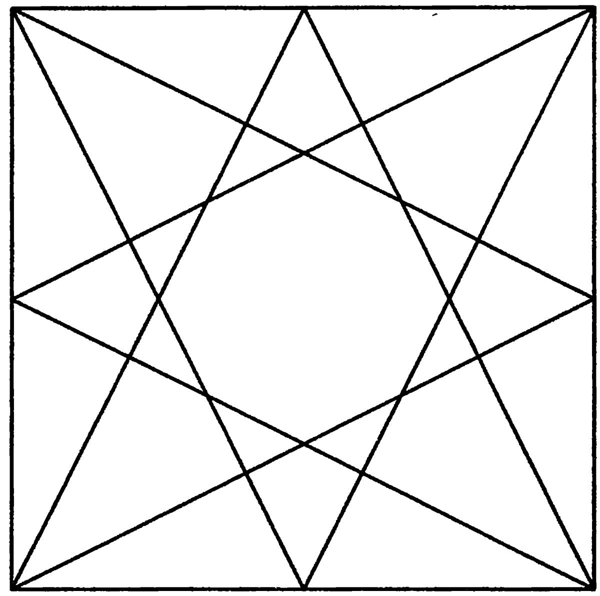


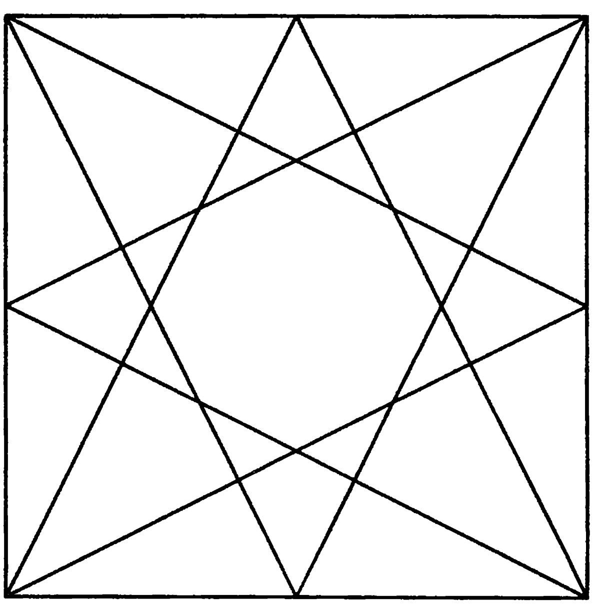
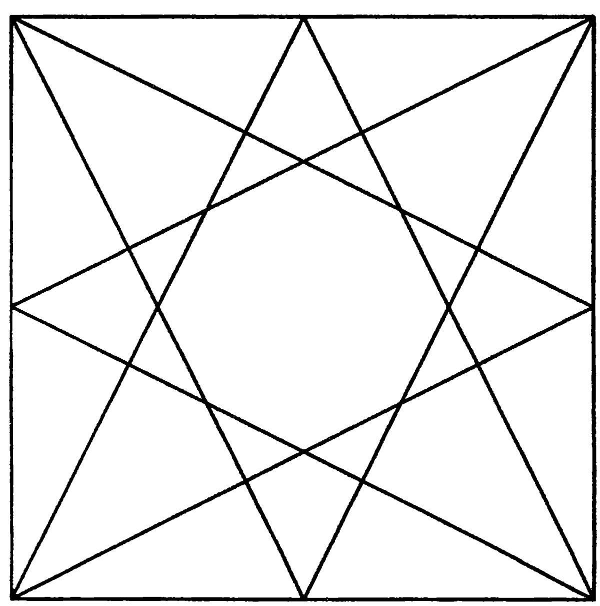




Use your pencil and straightedge to divide the squares below into 2x2, 3x3, 4x4, and 5x5 parts.

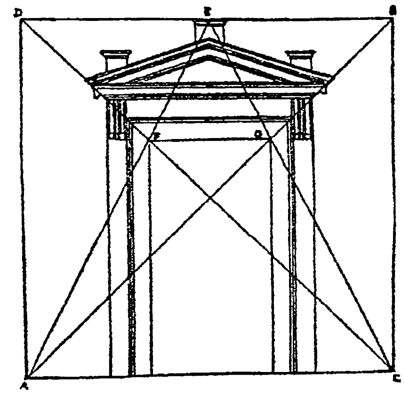





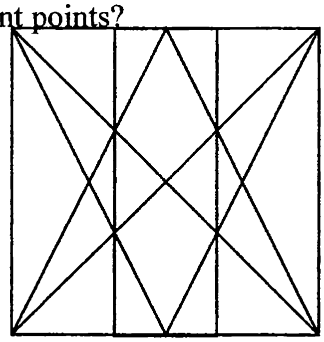
Serlio's Door

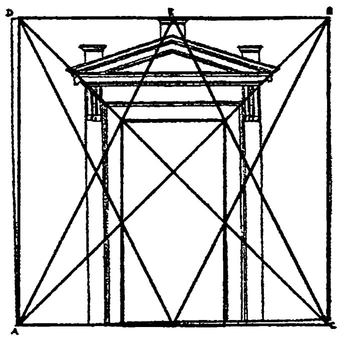
Sebastiano Serlio (1475-c1554) was an Italian Mannerist architect. His writing *Tutte l'opere d'archittura et prospetiva* is considered the most influential architectural treatise of the Sixteenth century. Serlio was interested in designing architectural harmony through geometry and number by modeling the harmony found in the simple mathematics of music.

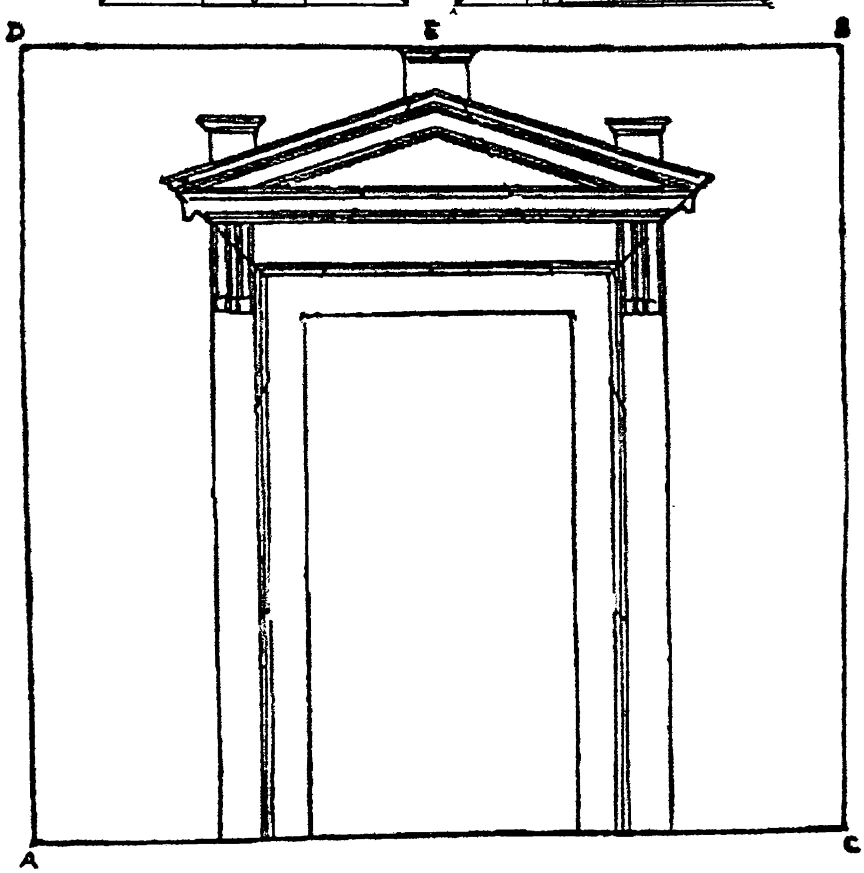


This drawing of a door appears in Serlio's *Libro primo d'architettura*. The width of the door, the height of the door and the side of the square are in the ratio 1:2:3, the "harmonic proportion" found in the ratios of musical concords. (See Volume 1 pages 14-17.)

You can analyze the door by dividing the overall square into thirds using the squares midpoints and corners. Notice where they cross. What other elements of its composition can you find by drawing lines between important points?







Divide Root Rectangles Into Equal Parts

When any "number" Root Rectangle is divided into that many equal parts, each part is a small, turned model of the whole Root Rectangle!

So, dividing a Root Two Rectangle into two equal parts, or a Root Three Rectangle into three identical parts, or a Root Four Rectangle into four equal parts, and a Root Five Rectangle into five equal parts àll produce small models of their whole.

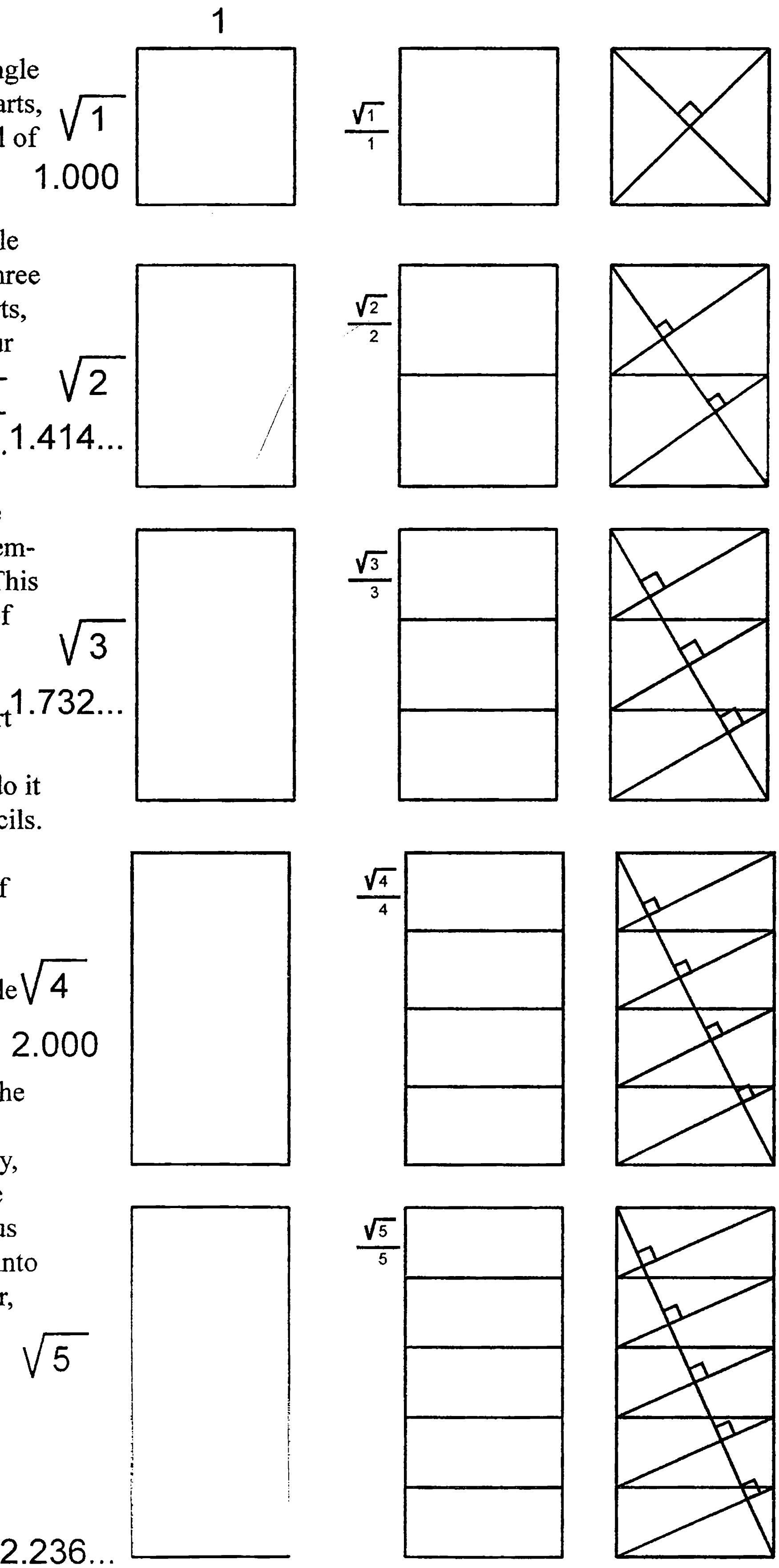
Each small rectangle may also be subdivided into smaller parts resembling other parts and the whole. This self-symmetry is really the core of dynamic symmetry.

Notice also that the long and short diagonals *cross at right angles*. Draw diagonals in the blanks to do it yourself. Shade with colored pencils.

Each crossing point is an "eye" of the rectangle, around which self-replicas whirl. We also see these properties in the Golden Rectangle (see Volume 3).

Each smaller rectangle is called the "reciprocal" of the next larger rectangle because, mathematically, its sides are in inverse ratio to the whole. The equation below tells us that any Root Rectangle divided into that many parts will make smaller, inverted models of the whole.

$$\frac{\sqrt{n}}{n} = \frac{1}{\sqrt{n}}$$



This *equal division resulting in self-similarity* doesn't happen for every rectangle.

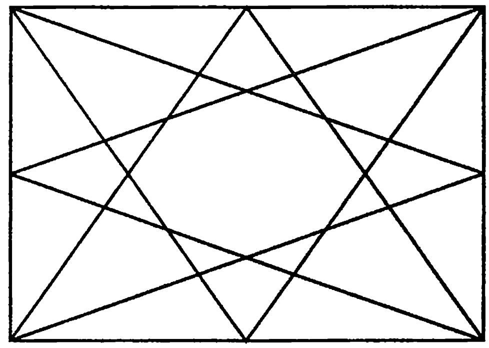
Those rectangles which are *not* Root Rectangles may also contain a turned, self-similar reciprocal within them, but *a whole number of them* will not exactly fit within the whole. There will always be a gap or overlap.

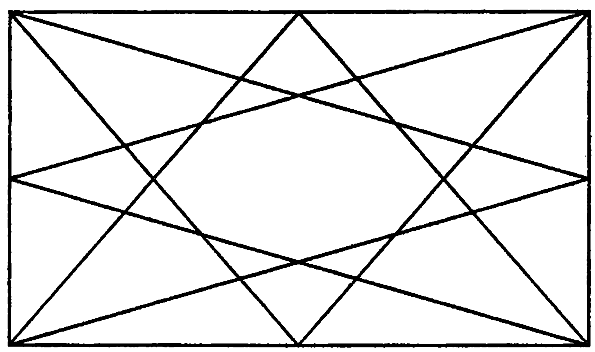
But Root Rectangles contain exactly a whole number of their own reciprocals within themselves.

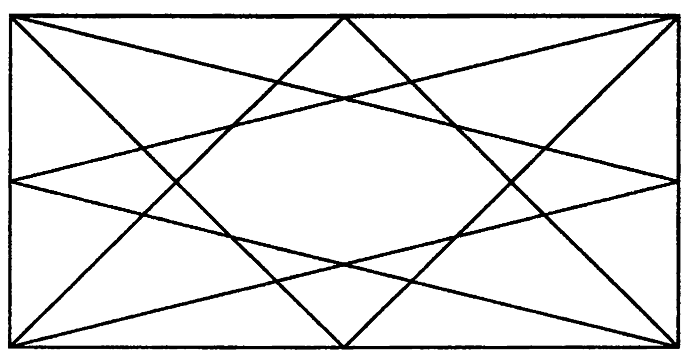
These properties of wholeness, equality and self-similarity are among the reasons that they are called harmonious, dynamic rectangles.

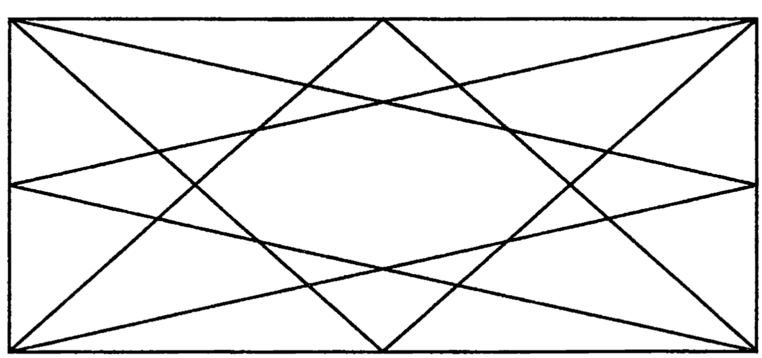
Use the techniques of the previous pages to divide these Root Rectangles (Root Two through Root Five) into their own number of identical parts which model the whole.

Then draw diagonals across the large and small rectangles to see that they cross at right angles.









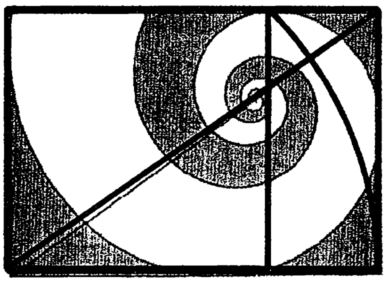
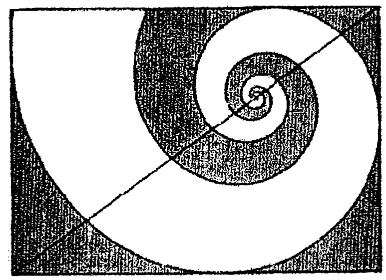
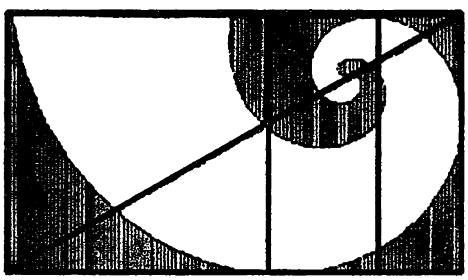
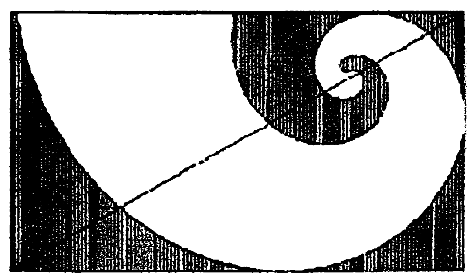
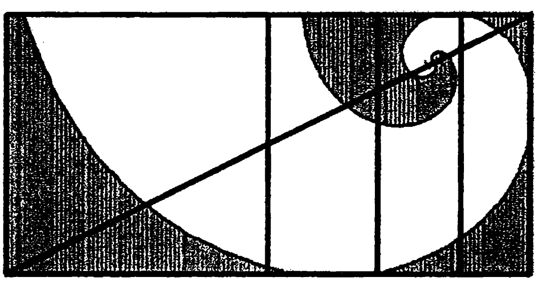
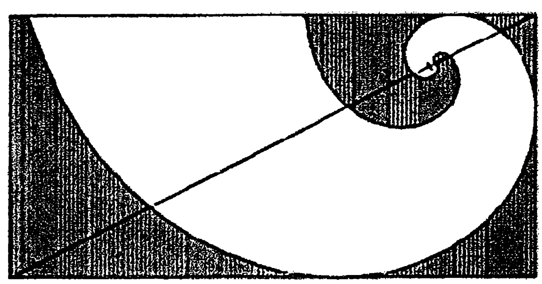
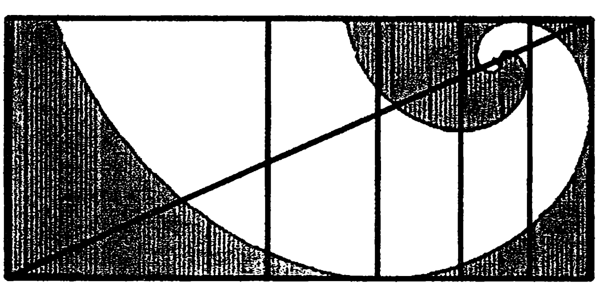
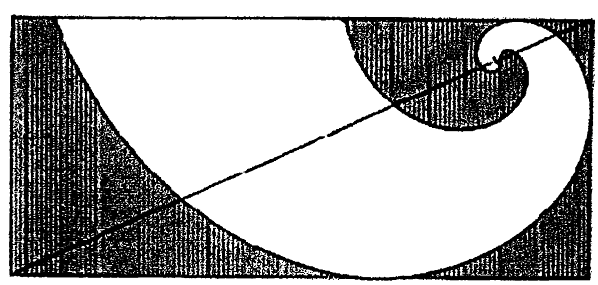
Spirals Within Root Rectangles

Because Root Rectangles are "dynamic" and contain smaller models of themselves, they will also contain beautiful and graceful spirals.

The Root Rectangles below are constructed by the process of expanding diagonals.

Notice how one full diagonal across each Root Rectangle passes through the spiral's eye.

Shade the spirals with colored pencils.

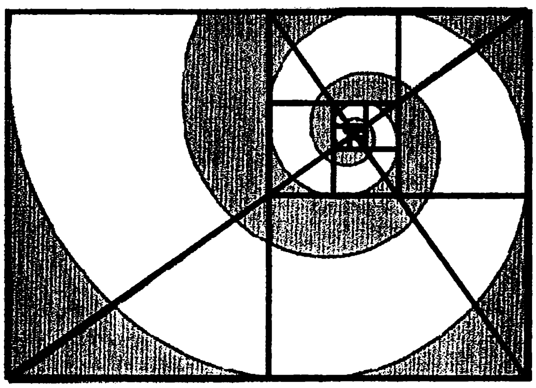









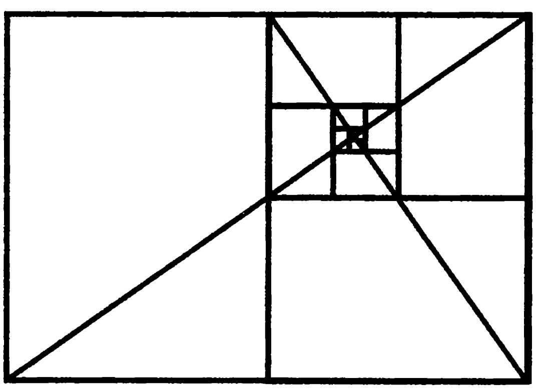
Below, each Root Rectangle is divided into self-similar parts. The smaller and smaller rectangles lead around and into the eye of the spiral. Notice how each diminishing Root Rectangle contains the same-shaped section of the spiral, only smaller.

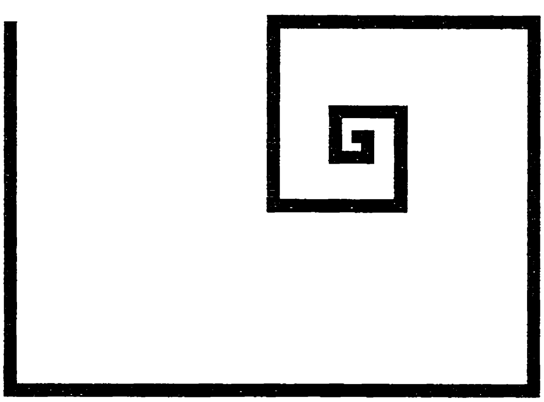
And notice how the long and short diagonals cross at the eye.
Can you find other diagonals which cross at the eye?

Self-similarity of parts interlock to produce a whole spiral. The "squared" spirals are produced by following around the sides of each smaller Root Rectangle.
Use thick colored pencils to highlight the curved and squared spirals.

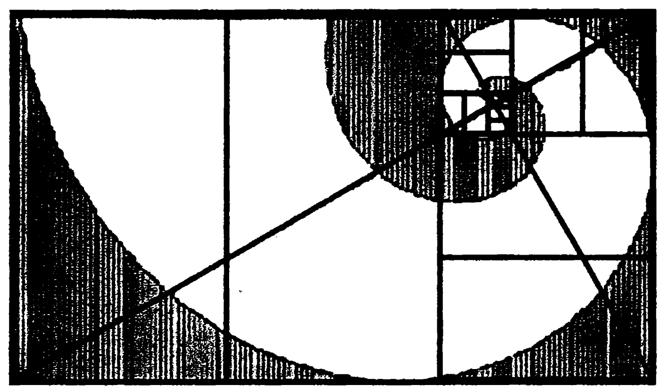
Each **Root Two Rectangle** is continually divided in half, making smaller and smaller Root Two Rectangles.

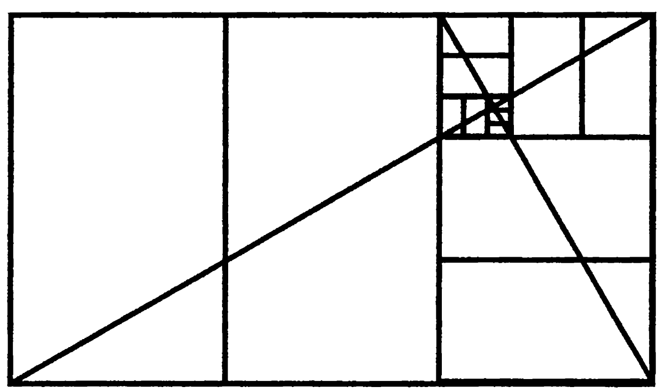


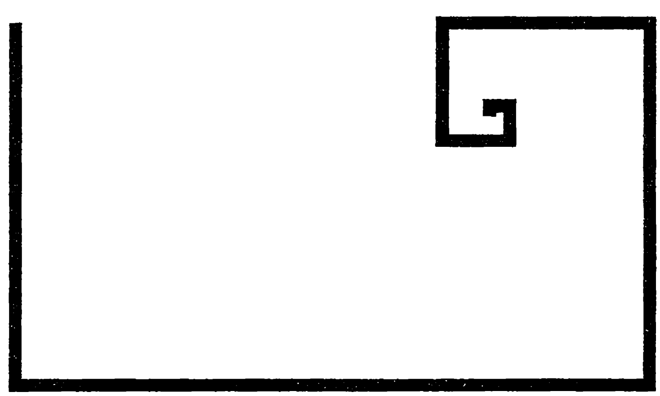




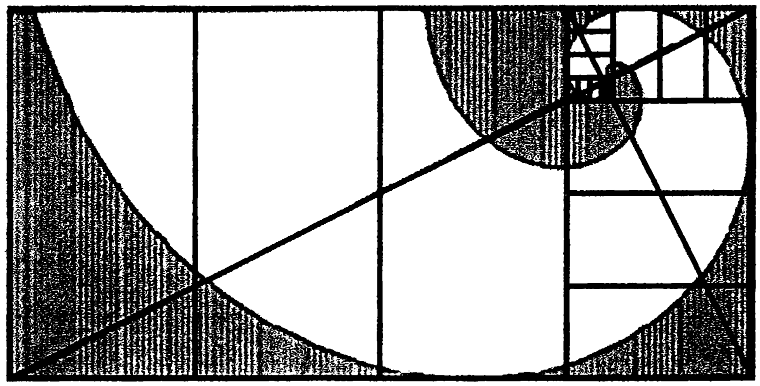
Each **Root Three Rectangle** is continually divided into thirds, making smaller and smaller Root Three Rectangles.

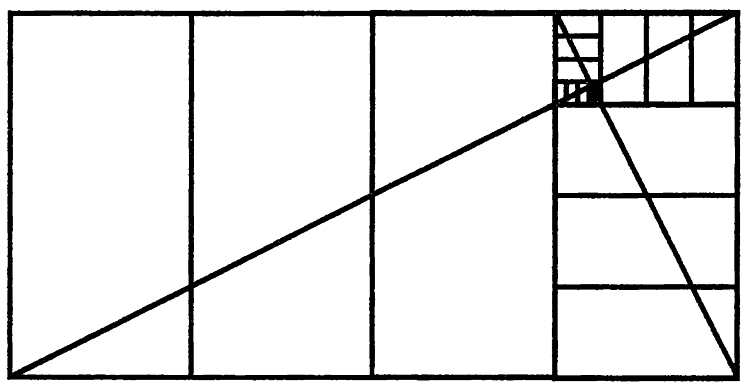




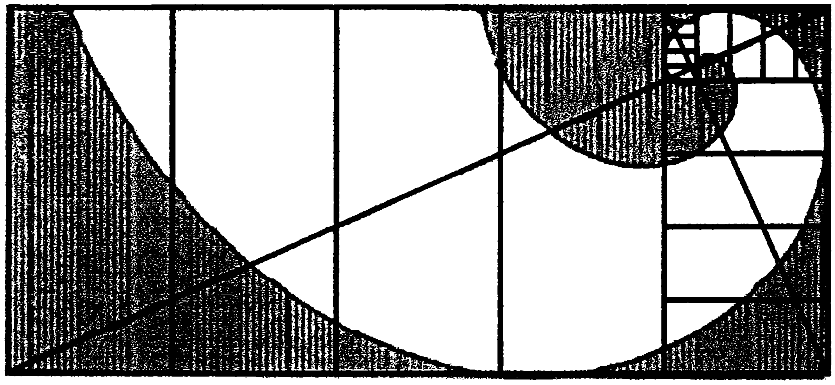


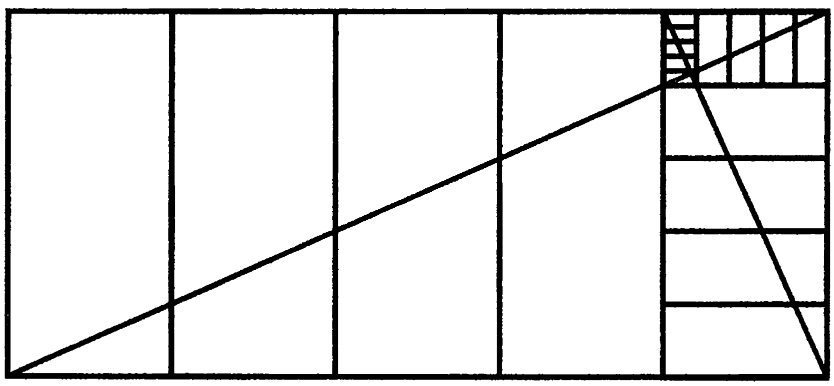
Each **Root Four Rectangle** is subdivided into fourths, making smaller and smaller Root Four Rectangles.





Each **Root Five Rectangle** is subdivided into fifths, making smaller and smaller Root Five Rectangles.

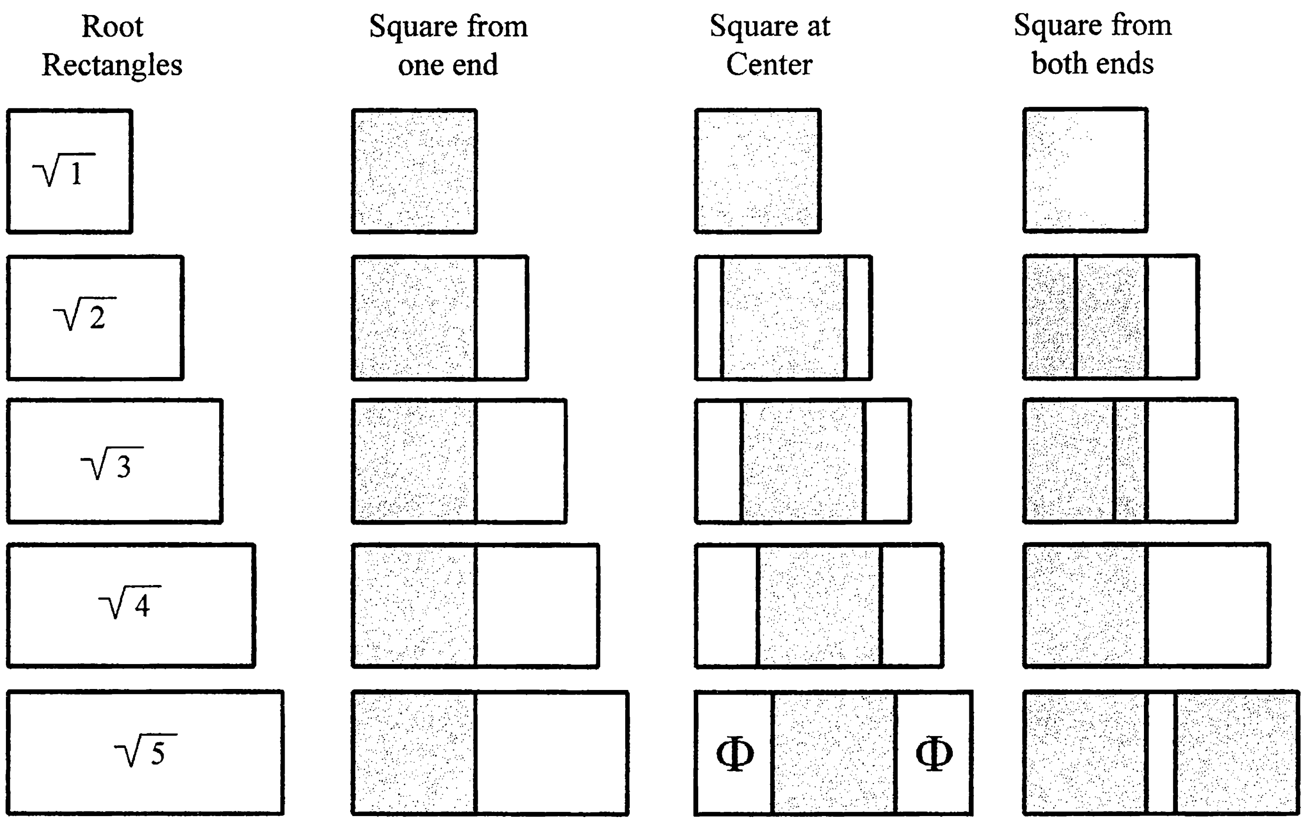




Root Rectangle Rabatment

Another very important geometric technique for exploring Root Rectangles is called “rabatment”. The word derives from the French for “folding back”, the way a cloth collar can be folded back.

Essentially, rabatment involves folding back or drawing a square at one or both ends of a rectangle, and sometimes at its center. This act reveals significant points, lines and areas within the root rectangle, and is very useful when analyzing Root Rectangle art. Here are the first five Root Rectangles showing the main rabatments or positions of squares. Notice that when squares are drawn from both ends they first overlap then grow apart.

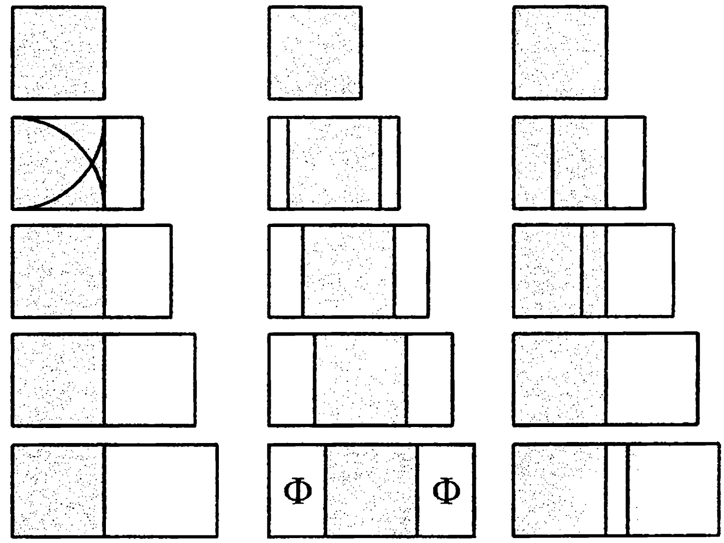


Notice that the Root Five Rectangle is actually a square with small Golden Rectangles at each end! (See Volume 3)

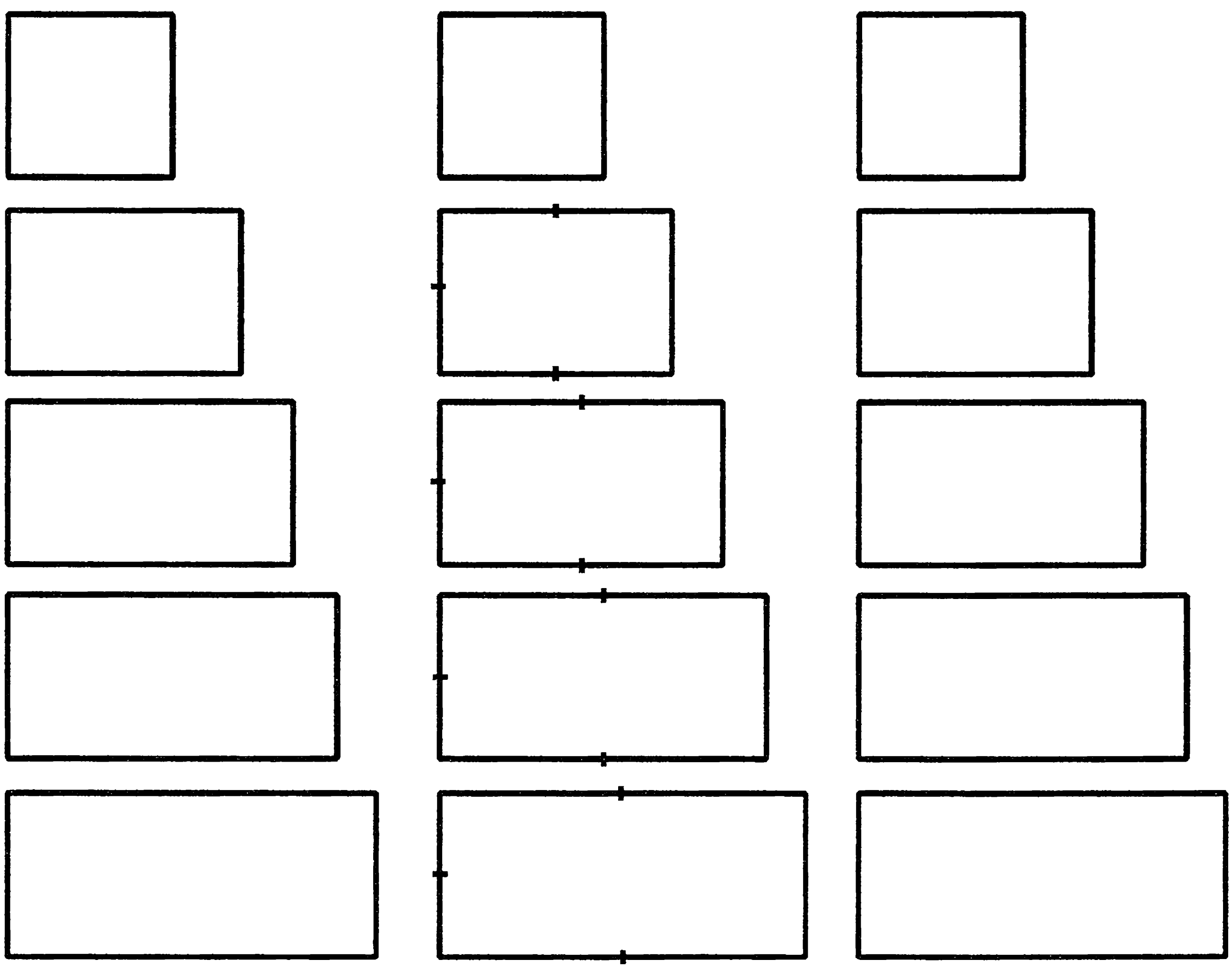
Rabat Squares In Root Rectangles

The way to draw a square at the end of a rectangle is to open the compass to the short side, then mark off this distance on both of the longer sides, and connect the points.

The way to draw a square at the center of a rectangle is to first place the compass point at the middle of the short side and open the pencil to the corner. Then put the compass point at the centers of the rectangle's long sides and mark the corners of the square on each side of it. The centers of the sides of the rectangles in the central column below are identified for you.



Use your compass, straightedge and colored pencils to apply Rabatment (as seen above) to the blank Root Rectangles below.



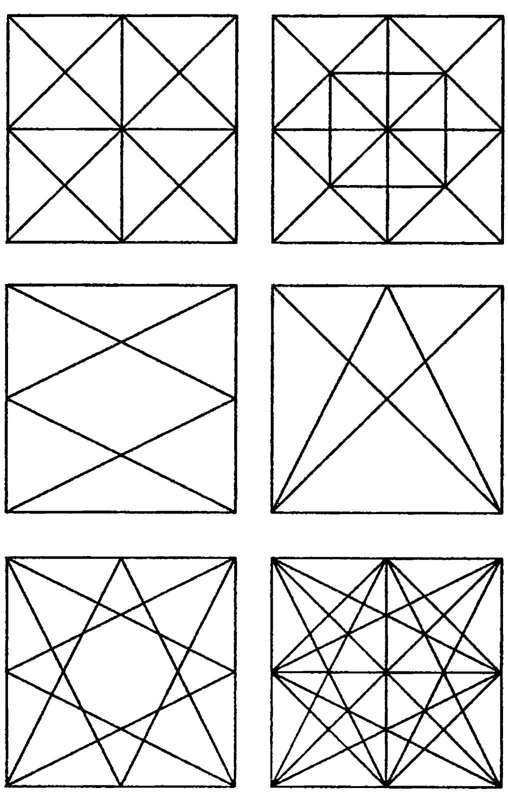
Hotspots: Points, Lines and Areas Of Visual Interest

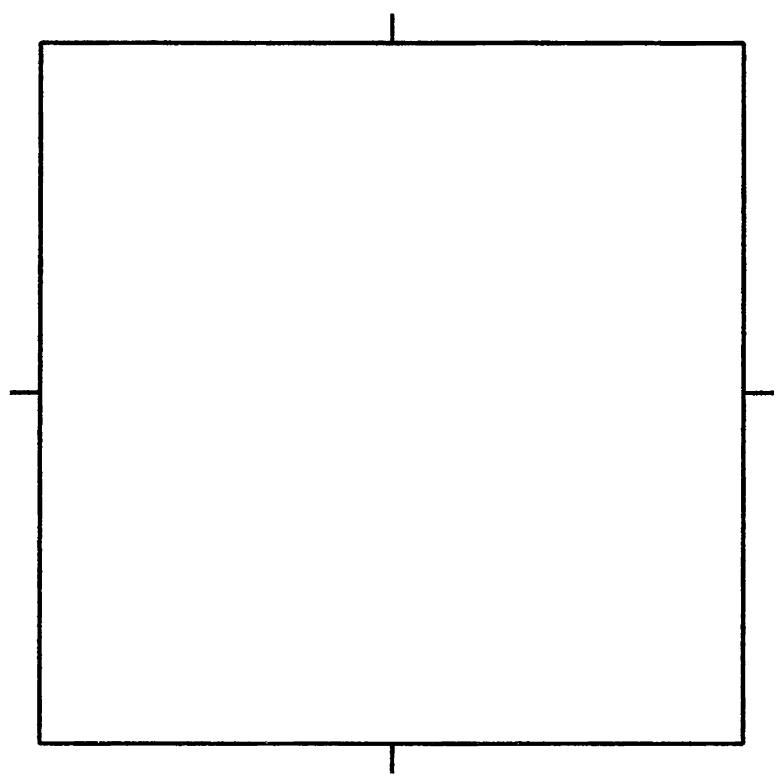
A rectangle is more than just four sides, corners and right angles. Inside is space. Enclosed space isn't empty but filled with geometric relationships which we can make visible by lines. But they exist whether we show them or not.

There are certain places within a rectangle that we're unconsciously sensitive to. For example, imagine two diagonals crossing in a square. We can feel when diagonals are crossing off-center. We can "feel" these points, lines and the areas they create without actually seeing them drawn in the rectangle. There are many such points, lines and areas within every rectangle. They're called various names including "visual points of interest", "points of attention", "visual centers of gravity", "points of visual peace", "lines of tension" and simply "hotspots". Our mind gives greater weight to these places because they occur where the rectangle's natural geometry would place them. When artists, architects, craftspeople and designers put objects or important elements in those places they feel correct, even powerful. Our mind *expects* to find something at one or more of these hotspots in geometry and art. Classical artists across cultures knew about this and we'll see many examples of how they applied this idea.

Below is a square with midpoints marked on its sides.

Use your straightedge with colored pencils to connect them in different ways and reveal visual points, lines and areas of interest to shade.

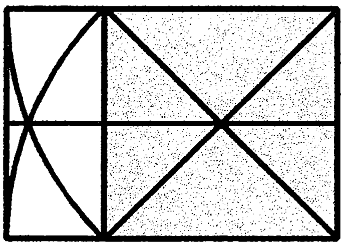


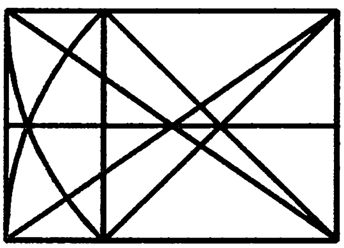


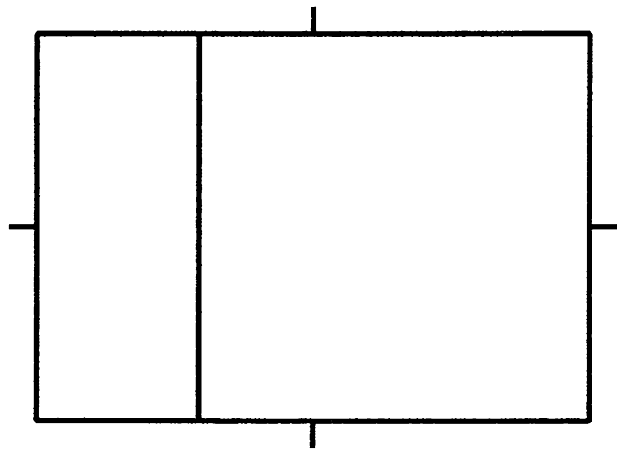
Along with lines connecting corners and midpoints, the points, lines and areas created by the sides of squares of rabatment also play roles identifying points of visual interest.

Arcs drawn with a compass between significant points will also identify interesting places which can be emphasized in a composition.

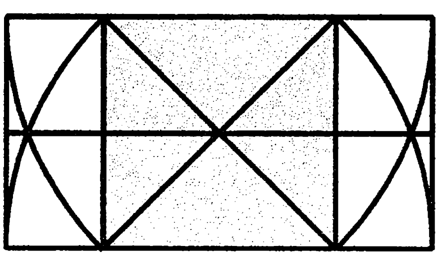
Below are Root Two Rectangles with a square drawn at one end. Midpoints of the sides are also identified. Use your compass and straightedge (and colored pencils) to draw lines and arcs among these points. Identify them as hotspots.

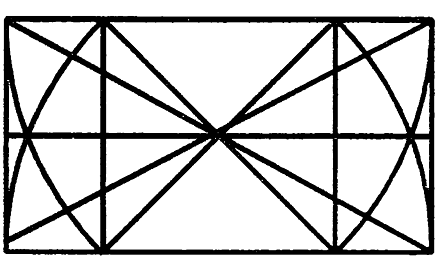


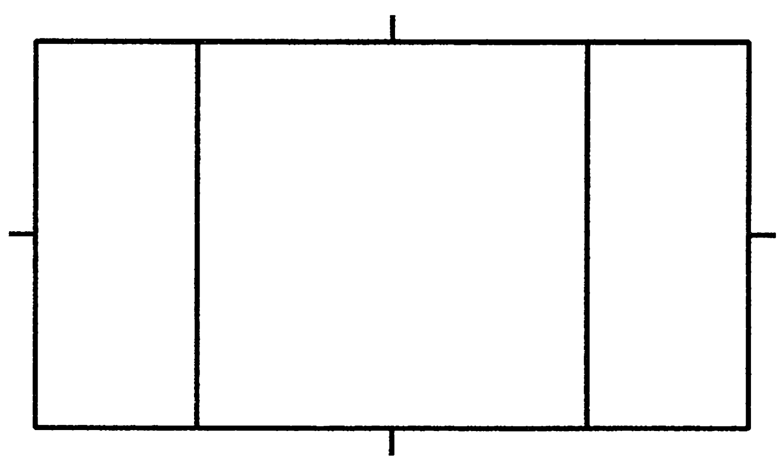




Below are two Root Two Rectangles which overlap at their central square. Use your compass and straightedge (and colored pencils) to draw lines and arcs among these points, and find new ones. Identify them as hotspots.

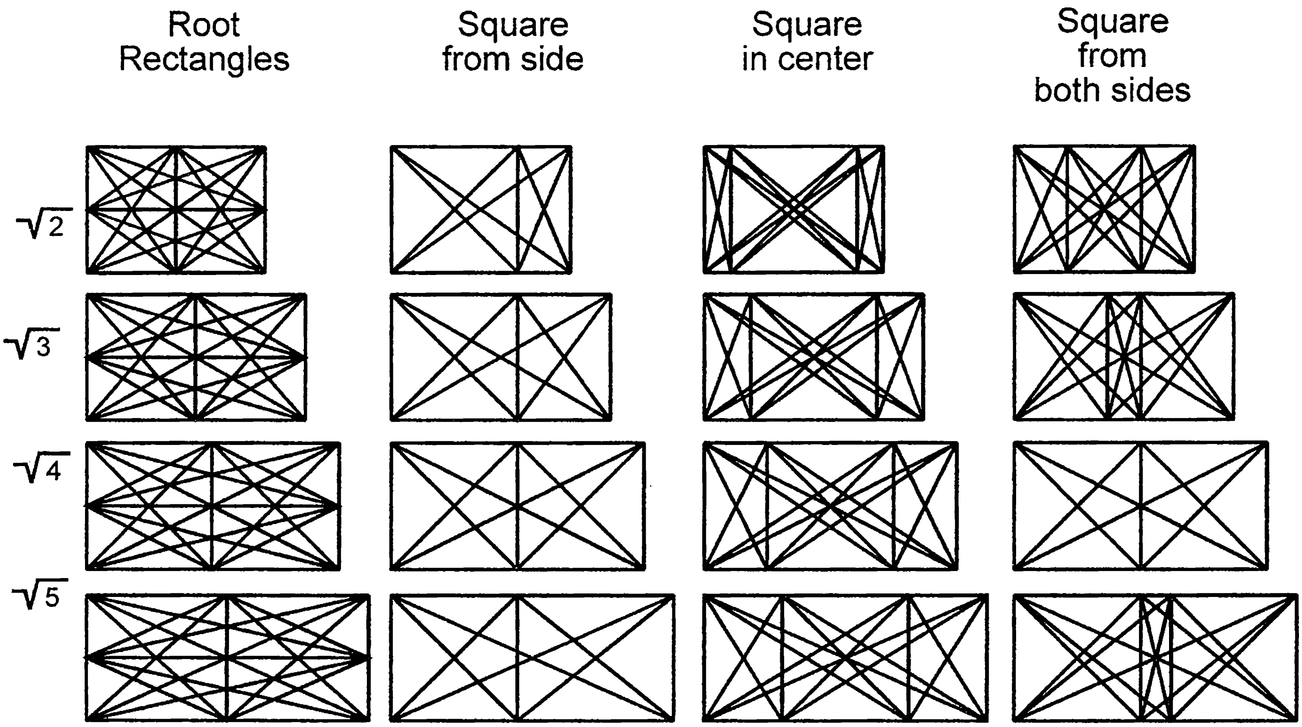


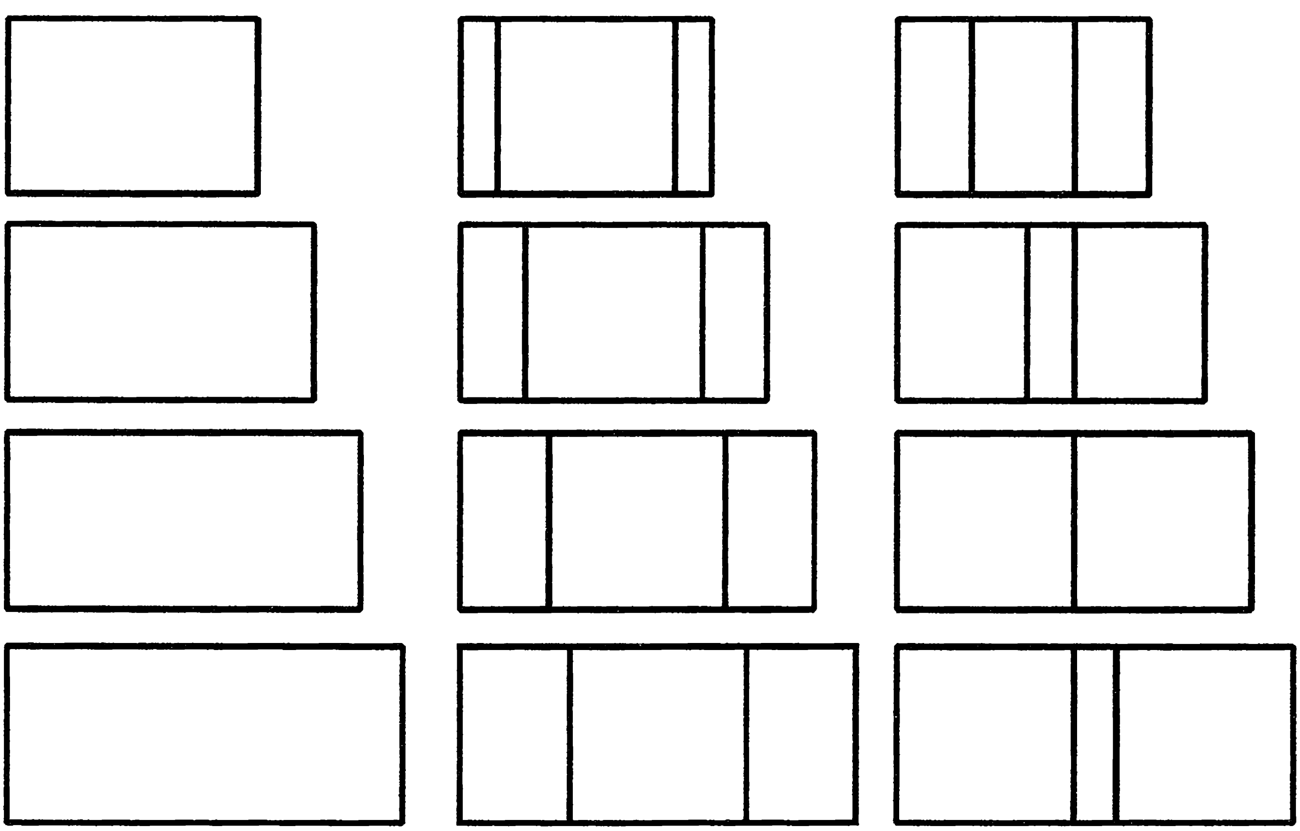




Locate More Visual Points Of Interest

Rabatment of squares within Root Rectangles give an artist many opportunities to identify interesting points, lines and areas of interest to where a viewer's eye will be drawn and comfortably dwell. Here are some possible lines to draw. Use colored pencils to shade areas. In the blank Root Rectangles below draw lines but also arcs to find significant points of interest.

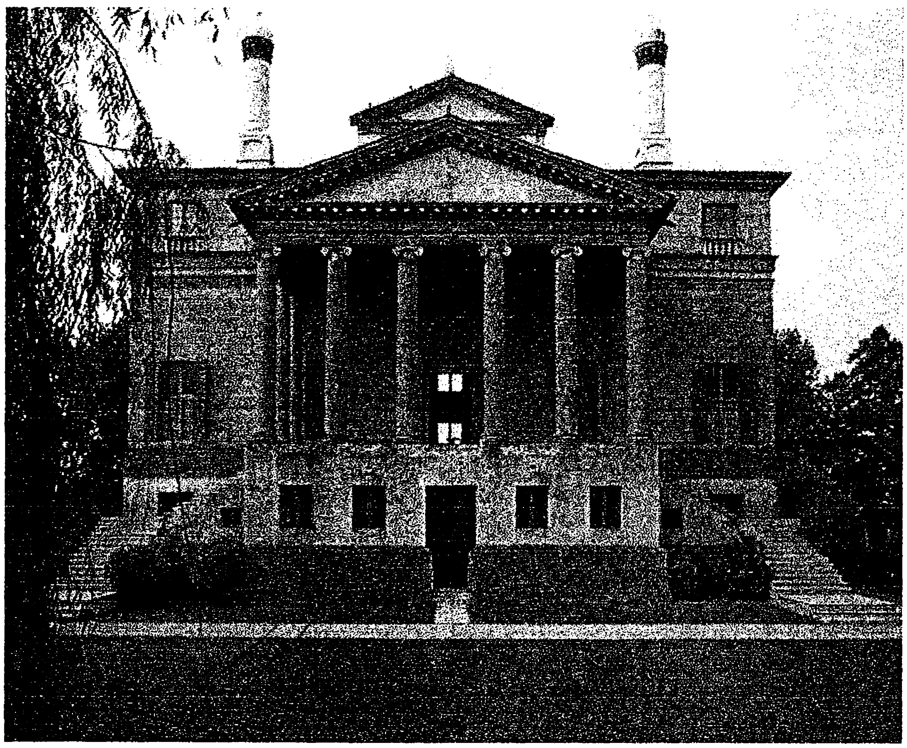




That has been our general introduction to the geometry of Root Rectangles.

We've seen how to construct a foundation square and then unfold the infinite series of Root Rectangles from it. We also learned how to divide and subdivide these Root Rectangles in various ways. These will be useful tools for exploring these rectangles in further detail, and to see how great masters of art, crafts, architecture and design made powerful use of them and these techniques for developing them.

Now let's look more directly at how the Root Two Rectangle has been applied to art.



Palladio's *Villa Foscari*

Root Two Rectangles

But, first, a word about the square root of two.

Computers can calculate square roots to millions of decimal places. The square root of two (to only 100 places) is:

1.4142135623730950488016887242096980785696718753769480731766797379907324784621070388503875343276415727...

The square root of any number multiplied by itself equals that number, and it's true for the square root of two. But try putting any *limited* number of its neverending digits, even the 100 digits above, into a calculator or computer, the answer will always fall short, something like 1.99999999999999999999998...

There are some useful approximations of the square root of two in simple fractions. More than forty centuries ago the Egyptians knew about 99/70 which they wrote as the sum of "unit fractions", fractions whose numerator is always one:

99/70 = 1 + 1/5 + 1/7 + 1/14 = 1.414285... quite close!

Thirty seven centuries ago the Babylonians sought an accurate value for the square root of two and they came up with this very good approximation (in their base-60 system, the origin of our timekeeping and circular measure):

1;24,51,10 = 1 + 24/60 + 51/3600 + 10/216000 = 1.41471... not too bad an approximation.

Thirty centuries ago in Vedic India, books of cord-stretching proportions used in sacred architecture and sculpture called the *Shilpa Shastras* approximated a closer square root of two this way:

$$1 + \frac{1}{3} + \frac{1}{3 \cdot 4} - \frac{1}{3 \cdot 4 \cdot 34} = \frac{577}{408} \approx 1.414215686.$$

But if you don't need to be very exact, the simpler fraction 17/12 = 1.41666... comes fairly close.

When these numbers are translated into geometry (as the diagonal of a square) then we can gain insight into why they were used in art, crafts, architecture and design. Certainly different proportions have different feels which can be taken advantage of using points of visual interest. But there is also a symbolism associated with shapes and proportions used more or less the same worldwide.

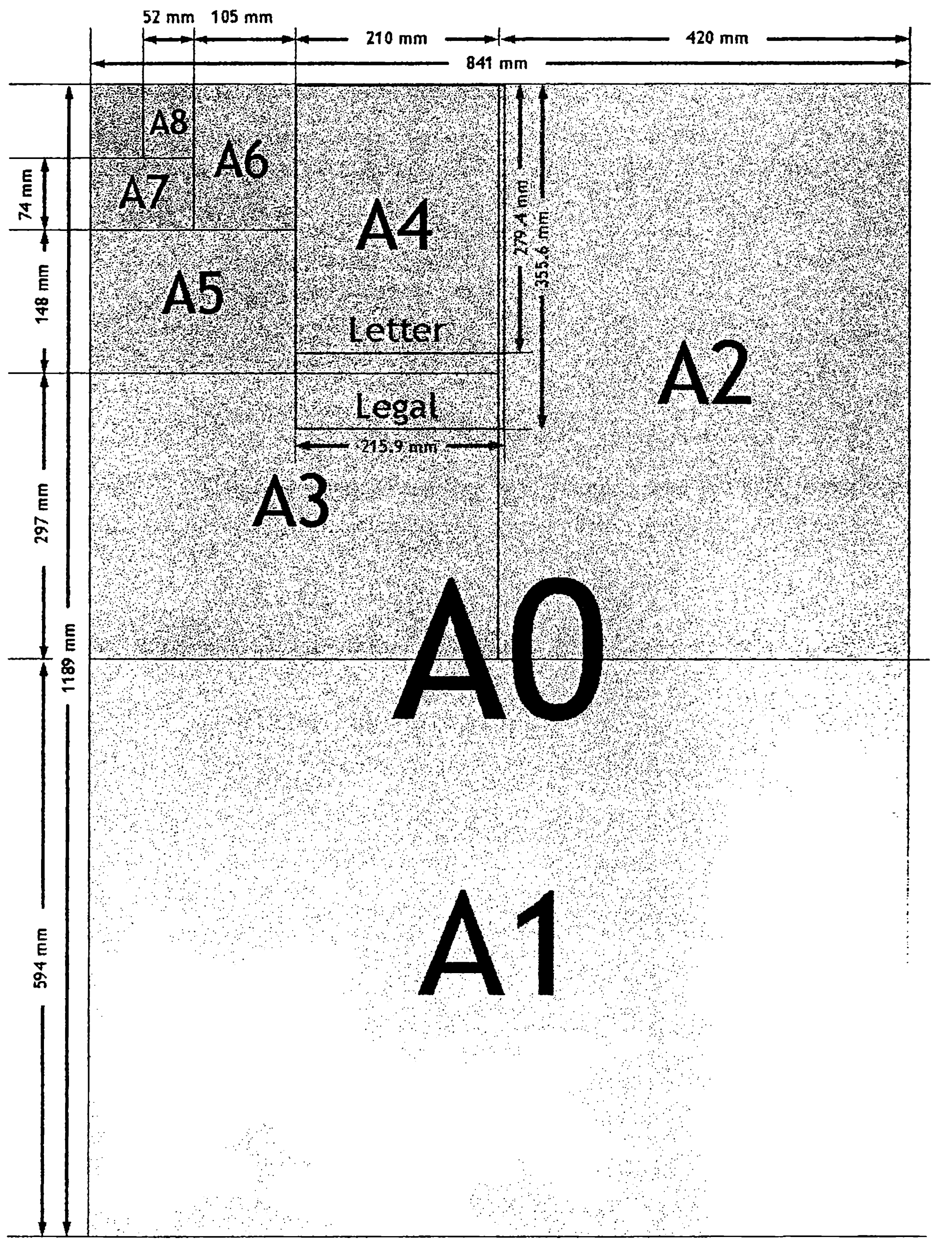
The circle everywhere is a symbol of wholeness, equality and heavenly perfection. It contrasts with the square, a worldwide symbol of earth, substance, matter (the four states of matter) and also the four directions. The octagon resembles a turning square, and so is a symbol of being intermediary between heaven and earth. This is why we see it so much in the architecture of sacred buildings across cultures. The Root Two Rectangle, being the first extension from the square, usually symbolizes birth and new beginnings. Thus we see it in the entrances to cathedrals and the shape of diplomas. Keep symbolism in mind as we look at each example. For more about this see "A Beginner's Guide To Constructing The Universe".

The International Paper Standard

A mathematical characteristic of the Root Two Rectangle has a practical application: when cut or folded in half, a Root Two Rectangle always produces smaller, turned Root Two Rectangle. Halving a Root Two Rectangle reveals its self-similarity.

This mathematical Truth appears in a way familiar to most of the world (except the United States and Canada) in the dimensions of writing paper and envelopes. It's the international paper size standard, called ISO 216. The system is metric, so the largest standard size, called A0, has an area of 1 square meter. Yet it's in the shape of a Root Two Rectangle. The importance is not in the man-made metric system, but in the eternal geometry.

Folding an A0 sheet in half creates a smaller Root Two Rectangle, called A1 size. When that is folded in half, size A2 is created, still another Root Two Rectangle. The process continues officially until A8, the smallest size.



The value of using a Root Two Rectangle is that it allows scaling without loss of image from one size to another. Thus an A3 page can be enlarged to A2 and retain the exact proportions of the original document, a fact used by photocopiers. Scalability means that less paper (and paper money) is wasted by printing companies. And if you fold you letter enough times, there will always be a proper envelope for it.

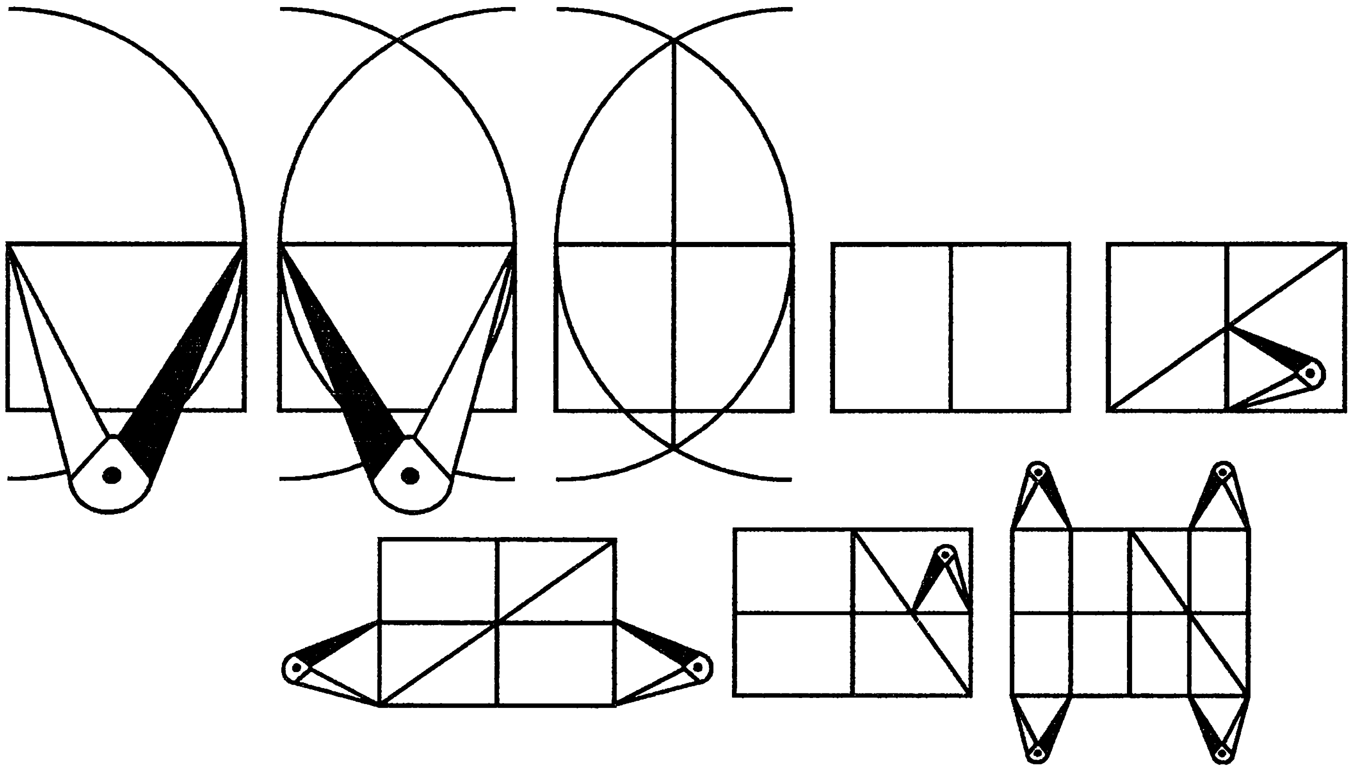
Trace this Root Two Rectangle and keep folding it in half to see that it remains a Root Two Rectangle.

It seems to be true that no piece of paper can be *folded in half* more than seven times. Try it with different sizes, shapes and thicknesses of paper.

Divide A Root Two Rectangle In Half

To easily divide any line segment or rectangle side in half we use the technique of the "almond" (see Volume 1 Chapter 2). Here's how:

Just open the compass to the length to be divided and swing crossing arcs to make the almond shape. Draw a line connecting the crossings which should halve the original line. Further subdivisions don't necessarily require the almond. Notice that a diagonal across each root rectangle crosses the division between root rectangles at their center. Just open your compass along the edge to this distance and transfer it to the sides. Connect the points and divide the rectangle in half. Continue the process.

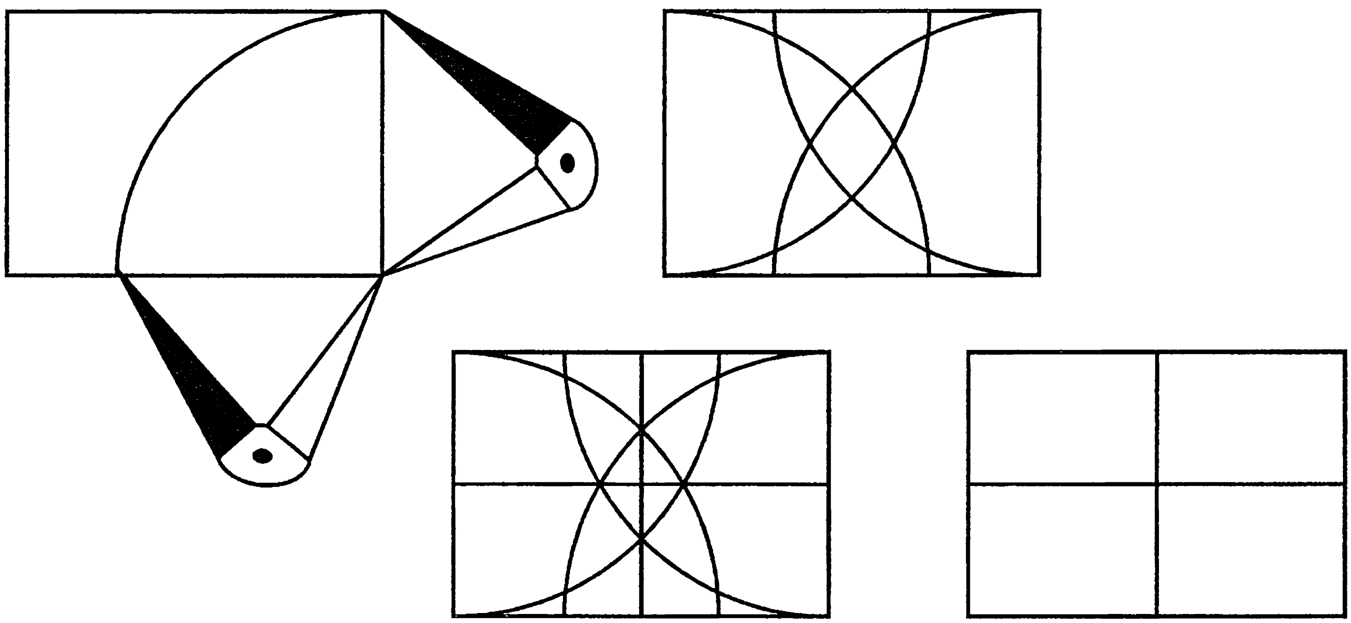


Another Way To Halve A Root Two Rectangle

This is done simply with arcs made from the shorter side.

Use your straightedge to connect points and extend the line segments to cross the longer sides.

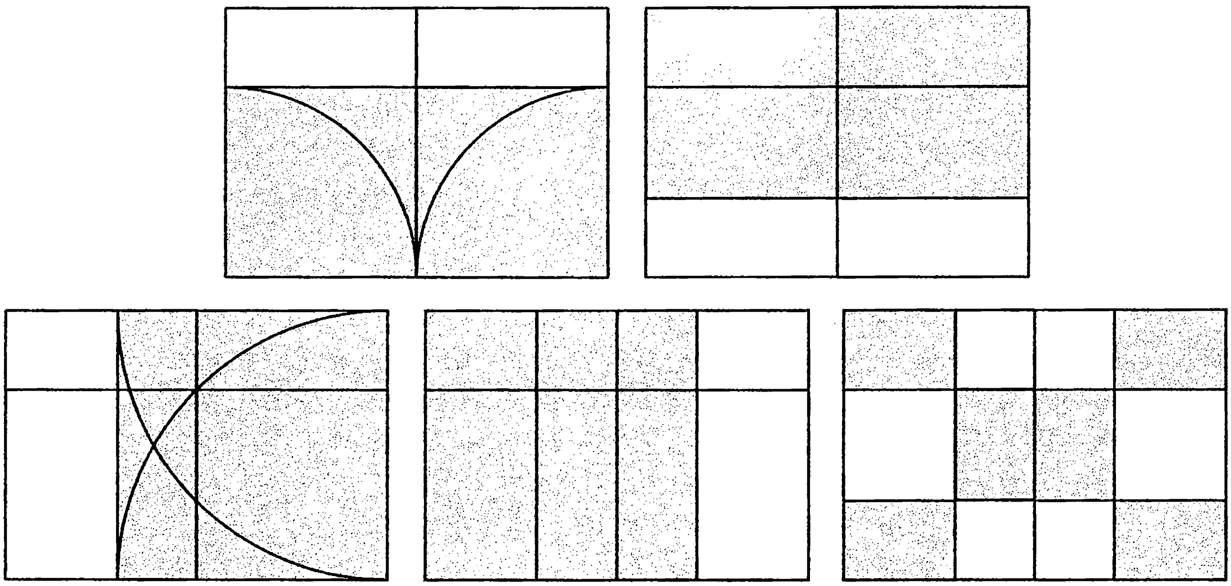
Repeat it in the next smaller Root Two Rectangles.



Subdivide A Root Two Rectangle By Rabatment

A Root Two Rectangle has been divided in half and a square has been drawn from the bottom of each. Another square is drawn inside the top of each half root Rectangle. More squares have been drawn inside the left and right ends which overlap.

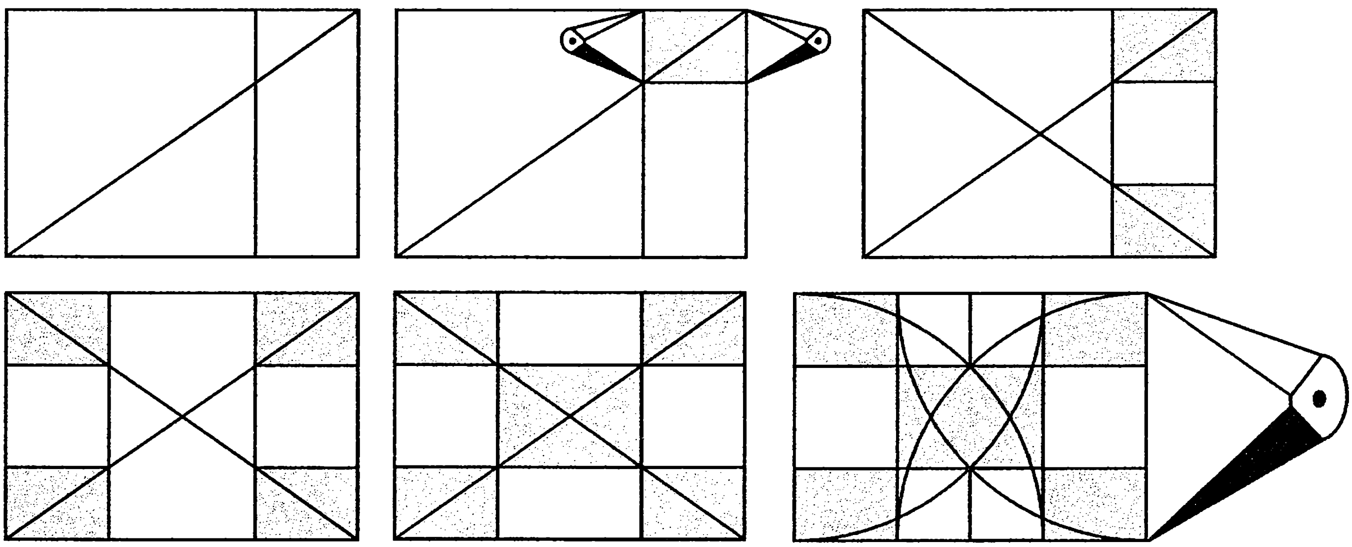
This divides the Root Two Rectangle into six squares (of two sizes) and six small identical Root Two Rectangles. Try it below.



Another Way To Subdivide The Root Two Rectangle

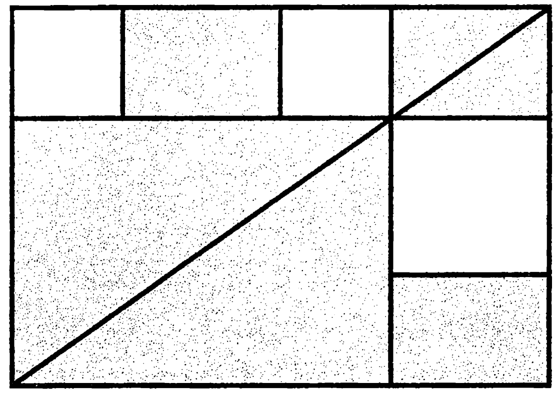
Here's a different way to subdivide a Root Two Rectangle into the same pattern. Start by rabatment. Use your compass to draw a square inside one end. Draw diagonals and notice where they cross the square's side. Use your compass to measure this distance from the side and mark it at the four corners, making four small Root Two Rectangles. Connect these crossings to make a larger Root Two Rectangle at the center. You might also want to use your compass to divide the rectangle in half and divide the top and bottom central rectangles into four small squares.

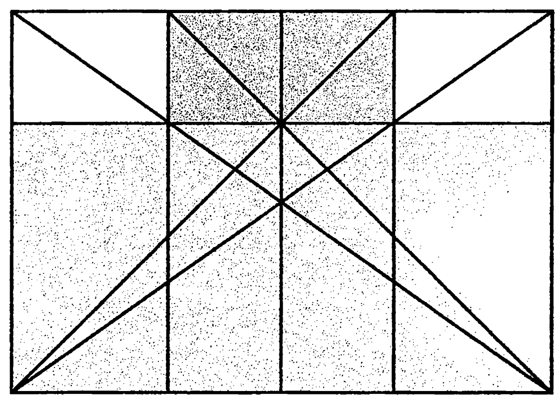
Do this in the Root Two Rectangle below. Use colored pencils to shade the areas.

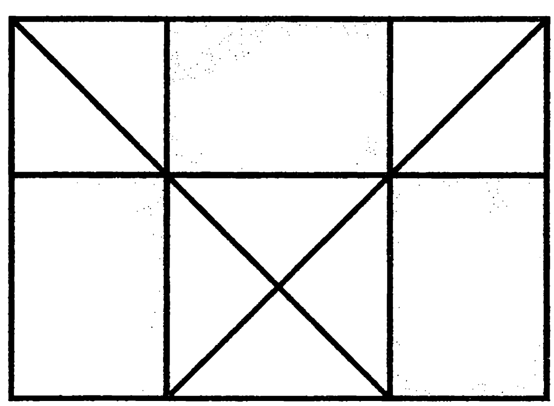


Replicate These Subdivisions of the Root Two Rectangle

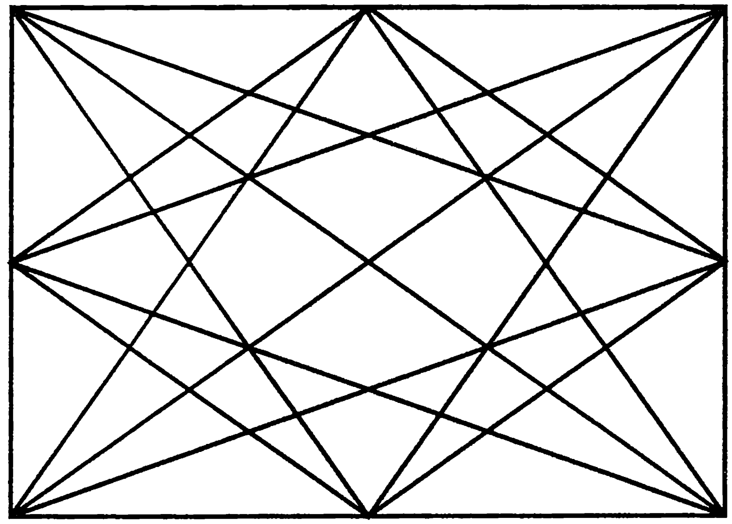
Each results in smaller Root Two Rectangles and squares. Start each by rabatment of a square from one or both ends. Notice whether the diagonal is for the whole Root Two Rectangle, or of a square. Use your compass to mark distances and connect points. Decide on the order of your steps.







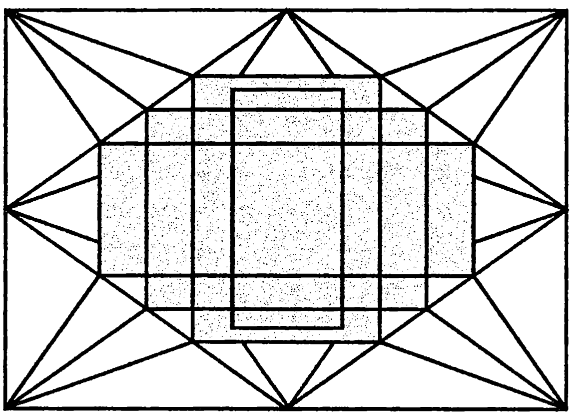
A Bouquet Of Harmonious Rectangles

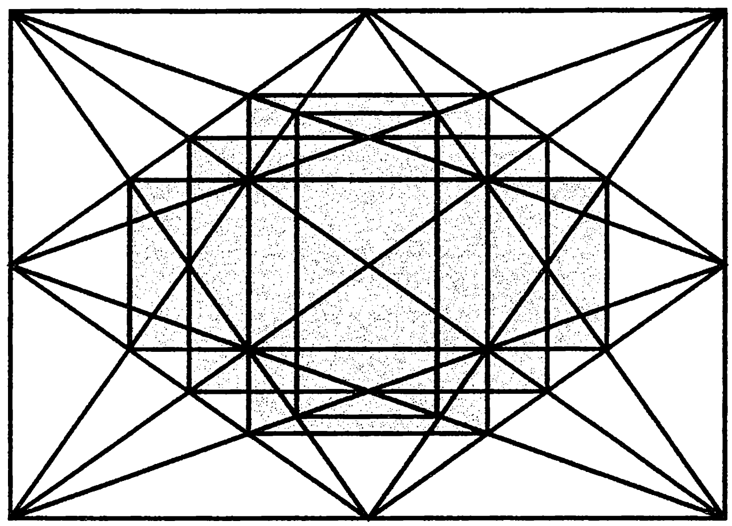


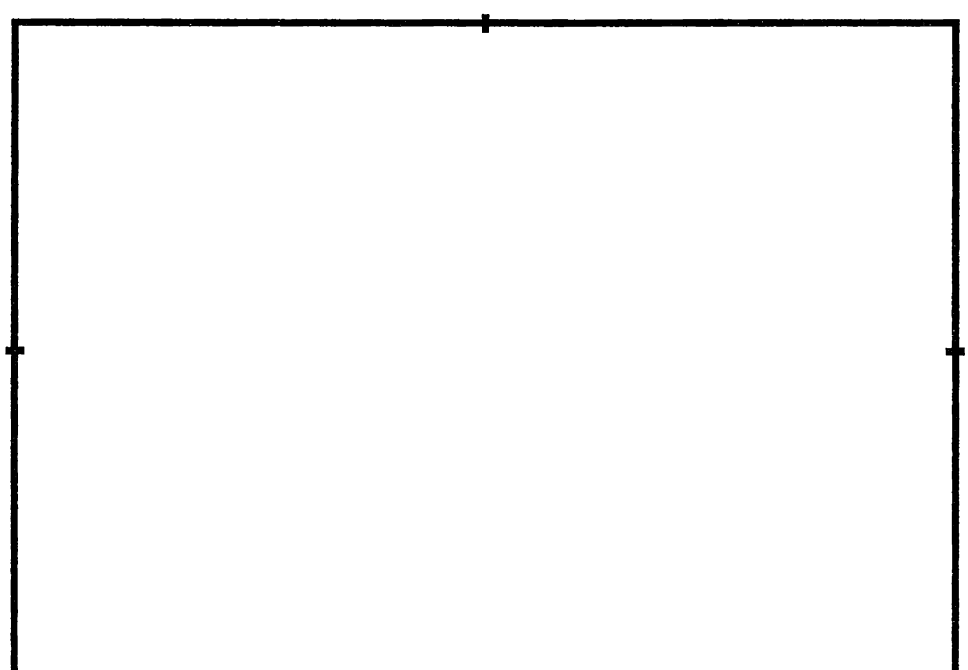
By simply drawing diagonals between a Root Two Rectangle's corners and the midpoints of its sides we create many crossing points.

Duplicate this design in the rectangle below and connect various crossing points to create this bouquet of interrelated rectangles.

Use colored pencils to shade the areas in your construction and the examples on this page.



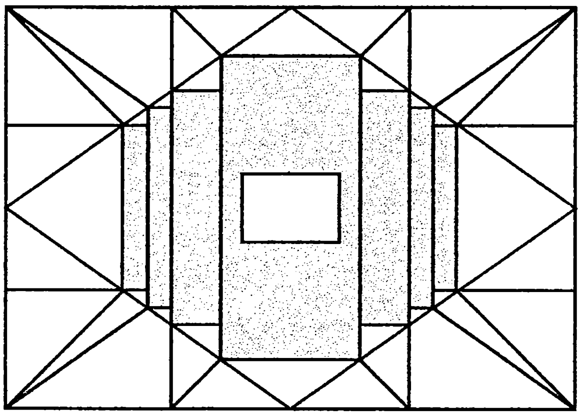


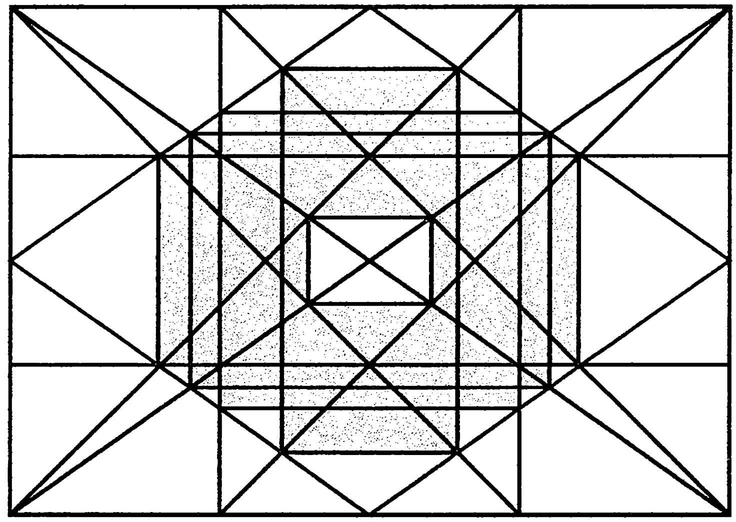


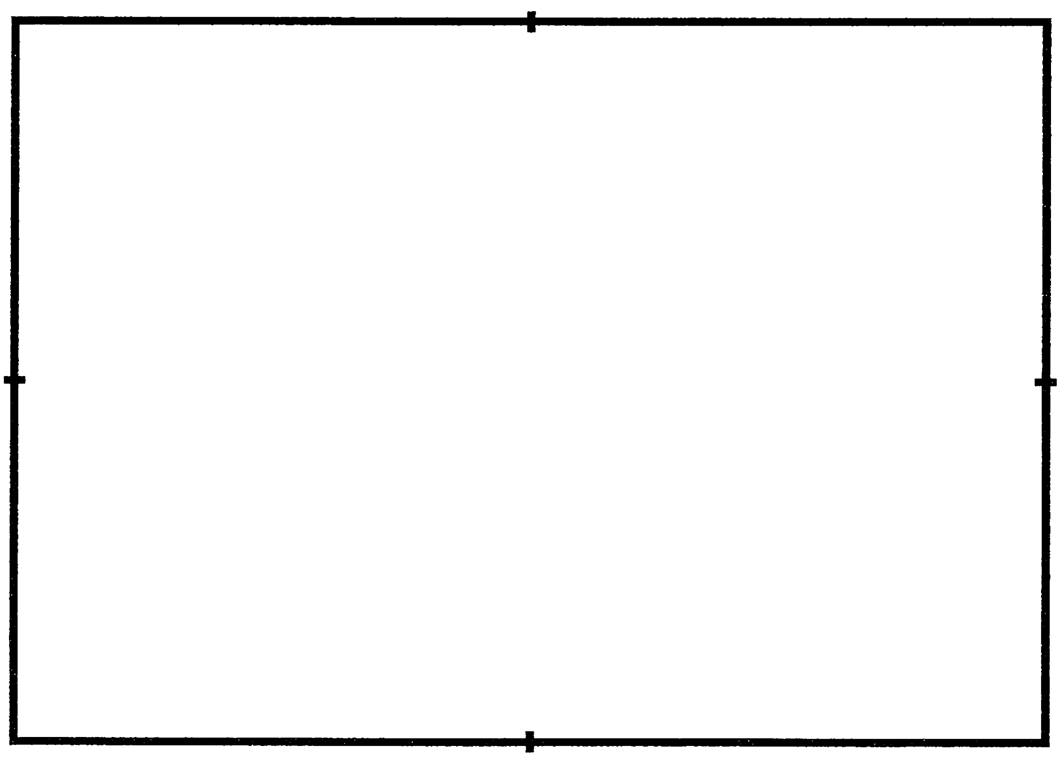
Another Bouquet Of Harmonious Rectangles

Rabatment of a square from each side in this Root Two Rectangle provides more points to connect and create another set of interesting rectangle relationships.

Of course, shade areas with colored pencils.

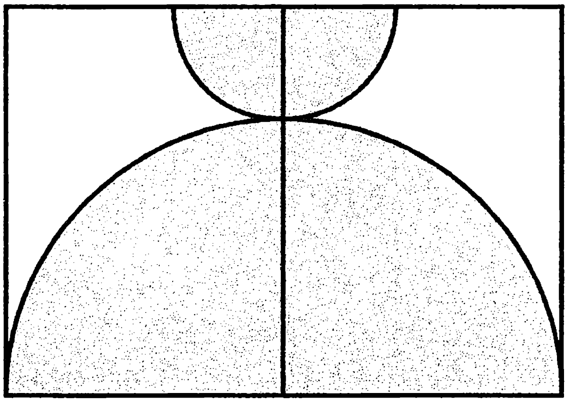


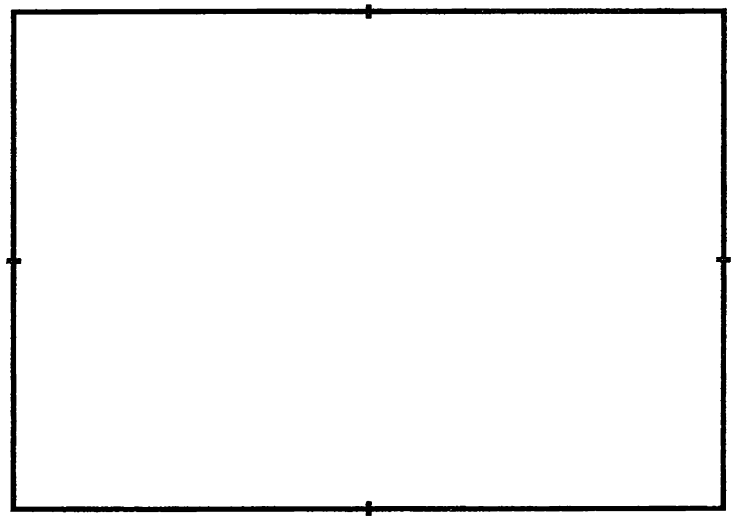




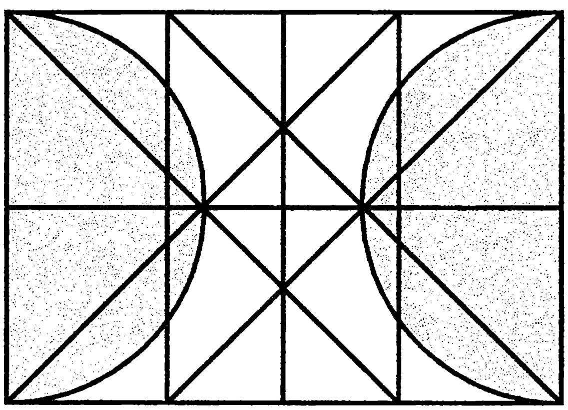
Patterns With Circles

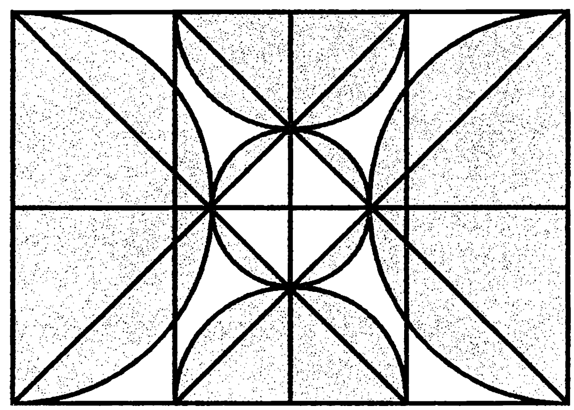
Use your compass to turn these arcs. You only need to realize where the legs of the compass go.

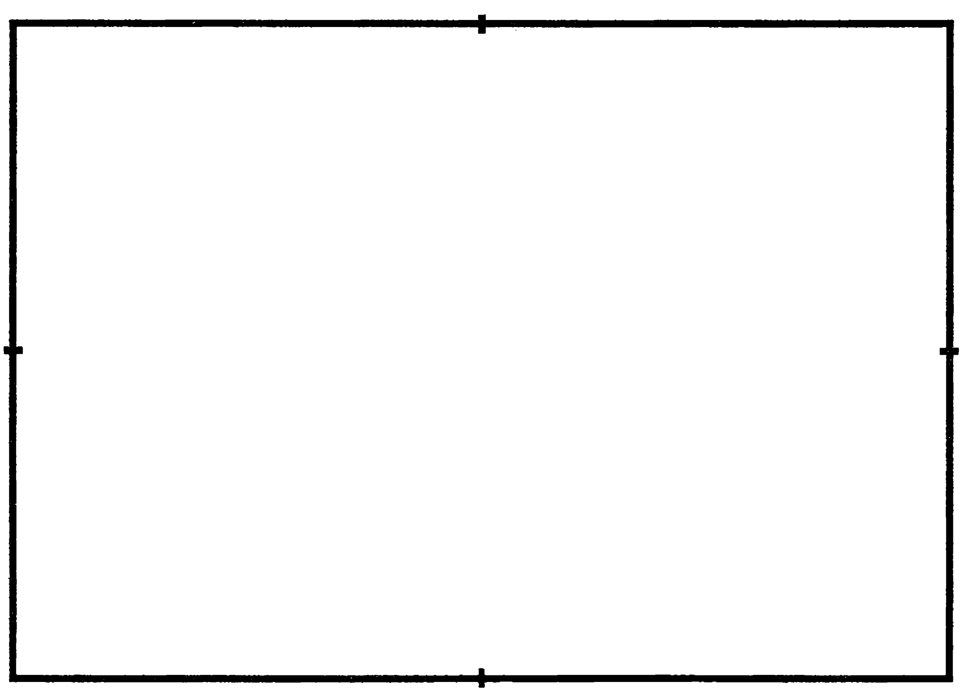




These below require the rabatment of squares from each end.



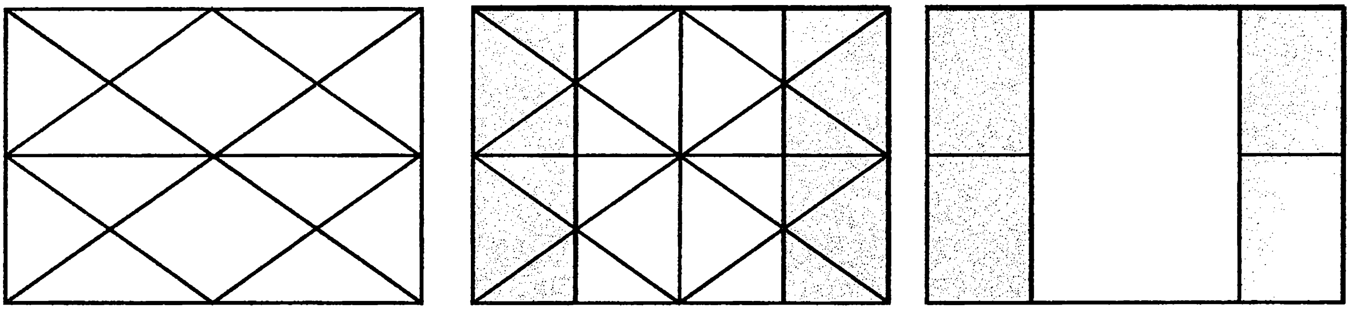


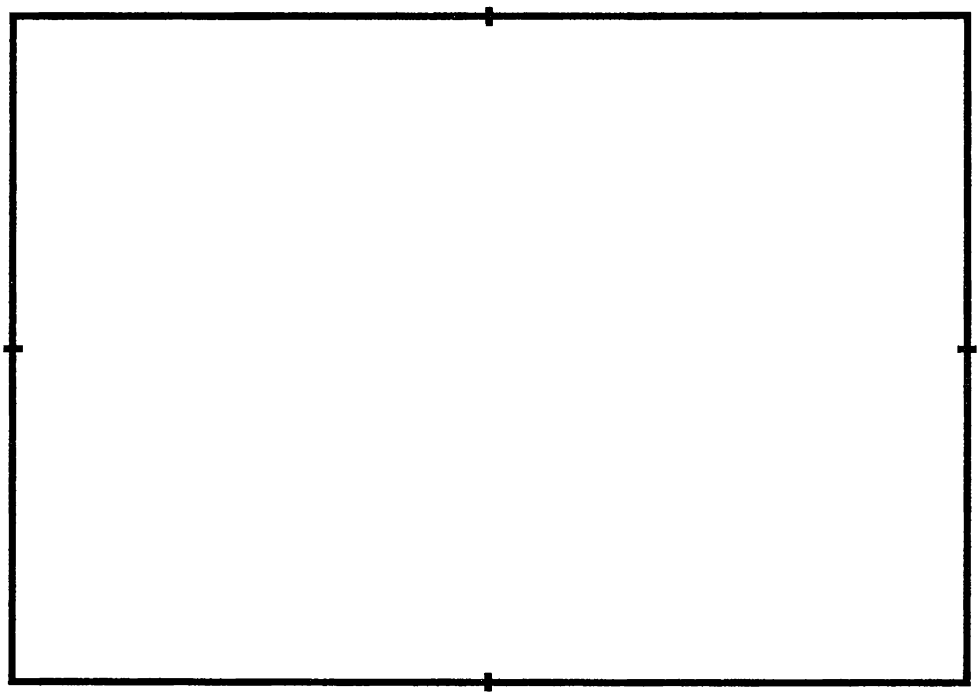


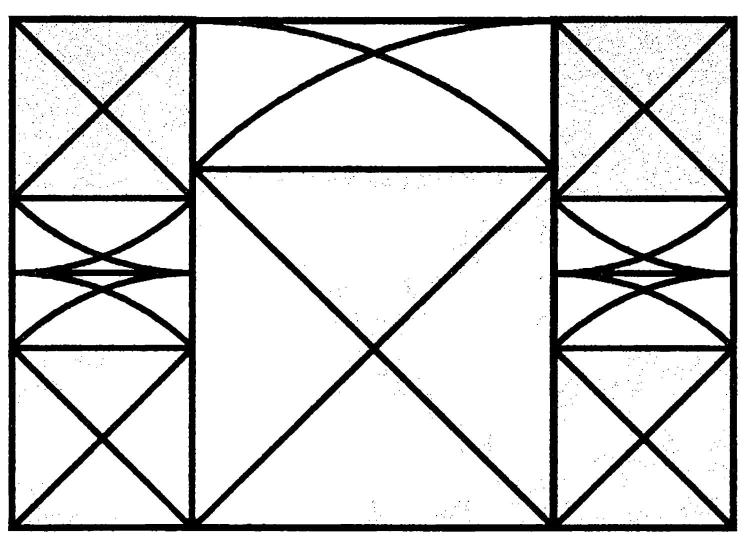
Division Into *Only* Root Two Rectangles

This construction divides a Root Two Rectangle into eight small Root Two Rectangles, or four small and one large Root Two Rectangles.

Start with this pattern and use your compass to transfer lengths. (It *doesn*'t require rabatment of squares.)





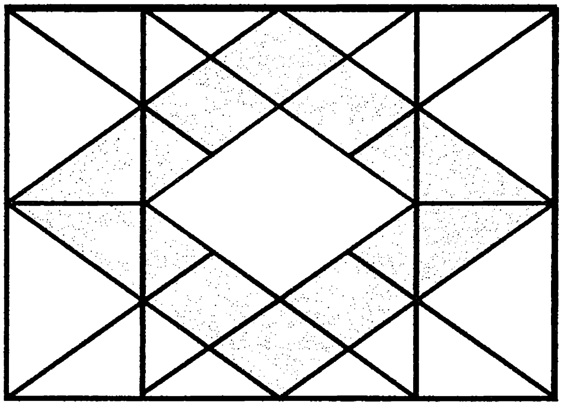


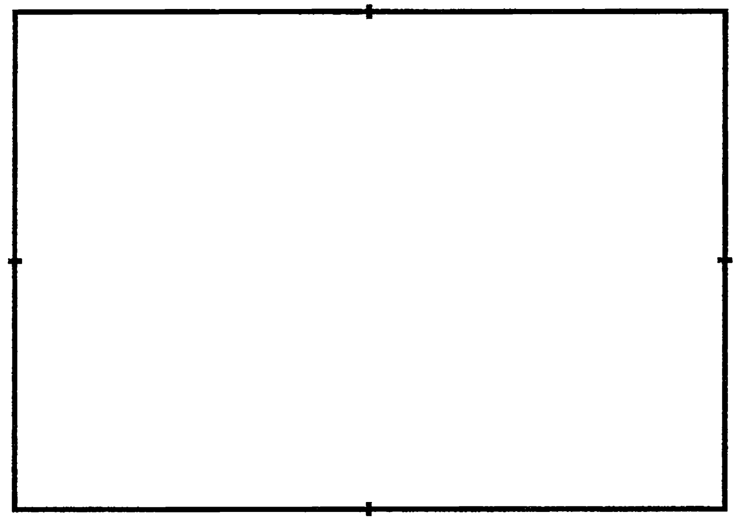
You can prove that each section is a Root Two Rectangle by marking off a square within it. Then open your compass across each square's diagonal and make an arc which should reach the rectangles' tops.

This is an important construction used often in worldwide art and architecture, as we shall see.

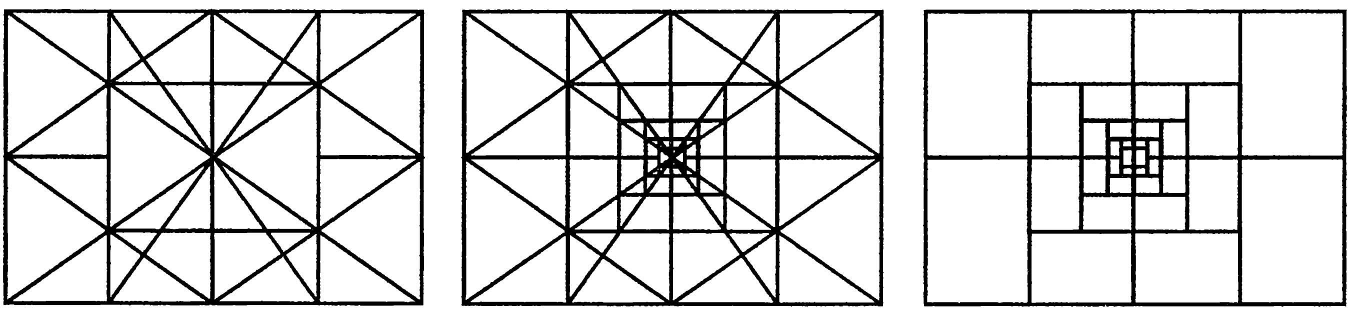
Replicate These Patterns

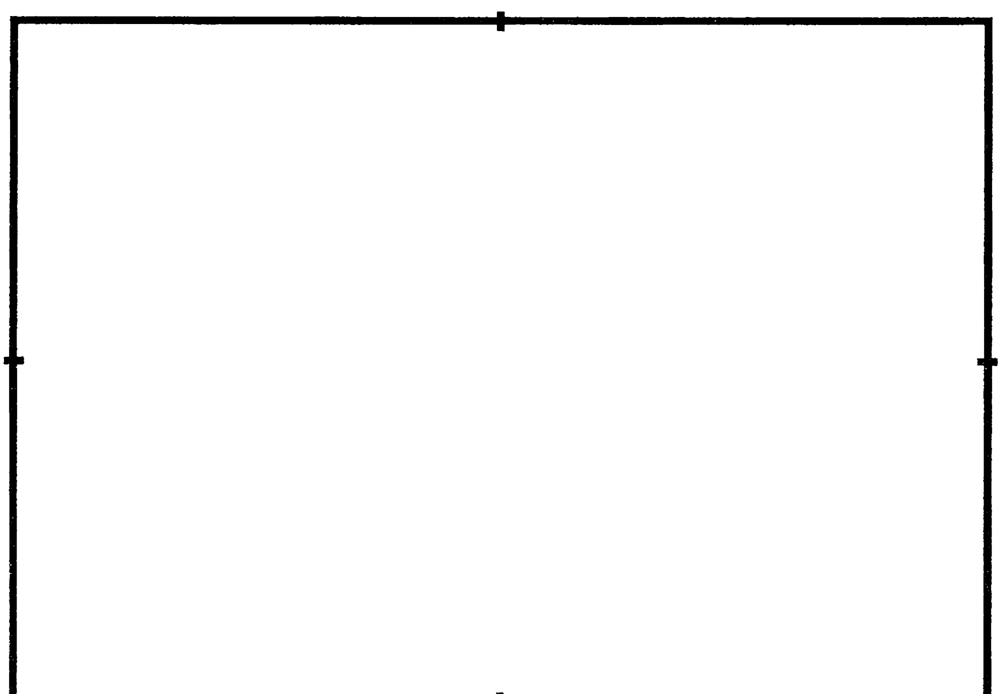
Use the previous construction to connect crossings and create these patterns. Shade with colored pencils.





This pattern below is made only of Root Two Rectangles of diminishing sizes.



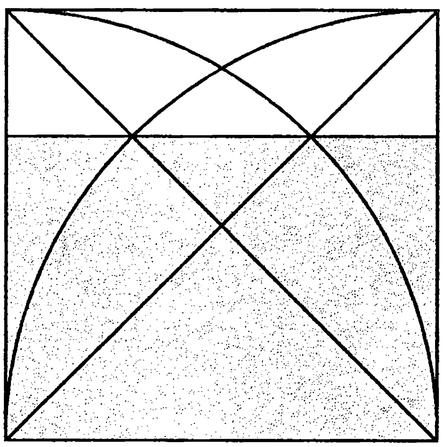


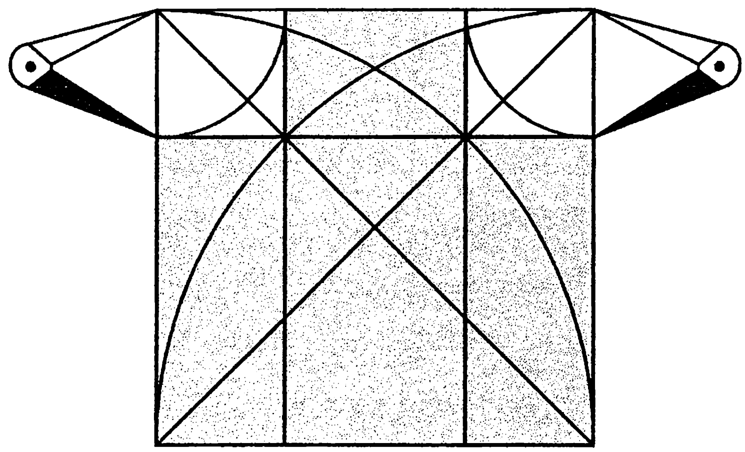
Construct Root Two Rectangles *Within* A Square

We saw how to expand a square by its diagonal to construct a Root Two Rectangle.
And the Root Two Rectangle also resides *within* a square.

Open your compass to the bottom of a square and make arcs to each upper corner. Then draw the square's diagonals. Notice where they cross and use your straightedge to connect these points, and extend the line. The area below this line is a Root Two Rectangle (shaded left).

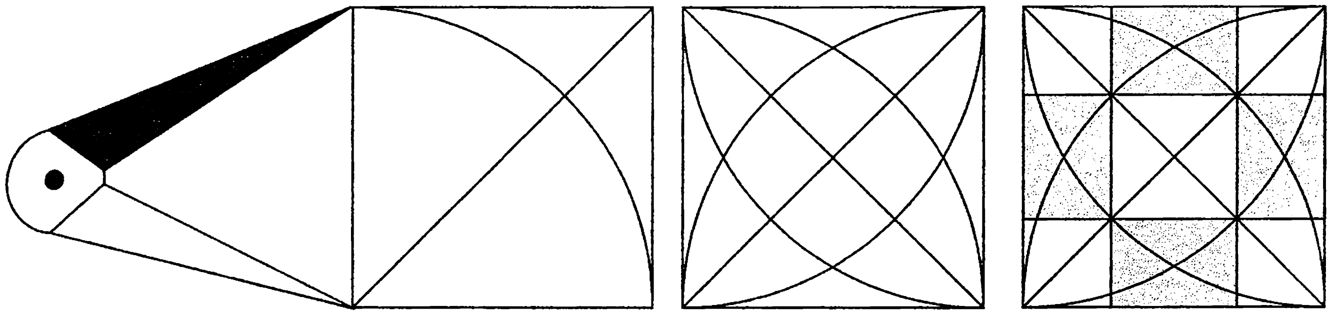
Make arcs at the top corners with your compass to divide the area above the Root Two Rectangle into a small central Root Two Rectangle flanked by two corner squares.

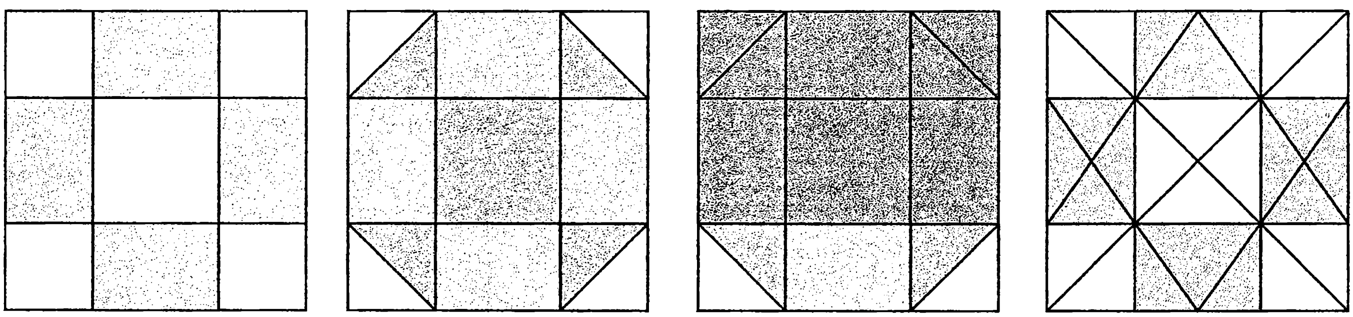


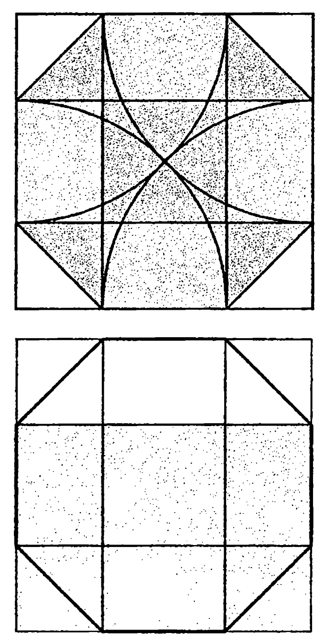


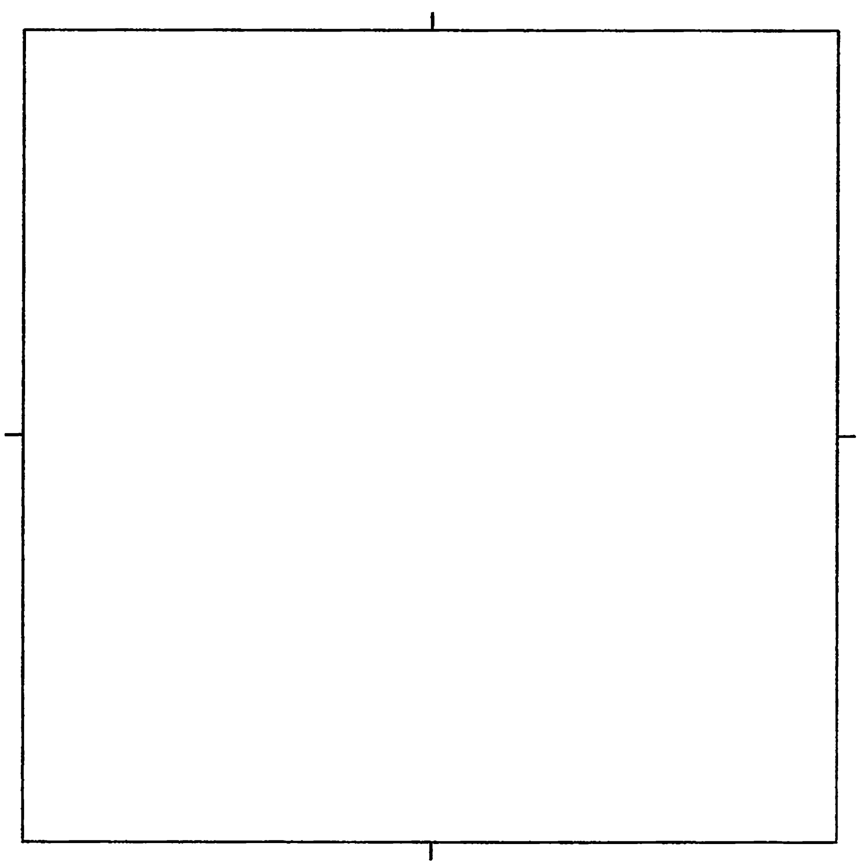
Root Two Rectangles In The Octagon

The previous construction can be developed in four directions. Connect the crossings of arcs and diagonals within a square to subdivide it into four identical Root Two Rectangles, four small corner squares, and a larger central square. This very important construction holds the proportions of a regular octagon seen in worldwide art and architecture. Use colored pencils to shade areas.









A large Root Two Rectangle also resides in the octagon's surrounding square.

Construct Overlapping Root Two Rectangles

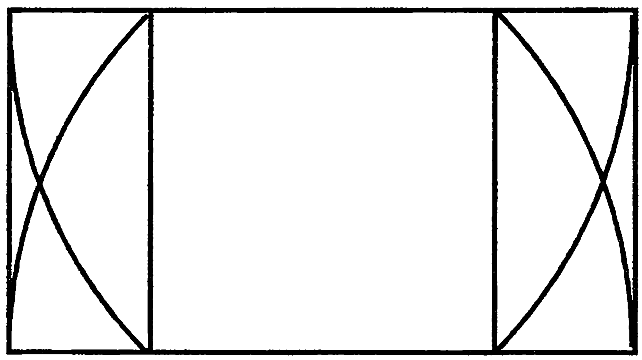
A useful construction appears when two Root Two Rectangles overlap on their common square.

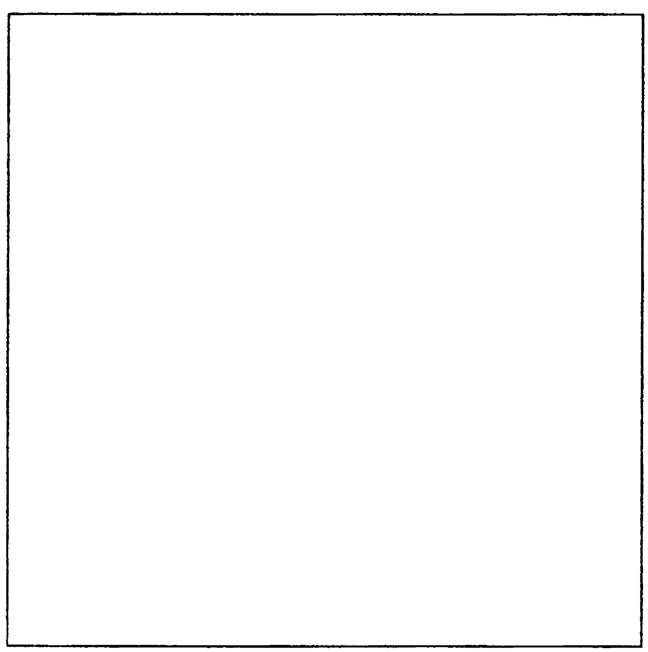
Start with the square below and swing your compass from its diagonals to extend it to the right making a Root Two Rectangle.

Do the same extending the square to the left.

Explore this elongated rectangle with your compass and straightedge, subdividing it further.

Can you prove mathematically why it's called a $2\sqrt{2} - 1$ Rectangle?





Root Two Rectangle Art

"The root two proportion is always generous and sturdy and has an aspect of reliability."
-- Christine Herter (*Dynamic Symmetry -- A Primer)*

Until a few of centuries ago most arts, crafts and architecture were intentionally designed using the practical and symbolic language of geometry. In part, this was because certain proportions appeal more powerfully to viewers and visitors to carefully crafted spaces. Root Rectangles are especially potent in this regard. Awareness of the Root Two Rectangle construction occurs in every civilization which knew the construction of the square. Another reason art was designed this way, especially sacred art, was because geometry was considered to be a sacred language, not created by anyone but discovered, revealed from timeless, unchanging mathematical principles. Plato refers to it as "perfect Truth".

The examples here are culled from many cultures which are presented approximately in chronological order although they actually overlap in time. The examples presented within each culture aren't necessarily presented chronologically but from the simplest geometry to more complex. The reason is that this book is a teaching tool whose main interest is in teaching geometry and art analysis, and this approach serves that purpose. It's hoped that you enjoy reading about, seeing and geometrically exploring these examples from a variety of different cultures which share exactly the same geometry.

It's also hoped that these examples inspire creative readers. Modern artists and art teachers today generally disdain and discourage the use of mathematics or any rational scheme in the creative process. It's thought that using geometry limits the artist, confines him to a dull, overly-logical and lifeless composition. But exploring the works of ancient artisans demonstrates that they knew otherwise. The proper use of geometry gives the artist suggestions for proportioning and placement of elements for a pleasing, even uplifting effect. The reader is reminded to see not only the overall proportioning of objects, but how internal points, lines and areas of interest, "visual centers of gravity" are made best use of. Important too are the symbolic uses of the circle, square, octagon and root rectangles. A few quotes about mathematics and art will set us on our way:

"Mighty are numbers; joined with art, resistless." -- Euripides (c 484-406 BC, Greek playwright)

(Artists should) "... fix their eyes on perfect Truth as a perpetual standard of reference, to be contemplated with the minutest care, before they proceed to deal with earthly canons about things beautiful."
"The good, of course, is always beautiful, and the beautiful never lacks proportion." -- Plato

"Sacred architecture is not, as our time chooses to see it, a 'free' art, developed from 'feelings' and 'sentiment', but it is an art strictly tied by and developed from the laws of geometry." -- Fredrik M. Lund

"Regulating lines ... are ... a springboard and not a straightjacket. They satisfy the artist's sense ... and confer on the work a quality of rhythm." -- Le Corbusier

"Knowledge of a basic law gives a feeling of sureness which enables the artist to put into realization dreams which otherwise would have been dissipated in uncertainty." -- Jay Hambidge

"This symmetry cannot be used unconsciously although many of its shapes are approximated by designers of great native ability whose sense of form is highly developed." -- Jay Hambidge

The ancient Egyptians were magnificent geometers who used mathematics for practical and symbolic purposes. Geometry was part of a "magical, scientific formula" providing structure at all levels of their civilization. To design anything without its proper geometry would have been unthinkable. Keeping as close to eternal principles as possible was partly responsible for the long duration of Egyptian civilization over thirty centuries, even influencing ideas and designs today.

Offering Tablet

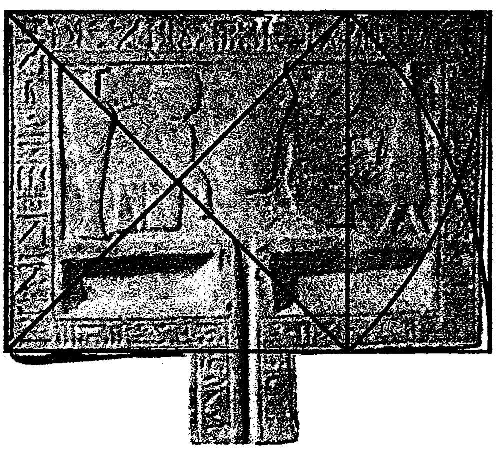
12th Dynasty, circa 1900 BCE

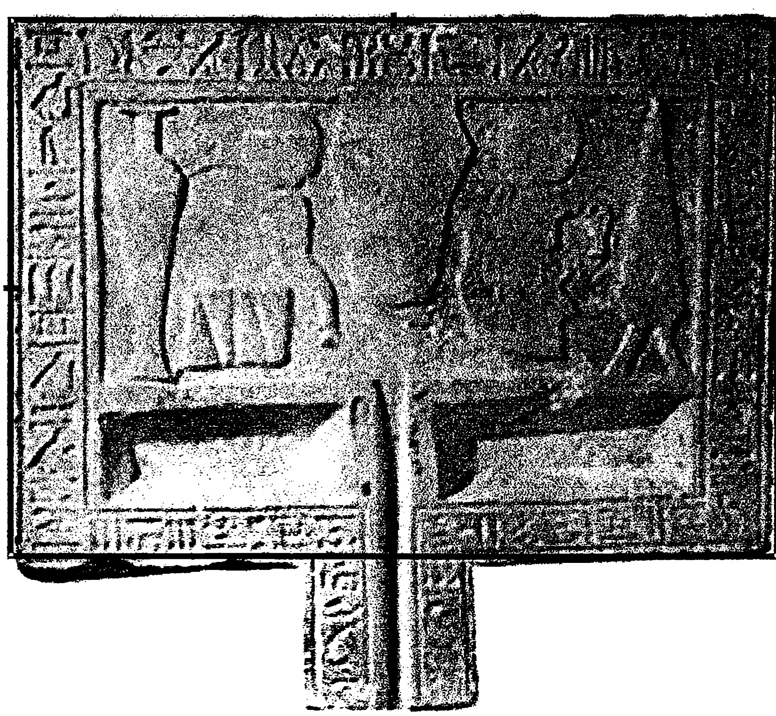
This table held offerings to a deity. Pictures on it depict oil, bread and vegetables. Two rectangles held liquid which was poured over the offering, then drained through the central channel

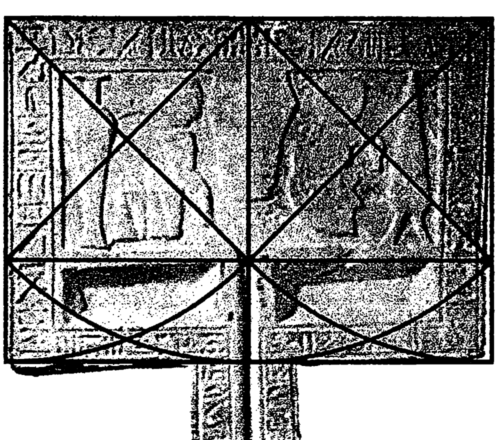
Simple geometric construction confirms that the table is a Root Two Rectangle, yet this alone doesn't show us much about its design. But if we remember that when a Root Two Rectangle is divided in half each part is a smaller Root Two Rectangle, we'll see it.

Rabatment within them reveals how the liquid rectangles are divided from the offering area.

A Root Two Rectangle has been drawn with midpoints on the sides to get you started.



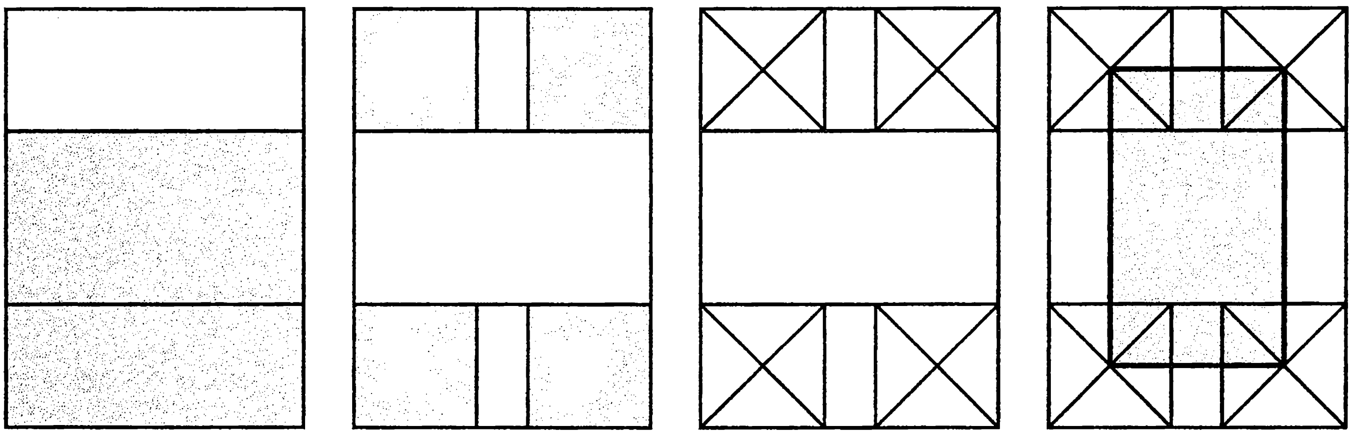




Box Lid From Tutankhamun's Tomb

18th Dynasty

This small box lid was found in the antechamber of Tutankhamun's tomb, although the box itself has never been found. The lid is inlaid with both the name and image of Nefernefrure, the fifth daughter of Akhenaten, suggesting that this lid was originally part of the burial treasure of the predecessor of Tutankhamun, Smenkhkare ("Strong is the Soul of Ra"), .



The lid is a Root Two Rectangle.

To see how the thickness of the frame and size of the central picture were determined, do this:

First rabat a square from the top and bottom. (Notice how Nefernefrure sits upon a line.)

Then, rabat a square inside each corner.

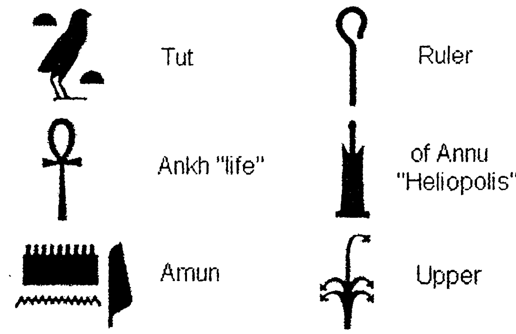
Draw diagonals in the four small squares.

Finally connect the center of each square to frame the inner rectangle.

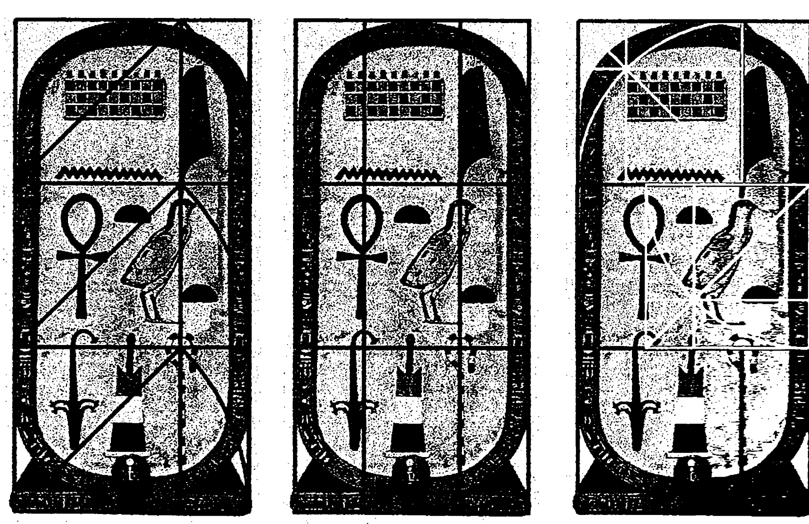
Tutankhamun's Cartouche Jewelry Box

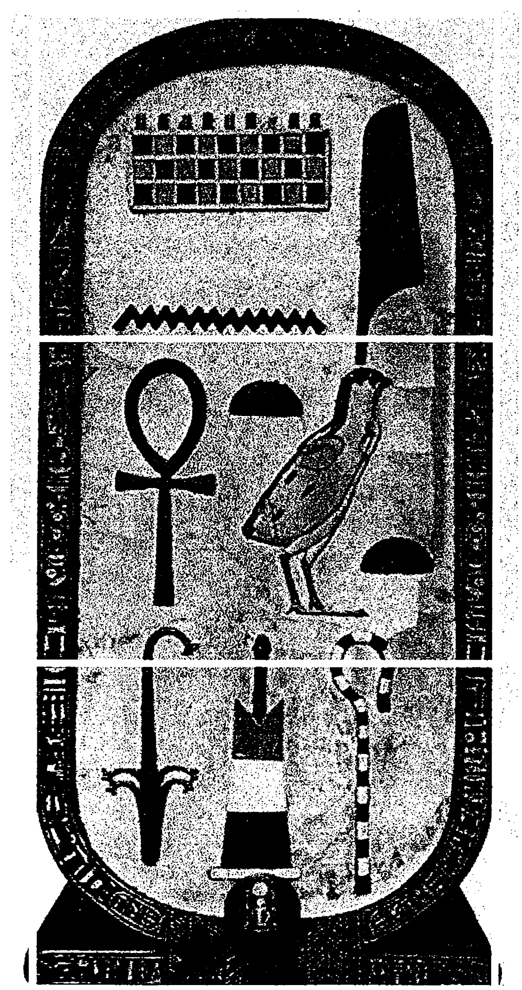
This box, also found in Tutankhamun's tomb, contained his jewelry and other personal items. Tutankhamun was the twelfth ruler (1334-1325 BCE) of fourteen during the Eighteenth Dynasty. He ruled from the ages of nine to nineteen. The tenth pharaoh, Akhenaten, had overturned the wise, traditional religious and social system of Egypt in favor of worship only of the physical sun disc, Aten. Everyone else worshiped Akhenaten and his family, who would intercede with Aten on their behalf. So when the boy Tutankhamun became pharaoh, his name was Tutankh*aten*, the "Living Image of Aten" the sun disc. But during his reign the system returned to the traditional system and his name was changed to the now-familiar Tutankhamun, "the Living Image of Amun", the traditional one hidden deity "behind" all manifestation.

The box's oval ring shape (now called a "cartouche" (from the French for "cartridge") and is itself a hieroglyph containing his later name. (Indeed, his name is written in the middle and top rows: Reading from right to left, and upward, the half-circle represents the sound "T", the chick the sound "OOH", the oval/cross "ANKH", and the feather "AH", the checkered rectangle "MEN" and wavy water lines a final "N" for emphasis.)



Geometrically, the box is proportioned as a stack of three identical Root Two Rectangles. Symbolically, three of anything indicates that it is sacred. When rabatment is applied and squares are drawn inside the ends of each rectangle, we see two vertical lines appear which align the glyphs. Three Root Two Rectangles are given for you to start with. Further geometric exploration will reveal even more about the precise placement of the glyphs.



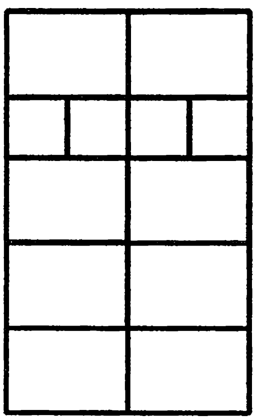


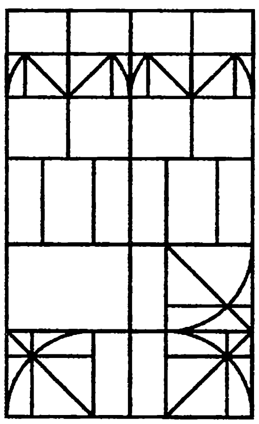
Tutankhamun's Double-Cartouche Perfume Box

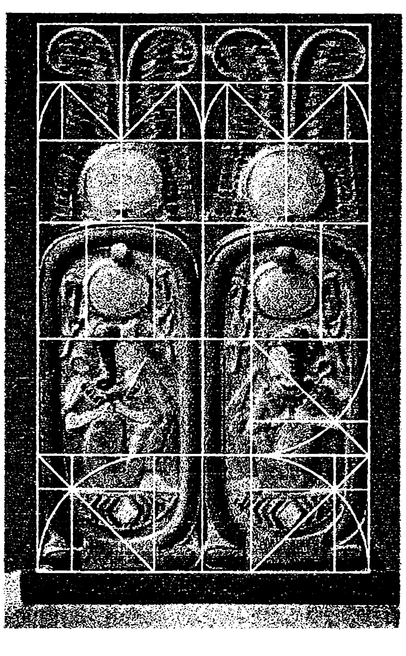
Also found in Tutankhamun's tomb is his perfume box shaped as two cartouches with sun-disc adorned plumes for lids. It's made of inlaid gold-plated wood on a silver-plated pedestal which is incised with a frieze of *ankh* (life) and *was* (dominion) glyphs. The larger inlays consist of colored glass paste while the smaller ones are stone. On the front and back are four slightly differing images of the young pharaoh represented at different stages of his life, where he squats on the basket glyph *Heb* ("festival").

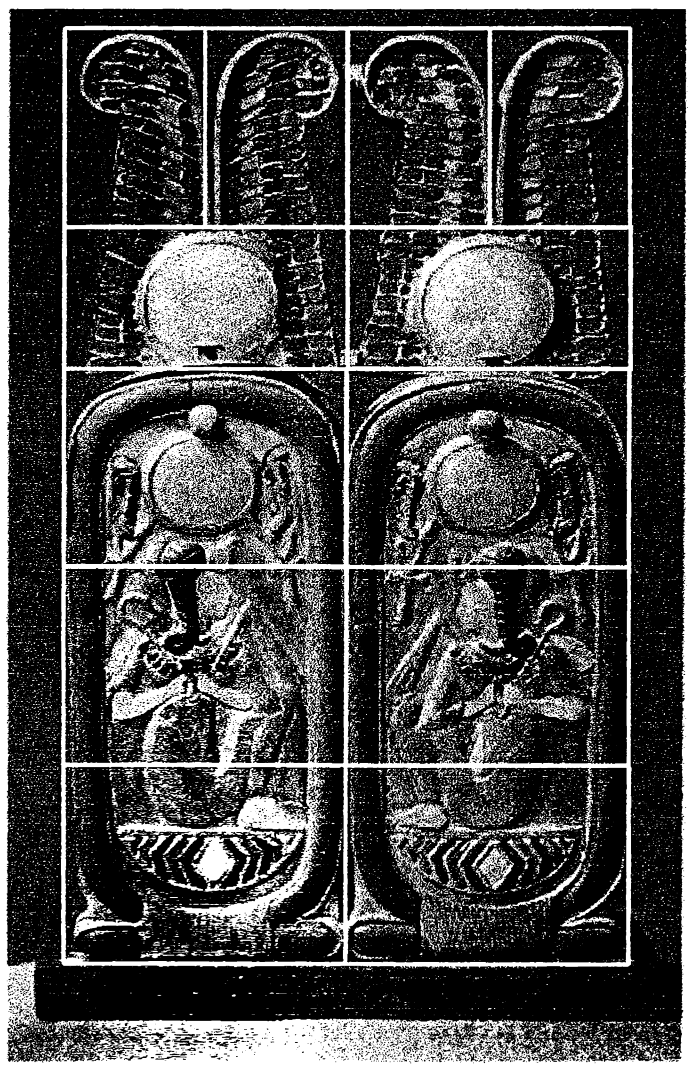
The hieroglyphs written in the cartouches spell Tutankhamun's throne name Neb-Kheperu-re ("Ra is the Lord of Manifestations") but indirectly, as a punning cryptogram. Instead of the simple disk of the sun signifying the god Ra, above Tutankhamun is a solar disk flanked by hooded cobras wearing ankhs around their necks. In place of the *Neb* basket meaning "Lord," there is a *Heb* basket. A traditional beetle *khepera* ("manifestations" or "transformations) is replaced by a winged beetle.

The geometry of each cartouche is the same as the geometry of the previous cartouche box: three Root Two Rectangles. But now the plumes above them continue the geometric scheme with similar rectangles and squares. A framework has been supplied. Apply rabatment of squares to explore it further.









Baboons Adoring Sunrise

Egyptian National Museum, Cairo

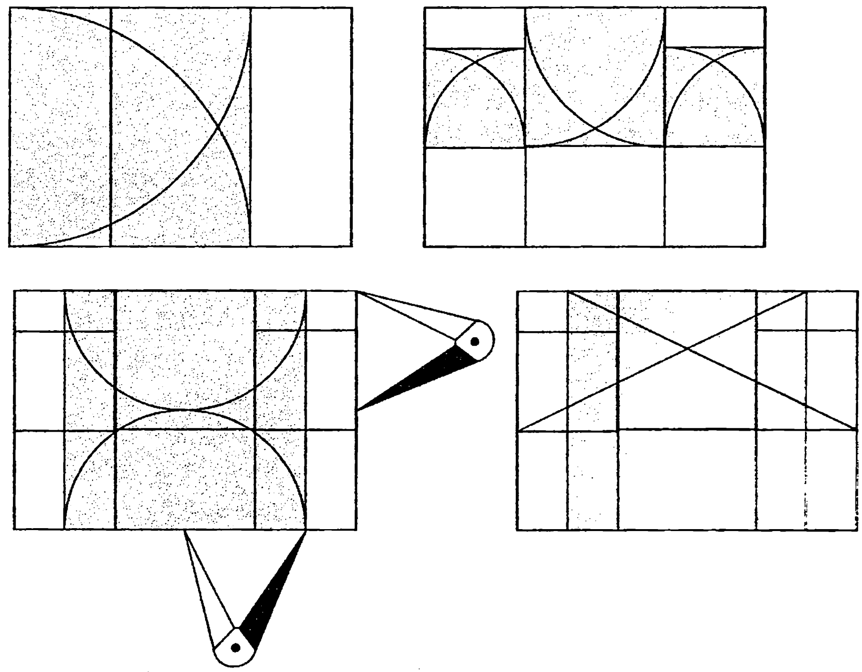
This statuette depicts two baboons in a prayerful position adoring the sunrise. Baboons were well known to begin shrieking at daybreak, and the ancient Egyptians saw this as homage, singing a hymn to the rising sun. Baboons were symbols of the god Djeheuti ("Thoth"), patron of scribes, writing and numbers.

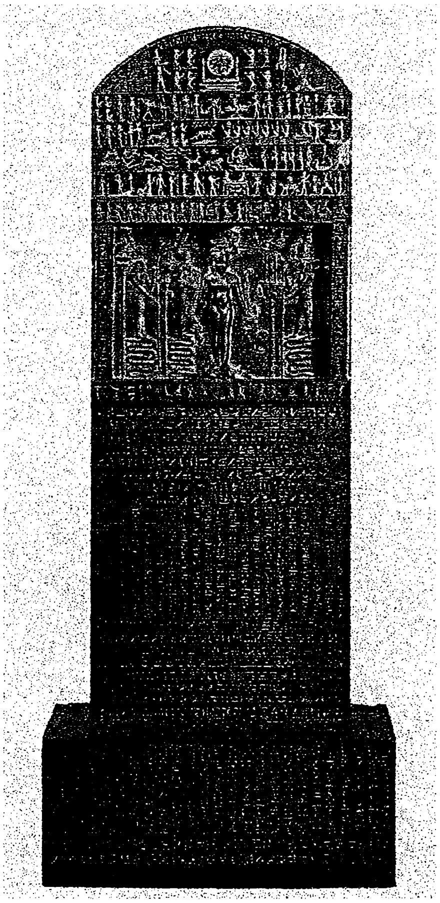
The scene is framed in a Root Two Rectangle, an appropriate symbol for the birth of the day. Rabat squares in each end.

Rabat another square in the top of the central rectangle. Extend its bottom and rabat two squares as shown.

Construct a large square in the center of the rectangle. Transfer half the length of the short side to the middle of the top and bottom to mark its corners.

Various diagonals using all these points will reveal more about their posture.





Metternich Magical Stela

Late Period, Dynasty 30, reign of Nectanebo II, c360–343 BCE
Greywacke stone
Metropolitan Museum Of Art, Fletcher Fund, 1950 (50.85)

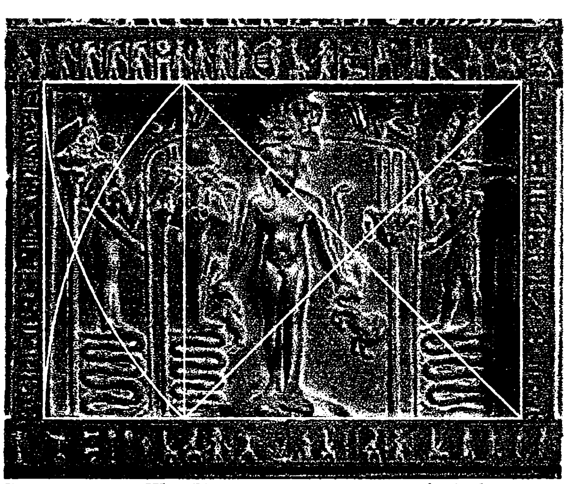
The top half of this stela was masterfully carved in a hard dark stone. Below are rows of hieroglyphs recording thirteen magic spells to protect against poisonous bites and wounds and to cure the illnesses caused by them.

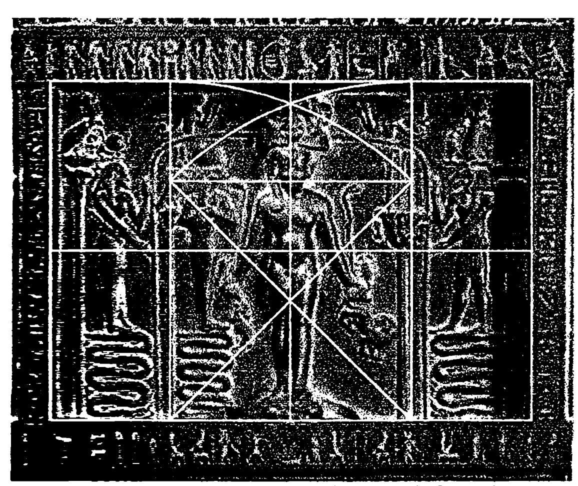
It was commissioned by the priest Esatum to be set up in the public part of a temple for people to recite the phrases and drink water that had been poured over its magic words and images.

Within a rectangle we see the child Heru (Greek "Horus") emerging from the stela as a three-dimensional statue with his left leg striding forward (symbolizing stepping into the spiritual realm) and directly facing the viewer. He is in a sacred canopy held up by the deities Djeheuti ("Thoth") and Aset ("Isis"). Above the canopy are the two *Utchats* with human hands and arms. Horus stands together with a figure of *Re-Harakhty*, god of the rising sun, and two standards in the form of papyrus and lotus columns. The lotus standard supports the two feathers of Osiris' headdress.

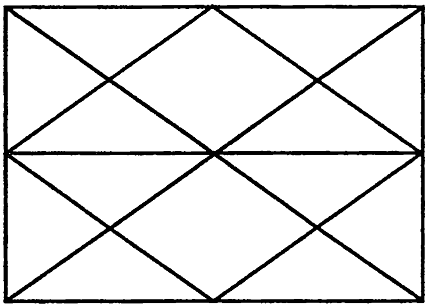
To symbolize his magic powers for overcoming death, Heru grips snakes and scorpions (foes of life and light) as well as an antelope (by its horns) and a lion (by its tail). His feet rest on two crocodiles (symbolizing the overcoming of death). Above him is the head of Bes, the dwarf deity who traditionally protects births. Heru is flanked by three deities who stand upon coiled snakes which have a knife stuck in their forehead.

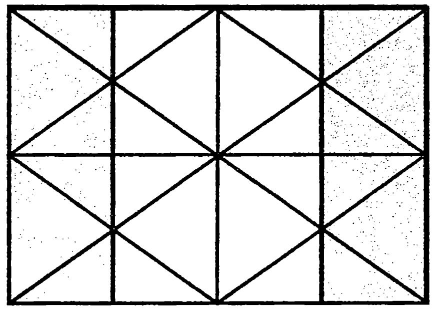
The rectangle in which Heru stands is a Root Two Rectangle, appropriate symbol of birth. You can prove it by marking off a square from one side and swinging your compass open to the square's diagonal. It can be further subdivided as shown. You can discover even more by exploring it on the next page with rabatment, diagonals and arcs.

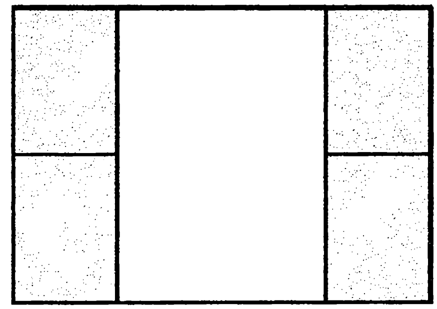


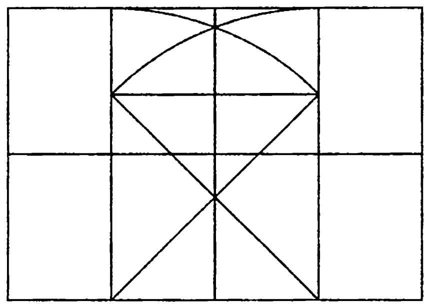


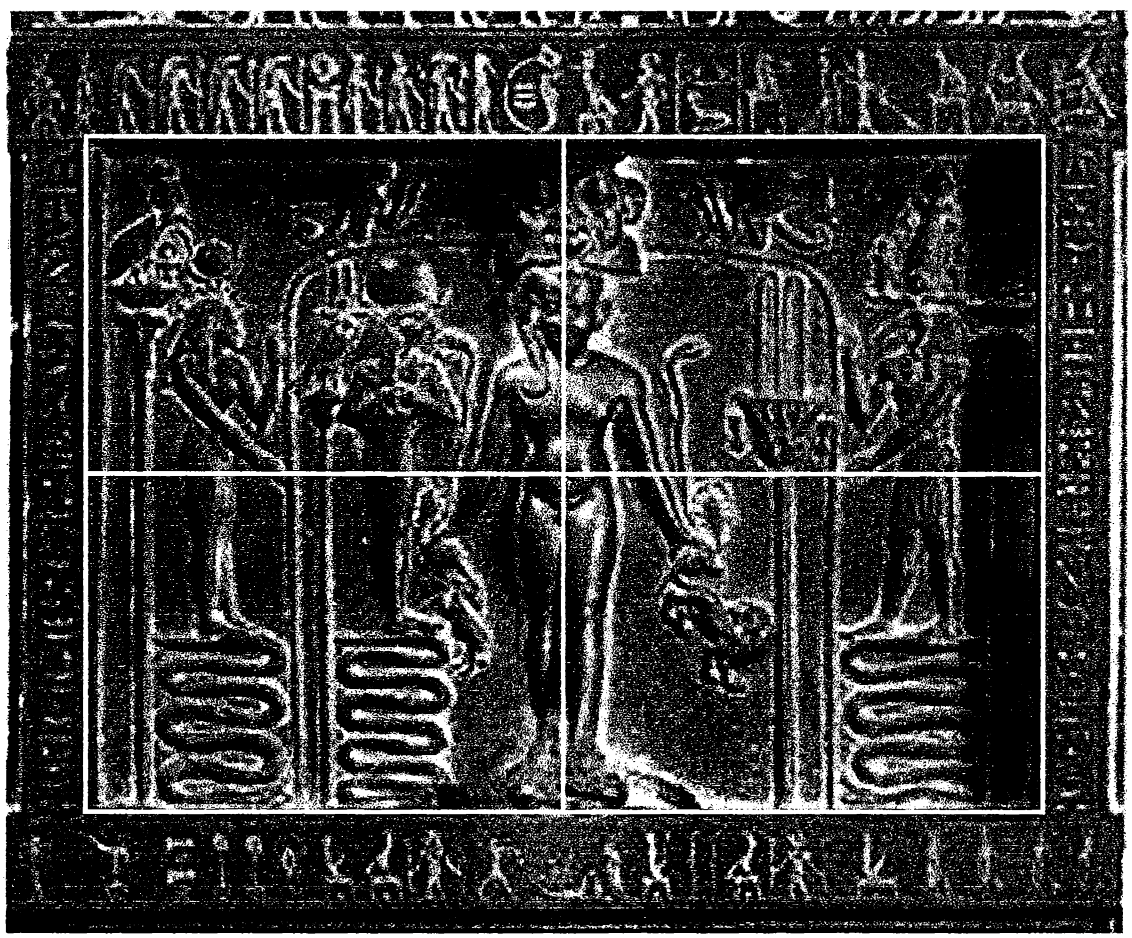
We've seen this geometric construction before, and we'll see it again through many centuries. Notice how the construction is centered on Heru's navel, a design characteristic which appears in the art and mythology of other cultures as well (see Volume 2 Chapter 6 page 127 -- Hindu deity Vishnu).









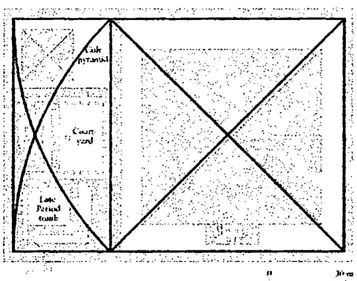


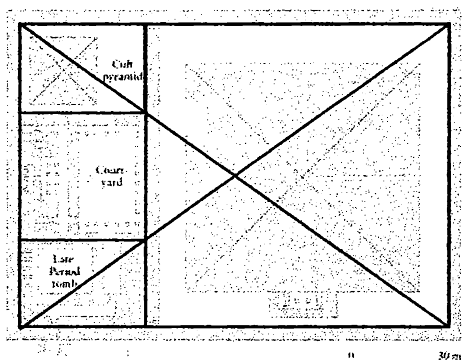
The Pyramid Complex Of Userkaf

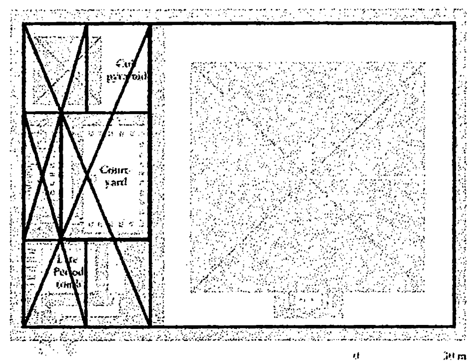
Fifth Dynasty, north of Saqqara

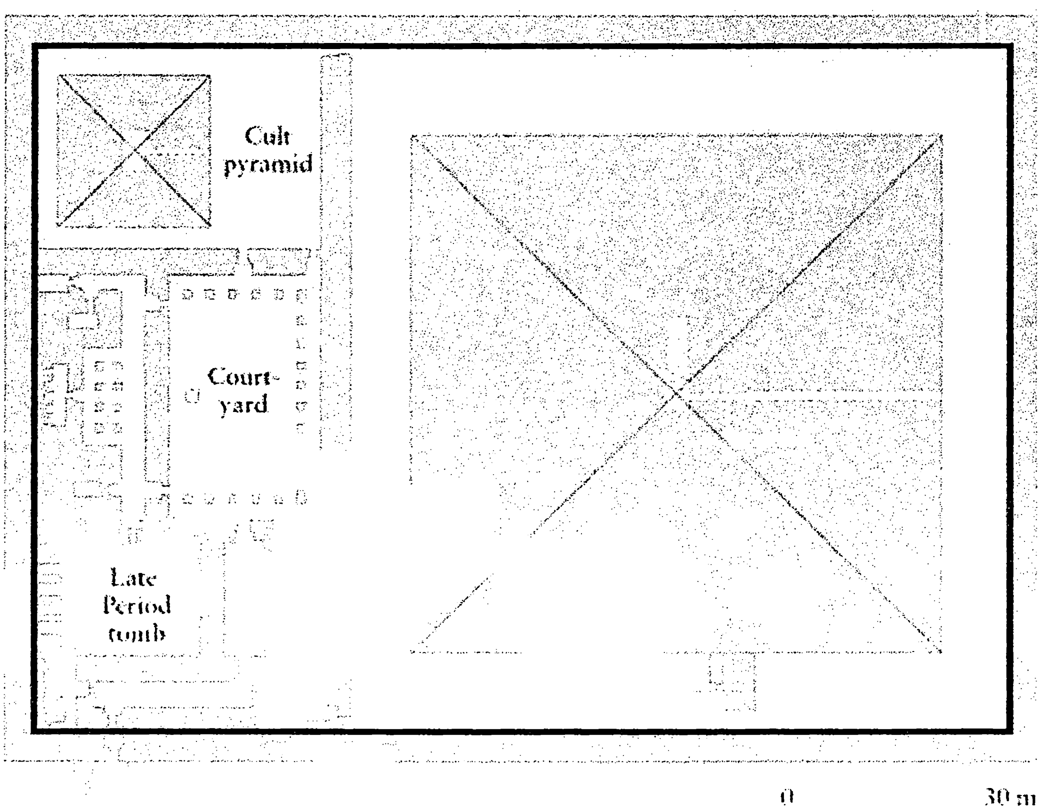
Userkaf's pyramid complex was named *wab-s.wt* ("the purest of places"). It is situated in the corner of the earlier 3rd Dynasty complex of Djoser. The complex consists of a main pyramid, to the east of which is located a small offering chapel. The main pyramid and offering chapel are surrounded by an enclosure wall. The structure of Userkaf's funerary complex deviates from tradition since the mortuary temple oriented towards the south instead of east, probably related to the dynasty's focus as a solar cult.

The whole funerary complex is a Root Two Rectangle, symbol of birth. Do the geometric construction shown to see how its elements are sited.







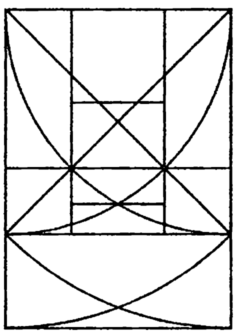


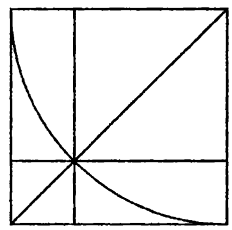
Menorah From The Jewish Temple In Jerusalem

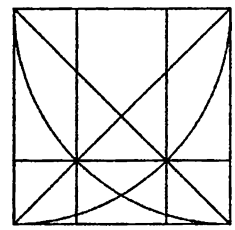
The actual seven-lamp gold candelabrum, or *Menorah* in Hebrew, was looted by the Romans in the year 70 from the great Jewish temple in Jerusalem after sacking the city. Sculpture from the Arch of Titus in Rome (below), which commemorates the deed, depicts the carrying off of the Menorah and other booty. This is how the Romans paid for the Coliseum, begun two years later.

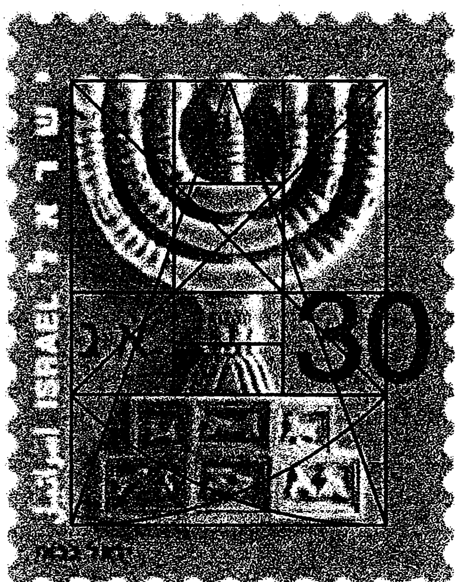
The temple's Menorah is remembered today on a modern Israeli stamp. A geometric examination of the Menorah with its stand reveals that its proportions can be understood in a divided and subdivided Root Two Rectangle.

First construct a large square inside the top of the rectangle, then use arcs and diagonals to find the Root Two Rectangle within it. Apply rabatment folding back squares to frame more elements.









A Greek *Oinochoe*

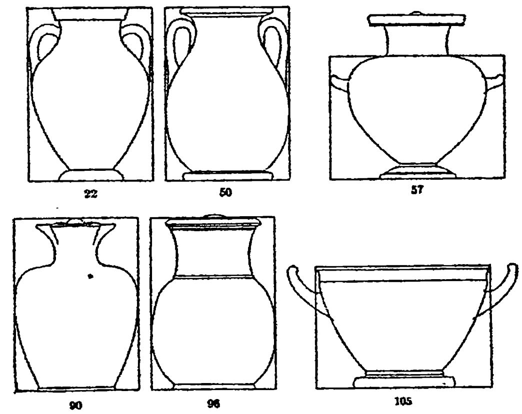
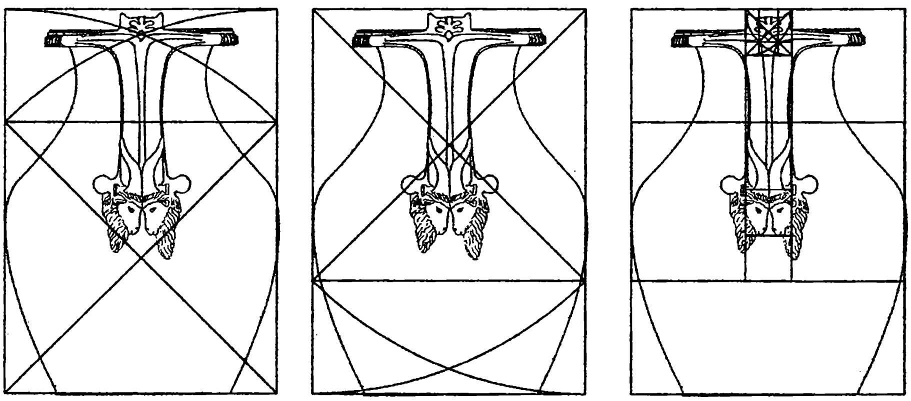


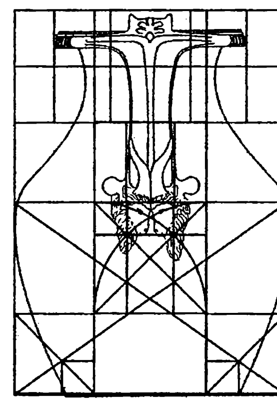
Diagram XL. Examples of the $\sqrt{2}$ rectangle (1.4142 = .7071)

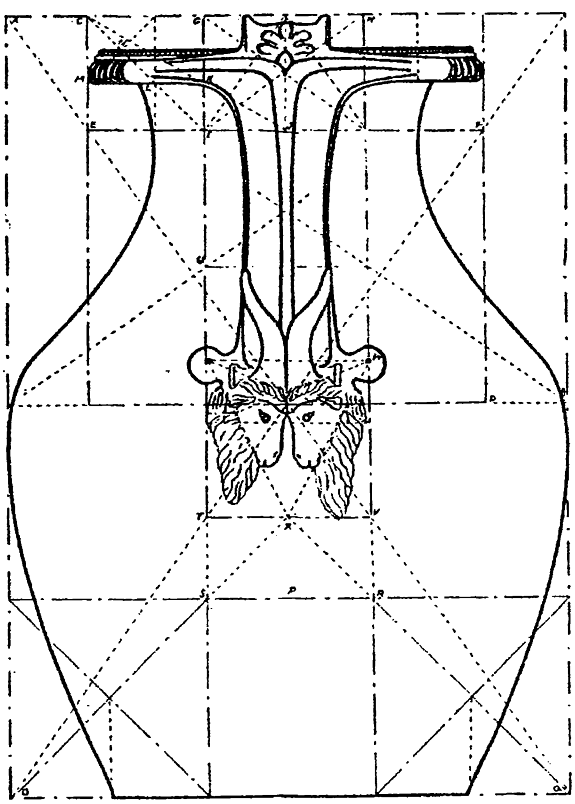
Greek pottery, like their architecture and sculpture, is well known for its geometric beauty. Many examples, like the *oinochoe* (water pitcher) below, in the Boston Museum, can be understood as a Root Two Rectangle. There are a few ways to explore it. You can draw a square from inside the top and bottom, then apply rabatment of squares to see more (three illustrations below, left).

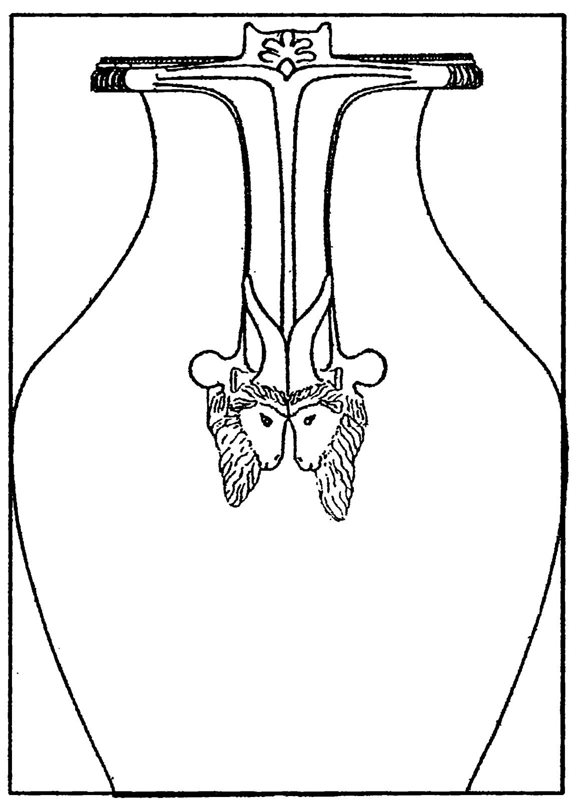
Or, by dividing it in half, you can apply rabatment and find different elements of its design (below, right)

An analysis by Jay Hambidge from his "Dynamic Symmetry: The Greek Vase" is seen at the bottom, right.





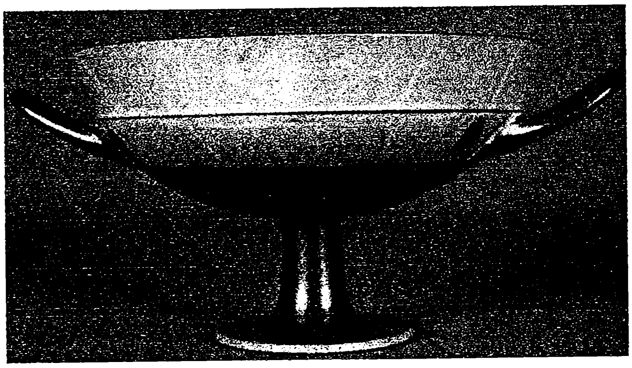


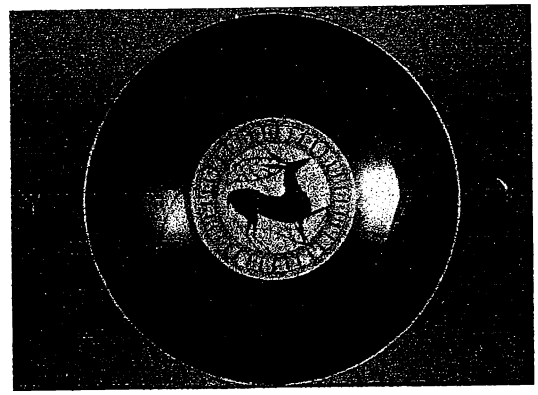


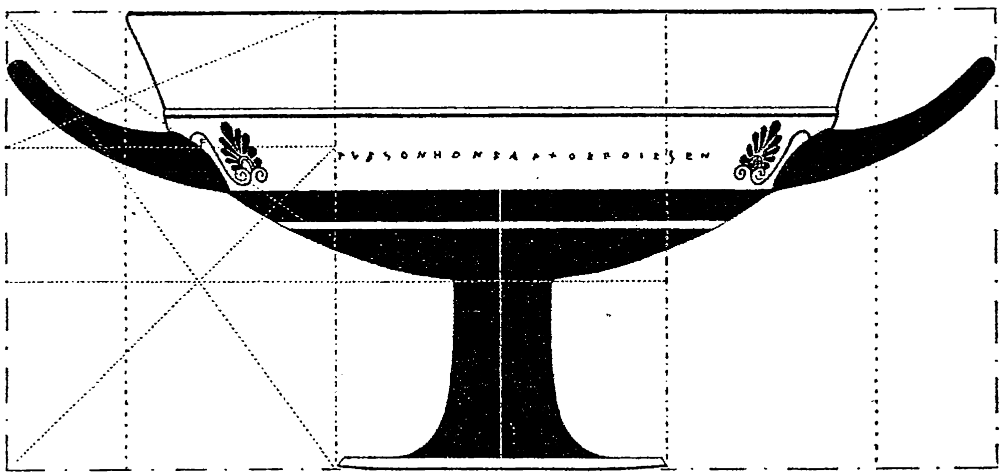
A Greek *Kylix*

Hambidge also studied this *kylix* or wine drinking cup. He found it to be framed by three vertical Root Two Rectangles. His measurements and analysis are found below.

On the next page is its illustration which you can explore too.







124

124 Kylix. Inv. 98.920. Hambidge, p. 53. Signed by Tleson, son of Nearchos, as potter. Interior picture, a wounded stag.

Average height, 0.14235 m. Width (handle to handle), 0.30 m. Diameter of bowl, 0.227 m. Diameter of foot, 0.10 m.

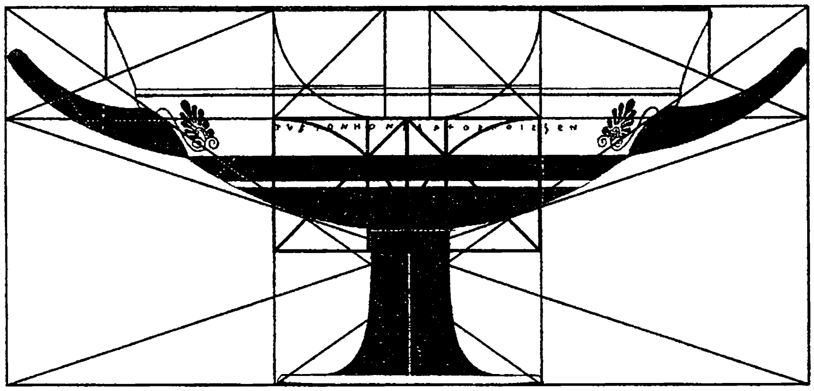
The enclosing area is made up of three $\sqrt{2}$ rectangles placed vertically side by side, the ratio being 2.1213, or $.7071 \times 3$. It can readily be seen that if the width of the over-all area is taken as 30 cm., the width of each of the $\sqrt{2}$ rectangles is 10 cm., and their height is 14.142 cm. This comes within a millimetre of the average height obtained from eight measurements (.1425, 141, 142, 1435, 142, 1435, 143, 1415). If a square be applied at the bottom of the three $\sqrt{2}$ rectangles the intersection of its diagonal with the diagonal of the $\sqrt{2}$ rectangle determines the height of the stem. The diameter of the bowl is determined by the intersection of the diagonal of the area which is in excess of the applied square with the diagonal of half the $\sqrt{2}$ rectangle.

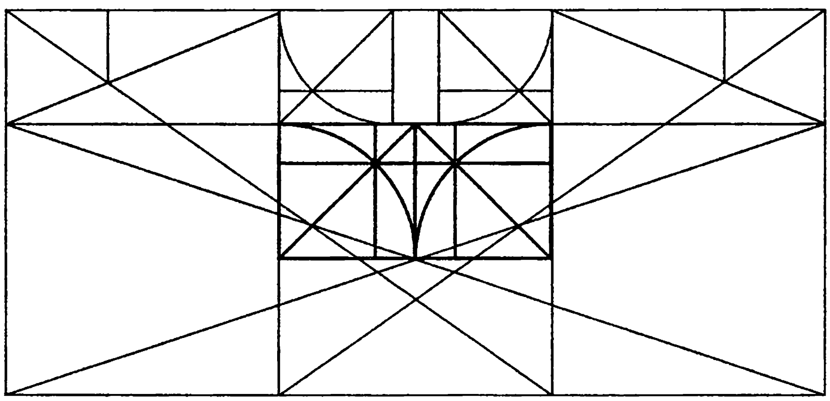
The ratios are:

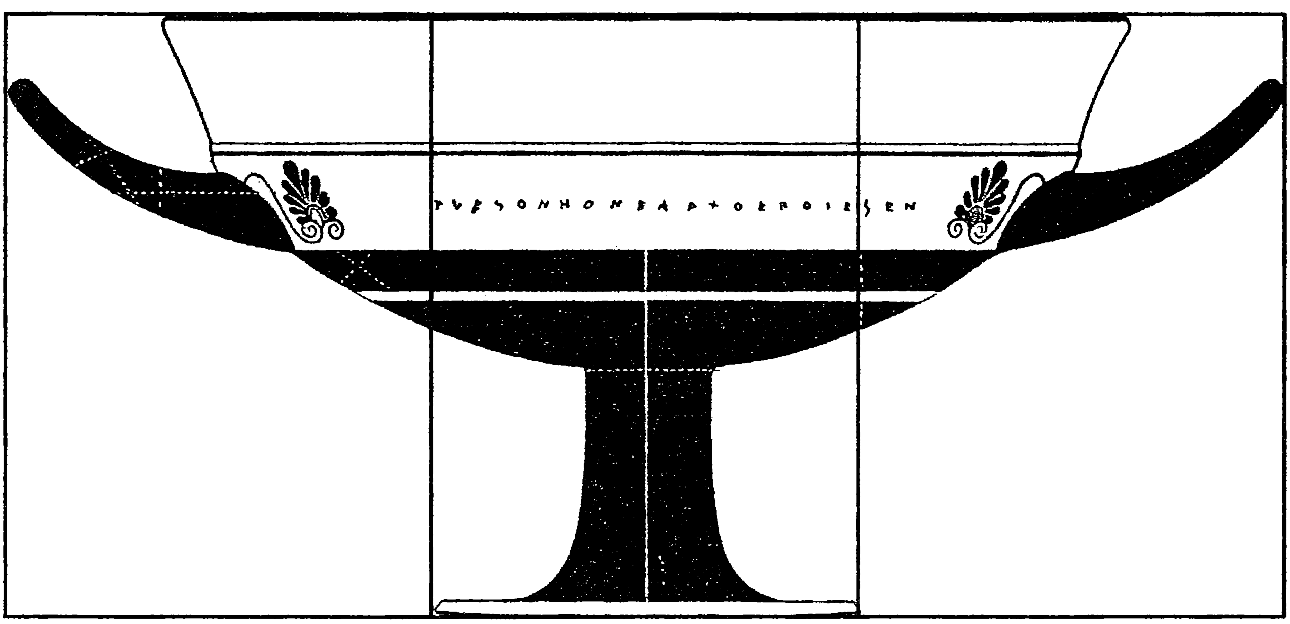
Height	1.000	Width	2.1213
Height of bowl	.5858	Diameter of bowl	1.5988
Height of stem	.4142	Diameter of foot	.7071

Its geometric analysis has been started by dividing the whole rectangle into thirds, three Root Two Rectangles. Inscribe a square inside the bottom of each of the three rectangles.

By applying rabatment of squares and drawing diagonals for their crossing points, as seen below, you can reproduce Hambidge's discoveries.

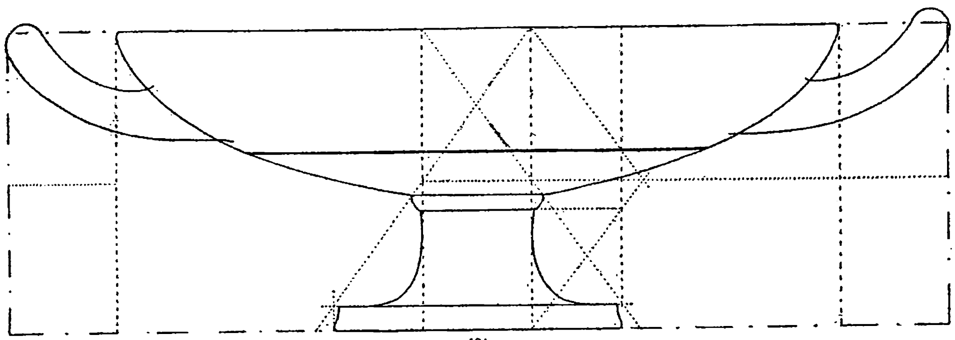






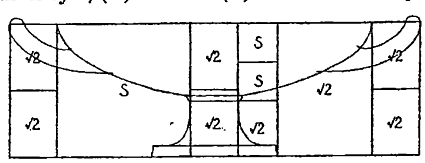
A Red Figure *Kylix*

Another analysis by Hambidge concerns this *kylix*. Its geometry is framed as two large squares flanked by two small vertical Root Two Rectangles (shaded), sharing one pair in-between them at the center.



134

134 Kylix. Inv. 13.83. Hambidge, p. 116. Painted decoration red-figured, except for the large eyes. Interior, warrior running. Exterior, between a pair of eyes, (*A*) Archer. (*B*) Warrior. Beazley, *V. A.*, p. 10, no. 17, assigns the painting to Oltos.

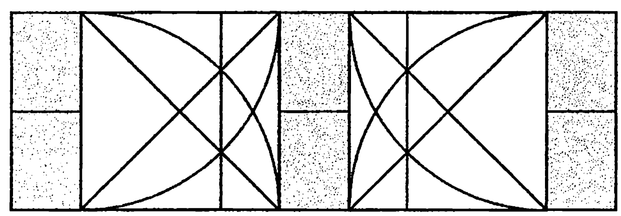


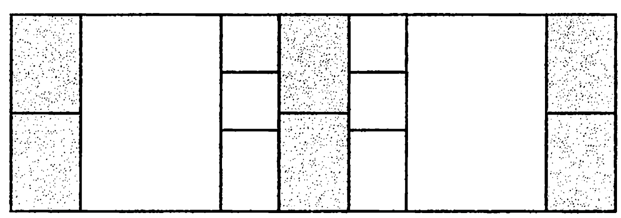
Height, 0.1305 m. Width, 0.402 m. Diameter of bowl, 0.307 m. Diameter of foot, 0.1225 m.

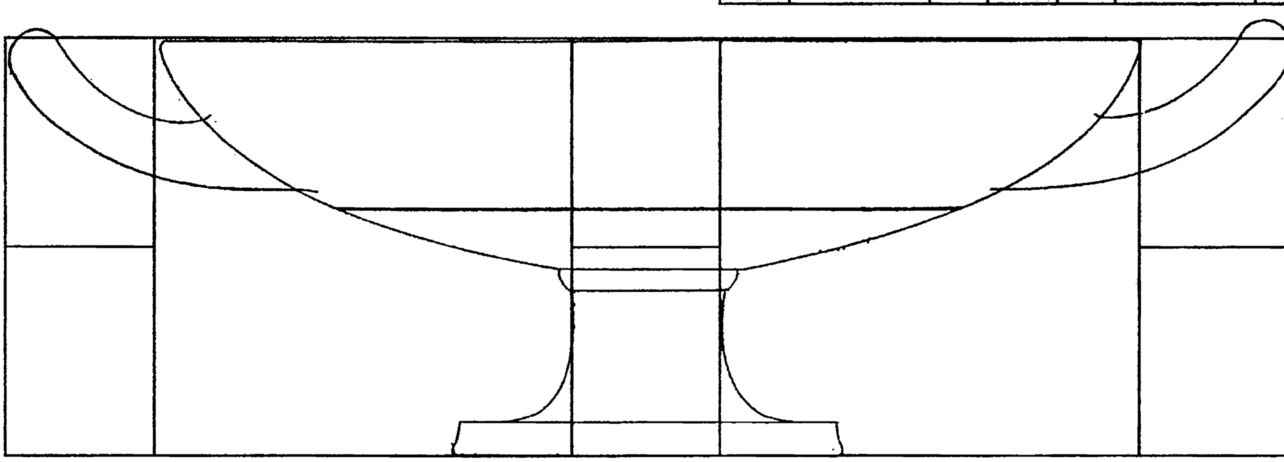
The ratio of height to diameter of bowl is exactly, 2.3535, i. e., $2+\frac{\sqrt{2}}{4}$. If .3535 be added to each end of this rectangle, the ratio becomes 3.0606. This ratio multiplied by the actual height, 0.1305 m., gives .3994083 m., as the total width. This is less than the actual width by .0025917 m. The geometrical analysis is very simple. Cf. the small diagram. The ratios are:

Height	1.000	Width	3.0606	Smallest diameter of stem	.3535
Height of stem to junction with bowl	.4571	Diameter of bowl	2.3535	Diameter of top of stem	.4142
Height of bowl alone	.5429	Projection of each handle	.3535	Diameter of foot	.9393

Implied in Hambidge's analysis, the central Root Two Rectangles are flanked by a rectangle made of two squares and a small Root Two Rectangle. You can construct them by opening your compass to the side of each large square and turn arcs. Draw vertical lines through where they cross, and extend them to reach top and bottom. Then just draw two squares in each rectangle. Use this to further explore the *kylix* design.

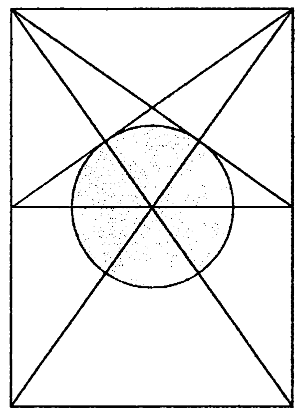


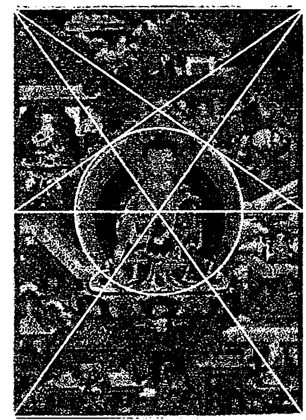




Shakyamuni Buddha Thangka

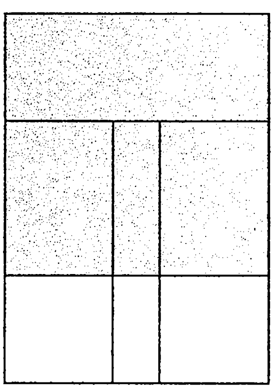
The historical Buddha and spiritual teacher Gautama Buddha lived about 2,500 years ago in India. He was born with the name Siddhartha Gautama and, after a quest for the truth behind life, suffering and death, underwent a spiritual enlightenment that led him to be called Buddha ("Awakened"). He's also known as Shakyamuni ("Sage of the Shakya clan"). Here he sits in the "earth witness" position amidst scenes of former lives. All five fingers of his right hand are extended to touch the ground, summoning the earth goddess to bear witness to his attainment of enlightenment. His left hand held flat in his lap symbolizes the union of method and wisdom. It's in this posture that he overcame obstructions by meditating on Truth.

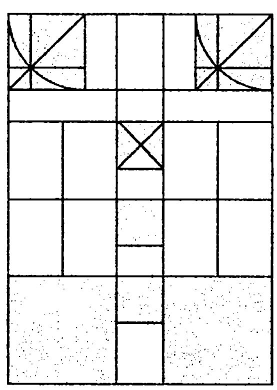


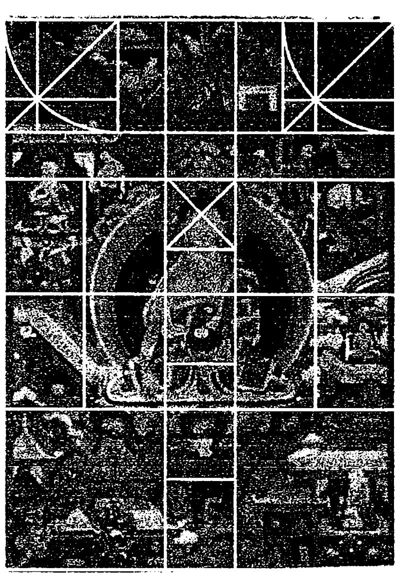


The geometry for this image of spiritual birth is a Root Two Rectangle. It can be understood by dividing it in half. Full and half diagonals define the center and radius of the circle of his aura.

A square drawn from the top of the full rectangle downward reveals that he isn't floating but sitting on the stable invisible bottom of the square. We don't see the line but we "feel" it. Further rabatment of squares from the bottom and within rectangles will reveal more of its design.

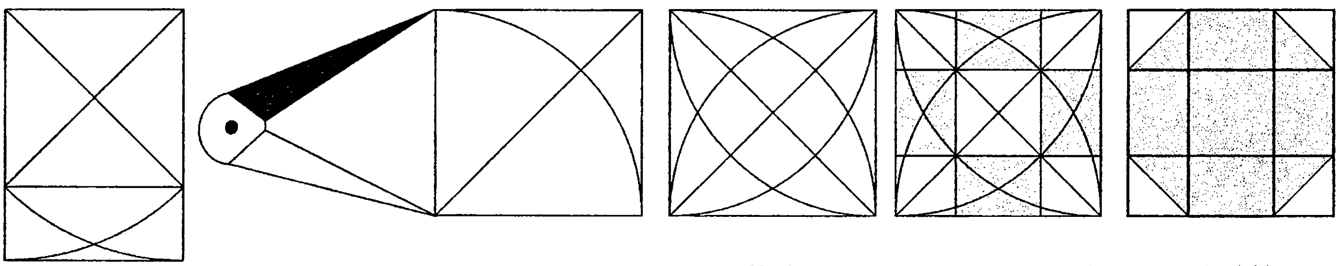


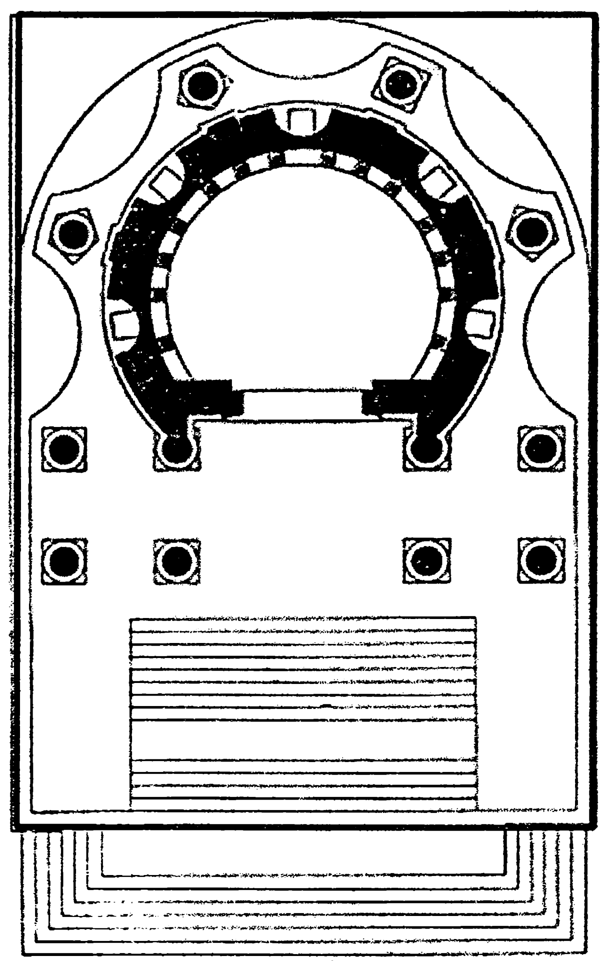




Temple of Venus At Baalbeck

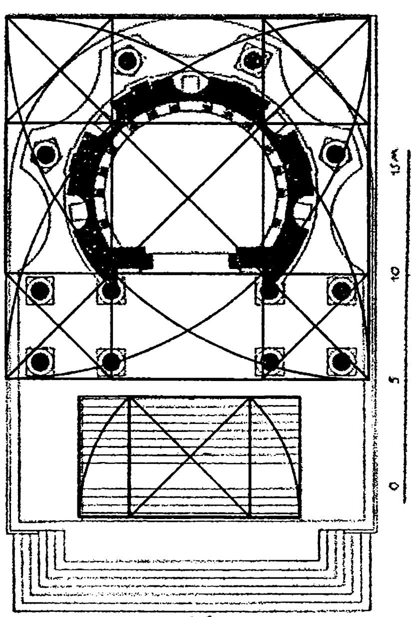
In 331 BCE the Greeks conquered a Phoenician town Baalbeqqa ("Lord of the fertile Bekaa Valley" Lebanon) and named it Heliopolis ("City of the Sun"). In 16 BCE it became a Roman colony under the Emperor Augustus. Over the next three centuries the Romans constructed a monumental temple complex of three temples (to Jupiter, Bacchus and Venus) within an enclosing wall built of stones weighing up to 800 tons each. The Temple of Venus is a small circular structure and was probably finished during the Third Century. When Christianity was declared an official religion of the Roman Empire in 313, Byzantine Emperor Constantine officially closed the Baalbeck temple complex which became a church dedicated to Saint Barbara, the patron saint of Baalbeck to this day.



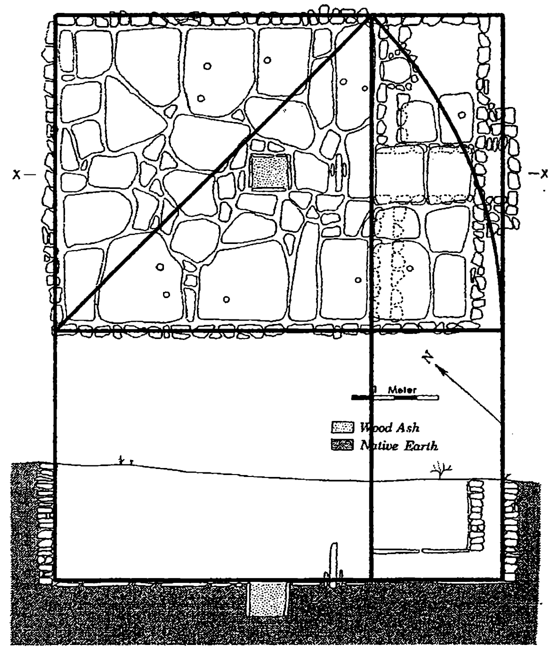


Begin by marking a square inside the top of the rectangle. Use your compass to verify that it's a Root Two Rectangle.

Then follow the steps above to construct an Octagon in the square. Note the crossings, and also the altar made by overlapping Root Two Rectangles.



Kiva at Kawaika-A

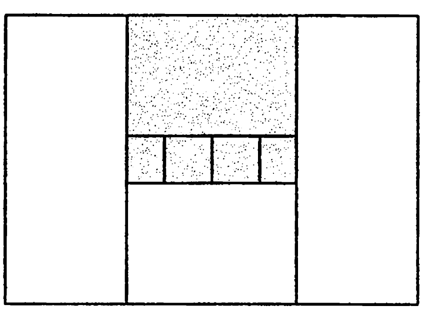


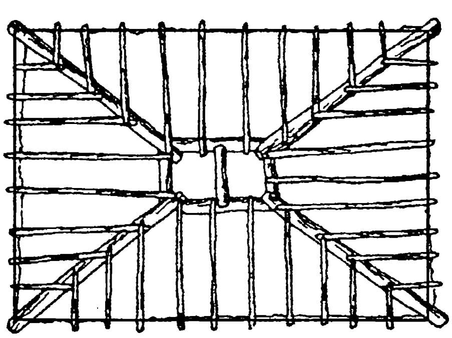
Kawaika-A is in Navajo County, Arizona (35°44’41"N 110°13’12"W). The rectangular shape of this kiva is partly below ground and partly above. The illustrations show the kiva as seen from above and also underground from the side. Notice its paving stone pattern. Do you see geometry in it?

An open-sided rectangle has been drawn along the outline of its stones. Draw a square in its left side. Open your compass to its diagonals and swing downward (and upward) to find two points on the lines to connect and make a Root Two Rectangle. Notice that the paving stones distinguish the square from its extension. Curiously at first, the rectangle doesn’t include the rightmost stones. But closer inspection shows that it forms the edge of the rectangular air hole. By extending the vertical lines downward we see that the Root Two Rectangle precisely defines the edges of the sacred pit dug into the earth.

Geometry of the square and its natural extensions were well known to all ancient peoples close to the earth.

The kiva’s roof is also a Root Two Rectangle. Draw squares from each end, and from top and bottom of the center rectangle, which overlap on the opening.





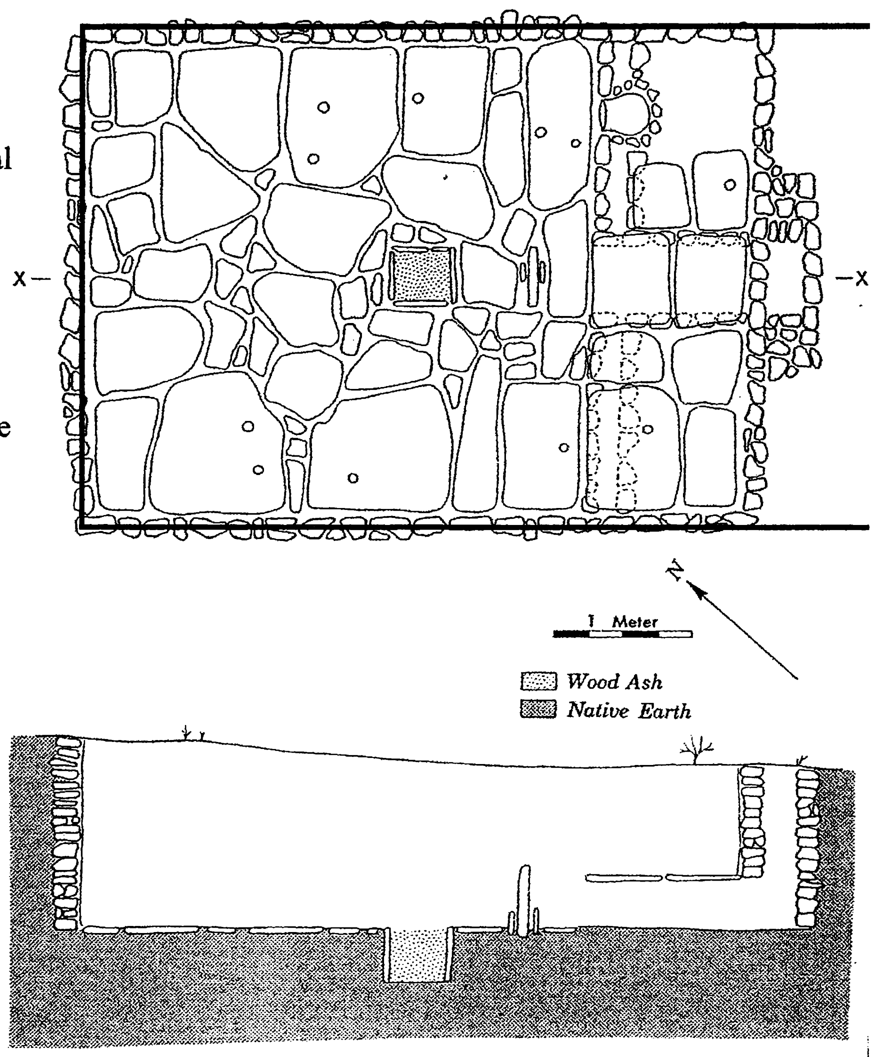
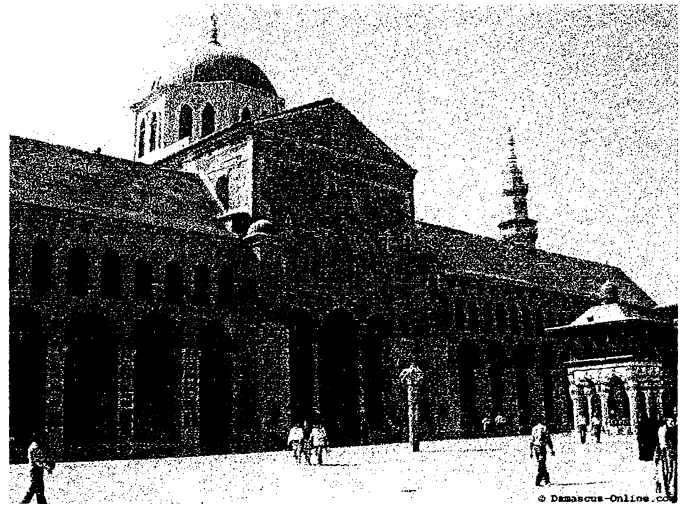


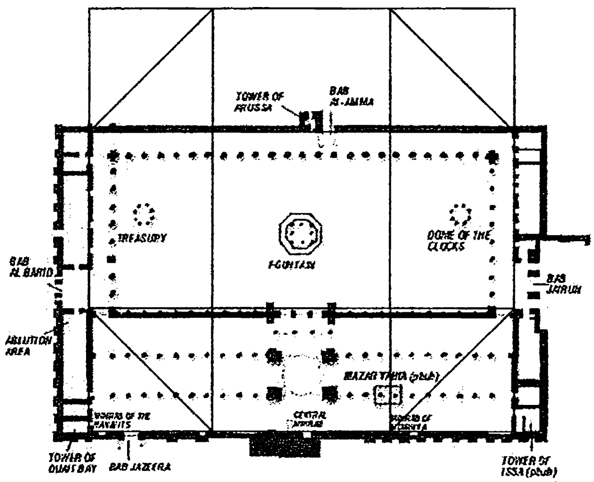
Fig. 49. Plan and profile of Test 4, Room 5, at Kawaika-a, showing the earlier front face of the rear bench, and the emplacement of a large utility jar beneath the right-hand side of the bench.

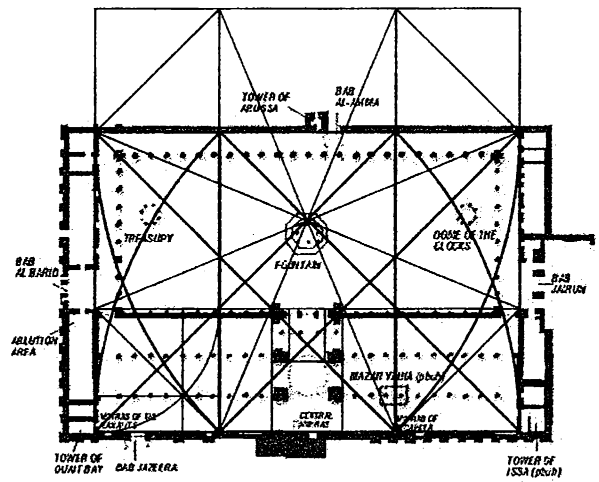
Omayyad Mosque of Damascus



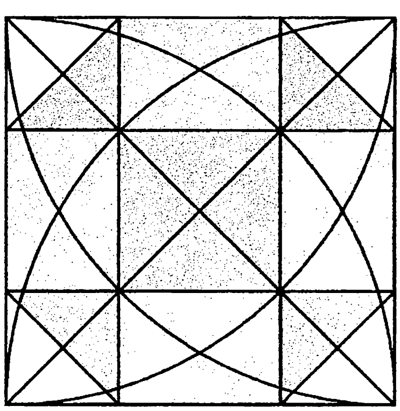
Built 709-715; restored 1970

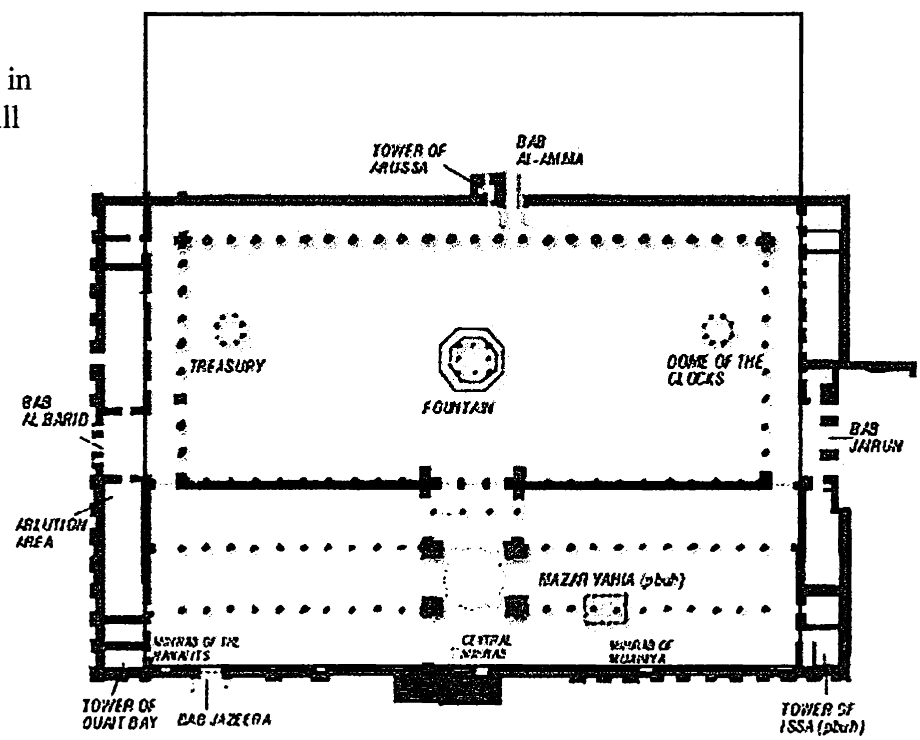
At first this was an ancient Aramaic temple dedicated to the god Hadad. Later it became a Roman temple to Jupiter, and then in the fourth century became a church (the Cathedral of St. John). Now the Great Omayyad Mosque of Damascus, *al-Masjed al-Umawi*, is known as the first monumental work of architecture in Islamic history. It served as a central gathering point (after Mecca) for faith and conquest of the surrounding territories under the Umayyad Caliphate.



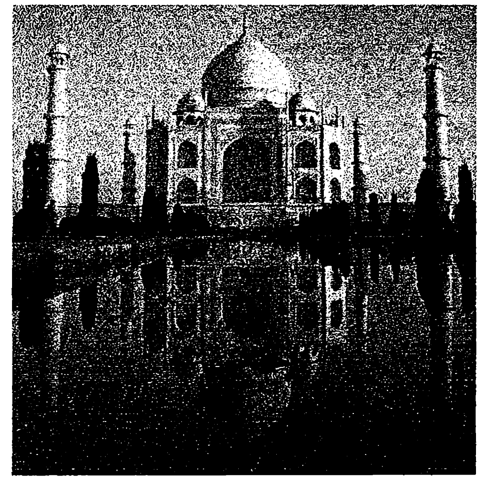


Begin by constructing an octagon in the square. Diagonals and arcs will reveal more.



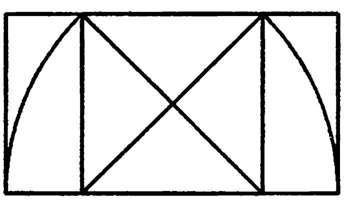


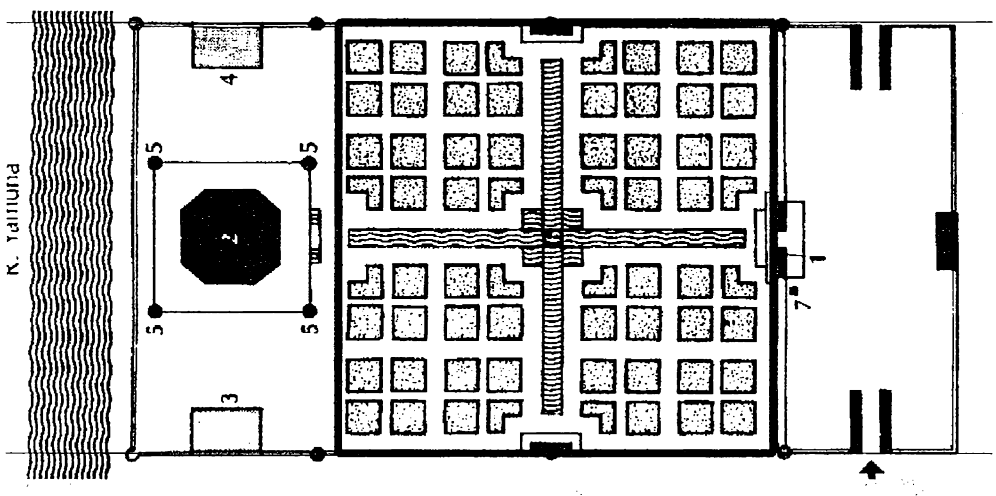
Taj Majal And Garden

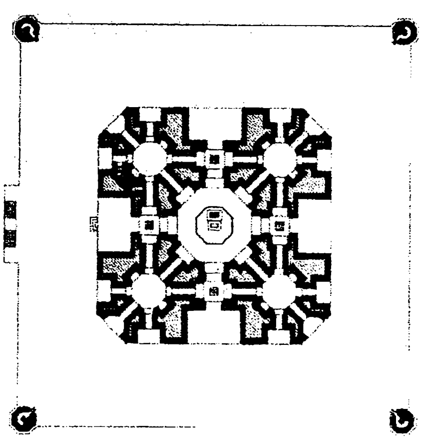


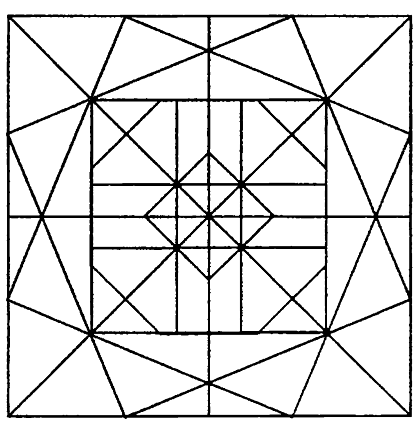
The Taj Mahal is set in the city of Agra, in the northern Indian state of Uttar Pradesh, on the banks of the Yamuna River. It was built by Shah Jahan ("King of the World") in memory of his wife, the beautiful Arjumand Bano Begum, known as Mumtaz Mahal ("Exalted of the Palace"). She was married at 21 but at the age of 39, after fourteen children, died in childbirth. Shah Jahan was inconsolable and for two years. There was no music, feasting or celebration of any kind. But as a passionate builder, he decided to erect a spectacular mausoleum for her. The site was chosen because it was unshaded an located on a bend in the river, so could be seen from Shah Jahan's personal palace in Agra Fort further upstream. He viewed it through the lens of a crystal still placed in the balcony latticework. Construction began in 1633 by 20,000 of the most skilled architects, inlay craftsmen, calligraphers, stone-carvers and masons from all across India, Persia and Turkey. Still it took 17 years to build. Yemen sent agates, the corals came from Arabia, garnets from Bundelkhand, and onyx and amethyst from Persia. Mumtaz Mahal's final resting-place was ornamented like a Queen's jewel-box.

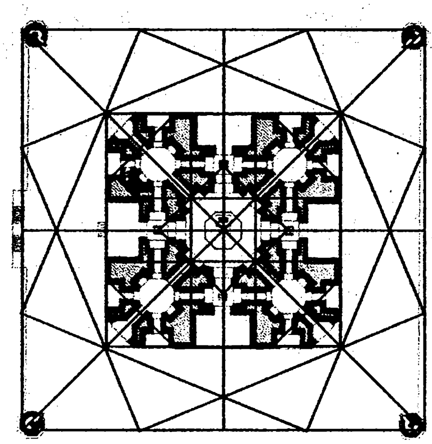
The geometry of the entire grounds can be drawn from its central square garden, by expanding the square as overlapping Root Two Rectangles to the left and right. The mausoleum itself is understood by the construction of a subdivided octagon.









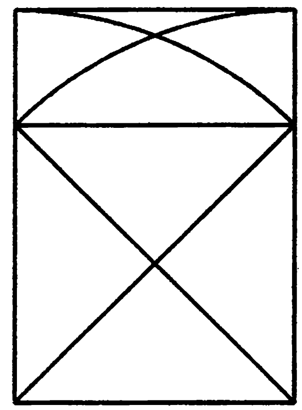


Fatehpuri Mosque Gate

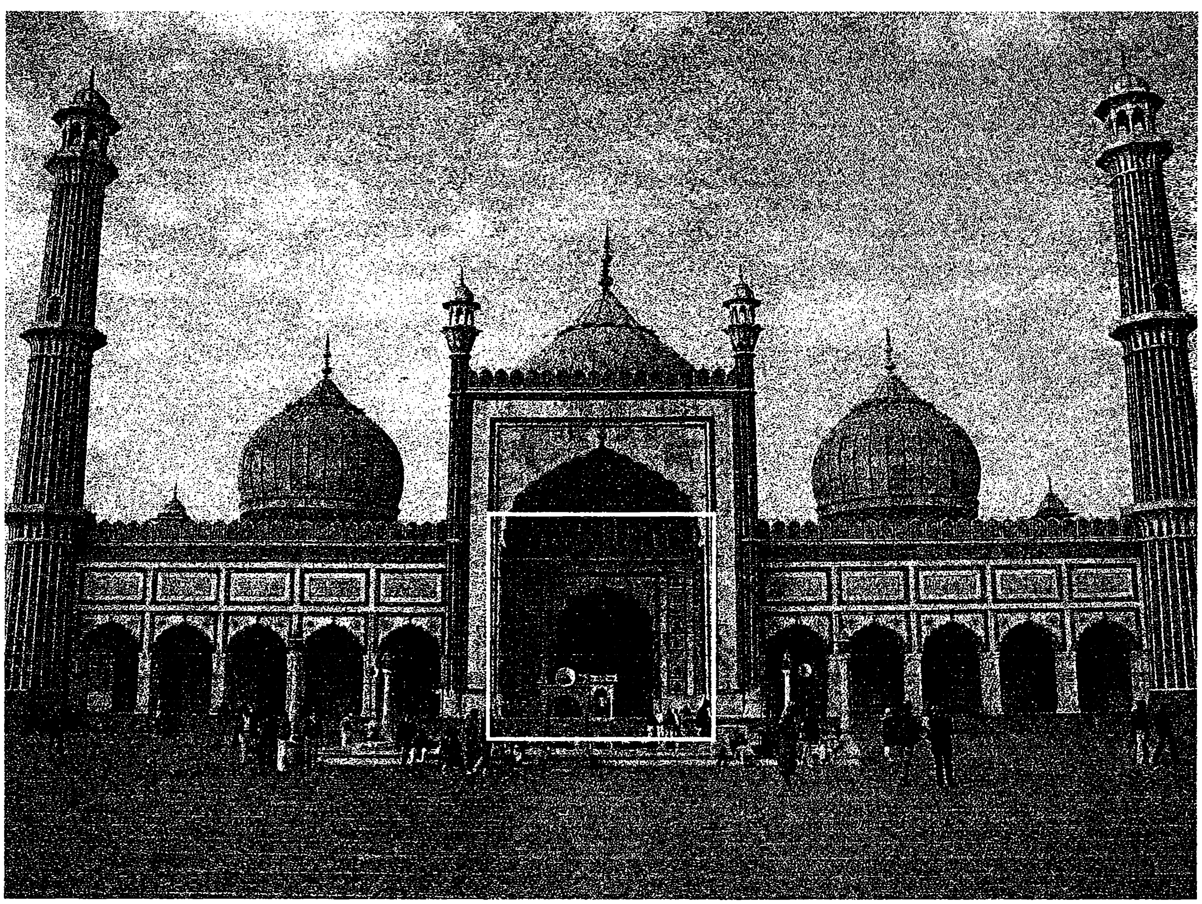
Delhi, India

The Fatehpuri Mosque, the *Fatehpuri Shahi Masjid*, was built in 1650 by Begum Fatehpuri, a later wife of the Mughul Emperor Shah Jahan and so it is named after her. It has two towering minarets on both sides of a large dome, and three gates: one is right in front of the Red Fort and the remaining two are in the north and south. *Kangoorahs (*parapets) of red stone frame the gates, and small domes are at the end. Inside is a large courtyard.

For our interests, the gate itself is framed as a Root Two Rectangle.



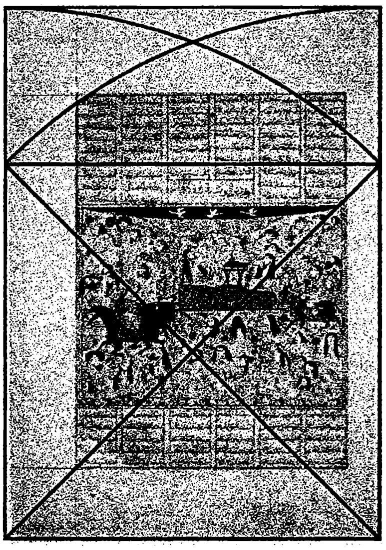
A square has been drawn for you to expand upward and see the Root Two Rectangle.

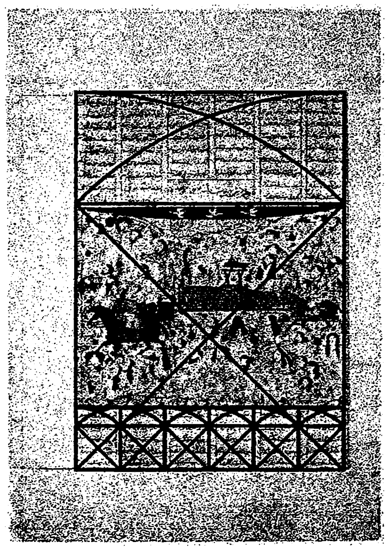


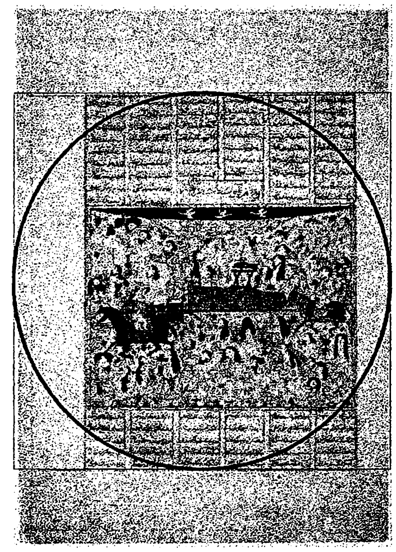
The Funeral Of Isfandiyar

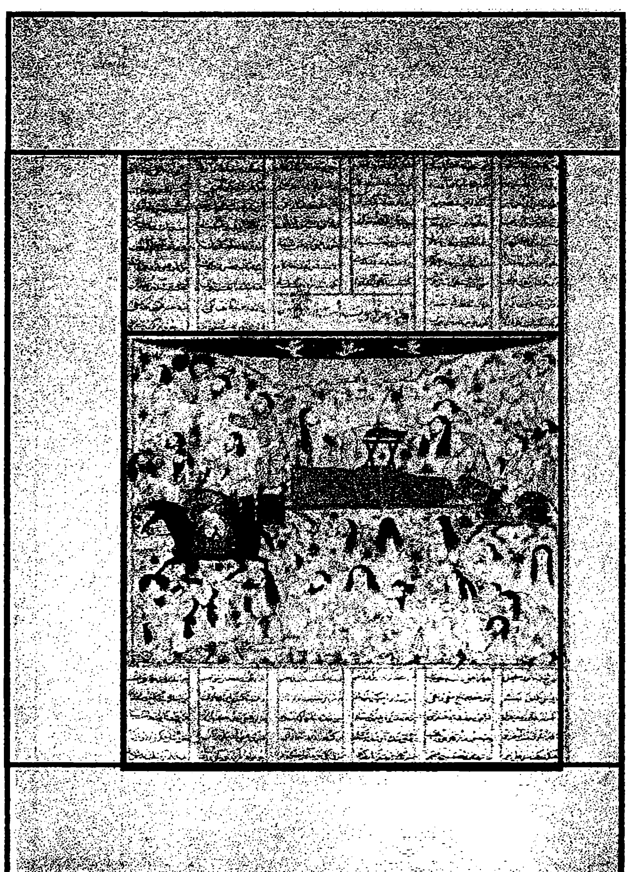
Persia, c1330-1335
Metropolitan Museum of Art, New York, Joseph Pulitzer Bequest, 1933 (33.70)

This detached page is from the Persian "Manuscript of the Shahnama" ("Book of Kings"). The overall page is a Root Two Rectangle, as is the inked illustration with text. Notice that the circle in the square shows that the rectangle is as tall as the page is wide. The text at the bottom of the illustration is structured in six small Root Two Rectangles. Verify these proportions for yourself on the illustration below.

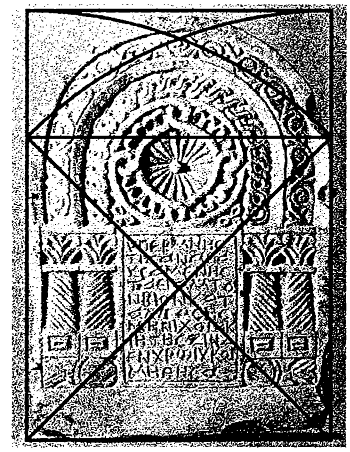








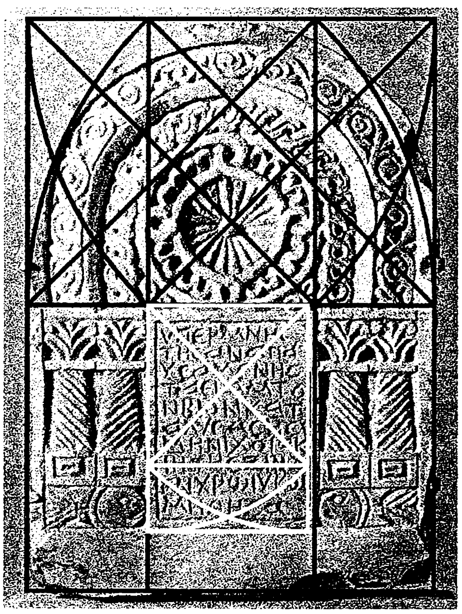
Coptic Christian Funerary Stela

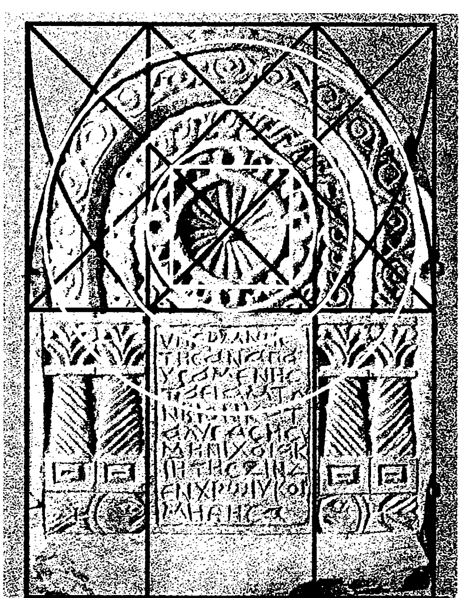


Byzantine, probably from Armant, Egypt, 500–700
Limestone with red, green, and black paint
20 11/16" x 14 9/16" = 1.420, and 52.5 x 37 cm = 1.418...
Metropolitan Museum of Art, New York, Rogers Fund, 1936 (36.2.6)

Inscribed in Coptic in an architectural frame: *To the memory of the deceased, Taeiam, who departed from this life on the eighteenth of Choiak [December] of the seventh indiction. She sleeps in Christ.*

Inscribing a square at one end of its overall rectangle and swinging its diagonal shows us that this stone fits in a Root Two Rectangle, a symbol of birth often seen in funerary stelae. Yet this simple construction doesn't reveal its design. But if we realize that we can divide any Root Two Rectangle in half to produce two smaller Root Two Rectangles, we see the true framework of its composition. Apply rabatment by drawing a square from each end of both Root Two Rectangles. Another Root Two Rectangle encloses the inscription. Diagonals drawn between significant points reveal the center and sizes of the concentric circles.





The Divine Geometer

French *Bible Moralisee* circa 1250

Österreichischen Nationalmuseum in Vienna, ONB 2554

This illumination is from one of the two oldest examples of a "moralized Bible", one which conveys ideas through pictures, with words as captions. The picture is sometimes called "The Divine Architect Of The Universe". It depicts Deity creating the universe as a golden sphere using a geometer's divider. Within the sphere of the living cosmos we see the World Soul.

We can read one possible source of this in Proverbs 8:27 -

> "When he prepared the heavens, I was there:
> when he set a compass upon the face of the depth..."

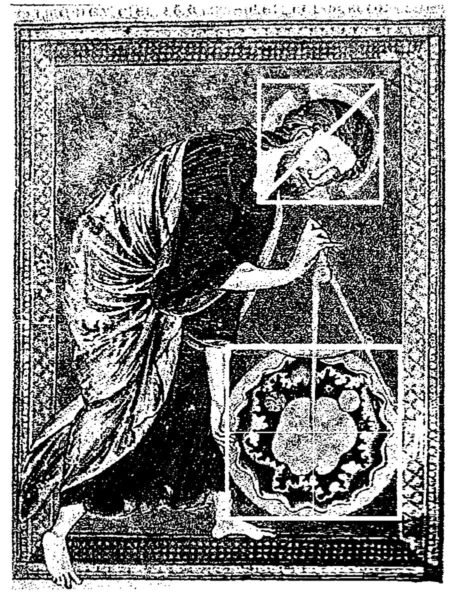
In great detail Plato described this geometric creation of cosmos by the Demiurge sixteen centuries earlier in his *Timaeus*, which was influential in the medieval Church:
[34a] "He spun it round uniformly in the same spot and within itself and made it move revolving in a circle; [34b] He made it smooth and even and equal on all sides from the center, a whole and perfect body compounded of perfect bodies, And in the midst thereof He set Soul, which He stretched throughout the whole of it, and therewith He enveloped also the exterior of its body; and as a Circle revolving in a circle... ."

So one might wonder whether this Biblical illumination honoring geometry might *itself* have geometry purposefully applied to its composition. A simple swing of our compass (seen above) shows us that it's a Root Two Rectangle, an appropriate symbol for the birth of the spherical cosmos.

Further divisions and subdivisions by rabatment, diagonals and arcs reveal more of its composition. The square's diagonal neatly skims the new spherical world, fixing its location in the scene. The top of the square runs horizontally through Deity's intent eye which straddles the space above and below.

When the whole Root Two Rectangle is replicated within the square its side runs from the tip of the nose down through the hand, through the vertical divider leg to become the *axis mundi* through the center of the newly forming universe.

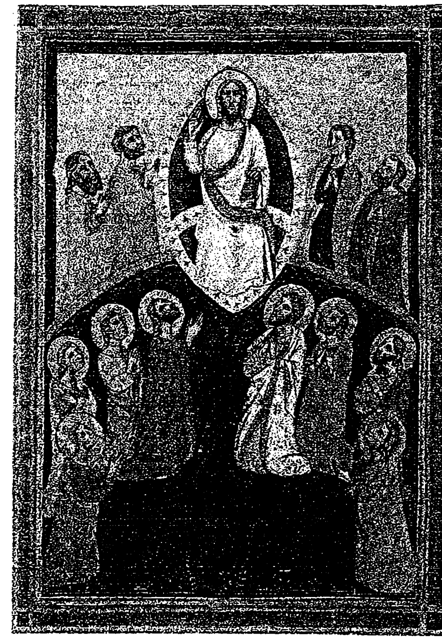
Further exploration on the next page will reveal more. Curiously, the painting's only two distinct circles (the divine halo above and the newborn universe below) are geometrically related: the diagonal running across the Divine mind equals the *side* of the square around the cosmos. They share the same square-root-of-two relationship as the sides of the whole painting. Interpreted symbolically, the Divine idea which comes from within translates to become the outer measure of the worldly square.



See what you can discover by applying geometric analysis to it.

Start by constructing squares inside the top and bottom of the whole picture, and in each half of the subdivided Root Two Rectangles given here.

Christ With Apostles

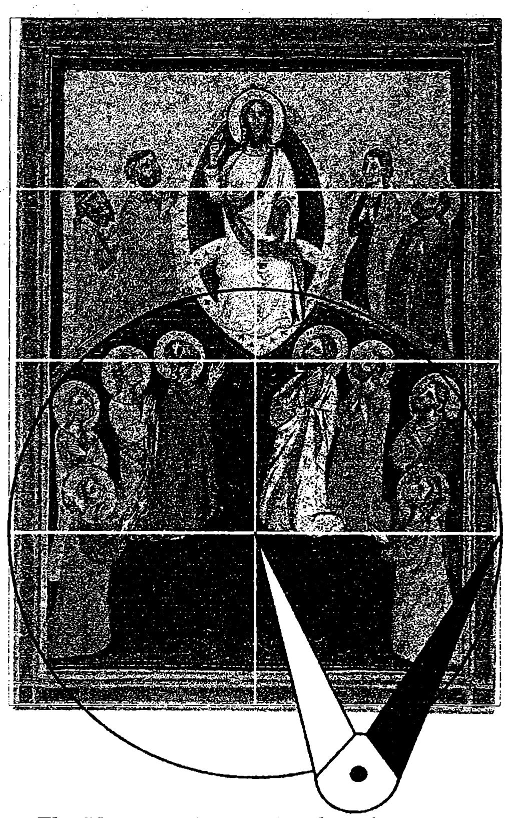


Pacino di Bonaguida (1302-c1340) spent his entire career in Florence as a painter in the middle of the Italian Proto-Renaissance period (c1100 - c1400). In addition to altarpieces he painted miniatures and decorations for illuminated manuscripts, and is now considered the inventor of miniaturism, a style which organizes the whole painting into multiple small-scale scenes. He's known for a strong sense of expressiveness and drama. His only signed work is the polyptych of the *Cruciifixion with SS Nicholas, Bartholomew, Florentius and Luke* now in the Accademia Gallery in Florence but originally on the high altar of S Firenze in Florence. Pacino is last documented *c*1330 when he enrolled in the Arte dei Medici e Speziali in Florence. In the Twentieth century scholars reconstructed his career and attributed more paintings to him, like this one.

Nothing is known about any training in geometry he may have had, but he certainly used it to compose his works. The whole of this painting is a Root Two Rectangle. But to understand it we must first divide it half, and half again, until we have a grid of eight small Root Two Rectangles (1). Right away we can place our compass point where two lower Root Two Rectangles meet, open it to the side of the painting, and turn a circle. This encloses the lower eight Apostles.

To proceed (2), we subdivide the top half's four Root Two Rectangles each in half, and half again, until we have sixteen smaller Root Two Rectangles. We also draw the square (black line) up from the bottom of the Root Two Rectangle. Now we're ready to understand the placement of Christ in the almond shaped *Vesica Piscis* (see Volume 1 Chapter 2).

1



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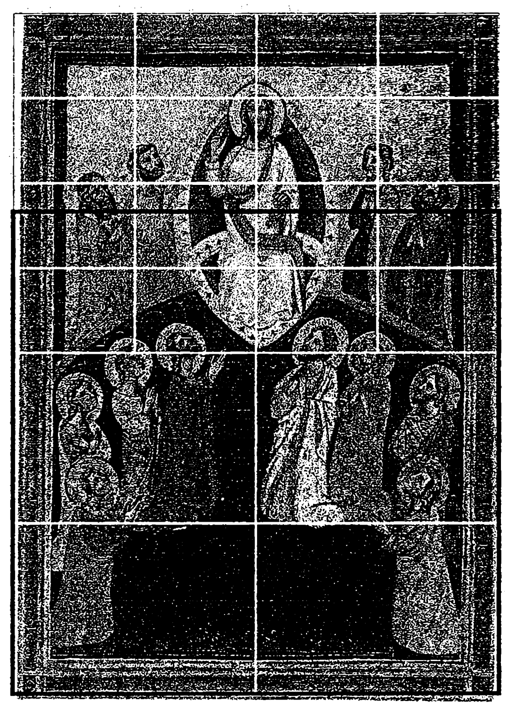
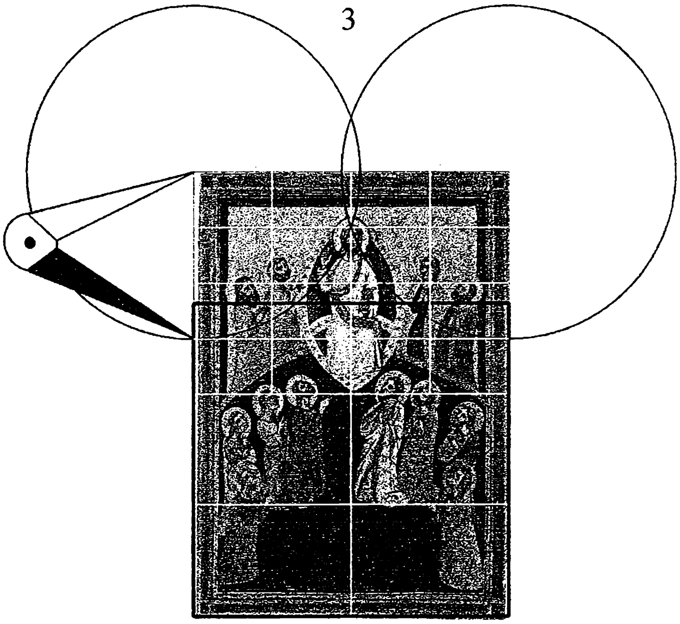
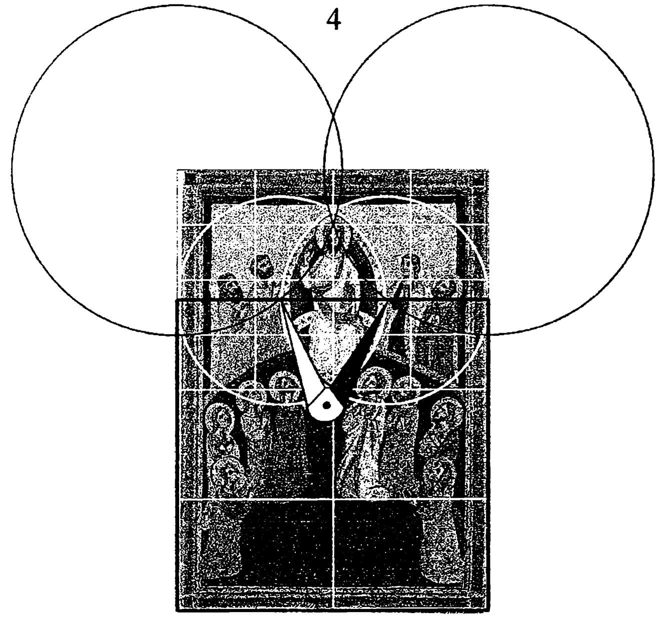


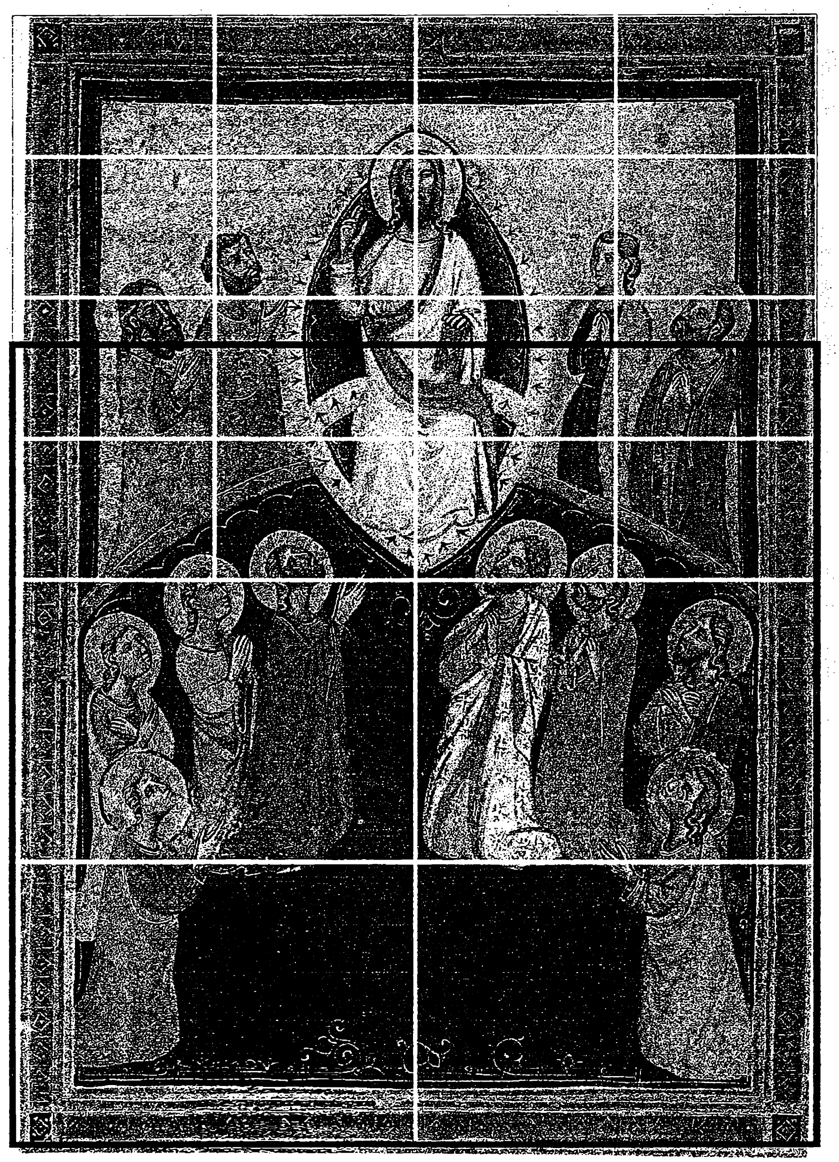




(3) Place the compass point at the top corner of the whole rectangle and place its pencil at the bottom of the third small Root Two Rectangle as shown. Turn a circle. Repeat this by turning this same circle from the upper right corner.

(4) Notice where these two large circles cross the top of the square made from the bottom. Open your compass between these two points and turn circles centered at each of these points. They'll reach the sides of the painting. This creates the *Vesica Piscis* which neatly contains the golden almond shape in which Christ sits, and groups the two pair of remaining Apostles.

Here is a larger version of the painting with the necessary Root Two grid (white) and square (black) which will allow you to replicate the described construction, and to investigate its composition further.



Page From An Ethiopian Illuminated Gospel

Late 14th to early 15th century, Ethiopia, Amhara region.
Metropolitan Museum of Art, New York, Rogers Fund, 1998 (1998.66)

The illuminated manuscript of the Four Gospels that this page came from was written in Ge'ez, the classical Ethiopian language and created at the Dabra Hayg Estifanos monastery in the Amhara region of Ethiopia. Its full-page paintings depict New Testament scenes from the life of Christ and portraits of the evangelists. Typical of Ethiopian painting, the imagery is two-dimensional and linear. Heads are seen frontally while bodies are often in profile.

The Four Gospels are the essence of Ethiopian Christianity. This manuscript was likely kissed, displayed during processions, and placed on the altar to mark important events on the church calendar such as feasts or holy days. Recent research suggests that a member of Ethiopia's ruling elite may have commissioned this manuscript for presentation to his or her favored church or monastery. Brief notations indicate that the church in question was dedicated to the Archangel Michael.

In the Fourth century the Ethiopian king Ezana converted to Christianity. Christianity became the official religion of the state whose legacy endured in various forms until the twentieth century. During the sixteenth century, Islamic incursions devastated the region and most Christian Ethiopian art that predates the seventeenth century was destroyed. This illuminated gospel is a rare survival.

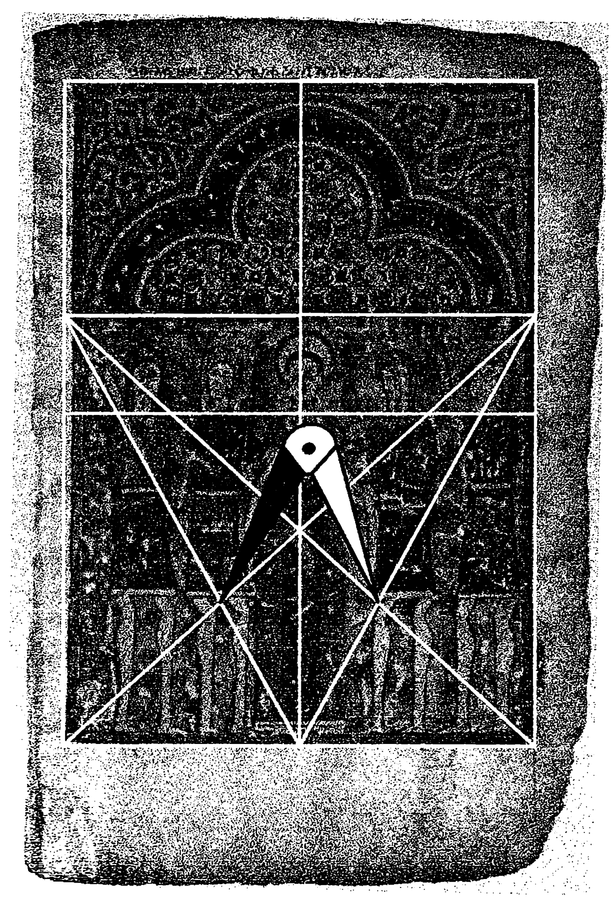
The painting itself is a Root Two Rectangle.
(Prove it with your compass.)

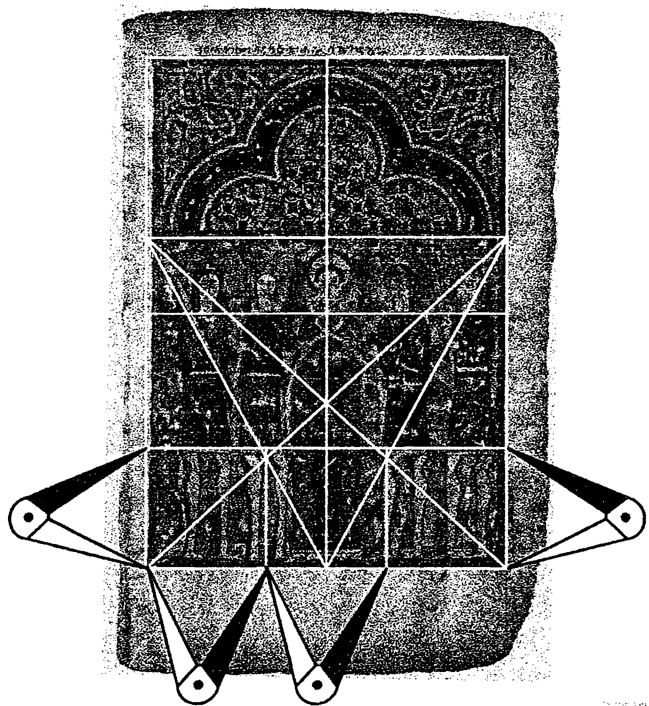
The way to understand its scheme is to divide it into smaller Root Two Rectangles, as has been done for you on the next page.

First, draw squares in the tops of each of the two small Root Two Rectangles to distinguish the painting's most holy upper section.

Next, three squares must be drawn along the bottom. There are various ways to do this. Here's one: divide the bottom side into thirds by drawing the four diagonals as shown.

Then open the compass between their crossing points. This is one-third the horizontal distance across the bottom.





Use this opening of the compass to measure three squares along the bottom, as shown above. Draw lines connecting the points across the hem of the garments.

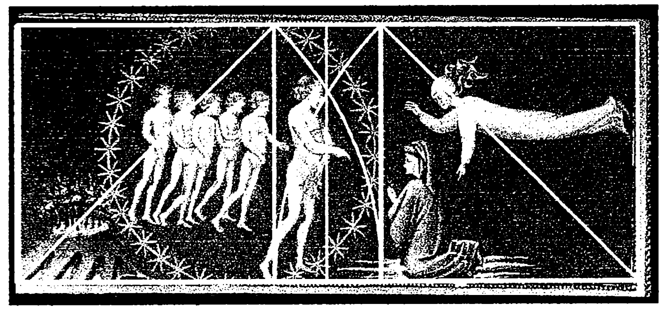
Notice that the horizontal line doesn't go through the diagonals' crossing points. But it would if the diagonals came from the corners of a square drawn from the whole bottom.
Next, apply rabatment and inscribe squares from each end of the middle section of the painting (above, right). The squares will overlap along each side of the central figure.

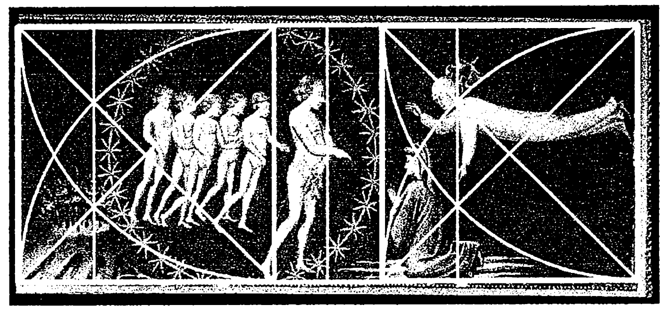
There's more in this for you to explore.

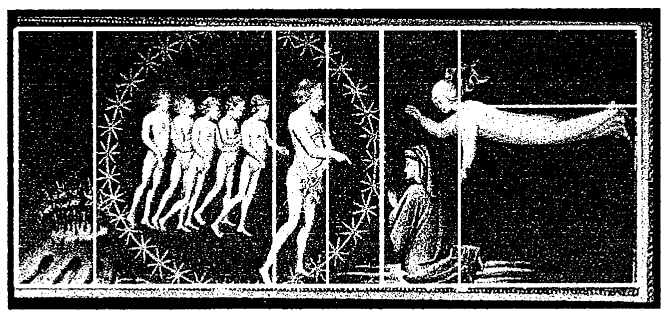
Can you find how to draw the circles in the upper section?

The Birth Of Adam

Giovanni di Paolo, Fifteenth Century
British Museum, London



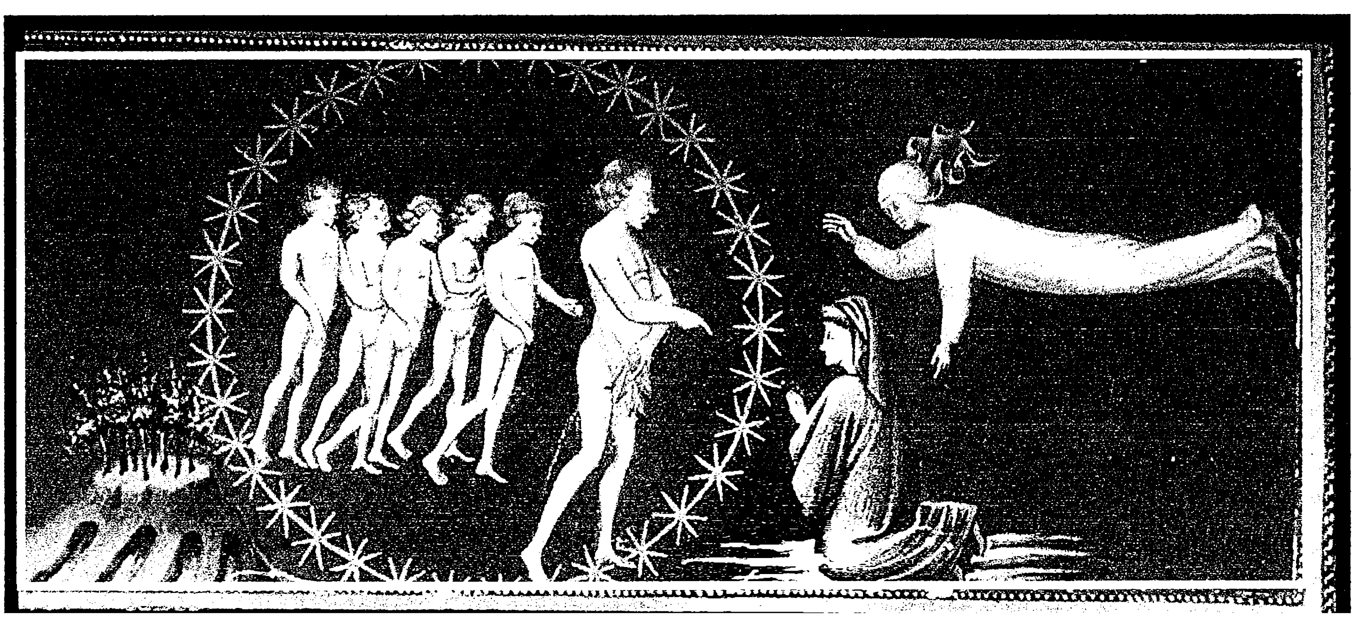




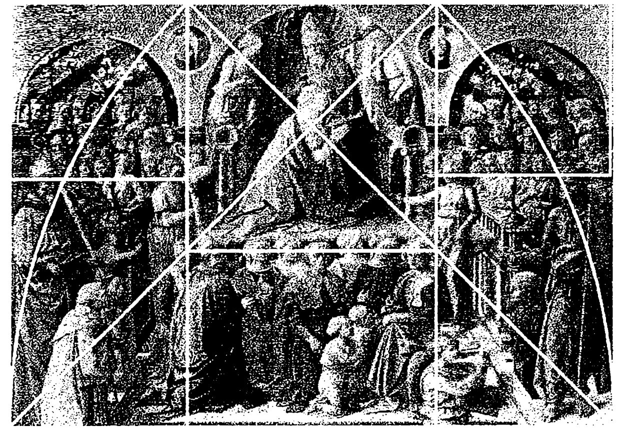
This illumination is one of sixty-one miniatures depicting scenes from Dante's Divine Comedy painted by Giovanni di Paolo (1395-1482), a prolific Italian Early Renaissance painter and illustrator of manuscripts of the Sienese School. This illumination depicts Beatrice floating above a kneeling Dante, showing him the birth of Adam (followed by whom?) in a circle of stars. The Garden of Eden is on a circular hilltop at lower left.

The whole painting is longer than a Root Two Rectangle. In fact, it is made of two Root Two Rectangles overlapping on their shared extension. To prove it, fold a square back from the right end of the added white rectangle. This places Dante in the square world, and Adam in a Root Two Rectangle, symbol of birth. Also draw a square inside the left end to frame the five smaller figures.

Next, draw diagonals and arcs in each square and notice where they cross. Use this to construct a vertical Root Two Rectangle within each square to see more about the placement of figures.

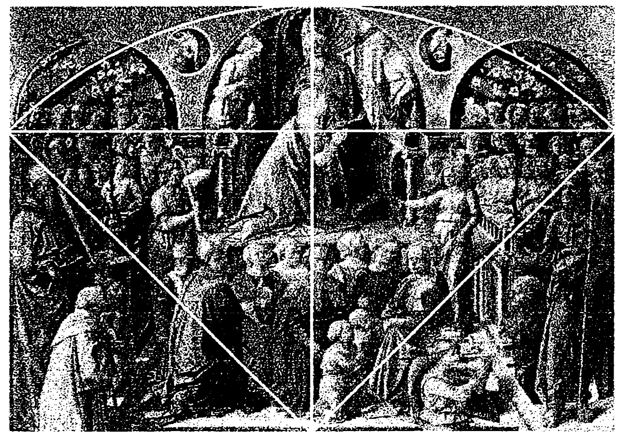


Coronation Of The Virgin



Filippo (Fra Lippi) Lippi, painted 1441-1447
Tempera on wood panel
Uffizi Gallery, Florence

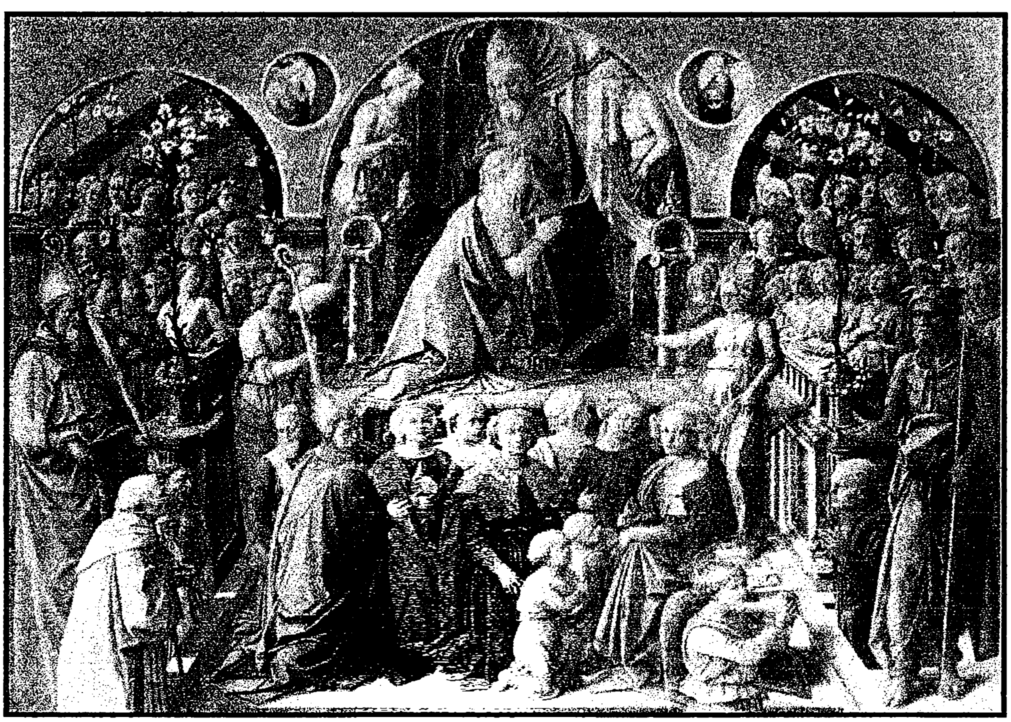
Fra Filippo Lippi (c1406-1469) was an orphan brought up in a convent. But instead of taking his vows, he caused a scandal by running off with a nun. He was recognized for his painting skills by the Medici family and they helped get the couple laicized before he was commissioned to paint some of the walls of their palace in Florence. (Their child, Filippino Lippi (1457-1504) also became a highly successful painter.) Fra Lippi is known for crowd scenes and serene Madonnas.



This "Coronation of the Virgin" was commissioned for Sant'Ambrogio in 1441 and contains both his self-portrait and that of the patron.

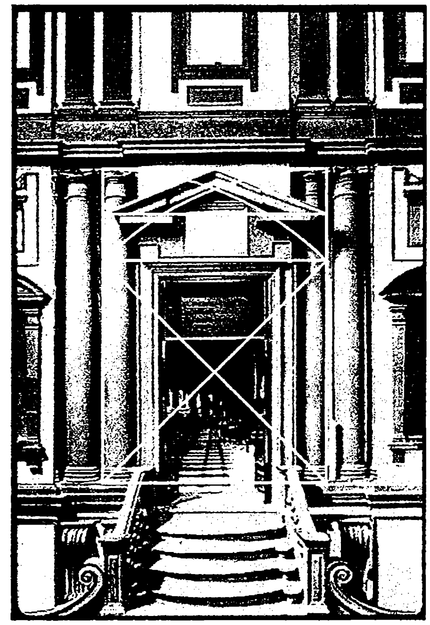
You can prove it's a Root Two Rectangle by constructing a square from one end and swinging arcs from its diagonal. Notice how the square's side defines the placement of a column. Draw another square from the other end to indicate the other column. We'll see this creation of a larger central opening flanked by two smaller openings later at cathedral entrances.

Next, draw a square in the top of the central rectangle to frame the upper scene, and the people below are framed in their own small Root Two Rectangle.



Finally, divide the whole picture in half and we have two smaller Root Two Rectangles. Draw a square from the bottom of each to further distinguish the columns' capitals. Further geometric exploration will reveal more of the painting's composition.

Laurentian Medici Library Entrance

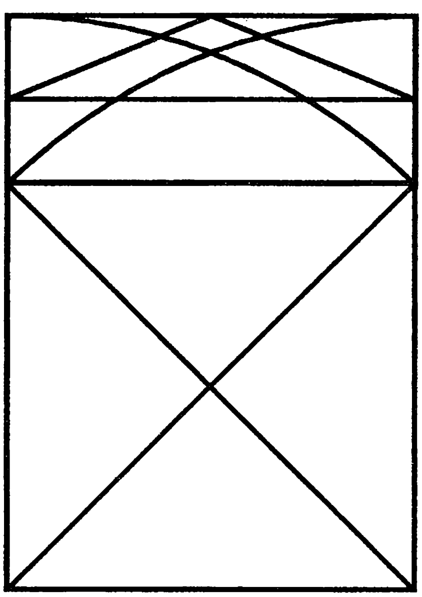


The *Biblioteca Medicea Laurenziana* in Florence, Italy, was a gift to the public in 1571 of the Medici's private library by the Great Duke Cosimo de'Medici. It contains a collection of nearly 11,000 ancient manuscripts, including 2,500 papyri,. The architecture was planned and partly realized by Michelangelo Buonarroti. The Library still continues to acquire books of a highly textual or esthetical quality.

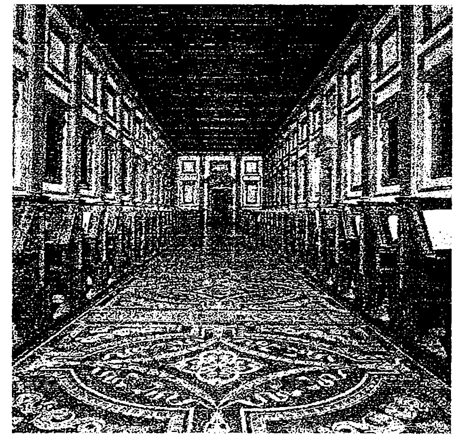
Approaching the Library we come to Michelangelo's entrance. A white rectangle has been drawn around it which includes the adjacent columns. Draw a square up from the bottom, using your compass to mark the length of the bottom side up onto the longer side. Notice how it shows us the top of the door itself.

Swinging your compass along the square's diagonal confirms that this is a Root Two Rectangle, inviting us in to a birth of knowledge. The slanted top of the doorway points to the center of the sides of the rectangle extended beyond the square.

You can see more of the harmonious design of the entrance by exploring it further with your compass and straightedge.

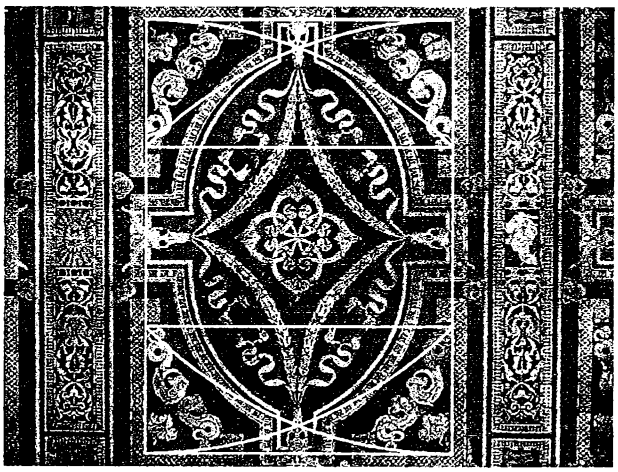


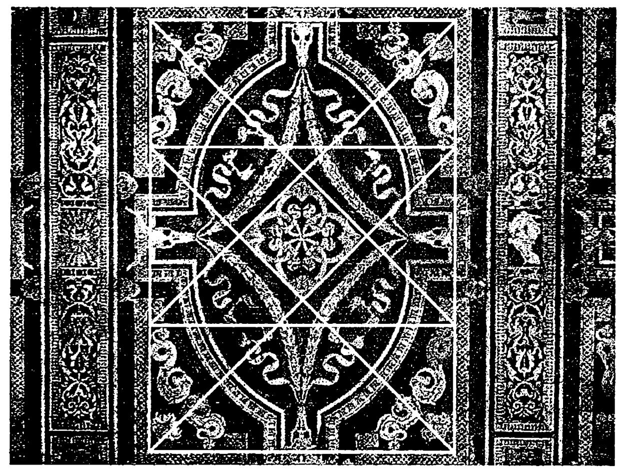
Laurentian Medici Library Floor Tile



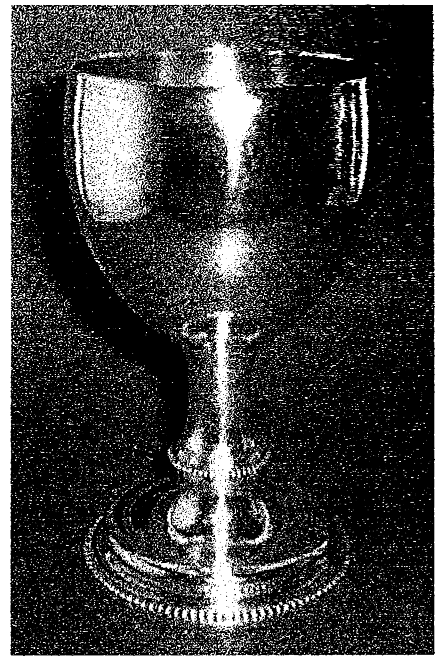
Once inside the Library we notice the design of the large floor tiles.

A white rectangle has been drawn on one tile. Draw a square from one end, then make an arc of its diagonal to show that it's a Root Two Rectangle. Apply rabatment to draw a square from the other end too. Then draw the two diagonals for each square. Notice how they form a central square framing the tile's design. Explore the tile to see further placement and dimensions of its elements.





Silver Wine Goblet



Height 6", Weight 7.2 Troy oz.

This George III silver wine goblet was made in London in 1777 by the silversmith James Stamp.

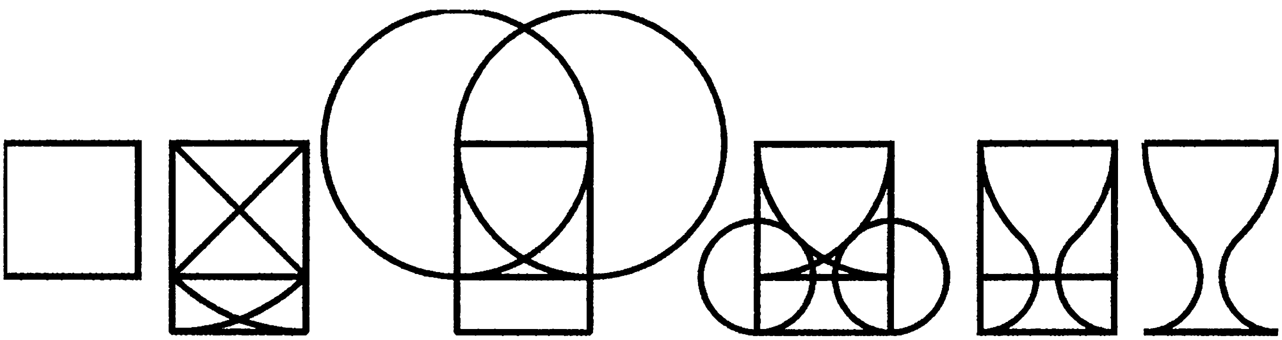
Its geometry (see steps below) begins with a square, extended by its diagonal downward to create a Root Two Rectangle which has been provided.

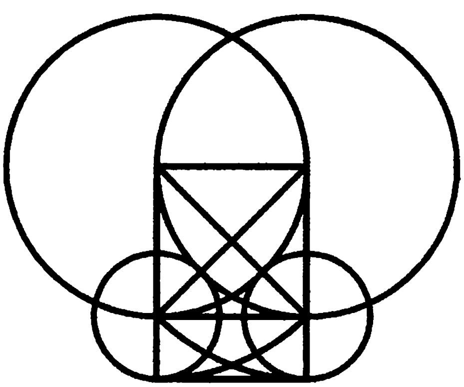
Open your compass along its top side and turn two large circles.

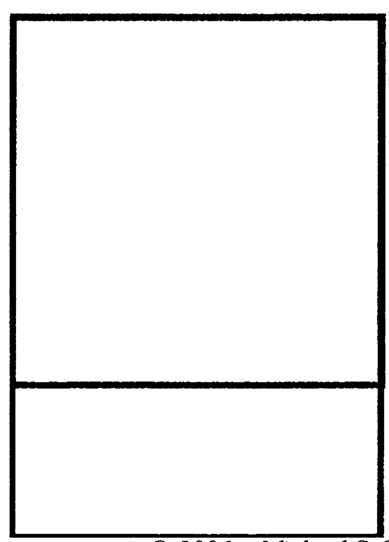
Then place your compass point at the bottom corner of the square and open the pencil to the bottom of the rectangle. Turn a small circle. Do the same with the compass at the other corner of the square and turn another small circle.

The curves made by the circles give the outline of the goblet which you can shade and decorate with colored pencils.

See also the *Rotating Goblets Construction* in Volume 1, Chapter 4 page 62.





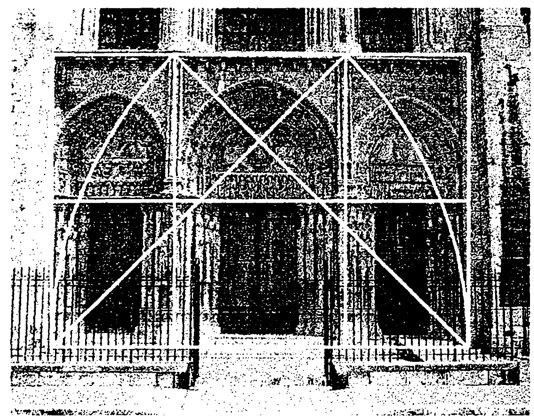


West Entrance To Chartres Cathedral

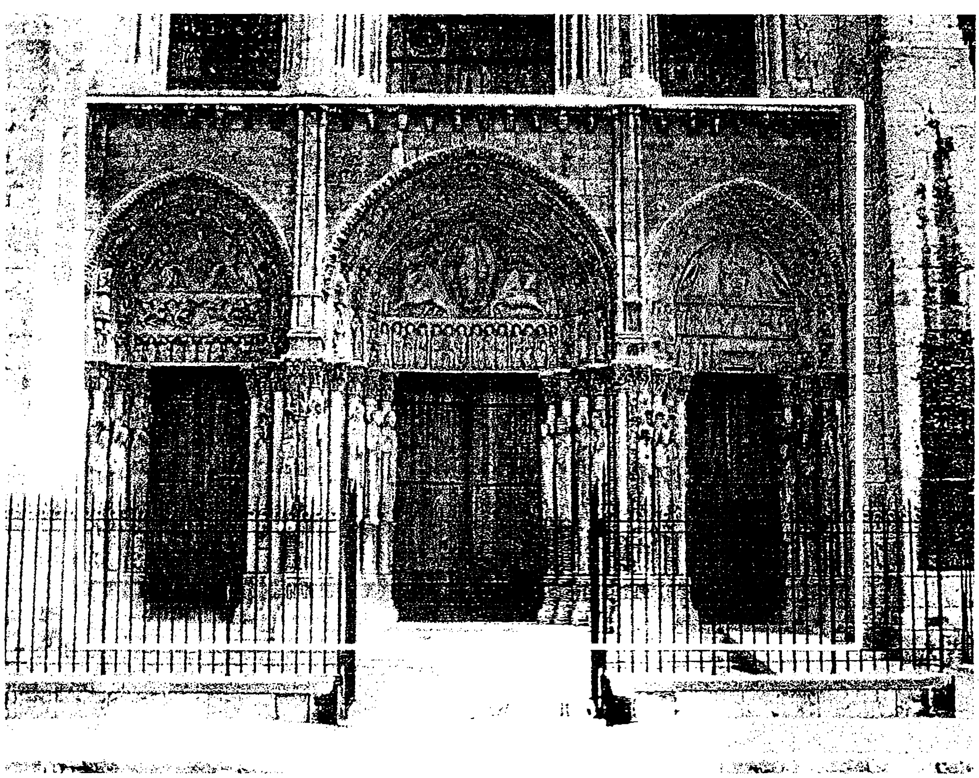
The "Cathedral of Our Lady of Chartres" is located about 50 miles from Paris. It's considered the finest example in all France of the "High Gothic" style of architecture.

Built on the site of a Romanesque church founded on an even earlier market and festival meeting place near a sacred spring, the cathedral was begun in 1145. Its central entrance, called "Royal Portal" was completed about 1155. In 1194 most of the cathedral was destroyed by a fire started by lightning, all except the west front which retains its "early Gothic" style. The body of the cathedral was rebuilt between 1194 and 1220. Its magnificent rose window was built soon after in early 13th century (see Volume 2, Chapter 12 page 217).

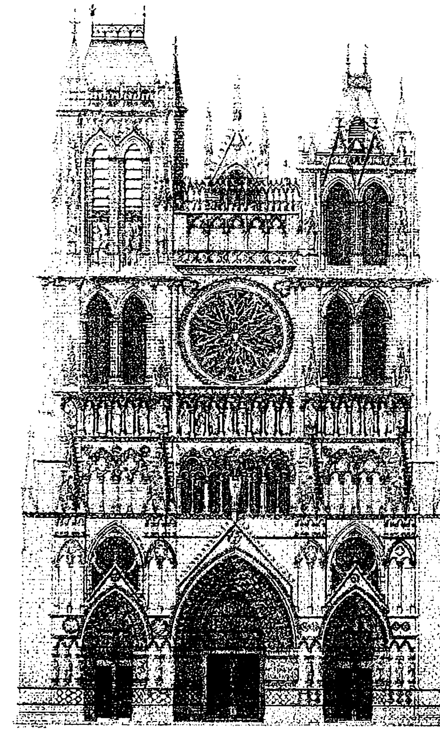
We're interested in the proportions of the Portal Royal. A rectangle enclosing the three entrances is a Root Two Rectangle. You can prove this by drawing a square inside one end, open your compass to its diagonal and swing an arc which reaches the other side. Draw a square in the other end and notice how their vertical lines divide the columns between the doors.



Divide the whole rectangle horizontally in half to reveal more about the architecture.



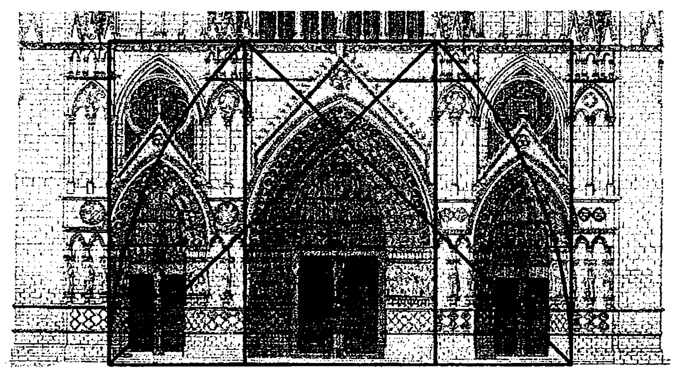
Cathedral Of Notre-Dame of Amiens
(West Facade and Floor Plan)



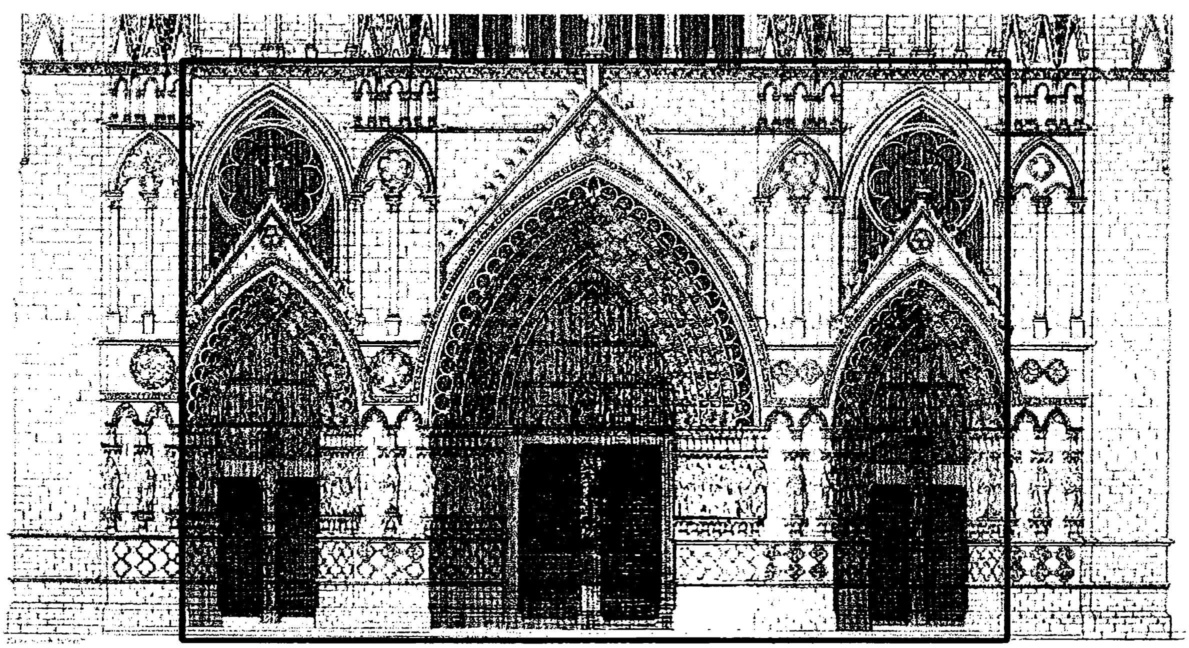
A Romanesque cathedral built on this site in 1152 was struck by lightning and burnt down in 1218. The current cathedral, in Gothic style, was begun in 1220 and mostly completed by 1245. The choir was started in 1238 and completed in 1269. It's labyrinth (see next page) was installed in 1288. The south and north towers were completed in 1366 and 1401.

Cathedral Amiens has the tallest and has greatest interior volume of all French Gothic Cathedrals. It's said to contain relics of St. John the Baptist (allegedly including his head) brought to Amiens in 1206 from the Fourth Crusade.

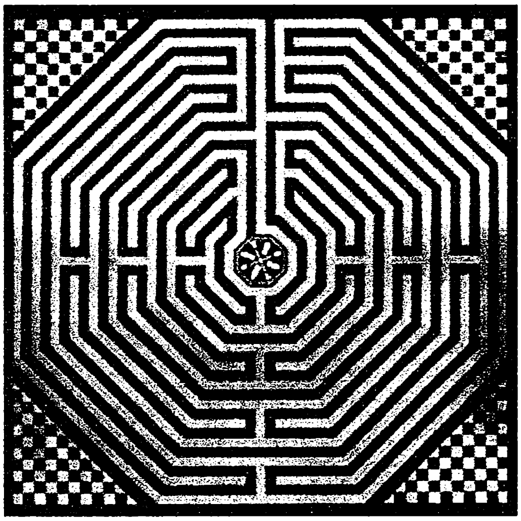
Considering only the three doors of its entrance, we see that their frame is a Root Two Rectangle.

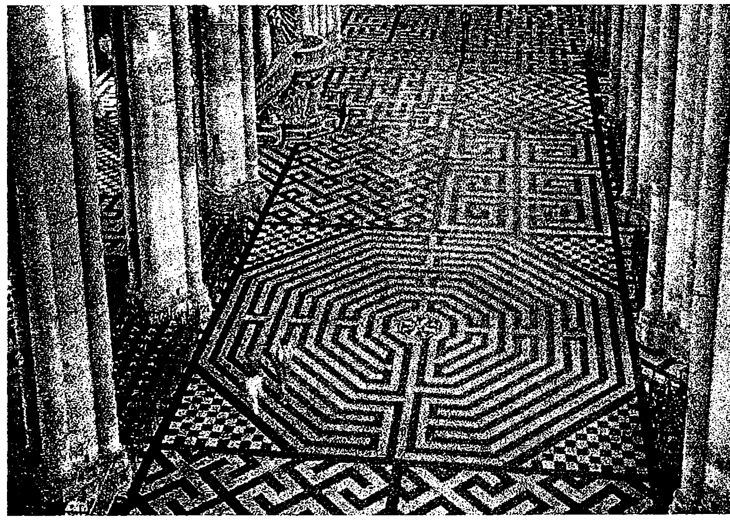


Apply rabatment to the enlargement below and draw a square from each end to notice what they reveal.

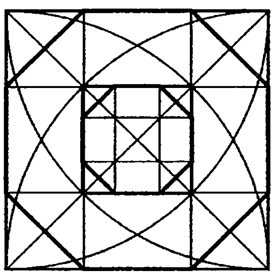


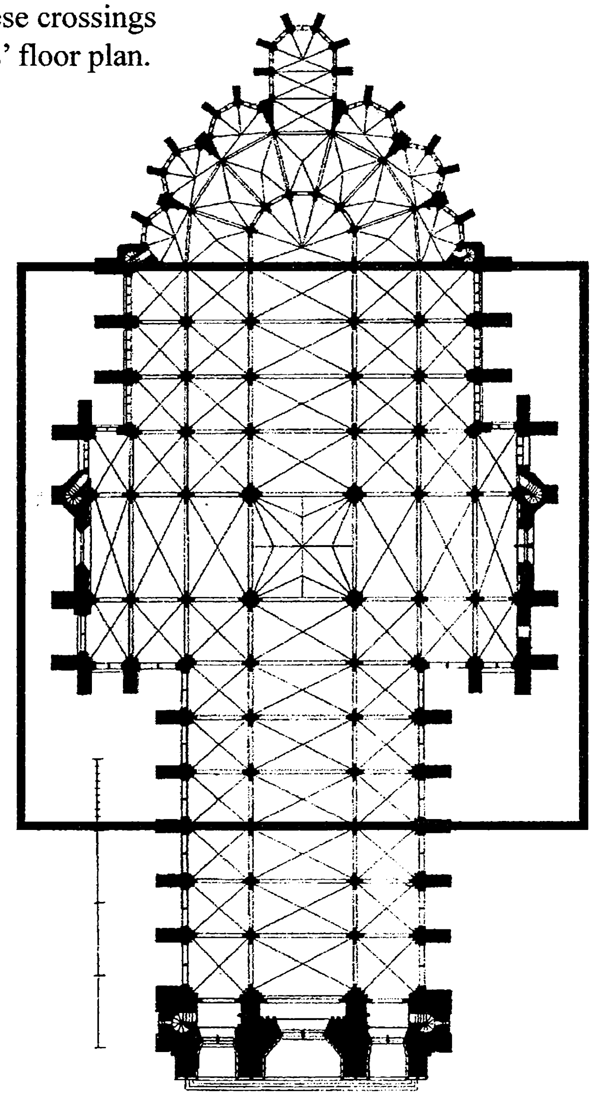
Inside Amiens Cathedral an octagonal labyrinth in the floor tiles gives us a clue to the geometric scheme of the whole cathedral's floor plan.





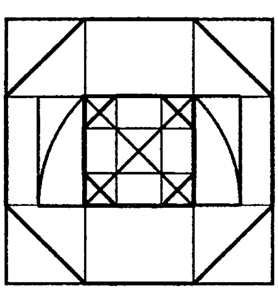
Open your compass along each side of the square overlaid on the floor plan and swing four arcs. Draw two diagonals and notice where they cross the arcs. Use these crossings to construct an octagon in the square overlaid upon Amiens' floor plan.

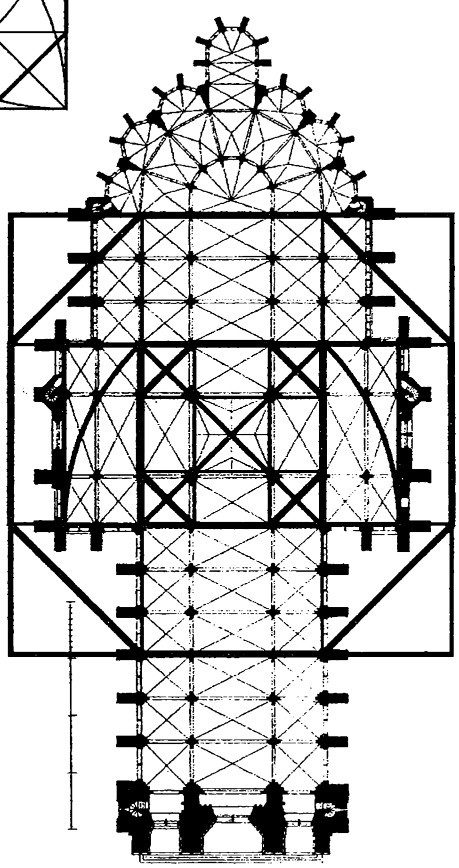




3. AMIENS.

Then open your compass across the diagonals of the small square at the octagon's center and swing arcs to the left and right to find the arms of the crossing.

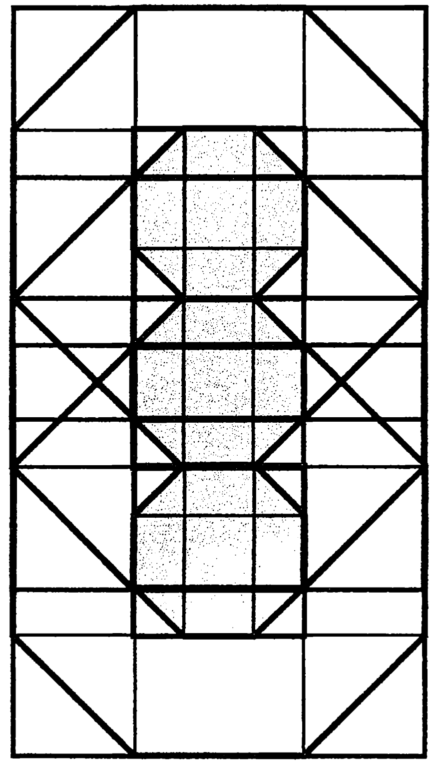


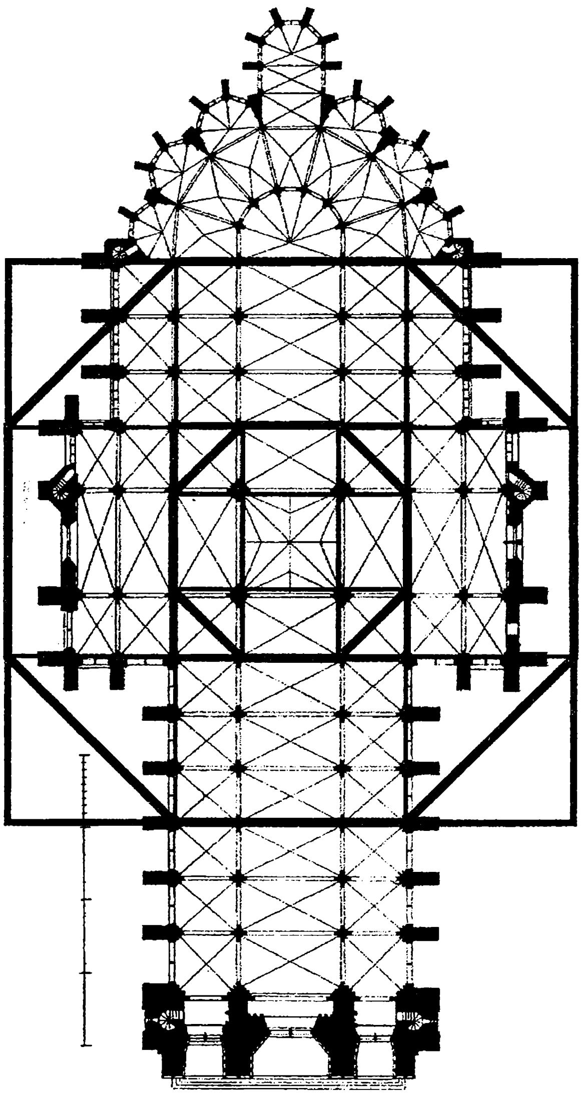


To complete the Cathedral's floor plan requires three such octagons overlapping and extending from the front entrance to the back of the chapels.

Extend the vertical sides of the square on the floor plan below and build the construction making three overlapping, divided octagons. Notice how the central octagonal spaces (shaded) are linked in sequence.

It is interesting to note that three octagons symbolize a number, three eights -- 888 --- a number significant in early Christianity. Since the letters of the Greek alphabet were also their numbers, every word has a number equal to the sum of its "letters". (This is called *gematria* in Latin and *isosephia* in Greek.) The letters of the Greek word *Iesous* -- Jesus -- add to 888. Other significant sacred words and phrases have their numbers which were translated into the dimensions of sacred art and architecture. This has been done across cultures from time immemorial.





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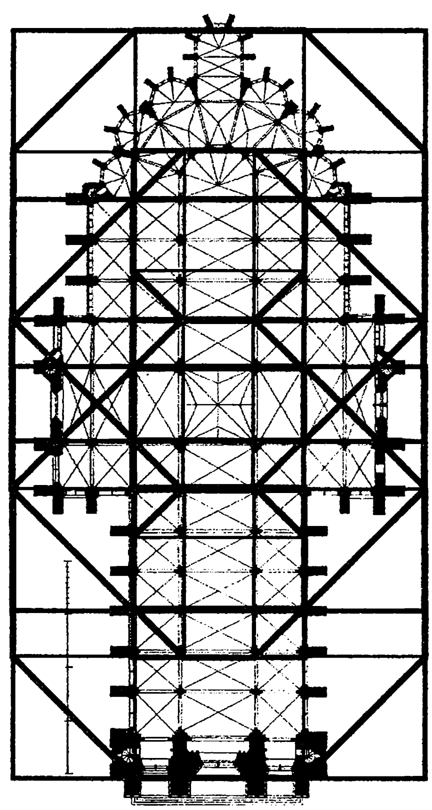

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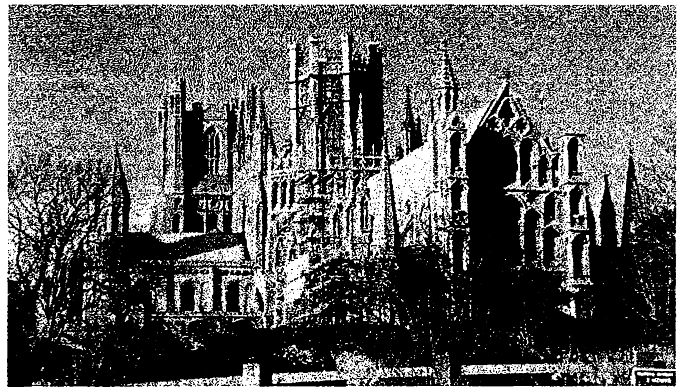
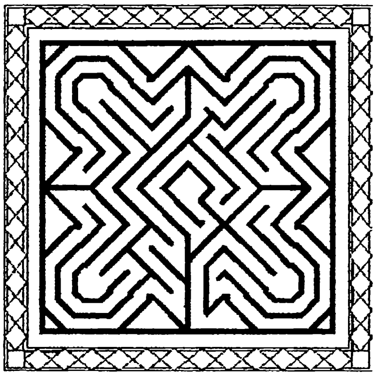
Ely Cathedral

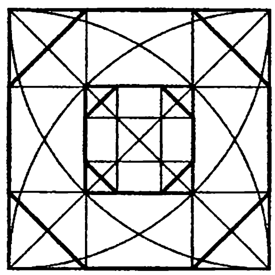


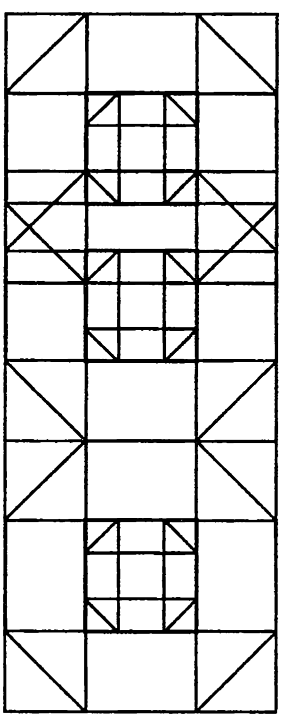
The tiny settlement in Cambridgeshire now known as Ely was founded in 672 by a Saxon princess St Etheldreda. In 970 it was refounded as an abbey and grew to became one of the greatest monasteries in England. The present cathedral was begun in 1083 and is considered one of the best examples of Norman architecture.

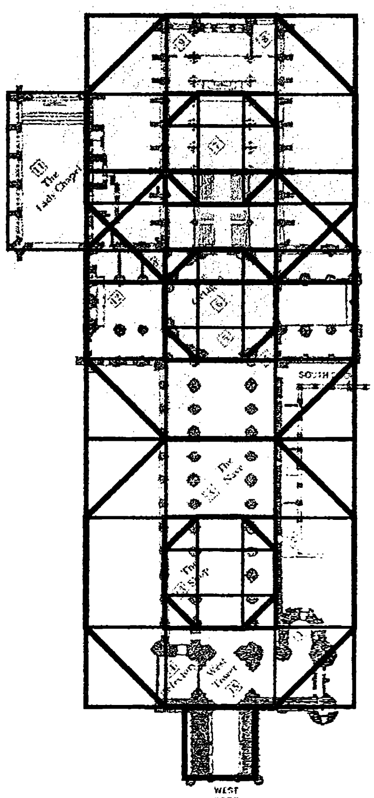


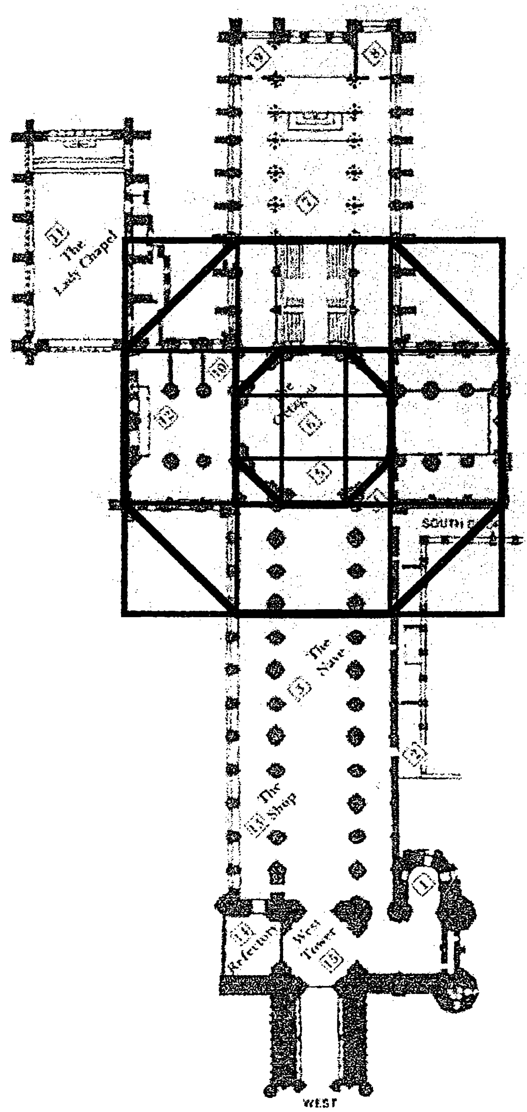
Ely Cathedral has a labyrinth at its entrance under the west tower, designed by Sir Gilbert Scott in 1870. The total length of the labyrinth path is 215 feet, exactly the height of the Cathedral's tower.

The geometry of its floor plan begins as an octagon around the crossing. In the tradition of three eights, extend it to become three octagons, two of which overlap, and two do not.







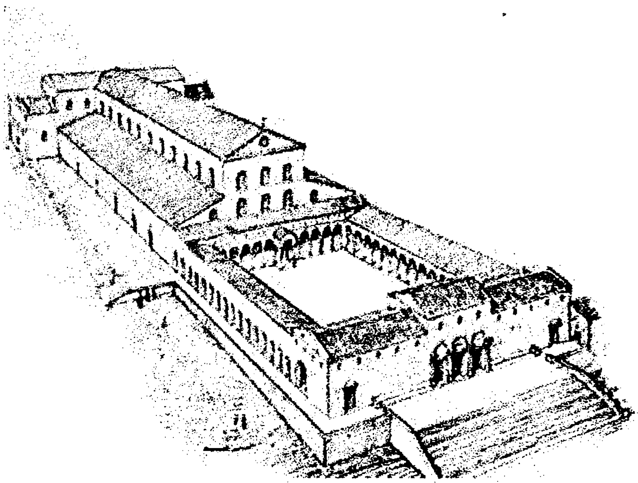


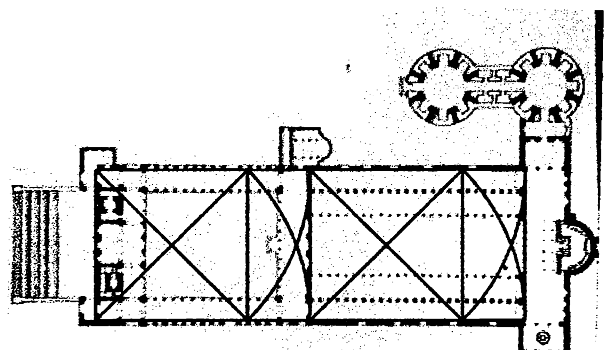
The Old Basilica Of Saint Peter, Rome

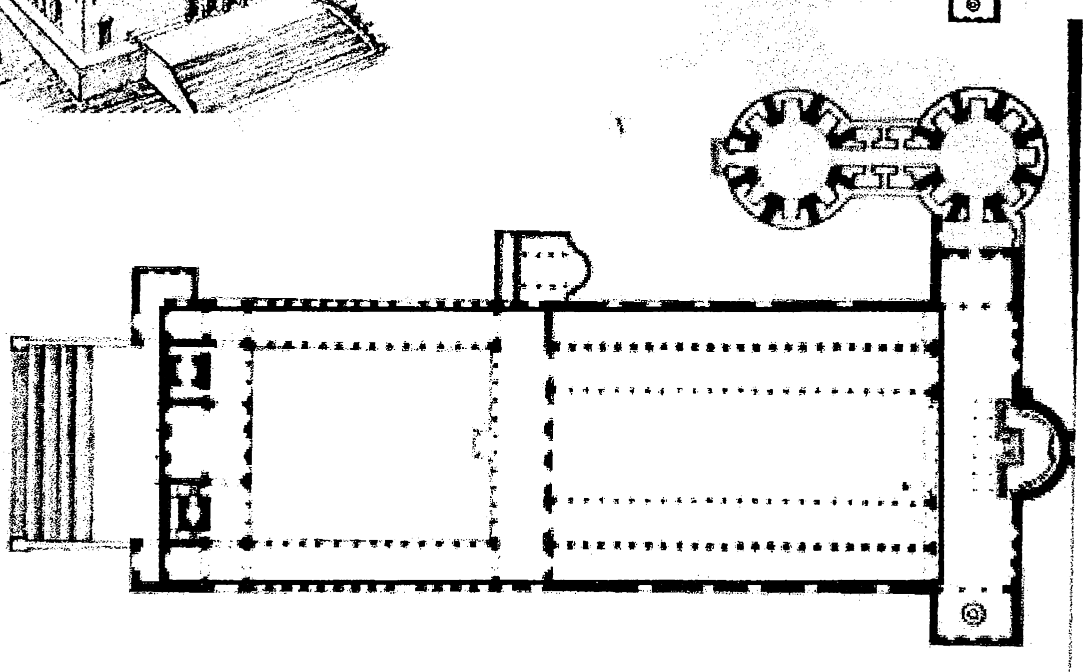
The present Church of St. Peter stands upon the site of the first century gardens of Agrippina. Her son, Caius Caligula, later built a circus (oval ring stadium) there. In it he erected an obelisk brought from Heliopolis, Egypt, which still stands today in the Piazza di S. Pietro. The exact spot in the circus of the crucifixion of St. Peter was preserved by tradition through the centuries, marked in the present Church of St. Peter by the altar. But before today's building was Old St. Peter's Basilica, built during 315-349 by Constantine at the request of Pope St Sylvester I.

The basilica was about half the size of the present St. Peter's. The nave and double aisles were divided by four rows of marble columns, twenty two in each. It contained numerous monuments of popes and emperors, was decorated with frescoes and mosaics, and was visited by pilgrims from throughout Europe. In the court there was a large canopy covering the huge pinecone taken from the Pantheon which now stands in the court of Vatican Museum. The basilica was lit by 100 lamps with oil from 56 dedicated olive orchards. In 1451 Pope Nicolas V decided to rebuild the old St Peter's, but it was Julius II who actually started reconstruction by employing Bramante to begin work in 1506.

Below is the plan for the Old Basilica, showing that it's designed as two horizontal Root Two Rectangles (which equals a Root Eight Rectangle). Mark a square inside each and swing their diagonals with your compass to verify that they're Root Two Rectangles.



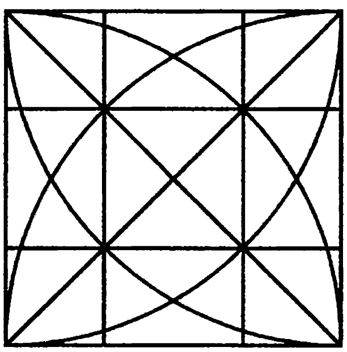


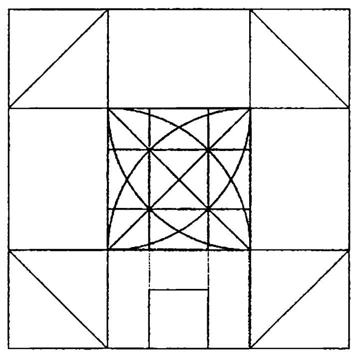


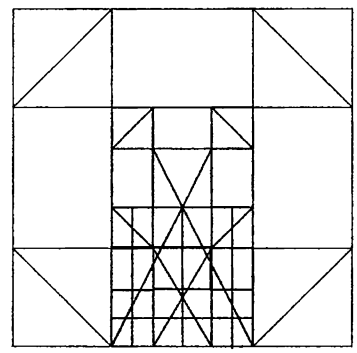
St. Patrick's Cathedral, New York City

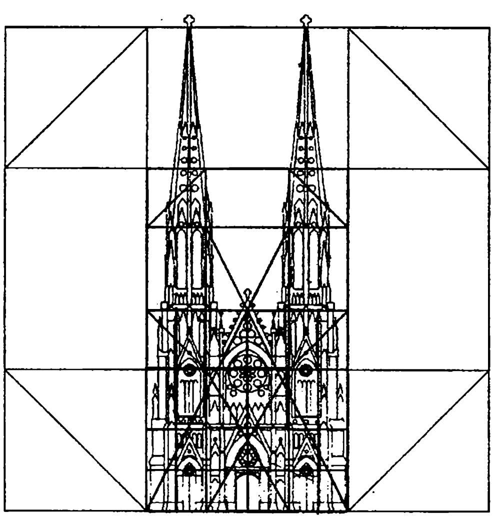
St. Patrick's Cathedral is a beautifully decorated Gothic style Catholic Cathedral near the center of Manhattan Island. It's the seat of the archbishop of the Roman Catholic Archdiocese of New York, and also a parish church. Originally, it stood at the intersection of Prince and Mott Streets to Mulberry Street, but after a fire in 1858 was rebuilt on the site of an orphanage on 50th Street and Fifth Avenue across from what would become Rockefeller Center. Work was halted during the Civil War but it was completed in 1878, although its 330 foot towers were added in 1888. It seats about 2,200 and its pietà is three times larger than the Michelangelo's Pietà.

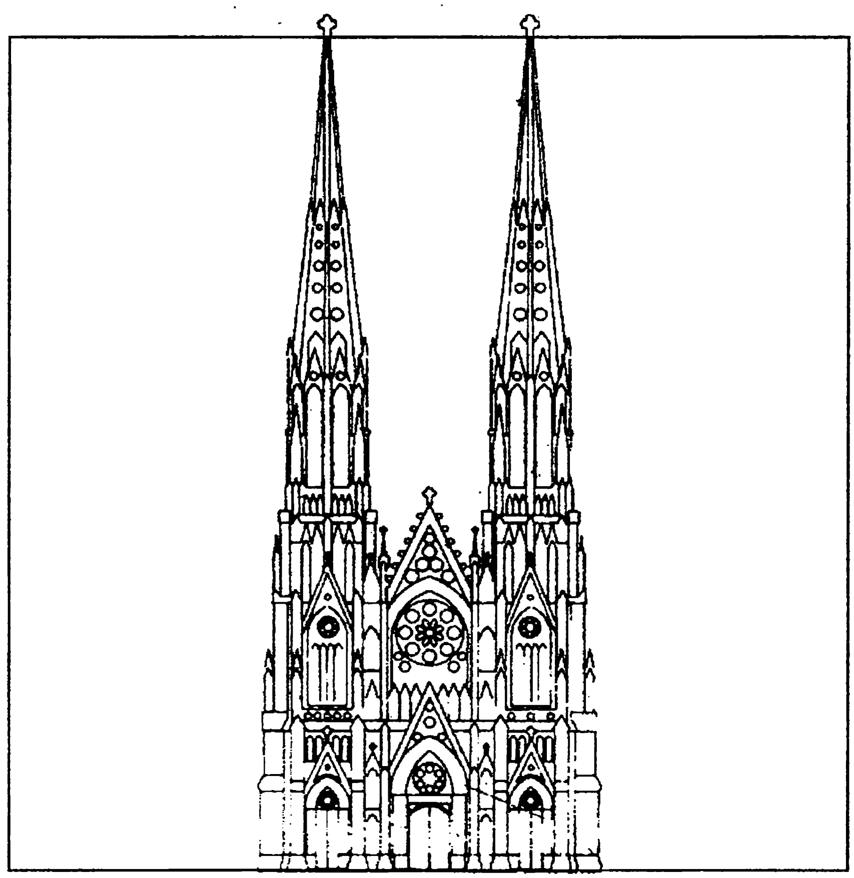
A square has been drawn around the Cathedral's front elevation. Notice how the crosses atop the towers are beyond the earthly square. To see its geometry, start by swinging arcs and drawing diagonals to construct an octagon in the square. Further divisions by rabatment of squares within its Root Two Rectangles, along with diagonals, arcs and crossings will reveal more of its harmonious plan.









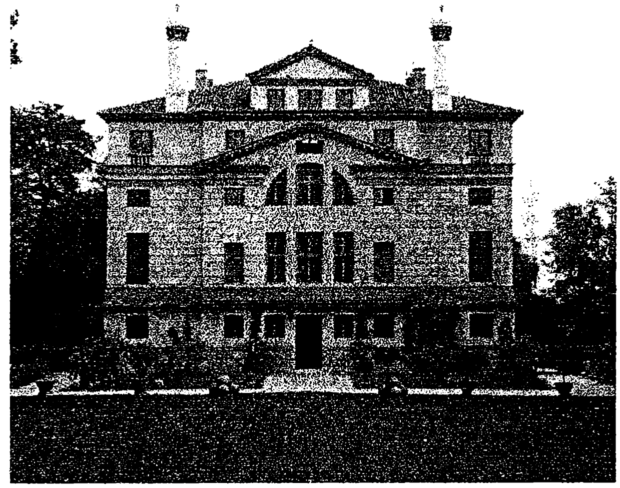


Palladio's Villa Foscari

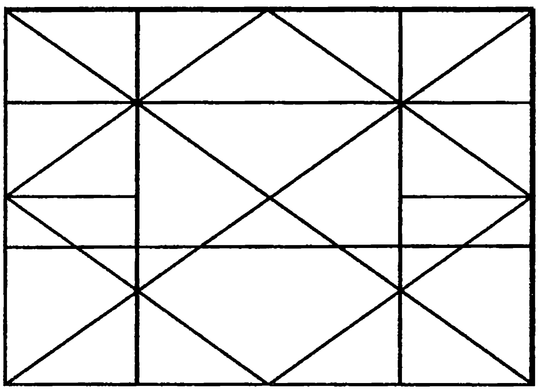
Venice, 1559

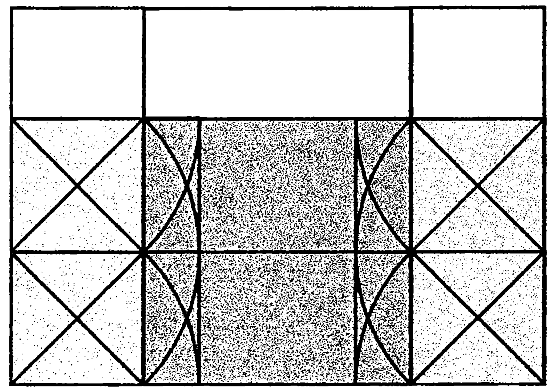
Leaving Cathedrals, we come this villa which Palladio designed and executed for the Foscari brothers whose family was one of the city's most powerful. Hence, its majestic, regal character which is virtually unknown in Palladio's other villas.

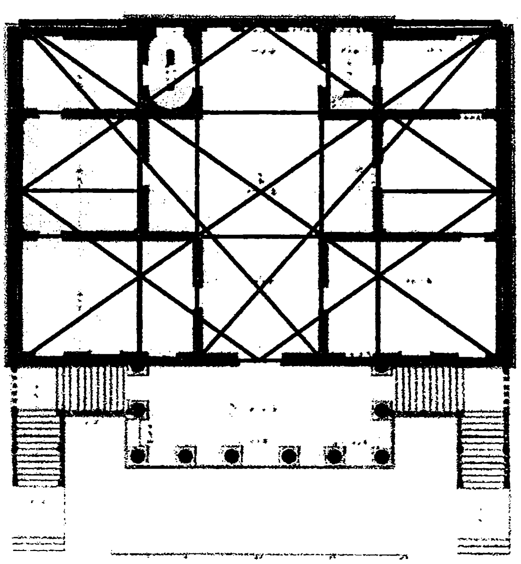
The geometry of its floor plan is a subdivided Root Two Rectangle. We've seen this scheme before. Apply this construction onto the plan below to see how Palladio harmonized the Villa's spaces.

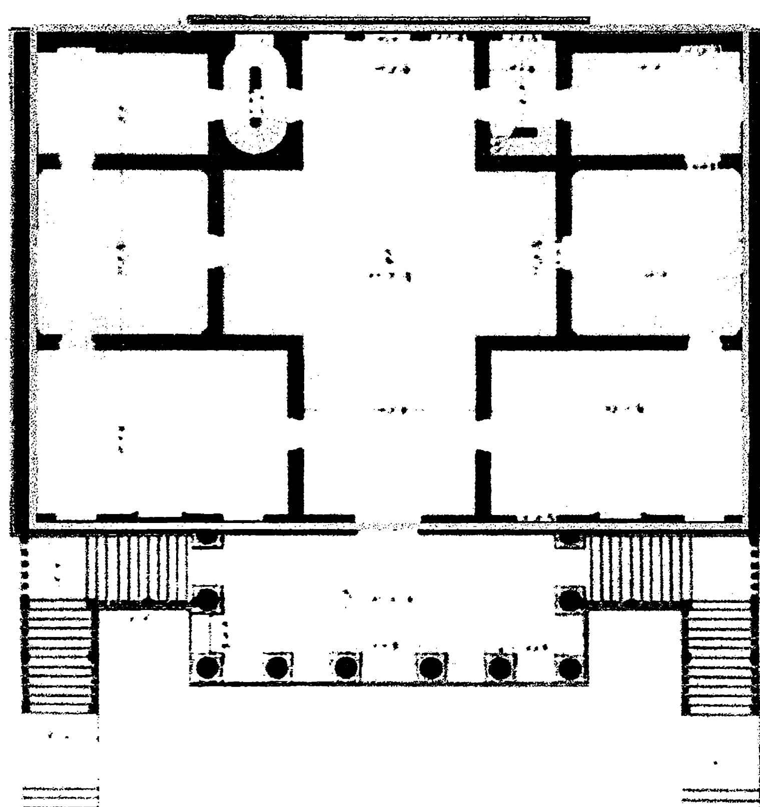


Villa Foscari East Facade

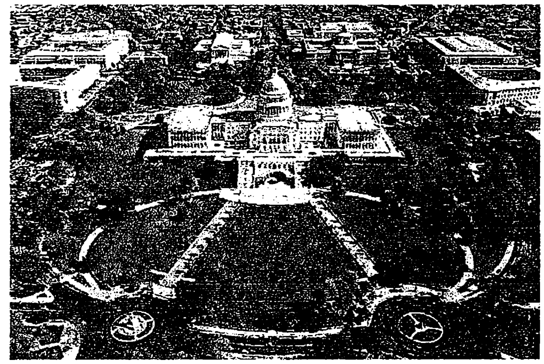






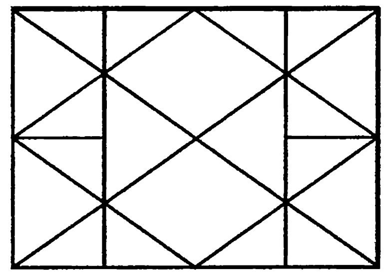


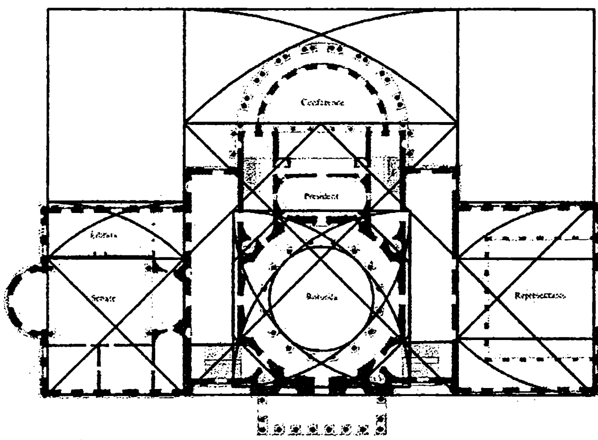
The United States Capitol Building

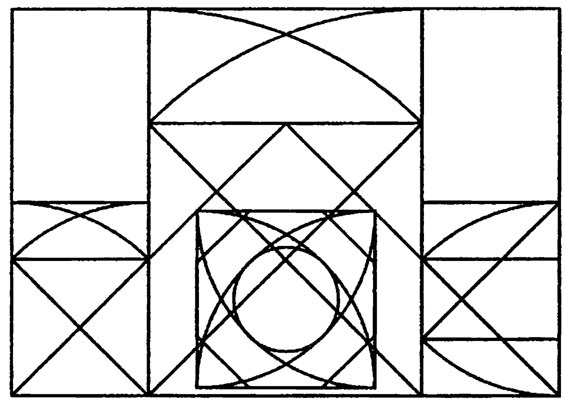


Although the competition to design the Capitol Building had already closed, Dr. William Thornton, a Scottish-trained physician living in Tortola, British West Indies, presented his plan which President Washington commended for its "grandeur, simplicity and convenience". He won the $500 award.

Thornton's plan depicted a building composed of three sections. The central section, topped by a low dome, is flanked on the north and south by two rectangular wings (one for the Senate and one for the House of Representatives). The cornerstone was laid on September 18, 1793 in a Masonic ceremony by President Washington at the building's southeast corner. In November 1800, Congress and the Library of Congress moved into the first part of the Capitol built. Much more about the history and construction of the Capitol can be found at http://www.aoc.gov/cc/ .

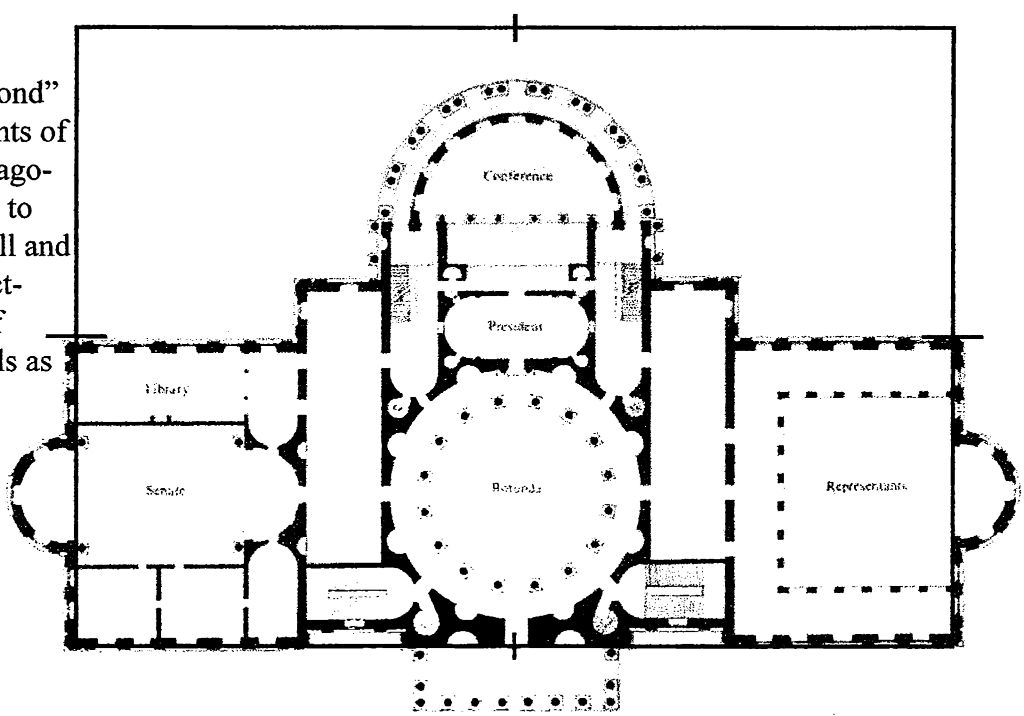






The geometry of its plan encloses it in a Root Two Rectangle. Make a "diamond" by connecting the midpoints of its sides. Then draw its diagonals and use the crossings to subdivide it into four small and one central Root Two Rectangle. Apply rabatment of squares, arcs and diagonals as shown to define much of the Capitol's architecture.

The plan seems to be part of a larger octagon which encompasses the grounds surrounding the building.

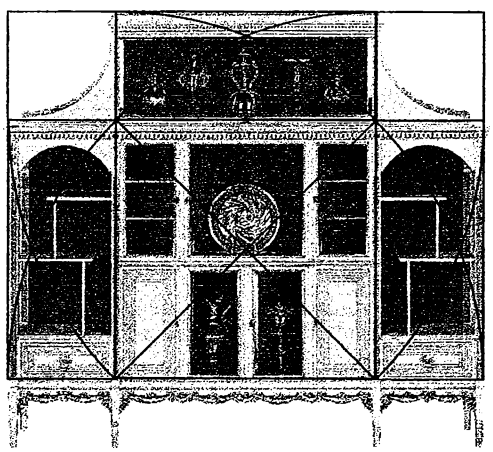


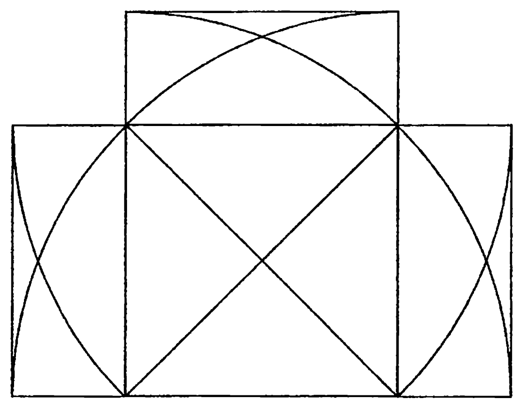
Herter Brothers Cabinet

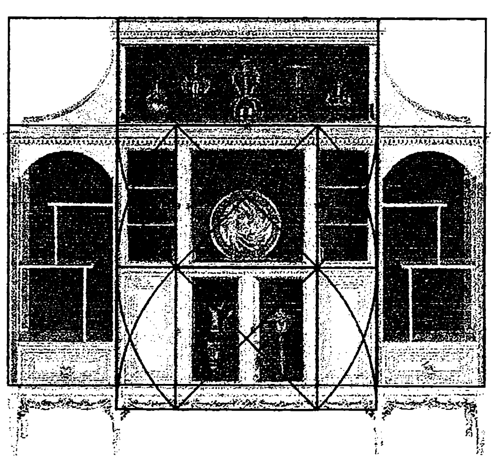
Cabinet for Oliver Ames, Boston, c1883
Metropolitan Museum Of Art, New York (1999.79)

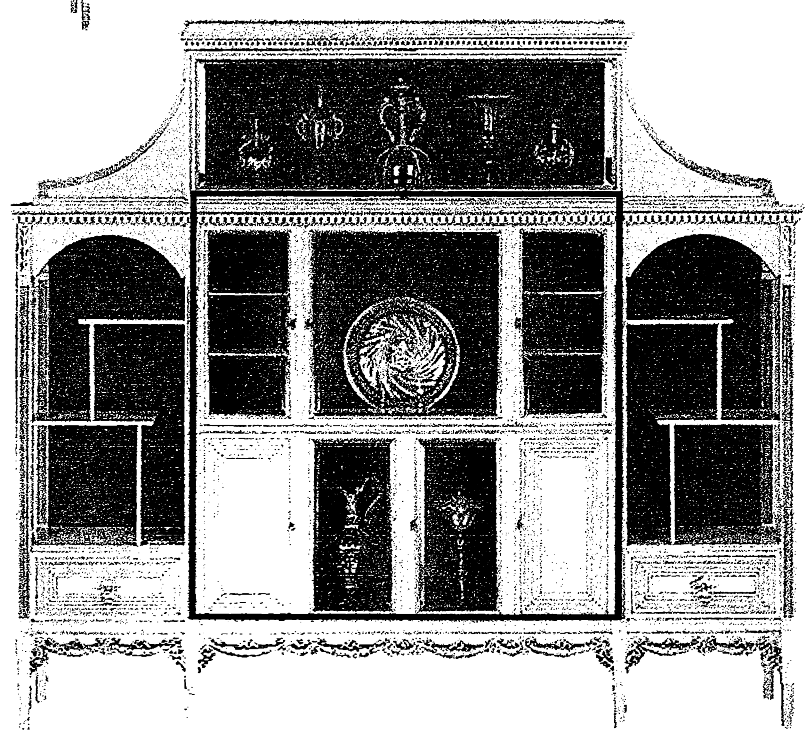
During the late nineteenth century (1864 to 1906), Herter Brothers, the New York firm of the German-born brothers Gustave and Christian Herter, was arguably the leading cabinetmaking and interior decorating firm in the United States. Their work was in great demand by the most affluent clients of this opulent era. This rare cabinet, made for Oliver Ames (a Boston industrialist and soon to be governor of Massachusetts) is made of painted and gilded maple wood, glass, brass and silk velvet.

One approach to its geometry starts with a square within the cabinet. Open the compass legs between the opposite corners of its diagonal, and swing an extension to make it a Root Two Rectangle. Do this to the left, right and above the square. Within this square are two smaller squares which have Root Two extensions to their left and right. Extend the sides of each rectangle to make a large rectangle around the body of the cabinet. Further subdivisions, some shown here, will define the dimensions of all its elements, the cabinets, drawers, shelves and legs integrated as one harmonious whole.









Slave Market With The Disappearing Bust Of Voltaire

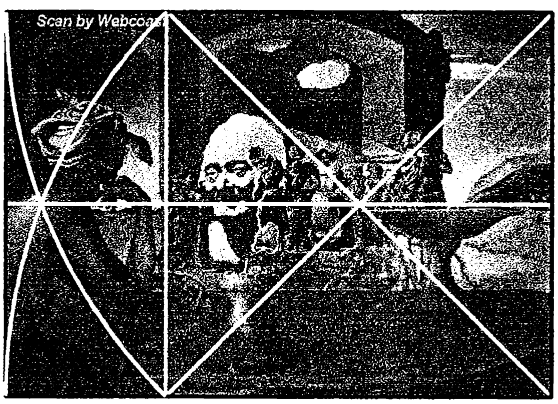
Salvador Dali, 1940
Oil on canvas, 18 1/4 x 25 3/4 in (= 1.4109)
Collection Mr. and Mrs. A. Reynolds Morse, on loan to Salvador Dali Museum, St.Petersburg, Florida

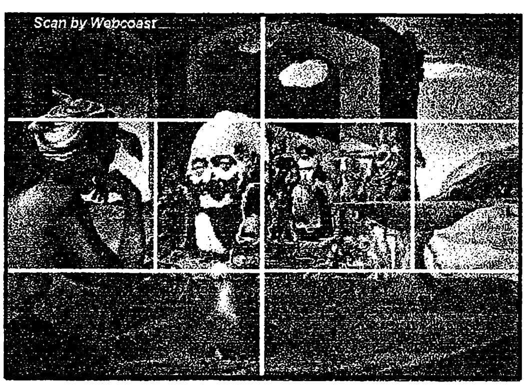
Dali delighted in double imagery. Are those women under an arch, or is that a bust of the philosopher Voltaire? He credited his ability to switch between such levels for the surreal visions he painted.

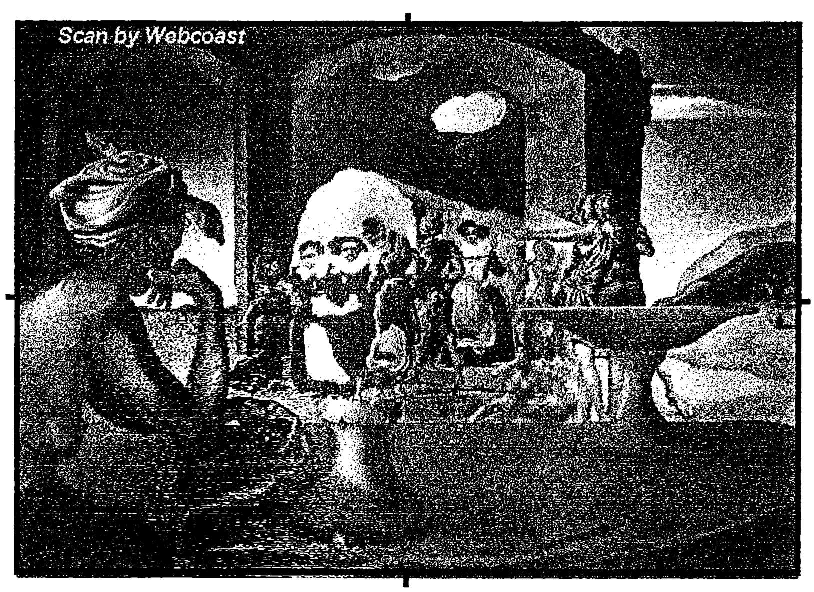
Dali wrote in his books that he was well versed in techniques of classical geometric composition. This painting is a Root Two Rectangle.

Mark a square in the right side and see how it defines the architecture. Divide it in half horizontally to find the horizon. (Midpoints of each side are marked.)

Now divide the whole into two side-by-side vertical Root Two Rectangles. Mark off squares at their tops to skim above the table. Further rabatment of squares will show how he harmoniously framed each area.

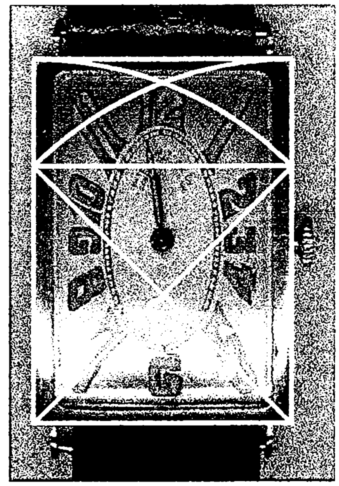


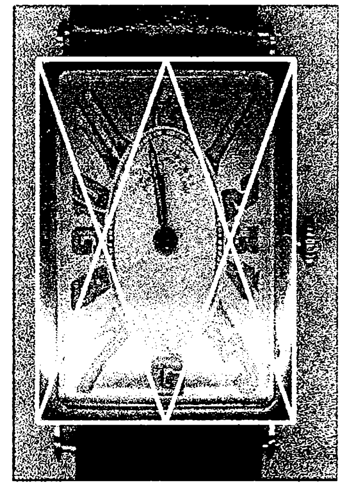




Classic Wrist Watch

The face of this classic wrist watch is shaped as a Root Two Rectangle.

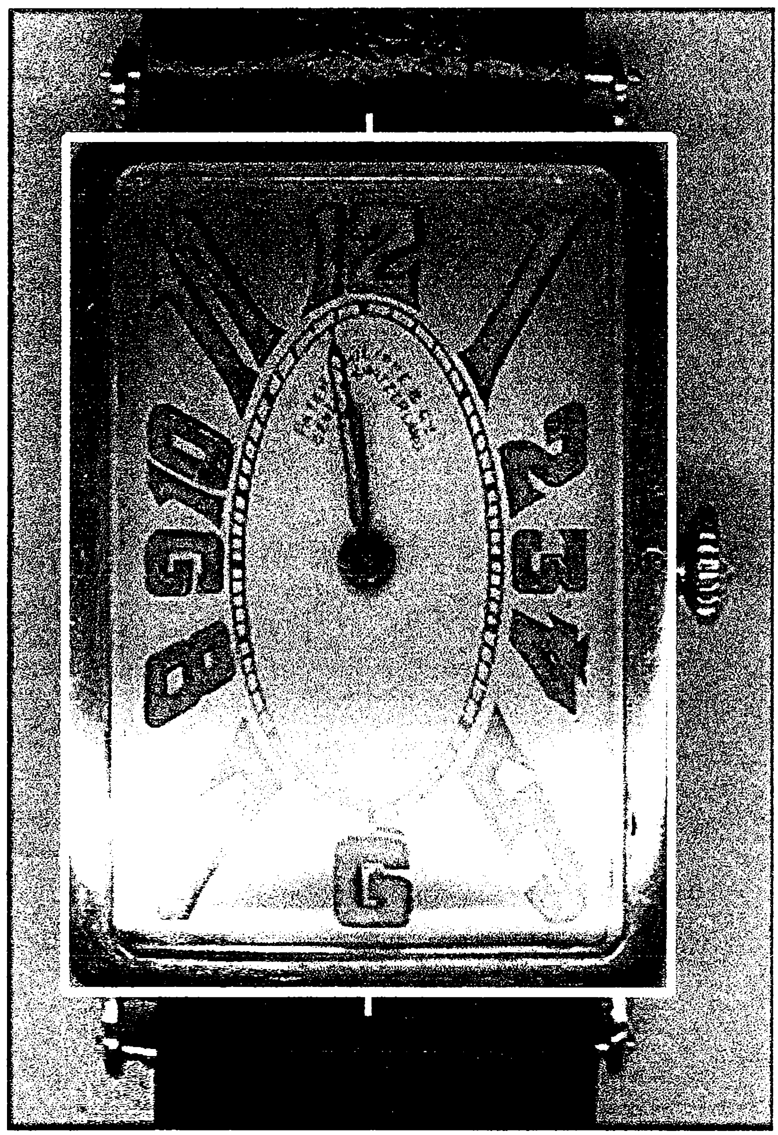




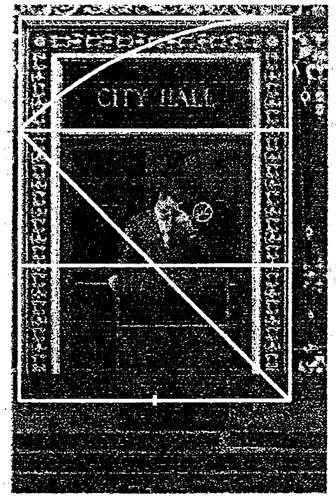
Draw a square inside the bottom and swing the arcs of its diagonal to prove it.

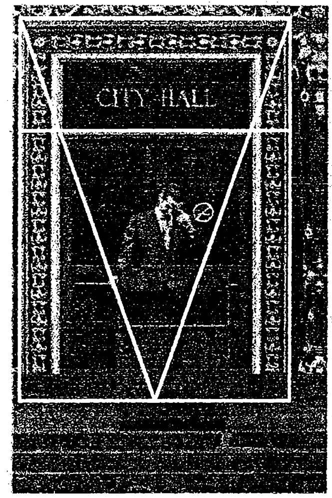
Notice that the small hour-hand just reaches the top of the square.

The oval at the center rests within the "diamond" of lines connecting corners with the opposite sides.



Entrance To City Hall (San Francisco)





San Francisco City Hall opened in 1916 after the old City Hall was destroyed in the 1906 earthquake. It's considered one of the world's best examples of Beaux Arts (French for 'fine art') architecture, a style which combines classical architecture from ancient Greece and Rome with Renaissance ideas, characterized by order, symmetry, formal design, grandiosity, and elaborate ornamentation. It took only two years for the new City Hall to be built and yet has a dome which is the fifth tallest in the world, taller than that of the Capitol in Washington DC.

The original design for City Hall was inspired by the gilded lead-plated dome and spire of Les Invalides in Paris, where Napoleon was laid to rest in 1861. City Hall's dome was originally covered with gold leaf and gilded copper. But the gilding was applied incorrectly and the copper eventually emerged with its familiar green patina. Today's restored finish is gold leaf on a special paint.

The doors to City Hall in San Francisco, like those of many civic and religious buildings around the world, are designed as Root Two Rectangles.

First, draw a square within its bottom.
The top of the square will define the actual doors.

A line drawn halfway across the square will show the doors' own division.

What about the thickness of the frame around the entrance?

Draw a V from the top corners of the whole rectangle to the middle of the bottom (marked). These lines cross the top of the square to define the width of the frame on each side.

Paper Money Geometry

Paper money all over the world, of every country, is very intentionally designed. This is not only in the complex technology developed to thwart counterfeiting, but also in the most simple aspect of its design: its dimensions.

Currency of the United States seems to be designed to fit in the extension beyond a square which makes it a Root Two Rectangle. This rectangle is equal to a small Root Two Rectangle plus a square. (We saw it before in the scheme of The Birth Of Adam by DiPaolo.)

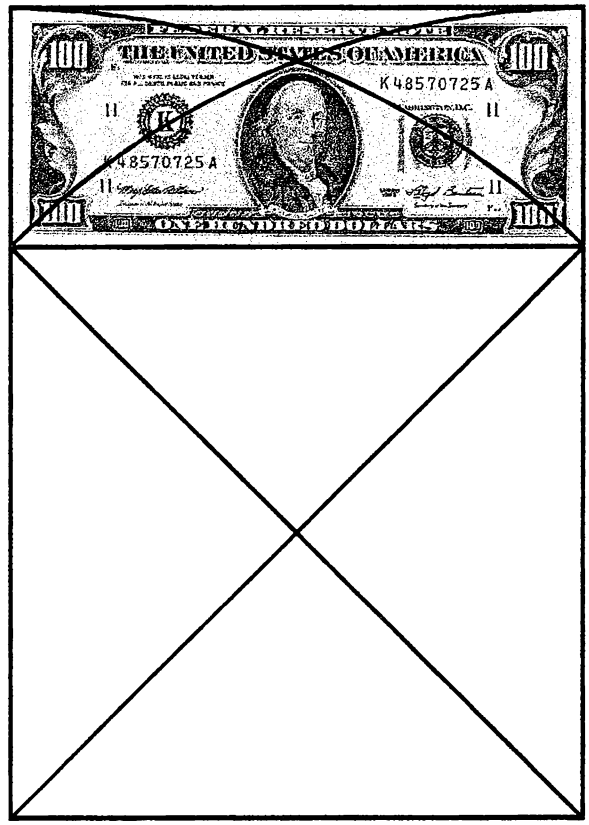
You can prove this by drawing a square inside each of its ends. Open your compass to their diagonals to see that their arcs cross and overlap between them over the portrait on the bill.

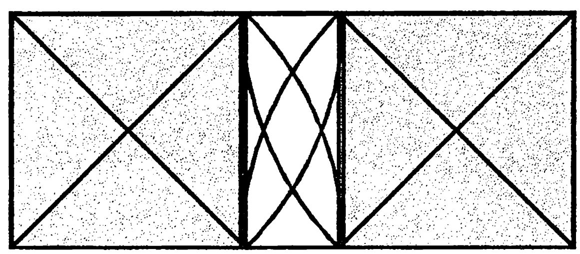
Notice how the vertical center of the bill crosses directly through the person's eye. This is an unspoken rule about traditional portraits, from the dollar bill to the Mona Lisa. The center goes through one eye.

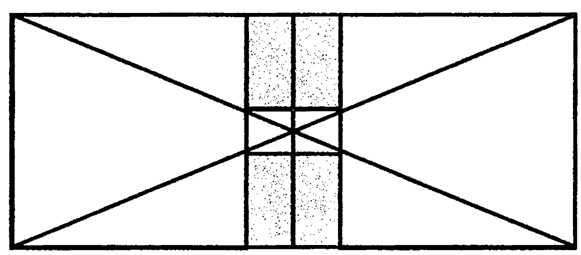
You can further explore the design of money by applying rabatment of squares in each created rectangle. Extending lines and adding diagonals too will reveal more. Notice, for example, how the serial number and the key in the seal lie along horizontal lines you can extend from small squares at the center.

The money of whatever country you live in, or have paper money for, can be explored by simple geometry to see how they were designed.

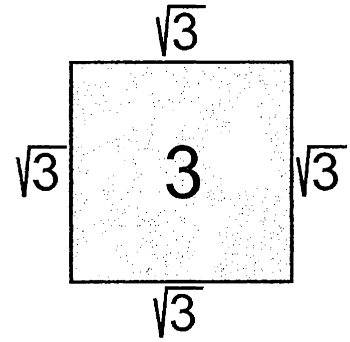
Apply your geometric skills to the bill below to see what you can find.







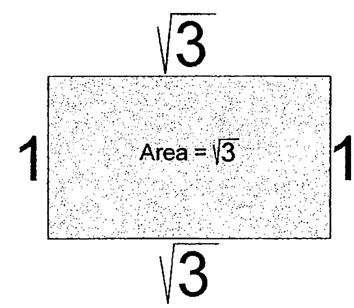
Root Three Rectangles



The Square Root of Three is, like the Square Root of Two, an irrational number which, when multiplied by itself, will exactly equal three. If the area of a square is three, then the length of each side must be the square root of three. It's value *approximately* equals

1.7320508075688772935274463415058723669428052538103806280558
069794519330169088000370811461867572485756756261414154067030299
699450949989524788116555120943736485280932319023055820679748201010846
749232650153123432669033228866506722546689218379712270471316603678615880
190499865373798593894676503475065760507566183481296061009476021871903
250831458295239598329977898245082887144638329173472241639845878553976
67...

If its infinite digits could be multiplied by themselves, the answer equals exactly three. But self-multiplying any limited number of its digits will always fall short of three. The more decimals used, the more exact it will be, but at some digit it will go awry.



A Root Three Rectangle has sides in the ratio of square root of three.

People have used simple rational approximations for it which include

26/15 = 1.733...

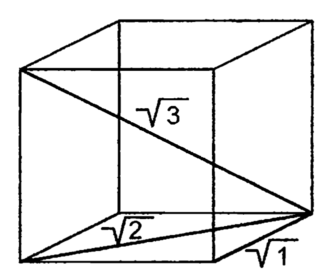
97/56 = 1.73214...

71/41 = 1.731707...

265/153 = 1.7320261... the most accurate.

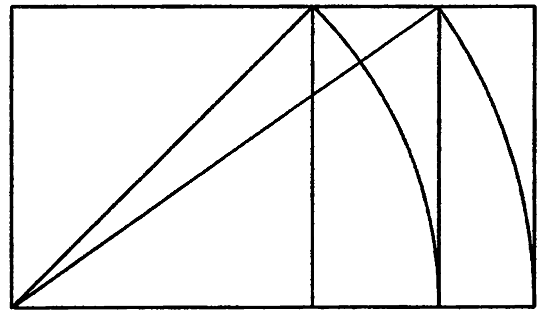
The square root of three occurs in nature. For example in seismology, the study of earthquakes and waves that move through and around the Earth. It describes the relationship between different types of waves traveling at different speeds. "Compression waves" move directly through rocks and liquids *the square-root-of-three times faster* than wavy "shear waves" which travel side-to-side and up-and-down. It has, in part, to do with the "triangular" shape of the waves.

Used symbolically in art, the square root of three is similar in significance to that of the number three: whole, complete, three-weave-as-one (trinity = tri-part unity), sacred and divine. Keep this in mind as we examine various examples of art.

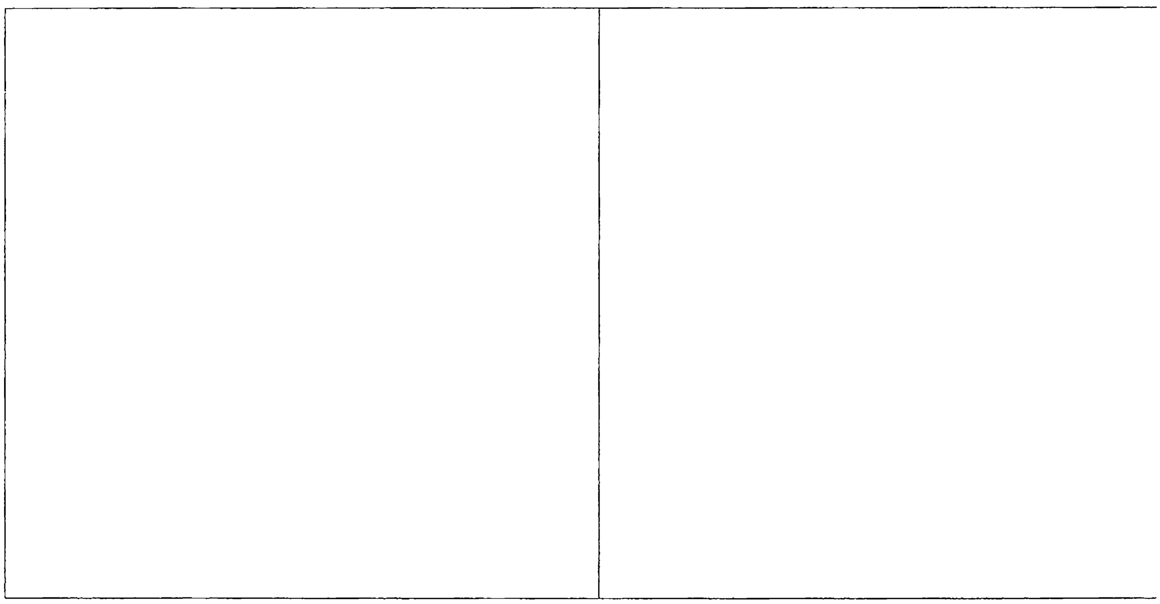


The square root of three appears as the diagonal of a cube compared with its edge. A square made on its edge would have an area of three.

Construct A Root Three Rectangle



Here's another opportunity four you to construct a Root Three Rectangle starting with the diagonals of a square and then a Root Two Rectangle.

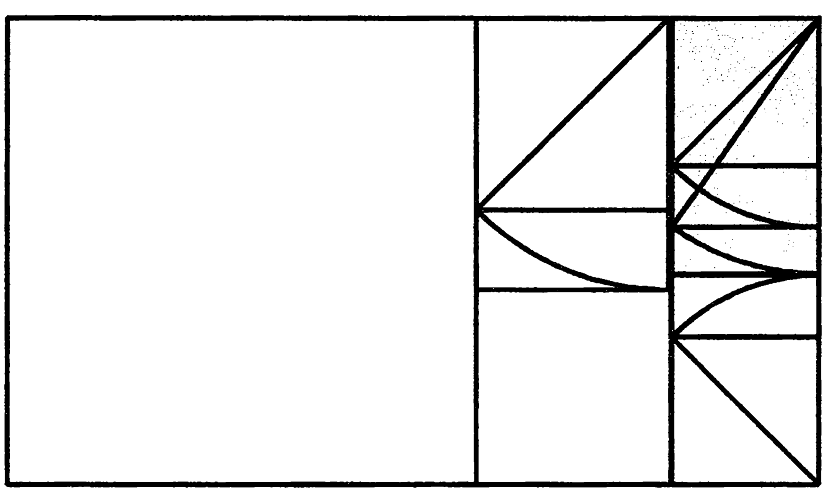


Subdivide Its Extensions

Each rectangular extension of the root rectangles is itself made of consecutive root rectangles.

Because they are linked this way they have harmonious "dynamic" properties.

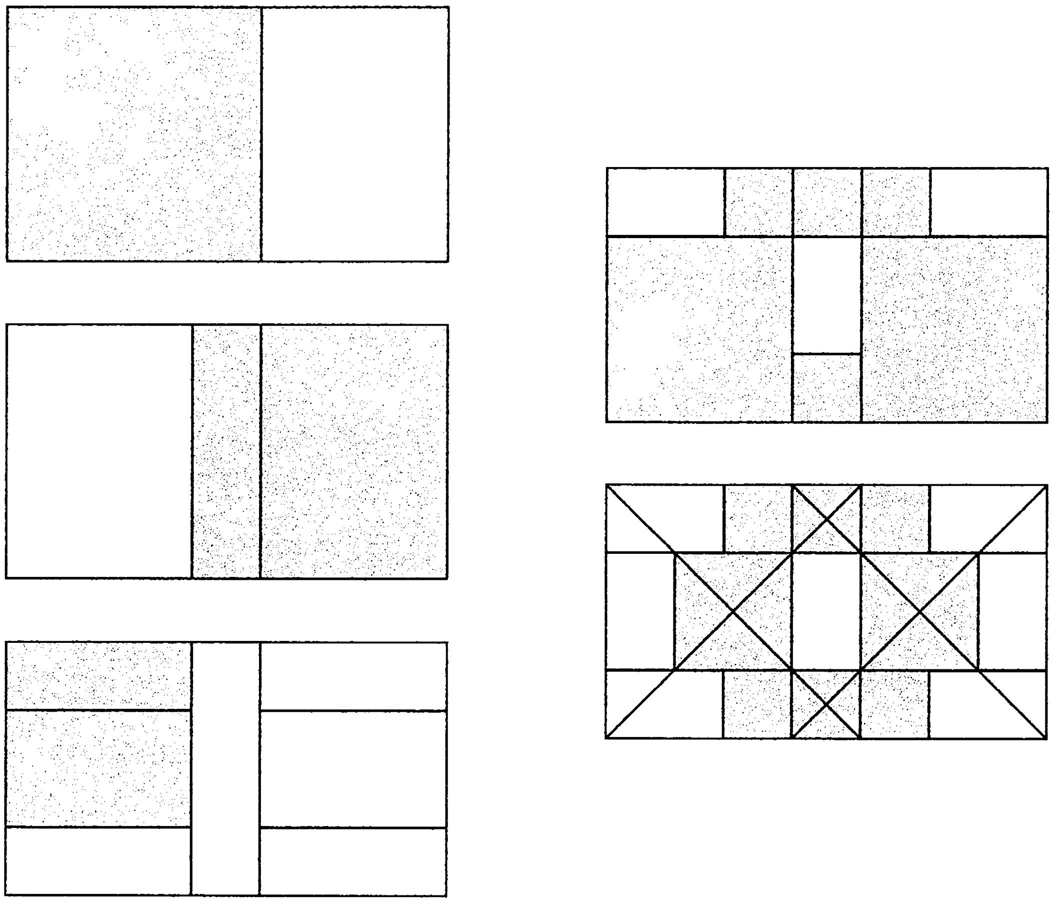
Replicate this geometry in your construction above to verify it, and test your geometric accuracy.



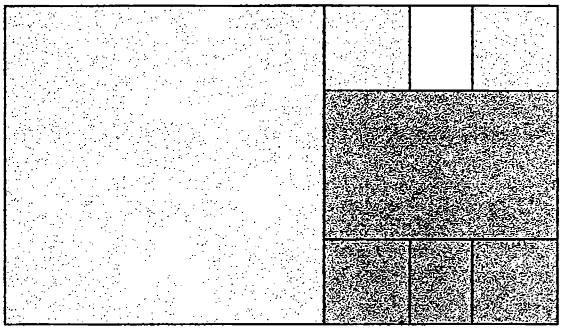
Rabatment In A Root Three Rectangle

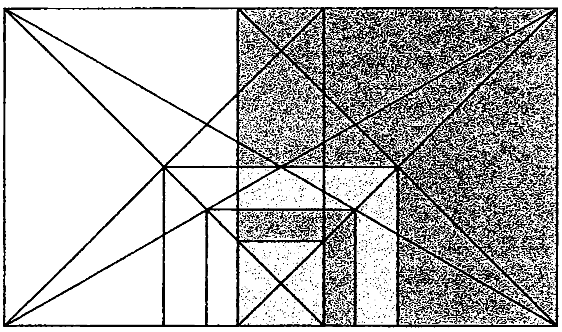
The technique of rabatment, "folding back" a square inside the ends of rectangles, remains a valuable tool for exploring geometry and art. Using rabatment, replicate the steps of this construction in the Root Three Rectangle below.

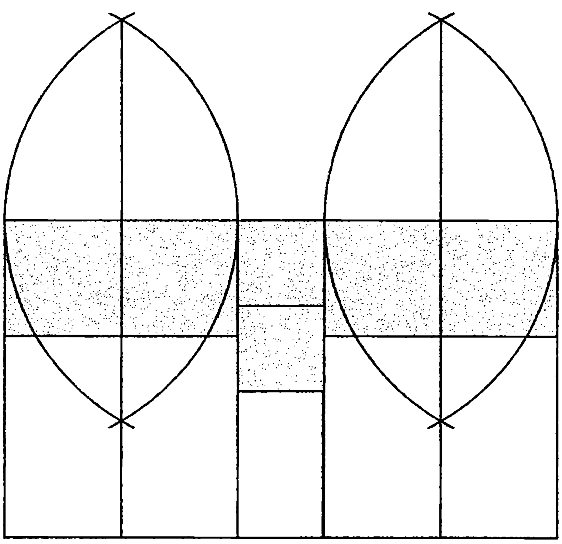
Use colored pencils to shade sections.

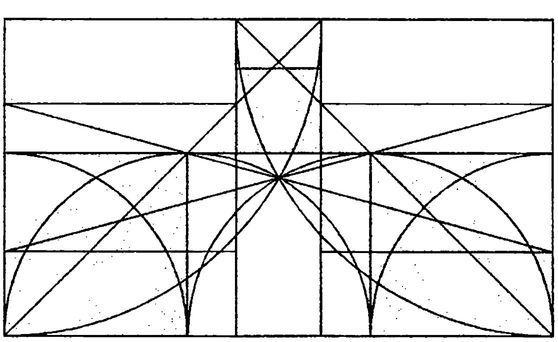


These also use rabatment of squares. But now, diagonals and their crossing points become valuable to show us where to connect other lines.









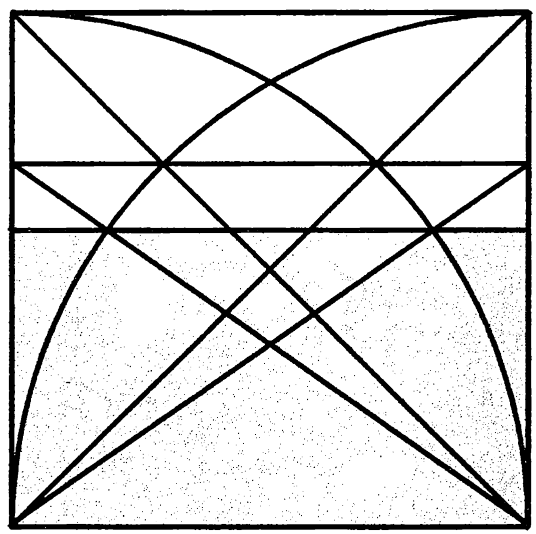
Construct A Root Three Rectangle In A Square

Like other root rectangles, the Root Three Rectangle can also be constructed within a square.

Start by making the arcs and diagonals. Connect where they cross to produce a Root Two Rectangle.

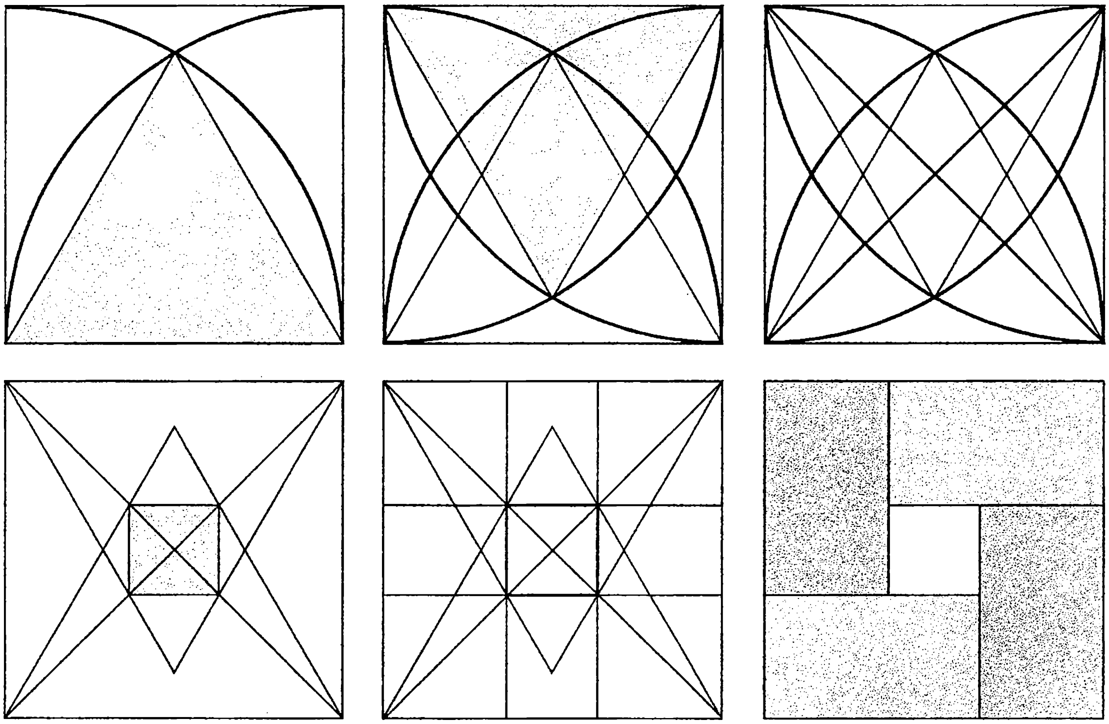
The diagonals of the Root Two Rectangle cross the arcs at the top of a Root Three Rectangle.

Extra: Construct another Root Three Rectangle from the top of the square. See how they overlap. Explore this construction with rabatment of squares.



Rotating Root Three Rectangles

Raising triangles on two opposite sides of a square leads to four Root Three Rectangles rotating within a square and around a square. This construction takes advantage of the fact that the height of an equilateral triangle compared to half its side is the square root of three. Use colored pencils to shade areas in these steps and in your construction.



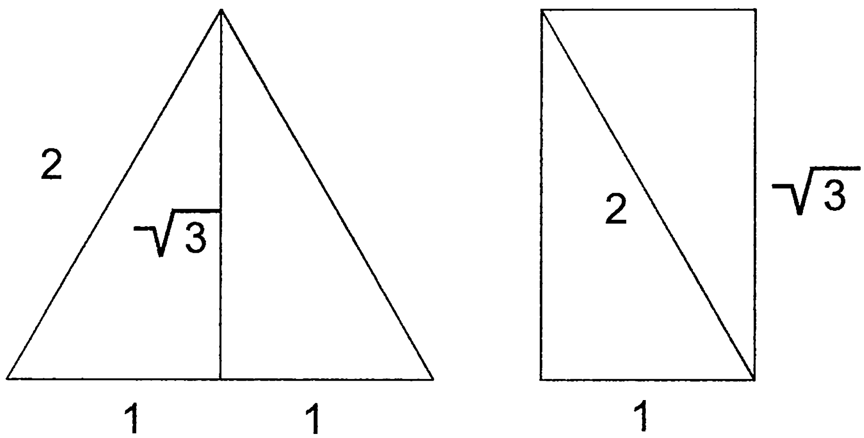
Root Three In The Triangle

The square root of three appeared in the previous construction because it naturally resides in the height of an equilateral triangle (compared to half its side).

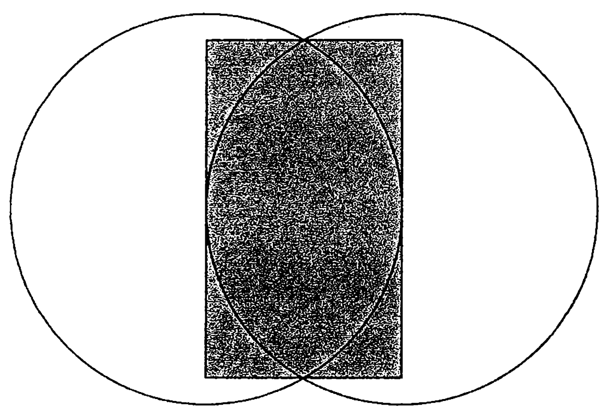
Try this: Construct or trace an equilateral triangle (see Volume 1 Chapter 3: draw a line segment, open the compass between its ends and swing arcs up to cross. Then connect the three points).

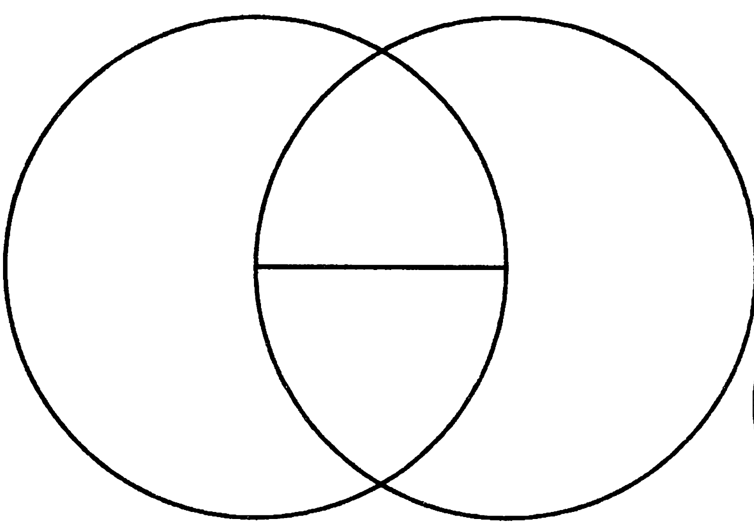
Draw a line from the top point to the middle of the base. Cut the triangle into two pieces along it.

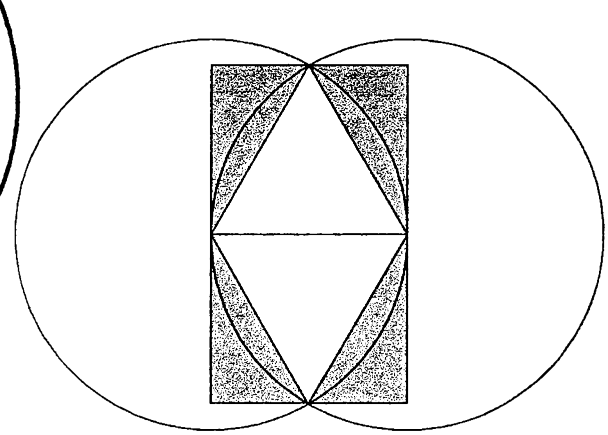
Then rearrange the pieces to make a Root Three Rectangle!



Because the square root of three is present in equilateral triangles, it certainly occurs in the *Vesica Piscis* ("Bladder of the Fish") or "almond" construction of two circles which touch each others' center. This creates the space for two mirror equilateral triangles. It's possible to build a Root Three Rectangle around the almond by constructing perpendicular lines.



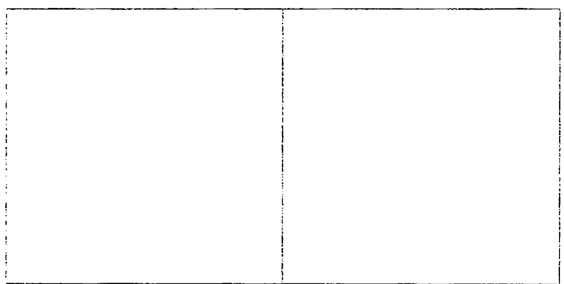


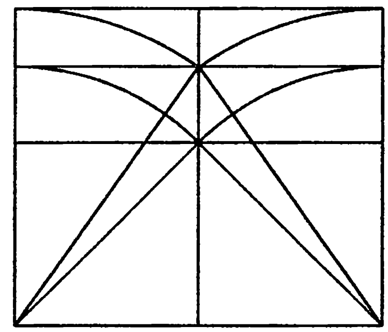


Construct A *Double* Root Three Rectangle

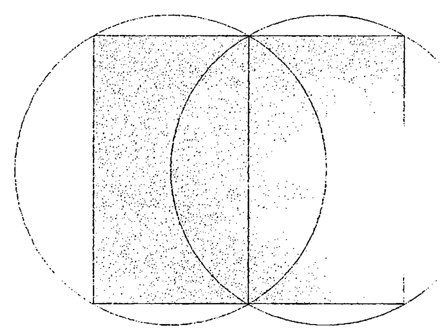
The double Root Three Rectangle is an interesting form and will be useful for examining art.

One way to create it is to start with two squares. Use their diagonals to build a double Root Two Rectangle upward, then a double Root Three Rectangle.

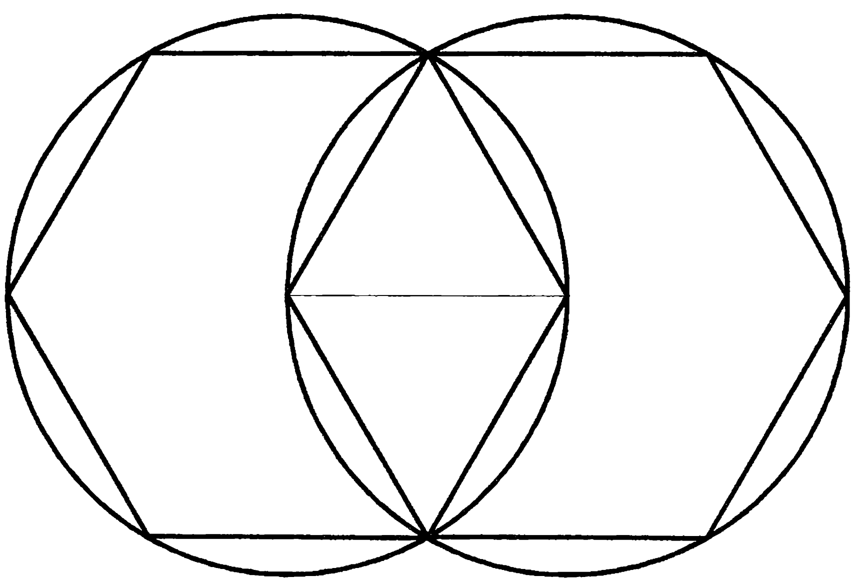


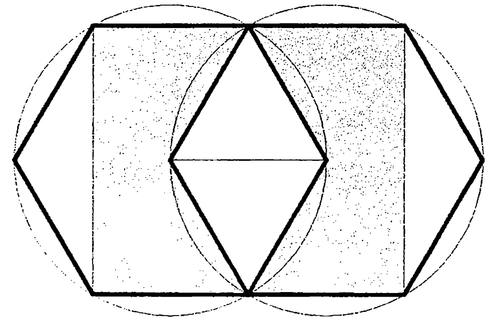


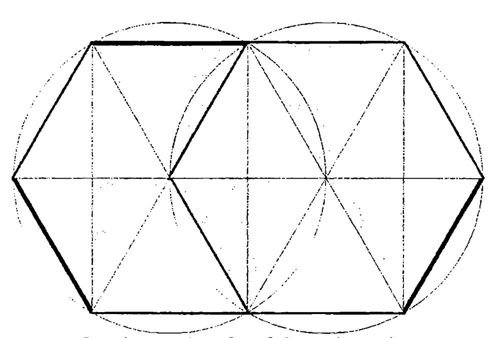
Overlapping Hexagons



Another way begins with the two circles and almond construction. Use the same compass opening which turned the circles to "walk" six points around each circle. This creates the points for two hexagons overlapping in the almond (see Volume 2 Chapter 6). You can fill them completely with ten equilateral triangles.

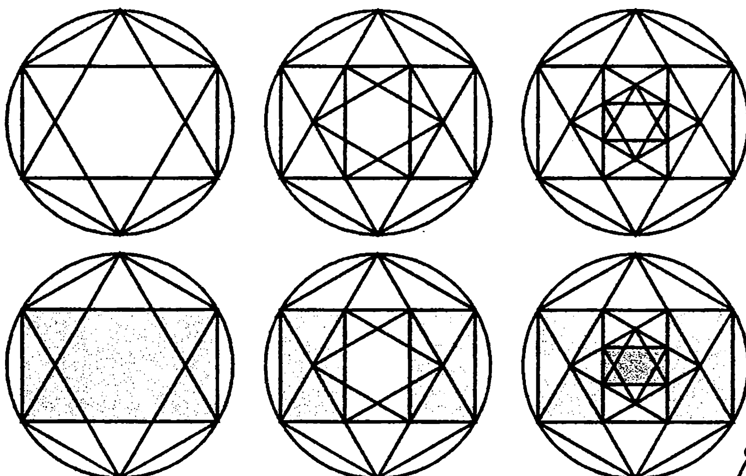






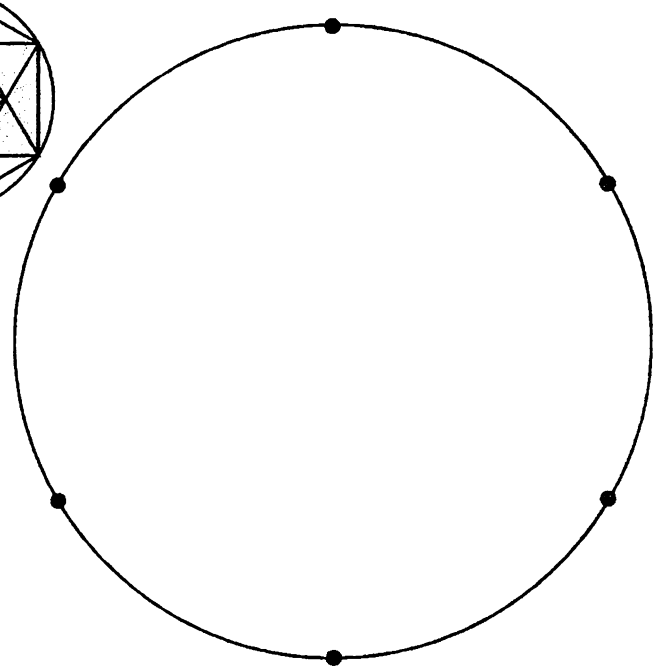
Subdivide Hexagrams For Root Three Rectangles

The hexagram star clearly holds a Root Three Rectangle (shaded, below left). And when the six corners of its central hexagon are connected to create a smaller hexagram within it, the whole Root Three Rectangle divides into three smaller, turned Root Three Rectangles. Every new hexagram has a hexagonal center which can be similarly subdivided into smaller and smaller hexagrams and Root Three Rectangles. The hexagon provides a powerful root-three proportioning system which conveys a feel of unity within diversity.



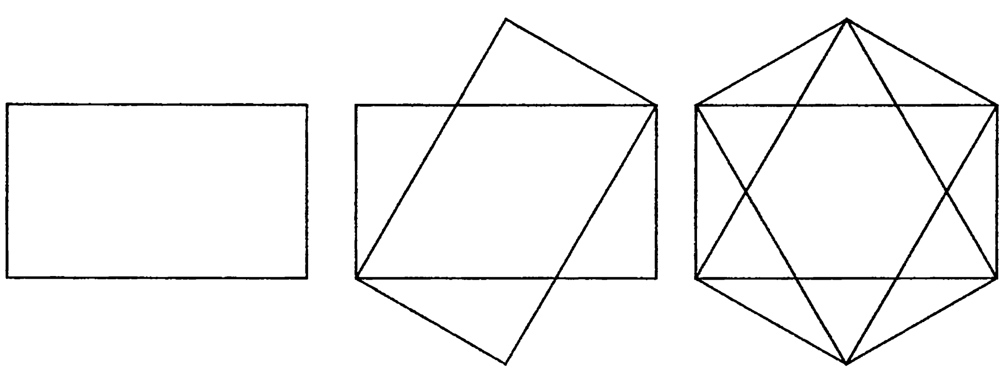
Use this circle with six points and your straightedge and colored pencils to draw hexagrams within hexagrams.

Shade the different sizes of Root Three Rectangles.



Overlap Root Three Rectangles

Cut out three identical Root Three Rectangles. Overlap them as shown to form a hexagon.
Use clear plastic rectangles to see their hexagram.
Challenge a friend to create a regular hexagon with the three rectangles.



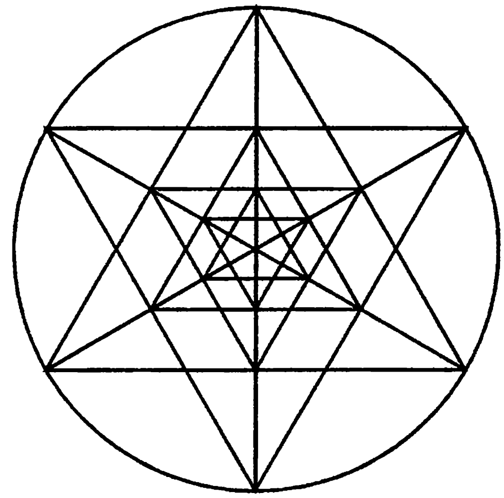
A Different Subdivision Of Hexagrams

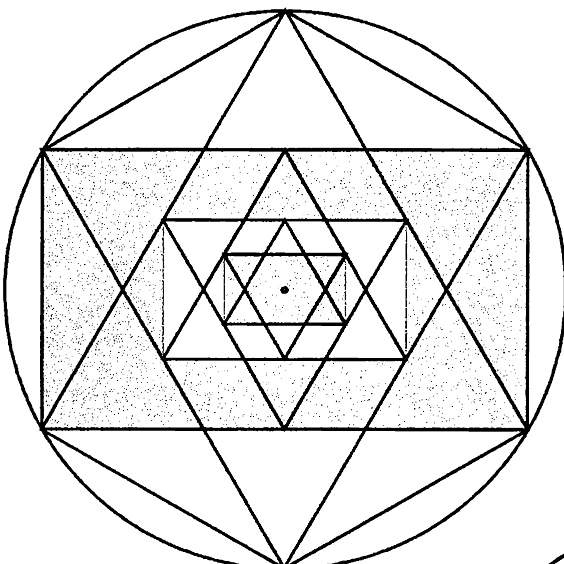
Instead of connecting the six corner points in each inner hexagram to create each smaller hexagram, they can also be drawn from the center of each inner hexagon's six sides, where the diagonals cross them.

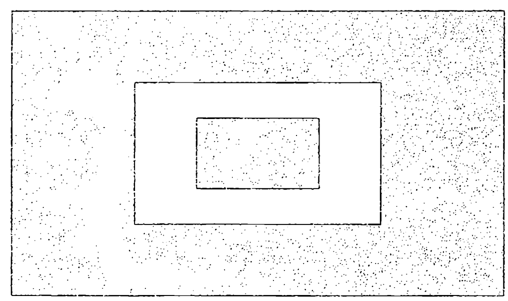
This also produces a series of smaller Root Three Rectangles, but they're all horizontal, not turned.

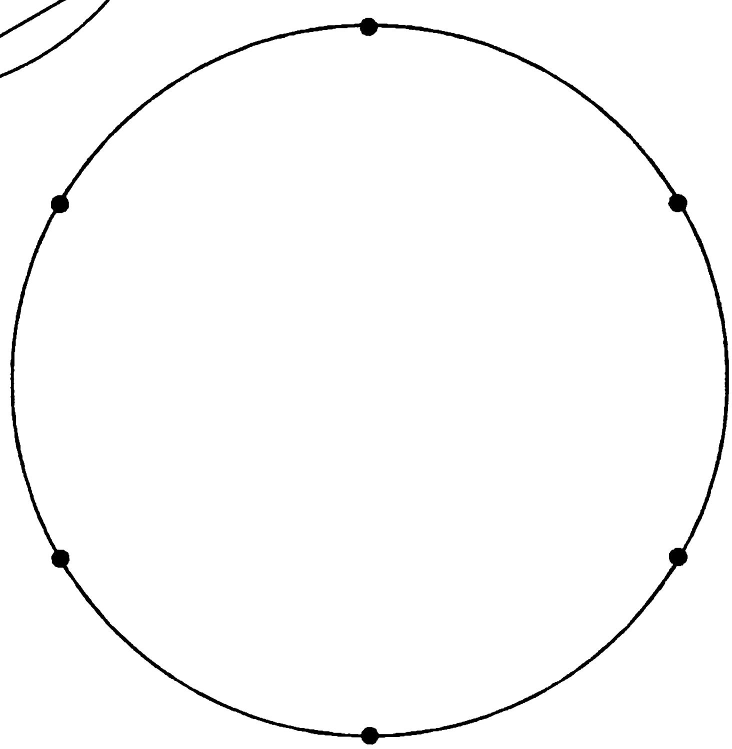
Replicate this construction in the circle below.

Use colored pencils to shade the different levels of Root Three Rectangles.

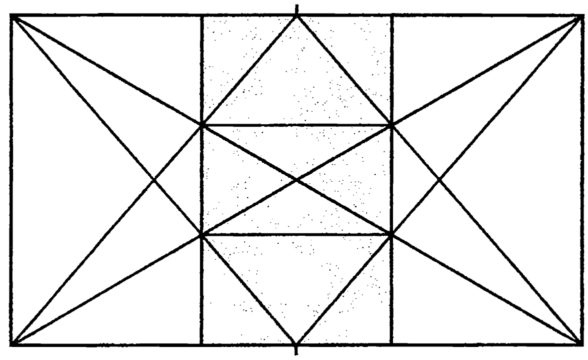






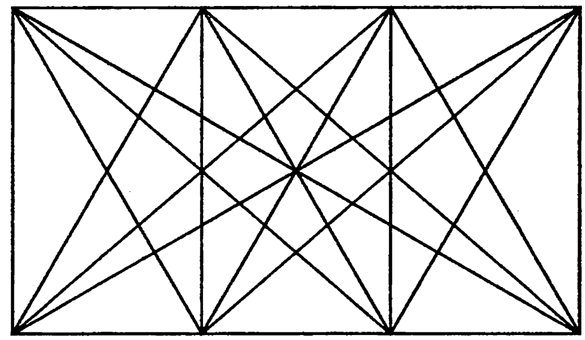


Draw Diminishing Root Three Rectangles



Recall the method for dividing any rectangle into three equal parts by connecting the crossings of diagonals.

When applied to a Root Three Rectangle, the smaller parts are themselves Root Three Rectangles.

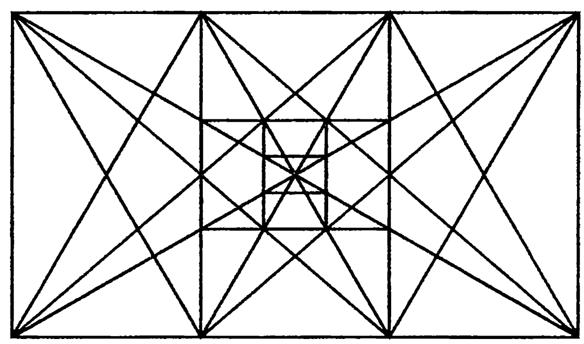


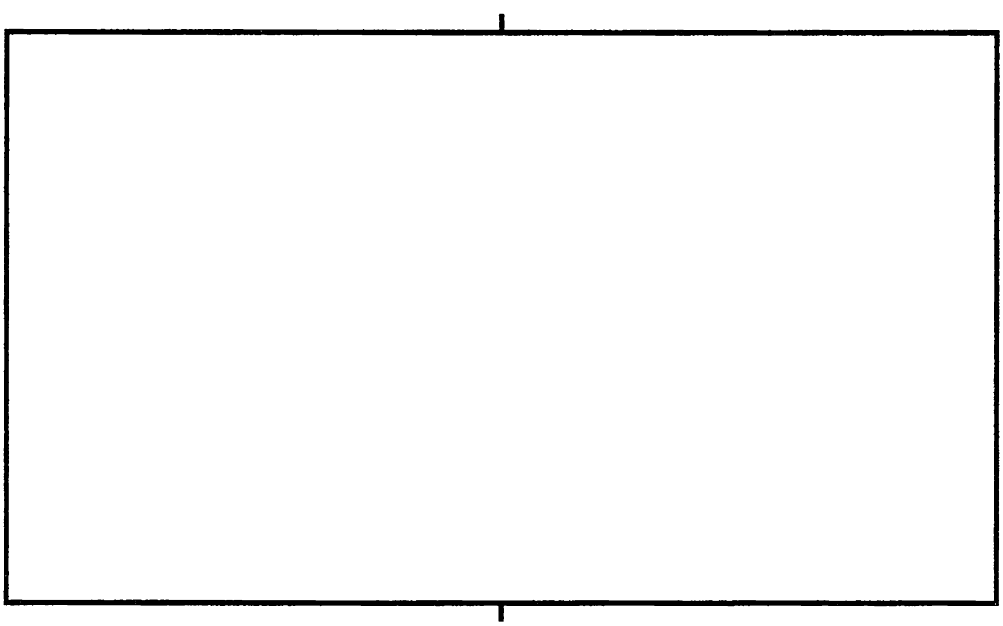
By drawing more diagonals as shown we can find crossing points which allow us to keep subdividing them into smaller and smaller versions of itself.

Divide the Root Three Rectangle below into three equal parts.

Then subdivide them into smaller and smaller Root Three Rectangles.

Use colored pencils to shade them.

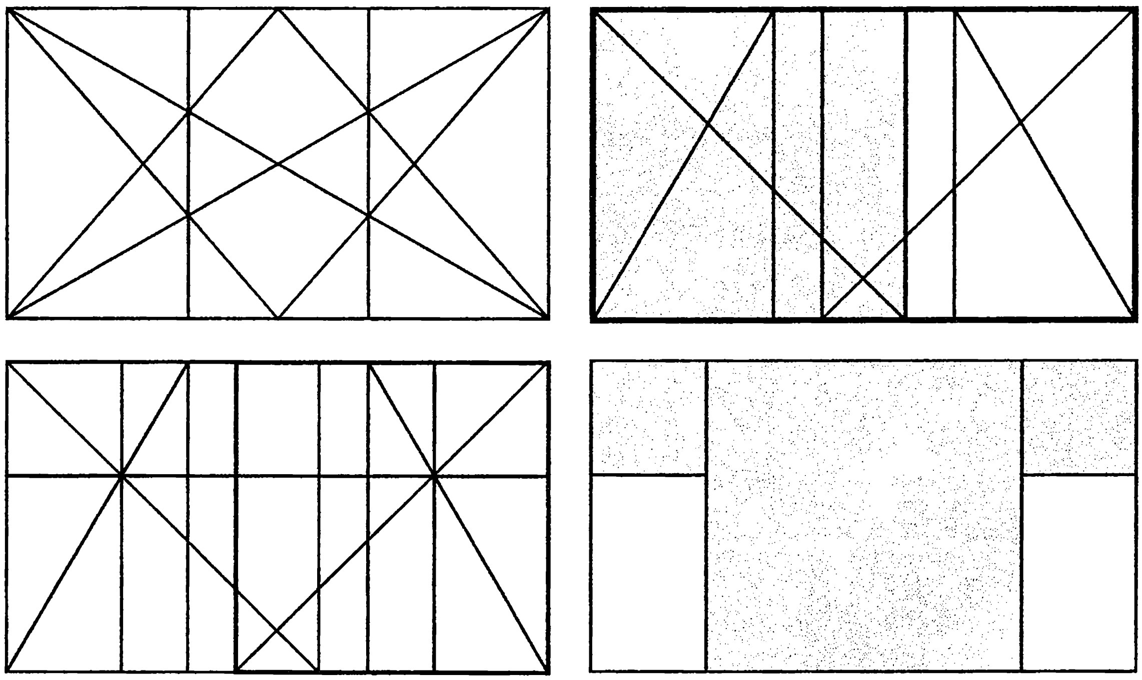


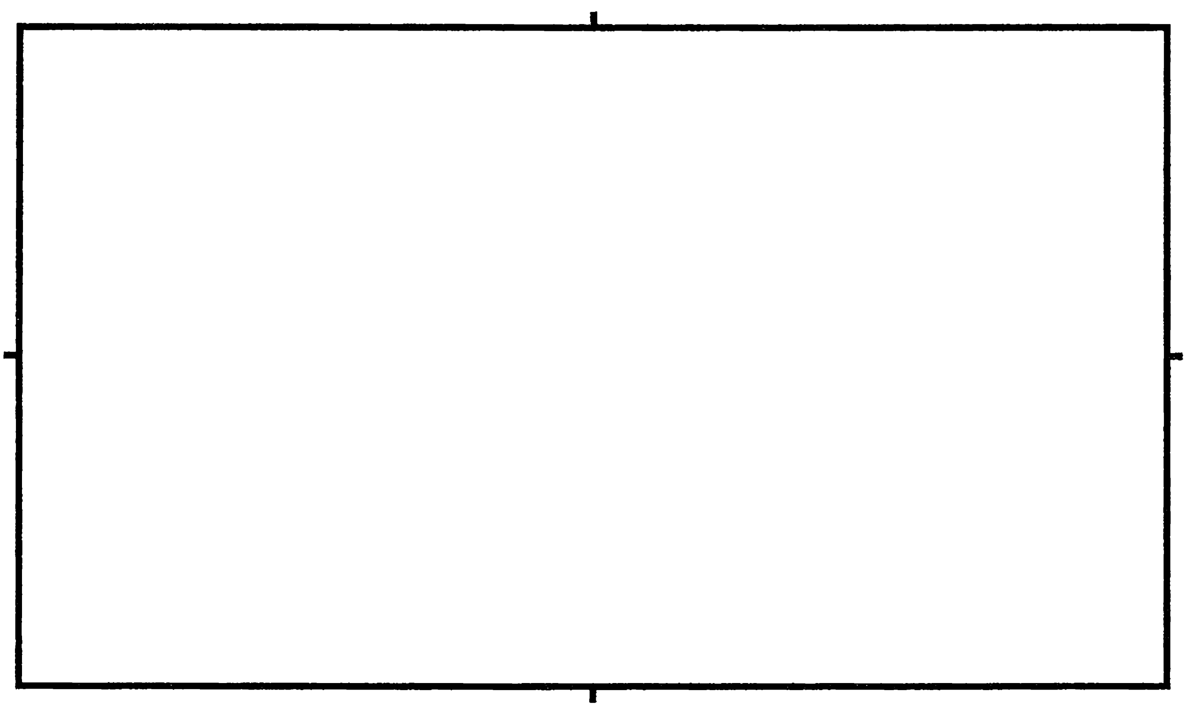


Replicate These Root Three Rectangle Constructions

First divide the Root Three Rectangle below into three equal rectangles.

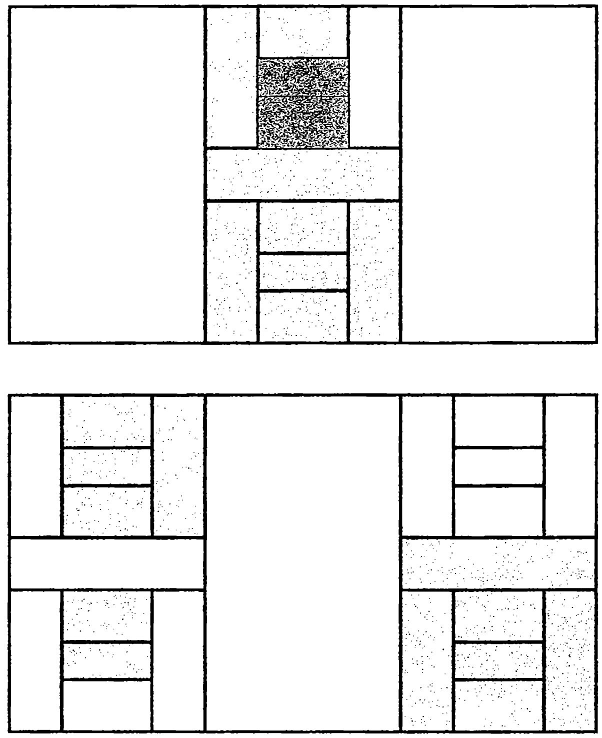
Then apply rabatment of squares from the ends to replicate this pattern of two small squares, one large central square, and two identical Root Three Rectangles.

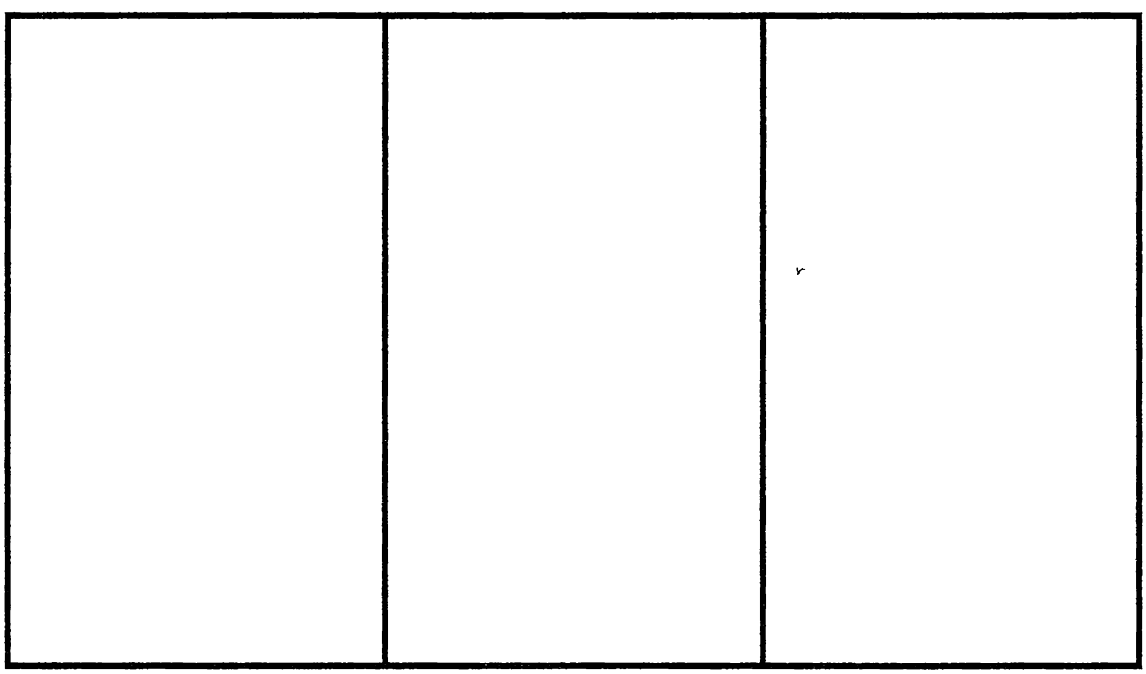




A Root Three Rectangle has been divided into three identical parts below.

Apply rabatment of squares (some shaded below) within them to replicate this pattern.

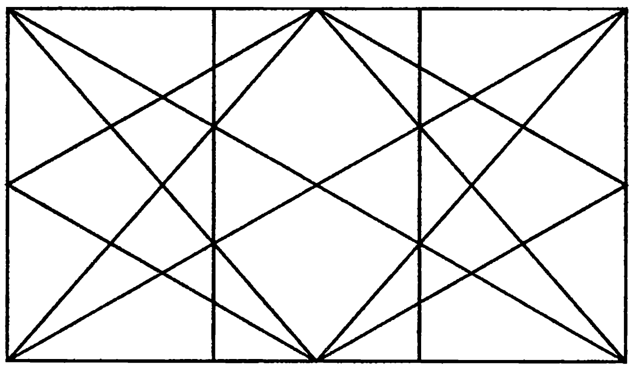


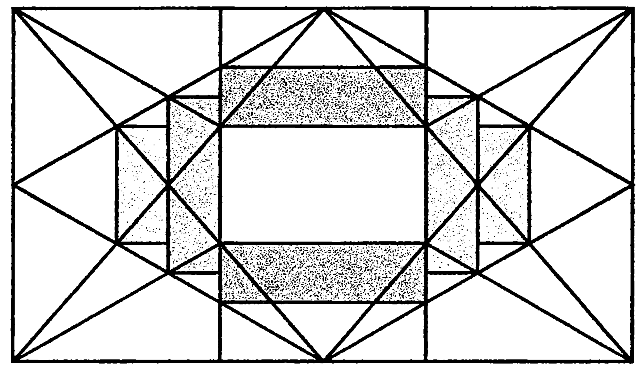


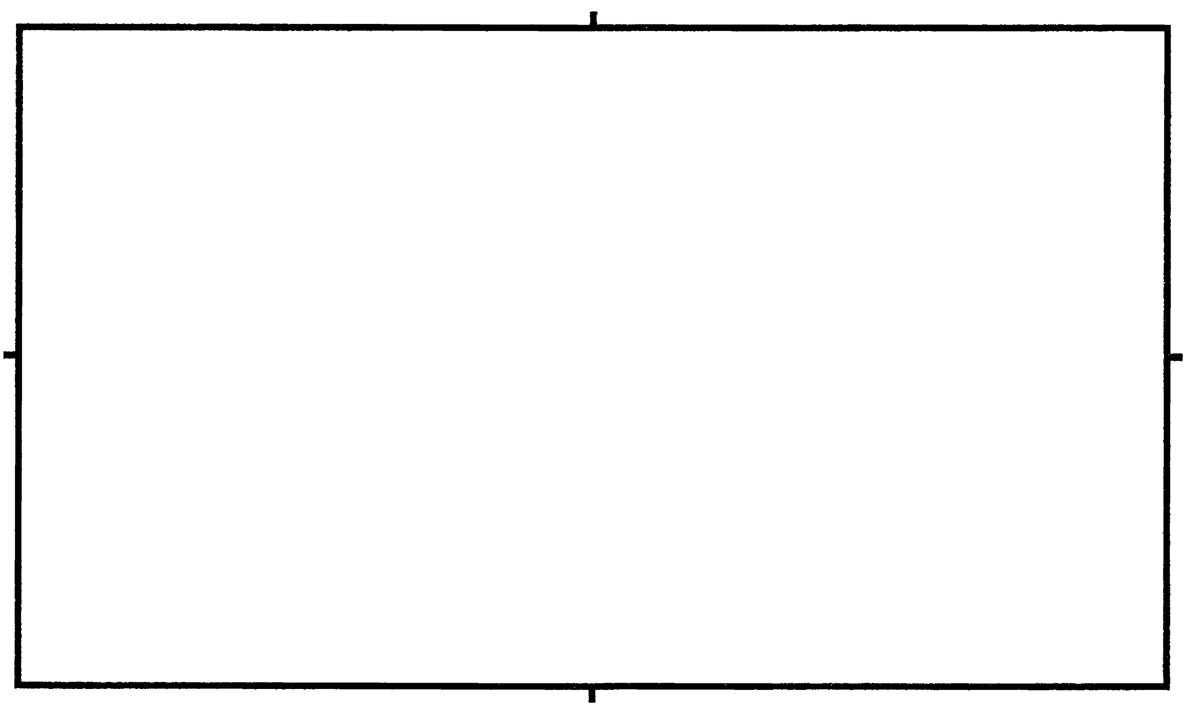
Draw diagonals and connect their crossing points to create rectangles in this pattern.

Develop it further.

Shade it any way you like.







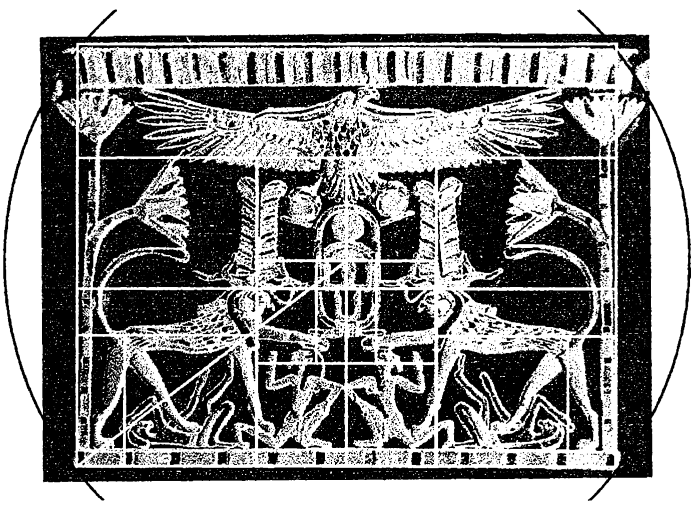
Root Three Rectangle Art

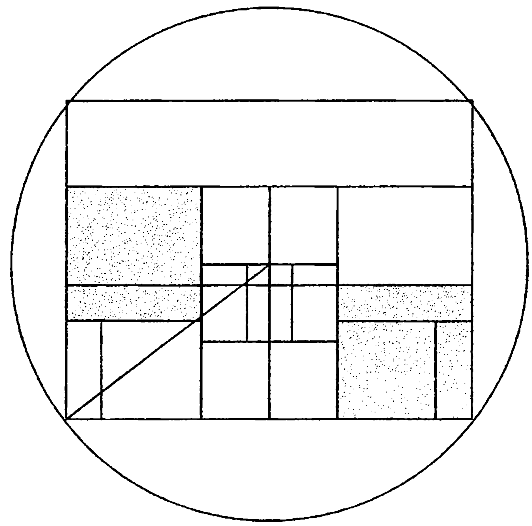
Pectoral Of Princess Mereret

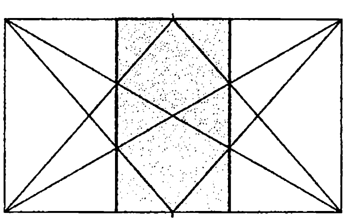
Middle Kingdom, 12th Dynasty (c1810 BCE)
Gold, Turquoise, Carnelian, Amethyst, Lapis Lazuli

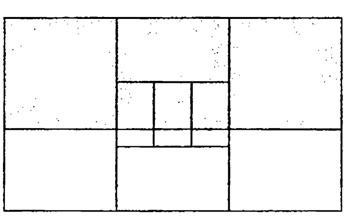
This openwork pectoral hung upon a necklace belonging to princess Mereret, daughter of Senusert III and sister of his successor Amenemhat III. The pectoral is set within the framework of a "chapel" where the vulture-goddess with outstretched wings hovers protectively over the scene of the victorious king, depicted as a griffin, dispatching his enemies, Libyans and Nubians. The griffin combines here the power of the falcon with the strength of the lion.

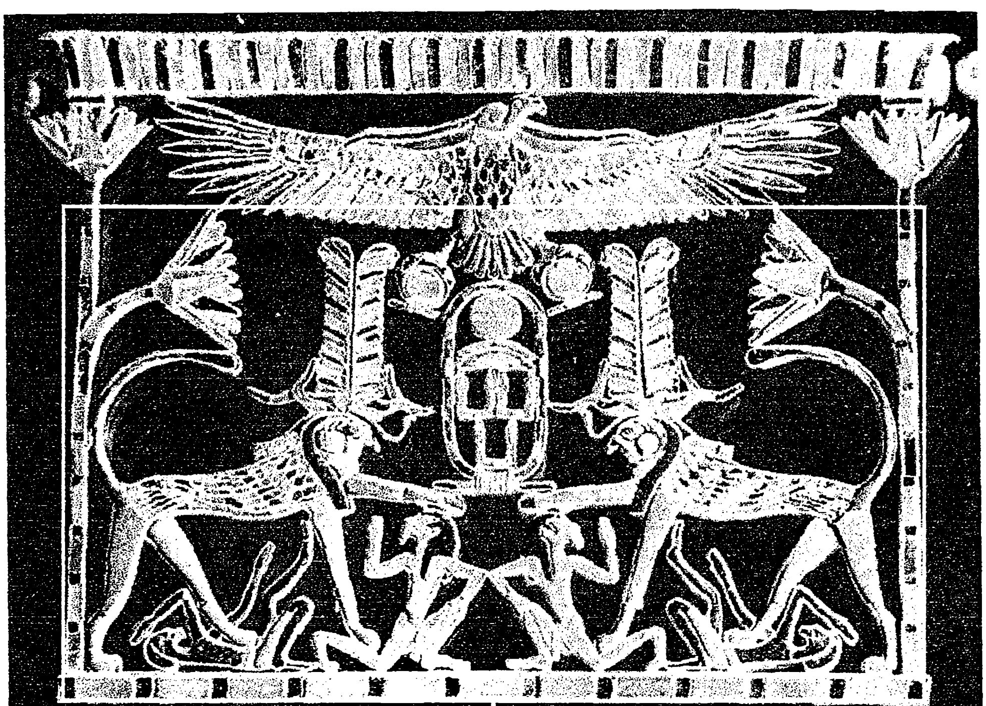
First, divide the Root Three Rectangle into three smaller Root Three Rectangles. Notice how they divide the scene precisely through the double-feather headdresses. Then subdivide the scene further as shown using rabatment of squares.







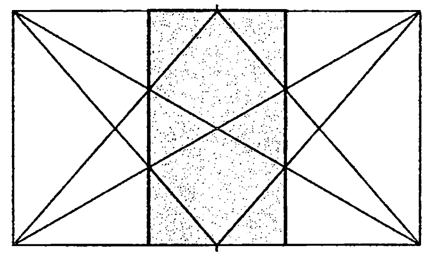


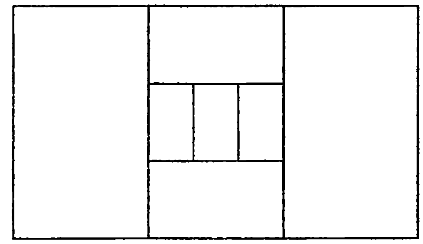


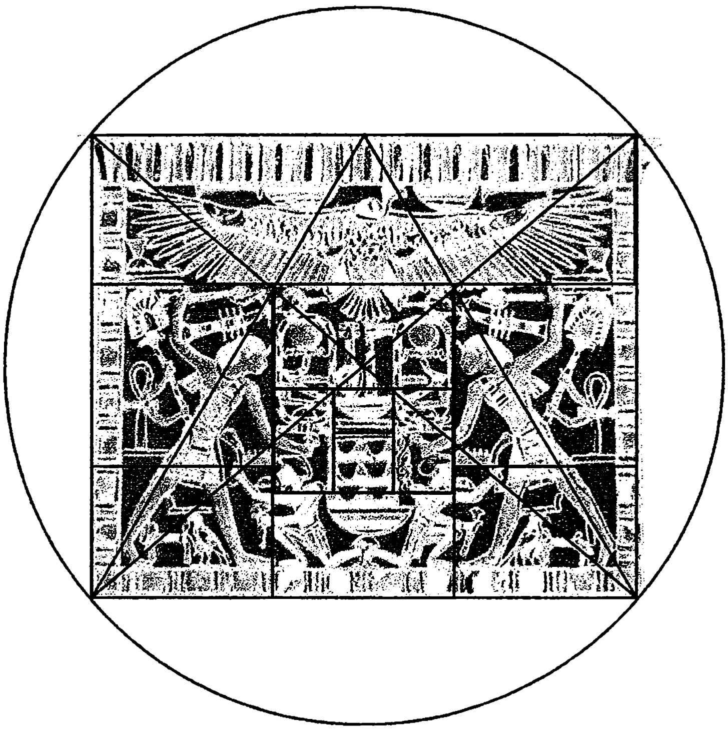
Another Pectoral Of Mereret

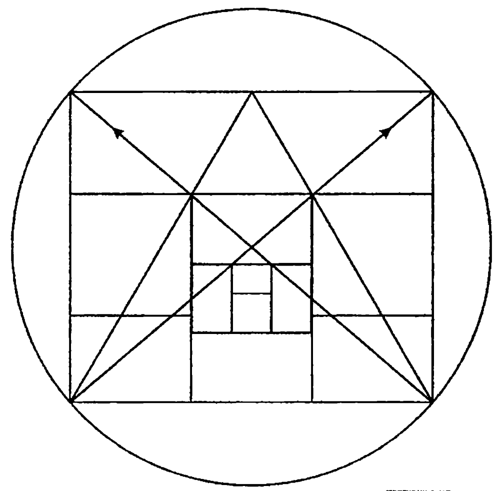
Subdivide the pectoral as shown to see its geometric framework.

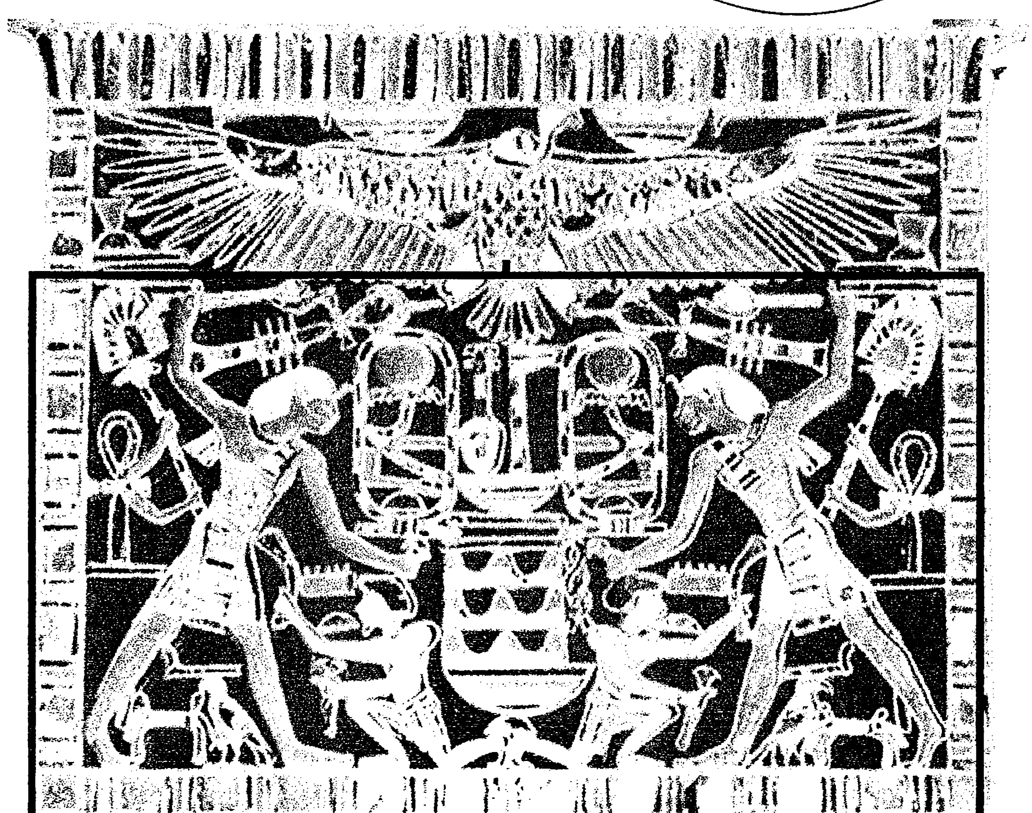
Notice how the geometry frames different elements of the scene.



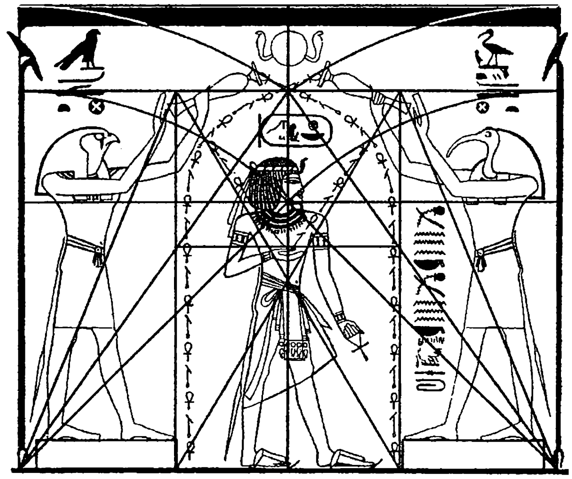






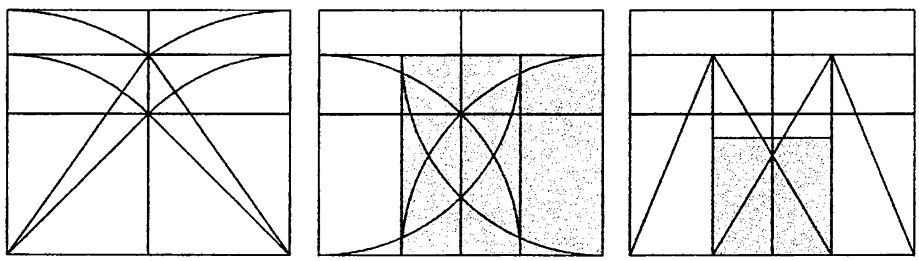


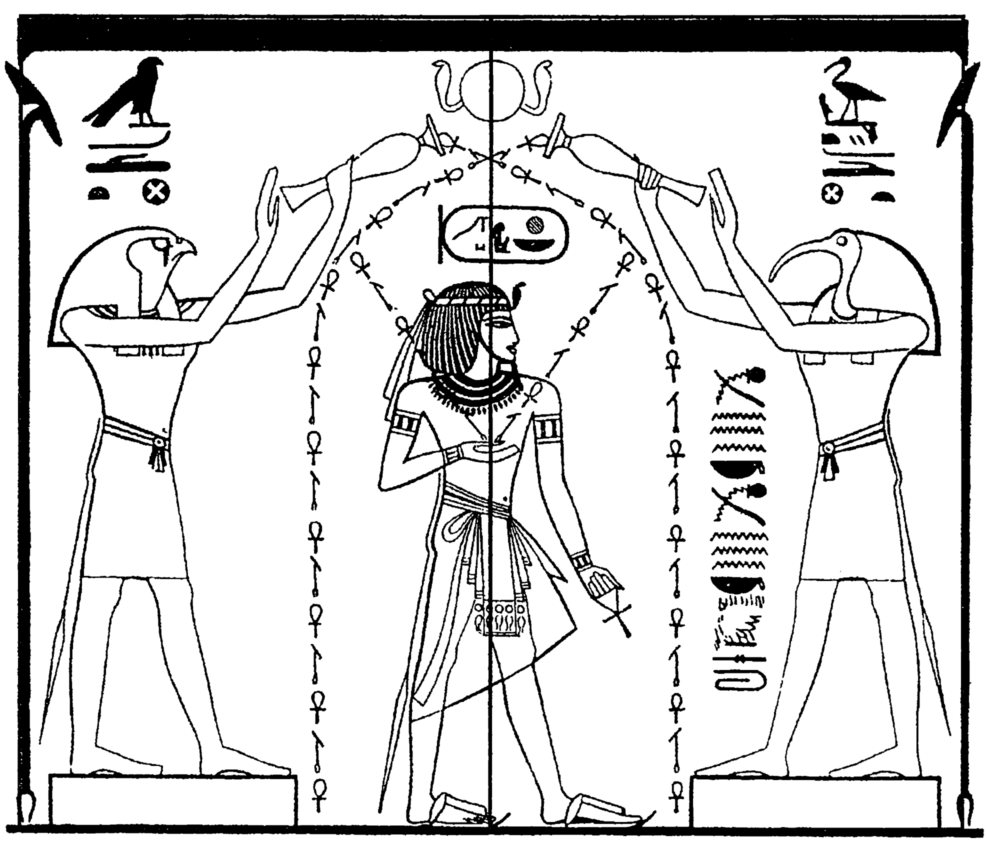
Stela Of Amenhotep's Purification



Amenhotep II was the seventh pharaoh of Egypt's 18th Dynasty. His name means "Amun is Pleased," or "Amun is Peaceful". As a young man he was known for his athletic abilities. He is said to have shot arrows through a copper plate while driving a chariot with the reins tied about his waist. Getting his military adventures done early in his reign, his regency is noted for its peacefulness and expansion of temples.

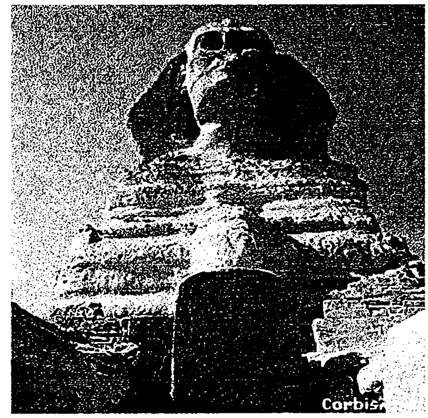
This scene, drawn from a stela, shows deities Heru (Greek Horus, hawk headed) and Djeheuti (ibis headed "Thoth") purifying the pharaoh by pouring the "waters of life" over Amenhotep. Notice that they are pouring the symbol "ankh" which signifies "life". Its geometry is that of a double Root Three Rectangle. Divide it as shown to see its proportioning.





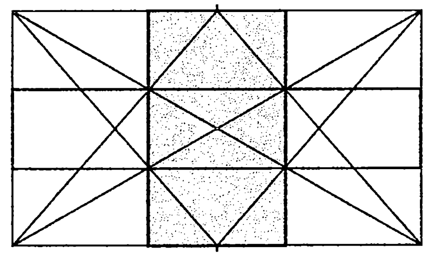
Horus and Thoth of Hat purifying Amenophis II

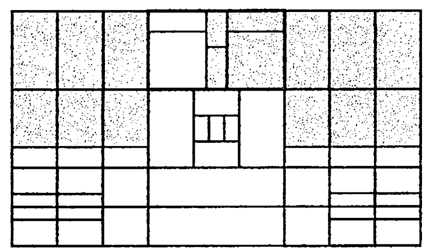
The Sphinx Dream Stela

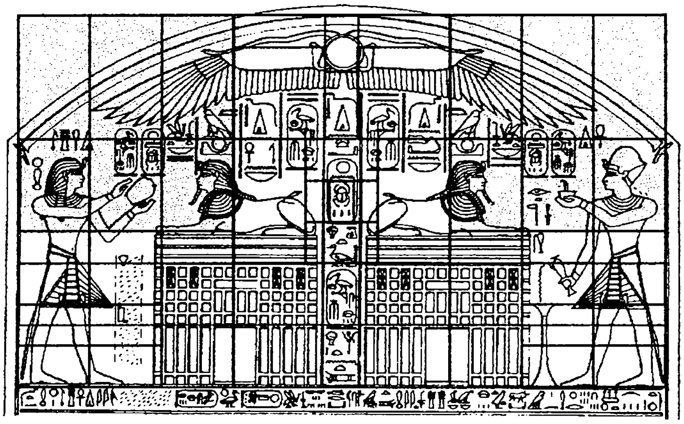


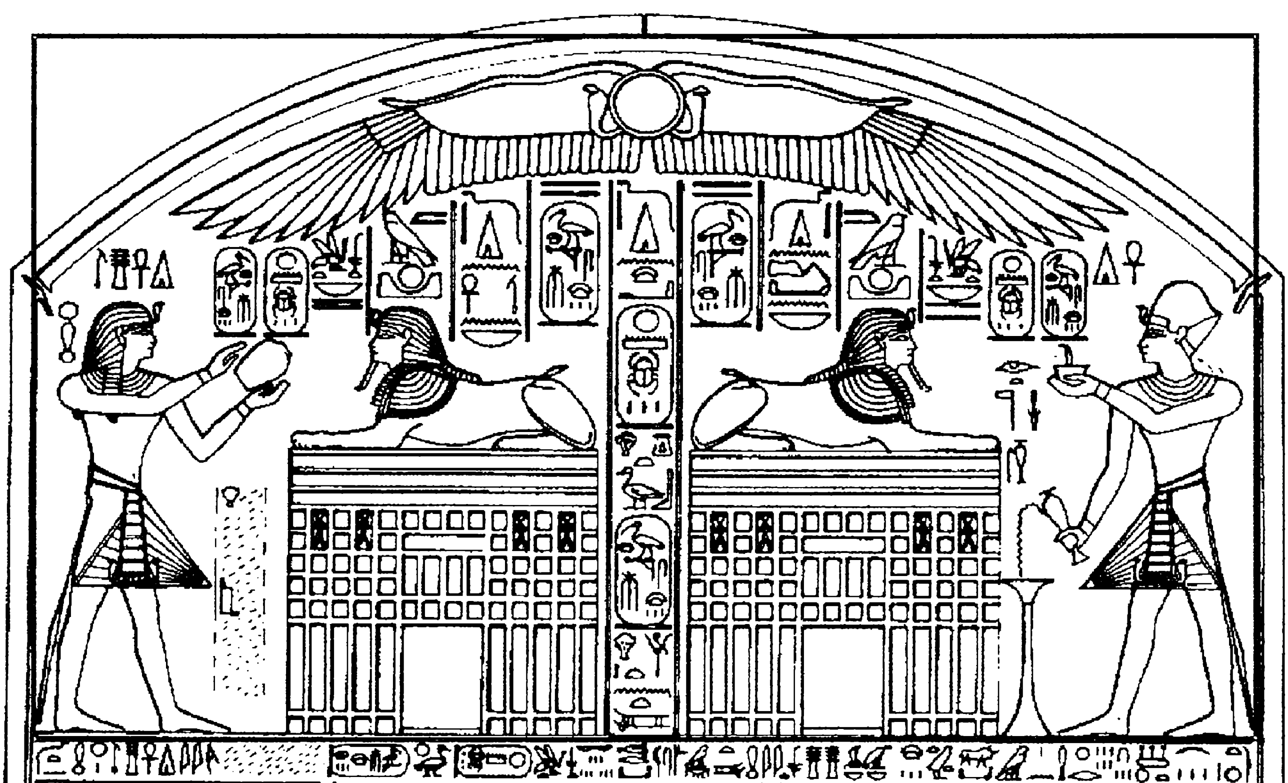
During the Eighteenth Dynasty young Prince Tuthmosis (son of Amenhotep II) was hunting in the desert. Although much of the sphinx was covered with sand, he fell asleep between its paws and dreamt that the sphinx spoke to him, promising to make him pharaoh if he cleared the sand from around it. Tuthmosis cleared the sand and through a series of coincidences became pharaoh of Egypt for nine years.

The top of the stela is framed in a Root Three Rectangle. Subdivide it into nine smaller Root Three Rectangles. Mark a square (shaded) inside the top of each. Also draw overlapping squares inside the small Root Three Rectangle at top center. Subdivide some Root Three Rectangles into smaller ones to see its hidden framework. The center of it all is the tail of the scarab beetle where the egg of the universe first emerged.









Tutankhamun's Scarab Pectoral

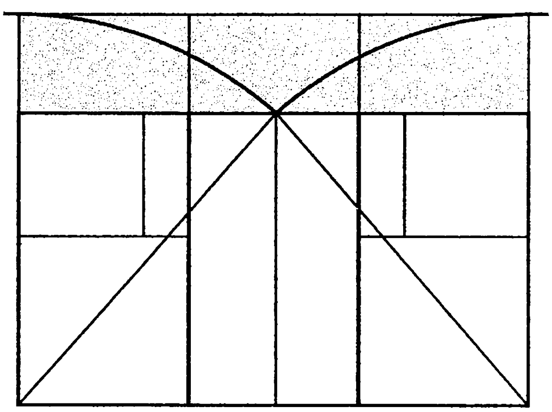
New Kingdom, 1570-1070 BCE

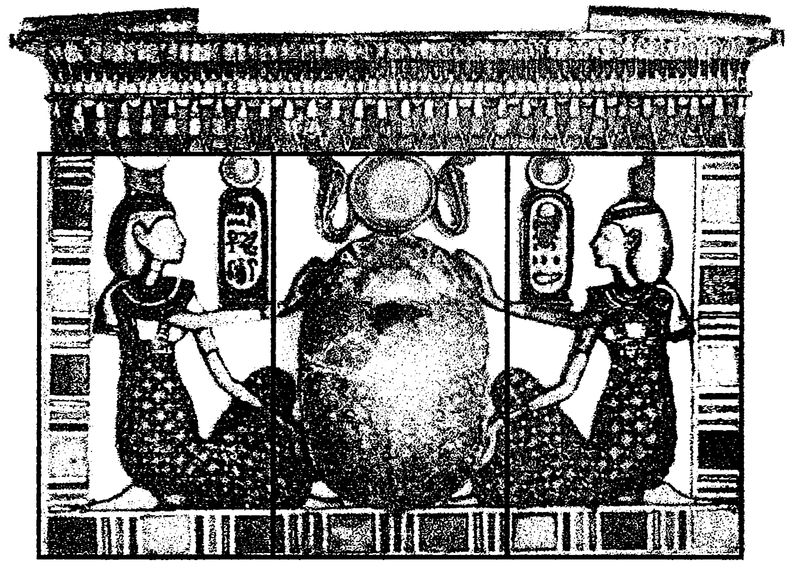
This pectoral, found in the tomb of Tutankhamun, depicts the goddesses Nebt-Het (Greek "Nepthys") and Aset ("Isis") on each side of the *khepera* scarab beetle. The scarab pushes or "transforms" the sun, flanked by sacred snakes, across the sky.

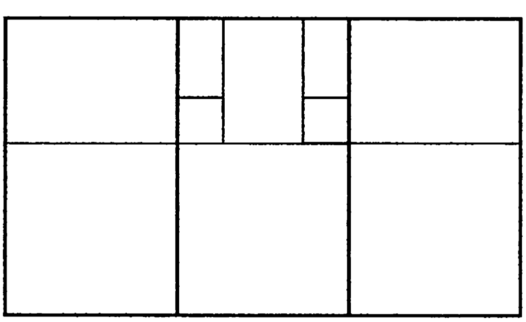
There are two ways to see its Root Three Rectangles framing the scene. Both are given for you to explore.

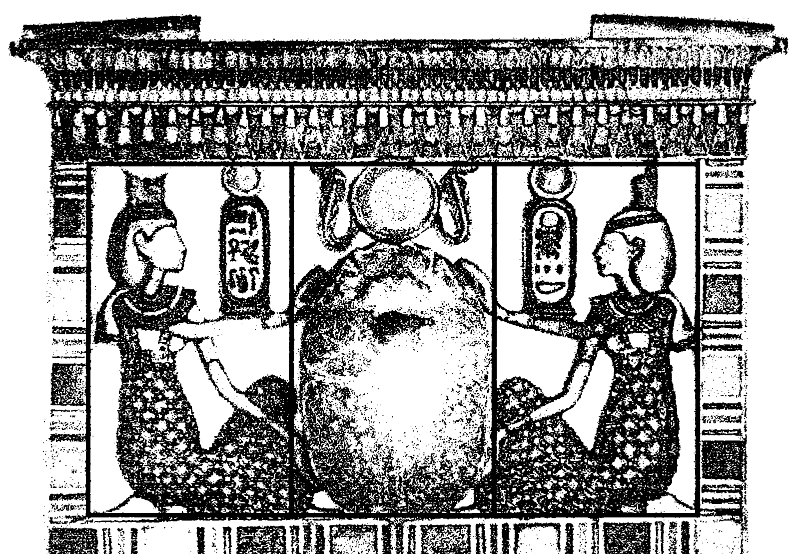
Note that the dimensions of the top of this sacred "chapel" are determined by swinging arcs from the top center of the Root Three Rectangles.

Explore it by rabatment of squares and diagonal crossings.







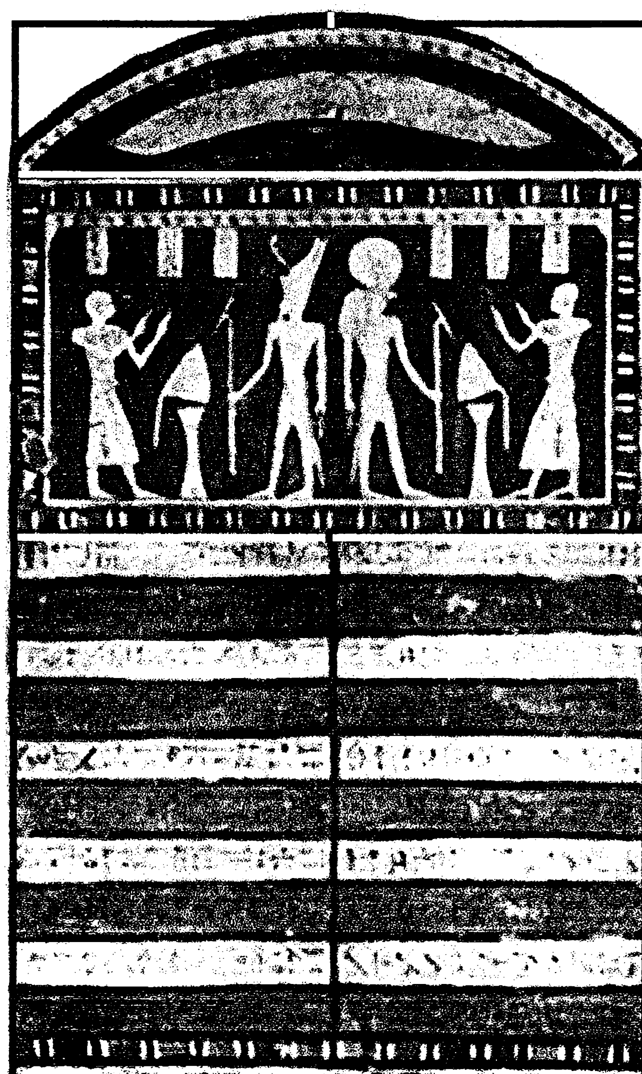


Stela Of Ha-hat

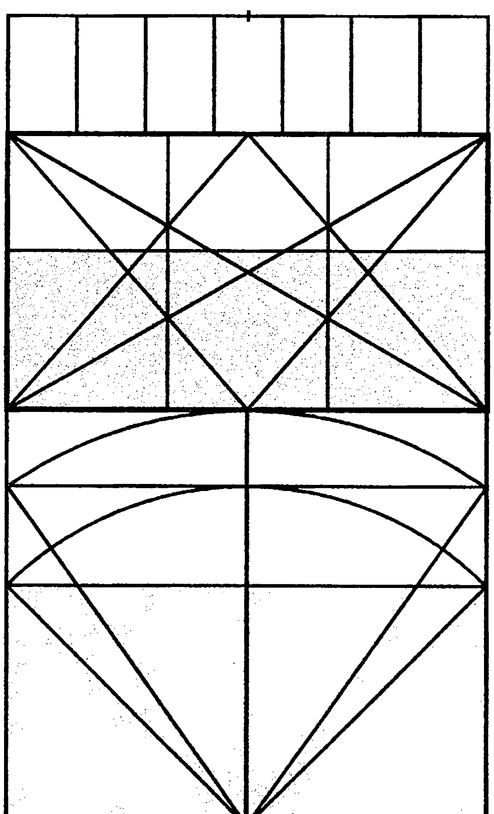
Wood with painted stucco layer
Thebes, 26th Dynasty c640 BCE
Kunsthistorisches Museum Vienna

The Twenty-Sixth dynasty of Egypt was the last native dynasty to rule Egypt before the Persian conquest (525 BCE) whose king Cambyses II carried Pharaoh Psammetichus III to Susa in chains. This stela was made for the tomb of Ha-hat, a "Month priest". The scene on the right shows homage being paid to the falcon-headed Harakhte, the morning sun, and on the left to Atum, the evening sun. The text, divided in half, are hymns to both gods. Ha-hat hoped to become one with the sacred sun.

The geometry consists of two side-by-side Root Three Rectangles framing the hymns. Atop this is a turned Root Three Rectangle for you to divide by diagonals into three vertical Root Three Rect-angles. Then draw a square in each of their bottoms.

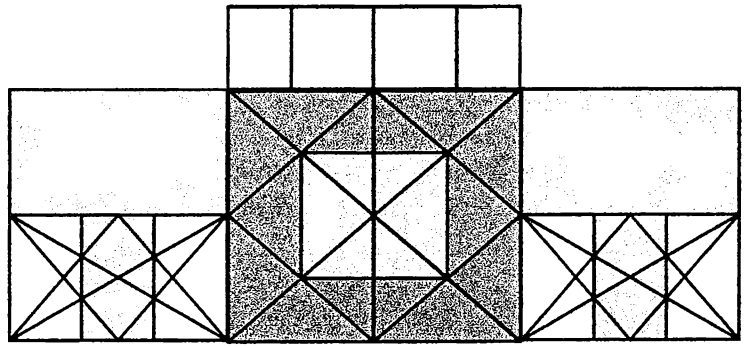


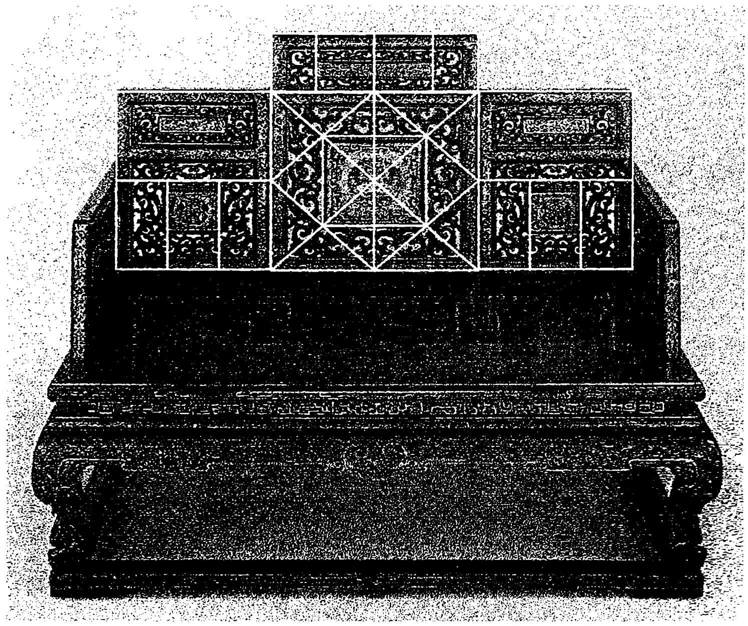
Atop this are seven small Root Three Rectangles. Midpoints at the top and bottom are given. Divide and subdivide each rectangle as shown to enjoy its construction. The symbolism of three -- sacred -- is emphasized.



Chinese Imperial Throne

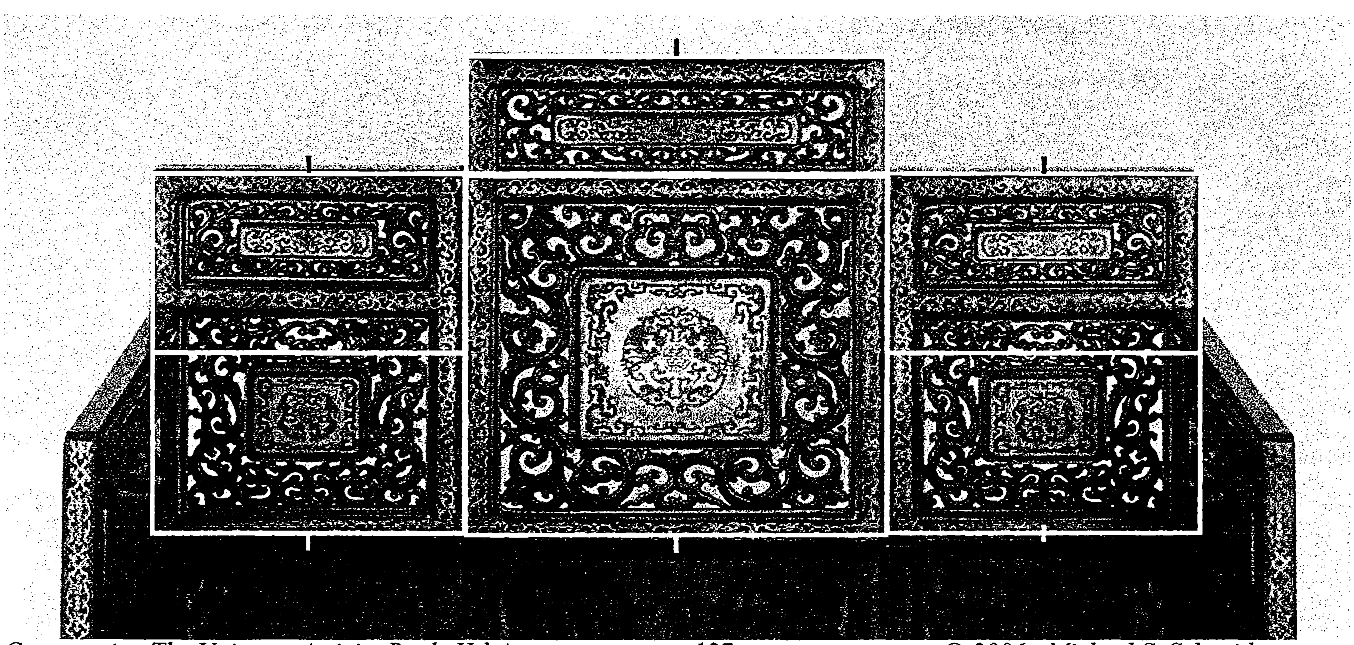
Polychrome lacquer over a softwood frame
Ch'ing dynasty, Ch'ien-lung period, China 18th century
Minneapolis Institute of Arts





"Made during the Ch'ien-lung period (1736-95), this rare piece of court furniture is one of the larger and more fully decorated thrones outside of China. Few lacquered thrones have dragons and celestial landscapes like those found here painted in gold lacquer across the entire expanse of the seat. The composition and iconography of the five-clawed imperial dragons cavorting amongst clouds and flaming pearls above the ocean is an official insignia and is similar to the decoration of court robes and other official court textiles of the period. The cabriole legs, aprons, and openwork back and side panels are all carved in relief with scrolls and lacquered in green, red, and gold. The panels of the removable back and side rails are decorated with stylized dragons and *shou* medallions emblematic of imperial rule and long life." [-- from the MIA catalog]

Two large central Root Three Rectangles are topped by two small horizontal R3Rs (mark a square in each) and flanked by pairs of R3Rs atop one another. Divide as shown.



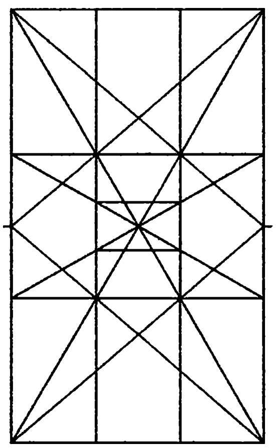
Mourning Athena

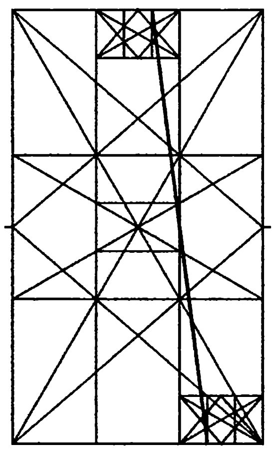
Marble, c640 BCE, Athens, Greece. The Parthenon Museum

Athena was the protector of the city of Athens and helper of heroes, including Heracles, Jason, and Odysseus. She was the Greek goddess of wisdom, weaving, crafts, justice and war, born fully formed and clothed for battle from the head of her father Zeus.

This votive relief was found on the Acropolis at the Parthenon, her most famous temple. We observe her clad in an Attic *peplos* with a belt, leaning on her staff and slightly bending her head towards the stele in front of her. Some interpret this as contemplation, or perhaps mourning for heroes fallen in war whose names are listed on the stela before her.

The whole is a Root Three Rectangle, signifying its sacredness. The centers of two sides are given for you to draw diagonals and divide it into nine smaller Root Three Rectangles. Subdividing further will reveal some of its design, including the slant of her staff.

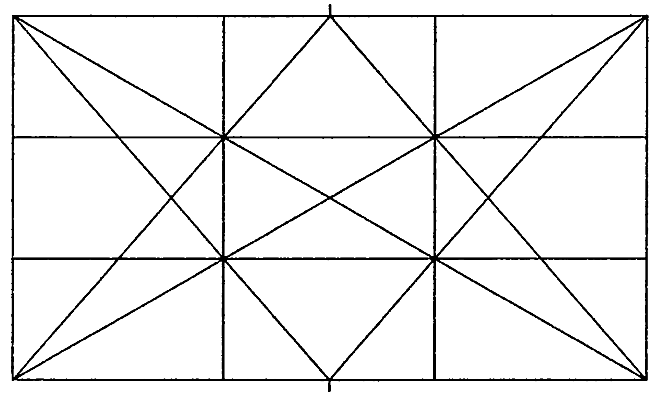




Greek Cup

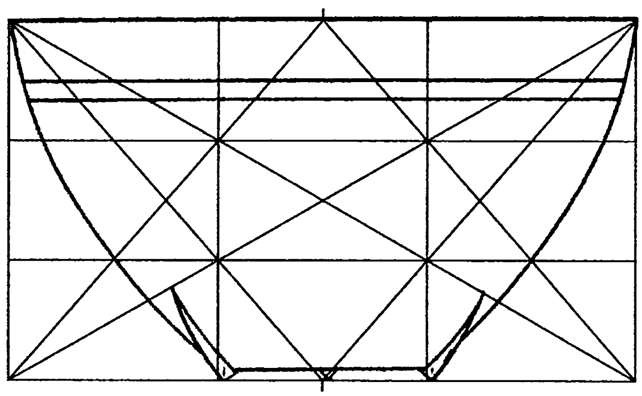
Throughout the ancient world, it was standard to design even ordinary household objects with beauty and harmony in mind. People exposed to harmony will develop a feel and desire for it.

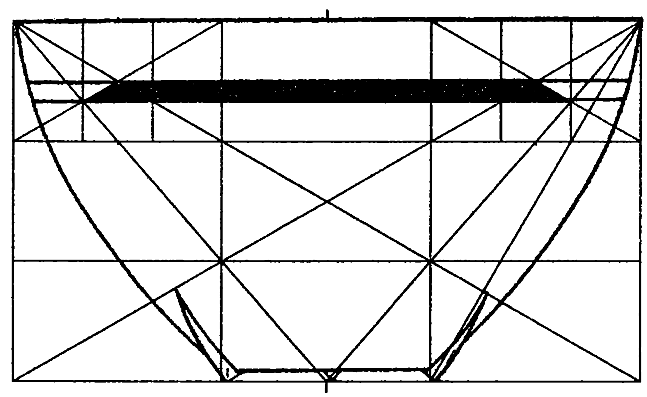
This cup fits in a Root Three Rectangle. Draw diagonals as shown to divide it into nine smaller Root Three Rectangles. Notice how they cross the upper limits of its three "feet".

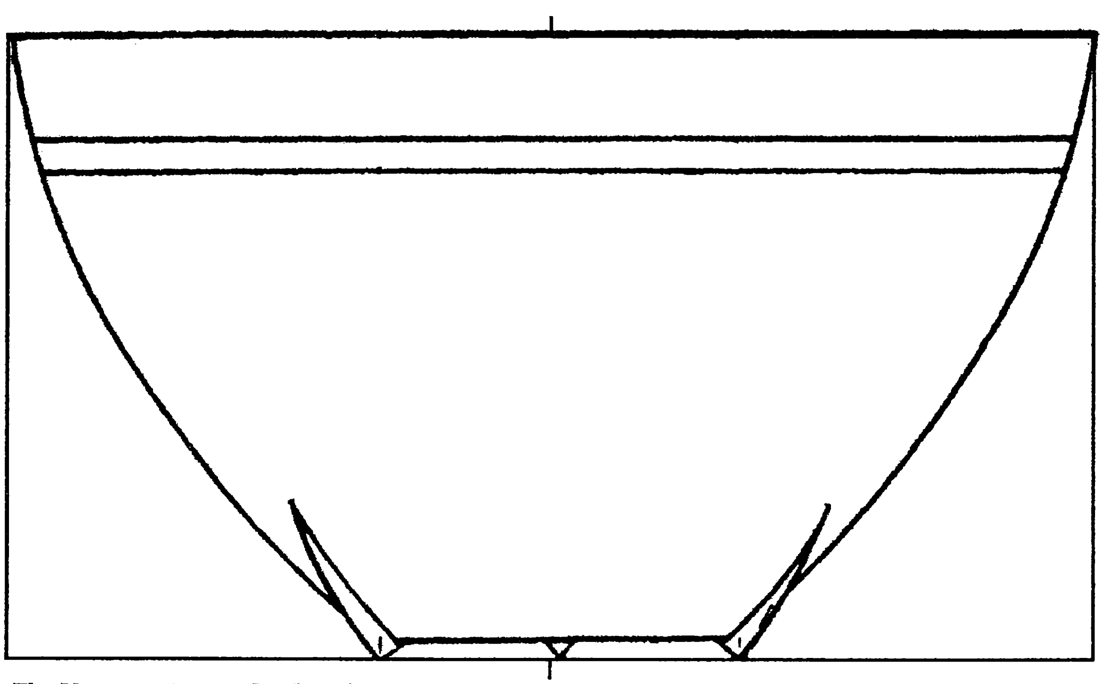


Notice the band across the top of the cup. Its two parallel lines are created by connecting different pairs of diagonal crossing points. Look closely to find them.

Use these lines and others to develop your own original design on the cup.

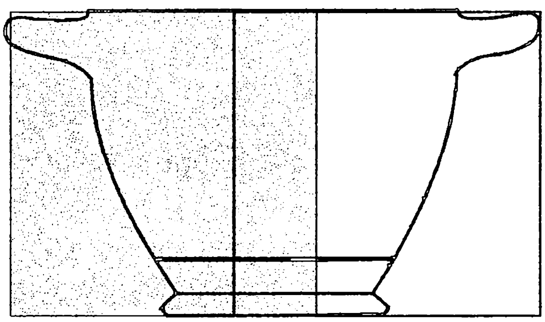


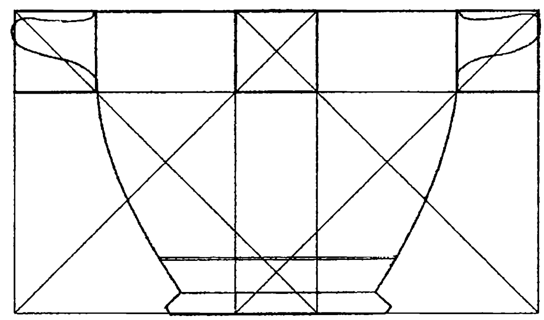


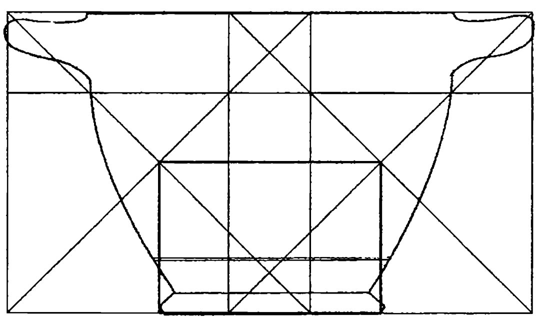


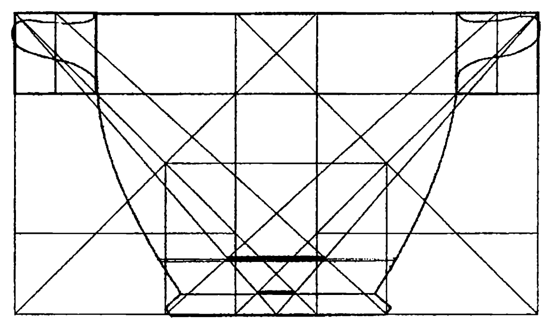
Greek *Skyphos*

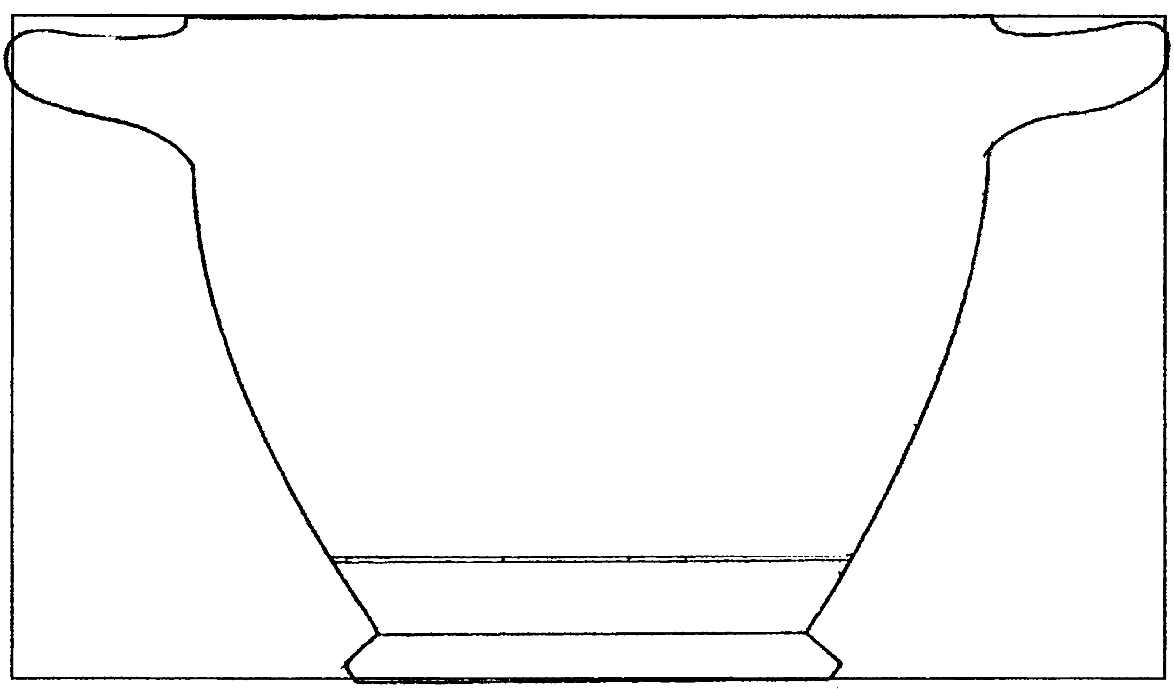
A *skyphos* was used to mix water with dehydrated wine (which was lighter to ship). Hambidge's analysis was developed to this construction. The whole is a Root Three Rectangle. Draw a square from each end. Draw a small square in the top of the central column where they overlap. Transfer this measure to the end handles. Diagonals in both squares indicate a rectangle which defines the base. More diagonals indicate the harmonious placements of bands near the bottom.







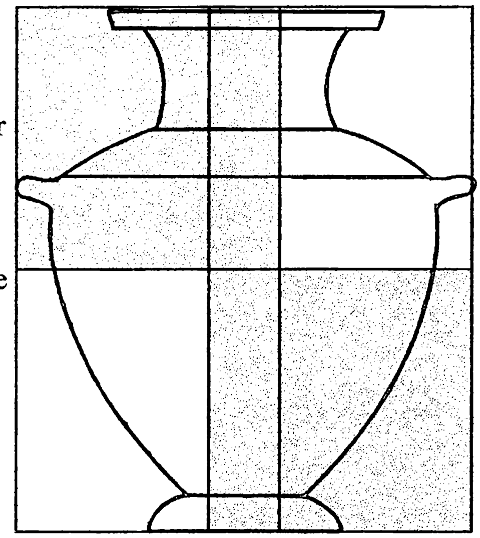




Greek *Hydria*

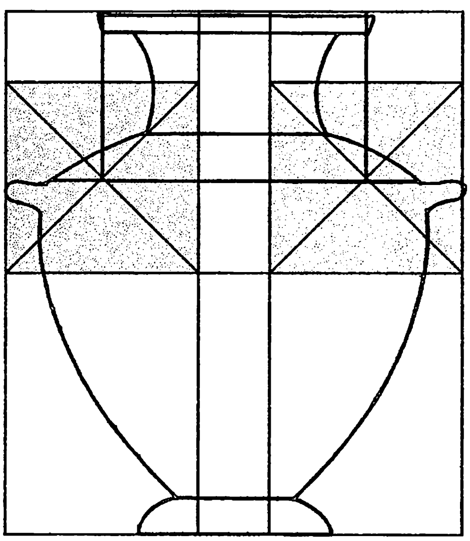
A *hydria*, as it name suggests, was used to store water. The geometry of one (again, analyzed by Jay Hambidge) is framed within two horizontal, stacked Root Three Rectangles.

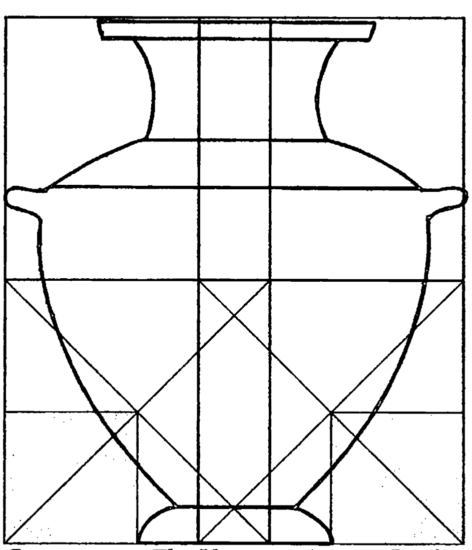
Apply rabatment by drawing squares from each end of both rectangles. In the upper rectangle, draw a square (shaded below) at the bottom of each rectangle to the left and right of the central overlap column.

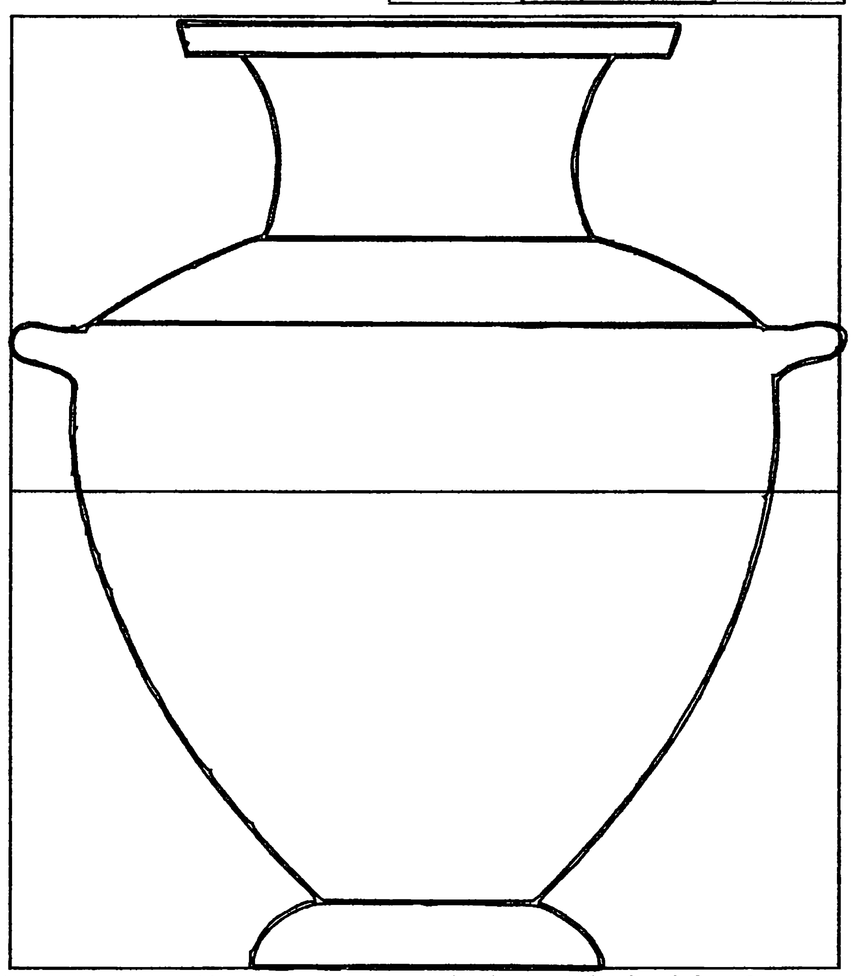


Draw the small squares' diagonals. Connect their intersections to show the level of the line across the *hydria*'s shoulder. Then transfer the measure of their intersection from the central column up to define the edge of the *hydria*'s mouth.

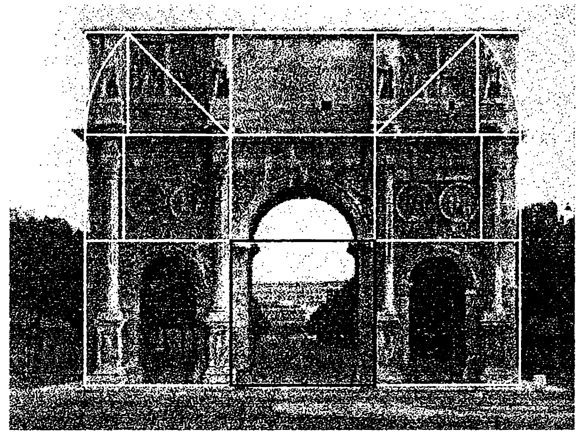
Draw diagonals in the overlapping squares in the bottom Root Three Rectangle. Draw a square from their intersections to each bottom corner and define the edges of the base.





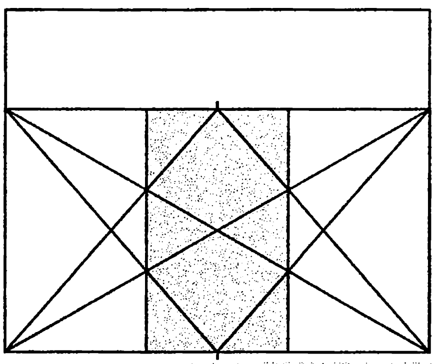


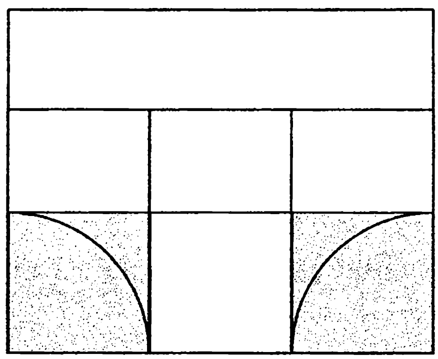
Arch Of Constantine

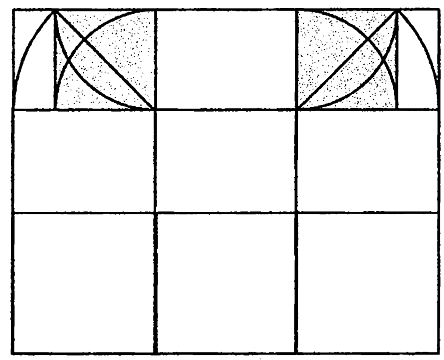


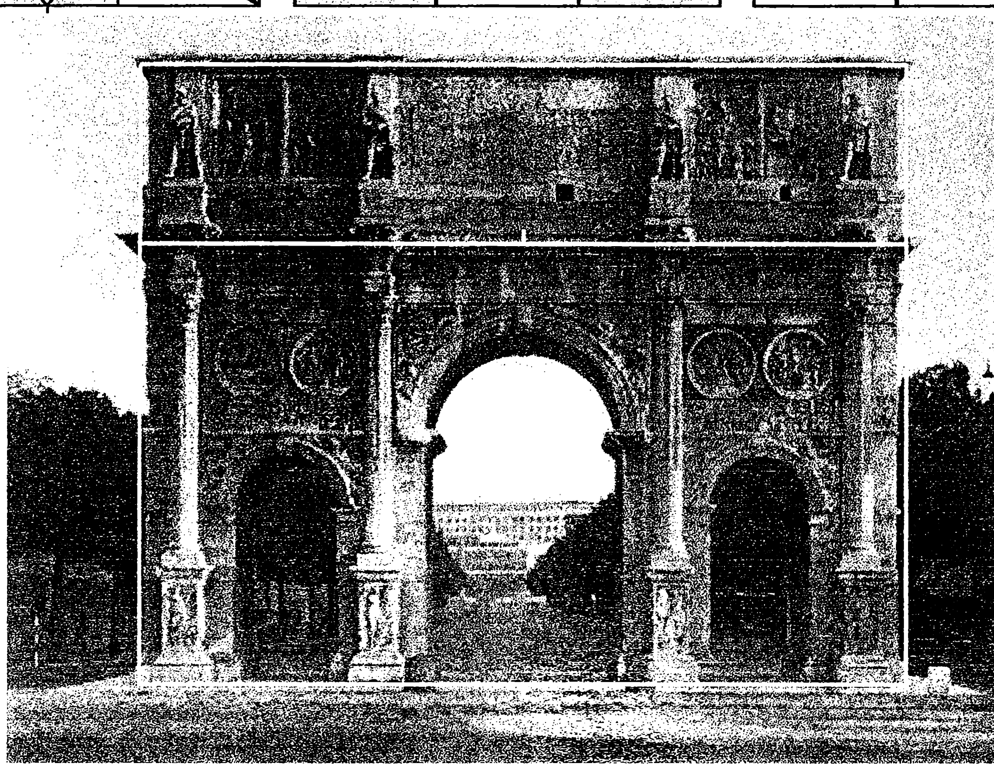
The Arch of Constantine is a triumphal arch located on the grounds of the Colosseum near the Palatine Hill. The *Via Triumphalis*, the road taken by emperors when they entered the city in triumph, passes through it. It was erected to commemorate Constantine's victory over Maxentius at the Battle of Milvian Bridge on October 28, 312, and was dedicated in 315. The lower part of the monument is built of marble blocks, while the top (called *attic*) is lighter brickwork revetted with marble. It has inscriptions on both sides, and its columns, of the Corinthian order, are made of yellow marble.

The geometry of the lower part is a horizontal Root Three Rectangle. A long rectangle rests atop it. First, draw diagonals in the Root Three Rectangle to divide it into three smaller Root Three Rectangles. Then draw a square in the bottom of each. Extend the vertical lines upward through the long rectangle. In each of the three upper rectangles, draw a square. A swing of the compass along each square's diagonal shows that this long rectangle is actually three horizontal Root Two Rectangles!

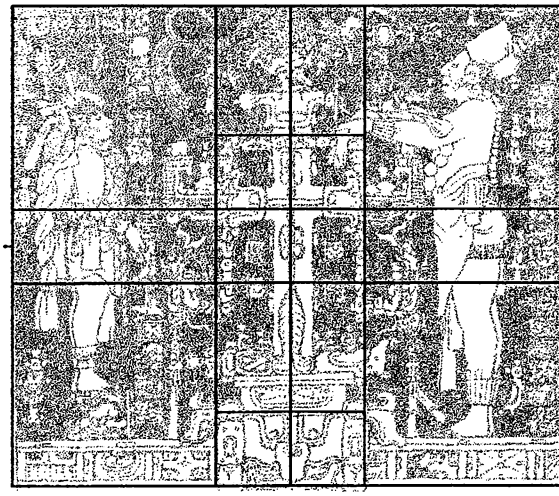








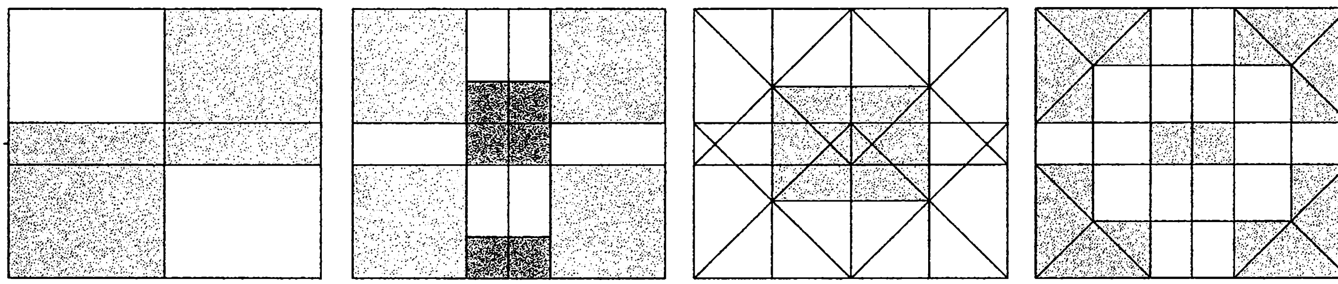
Mayan Stela of the Maize (Foliated) Cross

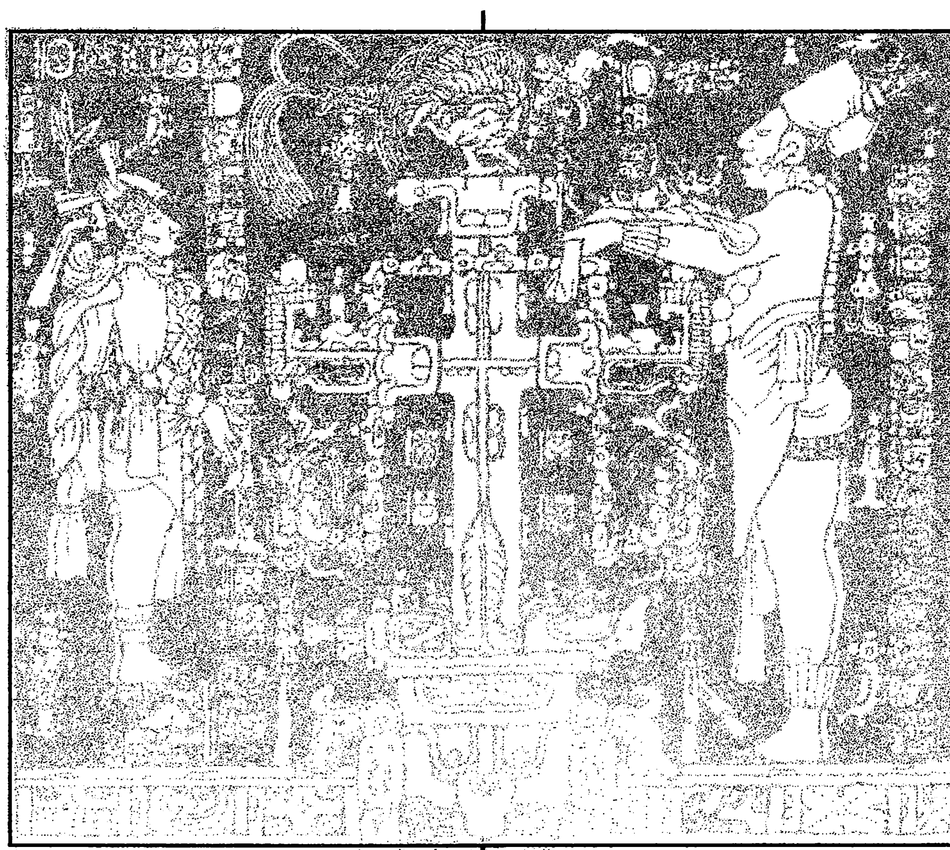


Classic Mayan site at Palenque, Chiapas, Mexico, 672

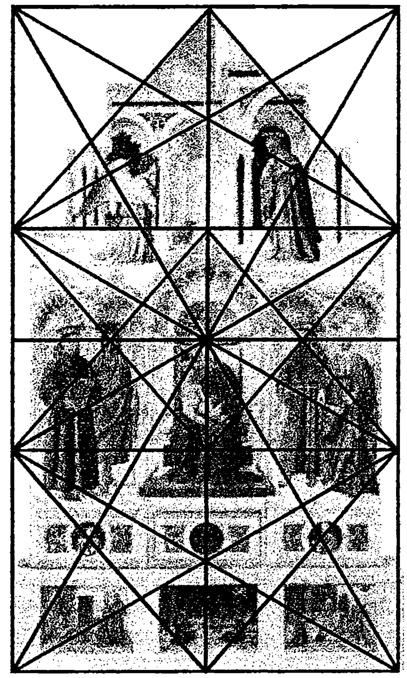
This stela is found in the Temple of the Foliated Cross at Palenque, a Mayan ceremonial center known for its harmonious architecture blending with the earth. The ruler-priests of Palenque made offerings to maize deities in the form of an elaborate maize cross or tree. The maize cross represents the reborn maize god at the place where the first human beings were formed from maize dough.

Its geometry is that of two Root Three Rectangles, signifying its sacred nature. Start by dividing it in half and drawing squares inside the top and bottom of each Root Three Rectangle. Draw squares inside the four corners. The crossings of diagonals in squares indicate the corners of central rectangles. Keep constructing squares and draw diagonals to see more.





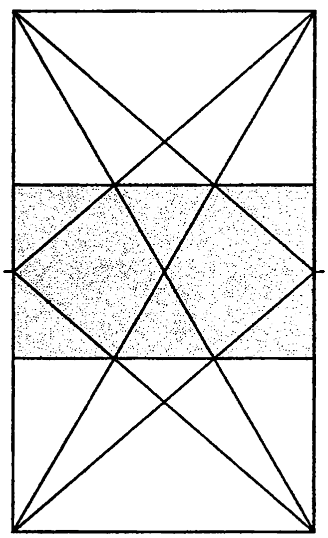
Polyptych Of Saint Anthony

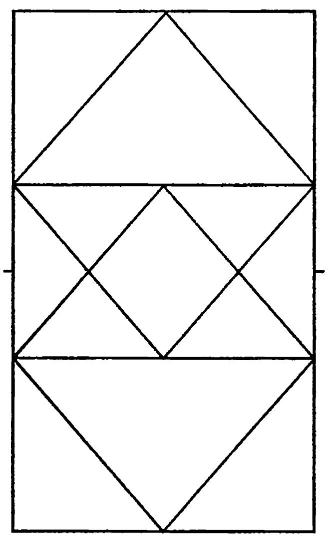


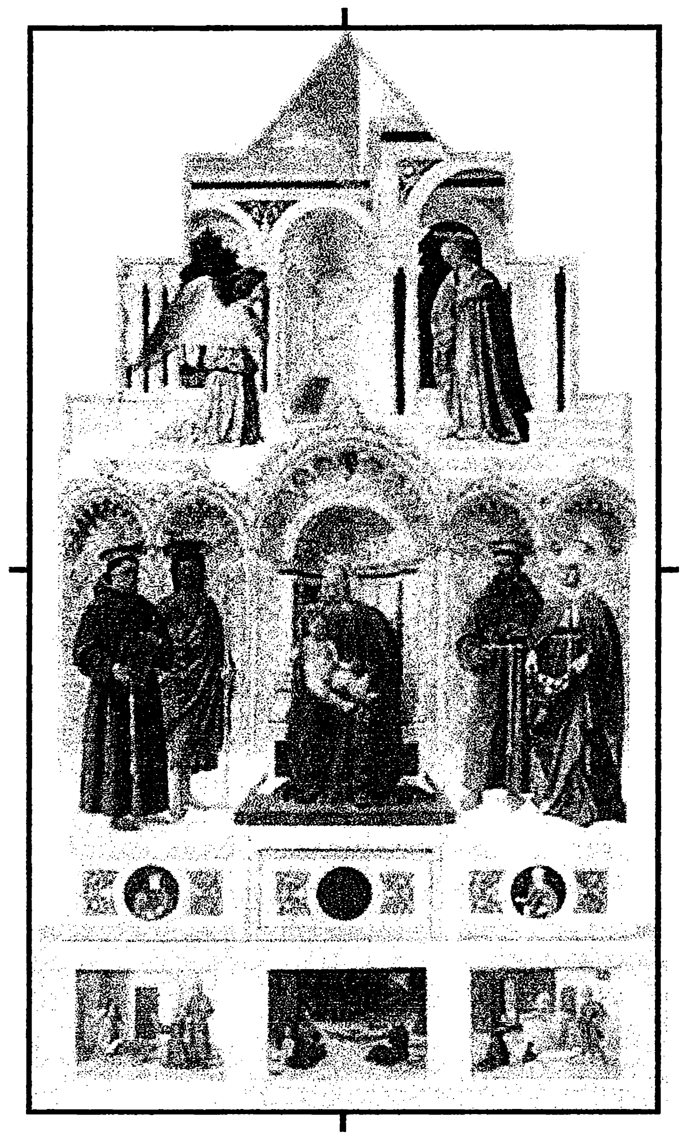
Mixed technique on poplar panel, 1460-70
Galleria Nazionale dell'Umbria, Perugia, Italy

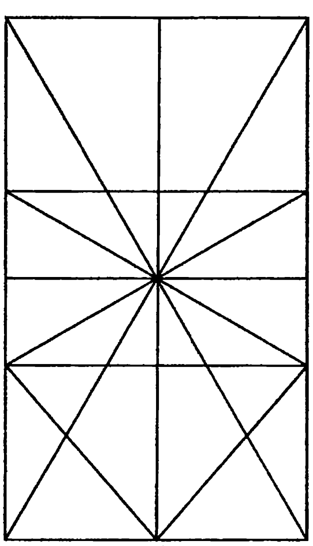
Piero della Francesca (1420-1492) was an Italian artist of the Early Renaissance known also to his contemporaries as a mathematician and geometer since he wrote books on mathematics and art concerning perspective and the five regular (Platonic) solids. This polyptych (also known as Madonna and Child With Saints) was commissioned by the nuns of the convent of Sant'Antonio da Padova. The central part of the composition shows the Madonna and Child with Saints Anthony, John the Baptist, Francis and Elisabeth.

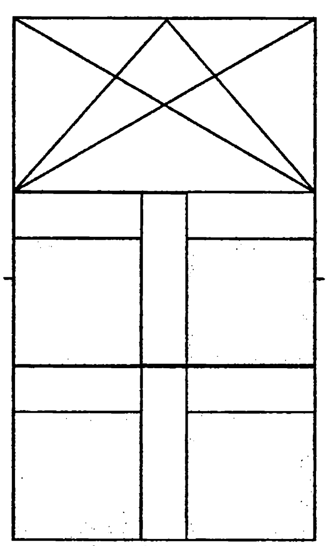
It should be no surprise that the whole polyptych has a structural unity based on the geometry of its overall Root Three Rectangle. To see its invisible framework, first subdivide the whole into three smaller Root Three Rectangles. Squares and diagonals will reveal the rest.









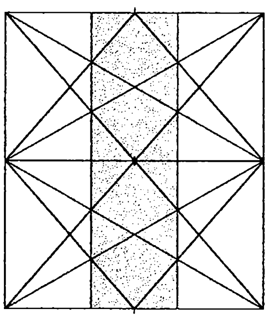


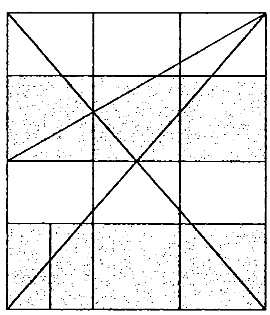
Gunzo's Dream

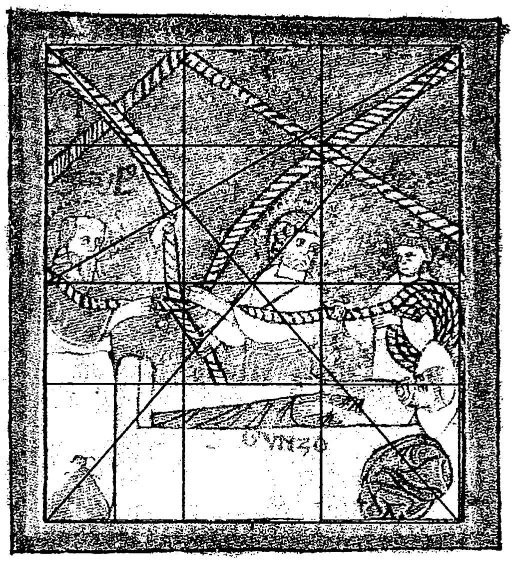
Manuscript detail c1180, Biblioteque National de France, Paris

The Benedictine Cluny Abbey was founded in 909 by St. Berno and dedicated to Saint Peter and Saint Paul. The third abbey church, Cluny III, was built in the Romanesque style on the site in 1088-1130. Until the 16th century, when Saint Peter's of Rome was built, Cluny III was the largest church in the Christian world. One of its innovations was pointed barrel vaults. Its architect was the monk Gunzo, abbot of Baume who was a well-educated scholar of mathematics, architecture and music. The project was executed by Hézelon, another architect and mathematician. They were both familiar with Vitruvius' books on architecture, *De Architectura*, and Boethius' books on music and arithmetic, *De Musica et Arithmetica*. According to Gunzo, St. Peter and St. Paul had come to him in a dream and they showed him how to mark out the plan of the church on the ground with ropes. After the abbey was built its dimensions were found to be harmoniously related to each other by the mathematics of musical ratios and of "perfect numbers" like 6 which equals the sum of its divisors (1+2+3=6). Hence, the ratio of its length (600 feet) to its height (100 feet) is six-to-one. The abbey was built with the same mathematics as the chants sung within it. The abbey was mostly destroyed during the French Revolution.

In this painting detail we see Gunzo dreaming of saints and ropes. Its geometry is two Root Three Rectangles. Draw diagonals to divide each into three smaller Root Three Rectangles. Then draw a square at the bottom of each of these six rectangles. The bottom squares define Gunzo's bed. The left bottom square, divided in half, defines the foot of the bed. A few diagonals reveal other significant points, lines and areas of the painting.





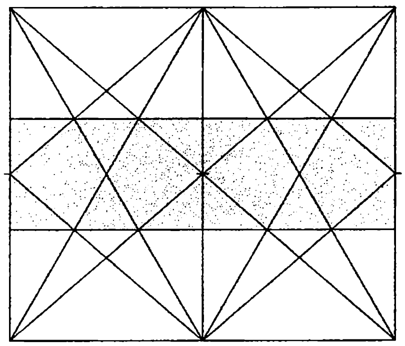


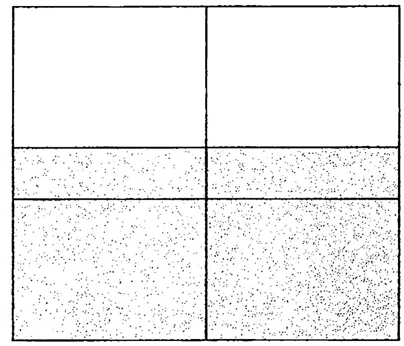
Gunzo Directing The Monks

Here we see Gonzo teaching other monks, presumably discussing mathematics, music and architecture.

Two vertical Root Three Rectangles frame the painting.

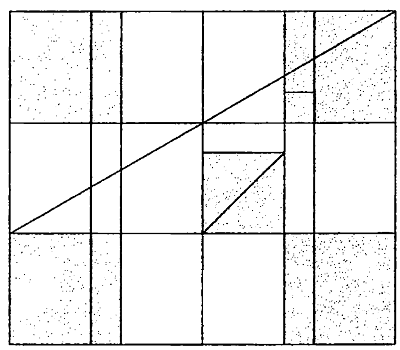
Draw diagonals to divide each into three smaller Root Three Rectangles.

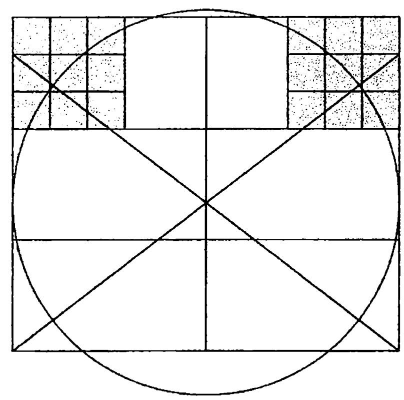




Draw a square at each end of each small rectangle.

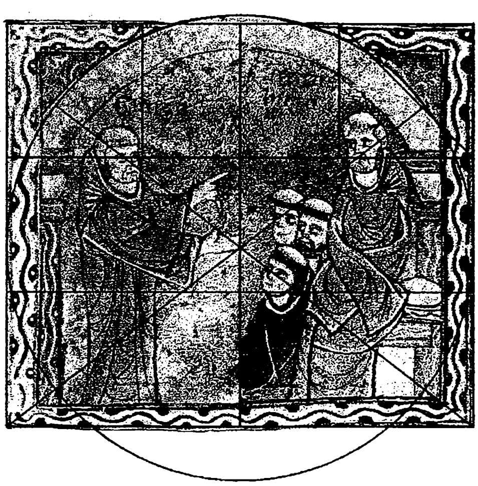
More squares and diagonals will position major elements, including Gunzo's pointing finger.

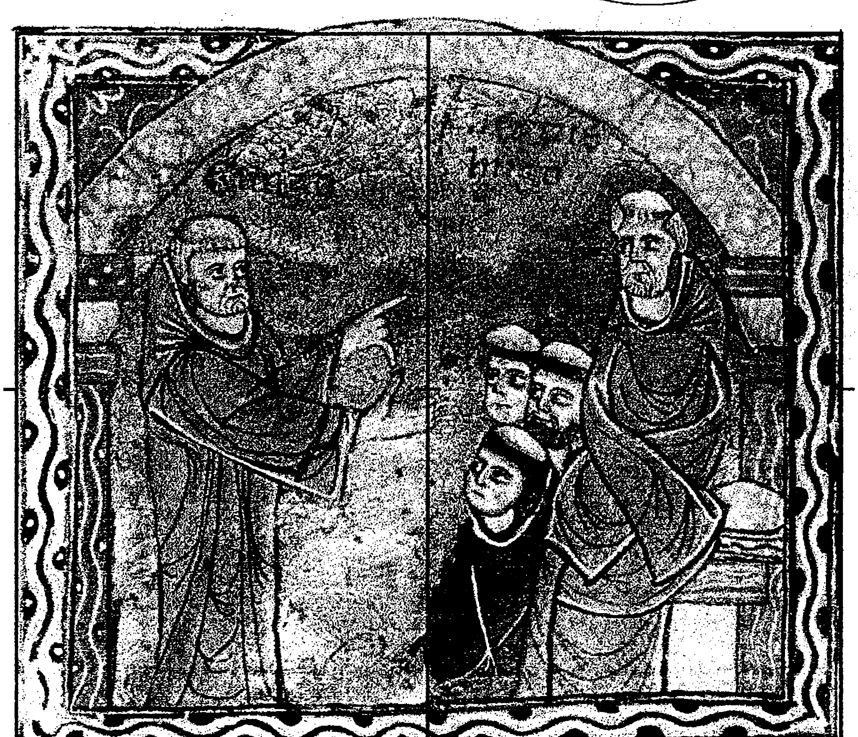




The center of the architectural circle can be found by drawing a small square in the two top corners. Then divide each square into a 3x3 grid. Diagonals from the bottom corners of the painting to the part of the grid as shown will cross at the circle's center.

Place your compass point where the lines cross at the bottom of Gunzo's lower hand. Place the pencil point at the center of the painting's side and turn the outermost circle of the arch.



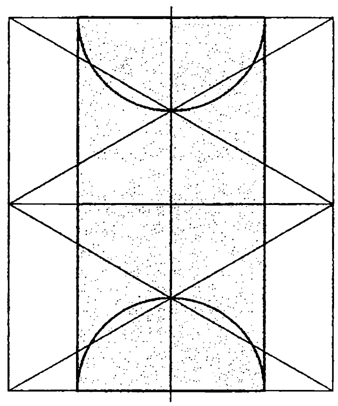


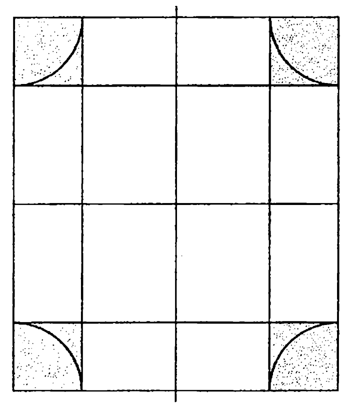
The Throne Of Grace

Jean Le Noir, Jacquemart de Hesdin and four anonymous artists
c1372-1390, from the *Petites Heures de Jean de Berry*
Bibliothéque Nationale de France, Paris

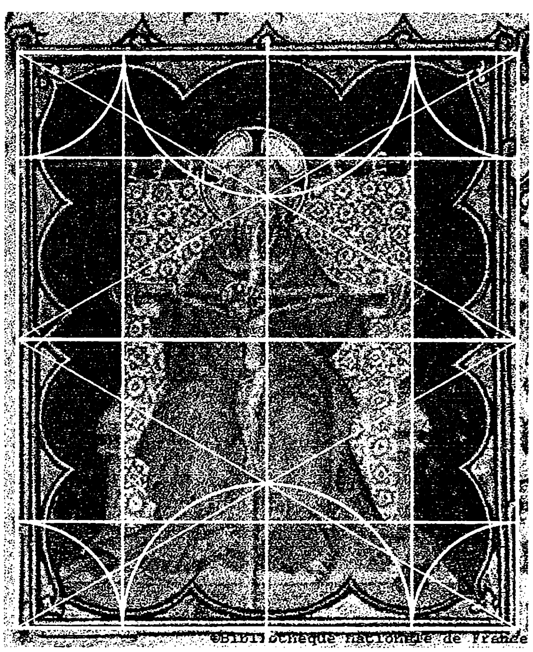
The "Throne Of Grace" is a classic theme appearing often in Medieval painting. This illumination is from a small prayer book and great work of devotional art commissioned by Jean de France, Duke of Berry, a collector and great patron of the arts. It's magnificently illuminated with 119 miniature paintings, profusely embellished with gold and silver and lavish scrollwork of birds and butterflies.

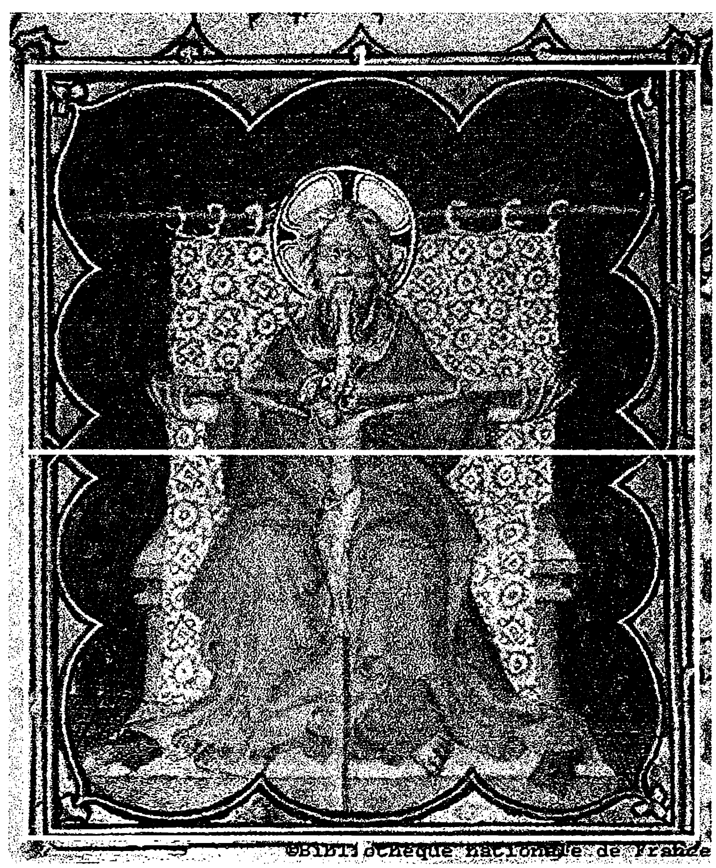
The collaborating artists apparently agreed upon a double Root Three Rectangle. To see its scheme, first draw diagonals in each rectangle. (Notice that the upper diagonals define the cross.) Place your compass point at the top middle and open the pencil to the crossing of the diagonals. Turning an arc will show two points at the top to mark. Repeat this in the lower rectangle and mark two points on the bottom. Connect the points with two vertical lines. (This creates a square at the center of each Root Three Rectangle.)





Finally, place your compass point at each corner and open the pencil to the marks just made at top and bottom. Swing it to the sides and mark these four points. Connect them with horizontal lines to see what they reveal.





Strasbourg Cathedral Altar

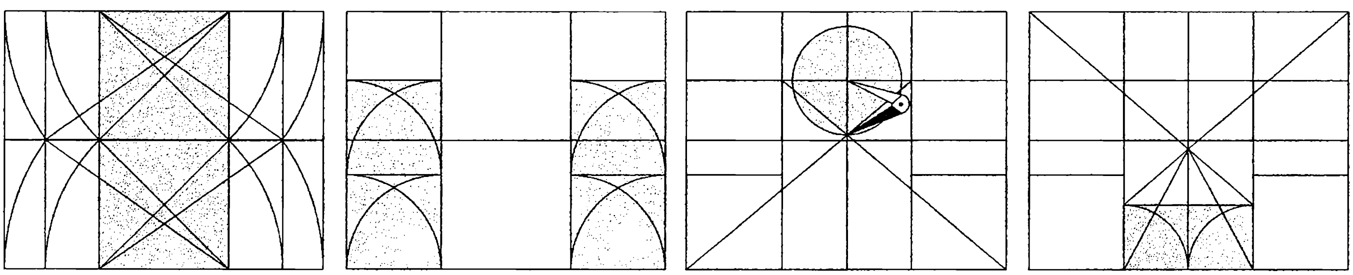
Strasbourg was established in the year 12 BCE, growing out of the Roman military camp Argentoratum and the neighboring fishing and hunting village, Strateburgum ("City of Roads"), a crossroads of Europe from which the city takes its name. Strasbourg Cathedral de Notre-Dame stands on the exact site of a Roman temple built on a little hill. Its first Romanesque version was begun 1015, but was destroyed by fire. It was rebuilt as a Gothic Cathedral, one of the most beautiful in Europe with its remarkable stained glass, murals, sculpture and astronomical clock.

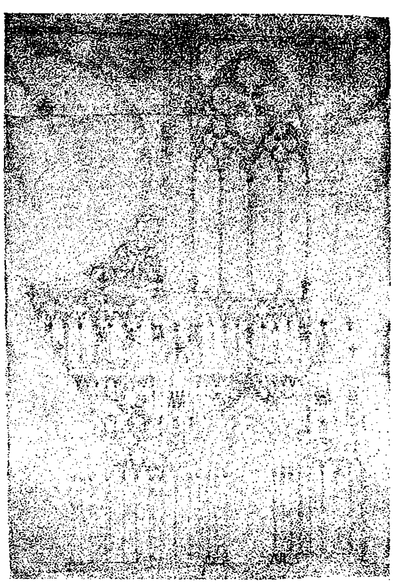
The geometry of its altar is especially interesting, being two pairs of overlapping Root Three Rectangles. To construct them, start with the central two stacked squares given. Use the diagonals of each square to construct a Root Three Rectangle to their left and to their right.

At the two bottom corners swing arcs upward to construct a stack of two squares at each side.

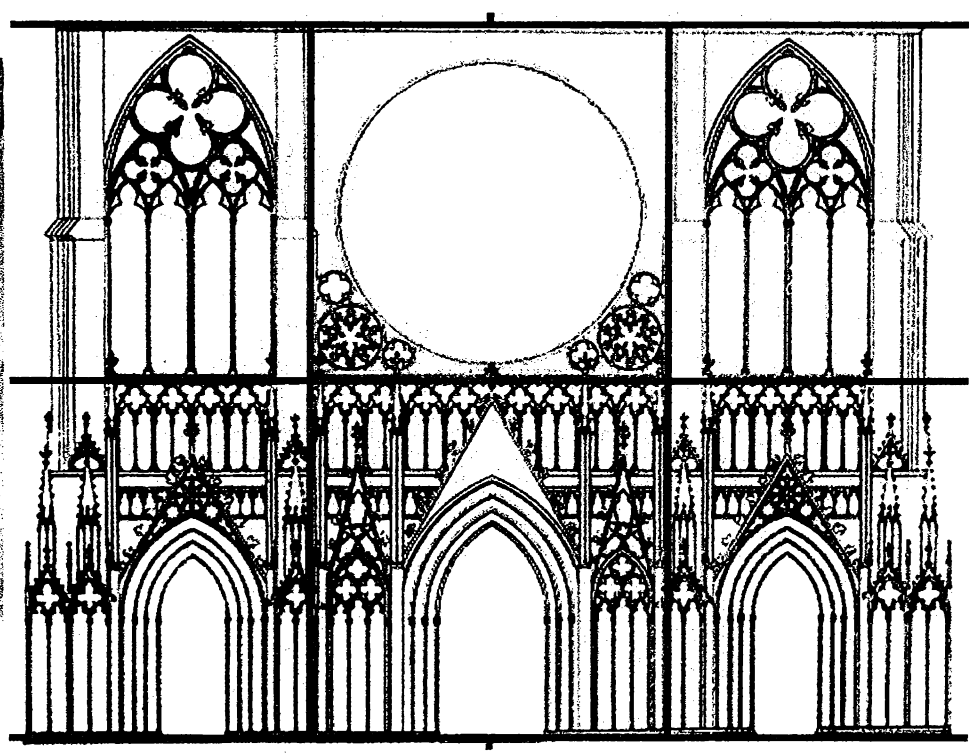
Draw squares and lines as shown to find the center and radius of the altar's central circle.

Draw the vertical center line and swing arcs to construct two squares at the bottom center.
Then draw lines as shown which cross at the apex of the gable over the central "door".
You can discover more on your own by starting with this geometric framework.





Original drawing of
the altar plan

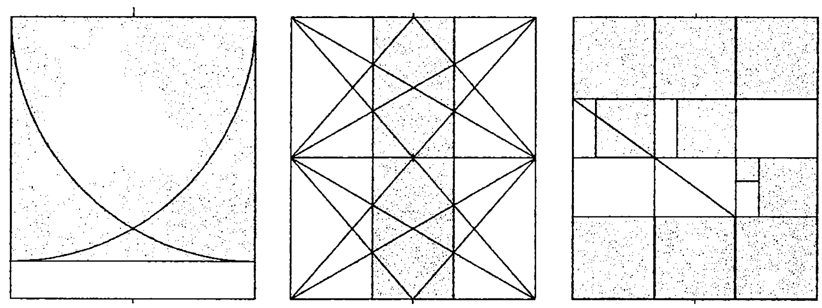


A Young Woman Standing At A Virginal

Johannes Vermeer, c1670
The National Gallery, London

Johannes Vermeer (1632-1675) is truly one of the great Dutch masters, though only about thirty paintings by him are known. Through his play of color in pearly light he's known for creating a crystalline atmosphere in his interior spaces. He's also known for subtle symbols which imbue scenes with moral significance. And part of his effect is due to geometry.

Here we see a woman standing under a portrait of cupid. She pauses from playing a virginal to look at us. A virginal was a keyboard instrument for which music was written extolling the solace that only love can bring. Latin text on the instrument lid reads "Music: companion of joy, balm for sorrow". The painting is a tribute to faithful love.

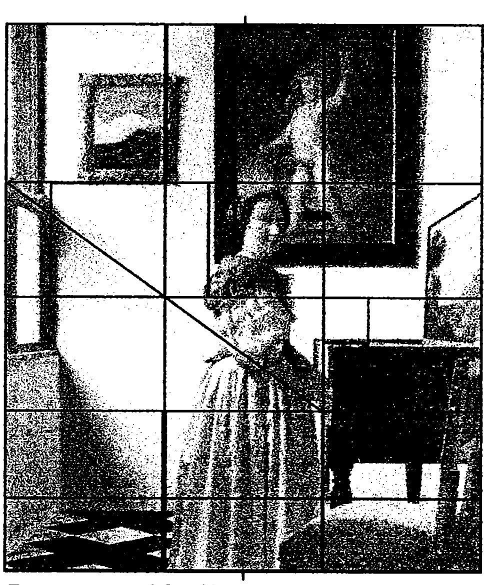


Its simple but carefully constructed geometric scheme begins with two Root Three Rectangles stacked horizontally.

Draw a large square from the top to see the line along the chair's seat.

Divide each Root Three Rectangle into three smaller Root Three Rectangles.

Turning arcs between parallel lines to draw squares in the places shown will reveal some of the framework upon which Vermeer aligned the elements of his painting. Draw lines between points to see more, including the slant of the woman's arm.

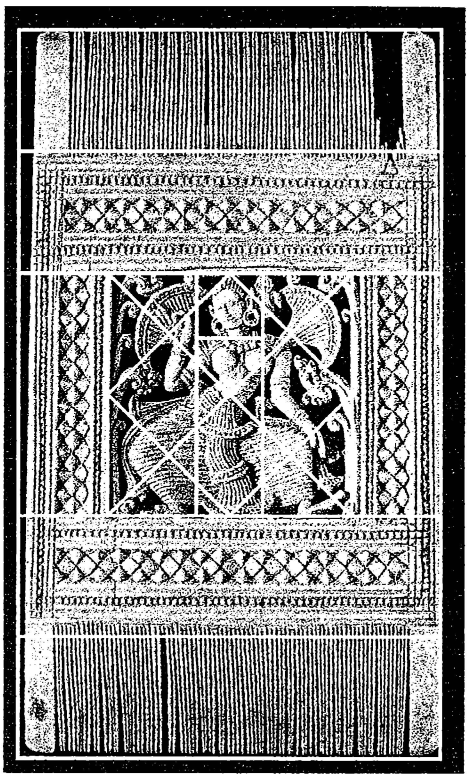


Ivory Comb From Sri Lanka

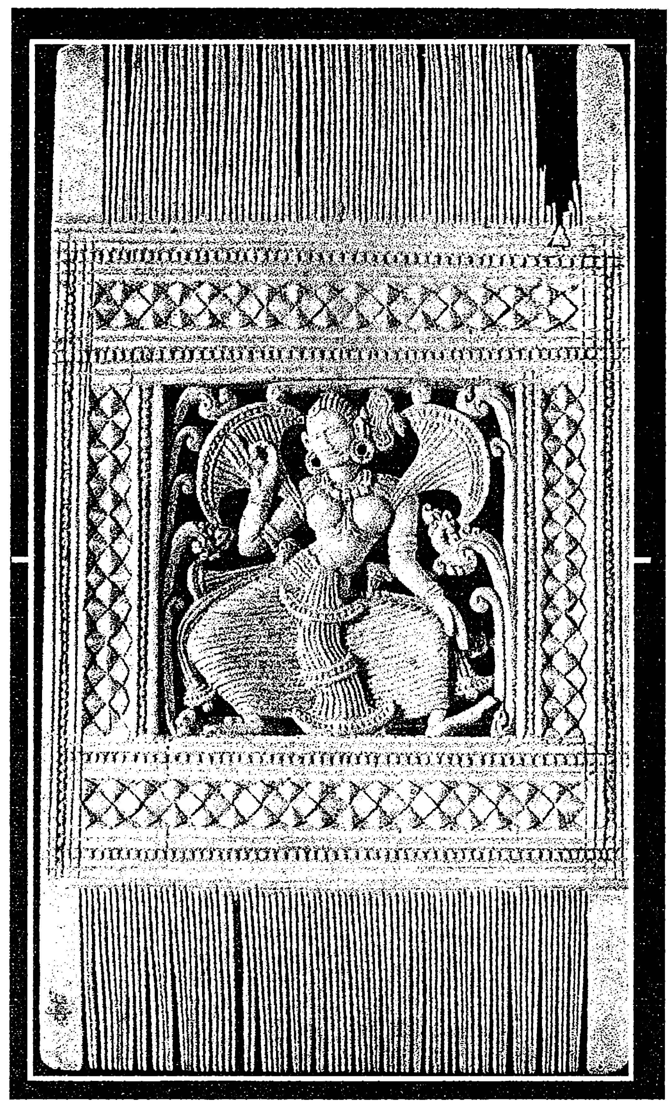
Ivory, Sri Lanka, 18th Century
The Asian Art Museum of San Francisco

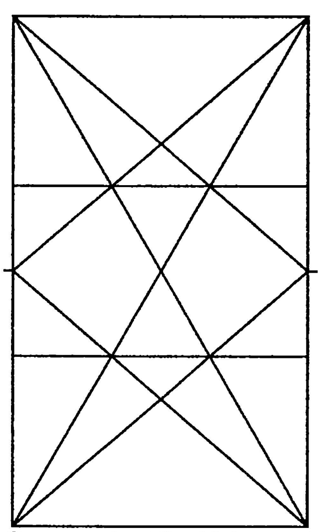
Ivory carvers were highly regarded in Sri Lankan society. This *panava* or comb has teeth at both ends. In the center is a superbly carved dancer who seems to be moving harmoniously while tapping her feet in rhythmic motion.

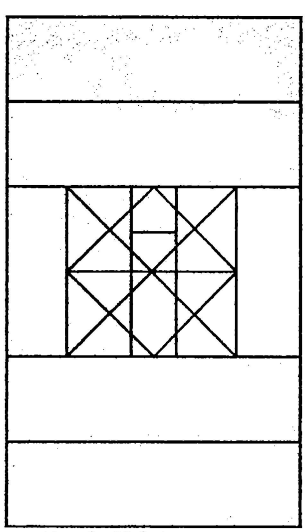
Care has been also given to its geometric scheme. The whole is a Root Three Rectangle. Start by drawing diagonals to divide it into three smaller Root Three Rectangles. In the center rectangle, draw a square from each end. Subdivide the central square containing the dancer as shown (see Volume 1 Ch 4).



Finally, divide the top and bottom Root Three Rectangles in half horizontally to separate the comb's teeth from the frame around the central scene.







Final Word

It's hoped that this brief tour of the dynamic Root Rectangles has informed and inspired you to want to learn more about them and to explore the delights of mathematics and art more deeply. Perhaps it has developed your interest in the subjects of geometry, art and art history, and that you'll want to explore particular pathways further. The topics are rich and lead in many directions, each one of which may take you to something of greater interest. At the very least, may this study increase your love of learning.

These works of art are just a few examples from among very many possible to have chosen from. We could go on and on, but your favorite examples of art are left for you to analyze, now having the basic tools needed to approach them. It's hoped that these activities kindle a greater appreciation of the role of geometry in great art, crafts, architecture across cultures and time. The wisdom of the eternal Truth of mathematics has always been, and is still available, to anyone interested in seeking it. Apparently this knowledge was known, respected and applied to art, especially sacred art worldwide to remind people of harmony, beauty, order, and higher ideals than those we're generally presented with today. These books may help you learn more:

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