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Oliver Sauter

Monetary Policy under Uncertainty

Historical Origins, Theoretical Foundations, and Empirical Evidence

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Stuttgart, Germany

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Stuttgart,
Oktober 2013 *Oliver Sauter*

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Introduction

Background

To describe the conduct of monetary policy a commonly used metaphor is the example of driving an automobile. If the car runs faster than the desired level of speed, the driver has to decelerate by exerting the brakes. If, however, the car runs too slow, the driver must accelerate his car. Applied on monetary policy this implies if the economy falls behind the central bank's desired target level – as measured for instance by the output – the central bank has to stimulate the economy. Hence, she will promote spending and investment by a reduction of the main refinancing rate. If, however, the economy picks up too much speed, the interest rate needs to be raised in order to moderate economic activity.[1]

The driver of a car should – indeed – steer his vehicle in a forward-looking way. This does not only hold because a backward-looking driver is less trustable. To minimize the loss of his car's worth, such as the degeneration of tires and brakes, as well as to minimize severe accidents, which trigger costly and time-consuming repairs, the driver should anticipate upcoming events like the change of the traffic lights ahead or the upcoming road conditions, as this foresight will help him to avoid abrupt maneuvers. Like car drivers, central banks seek to minimize their losses as well. Consequently, it is not surprising that most central banks in the advanced economies act in a forward-looking manner. Hence, to come to a decision about accelerating or decelerating the current speed of the economy possible future realizations and developments are already taken into account today.

Whilst this forward-looking behavior is of special importance for central banks that pursue an inflation targeting regime, central bank policy in general can benefit from this policy. Against this background Keynes (1923) argues that "[i]f we wait until a price movement is actually afoot before applying remedial measures, we may be too late" (Keynes 1923:187). Moreover, to stop a process which has already gained some momentum – such as the acceleration of the car – it may even be costlier to wait and only react when the effect actually materializes. Thus, a forward-looking policy always aims to act preemptively

[1] The analogy of monetary policy and car driving is well-worn, see, e.g. Macklem (2005) or Redrado (2012). However, the 'invention' of this metaphor should be ascribed to Bernanke (2004).

and by that must do its best to anticipate events, which are likely or only contingent to occur in the future (Mishkin 2000, Woodford 2000, 2012a).

The main argument for a forward-looking behavior, however, is the time lag between a monetary policy action and the intended effect on the economy. Unlike the brakes or the steering wheel of a car, the central bank's instruments come with significant lags attached. Thus, on the one hand forward-looking central banking opens the possibility of a preemptive and timely reaction, but on the other hand it is this very behavior which bears the dangers of a wrong assessment, and consequently an inappropriate policy reaction. In fact, it is exactly the time difference between action and consequence, which allows unforeseen events to unfold, and as such can question the previously taken decision.

A further problem arises if expectations of private agents are taken into account. Just like the mere pressure on the gas pedal that does not tell the driver how fast he is the interest rate *per se* does not tell the monetary authority how tight her policy stance actually is. Expectations become a crucial element in the monetary strategy. They can alter the monetary transmission process considerably, and may consequently amplify or dampen any monetary operation. Therefore, a merely forward-looking policy must be accomplished by a policy of forward-guidance, which not only accounts for the future path of expectations, but tries to actively steer them.

Monetary policy may thus be as easy as driving a car, but "the car is one that has an unreliable speedometer, a foggy windshield, and a tendency to respond unpredictably and with a delay to the accelerator or the brake" (Bernanke 2004).

In sum, it is the complexity of the economic system and especially the lack of knowledge about it, which prevents the central bank from foretelling future realizations of key variables and parameters to an acceptable degree, and thus, to adjust the necessary policy action properly. This holds especially for a forward-looking monetary policy, which besides its important benefits, also comprises the major shortcoming that the specific outcome and consequence of a particular policy action is to a large extent unpredictable. It is *uncertainty* about the nature, reliable data, and the functioning of the economy that prevails and which makes a proper policy course such a difficult task (Davidson 1991, Orphanides and Wieland 2013). The significance and prominence of uncertainty in monetary policy has been highlighted by many authors. Exemplary, Svensson and Woodford (2003b:1) comment that

> "[m]onetary policy is inevitably conducted under considerable uncertainty about the state of the economy and the nature of recent disturbances. Analyses of optimal policy that take no account of this are therefore of doubtful practical utility".

Uncertainty takes center stage in a well-designed monetary policy strategy. A deeper understanding is a major concern and serves as the motivation for this work.

Plan of the Book

At the 2003 Jackson Hole Symposium, Alan Greenspan declared that

> "[u]ncertainty is not just an important feature of the monetary policy landscape; it is the defining characteristic of that landscape. As a consequence, the conduct of monetary policy in the United States at its core involves crucial elements of risk management, a process that requires an understanding of the many sources of risk and uncertainty that policymakers face and the quantifying of those risks when possible" (Greenspan 2003).

While this citation foremost highlights the pervasion and comprehensiveness, and thus, importance of uncertainty in monetary policy decisions it also hints towards three important aspects. The first of which is the distinction between risk and uncertainty. The second aspect is the proper examination and incorporation of uncertainty in a monetary policy framework. The third aspect focuses on the quantification of uncertainty. These three aspects of uncertainty reflect the three parts of my work.

In Part I, I consider the distinction between risk and uncertainty. According to Greenspan (2003) it seems like these two terms can be used interchangeably without any further explanation. In consideration of *uncertainty* being virtually omnipresent with the next breath of air Greenspan regards *risk* management as the main task of monetary policy. This apparent equalization, however, is actually not admissible – although not only Greenspan gives reason to think so. Confusing risk and uncertainty is not a mere outcome of a sloppy terminology. Both concepts exhibit fundamentally different properties. Thus, when writing about monetary policy under uncertainty one should first and foremost make a clear distinction between what is meant by uncertainty on the one hand, and risk on the other. Therefore, Part I studies the historical origins of uncertainty in economic analysis. For this purpose I will draw on the two main references in economics within this field, namely Knight (1921) and Keynes (1921).

Part II deals with the understanding, and thus, the adequate policy response of the central bank in an uncertain environment. The sources and forms of uncertainty are numerous and the appropriate assessment and reaction are crucial tasks. Employing inaccurate estimates of parameters and key macro variables, such as the real-time estimates of the interest rate elasticity of investment or the current inflation rate, could cause serious overreactions of the policy instrument, which could provoke considerable harm to the economy. Most notably Brainard (1967) and Poole (1970) have laid the cornerstone analysis when considering uncertainty in a monetary policy framework. Thereby, they have extended the prevailing analysis of decision theory of Simon (1956) or Theil (1957), which had especially focused on decisions under certainty or certainty equivalence. Poole (1970) and Brainard (1967) have demonstrated that uncertainty alters the behavior of the monetary authorities significantly. This might be due to the *origin* of the shock term, that is, whether the shock originates from the money or goods market, or due to the fact that uncertainty concerning the transmission parameters changes the optimal reaction function. To give a better understanding of the sources and forms of uncertainty, and the implied recommendations, Chapter 2 provides a systematic array and overview with the supplement of examples to underpin the argumentation.

To show the effects of uncertainty, Chapter 3 develops firstly the benchmark case in a New Keynesian macro model. Subsequently, uncertainty is implemented with a special focus on model uncertainty. Model uncertainty can be understood as a superior form of uncertainty under which other forms of uncertainty can be subsumed. To handle model uncertainty different solution techniques such as robust control, or processes with Markov-switches can be applied. Especially the latter offers various opportunities to implement uncertainty, such as uncertainty about financial frictions or the existence of heterogeneous agents.

Part III of my work is split into two separate chapters, both dealing with an active assessment of uncertainty. Whilst the acknowledgment of uncertainty and the deduction of a proper response is a crucial task, it nevertheless treats uncertainty almost in an administrative way. However, beside this rather passive treatment, central bank policy must aim for an active evaluation and reduction of uncertainty. Hence, the question changes form how monetary policy should react on uncertainty into how monetary policy can influence uncertainty. Chapter 4 and 5 provide two assessments of this task.

Due to the fact that expectations are central to the monetary transmission process, the evaluation of these expectations is an essential first task to reduce uncertainty in monetary policy, see, e.g., Woodford (2003) or Boivin et al. (2010), which is elaborated in Chapter 4. To acquire this information survey-based measures as well as market-based measures can be applied. With the help of a factor analysis I take these measures and condense them in order to find general patterns and forces that drive uncertainty. The identification of such common forces would entitle one to infer from the development of uncertainty concerning one variable to the development of another variable. This analysis is done for the euro area and the US. First separately, afterwards for both regions combined.

The object of Chapter 5 is the evaluation of the communication of the European Central Bank. According to the famous Tinbergen Principle there should be at least as many instruments as objectives. Thus, if the reduction of uncertainty can be seen as an important sub-target of central banking, besides the traditional instruments of monetary policy, foremost the main refinancing rate, further tools need to be considered. This tool can be the communication of the central bank.

To reduce uncertainty actively, communication policy has developed to a leading instrument in the central bank's toolkit. The importance of communication emerged from the academic discussion, which started in the late sixties with respect to the formation of private expectations in response to monetary policy action. Since then, it has created a research field of its own covering the communication, credibility, and flexibility of the central bank. This subject of interest may be summarized by the term of *transparency*, which can be understood as "the absence of asymmetric information between monetary policy makers and other economic agents" (Geraats 2002:533). The transformation to more openness and transparency, which most of the big central banks have undergone during the last two decades underlines the importance of this issue (see, e.g., Issing 1999, Šmídková 2003, Kohn 2006).

Yet, communication is not only an instrument to justify possible deviations from previously announced targets, which otherwise would be fully ascribed to the inability of the central bank, but also an important tool to steer expectations in a desired direction. Hence, communication can serve *inter alia* as an important instrument to reduce uncertainty on behalf of the private agents, and thus, helps to assure the proper transmission of monetary policy. Utilizing periodical press statements, I analyze the communication of the European Central Bank of the last decade in Chapter 5 of my work. A special focus is on the most recent developments of the financial crisis and the European sovereign crisis. Applying a Markov-switch analysis the past ten years of communication policy are reconsidered, and it is demonstrated how the ECB has significantly changed her wording concerning uncertainty during the years into a more pronounced verbalism.

Finally, a conclusion summarizes my work and gives an outlook on possible future research.

Part I.

Historical Origins of Uncertainty in Economic Theory

1. Concepts of Uncertainty in Economic Literature

1.1. Background

Many authors have emphasized the importance of uncertainty in monetary policy making. This is true from a practical position (see, e.g., Issing 2002, Bernanke 2007, Bini Smaghi 2011), as well as from a theoretical position (see, e.g., Goodhart 1999, Williams 2012, Woodford 2012b). However, what is loosely called *uncertainty* is a matter of discussion and many definitions and interpretations come up in this context. A common confusion is certainly made between *risk* and *uncertainty*. In daily life risk and uncertainty are often used in order to express some favorable or unfavorable situation rather than to distinguish between different circumstances. We often speak of the *risk* of a loss and the *uncertainty* of a gain. In monetary policy the discussion of this discrepancy becomes especially apparent, when comparing comments and speeches of practical engineers with the theoretical body of literature. Most of the time central bank affiliated persons highlight the importance of so-called Knightian uncertainty. Accordingly, modern macro models are often said to account for uncertainty in their analysis. Yet, most of the time these models make explicit use of known probability distributions by applying statistical inference to calculate future states. This qualifies them rather as models of risk than uncertainty. These different interpretations and conceptions make it necessary to take a closer look at what is actually meant when talking about uncertainty.

No matter which school of thought may be considered, concepts of uncertainty are mostly rooted on the respective concepts of probability and knowledge – or rather their absence. Hence, probability theory is a fundamental building block on the analysis of uncertainty (Gerrard 1995). The philosophy of probability distinguishes three main theories.

First of all, the *relative frequency* concept defines probability as a proportion of a specific outcome relative to the number of trials. Hence, probability is used as a synonym for proportion. Probabilities, thus, are only calculable *a posteriori*. Although a crucial prerequisite of the frequency approach is the identical repetition the true probability can only be approximated. Yet, when the number of repetitions is increased the measured

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and true probability might be able to coincide (Ramsey 1926, O'Donnell 1989, Davidson 1991).

Secondly, the *logical* or necessarian theory of probability presumes relations between propositions which support or depend on each other. Opposed to the frequency approach the logical approach allows for *a priori* statements about some probability of relations, hence, predictions can be made even before the actual occurrence. Probability is an attribute of a proposition or hypothesis, and not of things themselves. The logical approach allows for non-numerical probabilities, which would not be conceivable for the frequency approach (O'Donnell 1989, McCann 1994).

The third concept is called *subjective* or personalistic approach. Like the name suggests and opposed to the two previously mentioned concepts probabilities are defined as individual evaluations of propositions. New evidence alters the assessment of a situation and probabilities remain subjective and vary – even on the same proposition and with the same evidence – between individuals. To turn personal sensations into numerical probabilities, and thus, make them comparable across individuals, Ramsey (1926) proposes to find out the odds one is accepting when betting on a specific outcome (Lawson 1988, O'Donnell 1989, Davidson 1991, McCann 1994).

In economic theory the work of Lawson (1985, 1988) delivers one of the most popular distinctions on probability theory (see, e.g., Dow 1995, Gerrard 1995, Perlman and McCann 1996). According to Lawson (1985, 1988) two main traditions in probability theory can be stated. A rather conventional view which is called *aleatory* probability, and a second one called *epistemic* probability notion. This twofold separation is the most broad and fundamental distinction of probability assessment. Following *inter alia* Lawson (1985), the aleatory approach is deduced from the paradigm of chance, and thus, is strongly connected to the frequency view of probability, whereas the epistemic approach rather stands for a description of our knowledge. In an aleatory world, probability is a feature of the objective world. It can be "approximated by the frequency with which the outcome does occur when a large number of 'trials' of the event are observed" (Shafer 1978:312). Despite the fact that all phenomena are governed by the laws of chance, which are given by the fundamental facts about nature, probability is not equivalent to determinism (Shafer 1978).

In contradiction to this concept probability from an epistemic view "describes our knowledge" (Shafer 1978:312) or belief about the material world. This shows the strong connection to the logical and subjective view of probability. In an epistemic view probability is rather a form of knowledge, and not like in the aleatory view an object of knowledge. According to this distinction in an aleatory world probabilities have to be discovered or learnt about. In an epistemic world, however, probabilities are attached to certain events or hypotheses (Lawson 1988).

Two of the most prominent authors in economic literature concerning the distinction of an aleatory or epistemic theory of probability are Frank H. Knight and John M. Keynes, the former being associated with an aleatory, the latter with an epistemic view (Lawson 1988). Both, Knight (1921) and Keynes (1921), published their works *Risk, Uncertainty and Profit*, respectively *A Treaties on Probability*, in 1921. Although they came from very different schools of thought, Knight a Chicago economist, supporter of the free market and advocate of a laissez-faire policy, and Keynes as a supporter of state intervention, both focused their critique on the feasibility of classical theory. In particular, by applying their critique on the way of how uncertainty was – or was not – dealt with in classical theory.

To Keynes uncertainty effects the economic system by altering the function of money. Uncertainty changes the *store of value* function and increases the liquidity preferences. This rise is followed by a change of the interest rate, and thus, the propensity to invest. The inhibited investment behavior combined with a low propensity to spend – both due to uncertainty – causes the economic downturn, and are followed by involuntary unemployment. To Knight, who was primarily concerned with the distinction between *risk* and *uncertainty*, uncertainty leads to imperfect competition, which is the source of benefits for the entrepreneur. The investor who makes the right decision, even in a situation of uncertainty, will be rewarded with an extra profit. Perfect competition is only susceptible to situations of risk (Keynes 1921, Spahn 2007).

Knight and Keynes rooted their ideas of uncertainty on their respective conception of probability. Dealing with uncertainty thus implies to reconsider the underlying reasons of the respective probability theory like the formation of knowledge. Therefore, next, the respective views of Keynes and Knight concerning their concepts of probability, and thus, uncertainty are elaborated. This is mainly done with respect to their major publications from the year 1921, Knight (1921) and Keynes (1921), respectively. In addition, a comparison of both concepts is conducted. In a last step the uncertainty concepts of Davidson, Shackle, and Ellsberg are briefly introduced and an attempt is made to integrate them into the basic concepts of Knight and Keynes.

1.2. Keynes' Epistemic View of Probability and Uncertainty

1.2.1. Obtaining Knowledge and Processing a Rational Belief

Keynes (1921) follows a *logical* theory of probability. This probability theory is embedded in an epistemic view of the world, hence, probability is a *form of knowledge*, a feature of how we think about the world. At the same time, Keynes does not have a subjectivist view which will become apparent in the following discussion (Lawson 1988, Dow 1995).

The starting of Keynes' theory of probability is the formation and processing of a *rational belief*. This rational belief does not necessarily have to be true. It could also be false as long as the processes, which accomplished this belief are based on rational reasons. Hence, the difference between a *rational* belief and mere belief is apart from the difference between true and false, but whether it is attained on reasonable grounds (Keynes 1921).

This rational belief can be obtained and formed in two ways, *direct* or *by argument*. In other words, part of our knowledge about a proposition, say A, we obtain direct and part indirect, which means by argument. The direct part of our knowledge comes from direct acquaintance about propositions of which we have sensations or understandings. Hence, direct acquaintances are our own experiences, understandings, or perceptions. By perceiving their logical relationship, which is due to our direct acquaintance, we can pass on the knowledge of one proposition to the knowledge of another proposition by assuming a logical relationship between them. This newly formed indirect knowledge, which is obtained by relating propositions on each other, is therefore obtained *by argument*; although Keynes admits that this mental process is only to some degree capable. Knowledge, obtained directly or by argument, entitles us to form a rational belief. Figure 1.1 captures the idea of this process (Keynes 1921).

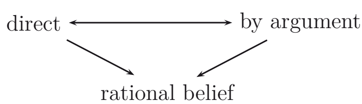

Figure 1.1.: Obtaining a rational belief

A distinction whether one is dealing with a rational belief, which is formulated due to knowledge obtained directly or by argument is somewhat difficult. Consequently, a distinction of what is direct and what is indirect knowledge seems to be unclear and maybe not even possible at all. To Keynes we have direct knowledge, e.g., about our own existence or some logical ideas. In contrast to the law of gravity which is obtained by argument. Another nebulous factor is the role of memory, for which the boundaries which make a sharp distinction are not feasible. Therefore, the status of instinct or habit is not clear and blurs the boundaries (Keynes 1921, Lawson 1985).

The characteristic of direct knowledge, however, is that we do not have a primary proposition, say a, but a secondary, say A, which involves the primary one. This secondary proposition can only be known directly. Thus, knowing something by argument always involves a direct part. From the knowledge of some secondary proposition a probable rational belief in a corresponding proposition can be formulated.

The relation of a *primary* proposition a at a given background or information set h, which leads to a probability α constitutes itself a *secondary* proposition A. Hence, we can say we have a rational belief in a, given the information h, of the degree α.

$$A: \quad a|h = \alpha \tag{1.2.1}$$

Equation (1.2.1) is called *probability relation*, and is crucial to the understanding of probability and uncertainty in the writings of Keynes (1921). The probability relation expresses the degree of a rational belief into an argument. That is, α expresses the degree of reasonability of the argument or hypothesis $a|h$.[1] Rational beliefs are thus capable of degrees (O'Donnell 1989). By processing direct or indirect knowledge we can therefore derive different degrees of rational beliefs *of* or merely *about* a certain proposition. Three different cases can be distinguished

$$\begin{aligned} a|h &= 1, \\ 0 \leq a|h &\leq 1, \\ a|h &= 0. \end{aligned}$$

In the first case, α equals unity, which means we have certainty about A. This leads to knowledge *of* the primary proposition a. The conclusion follows directly from the premises. If only a degree lower than unity is involved we merely have knowledge *about* a. It is only rational to believe a conclusion to the degree of α, and not fully, like it would be the case under certainty. The third case indicates that we have no good reason for the hypothesis $a|h$, but this means we have certainty in the exact opposite primary proposition $-a$, hence, $-a|h = 1$.[2] Most important, in any case we can build at least a

[1] A different writing of Equation (1.2.1), which will be picked up later expresses more clearly the hypothesis character of the secondary proposition as $P_e(\langle \omega, k, \rangle)$, with ω as a specific situation and k a possible outcome (Rohwer and Pötter 2002).

[2] Thus, if the proposition on the weather $a = \textit{rain}$ can not be supported at all, $-a = \textit{no rain}$ must be supported.

hypothesis and it is merely a matter of quantity or degree what will be the reasonability of this hypothesis, but the quality remains the same (Keynes 1921, O'Donnell 1989).

Certainty equals the highest degree of a rational belief. Opposed to only probable propositions a proposition which is certain must always be true. Additionally, if the knowledge is obtained directly, it is called self evident. Due to this relation, Keynes gives a prominent role to a certain rational belief. This is regarded as some outstanding form of knowledge, which is fundamental and serves as a reference from which probability and uncertainty can be deduced. Because certainty or knowledge is the highest degree of a rational belief, it can also be called maximum probability (Keynes 1921).

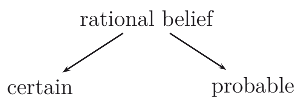

Figure 1.2.: Degrees of a rational belief

In this mindset a rational belief is therefore certain or only probable depending on our knowledge, see, Figure 1.2 which resumes Figure 1.1. If a proposition is only probable and therefore the rational belief is of a lower degree than certainty, we must rely our belief on knowledge of a related secondary proposition.

Because probability and knowledge are always relational the level or degree of a belief in a certain proposition is always defined with respect to a given level of knowledge. This emphasizes the relative character of a rational belief. Keynes uses the example of distance to a certain place to make this point clear. Accordingly, the distance to a place $\mathcal{A}$ is always defined relative to another place $\mathcal{S}$, which in most cases is the place a person actually is. The same holds for a rational belief, which is always relative to the given knowledge or information the person actually holds. In both cases the reference is often skipped for the sake of shortness, however, it should always be kept in mind.

Consequently, new information does not alter a certain proposition, it creates a new one. Moving further away from $\mathcal{S}$ does not discredit the old proposition of a certain distance between $\mathcal{S}$ and $\mathcal{A}$, rather it creates a new distance. If the proposition A is proved wrong due to new information, in fact, A is not wrong but irrelevant. Given the information h_0 it is quite reasonable to hold this rational belief with a degree of α. Given the new information h_1 ($h_0 \in h_1$) the proposition B, with a new degree of rational belief β, becomes reasonable to hold, superseding A. Both propositions, (1.2.2) and (1.2.3), exist in parallel (McCann 1994).

$$A: \quad a|h_0 = \alpha \tag{1.2.2}$$

$$B: \quad b|h_1 = \beta \tag{1.2.3}$$

To Keynes probability is always *objective*. Of course, knowledge can differ between people, hence, the conclusions found by one person can be different from one another. In this sense, knowledge can be called subjective as it is banded to a specific person. However, any two persons who hold the same *corpus of knowledge* must come to the same conclusion, i.e., the same probability assessment. It is mandatory as long as the process of finding any proposition is determined by logical relations, and this is the reason why Keynes characterizes probability as being objective rather than subjective. Given the level of knowledge the deduced probability, or in other words, the degree of a rational belief to hold concerning a proposition must be determined and fixed. Equation (1.2.2) can be written more generally as

$$A: \quad a|h_{ti} = \alpha, \tag{1.2.4}$$

while t denotes the time and i the individual. It is notable that the degree of the rational belief α has no subscript. This indicates that the knowledge may be different between agents and time, however, with the same knowledge at the same time there can only be one admissible conclusion.

Summing up, a rational belief is formed *directly* or *by argument* in a rational, relational, and objective manner. Depending on whether knowledge is *of* or only *about* a proposition, this proposition is known with certainty or only to a probable degree. Additionally, from Equation (1.2.2) and (1.2.3) it is obvious that probability is not something that can be learned. New information allows to build up a new proposition with a new probability that is reasonable to hold. This is equivalent to a different degree of a rational belief. However, new information does not shed light on the old relation which remains untouched. Thus, probability can only represent a level of knowledge, and varying knowledge entitles us to hold different probabilities about some propositions. In this sense, probability can not be a feature of the external world, which is fixed and we discover it as soon as new information is available. Probability is a *form* of a rational belief, which is in fact a way we *think about the world*. This, however, does not include that any proposition is more or less probable because we think so (Keynes 1921, Lawson 1985, 1988).

1.2.2. Uncertainty

Although uncertainty is assumed to be "the most fundamental element of the 'Keynesian revolution' " (Greer 2000:41)[3], especially considering long-term expectations or decisions about cash holding and investment, Lawson (1985) states that Keynes actually never explicitly defines uncertainty, at least not in his 1921 book. Nevertheless different interpretations have been derived out of his theory of probability. Thereby, it is not the case that a less probable situation is equivalent with a situation of uncertainty or whatsoever. The link between *uncertainty* and *probability* is not drawn from a 'more probable, the less uncertain' manner. The key question is, whether it is possible to perform a probability analysis at all, or in other words, to build a secondary proposition like Equation (1.2.1) (Lawson 1985, Greer 2000). Admissible reasons for the impossibility or at least improbability to define a probability relation are listed in an exhaustive manner in Keynes (1921:21ff), which he concludes to be (Keynes 1921:33):

1. There is no probability at all.
2. Different probabilities do not belong to a single set of magnitudes measurable in terms of a common unit.
3. Measures exist but probabilities are and remain unknown.
4. Probabilities exist and are capable of being determined by us, but we are not able to determine them in practice.

According to Lawson (1985) and O'Donnell (1989) there seem to be two broad directions leading to Keynesian uncertainty.

(1) The first would be simply out of an *unknown* probability. Keynes clarifies what he means by an *unknown* probability

> "Do we mean unknown through lack of skill in arguing from given evidence, or unknown through lack of evidence? The first is alone admissible [. . .]" (Keynes 1921:33).

Recalling what has been said about the concept of probability enlightens this position. New evidence that could compensate the lack of evidence in the latter case would not alter the original probability relation $A : a|h_0 = \alpha$, and thus, the original problem of arguing from given evidence. In fact, it would deliver a new probability, based on a different, a new information set and resulting in a new probability relation $B : b|h_1 = \beta$. Hence, the fact that a certain proposition A remains unknown is due to our lack of skill in arguing

[3] Greer (2000) draws on Davidson (1995, 1996).

from given evidence h_0, and not because of the lack of evidence. It is simply impossible to formulate a probability relation (Keynes 1921).

Also, it should be clear that even if one could argue from the given evidence the outcome could be quite questionable. "Of course, as a computational matter, mechanical use of formulas permits one to calculate a value for an arithmetic mean, standard deviation, and so on, of any data set collected over time. The question is what meaning the values calculated in this way should carry in interpreting the past, and in using to forecast the future" (Davidson 1991:131). Moreover "[t]he future is not calculable, even if the decision maker is competent to perform the mathematical operations necessary to calculate probabilities of conditional events given the necessary information. This is uncertainty [...] in the sense of Keynes and the Post Keynesians" (Davidson 1994:89). Davidson (1991) clarifies this issue by arguing that even if there is no lack of skill in arguing from given evidence, i.e., one is theoretically capable of calculating a probability relation, the outcome could be so questionable that it is equivalent to uncertainty.

(2) On the other hand, it is often argued that the Keynesian uncertainty refers to a situation where no information is attainable to formulate a *numerical* probability relation, i.e., a secondary proposition (see, e.g., Lawson 1985, Davidson 1991, Greer 2000). If such a relation is only less than unity not a situation of uncertainty is prevailing, but one of probability, because

> "By "uncertain" knowledge, let me explain, I do not mean merely to distinguish what is known for certain from what is only probable. The game of roulette is not subject, in this sense to uncertainty; nor is the prospect of a Victory bond being drawn. Or, again, the expectation of life is only slightly uncertain. Even the weather is only moderately uncertain. The sense in which I am using the term is that in which the prospect of a European war is uncertain, or the price of copper and the rate of interest twenty years hence, or the obsolescence of a new invention, or the position of private wealth owners in the social system in 1970. About these matters there is no scientific basis on which to form any calculable probability whatever. We simply do not know." (Keynes 1937:213f).

This second source of uncertainty, the impossibility to form a numerical relation, stands in focus of Keynes' further analysis. Thereby, it is not so much the question whether we need a numerical appraisal in order to find a probability. In fact, Keynes explicitly accounts for probabilities which can not be expressed in numerical values, except saying that they must lie between a range of zero and one. Yet, there is no possible comparison which characterizes these situations, and thus, creates uncertainty.

Figure 1.3 plots different probabilities, all between impossibility O and certainty I, whereby measures of a common unit lie on the same path (such as T and V). One could say that V is bigger than T, hence it is more probable. Even if one would say that V is greater then U, a comparison between both is still not possible due to their affiliation of a distinct subset (Keynes 1921). So uncertainty arises out of a numerically indeterminate or non-comparable probability relation. For a comparison it must not hold that it can only be done by probabilities lying on the same path, but only on an *ordinal* scale. V could be more probable, i.e., closer to certainty as T, but not necessarily twice as probable. Hence, a comparison could be possible even without knowing the exact magnitude of probabilities, as long as both assertions lie on the same path (O'Donnell 1989:50ff).

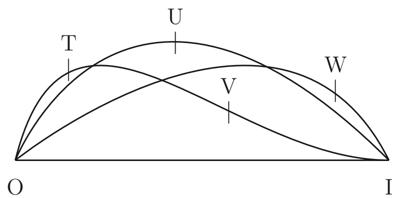

Figure 1.3.: Comparison of probabilities, based on Keynes (1921:42)

If, from an epistemic view, probability represents the knowledge or the way we think about the world, uncertainty must be the *absence* of this knowledge or the impossibility to form any such probability relation. This impossibility of forming a secondary proposition is the key feature of uncertainty in the sense of Keynes. Either, because it is simply unknown or because it is numerically immeasurable, although Keynes favors the second interpretation (Lawson 1988:913).

1.2.3. Weight of Argument

This so far said on probability, it is extended by Keynes with a further element called the *weight of argument*. The weight stands for the completeness of information.[4] The weight rises if new information is attained. Therefore it is irrelevant whether the information fosters or rebuts the proposition as long as its extends the basis of knowledge (Keynes 1921). With the introduction of the weight of argument a second dimension for the comparison of arguments is introduced. Comparing arguments against or in favor of a proposition is now complemented with the total amount of information that is used to formulate a

[4] Runde (1990) argues that Keynes actually defines three definitions of weight. First, as amount of relevant evidence, second as absolute amount of evidence, and third as degree of completeness of information. I account for the second interpretation.

probability relation. Consequently Dow (1995) argues in favor of another dimension of uncertainty. This second dimension, which increases if the weight decreases, would also foster the above mentioned argumentation of Davidson (1994), who questions the reliability of any proposition if the evidence is too remote, and thus, equates it with uncertainty. In the same direction Coddington (1982) argues that the price of cooper in 20 years might be uncertain, but it is very reasonable to assume it will be between a specific price range. Even the relaxation going from a particular price to a range – and may be even further – involves so many hard to predict forces, which puts any forecast on very shaky grounds.

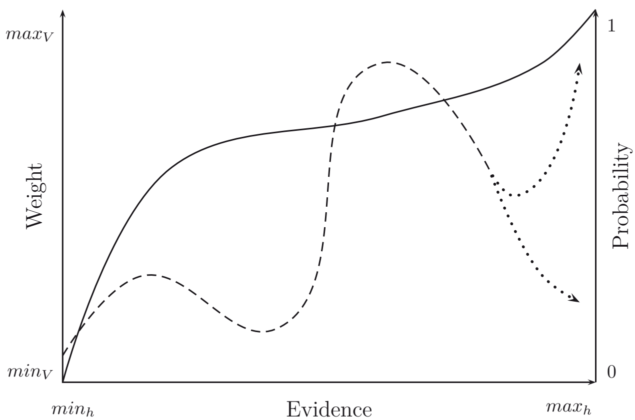

Figure 1.4.: Weight of argument, based on O'Donnell (1989)

Figure 1.4 plots a theoretical path of weight (solid line) and probability (dashed/dotted lines). Weight is measured on the left-hand scale and is strictly monotonically increasing. New relevant information, no matter what their content is, always increases the weight of the argument. Its lowest value corresponds to an *a priori* probability. Probability (dashed line) is plotted with respect to the right-hand scale and can increase as well as decrease, depending whether new information fosters or weakens the proposition. It is not essential that the same degree of belief comes with the same amount of weight. The outcome, i.e., the probability of an argument (say, 80% of all men have dark hair) with only n=10 participants seems to be more questionable than with n=1000 participants, although, the probability of the underlying hypothesis might be higher in the first case, compared to the second. Even though in the first case the probability might be larger, i.e., the reasonability assigned to the proposition

$$a|h_0 > b|h_1, \tag{1.2.5}$$

the amount of information could be quite low so that

$$V(a|h_0) < V(b|h_1). \tag{1.2.6}$$

The discussion concerning the weight of argument is quite controversial (see, e.g., Lawson 1988). Figure 1.4 illustrates a very common interpretation on the relation between weight and probability. But in fact, this stands in contradiction to the previous analysis, whereby new information always creates new propositions rather than enforcing existing propositions. As has been mentioned before, new information whether in favor or against it does not alter the first probability relation, but constitutes a new relation with a new proposition, given the new evidence. The probability line of Figure 1.4 can not refer to the same proposition as any proposition is tied to a specific amount of information.

It is tempting to include the weight of argument into the underlying information set. Yet, both concepts consider different aspects. The only thing that can be said is that there is something which increases as new information is gathered. This factor might be called *weight of argument*. A backdoor to this dilemma could be the interpretation of the horizontal axis as listing several different probability relations of the form $a|h_1 = \alpha_1$, $a|h_2 = \alpha_2, \ldots$, each capturing a different proposition with a different amount of information but with the same primary proposition a. Hence, Figure 1.4 would depict just a series of snap-shots all relating to different amounts of evidence.[5]

So while at first sight the weight of argument seems to be a reasonable and easy extension of the probability concept, it does not really fit into the developed ideas of Keynes (1921). Even Keynes remains uncertain about this feature and does not really consider the importance of his new concept (Keynes 1921, O'Donnell 1989). However, weight might be best understood as an additional dimension in the process of forming beliefs or probability statements.

Following this argumentation, it seems reasonable to follow O'Donnell (1989) and divide uncertainty into distinct forms, taking different aspects into account. While O'Donnell (1989) identifies three different forms of uncertainty concerning a, h, and the probability

[5] In a letter to Hugh Townshend, Keynes connects the risk premium to the probability and the liquidity premium to the weight of argument. Clearly the liquidity premium can rise and fall, this stands in contradiction to a monotonically increasing weight of argument if weight is interpreted as the completeness of information or the total amount of (relevant) evidence (Keynes 1921, Hoogduin 1987, O'Donnell 1989, Runde 1990).

relation itself, I would only distinguish two forms. The first associated with the probability relation as it has been discussed previously. The second form, however, with the absence of sound evidence. This would be in line with Davidson (1994) who questions the quality of the argumentation and would also account for two distinct concepts, one relating to the content of the probability relation itself, and one questioning the formation of this relation. These two dimensions are independent of each other, but nevertheless are eliminating competitors. It could be that the probability out of any proposition is close to one, thus, a probability judgment (in this case even a high one) can be performed, but only very little knowledge is available, which makes it meaningless altogether.

1.3. Knight's Aleatory View

1.3.1. Three Types of Probability

Frank Knight is probably the most cited author in economic literature, especially when it comes to uncertainty, and like Runde (1998) correctly mentions probably more people have quoted than read his 1921 publication. Analogous to Keynes (1921), Knight (1921) first develops a theory of knowledge and probability, and then concludes from this position to the meanings and characteristics of uncertainty. Knight stands for an *aleatory* view of the world in which probabilities are part of the external reality, and thus, can be learned or discovered (Lawson 1988).

To Knight the starting point of his analysis is the future-oriented acting of human beings in an ever changing environment. He is aware of the fact that we do not have omniscience about the future. However, not the fact of change is his concern, but rather the ignorance about it. Change *per se* could be in accordance with common laws which could be foreseen and would thus allow a perfect reaction and adjustment. Hence, we only know to some extent future developments or consequences of change. We do not know nothing, which means entire ignorance, nor do we have complete and prefect information. Yet, due to our conscious, we are able to form expectations about future events, even before they actually happen. “We *perceive* the world before we react to it, and we react not to what we perceive, but always to what we *infer*” (Knight 1921:201). By this future-oriented acting, we have to estimate the given factors, changes, and interdependences, as well as the probability of their occurrence (Knight 1921).

Because “the world is made up of *things*, which, *under the same circumstances* always *behave in the same way*” (Knight 1921:204), some things can be inferred by past data. This shows the connection of Knight's conception to the relative frequency approach mentioned before, albeit it is not the same. However, the citation also reveals a fundamental property of the aleatory probability view and demonstrates the origins of this school. It incorporates the fact that things can have an own probability, and thus, probability is part of the external reality. To predict, for example, the outcome of tossing a coin only the object itself will tell us what probability we can assign to the possibility that heads comes up in the next trial. No history whatsoever would be needed to form this proposition.

The question under which circumstances things behave in the same manner becomes crucial. A key step for this problem is the sorting of situations into groups of nearly homogeneous elements. Of course, this can only be done to a limited extent and it might be quite questionable up to which degree the human mind can handle this fragmentation. Dealing with these fragmented classes can be done in two ways. The first one, called *a priori* is applicable in the case of, e.g., calling the probability of a certain number when throwing a perfect dice. This probability can be easily calculated with mathematical methods based on general principles. Throwing a six on a perfect dice would therefore have the probability of 1/6, which we can assess even before we wage and which would not altered by repetition which is therefore needless. The probability of throwing a dice would be 'really' or 'truly' 1/6 for each number (Knight 1921).

A second type of probability is labeled *empirical* or *statistical*. It is obtained not by *ex ante* calculation, but by applying statistical methods on instances. Therefore sufficient reliable data must be existent to infer on the underlying probability of the object of investigation. Discovering for example the load of a manipulated dice by a constantly repetition of throwing would fall into this category. According to Knight, in most cases *a priori* judgments are not possible, and why this second case is much more relevant in daily business, e.g., in insurance affairs (Knight 1921).

This first and second category of probabilities differ in several ways. One aspect is the accuracy of classification. *A priori* judgments are based on perfect homogeneous groups.[6] Empirical judgments are only based on the behavior of nearly, but not perfect homogeneous groups. Hence, a *statistical* judgment can not and does not represent the real probability, but only the probability of an empirical class. Eventually one could say, whether or not a specific probability lies above or under the group probability. By tabbing the instances further and further to a case of less heterogeneity one would end up with nearly homogeneous groups distinguishable only by indeterminate factors.

In the first case, it is possible to give a probability even *before* the realization, hence, *a priori* due to the perfect homogeneity of instances. If this accuracy is relaxed it is no longer possible to give an *a priori* statement. So the difference between applying

[6] In this case Knight admits that by perfect homogeneous groups, we would not have probability but rather certainty. Yet, also if we have a homogeneous group we would also be confronted with indeterminable factors differing each item to one another, which is beyond our scope.

methods of the first or the second method depends on the accuracy of classification. In this view, probability situations can be ordered in some form of continuum depending on the homogeneity of the situation, which is shown in Figure 1.5. One extrema is the perfect homogeneous group, which is equivalent to *a priori* judgments. On the other side, there is the case of insufficient grouping with no possible judgment.[7]

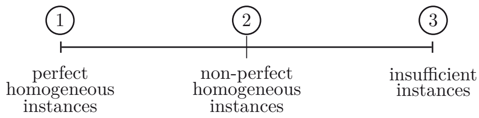

Figure 1.5.: Matter of degree

A priori and *empirical* decisions are, what is often referred to as 'situations of risk' or 'measurable uncertainty' (Schmidt 1996). Although they differ, they have much in common that it justifies a joint treatment. In both kinds of decisions the probability can be measured and the probabilities can be obtained *objectively* due to a known distribution. In general, these instances can be covered by insurance contracts turning risk into a fixed amount of money with no uncertainty left over (Hoogduin 1987).

Following Figure 1.5, beside a perfect homogeneous group which would justify an *a priori* statement and a second group justifying an empirical probability statement, there is also a third category labeled insufficient instances. This last type of probability judgment is called *estimate* and differs significantly from the first two mentioned groups.

1.3.2. Uncertainty

Although, technically, an *estimate* has the same form as an *a priori* or *empirical* judgment, e.g., a fraction of x also being y, an estimate has no scientific basis on which to classify instances such as an *empirical* distribution. Additionally, there is no possibility to formulate an objective judgment based on common mathematical laws and rules as it is the case with *a priori* judgments. This could be, for example, due to its uniqueness which precludes any categorization. Obviously, probability judgments are also possible in this third case. This is done by people every day. Yet, the correctness of these judgments

7 Runde (1998) proposes a reformulation regarding the distinction of *a priori* and *statistical* situations, because there is a second possibility of reading. There should rather be a distinction between an *a priori* and an *a posteriori* judgments rather then a statistical one. A loaded dice would be, with respect to Knight, the case of an *a priori* judgment because of the perfect homogeneity of, say, 100 trials. But if or if not the dice is loaded and on what number can only be said after 100 trials. Hence, according to Runde (1998) not the trial should be the criterion but rather the probability of the outcome.

is purely subjective. Thus, an objective measurement of the probability is excluded. To capture this source of looseness an extended discussion is needed.

Let $P(x)$ denote the probability that the event x will occur and V_i the value person i is attaching to that probability. This opens an extended treatment of probabilities. One could think of a probability of $P(x) = 0.5$, where only the value attached to that probability tells us, whether this is a situation of probability or rather uncertainty. If this value is between 0 and 1, a situation of probability is prevailing. For the special case of $V = 1$ an *a priori* judgment is done. There is full confidence in the probability judgment. Returning to the dice example, the probability of throwing a six is $P(x = 6) = 1/6$, which – in the case of a perfect dice – can be said with full confidence. If there is no confidence, i.e, $V = 0$ a situation of uncertainty is prevailing. There is no chance of giving a reliable probability judgment.

The following example helps to clarify. When assuming an urn with 50 red and 50 black balls, with $P(x)$ denoting the probability of drawing a red ball and $P(y)$ the probability of a black ball. If the exact proportion of red and black balls is given, an *a priori* situation would prevail with a confidence of $V = 1$. If only the total amount of balls is given, but the fraction is unknown, the observer can only make an estimate. In this case, the estimate would have such an unreliable basis that practically no probability judgment could be made. V would equal zero and the situation would be uncertain. If only a quick glance is allowed, the observer could infer from this on the proportions, based on a given level of confidence. A situation of a statistical probability would prevail. Table 1.6 summarizes these findings. Although in each case the probability is 0.5, they differ by the amount of confidence which brings them into different categories (Knight 1921, Schmidt 1996)

Measurement	Value of confidence
$P(x) = 0.5; P(y) = 0.5$	$V = 1$
$P(x) = P(y) = 0.5$	$V = 0$
$P(x) = 0.5; P(y) = 0.5$	$1 > V > 0$

Figure 1.6.: Two stage estimate, based on Schmidt (1996:71ff)

This process of formulating a probability judgment is called a two-stage process. It involves a subjective touch. The first level estimate of probability is purely objective, however, the second stage estimate – attaching the value of confidence – is of a subjective manner. Due to that there are "two separate exercises of judgment, the formation of an estimate and the estimation of its value" (Knight 1921:227). These are connected. If it is not possible to determine a probability relation, then the value of confidence must be zero. For this reason Netter (1996) states that only uncertainty is subjective, whereas probability and risk are purely objective. And in fact, Knight only talks of subjectivity in cases of estimates. He connects objective probability assessments with risk and subjective assessments with uncertainty (Knight 1921:233).

Nevertheless, what is meant by uncertainty in a Knightian sense is a matter of discussion. Schmidt (1996) points to two different accounts of uncertainty related to each other but viewed from a different stance. These two views manifests themselves in the Chapters VII and VIII of Knight (1921). Thereby, Chapter VII deals with uncertainty derived as a continuum from the subjective evaluation. Uncertainty is the inverse of the reliability ascribed to information, hence, all gradations are possibly covered by the extremes of perfect homogeneous and insufficient instances as it is shown in Figure 1.5. Accordingly, Chapter VII deals with the issue of *measurement*. It questions, whether some knowledge is sufficient in order to justify a reliable probability. Thus, uncertainty prevails if a probability judgment can not be classified in a more or less homogeneous manner. Then it is the inverse of reliability which is the result of a subjective evaluation (Schmidt 1996). If a reliable measurement can be done, an *a priori* or empirical judgment is feasible. Because theses situations are so radically different from the third one, Knight labels them *risk* rather then uncertainty.[8] True uncertainty holds if there is no objectively measurable probability attainable. This is the emphasis of Chapter VII of Knight (1921).

Chapter VIII focuses on uncertainty from the viewpoint of contingency. It dismisses the aspect of continuity. If an event is contingent it can not be uncertain. If events or "*such* things" (Knight 1921:247) have happened before, we can say that they are contingent, a class of 'such things' can be founded, and probability judgments can be made. Hence, contingency precludes uncertainty. A contingent event is – in general – insurable, even if it is held to be unique. The German unification for example, could have been seen as contingent, although it was unique. One could at least theoretically have contracted insurance.[9] The main aspect of Chapter VIII is therefore contingency (Schmidt 1996, Davidson 2010).

In both readings, uncertainty belongs to a group which is not insurable because it is not measurable. This can be due to the uniqueness brought forward in Chapter VII, or it is in the reading of Chapter VIII not appointed to be contingent (Schmidt 1996:69).

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8 Although Knight labels the third concept also *probability* due to the "established linguistic usage" (Knight 1921:231), but insisting on the difference against the others.

9 Yet, even these unique, never-happend-before-events are in line with the hitherto established argumentation. It could be that some things are guided by some cosmic law, but just have not occurred until now. Therefore, their first occurrence would label them as unique, although agents have just lacked of cognitive power to foretell them.

1.4. Further Concepts of Uncertainty

Despite these two concepts of probability and uncertainty several other concepts concerning the definition of uncertainty have emerged. Some of them show strong similarities to the so far mentioned concepts of Knight and Keynes, such as the discrimination imputed to Davidson between ergodic and non-ergodic circumstances, or the importance of so-called crucial events by Shackle. In the following, three further concepts of uncertainty are presented.

1.4.1. Davidson's True Uncertainty

Davidson wants to distance himself from the mainstream usage of the term uncertainty. Therefore, he goes one step further in defining what he calls *true uncertainty* in the sense of Keynes. According to him, if there is no information regarding future developments at the moment one comes to a decision, even if frequencies or whatsoever have existed in the past, a situation of true uncertainty is prevailing. The longer the time lapse is between choice and the resulting consequence, the more likely is that an agent is confronted with a situation of true uncertainty. Hence, the aspect of time becomes crucial in his assessment (Davidson 1991).

The key of this interpretation lies in the conception of the external reality. In a what Davidson calls it, immutable or ergodic system agents can form reliable expectations about future developments – at least in the short run – by analyzing past and current data. Agents thus, become merely backward-looking actors. Yet, this objective probability environment as it is supported by classical certainty models or the rational expectation hypothesis lacks of capturing so called *breaks* or *changes*. These types of phenomenons are not deducible from past data. By that he borrows from Knight (1921), when considering the unpredictability of highly unique events where only estimates and no longer *a priori* or empirical statements are feasible.

On the other hand, he also negates the subjectivists view. What might happen is that there is not always an exhaustive list of alternatives, which, even subjective, can not be arranged and ordered at all. If a situation is so unique or not held to be contingent, even subjective probabilities can not be formed. Hence, in a (Post-) Keynesian theory of uncertainty probability calculus is not applicable on an objective as well as on a subjective level. True uncertainty is neither deducible from past data, nor can always be an exhaustive list of possible outcomes be generated or ordered. In any case the transmutable character of the external reality does not provide the agent with a sufficient amount of information at the moment of choice (Davidson 1991, 1994, 1995).

In the words of Davidson (1991) true uncertainty prevails if

> "no expenditure of current resources on analyzing past data or current market signals can provide reliable statistical or intuitive clues regarding future outcomes" (Davidson 1991:130).

Davidson refers to Keynes (1921), who states that uncertainty is characterized by a situation where "there is no scientific basis on which to form any calculable probability whatever" but extends it. His definition of true uncertainty explicitly accounts for the absence of an *objective* probability (statistical) and a *subjective* (intuitive) probability (Davidson 1991).

There might be a chance to combine the concept of Knight (1921) and the Post-Keynesian view. In Knights world uncertainty is not a *form of knowledge*, but an *object of knowledge*. This view implies "that the world is made up of *things*, which, *under the same circumstances*, always *behave in the same way*" (Knight 1921:204). Yet, because the world is made up of so many things, even if we subsume things of the same kind, and so many circumstances exist, no classification would be thinkable which brings us to a point where we can manage those instances. To put it differently, no kind of finite intelligence could manage all possible combinations and outcomes. To deal with the reality we must simplify it. This means we build classes on things which fairly do behave the same way under nearly the same conditions. But by doing so we can never have exact knowledge.

The distinction of Davidson of ergodic and non-ergodic processes seems helpful in interpreting Knight. For any ergodic process analyzing past data helps to predict future developments. This would be the case for *a priori* or statistical judgments in the sense of Knight (1921). However, some processes are non-ergodic and therefore not capable of such methods. In these situations Knight speaks of uncertainty due to its uniqueness. Also Davidson speaks of (true) uncertainty in an environment where there is no information to predict future developments due to breaks and changes.

Drawing this parallel seems to be appropriate because of the emphasis of both authors, whereas ergodic processes seem to be very rare and nearly non-existent in daily business. Uncertainty, however, unfolds for highly unique events which appear to be numerous. Uncertainty, where there is no actual data, no expenditure, and no past data or whatsoever to formulate a probability, Davidson (e.g., 1991) calls *true* uncertainty. This would also hold for considering Knight, because it does not matter whether one can not perform an estimate due to its uniqueness or it is held not to be contingent.[10] Due to these similarities,

[10] As has been discussed early in this paper the distinction between ergodic and non-ergodic is not that clear as it might seems to be. Knights definition of uncertainty would also hold in an ergodic world. In this context uncertainty would just rely to cases where the relevant event has just not yet happened till now. Nevertheless this event could take place in accordance with ergodic, general laws. Taking into account the time aspect short time lapses thus happen in accordance with common

Lawson (1988) interprets Davidson's conception as a reformulation or development of Knight.

1.4.2. Shackel's True Uncertainty

Shackles concept of uncertainty, or what he (as well) calls 'true uncertainty' has also a close connection to the works of Knight (1921) and Keynes (1921). In Shackles conception, two forms of probability judgments exist. The first of which is called *a priori*. The similarity to Knight is captured not only in a linguistic way. These probabilities can be attached to events even before they actually happen. The probability which underlies this judgment is deduced from a frequency ratio, which one has not necessarily experience in person. Yet, to obtain this ratio several preconditions have to be met, such as events which are categorized in this manner need to be sufficiently uniform and numerous. If these conditions are met the process or experiment can be called *divisible*. Tossing a fair coin would deliver a ration of 50% that heads would be tossed next. Although one could not say what the next toss would bring, the experiment can be repeated under fairly the same conditions as often as one wishes and it would thus meet the requirements of a divisible experiment. The outcome – heads or tails – would always be held contingent. This attained frequency ratio can be called *knowledge* and has thus nothing to do with true uncertainty (Shackle 1949, 1952).

Yet, exactly these preconditions are only met in a very small number of cases. Most decisions are so unique, and thus, of a never-to-be-repeated-again manner that they can be called *crucial*. They are labeled crucial, because actions today change the behavior of the future. Like a game of chess, one move changes the whole progress of the game. There is no repetition under the same circumstances. These situations of non-divisible, non-serial experiments can not even be broken down into a divisible experiment, e.g., by tabbing instances. There is no chance of reaching a probability or frequency ratio for this class of experiments (Shackle 1949).

In fact, as has been previously mentioned in both cases one might not be capable of predicting what might be the next outcome of a coin toss or a chess move. The difference, however, is that in the former case a coin toss would be just one element in a divisible series which can be aggregated. In the latter case this aggregation is not admissible, due to the fact that the environment has changed decisively. This leads to the well know description of a *kaleidic* world, which is constantly changing and individuals are not capable of covering all possible states or outcomes. In these situations it is simply impossible to

laws, yet, the longer the time horizon, the more likely a non-ergodic situation is prevailing, see, e.g., Davidson (2010).

assign probabilities and it is this inability, whether objective or subjective, which Shackle entitles to call these situations *true uncertainty* (Rosser 2001, Basili and Zappia 2010).

The parallels to Knight (1921) seem to be obvious. Like Knight, Shackle calls situations where one can perform a probability analysis one of knowledge or risk rather than uncertainty. These situations are in principle insurable, which reduces risk to a fixed amount of money, and thus, uncertainty vanishes. A situation of true uncertainty is, like it is to Knight, one when there is no chance of reducing this uncertainty, for example, by tabbing into smaller categories. This could be due the uniqueness of a situation, whether it has never happen before or it is not repeatable in the future (Basili and Zappia 2010).

1.4.3. Ellsberg's Ambiguity

Even in situations of uncertainty – defined as a situation where no numerical probabilities can be attached to certain events – at least some people do assign probabilities. They assign *subjective* probabilities and by that turn former situations of uncertainty into situations of risk.[11]

The transformation from uncertainty to probability, and thus, from uncertainty to risk, is conducted by simply questioning people about their personal feelings or by presenting them bets on different outcomes on which they have to decide. If the respondents behave according to the Savage axioms, this simple game should reveal their preferences and personal probabilities on different situations. So in this case, every uncertainty in a Knightian sense can be turned into risk simply by offering bets and inferring from the given answers. Yet, and this is the main shortcoming of this theory, only if the Savage axioms are fulfilled[12] (Ellsberg 1961).

If, however, these axioms are violated it is impossible to infer probabilities from the given choices, because they would deliver not-coherent answers. According to Ellsberg (1961), this would be a situation where not certainty or risk, but also not uncertainty or complete ignorance prevail. In these cases people do not follow classical assumptions such as a minimax approach. This is a situation somewhat between risk, where probabilities and the future states of the world are known, and uncertainty, where no numerical probability can be assigned and possible future states are said to be unknown. This new situation evolves due to the ambiguity concerning the information in terms of amount, type or reliability and is thus called *ambiguity*. Although these situations where information is not reliable could be identified as objective, i.e., one would find situations where, objectively,

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[11] Although it is very questionable if a personal probability judgement could convince an insurance company to conclude a contract to cover this – now – risk.

[12] The most notably axioms are P(1) the complete ordering and P(2) the sure-thing-principle.

information is of bad quality, the final assessment is clearly subjective as it covers, e.g., personal experiences.

The difference between ambiguity and uncertainty lies in the knowledge of the possible future states. Under ambiguity these possible states are at least theoretically known, but there is no way of identifying some probability judgments. Under uncertainty, however, these future states remain unknown, hence, there is no way of formulating a probability relation on an unknown state. The future can not be anticipated fully, while it is yet to be created – needless to say. In other words, while under uncertainty the list of possible states is not exhaustive, under ambiguity it is. In both cases the probability remains unknown. A shortcoming of the concept of ambiguity is, however, the exclusion of a changing environment or the occurrence of structural breaks (Dequech 2000, Basili and Zappia 2005). Furthermore, under ambiguity it is worth to mention that the probability could be known, but the information is hidden rather then absent at the moment of decision. Under uncertainty this information is not existent. Again this is due to the fact that information regarding the future can not be known because the future is created at the moment of decision. So events not imaginable at the moment can not be assigned with probabilities – not even in principle. The information is not attainable even *ex post*, but it could be under ambiguity (Dequech 2000, Basili and Zappia 2005).

There has been a discussion about the change in Keynes writings concerning uncertainty, which can also be viewed from an ambiguity vs. uncertainty perspective. Especially one could distinguish, as has been done previously in this chapter, between uncertainty on different levels, namely uncertainty about the feasibility of creating numerical probabilities and uncertainty about the weight of argument. Within this distinction risk would be a situation where weight equals 100% and a probability is less than one. Certainty would claim for a weight and probability of 100% each, and under uncertainty a probability relation can not be made at all. Ambiguity would be a situation where no probability can be made due to the lack of knowledge, due to a small amount of weight (Dequech 2000). This view can also be found in Ellsberg (1961) where he confirms that under ambiguity it is not the actual proposition that is in question, but the relative support that a proposition can be used, that is their weight or their confidence. If "the confidence in a particular assignment of probabilities [...] is very low. We may define this as a situation of high ambiguity" (Ellsberg 1961:660).

Additionally, if the different reading of Chapter VII and VIII of Knight (1921) is acknowledged, whereby Chapter VII deals with the problem of measurement and Chapter VIII with the problem of contingency, one could read Chapter VII as well as situations under ambiguity and only Chapter VIII would be left over for situations of uncertainty. This depends on the question whether the unknown probabilities of Chapter VII are really absent or rather hidden. Only if they are merely hidden, Chapter VII could be interpreted as ambiguity, otherwise uncertainty prevails.

1.5. Taxonomy and Summary

The previous sections have especially highlighted the positions of Knight and Keynes and their understanding of probability. It should have become apparent that to a great extent the differences between Knight's and Keynes's conception of probability, and hence, to some extent as well on uncertainty, originate from their respective view of the world. Additionally, the theories of Davidson, Shakle, and Ellsberg have been elaborated.

Until today, there have been attempts on a taxonomy of probability and uncertainty in the writings of Knight and Keynes. On the other hand, there have been suggestions whereas both concepts should not be expressed in terms of the other or compared with each other due to their fundamental differences like the distinction of Lawson (1985, 1988) of an aleatory or epistemic theory.[13]

In what follows, I will pick up some criteria which have already been used to give a discrimination on the perception of probability, and thus, uncertainty of both authors. The main discrimination line is given by Lawson (1988), yet, other concepts are implemented.[14]

Table 1.1, which is taken from Lawson (1988), systematizes the discrimination between epistemic and aleatory views, depicted on the left-hand and right-hand side, respectively. In this distinction Knight is associated with an aleatory view of the world. Under this conception probability is an *object of knowledge* and part of the external reality. The world, or rather events taking place in this world are ascribed to the reality and can be understood as being provided with a *true* or *real* distribution. Those probabilities need to be discovered or learned. If, however, frequencies are revealed one is entitled to name a probability, thus Knight labels these situations risk. Due to the underlining assumption of a random generator which produces outcomes, probability statements are somehow statements about this generator, hence, probability can be defined as $Pr[G](k)$. A probability statement is thus a statement about the probability of realizing a certain outcome

[13] The dichotomy of aleatory and epistemic is strongly connected to the work of Lawson (1985, 1988), especially when applied on Knight and Keynes. Yet, it is not a creation of Lawson, but has a long tradition in probability theory. See, e.g., Carnap (1950) who discriminates between Probability1 and Probability2, which is nearly equivalent to the discrimination of epistemic and aleatory (McCann 1994). Or Shafer (1978), who shows how the works of Bernoulli and Lambert can be connected to this relatively 'new' discrimination approach.

[14] For further discriminations, one is, e.g., relegated to Dequech (2008), who discriminates between weak uncertainty (Knightian risk) and strong uncertainty (uncertainty in the sense of Knight and Keynes), or Fontana and Gerrard (2004) who make a distinction between a stable/unstable external environment (aleatory dimension) and the understanding of the causal structure (epistemic dimension).

	Probability is a property of knowledge or belief	Probability is also an object of knowledge as a property of the external material reality
Uncertainty corresponds to a situation of numerical measurable probability	Subjectivists	Rational expectation
Uncertainty corresponds to a situation of numerically immeasurable probability	Keynes	Knight

Table 1.1.: Discrimination of probability according to Lawson (1988)

or situation k, given the underlying generator G (Rohwer and Pötter 2002). If it is not possible to formulate a probability statement, for example due to the uniqueness of situations, uncertainty in the sense of Knight is prevailing. Hence, under these circumstances the underlying random generator is not known to the decision maker and there is consequently no way of defining an objective probability, although subjective probabilities can be formed. Thus, the event is not insurable and the uncertainty can not be eliminated, which in turn is the source of profit. This aleatory view underlies as well the rational expectation school where subjective probabilities of agents coincide with the objective, i.e., true probabilities. Yet, opposed to Knight, uncertainty is still numerical measurable (Hoogduin 1987, Lawson 1988, Greer 2000).

On the left-hand side, Keynes is classified who adheres to the conviction that probabilities are a *form of knowledge*. If probabilities can be given, these are derived in an objective and logical manner. Given the evidence, no matter the person who holds this belief, the same probabilities must be derived. However, if no such probability statement of the form $P_e(\langle\omega, k\rangle)$, with ω describing the situation and k the possible outcome, can be given, uncertainty prevails. Hence, uncertainty can never be captured by numerical probabilities. The subjectivists build probabilities no matter the external environment, therefore even uncertainty can be attached with numerical relations. Moreover, Table 1.1 shows the communalities of Knight and Keynes. To both, probabilities are derived in an *objective* process. This discriminates them from any subjective stance. Furthermore, any (meaningful) numerical assessment, precludes uncertainty (Lawson 1988).

The differentiation of Davidson (see, e.g., 1991, 1996 or more recently 2010) is quite close to that of Lawson (1988). Knight is connected with an ergodic world, while Keynes is connected to a non-ergodic, mutable environment. This view is in line with the aleatory–epistemic distinction of Lawson. Due to the same behavior of things over time, past data is a reliable source of forecasting the future in an immutable world. It is theoretically possible to predict at least to some account the future by revealing, discovering or learning probabilities (Davidson 1991, 1994). Also situations which can be described as "structural breaks" or "changes in regime" (Davidson 1991:113) can be implemented. Davidson (2010) admits that uniqueness of events does not stand in contrast to an ergodic or aleatory world. Although some events may be governed by the law of some cosmos the knowledge about these regularities can be absent at the moment of choice. Those events seem to be unique as they have not appeared yet, maybe due to a short period of time (Davidson 2010).

The distinction of Davidson is as well in accordance with the distinction of Shackle, who discriminates between divisible and crucial situations. The first one, described by Shackle as sufficiently uniform and numerous situations, can only be applied to an ergodic world, which in turn fits nicely on the aleatory distinction of Lawson. On the other hand, crucial events appear as non-divisible, which matches the circumstances of a non-ergodic or kaleidic world.

Assuming that probabilities of events do not change in a Knightian world, i.e., $Pr[G](k)$ does not change, and one can rely on relative frequencies in determining probabilities of this object, this deductive process, however, is only knowledge conserving but never knowledge creating. Therefore Keynes rejects this view of repetitions and replaces it with an expression of conviction. It is only the probability of an argument that a frequency or occurrence is probable, which can be made (Hoogduin 1987). Probabilities are statements about assertions about reality, not about reality itself. One can only give a statement about the probability of a hypothesis $P_e(\langle \omega, k \rangle)$. Thus in Keynes' world probability is a property or *form of knowledge*, which agents posses or attach to certain propositions or hypotheses. The formation of a belief and later a proposition exists only on the level of knowledge. This knowledge entitles us to hold a specific belief, which is supported by the evidence. The connection between both concepts of probability can be express by

$$P_e(\langle \omega, k \rangle) := Pr[G](k). \tag{1.5.1}$$

The left-hand side of Equation (1.5.1) depicts the epistemic view, the right-hand side depicts the aleatory view. To combine both views it is assumed that a hypotheses can be made about each situation a random generator brings out (Rohwer and Pötter 2002).[15]

In both cases, uncertainty is the absence of numerical measurable probability. Uncertainty precludes probability (Lawson 1988, Greer 2000). Uncertainty is a situation charac-

[15] To give a better understanding, here a following example. A random generator produces the situation of bad weather with the specific characteristic of rain. This situation, however, can be used to describe circumstances, i.e., hypotheses containing the situation and the specific characteristic. About these circumstances an epistemic probability judgment can be made which expresses the reasonableness of these circumstances.

terized by the absence of probabilistic knowledge. Due to the non-ergodic, ever changing character of the world, that is, for example, the absence or impossibility of identifying a random generator, future events are unpredictable. If probability is defined as a property of knowledge, uncertainty must be the total absence of this knowledge because knowing at least little would preclude uncertainty. In contrast to Knight, where agents lack the knowledge they need to formulate an estimate, to Keynes such estimates simply do not exist. He is not concerned about the probability of events, but the probability of arguments and if these arguments, which are logical relationships, can not be expressed uncertainty prevails (Shafer 1978, Hoogduin 1987, Lawson 1988, Perlman and McCann 1996, Greer 2000).

A vivid way of defining uncertainty has been made by the US politician Donald Rumsfeld who declares in 2002

> “There are known knowns, there are things we know we know. We also know there are known unknowns, that is to say we know there are some things we do not know. But there are also unknown unknowns, the ones we don’t know we don’t know.”

The same way of thinking is supported by an approach called *KuU* (Diebold et al. 2010). This acronym stands for Known, unknown, Unknowable. The *known* represents a situation of risk in the terminology of Knight. Probabilities as well as outcomes, also called events, are known to the decision maker. If there is no way of determining a probability distribution or they can not be assigned to events, the situation is labeled as *unknown*. This would equal a situation of ambiguity. Finally, if neither probabilities can be assigned, nor any event identified the situation is labeled as *Unknowable*, which would be equivalent to uncertainty in a Knightian sense. If the event actually occurs, it changes into the domain of *unknown* (Weisman 1984, Diebold et al. 2010). In the reading of Knight, *unknown* would be the essence of Chapter VII, whereas Chapter VIII treats *Unknowable*. Diebold et al. (2010), however, only see *unknowns* as being in accordance with Knight’s concept of uncertainty. In my opinion this is insufficient. Especially as unknowns still not account for the aspect of contingency. Furthermore, due to the fact that Knight always emphasized the importance of unique events, *Unknownable*, indeed, should be in accordance with Knight’s concept of uncertainty.

To sum up, this chapter has elaborated different accounts on the discrimination of probability, risk, and uncertainty. While demonstrating that various types and definitions of what is loosely called ‘uncertainty’ exist, especially the notions of Keynes, Knight, Davidson, Shackle, and Ellsberg have been taken into account. Moreover I have shown that, despite several linguistic differences, all authors share a very similar tenor at least on the definition of uncertainty.

Situations where any kind of probability relation can be formed, and (all) the future events can be entitled, are situations of risk in the sense of Knight (1921). Under these situations *a priori* as well as *empirical* statements can be given. These situations are encompassed by tabbing instances further and further into more or less homogeneous groups. However, these situations can only be found in an *ergodic* environment where time and space average coincide (Davidson 1991, 1994). These events are said to be *divisible* as they are uniform and numerous enough to be repeated for a meaningful statement (Shackle 1949).

If, however, only the possible future states are known, and the probability relation related to any specific state is only theoretically to be known, a situation of *ambiguity* is prevailing rather than risk.

In most decision problems neither of these situations can be found. Hence, probability judgments can not be given at all. Due to several reasons, for such situations only an *estimate* can be given. As a matter of fact, the world is ever-changing. Space and time average do not coincide due to the *non-ergodic* character of most processes (e.g., Davidson 1991). Every action today changes immediately and inevitable the whole set up of possible future states like a picture in a kaleidoscope. These events that have not happened before, but are of such high importance are said to be *crucial*. For such events no probability relation can be given at all and uncertainty prevails. This is either due to the impossibility of measurement or due to the fact that a situation is not held to be contingent.

Part II.

Theoretical Foundations of Monetary Policy under Uncertainty

2. Effects of Uncertainty on the Policy Design

2.1. From Uncertainty to Uncertainty

The previous chapter has elaborated different accounts on the discrimination of risk and probability on the one hand and uncertainty on the other. It has been shown that various types and definitions of what is loosely called 'uncertainty' exist. Especially the notions of Keynes, Knight, Davidson, Shackle, and Ellsberg have been taken into account. Despite several linguistic differences all authors share a very similar tenor, at least regarding their definition of uncertainty. The major part of literature defines uncertainty with respect to the notion and definition of Knight (1921). Accordingly, situations where any kind of probability relation can be formed and (all) the future events can be entitled are situations of risk. In these situations *a priori* as well as *empirical* statements can be given. These situations can be encompassed by tabbing instances further and further into more or less homogeneous groups (Knight 1921).[1]

Yet, in most decision problems neither of these situations occur. Hence, there is no chance of achieving a probability judgment. For such situations only an *estimate* can be given. This might be due to several reasons. To Knight (1921) the two main reasons why probability relations can not be given are due to measurement problems and due to the fact that some events are so unique that they are not even held to be contingent before they actually occur. This definition of uncertainty, whereby it its not possible to give a well defined probability distribution, is supported by various others authors, especially Keynes (1921).[2]

Despite the findings of Knight (1921) and other to cope with uncertainty, modern macroeconomics utilize well defined probability distributions, which are characterized by a known mean as well as a known variance. The assumption of agents who behave rational and live in a world of calculable risk "is by now a bedrock assumption in the social sciences"

[1] If, however, only the possible future states are known and the probability relation related to any specific state is only theoretically given a situation of *ambiguity* is prevailing rather than risk (Ellsberg 1961).

[2] Albeit the reasoning might differ to some extent as shown before.

O. Sauter, *Monetary Policy under Uncertainty,*
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(Nelson and Katzenstein 2011:10). Any modern treatment which incorporates uncertainty in fact only claims to deal with uncertainty, but *de facto* uncertainty is replaced merely by a risk analysis and uncertainty has been more or less relegated to the margin of the economic discipline.

An explanation for this development is hard to tell, although, a key role plays the decision theory of the late forties and early fifties, which is strongly connected with the works of von Neumann and Morgenstern (1944) and Savage (1954). While the former have introduced the concept of expected utility with the help of lottery bets, the latter has refined expected utility theory into a theory of *subjective* expected utility, which presumes that people always act ‘as if’ they have a well defined probability distribution in mind. This led to a subsequent warping of uncertainty in the notion of Knight (1921) and Keynes (1921) by most economists (Camerer and Weber 1992, Nelson and Katzenstein 2011).

On the other hand, Post-Keynesian literature accuses Tobin (1958) and the following rational expectation literature for the issue of a marginalization of uncertainty in (economic) thinking (Weisman 1984, Rosser 2001). Tobin (1958) is said to have “hijacked” (Rosser 2001:545) the concept of uncertainty developed by Keynes (1921), and has turned it into a concept of risk by equalizing risk and uncertainty. Thereby, Tobin has paved the way for the rational expectation literature, whereby people act as if their subjective probability distribution equals the objective distribution. This simplification, however, actually rules out uncertainty in the notion of Knight, vividly depicted in Table 1.1 of the previous chapter (Lucas 1977). In fact, what is not covered by rational expectation models is uncertainty arising out of the non-stationarity of the world, which includes situations or events which are not held to be contingent – like the occurrence of the *black swan* of Taleb (2008) (Davidson 2010). Knight’s and Keynes’s 1921 concept of uncertainty permits this non-stationarity (Meltzer 1982). These situations, the *unknowables*, however, are different from the *unknown* – which are both said to be uncertainty situations, in common literature.

No matter who to blame for the intermixture of risk and uncertainty, from a practical point of view uncertainty is simply ruled out and replaced by probability statements, because it is impossible to work with situations one could even not imagine. This practical problem is not objected by Keynes, it is rather confirmed by acknowledging that “[n]evertheless, the necessity for action and for decision compels us as practical men to do our best to overlook this awkward fact and to behave exactly as we should if we had behind us a good Benthamite calculation [. . .] multiplied by its appropriate probability, waiting to be summed” (Keynes 1937:214). Also, on the other side, the rational expectation theory indeed acknowledges the uncertainty concept of Knight (1921), but mentions that “[i]n cases of uncertainty, economic reasoning will be of no value”(Lucas 1977:15). Hence, both protagonists of thought appreciate the difference between probability and uncertainty, but also acknowledge that in daily business the concept of uncertainty in the spirit of Knight (1921) and Keynes (1921) can hardly be applied.

In what follows, I give a brief overview of possible sources, forms, and consequences of uncertainty. Accordingly, some aspects are examined exemplary to show the effects caused by uncertainty. For the mathematical part of this work I follow the established linguistic usage and speak of uncertainty, although in fact risk is assumed.

The variety of possible sources and forms of uncertainty is large. The most trivial case would be just an exogenous shock which hits the economic system, and thus, causes variables to deviate from their desired values. Those shocks are unforeseen and may arise due to different reasons which make them even harder to predict. Additionally, it is possible for them to unfold only during the current period, but have originated long before. This again complicates an identification and thus a proper handling.

Apart from this, economists have to predict the future development of key macro variables. Essential for a good forecast is the proper evaluation of the current state. Yet, this could cause another problem. Values and data of macro variables, for example, GDP, inflation, unemployment, or capacity utilization, are in most cases only available with a considerable time lag and are revised several times later on. During this lag, policy must rely on imprecise estimates of those variables. Moreover, some variables such as potential output are not observable at all, likewise trend output or the real interest rate. These synthetic variables need to be estimated or inferred by variables which are themselves subject of noise. Against this background, policy rules such as the famous Taylor rule are particularly vulnerable due to the fact that they may contain uncertain variables like the output gap as arguments. Both components of the output gap, the actual output and the potential output, are to a high degree uncertain, especially at the moment they are needed the most. As a result, policy settings which have been appropriate at one point in time can be inappropriate given the new, revised information set.

Yet, even if there are no exogenous disturbances and the state of the economy would be known perfectly, there would be problems inferring from one point in time to the other, due to the fact of an only partly known transmission channel. Generally speaking, the effect of one parameter on to the other is uncertain and may vary over time. Reasons for this can be found in the fact that different economic situations produce different transmission behaviors. For example, the process of financial intermediation has changed the transmission of policy signals significantly. This again bears the possibility of a formerly optimal response to be no longer appropriate due to changes in the functional chain. Closely related to the issue of a changing transmission channel is that one can not be sure about the reaction of private agents or the 'financial market' in response to particular policy actions. It might be that some action undertaken by the central bank could itself change relations, which have been assumed to be constant over time. Especially the formation and change of expectations are at the core of this kind of uncertainty.

Finally, if all the information about the impact or size of the transmission parameters would be known, the right data basis is employed, and it is accounted for unforeseen, exogenous shocks, one could even not be sure about the correctness of the assumed underlying model structure. Different schools of thought produce different models and approaches, some of them competing against each other. Hence, without shocks, valid data, and the correct estimation of parameters, there would still be no guarantee of whether the functional form of the workhorse model is correct or not.

This walk through a set of possible sources of uncertainty has revealed the most common classification of uncertainty. The first being *additive* uncertainty, relying on exogenous shocks that hit the system. The second is called *data* uncertainty, capturing measurement as well as estimation errors. *Multiplicative* or *parametric* uncertainty is prevailing if the intensity of the transmission process, that is, the effect of one variable onto the other can not be said with certainty. Thereby, it is often distinguished between a policy which is uncertain about the structure she is implemented into or about the effect she has on the structure, the latter being also called *strategic* uncertainty. This could be the case, for example, if reactions of private agents are incorporated into the decision of whether tightening or easing a respective policy stance. Extending parametric uncertainty to uncertainty covering not only the values of parameters, but also the functional form of models leads to *model* uncertainty. While model uncertainty can be simply understood as another type of uncertainty, it could also be used as a superior type, covering all other forms of uncertainty as sub-forms.

Although this classification seems to be common, see, e.g., Blinder (1998), Dow (2004) or Spahn (2009), a more comprehensive classification distinguishes only between *data* and *model* uncertainty, the former covering 'classical' concerns about the availability of data and the problem of data revisions, the latter covering general model, as well as additive or shock, and multiplicative uncertainty concerning the transmission process (see, e.g., Bundesbank 2004, González-Páramo 2006). Another categorization could be between uncertainty about the state, the structure of the economy, and strategic uncertainty. Thereby, uncertainty about the 'state' is defined as covering data and additive uncertainty. Uncertainty about the structure of the economy covers parameter and model uncertainty. Strategic uncertainty deals with interactions or expectations of market participants (Issing 2002). Apart from these different arrangements, in general one can identify four main categories of uncertainty, namely additive, data, parameter, and model uncertainty. This is the categorization used in this work.[3]

Yet, knowing about the existence of uncertainty does not mandatorily tell the respective policy maker how to react correctly. Although it is accepted that under a linear framework with a quadratic loss function *certainty equivalence* prevails, thus the policy maker should act 'as if' everything would be known with certainty, the advice might not be that clear for parametric uncertainty. If uncertainty about the transmission of monetary signals prevails it could be that only a relatively small reaction is needed in order to change, for example, expectations about the future path of inflation. The more caution stance has found its expression in the literature as the *Brainard principle*, with respect to the seminal work of Brainard (1967). It suggests that under parametric uncertainty the monetary authority should act less aggressive than she would under certainty. However, this concept has been criticized, for example, by Söderström (2002) who singles out that in some constellations it could be superior to react more aggressive than being gradual. According to Söderström (2002) it is crucial whether there is uncertainty about the impact or persistence parameters of the model.

In addition, to induce the intended reaction it could be necessary to give a rather strong signal to the market by cutting rates more drastic than under certainty (Spahn 2009). Acting too cautious could lead to a situation where the central bank runs into the risk of what is called 'falling behind the curve' (Goodhart 1999). In this case, if, e.g., the economy is picking up, an only gradual policy stance could boost price developments, calling for an even stronger policy reaction in the future with all its consequences on investment and employment (González-Páramo 2006).

Orphanides (2003a) shows how the presence of noise may lead into exaggerations when setting the interest rate in accordance with mismeasured data. Thus, when faced with data uncertainty it could be useful to assign less weight on uncertain variables – if they can be identified at all. This implies if the central bank follows a Taylor rule with output-gap and inflation-gap as arguments, and output data is noisy, she should assign less weight on the output-gap and respectively more on the inflation-gap in order to reduce excess volatility. In extreme, she should not account for measured output deviations at all (Orphanides 2003a).

With regard to model uncertainty there has been a development onto two approaches. Hansen and Sargent (see, e.g. 2001, 2003b, 2008) have transfered the problem of model uncertainty into the robust control literature. Given a true but unknown model the pur-

[3] This categorization excludes strategic uncertainty as a single source and rather covers this issue under the aspect of model or parameter uncertainty. This seems to be reasonable, as changes in the behavior or expectations of agents in fact change the behavior of the transmission channel, which is covered by the analysis of the already mentioned sources of uncertainty.

pose is to find a strategy which performs best given the worst conceivable consequences of different models surrounding the reference model of the central bank. In this setting they construct a game theoretic context with an evil agent playing against the central bank. The advantage of this type of modeling lies in the possibility to account for unquantifiable Knightian uncertainty, although there must be some agreement about the boundaries of the different models (Dow 2004, Dennis 2005).

Alternatively one could choose between a range of models, not so much focusing on one specific model. If so, one is able to calculate the 'cost' of a strategy which is made on the basis of a specific model, whilst the prevailing correct model is actually a different one. To work with both alternatives it is necessary to constrain the model somehow. In the first case, it is the budget of the malevolent player, in the second case, its the range of models being compared, which limits the analysis (Dow 2004).

A second approach to model uncertainty follows Bayesian principles, and targets a policy rule that works well over a range of models with different set-ups. This model averaging approach was mainly brought forward due to the impossibility to come to a compromise regarding the best strategy in dealing with model uncertainty. Within this category is also the idea of implementing possible switches between a set of models. The monetary authority must then calculate her optimal response with respect to a possible regime switch. However, both approaches have some strong limitations. Most notably, in either case the set of alternative models must be fully specified and probabilities need to be assigned, which precludes Knightian uncertainty (Issing 2002, Dennis 2005, Svensson and Williams 2005, Wagner 2007, Blake and Zampolli 2011).

Table 2.1 is taken from Dow (2004). It lists four different kinds of uncertainty, summarizing the above mentioned classification. Actually, only three *sources* are listed, because the latter two are both attached to model uncertainty, depending on whether Knightian uncertainty is precluded or not.

U	Type of uncertainty	Measure
U_1	additive	σ_ϵ^2 of equation
U_2	multiplicative	σ_ϵ^2 of parameter
U_{3a}	model	S_w
U_{3b}	model	unmeasurable

Table 2.1.: Classification of uncertainty

Uncertainty enters the model set-up either by the variance of the error term or the variance of the parameter values, whether additive or multiplicative uncertainty is prevailing. U_{3a} covers model uncertainty, where, although the model specification is unknown, the error terms distribution can be specified by S_w. For these three types, uncertainty can be more or less measured and expressed formally.[4] It is important to distinguish what is known from what is unknown. In Table 2.1 uncertainty can be identified and assigned to a specific source. If this is not the case, i.e., if there is no way what so ever to grasp what is known and what is not U_{3b} is prevailing. In this case uncertainty predominates in a Knightian sense. In reference to the previous chapters, in this situation it does not matter, whether this impossibility to assign probabilities is due to the real world or the inability of understanding, i.e., not matter if following Knight's aleatory or Keynes' epistemic notion of probability and uncertainty (Dow 2004).

The following sections take a closer look on the briefly mentioned forms of uncertainty and provide examples to underpin the argumentation.

2.2. Additive Uncertainty and Certainty Equivalence

The previous section has highlighted the seminal work of Brainard (1967). Together with Poole (1970) these two publications have notably shaped the discussion on monetary policy under uncertainty during the last century. Brainard (1967) demonstrates how *parametric* uncertainty changes the behavior of the monetary authority, and Poole (1970) analyzes, which instrument may serve best and delivers thus a smaller loss if *additive* uncertainty prevails, i.e., the possibility of an exogenous shock hitting the system.

Additive uncertainty summarizes all kinds of exogenous shocks, which could hit the economic world and affect either demand or supply factors or both. These stochastic shocks, which can not be anticipated, occur from time to time and prevent the system from reaching its desired level of the target variables. Thereby, supply side shocks – opposed to demand driven shocks – are of special concern due to the trade-off situation they create. Reducing the deviation of one target variable, say inflation, can only be done at the expense of missing the target of the output (Goodhart 1999, Spahn 2009).

In a taxonomy of potential sources of uncertainty additive uncertainty would be the most trivial form, especially when it comes to modeling. However, it offers some first valuable insights concerning properties and the general assessment of conducting policy under uncertainty, most notably the concept of *certainty equivalence*. Given the model set up additive uncertainty is introduced by adding a stochastic term with zero mean and positive but limited variance to the demand and/or supply equation. The variance of the added error term affects the dependent variables, hence, adding several stochastic terms,

[4] I would suggest to add data uncertainty by defining it as the variance of a specific variable. Yet, following Issing (2002) or Swanson (2000), data uncertainty would be already covered by additive uncertainty. This treatment, however, is only in part true, as I will show in Section 2.4.

all with a zero mean, equals 'adding variance' to the actual dependent variable which is object of a minimization procedure (Dow 2004).

The model in the subsequent paragraphs relies on the work of Poole (1970). He studied the different consequences of the choice of the instrument – in his case money supply or interest rate policy – in response to an additive goods or money market shock. For a better understanding, first of all the concept of *certainty equivalence* is briefly introduced. Afterwards the classical model of Poole (1970) is examined graphically as well as analytically.

2.2.1. The Concept of Certainty Equivalence

If only additive shock uncertainty prevails, the central bank is told to ignore these uncertainties, that is, the decision rule should be equivalent to the certainty case. The central bank behaves 'as if' she knows everything with certainty, although she is actually confronted with uncertainty. However, this rule is only applicable to a very limited amount of models and forms of uncertainty. To see why this proposition holds suppose an environment which can be described by a linear relationship of the form

$$y = a_1 r + u. \tag{2.2.1}$$

In Equation (2.2.1) the target variable is given by y, which could be, for example, a desired level of output. The policy instrument is given by r, and $a_1 < 0$ determines the response of the target variable to a policy action. For the additive shock vector u it holds that the expected value equals zero and the variance is limited and positive, $u \sim N(0, \sigma_u^2)$. Yet, any value of $u > 0$ affects the value of the dependent variable with the distribution now given as $y \sim N(y_f, \sigma_u^2)$. The deterministic character of the target variable has changed into a stochastic, due to the stochastic noise term. Hence, no guarantee can be given that $y = y_f$, that is, output equals its target or optimal level y_f.

The variance of the target variable is determined by the variance of the additive shock term. No matter what action is made by the monetary authority it can not alter the distribution of y, but only shift it, so that $y \sim N(a_1 r, \sigma_u^2)$. The uncertainty about the the target variable is in the system and it can not be affected by any central bank action. Indeed, the additive shock vector has added some variance to the dependent variable, however, it has not altered the mean of it. This explains why the best advice one could give is to behave 'as if' one would act under certainty.[5].

[5] The picture changes, though, if no longer additive uncertainty is prevailing, but rather uncertainty about the transmission parameter a_1. In this case the monetary authority would not only shift, but also alter the distribution of the target variable.

It is further supposed that the central bank follows a quadratic loss function, which penalizes output deviations from its target of the form

$$L = (y - y_f)^2 . \tag{2.2.2}$$

Since the policy parameter has to be set before the shock term reveals, the central bank has to make an estimate about the target variable in order to achieve a minimal loss.[6] The expected loss is given by

$$\begin{aligned} L &= E\left[(y - y_f)^2\right] && (2.2.3)\\ &= E\left[(a_1 r + u - y_f)^2\right] \\ &= a_1^2 r^2 + E\left[u^2\right] + y_f^2 + 2a_1 r E\left[u\right] - 2a_1 r y_f - 2y_f E\left[u\right] \\ &= a_1^2 r^2 + \sigma_u^2 + y_f^2 - 2a_1 r y_f . && (2.2.4) \end{aligned}$$

Equation (2.2.4) reveals that the loss of the monetary authority is determined *inter alia* by the variance of the shock vector, σ_u^2.

Due to the fact that the expectation operator only applies to the shock term, but the parameter and the variables a_1, r, and y_f are supposed to be known, the expression $E\left[L(r, y)\right]$ equals $L\left(r, E[y]\right)$. In order to achieve the minimal loss defined in Equation (2.2.2), the derivative of (2.2.4) with respect to the policy instrument delivers the optimal setting of the policy parameter r^* according to

$$\begin{aligned} \frac{\partial L}{\partial r} &= 2a_1^2 r - 2a_1 y_f \stackrel{!}{=} 0 \\ r^* &= \frac{y_f}{a_1} . && (2.2.5) \end{aligned}$$

Equation (2.2.5) shows that the decision rule is independent of what so ever the variance of the disturbance vector might be, as long as its zero mean remains, the constraint is linear, and the loss function is quadratic. This is known as the *certainty equivalence* principle, a concept ascribed to the work of Tinbergen (1952), Simon (1956), and Theil (1957). It states that, although the objective function is altered in its value by the additive disturbance vector, or rather its (co-) variance, the policy maker can ignore this uncertainty and go on in setting his policy 'as if' everything is known with certainty. Hence, to proceed the uncertain additive variable is merely replaced by its expected value (Simon 1956, Ljungqvist and Sargent 2004).[7]

6 With a time subscript the above system must be written as $y_{t+1} = a_1 r_t + u_{t+1}$ to highlight the chronological order.

7 Taking expected instead of true values indicates already a related issue which is discussed later in this work but should be mentioned already at this point. The concept of certainty equivalence also involves

2.2.2. Additive Uncertainty: The Poole-Model

The concept of certainty equivalence has revealed that additive uncertainty does not affect the policy reaction function of the monetary authority, no matter how large the variance of the shock might be. The conclusion of this finding is that a central bank does not have to care about additive uncertainty, due to the fact that she can not actively reduce the impact of uncertainty on the target variable. However, Poole (1970) shows that if the variance of the target variable is included into the loss function policy action must be set very well under consideration of the noise term, even if it is additive. The underlying question of Poole (1970) is whether the monetary authority should act through interest rate changes or money supply changes. Depending on the situation one or the other instrument might be superior.

Interestingly enough, from today's point of view and through the experiences gathered throughout the recent financial turmoil, Poole (1970) picks the interest rate as a representative for overall "money market conditions" (Poole 1970:199) and mentions that the conditions also cover "the level of free reserves in the banking system, the rate of growth of bank credit [. . .], or the overall "tone" of the money market" (Poole 1970:199). However, the interest rate is selected as being the best single variable to represent those conditions in an analytical analysis. This is due to the fact that Poole was not so much interested in the issue of uncertainty *per se*, but rather in the question of controlling quantities vs. controlling rates in conducting monetary policy. His primary goal is not to describe how central bank behavior changes when confronted with uncertainty – as it is the case, e.g., in Brainard (1967) – but rather to find reasons in favor or against different strategies in monetary policy. Nevertheless, his 1970 work is referred to as one of the major works in the field of monetary policy under uncertainty. Moreover, some authors, see, e.g., Collard and Dellas (2005), have translated the original model of Poole into a New Keynesian framework to show that the basic conclusions continue to hold under certain assumptions (Schellekens 2000).

To show why a consideration of the policy instrument matters the previous equations are extended and complemented by a money market equation. In the nomenclature of

the so-called *separation principle*. If the true value of any variable is hidden or the measurement is noisy, accordingly, the estimation problem is separated from the optimization problem. Hence, in a first step the central bank forms the best possible estimate of the unobserved variable and in a second step she optimizes with this estimate as if the values are certain. Forecasting and optimization need to be done separately and are thus independent from each other. This leads to the situation that the response of the optimization problem is independent of the degree of completeness of information, i.e., the variance as long as the best estimate is utilized. If this procedure is adopted to the problem stated here, the best estimate one could make for the value of the additive shock is zero. Hence, any optimal policy should go on by treating the exogenous shock as not existent when optimizing (Svensson and Woodford 2003b).

Poole (1970) the goods and money market equation, as well as the loss function are given respectively by

$$y = a_0 + a_1 r + u, \tag{2.2.6}$$

$$m = b_0 + b_1 y + b_2 r + v, \tag{2.2.7}$$

$$L = E\left(y - y_f\right)^2. \tag{2.2.8}$$

It holds that $a_1 < 0, b_1 > 0, b_2 < 0$. Hence output, y, depends negative from the interest rate, r. The money stock level, m, depends positive on the output and negative on the interest rate. The goods and money shock terms, u and v, exhibit zero mean and a positive variance. In accordance with the previous equations the target output is denoted by the subscript f, whereas the instrument of choice is denoted by the asterisk superscript.

Non-Stochastic Case

The aim of Poole (1970) is to show why identifying the shock properly matters for the right choice of the policy instrument, and thus, for the size of the realized loss. Yet, before the situation of uncertainty is considered, the non-stochastic case is taken into account. To minimize her loss function, which constitutes itself as deviations from the target output level, the central bank is equipped with two different instruments, namely the interest rate and the money supply.

(1) First, it is assumed that the interest rate is picked as the instrument of choice, i.e., $r = r^*$. Output is thus determined via the goods market equation as

$$y_{r*} = a_0 + a_1 r^*. \tag{2.2.9}$$

The money stock which is demanded to finance this level of output, given r^*, is determined by the money market equation as

$$\begin{aligned} m &= b_0 + b_1 y + b_2 r^* && (2.2.10)\\ &= b_0 + b_1\left(a_0 + a_1 r^*\right) + b_2 r^* \\ &= b_0 + b_1 a_0 + \left(b_1 a_1 + b_2\right) r^*. && (2.2.11)\end{aligned}$$

For this deterministic setting one would simply move along the goods market curve by varying the interest rate until the desired level of output, y_f, is met. Money stock and output evolve endogenously in oder to fulfill the optimal interest rate level, which is the only exogenous variable in this setup. Hence, the money market curve becomes horizontal.

(2) On the other hand, the instrument of choice could be the money supply or money stock change, $m = m^*$. The interest rate is determined by m^* according to

$$\begin{aligned} m^* &= b_0 + b_1 y + b_2 r \quad (2.2.12) \\ &= b_0 + b_1 a_0 + (b_1 a_1 + b_2)\, r \\ r &= (a_1 b_1 + b_2)^{-1} (m^* - b_0 - a_0 b_1)\,. \quad (2.2.13) \end{aligned}$$

The corresponding output level can be found by combining Equation (2.2.6) with the resulting interest rate of Equation (2.2.13) to get

$$\begin{aligned} y_{m*} &= a_0 + a_1 r \\ &= (a_1 b_1 + b_2)^{-1} \left[a_o b_2 + a_1 (m^* - b_0)\right]. \quad (2.2.14) \end{aligned}$$

If the instrument of choice is the money supply, one would move the money market curve, which is no longer a horizontal, until y_f is met. Money supply is exogenous, whereas output and interest rate evolve endogenously.

For convenience the relevant equations for an interest rate and money supply policy are respectively

$$\begin{aligned} y_{r*} &= a_0 + a_1 r^*, \quad (2.2.15) \\ m &= b_0 + b_1 a_0 + (b_1 a_1 + b_2)\, r^*, \quad (2.2.16) \\ y_{m*} &= (a_1 b_1 + b_2)^{-1} \left[a_o b_2 + a_1 (m^* - b_0)\right], \quad (2.2.17) \\ r &= (a_1 b_1 + b_2)^{-1} (m^* - b_0 - a_0 b_1)\,. \quad (2.2.18) \end{aligned}$$

The optimal values of the interest rate and the money stock for a given level of output can be found by the reduced Equations (2.2.15), (2.2.17), respectively as

$$\begin{aligned} r^* &= a_1^{-1} (y_f - a_0)\,, \quad (2.2.19) \\ m^* &= a_1^{-1} \left[y_f (a_1 b_1 + b_2) - a_0 b_2 - a_1 b_0\right]. \quad (2.2.20) \end{aligned}$$

In a deterministic world where all additive shocks equal zero, it would not matter which instrument is chosen to achieve the desired level of output. If the interest rate is the instrument of choice it is easy to see that one would just move along the goods market curve in order to meet the target output. The money supply would evolve endogenously in order to fulfill the interest rate target. Alternatively, if money supply is the chosen instrument, one just ‘moves’ the money market curve along the goods market curve, until the desired level of output is achieved. Given the goods market curve, the interest rate compatible with the target output level, y_f, would merely be a residual.

Because r^* is the result if money supply is the instrument of choice and money demand will be m^* if the interest rate is the chosen instrument, in this deterministic setting the choice of instrument would be “a matter of convenience, preference, or prejudice, but not of substance” (Poole 1970:204).

Due to the fact that the target output y_f is achieved anyway, the loss function would in both cases deliver a value of zero. Hence, even under loss aspects, without uncertainty the chosen policy – whether the interest rate or the money supply is the leading instrument – is equivalent (Poole 1970, Froyen and Guender 2007).

Stochastic Case

The assumption of certainty is relaxed by permitting stochastic shocks to hit the system. Because of certainty equivalence, the minimization problem should have the same solution under certainty, as well an under additive uncertainty. If it holds that $E(y) = y_f = y$, minimizing the loss function is equivalent to minimizing the variance of the output, *Var(y)*. By permitting additive uncertainty to enter the equation setup the dependent variable output becomes a random variable as well, due to the newly shock component. However, once the instrument has been chosen the situation should equal one of certainty and is thus certainty equivalent.

(1) Again, first the interest rate is chosen to be the instrument of choice. The loss function

$$L_{r*} = Var(y), \tag{2.2.21}$$

is equipped with Equation (2.2.15), but now complemented with the random disturbance vector u,

$$y_{r*} = a_0 + a_1 r^* + u. \tag{2.2.22}$$

The variance of output is given by

$$\begin{aligned} Var(y) &= E\left[(y - E[y])(y - E[y])\right] && (2.2.23)\\ &= E\left[(a_0 + a_1 r^* + u - a_0 + a_1 r^*)^2\right] \\ &= E\left[u^2\right] && (2.2.24)\\ L_{r*} = Var(y) &= \sigma_u^2 && (2.2.25) \end{aligned}$$

taking into account that $E[u] = E[v] = 0$, $E[u^2] = \sigma_u^2$, and $E[v^2] = \sigma_v^2$.

Equation (2.2.25) shows if the interest rate is set by the monetary authority and money evolves endogenously only goods market shocks enter the loss function. For a money market shock the value of the loss function remains zero.

(2) If the money stock is the instrument of choice, the corresponding loss function L_{m*} is derived by taking the reduced form Equation (2.2.17), which displays the optimal output level with money stock as the instrument, together with Equation (2.2.20), which equals the reaction function in this setting and therefore is needed in order to find the appropriate money supply to achieve y_f.

$$y_m = \underbrace{(a_1 b_1 + b_2)^{-1} \left[a_o b_2 + a_1 (m^* - b_0)\right.}\left. + b_2 u - a_1 v\right] \tag{2.2.26}$$

$$y_m = y_f + (a_1 b_1 + b_2)^{-1} (b_2 u - a_1 v) \tag{2.2.27}$$

The first part of Equation (2.2.26), covered by the brace, equals Equation (2.2.17), which delivers the optimal output level if money supply is the instrument. If the money supply is the chosen instrument, the variance of the output is given by

$$\begin{aligned} Var(y) &= E\left[(y - E[y])(y - E[y])\right] & (2.2.28) \\ &= E\left[\left(y_f + (a_1 b_1 + b_2)^{-1}(b_2 u - a_1 v) - y_f\right)^2\right] \\ &= E\left[\left((a_1 b_1 + b_2)^{-1}(b_2 u - a_1 v)^2\right)\right] \\ L_{m*} = Var(y) &= (a_1 b_1 + b_2)^{-2}\left(a_1^2 \sigma_v^2 + b_2^2 \sigma_u^2\right). & (2.2.29) \end{aligned}$$

Opposed to the previous finding a clear conclusion concerning the influence of uncertainty on the loss can not be drawn. The difference between L_{r*} and L_{m*} results from the fact that reaching the target level of output, y_f, with the interest rate as instrument only involves the goods market. Walking along the goods market curve is sufficient for an equilibrium. On the other hand, using the money stock as instrument would involve both sides of the model, the goods market as well as the money market (Poole 1970, Froyen and Guender 2007)

Comparing Loss

To find an answer to the question whether or not it is useful to steer the economy by money supply or via the interest rate two kinds of shocks (goods market and money market) are allowed to hit the system. They are compared in terms of losses they create.

For convenience, the respective loss functions under an 'interest rate regime' or 'money supply regime' are repeated here

$$L_{r*} = \sigma_u^2, \tag{2.2.30}$$
$$L_{m*} = (a_1 b_1 + b_2)^{-2} \left(a_1^2 \sigma_v^2 + b_2^2 \sigma_u^2\right). \tag{2.2.31}$$

For a goods market shock ($\sigma_u^2 > 0$) it holds that $L_{r*} > L_{m*}$, hence, the appropriate instrument should be the money supply, due to the smaller loss. On the other hand, for a money market shock ($\sigma_v^2 > 0, \sigma_u^2 = 0$) it holds that $L_{r*} < L_{m*}$, thus, in this case the interest rate as instrument is superior to the money supply.[8] These findings can also be traced in Figure 2.1. On the left-hand side, a money market shock is displayed which shifts the money market curve to the left or right, depending whether this is a positive or negative shock. Setting the interest rate and letting the money supply evolve endogenously makes the money market curve horizontally and the loss equals zero. Instead, using the money supply as instrument creates a loss greater than zero. On the right-hand side a possible goods market shock is demonstrated, which shifts the goods market curve to the left or right depending on the sign of the shock. As can be seen clearly, a horizontal money market curve, caused by an endogenous money supply, creates a larger loss compared to the case using the money supply as an instrument and letting the interest rate fluctuate as it likes.

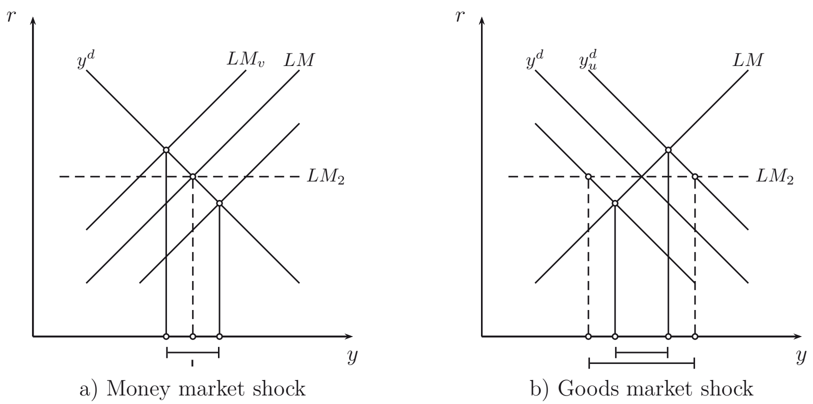

Figure 2.1.: Additive uncertainty in the model of Poole (1970)

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8 The pronunciation of the relative loss difference depends of course also on the parameter values. The closer b_2 is to zero, the smaller is the difference of Equation (2.2.30) and (2.2.31).

The result is not surprising. Equation (2.2.30) shows that a positive money market shock does not even enter the loss function, in case the interest rate is the chosen instrument. This is due to the fact that an increase in money demand is completely accommodated by the central bank, so that output is not affected at all. The interest rate remains at its previous level. The money market curve is horizontal and the output is purely defined by the goods market. If, however, the money supply is being fixed, a rise in money demand would lift the interest rate, causing a decline in output and thus, causing a higher value of the loss function.

If on the other hand, a positive goods market shock occurs, output rises above its target level so that $y > y_f$. The rise in money demand will, due to its money supply constraint, force the interest rate to rise, causing a dampening effect on demand. For an endogenous money supply, that is, the interest rate is pinned down on a fixed level, this effect will be absent with the consequence of an even larger deviation of output from its desired level. The rise of the interest rates thus provides a stabilizing effect on the economy which induces a smaller loss (Poole 1970, Spahn 2009).

With this simple model, Poole demonstrates that, while the reaction functions (2.2.19) and (2.2.20) remain the same, whether uncertainty is prevailing or not, the value of the loss functions, Equation (2.2.30) and (2.2.31), vary as well. However, once the instrument is being fixed, *certainty equivalence* holds. But, although *certainty equivalence* prevails and the reaction function is not altered by the source and size of the shock, it could be utile to decide which instrument to choose in order to minimize the overall loss, given the relative importance of the random shock and the parameters of the model.

The choice of instrument, between the short-term interest rate and some monetary aggregate, is in fact a matter of discussion. As Bindseil (2004) mentions there is nearly no way of controlling monetary aggregates, even in a very narrow band of, say, base money, than using the interest rate. Accordingly, the monetary base is often rather seen as an intermediate target than a direct policy instrument. Controlling the monetary aggregate would thus always imply the control of the nominal short-term interest rate. Accounting for that, there is no operational choice in conducting monetary policy via rates or quantities. Thus, the question changes from whether reacting with some monetary aggregate or the interest rate, to the question of how strong and how fast a reaction should be conducted (Bindseil 2004).

2.3. Parametric Uncertainty: The Brainard Example

The most prominent form of uncertainty is labeled parametric or multiplicative uncertainty. It is strongly connected to the seminal work of Brainard (1967), who analyzed the effectiveness of monetary policy in the presence of parametric uncertainty. His result, which has later been labeled *Brainard conservatism* (Blinder 1998), suggests that under parametric uncertainty the central bank must react less intense in response to a shock compared to the situation of certainty or certainty equivalence. Nevertheless, this result has been a matter of discussion during the years and it has been shown that the opposite reaction – to be more aggressive than under certainty – can also be supported (Craine 1979, Söderström 1999). An important feature of parametric uncertainty is the vaporization of *certainty equivalence*. While under additive uncertainty the policy rule is independent from the uncertainty added to the model, this no longer holds for uncertainty concerning the parameters of the model.

The original problem of Brainard (1967) can be illustrated as follows. It is assumed that the central bank is uncertain about the impact of her action concerning her target variable y_f (e.g., output or inflation). The target variable shall be controlled by an instrument variable, r. However, the impact from the control variable to the target variable is determined by some response or transmission coefficient, a. As before the model economy is hit by an exogenous shock, u. This relation is given as

$$y = ar + u. \tag{2.3.1}$$

For convenience, the nomenclature differs from Brainard (1967) and opposed to the original problem I presume the transmission variable and the shock being independent from each other for the rest of this work. In Equation (2.3.1) two distinct sources of uncertainty exist. First of all, the influence of the shock variable on output, which leads to the problem of a proper reaction of the policy instrument. This form of uncertainty has already been examined in the previous section. It has shown that due to certainty equivalence this poses no challenge to the monetary authority. The second source of uncertainty arises from the reaction of the monetary authority itself, and pertains its uncertain influence on the target variable.

It is assumed that the exogenous disturbance is represented by an i.i.d. error term with zero mean and constant, positive variance. The mean of the transmission parameter, however, is non-zero and said to be known to the central bank. The variance is constant and positive as well. The uncertainty surrounding the target variable must be a combination of the uncertainty – expressed by the variance – of the transmission parameter and the exogenous shock vector. It is given by

$$
\begin{aligned}
Var(y) &= Var(ar + u) && (2.3.2)\\
&= Var(ar) + Var(u)\\
\sigma_y^2 &= \sigma_a^2 r^2 + \sigma_u^2. && (2.3.3)
\end{aligned}
$$

Compared to the former case of additive uncertainty the variance of the target variable depends not only on the variance of the additive shock vector, but also on the variance of the transmission parameter. The influence of the latter, however, depends crucially on the size of the reaction parameter, which is under the control of the monetary authority. This already indicates that the monetary authority must take into account the uncertainty of the transmission parameter when calculating her optimal policy response. Certainty equivalence ceases to hold.

For further investigation it is assumed that the monetary authority wants to minimize a loss function, which penalizes positive as well as negative deviations from the target variable and is thus of the form

$$L = (y - y_f)^2. \tag{2.3.4}$$

The expected loss is attained by plugging Equation (2.3.1) into (2.3.4) to get

$$
\begin{aligned}
E[L] &= E\left[ar + u - y_f\right]^2\\
&= E\left[a^2\right] r^2 + E\left[u^2\right] + y_f^2 - 2E[u]\, y_f - 2E[a]\, r y_f + 2E[a]\, r E[u]\\
&= E\left[a^2\right] r^2 + E\left[u^2\right] + y_f^2 - 2E[a]\, r y_f. && (2.3.5)
\end{aligned}
$$

The expectation operator only applies to the transmission parameter a and the shock vector u. Furthermore, as has been noted before for the mean and the variance of those variables it holds that $E[u] = 0$, $E[u^2] = \sigma_u^2$, and $E(a) = \hat{a}$, $E[a - \hat{a}]^2 = \sigma_a^2$. Making use of the computational formula for the variance one can rewrite $E[a^2] = \sigma_a^2 + (E[\hat{a}])^2$.

Taking into account Equation (2.3.3) the expected loss can be simplified to

$$
\begin{aligned}
E[L] &= \left(\sigma_a^2 + E\left[\hat{a}^2\right]\right) r^2 + \sigma_u^2 + y_f^2 - 2E[a]\, r y_f\\
&= \sigma_a^2 r^2 + \sigma_u^2 + y_f^2 - 2E[a]\, r y_f + E\left[\hat{a}^2\right] r^2\\
&= \sigma_y^2 + y_f^2 - 2E[a]\, r y_f + E\left[\hat{a}^2\right] r^2. && (2.3.6)
\end{aligned}
$$

Hence, the loss is defined as

$$\begin{aligned} L &= \sigma_y^2 + (\hat{a}r - y_f)^2 \\ &= \sigma_a^2 r^2 + \sigma_u^2 + (\hat{a}r - y_f)^2, \end{aligned} \tag{2.3.7}$$

which is the sum of the variance of the target variable, plus the expected deviation from its target level. In other words, the expected value of the random target variable equals its variance plus the squared bias. Equation (2.3.7) shows how the policy instruments affects the variance as well as the deviation of the target variable. The minimization of both parts with only one instrument can only be done by taking into account some trade-off between those two parts (Martin 1999).

Differentiating Equation (2.3.7) delivers the optimal rule for the policy instrument under parametric uncertainty

$$r^* = \frac{\hat{a} y_f}{\sigma_a^2 + \hat{a}^2}. \tag{2.3.8}$$

With the emergence of uncertainty a smaller reaction of the interest rate compared to the certainty equivalence case is demanded. This more reluctant policy stance is called *conservatism*. Reformulating Equation (2.3.8) makes this conservatism even more clear. Replacing $\sigma_a/\hat{a}$ with ζ and define g as $1/(1+\zeta^2)$ delivers

$$r^* = g \frac{1}{\hat{a}} y_f. \tag{2.3.9}$$

Equation (2.3.9) shows how the optimal setting of the policy parameter is affected by the size of uncertainty concerning the transmission, represented by σ_a^2. The policy reaction is only a fraction of the certainty case, due to the fact that g is less than one. This becomes most obvious if Equation (2.3.9) is compared to the certainty (equivalence) reaction function

$$r^* = \frac{1}{a_1} y_f, \tag{2.3.10}$$

where the g is missing.

Growing uncertainty, indicated by a growing variance, calls for a less active policy stance, hence, a smaller reaction via r^* is demanded. For the extreme case of unlimited uncertainty, $\sigma_a^2 \to \infty$, r should be set to zero. For the case of no uncertainty concerning the response coefficient, i.e., $\sigma_a^2 = 0$, r equals the certainty equivalence case of Equation (2.3.10) as g approaches one (Brainard 1967, Martin 1999, Martin and Salmon 1999).

A more conservative policy in the presence of multiplicative uncertainty comes as no surprise. The expected error which is intended to be minimized consists of two sources. The first one, the deviation of the target level from its realized level, is minimized by the standard certainty equivalence reaction. The second source, the variance of the target variable, is minimized by a zero action policy. Thus, the optimal reaction must be somewhat in between these two policy rules. The actual reaction, however, depends on whether the policy is set rather with respect to the the target breach or the variance concerns (Onatski 2000).

Furthermore, Martin (1999) and Martin and Salmon (1999) identify two additional consequences of uncertainty, which they label *gradualism* and *caution*. By gradualism, they define the smoothing of the response to a shock. Due to the fact that only a part of the shock is offset, each period of the target variable becomes autocorrelated and since the interest rate is set in response to this lasting deviation, it becomes autocorrelated as well.[9] With a lower variance of the transmission parameter the degree of autocorrelation thus declines as well and gradualism disappears in the limit. It is absent if only additive uncertainty is prevailing (Martin and Salmon 1999).

Caution measures the magnitude of the total response, i.e., the cumulative interest rate response. Under certainty this is clearly $1/a$. The shock is completely offset in one period as it is costless to do so. Under parametric uncertainty, this response is smaller, due to the natural decay as time goes by. However, this does not hold for output and inflation. So in sum, under parametric uncertainty, the interest rate deviation from the desired – neutral – level is smaller, more phased in and the cumulative response is less than under certainty (Martin and Salmon 1999).

The example of Brainard (1967) shows how *certainty equivalence* ceases in the presence of parametric uncertainty. The policy rule is now affected by the size of the variance of the unknown parameter. Offsetting deviations of the target variable by large interest rate steps induces more variance. Changing the interest rate is not costless anymore, and thus, it is no longer optimal to completely offset a shock in one period as it is the case under certainty. On the other hand, the size of the disturbance vector is not taken into account in setting the policy instrument. This confirms the previous finding that additive disturbances, while they might affect the loss function, they do not affect the policy rule (Brainard 1967, Martin 1999).

The so far mentioned findings of this section gave birth to the *Brainard conservatism* and have been restated various times, underpinning the early findings of Brainard (1967). During the years there have been counter examples, which show that also a more aggressive

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9 Martin and Salmon (1999) further distinguish between *nominal* and *real* interest rate responses/ paths, although the conclusion is not significantly altered.

policy could be appropriate, especially when there is uncertainty about the *dynamics* of the economy rather than the *impact* of some policy action (see, e.g., Söderström 2002).

Beside technical reasons for a more or less aggressive policy reaction in the presence of uncertainty, compared to an environment of certainty equivalence, Caplin and Leahy (1996) highlight that a too cautions policy may indeed prove to be ineffectual to stimulate economic activity.

In the recessionary set up of Caplin and Leahy (1996) the policy maker faces the problem that she does not know, whether the intended interest rate cut is sufficient to stimulate the economy. Two possibilities emerge. In the first case, a too cautious policy reaction will pursue the ongoing recession and makes further interest rates cuts necessary. In the second — more aggressive — case, inflationary pressure will emerge. The task is thus to find an optimal policy in the sense that it should be as aggressive as needed but also as cautious as possible, considering that the central bank as well as the agents learn from the others action and reaction.

As an outcome of this game monetary policy authorities indeed should react more aggressive than actually needed for the intended outcome. It results from the fact that agents will know that any failure in stimulating the economy will be followed by further rate cuts, which gives them an incentive to wait and postpone action in the prospect of even better future conditions. Additionally, any policy failure will reduce the probability of a successful future policy as it reveals the prevailing bad condition of the economy.[10]

If, however, a more aggressive rate cut is considered, a situation which promotes economic activities can be achieved due to the fact that the agents of the economy expected this cut to be successfully and thus only of a temporary nature. This in turn creates a climate of urgency to invest and to benefit from the extraordinary financing conditions.

Never-the-less, despite these counter findings the *Brainard conservatism* represents still the most prominent guideline in practical policy making (Blinder 1998, Goodhart 1999).

Comparison between Additive and Parametric Uncertainty

To show the difference between additive, i.e., certainty equivalence, and parametric uncertainty, the following example is considered. No matter what kind of uncertainty prevails, the social loss is defined as

$$E[L] = (y - y_f)^2 \tag{2.3.11}$$

$$= \sigma_y^2 + (E[y] - y_f)^2 \tag{2.3.12}$$

[10] This result can also be applied to other situations of uncertainty, such as fiscal or tax policy issues or even private selling and bargaining activities (Caplin and Leahy 1996).

with

$$\sigma_y^2 = Var(ar + u), \tag{2.3.13}$$
$$E[y] = E\left[ar + u\right]. \tag{2.3.14}$$

However, σ_y^2 and $E[y]$ differ, depending on which kind of uncertainty is assumed.[11] Under additive uncertainty the expected loss and the resulting optimal rate of interest are given by

$$E[L] = \sigma_u^2 + \left(ar + \hat{u} - y_f\right)^2, \tag{2.3.15}$$
$$r^* = \frac{y_f - \hat{u}}{a}. \tag{2.3.16}$$

Under parametric uncertainty the loss as well as the optimal interest rate are given by

$$E[L] = r^2\sigma_a^2 + \sigma_u^2 + \left(\hat{a}r + \hat{u} - y_f\right)^2, \tag{2.3.17}$$
$$r^* = \frac{\hat{a}\left(y_f - \hat{u}\right)}{\sigma_a^2 + \hat{a}^2}. \tag{2.3.18}$$

For the example the following parameter values are assumed, $y_f = 0$, $\hat{u} = 6$, $\sigma_u = 1$, $\hat{a} = -2$. Under additive uncertainty the variance of the transmission parameter is set zero, $\sigma_a = 0$. Yet, under parametric uncertainty it holds that $\sigma_a = 2$.

Thus, according to Equation (2.3.12), the social loss is defined with two components, the variance of the output and the expected deviation form the target output level. Figure 2.2 plots in the upper part the expected output deviation (see, Equation (2.3.14)) on the vertical axes, and the variance of the output (Equation (2.3.13)) on the horizontal axes.[12]

In both situations the starting point is given by $E[y] = 6$ and $\sigma_y = 1$, where no interest rate action is taken at all. Subsequently, the interest rate is raised in order to contain the loss caused by the additive shock. Taking into account Equation (2.3.13) and (2.3.14), it becomes obvious that under additive uncertainty the variance of output stays the same no matter the size of the interest rate. Thus changing the interest rate only affects the expected deviation from the target output. Hence, if additive shock terms are the only source of uncertainty, a rising interest rate is equivalent to moving down the dotted line. It shows that only the variance of the error term affects the value of the target variable. There is no trade off situation.

On the other hand, under parametric uncertainty a change in the interest rate affects the expected deviation of the target variable as well as the variance. A rise in the interest

[11] It is assumed that $\sigma_{au}^2 = 0$

[12] For plotting reasons actually the square root of the variance, the standard deviation, is taken.

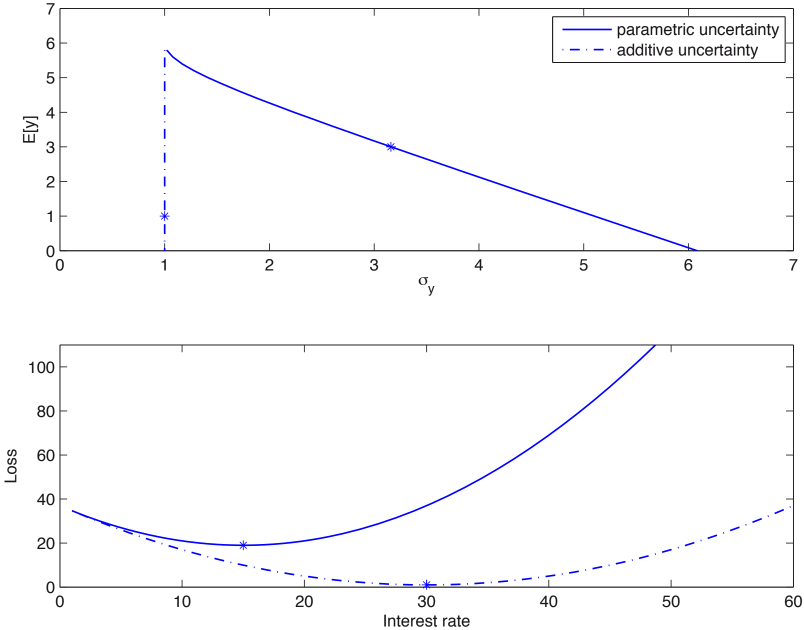

Figure 2.2.: Additive vs. parametric uncertainty

rate is equivalent to moving down the solid line. In Figure 2.2 a rising interest rate lowers the expected deviation of the target variable, but at the same time increases the social cost by a rising variance due to the parametric uncertainty.

The lower part of Figure 2.2 depicts the loss associated with both uncertainty situations. Again, the solid line represents the situation under parametric uncertainty, while the dotted line represents the situation under additive uncertainty only. The loss is calculated by Equation (2.3.15) and (2.3.17), respectively. Accordingly, the minimal loss under additive uncertainty is achieved by an interest rate of 3%, whilst under parametric uncertainty the minimal loss is achieved with an interest rate of only 1.5%. Both optimal situations are depicted by the asterisk. The interest rates levels which are associated with the optimal trade-off situations are also marked in the upper part of Figure 2.2 (Brainard 1967, Kähler and Pinkwart 2009).

This example offers insights on various aspects, which have already been suggested. Firstly, the so-called Brainard conservatism concept is confirmed. The optimal interest rate is lower under parametric uncertainty, compared to a situation with only additive disturbances. Secondly, the upper part of Figure 2.2 shows that under parametric uncertainty, when it is utile for the central bank to react less aggressive, the central bank accepts a higher deviation from the the target level in order to keep the variance low. Thirdly, under parametric uncertainty the loss is always higher compared to a situation under additive uncertainty as long as the interest rate reaction is larger than zero.

2.4. Data Uncertainty: The Use of the Optimal Estimate

2.4.1. Data Revisions: Real-Time and Final Estimates

Most economic key indicators are released as a preliminary forecast, being revisited several times in the subsequent periods. Especially output data – no matter if actual output, or potential, or trend output – is susceptible to measurement errors. This comes as no surprise, due to the difficulties in the measurement and calculation of production and income. Koske and Pain (2008) find that not so much the wrong estimation of potential output causes data revisions of the output gap, but rather the estimation of actual data. Likewise Marcellino and Musso (2011) list potential sources of uncertainty for the estimation of the output gap. They especially highlight errors due to model uncertainty, parameter uncertainty, and parameter instability. Model uncertainty causes measurement errors due to the variety of alternative models resulting in significantly different estimates. Parametric uncertainty as well as instability causes wrong estimates due to the change of parameters over time. This is of special concern when different vintages are used to calculate, e.g., recursively the development of the output gap. However, this can not be an excuse as monetary policy must act, even though the working data might be wrong or no more than approximations to the true data, which are only available with a considerable delay.

Figure 2.3 plots data from OECD (2008) concerning the assessment of the output gap during the time 1991 until 2007.[13] The upper panel plots German data, the lower panel US data. For both regions it can be seen that there is a significant difference between the first estimate (real-time data), the revision two years later, and the final data, which is defined as vintage at least three years after the first inquiry. The difference between the second year revision and the final data is mostly negligible. However, the difference between the real-time data and second estimate is considerable. For the US information on data revisions concerning inflation and output prior to 1991 can be found in Orphanides

[13] The output gap is defined according to OECD (2013) as the difference between actual and potential GDP as a per cent of potential GDP.

(2003b). For the time period 1966 until 1994 he shows that the US output gap is constantly underestimated during the first estimation, from which he concludes that policy failures during the late sixties and seventies are *inter alia* owed to a wrong assessment of the output gap associated with the productivity slowdown (Orphanides 2003b).

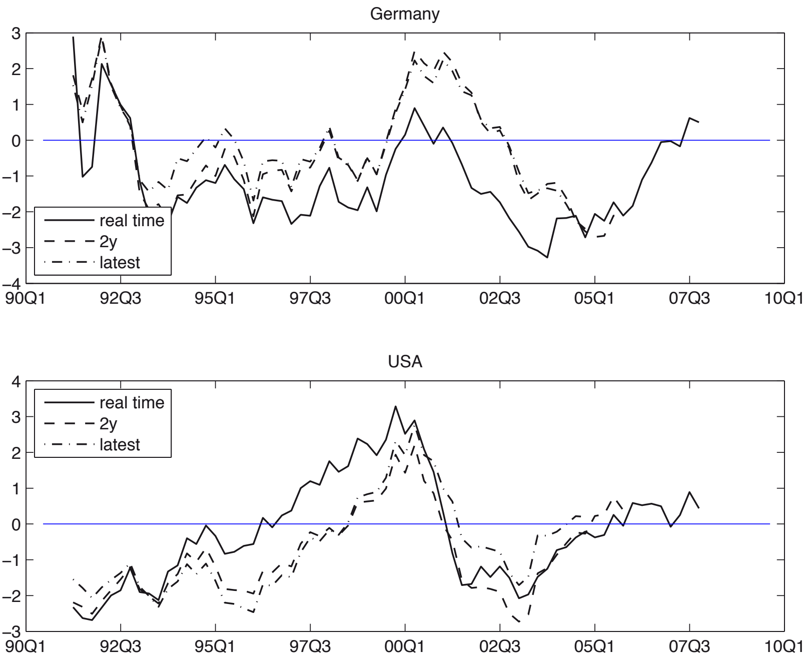

Figure 2.3.: Output gap estimates, Germany (top) and USA (bottom)

According to the OECD (2008) data set an underestimation of the output gap is also true for Germany, with respect to the observed period. The picture changes, however, for the US. From 1994 onwards the output gap is significantly overestimated before it switches back again around the year 2001. Figure 2.4 especially highlights the size and direction of data revisions in both regions. It shows the more or less persistent under- and overestimation of the output gap in Germany and the USA.

Goodfriend (2002) attest the Federal Reserve a successfully tightening of her monetary policy during the early nineties, which has followed an expansive stance in the mid-nineties. However, it could also be that the Fed was just very lucky. While she was reacting on real-time data, which shows a high output gap, in fact, she was only reacting

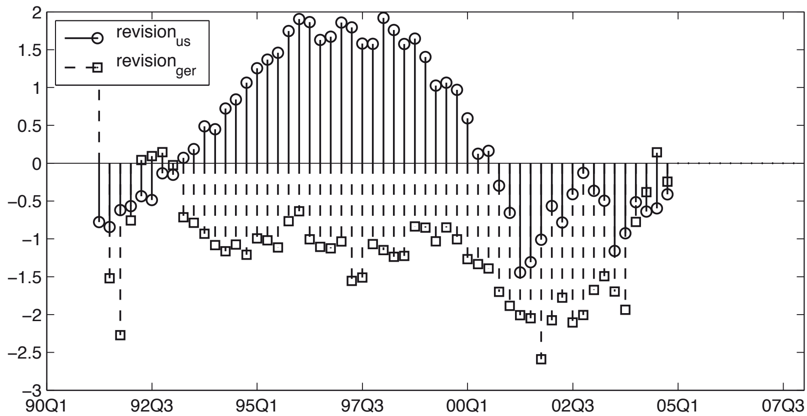

Figure 2.4.: Output gap revisions, Germany (top) and USA (bottom)

to a very low output gap according to the revised data. Hence, her policy was supported by a wrong estimation of the output gap. Against this background, the expansive policy of the Fed at the beginning of the millennium would be appropriate in the light of an underestimation of the output gap. Yet, these examples are highly speculative in many ways. For example, the Federal Reserve has a multidimensional target, and as such a high discretionary leeway. The output gap is just one of many factors that might influence the decision of the open market committee, but it again underlines the importance of data uncertainty and particularly its possible consequences.

The good news is, Tosetto (2008) shows that the correlation between preliminary estimates and further revision, indeed, exhibits a high value of at least 0.75, and Koske and Pain (2008) conclude that even preliminary estimates give a good picture of the business cycle. Only turning points pose a big challenge. During such times data revisions are more likely and also of greater extent, compared to other times (Orphanides and van Norden 1999, 2002). Figure 2.3 and 2.4 support this finding. For the years around 2001 the revision of the German data seems to be extra large. Yet, for the US data, the hypothesis can not be supported. The largest deviations of the sample period are in the mid-nineties. Although, other reasons might be responsible for this period.

For some periods it even seems like an improvement of the estimates, see, e.g., the period starting at the year 2004 in the US, has been realized. Yet, Chiu and Wieladek (2012) find for the time span 1991 until the end of 2005 for 15 OECD countries no evidence for an improvement of estimates, rather the contrary is true. Given the fact of a better inclusion of contemporaneous information, and thus, less noisy forecasts the authors call difficulties in disentangling trend and cycle to account for the bad outcome (Chiu and Wieladek 2012).

2.4.2. Imperfect Observation vs. Signal Extraction

Dealing with a model which accounts for data uncertainty, the formulation of the problem becomes crucial. If the system is defined as a linear relationship of the form $y = a_1 r + u$ and the loss function penalizes positive and negative deviations from the target output y_f, the policy rule is given by

$$r^* = \frac{1}{a_1} y_f. \tag{2.4.1}$$

This is the standard certainty solution, which has been derived before. For this setting, even though there is uncertainty, certainty equivalence prevails, i.e., the policy rule is impervious of an additive shock.

If, however, data uncertainty enters the set-up, some pre-considerations need to be done. In a first scenario the monetary authority observes a level of output, and it is assumed that

$$y_t = y_{t|t} + \eta_t. \tag{2.4.2}$$

Thus, the true value of output, y_t, equals the observed value, $y_{t|t}$, plus a zero mean random variable, η_t. Hence, the true value of output is measured with noise. From Equation (2.4.2) it can be inferred that $E_t(y_t) = y_{t|t}$. So the real-time or observed value equals the expected value of output, which is equal to the best estimate one can make.

Using the method described above and substituting output by its best estimate one will quickly see that there is nearly no difference to the example of additive uncertainty or in other words, again certainty equivalence holds. Given the loss function which penalizes deviations from potential output, but now working with estimates instead of true values, would deliver

$$L = E\left[(a_1 r + u + \eta - y_f)^2\right]. \tag{2.4.3}$$

Due to the zero mean of the disturbance vector and the (best) expectation of output, which equals the observed or real-time variable, the optimal interest rate is again defined as

$$r^* = \frac{1}{a_1} y_f, \tag{2.4.4}$$

which confirms the certainty equivalence proposition of the previous section. From this setting it becomes obvious why some authors call data uncertainty a form of additive uncertainty. If data uncertainty is merely understood as a somewhat sloppy observation of the current values the simple replacement of the actual variable by its best estimate, in fact, conveys the problem to a much easier one (see, for example, Swanson 2000).

Yet, this treatment of data uncertainty falls short of some important aspects. To say that data uncertainty implies certainty equivalence would be to hasty – at least with no further explanation. In the above calculation it has been implicitly assumed that the estimate of output is also the *best* estimate, hence, it is not biased.

The situation changes if instead of Equation (2.4.2) it is assumed that

$$y_{t|t} = y_t + \eta_t, \tag{2.4.5}$$

where y_t is the true value underlying the signal, hence, is not directly observable but can only be inferred by other data. The noise is again represented by η_t.

Although the difference seems to be marginal and maybe negligible it changes the whole character of the situation. Whilst before $y_{t|t} = E_t(y_t)$, this no longer holds for Equation (2.4.5). In fact, $y_{t|t} \neq E_t(y_t)$ but $E_t(y_t) = \left[\sigma_y^2/(\sigma_y^2 + \sigma_\eta^2)\right] y_{t|t}$. An increase in the variance of η_t now poses an effect on the best estimate. The reason for this change originates from the different way the problem is tackled now (Swanson 2004).

In Equation (2.4.2) the estimate was already declared as being *best*. In this case data uncertainty collapses merely into additive uncertainty, i.e., certainty equivalence. Data uncertainty does not impose any restriction or constraint to the policy maker. In fact, to her there is no problem at all as she behaves 'as if' everything is known with certainty. Working with observed variables which fulfill the condition of being the best estimate vanish somewhat the problem of uncertainty.

However, if instead the formulation of Equation (2.4.5) is used, the problem becomes one of *signal extraction*. Now, the policy maker is faced with the problem of inferring the true value of a variable which can not be observed but must be deduced from a signal which stands in an assumed relationship to the the variable of concern (Orphanides 2003a, Swanson 2004).

If this problem of potentially noisy data is ignored and the interest rate policy is conducted in the way of Equation (2.4.1), i.e., utilizing $y_{t|t}$ instead of the *best* estimate, which leads to a distorted policy reaction function. Working with the observed signal would deliver over- or under-reactions depending on the noise, leading to a non-optimal policy response, and thus, to a non-efficient situation. This becomes obvious if a simple rule in which the interest rate is set in accordance only with the observed state variable gap of the form

$$i_t = \gamma_x x_{t|t} \tag{2.4.6}$$

is assumed.

If Equation (2.4.5) holds, that is, the noisy observation is used instead of the best estimate the interest rate is set according to

$$\begin{aligned} i_t &= \gamma_x x_{t|t} && (2.4.7) \\ &= \gamma_x x_t + \gamma_x \eta_t. && (2.4.8) \end{aligned}$$

The underestimation of the output gap, for example, due to an overestimation of the potential output, could thus lead to an expansionary policy, which is in fact not adequate. Clearly this naive policy is absolutely adequate if there is no noise. In this case the problem collapses into a simple optimization exercise (Orphanides 2003a, Walsh 2003b).

In the presence of noisy data the central bank reacts inadvertently to the noise. The greater the noise is the more volatile interest rate changes become. Taking further noisy variables into account could exacerbate the problem even more. If for example the interest rate is additionally set as well in accordance with the inflation gap

$$\begin{aligned} i_t &= \gamma_x x_{t|t} + \gamma_\pi(\pi_{t|t} - \pi^*) && (2.4.9) \\ &= \gamma_x x_t \ + \ \gamma_\pi(\pi_t - \pi^*) + \gamma_x \eta_x + \gamma_\pi \eta_\pi, && (2.4.10) \end{aligned}$$

the error extends to $\gamma_x \eta_x + \gamma_\pi \eta_\pi$, consisting of the noise related to the output gap and the noise related to the inflation gap.

Orphanides (2003b) shows how the the misconception of the US production capacity may have led to an inflationary environment during the 1970s by restricting the ex post analysis to the information set up of those days. For a period from the mid eighties until the mid nineties Orphanides (2001) highlights that, despite output and inflation revisions, which may have contributed to the difference between the actual federal fund rate and a hypothetical Taylor rate based on final data, historical interpretations may also have fostered this effect. Thus, deviations of the actual rate compared to a Taylor rate with final data may have been also undergone evaluations which were interpreted differently these days than it would be today.

Figure 2.5 plots the actual federal fund rate together with a hypothetical Taylor-rule type interest rate.[14] The latter has been calculated using two different vintages of output gap estimates. What matters is not the comparison of the estimated rate and the actual interest rate, but the difference between using real-time data (first estimate) and revised data (final estimate). The wrong appraisal of the output gap (see the lower part of Figure 2.3) transmits itself into a possibly wrong interest rate setting. Thus, quite illustrative for the period 1994 until 2000, it can be seen that an over-estimation of the output gap is accompanied by a higher Taylor rate compared to the rate which would be realized if the final values for the output gap were employed.[15]

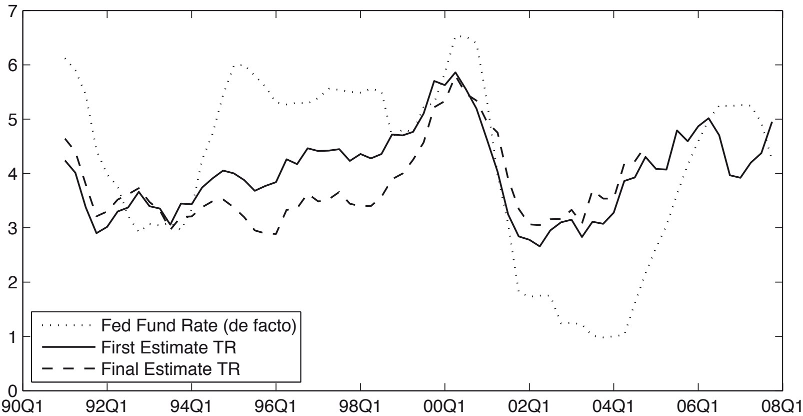

Figure 2.5.: Taylor rates with different vintages of output gap estimates

The more noisy variables are taken into account, the larger the possible error which could be made by the monetary authority. If, however, less variables can serve a less volatile interest rate, due to a smaller additional reaction term, a generally good advice would suggest the central bank merely 'less activism' in her policy, especially when the noise is plenty.

14 The assumed Taylor rule has the form $i_t = 4 + 0.5(\pi_t - 2) + 1.5y_t$ with π_t the actual inflation rate and y_t the OECD output gap estimate (Woodford 2001). It is acknowledged that there are several ways to calculate a Taylor interest rate using, e.g., different values for target variables or utilizing average values for a smoother character. However, the present rule is sufficient for this analysis.

15 Of course it is again noticed that several factors constitute the decision of the Fed and the output gap is just one of many, which can explain as well the difference between the hypothetical Taylor rate and the actual Fed rate like Orphanides (2003b) mentions.

With respect to a stabilizing policy, Orphanides (2003b) brings up two alternative rules to overcome the information problem of level values. In the case of high uncertainty over a state variable estimate, it could be utile to exclude this variable from the policy rule at all. In case of a noisy output gap, it could thus be superior to head for a strict inflation targeting, and by that ignoring GDP measurement errors at all. The effect of an exclusion of one variable can be seen by comparing Equation (2.4.7) with (2.4.9) where the first equation excludes the inflation noise by excluding inflation from the target catalog. One possible outcome of this procedure could be a smoothing of the interest rate and in fact, Rudebusch (2002) argues that, interest rate smoothing can be ascribed to incorrectly excluded variables from the reaction function.

Another possible way, could be the replacement of the output level by its growth rate due to the fact that the change of a gap is less sensitive to measurement problems. Orphanides (2003b) labels those alternatives as ‘prudent’ reaction functions. Under perfect information, however, those rules are not superior. They unfold only if noisy data is taken into account.

2.5. A Further Note on Certainty Equivalence

2.5.1. Certainty Equivalence in Risk Theory

If a system exhibits some special properties, it is said to be certainty equivalent. These necessary conditions are a quadratic loss function in combination with a linear constraint. However, due to the fact that researchers who come from risk theory and those who come from monetary policy theory might have a different sentiment about it, I will discuss this concept briefly at this juncture, and show the interrelations it has to the so far discussed forms of uncertainty.

Since the utility of consumption is a non-linear function it can not be assumed that $E[u(C)] = u[E(C)]$, that is, the expected utility of a given consumption level does not equal the utility of an expected consumption level. A rising curvature of the utility function is accompanied by a rise of this discrepancy. In risk theory this curvature represents the risk aversion of the agent. In general, an agent is risk avers if $u'' < 0$, risk neutral if $u'' = 0$, and risk loving if $u'' > 0$.

Figure 2.6 captures the idea. On the vertical axis utility, and on the horizontal axis consumption is depicted. Both consumption levels, C_1 and C_2, deliver a utility level via the utility function of $u(C_1)$ and $u(C_2)$, respectively. If uncertainty about the attainable level of consumption prevails, the expected level of consumption would be merely the arithmetic average of C_1 and C_2. If both consumption levels are equiprobable, hence,

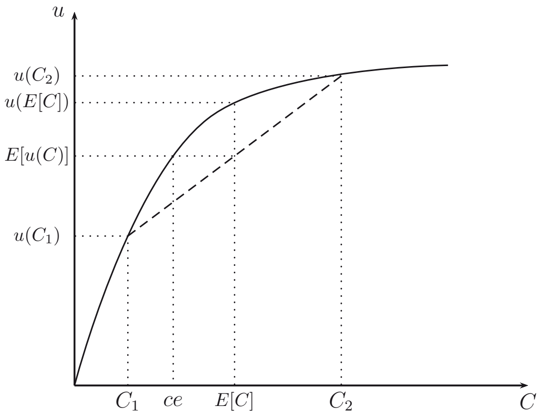

Figure 2.6.: Non-linear utility function

$p = 0.5$, the expected level of consumption lies right in the middle of C_1 and C_2, designated by $E[C] = p \times C_1 + p \times C_2$.

The attained utility from this expected consumption level is achieved by connecting the the two possible extreme outcomes of consumption on the utility function with a straight line, and plumbing it to the vertical axis. It delivers the expected utility $E\left[u(C)\right] = p \times u(C_1) + p \times u(C_2)$. As can be seen in Figure 2.6, the expected utility level deduced from the connecting line lies below the utility that would be achieved if the hypothetical consumption level would be actually realized, $E\left[u(C)\right] < u\left(E[C]\right)$. The opposite holds for a risk loving agent, i.e., $E\left[u(C)\right] > u\left(E[C]\right)$. This implies that a risk avers agent would rather take the expected level of consumption than to play.

To make this point clear consider the following numeric example. The utility function is given by $u(C) = \sqrt{C}$. Let $C_1 = 1$ and $C_2 = 9$ and the probability of each outcome be 0.5. Hence, $E\left[u(C)\right] = E[0.5\sqrt{1} + 0.5\sqrt{9}] = 2$. Yet, $u\left(E[C]\right) = \sqrt{5} = 2.24$, which states that this agent is clearly risk avers. The difference between these two utility levels is that one represents a certain outcome, whilst the other represents merely an expected, and thus, uncertain outcome. The question arises how much consumption with certainty would be needed to be exactly as well off as if the risky opportunity would be chosen, not knowing whether C_1 or C_2 is obtained.

The agent would be indifferent if both utility levels equal each other, that is, the utility received in the certainty case and the utility received in the uncertainty case. However, in this case the level of consumption would differ. The difference between the certain consumption level, marked as ce, and the expected level, $E(C)$, which give both the same utility, is known as the risk-premium. It can be stated that if $ce < E[C]$ the agent is risk averse, hence, she is willing to accept a less but certain level of consumption in favor of an expected higher but uncertain level of consumption. Clearly the opposite holds for a risk loving agent, i.e., $ce > E[C]$.

The more curvature the utility function exhibits the larger is the discrepancy between the certain payoff and its expected counterpart. The curvature represents the risk awareness of the consumer. As she becomes more and more risk averse she is more and more willing to accept a smaller amount of certain consumption, compared to the expected, but uncertain amount of consumption. The less curvature the utility function exhibits, the smaller the risk premium gets. In the extreme case of no curvature, i.e., the utility function is a straight linear line, the risk premium approaches zero and it always holds that $E[u(C)] = u(E[C])$.

It can easily be understood that a high risk awareness is more sensitive to uncertainty than a low one, and that risk neutrality is not affected by uncertainty at all. Figure 2.7 displays two different utility functions, U_1 and U_2. Further, different amounts of uncertainty, expressed by a different variance of the consumption level, are given. It can be seen that a higher amount of uncertainty calls for a higher risk premium to compensate the agent if U_1 prevails. The higher the variance, the greater the discrepancy between the certainty equivalence value and $E[C]$.

If, however, the agent becomes less risk averse, the utility function U_1 shapes more and more towards U_2. If the utility function is actually linear, i.e., the agent is risk-neutral, the degree of uncertainty about the consumption level has no influence on the expected utility of consumption at all. Considering the same choice as before, i.e., between a certain payoff and a risky one, ce and $E[C]$ of Figure 2.6 are always congruent if U_2 prevails. A rising degree of uncertainty, which is equivalent to a wider range between the two possible payoffs, has no influence any more (Varian 2000).

2.5.2. Parametric Uncertainty and Certainty Equivalence

Following the previous argumentation, yet, applied on monetary policy under uncertainty yields the following. If the transition equation of the economy is linear, and the loss function has a quadratic form (positive and negative deviations are penalized equally), the degree of uncertainty in the system does not influence the reaction function except that only expected values of the system are taken into account. Nevertheless the utility is certainly affected.

To see why certainty equivalence ceases to hold in the case of multiplicative uncertainty, opposed to additive uncertainty, Figure 2.9 covers the case of additive uncertainty and

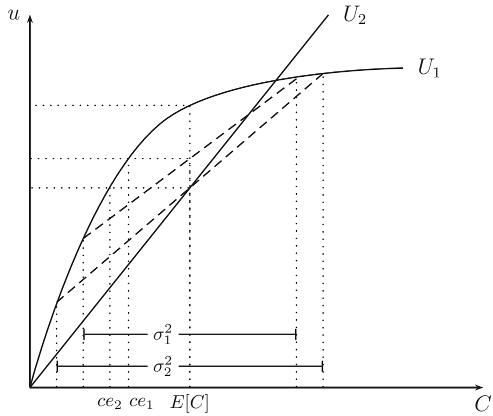

Figure 2.7.: Utility function and uncertainty

Figure 2.10 the case of multiplicative uncertainty. In both figures the policy instrument r is given on the horizontal axis, and the target variable y on the vertical axis. The relation between the policy variable and its target variable shall be given by the solid, linear, and negative sloped line and is of the form $y = c - ar$ with c being a constant and a some sort of reaction coefficient. The intuition is, given some value for c a rising interest rate depresses the overall economic output, i.e, a lower y. Hence, there is a negative connection between the policy instrument and the target variable.

In a certain environment each change in the policy variable would induce a predictable change in the target variable. Or in a revers order, to find a desired level of output, the necessary interest rate level can be easily deduced via the linear relationship. This is shown in Figure 2.8. It is assumed that the level of output is below its desired level, marked by the subscript f. In order to achieve this desired level the interest rate must be lowered on $r^* < r$.

If additive uncertainty enters the set up the expected value of the target variable and the realized value may differ. The simple linear relationship of before is amended by an exogenous, zero mean shock vector and changes into $y = c - ar + \epsilon$. In Figure 2.9 the expected level of output, given some interest rate level r, is marked by the solid line. The actual realized value of output could be within a band around its expected value which is suggested by the dashed lines. Hence, setting the policy instrument at r_1 would on average result in y_1^E, but could also be somewhere within the range y_{11} and y_{12}. This range is also shown by the vertical solid lines.

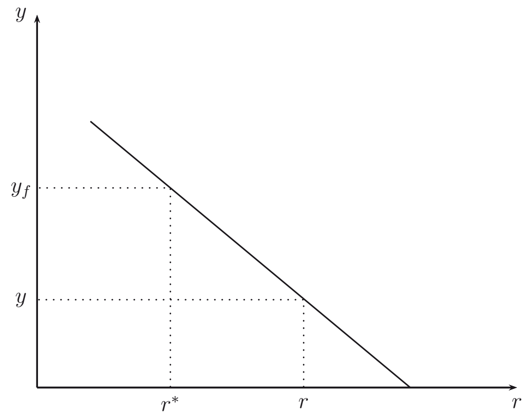

Figure 2.8.: Certainty in a linear framework

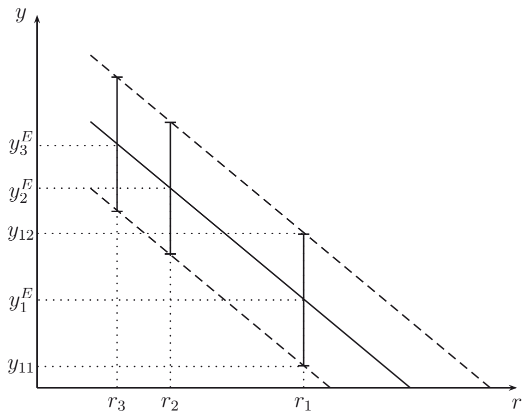

Figure 2.9.: Additive uncertainty in a linear framework

If again the output is lower than desired, the policy maker would implement the same policy as under certainty to reach the desired level of output. On average she achieves with the interest rate r_2 the output y_2^E, but again she could also end up in the range marked by the middle horizontal solid line. Figure 2.9 shows very nicely how the variance of the error term is transmitted one-to-one to the target variable, which itself has now become a random variable with $\sigma_y^2 = \sigma_\epsilon^2$.

What is striking is the fact that the uncertainty can not be reduced actively by the policy maker. Setting the interest rate from r_1 to r_3 would not be different from a change of r_1 to r_2 in terms of potential output deviations. The variance of output remains the same no matter the change of the control variable. The variance of the target variable is thus independent of the policy parameter setting. The policy maker acts in a situation of certainty equivalence. Although the setting of the policy parameter is not different under certainty than under additive uncertainty, clearly the policy maker is worse-off under uncertainty due to the potential output deviations from its desired level (Sack and Wieland 2000).

With multiplicative uncertainty the picture changes. Due to the fact that the transmission from the policy instrument to the target variable, given by the parameter a, is uncertain, the variance of the target variable is now $\sigma_y^2 = \sigma_a^2 r^2 + \sigma_\epsilon^2$. Thus, the variance of output depends from the uncertainty of the slope, but is as well increasing with the size of the policy reaction parameter. The comparison in Figure 2.10 shows, the size of an interest rate change matters for the potential output deviation marked again by the solid horizontal lines. In case of a wrong estimation concerning the transmission parameter, setting the interest rate from r_1 to r_3 creates a potentially larger output deviation than raising the interest rate only from r_1 to r_2. It becomes evident that certainty equivalence ceases to hold. The variance of the output can be actively reduced by the setting of the policy instrument. From an analytical point of view the expected value of the value function depends on the variance-covariance matrix of the state vector. If all parameters are non-stochastic, this matrix must coincide with the variance matrix of the disturbance vector and is thus independent of the instrument variable. If on the other hand the variance-covariance matrix depends also on the state or instrument, this matrix can be influenced by the central bank (Sack and Wieland 2000, Söderström 2002).

This setting shows the earlier discussed trade-off situation for the policy maker. On the one hand an aggressive policy reaction will move output closer to the desired target level on average, but on the other hand this will also create a higher uncertainty. Hence, a higher variance of the resulting level of output. A possible solution to this problem would be the partition of one big step into several small steps as shown in Figure 2.11. This confirms the previous finding, whereby under parametric uncertainty the optimal policy reaction does not yield a complete target achievement. The underlying idea of this behavior is a 'learning' policy maker. Given that some desired level of output is only achievable with tremendous uncertainty concerning the transmission, and thus, the resulting output levels, sequential steps could improve the situation. Instead of one big interest rate step, the policy maker only sets the rate from r_1 to r_2. After that she observes the consequences of his action and takes these into account for his second assessment, setting the rate from

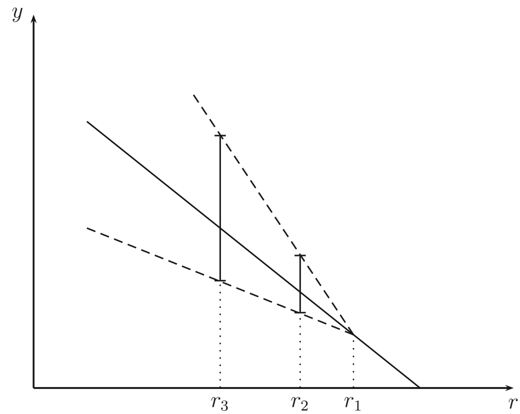

Figure 2.10.: Multiplicative uncertainty in a linear framework

r_2 to r_3. By that she narrows the potential uncertainty of missing the desired level of output y_f (Sack and Wieland 2000).

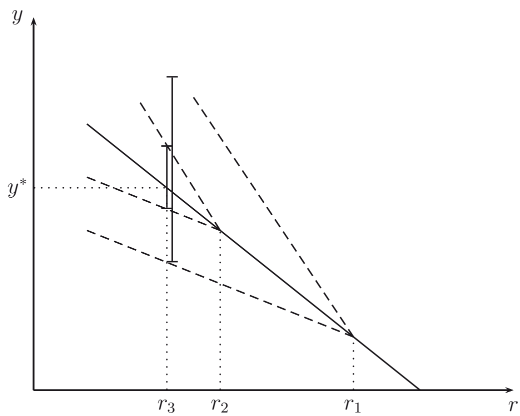

Figure 2.11.: Multiplicative uncertainty, several vs. one step

2.5.3. Data Uncertainty and Certainty Equivalence

Confronted with the problem of signal extraction and not merely with an additive error term, the monetary authority needs to work with the *best* estimate – or, in other words, a not biased estimate – to react 'optimal'. Yet, the question arises whether the forming of an optimal estimate can be done without any reference to the control problem. If so certainty equivalence should prevail and monetary policy can be conducted the standard way, replacing uncertain variables simply by their best estimates.

In fact, this separation is admissible. The *separation principle* states that in order to behave optimal, the monetary authority should first optimize 'as if' everything is known with certainty and should secondly substitute her optimal forecast for the unknown variable (Hansen and Sargent 2005). Figure 2.12 captures this idea.

$$\text{observation} \xrightarrow{\text{filter}} \text{filtered signal} \xrightarrow[\text{optimization}]{\text{replace \&}} \text{policy reaction}$$

signal & noise	signal (unbiased)	best estimate

Variance of noise term matters w.r.t. finding the right k

Variance of noise term does not matter any longer w.r.t. expectations

Figure 2.12.: Separation principle

It is assumed that the observation of a variable is contaminated by noise. This noise is filtered out by an appropriate filter such as the Kalman routine which uses all available measurement data to calculate in a predictor-corrector manner the best estimate. Using an initial guess or measurement, which is compared with a second observation, delivers a more accurate data point, which can be used again to be compared with a third measurement and so on. The goal of the process is to find the best estimate of the unobservable or noisy variable. Thereby best is defined as the minimum variance estimate. At the end there should be an estimate of an unobserved variable, which indeed has the lowest, that is best, variance.

The big advantage of this filter algorithm is that not every past data must be kept in storage. All past data are included in the latest best estimate. Hence, the problem can always be treated as a two period problem with 'old' and 'new' information. This improves the filter also from a technical point of view and contributes to its popularity (Maybeck 1979).

Not to go too much into detail, Equation (2.5.1) shows that the assessment of the variable $x_{t|t}$ depends on the past estimate $x_{t|t-1}$, and the new information gathered by observing z_t.

$$x_{t|t} = (1 - k_t)x_{t|t-1} + k_t z_t \tag{2.5.1}$$

So the best estimate of $x_{t|t}$ is constituted by the estimate made without the information of the new observation, $x_{t|t-1}$, and the new information, z_t. Thereby, more weight is given to the less uncertain part. The weight parameter k_t is obtained by differentiating Equation (2.5.1). Equipped with the new information of the observation z_t and the optimal weight k_t the posterior variance is calculated. This is in turn used to form an estimate and the according variance for the next period.

In general, the so-called *Kalman gain* which minimizes the variance of the estimate is given as

$$k_t = \frac{p_{t|t-1}}{(p_{t|t-1} + \sigma_v^2)}. \tag{2.5.2}$$

with $p_{t|t-1}$ denoting the prior variance and σ_v^2 the noise of the observation.

The general prior variance is

$$\begin{aligned} p_{t|t-1} =& \rho^2 p_{t-1|t-1} + \sigma_u^2 \\ =& \rho^2 \left(\frac{1}{p_{t-1|t-2}} + \frac{1}{\sigma_v^2} \right)^{-1} + \sigma_u^2, \end{aligned}$$

with ρ depicting the parameters, $p_{t|t}$ the posterior variance, and σ_u^2 some additive noise term of the process of interest. It can be seen from Equation (2.5.2) if the noise of the measurement becomes zero the Kalman gain equals one, that is, the observations are fully taken into account in Equation (2.5.1). If, however, the prior variance gets smaller compared to the noise variance, the Kalman gain gets smaller, and thus, the newly measurements become less important compared to the prediction based on the past observations. The estimate gets better and better the more observations are used. At the end of this process stands the best estimate of the unobserved variable. This estimate entitles the decision maker to form unbiased expectations regarding the unobservable variable.

The proper working of this filter is shown by Figure 2.13. For 200 periods it is assumed that the true process follows a state equation of the form $x_{t+1} = ax_t + u_{t+1}$. The observation equation is given by $z_t = cx_t + v_t$.[16] The simulated true time series is observed with noise. Yet, despite the noise in the observation, the Kalman filter delivers a series, which is most of the time quite close to the true values.

For the calculations of the filtered variables the size of the variance of the disturbance is absolutely relevant. It defines the size of the Kalman gain vector k, and thus, the weight placed on the new information, seen in Equation (2.5.2) and (2.5.1). However, after the

16 The parameterization, albeit not crucial if in certain ranges, is $a = 0.8$, $c = 0.7$, $\sigma_u^2 = 0.001$, $\sigma_v^2 = 0.01$. The Kalman filter can be used to overcome data uncertainty, however, the most popular usage is the determination of model parameter in combination with a maximum likelihood estimation.

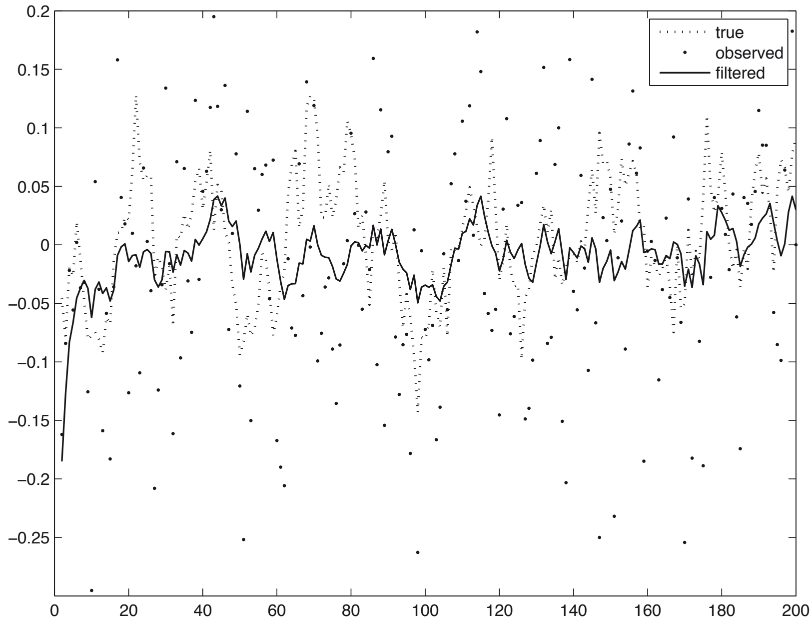

Figure 2.13.: Simulated times series with Kalman filter

observations have undergone the filter process the resulting best estimate entitles the decision maker to calculate the optimal policy reaction (Kalman 1960, Hamilton 1994).

Orphanides (2003a) clarifies with respect to data uncertainty that it is not true, whereas this 'additive' uncertainty does not matter at all. The uncertainty, measured by the variance of the error term, matters with respect to finding the optimal weight, which varies with the size of the error term. But the measurement error, or better its variance, does not matter with respect to the conditional expectations once the noise is filtered from the data.

The *separation principle* states the separation between optimizing and filtering. The order of these two steps is irrelevant. Whether first the optimization is conducted and second expectations are formed or the other way around does not matter. It is admissible due to the fact that an optimization is feasible even without knowing the true value of the exogenous variable by solving the decision makers Euler equation for the control variable. After that, in a second step, the conditional expectation of the exogenous variable can be deduced, given the distribution function of the shock and replacing the variable from the first step (Svensson and Woodford 2003b, Hansen and Sargent 2005).

Hence, it can be said that data uncertainty *per se* does not exhibit certainty equivalence. It depends whether the issue is tackled merely as an additive uncertainty problem or as a signal extraction problem, i.e., if the estimate of a given variable is biased or not. An unbiased estimate changes the problem into one of additive uncertainty, where the principle of certainty equivalence is fulfilled. If the problem is one of signal extraction and one has to infer a signal from noisy observations, the uncertainty or noise surrounding this signal is crucial and must be filtered out. In this sense, the filtering process is sensitive to uncertainty, however, once the noise has been removed, a standard optimization problem remains.

2.6. Interest Rate Smoothing due to Uncertainty

To give a short summary, this chapter has dealt with three different forms of uncertainty – namely additive, parametric, and data uncertainty – and already quarried some important insights. It has been shown that under additive uncertainty the policy rule is impervious to the exogenous shock vector. This finding, which only applies to very restrictive cases, is known as the *certainty equivalence* principle. On the other hand, Poole (1970) has shown that while the policy rule might be unaffected by additive uncertainty, the loss of the central bank in terms of deviations from a given target variable is not. Poole (1970) demonstrates that depending on the source of the shock, it could be more or less favorable to utilize the interest rate or the money supply as appropriate instrument of choice, due to the different impact they have on the loss function.

The most prominent example on monetary policy under uncertainty is by all means Brainard (1967). His seminal work demonstrates how the optimal rule for setting the interest rate is affected by the uncertainty surrounding the model parameter. He shows that under multiplicative uncertainty the concept of certainty equivalence ceases, thus the monetary authority must take account of the uncertainty compared to the case with additive uncertainty. The so-called *Brainard conservatism* suggests a rather moderate reaction to shocks in an uncertain environment. Although this concept has been criticized in many ways it still prevails in conducting monetary policy today.

Furthermore, the example of data uncertainty has revealed some important findings regarding the fact that most of the underlying economic data is only available with a significant lag and object of several data revisions. It has been shown that data uncertainty *per se* is not sufficient to tell whether or not certainty equivalence prevails. If the estimate of an uncertain variable is called to be *best*, the monetary authority can go on in setting her policy working with this estimate instead of the true but hidden value. Nevertheless, in achieving this *best* estimate an appropriate filter algorithm – like the Kalman filter routine – is needed, which definitely takes account for the uncertainty.

These findings are mirrored in the path of the main refinancing rate of the most important central banks such as the Federal Reserve, the Bank of England or the European Central Bank, all of which plotted in Figure 2.14. They all offer a similar pattern. Most of the time interest rates (i) move for a long time into the same direction, that is reversals are kept on a minimum. (ii) Steps into one direction are rather parceled into small but many than big but few steps. (iii) Interest rates are often kept constant for several subsequent periods. This pattern of partial adjustment has often been labeled gradualism or interest rate smoothing. The literature offers several explanations for this property, which can be divided into two big classes. The first class can be described by the expressions 'shaping expectations'. The second can be labeled 'uncertainty' (Goodhart 1999, Sack and Wieland 2000, Gerlach-Kristen 2004).

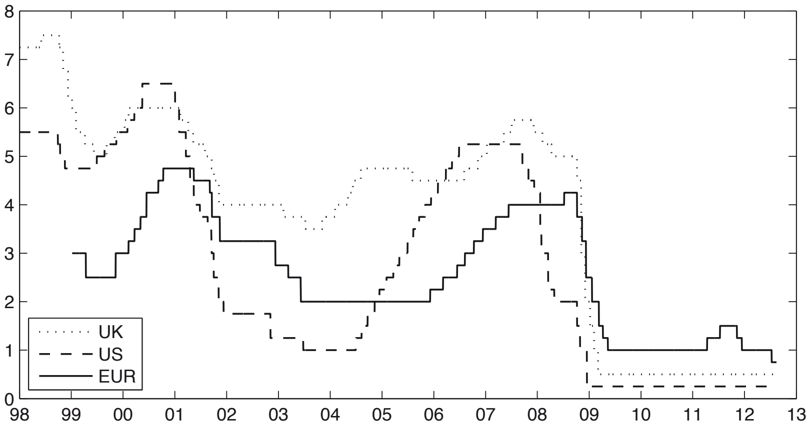

Figure 2.14.: Interest rates of major central banks

For the first class it can be stated that, interest rates move slow due to some sort of dealing or utilizing a forward-looking behavior of market participants. The agents infer future steps of the central bank by observing past and present decisions. Small steps into one direction (for a sufficient long period) would thus shape expectations of future steps and provide a clear guidance to the public. By that, controlling the short-term interest rate would trigger off long-term interest rates and enforcing the power of the short-term nominal interest rate. Combined with a sufficient communication this policy would increase the effectiveness of monetary policy even though only small interest rate changes are conducted (Goodfriend 1991, Goodhart 1999).

Another explanation within this class is given by Goodfriend (1987). To him central banks smooth interest rates in order "to maintain 'orderly money markets' " (Goodfriend 1987:339). This behavior would preserve the economy of large swings in asset prices followed by a higher risk of bankruptcies and banking crises. These kinds of factors, where the central bank actively manages the interest rate in order to shape expectations, are also called *intrinsic* or *endogenous*. A typical setting would be the partition of a desired interest rate change into several small steps, dragging the adjustment process over a longer period (Rudebusch 2006). Other endogenous reasons might be the building up of reputation and the consensus decision-making process on part of the central bank (Sack and Wieland 2000).

Despite these explanations, and first and foremost to a greater extent relevant for this work, uncertainties can provide a good reasoning for a gradual stance of monetary policy. This has been shown in the previous sections. If the central bank lacks of information, and thus, is unable to set its interest rate in an accurate and appropriate manner it creates a persistence of the policy rate. This kind of interest rate inertia is said to be *extrinsic* or *exogenous*. The policy rate reacts with respect to the validity of given parameters and is set with caution. As the information level changes policy actions may change as well (Rudebusch 2006).

For the case of data uncertainty most of the literature considers a less aggressive interest rate policy as appropriate. These findings hold under simple rules, such as the Taylor rule in form of reduced response coefficients, as well as under optimal policy rules (see, e.g., Cagliarini and Debelle 2000, Sack and Wieland 2000, Rudebusch 2006). This chapter has shown, if noise prevails in the data, a monetary authority would inadvertently react on this noise with the consequence of a higher interest rate volatility. A reaction function based on noisy data is likely to over- or underestimate the proper interest rate reaction, also inducing unnecessary interest movements. So the mere awareness of noise in the data should lead any central bank to be react more caution which could be the explanation for a smooth interest rate path (Cagliarini and Debelle 2000, Smets 2002, Orphanides 2003a).

Under parametric uncertainty, as the Brainard example has demonstrated, uncertainty enters the reaction function of the central bank, recommending a more cautious policy than under certainty due to the fact that certainty equivalence does not apply. This was demonstrated before in Section 2.5.1. Not knowing the exact transmission of interest movements should call for a less aggressive stance in monetary policy in order to avoid exaggerations due to unpredicted consequences. Although, this result, as shown for example in Craine (1979), Cagliarini and Debelle (2000), or Söderström (2002), does not hold in general. The question, whether one should be more or less aggressive cumulates in the case of model uncertainty, which some authors interpret as parameter uncertainty as well. However, in this case nearly no general answer about how the optimal interest rate rule should be set can be given, due to the fact that the right answer depends on so many factors (Rudebusch 2006).

Concluding, interest rate inertia maybe due to an active stance of monetary policy aiming for some sort of expectation shaping. Nevertheless, without assuming an active, or endogenous interest rate smoothing, uncertainty may be the main cause for smoothing interest rates. This again highlights the very early mentioned two challenges of monetary policy, which have been on the one hand the appropriate handling of uncertainty and on the other the active shaping and control of expectations.

3. Implementing Uncertainty into Monetary Models

3.1. The Benchmark Model

The previous chapter has enlightened the notion of uncertainty – which is actually risk – and has given some very common and illustrative examples of the consequences of uncertainty and the implications to monetary policy in general. Especially for the cases of additive, parametric, and data uncertainty, this has been shown. However, this has been done without making use of any kind of macro model, but in a rather simple equation context.

In what follows, I introduce uncertainty within the context of the New Keynesian macro model. Therefore, briefly, the certainty case is demonstrated. Furthermore, to show how uncertainty is implemented into this context, and if the previous findings can be confirmed, exemplary, parametric uncertainty is picked up again.

For a more general approach, the New Keynesian Model is transferred into the so-called *state space* notation, which gives a compact matrix representation of the system. This way of representation allows a convenient implementation of a so far not mentioned form of uncertainty – namely, model uncertainty – by discriminating between *structured* and *unstructured* uncertainty. Tackling model uncertainty can be achieved in several ways. The most popular method is robust control. However, particularly during the last years a rather new method has emerged, which especially allows to account for regime switches.

3.1.1. Basics of the New Keynesian Benchmark Model

Generally accepted for the analysis of monetary policy is the use a New Keynesian model framework. The structure of this model consists of two equations deduced from a so-called micro founded framework. While the explicit deduction is not done here, the model will be introduced in order to fulfill a benchmark analysis for further comparison. The deduction of the model relies heavily on Clarida et al. (1999), McCallum and Nelson (2004), Gali (2008), and Walsh (2010), but can also be found in other publications.

O. Sauter, *Monetary Policy under Uncertainty,*
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The New Keynesian framework is mainly characterized by two equations referred to as *New Keynesian Phillips Curve* (NKPC) and dynamic *IS Curve* (IS). Both equations are derived from the optimal behavior of a representative household and/or a representative firm. The household compares his utility of consumption with the disutility of labor, thus he maximizes his expected present value of utility. The optimality condition of this intertemporal allocation decision is given by the Euler equation of the household, which relates todays consumption to the (discounted) level of future consumption. Expected higher consumption in the future raise already today's consumption level due to consumption smoothing. This is equivalent with a higher demand today. By choosing the optimal level of consumption, the household also decides on the amount of labor he is willing to offer. This decision depends on the real wage as well as the amount of money he wants to hold, given the respective opportunity costs of holding money.

Accordingly, firms maximize their profits. It is assumed that the representative firm works under a situation of monopolistic competition, therefore, the firm does not only consider technical constraints of the production function, but also market constraints given by the demand for the specific good she produces. In the end, firms are not allowed to change prices fully flexible. Rather each period only a stochastic determined fraction of firms is allowed to change their product prices. Against this background, setting the price of her goods each firm has to consider current as well as future cost developments, due to the fact that it is random whenever she is able to change her prices again. This pricing is referred to as Calvo-pricing (Calvo 1983). When optimizing her behavior, the representative firm minimizes her costs, constrained by the production function. Yet, since she is acting under monopolistic competition and households do not only decide on the optimal overall consumption level, but also on the composition of the respective consumption basket, she must additionally consider the optimal price for the goods she is providing,with respect to the overall price-level of the economy.

Combing the optimality conditions of the representative household with those of the firm delivers an economy equilibrium where goods markets are cleared. The IS-Curve is given by the linearized version of the optimality condition of the household – the Euler equation – of the form

$$y_t = E_t y_{t+1} - \frac{1}{\sigma}(i_t - E_t \pi_{t+1}) + e_t^y. \tag{3.1.1}$$

Equation (3.1.1) already captures only deviations from the steady-state values, hence, y_t represents the output gap of the economy. Accordingly, i_t denotes the deviation of the nominal interest rate from its steady-state value. The current output gap is related to the expected one period ahead gap, as well as the expected real interest rate. The intertemporal elasticity of substitution is given by $1/\sigma$. Equation (3.1.1) can be understood as the economies demand side, thus, e_t^y, represents a demand shock to the system.[1] In sum, goods demand depends positive on future expectations and negative on the (expected) real interest rate (Clarida et al. 1999, Gali 2008, Walsh 2010).

The second equation, called the *New Keynesian Phillips Curve*, is derived by the optimal pricing decision of the representative firm. This decision depends on the size of the fraction of firms that are allowed to change their price in the current period as well as the overall price level, which consists of the prices of goods that are allowed to change in the current period and those that are not. Additionally, marginal cost are related positively to the size of the output gap. A rising discrepancy between actual output and steady state output (positive output gap) impose pressure on the marginal costs and causes a rise of the inflation rate. The connection between inflation and output is represented by

$$\pi_t = \beta E_t \pi_{t+1} + \kappa y_t + e_t^\pi . \tag{3.1.2}$$

As before, Equation (3.1.2) captures only deviations from the steady-state. Current inflation π_t is determined by future expectations and the current output gap x_t. Due to the forward-looking character of Equation (3.1.2) it becomes clear that past inflation deviations do not matter in determining the current level of inflation. Or, in other words, past inflation has no influence on the pricing decision, which is only driven by expected marginal costs and the possibility of future price adjustments. The fraction of firms that are allowed to change their price is given by $1 - \omega$. A rising ω implies a higher price rigidity, since less firms are allowed to change their price in the current period. For $\omega = 0$ every firm is allowed to change her price. This is equivalent with no price stickiness at all.

The discount factor of future inflation is given by β. A higher discount factor would weight future price developments more. Due to $\kappa = (1-\omega)(1-\beta\omega)(\sigma+\varphi)/\omega$, the discount factor as well as the fraction of firms that are allowed to change their prices influence the transmission from output to inflation. A rising discount factor implies a decreasing κ. Hence, current positive output deviations (i.e., higher marginal costs) are weighted less and inflation becomes less sensitive to changes of the current output. Inflation is thus determined by current and expected future marginal costs and the probability of being the lucky one who is allowed to change the price of her offered goods.

The NKPC represents the supply side of the system, thus, e_t^π can be understood as a supply shock or alternatively cost-push shock, which covers all effects on marginal cost. The exact form of the demand and supply shock vector, as well as their moments, will

[1] The specific form is postponed.

be defined later as they could have a significant influence on the outcome of the model (Clarida et al. 1999, Gali 2008, Walsh 2010).

Yet, the system so far is not stable. This is clear from an intuitive view as well as it can be proven mathematically. If, e.g., government expenditure rises, represented by a positive demand shock vector, the actual demand becomes greater than the actual potential output. A widening of the output gap in the same period is the consequence. This widening corresponds to higher marginal costs and is transmitted to inflation via Equation (3.1.2). Increased inflation leads to a higher expected inflation for the next period, and thus, a negative expected real interest rate, $i_t - E_t\pi_{t+1}$. This enlivens the output gap again. The effect is amplified by a strong κ, for example, due to a small ω, which means a high price flexibility due to the fact that many firms are allowed to change their prices.

To close the system a third equation is needed. This third condition, which should ensure the stability of the system, is a monetary policy rule that can take on two general forms, an *instrument rule* or a *targeting rule*. Both types are shown below, first the popular Taylor-type instrument rule, afterwards, the solution under discretion and commitment, i.e., under a targeting rule regime.

Instrument rules express the instrument, typically the interest rate, as a prescribed function of the available information. If only a few variables are taken into account, or only intuitions of the monetary authority, like how she should react on disturbances, this is called a *simple* or *restricted instrument rule*. It can also be further distinguished between explicit or implicit instrument rules depending on whether the instrument is set as a function of predetermined or non-predetermined variables. Although these kinds of rules are of minor political relevance they serve mainly for baseline scenario estimates. The most prominent example is given by Taylor (1993) and can be depicted as

$$i_t = \delta_\pi \pi_t + \delta_y y_t. \tag{3.1.3}$$

Equation (3.1.3) could be further extended to react as well on past interest rates in order to smooth the path and to avoid large changes. The interest rate rule is plugged into Equation (3.1.1) of the benchmark model. Together with Equation (3.1.2) the model is solved.

According to Equation (3.1.3), the interest rate is set in response to output and inflation target violations. If output or inflation rise above the desired target value, the interest rate is increased as well. To achieve a stable system, however, Equation (3.1.3) has to fulfill some preconditions with respect to its coefficients δ_π and δ_y, the respective reaction parameters. In general, for a determinate system the inflation rate reaction parameter δ_π

should be greater than one. This is referred to as the *Taylor principle*. It holds even if the reaction coefficient of output violations is set zero, thus the monetary authority only accounts for inflation deviations.

3.1.2. Optimal Policy Solutions

Beside the use of an instrument rule like the Taylor rule the use of a targeting rule shall be briefly discussed. A targeting rule is intended to minimize a certain loss function with respect to the model constraints, which represent the state of the economy. Possible and most often used target variables are inflation and output. Thereby, forecast values, current values, as well as lagged values can be utilized. (Rudebusch and Svensson 1998, Svensson 1999).

Equation (3.1.1) and (3.1.2) have of a forward-looking character. The public as well as the central bank form (rational) expectations about the possible future realizations of variables, but also abut possible future steps of the central bank. Hence, today's policy decisions must always be made under the consideration of these expectations. In this context the central bank could commit herself once and for all to a given reaction function, or she could re-optimize each period by setting her instrument in a day-to-day manner. The first strategy is known as the *commitment* policy. The latter is called the *discretionary* policy. In both cases, however, given the environment, i.e., the realization of the (target) variables, the setting of the policy instrument is done in an optimal manner.

The loss function of the central bank is the objective function of the optimization problem. It is typically of the form

$$L_t = \frac{1}{2} E_t \sum_{t=0}^{\infty} \beta^t (\pi_t^2 + \alpha_y y_t^2). \tag{3.1.4}$$

The variables π and y represent deviations from the target inflation rate and target output. The weight the central bank assigns to inflation target violations relative to output violations is controlled by α_y. Hence, the loss function penalizes deviations from the target variables. It can be shown that even the loss function can be deduced from a micro-founded framework in which $\alpha_y = \kappa/\xi$, with ξ being the elasticity of substitution of goods. Hence, deviations from output and inflation cause losses to the representative household, which are proportional to the losses of Equation (3.1.4). Minimizing Equation (3.1.4), thus, minimizes the overall loss of the society (Rotemberg and Woodford 1998).

Additionally to the penalization of inflation and output deviations, in some cases it might also be desirable to penalize too large interest rate changes. Reasons for this behavior can be found in Chapter 2. If the central banks accounts as well for interest rate changes, the loss function must be extended by an additional term of the form $\alpha_i(i_{t-1}-i_t)$. Hence, big steps of the interest rate from one period to the next rise the loss of the central bank in the same way deviations of the previously mentioned target variables do. Accordingly, α_i represents the relative weight placed on these deviations. Acknowledging for interest rate changes the objective of the monetary authority changes into minimizing the loss of Equation (3.1.4), by setting the policy instrument in a way that drives output and inflation in the most efficient manner keeping in mind not to implement too large interest rate steps (Gali 2008, Walsh 2010).

3.1.2.1. Discretion

Considering the *discretionary* case, the monetary authority does not commit herself on an everlasting policy rule. Instead she re-optimizes her behavior by setting the policy instrument in a day-to-day manner. Because she does not precommit herself, expectation about future developments are not taken into account. They are treated as fixed and given. Nevertheless there is a formation of expectations on behalf of the private sector.

It is assumed that the central bank has the total control over the demand curve, which maps the interest rate to the output gap.[2] Thus, the optimization of the loss function is only subject to Equation (3.1.2). The 'one period' loss or objective function shrinks to

$$L_t = \frac{1}{2}\left[\pi_t^2 + \alpha_y y_t^2\right] + F_t, \tag{3.1.5}$$

which is minimized subject to

$$\pi_t = g_t + \kappa y_t + e_t^{\pi}. \tag{3.1.6}$$

Because expectations are treated as given, the expression $\beta E_t \pi_{t+1}$ can be substituted by the constant term g_t, and accordingly $F_t = \frac{1}{2} E_t \sum_{t=1}^{\infty} \beta^t(\pi_t^2 + \alpha_y y_t^2)$ covers all losses beyond the current period.

The first order conditions deliver the *optimal targeting rules* for the variables output and inflation

$$\pi_t = -\frac{\alpha_y}{\kappa} y_t, \tag{3.1.7}$$

$$y_t = -\frac{\kappa}{\alpha_y} \pi_t. \tag{3.1.8}$$

[2] This simplification is admissible since it does not change the qualitative results (Sauer 2007).

These optimality conditions state a connection between output and inflation, which is known as a *leaning against the wind* policy. If prices tend to rise above the desired level, the monetary authority has to repress output below its steady state level in oder to relive marginal costs. Thereby, the speed or aggressiveness of adjustment depends on the fraction $-\kappa/\alpha_y$. As long as κ remains untouched, more weight placed on the output target causes a higher inflation variability if $\alpha_y \to 0$, the variability of inflation becomes zero. The opposite holds for output variability (Clarida et al. 1999).

To form the rational expectation solution of the model, the targeting rule of Equation (3.1.8) is combined with Equation (3.1.6), the constraint of the optimization problem. This delivers a difference equation for inflation in the current period

$$\pi_t = \beta E_t \pi_{t+1} + \kappa y_t + e_t^\pi \tag{3.1.9}$$

$$\pi_t = \frac{\beta \alpha_y}{\alpha_y + \kappa^2} E_t \pi_{t+1} + \frac{\alpha_y}{\alpha_y + \kappa^2} e_t^\pi . \tag{3.1.10}$$

The rational expectation solution states how inflation and output behave in equilibrium, when the central bank behaves according to the optimal targeting rule. It is obtained by utilizing the *minimum state variable* (msv) solution. Under discretion the only relevant state variable is the shock vector, e_t^π. Hence, the following guess is made, with Ψ_π being an undetermined coefficient,

$$\pi_t = \Psi_\pi e_t^\pi . \tag{3.1.11}$$

Equation (3.1.11) is called the *perceived law of motion* (plm).[3]

It is assumed that the shock follows an AR(1) process, $e_{t+1}^\pi = \rho e_t^\pi + \epsilon_{t+1}$, with ϵ_{t+1} being white noise with constant variance. Expectations from Equation (3.1.10) can be substituted with the use of the plm in Equation (3.1.11) according to

$$E_t \pi_{t+1} = \Psi_\pi e_{t+1}^\pi = \Psi_\pi \rho e_t^\pi \tag{3.1.12}$$

[3] By construction the minimum state variable solution of McCallum (see, e.g., 1983, 1999) excludes bubble solutions. This is achieved by limiting the set of possible solutions to "linear functions [...] of a minimal set of "state variables," i.e., predetermined of exogenous determinants of the current endogenous variables" McCallum (1999:626). Equation (3.1.10) only depends on $E_t y_{t+1}$ and e_t^π. Due to the fact that the presence $E_t y_{t+1}$ brings no additional variable into play, the only determinant left is e_t^π (McCallum 1989).

The difference equation for the inflation process (3.1.10) yields the actual law of motion (alm), which contains the expectation formation process of the private sector who is aware of his own influence on the target variables. The *actual law of motion* is given by

$$\pi_t = \frac{\beta\alpha_y}{\alpha_y + \kappa^2}\pi_t\rho + \frac{\alpha_y}{\alpha_y + \kappa^2}e_t^\pi. \tag{3.1.13}$$

The comparison between the perceived law of motion and the actual law of motion, Equation (3.1.11) and (3.1.13), delivers for the undetermined coefficients

$$\Psi_\pi = \frac{\alpha_y}{\kappa^2 + \alpha_y(1-\beta\rho)}, \tag{3.1.14}$$

$$\Psi_y = \frac{-\kappa}{\kappa^2 + \alpha_y(1-\beta\rho)}. \tag{3.1.15}$$

The reduced form Equations (3.1.16) and (3.1.17) expresses how inflation and output gap behave in the rational expectation equilibrium.

$$\pi_t = \frac{\alpha_y}{\kappa^2 + \alpha_y(1-\beta\rho)}e_t^\pi, \tag{3.1.16}$$

$$y_t = \frac{-\kappa}{\kappa^2 + \alpha_y(1-\beta\rho)}e_t^\pi. \tag{3.1.17}$$

The corresponding rational equilibrium interest rate is obtained by using the rational equilibrium values of output and inflation together with the re-ordered Equation (3.1.1),

$$i_t = E_t\pi_{t+1} + \sigma(E_t y_{t+1} - y_t + e_t^y). \tag{3.1.18}$$

Equivalently, Equation (3.1.18) can be solved by using the rational expectation solution of the msv solution (3.1.12) to get

$$i_t = \left(\frac{\alpha_y\rho + (1-\rho)\sigma\kappa}{\alpha_y(1-\beta\rho) + \kappa^2}\right)e_t^\pi + \sigma e_t^y. \tag{3.1.19}$$

Equation (3.1.19) shows how the interest rate evolves under an optimal discretionary policy in the equilibrium. It can also be displayed as

$$i_t = 1 + \frac{(1-\rho)\kappa\sigma}{\rho\alpha_y}\rho\pi_t + \sigma e_t^y \tag{3.1.20}$$

$$= \gamma_\pi\rho\pi_t + \sigma e_t^y, \quad \text{with} \quad \gamma_\pi = 1 + \frac{(1-\rho)\kappa\sigma}{\rho\alpha_y} > 1. \tag{3.1.21}$$

Although Equation (3.1.21) looks like an instrument rule like the previously mentioned Taylor rule, in fact it is not. An instrument rule is set in response to some exogenous shocks. For a ‘targeting regime’ the central bank rather ’sets’ the interest rate optimal in a way that an optimal relation between the targets of the loss function is achieved. Hence, the interest rate follows the rational equilibrium solution of the variables. It is thus, always a rational equilibrium interest rate. The inflation reaction coefficient is larger than one. This confirms the previous finding of the so-called *Taylor principle* after which the nominal rate must be set in a more than one-to-one manner to over-compensate the inflation rate acceleration (Clarida et al. 1999, Gali 2008, Walsh 2010).

Yet, Equation (3.1.19) offers some more insights, which have partly been elaborated on previous examples as well. Hitherto, it has been assumed that this is an example of monetary policy under certainty. This is only partly true. Some uncertainty is already implemented by the additive disturbance factors e_t^π and e_t^y. So speaking of certainty is not completely correct, although all parameters are assumed to be known with certainty, hence no parametric or data uncertainty is assumed. Despite this additive disturbance term, the broad literature treats this example as one of certainty. This stems from the well known fact of *certainty equivalence*, which prevails under additive uncertainty. It has been shown in Chapter 2. Under additive uncertainty the policy rule (3.1.19) is impervious of the the size of the noise, that is, the size of the shock variance.

Another well known fact is also illustrated in Equation (3.1.19). Additive, demand driven shocks widen the output gap. This widening is completely offset by the interest rate. There is no trade-off due to the fact that an expansive output growth as well as an expansive inflation rate can be overcome with the same interest rate reaction. This is not true for supply side shocks, which cause a short-run trade-off between stabilizing inflation and output (Clarida et al. 1999).

It should be noted that a discretionary policy is regarded as *time consistent*. This is true, because setting the policy instrument follows the same pattern each period. Equation (3.1.19) holds from period one until infinity. In a rational expectations equilibrium there is no incentive for the central bank to change her behavior in an unexpected way.

One significant drawback of the discretionary solution is the so-called inflation bias. It is assumed that under discretion the central bank is always tempted to push output above the natural level. Reasons for this behavior can be found, e.g., in a social desirability of a higher output or to overcome other output depressing effects likes taxes. However, it can be shown that trying to achieve a higher level of output only causes inflation to rise above the target. Thus, under discretion it could be that inflation is persistently higher, though no output gain is achieved. This inferior solution emerges from the fact that the central bank always tries to trick the public. Yet, private agents infer this behavior and react accordingly.[4] This situation gave birth to several adjustments and advices to overcome the problem, such as the idea of Rogoff (1985), who proposes a 'conservative' central banker, i.e., one who is not attempted to sacrifice price stability for a would-be output improvement (Kydland and Prescott 1977, Barro and Gordon 1983, Clarida et al. 1999, McCallum and Nelson 2004).

Discretion with no Persistence in the Exogenous Shock

Often, the above calculation is shortened by assuming a different process of the exogenous shock vector. Hitherto, is has been assumed that e_t^π follows an AR(1) process. For simplification it can be assumed that e_t^π already is white noise, hence $e_t^\pi = \epsilon_t^\pi \sim N(0, \sigma^2)$. This changes the behavior of the system as well as it simplifies the calculation.

The targeting rules of output and inflation remain the same as Equation (3.1.7) and (3.1.8), respectively. The formation of future expectations, however, changes. Expected inflation becomes

$$E_t \pi_{t+1} = \epsilon_{t+1}^\pi = 0. \tag{3.1.22}$$

The msv solution – although not really necessary in this case – delivers for the reduced from expression of inflation and output

$$\pi_t = \frac{\alpha_y}{\kappa^2 + \alpha_y} \epsilon_t^\pi, \tag{3.1.23}$$

$$y_t = \frac{-\kappa}{\kappa^2 + \alpha_y} \epsilon_t^\pi. \tag{3.1.24}$$

The optimal interest rate setting in the rational expectation equilibrium shrinks to

$$i_t = \left(\frac{\sigma \kappa}{\alpha_y + \kappa^2} \right) \epsilon_t^\pi + \sigma \epsilon_t^y. \tag{3.1.25}$$

This example shows quite illustratively the previous mentioned elaboration of the msv solution, by picking only the very necessary state variables to formulate an educated guess about the behavior of the system. The previous results, e.g., a short-run trade-off in case of a supply side shock, however, keep their relevance. Changes only appear in form of a less smooth equilibrium path of the target variables, due to the fact that the higher the persistence factor of the AR(1) process is, the smoother the adjustment process becomes.

[4] This also shows the often mixed up assumption about expectations under a discretionary system. In both systems, under discretion as well as under commitment, expectations are built. The difference emerges from the recognition of these expectations on behalf of the central bank.

Equation (3.1.23) and (3.1.24) can thus be interpreted as a special cases of Equation (3.1.16) and (3.1.17) with $\rho = 0$ (Walsh 2010).

3.1.2.2. Commitment

If the central bank does not re-optimize every period, but commits herself once-and-for-all to a policy plan, the optimization problem slightly changes. Still the central bank optimizes over a loss function with a given set of constraints, yet, expectations are not taken as exogenous anymore. Hence, they must be considered 'actively'. If the central bank sets her control variable with a strategy of commitment, she has to consider changes in expectations, which could help her to achieve her goal of a return to the steady state values of the target variables.

The optimization procedure is similar to the discretionary case. Again the Lagrangian needs to be set up. The loss function is given by Equation (3.1.4) and remains to be the objective function of the minimization problem. In contrast to the discretionary case, however, the central banks does not optimize for a given single period and reconsiders thereafter, but has to find a reaction policy which is suitable for all periods.

The Lagrangian of the commitment solution covers the loss function

$$L_t = \frac{1}{2} E_t \sum_{t=0}^{\infty} \beta^t \left(\pi_t^2 + \alpha_y y_t^2 \right), \tag{3.1.26}$$

subject to Equation (3.1.2). Opposed to the discretionary case, all periods from now until infinity are taken into account. It is given as

$$\mathcal{L} = E_t \sum_{t=0}^{\infty} \beta^t \left[\frac{1}{2} \left(\pi_t^2 + \lambda_y y_t^2 \right) - \phi_t \left(\pi_t - \beta \pi_{t+1} - \kappa y_t - e_t^{\pi} \right) \right], \tag{3.1.27}$$

where the conditional expectations part of Equation (3.1.2), $E_t \pi_{t+1}$, has been ruled out by the law of iterated expectations.

To find the first order conditions it is helpful to write out Equation (3.1.27) for the first three periods in detail. This facilitates to see how the periods are interlinked, and shows the difference between the discretionary case and the commitment solution.

$$\begin{aligned} \mathcal{L} = E_t \Big[& \frac{1}{2} \beta^0 \left(\pi_{t+0}^2 + \lambda_y y_{t+0}^2 \right) - \phi_{t+0} \beta^0 \left(\pi_{t+0} - \beta \pi_{t+1} - \kappa y_{t+0} - e_{t+0}^{\pi} \right) + \\ & \frac{1}{2} \beta^1 \left(\pi_{t+1}^2 + \lambda_y y_{t+1}^2 \right) - \phi_{t+1} \beta^1 \left(\pi_{t+1} - \beta \pi_{t+2} - \kappa y_{t+1} - e_{t+1}^{\pi} \right) + \\ & \frac{1}{2} \beta^2 \left(\pi_{t+2}^2 + \lambda_y y_{t+2}^2 \right) - \phi_{t+2} \beta^2 \left(\pi_{t+2} - \beta \pi_{t+3} - \kappa y_{t+2} - e_{t+2}^{\pi} \right) + \ldots \Big] \end{aligned} \tag{3.1.28}$$

For the output gap the first order conditions do not change no matter what the period is which is under consideration. This does not hold for the first order conditions with respect to inflation. In this case, it obviously matters whether the initial period or one of the subsequent periods is taken into account. In short the first order conditions for inflation and output for $t = 0, 1, 2, \ldots$ can be summarized to

$$\frac{\partial \mathcal{L}}{\partial \pi_t} = \pi_t - \phi_t \stackrel{!}{=} 0 \qquad \text{for } t = 0, \tag{3.1.29}$$

$$\frac{\partial \mathcal{L}}{\partial \pi_t} = \pi_t - \phi_t + \phi_{t-1} \stackrel{!}{=} 0 \qquad \text{for } t = 1, 2, \ldots, \tag{3.1.30}$$

$$\frac{\partial \mathcal{L}}{\partial x_t} = \alpha_y y_t + \kappa \phi_t \stackrel{!}{=} 0 \qquad \text{for } t = 0, 1, 2, \ldots. \tag{3.1.31}$$

Hence, the corresponding optimal targeting rule in the commitment solution differs as well, depending on the chosen time period between

$$\pi_t = -\frac{\alpha_y}{\kappa} y_t, \qquad \text{for } t = 0, \text{ and} \tag{3.1.32}$$

$$\pi_t = \frac{\alpha_y}{\kappa} y_{t-1} - \frac{\alpha_y}{\kappa} y_t, \qquad \text{for } t = 1, 2, \ldots. \tag{3.1.33}$$

In the initial period the targeting rule is the same whether discretion or commitment is presumed. But this pattern changes for the subsequent periods. The setting in the initial period is done according to Equation (3.1.32), with the commitment of setting its policy in the subsequent period according to Equation (3.1.33). But when the period actually arrives, the optimality conditions suggest to behave again according to Equation (3.1.32) (Clarida et al. 1999, Sauer 2007).

However, this strategic problem is already known to the central bank in the initial period. The literature has labeled this issue the time inconsistency problem. Woodford (2003) offers a solution, which emphasizes a 'timeless perspective'. Accordingly, the central bank should behave as if she had committed herself a long time before, neglecting the initial period, and conduct her policy according to Equation (3.1.32) in all periods. Obviously, by that she would create a loss due to the fact that she does not behave optimal. This loss, however, should be overcompensated by the timeless perspective of the future periods (Sauer 2007).

The rational expectation equilibrium is achieved analogous to the discretionary case by the msv solution of McCallum (1983, 1999), though two relevant state variables, e_t^π and y_{t-1}, must be taken into account this time. The two relevant difference equations are

$$\pi_t = \beta E_t \pi_{t+1} + \kappa y_t + e_t^\pi, \tag{3.1.34}$$

$$\pi_t = \frac{\alpha_y}{\kappa} y_{t-1} - \frac{\alpha_y}{\kappa} y_t. \tag{3.1.35}$$

The educated guess for the commitment solution (plm) is of the form

$$\pi_t = \Psi_{11} y_{t-1} + \Psi_{12} e_t^\pi, \tag{3.1.36}$$

$$y_t = \Psi_{21} y_{t-1} + \Psi_{22} e_t^\pi. \tag{3.1.37}$$

Expectation on inflation are formed according to

$$\begin{aligned} E_t \pi_{t+1} &= \Psi_{11} y_t \\ &= \Psi_{11}(\Psi_{21} y_{t-1} + \Psi_{22} e_t^\pi). \end{aligned} \tag{3.1.38}$$

The solution is, compared to the simple case of a discretionary policy, quite messy. It can be found, for example, in McCallum and Nelson (2004) or Sauer (2007). The rational expectations equilibrium path for inflation and output, however, are given as

$$\pi_t = \frac{\alpha_y(1-\delta)}{\kappa} y_{t-1} + \frac{1}{\gamma - \beta(\rho + \delta)} e_t^\pi, \tag{3.1.39}$$

$$y_t = \delta y_{t-1} - \frac{\kappa}{\alpha_y(\gamma - \beta(\rho + \delta))} e_t^\pi. \tag{3.1.40}$$

with $\gamma = 1 + \beta + \kappa^2/\alpha_y$, and $\delta = (\gamma - \sqrt{\gamma^2 - 4\beta})/(2\beta)$.

Exemplary, Figure 3.1 shows the equilibrium path after a supply shock for the discretionary (dashed line) as well as the commitment case (solid line).[5]

No matter the policy, the supply shock pushes inflation above the target value which in turn calls for a higher interest rate, and thus, unfortunately depresses output. This expresses the well known trade-off the central bank is faced between closing the inflation or the output-gap. Beside these stylized facts, however, Figure 3.1 shows that inflation and output find different ways back to their initial values depending on the prevailing central bank policy.

[5] The parameter values to create these figures are $\beta = 0.99, \kappa = 0.17, \sigma = 1, \rho_\pi = 0.5$. Obviously, different parameter values change the picture as well as different assumptions about, e.g., the shock process do. Yet, the qualitative conclusion is nearly not affected by these changes (Gali 2008:99f).

Under discretion, the widening of the output and inflation is proportional to the shock value, see Equation (3.1.16) and (3.1.17). Those gaps start to close right after the shock begins to fade away. If no correlation of the shock is assumed, hence, the supply shock has a purely transitory character, the target values would be achieved right after the period the shock occurs, i.e., period two.

This not true under commitment where the inflation and output gap remain open for several periods even if the shock impact has nearly totally vanished, which happens approximately in period ten. This highlights the discrimination between discretion and commitment. In the latter, the central bank allows for the deviation to persist in order to achieve a better output–inflation trade-off when the shock actually occurs. The supply shock generates a lower initial impact on inflation and output under commitment than under discretion due to the forward-looking character of Equation (3.1.2). This, however, holds only for a credible policy commitment of the central bank (Gali 2008).

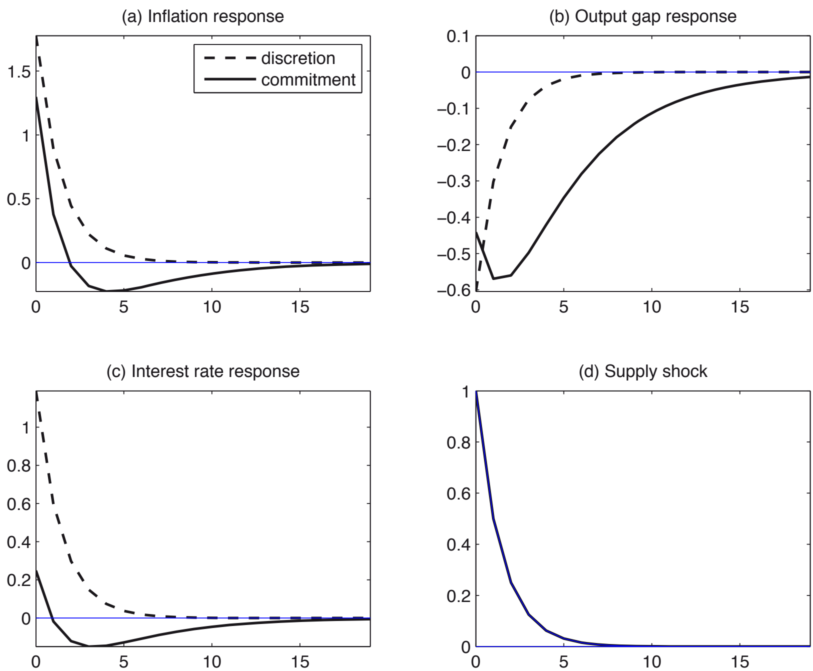

Figure 3.1.: Irf under discretion and commitment

3.1.3. Uncertainty in the New Keynesian Model

The same setting of the certainty analysis can be applied to a situation with uncertainty. To show how uncertainty evolves in the New Keynesian framework I choose parametric – also called multiplicative – uncertainty. This kind of uncertainty is chosen due to several reasons.

Taking additive uncertainty into consideration does not change the situation. As I have shown before, under additive uncertainty certainty equivalence holds, hence, the policy rule is impervious to the size of the additive noise term. In fact, additive uncertainty has just been discussed. On the other hand, data uncertainty, especially when it comes to estimating the model, can be overcome by utilizing appropriate filter techniques like the Kalman filter.

Yet, beside data uncertainty, multiplicative uncertainty is often regarded as the most prevalent form of uncertainty. Especially monetary policy in 'real life' suffers from uncertainty concerning the interrelations of variables or the impact of monetary actions, i.e., the monetary transmission process Issing (1999).

If parametric uncertainty prevails, the central bank can not be sure if, for example, a change of the interest rate is transmitted in her originally intended way to the target variables. Thereby, 'originally intended' is mostly related to the appropriate amplitude. The direction – at least – should be clear, although this is not mandatory. Hence, a rising interest rate in response to a positive output gap should be expected to close the gap and not to widen it even more. Yet, the speed of adjustment is mostly a matter of uncertainty.

The Brainard example of Chapter 2 has demonstrated that the reaction function is not independent of the variance of the uncertain transmission parameter. This has led to the conclusion that under parametric uncertainty the certainty equivalence principle is not fulfilled anymore, and thus, policy actions must be carried with more cautiousness to serve the trade-off between minimizing at the same time the deviation and the variance of the target variables.

For further analysis the supply and demand equations of Section 3.1 are utilized. They are repeated here for convenience with the interest rate elasticity of output abbreviated by $s = 1/\sigma$. Another change is the assumption concerning the noise term. Now the simpler version is assumed where both additive shock terms are already said to be white noise with zero mean and limited, but positive variance. This helps to simplify the calculation. The relevant equations are

$$\pi_t = \beta E_t \pi_{t+1} + \hat{\kappa}_t y_t + \epsilon_t^{\pi}, \tag{3.1.41}$$

$$y_t = E_t y_{t+1} - \hat{s}_t (i_t - E_t \pi_{t+1}) + \epsilon_t^{y}. \tag{3.1.42}$$

Working with parametric uncertainty, it follows that the transmission variables κ and s are now assumed to be random. This is marked by a 'hat' above the respective variables, which indicates estimated variables. Additionally, these random transmission variables become time dependent, i.e., they might change their value each period. As far as there is no learning in this model the distribution of these variables is equal in each period. The discount factor β remains non-stochastic, therefore it does not exhibit a time subscript.

Again the central bank is heading a goal of minimizing her loss, defined as deviations from the desired target values of inflation and output, π^* and y^*, respectively. Opposed to the certainty case, however, she can not be sure about the transmission of the output gap on inflation due to the uncertain transmission parameter $\hat{\kappa}$ or the reaction of the output gap in response to an increase of the nominal interest rate, i.e., due to the uncertainty surrounding $\hat{s}$.

In the following, it is assumed that the central bank follows a discretionary policy. Hence, expectations about future realizations of variables are taken as given. The uncertainty parameters are independent of each other with constant mean and limited variance, $\hat{\kappa} \sim N(\bar{\kappa}, \sigma_\kappa^2)$ and $\hat{s} \sim N(\bar{s}, \sigma_s^2)$.

It is supposed that the shock vectors can be observed by the central bank before taking any action. Yet, this assumption does not hold for the transmission parameters $\hat{\kappa}_t$ and $\hat{s}_t$. Thus, the central bank observes the exogenous supply or demand shock and considers what response serves best for her desired goal. In the course of this, she has to keep in mind that the transmission of her policy remains vague due to the uncertainty about the parameters of which she does not know the true values. This of course implies that the central bank knows that she is actually acting under uncertainty and furthermore which variables are effected. Analogous to the previous sections the discretionary loss function remains nearly unchanged as

$$L_t = \frac{1}{2} E_t \left[\pi_t^2 + \alpha_y y_t^2 \right]. \tag{3.1.43}$$

The monetary authority acknowledges the uncertainty and therefore – opposed to the certainty case – only the *expected* loss is minimized during the optimization procedure. In the certainty example of Section 3.1.2 another simplification was made by assuming that the central bank controls output in a perfect way. In this setting the IS curve was no longer a constraint to the system, but merely an extended policy instrument. Consequently, the minimization was foregone only with respect to the supply constraint of Equation (3.1.41).

Now, the control of output is no longer perfect due to the uncertain transmission process. Thus, for further analysis the IS constraint has to be taken into account as well.

Discretionary policy discards the control of expectations, $E_t\pi_{t+1} = E_t y_{t+1} = 0$, thus the respective Lagrangian under uncertainty becomes

$$\mathcal{L}_t = E_t \left[\frac{1}{2} \left(\pi_t^2 + \alpha_y y_t^2 \right) - \phi_\pi \left(\pi_t - \hat{\kappa}_t y_t - \epsilon_t^\pi \right) - \phi_y \left(y_t + \hat{s}_t i_t - \epsilon_t^y \right) \right]. \tag{3.1.44}$$

The optimal targeting rules are analogous to the certainty Equations (3.1.7) and (3.1.8) achieved via the first order conditions as

$$E\left[\pi = -\frac{\alpha_y}{\hat{\kappa}_t} y_t\right], \tag{3.1.45}$$

$$E\left[y = -\frac{\hat{\kappa}_t}{\alpha_y} \pi_t\right]. \tag{3.1.46}$$

The targeting rules under uncertainty look pretty much the same compared to the certainty counterparts except there is still an expected, uncertain value, namely $\hat{\kappa}_t$, in it. The rational expectations equilibrium values for inflation and output are

$$\pi_t = \frac{\alpha_y}{(\bar{\kappa}^2 + \alpha_y + \sigma_\kappa^2)} \epsilon_t^\pi, \tag{3.1.47}$$

$$y_t = \frac{-\bar{\kappa}}{(\bar{\kappa}^2 + \alpha_y + \sigma_\kappa^2)} \epsilon_t^\pi. \tag{3.1.48}$$

Beside the parameters which determine the certainty outcome, the equilibrium path of the target variables depends now as well on the size of the noise surrounding the uncertain transmission parameter κ. The bar depicts mean values. For any value of σ_κ^2 larger than zero the same shock leads to a less pronounced reaction of inflation and output in the equilibrium. If $\sigma_\kappa^2 = 0$, no uncertainty prevails, hence, Equation (3.1.47) and (3.1.48) equal Equation (3.1.23) and (3.1.24).

The optimal interest rate in the rational equilibrium with parametric uncertainty follows via the first order condition as[6]

$$i_t = \frac{\bar{\kappa}\bar{s}}{(\alpha_y + \bar{\kappa}^2 + \sigma_\kappa^2)(\bar{s}^2 + \sigma_s^2)} \epsilon_t^\pi + \frac{\bar{s}}{\bar{s}^2 + \sigma_s^2} \epsilon_t^y. \tag{3.1.49}$$

Equation (3.1.49) looks quite similar compared to the Brainard calculation and in fact the interpretation is straight forward as well. The effect parametric uncertainty unfolds is shown best if the certainty case is taken as reference. Under certainty mean values equal

6 Set $E\left[-\hat{\kappa}_t / \left(\hat{\kappa}_t^2 + \alpha_y + \sigma_\kappa^2\right) = \hat{s}_t i_t\right]$ and solve for the interest rate. The fraction should then be expanded with $\hat{s}/\hat{s}$ and solved for expectation to get Equation (3.1.49). A hat denotes expected values, a bar denotes the mean values.

their *de facto* realization and the noise vanishes such that the interest rate is set according to

$$i_t = \frac{\bar{\kappa}}{(\alpha_y + \bar{\kappa}^2)\bar{s}}\epsilon_t^\pi + \frac{1}{\bar{s}}\epsilon_t^y. \tag{3.1.50}$$

If there is no uncertainty about the interest rate elasticity of output, $\sigma_s^2 = 0$, but an increasing uncertainty about κ, which is expressed by an increasing σ_κ^2, the impact of a supply shock on inflation and output is watered down. Accordingly, the optimal interest rate path is less pronounced, which can be deduced from Equation (3.1.49). The same consequences follow from a rising uncertainty about the interest rate elasticity of output, σ_s^2. Rising uncertainty also dampens the response of the interest rate in case of a supply or demand shock.

For the case of an unlimited uncertainty, either of κ or the interest rate elasticity s, it is the best not to react at all in response to a what-so-ever shock. From Chapter 2 it is clear that under these circumstances the size of the variance of the target variable becomes prohibitive. If the uncertainty concerning κ and s is decreasing up to the point where $\sigma_\kappa^2 = \sigma_s^2 = 0$, Equation (3.1.49) and (3.1.50) become more and more congruent. For this situation, as Equation (3.1.50) shows, the interest rate rule is no longer impervious to uncertainty. However, as long as there is at least some kind of uncertainty concerning the transmission parameters the optimal interest rate setting changes – the larger the uncertainty, the more caution.

In both cases, as the Brainard example has already suggested, uncertainty concerning κ as well as the interest rate elasticity of the output gap s, dampens the response of the interest rate to supply shocks. Thereby, a smaller output weight will lead to a larger effect of the supply shock. But even if there uncertainty prevails and no weight is given to output stabilization at all, the reaction will still be less than under certainty. This finding again supports the Brainard recommendation to behave more cautiously in the presence of parametric uncertainty (Söderström 2002, Walsh 2003b,a, Gali 2008).

Figure 3.2 gives an example of parametric uncertainty. It shows the impulse response functions of the key variables in a purely backward-looking model of the economy where the policy acts under discretion.[7] The supply and demand curve are

$$\pi_{t+1} = \delta_{t+1}\pi_t + \kappa_{t+1}y_t + \epsilon^{\pi}_{t+1}, \tag{3.1.51}$$

$$y_{t+1} = \alpha_{t+1}y_t - s_{t+1}(i_t - \pi_t) + \epsilon^{y}_{t+1}. \tag{3.1.52}$$

The parameter values are set to $\delta = 1, \kappa = 0.4, \alpha = 0.85, s = 0.35$ (Söderström 2002). The loss function exhibits $\lambda = 1$. More precisely, Figure 3.2 plots the case where only s is uncertain, hence $\sigma_s^2 = 0.2$.[8] This is an *impact* parameter in the terminology of Söderström (2002). The interest rate response is given in the last row – clearly the Brainard (1967) result is confirmed. As I have already depicted in Chapter 2, the central bank favors to reduce her reaction, i.e., behaves less aggressive than under certainty, and thus, allows for a longer deviation of the target variables from their desired values in order to keep the possible variance low.

Opposed to the findings shown in Figure 3.2, Figure 3.3 gives the interest rate response if one of the *dynamic* parameters, α and δ, is uncertain. For the demand shock it is difficult so see, however, in both cases the reaction coefficient of the interest rate is larger under uncertainty than under certainty. Hence, with this parametrization the opposite policy recommendation under uncertainty is achieved, that is, the interest rate reacts more aggressive then under certainty to deviations of the target variables from the desired values.

The reasons for this behavior is the fact that under parametric uncertainty the central bank aims at a minimization of the deviation from the target variable as well as of the variance. This variance is high, when the deviation is large (which is not the case under additive uncertainty). If the inflation persistence is uncertain ($\sigma_\delta^2 > 0$), the uncertainty about future deviations can be reduced if the actual deviations are reduced. This is done by acting more aggressive, albeit this strategy is only admissible due to the fact that solely dynamics are affected (Söderström 2002).

---

7 With respect to Söderström (2002) instead of a forward-looking or hybrid model, a backward -looking model is taken due to computational reasons. This might seems to be unrealistic, yet, the results of hybrid models are quite comparable to those capturing only backward-looking behavior. Moreover, backward-looking models seem to fit better to the empirical data. In any case, these circumstances, however, do not alter the qualitative results.

8 The findings are qualitatively the same, no matter if κ or s is said to be uncertain.

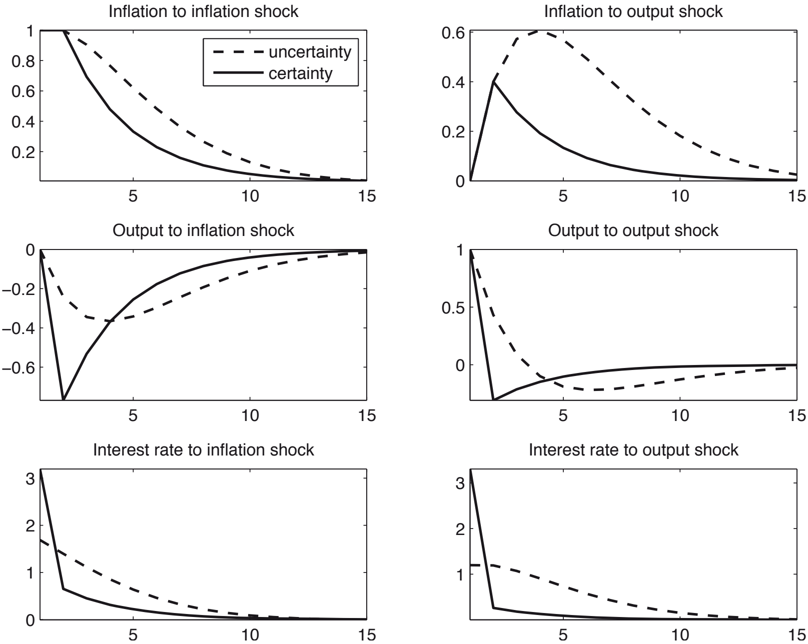

Figure 3.2.: Irf under parametric (impact) uncertainty ($\sigma_s^2 > 0$)

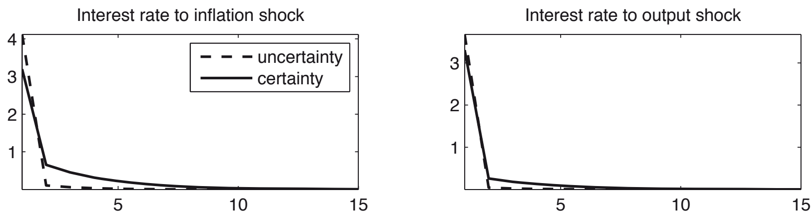

Figure 3.3.: Irf under parametric (dynamics) uncertainty ($\sigma_\delta^2 > 0$)

3.2. State Space Representation

The so far presented approach has been used to show how methods and findings of the simple models, e.g., of Brainard (1967), can be transferred into a more sophisticated model. The implications, such as the existence of certainty equivalence under additive uncertainty and the cease of it under parametric uncertainty have been confirmed.

However, during the last years it has become convenient in monetary policy analysis, to express the underlying model in the so-called *state space* representation. This rather compact form of writing allows especially for a better computational implementation.

The supply and demand equations

$$\pi_t = \beta E_t \pi_{t+1} + \kappa y_t + e_t^{\pi}, \tag{3.2.1}$$

$$y_t = E_t y_{t+1} - \frac{1}{\sigma}(i_t - E_t \pi_{t+1}) + e_t^{y}, \tag{3.2.2}$$

are transferred into the compact writing

$$x_{t+1} = Ax_t + Bu_t + C\epsilon_{t+1}, \tag{3.2.3}$$

where the vector x_t covers the target variables of the model and u_t is the vector of the control variables. The shock vector e_t follows an AR(1) process according to $e_{t+1}^i = \rho^i e_t^i + \epsilon_{t+1}^i$, with $i = \pi, y$. The matrices A, B, C comprise the model parameters.

This compact and efficient form, however, also exhibits some disadvantages. One of these is the effort that is needed to bring the given (first order) equations into the desired state space form, which actually can be considerable. Additionally, the distinction between *predetermined* and *non-predetermined* variables also called forward-looking or jump variables must be made. According to Blanchard and Kahn (1980), predetermined variables, $x_{1,t+1}$, are only a function of variables known at time t. This implies that they are a function only of lagged variables and contemporaneous shocks. For the information set I_t, with $I_t \supseteq I_{t-1}$, it must hold $x_{1,t+1} = E_t x_{1,t+1}$, no matter the realization of the target variables, hence, I_{t+1}. Thus, predetermined variables are not allowed to respond to news, shocks or what so ever becoming evident in period $t+1$.

Non-predetermined variables, $x_{2,t+1}$, are a function of current anticipations of future values of variables, hence, they are a function of any variable in I_{t+1}. If no shock occurs, it holds that $x_{2,t+1} = E_t x_{2,t+1}$, thus all realizations in I_{t+1} are equal to their expectations conditional on I_t. Yet, the equality between realization and expectation does not hold if any shock occurs in I_{t+1}, hence, one can write $x_{2,t+1} - E_t x_{2,t+1} = \epsilon_{t+1}$ (Blanchard and Kahn 1980, Buiter 1982, Klein 2000, Dennis 2001).

At the beginning of period t the shock vector ϵ_t^{π} and the predetermined variables $x_{1,t}$ are realized. In response the instrument variable u_t is set by the monetary authority. Consequently the non-predetermined variables $x_{2,t}$ result and the period finishes. In the full information case all variables are observable, thus the information set in t, on which, e.g., the interest rate decision depends on, is given as $\{\epsilon_t, x_{1,t}, \epsilon_{t-1}, x_{1,t-1}, i_{t-1}, \dots\}$. Expectations on the variables value of the next period are formed at the very end of the contemporaneous period, hence, $E_t x_{2,t+1}$ contains all the information available at the end of period t, that is, past and contemporaneous information (Svensson 1999).

Given the model utilized so far the vectors and matrices according to Equation (3.2.3) are thus

$$x_{1,t} = \begin{pmatrix} e_t^{\pi} \\ e_t^{y} \end{pmatrix}, x_{2,t} \begin{pmatrix} \pi_t \\ y_t \end{pmatrix}, u_t = i_t, \epsilon_t = \begin{pmatrix} \epsilon_t^{\pi} \\ \epsilon_t^{y} \end{pmatrix}. \tag{3.2.4}$$

The complete model based on Equation (3.2.3) is

$$A_0 \begin{pmatrix} x_{1,t+1} \\ E_t x_{2,t+1} \end{pmatrix} = A_1 \begin{pmatrix} x_{1,t} \\ x_{2,t} \end{pmatrix} + Bu_t + C \begin{pmatrix} \epsilon_{t+1} \\ 0 \end{pmatrix}, \tag{3.2.5}$$

with

$$A_0 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \beta & 0 \\ 0 & 0 & 1/\sigma & 1 \end{pmatrix}, A_1 = \begin{pmatrix} \rho^{\pi} & 0 & 0 & 0 \\ 0 & \rho^{y} & 0 & 0 \\ 1 & 0 & 1 & -\kappa \\ 0 & 1 & 0 & 1 \end{pmatrix}, B = \begin{pmatrix} 0 \\ 0 \\ 0 \\ -(1/\sigma) \end{pmatrix}, C = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}. \tag{3.2.6}$$

Multiplying Equation (3.2.5) with A_0^{-1} delivers the final state space form already stated in (3.2.3)

$$x_{t+1} = Ax_t + Bu_t + C\epsilon_{t+1}. \tag{3.2.7}$$

Equation (3.2.7) contains all the necessary information about the state of the economy analogously to the main equations of Section 3.1, (3.1.1) and (3.1.2).[9]

The solution follows the same steps as before. To achieve the stability of the system the saddle-path criterion of Blanchard and Kahn (1980) must be fulfilled, i.e., the number of non-predetermined variables must be equal to the number of eigenvalues in A bigger than one ($n_{np} = n_o$, or equivalently $n_p = n_i$).

In this set-up two non-predetermined variables, namely π_{t+1} and y_{t+1}, can be identified, hence, two eigenvalues outside the unit circle (n_o) are demanded. Certainly the stability

[9] The here presented way of writing down the system equations is only one possibility. To my knowledge, this is the most common one, see, for example, Söderlind (1999), Svensson and Woodford (2003a). Despite this form one could also write the model as $\Psi_{t+1} = A\Psi_t + Bi_t + w$. Yet, doing so, it is assumed that $\Psi_t = [x'_{1,t} x'_{2,t}]'$ and $w = [(C\epsilon_{t+1})'(x_{2,t+1} - E_t x_{2,t+1})']'$ including already the expectational error (Gerali and Lippi 2008).

of A depends on the actual numbers being assign to the parameters β, σ and κ, but under the typically assigned values the stability of the system is not given.

Ignoring for now the possible instability the solution is found via the (generalized) *Schur decomposition* (see, e.g., Söderlind 1999). This fragmentation rules out unstable solutions and delivers the rational equilibrium solution of the system. Accordingly, the predetermined variables depend on their own lagged values as well as the shock parameters. Hence, they can be described as a VAR(1) process. The solution for the forward-looking variables states a linear dependency of the predetermined variables in the same period, hence, one can write the solution according to

$$x_{1,t+1} = Mx_{1,t} + \epsilon_{t+1}, \tag{3.2.8}$$

$$x_{2,t} = Nx_{1,t}, \tag{3.2.9}$$

where the matrices M and N need to be determined.

As easy as it is done in Section 3.1, an instrument rule à la Taylor can be implemented. A rule of the type $i_t = \delta_\pi \pi_t + \delta_y y_t$ merely states a relationship between the policy instrument and the variables of the system. This can be expressed in matrix notation as

$$u_t = Fx_t, \tag{3.2.10}$$

with $F = \begin{pmatrix} 0 & 0 & \delta_\pi & \delta_y \end{pmatrix}$, capturing the coefficients of the policy rule. For the moment, F is kept undetermined. To solve the model this is not necessary at this point in time.

The solution of the system follows the same steps as before except the implementation of the general form of the policy rule delivers for the general state space form

$$\begin{aligned} x_{t+1} &= Ax_t + BFx_t + C\epsilon_{t+1} && (3.2.11) \\ &= (A + BF)x_t + C\epsilon_{t+1} \\ &= Gx_t + C\epsilon_{t+1}. && (3.2.12) \end{aligned}$$

A simple rule is implemented without any optimization routine. Equation (3.2.12) is solved in the same way as before, i.e., with the Schur decomposition. Choosing the parameters values rightly stability of the system is achieved, hence, the Blanchard-Kahn-conditions are fulfilled.

3.2.1. Optimal Policy

To conduct an optimal policy, again, the minimization of a loss function under discretion and commitment is considered. Depending on the policy objective, however, different solutions techniques are applied.

Under commitment the policy maker finds one strategy at the beginning of the time period under consideration, which is optimal once and for all. No re-optimization is done. Thus, the problem has a non-recursive character. To solve it most authors utilize the Lagrange technique. This solution procedure is based on the works of Currie and Levine (1985, 1993) (continuous time) and Backus and Driffill (1986) (discrete time). Opposed to this, e.g., Gerali and Lippi (2008), based on the work of Ljungqvist and Sargent (2004) who 'recursify' the non-recursive problem, utilizes the dynamic programming approach. Svensson (2010), beside the Lagrange method, makes use of the recursive saddlepoint method of Marcet and Marimon (1994, 2011), who reformulate the problem in order to as well apply dynamic programming solution techniques.

Some authors, see, e.g. Oudiz and Sachs (1985) declare the pre-commitment of any authority as an "unsatisfactory assumption" (Oudiz and Sachs 1985:286). Consequently they favor the more realistic case of a Markov-perfect Stackelberg-Nash equilibrium, or short, optimal discretionary policy. Here the policy maker re-optimizes every period.[10] The problem becomes recursive. For this case, popular solutions, see, e.g., Söderlind (1999), are based on the works of Oudiz and Sachs (1985) and Backus and Driffill (1986) using dynamic programming techniques, i.e., a value function which satisfies the Bellman equation. This algorithm is based on the iteration and convergence of the Ricatti equation. However, it is not clear, whether a closed form solution exist; Or as Söderlind (1999) puts it, "[t]he general properties of this algorithm are unknown. Practical experience suggests that it is often harder to find the discretionary equilibrium than the commitment equilibrium. It is unclear if this is due to the algorithm" (Söderlind 1999:819).

In any case, whether a commitment or discretionary policy is conducted, the loss function is given as

$$L_t = Y_t W Y_t'. \tag{3.2.13}$$

10 One should notice that, opposed to the normal commitment solution, i.e., no timeless-perspective, optimal discretionary policy is by definition time-consistent (Dennis 2001). Defined by Oudiz and Sachs (1985) "[a] *time consistent* equilibrium is one in which each government optimizes its policy choice taking as given the policy rules (or specific policy actions) that *future* governments will use" (Oudiz and Sachs 1985:280).

It is quadratic and penalizes positive and negative deviations from the target variables, which are captured by Y_t as

$$Y_t = C_x \begin{pmatrix} x_{1,t} \\ x_{2,t} \end{pmatrix} + C_u u_t. \tag{3.2.14}$$

Due to the simplification of assuming no interest rate smoothing the last part of Equation (3.2.14) vanishes, otherwise the interest rate goal of, say, minimizing interest rate changes relative to the prior period, $i_t - i_{t-1}$, would be captured by $C_u u_t$. The relative weights of the target variables are represented by the diagonal elements of the matrix W.

The intertemporal loss function without interest-smoothing becomes

$$L = E_0 \sum_{t=0}^{\infty} \beta^t L_t \tag{3.2.15}$$

$$= E_0 \sum_{t=0}^{\infty} \beta^t \left[x_t' Q x_t \right]. \tag{3.2.16}$$

with $Q = C_x' W C_x$[11].

Considering the commitment policy the first order conditions are achieved via the Lagrangian which is set up as

$$min\ E_0 \sum_{t=0}^{\infty} \beta^t \left[x_t' Q x_t + \phi_{t+1} \left(A x_t + B u_t + C \epsilon_{t+1} - x_{t+1} \right) \right]. \tag{3.2.17}$$

The resulting first oder conditions of Equation (3.2.17) need to be reordered by placing first predetermined variables (k_t) and second forward-looking variables (λ_t). The resulting set-up is similar to the the state space notation of the model Equation (3.2.5) and of the form

$$G E_t \begin{pmatrix} k_{t+1} \\ \lambda_{t+1} \end{pmatrix} = D \begin{pmatrix} k_t \\ \lambda_t \end{pmatrix}. \tag{3.2.18}$$

Opposed to the simple rule case, where no optimization has taken place, now the above system of optimality conditions is solved. This is done in the same way as the simple rule policy, i.e., with the help of the generalized Schur decomposition (Söderlind 1999, Klein 2000).

[11] Not at all it is taken for granted that the loss function is of the form given here where deviations from target variables are (more or less) equally penalized. Onatski and Stock (2002), e.g., give a loss function of a monetary authority who is concerned with worst-case scenarios. However, the authors admit that quadratic loss functions are by far the most common ones due to their tractability.

The solution gives the optimal commitment policy rule for the steady state. Like before predetermined variables are described by a VAR(1) process and forward-looking variables exhibit a linear relationship to the contemporaneous predetermined variables like Equation (3.2.8) and (3.2.9). However, opposed to the simple rule solution the matrices M and N are now obtained from an optimization routine such that their values are now 'optimal' as well (Söderlind 1999).

Under discretion the central bank re-optimizes every period by taking the state of the world, as well as expectations, as exogenous and thus given. With the arguments of today at hand, at each point in time the policy instrument is set in a way, which produces the optimal value for now and the future. Due to the sequential character, that is, a series of one-period problems, dynamic programming techniques which make use of the recursive structure can be applied. Based on Backus and Driffill (1986) most authors, see, e.g., Söderlind (1999), Söderström (2002), solve the discretionary problem via the value function. The value function gives the optimal value of the problem represented by the objective function and its constraints. This implies that the control variable has been set optimal. Since the objective function is quadratic and the constraint linear the value function can be guessed as a being also a quadratic function of the general form $V(x) = x'Px + p$.

To solve the optimization problem the primary guess of the value function is updated by the use of the Bellman equation to get the value function one period ahead. Subsequent a reaction function is derived, which can be substituted into the Bellman equation to solve for the value function. The newly value function is in turn used to evaluate whether an improvement can be achieved. For a large number of iterations it should be possible to find a value function, which is in accordance with an optimal control rule.

For the problem at hand the value function is given as

$$J(x_t) = x_t' V_t x_t + v_t, \tag{3.2.19}$$

which can be carried over to the Bellman equation by inserting the control rule as

$$\begin{aligned} x_t' V_t x_t + v_t &= min \ \left\{ x_t' R x_t + \beta \left[x_{t+1}' V_{t+1} x_{t+1} + v_{t+1} \right] \right\} && (3.2.20) \\ &= min \ \left\{ x_t' R x_t + \beta \left[(A x_t + B u_t)' V \left(A x_t + B u_t \right) + v \right] \right\}. && (3.2.21) \end{aligned}$$

For convenience no interest rate smoothing behavior is considered. In the optimum it must hold that

$$V(x_0) = \left\{ x_t' R x_t + \beta V\left(x_{t+1}\right) \right\}. \tag{3.2.22}$$

Notice that Equation (3.2.22) represents an optimal equation. It has been achieved by choosing the instrument variable u_t optimal, which does not mean that any value of u_t is actually known. The first order conditions can be derived and rearranged to obtain the *optimal targeting rule* of the form $u_t = Fx_t$, with

$$F = -(B'VB)^{-1}(B'VA). \tag{3.2.23}$$

The optimal rule of Equation (3.2.23) is replaced into Equation (3.2.21) to get the *algebraic matrix Riccati* equation which finally solves for V. Taking a first guess of the value function, which can be zero or a more sophisticated guess, V_t is achieved.

$$V_t = R + \beta\,(A + BF_t)'\,V_t\,(A + BF_t)\,. \tag{3.2.24}$$

After the values for the reaction function have been inserted the process starts again, utilizing a new, better guess for $V(x_0)$.[12] The solution process should converge for a large enough number of repetitions such that $\lim_{n\to\infty} |V_{n+1} - V_n)| < \zeta$, with ζ declaring a tolerable error degree. Thus, solving the Ricatti equation will solve the discretionary optimization problem. The solution, again, offers the result of certainty equivalence. The policy rule is independent from any additive shock vector. The optimization problem can be solved with or without the shock it delivers the same decision rule. However, the value function is affected by the shock term. Hence, whilst the policy rule is impervious to the degree of uncertainty the value function is not (Söderlind 1999, Söderström 1999, Ljungqvist and Sargent 2004).

If parametric uncertainty is introduced, the expected value function, and thus the optimal targeting rule and the Riccati equation change, indicating the non-certainty equivalence character. The resulting policy rule is given by

$$F = -\left(B'\left(V_u + V_u'\right)B + 2v_u\Sigma_B\right)^{-1}\left(B'\left(V_u + V_u'\right)A + 2v_u\Sigma_{AB}'\right). \tag{3.2.25}$$

The derivation of Equation (3.2.25) is somewhat messy and can be reconstructed elsewhere (see, e.g., Söderström 1999). The actual appearance of variables may differ if further target variables are implemented. Equation (3.2.23) and (3.2.25), however, build on the same assumptions, except that parametric uncertainty is assumed in the latter. The crucial difference between both reaction functions is the appearance of the noise which surrounds the parameters Σ_B, Σ_{AB}; v_u indicates elements of the matrix V_u, the position depends on the problem at hand. Equation (3.2.25) again shows the vaporization of certainty equiva-

12 This better guess is assumed to be V_t such that $V(x_0) = V_t$.

lence under parametric uncertainty due to the fact that the policy rule is affected by the prevailing uncertainty.

3.2.2. Implementing Uncertainty: A Compact Approach

It goes without saying, uncertainty can also be implemented into the state space representation. The previous section has already shown possible effects of parametric uncertainty on the formation of the optimal policy decision. Based on the distinction of *inter alia*, Tetlow and von zur Muehlen (2001), Levine and Pearlman (2008), Williams (2008), the so far discussed forms of uncertainty – namely, additive, data, and parametric or multiplicative uncertainty – are said to be kinds of *structured* uncertainty. Opposed to this, there is *unstructured* uncertainty. The difference between structured and unstructured uncertainty is between whether one can identify the source an especially form of the misspecification or whether this is not possible (Tetlow and von zur Muehlen 2001). A very nice approach to implement structured uncertainty into the state space writing can be found in Walsh (2003b) and (Williams 2008). According to Walsh (2003b) the true model of the economy is given by

$$x_{t+1} = Ax_t + Bu_t + C\epsilon_{t+1}. \tag{3.2.26}$$

The central banks perceived model, however, is

$$x_{t+1} = \hat{A}x_t + \hat{B}u_t + \hat{C}\epsilon_{t+1}, \tag{3.2.27}$$

the hat above the coefficient matrices depicts estimates of the central bank. The difference between Equation (3.2.26) and (3.2.27) must thus be the estimation error of the perceived model made by the central bank. The perceived model, however, can be written as the true model by accounting for this divergence such that the true model Equation (3.2.26) can also be written in the form 'estimate + error' as

$$x_{t+1} = \hat{A}x_t + \hat{B}u_t + \hat{C}\epsilon_{t+1} + w_{t+1}. \tag{3.2.28}$$

Hence, the true model can be displayed in possible wrong central bank estimates by adding a correction term for this misspecification. For the case $w_{t+1} = 0$ the true models coincides with the presumed model of the central bank, thus certainty prevails. The additional shock vector can be understood as a parameter, which accounts for different models all exhibiting a different form of uncertainty. Hence, depending on the design of the error term various kinds of uncertainty are thinkable (Williams 2008).

In the simplest version, the perceived model differs from the true model only due to additive uncertainty. Uncertainty enters the model via

$$w_{t+1} = \eta_{t+1}. \tag{3.2.29}$$

The error term just adds some variance to the model. The total variance of the model is now constituted by the already assumed shock vector ϵ_{t+1} plus the additional uncertainty component η_{t+1}. Both shocks are assumed to have zero mean and limited variance. The total shock vector thus exhibits zero mean and 'two times variance'.

Because all parameter estimates coincide with the true values, Equation (3.2.28) could also be written as

$$x_{t+1} = Ax_t + Bu_t + C\epsilon_{t+1} + \eta_{t+1}. \tag{3.2.30}$$

This does of course not alter the policy analysis. The policy is independent of the amount of uncertainty that enters the model additive, that is, the size of the variance of the noise term.

Within the setting of Equation (3.2.28), to include parametric uncertainty, the error term must be given as

$$w_{t+1} = (A - \hat{A})x_t + (B - \hat{B})u_t + (C - \hat{C})\epsilon_{t+1}. \tag{3.2.31}$$

The perturbed model with possibly wrong parameter estimates is then again given as

$$x_{t+1} = \hat{A}x_t + \hat{B}u_t + \hat{C}\epsilon_{t+1}. \tag{3.2.32}$$

The difference to the true model of Equation (3.2.26) can be found in the error term Equation (3.2.31), and the true model can be written in terms of the perceived model again as

$$x_{t+1} = \hat{A}x_t + \hat{B}u_t + \hat{C}\epsilon_{t+1} + w_{t+1}. \tag{3.2.33}$$

Hence, the difference between the true model and the distorted one, again, can be implemented by a single additional term. This term now captures discrepancies of the persistence and impact parameters (Walsh 2003b, Williams 2008).

The deduced optimal rule depends on the uncertainty about the matrices A, B, and C. If a shock hits the system, the monetary authority must reconsider her decision rule, taking explicitly into account the uncertainty surrounding the coefficient estimates of her model. However, without knowing the exact model as well as the parameter values, nothing can be said whether or not the monetary authority should act with more caution or more aggressively. A possible reaction function has been given in the previous section. The example of parametric uncertainty, represented here by the error term w_{t+1}, highlights an important implication. Although the error term is 'additive', it can not be treated as an additive error term, which exhibits certainty equivalence.

To capture data uncertainty within this set up, Equation (3.2.26) needs to be slightly extended. Because the state of the economy can not be determined with certainty it must be accounted for the information, which has been utilized to form the state estimate. Thus, the former notation changes into

$$x_{t+1} = A^1 x_t + A^2 x_{t|t} + B u_t + C\epsilon_{t+1} \tag{3.2.34}$$

where $x_{t|\tau}$ represents the best estimate for the state x at time t, given the information set I at time τ. If the state of the economy is observed perfectly, it holds that $x_t = x_{t|t}$ and $A^1 + A^2 = A$.

Because all parameters of the model are assumed to be known, but only the state is observed with noise, the difference between the true model (3.2.34) and the one perceived by the monetary authority, Equation (3.2.27) can be expressed by an additional term capturing measurement problems. In order to find a coherent treatment, the difference between the true model and the one perceived by the monetary authority, i.e., the error term, is extended into the more general form as well. The true model is reformulated by using the perceived values as well as the observation error, such that

$$x_{t+1} = A^1 x_t + A^2 \hat{x}_{t|t} + B u_t + C\epsilon_{t+1} + w_{t+1}. \tag{3.2.35}$$

The difference between the true model and the one using perceived values is given by the error term

$$w_{t+1} = A^2(x_t - \hat{x}_{t|t}). \tag{3.2.36}$$

Summarizing, all sorts of uncertainty dealt with hitherto can be captured by one additive term.[13] This gives a pretty way of systematizing uncertainty in monetary policy models by letting the model in its original form, but only changing the form of the additive component. This, however, needs to be further specified to account for an explicit form of

[13] With this set-up also several other forms like asymmetric information can be implemented in a pretty way, see, Walsh (2003a).

uncertainty. For a convenient representation, additive, parametric, and data uncertainty can be summarized according to

$$\begin{aligned} w_{t+1} = \eta_{t+1} \\ + (A - \hat{A})x_t + (B - \hat{B})u_t + (C - \hat{C})\epsilon_{t+1} \\ + A^2(x_t - \hat{x}_{t|t}). \end{aligned} \tag{3.2.37}$$

In general it seems like almost every uncertainty can be broken down to a single additive term capturing all variations of uncertainty. However, there are some things that need to be taken into account. With this notion of uncertainty the structure of uncertainty as well as the source of uncertainty have always been kept in mind. This is important due to the fact that, e.g., if $w_{t+1} = \eta_{t+1}$ no further policy reconsideration needs to be done due to certainty equivalence. This is not the case if w_{t+1} represent parametric or data uncertainty. This type of representing uncertainty is called *structured uncertainty*, even though the structure is somewhat hidden at first sight (Williams 2008).

The so far discussed treatment assumes that always some sort of structure and numerical distribution must have been given. Hence, approaches have been of a Bayesian nature. This seems unrealistic. Under these circumstances (i) priors have to be made as well as (ii) the type of misspecification is limited to those already known to the policy maker (Onatski and Stock 2002). Modeling structured uncertainty is not suitable in dealing with Knightian uncertainty, due to the fact that 'uncertainty' always presumes a probability distributions, but these are "problems of *risk*, and risks can be priced" (Tetlow and von zur Muehlen 2001:913).

However, what if the structure is unknown? In this case the monetary authority can not be sure about what kind of mismeasure of variables or misjudgment of coefficients she has been a victim of. Under these circumstances, when the form is not known to the decision maker, *unstructured* uncertainty prevails. The concept of unstructured uncertainty and the possible solution methods are mainly connected with the numerous works of Hansen and Sargent on robust control, see, especially, Hansen and Sargent (2008). This approach allows to capture uncertainty where neither the origin of uncertainty nor any probability distribution needs to be identified. By that, omitted variables can also be taken into account, and thus, it is also suitable to deal with Knightian uncertainty (Tetlow and von zur Muehlen 2001, Levine and Pearlman 2008).

The key feature of unstructured uncertainty is the collapse of all kinds of uncertainty into one additional shock vector w_{t+1} which forms the general model into

$$x_{t+1} = \hat{A}\hat{x}_{t|t} + \hat{B}u_t + \hat{C}\epsilon_{t+1} + w_{t+1}. \tag{3.2.38}$$

All variables are only estimates which are susceptible to errors. Although Equation (3.2.38) looks quite similar to the equations utilized in the analysis of structured uncertainty, it bears an important difference. Now, w_{t+1} can not be partitioned any more. Hence, if $w_{t+1} > 0$ there is no way of determining whether this originates, for example, due to parametric perturbations or due to problems in ascertaining the correct values of the state variables. It only states that some misspecification distorts the reference model and prevents it to behave in the expected manner. The only restriction is given to the size of w_{t+1} for which the lower bound is zero. This would equal certainty.

The timing of w is treated somewhat mixed. Hansen and Sargent (2008) use w_{t+1}, while Hansen and Sargent (2001) use the instantaneous variant with w_t. In the first case, the shock and the misspecification superpose each other, whereas in the latter the misspecification reveals before the shock hits the system. In general, and especially within the robust control approach, the first variant is dominant. This is done in order to mask the misspecification error with the shock vector, which makes an identification of the form and source even harder (Giordani and Söderlind 2004, Leitemo and Söderström 2004, Hansen and Sargent 2008).

Indeed, although it might look like an additional shock process, certainty equivalence does not hold due to the possible forms of w_{t+1}. Of course, it could be that w_{t+1} equals a simple additive shock vector. In this special case again certainty equivalence holds. Yet w_{t+1} could also be a linear function of (lagged) state variables as well as a non-linear function which would preclude certainty equivalence. It is important to mention that, although it looks similar the story behind w_{t+1} is totally different from the 'normal' shock vector e_{t+1}. While e_{t+1} simply hits the system as an exogenous shock that vanishes or at least diminishes after some time, w_{t+1} can represent persistent model misspecification (Hansen and Sargent 2008).

The advantage of the unstructured uncertainty approach lies in the little knowledge the policy maker needs to hold. Despite its simplicity it covers various kinds of uncertainty by defining simply one additional term. Only the total amount of uncertainty needs to be restricted by limiting the size of w_{t+1}. Additionally the concept of unstructured uncertainty allows to compare models differing in their (uncertain) values with those having a different structural construction. On the other hand, these simplifications can also come as a drawback if, e.g., some source of uncertainty is of special interest. Moreover, depending on the underlying but hidden uncertainty the control rule may vary significantly, which could induce involuntary volatilities (Giordani and Söderlind 2004).

A compromise between these two extreme cases could be to distinguish at least between uncertainty concerning the parameters or the dynamics of the system. The first case would thus be pretty standard, except no probability distribution could be given. Thus it extends, for example, the standard parametric uncertainty case but, however, allows for a broader treatment. Alternatively allowing for uncertain dynamics would also include omitted or unmodeled dynamics which imply shocks others than white noise. This middle way seems to be the most realistic way, thinking about the efforts of central banks in calibrating forecasting models (Tetlow and von zur Muehlen 2001, Levine 2010).

3.3. Model Uncertainty

Dealing with uncertainty in the so far discussed ways has several drawbacks. Foremost, the source of uncertainty is very difficult to discover. This already makes the detection of uncertainty itself quite difficult. Moreover, if located, the specific properties, i.e., the exact mean and variance of the parameters in question are hard to determine. To deal with these problems a new field of interest comes up which on the one hand gives a more realistic picture of the economy and 'real life' problems, but on the other hand questions the so far elaborated standard optimizations (Tetlow and von zur Muehlen 2001).

During the last years, research on monetary policy has especially focused on modeling *model* uncertainty (see, among others, Dennis 2008, 2010, Levine 2010). Model uncertainty can be understood as uncertainty about competing models. These uncertainties may arise due to different structural forms, as well as uncertainty about the same structural form, but with different parameter constellations and values (Onatski and Williams 2002, Levine and Pearlman 2008). The analysis of model uncertainty has also raised some new solution techniques, most prominent the robust control approach of Hansen and Sargent (2008) which shall be presented in the following.

Dealing with model uncertainty the question arises, whether model uncertainty can be understood as a 'super' form of uncertainty covering all other so far elaborated types of uncertainty? On the one hand, this is definitely true. It has been shown in the previous section that model uncertainty can be understood as the same model but with different – uncertain – parameter values. Hence, parametric uncertainty would be just a sub-form of model uncertainty. Accordingly, different estimates of, say the output gap, would maintain the structure of the model, but deliver different values of the key variables. Hence, data uncertainty can be taken as well as a sub-form of model uncertainty. On the other hand, model uncertainty can be understood as a set of competing models not only different by their assigned values, but also due to a different structural form. For this set up, model uncertainty clearly covers more than merely parametric or data uncertainty.

In sum, there seems to be two different approaches on modeling model uncertainty, whether the structure remains and merely values of parameters are uncertain, or if also the model structure is object of uncertainty. Hence, the 'correct' specification of the error is crucial (Tetlow and von zur Muehlen 2001, Onatski and Stock 2002, Levine 2010).

3.3.1. Measuring Misspecifications with the Concept of Entropy

The most popular approach to solve problems of model uncertainty is called robust control. The technique of robust control builds on the concept of entropy, which has been established in the economic literature by Hansen and Sargent (2008) as a relative distance measure between the true data generating process and the one estimated by the decision maker. The idea is thus to measure the discrepancy between different models.

It is assumed that the true data generating process can be described by the model f. The decision makers model, however, is f_{α_0}, with $\alpha_0 \in A$. A denotes the possible parameter space of the decision maker, hence, α_0 is one set of parameters of this space, which is, e.g., found by an estimation routine. This idea is depicted on the left-hand side of Figure 3.4. It becomes clear from Figure 3.4 that the true model parametrization must not be part of A. Hence, estimates of the specific parameter of the model can be made without taking into account the true values. The relative distance between the model equipped with the estimated values f_{α_0} and the true model f is measured by *entropy* $I(f_{\alpha_0}, f)$. As long as the estimated model is misspecified the entropy is positive (Hansen and Sargent 2008).

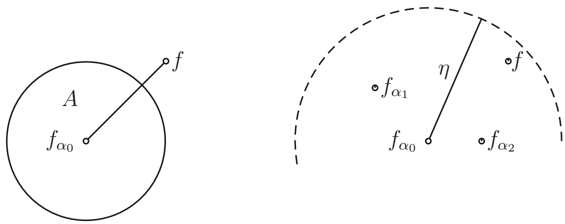

Figure 3.4.: Entropy (lhs) and robust policy (rhs) (Hansen and Sargent 2008)

To capture decision making under (model) uncertainty the above analysis is turned upside down. The decision maker, hence, the central bank, takes f_{α_0} as the reference model, which is surrounded by a set of alternative models, $f_{\alpha,i}$, with $i = 1, 2, \ldots$. One of these models, f, covers the true data generating process. The central bank is aware that her model might be misspecified and wants to find a policy rule which performs well over a range of models. The only restriction she makes herself is the amount of entropy she is willing to admit, that is, $I(f_{\alpha_0}, f) < \eta$, with η measuring the set of possible models surrounding the reference model.[14] This is shown on the right-hand side of Figure 3.4. The idea of a robust policy is to find a policy rule which fits well over a range of models, that is, a rule which minimizes the loss function of the central bank and thereby takes into account that the reference model might actually not be the date generating one (Hansen and Sargent 2008).

Although, the concept of unstructured uncertainty of Hansen and Sargent is used in the context of deducing a robust monetary policy, the idea can even be applied as a general form of additive, data, parameter, or model uncertainty, depending on the unknown variables (see, for example, Walsh 2003b, Williams 2008). In what follows the concept of un-/structured uncertainty of the previous section is pasted to the concept of entropy. As mentioned, the idea is to collapse various sources of uncertainty into one additional *shock* vector covering all misspecifications. However, this additive component is in general not equivalent to a merely additive shock vector.

The writing of Walsh (2003b) and Hansen and Sargent (2008) differ a bit. Both authors take the model estimated by the central bank as the starting point but from this point they argue nearly opposing. To Walsh (2003b) the perceived model is 'corrected' by the error term to achieve the true model. Hence, the error term displays the misspecification between the true model and the one assumed by the central bank. This has been demonstrated before. In the corresponding analysis of Hansen and Sargent (2008), however, they flip the direction. Thus the perceived model becomes somewhat the true model, which is surrounded by misspecified models – of which one can strangely be the true model. Their aim is to find an optimal policy rule expressed in terms of this approximated model. Hence, the difference between both is merely the direction of argumentation.[15]

In the following, to keep the spirit of the Hansen and Sargent robust control idea, the approximated model of the central bank is given by

$$f_{\alpha_0}: \quad x_{t+1} = Ax_t + Bu_t + C\epsilon_{t+1}. \tag{3.3.1}$$

The surrounding models which exhibit different parameter constellations f_α are thus given by

$$f_\alpha: \quad x_{t+1} = Ax_t + Bu_t + C(\epsilon_{t+1} + w_{t+1}). \tag{3.3.2}$$

[14] Hansen and Sargent (2008) agree to be silent about the way f_α is found. I will preserve the silence.

[15] With respect to the previous equations, Equation (3.2.38) is written with hats due to notation of Walsh (2003b). However, to follow the literature, see, e.g., Gerke and Hammermann (2011), Dennis (2010), Hansen and Sargent (2008) and to give a convenient writing, in the following the reference model is written without hats, thereby the notion of Hansen and Sargent (2008) is followed

It should be noted that it could be the case that for a certain parameter constellation α^*, Equation (3.3.2) equals the true date generating process. The argumentation, however, remains the same. As long as $w_{t+1} > 0$, the approximated reference model of the central bank and the surrounding models differ. These errors or misspecification enter the model through the conditional mean of the classical additive shock vector ϵ. The effect of w_{t+1}, which is of the general form $w_{t+1} = f_t(x_t, \dots x_{t-n})$, is thus a change of the conditional mean of the shock vector. The error term of Equation (3.3.1) is now distributed $N(w_{t+1}, I)$ rather than before $N(0, I)$. Because w_{t+1} is premultiplied by the matrix C, which contains the standard deviations of the original innovation, more volatile shocks provide greater room for misspecification. This underlines again the desired masking of the misspecification, which is achieved not only by a time synchronization, but also by a masking of the statistical properties (Hansen and Sargent 2001, 2002, Walsh 2003b, Dennis et al. 2009).

The decision maker has the freedom to choose the maximum size of the entropy. Thus she can limit the amount of uncertainty, which is equivalent to restraining the number of possible alternative models. Choosing a rather large amount of uncertainty leads as a consequence to a policy rule which is also robust over a large range of possible models. Yet, it might be only a very poor rule to the most of them.

A more specific rule is achieved on the cost of robustness by reducing the size of the entropy. In any case it is assumed that even for $\eta_0 > 0$ the difference between the true model and the distorted one will never be big enough to provoke the policymaker to improve his specification by learning. Thus, to circumvent the temptation of learning only marginal amounts of model misspecification are considered (Leitemo and Söderström 2004, Hansen and Sargent 2008).

To restrain the range of possible models boundaries are given to w_{t+1} in the way

$$E_0 \sum_{t=0}^{\infty} \beta^{t+1} w'_{t+1} w_{t+1} \leq \eta_0. \tag{3.3.3}$$

A nice way to think about η_0, is to look at it, as if it is the budget of some evil agent. This agent chooses w within his boundaries for a maximum error disturbance. For $\eta_0 = 0$ the malevolent player is out of power, hence, there is no uncertainty within the decision process. (Hansen and Sargent 2008, Dennis et al. 2009).

On the other hand, one could also think of restraining the uncertainty more directly by defining the error process more accurate. This offers again the difference between structured and unstructured uncertainty. Specifying the error term more precisely would lead from an unstructured treatment into a structured treatment of uncertainty. Clearly, making use of a more specific form of uncertainty, say, concerning the parameters of the model, would give the policy maker also a more specific reaction function, but at the same time would require a specification which can often not be given (see, e.g., Onatski and Williams 2002).

However, while technically it is a small step to go from unstructured to structured uncertainty or the other way around, this does not hold for the intension behind the two different accounts. In the structured uncertainty case the probability distribution, as well as the form of uncertainty, must be given. Thus the source of uncertainty must be known to the policy maker. This, as Tetlow and von zur Muehlen (2001) mention, is merely a situation of risk which can be priced and not one of uncertainty. This conclusion does not hold for the unstructured case. Because of the missing form and the absence of any probability prediction, unstructured uncertainty is often associated with uncertainty in the sense of Knight (1921) where the policy maker is unable to form any probability statement concerning the shock process, and thus is unable to identify the specific form of uncertainty she faces.

It has been said that only marginal model misspecification are considered in order to circumvent the question why the decision maker does not learn or at least reconsiders the parameters of the approximated model. But there is another reason. Beside the right timing of the error term, and the premultiplying of it with the coefficient matrix of the additive shock vector, keeping the error small makes it again harder to be detected at all by the decision maker. Hence, in sum, the structure as well as the source of the misspecification can hardly be found. This enforces the view of Knightian uncertainty where no probability judgment can be made (Hansen and Sargent 2008, Leitemo and Söderström 2008).

3.3.2. Escaping Model Uncertainty by Robust Control

Suppose a policy maker who is uncertain about the correctness of his model of the economy. Thereby, she can not specifically locate the uncertainty, for instance, if it is due to some parametric imprecisions or if the data used in his model might be contaminated by noise. Moreover the inaccuracy of his model compared to reality could also be due to some omitted variables, that is, influences not even noticed before and therefore not considered anyhow.

The great advantage of this method of modeling uncertainty is the mentioned possibility to deal with Knightian uncertainty, hence, no probability assignment needs to be done. This gives a wide range of possible misspecification to work with. But at the same time, this freedom can come as a big disadvantage. The policy maker follows a rule which, even designed originally for a different model, should also perform well in a different setup then intended. Yet, due to the fact that the source or structure of uncertainty is not specified precisely, it could nevertheless be the case that a policy reaction is quite misleading in a different set-up than the intended one. Especially concerns about the stability of the system arise, but also the fear of exaggerated and not appropriate reactions (Onatski and Williams 2002, Williams 2008).

Some of the basic assumptions of robust control haven been barely broached in the previous sections, discussing the difference between structured and unstructured uncertainty and the concept of entropy. The idea of robust control is modeled in the following way. The central bank is equipped with a model which she assumes to be the best approximation of the real world. She knows that it might not be the 'right' model, indeed she knows that is only an approximation, but she is also unable to specify the difference between her model and some other possible model in terms of a probability distribution. On the other hand the previously mentioned evil agent is introduced. This vehicle serves to model the misspecification the central bank faces. The evil agent is equipped with a budget, which enables him to disturb the central banks model in the best way he can. The disturbance enters the model of the central bank via an additional shock term leading into an *unstructured* uncertainty momentum. This additional shock term is constructed in a way that it can cover a wide range of misspecification; w_{t+1} can be a linear function of the contemporaneous state variables or lagged variables or could even feed back in a non-linear manner.

Robust control aims to find a policy rule, which minimizes a given loss function, considering the possibly worst case scenario, which equals a rather bad parameter constellation of the constraints. This worst case scenario is achieved by equipping the evil agent with noise to distort the central banks reference model. Hence, while the central bank *minimizes* her loss under her constraints, the evil agent *maximizes* the loss, given his constraint, that is, his budget. The resulting policy rule fits for the standard 'certainty case' as well as for the most undesirable parameter constellation.[16] This approach, with respect to Hansen and Sargent (2008) who can be seen as the 'inventors' of robust control in monetary policy design, is called *max-min* robust control approach (Giordani and Söderlind 2004).

In this context, different kinds of policy goals and constraints can be discussed, such as the discrimination between a discretionary policy or a commitment solution. Further

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[16] The so-called 'certainty case', of course, is not really a certainty case in the terminology of the previous chapters. Explained before, the model in question is just the 'approximated' model of the central bank. Here the terminology of Hansen and Sargent (2008) may be misleading. Other authors depart from this expression, see, e.g., Giordani and Söderlind (2004) and give the model the term 'reference model', which in my opinion fits better. In any case, the model in question – even if called certainty model – is not the true, i.e., the data generating model, but just an approximation to the unknown true model, which serves as the reference model for further policy analysis. For this analysis I will use the expressions 'reference model' and 'approximated model' interchangeable depending on the aim of the argumentation.

the model can be backward or forward-looking. Luckily, standard rational expectation solution techniques can be applied, which facilitate the calculation. This may also be a reason for the popularity of the approach.

The full optimization problem for the forward-looking case is given by

$$\min_u \max_w E_0 \sum_{t=0}^{\infty} \beta^t (x_t' Q x_t + u_t' R u_t + 2x_t' U u_t - \theta w_{t+1}' w_{t+1}), \tag{3.3.4}$$

subject to

$$x_{t+1} = A x_t + B u_t + C(\epsilon_{t+1} + w_{t+1}). \tag{3.3.5}$$

All variables keep their meanings and the multiplier θ defines the the set of models available to the evil agent.

Giordani and Söderlind (2004) show that under commitment, no matter if backward or forward-looking, the solution can be found via standard rational expectation solution techniques such as the (generalized) Shur decomposition, shown, e.g., in Söderlind (1999). This is due to the fact that the first order conditions are the same under maximization and minimization.[17] In the backward-looking case, the policy rule is – like the standard certainty case – a linear function of the state vector. But despite its additive character the solution is not certainty equivalence. Moreover the volatility matrix C in fact affects the policy rule (Hansen and Sargent 2008).

Under the forward-looking regime the dependencies are slightly more complicated. The discretionary solution equals the commitment solution if only backward-looking behavior is assumed. The last case left over – discretionary policy with forward-looking behavior – is discussed now in detail. Due to the fact that this case has also been considered under certainty, as well as under parametric uncertainty, a comparison between all three different scenarios can be given at the end of the section.

To show the effects of a robust policy more clearly I present the solution in the equational writing, rather than the more compact state space notation. This allows an easier way to show the differences arising between the benchmark case with no uncertainty, the robust control approach, and later on the parametric uncertainty case. The corresponding

[17] For this it must hold that in the forward-looking scenario the private sector and the central bank share the same information, have the same loss function, and the same preference for robustness (Giordani and Söderlind 2004).

supply and demand equations are nearly standard despite the newly perturbation element w_{t+1},

$$\pi_t = \beta E_t \pi_{t+1} + \kappa_t y_t + \Sigma_\pi(\epsilon_t^\pi + w_t^\pi), \tag{3.3.6}$$

$$y_t = E_t y_{t+1} - s(i_t - E_t \pi_{t+1}) + \Sigma_y(\epsilon_t^y + w_t^y). \tag{3.3.7}$$

The shock vectors ϵ^j, with $j = \pi, y$, are assumed to be white noise. Opposed to previous writings they are distributed $N(0,1)$, with Σ_j covering the standard deviations, such that the variance is given as Σ_j^2. This allows to mask the newly shock w_{t+1} and make it hard to detect.

Equation (3.3.6) and (3.3.7) show how the misspecification is disguised by the 'traditional' shock vector, due to the fact that the sum is multiplied by Σ_π and Σ_y. The distribution of the expression $\tilde{\epsilon} = (\epsilon + w)$ changes from the standard certainty case with $\tilde{\epsilon} \sim N(0,1)$, into $\tilde{\epsilon} \sim N(w,1)$. The mean of the additional error term has changed and is now unequal zero. If the classical error term would have a zero variance the misspecification could be detected immediately.[18] Again, despite the fact that the misspecification vectors w^π and w^y enter the model additive, it should not be confounded with the additive shock vectors (Hansen and Sargent 2001, Giordani and Söderlind 2004, Leitemo and Söderström 2004, Hansen and Sargent 2008).

Furthermore the timing of the interaction between the central bank and a malevolent player is crucial. Leitemo and Söderström (2004) point out that the outcome differs, whether a Nash game is played or the central bank acts as a Stackelberg leader.[19] In most cases (see, e.g., Onatski and Williams 2002, Hansen and Sargent 2008, Williams 2008), the Nash solution of best answers seems to be favored. The solution is mainly straight forward and very similar to the certainty case. Nevertheless, some changes obviously must occur.

The budget of the malevolent player is given and restricted by

$$E_0 \sum_{t=0}^{\infty} \beta^t (w_t^\pi)^2 \leq \eta, \tag{3.3.8}$$

$$E_0 \sum_{t=0}^{\infty} \beta^t (w_t^y)^2 \leq \eta. \tag{3.3.9}$$

18 In case of $\Sigma > 0$ the distribution of $\hat{\epsilon}$ changes from $N(0,\Sigma^2)$ into $N(w,\Sigma^2)$ due to the misspecification. In case of $\Sigma = 0$, the distribution changes from $N(0,0)$ into $N(w,0)$, hence, the specification error could be easily detected (Leitemo and Söderström 2004)

19 Under the Stackelberg solution the central bank first seeks for the optimal size of the evil agent disturbance and takes this optimal setting into account while she optimizes her policy reaction.

Under discretion the Lagrangian is set up in the usual way, incorporating now beside the normal additive shock vector the newly disturbance, which represents possible model deviation between the true data generating process and the reference model of the central bank.

The Nash solution – contrary to the Stackelberg-leader solution – does not take into account the optimal size of the specification error, hence, the Lagrangian is given as

$$\begin{aligned}\mathcal{L}_t = E_0 \sum_{t=0}^{\infty} \beta^t \Big[\pi_t^2 + \alpha_y y_t^2 - \theta \left((w_t^{\pi})^2 + (w_t^{y})^2 \right) \\ - \phi_t^{\pi} \left(\pi - \beta E_t \pi_{t+1} - \kappa_t y_t - \Sigma_{\pi} \left(\epsilon_t^{\pi} + w_t^{\pi} \right) \right) \\ - \phi_t^{y} \left(y_t - E_t y_{t+1} + s(i_t - E_t \pi_{t+1}) - \Sigma_y (\epsilon_t^{y} + w_t^{y}) \right) \Big]. \end{aligned} \quad (3.3.10)$$

The solution of the optimization problem changes from an standard minimization problem into a minimization-maximization problem. The central bank still minimizes her loss function, whereas the evil agent maximizes his damage in order to maximize the loss of the central bank.

The multiplier θ can be seen as the reciprocal of the evils budget. A rising θ is equivalent to a shrinking budget of the evil agent. Hence, a falling θ is equivalent to a higher budget, which increases the number of potential models against the central banks wants to be robust. If θ approaches its lower limit the largest possible set of models is achieved. This would give the most robust decision rule. If θ approaches infinity, the budget of the evil player equals zero. In this scenario the penalization of possible different models becomes maximal. There is no misspecified model on behalf of the evil agent, hence, there is no specification left against which the policy maker wants to be robust. This equals the standard rational expectation case (Leitemo and Söderström 2004, Hansen and Sargent 2008, Williams 2008).[20] See Table 3.1 for a summary.

Budget	Multiplier	Distortion	Robustness
max	$\theta \to 0$	max	high
min	$\theta \to \infty$	min	low

Table 3.1.: Robustness

If θ enters the policy function its size is crucial. The typical approach to define θ, could be to increase the preference for robustness by starting with the limiting case $\theta = \infty$ and building up more robustness by decreasing θ in small steps (Leitemo and Söderström

[20] According to Walsh (2003b) there could be also a different interpretation of θ, namely as the policy maker's attitude towards risk. A risk sensitive central bank would choose a very robust policy, while a less risk sensitive central bank would choose a less robust policy.

2004). On the other hand Hansen and Sargent (2003a) develop a so-called *detection error probability* approach which basically tries to compute θ by using a log likelihood ratio, given the background that one is not sure whether the approximating model or the worst case model generates the observed data (Hansen and Sargent 2003a, Giordani and Söderlind 2004). Yet, this question can be postponed as it does not affect the deduction of the first order conditions.

The first order conditions of the optimization problem under discretion now cover the optimal targeting rule of the variables output and inflation as well as it additionally gives the optimal size of the misspecification chosen by malevolent player

$$y_t = -\frac{\kappa}{\alpha_y}\pi_t, \tag{3.3.11}$$

$$w_t^\pi = \frac{\Sigma_\pi}{\theta}\pi_t, \tag{3.3.12}$$

$$w_t^y = 0. \tag{3.3.13}$$

The striking finding of these first order conditions is the unchanged targeting rule in Equation (3.3.11), which equals exactly the certainty benchmark case under discretion. The optimal trade off between inflation and output deviation is not affected by the misspecification of the model. This might sounds odd at a first sight, but becomes clear at a second glance. Under a robust policy inflation or output might be more volatile, nevertheless they are reduced by the same means as under certainty, namely reducing output in case of a rising inflation and vice versa (Lees 2004, Leitemo and Söderström 2004, Hansen and Sargent 2008).[21]

If the will for robustness rises (θ is decreasing) the misspecification term in Equation (3.3.12) gets bigger, due to an increasing budget of the evil agent. The optimal size of the misspecification varies positively with inflation and the standard deviation of the cost shock. This seems plausible, as the potential damage rises if inflation is high, as well as a high standard deviation enlivens the misspecification error even more. Interestingly enough, the optimal error of Equation (3.3.13) is always zero no matter the size of any variable of the system. This stems from the fact that any misspecification of Equation (3.3.7) can be immediately counteracted by the central bank by simply adjusting the interest rate (Leitemo and Söderström 2008).

[21] It should be noted that these results not necessarily have to hold under a different policy rule. In case of a simple rule, for instance, this finding does not hold.

Combining the first order conditions with the misspecified model Equations (3.3.6) - (3.3.7) gives the law of motion for the worst case scenario system as

$$\pi_t^{wc} = \frac{\alpha_y}{\alpha_y(1-\Sigma_\pi^2/\theta)+\kappa^2}\Sigma_\pi\epsilon_t^\pi, \tag{3.3.14}$$

$$= a_\pi^{wc}\Sigma_\pi\epsilon_t^\pi, \quad \text{with} \quad a_\pi^{wc} > 0, \tag{3.3.15}$$

$$y_t^{wc} = \frac{-\kappa}{\alpha_y(1-\Sigma_\pi^2/\theta)+\kappa^2}\Sigma_\pi\epsilon_t^\pi, \tag{3.3.16}$$

$$= a_y^{wc}\Sigma_\pi\epsilon_t^\pi, \quad \text{with} \quad a_y^{wc} < 0. \tag{3.3.17}$$

On behalf of the evil agent the optimal amount of misspecification is achieved by combining Equation (3.3.12) with (3.3.14) to get

$$w_t^\pi = \frac{\Sigma_\pi}{\theta}\pi_t$$

$$= \frac{\alpha_y}{\alpha_y(\theta-\Sigma_\pi^2)+\theta\kappa^2}\Sigma_\pi^2\epsilon_t^\pi. \tag{3.3.18}$$

The interest rate rule is achieved via the worst case equation for output (3.3.16), the optimal error misspecification size (3.3.13), and the IS equation (3.3.7).

$$i_t^{wc} = \frac{s^{-1}\kappa}{\alpha_y(1-\Sigma_\pi^2/\theta)+\kappa^2}\Sigma_\pi\epsilon_t^\pi + s^{-1}\Sigma_y\epsilon_t^y \tag{3.3.19}$$

represents the optimal interest rate response at the worst-case scenario. Again, this equation looks quite familiar to the certainty case. And in fact, if the preferences for robustness goes to infinity ($\theta \to \infty$), uncertainty diminishes as the bracketed term approaches one.

Opposed to other Bayesian methods there is no weighting taken into account here, hence, the policy-rule must be also robust against some really worse, but quite improbable situations. Due to reasons – which are kept silence – the central bank has chosen her approximating model in the best way she could, hence, one could say she has already incorporated weights into this selection. However, to conduct a robust policy she uses the worst case interest rate rule, even if there is no misspecification at all or the probably for a misspecification is very low. Hence, the worst case interest rate rule is plugged into the approximating model with zero misspecification to get the solution path under the most probable assumed model. This law of motion must be compared with the certainty case to get the 'cost' of conducting a robust policy – even if there is no misspecification.

Because there is no misspecification concerning the IS curve ($w_t^y = 0$), there is no change in the equilibrium path of output, whether the policy is conducted under certainty or under a robust policy. Yet, the solution for the supply equation changes so that

$$\pi_t^{apx} = \left(1 - \frac{\kappa^2}{\alpha_y(1 - \Sigma_\pi^2/\theta) + \kappa^2}\right)\Sigma_\pi \epsilon_t^\pi, \tag{3.3.20}$$

$$= a_\pi^{apx}\Sigma_\pi \epsilon_t^\pi, \quad \text{with} \quad a_\pi^{apx} > 0, \tag{3.3.21}$$

$$y_t^{apx} = y_t^{wc} = \frac{-\kappa}{\alpha_y(1 - \Sigma_\pi^2/\theta) + \kappa^2}\Sigma_\pi \epsilon_t^\pi, \tag{3.3.22}$$

$$= a_y^{apx}\Sigma_\pi \epsilon_t^\pi, \quad \text{with} \quad a_y^{apx} < 0. \tag{3.3.23}$$

Some drawbacks of the robust control approach might by worth mentioning. For example, the importance of determining the size of the robustness parameter θ is a crucial one. As has been mentioned before, there is no weighting between different models. That is, the policy rule is affected in the same way by very improbable situations, as it is by very probable constellations. Hence, a rising θ includes more and more models away from the best guess model. As these models affect the policy rule in the same way like models which are closer to the reference models or more probable, a trade-off between robustness and a policy rule which still performs quite ideal for the intended reference model arise. The more models are included, the more robust the policy becomes. Coevally, the policy rule might gets more and more inadequate for the reference model, which has been chosen to be the most probable model. Another drawback is the fact that models on the boundary of θ shape the policy function in the same way as all other possible misspecification within the circle around the reference model do. Yet, at the same time, models only a nuance away from θ do not find any influence in finding the right policy decision.

Furthermore, it has been assumed that no learning is conducted. This can also be seen as a drawback. For example, under commitment the malevolent player as well as the central bank have to commit on a certain behavior. This seems to be quite inappropriate especially if the central bank performs poor with the used rule. One would in this case rather expect the central bank to change her policy such that better results are achieved. The same should then hold for the malevolent player (Giordani and Söderlind 2004, Svensson 2007, Hansen and Sargent 2008).

3.3.3. Comparison of Equilibrium Paths under Certainty, Parametric Uncertainty, and Robust Control

Concluding, the so far elaborated rational expectations equilibrium paths for the policy rule as well as the state variables under discretion shall be compared. The selection criteria follows the same reasons outlined before. The certainty case serves as a benchmark. Parametric uncertainty precludes certainty equivalence. Additionally, some authors highlight the practical relevance of parametric uncertainty (Issing 1999), and other treat parametric uncertainty even in the same way as model uncertainty (Giordani and Söderlind 2004). Therefore it is interesting to see if at all, differences between parametric uncertainty and model uncertainty can be identified. This is of special concern, due the fact that parametric uncertainty could be just a subform of model uncertainty if treated in the way of Section3.2.2.

The following equations are taken from the respective preceding sections and are repeated here for convenience only. The equilibrium paths for inflation, output, as well as the interest rate under discretion are considered. The superscripts *ce*, *pu*, *wc*, and *apx* denote the respective situations of certainty equivalence, parametric uncertainty, worst case realization, and realization of the assumed central bank model, i.e., the model, which according to the central bank, approximates the true data generating process best, combined with the policy rule which also serves well for a worst case scenario.

$$\pi_t^{ce} = \frac{\alpha_y}{(\alpha_y + \kappa^2)}\epsilon_t^{\pi}, \tag{3.3.24}$$

$$\pi_t^{pu} = \frac{\alpha_y}{(\alpha_y + \bar{\kappa}^2 + \sigma_{\kappa}^2)}\epsilon_t^{\pi}, \tag{3.3.25}$$

$$\begin{aligned} \pi_t^{wc} &= \frac{\alpha_y}{\alpha_y(1-\Sigma_{\pi}^2/\theta) + \kappa^2}\Sigma_{\pi}\epsilon_t^{\pi}, \qquad (3.3.26)\\ &= a_{\pi}^{wc}\Sigma_{\pi}\epsilon_t^{\pi}, \quad \text{with} \quad a_{\pi}^{wc} = \frac{\alpha_y}{\alpha_y(1-\Sigma_{\pi}^2/\theta) + \kappa^2} > 0, \end{aligned}$$

$$\begin{aligned} \pi_t^{apx} &= \left(1 - \frac{\kappa^2}{\alpha_y(1-\Sigma_{\pi}^2/\theta) + \kappa^2}\right)\Sigma_{\pi}\epsilon_t^{\pi}, \qquad (3.3.27)\\ &= a_{\pi}^{apx}\Sigma_{\pi}\epsilon_t^{\pi}, \quad \text{with} \quad a_{\pi}^{apx} = 1 - \frac{\kappa^2}{\alpha_y(1-\Sigma_{\pi}^2/\theta) + \kappa^2} > 0. \end{aligned}$$

Compared to the benchmark case without uncertainty in Equation (3.3.24), parametric uncertainty in Equation (3.3.25) dampens the equilibrium path of inflation. This is due to the appearance of the noise variance of the uncertain variable. The worst case scenario of Equation (3.3.26) converges to the certainty case if the robustness parameter θ goes into infinity. This is equivalent to the very low preference for robustness or a very small budget of the malevolent player. A convergence is thus reasonable. A decrease in the preference for robustness thus causes a^{wc} to fall, resulting in a less volatile inflation rate due to a cost shock. The opposite is true for an increasing preference for robustness where inflation becomes more volatile. Yet, to realize these predictions the worst case model has to become actually true, which may be in fact quite improbable.

The most probable situation is attained if the benchmark or approximated model of the central banks becomes true. In this situation, Equation (3.3.27) governs the inflation rate due to the fact that the central bank applies the worst case policy rule, although the reference model becomes true. Now, opposed to the worst case scenario, a decrease in the preference for robustness (θ becomes larger) makes inflation even more volatile. Hence, the opposite is true for an increasing preference for robustness, where inflation becomes less volatile. Table 3.2 summarizes these findings for the inflation rate (Leitemo and Söderström 2008).

Pref. for robustness	θ	π^{wc}	π^{apx}
high	low	more volatile	less volatile
low	high	less volatile	more volatile

Table 3.2.: Inflation volatility

Taking the consequences of output into account the relevant equations are

$$y_t^{ce} = \frac{-\kappa}{(\alpha_y + \kappa^2)} \epsilon_t^{\pi}, \tag{3.3.28}$$

$$y_t^{pu} = \frac{-\bar{\kappa}}{(\alpha_y + \bar{\kappa}^2 + \sigma_\kappa^2)} \epsilon_t^{\pi}, \tag{3.3.29}$$

$$\begin{aligned} y_t^{wc} &= \frac{-\kappa}{\alpha_y(1 - \Sigma_\pi^2/\theta) + \kappa^2} \Sigma_\pi \epsilon_t^{\pi}, \\ &= a_y^{wc} \Sigma_\pi \epsilon_t^{\pi}, \quad \text{with} \quad a_y^{wc} = \frac{-\kappa}{\alpha_y(1 - \Sigma_\pi^2/\theta) + \kappa^2} < 0, \end{aligned} \tag{3.3.30}$$

$$\begin{aligned} y_t^{apx} &= \frac{-\kappa}{\alpha_x(1 - \Sigma_\pi^2/\theta) + \kappa^2} \Sigma_\pi \epsilon_t^{\pi}, \\ &= a_y^{apx} \Sigma_\pi \epsilon_t^{\pi}, \quad \text{with} \quad a_y^{apx} = \frac{-\kappa}{\alpha_y(1 - \Sigma_\pi^2/\theta) + \kappa^2} < 0. \end{aligned} \tag{3.3.31}$$

Again, the path of the target variable is dampened under parametric uncertainty due to the trade-off the central bank faces between a fast recovery and a low variance, shown in Equation (3.3.29). Concerning the worst case scenario an increase in the preference for robustness leads to an increase in a_y^{wc}, which causes a more volatile output reaction. Opposed to that, a decrease calls for a less volatile output. The approximated model utilizes again the worst case scenario interest rate rule although the worst case scenario does not materialize. Exactly the same equilibrium paths, no matter if worst case or approximated model, can be expected. This is due to the above mentioned effect that output deviations can be immediately off-set by the central bank and therefore the optimal size of noise w_t^y is equal to zero. Table 3.3 summarizes the results for output.

Pref. for robustness	θ	y^{wc}	y^{apx}
high	low	more volatile	more volatile
low	high	less volatile	less volatile

Table 3.3.: Output volatility

The interest rate paths are given as

$$i^{ce} = \frac{\sigma\kappa}{(\alpha_y + \kappa^2)}\epsilon_t^{\pi} + \sigma\epsilon_t^{y}, \tag{3.3.32}$$

$$i^{pu} = \frac{\bar{s}\bar{\kappa}}{(\alpha_y + \bar{\kappa}^2 + \sigma_\kappa^2)(\bar{s}^2 + \sigma_s^2)}\epsilon_t^{\pi} + \frac{\bar{s}}{\bar{s}^2 + \sigma^2}\epsilon_t^{y}, \tag{3.3.33}$$

$$\begin{aligned} i_t^{wc} = i_t^{apx} &= \Big(\frac{\sigma\kappa}{\alpha_y(1-\Sigma_\pi^2/\theta) + \kappa^2}\Big)\Sigma_\pi\epsilon_t^{\pi} + \sigma\Sigma_y\epsilon_t^{y}, \qquad (3.3.34)\\ &= a_i^{wc}\Sigma_\pi\epsilon_t^{\pi} + \sigma\Sigma_y\epsilon_t^{y}, \quad \text{with} \quad a_i^{wc} = \frac{\sigma\kappa}{\alpha_y(1-\Sigma_\pi^2/\theta) + \kappa^2}. \end{aligned}$$

The Brainard (1967) result is depicted in Equation (3.3.33).[22] Under robust control the interest rate is set in a way which is optimal, even if the worst case scenario happens. Hence, it is designed to fit even in the most undesirable and maybe also most improbable case. However, this interest rate setting is also applied if the worst case scenario does not unfold but the most likely model – which is the approximated or reference model. Therefore, the interest rule is the same for the worst case scenario as well as for the approximated model.

Demand shocks are treated in the same way under certainty as well as under the robust control setting. The preference for robustness has no influence on the interest rate reaction which is the same under robust control compared to certainty. This is not the case under parametric uncertainty for which the interest rate response is clearly dampened due to uncertainty. Thus, although, parametric uncertainty is sometimes handled as a sub-category of model uncertainty, it produces a different interest rate equilibrium path than the robust control example.

Supply side shocks offer another important difference between parametric uncertainty treated in the Brainard way and robust control. An increase in the preference for robustness is equivalent with a decreasing θ, or an increasing budget of the evil agent. It calls for a more aggressive policy on behalf of the central bank. A larger budget of the malevolent player, however, is equivalent to larger uncertainty for which Brainard (1967) would recommend a less aggressive policy. Nevertheless, as has been demonstrated this is also a suitable result (Söderström 2002, Leitemo and Söderström 2008).

[22] It holds that $\sigma = 1/s$.

This comparison has shown, considerable differences between parametric uncertainty and robust control exist. It highlights the mentioned aspect, whereas it matters if uncertainty is treated in a *structured* or in an *unstructured* way. As shown, the results can differ significantly. That is, it is not the same whether the difference between some reference model and the data generating process is due to 'some misspecification' or due to a concrete reason, like the uncertainty of specific parameters. It could be utile for the central bank to put some effort into the investigation of the source of uncertainty. Parametric uncertainty can thus only partly be treated as a subform of model uncertainty.

3.3.4. Markov-Switching and Model Uncertainty

Most recently another method to tackle the problem of model uncertainty has found its way into the academic discussion, namely Markov-switching or Markov-jump models. Opposed to the previously mentioned methods Markov-switch models do not aim to analyze local deviations from a steady state of *one* linearized model. Their purpose is to analyze various separate models, which all differ from another in some aspect. The tempting of this approach is its wide field of application, not only within the filed of uncertainty in a narrow sense, but also in combination with problems such as heterogeneous agents or financial frictions.[23]

While first used by Hamilton (1989) to estimate real GNP growth in the US business cycle, Markov-switching models which are related to problems in monetary policy have by far a shorter history. Only a very small group of authors have published in this field of research until today. Svensson and Williams (2005) describe a solution algorithm for backward as well as forward-looking models under possible future regime switches. In this context certainty equivalence ceases to hold and mean forecasts of variables are replaced by distribution forecasts which include switching probabilities into the policy reaction function. Blake and Zampolli (2011) use a semi-structural model to incorporate asymmetric beliefs on behalf of the central bank and agents into a Markov-switching setup. Quite recently, Williams (2012), based on Svensson and Williams (2005) and Curdia and Woodford (2010), incorporates the possibility of financial disruptions by distinguishing between normal times and times of crisis where credit spreads matter for inflation and output development.

The idea of Markov-switching models in the field of monetary policy captures the fact that economies occasionally exhibit changes in their behavior. Those changes can be associated with different reasons such as shocks like a financial crisis or due to a general arbitrary environment. For the latter case, one could think of a different behavior on

[23] Furthermore, Chapter 5 will offer an application of Markov-switching to analyze different communication regimes of the European Central Bank.

behalf of the agents, e.g., due to a different assessment of future price developments, a change in the propensity to consume, a different interest rate elasticity of investments, or whether the economy is experiencing a rise or drop of the economic activity.

First, I will show the general idea of Markov-switch models. Let the system, which describes the economy in a prosper time, be given as

$$y_t = a_1 r + e. \tag{3.3.35}$$

However, in times of stagnation the system is given by

$$y_t = a_2 r + e, \tag{3.3.36}$$

with $a_1 \neq a_2$. Hence the transmission of the control variable on the target variable is different depending on the prevailing regime. For example, regime one could be a healthy environment and regime two an unhealthy environment. Under each regime the interest rate might effect key variables in a different way.

Depending on whether what regime is prevailing the system behaves according to Equation (3.3.35) or (3.3.36). It is said to take the form of two different *regimes*, *states*, or *modes*. These expressions are used interchangeably during the work. Equation (3.3.35) and (3.3.36) can be summarized in one equation as

$$y_t = a_{s,t} r + e, \tag{3.3.37}$$

where s_t denotes the current state at time t. According to the example economy, s can take the value 1: healthy or 2: unhealthy.

A switch between these (two) states follows the realization of a Markov chain. In general, a Markov chain is just a stochastic process. The defining feature of this process is the property that the knowledge of the complete history of realizations contains as much informational content as the knowledge of only the last n realizations, that is only a few observations. For the special case of an order one Markov process the realization in the current period t depends only on the realization of the past period, $t-1$.

For more than two possible states the order one Markov chain, however, is

$$Pr(s_t = j | s_{t-1} = i, s_{t-2} = k, \dots) = Pr(s_t = j | s_{t-1} = i) = p_{ij}, \tag{3.3.38}$$

thus, p_{ij} denotes the probability to move from the prevailing regime in $t-1$, i, into the regime j in period t. The information that in $t-2$ regime k was prevailing becomes irrelevant.

The probability to change from one regime into the other can be displayed as a probability transition matrix where the diagonal elements show the probability to linger in the current state and the off-diagonal elements the probability to change the current state. In general it must hold $\sum_{j=1}^{N} p_{ij} = 1$ for each $i = 1, \ldots, N$.[24] Hence, the probabilities to linger in or change the regime must add up to one. The two regime case thus exhibits $p_{11} + p_{12} = p_{21} + p_{22} = 1$. Probabilities are assumed to be time-invariant and exogenous. If it holds that $p_{ii} = p$ it follows that $p_{ij} = 1 - p$. Hence, from $p_{jj} = q$ it follows that $p_{ji} = 1 - q$, such that the complete transition matrix can be written as

$$P = [p_{ij}] = \begin{pmatrix} p_{ii} & p_{ij} \\ p_{ji} & p_{jj} \end{pmatrix} = \begin{pmatrix} p & (1-p) \\ (1-q) & q \end{pmatrix}. \tag{3.3.39}$$

Figure 3.5 illustrates the switching probabilities. It becomes clear that if state one prevails there is the probability to linger in this state – given by p_{ii} – or to migrate into state two with the probability p_{ij}. As these are the only possible choices, both probabilities must add up to one. For the special case $P = I$ no switching between the regimes is allowed. Then it holds that $p_{ii} = 1, p_{ij} = 0$ and the economy rests for ever in her initial state.

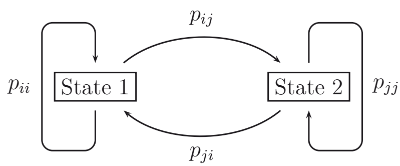

Figure 3.5.: Transition probabilities

For further analysis the following state space model is assumed which could cover the backward looking-model of Section 3.1.3,

$$x_{t+1} = A_{s,t+1} x_t + B_{s,t+1} u_t + C_{s,t+1} \epsilon_{t+1}. \tag{3.3.40}$$

The matrices $A_{s,t}, B_{s,t}, C_{s,t}$ denote the respective coefficient matrices at time t, depending on the prevailing state s. Due to the fact of changing coefficient matrices, the same shock vector has a different impact depending on the prevailing regime. That is, a shock unfolds and creates different adjustments whether the economy faces, e.g., a boom or a bust.

[24] This notation is possibly used different. See, e.g., Zampolli (2006).

The solution is found via the algorithm discussed in Oudiz and Sachs (1985) and Backus and Driffill (1986) which is based on value function iterations. Given one regime, certainty equivalence is prevailing with respect to the additive shock term, hence, the value function might be affected by the additive noise term, the decision rule, however, is not.

For a certain regime $s_t = i$ the value function can be arranged as

$$v(x_t, i) = x_t' V_i x_t + v_i, \tag{3.3.41}$$

which is equivalent to the linear quadratic certainty case, but including the state index i. Accordingly, the decision rule can also be assumed to be of the well known form

$$u(x_t, i) = F_i x_t, \tag{3.3.42}$$

for which the coefficients are again regime depended. If, however, more than one regime is considered, possible regimes switches in the future must be reconsidered as well. Therefore all value functions need to be solved jointly. The solution is again found with the set of interrelated Ricatti equations of the form (below without state dependency)

$$\begin{aligned} V_i &= R + \beta G\left[A'VA|_{s=i}\right] - \beta^2 G\left[A'VB|_{s=i}\right]\left(\beta G\left[B'VB|_{s=i}\right]\right)^{-1} G\left[B'VA|_{s=i}\right], \\ V &= R + \beta A'VA - \beta^2 A'VB\left(\beta B'VB\right)^{-1} B'VA. \end{aligned} \tag{3.3.43}$$

The policy coefficient of the policy rule of Equation (3.3.42) is determined as

$$\begin{aligned} F_i &= -\left(G\left[B'VB|_{s=i}\right]\right)^{-1}\left(G\left[B'VA|_{s=i}\right]\right), \\ F &= -\left(B'VB\right)^{-1}\left(B'VA\right). \end{aligned} \tag{3.3.44}$$

Both sets of equations, the Matrix Ricatti equations as well as the policy coefficients, are very similar to the certainty case given below in Equation (3.3.43) and (3.3.44). In fact, the only difference is given by the matrix G which introduces the probability of a regime switch into the system. It is derived according to

$$G\left[X'VY|_{s=i}\right] = \sum_{j=1}^{N} X_j'(p_{ij}V_j)Y_j. \tag{3.3.45}$$

So clearly, a the possibility of a regime switch enters the system. The value function as well as the decision rule are regime dependent and depend on the probability of a future regime switch. The latter, however, is independent of the shock vector (Svensson and Williams 2005, Blake and Zampolli 2006, Zampolli 2006).

Two special cases can be identified

- $P = I$: If the transition probability equals the identity matrix no regime switches are permitted. Given the regime, it is assumed to last forever. For n different regimes, n distinct policy rules exist, each of which corresponding to the policy rule under certainty. Due to the fact that switches are said to be impossible, the possibility of other regimes is not taken into account when calculating each rule.

- $A_i = A_j$: Theoretically a number of 'different' models exists, which are all the same due to the same coefficient matrices. Hence, in fact, there is only one model. Despite any possible transition from one model to the other the policy rule stays always the same.

On the first sight, Markov-switching modeling just seems like an equivalent representation of parametric uncertainty. However, this inference would be to hasty.

Parametric uncertainty treats parameter of the model as uncertain. That is, monetary policy takes into account not only the existing model, but also the uncertainty surrounding one or more parameters of the model, which is typically expressed by the variance. However, the set-up is always the same.

The Markov-switch method gives every single possible realization of one parameter its own model or equivalently its own state. Hence, *per se*, every single state – or equivalently model – is treated as a certainty model. However, it is uncertain whether this state is prevailing or not. This allows for a more specific evaluation. Utilizing the transition probability allows to give some parameter constellations a high probability to occur and others a rather low probability, yet, for each case a precise model must be formulated.

To show the difference between parametric uncertainty and the Markov approach the following section contains an example on that issue. Further, other examples utilizing the Markov-switch approach are contemplated, such as heterogeneous agents or financial frictions.

3.3.4.1. Application: Parametric Uncertainty

It is well known and documented in Section 3.1.3 of this work that uncertainty concerning the (impact) parameters of an model economy causes the central bank to act in a more cautions way than she would under certainty.

A closer look at the control rule, $u_t = Fx_t$, reveals how the parametric uncertainty enters the policy makers decision as

$$F = -\left[B'(V+V')B + 2v_{11}\Sigma_B^{11}\right]^{-1}\left[B'(V+V')A + 2v_{11}\Sigma_{AB}^{11}{}'\right]. \tag{3.3.46}$$

The policy rule incorporates the variances of the coefficient matrices of the model describing the economy, offsetting certainty equivalence and causing the monetary authority to react less aggressive if impact parameters are concerned and more aggressive if dynamic parameters are concerned (Söderström 2002).

A quite fruitful extension was the implementation of parametric uncertainty into a more general framework of model uncertainty. In this setup, the monetary authority is assumed to be confronted by a number of models not knowing which model might be the most appropriate description of the environment. With respect to the right policy response different solutions techniques have been established.

The concept of robust control is applied, e.g., if the central bank has a strong guess about the right model but admits that their might be deviations from the reference model she also wants to account for. Hence, there is a fair amount of trust in the general set up of the respective model, but the exact specification remains uncertain (Hansen and Sargent 2003b, 2008).

On the other hand, if the central bank faces a number of plausible reference models also Bayesian approaches can be utilized to achieve a good performance 'on average'. In any case, uncertainty remains *across* a finite number of models and not, as with the robust control approach, *within* the model.[25] Under these circumstances, parametric uncertainty remains a special case of model uncertainty (Cateau 2007).

If parametric uncertainty can be understood as a special case of model uncertainty and the approach of Markov-switching can be used in dealing with model uncertainty, hence, it should also be suitable to trace the problems of parametric uncertainty. For that reason, again, I chose the form and parameterization of the simple backward-looking model already used in Section 3.1.3 to show the effects of parametric uncertainty compared to the certainty case (Svensson and Williams 2005).

For further investigation I exemplarily choose the parameter s to be uncertain. I create two scenarios, the first is deduced from an uncertain parameter set up. Therefore σ_s^2 is set to 0.1. For the same parametrization, I set up $n_s = 10$ different regimes. All regimes are equal except their value of s which is drawn from a random normal distribution with mean 0.35 and variance 0.1. The system is simulated 15000 times. The results of a one standard deviation shock are shown in Figure 3.6[26]. The dashed line shows the impulse response of the uncertainty case where the policy reaction function, see Equation (3.3.46), includes the (co-) variance of the uncertain parameters. The solid line delivers the mean response of a 15000 times simulated Markov-switching model with all parameters certain. For each

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[25] One should not take 'within' the wrong way. Technically, a continuum of models is created, which figuratively orbit the reference model.

[26] Figures are based on a modified model version of Svensson and Williams (2005).

regime, however, s is filled with a random value of the mentioned normal distribution. The transition probability for each state is $1/n_s$, hence, equal.

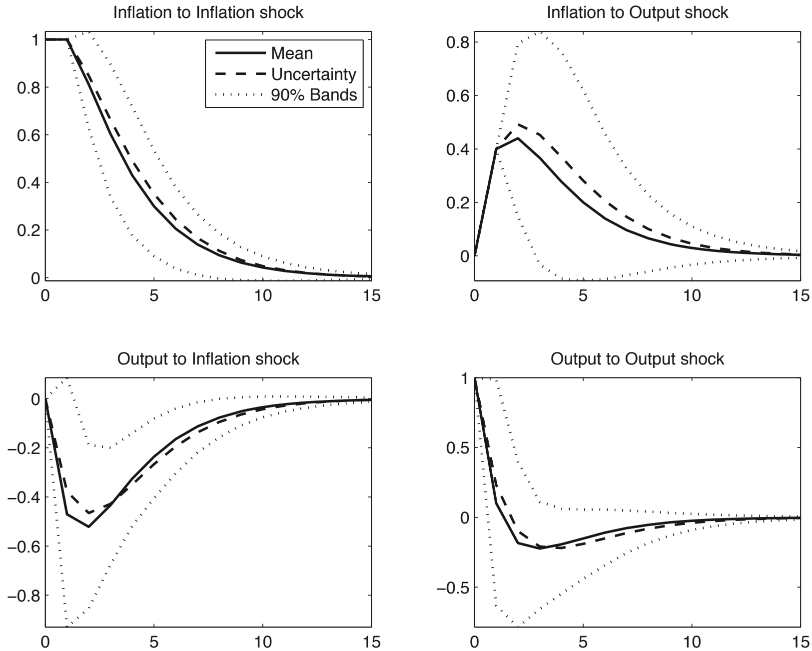

Figure 3.6.: Irf under Markov-switching and parametric uncertainty

Figure 3.6 shows how inflation and output behave due to a supply or demand shock. It can be seen that simulating the model a sufficient times, the mean of the Markov-switch models approaches the uncertainty path. Yet, opposed to the certainty – uncertainty comparison, there is no longer only a mean forecast of the development of each variable, but also the 90% interval bands; setting the interest rate thus accounts for the whole distribution. If simulated only a few times, irfs can be produced as well, which show a nearly congruent behavior of the simple uncertainty case and the Markov approach.

Concluding, it can be stated that there are similarities between both approaches and analyzing parametric uncertainty can also be done by a Markov-switching approach. However, Markov-switching can not only reproduce the parametric results, but can also go further by taking into account possible different future scenarios. Therefore, Svensson and Williams (2005:1) denote it as "distribution forecast targeting" to distinguish themselves from a merely mean forecast targeting. Beside others, within the field of model uncertainty, possible extensions to this account could cover different forms of uncertainty which feedback in different ways or different information set-ups on behalf of the central bank, e.g., whether the current mode is actually observable, or only with delay, or learnable.

3.3.4.2. Application: Heterogeneous Agents

During the most recent years, a growing body of literature has dedicated themselves to the problem of heterogeneous agents (see, e.g., Brock and Hommes 1997, Berardi 2009, Geiger and Sauter 2009, Branch and Evans 2010). Most recently, DeGrauwe (2010) has built a model with two kinds of agents who distinguish themselves in the appraisal of future variables. Those agents do not have to (and in fact do not) form rational expectations, which is the most common approach in the modeling literature. Instead, those two groups of agents are said to follow some sort of heuristic regarding their assessment of future variables.

Stylized, the resulting model structure can be displayed for the supply curve as

$$\pi_t = \beta E_t^{av} \pi_{t+1} + \kappa y_t + e_{t+1}^{\pi}, \tag{3.3.47}$$

with

$$E_t^{av} \pi_{t+1} = \alpha E_t^1 \pi_{t+1} + (1-\alpha) E_t^2 \pi_{t+1}. \tag{3.3.48}$$

$$= \alpha g_t - (1-\alpha) g_t \tag{3.3.49}$$

In this set-up, the switching of agents is a gradual process, where theoretically all combinations between 0% and 100% of agents belonging to one certain group are possible. The exact fraction size is determined by an endogenous switching process that enlarges the group which is encouraged by the actual situation.

For the central bank it can be important to know the size of the fraction parameter α, due to the fact that it determines significantly the transmission process. If the exact size of the parameter is uncertain, the central bank in fact acts under parametric uncertainty. This is the case, because in the end is does not matter, whether the parameter itself or any underlying parameter is uncertain, like an uncertain fraction of agents who hold a specific future prospect. However, this is not fully true. DeGrauwe (2010) emphasizes the difference between typical models of uncertainty and the model presented by him. The most significant difference arises out of the fact that the state under which the economy is hit by a shock now matters for the future development. The future transmission process initiated by a random shock thus depends on the timing of the shock.

Albeit, if the central bank can at least identify a limited number of possible compositions, she could generate a model for each of these and make use of a Markov-switch analysis. The advantage of the Markov approach lies in the specification of the transition probabilities. For example, it could be the case that it is quite easy for any agent to switch his expectation behavior from a rather complicated mechanism into a simpler version given the economic developments. One could think, e.g., on the usefulness of a sophisticated model of determining inflation expectations in an environment of hyperinflation. On the other hand, it might be very difficult for any agent to switch his forecast mechanism back. This offers another extension of the DeGrauwe (2010) model, in which agent switches can be explained by some sort of herding behavior. Markov-switch enables the construction of a model, which explicitly accounts for the switching process and, moreover, is able to generate as well an 'asymmetric' switch procedure, i.e., it is easy to get from mode one to two, but hard to get back.

3.3.4.3. Application: Financial Market

A very similar problem could arise if the model is extended by financial frictions. Encouraged by the developments on financial markets of the past years, Williams (2012) has developed a model which embeds the possibility of financial turmoil into a standard policy model. Based on Svensson and Williams (2005, 2007) he models the world with two different states. The first state represents a 'normal' environment. During these times, monetary policy is primarily engaged in setting the interest rate in a way that suits inflation and output very well. That is, when ever an exogenous shock occurs the central bank reacts at least in a certainty equivalence manner to offset the consequences.

However, the model economy is occasionally hit by a 'crisis' . This is the second state or regime. The crisis is defined as some sort of financial friction under which initially exclusive financial market shocks now affect the whole economy. Nevertheless, under these conditions the central bank still aims to stabilize inflation and output around its target level, though under more difficult circumstances as shocks now change the established transmission channels. The transition between the two states follows a Markov-switching process. The future state is thus hard to predict, that is, the policy maker is uncertain which state will prevail next.

Opposed to other papers that include financial frictions and its consequences on the economy, the approach of Williams (2012) allows the central bank to concentrate more – tough, not exclusive – on the standard policy during calm times and only takes financial frictions into account if they cause trouble.

The model is set up in a way already developed in Svensson and Williams (2005, 2007), that is, a forward-looking structure which is applicable to $j = 2$ different modes. The central bank minimizes her quadratic loss function. In normal times the model of Linde (2005) is assumed, which is a pretty standard New Keynesian model. In times of crisis, however, the assumptions of Curdia and Woodford (2010) are met, that is, credit spreads matter for the evolution of inflation and output. This spread is due to a difference between deposit and borrowing rate that induces a gap between the marginal utility of savers and borrowers. For simplicity, this spread is said to be exogenous.

This financial friction enters the model via the inflation and output relation in form of the credit spread ω and the marginal utility gap Ω such that

$$\pi_t = \omega_f E_t \pi_{t+1} + (1 - w_f)\pi_{t-1} + \gamma y_t + \xi_{jt}\Omega_t + c_\pi \epsilon_{\pi t}, \tag{3.3.50}$$

$$\begin{aligned} y_t = {} & \beta_f E_t y_{t+1} + (1 - \beta_f)\left[\beta_y y_{t-1} + (1 - \beta_y) y_{t-2}\right] \dots \\ & - \beta_r (i_t - E_t \pi_{t+1}) + \theta_{jt}\Omega_t + \phi_{jt}\omega_t + c_y \epsilon_{yt}. \end{aligned} \tag{3.3.51}$$

$$\Omega_t = \delta E_t \Omega_{t+1} + \omega_t, \tag{3.3.52}$$

$$\omega_{t+1} = \rho_{\omega,jt+1}\omega_t + c_{\omega,jt+1}\epsilon_{\omega t+1}. \tag{3.3.53}$$

The crucial differences compared to the inflation and output equation of the standard model are the terms $\xi_{jt}\Omega_t$ of the inflation equation and $\theta_{jt}\Omega_t + \phi_{jt}\omega_t$ of the output equation. Additionally a Taylor rule that accounts for the credit spread is assumed, but not displayed here.

It can be seen nicely in Equation (3.3.50) and (3.3.51), the size/occurrence of the financial distress depends on the prevailing mode j. For calm or normal times, $\xi = \theta = \phi = 0$. However, during crisis times those coefficients are non zero and the credit spread unfolds its impact. The credit spread *per se* is always existent, yet, its impact depends on the mode.

Williams (2012) calibrates the model and assumes different behaviors such as the observability or the non-observability combined with an adaptive or Bayesian learning of the mode. However, this discrimination plays only a minor role in the end. The result states that uncertainty matters, but only to a very limited extent. This result, of course, is very dependent on the model and the parametrization. Therefore I want to highlight not so much the result of the model, but the idea of modeling uncertainty in a Markov-switching environment.

Unfortunately, the big drawbacks of the Markov modeling become obvious also in this example. The transition equation must be defined as well as the mode specific model. Hence, private agents as well as policy makers are assumed to, and have to form, for example, expectations about the type of crisis, is severity, and its frequency of occurrence. However, some of the weakness of the model might also be accountable to the modeling of the financial friction and not so much the modeling of uncertainty. A thinkable way could be the modeling of several forms of financial ‘anomalies’ such as different frictions and different behaviors and reactions all connected by a Markov-switching process (Williams 2012).

3.4. Summary of Chapter 2 and 3

The last two chapters, which constitute the second part of my work, have demonstrated how the introduction of uncertainty changes the optimal behavior of the central banks.

Opposed to Part I, which highlights the importance of uncertainty, Part II, in fact, deals with risk rather than uncertainty. The reasons for this curiosity have been laid down at the beginning of Chapter 2. Subsequently, different forms of uncertainty have been introduced to show the effect of uncertainty on the optimal policy behavior. To make these effects as clear as possible, I have started with rather simple models. The results can be summarized in the following way. Additive uncertainty poses no challenge to the monetary authority, due to the fact that certainty equivalence prevails, hence, the policy rule is impervious to the uncertainty. If parametric uncertainty prevails, in general a less aggressive policy stance is recommended, although counter examples can be given. Data uncertainty can be overcome if appropriate filter techniques are applied.

Chapter 3 has focused on the treatment of model uncertainty. Therefore, I have introduced a common benchmark model. Model uncertainty can be understood as some sort of super form of uncertainty, covering several variations as subcategories. Model uncertainty comes closest to the concept of uncertainty in the sense of Knight (1921) and Keynes (1921), depicted in Chapter 1. Furthermore, it offers some fruitful extensions and handling prospects, like robust control or Markov-switch. However, despite the equalization of model and parametric uncertainty by some authors, the results can differ significantly depending on the method uncertainty is dealt with.

Some general advice how to act under certainty, still the so-called Brainard conservatism can be regarded as the most prevailing policy guideline. From a practical point of view, this advice is reinforced when looking at the interest rate setting of the major central banks. To pick up the car example of the introduction, the adversities of the environment combined with the ignorance about the exact functioning of the vehicle, force the driver to handle his gas and break paddle with care.

Every method of Chapter 2 and 3 is restricted to the task, how to deal with uncertainty. That is the question, what is the optimal reaction of a central bank when she faces uncertainty. The chapters so far have thus elaborated on a rather *passive* stance of monetary policy, where no attempt has been made to reduce uncertainty actively. These investigations and its findings have shaped the field of research during the last fifty years.

Its understanding is crucial to any practical policy recommendation. Yet, this stance of policy is of course only possible if uncertainty is actually detected. The next part of my work will deal with this issue of uncertainty measurement, hence, the detection, as well as the active containment of uncertainty.

Part III.

Empirical Investigations on Uncertainty

4. Assessing Uncertainty in the Euro Area and the USA

4.1. Introduction

Since the beginning of the the financial crisis, which is dated roughly around the years 2007 and 2008, five years have passed. Starting as a subprime mortgage crisis it evolved into ever expanding dimensions and in its aftermath even unveiled the massively accumulated macroeconomic imbalances within the euro area, which keep us busy until today. Since then, considerable uncertainty with respect to resolving the crisis, but also possible future developments exists. Therefore the European Central Bank (ECB) as well as the Federal Reserve (Fed) face a multiplicity of challenges. While uncertainty might be a problem of general concern, during times of turmoil it becomes of special interest. During such periods market participants might change their usual behavior with the effect of a break up of established transmissions. This implies an even higher degree of uncertainty for the monetary authority (González-Páramo 2008).

The past two chapters have shown how to deal with uncertainty when it is actually acknowledged. However, before this management of uncertainty takes place, there must be an identification of the actual degree of prevailing uncertainty. Despite the fact that central banks must do their best to asses the value of key variables, a great problem is posed by the uncertainty about the effect of their policy, that is, the functioning of the monetary transmission channel. At heart of this channel are the expectations of the private agents (ECB 2006).

The importance of expectations has been highlighted in numerous publications, see, e.g., Taylor (1982), ECB (2006, 2009, 2011b), or Woodford (2012a). Moreover it has created a new field of research which takes, for example, learning explicitly into account, see, especially, Evans and Honkapohja (2008). The great interest in expectations is due to the mentioned fact that expectations are at the core of the monetary transmission process. A wrong appraisal of expectations can generate serious problems as it may accidentally dampen or amplify central bank actions. Therefore, assessing the expectations of private agents can serve as a valuable assistant in the conduction of monetary policy.

O. Sauter, *Monetary Policy under Uncertainty,*
DOI 10.1007/978-3-658-04974-4_4,

Since expectations can not be observed directly they need to be extracted from other measures. To get an idea of how market participants expect future values to be, surveys as well as stock market data can be a valuable source of information. On the one hand, survey measures catch expectations about future realizations, because they explicitly question expected values of key variables for different horizons ahead. On the other hand, stock market data covers expectations implicitly, due to the fact that expected future developments are the main determinant of today's asset value (ECB 2011b, Guerron-Quintana 2012).

Past works on capturing expectations and identifying uncertainty via stock market or survey-based measures can be found, for example, in Giordani and Söderlind (2003), Söderlind (2008), Galati et al. (2009), Geiger et al. (2009). However, literature determining uncertainty during the financial crisis with the help of survey-measures is still meager.[1] Additionally, until now, there has been no joint analysis on uncertainty covering the euro area and the US.

The objective of this chapter is to extract common factors that drive the development and dynamics of uncertainty within the euro area, as well as the Unites States, during the period 2001 until today. Hence, the purpose of this chapter is to find common forces and patterns, which underlie the individual measures of uncertainty. Thus, this work differs in two main aspects from any mentioned before, as it conducts a factor analysis which includes (a) the financial crisis as well as (b) US and European data jointly. If such common forces can be identified, this would give reason to infer that high uncertainty concerning one variable leads to uncertainty of another variable, not necessarily within the same country or region, a question which has currently been raised by Conflitti (2012).

This extraction of latent, but invisible factors, is conducted with the help of an explanatory factor analysis, with respect to survey and market-based uncertainty measures. Therefore, I will firstly introduce general methods of identifying and measuring uncertainty, which are grouped into survey-based measures and stock market-based measures. Further on, a factor analysis, which is firstly constricted to the European data set and secondly to the US data set is conducted. Thirdly, the European and US data are combined in order to perform a global factor analysis with the objective of identifying transnational uncertainty forces.

[1] The body of literature concerning this issue is however growing rapidly. A notable publication concerning measures of uncertainty is Baker et al. (2012a), although their approach differs slightly to my own as will be become apparent later on. Further, some publications which focus mainly on expectations, touch the aspect of uncertainty as well, see, e.g., ECB (2011b).

4.2. Empirical Evidence of Uncertainty Measures

Dealing with uncertainty presumes the identification of uncertainty, which is primarily achieved by measuring expectations about the future development of key variables. The degree of uncertainty concerning each variable is typically deduced by the standard deviation of the expected values (Giordani and Söderlind 2003, Guerron-Quintana 2012). Of special interest is the perception of future values by market participants. To obtain these perceptions mainly two methods are utilized. The first group of measures is based on surveys such as the Survey of Professional Forecasters (SPF) provided by the European Central Bank and the Federal Reserve Bank of Philadelphia.[2] The second group consists of financial market-based measures which rely, for example, on stock market data, inflation-linked bonds, or swap rates.

To get a first impression of the data I firstly present some stylized time series of the standard deviation and mean of the variables of the Survey of Professional Forecasters. Furthermore, a brief look is taken on the option implied indices *Vstoxx* and *Vix* which represent the market-based uncertainty measures.

4.2.1. Measuring Uncertainty via Surveys

Despite the fact that, "there is no clear theoretical link between current disagreement and current uncertainty about the future ... ", Bomberger (1996) states, "... but a large number of papers have used this proxy" (Bomberger 1996:382). Bomberger (1996) himself shows that for the period 1946 until 1994 survey measures of disagreement in fact provide a reliable basis for assessing uncertainty. Ever since then a huge body of literature has used survey-based measures to investigate the perception of the future development of key macro variables as a measure of uncertainty, see, for example, Mankiw et al. (2003), Galati et al. (2009), Leduc et al. (2009).[3]

Although surveys deliver *direct* information on inflation expectations, compared to financial market data they have several shortcomings. As mentioned, for example, by Croushore (1993) or Giordani and Söderlind (2003), there is no guarantee that the respondents give their *best* estimate. This is an attitude which could be due to *tactical games* or the *fear of consequences*, and which can cause biased or exaggerated answers. An attempt to reduce this source of error has been made by making the publication of

[2] Surveys can be further distinguished whether professional agents, i.e., bank manager, hedge fonds etc., or rather normal consumers are taken as respondent. For the latter, the University of Michigan, US, provides the relevant time series. However, to keep the things comparable, for further investigation I rely only on the professional forecasts.

[3] It should be mentioned that in spite of Bomberger (1996) there is also research which holds only little evidence that disagreement measured by surveys serves well as proxy for uncertainty. However, the result is mixed for inflation expectations (Rich and Tracy 2010).

respondents of the survey anonymous. However, this anonymity may reduce the risk of 'wrong' forecasts and might as well encourage the respondent to report what she really believes, yet, at the same time it also gives rise to less precise forecasts, due to the fact that there is no accountability for the predictions. Additionally, due to the low investigation frequency of surveys, ranging from monthly up to biannually reports, they can not reflect behavioral changes properly. In the end, they suffer from the well-known problems with the design of questionnaires (Galati et al. 2009).

The ECB provides with the Survey of Professional Forecaster (SPF) a survey which measures future expectations of the euro area. It was established in 1999 and is based on a quarterly questionnaire. Participants are experts affiliated to financial and non-financial institutions within the euro area. Since 1968 the Federal Reserve Bank of Philadelphia, the American Statistical Association, and the National Bureau of Economic Research have been conducting alternately a comparable survey for the US. During this time various changes have occurred, such as variables haven been added, dropped or been replaced by others. Additionally different forecast horizons have been added or dropped. Comparable data of the ECB has thus only been available since 2001, whereby US data reaches back until the late sixties.[4]

Unfortunately, surveys are not coherent. Neither across the US and European data nor within one region. This is to some extent due to the specific nature of each variable. Unemployment as well as inflation data, for example, are available on a monthly basis whereas GDP data is only available on a quarterly basis. In addition, the US survey takes various variables into account which are not even inquired by the ECB, e.g., expectations about real consumption, new housings or corporate profits. Therefore, to obtain a comparable picture of the US and euro area data I will rely on a subset of each survey.

4.2.2. Survey of Professional Forecaster Data

European Survey Data

The European survey data is taken from the Survey of Professional Forecasters which is provided by ECB (2012). Table 4.1 lists the variables taken into account, the last column gives the abbreviations used in the following analysis.[5]

[4] An overview of the European survey is given by Bowles et al. (2007). Respectively, Croushore (1993, 2010) offers good manuals for the American survey.

[5] The SPF is conducted on a quarterly basis. If questions are constructed as a *rolling window*, survey date and forecast horizon are of a constant relation. For example, if the survey date is 2009q1 the rolling 1 year ahead forecast is for 2010q1, if the survey is conducted in 2009q2 the forecast is for 2010q2. On the other hand, no rolling window forecasts are of no constant relation. If, for example, the survey date is 2009q1, the 5 year ahead forecast is for the year 2014; If the survey date is 2009q2, the forecast remains for the year 2014.

GDP 1 year ahead (rolling)	gdp_1
GDP 2 years ahead (rolling)	gdp_2
GDP 5 years ahead	gdp_5
Inflation current year	inf_c
Inflation 1 year ahead (rolling)	inf_1
Inflation 2 years ahead (rolling)	inf_2
Inflation 5 years ahead	inf_5
Unemployment 1 year ahead (rolling)	une_1
Unemployment 1 years ahead (rolling)	une_2
Unemployment 5 years ahead	une_5

Table 4.1.: Survey data, EUR

A closer look at the European survey data reveals that all variables show more or less the same pattern during the last decade, especially at the beginning of the financial crisis in the year 2007 and its intensification in the year 2008 with the fall of Lehman Brothers. Figure 4.1 shows the standard deviations of all three considered variables GDP, inflation, and unemployment. The standard deviation of all variables has become larger during the financial crisis, whereby it has been relatively low in the years before. This holds – even though to a different extent – for all forecast horizons. It is especially pronounced for short-term GDP uncertainty. Additionally, the long-term (5 years) forecast of unemployment which was relatively high even in the years before 2008, and, compared to other forecast horizons, did not really recover in the years after the peak. It is important to notice that Figure 4.1 depicts standard deviations. Hence, one can not conclude that short-term expectations of, say, inflation are low or lower than long-term expectations, but only that the disagreement of the forecast is higher or lower.

Reasoning a high degree of uncertainty from a high standard deviation among the evaluation of the participants of the survey, I conclude that uncertainty has been on an all-time high during the financial crisis only slowly recovering during the most recent years with values close to the pre-crisis level. In general this does not seem to be surprising.

Interestingly enough, however, long-term GDP uncertainty remains quite low even during the crisis. Only short-term uncertainty hikes around the year 2009. Nevertheless, all horizons recover back to their pre-crisis level. Opposed to that, inflation uncertainty offers a rise in all considered horizons during the crisis. Especially long-run inflation expectations are said to reflect the ability of the central bank to anchor expectations well. Against this background, the picture of accelerating rates, no matter their time-aspect, but especially long-term, has raised the question whether inflation expectations are really well anchored in the euro area. In fact, Gerlach et al. (2011) find evidence that long-term expectations

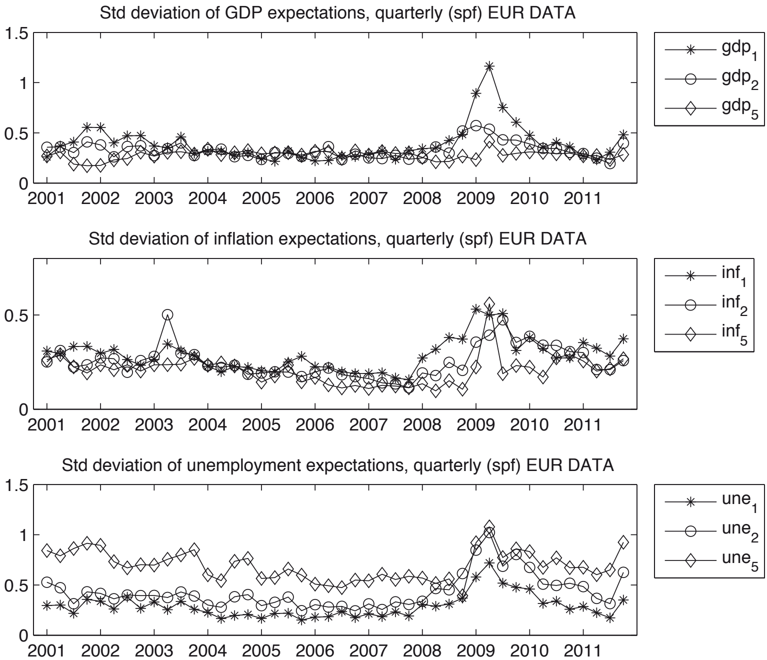

Figure 4.1.: Standard deviation of GDP, inflation and unemployment, EUR

seem to be less firmly anchored than at first sight presumed. This thought is fostered even more as the dispersion of inflation expectations has not recovered totally since its peak in the year 2009.

A high standard deviation reflects a severe divergence, whereas a low standard deviation reflects a consonant appraisal of future rates among forecasters. A possible interpretation for the high standard deviation around the year 2009 can be found in Geiger and Sauter (2009) and Geiger et al. (2009). The rising standard deviation could thus be the consequence of a break up between different camps of agents who follow different approaches in predicting future developments. The first camp consists of agents who can be called *monetary believers*. They form expectations according to the observation of monetary dynamics. According to this position, the immense liquidity provision on behalf of the ECB therefore leads to a rising inflation rate in the long run.

On the other hand, there is a second camp whose supporters focus their assessment on *real* factors such as capital utilization or GDP growth. In their opinion, despite the liquidity rush, the poor output development delivers no reason for inflationary pressure in the future. These camps are nearly congruent in calm times, but break apart if things start shaking, which was definitely the case around the year 2009.

Uncertainty concerning the development of unemployment has also been risen during the crisis no matter the time aspect may be. However, the acceleration was very moderate and, except the long-term perception, uncertainty has nearly recovered the years after, even with a slight rise during the most recent quarters of 2011.

Despite the acceleration of uncertainty, the mean forecast shows a different picture, see Figure 4.2. While short-term expectations vary during the crisis, long-term expectations of GDP growth and inflation remain nearly unchanged. This finding is not in contradiction to the standard deviation findings; a constant mean is in line with a high standard deviation where extrema net each other out.

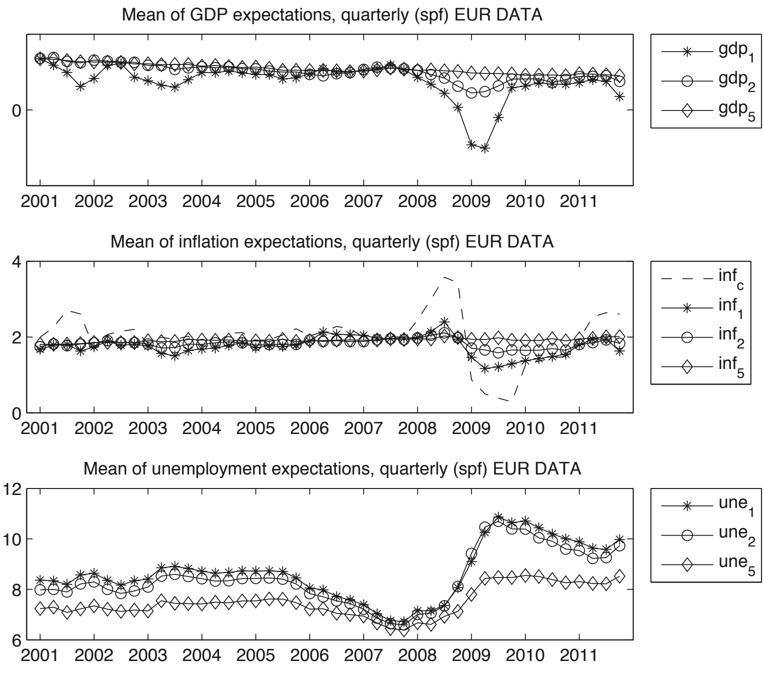

Figure 4.2.: Mean values of GDP, inflation and unemployment, EUR

Ball et al. (1990) relate the different behavior of short and long-term variables to the distinction whether shocks are only of a temporary or permanent nature. In the first case, only the variance of short-term variables is affected, while in the latter case long-term deviations from the trend can be observed as well. Due to the fact that long-term variables remain close to their historical trend and only deviations of short-term variables can be observed, Figure 4.2 suggests that the majority of professional observers regarded the distortions during the financial crisis rather to be of a temporary nature. This holds at least for inflation and GDP growth.

The somewhat strange behavior of current inflation expectations is repeated to a minor degree in the one year forecast. This behavior is also reflected in other measures, such as inflation linked bonds, see, e.g., ECB (2011b). In accordance with Geiger et al. (2009), this may reflect a confusion in the appreciation of monetary measures vs. the expected economic downturn. This explanation is fostered by ECB (2011b), whereby the sharp rise and decline is mainly driven by a drop in confidence, and thus an insecurity about the future development.

US Survey Data

Figure 4.3 and 4.4 plot the standard deviation and mean of the US variables listed in Table 4.2.[6] Interpreting the US data is more difficult than the European data, due to its less uniform picture. Even an outstanding event such as the financial crisis is less obvious in the US data and can merely be detected in Figure 4.3.

RGDP 1 year ahead (rolling)	$RGDP_1$
RGDP 10 years ahead	$RGDP_{10}$
Inflation 1 year ahead (rolling)	CPI_1
Inflation 10 years ahead	CPI_{10}
Unemployment 1 year ahead (rolling)	UNE_1

Table 4.2.: Survey data, US

Standard deviation of real GDP growth expectations is marked by three significant humps. The first hump is a heightened uncertainty due to the recession lasting from March 2001 until November 2001, followed by the second hump in the year 2003, which corresponds to the corporate scandals and the beginning of the Iraq war. The last, most significant hike starts in 2007 with the beginning of the financial crisis. During all these periods uncertainty on the ten year outlook remains quite low, and only short-term variations occur. Inflation uncertainty seems to be rather erratic. Short and long-term uncertainty perform a slow acceleration, already starting around the year 2006. However, there seems to be no scientific explanation for this behavior except that maybe the upcoming change

6 For a better discrimination, US values are abbreviated with capital letters.

at the head of the Fed (Bernanke succeeding Greenspan in 2006) might has driven some uncertainty in the market, as elections in general do. Even though one can identify a peak of uncertainty during 2009, it is more or less 'one of many' humps regarding the last years. Unemployment uncertainty rises vis-à-vis the evolving financial turmoil and has still not yet recovered to the pre-crisis level.

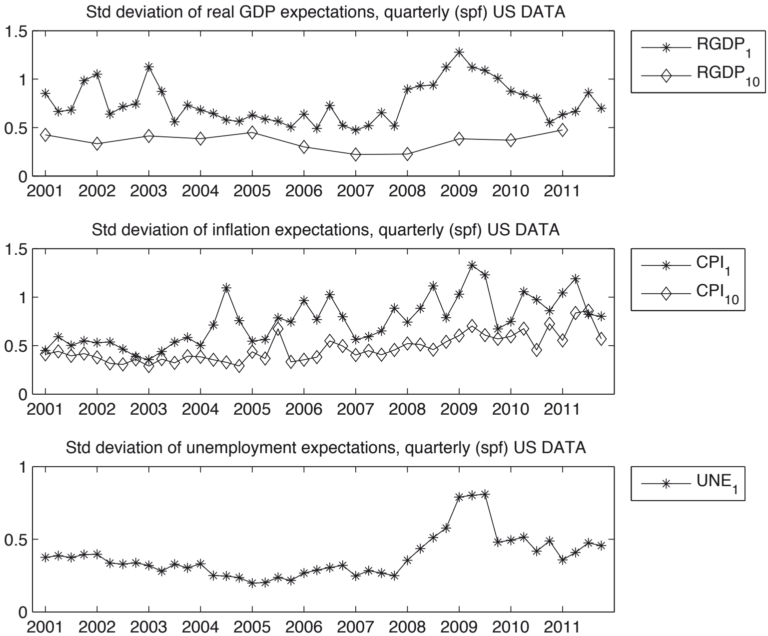

Figure 4.3.: Standard deviation of real GDP, inflation and unemployment, US

Taking into account the mean values plotted in Figure 4.4 the drop of expectations regarding real GDP growth becomes obvious. The ground is hit at the first quarter of 2009, at the same time when uncertainty, measured by the standard deviation, hits the ceiling. However, the mean recovers fast until it reaches pre-crisis levels in the year 2011, which is also true for real GDP uncertainty, shown in Figure 4.3.

Comparing mean and standard deviation of inflation, it looks like deflationary scares such as around the years 2004 and 2009 come along with heightened uncertainty. Nevertheless, long-term expectations of inflation and real GDP seem to be well anchored, mean and standard deviation remain quite constant during all the years. Unemployment shows the same wave pattern as in Europe with uncertainty still not fully recovering to the pre-crisis level and with a significant higher mean forecast.

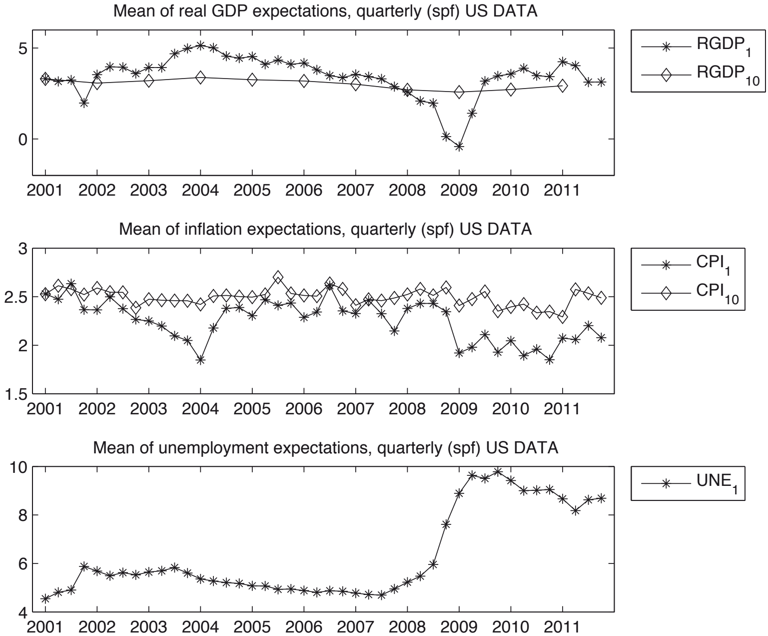

Figure 4.4.: Mean values of real GDP, inflation and unemployment, US

4.2.3. Measuring Uncertainty via Stock Market Volatility

Uncertainty measures derived from financial data are predominantly based on information given by stock market volatility, more precisely on option volatility. These data are available in high frequency which allows the examination of changes in behavior quite well. Due to the fact that this is only an indirect measure, there is a potential risk of swapping information. Yet, despite this shortfall, volatility indices are a good proxy for uncertainty and are commonly used in academic literature (ECB 2011a, Bekaert et al. 2010).

To see how option volatility can replicate stock market uncertainty consider the pricing of a call option. Only if the current value of the underlying asset (S_t) exceeds the strike price of the asset (X), the call option goes *into the money*. This is due to the fact that a call option guarantees the holder to buy the underlying asset still at the lower strike price. Otherwise, if the asset price does not exceed the strike price, the option is worthless. Thus its value is zero. Hence, the value of a call option at expiry is $(S_T - X) \vee 0$. Therefore, the price of a call option is open ended in one direction, as it rises with the underlying asset, but it is chopped off at the bottom due to the fact that it can not be worth less than zero.

Any foreseen price change would make an option obsolete. The value of an option is thus attained by the uncertainty surrounding future price developments, i.e., the variance of the price of the underlying. Due to the chopped minimum value of an option, options with a higher variance of the underlying must – compared to options with a lower variance of the underlying – gain a higher price. This is due to the fact that they can attain a potentially greater payoff, while at the same time bear the same (limited) downside risk (Hull 2009, Neely 2005).

The volatility of an option at one point in time is not observable, but it can be estimated using past observations. Yet, in reality, most market participants make use of the so-called *implied* volatility, which is deduced by the Black-Scholes formula (Black and Scholes 1972, 1973). For a call option it holds that

$$C = S_0 N(d_1) - X e^{-rT} N(d_2), \tag{4.2.1}$$

with

$$d_1 = \frac{ln(S_0/X) + (r + \sigma^2/2)T}{\sigma\sqrt{T}}, \quad d_2 = \frac{ln(S_0/X) + (r - \sigma^2/2)T}{\sigma\sqrt{T}}.$$

The price of the option (C) is determined as a function of its strike price (X), the underlying asset value (S), the risk-free interest rate (r), the time to expiry (T), the cumulative normal density function ($N(\cdot)$), and its variance (σ^2). Except for the variance all variables are observable in real-time so that the volatility which is necessary to match the current market price of the option can be calculated easily. In contrast to historical volatilities, implied volatilities are forward-looking due to the fact that they include expectations and assumptions of market participants about future stock movements. In the following, I will use the implied volatility index Vstoxx for the European market and the Vix for the US market.

4.2.4. Stock Market Data

Figure 4.5 captures the implied volatility of the European and the US stock market, measured by the Vix and Vstoxx. Both indices are calculated as a weighted average of options relying on the S&P 500 and the EURO STOXX 50, respectively. The higher the value of the index rises, the higher is the expected variance of the underlying. Due to the explained constitution of the index which chops off one side of the risk this serves as an indicator of uncertainty.

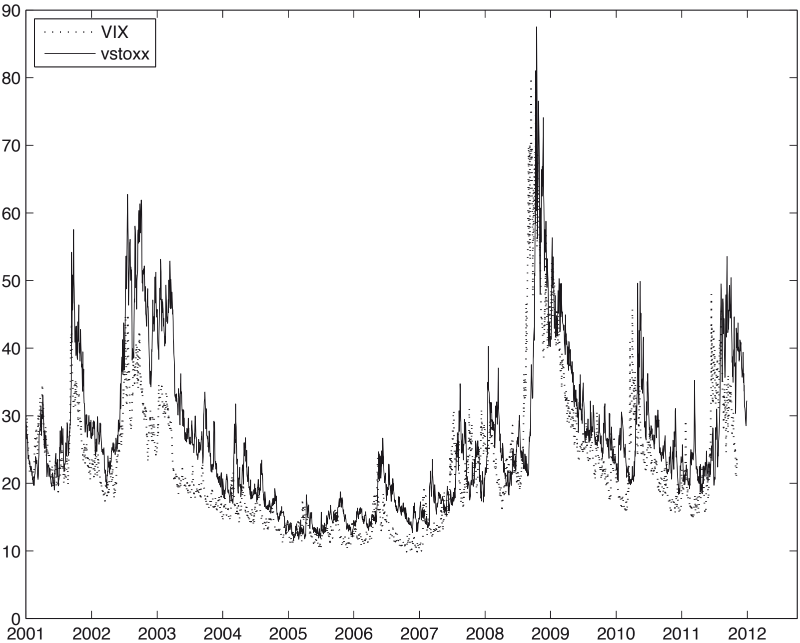

Figure 4.5.: Vix and Vstoxx, daily

The picture corresponds to the findings of the SPF. What is most striking is the nearly perfect synchronized course of the Vix and Vstoxx, which already indicates a strong connection and interrelation of uncertainty concerning the European and American stock market. Compared to the survey measure, the stock market measure is more sensitive to unforeseen shocks. This is due to several reasons. Most importantly, stock prices react much faster than nearly every other economic variable. News, which would fall into oblivion if contracts could only be made on a quarterly basis, can be incorporated nearly immediately. Squeezing the stock market data into a quarterly scheme a smoother and less volatile picture is achieved, which corresponds quite well to the survey-based graphs presented before.

With the Vix and Vstoxx on hand, it is easy to recap the developments of the past ten years in detail. Some crucial events are the first peak at the end of 2001 which is owed to the terror attacks of 9/11. The mid-2002 hump corresponds to the US corporate scandals (Enron, WorldCom, AOL, etc.) on the basis of which the Sarbanes-Oxley Act was passed. The following smaller hump can be associated with the Iraq invasion in the first quarter of 2003.

The second major group of events start around the year 2008 with the collapse of Bear Stearns followed by the second-to-none rise and fall of uncertainty due to the financial crisis and the following (unconventional) measures undertaken by the Fed and the ECB. The last humps around mid 2010 and 2011 are due to the so-called Euro crisis reflecting the risk of default in various European countries, most notably Greece (Bekaert et al. 2010, Baker et al. 2012a,b).

Further insights are offered if the market-measure is decomposed. This unbundling is admissible due to the fact that the Vstoxx and Vix capture the perceived price level, as well as the level of uncertainty of the future stock prices. ECB (2011a), relying on Bekaert et al. (2010), shows for the Vix that uncertainty relative to risk has been extremely low during the years 2005 until 2007 and rises thereafter. In general, however, a rise of the Vix is mainly driven by uncertainty changes rather than a change in risk aversion.[7] For the European measure Vstoxx, I have adopted this measure. The picture is shown in Figure 4.6 and corresponds quite well to the findings of ECB (2011a) for the Vix decomposition. Analogously, uncertainty (solid line) is relative low during the years 2005 until 2007. However, afterwards uncertainty rises. Compared to Figure 4.5 the rise and fall of the Vstoxx can be ascribed to a large extent to a rise and fall of uncertainty, shown in Figure 4.6. This becomes most obvious for the humps in late 2008, 2010, and 2011, but also for the smaller eruptions like 2004 or 2006. Therefore, for convenience, I will rely for the further analysis on the original Vix and Vstoxx data.

[7] Risk-aversion is defined as the difference between the squared Vix and the conditional variance, which is defined as uncertainty.

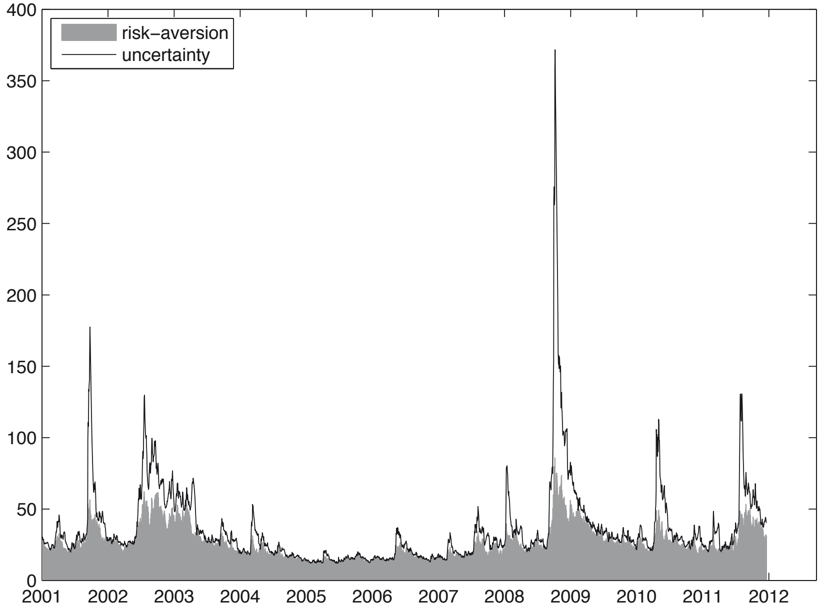

Figure 4.6.: Decomposition of Vstoxx

4.3. Factor Analysis of Empirical Data

4.3.1. Factor Analysis of European Data

In what follows, I first conduct a factor analysis of the European data and afterwards of the US data. The purpose of this analysis is to evaluate region specific factors and patterns which drive uncertainty. I will show that if taken separately, uncertainty in Europe follows different patterns and forces than in the US. In a next step I combine both data sets to evaluate whether a region independent uncertainty factor exists. If so, this factor could be interpreted as a transnational US-euro area uncertainty factor. Hence, the first analysis delivers the underlying *structure* and *characteristics* of uncertainty in terms of time and variable in each region, whilst the second analysis focuses on the question whether uncertainty is regional or transnational between Europe and the US.

Both measures – standard deviation of survey forecasts and the stock market-based volatility index – account for uncertainty. I have shown in the previous sections that both measures share the same ups and downs which suggests a common underling factor that drives this uncertainty. The common driving force behind these measures of uncertainty appears to be some kind of general, macroeconomic uncertainty. This latent but not direct observable force can become visible with the help of factor analysis.

The euro area data set consists of the uncertainty measures taken from the SPF and the Vstoxx; they are listed in Table 4.3. The first column indicates the source of the uncertainty measure, the second contains the respective variable, the last column gives the abbreviations used for further analysis. All variables are taken from the period first quarter 2001 until fourth quarter 2011.

Stock market data	Vstoxx	*vstoxx*
Survey data	GDP 1 year rolling	gdp_1
	GDP 5 years ahead	gdp_5
	Inflation 1 year rolling	inf_1
	Inflation 5 years ahead	inf_5
	Unemployment 1 year rolling	une_1
	Unemployment 5 years ahead	une_5

Table 4.3.: Data set of factor analysis, EUR

The data set offers a Kaiser-Meyer-Olkin (KMO) criterion value of 0.75. This is according to Kaiser and Rice (1974) “middling”, and thus, acceptable.[8] All individual measures of sampling adequacy (MSA) are at least greater than 0.6 and most of them larger than 0.7. The only exception is the standard deviation of long-term GDP growth, which is 0.22. Nevertheless, and for the sake of completeness, I keep this variable for further analysis.

The eigenvalues are listed in Table 4.4. To find the right number of underlying driving factors, I follow Kaiser and Rice (1974) and assume as many factors as eigenvalues greater than one. Two eigenvalues can be identified which fulfill this criterion, hence, two common factors are extracted.

Eigenvalues
4.05
1.34
0.70
0.46
0.22
0.15
0.07

Table 4.4.: Eigenvalues of factor analysis, EUR

[8] Kaiser and Rice (1974) propose a scale of sampling adequacy, according to values greater 0.5 are said to be “miserable”, greater 0.6 “mediocre”, greater 0.7 “middling”, greater 0.8 “meritorious”, and greater 0.9 “marvelous”. Values smaller than 0.5 are “unacceptable”.

Table 4.5 lists the respective factor loadings of the two factors as well as the communality and the specific variance, i.e., the uniqueness. The loadings are generated using *promax* rotation.[9] For a better representation all factor loadings absolutely smaller than 0.25 have been removed from the table. Column 4 and 5 comprise the communality and uniqueness, no matter their value. The loadings are also plotted in Figure 4.7.

Variable	Factor 1	Factor 2	Communality	Uniqueness
$vstoxx$	0.6584	-	0.4467	0.6269
gdp_1	0.9095	-	0.8327	0.1012
gdp_5	-	0.4780	0.2727	0.8252
inf_1	0.8445	-	0.7133	0.2944
inf_5	-	0.9718	0.9032	0.0050
une_1	0.9572	-	0.9168	0.0616
une_5	0.4017	0.5225	0.4344	0.3612

Table 4.5.: Loadings EUR, promax rotation (cropped)

It can be seen very clearly, even from the table, that there is a distinction between long-term and short-term uncertainty. All short-term variables (gdp_1, inf_1, and une_1), as well as the volatility index ($vstoxx$) are highly loaded on Factor 1. Opposed to that, long-term forecasts (gdp_5, inf_5, and une_5) are loaded predominately on Factor 2.

This loading pattern is in line with an economic interpretation of a separation between short and long-term drivers of uncertainty, see, for example, Caporale and Kontonikas (2006), Caporale et al. (2010), where the authors distinguish between the dynamics of short-run and long-run inflation uncertainty for different European countries. Mentioned already before, Ball et al. (1990) reason the separation to the fact that uncertainty about short-term inflation development depends mainly on temporary shocks, whereas uncertainty about inflation development far in the future depends on the variance of permanent shocks.

Uncertainty about the future development of a variable and the anchoring of expectations are just two sides of the same coin. When faced with disruptions, low uncertainty concerning a variable can be put on a level with well anchored expectations concerning the same variable and vice versa. Hence, uncertainty which is driven by two latent forces can equally be interpreted as expectations which are anchored in two different ways; one rather on a short-term, the other rather on a long-term future horizon. This reasoning is also supported by the findings of Figure 4.1. The standard deviation of long-term expectations seems to be decoupled from short-term movements. This holds especially under

[9] The varimax-method delivers a very similar picture. For interpretation and presentation, however, promax seems to be more appropriate as it highlights the specific loadings more clearly.

turbulent times such as the financial crisis where the standard deviation of long-term expectations remains quite low compared to short-term movements, which is equivalent to well anchored long-term expectations and at least less anchored short-term expectations.

The pattern deduced from factor analysis and Figure 4.1 is especially pronounced for inflation uncertainty, hence, from the opposite point of view, the anchoring of inflation expectation. According to Figure 4.1 this separation should also hold for GDP growth. Yet, factor analysis does not support this view. Whilst short-term GDP growth uncertainty is as well highly loaded on Factor 1, the long-term perspective only offers a rather weak loading of Factor 2. However, due to the poor MSA value and the high specific variance of long-term GDP growth uncertainty (0.83), factor analysis is not meaningful enough for this variable and I rely rather on the graphical standard deviation representation.

Unemployment uncertainty, neither from Figure 4.1, nor from factor analysis can be differentiated in this sharp distinction. Long-term uncertainty seems to follow its own force until the year 2007 and co-moves with short-term uncertainty from then on. However, this view is supported by the factor analysis, which extracts one factor for short-term unemployment uncertainty with a high loading of 0.95. Long-term uncertainty concerning unemployment is opposed to inflation and GDP uncertainty not only or mainly driven by a second Factor, but nearly equally driven by both factors which seem to fit well into the argumentation of Figure 4.1.

The data of Table 4.5 is plotted in Figure 4.7. Factor 1 is depicted on the horizontal axes and Factor 2 on the vertical axes. Figure 4.7 highlights explicitly the discussed findings.

Considering this pattern, I declare Factor 1 to be the short-term uncertainty force, and Factor 2 to be the long-term uncertainty force. The results suggest that one can not conclude from uncertainty concerning the development of short-term variables to the development of uncertainty in a long-term prospect. Yet, the inference from a short-term variable to another short-term variable seems to be admissible. This classification supports the distinction which is found in the literature, between drivers of short and long-term uncertainty. It is especially pronounced for inflation uncertainty, and only to a minor amount for GDP growth and unemployment uncertainty.

4.3.2. Factor Analysis of US Data

Analogously to the European analysis, the US data set is constituted by the SPF provided by the Federal Reserve Bank of Philadelphia and the Vix provided by the Chicago Board Options Exchange (CBOE). Unfortunately – with the exception of inflation – for the most variables the US survey only offers three horizons: four quarters ahead, annual average current year, and annual average next year. For factor analysis I use the data shown in

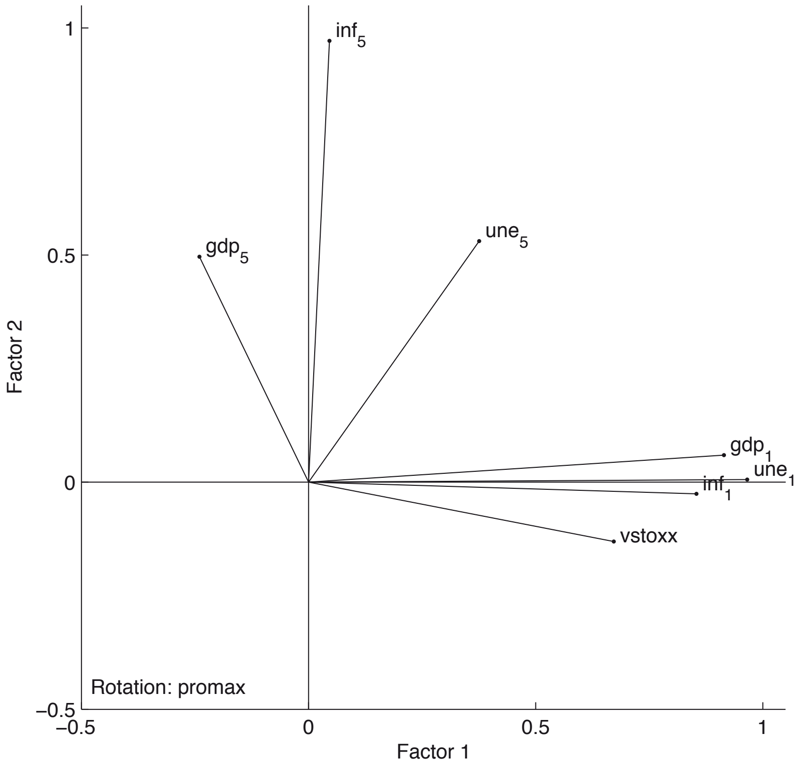

Figure 4.7.: Loadings EUR, promax rotation

Table 4.6. Except for the standard deviation of long-term inflation expectations, which is a ten year forecast, all survey variables are the standard deviation of a four quarters ahead forecast.

Stock market data	Vix	VIX
Survey data	Inflation 1 year rolling	CPI_1
	Inflation 10 years ahead	CPI_{10}
	Real Consumption 1 year rolling	$RCONSUM_1$
	Real GDP 1 year rolling	$RGDP_1$
	Unemployment 1 year rolling	$UNEMP_1$

Table 4.6.: Data set of factor analysis, US

The KMO criterion offers a value of 0.75 for the given data set which is according to Kaiser and Rice (1974) "middling". All MSA values are at least larger than 0.5, hence "acceptable". Again, two eigenvalues larger then one can be identified (3.47 and 1.46), thus, factor analysis is conducted with the assumption of two factors.

The deduced loadings are given in Table 4.7 where all values smaller than 0.17 have been removed for convenience. I have conducted the orthogonal varimax rotation, the extracted factors are thus independent from each other. Nevertheless, the results using promax are comparable. Again, the last two columns give the communality and uniqueness, respectively.

Variable	Factor 1	Factor 2	Communality	Uniqueness
VIX	0.8195	-	0.6774	0.3226
CPI_1	-	0.7793	0.6115	0.3885
CPI_{10}	-	0.7661	0.6059	0.3941
$RCONSUM_1$	0.8484	-	0.7317	0.2683
$RGDP_1$	0.8877	-	0.8164	0.1836
$UNEMP_1$	0.7244	0.6256	0.9162	0.0838

Table 4.7.: Loadings US, varimax rotation (cropped)

According to Table 4.7, stock market volatility, real consumption, and real GDP are solitary loaded on Factor 1. Inflation uncertainty – short and long-term – is loaded solitary on Factor 2. Unemployment uncertainty seems to be nearly equally dependent on both factors.

Analogous to the euro area, stock market and short-run GDP uncertainty are loaded by one common factor which as well drives real consumption. Also, short-run uncertainty concerning unemployment is driven by this factor to some extent. Yet, opposed to the European data, short-run unemployment uncertainty is also driven by a second factor.

Most importantly, the strong distinction of the European data between short and long-term uncertainty of inflation expectations can not be found in the US data. Both inflation horizons are loaded by the same factor which explains for both variables around 60% of the variation. Hence, the distinction made for the European data, which was encouraged by the literature, can not be approved for the US data with the use of factor analysis. Moreover, due to the fact that varimax is an orthogonal rotation, both factors are independent from each other. Hence, there is a strong difference between short and long-term inflation uncertainty on the one hand and (nearly) all other variables on the other hand.

With the exception of unemployment uncertainty the US data taken into account offers a distinction, which reminds of the 'classical dichotomy', i.e., between *real* and *nominal* values. Due to the fact that the volatility index is based on the stock market and stock prices are based indirectly on real developments, the classification of the Vix fits very well into this categorization.

The results of Table 4.7 are plotted in Figure 4.8. Factor 1 is depicted on the horizontal axes and Factor 2 on the vertical axes. Quite illustrative Figure 4.8 shows how all real values gather around the horizontal axes, while opposed to that, CPI_1 and CPI_{10} have a strong vertical orientation. Finally, unemployment is right in the middle.

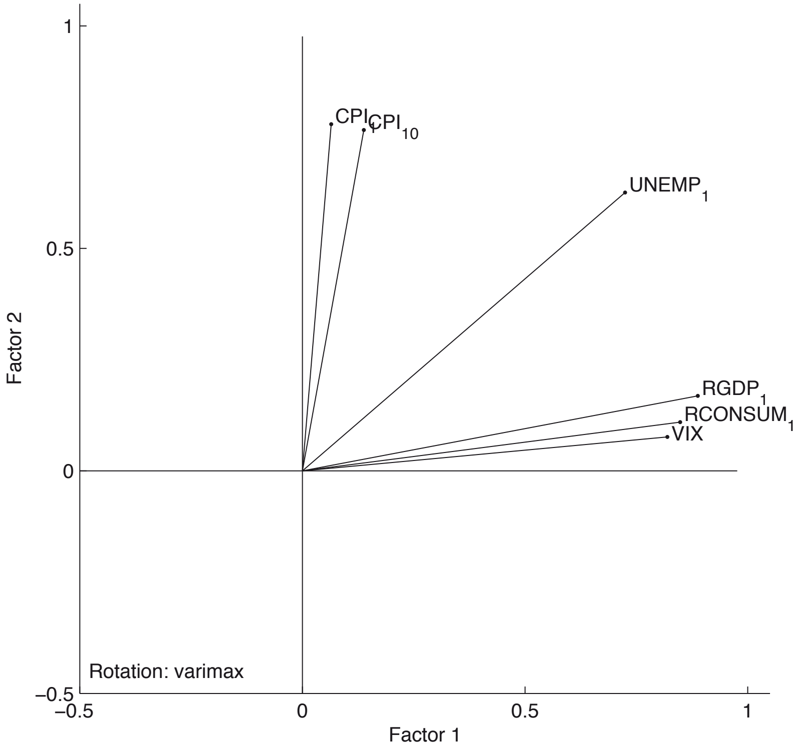

Figure 4.8.: Loadings US, varimax rotation

Especially the graphical representation highlights the different pattern of the US results, compared to the European findings. Hence, to conclude the region specific analysis, it can be stated that the regions taken into account follow distinct patterns concerning the forces of uncertainty.

The euro area exhibits a distinction between the loadings of short-term and long-term uncertainty which holds especially for inflation uncertainty and, though less, for GDP growth and unemployment uncertainty. Opposed to this pronounced distinction, the US factor analysis suggests no distinction between short and long-term uncertainty on inflation. Long and short-term uncertainty are driven by one common force. Thus, one could argue that inflation expectations, deduced from the Survey of Professional Forecasters, no matter their time horizon are anchored nearly equally strong. Further, the pattern gives reasons to conclude that a high uncertainty concerning the development of real variables is decoupled from the uncertainty concerning the nominal developments. Hence, any inference from the uncertainty surrounding price developments, e.g., whether rather inflationary or deflationary tendencies are expected, on uncertainty concerning the economic development is not admissible. A finding which is astonishing. Moreover this reminds on the classical distinction between uncertainty about monetary values – no matter their time aspect – and real variables including stock market uncertainty.

4.3.3. Combined Factor Analysis of US and European Data

After both regions have been analyzed separately, in what follows a global factor analysis is conducted which covers both data sets – the European as well as the US. The objective is to identify, whether Europe and the United States follow one common factor in terms of uncertainty or if uncertainty is region driven.

For the global factor analysis both data sets are combined. Due to the poor MSA value of US unemployment and the long-term European GDP growth uncertainty, I have dropped these values for the further analysis. The remaining eleven variables exhibit an MSA value at least greater than 0.5 and are listed in Table 4.8. The overall KMO criterion is 0.8, which is according to Kaiser and Rice (1974) “meritorious”.

Stock market data	VSTOXX	$vstoxx$
	VIX	VIX
Survey data US	Inflation 1 year rolling	CPI_1
	Inflation 10 years ahead	CPI_{10}
	Real Consumption 1 year rolling	$RCONSUM_1$
	Real GDP 1 year rolling	$RGDP_1$
Survey data EUR	GDP 1 year rolling	gdp_1
	Inflation 1 year rolling	inf_1
	Inflation 5 years ahead	inf_5
	Unemployment 1 year rolling	une_1
	Unemployment 5 years ahead	une_5

Table 4.8.: Data set of factor analysis, US & EUR

The eigenvalue criterion suggests that three common factors load uncertainty in the US and Europe. Loadings, communality, and uniqueness, which are produced by the *varimax* rotation are listed in Table 4.9. The upper panel of Table 4.9 shows the US data, the lower panel the euro area values. For an easier interpretation Table 4.10 dismisses all factor loadings with a value less than 0.3.

Variable	Factor 1	Factor 2	Factor 3	Communality	Uniqueness
VIX	0.9839	0.1462	0.0743	0.9950	0.0050
CPI_1	-0.0342	0.0853	0.9604	0.9308	0.0692
CPI_{10}	0.1603	0.0814	0.6227	0.4201	0.5799
$RCONSUM_1$	0.6467	0.3967	0.2188	0.6234	0.3766
$RGDP_1$	0.6500	0.4849	0.1714	0.6870	0.3130
$vstoxx$	0.9183	0.1896	-0.0201	0.8797	0.1203
gdp_1	0.4527	0.7947	0.2532	0.9006	0.0994
inf_1	0.4771	0.6165	0.3886	0.7586	0.2414
inf_5	0.0878	0.6633	0.0712	0.4528	0.5472
une_1	0.4747	0.7869	0.2754	0.9204	0.0796
une_5	0.1337	0.7917	-0.0927	0.6533	0.3467

Table 4.9.: Loadings US and EUR, varimax rotation

Variable	Factor 1	Factor 2	Factor 3	Communality	Uniqueness
VIX	0.9839	-	-	0.9950	0.0050
CPI_1	-	-	0.9604	0.9308	0.0692
CPI_{10}	-	-	0.6227	0.4201	0.5799
$RCONSUM_1$	0.6467	0.3967	-	0.6234	0.3766
$RGDP_1$	0.6500	0.4849	-	0.6870	0.3130
$vstoxx$	0.9183	-	-	0.8797	0.1203
gdp_1	0.4527	0.7947	-	0.9006	0.0994
inf_1	0.4771	0.6165	0.3886	0.7586	0.2414
inf_5	-	0.6633	-	0.4528	0.5472
une_1	0.4747	0.7869	-	0.9204	0.0796
une_5	-	0.7917	-	0.6533	0.3467

Table 4.10.: Loadings US and EUR, varimax rotation (cropped)

Table 4.10 shows that despite other interrelations there is a strong connection between the European and US stock market-based volatility index. Factor 1 loads both of them very high with values larger than 0.9. This factor accounts for more than 99% of the US stock market uncertainty and nearly 90% of the European stock market uncertainty. Accordingly both variables exhibit a specific variance (uniqueness) close to zero. A specific variance of zero would indicate that the respective variable is solely driven by the common factors. Given this result, I declare Factor 1 as the 'transnational stock market uncertainty factor'.

Factor 2 is said to be the 'European uncertainty factor'. Beside the stock market index all European variables no matter their time horizon are at least loaded with a value larger then 0.6 by this latent force. Due to the fact that especially uncertainty about GDP growth and unemployment is compared to inflation higher loaded by this factor, I shall refine the Factor 2 to be a 'European *real* variables uncertainty factor'.

Factor 3 loads to a high amount US inflation uncertainty and to a less extent short-term European inflation uncertainty. I declare Factor 3 as the 'US inflation uncertainty factor', although long-term US uncertainty also shows a high amount of uniqueness.

Despite these 'first-look' explanations there are several connections between variables and factors which shall be discussed briefly. For example, Factor 1 not only loads stock market data but – to a smaller extent – also US real variables $RGDP_1$ and $RCONSUM_1$ and to even less extent European short-term variables gdp_1, inf_1 and une_1. That is, uncertainty concerning the stock market here and abroad is also connected to uncertainty about the real development of the US and European economy. Due to the amount of the respective loading and the affected variables, I would reason that uncertainty on behalf of the development of the US economy also dominates uncertainty on the short term-development of the European economy. However, causalities can not be delivered by this analysis method.

The opposite picture is valid for Factor 2. Factor 1 strongly loads US values and only weakly European values. On the contrary, Factor 2 loads strongly European values, but only to a minor extent US values. This finding confirms the previous separation between an US and European uncertainty factor and shows the strong connection of the European and US economy – at least for the short-term horizon.

Yet, Factor 3 should be refined into 'US *short-term* inflation uncertainty factor'. Given the data set, Factor 3 reveals the second highest loading of the data set on short-term inflation uncertainty. It also confirms the previous finding, see, Table 4.7, whereby US inflation uncertainty is decoupled of any other variable.

The previous pattern of a short–long distinction for the European data and a real–monetary distinction of the US data fades, when both data sets are combined. This is particularly true for the European analysis. The US data, however, still provides a distinction in the spirit of the classical dichotomy with a real factor on the one hand and a monetary factor on the other. The previously sharp European distinction between short and long-term uncertainty can not be found in the global factor analysis. All survey-based values are loaded on Factor 2, no matter their forecast horizon.

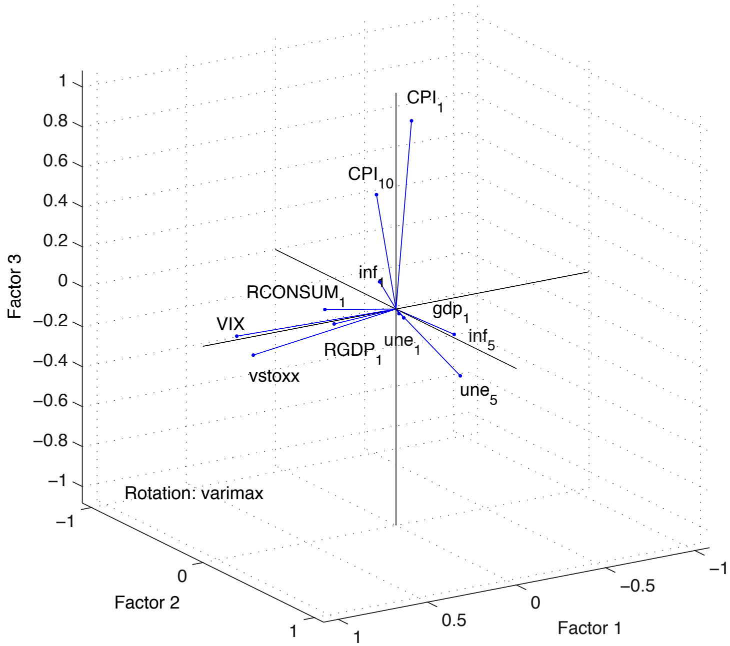

Figure 4.9.: Loadings US and EU, varimax rotation

The findings of Table 4.9 are illustrated in Figure 4.9. The left-to-right axes plots Factor 1 with the leading stock market variables Vstoxx and Vix, as well as the real values of the US. Factor 2 is depicted on the back-to-front axes with the European values. Finally, on the vertical axes Factor 3 is depicted. It shows the pronounced role of US inflation uncertainty, which is on both horizons nearly exclusively loaded by this third factor.

An important implication of the above picture is the fact, that the ECB must not consider possible uncertainty developments of the US in her policy stance, or only to a very limited amount. This would not be the case if uncertainty measures of the euro area and the US have been loaded high by the same factor. The only exception are the uncertainty measures Vstoxx and Vix.

Interpreting these findings must be done with caution. The different pattern reflects differences between both economic regions. Such differences can arise out of the different cultural or institutional backgrounds. However, from a monetary policy perspective, these findings may arise out of the differences between the leading policy actors of both regions. When compared to the Federal Reserve System the ECB is a fairly young institution with a very short history. The ability to adequately form expectations on macro variables that are influenced by policy actions, thus, might be different in both regions. Additionally, the ECB is only in charge of price stability and not – like the Fed – mandated to promote maximum employment and moderate long-term interest rates. Hence, the different uncertainty forces of real and monetary values could originate from the threefold mandate of the Fed whereas the common European factor indicates the unique price stability goal of the ECB. If so, one must admit that the public obviously does not believe in a control of the stock market on behalf of the ECB, and thus, uncertainty concerning the stock market is not driven by European, but, American forces.

4.4. Summary of Chapter 4

This chapter attempted to explore the evolution and interrelations of uncertainty measures during the last decade. This analysis has been conducted with the help of an explanatory factor analysis in order to find common forces which underly these individual measures of uncertainty. The data constitutes itself by the results of the Survey of Professional Forecasters (US and Europe), as well as the stock market volatility measures Vix and Vstoxx. First, the euro area and the US data have been investigated separately, and later on jointly.

The European survey data indicates low uncertainty during the pre-crisis years, measured by the standard deviation of the individual forecasts. All variables show an acceleration of uncertainty during the crisis over nearly all forecast horizons. Nevertheless, mean expectations remain nearly constant or at least quickly return to their pre-crisis values. For the US survey data an acceleration of inflation uncertainty can be found, starting around the year 2006. This is true for short and long-term expectations. Additionally, deflationary tendencies seem to promote a rise in uncertainty. Stock market data shows a nearly perfect synchronized run of both economic regions.

The main finding is that in fact, patterns and relations between the uncertainty of different variables exist. However, they differ between the regions under investigation. For the euro area, factor analysis shows a clear separation between uncertainty concerning short-term forecasts and uncertainty concerning long-term forecasts. This holds more or less for all variables taken into account, and is especially pronounced for inflation expectations. This finding is in line with other research, whereby short-term inflation uncertainty is driven by temporary shocks and long-term uncertainty by permanent shocks. According to my findings, this result can be assigned also to other variables such as GDP growth or unemployment expectations, yet, only to a minor amount.

In contrast to the European finding of a short–long distinction, US factor analysis delivers a rather 'classical' picture of a 'real' and a 'monetary' factor. On the one hand, there is a factor that drives short and long-term inflation uncertainty. While, on the other hand, uncertainty concerning stock market movements, as well as real consumption and GDP growth, are governed by a different factor. Unemployment seems to be somewhat in-between.

Combining both data sets yields three distinct factors which load uncertainty. Firstly, an international stock market uncertainty factor exists, which loads especially the Vix and Vstoxx, and to a smaller extent real US variables. Secondly an European uncertainty factor can be identified, loading mainly European uncertainty measures disregarding the forecast horizon. Thirdly, as the US analysis already revealed, US short and long-term inflation uncertainty follows a distinct common factor, which only to a very limited amount also affects euro inflation uncertainty. Reasons for these patterns I ascribe to the different policy mandates as well as the unequal age of the monetary institutions here and abroad.

4.A. Appendix: General Introduction to the Method

The aim of a factor analysis is to reduce the complexity of the original data set and reveal latent, common forces, which drive the empirical data.[10] Therefore, a theoretical model where each variable, y_j, with $j = 1, \ldots, m$, is a linear combination of the underlying factors, f_k, with $k = 1, \ldots, p$, and some residual component, e_j, is presumed. Each factor is loaded with λ, in which λ_{jk} is the weight of the k-th factor on the j-th variable. This relationship is written as

$$y_j = \lambda_{jk} f_k + e_j, \tag{4.A.1}$$

and is equivalent to the matrix notation

$$y = \Lambda f + e, \tag{4.A.2}$$

[10] The general method of factor analysis can be found in several textbooks. This appendix section serves as a short introduction into the general method. For the rest of this section I rely mainly on Rinne (2000).

with $y = (y_1, \ldots, y_m)'$, $f = (f_1, \ldots, f_p)'$, and $e = (e_1, \ldots, e_m)'$. The respective loadings are summarized according to

$$\Lambda := \begin{pmatrix} \lambda_{1,1} & \cdots & \lambda_{1,p} \\ \vdots & \ddots & \vdots \\ \lambda_{m,1} & \cdots & \lambda_{m,p} \end{pmatrix}. \tag{4.A.3}$$

Due to the fact that, the aim of factor analysis is the reduction of complexity of the original data it must hold that $p < m$, i.e., the number of deduced factors is less than the number of variables of interest.

Under particular assumptions the covariance structure of the hypothetical model can be calculated. The variance of y_j is

$$\begin{aligned} Var(y_j) &= \sum_{k=1}^{p} \lambda_{jk}^2 Var(f_k) + Var(e_j) = 1 \\ &= \sum_{k=1}^{p} \lambda_{jk}^2 + \sigma_j^2. \end{aligned} \tag{4.A.4}$$

The first part on the right-hand side of Equation (4.A.4) is called *communality* of the variable y_j. A convenient writing is $h_j^2 := \sum_{k=1}^{p} \lambda_{jk}^2$. It is the part of a variance of each variable, which is explained by the common factors. For two common factors the communality would be given as $h_j^2 = \lambda_{j1}^2 + \lambda_{j2}^2$, where the first part is attached to the first common factor and the second part to the second common factor, respectively. The sum of all communalities, no matter the variable, is defined as $h^2 := \sum_{j=1}^{m} h_j^2$.

The second part on the right-hand side of Equation (4.A.4) is called *specific variance*. It accounts for the part of variance which is not explained by the common factors, hence, its factor specific. If in the hypothetic model of Equation (4.A.1) y_j could be fully explained by $\lambda_{jk} f_k$, i.e., the common factors, the error term and thus its variance must equal zero.

For each two variables the covariance can be calculated so that

$$Cov(y_j, y_i) = \sum_{k=1}^{p} \lambda_{ik}\lambda_{jk} = \Lambda\Lambda', \quad \text{with } j \neq i. \tag{4.A.5}$$

Combining Equation (4.A.4) and (4.A.5) the correlation matrix R of the original data becomes

$$R = \Lambda\Lambda' + \Psi, \tag{4.A.6}$$

with Ψ representing the specific variance on its diagonal elements and all off-diagonal elements zero.

Equation (4.A.6) is said to be the *theoretical variance covariance matrix*. In order to remove specific variances $R_h = R - \Psi = \Lambda\Lambda'$ is calculated. The reduced correlation matrix R_h contains the communalities on its diagonal and the correlations coefficients on the off-diagonal positions.

The following example will underpin the above. If two variables, inflation (*inf*) and GDP growth (*gdp*), and two common factors are assumed, Equation (4.A.1) can be written as

$$y_{inf} = \lambda_{inf,1} f_1 + \lambda_{inf,2} f_2 + e_{inf}, \tag{4.A.7}$$

$$y_{gdp} = \lambda_{gdp,1} f_1 + \lambda_{gdp,2} f_2 + e_{gdp}. \tag{4.A.8}$$

Hence, the development of inflation given by Equation (4.A.7) is governed by two common factors which also influence the development of GDP growth, and further by a specific factor which governs solely inflation but not GDP growth. The opposite holds for Equation (4.A.8).[11] In matrix notation the above stated can be written as

$$y = \Lambda f + e, \tag{4.A.9}$$

with $y = (inf, gdp)'$, $f = (f_1, f_2)'$, and $e = (e_1, e_2)'$. The loadings are summarized according to

$$\Lambda := \begin{pmatrix} \lambda_{inf,1} & \lambda_{inf,2} \\ \lambda_{gdp,1} & \lambda_{gdp,2} \end{pmatrix}. \tag{4.A.10}$$

The variance of each variable (*inf* and *gdp*) is respectively given as

$$\begin{aligned} Var(y_{inf}) &= Var(\lambda_{inf,1} f_1) + Var(\lambda_{inf,2} f_2) + Var(e_{inf}) \\ &= \lambda_{inf,1}^2 + \lambda_{inf,2}^2 + \sigma_{inf} \\ &= \sum_{k=1}^{p} \lambda_{inf,k}^2 + \sigma_{inf}^2, \end{aligned} \tag{4.A.11}$$

$$\begin{aligned} Var(y_{gdp}) &= Var(\lambda_{gdp,1} f_1) + Var(\lambda_{gdp,2} f_2) + Var(e_{gdp}) \\ &= \lambda_{gdp,1}^2 + \lambda_{gdp,2}^2 + \sigma_{gdp} \\ &= \sum_{k=1}^{p} \lambda_{gdp,k}^2 + \sigma_{gdp}^2. \end{aligned} \tag{4.A.12}$$

11 In fact, the example depicts a situation of $p = m$. This, however, shall be admissible for the demonstration purpose.

The covariance is accordingly given as

$$
\begin{aligned}
Cov(y_{inf}, y_{gdp}) &= \lambda_{inf,1}\lambda_{gdp,1} + \lambda_{inf,2}\lambda_{gdp,2} \\
&= \sum_{k=1}^{p} \lambda_{inf,k}\lambda_{gdp,k} \quad (inf \neq gdp) \\
&= \begin{pmatrix} \lambda_{inf,1} & \lambda_{inf,2} \\ \lambda_{gdp,1} & \lambda_{gdp,2} \end{pmatrix} \begin{pmatrix} \lambda_{inf,1} & \lambda_{inf,2} \\ \lambda_{gdp,1} & \lambda_{gdp,2} \end{pmatrix}'.
\end{aligned}
\tag{4.A.13}
$$

Combining variance and covariance delivers a picture of

$$
R = \begin{pmatrix} \sum_{k=1}^{p} \lambda_{gdp,k}^2 + \sigma_{gdp}^2 & \sum_{k=1}^{p} \lambda_{gdp,k}\lambda_{inf,k} \\ \sum_{k=1}^{p} \lambda_{inf,k}\lambda_{gdp,k} & \sum_{k=1}^{p} \lambda_{gdp,k}^2 + \sigma_{gdp}^2 \end{pmatrix} \tag{4.A.14}
$$

$$
= \begin{pmatrix} \lambda_{inf,1}^2 + \lambda_{inf,2}^2 & \lambda_{gdp,1}\lambda_{inf,1} + \lambda_{gdp,2}\lambda_{inf,2} \\ \lambda_{inf,1}\lambda_{gdp,1} + \lambda_{inf,2}\lambda_{gdp,2} & \lambda_{gdp,1}^2 + \lambda_{gdp,2}^2 \end{pmatrix} + \begin{pmatrix} \sigma_{inf} & 0 \\ 0 & \sigma_{gdp} \end{pmatrix}
$$

$$
= R_h + \Psi. \tag{4.A.15}
$$

The variance is depicted by the diagonal elements and the covariance by the off-diagonal elements. Yet, R still contains the specific variances due to the variance calculation. Hence, $R_h = R - \Psi$ covers only the loadings so that $R_h = \Lambda\Lambda'$. Factor analysis is only interested in the common factor loadings, thus it is appropriate to split R into $\Lambda\Lambda'$ and Ψ.

The diagonal element of R_h are of the form

$$
\sum_{k=1}^{p} \lambda_{inf,k}^2 = \lambda_{inf,1}^2 + \lambda_{inf,2}^2 \tag{4.A.16}
$$

$$
\tag{4.A.17}
$$

which equals the communality h_{inf}^2. Hence, all diagonal elements cover the communality. The same holds for the variable *gdp*.

Now, despite the fact that the symmetric *theoretical variance covariance matrix* has be been created, the loadings λ are still undetermined! Thus, the objective must be to find the right loadings $\hat{\lambda}$ which make the total communality ($\hat{h}^2$) as close as possible to the sum of the observed total variance of the variables.[12] The loadings are thus chosen optimal if the commonalties, i.e., the sum of the variance explained by the common factors, explains the possibly largest part of the total variance.

Finding the right loadings is equivalent to a factorization of R_h. However, the solution is not trivial and multiple solutions may exist. The number of extracted factors is deduced via the Kaiser criterion (Kaiser 1970, Kaiser and Rice 1974). For $m = p$ no further

[12] The hat displays estimated values.

information can be generated and no reduction of the data complexity is achieved. Thus, as an upper boundary it must hold $p < m$. If the eigenvalue of a factor is greater than one, it explains more of the total variation than any other single variable. Vice versa, if the eigenvalue is smaller than one, the factor explains less than any single variable. Subsequently, the lower bound to find the right number of factors should be the number of eigenvalues larger than one. In general, one would actually choose as many factors as eigenvalues greater one, which is according to Kaiser (1970) called the *natural* number of factors. However, an increase in the number of factors does not substantially affect the value of major factors. In any case, these criteria can only serve as a guideline. To find the right number of factors there should always be an economic reasoning; either ex ante or ex post (Kaiser 1970).

If more than one common factor is assumed, multiple solutions may exist for Equation (4.A.6). This implies, as long as Equation (4.A.6) is fulfilled any kind of loadings is allowed. This property is used to get a clearer and much better interpretable picture of the extracted factors. The underlying process of this transformation is called *rotation*.

Any rotation of the factor loadings is done with the help of a transformation matrix P (with $PP' = I$), which delivers

$$
\begin{aligned}
R &= (LP)(LP)' + \Psi && \text{(4.A.18)} \\
&= LPP'L' + \Psi \\
&= LL' + \Psi. && \text{(4.A.19)}
\end{aligned}
$$

Hence, despite an arbitrary loading, the solution to the correlation matrix is still correct. Figure 4.10 shows this rotation in a graphical way. On the left-hand side, the ellipse stands for a group of several loading combinations in the two factor space, spanned by the horizontal and vertical axes. The axes are now rotated counter clockwise to fit better into the 'scatterplot' of loadings. This is depicted on the right-hand side of Figure 4.10. However, it is important to notice, the data has not changed.

The rotation enables the researcher to draw a picture of the data, which fits best according to the purpose of his research goal. Due to the fact that the factors as well as their respective loadings are rotated, the initial Equation (4.A.6) is not violated, but only represented in a different, more convenient way. Among others, the most prominent orthogonal rotation form is *varimax*, the most prominent oblique rotation form is *promax*. For an oblique transformation the extracted factors are not independent from each other and can be interpreted as correlations. The opposite is true for an orthogonal rotation. In the upcoming analysis I will make use of both methods.

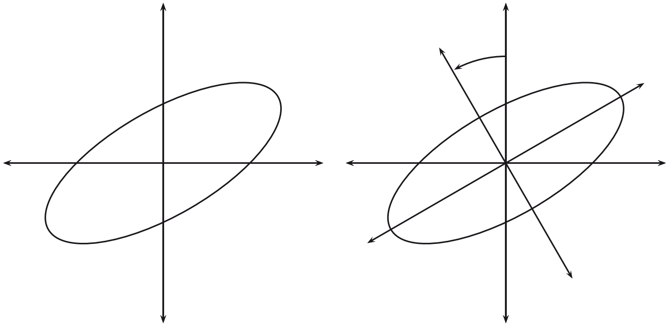

Figure 4.10.: Factor rotation

It should be mentioned that factor analysis offers a lot of space in terms of execution and interpretation. This is true for finding the right number of factors – as there is no compulsory rule, or the question when to chose which rotation method, and if so, to which amount a rotations should be conducted. All these degrees of freedom should not be mistaken for 'data manipulation'. Rather this should be taken as an advantage of this method, to illustrate the data in a convenient way, which helps the author to make his idea more conspicuous.

5. Communicated Uncertainty of the ECB

5.1. Introduction

This section is linked to the previous empirical investigation of uncertainty. However, nearly the opposite position is taken. Whilst before, uncertainty on behalf of private agents has been taken into account this chapter introduces a new approach to measure the communicated uncertainty of the European Central Bank.

During the last two decades, central banks all over the world have followed a path to more openness, i.e., more transparency, which comes along with a higher degree of communication towards the public. This process started roughly around the mid-nineties. During this time, the Federal Reserve started to announce her decision on the federal funds rate target. This first step was accomplished by other central banks, e.g., by the publication of macro forecasts and reached today's level of transparency, where some central banks even provide the public with minutes of the most recent committee meetings. These developments gave birth to a new field of research which investigates market movements as an answer to central bank communication, by examining, for example, speeches, interviews, press statements, and other publications.

To a great extent, these developments towards a greater transparency have also had influence in the constitution of the ECB, and its self-conception of communication policy. From the very beginning, the ECB accompanied her meetings and interest rate decisions by a press conference, which follows immediately after the announcement of the upcoming level of the main refinancing rate. Nevertheless, the ECB still does not publish the minutes of the meetings of the governing council which make a detailed analysis impossible.

However, this caginess on behalf of the ECB allows me to investigate how much the ECB is *willing to communicate* her perceptions concerning the current state and the future development of the economy to the public. This is of special interest, when it comes to the communication of uncertainty. The ECB communicates her interest rate decisions mainly through an official press statement right after the meeting. Hence, no matter the heaviness of the discussion among the participants of the board during the meeting,

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she can determine exactly to what degree she is willing to communicate the public, not only her stance in monetary policy, but also the uncertainty that comes along with her estimates. This gives her the power to steer expectations in a narrow way. A feature which would vanish if the minutes of her meetings would be published, as these publications can make previous statements unreliable if they both do not share the same tenor. Hence, analyzing the ECB press statements gives a picture of how much the ECB is willing to admit uncertainty regarding the assessment of the overall situation.

To analyze these statements a tool already used in the field of psychology and lately particularly in finance literature is applied. It is called sentiment analysis and its objective is to capture the tone of a certain publication. The textual analysis is conducted with the help of a statistical software (SAS), which counts the occurrence of signal words in a predefined text passage. This tool has already achieved success, for example, in analyzing the sentiment or tone of a news articles and should thus be also applicable for the data on hand.[1]

5.2. The Importance of Central Bank Communication

In 1987 William Greider published his bestseller book ‘Secrets of the Temple: How the Federal Reserve Runs the Country’. Already the title shows quite striking the perception and conception of central banking before the mid-nineties. At this time, monetary policy at the Federal Reserve and other central banks was understood as a mystic or esoteric art with the chairman as its high priest in the center (Woodford 2005, Blinder et al. 2008).

Ever since then, greater openness, i.e., more transparency and more communication on behalf of the central bank have marked modern monetary policy. For example, not before 1994 the Federal Open Market Committee (FOMC) started to announce decisions concerning the federal fund rate; in July 1995 the FOMC firstly announced a numerical target of the federal fund rate which was later, in 1999, extended by publishing possible future scenarios of the rate, the so-called ‘bias’ or ‘tilt’. Further developments up to more transparency and openness followed such as the publication of detailed forecasts (no matter if verbal or quantitative) and precise minutes of the meetings (Blinder et al. 2008, Rudebusch and Williams 2008).

These changes in central bank communication are mainly ascribed to the former Fed chairman Alan Greenspan. His understanding of conducting central bank policy has

[1] Recently this method has also been applied to the extraction of information for the process of financial decision making, see FIRST (2013).

shaped the picture of the Federal Reserve System we witness today and has been a determining factor in the constitution of other central banks around the globe, most notably, the European Central Bank (Blinder et al. 2008).

The growing importance of this issue is accompanied by a growing body of literature concerning communication, transparency, and openness. Very good surveys can be found, e.g., in Ehrmann and Fratzscher (2005) and Blinder et al. (2008). The influence and importance of a proper central bank communication has also been highlighted by numerous officials and representatives of the ECB, see, e.g., Issing (2006), Trichet (2008a,b), Asmussen (2012). Additionally, a variety of publications on various related themes exists, for example, how central bank communications affects asset prices (see, e.g., Lucca and Trebbi 2009, Born et al. 2011), or how communication strategies change under uncertainty and stressful times (see, e.g., Bulir et al. 2012).

Furthermore, there has always been a lively debate concerning to which degree and in which way openness and transparency are favorable for the conduct of monetary policy, or, whether too much transparency actually harms a sound policy. An example, is the influence on asset prices via a pronounced communication, which emerges precisely due to the *creation* of, or *focusing* on, specific news. For the case of published minutes it may be favorable, for example, only to publish majority votes and to keep disagreement under the rug to circumvent such possible distortions (ECB 2002, Issing 2005, Rudebusch and Williams 2008, Güler 2013).

In many cases the transparent policy of the central bank was not practiced as an end in itself. Due to the change in the way central banks act and behave a new instrument was born, namely, the *management of expectations*. This instrument is as least as important as any so called direct instrument such as the main refinancing interest rate. As Woodford (2005) puts it, "...not only do expectations about policy matter, but, at least under current conditions, very little else matters" (Woodford 2005:3).

The understanding of central bank policy on behalf of rational thinking agents enhances current policy actions enormously, sometimes even if no direct action is conducted, but only future perceptions are communicated. Today's interest rates, in fact, do not matter at all. It is the expected future path of interest rates that governs financial market prices and ultimately spending and pricing decisions. Thus, the management of these expectations of future developments should be the aim of each modern central bank (Woodford 2005, 2012a).

However, a fruitful management of expectations can only be applied if the communication policy is absolutely clear, hence, an explicit language must be relied upon. Therefore it is necessary to choose words wisely as market participants hang on the lips of central bankers and interpret every single word. A wrong announcement, the wrong answer in an interview, and even the wrong facial expression could cause consequences which are not intended by any central bank. For example, in October 2000 the former ECB chairman Duisenberg triggered an immediate depreciation of the Euro by hinting that there would be no further support for the Euro on behalf of the ECB (Blinder et al. 2008). This example demonstrates the importance and perils of this newly created instrument and shows that even central bankers have to cope, and learn, how to work with this new power. Furthermore, not only statements *per se* have been under constant observations, but also changes and substitutions of words and phrases. Likewise, Woodford (2012a) lists examples of a change in the Fed funds rate expectation, which are not accompanied by a change in the current target rate, but a change in the wording of communication.

Due to the fact, that speeches, interviews, and press releases are a very sensible field in terms of communication, one can imagine that there is no word without a reason in the official statements of a central bank. One of the most important organs of communication and certainly the one which newspapers and other financial actors pay great attention to, are the press announcements following each meeting of the monetary policy committee and which are (mostly) read out by the president of the ECB himself. The pronounced role of these statements gives the above stated even more weight. Thus, the wording of these statements is chosen extremely carefully and always with a reason. Keeping in mind these restrictions, the analysis of official ECB speeches allows me to infer, to what extent the European Central Bank is willing to reveal her position and knowledge concerning uncertainty.

5.3. Traditional Measures of Uncertainty

I have already shown in Chapter 4 that mainly two methods exist to measure uncertainty. The first of which, a direct measure, relies on surveys such as the Survey of Professional Forecaster provided by the ECB or the FRB of Philadelphia. The other measure is deduced from financial indicators, most notably the Vix for the US or the Vstoxx for the European market. The advantages and disadvantages of these measures have been discussed previously.[2]

Both measures display the uncertainty of the market. The Vstoxx and the Vix reflect the market behavior of stock options, which are mainly traded by private agents. In the same way surveys reflect the dispersion, and thus uncertainty, of actors of the financial sphere, such as banks or research institutions, but also non-financial participants, although this is the minority. Hence, both measures consider the uncertainty of only one side of the

[2] See as well, Croushore (1993), Giordani and Söderlind (2003), Bowles et al. (2007), Galati et al. (2009), Bekaert et al. (2010).

expectation formation process, which is the private agent side (Croushore 1993, Bowles et al. 2007).

However, none of these measures covers uncertainty on behalf of the central bank. And in fact, none of the above presented measures is applicable to get a picture of how the central bank deals with uncertainty. Although, this is not fully true. During the last years central banks have started to publish more and more information about the discussion within their committees, so-called minutes. Theoretically, these minutes could be analyzed with the same technique as survey results to get a picture of how large the dispersion, and thus, the uncertainty within the committee, as a representative organ of the central bank, is. Analyzing more or less detailed minutes allows for the opportunity to investigate the amount of uncertainty *within* the central bank committee, that is, how members of the committee disagree about the perception of the current situation and their proper response. Additionally, one could eventually infer about the assessment of key macro variables on behalf of the central bank, and thus, also about the uncertainty of these assessments by analyzing the minutes.

Due to the fact that the ECB does not publish detailed information, a different analysis must be conducted, which is not less interesting and which shall be discussed in this work. The retention of detailed information on behalf of the ECB gives the opportunity to analyze by how much the ECB is willing to admit how uncertain she is with respect to actual data as well as future prospects of key variables and political developments. The power of this approach is that any discussion at a meeting of the ECB governing council can be held as lively as it should be, at the end, the press statement given by the president of the ECB only reflects the amount of uncertainty the ECB is willing to communicate to the public. Despite the similar analysis technique, this approach stands in a stark contrast to earlier research which focuses on uncertainty within a central bank committee. The next section introduces the utilized approach.

5.4. Sentiment Analysis: Talking by Numbers

5.4.1. Literature

In the field of economics, analyzing a qualitative text and turning it into a quantitative assessment can be found especially in finance literature. Tetlock (2007) would be the starting point of a literature review in this field. In this paper the interaction between the stock market and the popular daily *Wall Street Journal* column 'Abreast of the Market' is analyzed. On the one hand, Tetlock (2007) finds a strong link between media pessimism and downward pressure on market prices, and the trading volume. On the other hand he finds a link from low returns to media pessimism. From his analysis he reasons that media content has an unconsidered influence on the the behavior of investors, and thus, on the stock market.

For his analysis, Tetlock (2007) makes use of the so-called ‘General Inquire’ content analysis program to count for signal words. This analysis relies on the ‘Harvard psychosocial dictionary’. However, Loughran and McDonald (2011) question the use of the Harvard dictionary, specifically the Harvard-IV-4 TagNeg (H4N). Due to the fact that the H4N was originally developed for the analysis of psychological and sociological contents, they reason that it is hardly applicable in the realm of business and finance literature. To overcome this handicap and to capture the tone of corporate 10-K reports, Loughran and McDonald (2011) created a new list of negative-meaning words which fits better into the financial environment.

Making use of the technique elaborated in these papers a handful of literature has very recently started to analyze central bank minutes. Despite many commonalities, yet, they focus on different aspects. For example, Apel and Grimaldi (2012) measure the sentiment of of the Swedish Riksbank minutes to predict future policy decisions. For this sentiment detection they construct an alternative list to capture the tone of the minutes.[3] Within their analysis they find a positive predicting power from minutes on future policy actions. Grimaldi (2011) uses the approach of Loughran and McDonald (2011) to measure and interpret financial stress making use of the ECB monthly bulletins. Together with other possible indicators, such as surveys or financial variables, these results can be useful to build an overall financial stress indicator. My work resembles as well Mayes and Montagnoli (2011). They use the minutes of three central banks, namely the Bank of England, the Czech National Bank, and the Sveriges Riksbank, to analyze how these institutions communicate uncertainty within their discussion. Furthermore, important for this work serves Baker et al. (2012a,b). They have created an uncertainty indicator which is based on three single indicators: newspapers, tax code provisions, and forecaster disagreement with the aim to estimate, for example, GDP losses due to uncertainty. Concerning the data set, Jansen and deHaan (2010) have as well evaluated the introductory statements of ECB, yet, with the aim of identifying whether the ECB communicates in a consistent manner.

Within the so far presented literature, the here presented work is somewhat in between. Certainly, Tetlock (2007) and Loughran and McDonald (2011) lay the corner stone of the textual analysis. Due to the fact that until today the European Central Bank has

[3] To get a feeling of such a list: The Loughran and McDonald (2011) ‘negative list’ contains 2337 words, whereas Apel and Grimaldi (2012) utilize 7 words together with 8 modal words like ‘high’ or ‘fast’.

not provided any minutes of the monetary policy committee meetings, detailed analyses like Grimaldi (2011), Mayes and Montagnoli (2011), or Apel and Grimaldi (2012) are not possible for the European market. In line with Baker et al. (2012a,b) who build an uncertainty indicator which is partly based on newspaper evaluations, and Jansen and deHaan (2010) who take the introductory statements, but focus on a different research question, my research borrows from both as it builds an uncertainty indicator deduced from the introductory statements.

5.4.2. Theoretical Underpinning

The idea of a sentiment analysis is to capture the tone of a specific text or only text phrase. Generally, it is possible to differentiate between a sentiment which is extracted from *structured* and *unstructured* data sources. Structured data is already available in a quantitative manner like financial ratios or specific time-series data. Unstructured data, however, needs to be analyzed semantically before a quantification can be conducted. Therefore it is necessary to convert a *qualitative* source into a *quantitative*, i.e., an evaluable data set. The transformation is done by coding the respective text with respect to key words or phrases. This implies a counting of words which are related to a specific content. For example, analyzing a text passage with regard to uncertainty would be conducted by counting words such as, of course, 'uncertainty', but also 'risk', 'ambiguity', and sub-forms such as 'risked', 'riskier', 'riskiness', and so on. The result is a specific number of phrases or words with respect to a specific text.

Yet, this method has some shortcomings. Most obvious is that the meaning of a sentence is reversed if a negation is placed in front of it. Hence, the situation is very different, whether we have a recession or *not* a recession. The same holds if, for example, the 'recession' is preceded by the word 'weak' or 'strong', which is even harder to distinguish by a computational routine. Despite this drawback the so-called 'bag of words' approach, i.e., the comparison of a text passage with a list – or bag – of words, is the most common used method in textual analysis and has served well in previous publications, see, e.g., Tetlock (2007), Loughran and McDonald (2011), Apel and Grimaldi (2012) or Baker et al. (2012a,b). To see why this is true consider the following aspects.

Right list: As cited in Loughran and McDonald (2011), the foremost criteria for a good content analysis lies in the right criteria list, i.e., the right signal words must be identified. For this reason, Loughran and McDonald (2011) modified the existing Harvard-IV-4 tag list by extending and cutting it to fit proper into a financial, rather than the originally intended sociological and psychological context. Fortunately, they also created a dictionary containing 291 entries within the sphere of uncertainty.

Relative frequency: The idea behind the 'bag of words' approach is that a high frequency of words must be correlated with the current market condition, or, at least with the by the author intended tone of the document. Hence, each representative of the European Central Bank, and most notably the president in his introductory statement following each interest rate decision, would not loosely use, e.g., the word 'uncertainty' if there is the possibility of a misinterpretation on behalf of the private sector. If so, certainly a different phrase is used. Accordingly, it can be assumed that rather the word 'boom', instead of 'no recession' is utilized to indicate a sound environment. Thereby, one can assume that it only matters to a very limited amount, whether the word combination 'less uncertainty' or 'high uncertainty' is utilized. As long as the word 'uncertainty' pops up the intention of the speaker is to draw attention on an uncertain issue. If not, he would have chosen a different phrase. Hence, no matter if uncertainty is low or high, uncertainty prevails anyway (Grimaldi 2011).

Negation: With respect to the previously mentioned example of negation, the 'bag of word' technique is still applicable. This is admissible due to the fact that it is more likely that a positive expression will be negated, e.g. 'we have no certainty' rather than 'we have no uncertainty', or 'a recovery is not certain' rather than 'a recovery is not uncertain'. Hence, it must not be accounted for negations, as it would not change the result significantly (Grimaldi 2011, Loughran and McDonald 2011).

Weighting: To improve the results and to circumvent the overweighting of frequently used words, a weighting scheme could be introduced which gives less important but highly used words only a very small amount in terms of scoring. However, this is not essential or mandatory and if conducted at all, this should be done with caution (Loughran and McDonald 2011).

5.5. Sentiment Analysis: Empirics

5.5.1. Data & Methodology

As I have shown in Section 5.4.1 that on the one hand, there is literature which measures the sentiment of newspapers (see, e.g., Tetlock 2007, Loughran and McDonald 2011), and on the other hand literature exists, which focuses explicitly on central bank communication – utilizing published minutes of committee meetings (see, e.g., Grimaldi 2011, Mayes and Montagnoli 2011, Apel and Grimaldi 2012).

However, detailed transcripts of meetings, which have been used in the above mentioned works, do not exist – or least have not been published – for the European Central Bank. Hence, for this analysis, which focuses especially on the communication of uncertainty on behalf of the ECB, I will use the introductory statements of the ECB. Although these statements do not have the informational content that minutes provide they have the advantage of a nearly standardized framework, which can be separated into the categories: general introduction, economic analysis (second pillar), monetary analysis (first pillar), and concluding remarks. In fact, to observe the desired level of communicated uncertainty on behalf of the ECB this framework works even better than a detailed minute.

The focus of this chapter is not how much uncertainty prevails in the market. Moreover, this analysis can not quarry how uncertain the ECB actually is. In fact, the focus is the desired level of *communicated* uncertainty on behalf of the central bank. That is, how much is the central bank willing to admit uncertainty – in her own perspective as well as she is ascribing to the market. The ECB herself admits that well structured information is a key element for a proper communication with the public (see, ECB 2002). For this reason, the introductory statement works perfectly as it is an official, well reconsidered statement of the ECB. One can assume every word and sentence – and especially its absence – is done for a reason. For the same reason, I also exclude the Q&A following each session. Questions and (at least to some extent) answers can not be that well considered compared to the prior assessment.[4]

For the textual analysis all introductory statements preceding each interest rate decision from 2002 until 2012 are considered. Although statements are available ever since 1998, not before November 2001 the Governing Council decided to asses the stance of the monetary policy only at their first meeting of the month. Hence, only since 11/2001 and beyond a well structured monthly documentation is available. All of the following changes, such as the new ordering and re-naming of the first and second pillar in mid 2003, pose no problem for the analysis. This gives an overall sample size of 129 observations which is comparable to other works (see, e.g., Grimaldi 2011, Brissimis et al. 2012).[5]

Opposed to other works, I will not consider newspaper articles to improve or contrast my results. The previously published work of Baker et al. (2012a) creates an uncertainty indicator for the US and the European market which is based *inter alia* on a sentiment analysis of newspaper articles. In my opinion this is very questionable. A newspaper analysis may be feasible for a region which shares the same language and background such as the US. However, for a proper analysis of the European newspaper market one would have to account for every prevailing language of the EU with all its specific wordings and meanings.

[4] Of course, one can assume that the Q&A session is part of some strategic game, well included into an overall strategy of the ECB, which serves just as a different communication channel.

[5] I will not conduct this analysis for the FOMC of the Federal Reserve. Among others, reasons are the significantly shorter period of available statements and foremost a different, more elaborated system which is not comparable to the European one.

The ‘bag of words’ constitutes itself from the uncertainty word list of Loughran and McDonald (2011). This is to my knowledge the most exhaustive list within the financial realm. Of course there have been other works that apply the ‘bag of words’ technique, however, these works, see, e.g., Apel and Grimaldi (2012), Baker et al. (2012a,a) consider other research questions or slightly different analysis techniques which do not make them suitable for this analysis.

The following sections provide the findings of the sentiment analysis. For convenience, I have subdivided them according to the analysis method. Therefore, Section 5.5.2 provides a descriptive analysis of the developed uncertainty measure compared to other uncertainty indicators like the European volatility index Vstoxx. Section 5.5.3 shows the results of a Markov-switch analysis.

5.5.2. Descriptive Findings

To give an intuition about the textual analysis Table 5.1 lists exemplary the findings for May 2011 and 2012. The upper part lists the overall number of words of Mario Draghi’s introductory statements for both dates. It is subdivided into the parts: introduction, economic analysis, monetary analysis, conclusion, and the total sum of all categories. It can be stated that the total amount of words for each category remains roughly at the same level during the time of investigation. For example, the overall word count over all categories is around 1400 words for each statement.

The lower part of Table 5.1 delivers the number of uncertainty related words in each category, respectively. In parenthesis the according relative value is given.[6]

Date	Intro.	Econ.	Mon.	Con.	Tot.
			All words		
May 2012	297	361	333	402	1391
May 2011	373	543	357	672	1563
			Uncertainty related		
May 2012	4	10	1	1	16
	(1.3%)	(2.8%)	(0.3%)	(0.2%)	(1.2%)
May 2011	5	25	4	7	39
	(1.3%)	(4.6%)	(1.1%)	(1.0%)	(2.5%)

Table 5.1.: Result of textual analysis (May 2011, 2012)

[6] It can be shown that differences between absolute and relative figures are neglectable during most part of the time span. Especially during the last years – marked by the financial turmoil – there is nearly no difference at all. Yet, for the further analysis I take the relative values rather than the absolute figures.

Figure 5.1 plots the Vstoxx as a dashed line.[7] For further analysis, I will use the Vstoxx as a benchmark measure of uncertainty for the European market. On the first sight, the Vstoxx offers four characteristic hikes which represent a heightened market uncertainty and which have been already mentioned in Chapter 4. These peaks are the ones around the years 2003, 2008, mid 2010 and late 2011. In the search of events that contribute to these significant peaks, the literature links the 2003 hike to the 2$^{\text{nd}}$ Gulf War (Iraq invasion); late 2008 to the Lehman bankruptcy; May 2010 to the Greek downgrading and bailout request, and October 2011 to the Greek bailout referendum and later Papandreou resignation (see, especially, Baker et al. 2012a).

The solid line gives the European Policy Uncertainty Index (EPU) of Baker et al. (2012a, 2013), which is based on two distinct sources. The first is a newspaper sentiment analysis of five European economies containing 10 different newspapers. The second source is based on Consensus Economics which delivers forecast dispersion as a measure of uncertainty (Baker et al. 2013).

Vstoxx and EPU show roughly the same picture. This is not astonishing due to the fact that the EPU is a mixture of several uncertainty indicators deduced from the perception of market participants. Due to this congruence, the aggregation of different measures of uncertainty in Baker et al. (2012a, 2013) is not problematic. However, this does not hold for the uncertainty index deduced from the ECB statements. A combination with the Vstoxx, in order to get an overall picture of uncertainty in the Euro zone, would mix to different kinds of investigation and is thus not admissible. Differences between the Vstoxx and the EPU arise around the years 2003 and 2009, where the Vstoxx is more accentuated. From the beginning of 2012 until the middle of 2012 the picture is reversed, thus the EPU remains at a relatively high level whereas the Vstoxx declines. Despite these minor difference, however, it can be said that both measures show the same tenor concerning uncertainty in the euro area.

Figure 5.2 shows again the Vstoxx, but now in combination with the developed ECB uncertainty index (ECBU) marked by the cross-line. The ECBU index covers the overall occurrence of uncertainty related words and has been standardized as well. Compared to Figure 5.1, which delivers a quite uniform picture, Figure 5.2 shows that there are significant differences between the Vstoxx, as the standard market measure, and the communicated ECB uncertainty measure ECBU. The comparison between both measures can be separated into two parts. The first part covers the period from 2002 until 2008 and the second part captures the horizon from 2008 until today. Hence, the discrimination is between *before* and *after* the Lehman bankruptcy in September 15, 2008.

[7] This and all further series have been standardized according to $z = (x - \mu)/\sigma$.

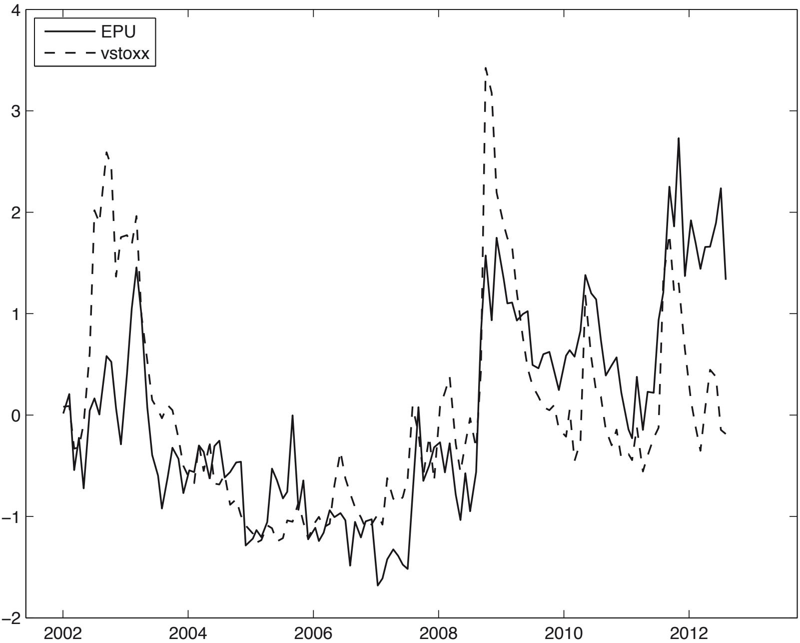

Figure 5.1.: European Policy Uncertainty (EPU) and Vstoxx

Looking at Figure 5.2 it becomes obvious that approximately until the year 2004 the ECB is below the market measure in her communication of uncertainty, thereafter she is above the benchmark measure Vstoxx. However, this is merely a level effect, in any case, the ECB measure as well as the Vstoxx have a very similar shape in the years after 2004 until 2007. What strikes is the sharp rise of the ECBU starting at the beginning of the year 2007 with its peak in September 2007. This behavior is of special interest as this pronounced hike is – at least not in the same amplitude – not accompanied by the Vstoxx. The peak of the ECBU must be ascribed to the beginning of the financial crisis, which is often said to be August 9, 2007 (see, e.g., Borio 2008, Khandani and Lo 2011).

Hence, whilst the Vstoxx has not reacted to the "Quant Meltdown" or "Quant Quake" (Khandani and Lo 2011:2) in 2007 it seems like the communication of the ECB has already taken up this issue. And in fact, a closer look at the statements reveals that the ECB took notice of the disruption and is possible, uncertain, consequences. This event is represented only to a very limited amount in the Vstoxx data, which shows only two

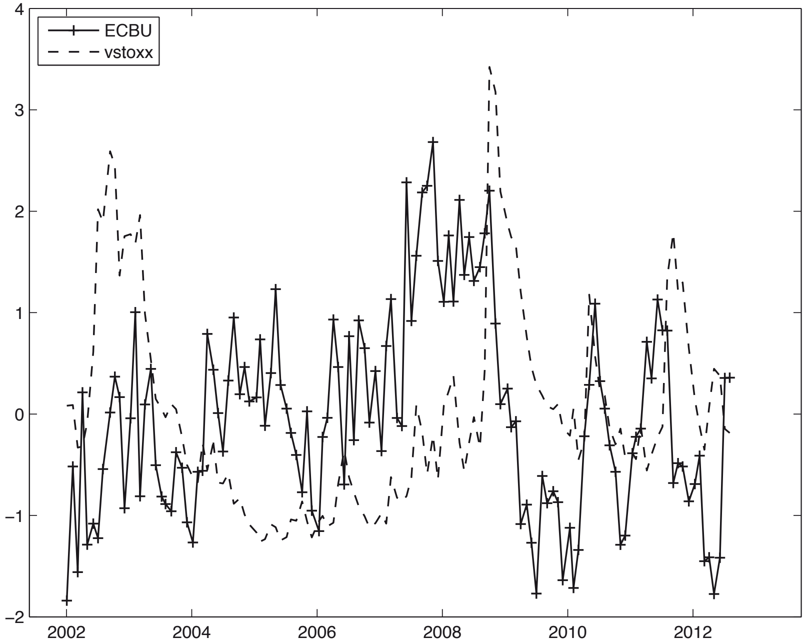

Figure 5.2.: ECB uncertainty (ECBU) and Vstoxx

little humps around the end of 2007 and the beginning of 2008. Its peak is reached not until September 2008, which coincides with the Lehman bankruptcy. Striking is the fact, that at the same time the Vstoxx rises to its never-been-before high, the ECBU, i.e., the amount of communicated uncertainty of the ECB, experiences a stark fall down to a never-been-before low. This drop also marks the beginning of the second period of my distinction of 'before' and 'after' the crisis.

This first descriptive analysis of the time *before* the Lehman bankruptcy offers a somewhat random ECB communication especially compared to the second part of my interpretation. This behavior may be owned to several factors such as a rather calm economic environment or maybe just the simplemindedness of the ECB in her starting years.

The years *after* 2009 show a different picture. The sharp drop of uncertainty communication around the years 2008/2009 may be due to a realization on behalf of the ECB about the impact of her wording. The year 2008 marks the first severe crisis in the history of the ECB. It seems like she suddenly becomes aware of her communication tool and discards all uncertainty potential from her official communication in order to appease the market from the fear of the observed turmoil. Whilst the ECBU is relatively high at the beginning of the financial crisis in 2007, it is nearly demonstratively low in the aftermath of Lehman. Together with other measures, such as the enormous liquidity provisions which have been part of the crisis management of the ECB, this may have caused overall uncertainty to decline in the following months, demonstrated likewise by a fall of the Vstoxx in the months after. Hence, the ECBU reacts somewhat around one year earlier as the Vstoxx does. It rise already in 2007, but the Vstoxx follows only in 2008. Thereafter it falls already in 2008, whereas the Vstoxx declines not before the beginning of 2009. A similar pattern can be observed when looking at the most recent events of the Euro crisis around the years 2011/2012. Moreover, uncertainty measured by the Vstoxx is relatively high compared to the time period between the years 2004 until 2007, communicated uncertainty on behalf of the ECB is most of the recent time even lower then in mid 2000, although, one would testify these days an objectively higher uncertainty environment than at the beginning of the century.

This adds a new view on the crisis management of the ECB. Existing literature, see, e.g., González-Páramo (2009), highlight the appeasing effect of non-standard measures in the aftermath of the crisis. As well there has been a discussion to what extend a proper communication can reduce overall uncertainty, see, e.g., Born et al. (2011), Asmussen (2012). The importance of a decrement of uncertainty for economic growth has been shown lately by Baker et al. (2012a,b). Yet, there has been no discussion to what extend a shift in the wording of communication concerning uncertainty has contributed to the decline in overall uncertainty, and thus, to a speedy convalescence of the European economy in the months and years after the outbreak of the crisis in the year 2007.

It is not surprisingly, that the developed ECBU differs as well from the Vstoxx as from the EPU. Again it becomes obvious that communicated uncertainty on behalf of the ECB and market uncertainty have not to be the same. Differences can arise if the ECB wishes to draw a different picture of the economy in order to steer expectations of the market. Hence, could it be that if the central bank denies uncertainty by excluding it from her official statements, overall uncertainty in fact declines as well? This is a difficult question to answer. As has been presented above, there is evidence that communication helps to steer the economic development. One example of this is the development of asset prices. However, as has been said before, for today the limited amount of data makes this question almost impossible to answer.

Yet, the thesis of a strong connection between the uncertainty communication of the ECB and market uncertainty finds support in the subsequent periods of 2009. After the drop of the Vstoxx and the ECBU in 2009 they again show a very similar pattern sharing ups and downs in the following three years, which are marked by the problems of the European monetary union. Compared to the periods before 2009 the Vstoxx as well as the ECBU show a much more pronounced course. Especially the ECBU, which shall be in focus of this discussion, seems to be more explicit than before. The pattern around the Lehman crisis, where it seemed like the ECB has demonstratively reduced her uncertainty wording, can be found here again.

Twice there was a sharp reduction of uncertainty words in the official communication of the ECB. This reduction followed events that might have beard the possibility to create a higher uncertainty in the market, namely the 2010 peak to the Greek bailout request and the 2011 peak to the Greek bailout Referendum and Papandreou resignation. For the second of these, the ECB even retracted her communication to an all time minimum by reducing her uncertainty wording significantly during the time right before 2012, and then even more throughout the year 2012.

Figure 5.3 offers another interesting feature. The diagram plots the relative appearance of uncertainty related words on the vertical axis. This is shown for the economic analysis, as well as for the monetary analysis part of the statements. Until the year 2007 both lines are roughly around the same value, although the monetary values seem to be more erratic than the economic figures. However, starting approximately in 2007 we can observe a divergence between both parts. Whilst the economic analysis remains more or less at the same level of 3-4 uncertainty related words per 100 category words, uncertainty related words in the monetary part are getting fewer and fewer. The overall decline of the ECBU (excluding the introduction and the conclusion part) can thus be attributed nearly exclusively to a decline of mentioning uncertainty in the monetary analysis. It is hard to tell, what the reason for this finding might be. However, it could be that the decline can be interpreted as the complement to the starting of the massive liquidity provision, which may be accompanied by an increased fear and uncertainty concerning inflationary tendencies.

A general drawback of the analysis is the fact that one can not say whether the ECB follows the market or whether the market follows the ECB. An analysis of this question is not possible due to several reasons, first of which is the limited amount of existing data. However, this problem may resolve itself as more and more data becomes available as time goes by. Additional, the frequency of the data generating process remains too low. Interest rate decisions are announced every first Thursday of the month, hence, it is very difficult to say if and to what extent these statements cover past events or anticipate future ones.

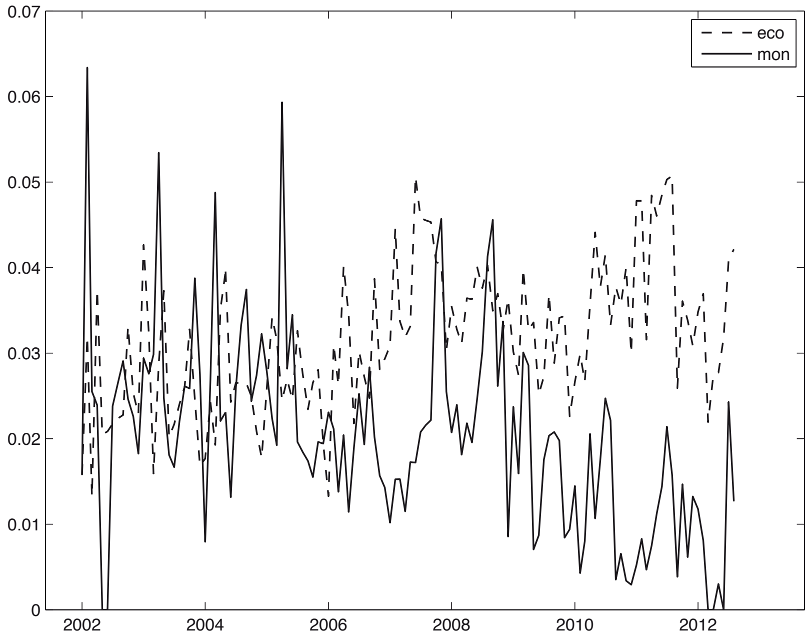

Figure 5.3.: ECB uncertainty (ECBU) split

5.5.3. Markov-Switch Analysis

To underpin my argumentation, whereby the year 2008 marks an important turning point in the communication policy of the ECB, a further investigation of the EMPU is needed. To validate the descriptive results in the following I will apply a Markov-switch analysis. This method has been introduced already in Chapter 3 of Part II, hence, only some points are repeated here.

Markov-switch analysis can be a powerful tool to identify regime switches within a given time series. If two regimes are assumed and 200 periods are under consideration, it could thus be that from period 0 until 100 the model is said to follow the rules of regime 1, from 101 until 150 it is said to follow regime 2, and from 151 until 200 regime 1 again. Hence, it can be used, e.g., to distinguish a time series into a bear and bull market. The purpose of conducting a Markov-switch analysis with the ECBU index is to distinguish different periods by means of the ECB communication.

In general, it is assumed that a times series can be explained by a model of the form

$$y_t = x_t + \epsilon_t. \tag{5.5.1}$$

Due to the fact that $\epsilon \sim N(0, \sigma^2)$, the expected value of y_t is x_t.

Furthermore, it is assumed that the given times series can be distinguished in s different regimes which have their own specific properties. Hence, the general writing of Equation (5.5.1) is

$$y_t = x_{t,s} + \epsilon_{t,s}. \tag{5.5.2}$$

If it is assumed that $s = 1, 2$, it must hold that regime one differs from regime two by a different mean and/or variance. For the case $\epsilon_1 = \epsilon_2$ only the mean differs while the variance remains the same. The same holds for the opposite case where only the variance changes while the mean remains the same. Between both regimes a switching matrix P exist which after each time-step gives the probability of a jump into a different regime or to remain in the present one (Hamilton 2005, Perlin 2012).

In what follows, an identifying routine is applied on the developed ECB Uncertainty Indicator ECBU. Thus, the series created by the 'bag of words' technique is divided into parts where each part has its own characteristic. Due to the fact, that the ECBU index measures the relative frequency of uncertainty related words, it suggest itself to presume two different states in the communication policy of the ECB, namely, a 'normal' state and an 'uncertainty' state of communication. As the name already indicates, the normal state is assumed to prevail under a fairly stable economic environment where the ECB acts plain vanilla. However, if times start to get wild, the communication policy switches into the 'uncertainty' state of communication. The different states shall be identified by analyzing the ECBU index. Thereby it is possible that the two expected states discriminated themselves by a different mean or variance.

Making use of such an identifying routine (see, Perlin 2012), Figure 5.4 is generated, which shows the probability weight of a certain state, reaching from zero, not probable, to one, highly probable. With respect to the period taken into account Table 5.2 gives the corresponding transition matrix in the upper part and the expected duration of one regime in the lower part.

The most suitable result is achieved by assuming two distinct states, although theoretically more than two states can be assumed.[8] Figure 5.4 depicts that swings between the two different regimes can be identified. The difference between both regimes lies in the

[8] To find the right number of regimes, I have also considered more than two possible regimes. Yet, beside an economic reasoning, the results of this check as well hint towards two regimes.

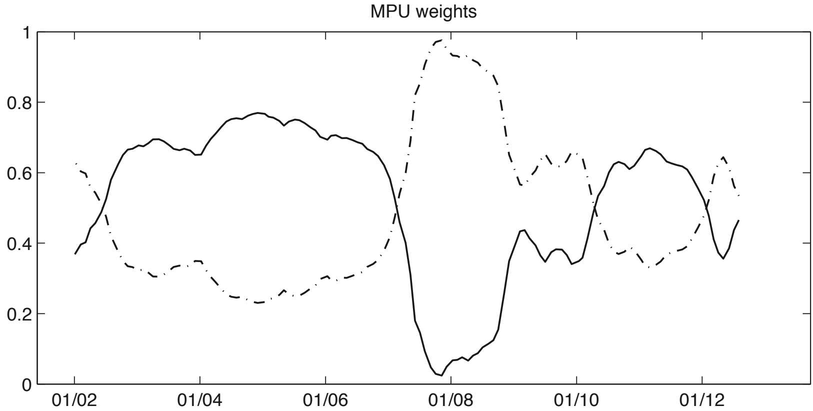

Figure 5.4.: Markov-switch analysis of ECB uncertainty indicator (ECBU)

Transition probabilities			Exp. duration of regimes	
	s=1	s=2		
s=1	0.97	0.06	s=1	31.42
s=2	0.03	0.94	s=2	17.68

Table 5.2.: Switching matrix and expected duration of regimes

variance of the error term, hence, both states exhibit the same mean but show a different variance.[9] However, as both series offer a different variance, state 1: 0.48, and state 2: 1.84, it is still an open question whether the discrimination is a reasonable one. Therefore a closer look has to be taken at the timing of the two states.

Figure 5.4 allows the following rough discrimination of regimes for the period 2002-01 until 2012-08

From	Until	Regime
05/2002	04/2007	1
05/2007	03/2010	2
04/2010	02/2012	1
03/2012	end of sample	2

Table 5.3.: Discrimination of regimes

9 This is of course plausible, due to the fact that standardized values have been used

This discrimination of Table 5.3 fits well into the above argumentation. I call regime 1 (solid line) to be the 'normal' on. This regimes prevails under a rather calm or stable environment. Therefore the probability of this regime is high, roughly until spring 2007. At this point in time, the Markov analysis gives a probability switch, turning away from state 1 into state 2 (dashed line). From then on, the probability of regime 2 rises and has reached its maximum at the end of the year 2007, which is the initiation of the financial crisis. That is, the Markov analysis indicates a regime switch in the communication of the ECB which starts around 2006/2007 and lasts until the beginning of 2010. This regime 2 is said to be the 'uncertainty' regime, which prevails under times of stress and turmoil. One can see, already at the end of 2008 there seems to be a tendency away from regime 2 to regime 1. However this development is stopped by the Lehman bankruptcy, which stretches the uncertainty period until the year 2010.

In 2010, we observe the reintroduction of the normal environment indicated by a probability switch of state 1 and 2, which lasts until the beginning of 2012. This time span is also accomplished by a good development of the stock market, most notably the German DAX. The rest of 2012 is somewhat in between the regimes 1 and 2.

The special character of the period between the years 2007 and 2010 is also reinforced by other works. Brissimis et al. (2012) aim to identify periods, in which inflation expectations do not follow a rational expectation hypothesis. For this reason they employ a Markov-switch analysis as well. As a result, they identify the same above mentioned period, for which they state a non-rational behavior. According to Brissimis et al. (2012) the characteristic of a non-rational environment is a persistent forecast error. They reason that extraordinary measures of the ECB have been useful to restore the long-run relationship between inflation and inflation expectations. However, they do not take into account a change in the communication behavior of the ECB.

Concluding briefly my empirical findings, the Markov analysis combined with the descriptive analysis delivers a unanimous picture. The Markov discrimination is based on times of a different variance depending on the prevailing regime. In times of stress the ECBU shows a higher variance compared to normal periods. In other words, the ECB policy is more pronounced during times of stress. This confirms the picture elaborated in the previous section. It is shown in Figure 5.2 that the communication before 2007 was marked by a rather 'random' communication of the ECB with no specific pronunciation. The pictures changes after the year 2007, where it seems like that the ECB becomes fully aware of her communication instrument, and thus, communicates in times of stress in a more accentuated way then in normal times resulting in a higher variance.

5.6. Summary of Chapter 5

This chapter aimed for an assessment of central bank communication, by focusing on signal words in the press conference statements of the ECB President, over the last 10 years. The importance of proper communication has been discussed intensively during the last years and has been accomplished by many central banks opening themselves to the public in terms of transparency and communication.

Until today, when evaluating uncertainty empirically, primarily two approaches have been considered in the discussion. The first of them covers market perceptions, evaluated either by a stock market indicator like the Vstoxx, or by survey measures like the the Survey of Professional Forecasters. The second approach exploits central bank minutes. This approach hinges strongly on the properties of these minutes, i.e., how much is published and when it is published.

My analysis, however, differs from both approaches. Whilst, survey data, stock market data, or newspapers always capture the market perception of uncertainty, minutes provide an assessment of the uncertainty within the central bank committee. The indicator presented in this work, however, expresses the *willingness* of the central bank to communicate uncertainty to the public, by explicitly making use of uncertainty related words. Hence, neither the uncertainty of the market, nor the uncertainty of the central bank is examined. The central bank is free to choose, when, and to what extent, she wants to communicate uncertainty to the market. While this might be neglectable under normal times, it can be decisive under times of economic turmoil such as the recent financial crisis, due to the fact that communication can become a powerful tool to reduce market uncertainty.

Indeed, the analysis shows that measures which capture market expectations show a very similar picture independent of their origin. Actually for most of the sample period they are nearly congruent. This picture changes if these measure are compared with the index of the communicated ECB uncertainty, ECBU. In general, the stance of communicated uncertainty on behalf of the ECB follows closely the market measure Vstoxx. However, the communicated uncertainty reacts earlier, and is significantly reduced during major shocks, such as the Lehman bankruptcy in late 2008. I reason this to an appeasement of the market on behalf of the ECB, in order to bring down overall uncertainty. The same pattern can be observed for the ongoing Euro crisis, where communicated uncertainty is most of the time even lower then under the comparatively stable mid 2000s. Yet, it is not clear whether which index takes a leading or following position.

With the help of a Markov-switch analysis, I identify two different communication regimes, which I label ‘normal’ and ‘uncertainty’ regime. The Markov analysis fits well into the descriptive assessment. Under turbulent times, the second – uncertainty – regime prevails, which exhibits a higher variance with respect to the occurrence of uncertainty related words.

In fact, my analysis highlights the importance of transparency and especially communication of central banks during times of stress. It fosters the assumption whereby communication is a powerful tool to reduce uncertainty, and thus, permits a more accurate and calculable adoption of traditional monetary instruments.

Conclusion and Outlook

In fact, uncertainty is not only an important feature of monetary policy, but “it is the defining characteristic” (Greenspan 2003). Yet, whilst the general importance of uncertainty has not changed over the past decades, the assessment of uncertainty has changed very well. The aim of my work is a threefold consideration of monetary policy under uncertainty. This separation follows the changes, which have occurred during the last years, both, with respect to the notion of uncertainty, and with respect to the self-conception of monetary policy. The three parts of my work reflect this change and the different perceptions, as each part covers one major aspect.

Early publications on uncertainty in economics made a clear distinction between uncertainty on the one hand, and probability or risk on the other hand. Accordingly, uncertainty is a defining characteristic in the process of decision making and it is given a pronounced role. Moreover, uncertainty is the driving force of several, sometimes conflicting, peculiarities, like the genesis of profits in the writings of Knight or the trigger for a phase of stagnation according to Keynes.

Yet, despite this special character of uncertainty, a first change emerged, when uncertainty analysis was superseded by risk analysis. What might look like a negligible confusion, actually changes the picture entirely. Risk can be insured, and thus, theoretically it can be turned into a fix cost factor.[1] Though, thereby all the defining implications of uncertainty simply vanish. Nevertheless, the common linguistic usage of the term uncertainty, when actually risk is supposed, has survived this change. Consequently it is not surprising that modern macro models claim to act under ‘uncertainty’, but actually make use of well defined probability distributions. Given this change in the perception of uncertainty, the last century has been marked by seeking optimal policy responses in the presence of ‘uncertainty’. The understanding of these issues and the deduced recommendations have found its way into the theoretical as well as into the practical realm of monetary policy design and have indeed shaped the landscape significantly. Moreover, this strand of literature has shown that ignoring these aspects discredits any serious policy assessment.

[1] This holds at least to some extent, due to the fact that systemic or systematic risk can not be insured.

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However, during the last decades, a second change occurred. This time the change affected the strategy of monetary policy. Due to several reasons, the monetary policy stance in general changed from a passive, that is a more or less administrative role of monetary policy, into a rather active position. The management of expectations became a crucial aspect in the conduct of monetary policy. This change was accomplished by several developments, like a higher transparency on behalf of the central bank. Consequently, the treatment of uncertainty has undergone this change as well. Central banks shifted away form merely accepting and coping of uncertainty, towards an active assessment and containment of uncertainty.

To mirror these different developments, in Part I I focus on a detailed analysis of the actual meaning of uncertainty. This is important due to the fact that several versions and interpretations of the 'true' meaning of uncertainty exist. Moreover, a common linguistic usage has been established, which often even equalizes uncertainty with risk. To recognize what is meant by uncertainty, the understanding of the philosophical background of probability theory is an essential building block and a necessary precondition for a thorough assessment of the entire scope of uncertainty.

Therefore, I investigate the origins of probability and decision theory in economic literature. A suitable distinction between risk and uncertainty is deduced from two different concepts of probability, namely the aleatory concept and the epistemic concept of probability. In economics, these positions can be ascribed to the perceptions of Knight and Keynes. In Keynes' epistemic understanding of probability, statements are a property of knowledge. Yet, in the aleatory framework of Knight, probability is a property of the external reality. My work shows that both authors, despite their differences in probability theory, share the same attitude towards the perception of uncertainty, which is also shared by other differentiations like those of Davidson or Shackle. Accordingly, uncertainty prevails if no probability statement can be made. Reasons for such situations are foremost the impossibility of measurement and the fact that some situations are not even considered to be contingent. The change from uncertainty to a risk treatment can be ascribed to several developments, culminating in the rational expectation literature. Despite its fundamental differences, it has become generally accepted in modern macroeconomics, due mainly to practical and especially computational reasons.

Part II provides a comprehensive and structured treatment of various different tracts on monetary policy and uncertainty.[2] This systematization is backed with theoretical and empirical examples to give an as focused picture as possible of the effects of uncertainty and the recommended remedies.

[2] I have adopted the common linguistic usage of uncertainty which is actually risk.

The assessment of uncertainty with all its specific forms is of special importance. Depending on the source and form of uncertainty, different policy recommendations must be given to the monetary authority. My categorization of uncertainty distinguishes between additive, parametric, data, and model uncertainty. I show that under additive uncertainty, *certainty equivalence* prevails, i.e., the central bank conducts her policy as if everything is known with certainty, ignoring the possibility of shocks. Although additive uncertainty might be policy neutral, this is accompanied by a non-neutrality with respect to utility. Under parametric uncertainty, the decision maker is faced with a trade-off situation. On the one hand, she tries to reduce the deviation from the target variable. Yet, pursuing this minimization, she must on the other hand consider the effect she exerts on the variance of the target variable. Under parametric uncertainty, certainty equivalence ceases to hold and the optimal reaction magnitude must be lower than under certainty. This less aggressive policy stance in the presence of parametric uncertainty – the so-called Brainard conservatism – still seems to be the most valuable guideline in the conduct of monetary policy. Moreover, this advice is also suitable if uncertainty with respect to the estimation of key variables is considered, due to the fact that imprecise estimates might cause an inadvertent reaction to the data noise. An attenuated reaction can mitigate this problem.

The New Keynesian framework likewise highlights the basic findings and offers further insights, e.g., on the rational expectation equilibrium path under uncertainty. I show how several forms of uncertainty can be represented in a compact form, by just incorporating one additional term. This leads me to model uncertainty. Two distinct possibilities of how to treat model uncertainty are discussed, namely robust control and a Markov-switch analysis. Both methods are used to tackle the equilibrium paths of the target and instrument variables in order to compare them with the optimality conditions of the certainty and the parametric uncertainty case. The analysis shows that despite an often made equalization of parametric uncertainty and model uncertainty, the policy recommendations can differ significantly.

Whereas Part II shows the impact of uncertainty on monetary policy decisions, Part III considers, how monetary policy can influence uncertainty. Hence, the direction of investigation is almost contrary. To show, how this *active* assessment and further on containment can be done, I firstly rely on traditional measures to evaluate patterns and commonalities of uncertainty in Chapter 4. Thereafter, in Chapter 5 I combine the findings of the transparency debate in central banking with the aspect of an active fighting of uncertainty via a more pronounced communication policy.

Chapter 4 investigates uncertainty measures in the euro area and the US. These measures depict the uncertainty of market participants on future developments. In the search for common forces, a factor analysis reveals that uncertainty about the future development of key macro variables follows distinct forces, depending on the time aspect as well as the kind of variables taken into account. The main finding is that in fact relations between the uncertainty of different variables exist. For the European data set, uncertainty can be separated into forces which drive long or short-term uncertainty. This holds especially for inflation expectations. For the US, the data is mixed, however, the results suggest a 'classical' distinction between real and nominal variables. A combined analysis of both regions delivers three distinct factors that drive uncertainty. A European factor, a US inflation factor, and a factor which loads US real variables as well as the stock markets in both regions.

The contribution of Chapter 5 is the introduction of a newly created uncertainty indicator. This indicator stands in stark contrast to other already existing indicators, which are utilized in literature so far. Opposed to these established measures, I evaluate the *communicated* uncertainty of the European Central Bank (ECB) and thereby add a new view to the discussion of communication and transparency in central banking. With the help of a sentiment-analysis, I show that the ECB has significantly changed her communication pattern since the beginning of the financial crisis, especially in the years following 2008. The ECB makes use of the power of communication and by that appeases the uncertainty of the market participants. The importance of this field of research becomes particularly apparent when traditional instruments of monetary policy lose their effectiveness, which might be the case when the interest rate is at its zero lower bound, and thus can hardly be used for a further stimulation of the economy. Moreover, communication can serve as a valuable tool to achieve an orderly functioning of the monetary transmission channel, by reducing market uncertainty.

Especially the last two chapters offer room for future research. Within the field of Chapter 4, future work should initially concentrate on the extension of the data basis to enhance the robustness of the analysis. Possible extensions are further uncertainty measures, the inclusion of other countries or regions, and the expansion of the investigation period. In addition, further analysis methods could employed to foster my findings. However, all of these issues come along with significant obstacles. The most obvious example is the time aspect, which is difficult to extend at once.

The extension of the sentiment analysis of Chapter 5 is probably the most fruitful avenue for future research. The prevailing analysis could be backed with the help of other dictionaries or an own 'central bank policy bag of words', which could even cover other aspects of monetary policy communication as well. To foster the results concerning the ECB an application to other central banks, if possible, would be of interest. Another suitable direction could be the extension of the sentiment analysis on further forms of publications. As this would definitely change the original idea of my analysis, it could on the other hand deliver the possibility of an overall picture of the tone of official publications concerning uncertainty. Overall it should be possible, to create an index in the spirit of Baker et al. (2013), but with a focus particularly on the communication of central banks. Furthermore publications of the national central banks of the euro area could be included into the analysis, foremost the Bundesbank. With this setup it could be investigated, whether national central banks and the ECB share the same tone in their communication policy. This could be of special importance during the most recent turmoil, where especially the Bundesbank has shown her different point of view on several ECB decisions. Thus the possible discord within the ECB council could be approximated by the tone of the publications.

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I was a writer, can't write another book.