Heinrich von Stackelberg

Market Structure and Equilibrium

Translated by Damien Bazin (Scientific Director), Lynn Urch and Rowland Hill

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To my wife, Isabelle, with love.

About the Translators

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Originally from Northern Ireland, **Rowland Hill** is an economics graduate who has managed a small translation and training agency on the west coast of France since 2002. Before that he was head teacher of a special school in London.

Comments On The Translation

The idea of translating the work of German economist Heinrich Freiherr von Stackelberg (1905–1946) came to me as I was teaching international economics and economic game theory in a university context. My students constantly confound the concepts of monopoly, oligopoly (simple, bilateral or contested) and oligopsony, but also the concept of perfect competition. This inability to classify the different markets reappeared when I was teaching abroad. Rather than cast doubt on the aptitudes of my students to understand, I chose to examine my own approach as a teacher. On asking my learned colleagues if they too had experienced this phenomenon in their teaching, it came as no surprise to me when they responded that unfortunately Stackelberg's famous market classification table was only memorised by their students for a limited period. It is true that our memories are selective and that something learned out of context is not internalised and indeed often quickly forgotten. If we support Nicolas Boileau's view then, "Whatever we conceive well we express clearly, and words flow with ease" (an extract from *Art poétique*). This quote contains a degree of truth. My students did not have the means to assimilate how and why the problems associated with competition and the effective organisation of markets were of key importance and remain vital to this day.

When we undertook to translate *Marktform und Gleichgewicht* my first goal was to make the founding concepts of Heinrich Freiherr von Stackelberg more accessible. Such an access would have to stem from the contextualisation of the author's thinking as this would enable us to find the paths he followed and the intrinsic reasons that motivated him to present his own particular view of modern economics. In a more specific sense, his vision of the world in relation to what influences competition was refined by way of dyopoly (duopoly) models. Stackelberg both revisits the work of the French mathematician-economists Antoine Augustin Cournot and Joseph Louis François Bertrand and provides a perspective on the subject of conjectural interdependence (situational issues involving company leaders and satellite companies).

If the intention of this translation project is to highlight one of the cornerstone pieces of work in Economic Science, it has to be added that this book would not have seen the light of day but for the skill and enthusiasm of other professionals who from the outset were willing to participate in the project. I would like to acknowledge the work of Lynn Urch and Rowland Hill who collaborated fully on the translation of the book. Indeed, the scale of the work, its conceptual richness, the use of very specific and technical terms, but in addition the difficulty in interpretation led us to work together as an academic team throughout. Their valuable help with the German and the English has more than amply contributed to the success and to the fullness of this translation, which could not have been achieved without regular exchange, suggestions and necessary improvements. I would also like to thank Barbara Fess, Dr. Werner A Müller and Dr. Martina Bihn from *Springer Publishing Company* for their confidence. Naturally, as a science researcher I had to make both scientific and linguistic decisions in order to preserve the wholeness of the book; I take full responsibility for these choices. By deciding to translate this pivotal text from German to English and hence make it available to many more readers, we hope in some small way to be able to offer a new insight.

Nevertheless, the reader needs to be aware of the risks involved in translating any work. To translate is to unequivocally accept the likelihood of subjectivity and hence the risk of a subjective interpretation of the author's words, even if all attempts have been made to remain as faithful as possible to the original text. We aimed to "stay true" to the German on every occasion and where lexical choices however did have to be made they were made in order to facilitate the ease of access to the book by the reader. In a spirit of candidness we wish to make this clear right from the start.

Dr. Damien Bazin

Translators' Commentary

We have assumed that the target audience of this English translation will include international academics and students whose mother tongue may not be English. Therefore, on occasion we have opted for a lower register lexical choice to try to optimise the accessibility of this translation e.g. "*gave up* his own market dominance 'rather than' renounced" pp. 18. Furthermore, where Stackelberg has used synonyms for stylistic reasons, we have intentionally repeated a lexical noun choice in our translation firstly to avoid confusion ("circumstance(s)" (Umständen; Tatbestände), see pp. 9) and secondly as a reaction to the conceptual richness of the work, especially in the Mathematical Appendix e.g. (Voraussetzung pp. 6; Prämisse pp. 2: "assumption"). The verb "sich richten" is used frequently by Stackelberg in Chapter 2 (e.g. pp. 16) and although Leontief (pp. 555) translates this as "to follow", we have decided to use the literal translation, "to orientate oneself". It is the translators' subjective decision to stay closer to the text than Leontief. The decision has also been taken to split some lengthy paragraphs appearing in the original text to increase accessibility to the modern reader.

It has been possible to maintain a high degree of lexical consistency and thus cohesion throughout the book by using a computerised translation memory which prompts target text selections through recognition of individual words and phrases chosen earlier in the text by the translator, and even manages multiple meanings. As early as 1934, *Marktform und Gleichgewicht* was reviewed in English and a corpus of these and other relevant publications was constructed as a primary source of terminology for translation. These lexical choices have been clearly cited in the text as bracketed translator's notes citing the source text and page number, see pp. 2. Where possible, 1930s technical terminology has been used even if this has been superseded by later additions. As the first of these reviewers, Leontief has been given precedence over the others and his terms have been used even in instances where an alternative spelling has become common nowadays, for example, a translator might have decided on "duopoly", but Leontief (pp. 555) states that Stackelberg uses "dyopoly" in English, see pp. 16. The internet has been selectively used as a corpus of material to supplement other valuable online resources, notably

the Oxford English Dictionary and the British National Corpus (100 million word collection of samples of spoken and written English), especially to check collocation.

Frustratingly, despite the extensive online resources available now, the translators have been unable to entirely confirm the translation of certain terms, such as "directional tangent vector" (Richtungstangenz) pp. 121, "quantity independence point" (Mengenunabhängigkeitspunkt) and "price independence point" (Preisabhängigkeitspunkt) pp. 58, "comparison combination" (Vergleichskombination) pp. 54. as well as "with the highest index" (mit dem absolute höchsten Index) pp. 42. Terms that were queried and the translations confirmed include "marginally" (auf dem Rande) pp. 42 and "combinations" (Kombinationen) pp. 44. Examples of those that were supplied by other members of the translation team include "non-negative part" (dem nicht negativen Teil) pp. 42 and "sequence of reactions" (Reaktionsreihe) pp. 59.

In the course of such online searches, it was interesting to note that Stackelberg's book is cited in very recent publications, the economic crisis of 2007 in some ways mirroring that of the 1930s when the author was writing. Aside from his ideas, the richness of Stackelberg's language is also impressive. He was a polyglot, able to communicate in multiple European languages which he did as a lecturer in various universities across Europe. He clearly expected his reader to read many languages too, as evidenced by his inclusion of quotations in this book, some lengthy, in Greek (pp. 2), French (pp. 65), Italian (pp. 71; 72), Danish (pp. 77) and English (pp. 80), and by the fact that he did not offer a single translation of any foreign title, quotation, etc. into German.

In order to help guide the reader in his or her bibliographical research we took the decision to provide the title of a foreign work in English where this title has already been translated. These translations, in italic, are located in square brackets immediately after the original title (only included after the first appearance of the title).

It has been a pleasure and a privilege to be involved in the translation of such a work.

Damien Bazin, Lynn Urch and Rowland Hill

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Chapter 1
Stating the Economic Problem and Basic Principles

§ 1. Stating the Economic Problem

Classical theory – which also includes classical doctrines from both the marginal utility and in particular the mathematical schools of economic thought – has primarily focused on a market structure characterised by a large number of independent economic agents, or market participants, competing with one another. Here price is rightly regarded as a variable, independent of the behaviour of the individual. It appears that a well-defined equilibrium under this market can be derived from the given premise that such a market is therefore a functioning mechanism in the whole structure of the economy. In addition, the problem of monopoly in its narrowest sense has also been thoroughly investigated in some detail. It appears that when the supply sides or the demand sides are combined in the hand of a single economic agent, a similarly well-defined but different equilibrium occurs.

However, the clash of opinions concerning treatment of the hybrid form has not yet been resolved. There is also no systematic investigation of the problem that encompasses all conceivable market structures and analyses the relationships of mutually interdependent markets. In the following work we will seek to establish the principles to fill in these gaps. To begin with we will immediately determine the market structures themselves. We therefore do not intend to examine how the various market structures can come about due to specific development trends that have a dynamic impact on the economy. The results of our study always arise whenever and wherever specific market structures occur, for whatever reason. Thus preliminary work is carried out in order to review these remodelling tendencies. We will be able to directly extract from this study the final link in a logical chain of conclusions reached for a particular market structure.

H. von Stackelberg, *Market Structure and Equilibrium*,
DOI 10.1007/978-3-642-12586-7_1,
Translation from the German language edition: "Marktform und Gleichgewicht"

§ 2. Systems of Market Structures

Of the various criteria which enable market structures to be distinguished from one another the only two of interest here are the number of supplying and demanding economic agents and the size of volume turnover achieved by the individual. These are the only two important factors in the question of the economic law of motion and the functioning of the market mechanism, as will become more obvious in the following pages. We will initially focus on the number of economic agents. At first glance this criterion may lead to a great many possible combinations since each side of the market can have any number of economic units. However it will be shown in the course of the study that the structures of each side of the market can be summarised into three typical categories covering all situations: where one side of the market consists of only one economic agent so this is a monopoly, where it consists of very few economic units this is hence an oligopoly[1)] and where very many economic agents are present, then "free competition" predominates[2)]. Neither of these latter categories differs from the other by any definite distinction. However, in their consequences they present two fundamentally differing cases. How they are transformed from one into the other will become apparent in the course of our study.

Since both sides of the market can exhibit each of the three categories as defined above we can thus say that there are nine typical market structures in total. Each of these market structures must be described correctly.

For this purpose let us next state that if nothing is stipulated about one side of the market, we should assume a structure of free competition. Where monopoly is present on both sides of the market, this market structure is thus referred to as a

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[1)] By using this word we endorse Chamberlin's terminology (Edward Chamberlin, *The Theory of Monopolistic Competition Cambridge* [Mass.], 1933). No particular term has yet emerged for this instance. "Monopolistic competition" is not a very fortunate linguistic term. "Restricted competition" (translator's note: ('Beschränkte Konkurrenz') see Pribram: 445–446 in Senn, *History of Economic Ideas*: 15) is too general. The term "polipoly" (translator's note: (Polypol), see Maks and Haan: 47) would cover the situation almost as well as the term "oligopoly" and has been used by U. Ricci however, who first coined it (*Dal protezionismo al sindicalismo*, Bari, 1926: 131) for a different instance. By "polipolio" Ricci understood the state in which each branch of production or at least each species of a class of goods is monopolised – and so the case which Schneider describes as a "universal monopoly" (Schneider, *Reine Theorie monopolistischer Wirtschaftsformen*, Tübingen, 1932: 83).

[2)] The terms "monopoly" and "oligopoly" really occur linguistically only on the supply side since "-pol" is derived from $\pi\omega\lambda\varepsilon\tilde{\imath}\nu$ = "to sell". Since "competition" also exists with oligopoly we should thus describe the third case with "polypoly". Correspondingly for the demand side we can coin the terms "monoon", "oligoon", "polyon" (from $\grave{\omega}\nu\varepsilon\tilde{\imath}\sigma\tau\alpha\iota$ = "to buy", $\acute{o}$ $\tilde{\omega}\nu\circ\varsigma$ = "the purchase"). The term "polypoly" is no longer available however (see footnote 1). Moreover it is not appropriate to burden an already complicated account with new terms where that can be avoided. Translator's note: English terminology "free competition" (freie Konkurrenz) taken from Leontief: 555.

"bilateral monopoly"[1)] and the term "bilateral oligopoly" similarly emerges (translator's note: where relevant, the English terminology used in this translation follows that of the review entitled "Heinrich von Stackelberg's Marktform und Gleichgewicht", *Journal of Economic Studies* 23, 5/6, 1996, 58–70, Düsseldorf, 1993 by F. M. Scherer of Harvard University on pp. 65). Where both sides occur in "free competition" this will thus be referred to from now on quite simply as "free competition". Where a "demand oligopoly" is confronted with a "monopoly", a "reduced monopoly" is present. Correspondingly the term "reduced monopsony" emerges. Thus we arrive at the following overview of the system:

Supply-side / Demand-side structure	Free competition	Oligopoly	Monopoly
Free competition	Free competition	Oligopoly	Monopoly
Oligopoly	Demand oligopoly	Bilateral oligopoly	Reduced monopoly
Monopoly	Monopsony	Reduced monopsony	Bilateral monopoly

Throughout the following study we will initially assume, provided nothing else arises in connection therewith, that economic agents on one side of the market do not essentially differ from one another in terms of "assets" or "variables".

§ 3. Elements of the Economic Model

Goods, natural individuals[2)] and companies are the elements of the theoretical economic model being applied to the case in hand.

1. The term "goods" requires only a few additional remarks. This term is interpreted very broadly and includes all consumer and producer goods, including services from people and things. Here we can avoid the difficulty that arises by the introduction of capital as an abstract calculable variable. We will assume that the individual economic agents are not owners of amounts of purchasing power, but are owners of particular capital goods. Therefore in our economic model the individual company should not take up capital in the form of purchasing power but instead borrow the means of production needed directly from the owners[3)] of these means.

Similarly, the repayment obligation should not refer to a particular sum of money but should in substance mean reimbursement of the means of production

[1)]See Pigou, "Equilibrium Under Bilateral Monopoly" in: *The Economic Journal*, Vol.18 (1908: 205 et seq.). Schneider, Reine Theorie, ibid.: 59.

[2)]We could substitute the term "natural individual" for "family".

[3)]For the most part, we have Walras to thank for this useful simplification. See Walras, "Eléments d'économie politique pure" [*Elements of Pure Economics*], Paris, 1926, Leçons 17 and 18.

in question "in equal goods and quantity"[1)]. Interest then appears simply as the price of borrowing a particular means of production.

We must be careful to observe the conceptual distinction between the goods themselves and their service. Thus for example the automobile is not merely a good but a service too. Each of these goods has its own particular market which fundamentally differentiates it from another. Two types of goods respectively are therefore available, each specially appearing in a detailed and numbered register of all goods in a social economy and each with its own unique number.

2. Each natural individual has a purpose – this will not be examined in great detail here – that they can achieve with the help of goods. From this purpose comes a demand for goods. The particular form of this demand is described in the greatest detail by the Pareto system of indifference curves[2)]. Since we can surely assume that demand is simultaneously comprised of several types of goods and is independent of any ranking in the actions of the consumer, a system for indifference diversity thus arises that allows the construction of an "ordinal utility index function" in the Pareto sense. Each natural individual is entirely defined within his requirement levels by his "index function". Furthermore, Pareto's investigations allow the more common term "utility" (translator's note: (Nutzen) see Scherer: 60) to replace "index function", thereby avoiding any inferences from this term that cannot also be drawn from the precise term "ordinal utility index function".

3. Production, which is understood to mean the transformation of goods into other goods, forms a significant part of the economy. The term transformation is interpreted very broadly here and describes changes in goods according to the range, time and structure. We will describe an economic agent which performs the task of production as a company. Every company has certain opportunities to transform a random "assortment" of different goods into other goods.

All these possibilities can be expressed mathematically by a production function. In the following study we will mainly focus on the issue of simple production and ignore that of production. This is therefore also justifiable because it lets us see that the basic problems of this production can be reduced to simple production[3)]. So for every good and every company, a production function is established that allocates that quantity of the product to notional variable quantities of the means of production that can be technically manufactured in the most rational way[4)].

[1)]In terms of "technical progress" we can admit that repayment of creditors' demands in a different means of production occurs where their procurement appears to the debtor to be the formal equivalent of the outstanding return obligation.

[2)]V. Pareto, *Manuel d'économie politique* [*Manual of Political Economy*], 2^{ième} éd. Paris, 1927, chap. III, No. 29–38, and appendix, No. 1 et seq.

[3)]v. Stackelberg, *Grundlagen einer Reinen Kostentheorie*, Vienna, 1932, chap.3.

[4)]See e.g. the formulation of the production function by Erich Schneider, "Zur Interpretation von Kostenkurven", *Die Archiv für Sozialwissenschaft und Sozialpolitik*, Bd. 65 (1931: 272, formula (3)).

§ 4. Basic Principles of the Free Market Capitalist Economy

The following chapters examine the extent to which typical market structures are compatible with the basic principles of the free market capitalist economy and – insofar as there is proof of compatibility – which economic laws of motion are valid in this context for individual market structures.

The basic principles which are arrived at here are:

1. The principle of maximum utility. This applies for natural individuals (consumers) and implies that the individual person is anxious to make his "utility", or more precisely his "ordinal utility index", as large as possible.

2. The commercial principle. This applies for single units of production, and therefore for companies, and implies that the highest priority of a capitalist company is to obtain maximum net profit. It may be understood as a special form of the principle of maximum utility.

3. The freedom of pricing: prices are set as a result of the behaviour of single individuals in the market.

Both of the first two principles are the "engine" that continually drives the actions of individuals. Together with the third principle they result in a particular rule for the behaviour of each economic agent in every market. This rule governs the economic law of motion for the market in question. To avoid misunderstandings we expressly emphasise here that the term "economic law of motion" is meant in a "static" sense. This will emerge from the argument as a whole. Where such a law of motion leads to an economic equilibrium, then the aforementioned basic principles of the free market economy are compatible with the relevant market structure, but not otherwise.

The relevant market structures, economic elements and basic principles form the socioeconomic model in the following study.

§ 5. The Classical General Equilibrium Model[1)]

1. The complexity of the subject necessitates the use of the technique described by Marshall and referred to as the "isolation method". The initial position forms the classical equilibrium model. Here the freedom of price formation is partly restricted. In particular it is assumed at the outset that no single economic agent is able to influence the market price, but employs this as a constant variable in its calculation. Prices are indeed not regarded as variables determined by external

[1)] See the basic works of the Lausanne School. In addition: Luigi Amoroso, *Critica del sistema capitalista. Corso professato nell'anno academico*, 1931–32, Roma, 1932 (presentation in the simplest possible form without any mathematical symbols).

forces. It is initially assumed that each individual likes to see each price as being independent of his individual behaviour – this might however be adjusted as part of the general equilibrium – and he therefore does not take any measures aimed at changing this price. To use Pareto's words: The behaviour of a single economic agent here belongs to "type I"[1)]. Under these conditions and under other additional assumptions that are in no way self-evident that a constant price limits both the supply and the demand of any individual, as a rule a "general economic equilibrium" emerges which is clearly described in detail by Walras and Pareto.

If the assumption of an independent price can be relaxed then the analysis of individual market structures shows the extent to which this condition now appears as a consequence of the premise of the single markets and also how it is effectively supplanted – in particular in other market structures – by different pricing mechanisms.

2. The individual equilibrium of the natural individual results from the circumstance of the "ordinal utility index function" based on the principle of maximum utility and the requirement to balance revenue and expenditure with each other in respect of the given prices of the individual goods.

This latter requirement, expressed by the so-called "budget equation", selects from every conceivable combination of quantities of goods that the individual in question would demand and supply, all those that achieve a "balance equilibrium" at the given prices. Under these "correctly balanced" combinations the individual selects the one that has the highest "ordinal utility" index based on the principle of maximum utility. Where the goods are divisible arbitrarily – something we can assume approximately and simplistically – it is precisely those that are favourable under the "correctly balanced" combinations that the "weighted marginal utility" evens out. That is, the "ordinal utility element" or "straight line marginal utility" of each of the favourable combinations is proportional to the associated prices. Where prices change in the same proportion, the favourable combination does not. Where prices change in different proportions, other combinations now become "correctly balanced" and the "weighted marginal utility level" is fulfilled for another combination. We therefore see that a particular combination of a partly supplied and partly demanded quantity of goods that is most favourable corresponds to every system of prices that is realised by the natural individual for the relevant price system. The quantities demanded and supplied of the various goods by every individual thus appear to be functions of all prices. These are the so-called individual supply and demand functions first derived in this form by Walras.

[1)]V. Pareto, *Manuel* (ibid.), chap. III, No. 39–51.

The question of how supply and demand of a natural individual for a good is related to the price of this good is generally impossible to answer[1)]. Pareto has shown however[2)] that usually it is possible to calculate that a price increase serves to increase individual supply and decrease individual demand, while a decrease in price produces the opposite effect. Furthermore we can also generally establish this thesis which is all too often seen as self-evident "on a large scale" since a large price increase must at any rate ultimately have an effect that is both prohibitive on demand and on supply.

3. The individual equilibrium of the company relies on the production function and the commercial principle. A certain quantity of the product to be produced can be manufactured with the help of various combinations of the means of production. As soon as prices are given to the means of production there is a means of production combination that gives rise to the lowest costs. In precise terms it is those for which marginal productivity is proportional to the price of the means of production. Their costs are described as the costs of the relevant quantity of the manufactured product. The profit of the company is the difference between the revenue for that product and its costs. Once the price of the product is given there is one quantity of goods out of all the quantities it is possible to manufacture that is the most favourable and maximises profit. It is that for which marginal costs are equal to the price of the product. A quantity of every means of production demanded by the company thus corresponds to each price system and likewise the quantity of product that is supplied by the company. Proportional changes in the price level do not change these quantities. They are therefore only dependent on the price proportions in the same way as the quantities supplied and demanded by the natural individual. For this reason the individual supply and demand functions of the company are also determined. On the assumption that no joint production occurs other than by the natural individual, the validity of the established thesis regarding the dependence of the quantity of goods supplied and demanded by the associated prices can generally be substantiated here.

4. The general equilibrium arises as a result of the individual equilibrium. For every good, an overall quantity demanded in unit time by the individual demand functions and an overall quantity supplied in unit time by the individual supply functions is assigned to each price system. Under all possible price combinations there is one that links the quantity supplied of any good with the quantity demanded of this good up to an indefinite proportionality factor. This price combination is the

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[1)]The evidence that U. Ricci presents in his essay "Può una curva di domanda esser cressente" (*Giornale degli Economisti e Annali di Economia*, 1932) for the proposition that the quantity demanded is a decreasing function of the market price is only valid under particular conditions as e.g. Pareto posited in detail (c.f. the next footnote). Translator's note: v. Stackelberg used *Giornale degli Economisti* to denote the title of this journal when the title is in fact *Giornale degli Economisti e Annali di Economia*. For the purposes of accuracy we have chosen to give the full title of all the journals referred to in this book.

[2)]V. Pareto, *Manuel*, Appendix No. 52–55; in addition ibid., "Economie mathématique" in *Encyclopédie des sciences mathématiques pures et appliquées*, Tome I, arithmétique et algèbre:. Vol. 4: 626 et seq., calcul des probabilités. Théorie des erreurs. Applications diverses (Reference completed by the translators).

price system of the equilibrium. The forces that induce the realisation of this price combination in the economy have been documented. Where the supply of a good is outweighed by the demand, its price falls due to competition. As a result supply decreases and at the same time demand increases. Such a price change brings about displacements in other markets. Here however a corresponding correction occurs through a movement in the associated prices. Since supply and demand of any good is most strongly linked just to the price of this good, the corrective movements of the individual prices ultimately prevail in their markets and the equilibrium state is achieved. The indefinite proportionality factor of the price combination is determined by the creation of money.

Every good has its specific market. The individual markets are related to one another and "uno actu" form our socioeconomic model. Each of these markets can clearly display the market structures defined above to a greater or lesser extent. The study in the following chapters will focus on one of these markets and subject it to the effects of various alternative premises. Subsequently several markets will be analysed for their interrelationship and finally the different market structures will be placed within the general socioeconomic context.

§ 6. The General Notion of Individual Equilibrium in a Market[1)]

According to the isolation method we will now assume that an individual in a particular market is subject to no conditions whatsoever in respect of price and market structure while as before in the other market the classical equilibrium is achieved with constant prices. For this market we will therefore assume a representative market where the individual gives money to receive commodities or gives commodities and receives money. The general rules that arise from the basic principles in respect of the relationship of the individual will now be determined.

1. We initially turn to the natural individual. Let us assume in our example that a particular quantity of goods is obtained in the market and that a specific amount of money is spent. Demand or supply in relation to this individual in the other market is then entirely determined by the considerations set out in the previous paragraph. Every increase in the quantity of goods received improves his situation i.e. increases his "ordinal utility index" or his "utility". Against this the individual incurs a disadvantage by parting with his money as consideration – his "utility" is decreased because every increase in the money spent requires the individual to reduce his other demand or to increase his supply. Mathematically speaking therefore the "utility" of the individual appears as an increasing function of the quantity of goods to be received and as a decreasing function of the quantity of money acquired.

[1)]See here: Pareto, *Manuel* (ibid.), chap. III, § 75 et seq. and Appendix, § 22 et seq.

Where the individual in our example appears in our market as a supplier, corresponding considerations apply except that the amount obtained takes the place of the goods and the quantity of money acquired takes the place of the amount spent. Here therefore the "utility" is an increasing function of the quantity of money and a decreasing function of the quantity of goods.

2. The general market situation of the company is determined accordingly. Where the good in question is a means of production for the company, it is therefore demanded in a specific quantity and for consideration of a specific quantity of money, and supply and demand by the company in the other market is therefore determined for the reasons set out in the previous paragraph whereas the quantity of money only influences the profit from which it is simply deducted. Each increase in the quantity of goods brings with it an opportunity to produce more and so to increase profit[1)]. Every increase in the quantity of money spent reduces the profit by the same amount. Profit under these circumstances is therefore a monotonically increasing function of the quantity of goods and a monotonically decreasing function of the quantity of money.

By contrast, where the company is the supplier of our goods, every increase in the quantity of goods is linked to increased costs, thereby reducing the profit, while the amount obtained represents revenue and its increase thus increases the profit. In this example profit is an increased function of the quantity of money and a decreased function of the quantity of goods.

3. For general analysis the position of the natural individual and the company appear to be completely comparable in form. We can therefore dispense with a general analysis of the difference between the two types of economic agents. The natural individual can also be seen as representative of the company in which the ordinal utility indexes are merely regarded as a net profit in cash[2)].

4. For each economic agent the market situation results in a specific relationship between the sum of money given and the quantity of goods obtained or between the quantity of goods given and the sum of money obtained. In the first case it is because the quantity of goods increases utility and in the second, the sum of money, then the sum of money or the quantity of goods, reduces utility. Thus, in accordance with the "basic principles", the intention will subsequently be at all times to acquire the greatest "yield" of goods or money with the least "expenditure" of money or goods possible. The greatest possible quantity of goods or money obtained corresponds to an indefinite sum of money or quantity of goods given. An inverse functional relationship exists between goods and money.

These functions monotonically increase within their domain i.e. a larger quantity of goods should also be obtained for a larger sum of money or a larger sum of

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[1)]At the very least however the increase in quantity of the means of production may mean there will be no reduction in profit since the company will only use the means of production to the extent that it can increase profit.

[2)]A detailed form of the profit function is to be used for special analysis in the event that the company is the supplier. Profit is the difference between revenue and the total costs that are a function of the quantity of goods delivered.

money should also be obtained for a larger quantity of goods. As these functions become established, it all depends on the market structure and this will form an essential part of the current study. Based on previous results we can however already describe the behaviour of the economic agent in general terms. Each individual strives to use his "utility" to the maximum with respect to the functional dependence as previously characterised. The link that exists in the market for our individual between money and goods – regardless of the particular cause that has led to it – selects from all the conceivable pairs of "sums of money – quantity of goods" those that are actually possible under the given circumstances. Under these circumstances the individual seeks out the favourable pair according to both the first basic principles outlined above. Consequently the behaviour of the individual – his supply and demand – has already been determined by virtue of that general link – the notional availability in every market or at least by the individual – between interchangeable sums of money and quantities of goods.

Where there is a price present in the market that is constant and independent of the behaviour of a single individual, then the sum of money is proportional to the associated quantity of goods, where price is the proportionality coefficient, and thereafter the circumstances outlined in part 5 simply ensue.

Chapter 2
General Analysis of Typical Market Structures

§ 1. Methods

The general question to be answered in this chapter is how individual economic agents behave in a given market structure and what is the overall picture that emerges? This question is quite difficult to answer. We cannot directly deduce either the individual's behaviour or the overall picture from a particular premise. Our critical analysis must pursue an indirect route. This indirect method, as it will be applied here, is new in the field of economic theory research. Therefore, to begin with we ought to describe its basic principles without reference to the individual subject under investigation.

The structure of the classical general equilibrium relies on a restriction of the principle of free price formation. The independence (translator's note: (Unabhängigkeitsposition) Scherer: 64) of price from the behaviour of each individual has been assumed from the first – while in reality the relationship between individual and price only results from the market structure. Where this assumption would indeed have existed from the first, individual market structures would thus not have differed from one another in respect of their price formation and they would all have exhibited the same economic law of motion.

For the market structure we are about to examine, the validity of this restrictive hypothesis is initially assumed for all economic agents, with the exception of a few individuals. The unrestricted validity of the principle of free price formation is re-established however for the economic agent in our example. We then ask how this individual will behave on the strength of the established basic principles of the free market capitalist economy and where there is complete market transparency[1)]. This examination is carried out for each individual in the relevant market.

Depending on the market structures, special considerations are then necessary in circumstances that cannot be expressed here in a general statement.

[1)]I would like to thank my teacher, Professor Erwin von Beckerath, for this useful term.

H. von Stackelberg, *Market Structure and Equilibrium*,
DOI 10.1007/978-3-642-12586-7_2,
Translation from the German language edition: "Marktform und Gleichgewicht"

The purpose of this is either to find a stable price formation in the relevant market or to demonstrate that such a stable price formation cannot exist. The price formation for a particular market structure is stable when it exhibits the following properties:

(a) Where the price formation structure is set as the assumption and the unrestricted freedom of price formation is established for any economic agent, its behaviour is then unchanged.

(b) The behaviour of any individual is not aimed at changing the actual price formation structure.

Where a stable equilibrium is not present, either the price formation structure or the market structure must change in some way and determining this more precisely is the subject of the sixth chapter.

§ 2. Classical Equilibrium by Free Competition

We are initially applying our methods to market structures that fall within an already well-researched area of economics. The key evidence to be provided is that the classical general equilibrium is in fact achieved in the market with free competition, and an independence of prices from the behaviour of each individual is therefore a consequence of the market structure stated above. The hard evidence can of course only be gathered using mathematical techniques and also however allows this critical analysis and its conclusions to be expressed in a very logical fashion.

1. The classical equilibrium has already been described in the first chapter where the independence of prices was accepted as an assumption. We will now abandon this assumption for an individual, i.e. a demander. This demander is therefore faced with a market in which a large number of demanders still compete with him and an equally large number of suppliers make up the other side. We will refer to the other demanders collectively as "the rivals" (translator's note: ("die Konkurrenz"), see Leontief: 555). The independence of price is initially still a requirement for "the rivals" and the supply side. Owing to the known market mechanism, immediately our demander demands a particular quantity of goods, then a specific demand by a rival arises with a specific supply and a specific price on the other side. So price is linked through the market mechanism to the quantity of goods demanded by our demander. That is its function. Once we have determined that function definitively, we also know the link between the quantity of goods demanded by our individual and the sum of money given in exchange since this is simply the product of price and quantity.

2. It can now be shown that the price is effectively almost independent of our individual's demand. We can explain this clearly as follows. Let us assume that our demander demands the same quantity he would demand if he regarded the price as being entirely independent and then let us ask ourselves how the price would change if our demander suddenly demanded nothing at all. At this level of

demand a smaller percentage of total demand would disappear and so the price would have a tendency to decrease by a small percentage too[1)]. An increase in competitive demand linked to the price decrease however counteracts this tendency. These responses to changed market conditions would result in a new equilibrium whereby a price that is only slightly less than the original price would reduce the quantity by a negligible amount. It appears that even the largest change in demand that our demander could make has hardly any effect on the price. Therefore, where the assumption of independent prices is rejected for a demander they will however consider the demander to be independent since this independence is a result of the free competition market structure. Where this assumption is rejected for a second demander, the same conclusion will be reached and so on. The same also applies to supply, such that at the end of this process of logical deduction the independence of prices as an assumption entirely disappears and instead emerges as a consequence of the market structure and the basic principles.

§ 3. Monopsony and Monopoly[2)]

The deliberations in the previous paragraphs have not led to any new findings, but have merely presented the usage of our analytical methods and explained the critical analysis of the exact evidence for notions that incidentally are standard for price formation in a market with an atomised demand side and supply side. Also we will not establish any new territory in these paragraphs but instead test our line of argument on two similarly very established and well-defined market structures, monopsony and monopoly.

1. Classical price formation does not apply to the case of monopsony or monopoly. Once the assumption of independent prices is relaxed, the monopolist changes his behaviour. It is a fact that the monopolist notes the responses of the other side to the price changes, and so for him price is a variable that is dependent on his demand or supply. If he were to take no notice of this dependence, then a classical price formation would simply be achieved. In fact, he takes this dependence into account according to the principle of maximum utility and thus achieves the "monopolistic price formation" (translator's note: (monopolistische

[1)]Chamberlain presents the same evidence (ibid.: 16 et seq.). (*Translator's note: v. Stackelberg is referring here to the American economist, Chamberlin*)

[2)]See Auspitz and Lieben, *Untersuchungen über die Theorie des Preises*, Leipzig, 1889: 361 et seq. Cournot, *Untersuchungen über die mathematischen Grundlagen der Theorie des Reichtums*. German ed., Jena, 1924, Capt. V. A. Marshall, *Principles of Economics*, 8th ed. London, 1925 [*Research into the Mathematical Principles of the Theory of Wealth*], Book V, chap. 14. E. Schneider, *Reine Theorie*, ibid.: 5 et seq. L. Amoroso, "La teoria matematica del monopolio, trattata geometricamente". *Giornale degli Economisti e Annali di Economia*, Ser. 2, Vol. 43 (1911: 207 et seq.).

Preisbildung), see Maks and Hahn: 40) that has been accurately described in detail by Cournot, Edgeworth, Marshall and others.

2. The fact that the monopolist changes his behaviour as soon as the restrictive assumptions for him are reversed, while every other economic agent in that market persists with his behaviour, as shown in the previous paragraphs and according to our method, this leads to the replacement of one classical price formation set as an assumption by another. This other price formation structure can be briefly formulated as follows: the monopolist dominates the market and modifies the price. The economic agents on the other side regard price as a variable independent of their behaviour, meaning that they behave just as they do under free competition. Evidence for this assertion using our method is to be stated and so we initially set the price formation structure described just now as the assumption for the whole market and then re-establish the independence of price formation for every individual economic agent (in order to maintain the assumption for the other economic agents) and investigate whether that kind of market behaves as the assumption states or whether it is possible that maximum utility is achieved in another way. Where its behaviour does not change, the validity of the price formation structure that was initially only proposed as a hypothesis is thus conclusively established. Where the assumption ceases to be valid for the "freed" economic agent, the hypothesis is thus invalid and must be replaced by another.

In our case, validity of the hypothesis for the monopolist is already emerging from the deliberations as outlined at the beginning. By contrast, no single individual is able to influence the price of an atomised side of the market, regardless of whether this is established in the competition mechanism on the demand or the supply side or whether it is set or manipulated by the monopolist. The evidence is quite similar to that cited in the previous paragraphs, only here "elasticity of the other side" ceases to exist – since even this is monopolised.

We must take into consideration that the monopolist can take various steps to dominate the market. He can set the quantity of goods sold and leave the other side to outcompete on price. On the other hand he can however also spontaneously set the price and leave it to the other side to set a particular quantity to sell on the strength of this price. In such cases, the monopolist therefore determines the quantity sold and achieves precisely the price he wants through the response of the other side. In other cases, he sets the price and by the response of the other side he obtains exactly the quantity sold that he wants to obtain[1)]. In both cases, he acts with the intention of achieving the highest ordinal utility index or largest profit possible. Where all other markets exhibit the classical price formation structure, no difference thus exists between the two opportunities of the monopolistic market policy. In both cases, precisely the same quantity is sold and the same price occurs. Things are different however when several markets are organised in

[1)]A. Marshall, *Principles* (ibid.) deals with the monopoly form of equilibrium quantity; Cournot (ibid.) shows the structure of price equilibrium; Maffeo Pantaleoni (*Manuale di economia pura*, English edition: "Pure Economics", London, 1898) contrasts these two structures (pp. 153) of monopolistic market policy.

a monopolistic way. As we will see in the third and fourth chapters, in this case and in certain circumstances, the equilibrium quantity leads to a different outcome to that of the price equilibrium. To simplify the way this is expressed, from now on we will refer to the monopolist's reaction when he sets the quantity as his "quantity adjustment policy" (translator's note: (Mengenpolitik), see Leontief: 555) and when he sets the price, as the "price adjustment policy" (translator's note: (Preispolitik, ibid.)).

As our ultimate conclusion we can state that according to the basic principles of the free market capitalist economy the monopolistic price formation structure newly described results from the market structure of the monopsony or monopoly. Compatibility of these basic principles with the market structures just examined is thus proven.

3. Monopoly theory can be immediately applied when it is a question of a so-called "imperfect" (translator's note: see Leontief: 559) monopoly. This is present when a demander (or supplier) achieves a part of the demand (or supply), a part not to be underestimated, while each of his rivals are also involved to a small extent, such that each of them views the price as an independent variable. The difference between the "perfect" and the "imperfect" monopoly is only slight[1)].

§ 4. Oligopolistic Supply and Demand

1. The three market structures discussed so far have been easily dealt with since no differences in opinion exist regarding their price formation, and their description only had to be given for the sake of completeness and to illustrate the method used here. In the paragraphs that follow, we are entering a sphere that has indeed been much discussed and about which there are still many differences of opinion, as we will see in the fifth chapter. The equivalence that exists between the similarly well-described demand and supply structures may only be discussed here in extenso with regard to a structure, e.g. a supply oligopoly, and to make deductions about the other form by analogy. The complexity of the subject makes it necessary to initially discuss a special kind of oligopoly, namely dyopoly (translator's note: Leontief: 555 says that Stackelberg himself uses "duopoly" spelled with a "y").

2. In the supply dyopoly two suppliers are faced with an atomised demand. The assumption that governs the theoretical starting point is as before that price is seen from the behaviour of all the market participants as an independent variable and thus the classical price formation occurs.

[1)] See Karl Forchheimer: "Theoretisches zum unvollständigen Monopol". *Schmollers Jahrbuch*, Jg. 32 (1908, 1: 1–12).

The next step in the analysis frees the first of the two suppliers of these shackles. It can now be seen that this will change his behaviour since the more the price effectively depends on a supplier's supply, the larger this supplier's share of total supply. The first supplier will thus have a similar, but gradually weaker position[1)] than a monopolist. If we also now reject assumptions of a constant price for the second supplier, the following situation will thus be seen: his rival offers a specific quantity of goods on the strength of a specific market policy observation. Just as when we were considering the monopoly, let us initially retain this observation, even if the real assumptions change. The second supplier will then view his rival's supply as being independent of his own supply. Despite this, he will now change his previous behaviour as the demand side responds noticeably to changes in the supply of the second supplier, provided they deliver a noticeable part of the total supply. The second supplier will to the best of his ability therefore look to exploit the reaction of the demander to his changes in supply as far as the given supply of the first supplier is concerned and consequently achieve a similar position to the monopolist.

Let us return to the first supplier. He will establish that assumptions about his behaviour have changed because his rival for his part pursues a specific market policy, meaning – to use Pareto's terminology – he trades according to "type II". And indeed the first supplier will furthermore determine that his rival always views the supply of the first supplier as a given variable which he has to take into account. Expressed more precisely, the second supplier matches any supply set by his rival with a specific supply level of his own that generates him the biggest profit under the given circumstances. The supply of the second supplier therefore appears as a function of the supply of the first supplier. This first supplier will make use of this fact. In accordance with his rival's dependence, as outlined above a moment ago, he will examine all profit maximisation options and implement the best one. We can say that the first supplier dominates the market, while the second is his follower (translator's note: (Mitläufer) Leontief: 555). It should be very clear from the explanations in the previous paragraphs that the atomised demand side will always consider price to be a variable, independent of its individual behaviour.

The strange thing about the provisional result we have noted is the fact that both suppliers have two completely different positions in the market. Also, it is not difficult to appreciate that you cannot effectively adopt a system of equal rights between two suppliers based on real assumptions by yourself because the behaviour of your rival is also governed by the real assumptions on which an individual bases his behaviour. It can now be shown with the help of logical deduction that the assumption about the behaviour of the rival cannot be the same for every supplier. To simplify our presentation we will refer to the first supplier as A and the second as B. This then leads to the following logical conclusions:

[1)] He has an "imperfect" monopoly here.

A views the behaviour of B either as dependent on or independent of his own behaviour:

1) Assuming that A views the behaviour of B as being independent of his (A's) behaviour, in this case A regards B's supply as a given variable and he orientates himself to it. Thus the behaviour of A is dependent on the behaviour of B. If B therefore trades based on assumptions that match reality he thus sees the behaviour of his rival A as being dependent on his (B's) behaviour.

2) Assuming that A views the behaviour of B as being dependent on his (A's) behaviour, if that matches reality, then it means that B is orientating himself to A's behaviour. That is only possible however if B always views A's behaviour as being a given situation, that is, he views it as being independent of his (B's) behaviour.

The real problem of dyopoly and moreover, of oligopoly, lies in this strange difference between the actual positions of both suppliers. Since each supplier can have each of the two types of position, the price formation structure outlined above is thus imperfect. Each of the two rivals will strive to achieve (Scherer: 64) the most favourable situation. It is possible that the more favourable position is independence. The opposite can however also happen. Namely, the supplier would favour being dependent, that is, being the "follower" because, due to the special price and cost conditions, his profit is even larger if he orientates himself according to his rival and "dominates" this market as if it were the other way around.

In a purely theoretical sense it is not possible to distinguish which of these two scenarios is the more probable. Only empirical research, in particular, statistical analysis of the demand curve, can reveal something conclusive about the greater or lesser probability of these two possibilities. We can however, as is to be explained again in more detail in the fourth chapter, put forward the assumption based on general experience that the favourable position is generally independence. And we can further assume that the question about which position is the more favourable as a rule has the same answer for both dyopolists. These two assumptions are valid whenever the demand function is close to being a straight line.

Thus for the price formation under dyopoly we have three cases:

1) First case: Each of the two dyopolists is striving for "market dominance", (translator's note: (Marktherrschaft) Scherer: 64), that is, a position of independence, because this promises the greatest profit. Then e.g. the first dyopolist will try to convince the second dyopolist that the former's actual supply has to be seen as an independent variable. The first dyopolist can achieve this by no other means than simply "blindly" supplying a particular quantity and indeed precisely the quantity he wants his rival to orientate towards, that is, the quantity of goods the first dyopolist would supply if he already dominated the market and the second dyopolist was his follower. We refer to this quantity as the first dyopolist's "independent supply".

The second dyopolist will also behave in exactly the same way because for him at the outset, there is no reason whatsoever to give way to the first dyopolist. Therefore he will also achieve his "independent supply".

We refer to this situation as the "Bowley dyopoly" because Bowley was the first to describe this scenario[1]. Total supply in the "Bowley dyopoly" is the sum of both "independent supplies". The price formation is, as we can immediately see, not a stable one since the behaviour of the two dyopolists is not oriented around achieving the greatest profit under the "given circumstances" when you consider that for the dyopolist's rival this is the actual supply under a "given circumstance". A reaction to the "given circumstances" is not even possible here. If each dyopolist considered the other's supply as a "given circumstance", then at the same time, he would ignore another "given circumstance", namely the rival's "dependence" associated with such a reaction. The reaction of each dyopolist is much more oriented here around the intention of changing the existing price formation and so this contradicts the second condition that we described earlier for the stable price formation of a true general equilibrium[2]. Just to achieve this change, the dyopolist has accepted the disadvantages of the battle – that is what he is clearly dealing with here – for himself. If he accepted the market dominance of his rival, oriented himself to the reaction of his rival and gave up his own market dominance, he would do better than he would in a "Bowley dyopoly" even if the greater advantage of such a surrender would fall to his rival. It is possible that the "Bowley dyopoly"

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[1]A. L. Bowley, *Mathematical Groundwork of Economics*. Oxford, 1924: 38, Explanations regarding dyopoly, in particular the two reaction functions for X_1 and X_2. We cannot however agree with the final conclusion. The price in the "Bowley dyopoly" needs by no means to be approximately equal to the marginal costs, and therefore lies in the proximity of the competitive price.

Although Bowley, after whom we have named the dyopoly structure, is to our knowledge the first person to have constructed the two definitive equations, no particular originality can be attributed to him because his deduction should essentially be defined by Edgeworth's fundamental publication: "La teoria pura del monopolio", *Giornale degli Economisti e Annali di Economia*, July, 1897, in particular through the publications on pp. 26 et seq. (Published in English: "The Pure Theory of Monopoly", Papers Relating to Political Economy, London, 1925, Vol. 1: lll et seq.: compare pp. 122 et seq.). The facts described by Edgeworth are indeed materially different from those of the "Bowley dyopoly", but formulaically they contain all the necessary information. It would have been possible to transfer the Edgeworth deduction to our case by a simple argument by analogy. This circumstance will be further described at a later stage (see Chaps. 3 and 4).

Henry L. Moore further achieved an important prior publication on the concept of "Bowley dyopoly" in his publication "Paradoxes of Competition", *Quarterly Journal of Economics*, Vol. 20 (1905/6: 211 et seq.).

Kurt Sting presents a clearer account ("Die polypolitische Preisbildung", *Jahrbücher für Nationalökonomie und Statistik*, Vol. 134 (1931,1): 761 et seq.). The situation referred to by us as the "Bowley dyopoly" then arises if the two dyopolists react in a hyper-political way in the sense of a sting.

For futher reading, see my *Grundlagen einer reinen Kostentheorie*, Vienna, see 1932: 87 et seq. and my publication "Sulla teoria del dyopolio e polipolio", in: *Rivista Italiana di Statistica, Economia e Finanza*, June, 1933. In these two publications, I have referred to the dyopoly situation in question as the "Pareto dyopoly". I consider this description today to be inappropriate since the facts assumed by Pareto (*Manuel*, ibid. Appendix No. 69 and 70) are appreciably different to those for the "Bowley dyopoly".

For further reading, see chap. 5.

[2]Chap. 2, § 1.

ultimately becomes market dominance for one of the two dyopolists. It is however not possible to determine in theory which of the two dyopolists will win and it is always possible that eventually the dyopolist who initially gave in will make a new attempt to regain market dominance – so that in the end, the "Bowley dyopoly" re-occurs[1)].

2) Second case: each of two dyopolists wants to be the "follower" because that is best for them respectively. Then, for example, the first dyopolist will try to convince the second dyopolist that the former's actual supply has to be seen as a dependent variable. The first dyopolist is unable to do this in any way other than treating it as though his rival's supply was always a given variable, meaning that he matches his own quantity supplied to each individual quantity set by his rival that would provide him (the first dyopolist) with the greatest profit when his rival's supply remained unchanged. Expressed another way, he thus reacts to each of his rival's supply levels as described above for the "follower".

The second dyopolist will also behave in exactly the same way because for him, from the outset, there is no reason not to give way to the first dyopolist. He will therefore also deliberately play the role of the "follower".

We refer to this kind of dyopoly as a "Cournot dyopoly" because Cournot was the first to clearly and correctly describe this situation[2)], without however explaining the conditions under which it occurs. Here too, total supply occurring as a result of the behaviour of the two dyopolists outlined above is a well-defined variable. Generally there is just one particular supply combination for the two dyopolists where the first dyopolist's supply is exactly the same supply response to the supply of the second and this in turn at the same time is the supply reaction of the second to the first dyopolist's supply. Here again the price formation is not a stable one since neither of the dyopolists are looking to achieve the largest profit "under the given circumstances", but rather a specific modification of these "circumstances". If he were to abandon trying to obtain the most favourable position, he would accept the circumstances of his rival's dependence and achieve his "independent supply". In this way he would be already doing what his rival would like him to do, but would however also be better off than in the "Cournot dyopoly". It is therefore possible that the "Cournot dyopoly" ultimately becomes the "market dominance" of a dyopolist. However it can then suffer a degeneration at any time. We see that neither the "Cournot dyopoly" nor the "Bowley dyopoly" constitutes an equilibrium state because here the second condition of a stable price formation is not fulfilled.

The difference between these two types of dyopoly can be emphasised as follows: two circumstances are fulfilled by the behaviour of a dyopolist. Firstly, the fact that he supplies a particular quantity of goods and secondly, the fact that he reacts in a particular way to a rival supply that he considers to be unalterable. In the "Bowley dyopoly" each dyopolist is oriented around the change possibilities of the rival supply and does not note the actual rival supply itself, whereas in the "Cournot

[1)]A numerical example for a case where the position of independence for every dyopolist is the most favourable one, see Chap. 4., § 6. It is recommended that the reader reads this example.

[2)]Cournot, ibid. chap. VII.

dyopoly" each dyopolist is oriented around the actual rival supply and ignores its change possibilities. A dyopolist can only succeed in obtaining the most favourable position when he disregards one or other of the circumstances and when this is quite deliberate in both cases. We ought finally to mention here again as a particularly noteworthy fact that one trade from all the "given circumstances" is absolutely impossible because both the circumstances that were described a moment ago are "given circumstances" that a dyopolist cannot simultaneously determine his reaction to. In any case, the dyopolist – whether he wants to fight or surrender – must therefore single out one circumstance from all of the given ones that he wishes to orientate himself to[1)].

3) Third case: one dyopolist strives towards a position of independence when the second favours a position of dependence. Here it is in the interest of each dyopolist to simply do what the other would like. A real equilibrium thus occurs since everyone immediately orientates his behaviour to what offers him the greatest profit maximisation and no one has an interest in modifying the actual price formation structure. We refer to this dyopoly scenario as "asymmetric dyopoly" (translator's note: (asymmetrische Dyopol) see Senn: 15). This is also achieved, as shown, when one of the two dyopolists in the "Bowley" or "Cournot" dyopoly gives up the most favourable position. This does not then produce a stable equilibrium, but rather an unstable one, since the "Bowley" or "Cournot dyopoly" can re-establish itself from this again at any time.

From these three scenarios, on the strength of the assumption outlined earlier and which is still to be explained in more detail later on, we can describe the "Bowley dyopoly" as the rule and the "Cournot dyopoly" as the exception and the "asymmetric dyopoly", where it constitutes a stable equilibrium, as an exceptional case. This "exceptional case" can then clearly only occur if the individual relationships, in particular the cost and production functions of the two dyopolists, differ from each other considerably.

3. Analysis that was carried out for the supply dyopoly is now also extended to the general supply dyopoly. Let us assume that the first supplying individual sees his rival's supply as a given variable that he orientates himself towards. He then behaves "monopolistically" (translator's note: (monopolistisch), see Scherer: 68) towards demand so that he manipulates the price "under the given circumstances". The second supplier can now influence the supply of the first supplier since this also depends on the supply of the second supplier.

Similarly however, the second supplier regards the supply of his other rival as given. The third oligopolist also even manipulates the supply of the first and second oligopolist as well as their demand, whereas he himself depends on the remaining supply and orientates himself towards it, and so on. Therefore a continuous chain occurs that ends with the last supplier. This last supplier dominates the whole

[1)]Compare the numerical example in Appendix, IX, 3.

market in the way that was outlined above. We thus obtain a supplier ranking that follows logically from the asymmetry of the rivals' interdependence outlined above and shows all the possible positions.

Each oligopolist can now recognise any of the positions described as being the most favourable for him.

1) As a rule each pursues the position of "last" oligopolist and so supplies exactly the quantity he would supply if he already had this position of market dominance. We refer to this market situation as a "Bowley oligopoly" in the same way as a "Bowley dyopoly".

2) In special circumstances each supplier seeks to obtain the position of first oligopolist. This results in the "Cournot oligopoly" described by e.g. Amoroso[1)] and Schneider[2)]. It is merely an aside, as mentioned above, that the special circumstances should occur less often under oligopoly than under dyopoly.

In general oligopoly theory, many "mixed cases" are mixed dyopoly cases. This occurs when at least two oligopolists aspire to different positions. Here, an asymmetric dyopoly would be a situation where just one oligopolist aspires to each of the possible positions. Such a situation might only become reality by an extraordinary and improbable coincidence. Furthermore we should mention a situation in which all the oligopolists apart from one aspire to the "first" position and thus complete dependence, whereas the most favourable situation is where the oligopolist is dominating his rival's behaviour. Generally however, the "mixed cases" lead to completely chaotic, confused market conditions. The same also applies to the "Bowley oligopoly", where individual oligopolists give up trying to achieve absolute market dominance and aspire to dominate sectors of the market and so to fight for the position of "first" and "last" oligopolist. A systematic review of all possibilities would take us too far from our subject and is also not of interest.

4. The observations that were made about oligopolistic supply can be repeated almost word for word for oligopolistic demand and lead to the same conclusion. This can be stated for oligopoly as follows.

The market structures of demand and supply oligopolies are rarely consistent with the basic principles of the free market capitalist economy, and then only for exceptional cases, that is, in cases where a stable "asymmetrical price formation" occurs. As a general rule, the "Bowley oligopoly" and exceptionally, the "Cournot oligopoly" occur. In fact, both exhibit a well-defined total demand or well-defined total supply, not in a steady state but where the solution inclines towards warfare. However, this war cannot be resolved by the market mechanism. The different mixed oligopoly structures and the "Bowley" or "Cournot" oligopolies are very confused and chaotic. A small number of oligopolists decide not to try to achieve the best market position and aspire instead to the intermediary position.

[1)] L. Amoroso, "La curva statica di offerta", *Giornale degli Economisti e Annali di Economia*, Vol. 70, 1930: 10 et seq.

[2)] E. Schneider, *Reine Theorie* (ibid.): 149 et seq.

5. The more economic entities an oligopolistically organised side of the market has, the closer the picture of the market becomes to the market structure of "free competition" (translator's note: (freie Konkurrenz), see Leontief: 555). This is effectively proven by the "convergence" of the oligopoly with the number of individuals. The market position for each individual is ultimately such that it makes almost no more difference to their profit or utility whether they dominate the market or are a follower or even whether they view price as being independent of their behaviour. The rivalry applies increasingly only to fractional amounts. The discrepancies between price and costs caused by this rivalry become increasingly irrelevant, so that the unstable (translator's note: (gleichgewichtslose) Senn: 16) oligopoly – with the increasing number of rival oligopolists and the decreased size of their individual turnover – becomes the classical general equilibrium of free competition.

§ 5. Bilateral Monopoly[1)]

1. In a bilateral monopoly a demander and supplier are opposites. Where the assumption of independent prices is set for both this culminates in the classical price formation structure. Each of the two market participants always view price as a given variable and match it with a particular demand or a particular supply. The higher the price, the less is demanded or the more is supplied and vice versa. For every quantity of goods sold there is a specific price at which this quantity of goods is demanded (the smaller the price, the greater the quantity of goods) and another specific price at which this quantity of goods is supplied (the higher the price, the greater the quantity of goods). Therefore each quantity of goods is matched by a specific "demand price" and a specific "supply price". Where the assumption of an independent price applies to both market participants, the quantity sold is exactly that which is consistent with their "demand price" and "supply price". We refer to this quantity as the "standard quantity" and the price as the "standard price" (translator's note: (Normalpreis), see Scherer: 68).

2. Where we now reject the assumption of an independent price for the demander, so the suppliers' reactions to price changes will take this into account in exactly the same way as a monopsonist faced with a supply that is economically and competitively determined. He will no longer trade based on his "demand price" but choose the most favourable of the "supply prices" and simply set that, or else demand exactly the quantity that is offered at this supply price. Here, the price and the quantity would be less than the "standard price" and "standard quantity". It also follows that the "demand price" for this quantity sold, that is of course no longer in evidence, is higher than the "standard price", and is only just higher than the "supply

[1)]See F. Y. Edgeworth, *Mathematical Psychics*, London, 1881 (ed. 1932): 16 et seq. A. Marshall, *Principles*, ibid. Appendix F.: "Barter": 791; Mathematical App., Note XII bis: 844 et seq.

price" that occurs in the market. The difference between the notional demand price and the "supply price" achieved is kept by the monopsonist as his monopoly profit. We refer to the market situation described a moment ago as "demand dominance".

3. Where we reject the assumptions of an independent price for the supplier, they do however exist for the demander and this therefore results in a very similar picture. Now the supplier dominates the market; he calculates the demander's responses and determines how they are expressed in the relationship between the quantity sold and the demand price. The quantity of goods sold is likewise smaller than the "standard quantity". The market price that occurs, in this case, the demand price of the quantity sold, is higher than the standard price however and definitely higher than the supply price – that is not the case here – of the quantity of goods sold. The monopolist "pockets" the difference between the demand price achieved and the notional supply price as a monopoly profit. We refer to this market situation as "supply dominance".

4. The market situation described under 2 and 3 is no different to the monopsony or monopoly that were described in § 3. However, an exaggeration of the market dominance of one side or the other is still conceivable here and probably cannot occur if the other side is atomised.

For each demander such a large sum of money is assigned to each quantity of goods demanded that if it were to be spent on this quantity of goods, it would devalue the purchase of this quantity of goods. The quantity of money forfeited decreases their ordinal utility index by the same amount as the purchase of the quantity of goods increases it. Through this correlation, we achieve a specific sum of money as a function of the quantity of goods. We refer to this function as the "demander's equivalence function", that is, the sum of money thus assigned to any quantity of goods as being the "equivalent amount" of the quantity demanded.

Furthermore, such a small sum of money is assigned to each quantity of goods supplied that if the demander merely achieved it, it would exactly represent compensation for the quantity of goods given, that is, the sum of money that the supplier's ordinal utility index exactly increases the amount by and that the giving of the quantity of goods decreases it by. We refer to this sum of money as the "equivalent amount of the quantity supplied" and its resulting function as the "supplier's equivalence function"[1)].

Where the supplier is a company, his "equivalence function" is hence the same as the "total cost function" (translator's note: (Gesamtkostenfunktion), see Leontief: 556).

So where the demander only has the choice either not to purchase anything at all or to purchase a specific given fixed quantity of goods for which he has to pay an amount that is less than the corresponding equivalent amount, he will make a

[1)] These equivalence functions are none other than the cost and utility curves constructed by Auspitz and Lieben, *Untersuchungen über die Theorie des Preises* (pp. 5 et seq. and pp. 8 et seq.).

decision to purchase. Similarly, the supplier will sell if he receives an amount for his quantity supplied that is higher than the corresponding equivalent amount. Where e.g. the demander dominates the market he can present the supplier with the alternative, either not to sell anything or to sell a quantity of goods that the demander determines, multiplied by the price also stipulated by the demander and resulting in an amount that lies just above the supplier's equivalent amount. The demanders will now compare all combinations of quantities of goods and corresponding supplier equivalent amounts with each other and choose the one that is the most favourable, meaning the one that uses the ordinal utility index to the maximum. He will then consistently demand this quantity and therefore pay an amount that lies just above the supplier's corresponding equivalent amount. We refer to this situation as the "supplier's utility".

Accordingly, the supplier can include the demander's "equivalence function" in his calculations and then exploit the demander.

In both cases he will pocket the largest possible profit by dominating part of the market and in both cases the price differs substantially from the standard price, either above or below. By contrast, the quantity sold is always equal to the standard quantity when both are monopolist firms.

Where the other side is atomised, "exploitation" is generally therefore not possible because the monopolist would be compelled to impose a fixed quantity sold on countless of each of his partners.

5. Where we ultimately resolve the assumption of independent prices for both market participants, we immediately see that an automatic equilibrium is impossible[1)] because the favourable position here is in any case one of market dominance[2)] i.e. the definite opportunity to exploit the other side. For standard quantities sold each market participant would perhaps attempt to secure the margin between the demand price and the supply price in the case of less than the standard quantity sold or even the difference between the equivalent amount of the demand and the equivalent amount of the supply. A power struggle would emerge, the outcome of which would have a different result according to the actual state of the market in question but that would have nothing in common with the automatic self-regulation of the market. We thus see that the market structure of a bilateral monopoly is irreconcilable with the basic principles of the free market capitalist economy.

[1)] F. Edgeworth, *Mathematical Psychics* (ibid.), Marshall, *Principles* (ibid.), A. C. Pigou, "Equilibrium under bilateral monopoly", *The Economic Journal*, Vol. 18 (1908): 205 et seq. A. L. Bowley, "Bilateral monopoly". *The Economic Journal*, Vol. 38 (1928): 651 et seq.

[2)] The opposite would only be able to occur where there is a partially increased demand price function or a partially decreased supply price function. Both could be described as impossible.

§ 6. Bilateral Oligopoly, Reduced Monopsony, Reduced Monopoly

Detailed analysis of the market structures dealt with so far allows us to directly refer to the last three types still to be described without our methods having to break new ground.

1. A bilateral oligopoly[1)] occurs when both sides of the market consist of only a few economic agents. Its structure is a hybrid form, something between a single-sided oligopoly on the one hand and a bilateral monopoly on the other. We can already infer from this fact that a specific automatic equilibrium is not possible in a bilateral oligopoly under the basic principles of the free market capitalist economy.

There is an antagonism underlying each of the two sides of the market as described for single-sided oligopolies. Add to that the contradiction, even if only a small one, that is characteristic of a bilateral monopoly. This also destroys the opportunity for the bilateral oligopoly to bring about an equilibrium that would correspond to that of the "asymmetrical oligopoly", since even if oligopolists from each side of the market were to complement each other there would remain a power struggle between the two sides of the market. Indeed it would not be possible for the individual to achieve the market position of "utility" as his aim, but achieving market dominance would however remain the pursuit of any individual market participant. As a rule, an equilibrium would only be conceivable if all the market participants, except one at the most, would consider price to be a variable independent of their individual behaviour. This requirement would however contradict reality, as the description of the single-sided oligopoly shows.

As a consequence we can state that the free market capitalist economy is irreconcilable with the market structure of a bilateral oligopoly.

2. Restricted monopsony and restricted monopoly are hybrid forms, somewhere between bilateral oligopoly and simple monopsony or monopoly, or even between both the latter market structures and bilateral monopoly. Here a monopolist on one side is confronted with an oligopoly on the other side. A true automatic equilibrium – in the strict sense of the word – does not occur here either. However we will not easily be able to say that they are unstable, in the same way as a single-sided oligopoly, a bilateral monopoly or a bilateral oligopoly for instance. The power struggle between the two sides of the market within these market systems will essentially occur due to the superior position of the monopolist. Here however the monopolist is the one with the advantage, so we can calculate that he will dominate the market – something that cannot be said about either a bilateral oligopoly or a bilateral monopoly on either side of the market. To appreciate this we will summarize the different methods of market policy in other market structures. In free competition each market participant had to restrict themselves to a price that was independent of their behaviour to achieve a particular quantity sold. In the case of

[1)] See F. T. Edgeworth, *Mathematical Psychics*, ibid.: 34 et seq.

single-sided oligopolies each oligopolist tried to influence the price, not directly however, but instead similarly by realising a specific quantity sold that was generally imposed on the rival and was dependent in special circumstances on the behaviour of the rival. In the case of a single-sided monopoly either the quantity sold or the price can be autonomously set by the monopolist.

In a bilateral monopoly the battle was eventually fought either by setting a price (translator's note: (Setzung eines Preises) Senn: 16) or even by fixing a price and a quantity sold. The situation was similar in a bilateral oligopoly, even if price setting by one of the market participants only offered a reduced chance of success. In a restricted monopoly, precisely this path pursued by the monopolist could however lead to the desired result since the oligopolist that is faced with price setting by the monopolist would run the risk of being eliminated by a rival in each counter-attack. It is thus to be expected that the oligopolists acknowledge the price set by the monopolist and so will not otherwise behave as the economically competitively organised side in a simple monopoly. The result would be an equilibrium that can be described as the market dominance of the monopolist. Of course this equilibrium is continually threatened with possible counteraction by the oligopolists and the stronger these are the fewer oligopolists oppose the monopoly. Where e.g. the oligopolistic side of the market is a dyopoly, so here too the market dominance of the monopolist is indeed to be expected. However, the instability of the general equilibrium is considerable here.

In conclusion, we can thus state that the free market capitalist economy is consistent with a restricted monopoly to a certain degree. An imperfect equilibrium occurs. The functioning of these market structures is however vulnerable to shocks which are impossible in market structures with a true equilibrium.

Chapter 3
Analysis of the Relationship Between Dual Markets and Multiple Markets

§ 1. System of Relationships Between Two Markets

1. All markets in a social economy are interlinked within some sort of relationship. Depending on how close the relationships are, they can be divided into several types. Where economic entities that enter market *A* simultaneously participate wholly or partially in another market *B*, we can thus say that a primary relationship exists between these two markets *A* and *B*. Where economic entities that meet other economic entities in a third market participate in market *A* and are also market participants in market *B*, we can thus say that a secondary relationship exists between the two markets *A* and *B*, etc.

It is possible that different relationships exist between two markets.

The similarity with this division of relationships between two markets to Menger's theory of goods is immediately obvious and is also by no means accidental.

2. In the following pages we will only focus on the primary relationships. We can distinguish three types of relationships:

1) Where demanders in market *A* are demanders in another market *B* at the same time, a "primary demand relationship" thus exists between those two markets.

2) Where suppliers in market *A* are suppliers in another market *B* at the same time, a "primary supply relationship" thus exists between those two markets.

3) Where demanders in market *A* are suppliers in another market *B* at the same time, a "vertical primary demand relationship" thus exists between those two markets. Market *A* is the "leader" (translator's note: (vorgelagerte) the English terminology is used by Peter R. Senn, "Heinrich von Stackleberg in the History of Economic Ideas", (1996) *Journal of Economic Studies 23*, *5/6:16*, Evanston, Illinois: MCB University Press) and market *B* is the "follower" market.

Two links can exist between these three types:

a) A primary demand and a primary supply relationship can exist between two markets at the same time. Then we refer to a "horizontal double relationship".

H. von Stackelberg, *Market Structure and Equilibrium*,
DOI 10.1007/978-3-642-12586-7_3, © Springer-Verlag Berlin Heidelberg 2011
Translation from the German language edition: "Marktform und Gleichgewicht"
by Heinrich von Stackelberg, © Springer-Verlag Wien New York 1934, all rights reserved

b) Demanders in market *A* can be suppliers in another market *B* and suppliers in market *A* can be demanders in market *B* at the same time. Then we refer to a "cyclical relationship" between these two markets.

Various mixed relationships can also still occur, so economic entities, for example, that are each other's demander/supplier in one market can be rivals in another market. A precise classification of these mixed relationships that ought to be defined in a special theory of market relationships can be disregarded here since what matters to us is only an analysis of typical market relationships.

3. The question that is the subject of this chapter can be briefly drafted as follows: How are the conclusions arrived at so far in our theory about market structures to be modified where two or more markets of the same or different market structure are interlinked in a primary relationship? Consequently, in order to lead the way, we should thus now discuss all types of primary relationships between all the possible market structures. It is however possible to avoid such an excessively elaborate description.

Initially we will not refer to any of the market types which exhibits one of the two forms of free competition since these links have been made implicit in the behaviour of market structures. The general assumption has been made in this case that all "other" markets should display a structure of free competition. These "other" markets however also include all those where the respective market that appears is linked by a primary relationship. For an analysis of other combinations we will initially study the relationships between markets with a simple monopoly. We will restrict our comments to the behaviour of markets which only include companies because these markets are effectively the most important and can also be described more easily. Correspondingly the general situation can also be inferred by analogy with some certainty.

4. Between two monopolistic markets a total of 16 types of primary relationships can exist, namely the three simple and the two double relationships between two monopsonies, likewise between two monopolies and also between a monopsony and a monopoly, whereby however the vertical relationship occurs twice in the third scenario, once with the leader monopsony and again with a leader monopoly. Of these 16 types only three are of interest in our scenario. The situation can be seen clearly in the case of the other 13. Therefore they are listed below without a detailed study and briefly characterised.

1) Where a primary demand relationship exists between two markets with monopsonies this simply means that the monopsonist in one market and the monopsonist in the other are one and the same economic agent. This agent regulates his demand for each good so that maximum profits are achieved. We have nothing here but a generalisation of the simple monopsony with characteristics that have not been modified in any way.

2) For the primary supply relationship between two monopolies the same applies in relation to the simple monopoly.

3) In a horizontal double relationship between two monopsonies we also have a situation that does not essentially differ from the ordinary primary demand relationship between two monopsonies. Here there is just the collapse of the

competitively organised side of the market where however nothing essential has changed.

4) The same also applies mutatis mutandis for the horizontal double relationship between two monopolies.

5) A vertical relationship between two monopsonies then exists when a monopsonist appears as a supplier in the market of the other monopsonist since supply here is competitively organised and thus this fact can have no importance whatsoever. The monopsonist plays no fundamental role as a rival in the supply market.

6) The same also applies mutatis mutandis for the vertical relationship between two monopolies.

7) A cyclical relationship between two monopsonies does not differ much from the vertical one.

8) The same also applies mutatis mutandis for the cyclical relationship between two monopolies.

9) A primary demand relationship between a monopsony and a monopoly can only exist where the monopsonist in one market is a rival in the demand of the other market. It is clear that his participation here can have no measurable effects whatsoever.

10) The same also applies mutatis mutandis for the supply relationship between a monopsony and a monopoly.

11) Also a horizontal double relationship between a monopsony and a monopoly does not essentially differ from the two previous cases.

12) Where a vertical relationship exists between a monopsony and a monopoly with a leader monopsony, the implication is therefore that an economic agent regulates both a monopoly as well as a monopsony. He is a monopolist so to speak on each side of the market. It is apparent that his position with respect to a simple monopsonist or monopolist is merely slightly different, rather than different in principle. Our monopolist simply dominates both markets and can obtain the most favourable position in both markets. The general monopolistic equilibrium is achieved in both markets.

13) Where a cyclical relationship exists between a monopsony and a monopoly, so a monopolist is confronted with rival individuals on both the demand and the supply side. Here the monopolist can exploit his other side in a way that can be described as "total exploitation". Where the rival individual's behaviour in one market is independent of his behaviour in the other we have a situation that is not essentially very different in structure from the one just described in 12). Where by contrast, the rival's supply depends on how much they as the demander receive, the monopolist can thus regulate his behaviour so that the last quantity of product given by him as a supplier to the rival is exactly equal in value to the rival's resultant achieved quantity supplied and that received by the monopolist as a demander.

It is definitely worthwhile studying this market type in more detail, but it is not of any further interest for our purposes. For us, it is sufficient to state that here also a well-defined, if not also very "unfair" equilibrium occurs.

Of these 13 types of market relationships that are essentially immediately assessable as regards their main characteristics, there are three that call for special study. They are the following:

a) Type I: Supply relationship between two monopsonies. Here two monopsonists are faced with tied supply.

b) Type II: Demand relationship between two monopolies. Here two monopolists are faced with tied demand.

c) Type III: Vertical relationship between a monopoly and a monopsony with a leader monopoly. Here a monopolist supplies a group of numerous economic entities that are competing with each other and who in their turn are supplying a monopsonist. We refer to this market relationship as an intermediary bilateral monopoly.

These three types are analysed in subsequent paragraphs where we assume that all economic entities are companies, unless it is specifically stated otherwise.

§ 2. Two Monopsonies with Tied Supply (Type I)

1. The market type we are to study here shows two monopsonistic companies that are confronted with competing supply sides composed wholly or partially of one and the same companies. The companies that are common to the two markets thus supply both goods. Combining these functions is however not yet sufficient to establish a real relationship between the two markets. Where each good is produced completely independently of the other so that the market relationships of one good can have no influence whatsoever on the supply of the other goods, so the commonality is an economically immaterial fact. Both markets function as if they were also officially independent of one another.

However, it is a different situation when the production of the two goods is tied in some way, i.e. if production of one of the goods is dependent in some way – technically or organisationally – on the production of the other good. We can then speak about "tied production" and "tied supply". Tied production[1)] occurs in two types:

1) Complementary production. Here the production of one product is linked with the production of the other in more or less rigid proportions. An expansion in the production of one good at the same time improves the production possibilities of the other. Classic examples are grain and electricity, meat and wool, and gas and coke.

[1)]See Auspitz and Lieben, ibid.: 234 et seq. A. Marshall, *Principles*, ibid. Book V, chap. VI and Appendix. M. Pantaleoni, "Una visione cinematographica del progresso della scienza economica" in: *Erotemi di economia* [*Principles of Economics*], Vol. 1: 202 et seq. (Prezzi connessi). Marco Fanno, "Contributo alla teoria dell'offerta a costi congiunti". Supplemento al *Giornale degli Economisti e Annali di Economia*, October 1914. Josef Schumpeter, Article "Angebot" in *Handwörterbuch der Staatswissenschaften*, 4. ed., Vol. 1, Jena, 1923: 3001.

2) Alternative production. Here one and the same company can produce either one good or the other, however, where one of the goods is no longer a by-product of the other, but on the contrary, an expansion of the production of one good reduces the production opportunities of the other good. As a general example, this is where different types of a product are referred to as being produced in one and the same company.

The response of a producer of complementary products to price changes is completely different to that of a producer of alternative products. Indeed, the supply of each good changes in the same direction as the price. Moreover a change in the price of one good however brings about a response regarding the production of the other good. So where complementary production is concerned, the supply of a good increases with the increase in the price of another good and vice versa. However, where both the goods are alternative products, then an increase in the price of one good brings about a decrease in the supply of the other and conversely a decrease in the price causes an increase in the supply of the other good. It is only when both goods are produced independently that a price change in a good has no influence on the supply of the other good.

These reactions by the individual supplier accordingly affect the overall reaction of the supply sides of both markets. The total supply of any good depends on both prices. The relationship of dependence, whether it is a question of complementary or alternative products as the case may be, is different in the ways just described. On the other hand, the price is therefore the one that a monopsonist can obtain for a particular quantity of goods and this is also dependent on the quantity demanded by the other monopolist. Where complementary production is concerned, the higher this price is the less the other monopolist demands.

By contrast, the higher the price of a good that is an alternative product, the more of the other good is demanded.

2. Assuming that one monopolist regards the quantity demanded by the other monopolist as a given variable, he will then behave towards the supply side of his market in exactly the same way as an ordinary monopolist. For a given quantity demanded in another market he will produce a demand that secures him the largest profit in the way that has been discussed. He will thus orientate himself to the current quantity demanded by the other monopolist.

This other monopolist for his part will exploit this first monopolist's dependence if he acts accordingly in the real, overall situation. He will establish that his quantity demanded not only directly influences the price in his market but also indirectly in the way that he leads the first monopolist to a specific demand which has a further effect on the price in the second market. In the final assessment of his demand he will thus include in his calculation not only the conditions of his market, i.e. the second market, but also the repercussions on the first market.

We thus see that the situation of the two monopsonists with tied supply is exactly equivalent to the demand dyopoly. In the "preliminary" price formation just described, the first monopolist has the "position of dependence" and is the follower of the second monopolist, whereas the latter takes the "position of independence"

and dominates the whole market. The final price formation resulting from our market equilibrium thus depends on which of these two positions is the most favourable, and will generally not result in an equilibrium. The explanations made in extenso for the study of the demand dyopoly can be repeated here almost word-for-word. Another new criteria that is lost in the ordinary dyopoly can be added however, namely that it is a question here of two different goods.

3. So far we have looked at a situation in which the quantity demanded by one or other of the monopolists was seen as an independent variable. A correspondingly similar observation arises when we substitute a monopsonist's price for their quantity demanded. Assuming a monopolist does not regard the quantity demanded by the other monopolist as an independent variable, but instead the price, then he too will behave in exactly the same way as an ordinary monopolist who is not communicating directly or indirectly with any other monopolistic market. He will thus look to achieve his maximum profit at the given price of the other monopolist and so will comply with the price set by the other monopolist.

Then the second monopolist will exploit the dependence of the first monopolist. He will determine that the direct reaction of his supplier depends on his price adjustment policy and not only that, but also on the reaction of the first monopolist who responds again indirectly in the second market. Consequently the second monopolist will take the response of the first monopolist into account and now for instance not fix his quantity, but his price instead. We see an absolute similarity with dyopoly theory except that fixed prices simply replace the quantity of goods. Here the position of independence or alternatively, dependence, can also be the most favourable. In effect, both the Bowley and the Cournot dyopolies can thus also adapt to a similar situation where an asymmetrical dyopoly structure is also possible. In order to describe these different circumstances we must introduce a whole series of new descriptions:

a) Where a monopolist regards the quantity sold by the other monopolists as an independent variable which he orientates himself to, we can therefore refer to the position of the first monopolist as "quantity dependent" and the position of the second as "quantity independent".

b) Where a monopolist does not regard the quantity sold, but instead the price of the other monopolists as an independent variable, we can therefore refer to the position of the first monopolist as "price dependent" and the position of the second as "price independent".

It is impossible that a monopolist views both the quantity sold and the price of the other monopolist as independent. Either e.g. the first monopolist considers the quantity sold by the second as being independent, then he can see their price only as a result of the general pricing structure process which he (the first monopolist) amongst others is also involved in or the first monopolist sees the price set by the second monopolist as a given variable. Then the quantity sold by the second monopolist appears to him to be the result of the general market mechanism that is also influenced by him (the first monopolist).

It is also revealing that in a simple dyopoly (translator's note: (einfachen Dyopol) Leontief: 555) "price dependence" cannot exist when it is hence only a question of a

homogenous good, in a similar way to the alternative of "price independence". Just as in free competition, a demander or a supplier can set a discounted price in the dyopoly. In a market where a standardised good is sold, only one price is ever possible. A price increase by a demander will immediately entice all suppliers to compete the price down again. A price reduction would immediately involve a loss for all the suppliers who must push through another price increase. In a dyopoly a single price can only be influenced by the quantity sold by individual dyopolists.

Something similar applies for the extreme case of complementary goods, in which two goods are supplied (or demanded) in wholly fixed proportions. These two complementary products can be described as joint products. Two monopsonists for joint products can only achieve positions of price dependence or price independence. Here the independent setting of the quantity sold is not possible[1)].

4. Whereas we had two possibilities in the simple dyopoly for each dyopolist, either the position of dependence or independence was the most favourable for him. Four alternatives are now available. For each monopolist the most favourable position is either quantity independence or price independence, or quantity dependence or price dependence. In particular it is a strange as well as an interesting fact that as a rule these four positions all differ from each other.

Where the different possibilities for the most favourable position are combined this gives a total of ten different configurations of which only two display a true equilibrium.

1) Where each monopolist seeks to obtain quantity independence – because this position is now the most favourable for him out of the four possibilities – so we obtain a situation already described as the Bowley dyopoly where each achieves his independent volume of sales. There are no concerns about broadening the description of the "Bowley dyopoly" to also include situations of tied production.

2) Where quantity dependence is the most favourable for each monopolist we thus also obtain a well-documented situation that we want to name the "Cournot quantity dyopoly" for tied markets.

3) Where each monopolist seeks to obtain price independence, so each autonomously sets their independent price. Edgeworth[2)] was the first to recognise this situation that is similar to the Bowley dyopoly. Therefore we will refer to it as the "Edgeworth dyopoly".

4) Where each regards the price set by the other monopolist always as given and independent – in order to obtain price dependence – we thus obtain a situation similar to the "Cournot dyopoly". Since Cournot himself first described this situation[3)] we thus refer to it as the "Cournot price dyopoly" as distinct from the "Cournot quantity dyopoly". In the simple dyopoly and for joint production we

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[1)]Edgeworth has already pointed to this circumstance (*La teoria pura*, ibid.: 27, or Papers, ibid. Vol. 1: 122, Remarks) and also given the basis for this.

[2)]"La teoria pura" (ibid.), *Giornale degli Economisti e Annali di Economia*, 1897: 27/28; Papers (ibid.), Vol. 1: 123.

[3)]ibid. chap. IX.

can easily refer to the "Cournot dyopoly" because here only one form of this dyopoly is ever a possibility.

5) Where one monopolist is looking for quantity independence, and the other quantity dependence, we thus obtain the "asymmetrical quantity dyopoly".

6) Similarly we coin the term "asymmetrical price dyopoly".

7) Ultimately there are still four possible combinations:

a) "quantity independence – price independence", b) "quantity independence – price dependence", c) "quantity dependence – price independence" and d) "quantity dependence – price dependence" and we want to refer to these as "mixed cases". We should not be concerned with these further. They are unstable compared with the two "asymmetrical dyopolies".

Since the two asymmetrical dyopolies that would achieve a true equilibrium might only occur as a rare exception, we can thus conclude that in the market equilibrium studied just now the basic principles of the free market capitalist economy will not lead to a general equilibrium. These basic principles are thus irreconcilable with this market relationship (type I) in the same sense as with the market structure of a demand dyopoly.

§ 3. Two Monopolies with Tied Demand (Type II)

1. The critical analysis of the previous paragraphs can be repeated here almost word for word. The common feature on the demand side of the two markets then has also only one real meaning here when the use of both goods by the individual demander displays a specific relationship[1)] to both goods. These goods can either behave as complementary[2)] or alternatives (rivals)[3)] to each other. Where the price of one good increases so the demand for a complementary good decreases and the demand for a rival good increases if the demander is a company. Thus, for the supplier a quite specific link between the prices set and the saleable quantities

[1)]See Menger, *Grundsätze der Volkswirtschaftslehre*, 2nd ed., Vienna, 1923: 23 et seq. Auspitz and Lieben, ibid.: 170 et seq. A. Marshall, ibid. M. Pantaleoni, ibid. V. Pareto, *Manuel* (ibid.), chap. IV. Amoroso, *Lezioni di economia matematica*, Bologna (1921: 82 et seq.). Strigl, Article "Nachfrage" in *H.d.St.*, 4th ed., Vol. 6 (1925). R.G.D. Allen, "A Comparison between Different Definitions of Complementary and Competitive Goods", *Econometrica*, Vol. 2, No. 2 (1934).

[2)]The exceedingly diligent paper by Rosenstein-Rodan, "La complementarietà prima delle tre tappe del progresso della teoria economica pura" in: *La Riforma Sociale*, Vol. 44 (1933: 257 et seq.), unfortunately adds very little to the subject of complementary goods (in the narrowest sense, compared with "alternative" or "supplementary" goods).

[3)]See Marco Fanno, "Contributo alla teoria economica dei beni succedanei", *Annali di economia*, Vol. 2 (1926: 337 et seq.). Fanno deals also with the supply of two rival goods demanded by two monopolists (pp. 410 et seq.). He therefore affiliates himself essentially with Edgeworth and Pigou.

results. Alternatively, there is similarly a link between the quantities supplied and the prices that are achievable. Where the two goods are complementary the price thus appears as a decreased function of the good that the supplier is referring to and as an increased function of the other good. Where the two goods are rivals so the prices are a decreased function of both goods.

For the two monopolists we obtain the four possible positions described in the previous paragraphs, namely quantity independence, quantity dependence, price independence and price dependence. No matter which position appears to be the most favourable to the individual monopolist, the same price formation structures can occur as with "type I", namely the "Bowley dyopoly", "Edgeworth dyopoly", "Cournot quantity dyopoly", "Cournot price dyopoly", the two asymmetrical (general equilibrium) dyopolies and the four (unstable) mixed structures. The demand relationship of the two monopolies is therefore also as irreconcilable in the same sense with the basic principles of the free market capitalist economy as the straightforward supply dyopoly.

2. The questions about which of the four positions stated above for the individual monopolists is the most favourable and which price formation structure actually occurs as a general rule is even more difficult to determine in market relationships than in a simple dyopoly. The different possibilities are linked to preconditions that can only be revealed through detailed mathematical analysis. We will be able however to determine the following regularities – with the same caveats for the corresponding assumptions in the theory of the simple dyopoly:

1) Where rival goods are concerned, quantity independence is hence generally more favourable than quantity dependence. This assertion corresponds to the assumption made for a simple dyopoly, namely a situation where both suppliers supply a completely homogenous good and are thus dyopolists, can be seen as a special case of the supply of rival (supplementary) goods. The same also applies for the monopsonist.

In contrast, price dependence is generally more favourable than price independence. We can thus distinguish three scenarios in the case of rival goods:

a) As a general rule either the "Bowley dyopoly" or the "Cournot price dyopoly" occurs.

b) In special circumstances the "Cournot quantity dyopoly" or the "Edgeworth dyopoly" occurs.

c) In exceptional cases an asymmetrical dyopoly or a mixed case arises.

2) For complementary goods quantity dependence is generally more favourable than quantity independence and price independence more favourable than price dependence. We thus obtain:

a) As a general rule, the "Cournot quantity dyopoly" or the "Edgeworth dyopoly"

b) In special circumstances, the "Bowley dyopoly" or the "Cournot price dyopoly"

c) In exceptional cases, a mixed structure

3. We now still have to decide whether it is the "Bowley dyopoly" or the "Cournot price dyopoly" that governs rival goods and whether "the Cournot quantity dyopoly" or "the Edgeworth dyopoly" governs complementary goods. Under

the same assumptions as before it will be shown that the most favourable position for rival goods is quantity independence and for complementary goods, it is price independence.

Consequently, for cases of rival goods or products, as a rule it is the "Bowley dyopoly" while for complementary goods or products it is the "Edgeworth dyopoly". The evidence for the accuracy of this assertion can only be examined in the next chapter.

It is worth noting that these conclusions fit comfortably into the previous framework of our theory:

a) Where two goods are almost identical, so it is indeed still possible for both monopolists = dyopolists to pursue a price adjustment policy rather than quantity adjustment policy. But quantity adjustment is more advantageous than price adjustment since quantity independence is more favourable than price dependence. In the simple dyopoly we therefore see the circumstance where only the quantities vary and where the equilibrium can also be substantiated en route.

b) In the same way, evidence therefore emerges that quantity adjustment for complementary goods that are linked with one another in rigid proportions is out of the question here.

c) Where the two goods are neither complementary nor rivals, the two monopolists are thus independent of each other, so it is immaterial to them whether they pursue quantity adjustment or price adjustment. This agrees with the results we have found for the simple monopoly.

4. The grounds for this whole hierarchy of relationships can be seen in Chapter 4[1)].

§ 4. The Intermediary Bilateral Monopoly (Type III)

There is a vertical relationship between a monopoly and a monopsony with a leader monopoly when a monopolist supplies a competing number of individuals that for their part are suppliers to another monopolist. The two monopolists have the rival economic entities in their grip.

1. Where the supply of these rivals is independent of the outcome of their demand, so only an individual relationship is present between the two markets, but not a financial one, and legislation for the simple monopoly applies for each of the two markets. As a rule however and in particular, every time companies are involved, the more that is supplied by an economic agent, the more advantageously it can be purchased. A cheap means of production enables a greater supply at the given price of production, rather than a more expensive one.

The companies on the competitive side of the market regard both prices as a given variable and assign a specific quantity demanded and a specific quantity

[1)]See the algebraic example in Appendix X.

supplied to each pair of variates for these prices. As a result of this, for each monopolist there is a price he can obtain for a specific quantity of goods supplied or demanded or that he has to pay, that depends not only on this quantity but also on the quantity sold of the other good. And indeed these interrelationships are also entirely clear. The monopolist receives a higher price the less he supplies or the more the monopsonist demands. The monopsonist must pay a higher price the more he demands or the less the monopolist supplies.

On the other hand the competing companies will demand and supply more the lower the monopolist's price is and the higher the monopsonist's price is.

2. It can easily be seen from the conclusions taken from previous studies that the intermediary bilateral monopoly also presents a very similar situation to the simple dyopoly. Where the monopsonist sees the quantity supplied by the monopolist as a given variable, so he now orientates himself towards the monopolist. He then takes into account not only the demand of the competing companies that he is facing but also the reactions of the monopsonist. He thus dominates the overall situation of the markets and the monopsonist is his follower. The situation is reversed if the monopolist orientates himself according to the monopsonist's demand. The same applies if the behaviour of the monopolist does not relate to the quantity sold, but rather to the price set.

The market relationship is quite similar to that of complementary goods, as analysis of the reactions of competitively organised market participants shows. We therefore only need to refer to the extensive conclusions outlined in this case. The assumption exists that "type III" equates especially to the Edgeworth dyopoly. An acceptance of this market relationship with the principles of the free market capitalist economy must be disputed in the same way as for the two previous types.

§ 5. Market Equilibrium and Equilibrium

In accordance with previous explanations we can make a set of assertions whose accuracy is not readily apparent.

1. Relationships between two markets, one of which presents a structure exhibiting free competition, cannot modify the laws of price formation for these markets.

2. Relationships between unstable market structures can weaken barriers that face the equilibrium. The weakened effect especially occurs if it concerns rival goods since the markets for two rival goods – regardless of whether this is "rivalry" in terms of production or consumption – behave in a similar way to the two parts of the same market.

3. Relationships between monopolistic markets, that in isolation would display an equilibrium, are able to cancel out the equilibrium.

4. These conclusions that are based on studies of primary relationships between two markets allow generalisations to be made logically in respect of primary relationships between multiple markets, where conclusions will similarly be reached about the related market structures, so e.g. the demand relationship between multiple monopolies compares with the picture of a supply oligopoly.

Where the number of monopolists is very large, a situation can thus occur where the individual takes no account of their influence on the behaviour of the other monopolists because this influence is very slight. Then a similar but stable relationship to the one between the supply oligopolists in the case of the "Cournot oligopoly" occurs between monopolists. Each restricts himself to the monopolistic domination of his market. In a similar way, this also applies to the supply relationship between multiple monopsonies and the relationships between multiple monopolies and monopsonies. Where someone thinks they are monopolising the supply side of all the markets in a social economy so a market system occurs for which Umberto Ricci first coined the term "polypoly"[1)]. According to Ricci a general equilibrium occurs here when the individual market displays the price formation structure of the monopoly. Schneider[2)] also argues in favour of the same view. We can add two further conditions to this opinion. These conditions emerge directly from studies of the relationships between two monopolies:

1) In this polypoly the demanders for each product must be very numerous for each product, even if each of them as a supplier has a monopoly at their disposal.

2) Demand relationships are only allowed to occur between a large number of markets at once. A relationship between two markets may only be slight.

5. It is also possible to deduce higher level relationships from the analysis of primary relationships. As a rule, these can be regarded as weak links to primary relationships as follows. A brief observation of the secondary demand relationship between two monopolies may clarify this. We have three markets here: *A*, *B* and *C*. Markets *A* and *C* show a monopoly. Demanders in market *A* also partly occur in the competitively organised market *B*. Other participants in market *B* are simultaneously demanders in market *C*. The monopolistic price adjustment of market *A* influences the price in market *B*. The pricing structure in market *B* in turn has an influence on market *C*. The same applies the other way around. Thus the supplier in market *A* cannot be indifferent to the behaviour of the suppliers in market *C* and vice versa. We thus have a situation that is similar to the primary demand relationship between two monopolies, only here the interdependencies are generally weaker. The same also applies essentially for the other secondary relationships and correspondingly for those at a higher level.

6. The supply relationship between multiple monopsonists or the demand relationship between multiple monopolists can be seen overall as a general form of market analysis. In reality only a few goods are completely standardised. Generally, e.g. one company's product is not completely identical to the product of another company. Each supplier thus has a quasi-monopolistic position that is restricted by monopolies of related markets. Therefore the model of market relationships between monopolies is of fundamental importance, just for the practical analysis

[1)]U. Ricci, *Dal Protezionismo al Sindacalismo*, Bari, 1926: 131 et seq.

[2)]E. Schneider, *Reine Theorie*, ibid.: 83 "Universelles Monopol".

of economic theory in general and of market analysis in particular[1]. So it can only explain why in practice "quantity determination" is often not pursued at all as it would instead produce "price determination" from the assumptions of "free competition" (perfect competition) or even a homogenous market. Hence e.g. the policy of the producers that often occurs in practice "not to lose any clients" also fits into the logical context of the theory.

[1]See Piero Sraffa, "The Laws of Returns under Competitive Conditions", *The Economic Journal*, Vol. 36 (1926: 535 et seq., especially pp. 544). Hotelling ("Stability in Competition", *The Economic Journal*, Vol. 39 [1929: 41–57]) is credited with being the first to accurately examine the rivalry between suppliers of an approximately homogenous good (heterogeneous only in terms of business location) in the schematic structure of the market relationship between monopolists. Chamberlin (ibid.) strongly advises putting the theoretical premises in concrete terms. Joan Robinson (*The Economics of Imperfect Competition*, London, 1933. Introduction) expresses a similar view. None of the researchers mentioned above however go as far as a systematic analysis of the price formation structure and its strength.

Chapter 4
Dyopolistic Market Share Ranking

§ 1. General Analysis

1. The broad similarity between the demand dyopoly, the supply dyopoly, the supply relationship between two monopsonists, the demand relationship between two monopolists and the intermediary bilateral monopoly allows us to summarise some problems specific to these market structures and market relationships. In particular, any such summary of the results should solve the question of when the individual positions are most favourable for the individual dyopolists and monopolists respectively and when the individual dyopolistic structures occur.

2. The characteristic of the five market configurations stated above is the entrance onto the market of two economic entities that each sells a quantity of goods or sets a market price, and in doing so will influence either the ordinal utility index or profit of the other. We refer to the two economic entities as A and B, and the quantity of goods they sell or the price set by them as ξ and η and we can say that the ordinal utility index of either of the economic entities depends on these two variables. On this basis we can provide a general analysis that is relevant for both quantity as well as price adjustment. Since ξ and η can mean both quantities and prices, from now on we will thus simply refer to two "values" ξ and η, where we use the term "value" in its mathematical sense[1)].

3. As this only concerns two economic individuals, we can represent the problem geometrically. This is necessary because the relationships to be examined would otherwise be very complex. We construct a rectangular coordinate system and mark the ξ values on the x-axis and the η values on the y-axis. Each point on the first quadrant of the (ξ, η) plane then presents a special combination of the two values. An ordinal utility index (a profit) is thus assigned to each point for individual A and another for individual B. All points exhibiting the same ordinal utility index draw a curve that Pareto terms an "indifference curve" (translator's note:

[1)]This "value" therefore has nothing to do with "economic value".
Translator's note: (Wert), see Leontief: 557).

H. von Stackelberg, *Market Structure and Equilibrium*,
DOI 10.1007/978-3-642-12586-7_4,
Translation from the German language edition: "Marktform und Gleichgewicht"

(Indifferenzkurve), see Scherer: 60), hence the (ξ, η) plane in its first quadrant is overlaid with two indifference curves. One curve relates to individual A, the other to individual B. These indifference curves can a priori have any conceivable shape. For our purposes, we are permitted however to make two restrictive assumptions:

a) A combination (one point) with the highest index can only lie marginally on the first quadrant and thus either on the non-negative part of the x-axis or on the non-negative part of the y-axis or at infinity.

b) The index of either of the two indifference curves is greater than the indexes of all of the indifference curves in between.

4. Where A looks at the value η achieved for B as an independent variable, he assigns the value ξ that achieves the largest ordinal utility index for him with the given η to each value of η. This means that he chooses the most favourable point parallel to the ξ-axis at distance η. We can rule out a scenario where this most favourable point lies at infinity since there can never be any question of an infinitely large turnover or price. Where this point does not additionally lie on the η-axis, it can therefore only be the point where the parallel is tangential to A's indifference curve. This is a consequence of condition b). On the other hand, since we can always presuppose the occurrence of a maximum in the cases that interest us we must thus postulate that each indifference curve for *A* has a point with a tangent parallel to the ξ-axis, which incidentally also passes completely on one side of this tangent (either above or below). And we must further assume that the ordinal utility indexes slope straight up on this side, otherwise it would not be an ordinal utility maximum at the point of tangency (translator's note: see Scherer: 63), but a minimum instead. Thus the indifference curves for *A* are at least near their points with a horizontal tangent either convex downwards – where the indexes increase upwards – or convex upwards – where the indexes decrease downwards.

A therefore assigns the point of tangency of the parallels to each value of η with an indifference curve. All of these points of tangency, and therefore all points in which the indifference curves have horizontal tangents, draw *A*'s "reaction curve" (translator's note: (Reaktionskurve), see Scherer: 63) for the value η for *B*. This reaction curve assigns a ξ to each η, which means that it defines ξ as a function of η.

Since one point of this curve equates to each η, the reaction curve for *A* thus runs at all events from the bottom to the top. Incidentally, depending on the specific shape of the indifference curve, it can however decrease both from left to right (meaning it draws an acute angle with the negative direction of the ξ-axis) as well as increase (meaning it draws an acute angle with the positive direction of the ξ-axis) or even run vertically[1)].

Accordingly, we obtain the following four typical indifference curve shapes for economic agent A, which of course include an indefinite number of mixed structures (see Fig. 4.1).

[1)]For the direction of the reaction curves for A and B in relation to each other see Cournot, ibid.: 70 and 71, also 88 and 89. Schneider, *Reine Theorie*..., ibid.: 145 and 146, also 41.

Type α: ordinal utility indexes decreasing from the bottom to the top, negatively sloping reaction curve.

Type *β:* like α, but reaction curve positively sloping.

Type y: like α, but ordinal utility indexes increasing from the bottom to the top.

Type δ: like *y*, but reaction curve positively sloping.

5. What we have stated also applies word-for-word to economic agent B if the following terms are substituted for each other: ξ and η, "above" and "right", "below" and "left" "horizontal" and "vertical", "x-coordinate" and "y-coordinate".

As a result, the corresponding types of indifference curve have the following appearance for *B* (see Fig. 4.2).

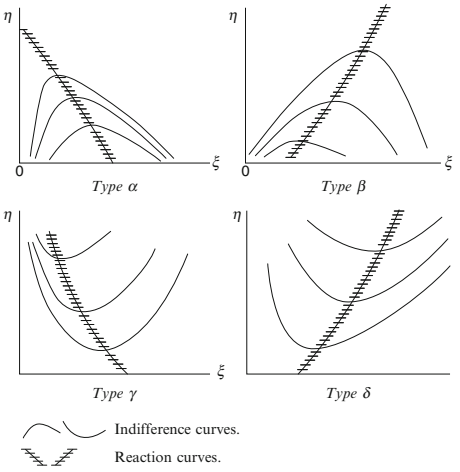


Fig. 4.1

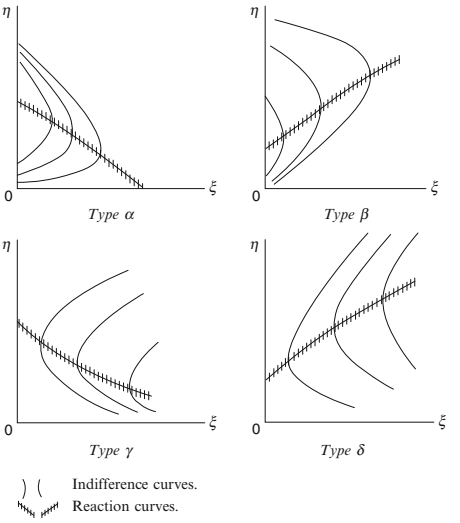


Fig. 4.2

6. We refer to the reaction curve for *A* with α and the reaction curve for *B* with *b*. Where economic agent B always regards value ξ for economic agent A as a given independent variable, so a value η corresponding to the value ξ is always achieved by B's reaction curve. Then be that as it may, *A* will not look for the most favourable point on a parallel to the ξ-axis, but instead the most favourable point along reaction curve *b*, meaning however the point where this reaction curve *b* is tangential to one of the indifference curves for *A*. This point lies on reaction curve *a* only if reaction curve *b* runs horizontally to the point of tangency. We refer to this point as the independence point for *A*. Accordingly, we define the independence point for *B*. Where each of the two economic entities is trying to achieve their independence point, we obtain the point for the independence value for *A* and the independence value for *B*. We refer to this point as the "Edgeworth–Bowley dyopoly point". We refer to the intersection point of two reaction curves as the "Cournot dyopoly point". The combination described by it is achieved

if the two economic entities orientate themselves towards the behaviour of the other and thus strive to achieve (Scherer: 64) a position of dependence (translator's note: see Heertje: 50).

The position of the independence point for *A* is dependent on the shape of *A*'s indifference curves and on the direction of B reaction curve. It lies to the left of the Cournot dyopoly point if the indifference curves for *A* are convex upwards and reaction curve *b* increases from left to right, or if A indifference curves are convex downwards and reaction curve *b* decreases from left to right. In the two contrasting cases it lies to the right of the Cournot dyopoly point. If we refer to the possible combinations of the four types of indifference curves for each economic agent, hence we only cite the type applicable for *A* and then for *B*, we can thus formulate the above as follows: the independence point for *A* (that always lies on *B*'s reaction curve *b*) is:

1) Left of the Cournot dyopoly point in the combinations:
$\alpha\beta$, $\alpha\delta$, $\beta\beta$, $\beta\delta$ and $\gamma\alpha$, $\gamma\gamma$, $\delta\alpha$, $\delta\gamma$;
2) Right of the Cournot dyopoly point in the combinations:
$\alpha\alpha$, $\alpha\gamma$, $\beta\alpha$, $\beta\gamma$ and $\gamma\beta$, $\gamma\delta$, $\delta\beta$, $\delta\delta$.

Accordingly, the independence point for *B* is:

1) Below the Cournot dyopoly point in the combinations:
$\beta\alpha$, $\delta\alpha$, $\beta\beta$, $\delta\beta$ and $\alpha\gamma$, $\gamma\gamma$, $\alpha\delta$, $\gamma\delta$;
2) Above the Cournot dyopoly point in the combinations:
$\alpha\alpha$, $\gamma\alpha$, $\alpha\beta$, $y\beta$ and $\beta\gamma$, $\delta\gamma$, $\beta\delta$, $\delta\delta$.

We refer to:

The Cournot dyopoly point *C*;

The Edgeworth–Bowley dyopoly point π;

The independence point U_1 for *A*;

The independence point U_2 for *B*.

Overall, we have to analyse the 16 combinations below. Therefore we will condense the combinations that show the same result because the account will be shorter and clearer as a result. In paragraph 7 we deal with the combinations $\alpha\alpha$, $\gamma\gamma$ and $\beta\delta$ ($\delta\beta$); in paragraph 8, the combinations $\beta\beta$, $\delta\delta$ and αy ($y\alpha$) and in paragraph 9, the combinations $\alpha\beta$, ($\beta\alpha$), $\beta\gamma$, ($\gamma\beta$), $\gamma\delta$, ($\delta\gamma$), $\delta\alpha$, ($\alpha\delta$). The bracketed combinations do not need to be described again in detail because the combinations can be seen immediately, just by substituting *B* for *A*.

7. For the combinations $\alpha\alpha$, $\gamma\gamma$ and $\beta\delta$, ($\delta\beta$), as we immediately see from the geometric interpretations, their particular independence point is the most favourable for each of the two economic entities (see Fig. 4.3).

If we observe the direction in which the ordinal utility indexes increase or decrease we then see that for *A* the point U_2 is less favourable in all four cases than point *C*. On the other hand, since point U_1 for A is always more favourable than *C* (not just in these four cases), so U_1 is by consequence the most favourable point for *A*. Similarly for *B*, point U_2 *is* the most favourable. Thus in effect the "Edgeworth–Bowley dyopoly" occurs in π in all four combinations, where it can be easily seen that π is the least favourable of the four points for both economic entities.

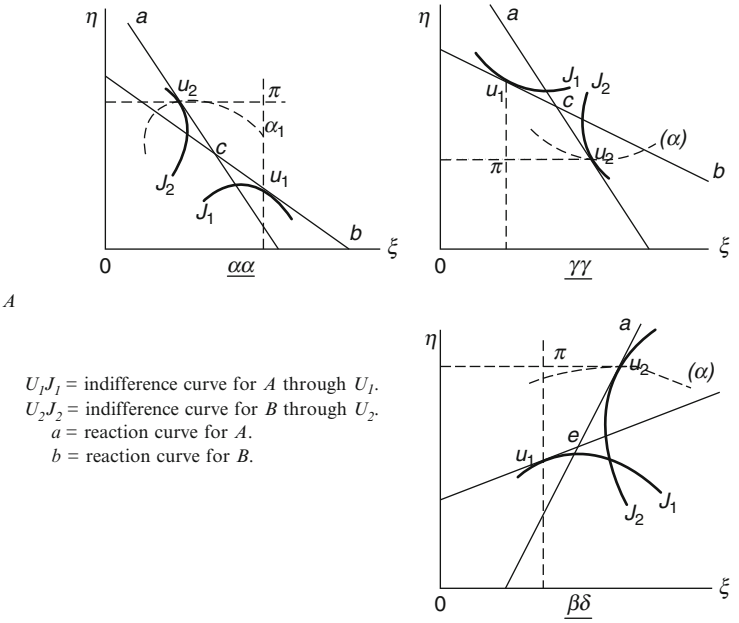


Fig. 4.3

8. For the combinations $\beta\beta$, $\delta\delta$ and $\alpha\gamma$, $(\gamma\alpha)$ the other's independence point is generally the most favourable for the two economic entities, since A indifference curve through U_2 shows a higher index than his indifference curve through U_1, and for B the opposite applies (see Fig. 4.4).

In effect the Cournot dyopoly occurs in C where each strives to achieve the position of dependence and responds to the behaviour of the other according to his own reaction curve. It should be noted that C is the least favourable point for both individuals.

There are also however two exceptions:

a) U_2 can be between point C and A's indifference curve through U_1. For A the most favourable position is then independence and the stable equilibrium of the asymmetrical dyopoly in U_1 occurs.

b) U_1 can lie between point C and B's indifference curve through U_2. Then the asymmetrical duopolistic equilibrium occurs in U_2 for similar reasons.

It is however impossible that U_2 simultaneously lies between C and A's indifference curve through U_1, and that U_1 lies between C and B's indifference curve through U_2. Thus the "Edgeworth–Bowley dyopoly" cannot occur for these four combinations.

9. In the combinations $\alpha\beta$, $\beta\gamma$, $\gamma\delta$ and $\delta\alpha$, for A, U_1 is always the most favourable point (see Fig. 4.5).

Here U_2 is less favourable for A than C so A will look to achieve his independence point.

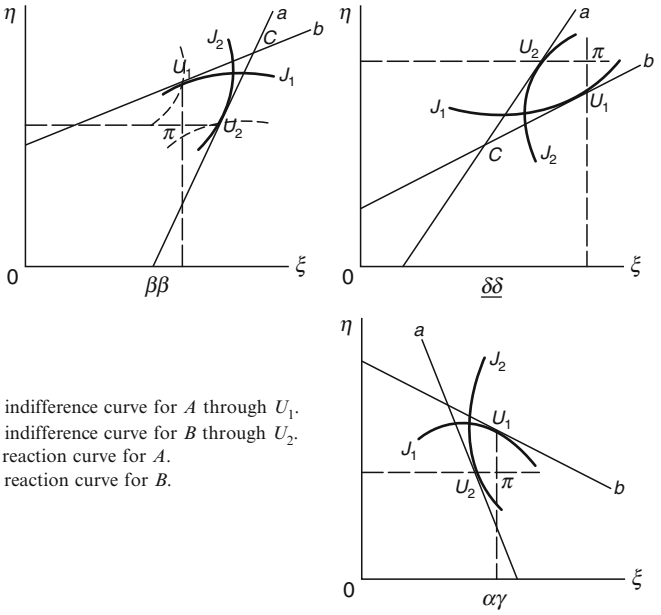


Fig. 4.4

Two possibilities are available for *B*:

a) Where his indifference curve through U_2 lies between U_1 and C_1, the most favourable point for him is thus U_1. The asymmetrical dyopoly therefore occurs at U_1.

b) Where point U_1 lies between C and his indifference curve through U_2, for *B* his independence point U_2 is thus the most favourable. The "Edgeworth–Bowley dyopoly" then occurs in π. In this case π is the least favourable point for both economic entities and could be especially unfavourable for *A*.

These four combinations thus show (possibly as a rule) the equilibrium of the asymmetrical dyopoly at U_1 or (possibly as an exception) the "Edgeworth–Bowley dyopoly".

The same applies for combinations $\beta\alpha$, $y\beta$, $\delta\gamma$ and $\alpha\delta$, if *A* is interchanged with *B*.

10. We can summarise the results of our analysis:

1) The "Edgeworth–Bowley dyopoly" occurs:

a) Without exception for the configurations $\alpha\alpha$, yy, $\beta\delta$ and $\delta\beta$.

b) As a special case for the configurations:

I. $\alpha\beta$, $\beta\gamma$, $\gamma\delta$ and $\delta\alpha$, where *A* always strives to achieve the position of independence (translator's note: see Heertje: 50) and is indeed especially disadvantaged by the "Edgeworth–Bowley dyopoly".

II. $\beta\alpha$, $\gamma\beta$, $\delta\gamma$ and $\alpha\delta$, where *B* always strives to achieve the position of independence and is indeed especially disadvantaged by the "Edgeworth–Bowley dyopoly".

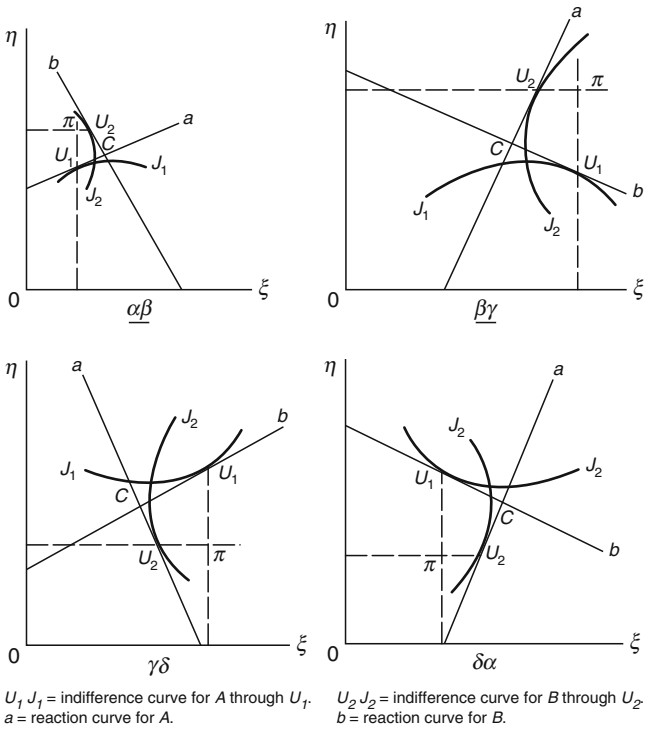


Fig. 4.5

2) The "Cournot dyopoly" occurs as a rule for the configurations $\beta\beta$, $\delta\delta$, $\alpha\gamma$ and $\gamma\alpha$.

3) The "asymmetrical dyopoly" occurs:

a) As a special case, for the configurations:

I. $\alpha\beta$, $\beta\gamma$, $\gamma\delta$ and $\delta\alpha$ with independence for *A* and dependence for B.

II. $\beta\alpha$, $\gamma\beta$, $\delta\gamma$ and $\alpha\delta$ with dependence for *A* and independence for *B*.

b) As an exceptional case for the configurations $\beta\beta$, $\delta\delta$, $\alpha\gamma$ and $\gamma\alpha$, and indeed in both forms (independence for *A*, dependence for *B* and vice versa).

11. The analysis carried out a moment ago allows us to assess the alternative price formation structures for the dyopoly-like, or as we can also say, for the "dyopoloid" market situations. To this end we must simply establish:

1) Whether the ordinal utility indexes (profits) of the two dyopolists or monopolists increase or decrease from the bottom to the top or from left to right respectively;

2) Whether the two reaction curves have a positive or a negative slope.

By answering these two questions we are placed in a position of:

1) Selecting one from all possible "quantity adjustment" options

2) In the same way selecting one from all possible "price adjustment" options

3) Ultimately making the last selection by comparing "quantity adjustment" and "price adjustment" positions that are possible and determining the actual price formation structure that occurs.

§ 2. The Simple Dyopoly

In the following study we are restricting ourselves to an examination of companies. For companies, profit has to measure, in monetary units, the size of the ordinal utility index.

I. The Demand Dyopoly

Here A and B are the two demand dyopolists. Let ξ and η be the two quantities demanded of the same good by A and B.

1. Where A demands a specific quantity ξ so the price he has to pay is lower and the greater his profit is, the less his rival B demands. Consequently his profit declines with increasing η i.e. it declines from the bottom to the top.

For the same reason B's profit declines from left to right (i.e. with increasing ξ), so for the demand dyopoly the only possible combinations are $\alpha\alpha$, $\alpha\beta$, $\beta\alpha$ and $\beta\beta$.

2. The more B demands, the smaller the share of the market that remains for A, and the higher the values are for the price coefficient region for A the more sacrifices he must make for the acquisition of the good. It will thus generally restrict his demand and so it can be assumed as a rule that demand for *A* decreases on the reaction curve for *A* when the demand for B increases and vice versa. The reaction curve for *A* therefore generally declines from left to right i.e. it has a negative slope. For the same reason, the reaction curve for *B* also has a negative slope. There may be exceptions however.

Consequently, for the demand dyopoly we can look at the configuration $\alpha\alpha$ as a general rule, while the configuration $\beta\beta$ is a special circumstance and the unsymmetrical configurations $\alpha\beta$ and $\beta\alpha$ are exceptions.

II. The Supply Dyopoly

Here, A and B are the two supply dyopolists. Let ξ and η be the two quantities supplied of the same good by A and B.

1. In the supply dyopoly, profit also declines from the bottom to the top and from left to right respectively because the more B offers for A's given supply, the lower

the price and hence the profit for A. The same applies mutatis mutandis for B. Thus here only the configurations $\alpha\alpha$, $\alpha\beta$, $\beta\alpha$ and $\beta\beta$ are also a possibility.

2. For the same reasons as for the demand dyopoly the reaction curves of the two suppliers generally have a negative slope. An increased supply by B narrows the market for A and applies pressure to the price, hence A will generally reduce his supply. The same also applies mutatis mutandis for B. There may be exceptions however.

Consequently, for the supply dyopoly the configuration $\alpha\alpha$ is likewise the rule, whereas the combinations $\alpha\beta$, $\beta\alpha$ and $\beta\beta$ are indeed already exceptions to the stated market share ranking.

It shows that for a dyopoly, as has already been explained in the second chapter, the "Bowley dyopoly" is the rule, the "Cournot dyopoly" is the special circumstance (for reasons of symmetry) and the "asymmetrical dyopoly" is the exception.

§ 3. The Supply Relationship Between Two Monopsonies

Here A is one demander and B is another.

I. Alternative Products

1. Where two demanded goods are alternative products there is then a similarity between this situation and the ordinary demand dyopoly. We initially analyse the effects of a change in B's behaviour on A's position. The results obtained can be immediately applied to B by simply substituting B for A.

Therefore, for alternative products, where B increases his demand or his price, the production possibilities for the good demanded by A are lessened. From this it emerges that A would otherwise have to pay a higher price for an unchanged quantity demanded or would obtain a reduced quantity of goods at an unchanged price.

In general terms, A's market contracts. By contrast, where B decreases his demand or his price, A's market expands. A can reduce his price for a constant quantity of goods demanded or obtain a greater quantity of goods with a constant price. Where A leaves his quantity demanded unchanged so as a result the greater his profit is, the lower the price set by B or the smaller the quantity demanded by B. On the other hand, where A leaves his price unchanged, the same relationship only applies between his profit and the price or the quantity demanded by B where the marginal return that A receives from the quantity of goods purchased is smaller than the price A pays for his good (per quantity unit). From now on we will restrict our study to the area in which price does not exceed marginal return in so far as it concerns the tendency of the rate of profit to change. This restriction is purely theoretical and does not restrict our investigation to any real extent. Namely, we can

be sure at the outset that the marginal return is always greater than the price[1)] both on the reaction curve for *A* as well as at its independence point, provided this does not lie on the ξ-axis.

We can extract the following conclusions from our deliberations:

1) A's profit is a decreased function of the quantity demanded by B and within the scope of our study it is also a decreased function of the price set by B.

2) In general A will respond to an increase in demand or a price increase by B – because his market will contract – such that he restricts his quantity demanded and increases his price. Usually, the quantity that A ultimately demands lies between the quantity originally demanded by A and the quantity A would obtain if he left his price unchanged. The price that A ultimately pays generally lies between the original price and the price that A would otherwise have to pay with an unchanged quantity demanded.

The same applies for B.

With the appropriate interpretation of the values ξ and η, these conclusions allow us to immediately show the individual shares of the market that are possible and indeed their established ranking, namely "the general rule", "special circumstances" and "exceptional cases".

2. Quantity adjustment. Let ξ and η be the quantity of goods demanded by both monopolists. As we saw a moment ago, A's profit declines if B increases his demand. The same applies to B, meaning that in our geometric representation both profits decrease from the bottom to the top and from left to right respectively.

A responds in general to an increase in demand by B by reducing his quantity demanded ξ. The same applies for B i.e. both reaction curves generally have negative slopes. The configuration $\alpha\alpha$ is thus the general rule and the combinations $\alpha\beta$, $\beta\alpha$, $\beta\beta$ are the exceptions. As a result, in our example quantity independence is generally more favourable than quantity dependence.

3) Price adjustment. Let ξ and η be the prices set by both monopolists. As we saw in 1, A's profit declines with a price increase by B and as a rule A likewise responds to this by a price increase. Vice versa, the same applies to B. Thus we generally obtain the combination $\beta\beta$, and $\alpha\alpha$, $\alpha\beta$, $\beta\alpha$ in exceptional cases. With regard to the demand for alternative products, price dependence is thus generally more favourable than price independence.

[1)]This follows from general monopoly theory where the difference between marginal return and price for the monopsonist, and between price and marginal costs for the monopolist respectively, is equal to price divided by the (positively correlated) elasticity of supply or demand or equal to the product of price and its flexibility (likewise positively correlated). In our example, the actual or underlying price response by the relevant demander to the demand or supply for the price he sets, that is taken as the basis for his calculation, can be plotted as a supply curve. See also: Cournot, ibid.: 49 and 50. Amoroso, *La curva statica*..., ibid.: 10. J. Robinson, ibid.: 54 and 55. Also: my *Grundlagen einer reinen Kostentheorie*, ibid.: 17, 47 and 48. Especially Satz (XXI).

4) Comparison between quantity independence and price dependence. A moment ago we saw that for A quantity independence or price dependence is the most favourable position. Similarly, in order to be able to judge this alternative, let ξ be the quantity demanded for A and η be the price set by B.

Where B increases his price so A must also pay a higher price if his quantity demanded remains unchanged, meaning A's profit declines from the bottom to the top. Where A increases his quantity demanded, so B, if he leaves his price constant, only obtains a reduced quantity of goods, i.e. his profit also declines from left to right. We also saw that with a price increase by B, the demand by A is restricted i.e. the reaction curve for *A* has a negative slope. On the other hand, B increases his price with an increase in demand by A i.e. the reaction curve for *B* has a positive slope. We thus obtain the combination $\alpha\beta$.

The situation must now be briefly explained once again. Since we have stated that ξ is the quantity demanded for A, thus U is obtained when B regards the quantity demanded by A to be an independent variable i.e. U_1 is the quantity independence point for A. On the other hand, U_2 occurs when A considers the price set by B to be an independent variable i.e. point U_2 is a price dependence point for A. However we now know that A's point U_1 is always more favourable than point U_2 for the combination $\alpha\beta$. As a result, in our example we can conclude that for A quantity independence is more favourable than price dependence. Exactly the same evidence emerges for B.

We therefore obtain the result that the "Bowley dyopoly" occurs as a rule here in regards to the demand for alternative products.

In conclusion it should be pointed out that by comparing the quantity and price adjustment situation we can also clearly deduce the form of the configuration that occurs. A price increase by B has an effect on the situation for A in exactly the same way as an increase in demand, and vice versa the same applies for B. Where we then have quantity adjustment situation $\alpha\alpha$ and price adjustment situation $\beta\beta$ so we obtain here, where ξ is the quantity demanded by A and η is the price set by B, the combination $\alpha\beta$, namely the quantity adjustment situation for A combined with the price adjustment situation for B.

II. Complementary Products

We can shorten our account now as the method of examination we use is the same as in the previous section.

1. Where B increases his demand or his price, so the production possibilities for the good demanded by A for complementary products are improved. From this it emerges that A only needs to pay a reduced price for an unchanged quantity demanded or can obtain a greater quantity of goods for an unchanged price. Expressed in general terms, A's market has expanded. By contrast, where B decreases his demand or his price, so A's market is constricted. A must increase

his price for a constant quantity demanded or reduce his quantity demanded for a constant price. Where A leaves his quantity demanded unchanged, his profit is thus greater the more B demands or the higher B sets the price. By contrast, where A leaves his price unchanged the same applies only if the marginal return for A is not less than the price he must pay. In order to bring about a simple formulation of our theory here, we can also restrict the study to the area in which price does not exceed marginal return and where it is restricted in an absolutely methodical way for the same reasons as in the previous section[1)].

We can hence establish the following regularities:

1) A's profit is an increased function of the quantity demanded and price for B.

2) As a general rule, A will respond to an increase in demand or a price increase from B with an increase in demand and a decrease in price. The new quantity demanded will be greater than for the original price and smaller than it would have been with an unchanged price. The new price is lower than the original one and higher than it would have been with an unchanged quantity demanded.

The same applies to B.

2. Quantity adjustment. Where ξ is the quantity demanded by A and η is the quantity demanded by B so we immediately see that here the combination $\delta\delta$ represents the general rule, meaning that quantity dependence is generally more favourable for both monopolists than quantity independence.

3. Price adjustment. Where ξ is the price set by A and η is the price set by B we thus obtain the combination $\gamma\gamma$. Here price independence is hence generally more advantageous than price dependence for both monopolists.

4. Comparison. Let ξ be the price set by A and η the quantity demanded by B. As a general rule, A responds to an increase in demand by B with a decrease in price, meaning his reaction curve has a negative slope. As a general rule, B responds to a price increase by A with an increase in demand, meaning his reaction curve has a positive slope. Since A's profit is an increasing function of η, B's profit is an increasing function of ξ and so as a rule we obtain $\gamma\delta$ as the "comparison combination". Here U_1 is the price independence point for A because ξ is A's price. Correspondingly U_2 is the quantity dependence point for A. In the combination $\gamma\delta$ for A, point U_1 is more favourable than point U_2. Hence price independence is usually the most favourable position for A for complementary products. The same is true for B if ξ is the quantity demanded by A and η, the price set by B, consequently we see that the "Edgeworth dyopoly" is the rule in our example.

[1)] See pp. 50, text and footnote.

§ 4. The Demand Relationship Between Two Monopolies

Here A is one supplier and B is the other.

I. Rival Goods

1. Where B increases his supply or decreases his price, the market for A will contract. Where B decreases his supply or increases his price, the market for A will expand. We restrict the scope of our study to those points where price is not less than marginal costs[1)]. Then we can set out the following rules:

1) A's profit is a decreased function of supply and an increased function of the price set by B.

2) As a general rule, A will respond to an increase in supply or a decrease in price from B by a decrease in supply and price where both variables will be intermediate values in the same sense as in the description from the previous paragraph.

The same applies for B.

2. Quantity adjustment. Where ξ is the quantity supplied by A and η is the quantity supplied by B we see from our geometric representation 1) that profit declines from the bottom to the top and from left to right respectively and 2) that generally both reaction curves have a negative slope. The combination $\alpha\alpha$ thus usually occurs where quantity independence is preferred to quantity dependence by both monopolists.

3. Price adjustment. Where ξ is the price set by A and η the price set by B we see 1) that profit increases from the bottom to the top and from left to right respectively and 2) that generally both reaction curves have positive slopes. The combination $\delta\delta$ applies as a rule i.e. each monopolist prefers price dependence to price independence.

4. Comparison. Let ξ be the quantity supplied by A and η the price set by B. Where A's profit is an increased function of the price set by B thus it increases here from the bottom to the top. As a general rule, A responds to a price increase by B by increasing his supply i.e. his reaction curve has a positive slope. Situation δ thus applies for A. B's profit is a decreased function of supply by A and as a general rule, B responds to an increase in supply by A with a price reduction i.e. type α applies here for B. We consequently have the "comparison combination" $\delta\alpha$ and see that quantity independence is more favourable for A as a general rule than price dependence. The same also applies for B where ξ is the price set by A and η is the quantity supplied by B. We see that the "Bowley dyopoly" generally occurs for rival goods.

[1)] See pp. 50 text and footnote. Here the response of prices to changes in supply or demand to price changes can be plotted as a demand curve.

II. Complementary Goods

1. The following relationships apply here, based on similar observations to the previous section and with the same restrictions to the scope of our study:

1) A's profit is an increasing function of the quantity supplied by B and a decreasing function of the price set by B.

2) A generally responds to an expansion in supply or a price reduction by B with an expansion in supply and a price increase.

Conversely the same applies to B.

2. Quantity adjustment. Where ξ is the quantity supplied by A and η is the quantity supplied by B, we thus obtain the combination $\delta\delta$, as can be easily seen, and we can note that quantity dependence here is more favourable than quantity independence for both monopolists.

3. Price adjustment. Where ξ is A's price and η is B's price we thus obtain the combination $\alpha\alpha$ as a general rule, where price independence is more favourable than price dependence for both monopolists.

4. Comparison. Let ξ be the price set by A and η the quantity supplied by B. A's profit increases with increased η and thus from the bottom to the top. The profit for B decreases with increasing ξ and thus from left to right. The reaction curve for A has a positive slope and the reaction curve for B has a negative slope. We obtain the "comparison combination" $\delta\alpha$ from which we learn that for A price independence is generally more favourable than quantity dependence. We obtain the same result for B where ξ is the quantity supplied by A and η is the price set by B. For complementary goods the "Edgeworth dyopoly" thus usually occurs.

§ 5. The Intermediary Bilateral Monopoly

Here A is the monopolist and B the monopsonist.

1. Where B increases his demand or his price, there is an increased stimulus for production for the "middle tier" of rival companies that will have an impact in an expanding market for A, namely A's profit increases if A's supply is constant and we restrict ourselves to those points where A's marginal costs do not exceed A's price, and also where A's price is constant. A's profit is thus an increasing function of both the quantity demanded and B's price. With an increase in demand or a price increase by B, as a general rule A will increase both his quantity supplied as well as his price and achieve "intermediate values" in the same sense as already described earlier.

Where A increases his supply or he reduces his price, the production possibility of the good that B demands is thus improved and the market for B expands. Where B leaves his price unchanged he can now obtain a greater quantity of goods. As a result his profit increases if we restrict ourselves to the area where the price B has to pay does not exceed his marginal return[1)]. Where B leaves his quantity demanded unchanged

[1)] See pp. 50 and pp. 53 and Remarks and pp. 59, Remarks.

he now only needs to pay a reduced price to similarly increase his profit. B will generally respond to an increase in supply or a decrease in price by A and also in point of fact, to "intermediate values" by both increasing his demand and reducing his price.

2. Quantity adjustment. Where ξ is the quantity of goods supplied by A and η is the quantity of goods demanded by B, so A's profit is an increasing function of η and B's profit is an increasing function of ξ. Furthermore both reaction curves generally have a positive slope, meaning that we usually obtain the combination $\delta\delta$ where quantity dependence for both monopolists is more favourable than quantity independence.

3. Price adjustment. Where ξ is the price set by A and η the price set by B, A's profit is an increasing function for η and B's profit is a decreasing function for ξ. A responds to a price increase by B with a price increase and the same applies vice versa for B meaning that both reaction curves have positive slopes. The relevant combination is thus $\delta\beta$, where price independence for both monopolists is more favourable than price dependence.

4. Comparison.

a) Where ξ is the price for A and η the quantity demanded by B, A's profit is an increasing function of η and B's profit is a decreased function of ξ. A responds to an increase in demand by B with a price increase. His reaction curve thus has a positive slope. B responds to a price increase by A with a reduction in demand. His reaction curve thus has a negative slope. The "comparison combination" is consequently $\delta\alpha$, meaning that price independence for A is more favourable than quantity dependence.

b) Where ξ is the quantity supplied by A and η is the price set by B, A's profit is an increasing function of η and B's profit is an increasing function of ξ. Where B increases his price, A generally responds with an increase in supply meaning his reaction curve has a positive slope. Where A increases his supply, B generally responds with a reduction in the price meaning his reaction curve has a negative slope. The "comparison combination" here is $\delta\gamma$, from which we infer that for B price independence is also more favourable than quantity dependence.

We can conclude from this that the "Edgeworth dyopoly" occurs in the intermediary bilateral monopoly.

§ 6. A Numerical Example

1. This study has so far shown that dyopoly theory is of central importance within the subject as a whole because a dyopoly is not only a concrete and simple version of an oligopoly, but rather the findings of any investigation into a dyopoly also apply to the three most important types of market relationship. Therefore it is appropriate at this point to present a numerical example with simplified assumptions in order to convey a clear picture of the basic facts about a dyopoly.

2. We will look at a supply dyopoly which has two companies supplying it, A and B. We will assume that both companies only have fixed and marginal costs[1)].

[1)]See Schmalenbach, *Grundlagen der Selbstkostenrechnung und Preispolitik*, 5th ed. Leipzig, 1930: 32 et seq. Translator's note: (Proportionaler Satz), see Scherer: 65).

Firm A has fixed costs of RM 500.00 (Scherer: 62 Reichsmark) per day. Its "marginal cost"[1)] was RM 2.00 per zentner (translator's note: one zenter is equivalent to approximately 50 kg) for the product. Firm B has fixed costs of RM 600.00 per day and marginal costs of RM 1.50 per zentner for the product. In this way we obtain the following pay-off matrix (Scherer: 62) for A and B:

Daily production in zentners	Daily costs for A in RM	Daily costs for B in RM
100	700	750
200	900	900
300	1,100	1,050
400	1,300	1,200
500	1,500	1,350
600	1,700	1,500
700	1,900	1,650

Both these companies experience demand for specific quantities of the product per day at specific prices. The price of RM 10.00 should be prohibitive in such a way that nothing is sold at this price. The quantity of 1,000 zentners per day is believed to be able to entirely satisfy demand so that this quantity could only be fully accommodated for free. Apart from that, the following matrix is believed to apply:

Price in RM	9	8	7	6	5	4	3	2	1
Demand per day in zentners	100	200	300	400	500	600	700	800	900

We are now able to determine the profit for A and B in relation to each supply for A and B. For example, where A supplies 200 zentners and B supplies 300 zentners per day, total output (Scherer: 65) is therefore 500 zentners per day. The price is then RM 5.00 per zentner. Daily revenue for A amounts to RM 1,000.00 with a daily profit (revenue minus costs) of RM 100.00. Daily revenue for B is RM 1,500.00 with a daily profit of RM 450.00. We thus obtain a profit table for A and B respectively:

A's profit table (translator's note: (Gewinntabelle), see Scherer: 61) for every supply combination.

Supply by B	Supply by A						
	0	100	200	300	400	500	600
0	−500	200	700	1,000	1,100	1,000	700
100	−500	100	500	700	700	500	100
200	−500	0	300	400	300	0	−500
300	−500	−100	100	100	−100	−500	−1,100
400	−500	−200	−100	−200	−500	−1,000	−1,700
500	−500	−300	−300	−500	−900	−1,500	–
600	−500	−400	−500	−800	−1,300	–	–

In the same way we obtain a profit table for B:
B's profit table.

Supply by B	Supply by A						
	0	100	200	300	400	500	600
0	−600	−600	−600	−600	−600	−600	−600
100	150	50	−50	−150	−250	−350	−450
200	700	500	300	100	−100	−300	−500
300	1,050	750	450	150	−150	−450	−750
400	1,200	800	400	0	−400	−800	−1,200
500	1,150	650	150	−350	−850	−1,350	–
600	900	300	−300	−900	−1,500	–	–
700	450	−250	−950	−1,650	–	–	–

There needs to be a brief explanation of these tables. Negative profits means losses. Where a company has no production and thus lies inactive, its loss is thus equal to the fixed costs because we are assuming for simplicity's sake that fixed costs and idle time costs are the same. Profit is simply read off as soon as the supply for A and B are known e.g. if the profit that A or B obtains when A supplies 300 and B supplies 400, then the amount in the column is 300 and the amount in the row is 400. For A this therefore makes RM – 200.00 (a loss of RM 200.00), while for B it makes RM 0.0.

3. Where B supplies 100 zentners, A will supply the quantity that provides the largest profit for A. We can read off the table that this quantity lies between 300 and 400 zentners and it is indeed exactly 350 zentners since the profits are equal at 300 and 400 zentners. The calculation of the quantity of goods supplied by A is to be conducted using sequences of statistics as follows, in exactly the same way as the calculation for the place of greatest (most frequent) density[1)]. In this way we can plot the reaction curve (reaction function (translator's note: (Bestimmungsgleichung), see Heertje: 53)) for A as a sequence from which we read off the most favourable supply by way of A from A's profit table for each supply by B. We then obtain:

A supplies in zentners	400	350	300	250	200	150	100	50
for a supply by B in zentners	0	100	200	300	400	500	600	700

Similarly, the reaction curve (reaction function) for B is presented as a sequence:

B supplies in zentners	425	375	325	275	225	175	125	75
for a supply by A in zentners	0	100	200	300	400	500	600	700

The quantities supplied for B cannot easily be extracted from the profit table because true maximum profits lie between these quantities.

4. The Cournot dyopoly then results if each company orientates itself to its rival's supply. The quantities supplied for A and B that occur in the Cournot dyopoly are

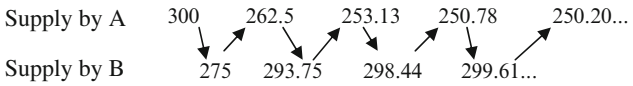
[1)]See instead all of: W. Winkler, "Statistik", Leipzig, 1925 (*Wissenschaft und Bildung*, Vol. 201: 59–61).

those that simultaneously occur in the sequence of reactions for A and the sequence of reactions for B. In our example, this is 250 zentners for A and 300 zentners for B. We can immediately extract from the sequence of reactions for A that A supplies 250 zentners when B supplies 300 zentners. Conversely it is easy to see from the sequence of reactions for B that B supplies 300 zentners when A supplies 250 zentners.

Total output is 550 zentners here and the price is RM 4.50, the profit[1)] for A is RM 125.00 and the profit[2)] for B is RM 300.00.

Cournot[3)] has very clearly shown that if each dyopolist repeatedly adapts to the current behaviour of his rival and thus also repeatedly responds to the manner in which his sequence of reactions stipulates, then the Cournot dyopoly is stable. Then (and only then) does a divergence by him trigger an automatic correction mechanism. Let us assume that by chance A does not offer 250 but instead 300 zentners.

Then the following successive supply for both rivals can be read off from both sequences of reactions:



We see that both sequences very quickly converge on the pair of variates (250, 300). The difference between a random level taken from these intermediary supply levels with respect to the corresponding supply of the Cournot dyopoly is always a quarter of the difference in the previous values.

5. The position of independence for A occurs if B orientates himself towards the supply for A and A takes this fact into account. This means that A realises that B responds in this way to each supply level for A as can be seen from B's reaction sequence. A will thus initially establish that under these circumstances only the supply combinations can be achieved that occur in B's reaction table. He will then choose those combinations that achieve the greatest profit for him (A). He thus establishes the following calculation:

Supply by A	100	200	300	400	500	600
Supply by B according to its reaction sequence	375	325	275	225	175	125
Total supply	475	525	575	625	675	725
Market price	5.25	4.75	4.25	3.75	3.25	2.75
A's profit[4)]	−175	50	175	200	125	−50

[1)] The revenue for A is: 250 . 4.50 = RM 1,125.00
his costs: 500 + 250 . 2 = RM 1,000.00

[2)] The revenue for B is: 300 . 4.50 = RM 1,350.00
his costs: 600 + 300 . 1.50 = RM 1,050.00

[3)] Ibid.: 70 and Fig. 2. See also Schneider, *Reine Theorie*, ibid.: 143.

[4)] A's revenue is always its supply multiplied by price. A's costs that need to be deducted can be directly extracted from the cost table.

The greatest profit for A is in A's supply of 375 zentners. B then supplies 237.5 zentners in line with his reaction sequence. The total output amounts to 612.5 zentners and the price is RM 3.875 per zentner. A's profit here is RM 203.13 and thus RM 78.13 higher than its profit in the Cournot dyopoly. By contrast, B incurs a loss of RM 35.94. He cannot improve his position in any way however if A persists in supplying 375. The amount of RM 35.94 is still the minimum loss he has to suffer under the given market conditions.

6. The position of independence is correspondingly similar for B. Where A orientates himself according to his sequence of reactions towards B's current supply so B will make the following calculation:

Supply by B	100	200	300	400	500	600	700
Supply response by A	350	300	250	200	150	100	50
Total supply	450	500	550	600	650	700	750
Market price	5.50	5	4.50	4	3.50	3.50	2.50
Profit made by B	−200	100	300	400	400	300	100

We immediately see that B retains the greatest profit if he supplies 450 zentners. In this case A supplies 175 zentners. Total output amounts here to 625 zentners and the price is RM 3.75. B's profit is RM 412.50 and thus higher by RM 112.50 than in the Cournot dyopoly. By contrast, A incurs a loss of RM 193.75. He is incapable of improving his position however with a supply by B of 450 zentners. In the given market conditions RM 193.75 is the smallest loss A has to suffer.

7. As each of the two suppliers has the best terms if he achieves his position of independence, thus each will strive to achieve his position of independence. He will try to convince his rival that his supply is a given variable that the other has to take into account. A and B will thus simply "have a go" at supplying their independent supply of 375 zentners and 450 zentners respectively and so we obtain the Bowley dyopoly. Total output is 825 zentners here. The price is RM 1.75. A's loss amounts to RM 593.75 and is thus higher than his fixed costs. B's loss is RM 487.50. This situation is unsustainable for both of them. But the surrender one of them would give the other the position of independence and mean the destruction of both of them since we have seen that when one supplier achieves his position of independence the other suffers a loss and therefore must permanently abandon his operations. If they do not come to an agreement beforehand, each of them will thus battle with the other until they give way. In our example, Firm A will probably be the loser because it incurs the greater losses. This is also in line with the structure of our example in which B was assumed to be a modern company (with higher costs and lower marginal costs).

8. The task of previous studies was to answer the question about the structural conditions the free market capitalist market functions under and which it does not. A discussion about the way to resolve the antagonism associated with unstable market structures and market equilibriums should be left for the last chapter. However we want to anticipate this chapter by making one point here.

Rather than entering into a war that must end with the destruction of one supplier, both companies can also come to an agreement and set up a monopoly together. Then in our example, Firm A's plant would be shut down because modern Firm B produces every quantity of product with reduced marginal costs whereas fixed costs must be borne by both companies anyway (since we have given them the same idle time costs). Then exactly that quantity for B is produced and supplied that would make a maximum profit for B with a supply of zero for A. Fixed costs for A are simply deducted from this profit. The remainder constitutes net profit.

We can immediately see from the profit table for B that this quantity supplied would amount to 425 zentners. The price would be put at RM 5.75. Net profit would amount to RM 706.25. It is considerably higher here than the sum of the independence profit of the two companies, namely RM 203.12 + RM 412.50 = RM 615.63. It would increase by the same magnitude in which the plant that was shut down by A would be written back and amortized, and ultimately increase by the amount of A's fixed costs to RM 1,206.25. It is thus to be assumed that both companies, rather than becoming involved in a risky war involving heavy losses, come to an agreement and share the monopoly profit according to a ratio of some sort. It is clear and we can easily see from the figures that the demand side would be the one to suffer in this case.

9. If we compare our example with the "general analysis" in §1 we can immediately see that the conditions of combination $\alpha\alpha$ have been met. The profit table shows that profits decrease from the top[1)] or the right. Both reaction sequences also show that both reaction curves have a negative slope. The numerical results also conform to the General Principles theory[2)]. We would not have expected any other result.

[1)]In the profit table for A, profits decrease from the "top" to the "bottom". However, this only occurs because the quantities supplied by B in the table have been reduced from the "top" to the "bottom" and conversely from the "bottom" to "top" in the representation of the coordinates.

[2)]Determining the output of both companies if they were to agree to a cartel with the purpose of forming a monopolistic domination of the market would generally happen in such a way that the two profit tables would be added together (overlaid) and this determines the point of maximum total profit. In our example, this point is (0; 425). We have determined this using a simple technique since the method is not important here.

Chapter 5
Landmark History of Economic Thought

Our theory shows that we have two problem areas to deal with as regards "unstable market structures" (translator's note: (gleichgewichtslose), see Senn: 15). At the heart of a problem that is as interesting as it is important is the dyopoly, encompassing both the oligopoly and dyopoloid market relationships. The second problem area is that of the bilateral monopoly and the bilateral oligopoly. We now turn to the history of the theories which these two areas have focused on. In the first paragraph we will give an outline of dyopoloid theory, in the second we will show its development to date and in the third we will refer to scholarly opinion about bilateral monopolies.

§ 1. First Attempt to Solve the Problem of Dyopoly

I. August Cournot 1838

1. The problem of the supply dyopoly and the supply oligopoly was first examined by Cournot in the area of exact theory. In 1838 his principal work was published[1)], including the seventh chapter, "Of the Competition of Producers". With his famous example of the two mineral water sellers he explains his theory of supply dyopoly and then expands it for the theory of supply oligopoly. We have already encountered his attempt at a solution as the "Cournot dyopoly" and the "Cournot oligopoly". His error is that he fails to probe into the assumptions under which the "Cournot dyopoly" occurs but instead assumes from the outset that each supplier views the quantity supplied by their rival as an independent variable that is always constant. We have seen that if this assumption were to be applied in exactly this way, Cournot's solution would be correct and would produce a stable price formation structure. However the behaviour assumed by Cournot for each supplier

[1)] Ibid.

H. von Stackelberg, *Market Structure and Equilibrium*,
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does not follow from this problem and even if – based on the supplier's pursuit of a position of dependence – the "Cournot dyopoly" occurs, this does not, as we have seen, produce a stable equilibrium. Cournot has in fact shown that his solution produces a stable equilibrium when each supplier follows his rival's current supply. However he has not investigated whether the behaviour of the supplier himself is stable. We have seen that in reality the Cournot dyopoly does not produce a state of equilibrium, but rather one of warfare.

In Cournot's Chap. 2, which is meant to illustrate his dyopoly theory, he assumes that both reaction curves have a negative slope.

This configuration is, as we have shown, in fact not the only possible one, but is the more probable, especially when the position of independence and not of dependence is, as we know, the more favourable for each supplier and especially therefore under the "Bowley dyopoly" rather than the "Cournot dyopoly".

2. Cournot also examines the demand relationship between two monopolies in the study stated above. In Chapter IX he writes about "The Mutual Relations of Producers". He examines the production of brass from zinc and copper and makes the assumption that both these metals are only useful for the production of brass and are only alloyed in fixed proportions. Cournot rightly assumes that here it is only a question of price adjustment. His attempt at a solution is the "Cournot price dyopoly" that we are already familiar with. He assumes that each supplier considers the price set by the other as an independent variable that is always constant and sets his price for the given price of the complementary good such that his profit is at a maximum. In addition to this the same can be said for the "Cournot quantity dyopoly". Cournot does not test the conditions under which such behaviour by the supplier occurs, so he does not realise that his solution is not the only possible solution and that – even if it were – it would not produce an equilibrium but lead to warfare instead.

As both Chaps. 7 and 8 show, Cournot calculates the possibility of price reaction curves with a positive as well as a negative slope. We have seen that in the first scenario the "Cournot price dyopoly" is for all practical purposes to be expected. By contrast, in the second, which ought to represent the rule for complementary goods, the resulting price formation structure is not the "Cournot price dyopoly", but the "Edgeworth dyopoly" instead.

II. Joseph Bertrand 1883

The French mathematician Bertrand undertook a review[1)] of Cournot's attempt at a solution for the simple dyopoly and this also resulted in a second attempt at a solution. Bertrand's initial position can be easily described with the help of our terminology. Whereas Cournot started out from the assumption that both dyopolists

[1)] *Journal des Savants*, Year 1883, Paris: 503.

pursue quantity adjustment, Bertrand argued in contrast that for them it was merely a question of price adjustment. Bertrand's argument was as follows: "Cournot supposes that one of the rivals will reduce his prices to attract purchasers to him and the other will in turn reduce them further to entice them back and they will not stop doing so until one of them would no longer have anything to gain from lowering his prices, even if his rival were to surrender. One incontrovertible objection appears. In this hypothesis only one solution is possible – the reduction would have no end. Indeed, regardless of the common price agreed by them both, if one of the rivals were to reduce his price of his own accord, he will attract all the sales, disregarding any trivial exceptions, to him and will double his revenue if his rival allows him to. If Cournot's rules conceal this obvious result, it is because, by a strange oversight, he introduces the quantities sold by the two rivals under the name of D and D', and by treating them as independent variables he supposes that if one is changed by the decision of one of the company owners, the other should remain constant. The contrary is obviously true" (translator's note: quotation translated from the original French).

Cournot's assumptions are different to the way Bertrand has apparently understood them. In Cournot's problem each supplier regards the rival's output quantity as being independent of their behaviour (not as an absolute constant). By contrast, Cournot assumes that price is always the same for both dyopolists. For Cournot both dyopolists look to influence price only by achieving a corresponding turnover or by quantity adjustment, rather than, by contrast, directly through price setting, which in our terminology would be "price adjustment".

According to Bertrand each dyopolist can pull all demand towards him by another similar minor reduction in his price to below the price set by his rival. It would therefore start a chain of undercutting that must be absorbed by each profit. We see that the accusation he levelled against Cournot can also be levelled at him. Whereas for Cournot a dyopolist initially regards the turnover of his rival as a constant variable, according to Bertrand he initially regards the price set by the rival as a constant. Only then does undercutting the other side's price make sense. If the first dyopolist studies Bertrand's theory closely and understands the futility of undercutting on price and then renounces any price setting, simply launching his independent supply on the market. If he simultaneously delivers the assertion that he wants to sell this quantity of goods at whatever arbitrary price via the market mechanism – by competing on demand – we can now ask, how would this happen? It would make no sense then for the second dyopolist to undercut the first dyopolist's price because no such price exists. The second dyopolist would notice that with an arbitrary price he can always just deduct the difference between total demand and the output of the first dyopolist, which is now effectively constant. He will thus set the price or his quantity supplied – in this case leading to the same result – in such a way that he achieves maximum profit for the given supply by the first dyopolist. As a result however we obtain an asymmetrical dyopoly showing the quantity independence of the first dyopolist and the quantity dependence of the second that we are familiar with. We do not need to explain in more detail here that the second dyopolist will furthermore also try to achieve quantity independence by

which the "Bowley dyopoly" eventually occurs. It is enough for us to have indicated the content of the Bertrand attempt at a solution and why we have to disagree with him. In our opinion Bertrand's attempt at a solution even takes a step backwards compared with Cournot because Cournot has at least described a possible situation, whereas in our view Bertrand's notional price war cannot form any part whatsoever of a correct theory of dyopoly.

Nevertheless, Bertrand's attempt at a solution has indeed played a large role in economic theory because authorities like Marshall and Edgeworth[1)] have approved Bertrand's critical analysis. Marshall applies Bertrand's theory to the example of production with increased yield[2)] whereas Edgeworth[3)] applies it to a borderline case of production with decreased yield, namely for a situation where the production capacity of any producer is permanently limited.

Edgeworth's argument is as follows: Where the production of both suppliers is strictly limited, this initially exerts downward pressure on the price to such an extent that each can sell to the limit of his productive capacity. However, the first of the two dyopolists will then increase his price again. He is always left with the difference between the total sales at the respective price and the maximum sales of his rival. In this way he will set his price so that he maximises profit under these circumstances. The second dyopolist is then left with no other course of action than to follow the first dyopolist's price increase and increase his price to the same level. After that, reciprocal undercutting begins again. Edgeworth thus asserts that the price will fluctuate up and down in the way described.

We can advance the same arguments against this opinion as we did for Bertrand's attempt at a solution. One dyopolist needs only to launch his independent supply at an arbitrary price on the market to force his rival to set his price in such a way as to achieve the rival's asymmetrical dyopoly with quantity dependence.

III. Francis Isidro Edgeworth 1897

In his pivotal essay "La teoria pura di monopolio"[4)] Edgeworth has linked the attempt at a solution described a moment ago as a special case, with a general observation that we must consider to be an outstanding source of help with the problem of dyopoloids, which is curiously left unheeded in the subsequent discussion of this subject. For Edgeworth the simple dyopoly is a special case in the

[1)]Compare even in 1891 in *Revue d'économie politique*: 10 et seq.: "La théorie mathématique de l'offre et de la demande et le coût de production", especially pp. 16 et seq.

[2)]A. Marshall: *Principles of Economics*, 1st ed. London, 1890, Footnote 2 on pp.485. See also 8th ed., London, 1925, Book V, Chap. XIV, § 9.

[3)]*La teoria pura di monopolio*, ibid.: 21–26.

[4)]Ibid.

demand relationship between monopolies. He examines the price formation occurring here in which he also assumes price adjustment. We have seen that this is possible when goods supplied by both producers are not identical. The incorrect (in our view) account of simple dyopolies is a result of the improper use of an examination of price adjustment for solving this problem. By contrast Edgeworth has made a contribution of lasting importance to the theory of tied demand for monopolised products.

Edgeworth looks at the supply of two monopolised goods[1)]. The quantity demanded of any good depends on both prices. The goods are thus "tied" and in fact he assumes they are complementary i.e. the quantities demanded are a decreased function of both prices. Initially Edgeworth does not describe the general economic problem but looks in more detail at the marginal case in which products are demanded in fixed proportions, hence exactly the problem that Cournot focussed on in the IXth Chapter of his "Recherches" (translator's note: "Recherches" is 'Research' in English). Referring specifically to Cournot he thus presents his attempt at a solution and presents the "Cournot price dyopoly". He then criticises this solution in which he remarks on the possibilities of the "position of independence". We can verbalise his critical analysis that he expressed in mathematical formulae[2)] in the following way, but this is unfortunately not up to a high standard in terms of clarity, brevity and rigour.

Where the first monopolist behaves according to his reaction function he thus always views the price set by the second monopolist as a given variable which he follows and so the other monopolist will no longer remain on his reaction curve. On the contrary he will increase his profit if he includes the responses of the first monopolist in his calculations and sets a price that provides him with the most profit, taking into consideration the responses of the first monopolist. This means

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[1)] *La teoria pura*, ibid.: 26 et seq., or "The Pure Theory",. . . ibid.: 121 et seq.

[2)] Edgeworth initially shows that the profits U and V of the two monopolists depend on the demand functions of both prices. Then he continues (from above): "According to Cournot, prices p_1 and p_2 are determined by the equations

(1) $\frac{\partial U}{\partial p_1} = 0$ (2) $\frac{\partial V}{\partial p_2} = 0$

As an explanation we note that the first of these equations with the given p_2 determines value p_1 and maximises the profit U with this p_2. This therefore applies for the first supplier and is simply the equation for his reaction curve. The same applies for the second equation in relation to the second monopolist". Edgeworth then continues (see above reference): "Conversely, it can be pointed out that these equations cannot apply simultaneously, since where one of these e.g. the first, applies, then the second no longer does. Why should the second monopolist stop at the point at which $\partial V/\partial p_2 = 0$?

By making changes to his price he will go to the point at which the differential quotient is:

$\partial V/\partial p_2 + \partial V/\partial p_2 \cdot dp_1/dp_2 = 0$

where dp_1/dp_2 arises from equation (1) as a result. Together with (1) this equation will determine p_1 and p_2".

that if the first monopolist follows the price set by the second then the latter will achieve his position of independence.

Edgeworth applies this decision to the general case of complementary demand and expresses the view that in the end neither the "Cournot price dyopoly" nor the price independence point would be achieved by the monopolist. Edgeworth did not examine the problem further but instead formed the opinion that an equilibrium definitely does not occur for rival products and probably does not occur for complementary ones either.

Our theory shows that this latter conclusion by Edgeworth incorrectly describes the real situation since in both cases the final conclusions are similar. Edgeworth's critical analysis is however very important since it introduces the term "price independence" into the discussion. Therefore we have a situation in which each of the two monopolists are striving to achieve price independence as described by the "Edgeworth dyopoly" even if Edgeworth himself does not really probe into this market situation.

IV. Vilfredo Pareto 1909

Although Pareto in 1896 in his *Cours*[1)] is essentially still adding to the Bertrand solution, he relies on a completely different train of thought as a basis for his subsequent attempt at a solution to the problem of the simple dyopoly. In his "Manuel"[2)] published in 1909 he examines the "monopoly of two individuals and one good".

In 1911 he analyses the same problem in his "Economie mathématique"[3)]. We base our illustration of his attempt to solve the problem on this second version because the description is better and does not differ from the first in principle. Pareto argues as follows: Where the market price is dependent on the output quantities of both "monopolists", these two quantities are the independent variables which both sets of profits also depend on. Now Pareto deals with the whole problem as if both outputs were independent variables from the point of view of either of the two "monopolists". In other words, he assumes that it belongs to the essence of the problem that each "monopolist" can vary both quantities independently and since the profit in monetary terms or the ordinal utility index of each of the "monopolists" is dependent on both quantities, each would subsequently strive to set both quantities such that his profit would be at a maximum. Pareto tries to show that three independent equations for determining the two unknown variables are eventually

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[1)] *Cours d'économie politique*, Lausanne, 1896: 67 and 68.

[2)] *Manuel d'économie politique* [*Manual of Political Economy*], Paris, 1009: 595 et seq. (similarly 2nd ed. 1927).

[3)] "Economie mathématique" in: *Encyclopédie des sciences mathématiques*, Paris-Leipzig, 1911, Tome 1, Vol. 4: 606/7.

produced from striving to do so and thus the problem is generally overdetermined. What does this result mean now?

The assumption that the first monopolist could set both quantities in whatever way he likes only means in practice that he can firstly change his own output as he wishes – this is also our view – and secondly that he can command his rival to achieve a particular output – something that we in fact hold to be possible but inconsistent with the assumption of free market price formation. It is however immediately obvious that it would be absurd to simultaneously make these assumptions for both monopolists. Amoroso[1)] has clearly and accurately examined and criticised Pareto's reasoning. He writes (see earlier reference), "It is an overdetermined system. What does this overdetermination mean? It means that it is impossible to accept that A forces B to do his will and that simultaneously B forces A to do what he wants. This impossibility is obvious however and if that is the meaning of the monopolists' problem then it would be pointless to address it. If B is A's slave then is it clearly impossible to assume that A is simultaneously B's slave. . ."

The Pareto attempt at a solution thus clearly does not achieve its main goal. In reality Pareto does not "solve" (translator's note: (löst), see Senn: 16) an economic problem but a logical one instead, which does not require a solution but presents a complete contradiction[2)].

In contrast the problem of the dyopoly is as follows: What occurs when two suppliers enter a market and both can randomly vary their own supply? Cournot, Bertrand and Edgeworth try to provide the answer to this question. In his analysis of this attempt at a solution, Pareto fails to recognise this and deals with a completely different problem himself, one that does not fall within economic theory.

It is significant and at the same time it highlights Pareto's error in the treatment of the simple dyopoly that he only deduces the "Cournot price dyopoly" for the case of a "monopoly of two individuals and two goods" (Appendix No. 71 et seq.) and maintains that it is a true equilibrium. Where two different goods are concerned, he regards both prices as the independent variables which the output quantities and profits of both suppliers depend on and which all other variables in the system depend on too. He now leaves each supplier to vary his own price and to consider the price of the others respectively as given. However, we have shown that this solution is only implicitly correct and produces an equilibrium where there is no relationship between the two markets themselves, or to express this another way, where there is no dyopoloid. Pareto's attempt at a solution to the dyopoloid problem thus represents a step backwards compared with Edgeworth's attempt.

[1)] *La curva statica*, ibid.: 18, 19.

[2)] It shows that the treatment of this logical problem by Pareto is also flawed.

§ 2. Development of Dyopoly Theory

I. The Bertrand–Edgeworth School

Marshall and Edgeworth's authority initially helped Bertrand's attempt at a solution to gain general respect. We already mentioned that in 1896 Pareto also adopted it. As early as 1892 in his article "Monopoly" in the first edition[1)] of his *Handwörterbuch* (*Concise Dictionary*), W. Lexis had advanced a similar view to Bertrand only nine years earlier and perhaps without knowing Bertrand. In January 1898 Irving Fisher[2)] endorsed Edgeworth's version of Bertrand's critical analysis. An outstanding analysis of the silent assumptions made by Cournot leads to Henry Ludwell Moore writing his article "Paradoxes of Competition"[3)] in February 1906. In his positive remarks he essentially endorses the Bertrand–Edgeworth attempt at a solution for the simple dyopoly. In 1906 Achille Loria[4)] criticises the Bertrand–Edgeworth theory, but by completely misunderstanding the problem.

In his opinion, if no equilibrium were to occur, both dyopolists would communicate with each other and act in unison so that there could be no circumstances where there was uncertainty about price. No one will want to dispute this, but what is the answer to the question of whether or not an equilibrium occurs when the dyopolists do not communicate? Umberto Ricci[5)] correctly repulses the attacks by Loria.

Joseph Schumpeter, who also subsequently shows support for Cournot's solution, specifically aligns himself with the Bertrand–Edgeworth view in his 1908 *Wesen und Hauptinhalt* (*Introduction and Main Contents*)[6)].

Arthur Cecil Pigou supports the same viewpoint. He presented a description of Edgeworth's opinion in his *Wealth and Welfare* (1912)[7)] and in it maintains that Edgeworth's view had been adopted by all mathematical economists. At the

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[1)] *Handwörterbuch der Staatswissenschaften*, 1st ed., Vol. 4, Jena, 1892: 12121/13, see also 2nd and 3rd ed.

[2)] J. Fisher: "Cournot and Mathematical Economics", *The Quarterly Journal of Economies*, Vol.12 (1897/18: 126 and 127).

[3)] Ibid.

[4)] *The Economic Journal*, Vol.16: 365–371.

[5)] U. Ricci: "Rassegna del movimento scientifico", *Giornale degli Economisti e Annali di Economia*, February 1907 (Serie 2, Vol. 34: 152). In addition, see: Amoroso: "La teoria dell'equilibrio economico secondo il Prof. Vilfredo Pareto", *Giornale degli Economisti e Annali di Economia*, Serie 2, Vol. 39, 1909: 364. Edgeworth himself answers Loria's Einwand in *The Economic Journal*, Vol. 17 (1907). See "Papers", ibid.: Vol. 2: 329 et seq.

[6)] Joseph Schumpeter: *Das Wesen und der Hauptinhalt der theoretischen Nationalökonomie* [*The Nature and Essence of Economic Theory*], Leipzig, 1908: 269.

[7)] London, 1912: 193 et seq. Similarly: *Economics of Welfare*, 2nd ed. 1924, pt. II, Ch. 14, 3rd ed. 1929, pt. II, Ch. 15.

1922 conference[1)] to discuss Amoroso's "Lezioni di economia matematica"[2)] Edgeworth, who had in the meantime adopted the basic principles of Cournot's solution, could thus explain that Amoroso was the only one of the prominent economists to defend Cournot's opinion. Actually, around this time Edgeworth's opinion was prevalent not only in the Anglo-Saxon world. In 1925/26 in his "Contributo alla teoria economica dei beni succedanei"[3)] Marco Fanno briefly presents all the controversies, mentioning the dissenting opinion of Amoroso and referring to the Edgeworth solution again; he then writes word-for-word, "Such was Edgeworth's solution, accepted by Pigou and the others and also approved by us, because it is undoubtedly correct" (Quotation translated from the original Italian). In his article "Preis (Monopolpreis)" (translator's note: meaning "Price (Monopoly Price)") in the *Handwörterbuch der Staatswissenschaften*[4)] Hans Mayer produces a theory of the "imperfect monopoly" that Edgeworth adheres to.

In his publication, "Der Erkenntniswert der funktionellen Preistheorie"[5)] Mayer also tries to solve the second Cournot problem – the total complementarity of monopolised goods. However, rather than using the correct critical analysis by Edgeworth, for this case he essentially transfers[6)] the Bertrand–Edgeworth solution for the simple dyopoly problem to the case of complementary goods and so achieves an incorrect result.

We can thus say that until very recently prevailing opinion accepted the theory of the simple dyopoly as developed by Bertrand, Marshall and Edgeworth. Until the publication of the *Manuel* by Pareto this attempt at a solution in fact remained unopposed. It must be stressed especially that precisely the correct part (in our opinion) of the Edgeworth theory of the dyopoloid, namely the treatment of monopolised complementary goods, would go completely unnoticed.

II. The Lausanne School

The first objection to the Edgeworth solution was raised by Pareto in 1909 in his *Manuel* where he actually rejects Cournot's description and instead treads new paths himself. We have already explained in detail why we must consider his opinion to be mistaken compared with those of Cournot on the one hand and Bertrand–Edgeworth on the other – they at least contain part of the truth, yet

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[1)] *The Economic Journal*, Vol. 32 (1922: 400 et seq.).

[2)] Ibid.

[3)] *Annali di economia*, Vol. 2 (1925/26: 410–414).

[4)] 4th ed., Vol. 6, Jena, 1925: 1032.

[5)] "Die Wirtschaftstheorie der Gegenwart", Vol. 2, Wien, 1932: 162, 163 and Comments. Translator's note: published in English as, "The Cognitive Value of Functional Theories of Price".

[6)] If we have interpreted his explanations correctly.

followers of the "Lausanne School" sided with Pareto and praised the rigour of his solution.

Luigi Amoroso, subsequent champion of Cournot's solution, was the first to adopt Pareto's critical analysis. He writes[1)], "The precise solution to the problem is given in the *Manuale*" (quotation translated from the original Italian), and later[2)], "The solution has been given by Pareto in the Appendix of his *Manuel d'Economie Politique*" (quotation translated from the original Italian).

Likewise Wladimir Zawadzki endorses Pareto's dyopoly theory. He first presents[3)] Cournot's theory of monopolised complementary goods, which he readily accepts, thereby staying true to his former teacher Pareto. He then also presents Cournot's dyopoly theory and contrasts it with Pareto's critical analysis.

Zawadzki writes [in French] among other things (pp. 72), "Cournot's error has been discovered and the impossibility of an equilibrium in the case of two monopolists has been revealed through the work of F. Y. Edgeworth and V. Pareto", and strangely he does not concern himself with the basic difference between Edgeworth and Pareto's reasoning. The approach to science at that time was quite often allowed to be like this. The leading authorities rejected the possibility of a dyopolistic equilibrium – that was the main issue. The theoretical differences were considered to be trivial and were not of any concern. It was thus felt that the assertion was accurate and that an equilibrium did not occur, although they did not try very hard to find definite theoretical grounds to support this. The Bertrand–Edgeworth critical analysis may have seemed plausible, but Edgeworth himself indeed considered Pareto's opinion not to be that different from his own. He writes[4)], "... from the writings of Professor Pareto with which we are acquainted we do not gather that he differs seriously from the conclusions reached in the article on Monopoly to which we have referred" (quotation originates in English).

III. The Rebirth of Cournot's Theory

Indeed the first to advocate the general rejection was Knut Wicksell, who as early as 1913 in his "Vorlesungen über Nationalökonomie"[5)] presents the Cournot solution and criticises Bertrand and Edgeworth, especially the latter, with arguments that we must wholly praise. In 1921[6)] Amoroso joins forces with Knut

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[1)]Luigi Amoroso: "La teoria dell'equilibrio economico secondo il Prof. Vilfredo Pareto", *Giornale degli Economisti e Annali di Economia*, October 1909: 364.

[2)]L. Amoroso: "La teoria matematica del monopolio", *Giornale degli Economisti e Annali di Economia*, August 1911: 229.

[3)]W. Zawadzki: *Les mathématiques appliquées à l'économie politique*, Paris, 1914.

[4)]*The Economic Journal*, Vol. 32 (1922: 405).

[5)]Jena, 1913: 152 and 153 Remarks. Translator's note: published in English as, "Lectures on Political Economy".

[6)]"Lezioni di economia matematica", ibid.: 254 et seq. ("Problema di n monopolisti").

Wicksell, who in the meantime had changed his mind, initially without openly declaring this. In 1925 Wicksell writes an article on "Mathematische Nationalökonomie" (translator's note: "Mathematical Economics") that is included in 1927 by Joseph Schumpeter in the Heidelberg Archive[1]. Here, following an analysis of Bowley's *Mathematical Groundwork* he presents Cournot's solution for the simple dyopoly and the second problem – monopolised complementary goods – at length and with a very clear formulation and presentation, and criticises Bertrand, Edgeworth and Bowley, siding with Cournot. It is interesting that he ostensibly maintains that "Cournot's quantity dyopoly" along with "Cournot's price dyopoly" is generally possible for tied demand for monopolised goods and describes the solution as uncertain[2].

In his *Wesen und Hauptinhalt* (translator's note: published in English as: *Nature and Essence*) Joseph Schumpeter too still supported Edgeworth, who in his introduction[3] to the above-mentioned publication written by Wicksell, now declared his support for Cournot in his essay, "The Instability of Capitalism"[4] and in his foreword to the *Problems of Monopoly and Economic Warfare* by Zeuthen[5].

Independently of this development we find Cournot's views in the work of two North American mathematicians who focus on problems of economic theory and especially on issues relating to dynamic economics. These two theoreticians are Griffith C. Evans[6] and C. F. Roos[7].

1929 and 1930 were especially important years for establishing Cournot's view. A whole series of very notable contributions were swept away by the Edgeworth theory of the simple dyopoly which established the Cournot solution to be the correct one. In March 1929 Harold Hotelling[8] published a very significant article, "Stability in Competition" – prompted on the one hand by the work of his fellow countryman mentioned a moment ago and on the other by Piero Sraffa's

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[1] *Archiv für Sozialwissenschaft und Sozialpolitik*, Vol. 58 (1927: 252 et seq.).

[2] Ibid.: 273.

[3] Joseph Schumpeter: "Zur Einführung der folgenden Arbeit Knut Wicksells", *Arch. f. Sozialwissenschaft und Sozialpolitik*, Vol. 58 (1927: 238 et seq., especially pp. 248/249).

[4] *The Economic Journal*, Vol. 38 (1928: 361 et seq., especially pp. 369 et seq.).

[5] F. Zeuthen: *Problems of Monopoly and Economic Warfare*, with a preface by Prof. Joseph A. Schumpeter, London, 1930.

[6] C. G. Evans: "A Simple Theory of Competition", *American Mathematical Monthly*, Vol. 29 (1922: 371 et seq.). id.: *Mathematical Introduction to Economics*, first ed. New York 1930, Chap. III: 22 et seq.

[7] C. F. Roos: "A Mathematical Theory of Competition", *American Journal of Mathematics*, Vol.47 (1925: 103 et seq.). id.: "A Dynamical Theory of Economics", *The Journal of Political Economy*, Vol.35 (1927: 632 et seq., especially pp. 635/636). id.: "Generalized Lagrange Problems in the Calculus of Variations." *Transactions of the American Mathematical Society*, Vol.30 (1928: 360 bis 384, especially pp. 360/361). id.: "A Mathematical Theory of Price and Production Fluctuations and Economic Crisis", *The Journal of Political Economy*, Vol. 38 (1930: 501 et seq., especially pp. 513–518).

[8] "Stability in Competition", *The Economic Journal*, Vol. 39 (1929: 41 et seq.).

explanations[1)] about the inhomogenous nature of most products produced by multiple producers.

The remarkable thing about his explanations is that he adds the Cournot theory of simple dyopolies to the analysis by Bertrand–Edgeworth, but in contrast to Sraffa, mentions that it would focus on a completely homogenous standardised good for multiple producers only in the rarest of cases. A more or less fixed group of customers would exist just for each producer so that we would only be able to say that the turnover of a dyopolist in the first instance depends on the price he sets and in the second, on the price set by his rival, without both prices needing to be equal. We see that through these undoubtedly realistic assumptions, the general situation, namely monopolising two rival goods, takes the place of simple dyopolies. Here however Hotelling maintained the existence of a true general equilibrium. The equilibrium that he then used in his example – the production of a homogenous good at two different locations and the inclusion of transport costs in the study[2)] – is merely the "Cournot price dyopoly". The nature of the Bertrand–Edgeworth attempt to solve the simple dyopoly becomes especially significant to Hotelling's view. From his analysis Edgeworth shows only the impossibility of the "Cournot price dyopoly" in the simple dyopoly. The assumptions are appropriate since each dyopolist in the Edgeworth example changes his price under the assumption that his rival leaves his price unchanged at first[3)]. It is clear then that an equilibrium of the "Cournot price dyopoly" type cannot occur. However, where both goods supplied are heterogenous, a "Cournot price dyopoly" is possible, as we have seen, and Hotelling's claims occur, thus adding to the Cournot basic principles. He appears not to have even noticed Edgeworth's explanations about price independence. In any case his problem is of great importance since it shows the considerable area of application of the theory of monopolised rival goods and in the same way refers to the possibility of a much better alignment of theory to reality.

In January 1930 a magnificent essay written by Amoroso appeared, *La curva statica di offerta*[4)] in which he[5)] tackles the dyopoly problem in the Cournot sense, expands it to include general oligopolistic supply and deduces a conditional equation for the output of oligopolists that includes the formulae for the simple

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[1)]Piero Sraffa: "The Laws of Returns under Competitive Conditions", *The Economic Journal*, Vol. 36 (1926: 535 et seq.). On pp. 544 he writes, "... the chief obstacle which hinders the free play of competition, even where this appears to predominate, and which at the same time renders a stable equilibrium possible, even when the supply curve for the products of each individual firm is descending – that is, the absence of indifference on the part of the buyers of goods as between the different producers" (quotation originates in English), and a few lines later on pp. 545: "When each of the firms producing a commodity is in such a position, the general market for the commodity is subdivided into a series of distinct markets" (quote originally in English).

[2)]This example has already been found in a different context, see John Bates Clark and John Maurice Clark: *The Control of Trusts*, New York, 1914: 106, footnote.

[3)]Wicksell, in both his works cited above, rightly points to this implicit assumption by Edgeworth.

[4)]Ibid.

[5)]pp. 9 et seq.

monopoly and for free competition[1)] as marginal cases. Here Amoroso argues explicitly, and in our view entirely correctly, with Bertrand–Edgeworth and Pareto and belatedly justifies his change of opinion that was mentioned previously.

At the end of 1930 Erich Schneider published[2)] a paper in which he adds the analysis of Wicksell to Edgeworth and presents an accurate analysis of Pareto. He then advocates the Cournot solution and likewise generalises it to the generic case of oligopoly, ostensibly without being familiar with Amoroso's publication mentioned above. We have already set out the reasons as to why we cannot agree with Amoroso and Schneider's positive expositions. The criticism that Paul Braess[3)] levelled at Schneider and Wicksell appears to us of course to be hardly convincing.

A paper by Tinbergen[4)] also tries among other things to corroborate the Cournot dyopoly theory by empirical methods. It does not contradict our theory however, even if Tinbergen shows that the assumption of a price formation accordingly better suits the Cournot theory for his example than the picture of free competition or the simple monopoly.

Erich Schneider, who again advocates the Cournot solution in a special case[5)], expands this view to a general theory in his *Reinen Theorie monopolistischer Wirtschaftsformen*[6)] (which we will return to for the treatment of the development of the theory of the bilateral monopoly).

Here he advanced the Cournot solution, not only for the simple dyopoly[7)], but he also derived the "Cournot price dyopoly" for all forms of monopoly demand relationships and linked it with his view of the bilateral monopoly in the treatment of the "universal monopoly".

IV. The Term, "The Position of Independence"

As early as 1897, Edgeworth, as we showed earlier, had found the term "price independence" for monopolised complementary goods without this important theoretical discovery exerting any influence on the development of the theory of

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[1)]A deduction of this formula through an elementary method and a similarly simple description of Amoroso's "Critica del sistema capitalista", ibid.: Lezioni XXXIV and XXXV.

[2)]Erich Schneider: "Zur Theorie des mehrfachen Monopols, insbesondere der des Dyopols", *Die Archiv für Sozialwissenschaft und Sozialpolitik*, Vol. 63 (1930: 539 et seq.).

[3)]Paul Braess: "Kritisches zur Monopol-und Dyopoltheorie", *Die Archiv für Sozialwissenschaft und Sozialpolitik*, Vol. 65 (1931: 525 et seq.).

[4)]J. Tinbergen: "Bestimmung und Deutung von Angebotskurven". An example. *Zeitschrift für Nationalökonomie (Journal of Economics)*, Vol. 1 (1930: 669 et seq., especially pp. 6761).

[5)]Erich Schneider: "Drei Probleme der Monopoltheorie", *Zeitschrift für Nationalökonomie*, Vol. 2 (1931: 376 et seq., especially pp. 382 et seq.).

[6)]Ibid.

[7)]The same view is also shown in Schneider's investigation into "Das Verteilungs-und Kostenproblem in einer vertrusteten Industrie", *Schmollers Jahrbuch*, Vol. 56 (1932/1: 361 et seq.).

the dyopoloid until now. In his publication, the "Paradoxes of Competition" (1906) cited above, Moore analysed Cournot's implicit assumptions and showed that Cournot tacitly makes the assumption that each dyopolist regards the output of his rival as an independent variable. This accurate assessment might suggest that the term "quantity dependence" opposes the term "quantity independence". Moore does not however reach this conclusion.

The Mathematical Groundwork of Economics, the introduction to economic theory published by Bowley[1)] in 1924, presents the supply dyopoly at the end of the paragraphs on "Several manufacturers, one commodity" (thus the "supply oligopoly")[2)]. Here each dyopolist regards his rival's output as being dependent on his own output, meaning therefore that it is a function of his own quantity supplied. This function "depends on what each producer thinks the other is likely to do". It is clear that these two functions cannot exist simultaneously in reality. Here each dyopolist has the "position of independence", meaning that the market situation described by Bowley is the dyopolistic structure we have already analysed under the name, the "Bowley dyopoly". Bowley himself, who might have been prompted to this formulation by Edgeworth, appears not to have clearly seen all of the consequences since he then continues to write, "There is then likely to be oscillation in the neighbourhood of the price given by the equation marginal price for each selling price. . ." (translator's note: quotation originates in English). This assertion appears to us to be without foundation. It is not a consequence of the problem and might just be accidentally true. It is also strange that Bowley has no answer to Wicksell's attacks[3)] on his account of dyopolies, whereas he comments on Wicksell's view[55] of bilateral monopolies at length[4)]. In any event Bowley must be primarily attributed with conceiving "quantity independence" which is why we have also named the market situation in which oligopolists aspire to "quantity independence" after Bowley.

The first to use the term "quantity independence" quite deliberately is Kurt Sting[5)]. By employing what in our opinion is rather unfortunate terminology he provides a clear account of "hyper-political price setting" that is really only our "quantity independence".

We have cited our own contributions above[6)] and mention them here just for the sake of completion.

[1)] Oxford, 1924.

[2)] pp. 38.

[3)] *Mathematische Nationalökonomie*: ibid.

[4)] *The Economic Journal*, 1928: 651 et seq., *Bilateral Monopoly*.

[5)] "Die polypolitische Preisbildung. Ein Kapitel der Preistheorie." *Jahrbücher für Nationalökonomie und Statistik*, 3rd F., Vol. 79 (1931: 761 et seq., especially pp. 773/74).

[6)] Chap. 2, § 4.

V. The Eclectic School

1. The increasing importance of "Imperfect Competition" (translator's note: Stackelberg uses "imperfect competition" in English) theory was as a result already clearly visible to the outside world in recent times since several monographs had been devoted to it in quick succession. Besides the book already mentioned by Erich Schneider that was based entirely on the Cournot solution, we will mention two additional papers here. In 1930 the Dane, F. Zeuthen, published his *Problems of Monopoly and Economic Warfare*[1)] and in 1933 the American Edward Chamberlin followed him with his book, *The Theory of Monopolistic Competition*[2)]. Both papers unite in their efforts to reconcile repeatedly argued opinions about the market structure of dyopolies with one another. This was supposed to be achieved by adding still further conditions to the general assumptions about dyopoly. Depending on the meaning of these additional conditions, one or other of the solutions is believed to be correct. It would become clear from the actual situation at the time which of these assumptions applies. Chamberlin[3)] thus states, "Duopoly is not one problem, but several" (translator's note: quotation originates in English).

a) In addition, these assumptions are referred to by Zeuthen, who only considers the possibility of "price adjustment" for dyopolists in effecting price changes in the market. Zeuthen distinguishes the following possibilities: a smaller price reduction by one dyopolist – with an unchanged price by the other – can still attract the unsupplied part or the part of the market supplied by the rival or both parts or neither of them, and in fact all of them in various quantities. It all depends on which effects would produce a specific price formation.

We cannot explore Zeuthen's theory in detail but instead will focus only on one point and use it to explain why we cannot agree with Zeuthen's solution. First of all, in our view, only one assumption is possible with a homogenous notional good, namely, an individual price reduction on the part of one dyopolist, whose turnover immediately extends to all of his clients' rivals, and the part of demand that is as yet impossible at these new prices, but is becoming significant. Here, according to Zeuthen, Bertrand's solution occurs and he only modifies this with respect to Edgeworth, since according to him price applies downward pressure on costs and

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[1)] Ibid. Zeuthen has his opinion on the dyopoly problem even in 1928 in his book *Den Økonomiske Forderling* and in 1929 appearing in a publication entitled: "Mellem Monopol og Konkurrence" in the *Nationaløkonomisk Tidskrift*. Neither paper was available to the author on linguistic grounds.

[2)] *Harvard Economic Studies*, Vol.38, Cambridge (Mass.), 1933. Chamberlin had published his dyopoly theory as early as November 1929 in: *The Quarterly Journal of Economics*: 63 et seq.: "Duopoly: Value, where Sellers are few."

[3)] Ibid.: 53. A similar eclectic view is found from John Maurice Clark in his article "Monopoly", *Encyclopaedia of the Social Sciences*, Vol. 10, New York, 1933, especially pp. 627 and 628. Pigou again also appears to be more inclined in this direction. See *The Economics of Welfare*, 4 ed. (1932), pt. II, Ch. XV. Also: "A Note on Imperfect Competition", *The Economic Journal*, Vol. 43 (1933: 108 et seq.).

then an equilibrium is achieved. We must raise the same objections to this solution as with Bertrand–Edgeworth's attempt at a solution.

All other cases that Zeuthen makes have to be dealt with in exactly the same way as monopolised rival goods that are however different from each other (as e.g. the Hotelling problem). We have tried to show these solutions in earlier chapters[1)].

Zeuthen also focuses on the case of monopolised complementary goods, which he strangely refers to as a "bilateral monopoly"[2)]. He tries to present the market structure known to us as "bilateral monopoly" as a special case of his "bilateral monopoly". The conclusion for his price formation theory of monopolised complementary goods essentially agrees with Cournot and is thus the straightforward "Cournot price dyopoly".

a) In contrast Chamberlin[3)] puts forward additional assumptions about the ideas that both dyopolists have about their respective behaviour.

He initially analysed the simple dyopoly and here distinguished the following possibilities:

1) Each dyopolist always regards the output of his rivals as a given variable. Chamberlin deduces the "Cournot quantity dyopoly" from this assumption – and we can agree with this.

2) Each dyopolist always regards the price set by his rival as a given variable. According to Chamberlin the Bertrand–Edgeworth solution occurs here, but with some modifications.

3) Both dyopolists are aware of their mutual dependence. Here Chamberlin claims the occurrence of the same price formation as in the simple monopoly, meaning in the case of both dyopolists joining together. We do not consider this outcome to be correct. Instead, all the possibilities that we have derived in our theory occur here.

4) Each dyopolist always regards price as an independent variable. That is the assumption of the "classical price formation". Therefore we agree with Chamberlin when he claims that the price formation structure of free competition always results from this assumption without consideration being given to the number of suppliers.

5) Ultimately Chamberlin investigates a series of cases in which uncertainty about the behaviour of the rival predominated for one dyopolist.

What do we now have to consider for these assumptions that should be shown by the variety of dyopoly problems? From the explanations in our second chapter we can say that for the subsidiary assumptions mentioned, only 3) and 5) are a

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[1)] A similar opinion to that of Zeuthen is seen by W. G. Waffenschmidt: "Cournotsche Gleichungen für beschränkten Wettbewerb", *Die Archiv für Sozialwissenschaft und Sozialpolitik*, Vol. 67 (1932: 513 et seq.).

[2)] Ibid.: 63 et seq. Translator's note: (bilaterales Monopol), see Heertje: 48).

[3)] Ibid.: 30 et seq.

possibility. Assumption 3) forms the basis of our theory. From it we can also easily deduce that assumption 5) is possible and we can refer to it as an "asymmetrical assumption". It should be plausible for the time being that neither of the two dyopolists can ever consider the price set by his rival as invariable since he would quickly be informed by the actual behaviour of both in setting a better price. He is also unable – as we have tried to point out – to consider the market price as independent. It is only a question of not knowing the response possibilities of the rival. Not knowing this can however only occur when the rival does not "respond" (translator's note: (reagiert), see Heertje: 48) i.e. when he holds the position of independence. Thus in our view only those assumptions are possible 5) that would also lead to an asymmetrical quantity dyopoly. In our view the function of dyopoly theory is to point to those "additional assumptions" which are actually possible for the dyopolistic market. Chamberlin has not done justice to this task.

Chamberlin also applies his dyopoly theory to the case of monopolised rival goods[1)]. His explanations about this subject are therefore also to be opposed using the same objections.

2. Whereas both of the works discussed a moment ago strive to combine the conflicting theories, the Englishwoman, Joan Robinson, in her newest book, *The Economics of Imperfect Competition*[2)] tries, so to speak, to eliminate this problem. She simply assumes that each supplier opposes a particular sales function that gives him the price he can sell a particular quantity at or the quantity he can sell at a particular price. This sales function, described by her as an "individual demand curve" (translator's note: terminology consistent with original text written in English), is thus believed to show all the effects that the realisation of a particular sale has on the supplier in question. Each supplier appears – according to Sraffa's suggestion and Hotelling's experiment – to be a monopolist of a precisely determined good and so the entire economic production setup consists of a large number of "rival monopolists". Meanwhile Robinson expresses the concept of the general equilibrium – the absorption of all exceptional profits, or put another way, the levelling of prices to average (unit) costs, is to be applied to this economic picture and it ultimately becomes an apparently closed theory that likewise closely matches reality in a "World of Monopolies".

This, the entire logical and geometric machinery of theoretical socio-economic investigation, handled in such a masterly manner, rests thus on the term, the "individual demand curve" and retains its true meaning from the assessment of this one assumption. However strange it appears, Robinson did not analyse this

[1)]Ibid.: Chap. V, especially pp. 100.

[2)]London, 1933. See also: J. Robinson, "Imperfect Competition and Falling Supply Price", *The Economic Journal*, Vol. 42 (1932: 544 et seq.), and the discussion between Pigou, Shove, Harrod and the author about this publication, in the March, June and September 1933 editions of the same journal.

term at all closely, but instead, from the outset she assumed the possibility of constructing such curves and their findings would simply relate to such curves, no matter how they were defined.

Robinson writes (pp. 21),

"The phrase 'individual demand curve' means, not the demand curve of an individual buyer, but the demand curve for the product of an individual firm. Complications are introduced into the problem of the individual demand curve by the existence of advertising, but these have been ignored. It may be assumed that expenditure on advertisement necessary to increase the sales of a firm can be treated as equivalent, from the point of view of the entrepreneur, to a reduction in price having the same effect upon sales.

In an industry which is conducted in conditions of imperfect competition a certain difficulty arises from the fact that the individual demand curve for the product of each of the firms composing it will depend to some extent upon the price policy of the others. Thus if one raises its price, the demand curves for the others will be raised. This may cause them to raise their prices also, and the rise in their price will react upon the demand for the commodity of the first firm. In drawing up the demand curve for any one firm, however, it is possible to take this effect into account. The demand curve for the individual firm may be conceived to show the full effect upon sales of that firm which results from any change in the price which it charges, whether it causes a change in the prices charged by the others or not. It is not to our purpose to consider this question in detail. Once the demand curve for the firm has been drawn, the technique of analysis can be brought into play, whatever the assumptions on which the demand curve was drawn up.

It is frequently convenient to refer to a demand curve as the 'average revenue curve' of a seller" (translator's note: quotation originates in English).

This is the Robinson definition of the "individual demand curve". It is thus supposed to show the full effects of a price change by the company. This also has the effect of, amongst other things, causing another company's "individual demand curve" to shift. In order to assess the effect of this shift, we must know the second company's "sales curve" (translator's note: (Absatzkurve), see Maks and Haan: 44). However, according to Robinson this is also to be defined like the first company's "sales curve", that is, assuming a knowledge of the first company's "sales curve". We see that the Robinson definition of the "individual demand curve" already assumes a knowledge of this curve which follows a circular logic.

Let us now examine whether or not it is at all possible to define an "individual demand curve" (translator's note: Stackelberg's terminology is in English). If this were possible another way, the new definition would simply replace the one given by Robinson without the explanation being changed to a different one.

Let us consider two monopolists who produce rival or complementary goods. We know that these two monopolists cannot simultaneously hold exactly the same market position. Where the first monopolist has a position of independence and the second has a corresponding position of dependence, so "individual demand curves"

can be constructed for both of them, as we will now explain. First of all, the output quantity (or price) for the first monopolists is given.

Then the second monopolist is confronted with a very specific link between turnover and price and thus a specific demand curve. Where the first monopolist adjusts his output (or his price) so we obtain a new "demand curve" for the second monopolist. Therefore a family of demand curves face the second monopolist, whose parameter is the output (or price) of the first monopolist. From this, the "demand curve" of the first monopolist can now also be deduced. Where he supplies a specific quantity of goods (or sets a specific price) the second monopolist is faced with a specific demand curve from his family of curves. He then achieves a specific turnover at a specific price and the maximum profit provides him with this demand curve. This means that a very specific price (or turnover) corresponds to the first monopolist's quantity of goods supplied (or price). Thus ultimately the result is the "individual demand curve" for the first monopolist. He achieves the "Cournot monopoly point" on his demand curve, whereby the second monopolist is faced with a special demand curve from his family of curves on which he likewise achieves the "Cournot monopoly point". It is important to note that we obtain different results depending on whether or not quantity or price independence is assumed for the first monopolist. We therefore see that for an asymmetrical dyopoly two specific definitions of real "sales curves" are possible. These definitions are however different for the two suppliers. No definition is possible with sales curves that are the same for both monopolists and that reflect reality.

Now we know that we must essentially take this into account, not with an asymmetrical dyopoly, but with a "Bowley" (or "Edgeworth" dyopoly or if need be with a "Cournot dyopoly" instead. In the "Bowley" (or "Edgeworth") "dyopoly" each monopolist acts as if he was confronted by the "sales curve" for the position of independence. These curves are however both imaginary, meaning that the actual price (or turnover) that occurs does not agree with the one shown by the curve. In the "Cournot dyopoly" each monopolist acts as if he was faced with a family of sales curves whose parameters are different to the output (or price) of the other monopolist. Indeed, in the outcome each monopolist achieves the Cournot monopoly point on a special sales curve. However the reactions of the price shown by this curve to changes in supply (or turnover to changes in price) do not agree with reality because they do not express the respective responses of the two suppliers.

We therefore see that where the "individual demand curves" accurately reflect reality, they cannot be defined in the same way.

In the same way, the definition is only of imaginary sales curves that have unstable dyopolistic (and oligopolistic) structures and do not reflect reality.

Taking this fact into consideration, the analysis carried out by Robinson is indeed still of great value. Overall, it must however break down with the introduction of equilibrium terminology. So far as unstable market structures in particular exist it is not possible to use ratios that only apply to an economic equilibrium (absorption of profits, etc.) – although in a "World of Monopolies" that might be the rule.

§ 3. Overview of the Development of Bilateral Monopoly Theory

The credit for being the first to accurately tackle the problem of both the bilateral monopoly and the bilateral oligopoly goes to Edgeworth, who as early as 1881 in his *Mathematical Psychics*[1)] not only establishes proof of the error of a general equilibrium but also precisely defines the area within which the monopolists' or oligopolists' war would play out. However ten years before him, Karl Menger[2)] had demonstrated in an elementary way the uncertainty of prices in an "isolated exchange". In the following period the whole "Viennese School"[3)] brought an end to Menger's view, and Edgeworth's version of the theory prevailed in the literature that was published abroad. A. Marshall[4)] essentially adopts it, as his famous example of the exchange of apples and nuts shows.

A criticism that Edgeworth[5)] made of Marshall's reasoning is revealed by Arthur Berry[6)] to be an error based on a misunderstanding and is even acknowledged as such by Edgeworth[7)]. Pigou[8)] tries to investigate the market situation of the bilateral monopoly more closely and to narrow down the limits within which the price war plays out, but stands by the view that an equilibrium cannot occur as a matter of principle.

[1)]Ibid.: 16 et seq. See also: "On the Application of Mathematics to Political Economy", Papers. . . , ibid.: 306 et seq.

[2)]Karl Menger: *Grundsätze der Volkswirtschaftslehre*, Vienna, 1871, Chapter Five, § 1 (pp. 175 et seq.). See also Albert Eberhard Friedrich Schäffle: *Das gesellschaftliche System der menschlichen Wirtschaft*, new 3rd edition in two vols. – completely revised, Vol. 1, Tübingen, 1873: 193, § 112, 1. Schäffle believes that the price is set midway between its two extremes.

[3)]See instead all of E. v. Böhm-Bawerk: "Grundzüge der Theorie des wirtschaftlichen Güterwertes", second part, *Jahrbücher für Nationalökonomie und Statistik*, N. F., Vol. 13 (1886: 492 and 493). Ibid.: *Positive Theorie des Kapitals*, 4th ed., Jena, 1921, Book III, Section II, II A. (pp. 269 et seq.). Fr. v. Wieser wrote (in *Der natürliche Wert* [*Natural Value*], Vienna, 1889: 38), "Menger's price theory and additionally its exposition with Böhm-Bawerk ('Werth') can therefore serve as the starting point for our own description." Also: E. v. Philippovich: *Grundriß der Politischen Ökonomie*, 16th ed., Vol. 1, Tübingen, 1921: 256 and 257.

[4)]A. Marshall: *Principles of Economics*, 1st ed. (London, 1890 et seq.).

[5)]"Osservazioni sulla teoria matematica dell'economia politica con riguardo speciale ai Principi di Economia di Alfredo Marshall", *Giornale degli Economisti e Annali di Economia*, March 1891: 233 et seq. (Ser. 2, Vol. 2).

[6)]"Alcune brevi parole sulla teoria del baratto di A. Marshall", *Giornale degli Economisti e Annali di Economia*, June 1891: 549 et seq.

[7)]"Ancora a proposito della teoria del baratto", *Giornale degli Economisti e Annali di Economia*, October 1891: 316 et seq. Correspondingly, in his essay, "Osservazioni. . ." in his translation into English, Edgeworth omitted the sections identified by him as incorrect and published in this revised form under the title "On the Determinateness of Economic Equilibrium" in *Papers Relating to Political Economic*s, London, 1925, Vol. 311 et seq.

[8)]"Equilibrium under Bilateral Monopoly", *The Economic Journal*, Vol. 18 (1908: 205–220). See also "The Principles and Methods of Industrial Peace", London.

Initially even Joseph Schumpeter[1)] and Wicksell[2)] still categorically take the view that price formation is uncertain in the bilateral monopoly (isolated exchange). Pareto's view is no different however to the one that in our view was insufficiently substantiated. E. Schneider[3)] can rightly raise serious objections to his brief explanations in the *Cours*[4)]. One clear illustration of the bilateral monopoly (isolated exchange) in general and the company bilateral monopoly in particular leads to Bowley[5)] who likewise arrives at a conclusion of instability. In the same way we find Hans Mayer's view[6)] that "always, when two monopolies confront each other, [the price] is not able to be clearly determined". Leduc[7)] expresses a similar view.

Again objections are raised against this prevalent, and in our view, correct theory. We do not mean those schools of thought for which, for instance, the following sentence by Dobretsberger[8)] might be an example:

"The indeterminate nature of bilateral monopoly prices (i.e. the insufficiency of each theoretical price formation pattern relative to those price phenomena where rivalry on both sides is eliminated and that are incomplete in respect of continuously accumulating phenomena) shows that as an abstract basic assumption, abstract price theory has assumed rivalry to be an ideal for any structure." Such objections can ultimately be raised against any theory without being demonstrable in some way. By the same token we could say that the view that oval wheels will do for a wagon is based on the concept that the roundness of the wheels "as an ideal example in any sort of structure is assumed in the abstract basic assumptions". We do not need to explain ourselves for objections of this kind.

Here we come to the contributions of Wicksell, Schumpeter, Zeuthen and Schneider. Wicksell, who stated that price formation in the "isolated exchange" was generally uncertain in his *Vorlesungen*...[9)] and ostensibly so, even in his publication *Mathematische Nationalökonomie* (1927)[10)], is of the same view and in the same paper supports – even if with a certain restraint – the opinion that a definite specific price occurs in the bilateral monopoly for companies[11)]. He examines an example in which the monopolist (a worker) supplies a means of

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1)*Wesen und Hauptinhalt*, ibid.: 270.

2)*Vorlesungen*..., ibid. 99 et seq.

3)*Reine Theorie*..., ibid.: 120/121.

4)Ibid.: 65.

5)*Mathematical Groundwork*, ibid.: 5 et seq. Also pp. 62.

6)Article "Preis (Monopolpreis)" by Hans Mayer, *H. d. St*., Vol.6: 1036/37.

7)Gaston Leduc: *La Théorie des Prix de Monopole*. Doctoral thesis. Aix-en-Provence, 1927: 252.

8)Josef Dobretsberger: "Konkurrenz und Monopol in der gegenwärtigen Wirtschaft mit besonderer Berücksichtigung der österreichischen Industrie", Leipzig and Vienna, 1929 (*Wiener Studien*, Vol. 12: 26).

9)Ibid.

10)Ibid.: 261.

11)Ibid.: 275–278.

production that is demanded by only one demander (a manufacturer) who again in turn operates through a monopoly. A monopolist faces a company that has both a demand as well as a supply monopoly. Now Wicksell maintains that price is always set by the monopolist in question. To substantiate this assertion he writes, "If the manufacturer is able to set wages, then the worker does not have a monopoly, contrary to the assumption"[1]. This analysis is not sound in our view. The monopoly is not defined by the monopolist himself being able to set the price, but instead by the supplier or demander being the only one in the market. How price formation acts must only be concluded from all the market conditions and may not constitute part of the definition of the market structure. In order to understand the consequences of Wicksell's view as clearly as possible we want to assume that the monopsonist in our bilateral monopoly does not have a monopoly in the market for his product, but instead follows the prices therein.

We can imagine the circumstances that would result from the corresponding statutory or contractual restrictions in practice. Let us first assume that free competition similarly prevails in the market for the means of production. Then both the demander and supplier of the means of production follow the prevailing price and the classical price formation occurs. Where a monopsony prevails in the market for the means of production, then the demand company that is now in charge of a monopsony does not follow the price but sets it itself. Where a monopoly prevails in a market for the means of production, the supplier sets the price and the demand company follows him. So what happens where there is a bilateral monopoly? Wicksell maintains that the supplier now sets the price and the demander follows. Then no difference would exist between the bilateral monopoly and the monopoly, meaning that the demander would behave as if he was not operating through a monopoly. Similarly, Wicksell can also assume that the demander sets the price himself and the supplier follows the price. Then the supplier would behave as if he did not operate through a monopoly. Both assumptions are incorrect. Actually no automatic price occurs, as we attempted to demonstrate in our explanations in Chap. 2.

More radically, Schumpeter – who in his *Wesen und Hauptinhalt*[2] still maintains that price is uncertain in the bilateral monopoly – supports the existence of a general equilibrium in the bilateral monopoly in the introductory essay[3] to the above paper by Wicksell. Apart from accepting Wicksell's line of argument as explained a moment ago, which Schumpeter expressed in the course of an analysis of Wicksell's concept on the isolated exchange and that we must oppose with the same objections, Schumpeter introduces a new idea into the discussion. He writes,[4] "If a random price is called out (Walras' "prix crié par hasard" (translator's note: French expression which literally means "price called out at random [by the

[1] Ibid.: 276.

[2] Ibid.

[3] Ibid.: 249–251.

[4] Ibid.: 251, Remarks.

auctioneer])) and is such that employers have as much work at [this price] as the workers want to 'deliver', there thus exists an apparent equilibrium. Where it is higher or lower we do not indeed have pressure by the competition mechanism on either of the two sides, but may instead attempt to influence the other party through relevant sacrifices (of money or output). And, as we are convinced, this also leads to a stable . . . equilibrium in the sense of the theory when you reflect upon the possible scenarios.

Naturally it lies at another point for free competition compared to conditions that are otherwise the same". He makes a very similar argument in his paper on "The Instability of Capitalism"[1)].

The following can be said to counter this. The "prix crié" (translator's note: French expression which literally means "price called out") is a price that is always regarded by both monopolists as a given, independent variable. An equilibrium ultimately occurs without doubt for a "prix crié" that incidentally – despite Schumpeter's remarks to the contrary – lies at the same point as for free competition. However we cannot just assume a "prix crié" on examination of the "bilateral monopoly" because this would be synonymous with the assumption of an independent price and would contradict the basic principles of free price formation. On the free capitalist market there is now no authority that announces a price that market participants follow.

Wicksell's concept is very clearly expressed by Erich Schneider[2)]. According to Schneider, the monopolist always sets the price and the monopsonist follows him. Schneider gives reasons for his point of view just like Wicksell. Thus we only need to refer to our objection to the latter.

Zeuthen[3)] tries, as was already mentioned above, to show the bilateral monopoly as a special case for monopolised complementary goods and also claims the occurrence of a general equilibrium. We consider this analogy to be out of the question. The difficulty in a bilateral monopoly is that it does not appear to be at all possible to set a price for the good in question because suppliers and demanders aspire to different prices. This occurrence is completely absent in the market relationships stated above. Therefore the instability of bilateral monopolies is also very different and much clearer than in the dyopoly.

The criticism by the social scientists mentioned above towards the prevalent theory of bilateral monopoly is therefore based on biased, selective and therefore unrealistic assumptions, as Bowley[4)] has alleged with outstanding clarity in regard to Wicksell.

[1)] Ibid.: 370–372.

[2)] Erich Schneider: "Drei Probleme der Monopoltheorie", *Zeitschrift für Nationalökonomie*, Vol. 2 (1931: 376 et seq.). Ibid.: *Reine Theorie*. . . , ibid.: 59 et seq.

[3)] Ibid.: 89 et seq.

[4)] *Bilateral Monopoly*, ibid. (1928).

Chapter 6
Market Structure and Economic Policy

§ 1. The Significance of the Unstable Market in the Real Economy

1. It actually goes without saying that the theoretical picture inferred in the first four chapters of this book does not directly describe reality. It shows in fact the relationships between market participants, prices and quantities sold that potentially also exist in the actual economy. However, economic agents, as we encounter them in reality, do not behave according to abstract, rational calculated patterns that follow our "individua oeconomica". They are more or less unaware of rivals' responses to them or to the difference between the "position of dependence" and "position of independence" or the marginal curves for bilateral monopolies, etc. The basic principles behind the free market capitalist economy also apply in reality and likewise are by no means unrestricted, as in the theoretical model – ideas such as a "fair price", a fair profit mark-up expressed as a percentage of the price, frowning on "exorbitant prices", necessity, paying "decent" wages, making it impossible to put workers out on the street, etc. more or less dominate real economic life and achieve a "similar equilibrium" state where abstract theory deduces instability from the assumptions of free market capitalist economy.

However this does not in any way mean undermining findings obtained theoretically. In reality this is only just the evolution of circumstances that have been decided from notional assumptions begun at the outset. The conclusions of our theory also apply to the same extent and to the same degree as those asserted for the application of the basic principles of the free market capitalist economy and not unlike those in the theory of the classical general equilibrium, except that our theoretical picture of economic life is opposed to the classical idea. The classical model of the economy for a general equilibrium is an ideal picture of a functioning system and the real economy differs from it to a greater or lesser extent due to the "restraints" stated above and likewise shows signs of trouble along the way that can occasionally very seriously impair the economic mechanism. By contrast, where the conditions of the classical equilibrium – general free competition – do not exist,

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the theoretical model thus shows a completely unstable picture that is fundamentally chaotic and forever undergoing shocks, i.e. that operates diametrically opposed to a trouble-free functioning economy. The real economy, insofar as it contains oligopolies and bilateral monopolies, and fulfils the basic principles of free market price formation therefore functions because of the "restraints" outlined above, not despite them. The more the free capitalistic economic system is threatened, the more realistically it displays the basic principles that characterise it. Consequently we can say that in the age of company consolidations, the less stable the free market capitalist economy is, the more the rationalisation of all economic entities is pushed through.

Before we go on to characterise the remodelling of the economy, which leads to the instability of the market, we must clarify our position regarding the opportunities of this rationalisation process insofar as it relates to the free capitalist market.

2. In a bilateral monopoly, the real market situation is hardly different to the theoretical one. Where the two market participants are tied by conventional factors, in reality an equilibrium occurs that can only be distinguished from economic theory but not explained by it, to the extent that non-commercial factors become less important for both market participants while the basic principles of the free market capitalist economy simultaneously fade into the background and must show the instability of the bilateral monopoly to advantage. The situation is different in the oligopolistic market. Existing relationships have now become so complicated that they cannot also be easily understood by the economic agents who are behaving rationally. Where the "restraints" are eliminated by a rationalisation process it cannot yet be easily said that the price formation deduced theoretically really occurs. This is merely just the ideally typical consequence of entirely rationalised behaviour by market participants and of complete all-round market transparency. Where these conditions are not fulfilled – and in reality they are never completely met – then the behaviour of actual market participants must be investigated and the effective price formation firstly deduced from it.

So long as oligopolists consider price to be a given variable in terms of ethics, traditions, conventions or just in terms of the market, an oligopoly presents no problem. It is only when oligopolists notice each other's dependence with the achievable price and the saleable quantity of goods, and investigate this dependence more closely that one of them enters into something we wish to refer to as the "market rationalisation process". This rationalisation process consists of each oligopolist trying to establish the correlation between price and turnover for his own situation and applying the conclusions he obtains within his market policy. Our theory – and to some extent its practical meaning – now allows us to understand that the conclusions drawn from this market analysis must be under different conditions.

3. To simplify the analysis we will examine a simple supply dyopoly. Generalisations can be easily made about the conclusions if we replace the "quantity of goods supplied" with the term "value" using the sense as defined in our fourth chapter.

We will therefore imagine two companies supplying a homogenous good. Both will initially analyse the market onto which they are releasing different quantities

and calculate the change to the prices and profits obtained. Which picture will they obtain from the market in this way?

a) Let us assume that the first dyopolist provides four different quantities of goods for the market on Monday, Tuesday, Wednesday and Thursday, and establishes their associated prices and profits, thus calculating the most favourable output and supplies for the Friday and Saturday. The second dyopolist by contrast adjusts his output each Monday in the course of a month and calculates his average weekly price and weekly profit, and the most favourable output for him for the trial week when he entered the market. What price formation would occur?

After setting his output the first time the second supplier will notice a fluctuation in the price caused by the first supplier's fluctuating daily output that comes to a halt on Friday. The next Monday the second supplier adjusts his output and as a result the price and the profit of the first supplier changes, thereby compelling a new market analysis to be conducted so that the change in the second supplier's supply undergoes a new fluctuation in price at the start of the second week and stabilises to a new level at the end of the week. Each week therefore leads to a new "sales curve" for the first supplier and a new point on his "sales curve" for the second supplier. After the fourth week the second supplier will be able to read off his "sales curve" from the four points and choose his most favourable supply from it. On the Monday of the fifth week he will release this most favourable supply onto the market and at the end of this week the first supplier will also know his "most favourable point" and will have achieved it so that the final stabilisation occurs. This stabilisation occurs, as we can easily see, at the independence point of the second supplier because the first supplier, who adjusts his supply every day, does so in order to achieve a constant supply relative to his rival and thus obtains a sales function "for his rival's unchanged supply". The second supplier, by contrast, obtains a sales function that also shows the effects of the reactions of the first supplier. A sales curve for the first supplier corresponds to each point on the sales curve for the second.

We therefore see that the difference described between the two suppliers in the market analysis method produces this asymmetrical result. Conversely, if the first supplier conducted his experiment in the long-term, and the second in the short-term, in effect the first supplier's independence point would be realised.

b) What happens now if no ranking whatsoever is established in the experiment and market analysis is merely carried out by each company independently? No definite outcome can then emerge at all since different prices are to be assigned to a dyopolist's arbitrary output quantity depending on his rival's behaviour. At any rate we will perhaps be able to establish a certain correlation. We will assume that each of the two dyopolists will be interested in the area of his current maximum profit. These relative profit maximums will therefore be attractive to a certain extent in the sense that they will be the most densely occupied in a correlation pattern or at least the mean variation will be fairly symmetrical.

We will imagine a statistical table in the following arrangement where output quantities for the first supplier are entered in the first rows and output quantities for the second supplier in the first columns, making a symmetrical separation into classes. The number of days on which a quantity combination was supplied

that falls into both class intervals is entered into each field in the table. Then according to our assumption we would expect that the two regression curves for this correlation pattern approximately match the "reaction curves" of the two dyopolists.

Now of course this correlation pattern will not be available to either of the two dyopolists. What will be available to him is the correlation between his output quantity and prices. This results from the following arrangement where the dyopolists' output quantities are entered in the first rows with prices in the first columns – both are separated into classes. The number of days is given in each field on which an output belonging to a class has been sold at a price that also belongs to a class. A price regression curve that can be regarded as a "sales curve" then occurs with regard to output quantities. According to our explanations, each dyopolist's output is now to be occupied at its densest point for those prices that provide the rivals with a relative maximum profit, meaning that the "sales curve" that was statistically derived a moment ago is the sales curve for the position of independence. The effect of this sort of market analysis is therefore expected to take the shape of the Bowley dyopoly.

c) The Bowley dyopoly would of course display an outcome that would contradict the market analysis data of both dyopolists, so they might possibly continue their market analysis. Where they do so independently of each other, in the sense that soon one of them and then the other varies his output to establish the most favourable output and finally achieves this in such a way – which can now easily be seen – that the Cournot dyopoly therefore occurs due to the reaction mechanism described at the end of the fourth chapter. We no longer need to explain in detail that this is also not at all stable.

d) Generally it can therefore be said that the current output combination falls somewhere within the diamond-shaped area, and that the Bowley and Cournot dyopoly point is formed by both of the independence points. Where additional rationalisation of the market is pushed through there will be an analysis of the aggregate demand function, insight into interrelations within the dyopoly market, etc. and price formation shifts according to the Bowley dyopoly point, as the hypotheses for our theoretical deduction are now proven in increasingly larger numbers. We therefore see that the theoretical instability of oligopolistic markets is also of great importance for real life situations. This conclusion applies generally for oligopolies and must be pushed through more strongly when the oligopolistic side of the market contains fewer rivals.

§ 2. Safeguarding the Equilibrium as a Function of Economic Policy

1. The conclusion of our second chapter allows us to formulate the following four market structures: oligopolistic demand, oligopolistic supply, bilateral oligopoly and bilateral monopoly, which are incompatible with both of the basic principles of

the free market capitalist economy, namely "individual striving for maximum utility (maximum profit)" and "freedom of price formation". The third chapter has further demonstrated that the coexistence of multiple markets with simple or restricted monopolies or monopsonies is then also incompatible with the basic principles outlined previously if effective relationships exist between these markets for Types I, II or III. Since the existence of such market relationships in an economy must be assumed to be the rule, so we can generally say that to the extent that the economy's market structures diverge from "free competition", the incompatibility of the economy's exchange mechanism shows the basic principles mentioned above to advantage.

In practice, this incompatibility means jeopardising and impairing the entire economic system, possibly to an unlimited extent. At the same time, it is a force that tends to remodel the organisation of the economy. The purpose of these paragraphs is to create an overview of those remodelling events in economic life that cause this instability to be eliminated. This is specifically mentioned in advance for the explanations below; it is therefore by no means assumed that the economic policy ideas and activities to be named have been caused consciously or unconsciously by those impairments. This can be the case, but does not need to be. We will not be examining this question now. The remodelling tendencies should instead be investigated as to whether they bring about an elimination of that instability or not. What matters to us here is simply the function of this series of events, rather than their cause.

The following instability due to the incompatibility referred to previously can only be removed by eliminating at least one of the factors that is incompatible with the economic reconstruction. Thus we can categorise the following possibilities:

1) Elimination of unstable market structures

2) Elimination of free market price formation

3) Elimination of the principle of maximum utility (in particular of the commercial principle)

The measures outlined above to eliminate this can firstly be taken by the economic subject himself and secondly by the state (translator's note: (Staat), see Scherer: 68).

2. Overall, where there is traditional rather than rational trading – as we have already seen in the first paragraphs of this chapter – instability does not exist in the sense described previously. Here we can speak about an elimination or at the very least a modification of the principle of maximum utility. To a certain degree, the same applies to a new cooperative movement that indeed also represents a removal of the free market price formation. By contrast, it is not possible to eliminate the commercial principle in the capitalist business sector since this is generally regarded as the essential characteristic of the capitalist economic system. Therefore we only need to take this elimination into account for unstable market structures and free market price formation. It has been emphasised by all the theorists working on the problem of oligopoly that it is in the interests of oligopolists to amalgamate and to convert the oligopoly into a monopoly.

We can thus regard the formation of cartels, companies and trusts on the market with oligopolistic supply or demand to be the conversion of these unstable market structures into those of simple monopolies or monopsonies. However we immediately recognise that this process of consolidation is not appropriate for guaranteeing an economic equilibrium since the creation of monopolies in fact eliminates the instability of the simple oligopoly, simultaneously producing instability however that is linked to the established market relationships between monopolies. Furthermore, rival monopolies and complementary goods (or products) must also be combined – a process that can only be achieved by completely merging all production – and this can never occur in an economy based on private initiatives from private individuals. Moreover this consolidation must lead to bilateral monopolies, as we hinted earlier. The instability of these market structures can be eliminated if both monopolists combine together in a vertical structure. The danger of an irreconcilable war exists however to a much greater degree than in the oligopoly, because whilst the consolidation of oligopolists into a monopoly does not just neutralise the instability of the oligopoly but also offers the possibility of larger profits, overall the vertical merger represents a compromise, with both sides deciding to content themselves with that. An increase in their common profits now only occurs if the demander in the bilateral monopoly sells his products simultaneously in the market by way of a simple monopoly[1)]. The war between labour unions and employers' associations that occasionally relates class war ideology to real life can in our context be seen as one such irreconcilable example, i.e. by the participants not working together on a voluntary basis to remove antagonism in the bilateral monopolised labour market.

We can interpret the cartel with its many agreements on price setting, volume of production, sale and purchase conditions, etc. as a private enterprise sector form of elimination of free market price formation that in the end can also serve as long-term delivery contracts between leader and follower companies and groups of companies, thereby in fact eliminating instability in the market in question. However, such agreements are difficult to reach and improbable therefore when it concerns multiple monopolised markets with these relationships. So finally we must take into account that each private market organisation is threatened by outsiders, whereby the equilibrium is inactively and sometimes also actively disturbed.

So we can generally say that the equilibrium safeguards set up by the above organisations are no guarantee of a trouble-free exchange mechanism, apart from the fact that the commercial principle, which in the end produces and supports the measures mentioned above, at the same time causes the monopolistic tendency to exploit, which reduces economic productivity and generally leads to an unchecked increase in any resistance to signs of trouble in the whole economic process. We should have sufficiently verified the accuracy of these assertions here.

[1)]See: v. Stackelberg: *Grundlagen einer reinen Kostentheorie*, ibid.: 74, Satz (XXXVIII).

2. Liberal economic policy has always rejected interference by the state in economic life. If we used the knowledge gained in previous chapters relating to the reasons for an economic policy, we would arrive at just the opposite conclusions. Here however we merely want to briefly examine the extent to which the state's economic policy can operate an equilibrium.

The formation of compulsory cartels regulated by the state can be implicitly judged to eliminate unstable market structures according to the criteria alluded to a moment ago. The price regulation that is mostly associated with this means simultaneously abolishing free market price formation. Most economic policy measures for domestic policy are capable of having this function. Price controls, wage tariffs, working time regulations, the volume of production regulations, usury laws, unfair competition and the law against unfair competition may all be afforded this meaning. But the random interventionism of liberal states that is mostly an outcome of parliamentary compromises cannot be allowed to have its effect, not unlike all of the private market policy measures mentioned briefly before. The partial elimination of the instability limited by the market structure often produces an increase in the signs of trouble so that individual interventions with others are not co-ordinated from a lofty standpoint.

The situation is different however if a stronger state imposes its desire for the reform of a single goal from within economic life. A possibility exists here for supplementing the automatic economic events as is appropriate in terms of sense and meaning, that do not function or conform with the state's intentions, under certain conditions via economic policy. This economic policy then has two functions:

1) Supplementing the automatic economic forces where they cannot achieve a "natural"[1)] equilibrium such that a "conventional"[2] equilibrium occurs

2) Establishing an economic system that does not only function, but functions in the manner desired by the state

It is plausible that the state, which itself also influences an automatic functioning of the economy according to its own goals will never give up the achievement of its objectives where its intervention is already necessary due to real technical and economic circumstances.

Italy shows an interesting example of integral market regulation by a state. The corporatively organised market as it is found in the fascist economic constitution has already been developed in the course of the year so that a short theoretical description of its possible price formation appears. We have therefore devoted the following concluding paragraphs of our book to a theoretical analysis of the economic constitution for the corporate market organisation.

[1)] I would like to thank my supervisor, Professor Erwin von Beckerath for these terms. Translator's note 1 (natürlichen Gleichgewicht), see Heertje: 56. Translator's note 2: (konventionelles Gleichgewicht), see Leontief: 559).

§ 3. The Theoretical Model of Fascist Corporative Markets

1. The fascist economic system in Italy has experienced various interpretations and we now want to focus on three typical ones.

The school of thought that at its most radical rejects not only the findings, but also the methods of previous economic theory, is Ugo Spirito's theory that culminates in the doctrine of the "identity of the individual and the state". This means that the individual only has consciousness and the right to exist insofar as he is a functioning part of the state. In consequence all his actions are fundamentally actions of the state.

Another school of thought is that of Gino Arias who coined the term the "corporative conscience" ("coscienza corporativa"). Here the individual member of the corporative state acts freely, because he can never enter into conflict with the state or the overall interests of the nation due to the "corporative conscience" inside him.

These two normative ideas – since they just concern such an obviously different reality – would in our context mean abolishing the principle of maximum utility in the sense it has been used until now. If people were to translate Spirito's dogma into reality into reality through a process of development and education, instability in the markets – insofar as it still existed – would therefore not occur, because the state, whose individuals would be instruments carrying out their business, cannot otherwise enter into conflict with itself. If Aria's theory did actually become reality, instability would be eliminated as a result of the "corporative conscience" preventing those individuals concerned with doing something on their own responsibility and on their own initiative that could bring disorder to the national economic system.

Both ideas are just dogmas, utopias that in no way should be seen as describing the current state of the situation. Both of the economists, Alberto De'Stefani and Luigi Amoroso, working together in this field, were likely to come to the actual relationships next[1)]. They explain that the free play of the forces under today's relationships does not guarantee either a "desired" equilibrium or a general equilibrium and the economy therefore needs an ordered, connected and balanced leadership from the state. This is the economic function of the corporative state.

We can completely agree with this view. We now want to briefly examine how this balancing and target setting function for the corporative state appears in the markets.

2. A short schematic description of the market organisation that it preformed and programmed into the fascist economic constitution and in practice is partly

[1)]A. De'Stefani and L. Amoroso: "Lo Stato e la vita economica", *Rivista Italiana di statistica, economia e finanza*, Anno IV (1932: 201 et seq.).

The same: "La logica del sistema corporativa", *Rivista internationale di scienze sociale e discipline ausiliarie*, Anno 41, Ser. 3, Vol. 4 (1933: 393 et seq.).

reality and is partly still in the making, was stated in advance of the theoretical analysis[1)].

The market participants in the labour markets for the individual lines of production are divided into employers and employees and brought together in unions. The unions are self-governing bodies under public law and have the function of regulating labour market relations, and wages in particular. Furthermore, the corporation for a branch of production is formed by both unions involved with that branch of production. This is not a self-governing body, but an instrument of the state.

Regulation of the labour market is carried out in such a way that both unions sit around the negotiating table and try to conclude a contract that is then binding on all the branches of production in question. Where a contract is not agreed, a higher authority will try to attempt to reach an agreement. If mediation does not succeed, an industrial tribunal decides and this does not just counterbalance the interests of the participants, but above all, it must promote the aggregate productivity of the economy. From this point of view the state can also encourage the industrial tribunal to intervene. An agreement between the parties in the market should not only be achieved by these institutions but also be in line with the intentions of the state as trustee of the nation.

In the same way that the unions regulate relationships in the labour market, it is the job of corporations to organise relationships between the different branches of production. The corporative rules for these material goods markets (markets for intermediate products in the widest sense) similarly follow the union rules for the labour markets. It is characteristic of the employee's participation in the regulation of relationships between the branches of production and also the fact that this regulation, being a responsibility of the corporations, is a function of the state.

3. According to our market theory, in order to now be able to state the results on balance from the picture of the corporative market described previously, we just need to lay forth the very brief but succinct words of Erwin von Beckerath[2)],

"This regulation appears to be mechanical, such that both sides of the labour market, company and worker, are combined in recognised unions and these monopolise the organisation of labour relations. So, according to this theoretical economist, two monopolistic organisations face each other but no natural equilibrium is established between them. When Carta del Lavoro initially leaves it to the representatives of the two union groups to find the artificial (intended) equilibrium, this means taking advantage of the participants' insight into market relationships for wage setting. Still, the state nevertheless reserves the right to make the decision if the parties do not come to an agreement or the outcome they arrive at is not in line with those three requirements that Carta del Lavoro set for wages. Briefly, these are

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[1)]For closer guidance we refer to:

E. v. Beckerath: "Die Wirtschaftsverfassung des Faschismus", *Schmollers Jahrbuch*, Vol. 56 (1932, II), (Festgabe für Werner Sombart), pp. 347.

Also: Walter Heinrich: *Der Faschismus. Staat und Wirtschaft im neuen Italien*. 2nd, revised ed. Munich (1932: 46–112). Hans Reupke: *Das Wirtschaftssystem des Faschismus*, Berlin, 1930.

[2)]Ibid.: 359.

that the state has the power to impose its will in an agreement in which it authorises publication or imposes a different equilibrium on the participants.

In this case the path leads to an industrial tribunal, but has not often been followed so far.

According to Carta, other economic markets can also be organised in a similar way to the labour market".

According to the notion of corporative market organisation, all markets are therefore converted into the structure of the bilateral monopoly corporative market. Here however free market price formation is abolished and instead a price is ultimately set by the state that is based however on the participants' knowledge of and interest in the facts. This "conventional" price[1)] appears to each of the market participants to be a given variable that is independent of his individual reaction (translator's note: "independent variable" (unabhängige Größe), see Leontief: 559). Where no balance is achieved at that price for the quantity of goods supplied and demanded, the state can correct the price at any time. The state's commitment to the price formation is decisive. It may be that the price differs from the "standard price" for political or socio-political reasons (e.g. in the labour market). However, where there are no such reasons, normal sales occur at the standard price based on the principle of maximum productivity in the economy (as Carta del Lavoro repeatedly states) in the ideal scenario.

When we compare the "conventional" equilibrium from the bilateral monopolised corporative market, which is diametrically opposed to free competition – an equilibrium that occurs due to repeated state intervention – with the "natural" equilibrium – in practice this never occurs in reality – in a market with "free competition", we see that in principle the corporative market leads to the same outcome as with "free competition". The actual differences in the corporative general equilibrium in an ideal situation are not to be judged differently in the end to the actual differences in the approximately competitively organised markets in the free market capitalist economy of the past compared to its theoretical ideal. Thus it can be seen that the corporative organisation of the market neutralises among other things precisely those structural changes that have led to the destruction of the natural equilibrium in the free market capitalist economy, and it achieves a new equilibrium.

[1)] This price has a certain similarity with Schumpeter's (ibid.) "prix crié" established in the bilateral monopoly problem. Translator's note: (konventionelle Preis), see Leontief: 559.

Mathematical Appendix

I. The General Structure of the Individual Equilibrium in a Market. (First chap., § 6.)

1. We denote:
a) For the demander:
The given sum of money in unit time with h,
The quantity of goods obtained for it with x;
b) For the supplier:
The given quantity of goods in unit time with y,
The quantity of money obtained for it with k;
c) An economic agent's ordinal utility index with I.

Where the economic agent is a company, his ordinal utility index is simply the profit for the company, measured in monetary units.

2. Based on the explanations in the text, for the demander, I is a monotonically increasing function of x and a monotonically decreasing function of h, while for the supplier, it is a monotonically decreasing function of y and a monotonically increasing function of k.

We therefore derive:
for the demander:

$$\left.\begin{aligned} &\text{a)}\ I = \varphi(x,h), \\ &\text{b)}\ \frac{\partial I}{\partial x} = \varphi_x > 0, \quad \text{c)}\ \frac{\partial I}{\partial h} = \varphi_h < 0; \end{aligned}\right\} \tag{1}$$

for the supplier:

$$\left.\begin{aligned} &\text{a)}\ I = \psi(y,k), \\ &\text{b)}\ \frac{\partial I}{\partial y} = \psi_y < 0, \quad \text{c)}\ \frac{\partial I}{\partial k} = \psi_k > 0. \end{aligned}\right\} \tag{2}$$

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3. Where the demander is a company, his ordinal utility index = profit is the difference between the monetary revenue E, that the company – in return for the other given capital goods – has x to thank for the quantity of capital goods gained, and the sum of money given h; E is therefore a function of x, and using (1) we derive the particular terms:

$$\left.\begin{aligned} &\text{a) } I = \mathrm{E}(x) - h, \\ &\text{b) } \frac{\partial I}{\partial x} = E' > 0, \quad \text{c) } \frac{\partial I}{\partial h} = -1 < 0. \end{aligned}\right\} \tag{3}$$

Where the supplier is a company, his ordinal utility index = profit is the difference between the sum of money k gained for y and total costs K for y. K is a function of y, and using (2) we derive the particular terms:

$$\left.\begin{aligned} &\text{a) } I = k - K(y), \\ &\text{b) } \frac{\partial I}{\partial y} = -K' < 0, \quad \text{c) } \frac{\partial I}{\partial k} = 1 > 0. \end{aligned}\right\} \tag{4}$$

4. The economic agent's behaviour is based on his idea of the link that exists between x and h or between y and k. This idea can, as we will see again, correspond to the actual market relationships in some circumstances and not in others. In any case however h is a monotonically increasing function of x and likewise k is a monotonically increasing function of y, because the individual will never give away a lesser quantity of goods or quantity of money for a greater quantity of money or quantity of goods. Let the following relationships be:

for the demander:

$$\text{a) } h = h(x), \quad \text{b) } \frac{dh}{dx} = h' > 0; \tag{5}$$

for the supplier:

$$\text{a) } k = k(y), \quad \text{b) } \frac{dk}{dy} = k' > 0. \tag{6}$$

These functions are defined for ($x = 0$; $x = x_{\max}$) or ($k = 0$; $k = k_{\max}$).

5. According to the basic principles of the capitalist economy each individual tries to set his ordinal utility index at a maximum in accordance with the constraints (5a) or (6a). Let the following equations for his behaviour thus be:

for the demander:

$$\left.\begin{aligned} &\text{a) } \varphi_x + \varphi_h \cdot h' = 0, \\ &\text{b) } \varphi_{xx} + 2\varphi_{xh} \cdot h' + \varphi_{hh} \cdot [h']^2 + \varphi_h \cdot h'' < 0, \end{aligned}\right\} \tag{7}$$

for the supplier:

$$\left.\begin{aligned} &\text{a)}\ \psi_y + \psi_k \cdot k' = 0, \\ &\text{b)}\ \psi_{yy} + 2\psi_{yk} \cdot k' + \psi_{kk} \cdot [k']^2 + \psi_k \cdot k'' < 0. \end{aligned}\right\} \tag{8}$$

6. The quotient from the sum of money and quantity of goods, i.e. the quantity of money therefore that is paid or obtained is denoted as being the price of this good. Let p be the price:

$$\text{a)}\ p = \frac{h}{x} \quad \text{or} \quad \text{b)}\ p = \frac{k}{y}. \tag{9}$$

II. The Classical General Equilibrium in a Market with Free Competition. (Second Chap., § 2.)

1. The number of demanders in our market is m; the number of suppliers is n. We allocate the economic entities on each side of the market and assign the functions and variables with the corresponding numbers as indexes. We initially assume, in line with our methods set out in the 2nd chapter, that each individual regards price as a variable independent of his behaviour. Then each individual tries to make his ordinal utility index reach a maximum for each given price; p is therefore regarded as a constant. Each individual pays or obtains the same price in this market since it concerns a homogenous good. Using I (9), we then derive:

$$h' = \mathrm{k}' = p. \tag{1}$$

From (1) and I (7), the following applies for the demander:

$$\left.\begin{aligned} &\text{a)}\ \varphi_x + \varphi_h \cdot p = 0, \\ &\text{b)}\ \varphi_{\mathrm{xx}} + 2\varphi_{\mathrm{xh}} \cdot p + \varphi_{\mathrm{hh}} \cdot p^2 < 0 \end{aligned}\right\} \tag{2}$$

Similarly, for the supplier the following applies from (1) and I(8):

$$\left.\begin{aligned} &\text{a)}\ \psi_y + \psi_k \cdot p = 0, \\ &\text{b)}\ \psi_{yy} + 2\psi_{yk} \cdot p + \psi_{kk} \cdot p^2 < 0 \end{aligned}\right\} \tag{3}$$

In the equilibrium the price must be set so that the total quantity of goods demanded at this price is equal to the total quantity of goods supplied. Therefore, let:

$$\sum_{\mu=1}^{m} x_\mu = \sum_{\nu=1}^{n} y_\nu. \tag{4}$$

We derive m unknown quantities of goods demanded x_μ and n unknown quantities of goods supplied y_ν and an unknown price p, in total $m + n + 1$ unknowns. We derive just as many equations, namely m from form (2a), n from form (3a) and the equation (4). Thus the equations listed completely determine the equilibrium.

We denote total demand with X and total output with Y. We thus derive:

$$\text{a)} \sum_{\mu=1}^{m} x_\mu = X, \quad \text{b)} \sum_{\nu=1}^{n} y_\nu = Y. \tag{5}$$

2. Where we abandon the assumption of an independent price for demand individual No. 1, so the first of the equations (2a) initially does not apply. Demander No. 1 is confronted with a market in which the supplier behaves according to equations (3) and his rivals according to equations (2) and to which equation (4) applies. This market situation sets the quantity demanded x_1 for demander No. 1 and the sum of money to be paid for it h_1 within a very specific relationship and this is the purpose of the following analysis. Equation (4), the $m - 1$ equations (2a) and equations (3a) define x_1 as a function of the price p. And in fact the quantities demanded $x_2 \ldots x_m$ as functions of p determine $m - 1$ equations (2a) and the quantity supplied $y_1 \ldots y$ as functions of p likewise determine the n equations (3a), and the quantity demanded x_1 as a function of $x_2 \ldots x_m$ and the equation $y_1 \ldots y_n$ (4). If we put the price p in brackets after the quantity of goods that are dependent on it to indicate the functional interrelationship, we obtain:

$$x_1 = \sum_{\nu=1}^{n} y_\nu(p) - \sum_{\mu=2}^{m} x_\mu(p) = Y(p) - \sum_{\mu=2}^{m} x_\mu(p) = x_1(p).$$

Function $Y(p)$ shows the dependence of the total output by price and is therefore the familiar supply function.

$\sum_{\mu=2}^{m} x_\mu(p)$ is the demand function of rival demand with demander No. 1. We denote the quantity of goods demanded (or supplied) by the rival of a demander No. x (or supplier No. λ) with X_x (or Y_λ) and we eventually obtain the following term for demander No. 1:

$$x_1(p) = Y(p) - X_1(p). \tag{6}$$

The function $x_1(p)$ indicates the quantity of goods that the demander No. 1 in the market in question can obtain if he offers price p.

2. We denote the elasticity of demand by ε and the elasticity of supply by η; in accordance with the definition that has again become customary we formulate:

$$\left.\begin{aligned} \text{a)}\ \varepsilon &= \frac{dX}{X} : \frac{dp}{p} = \frac{dX}{dp} \cdot \frac{p}{X}, \\ \text{b)}\ \eta &= \frac{dY}{Y} : \frac{dp}{p} = \frac{dY}{dp} \cdot \frac{p}{Y}. \end{aligned}\right\} \tag{7}$$

Similarly, the elasticity of demand ε_x of rival demander No. x and the elasticity of supply η_λ of rival supplier No. λ are defined:

$$\text{a)}\ \varepsilon_x = \frac{dX_x}{dp} \cdot \frac{p}{X_x}, \quad \text{b)}\ \eta_\lambda = \frac{dY_\lambda}{dp} \cdot \frac{p}{Y_\lambda}. \tag{8}$$

3. Using I, let (9) be demander No. 1:

$$h_1{}' = \frac{dh_1}{dx_1} = p + x_1 \cdot \frac{dp}{dx_1} = p + x_1 : \frac{dx_1}{dp}.$$

Using (6), (7b) and (8a) we thus obtain:

$$h_1{}' = p + \frac{x_1}{\frac{dY}{dp} - \frac{dX_1}{dp}} = p.\left(1 + \frac{x_1}{\eta.Y - \varepsilon_1.X_1}\right);$$

from here it follows, using (4) and (5) that:

$$h_1{}' = p \cdot \left[1 + \frac{x_1}{X} \cdot \frac{1}{\eta - \varepsilon_1.\left(1 - \frac{x_1}{X}\right)}\right]. \tag{9}$$

The coefficient for p in (9) differs from the unit in that the smaller it is, the smaller the share the first demander has of aggregate demand (the smaller the value of x_1/X is). Since the elasticity η of supply is generally positive (and always positive for companies) and the elasticity of demand ε_1 is generally negative (and always negative for companies); as a rule both are different for zero so that the fraction

$$\frac{1}{\eta - \varepsilon_1\left(1 - \frac{x_1}{X}\right)}$$

generally has a positive finite value. Generally we derive

$$\lim_{\frac{x_1}{X} \to 0} h_1{}' \to p, \tag{10}$$

i.e. The smaller the share that demander No. 1 has of aggregate demand, the smaller the error that results if the price is set for $h_1{}'$ that is however, if demander No. 1 regards price as a variable that is independent of his reaction and acts according to equation (1). This also applies if η or ε_1 is zero (but not simultaneously). Where $\eta = 0$, so we see that the way the other side reacts to price changes is of no significance in principle for the behaviour of rival economic entities between themselves. There is also therefore the proviso that the rivalry of many economic entities makes price an independent variable for one rival in particular.

4. The reasoning described here for demander No. 1 applies in the same way for each other demander and similarly, each supplier, with the following modifications:

Instead of (9) for supplier No. λ, the following applies:

$$k_\lambda' = p \cdot \left[1 + \frac{y_\lambda}{Y} \cdot \frac{1}{\varepsilon - \eta_\lambda\left(1 - \frac{y_\lambda}{Y}\right)}\right]. \tag{11}$$

ε is generally negative and η_λ is positive.

III. The Monopoly. (Second Chap., § 3.)

1. For the monopsonist a special case of formula II (9) applies. His demand is equal to aggregate demand. He has no rivals; there is also no elasticity ε_1 of competitive demand. Therefore, the following simple formula applies for the monopolist:

$$h' = p\left(1 + \frac{1}{\eta}\right), \tag{1}$$

which is put into in equation I (7). Where the monopsonist is a company, from I (3), I (7) and (1), we derive:

$$E' - p = \frac{p}{\eta} > 0. \tag{2}$$

2. Similarly, for the monopolists instead of (1):

$$k' = p\left(1 + \frac{1}{\varepsilon}\right) \tag{3}$$

and instead of (2):[1)]

$$K' - p = \frac{p}{\varepsilon} < 0. \tag{4}$$

3. In the imperfect monopoly (in the sense of Forchheimer[2)]) the formulae II (9) or II (11) apply with the proviso that $\frac{x_1}{X}$ or $\frac{y_1}{Y}$ may not be ignored.

[1)] This formula is identical to Amoroso's formula: $p - m = \frac{p}{\varepsilon}$ (*La curva statica*..., ibid.: 10), the difference being that Amoroso, like Marshall, has positively correlated ε.

[2)] Ibid.

4. We still have to prove that it is immaterial whether the monopolist sets a specific quantity sold or a specific price (and deals with it as an independent variable). To do this, we only need to completely differentiate I (1a) or I (2a) with respect to p, noting I (5a) or I (6a) and the dependence of X or Y on p (see II (6)).

We obtain

$$\frac{dI}{dp} = \varphi_x \cdot \frac{dx}{dp} + \varphi_h \cdot h' \cdot \frac{dx}{dp}. \tag{5}$$

I (7a) again results from (5) being set to zero. A similar scenario applies for supply.

5. From the concluding remarks in section II, 3, it emerges that the monopolist's opposite number considers price to be an independent variable.

IV. The Oligopoly. (Second Chap., § 4.)

1. The demand function (see II, 2) is:

$$X = N(p). \tag{1}$$

Its inverse is:

$$p = F(X). \tag{2}$$

The reversibility is given, since the demand function decreases monotonically.

2. We initially look at a supply dyopoly for companies. Using I (4a), I (9b) and (now also applicable of course) II (4) and II (5a), the profits of both companies are:

$$\left.\begin{aligned} &\text{a)}\ G_1 = y_1 \cdot F(y_1 + y_2) - K_1(y_1), \\ &\text{b)}\ G_2 = y_2 \cdot F(y_1 + y_2) - K_2(y_2). \end{aligned}\right\} \tag{3}$$

Where the first company regards the second company's supply y_2 as a given variable, it looks to determine its own supply y_1 for a given variable y_2 which is a maximum G_1. The following therefore applies:

$$F + y_1 \cdot F' - K_1' = 0. \tag{4}$$

The second company then has the position of independence because y_1 depends on y_2 through equation (4). Here the following applies for the second company:

$$F + y_2 \cdot F' - K_2' + y_2 \cdot F' \cdot \frac{dy_1}{dy_2} = 0.$$

$\frac{dy_1}{dy_2}$ therefore results from (4) and in fact is:

$$\frac{dy_1}{dy_2} = -\frac{F' + y_1 \cdot F''}{2F' + y_1 \cdot F'' - K_1''}.$$

Thus for the second "independent" company, this equation applies:

$$F - K_2' + y_2 \cdot F' \cdot \frac{F' - K_1''}{2F' + y_1 \cdot F'' - K_1''} = 0. \tag{5}$$

If the numbers 1 and 2 are interchanged in equations (4) and (5), a pair of equations is obtained that describes the position of independence of the first company or the position of dependence of the second.

1) Where both companies are striving to achieve the position of independence, the "Bowley dyopoly" thus exists so that the second company's supply has a value of y_2 resulting in the equations (4) and (5), if these are solved according to y_1 and y_2, and similarly, the first company's supply results from the equations (4) and (5) when the numbers 1 and 2 are interchanged.

2) Where both companies are striving to achieve the position of dependence, the "Cournot dyopoly" exists and each company behaves as if the rival company's supply was always a given variable, and correspondingly with the first company, according to equation (4) and the second according to the same equation, but after swapping the numbers 1 and 2. We thus obtain two equations for determining both quantities supplied in the "Cournot dyopoly".

3) The two asymmetrical dyopolies are described directly by the two equations (4) and (5) or by the same equations after interchanging the numbers 1 and 2.

3. Similar observations lead to an analysis of the general demand dyopoly or supply dyopoly. This should be carried out below for the general demand oligopoly.

We have m demanders with supply functions (see II, 2):

$$Y = A(p) \tag{6}$$

or – since $A\ (p)$ is monotonically increasing – whose inverse

$$\rho = \Phi(y) \tag{7}$$

is the opposite. Using I (9) and II (4) for demander No. μ, function I (5) now has the form:

$$h_\mu = x_\mu \cdot \Phi\left(\sum_{\varrho=1}^{m} x_\varrho\right). \tag{8}$$

1) Let us assume that the first demander always regards his rival's demand as a variable that is not dependent on his behaviour, then let I (7) be this conditional equation for his quantity demanded:

$$\frac{\partial \varphi_1}{\partial x_1}\{x_1, h_1\} + (\Phi + x_1 \cdot \Phi') \cdot \frac{\partial \varphi_1}{\partial h_1}\{x_1, h_1\} = 0. \tag{9}$$

Using (8) for $\mu = 1$ and using (9), x_1 is defined as a function $\sum_{\varrho=2}^{m} x_\varrho$.
With Schneider[1)] we denote this function to be a reaction function and write:

$$x_1 = R_1\left(\sum_{\varrho=2}^{m} x_\varrho\right). \tag{10}$$

2) We can now further assume that demander No. 2 takes dependence (10) into greater account. By contrast he always regards the demand $\sum_{\varrho=3}^{m} x_\varrho$ as a given variable, then the conditional equation for his quantity demanded is:

$$\frac{\partial \varphi_2}{\partial x_2}\{x_2, h_2\} + [\Phi + x_2 \cdot \Phi' \cdot (1 + R_1')] \cdot \frac{\partial \varphi_2}{\partial h_2}\{x_2, h_2\} = 0. \tag{11}$$

Equation (11), together with (8) and (10), defines the second demander's reaction function, which expresses the dependence of his quantity demanded for the demand of demanders No. 3 to No. *m*. We thus derive:

$$x_2 = R_2\left(\sum_{\varrho=3}^{m} x_\varrho\right). \tag{12}$$

3) The third demander behaves according to the same conditions as the second demander, i.e. function (12).
The same conditional equation applies for his quantity demanded:

$$\frac{\partial \varphi_3}{\partial x_3}\{x_3, h_3\} + [\Phi + x_3 \cdot \Phi' . (1 + R_1')(1 + R_2')] \cdot \frac{\partial \varphi_3}{\partial h_3}\{x_3, h_3\} = 0, \tag{13}$$

from which we derive the reaction function:

$$x_3 = R_3\left(\sum_{\varrho=3}^{m} x_\varrho\right)$$

[1)] *Reine Theorie*..., ibid.: 31.

For now we can easily ignore how this development occurs. Demand x_μ of the μ^{th} demander now follows from this equation:

$$\frac{\partial \varphi_\mu}{\partial x_\mu}\{x_\mu, h_\mu\} + \left[\Phi + x_\mu \cdot \Phi' \cdot \prod_{\varrho=1}^{\mu-1}(1 + R_\varrho{}')\right] \cdot \frac{\partial \varphi_\mu}{\partial h_\mu}\{x_\mu, h_\mu\} = 0. \tag{14}$$

The m^{th} demander dominates the entire market. He sets his quantity demanded. The penultimate demander orientates himself towards this, the third towards the demand of both of these, etc.

Equation (14) primarily indicates how excessively complicated the relationships are that occur here. As there are $m!$ possibilities for numbering the demanders, whereby each time each demander comes into another market situation, we can state that each demander will compare $m!$ positions with each other to their greater or lesser advantage. He will aspire to the most favourable of these $m!$ positions. This produces all the possibilities that are carefully described in the text.

4. Using II (4), II (7b) and (7) is:

$$\Phi' = \frac{\Phi}{X \cdot \eta} = \frac{p}{X \cdot \eta}.$$

If we put this term into (14) we thus obtain:

$$\frac{\partial \varphi_\mu}{\partial x_\mu} + p \cdot \left[1 + \frac{x_\mu}{X} \cdot \frac{1}{\eta} \cdot \prod_{\varrho=1}^{\mu-1}(1 + R_\varrho')\right] \cdot \frac{\partial \varphi_\mu}{\partial h_\mu} = 0. \tag{15}$$

From (15) it is easy to see that the closer the conditional equation for a demander's demand in any market position gets to the rival economic form II (2a), the smaller the share of aggregate demand that the demander in question has, because then x_μ/X is smaller. The convergence of $\Pi_{\varrho=1}^{\mu-1}(1 + R_\varrho{}')$with increased μ however can only be proven for $-1 < R_p{}' < 0$; this should however also be the general rule since a demander generally restricts his demand when a rival increases demand and in doing so increases the price, and on the other hand this restriction is not as great as his rival's increase in demand.

V. The Bilateral Monopoly. (Second Chap., § 5.)

With the bilateral monopoly, where the assumption of an independent price applies to the market, equation II (2) applies for the demander with equation II (3) and equation II (4) for the supplier that in this instance has a simple form since only a demander and a supplier are present:

$$x = y. \tag{1}$$

Equation II (2) defines demand price p_1 at which a specific quantity of goods x is demanded as a function of this quantity. This function is just demand function IV (2) that we can write as:

$$p_1 = F(x) \tag{2}$$

Similarly equation II (3) defines the supply price p_2 at which a specific quantity of goods supplied y is a function of this quantity; this is supply function IV (7) that we write as:

$$p_2 = \Phi(y) \tag{3}$$

Function (2) is generally monotonically decreasing and function (3) is monotonically increasing.

The standard quantity that we want to denote with a, we obtain by equating p_1 and p_2 in accordance with (1) from the equation:

$$F(a) = \Phi(a). \tag{4}$$

We obtain the standard price results by putting a into (2) or (3).

1. Where the assumption of an independent price is only abolished for the demander, this occurs if he is just limited to setting the price or the quantity sold, acting as a monopsonist towards the supplier. The same applies mutatis mutandis if the assumption of an independent price is abolished just for the supplier. We therefore only need to refer to the explanations in Sect. III for both these cases.

2. The "equivalence function" is the geometric region for all the points of the (x, h) or (y, k) plane in which the ordinal utility index is the same as in the points $(x = 0, h = 0)$ or $(y = 0, k = 0)$.

The following equation for the demander's equivalence function thus applies using I:

$$\varphi(x, h) = \varphi(0, 0). \tag{5}$$

The value of h, that fulfils x as specified in equation (5), is the equivalent amount to this x belonging to the demander. Denoted by D, the demander's equivalence function[1)] is thus defined by (5):

$$D = D(x) \tag{6}$$

[1)] This is just the demander's indifference curve going through point (0, 0) on the (x, h) plane.

And similarly through the equation:

$$\psi(y, k) = \psi(0, 0) \tag{7}$$

the supplier's equivalent amount S as the function[1)]

$$S = S(y) \tag{8}$$

is defined by y.

3. Where the demander "exploits" the supplier so the following relationship results for him from (1) and (8):

$$h = S(x).$$

Where this term is put into I (7a) we obtain an equation for the quantity $x = t_1$ that the demander permanently demands. In accordance with (7) this equation is:

$$\varphi_\chi(t_1, S(t_1)) \cdot \psi_k(t_1, S(t_1)) - \varphi_h(t_1, S(t_1)) \cdot \psi_y(t_1, S(t_1)) = 0. \tag{9}$$

Where the supplier "exploits" the demander, the following relationship results for him from (1) and (6):

$$k = D(y).$$

Where this term is put into I (8a) we obtain an equation for the quantity t_2 that the monopolist permanently supplies. In accordance with (5) this equation is:

$$\psi_y(t_2, D(t_2)) \cdot \varphi_h(t_2, D(t_2)) - \psi_k(t_2, D(t_2)) \cdot \varphi_\chi(t_2, D(t_2)) = 0. \tag{10}$$

The price at which the demander permanently demands quantity t_1 is

$$\frac{S(t_1) + \vartheta_1}{t_1}$$

where θ_1 is a much smaller positive quantity that is just sufficient to cause the supplier to provide this quantity for the necessary consideration. The price at which the supplier accordingly permanently supplies the quantity t_2 is: $\frac{D(t_2) - \vartheta_2}{t_2}$. The small positive quantity θ_2 is just sufficient to cause the demander to buy the quantity for the desired consideration.

4. a) Where both monopolists are companies, (2) has the special form I (1a) and I (3a) using II (2):

$$p_1 = E'(x) \tag{11}$$

[1)] This is the supplier's indifference curve through point (0, 0) of the (y, k) plane.

and likewise (3) has the special form I (2a) and I (4a) using II (3):

$$p_2 = K'(y). \tag{12}$$

Function (11) is monotonically decreasing using II (2b) in accordance with I (3), and (12) is monotonically increasing using II (3b) in accordance with I (4). Equation (4) for standard quantity a is now:

$$E'(a) = K'(a). \tag{13}$$

b) Where the assumption of the independent price is only abolished for the demander this applies for him if he sets either only the price or only the quantity, using (1) and (12)

$$h = x \cdot K'(x). \tag{14}$$

His quantity demanded q_1 then results using 14, I (3a) and I (7) from the equation:

$$E'(q_1) - K'(q_1) = q_1 \cdot K''(q_1). \tag{15}$$

Since K'' (q_1) is positive according to the explanations at (11) and (12) and $E'(x) - K'$ (x) is a decreasing function thus

$$q_1 < a. \tag{16}$$

The price occurs by putting q_1 into (12) and K' (q_1) is hence smaller than the standard price K' (a) using $K'' > 0$.

c) Where the assumption of an independent price is only abolished for the supplier, we notice something entirely comparable occurring here. Equation (14) is relevant here:

$$k = y \cdot E'(y).$$

The quantity supplied q_2 fulfils the equation similar to (15)

$$E'(q_2) - K'(q_2) = q_2 \cdot E''(q_2).$$

Also using $- q_2 E''$ $(q_2) > 0$ (see (16))

$$q_2 < a.$$

By contrast, price E′ (q_2) is greater than standard price E' (a).

d) Using I (3a) and I (4a), here equations (9) and (10) derive the form:

$$E'(t_1) = K'\,(t_1) \tag{17}$$

and

$$E'(t_2) = K'(t_2). \tag{18}$$

Using (13), (17) and (18) for companies we thus derive

$$a = t_1 = t_2.$$

i.e. the quantities sold here are equal to the standard quantity. Also the functions D (x) and S (y) can be now be determined more precisely:

Using (5) and I (3a) we can derive

$$D = E(x) - E(0); \tag{19}$$

and similarly using (7) and I, (4a):

$$S = K(y) - K(0). \tag{20}$$

Where we compare (19) with (11) or (20) with (12), we see that the equivalence functions for companies are simply the integrals of their demand price functions or supply price functions.

5. Where assumptions about the independent price for both market participants is abolished, a power struggle occurs between them over the quantities sold q_1 and q_2 or t_1 and t_2, as well as over the sums of money $x[F(x)–\Phi\ (x)]$ or $D\ (x) - S\ (x)$. Where both parties in the market are companies, the war will essentially be decided on the sum of money E $(a) - E\ (0) - K\ (a) + K\ (0)$. The outcome is uncertain in all cases.

VI. Preliminary Remarks for the Third Chapter.

The market participants ought to be companies, as was already determined in the text. This restriction implies that participating economic entities do not aspire to the maximum ordinal utility index in general, but instead to maximum profit in monetary terms.

1. We will look at the relationships between the markets for goods A and B. We denote:

1)	the	individual	quantities demanded	of	good	A	with	x
	〃	〃	〃	〃	〃	B	〃	u
2)	〃	〃	quantities supplied	〃	〃	A	〃	y
	〃	〃	〃	〃	〃	B	〃	v

3) The corresponding total quantities with X or U or Y or V.

4) the price of good A with p
〃 〃 〃 〃 B 〃 q

2. Where a demand relationship between the two markets exists, it thus implies that profits G of a few or all demanders, at given prices, depend on the quantities of both goods. For this demander this term applies similarly to I (3a):

$$G = E(x,u) - x \cdot p - u \cdot q = G(x,u). \tag{1}$$

The same applies for the supply relationship. Here, according to I (4a) we can say:

$$G = y \cdot p + v \cdot q - K(y,v) = G(y,v). \tag{2}$$

Where we regard these functions without any consideration for whether they concern a supply side or a demand side, we denote the quantity of goods A sold to be ξ and the quantity of goods B sold by η. Hence the following equation generally applies: $G = G\,(\xi, \eta)$.

3. Where both goods are complementary (whether these are goods for complementary use or complementary production) it thus has the following meaning: an increase in ξ by the positive increment $\Delta\xi$ produces a stronger increase or weaker decrease in profits the greater η is. Therefore where the increment $\Delta\eta$ is positive, the following applies:

$$G(\xi + \Delta\xi, \eta + \Delta\eta) - G(\xi, \eta + \Delta\eta) > G(\xi + \Delta\xi, \eta) - G(\xi, \eta).$$

Where we now divide both sides by $\Delta\xi$ and allow $\Delta\xi$ to tend to zero, we obtain:

$$\frac{\partial G}{\partial \xi}\{\xi, \eta + \Delta\eta\} > \frac{\partial G}{\partial \xi}\{\xi, \eta\}.$$

The first derivative of G to ξ is thus an increasing function of η. The inequality is ultimately a mathematical term[1)] for demand complementarity or supply complementarity for both goods:

$$\frac{\partial^2 G}{\partial \xi \partial \eta} > 0. \tag{3}$$

4. Where both goods are independent of each other in their relationship to profit, the market relationship is therefore only an individual one and so the decrease or

[1)]Cf. V. Pareto, *Manuel*, Appendice: 576 et seq.

increase in profits by the increase in one good is not dependent on the quantity of the other good. We thus obtain the equation[1)]:

$$\frac{\partial^2 G}{\partial \xi \cdot \partial \eta} = 0. \tag{4}$$

5. Where the two goods are rival (alternative) goods, an increase in ξ produces a smaller increase or a stronger decrease in profits the greater η is. We now obtain therefore

$$G(\xi + \Delta\xi, \eta + \Delta\eta) - G(\xi, \eta + \Delta\eta) < G(\xi + \Delta\xi, \eta) - G(\xi, \eta),$$

from which ultimately follows a mathematical term[2)] for the fact that both goods are rivals or alternatives:

$$\frac{\partial^2 G}{\partial \xi \partial \eta} < 0. \tag{5}$$

6. We show the following helpful sentence:

1) The inequality $a_v.b_v - {c_v}^2 > 0$ applies for every v and

2) furthermore every a_v and every b_v derives the same signs and the following inequality applies:

$$\left(\sum_{v=1}^{n} a_v\right) \cdot \left(\sum_{v=1}^{n} b_v\right) - \left(\sum_{v=1}^{n} c_v\right)^2 > 0. \tag{6}$$

I. We show this sentence initially for $n = 2$. Let

$$a_1 \cdot b_1 - {c_1}^2 = a, a_2 b_2 - {c_2}^2 = b \text{ and}$$
$$a_1 b_2 + a_2 b_1 - 2c_1 c_2 = c.$$

Then:

$$c \geqq a_1 b_2 + a_2 b_1 - 2|c_1 c_2| > a_1 b_2 + a_2 b_1 - 2|\sqrt{a_1 b_1 a_2 b_2}|$$
$$= \left(\sqrt{a_1 b_2} - \sqrt{a_2 b_1}\right)^2 \geqq 0.$$

And consequently

$$(a_1 + a_2)(b_1 + b_2) - (c_1 + c_2)^2 = a + b + c > 0.$$

[1)]Cf. V. Pareto, *Manuel*, Appendice: 576 et seq.

[2)]Cf. V. Pareto, *Manuel*, Appendice: 576 et seq.

II. We now use the total induction method to prove this. We must prove that the assertion for n is correct if it is correct for $n-1$. Let

$$\sum_{v=1}^{n-1} a_v = a, \quad \sum_{v=1}^{n-1} b_v = \beta, \quad \sum_{v=1}^{n-1} c_v = \gamma,$$

and assertion (6) is identical to this assertion:

$$(a + a_n)(\beta + b_n) - (y + c_n)^2 > 0.$$

This assertion, as was shown in I, is correct if $a.\,\beta - \gamma^2 > 0$, that is, if (6) is correct for $n - 1$.

III. Since assertion (6) for $n = 2$ has been established in I, it applies generally using II.

VII. Two Monopsonies with Tied Supply. (Third Chap., § 2.)

1. Where a supply relationship exists between two monopsonies, equations VI (2) apply for the supplier. Each supplier tries to have his profit at a maximum where he regards the prices p and q as independent variables. Let us number the suppliers from 1 to n and assign the corresponding variables to the corresponding indices and so we obtain the two supply functions for supplier No. v by the differentiation of function VI (2) by y and v and by setting the derivatives to zero:

$$\text{a)}\ p = \frac{\partial k_v}{\partial y_v}\{y_v, \upsilon_v\}, \quad \text{b)}\ q = \frac{\partial K_v}{\partial \upsilon_v}\{y_v, \upsilon_v\}, \tag{1}$$

and where it is inevitable as a second maximum condition for G_v:

$$\left.\begin{aligned} &\text{a)}\ \frac{\partial^2 K_\upsilon}{\partial {y_\upsilon}^2} > 0, \quad \text{b)}\ \frac{\partial^2 K_\upsilon}{\partial {\upsilon_\upsilon}^2} > 0, \\ &\text{c)}\ \frac{\partial^2 K_\upsilon}{\partial {y_\upsilon}^2} \cdot \frac{\partial^2 K_\upsilon}{\partial {v_\upsilon}^2} - \left(\frac{\partial^2 K_\upsilon}{\partial y_\upsilon \partial \upsilon_v}\right)^2 > 0. \end{aligned}\right\} \tag{2}$$

The two equations (1) define y_v and v_v to be functions of p and q. If we denote the left side of (2c) to be δ_v under (2), by the inversion rule we obtain:

$$\left.\begin{aligned} &\text{a)}\ \frac{\partial y_v}{\partial p} = \frac{1}{\delta_v} \cdot \frac{\partial^2 K_v}{\partial {v_v}^2} > 0, \quad \text{b)}\ \frac{\partial v_v}{\partial q} = \frac{1}{\delta_v} \cdot \frac{\partial^2 K_v}{\partial {y_v}^2} > 0, \\ &\text{c)}\ \frac{\partial y_v}{\partial q} = \frac{\partial v_v}{\partial p} = -\frac{1}{\delta_v} \cdot \frac{\partial^2 K_v}{\partial y_v \partial v_v}. \end{aligned}\right\} \tag{3}$$

2. Using $Y = \sum_{v=1}^{n} y_v$ and $V = \sum_{v=1}^{n} v_v$ follows from (3) for the total output of both goods:

$$\left.\begin{aligned} &\text{a)}\ \frac{\partial \mathrm{Y}}{\partial p} = \sum_{v=1}^{n} \frac{1}{\delta_v} \cdot \frac{\partial^2 K_v}{\partial {v_v}^2} > 0, \quad \text{b)}\ \frac{\partial V}{\partial q} = \sum_{v=1}^{n} \frac{1}{\delta_v} \cdot \frac{\partial^2 K_v}{\partial {y_v}^2} > 0, \\ &\text{c)}\ \frac{\partial V}{\partial q} = \frac{\partial \mathrm{Y}}{\partial p} = -\sum_{v=1}^{n} \frac{1}{\delta_v} \cdot \frac{\partial^2 K_v}{\partial y_v \partial v_v}. \end{aligned}\right\} \tag{4}$$

We denote the determinants $\frac{\partial \mathrm{Y}}{\partial p} \cdot \frac{\partial V}{\partial q} - \frac{\partial \mathrm{Y}}{\partial q} \cdot \frac{\partial V}{\partial p}$ with D. The following allows us to easily prove that D is positive. D fulfils the assumptions of inequation VI (6) in particular. We thus derive:

$$\text{d)}\ \frac{\partial \mathrm{Y}}{\partial p} \cdot \frac{\partial V}{\partial q} - \frac{\partial \mathrm{Y}}{\partial q} \cdot \frac{\partial V}{\partial p} = D > 0. \tag{4}$$

The integration of the terms (4a) and (4c) or (4b) and (4c) leads to these supply functions:

$$\text{a)}\ Y = Y(p, q) \quad \text{and} \quad \text{b)}\ V = V(p, q), \tag{5}$$

which give the quantity of goods Y and V that are supplied at specific prices p and q.

The use of the inversion rule under (4) leads to the terms:

$$\left.\begin{aligned} &\text{a)}\ \frac{\partial p}{\partial Y} = \frac{1}{D} \cdot \sum_{v=1}^{n} \frac{1}{\delta_v} \cdot \frac{\partial^2 K_v}{\partial {y_v}^2} > 0, \\ &\text{b)}\ \frac{\partial q}{\partial V} = \frac{1}{D} \cdot \sum_{v=1}^{n} \frac{1}{\delta_v} \cdot \frac{\partial^2 K_v}{\partial {v_v}^2} > 0, \\ &\text{c)}\ \frac{\partial p}{\partial V} = \frac{\partial q}{\partial Y} = \frac{1}{D} \cdot \sum_{v=1}^{n} \frac{1}{\delta_v} \cdot \frac{\partial^2 K_v}{\partial y_v \partial v_v}. \end{aligned}\right\} \tag{6}$$

The functional determinant $\frac{\partial p}{\partial Y} \cdot \frac{\partial q}{\partial V} - \frac{\partial p}{\partial V} \cdot \frac{\partial q}{\partial Y}$ is the reciprocal of D and thus positive using (4d). The terms (6) give the size of the changes in price that are necessary to produce specific changes in supply. Its integration results in the inversion functions at (5):

$$\text{a)}\ p = p(Y, V), \quad \text{b)}\ q = q(Y, V), \tag{7}$$

that give the prices p and q at which the specific quantities of goods Y and V will be supplied.

3. We denote the monopsonists' profit for good A with A and the monopsonists' profit for good B with B and put (3a) according to I and using $X = Y$ and $U = V$:

$$\text{a)}\ A = E_1(Y) - Y \cdot p, \quad \text{b)}\ B = E_2(V) - V \cdot q. \tag{8}$$

a) Where the first monopolist regards the second monopolist's quantity demanded V as a given variable independent of his behaviour, for him the prices p and q under the functions (7) are only dependent on his quantity demanded Y or alternatively, his quantity demanded Y and price q under (5) are only dependent on his price set p. Where the first monopolist pursues quantity adjustment, the conditional equation for Y results, in which (8a) is differentiated for constant V by Y and the derivative is equal to zero, thus:

$$\underset{(V=\text{const.})}{\frac{dA}{dY}} = E_1'(Y) - p - Y \cdot \frac{\partial p}{\partial Y} = 0. \tag{9}$$

By contrast, where he pursues price adjustment, (8a) must be differentiated by p under (5) with constant V and the derivative must be equal to zero, thus:

$$\underset{(V=\text{const.})}{\frac{dA}{dp}} = [E_1'(Y) - p] \cdot \left(\frac{\partial Y}{\partial p} + \frac{\partial Y}{\partial q} \cdot \frac{dq}{dp}\right) - Y = 0. \tag{10}$$

We obtain the differential quotient $\frac{dq}{\partial p}$ from (5b), in which we let $V = \text{const.}$ It is thus:

$$\frac{\partial q}{\partial p} = -\frac{\frac{\partial V}{\partial p}}{\frac{\partial V}{\partial q}}. \tag{11}$$

Where we observe (11), (4d) and then (4b) and (6a) we see that

$$\frac{\partial Y}{\partial p} + \frac{\partial Y}{\partial q} \cdot \frac{dq}{dp} = \frac{1}{\frac{\partial p}{\partial Y}} \tag{12}$$

is true.

If we put (12) into (10) and then multiply (10) with $\frac{\partial p}{\partial Y}$, we again obtain equation (9). That means that where the first monopolist always regards the second monopolist's quantity demanded to be given, he then responds to the behaviour of the second monopolist in an unambiguous way no matter whether he pursues price or quantity adjustment.

b) By contrast, where the first monopolist does not regard the second monopolist's quantity demanded V as a given fact but rather the price the latter sets, this is a

different situation altogether. Then for him the quantities Y and V over (5) appear as functions of p alone or variables p and V over (7) as functions of Y alone. Where the first monopolist pursues price adjustment, his behaviour is determined by the equation:

$$\underset{(q=\text{const.})}{\frac{dA}{dp}} = [E_1{}'(Y) - p] \cdot \frac{\partial Y}{\partial p} - Y = 0. \tag{13}$$

Where he pursues quantity adjustment, likewise equation (13) eventually occurs in a similar way to a) (equations (10) to (12)), from which we conclude that the first monopolist's behaviour is unambiguous if he regards the price the second one sets to be a given. Comparison of the equations (9) and (13) shows that they are fundamentally different. This becomes especially significant to the term when (13) is divided by $\frac{\partial Y}{\partial p}$. We then derive:

$$E_1{}'(Y) - p - Y \cdot \frac{1}{\frac{\partial Y}{\partial p}} = 0 \tag{14}$$

The variables $\frac{\partial p}{\partial Y}$ and $\frac{1}{\frac{\partial Y}{\partial p}}$ are known to be fundamentally different from each other however, something that can also be immediately seen from (6a) and (4a).

c) Where the first monopolist follows the second monopolist's current quantity demanded, a link thus exists for the first monopolist in (9) and (7a) between Y and V.

Where (7a) is put into (9) as follows we thus obtain Y as an implicit function of V. This is the reaction function for the first monopolist. We can therefore state:

$$Y = R(V). \tag{15}$$

Where the second monopolist pursues quantity adjustment there consequently occurs a conditional equation for his behaviour through differentiation for (8b) by V under (7b) and (15) and setting the derivative to zero:

$$\underset{(Y=R(V))}{\frac{dB}{dV}} = E_2{}'(V) - q - V \cdot \left(\frac{\partial q}{\partial Y} \cdot R' + \frac{\partial q}{\partial V} \right) = 0. \tag{16}$$

Thus, in order to determine his behaviour when he pursues price adjustment we must observe that the following equation applies using (5a), (5b) and (15):

$$R(V(p,q)) = Y(p,q) \tag{17}$$

by which p is implicitly defined as a function of q. By the differentiation of (8b) by q under (5b) and (17) and by setting the derivative to zero position we then

obtain the conditional equation for the pricing policy behaviour of the second monopolist:

$$\underset{(Y=R(V))}{\frac{dB}{dq}} = [E_2'(V) - q].\left(\frac{\partial V}{\partial q} + \frac{\partial V}{\partial p} \cdot \frac{dp}{dq}\right) - V = 0. \tag{18}$$

We obtain the differential quotient $\frac{dp}{dq}$ from (17):

$$\frac{dp}{dq} = -\frac{R' \cdot \frac{\partial V}{\partial q} - \frac{\partial Y}{\partial q}}{R' \cdot \frac{\partial V}{\partial p} - \frac{\partial Y}{\partial p}}. \tag{19}$$

Where we observe (19) and (4c) and also (4c) and (6c) as well as (4a) and (6b), we thus see that the following is true:

$$\frac{\partial V}{\partial q} + \frac{\partial V}{\partial p} \cdot \frac{dp}{dq} = \frac{1}{R' \cdot \frac{\partial q}{\partial Y} + \frac{\partial q}{\partial V}} \tag{20}$$

If we put (20) into (18) and multiply equation (18) with $R' \cdot \frac{\partial q}{\partial Y} + \frac{\partial q}{\partial V}$, we again obtain equation (16), meaning that where the first monopolist regards the second monopolist's quantity demanded to always be a given, the reaction of the second monopolist – no matter whether he is pursuing quantity adjustment or price adjustment – is clearly determined by equation (16).

d) Where the first monopolist follows the second monopolist's current price set, a link thus exists for the first monopolist in (13) and (5a) between p and q. This implicit function is the reaction function for the first monopolist. We can therefore state:

$$p = r(q). \tag{21}$$

The pricing policy behaviour of the second monopolist is then determined using (8b), (5b) and (21) from this equation:

$$\underset{(p=r(q))}{\frac{dB}{dq}} = [E_2'(V) - q].\left(\frac{\partial V}{\partial p} \cdot r' + \frac{\partial V}{\partial q}\right) - V = 0. \tag{22}$$

The evidence for believing that his reaction is determined by the same equation when he pursues quantity adjustment occurs under (21) and (7) in an entirely comparable way to the reasoning in c) (equations (17) to (20)).

It still has to be shown here that the two equations (16) and (22) are different to each other, meaning that the behaviour of the second monopolist is different depending on whether the first monopolist regards the quantity demanded or the

price set by the second monopolist to be independent. To appreciate this we will compare (18) and (22).

These equations are the same except for the importance of $\frac{dp}{dq}$ or r'.

$\frac{dp}{dq}$ is derived in (18) by (17) from (9) and the derivative of p by q from (13) is in (22)r'. Since (9) and (13) are however fundamentally different, as we saw, from each other, thus the variables $\frac{dp}{dq}$ and r' in (18) and (22) are also different.

The equations (9), (13), (16) and (22) describe the four possible market policies. (9) is the position of quantity dependence, (16) is the corresponding position of quantity independence. (13) is the position of price dependence, (22) is the corresponding position of price independence. None of these four positions depends on whether the monopolist holding it is pursuing quantity or price adjustment.

The four equations outlined previously are only then the same i.e. the four positions are only identical, when both goods are neither alternative or complementary, meaning if $\frac{\partial p}{\partial V} = \frac{\partial q}{\partial Y} = 0$ is true.

Then, as follows $\frac{1}{\frac{\partial Y}{\partial p}} = \frac{1}{\frac{dY}{dp}} = \frac{dp}{dY} = \frac{\partial p}{\partial Y}$, etc. (13) or (14) thus obtain the same form as (9), (16) and (22).

VIII. The Intermediary Bilateral Monopoly. (Third Chap., § 4.)

1. The market relationship analysed in detail a moment ago indicates all of the main factors for the two other market relationships. The demand relationship for two monopolies corresponds completely to those described a moment ago and can be similarly applied almost word-for-word and step-by-step. The intermediary bilateral monopoly is repeated here in terms of method.

2. The relationship between a company's demand and supply can be explained as follows: where a company demands quantity x of good A and supplies quantity v of good B, its revenue is the product of price q of good B and quantity v while its costs are the product of price q of good A and quantity x plus other costs that we will denote here with L. L depends both on x as well as on v and in fact the greater L is, the greater v is and the smaller x is (because A is a means of production for the company that allows greater use or savings to be made in respect of "other" means of production.) This company's profit is then:

$$G = v \cdot q - x \cdot p - L(x, v), \tag{1}$$

where, as was just shown,

$$\text{a) } \frac{\partial L}{\partial x} < 0 \text{ and b) } \frac{\partial L}{\partial v} > 0 \text{ are true.} \tag{2}$$

Where we number the companies that compete as demanders in the market for good A and suppliers in the market for good B from 1 to r, thus for each company

No. ϱ the equations apply, because of the second basic principle of the capitalist economy where companies attempt to maximise their profit and therefore regard prices p and q as independent variables:

$$\text{a) } p = -\frac{\partial L_\varrho}{\partial x_\varrho}\{x_\varrho, v_\varrho\}, \quad \text{b) } q = \frac{\partial L_\varrho}{\partial v_\varrho}\{x_\varrho, v_\varrho\}, \tag{3}$$

where the second maximum condition is simultaneously fulfilled:

$$\left.\begin{aligned} &\text{a) } \frac{\partial^2 L_\varrho}{\partial {x_\varrho}^2} > 0, \quad \text{b) } \frac{\partial^2 L_\varrho}{\partial {v_\varrho}^2} > 0, \\ &\text{c) } \frac{\partial^2 L_\varrho}{\partial {x_\varrho}^2} \cdot \frac{\partial^2 L_\varrho}{\partial {v_\varrho}^2} - \left(\frac{\partial^2 L_\varrho}{\partial x_\varrho \partial v_\varrho}\right)^2 > 0. \end{aligned}\right\} \tag{4}$$

The inverse of (3) follows if we denote the determinants (4c) by θ_ϱ:

$$\left.\begin{aligned} &\text{a) } \frac{\partial x_\varrho}{\partial p} = \frac{1}{\vartheta_\varrho} \cdot \frac{\partial^2 L_\varrho}{\partial {v_\varrho}^2} < 0, \quad \text{b) } \frac{\partial v_\varrho}{\partial q} = \frac{1}{\vartheta_\varrho} \cdot \frac{\partial^2 L_\varrho}{\partial {x_\varrho}^2} > 0, \\ &\text{c) } \frac{\partial x_\varrho}{\partial q} = -\frac{\partial v_\varrho}{\partial p} = -\frac{1}{\vartheta_\varrho} \cdot \frac{\partial^2 L_\varrho}{\partial x_\varrho \partial v_\varrho}. \end{aligned}\right\} \tag{5}$$

The examination of (5c) indicates that

$$\frac{\partial^2 G_\varrho}{\partial x_\varrho \partial v_\varrho} = -\frac{\partial^2 L_\varrho}{\partial x_\varrho \partial v_\varrho} > 0$$

is true; because the higher price q is for the product, the more is demanded at a constant price p for the means of production; or, the higher price p is for the means of production, the less is supplied for this at a constant price q for the product. (This assertion, that is self-explanatory, can also be proven analytically by rather complicated methods from the rival economy's production term.)

Using (5), the determinant is:

$$\Theta = \frac{\partial X}{\partial p} \cdot \frac{\partial V}{\partial q} + \frac{\partial X}{\partial q} \cdot \frac{\partial V}{\partial p} < 0,$$

where the conditions of VI (b) for $-\Theta$ are fulfilled in particular. The additional analysis is relevant in every detail to the reasoning in section VII.

IX. The Dyopoly. (Numerical Example.)

1. The geometric illustration of the fourth chapter does not require an analytical supplement because a greater degree of accuracy is not to be obtained. We can therefore immediately switch to discussing specific examples.

2. Our numerical example in the text is at the heart of the following analytical terms:

Demand function: $p = 10 - \frac{x}{100}$,

Cost function for A: $K_1\ (y_1) = 500 + 2\ y_1$,

Cost function for B: $K_2\ (y_2) = 600 + 1.5\ y_2$.

All the functions that occur are either linear or quadratic so that the interpolation methods applied agree in every respect. The calculation of the individual values occurs simply by putting the given functions or their derivatives into the formulae IV (3) to IV (5). We will therefore abandon any analytical wording of the text and instead work through another numerical example that in contrast to the text example does not illustrate the configuration $\alpha\alpha$, but rather $\beta\beta$.

3. We will look at the demand function

$$p = F(x) = 100e^{-\frac{1}{10}\sqrt{x}} \tag{1}$$

and assume that the two suppliers A and B only have fixed costs. We do not need not to take these into account at all and can replace revenue with profits, because profit is now always at a maximum when revenue is at a maximum, and vice versa. Revenue is now identical to gross profit.

We denote A's supply with x_1 and B's supply with x_2 and then derive the following relationships:

a) Total output:

$$x = x_1 + x_2, \tag{2}$$

b) Revenue = A's profit:

$$G_1 = x_1 \cdot 100e^{-\frac{1}{10}\sqrt{x}} = x_1 \cdot F(x), \tag{3}$$

c) Revenue = B's profit:

$$G_2 = x_2 . 100e^{-\frac{1}{10}\sqrt{x}} = x_2 \cdot F(x). \tag{4}$$

The two profit functions are completely symmetrical and therefore all the statements about A also apply for B if the indices 1 and 2 are interchanged.

4. The reaction functions R_1 and R_2 for A and B result in such a way that the partial derivatives for G_1 by x_1 or G_2 by x_2 are equal to zero:

a) Reaction function for A:

$$\frac{\partial G_1}{\partial x_1} = \left(1 - \frac{x_1}{20\sqrt{x}}\right) \cdot F(x) = 0,$$

from which, using (2):

$$x_1 = 20\sqrt{x_1 + x_2} \tag{5}$$

and ultimately, since x_1 and x_2 can only be positive:

$$x_1 = 200 + 20\sqrt{x_2 + 100} = R_1(x_2). \tag{6}$$

b) Correspondingly, we obtain the reaction function for B:

$$x_2 = 20\sqrt{x_1 + x_2} = 200 + 20\sqrt{x_1 + 100} = R_2(x_1). \tag{7}$$

5. Using

$$\frac{\partial G_1}{\partial x_2} = -\frac{x_1}{20\sqrt{x}} \cdot F(x) < 0 \tag{8}$$

where the ordinal utility indexes (profits) for A decrease from the bottom to the top and similarly, the ordinal utility indexes for B from left to right, the derivative reaction function for A is:

$$R_1'(x_2) = \frac{1}{40\sqrt{x_2 + 100}}. \tag{9}$$

It is positive. The same applies for B. We thus derive the configuration *ßß* and may expect the outcome to be the occurrence of the Cournot dyopoly.

6. We still want to show that the indifference curves are essentially convex upwards or to the right, meaning that in the entire field of definitions they have only one extreme and are convex upwards in that vicinity.

We obtain the directional tangent vector for A's indifference curve through differentiating (3) for constant G_1. It is here:

$$\frac{dx_2}{dx_1} = -\frac{\frac{\partial G_1}{\partial x_1}}{\frac{\partial G_1}{\partial x_2}} = \frac{\left(1 - \frac{x_1}{20\sqrt{x}}\right) \cdot F(x)}{\frac{x_1}{20\sqrt{x}} \cdot F(x)} = 20\frac{\sqrt{x}}{x_1} - 1. \tag{10}$$

Term (10) is only zero on A's reaction curve (5).

The curvature of the indifference curve gives the second derivative:

$$\left.\begin{aligned}\frac{d^2x_2}{dx_1{}^2} &= \frac{\partial \frac{\partial x_2}{\partial x_1}}{\partial x_1} + \frac{\partial \frac{\partial x_2}{\partial x_1}}{\partial x_2} \cdot \frac{dx_2}{dx_1} \\ &= \frac{10 \cdot \frac{x_1}{\sqrt{x}} - 20\sqrt{x}}{x_1{}^2} + \frac{10}{x_1\sqrt{x}}\left(20\frac{\sqrt{x}}{x_1} - 1\right) = \frac{200 - 20\sqrt{x}}{x_1{}^2}.\end{aligned}\right\} \tag{11}$$

It is only positive in the area where $x_1 + x_2 < 100$. In particular, it is negative at the zero point for (10), that is, on A's reaction curve. Since this increases monotonically, the smallest reaction function value is

$$R_1(0) = 400.$$

7. The Cournot dyopoly occurs if each party regards the rival's supply to be independent and follows his reaction function. Then the two equations (6) and (7) determine the two quantities supplied and together result in the Cournot supply. The two values that simultaneously fulfil (6) and (7) are:

$$\text{a) } x_1 = 800, \quad \text{b) } x_2 = 800 \text{ and therefore } \text{c) } x = 1,600. \tag{12}$$

The price here is:

$$p = 100e^{-\frac{1}{10}\sqrt{1600}} = 100e^{-4} = 1.83. \tag{13}$$

Profits amount to:

$$G_1 = G_2 = 800 \cdot 1.83 = 1,464. \tag{14}$$

8. A's independence point occurs when A regards the rival supply x_2 on the reaction function $R_2(x_1)$ as a function of his own supply x_1. Using (7) we then derive:

$$\sqrt{x} = \sqrt{x_1 + x_2} = \sqrt{x_1 + 100} + 10$$

and thus

$$G_1 = x_1.100.\ e^{-\frac{1}{10}\sqrt{x_1+100}-1}$$

whereby the conditional equation for A's independent supply follows:

$$\frac{dG_1}{dx_1} = 100e^{-\frac{1}{10}\sqrt{x_1+100}-1} \cdot \left(1 - \frac{x_1}{20\sqrt{x_1 + 100}}\right) = 0.$$

From this we obtain $x_1 = 20\sqrt{x_1 + 100}$ and ultimately

$$\text{a) } x_1 = 200 + \sqrt{80,000} = 482.8 \tag{15}$$

and using (7)

$$\text{b) } x_2 = 682.8. \tag{16}$$

Total output here is:

$$x = 1,165.6, \tag{17}$$

that is sold at a price of

$$p = 100e^{-\frac{1}{10}\sqrt{1,165.6}} = 3.29 \tag{18}$$

Profits amount to:

$$\left.\begin{aligned} &\text{a) } G_1 = 482.8 \cdot 3.29 = 1,588.41, \\ &\text{b) } G_2 = 682.8 \cdot 3.29 = 2,246.41. \end{aligned}\right\} \tag{19}$$

For reasons of symmetry, precisely the same figures apply for B if indices 1 and 2 are interchanged.

9. The Bowley dyopoly occurs when each tries to achieve his own independent supply. Here using (15a), both A as well as B supply quantity 482.8. Total output then is:

$$x = 965.6, \tag{20}$$

the price

$$p = 100e^{-\frac{1}{10}\sqrt{965,6}} = 4.47. \tag{21}$$

Profits

$$G_1 = G_2 = 482.8 \cdot 4.47 = 2,158.12. \tag{22}$$

10. Where the four possibilities – the Cournot dyopoly, A's independence point, B's independence point and the Bowley dyopoly – are compared with regard to the advantage provided to supplier A we immediately see that A manages best when B's independence point is achieved. Using (19b) A would obtain a profit here of 2,246.41 whereas his profit would amount to 2,158.12 in the Bowley dyopoly, 1,588.41 with his own independence point and just 1,464.00 in the Cournot

dyopoly. In order to achieve B's independence point, A can only follow a single path. He must convince B that he (A) always follows B's supply and responds to each supply from B in such a way that it dictates his (A's) reaction function (6). He can only do so however if he simply responds in this way to each supply x_2 i.e. that he voluntarily undertakes a relationship of "dependency" with B.

Since, on the other hand, it is effectively the case that if A always behaves according to his reaction function $R_1\,(x_2)$, then the most favourable point for B is his independence point. Thus when B accepts A's "dependency" to be an unchanging situation A has then achieved his objective.

Now however B will think and behave exactly the same and thus for his part accept A's "dependency". Meanwhile each party now behaves according to his reaction function $\mathrm{R}_1(x_2)$ or $R_2(x_1)$ and the "Cournot dyopoly" effectively occurs, that is, the scenario for $\beta\beta$. This clearly unstable Cournot price formation will only be able to remain as long as a dyopolist has given up achieving the most favourable position, takes the reaction of the other into account as a "given circumstance" and is now content to manage this as best he can in the given circumstances, that is, to achieve his own independence point.

X. The Demand Relationship Between Monopolies. (Algebraic Example)

1. In the following we will work through a simple algebraic example for the demand relationship between monopolists. Here the special features of these market equilibriums that we were able to carry out in Sect. IX will at the same time become clear using an argument by analogy based on the supply relationship between monopsonists.

The (variable) quantities of the two goods A and B are x and y, profits = revenue for the two suppliers is A and B and the two prices are p and q. There are no variable costs.

2. Following Edgeworth[1)] let us assume these demand functions:

$$\text{a)}\ x = 1 - p - a_1 q, \quad \text{b)}\ y = 1 - a_2 p - q.$$

Since the equation stated in VII (4c) $\frac{\partial Y}{\partial q} = \frac{\partial V}{\partial p}$ also applies for tied demand (see VIII, 1) thus we inevitably derive: $a_1 = a_2 = a$, so that our two demand functions take the final form:

$$\text{a)}\ x = 1 - p - aq, \quad \text{b)}\ y = 1 - ap - q. \tag{1}$$

[1)] *La teoria pura*... ibid.: 26, or *The Pure Theory*... ibid.: 122.

We derive

$$\frac{\partial x}{\partial q} = \frac{\partial y}{\partial p} = -\alpha.$$

An increase in q produces a decrease in x with positive a and an increase with negative α. Accordingly, changes in p have an effect on demand for y, meaning, where a is positive, goods are complementary and where α is negative, they are rivals.

Using VII (4d) the positive functional determinant D for (1) is:

$$D = 1 - \mathrm{a}^2 > 0. \tag{2}$$

From this follows:

$$|\alpha| < 1. \tag{3}$$

Where we rearrange the functions under (1), i.e. we solve them for p and q we thus obtain under (2):

$$\text{a) } p \cdot D = 1 - \alpha - x + ay \quad \text{b) } q \cdot D = 1 - \alpha + ax - y.$$

We can state:

$$\text{a) } 1 - \alpha = \beta, \quad \text{b) } p \cdot D = P, \quad \text{c) } q \cdot D = Q, \tag{4}$$

thus we derive:

$$\text{a) } \mathrm{P} = \beta - x + ay, \quad \text{b) } Q = \beta + ax - y. \tag{5}$$

For the A and B profits we obtain the following terms:
from (1):

$$\text{a) } A = p - p^2 - apq, \quad \text{b) } B = q - apq - q^2 \tag{6}$$

and from (5) allowing for (2) and (4):

$$\text{a) } D \cdot A = \beta x - x^2 + axy, \quad \text{b) } D \cdot B = \beta y + axy - y^2. \tag{7}$$

Since the conditions for the two monopolists are entirely symmetrical, we want to determine the position of dependency for the first monopolist and the position of independence for the second in the following pages.

3. Price dependence – price independence.
1) The reaction function of the first monopolist (price dependence):
Through setting the zero position for

$$\frac{\partial A}{\partial p} = 1 - 2p - \alpha q$$

the following is true:

$$p = r_1(q) = \frac{1}{2} - \frac{\alpha}{2} q. \tag{8}$$

2) Setting the price for the second monopolist (price independence):
Through setting the zero position for:

$$\frac{dB}{dq} = \frac{\partial B}{\partial q} + \frac{\partial B}{\partial p} \cdot r_1{}' = 1 - \alpha p - 2q + \frac{\alpha^2}{2} q = 1 - \frac{\alpha}{2} + \alpha^2 q - 2q$$

the price set by the second monopolist is:

$$q = \frac{2 - \alpha}{2(2 - \alpha^2)} \tag{9}$$

3) By substituting (9) in (8) we obtain the price for the first monopolist:

$$p = \frac{4 - 2\alpha - \alpha^2}{4(2 - \alpha^2)} \tag{10}$$

4) The quantities supplied are then taken from (1a) and (1b):

$$\text{a) } x = \frac{4 - 2\alpha - \alpha^2}{4(2 - \alpha^2)}, \quad \text{b) } y = \frac{2 - \alpha}{4}. \tag{11}$$

5) We obtain the profits by multiplying (10) and (11):

$$\text{a) } A = \frac{(4 - 2\alpha - \alpha^2)^2}{16(2 - \alpha^2)^2}, \quad \text{b) } B = \frac{(2 - \alpha)^2}{8(2 - \alpha^2)}, \tag{12}$$

4. Quantity Dependence – Quantity Independence.
1) The reaction function for the first monopolist (quantity dependence):
By setting the zero position of $\frac{\partial A}{\partial x}$ and using

$$D \cdot \frac{\partial A}{\partial x} = \beta - 2x + \alpha y$$

the reaction function of the first monopolist is:

$$x = R_1\ (y) = \frac{\beta}{2} + \frac{\alpha}{2}\ y. \tag{13}$$

2) Supply for the second monopolist (quantity independence):
By setting the zero position of $\frac{\partial B}{\partial y}$ and using

$$\begin{aligned} D \cdot \frac{dB}{dy} &= D \cdot \frac{\partial B}{\partial y} + D \cdot \frac{\partial B}{\partial x} \cdot R_1{}' = \\ &= \beta + \alpha x - 2y + \frac{\alpha^2}{2} y = \beta + \frac{\alpha\beta}{2} + \alpha^2 y - 2y \end{aligned} \tag{14}$$

the following is true

$$y = \beta \cdot \frac{2 + \alpha}{2(2 - \alpha^2)}.$$

3) By substituting (14) into (13) we obtain the supply for the first monopolist:

$$x = \frac{\beta}{2} \cdot \frac{4 + 2\alpha - \alpha^2}{2(2 - \alpha^2)}. \tag{15}$$

4) Prices result by substituting (14) and (15) into (3):

$$\text{a)}\ P = \beta \cdot \frac{4 + 2\alpha - \alpha^2}{4(2 - \alpha^2)}, \quad \text{b)}\ Q = \beta \cdot \frac{2 + \alpha}{4}. \tag{16}$$

5) The following formulae then apply for profits:

$$\left.\begin{aligned} \text{a)}\ A &= \frac{x \cdot P}{D} = \frac{\beta^2}{D} \cdot \frac{(4 + 2\alpha - \alpha^2)^2,}{16(2 - \alpha^2)^2} \\ \text{b)}\ B &= \frac{y \cdot Q}{D} = \frac{\beta^2}{D} \cdot \frac{(2 + \alpha)^2}{8(2 - \alpha^2)}. \end{aligned}\right\} \tag{17}$$

5. We now compare the position of price dependence with the position of price independence. For this purpose we divide the price dependence profit (12a) by the price independence profit (12b). Depending on whether the quotient is greater or smaller than 1, either price dependence or price independence will be the more

favourable position. We describe the difference $\frac{A}{B} - 1$ for price adjustment with π, thus using (12), the following applies:

$$\pi = \frac{A}{B} - 1 = \frac{(4 - 2\alpha - \alpha^2)^2 \cdot 8 \cdot (2 - \alpha^2)}{16 \cdot (2 - \alpha^2)^2 \cdot (2 - \alpha)^2} - 1$$
$$= \frac{(4 - 2\alpha - \alpha^2)^2}{2 \cdot (2 - \alpha^2) \cdot (2 - \alpha)^2} - 1 = \frac{16 - 16\alpha - 4\alpha^2 + 4\alpha^3 + \alpha^4}{16 - 16\alpha - 4\alpha^2 + 8\alpha^3 - 2\alpha^4} - 1$$

and ultimately:

$$\pi = -\alpha \cdot \frac{\alpha^2(4 - 3\alpha)}{2(2 - \alpha^2) \cdot (2 - \alpha)^2} = \alpha \cdot \frac{Z}{N}. \tag{18}$$

Using (3), Z as well as N is positive. The sign for π is thus opposed to the sign for a. Our example comes from this:

a) Where both goods are complementary, that is, $\alpha > 0$ price independence is thus more favourable than price dependence, since then $B > A$.

b) Where both goods are rivals, that is, $\alpha < 0$, price dependence is more favourable than price independence, since then $A > B$. If the two goods are demanded independently of each other, $a = 0$ therefore (see VI (4)) the two positions coincide. From the formulae (9), (10), and (12) we see as follows for $\alpha = 0$ that

$$p = q = \frac{1}{2}, x = y = \frac{1}{2} \text{ and } A = B = \frac{1}{4}.$$

6. We now compare the quantity position of dependence with the quantity position of independence in which we divide quantity position of dependence profit (17a) by quantity position of independence profit (17b). We use the same methods as in 5 to describe the difference for quantity adjustment with μ. Then using (17), the following applies:

$$\mu = \frac{A}{B} - 1 = \frac{D \cdot \beta^2 \cdot (4 + 2\alpha - \alpha^2)^2 \cdot 8 \cdot (2 - \alpha^2)}{D \cdot \beta^2 \cdot 16 \cdot (2 - \alpha^2)^2 \cdot (2 + \alpha)^2} - 1$$
$$= \frac{(4 + 2\alpha - \alpha^2)^2}{2.(2 + \alpha^2) \cdot (2 - \alpha^2)} - 1 = \frac{16 + 16\alpha - 4\alpha^2 - 4\alpha^3 + \alpha^4}{16 + 16\alpha - 4\alpha^2 - 8\alpha^3 - 2\alpha^4} - 1$$

and ultimately:

$$\mu = \alpha \cdot \frac{\alpha^2(4 + 3\alpha)}{2(2 + \alpha)^2(2 - \alpha^2)} = \alpha \cdot \frac{Z}{N}. \tag{19}$$

Using (3) Z is positive as well as N. The sign for μ thus agrees with the sign for a. From this comes our example:

a) Where the two goods are complementary, thus $\alpha > 0$ and quantity dependence is more favourable than quantity independence, since then $A > B$.

b) Where the two goods are rivals, thus $\alpha < 0$ and quantity independence is more favourable than quantity dependence, since then $B > A$. Where the two goods are demanded independently of each other, thus $\alpha = 0$ and the two positions coincide. $D = 1$ initially comes from formula (2) and therefore from (4) for $\alpha = 0$: $\beta = 1$, $P = p$ und $Q = q$. From (14), (15), (16) and (17) then comes: $x = y = \frac{1}{2}, p = q = \frac{1}{2}, A = B = \frac{1}{4}$.

We also see therefore that price adjustment and quantity adjustment then lead to the same economic outcome.

7. It now still remains for us to compare each of the most favourable pricing policy positions with each of the most favourable quantity policy positions, thus (for complementary goods) price independence with quantity dependence and (for rival goods) price dependence with quantity independence.

a) For a comparison of price independence with quantity dependence we divide quantity dependence profit A from (17a) by price independence profit B from (12b) and denote the difference to be $\frac{A}{B} - 1$ in this example with π^*. Then the following applies using (2) and (4):

$$\pi^* = \frac{A}{B} - 1 = \frac{(1-\alpha)^2 \cdot (4 + 2\alpha - \alpha^2)^2 \cdot 8 \cdot (2 - \alpha^2)}{(1-\alpha^2) \cdot 16 \cdot (2-\alpha^2)^2 \cdot (2-\alpha)^2} - 1$$

$$= \frac{(1-\alpha) \cdot (4 + 2\alpha - \alpha^2)^2}{(1+\alpha) \cdot 2 \cdot (2-\alpha^2) \cdot (2-\alpha)^2} - 1 = \frac{16 - 20\alpha^2 + 5\alpha^4 - \alpha^5}{16 - 20\alpha^2 + 4\alpha^3 + 6\alpha^4 - 2\alpha^5} - 1$$

and ultimately:

$$\pi^* = -\alpha \cdot \frac{\alpha^2(4 + \alpha - \alpha^2)}{2(1+\alpha)(2-\alpha^2)(2-\alpha)^2} = -\alpha \cdot \frac{Z}{N}. \tag{20}$$

Using (3), Z and N are positive. Therefore the sign for π^* is opposed to the sign for a. For the complementary goods we are interested in here, thus for $a > 0$, π^* is negative, i.e. price independence is more favourable than quantity dependence for complementary goods. This is unquestionably the most favourable position.

b) For a comparison of price dependence and quantity independence we divide the price dependence profit A from (12a) by the quantity independence profit B from (17b) and denote the difference $\frac{A}{B} - 1$ in this case with μ^*. Then the following applies using (2) and (4):

$$\mu^* = \frac{A}{B} - 1 = \frac{(1-\alpha)^2 \cdot (4 - 2\alpha - \alpha^2)^2 \cdot 8 \cdot (2 - \alpha^2)}{(1-\alpha^2) \cdot 16 \cdot (2-\alpha) \cdot (2+\alpha)^2} - 1$$

$$= \frac{(1+\alpha) \cdot (4 + 2\alpha - \alpha)}{(1-\alpha) \cdot 2 \cdot (2-\alpha^2) \cdot (2+\alpha)^2} - 1 = \frac{16 - 20\alpha^2 + 5\alpha^4 + \alpha^5}{16 - 20\alpha^2 - 4\alpha^3 + 6\alpha^4 + 2\alpha^5} - 1$$

and ultimately

$$\mu^* = -\alpha \cdot \frac{\alpha^2(4 - \alpha - \alpha^2)}{2(1-\alpha)(2-\alpha^2)(2+\alpha)^2} = -\alpha \cdot \frac{Z}{N}. \tag{21}$$

Using (3), Z and N are positive. Consequently the sign for μ^* agrees with the sign for α. For the rival goods we are interested in here, thus for $\alpha < 0$, μ^* is negative, i.e. quantity independence for rival goods is more favourable than price dependence. This is unquestionably the most favourable position.

8. To determine the absolutely most favourable positions we have pursued this somewhat circuitous route because it methodically follows the general reasoning in the text. We would have been able to achieve the objective in this particular example more easily if we had compared the position of price dependence with that of quantity dependence. In this way we would otherwise have been able to simultaneously obtain the total ranking of the market policy positions that of course only apply in this example but which cannot be substantiated using our general method.

If we denote price dependence profit with A *(p)*, quantity dependence profit with A (x) and the difference $\frac{A(p)}{A(x)} - 1$ with v, then the following applies using (12a) and (17a) as well as (2) and (4):

$$v = \frac{A(p)}{A(x)} - 1 = \frac{(1-\alpha^2) \cdot (4 - 2\alpha - \alpha^2) \cdot 16 \cdot (2-\alpha^2)^2}{(1-\alpha)^2 \cdot 16 \cdot (2-\alpha^2) \cdot (4 + 2\alpha - \alpha^2)^2} - 1$$

$$= \frac{(1+\alpha) \cdot (4 - 2\alpha - \alpha^2)^2}{(1-\alpha) \cdot (4 + 2\alpha - \alpha^2)^2} - 1 = \frac{16 - 20\alpha^2 + 5\alpha^4 + \alpha^5}{16 - 20\alpha^2 + 5\alpha^4 - \alpha^5} - 1$$

and ultimately

$$v = \alpha \cdot \frac{2\alpha^4}{(1-\alpha)(4 + 2\alpha - \alpha^2)^2} = \alpha \cdot \frac{Z}{N}. \tag{22}$$

Since Z and N are also positive here, the sign for v agrees with the sign for a.

Consequently, for complementary goods ($a > 0$) price dependence is more favourable than quantity dependence ($A\ (p) > A\ (x)$) and conversely for rival goods ($a < 0$) quantity dependence is more favourable than price dependence. Where we further denote the price independence profit with B (q) and the

quantity independence profit with $B\ (y)$ we obtain the following from (18), (19) and (22),

a) For complementary goods:

$$B\ (q) > A\ (p) > A(x) > B\ (y),$$

i.e. price independence is more favourable than price dependence; this is more favourable than quantity dependence and this is more favourable than quantity independence.

b) for rival goods:

$$\mathrm{B\ (y) > A(x) > A(p) > B(q)},$$

i.e. quantity independence is more favourable than quantity dependence; this is more favourable than price dependence and this is more favourable than price independence.

XI. Additional Item.

R.F. Harrod's attempt at a solution

While this book was going to press, an essay appeared by R. F. Harrod[1)] on the subject of the dyopoly problem. This contribution is therefore important because Harrod undertakes to try to analytically formulate the "individual demand curve" in the same sense as Joan Robinson[2)]

Harrod describes aggregate demand with $F\ (x)$, the "individual demand functions" which the two producers are confronted with using $f_a\ (x_a)$ and $f_b\ (x_b)$, their (marginal) cost functions with $\varphi_a(x_a)$ and $\varphi_b(x_b)$. The following are given: $F\ (x) = F\ (x_a + x_b)$, $\varphi_a(x_a)$ and $\varphi_b(x_b)$, whilst the derivatives $f_a'\ (x_a)$ and $f_b'(x_b)$are the individual demand functions that they have strived for.

1. Harrod assumes from the outset that f_a and f_b are linear. There is a "petitio principii" in this, because f_a and f_b depend on F, φ_a and φ_b and we cannot know in advance whether or not the assumption of linearity we are looking for contradicts the given functions or even the entire problem for the functions. This "petitio" is however not decisive in terms of the real problem because this linearity is not necessary for Harrod's reasoning, as he himself notes at the end of his explanations.

2. Harrod allows the first dyopolist to increase his supply x_a by $\Delta\ x$ and tries to calculate the amount by which the second dyopolist in this example will reduce their supply x_b. He thus assumes here that the second dyopolist responds in a particular way to changes in supply by the first. The response results as a consequence of a shift that is produced for f_b by the change in x_a. Harrod determines this

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[1)]"The Equilibrium of Duopoly", *The Economic Journal*, June 1934: 336–337

[2)]Ibid.

shift as if f_b was simply the unused part of the first dyopolist's aggregate demand curve, i.e. ostensibly without noticing he has done so, Harrod tacitly invents the method for calculating this shift:

$$f_b(x_b) = F(x_a + x_b). \tag{1}$$

In fact, this equation is entirely accurate. x_a is the parameter for the second supplier's family of demand curves and they apparently hold the position of dependence. However, Harrod tries another formula for f_b. Hence, equation (1), although it is objectively correct, appears as a "petitio principii" within Harrod's reasoning.

3. For the second dyopolist's reduction in supply produced by the first dyopolist's increase in supply by Δx, Harrod obtains the term:

$$\frac{2f_b'(x_b)}{2f_b'(x_b) - \varphi_b'(x_b)} \cdot \frac{1}{2}\Delta x.$$

From this, the following formula ultimately occurs for $f_a'(x_a)$:

$$f_b'(x_a) = F'(x)\frac{f_b'(x_b) - \varphi_b'(x_b)}{2f_b'(x_b) - \varphi_b'(x_b)}. \tag{2}$$

This is Harrod's formula (1). It precisely equates to the coefficient of y_2 in our formula IV (5) if we look at equation (1) and assume $F(x)$ to be linear.[1)]) Formula (2) thus describes the first dyopolist's position of independence. So, however, by exchanging a and b Harrod also allows the same formula (2) to apply for the second dyopolist. In doing so, Harrod however adjusts the assumptions of his critical analysis by implication because now the second dyopolist varies his supply and the first follows him. Formula (2) and the same formula with the interchanged indices a and b cannot both apply simultaneously, as we tried to show in dyopoly theory. As we saw, (2) applies for f_a' only if equation (1) applies for f_b and $f_b = F'$ is thus true. Harrod's entire reasoning breaks down because of this fact.

4. To make Harrod's error as clear as possible, we will use his equations for the numerical example in our text[2)] that underlie the functions in Mathematical Appendix IX, 2. Here is the demand function:

$$F(x_a + x_b) = 10 - \frac{x_a + x_b}{100}, \tag{3}$$

[1)] φ is the marginal cost function and thus $\varphi' = K''$.

[2)] 4th Chap., § 6.

the (marginal) cost functions:

$$\varphi_a(x_a) = K'_1 = 2, \tag{4}$$

$$\varphi_b(x_b) = K'_2 = 1.5. \tag{5}$$

Then the following is true according to Harrod (see (2)):

$$f_a'(x_a) = -0.01 \cdot \frac{f_b'(x_b)}{2f_b'(x_b)} = -0.005, \tag{6}$$

$$f_b'\ (x_b) = -0.01\ .\ \frac{f_b'\ (x_b)}{2\ f_b'\ (x_b)} = -0.005, \tag{7}$$

Harrod's conditional equations (3), (4) and (5) for x_a and x_b are:

$$F(x_a + x_b) = \psi_a(x_a) - x_a \cdot f_a'(xa), \tag{8}$$

$$F(x_a + x_b) = \psi_b(x_b) - x_b \cdot f_b'(xb), \tag{9}$$

We thus derive for (8) using (3), (4) and (6):

$$8 - 0.015x_a - 0.01x_b = 0 \tag{10}$$

and for (9) using (3), (5) and (7):

$$8.5 - 0.01x_a - 0.015x_b = 0. \tag{11}$$

Both of these equations fulfil the pair of variates:

$$x_a = 280(\text{ztr.}),\ x_b = 380(\text{ztr.}). \tag{12}$$

Total output is $x_a + x_b = 660$ ztr., the price using (3): $p =$ RM 3.4. The two profits G_a and G_b using (3) and IX, 2 are:

$G_a =$ RM – 108 (loss of 108 RM),
$G_b =$ RM + 122.

It is easy to see from the data in the example that (12) definitely does not produce an equilibrium or a solution to the problem at all. Where the first dyopolist permanently supplies the quantity of 280 ztr. it is more advantageous for the second dyopolist not to supply 380 ztr., but just 285 ztr. since he then obtains a profit of RM 212.05. By contrast, where the first dyopolist follows, according to formula (10), the second dyopolist will not supply 380 ztr. but 475 ztr. instead. Then the first

producer's supply is only 216.4 ztr. according to (10) and the second dyopolist obtains a profit of RM 152.08. On the other hand, where the second dyopolist permanently supplies the quantity of 380 ztr., it is more advantageous for the first to supply only 210 ztr. instead of 280 ztr. since he then only has to suffer a loss of RM 59. However, where the second dyopolist follows formula (11) the first dyopolist will supply 350 ztr. His loss then amounts to RM 91.67.

We can see that no matter how the assumptions for the dyopolists' reactions are made, the supply events as Harrod calculates them will never occur. The explanations set out in our example in the 4th Chap., § 6 show how the market will in fact be organised and also show that none of the possibilities explained a moment ago are possible.

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