NEGATIVE MASS PROPULSION

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Schrödinger's analysis of the Dirac equation gives a hint for the existence of negative masses hidden behind positive masses. But their use for propulsion by reducing the inertia of matter for example, in the limit of macroscopic bodied with zero rest mass, depends on a technical solution to free them from their imprisonment by positive masses. It appears that there are basically two ways this might be achieved: 1. By the application of strong electromagnetic or gravitational fields or by high particle energies. 2. By searching for places in the universe where nature has already done this separation, and from where the negative masses can be mined. The first of these two possibilities is for all practical means excluded, because if possible at all, it would depend on electromagnetic or gravitational fields with strength beyond what is technically attainable, or on extremely large likewise not attainable particle energies. With regard to the 2nd possibility, it has been observed that non-baryonic cold dark matter tends to accumulate near the center of galaxies, or places in the universe which have a large gravitational potential well. Because of the equivalence principle of general relativity, the attraction towards the center of a gravitational potential well, produced by a positive mass, is for negative masses the same as for positive masses, and large amounts of negative masses might have over billions of years been trapped in these gravitational potential wells. Now it just happens that the center of the moon is a potential well, not too deep that it cannot be reached by making a tunnel through the moon, not possible for the deeper potential well of the earth, where the temperature and pressure are too high. Making a tunnel through the moon, provided there is a good supply of negative mass, could revolutionize interstellar space flight. A sequence of thermonuclear shape charges would make such tunnel technically feasible.

Keywords:

1. INTRODUCTION

If we extend the law of gravity to negative masses, but hold onto the equivalence of inertial and gravitational mass, we have to distinguish in between the following four cases, if a test particle is placed near a gravitational field producing mass (Table 1).

Under the principle of equivalence if a negative test mass particle would be placed in the gravitational field of earth, it would not fall upwards. A test particle, regardless whether it has positive or negative mass, would there always fall down. It would fall upwards only if placed in the field of a large negative mass.

A somewhat different situation arises if both masses, the field producing mass and the mass of the test particle, have the same absolute value but are permitted to have different signs. There we have to distinguish between the following cases (Fig. 1).

If both masses are positive, we have the usual Newtonian attraction. For negative masses, the force has the same magnitude but is repulsive. A quite different situation exists if one mass is positive and the other one is negative. With both forming a mass dipole, the system becomes self- accelerating, because one mass is repelled and the other one attracted. With the two masses having opposite sign, the total energy and momentum of the combined system remains zero for all times, leaving intact the conservation laws of energy and momentum. Under its self-acceleration, the mass dipole would eventually reach the velocity of light. It is this property of self-acceleration without expenditure of energy which has intrigued many researchers and raised the question for a propulsion system without limits. We remark that even without an appreciable gravitational interaction, a mass dipole with zero, or close to zero inertial mass, could be accelerated to very high velocities with negligible jet power and energy.

TABLE 1: *Example Cases for Test Particles.*

Case	Gravitational field producing mass	Mass of test particle	Motion of test particle
1	+	+	Attraction
2	+	-	Attraction
3	-	+	Repulsion
4	-	-	Repulsion

No matter how strange the properties associated with negative masses appear to be, there can be little doubt that they can be incorporated into Einstein's gravitational field theory as long as they do not violate the principle of equivalence. In particular, the well known Schwarzschild solution for a positive mass M [1]

$$ds^2 = \frac{dr^2}{1-2\gamma M/c^2 r} + r^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right) - \left(1-2\gamma M/c^2 r\right)c^2 dt^2 \tag{1}$$

can be extended to a negative mass, simply by replacing M with –M:

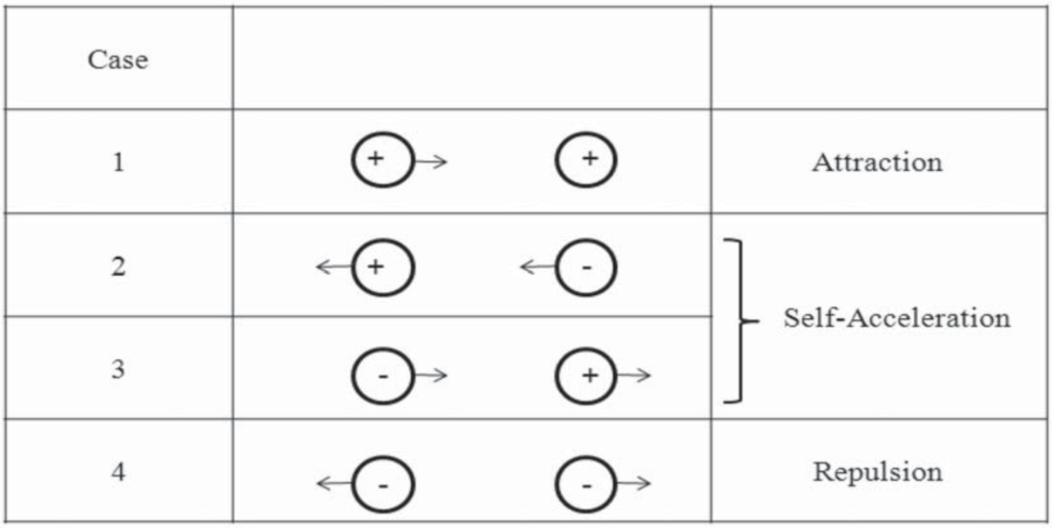

Fig. 1 Example cases when a test particle and a near-by field producing mass have equivalent mass.

$$ds^2 = \frac{dr^2}{1+2\gamma M/c^2 r} + r^2(d\theta^2 + \sin^2\theta d\varphi^2) - (1+2\gamma M/c^2 r)c^2 dt^2 \tag{2}$$

where γ is Newton's constant.

One therefore has to raise the question if nature has not made use of negative masses somewhere. Over and over again we have found that what within the framework of the fundamental laws of physics is possible exists. Only one important physical set of laws, Einstein's special theory of relativity appears to forbid the existence of negative masses. Because in a relativistic quantum field theory the particle number is not a conserved quantity, the existence of negative masses would make all matter unstable against a decay into negative masses, unless there is a fundamental symmetry for the existence of negative masses.

Apart from Einstein's purely kinematic interpretation of special relativity, being the expression of a Minkowskian space-time structure, there is an older alternative dynamic interpretation by Lorentz and Poincaré, which assumed the existence of an ether. While for (1) the geodesic of a test particle is approximately a cone cut where m is positioned in one focus, it is for (2) approximately a cone cut where m is positioned in one "virtual" focus. In it space and time are absolute and it can explain all relativistic effects as well, with all objects in absolute motion through the ether suffering a Lorentz contraction and time dilation. Precision measurements of the black body radiation led Planck to replace his E=nhυ with E=(n+1/2)hυ, implying that each oscillator has the vacuum a zero point energy E(0)=(1/2)hυ which was the reason why Nernst called the zero point energy of the vacuum a kind of an ether. If this ether has a grainy structure, characterized by some smallest length, then according to Heisenberg's uncertainty principle relativity would ultimately break down at a high energy. Depending on how small this length is, this energy can be so high as to be far beyond the capabilities of any existing particle accelerator or even beyond the high energy of cosmic ray particles, making both interpretations of special relativity experimentally indistinguishable at the energies presently available.

2. THE THEORY OF BONDI

The first attempt to introduce negative masses into general relativity to describe a mass dipole, was made by H. Bondi [2]. For a uniformly accelerating mass dipole Bondi uses the axially symmetric metric by Weyl and Levi-Civita [3]:

$$ds^2 = e^{2\varphi}dt^2[e^{2\sigma}(dr^2 + dz^2) + r^2 d\theta^2] \tag{3}$$

where $\varphi = \varphi(r, z)$ and $\sigma = \sigma(r, z)$ satisfy

$$\left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{\partial^2}{\partial z^2}\right)\varphi = 0 \tag{4}$$

$$\frac{\partial \sigma}{\partial r} = r\left[\left(\frac{\partial \varphi}{\partial r}\right)^2 - \left(\frac{\partial \varphi}{\partial z}\right)^2\right] \tag{5}$$

$$\frac{\partial \sigma}{\partial z} = 2r\frac{\partial \varphi}{\partial r}\frac{\partial \varphi}{\partial z} \tag{6}$$

Inserting (3-6) into Einstein's nonlinear gravitational field equations, one obtains four nonlinear partial differential equations ($\kappa = 8\pi\gamma/c^4$) given by

$$-\kappa\rho = -\kappa T_0^0 = e^{2(\varphi-\sigma)} \left[-2\nabla^2\varphi + \nabla^2\sigma - \frac{1}{r}\frac{\partial\sigma}{\partial\tilde{r}} + \left(\frac{\partial\varphi}{\partial\tilde{r}}\right)^2 + \left(\frac{\partial\varphi}{\partial\tilde{z}}\right)^2\right] \tag{7}$$

$$-\kappa p_{11} = -\kappa T_1^1 = \kappa T_2^2 = \kappa p_{22} = e^{2(\varphi-\sigma)} \left[\frac{1}{r}\frac{\partial\sigma}{\partial\tilde{r}} - \left(\frac{\partial\varphi}{\partial\tilde{r}}\right)^2 + \left(\frac{\partial\varphi}{\partial\tilde{z}}\right)^2\right] \tag{8}$$

$$-\kappa p_{33} = -\kappa T_3^3 = e^{2(\varphi-\sigma)} \left[\nabla^2\sigma - \frac{1}{r}\frac{\partial\sigma}{\partial\tilde{r}} + \left(\frac{\partial\varphi}{\partial\tilde{r}}\right)^2 + \left(\frac{\partial\varphi}{\partial\tilde{z}}\right)^2\right] \tag{9}$$

$$-\kappa T_{12} = 2\frac{\partial\varphi}{\partial\tilde{r}}\frac{\partial\varphi}{\partial\tilde{z}} - r\frac{\partial\sigma}{\partial\tilde{z}} \tag{10}$$

In solving these equations Bondi assumes that φ is small, which then also implies that because of (5) and (6) σ is small by the second order. Neglecting second and higher order terms, the solution of the problem is reduced to the solution of the linear Laplace equation for the scalar potential φ, which turns out to be the Newtonian potential in empty space. Under this assumption Bondi can reproduce the uniform acceleration of the mass dipole, as it is expected from the elementary analysis[1]. It is here that we will depart from Bondi[2], with the nonlinearity of the gravitational field equations leading to a very different result. The nonlinearity also sheds light on why it is so difficult to separate negative from the positive masses, whereby negative masses are all around us, but imprisoned by positive masses.

3. HUND'S NONLINEAR NEWTONIAN THEORY OF GRAVITY

As explained by Hund [4], already Newton's theory of gravity, in conjunction with the postulates of special relativity, leads to a nonlinear theory of gravity. With this model theory of gravity the nonlinearity of the gravitational field can be much better explained than with Einstein's theory.

Hund begins with the force **F** acting on a mass m moving with the velocity **v** on a revolving circular platform

$$\mathbf{F} = m\mathbf{g} + \frac{m\mathbf{v}}{c} \times \mathbf{G} \tag{11}$$

where **G** is the Coriolis force field. If **g** is the gravitational acceleration by a real masses with the density ρ, one has in Newton's theory

$$div\mathbf{g} = -4\pi\gamma\rho \tag{12}$$

On the revolving circular platform **g** has a vertical component from the earth gravitational field, but also a radial component by the radial centrifugal acceleration which is not source-free. With the radial component $|g| = \omega^2 r$, where $\omega = 2\pi/T$, with T the time of revolution, and r the radial distance from the center of the revolving circular platform, one has

$$div\mathbf{g} = 2\omega^2 \tag{13}$$

Comparing (13) with (12) one sees that the centrifugal force corresponds to a gravitational repulsion of a homogeneous mass density

$$\mu = -\frac{\omega^2}{2\pi\gamma} \tag{14}$$

For a typical "merry-go-round" one has T = 10 sec and hence $\omega = 0.6 s^{-1}$. For this example one obtains from (14) $\mu = -10^6 g/cm^3$, taken as an absolute value about equal to the mass density of a white dwarf.

The mass density (14) is not fictitious but represents physical reality. According to Einstein's $e = mc^2$ one obtains for the electric field **E** the mass density

$$u = \frac{\varepsilon_0 E^2}{8\pi c^2} > 0 \tag{15}$$

Replacing ε0 with –(1/γ), one obtains the energy density of the gravitational field

$$u = -\frac{g^2}{8\pi\gamma} < 0 \tag{16}$$

It possess the (negative) mass density

$$\mu = -\frac{g^2}{8\pi\gamma c^2} \tag{17}$$

Besides the field **g**, we have on the revolving circular platform the Coriolis force field

$$\mathbf{G} = 2c\boldsymbol{\omega} \tag{18}$$

By setting **G** = **g** and inserting **G** into (17) with $\mathbf{g}^2 = \mathbf{G}^2$, one obtains the mass density (14). This means the centrifugal force is the gravitational force by the mass density of the Coriolis field.

But where is this huge negative mass coming from? The obvious answer is by the very large vacuum energy, making itself felt by going to an accelerated frame of reference.

In Mach's principle the motion of the distant galaxies as seen in an accelerated frame of reference, is responsible for the inertial forces. But this idea is wrong, because if by some miracle the distant galaxies would be set into motion, it would take a long time before their field propagating with the velocity of light reaches the Earth [5].

Adding the mass of the Coriolis field on the right side of the equation (12), we have by putting it on the left hand side

$$div\mathbf{F} - \frac{1}{2c^2}\mathbf{G}^2 = -4\pi\gamma\rho \tag{19}$$

In one further step one should have

$$div\mathbf{F} - \frac{1}{2c^2}(\mathbf{F}^2 + \mathbf{G}^2) = -4\pi\gamma\rho \tag{20}$$

or if **G** = 0 and **F** = -∇ϕ where ϕ is the Newtonian gravitational potential, one has for Poisson's equation:

$$\nabla^2\phi = 4\pi\gamma\rho - \frac{1}{2c^2}(\nabla\phi)^2 \tag{21}$$

According to (21) the positive mass as the source of the Newtonian potential is reduced by the negative mass of its field. Einstein's theory leads to almost the same, except that there the negative gravitational mass density is twice as large. One can there write

1. To reach 1 g the positive and negative mass must be about the mass of the earth. Much larger accelerations are possible for much smaller masses if the interaction between the positive and negative mass is not gravitational, for example if both are charged up to a high electric potential.

2. The author had the pleasure to meet Prof. Bondi on a flight from Graz, Austria in 1993 (we are both members of an academy which had a meeting in Graz that year), and I asked him how his solution can be correct since it does not include the positive mass of the gravitational field of a mass dipole. This problem will be analysed in the next chapter, and its solution has far reaching consequences.

$$\nabla^2\phi + \frac{1}{2c^2}(\nabla\phi)^2 = 4\pi\gamma\rho \quad (22)$$

The earth, for example, is embedded in a sea of negative mass.

Making the substitution

$$\psi = e^{\phi/c^2} \quad (23)$$

transforms (22) into

$$\nabla^2\psi = \frac{4\pi\gamma\rho}{c^2}\psi \quad (24)$$

If ρ is a delta function at r = 0 where

$$m = \int 4\pi r^2 \delta(r)\rho dr \quad (25)$$

One obtains from (24):

$$\frac{d}{dr}\left(r^2\frac{d\psi}{dr}\right) = \frac{m\gamma\psi}{c^2} \quad (26)$$

or

$$\phi = \frac{m\gamma}{r} \quad (27)$$

the same as in Newton's theory.

For a sphere of constant density ρ one obtains the solution

$$\psi = \frac{\sinh kr}{kr}, k^2 = \frac{4\pi\gamma\rho}{c^2} \quad (28)$$

hence,

$$\phi = c^2 \ln\left[\frac{\sinh kr}{kr}\right] \quad (29)$$

For small values of r one has

$$\phi = (2\pi/3)\gamma\rho r^2 \quad (30)$$

with the force per unit mass

$$|\mathbf{g}| = -|\nabla\phi| = (4\pi/3)\gamma\rho r \quad (31)$$

as in Newton's theory.

In general one has

$$|\mathbf{g}| = c^2\left[\frac{1}{r} - \frac{\sqrt{4\pi\gamma\rho}}{c} ctnh\left(\frac{\sqrt{4\pi\gamma\rho}}{c} r\right)\right] \quad (32)$$

which in the limit r → ∞ becomes a constant:

$$|\mathbf{g}| = -\sqrt{4\pi\gamma\rho}c \quad (33)$$

This result implies the self shielding of a large positive mass by the negative mass of its own gravitational field surrounding the mass[3]. The shielding becomes important at the distance

$$R = c/\sqrt{4\pi\gamma\rho} \quad (34)$$

Inserting into (34) the average mass density of the universe, R becomes the radius of the universe. The negative gravitational mass of the universe there shields its positive mass.

For a complete symmetry between positive and negative masses, where the negative mass would in a likewise way shielded by a positive mass, the sign of the gravitational constant would have to be changed.

4. THE THEORY OF BONDI REVISITED

We are now in a position to revisit the theory by Bondi. In his treatment of the positive-negative-mass dipole two-body-problem, he did not take into account the mass of the gravitational field for this configuration. According to (27) the gravitational potential of a point mass remain the same as in Newton's theory. This means the gravitational potential interaction energy for two positive equal masses

$$E_{pot} = -\gamma\frac{m \times m}{r} = -\frac{\gamma m^2}{r} \quad (35)$$

is changed for a mass dipole into

$$E_{pot} = -\gamma\frac{m \times (-m)}{r} = \frac{\gamma m^2}{r} \quad (36)$$

In the theory of Bondi this positive field mass must be added to the positive mass, resulting in a mass pole-dipole, which is a mass pole with a superimposed mass dipole.

5. THE ZITTERBEWEGUNG PHENOMENON AS A MANIFESTATION OF NEGATIVE MASSES

There is no fundamental physical principle standing in the way which forbids the existence of negative masses. If this is true, the question is: Where are these negative masses? The recently noticed large bubbles or voids observed in intergalactic space could possibly be explained by the repulsive force of negative masses assumed to occupy the voids, but alternative, less exotic, explanations have been offered as well. However, there is at least one fundamental phenomenon which strongly speaks for the existence of negative masses. It is the spin of the fermions, like the spin of the electron. Fermions are described by Dirac's relativistic wave equation. This equation has both positive and negative energy components and because of the mass energy relation, it therefore must have negative mass components. According to Schrödinger [6,7], it is these negative mass components which lead to the phenomenon of the spin. Since the overall mass of electron is positive, the occurrence of negative masses in the Dirac equation must mean that the electron is a mass pole with a superimposed mass dipole [8,9].

The spin is definitely not an intrinsic rotational motion of a finite size particle, as older models have suggested. The original model by Uhlenbeck and Goudsmit, for example,

3. Reversing the sign of the gravitational constant γ, a negative mass would be shielded by the positive mass of its gravitational field.

cannot possible be correct because it requires superluminal rotational velocities for an electron with the classical radius $r_0 = e^2/mc^2$.

Figure 2 illustrates how the translation of a mass dipole leads to the occurrence of angular momentum, with m^-v opposite to the direction of m^+v. Making the positive mass slightly larger, with the center of mass S outside the line connecting both masses as shown in Fig. 3, such a pole-dipole particle moves on a circle.

Because the circular motion is self-accelerating it will eventually reach the velocity of light. The connection with Schrödinger's analysis is reached if one puts ($\hbar$ Planck's constant, m in the Dirac equation)

$$r_c = \frac{\hbar}{2mc} \tag{37}$$

whereby the circular motion around S produces just the same angular momentum $2mvr_c = (1/2)\,\hbar$ as in Dirac's equation.

The result first obtained by Breit [10], was that according to Dirac's equation the velocity of the particle of mass m described by the Dirac equation should be equal to the velocity of light. The interpretation of this result is that a electron, presented by the pole-dipole configuration, makes a circular luminal motion onto which a subluminal motion of the center of mass S is superimposed. The trajectory of the resulting motion is a screwline, but it is the subluminal motion of S which one identifies with the (time averaged) velocity. The result derived from this simple pole-dipole model is in beautiful agreement with Schrödinger's analysis of the Dirac equation, in which the luminal rotational motion emerges as a fluctuation of the particle coordinate, called by Schrödinger Zitterbewegung (German for quivering motion).

We may analyze the simple pole-dipole model in more details as follows. If the positive and negative masses are m^+, m^-, with

$$m^+ - \left|m^-\right| << m^+$$

and with the mass pole m is given by

$$m = m^+ - \left|m^-\right| \tag{38}$$

The mass dipole is

$$p = m^+ r \simeq \left|m^-\right| r \tag{39}$$

where $r << r_c$ is the distance of separation in between m^+ and m^-. The center of mass is determined by

$$m^+ r_c = \left|m^-\right| (r_c + r) \tag{40}$$

The angular momentum J then becomes (ω angular velocity of circular motion, with $r_c\omega \to c$):

$$\begin{aligned} |J| &= \left| \left[m^+ r_c^{\,2} - \left|m^-\right| (r_c + r)^2 \right] \omega \right| \\ &= m^+ r_c r \omega \\ &= m^+ r c \end{aligned} \tag{41}$$

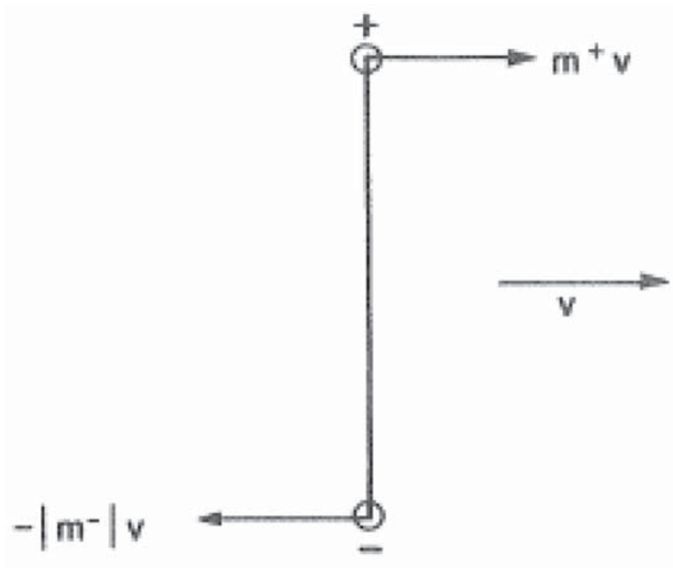

Fig. 2 Translation of mass dipole.

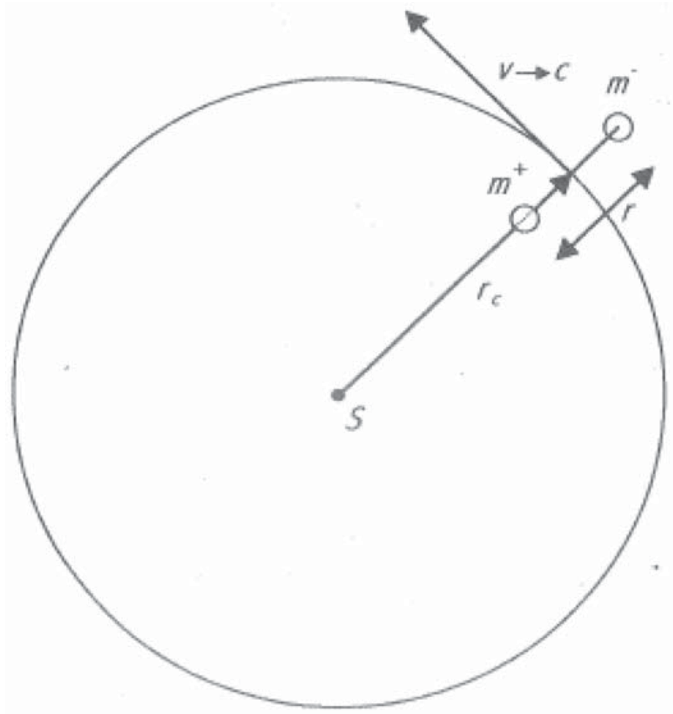

Fig. 3 Circular motion of a pole-dipole particle.

Comparing (40) with (37) shows that

$$2m^+ rc = \hbar \tag{42}$$

and

$$m/m^+ = r/r_c \tag{43}$$

Experimentally, a particle described by the Dirac equation is indistinguishable from a point. This would make r = 0. In reality its size must be a finite but in principle can be very small. This means that m^+ (and

$$\left|m^-\right|)$$

are likely to be very much larger than m.

It has been conjectured by Hönl and Papapetrou [8,9] that in the pole-dipole particle the surplus positive mass comes from the positive gravitational interaction energy of a very large positive m^+ mass with a likewise very large negative

$$m^- = -\left|m^+\right|$$

mass. According to this hypothesis one would have for the rest mass energy

$$mc^2 = \frac{\gamma\left|m^+\right|^2}{r} \tag{44}$$

Combining (44) with (42) one can compute m^+. The result is [8,9]:

$$m^+ = \sqrt[3]{\frac{mc\hbar}{2\gamma}} \simeq 6\text{x}10^{-13} g \tag{45}$$

larger by factor 3.6 x 10^{11} than the mass of the proton.

We therefore see that there are huge amounts of negative masses bound to positive masses in Dirac spinors. It shows that it cannot be a simple matter to free the negative masses from the positive masses. And is explains why the masses of the elementary particles are so much smaller than the Planck mass, $m_p \approx 10^{-5}$g.

6. PLANCK MASS PLASMA HYPOTHESIS [11,12][4]

We make here the proposition that the fundamental group is SU(2) [13], and that by Planck's conjecture the fundamental equations of physics contain as free parameters only the Planck length r_p, the Planck mass m_p and Planck time t_p:

$$r_p = \sqrt{\frac{\hbar\gamma}{c^3}} \approx 10^{-33} cm$$

$$m_p = \sqrt{\frac{\hbar c}{\gamma}} \approx 10^{-5} g$$

$$t_p = \sqrt{\frac{\hbar\gamma}{c^5}} \approx 10^{-44} s$$

The assumption that SU(2) is the fundamental group means nature works like a computer with a binary number system. Since SU(2) is isomorphic to SO(3), the rotation group in R3, explains why natural space is three-dimensional [13].

The Planck's mass plasma conjecture is the assumption that the vacuum of space is densely filled with an equal number of positive and negative Planck mass particles, with each Planck length volume in the average occupied by one Planck mass, with the Planck mass particles interacting with each other by the Planck force over a Planck length, and with Planck mass particles of equal sign repelling and those of opposite sign attracting each other. The particular choice made for the sign of the Planck force is the only one which keeps the Planck mass plasma stable. While Newton's actio-reactio remains valid for the interaction of equal Planck mass particles, it is violated for the interaction of a positive with a negative Planck mass particle, even though globally the total linear momentum of the Planck mass plasma is conserved, with the recoil absorbed by the Planck mass plasma as a whole.

It is the local violation of Newton's actio=reactio which leads to quantum mechanics at the most fundamental level, as can be seen as follows: Under the Planck force $F_p = m_p c^2/r_p$, the velocity fluctuation of a Planck mass particle interacting with a Planck mass particle of opposite sign is $\Delta v = (F_p/m_p)t_p = (c^2/r_p)(r_p/c) = c$, and hence the momentum fluctuation $\Delta p = m_p c$. But since $\Delta q = r_p$ and because $m_p r_p c = \hbar$, one obtains Heisenberg's uncertainty relation $\Delta p \Delta q = \hbar$ for a Planck mass particle. Accordingly, the quantum fluctuations are explained by the interaction with hidden negative masses, with energy borrowed from the sea of hidden negative masses.

According to Newtonian mechanics, the interaction of $\dot{\delta} = a_p t_p = c$ a positive with a negative Planck mass particle leads to a velocity fluctuation with a displacement of the particle equal to $\delta = (1/2) a_p t_p^2 = r_p / 2$ where $a_p = F_p/m_p$. Therefore, a Planck mass particle immersed in the Planck mass plasma makes a stochastic quivering motion (Zitterbewegung) with the velocity

$$v_D = -\left(r_p c/2\right)\left(\nabla n/n\right) \tag{46}$$

where n is the number density of positive or negative Planck mass particles, with the average equal to $n = 1/r_p^3$ .The kinetic energy of this diffusion process is given by

$$\left(\frac{m_p}{2}\right)v_D^2 = \left(\frac{m_p}{8}\right)r_p^2 c^2 \left(\frac{\nabla n}{n}\right)^2 = \left(\frac{\hbar^2}{8m_p}\right)\left(\frac{\nabla n}{n}\right)^2 \tag{47}$$

Putting

$$\mathbf{v} = \frac{\hbar}{m_p}\nabla S \tag{48}$$

where S is the Hamilton action function and **v** is the velocity of the Planck mass plasma, the Lagrange density for the Planck mass plasma in the potential U is

$$L = n\left[\hbar\frac{\partial S}{\partial t} + \frac{\hbar^2}{2m_p}\left(\nabla S^2\right) + U + \frac{\hbar^2}{8m_p}\left(\frac{\nabla n}{n}\right)^2\right] \tag{49}$$

Variation of (49) with regard to S according to

$$\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial S/\partial t}\right) + \frac{\partial}{\partial t}\left(\frac{\partial L}{\partial S/\partial r}\right) = 0 \tag{50}$$

leads to

$$\frac{\partial n}{\partial t} + \frac{\hbar}{m_p}\nabla(n\nabla S) = 0 \tag{51}$$

or

$$\frac{\partial n}{\partial t} + \nabla(n\mathbf{v}) = 0 \tag{52}$$

4. To avoid a misidentification with the luminiferous ether of 19th century physics, I had chosen for it the word "Planck aether", following a suggestion by Nernst to call the zero point vacuum energy, discovered by Planck (with (1/2)hv for each oszillator) an aether. I have henceforth chosen the perhaps better wording: "Planck mass plasma".

which is the continuity equation of the Planck mass plasma. Variation with regard to n according to

$$\left(\frac{\partial L}{\partial n}\right)-\frac{\partial}{\partial r}\left(\frac{\partial L}{\partial n/\partial r}\right)=0 \qquad (53)$$

leads to

$$\hbar\frac{\partial S}{\partial t}+U+\frac{\hbar^2}{2m_p}\left(\nabla S^2\right)+\frac{\hbar^2}{4m_p}\left[\frac{1}{2}\left(\frac{\nabla n}{n}\right)^2-\frac{\nabla^2 n}{n}\right]=0 \qquad (54)$$

or

$$\hbar\frac{\partial S}{\partial t}+U+\frac{\hbar^2}{2m_p}\left(\nabla S^2\right)+\frac{\hbar^2}{2m_p}\frac{\nabla^2\sqrt{n}}{\sqrt{n}}=0 \qquad (55)$$

With the Madelung transformation

$$\psi=\sqrt{n}e^{iS},\psi^*=\sqrt{n}e^{-iS} \qquad (56)$$

(51) and (55) is obtained from the Schrödinger equation

$$i\hbar\frac{\partial\psi}{\partial t}=-\frac{\hbar^2}{2m_p}\nabla^2\psi+U\psi \qquad (57)$$

In the Planck mass plasma hypothesis all particles, save and except the Planck mass particles, are quasi-particles of the Planck mass plasma, like the phonons, rotons, excitons etc. of condensed matter physics, and by the in wave structure are Lorentz invariant. In forming quantized vortices, the Planck mass plasma also has vortex waves, simulating Maxwell's and Einstein's electromagnetic and gravitational waves. Dirac spinors are made possible by the negative masses of the Planck mass plasma.

Quantum mechanics predicts for each harmonic oscillator the zero-point energy $(1/2)\hbar\omega$, which has to be multiplied with the volume element in frequency space $4\pi\omega^2 d\omega$, to obtain the zero-point energy spectrum:

$$f(\omega)d\omega=\text{const.}\omega^3 d\omega \qquad (58)$$

Now (58) turns out to be just the only spectrum that is Lorentz invariant. But (58) is also the only one which does not lead to a friction force on a charged particle moving through an electromagnetic spectrum with this frequency dependence. This means, special relativity is a consequence of quantum mechanics, because quantum mechanics leads to the zero point vacuum energy. One may also say that the zero-point vacuum energy "generates" the Minkowski space-time.

7. DYNAMIC INTERPRETATION OF LORENTZ INVARIANCE

A cut-off at the Planck frequency generates a distinguished reference system in which the zero-point energy spectrum is isotropic and at rest. In this distinguished reference system, the scalar potential from which the forces are to be derived satisfies the inhomogeneous wave equation:

$$-\frac{1}{c^2}\frac{\partial^2\Phi}{\partial t^2}+\nabla^2\Phi=-4\pi\rho(\mathbf{r},t) \qquad (59)$$

where $\rho(\mathbf{r}, t)$ are the sources of this field. For a body in static equilibrium at rest in the distinguished reference system for which the sources are those of the body itself one has

$$\nabla^2\Phi=-4\pi\rho(\mathbf{r}) \qquad (60)$$

If set into absolute motion with the velocity v along x-axis, the coordinates of the reference system at rest with the moving body are obtained by the Galilei transformation:

$$\begin{aligned} x' &= x-vt,\\ y' &= y,\\ z' &= z,\\ t' &= t \end{aligned} \qquad (61)$$

Transforming (59) into (see Appendix)

$$-\frac{1}{c^2}\frac{\partial^2\Phi'}{\partial t'^2}+\frac{2v}{c^2}\frac{\partial^2\Phi'}{\partial x'\partial t'}+\left(1-\frac{v^2}{c^2}\right)\frac{\partial^2\Phi'}{\partial x'^2}+\frac{\partial^2\Phi'}{\partial y'^2}+\frac{\partial^2\Phi'}{\partial z'^2}=-4\pi\rho'(\mathbf{r}',t') \qquad (62)$$

After the body has settled into a new equilibrium in which $\partial/\partial t'=0$, one has instead of (60)

$$\left(1-\frac{v^2}{c^2}\right)\frac{\partial^2\Phi'}{\partial x'^2}+\frac{\partial^2\Phi'}{\partial y^2}+\frac{\partial^2\Phi'}{\partial z^2}=-4\pi\rho(x',y,z) \qquad (63)$$

Comparison of (63) with (60) shows that the l.h.s. of (63) is the same if one sets **Φ'** = **Φ** and

$$dx'=dx\sqrt{1-v^2/c^2}$$

This implies a uniform contraction of the body by the factor

$$\sqrt{1-v^2/c^2}$$

because the sources are contracted by factor

$$\sqrt{1-v^2/c^2}$$

as well, whereby the r.h.s. of (63) becomes equal to the the r.h.s. of (60). Since the zero-point energy is invariant under a Lorentz transformation, the quantum potential changes in the same way as **Φ**. The body therefore sustains its static equilibrium under a contraction by the factor

$$\sqrt{1-v^2/c^2}$$

if set into absolute motion, explaining the Lorentz contraction dynamically.

The clock retardation effect can be derived from the contraction effect, and from there the Lorentz transformation. Following Builder [14,15] this original interpretation of Lorentz invariance by Lorentz and Poincaré has been worked out in every detail by Prokhovnik [16]. To derive the clock retardation effect from the contraction effect one considers a light clock, which is a rod with mirrors attached to its two ends in between light signal, is sent forth and back. If the length of the rod is *l*, and if the rod rests in the distinguished reference system, the time needed for the light signal to be sent forth and back is

$$t_0 = 2l/c \tag{64}$$

If prior to be set into motion the rod is inclined against the x-axis by the angle φ, it appears to be inclined against the x-axis by the different angle ψ after set into motion, with ψ expressed through φ by

$$\tan\psi = \gamma\tan\varphi$$
$$\gamma = \left(1 - v^2/c^2\right)^{-1/2} \tag{65}$$

The absolute motion then contracts the rod from l to l':

$$l' = l\sqrt{1-\left(v^2/c^2\right)\cos^2\varphi} = \frac{l}{\gamma\sqrt{1-\left(v^2/c^2\right)\sin^2\psi}} \tag{66}$$

Relative to the moving rod the velocity of light is anisotropic, and for the to and fro directions given by

$$c_+ = \sqrt{c^2 - v^2\sin^2\psi} - v\cos\psi$$
$$c_- = \sqrt{c^2 - v^2\sin^2\psi} + v\cos\psi \tag{67}$$

with the time t' for a to and fro signal given by

$$t' = l'/c_+ + l'/c_- = \gamma t_0 \tag{68}$$

Therefore, as seen from an observer at rest in the distinguished reference system the clock goes slower by the factor

$$\gamma = 1\big/\sqrt{1-v^2/c^2}$$

independent of the inclination of the rod making up the clock. With solid bodies held together by electromagnetic forces, clocks made from solid matter should behave like light clocks. As it was claimed by Poincaré, it should for this reason be possible to obtain the Lorentz transformations solely from the contraction effect with a proper convention about the synchronization of clocks.

According to Einstein, two clocks, A and B, are synchronized if

$$t_B = \frac{1}{2}\left(t_A^1 + t_A^2\right) \tag{69}$$

where t_A^1 is the time a light signal is emitted from A to B, reflected back at B back to A, arriving at A at the time t_A^2, and where it is assumed that the time t_B at which the reflection at B takes place is equal the arithmetic average of t_A^1 and t_A^2. Only by making this assumption does the velocity of light turn out always to be isotropic and equal to c. from an absolute point of view, the following rather is true. If t_R is the absolute reflection time of the light signal at clock B, one has for the out and return journeys of the light signal from A to B and back to A, if measured by an observer in an absolute system at rest in the distinguished reference system:

$$\gamma(t_R - t_A^1) = d/c_+ ,$$
$$\gamma(t_A^2 - t_R) = d/c_- \tag{70}$$

Where d is the distance between both clocks, and where c_+ and c_- are given by (67). Adding the equations (70) one obtains

$$c(t_A^2 - t_A^1) = 2\gamma d\sqrt{1-\left(v^2/c^2\right)\sin^2\psi} \tag{71}$$

If an observer at rest with the clock wants to measure the distance from A to B, he can measure the time it takes a light signal to go from A to B and back to A. if he assumes that the velocity of the light is constant and isotropic in all inertial reference systems, including the one he is in, moving together with A and B with the absolute velocity v, the distance is

$$d' = (c/2)(t_A^2 - t_A^1) \tag{72}$$

And because of (71)

$$d' = \gamma d\sqrt{1-\left(v^2/c^2\right)\sin^2\psi} \tag{73}$$

Comparing this result with (66), one sees that he would obtain the same distance d′, if he uses a contracted rod as a measuring stick, or Einsteins's constant light velocity postulate. The velocity of light between A and B by using a rod to measure the distance and the time it takes a light signal in going from A to B and back to A, of course, will turn out to be equal to c, because according to (72)

$$\frac{2d'}{t_A^2 - t_A^1} = c \tag{74}$$

Rather than using a reflected light signal to measure the distance d', the observer at A may try to measure the one-way velocity of light by first synchronizing the clock B with A and then measure the time for a light signal to go from A to B. However, since this synchronization procedure also uses reflected light signals, the result is the same. For the velocity he finds

$$\frac{d'}{t_B - t_A^1} = \frac{d'}{1/2(t_A^1 + t_A^2) - t_A^1} = \frac{2d'}{t_A^2 - t_A^1} = c \tag{75}$$

By subtracting the equations (70) one finds that

$$t_R = t_B + \left(\gamma/c^2\right) vd\cos\psi \tag{76}$$

which shows that from an absolute point of view the "true" reflection time t_R at clock B is only then equal to t_B if v = 0. From an absolute point of view the propagation of light is isotropic only in the distinguished reference system, but anisotropic in a reference system in absolute motion against the distinguished reference system. This anisotropy remains hidden due to the impossibility to measure the one way velocity of light. The impossibility is expressed in the Lorentz transformations themselves, containing the scalar c^2 rather than the vector c, through which an anisotropic light propagation would have to be expressed.

8. NEGATIVE MASS INTERPRETATION OF THE AHARONOV-BOHM EFFECT [17]

In Maxwell's equations the electric and magnetic fields can be expressed through a scalar potential **Φ** and a vector potential **A**:

$$\mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{A}}{\partial t} - grad\Phi$$
$$\mathbf{H} = curl\mathbf{A} \tag{77}$$

E and **H** remain unchanged under the gauge transformation of the potentials

$$\Phi' = \Phi - \frac{1}{c}\frac{\partial f}{\partial t}$$
$$\mathbf{A}' = \mathbf{A} + \text{grad}f \tag{78}$$

where f is called the gauge function. Imposing on **Φ** and **A** the Lorentz gauge condition

$$\frac{1}{c}\frac{\partial \Phi}{\partial t} + div\mathbf{A} = 0 \tag{79}$$

the gauge function must satisfy the wave equation

$$-\frac{1}{c^2}\frac{\partial^2 f}{\partial t^2} + \nabla^2 f = 0 \tag{80}$$

In an electromagnetic field the force on a charge e is

$$F = e\left[\mathbf{E} + \frac{1}{c}\mathbf{v}\times\mathbf{H}\right] = e\left[-\frac{1}{c}\frac{\partial \mathbf{A}}{\partial t} - grad\Phi + \frac{1}{c}\mathbf{v}\times curl\mathbf{A}\right] \tag{81}$$

By making a gauge transformation of the Hamilton operator in the Schrödinger wave equation, the wave function transforms as

$$\psi' = \psi \exp\left[\frac{ie}{\hbar c}f\right] \tag{82}$$

leaving invariant the probability density $\psi^*\psi$.

To give gauge invariance a hydrodynamic interpretation, we compare (81) with the force acting on a test body of mass m placed into the moving Planck mass plasma. This force follows Euler's equation and is

$$F = m\frac{dv}{dt} = m\left[\frac{\partial v}{\partial t} + grad\left(\frac{v^2}{c}\right) - \mathbf{v}\times curl\mathbf{v}\right] \tag{83}$$

Complete analogy between (81) and (83) is established if one sets

$$\mathbf{\Phi} = -\frac{m}{2e}v^2$$
$$\mathbf{A} = -\frac{mc}{e}\mathbf{v} \tag{84}$$

According to (78) and (82), **Φ** and **A** shift the phase of a Schrödinger wave by

$$\delta\varphi = \frac{e}{\hbar}\int_{t_1}^{t_2}\Phi dt$$
$$\delta\varphi = -\frac{e}{\hbar c}\oint \mathbf{A}.d\mathbf{s} \tag{85}$$

The corresponding expressions for a gravitational field can be directly obtained from the equivalence principle [3]. If $\partial\mathbf{v}/\partial t$ is the acceleration and w the angular velocity of the universe relative to a reference system assumed to be a rest, the inertial forces in this system are

$$F = m\left[\frac{\partial \mathbf{v}}{\partial t} + \dot{\boldsymbol{\omega}}\times\mathbf{r} - \boldsymbol{\omega}\times(\boldsymbol{\omega}\times\mathbf{r}) - \dot{\mathbf{r}}\times 2\boldsymbol{\omega}\right] \tag{86}$$

For (86) we write

$$F = m\left[\hat{\mathbf{E}} + \frac{1}{c}\mathbf{v}\times\hat{\mathbf{H}}\right] \tag{87}$$

where

$$\hat{\mathbf{E}} = \frac{\partial v}{\partial t} + \dot{\boldsymbol{\omega}}\times\mathbf{r} - \boldsymbol{\omega}\times(\boldsymbol{\omega}\times\mathbf{r})$$
$$\hat{\mathbf{H}} = -2c\boldsymbol{\omega} \tag{88}$$

With

$$curl(\dot{\boldsymbol{\omega}}\times\mathbf{r}) = 2\dot{\boldsymbol{\omega}}$$
$$div(-\boldsymbol{\omega}\times(\boldsymbol{\omega}\times\mathbf{r})) = 2\omega^2 \tag{89}$$

one has

$$div\hat{\mathbf{H}} = 0$$
$$\frac{1}{c}\frac{\partial \hat{\mathbf{H}}}{\partial t} + curl\hat{\mathbf{E}} = 0 \tag{90}$$

$\hat{\mathbf{E}}$ and $\hat{\mathbf{H}}$ can be derived from a scalar and vector potential

$$\hat{\mathbf{E}} = -\frac{1}{c}\frac{\partial \hat{\mathbf{A}}}{\partial t} - grad\Phi$$
$$\hat{\mathbf{H}} = curl\hat{\mathbf{A}} \tag{91}$$

Applied to a rotating reference system one has

$$\hat{\Phi} = -\frac{1}{2}(\boldsymbol{\omega}\times\mathbf{r})^2$$
$$\hat{\mathbf{A}} = -c(\boldsymbol{\omega}\times\mathbf{r}) \tag{92}$$

Or

$$\hat{\Phi} = -\frac{v^2}{2}$$
$$\hat{\mathbf{A}} = -c\mathbf{v} \tag{93}$$

Apart from the factor m/e, this is same as (84).

For weak gravitational fields produced by slowly moving matter, Einstein's linearized gravitational field equations permit the gauge condition (replacing the Lorentz gauge)

$$\frac{4}{c}\frac{\partial \Phi}{\partial t} + div\hat{\mathbf{A}} = 0$$
$$\frac{\partial \hat{\mathbf{A}}}{\partial t} = 0$$

With the gauge transformation for **Φ** and **A**

$$\Phi' = \Phi \qquad \hat{\mathbf{A}}' = \hat{\mathbf{A}} + \mathrm{grad} f \tag{94}$$

where f has to satisfy the potential equation

$$\nabla^2 f = 0 \tag{95}$$

For a stationary gravitational field the vector potential changes phase of the Schrödinger wave function according to

$$\psi' = \psi \exp\left[\frac{im}{\hbar c} f\right] \tag{96}$$

leading to phase shift on a closed path

$$\delta\varphi = -\frac{m}{\hbar c}\oint \hat{\mathbf{A}}.d\mathbf{s} \qquad \delta\varphi = -\frac{m}{\hbar}\oint \mathbf{v}.d\mathbf{s} \tag{97}$$

In the hydrodynamic interpretation suggested by the Planck aether hypothesis, the phase shifts caused by either the magnetic or the gravitational vector potential result from a circular flow of the Planck aether. The principle of equivalence can precisely relate this circular flow to the angular velocity of a rotating platform. According to (93), one has for the gravitational vector potential in a rotating frame of reference

$$\hat{\mathbf{A}} = -\boldsymbol{\omega} cr \tag{98}$$

with to the phase shift given by (97). One can apply (97) to the Sagnac effect [18] for photons of frequency υ. By putting $mc^2 = h\upsilon = 2\pi\hbar\upsilon$, with the result that

$$\delta\varphi = -\frac{2\pi v}{c^2}\oint \mathbf{v}.d\mathbf{s} \qquad \delta\varphi = 2\omega\pi r^2 \frac{2\pi v}{c^2} \tag{99}$$

the same as predicted without quantum mechanics.

We now compute the phase shift (85) by a magnetic vector potential. To make a comparison with the gravitational vector potential in the Sagnac effect, we consider the magnetic field produced by an infinitely long cylindrical solenoid of radius R. Inside the solenoid the field is constant, vanishing outside. If the magnetic field inside the solenoid is H, the vector potential is

$$A_\varphi = \frac{1}{2}Hr \quad r < R \qquad A_\varphi = \frac{1}{2}\frac{HR^2}{r} \quad r > R \tag{100}$$

According to (85) the vector potential on a closed path leads to the phase shift

$$\delta\varphi = -\frac{e}{\hbar c}H\pi r^2 \quad r < R \qquad \delta\varphi = -\frac{e}{\hbar c}H\pi R^2 \quad r > R \tag{101}$$

As noted by Aharonov and Bohm [17], there is a phase shift for r > R, even though for r > R, H =0 (because for r > R, curl **A** = 0).

Expressing **A** by (84) through **v**, by a hypothetical circular aether velocity, one has

$$v_\varphi = -\frac{e}{2mc}Hr \quad r < R \qquad v_\varphi = -\frac{e}{2mc}\frac{HR^2}{r} \quad r > R \tag{102}$$

One sees that inside the coil the velocity profile is the same as in a rotating frame of reference, having outside the coil the form of potential vortex. If expressed in terms of the aether velocity the phase shift becomes

$$\delta\varphi = -\frac{m}{\hbar}\oint \mathbf{v}.ds \tag{103}$$

the same as (97) for the vector potential created by a gravitational field, and hence the same as in the Sagnac experiment and neutron interference experiment. But for the magnetic vector potential the aether velocity can easily become much larger than in any rotating platform experiment. According to (102), the velocity reaches a maximum at r = R, where it is

$$\frac{|v_{\max}|}{c} = \frac{eHR}{2mc^2} \tag{104}$$

For electrons this is $|v_{\max}|/c = 3\times10^{-4}HR$, where H is measured in Gauss. For H =10^4 G, this would mean that $V_{\max} \geq c$ for R >0.3 cm. If this would be same velocity as on a rotating platform, it would lead to an enormous centrifugal and Coriolis field inside the coil, obviously not observed.

The Planck mass plasma hypothesis model can give a simple explanation for this paradox. The Planck aether consists of two superfluid components, one composed of positive Planck masses and the other one of negative Planck masses. The two components can freely flow through each other, making possible two configurations, one where both components are corotating and one where they are counterrotating. The corotating configuration is realized on a rotating platform, where it leads to the Sagnac and neutron interference effects. This suggests that in the presence of the magnetic vector potential the two superfluid components are counetrrotating. Outside the coil, where curl **A** = 0, the magnetic energy density vanishes, implying that the magnitudes of both velocities are exactly the same. Inside the coil, where curl **A** ≠ 0, there must be a small imbalance in the velocity of the positive over the negative Planck masses to result in a positive energy density.

9. NEGATIVE MASSES IN COSMOLOGY

In the Planck mass plasma theory, the negative gravitational field energy surrounding a mass is due to an excess of negative over a positive mass, and the negative mass surrounding a highly collapsed spherical body or black hole is of the same order of magnitude as the positive mass accumulated inside the collapsing body. An assembly of interacting positive and negative masses can, in general, not lead to a thermodynamic equilibrium. If all the positive masses are separated from the negative ones, it is sufficient to require that each mass species reaches thermodynamic equilibrium. It was shown by Vysin [19] that an assembly of negative masses can acquire thermodynamic equilibrium provided the temperature is negative. The kinetic energy of a negative Planck mass is negative, and an assembly of negative Planck masses has, for this reason, a negative temperature. It therefore can reach thermodynamic equilibrium.

We now make the following hypothesis:

> *If an assembly of positive and negative masses, with the total energy equal to zero, is brought together, the temperature and hence entropy of the mixture will go to zero.*

This hypothesis is the only one consistent with Nernst's theorem. It, of course, implies that an assembly of positive and negative masses can perfectly mix because otherwise no equilibrium can be reached. To satisfy this hypothesis, we assume that the negative masses have negative entropy. Only the assumption that an assembly of negative masses has negative entropy permits an analytic continuation of the entropy from positive to negative temperatures. If the entropy for negative temperatures would be counted positive, the function dS/dT would be discontinuous at T = 0.

For the entropy of a mixture of positive and negative masses to become zero requires an exact correlation in the disorder of the positive mass gas with the disorder of the negative mass gas. This certainly true if the negative mass is equal to the negative mass of the gravitational field of the positive mass, because the Newtonian gravitational field of each particle, all the way down to the smallest dimension, is precisely correlated to the position of the particle. The entropy of the positive mass of matter and the entropy of the negative mass of its gravitational field (correlated to the entropy of the positive mass which is the source of this field) might therefore be called complementary (like a positive and negative photographic image). The expansion from a very small phase space volume would then be possible, because if the positive and negative masses are densely packed within the same volume, not only their energy, but also their entropy would cancel.

The time needed to bring back the universe to its original low entropy state, is the Poincaré recurrence time[5]. While under normal conditions this time is huge, it may in a dense mixture of positive and negative masses with a divergent acceleration become quite small. This might explain why the initial entropy of the universe is very small. The Planck mass plasma hypothesis gives a plausible explanation for the observed vanishing of the sum of all charges: The electric, color and weak charges. With the phenomenon of charge explained to result from the zero point fluctuations of Planck masses bound in vortex filaments and with an equal number of positive and negative Planck masses, the sum of all the charges must vanish. That this should be also be true for the gravitational charges finds its expression in the compensation of the positive energy in the universe by its negative gravitational energy. This compensation explains why the flatness parameter is $\Omega = 1$.

5. The Poincare recurrence time is the time needed for a large number of particles to return to their initial low entropy state in configuration space. This time is huge, but it might perhaps become short in the course of gravitational collapse, because an infinite time outside the event horizon takes a finite time inside of it.

Furthermore, with the sum of all gravitational charges equal to zero, the cosmological constant Λ playing the role of a kind of charge, demands that $\Lambda = 0$.

Finally, with the negative entropy of the negative Planck masses playing the role of a kind of photographic negative for the positive entropy of the positive Planck masses, the total entropy, made up from the sum of the positive and negative Planck masses, would also be equal to zero.

In summary we may write

$$\sum \text{charges} = 0 \tag{105}$$

with the cosmological consequence

$$\Omega - 1 = \Lambda = S = 0 \tag{106}$$

The horizon problem is here resolved by superluminal electromagnetic and gravitational shock waves during the high density phase of the cosmological evolution, not by an inflationary expansion of space.

10. THE CUSP/CORE PROBLEM IN GALACTIC HALOS

We have seen there appears to be strong evidence for the existence of negative mass matter in the universe. And we have also given reasons that negative mass matter might be hidden behind positive matter, forming pole-dipole Dirac spinor configurations neutralizing the negative mass matter. This still leaves open the question if in certain regions of space there might be a surplus of negative over positive mass matter, and if negative mass matter can be "mined" from such regions.

It has been conjectured by Forward [20], that negative mass matter might be located in the intergalactic voids, explaining the "bubble" structure of the metagalaxy. The negative mass in the voids would produce gravitational potential hills repelling all matter, positive or negative, like the positive matter of the galaxies produces gravitational potential wells attracting all matter, positive or negative. But the accumulation of negative mass matter in the gravitational wells of positive matter reduces and flattens the depth of the wells. This simple fact might explain the unsolved cusp/core problem of galactic halos [21].

But what happens in the center of galaxies must also happen to a lesser degree in the center of the sun, the planets, the earth and the moon. To "mine" negative mass matter, if it should there exist, excludes the sun, but also all planets, who have a hot molten core. This does not exclude the moon, having the deepest potential well near the earth, with only hot rocks in its center accessible by advanced nuclear mining technology, permitting to make a tunnel through the Moon [22].

11. SEARCHING FOR NEGATIVE MASS MATTER IN THE GRAVITATIONAL POTENTIAL WELL OF THE MOON

The radius of the moon is $R = 1.74 \times 10^8$ cm and the gravitational acceleration at its surface. At a distance r < R from its center the gravitational acceleration it is

$$g(r) = -g_0(r/R) \tag{107}$$

and the pressure balance equation is

$$\frac{dp}{dr} = -\rho g_0 \frac{r}{R} \tag{108}$$

where $p = p(r)$ is the pressure for r < R. The pressure at the center therefore is

$$p_{\max} = \frac{\rho g_0}{R}\int_0^R r dr = \frac{1}{2}\rho g_0 R \tag{109}$$

With r ≅ 3.33g/cm^3 (n~10^{23} cm^{-3} particle number density) the average density of the moon, one finds that $p_{\max} \cong 5 \times 10^{10}$ $dyn/cm^2 \cong 50{,}000 atm$.

The temperature T in the center of the moon can in the lowest approximation be estimated by the ideal gas equation $p_{\max} = nkT$, ($k \equiv 1.38 \times 10^{16}$ erg/K, Boltzmann constant), where $n = 1/r_B^3$ (rB Bohr radius) is the number density of atoms/cm3. For liquid hydrogen $n = 5 \times 10^{22}$ cm^{-3}, and for uranium metal $n = 4 \times 10^{22}$ cm^{-3}, which shows that for condensed matter n is about the same for all elements. With $n = 10^{23}$ cm^{-3} for compressed lunar rocks, and $p = 5 \times 10^{10}$ dyn/cm^2 one finds that $T = 3500K$. But as one already can see from the van der Waals equation of state, this estimate ignores the inner-atomic cohesive forces, which for a given pressure reduces the temperature. Consistent with this expected reduction in the temperature, magnetic response measurements by the Apollo 12 lunar mission have obtained for the center of the moon a temperature of T = 1240 K [23]. This means the center of the moon consists of hot rocks.

If appreciable amounts of negative mass matter have accumulated over billions of years in the center of the moon, it is more likely that this matter is in the form of ultra-light matter, perhaps by order of magnitude lighter than ordinary matter. There are indications that a Swedish research group has found evidence for the existence of an ultra-dense phase of deuterium, about more than 100,000 more dense than water [24]. Suppose, that in the center of the moon the accumulation of negative mass matter has led to a form of matter which is 100,000 lighter than steel, but still has the strength of steel, this would not lead to be negative-positive mass self-chasing mass dipole, as it is envisioned by Forward [20], but to something very important for space-flight, because it would drastically reduce the energy requirements to accelerate a space craft made from such ultra-light material.

The question if there is such an unusual substance in the center of the moon can probably be answered by a seismic wave tomography, obtained by nuclear explosions set off on the surface of the moon.

12. MAKING A TUNNEL THROUGH THE MOON [22]

The cohesive energy of the rocks is of the order $\varepsilon_r \approx 10^{10}$ erg/cm^3. Therefore, the explosive yield needed to shatter a spherical volume of radius r is

$$E \cong (4\pi/3)\varepsilon_r r^3 \tag{110}$$

The energy released in a kiloton nuclear explosion is $E \approx 4 \times 10^{19}$ erg. With this energy, the radius of the crushed rocks would be $r \approx 10^3$ cm (= 10 m), and with a 10kt explosion it would be twice as large.

To make a tunnel, a cylindrical, rather than spherical, volume of crushed rocks is desired. For this reason a thermonuclear shape charge or an explosive lens is better suited to shatter the rocks.

In the center of the moon the temperature T is about one thousand degree, and the heat diffusion equation given by

$$\frac{\partial T}{\partial t} = \chi \nabla^2 T \tag{111}$$

where χ is the heat diffusion coefficient, the diffusion time for a layer of thickness x is

$$\tau = x^2/\chi \tag{112}$$

For rocks one has $\chi \cong 4 \times 10^{-3}$ cm^2/s [25]. Taking the example $x = 20m = 2 \times 10^3$ cm, one finds that $\tau = 10^9$ $s \cong 30yr$, and at the high rock temperatures the heat diffusion time would be uncomfortably long.

The situation is drastically changed for a layer of crushed rocks, because there it is possible to remove heat by a coolant pumped through the porous medium of the crushed rocks. At the high temperatures of several thousand centigrade, a liquid alkali metal, for example lithium, abundantly available on the moon, could be used as a coolant. The velocity the coolant diffuses into the crushed rocks is determined by Darcy's law

$$v = -D \text{ grad } p \tag{113}$$

where p is the pressure, $D = \kappa/\rho g$, with $\kappa \sim 1cm/s$ a typical value. If the pressure gradient is provided by the gravitational force one has grad$p = \rho$g and hence

$$|v| = \kappa \sim 1cm/s \tag{114}$$

The time needed for the liquid metal to pass through a ~ 20 m thick layer is then ~ 2 x 10^3 s ~ 1 h. The specific heat per unit volume of the coolant is $\rho c_v \sim 3 \times 10^7$ erg/cm^3K. For T ~10^3 K, one has $\rho c_v T \sim 10^{11}$ erg/cm^3.

The heat per unit volume which has to be removed from the crushed rocks is of the order p, where p is the rock pressure. In the center of the moon where p = 5 x 10^{10} dyn/cm^2, this energy is 5 x 10^{10} erg/cm^3. It thus follows that the volume of the liquid coolant must be about ½ of the rock volume to be cooled. For a rock volume of (20 m)^3 ~10^4 m^3, a coolant volume of about 5 x 10^3 m^3 would be needed. The same coolant can be used many times over after the heat is removed from it, which could be done on the surface of the moon by radiation or perhaps better by heat exchangers transferring the heat to lunar sand. Without a thick layer of shattered rocks surrounding the tunnel, the pressure acting on the tunnel wall would be large, in particular, in the center of the moon. Because of friction between particles of the shattered rock, large shear stresses can be sustained changing the pressure distribution in the rock reducing the pressure gradient and hence the pressure on the tunnel wall.

A more detailed calculation [22] for the pressure distribution in the shattered rock tunnel wall gives

$$p = (r/r_0)^9 \tag{115}$$

where r_0 is the radius of the tunnel.

Integrating eqn (108) one obtains for the pressure distribution in the moon

$$p(r) = -\frac{\rho g_0}{R}\int_R^r r dr = -\frac{\rho g_0}{R}(R^2 - r^2) \quad (116)$$

for which one can also write

$$p(r) = p_{\max}\left(1-\left(\frac{r}{R}\right)^2\right) \quad (117)$$

If r_s is the horizontal radius up to which the rocks at a certain depth have to be shattered (cylindrical case), one finds by equating p(r) in (115) and (117)

$$r_s = r_0\left(p_{\max}/p_0\right)^{1/9}\left(1-\left(\frac{r}{R}\right)^2\right)^{1/9} \quad (118)$$

The total energy required to shatter the rocks to make a tunnel from the center at the moon where r = 0, to its surface where r = R, is then given by summing up over the slices with radius r_s and thickness dr

$$E = \pi r_0^2 \varepsilon_r \left(\frac{p_{\max}}{p_0}\right)^{2/9}\int_0^R\left(1-\left(\frac{r}{R}\right)^2\right)^{2/9} dr \quad (119)$$

where as in eqn (110) $\varepsilon_r \cong 10^{10}$ erg/cm^3 is the cohesive binding energy of the rocks. For (119) one can write

$$E = \pi r_0^2 R\varepsilon_r \left(p_{\max}/p_0\right)^{2/9}\int_0^1\left(1-x^2\right)^{2/9} dx \quad (120)$$

With the help of Euler's betafunction one has

$$\int_0^1\left(1-(x)^2\right)^{2/9} dx = (1/2)B\left(\frac{1}{2},\frac{11}{9}\right) \cong \frac{\sqrt{\pi}}{2} \quad (121)$$

Hence

$$E \cong (\pi^{3/2}/2) r_0^2 R\varepsilon_r \left(p_{\max}/p_0\right)^{2/9} \quad (122)$$

Inserting $r_0 = 2 \times 10^3$ cm, $R = 1.74 \times 10^8$ cm, $\varepsilon_r \cong 10^{10}$ erg/cm^3, $(p_{max}/p_0) = 5 \times 10^4$, one finds that $E \cong 2 \times 10^{24}$ $erg \cong 5 \times 10^4$ $kt = 50$ Mt

It must be emphasized that this energy must be quite nonuniformly released along the tunnel shaft. Nuclear fusion explosions below a yield of 10kt become uneconomical with only a fraction of the energy in the fissionable material (needed to make a critical assembly) released. For a 10kt fission explosion the shatter radius computed from (110) is ~ 20 m. With a tunnel radius $r_o \sim 10$ m one would have $(r_s/r_0) \sim 2$ and from (118) that

$$1 - (r/R) \cong 2^9 (p_0/p_{\max}) \quad (123)$$

Putting for the depth of the tunnel (if measured from the surface of the moon) $\delta = R - r$, with

$$2d/R \cong 2^9 (p_0/p_{\max}) \cong 10^{-2} \quad (124)$$

one has $\delta \cong 10$ km.

For a depth $\delta \leq 10$ km the nuclear explosion with a yield ≤ 10 kt would suffice, but which is uneconomical. It is for this reason suggested to use altogether thermonuclear explosive devices where the cost per yield is much lower. To penetrate and shatter the rocks more efficiently, jet–generating thermonuclear explosive lenses could be used. The thermonuclear detonation wave ignited at one point is there shaped into a jet producing conical explosion by placing obstacles into the path of the wave. The ignition can be done by a fission explosive, but conceivably also by a powerful laser beam, with the laser beam projected down the tunnel shaft, triggering the thermonuclear explosive positioned at the lower end.

With the above-given estimate of ~ 50 Mt needed to dig the tunnel shaft, the number of thermonuclear explosive devices making use of the detonation wave lens technique could for this reason be quite reasonable, and certainly much less than the number of required fission explosives.

After nuclear explosions have crushed the rocks and the heat is removed, the tunnel wall has to be made from some kind of ceramic material, since water with which to make concrete is only sparsely available on the moon. But for the wall to last, its temperature must be kept low. The low heat conductivity of rocks, requiring little cooling, is there of considerable help. For the crushed rocks the heat conduction coefficient should not be very different than for solid rocks. According to eq. (112) the heat diffusion time for a 20 m layer of rocks is ~ 30 yr. this means that a small, continuous removal of the heat through the injection and circulation of a liquid metal into the crushed rocks should keep down the temperature of the tunnel wall and its environment.

13. CONCLUSION

The purpose of this study is the question if negative mass might exist, and if negative mass propulsion is possible at all. It is unlikely to be possible in the fashion speculated by Forward [20] (but also see Winterberg [26]), where a negative mass is chasing a positive mass without the expenditure of any energy. Rather, it might perhaps be possible through the existence of an ultra-light form of matter with the tensile strength of ordinary matter on a macroscopic scale where positive matter is bound to negative matter, as it happens with Dirac spinor particles on a microscopic scale. This is the speculative existence of macroscopic bodies approaching zero rest mass, of importance for space flight because such matter would greatly reduce the energy requirements.

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APPENDIX DERIVATION OF EQ. (62):

For the transformation of the wave equation

$$-\frac{1}{c^2}\frac{\partial^2\phi}{\partial t^2}+\frac{\partial^2\phi}{\partial x^2}=0$$

by a Galilei transformation

$$x' = x - vt$$
$$t' = t$$

one has to set

$$\frac{\partial}{\partial t}=\frac{\partial}{\partial t'}+\frac{\partial}{\partial x'}\frac{\partial x'}{\partial t}=\frac{\partial}{\partial t'}-v\frac{\partial}{\partial x'}$$
$$\frac{\partial}{\partial x}=\frac{\partial}{\partial x'}\frac{\partial x'}{\partial x}+\frac{\partial}{\partial t}\frac{\partial t'}{\partial x}=\frac{\partial}{\partial x'}$$

hence

$$\frac{\partial^2}{\partial t^2}=\left(\frac{\partial}{\partial t'}-v\frac{\partial}{\partial x'}\right)^2=\frac{\partial^2}{\partial t'^2}-2v\frac{\partial^2}{\partial x'\partial t'}+v^2\frac{\partial^2}{\partial x'^2}$$

thus obtaining eq. (62).

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