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Urban Unemployment and City Formation. Theory and Policy Implications*

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Abstract: We study the role of unemployment in the context of the endogeneous formation of a monocentric city where firms set efficiency wages to deter shirking. We first show that in equilibrium the employed locate at the vicinity of the city-center, the unemployed reside at the city-edge and firms set up in the city-center. We then establish conditions that ensure existence and uniqueness of both the labor market equilibrium and the (monocentric) equilibrium urban configuration. Last, we perform different comparative statics analyses and derive some policy implications. We show in particular that a policy subsidizing the commuting costs of both the employed and unemployed workers reduces urban unemployment, increases utilities of all workers but raises inequality whereas a policy that subsidizes only unemployed workers' commuting costs increases urban unemployment, does not always raise workers' utilities but cuts inequality.

Keywords: efficiency wages, agglomeration economies, endogeneous location of workers and firms, urban unemployment, subsidizing commuting costs.

JEL Classification: J64, R14.

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1. Introduction

Urban unemployment is one of the growing problems of our society due to its implications in terms of poverty, ghettos and segregation. Even though this has been recognized for a long time by sociologists and is well documented by empirical studies, few theoretical models have been proposed by economists. In a recent survey article, Zenou (1999) identifies three causes of urban unemployment:

(i) Too high and rigid urban efficiency wages. Since workers are tempted to shirk and since it is costly to monitor workers, firms set a self enforcing contract by paying their workers an efficiency wage that induces them not to shirk and to stay in the city. This (efficiency) wage is greater than the market clearing wage and thus, since in equilibrium all firms behave in the same way, there will be a durable level of (involuntary) unemployment in the city. Here the introduction of space increases the efficiency wage and thus the level of unemployment.

(ii) Urban search frictions. It has been observed that workers who are the furthest away from jobs, have poor information and thus their probability of finding a job is low. In a model where job search is adversely affected by distance to the employment center and where location is an endogeneous variable, it can be shown that urban unemployment exists because of search frictions and stochastic rationing that cannot be eliminated by price adjustments (see Wasmer and Zenou, 1997, and Coulson, Laing and Wang, 1997).

(iii) Spatial mismatch. First pointed out by Kain (1968), this hypothesis highlights the fact that, because of firms' relocation towards the city periphery, (black) workers, who generally reside in inner cities, face strong geographic barriers to finding and keeping well-paid jobs. There is a "spatial mismatch" between workers' residence and their workplace yielding urban unemployment that persists because of housing discrimination (see Brueckner and Martin, 1997[1], and Coulson, Laing and Wang, 1997).

In all these approaches, firms' location is assumed to be fixed and the employment center is thus prespecified (even in the spatial mismatch literature where the main employment center is predetermined). There is in fact another literature that deals with the endogeneous location of firms and formation of cities by explaining why cities exist, why cities form where they do and why economic activities ag-

[1] In their model, there is no unemployment. However, it is the first paper that gives a theoretical explanation of the spatial mismatch.

glomerate in a small number of places. In their very complete survey, Fujita and Thisse (1996) give three main reasons for agglomeration economies: externalities under perfect competition (see e.g. Beckmann, 1976, Borukhov and Hochman, 1977, Fujita and Ogawa, 1982, Papageorgiou and Smith, 1983, among others), increasing returns under monopolistic competition (see e.g. Abdel-Rahman and Fujita, 1990, Krugman, 1991, Fujita and Krugman, 1995, Fujita and Mori, 1997,...) and spatial competition under strategic interaction (Hotelling types of models). However, in all these urban models unemployment is absent.

In the present paper, we bring together these two strands of literature by proposing a framework where urban unemployment is due to efficiency wage and where firms and workers are allowed to choose optimally their location so that the employment center is endogeneous and determined in equilibrium. The main force of agglomeration consists of firms' externalities such as face to face communication so that firms want to be together in order to save transaction costs. To the best of our knowledge, *this is the first paper that studies urban unemployment in the context of perfectly mobile firms and endogeneous employment center.*[2].

Our results are the following. We first show that in equilibrium the employed locate at the vicinity of the city-center, the unemployed reside at the city-edge and firms set up in the city-center. We then establish conditions that ensure existence and uniqueness of both the labor market equilibrium and the (monocentric) equilibrium urban configuration. Last, we perform different comparative statics analyses and derive some policy implications. We show in particular that a policy subsidizing the commuting costs of both the employed and unemployed workers reduces urban unemployment, increases utilities of all workers but raises inequality whereas a policy that subsidizes only unemployed workers' commuting costs increases urban unemployment, does not always raise workers' utilities but cuts inequality.

The remainder of the paper is as follows. In section 2, we present the basic model. Sections 3 and 4 are devoted to the equilibrium urban configuration and the labor market equilibrium analyses. In sections 5 and 6, comparative statics and policy implications of the model are derived. Finally, section 7 concludes.

[2]Smith and Zenou (1997) and Coulson, Laing and Wang (1997) have a model of urban unemployment where only a part of the firms are mobile since the main employment center (located in the city-center) is exogeneously fixed.

2. The model

A. The city

The city is *closed* (utility and profit levels are endogeneously determined while the number of workers and firms are exogeneous), *linear* and *symmetric*. The middle of the city is normalized to 0 and the length of the city is denoted by f on its right and by $-f$ (symmetry) on its left. There is no vacant land and no cross-commuting (workers cannot cross each other when they go to work) in the city. All the land is owned by absentee landlords.

B. Workers

There are two types of workers, the employed (group 1) and the unemployed (group 2). We will study later the endogeneous formation of unemployment. There is a continuum of workers of each type whose mass is given by N_1 and U respectively (with $N_1 + U = \overline{N}$).

Assumption 1. Land consumption.
All workers (employed and unemployed) consume the same amount of land, which is normalized to 1 *for simplicity.*

We further assume that the density of workers $h(.)$ in each location of the city within a *residential area* is equal to 1 (a residential area is an area when only households locate). Assumption 1 is quite common in urban economics especially when workers are heterogeneous since it allows us to determine the exact location of each worker in the city and to obtain closed-form solutions. Even though workers and non-workers consume the same amount of land, they differ by their revenue and commuting costs. Let us denote by x_l, $w(x_l)$ and b, the location of firms (or equivalently workers' workplace which will be determined endogeneously in equilibrium), the wage at x_l and the unemployment benefit exogeneously financed by the government.

Concerning commuting costs, employed workers bear them for two reasons: to work and to buy goods. The unemployed bear commuting costs only to buy goods. For simplicity, we assume that the shopping center is always located exactly in 0 the middle of the city. Observe that the shopping center is where consumers buy goods but not where production takes place, goods being produced by firms in the workplace. The latter will be determined endogeneously in equilibrium but since we focus on a monocentric city, it will be in the city-center. Employed workers incur a (weekly) commuting cost of t dollars per unit of distance, and in addition, take $\alpha > 0$ shopping trips for every commuting trip. Unemployed workers incur only shopping costs of αt per unit of distance. If we denote by x, the distance to 0, the middle of the city, we have therefore:

Assumption 2. Commuting costs.

The total commuting cost of an employed worker residing in x and working in x_l is equal to: $\alpha tx + t\left|x - x_l\right|$.

The total commuting cost of an unemployed worker residing in x is equal to: αtx.

We are now able to write the budget constraint of an employed worker residing in x and working in x_l. It is given by:

$$w(x_l) = R(x) + z_1 + \alpha tx + t\left|x - x_l\right| \tag{2.1}$$

where $R(x)$ is the land rent market and z_i $(i = 1, 2)$, the composite good (taken as the numéraire) consumed by group i. The unemployed located at x has the following budget constraint:

$$b = R(x) + z_2 + \alpha tx \tag{2.2}$$

We assume that all workers have the same utility function (same preferences) that depends on housing and composite good consumptions. Since all workers consume one unit of land, we can write these functions as indirect utilities. Therefore, each employed and unemployed worker solves respectively the following programs:

$$\max_{x, x_l} \; z_1 = w(x_l) - R(x) - \alpha tx - t\left|x - x_l\right| \tag{2.3}$$

$$\max_{x} \; z_2 = b - R(x) - \alpha tx \tag{2.4}$$

In equilibrium, all workers of the same type enjoy the same utility level or equivalently the same level of composite good consumption (we denote them respectively by z_1^* et z_2^*).[3] Bid rent functions (defined as the maximum rent that workers are ready to pay in order to reach the equilibrium utility level) are respectively equal to:

$$\Xi_1(x) = w(x_l) - z_1^* - \alpha tx - t\left|x - x_l\right| \tag{2.5}$$

$$\Xi_2(x) = b - z_2^* - \alpha tx \tag{2.6}$$

[3] All variables with a star as a superscript are equilibrium variables.

C. Firms

There exists a continuum of identical firms, which allows us to treat their distribution in the city in terms of density. The firms' density in each point x of the city is denoted by $m(x)$ and the mass of firms is equal to M.

Assumption 3. Production.

Each firm uses a fixed quantity of land $\overline{Q}$ and a variable quantity of labor L to produce Y. The production function is thus given by:

$$Y = f(\overline{Q}, L) \qquad \textit{with} \qquad f(\overline{Q}, 0) = f(0) = 0 \ , \qquad \frac{\partial f(.)}{\partial L} > 0 \qquad \textit{and} \qquad \frac{\partial^2 f(.)}{\partial L^2} \leq 0 \ ,$$

and the Inada conditions, i.e., $f'(0) = +\infty$ and $f'(+\infty) = 0$.

The labor demand of each firm, L, is determined by profit maximization. Since all firms are identical, we have $L = N_1/M$ and the aggregate production function is given by: $F(\overline{Q}, L) = Mf(\overline{Q}, \frac{N_1}{M})$. Moreover, since $F'(\overline{Q}, L) = f'(\overline{Q}, L)$, the labor demand can be determined by the profit maximization of one (representative) firm.

We have to model agglomeration forces. In our framework, the main force of agglomeration is the fact that production needs transactions between firms (information exchanges, face to face communication...). There are different ways to model these transactions. Since we want to focus on the endogeneous formation of a monocentric city, we have chosen the following one.

Assumption 4. Transaction costs.

The total transaction cost between a firm located at x and all the other firms in the city is equal to:

$$\tau T(x) = \tau \int_{-f}^{f} m(y) \left| x - y \right| dy = \tau \left[\int_{-f}^{x} m(y)(x - y)dy + \int_{x}^{f} m(y)(y - x)dy \right]$$

where τ denotes the transaction cost per unit of distance, $m(x)$, the density of firms at x, and $T(x)$, the total distance of transaction for a firm located at x.

This assumption is very important for the urban equilibrium configuration since it affects both workers and firms' bid rents. For example, with this type of function we cannot obtain a duocentric city (see Fujita, 1990, for an extensive discussion of this issue). In fact, it is essentially the second derivative of $T(x)$ that plays a fundamental role. We further assume that within a *business area* the density of firms $m(x)$ is constant and equal to $1/\overline{Q}$ (a business area is an area when only firms locate). We have therefore:

$$T'(x) = \int_{-f}^{x} m(y)dy - \int_{x}^{f} m(y)dy = 2xm(x) \tag{2.7}$$

$$T''(x) = 2m(x) = \frac{2}{\overline{Q}} \geq 0 \tag{2.8}$$

where $T(x)$ is a convex function inside an area where firms are concentrated (business area), i.e., $m(x) > 0$, and is linear in residential areas, i.e., $m(x) = 0$. We are now able to write the profit function of each firm as follows:

$$\Pi = pY \ - R(x)\overline{Q} - w(x)L - \tau T(x) \tag{2.9}$$

where $w(x)$ is the wage profile that will be defined below. The objective of each firm is to chose a location x that maximizes its profit (2.9). Its bid rent, which is the maximum land rent that a firm is ready to pay at location x to achieve profit level Π^*, given the distribution of firms $m(x)$, is therefore given by:

$$\Phi(x) = \frac{1}{\overline{Q}} \left[pY \ - w(x)L - \tau T(x) - \Pi^*\right] \tag{2.10}$$

where Π^* is the equilibrium profit level common to all firms.

Last, by using the following definition: two firms x and x' are connected if $|x_l - x_l'| = 0$, we can spell out our last assumption.

Assumption 5. There are no commuting costs for workers within connected firms.

This assumption is made for simplicity but does not affect the main result. It can be relaxed in two ways. First, workers can bear positive commuting costs within connected firms (as in Fujita and Ogawa, 1980). Second, all workers can have the same total commuting cost whenever they enter the interval of connected firms which is equal to a fixed cost times the average size of the interval. However, both cases complicate the analysis (the second one being easier) without altering the main results (see Zenou, 1998). In equilibrium, we will focus only on a monocentric configuration so that all firms will be connected in the middle of the city. In this context, a natural interpretation of assumption 7 is that this connected interval corresponds to a shopping mall so that workers have a positive commuting cost to go there but then, within the mall, no commuting cost.

3. The endogeneous formation of the monocentric city

We want to find equilibrium conditions for the endogeneous formation of a *linear* and *monocentric* city. We have assumed that the city is symmetric so that we can consider only the right side of it, i.e., the interval $[0, f]$. A monocentric city is such that (on the right of 0):

$$\begin{array}{llll} h(x) = 0 & \text{and} & m(x) = 1/\overline{Q} & \text{for} \quad x \in [0, e] \\ h(x) = 1 & \text{and} & m(x) = 0 & \text{for} \quad x \in [e, f] \end{array}$$

which means that firms locate in the CBD, i.e., the interval $[-e, e]$, and workers reside outside of it.

With assumption 5 and with the assumption of no cross-commuting for workers (so that between 0 and e individuals commute to firms that are situated on their left), in a monocentric city the equilibrium wage profile is given by:

$$w(x_l) = w_1^* \tag{3.1}$$

where w_1^* is the efficiency wage that will be determined later. Equation (3.1) means that *there is no wage gradient in the city* since wages do no depend on distance. By using (2.10), this implies that the bid rent of firms is equal to:

$$\Phi(x) = \frac{1}{\overline{Q}} \left[pY \ - w_1^* L^* - \tau T(x) - \Pi^* \right] \tag{3.2}$$

with

$$\Phi'(x) = -\frac{\tau T'(x)}{\overline{Q}} \leq 0 \tag{3.3}$$

$$\Phi''(x) = -\frac{\tau T''(x)}{\overline{Q}} \leq 0 \tag{3.4}$$

In this context, we have

$$\Phi'(x) = \begin{cases} -2\tau x / \overline{Q}^2 < 0 & \text{for } x \in [0, e] \\ 0 & \text{for } x \in \,]e, f] \end{cases} \tag{3.5}$$

and

$$\Phi''(x) = \begin{cases} -2\tau / \overline{Q}^2 < 0 & \text{for } x \in [0, e] \\ 0 & \text{for } x \in \,]e, f] \end{cases} \tag{3.6}$$

We are now able to locate all workers in the city. By using (2.5) and (2.6), the employed workers have the following bid rent:[4]

$$\Xi_1(x) = w_1^* - z_1^* - (1+\alpha)t(x-e) \tag{3.7}$$

while the unemployed workers' bid rent is given by:

$$\Xi_2(x) = b - z_2^* - \alpha t(x-e) \tag{3.8}$$

Because of Assumption 5, workers take only into account the commuting cost to the CBD fringe, e, since between e and 0, it is nul. The slopes of (3.7) and (3.8) are respectively equal to:

$$\Xi_1'(x) = \begin{cases} 0 & \text{for } x \in [0,e] \\ -(1+\alpha)t < 0 & \text{for } x \in]e,f] \end{cases} \tag{3.9}$$

and

$$\Xi_2'(x) = \begin{cases} 0 & \text{for } x \in [0,e] \\ -\alpha t < 0 & \text{for } x \in]e,f] \end{cases} \tag{3.10}$$

Proposition 3.1. *The unemployed reside at the outskirts of the city whereas the employed workers locate at the vicinity of the city-center.*

This result is quite intuitive. Since the employed work at the city-center, they outbid the unemployed to the periphery in order to save commuting costs. Observe that Proposition 3.1 is valid only if $\Xi_1(0) > \Xi_2(0)$ which, by using (3.7) and (3.8), is equivalent to:

$$w_1^* - b + t.e > z_1^* - z_2^* \tag{3.11}$$

We will show that this condition is always true in equilibrium. Observe also that Assumption 1 drives partly this result. Indeed, if housing consumption were endogeneously chosen, then the employed who are richer than the unemployed would consume more land (land being a normal good) and would be attracted to the periphery where the land is cheaper. However, this would complicate the analysis without changing the basic results since we could always find conditions that guarantee that the employed live at the outskirts of the city.

[4] Observe that z_1^* and z_2^* do not depend on workers' location x since in equilibrium all workers of the same type reach the same utility level whatever their location.

Let us denote by g on the right of 0 (and thus $-g$ on the left of 0) the border between the employed and the unemployed. This means that the employed reside between e and g (on the right of 0) and the unemployed between g and f (see Figure 1).

[*Insert Figure* 1 *here*]

The monocentric urban equilibrium configuration is when firms outbid workers outside the CBD. Consequently, let us write the equilibrium conditions for a monocentric city. As stated above, all firms are located in the CBD between $-e$ and e (0 being in the middle of this interval), the employed workers reside between $-g$ and $-e$ (on the left of 0) and between g and e (on the right of 0) and the unemployed workers reside between $-f$ and $-e$ (on the left of 0) and between e and f (on the right of 0), as described by Figure 1. Since the equilibrium is symmetric, the analysis can be performed only on the right side of the city, i.e., between 0 and f. If we denote by R_A the agricultural land rent (outside the city), the equilibrium conditions are given by:

Labor Market

$$L.m(x) = \int_0^e h(x)dx \qquad \text{for each } x \in [0, e] \tag{3.12}$$

Land Market

$$R(x) = Max\ \{\Xi_1(x), \Xi_2(x), \Phi(x), R_A\} \qquad \text{for } x \in [0, f] \tag{3.13}$$

$$R(x) = \Phi(x) \geq \Xi_1(x) \qquad \text{for } x \in [0, e[\tag{3.14}$$

$$R(x) = \Phi(x) = \Xi_1(x) \qquad \text{at } x = e \tag{3.15}$$

$$R(x) = \Xi_1(x) \geq \Phi(x) \qquad \text{for } x \in]e, g[\tag{3.16}$$

$$R(x) = \Xi_1(x) = \Xi_2(x) \qquad \text{at } x = g \tag{3.17}$$

$$R(x) = \Xi_2(x) \geq \Xi_1(x) \qquad \text{for } x \in]g, f[\tag{3.18}$$

$$R(x) = \Xi_2(x) = R_A \qquad \text{at } x = f \tag{3.19}$$

$$\overline{Q}.m(x) + h(x) = 1 \qquad \text{for } x \in [0, f] \tag{3.20}$$

Constraints

$$\int_0^e Lm(x)dx = \frac{L.M}{2} \qquad \text{for } x \in [0, e] \tag{3.21}$$

$$\int_e^g h(x)dx = \frac{L.M}{2} \qquad \text{for } x \in [e, g] \tag{3.22}$$

$$\int_g^f h(x)dx = \frac{U}{2} \qquad \text{for } x \in [g, f] \tag{3.23}$$

Let us comment these equilibrium conditions. The labor market condition just states that within the CBD, labor supply equals labor demand. The land market conditions ensure that landlords offer land to the highest bid rents, that in the CBD firms outbid workers and outside the CBD the employed outbid the unemployed and that the land rent market is continuous. The last three equations are the standard population constraints.

By solving (3.21), (3.22) and (3.23), we easily obtain:

$$e^* = -e^* = \frac{\overline{Q}M}{2} \tag{3.24}$$

$$g^* = -g^* = \frac{(L^* + \overline{Q})M}{2} \tag{3.25}$$

$$f^* = -f^* = \frac{\overline{N} + \overline{Q}M}{2} \tag{3.26}$$

Observe that e^* and f^* are equilibrium values that are not affected by the labor market equilibrium. Indeed, e^* is just half of the size of the CBD, which is equal to the number of firms, M, times their land consumption, $\overline{Q}$. Since the city is closed, $\overline{N}$, the active population, is exogeneous and the city size f^* is thus equal to the size of the CBD, $\overline{Q}M$, plus the size of $\overline{N}$. Since we focus on the right size of the city, we have to divide everything by 2. However, this is no longer true for g^*, the border between the employed and the unemployed workers, since it depends crucially of the size of employment, L^*, and of unemployment, $U^* = \overline{N} - L^*M$, that will be determined in the labor market equilibrium.

We are now able to determine the equilibrium utility and profit levels. By using equations (3.15), (3.17) and (3.19), we easily obtain:

$$z_1^* = w_1^* - \frac{t}{2}\left(\alpha\overline{N} + LM\right) - R_A \tag{3.27}$$

$$z_2^* = b - t\frac{\alpha\overline{N}}{2} - R_A \tag{3.28}$$

$$\Pi^* = pY^* - w_1^* L^* - t\frac{\overline{Q}}{2}\left(\alpha\overline{N} + L^* M\right) - \tau\frac{\overline{Q}M^2}{2} - R_A\overline{Q} \tag{3.29}$$

where L^* is the equilibrium employment level for each firm, $Y^* = f(\overline{Q}, L^*)$, the corresponding production level, and $\overline{N} = L^*M + U$. It is useful to identify the *equilibrium space costs,* i.e., land rent plus travel costs plus transaction costs (the latter is only for firms) for the employed, the unemployed and firms (identified by the subscript F) which are respectively given by:

$$SC_1^* = \frac{t}{2}\left(\alpha\overline{N} + L^* M\right) + R_A \tag{3.30}$$

$$SC_2^* = t\frac{\alpha\overline{N}}{2} + R_A \tag{3.31}$$

$$SC_F^* = \left[t\left(\alpha\overline{N} + L^* M\right) + \tau M^2 + 2R_A\right]\frac{\overline{Q}}{2} \tag{3.32}$$

This yields the following *space-cost differential* between the employed and the unemployed:

$$\Delta SC^* = SC_1^* - SC_2^* = \frac{tL^*M}{2} \tag{3.33}$$

We are now able to demonstrate that $\Xi_1(0) > \Xi_2(0)$. Indeed by using (3.27) and (3.28), (3.11) rewrites $t.e > -tL^*M/2$, which is obviously always true whatever the value of L^*.

4. The labor market equilibrium

Concerning the firms' wage policy, we develop an efficiency wage model based on shirking (see Shapiro and Stiglitz, 1984 or Zenou and Smith, 1995). We assume that there is a moral hazard problem: workers know exactly their effort level whereas firms don't. For simplicity, θ, the effort level, takes only two discrete values: either the worker shirks, $\theta = 0$ or he does not shirk and $\theta > 0$. Thus, the utility of a shirker is given by:

$$z_1^S = z_1^* \tag{4.1}$$

where z_1^* is defined by (3.27) and the one of a non-shirker is equal to:

$$z_1^{NS} = z_1^* - \theta \tag{4.2}$$

We further assume that *firms cannot perfectly monitor workers* so that there is a probability of being detected shirking, denoted by c, which is less than 1 (firms can for example control randomly a fraction of workers). If a worker is caught shirking, he is automatically fired. In this context, firms propose to their employees a self-enforcing contract that induce workers not to shirk. This will determined the efficiency wage which is defined such that the expected utility of non-shirking is always greater than the one of shirking. We have therefore:

$$c\left[\gamma.z_1^S + (1-\gamma).z_2^*\right] + (1-c)\ z_1^S \geq z_1^{NS} \tag{4.3}$$

where z_1^S, z_1^{NS} and z_2^* are respectively defined by (4.1), (4.2) and (3.28), and $\gamma = LM/\overline{N}$ is the probability to find a job for an unemployed worker. Thus condition (4.3) means that when caught shirking (with exogeneous probability c), a worker can find another job with probability γ, in this case he will always shirk since $z_1^S > z_1^{NS}$, and can stay unemployed with probability $1-\gamma$. If he does not shirk, he is sure to stay employed. In equilibrium the constraint (4.3) is biding so that it can be rewritten as:

$$z_1^* - z_2^* = \frac{\theta}{c(1-\gamma)} \tag{4.4}$$

which by using (3.27) and (3.28) leads to the following efficiency wage:

$$w_1^* = b + \frac{\theta}{c(1-\gamma)} + t\frac{LM}{2} \tag{4.5}$$

Then by using the fact that $\gamma = LM/\overline{N}$, we obtain:

$$w_1^* \equiv w_1(L) = b + \frac{\theta}{c}\left(\frac{\overline{N}}{\overline{N}-LM}\right) + t\frac{LM}{2} \tag{4.6}$$

or equivalently

$$w_1^* \equiv w_1(U) = b + \frac{\theta}{c}\left(\frac{\overline{N}}{U}\right) + \frac{t}{2}\left(\overline{N}-U\right) \tag{4.7}$$

Equation (4.6) is referred to as the Urban Non-Shirking Condition (UNSC hereafter), i.e., the (efficiency) wage that firms must pay for each level of employment in order to induce workers not to shirk and to stay in the city. The interpretation of (4.6) or (4.7) is quite intuitive. First, we obtain the standard effects of efficiency wages in a non-spatial framework. Indeed, the unemployment benefit, b, and the effort level, θ, affect positively w_1^* whereas c, the detection probability has a negative impact on it. Second, an increase in the level of unemployment, U, reduces the efficiency wage (see (4.7)). This captures the fact that unemployment serves as a discipline device for workers (Shapiro and Stiglitz, 1984) since when unemployment is high, workers will be reluctant to shirk because of a lower probability of finding a job if caught shirking, and thus firms can set lower efficiency wages. Last, when t, the commuting cost per unit of distance, increases firms must increase their wage in order to induce workers to stay in the city. Thus, the introduction of space leads to an increase of $tLM/2$ in the efficiency wage (compared with the one in Shapiro-Stiglitz). In fact, $LM/2 = g^* - e^*$ so that firms compensate the employed worker who is the furthest away from the CBD fringe (i.e. e^*) residing exactly in g^*. Moreover, by using (3.33), one can see that $tLM/2 = \Delta SC$, i.e., the space cost differential between the employed and the unemployed. This means that when they set efficiency wages, *firms take into account the employed workers' commuting costs* (remember that the space cost differential between workers and non-workers is exactly equal to the commuting cost of the employed).

To sum-up, when firms set their efficiency wage they consider three elements. The first one is b, the unemployment benefit since they must induce the unemployed to leave welfare. The second one is $\theta/[c(1-\gamma)]$ since they must induce workers not to shirk (these are the standard effects already obtained by Shapiro-Stiglitz). The third and last one, $tLM/2$, is the spatial element since firms must induce their workers to stay in the city. The urban efficiency wage thus has two main roles: to *deter* shirking and to *compensate* for commuting costs.

Let us study how w_1 behaves with L. By using (4.6), we obtain:

$$\frac{\partial w_1(L)}{\partial L} > 0 \quad ; \quad \frac{\partial^2 w_1(L)}{\partial L^2} > 0 \tag{4.8}$$

$$\lim_{L \to \overline{N}/M} w_1(L) = +\infty \tag{4.9}$$

$$w_1(L=0) = b + \frac{\theta}{c} \tag{4.10}$$

Inequality (4.8) states that the efficiency wage is an increasing and convex function of employment (see Figure 2); this is because when employment increases the threat of being fired is less important and firms must increase their wage to induce workers not to shirk. The second equation (4.9) is very important since it says that full employment is not compatible with efficiency wages. Indeed, if this was not true, then firms could always set an efficiency wage at the full employment level. In this context, workers would always shirk because even if they are caught shirking they can always find a new job. This is in contradiction with the nature of efficiency wages. The last equation (4.10) just states that at zero employment level, firms can set a positive (efficiency) wage.

[*Insert Figure* 2 *here*]

More generally, *the urban unemployment is involuntary* since the unemployed workers are ready to work for a lower wage in order to get a job but firms will never accept this offer because the UNSC will not be respected and all workers will shirk. Therefore it is the presence of high and sticky wages that create (involuntary) unemployment. In this context, *taking into account space increases the level of unemployment since urban efficiency wages are higher.*

The labor market equilibrium is now described. Each firm solves the following program:

$$\max_L \Pi^* \qquad s.t. \quad w \geq w_1^* \tag{4.11}$$

where Π^* is defined by (3.29). By using (3.29), the solution of (4.11) is such that:

$$w_1^* = pF'(\overline{Q}, L) - t\overline{Q}M/2 \tag{4.12}$$

which defines the labor demand curve. At this stage, it is important to observe that the labor demand curve is affected negatively by t the commuting cost (per unit of distance)

Theorem 4.1. *There exists a unique labor market equilibrium, where* w_1^* *is given by:*

$$w_1^* = b + \frac{\theta}{c}\left(\frac{\overline{N}}{\overline{N} - L^*M}\right) + t\frac{L^*M}{2} \tag{4.13}$$

and where L^* *is defined by:*

$$b + \frac{\theta}{c}\left(\frac{\overline{N}}{\overline{N} - L^*M}\right) + \frac{tM}{2}(L^* + \overline{Q}) = pF'(\overline{Q}, L^*) \tag{4.14}$$

Proof. On one hand, by (4.8), (4.9) and (4.10), we know that $w_1(L)$ is an increasing and convex function of L, whose intercept is a positive constant $(b+\theta/c)$ and has a tangent at $L = \overline{N}/M$. On the other, by Assumption 3, $F'(\overline{Q}, L) - t\overline{Q}M/2$ is decreasing and convex in L (since $F(.)$ is increasing and concave in L

and $t\overline{Q}M/2$ is the constant that does not depend on L), and $F'(L=0)=+\infty$ and $\lim_{L\to+\infty}F'(\overline{Q},L)=0$ (Inada conditions). In particular, $\lim_{L\to+\infty}F'(\overline{Q},L)=0$ means that $F'(L=\overline{N}/M)$ is equal to a positive constant. In this context, there exists a unique labor market equilibrium with a unique value of w_1^* and L^* (see Figure 2). ■

Observe that this theorem is contingent on the existence and uniqueness of the urban spatial configuration equilibrium (we check that below). We can now examine how L^* varies with the different parameters. By totally differentiating (4.14), we easily obtain:[5]

$$\begin{aligned} \frac{\partial L^*}{\partial t} &< 0 \quad ; \quad \frac{\partial L^*}{\partial \overline{Q}}<0 \quad ; \quad \frac{\partial L^*}{\partial M}<0 \\ \frac{\partial L^*}{\partial c} &> 0 \quad ; \quad \frac{\partial L^*}{\partial \theta}<0 \quad ; \quad \frac{\partial L^*}{\partial b}<0 \end{aligned} \tag{4.15}$$

so that we can write L^* as $L^*(t,\overline{Q},M,b,c,\theta)$. This result is quite intuitive since when the efficiency wage is positively (negatively) affected by a parameter, the UNSC shifts leftward (rightward) so that the level of L^* decreases. For $\overline{Q}$ it is because the labor demand curve shifts downward when it increases. Since τ or α does not affect the efficiency wage or the labor demand curve, it has no impact on L^*.

One can argue that we don't need efficiency wages in this model since the key element in the wage is the commuting cost compensation. Assume now that there is no moral hazard problem so that $\theta=0$. We have a competitive model with the same labor demand curve but with a different labor supply one (below the UNSC) and still unemployment. Thus, $L^C>L^*$ (where superscript C stands for the competitive model) and $w^C=b+tL^CM/2<w^*$ and of course $z_1^*-z_2^*=0$. However, the key question for the relevance of the efficiency wage is about the interaction between effort, commuting cost and equilibrium employment level. By using (4.14) and (4.15), it is easily checked that:

$$\frac{\partial^2 L^*}{\partial t\partial\theta}=\frac{\overline{N}M^2(L^*+\overline{Q})}{2c\left[\left(\frac{\theta}{c}\frac{\overline{N}M}{(\overline{N}-L^*M)^2}+\frac{tM}{2}-pF''(.)\right)(\overline{N}-L^*M)\right]^2}>0$$

[5] In order to obtain $\partial L^*/\partial\overline{Q}>0$, we assume that $pF''(\overline{Q},L^*)>tM/2$.

and

$$\frac{\partial\left(L^C - L^*\right)}{\partial t} = -\frac{M(L^C + \overline{Q})}{tM - 2pF''(.)} + \frac{M(L^* + \overline{Q})}{\frac{2\theta}{c}\frac{\overline{N}M}{(\overline{N}-L^*M)^2} + tM - 2pF''(.)} < 0$$

The first inequality shows that the effort level affects positively the (negative) impact of commuting costs on employment level in an efficiency wage economy. In other words, when θ increases, the impact of t on L^* becomes more important. The second inequality shows that the competitive and the efficiency wage models yields different implications in terms of the impact of t on employment levels. In particular, when commuting costs increase the marginal variation of employment level in lower in the competitive case than in the efficiency wage one since $\frac{\partial L^C}{\partial t} < \frac{\partial L^*}{\partial t}$. All these results demonstrate the importance of the efficiency wage analysis and how it differ with the competitive case.

We now have to check that there exists a unique urban equilibrium as described by Figure 1. By plugging (4.6) in (3.27), (3.28) and (3.29), we obtain:

$$z_1^* = b - \frac{\alpha t\overline{N}}{2} + \frac{\theta}{c}\left(\frac{\overline{N}}{\overline{N} - L^*M}\right) - R_A \tag{4.16}$$

$$z_2^* = b - \frac{\alpha t\overline{N}}{2} - R_A \tag{4.17}$$

$$\begin{aligned}\Pi^* \;\; = \;\; & p.f(\overline{Q}, L^*) - \left[b + \frac{\theta}{c}\left(\frac{\overline{N}}{\overline{N} - L^*M}\right)\right] L^* - \tau\frac{\overline{Q}M^2}{2} \\ & -\frac{t}{2}\left[\alpha\overline{N}\,\overline{Q} + L^*M\left(L^* + \overline{Q}\right)\right] - R_A\overline{Q}\end{aligned} \tag{4.18}$$

where L^* is defined by (4.14) and can thus be written as $L^*(t, \overline{Q}, M, b, c, \theta)$. It is easy to verify that in equilibrium, $z_1^* > z_2^*$, i.e., the employed are better off than the unemployed, since

$$z_1^* - z_2^* = \frac{\theta}{c(1-\gamma)} = \frac{\theta}{c}\left(\frac{\overline{N}}{\overline{N} - L^*M}\right) \tag{4.19}$$

which is the surplus for the employed workers. Moreover, we assume that b and p are large enough so that z_2^* and Π^* are always strictly positive. We have also:

$$g^* = -g^* = \frac{(L^* + \overline{Q})M}{2} \tag{4.20}$$

where L^* is defined by (4.14). In this context, by using (4.16), (4.17) and (4.18), and (3.7), (3.8), (3.2) and (3.24), the equilibrium land rent is given by:

$$R^*(x) = \begin{cases} t\left(\alpha\overline{N} + L^*M\right)/2 + \tau\left(\frac{M^2}{4} - \frac{x^2}{\overline{Q}^2}\right) + R_A & \text{for } x \in [-e^*, e^*] \\ t\left[(L^* + \overline{Q})M + \alpha(\overline{N} + \overline{Q}M) - 2(1+\alpha)\left|x\right|\right]/2 & \text{for } x \in [-g^*, -e^*] \\ +R_A & \text{and } x \in [e^*, g^*] \\ \alpha t\left[\left(\overline{N} + \overline{Q}M\right) - 2\left|x\right|\right]/2 + R_A & \text{for } x \in [-f^*, -g^*] \\ & \text{and } x \in [g^*, f^*] \\ R_A & \text{for } x \in]-\infty, -f^*] \\ & \text{and } x \in [f^*, +\infty[\end{cases}$$

We must now find conditions that guarantee the existence of a monocentric city as depicted by Figure 1. Observe from Proposition 3.1 that workers' bid rents are both linear and decreasing and that the unemployed have a flatter bid rent than the employed (within the CBD both bid rents are constant). From (3.5) and (3.6), we also know that firms' bid rents are decreasing (concave in the CBD and then linear). We therefore have the following result.

Theorem 4.2. *The monocentric city is an equilibrium configuration if the following condition holds:*

$$t \leq \frac{\tau M}{2(1+\alpha)\overline{\overline{Q}}} \equiv D_2 \tag{4.21}$$

Proof.

First, if condition (3.15) is satisfied, then condition (3.16) can be replaced by:

$$\Phi'(e^*) < \Xi_1'(e^*) \tag{4.22}$$

which, by using the equilibrium land rent, is equivalent to:

$$t < \frac{\tau M}{(1+\alpha)\overline{\overline{Q}}} \equiv D_1 \tag{4.23}$$

In the same way, if condition (3.17) is satisfied, then condition (3.18) can be replaced by:

$$\Xi_1'(g^*) < \Xi_2'(g^*) \tag{4.24}$$

which is always true by Proposition 3.1.

We must now check that (3.14) is verified. If condition (3.15) is satisfied then, because of the strict concavity of $\Phi(x)$ in the interval $[0, e^*]$, (3.14) can be replaced by (using the equilibrium land rent) :

$$\Phi(0) \geq \Xi_1(0) \tag{4.25}$$

which is equivalent to (4.21). Notice that if condition (4.21) is verified then (4.23) is also satisfied since $D_2 < D_1$. ■

The following comments are in order. First, the endogeneous formation of a monocentric city is possible only if workers' commuting cost t (per unit of distance) is low and firms' transaction cost τ (per unit of distance) is important. This is quite intuitive since the transaction cost is the *agglomeration force* to the CBD for firms (via $\tau T(x)$), and the commuting cost is the *dispersion force* for firms (via the efficiency wage) and the *attraction force* for workers. Thus in order to have a monocentric city it must be that firms bid away workers from the CBD so that the agglomeration force dominates the dispersion force. Second, the augmentation of $\overline{Q}$, firms' land consumption, has a negative impact on the city formation $\overline{Q}$ since it affects negatively profits and thus firms' bid rent. Third, the endogeneous monocentric city formation is more likely to occur when M, the number of firms, is large since transaction costs increase with M. Last, α, the number of trips devoted to shopping has to be small enough in order for (4.21) to be satisfied. Indeed, if workers are going too often to the city-center where the shopping center is located, they will obviously bid away firms to the periphery.

5. Comparative statics analysis

It will be interesting to perform comparative statics analyses to examine how equilibrium variables (profits and utilities) vary with changes in exogeneous parameters. If we start with the *spatial parameters*, α (the number of trips devoted to shopping), t (the commuting cost per unit of distance), τ (firms' transaction costs per unit of distance) and R_A (the agricultural land rent), then by differentiating (4.16), (4.17) and (4.18) and by using (4.15), we easily obtain:[6]

$$\frac{\partial z_1^*}{\partial \alpha} < 0 \quad ; \quad \frac{\partial z_1^*}{\partial t} < 0 \quad ; \quad \frac{\partial z_1^*}{\partial R_A} < 0 \tag{5.1}$$

[6] To calculate $\partial \Pi^* / \partial t$, we use the fact that in equilibrium w_1^*, defined by (4.13), is equal to $pF'(\overline{Q}, L^*)$.

$$\frac{\partial z_2^*}{\partial \alpha} < 0 \quad ; \quad \frac{\partial z_2^*}{\partial t} < 0 \quad ; \quad \frac{\partial z_2^*}{\partial R_A} < 0 \tag{5.2}$$

$$\frac{\partial (z_1^* - z_2^*)}{\partial t} < 0 \tag{5.3}$$

$$\frac{\partial \Pi^*}{\partial \alpha} < 0 \quad ; \quad \frac{\partial \Pi^*}{\partial \tau} < 0 \quad ; \quad \frac{\partial \Pi^*}{\partial R_A} < 0 \tag{5.4}$$

$$\begin{aligned} \frac{\partial \Pi^*}{\partial t} &= -\frac{1}{2}\left[\alpha \overline{N}\,\overline{Q} + L^* M (L^* + \overline{Q})\right] \\ &\quad - \frac{\partial L^*}{\partial t}\left[\frac{\theta}{c}\frac{\overline{N}\,L^* M}{(N - L^* M)^2} + \frac{tM}{2}(L^* + \overline{Q})\right] \end{aligned} \tag{5.5}$$

The following comments are in order. When α or R_A increases, all equilibrium profits and utilities are cut. The first effect is due to the fact that the location of the shopping center is fixed exogeneously in the middle of the city 0. So when workers (employed and unemployed) go there more often to buy goods, their commuting costs and thus their bid rents increase. This leads to a decrease in their utility levels. Moreover, since firms must bid away workers in order to occupy the CBD, they must raise their bid rent when workers' bid rents increase, yielding a decrease in their equilibrium profits. The interpretation of R_A is similar since increasing R_A augments workers' bid rent (the competition in the land market becomes fiercer). When τ increases, equilibrium profits are reduced since costs are higher for firms but it does not affect the employed and unemployed utilities since there are no spatial interactions between workers. The most interesting effect is the one of t. Not surprisingly, when t increases, equilibrium utilities of all workers in the city decrease. However, according to (5.3), *increasing t cuts inequality since the difference in utilities between employed and unemployed workers becomes less important.* The main reason is that employed and unemployed workers don't support the same commuting costs since the former travel more often than the latter. As a consequence, an increase in t affects more the employed than the unemployed. This has important policy implications that we will investigate in the next section. Observe also that t has an ambiguous effect on Π^*. In fact, inspection of (5.5) shows that there are two distinct effects. The first term of the RHS of (5.5) is *negative* and reflects the *direct* impact of t on Π^*. The second term, *positive*, reflects an *indirect* effect through L^*. Indeed, when t increases, profits decrease since firms' bid rents must be higher to bid away workers outside the CBD (*direct* or *urban* effect); but at the same time, the employment level is cut since employers must increase their efficiency wage in order to meet the UNSC (see (4.15)) and thus profits are augmented (*indirect* or *labor* effect). The net effect is thus ambiguous. In this context *both land and labor markets interact* since when t varies it affects the land market by increasing commuting costs for all workers but it also affects the labor market by modifying the wage policy and thus the equilibrium level of unemployment. This suggests that a policy aiming at subsidizing commuting costs have an impact not only in the land market but also in the labor market. We will analyze this issue in the next section.

Let us now study the *labor market* parameters, b (the unemployment benefit), θ (the effort level) and c (the monitoring technology). We focus only on z_1^* and z_2^* since the comparative statics analysis on Π^* is extremely messy. We obtain:

$$sgn\frac{\partial z_1^*}{\partial b} = sgn\left[1 + \frac{\theta M\ \overline{N}}{c(\overline{N} - L^*M)^2}\frac{\partial L^*}{\partial b}\right] \tag{5.6}$$

$$sgn\frac{\partial z_1^*}{\partial \theta} = sgn\left[1 + \frac{\theta M}{\overline{N} - L^*M}\frac{\partial L^*}{\partial b}\right] \tag{5.7}$$

$$sgn\frac{\partial z_1^*}{\partial c} = sgn\left[\frac{M}{\overline{N} - L^*M}\frac{\partial L^*}{\partial c} - \frac{1}{c}\right] \tag{5.8}$$

$$\frac{\partial z_2^*}{\partial b} > 0 \tag{5.9}$$

The interpretation of z_2^* is easy and straightforward. When the unemployment benefit increases, the unemployed are better off; without surprise θ and c do not affect z_2^*. Things get a little bit more complicated for z_1^*. Indeed, in all results two effects (affecting the efficiency wage) are present: a direct effect and an indirect one through the equilibrium employment level L^*. Let us study for example the case of the unemployment benefit b. On the one hand, when b rises, the employed have better outside opportunities and firms must increase their efficiency wage to avoid shirking (the UNSC shifts upwards), yielding a rise in z_1^* (direct effect). On the other, when b increases, L^* is cut (the labor demand shifts downwards) and thus U^* is augmented, making the threat of unemployment more severe: this leads to a reduction in the efficiency wage and thus in z_1^* (indirect effect). The net outcome is therefore ambiguous. We have exactly the same interpretation for θ and c. It is important to observe that the interaction between land and labor markets yields unusual results in urban economics. In general, increasing b raises z_1^* (see e.g. Miyao, 1975, Hartwick, Schweizer and Varaiya, 1976, Fujita, 1989, ch.4, Gannon and Zenou, 1997); this is true in a "pure" urban model where there is *no* labor market.

Finally, the *demographic* parameters M (the number of firms), $\overline{N}$ (the active population) and $\overline{Q}$ (firms' land consumption) also have interesting implications. We have indeed:

$$sgn\frac{\partial z_1^*}{\partial M} = sgn\left(L^* + \frac{\partial L^*}{\partial M}M\right) \tag{5.10}$$

$$\frac{\partial z_1^*}{\partial \overline{N}} < 0 \quad ; \quad \frac{\partial z_1^*}{\partial \overline{Q}} < 0 \tag{5.11}$$

$$\frac{\partial z_2^*}{\partial \overline{N}} < 0 \tag{5.12}$$

When $\overline{N}$ increases, the city size f^* rises so that efficiency wages are reduced (unemployment increases) and commuting costs are augmented (the distance to the city-center is larger): these two effects lead to a cut in both z_1^* and z_2^*. When $\overline{Q}$ increases, all borders in the city (e^*, g^* and f^*) are increased but not $g^* - e^*$ nor $f^* - g^*$. Thus the main effect of this increase is that L^* decreases and thus U^* increases. This leads to a cut in the efficiency wage and therefore in z_1^*; z_2^* is not affected. Concerning M we have a similar effect. When it rises, L^* decreases so that the sign of L^*M is ambiguous. Since $U^* = \overline{N} - L^*M$, no prediction can be made on z_1^*.

6. Policy implications

As discussed in the previous section, we would like to analyze a policy that subsidizes t the commuting cost per unit of distance. We keep things rather simple by not considering the objective function of government and its budget constraint, assuming that unemployment benefits and commuting costs are exogeneously financed.

6.1. Subsidizing all commuting costs

Let us start with a policy that subsidizes all workers' commuting costs (both employed and unemployed workers), where $0 < \delta < 1$ is the *ad valorem* subsidy.

This policy *shifts downwards* the UNSC since:[7]

$$w_1(L) = b + \frac{\theta}{c}\left(\frac{\overline{N}}{\overline{N} - LM}\right) + (1-\delta)t\frac{LM}{2}$$

and *shifts upwards* the demand curve for labor since:

$$w_1 = pF'(\overline{Q}, L) - (1-\delta)t\overline{Q}M/2 \tag{6.1}$$

It is important to observe that labor demand increases because of the land market. Indeed, when commuting costs are subsidized, the employed workers' bid rent decrease and the competition for land between workers and firms becomes less fierce. Thus, firms who want to set up in the city-center and to bid away workers can propose lower bid rents and thus increase their profits. As a result employment increases. The net effect of this policy is therefore *a reduction of urban unemployment but an ambiguous effect on efficiency wages* (see Figure 3). The intuition for employment is quite clear. Indeed, when commuting costs are subsidized, firms, which take into account workers' commuting costs when setting their efficiency wage, set a lower wage for each level of employment, thus shifting downwards the UNSC. At the same time, subsidizing commuting costs implies more employment since commuting costs negatively affect firms' profits. As a result, $L_\delta^* > L$ and urban unemployment is cut. Concerning efficiency wages, let us calculate the space-cost differential between the employed and the unemployed workers. By using (3.33), we easily obtain:

$$\Delta SC_\delta^* = (1-\delta)t\frac{LM}{2} = \Delta SC^* - \delta.t\frac{LM}{2} \tag{6.2}$$

which is less important than in the case with no-subsidy. In this context, the efficiency wage is given by:

$$w_{1,\delta}^* = b + \frac{\theta}{c}\left(\frac{\overline{N}}{\overline{N} - L_\delta^* M}\right) + (1-\delta)t\frac{L_\delta^* M}{2} \gtrless w_1^*$$

It is easy to see that two effects are present. On one hand, with the δ policy, since $L_\delta^* > L$, the threat of unemployment is lower and thus firms must induce workers not to shirk by increasing their efficiency wage. On the other, because of

[7] The subscript δ refers to the model where employed and unemployed workers' commuting costs are subsidized.

lower commuting costs, firms compensate workers less for space cost differential (see (6.2)) and thus efficiency wages decrease. The net effect is thus ambiguous and depends on the relative slopes of the labor demand curve and the UNSC (see Figure 3).

It is also interesting to analyze the effect of this policy on equilibrium utilities and profit. We have:

$$z_{1,\delta}^{*}=b-\frac{\alpha(1-\delta)t\overline{N}}{2}+\frac{\theta}{c}\left(\frac{\overline{N}}{\overline{N}-L_{\delta}^{*}M}\right)-R_{A}>z_{1}^{*} \tag{6.3}$$

$$z_{2,\delta}^{*}=b-\frac{\alpha(1-\delta)t\overline{N}}{2}-R_{A}>z_{2}^{*} \tag{6.4}$$

$$z_{1,\delta}^{*}-z_{2,\delta}^{*}=\frac{\theta}{c}\left(\frac{\overline{N}}{\overline{N}-L_{\delta}^{*}M}\right)>z_{1}^{*}-z_{2}^{*}=\frac{\theta}{c}\left(\frac{\overline{N}}{\overline{N}-L^{*}M}\right) \tag{6.5}$$

$$\begin{aligned}\Pi_{\delta}^{*} &= p.f(\overline{Q},L_{\delta}^{*})-\left[b+\frac{\theta}{c}\left(\frac{\overline{N}}{\overline{N}-L_{\delta}^{*}M}\right)\right]L^{*}-\tau\frac{\overline{Q}M^{2}}{2} \\ &\quad -\frac{(1-\delta)t}{2}\left[\alpha\overline{N}\,\overline{Q}+L_{\delta}^{*}M\left(L_{\delta}^{*}+\overline{Q}\right)\right]-R_{A}\overline{Q}\gtrless\Pi^{*}\end{aligned} \tag{6.6}$$

Utilities of both the employed and the unemployed increase but inequality also increases (see (6.5)). This is due to the fact that the employed have more commuting costs than the unemployed since they commute for both working and shopping while the unemployed commute only for shopping. The equilibrium profit Π_{δ}^{*} can be greater or lower than Π^{*} depending on the value of the production $p.f(\overline{Q},L_{\delta}^{*})$ which increases since $L_{\delta}^{*}>L^{*}$ compared with all costs (which also increase).

[*Insert Figure* 3 *here*]

6.2. Subsidizing only the unemployed workers' commuting costs

Let us now focus on the second policy where the government subsidizes only unemployed workers' commuting costs (a policy that is frequently advocated by policy makers) so that the employed and unemployed workers' commuting costs are respectively equal to $t_1 = t$ and $t_2 = (1-s)t$, where $0 < s < 1$ is the *ad valorem* subsidy. Bid rents can be written as:[8]

$$\Xi_{1,s}(x) = w_1^* - z_1^* - (1+\alpha)t(x-e) \tag{6.7}$$

$$\Xi_{2,s}(x) = b - z_2^* - \alpha(1-s)t(x-e) \tag{6.8}$$

By using these values, firms' bid rent and the land market equilibrium conditions, we easily obtain:

$$z_{1,s}^* = w_{1,s}^* - \frac{t}{2}\left[\alpha\overline{N} + L_s^*M - \alpha s\left(\overline{N} - L_s^*M\right)\right] - R_A \tag{6.9}$$

$$z_{2,s}^* = b - t\frac{\alpha(1-s)\overline{N}}{2} - R_A \tag{6.10}$$

$$\Pi_s^* = pY^* - w_1^*L_s^* - t\frac{\overline{Q}}{2}\left[\alpha(1-s)\overline{N} + (1+\alpha s)L_s^*M\right] - \tau\frac{\overline{Q}M^2}{2} - R_A\overline{Q} \tag{6.11}$$

For the employed, the unemployed and firms, their space costs are respectively equal to:

$$SC_{1,s}^* = \frac{t}{2}\left[\alpha\overline{N} + L_s^*M - \alpha s\left(\overline{N} - L_s^*M\right)\right] + R_A \tag{6.12}$$

$$SC_{2,s}^* = \frac{t}{2}\left[\alpha(1-s)\overline{N}\right] + R_A \tag{6.13}$$

$$SC_{F,s}^* = \left\{t\left[\alpha(1-s)\overline{N} + (1+\alpha.s)L_s^*M\right] + \tau M^2 + 2R_A\right\}\frac{\overline{Q}}{2} \tag{6.14}$$

so that the space-cost differential between workers and non-workers is given by (using (3.33)):

$$\Delta SC_s^* = (1+\alpha.s)\frac{tLM}{2} = \Delta SC^* + \alpha.s.t\frac{LM}{2} \tag{6.15}$$

This means that, compared to the non-subsidy case, (for any level of L) *the space-cost differential has increased.* In this context, the urban non-shirking constraint (UNSC) is equal to:

$$w_{1,s} = b + \frac{\theta}{c}\left(\frac{\overline{N}}{\overline{N} - LM}\right) + (1+\alpha s)t\frac{LM}{2} \tag{6.16}$$

[8] The subscript s refers to the model where only the unemployed workers' commuting costs are subsidized.

so that, compared to the case with no-subsidy, the UNSC *shifts upwards* by exactly $\alpha stLM/2$. Furthermore, by using (6.11), the labor demand curve is now defined by

$$w_{1,s} = pF'(\overline{Q}, L) - (1 + \alpha.s)t\overline{Q}M/2$$

so that it *shifts downwards* compared to the no-subsidy case. Indeed, with this policy, the difference in commuting costs between the employed and the unemployed increases so that to occupy the city-center firms must increase their bid rent, yielding a cut in profits. As a result, they hire less workers, thus reducing urban employment. Consequently, subsidizing only the unemployed workers' commuting costs leads to *an increase in urban unemployment while the effect on efficiency wages is ambiguous* (see Figure 4) and we have: $L_s^* < L^*$. The intuition behind this seemingly counter-intuitive result is the following. When the government subsidizes only the unemployed workers' commuting costs, the employed workers' commuting costs are relatively higher and the space cost differential rises. Thus, in order to meet the UNSC firms must compensate for the employed workers' commuting costs and thus wages must be higher for each level of employment, leading to a reduction of employment. At the same time, profits are reduced because of the effect described above on the land market and labor demand decreases. These two effects have the same sign and urban unemployment rises. Concerning efficiency wages, we have:

$$w_{1,s}^* = b + \frac{\theta}{c}\left(\frac{\overline{N}}{\overline{N} - L_s^* M}\right) + (1 + \alpha s)t\frac{L_s^* M}{2} \gtrless w_1^*$$

Here also the efficiency wage can be higher or lower than the one with no subsidy. Indeed, on one hand firms need less to deter shirking since the employment level is lower and thus the threat of unemployment is greater. On the other, they need more to compensate for commuting costs since the space cost differential has increased (see (6.15)). The net effect is therefore ambiguous and depends on the relative slopes of the two curves (see Figure 4).

[*Insert Figure* 4 *here*]

In this context, equilibrium utilities and profit are equal to:

$$z_{1,s}^* = b - \frac{\alpha(1-s)t\overline{N}}{2} + \frac{\theta}{c}\left(\frac{\overline{N}}{\overline{N} - L_s^* M}\right) - R_A \gtrless z_1^* \tag{6.17}$$

$$z_{2,s}^{*}=b-\frac{\alpha t(1-s)\overline{N}}{2}-R_{A}>z_{2}^{*} \tag{6.18}$$

$$z_{1,s}^{*}-z_{2,s}^{*}=\frac{\theta}{c}\left(\frac{\overline{N}}{\overline{N}-L_{s}^{*}M}\right)<z_{1}^{*}-z_{2}^{*}=\frac{\theta}{c}\left(\frac{\overline{N}}{\overline{N}-L^{*}M}\right) \tag{6.19}$$

$$\begin{aligned}\Pi_{s}^{*} &= p.f(\overline{Q},L_{s}^{*})-\left[b+\frac{\theta}{c}\left(\frac{\overline{N}}{\overline{N}-L_{s}^{*}M}\right)\right]L_{s}^{*}-\tau\frac{\overline{Q}M^{2}}{2} \\ &\quad -\frac{t}{2}\left[\alpha\overline{N}\,\overline{Q}+L_{s}^{*}M\left(L_{s}^{*}+\overline{Q}\right)\right]-R_{A}\overline{Q}\ \gtrless\ \Pi^{*}\end{aligned} \tag{6.20}$$

Contrary to the previous case, this policy does not always increase employed workers' utility since, on one hand, their commuting costs are reduced (direct effect that increases their utility) but, on the other, firms must compensate employed workers to induce them to stay in the city. As we have seen below, the net effect will depend on if the efficiency wage increases or decreases after this policy. Observe also that inequality decreases since the unemployed that commute less than the employed have now lower costs.

Proposition 6.1. *A policy that subsidizes the commuting costs of both the employed and unemployed workers reduces urban unemployment, increases utilities of all workers but raises inequality whereas a policy that subsidizes only unemployed workers' commuting costs increases urban unemployment, does not always raise the employed workers' utility but cuts inequality.*

The intuition behind these results is quite clear. *In the first policy (where all workers' commuting costs are subsidized) the space-cost differential decreases and labor demand increases whereas in the second one (where only the unemployed workers' commuting costs are subsidized), we have exactly the opposite result.* Since firms compensate for space cost differential, a decrease (an increase) of it raises (cuts) employment. Since firms' profit is affected by employed workers commuting costs through bid rents, labor demand increases (decreases) when space cost differential is cut (augmented).

The main message of this result is that it is crucial not to have a partial equilibrium framework when dealing with policies aiming at reducing unemployment. Already in their seminal paper, Albrecht and Axell (1984) have pointed out the importance of a general equilibrium rather than a partial analysis for the study of the labor market. In a general equilibrium model with sequential search, they show that an increase in the unemployment benefit can (in certain cases) decrease unemployment, a result that can never happen in the standard partial equilibrium search model. In our model, the introduction of a land market in an efficiency wage model demonstrates that spatial policies (such as subsidizing commuting costs) can have unusual effects because they affect both land and labor markets. The other important message of this result is that the location of firms and thus of the employment center(s) must not be exogeneous but rather determined optimally. Indeed, if firms were not mobile, then subsidizing commuting costs would not affect labor demand (since firms would not compete with workers for land) and some results would be changed. In fact, it is easy to see that our results on urban unemployment would not change but the impact on urban efficiency wages would be different since in the first policy they would decrease whereas in the second one they would increase. Last, observe that a policy that increases the unemployment benefit b shifts upwards the UNSC and thus increases both urban unemployment and efficiency wages but does not affect the land market and thus labor demand. This highlights the fact that subsidizing commuting costs or increasing unemployment benefits are two distinct policies that have different mechanisms and implications.

7. Conclusion

In this paper, we have developed a model of urban unemployment where the location of all workers and firms was endogeneous and determined in equilibrium. In the land market, all agents bid for rents in order to occupy some space in the city. We find conditions ensuring that a unique urban equilibrium configuration exists in which firms locate at the city-center (CBD), the employed at the vicinity of the CBD and the unemployed at the periphery of the city. In the labor market, firms set efficiency wage to deter shirking and to induce workers to stay in the city. We also show that there exists a unique labor market equilibrium that is compatible with the urban equilibrium one. We then perform some comparative statics analyses and derive policy implications. The most striking result obtained is that a policy that subsidizes the commuting costs of both the employed and unemployed workers reduces urban unemployment whereas a policy that subsidizes only the unemployed workers' commuting costs increases urban unemployment.

This result is interesting because it contradicts the common view that subsidizing unemployed workers' commuting costs cuts unemployment. This reinforces our belief that the study of urban unemployment is extremely important for policy makers since it introduces another market and since unemployment policies are rarely global but rather specific.

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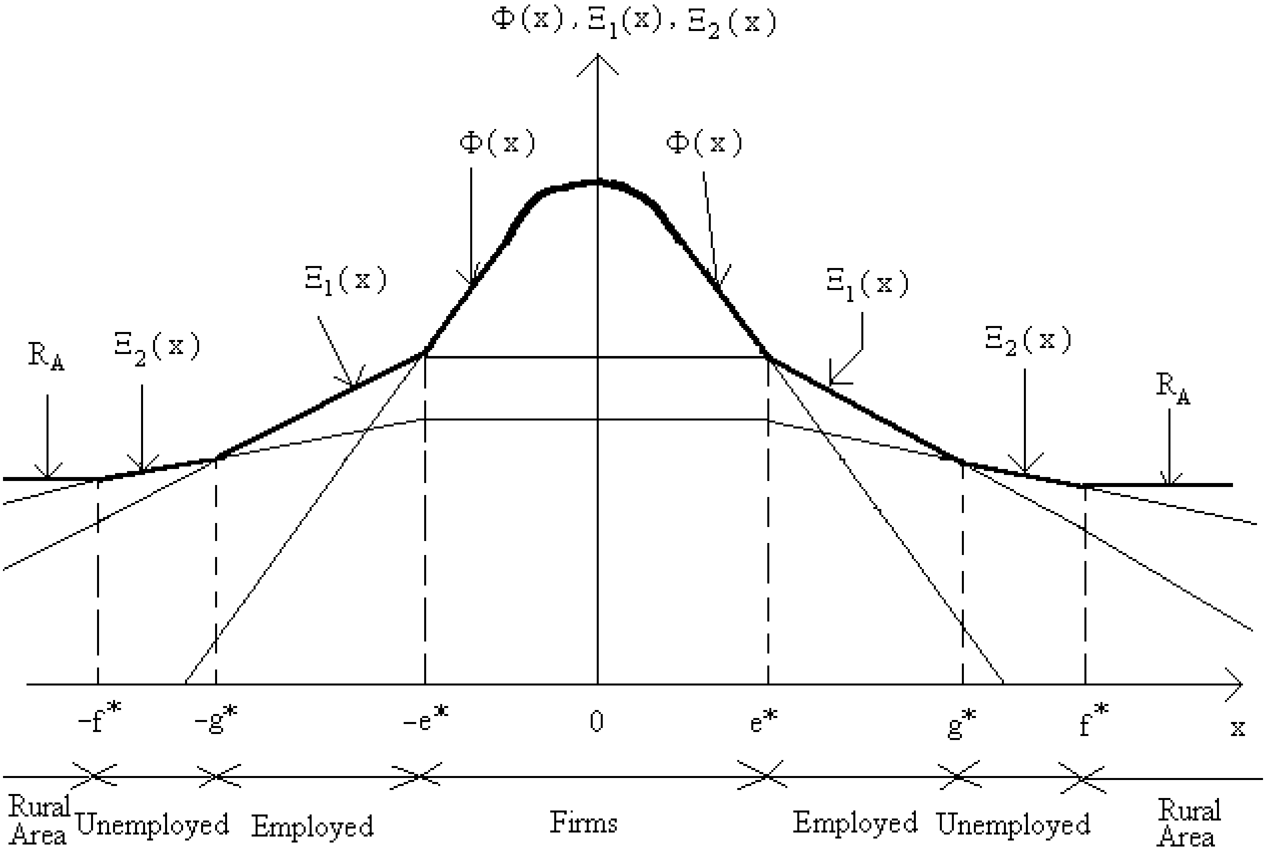

Figure 1 : Urban equilibrium configuration

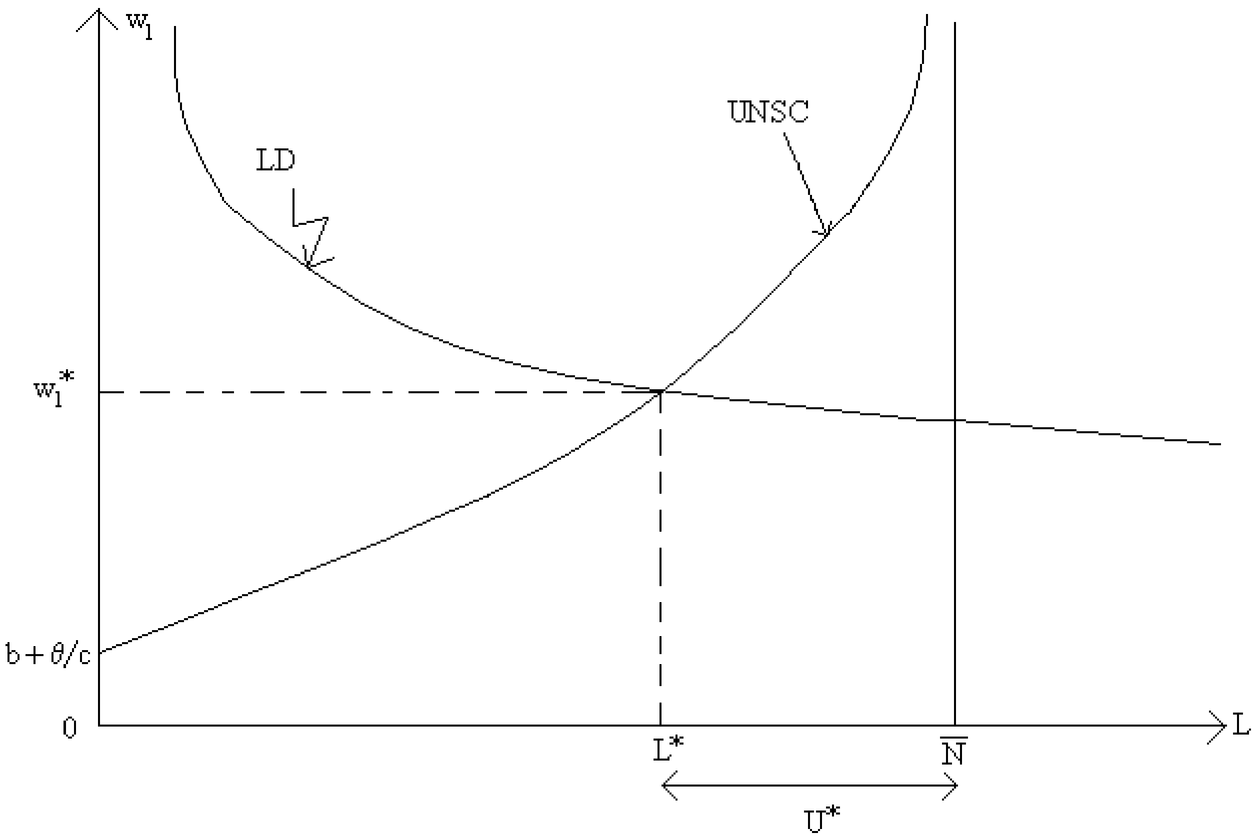

Figure 2 : The labor market equilibrium

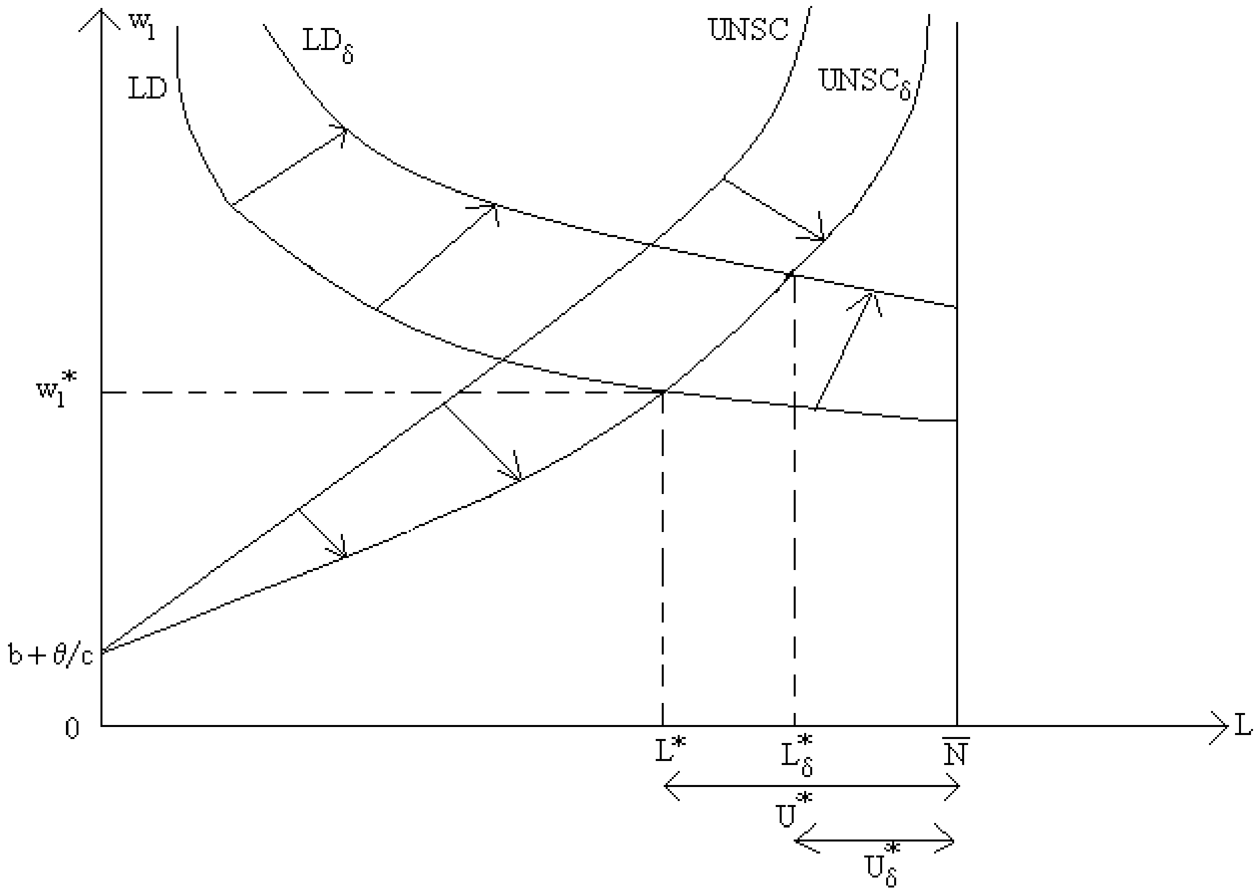

Figure 3 : Subsidizing all commuting costs

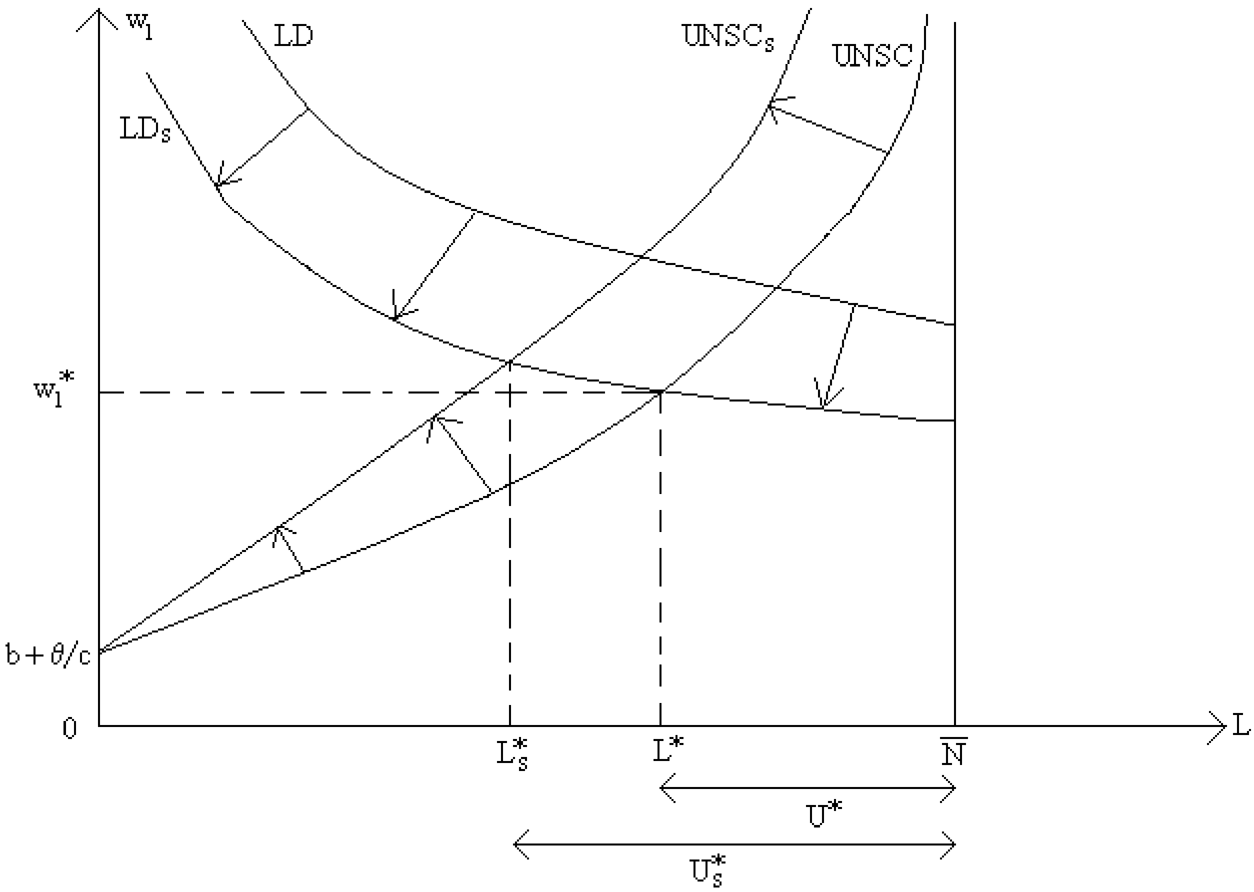

Figure 4 : Subsidizing the unemployed workers' commuting costs

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