

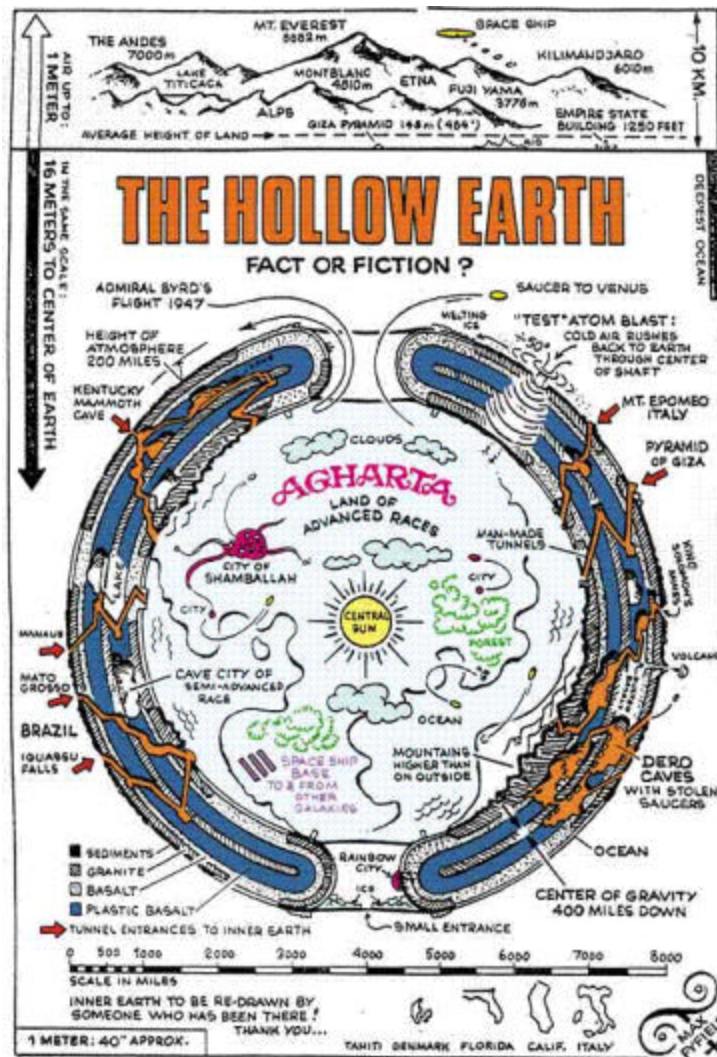
How Euler Did It

by Ed Sandifer



Euler and the Hollow Earth: Fact or Fiction?

April 2007



Is the earth hollow? Is there a sun 600 miles in diameter at the center of the hollow earth? Is the inside of the shell of the hollow earth covered with mountains larger than the ones we see on the outside? Is there a hole in the shell of the hollow earth through which flying saucers from Venus and space ships from other galaxies fly to get to their bases inside the hollow earth? Are there secret

passages from the bases of the Great Pyramid and other locations around the earth that connect the outside to the inside of the earth? Is the mushroom cloud of an atomic bomb really caused when the bomb pokes a hole through the shell and the gasses inside the earth rush through to escape?

Some people claim to believe all of this, and they even give us detailed maps of what is inside the earth. See, for example, the extravagant map above, drawn by Max Fyfield and available on scores of pages on the World Wide Web. [Fy] Imagine my surprise when I found apparently reputable sources that said that Euler also endorsed a hollow earth theory, and that Fyfield's map was based on Euler's theories. This, of course, piqued my curiosity, so I decided to look into the question of Euler and the Hollow Earth.

Here are a few excerpts from some websites I found that credit Euler with a hollow earth theory.

Leonhard Euler

Later theorists came up with variations to Halley's [sic] model. In the seventeenth century, **Leonhard Euler** proposed a single-shell hollow Earth with a small sun (1.000 km across) at the centre, providing light and warmth for an inner-Earth civilisation. Others proposed *two* inner suns, and even named them: Pluto and Proserpine.

<http://strangemaps.wordpress.com/2007/03/01/85-inside-the-hollow-earth/>

Leonhard Euler

In the eighteenth century A Swiss mathematician Leonhard Euler took the multiple spheres theory and replaced it with a single hollow sphere that contained a sun 600 miles wide. He said the sun maintained heat and light for an advanced civilization that he said lived there. A Scottish mathematician Sir John Leslie suggested that there was not one sun but in fact two he named these Pluto and Proserpine.

http://tinwiki.org/wiki/Hollow_Earth

Leonard Euler

Leonard Euler (1707-1783), noted mathematician and one of the founders of higher mathematics. He stated that "mathematically the Earth has to be hollow". He also believed there was a center sun inside the Earth's interior, which provided daylight to a splendid subterranean civilization.

<http://www.xenophilia.com/zb0008d.htm>

Even the usually reliable John Lienhard, a historian of science at the University of Houston and the creator of the NPR feature Engines of our Ingenuity, got in on the act, or was taken in by the deception: [L]



No. 2180:
HOLLOW EARTH

by John H. Lienhard

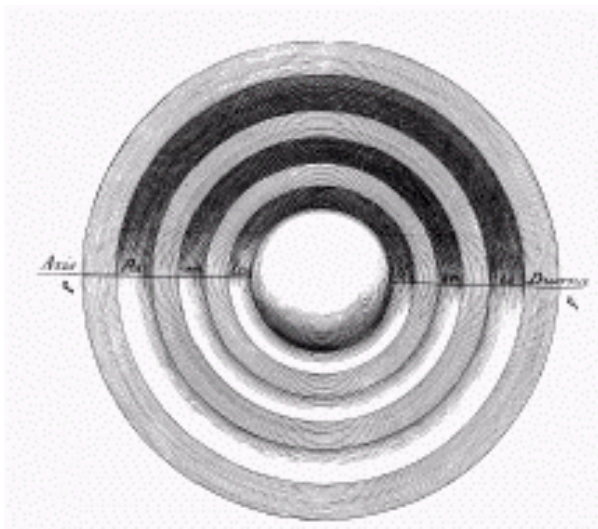
One person who picked up on that idea was Leonhard Euler, the great mathematician of the 18th century. Euler proposed that Earth was completely hollow (no concentric shells) with a six-hundred mile diameter sun in the center. His hollow interior could be reached through holes at the North and South Poles.

<http://www.uh.edu/engines/epi2180.htm>

The earliest hollow earth theory seriously set forth by an important scientist seems to be by Edmond Haley. [H] In about 1691, Haley was trying to explain why the earth has a magnetic field, and why it varied. He proposed that the earth might be composed of concentric shells, separated by fluids so that one shell could move relative to another. It was this relative motion, he said, that caused the magnetic field, and variations in the motion caused the variations in the field. Simanek [Si] offers the illustration at the right, which he says comes from Haley's 1681 paper.

Modern readers might be tempted to mock Haley's gullibility and naivety in proposing such a theory, but they would be unfair to do so. It was a well-reasoned effort to explain an observed scientific phenomenon. The theory conformed to the facts as the scientists of his time knew them, and, as new facts emerged that contradicted the theory, they abandoned the theory. That is the way the scientific method is supposed to work.

Note that Haley's theory involved no flying saucers, hidden central sun, or secret tunnels from the base of the Pyramid of Giza.



Let us move on to the next century. In the 1730s, one of the open questions of science concerned the shape of the earth. Some people thought that the earth would bulge at the poles, and be narrower at the equator so that it could spin more efficiently. Euler joined Newton and others in believing that the earth would bulge at the equator and be flatter at the poles. In 1738 he published a paper [E32] "On the

shape of the Earth” in which he considered the earth as a fluid mass and predicted that it would bulge at the equator rather than at the poles. Cassini had made measurements a few years earlier that suggested that there was a bulge at the poles. At about the same time as Euler was writing E32, Maupertuis was planning a pair of expeditions, one to Peru and the other to Lapland, to make more accurate measurements that would show that Euler and Newton were correct. For this, Maupertuis became famous as “the Man who Flattened the Earth.”

Through this, Euler never suggested that the earth was hollow. He considered it as a fluid with a crust on it, not too different from modern theories.

Euler passed up another opportunity to propound a hollow earth theory in the early 1750s. The topic of interest was the precession of the orbit of the moon. The moon has an elliptical orbit around the earth, and each month the axis of that ellipse moves about 3° . That wandering of the axis is called *precession*. Euler, D’Alembert and Clairaut studied this, but their analyses could only explain about half of the observed precession. Being faithful to the scientific method, they examined their assumptions to try to find ways to explain the other half of the precession. We’ll mention two of their efforts.

They considered the possibility that Newton’s inverse square law for gravity was not quite right, and that for shorter astronomical distances, gravity was a little stronger than predicted. Perhaps the force of gravity between two masses M and m , was not $Mm\frac{G}{r^2}$, where G is the gravitational constant.

Perhaps it was a little more than that, say $Mm\left(\frac{G}{r^2} + \frac{g}{r^4}\right)$, where g is another gravitational constant.

They also considered the possibility that the moon was somehow more massive than they thought. Perhaps it even had two parts and was shaped like a dumbbell, with the nearer part always hiding the more distant part from our view.

They *could* have tried to explain the phenomenon by finding a way to make the earth less massive than they thought, perhaps by being hollow, but from the evidence that is currently available, it seems that they did not consider this possibility.

Eventually, Clairaut found a way to improve the analysis by considering more terms in certain series expansions, and he explained the other half of the precession. Like Haley, everyone involved followed good scientific method. When the predictions didn’t fit the observations, they tried to improve their theories and to refine their analysis until they could explain the discrepancies.

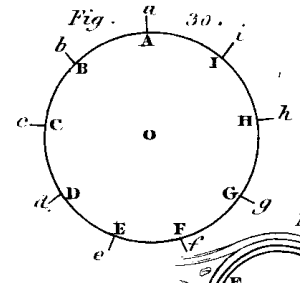
Simanek [Si] points us to a third place people might think they find a Hollow Earth theory in Euler’s work:

Some books and websites say that Leonhard Euler proposed a simpler hollow earth model. Some give details, but few provide a reference. One that does cites: Euler, Leonhard; **Letters of Euler on Natural Philosophy**, Vol 2, Letter LVIII, pp 202-203, 1835. However, that references his comments on an interesting mathematical problem: "If you drilled a hole all the way through the earth, and dropped a stone in the hole, what would happen?" It's a "thought experiment" and someone may have misread Euler, supposing Euler really thought there was hole all the way through the earth. Then others picked it up without checking sources.

Though Simanek apparently means Volume I, letters XLIX and L (and I think there may be problems with his citation of Haley as well), he is accurate in his account of Euler's *Lettres à un Princess d'Allemagne* [E343] and in his warning against believing such things "without checking sources."

What did Euler really say? In Letter XLIV, he gives us the illustration at the right that he calls "Fig. 30."¹ Here, he explains that the direction "down" changes at different locations on the earth. At one point, it may be in the direction *aA*, another *bB*, etc., but always, "down" means "towards the center of the earth," marked *O*. He writes:

"In fact, were you to dig a hole in the earth, at whatever place, and to continue your labour incessantly, digging always downward and downward perpendicularly, you would at length reach the centre of the earth. ... It is true, such a project could never be executed, as it would be necessary to dig to the depth of 3956 English miles; but there is no harm in supposing it, in order to discover what would be the result.

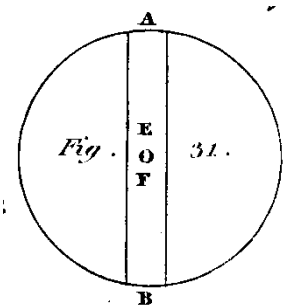


Having explained what "down" means at the surface of the earth, he invites us into his hypothetical hole to see what "down" means inside the earth. He gives us "Fig. 31," with his hole clear through the earth from *A* to its antipode at *B*. He explains that, whether one falls from point *A*, *B*, *E* or *F*, one would always fall towards the center of the earth, that is, towards point *O*.

Euler continues his discussion in Letter L, writing:

"Let us now return to the aperture made in the earth through its centre; it is clear, that a body at the very centre must entirely lose its gravity, as it could no longer move in any direction, all those of gravity tending continually toward the centre of the earth. ...

"Having travelled, in idea, to the centre of the earth, let us return to its surface, and ascend to the summit of the loftiest mountains."



Thus it is clear that Euler is, indeed, doing a thought experiment. There is no real hole to the center of the earth, and he isn't even considering a hollow earth, just one with a hole in it.

We conclude that Euler did *not* propose a theory that the earth is hollow.

This column has relied rather too heavily on web resources, so some readers might think that the pseudo-scientific ideas set forth here circulate only because of the Internet. This is not the case, as the remarkable bibliography [Fr] that Ruth Freitag prepared for the Library of Congress demonstrates. Among over sixty books and articles on her list, there are five from the 1820's that endorse the Hollow Earth theory. The list further suggests that the theory enjoyed resurgences in the 1880's and again in the 1930's. The Internet did not create these theories. It only makes them easier to find.

¹ In Euler's time, it was difficult to include figures in the text of a book, so usually all the illustrations were gathered together and printed on just a few sheets, which were bound in the back of the book. The lines in the lower right of this illustration are part of a different figure, one about the anatomy of the eye, not part of his figure of the earth.

It is quite difficult to separate fact from fiction when trying to write about the history of scientific fantasies and hoaxes.² I believe that three of the sources cited here, [Fr, L, Si], should be taken particularly seriously, and they should be forgiven if a band of hoaxers were able to mislead them about Euler's role in the hollow earth theories. Of course, maybe I was fooled, too. I hope that we've set the record straight, and we can treat it all as a good April Fool's Day joke.

References:

- [Fr] Freitag, Ruth S., "Hollow Earth Theores: A List of References," <http://www.loc.gov/rr/scitech/SciRefGuides/hollowearth.html>, 1997.
- [Fy] Fyfield, Max, "The Hollow Earth: Fact or Fiction," online at various sites, for examples: <http://www.promedia.net/users/vtown/map.html>, <http://www.erenouvelle.com/dossagarcreu.php>, and many others.
- [E32] Euler, Leonhard, Von der Gestalt der Erden, *Anmerck ungen üver die Zeitungen*, 1738. Reprinted in *Opera Omnia* Series III vol. 2 pp. 325-346.
- [E343] Euler, Leonhard, *Lettres à un Princess d'Allemagne*, 2 vols., St. Petersburg, 1768. Reprinted in *Opera Omnia*, Series III, vols. 11 and 12. Third English edition edited by David Brewster and published under the title *Letters of Euler on Different Subjects in Natural Philosophy Addressed to a German Princess*, Edinburgh, 1823. English edition available online at EulerArchive.org.
- [H] Halley, Edmond, An account of the cause of the change of the variation of the magnetical needle with an hypothesis of the structure of the internal parts of the earth: as it was proposed to the Royal Society in one of their later meetings. *Philosophical Transactions of the Royal Society of London* **16**:563-578. Available on JSTOR.
- [L] Lienhard, John H., "Hollow Earth," *Engines of Our Ingenuity* No. 2180, <http://www.uh.edu/engines/epi2180.htm>.
- [Si] Simanek, Donald E., "Turning the Universe Inside-Out: Ulysses Grant Morrow's Naples Experiment," <http://www.lhup.edu/~dsimanek/hollow/morrow.htm>.

Ed Sandifer (SandiferE@wcsu.edu) is Professor of Mathematics at Western Connecticut State University in Danbury, CT. He is an avid marathon runner, with 34 Boston Marathons on his shoes, and he is Secretary of The Euler Society (www.EulerSociety.org). His new book, *The Early Mathematics of Leonhard Euler*, was published by the MAA in December 2006, as part of the celebrations of Euler's tercentennial in 2007. The MAA will be publishing a collection of the *How Euler Did It* columns during the summer of 2007.

How Euler Did It is updated each month.
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² I'm trying to make a distinction here. I am taking a "fantasy" to be something that a person makes up and believes. A "hoax," on the other hand, the person who makes it up does not believe it, but wants other people to believe it. If he does not mean other people to believe it, then it is just "fiction."