images of neighbouring stars of equal magnitude. This peculiarity of the Nova's disc was first remarked by Flammarion and Antoniadi, and later by Max Wolf, Kostinsky, and von Gothard. It owes its origin probably to the exceptionally strong ultra-violet rays emitted by the Nova, which are not brought to the focus for which the objective is corrected.

The recent photographs have been taken by Mr. Butler and Mr. Rolston. The visual observations have chiefly been made by Messrs. Fowler and Butler. Mr. Baxandall has undertaken the reduction to wave-lengths and the discussion of the lines in the photographic spectrum, while Dr. Lockyer and Mr. Baxandall have assisted in the preparation of the present paper.

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" An Attempt to ascertain the Date of the Original Construction of Stonehenge from its Orientation." By Sir NORMAN LOCKYER, K.C.B., F.R.S., and F. C. PENROSE, F.R.S. Received October 21, 1901.

This investigation was undertaken in the spring of the present year, as a sequel to analogous work in Egypt and Greece, with a view to determine whether the orientation theory could throw any light upon the date of the foundation of Stonehenge, concerning which authorities vary in their estimate by some thousands of years. We beg to lay before the Royal Society the results derived from a careful study of its orientation for the purpose of arriving at the probable date of its foundation astronomically. This is not, indeed, the first attempt to obtain the date of Stonehenge by means of astronomical considerations. In Mr. Godfrey Higgins' work* he refers to a method of attack connected with precession. This furnished him with the date 4000 B.C.

More recently, Dr. W. M. Flinders Petrie,† whose accurate plan is a valuable contribution to the study of Stonehenge, was led by his measures of the orientation to a date very greatly in the opposite direction, but, owing to an error in his application of the change of obliquity, clearly a mistaken one.

As the whole of the argument which follows rests upon the assumption of Stonehenge having been a solar temple, a short discussion of the grounds of this view may not be out of place ; and, again, as the approximate date which we have arrived at is an early one, a few words may be added indicating the presence in Britain at that time of a race of men capable of designing and executing such work.

* ' The Celtic Druids.' 4to. London, 1827.
† ' Stonehenge,' &c., 1880.

As to the first point, Diodorus Siculus (ii, 47) has preserved a statement of Hecatæus in which Stonehenge alone can by any probability be referred to.

" We think that no one will consider it foreign to our subject to say a word respecting the Hyperboreans.

"Amongst the writers who have occupied themselves with the mythology of the ancients, Hecatæus and some others tell us that opposite the land of the Celts [ἐν τοῖς ἀντιπέραν τῆς κελτικῆς τόποις] there exists in the Ocean an island not smaller than Sicily, and which, situated under the constellation of The Bear, is inhabited by the Hyperboreans; so called because they live beyond the point from which the North wind blows. . . . If one may believe the same mythology, Latona was born in this island, and for that reason the inhabitants honour Apollo more than any other deity.   A sacred enclosure [νῆσον] is dedicated to him in the island, as well as a magnificent circular temple adorned with many rich offerings. . . . The Hyperboreans are in general very friendly to the Greeks."

The Hecatæus above referred to was probably Hecatæus of Abdera, in Thrace, fourth century B.C.; a friend of Alexander the Great.   This Hecatæus is said to have written a history of the Hyperboreans : that it was Hecatæus of Miletus, an historian of the sixth century B.C., is less likely.

As to the second point, although we cannot go so far back in evidence of the power and civilisation of the Britons, there is an argument of some value to be drawn from the fine character of the coinage issued by British kings early in the second century B.C., and from the statement of Julius Cæsar (' De Bello Gallico,' vi, p. 13) that in the schools of the Druids the subjects taught included the movements of the stars, the size of the earth and the nature of things (Multa præterea de sideribus et eorum motu, de mundi magnitudine, de rerum natura, de deorum immortalium vi ac potestate disputant et juventuti tradunt).

Studies of such a character seem quite consistent with, and to demand, a long antecedent period of civilisation.

The chief evidence lies in the fact that an " avenue," as it is called, formed by two ancient earthen banks, extends for a considerable distance from the structure, in the general direction of the sunrise at the summer solstice, precisely in the same way as in Egypt a long avenue of sphinxes indicates the principal outlook of a temple.

These earthen banks defining the avenue do not exist alone.   As will be seen from the plan which accompanies this paper, there is a general common line of direction for the avenue and the principal axis of the structure, and the general design of the building, together with the position and shape of the Naos, indicate a close connection of the whole temple structure with the direction of the avenue.   There may have been other pylon and screen equivalents as in ancient temples,

which have disappeared, the object being to confine the illumination to a small part of the Naos. There can be little doubt, also, that the temple was originally roofed in, and that the Sun's first ray, suddenly admitted into the darkness, formed a fundamental part of the cultus.

While the actual observation of sunrise was doubtless made within the building itself, we seem justified in taking the orientation of the axis to be the same as that of the avenue, and since in the present state of the S.W. trilithon the direction of the avenue can probably be determined with greater accuracy than that of the temple axis itself, the estimate of date in this paper is based upon the orientation of the avenue. Further evidence will be given, however, to show that the direction of the axis of the temple, so far as it can now be determined, is sufficiently accordant with the direction of the avenue.

The orientation of this avenue may be examined upon the same principles that have been found successful in the case of Greek and Egyptian temples—that is, on the assumption that Stonehenge was a solar temple, and that the greatest function took place at sunrise on the longest day of the year. This not only had a religious motive; it had also the economic value of marking officially and distinctly that time of the year and the beginning of an annual period.

It is, indeed, probable that the structure may have had other capabilities, such as being connected with the equinoxes or the winter solstice; but it is with its uses at the summer solstice alone that this paper deals.

There is this difference in treatment between the observations required for Stonehenge and those which are available for Greek or Egyptian solar temples—viz., that in the case of the latter the effect of the precession of the equinoxes upon the stars, which as warning clock stars were almost invariably connected with those temples, offers the best measure of the dates of foundation; but here, owing to the brightness of twilight at the summer solstice, such a star could not have been employed, so that we can rely only on the secular changes of the obliquity as affecting the azimuth of the point of sunrise. This requires the measurements to be taken with very great precision, towards which care has not been wanting in regard to those which we submit to the Society.

The main architecture of Stonehenge consisted of an external circle of about 100 feet in diameter, composed of thirty large upright stones, named sarsens, connected by continuous lintels, and an inner structure of ten still larger stones, arranged in the shape of a horseshoe, formed by five isolated trilithons. About one-half of these uprights have fallen and a still greater number of the lintels which they originally carried. There are also other lines of smaller upright stones, respecting which the only point requiring notice in this paper is that none of them would have interrupted the line of the axis of the avenue. This

circular temple was also surrounded by an earthen bank, also circular, of about 300 feet in diameter, interrupted towards the north-east by receiving into itself the banks forming the avenue before mentioned, which is about 50 feet across. Within this avenue, and, looking north-east from the centre of the temple, at about 250 feet distance and considerably to the right hand of the axis, stands an isolated stone, which from a mediæval legend has been named the Friar's Heel.
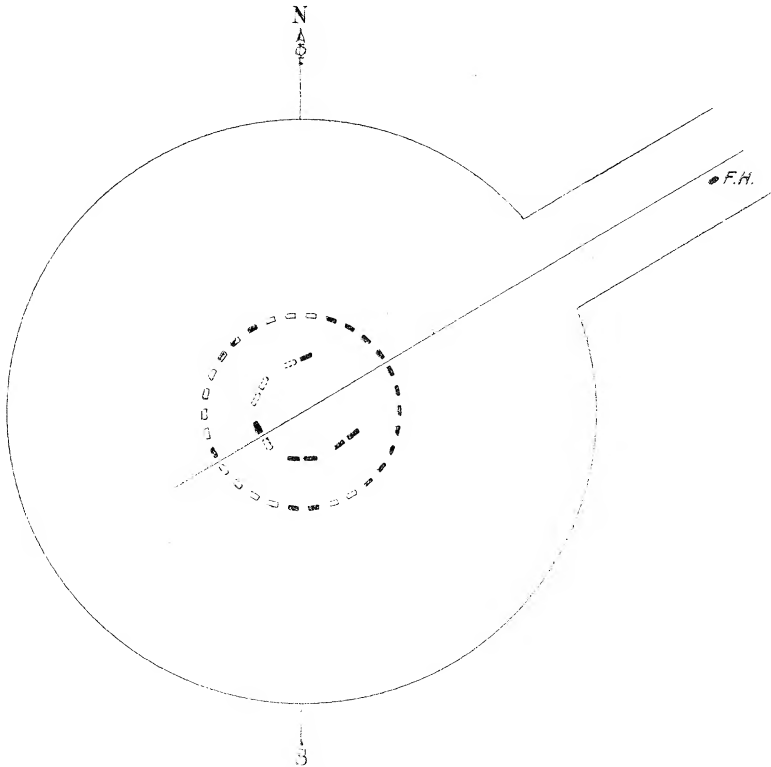

Fig. 1.—General plan of Stonehenge.

The axis passes very nearly centrally through an intercolumniation (so to call it) between two uprights of the external circle and between the uprights of the westernmost trilithon as it originally stood. Of this trilithon the southernmost upright with the lintel stone fell in the year 1620, but the companion survived as the leaning stone which formed a conspicuous and picturesque object for many years, but happily now restored to its original more dignified and safer condition of verticality. The inclination of this stone, however, having taken place in the direction of the axis of the avenue,

and as the distance between it and its original companion is known both by the analogy of the two perfect trilithons and by the measure of the mortice holes on the lintel they formerly supported, we obtain by bisection the measure (viz. 11 inches) from its edge of a point in

FIG. 2.—The stake placed on the fallen stone indicates the axis shown on FIG. 1, in relation to the leaning stone (on the left), and the centre of the N.E. trilithon.

the continuation of the central axis of the avenue and temple, and which has now to be determined very accurately. The banks which form the avenue have suffered much degradation. It appears from Sir ¡Richard Colt Hoare's account that at the beginning of the last century they were distinguishable for a much greater distance than at present, but they are still discernible, especially on the northern side,

for more than 1300 feet from the centre of the temple, and particularly the line of the bottom of the ditch from which the earth was taken to form the bank, and which runs parallel to it. Measurements taken from this line assisted materially those taken from the crown of the bank itself. With this help and by using the southern bank and ditch whenever it admitted of recognition, a fair estimate of the central line could be arrived at. To verify this, two pegs were placed at points 140 feet apart along the line near the commencement of the avenue, and four others at distances averaging 100 feet apart nearer the further recognisable extremity, and their directions were measured with the theodolite, independently by two observers, the reference point being Salisbury Spire, of which the exact bearing from the centre of the temple had been kindly supplied by Colonel Johnston, R.E., the Director-General of the Ordnance Survey. The same was also measured locally by observations of the Sun and of Polaris, the mean of which differed by less than 20″ from the Ordnance value. The resulting observations gave for the axis of the avenue nearest the commencement an azimuth of 49° 38′ 48″, and for that of the more distant part 49° 32′ 54″. The mean of these two lines drawn from the central interval of the great trilithon, already referred to, passes between two of the sarsens of the exterior circle, which have an opening of about 4 feet, within a few inches of their middle point, the deviation being northwards. This may be considered to prove the close coincidence of the original axis of the temple with the direction of the avenue.

This value of the azimuth, the mean of which is 49° 35′ 51″, is confirmed by the information, also supplied from the Ordnance Survey, that from the centre of the temple the bearing of the principal bench mark on the ancient fortified hill, about 8 miles distant, a well-known British encampment named Silbury or Sidbury, is 49° 34′ 18″, and that the same line continued through Stonehenge to the south-west strikes another ancient fortification, namely, Grovely Castle, about 6 miles distant and at practically the same azimuth, viz., 49° 35′ 51″. For the above reasons 49° 34′ 18″ has been adopted for the azimuth of the avenue.

The present solstitial sunrise was also watched for on five successive mornings, viz., June 21 to 25, and was successfully observed on the latter occasion. As soon as the Sun's limb was sufficiently above the horizon for its bisection to be well measured, it was found to be 8′ 40″ northwards of the peak of the Friar's Heel, which was used as the reference point, the altitude of the horizon being 35′ 48″. The azimuth of this peak from the point of observation had been previously ascertained to be 50° 39′ 5″, giving for that of the Sun when measured, 50° 30′ 25″, and by calculation that of the Sun with the limb 2′ above the horizon should be 50° 30′ 54″. This observation

was therefore completely in accordance with the results which had been obtained otherwise.

The time which would elapse between geometrical sunrise, that is, with the upper limb tangential with the horizon, and that which is here supposed, would occupy about 17 seconds, and the difference of azimuth would be 3′ 15″.

The remaining point is to find out what value should be given to the Sun's declination when it appeared showing itself 2′ above the horizon, the azimuth being 49° 34′ 18″.

The data thus obtained for the derivation of the required epoch are these :—

(1.) The elevation of the local horizon at the sunrise point seen by a man standing between the uprights of the great trilithon (a distance of about 8000 feet) is about 35′ 30″, and 2′ additional for Sun's upper limb makes 37′ 30″.

(2.) − Refraction + parallax, 27′ 20″.

(3.) Sun's semi-diameter, allowance being made for greater eccentricity than at present, 15′ 45″.

(4.) Sun's azimuth, 49° 34′ 18″, and N. latitude, 51° 10′ 42″.

From the above data the Sun's declination works out 23° 54′ 30″ N., and by Stockwell's tables of the obliquity, which are based upon modern determinations of the elements of the solar system,* the date becomes 1680 B.C.

It is to be understood that on account of the slight uncertainty as to the original line of observation and the very slow rate of change in the obliquity of the ecliptic, the date thus derived may possibly be in error by ± 200 years.

In this investigation the so-called Friar's Heel has been used only as a convenient point for reference and verification in measurement, and no theory has been formed as to its purpose. It is placed at some distance, as before mentioned, to the south of the axis of the avenue, so that at the date arrived at for the erection of the temple the Sun must have completely risen before it was vertically over the summit of the stone. It may be remarked, further, that more than 500 years must yet elapse before such a coincidence can take place at the beginning of sunrise.

We give in an appendix certain details of the observations.

We have to express our thanks to Sir Edmund Antrobus, Bart., for much kind assistance during our survey ; and to A. Fowler and Howard Payn, Esqs., for skilful and zealous co-operation in the measurements and calculations. As already stated, Colonel Duncan A. Johnston, R.E., Director-General of the Ordnance Survey, has also

* 'Smithsonian Contributions to Knowledge,' vol. 18, No. 232, table 9. Washington, 1873.

been good enough to furnish us with much valuable information, for which our best thanks are due.

APPENDIX.

The instrument chiefly employed was a 6-inch transit theodolite by Cooke with verniers reading to $20''$ in altitude and azimuth. Most of the observations were made at two points very near the axis, which may be designated by a, b. Station a was at a distance of 61 feet to the south-west of the centre of the temple, and b 364 feet to the north-east. The distance from the centre of Stonehenge to Salisbury Spire being 41,981 feet, the calculated corrections for parallax at the points of observation with reference to Salisbury Spire are :—

$$\text{Station } a \quad + \quad 4' \quad 12''.$$
$$,, \quad b \quad - 25 \quad 20.$$

(1.) *Relative Azimuths.*—Theodolite at station a—

Salisbury Spire	0	0	0
N. side of opening in N.E. trilithon of the external ring	237	27	40
Tree in middle of clump on Sidbury Hill	237	40	20
Highest point of Friar's Heel..............	239	47	25
S. side of opening in N.E. trilithon	240	14	40
Middle ,, ,, ,, ...	238	51	10

(2.) *Absolute Azimuths.*—All the azimuths were referred to that of Salisbury Spire, the azimuth of which was determined by observations of the Sun and Polaris.

(*a*.) *Observation of Sun, June* 23, 1901, 3.30—3.40 P.M.

Mean of observed altitudes of Sun	41°	26'	35''
Refraction .... $-1'\ 4''$			
Parallax .... $+\ 6$	0	0	58
True altitude of Sun's centre	41	25	37

Latitude $= 51°\ 10'\ 42''$.   Sun's declination $= 23°\ 26'\ 43''$.
Using the formula

$$\cos^2 \tfrac{1}{2}A = \frac{\sin \tfrac{1}{2}(\Delta + c - z)\sin \tfrac{1}{2}(\Delta + z - c)}{\sin c \,.\, \sin z}$$

where $A =$ azimuth from south, $\Delta =$ polar distance, $c =$ co-latitude, and $z =$ zenith distance,

we get

Azimuth of Sun ....S. 75° 30′ 30″ W.
Mean circle reading on Sun 84   38   35

Azimuth of Salisbury Spire ...S. 9   8   5 E.

b. Observations of Polaris.—June 23, 1901. Time of greatest easterly elongation, calculated by formula $\cos h = \tan \phi \cot \delta$ is G.M.T. 1.34 A.M.

Azimuth at greatest easterly elongation, calculated by the formula

$$\sin A = \cos \delta \sec \phi,$$

is 181° 57′ 0″ from south.

Observed maximum reading of circle...... 256° 33′ 0″
True azimuth of star 181   57   0

Meridian (S.) reading of circle   74   36   0
Circle reading on Salisbury Spire   65   28   0

Azimuth of Salisbury Spire ......S.   9   8   0 E.

The mean of the two determinations gives for the azimuth of Salisbury Spire S. 9° 8′ 2″ E. This result agrees well with the value of the azimuth communicated by the Ordnance Survey Office, namely, 9° 4′ 8″ from the centre of the circle, which being corrected by + 4′ 12″ for the position of station *a*, is increased to 9° 8′ 20″.

Hence from the point of observation *a* 9° 8′ 20″ has been adopted as the azimuth of Salisbury Spire.

We thus get the following absolute values of the principal azimuths from the point *a* :

Highest point of Friar's Heel ....   239° 47′ 25″
                                             − 9   8   20

                                             230   39   5
                                  or N.   50   39   5 E.

Middle of opening in N.E. trilithon......   238   51   10
                                             − 9   8   20

                                             229   42   50
                                  or N.   49   42   50 E.

The difference of $8\frac{1}{2}′$ between this and the assumed axis 49° 34′ 18″ is so slight, that considering the indirect method which has necessarily been employed in determining the axis of the temple from the position

of the leaning stone, and the want of verticality, parallelism, and straightness of the inner surfaces of the opening in the N.E. trilithon, we are justified in adopting the azimuth of the avenue as that of the temple.

Next, with regard to the determination of the azimuth of the avenue as indicated by the line of pegs to which reference is made in the body of the paper. The small angle between the nearest pegs A and B (which are supposed to be parallel to the axis of the avenue), observed from station *a*, was measured, and the corresponding calculated correction was applied to the ascertained true bearing of the more distant peg B.

Thus

True bearing of peg B = ...	238°	35′	0″
Calculated correction to peg A = ...	0	12	8
True bearing of line AB	238	47	8
Bearing of Salisbury Spire	189	8	20

True bearing of a line parallel to the
  axis of near part of avenue ....N. 49 38 48 E.

The mean of the three independent determinations by another observer was 49° 39′ 6″.

The calculated bearing of the more distant part of the axis of the avenue determined in the same manner by observations from station *b* is 49° 32′ 54″. The mean of the two, namely, 49° 35′ 51″, justifies the adoption of the value 49° 34′ 18″ as given by the Ordnance Survey for the straight line from Stonehenge to Sidbury Hill.

(3.) *Observation of Sunrise.*—On the morning of June 25, 1901, sunrise was observed from station *a*, and a setting made as nearly as possible on the middle of the visible segment as soon as could be done after the Sun appeared.

The telescope was then set on the highest point of the Friar's Heel, and the latter was found to be 8′ 40″ south of the Sun.

Sun's declination at time of observation...	23°	25′	5″
Elevation of horizon at point of sunrise ...	0	35	48
Assuming 2′ vertical of Sun to have been visible at observation, we have apparent altitude of Sun's upper limb......	0	37	48
Refraction.............. − 27′ 27″ ⎫ Parallax .... + 0 9 ⎭	− 0	27	18
True altitude of upper limb..................	0	10	30
Sun's semi-diameter	0	15	46
True altitude of Sun's centre	− 0	5	16

From this it results that the true azimuth
   of the Sun at the time of observation = N. 50° 30′ 54″ E.
  And since azimuth of Friar's Heel ...... =   50   39   5

2′ of sunrise should be N. of Friar's Heel   0   8 11
Observed difference of azimuth .... =   0   8 40

     Observed—calculated .... =   0   0 29

The observation thus agrees with calculation, if we suppose about 2′ of the Sun's limb to have been above the horizon when it was made, and therefore substantially confirms the azimuth above given of the Friar's Heel and generally the data adopted.

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" The pear-shaped Figure of Equilibrium of a Rotating Mass of
    Liquid." By G. H. DARWIN, F.R.S., Plumian Professor and
    Fellow of Trinity College, Cambridge. Received October 21,
    1901.

(Abstract.)

This is the sequel to a paper on " Ellipsoidal Harmonic Analysis," presented to the Royal Society in June, 1901.

Rigorous expressions for the harmonics of the third degree may be found by the methods of that paper, and the processes are carried out here. The functions of the second kind are also found, and are expressed in elliptic integrals.

So much of the results of M. Poincaré's celebrated memoir[*] on rotating liquid as relates to the immediate object in view is re-investigated, with a notation adapted for the use of the harmonics already determined. The general expressions for the coefficients of stability having been found, those for the seven coefficients corresponding to the harmonics of the third degree, as applicable to the Jacobian ellipsoids, are reduced to elliptic integrals.

The principal properties of these coefficients, as established by M. Poincaré, are enumerated. He has shown that the ellipsoid can bifurcate only into figures defined by zonal harmonics with reference to the longest axis of the Jacobian ellipsoid; that it must do so for all degrees; and that the first bifurcation occurs with the third zonal harmonic.

A numerical result given in the paper seems to indicate that as the ellipsoid lengthens, it becomes more stable as regards deformations of the third degree and of higher orders, and less stable as regards the lower orders of the same degree.

FIG. 2.—The stake placed on the fallen stone indicates the axis shown on Fig. 1, in relation to the leaning stone (on the left), and the centre of the S.E. trilithon.