ARTICLE 31

The Musical Nature of the Mereon System

by

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Abstract

The intrinsic, perpetually cyclic nature of the Mereon System is demonstrated by correlating it with the eight Church musical modes, which constitute a cyclic, holistic system. These musical scales, which start and finish with the D scale, comprise 48 notes between the tonic and octave — a pattern of 50 notes that is represented by the 50 corners of the first (6+6) regular polygons of the inner Tree of Life. They comprise 14 different notes (eight with tone ratios of the Pythagorean scale, six with non-Pythagorean tone ratios). The eight scales have 168 rising intervals and 168 falling intervals between notes above the tonic. These intervals are symbolised by the (168+168) yods in the first (6+6) polygons other than their corners. These are the musical and Tree of Life counterparts of the 168 automorphisms and 168 antiautomorphisms of the Klein Quartic. Its symmetry group PSL(2,7) is isomorphic to the symmetry group SL(3,2) of the Fano Plane representing the multiplication of octonions. Their group of automorphisms is the exceptional group G_2. Its seven pairs of roots correlate with the seven pairs of notes in the seven musical scales, explaining why the correspondence exists: Pythagorean music and physics based upon octonions share the same principles. The 48 'beams of energy' emitted from the 48 vertices of the 144 Polyhedron in the Mereon System express the 48 basic degrees of freedom manifested by a holistic system. In the context of the musical scales, these are notes, the eight bundles of six rays representing the eight musical scales, each with six notes between the tonic and octave. The icosahedron with 12 B vertices in the 120 Polyhedron represents the 12 different notes between the tonic and octave of these scales. Eight of them consist of Pythagorean notes and their complements. The eight bundles of six rays converge as eight single rays to their corresponding vertices. The 216 edges of the 144 Polyhedron represent the 216 intervals other than octaves between the notes of the eight Church musical modes. The 180 edges of the 120 Polyhedron represent the 180 intervals between their notes other than the octave. The 13 rising and 13 falling intervals between the tonic and the other types of notes in the Church modes are the musical counterparts of the 13 Catalan solids and their duals. Just as the 120 Polyhedron has 26 sheets of B & C vertices and 33 sheets of A, B & C vertices, so it is the 26th member of the family of Archimedean and Catalan solids and the 33rd stage in the development of polyhedra. The 120 Polyhedron contains 1680 vertices, edges & triangles, where 1680 is the number of yods in the lowest 33 Trees of Life constructed from tetractyses. The 28 polyhedra defined by its 62 vertices have 3360 hexagonal yods in their faces. This is the number of yods needed to construct the inner Tree of Life from 2nd-order tetractyses. It is also the number of helical turns in one revolution of the ten whorls of the 'UPA' described by Annie Besant & C.W. Leadbeater, showing how the 120 Polyhedron embodies geometrically the structural parameter of this object, identified in earlier work by the author as the $E_8 \times E_8$ heterotic superstring constituent of up and down quarks.

1. The Ancient Musical Scales/Church Modes

Historically speaking, musical scales were always divided into eight notes because the ancient Greeks regarded them as composed of two tetrachords (sets of four notes). If the pitch, or ‘tone ratio,’ of the starting note (‘tonic’) of a scale is given the value of 1, the eighth note of the scale (‘octave’) has a tone ratio of 2, that is, it has twice the frequency of the tonic and is the tonic of the next higher set of eight notes. The arithmetic mean of these two frequencies is (1+2)/2 = 3/2. This is the tone ratio of the ‘perfect fifth,’ so-called because it is the fifth note in the ascending scale, counting from the tonic. The musical scale based entirely on octaves and fifths is called the ‘diatonic scale.’ The tone ratios of the eight notes making up an octave of this scale are:

1	9/8	$(9/8)^2$	4/3	3/2	27/16	243/128	2

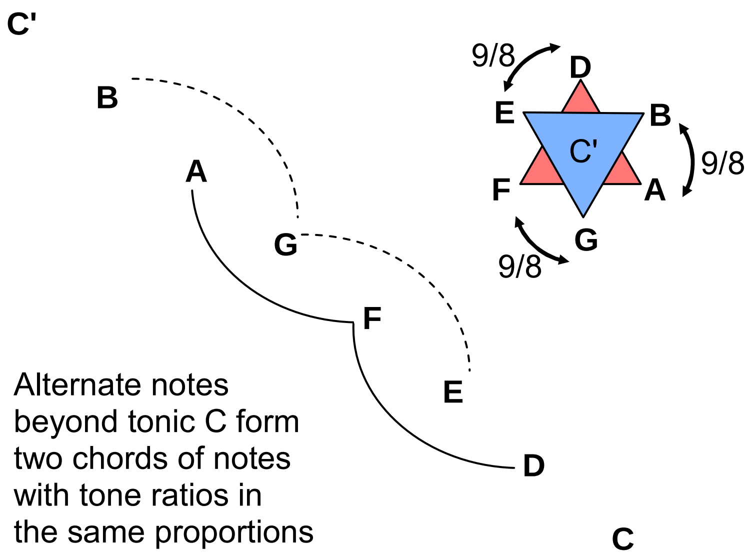
This diatonic scale is also called the ‘Pythagorean scale' because Pythagoras is generally thought to have discovered its mathematical basis. It comprises five tone intervals (T) of 9/8 and two intervals (L) of 256/243, called in Greek the *leimma*, or ‘left over,’ which corresponds to the modern semi-tone, although slightly less than it. The interval pattern of the Pythagorean scale is:

tone–tone–leimma–tone–tone–tone–leimma.
T T L T T T L

In the C scale, the tonic is labelled ‘C’ and subsequent notes in the scale are labelled D, E, F, G, A & B, the octave being written as C'. Their tone ratios are:

1	9/8	$(9/8)^2$	4/3	3/2	27/16	243/128	2
C	D	E	F	G	A	B	C'

As proved in Article 14,[i] the six notes above the tonic of the C scale can form only two triplets of notes with the same proportions of their tone ratios. They are (D, F, A) and (E, G, B), corresponding members of which are separated by a tone interval (Fig. 1).



Arrows connect corresponding notes a tone interval (9/8) apart

Figure 1. The pairs of notes (D, E), (F, G) and (A, B) have the same relative tone interval of 9/8. Note E corresponds to note D, G corresponds to F and B corresponds to A.

The seven notes D–C' above the tonic of the C scale therefore comprise three pairs of notes (D, E), (F, G) & (A, B) and the octave C' as well as a (3:3:1) pattern. The two triplets (D, F, A) and (E, G, B) correspond in the Tree of Life (Fig. 2) to the two triads of Sephiroth of Construction: Chesed-Geburah-Tiphareth and Netzach-Hod-Yesod, whilst the last note of the C scale, the octave C', corresponds to Malkuth, the last Sephirah of Construction, which completes the emanation of the Tree of Life. The three pairs of notes (D, E), (F, G) & (A, B) separated by a tone interval correspond to the pairs of Sephiroth of Construction on the three pillars of the Tree of Life. This parallelism suggests that the musical scales, both collectively and in their mathematically perfect version — the Pythagorean scale — conform to the Tree of Life, the Kabbalistic representation of 'Heavenly Man.' This article will add to the evidence presented in

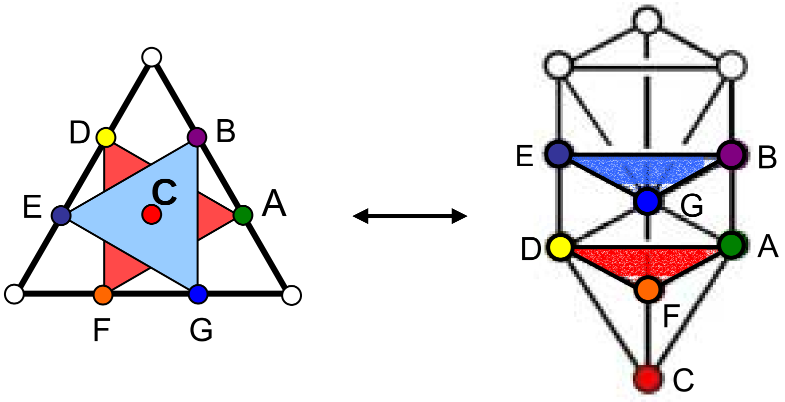


Figure 2. Equivalence between the Tree of Life and the tetractys allows the octave and the three pairs of notes in the Pythagorean scale separated by a tone interval to be assigned to the seven Sephiroth of Construction and to the seven hexagonal yods.

Article 14 for this bold assertion by showing that a remarkable similarity exists between the intervallic composition of the musical scales and the disdyakis triacontahedron, known to researchers in the Mereon Project as the '120 Polyhedron' and proven in

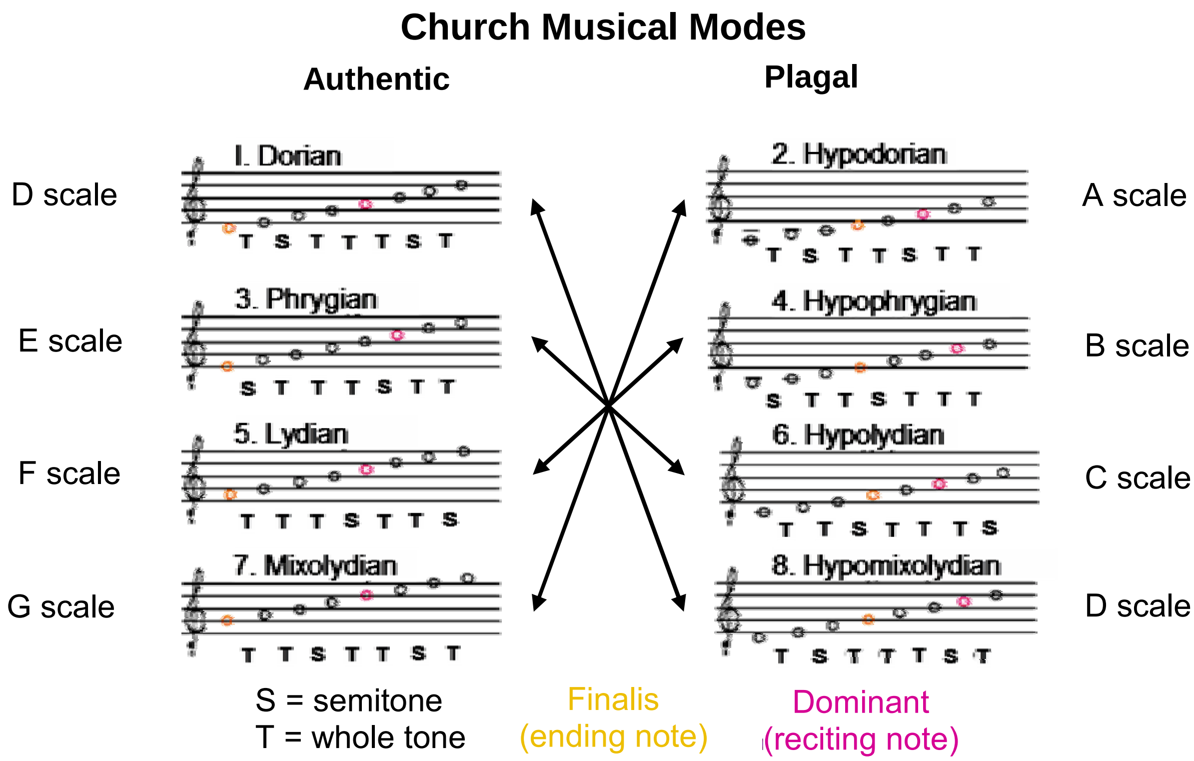


Figure 3. The pattern of intervals of each Authentic Mode is the reverse, or mirror image, of a Plagal Mode linked by an arrow.

Articles 22–30 to be the polyhedral representation of what the author has called the "inner Tree of Life" (to be discussed later).

Many modern musical scholars hold the view that the ancient Greek musical modes, such as the Dorian and Phrygian modes, were not different octave species but different keys of the same scale. Article 16 refuted this belief with two arguments:

1. the ancient Greeks would not have used two different musical terms — ‘harmonia’ and ‘tonos’ — to mean the same thing, namely, key, instead of scale and key;
2. the Pythagorean mathematician Nichomachus stated[ii] (quoting Pythagoras and Plato) that the ancient Egyptians ascribed 28 sounds to the universe, indicating that they were aware of the seven octave species, which have 28 notes with tone ratios belonging to the Pythagorean scale. As it is known[iii] that many musicians of ancient

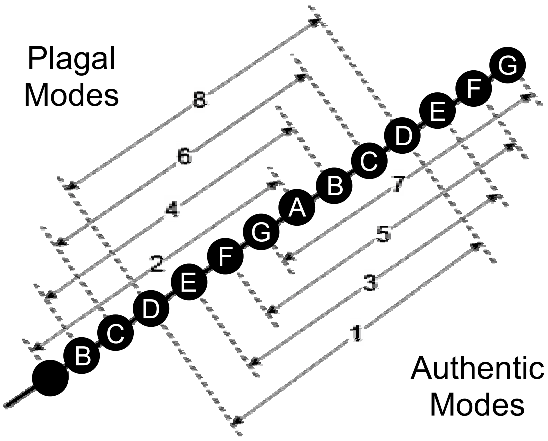


Figure 4. The four Authentic Modes and the four Plagal Modes of Church plainsong.

Greece, as well as learned men like Pythagoras and Plato, studied music with the Egyptians, it is highly unlikely that they neither learnt of the existence of the seven octave species nor played music in various modes based upon them.

The musical modes that have served as the basis of plainsong in the Roman Catholic Church originated not from the ancient Greek modes described by Plato and Aristotle but from the Byzantine octoechoi, which were four pairs of melodic motives that expressed different feelings, or *ethos*. The church modes consist of four ‘Authentic Modes’ (so-called because they were given the ancient Greek names of Dorian, Phrygian, Lydian and Mixolydian) and four ‘Plagal Modes’ (Fig. 3). The latter are separated from the former by a perfect fourth (Fig. 4). By dividing a circle into seven arcs representing successive intervals of the Pythagorean scale and selecting consecutive sequences of seven intervals as each scale, it is readily seen that there can be only seven different musical scales because there are seven intervals between successive notes in a scale and so the eighth sequence merely repeats the first one:

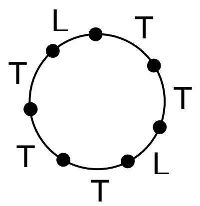


Figure 5. The circle of five tone intervals (T) and two leimmas (L).

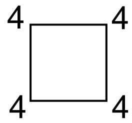
Hypolydian	C' scale								T	T	L	T	T	T	L
Hypophrygian	B scale							L	T	T	L	T	T	T	
Hypodorian	A scale						T	L	T	T	L	T	T		
Mixolydian	G scale					T	T	L	T	T	L	T			
Lydian	F scale				T	T	T	L	T	T	L				
Phrygian	E scale			L	T	T	T	L	T	T					
Dorian	D scale		T	L	T	T	T	L	T						
Hypolydian	C scale	T	T	L	T	T	T	L							

Note: C D E F G A B C' D' E' F' G' A' B' C''
Tone interval: T T L T T T L T T L T T T L

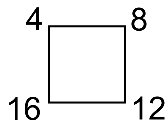
Moreover, as all intervals are selected, their sequences do not depend upon the starting point on the circle. The eight Church modes are labelled 1–8 (see Fig. 4) and start with the D scale. It is called the ‘Dorian mode,’ although whether it was what the ancient Greeks understand as this mode is unknown. It turns out that the numbering of modes is not arbitrary because, starting with the tonic of the D scale, the last of the different scales is the Pythagorean scale (C scale), the only scale with *all* its notes having tone ratios with Pythagorean values. As it displays the most mathematical harmonies, It is intuitively natural that the last of any sequence of these scales should be the Pythagorean scale. The D scale, however, is the only scale whose pattern of intervals is symmetric. Fig. 3 indicates that it is the same as its mirror image and that Modes 3, 5 & 7 are the mirror image of Modes 6, 4 & 2. This unique property of the D scale gives it a pivotal role in establishing the connection between the musical scales and the Mereon System, as will be explained later. If each rising interval in a scale is replaced by its falling counterpart, i.e., $T \rightarrow T^{-1}$ and $L \rightarrow L^{-1}$, an ascending scale, e.g., TLTTLTT, becomes $T^{-1}L^{-1}T^{-1}T^{-1}L^{-1}T^{-1}T^{-1}$, which is the descending version of the scale TTLTTLT. Inverting each interval of a scale creates another scale whose pattern of intervals is its mirror image. Only the D scale is unchanged when its intervals are inverted.

The eight C–C' scales shown above (or eight consecutive scales starting with any note) span 15 successive notes C–C'' and 14 intervals, showing how the Godname Yah with number value 15 prescribes the range of notes needed to define the complete set of four Authentic Modes and four Plagal Modes.

Their (8×7=56) intervals comprise 16 leimmas (L) and 40 whole tones (T), where

$16 = 4^2 =$ 

and

$40 = 4\times10 = 4(1+2+3+4) =$ 

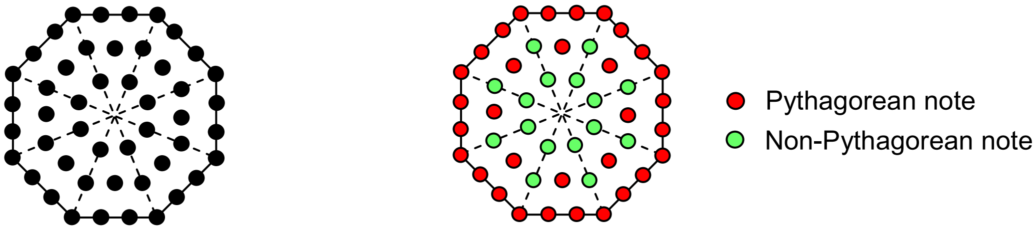
This demonstrates how the Tetrad (4) symbolised by the square expresses the tone interval and leimma composition of the eight scales. Notice that, as each scale has the same number of tone intervals and the same number of leimmas, this composition is the same whatever the scale chosen to start the sequence, e.g., 40 whole tones and 16 leimmas are spanned by the eight scales that start and end with the D-scale.

Table 1. The tone ratios of the eight scales C–C'.

Musical scale	**Tone ratio**							
C' scale	1	9/8	81/64	4/3	3/2	27/16	243/128	2
B scale	1	256/243	32/27	4/3	1024/729	128/81	16/9	2
A scale	1	9/8	32/27	4/3	3/2	128/81	16/9	2
G scale	1	9/8	81/64	4/3	3/2	27/16	16/9	2
F scale	1	9/8	81/64	729/512	3/2	27/16	243/128	2
E scale	1	256/243	32/27	4/3	3/2	128/81	16/9	2
D scale	1	9/8	32/27	4/3	3/2	27/16	16/9	2
C scale	1	9/8	81/64	4/3	3/2	27/16	243/128	2

(White cells denote Pythagorean notes; red cells denote non-Pythagorean notes).

The eight C–C' musical scales have 48 notes with Pythagorean tone ratios (white cells in Table 1) and 16 notes with non-Pythagorean tone ratios (red cells). Table 2 lists their numbers in each scale. These notes can be represented by an octagon whose sectors



The 48 (●) yods surrounding the centre of an octagon divided into 8 tetractyses symbolize the 48 notes between the tonic and octave in the eight Church musical modes. Each tetractys denotes a mode, the 6 yods per tetractys denoting the 6 notes between the tonic and octave.

The 32 (●) yods symbolize the 32 Pythagorean notes between the tonic and the octave.

The 16 (●) yods symbolize the 16 non-Pythagorean notes between the tonic and the octave.

Figure 6. The Pythagorean division of the octagon generates the Pythagorean and non-Pythagorean notes of the eight C–C' musical scales.

are tetractyses (Fig. 6) because 48 yods surround its centre. The Pythagorean notes consist of eight tonics (1), eight octaves (2) and 32 notes between the tonic and octave with intermediate tone ratios. They comprise 26 notes of the seven scales C–B and the

Table 2. Number of Pythagorean/non-Pythagorean tone ratios in the musical scales.

Musical scale	**Number of Pythagorean tone ratios (white cells)**	**Number of non-Pythagorean tone ratios (red cells)**
C' scale	8	0
B scale	3	5
A scale	5	3
G scale	7	1
F scale	7	1
E scale	4	4
D scale	6	2
C scale	8	0

Total = 48

six notes of the repeated C' scale. The number of independent (non-repeated) Pythagorean notes is 26, which is therefore prescribed by the Godname Yahweh (YHVH), the values of the letters of the Godname denoting the numbers of tones with tone ratios of two possible values:

$$26 = \underset{\substack{5\\H}}{5(9/8)} + \underset{\substack{6\\V}}{6(3/2)} + \underbrace{3(81/64) + 2(243/128)}_{\substack{5\\H}} + \underbrace{6(4/3) + 4(27/16)}_{\substack{10\\Y}}$$

The letter values of Yahweh also define the numbers of Pythagorean notes between the tonic and octave of the seven independent scales:

F scale	5	——	H
C scale	6	——	V
G scale	5	——	H
D scale	4	}	
A scale	3	} 10	Y
E scale	2	}	
B scale	1	}	

Total = 26 = YHVH.

Including their octaves, there are 33 Pythagorean notes above the tonic of the seven scales. The numbers 26 and 33 measure, respectively, the number of combinations and the number of permutations of 10 objects arranged in a tetractys:

	Number of combinations	Number of permutations
a	$2^1 - 1 = 1$	$1! = 1$
b c	$2^2 - 1 = 3$	$2! = 2$
d e f	$2^3 - 1 = 7$	$3! = 6$
g h i j	$2^4 - 1 = 15$	$4! = 24$
	Total = 26	Total = 33

Indeed, Table 1 indicates that those types of Pythagorean and non-Pythagorean tones above the tonic that occur more than once are *themselves* 10 in number:

9/8
81/64 4/3
3/2 27/16 243/128
256/243 32/27 128/81 16/9

The seven distinct octave species have notes above the tonic with 13 different tone ratios listed below in increasing order of magnitude:

256/243 9/8 32/27 81/64 4/3 1024/729 729/512 3/2 128/81 27/16 16/9 243/128 2

(numbers in red denote notes not belonging to the Pythagorean scale (C-scale)). Including the tonic, they consist of seven *pairs* that, as intervals, span an octave. In order of increasing magnitude, they are:

1.	1	2	1×2 = 2
2.	256/243	243/128	256/243×243/128 = 2
3.	9/8	16/9	9/8×16/9 = 2
4.	32/27	27/16	32/27×27/16 = 2
5.	81/64	128/81	81/64×128/81 = 2
6.	4/3	3/2	4/3×3/2 = 2
7.	1024/729	729/512	1024/729×729/512 = 2

The 14 notes consist of two pairs of Pythagorean notes, four pairs of a Pythagorean and a non-Pythagorean note and one pair of non-Pythagorean notes. The significance of this structure will emerge later.

As demonstrated in Article 16[iv], every one of the 28 intervals between two notes belonging to each of the seven different musical scales is equal to the tone ratio of one of the 13 different notes above the tonic that can be found in these scales. In other words, all their intervals are just repetitions of this basic set.

Table 3 lists the tone ratios of the eight D–D' scales. They comprise 30 Pythagorean notes and 18 non-Pythagorean notes. Table 4 compares the number of notes in the

Table 3. The tone ratios of the notes of the eight D–D' musical scales.

Musical scale	Tone ratio							
D' scale	1	9/8	32/27	4/3	3/2	27/16	16/9	2
C scale	1	9/8	81/64	4/3	3/2	27/16	243/128	2
B scale	1	256/243	32/27	4/3	1024/729	128/81	16/9	2
A scale	1	9/8	32/27	4/3	3/2	128/81	16/9	2
G scale	1	9/8	81/64	4/3	3/2	27/16	16/9	2
F scale	1	9/8	81/64	729/512	3/2	27/16	243/128	2
E scale	1	256/243	32/27	4/3	3/2	128/81	16/9	2
D scale	1	9/8	32/27	4/3	3/2	27/16	16/9	2

(White cells denote Pythagorean notes; red cells denote non-Pythagorean notes).

C–C' scales and the D–D' scales. The latter has the same numbers of notes and their octave complements. It is straightforward to verify that, like the C–C' scale, none of the other scales exhibit this symmetry possessed by the D scale, the Dorian musical mode. This is because it is unique among the scales in that its pattern of intervals — TLTTTLT — is its own mirror reflection.

Table 4. Numbers of different notes in the eight C–C' and the eight D–D' scales

C scale–C' scale		D scale–D' scale	
6(9/8)	5(16/9)	6(9/8)	6(16/9)
4(81/64)	3(128/81)	3(81/64)	3(128/81)
7(4/3)	7(3/2)	7(4/3)	7(3/2)
5(27/16)	4(32/27)	5(27/16)	5(32/27)
3(243/128)	2(256/243)	2(243/128)	2(256/243)
729/512	1024/729	729/512	1024/729
Total = 26	Total = 22	Total = 24	Total = 24

The tonic, octave and the 24 pairs of notes and their complements can be assigned to the 50 corners of the first (6+6) regular polygons of the inner Tree of Life (Fig. 7). Article 4 showed that this set of polygons constitute a Tree of Life pattern because the number values of the ten Godnames define its geometrical properties.[v] The two endpoints of the shared root edge denote the tonic and the octave, and a corner and its mirror image in

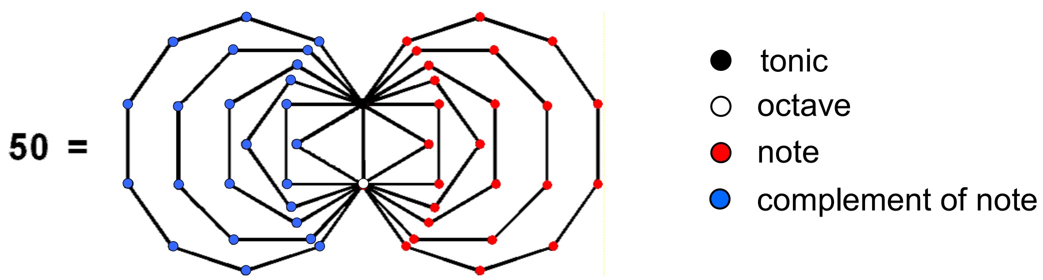
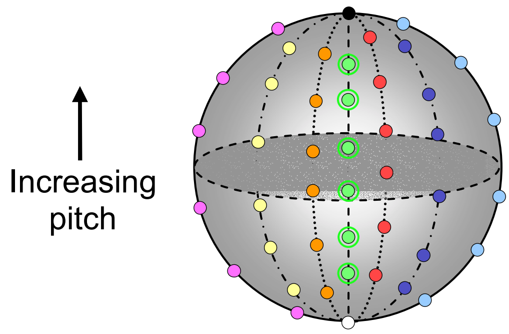


Figure 7. The first (6+6) polygons constitute a Tree of Life pattern. The endpoints of the root edge denote the tonic and octave, the 24 external corners on one side denote 24 notes of the eight D–D' scales between the tonic and octave and their mirror images denote their complementary notes.

the other similar set of polygons denote a note and its complement.

Suppose that the eight notes of a scale are denoted by points on a great circle of a sphere passing through the South Pole (tonic) and North Pole (octave) (Fig. 8). As the intervals of the E, F & G scales are the mirror images of, respectively, the C, B & A scales, a note is diametrically opposite every other note on the circle. As the pattern of



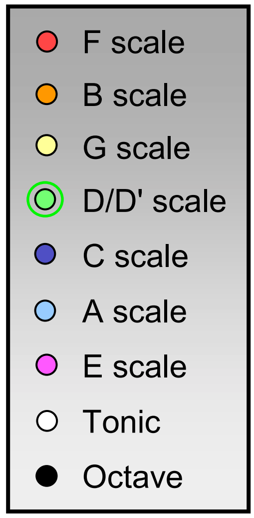


Figure 8. The spherical geometry of the 48 notes between the tonic and the octave of the eight D–D' scales. Each point on a great circle denoting a note is diametrically opposite another point on the same circle because to each Authentic Mode there corresponds a Plagal Mode whose pattern of intervals is its mirror image. The notes of the D and D' scale lie along the vertical axis of the sphere.

intervals of the D scale is its own mirror image, its notes can be represented by points on the central vertical axis of the sphere, each point being the same distance from the centre as its mirror image. This means that the eight scales D–D' can be represented by (2+8×6 = 50) notes on the sphere and its axis, 12 of them lying on the latter as six different pairs of identical notes assigned to the six points between the North and South Poles. It demonstrates how the Godname Elohim with number value 50 prescribes this sequence of scales. It also demonstrates the power of the Tetrad (4) to express properties of holistic systems such as the seven musical scales, for the number of points symbolising their notes is 44. The Divine Name Eloha assigned to Geburah with number value 36 prescribes the (6×6=36) notes between the tonic and the octave of the E–C scales that lie on the surface of the sphere and surround the 14 notes on its axis.

The 42 notes between the tonics and octaves of the seven different musical scales consist of the following pairs:

A = 5	×5 —	5(9/8)	5(16/9)	9/8×16/9 = 2
I = 10	×10	6(4/3)	6(3/2)	4/3×3/2 = 2
		4(27/16)	4(32/27)	27/16×32/27 = 2
H = 5	×5	3(81/64)	3(128/81)	81/64×128/81 = 2
		2(243/128)	2(256/243)	243/128×256/243 = 2
A = 1	×1 —	729/512	1024/729	729/512×1024/729 = 2

The Godname Ehyeh (AHIH) with number value 21 prescribes the 21 notes and their 21 complements, its letter values specifying different combinations of notes. As pointed out earlier, the Godname Yahweh prescribes the 26 Pythagorean notes in this set of 42 notes.

The 50 notes of the eight scales D–D' represented in Fig. 8 consist of seven sets of notes and their seven sets of complementary notes:

25	1(1)	1(2)	25
	6(9/8)	6(16/9)	
	3(81/64)	3(128/81)	
	7(4/3)	7(3/2)	
	5(27/16)	5(32/27)	
	2(243/128)	2(256/243)	
	729/512	1024/729	

There have to be at least seven pairs of complementary notes because there are seven notes in a scale below the octave. The fact, however, that the eight scales contain *only* seven pairs is non-trivial, because, although they comprise 14 different notes when the tonic is included, there is no obvious reason why they should group into seven pairs of complementary notes. It becomes obvious only when all possible intervals between the notes in each scale are calculated and found to consist of the *same* set of tone ratios of their notes.[vi] This is because every interval has its complementary interval and therefore 14 different intervals must be grouped into seven pairs. As there is a note in the seven scales whose tone ratio equals one of these intervals, all their types of notes can be grouped into seven pairs.

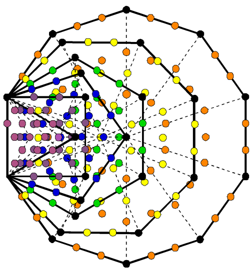


Figure 9. Associated with each set of six enfolded polygons are 168 yods other than their 26 corners. They symbolise the 168 intervals between the notes of the eight D–D' scales above the tonic.

As pointed out earlier, the eight D–D' scales are the only set of eight scales that possess the same number of notes and their complementary notes. The D scale is not only uniquely symmetric in its sequence of intervals but also the only scale whose 36 intervals have the values of 26 Pythagorean tone ratios.[vii] This prescription by the Godnames Eloha with number value 36 assigned to Geburah and Yahweh with number value 26 assigned to Chokmah reflects its status in Church music as the Dorian Mode and Mode 1 from which all the other scales (ending with Mode 7, the mathematically most harmonious Pythagorean scale) can be generated by shifting one interval at a time. Furthermore, the D scale is the only one with 10 intervals with non-Pythagorean values.[viii] As the seven octave species have 66 non-Pythagorean intervals,[ix] the eight D-D' scales have (66+10=76) non-Pythagorean intervals. This is how the Godname Yahweh Elohim with number value 76 prescribes the set of eight Church musical modes: it measures all its non-Pythagorean intervals.

Excluding the interval 1 between a note and itself, the seven different scales D–C have 130 Pythagorean intervals.[x] Excluding their seven octaves, they have 123 Pythagorean intervals. The eight notes of the D scale have 26 Pythagorean intervals. They include eight ‘1’s and one 2, that is, (26–8–1=17) such intervals between 1 and 2. The number of intervals between 1 and 2 in the eight scales D–D' = 123 +17 + 76 = 216, which is the number value of Geburah. 48 of these are their notes between their tonic and octave, *leaving 168 intervals between the notes of the eight D–D' scales above the tonic*. This is the number of Cholem Yesodeth, Mundane Chakra of Malkuth. It is the superstring structural parameter discussed in many previous articles. They are symbolised in the first six enfolded, regular polygons as the 168 yods within them other than their 26 corners (Fig. 9), which, as we saw in Fig. 7, symbolise notes of the eight scales. They are symbolised in the spherical representation of the eight D–D' scales as the 168 intervals between the seven notes above the tonic in each scale.

We found in Article 16[xi] that the seven scales D–C have 84 rising, Pythagorean intervals that are repetitions of the six Pythagorean notes between the tonic and the

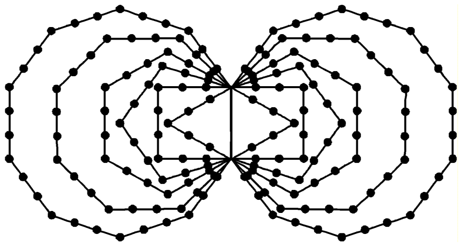


Figure 10. 168 yods lie along the edges of the first (6+6) polygons outside their shared edge. They symbolise the 84 Pythagorean rising intervals and their 84 falling counterparts (reciprocals) that are repetitions of the Pythagorean intervals in the seven musical scales.

octave. They have 84 falling intervals as their counterparts, making 168 in total. They are symbolised by the 84 yods along the edges of the first six enfolded polygons and their 84 mirror images in the other set of polygons (Fig. 10). Mirror symmetry of corresponding corners denotes the difference between a rising and a falling interval.

In terms of notes of the C scale, the composition of the 84 rising, Pythagorean intervals is

D	E	F	G	A
28	11	23	17	5

Therefore, there are (2×(28+11) = 78) rising and falling seconds and thirds and 2×(23+17+5) = 90 rising and falling fourths, fifths and sixths (the seventh never appears more than once in a scale). 78 is the number value of the Kabbalistic word 'Cholem' and 90 is the number value of the word 'Yesodeth.'

The seven different scales have 12 basic notes with different tone ratios between 1 and 2. All the intervals between notes in these scales have values found in this set. Hence, there are in the eight scales (216–12=204) repetitions of these notes and intervals. Including the tonic and the octave, the 50 notes of these scales arranged as in Fig. 8 and starting with the same note have 206 notes/intervals other than the basic set. Of these, 136 are Pythagorean notes/intervals and 70 are non-Pythagorean notes/intervals. It was shown in Article 29[xii] that the triakis tetrahedron, the simplest Catalan solid, has 136 vertices, edges and triangles surrounding its centre when its internal triangles are regarded as single tetractyses and its faces are divided into three tetractyses. There are as many geometrical elements surrounding this polyhedron as there are Pythagorean notes and intervals in the eight scales in addition to its basic set. The reason for this correspondence is that both the eight D–D' musical scales and the triakis tetrahedron conform to the archetypal pattern of the Tree of Life, as proved for the latter in Article 29, and therefore must display properties that numerically correspond to each other.

Ten Pythagorean notes (tonic, octave & two identical sets of the four notes E, G, A & B of the D scale) lie on the central axis of the spherical representation of the eight D–D' scales. Hence, the 206 intervals consist of (136–10=126) Pythagorean intervals of the six scales other than the two D scales and 80 intervals made up of 70 non-Pythagorean intervals and 10 Pythagorean notes arranged along the axis. As, according to Table 2 of Article 29, the triakis tetrahedron has 132 geometrical elements surrounding its axis, six of which are its vertices, 126 other vertices, edges and triangles surround its axis.

Once again, correspondence appears between different features of each system.

The counterpart in the triakis tetrahedron of the fact discussed above that 168 intervals exist between the notes above the tonic of the eight D–D' scales is that, when its internal triangles are divided into three tetractyses, it has 168 geometrical elements surrounding its axis. As the Pythagoreans taught, "music is geometry."

As commented upon earlier, the 14 different notes of the seven musical scales consist of seven pairs of notes and their complements:

1.	1	2	1×2 = 2
2.	256/243	243/128	256/243×243/128 = 2
3.	9/8	16/9	9/8×16/9 = 2
4.	32/27	27/16	32/27×27/16 = 2
5.	81/64	128/81	81/64×128/81 = 2
6.	4/3	3/2	4/3×3/2 = 2
7.	1024/729	729/512	1024/729×729/512 = 2

(the ordering is in terms of increasing size of the first member of each pair). Let us write, firstly, the octave 2 as '**1**' (using boldface to remind ourselves that **1** is not the

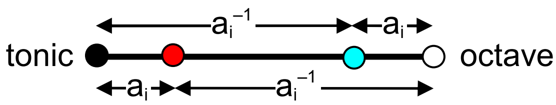
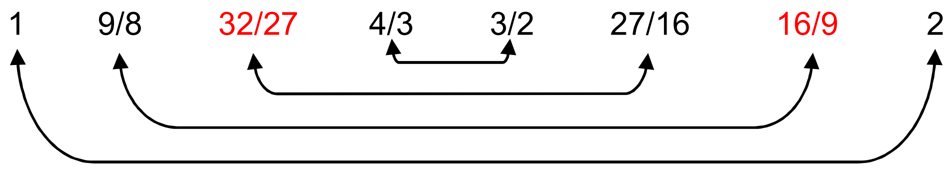


Figure 11. A note (●) and its complement (●) are equidistant from the ends of a musical scale.

number 1), secondly, each note of one set as a_i (i = 1–7) and, thirdly, its complement as a_i^{-1}. The product of an interval and its complement may then be expressed as

$$a_i a_i^{-1} = \mathbf{1}.$$

The complement of a given note is as far below the octave as that note is above the tonic — they are equally spaced from each end of the scale (Fig. 11). The complement can be regarded as the inverse of a note in the sense that their intervals span the octave. A rising musical interval a = n (n>1) has a complement with interval 2/n, *not* 1/n, which is the value of the falling interval. Every member of the set of 14 notes has a complement, or inverse, which is also a member of that set. As the D scale is unique in having a pattern of intervals that is identical to its mirror image, this means that it alone amongst the scales is made up of four pairs of notes and their complements:



In terms of the Pythagorean tone T and leimma L, the seven pairs of notes making up the musical scales have the form:

1.	1	T^5L^2 (= 2)
2.	L	T^5L
3.	T	T^4L^2
4.	TL	T^4L
5.	T^2	T^3L^2
6.	T^2L	T^3L
7.	T^2L^2	T^3

Eight notes are Pythagorean and six notes (written above in red) are non-Pythagorean.

By arranging the intervals between successive notes of the Pythagorean scale on a circle (Fig. 12), all possible scales are the sequences of seven intervals generated by starting at successive intervals until all of them have been selected. Their notes and

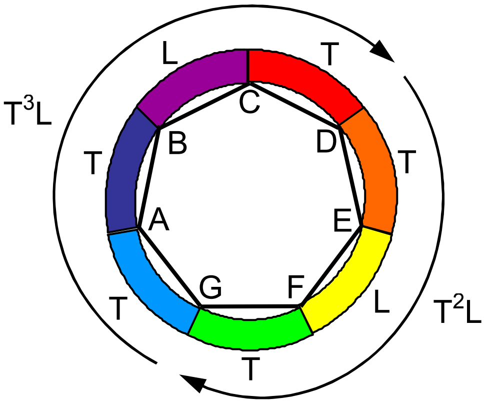
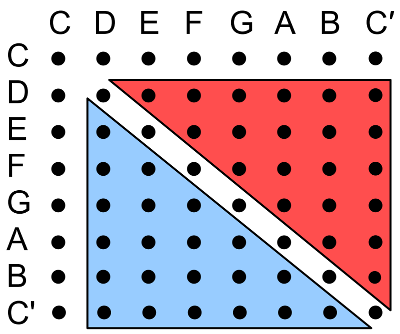


Figure 12. The seven musical scales are the seven possible consecutive sequences of intervals arranged on a circle. The 13 different intervals between their notes are all possible, different combinations of *consecutive* intervals.

intervals are all possible different combinations of *consecutive* intervals. They have the forms listed above. The complement of any note represented by an arc of the circle in Fig. 12 is that created by the opposing arc that completes the circular pattern of intervals. Just as every colour has its complementary colour, so, too, do the notes.

The eight notes of a musical scale have $\binom{8}{2}$ = 28 intervals, of which seven are those of the seven notes above the tonic. There are 21 intervals between these notes (Fig. 13). The eight D–D' scales have (21×8 = 168) such rising intervals between their (8×7 = 56) notes. Similarly, there are 168 falling intervals. They are symbolised in the inner Tree of



Each of the notes of the eight D–D' musical scales (the C scale is shown here) has 21 rising intervals (denoted by dots in the red triangle) between notes other than their tonics and 21 falling intervals (denoted by dots in the blue triangle). The eight scales have (8×21=168) rising intervals and 168 falling intervals.

Figure 13. The 56 notes of the eight D–D' musical scales above their tonics have 168 rising intervals and 168 falling intervals.

Life by the 168 yods in each set of the first six enfolded polygons (Fig. 14), whose corners denote the 50 notes of the eight D–D' scales arranged on the surface of the sphere and along its axis, as discussed earlier and shown in Fig. 8.

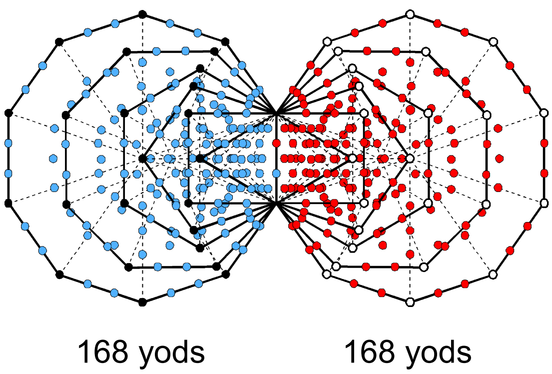


Figure 14. The (6+6) enfolded polygons have 50 corners symbolising the 50 notes of the eight musical scales arranged on a sphere. Associated with each set of six polygons are 168 yods other than their corners. One set of yods symbolises the 168 rising intervals between their notes. The other set symbolises their 168 falling intervals.

This remarkable analogy between the intervallic composition of the eight church musical modes and the inner Tree of Life would remain valid whatever set of eight scales was considered. The eight scales starting with the D scale have been chosen because: 1. it was the Dorian mode that was, historically speaking, made the first of the Authentic Modes, and 2. starting with the D scale means that the Pythagorean scale is the last different scale that can be generated from it — a sequence that is natural in view of its special significance as the most mathematically harmonious scale and its historical importance as the basis of Western musical theory. The interval composition of the scales does, of course, depend upon which set of eight scales is considered.

The seven pairs of notes and their complements consist of two pairs of Pythagorean notes (tonic/octave & perfect fourth/fifth) and five pairs of notes that each includes at least one non-Pythagorean note. The 2:5 division is another example of the basic pattern that starts in the seven scales, each with five tone intervals T with magnitude 9/8 and two leimmas L with magnitude 256/243.

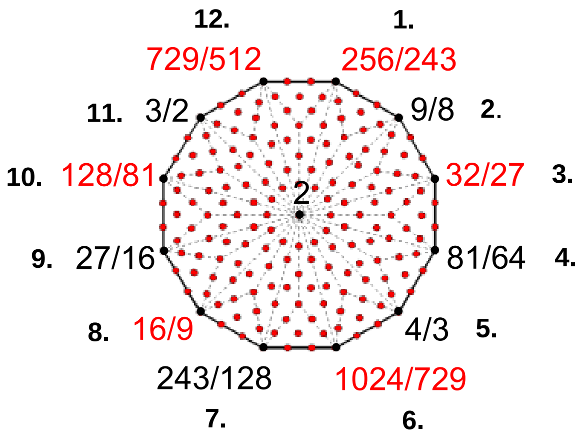


Figure 15. The 12 corners of the dodecagon symbolise the 12 different notes between the tonic and the octave of the eight D–D' scales. A note and its complementary note are diametrically opposite each other. The central yod denotes the octave and the 168 (•) yods surrounding it denote the 168 intervals between the notes above the tonic.

The last of the regular polygons in the inner Tree of Life is the dodecagon. As the tenth regular polygon, it represents the completion of the Pythagorean paradigm of wholeness symbolised by the tetractys. When its 12 sectors are each divided into three tetractyses, it contains 181 yods (Fig. 15). 13 yods are at the corners and centre of the dodecagon and 168 yods are generated by this transformation. The 12 types of musical notes between the tonic and the octave in the seven types of scales can be assigned to its corners, with the octave assigned to its centre. The 168 rising intervals between them can be assigned to the 168 yods. The 168 yods in the other dodecagon symbolise the 168 falling intervals. This demonstrates the remarkable connection between the sacred geometry of the inner Tree of Life and the musical potential of the eight scales.

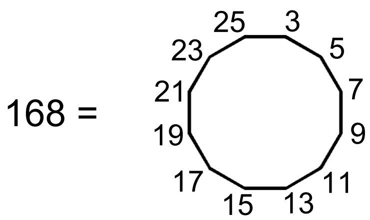


Figure 16. The sum of the first 12 odd integers after 1 is the number of intervals between the notes above the tonic of the eight D–D' musical scales.

As

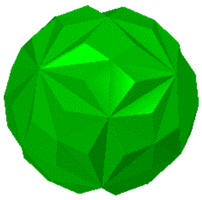
$$13^2 – 1 = 168 = 3 + 5 + \& + \ldots + 25,$$

the number 168 is not only the number of yods in a dodecagon other than its centre and corners but also the sum of the first 12 odd integers that can be assigned to its corners (Fig. 16). This demonstrates the deep arithmetic, geometric and musical connection between the numbers 12 and 168. It appears in the first four Platonic solids ending with the icosahedron with 12 vertices as the average number of yods needed to build them out of tetractyses.[xiii]

Inspection of Tables 3, 4, 5, 7, 8 & 9 in Article 16[xiv] shows that the C, E, F, G, A & B scales have (including octaves) 112 Pythagorean intervals and 56 non-Pythagorean intervals, that is, 168 intervals. Once again, the number value of Cholem Yesodeth measures the number of intervals — this time for the six scales other than the D scale and including octaves.

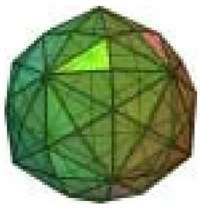
2. Connection between the 144/120 Polyhedra and the eight scales

We found earlier that the eight D–D' scales have 216 intervals other than octaves between their 64 notes. We also found that they have 12 types of notes other than octaves and 168 intervals between notes above the tonic, totalling 180 intervals other than the octave. Compare this with the fact that the 144 Polyhedron has 216 edges and the 120 Polyhedron has 180 edges (Fig. 17). In the first case, there are (8×6 + 168 = 216) intervals. The (8×6 = 48) notes between the tonic and octave of the eight scales



144 Polyhedron

216 edges → 216 intervals between tonic and octave of eight D–D' scales



120 Polyhedron

180 edges → 12 basic intervals in eight D–D' scales between tonic and octave + 168 intervals between notes above the tonic

Figure 17. The edges of the 144 & 120 Polyhedra correspond to all the possible intervals between the notes of the eight Church musical modes.

contain the basic set of 12 notes, that is, there are 36 repetitions of some of these notes, where 36 is the number value of Eloha, Godname of Geburah. These repetitions are represented by the 36 extra edges of the 144 Polyhedron. We see that the edges of the 144 Polyhedron represent all possible intervals other than octaves between the

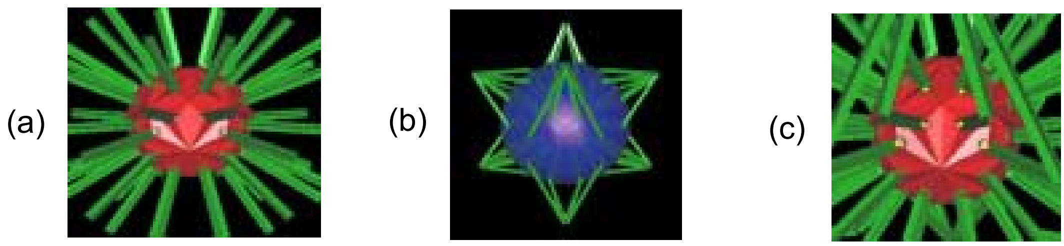


Figure 18. The 48 'beams of energy' (a) emitted from the 48 vertices of the 144 Polyhedron focus into eight bundles of six rays (b) on passing through the 'focussing sphere.' The six yellow dots in (c) mark vertices that form a bundle. (*From material provided by Lynnclaire Dennis*)

notes of the eight Church musical modes and that the edges of the 120 Polyhedron represent the basic set of 12 intervals and their 168 repetitions:

144 Polyhedron: $216 = 48 + 168.$

$\downarrow$

120 Polyhedron: $180 = 12 + 168.$

Let us now compare these details with how the 144 Polyhedron interacts with the 120 Polyhedron. According to Lynnclaire Dennis,[xv] an energy build-up within the former is released in the form of 'beams of energy' radiating out through the 48 vertices of the polyhedron. Each beam has a Y-shaped cross-section. They become bundled into eight groups of six beams. Figure 18c shows a set of six vertices (marked by yellow dots) of the 144 Polyhedron from which the six beams emerge before they become bundled on

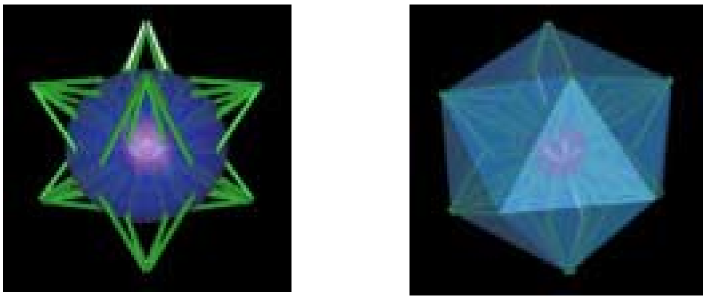
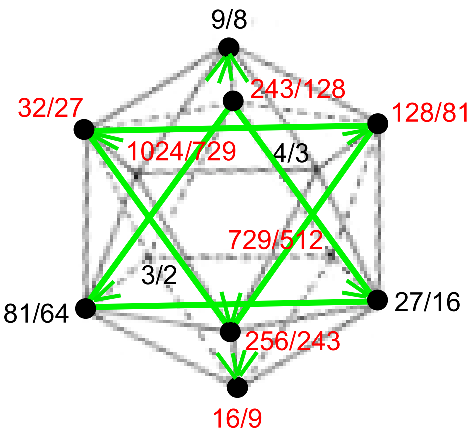
(a)  (b)

Figure 19. The eight bundles of six beams of energy (green lines) converge at eight vertices of an octahedron (a) within the icosahedron (b).
(From material provided by Lynnclaire Dennis)

passing through the focussing sphere. The 48 beams move out radially from the 144 Polyhedron in straight lines. As they pass out through the focussing sphere, they are bent but remain straight, retaining their Y-shaped cross-section. The bending occurs such that each of the eight groups of six beams converges into a different vertex of the



Green lines converge on the eight vertices (●) of the icosahedron representing the four pairs of Pythagorean notes and their non-Pythagorean complements.

Figure 20. The 12 notes between the tonic and octave of all the musical scales can be assigned to the vertices of an icosahedron. Diametrically opposite vertices denote a note n and its complement m, where nm = 2.

120 Polyhedron, each group of six beams melding into a point. These vertexes are eight of the 12 B vertices of the icosahedron (Fig. 19). The other four vertices into which beams do not focus lie in the same plane and form a great circle.

The eight bundles of six beams of energy correspond to the eight musical scales, each with six notes between the tonic and octave. Including the tonic and the octave, these scales contain eight different Pythagorean notes and six different non-Pythagorean notes. Just as these 48 notes consist of eight different Pythagorean notes, so the 48 beams of energy converge to eight vertices of the icosahedron.[1] Its 12 vertices symbolise the 12 types of notes belonging to the musical scales between the tonic and octave (Fig. 20). These notes are made up of four pairs of notes and their complements, one of which is a Pythagorean note and the other a non-Pythagorean note, one pair of Pythagorean notes (perfect fourth and perfect fifth) and one pair of non-Pythagorean notes:

8	256/243	243/128	256/243×243/128 = 2
	9/8	16/9	9/8×16/9 = 2
	32/27	27/16	32/27×27/16 = 2
	81/64	128/81	81/64×128/81 = 2
4	4/3	3/2	4/3×3/2 = 2
	1024/729	729/512	1024/729×729/512 = 2

Each diametrically opposite pair of B vertices denotes a note and its complement. The inversion symmetry of the icosahedron has its musical counterpart in the fact that the complement of any note in the seven different musical scales with tone ratio n is one with tone ratio 2/n that is also one of these notes. The eight vertices symbolising the four pairs of Pythagorean and non-Pythagorean notes indicated above are the eight vertices to which the eight bundles of beams converge. Notice that the four pairs of Pythagorean notes 9/8, 27/16, 81/64 & 243/128 are each separated by a perfect fifth. Similarly for their respective complements 16/9, 32/27, 128/81 & 256/243. The four vertices to which beams of energy do not converge signify the pair of Pythagorean notes (4/3, 3/2) & the pair of non-Pythagorean notes (1024/729, 729/512) at the *centre* of the set of 12 notes making up the seven different musical scales:

256/243 9/8 32/27 81/64 | 4/3 1024/729 729/512 3/2 | 128/81 27/16 16/9 243/128

The 30 Pythagorean notes of the eight D–D' scales between the tonic and the octave are assigned to the 30 A vertices of the 120 Polyhedron and its 18 non-Pythagorean

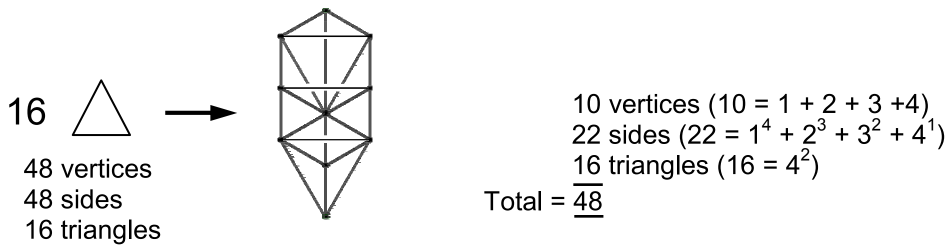


Figure 21. 16 triangles come together and join at 22 sides to create the Tree of Life with 48 vertices, edges and triangles.

notes, the tonic and the octave are assigned to the 20 C vertices of the dodecahedron. The musical basis of the 120 Polyhedron is summarised below:

Eight D–D' musical scales		120 Polyhedron
30 Pythagorean notes	→	30 A vertices
12 different notes of 7 types of musical scales	→	12 B vertices

[1] It may be relevant that an octahedron with eight faces has 48 rotational and mirror symmetries.

Tonic, octave and 18 non-Pythagorean notes	→	20 C vertices
180 intervals between notes of eight scales	→	180 edges
120 intervals between 48 notes between tonic and octave of eight scales	→	120 edges above faces of rhombic triacontahedron
12 basic intervals + 48 intervals between octave and six notes between tonic and octave	→	60 edges of rhombic triacontahedron

3. Comparison with the Tree of Life

Given that previous articles have demonstrated that the 120 Polyhedron is the polyhedral form of the inner Tree of Life, it should come as no surprise that other aspects of the Mereon system should appear in this context, as now explained. 16

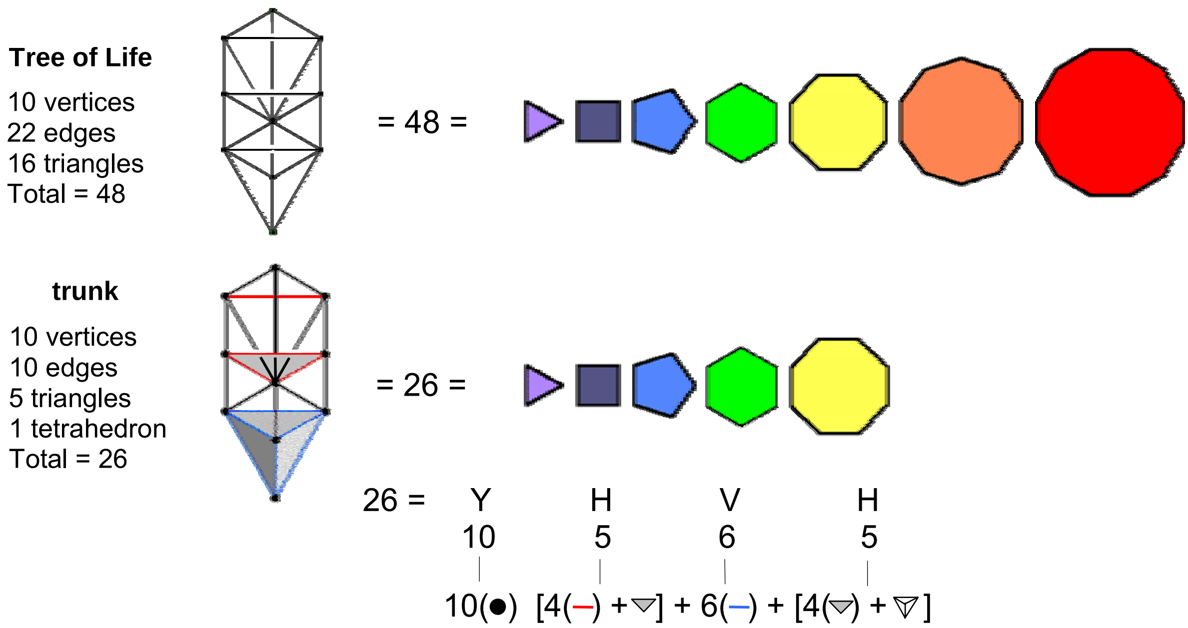


Figure 22. The outer Tree of Life is composed of 48 vertices, edges & triangles that correspond to the 48 corners of the seven regular polygons constituting its inner form. Yahweh with number value 26 prescribes the 26 geometrical elements forming the trunk of the outer tree. Its letter values denote different groups of geometrical elements. Yahweh also prescribes the first five polygons of the inner tree with 26 corners.

separate triangles combine by joining 41 of their edges to create the outer form of the Tree of Life with 10 vertices of 16 triangles with 22 edges (Fig. 22). 64 geometrical elements (38 vertices, 26 edges) disappear in this union, where 64 = 4^3. It comprises 48 geometrical elements made up of 10 vertices, 22 edges and 16 triangles. As the

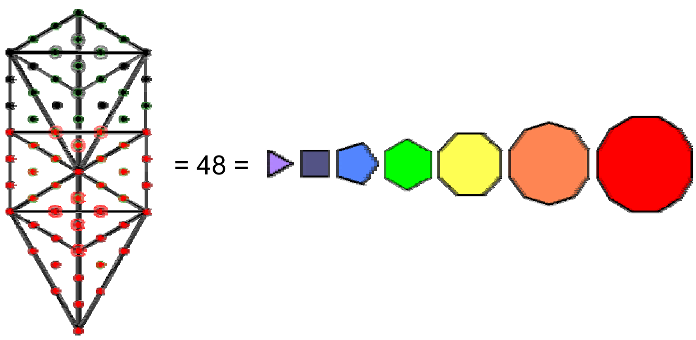


Figure 23. The seven separate, regular polygons comprising the inner Tree of Life have as many corners (48) as there are (●) yods in the 1-tree up to the level of Chesed, the first Sephirah of Construction.

geometrical representation of the integers 1, 2, 3 & 4, the ‘trunk’ of the Tree of Life consists of the point (Kether), line (Chokmah-Binah), triangle (Chesed-Geburah-Tiphareth) and tetrahedron (Netzach-Hod-Yesod-Malkuth). Fig. 22 shows that it is

1	L	T	TL	T^2	T^2L	T^2L^2	T^3	T^3L	T^3L^2	T^4L	T^4L^2	T^5L	T^5L^2
1	256/243	9/8	32/27	81/64	4/3	1024/729	729/512	3/2	128/81	27/16	16/9	243/128	2

14 tone ratios of notes in the seven musical scales

Archimedean solid	falling interval	rising interval	Catalan solid
truncated tetrahedron	243/256	256/243	triakis tetrahedron
cuboctahedron	8/9	9/8	rhombic dodecahedron
truncated cube	27/32	32/27	triakis octahedron
truncated octahedron	64/81	81/64	tetrakis hexahedron
rhombicuboctahedron	3/4	4/3	deltoidal icositetrahedron
snub cube	729/1024	1024/729	pentagonal icositetrahedron
icosidodecahedron	512/729	729/512	rhombic triacontahedron
truncated cuboctahedron	2/3	3/2	disdyakis dodecahedron
truncated icosahedron	81/128	128/81	triakis icosahedron
truncated dodecahedron	16/27	27/16	pentakis dodecahedron
rhombicosidodecahedron	9/16	16/9	deltoidal hexacontahedron
snub dodecahedron	128/243	243/128	pentagonal hexacontahedron
truncated icosidodecahedron	1/2	2	disdyakis triacontahedron

Figure 24. The 13 Catalan solids and their 13 duals are the polyhedral counterpart of the 13 different rising and 13 different falling intervals between the notes of the seven types of musical scales.

made up of 26 geometrical elements. This trunk is prescribed by the Godname Yahweh with number value 26, its letter values denoting different groups of elements. The counterpart of the 48 elements in the inner form of the Tree of Life is the 48 corners of the seven regular polygons. The counterpart of the trunk in the inner Tree of Life is the set of the first five polygons, which have 26 corners.

Transformed into tetractyses, the 19 triangles of the lowest Tree of Life of any set of overlapping Trees of Life contain 80 yods. There are 48 yods up to the level of the path joining Geburah to Chesed — the first Sephirah of Construction (Fig. 19). This further demonstrates that 48 fundamental degrees of freedom are needed to express just that part of the Tree of Life that contains the seven Sephiroth of Construction. This number is the basic structural parameter defining the form of the Tree of Life — whether geometrical elements in the case of its outer form or corners of polygons in the case of

48 =

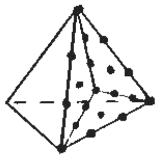


Figure 25. Division of each face of a tetrahedron into three tetractyses generates 48 hexagonal yods (only one face so constructed is shown).

its inner form. It also quantifies whatever holistic system is designed according to this blueprint. In the case of the eight Church musical modes, the number 48 is the number of notes between the tonic and octave of these modes. It should therefore come as no surprise that it figures so prominently in the construction of the geometry of the 120 Polyhedron. We saw earlier that, if the 12 notes of the eight musical scales are assigned to the corners of the dodecagon, the extra 168 yods needed to construct it from 12 tetractyses symbolise the 168 intervals between their notes. Thus formed, its centre is surrounded by 12 vertices, 24 edges and 12 triangles, that is, 48 geometrical elements are needed to construct the dodecagon, starting from a point. Once again, the number 48 defines the minimal number of bits of information required to create a complete Tree of Life pattern.

When its faces are divided into three tetractyses, the simplest Platonic solid — the tetrahedron — has 48 hexagonal yods in its 12 tetractyses (Fig. 25). The basic building block of polyhedra is made of 48 yods symbolising the seven Sephiroth of Construction. Any system whose structure conforms to the universal blueprint of the Tree of Life must necessarily possess this characteristic number of degrees of freedom.

We found in Article 27 that, when its interior triangles and its faces are each divided into three triangles, the 120 Polyhedron comprises 2400 vertices, edges and triangles.[xvi] As

$$49^2 - 1 = 3 + 5 + 7 + \ldots + 97,$$

this number is the sum of the first 48 odd integers after 1. Yet again, the number 48 defines arithmetically the very geometrical composition of the 120 Polyhedron — the polyhedral manifestation of the inner Tree of Life.

Just as the rhombic triacontahedron underlies the 120 Polyhedron as the disdyakis triacontahedron, so the Catalan solid with the next larger number of faces — the disdyakis dodecahedron — is the foundation of the 144 Polyhedron, which is the result of adding tetrahedra to its 48 faces and 26 vertices. In fact, it may be seen as the ‘trunk’ of this polyhedron. Just as Yahweh (YHVH) with number value 26 prescribes the trunk of the Tree of Life because the latter comprises 26 geometrical elements Fig. 22) and the disdyakis dodecahedron because it has 26 vertices, so Yah (YH) with number value 15 marks out this member of the family of 26 Archimedean and Catalan solids. This is because, starting from the simplest Catalan solid — the triakis tetrahedron — and counting back and forth between pairs of dual polyhedra, the disdyakis dodecahedron is the 15th polyhedron (see Fig. 24), whilst the 26th is the 120 Polyhedron. The Godname

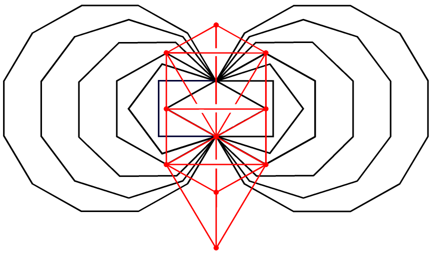


Figure 26. Of the 70 corners of the two sets of 7 regular polygons, eight are shared with the outer Tree of Life. The 62 unshared corners symbolise new degrees of freedom manifesting in 3-dimensional space as the 62 vertices of the 120 Polyhedron.

Yahweh therefore determines *both* polyhedra in the Mereon system, whilst Yah prescribes the family of 15 Archimedean solids, the family of 15 Catalan solids and the disdyakis dodecahedron as the 15th in the two families.

When the outer Tree of Life is projected onto the plane containing its inner form (Fig. 26), eight of the 70 corners of the two sets of enfolded polygons coincide with the projections of Sephiroth and Daath. The 62 corners unshared with the outer Tree of Life

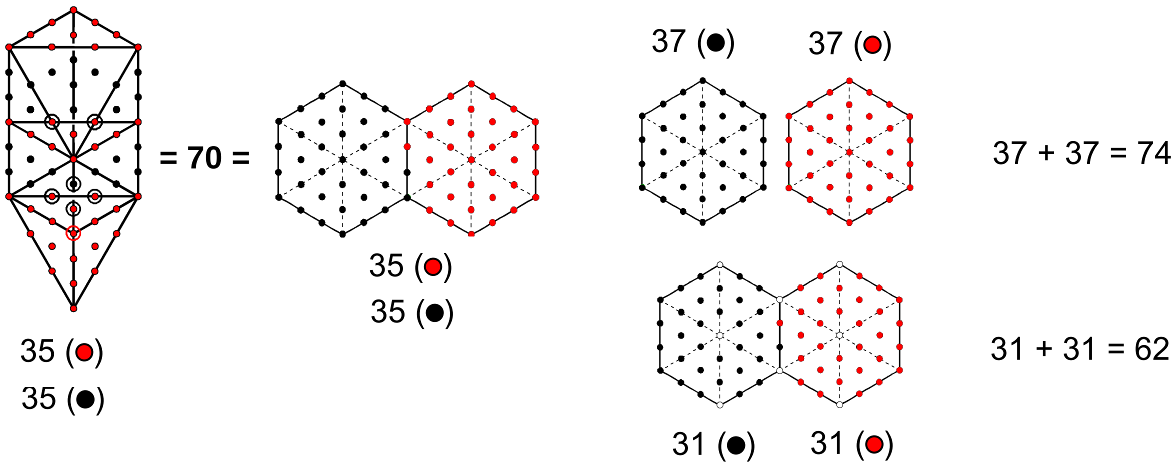
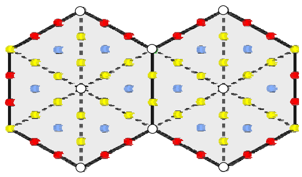


Figure 27. The pair of hexagons in the inner Tree of Life has as many yods as the Tree of Life, the 35 yods in its trunk being the number of yods associated with each polygon. The two separate hexagons contain 74 yods symbolising the 74 vertices of the 144 Polyhedron. The two joined hexagons have 62 yods other than the eight (○) yods shared with the Sephiroth of the Tree of Life. These symbolize the 62 vertices of the 120 Polyhedron.

represent new degrees of freedom. They are the 62 vertices of the 120 Polyhedron.

The hexagon is the fourth regular polygon. Constructed from tetractyses, a pair of separate hexagons is made up of 74 yods (Fig. 27). They symbolise the 74 vertices of the 144 Polyhedron. The 48 vertices that emanate the 48 beams of energy correspond to the 48 yods on the boundaries of the pair of hexagons and at the centres of their sectors. The 26 other vertices (vertices of the underlying disdyakis dodecahedron) correspond to the 26 radial yods. Joined together in the inner Tree of Life, the pair of hexagons has 62 yods other than those shared with the outer Tree of Life. They symbolise the 62 vertices of the 120 Polyhedron. The 12 yods at the centres of each sector of the hexagons denote its 12 B vertices, the 20 yods lying on their boundaries symbolise the 20 C vertices and the 30 radial yods symbolise its 30 A vertices.

The isomorphism between the pair of hexagons and the 120 Polyhedron provides a natural representation of the 12 notes of the seven different musical scales because we found earlier that they correspond to the 12 B vertices of the icosahedron. The six yods at centres of sectors in one hexagon (Fig. 28) denote (in order of increasing tone ratios)



30 (○) → 30 A vertices
12 (○) → 12 B vertices
20 (●) → 20 C vertices

○ yod shared with Tree of Life as a Sephirah.

Figure 28. The 30 radial (○) yods symbolize the 30 A vertices of the 120 Polyhedron, the 12 (○) yods at centres of sectors symbolize its 12 B vertices and the 20 (●) yods on the external boundaries denote its 20 C vertices.

the six notes 256/243, 9/8, 32/27, 81/64, 4/3 & 1024/729 and their mirror images in the other hexagon denote the complements of these notes. The 30 Pythagorean notes in the eight D–D' scales are symbolised by radial yods and the 20 notes consisting of the tonic, octave and 18 non-Pythagorean notes are represented by the 20 boundary yods.

168(•) =

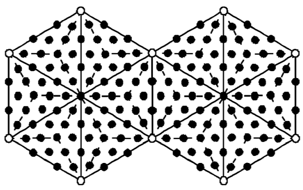


Figure 29. The two adjoined hexagons with their sectors divided into three tetractyses contain 168 yods other than their corners. They symbolize the 168 repetitions in the seven different musical scales of the basic set of 12 notes between the tonic and octave.

That this parallelism between the pair of hexagons and the 120 Polyhedron is not merely due to coincidence is indicated by the fact that they require 168 more yods to construct their sectors from three tetractyses (Fig. 29). As discussed earlier (see Fig. 10), this is the number of rising and falling *repeated* intervals between the notes between the tonic and the octave of the seven different types of musical scales.[xvii] The 84 yods associated with one hexagon denote their 84 rising intervals and the 84 yods in the other hexagon denote their 84 falling intervals.

4. The Holistic Character of the Number 33

Building upon the work of Robert Gray, who discovered[xviii] that the 62 vertices of the 120 Polyhedron are arranged in seven sheets of (2+28) A vertices, 11 sheets of 12 B vertices and 15 sheets of 20 C vertices, the correspondence between this pattern and the note composition of the seven different musical scales was mapped out in Article 26.[xix] It was pointed out in Section 1 that they comprise 26 Pythagorean notes between the tonic and the octave and seven octaves, that is, 33 Pythagorean notes. The tonal composition of the former is

$$26 = 5(9/8) + 6(3/2) + 3(81/64) + 2(243/128) + 6(4/3) + 4(27/16)$$

5 (H) + 6 (V) = 11; 5 (H) + 10 (Y) = 15

The letter values of Yahweh denote the numbers of notes of one or two types. Alternatively, the 26 Pythagorean notes between the tonic and the octave may be divided up in terms of the seven musical scales/church modes listed in Table 1:

Mode 1	D scale	4	10 = Y (Modes 1–4)		15 = YH (Modes 1–5)
Mode 2	A scale	3			
Mode 3	E scale	2			
Mode 4	B scale	1			
Mode 5	F scale	5	H		
Mode 6	C scale	6	V	11 (Modes 6–7)	
Mode 7	G scale	5	H		

However the composition of the 26 Pythagorean notes is decided, the crucial point is that the 7:15:11 pattern of sheets of, respectively, A, C & B vertices is *identical* to the pattern of different notes in the seven different scales, the seven sheets of A vertices corresponding to their seven octaves. The parallelism exists because both systems — one geometrical, one musical — are Tree of Life patterns that are prescribed by Divine Names such as Yah and Yahweh. The number 33 is always a measure of a *whole*.

The same pattern can also be seen in the building up of the regular and semi-regular polyhedra. The first stage of creation of form is the straight line, the second stage is the triangle and the third stage is the tetrahedron, the simplest of the five regular polyhedra, or Platonic solids. The dodecahedron, the last of the Platonic solids, is formed at the seventh stage and the disdyakis triacontahedron, the last of the 26 semi-regular polyhedra, represents the 33rd stage:

1. line
2. triangle
3. tetrahedron
4. octahedron
5. cube
6. icosahedron
7. dodecahedron

(3–7: Platonic solids) — 1–7: 7 → 7 sheets of 30 A vertices

8. truncated tetrahedron
9. cuboctahedron
10. truncated cube
11. truncated octahedron
12. rhombicuboctahedron
13. snub cube
14. icosidodecahedron
15. truncated cuboctahedron
16. truncated icosahedron
17. truncated dodecahedron
18. rhombicosidodecahedron
19. snub dodecahedron
20. truncated icosidodecahedron

(8–20: Archimedean solids)

21. triakis tetrahedron
22. rhombic dodecahedron

8–22: 15 → 15 sheets of 20 C vertices

23. triakis octahedron
24. tetrakis hexahedron
25. deltoidal icositetrahedron
26. pentagonal icositetrahedron
27. rhombic triacontahedron
28. disdyakis dodecahedron
29. triakis icosahedron
30. pentakis dodecahedron
31. deltoidal hexacontahedron
32. pentagonal hexacontahedron
33. disdyakis triacontahedron (120 Polyhedron)

(21–33: Catalan solids) — 23–33: 11 → 11 sheets of 12 B vertices

(22 → 28; 27 → 33; 28 → 144 Polyhedron)

Listed above are, firstly, the Archimedean solids and then the Catalan solids. Just as it exhibits 33 sheets of vertices, so the 120 Polyhedron is the 33rd stage in the generation of the semi-regular polyhedra. The 15th stage after the seventh generates the rhombic dodecahedron with 26 vertices & faces, 36 edges & faces and 50 vertices, edges & faces. With its geometry prescribed by four Godnames (Yah = 15, Yahweh = 26, Elohim = 50 and Eloha = 36), it should not be surprising that this polyhedron plays a fundamental role in the generation of the 144 Polyhedron. Sticking rhombic pyramids to its 12 faces creates the disdyakis dodecahedron with 48 faces and 26 vertices (shown above linked by a red arrow to the rhombic dodecahedron). It is *this* polyhedron (not the cuboctahedron considered by Gray) that is fundamental to the generation of the 144 Polyhedron because it plays the same role in generating this polyhedron as the rhombic triacontahedron does in creating the 120 Polyhedron. Sticking tetrahedra to its 48 faces creates the 144 Polyhedron with (48+26=74) vertices and (3×48=144) faces, whilst adding rhombic pyramids to the rhombic triacontahedron generates the disdyakis triacontahedron, the 120 Polyhedron.

It was shown in Article 24[xx] that the geometry of the Tree of Life generates the rhombic faces of the rhombic dodecahedron and the golden rhombic faces of the rhombic triacontahedron.

Confirmation that the rhombic dodecahedron is, as the 15th stage in the generation of the 120 Polyhedron, the polyhedral root of the Mereon System is provided by the letter values of Yahweh. In the last 11 stages, the rhombic triacontahedron is the fifth stage and the disdyakis dodecahedron is the sixth stage and the first of the last five stages. The rhombic dodecahedron is picked out by the first two letters of YHVH and the two polyhedra that determine the two polyhedra of the Mereon system are selected by the number values of the last two letters H and V of YHVH:

1. triakis octahedron
2. tetrakis hexahedron
3. deltoidal icositetrahedron
4. pentagonal icositetrahedron
5. rhombic triacontahedron

} 5 = H

6. disdyakis dodecahedron
7. triakis icosahedron
8. pentakis dodecahedron
9. deltoidal hexacontahedron
10. pentagonal hexacontahedron
11. disdyakis triacontahedron (120 Polyhedron)

} 6 = V

Notice also that the disdyakis dodecahedron is 21st in the list of semi-regular polyhedra and so is prescribed by the Ehieh, Godname of Kether with number value 21. The 21st stage marks the triakis tetrahedron, the simplest of the Catalan solids. It was shown in Article 29[xxi] that, if the holistic disdyakis triacontahedron is likened to the tetractys — the Pythagorean symbol of holistic systems — the triakis tetrahedron is its 'yod.' This is because the geometrical properties of the former are exactly *ten* times the corresponding one for the latter. The triakis tetrahedron is built from 137 vertices, edges & triangles, where 137 is the 33rd prime number and the number known to physicists to define the fine structure constant $e^2/\hbar c \approx 1/137$. The number 33 not only determines the stage of generation of the 120 Polyhedron — the last Catalan solid — but also the geometrical composition of the first Catalan solid, whose properties are multiplied tenfold in the latter!

The reason why 33 appears in the Pythagorean note composition of the seven musical scales and in the mathematical description of the 120 Polyhedron is that both are holistic systems whose components are 33 in number. Or, better, that they are manifestations of a universal cycle that requires 33 stages to complete itself. This is demonstrated par excellence in the tree level structure of ten overlapping Trees of Life, each representing one of the ten Sephirah of the Tree of Life. Fig. 30 indicates that the latter has seven horizontal divisions marking the stages of emanation of Sephiroth of Construction. Although Daath is not a Sephirah, this point in the Tree of Life is Yesod of the next higher, overlapping tree and therefore counts as a division unless there is no higher tree. These divisions are called 'tree levels.' Fig. 31 shows that ten overlapping Trees of Life have 33 tree levels (in general, n trees have (3n+3) tree levels). They measure the

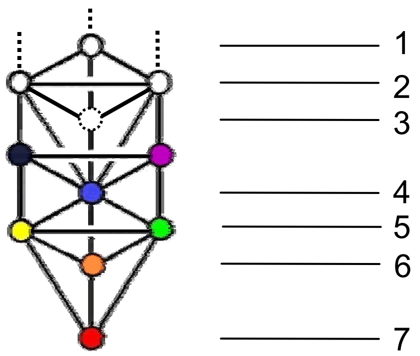


Figure 30. The seven tree levels.

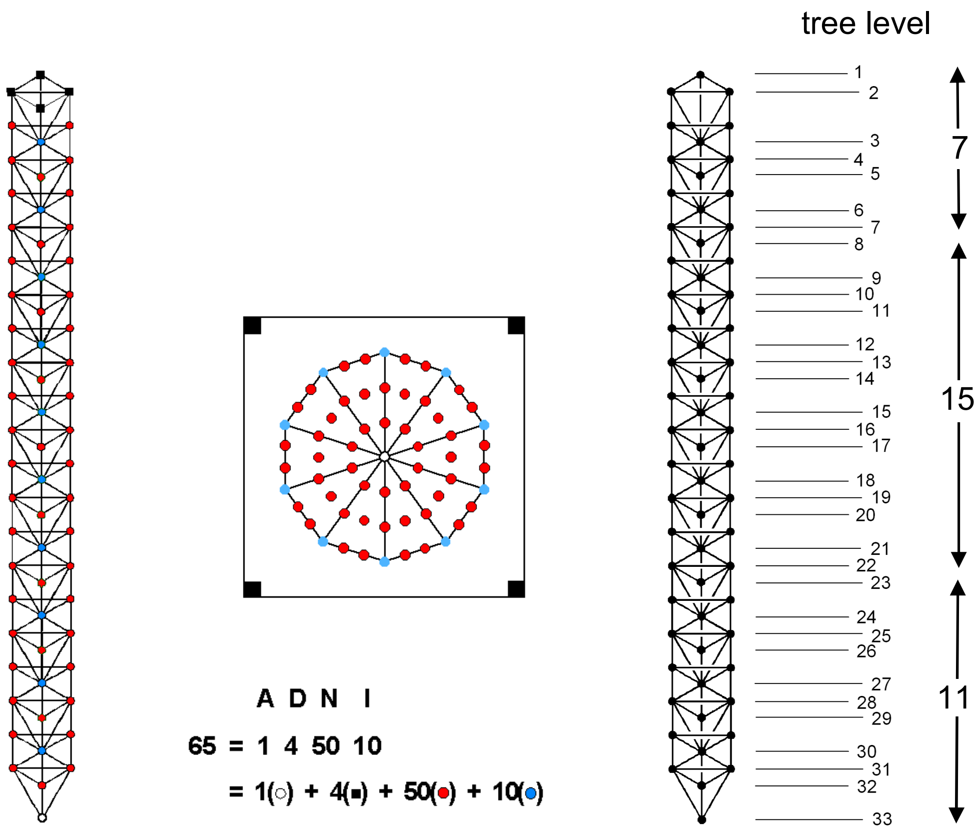


Figure 31. Equivalence of the 10-tree prescribed by the Godname Adonai and its tetractys representation. Ten overlapping Trees of Life have 33 tree levels.

complete cycle of emanation of 10 Sephiroth. In an abstract sense, the number 33 denotes the number of stages in the completion of a cycle of development of a system that culminates in an object that fully embodies the universal blueprint of the Tree of Life. The relationship between the 33 vertex sheets of the 120 Polyhedron and the 33 tree levels of 10 overlapping Trees of Life was analysed in Article 25.[xxii] Also discussed there is what the 7:15:11 vertex sheet composition of the 120 Polyhedron means for

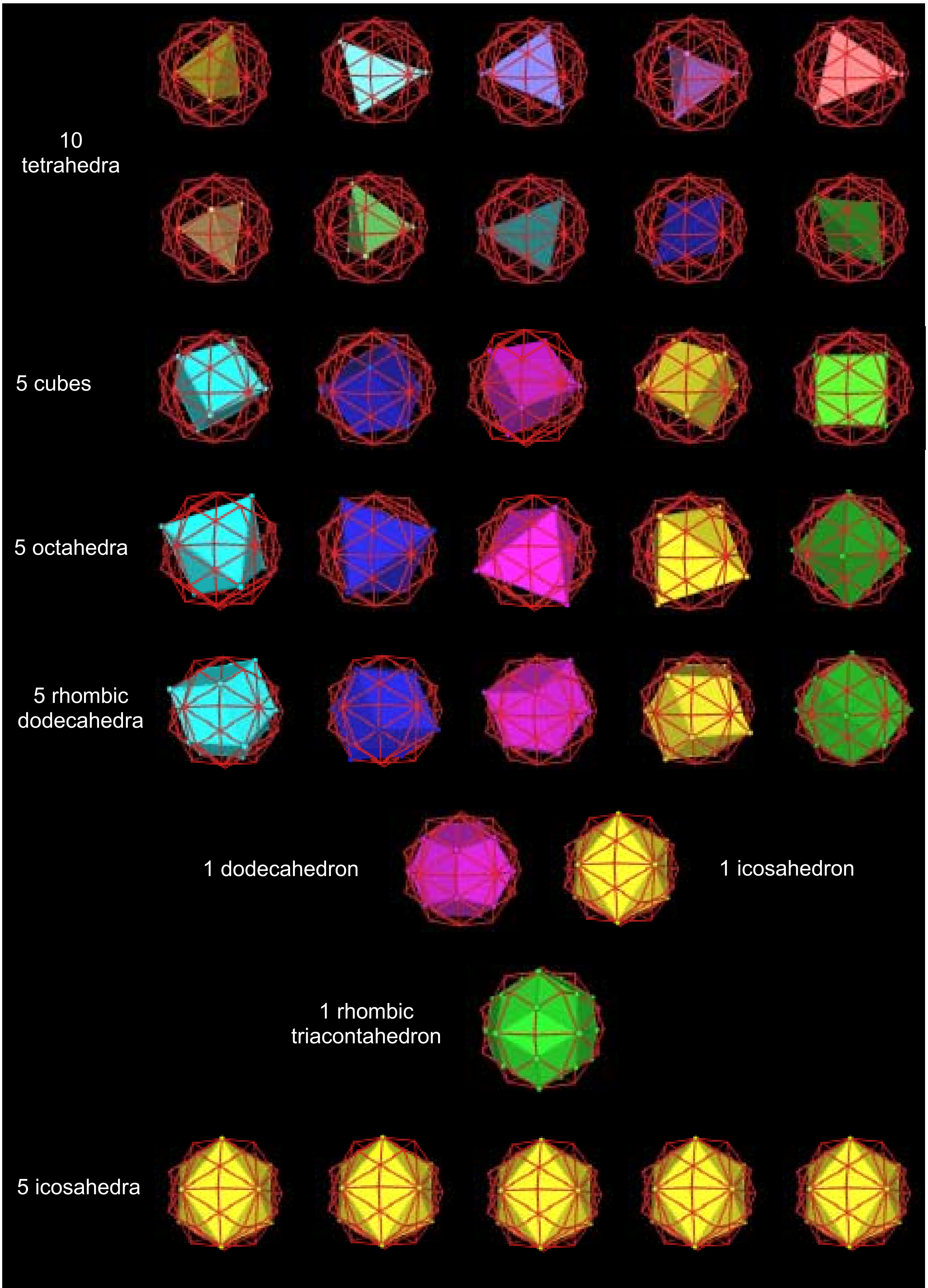


Figure 32. The 33 polyhedra making up the 120 Polyhedron and the mid-sphere.
(From http://www.rwgrayprojects.com/Lynn/P120BUP/buildup.html)

superstring physics. The 10-dimensions of space-time that are predicted by superstring theory is mapped by 10 overlapping Trees of Life. The Godname Adonai (ADNI) assigned to Malkuth prescribes the lowest 10 trees because its number value 65 is the number of their Sephirothic emanations. Fig. 31 shows how the letter values of ADNI specify different types of emanations. The number 33 appears here because 65 is the 33rd odd integer after 1. Once again, it quantifies a holistic system, because each tree represents one of the Sephiroth.

Counting from the tonic of the first octave of the Pythagorean scale, the tone ratio 24 is the 33rd note and the perfect fifth of the fifth octave (Table 5). Counting from the latter, the 33rd note is 576 = 24^2 and still the perfect 5th of the new fifth octave. This is the 65th note from the tonic of the first octave. The Godname Adonai with number value 65

Table 5. Every 33rd note in the Pythagorean scale increases in pitch by a factor of 24.

	C	D	E	F	G	A	B	Number of overtones
1	1	9/8	81/64	4/3	3/2	27/16	243/128	0
2	2	9/4	81/32	8/3	3	27/8	243/64	2
3	4	9/2	81/16	16/3	6	27/4	243/32	4
4	8	9	81/8	32/3	12	27/2	243/16	7
5	16	18	81/4	64/3	24	27	243/8	11
6	32	36	81/2	128/3	48	54	243/4	15
7	64	72	81	256/3	96	108	243/2	20
8	128	144	162	512/3	192	216	243	26
9	256	288	324	1024/3	384	432	486	32
10	512	576	648	2048/3	768	864	972	38
11	1024	1152	1396	4096/3	1536	1728	1944	39

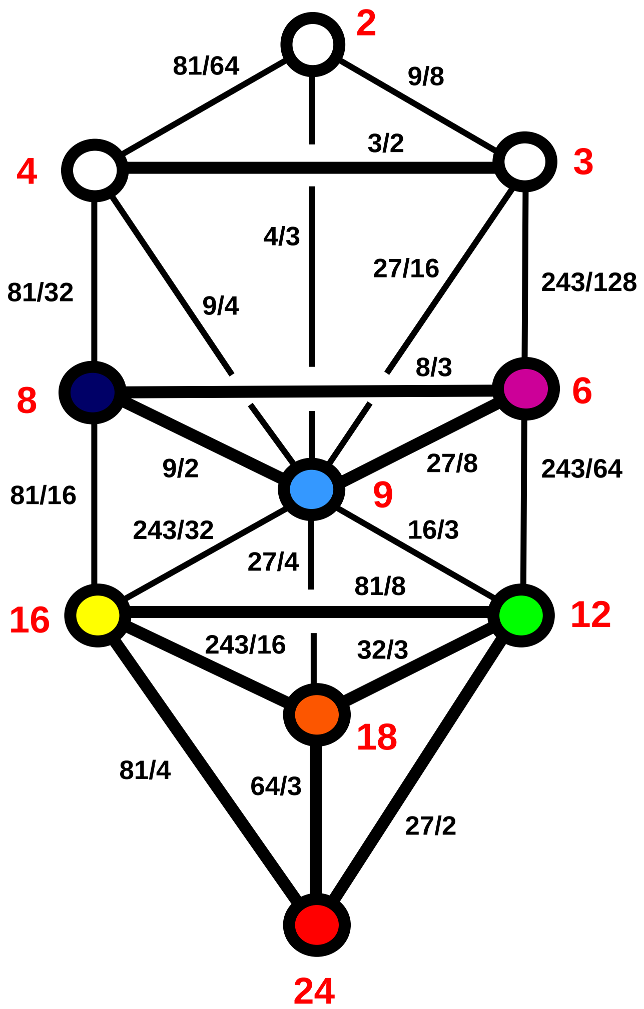
prescribes sequences of 33 notes in the Pythagorean scale whose last note has a tone ratio always 24 times that of the first note. This does not depend on which note is the starting point. Any sequence of 33 notes ends in one whose frequency is 24 higher than that of the first, thus demonstrating the cyclic character of this number.

The significance of the perfect 5th of the fifth octave is that it is the last of *eight* successive fifths (not all perfect fifths):

G_1	D_2	A_2	E_3	B_3	F_4	C_5	G_5
3/2	9/4	27/8	81/16	243/32	32/3	16	24

(*Subscripts denote the octave number*)

In general, every 33rd note is every eight successive fifth. Table 5 indicates that the 33rd note is the *tenth* overtone. 22 of the notes up to the perfect 5th of the fifth octave are fractional. They comprise 16 notes in the first three octaves (separated from the rest of the table by a thick line) and six notes in the fourth and fifth octaves up to the last fifth. This 6:16 division corresponds in the Tree of Life to the six Paths that are edges of the tetrahedron whose vertices are the four lowest Sephiroth and to the 16 Paths outside it (Fig. 33). The 32 fractional notes and overtones up to G_5 conform to the geometrical pattern of the Tree of Life, the ten overtones corresponding to the ten Sephiroth and the 22 fractional notes corresponding to the 22 Paths that connect them. The ordering of notes in Fig. 33 follows the traditional Kabbalistic numbering of Paths.



(Thick lines are Paths of the trunk of the Tree of Life)

Figure 33. The Tree of Life pattern of the first ten overtones.

As the tenth overtone, G_5 with tone ratio 24 corresponds to Malkuth, the lowest Sephirah, signifying the physical manifestation of any holistic system conforming to the Tree of Life blueprint. The significance of this number to superstring theory is discussed in Article 12.[xxiii]

The number of corners of the 7n regular polygons enfolded in n overlapping Trees of Life is

$$C(n) = 35n + 1,$$

where "1" denotes the uppermost corner of the hexagon enfolded in the tenth tree (the hexagon is the only one of the seven regular polygons that shares its corners with polygons enfolded in adjacent trees. This results in its being picked out by the above formula). Therefore, C(10) = 351 = 1 + 2 + 3 +…. + 26. This shows how Yahweh with number value 26 prescribes the inner form of 10 overlapping Trees of Life. 351 is the number value of *Ashim* ("Souls of Fire"), the Order of Angels assigned to Malkuth. The number of corners of the 7n polygons outside their n root edges = C'(n) – 2n = 33n + 1. Hence, C'(10) = 331. This is the number value of *Ratziel*, Archangel of Chokmah. C'(n+1) – C'(n) = 33. In other words, there are 33 corners per set of polygons outside their root edge (Fig. 34). *The emanation of successive Trees of Life generates 33 new geometrical degrees of freedom associated with their inner form*. This again

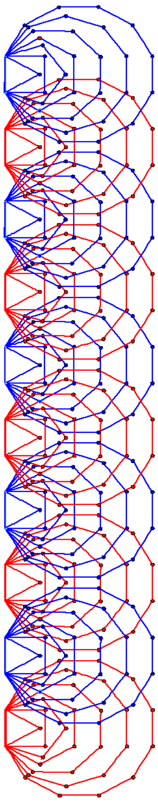
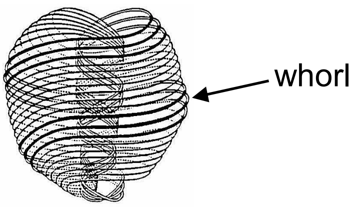
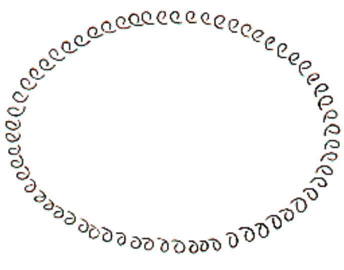


Figure 34. *Ratziel*, the Archangel of Chokmah, prescribes 10 overlapping Trees of Life because its number value is 331, which is the number of corners of the 70 polygons enfolded in 10 trees outside their shared edges. The total number of corners is 351, where 351 = 1 + 2 + 3 +… + 26. This is how the Divine Name Yahweh with number value 26 prescribes the inner form of 10 Trees of Life. There are 34 corners in each set of seven polygons outside their shared edge. As the uppermost corner of the hexagon coincides with the lowest corner of the hexagon enfolded in the next higher tree, there are 33 corners *per set of polygons*. These independent, geometrical degrees of freedom associated with each tree correspond to the 33 tree levels of 10 overlapping Trees of Life.

demonstrates the cyclic aspect of the number 33 in measuring the number of entities (polyhedra, musical notes, etc) in a sequence that represents a holistic system



UPA/heterotic superstring (From *Occult Chemistry,* Annie Besant & C.W. Leadbeater (1951)



Each helical whorl has 1680 coils.

Figure 35. Each of the 10 whorls of the UPA (identified by the author as the $E_8 \times E_8$ heterotic superstring constituent of a quark) is a helix with 1680 turns.

conforming to the archetypal pattern of the Tree of Life.

When its triangles are turned into tetractyses, the number of yods in the lowest n overlapping Trees of Life is given by

$$Y(n) = 50n + 30.$$

Therefore, Y(33) = 1680. As discussed in many previous articles, this is the number of turns in each helical whorl of the UPA (Fig. 35), the indivisible unit of matter described by Annie Besant and C.W. Leadbeater 110 years ago with the aid of a yogic siddhi. In other words, the lowest 33 Trees of Life are made up of as many yods as there are circularly polarized waves in a whorl. This is how the number 33 determines the structural parameter of superstrings inside atomic nuclei. Malkuth of the 33rd Tree of Life is the 65th emanation on the central Pillar of Equilibrium, showing how Adonai, the Godname of Malkuth with number value 65, prescribes this number.

As pointed out in Article 22,[xxiv] there are seven steps leading to the generation of the disdyakis triacontahedron, starting with the tetrahedron. The first five are the Platonic solids and the sixth is the rhombic triacontahedron, the Catalan solid that is compounded from the dodecahedron and the icosahedron. When constructed from tetractyses, the numbers of hexagonal yods in their faces are listed below:

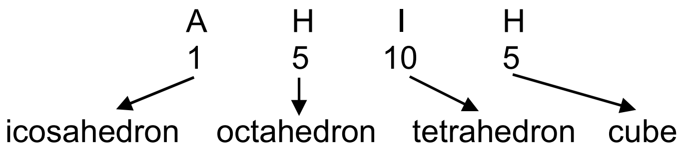
tetrahedron	48	240 (tetrahedron + octahedron + cube)
octahedron	96	
cube	96	
icosahedron	240	
dodecahedron	240	
rhombic triacontahedron	240 + 240	
120 Polyhedron	240 + 240	
	TOTAL = 1680	

As discussed earlier, the 120 Polyhedron is the 33rd stage in the cycle of development of regular and semi-regular solids. Once again, the number 33 is associated with the superstring structural parameter 1680. The five Platonic solids and two semi-regular solids correspond to the five whole intervals and two leimmas of the Pythagorean musical scale.

The 62 vertices of the 120 Polyhedron define the vertices of 28 regular and semi-regular solids: ten tetrahedra (two vertices at each C vertex), five cubes (two vertices at each C vertex), five octahedra (one vertex at each A vertex), one icosahedron (one vertex at each B vertex), one dodecahedron (one vertex at each C vertex), five rhombic dodecahedra (two vertices at each C vertex and one vertex at each A vertex) and one rhombic triacontahedron (vertices at B and C vertices). The middle sphere of the Mereon system consists of five icosahedra. The complete system therefore consists of 33 polyhedra. Constructed from tetractyses, the hexagonal yod populations of the faces of the 28 polyhedra definable within the 120 Polyhedron are:

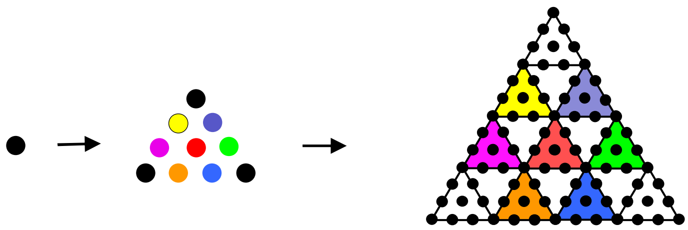
tetrahedron:	10×48 =	480	1680 (tetrahedron to icosahedron)
cube:	5×96 =	480	
octahedron:	5×96 =	480	
icosahedron:	1×240 =	240	
dodecahedron:	1×240 =	240	1680 (dodecahedron to rhombic triacontahedron)
rhombic dodecahedron:	5×192 =	960	
rhombic triacontahedron:	1×480 =	480	
	Total =	3360	

There are 21 Platonic solids of the four types that the ancient Greeks believed were the shapes of particles of the elements Earth, Water, Air and Fire. Their hexagonal yod population is 1680 — the same as the number of hexagonal yods in the seven polyhedra listed above. This is remarkable evidence of beautiful, mathematical design. The Divine Name Ehieh with number value 21 prescribes this polyhedral embodiment of the superstring structural parameter 1680, its letter values specifying the numbers of polyhedra of each type:



The remaining three types of polyhedra also have 1680 hexagonal yods in their faces, giving a total number of 3360. This is astounding for two reasons:

1. Divine Unity symbolised by the Pythagorean Monad, or mathematical point ("0th-order tetractys"), differentiates, firstly, into the familiar tetractys ("1st-order tetractys") with 10 yods (three corners, seven hexagonal yods), secondly, into the "2nd-order tetractys" with 85 yods (15 corners, 70 hexagonal yods), and so on:



3360 is the number of yods in the seven enfolded, regular polygons constituting the inner form of the Tree of Life when their 47 sectors are each turned into 2nd-order tetractyses (Fig. 36).[xxv]
2. 3360 is the number of circularly polarised wave oscillations made during each of the five revolutions of the 10 whorls of the UPA (see Fig. 35). Each yod making up the seven enfolded polygons constructed from 2nd-order tetractyses symbolises one

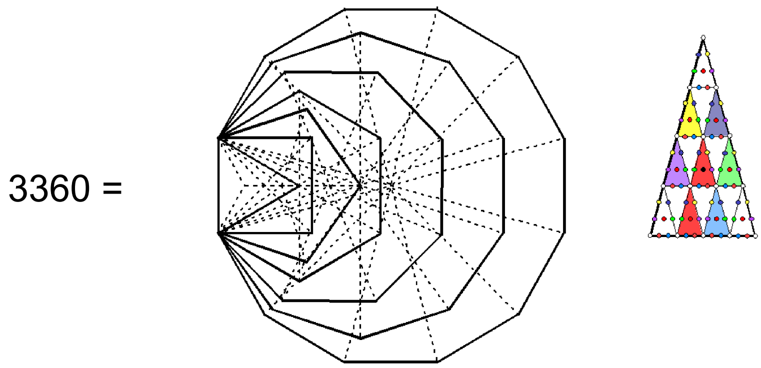


Figure 36. Constructed from 2nd-order tetractyses, the seven enfolded regular polygons constituting the inner form of the Tree of Life contain 3360 yods. This is the number of hexagonal yods in the 28 regular and semi-regular solids generated by the 62 corners of the 120 Polyhedron.

oscillation made in the 10 whorls during one complete revolution around the axis of the UPA. This is not coincidence but a demonstration of the equivalence between the 2-dimensional form of the inner Tree of Life — the seven enfolded polygons — and its polyhedral form — the 120 Polyhedron.

Binah, the third Sephirah in the Tree of Life and the one embodying the cosmic feminine principle (Kabbalists call it the "Great Mother"), has a gematraic number value of 67. The number of yods in n overlapping Trees of Life is

$$Y'(n) = 50n + 20.$$

Therefore, Y'(67) = 3370. There are 10 yods in the uppermost triangle whose base is the path joining Chokmah and Binah of the highest tree. Below Binah of the 67th tree are 3360 yods. Amazingly, the hexagonal yod population of the 28 polyhedra within the 120 Polyhedron is the number of yods below Binah of the very number of trees that equals the number value of Binah!

If we now include the five icosahedra of the middle sphere, the total number of hexagonal yods in the Mereon System of 33 polyhedra is 3360 + 5×240 = 4560. As Y'(91) = 4570, this is the number of yods below Binah of the 91st tree, which is the highest tree in CTOL ("Cosmic Tree of Life"), the map of all levels of reality.[xxvi] *This is profoundly significant*. It says that, below the Supernal Triad of the highest tree in CTOL

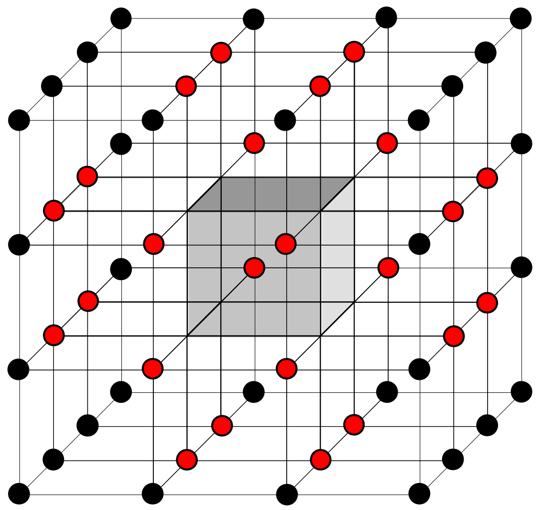


Figure 37. 168 faces of the 26 cubes in the 3×3×3 array surrounding the central (grey) cube define the 32 corners (●) along the 12 edges of the array. 288 faces define the 24 corners (●) within the 6 faces of the cube. (168+288=456) faces define the 56 corners on these faces.

are the same number of yods in all the tetractyses that form these trees as there are hexagonal yods in the faces of all the polyhedra in the mid sphere and the 120 Polyhedron of the Mereon System! Embodied in its polyhedral geometry is a measure that transcends all physics because it refers to the totality of *all* levels of existence — physical and superphysical. 4560 is the number of yods in 456 tetractyses, where 456 is the 228th even integer. 228 is the 227th integer after 1 and 227 is the 49th prime

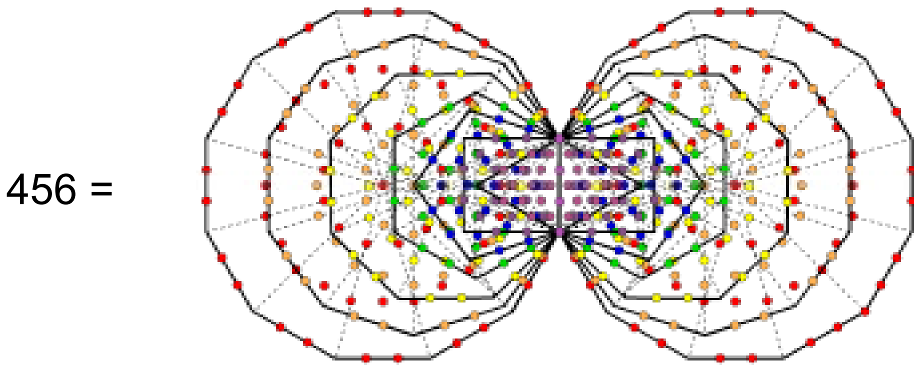


Figure 38. There are 456 yods in the inner Tree of Life that are not external corners of polygons.

number, where 49 is the number value of El Chai, Godname of Yesod. This demonstrates the mathematically archetypal nature of the Divine Names.

The number 456 was encountered in Article 21, where the Klein Configuration, the mapping onto the hyperbolic plane of the 168 automorphisms of the Klein Quartic, was shown to be isomorphic to a 3×3×3 array of cubes. Each of the 64 corners of this array of 27 cubes is the point of intersection of three orthogonal faces of cubes. 168 faces define the 32 corners along the 12 edges of the 3×3×3 array and 288 faces define the 24 corners within its six faces (Fig. 37). Therefore, (288+168=456) faces define the 56 corners on its faces. This has a counterpart in CTOL. As pointed out earlier, the lowest 33 trees of CTOL contains 1680 (= 168×10) yods when their triangles are turned into tetractyses, so that there are (4560–1680=2880=288×10) yods above these trees up to Binah of the highest tree. The corners in the edges of the cubic array are formed from 168 faces, just as the lowest 33 trees consist of the yods in 168 tetractyses, whilst the corners *inside* the edges are formed from 288 faces, just as the trees *above* the lowest 33 trees up to Binah of the highest tree are made up of the yods in 288 tetractyses. In the context of the Mereon System, there are 168×10 hexagonal yods in the 21 Platonic solids of the first four types and 288×10 hexagonal yods in the 12 remaining polyhedra of four types (six Platonic solids of two types and six Catalan polyhedra). This correspondence is remarkable.

The reason why all these systems should display this number is that the inner Tree of Life has 456 yods other than its shape-forming, external corners of polygons (Fig. 38). The number represents the extra number of yods needed to construct it from 94 tetractyses. For the Mereon System, it is the number of tetractyses whose yod population equals the number of hexagonal yods in the faces of the 33 polyhedra. For the 3×3 cubic array that is isomorphic to the Klein Configuration, it is the number of faces of cubes that define the 56 corners of cubes in the faces of the array. For CTOL, it is the number of tetractyses whose yod population is the number of yods below Binah of the highest tree. From the “Great Mother” of CTOL issues 4560 form-determining bits of information. In all three cases, the number determines the *form* of the system — the manifestation through number of the cosmic feminine principle.

5. The Musical Counterpart of G_2

The octonions form the fourth (and last) class of division algebras. An octonion has the form:

$$N = a_0e_0 + a_1e_1 + a_2e_2 + \dots + a_7e_7,$$

where the a_i (i = 0–7) are real numbers, e_0 is the identity element and the seven unit octonions e_i (i = 1–7) are imaginary numbers: $e_i^2 = -1$. Their multiplication is anti-commutative:

$$e_ie_j = -\,e_je_i \qquad (i\neq j)$$

non-associative:

$$e_i(e_je_k) \neq (e_ie_j)e_k$$

and follows the rule

$$e_ie_{i+1} = e_{i+3}.$$

Every unit imaginary octonion has an inverse, or conjugate, $e_i^* = -e_i$, where $e_ie_i^* = 1$. Similarly, the conjugate of an octonion N is N*, where

$$N^* = a_0 - a_1e_1 - a_2e_2 - \dots - a_7e_7.$$

The norm of an octonion is $||N|| = \sqrt{N^*N}$, where

$$||N||^2 = a_0^2 + a_1^2 + a_2^2 + \ldots + a_7^2.$$

The inverse of N is $N^{-1} \equiv N^*/||N||^2$, so that $N^{-1}N = NN^{-1} = 1$. It was shown in Article 28 that there are 13 classes of real, linear combinations of two or more unit octonions. This means that there are 13 classes of octonions that are inverses to the former, so that the 26 classes of octonions and their inverses are prescribed by Yahweh with number value 26.

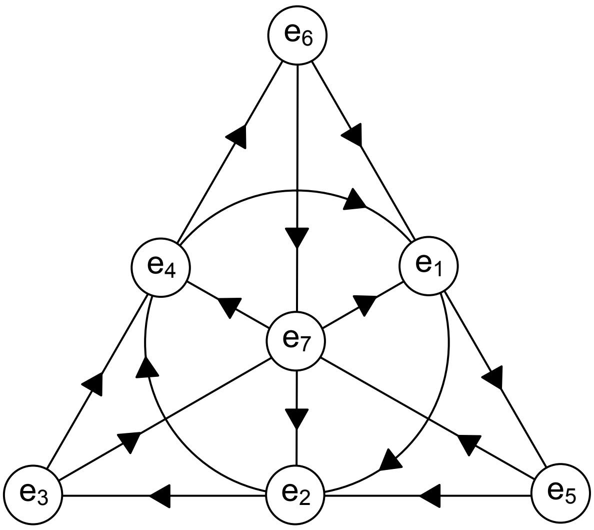


Figure 39. The Fano Plane represents multiplication of the unit imaginary octonions. Arrows connecting triplets of octonions indicate the order of multiplication that generates the third octonion on the same straight or curved line.

This pattern is analogous to the 13 Archimedean solids and their duals — the 13 Catalan solids. The counterpart of polyhedral duality in the world of octonions is the relationship of an octonion $N' = N^{-1}$ being the inverse of the octonion N, where $N'N = 1$. Its counterpart in music is the relationship between a note of tone ratio n and its complement of tone ratio m, where nm = 2. What is sufficiently remarkable in the analogy to take it seriously is that

1. just as there are 13 classes of octonions and 13 classes of their inverses, so the seven musical scales are made up of 13 different notes above the tonic, that is, they have 13 rising intervals and 13 falling intervals;
2. just as the seven imaginary octonions have seven inverses $e_i^* = -e_i$, so the 14 different notes of the seven scales are divided into seven notes and their complements:

imaginary octonion	**inverse**	**musical notes**	**complement**
e_1	e_1^{-1}	tonic (= 1)	octave T^5L^2 (= 2)
e_2	e_2^{-1}	L	T^5L
e_3	e_3^{-1}	T	T^4L^2
e_4	e_4^{-1}	TL	T^4L
e_5	e_5^{-1}	T^2	T^3L^2
e_6	e_6^{-1}	T^2L	T^3L
e_7	e_7^{-1}	T^2L^2	T^3

Moreover, just as the seven e_i form seven ordered sets of three, or 3-tuples: (e_i, e_{i+1}, e_{i+3}), where the product of two ordered members is the third in sequences represented by the circle and six lines of the Fano Plane shown in Fig. 39, so, too, the seven notes and their complements form seven different musical scales. The analogy between the imaginary octonions and the musical intervals is not perfect because multiplication of the former is non-commutative, whereas multiplication of the values of the latter is commutative. However, it is not the imaginary octonions per se that should be compared with musical intervals but, rather, the seven combinations of one, two and three octonions in each 3-tuple:

e_i, e_{i+1}, e_{i+3}

e_ie_{i+1}, $e_{i+1}e_{i+3}$, $e_{i+3}e_i$ (6 permutations)

$e_ie_{i+1}e_{i+3}$ (6 permutations)

Each 3-tuple has 12 permutations of two or three imaginary octonions. Similarly, as $e_{i+3}e_{i+1} = – e_{i+1}e_{i+3} = – e_i = (–e_{i+3})(–e_{i+1})$ and $e_i^{-1} = –e_i$, then $(e_{i+3})^{-1}(e_{i+1})^{-1} = (e_i)^{-1}$, that is, the inverses of each octonion in a 3-tuple also form a 3-tuple with 12 permutations of pairs or triplets. The seven octonions therefore have (7×12=84) permutations, as do their seven inverses. Each permutation reduces to one of the octonions. The seven

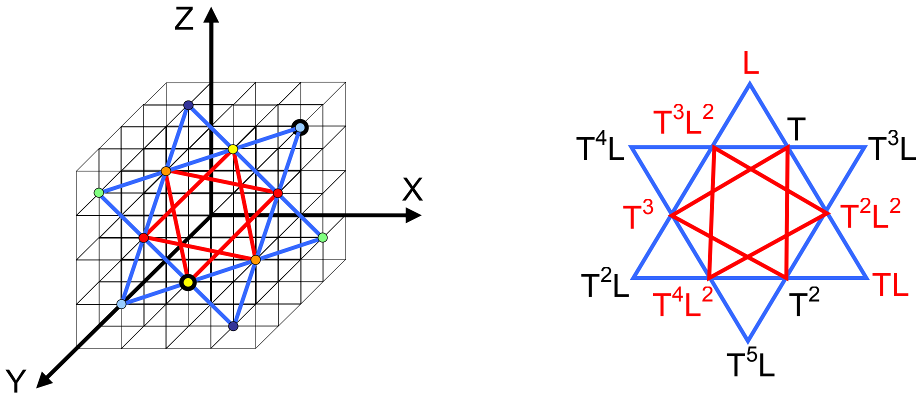


Figure 39. The 12 roots of G_2 (shown as coloured dots) form two nested Stars of David with its two simple roots at vertices of each star (shown as black dots). The 12 notes of the seven musical scales form a similar pattern with opposite star points denoting pair of notes that are complementary.

3-tuples of octonions and the seven 3-tuples of their inverses display (84+84=168) distinct permutations of their members. Compare this with the fact[xxvii] that the seven musical scales have 84 rising intervals between their notes that are repetitions of the basic set of Pythagorean intervals, which means that they also have 84 falling intervals, that is, intervals between two notes falling in pitch. The number of rising intervals in each scale that are repetitions of the Pythagorean intervals varies from scale to scale:[xxviii]

C scale	D scale	E scale	F scale	G scale	A scale	B scale
13	12	13	12	11	11	12

Total = 84.

Average = 84/7 = 12.

This is unlike the number of permutations of pairs and triplets of imaginary octonions in each 3-tuple, which is the same (12) in each one. In terms of notes, the composition of the 84 Pythagorean intervals is

D	E	F	G	A
28	11	23	17	5

The interval 243/128 for note B and the interval 2 for the octave are absent because they are the only ones that never appear more than once in a scale (in the former case, the C, E, F & B scales).

The smallest of the five exceptional groups, the rank-2 group G_2 has 14 roots. It has two simple roots (0,1,–1) and (1,–2,1) denoted by black dots in Fig. 39 and 12 roots consisting of six pairs:

red Star of David	(1,–1,0)	(–1,1,0)
	(–1,0,1)	(1,0,–1)
	(0,1,–1)	(0,–1,1)
blue Star of David	(2,–1,–1)	(–2,1,1)
	(–1,2,–1)	(1,–2,1)
	(–1,–1,2)	(1,1,–2)

They form two nested Stars of David. The red star shown in Fig. 39 denotes the first triplet of roots and their inversions and the blue star denotes the second triplet of roots and their inversions.

Compare this with the seven pairs of intervals and their complements between notes in the seven musical scales:

1	2 (= T^5L^2)
L	T^5L
T	T^4L^2
LT	T^4L
L^2T	T^4
T^2	T^3L^2
LT^2	T^3L
L^2T^2	T^3

The tonic and the octave play the role of the two simple roots and the six pairs of intervals and their complements correspond to the six pairs of roots of G_2 and their inversions. This correspondence permits a nested Star of David representation of the 12 intervals (Fig. 39). Complementarity of intervals corresponds to spatial inversion of

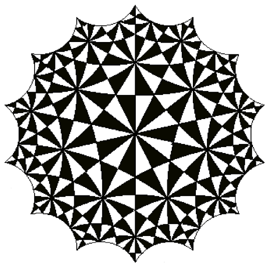


Figure 40. The 168 black triangles of the Klein Configuration represent the 168 automorphisms of the Klein Quartic. The 14 sectors represent the 14 roots of G_2.

roots of G_2. Diametrically opposite vertices of each star signify an interval and its complement. Whatever the assignment of intervals to star points, the members of one triplet have tone ratios in the same relative proportions as those of the other triplet because the latter are their complements with tone ratios that, being reciprocally related, are in the same proportions, although reversed in order. The two red triplets (T, T^2, T^3) and (T^2L^2, T^3L^2, T^4L^2) have tone ratios in the relative proportions 1:T:T^2, i.e., those of the perfect 4th, the perfect 5th and the major 6th of the C scale. What is non-trivial, however, is that the two blue triplets (L, LT, LT^2) and (T^3L, T^4L, T^5L) have tone ratios in the *same* proportion. If M-theory is isomorphic to music even more than previous articles have revealed, this appearance of the same relative proportions between intervals represented by different Stars of David may be telling us that subgroups of rank 3 such as SU(3) play a role that extends beyond that already known for it as the gauge symmetry group governing the colour force between quarks.

Perhaps the analogy between the root structure of G_2 and the 14 basic notes of the seven musical modes is indicating that there is a fundamental representation of SU(7) with 48 gauge fields consisting of an SU(3) triplet, an SU(3) antitriplet and two SU(3) singlets?

The Klein Configuration is the {7,3} tiling in the hyperbolic plane of the 168 automorphisms of the Riemann surface of the famous 'Klein Quartic':

$$x^3y + y^3z + z^3x = 0.$$

As a surface of genus 3, the Klein curve is a Hurwitz curve having the maximum number of automorphisms for a surface of genus 3. Figure 40 depicts these transformations as black hyperbolic triangles distributed over 14 sectors of the Klein Configuration, 12 per sector, to create 24 heptagons with 168 triangular sectors that cover a 3-torus (Fig. 41). As the 168 element PSL(2,7) of the Klein Configuration is isomorphic to the symmetry group SL(2,3) of the Fano Plane that describes octonion multiplication and as the 14-dimensional G_2 is the group of automorphisms of the octonions, the 12 triangles in each of the sectors of the Klein Configuration represent

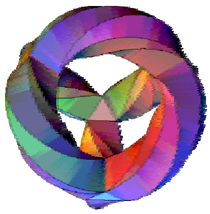
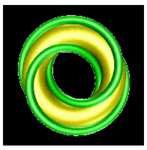


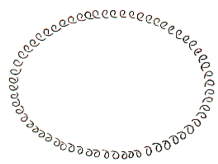
Figure 41. The 7-fold symmetry of the Klein Configuration and the 24-fold symmetry of the tetrahedral 3-torus creates the 168-fold symmetry of the Klein Quartic.

each of the 14 elements of G_2. These sectors correspond to the 14 notes of the seven

(a)

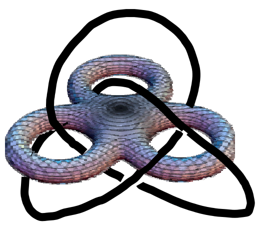


(2,3) torus knot wound on the 1-torus

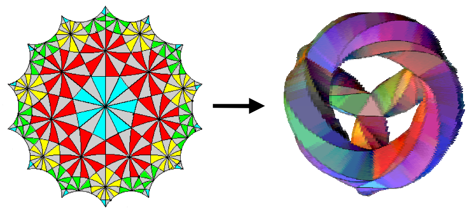


a whorl of the UPA is a helical coil with 1680 turns wound on a 1-torus.

(b)



(2,3) torus knot wound on the 3-torus



tiling on the 3-torus of the 168 automorphisms of the Klein Quartic.

Figure 42. The 1-torus (a) on which the (2,3) torus knot winds is the surface around which each whorl of the heterotic superstring coils 1680 times. The knot can also wind around a 3-torus (b), which is the surface in which can be embedded the Klein Configuration representing the Riemann surface of the Klein Quartic with 168 automorphisms denoted by the triangular sectors of 24 heptagons.

musical scales and their 168 triangles correspond to the 168 repeated, rising and falling Pythagorean intervals between these notes (or, alternatively, to the 168 intervals between the notes above the tonic of the eight D–D' scales).

The (2,3) torus knot that generates the geometry of the Mereon System can be wound on a 3-torus as well as on a 1-torus (Fig. 42). The latter is the surface around which each whorl of the UPA/heterotic superstring winds 1680 times. The former is the surface in which the Klein Configuration can be embedded. As the octonions provide a natural basis for the Lie algebra of E_8 and are connected through the Fano Plane to the Klein Quartic, the Mereon knot determines both the toroidal form and the winding number 1680 of the $E_8 \times E_8$ heterotic superstring.

Because it is well-known to mathematicians that G_2 is the automorphism group of the octonions, perhaps it should not come as a surprise that the pattern of basic intervals in the seven musical scales is similar to the root structure of G_2, given that we have already demonstrated a correspondence between intervals and the properties of octonions that is too rich to be merely coincidental. Nevertheless, it is still remarkable that the same mathematical structure should appear in the 8-component octonions and in the seven 8-note musical scales — even as far as the number of their repeated rising and falling intervals being the order of the symmetry group SL(3,2) for octonions! Moreover, the pattern of intervals of the musical scales is independent of the pitch of a particular octave, remaining true whether the pitch of the tonic is several hundred cycles per second or 10^{24} cycles per second — the frequency associated with subatomic processes, although such fast vibrations would no longer amount to the kind of sound that is detectable to the human ear, which is only sensitive to sounds spanning up to about ten octaves. The remarkable analogy between the musical scales and octonions strongly suggests that, far from being a now defunct musical scale that is of interest only to historians of science and music, the Pythagorean musical scale shares with octonions a vital role in the unified, holistic description of the real world. This is further strongly indicated by the correspondences analysed in this article between the Mereon System and the ancient musical modes of the Roman Catholic Church. The universality of this system, whose 2-dimensional counterpart is the outer and inner forms of the Tree of Life, means that it has many different layers of interpretation. Music is one such layer, and it provides insight into the general meaning of the Mereon System.

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[vii] Ibid. Compare Tables 3–9, pp. 9–11.

[viii] Ibid, p. 10.

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