ARTICLE 9

HOW THE SQUARE ENCODES THE SUPERSTRING PARAMETERS 168 & 248

by

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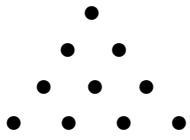
ABSTRACT

Previous articles showed that the superstring parameters 248 and 168 are embodied as numbers in the outer and inner forms of the Tree of Life, as well as in various sets of regular polygons belonging to the latter whose properties are determined by the number values of the Godnames of the ten Sephiroth. This article will prove that the prescription by the Godnames of 'sacred geometry' encoding such numbers of cosmic meaning applies also to single, regular polygon, particularly the square. The 25 yods of a square divided into four tetractyses symbolise the spatial dimensions predicted by the quantum mechanics of spinless strings, their geometrical differentiation corresponding to the distinction between bosonic and superstring dimensions of space. 248 is found to be the number of yods other than corners of tetractyses in a square whose sectors are transformed into the next higher-order tetractys. 168 is the number of yods surround the centre of a square whose sectors are divided into three triangles, the latter then being each divided into three tetractyses. As further illustration of the Tetrad Principle proposed in Article 1, the square, which symbolises the number 4, or Pythagorean tetrad, also represents in an arithmetic way beautiful properties of these transformed squares. Finally, equivalence (in the sense of sharing similar properties) is demonstrated between the square and the seven, enfolded, regular polygons constituting each half of the inner form of the Tree of Life.

1. INTRODUCTION

The Pythagorean mathematician Nichomachus of Gerasa said that the Pythagoreans called the number 4, or tetrad, “the greatest miracle,” “a God after another manner,” ”a manifold divinity,” and “holding the key of Nature.” Why did he (and his precursors) value this number so highly? Lacking insight into the mystical wisdom of ancient sages like Pythagoras and possessing only a few surviving written fragments of his teachings — themselves only elementary and distorted summaries of a now lost knowledge taught by word of mouth — modern scholarship has no satisfactory answer to this question. Historians of Greek mathematics realise that the early Pythagoreans placed special emphasis on the number 4 because their so-called ‘perfect number’ 10 is the fourth triangular number, otherwise known as the ‘tetractys’ (from *tetras*, four):



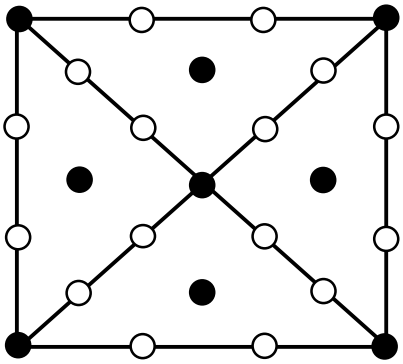
1
2
3
4

They knew also that the numbers 1, 2, 3 & 4 contain the symphonic ratios that make possible the musical scale, i.e. 1:2, the octave; 2:3, the perfect fifth; and 3:4, the perfect fourth. But the *true* reason why the Pythagoreans valued the tetrad so highly was far more profound. They intuitively recognised that a four-fold pattern or structure existed in the *mathematics* of the cosmos, generating not only the number 10 as the measure of divine perfection and the tetractys as its symbol of wholeness but also numbers that *quantify physical phenomena*. This rule was formulated as the ‘Tetrad Principle’ in Article 1, which showed how parameters of superstring theory have a natural expression in terms of the numbers 1, 2, 3 & 4 — even the number 4 itself.

In this article it will be explained how the square, the geometrical symbol of the number 4, embodies certain numbers of relevance to superstring theory (and beyond), thus amply justifying the assignment to the tetrad by the Pythagoreans of the title “holding the key of Nature.”

2. ENCODING OF 25 & 248 IN THE SQUARE

Constructed from four tetractyses, a square contains 25 yods:

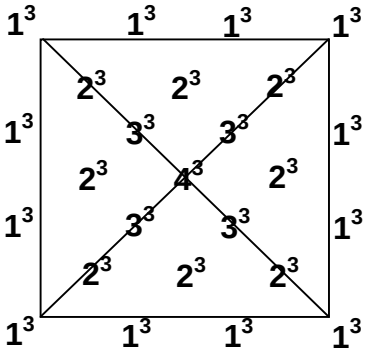
25 =  = 16(○) + 9(●).

They symbolise the 25 dimensions of space predicted by the quantum mechanics of spinless (bosonic) strings. The number 16 (4th square number) is the number of purely bosonic string dimensions of the 26-dimensional space-time whose compactification, according to the author's theory outlined in Article 2, generates superstrings with 9 remaining spatial dimensions (9 = 4th odd integer after 1). The central yod (●) symbolises the longitudinal dimension of a string, the surrounding 24 (= 1×2×3×4 = 4!) yods represent the 24 transverse dimensions of a bosonic string and the surrounding 8 yods (●) denote the 8 transverse, purely superstring dimensions (8 = 4th even integer). 25 is the arithmetic mean of the cubes of the first 4 integers:

$$25 = \frac{1^3 + 2^3 + 3^3 + 4^3}{4}$$

When the cubes of the integers 1, 2, 3 & 4 are assigned to the positions of the 25 yods in a square, their sum:

248 =



is the dimension of the unified, superstring gauge symmetry group E_8. In other words, the square not only encodes the number of spatial dimensions of space-time but also embodies the number of quantum states of the particle transmitting the unified force between superstrings!

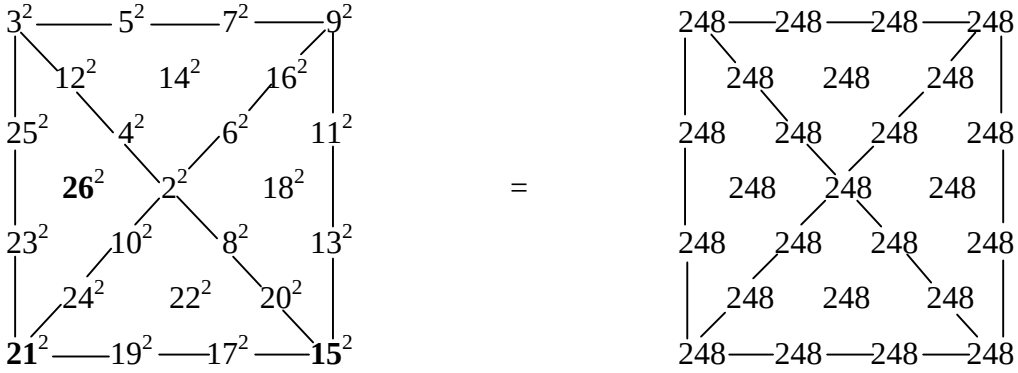
Using the identity:

$$6200 = 248\times25 = 2^2 + 3^2 + 4^2 + ... + \mathbf{26}^2,$$

where the first term, $2^2 = 4$, is the tetrad and the last term**,** $\mathbf{26}^2$, is the square of the sum (**26**) of the first *four* Mersenne numbers:

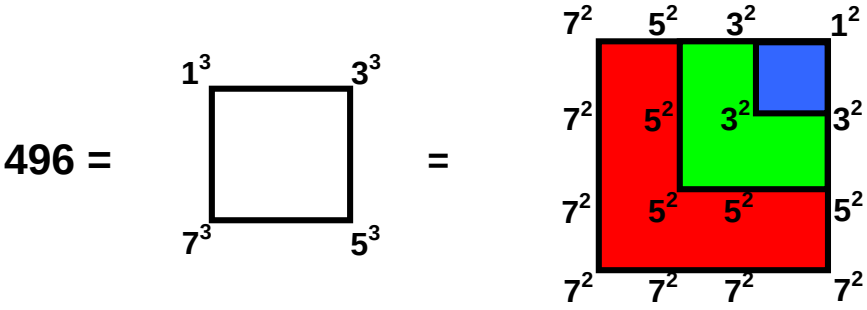
$$2^1 - 1 = 1$$
$$2^2 - 1 = 3$$
$$2^3 - 1 = 7$$
$$2^4 - 1 = \underline{\mathbf{15}}$$
$$\text{TOTAL} = \underline{\mathbf{26}}$$

we see that the square displays the following remarkable property of the number 248:



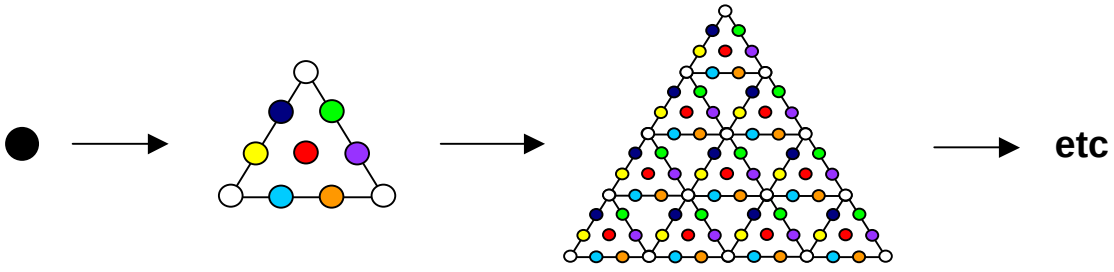
Starting with the tetrad, $2^2 = 4$, at the centre of the square and assigning the 24 (= 1×2×3×4) squares 3^2, 4^2, ... $\mathbf{26}^2$ to the surrounding 24 yods of the four tetractyses, with squares of even integers inside the square and squares of odd integers on its boundary, the sum of these 25 squares is that resulting from assigning the number 248 to each yod in the square. This demonstrates the unique significance of the superstring parameter 248 vis-à-vis the square.

The dimension **496** of the superstring gauge symmetry groups $E_8 \times E_8$ and O(32) that apply to the five superstring theories is the sum of the cubes of the first *four* odd integers:

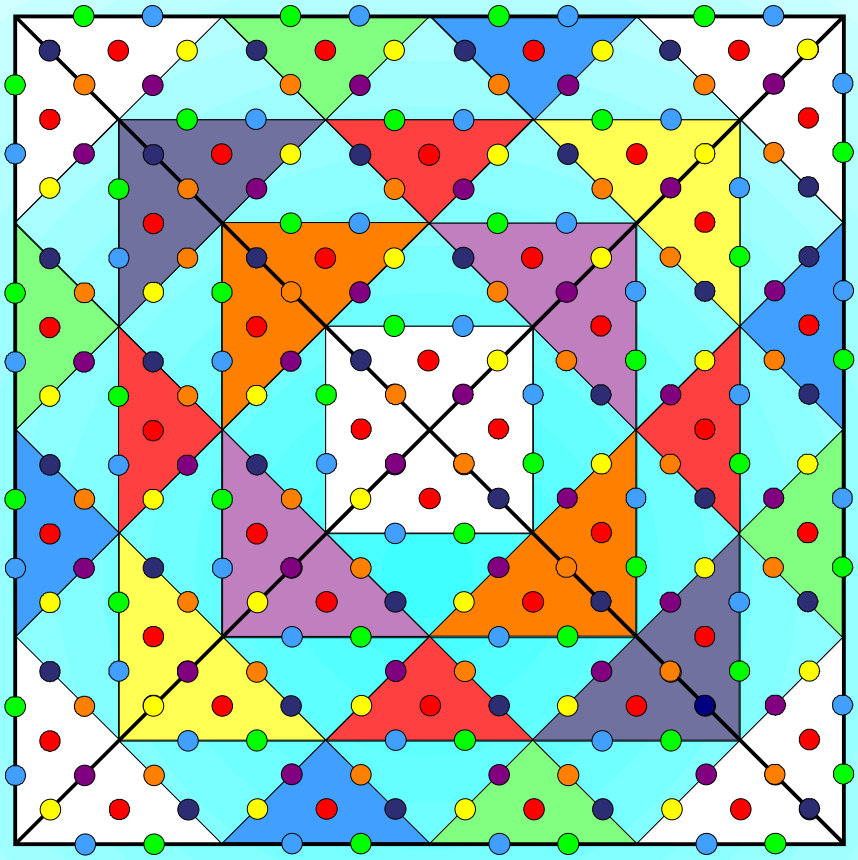


This means that it can also be represented by a 4×4 *square* array of the squares of 1, 3, 5 & 7.

The square encodes the number 248 geometrically as well as arithmetically, as now explained. Starting from the point, or Pythagorean monad, the tetractys has ten points (symbolised below by circles), each of which is a potential tetractys of ten points, and so on.



The number of yods in the next higher order tetractys shown above is:



The square, symbol of the Pythagorean tetrad, is formed from 40 tetractyses with 248 yods other than 41 corners

Figure 1

$$85 = 4^0 + 4^1 + 4^2 + 4^3.$$

The number of yods (○) at corners of the ten tetractyses is **15**, which is the number of combinations of *four* objects and the *fourth* Mersenne number. The number of coloured yods is

$$70 = \begin{array}{c} 7 \\ 7\ \ 7 \\ 7\ \ 7\ \ 7 \\ 7\ \ 7\ \ 7\ \ 7, \end{array}$$

where 7 is the *fourth* odd integer. 70 is the *fourth*, *4-dimensional*, *tetrahedral* number after 1, i.e., it is the number of balls that can be piled up in 4-dimensional space into the fourth tetrahedron.

Consider now a square divided into its four sectors, each of which is this higher order tetractys. The total number of tetractyses is

$$40 = \begin{array}{c} 4 \\ 4\ \ 4 \\ 4\ \ 4\ \ 4 \\ 4\ \ 4\ \ 4\ \ 4 \end{array}$$

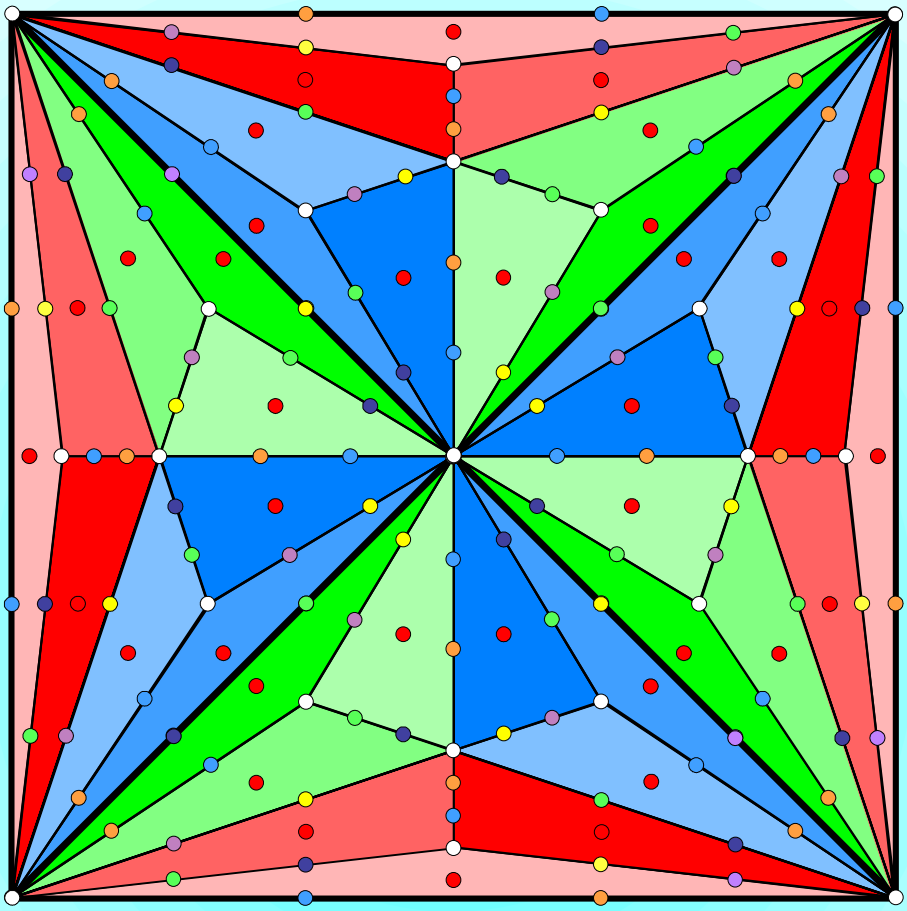
The total number of yods surrounding the centre of the square is

$$288 = 1^1 + 2^2 + 3^3 + 4^4.$$

The number of corners surrounding the centre is

$$40 = \begin{array}{c} 4 \\ 4\ \ 4 \\ 4\ \ 4\ \ 4 \\ 4\ \ 4\ \ 4\ \ 4 \end{array}$$

Notice how the tetrad determines the properties of both the higher-order tetractys and the square when the latter is constructed from the former. Figure 1 shows that there are 248 coloured yods (yods other than corners of tetractyses). They number seven per tetractys and are arranged at the centre and six corners of a hexagon. There are 32 yods lying on the diagonals of the square that are shared with adjacent tetractyses. 32 = 4×8, where 8 = 4th even integer. This leaves (248 – 32 = **216**) yods that are unshared with one another. Alternatively, there are 32 coloured yods on the sides of the square and **216** such yods inside its boundary. It is amusing that this is the number of years that, according to Androcydes, the Pythagorean writer of the work *On Symbols*, and others was thought in antiquity to elapse between successive lives of Pythagoras. **216** is the number value of Geburah, the fifth Sephirah (see table below). It is also the notorious number mentioned by Plato in a passage in his *Republic*, which is thought (1) to refer to its property as the sum of the cubes of the numbers 3, 4 & 5 measuring the sides of Pythagoras' famous right-angled triangle.



The square, symbol of the Pythagorean tetrad, is formed from 36 tetractyses with 21 corners and with 168 yods surrounding its centre

Figure 2

Of the **48** yods on the boundary of the square whose sectors are divided into the next higher-order tetractys, four are its corners, leaving 44 other yods, once again illustrating the influence of the tetrad in determining or expressing the properties of objects with sacred geometry.

3. ENCODING OF SUPERSTRING PARAMETER 168 IN THE SQUARE

In previous articles it was explained how the outer and inner forms of the Tree of Life encode the number **168**, which is the number value of Cholem Yesodeth, the Mundane Chakra assigned to Malkuth and the number of circularly polarised waves making up half a revolution of a string component of the superstring constituent of up and down quarks. We have already seen that the square encodes the dynamical parameter 248 of the superstring. It will now be shown how it embodies its *structural* parameter **168**.

Instead of turning each sector of a square into a higher-order tetractys, consider it divided into three triangles, each of which is then split into three more triangles turned into tetractyses (*fig*. 2). Both a single square and the pair of squares enfolded in the inner form of the Tree of Life will be considered. Their properties are listed below:

Table

(Numbers in shaded boxes appear in the discussion below)

Sephirah	Title	Godname	Archangel	Order of Angels	Mundane Chakra
Kether	**620**	**21**	**314**	**833**	**636**
Chokmah	**73**	**15, 26**	**331**	**187**	**140**
Binah	**67**	**50**	**311**	**282**	**317**
Chesed	**72**	**31**	**62**	**428**	**194**
Geburah	**216**	**36**	**131**	**630**	**95**
Tiphareth	**1081**	**76**	**101**	**140**	**640**
Netzach	**148**	**129**	**97**	**1260**	**64**
Hod	**15**	**153**	**311**	**112**	**48**
Yesod	**80**	**49**	**246**	**272**	**87**
Malkuth	**496**	**65, 155**	**280**	**351**	**168**

Single square

Square has four sectors, each with (3×3 = 9) triangles/tetractyses having 12 edges inside it.

Number of corners of tetractyses = 1 + 4 + 4×4 = **21** (19 outside root edge);
Number of sides of tetractyses = 4 + 4 + 4×12 = 56 (55 outside root edge, **48** not sides of sectors);
Number of tetractyses = 4×9 = **36**;
Number of corners & sides of tetractyses = **21** + 56 = 77;
Number of corners & triangles = **21** + **36** = 57 (55 outside root edge);
Number of sides & triangles = 56 + **36** = 92 (91 outside root edge);
Number of corners, sides & triangles = 77 + **36** = 113 (113 – 3 = 110 outside root edge, 113 – 8 = 105 inside square);
Number of yods = **21** + **36** + 56×2 = 169 (169 – 4 = 165 outside root edge, **168** surrounding centre);
Number of yods at corners of tetractyses = **21**;
Number of hexagonal yods = 169 – **21 = 148;**
Number of yods on edges of tetractyses = 169 – **36** = 133 (133 – 4 = **129** outside root edge or generated from square with four corners);
Number of hexagonal yods on edges of tetractyses = 133 – **21** = **112**;
Number of yods on boundary of square = 12;
Number of yods inside square = 169 – 12 = 157 (17 corners and **140** hexagonal yods);
Number of yods generated from square with 4 yods at corners = 169 – 4 = 165.

Two joined squares

Two squares have 8 sectors, each with (3×3 = 9) triangles/tetractyses having 12 edges inside it.

Number of corners of tetractyses = 2 + 2×19 = 40;
Number of sides of tetractyses = 1 + 2×55 = 111 (110 outside root edge);
Number of tetractyses = 2×**36** = **72**;
Number of corners & sides of tetractyses = 40 + 111 = 151;
Number of corners & triangles = 40 + **72** = **112**;
Number of sides & triangles = 111 + **72** = 183;
Number of corners, sides & triangles = 151 + **72** = 223 (223 – 3 = 220 outside root edge, 223 – 13 = 210 inside squares);
Number of yods = 4 + 2×165 = 334 (334 – 4 = 330 outside root edge)
Number of yods at corners of tetractyses = 2 + 2×19 = 40;
Number of hexagonal yods = 334 – 40 = 294 (292 outside root edge);
Number of yods on edges of tetractyses = 334 – **72** = 262 (262 – 4 = 258 outside root edge);
Number of hexagonal yods on edges of tetractyses = 262 – 40 = 222 (220 outside root edge);
Number of yods on boundary of squares = 4 + 2×(12 – 4) = 20;
Number of yods inside squares = 334 – 20 = 314 (34 corners and **280** hexagonal yods, **140** in each square);
Number of yods generated from two joined squares with 6 corners = 334 – 6 = 328.

Below are listed the ways in which the number values of the ten Godnames prescribe the properties of the square and the pair of joined squares:

Kether: **21** **36** tetractyses in square have **21** corners. Also, 210 corners, sides & triangles inside squares, where

$$210 = \begin{array}{c} \mathbf{21} \\ \mathbf{21\ \ 21} \\ \mathbf{21\ \ 21\ \ 21} \\ \mathbf{21\ \ 21\ \ 21\ \ 21} \end{array}$$

Chokmah: **15** Number of corners & sides of tetractyses = 151 = 150th integer after 1, where

$$150 = \begin{array}{c} \mathbf{15} \\ \mathbf{15\ \ 15} \\ \mathbf{15\ 15\ \ 15} \\ \mathbf{15\ \ 15\ \ 15\ \ 15} \end{array}$$

26 Tetractyses in square have 56 sides, of which 52 are generated from square, where 52 = **26**th even integer;

Binah: **50** Number of corners, sides & triangles in two squares separated by root edge = 3 + 2×113 = 229 = **50**th prime number;

Chesed: **31** Number of new yods outside root edge on edges of tetractyses in square = **129** – 2 = 127 = **31**st prime number. Also, number of yods outside root edge of two joined squares other than corners = 330 – 19 = **311** = 310th integer after 1, where

$$310 = \begin{array}{c} \mathbf{31} \\ \mathbf{31\ \ 31} \\ \mathbf{31\ \ 31\ \ 31} \\ \mathbf{31\ \ 31\ \ 31\ \ 31} \end{array}$$

Geburah: **36** Square has **36** tetractyses. Also, two joined squares have 151 corners and sides, where 151 = **36**th prime number;

Tiphareth: **76** Square has 77 corners and sides, where 77 = **76**th integer after 1;

Netzach: **129** Number of yods in two joined squares outside root edge on sides of tetractyses = 258 = **129**th even integer. Also, number of yods generated from square with four corners;

Hod: **153** Number of corners & sides of two separate squares = 2×77 = 154 = **153**rd integer after 1;

Yesod: **49** Tetractyses in two joined squares have 111 sides, of which 14 are sides of sectors other than root edge. Number of sides other than sector edges = 111 – 14 = **97** = **49**th odd integer;

Malkuth: **65** **129** yods outside root edge on sides of two squares, where **129 =** 65th odd integer**;**

155 Number of yods inside square surrounding centre = 157 – 1 = 156 = **155**th integer after 1.

The most important property listed above is that a square whose sectors are constructed from three triangles, each themselves constructed from three tetractyses, has **168** yods surrounding its centre. The number values of the Godnames of the ten Sephiroth therefore prescribe this structural parameter of the superstring.

The Pythagorean tetrad (4) determines the following properties of the square and the pair of joined squares present in the inner form of the Tree of Life:

1. Number of corners of tetractyses in two joined squares = 40

$$= \begin{array}{cc} 10 & 10 \\ & \square \\ 10 & 10 \end{array} \qquad (10 = 1 + 2 + 3 + 4)$$

2. Number of sides outside the root edge of the square = 55

$$= \begin{array}{c} 1 \\ 2\ \ 3 \\ 4\ \ 5\ \ 6 \\ 7\ \ 8\ \ 9\ \ 10 \end{array} \qquad = \begin{array}{cc} 2^2 & 3^2 \\ & 1^2 \\ 5^2 & 4^2 \end{array}$$

This is also the number of corners and triangles in the square outside the root edge.

3. Number of corners, sides & triangles in two joined squares outside the root edge = 220

$$= \begin{array}{cc} 4^2 & 6^2 \\ & 2^2 \\ 10^2 & 8^2 \end{array}$$

This is also the number of hexagonal yods outside the root edge in two joined squares.

4. Number of yods in the square = 169 = 13^2

$$= \begin{array}{cccc} 3 & 5 & 7 & 9 \\ 25 & & & 11 \\ & & 1 & \\ 23 & & & 13 \\ 21 & 19 & 17 & 15 \end{array}$$

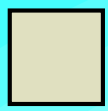
168 yods surround its centre. This is the sum of the 12 integers on the square's boundary.

5. Number of extra yods in the square with four yods at its corners = 165

$$= \begin{array}{cc} 3^2 & 5^2 \\ & 1^2 \\ 9^2 & 7^2 \end{array}$$

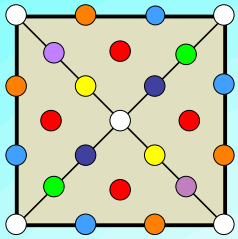
6. Number of yods outside the root edge of two joined squares = 330

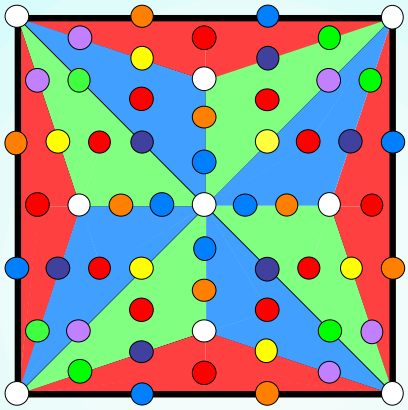
$$= \begin{array}{cc} 7^2 & 8^2 \\ & 6^2 \\ 10^2 & 9^2 \end{array}$$



TETRAD PRINCIPLE

The 4th member of the sequence of squares embodies the superstring parameter 168





168 yods surround the centre of the square

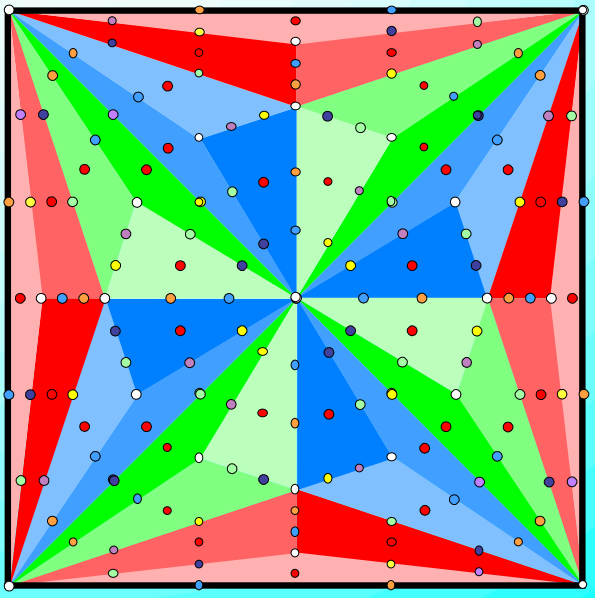


Figure 3

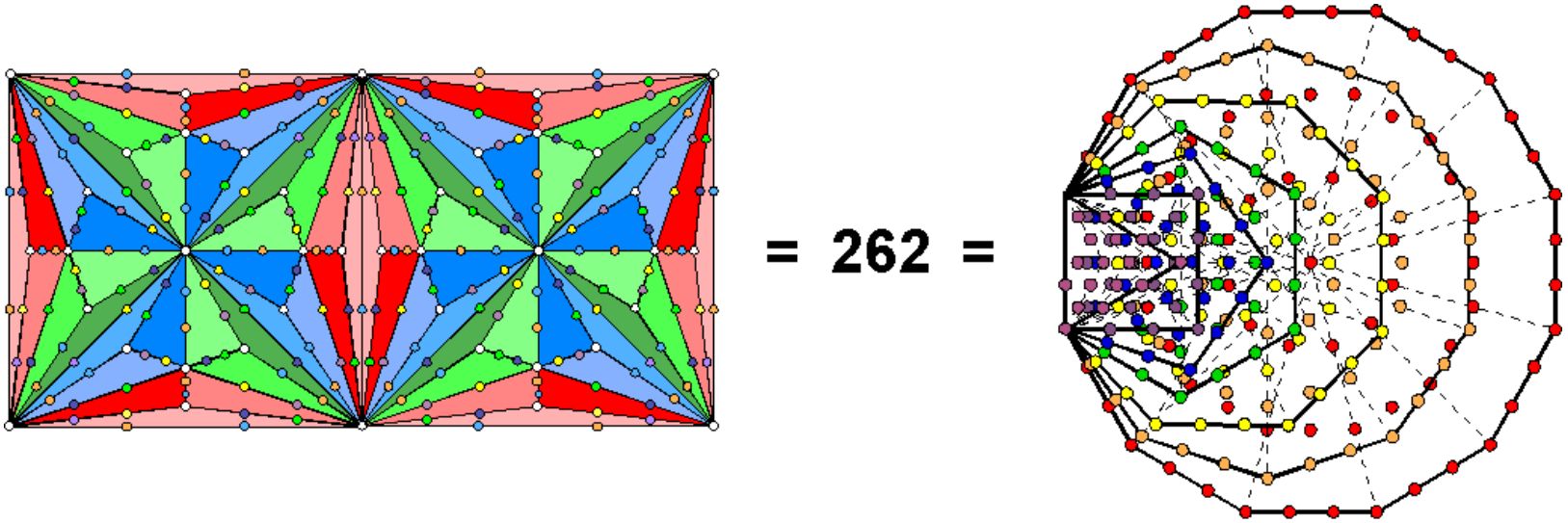


Figure 4

Two joined squares have as many yods (262) on the edges of their 72 tetractyses as the 7 enfolded, regular polygons have yods associated with them

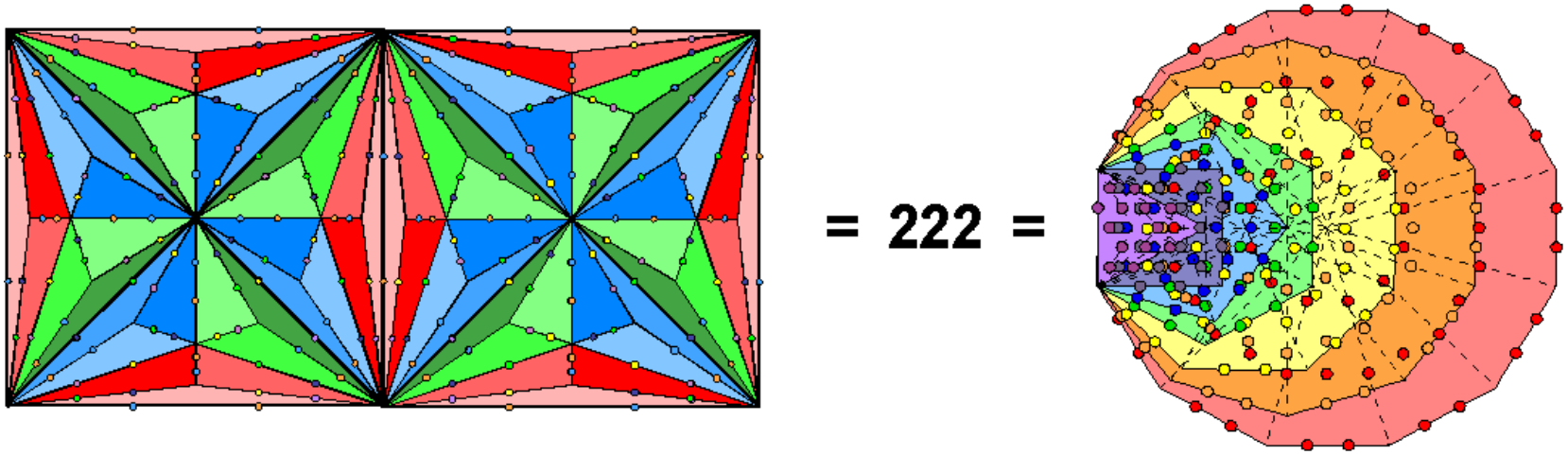


Figure 5

Two joined squares have as many hexagonal yods (222) on the edges of their 72 tetractyses as there are hexagonal yods associated with the 7 enfolded, regular polygons

The tetrad also determines the superstring parameter **168** because this is the number of yods surrounding the centre of the *fourth* member of the sequence of tetractys-generated squares, starting with the bare square itself (*fig. 3*). This is a remarkable illustration of the general Tetrad Principle discussed in detail in Article 1.

4. EQUIVALENCE OF SQUARE AND 7 ENFOLDED POLYGONS

According to the properties of the square listed in section 2, the pair of squares has 262 yods on the *boundaries* of their **72** tetractyses. The seven regular polygons enfolded in the Tree of Life have 264 yods, that is, 262 yods are associated with either set of polygons. The number of yods defining the *shapes* of the tetractyses making up the pair of squares is that needed to *fill up* each half of the inner form of the Tree of Life (f*ig.4)*. Furthermore, there are 222 hexagonal yods on the boundaries of the **72** tetractyses. These yods correspond to the Sephiroth of Construction above

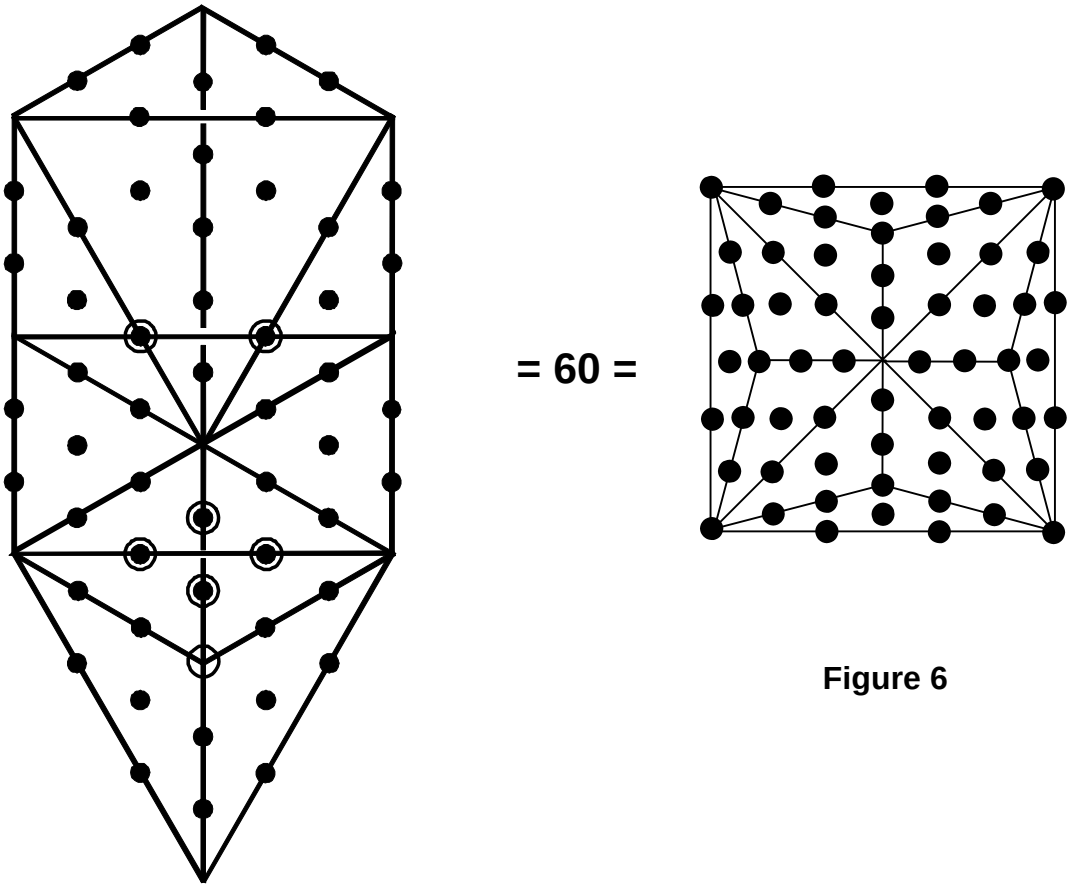


Figure 6

Malkuth, the lowest Sephirah of Construction. 222 is the number of *hexagonal* yods associated with each set of seven enfolded polygons *(fig. 5*). These two remarkable similarities demonstrate that the square possesses an implicit or hidden equivalence to the inner form of the Tree of Life.

The square is also equivalent in the same sense to the *outer* form of the Tree of Life. The reason for this is as follows: when the 16 triangles of the Tree of Life are turned into tetractyses, they contain 70 yods made up of the 10 corners of tetractyses at the positions of the Sephiroth and 60 hexagonal yods (*fig. 6*). When each sector of a square is divided into three triangles that are then turned into tetractyses, the square is found to contain 61 yods. We see that 60 extra yods are needed to construct such a transformed square from the point at its centre, just as 60 additional yods are required to construct the outer form of the Tree of Life from tetractyses.

5. CONCLUSION

The simple square symbolises the number 4, or tetrad, so highly esteemed by the Pythagoreans. The four integers 1, 2, 3 & 4 sum to 10, which is the number of Sephiroth of the Tree of Life and the number of points in the tetractys — an equivalent symbol of the ten-fold nature of divine unity. A square divided into tetractyses has 25 yods symbolising the 25 spatial dimensions of the universe. A square with each sector divided into three tetractyses has 60 yods surrounding its centre that correspond to the 60 extra yods making up the triangles of the Tree of Life when they are turned into tetractyses. A square with each sector divided into three triangles that are then divided into three tetractyses contains **168** yods surrounding its centre. This number characterises the manifestation of the Tree of Life in the subatomic world as the number of circularly polarised waves made by each one of the ten string components of a spinning superstring when it completes half a revolution in space, that is, one-tenth of the five complete revolutions of each component. The number **168** is defined arithmetically by a square because it is the sum of the twelve, successive odd integers after 1 that can be spaced along the sides of a square, four to each edge. A square with each sector turned into the next higher order tetractys contains 248 yods other than corners of tetractyses. These yods, which symbolise the seven Sephiroth of Construction, signify the 248 quantum states of the particle transmitting the unified force acting between superstrings of either ordinary or shadow matter. A 4×4 square array of the squares of the first four odd integers has as its sum the number (**496**) of such states responsible for the interactions of superstrings of *both* kinds of matter. Successive transformations of the square generate numbers corresponding to ever more *concrete* realisations of the cosmic blueprint. Truly, one must agree with the Pythagoreans in their calling the number 4 “the greatest miracle” and “holding the key of Nature.”

REFERENCES

1. J. Adams, *The Republic of Plato,* Cambridge University Press, Cambridge, 1929.