

# Geometrodynamics, Inertia and the Quantum Vacuum

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## ABSTRACT

Why does  $\mathbf{F}$  equal  $m\mathbf{a}$  in Newton's equation of motion? How does a gravitational field produce a force? Why are inertial mass and gravitational mass the same? It appears that all three of these seemingly axiomatic foundational questions have an answer involving an identical physical process: interaction between the electromagnetic quantum vacuum and the fundamental charged particles (quarks and electrons) constituting matter. All three of these effects and equalities can be traced back to the appearance of a specific asymmetry in the otherwise uniform and isotropic electromagnetic quantum vacuum. This asymmetry gives rise to a non-zero Poynting vector from the perspective of an accelerating object. We call the resulting energy-momentum flux the *Rindler flux*. The key insight is that the asymmetry in an accelerating reference frame in flat spacetime is identical to that in a stationary reference frame (one that is not falling) in curved spacetime. Therefore the same Rindler flux that creates inertial reaction forces also creates weight. All of this is consistent with the conceptualization and formalism of general relativity. What this view adds to physics is insight into a specific physical process creating identical inertial and gravitational forces from which springs the weak principle of equivalence. What this view hints at in terms of advanced propulsion technology is the possibility that by locally modifying either the electromagnetic quantum vacuum and/or its interaction with matter, inertial and gravitational forces could be modified.

## INTRODUCTION

Why does  $\mathbf{F}$  equal  $m\mathbf{a}$  in Newton's equation of motion,  $\mathbf{F} = m\mathbf{a}$ ?

How does a gravitational field produce a force?

Why are inertial mass and gravitational mass the same?

These are questions that are usually thought to be more appropriate for philosophers than for physicists, since the apparent facts of nature addressed in these questions are generally regarded as axiomatic.[1] If one assumes that one plus one equals two, plus a limited number of additional axioms, one can develop a self-consistent system of mathematics, but one has to start with such assumptions. In the realm of geometry, the discovery of non-Euclidean geometry in the 19th century taught us that other, non-intuitive assumptions are possible, and indeed, Riemannian geometry became the basis for physics in Einstein's general relativity (GR). Alternate foundational assumptions can lead to new insights.

Since 1991 we have been engaged in investigations involving the nature of inertia based on the foundational assumption that the electromagnetic quantum vacuum, also called the zero-point field or zero-point

fluctuations, is a real and underlying universal sea of energy capable of interacting with matter, and that this interaction may be described using the techniques of stochastic electrodynamics (SED).[2] Following a successful multiyear NASA-funded investigation at Lockheed Martin and California State University at Long Beach, the privately-funded California Institute for Physics and Astrophysics (CIPA) was established in 1999 specifically to study the electromagnetic quantum vacuum and its effects. A group of five postdoctoral fellows with expertise in quantum electrodynamics, superstring and M-brane theory, general relativity, plasma physics and Casimir effects are engaged in theoretical efforts to come to a deeper understanding of foundational questions in physics and to examine the possible nature and degree of electromagnetic quantum vacuum–matter interactions both within and beyond the SED approximations. CIPA has also funded projects by experts at several universities directed toward the general goal of explaining the origin of inertia and certain aspects of gravitation and other relevant physical and astrophysical effects as due to the electromagnetic quantum vacuum.

It appears that all three of the seemingly axiomatic foundational questions posed above have an answer involving an identical physical process: interaction between the electromagnetic quantum vacuum and the fundamental charged particles (quarks and electrons) constituting matter. All three of these effects and equalities can be traced back to the appearance of a specific asymmetry in the otherwise uniform and isotropic electromagnetic quantum vacuum. The key insight is that the asymmetry in an accelerating reference frame in flat spacetime is identical to that in a stationary reference frame in curved spacetime.

It was shown by Unruh [3] and by Davies [4] that a uniformly-accelerating detector will experience a Planckian-like heat bath whose apparent “temperature” is a result of quantum vacuum radiation. A tiny fraction of the (enormous) electromagnetic quantum vacuum energy can emerge as real radiation under the appropriate conditions. The theoretical prediction of Unruh-Davies radiation is now generally accepted and SLAC physicist P. Chen has recently proposed an experiment to detect and measure it.[5] Rueda and Haisch (hereafter RH) [6] analyzed a related process and found that as perceived by an accelerating object, an energy and momentum flux of radiation emerges from the electromagnetic quantum vacuum and that the strength of this momentum flux is such that the radiation pressure force on the accelerating object is proportional to acceleration. Owing to its origin in an accelerating reference frame customarily called the *Rindler frame*, and to the relation of this flux to the existence of a *Rindler event horizon*, we call this flux of energy and momentum that emerges out of the electromagnetic quantum vacuum upon acceleration the *Rindler flux*.

If the Rindler flux is allowed to electromagnetically interact with matter, mainly but perhaps not exclusively at the level of quarks and electrons, a reaction force is produced that can be interpreted as the origin of Newton’s  $\mathbf{F} = m\mathbf{a}$ . In this view, which we call the *quantum vacuum inertia hypothesis*, matter resists acceleration not because of some innate property of inertia, but rather because the quantum vacuum fields provide an acceleration-dependent drag force. In future attempts we and coworkers intend to examine the possible contributions of other components of the quantum vacuum besides the electromagnetic. (This is relevant to the issue of possible neutrino mass, which could not be due to the electromagnetic quantum vacuum, but might possibly be due to the vacuum fields of the weak interaction.)

GR declares that gravity can be interpreted as spacetime curvature. Wheeler coined the term geometrodynamics to describe this: the dynamics of objects subject to gravity is determined by the geometry of four-dimensional spacetime. What geometrodynamics actually specifies is the family of geodesics — the shortest four-dimensional distances between two points in spacetime — in the presence of a gravitating body. Freely-falling objects and light rays follow geodesics. However when an object is prevented from following a geodesic trajectory, a force is experienced: the well-known force called weight. Where does this force come from? Or put another way, how does a gravitational field exert a force on a non freely-falling, fixed, object, such as an observer standing on a scale on the Earth’s surface? *This proves to be the identical physical process as described in the quantum vacuum inertia hypothesis, due to a non-zero Rindler flux.*

In the SED approximation, the electromagnetic quantum vacuum is represented as propagating electromagnetic waves.[7,8] These should follow geodesics. It can be shown that propagation along curved geodesics creates the identical electromagnetic Rindler flux with respect to a stationary fixed object as is the case for an

accelerating object.[9] This is perfectly consistent with Einstein’s fundamental assumption of the equivalence of gravitation and acceleration. An object fixed above a gravitating body will perceive the electromagnetic quantum vacuum to be accelerating past it, which is of course the same as the perception of the object when it is doing the accelerating through the quantum vacuum. Thus in the case of gravity, it would be the electromagnetic Rindler flux acting upon a fixed object that creates the force known as weight, thereby answering the second question. The answer to the third question then immediately follows. Since the same flux would be seen by either a fixed object in a gravitational field or an accelerating object in free space, the force that is felt would be the same, hence the parameters we traditionally call inertial and gravitational mass must be the same. This would explain the physical origin of the weak principle of equivalence.

All of this is consistent with the mathematics of GR. What this view adds to physics is insight into a specific physical process creating identical inertial and gravitational forces. What this view hints at in terms of advanced propulsion technology is the possibility that by locally modifying either the quantum vacuum fields and/or their interaction with matter, inertial and gravitational forces could be modified and possibly one day freely controlled.

### THE ELECTROMAGNETIC QUANTUM VACUUM

The quantization of the electromagnetic field in terms of quantum-mechanical operators may be found in various standard textbooks, e.g. Loudon [10]: “The electromagnetic field is now quantized by the association of a quantum-mechanical harmonic oscillator with each mode  $\mathbf{k}$  of the radiation field.” This can be understood as follows: Application of the Heisenberg uncertainty relation to a harmonic oscillator requires that its ground state have a non-zero energy of  $h\nu/2$ . This reflects the fact that quantum mechanically a particle cannot simultaneously be exactly at the bottom of its potential well and have exactly zero momentum. There exists the same  $h\nu/2$  zero-point energy expression for each mode of the electromagnetic field as for a mechanical oscillator. (Formally, mode decomposition yields that each mode can be mathematically made into a harmonic oscillator in the sense that the same differential equation is obeyed as for a mechanical oscillator.)

Summing up the energy over the modes for all frequencies, directions, and polarization states, one arrives at a zero-point energy density for the electromagnetic fluctuations, and this is the origin of the electromagnetic quantum vacuum. An energy of  $h\nu/2$  per mode of the field characterizes the fluctuations of the quantized radiation field in quantum field theory. In the semi-classical representation of SED the quantum vacuum is represented by propagating electromagnetic plane waves,  $\mathbf{E}^{zp}$  and  $\mathbf{B}^{zp}$ , of random phase having this average energy,  $h\nu/2$ , in each mode.

The volumetric density of modes between frequencies  $\nu$  and  $\nu + d\nu$  is given by the density of states function  $N_\nu d\nu = (8\pi\nu^2/c^3)d\nu$ . Each state has a minimum  $h\nu/2$  of energy, and using this density of states function and this minimum energy — that we call the zero-point energy — per state one gets the spectral energy density of the electromagnetic quantum vacuum:

$$\rho(\nu)d\nu = \frac{8\pi\nu^2}{c^3} \frac{h\nu}{2} d\nu. \tag{1}$$

Writing this zero-point radiation together with ordinary blackbody radiation, the energy density is:

$$\rho(\nu, T)d\nu = \frac{8\pi\nu^2}{c^3} \left( \frac{h\nu}{e^{h\nu/kT} - 1} + \frac{h\nu}{2} \right) d\nu. \tag{2}$$

The first term (outside the parentheses) represents the mode density, and the terms inside the parentheses are the average energy per mode of thermal radiation at temperature  $T$  plus the zero-point energy,  $h\nu/2$ , which has no temperature dependence. Take away all thermal energy by formally letting  $T$  go to zero, and one is still left with the zero-point term. The laws of quantum mechanics as applied to electromagnetic

radiation force the existence of a background sea of electromagnetic zero-point energy that is traditionally called the electromagnetic quantum vacuum.

The spectral energy density of eqn. (1) was thought to be no more than a spatially uniform constant offset that cancels out when considering energy fluxes, but it was discovered in the mid-1970's that the quantum vacuum acquires special characteristics when viewed from an accelerating frame. Just as there is an event horizon for a black hole, there is an analogous event horizon for an accelerating reference frame. Similar to radiation from evaporating black holes proposed by Hawking [11], Unruh [3] and Davies [4] determined that a Planck-like radiation component will arise out of the quantum vacuum in a uniformly-accelerating coordinate system having constant proper acceleration  $\mathbf{a}$  (where  $|\mathbf{a}| = a$ ) with what amounts to an effective "temperature"

$$T_a = \frac{\hbar a}{2\pi c k}. \quad (3)$$

This "temperature" characterizing Unruh-Davies radiation does not originate in emission from particles undergoing thermal motions. <sup>a</sup> As discussed by Davies, Dray and Manogue [12]:

One of the most curious properties to be discussed in recent years is the prediction that an observer who accelerates in the conventional quantum vacuum of Minkowski space will perceive a bath of radiation, while an inertial observer of course perceives nothing. In the case of linear acceleration, for which there exists an extensive literature, the response of a model particle detector mimics the effect of its being immersed in a bath of thermal radiation (the so-called Unruh effect).

This "heat bath" is a quantum phenomenon. The "temperature" is negligible for most accelerations. Only in the extremely large gravitational fields of black holes or in high-energy particle collisions can this become significant. Recently, P. Chen at the Stanford Linear Accelerator Center has proposed using an ultra high intensity laser to accelerate electrons violently enough to directly detect Unruh-Davies radiation.[5]

Unruh and Davies treated the electromagnetic quantum vacuum as a scalar field. If a true vectorial approach is considered there appear additional terms beyond the quasi-thermal Unruh-Davies component. For the case of no true external thermal radiation ( $T = 0$ ) but including the acceleration effect ( $T_a$ ), eqn. (1) becomes

$$\rho(\nu, T_a) d\nu = \frac{8\pi\nu^2}{c^3} \left[ 1 + \left( \frac{a}{2\pi c\nu} \right)^2 \right] \left[ \frac{h\nu}{2} + \frac{h\nu}{e^{h\nu/kT_a} - 1} \right] d\nu, \quad (4)$$

where the acceleration-dependent pseudo-Planckian Unruh-Davies component is placed after the  $h\nu/2$  term to indicate that except for extreme accelerations (e.g. particle collisions at high energies) this term is negligibly small. While these additional acceleration-dependent terms do not show any spatial asymmetry in the expression for the spectral energy density, certain asymmetries do appear when the momentum flux of this radiation is calculated, resulting in a non-zero Rindler flux.[6] This asymmetry appears to be the process underlying inertial and gravitational forces.

## ORIGIN OF THE INERTIAL REACTION FORCE

Newton's third law states that if an agent applies a force to a point on an object, at that point there arises an equal and opposite reaction force back upon the agent. In the case of a fixed object the equal and opposite reaction force can be traced to interatomic forces in the neighborhood of the point of contact which act to resist compression, and these in turn can be traced to electromagnetic interactions involving orbital electrons of adjacent atoms or molecules, etc.

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<sup>a</sup> There is likely to be a deep connection between the fact that the spectrum that arises in this fashion due to acceleration and the ordinary blackbody spectrum have identical form.

Now a similar experience of an equal and opposite reaction force arises when a non-fixed object is forced to accelerate. Why does acceleration create such a reaction force? We suggest that this equal and opposite reaction force also has an underlying cause which is at least partially electromagnetic, and specifically may be due to the scattering of electromagnetic quantum vacuum radiation. RH demonstrated that from the point of view of the pushing agent there exists a net flux (Poynting vector) of quantum vacuum radiation transiting the accelerating object in a direction opposite to the acceleration: the Rindler flux.[6] Interaction of this flux with the quarks and electrons constituting a material object would create a back reaction force that can be interpreted as inertia. One simply needs to assume that there is some dimensionless efficiency factor,  $\eta(\omega)$ , that in the case of particles corresponds to whatever the interaction process is (e.g. dipole scattering). In the case of elementary particles we suspect that  $\eta(\omega)$  contains one or more resonances — and in the Appendix discuss why these resonances likely involve Compton frequencies of relevant particles forming a composite particle or object — but this is not a necessary assumption.

The RH approach relies on making transformations of the  $\mathbf{E}^{zp}$  and  $\mathbf{B}^{zp}$  from a stationary to a uniformly-accelerating coordinate system (see, for example, §11.10 of Jackson for the relevant transformations [13]). In a stationary or uniformly-moving frame the  $\mathbf{E}^{zp}$  and  $\mathbf{B}^{zp}$  constitute an isotropic radiation pattern. In a uniformly-accelerating frame the radiation pattern acquires asymmetries. There appears a non-zero Poynting vector in any accelerating frame, and therefore a non-zero Rindler flux which carries a net flux of electromagnetic momentum. The scattering of this momentum flux generates a reaction force,  $\mathbf{F}_r$ , proportional to the acceleration. RH found an invariant scalar with the dimension of mass quantifying the inertial resistance force of opposition per unit of acceleration resulting from this process. We interpret this scalar as the inertial mass,

$$m_i = \frac{V_0}{c^2} \int \eta(\nu) \rho_{zp}(\nu) d\nu, \quad (5)$$

where  $\rho_{zp}$  is the well known spectral energy density of the electromagnetic quantum vacuum of eqn. (1). In other words, the amount of electromagnetic zero point energy instantaneously transiting through an object of volume  $V_0$  and interacting with the quarks, electrons and all charges in that object is what constitutes the inertial mass of that object in this view. It is change in the momentum of the radiation field that creates the resistance to acceleration usually attributed to the inertia of an object.

Indeed, not only does the ordinary form of Newton's second law,  $\mathbf{F} = m_i \mathbf{a}$ , emerge from this analysis, but one can also obtain the relativistic form of the second law: [6]

$$\mathcal{F} = \frac{d\mathcal{P}}{d\tau} = \frac{d}{d\tau}(\gamma_\tau m_i c, \mathbf{p}). \quad (6)$$

The origin of inertia, in this picture, becomes remarkably intuitive. Any material object resists acceleration because the acceleration produces a perceived flux of radiation in the opposite direction that scatters within the object and thereby pushes against the accelerating agent. Inertia in the present model appears as a kind of acceleration-dependent electromagnetic quantum vacuum drag force acting upon electromagnetically-interacting elementary particles. The relativistic law for “mass” transformation involving the Lorentz factor  $\gamma$  — that is, the formula describing how the *inertia* of a body has been calculated to change according to an observer's relative motion — is automatically satisfied in this view, because the correct relativistic form of the reaction force is derived, as shown in eqn. (6).

## ORIGIN OF WEIGHT AND THE WEAK EQUIVALENCE PRINCIPLE

Einstein introduced the *local Lorentz invariance* (LLI) principle in order to pass from special relativity to GR. It is possible to use this principle immediately to extend the results of the quantum vacuum inertia hypothesis to gravitation (details discussed in two forthcoming papers [9]).

The idea behind the LLI principle is embodied in the Einstein elevator thought experiment. He proposed that a freely-falling elevator in a gravitational field is equivalent to one that is not accelerating and is far

from any gravitating body. Physics experiments would yield the same results in either elevator, and therefore a freely-falling coordinate frame in a gravitational field is the same as an inertial Lorentz frame. (This is rigorously only true for a “small elevator” since a gravitational field around a planet, say, must be radial, hence there are inevitably tidal forces which would not be the case for an ideal acceleration.) The device Einstein used to develop general relativity was to invoke an infinite set of such freely falling frames. In each such frame, the laws of physics are those of special relativity. The additional features of general relativity emerge by comparing the properties of measurements made in freely-falling Lorentz frames “dropped” one after the other.

This approach of Einstein is both elegant and powerful. The LLI principle immediately tells us that an object accelerating through the electromagnetic quantum vacuum is equivalent to an object held fixed in a gravitational field while the electromagnetic quantum vacuum is effectively accelerating (falling) past it. The prediction of GR that light rays deviate from straight-line propagation in the presence of a gravitating body — which Eddington measured in 1919 thereby validating GR — translates into acceleration (falling) of the electromagnetic quantum vacuum. An object accelerating through the electromagnetic quantum vacuum experiences a Rindler flux which causes the inertia reaction force. A fixed object past which the electromagnetic quantum vacuum is accelerating, following the laws of GR, experiences the same Rindler flux and the resulting force is what we call weight. That is why  $m_g = m_i$  and is the basis of the weak equivalence principle.

## CONCLUSIONS

Geometrodynamics is an elegant theoretical structure, but there is a very fundamental physics question that geometrodynamics has never satisfactorily addressed. If an object is forced to deviate from its natural geodesic motion, a reaction force arises, i.e. the weight of an object. Where does the reaction force that its weight come from? That same force would also be the enforcer of geodesic motion for freely falling objects. GR specifies the metric of spacetime from which geodesics can be calculated, but is there a physical mechanism to keep freely-falling objects from straying from their proper geodesics? Geometrodynamics does not provide a physical mechanism for this. It can only claim that deviations of an object from its proper geodesic motion results in an inertial reaction force. This is true but uninformative. The quantum vacuum inertia hypothesis provides a physical process generating inertia and weight.

Quantum physics predicts the existence of an underlying sea of zero-point energy at every point in the universe. This is different from the cosmic microwave background and is also referred to as the electromagnetic quantum vacuum since it is the lowest state of otherwise empty space. This sea of energy fills all of space and is absolutely the same everywhere as perceived from a constant velocity reference frame. But viewed from an accelerating reference frame, the radiation pattern of the energy becomes minutely distorted: a tiny directional flow is experienced by an accelerating object or observer, the Rindler flux. Importantly, the force resulting from that energy-momentum flow turns out to be proportional to the acceleration. When this energy-momentum flow — that arises automatically when any object accelerates — interacts with the fundamental particles constituting matter (quarks and electrons) a force arises in the direction opposite to the acceleration. This process can be interpreted as the origin of inertia, i.e. as the basis of Newton’s second law of mechanics:  $\mathbf{F} = m\mathbf{a}$  (and its relativistic extension).[6]

It has now been discovered that exactly the same distortion of the radiation pattern occurs in geometrodynamics when the metric is non-Minkowskian.[9] The curved spacetime geodesics of geometrodynamics affect the zero-point energy in the same way as light rays (because the zero-point energy is also a mode of electromagnetic radiation). The gravitational force causing weight and the reaction force causing inertia originate in an identical interaction with a distortion in the radiation of the zero-point energy field. Both are a kind of radiation pressure originating in the electromagnetic quantum vacuum. The underlying distortion of the radiation pattern is due to an event horizon-like effect and is related to Unruh-Davies radiation and Hawking radiation.

What the quantum vacuum inertia hypothesis accomplishes is to identify the physical process which is the enforcer of geometrodynamics or general relativity. The quantum vacuum inertia hypothesis appears

to provide a link between light propagation along geodesics and mechanics of material objects. Moreover, since the distortion of the zero-point energy radiation pattern is the same whether due to acceleration or being held stationary in a gravitational field, this explains a centuries old puzzle: why inertial mass and gravitational mass are the same: both are due to the same non-zero Rindler flux. This gives us a deeper insight into Einstein's principle of equivalence.

## APPENDIX: INERTIA AND THE DE BROGLIE WAVELENGTH

Four-momentum is defined as

$$\mathbf{P} = \left( \frac{E}{c}, \mathbf{p} \right) = (\gamma m_0 c, \mathbf{p}) = (\gamma m_0 c, \gamma m_0 \mathbf{v}), \quad (A1)$$

where  $|\mathbf{P}| = m_0 c$  and  $E = \gamma m_0 c^2$ . The Einstein-de Broglie relation defines the Compton frequency  $h\nu_C = m_0 c^2$  for an object of rest mass  $m_0$ , and if we make the de Broglie assumption that the momentum-wave number relation for light also characterizes matter then  $\mathbf{p} = \hbar \mathbf{k}_B$  where  $\mathbf{k}_B = 2\pi(\lambda_{B,1}^{-1}, \lambda_{B,2}^{-1}, \lambda_{B,3}^{-1})$ . We thus write

$$\frac{\mathbf{P}}{\hbar} = \left( \frac{2\pi\gamma\nu_C}{c}, \mathbf{k}_B \right) = 2\pi \left( \frac{\gamma}{\lambda_C}, \frac{1}{\lambda_{B,1}}, \frac{1}{\lambda_{B,2}}, \frac{1}{\lambda_{B,3}} \right) \quad (A2)$$

and from this obtain the relationship

$$\lambda_B = \frac{c}{\gamma v} \lambda_C \quad (A3)$$

between the Compton wavelength,  $\lambda_C$ , and the de Broglie wavelength,  $\lambda_B$ . For a stationary object  $\lambda_B$  is infinite, and the de Broglie wavelength decreases in inverse proportion to the momentum.

Eqn. (5) is very suggestive that quantum vacuum-elementary particle interaction involves a resonance at the Compton frequency. De Broglie proposed that an elementary particle is associated with a localized wave whose frequency is the Compton frequency. As summarized by Hunter [14]: "... what we regard as the (inertial) mass of the particle is, according to de Broglie's proposal, simply the vibrational energy (divided by  $c^2$ ) of a localized oscillating field (most likely the electromagnetic field). From this standpoint inertial mass is not an elementary property of a particle, but rather a property derived from the localized oscillation of the (electromagnetic) field. De Broglie described this equivalence between mass and the energy of oscillational motion... as '*une grande loi de la Nature*' (a great law of nature)."

This perspective is consistent with the proposition that inertial mass,  $m_i$ , may be a coupling parameter between electromagnetically interacting particles and the quantum vacuum. Although De Broglie assumed that his wave at the Compton frequency originates in the particle itself (due to some intrinsic oscillation or circulation of charge perhaps) there is an alternative interpretation discussed in some detail by de la Peña and Cetto that a particle "is tuned to a wave originating in the high-frequency modes of the zero-point background field." [8] The de Broglie oscillation would thus be due to a resonant interaction with the quantum vacuum, presumably the same resonance that is responsible for creating a contribution to inertial mass as in eqn. (5). In other words, the electromagnetic quantum vacuum would be driving this  $\nu_C$  oscillation.

We therefore suggest that an elementary charge driven to oscillate at the Compton frequency,  $\nu_C$ , by the quantum vacuum may be the physical basis of the  $\eta(\nu)$  scattering parameter in eqn. (5). For the case of the electron, this would imply that  $\eta(\nu)$  is a sharply-peaked resonance at the frequency, expressed in terms of energy,  $h\nu_C = 512$  keV. The inertial mass of the electron would physically be the reaction force due to resonance scattering of the electromagnetic quantum vacuum radiation, the Rindler flux, at that frequency.

*This leads to a surprising corollary.* It has been shown that as viewed from a laboratory frame, a standing wave at the Compton frequency in the electron frame transforms into a traveling wave having the de Broglie wavelength for a moving electron. [8,14,15,16] The wave nature of the moving electron (as measured in the Davisson-Germer experiment, for example) would be basically due to Doppler shifts associated with its Einstein-de Broglie resonance at the Compton frequency. A simplified heuristic model shows this, and

a detailed treatment showing the same result may be found in de la Peña and Cetto [8]. Represent a quantum vacuum-like driving force field as two waves having the Compton frequency  $\omega_C = 2\pi\nu_C$  travelling in equal and opposite directions,  $\pm\hat{x}$ . The amplitude of the combined oppositely-moving waves acting upon an electron will be

$$\phi = \phi_+ + \phi_- = 2 \cos \omega_C t \cos k_C x. \quad (A4)$$

But now assume an electron is moving with velocity  $v$  in the  $+x$ -direction. The wave responsible for driving the resonant oscillation impinging on the electron from the front will be the wave seen in the laboratory frame to have frequency  $\omega_- = \gamma\omega_C(1 - v/c)$ , i.e. it is the wave below the Compton frequency in the laboratory that for the electron is Doppler shifted up to the  $\omega_C$  resonance. Similarly the zero-point wave responsible for driving the electron resonant oscillation impinging on the electron from the rear will have a laboratory frequency  $\omega_+ = \gamma\omega_C(1 + v/c)$  which is Doppler shifted down to  $\omega_C$  for the electron. The same transformations apply to the wave numbers,  $k_+$  and  $k_-$ . The Lorentz invariance of the electromagnetic quantum vacuum spectrum ensures that regardless of the electron's (unaccelerated) motion the up- and down-shifting of the laboratory-frame spectral energy density will always yield a standing wave in the electron's frame.

It can be shown [8,15] that the superposition of these two oppositely-moving, Doppler-shifted waves is

$$\phi' = \phi'_+ + \phi'_- = 2 \cos(\gamma\omega_C t - k_B x) \cos(\omega_B t - \gamma k_C x). \quad (A5)$$

Observe that for fixed  $x$ , the rapidly oscillating “carrier” of frequency  $\gamma\omega_C$  is modulated by the slowly varying envelope function in frequency  $\omega_B$ . And *vice versa* observe that at a given  $t$  the “carrier” in space appears to have a relatively large wave number  $\gamma k_C$  which is modulated by the envelope of much smaller wave number  $k_B$ . Hence both timewise at a fixed point in space and spacewise at a given time, there appears a carrier that is modulated by a much broader wave of dimension corresponding to the de Broglie time  $t_B = 2\pi/\omega_B$ , or equivalently, the de Broglie wavelength  $\lambda_B = 2\pi/k_B$ .

This result may be generalized to include quantum vacuum radiation from all other directions, as may be found in the monograph of de la Peña and Cetto [8]. They conclude by stating: “The foregoing discussion assigns a physical meaning to de Broglie’s wave: it is the *modulation* of the wave formed by the Lorentz-transformed, Doppler-shifted superposition of the whole set of random stationary electromagnetic waves of frequency  $\omega_C$  with which the electron interacts selectively.”

Another way of looking at the spatial modulation is in terms of the wave function: the spatial modulation of eqn. (A5) is exactly the  $e^{ipx/\hbar}$  wave function of a freely moving particle satisfying the Schrödinger equation. The same argument has been made by Hunter [14]. In such a view the quantum wave function of a moving free particle becomes a “beat frequency” produced by the relative motion of the observer with respect to the particle and its oscillating charge.

It thus appears that a simple model of a particle as an electromagnetic quantum vacuum-driven oscillating charge with a resonance at its Compton frequency may simultaneously offer insight into the nature of inertial mass, i.e. into rest inertial mass and its relativistic extension, the Einstein-de Broglie formula and into its associated wave function involving the de Broglie wavelength of a moving particle. If the de Broglie oscillation is indeed driven by the electromagnetic quantum vacuum, then it is a form of Schrödinger’s *zitterbewegung*. Moreover there is a substantial literature attempting to associate spin with *zitterbewegung* tracing back to the work of Schrödinger [17]; see for example Huang [18] and Barut and Zanghi [19].

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