"On the Orbit of the Seventh Satellite of Jupiter."
Daniel Walter Morehouse.

Dissertation submitted in partial fulfillment of the requirements for the Degree of Doctor of Philosophy in the University of California, April 6th, 1914.

Approved by Sub-Committee in Charge.

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Accepted for Publication in Lick Observatory Bulletin, after revision by the Berkeley Astronomical Dept.

W. W. Campbell
Editor.

UNIVERSITY OF CALIFORNIA
DEAN OF THE GRADUATE SCHOOL
BERKELEY

April 7, 1914.

To the Sub-Committee in Charge of the Candidacy of Daniel Walter Morehouse for the Degree of Doctor of Philosophy:

Gentlemen:

Herewith, please find the dissertation of Mr. Morehouse "On the Orbit of the Seventh Satellite of Jupiter". While the work has not been carried through to the point originally contemplated, Mr. Morehouse has done a very large amount of ingenious work. He has met with unforeseen difficulties in the way of short period perturbations, but is now entirely on the right track for the ultimate solution of this difficult problem. The thesis forms a very acceptable record of the investigation necessary to overcome the initial difficulties. For publication, it should be condensed to about one-half its size.

The dissertation in itself does not reveal the enormous amount of work done by Mr. Morehouse, who has had two computers at his own expense for the past two years. I have no hesitation in approving the thesis on my part.

Very truly yours,

A. O. Leuschner

ii

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ON THE ORBIT OF THE SEVENTH SATELLITE OF JUPITER.

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At the time this discussion was started there had been computed three orbits known to the writer. The first was computed by Dr. C. D. Perrine at the Lick Observatory and published in the *Lick Observatory Bulletin, No. 78.* The second was by Dr. F. E. Ross and published in the *Lick Observatory Bulletin, No. 82.* This orbit was afterward recomputed on the basis of twelve observations, distributed uniformly over the observed arc from January 3rd, 1905, to September 25th, 1906, and was published in *Astronomische Nachrichten Band 174 Nr. 4175.* The third was computed by Dr. R. T. Crawford and at the Student's Observatory of the University of California, according to "Leuschner's General Theory of Satellite Orbits," which is published in volume VII of the *Lick Observatory Publications* and in *Theoretische Astronomie von Dr. W. Klinkerfuse, Neubearbeitung, von Prof. Dr. H. Buchholz. Anhang zu den Leuschner'schen Methoden der Bahnbestimmung.*

The fundamental positions upon which the third orbit is based are as follows:

Mean Place 1905.0

Date 1905	P. S. T.			R. A.			Dec.		
Jan. 3	7h.	12m.	0s.	1h.	16m.	23.68s.	+7°	13'	57".2
Feb. 8	7	10	0	1	34	40.59	+8	55	18.0
Mar. 6	7	40	0	1	54	24.15	+10	35	49.9

Place observ[ed]

With the velocities and accelerations in right ascension and declination computed for the middle date from these positions, the right ascensions and declinations for January 28.623611 and February 21.642361 G. M. T. 1905 were computed to see if the higher derivatives

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The fundamental positions upon which this orbit is based are as follows:

Mean Place 1905.0

Date 1905	P. S. T.			R. A.			Dec.		
Jan. 3	7h.	12m.	0s.	1h.	16m.	25.68s.	+7°	13'	57".2
Feb. 8	7	10	0	1	34	40.59	+8	55	13.0
Mar. 6	7	40	0	1	54	24.15	+10	35	19.9

With the velocities and accelerations in right ascension and declination computed for the middle date from these positions, the right ascensions and declinations for January 28.623611 and February 21.642361 G. M. T. 1905 were computed to see if the higher derivatives were negligible. The residuals showed that they were not negligible and that more observations must be taken into account, so that the third and fourth derivatives in the two coordinates with respect to the time could be utilized. Two more observations were therefore chosen:

Mean Place 1905.0

Date 1905	P. S. T.	R. A.	Dec.
Jan. 28	6h. 58m. 0s.	1h. 27m. 44.32s.	+8° 18' 10".7
Feb. 21	7 25 0	1 44 2.98	9 43 51.6

With these five observations new velocities and accelerations for the middle date were determined and a double solution for the orbit accomplished. A representation of an observation taken August 9, 1905, showed clearly that the retrograde orbit was not the true one and the direct orbit was therefore retained as the physical solution.

After the Special Perturbations(I)had been computed by Encke's Method, with eight day intervals over this period and the corresponding corrections to the satellites' coordinates made, the representations of the fundamental positions gave residuals corresponding to orbit I_a (Crawford):

		Jan. 3	Jan. 28	Feb. 21	March 6
(O-C)	$\Delta\alpha$	- 9".6	-2".3	+2".6	+ 1".0
	$\Delta\delta$	+23.9	-0.4	-0.6	-11.3

A differential correction was then made using the series formulae for f, g, δf, and δg, which gave for the first differentially corrected orbit the following elements:

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Mean Place 1905.0

Date 1905	P. S. T.	R. A.	Dec.
Jan. 28	6h. 50m. 0s.	1h. 27m. 44.32s.	+8° 18' 10".7
Feb. 21	7 25 0	1 44 2.98	9 45 51.6

With these five observations new velocities and accelerations for the middle date were determined and a double solution for the orbit accomplished. A representation of an observation taken August 9, 1905, showed clearly that the retrograde orbit was not the true one and the direct orbit was therefore retained as the physical solution.

After the Special Perturbations(I)had been computed by Encke's Method, with eight day intervals over this period and the corresponding corrections to the satellites' coordinates made, the representations of the fundamental positions gave residuals corresponding to orbit I_a (Crawford):

		Jan. 5	Jan. 28	Feb. 21	March 6
(O-C)	Δα	- 9".6	-2".5	+2".6	+ 1".0
	Δδ	+25.9	-0.4	-0.6	-11.5

A differential correction was then made using the series formulae for f, g, δf, and δg, which gave for the first differentially corrected orbit the following elements:

ORBIT I_b BASED ON PERTURBATIONS I.

Epoch	1905,	Feb.	8.600930	G. M. T.	
M_o	283°	4'	4″0		
Ω	288	19	59″4	Mean Equinox	
ω	187	29	41.4	and	
π	115	49	40.8	Equator	
i	25	39	23.5	1905.0	
log e		9.084645			
log a		8.893716			
μ		1.390269 per day.			
P		258.9424 days.			

The residuals for January 3rd and March 6th, 1905, were then reduced to:

		Jan 3.	March 6.
(O-C)	Δα	-3″5	-1″4
	Δδ	+3.8	-1.2

The Special Perturbations on the basis of these osculating elements were then continued at eight day intervals up to August 9th, 1905. The representation of the satellite's position for August 9.96 G. M. T. 1905 gave the following residuals:

(O-C) Δp=+1°4; Δs=+7″7

the original observations having been taken in position angle and distance.

It was at this point that the writer took up the problem of correcting the orbit. I now had reliable observations of the satellite in right ascension and declination for 1905 published in the *Lick Observatory Bulletin, No. 156.* In order to be sure that the

residuals obtained by Dr. Crawford were not spurious, I represented some August positions using his orbit I_b, the perturbations I, which he had computed, being included, and obtained the following residuals:

1905 G. M. T.

	Aug. 6.954824	Aug. 7.962500	Aug. 9.962500	Aug. 11.978125
(O-C) $\Delta\alpha$	-7' 5."8	-6' 59."1	-6'51."4	-6'37."0
$\Delta\delta$	-0'19."6	-0' 18."5	-0'14."9	-0' 8."6

	Aug. 12.989583
(O-C)	-6' 32."0
	-0' 7."2

These residuals do not seem consistent. I therefore made a recomputation using my second method (b) page 14 for the determination of the geocentric coordinates of Jupiter, the first method (a) having been used in the above residuals. These are:

1905 G. M. T.

	Aug. 6.954824	Aug. 7.962500	Aug. 9.962500	Aug. 11.978125
(O-C) $\Delta\alpha$	-7' 6."0	-6' 59."5	-6' 52."3	-6' 36."6
	-0'19."6	-0' 18."3	-0' 14."9	-0' 8."8

	Aug. 12.989583
(O-C) $\Delta\delta$	-6' 31."5
	-0' 7."2

In this work the dates were not corrected for aberration or the positions for parallax, it being intended only as a rough check and the residuals were not used in subsequent computation on the orbit. My plan was to correct Dr. Crawford's orbit by the application of differential corrections as developed in the "Leuschner's Short Method," using the Closed Expressions for f, g, δf, and δg. This method is published in residuals obtained by Dr. Crawford were not spurious, I represented some August positions using his orbit I_b, the perturbations I, which he had computed, being included, and obtained the following residuals:

1905 G. M. T.

	Aug. 6.954824	Aug. 7.962500	Aug. 9.962500	Aug. 11.978125
(O-C) $\Delta\alpha$	-7′ 5″.8	-6′ 59″.1	-6′ 51″.4	-6′ 37″.0
$\Delta\delta$	-0′ 19″.6	-0′ 18″.5	-0′ 14″.9	-0′ 8″.6

	Aug. 12.985763
(O-C)	-6′ 32″.0
	-0′ 7″.2

These residuals do not seem consistent. I therefore made a recomputation using my second method (b) page 12 for the determination of the geocentric coordinates of Jupiter, the first method (a) having been used in the above residuals. These are:

1905 G. M. T.

	Aug. 6.954824	Aug. 7.962500	Aug. 9.962500	Aug. 11.978125
(O-C) $\Delta\alpha$	-7′ 6″.0	-6′ 59″.5	-6′ 52″.5	-6′ 36″.6
	-0′ 19″.6	-0′ 18″.3	-0′ 14″.9	-0′ 8″.8

	Aug. 12.985763
(O-C) $\Delta\delta$	-6′ 31″.5
	-0′ 7″.2

In this work the dates were not corrected for aberration or the positions for parallax, it being intended only as a rough check and the residuals were not used in subsequent computation on the orbit. My plan was to correct Dr. Crawford's orbit by the application of differential corrections as developed in the "Leuschner's Short Method," using the Closed Expressions for f, g, ff, and gg. This method is published in

Theoretische Astronomie von Dr. W. Klinkerfues, neubearbeitung, von Prof. Dr. H. Buchholz; Zweiundachtzigste Vorlesung ab seite 477 bis 490.

Under the false impression that I should have a longer arc than the one included between Jan. 3rd and Aug. 9th, 1905, before attempting a differential correction I continued the computation of perturbations I from Aug. 16 to Dec. 30th, 1905, at eight day intervals, on the basis of Dr. Crawford's orbit I_b. I then represented a series of positions for December, published in the *Lick Observatory Bulletin, No. 156,* as follows:

1905 G. M. T.

		Dec. 2.931763	Dec. 3. 922627	Dec. 4.909664
(O-C)	Δα	-0' 4".8	-0' 2".9	-0' 0".5
	Δδ	+1' 17".5	+1' 15".6	+1' 15".9

		Dec. 22.825776	Dec. 23.831921	Dec. 25.361805*	Dec.29.836111
(O-C)	Δα	-1' 48".0	-2' 1".3	-2' 21".5	-3' 31".6
	Δδ	+1' 26".1	+1' 26".3	+1' 26".7	+1' 33".4

It was finally decided to retain Dr. Crawford's first and middle places and introduce as third place a position for August 9.962500 G. M. T. 1905, and make a differential correction to remove the residuals of Jan. 3rd and August 9th. The corrections for aberration and parallax were now applied to the residuals to be removed, on the basis of the satellite's distance obtained with the orbit to be corrected, i.e., the distance of the satellite from the earth as found by means of Dr. Crawford's orbit was used to correct the dates of Jan. 3rd and Aug. 9th for aberration, and the satellite's mean places for geocentric parallax.

*Greenwich observation.

The f's and g's for these corrected intervals were obtained by means of Closed Expressions, using both of the following sets of formulae:

(a) $f_t = \dfrac{x_t y'_0 - x'_0 y_t}{x_0 y'_0 - x'_0 y_0}$; $g_t = \dfrac{x_0 y_t - y_0 x_t}{x_0 y'_0 - x'_t y_0}$

(b) $f_t = 1 - \dfrac{\gamma^2}{r_0}$ $g_t^2 = (2 r_t r_0 - p\gamma_t^2)\gamma_t^2$ where

$\gamma = \sqrt{2a} \sin \bar{g}$, and $2\bar{g} = E_t - E_0$; and γ_t, $\sin \bar{g}$ ^{2}

and g must have the same sign and are negative for dates previous to the epoch. In the first set of formulae (a) the x, y, and z's are the satellite's rectangular equatorial coordinates referred to Jupiter, and uncorrected for perturbations. The f's and g's from these two formulae were found to check.

In the solution for the *sixteen differential coefficients I,* based on orbit I_b, the geocentric equatorial coordinates ξ_0 η_0 ζ_0 and the geocentric distance ρ_0 of the satellite for the middle date are needed, in addition to the jovocentric velocities x', y', and z'. Consider the equations:

$$\xi_0 = \rho_0 \cos\alpha_0 \cos\delta_0;\ \eta_0 = \rho_0 \cos\delta_0 \sin\alpha_0 \text{ and } \zeta_0 = \rho_0 \sin\delta_0$$

Since we have an approximate value of ρ_0 from our osculating orbit and since our osculating orbit must pass through the middle observed position we can obtain ξ_0, η_0, and ζ_0 from our observed right ascension and declination and our best value of ρ_0.

This was the method used in computing the first differential correction where Dr. Crawford's best value of $\log \rho_0 = 0.730278$ was employed, together with the satellite's observed mean place corrected for parallax on the basis of the same ρ_0. With these sixteen differential coefficients (See page 17) and the four residuals expressed in radians

The f's and g's for these corrected intervals were obtained by means of Closed Expressions, using both of the following sets of formulae:

$$(a)\quad f_t = \frac{x_t y_o' - x_o' y_t}{x_o y_o' - x_o' y_o} \; ; \qquad g_t = \frac{x_o y_t - y_o x_t}{x_o y_o' - x_o' y_o}$$

$$(b)\quad f_t = 1 - \frac{2\gamma^2}{r_o} \qquad g_t = (2r_t r_o - c\gamma^2)\gamma \quad \text{where}$$

$$\gamma = \sqrt{a}\,\sin\bar{\xi}, \text{ and } 2\bar{\xi} = E_t - E_o; \text{ and } \gamma_t, \sin\bar{\xi}$$

and g must have the same sign and are negative for dates previous to the epoch. In the first set of formulae (a) the x, y, and z's are the satellite's rectangular equatorial coordinates referred to Jupiter, and uncorrected for perturbations. The f's and g's from these two formulae were found to check.

In the solution for the sixteen differential coefficients I, based on orbit I_o, the geocentric equatorial coordinates ξ_o η_o ζ_o and the geocentric distance ρ_o of the satellite for the middle date are needed, in addition to the jovocentric velocities x', y', and z'. Consider the equations:

$$\xi_o = \rho_o \cos\alpha_o \cos\delta_o; \; \eta_o = \rho_o \cos\delta_o \sin\alpha_o \text{ and } \zeta_o = \rho_o \sin\delta_o$$

Since we have an approximate value of ρ_o from our osculating orbit and since our osculating orbit must pass through the middle observed position we can obtain ξ_o, η_o, and ζ_o from our observed right ascension and declination and our best value of ρ_o.

This was the method used in computing the first differential correction where Dr. Crawford's best value of log ρ_o=0.750278 was employed, together with the satellite's observed mean place corrected for parallax on the basis of the same ρ_o. With these sixteen differential coefficients (See page 17) and the four residuals expressed in radians for Jan. 3rd and Aug. 9th, the following equations were solved by Gauss's method of elimination:

$$a_i\delta\rho_o+b_i\delta x'_o+c_i\delta y'_o+d_i\delta z'_o=n_i (i=1,\ 2,\ 3,\ 4)$$

giving as a correction to the geocentric distance and velocities $\log\delta\rho_o=7.468111n$; $\log\delta x'_o=9.476219n$; $\log\delta y'_o=8.328423n$; $\log\delta z'_o=8.682262n$. To obtain the corrections to the jovicentric coordinates of the satellite for the middle date the relations

$$\delta x_o=\frac{\xi_o}{\rho_o}\delta\rho_o \qquad \delta y_o=\frac{\eta_o}{\rho_o}\delta\rho_o \qquad \delta z_o=\frac{\zeta_o}{\rho_o}\delta\rho_o$$

were used.

These equations are derived by differentiating the relations $(\xi)+x=\rho\cos\delta\cos\alpha$; $(\eta)+y=\rho\cos\delta\sin\alpha$; and $(\zeta)+z=\rho\sin\delta$ with respect to ρ considering x, y, and z as the variables. This is rigorous if the geocentric coordinates of Jupiter remain constant. But these remain constant only if the true middle date corresponding to the new geocentric distance of the satellite remain unchanged, or in other words, if the aberration time remain practically constant. An appreciable change in aberration time would require a small correction $\delta(\xi)$, $\delta(\eta)$, and $\delta(\zeta)$. We would then have $\delta x_o=\frac{\xi_o}{\rho_o}\delta\rho_o+\delta(\xi)$; $\delta y_o=\frac{\eta_o}{\rho_o}\delta\rho_o+\delta(\eta)$; and $\delta z_o=\frac{\zeta_o}{\rho_o}\delta\rho_o+\delta(\zeta)$. These corrections are negligible as will be shown on page 19.

The accuracy of the solution of the four equations

$$a_i\delta\rho_o+b_i\delta x'_o+c_i\delta y'_o+d_i\delta z'_o=n_i (i=1,\ 2,\ 3,\ 4)$$

was tested by substituting the computed values of the unknowns in the equations and proving them identities. These six corrected constants and the corrected geocentric distance gave for Jan. 3rd and Aug. 9th, the following equations were solved by Gauss's method of elimination:

$$a_i\delta\rho_0 + b_i\delta x'_0 + c_i\delta y'_0 + d_i\delta z'_0 = n_i \quad (i=1,\ 2,\ 3,\ 4)$$

giving as a correction to the geocentric distance and velocities $\log \delta\rho_0 = 7.4681111n$; $\log \delta x'_0 = 3.4706213n$; $\log \delta y'_0 = 8.5284229n$; $\log \delta z'_0 = 8.6822626n$. To obtain the corrections to the jovecentric coordinates of the satellite for the middle date the relations

$$\delta x_0 = \frac{\xi_0}{\rho_0}\delta\rho_0 \qquad \delta y_0 = \frac{\eta_0}{\rho_0}\delta\rho_0 \qquad \delta z_0 = \frac{\zeta_0}{\rho_0}\delta\rho_0$$

were used.

These equations are derived by differentiating the relations $(\xi)+x=\rho\cos\delta\cos\alpha$; $(\eta)+y=\rho\cos\delta\sin\alpha$; and $(\zeta)+z=\rho\sin\delta$ with respect to ρ considering x, y, and z as the variables. This is rigorous if the geocentric coordinates of Jupiter remain constant. But these remain constant only if the true middle date corresponding to the new geocentric distance of the satellite remain unchanged, or in other words, if the aberration time remain practically constant. An appreciable change in aberration time would require a small correction $\delta(\xi)$, $\delta(\eta)$, and $\delta(\zeta)$. We would then have $\delta x_0 = \frac{\xi_0}{\rho_0}\delta\rho_0 + \delta(\xi)$; $\delta y_0 = \frac{\eta_0}{\rho_0}\delta\rho_0 + \delta(\eta)$; and $\delta z_0 = \frac{\zeta_0}{\rho_0}\delta\rho_0 + \delta(\zeta)$. These corrections are negligible as will be shown on page 19.

The accuracy of the solution of the four equations

$$a_i\delta\rho_0 + b_i\delta x'_0 + c_i\delta y'_0 + d_i\delta z'_0 = n_i \quad (i=1,\ 2,\ 3,\ 4)$$

was tested by substituting the computed values of the unknowns in the equations and proving them identities. These six corrected constants and the corrected geocentric distance gave

ORBIT II BASED ON PERTURBATIONS I.

Epoch	1905	Feb. 8.600947		G. M. T.
M_o	297°	51'	11".1	
Ω	291	39	42.1	Mean Equinox
ω	177	41	31.5	and
π	109	21	13.6	Equator
i	26	10	25.3	1905.0.
e	0.195772			
log a	8.897738			
log μ	0.137067 per day			
P	262.56437 days.			

Constants for the Equator 1905.0

$$x=r(9.960048)\sin(201°\ 33'\ 50''.5+v).$$
$$y=r(9.994166)\sin(107\ \ 18\ \ 38.2+v).$$
$$z=r(9.644531)\sin(177\ \ 41\ \ 31.5+v).$$

The residuals obtained from this orbit are:

		Jan. 3.	Aug. 9.
(O-C)	Δα	+1' 2".1	-1' 31".8
	Δδ	+0' 14".4	+0' 31".3

Three of these residuals are larger than those to be removed, viz., the Δα and Δδ for January 3rd, and the Δδ for August 9th. When the residuals to be removed are as large as the Δα for August 9th, the linear relations are not satisfied and a second solution of the four equations,

$$a_i\delta\rho_o+b_i\delta x_o'+c_i\delta y_o'+d_i\delta z_o'=n_i\ (i=1,\ 2,\ 3,\ 4)$$

using the same differential coefficients and the remaining residuals, is generally advantageous. I therefore made a second solution of the above equations with new values of n_i from the above residuals. The

ORBIT II BASED ON PERTURBATIONS I.

Epoch 1905 Feb. 8.600947 G. M. T.

M_0	237°	51'	11".1	
Ω	291	59	42.1	Mean Equinox
ω	177	41	31.5	and
π	109	21	13.6	Equator
i	26	10	25.3	1905.0.
e	0.1957772			
log a	8.897758			
log μ	0.137067 per day			
p	262.56457 days.			

Constants for the Equator 1905.0

x=r(9.960048) sin (201° 55' 50".5 + v).

y=r(9.994166) sin (107 18 58.2 + v).

z=r(9.644531) sin (177 41 31.5 + v).

The residuals obtained from this orbit are:

		Jan. 3.	Aug. 9.
(O-C)	Δα	+1' 2".1	-1' 51".8
	Δδ	+0' 14".4	+0' 51".5

Three of these residuals are larger than those to be removed, viz., the Δα and Δδ for January 3rd, and the Δδ for August 9th. When the residuals to be removed are as large as the Δα for August 9th, the linear relations are not satisfied and a second solution of the four equations,

$$a_r\delta\rho_0+b_r\delta x'_0+c_r\delta y'_0+d_r\delta z'_0=n_r \quad (r=1, 2, 3, 4)$$

using the same differential coefficients and the remaining residuals, is generally advantageous. I therefore made a second solution of the above equations with new values of n_r from the above residuals. The result is interesting. The elements were changed to:

Epoch	1905	Feb. 8.600930		G. M. T.
M_0	277°	20'	44".3	
Ω	291	19	18.6	
ω	197	2	14.7	
i	25	7	47.0	
e	0.178696			
log a	8.894601			
log μ	0.141774 per day			
P	259.7341 days.			

The residuals for the same days were:

		Jan. 3	Aug. 9.
(O-C)	$\Delta\alpha$	-1' 22".4	+0' 51".5
	$\Delta\delta$	-0' 15".6	-0' 11".1

These residuals have exactly opposite signs to those above and are less satisfactory for Jan. 3rd. To test orbit II a little further, I represented Dec. 4 and 29, 1905, with the following residuals:

		Dec. 4	Dec. 29
(O-C)	$\Delta\alpha$	-38".3	+1' 25".2
	$\Delta\delta$	-19".1	+0' 27".1

If we compare these residuals for the four dates Jan. 3rd, Aug. 9th, Dec. 4th, and Dec. 29th, 1905, with the corresponding residuals computed with Dr. Crawford's orbit, we find that, excepting Jan. 3rd and Dec. 4th, the right ascensions represent much closer and the declinations are all improved with the exception of Jan. 3rd and Aug. 9th. The tabulation would be as follows:

result is interesting. The elements were changed to:

Epoch	1905	Feb. 8.600930	G. M. T.
M_0	277°	20'	44".5
Ω	291	19	18.6
ω	197	2	14.7
i	25	7	47.0
e	0.178696		
log a	8.591601		
log μ	0.141774 per day		
P	259.7541 days.		

The residuals for the same days were:

		Jan. 3	Aug. 9.
(O-C)	Δα	−1' 22".4	+0' 51".5
	Δδ	−0' 15".6	−0' 11".1

These residuals have exactly opposite signs to those above and are less satisfactory for Jan. 3rd. To test orbit II a little further, I represented Dec. 4 and 29, 1905, with the following residuals:

		Dec. 4	Dec. 29
(O-C)	Δα	−38".5	+1' 25".2
	Δδ	−19".1	+0' 27".1

If we compare these residuals for the four dates Jan. 3rd, Aug. 9th, Dec. 4th, and Dec. 29th, 1905, with the corresponding residuals computed with Dr. Crawford's orbit, we find that, excepting Jan. 3rd and Dec. 4th, the right ascensions represent much closer and the declinations are all improved with the exception of Jan. 3rd and Aug. 9th. The tabulation would be as follows:

ORBIT I_b AND PERTURBATIONS I.

		Jan. 3	Aug. 9	Dec. 4	Dec. 29
(O-C)	$\Delta\alpha$	-0' 3".5	-6' 31".4	-0' 0".5	-3' 31".6
	$\Delta\delta$	+0' 3".8	-0' 14".9	+1' 15".9	+1' 33".4

ORBIT II AND PERTURBATIONS I.

		Jan. 3	Aug. 9	Dec. 4	Dec. 29
(O-C)	$\Delta\alpha$	+1' 2".1	-1' 31".8	-0' 38".3	+1' 25".2
	$\Delta\delta$	+0' 14".4	+0' 31".3	-0' 19".1	+0' 27".1

It seems that the orbit I_b has been improved for the longer arc. New differential coefficients II based on Orbit II were now computed. The dates Jan. 3rd and August 9th were again corrected for aberration and the residuals for parallax. The f's and g's were obtained for the Closed Expressions. The corrections to the geocentric distance $\delta\rho_0$ and the velocities $\delta x'_0$, $\delta y'_0$, $\delta z'_0$, were $\log \delta\rho_0 = 7.737805$; $\log \delta x'_0 = 9.251133$; $\log \delta y'_0 = 9.266393n$; $\log \delta z'_0 = 8.807418$. Using the relations $\delta x_0 = -\frac{\xi_0}{\rho_0}\delta\rho$, etc., the corrections to the jovocentric equatorial coordinates were: $\log \delta x_0 = 7.694355$; $\log \delta y_0 = 7.336159$; $\log \delta z_0 = 6.928382$. The elements and residuals resulting from the correspondingly corrected constants are:

ORBIT III_a BASED ON PERTURBATIONS I AND DIFFERENTIAL COEFFICIENTS II.

Epoch	1905	Feb.	8.600915	G. M. T.
M_0	274°	8'	44".2	
Ω	291	34	17".9	Mean Equinox
ω	199	30	00.1	and
π	131	4	17.0	Equator
i	25	0	32.4	1905.0

ORBIT I_b AND PERTURBATIONS I.

		Jan. 3	Aug. 9	Dec. 4	Dec. 29
(O-C)	Δx	-0' 3".5	-6' 31".4	-0' 0".5	-3' 31".6
	Δδ	+0' 3".6	-0' 14".3	+1' 15".9	+1' 35".4

ORBIT II AND PERTURBATIONS I.

		Jan. 3	Aug. 9	Dec. 4	Dec. 29
(O-C)	Δx	+1' 2".1	-1' 51".8	-0' 38".5	+1' 29".7
	Δδ	+0' 14".4	+0' 51".3	-0' 19".1	+0' 27".1

It seems that the orbit I_b has been improved for the longer arc. New differential coefficients II based on Orbit II were now computed. The dates Jan. 3rd and August 9th were again corrected for aberration and the residuals for parallax. The f's and g's were obtained for the Closed Expressions. The corrections to the geocentric distance $\delta\rho_0$ and the velocities $\delta x'_0$, $\delta y'_0$, $\delta z'_0$, were log $\delta\rho_0$=7.757805; log $\delta x'_0$=3.251155; log $\delta y'_0$=9.266393n; log $\delta z'_0$=8.807416. Using the relations $\delta x_0=\frac{\xi_0}{\rho_0}\delta\rho$, etc., the corrections to the jovocentric equatorial coordinates were: log δx_0=7.694355; log δy_0=7.356159; log δz_0=6.928582. The elements and residuals resulting from the correspondingly corrected constants are:

ORBIT III_a BASED ON PERTURBATIONS I AND DIFFERENTIAL COEFFICIENTS II.

Epoch	1905 Feb. 8.00915 G. M. T.			
M_0	274°	8'	44".2	
Ω	291	54	17.9	Mean Equinox
ω	199	50	00.1	and
π	131	4	17.0	Equator
i	25	0	32.4	1905.0

e	0.175418
log a	8.893754
log μ	0.143043 per day
P	258.9765 days.

Constants to the Equator 1905.

$x=r(9.963541)\sin(223°\ 4'\ 10''.8+v)$

$y=r(9.994689)\sin(129\ \ 12\ \ 43.2+v)$

$z=r(9.626095)\sin(199\ \ 30\ \ 00.1+v)$

Residuals 1905 G.M.T.

		Jan 3.605636	Aug. 9.932841	Dec. 4.881934	Dec. 29.806383
(0-C)	Δα	-1' 38".9	+0' 36".5	+1' 19".3	+0' 47".3
	Δδ	-0' 10.3	-0' 8".7	-0' 8".0	-0' 25".1

These residuals are of the same order as those to be removed except that they are opposite in sign, showing a lack of convergency. It was thought worth while, however, to substitute these last found residuals in the equations $a_i\delta\rho_0+b_i\delta x'_0+c_i\delta y'_0+d_i\delta z'_0=n_i(i=1, 2, 3, 4)$ and solve by means of the differential coefficients II computed in the second differential correction. This proved quite successful, giving the following elements and residuals:

ORBIT III_D BASED ON PERTURBATIONS I AND DIFFERENTIAL COEFFICIENTS II.

Epoch 1905, Feb. 8.6009739

M_0	293°	56'	1".2	
Ω	291	4	59.7	Mean Equinox
ω	182	11	4.5	and Equator
π	113	16	4.2	1905.0
i	26	1	5.2	

e	0.193460
log a	8.895706
log μ	0.140115 per day
p	260.72830 days.

Constants for the Equator 1905.0

$x=r(9.960187)\sin(205^\circ\ 24'\ 15''.7+v).$

$y=r(9.994525)\sin(111\ \ 17\ \ 37.4+v).$

$z=r(9.642123)\sin(182\ \ 11\ \ \ 4.5+v).$

Residuals 1905 G. M. T.

		Jan. 3.605602	Aug. 5.930170	Aug. 6.925025	Aug. 7.932678
(O-C)	Δα	+0' 34".0	-0' 48".6	-0' 45".6	-0' 45".6
	Δδ	-0' 3".2	+0' 6".2	+0' 7".5	+0' 7".6
		Aug. 8.934825	Aug. 9.932772	Aug. 11.948613	Aug. 12.960060
(O-C)	Δα	-0' 56".3	-0' 52".0	-0' 46".4	-0' 51".1
	Δδ	+0' 5".1	+0' 9".4	+0' 12".5	+0' 12".2
		Oct. 29.570933*	Dec. 25.374269*	Dec. 29.836567	Dec. 30.337915*
(O-C)	Δα	-0' 19".9	-1' 8".9	-1' 12".5	-1' 12".7
	Δδ	+0' 4".4	+0' 42".0	+0' 39".4	+0' 41".4

This orbit is strikingly like the one computed by Dr. Ross referred to on page 1, and the residuals are of the same order. Those obtained by Ross' orbit for dates around Aug. 9th are:

1905 G. M. T.

		Aug. 7.96250	Aug. 8.96250	Aug. 9.96250
(O-C)	Δα	-0' 48".0	-0' 58".1	-0' 43".9
	Δδ	+0' 22".0	+0' 25".2	+0' 19".9

*Greenwich Observations.

These residuals have been carefully checked and the geocentric coordinates of Jupiter computed by both methods (a) and (b) given on pages 14 and 15. The inconsistency in the Aug. 8 residual will be noted later on. According to Dr. Ross' own statement his elements represent his fundamental position with the average error of $0\overset{\prime}{.}4$. An observation taken Nov. 23rd, 1906, by Dr. Perrine at the Lick Observatory gives a residual in right ascension (O-C) $-3\overset{\prime}{.}3$, and on Dec. 22nd, 1906, an observation by Dr. Wolf at Heidelberg represents in right ascension by (O-C) $+10\overset{\prime}{.}0$ (See *Astronomische Nachrichten Bond 174, Nr. 4175.*). An observation on Dec. 14, 1907, made at Greenwich is represented by the above orbit III_b with residuals in right ascension (O-C) $-25\overset{\prime\prime}{.}7$ and declination $+3\overset{\prime\prime}{.}2$.

Dr. Ross calls attention to the fact that on account of the large perturbations to which the satellite is subjected, the process of determining elements is necessarily a slowly converging one, and that from Jan. 3rd to March 6th, 1905, the perturbations were so unusually large as to render it almost impossible. In view of these statements it is interesting to note that the above elements were derived by differentially correcting an orbit originally obtained over this identical arc, when the solar perturbations were included in the direct solution.

Again it is seen that the residuals in right ascension for Jan. 3rd and Aug. 9th are opposite in sign to those removed and also a little smaller. This lack of convergency could be explained on the basis of three assumptions, as was pointed out by Prof. ~~A. O.~~ Leuschner: (1) Errors of observations; (2) Some error of computation or of handling the method; (3) Unavoidable inaccuracies in the perturbations, as these are still

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Dr. Ross calls attention to the fact that on account of the large perturbations to which the satellite is subjected, the process of determining elements is necessarily a slowly converging one, and that from Jan. 3rd to March 6th, 1905, the perturbations were so unusually large as to render it almost impossible. In view of these statements it is interesting to note that the above elements were derived by differentially correcting an orbit originally obtained over this identical arc, when the solar perturbations were included in the direct solution.

Again it is seen that the residuals in right ascension for Jan. 3rd and Aug. 5th are opposite in sign to those removed and also a little smaller. This lack of convergency could be explained on the basis of three assumptions, as was pointed out by Prof. A. O. Leuschner: (1) Errors of observations; (2) Some error of computation or of handling the method; (3) Unavoidable inaccuracies in the perturbations, as those are still based on Dr. Crawford's initial elements.

In representing the positions the geocentric equatorial coordinates of Jupiter must be known for the same instant of time as the computed coordinates of the satellite. This involves a question in aberration, since Jupiter's distance is not equal to the satellite's.

In the foregoing work the method of obtaining Jupiter's coordinates was as follows:

(a) First interpolate from the *American Ephemeris*, Jupiter's geocentric distance (ρ) geocentric apparent right ascension (α) and declination (δ) for the observed or uncorrected time($t+\delta t$). Reduce the right ascension and declination to the mean equinox for the beginning of the year including the aberration terms (i, H, h), then

$(\xi)=(\rho)\cos(\delta)\cos(\alpha)$; $(\eta)=(\rho)\cos(\delta)\sin(\alpha)$; $(\zeta)=(\rho)\sin(\delta)$.

There is a slight inaccuracy in this method owing to the fact that Jupiter's distance is not equal to the satellite's distance and the aberration time would differ for the two. This would introduce an error into ρ.

(b) A more accurate method would be to correct the observed time ($t+\delta t$) by δt obtained with the satellite's best ρ. With this true or reduced time interpolate from the *American Ephemeris* Jupiter's heliocentric longitude (λ) latitude (β) and distance (r), referred to the mean equinox of date. Then bring the longitude to the beginning of the year by applying the precession with opposite sign as interpolated from the *American Ephemeris* page 286. These heliocentric ecliptic coordinates (λ) and (β) are next converted into heliocentric equatorial coordinates (a) and (d) after which $(x)=(r)\cos(a)\cos(d)$; $(y)=(r)\cos(d)\sin(a)$; $(z)=(r)\sin(d)$. Now interpolate the solar based on Dr. Crawford's initial elements.

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$$\rho \cos\delta \cos\alpha=\xi;\ \rho \cos\delta \sin\alpha=\eta;\ \rho \sin\delta=\zeta$$

where α and δ are the satellite's mean place and must be compared with the observed mean place, i.e., the photographic mean place as published.

(c) As a check on this process, a third method is to obtain (x), (y), and (z) the same as in (b), viz., $(x)=(r)\cos(a)\cos(d)$, etc., But the Solar coordinates $X_{,}$, $Y_{,}$, and $Z_{,}$ are interpolated from the *American Ephemeris* for the reduced time (t), on the basis of the satellite's distance. Then $(\xi)=(x)+X_{,}$; $(\eta)=(y)+Y_{,}$; $(\zeta)=(z)+Z_{,}$; and $\xi=(\xi)+x$; $\eta=(\eta)+y$; $\zeta=(\zeta)+z$ where x, y, and z are the jovocentric equatorial coordinates for the reduced date (t) obtained from the osculating orbit and corrected for the perturbations. We can now write $\rho_{,}\cos\delta_{,}\cos\alpha_{,}=\xi$; $\rho_{,}\cos\delta_{,}\sin\alpha_{,}=\eta$; $\rho_{,}\sin\delta_{,}=\zeta$ where $\alpha_{,}$ and $\delta_{,}$ are the geocentric right ascension and declination of the satellite and are to be compared in obtaining the residuals with the observed apparent right ascension and declination of the satellite reduced to the beginning of the year exclusive of the aberration terms (i, H, h) the geocentric parallax having been applied.

When the residuals for Aug. 9th were recomputed by the two last methods they were found to check with each other, but to change the residuals obtained by the first method by less than 0."2 in right ascension and 0."1 in declination.

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(c) As a check on this process, a third method is to obtain (x), (y), and (z) the same as in (b), viz., $(x)=(r)\cos(a)\cos(d)$, etc., But the Solar coordinates X_1, Y_1, and Z_1 are interpolated from the *American Ephemeris* for the reduced time (t), on the basis of the satellite's distance. Then $(\xi)=(x)+X_1$; $(\eta)=(y)+Y_1$; $(\zeta)=(z)+Z_1$; and $\xi=(\xi)+x$; $\eta=(\eta)+y$; $\zeta=(\zeta)+z$ where x, y, and z are the jovocentric equatorial coordinates for the reduced date (t) obtained from the osculating orbit and corrected for the perturbations. We can now write $\rho_1 \cos\delta_1 \cos\alpha_1=\xi$; $\rho_1 \cos\delta_1 \sin\alpha_1=\eta$; $\rho_1 \sin\delta_1=\zeta$ where α_1 and δ_1 are the geocentric right ascension and declination of the satellite and are to be compared in obtaining the residuals with the observed apparent right ascension and declination of the satellite reduced to the beginning of the year exclusive of the aberration terms (i, B, h) the geocentric parallax having been applied.

When the residuals for Aug. 9th were recomputed by the two last methods they were found to check with each other, but to change the residuals obtained by the first method by less than 0.″2 in right ascension and 0.″1 in declination.

In all of the foregoing residuals by the different orbits both methods (a) and (b) of computing Jupiter's geocentric coordinates were used, and with the Greenwich observations usually all three methods (a) (b) and (c) were made to check. As a rule the residuals from the Greenwich observations are consistent with those derived from the Lick observations. This would seem to check the observations insofar as those made at Greenwich are for the same period as those taken at the Lick Observatory. However, as has been noted, the residuals in right ascension for Aug. 8, 9, and 12 are inconsistent with those for Aug. 5, 6, 7, and 11. Unfortunately there are no Greenwich observations for this period, and as the errors might be explained by errors of computation or of handling the method, it was decided to investigate this point.

A third differential correction (III) was then started. The intervals were corrected for aberration with the last found value of ρ_1 and ρ_3 viz., log ρ_1=0.681225 and log ρ_3=0.711153, the epoch being Feb. 8.6009739, thus giving for the first corrected date for aberration Jan. 3.605631 and for the third corrected date Aug. 9.932822 G. M. T. 1905. The f's and g's were obtained by means of formula (a) page 6, with the values of x_1 y_1 z_1, x_3 y_3 and z_3 obtained from the previous orbit for the newly corrected dates, but not corrected for perturbations and x_0 y_0 z_0 x'_0 y'_0 and z'_0 corrected by δx_0, δy_0 and δz_0, $\delta x'_0$, $\delta y'_0$ and $\delta z'_0$ resulting from the last differential correction (III). The value of ρ_0 was now log ρ_0=0.729660, which gave for ξ_0, η_0 and ζ_0 log ξ_0=0.686210; log η_0=0.328014; and log ζ_0=9.920238. Log (ρ) cos ψ= 0.724249. The jovocentric distance r_0 of the satellite for the middle date from the previous orbit gave log r_0=8.876174. The value of β

In all of the foregoing residuals by the different orbits both methods (a) and (b) of computing Jupiter's geocentric coordinates were used, and with the Greenwich observations usually all three methods (a) (b) and (c) were made to check. As a rule the residuals from the Greenwich observations are consistent with those derived from the Lick observations. This would seem to check the observations insofar as those made at Greenwich are for the same period as those taken at the Lick Observatory. However, as has been noted, the residuals in right ascension for Aug. 8, 9, and 12 are inconsistent with those for Aug. 5, 6, 7, and 11. Unfortunately there are no Greenwich observations for this period, and as the errors might be explained by errors of computation or of handling the method, it was decided to investigate this point.

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For the three differential corrections

	1st	2nd	3rd
From elements	I_b	II	III
log a_1 =	0.209302	0.228051	0.209361
" a_2 =	9.630998	9.645120	9.603564
" a_3 =	8.837917	8.401832n	8.701107n
" a_4 =	7.392481n	7.917837	8.251896
" b_1 =	8.237495n	8.295849n	8.314689n
" b_2 =	7.605651n	7.655845n	7.648449n
" b_3 =	7.637328	7.689386	7.702844
" b_4 =	6.805157	6.944326	6.887308
" c_1 =	8.453489	8.446592	8.457053
" c_2 =	7.847013	7.828469	7.822172
" c_3 =	7.953902	7.840132	7.792554
" c_4 =	7.258555n	7.184887n	7.088483n
" d_1 =	7.557297n	7.603697n	7.634492n
" d_2 =	7.654382n	7.660809n	7.663015n
" d_3 =	7.257385	7.293142	7.266201
" d_4 =	7.701089n	7.715046n	7.715180n
resulting in elements	II	III_a and III_b	IV

therefore was log cos β=9.946261. The sixteen differential coefficients as computed by the formulae on pages 1006 and 1007 in W. Klinkerfues' *Theoretische Astronomie* are tabulated in the third column of the following schedule; the first two columns contain the coefficients for first and second differential corrections respectively.

For the three differential corrections

		1st	2nd	3rd
From elements		I_b	II	III
log a_1	=	0.204502	0.228051	0.209561
" a_2	=	9.650598	9.649120	9.609564
" a_3	=	8.857917	8.401652n	8.701107n
" a_4	=	7.392131n	7.917857	8.251596
" b_1	=	8.237475n	8.295649n	8.314689n
" b_2	=	7.609651n	7.659845n	7.648449n
" b_3	=	7.657528	7.659586	7.702844
" b_4	=	6.805157	6.944526	6.887508
" c_1	=	8.455469	8.446592	8.457055
" c_2	=	7.347013	7.828469	7.822172
" c_3	=	7.955902	7.840132	7.792394
" c_4	=	7.256559n	7.164587n	7.088465n
" d_1	=	7.597257n	7.603697n	7.654492n
" d_2	=	7.645832n	7.660809n	7.663019n
" d_3	=	7.257585	7.295142	7.266201
" d_4	=	7.701069n	7.715046n	7.715180n
resulting in elements		II	III_a and III_b	IV

With the third set of coefficients and the residuals for Jan. 3rd and Aug. 9th scheduled on page 12 reduced to radians, I solved the equations

$$a_i\delta\rho_0+b_i\delta x'_0+c_i\delta y'_0+d_i\delta z'_0=n_i (i=1, 2, 3, 4)$$

and obtained log $\delta\rho_0$=6.941987, log $\delta x'_0$=7.309435, log $\delta y'_0$=8.591934n and log $\delta z'_0$=8.328896.

From the relations $\delta x_0=-\frac{\xi_0}{\rho_0}\delta\rho_0$ etc., the corrections to the jovicentric coordinates for the middle date were: log δx_0=6.898537, log δy_0=6.540341, and log δz_0=6.132565.

ORBIT IV BASED ON DIFFERENTIAL COEFFICIENTS III AND PERTURBATIONS I.

Epoch	Feb. 8.6009687			
M_0	291°	30'	46."7	
Ω	291	51	6.6	Mean Equinox
ω	184	14	23.0	and
π	116	5	29.6	Equator
i	25	48	5.1	1905.0
e	0.1950242			
log a	8.896734			
log μ	0.138573 per day			
P	261.655625 days.			

Constants for the Equator.

$$x=r(9.961311) \sin (208° 14' 57."0 + v)$$

$$y=r(9.994225) \sin (114 \quad 5 \quad 29.2 + v)$$

$$z=r(9.638742) \sin (184 \quad 14 \quad 23.0 + v)$$

Representation by means of these constants to the equator and the three above described methods of obtaining Jupiter's geocentric

With the third set of coefficients and the residuals for Jan. 3rd and Aug. 9th scheduled on page 12 reduced to radians, I solved the equations

$$a_i\delta\rho_0+b_i\delta x'_0+c_i\delta y'_0+d_i\delta z'_0=n_i (i=1, 2, 3, 4)$$

and obtained log $\delta\rho_0$=6.941987, log $\delta x'_0$=7.307435, log $\delta y'_0$=8.391754n and log $\delta z'_0$=8.328836.

From the relations $\delta x_0=\frac{\xi}{\rho_0}\delta\rho_0$ etc., the corrections to the jovicentric coordinates for the middle date were: log δx_0=6.898537, log δy_0=6.840341, and log δz_0=6.132565.

ORBIT IV BASED ON DIFFERENTIAL COEFFICIENTS III AND PERTURBATIONS I.

Epoch	Feb. 8.6003687			
M_0	291°	30'	46."7	
Ω	291	31	6.6	Mean Equinox
ω	184	14	25.0	and
π	116	5	29.6	Equator
i	25	48	5.1	1905.0
e	0.1390242			
log a	8.896754			
log μ	0.136373 per day			
P	261.653625 days.			

Constants for the Equator.

$$x=r(9.961311) \sin (208° 14' 37."0 + v)$$

$$y=r(9.994225) \sin (114 \quad 5 \quad 29.2 + v)$$

$$z=r(9.658712) \sin (184 \quad 14 \quad 25.0 + v)$$

Representation by means of these constants to the equator and the three above described methods of obtaining Jupiter's geocentric coordinates (see page 14) resulted as follows:

		(a)		(b)		(c)	
		Jan. 3	Aug. 9	Jan. 3	Aug. 9	Jan. 3	Aug. 9
(O-C)	$\Delta\alpha$	+9.″9	-15.″8	+10.″4	-15.″6	+10.″4	-15.″7
	$\Delta\delta$	+1.4	+6.2	+1.4	+6.3	+1.5	+6.2

The above results check the accuracy of Jupiter's geocentric coordinates for the first and third dates. I have continued to use the relations $\delta x_\circ = -\frac{\xi_\circ}{\rho_\circ}\delta\rho_\circ$ etc., to correct the satellite's jovicentric coordinates for the middle date, while the rigorous equation would be $\delta x_\circ = -\frac{\xi_\circ}{\rho_\circ}\delta\rho_\circ + \delta(\xi)$. I therefore recomputed the true geocentric coordinates of Jupiter by my method (b) page 14 for the observed middle date corrected for aberration on the basis of this last corrected ρ, to see if they had changed or if $\delta(\xi)$, $\delta(\eta)$ and $\delta(\zeta)$ were negligible. These values together with those previously used from Dr. Crawford's work are:

New Values.

$\log(\xi) = 0.681180$; $\log(\eta) = 0.322423$; $\log(\zeta) = 9.903184$

Crawford's Values.

$\log(\xi) = 0.681177$; $\log(\eta)$ 0.322421; $\log(\zeta) = 9.903183$

The changes in these values are so slight that there can be no question about the accuracy of the simpler form of the formulae.

As a further check I used the x, y, and z of the jovicentric coordinates of the satellite for the middle date as obtained by Orbit IV and Perturbations I and these new values of Jupiter's geocentric coordinates for the same instant in the equations

$(\xi)+x=\rho\cos\delta\cos\alpha$; $(\eta)+y=\rho\cos\delta\sin\alpha$; and $(\zeta)+z=\rho\sin\delta$

and solved for the right ascension and declination of the satellite.

The results are:

	R. A.	Dec.
C	23° 40' 9".6	+8° 55' 18".5
O	23° 40' 9".75	+8° 55' 18".8

This seems to be quite satisfactory.

With this orbit IV, I represented a series of observations using both the f's and g's and the constants for the equator in the computation of the jovocentric coordinates of the satellite and the second method (b) page 14 for the geocentric coordinates of Jupiter. The following residuals were obtained:

		Jan. 3.605622	Aug. 5.930170	Aug. 6.925025	Aug. 7.932678
(O-C)	Δα	+0' 10".4	-0' 12".7	-0' 10".2	-0' 9".9
	Δδ	+0' 1".4	+0' 7".3	+0' 6".5	+0' 6".6
		Aug. 8.935059	Aug. 9.932821	Aug. 11.948613	Aug. 12.96060
(O-C)	Δα	-0' 20".1	-0' 15".6	-0' 9".8	-0' 14".4
	Δδ	+0' 8".9	+0' 6".3	-0' 2".5	+0' 3".2
		Oct. 29.571277*	Dec. 25.374184*	Dec. 29.811540	Dec. 30.337969*
(O-C)	Δα	+0' 36".4	-0' 31".1	-0' 55".8	-0' 58".1
	Δα	+0' 15".7	+0' 5".8	+0' 3".0	+0' 5".7

These residuals are still on the basis of the perturbations I computed with Dr. Crawford's orbit I_b. O.R.

The inconsistency in the Δα for Dec. 25th, I have tried to remove. A new aberration time was determined and the jovocentric coordinates of the satellite were recomputed. These were used with the geocentric coordinates of Jupiter obtained by method (c) page 14, and the computed right ascension compared with the observed apparent place reduced to the beginning of the year exclusive of the h, H, i terms. This gave for Δα -31".3.

The results are:

	R. A.	Dec.
C	23° 40' 3".6	+8° 33' 18".5
O	23° 40' 3".73	+8° 33' 18".8

This seems to be quite satisfactory.

With this orbit IV, I represented a series of observations using both the f's and g's and the constants for the equator in the computation of the jovocentric coordinates of the satellite and the second method (b) page 14 for the geocentric coordinates of Jupiter. The following residuals were obtained:

		Jan. 3.605622	Aug. 5.350170	Aug. 6.325025	Aug. 7.332678
(O-C)	Δα	+0' 10".4	-0' 12".7	-0' 10".2	-0' 3".9
	Δδ	+0' 1".4	+0' 7".5	+0' 6".5	+0' 6".6
		Aug. 8.327099	Aug. 9.332621	Aug.11.348615	Aug.12.360060
(O-C)	Δα	-0' 20".1	-0' 15".6	-0' 9".8	-0' 14".4
	Δδ	+0' 8".9	+0' 6".3	-0' 2".5	+0' 3".2
		Oct.29.571277*	Dec.25.574164*	Dec.25.611540	Dec.30.537369*
(O-C)	Δα	+0' 36".4	-0' 31".1	-0' 59".8	-0' 36".1
	Δα	+0' 15".7	+0' 3".8	+0' 3".0	+0' 5".7

These residuals are still on the basis of the perturbations I computed with Dr. Crawford's orbit Ib. O.K.

The inconsistency in the Δα for Dec. 25th, I have tried to remove. A new aberration time was determined and the jovocentric coordinates of the satellite were recomputed. These were used with the geocentric coordinates of Jupiter obtained by method (c) page 14, and the computed right ascension compared with the observed apparent place reduced to the beginning of the year exclusive of the h, H, i terms. This gave for Δα -31".3.

It was now decided to recompute the Perturbations II on the basis of this last orbit (IV) at eight day intervals, retaining Feb. 8 as the date of osculation. The new constants of integration, determined by the method of Mechanical Quadrature, are practically the same as those obtained by Dr. Crawford. His values for the second determination are:

	$\bar{\xi}$	$\bar{\eta}$	$\bar{\zeta}$
'f	+ 86	+74	+ 9
"f	+120	+95	+14

As computed by the last elements they are:

'f	+ 85	+74	+ 9
"f	+119	+94	+13

The Perturbations(II)themselves are uniformly smaller and as is shown by the table of integration, are irregular and slowly converging for March and April.

When these new perturbations were substituted for the old ones the residuals were changed to:

		Jan. 3.605622	Aug. 5.930170	Aug. 6.925025	Aug. 7.932678
(O-C)	Δα	+0' 11."8	-0' 30."4	-0' 27."4	-0' 27."2
	Δδ	+0' 2."1	+0' 8."7	+0' 7."7	+0' 7."9
		Aug. 8.935059	Aug. 9.932821	Aug. 11.948613	Aug. 12.96060
(O-C)	Δα	-0' 37."9	-0' 31."1	-0' 27."1	-0' 31."5
	Δδ	+0' 10."6	+0' 7."4	+0' 3."5	+0' 3."9
		Oct. 29.571277*	Dec. 25.374184*	Dec. 29.811540	Dec. 30.337969*
(O-C)	Δα	+0' 33."5	-0' 1."9	-0' 47."0	-0' 50."4
	Δδ	+0' 16."4	-0' 25."7	-0' 23."0	-0' 20."2

*Greenwich observations.

It was now decided to recompute the Perturbations II on the basis of this last orbit (IV) at eight day intervals, retaining Feb. 6 as the date of osculation. The new constants of integration, determined by the method of Mechanical Quadrature, are practically the same as those obtained by Dr. Crawford. His values for the second determination are:

	$\bar{\xi}$	$\bar{\eta}$	$\bar{\zeta}$
f'	+ 86	+74	+ 9
f"	+120	+95	+14

As computed by the last elements they are:

f'	+ 85	+74	+ 9
f"	+119	+94	+13

The Perturbations(II)themselves are uniformly smaller and as is shown by the table of integration, are irregular and slowly converging for March and April.

When these new perturbations were substituted for the old ones the residuals were changed to:

	Jan. 3.603622	Aug. 5.930170	Aug. 6.925023	Aug. 7.932678
(O-C) Δα	+0' 11".8	-0' 30".4	-0' 27".4	-0' 27".2
Δδ	+0' 2".1	+0' 8".7	+0' 7".7	+0' 7".9
	Aug. 8.933539	Aug. 9.932821	Aug. 11.948613	Aug. 12.936060
(O-C) Δα	-0' 37".9	-0' 31".1	-0' 27".1	-0' 31".5
Δδ	+0' 10".6	+0' 7".4	+0' 3".5	+0' 3".9
	Oct. 23.571277*	Dec. 25.574184*	Dec. 29.811340	Dec. 30.357963*
(O-C) Δα	+0' 33".5	-0' 1".9	-0' 47".0	-0' 30".4
Δδ	+0' 16".4	-0' 25".7	-0' 25".0	-0' 20".2

*Greenwich observations.

These residuals are increased in both coordinates for all but the October and December observations. It is to be noted that the Aug. 8th, 9th, and 12th residuals bear about the same ratio to each other, and to the other August residuals as they have in all the representations. The Aug. 9th and August 12th residuals have been consistent with each other all the time. It would hardly seem that the same error of observation would be made in both positions unless they were referred to the same fundamental stars whose positions were in error.

Using the same differential coefficient (III) page 17 an attempt to remove the Jan. 3 Δα (+11"8), Δδ (+2"1) and Aug. 9 Δα (-31"1) Δδ (+7"4) residuals resulted in the following elements:

ORBIT V BASED ON PERTURBATIONS II AND DIFFERENTIAL COEFFICIENTS III.

Epoch	1905, Feb. 8.6009657		
M_o	286°	37'	43"5
Ω	291	52	10.0
ω	188	49	53.0
π	120	42	3.0
i	25	30	16.5
e	0.191387		
log a	8.896804		
log μ	0.138468 per day		
P	261.71888 days.		

Representation by the f and g formulae gives the residuals:

		Jan. 3.605622	Aug. 9.932824
(O-C)	Δα	-0' 27"9	-0' 0"9
	Δδ	-0' 6"1	+0' 1"7

A further test of this orbit made, using

$$x = r(9.962224) \sin (212^\circ \; 48' \; 24''.6 + v)$$

$$y = r(9.994340) \sin (118 \; \; 44 \; \; 42.2 + v)$$

$$z = r(9.634057) \sin (188 \; \; 49 \; \; 53.0 + v)$$

gave as residuals for

ORBIT V AND PERTURBATIONS III

		Jan. 3.605622	Jan.28.593554	Feb.21.610333	Mar. 6.619898
(O-C)	Δα	-0' 27".9	-0' 10".8	+0' 6".6	+0' 13".4
	Δδ	-0' 6".1	-0' 3".4	+0' 0".5	+0' 6".4
		Aug. 5.930099	Aug. 8.934825	Aug.11.948366	Aug.26.931126
(O-C)	Δα	+0' 0".2	+0' 10".0	+0' 7".7	+0' 9".4
	Δδ	+0' 1".6	+0' 3".2	-0' 4".6	+0' 1".5
		Oct. 3.904190	Oct. 29.570933*	Dec.29.811468	Dec.30.337933*
(O-C)	Δα	+0' 43".0	+0' 28".0	-0' 39".7	-0' 43".0
	Δδ	-0' 4".6	-0' 9".2	-0' 18".2	-0' 15".4

It is difficult to understand the residual in Δα for Jan. 3rd. The inconsistencies in the residuals for Aug. 8th and 11th as noted above are also apparent in this orbit. It would therefore seem quite evident that they are due to mistakes of observation.

It may be of interest to note that these last elements are not greatly different from the original elements by Dr. Crawford, except in the longitude of the node, the eccentricity, and the period. This is just the condition we might expect with the length of the arc used.

It is more than probable that short period perturbations which have not been considered in the successive differential corrections are the cause. It certainly would make a safe check to settle this point before a more complete differential correction based on normal

A further test of this orbit made, using

$$x = r(9.962224) \sin (212^\circ\ 18'\ 24''.6 + v)$$

$$y = r(9.974340) \sin (116\ \ 44\ \ 12.2 + v)$$

$$z = r(9.634057) \sin (138\ \ 49\ \ 33.0 + v)$$

gave as residuals for

ORBIT V AND PERTURBATIONS III

		Jan. 3.603622	Jan. 28.595534	Feb. 21.610535	Mar. 6.619898
(O-C)	Δx	-0' 27".5	-0' 10".3	+0' 6".6	+0' 15".4
	Δδ	-0' 6".1	-0' 5".4	+0' 0".5	+0' 6".1
		Aug. 5.950039	Aug. 8.951825	Aug. 11.948566	Aug. 26.951126
(O-C)	Δx	+0' 0".2	+0' 10".0	+0' 7".7	+0' 5".4
	Δδ	+0' 1".6	+0' 5".2	-0' 4".6	+0' 1".5
		Oct. 3.904190	Oct. 23.970935	Dec. 29.811468	Dec. 30.357935*
(O-C)	Δx	+0' 43".0	+0' 28".0	-0' 39".7	-0' 43".0
	Δδ	-0' 4".6	-0' 5".2	-0' 18".2	-0' 15".4

It is difficult to understand the residual in Δx for Jan. 3rd. The inconsistencies in the residuals for Aug. 5th and 11th as noted above are also apparent in this orbit. It would therefore seem quite evident that they are due to mistakes of observation.

It may be of interest to note that these last elements are not greatly different from the original elements by Dr. Crawford, except in the longitude of the node, the eccentricity, and the period. This is just the condition we might expect with the length of the arc used.

It is more than probable that short period perturbations which have not been considered in the successive differential corrections are the cause. It certainly would make a safe check to settle this point before a more complete differential correction based on normal places should be made. It was therefore decided to take Dr. Crawford's initial orbit (I_a) and recompute the perturbations for a four day interval or even a two day interval if this should seem necessary. The new constants of integration for a four day interval are:

	$\bar{\xi}$	$\bar{\eta}$	$\bar{\zeta}$
'f	-179	-137	-21
"f	- 35	- 27	- 5

as compared with Dr. Crawford's values on the bases of the same orbit for an eight day interval, which are:

'f	+ 86	+74	+ 9
"f	+120	+95	+14

For the sake of comparison both tables of perturbations are given herewith:

places should be made. It was therefore decided to take Dr. Crawford's initial orbit (I_a) and recompute the perturbations for a four day interval or even a two day interval if this should seem necessary. The new constants of integration for a four day interval are:

	$\bar{\xi}$	$\bar{\eta}$	$\bar{\zeta}$
'f	-179	-137	-21
"f	- 35	- 27	- 5

as compared with Dr. Crawford's values on the bases of the same orbit for an eight day interval, which are:

'f	+ 86	+74	+ 9
"f	+120	+95	+14

For the sake of comparison both tables of perturbations are given herewith:

PERTURBATIONS (III) IN X.

Date	"f	'f	f	f'	f"	f"'	f'V
Jan. 0	+20564		+228				
		-3750		-69			
Jan. 4	+16814		+297		-21		
		-3453		+48		+8	
Jan. 8	+13361		+345		-13		- 5
		-3108		+35		+3	
Jan. 12	+10253		+380		-10		- 6
		-2728		+25		-3	
Jan. 16	+ 7525		+405		-13		+12
		-2323		+12		+9	
Jan. 20	+ 5202		+417		- 4		- 9
		-1906		+ 8		0	
Jan. 24	+ 3296		+425		- 4		+ 4
		-1481		+ 4		+4	
Jan. 28	+ 1815		+429		+ 0		- 1
		-1052		+ 4		+3	
Feb. 1	+ 763		+433		+ 3		- 3
		- 619		+ 7		0	
Feb. 5	+ 144		+440		+ 3		+ 2
		- 179		+10		+2	
Feb. 9	- 35		+450		+ 5		- 4
		+ 271		+15		-2	
Feb. 13	+ 236		+465		+ 3		- 3
		+ 736		+18		-5	
Feb. 17	+ 972		+483		- 2		+ 1
		+1219		+16		-4	
Feb. 21	+ 2191		+499		- 6		- 6
		+1718		+10		-10	
Feb. 25	+ 3909		+509		-16		+ 2
		+2227		- 6		-8	
Mar. 1	+ 6136		+503		-24		- 1
		+2730		-30		-9	
Mar. 5	+ 8866		+473		-33		
		+3203		-63			
Mar. 9	+12069		+410				

In units of the eighth decimal place.

PERTURBATIONS (III) IN Y.

Date	"f	'f	f	f'	f"	f'"	f'V
Jan. 0	+17291		+193				
		-3121		+32			
Jan. 4	+14170		+225		+10		
		-2896		+42		-13	
Jan. 8	+11274		+267		- 3		+9
		-2629		+39		- 4	
Jan.12	+ 8645		+306		- 7		+4
		-2323		+32		0	
Jan.16	+ 6322		+338		- 7		-4
		-1985		+25		- 4	
Jan.20	+ 4337		+363		-11		+3
		-1622		+14		- 1	
Jan.24	+ 2715		+377		-12		+4
		-1245		+ 2		+ 3	
Jan.28	+ 1470		+379		- 9		-2
		- 866		- 7		+ 1	
Feb. 1	+ 604		+372		- 8		+3
		- 494		-15		+ 4	
Feb. 5	+ 110		+357		- 4		0
		- 137		-19		+ 4	
Feb. 9	- 27		+338		0		+3
		+ 201		-19		+ 7	
Feb.13	+ 174		+319		+ 7		-4
		+520		-12		+ 3	
Feb.17	+ 694		+307		+10		+3
		+827		- 2		+ 6	
Feb.21	+ 1521		+305		+16		-7
		+1132		+14		- 1	
Feb.25	+2653		+319		+15		+1
		+1451		+29		0	
Mar. 1	+ 4104		+348		+15		-9
		+1799		+44		- 9	
Mar. 5	+ 5903		+392		+ 6		
		+2191		+50			
Mar. 9	+ 8094		+442				

In units of the eighth decimal place.

PERTURBATIONS (III) IN Y.

Date	″f	′f	f	f′	f″	f‴	f^{iv}
Jan. 0	+17291		+193				
		-3121		+32			
Jan. 4	+14170		+225		+10		
		-2896		+42		-13	
Jan. 8	+11274		+267		- 3		+9
		-2629		+39		- 4	
Jan. 12	+ 8645		+306		- 7		+4
		-2323		+32		0	
Jan. 16	+ 6322		+338		- 7		-4
		-1985		+25		- 4	
Jan. 20	+ 4337		+363		-11		+3
		-1622		+14		- 1	
Jan. 24	+ 2715		+377		-12		+4
		-1245		+ 2		+ 3	
Jan. 28	+ 1470		+379		- 9		-2
		- 866		- 7		+ 1	
Feb. 1	+ 604		+372		- 8		+3
		- 494		-15		+ 4	
Feb. 5	+ 110		+357		- 4		0
		- 137		-19		+ 4	
Feb. 9	- 27		+338		0		+3
		+ 201		-19		+ 7	
Feb. 13	+ 174		+319		+ 7		-4
		+520		-12		+ 3	
Feb. 17	+ 694		+307		+10		+3
		+827		- 2		+ 6	
Feb. 21	+ 1521		+305		+16		-7
		+1132		+14		- 1	
Feb. 25	+2653		+319		+15		+1
		+1451		+29		0	
Mar. 1	+ 4104		+348		+15		-9
		+1799		+44		- 9	
Mar. 5	+ 5903		+392		+ 6		
		+2191		+50			
Mar. 9	+ 8094		+442				

In units of the eighth decimal place.

PERTURBATIONS (III) IN Z.

Date	"f	'f	f	f'	f"	f'"	f'V
Jan. 0	+3058		+ 53				
		-622		+13			
Jan. 4	+2436		+ 66		-5		
		-556		+ 8		0	
Jan. 8	+1880		+ 74		-5		+1
		-482		+ 3		+1	
Jan. 12	+1398		+ 77		-4		+1
		-405		- 1		+2	
Jan. 16	+ 993		+ 76		+2		-3
		-329		- 3		-1	
Jan. 20	+ 664		+ 73		-3		+3
		-256		- 6		+2	
Jan. 24	+ 408		+ 67		-1		+1
		-189		- 7		+3	
Jan. 28	+ 219		+ 60		+2		-2
		-129		- 5		+1	
Feb. 1	+ 90		+ 55		+3		0
		- 74		- 2		+1	
Feb. 5	+ 16		+ 53		+4		+1
		- 21		+ 2		+2	
Feb. 9	- 5		+ 55		+6		-1
		+34		+ 8		+1	
Feb. 13	+ 29		+ 63		+7		-3
		+ 97		+15		-2	
Feb. 17	+ 126		+ 78		+5		0
		+175		+20		-2	
Feb. 21	+ 301		+ 98		+3		0
		+273		+23		-2	
Feb. 25	+ 574		+121		+1		-5
		+394		+24		-7	
Mar. 1	+ 968		+145		-6		+4
		+539		+18		-3	
Mar. 5	+1507		+163		-9		
		+702		+ 9			
Mar. 9	+2209		+172				

In units of the eighth decimal place.

PERTURBATIONS (I) IN X (Crawford).

Date	"f	'f	f	f'	f"	f'"	f'V
Jan. 4	+16729		+1186				
		-6577		+333			
Jan. 12	+10152		+1519		-182		
		-5058		+151		+ 77	
Jan. 20	+ 5094		+1670		-105		+24
		-3388		+ 46		+101	
Jan. 28	+1706		+1716		- 4		-38
		-1672		+ 42		+ 63	
Feb. 5	+ 34		+1758		+ 59		-87
		+ 86		+101		- 24	
Feb. 13	+ 120		+1859		+ 35		-132
		+1945		+136		-156	
Feb. 21	+ 2065		+1995		-121		-198
		+3940		+ 15		-264	
Mar. 1	+ 6005		+2010		-385		
		+5950		-370			
Mar. 9	+11955		+1640				

In units of the eighth decimal place.

PERTURBATIONS (I) IN Y (Crawford).

Date	"f	'f	f	f'	f"	f'"	f'V
Jan. 4	+14120		+ 902				
		-5550		+324			
Jan. 12	+ 8570		+1226		--97		
		-4324		+227		- 66	
Jan. 20	+ 4246		+1453		-163		+ 76
		-2871		+ 64		+ 10	
Jan. 28	+ 1375		+1517		-153		+ 81
		-1354		- 89		+ 91	
Feb. 5	+ 21		+1428		- 62		+ 64
		+ 74		-151		+155	
Feb. 13	+ 95		+1277		+ 93		- 16
		+1351		- 58		+139	
Feb. 21	+ 1446		+1219		+232		-172
		+2570		+174		- 33	
Mar. 1	+ 4016		+1393		+199		
		+3963		+373			
Mar. 9	+ 7979						

In units of the eighth decimal place.

PERTURBATIONS (I) IN Z (Crawford).

Date	"f	'f	f	f'	f"	f'"	f'V
Jan. 4	+2420		+262				
		-1041		+ 47			
Jan. 12	+1379		+309		-65		
		- 732		- 18		+ 32	
Jan. 20	+ 647		+291		-33		+22
		- 441		- 51		+ 54	
Jan. 28	+ 206		+240		+21		- 1
		- 201		- 30		+ 53	
Feb. 5	+ 5		+210		+74		-34
		+ 9		+ 44		+ 19	
Feb. 13	+ 14		+254		+93		-61
		+ 263		+137		- 42	
Feb. 21	+ 277		+391		+51		-88
		+ 654		+188		-130	
Mar. 1	+ 931		+579		-79		
		+1233		+109			
Mar. 9	+2164		+688				

In units of the eighth decimal place.

DRAKE UNIVERSITY
DES MOINES, IOWA.

D. W. MOREHOUSE
DEPARTMENT OF PHYSICS AND ASTRONOMY

March twenty-nineth,
1 9 1 4.

My dear Prof. Leuschner:-

Your telegram and letters of March 19th and 24th received, also the computations and my first report. Immediately upon receiving your telegram I set to work to revising my report in accordance with the notations you had made and which I had copied on my duplicate report. I have re-computed and checked every residual that goes into the report, the coordinates of Jupiter have been checked by both my methods (a) and (b) and for the Greenwich observations by all three methods (a), (b), and (c). This has required no little time I assure you but I feel the work is very much more satisfactory than when I first presented my report. The thesis goes into the hands of the typewriter to-morrow morning. I will undoubtedly be able to send you the finished report not later than Friday or Saturday of this week.

Now as to the continuation of the short period perturbations I beg to say that I have no other thought than to continue these throughout the remainder of the year and I would not think of resting for a moment until another differential correction has been made upon the basis of three good normal places. There is one question upon which I am not certain and would like your advice before proceeding with the short period perturbations, that is, would you continue them on the basis of Dr. Crawford's orbit I_a; I_b; or my orbit IV? After this has been decided we will need some further conference concerning the formation of normal places before proceeding to the differential correction. I can assure you that this work will be completed before you are ready to print the thesis. I have no thought of stopping the work for sometime to come.

With regard to my being present to receive my Doctor's degree, it was my understanding that it would not be necessary for me to return to Berkeley but that the degree could be granted in absentia. However, I beg to say, Dr. Leuschner, that if this is against all of your rules and practices I will not ask this still further favor, but if in your judgment, it will be better for me to come to Berkeley in May and thus not cause you any further trouble over special favors for me, I will be glad to do so. I begin to feel that I have been one constant exception which has burdened your department and I hasten to assure you that the degree is worth coming after. Of course, if it is not necessary, I can ill afford to spend the amount of money necessary to make the trip for the personal satisfaction I would receive from it, as great as that certainly would be. You understand there are no rates to the coast at this time of the year, but that will not stand in my way in the least. Of course I could talk over the matter of continuing the work and gain very much by a personal interview unless, perchance, you should be coming East sometime this summer.

Trusting you will receive my report in due time and that you will find the revisions I have made in accordance with your wishes, I await your further suggestions.

I may say that I have not received the blanks from the recorder's office that I received last year and I am wondering if it will be necessary to fill this out.

Yours very truly,

D. W. Morehouse

Dean A. O. Leuschner,
University of California,
Berkeley, California.