ASTRONOMY OF STELLAR ENERGY AND DECAY

A general reader's outline of facts and theories about the life-history of stars, and a student's introduction to their radiation, steady or varying or catastrophic

by

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DOVER PUBLICATIONS INC.
NEW YORK, N.Y.

This new Dover edition first published in 1959 is an unabridged and unaltered republication of the First Edition. It is published through special arrangement with Faber and Faber.

Manufactured in the United States of America

Dover Publications, Inc.
180 Varick Street
New York 14, New York

PREFACE

There are many popular books which excellently describe the whole field of astronomy; but their need to spend many words turning general opinion into picturesque imagery, without any aid from mathematical symbols, is bound to leave the persistently enquiring reader wondering whether those opinions represent fact or guesswork. On the other hand, there are the rigorous treatises, Eddington's *Internal Constitution of the Stars* (1926), Rosseland's *Theoretical Astrophysics* (1936), and Chandrasekhar's *Stellar Structure* (1939), as well as the continentally published monographs by Milne, Lundmark, Strömgren and the Harvard, Yerkes, and other Astrophysical Journal contributors. These will initiate a young physics graduate into researching upon a star's Natural History. But it takes an experienced worker to read Chandrasekhar with profit. I propose here to attempt a bridge between the popular and the technical, in the hope of being useful to those of no official scientific training, but still profitable to the working physicist or undergraduate or teacher in science. For all types of reader I aim chiefly to distinguish between what astrophysicists observe and measure and what they infer and what they approach by rational speculations.

An attempt to cover the whole of astronomical enquiry at this level would be impracticable, and the particular topic here selected is the one, I think, most urgently needing such discrimination between factual knowledge and exploratory theory, the topic stated in the following questions: What is a star? What is its life-history? How is it kept in being? In particular, why do certain amounts of matter aggregate themselves into separate bodies, and what agency allows them the luminosity which is their sole access to our consciousness? Are the fluctuating brightnesses of 'variable' stars a temporary or a permanent weakness? Is the occasional outbreak of a seemingly 'new' star a catastrophe overhanging many familiar stars, possibly our own sun? Is the latter on the downgrade or upgrade of stellar life?

The present-day scientific student is particularly interested in finding whether atomic physics can contain the answer to such questions, and there has been some lack of accounts separating recorded facts of the observatory, laboratory research, and theoretical interpretation.

To interest the non-mathematical, as well as the large population nowadays possessing some acquaintance with that convenient language, I have divided this essay into a Part I and Part II. The former is a descriptive account of what is observed and what is surmised and why, discussed in terms not demanding previous technical acquaintance with physics. Part II is a simplified version of the quantitative steps linking the observations with the conclusions. The general reader with no pleasure in any equations may wish to keep mainly within Part I; but one of the aims of this treatment will be fulfilled if he occasionally wanders into Part II and discovers for himself how little armoury he needs for appreciating the beauty and the convincement of tracing an argument in symbols and actual quantities. He will be exploring the sole way in which critical minds can escape the endless ambiguity of mere verbal persuasion, and therefore the sole claim of the exact sciences to be knowledge. So Parts I and II are complementary, the former stating the results which are proved in the latter, and the latter filling in for the critical reader the justifications or warnings by which the notions of Part I must stand or fall. But this discrimination is not intended to make each Part quite independent of the other. I have expressed the hope that the general reader of Part I will want at least to glance over the kinds of argument employed in Part II; though the physical ideas may be novel and will be explained, even the insertion of all the steps in the final chapter rarely invokes more than a junior student's mathematics, and at risk of offending the learned I have even reminded those long out of touch by defining some very elementary functions. On the other hand, physicists wanting to start at once on Part II will find that much of the descriptive material must be sought in Part I in the corresponding chapters.

The experience behind the writing of Part II has included many years of attempting, with frequent success, to infect senior students of physics with the desire to see their laboratory knowledge exemplified in the large-scale processes of Nature: they will want to take their astrophysics without the careful avoidance of equations demanded by the general reader of Part I. Behind Part I is a background of many talks to general audiences, given under a University's Extra-Mural Board and elsewhere. The possibility of appealing to both non-technical and technical readers, in a single book, arose after the valuable experience of devising a course for science masters and mistresses—some not trained as physicists—on 'astronomy in the light of modern physics'. I am deeply indebted to all these audiences, who unite in demanding in varied language, 'What right has modern science to claim it can know anything about a star's constitution, life-history, and probable fate?'

BIRMINGHAM UNIVERSITY
December 5, 1948

FOREWORD

In his book on the Natural History of a star Dr. Martin Johnson has confined himself to existing masses of gas radiating energy into surrounding space. He has left to other writers or to a later book the question as to how these aggregations of matter have come into being—the pre-natal history of a star. But enough problems remain to form an interesting volume on the behaviour of the stars, steady or unsteady. Where does the energy come from which enables them to keep on shining? Why do some stars vary in brightness or size? What is the likely course of a star's evolution?

As a teacher with experience of classes of very different educational backgrounds, Dr. Johnson has seen the need for a straightforward story unencumbered with mathematical equations, but also for an account of the fundamental ideas on which the story is based. He has given himself the interesting, though difficult, task of dividing his book into two parts to meet these two needs. Part I is for the educated non-technical layman who wants to know what views astronomers hold today about the stars and something of the observational material on which those views are based; Part II is for the reader with a mathematical or physical training, who wants to know how the conclusions of the astronomers link up with modern physical theories.

As this group of readers, extends over a wide range of technical background, individuals will differ among themselves in their judgment of the success with which Part II has been written at the proper level of completeness or difficulty. The present writer, in welcoming this volume and the underlying scheme of presentation, expresses his own view that a well-judged line of demarcation separates Parts I and II and that Part II has been set at a very reasonable level. It is to be hoped that readers who buy the book mostly for Part I will stray into Part II here and there. They will find a further fund of interesting reading.

His method of approach to the subject has saved Dr. Johnson from a fault common among writers of popular books on science, an undue dogmatism in matters that are still open to dispute. The more cautious attitude is well justified in astrophysics, where the outlook has altered so frequently and so radically with the advent of new knowledge in the past fifty years. In the next edition of this book growing knowledge of the magnetic fields of stars and of radio-astronomy may have provided their own contribution to the Natural History of the stars. It is safe in any case to assume that, as should be the case in a living science, some present views will have been revised and fresh suggestions will have been put forward in accord with developing physical theories.

F. J. M. STRATTON

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Part I
GENERAL READER'S OUTLINE OF THE FACTS AND THEORIES

The brightness of a star is its first measurable property, but intrinsic luminosity needs to be separated from the effects of distance. From colour distribution and spectral lines the surface temperature is the next inference, and brightness compared with temperature is a first clue to size of the star. The mass contained within this size is obtainable, if attractions can be traced between a pair of associated stars.

(1) Luminosity, apparent and absolute.
(2) Temperature of surface.
(3) Mass.
(4) Size.

When this knowledge of Luminosity, Temperature, Mass, Size, has enabled the precise state of stellar material to be envisaged, the problem of energy arises: what maintains the luminosity, and for how long, in such bodies?

(1) Densities, and the condition of the visible gases.
(2) Energy of a steady star.

The distinction into Giant and Dwarf stars, from comparing brightnesses where spectra have revealed the temperature, was a first clue towards evolution; but it is no longer possible to describe all giants as young and dwarfs as old. The facts are here classified, including the fantastic density of White Dwarfs, but theories deferred for criticism in the final chapter.

The future clues to evolution may lie in those stars which are not constant in brightness, and either pulsate, or eject their atmospheres continuously or catastrophically. 'New' stars are the extreme of such eruption, and may denote collapse, with a White Dwarf or with nebulous surrounding halo, as aftermath. No one theory explains these, but there are irregularly varying stars which seem to bridge the gap between the Nova explosion and the milder local eruptions of the quiet sun.

Part II

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for Chapter 2 (Physical properties observed and inferred) for Chapter 3 (Sources of energy)

for Chapter 4 (Classifications)

for Chapter 5 (Unsteady stars)

for Chapter 6 (Models for distinguishing steady and catastrophic evolution).

ILLUSTRATIONS

ACKNOWLEDGMENT

The Plates are taken from photographs generously provided by the following authorities: Plate 1, Harvard Observatory; Plate 2, Mount Wilson Observatory; Plate 3, Cambridge Observatory; Plate 4, Dr W. Baade and Mount Wilson.

For permission to copy from published diagrams, it is a pleasant duty to thank also the following:

Miss Eddington, and the Cambridge University Press; Professor Stewart of Princeton and the publishers (Messrs Ginn and Co) of the Russell, Dugan, Stewart, treatise; Dr R. H. Baker and the D. Van Nostrand Co; Dr W. E. Williams and Messrs Methuen and Co; Professor Milne and the Royal Society and the Royal Astronomical Society; The British Astronomical Association; Professor Minnaert of Utrecht; Dr R. B. Baldwin and the publishers of the Astrophysical Journal; the authorities of the Lick, the Illinois, and the Harvard Observatories, and Dr Payne-Gaposchkin and Dr Leon Campbell.

In personal acknowledgment, I owe to Professor F. J. M. Stratton the inspiration of student days, and unfailing encouragement in all subsequent astronomical ventures, including the present book which he had the kindness to read in typescript. To Professor M. L. Oliphant I am very grateful for constant and friendly encouragement towards devoting time to astrophysics as a member of his staff.

Part I

GENERAL READER'S OUTLINE OF THE FACTS AND THEORIES

CHAPTER I

The Share of Observatory and Laboratory and Theory in Tracing the Natural History of a Star

Astronomy, as practised over many centuries and pursuing an increasing accuracy of measurement and prediction today, is often taken as the study of points of light in the sky, their position at precisely measured instants and therefore their motions real or apparent: the separation of real motions demands an exact knowledge of how the apparent motions are imposed by our earth's spin, its orbit around the sun, and the orientation of its axis. Astrophysics, a newer science opened by the first study of spectra or analysed light in the laboratories of the nineteenth century, accepts without question those locations and motions of stars and earth, and embarks on the less accurately soluble problem of what kind of body is implied in any of those luminous points. What maintains that luminosity, how hot or massive or dense is the body? How does the condition of its material compare with what we can handle in the physical and chemical laboratory? Where lies the sun—our only nearby star—along any sequence of physical evolution to be traced for stars in general? Is such evolution always quiet and orderly, or spasmodic and catastrophic?

It is worth pausing to survey the methods, and the kind of workers' training, required for any hope of answering such questions. This will determine to some extent whether anyone outside professional scientific circles is likely to discuss or even read with profit these somewhat fateful astrophysical topics; it will also settle the plan to be followed in the author's attempt to make clear some of these questions to a wider circle.

Astronomy always needed one kind of physicist, the optical designer and latterly the photographic expert concerned with, e.g., colour sensitivity of plates. But once the observatory and the scheme of visual and photographic recording and timing was established, a training in mathematics was the main or even sole gateway to knowledge. In the older science of Astronomy, this mathematics meant primarily trigonometry and dynamics, in order to compute the location of a star on the apparent sphere of the sky, and in order to predict exactly the position of moon or planets, which is determined by a single law of gravitation known since Newton. In Astrophysics, the requirements of the investigator are wider, but are often more capable of explanation in non-technical terms. The pioneer must certainly be able to read and create in mathematics, but more especially in those abstract laws which balance the loss and gain of heat and electromagnetic radiation by material systems, whether whole stars or groups of the atoms of which they are composed. These laws are included in the Thermodynamics of Radiation. The pioneer must in particular be able to apply thermodynamics to idealised spheres of gas comparable with stars, and to the exchanges of radiation which occur when electrons within the atom alter their energy levels. Most of such topics are handled by the powerful mathematical tool of the conception of 'equilibrium': for example, if a star is of constant behaviour, the loss of heat and light from its surface must be exactly balanced by an equal generation of energy by some mechanism in its interior, and if any zone in a star is not heating up or cooling down it must be re-emitting the energy it absorbs. These concepts of Thermodynamic Equilibrium and Radiative Equilibrium have been the starting point of most attempts to correlate theory with observed fact. In handling them today, the astrophysical pioneer must not only be familiar with the energy levels of electrons in the outer or chemical regions of an atom's structure, which underlie the optical spectra that he photographs, but he must be aware of the energy liberated when the core or nucleus of the atom is transmuted into another species. These liberations are contrived in the laboratories of the post-Rutherford scientific age by artificial acceleration of charged atoms; but if we apply to them the fruitful conception of equilibrium we are forced to admit that under the millions of degrees of temperature, and the millions of atmospheres of pressure at a star's centre, similar liberations of atomic energy must be proceeeding much faster than in any laboratory. They are indeed the only recognisable source of a star's light and heat.

But apart from the pioneer, the worker on routine astrophysical research can carry out much of his job with only second-hand acquaintance in atomic theory, and he is mainly an experimental physicist trained in the technique of adapting electronic devices to exact measurement of radiation. In particular, the subject has its whole observational basis in the quantity and quality of a star's radiation, and this makes the Natural History of a star a problem of experimental exploration today, rather than of mathematics. The situation emphasises itself sufficiently when we treat the primary 'observable' of a star's life, its luminosity. The first urgent necessity is for automatic recording to supersede personal error, and here Photo-Electricity, the branch of electronic science dealing with the emission of a metal's electrons under stimulus from the light it absorbs, has brought the study of a star's radiation to a stage of very exact measurement. It is a comment on the changing emphasis in astronomy that photo-electricity was unknown when many of the triumphs of the older dynamics were achieved, for instance the discovery of the planet Neptune by calculating gravitationally what must be the unknown disturber of the path of Uranus.

We are therefore today no longer in an age when only the scholarship candidate of brilliance, or the engineer, is able to claim 'The clue to the stars will be in my mathematics': there are tens of thousands familiar with electronic devices in home or employment, and millions whose cinema beguilement functions only by grace of a hidden photo-electric cell. Many of these might discover among the following pages how the familiar devices yield definite knowledge of the vastest Natural History of experience, when included in an observatory's equipment. A great variety of the devices and ideas of astrophysics could be intelligible to a far larger fraction of the public than were ever enabled by natural mathematical aptitude, or the fortune of training, to master the older science of astronomy.

It is advisable to stress this dependence of astrophysics upon a laboratory science now very widespread, since the most universally read books on stars have been the masterpieces of popularisation by the late Sir Arthur Eddington and Sir James Jeans. Both were workers with brain, library, and study table, rather than with laboratory apparatus; they were also philosophers who presented among their popular astrophysics important but highly individual opinions about the ultimate significance of the universe—concerning which physics itself, terrestrial or stellar, must be silent. There is accordingly a widespread impression, among physicists as well as among general readers, that astrophysics is a spinning of *a priori* speculation. Eddington and Jeans would have been the first to insist that the Natural History of a star is an idle tale unless founded upon the reliability of measurement, developed in laboratory and observatory through years of seemingly fanatical devotion to precision, correction, revision, criticism pitiless and unprejudiced. It is hoped that the emphasis throughout this essay on distinguishing between what is measured and what is inferred and what is surmised, will serve to exhibit astrophysics as always exploratory but as only in the most disciplined sense speculative.

The plan of our discussion is therefore to begin with the first of 'observables', the brightness of a star, both in quantity and in quality or colour of the radiation. Chapter II is spent on showing how this can lead to a settling of the principal physical properties discoverable about a star, if its mass or total contained amount of material can also be inferred. The latter requirement draws upon other 'observables' from the older dynamical astronomy, where the orbits of pairs of stars under mutually attracting gravitation is an important clue.

If a definite amount of matter is confined within a definite radius and radiates at a definite temperature, we can infer in Chapter III a coherent picture of the condition of that material; for instance it may reveal itself as chemically associated or dissociated into atoms, as an ionised gas, etc., in terms of laboratory discoveries of the energy required for such condition.

But this raises at once the question as to what supplies that energy in a star; the last part of Chapter III describes two possible answers which have attempted to explain what keeps the mass of the sun radiating at its observed luminosity, the older view based on gravitation and the new view based on energy of the atomic nucleus.

The stage of the argument already reached allows a generalised picture of what sort of body a star can be; but the Natural History of actual stars as they differ from each other demands some system of classifying them. This is developed in Chapter IV, from their spectra or analysed light. The vast science of spectroscopy is a closed book to many non-scientific readers, and Chapter IV is prefaced by an introduction intended for those entirely unacquainted with this subject. The result of scrutinising stars according to their spectra as well as according to their brightness led to the first attempt at an evolutionary order—the Giant and Dwarf theory. Although this has now lost trace of its original intention, it still contains features of use, which we here describe where they involve fact and not mere theory. At the end of Chapter IV is introduced the fantastic density of the White Dwarfs, which at tons to the cubic inch outrage most violently the older views of what 'matter' can be. The rigorous proof of this scandalous truth is outlined in Part II, but its simple and convincing explanation can be given in principle in Part I.

Hitherto these schemes of ageing or quietly evolving stars take no account of changes in habit, regularly periodic or violently catastrophic; but there are stars which oscillate or pulsate with very large fluctuation in their light, and others have been known to burst out with a ten-thousand-fold increase of light within a few days. These latter were known as 'new' stars, and, although catastrophe to an old star is undoubtedly what really occurs, the sudden presence of an apparently new luminous point in the sky is still given the name 'Nova'. Chapter V is an account of the actual observed facts about all these 'unquiet' stars, in rather more detail than has been available in any but specialist literature hitherto: they include some of the most startling phenomena of astronomy, and it is obvious that stellar evolution must always be an incomplete tale unless it assigns correct place in the sequence to these sudden explosions of a star, and also to the other or periodic habit of alternate swelling and shrinking. Much of the traditional discussion of these unsteady stars has tended to concern itself with speculation as to causes of the inconstancy; following our policy in this essay we state the facts in Chapter V and leave theory to Chapter VI. As final item in factual knowledge, it is intriguing to exhibit our familiar sun, to realise how quiescent it is compared with an oscillating or exploding star, and yet how vigorously erupting it is on a scale minute in cosmic significance but vast to our nearby sensitivity: the effects of the briefest solar flares upon our terrestrial upper atmospheric layers are drastic enough, but would require a separate treatise to discuss.

In Chapter VI, the possibility of theoretical explanation for the previous facts is centred round the notion of stability criteria; the explorer's duty is to envisage idealised or 'model' spheres of gas, and to calculate under what conditions they would be stable or unstable. Such models must then be compared with what is observed and inferred about actual stars, emending each model property according to check until the gap between theory and actuality begins to close. The process brings astrophysics to the verge of speculation, but speculation rationally based and disciplined if it is to serve a scientific purpose.

Towards that final chapter we now set out by discussing the primary observed fact of a star's brightness in Chapter II.

CHAPTER II

The Properties which can be Discovered in a Steady Star

(I) LUMINOSITY, APPARENT AND ABSOLUTE

The most fundamental of the 'observables', in fact the feature by which we become conscious of a star's existence, and from which any Natural History of it must begin, is its brightness; we shall adopt the accepted term 'luminosity', denoted by L with some appropriate subscript in brackets to show which particular meaning of luminosity is being used. This enforces the first demand upon an observatory, in principle just the attachment of some apparatus at the eye-end of the main telescope capable of grading the intensity of light received from any star compared with that from other stars. Electrical and optical devices for this will be detailed in Part II, together with a certain splitting of the definitions of L. The definitions are needed because quantity of light is not unambiguous unless quality is also watched: a photographic plate is much more sensitive to blue than to red, and the eye is most sensitive to yellow, while some devices for measuring radiation are most sensitive to red. Before listing one star as more bright than another, it is obviously necessary to state whether this is meant on the photographic or the visual or any other scale; the order might become inverted if a bluish star reported very bright from photographic measurement were re-examined with a thermo-electric recorder sensitive to red.

When these differing ways of grading luminosity are taken into account, the science of 'photometry' or measurement of brightness is just the establishment of a technique for eliminating the unreliability of the human observer, who used to guess 'that star is of first magnitude, and that other is of second or third magnitude'. This term 'magnitude', denoting some number which becomes larger as brightness diminishes, is still the basis of current terminology: stars differ over many millions-fold range in brightness, conveniently expressible on a scale of ten or twenty magnitudes with their decimal and centesimal subdivisions available to modern instruments. In Part II it will be shown that we can agree upon a scale such that a first magnitude star is exactly 2·512 times brighter than a second magnitude star. A difference of five magnitudes then denotes exactly a hundred-fold difference of brightness and ten magnitudes exactly ten thousands-fold.

But it can be realised at once that these measurements as they stand do not represent any intrinsic property of a star: some stars are faint because they actually emit little light, others because they are very distant. What the observatory has measured or the eye estimated has been merely *apparent* luminosity, usually written L (app.) on one or other of the photometric scales, whereas what we require is some measure of the star's own behaviour independently of its distance. If all the stars could be arranged at the same distance from us, the order of their brightness would differ extremely from the order in which they appear to shine at their actual distances which themselves differ many thousands-fold. So L (app.) is only the first step towards investigating absolute luminosity, L (abs.), which is the quantity needed before we can start to unravel a star's constitution and life-history. Since we require a property intrinsic to the star and independent of its situation, L (abs.) is conveniently defined as the luminosity which a star would exhibit if all stars were placed at a single distance from the observer; the selected distance is ten parsecs or just over two million times the sun's distance from the earth.

In the cases where the distance of a star is known, L (app.) can be converted unambiguously into L (abs.), because all radiation diminishes in intensity inversely with the square of the observer's distance from its source. But direct knowledge of a star's distance is available only if that star is near enough for the earth's varying position in its annual path round the sun to impose a measurable shift in apparent location of the star compared with more distant stars—the method of parallax employed in Part II. For stars whose shift due to this annual migration of the earth is too small to be accurately measured, for instance less than a hundredth of a second of arc, distance ceases to be an 'observable' and must be inferred from some other property known itself to vary with distance or with size or with luminosity.

In certain important cases it has been possible to obtain reliable information as to L (abs.) without the intermediate step of knowing first the distance; a particular kind of star, the Cepheid variables, described in more detail later, shows regularly periodic fluctuation in brightness, and it is found that the average brightness of each Cepheid, that is to say the mean between maximum and minimum in each star's fluctuation, is connected with the period of time between successive maxima. Fig. 1 exhibits this important relationship, and also emphasises from the scattered points on the graph that brightness cannot be *accurately* assessed from period but nevertheless is knowable within a fairly narrow margin of error. The name of Shapley of

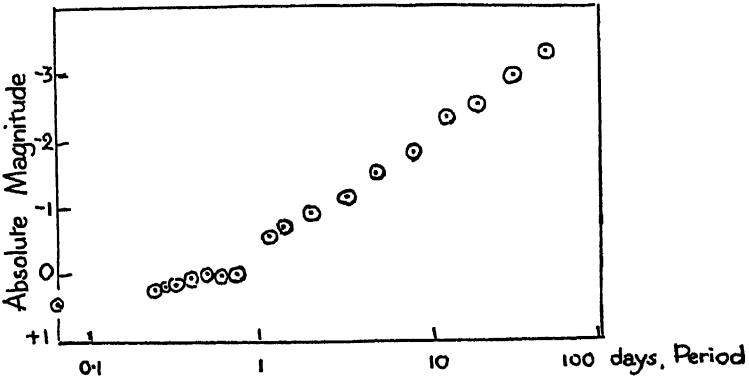

FIG. 1.

Cepheid pulsating stars show this relationship between their brightness and the periodic time-interval from maximum to maximum in their regular fluctuation of light. The eight short periods belong to Cluster variable stars, and the ten longer belong to Classical Cepheids. This form of the diagram is from Campbell and Jacchia's *Variable Stars* (Harvard).

Harvard will always be associated with the most critical and constructive work on this subject. However faint a star by reason of distance, it is always practicable to measure accurately whether its fluctuations follow each other at intervals of hours or days or weeks, and the Cepheid argument is based on cases where information about distance or *L* (abs.) is independently available: for cases where distance is otherwise unknown, *L* (abs.) is inferred from the periodic time of fluctuation, with the reliability of accepting Fig. 1 as representing a law of Nature although we may not be sure of the reason for the law. *L* (abs.) can thus be obtained from an 'observable', but the observed quantity is not a distance but a time-interval.

Other useful clues to *L* (abs.) are again from other empirical correlations with some observable property known to be connected with luminosity: the method of 'spectroscopic parallax' relies on the fact that the intensities of certain spectrum lines differ slightly according to density of the gases in a star's atmosphere, and therefore offer a clue to whether a given mass is distributed over large or smaller volume. Large volume confers high *L* (abs.) compared with that permitted by smaller volume at the same mass and temperature. The Mount Wilson list by 1935 had investigated more than 4000 stars for this particular type of inference, and where distance of star is separately known the check is satisfactory.

Since *L* (abs.) is thus a quantity to which one contributory factor is the area of the star's illuminated disc, any actual detection and measurement of this disc would be a valuable confirmation of all these clues. All stars must possess finite disc, but not even the nearest star can show it in even the largest telescope: the closest is at hundreds of thousands of times the sun's distance, and nothing but an infinitesimal point of light is seen or photographed. This is not a limitation imposed by instrument construction but by the wave-length of light. Nevertheless there has been developed since 1920 the technique of 'interference of light from differing paths' whereby the area of a star's surface can be assessed in a few cases; we refer to this later when discussing size of stars—it is a satisfying confirmation of the methods of approaching *L* (abs.), to find a few stars to which diameters hundreds of times the sun's can be assigned by actual measurement with this 'interferometer'.

By exploiting all these methods, direct and indirect, some of them overlapping and thus checking the reliability, the observatory's list of measured L(app.) can be converted into the L(abs.) of some thousands of typical stars. A graded order is thus obtained stating quantitatively the actual amount of light and heat given out by a star, a property of its dimensions and its internal constitution and not a mere accident of the distance which controls the faintness or brightness that we happen to see. Such grading in L (abs.) is the first stage towards a Natural History of a star by elimination of the adventitious factor of the observer's position; it reveals 'stars' as a class of celestial objects ranging up to ten thousand times the sun's brightness or light-capacity and down to a millionth of the sun's. The sun itself, if placed at the standard distance assumed for grading in L (abs.), is not a very bright star, between fourth and fifth magnitude and comparable with the faintest objects noticeable to the naked eye.

The major task of astrophysics is to allocate responsibility for these luminosities among the possible determining factors; temperature, size, and source of internal energy of the star will accordingly be quantities to seek, and we proceed next to discuss temperature. So far as direct observation goes, it is of course the temperature of a star's outside surface that is accessible, though implications as to internal inaccessible temperatures will be discussed later. It must also be remembered that 'surface' of so completely gaseous a body as a star will need careful defining, and this will be taken care of in the next section.

(2) TEMPERATURE OF SURFACE

The study of luminosity was the estimation of *quantity* of radiation emitted by a star, though we had to notice a proviso securing that differences in *quality* of the light should not confuse any quantitative list. But if in each star we now pursue the analysis of the light's quality, to the extent of finding the precise distribution of colour which physically is a distribution of intensity among the differing wave-lengths in a complex light, then an estimate of the temperature of the surface emitting the light can readily be made.

In principle, the method is intelligible from the commonest experience; a piece of wire and a thick poker each in a hot fire would give out very different total quantities of radiation because of their difference in size, but if we could estimate exactly the degree of redness or yellowness or other constituents of their glow this factor of 'quality' is a precise measure of the temperature. This is only true if the temperature is high enough for the light not to depend upon the particular chemical reactions of the particular substance as it does in a flame. The hottest terrestrial furnaces fulfil this condition, and so do most stars. Whether bright or faint, they differ in quality of light, ranging from the deepest red through orange and yellow to those whose white has a bluish tinge implying a temperature far higher than our terrestrially-contrived white-heat. In judging, possibly, that the white-hot wire has a higher temperature than the red-hot poker, although emitting less total quantity of radiation because of its size, we are utilising the fact embodied in Wien's law; this law we shall formulate in Part II, giving a numerical constant as product of absolute temperature and the wave-length of maximum intensity in a 'continuous' spectrum. For stellar spectra whose continuous band of colour is not too broken up by partial absorption, temperature estimation reduces thus to a single extraction of the distribution of brightness among the colours, either by applying a detecting instrument to the analysed light or by calibrating a photographic plate whose colour sensitivity must be experimentally determined.

Apart from actually pursuing the point of maximum intensity in the colour distribution, the same principle affords another way of defining a star's temperature by 'colour index' or 'heat index'. It will be remembered that, when we began the discussion of luminosity, there arose at once different grading scales according as the more blue-sensitive photo-plate or the more yellow-sensitive eye or red-sensitive thermo-electric detector were used: accordingly the difference between a star's 'magnitude' on these differing gradings is itself a measure of temperature. A star high on the photographic grading may be hotter than a star high on the red grading, and when quantitatively treated we can make a definition which gives in precise form our natural judgment that called a red star cooler than a white or blue star.

In addition to this 'continuous' spectrum or radiation distributed among all colours, a star shows a 'line-spectrum' or set of separate radiations each of a closely associated group of wave-lengths. These are attributed, from comparable laboratory experiments, to an atmosphere of somewhat cooler gases surrounding the body which emitted the colours. But 'Atmosphere' has not quite the easy terrestrial definition when the body below it is not solid or liquid, and the regions giving the colour spectrum and the superposed line-spectrum in a star must fade into one another along a steady density-gradient; it is a convention, rather than any recognition of a discontinuity in a star's structure, to say that the colour spectrum is emitted by the star's 'surface' and the line-spectrum by its atmosphere. The sun as a typical star presents an apparently sharp boundary at its edge, and an eclipse separates out the 'atmosphere' for a few brief moments, but even the sun's disc is hotter at the centre than near the edge, indicating that we are not regarding a completely impenetrable disc but seeing 'into' a glowing gas.

We have now to consider whether a star's line-spectrum can yield a temperature estimation, as well as its colour spectrum: we shall find that it does, and that quite correctly the atmospheric blanket is slightly cooler than the hot 'surface' which as source of the majority of the light is called the Photosphere. The argument is less obvious than that of 'temperature from colour', but can, I think, be made intelligible in principle without the equations of Part II. Pending the introduction to spectroscopy in Chapter IV, a line spectrum can be taken as a set of dark lines on a photo-plate. each the image of the spectroscope's slit in one particular wave-length of light. The position of these lines, measured with a microscope and compared with laboratory experiment, reveals what chemical elements or compounds in what physical state are present in the star at the particular atmospheric level from which the light has come. The essential difference, between the set of lines and the colour distribution against which they stand out, is that the latter is a property of temperature alone, while the lines are capable of indicating temperature but are also specific to particular substances, such as hydrogen or one of the metals. That a line spectrum should be a temperature indicator is a very important development in astrophysics due initially to the Indian, Megh Nad Saha (about 1920).

Saha's discovery was the recognition that any element, for instance calcium or magnesium, identified in sun or star by the position of its spectral lines, must have its atomic structural electrons in a particular stage of excitation for the possible set of lines to appear in any particular grouping of their intensities. The energy for successive excitations of different electrons in any atomic structure can be measured precisely in the laboratory. We actually liberate the requisite energies in our experiments electrically or optically, but we have to suppose that in the stars the comparable liberation is purely thermal and is a measure of temperature. Saha was the first to provide the equations for thus calculating these temperatures in terms of the electronic energies or 'excitation potentials and ionisation potentials' known from experiment, although physical chemists had exploited similar formulae in the 'cooler' problem of chemical reactions. The Saha theory does not mean that measurement of the intensities of certain stellar spectral lines gives by itself an unambiguous deduction of a star's atmospheric temperature, because more than one property is involved; in particular the pressure of the gases and the number of 'free' electrons, as well as the temperature and the ionisation potentials, decide which lines show most strongly in the spectrum. Great advances beyond Saha's first suggestion were made in England by Fowler and Milne and in America by Russell, and evidence had to be pieced together from many differing kinds of stellar atmosphere, before reliable results could be claimed as applicable both to the denser stars ('dwarfs') and more rarified stars ('giants'). It is, however, quite definite nowadays that the various grades of stars can be assigned atmospheres on a Saha basis, of which the coolest are at about

2500 degrees and the hottest at about 40,000. In cases where the temperature of the sub-atmosphere or Photosphere is also known, from the Wien law of the continuous colour spectrum, the atmospheric figure is slightly lower than the photospheric, as expected since the region of the line spectrum is an absorbing blanket cooler than its substratum. For instance the Saha temperature of the sun is round about 5500 while the Wien temperature of the lower strata emitting the colour is round about 6000. These figures are high compared with the hottest terrestrial furnaces at about 3000, but distinctly cool compared with the hottest star at about 40,000 and trivial compared with any star's centre: we shall show later that the inaccessible interior of a star is probably at a temperature only to be quoted in millions of degrees.

(3) MASS

The question as to the total amount of material in a star, its mass, is bound to be asked next, and to be answered in some way before we can form any convincing picture of 'what is a star?' We are careful here to use this term 'mass' which in non-scientific language is apt to be replaced by 'weight'; the connection between these two words is only unambiguous on our earth, where we deal with a very nearly constant magnitude of gravitational attraction so that 'amount of matter one pound' only differs from 'weight of one pound' by a very nearly constant factor. A mass of a pound, or any such fixed amount of material, will weigh differently on different stars according to the particular gravitational attraction there to be experienced, depending on the particular mass and radius of that star.

It will become apparent later in our argument that mass, or total amount of material contained, is what ultimately decides whether any celestial body can be a 'star'. Eddington liked to define 'star' as any aggregation of material between 10^{32} and 10^{37} grams: (using the shorthand of 10^{6} = million, 10^{12} = million million, 10^{18} = million million million, etc.). A body of less mass may be a planet or satellite or asteroid or meteorite, but cannot remain self-luminous as a star is. On the other hand, material aggregations exceeding 10^{37} grams would scarcely survive the instability due to radiation pressure which would tend to blow the object to pieces. If there is thus any prospect that there are natural limits to a star's mass, this essay's policy of separating fact from theory demands an account of how the actual amount of material in any star can be discovered.

Direct knowledge of the mass of any distant object is only obtainable from observing the gravitational pull which it exerts and experiences relative to some other mass. Since any star possessing planets, as our sun does, will certainly conceal them beneath its own brightness at so great a distance, we are limited for data to those stars which are 'binaries', consisting of a closely associated pair revolving round their common centre of gravity. There is no doubt that a considerable proportion of known stars, sometimes estimated at a quarter, are binaries, possessing each a companion which may be larger or smaller or brighter or darker but is also a star and not merely the tiny non-luminous scrap which would be a planet. But in many cases the 'double' is too distant from us, or its two components are too near to one another, for any telescope to reveal more than the single combined point of light. In fact many binaries are only studied by the indirect but very definite evidence that the two companions mutually eclipse each other, or that their spectral lines split to reveal the concealed motion of the one component relative to the other. Only in a minority can sufficient evidence be pieced together to make a precise account of what masses can have evolved what movements. Aitken's 1932 list of 17,000 represents a great master's work, but about 2000 visually detectable have already yielded some information about how the two partners move around each other. For a hundred or so the orbits are known with some precision. For perhaps more than fifty the separate masses are known. That is a minute number of stars among millions, but the achievement is of great importance where it includes a wide selection in range of star, cooler and hotter, larger and smaller, and of various spectral types, so that a sample cross-section of the universe seems to be obtainable.

Without trespassing on the details of Part II, we may mention here briefly the way in which binary stars reveal their mass. In the orbit or periodic path of two mutually attracting bodies around a common centre or focus, the mechanical law first discovered by Kepler gives the sum of the two masses if size of orbit and time to traverse it are known. To know each of the masses separately requires tracing of the relative velocities. Now binary star systems can yield these required data from the following kinds of observation:

(*a*) The two stars may be so far apart that telescopic seeing or photographing elucidates the orbit; recording over only a fraction of the whole circuit is needed, as the form is bound to be elliptic. Unfortunately at these large separations the velocities do not show up well; the star Procyon is a double with period 40 years, and Castor exceeds 300 years with its components more widely separated than the most distant menbers of our own solar system.

(*b*) The above 'visual binaries' have been supplemented when 'interference' methods, which we mention in speaking of a star's size, are employed for pairs too close for the telescope to separate them.

(*c*) 'Spectroscopic binaries' are those whose components are too close together for any telescope to separate them, but which reveal their binary nature when their spectral lines split periodically. The Doppler effect, described later, is a slight shift of a spectral line to one side or the other when the source of light is receding or approaching: it is obvious that if the orbit of a binary pair is sufficiently 'edge-on' to our line of sight there will recur periodically moments when one of the pair is receding and its spectral lines shift to one side while the other is approaching and its lines shift to the opposite. The combined spectral line appears as split, therefore, during that phase of the orbit, and coalesces again whenever the stars are crossing our line of sight. Microscopic measurement of the spectrum photo-plates reveals the velocity range. In favourable cases, where both stars are bright enough and the orientation of orbit is suitable, the data for calculating both masses become available (Fig. 2).

(*d*) Since the Doppler effect only implies the component of motion along our line of sight, the spectroscopically-found

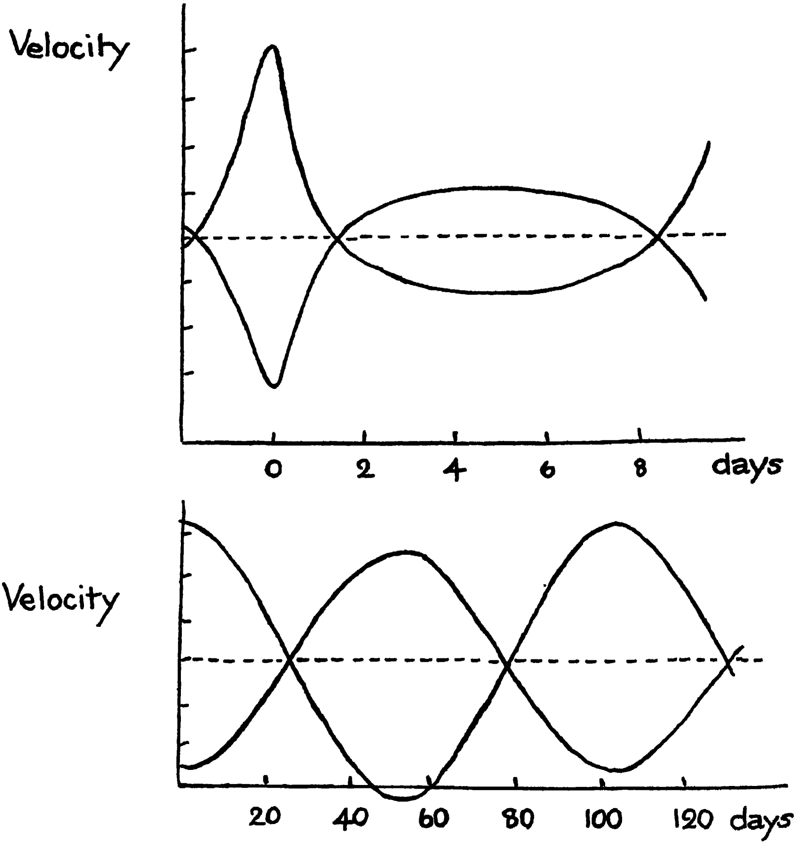

FIG. 2.

Two instances of spectroscopic binary stars: in each case the velocity of alternate approach and recession of the pair of companions is measured by the Doppler displacement of their spectral lines, and is here shown as it goes through its cycle while the mutual orbit of the pair is completed, in about ten days for one binary pair and about a hundred days for the other pair. Diagram from the Lick Observatory.

binaries only provide the lower limit to the actual velocities. But if it happens that the orientation of the orbit is very near to 'edge-on', each of the pair of stars alternately blots out or eclipses the other. Tracing of the combined luminosity then yields a picture like Fig. 3, the deepest dip occurring when the dimmer star comes between us and the brighter and the shallow dip implying that the fainter star is being eclipsed.

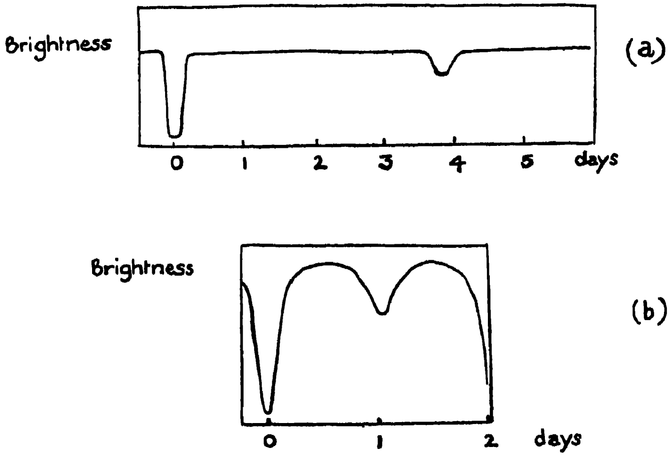

FIG. 3.

Two instances of eclipsing binary stars: in each case there is a deep fall in brightness when the dimmer of the pair comes between us and the brighter, and a slighter fall when the brighter component similarly eclipses the dimmer. In (*a*) the stars are well separated, but in (*b*) they are more nearly in contact, with widespread atmosphere between, as shown by the proportion of total orbit occupied by the eclipses. In the closer pair, the stars are also elongated by tidal distortion, while mutual radiation further complicates the light-curve.

(*a*) is from Stebbins at University of Illinois, (*b*) is from Lick Observatory.

Binaries that are visual, or which show both the spectroscopic clue to velocity and the eclipses which guarantee the orientation of orbit, enable the computer to assemble enough evidence for applying Kepler's laws, and the star's mass is found.

Although many thousands of binaries have been investigated, the majority of stars are probably single, as is our sun, and we do not know how large a majority. Indirect knowledge of a single star's mass depends on discovering some property dependent upon mass and itself observable or capable of reliable inference. An instance of great importance is the Mass Luminosity law, since on the whole it is commoner to have reliable clues to a star's Absolute Luminosity than to its Mass. For the comparatively few cases where both are definitely known, it is found that a simple relationship can be plotted between them. This is shown in Fig. 4, the scattering of the points indicating that it is a very definite relation but that the inaccuracies of determining L and M make the inference of either from the other only rough. This law is a representation of observed fact and seems to apply to most stars except certain

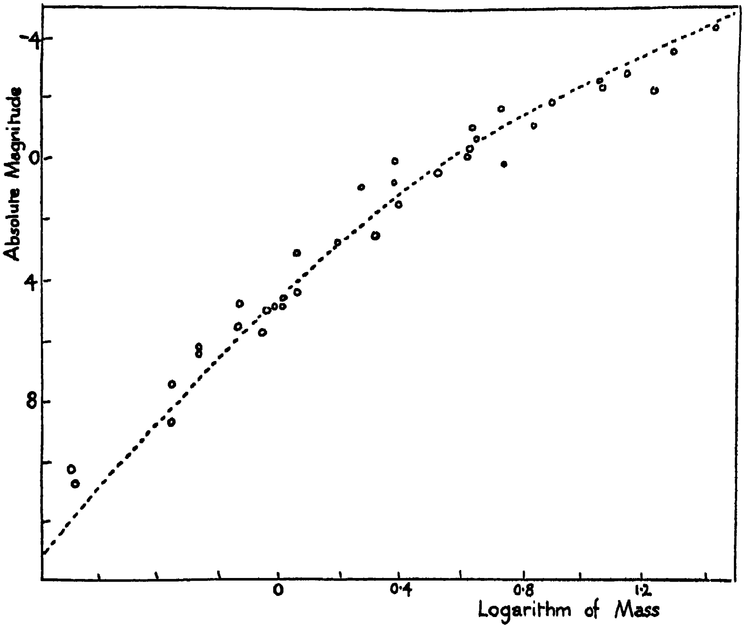

FIG. 4.
Eddington's form of the connection between a star's Mass and its Luminosity: the dotted line is from his theory (discussed in detail in Part II) and the points represent measured data from particular stars. The range in luminosity is from above a thousand times the sun's brightness, down to a two-hundredth of the sun.
Diagram from Eddington's *Internal Constitution of the stars.*

extremes: the White Dwarfs, which we discuss later, fall quite off the diagram and so do some Super-giants. But when Eddington first demonstrated that for model stars obeying some very restrictive conditions a Mass-Luminosity law will be an inevitable consequence of their structure, he was surprised to find what a large range of stellar types do actually exhibit the law. Since then, theorists have modified the restrictions until the factual relationship can hardly be taken to guarantee any particular set of assumptions, and its use is simply a 'backstairs' way of estimating stellar mass, the precise reason for which is not yet fully clarified.

Gathered from all these various degrees of indirect and direct inference, stellar masses do not appear to differ as widely from that of the sun as they might if just any aggregation of material in empty space could behave as a star. It would be idle to list such enormous masses in grams or tons, but realising that the sun contains 2×10^{33} grams or about 2×10^{27} tons, all the other stars can be stated as fraction or multiple of the sun's mass as convenient unit. The vast majority turn out to be between about a fifth and five times the solar mass, though a few rare instances of mass up to a hundred suns are known. Between 10^{32} and 10^{35} grams seems to cover practically everything recognised as a star, and we shall pursue later some possible reasons for this limitation. Meanwhile, having achieved the estimate of a star's range in amount of contained material, the next enquiry must consider the size; into what range of volume are those rather closely restricted amounts of matter compressed by Nature?

(4) SIZE

Direct measure of a planet's size would come at once when area of disc and the distance were found: but we have already had occasion to point out that even the nearest star (a hundred thousand times the sun's distance from us) exhibits no disc but only an infinitesimal point of brightness, even in the largest telescope. To see the disc of a star would be like seeing the round shape of a golf ball at a hundred miles. If, however, light from a star is led by a set of mirrors along two paths into the final instrument, and the rays recombined so as to form a pattern of 'optical fringes' or alternating dark and light bands, a delicate adjustment can be carried out which permits the angular distance from edge to edge of the invisible disc of the star to be calculated from the displacement of the controlling mirrors. The phenomenon of these 'interference fringes' is a long-known property of the wave-character of light, and is of similar origin to the coloured bands seen when a film of oil covers a pool of water; its first employment in astronomy was when a 20-foot frame carrying 6-inch adjustable mirrors was erected outside the aperture of the 100 inch telescope at Mount Wilson in 1920. We state the equations used in Part II. The American research was a triumph in difficult technique, and in a few instances where the star's distance was available the resulting angular measurement proved conclusively that certain stars possess bodies of diameter 200, 300, even nearly 500 times that of the sun. Since area of illuminated disc increases with square of radius, there are evidently some stars which present illuminated area a hundred-thousand-fold greater than the sun, a quantity which therefore must be accepted as ranging more extensively than either a star's mass or its temperature. The most striking achievement of the 'interferometer' was to detect an actual periodic variation in one of these enormous invisible discs: 'α' in Orion was found to alter its diameter regularly from about 330 to about 540 times the sun's, a type of unsteadiness we shall discuss in some detail in a later chapter.

A different and indirect attack on the problem of a star's size comes quite simply from comparing temperature and Absolute Luminosity. We may recollect the particular means of estimating the surface temperature of the star by Wien's law, a measurement of the colour or wave-length at which occurs the maximum intensity in the 'continuous' spectrum; the other experimentally verified law of such spectra of all colours is that of Stefan. Stefan's law identifies this particular kind of colour spectrum with the emitting of a quite definite amount of energy per second from each unit of area of the hot surface. Stefan's constant factor, yielding precise knowledge of this 'energy per area', is available to considerable exactitude from laboratory experiment, and only has to be combined with a known temperature. So combining Wien's law temperatures of a star with Stefan's numerical constant factor, we are able to calculate with confidence the energy emitted per unit area by a star. But the *total* energy emitted by the star is known if the Absolute Luminosity is known—brightness received from a standard distance was what defined L (abs.): simple dividing of 'energy per area' into total energy confers at once a knowledge of the area of the star's luminous surface.

The size of stars thus obtained from temperature agrees reasonably with the direct evaluation from the interferometer; but it must be noted that, in the huge distended balloon-like stars where such comparison of methods is possible because of the extreme size, the estimation of temperature by the Wien method happens to be particularly difficult. For those giant red stars are the coolest known; hence they exhibit bands of chemical compounds blurring awkwardly the continuous colours upon which the critical measurement depends.

As in the problem of finding a star's mass, there is special information to be obtained from binaries; when it is feasible to infer from fluctuations of the luminosity that a star has a close companion eclipsing it periodically, the duration of each eclipse compared with the duration of the whole cycle of the pair's revolution is a useful clue to the size of each star. If, in the diagram Fig. 3, the level portion representing steady luminosity lasts long compared with the dips which represent one star covering the other, then the distance the stars move in their orbit is large compared with their own size. But if the dip leaves little undisturbed light in between, the stars must be so close together that size of orbit is not much greater than size of star. For example, the double star β of the constellation Lyra seems to have its components almost in contact with each other as they revolve. If the two dips representing eclipse of each in turn have very different shape on the graph, intriguing differences between the two stars of the pair can be disentangled. Queer individualities appear in this very profitable line of research: if one star of a pair possesses an enormous distended atmosphere and the companion star is small and dense and sharply bounded, one of the dips in the luminosity curve will be shallow because the large star is never fully covered by the smaller. The other dip may be sharp when the small star is covered by the other's photosphere, but gradual and altering in colour while one star is still partially seen through the other's atmosphere. ζ in Auriga is a case where careful research has yielded remarkable information in spectrum photographs taken at successive instants when one component star is absorbing in its own atmosphere different colours of the other star. All this is obtainable from a star which telescopes alone can never reveal as anything but a single point of light.

This discussion of observable facts, in quality as well as in quantity of light from a single point source of the radiation, may serve to suggest how photometry and spectrum measurement supplements photography in astrophysics. The fundamental quantities, Luminosity, Temperature, Mass, Radius, are obtained with valuable confirmatory lines of argument where the several modes of attack overlap. These have brought down the concept of 'star' from infinitesimal point in the sky, to definite aggregations of matter, elements identifiable with those familiar terrestrially, distributed over the vast size of the stellar body revealed by the quantitative results. What is very much more unfamiliar is the high temperature and the extremely high and low densities which the facts demand: the familiar chemical elements must be in a state unfamiliar to laboratory experience, and the next task of astrophysics is to enquire what may become definitely known about the stellar condition of matter.

CHAPTER III

State of Matter and Source of Energy in a Steady Star

(1) DENSITIES, AND THE CONDITION OF THE VISIBLE GASES

A first sequel to any list of masses or amounts of material in a star, combined with a list of star sizes, yields at once the state of packing of the material, that is to say the density averaged over the whole body of the star. In common experience, a sphere of material in which our weighing gave us the mass in grams and a compass or caliper measurement gave the diameter, is characterised by a density which is the quotient of mass divided by volume; for iron this density is about 7·8 grams per cubic centimetre, among the lightest metals it is 2·65 for aluminium and 1·74 for magnesium, while among the heaviest it is about 18 for tungsten and about 19 for gold. In comparison, water has density of one, in similar units, with oils and light floating solids less than one. Gases as we know them weigh far less, hydrogen less than a ten-thousandth of one gram per cubic centimetre and oxygen more than one-thousandth. The earth's mean density is about 6 grams per cubic centimetre, heavy because of its metallic core. The mean density of the sun is about 1·4, rather heavier than water. But this word 'mean' emphasises that with objects of cosmic size the actual density must vary from outer boundary to central core; for a star the variation may be very great, the mean value obtained by simply dividing total mass by total volume being large compared with any property of the outermost layers accessible to observation.

With these comparisons it is interesting to notice the result of dividing stellar masses by the volumes obtained from the diameters in the previous chapter: few (except the White Dwarfs, freaks to be described later) are strikingly denser than the sun, and those of the large sizes measured by the interferometer are so very much less dense than the sun that they resemble the rarefied gases of our electric discharge lamps and other vacuum tubes, rather than any state of matter which could be seen or handled.

This is of course immediately inevitable when we find stars possessing diameters up to 400 times the sun's but masses only half a dozen times the sun's, since the volume ($\frac{4}{3}\pi r^3$) is proportional to the cube of the linear measure: a mass even so great as ten times the sun would be distended over thirty million times the volume at the diameter quoted. Such a star would occupy much of the solar system instead of being localised at the centre, if it replaced our sun; some of the brightest stars we see are of this nature, brilliantly luminous but of an extreme tenuity comparable with the densities studied terrestrially under the topic of 'vacuum'.

Now the spectra of all stars make it quite certain that they are constituted of the familiar terrestrial substances, including most metals as well as elements we ordinarily meet in the gaseous state such as hydrogen, oxygen, nitrogen, helium, etc. But these densities make quite as certain that even the metals are in a vaporised and even very rarefied condition in the stars. The same conclusion is enforced by all the estimates of temperature which we discussed for a stellar atmosphere, while inside the star the temperatures must rise further—how much further we shall consider in a later chapter. For the coolest stellar atmospheres (about 2500 degrees) are above the melting and even vaporising point of many well-known metals exhibiting therein their characteristic spectral lines.

All these trains of evidence converge in demanding that even the less rarefied and the cooler stars, and certainly our sun, are not to be taken as anywhere solid or liquid but as just balls of extremely hot gases and vapours, although consisting of the familiar substances which on earth happen to be condensed into liquid or solid form.

What is less familiar, however, is the chemical and physical separation into ultimate atomic and electronic components, which those substances undergo at stellar temperatures and densities: at this stage of the enquiry the astrophysicist more than repays what he has learnt from the laboratory, as his work reveals extremes to which terrestrial experiments can only point distantly. Chemically, one definite conclusion becomes at once insistent: in the terrestrial laboratory we deal on the whole with compounds, or matter in which the primitive atoms have become bound in molecular groupings such as water, carbon dioxide, the hydrocarbons of fuel, the carbohydrates of living bodies, the oxides of the metals, etc. But at high temperatures, such as stellar, all those homely compounds dissociate into their ultimate constituents, becoming 'free' atoms of hydrogen, carbon, silicon, etc. To produce these 'dissociations' in the laboratory requires deliberately planned effort, using definitely known liberations of energy from thermal or electrical or optical sources. In a star Nature is simpler than our contrivances, and the energy is purely thermal, so that the precise degree of dissociation into atoms is exactly calculable. For example, in the cooler stars certain compounds do survive, oxides, hydrides, etc., in very small fractional concentration, and their characteristic band spectra are readily detectable, whereas in the sun these are extremely faint and the condition of hydrogen and oxygen is predominantly atomic. In even hotter stars no trace of chemical compounds remains, and all substances exhibit in their spectra the purely atomic state.

We can pursue this type of argument further; at temperatures enforcing a hundred per cent dissociation into separate atoms, the energy available begins to be enough for the atoms themselves to lose the outermost of the electrons which make up their own structure, that is to say, they begin to be 'ionised'. This is detectable in the novel sets of lines appearing in the spectra. We evidently are here dealing with the inverse of Saha's problem referred to in the previous chapter. We can make comparison of stellar spectra with the progressive stages of dissociation and ionisation achieved in the laboratory, by increasing the stimulus from quiet flames to an electric arc and finally to a violent high voltage discharge, and we find that successively hotter stars achieve the same by the uniform distribution of thermal energy alone, so by Saha's methods we calculate the star's temperature. The state of matter which thus has to be envisaged is a reduction to simplicity; for at high temperature and low density, compared with the complexity of terrestrial occurrences, the stellar environment of Thermodynamic Equilibrium enables the available energies to be distributed in a uniform manner. Eddington rightly claimed that we may have more hope of understanding a star than anything terrestrial, and the laboratory physicist may well envy his astrophysical colleague. It is only in those vast and distant globes which underlie the celestial points of light that temperature is high enough for most terrestrial properties of matter to be eliminated, so that the elements respond calculably to a simple distribution of precisely assessable energy.

(2) ENERGY OF A STEADY STAR

The first stellar property which we investigated, the luminosity, implied that any star must be losing energy at a prodigious rate. For instance, the sun's radiation amounts to about 2×10^{33} ergs per second or about a million million million million horse-power. The more luminous stars emit their energy up to another million-fold. How does the star's economy afford this loss without cooling down?

A principal aim of this essay is to extract the contrast in behaviour between stable or steady stars and stars which are unstable enough to erupt or pulsate or explode: the fundamental distinction between the two kinds emerges as soon as these queries about energy are pursued. For if we make the assumption that a star is in a steady state, we can utilise the simplifying certainty that we know exactly how much energy is being generated inside that star—an amount just balancing the energy which it emits in radiation. A star which erupts or bursts catastrophically or periodically must have been exceeding that balance, and creating more energy than it was able at the immediately preceding time to get rid of; it must have been warming up, whereas if it had been radiating without sufficient internal generation of energy it must be cooling down. Now, considered over long enough time, say more than a million years, a star undoubtedly does warm up or cool down—that constitutes our problem of stellar evolution: even the sun, now fortunately steady with only minor fluctuations in total radiation to the extent of about one per cent, must have a future and a past unidentical with its present output of energy. But over the few years or centuries in which we or our ancestors have watched, the sun and other stars thus offering the simpler of astronomical properties have been steady. By definition, this constancy of temperature and brightness implies that the conception of equilibrium is applicable to them, and that the energy they create inside them is exactly that which we see being lost from their surface in radiated light and heat.

The scientific approach to a star's life is therefore first made from the point of view of equilibrium, and we ask to begin with 'Given the luminosity which we observe, how does that particular amount of matter comprised in that volume with that surface temperature create energy equal to such luminosity?' The red hot poker cools by its own loss of radiation unless the energy is replenished by putting it back into the fire: there is no external fire to replenish the vast outpouring of a star's energy, and the problem of its maintained life as a luminous object is a problem of some internal mechanism. Two suggested answers to this problem will now be outlined, the gravitational of Kelvin's nineteenth century, and the atomic of the present post-Rutherfordian generation, with in particular the contribution begun by Bethe.

(3) GRAVITATION AS SOURCE OF ENERGY

To a past century, the only large-scale source of energy imaginable for a star was a by-product of its own shrinkage under the universal force of gravitation: we have considered gravitation as controlling the effect of one body upon another, in binary star systems, but here we encounter the gravitational tendency of a single body to fall inward upon its own core. The difficulty (which will appear quantitatively in a simple calculation in Part II) is that at the rate needed to supply the energy for the luminosity the star would exhaust its possibilities of shrinkage in a 'short' time. 'Short' may mean a million years, but the sun has been giving out the light and heat necessary to terrestrial life for much longer than a million years, probably longer than a thousand million, judging by the age of the fossil-bearing rocks whose original life must have been nourished in sun-light.

The argument for gravitational maintenance of sun or any star is, briefly, that all diminution of potential energy means a reappearance in some other form of the same amount of energy, often in the form of heat; this principle of 'conservation of energy' generalises all our physical experience of Nature. In the particular case here, an object falling under gravity gains energy of motion, a fact demonstrable in the commonest of daily life; with a mass of gas on a large scale, this energy of motion is the energy of innumerable atomic collisions, namely heat.

A star, which we have shown is a vast sphere of gas, must be always experiencing the gravitational tendency of its material to shrink upon its centre, just as loose particles in the terrestrial atmosphere fall: but it must be remembered that the star is gaseous throughout. In fact only the pressure of the gas resisting further squeeze, and the outward pressure of radiation itself, prevents collapse into far higher densities than those found in our earlier argument. The White Dwarf stars, discussed later, show what happens when that radiation pressure fails and the star does collapse. Later we also face the question of whether a star may gain even more energy by shrinking than it is then losing by radiating, and thus rise in temperature. But at present we are only concerned to ask whether steady shrinkage over a long life could as much as just make up the loss of radiation and keep the star shining steadily. The problem of particular interest is, for how long could our sun radiate away enough light and heat to maintain terrestrial life without cooling at all, if slow shrinkage at approximately the present dimensions were the main source of replenishment of the lost energy? This problem, left to twentieth-century astrophysics as legacy by Kelvin and Helmholtz, was quite definitely soluble, since it is possible to decide within narrow limits how much of the energy of gravitational shrinkage goes to radiation and how much goes to internal heating of the star. In Part II we outline the classical proof that such a source could only keep the sun shining for ten million years at most, whereas geological evidence of past life on the earth shows conclusively that the sun has in fact radiated at not very different rate from the present for a hundred times as long, at least a thousand million. It is conceivable, and is sometimes argued, that the cooler red stars do keep themselves shining by gravitational contraction; but if so, their life in the phase in which we find them is very short, perhaps less than a million years. For the sun, there is no doubt whatever as to the total inadequacy of this source of energy.

In recent years, Hoyle and Lyttleton have drawn attention to the effects of a star's habit of gathering gas or cosmic dust from interstellar space; this may affect both replenishment of lost mass and lost energy, but there is not as yet universal agreement as to the extent. We discuss therefore here only the alternative to gravitational energy offered by atomic physics.

(4) ATOMIC SOURCES OF ENERGY

This very fruitful alternative began to be guessed at during the first quarter of this century, when the facts of radioactive or unstable chemical elements became known. But while it was easy to say vaguely 'there must be atomic transmutations going on in stars', only in the 1920's was any estimate reached as to amounts of energy likely to be liberated by particular atomic reactions. In 1939 Bethe in America and Weiszäcker in Europe carried ideas first mooted by Atkinson the Englishman and Houtermans the Dutchman, so far as to work out a mechanism for the sun's maintenance.

The essence of the situation could have been predicted at any time since 1905 when Einstein propounded his first theory of Relativity. With most of that theory we are not in this essay concerned, only with one feature—its identification of mass with energy. In one sense all our experience of Nature forces us to recognise that mass is 'conserved' like energy is conserved; chemical transformations do not create or destroy matter but only alter its behaviour. If, however, we recognise the possibility, becoming actuality since Rutherford's first experiments in the 1930's, that one species of atom can be transmuted into another quite different species—the alchemist's dream at last true in an exacter sense than he ever thought—then Einstein's equivalence of mass and energy becomes of vital importance. If the final aggregate of atoms comprises less mass than the initial atoms, the difference in mass reappears as liberated energy. The actual transformation factor, involving the square of the velocity of light, makes a very small loss of mass set free a vast store of energy; it was often remarked that the hydrogen from a thimbleful of water, transmuted to helium, would liberate the energy to drive a liner across the Atlantic. That this access of free energy on loss of mass is no mere dream was proved at its first 'practical' demonstration, the atomic bomb, about whose terrible potency the luckless Japanese cities could have no doubt.

In the centre of a star, temperature and gas pressure are immensely higher than in the outermost skin which alone we see, in spite of low densities: how much higher we shall consider later. Atomic transmutations must be proceeding at a rate which vastly exceeds the products of our most ingenious and costly experiments in the laboratory. There is no doubt at all that a highly prevalent example of such 'nuclear synthesis' in a star is the formation of the element helium by gradual destruction of the element hydrogen; Bethe and the other workers we mentioned calculate very convincingly the temperatures at which this 'reaction' reaches a significant rate. In Part II we give the simplified calculation showing that loss of quite a small percentage of the sun's hydrogen to form helium can provide ample energy to maintain the sun's luminosity for a thousand million years—a source thus a hundred-fold more effective than the gravitational shrinkage which until recent years seemed the only conceivable.

In a later chapter we shall return to ask whether such atomic processes can also explain the life history of other stars than the sun, and in particular we shall describe the modern view that a star's course of evolution may be principally just the course of gradual exhausting of its hydrogen—the atomic fuel for supplying large-scale cosmic energy. It will be found that one particular means of helium production out of hydrogen via the intermediacy of carbon and nitrogen, ('Bethe's cycle'), has proved so far the most valuable suggestion, and though not without its puzzles has brought nearer a definite picture of a star's life-history.

CHAPTER IV

The Classification which offers a Clue Towards a Star's Evolution

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(I) INTRODUCTION TO SPECTRA AND THEIR INTERPRETATION

In order not to omit an essential link during the most general survey of possibilities in Chapters II and III, some mentions of a star's spectrum have already been introduced, for instance under Saha's treatment of temperature and in quoting the Doppler displacement of lines from a moving star. But having now reached the need for a classification of stars, and the most significant classification being one based upon spectra, it will be appropriate to digress and to complete some of the necessary background. We will attempt, without assuming previous acquaintance with facts or terminology, to say what has become the meaning and the use of that extensive branch of physics known as spectroscopy.

The whole topic of spectra, originally optical, nowadays includes the analysis of all radiation, not only visible light but X-rays, heat radiation, radio waves, and at the other extreme the γ rays of radium. For all of these are particular cases of a transmission of energy through space by a wave-motion, in which the vibrating quantities are electrical and magnetic instead of material; the electromagnetic nature of light allows it to traverse interstellar space where there is almost complete absence of material, but wave motion of material itself has traditionally been easier to picture, for instance the atmospheric waves of sound, waves on the surface of water, and vibrations of elastic solids.

Wave motion of all kinds, whether of light in empty space or of sound and other distortions in material, is describable by three quantities, the distance from crest to crest of the undulation, or wave-length, written λ, the frequency or number of vibrations per second, and the velocity of unimpeded travel. In the case of light and all other electromagnetic radiation this velocity is found to have a fixed magnitude of 3×10^{10} cm. per sec. or about 186,000 miles per second in free space.

The term spectrum denotes the analysed spread or dispersion of the differing wave-lengths in any mixed radiation. In the particular range of λ called 'light' because our eyes are sensitive to it, the spectrum can be made visible as a spread of colours from violet to red, which are the analysed constituents of white light. The very small visible range in λ stretches from about 4000 to about 7000 in units of hundred millionths of a centimetre, out of a total range of all known electromagnetic waves whose λ reach from less than a tenth of that extremely small unit to many millions, indeed in radio to hundreds of kilometres. Beyond the limits of red and violet sensitivity of the eye, electrical and thermal means of detection become necessary, and even for visible light are more reliable.

Apparatus for 'dispersing' or separating the different λ's, for instance transparent prisms and their accessories, may be erected at the focus of an astronomical telescope or in the camera exposed to a laboratory source of light. Any selection of those spectrum colours can thereby be made to appear as a set of spaced-out dark lines on a photographic plate, each line denoting one wave-length or group of nearby wave-lengths. Each such 'spectral line' is actually the monochromatic image of the slit-shaped aperture by which the light is admitted to the spectroscope. In a 'continuous spectrum', mentioned in connection with Wien's law of temperatures, the lines are so close together as to merge into the continuously graded photographic shading which visually would imply the presence of *all* the colours, each fading into its neighbour, instead of a small selection with gaps between. The spectrum of sufficiently hot incandescent material shows a partial or complete range of such continuous grading, and it is its point of maximum intensity which in Wien's law denoted a particular temperature (Chapter II); whereas flames and gases electrically excited in the laboratory exhibit only isolated and characteristic groups of lines, which by comparison with other laboratory spectra are identifiable as arising from particular chemical species of atom or molecule. Most stars exhibit the continuous spectrum of the hot body, with superposed lines or bands denoting the elements or compounds in the somewhat cooler atmosphere; these lines or bands show dark against the bright colour background. In Chapter II some account was given of the temperatures inferred from these spectra.

Many astronomical conclusions can be drawn from slight displacements of the spectral lines relative to those photographed from a laboratory experiment; in particular the Doppler effect is of great value. This is a small but definite displacement of a line towards violet or towards red according as the source of light is moving towards or away from the observer. Wave-lengths are measurable to very high precision, so a star's plate under a microscope can provide accurate knowledge of a star's velocity along the line of sight: some uses of this were quoted in discussing binary stars. To take an example, a certain line due to atomic hydrogen in the laboratory has a wave-length of 4861·4 if the source of light is stationary; if in a star the same line shifts to 4877·6, or is 16·2 units displaced, then the star or such portion of its atmosphere which emits that light is moving away from us at 600 miles per second.

Ever since the pioneer work of the Dane, Niels Bohr, from 1913 onwards, these precise wave-lengths of spectral lines have been known to denote the energy exchanged when electrons become displaced within the structure of the atom. Experimental determination of the amounts of these energies can be made in the laboratory, for different chemical elements in the process of losing the successive electrons which control the grouping of their spectral lines; it is these experiments which enabled Saha and others to evolve a connection between the intensities of lines observed in stars and the temperatures at which the stellar atmosphere is glowing. Besides identifying the substances in a star, spectroscopy therefore adds information about physical condition, in particular temperature, and also knowledge of the velocity of the gases or of the whole star.

Photometry added to spectroscopy, referred to as spectrophotometry, is the most delicate and rewarding technique of all —the study of the distribution of intensities among the wavelengths, which is the study of what is going on in the atoms many million miles away under an infinitesimal point of light.

(2) THE SUN'S SPECTRUM

The unique situation of our only near star, the sun, creates an enormous excess of Apparent Luminosity relative to its rather meagre Absolute Luminosity; with such abundance of available light, instrumental equipment for solar physics can be concentrated on close analysis of detail instead of on economising the faint radiations from a distant star. Features changing and permanent, such as the contrast between 'sun-spots' and the main disc, are daily investigated at many observatories, and the exact wave-length of more than 20,000 lines in the solar spectrum has been measured. Not all have been identified, but we are no longer in the pioneer days when the important element helium was discovered in the sun before it was known terrestrially; unknown lines are likely to be rare transient compounds or unusual electronic states of the most familiar elements.

The sun's 'continuous' spectrum as coloured background yields by Wien's law of maximum intensity a temperature for the photosphere pretty close to 6000 degrees; the estimation can be made by passing the spectrum photo-plate through an electrical device for tracing the darkening across an image and allowing for the varied sensitivity of the emulsion. The 'line' spectrum superposed on the colour background is very rich in metals, over 2000 lines alone belonging to iron; from magnesium and other light metallic vapours, temperatures derived by Saha's type of argument are about 5500, exhibiting the expected gradient from hotter photosphere to cooler lines.

The lines due to elements which on earth are gaseous, hydrogen, oxygen, nitrogen, etc., are faint in the sun. This does not mean that they are rare; we have no reason to say the sun is 'made of iron' because metallic lines dominate the spectrum, any more than to suppose certain hotter stars showing mainly helium lines are made of that gas. It is highly probable that the sun and most stars do not differ very greatly in actual composition, and hydrogen is probably the most abundant element. The domination in any spectrum of any particular element implies more definitely that temperature happens to be such as to excite those particular electronic energies available at 5500, or at some other temperature in some other star. Thus in hotter or cooler stars the particular temperature 'brings out' to prominence some particular substance which may not be especially abundant, just as in the laboratory or in an open fire an extremely minute trace of sodium will dominate the whole colour by its orange-yellow.

This dependence upon temperature mainly, and on gas pressure secondarily, rather than on composition, becomes very evident when the disturbed 'sun-spot' areas on the sun's disc are compared spectroscopically with the main surface. The spots cover vortex or whirlpool areas and are cooler; hydrogen shows less, metals easily ionised show more strongly, chemical compounds show more strongly also. Applying the Saha methods, this all means a drop in temperature locally to below 5000 degrees.

By catching the brief moments of an eclipse, and by special techniques at other times, it is possible to isolate the various heights above the hot photosphere at which its atmospheric spectra originate; the result is not the stratification of heavy vapours below and lighter above, but a turbulent erupting of certain favoured substances, hydrogen and the metal calcium especially, flaring in outrush to 100,000 miles in a few hours. The sun is our prime example of a STEADY star, but that is in comparison with a wilder standard: at local spots it is violently unsteady compared with the most savage of terrestrial storms.

(3) SPECTRAL TYPES OF STARS

(*a*) Cooler than the sun. The current nomenclature for classifying stars is based upon their spectra, but the alphabetical lettering in use survives from an age before the precise relations of temperature and spectra were discovered. The lettering is therefore an agreed convention adhered to without intrinsic significance in any letter. The sun is a 'G' star, and a star whose spectrum resembles the sun-spot rather than the sun's disc is a 'K' star. These classes are subdivided by numerical subscripts in a way we need not detail here but which will be used in Part II. We can follow further the strengthening of lines of calcium and metals of low ionisation potential, the strengthening of hydrocarbon compounds, and the weakening of hydrogen lines, all of which develops in the cooler sun-spots; thus we find 'M' type stars still cooler at 3000–4000 degrees, showing increasing intensity of broad spectral bands indicating that the atmosphere is not too hot for chemical compounds to survive in some abundance. Metallic oxides dominate the 'M' star spectra, while a parallel temperature sequence of 'N' and 'R' stars are characterised by spectrum bands denoting compounds of gases with carbon. At 3000 degrees and below, these bands are so strong that it becomes very hard to distinguish the colours of the 'continuous' background; where the latter is at all clear, its weakness at the blue end and strength in red gives the reddish tinge visible even by eye in all these cooler stars. Below 2000 degrees only a star of extremely large surface or one very near to us would be detectable at all.

(*b*) Hotter than the sun. On the opposite side from 'K' and 'M', the solar or 'G' type fades into the 'F'. These stars show progressive dimming of the easily ionised metal lines, complete loss of the chemical compound bands, and strengthening of the hydrogen lines, as the colour sequence on Wien's law indicates successively higher temperatures. Visual observation records this as bringing in a whiteness in the previous transition from orange to yellow in the general colour. When 'F' is past and the 'A' stars are considered, at temperatures from about 7000 to 10,000 degrees, the spectrum shows very few lines above the continuous colours, which are in fact only noticeably crossed by the few strong broad hydrogen lines. These exhibit excellently the 'series' phenomenon which Bohr in 1913 was the first to explain as denoting the electronic levels in the hydrogen atom. By 'series' we mean that the distribution of the electrons in the atoms has spaced out the lines into an obvious pattern, the distance between successive lines on the photo-plate diminishing regularly towards the violet end. We emphasise again that dominance by hydrogen lines implies a temperature able to bring out that feature by the appropriate energy stage, and does not mean an excessive amount of hydrogen. The next stars, of 'B' type spectrum, are still hotter, and again of few lines; but hydrogen has become weak, and the most noticeable lines are of helium. Helium requires a higher temperature to excite its spectrum, and the 'B' stars on the Saha classification are around 16,000 degrees in the temperature of their atmospheres. The colour spectrum is by now white or bluish white as the Wien maximum has shifted away from the red. The hottest stars of all, the spectral type 'O', exhibit helium but not in the 'neutral' condition of the 'B' stars; helium in the 'O' stars has lost one electron and the lines are those imposed by the energy exchanges of the inner electron. The temperature at which this occurs is from 30,000 degrees upwards, and the corresponding laboratory exhibition of the ionised helium spectrum is only with the greatest difficulty achieved by the expenditure of electrical energy in sudden condenser discharges.

Although this classification by spectrum is in general based on the *dark* lines crossing a *colour* background, an intriguing puzzle is set by some stars, among them the coolest as well as the hottest, which exhibit their line spectrum as actually brighter than the continuous colour beneath. This probably denotes a lack of the sharply bounded photosphere which characterised the sun and the majority of stars. The few 'bright-line stars' are probably surrounded by vast glowing haloes spreading beyond what in our part of the universe would be the confines of the solar system.

In one particular sub-type of the 'O' stars, those known from their discoverers as the Wolf-Rayet, these bright lines are extremely broad and almost certainly include a great range of Doppler displacement; the inference is that these atmospheres are in incessant eruption, the whole atmosphere blowing off the star towards us at many miles per second. These will be considered later as a case of 'unsteady' star; it must be remembered that our own sun erupts over hundreds of thousands of kilometres, but only at odd points, for instance in the 'flares'

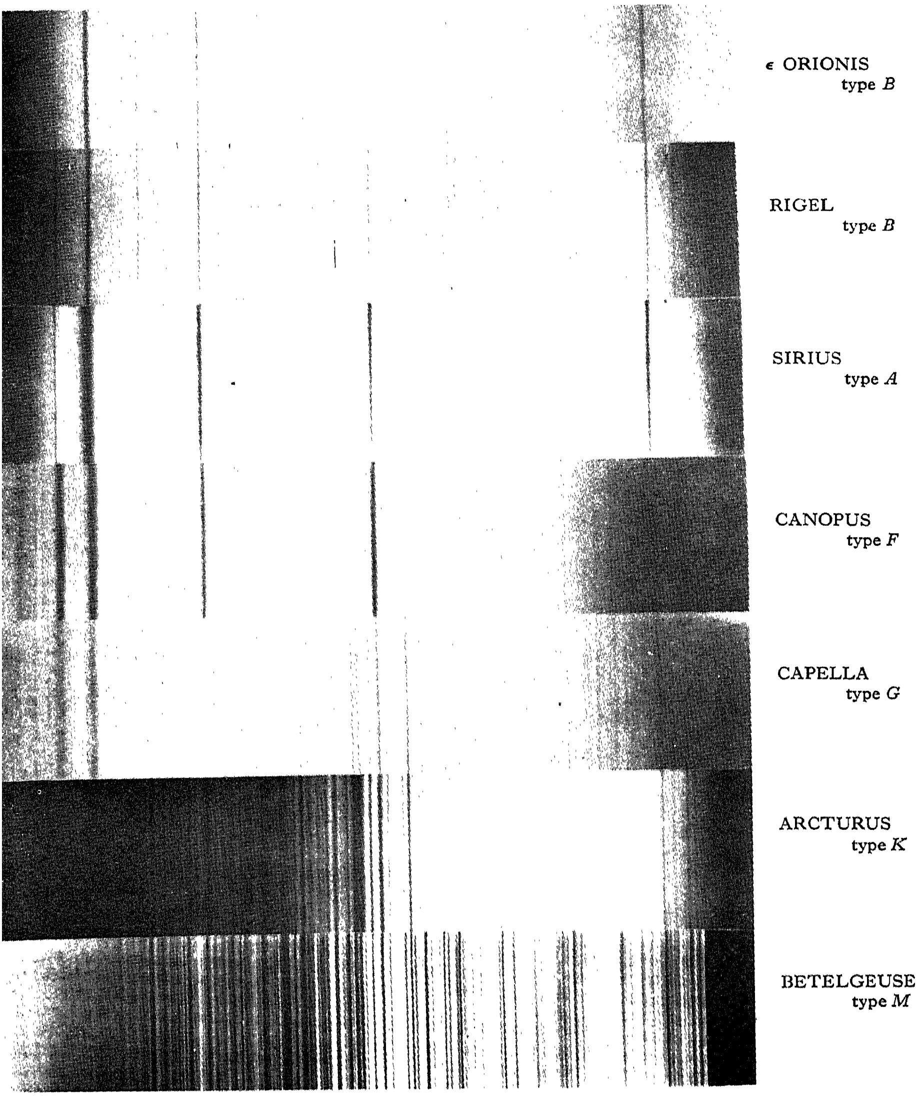

Plate 1. Types of Stellar Spectra

(*By courtesy of Harvard Observatory*)

These seven well-known bright stars were prominent among the specimens on which the spectroscopic classification of dark-line stars was first established by the Harvard researchers.

which disturb radio reception. Seen at a greater distance, the sun's spectrum would not be detectably disturbed, and the uprushing atmospheres of the Wolf-Rayet stars are a far vaster divergence from equilibrium.

(4) GIANTS AND DWARFS

We are now in a position to realise how there grew up the fundamental Giant-Dwarf classification, which has proved remarkably fruitful in *suggesting* evolutionary discussion of stars, although it was erroneous in its first attempts to identify

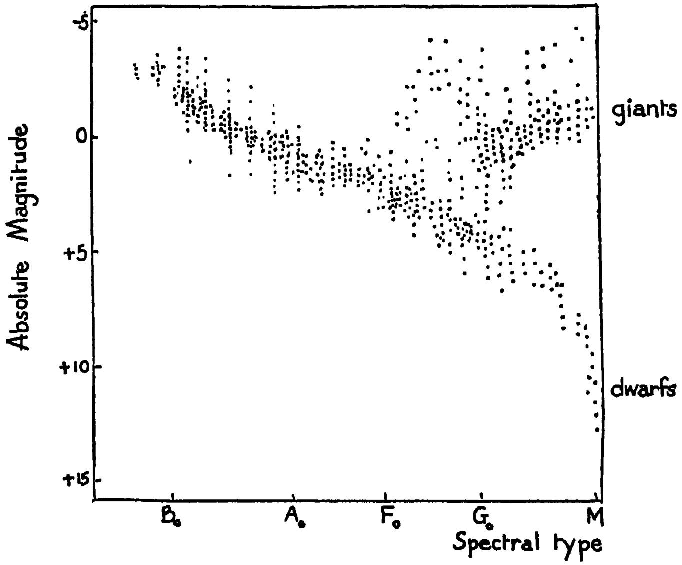

FIG. 5.

The relation between Luminosity and type of spectrum: this form of diagram was developed by Russell of Princeton, and shows the progression of brightness along the spectral sequence; it indicates the branching towards the cooler stars, where the giants separate from the less luminous dwarfs of the main sequence at the same temperature. A few White Dwarfs at the bottom of the diagram near letters A and F, and some super-giants at the top, stand apart from both branches of the scheme. This graph is adapted from Baker's reproduction of a large-scale plot by Gyllenberg of Lund which included nearly 7000 star points.

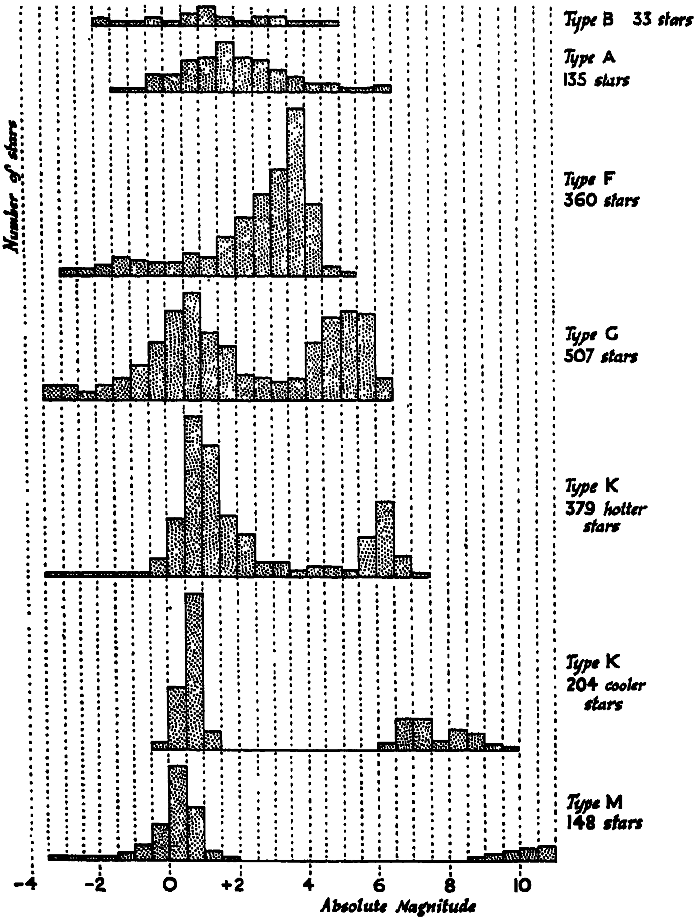

FIG. 6.

Danjon first employed this method of showing the branching of cool stars into Giants and Dwarfs, using the data of Adams and collaborators in America and of Jackson and Furner in England. For each spectral type the number of stars in any luminosity group is plotted against Absolute Magnitude; this indicates the distinction into very bright giants and very faint dwarfs, which develops most strongly among the coolest stars but is lost at the higher temperatures where the graphs become single. About 1700 stars contribute, and we have here redrawn into a step diagram the original data from which Danjon worked.

physical state and age of a star. We have already classified stars by their spectral type, which on Saha's principles is a temperature sequence from the coolest 'M' through 'K', 'G', 'F', 'A', 'B', 'O', to the hottest. We must also recollect here that the whole study of a star's physical constitution started from a classification of Absolute Luminosity which could be derived from the observable amount of Apparent Luminosity. The Giant-Dwarf view of stars arises when the task of putting the stars into an order of luminosity is compared with putting the same stars into an order of spectral type. The names of pioneers in this mode of analysis are principally Hertzsprung of Leiden in Holland, who saw the earliest clues to the situation as far back as 1905, and Russell of Princeton in U.S.A. who first devised 'spectrum-luminosity diagrams' in 1913. We reproduce a typical example of the latter in Fig. 5, and in Fig. 6 a statistical diagram which indicates in a different way the numbers of stars of all spectral types which possess any given luminosity.

Each of these classical diagrams enforces a definite conclusion from its widely assembled facts, that whereas the hotter stars show a single rough correlation between brightness and spectral type, denoting the not unexpected tendency for the hotter to be the brighter, the cool stars split into two distinct kinds, one kind extremely luminous and the other extremely faint. These two extreme species of cooler star were reasonably called the Giants and the Dwarfs. Consider how the facts of the diagrams justify the distinction and the picturesque naming, as follows.

The three quantities on which a star's luminosity can depend are its temperature, its mass, and its surface area; we had the fact reliably established that masses do not vary greatly from star to star, a ratio of ten to one covering the great majority, whereas the cool red stars exhibit disparity in luminosity up to ratios of ten thousand to one between a Giant and a Dwarf otherwise not dissimilar. In particular, this disparity is between stars of much the same temperature, at the red end of the scale or about 3000 degrees. The remaining possibility is that the striking difference in brightness is just a discrepancy in size or surface area, a Giant of similar mass and temperature having thousands of times the area of illuminated surface possessed by the Dwarf. Hundred-fold increase in stellar diameter would provide ten-thousand-fold increase in surface and therefore ten-thousand-fold luminosity if other factors were equal. We here meet another aspect of the fact reported under Chapter II (4), that the larger stars examined by the 'interference' method do reveal diameters hundreds of times that of our own sun, and those are just the luminous Giants. The sun is definitely a Dwarf, though not an extreme one of the red type.

The view of stellar evolution immediately suggested by these Giant-Dwarf *facts* was this. Since a star loses heat in giving out its normal radiation, a primitive theory had supposed the hottest 'O' and 'B' types to be the youngest, and the cool 'K' and 'M' types to be the oldest having lost the energy-radiating powers of youth. Such a view neglected the consideration (Chapter III, (3)) that a star may maintain or even increase for a while its internal heat by gravitational shrinkage. The revelation of Giant-Dwarf distinction among the cool red stars suggested that not all the coolest are old, but only the Dwarfs. The Giants, vast distended spheres of the most tenuous density, whose enormous surface area confers such high luminosity, might be the youngest and in a stage still capable of becoming hotter by the process of shrinkage. On this view the hot 'O' and 'B' stars are in middle age, not youth.

This has since been recognised to be an over-simplification; it is not likely that a young red 'M' star actually becomes in turn a 'K', 'G', 'F', 'A', 'B', 'O', star and after passing a maximum temperature cools down through the same sequence in reverse, until in old age it is a red dwarf of the same spectrum with which it began life. The 'O' stars are fundamentally more massive than the 'M', 'K', 'G'. Nevertheless it is unlikely that the cool dwarfs are in an early stage of heating up, and there are still many who regard the red giants as young, in spite of the suggestive fact that some of them in star clusters may be quite old. If they are young, their youth will be brief if maintained by gravitation; we know very little as yet about the duration of stages in stellar life. The initial breakdown of the giant-dwarf sequence as a *theory* of evolution came when Eddington found that even dwarfs had the constitution of a perfect gas, so that their contraction could not be regarded as having reached the limit. It is best to regard any theory with reserve, but to accept the *fact* of there being these two violently discrepant groups among the cooler stars, whether or not the one can grow out of the other.

A serious omission in any of the above theories is the lack of any rational place in the scheme to fit the White Dwarfs: these are stars of extremely high density, higher than anything of the cool red type, and nevertheless hot, for example of the 'A' spectrum type. They lie entirely off the graph in the luminosity-spectrum diagrams, and although they are often regarded as products of violent catastrophe rather than of quiet evolution we shall consider them next, before investigating unstable stars in Chapter V.

(5) THE WHITE DWARF STARS

The density which has to be assigned to White Dwarfs is so fantastically beyond anything realisable in terrestrial materials, that the dependence of the argument upon fact must be traced in some detail; it was not until later than the factual discoveries that theory made the state of things intelligible.

The first White Dwarf known was the 'companion to Sirius', and the tale unfolded as follows. Quantitative equations will be supplied in Part II.

Sirius, the Dog Star blazing over our winter skies below the unforgettable pattern of Orion, was in the 1830's found by Bessel to possess a periodic disturbance in its 'proper motion'. Proper motion is the slight but detectable drift in position which can be discovered by the most careful locating for most stars. Since no other star is near enough to cause this disturbance, it seemed probable that the single point of light covers a closely bound pair of stars circulating round their common centre of mass—Sirius is a binary. The two components, which we will call Sirius 'A' and Sirius 'B', are of strikingly different luminosity, the one being about ten thousand times the brightness of the other. It was no easy problem to isolate the light of the fainter, a feat achieved in 1862. In 1915 Adams of Mount

Wilson succeeded in obtaining the spectrum of Sirius 'B' separate from that of Sirius 'A', by devices to escape the blotting-out of the faint lines under the ten-thousand-fold excess of its brilliant neighbour. The astonishing feature immediately revealed was that Sirius 'B' has a 'hydrogen' spectrum only slightly cooler than Sirius 'A'. Now since the temperature of these hydrogen atmospheres is fairly precisely obtainable, the energy per unit of area can be inferred with considerable degree of accuracy from Stefan's law (Chapter II, (4)). Comparing such *energy per unit area* with *total energy* of Sirius 'B' which is extremely faint, only one ten-thousandth of the luminosity of its companion, a quite reliable comparison can be made between the inferred surface areas of Sirius 'A', Sirius 'B', and the sun. Sirius 'A' is just over one and a half times the radius of the sun, but Sirius 'B' is little more than one fortieth of the sun's radius. The densities are then obtainable from the mass of each star. The sun's density is accurately known, that of Sirius 'A' comes out to be slightly less than the sun's, but the mean density of Sirius 'B' works out at 68,000 grams per cubic centimetre or more than a ton per cubic inch.

Since the heaviest terrestrial materials, such as a few metals, Platinum, Tungsten, etc., are less than a thousandth of this fantastic figure, the whole argument had to be re-scrutinised with the most rigorously critical care (see Part II); but there remained after such scrutiny no doubt whatever as to the right order of magnitude, although of course there is no claim to precision, and 'thousands of times terrestrial densities' is all we are sure about.

It is not, however, so surprising a conclusion, when the atomic condition of matter in the interior of some stars is taken into account. Every element must, at the most conservative estimate of temperature, be 'ionised down to the X-ray level'; by this is meant that the atom must have been stripped of almost all its halo of electrons. All views of atomic structure since Rutherford's first triumphs about 1910 agree on one point, namely that an atom is an extremely 'open-work' structure—empty space over nine hundred and ninety-nine thousandths of its volume and only held together by the electrical forces of that halo of electrons.

Remove these, and little is left but the 'nucleus' which determines the main mass; you have thereby removed the impediment to compressibility of matter, and high gas pressure would assemble the bare nuclei into one thousandth of the normal atomic volume or less. This we cannot effect in the laboratory, but at the centre of a star only the pressure of radiation itself prevents such compression. The chain of argument revealing the enormous density of a White Dwarf is indirect but perfectly sound: it is also not at all at variance with what the interior of a star could produce if it ceased to be distended by radiation pressure. The obvious problem set is not whether the White Dwarf is reasonable, but why can some stars maintain enough radiation pressure to resist such compression, while others cannot.

Other White Dwarfs than Sirius 'B' have since been discovered, some of even more extreme density. Since their very low luminosity prevents our seeing any but the nearest, it is likely that far more of these nightmare laboratories of high stellar density exist than can ever be discovered. The White Dwarf state is a final state of material reaching our consciousness, since its only exit is into the 'Black Dwarf' which would be invisible.

But the problem of what reduces a star to this state is a puzzle which has thrown up numerous controversial opinions; we shall refer to it after discussing catastrophic stars, since collapse of an ordinary star into the White Dwarf state has mostly been regarded as marking a sudden crisis in life-history. We shall have to face the possibility that a star may suddenly fail in the radiation pressure which distends it against gravitation, leaving the latter free to enforce cataclysmic fall towards a highly condensed central core. Whatever the result of conflicting theories, the *facts* mark out the White Dwarfs as aberrations outside the relationship of luminosity and spectra exhibited by most stars.

CHAPTER V

Stars which are not Steady

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(1) PULSATING STARS

The phenomenon of the White Dwarfs had suggested that at some epoch in its life-history a star may undergo drastic change. We have no certain evidence that the particular change has ever been observed happening; but other changes in stars do occur before our eyes, some periodic from which the star regularly recovers, and some indicating a catastrophe which seems in general not repeatable. We shall in this Section consider the so-called 'Cepheid variables' or stars which fluctuate regularly in brightness, and to which the best-known example, δ in the constellation Cepheus, gives the name.

Luminosity is, of course, the first of all 'observables', and alteration in luminosity is therefore the primary source of any knowledge we may acquire about the unsteadiness of a star. So we must first rid ourselves of certain causes of the light's fluctuation which do not involve changes in a star's constitution. We have mentioned in Chapter II the behaviour of 'binaries' or closely associated pairs of stars, and we may recall that if the orientation of the plane of revolution of such a pair allows them to cross our line of sight one in front of the other, that occasion has some similarities to an eclipse in our own solar system when the moon has come between us and the sun. If one component of the binary pair is very bright and the other relatively faint, there will be alternating eclipses with a deep cut-off of light and with a shallower cut-off, and a type of 'light-curve' or time-diagram of fluctuating luminosity due to this cause was shown in Fig. 3. Binaries were also stated to be detectable through the periodic behaviour of their spectral lines in the Doppler effect (Chapter IV, (1)), which displaces a line slightly towards red (if the star is receding) or towards violet (if the star is approaching). When the pair of stars is in that part of its rotation which makes one recede and the other approach along our line of sight, both Doppler displacements will occur and the spectrum line from the combined light will split right and left on the photo-plate. Hours or days later the pair are instead crossing our line of sight, and as there is then no recession or approach the Doppler shift ceases and the spectral lines are temporarily single again.

But having set apart these commoner instances of light fluctuation, ascribable to the fact that the point of light conceals a pair of stars, we can consider the behaviour of another class of variable star, the Cepheid, whose periodic changes of luminosity denote constitutional fluctuation in a single body and not the mutual eclipsing of a pair.

At first, serious attempts were made to explain Cepheids as being also binaries, until the calculated size of orbit turned out to be impossibly small, indeed smaller than the star itself. Another view postulated an elongated star, stretching, contracting, spinning, and so accounting for regular change in total brightness. For a generation now, mainly due to Eddington, it has been universally recognised that the most likely situation is a spherical star pulsating, i.e. alternately swelling and shrinking, so that the contribution of sheer size to luminosity waxes and wanes periodically. If such contraction and expansion reaches considerable amplitude, temperature changes will of course also occur: it is not always clear which is cause and which is effect.

Close study of the spectral changes accompanying the luminosity change indicates a Doppler effect of shifting lines, but occurring at phases of the light change which definitely exclude the explanation of a binary star. For the fastest outward velocity of the gases in the star occurs at the star's brightest moment, but the fastest inward fall of the gases occurs at the dimmest moment; in any mutually eclipsing pair of stars, the dimming would occur when the bodies are crossing our line of sight and when therefore there is no Doppler effect at all.

It is useful nowadays to classify together all kinds of star which exhibit periodic fluctuation of light due thus to some constitutional unsteadiness and not due to being eclipsed by a companion. The periodicity of some is very exact, but others with the longest time-interval between brightening and dimming tend to be of less precise recurrence, and the classification here merges into the 'irregular variables'; of those latter the extremest only 'vary' occasionally or even only once—the so-called Novae or 'New Stars' which may in some cases be exploding stars.

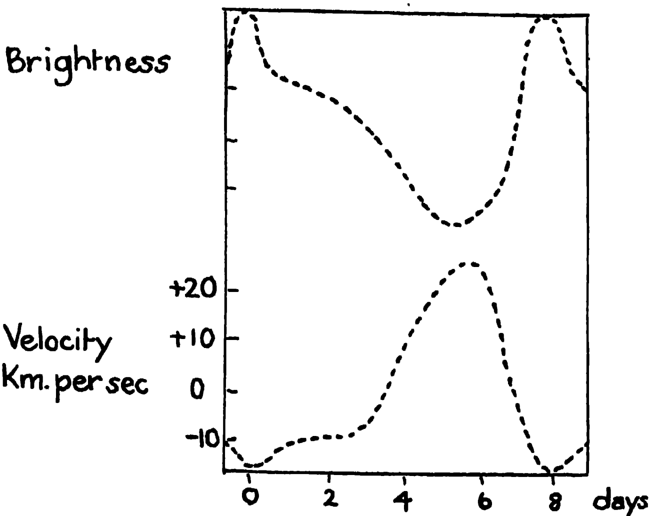

FIG. 7.

If the brightness, and the velocity of up- or down-rush of the gases in a Cepheid pulsating star, are compared over the cycle of days in which they regularly repeat, the maximum brightness occurs at greatest up-rush (negative velocity) and the minimum at greatest speed of fall (positive velocity). Diagram from Curtiss of Lick Observatory.

(*a*) Classical Cepheids: the particular relation mentioned between the phase of the light-change and the Doppler velocity of the up-rushing gases as the star expands, is expressed in Fig. 7. The 'period' or time from maximum brightness to next maximum is typically (in the case of the first known in Cepheus) about a week, but ranges up to about fifty days and down to a day or two. There is, as mentioned previously, a very useful correlation between period and the mean brightness of each Cepheid, enabling Absolute Luminosity to be inferred from period, a valuable clue to the distance of some of the furthest groupings in the universe. These classical Cepheids are mostly yellow stars, in the classification of Chapter IV, of spectral types 'F' and 'G'.

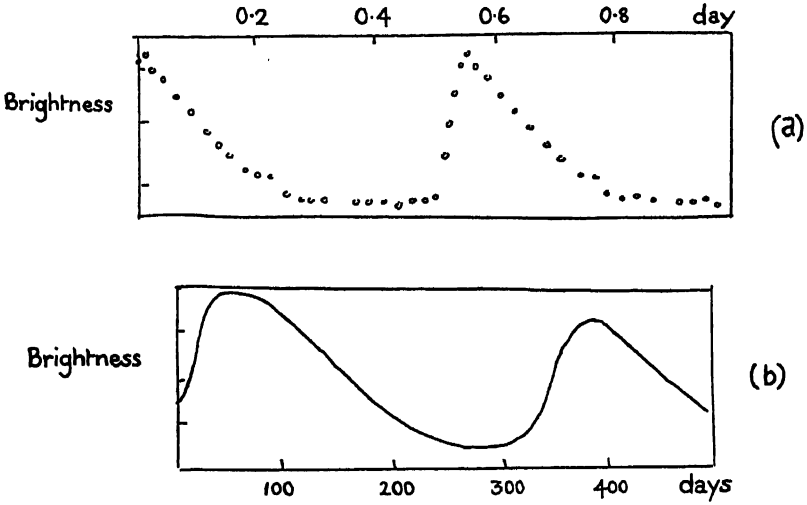

FIG. 8.

(*a*) Pulsating stars with very short period can double their brightness in an hour. This 'Cluster Cepheid' is from the publications of Harvard Observatory.
(*b*) The other extreme in length of period of pulsating stars: the star Mira takes nearly a year to repeat, and successive peaks are not identical as they were for Cepheids. Diagram from the British Astronomical Association observers.

(*b*) Cluster Cepheids: these are of very short period, about a day only, so that their peculiarly steep rise (Fig. 8) makes such a star able to double its brightness in an hour. They are blue stars of spectral class 'A'. The name is due to their occurrence in the Globular Clusters which contain many thousands of stars each. In contrast to the classical Cepheids, a possible but puzzling clue to their evolutionary status is that their individual motions are among the fastest known whereas the classical are among the slowest in 'proper motion'.

(*c*) Long-period variables: the mechanism is here again nowadays recognised as pulsation, but the period or interval between successive maxima is from about two months up to a year or two: common examples are about 300 days. In Fig. 8 it is seen that the shape of the light-curve repeats itself less exactly than for Cepheids, and the amount of the fluctuation is not always the same each time. These are cool red giants, and among them 'Mira' in the constellation Cetus has been watched for centuries as deserving its name of 'wonderful'. It may be noticed that as Mira is so low in the temperature scale a small rise in temperature means a startling shift of the point of maximum intensity in the spectrum, into the red to which our eyes are sensitive, and away from the infra-red to which we are not sensitive and which is often called 'heat-radiation'. Hence Mira rises from 9th magnitude to 3rd magnitude, a rise of 250-fold as measured on a visual scale, while the temperature rises only from about 1900 to about 2600 degrees. Part at least of the change in light is due to the swelling of the diameter by about 30 per cent.

These facts of stellar pulsation are among the most intriguing and suggestive in astronomy. In Chapter III, (3) the shrinkage of a star was considered as involving vast amounts of gravitational energy with possible evolutionary significance: so it will be necessary in the final Chapter VI, after introducing the criteria which decide whether a star's material can move without endangering the stability of the whole star, to consider the attempts made by Eddington and others to attack the mystery of why a star can pulsate, or expand and shrink in so precise a rhythm.

Before that discussion, other unsteady stars must be looked at: first those which instead of thrusting out and then withdrawing their atmospheres continuously eject them into space.

(2) ERUPTING STARS, AND THE PLANETARY NEBULAE

We here put together two kinds of observed fact already mentioned in other connections. The first fact was that some of the 'O' type stars exhibit very broad and bright spectral lines; the width of these lines indicates high speed Doppler displacements

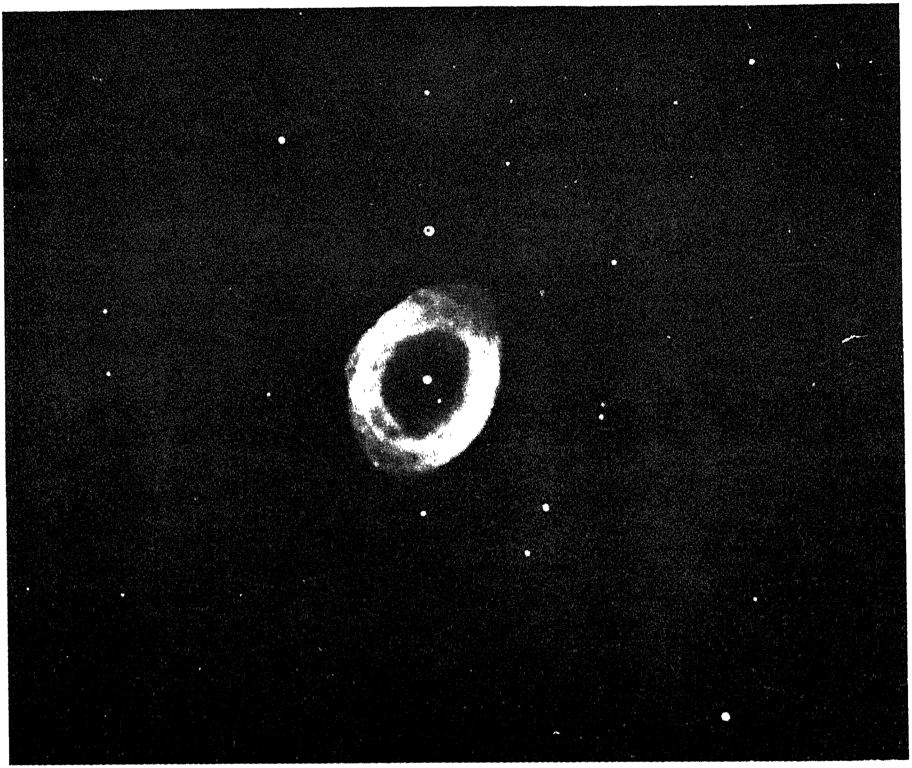

Plate 2. Ring nebula in Lyra

(By courtesy of Mount Wilson Observatory)

The detached glow is a fluorescence under the impact of ultra-violet radiation from the central star; in many such objects the halo contains familiar gases exhibiting an unfamiliar spectrum peculiar to the extremely low pressure, for example the green lines 4959 and 5007 of doubly ionised oxygen atoms and the violet 3726 and 3729 of singly ionised oxygen. The shell of gas was probably ejected from the abnormally hot star, and now spreads over a volume a million-fold exceeding that of our solar system.

implying a great range of velocities in out-rushing gases, and their brightness compared with the usual dark lines of most stars indicates a vast extension of atmosphere into a luminous cloud surrounding the star to a great distance. This cloud fluoresces under the impact of ultra-violet rays from the star itself. These stars formed the Wolf-Rayet division of the 'O' stars, themselves the hottest type with surface temperatures of 30,000 or 40,000 compared with our sun's 5,500 and the 'M' star's 3,000 degrees. Some estimates of Wolf-Rayet temperatures have exceeded 100,000.

The second fact has sometimes been regarded as one of the possible consequences of any such continuously erupting atmosphere. We refer to the so-called Planetary Nebulae, of which the best known is the 'Ring Nebula' in Lyra, but of which nearly 150 are known. In spite of the name they have nothing whatever to do with planets or any object so local as our solar system, but in a small telescope they look somewhat like a planet's disc—hence the first and now misleading name. Plate 2 is of the example in Lyra: it is clearly a semi-transparent shell of gas, seen thus as a detached ring in those cross-sections presenting to our line of sight the greatest thickness of material. A feature of the spectrum is the few bright lines which used to be called 'nebulium' because they seemed to have no laboratory counterpart; these lines are now known to come from oxygen in the atomic state and with its electrons at a definite energy level which could not persist at the less extreme rarefaction available in laboratory experiment. The nebular sphere is thus extremely tenuous, glowing under the impact of radiation from the central star.

The old problem as to whether nebulae, or luminous non-stellar clouds of gas, are parents or offspring of stars, has here one of its rarely convincing answers. These Planetary Nebulae, when found to be surrounding a hot star of Wolf-Rayet type, may well be the latter's offspring, in the sense of being the result of long-continued ejection of atmosphere from the star. Their Natural History is by no means settled, however, and we shall return later to some of the speculations attempting to connect Planetary Nebulae, Wolf-Rayet stars, White Dwarfs, and the Novae or catastrophic stars which we are next to describe. It is of some significance that out of the 200 or so Wolf-Rayet stars known, about forty are nuclei of Planetary Nebulae; but it is also true that other Wolf-Rayets are known without any surrounding halo beyond the actual attached atmosphere, which, however extensive, is on a much smaller scale than any nebula. Planetary Nebulae also exist whose nuclei are altogether 'tamer' stars than Wolf-Rayet, and some even appear to possess no nucleus at all, so far as can be observed. It must be remembered that a star at the centre of a nebula is not easily estimated in luminosity, as the surrounding cloud must introduce some obscuration which may be different for different instances according to the mechanism of fluorescence.

(3) NOVAE AND THE LESSER CATASTROPHE

This word, as meaning 'new star', denotes the situation in which stellar variability in luminosity reaches the extreme of a flare-up, allowing the terrestrial observer to experience a bright star of which nobody had previously been aware. But it is unlikely that we ever thus see before our eyes the sudden birth of a star—whatever that may mean. The 'new' star is often traced back to an obscure mark on previously exposed photographic plates, too faint to have been given individual character until the outburst; afterwards, after a year or so of violently changing history, the Nova returns to faintness, with some intriguing aberrations en route. These aberrations, and the partial resemblance to a Wolf-Rayet or a Planetary in the later stages, shows that we are not watching the evolutionary sequence of a normal star, shortened from millions of years to a few months, but a disease which more probably represents the loss of ordinary stability in an already ageing star.

The facts exhibit a remarkable uniformity: although about a hundred Nova outbursts have been known among the millions of stars of our Galaxy, most of these are listed from the present century, since fine instrumental detection was not available earlier and the great majority have been too distant to cause sensation among the non-scientific. Half a dozen in the last half century have reached a brightness about the first magnitude, and these 'naked-eye novae' tend to follow a similar course; the similarity is seen in Fig. 9, where the light curves of three are superposed. The rise to brilliance is very steep, the star brightening thousands-fold in a few days, and sometimes forcing itself upon the most careless attention a night or two after emerging from only photographically detectable faintness. The much slower decay covers months or a year or two, often with oscillations in luminosity recalling the Cepheids but here superposed upon a gradual dimming.

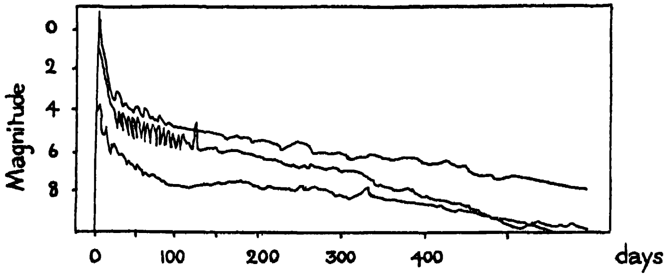

FIG. 9.

Nova Aquilae 1918, Nova Persei 1901, Nova Geminorum 1912. Typically, these brightened many thousands-fold in a few days, then decayed slowly with subsidiary oscillations of their luminosity. The superposed diagram is by Campbell of Lick Observatory, indicating the common features of such outbursts.

The following have been the most noticeable Novae of the twentieth century; the notation generally accepted is to write 'N' followed by the constellation in which the outbreak appears, and the year. We add the maximum brightness in 'magnitudes' and the rough ratio of brightest to faintest luminosity.

N	Persei	1901	+0·1	25,000
N	Geminorum	1912	+4	600
N	Aquilae	1918	-1·1	60,000
N	Cygni	1920	+1·8	3,000
N	Pictoris	1925	+1·2	10,000
N	Herculis	1934	+1·5	5,000

The approach to a common habit is apparent when we realise that the faint pre-nova state was at magnitude 10 or 11 or even fainter, far below visibility by eye, so that the rise is to be measured in factors which may well be many thousand: this is clearly a far more drastic change in a star's constitution than any Cepheid behaviour. Historical Novae which excited profound emotional reaction among contemporaries include 'Kepler's star' of A.D. 1604, 'Tycho's star' of A.D. 1572 visible in daylight and rivalling Venus at brightest, and perhaps another famous in history may have been the Star of Bethlehem.

The first discovery of any particular Nova is, of course, a haphazard chance; among people 'out that night' there has to be someone to whom the ordinary stars are so familiar that he at once recognises a newcomer to the sky. A postman on his route before dawn, accustomed almost subconsciously to recognise the familiar nightly distribution of stars, caught one Nova in South Africa: he was shrewd enough to realise the importance of getting accurate instruments set to work at once, and notified an observatory in time for the vital task of photometry and spectrum photography before the rapidly changing star had passed its most mysterious phases around the terrific maximum. It is fantastic, but true, to record that the man who first saw the great Nova of 1891 attempted the impossible task of catching the next one, and actually did so, ten years later.

It is of little use to force these catastrophic objects into any of the classification of our steady stars—they resemble several of the spectrum classes, not only in brief succession but even simultaneously. In their few weeks or months of violent life, Novae may pass through the stage of a 'B' type spectrum, (Chapter IV, (3)) giving place within twelve hours to the 'A' type, then to 'A' or 'F' around maximum brightness, and yet sometimes showing the hydrocarbon bands of the coolest 'R' or 'N' stars at a moment when the most violent expenditure of energy is scarcely past. After the maximum they show a hydrogen absorption spectrum, accompanied by lines resembling the high excitation spectra of the hottest stars. These absorption lines have Doppler displacements indicating unpredecented speeds of out-rushing gases, sometimes in successive outbursts at

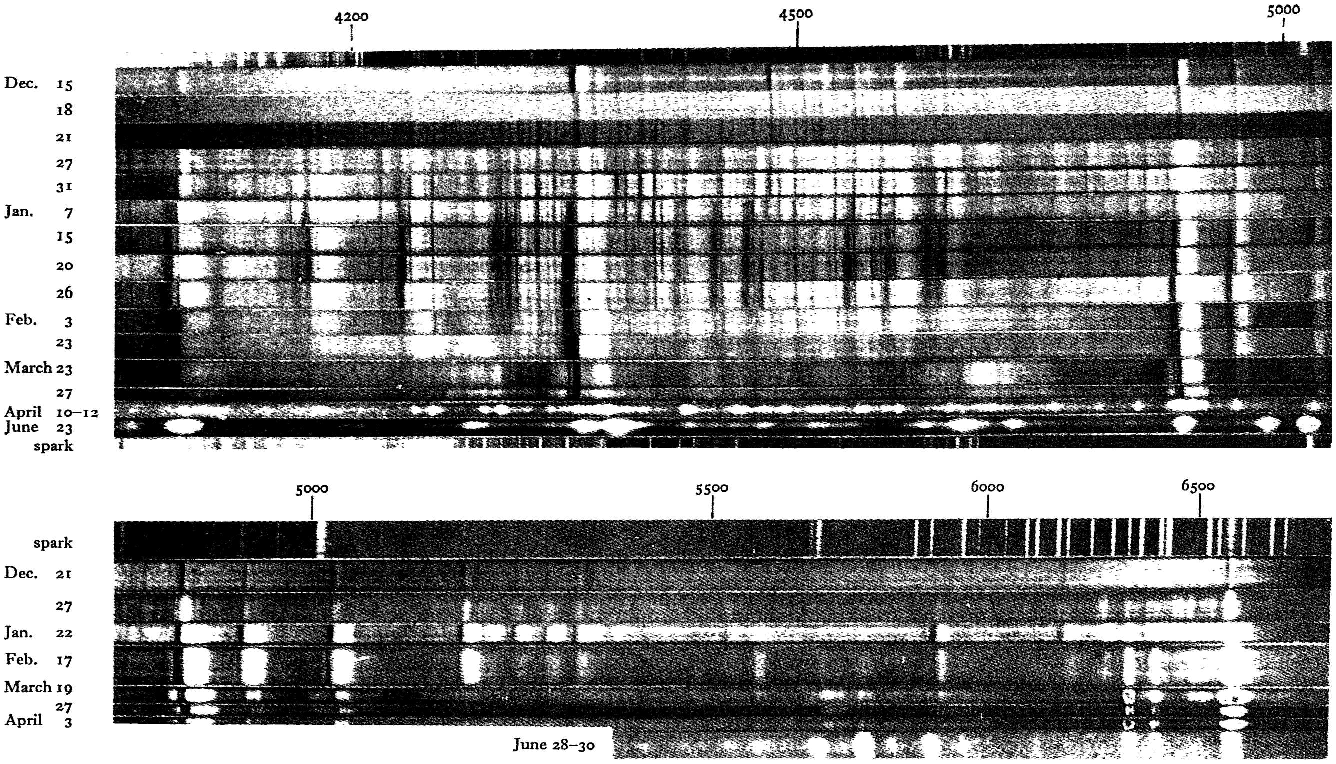

Plate 3. Day-by-day Evolution of Nova Herculis (1934)

(*By courtesy of Cambridge Observatory*)

See page opposite for description

1500 kilometres per second, 2000, and even more than 3000 km. per sec. These figures are far higher than anything that we can produce artificially as explosive velocities. As the star proceeds to fade, *bright* bands accompany the absorption lines and the spectrum reminiscent of a detached nebula appears. In some features the Wolf-Rayet spectrum is imitated. Finally the dimmed remnant of the brief blaze develops a luminous cloud surrounding the star, and still after years detectably receding from the site. The final stage therefore has some resemblance to a Planetary Nebula, reinforcing our view that the latter are products of gaseous ejection, but not necessarily proving that all Planetaries were once Novae; the accumulation of the Planetary's shell may have been more continuously and quietly achieved, perhaps by something more like the Wolf-Rayet process, an evaporation rather than an explosion.

Theories of what could cause a Nova outbreak are very various; they have ranged from the now discredited idea of a stellar collision, through the less unlikely possibility of thermal disturbance due to a star swimming into a region of drifting nebulosity, to several distinct but not always convincing grounds why a star might collapse from internal reasons. The extreme of the latter type of theory regards the post-Nova stage as White Dwarf, but it is doubtful whether this can be supported from actual spectrum and luminosity of the surviving

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Day-by-day Evolution of Nova Herculis (1934)

(See Plate 3 opposite)

During the winter 1934-5, this star erupted into a brief catastrophe involving high luminosity and dramatic spectrum modifications. These successive Cambridge spectra illustrate the rapid growth of broad bright bands bordered by dark edges on their violet side, indicating an exploding atmosphere with jets of gas moving in the line of sight at several hundreds of kilometres per second, as in the spectral line contour of Fig. 10*d*. The last picture, (June 1935), shows the Wolf-Rayet stage when the ejected gases begin to behave like a detached nebular shell. Top and bottom lines are the comparison spectra from laboratory sources of radiation. The numbers along the spectra, denoting wave-lengths, show the changes in the red (6500-5000 *A*) and in the yellow, green, blue, (5000-4000 *A*) in the two pictures respectively.

star; the still surrounding gases make detection of detail in the final stage very difficult. Moreover, the White Dwarf state is (Chapter IV, (5)) one from which no recovery or resurrection seems likely, whereas more than one instance is now known of a Nova repeating after some years of quiescence. We return to these theories in Chapter VI.

(4) SUPER-NOVAE AND THE GREATER CATASTROPHE

The Nova phenomenon was of sufficient violence: if our own sun were to behave so, even the first hours towards that thousands-fold increase of light and heat would destroy all terrestrial life. But in recent years there have been discovered a few very distant occurrences so much more violently catastrophic than the 'ordinary' Nova that they have had to be classed as Super-Novae, denoting a quite different scale. If the Novae we have been describing brightened ten-thousand-fold, the Super-Novae show in their faint distance a definite sign of million-fold or ten-million-fold brightening. Needless to say, no such phenomenon has occurred among the stars of our Galaxy since men have been watching and investigating. There is one possible exception to that statement: in A.D. 1054 Chinese records chronicle a violent outburst of light in Taurus, which most of our world was not then sufficiently civilised to document. And it is true that today there is in Taurus the strange 'Crab Nebula'. The Crab is an irregular (Plate 4) mass of luminous gas, and recent measurements have revealed that it is in constant expansion outwards. The present rate of expansion would suggest a starting point from a centre about 900 years ago, and many competent critical minds have been tempted to identify it with the still expanding shell of gas from a colossal Nova outbreak of much more than ordinary proportions.

But if the 'Crab' is indeed the wreck of our only 'local' Super-Nova in the centuries of human history, there are regions outside and beyond the Milky Way, and at distances very much greater than that of the furthest stars of our Galaxy. In those outer spaces are separate 'universes' such as the Spiral Nebulae, whose mystery is of a mentally and logically remoter nature, compared with problems of the individual stars. Their dis-

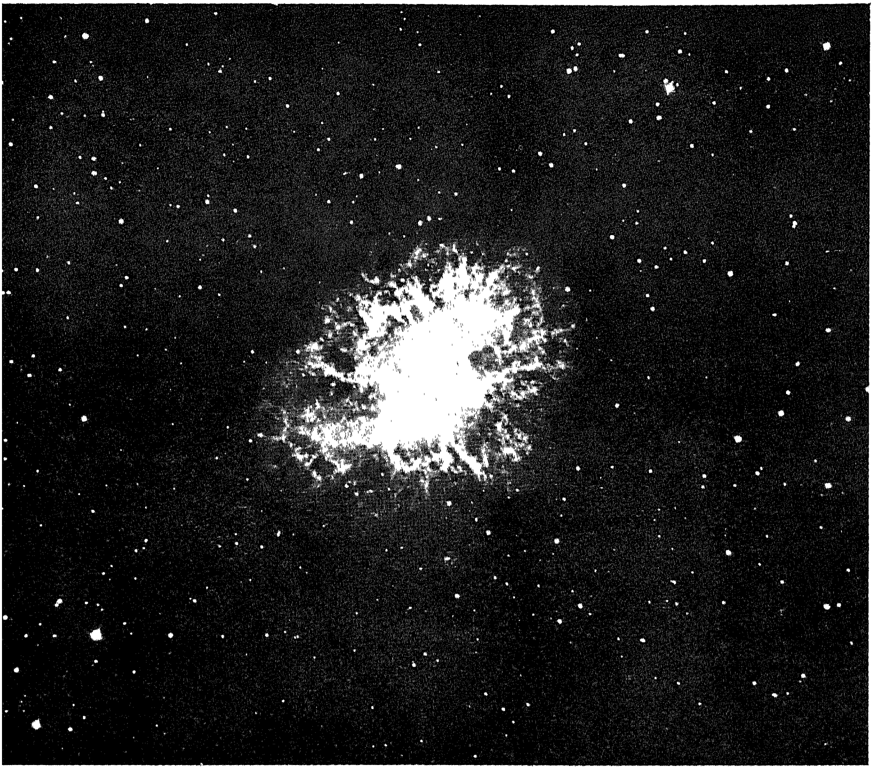

Plate 4. Crab nebula in Taurus

(*By courtesy of Mount Wilson Observatory, and Dr W. Baade who photographed in the red end of the spectrum*)

Successive photographs show that these gases are expanding and that their velocity implies a central origin about 900 years ago. It has been plausibly suggested that they represent the still spreading wreckage of a stellar outburst seen as a new star in that region by Chinese observers in A.D. 1054.

tances are only to be estimated in the millions of years that light at 186,000 miles per second takes to reach us thence, and their investigation involves fresh physical questions: the laws of gravitation and of atomic and spectroscopic science may there depend upon peculiarities scarcely detectable in the smaller scale of the laboratory or even of our Galaxy of stars. But those most distant aggregates, each totalling perhaps millions of stars too remote to be separately seen, do for a moment become relevant to the Nova problem, because from time to time one of those galaxies suddenly develops a 'bright spot'. Now when such a spot outshines the whole of the galaxy in which it appears, we only require the reasonable assumption that 'star' there means something of the same kind of object as the individuals of our own acquaintance, numbered as in our Milky Way by millions, to realise that if the bright spot is a Nova it must be an outburst of millions-fold, not merely thousands-fold as in our commoner Novae.

So far as any spectroscopy and spectrophotometry of so faintly distant objects has been possible, notably with the 100-inch reflector of Mount Wilson, these Super-Novae do follow something like the life of our nearer Novae. Of course the early and late stages are below the threshold of detection, and not much more than the peak of maximum luminosity is traceable in detail. A dozen or so of these Super-Novae have rewarded careful search in the first few years; this astonishingly large number is due to the fact that even if only once in several centuries (the Crab seems unique) a Super-Nova occurs in any one Galaxy, we have in those Spiral Nebulae many galaxies simultaneously to examine, and the harvest may become rich. It is tantalising that there are only one or two telescopes in the world of sufficient power to detect a Super-Nova, and most of the work has been done by Baade and Minkowski and a few others who have marked the problem as of very high priority.

In spite of the tremendous energy output of even the mildest Nova, it is worthy of note that the sudden eruption of only one ten-thousandth of an ordinary star's material would readily suffice to provide the observed facts of an ordinary Nova. It is the suddenness of production of the energy, or its lack of steady escape, which causes the luminosity to rise to so high a peak as thousands of times the normal. But if that is true for the 'ordinary' Nova, the Super-Nova which in any one galaxy is so much rarer, must represent more nearly the total disruption of an entire star. We will attempt below in Chapter VI some estimate of what has been considered to distinguish these two grades of stellar explosion, in terms of some general laws as to the limits of stable life of a star.

(5) PECULIAR FLUCTUATING STARS

There are a number of cases of stellar light fluctuation which do not come under the types we have been classifying; they have neither the regularity of the Cepheids nor the violence of the Novae. We introduce them here as emphasising that no one of these phenomena of unsteadiness is quite isolated, and that there are possibilities of tracing a graduation of instability, local or total, ranging from the mild eruptions of our sun to the most violent of Novae. The warning to watch for similarities and connections is a very necessary preface to any future view as to what decides the quiet or the catastrophic in stellar evolution; it has often been neglected both in research and in exposition, but it offers the main hope of advance in knowledge.

Among peculiarly varying stars which may fill the gaps between the types already mentioned, are the following: (*a*) stars typified by the example of 'S S' Cygni. These possess some of the habit of a very mild Nova, but also some of the habit of a rather irregular pulsating star. They are faint for a month or two and then very suddenly increase in luminosity by four or five magnitudes, that is, up to a hundred-fold, afterwards fading not quite so rapidly. Repetition, after a few months' interval, is not always equally spaced-out on each occasion, and the repeats last for longer and briefer peaks in alternate cycles. (*b*) Stars typified by 'R' in Corona Borealis. These are a curious inversion of the Nova sequence. The star remains steady, perhaps for years, then suddenly *dims* by eight or nine magnitudes or several hundred-fold, in a few weeks, recovering after months. Repetition after some years may be on a much larger or much smaller scale.

These, as also the better-known fluctuations of Cepheids and Novae, may involve factors of changing temperature but also factors of changing area of surface due to expansion or contraction of the whole star. This may also entail the added luminosity of a glowing cloud, or the loss of brightness due to an obscuring cloud; responsibility has not yet been completely assigned. If the surrounding clouds have themselves been shot out of the star, the situation verges upon the Wolf-Rayet phenomenon, and also contains the germ of the Planetary Nebula development.

All these cases suggest that what we may be observing is not solely a 'star', as body of fixed boundary, but a star surrounded by a gaseous 'envelope'. Many contemporary astrophysicists, notably Otto Struve in U.S.A., have found such 'enveloped' stars to present the most intriguing but baffling problems. Struve draws the primary distinction between 'static' envelopes such as possessed by α in Cygnus, and outwardly moving envelopes such as possessed by 'P' also in the constellation Cygnus which contains several peculiar stars. 'P' Cygni itself seems to be extruding its own atmosphere at 400 km. per sec. almost like a Nova or at least a Wolf-Rayet.

When we discuss below, in Chapter VI, the stability of a star's structure, the analogy of these more obviously enveloped stars supplies the suggestion that some of the red giants, quiet and invariable though they may be, are not so much stars as star-centred envelopes. It will appear that some of the mysteries of the red giant stage might be clarified if their great luminosity were not purely stellar like that of the sun.

(6) THE SUN AS STEADY OR AS UNSTEADY STAR

It will be an appropriate comment on the very unquiet stars which we have been describing, to contrast with them the sun's not quite perfect degree of steadiness; the approach to perfection is there of more than academic interest, as it is the title to our life and evolution as living creatures, and its security will be examined in the final chapter.

The sun's total brightness, considered over all the spectrum which is able to penetrate our outer atmosphere and reach the earth, fluctuates with periodicities annual and seasonal, but that is due to the orientation of the earth's axis and orbital travel; there is also an intrinsic fluctuation due to the sun's own constitution, of about one or two per cent in total spectrum reaching us. This intrinsic fluctuation has a periodicity of about eleven years, a period in which the sun-spots, or local whirlpool centres blotching the sun's surface, pass through a maximum in the fraction of that surface which they affect.

There is evidence that the extreme ultra-violet end of the sun's spectrum, caught and trapped in our upper atmosphere and never reaching our instruments (unless V2 rockets), fluctuates more wildly, perhaps up to a hundred per cent, over the sun-spot cycle of eleven years.

The briefest of solar disturbances, the so-called 'flares', are detectable when photographs of the solar surface are taken in light of a single wave-length; they are associated with almost simultaneous consequences of excess ultra-violet light impinging on our upper atmosphere and affecting the latter's power of reflecting radio transmission. These flares probably involve brief local increases of many thousands per cent in certain narrow ranges of the sun's emitted spectrum. The areas of solar surface affected (or 'infected') are, however, only small fractions at any one time, so that the sun would only count as fluctuating very slightly, even in those critical wave-lengths, if observed from a great distance as any star is observed by us.

But if the sun is thus 'non-variable' to an accuracy of a few per cent when its whole area is averaged, the local outbursts to which any small patches of the surface are subject do carry amounts of material and energy very large by terrestrial standards. At those rare and scientifically precious occasions of solar eclipse, when the moon conveniently blots out for a few minutes the intense brightness of the sun's disc, there can be seen and photographed the 'prominences' or clouds of glowing hydrogen in process of being thrown violently off the surface—some falling back in cascade, some breaking away to outer space.[1] These prominences cover regions hundreds of times the

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[1] By a technique we will not enter into here, prominences can nowadays be photographed without waiting for an eclipse.

surface area of the earth, they can be seen rising at speeds up to 300 miles per second, momentarily reaching heights of a quarter of a million miles above the sun. Part of the mechanism of their ejection, the mechanical pressure of absorbed radiation, they probably share in common with the gaseous ejections from Wolf-Rayet stars and even Novae; so the contrast between actual material in motion and yet total almost undetectable deviation from steady state of the sun, indicates by comparison what is implicit in the infinitely vaster disturbance of the Nova. It is also a healthy reminder that the sun exhibits incessantly some mild trace of the disease which if catastrophic would instantly destroy us all. This mild trace is tiny enough on the stellar scale, but sufficiently devastating to contemplate by any terrestrial standards.

We have now exhibited the characters of steadiness and unsteadiness of stars sufficiently to attack the fundamental scientific enquiry of what makes for stability of a star, and to approach the unsolved problem of why some stars do occasionally lose their stability and become Novae. Is quiet evolution or catastrophic the predictable life-history or life-end for any particular kind of star?

CHAPTER VI

Models and Theoretical Schemes Attempting to Distinguish Between Quiet and Catastrophic Evolution

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(1) POLYTROPES AND THE TEMPERATURE AT A STAR'S CENTRE

We have hitherto classified the facts of stellar behaviour, with a main emphasis on the observational data which force those facts upon our attention. Thus we began with the experimental sciences of photometry and spectrum photography, and ended in distinctions between the seemingly permanent features and on the other hand the fluctuations, periodic or single, mild or catastrophic, which show that a star is not necessarily or always a steadily evolving body. Until both the mild and the violent fluctuations are explainable, we shall not be sure whether the differing spectral types, their temperatures and densities, represent stages in quiet development or prelude to catastrophe.

To face the possibility of any tentative interpretation of all these facts, we turn from the observatory to the library and study table of the theoretical physicist. But we must recognise from the outset that the latter is not a completely independent worker—the best theoretical physicist is always in the closest association with his laboratory colleague; at this epoch in the history of science, a spinning of mathematical yarn from the armchair would be idle unless the mathematician were deeply immersed in the latest explorations and conclusions of the experimenter, although he need not take part in the actual experiments. Theoretical physics is the discovery of the pattern of ideas and equations by which the experimenter's measurements can be shown to be part of some coherent whole; the pattern is tested, justified, or ruthlessly rejected, not only according to its intrinsic rationality but according to whether it predicts further observable or measurable facts which can be verified in the laboratory.

Among the chief weapons towards this purpose is the theoretical physicist's skill in making models; by this we do not mean constructions of metal or other materials handy to the engineer-designer. By 'model' in astrophysics is meant a set of calculations about the behaviour which a supposed and idealised star would show if we could choose its properties. By adjusting our choice of such properties, the calculations of resulting behaviour come out differently. Many such models have to be computed, until the behaviour agrees with the behaviour observed in any actual star. It is then a reasonable hypothesis that the actual star possesses other characteristics also resembling those predictable for that particular model. It may commonly happen that more than one selected value for any specific property, such as a temperature distribution, will provide the same behaviour; in which case the characteristic we have chosen to vary is not sufficiently critical for decision, and other properties must be explored.

The way of applying this plan of campaign to the problem of a star's life is as follows. The researches described in Chapters II, III, IV and V demand that, whatever in detail we assume or discover about a star, it must be a sphere of very hot gas radiating at some observable temperature; if it is also in a steady state it must be generating energy within itself at the same rate as its loss in luminosity. We considered two of the ways, gravitational and atomic, whereby the star could generate that energy. The first model, the most primitive, and the nearest to a 'contained' gas in the laboratory, would be a sphere of gas whose temperature and density were the same throughout the whole mass—a 'uniform sphere model'. Very brief consideration shows this to be a totally inadequate model; a gravitating mass of compressible material on a sufficiently large scale is bound to pack more tightly towards the centre, a condensation resisted by a gas pressure increasing inwards which would go with a temperature also increasing inwards. Any arguable guess makes the interior of a star on the whole hotter than its exterior, and the 'uniform' model must be superseded at once by models with 'gradients' of temperature, pressure, and density. These gradients may reverse locally, but will have a single main direction, and different models will be distinguishable by the different steepness of these distributions of temperature and other properties along the star's radius from centre to boundary. A correct model must show that its distributions give rise to the actual mean density, and to temperature gradients capable of transporting to the surface the energy for the observed luminosity of any particular star.

The most valuable of these star models, worked out in the early years of this century by Emden and exploited in the 1920's by Eddington and all later workers, is the 'polytrope', which reduces the description of properties to a single number or 'polytrope index'. In Part II it will be shown that of all possible sizes of this number, only those within a narrow range around three can represent actual stars, and that models selected around 'index 3' can illustrate faithfully a wide range of the actual properties of stars.

An important use of the polytrope model is that, for each particular index, the ratio of temperature and pressure at the star's centre to temperature at other points in the star can be worked out mathematically with precision, if definite assumption as to material and the absorption of radiation is made. The facts known about a star (Chapters II and III) allow its mean density to be definitely known, and the corresponding polytrope then allows its *central* density, its central gas pressure, and for any given molecular weight the temperature at the centre also, to be calculated. To select a molecular weight seems like guessing the material throughout the star; but the assumption can be restricted into a definite and narrow range of probability, as it depends almost entirely on the state of ionisation, or loss of electrons from atoms, suffered by only the main constituents of the star. This becomes quite definite in the present state of atomic knowledge, and the main unknown factor remaining is the extent to which Hydrogen has become transformed into Helium. Stars may well differ mostly according to their progress along that transformation, and it is amazing how much can reliably be said about a star's evolution if the Hydrogen/Helium ratio is written down. For stars not too hot, such as the sun, the calculations about the state of the far interior become quite manageable, because the ordinary pressure known from universal gas laws is almost the only force opposing the equally known gravitation; for the hotter stars, there is also the pressure of radiation, and the effect of this upon a star's stability becomes decisive, as described in (2).

It is worth quoting one or two results at this stage. The well-known star Capella has the known mass $8{\cdot}3 \times 10^{33}$ grams and radius $9{\cdot}55 \times 10^{11}$ cm., so that its mean or average density is 0·00227 gm. per cm.^3—a diffuse giant. Now *if* Capella is correctly representable by a polytrope model of 'index 3', the central density is 54·36 times the mean density, and, for equilibrium with its gravitational tendency to shrink, the opposing tendency to distend necessitates a gas pressure at the star's centre of $6{\cdot}11 \times 10^{13}$ dynes per cm.^2, which is over ten million atmospheres. For a likely average molecular weight the central temperature is just exceeding ten million degrees. Suppose the polytrope index were taken as $2\frac{1}{2}$ instead of 3, the central density is then 24·1 times the mean instead of 54·36: but this still makes central pressures in millions of atmospheres and central temperatures in millions of degrees, and actually the polytrope index cannot wander far from three without the model becoming subject to instability and only representing something totally unlike any star.

In Chapter III, (4) we introduced atomic energy for stars, on the grounds that the temperature and pressure at a star's centre must be very high compared with conditions at its surface: the polytrope models state precisely just how high, for given conditions of molecular weight, radiation pressure, and ultimately some definite radiation-absorbing properties of atoms. The total range of uncertainty in such assumptions must render the *exact* model always a rough approximation to fact; but it leaves no doubt as to the narrow range of polytropes capable of providing a reliable picture of a star's inaccessible interior.

We shall refer in (3) to the possibility that some stars are not simple enough to be represented by single polytropes, and may contain matter in more than one state with definite bounding zones. A White Dwarf is such a star, and we found them already to be not amenable to the normal law of mass and luminosity. At the other extreme, the very luminous red giants are today sometimes regarded as incapable of polytrope representation, and perhaps maintained by a 'shell source' of energy inside which is a uniform core.

(2) CRITERIA FOR A STAR'S STABILITY

(*a*) The significance of 'gamma': The first thing which must be assured for the continued existence of a star, is that it shall not dissipate its substance to outer space. The borderline to stability is at once suggested by the fact that we found many stars do tend to evaporate their atmospheres, but only to the extent of a minute fraction of their total material. Physically, this requirement of stability means that pressure from within the star shall hold in check the gravitational tendency to shrink inwards, but shall not overwhelm it. We trace in Part II a simple form of the argument, resting on quite elementary and universally accepted facts of thermodynamic behaviour of gases, which shows that a quantity 'gamma' (γ) must not fall below the limiting value 4/3 if the star is to hold together at all. γ is in physics the ratio between the energy needed to raise the temperature of a gas under conditions of constant pressure,to the energy needed under conditions of constant volume—the 'ratio of specific heats'. γ is intimately related to the polytrope index which specified in the preceding paragraph any selected model for a spherical distribution of gases, so this index and the ratio of specific heats are subject to restricted ranges which are not unconnected. The reason why γ becomes such a crucial quantity for a star's stability is that it controls the extent to which an attempt to absorb excess heat can be relieved by a gas immediately moving out of its place by expansion; it thus decides the possibility of preventing tension from rising too high when a star alters its energy generation or its energy transmission.

(*b*) Fall of gamma in Cepheids and Novae: For the reason just mentioned, discussion of the conditions which deprive a star of its stability has often usefully been in terms of asking what could reduce γ at any particular depth below a star's surface, until the value of this quantity fell below the safety limit. One way in which this could occur is if at some point within the star the degree of ionisation—the extent to which atoms have been deprived of outer electrons—is altering steeply with only small change in depth: the outflowing stream of radiation is then expended on performing the work of ionisation, so that the region behaves like material of abnormal specific heat. The temperature gradient becomes distorted, and the situation can be represented as a local fall in the value of gamma. The stability criterion fails, and the gases rush outward. If this motion restores stability, the material settles down again, but if restoration is incomplete the out-rush may carry the gases beyond the boundary. It is arguable that these two possibilities may underlie what we observe as Cepheid pulsation and Nova explosion, respectively. In fact Eddington in 1941 invoked such a mechanism in a layer of Hydrogen just under a star's surface to explain some features of the Cepheid pulsation, and in 1939 Biermann had utilised the same principle at higher stages of ionisation deeper in a star as possible reason for a Nova to explode. Both these theories incur serious difficulty when pushed into detail, but there is no doubt such layers with their abnormal temperature gradients would cause motion of a star's gases, whether the latter becomes observable or restabilises itself.

(*c*) Radiation pressure, Mass, and Luminosity: A reason for instability more easy to apply in specific cases is the effect of the pressure exerted by radiation itself. That light, heat radiation, X-rays, etc., all electromagnetic radiations capable of travelling though empty space at 186,000 miles per second, do exert a mechanical pressure, is rigorously predictable in theory, but a fact hard to verify in the laboratory. Only by the utmost ingenuity at devising sensitive detectors of this pressure and at eliminating the masking effects of air currents and other terrestrial disturbances, is this very delicate phenomenon amenable to experiment. But since it increases with the fourth power of temperature, whereas gas pressure increases only in simple direct proportion to temperature, the high solar and stellar temperatures will bring it out to dominate the physical forces. For instance, a temperature rise ten-fold means only ten-fold rise in gas pressure, but it means a rise of 10^4 or ten-thousand-fold, in the radiation pressure, which accordingly becomes a feature not to be neglected in the hottest stars. It may well be the deciding factor in permitting a star to distend against gravitation.

To Eddington was first due, in the 1920's, the calculation proving that for simple models of a star, the more massive the star is the more luminous it is, and also the greater becomes the fraction $(1-\beta)$ which radiation pressure contributes to total pressure, compared with (β) the fraction contributed by ordinary gas pressure. It is also possible to correlate the growth of $(1-\beta)$ with a fall in the effective size of gamma, and so to bring the instability caused by excessive radiation pressure into line with the previous criterion of a star's safety. Some tabulated quantities in Part II will reinforce this, but the clearest picture of the situation can be drawn without the equations, as follows. A 'star', modelled as a polytrope compressed by gravitation and distended by gas pressure and radiation pressure, and existing only as these forces are in equilibrium, is a collection of matter which can only hold together if it is within a rather narrow range of mass. If the amount of material it contains is too small, the Mass-Luminosity relation forbids it sufficient brightness to be seen. If the amount of material it contains is too great, the rise of $(1-\beta)$ indicates that it will be in danger of being blown to pieces by the enormous radiation pressure. These considerations are embodied in the Mass-Luminosity law already referred to in an earlier chapter, which agrees with the facts for the main sequence of stars, but not for the White Dwarfs. Eddington was the pioneer in this work, and his inimitable gift for imaginative phrasing of a precise quantitative fact expresses the first main conclusions better than anyone else is ever likely to; we shall quote here a paragraph from his *Internal Constitution of the Stars*, particularly as it is a large and mathematical treatise which the non-scientific reader will be unwilling to open.

'We can imagine a physicist on a cloud-bound planet who has never heard tell of the stars, calculating the ratio of radiation pressure to gas pressure for a series of globes of gas of various sizes, starting with say a globe of mass 10 gms., 100 gms., 1000 gms., and so on, so that his n-th globe contains 10^n gms. The Table shows the more interesting part of his results.

Number of globe	*Radiation pressure*	*Gas pressure*
32	0·0016	0·9984
33	0·106	0·894
34	0·570	0·430
35	0·850	0·150
36	0·951	0·049
37	0·984	0·016
38	0·995	0·0049

The rest of the Table would consist mainly of long strings of 9's and 0's. Just for the particular range of mass about the 33rd to 35th globes the Table becomes interesting and then lapses back into 9's and 0's again. Regarded as a tussle between gas pressure and radiation pressure, the contest is overwhelmingly one-sided except between numbers 33–35 where we may expect something interesting to happen. What 'happens' is the stars. We draw aside the veil of cloud beneath which our physicist has been working, and let him look up at the sky. There he will find a thousand-million globes of gas nearly all of mass between his 33rd and 35th globes, that is to say between ½ and 50 times the sun's mass. The lightest known star is about 3×10^{32} gm. and the heaviest about 2×10^{35} gm. The majority are between 10^{33} and 10^{34} where the serious challenge of radiation pressure to compete with gas pressure is beginning.'

It is significant that the most massive stars are found to be the hot 'O' type stars, whose atmospheres are in a state of continuously being driven off into space—the visible result of this calculated excess of radiation pressure.

(*d*) Collapse of radiation pressure in Milne's and Chandrasekhar's Novae: But if a star too massive *would* burst, this might indeed account for why the actual stars we see surviving are within a safe mass range, but does it give any clue as to why some stars do appear to lose their stability and become Novae? We cannot suppose a Nova has *become* more massive and thereby reached a danger point?

We here meet the paradox that whereas the most massive are permanently liable to be disrupted by any small disturbance, their high radiation pressure keeping them near the danger limit, a widespread view has for a quite different reason attributed the Nova catastrophe to a sudden withdrawal of all radiation pressure from a star hitherto distended thereby. How can a diminution of radiation pressure effect the disaster which seemed most likely to occur from excess of that same property? The argument begins with Milne about 1930, was developed by the Indian Chandrasekhar, now professor at Chicago, in 1935, and applied by the Russian Gamow in U.S.A. between 1939 and 1945, also by Hoyle in England in 1946, extending the notion to Super-Novae. The ground common to these researches is to consider the breakdown of the balance between radiation pressure and gravitation; suppose a normal star whose outer regions are held off from the centre by the pressure of radiation acting in the opposite direction to gravity. If now this pressure were withdrawn with a rapidity which we could call collapse, gravitation would at once shrink up the star to a much smaller volume. By the considerations we outlined in Chapter III, (3), there will be a change of potential energy to heat at this shrinkage; but this was previously thought of as taking place slowly over millions of years. If the collapse is sudden, energy comparable with the sum total of all that the sun has radiated since long before there were men on earth would be transformed to heat in a few weeks or days. It is obvious that there is no need to seek further for the Nova's terrific output.

Milne and Chandrasekhar envisaged this collapse shrinking a distended 'envelope' on to a dense 'core', and suggested the White Dwarf density as the final stage. But we have seen that difficulties arise when the outburst repeats itself, as a few Novae have done, since the White Dwarf state of matter is not likely to be capable of recovery. Nor is there any proof that the nuclei of Planetary Nebulae, also candidates for the rôle of 'past Novae', are White Dwarfs. Gamow and Hoyle considered not merely the collapse on to a core whose mass may be nearly the whole original star, but a greater explosion leaving only a fragment to remain as the post-Nova, whether White Dwarf or other. Such explosions, therefore, would produce Super-Novae rather than the ordinary Novae of Milne and Chandrasekhar.

What was missing from Milne's and Chandrasekhar's theories was any physical reason why radiation pressure should cease; Gamow and Hoyle supply such a reason, to which we now turn. But even they fail to prove that the collapse must be sudden and complete instead of gradual and possibly interrupted or even reversed before becoming catastrophic.

(3) THE PROCESS OF HYDROGEN EXHAUSTION, AND ITS EVOLUTIONARY SIGNIFICANCE

The steady evolutionary cooling of a Dwarf must of course entail steady diminution of radiation pressure as luminosity declines, but that is no source of sudden collapse; the latter must imply something more rapid, and could only occur by some cessation of the star's supply of internal energy. Until about 1939, knowledge of the energy sources was too vague for any ground for cessation of supply to be pictured: in the era of Eddington's intensive exploration of stellar constitution, it was commonly said that 'the source of a star's heat and light is some transmutation of the elements,' but at the time the Rutherford school was only just discovering the basis of nuclear physics. We have, in Chapter III, (4), mentioned the loss of mass when Hydrogen is transmuted into Helium, equivalent in Relativity physics to a vast liberation of energy, and this is nowadays accepted as the reservoir from which the sun has drawn its supply for radiating throughout the history of terrestrial life. But Bethe's calculation, in 1939, of this energy as liberated through the operation of his Carbon-Nitrogen cycle, assumed that a considerable fraction of a star's material is Hydrogen available for Helium production. It is an inevitable sequel to ask what could happen, perhaps suddenly, when the supply of hydrogen becomes exhausted. Is the collapse of a star a consequence of the final running short of this 'fuel', and the resulting withdrawal of the radiation pressure which had hitherto supported the distended star? This is the starting point of Gamow's and Hoyle's work.

(*a*) Cowling's stability during Bethe's reactions: There were difficulties even in the prospect of a star's survival while its hydrogen was still plentiful; in fact there was a bar to supposing a star could exist for any length of time while atomic transmutations were developing in its interior. This bar was only disposed of by Cowling in the late 1930's, as follows. Even before much detail was available about nuclear atomic reactions, it became clear that when they took place by the stimulus of heat, as in a star, instead of by directed acceleration of high speed particles in the laboratory, their rate would be extremely sensitive to change in temperature. At a high enough temperature for such reactions as the Hydrogen-Helium, say a million degrees or so, the slightest further rise in temperature would speed up enormously the efficiency of the process. We express this in Part II as a high index of temperature dependence. It was argued up to the 1930's that unless this index is small, any change will reinforce itself so suddenly that the process will get out of hand and the star create energy faster than it can get rid of it. No star could remain stable under a high index. This caution made many critical workers disbelieve in a star's steady evolution by atomic reactions, until the important papers of Cowling made clear that under the most likely conditions a star can tolerate extremely high temperature coefficients of reaction, so long as the quantity which we labelled 'gamma' is not already dangerously low. Change of reaction rate in a region of low gamma is alone risky, and this reinforces the danger to the most massive stars, where radiation pressure has already reduced gamma, if the reaction speeds become accelerated. But, apart from those exceptional masses, Cowling's stability criteria allowed Bethe in 1939 to work out, without misgiving, his Carbon-Nitrogen process whereby Hydrogen transmutes into Helium with enough surplus energy to keep the sun hot: the supposition understood throughout was that there must arise no shortage of the Hydrogen.

(*b*) Gamow and the sun's future: It has been Gamow in U.S.A. and Hoyle in England who have explored most widely the possible fate which might overtake a star when the Bethe reactions have kept it shining so long that the original proportion of Hydrogen has diminished almost to exhaustion. There can be no doubt that energy generation must diminish, and with it the radiation pressure which had prevented gravitational shrinkage; so there is no denying that the largest single factor in determining the line of a star's evolution is the amount of unused Hydrogen it still possesses. But the assumption that the final exhaustion of supply comes suddenly may well be queried; most stars of all ages exhibit a healthy hydrogen spectrum. Gamow and his associates have built upon this hypothesis of suddenness in hydrogen exhaustion several theories of the Super-Nova outbreak, but, apart from the lack of proof that self-adjustment by the star cannot take place quietly, Gamow invokes even more doubtfully the shadowy properties of the 'neutrino' to transport his energy through the star. The neutrino is a particle playing considerable part in atomic hypothesis, but it has not the experimentally established confidence which we give to Bethe's even unstable nuclei and rare isotopes.

A feature of Gamow's exploitation of changing hydrogen content as a star's title to life, is that he applies it to the sun. On most previous theories our sun had always been regarded as now in a slowly decaying phase, for instance in the earliest Giant-Dwarf views; Gamow, without introducing here any catastrophic possibilities, gives reason for a slight but definite curvature in the line of development, the solar luminosity at present being, he considers, slightly on the increase. Both size and luminosity are due to pass a maximum with a growth of absorption coefficient as Hydrogen diminishes and Helium increases. It does not seem yet to have been realised that our ideas of past geological and climatic evolution may need altering if he is right.

Another reversal of traditional ideas required, if Gamow's papers of 1943 are accepted, is that he suggests the life of a Planetary Nebula as a regime reducing the mass of its central star until the latter is so light that it falls into the catastrophic fate envisaged by Chandrasekhar. These Nebulae would then represent the last phase preluding collapse into Super-Novae, and Gamow's paper ends in the hope that the author will catch one of these catastrophes in one of the well-known Planetary Nebulae. It is, however, by no means certain that these mysterious objects have not already completed the most violent phase of their lives.

(*c*) Hoyle and the massive star's collapse: Hoyle also works out some of the consequences of collapse following exhaustion of the Hydrogen supply, but his most important papers (1946–7) are concerned with the fate of the most massive stars. His mechanism suggests some detail of how the 'Crab', which we referred to earlier, may have gone through its Super-Nova phase. These papers, by no means universally accepted, have the striking feature of attacking seriously for the first time the mystery of how the heavier chemical elements were synthesised; other astrophysicists have talked almost as if the only atoms of importance to stellar evolution were Hydrogen and Helium. The temperatures postulated by Hoyle in his exploding stars are far in excess of any previously mentioned by anyone, and might well set up synthesis of the heavier elements; but experimental knowledge lags far behind at this stage, and the work has to be considered as only in its beginning.

(*d*) The red giants and the 'shell' source of energy: It is suitable to end this essay by reminding the reader that all the theoretical exploration outlined in this chapter constitutes an adventure in 'models'; we began with the device of polytropes. What way of calculating the distribution of temperature, density, pressure, in a gas sphere, will permit as inference the luminosity and radius and surface temperature of any actual star? Eddington's first polytrope model assumed the laws of a perfect gas and a simplified law of radiation absorption: gas pressure then had to share with radiation pressure the opposition to gravitational shrinkage, and the stars of great mass became obviously liable to instability. Other stars became liable to collapse under failure of radiation pressure when the Hydrogen was exhausted, and the White Dwarf state of matter might be the end point of evolution. But at that stage the star ceased to be representable by any single polytrope model.

The extension of theoretical model-building into composite types is demanded, however, not only by the White Dwarf situation; there have in recent research been several problems in which no single phase or state of material can represent the facts throughout the whole model. Much ingenuity has been expended lately on 'fitting' idealised atmospheres on to idealised cores, and the solution of the problem thus set to the theoretical astrophysicist depends upon where in the star he locates the processes of energy generation. It is obvious to begin with that the atomic reactions, so steeply dependent upon temperature, will not occur throughout the whole of the star, and a first guess is to suppose them localised near the centre—a 'point source' of energy. If, however, we have regard to the gradual exhausting of the materials for those reactions, especially hydrogen, that central core will be the first to fail in supply because it is where the most rapid transformation into other elements, such as helium, is taking place. Unless the hydrogen from the rest of the star can diffuse inwards to restore the supply, the zone of rapid reaction and source of energy will become displaced outwards; energy will be generated in a narrow region which gradually changes its site further and further out from the centre of the star, like a moving wall, with 'dead' regions both in front of it and behind. This has been called a 'shell source' of energy, and behind it the central core of the star no longer containing enough hydrogen to act as source of energy, settles down to a constant and uniform high temperature. The core of such a star would be the 'uniform polytrope of infinite index' which we dismissed at the beginning as impossible representation of an *entire* star, though it may correctly represent the 'dead' centre inside the 'shell'. Outside this 'isothermal core' there is not only the energy-generating shell, possibly quite thin, but outside that in turn a third 'phase' which is an atmosphere more like what earlier researches had postulated for the whole of any ordinary star.

It is a striking reversal of the earlier theories of Giant-Dwarf evolution that this shell source has recently been invoked for the red giants. Those vast distended, cool-surfaced stars have become on closer examination even more mysterious than their opposite extreme the White Dwarfs. The first Giant-Dwarf theories, 1910–20, considered them as the youngest, shrinking and raising their temperature thereby. But calculations such as we quoted for the solar case make clear that, if the red giants are maintaining their vast luminosity by gravitational contraction, then our ephemeral observation must have caught them in a very brief transient stage of their evolution; they may last so for a few thousands or tens of thousands of years. This is long compared with unsteady stars, the months of brilliance of a momentary Nova, or even the few centuries needed for a Wolf-Rayet to grow the halo of a Planetary Nebula, but it is only a thousandth of an ordinary star's life. Another difficulty about the supposed infantile character of the red giants is that if they are representable as a single polytrope their internal temperature is too low, even perhaps below a million degrees, for the helium-producing reaction which is the other alternative to gravitational maintenance.

Two attempts have been made to avoid these difficulties. Firstly, there are other atomic reactions besides the Bethe cycle: some, involving the next elements in the chemical series after hydrogen and helium, Lithium, Beryllium, Boron, can set in at slightly lower temperature, say a million degrees compared with Bethe's ten million. It has been suggested that the red giants 'live upon' those other light elements, which do certainly seem to have been reduced by exhaustion in later stars. An intriguing by-product of this theory suggests that transitions between the stages of exhaustion of these other elements in succession can set up the pulsation of Cepheids.

But the Lithium theory still regards the red giants as young stars, and it allows a time for infancy not much less brief than that offered by the gravitational theory. Gamow has characteristically overthrown the entire conventional approach by regarding red giants as created by a later stage in a star's life. He suggested in 1945 that what we called the 'point source' model will be liable to turn into what we called the 'shell source with isothermal core' when the hydrogen begins to fail at the centre of a hot and ageing star. The increase of radius as the shell develops within the star might push out a vast distended envelope, not unlike the huge exterior of the red giant, hundreds of times the sun's diameter. Gamow even suggests that when the outward-migrating shell source gets near the surface there will be eruptions of the Wolf-Rayet type; but the Wolf-Rayet has been allotted too many rôles in stellar evolution lately for any one of them to be finally convincing, and indeed the papers of Miss Harrison show Gamow's argument to be vulnerable.

The whole treatment of the 'shell-source' really awaits some understanding of how rapidly loss of hydrogen in a star's centre can be replaced by diffusion from the rest of the star. The problem now demanding solution is therefore the mixing of the proportions of the chemical elements throughout a star. This is a very disturbing problem, because it reminds us that a still more intractable query lies behind; when we do settle at all convincingly which are the youngest and which are the oldest stars, so complacently but so unlastingly settled by the first Giant-Dwarf theories, why do they all seem to contain so nearly the same materials? there seems no piece of matter in the universe which does not already contain a fairly full complement of the highly evolved chemistry of terrestrial matter, and the genesis of the elements is a problem scarcely touched even by those who have pioneered gallantly into the genesis of the star types.

Part II

STUDENT'S INTRODUCTION TO ANALYSIS OF THE FACTS AND ARGUMENTS OF THE THEORIES

Luminosity Relationships

EXPERIMENTAL METHODS FOR LIGHT-CURVES: PHOTO-CELLS AND VACUUM THERMOCOUPLE

We have shown in Part I that the factual basis in astrophysics is the observation of Apparent Luminosity. This entails comparison of the intensity of radiation actually received from the different stars, so we will attack the problem of making such observations reliable, before discussing how Apparent Luminosity can be turned into intrinsic or Absolute Luminosity; we will subsequently trace the argument whereby great or less great distance of a star allows any particular intrinsic brightness to show in our perception as low or high Apparent Luminosity.

Photometry, as the laboratory technique of comparing the intensities of light received on one instrument from differing sources, is adaptable to the observatory with certain special requirements. We want first to know the total light we are receiving from the star, relative to that received from other stars; we shall also want to know how that light is distributed along its spectrum, or concentrated perhaps in a few isolated wave-lengths. For each of these purposes we allow the light to act upon some sensitive surface, and we may choose either to record the effect electrically at the time, or to measure it at leisure after the light has made its mark on a photographic plate. These alternatives offer the following plans of action.

A. Photometry of total radiation from the star:
  1. Direct reception of light on sensitive surfaces, such as (*a*) photocells, or (*b*) thermocouples.
  2. Measurement of the darkening on a photographic plate on which the light has recorded itself.

B. Spectrophotometry of stellar light which has already been analysed: Methods as in *A*, especially *A*2, adapted to measuring the image of the spectroscope slit instead of the image of the star.

All these techniques represent the attempt to replace the unreliability of visual estimation, which differs from one occasion to another and from one person to another. Actually many amateurs who have trained themselves to the study of variable stars, do achieve an uncanny skill in recognising, night after night, how one star's light is altering in comparison with steadier stars in the same field of view; but this skill is rare and not to be expected from general personnel preoccupied with a wide range of laboratory controls. A first improvement to visual estimation is to use a 'null method', judging brightness by measuring how much artificial dimming is needed to cancel it; for instance, a wedge-shaped slide of dark glass can be made to traverse an eye-piece, and its position can be noted at the precise adjustment sufficient to blot out sight of the source of light. Nicol prisms or other polarising equipment will occur to optical experimenters as equivalent devices. The greatest difficulty is that these absorbers do not obscure different colours to the same extent—a situation complicated by the fact that stars differ extremely in their colour distribution, as any glance at the night sky can tell.

Consider now some of the above instrumental methods for obtaining reliable data independent of individual idiosyncrasy.

Under *A* 1. (*a*), the modern photo-electric cell has improved enormously the precision of stellar photometry; in the cell a sensitive metal surface emits electrons at a rate closely proportional, under given conditions, to the intensity of certain radiations absorbed by it. This rate can be measured or continuously recorded by a sensitive galvanometer, or in some cases through a valve amplifier differing from that of a radio set by its response to brief pulses instead of to high frequencies. A photocell particularly useful has employed a sensitive cathode surface of oxidised Caesium deposited on Silver, enclosed in a high vacuum or with a trace of Argon gas. Surfaces of Antimony-Caesium are also now in use, among others, as cathodes of photocells in astronomy. Circuits have been designed in which two or more photocells are balanced against each other, so as to have all conditions in common except the incidence of starlight from a large telescope upon one of the cells; it is thus possible to secure that the extremely small currents are due to the starlight alone, and not to stray causes affecting both cells. A recent research report, in the *Astrophysical Journal* for January 1948, estimated the practicable limit to direct measurement of a star's light by photocells as the 16th magnitude with a 12-inch telescope lens, and 20th magnitude with the 100-inch telescope. This extreme sensitivity, as will be seen from the definition of 'magnitude' scales below, means that the best instrument could measure the light from a star so faint as to give us only one hundred-millionth of what we receive from the bright star Sirius. At these extreme sensitivities, the chief trouble is disturbance due to general sky illumination.

Photocells of different metals, with a variety of ways of preparing the sensitive cathode surface, respond very variously to different colours, so that a laborious preliminary is the standardising of each kind by a diagram on which can be plotted the variation of response with wave-length, visible or ultra-violet or even infra-red.

In *A* 1. (*b*), the starlight generates a very minute heating, at the junction between two metals, setting up a corresponding electromotive force which drives a current capable of being recorded. Considering that such a 'thermojunction' or 'thermocouple' is commonly used to measure temperatures which we would call 'hot', the sensitivity needed for detecting the warming effect of light from a single star must evidently be very extreme, and the technique has only attracted a few bold workers of the highest experimental skill. Pettitt and Nicholson, of the Mount Wilson Observatory, built successful thermocouples and mounted them at the focus of the 100-inch telescope; one of their examples consisted of the junction between wires of Bismuth and an alloy of Tin, each $\frac{1}{30}$ of a millimetre diameter, drawn down to this thinness in glass which could be subsequently dissolved away in hydrofluoric acid. Mounted in a very minute vacuum tube, this thermocouple gave rise to a millimetre deflection of a reflected galvanometer beam, for a change in temperature estimated as only one hundred-thousandth of a degree. The galvanometer, sensitive to a ten thousand-millionth of an ampere of current, records its deflection on a moving photographic film, and a pair of the detectors are oscillated mechanically to and fro across the star's image, because it is easier to be certain of the excursions of a wavy line than of the size of an isolated quantity. The highest temperatures measured at the telescope focus were about $\frac{1}{20}$ of a degree, and the smallest about a millionth, screens being used for cutting down the radiation in some portions of the experimental range. The outfit constitutes one of the most delicate and yet reliable measuring devices known to physics, and the list of stellar luminosities obtained thereby is a triumph to be envied by the laboratory worker in any branch of science.

The alternative which we called *A* 2. is to make the eye-end of the telescope into a camera, and to measure at leisure after a long exposure the images or dark spots at which plate or film has recorded the impact of a star's light. Comparison of stellar brightnesses can be made by comparing the size of these dark spots, as the more intense radiation has spread more widely in the photographic emulsion. There are also useful devices, such as comparison by superposing other plates containing spots of a standardised darkness. In general the most accurate treatment of photographs, and particularly suitable for spectra, is to insert them in a 'microphotometer': placed between a lamp, a set of slits confining its light to an extremely narrow beam, and some photometric detector such as photocell or thermocouple, the intensity of the image can be accurately found by measuring the fraction of the lamp's beam blotted out by the dark spot on the picture. Traversing the photograph across the narrow beam, a faint image obstructs the transmission less than a bright image which had blackened the plate into a deeper spot.

It is seen that measurement of a plate becomes an example of the same technique as had dealt with direct starlight in *A* 1., with the advantage of transferring to the laboratory the whole paraphernalia of photometry with its delicate devices, instead of having them attached to the telescope and needing to catch the fleeting time of appearance of the star in an uncertain sky. In exchange for this advantage, one faces the added problem of determining the laws which connect blackness of photographed spot with intensity of the radiation causing it.

Photographic plates, of course, are far more sensitive to violet than to red light, unless 'doped' for some particular spectral region; on the other hand, thermocouples respond to the total radiation, and are commonly associated with the red or infra-red which dominate laboratory sources of heat. The oldest photocells were only sensitive to ultra-violet, but are nowadays to be had in almost any colour range. This means that any set of stars treated by different photometric methods will be assigned very different places in any order of graded brightness. Before we draw the consequent distinctions between Visual, Photographic, and other means of defining a star's magnitude, it will be appropriate here to digress into the special photometric problem which arises when the starlight has already passed through prisms or other devices for analysing the total radiation into its constituent colours. This is the problem which we classed as *B*, Spectrophotometric instead of merely Photometric, and it is here that the microphotometer has made contributions far more valuable than its rarer use on star images.

APPLICATION TO SPECTRUM LINES AND THEIR CONTOURS

If the starlight is arranged to illuminate a narrow slit at the focus of the telescope, and a set of prisms then disperses the light so that images of the slit appear on the photographic plate in as many wave-lengths as contributed to that particular star's light, the assemblage of dark lines on the plate exhibits the spectrum and its distribution of intensities according to the different degrees of blackening in each line. Visually, these lines would be seen in their colours, or would merge into the entire red-to-violet sequence of the 'continuous' spectrum if sufficient lines overlap; but it must be remembered that the photographic plate shows only degrees of shading, and that the location of each line on a scale of wave-lengths is its only clue to colour. But the strength or weakness of any colour can be accurately assessed by exactly the same devices as served to compare the total brightness of one star against another. The spectrum plate is traversed through the 'microphotometer' which we described under *A* 2., and the photocell is employed to decide what fraction of a lamp's beam is blotted out by the degree of blackening at each spectral line on the plate. The only difference is that in the photometry of a star's total light the blackened patch on the plate would be the image of the star with all its constituent colours superposed on the same spot, whereas in spectrophotometry there are multiple dark lines from each star; each line is the image of the slit of the spectroscope in one group of wave-lengths or colour. The use of the microphotometer in this case becomes the comparison of the darkening in the different lines, implying the comparison of how the starlight is distributed along its spectrum, strong in the blue, weak in the red, etc. We shall find it convenient to think in terms of the photographer's 'positive' picture, so that in our diagrams and descriptions a 'peak' implies a bright line and a 'trough' implies a dark line or a region where luminosity is absorbed.

The very important modern study of spectral line 'contours' arises as follows. These so-called 'lines' are not merely strong or faint, but possess considerable width as they spread over a finite range of wave-length, and their blackness or whiteness on the plate is not spread at all uniformly over their width. If instead of crudely pretending that the line has a sharp boundary at which its trace on the plate ceases abruptly, our microphotometer is delicate enough to outline the gradation of intensity as in the graphs shown in Fig. 10, a great deal of information is at once available. For example, a close pair of spectral lines showing darker than their background in the Sun would have intensity contours as in (*a*); they represent the absorption of certain radiations from the body of the Sun when passing through the cooler atmosphere enveloping it. In the laboratory, the same pair of lines appear brilliantly bright because we can provide no such overwhelming background, so the microphotometer trace appears as (*b*); the 'lines' might now be broad enough to overlap, because the laboratory's source of them is probably a gas at high pressure. For instance, it is a striking experience for the student, who is familiar with the two yellow Sodium lines in the laboratory flame, to recognise them fine and narrow and dark in the solar spectrum, but with their separation still the same: the 'centres' are found at 5891·6 and 5897·5 units of wave-length, whether they are dark or bright or narrow or wide.

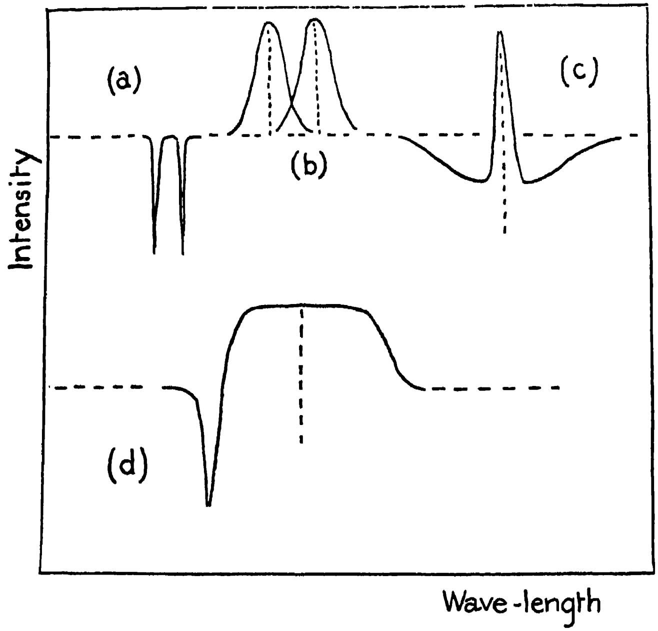

FIG. 10.

Contours of spectral lines: these are idealised types which underlie actual microphotometer tracings of intensity for different wave-lengths in the analysis of a spectrum photograph. (*a*) Two narrow absorption lines. (*b*) Two broader emission lines. (*c*) Narrow emission at the centre of a very broad and shallow absorption line. (*d*) Narrow absorption at the violet edge of a broad emission band such as occurs in a star which is ejecting its atmospheric gases.

Quite apart from inferring the chemistry and physics of a star's atmosphere from the peak-and-trough structure of adjacent line contours, a single 'line' may be spread over a range of wave-lengths, or even split into seemingly separate peaks, most commonly by the Doppler effect which shifts the exact wave-length through forward or backward motion of the radiating and absorbing gases. Example (*c*) shows a very broadened absorption line of Hydrogen, denoting that the star is rotating rapidly, and this broad darkening has a narrow centre of intense brightness caused by fluorescence of a halo of the same

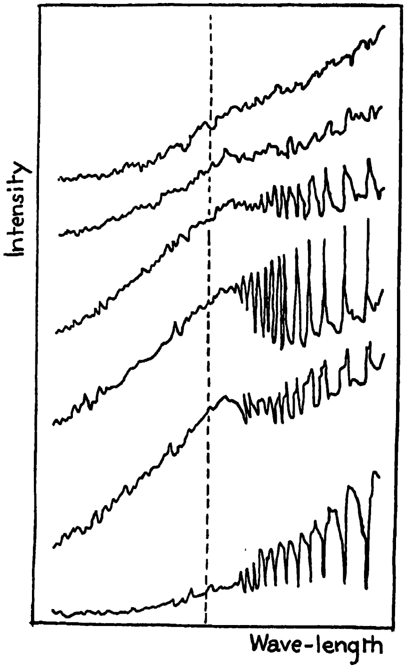

FIG. 11.

Actual microphotometer tracings for comparison with the idealised specimens of Fig. 10. These all cover the (dotted) region where the Balmer lines of atomic Hydrogen converge to the head of their series. In the bottom curve, several of these Hydrogen lines appear as absorptions, cutting deeply into the general distribution of light. In the other curves, denoting the successive appearances of a variable star, the same lines appear as emission peaks rising above a general depth of absorption.

(Photo-plate and microphotometer trace from Baldwin, in *Astrophysical Journal*.)

gas ejected far outside the star's 'boundary'. In example (*d*), the broadened bright line denotes a gas rushing in all directions from the star, with a much narrower dark line displaced towards the violet side only where the material erupting towards us obscures our line of sight to the hot stellar point itself. We shall refer to these remarkable inferences from the contours of spectral lines, when discussing exploding stars in a later chapter; the observational technique of their microphotometry is the most striking modern development of the science of estimating how degrees of darkening are graded in a 'black' mark on a photographic plate. One of the most fruitful applications will occupy the later section on how stellar temperatures are deduced. In Fig. 11 are some samples of microphotometer tracings from actual stars, with significance described under the Figure. Meanwhile we return to the inferences to be drawn if instead of spectrum lines there are simple dark spots representing images of entire stars on the plate. These spots, or in any alternative technique the recordings of light received on the telescope's photocell or thermocouple, must now be translated into 'magnitudes', in order to arrange stars in a convenient gradation of brightness.

VISUAL, PHOTOGRAPHIC, AND BOLOMETRIC MAGNITUDES, AND COLOUR INDEX

The term 'magnitude', as a precise and therefore reliable designation of a star's brightness, survives from the days when a cursory visual guess would call the brightest 'first magnitude' the next 'second magnitude' and so on, the numerical title rising as brightness falls. The system dates back to Hipparchus. The first step to put such designation on to a basis covering all degrees of faintness was the recognition, in the nineteenth century, that detectable change in any sensation of sight (or hearing, similarly) is proportional to the actual strength of the sensation—the Weber-Fechner law. Equivalent statements are that the intensity of a sensation varies as the logarithm of the stimulus producing it, or crudely, that *dividing* brightnesses can be expressed by *adding* 'magnitudes'. On this principle, the scale in use was defined by Herschel and by Pogson, adopting the convenient numerical factor 0·4 which is the logarithm to base 10 of 2·512. Starting from the fact that the log. of a number to base 10 is the power to which 10 must be raised to obtain that number, so that log 100=2, log 1,000,000=6, etc., we define magnitude so that five of such units imply a luminosity change of a hundredfold. If m_1 and m_2 are the magnitudes describing luminosities I_1 and I_2,

$$\log_{10} (I_2/I_1) = -0{\cdot}4\ (m_2 - m_1).$$

A star of first magnitude is then 2·512 times brighter than one of second magnitude, 6·31 times brighter than third magnitude, 15·85 times fourth magnitude, etc. The definition usefully makes a difference of 5 mag. for luminosities differing by the factor a hundred, 10 mag. for ten thousand, 15 mag. for a million. Fractional magnitudes denote stars of intermediate luminosities. Since a few stars are brighter than the original datum selected, 2·512 times brighter than mag. 1 is denoted by mag. 0 and further rise of brightness by the same factor is written $m = -1$, etc. The unaided eye can detect down to about $m = +6$ or a hundred times fainter than first magnitude, but the longest practicable exposure of a photographic plate can reach $m = +21$ or a hundred million times fainter. We mentioned previously the estimated limit of photo-electric cell detection with the largest telescope as $m = 20$.

As example of applying the above equation, if Sirius has mag. $-1{\cdot}6$ it is a thousand million times brighter than a star of mag. 21, because,

$$\log (I_2/I_1) = -0{\cdot}4\ (-1{\cdot}6 - 21) = +9{\cdot}04$$
$$I_2/I_1 = 1{\cdot}1 \times 10^9.$$

Again, the enormous brightening of a Nova or exploding star may be of the order of forty-five thousand-fold: in that case,

$$\log (I_2/I_1) = \log 45{,}000 = 4{\cdot}69 = -0{\cdot}4\ (m_2 - m_1)$$
$$m_2 - m_1 = -11{\cdot}7 \text{ magnitudes.}$$

Accurate photometry can list stars to one per cent of a magnitude, but as soon as any precision at all is attempted we have to begin to distinguish between Visual, Photographic, Photovisual, and Bolometric magnitudes, as follows. Since the eye is most sensitive to green-yellow, but the photographic plate to violet (unless doped to red or other sensitivity), any set of stars ranges itself differently according as predominant colours give any of them a favourable position on a scale of visually or photographically measured magnitudes. To eliminate personal differences among the former, instrumental photometry is said to determine a Photovisual scale when plates of isochromatic sensitivity are exposed behind a yellow filter; the camera thus biased defines a scale capable of all the accuracy of photographic method but with a colour sensitivity artificially adjusted to be very similar to that of the human eye. When total radiation is being measured, without the plate's or the eye's preference for colour, a third scale of magnitudes emerges, called Bolometric since the bolometer was an early instrument yielding electrical detection of the total energy or heat in radiation.

It is obvious that, on the principles of Chapter II in Part I, a cool and therefore reddish star will appear brighter and be denoted by a lower magnitude number on the bolometric scale, but fainter and of greater magnitude number on the photographic scale, in comparison with a hotter and therefore bluer star, even though visually or on the photovisual scale they might seem equally bright. In fact, differences between the magnitude of a star on the three scales is a useful measure of a star's temperature. Two such differences are commonly used, defining 'colour index' and 'heat index':

Colour index = photographic mag. – photovisual mag.

This index is taken as zero for a certain type of star (A_0), and increases as a negative quantity the hotter and more blue the star, but increases as a positive quantity the redder the star. Similarly,

Heat index = photovisual mag. – bolometric mag.

PARALLAX, SPECTROSCOPIC PARALLAX, DYNAMICAL PARALLAX

We emphasised in Part I that the whole elaborate observational technique for finding Apparent Luminosity L (app.) is only the preliminary to what we most need to know about a star, its intrinsic brightness L (abs.) It is no final clue to a star's constitution to know merely how bright it appears to us, until we disentangle its intrinsic light-giving and heat-giving rate of energy emission from the accident of whether it is near or distant. Hence all measured brightness must be turned by some calculation into answering the question: 'What brightness would the star have if it were at some standard distance the same for all stars?'

The definition of L (abs.) selects as this standard distance 10 'parsecs'. At that distance our Sun instead of outshining the whole sky would be a star of only magnitude 4·85, among the fainter seen with unaided eye; this is its real status among other stars with a common Natural History, and the status which we have to accept as astrophysicists, however impressed we remain by our debt to its location as the nearest of all stars.

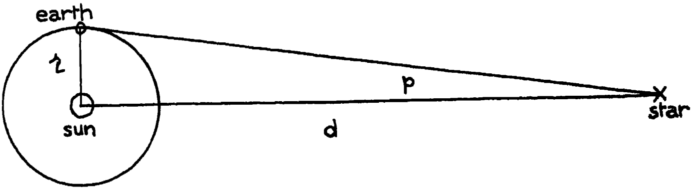

FIG. 12.

Parallax: the angle p represents the shift of direction in the light from a star as the terrestrial observer's annual path moves him round the sun. Measurement of p confers knowledge of the star's distance d since r is known, but can only be accurate to 0·005 so that stellar distances exceeding 100 parsecs or 300 Light-Years need to be estimated by less direct means.

The term 'parsec' introduces the unit of distance based on *angular* measurement. It is the distance at which the radial separation of earth from sun subtends an angle of one second of arc, i.e. an object at that distance exhibits *parallax* of one *second*, a shift of that amount in position as the terrestrial observer moves a quarter of his annual path round the sun. The words 'Parallax' and 'Second' together make the convenient Parsec unit of distance. Since the earth is 92 million miles from the Sun, one parsec is equal to $3{\cdot}083 \times 10^{13}$ kilometres, as can be seen from the diagram (Fig. 12).

In this picture, the angle p in radian measure is r/d, so that as the number of seconds of arc in a radian is 206,265 and r is $1{\cdot}495 \times 10^8$ kilometres, the distance of a star is, in km.,

$$d = (206{,}265 \times 1{\cdot}495 \times 10^8)/p \text{ in seconds.}$$

The other unit of distance convenient for large astronomical quantities is the Light-Year, or distance that light travels in a year ($3{\cdot}156 \times 10^7$ sec.) at its speed of $2{\cdot}998 \times 10^5$ km. per sec. It is obviously the product of this velocity and the number of seconds, so that one parsec is 3·258 Light-Years.

If now a star's parallax (usually written π seconds) is known from the change of apparent location in the sky over the year, the distance is calculable which can turn Apparent brightness into the Absolute brightness or luminosity the star would possess at 10 parsecs. The actual relation is,

$$\text{Absolute magnitude} = \text{Apparent magnitude} + 5 \log(10\,\pi)$$

The nearest star has $\pi = 0''{\cdot}76$ and is at distance 4·3 Light-years; using the most refined methods of tracing exceedingly small displacements in the annual parallactic shift, the errors can be reduced to within $0''{\cdot}005$. But there are only about two dozen stars so near to us that their parallax exceeds $0''{\cdot}25$, and the error becomes unmanageable for distances greater than 300 Light-years. For all the more distant stars, the vast majority, indirect methods have to be adopted, by which Absolute Luminosity can be estimated roughly but with confidence within that roughness. Among these indirect approaches are the study of 'Spectroscopic parallax' and 'Dynamical parallax'. These terms are used, although they are really methods of obtaining access to L (abs.) without the intermediate step of knowing the parallax: in fact they offer substitutes for a parallax which would at great distance become too small to measure.

With regard to Spectroscopic Parallax, use is made of the fact that some spectral lines appear in definitely different intensity for a compact small star and for a grossly distended star of similar mass and temperature and other physical properties. Adams at Mount Wilson, from 1914 onwards, first made accurate comparison between pairs of stars with very similar character and with known parallax but differing greatly in

Absolute magnitude. For instance, take the stars α Tauri and 61 Cygni, of *L* (abs,) 0·4 and 8·0 respectively, the former being therefore 1100 times brighter than the latter; the Calcium spectrum line at wave-length 4435 is very intense in 61 Cygni, but the Strontium line at 4216 is stronger in α Tauri. The strength of these two lines was traced through a long sequence of stars of high and low luminosity, and a graph constructed relating the difference in intensity between these lines to the quantity *L* (abs.). Other lines, of Iron, Titanium, etc., yield similar results, and by 1935 the list at Mount Wilson contained already over 4000 stars thus classified in their gradation of Absolute brightness through the measured intensities of certain of their spectral lines. For the cases among them where parallax is actually known through direct measurement, the reliability of the device is checked, and there seems every reason for accepting the correctness of *inferred* *L* (abs.) as substitute for parallax where the direct check is not possible.

Dynamical Parallax is a term used when a visual binary, or detectable pair of stars, yields evidence as to the combined mass of the two, the periodic time taken to revolve around each other and their distance apart. It becomes possible, by making some quite rational assumptions as to their mutual orbit, to obtain a rough estimate of the distance from us or an equivalent to the inaccessible parallax.

INDIRECT INFERENCES AS TO ABSOLUTE LUMINOSITY

Fig. 1 of Part I showed the empirical relationship found between Period of the Cepheid pulsating stars and their *L* (abs.) enabling the latter to be inferred with considerable security if the time from maximum to maximum in the cycle of fluctuation has been observed. Again, in Fig. 5 of Part I the result of plotting on a diagram the spectral types of stars and their *L* (abs.) enables at once a rough estimate of the latter to be made as soon as the spectrum is decipherable. The third important empirical relation (Fig. 4 of Part I) connects *L* (abs.) with a star's Mass; but this graph is more often used for estimating Mass in cases where luminosity has already been extracted by one of the direct or indirect methods. It must be noticed that in Fig. 5 not only is there much scattering of the points which build up the general shape of the picture relating spectral type to brightness, but also that there are the White Dwarfs which we discussed in Part I as having no place in the general trend of the picture at all; furthermore, there is the ambiguity that a cool red star may be either extremely bright (a Giant owing its luminosity to large surface area) or extremely faint (a Dwarf showing little light because of small surface area). Deduction of a star's absolute brightness and distance from its spectral type is in any case rough, and only reliable along the main sequence of the diagram; apart from the White Dwarf problem introduced in Part I, we also at the end in Chapter VI confessed that the structure of extreme red Giants is still too much of a mystery for the edge of the diagram to have any accepted meaning.

We have now completed, as far as possible for an account of this size, the assessment of how observation is made and how inferences are drawn, in the science of stellar photometry and spectrophotometry. Since all these measurements converge upon the decision as to how much energy a star is emitting (and how much it must therefore replenish for maintenance), we have laid somewhat laborious stress upon the distinction of fact and deduction in this topic, before embarking on the remaining questions of how astrophysics can associate this energy with temperature and other characteristics of a star. When those have been settled the query as to source or origin of the energy can be approached, together with the conditions for growth or decay of a star according to lack or abundance of the energy source.

Black Body Temperatures

ESTIMATES FROM WIEN'S LAW OVER THE WHOLE STELLAR RANGE

The problem of finding the temperature of a star's surface involves a particular set of the many inferences which can be drawn from spectrophotometry, so the methods of obtaining the observational data have already been outlined in the preceding section; the microphotometric apparatus is not very different from that used in stellar magnitudes and the contours of spectral lines. To obtain temperatures from Wien's law, the microphotometer must record for us the distribution of intensity among the wave-lengths which range from the violet end to the red end of the sequence of colours making up the more or less white light from the star; this white light has been dispersed by the analysing power of the prisms in the spectroscope, before reaching the photometer. Since this particular method depends on the properties of a 'continuous' spectrum, we ignore the spectral lines here, and pretend that the dark chasms which cross the band of colour and denote chemical substances are negligible interruptions to the graded colour sequence. The lines themselves offer a quite different way of inferring temperatures, which we discuss in a later section. To the extent to which the star's colours thus submerge its absorption lines, it will approximate to the ideal 'Black Body', so-called because if it emits all wave-lengths it must also absorb all wave-lengths—a property to which a black covering (or a white hot furnace) is a near imitation. But although the ideal Black Body emits all wave-lengths, it emits them with greatly differing relative intensities; it is this precise distribution of intensities which indicates the temperature, infallibly if the spectrum is purely due to heat and not to particular substances which would give line spectra. Wien's law of Black Body spectra, applicable thus to very hot objects, can be stated,

$$\lambda_{\max} T = 0{\cdot}288 \text{ cm. degree on Absolute scale}$$

and is illustrated in Fig. 13. The inverse proportionality of temperature to $\lambda_{\max}$ the wave-length at which the maximum intensity occurs in the spectrum, causes the peak of each curve to shift from red towards blue as the temperature is raised. We watch a hot iron turn from dim reddish to a purer red and to white, as the yellower and greener constituents of the white heat begin to play their part. The bluish tinge in some stars denotes their extremely hot end of the temperature scale, as the dull red of others denotes that they are among the coolest. Wien's law is the exact expression of those vaguer experiences, and extracts a definite temperature from any spectrum plate from which the microphotometer can pick out the location of the point of maximum intensity. It must be recollected that this point, at which the intensity of the radiation along the continuous spectrum is greatest, is not necessarily the darkest place on the photographic plate; but that adjustment can be made by auxiliary experiments to find the exact colour sensitivity of that particular brand of plate. A more serious difficulty is that no star is a perfectly 'Black radiator'; we refer below to the difficulty of eliminating the gaps in the continuous colours due to dark lines of the absorbing atmosphere outside the star. Both the coolest and the hottest stars are bad cases for applying Wien's law, as the colour band is cut and blurred by atomic lines and by selective colour absorptions, often widened by the Doppler effect which spreads the wave-lengths according to velocity. What can be decided without any doubt by the Wien method, is that if we omit the intractable hottest and coolest, the remaining stellar surfaces range from about 3000° to 30,000°. This range is sufficient to establish beyond question the statements of Part I about the fluidity, gas pressure, state of chemical dissociation and atomic ionisation, etc., of stellar material.

Between those extremes, our Sun is a good subject for Wien's law: the estimates of maximum intensity along its continuous spectrum yield somewhat above 6000° and somewhat below 6000°, according as the light is drawn from the centre or near the edge of the glowing disc, since we see into different depths of the hot gas at different points.

THE ESTIMATE FROM STEFAN'S LAW FOR THE SUN

The diagram (Fig. 13) of light intensities for all the wave-lengths of a continuous spectrum embodies also the other law, Stefan's, associated with Wien's as characterising Black Bodies and the very hot materials which lose their chemical line spectra and approximate to Black Bodies. Either Wien's or

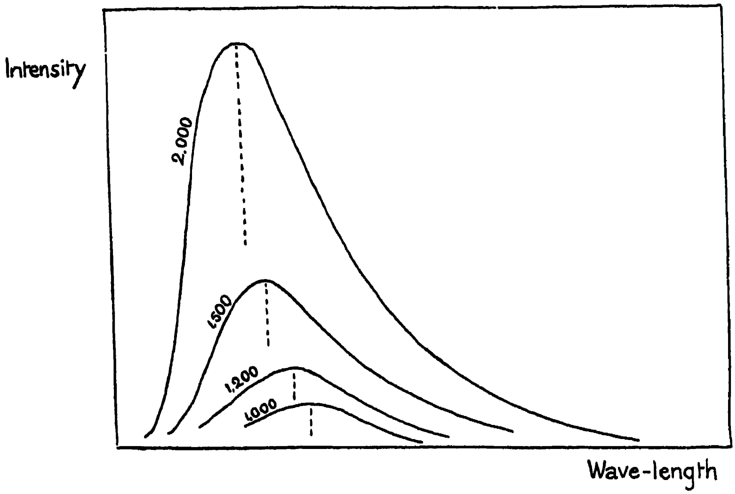

FIG. 13.

Intensity distribution in a Black Body radiator at temperatures from 1000 to 2000 degrees: Wien's law describes the shift of the maximum point (dotted) towards shorter wave-lengths as temperature rises. The shape of the whole curve is described by Planck's equation which gives the energy at any frequency ν and temperature T as,

$$E_\nu = \frac{8\pi h\nu^3}{c^3}\left(\frac{1}{e^x - 1}\right) \text{ where } x = \frac{h\nu}{\kappa T} \text{ and } h \text{ is Planck's constant.}$$

Stefan's law can be extracted by isolating particular features in Planck's equation, quoted under the Figure. Instead of stating with Wien the shift of the point of maximum intensity as temperature rises, Stefan's law states the area covered by each curve, or the total energy emitted in the entire aggregate of wave-lengths. The vast increase of area for small rise of temperature in Fig. 13 implies that energy output rises much faster than merely proportionally to temperature, and in fact Stefan's law is,

$$E = 5{\cdot}32 \times 10^{-5}\, T^4.$$

Here E is total energy emitted per cm.^{2} per second, in ergs. The numerical constant has been obtained by a large variety of laboratory experiments, which agree to considerable precision when all precautions are elaborated to guard against loss of light.

Stefan's law is not easily applicable to any celestial object except the Sun, where the enormous brilliance due to its proximity makes considerable accuracy possible in estimating the amount of energy received per cm.^{2} on a terrestrial absorbing surface. This latter quantity, when corrected for loss in transmission through clouds and atmosphere, is called the 'solar constant', and has inspired a number of special photometric and calorimetric devices. It is commonly stated as 1·93 calories per minute per cm.^{2}, in units of heat received. It is subject to small fluctuations, apart from those arising in the earth's atmosphere, indicating that the sun is not a perfectly steady star, as we mentioned in Part I. This solar constant, multiplied by $4{\cdot}18 \times 10^7$ to convert the calories into the energy unit of ergs, and divided by 60 to agree with the statement in seconds, can be made to yield as follows an estimate of the solar temperature.

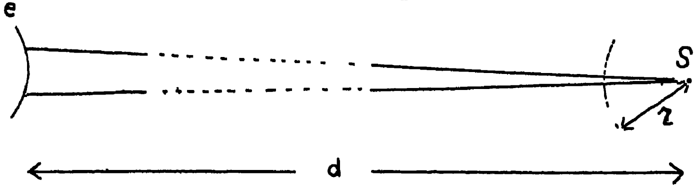

FIG. 14.

Inverse square diminution of radiation with distance: from measured intensity, or Solar Constant, at the earth's surface *e* a known distance *d* from the sun's centre *S*, the intensity at the sun's surface *r* from the centre, is calculable.

Consider (Fig. 14) a cone of light of axial length *d*, centred at the sun's centre *S* and cutting the earth's surface *e* at a point where the heat is measured in the solar constant. If *r* marks off the sun's surface— and of course to make the cone visible on the picture at all the earth and sun and distance must be quite out of proper scale—the energy crossing unit area at the sun's surface exceeds that crossing unit area at the earth by the factor d^2/r^2; this is the purely geometrical scale-up due to the fact that radiation is bound to diminish in intensity by the square of the distance. This factor is 46,000 for distance and size of the sun. We obtain therefore from the observed solar constant, converted to the appropriate units and multiplied by the distance factor to give rate of energy flow per cm.^{2} at the sun's surface,

$$(1{\cdot}93 \times 4{\cdot}18 \times 10^7 \times 4{\cdot}6 \times 10^4)/60$$

But in Stefan's law this is just the emission from an object at temperature T on the absolute scale, if it is behaving as a Black Body radiator, and so can be equated to,

$$5{\cdot}32 \times 10^{-5}\ T^4$$

Putting together these two sides of the same facts, it is found that,

$$T^4 = 0{\cdot}116 \times 10^{16} \quad \text{and} \quad T = 5840$$

Provided that the meteorologists can make the correct allowance for losses in our atmosphere, this affords a rather more accurate estimate of the solar surface temperature than Wien's law, which depended upon measuring the exact wave-length at which the maximum intensity occurred in the colour spectrum. The confirmation of a figure near 6000° from two methods gives considerable confidence, but it must be remembered that starlight is far too faint for extending to other bodies than the sun any treatment except the implications of Wien's law. Both Wien and Stefan depend on the close resemblance of a hot celestial body to the ideal 'Black' radiator, and before leaving the topic of such continuous spectra we must regard somewhat more seriously this limitation.

DISTURBANCES OF A CONTINUOUS SPECTRUM

We have pointed out that a true continuous spectrum would show no dark absorption lines crossing its colours, even the narrow lines of the sun. Fig. 15 (*a*) and (*b*) shows some photometric tracings of the solar spectrum, not idealised as when in Fig. 13 we wanted to show only the smooth curve of the 'Black' laws of Planck, Stefan and Wien, but with the actual denser absorption lines cutting into the colour distribution where

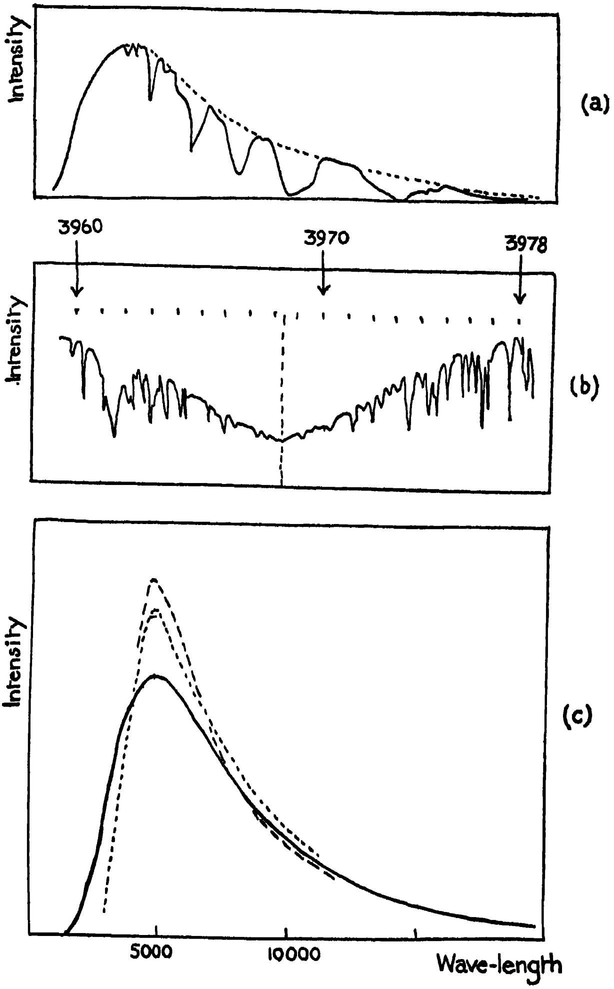

FIG. 15.

Distribution of intensity in the solar spectrum:

(*a*) Langley's pioneer curve, showing the deep and broad cuts due mainly to absorption in the infra-red wave-lengths by vapours of the earth's atmosphere.

(*b*) Highly magnified analysis of the region around the violet Calcium line at wave-length 3968, showing very wide-spreading absorption due to the ionised Calcium, with a great many narrow lines superposed. Diagram from the 'Utrecht Solar Atlas' reproduced in Goldberg and Aller *Atoms, Stars and Nebulae*.

(*c*) Milne's graph of observed solar intensities (dotted) compared with his theoretical curve based on Planck's equation and an assumption of constant absorption coefficient.

chemical species in the solar atmosphere absorb certain wave-lengths selectively. It is by the location of these lines that the composition of the outer regions of the sun is known, whereas a purely continuous spectrum can denote a temperature but is independent of particular substances. It can be seen that, if the deepest of the lines or cuts happened to be near the point of maximum intensity, it would be impossible to apply Wien's law with any hope of accuracy. In the cooler red stars dark bands representing hydrocarbon compounds and oxides blur much of the continuous spectrum; also at the hotter end of the stellar range, the Wolf-Rayet stars described in Part I have their continuous spectra very much smothered by *bright* lines broadened into wide patches along the spectrum by the Doppler effect of their violently out-rushing gases. We indicated, in the paragraphs on microphotometry of line contours, some of the measurements and inferences relevant to these widened 'lines'.

In some stars there is the added complication that the capture of electrons at high speed by hydrogen atoms can generate another species of continuous spectrum, quite different to the 'heat' spectrum of the Black radiator, especially to the violet side of the wave-length 3646.

On Fig. 15 (*c*), for comparison, the graph is of Planck's equation of the Black radiator for 6000° and serves to indicate that around the peak the error is considerable even apart from absorption lines. Variations in theoretical curves can be inserted according to laws of how absorption alters with wave-length. The deepest solar lines, due to Calcium, occur at 3934 and 3968 at the violet end of the spectrum, and one of them is analysed in Fig. 15 (*b*).

Ionisation Temperatures

THE SAHA THEORY AND MEASUREMENT

In Part I a brief general account was given of the principle on which intensities of spectral lines, as distinct from continuous spectra, can be made to yield information about a star's temperature. It was emphasised that since the absorbing gases giving rise to those lines constitute a cooler atmosphere, surrounding the hot photosphere or source of the continuous spectrum, any temperature inferred from the lines may be checked as being somewhat lower than the Black Body temperature of the *same* star. But it is obvious that any theory must be suspect if it deduced a line spectrum cooler than the sun's to overlie a 30,000° photosphere: the temperatures from continuous spectra and from line spectra must form a roughly parallel sequence throughout the range of stars if we are to regard these methods with any confidence.

The method of Saha was described in Part I as correlating observed intensities in the several spectra possible to any chemical element, with (*a*) the energy required to excite 'levels' in the atom's electronic structure, and with (*b*) the temperature and gas pressure of that atom's environment. The energy required to change the electronic distribution within an atom is termed 'excitation potential' in units of electron-volts, and the greater energy required to ionise the atom or detach a first, second, third, etc. electron, is the first, second, etc. 'ionisation potential'. At a given absolute temperature T, the thermal energy available in the gas can be assessed in terms of kT, where k is Boltzmann's constant in the mechanics of gas molecules; so the fraction E/kT, where E is an excitation potential or ionisation potential, is a crucial quantity. The diminishing of E/kT as temperature rises allows electrons to be excited or finally detached from their atomic structures. In fact it is possible to show from very general and incontestable argument that if a gas has reached a state of equilibrium,

$$\frac{n\,(\text{ionised}) \times n\,(\text{electrons})}{n\,(\text{neutral})} = ce^{-E/kT}$$

Here e is the number 2·718, base of the natural logarithms, and the n's are the concentrations of ionised and neutral atoms and of the electrons thereby liberated. The constant of proportionality c is made up mainly of,

$$(2\pi mkT)^{3/2}/h^3$$

where h is Planck's constant determining the quantum or unit in multiples of which energy can be exchanged, and m is the mass of the electron. There is also involved the 'statistical weight' connected with the intrinsic probability of electrons occupying states of definite energy.

It may be seen that, as T rises, the proportion of ionised to neutral atoms rises in the equation, so that observation of this proportion from the relative intensities of the appropriate spectral lines can allow inference of the temperature: this is the germ of the Saha method. But Saha's pioneer attempt in 1920, in close analogy with the physical chemists' calculation of dissociation of substances into their elements at lower temperatures, suffered from serious disadvantages. In particular it required one to estimate the concentration of ionised atoms at which the characteristic spectrum would just begin to be visible. This made the calculation of temperature terribly sensitive to the unknown 'relative abundance' of the different substances in the star's atmosphere.

FOWLER AND MILNE'S METHOD OF MAXIMA

Much of the disadvantage preventing wide application of the Saha theory was removed in 1923–4 by Fowler and Milne. They utilised the fact that, as temperature rises, the higher states of electronic energy needing an excitation to the amount E will become progressively more populated, but since these excited atoms can only be derived from neutral atoms which themselves become less populous as temperature rises, the total concentration in higher states of the neutral atom must pass through a maximum and fall again as temperature rises further. The same thing will happen to the spectral excitation stages of the ionised atom at still higher temperature, etc. Accordingly, instead of searching for the first visibility of a given spectrum among a list of stars, Fowler and Milne set the astronomers searching for the star in any list, at which a given spectrum has its maximum intensity, relative to its weaker appearance in other stars both hotter and cooler. The association of a maximum strength with Temperature, gas pressure, excitation potential E_r and ionisation potential E_1, can be stated in the following equations. The growth of x the fraction of atoms ionised in an environment where the free electrons themselves contribute a gas pressure P_e is,

$$\log \frac{x}{1-x} P_e = -\frac{E_1}{kT} + \frac{5}{2} \log T + \log\left(\frac{[2\pi m]^{3/2} k^{5/2} \sigma}{h^3}\right) - \log b(T)$$

$b(T)$ is an important function on the value of which much of the earlier discussion centred, and Fowler and Milne's advance was facilitated by finding that instead of rendering the calculation intractable it reduces to 1 or 2 in many cases. Similarly the quantity σ is 1 or 2 according to the stage of ionisation, single or double, etc. The use of P_e instead of ordinary gas pressure was due to Russell of Princeton. The fraction of non-ionised or neutral atoms in any state of excitation r is,

$$f_r = q_r \, e^{-(E_1 - E_r)/kT} \Big/ b(T)$$

where q is a simple quantity contributing to $b(T)$. The fraction of the total number of atoms that remain neutral and also are in the r state is,

$$n_r = f_r \, (1 - x)$$

so that putting together these three equations,

$$n_r = \frac{q_r e^{-(E_1 - E_r)/kT}}{b(T) + a T^{5/2} e^{-E_1/kT}}$$

In this expression a is condensible into a numerical quantity,

$$a = \frac{(2\pi m)^{3/2} k^{5/2} \sigma}{h^3 P_e} = \frac{0{\cdot}332\sigma}{P_e}$$

We require the conditions under which n_r is at a maximum, as this is when the associated spectral lines will show at their strongest. n_r is maximum at the temperature for which,

$$\frac{e^{E_r/kT}}{b(T)\,e^{E_1/kT}+aT^{5/2}}$$

is maximum, and this occurs when,

$$P_e=\frac{0\cdot332\sigma}{b(T)}\,\frac{E_r+\frac{5}{2}kT}{E_1-E_r}\,T^{5/2}e^{-E_1/kT}$$

The final form of this states the pressure for a given spectrum to reach its greatest strength at the temperature T, and completes the method of calculating the star's temperature and pressure if the observer's photographs can be measured sufficiently accurately to establish a list of stars with graded intensity of their spectral lines. The latter throws the ultimate responsibility back upon the expert in experimental spectrophotometry. We show here (Fig. 16) a simple set of Fowler and Milne's graphs, for an electron pressure of $1\cdot31\times10^{-4}$ atmosphere.

STELLAR TEMPERATURES FROM THE ABSORPTION LINES

We quote a list of Fowler and Milne's temperatures for the pressure mentioned which is a likely average. The second column gives the letter of the spectral type of the star whose atmosphere is at the stated temperature, and the third column the atomic species whose spectrum was utilised for that star. We add finally a list of the more relevant ionisation potentials, in electron-volts. These calculated temperatures are similar to, and possibly more reliable than, the estimates from Wien's law where the spectrum is the continuous colour distribution lying slightly deeper in the star than the gases which show the absorption lines. Fowler and Milne did not extend their work down to the coolest stars, where chemical compounds rather than atomic spectra dominate the radiation; it is in those coolest stars that we stated the weakness of the Wien method also. A range from below 3000° to above 30,000° covers both the line spectra and the colour temperatures, with failing reliability at both extreme ends of the scale; only near the

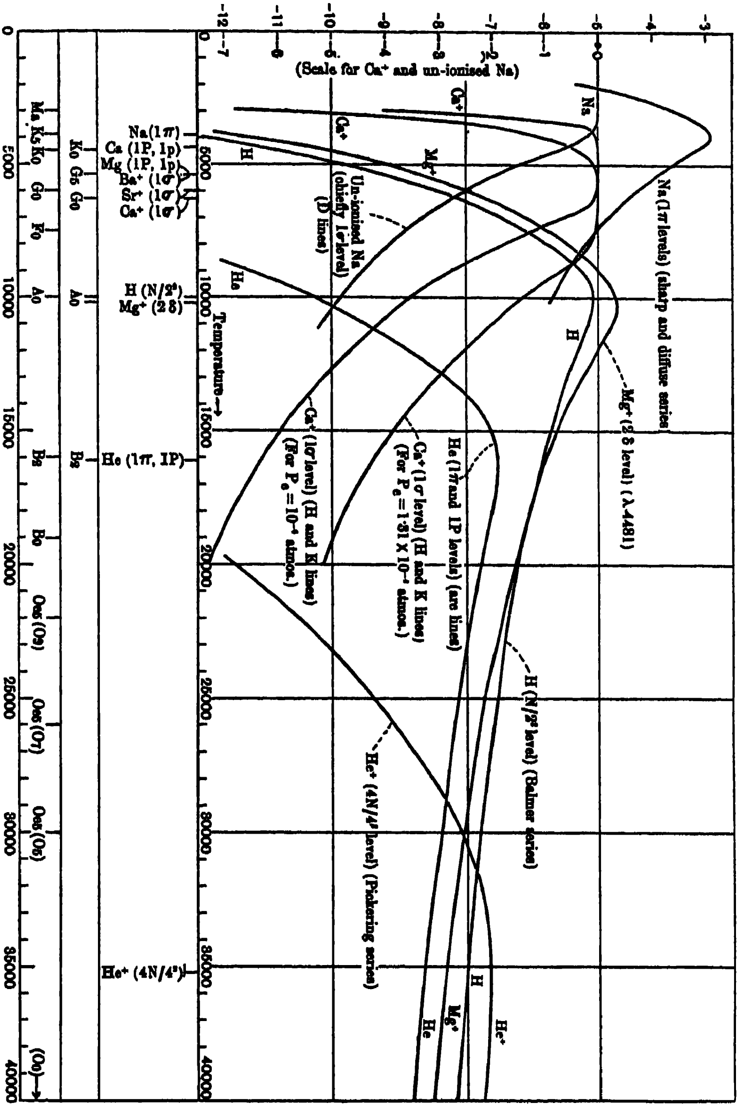

FIG. 16.

The development, by Fowler and Milne, of the Saha theory of stellar temperatures: their diagram shows how various spectra begin to appear in cooler stars, reach their greatest strength at a certain range of stellar temperature, and become weaker progressively as hotter stars are reached. These theoretical curves enable temperatures to be assigned to any star in which a given spectrum attains its greatest intensity, if reasonable knowledge of the gas pressure can be assumed.

middle is accuracy sufficient to check that the line spectrum is, as it ought to be, 'cooler' than the continuous spectrum, for instance in the sun.

Temperature	*Spectral Type*	*Element*
3,900	K_5	Na
4,270	K_5	Ca
4,420	K_5	Ca
5,250	G_5	Mg
5,440	G_5	Mg
5,450	K_0–K_5	Ba+
5,970	G_0–K_0	Sr+
(5,300)	K_0	Ca+
10,000	A_0	H
10,220	A_0	Mg+
16,100	B_2	He
35,200	O	He+

Ionisation potentials, in electron-volts:

Na	Ca	Fe	Ca+	H	C+	He	O+	Si+++	N++	He+
5·1	6·1	7·8	11·8	13·5	24·3	24·5	35·0	45·0	47·2	54·2

We have quoted the pioneer work of Fowler and Milne, but the subject has been much developed since, notably by Russell in U.S.A. and Pannekoek in Holland, many of whose diagrams of temperature maxima of spectral lines carry the problem into further detail.

The Mass of a Star

BINARY ORBITS

In Part I the method was outlined whereby the mass of a star can be discovered, in cases where the 'double star' or binary pair is within reach of certain kinds of investigation. The pair may be near enough to separate visually; or only the splitting of their spectral lines, or the dimming when one of the pair eclipses the other, may betray the existence of more than a single star underlying the single point of light. It remains here to list the actual quantities which need observing, and to show how the star's mass can be manœuvred into being the only unknown and therefore calculable from the known data.

The mechanics of orbits, i.e. of the path periodically traversed by a planet round the sun, a moon round a planet, or a star and its companion round their common centre of mass, has been based on Kepler's laws since the seventeenth century. For the present purpose the most relevant of the Kepler laws states that 'The ratio between the squares of the periods (*P*) for any two planets is the ratio between the cubes of their mean distances from the sun, the sun being at the focus of the elliptic path of a planet.' *P* is the time taken in the planet's periodic circulation of its orbit.

The force of gravitational attraction, known since Newton, between any two masses M and M_1 at distance d apart, is,

$$G\frac{M\,M_1}{d^2}$$

where G is the universal constant of gravitation. The system 'holds together', or maintains a stable orbit, only because this gravitational attraction just balances the centrifugal tendency of the circulating mass M_1 which is,

$$M_1\frac{4\pi^2}{P^2}d$$

If the two expressions are equated, for two planets M_1 and M_2 at distances d_1, d_2,

$$P_1^2/P_2^2 = d_1^3/d_2^3$$

which shows the earlier Kepler statement as a consequence of the later Newtonian gravitation.

Since the planet attracts the sun as well as vice versa, the attraction on unit mass is not strictly GM/d^2 but $G(M+M_1)/d^2$, so that a more accurate form of the law becomes,

$$\frac{(M+M_1)\,P_1^2}{(M+M_2)\,P_2^2} = \frac{d_1^3}{d_2^3}$$

This correction is slight if M is the massive sun and M_1, M_2, are light planets, but becomes essential if the two masses are stars not differing greatly in mass. For a binary pair, if the units are chosen, the sum of the two masses can be known from the cube of their separation, divided, as in this equation, by the period of mutual revolution.

Selecting therefore the units, take the masses m_1 and m_2 as multiples of the sun's mass, the separation a as semi-major axis of their ellipse in seconds of arc, and P the period in sidereal years, the relation becomes,

$$m_1 + m_2 = a^3/P^2\pi^3$$

where π is the parallax of the system in seconds of arc. This equation justifies the term 'dynamical parallax' used in a previous section, as π is obtained if a and P are known, since in many cases the sum of the masses does not differ greatly from twice the sun's mass, while the value of π is not sensitive to such differences.

In addition to this possibility of finding the sum of the masses if the parallax is otherwise known, the second expression above can give the ratio between the masses as inversely the ratio of their distances from the common focus of the orbit. If both possibilities can be realised, sum and ratio together provide the individual masses. This is feasible if the revolution of the pair can be observed relative to more distant and more nearly 'fixed' stars in the same field of view. Binaries near enough to be separated visually are rare, and about fifty have yielded the required information to obtain the individual masses, but when the separation is less than $\frac{1}{10}$ sec. of arc the quest becomes nearly hopeless. Fortunately, valuable evidence comes from observing the split of the spectral lines as each of the pair in turn advances and retreats—we discussed this in Part I, and illustrated the velocity curves of a binary which can afford knowledge of m_1/m_2 as the inverse ratio of the range in velocity of the two bodies. Solution of the problem of the separate masses is also in principle within reach, but with the uncertainty that the inclination of the orbit to our line of sight is unknown; we may only be watching an inclined component of the velocities, not their maximum 'edge-on'. Knowledge of this angle of inclination can be obtained if the binary pair eclipse each other, as described in Part I, a full eclipse guaranteeing that the orbit is orientated 'edge-on'.

The results, remarkable in the convergence of so many lines of evidence about a single indivisible point of light, are sufficiently widespread to establish the conclusion of Part I, that the great majority of stars are between $\frac{1}{5}$ and 5 times the solar mass. The Mass-Luminosity law which crystallises these data we shall try to make significant under Chapter VI.

The Size of a Star

It is worth prefacing this section with the reminder that even the largest telescope affords no *direct* data as to stellar size: the nearest star would no more exhibit a detectable disc than a golf ball seen 100 miles away, so we are inevitably confined to seeking indirect inferences from the infinitesimal point of brightness that covers even the largest star.

We pointed out in Part I that if the temperature of a star with a dominating continuous spectrum is known, and also its Absolute Luminosity, the former gives knowledge of energy emission per unit area by Stefan's law, while the Absolute Luminosity gives the total energy emission from the entire star. Dividing the one quantity by the other, yields in principle the area $4\pi r^2$, hence r or the diameter or volume; if we also know the mass the mean density is known, $M/\frac{4}{3}\pi r^3$. These principles often meet difficulty in application, but they underlie the fact that a star very bright but also very cool must possess an enormously distended size—the 'giant' which was described in Part I.

In the discussion of binary pairs in Part I (Fig. 3), it was also pointed out that if the two stars of the pair eclipse each other, the time over which the resultant dimming of light can last is a fraction of the total period, a fraction denoting the portion of the whole orbital traverse in which one star covers the other: this gives a simple measure of the star's size in terms of its orbital travel. Some binaries, indeed, indicate in this way that they revolve around each other almost in contact, and the glowing atmosphere may well be found to enwrap both stars in a single halo. The situation is complicated if so near an approach to one another distorts the spherical shape, and elongates a star into an ellipsoid at times.

But these sources of information about stellar size represent hints rather than precise inferences, and a quantitative treatment more profitable to pursue here is the 'interferometer' deduction of a star's diameter from the disappearance of dark fringes in the eyepiece of a novel form of telescope.

THE INTERFEROMETER MEASUREMENTS AT MOUNT WILSON

Since light is a form of wave motion, two sequences of waves can reinforce each other or cancel, according to their phase difference, i.e. according as troughs of one wave system are superposed upon troughs or upon crests of the other. The image formed by an optical instrument is a pattern of such 'interference' between superposed waves, and this sets the limit to the smallness of objects which can be seen in any wave-length of illumination. We are unable to distinguish objects whose size is insufficient to have a certain definite arithmetical relation to the wave-length of the illumination. Michelson, so far back as 1890, pointed out the possibilities of admitting starlight through two small apertures in a screen *AB*, (Fig. 17) in front of the object glass of a telescope, and observing in the focal plane *F* the 'fringes' or alternating pattern of dark and light due to any extremely small source of luminosity. The latter condition is satisfied by an object so distant as a star. Consider two luminous points *x* and *y* subtending angle θ at the lens, and mark *p* and *r* as the bright fringes due to *y*, and *q* a bright fringe due to *x*. When *q* is midway between *p* and *r*, the two sets of fringes

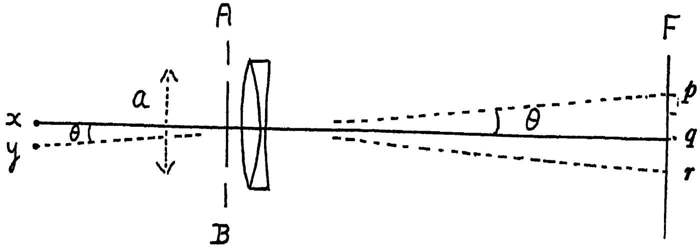

FIG. 17.

Principle of formation of interference pattern *p*, *q*, *r*, on a focal plane *F*, when the lens is covered by screen *AB* containing two orifices at distance *a* apart. Diagram from W. E. Williams, *Applications of Interferometry*.

oppose each other, giving uniform illumination, and the condition for this to occur is,

$$\theta = \lambda/2a \text{ or } 3\lambda/2a \text{ or } 5\lambda/2a \ldots, \text{ etc.}$$

since λ the wave-length is equal to the difference of optical path for the channels A and B whereby light reaches the focal plane F.

If xy, instead of being two isolated points, is a very small but continuous line of illumination, the focal plane will appear uniformly bright, the dark and light fringes lost, if,

$$\theta = \lambda/a \text{ or } 2\lambda/a \text{ or } 4\lambda/a \ldots, \text{ etc.}$$

If xy is a circular disc, as the inaccessible shape of a star will be, a fraction modifies this through the geometry of the circle, and,

$$\theta = 1{\cdot}22\lambda/a$$

for the first disappearance of fringes. Michelson and other experimenters tried this method, for deducing the size of xy, in the case of Jupiter's moons; but they were unable to get down as far as the infinitesimal disc of a star—the distance we called 'a' would have to be 20–40 feet, and no telescope lens is as wide.

The profitable solution, instead of using the two apertures at opposite edges of a telescope lens, was to make the distance 'a' the separation between two mirrors mounted on a steel girder across the 100-inch telescope of Mount Wilson. The path of light, modified from Fig. 17, is then seen in Fig. 18.

The inner mirrors are carefully adjusted until the beams of light overlap in the field of view of the eyepiece (we omit all the telescope, which is the usual reflector), and what is seen is the set of 'fringes' or dark and bright parallel lines. The outer mirrors are then moved symmetrically in or out until the fringes merge into a uniform brightness. If a is the smallest separation of mirrors allowing this, the angle subtended at the star is,

$$\theta = 1{\cdot}22\lambda/a \text{ radians}$$

The same thing happens at the appropriate multiples of the angle. If the star is less bright at its edge than centre, as in the case of our sun, the numerical factor enlarges to 1·43.

The linear diameter of the star, D, is obtained by transforming the angle θ: expressed as a multiple of the sun's diameter,

$$D = 107\theta/\pi,$$

where π is the star's parallax.

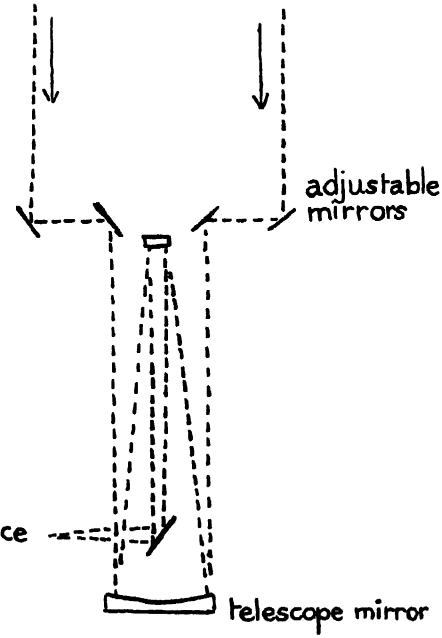

FIG. 18.

Application of the principles of Fig. 17 to reflecting telescope at Mount Wilson: four plane mirrors are mounted on a 20-foot girder across the main aperture, the two outer ones adjustable until the interference fringes vanish in the eye-piece. The calculation is in the text, and this diagram is from Russell, Dugan, and Stewart's *Astronomy* in their description of the apparatus of Pease and his collaborators.

As an example, we will quote the classic measurement of the diameter of the giant red star Betelgeuse in Orion, in 1920. The fringes were found to disappear when the mirrors had been gradually separated to 307 cm. with an effective wave-length of light 5750 Angstroms or units of 10^{-8} cm. Then,

$$\theta = 1{\cdot}22\lambda/a = 1{\cdot}22 \times 5{\cdot}75 \times 10^{-5}/307 = 2{\cdot}28 \times 10^{-7} \text{ radians}$$

Transforming to seconds of arc, this becomes $0''\cdot047$. The parallax of this star is $0''\cdot017$, so that the star's diameter compared with the sun's is,

$$D = 107 \times 0\cdot047/0\cdot017 = 295$$

Betelgeuse is therefore 260 million miles in diameter, compared with the sun's less than a million.

In the hands of Pease and collaborators at Mount Wilson, this very delicate and difficult experiment has yielded the important knowledge that the giant red stars have diameters up to several hundred times that of the sun, Mount Wilson having built a 50-foot interferometer in addition to the 20-foot. But only the largest stars are amenable to the technique.

One of the most intriguing results is that repeated measurements detect the actual swelling and shrinking of some of the red giants, the physical cause previously merely guessed as cause of the oscillation in light which we discussed in Part I, Chapter V.

Since all the data on stellar masses confine the range of this quantity for most stars to between $\frac{1}{5}$ and about 5 times the solar mass, but we now find volumes ranging up to $(500)^3$ or a hundred million times the sun's, we possess at once the picture outlined in Chapter III of Part I; we spoke there of vast distended red giants whose mean density is so extremely low that their state of material resembles that of our vacuum tubes (if the latter could be hot enough), rather than any solid or liquid matter as we are acquainted with it in the laboratory.

We have now added to the verbal descriptions of Chapters I, II and part of III, from Part I, some justification for the statements there made. We have outlined the more essential steps in the arguments by which are inferred the main physical data concerning a star, its Apparent and real Luminosity, its Black Body and Line temperatures, its Mass, Size, and hence Density. We have now to face the consequent enquiry of how the star can maintain its existence as emitter of so enormous an outflow of radiant energy. This is the central problem of stellar Natural history, and we have to attempt some quantitative justification for the suggestions made in Part I as to the star's source of life or energy.

Gravitational Energy of a Star

THE KELVIN-HELMHOLTZ CALCULATION FOR MAINTENANCE OF THE SUN

In Chapter III of Part I an approach was made in general terms, to the problem of the maintaining of a star's luminosity. We pointed out that the situation of the *steady* star of constant luminosity presents a certain simplification, in spite of the wide-ranging queries in physics implied; since if we know L (abs.) we know that the star is emitting a calculable amount of energy per second, and if the star is really a steady one, it must therefore be generating an equal amount of energy per second.

What is the source of this generation of energy—the vitality of the star? In the particular case of the sun, what maintains its enormous daily loss of radiated energy, and why does it not cool down as result of that loss? How long has it been shining at its present luminosity? Geological evidence here becomes relevant, as it supplies data as to approximately the number of millions of years during which the earth has benefited from a not very dissimilar rate of radiation.

We selected in Part I two most promising sources of solar energy, one arising from gravitation and the other from atomic nuclei, and we stated verbally what was meant by the transformation of potential energy when a large body shrinks under gravitational attraction towards its centre. We noted also that these two sources of energy are not the sole possibilities, mentioning the important work of Hoyle and Lyttleton in drawing attention to the energy which a star could acquire by gathering interstellar gas or dust, or 'sweeping up the cosmic debris of space'.

We shall now give a modernised version, based mainly on the writings of Chandrasekhar the Indian now at the Yerkes Observatory of Chicago University, of the gravitational argument about energy gained by the sun in shrinking. The essential physical ideas are those of Kelvin and of Helmholtz, late in the nineteenth century before physics had progressed sufficiently for the alternative of atomic transformations to be considered.

It may be helpful to state baldly at first the final result, and thereafter to trace the argument from the beginning, as the rarity of any complete treatment makes it worth while to insert all the steps in such a way as can be followed from an elementary acquaintance with physics. It is a regrettable lack that this classical and not abstruse argument, excellently typical of what can be done in applying conventional concepts to Nature at large, has usually been kept in learned treatises and papers not touched by the everyday student, and in those papers of course the steps needed by the student are omitted.

If L is the average luminosity of the sun over all geological history, say time t, and at present this luminosity is roughly 4×10^{33} ergs per second, while the solar radius R is 7×10^{10} cm. containing its mass M of 2×10^{33} gm., and the gravitational constant G is $6{\cdot}7 \times 10^{-8}$ in cm. gm. sec. units,

Lt = some known fraction of the change in potential energy of M while it has been shrinking down to radius R

$$= q\,\frac{GM^2}{R}\left(\frac{3\gamma-4}{3(\gamma-1)}\right)$$

q is a small quantity between $\frac{3}{2}$ and 1, and γ is the ratio of specific heats of a gas, whose highest value is $1{\cdot}67$ which makes the bracket equal to $\frac{1}{2}$. Inserting the values we have quoted, and solving for t,

$$t = 2 \times 10^7 \text{ years}$$

This at once reveals the inadequacy of the gravitational source of energy, since the duration of geological evolution of the earth, as well as the analysis of the rocks for their Uranium/Lead ratio and Uranium/Helium ratio, demand that the sun shall have shone at not very much less than its present rate for more than 10^9 years: gravitational shrinkage can only have supplied the energy for not much more than 1 per cent of the time.

The argument leading to the above equation and its far-reaching implications will now be traced. We shall need to utilise only the notions familiar to the undergraduate in science, the Thermodynamics taught to junior students of Physics, Chemistry, or Engineering.

It is convenient to distinguish between U the internal energy of a gas depending on its heat capacity and temperature, Ω the potential energy of a sufficiently large sphere such as a star due to the gravitational attraction to its centre, and a total energy E. The principle of conservation of energy during any change is conveniently applied through the so-called 'virial' theorem,

$$\text{Twice the Kinetic Energy} + \Omega = 0$$

If K is the gas constant per unit mass, T the temperature, and C_p and C_v the specific heats or energy needed for unit temperature rise under conditions of constant pressure and constant volume respectively, then for any small item of mass denoted by dm,

$$d(\text{Kinetic energy}) = \tfrac{3}{2}\, KT\, dm = \tfrac{3}{2}\, (C_p - C_v)\; T\, dm$$

This expresses the energy employed in expanding the material. But also, for unit mass,

$$U = C_v\, T \quad \text{so that} \quad dU = C_v\, T\, dm$$

so that we can write,

$$d(\text{Kinetic energy}) = \tfrac{3}{2}\, (\gamma - 1)\; dU$$

where $\gamma = C_p/C_v$ the ratio between the specific heats. For the whole star,

$$\text{Kinetic energy} = \tfrac{3}{2}\, (\gamma - 1)\; U$$

Hence the equation denoting conservation of energy becomes,

$$3(\gamma - 1)\; U + \Omega = 0$$

The total energy, which is E, is,

$$E = U + \Omega$$

and can therefore be expressed, substituting for Ω and for U in turn, as,

$$E = U - 3\,(\gamma - 1)\; U = -\,(3\gamma - 4)\; U = [(3\gamma - 4)\,/3(\gamma - 1)]\;\Omega$$

It will be important to recognise, in our later chapter on the stability of stars, that this quantity changes its sign as γ passes through the critical value $\tfrac{4}{3}$, making the star scatter its contents to the sky if the ratio of specific heats falls below this value.

For the present purpose it is sufficient to notice that during any contraction of the star there is a *change* of energy dE, and that,

$$dE = \frac{3\gamma - 4}{3(\gamma - 1)}\, d\Omega$$

This fraction $3\gamma - 4/3\ (\gamma - 1)$ of the mechanical work done in shrinking $d\Omega$, is lost to the star in radiation emitted, while the remaining fraction,

$$1 - \frac{3\gamma - 4}{3(\gamma - 1)} \text{ or } \frac{1}{3(\gamma - 1)}$$

is added to the internal energy as dU and raises its temperature.

We have so far considered only how these changes are interconnected, and must now fix a definite value to $d\Omega$ the change due to shrinkage of the star. This is the work done by the force which in Newtonian mechanics is, for unit mass,

$$GM/R^2$$

giving, for actual mass M of the star,

$$\Omega = -qGM^2/R$$

q is a characteristic of the particular way in which the star's material is distributed, and is equal to $3/(5 - n)$ where n is the polytrope index discussed in Chapter VI. For most stars, where n is about 3, $q = \frac{3}{2}$, nearly. The loss of energy in radiating heat and light is therefore, as the above fraction of the energy supplied by gravity while the star has shrunk from widespread diffusion into its present radius,

$$\text{Energy emitted as radiation} = -\frac{3\gamma - 4}{3(\gamma - 1)}\,\Omega$$
$$= q\,\frac{GM^2}{R}\,\frac{3\gamma - 4}{3(\gamma - 1)}$$

We noted that γ cannot be less than $\frac{4}{3}$ if the star is to 'hold together'; its maximum value of $\frac{5}{3}$ making half the gravitational energy turn into radiation. Inserting the actual values for the sun, as we did in our statement of the result, and remembering that this total of energy emitted is the luminosity multiplied by the duration of shining, Lt, we have all the quantities in the equation known except t; the latter is thus found to be not more than 20 million years, only 1 or 2 per cent of the time during which terrestrial evolution has definitely been nourished in that sunshine.

This proved inadequacy of the Victorian conception of whence comes the solar energy, constituted one of the major puzzles of Natural History; the puzzle became soluble in the twentieth century through convergence of two main streams of research, the Relativity begun by Einstein and the experimental probing of sub-atomic Nature begun by Rutherford, to which we now turn.

Atomic Sources of Energy

THE EINSTEIN EQUIVALENCE OF MASS AND ENERGY

During the first quarter of this century, when it became realised that energies exceeding the thermal energy of our highest available temperatures are needed to detach even the chemist's valency electrons from some atoms, it was surmised vaguely that disruption of the more intimate structure of the positive core of an atom must require energy expenditure on a scale vaster than any hitherto explored. Rutherford and his pupils, elucidating between 1910 and 1925 the first known properties of the nucleus of the atom, were able to turn such surmises into quantitative estimates. The clue lay in the identification of mass or inertia with energy; this identification brings together two concepts previously distinct and representing the two sides of the 'matter and energy' classification of the older sciences. The equivalence of mass and energy dates from Einstein's earliest Relativity theorems of 1905, in which,

$$E = mc^2$$

where E is an energy in ergs, m is a mass in grams, and c is the velocity of light, 3×10^{10} cm. per sec. Owing to the enormous scale of light velocity, 186,000 miles per sec., a very small amount of mass would if it disappeared imply the corresponding creation of this vast amount of energy. If we recognise that in some transmutations of atomic species resulting from disruption of the nucleus, the final product has less mass than the original constituents, this 'lost' mass reappears as energy which in general exceeds that liberated in any chemical reaction. For in chemistry only the lesser energy difference between electronic configurations of molecules is liberated, in combustion and other oxidations or even explosions: in explosions there is no loss of mass or transmutation of elements, but only a rearrangement of permanently conserved mass into a gaseous instead of solid state with new molecular attachment. In contrast, on the scale of changing one atom into another, it was commonly remarked that transformation of the hydrogen from a cupful of water, in becoming helium, would liberate the energy to drive a liner across the Atlantic.

THE QUANTITY OF ENERGY LIBERATED AT FORMATION OF HELIUM FROM HYDROGEN

The amount of the quantity m, lost when helium comes into being from hydrogen, and capable of yielding E in the above equation, can be readily estimated, since the relative masses of hydrogen and helium atoms are,

$$m_{\mathrm{H}} = 1{\cdot}008 \qquad m_{\mathrm{He}} = 4{\cdot}00$$

Hence the simplest of all transmutations of elements, the synthesis of one helium from four hydrogens or an α particle from four protons, entails the disappearance of 0·008 grams for every gram of hydrogen used up. Applying the equation quoted,

$$E = mc^2 = 0{\cdot}008 \times (3 \times 10^{10})^2 = 7 \times 10^{18} \text{ ergs}$$

This is equivalent to about 200,000 kilowatt-hours, liberated from the single gram, a small fraction of one ounce, compared with the vast number of tons of coal burnt in a power station to liberate the same energy.

The sun's mass is 2×10^{33} grams, and if 10 per cent of this were originally hydrogen, and then 10 per cent of that 10 per cent or $\frac{1}{100}$ of the whole mass, became helium in the course of evolution, the energy liberated would be,

$$7 \times 10^{18} \times \tfrac{1}{100}\,(2 \times 10^{33}) = \text{more than } 10^{50} \text{ ergs}$$

Recollecting the solar luminosity, 4×10^{33} ergs per second, this source would supply the present rate of the sun's emission of heat and light for the 10^9 years and more, required by the geological evidence. The atomic source thus turns out to be 100 times as powerful as the gravitational attraction previously considered the only likely mode of maintenance of solar activity.

The actual transmutations of various elements, effected in the 1920's and 1930's by pupils of the Rutherford school, and other pioneers all over the world, demonstrated conclusively that this enormous gain of energy through loss of mass is a reality, and not merely a prediction of Relativity theory. Nowadays, just as chemists have always classed as 'exothermic' a reaction which liberates energy by readjustment of electronic orbits, the Principle of conservation of energy requiring electronic losses to reappear as heat or radiation, so the equivalence of mass and energy in modern physics can state with precision the far greater gains of energy on loss of mass for a 'reaction' between atomic nuclei.

That this new source of natural energy is more than dream, can be of no doubt to the unhappy Japanese cities, upon which was tried its first 'practical' application; the civilisation of this day may well be blamed that the first use of this atomic energy was in warfare rather than in peace-time fuel economy.

BETHE'S CARBON-NITROGEN CYCLE, AND OTHER LIGHT ELEMENT REACTIONS

In the early 1930's some calculations were made by various workers, notably Atkinson now of Greenwich Observatory, attempting to decide how these transmutations of atomic nuclei might be occurring in stars. If 'exothermic' in the sense we used in the preceding paragraphs, the transmutations are certainly adequate to maintaining the heat and light. Nuclear reactions of the kind postulated were already being produced in laboratories, but in all these artificial experiments the initial disruption was caused by the radiations emitted spontaneously from Radioactive substances, or by the impact of ions or electrons accelerated by high voltages in apparatus such as that of Cockcroft and Walton in England or of Livingstone in California. One had to suppose that in the stars the thermal energy of particles in a gas due entirely to its high temperature could play the part of disrupting the nuclei, and liberating the energy at loss of mass in the resulting reactions. Eddington and others had already shown that at the centre of a star, temperatures of millions of degrees would be inevitable, and it seemed probable that at such temperatures nuclear reactions must be proceeding spontaneously at considerable rate: in particular, helium must be in process of being continuously created from hydrogen.

But it was not until 1938–9 that several workers, notably Bethe in U.S.A. and Weiszäcker in Germany, were able to trace convincing reasons for the main stellar energy to be ascribed to one particular kind of reaction, with a very few others of subsidiary importance. In a paper which has become classic, in the American *Physical Review* for 1939, Bethe developed into detail some calculations based on previous work by himself and by Critchfield, Teller, and ultimately the Atkinson-Houtermans researches of the earlier years. He examines the intrinsic probability of collisions between protons (hydrogen nuclei) and all the other light elements in the chemists' periodic Table, and of reaction resulting from collision. He shows convincingly that only the following list are at all likely to occur at the temperatures between one million and 30 million degrees, which covers the range of stellar interiors.

(*a*) Low temperature reactions. 'Low' here means not greatly exceeding 10^6 degrees:

$$
\begin{aligned}
{}^{1}_{1}\mathrm{H} + {}^{1}_{1}\mathrm{H} &= {}^{2}_{1}\mathrm{H} \\
{}^{2}_{1}\mathrm{H} + {}^{1}_{1}\mathrm{H} &= {}^{3}_{2}\mathrm{He} \\
{}^{7}_{3}\mathrm{Li} + {}^{1}_{1}\mathrm{H} &= 2\,{}^{4}_{2}\mathrm{He} \\
{}^{9}_{4}\mathrm{Be} + {}^{1}_{1}\mathrm{H} &= {}^{6}_{3}\mathrm{Li} + {}^{4}_{2}\mathrm{He} \\
{}^{11}_{5}\mathrm{B} + {}^{1}_{1}\mathrm{H} &= 3\,{}^{4}_{2}\mathrm{He}
\end{aligned}
$$

Following the modern convention, the upper index number denotes atomic weight and the lower index number denotes atomic number, the differences among the former for any given figure among the latter indicating the several 'isotopes' of which any terrestrial sample is a mixture. The above reactions are all instances of protons (${}^{1}_{1}\mathrm{H}$) being 'captured' by other protons or by the double-mass hydrogen isotope Deuterium, or by Lithium, Beryllium, or Boron, each reaction contributing by its main product or a by-product to the creation of one or other of the isotopes of helium. Disappearance of hydrogen and creation of helium will liberate energy to the amount we have already calculated.

The astronomical problem now raised, is that since all Bethe's reactions increase their rate enormously for any rise of temperature within the range mentioned, at temperatures much exceeding a million degrees all the Li, Be, B, in the star will 'burn up' very rapidly, so that the star's life will be short. That would certainly be true in the case of the sun, though it is possible that the cooler red giants do 'live upon' these elements. We return to this problem in Chapter VI.

(*b*) Reactions at intermediate temperature, say 1–10 million degrees: It is likely that reaction between protons, deuterons, and helium predominate in this range.

(*c*) 'High' temperature reactions, between 15 and 25 million degrees: Bethe's most notable contribution is his theory that here Carbon with Nitrogen plays the part called 'catalytic' by chemists; that is to say, they take part in a sequence of reactions the net effect of which is always hydrogen loss and helium gain, but the Carbon itself reverts to its original state, and so is not consumed as would be Li, Be, and B. The sequence of this 'Bethe cycle', as it has come to be called, is as follows:

$^{12}_{6}C + ^{1}_{1}H = ^{13}_{7}N$ + emission of γ ray.

$^{13}_{7}N = ^{13}_{6}C$ + emission of positive β ray

$^{13}_{6}C + ^{1}_{1}H = ^{14}_{7}N + \gamma$ ray

$^{14}_{7}N + ^{1}_{1}H = ^{15}_{8}O + \gamma$ ray

$^{15}_{8}O = ^{15}_{7}N$ + positive β ray

$^{15}_{7}N + ^{1}_{1}H = ^{12}_{6}C + ^{4}_{2}He$

It is seen that the net consequence is conversion of H to He, but that the original C is in the end re-formed to begin the 'cycle' once more. Chemists will notice that some very unfamiliar 'isotopes' are postulated in the cycle:

(1) In terrestrial Carbon, 99 per cent is $^{12}_{6}C$ with the isotope $^{13}_{6}C$ rare but stable. In terrestrial Nitrogen, 99 per cent is $^{14}_{7}N$ with the isotope $^{15}_{7}N$ rare but stable. In terrestrial Oxygen, 99 per cent is $^{16}_{8}O$ with isotopes $^{17}_{8}O$ and $^{18}_{8}O$ rare but stable.

(2) There are also the unstable emitters of β rays discussed by Fermi and his collaborators, for instance $^{16}_{7}N$, and the unstable emitters of positrons, or β rays with reversed sign, $^{13}_{7}N$ and $^{15}_{8}O$, discussed by Curie and Joliot.

The sun's temperature, at its centre, is probably about 19 million degrees and some estimates suggest over 20; Bethe and his associates, also Sen and Burman, have shown that with assumptions intrinsically likely to be fulfilled the Bethe cycle does indeed supply the correct amount of energy to keep the sun shining as it has done for its long life.

The very piquant situation which may arise when a star has consumed most of its hydrogen, by this conversion into helium, is a problem vital and indeed central nowadays, in any notions of stellar evolution. We will raise it under Chapter VI, where we have to face the question whether abnormal behaviour of erupting or exploding stars may denote the exhaustion of this atomic source of steady life. Other disturbing queries will also there arise, as to how a star keeps stable even when fuelled with hydrogen; for instance the fact that rates of reaction increase so enormously with temperature had suggested in the 1930's that no star could possibly 'live upon' consuming its hydrogen—as soon as any of those reactions set in, the consequent heating would accelerate the reaction rate until it disrupted the star as a celestial atomic bomb. Bethe's reactions were of no lasting use to Astrophysics until criteria were devised, by Cowling and others, for the stability of a star in which such nuclear transformations occur.

Statistics on Luminosity and Spectral Type

The large branch of modern astronomy comprising stellar statistics has not, at first sight, very direct relevance to the problem of this book, which is confined to the conditions of steady and unsteady behaviour of stars and their production and emission of energy. The aim of statistics is surely the understanding of the Galaxy, not of the individual star. But it is essential for our enquiry to know at least whether some definite range in brightness or spectrum can cover any proportion of the stars which we have to fit into a Natural History.

SPECTRAL TYPES

We begin by extending somewhat the very rough classification described in Chapter IV of Part I, in which from hottest to coolest the stars were classified in an arbitrary set of alphabetical types, 'O', 'B', 'A', 'F', 'G', 'K', 'M'; this was a sequence along which the spectral lines of ionised helium gave way to neutral helium, then to hydrogen, then the metallic vapours, and finally oxides and carbon compounds.

We emphasised in Part I that the dominance of any star's spectrum by some element is only rarely evidence for the great abundance of that element, and far more often a sign that the temperature and pressure is just such as to permit material of that particular ionisation potential to exhibit visible lines. In fact we have already correlated the strength and fading of those lines along the stellar sequence with the temperatures obtained from the Saha theory. It is convenient to subdivide those lettered types by subscripts 0—9, the lower number denoting proximity to the next 'earlier' or hotter type; thus an A_0 star is among the hottest of the 'A' type, only just cooler than the B_9 stars, while A_5 is half-way to the hotter end of the 'F' type.

We remarked in Part I that the 'O' type at the highest temperatures include some stars with *bright* lines instead of the usual dark, discovered by Wolf and Rayet, just as at the lowest temperatures the 'M' stars with metallic oxide spectra also include some with bright lines. The dark line 'O' stars may be very different bodies from the Wolf-Rayet, and it is also likely that among the coolest stars the 'M' stars are paralleled by a Carbon type differing in composition rather than merely in temperature.

In the commission on classification organised by the International Astronomical Union (Report 1935), it was recommended that two parallel sequences should be recognised even among the Wolf-Rayet group or Bright line 'O' type: (*a*) there is, therefore, the Carbon sequence, in which helium and ionised helium are accompanied by bright bands of C^+, C^{++}, C^{+++} and O^+, O^{++}, O^{+++}, O^{++++}, O^{+++++}, that is to say, Carbon and Oxygen in an environment of such high temperature that the atom has lost electrons 1, 2, 3 and even 4 and 5. Since loss of one electron begins a second spectrum, compared with the first spectrum of a neutral element, these successive spectra are usefully written O II, O III—O VI. In this notation we describe these particular stars as dominated by C III at wave-length 4650, and 5696, C IV at 5812, and He II at 4686, the numbers denoting the wave-length in Angstroms. (*b*) There is the parallel Nitrogen sequence, where He I and He II are accompanied by N III, N IV, N V, or nitrogen atoms which have been stripped of as many as 2, 3, or 4 electrons. These two kinds of Wolf-Rayet stars are now called W(C) and W(N) respectively.

The other very high temperature stars retain their 'O' label, and there has been more than one attempt to arrange them into a sequence with physical significance; the whole type has sometimes been designated O_a, O_b,—O_e, of which the first three letters would include the Wolf-Rayet. H. H. Plaskett gave them a thermal sequence, assigning the extreme limit O_0 to a hypothetical star whose lines are completely swamped by the continuous spectrum, and in which O_5—O_9 develop the characteristics merging into the 'B' type. All these stars showing multiple ionisation of N, O, etc. and He II, must have surface temperatures exceeding 30,000 degrees.

The type B_0—B_9 covers the stage of weakening of He II and domination of the spectrum by the dark lines of neutral helium (He I). Single ionisation of Silicon and Magnesium appear. Type 'A', beginning with A_0, is characterised by dominance of the great strength of the Hydrogen (Balmer) lines, and there begin to be noticeable the lines of ionised Calcium which dominate the violet end of the metallic stars. These lines, called 'H' and 'K' by the early workers, are only one tenth as intense as the H of hydrogen in an A_0 star, but have exceeded the hydrogen strength in A_5. In the 'F' type the hydrogen is still weaker, while metals Ca, Fe, Ti, Sr, strengthen, and the first bands signalling chemical compounds (the hydrocarbon CH) appear. The Sun is a G_0 star, but the sun-spot spectrum resembles the cooler K_0 stars, in which the Ca II lines are at their strongest. The prominent stars Arcturus and Aldebaran are 'K' stars, Procyon is an 'F' star, and the bluish-white Sirius Vega, and Castor, are 'A' type stars.

At the stage 'G' the distinction of Giant from Dwarf star begins to be important, recognising the fact discussed in Part I that cooler stars may be distended or compact, of high or low luminosity according to the size into which their mass is packed. Fig. 5 of Part I exhibited this split of the luminosity-type diagram into two branches. In classifying spectra, this distinction is recognised by '*d*' and '*g*', so that the yellow giant Capella is gG_0 to distinguish it from the more compact Sun of similar temperature dG_0.

Over 2000 of the sun's 18,000 spectral lines are due to Iron, typical of the 'many-lined' types 'F', 'G', 'K', contrasted with the few simple lines of the hotter hydrogen and helium stars. For all these middle ranges of the stellar sequence, the classification based on the Henry Draper Catalogue of Harvard Observatory has been the most profitable.

The red stars at the coolest extreme were originally divided into 'M', 'R', 'N', 'S', lettering. But a great deal has been done in the last few years to rearrange them according to some rational basis such as temperature; actually in these cool stars, 4000–2500 degrees probably, temperature ceases to be the only main discriminant. Not only does '*g*' and '*d*' enter as separating distended and compact stellar structures more completely, but it is suspected that actual chemical composition or relative abundance of elements may play a great part. In the hands of Keenan and others it has become possible to separate parallel sequences, of similar temperature but richer or poorer in Carbon. The 'M' type, dominated by bands of Titanium oxide which intensify as temperature diminishes, has to be paralleled by what used to be the 'N', 'R', 'S' stars which seem to possess more available Carbon. Some of these latter can be graded according to the strength of the hydrocarbon band of CH which is detectable even as far towards high temperatures as the Sun's atmosphere. The poisonous gas Cyanogen (CN) and the diatomic molecule of Carbon (C_2) are other essential spectroscopic features of the cooler stars, and we pointed out in Part I how difficult these bands make the task of measuring the continuous spectrum of the cooler stars. It may also be remarked that CH, C_2, CN, though well-known in the mass-spectrographs of low-pressure electric discharge apparatus, are not capable of being isolated for stable existence in the chemical laboratory: they play their terrestrial part as attached portions of larger and more complex molecules which could not survive at stellar temperatures. In fact, stellar chemistry is a study of 'radicals' or fragments of molecules, the primitive attachments between atoms, blurred at our terrestrial low temperatures which allow liquid and solid aggregation.

COLOUR TEMPERATURES OF THE SPECTRAL TYPES, GIANT AND DWARF, COMPARED WITH THE IONISATION TEMPERATURES

In discussing the Saha-Fowler-Milne treatment of ionisation, accounting for the dark lines in the stellar atmospheres, we listed some of the temperatures which could thereby be inferred; for comparison we here quote some stages along the decimal subdivision of the spectrum types, worked out from the main colour distribution instead of from the intensities of the lines. The temperatures are therefore representative of the stars' photospheres, a slightly lower level in the material structure than the seat of the absorbing layers giving rise to the dark lines. The distinction of Giant and Dwarf can also be introduced here, a distended star being in general somewhat cooler than a more compact star at the same temperature. The figures are from the researches of King at Harvard and Seares at Mount Wilson, and utilise as factual data the 'colour indices' which we defined earlier as difference between photographic and photovisual luminosity—a fair measure of the blue or red tendency of the spectral intensity distribution in Wien's law.

Spectral type	*Colour index*	*Temperature*				
O	—	35,000–25,000				
B_0	−0·33	21,000				
B_5	−0·18	14,000				
A_0	0·00	10,600				
A_5	+0·20	8,200				
F_0	0·33	7,100				
F_5	0·47	6,300				
G_0	0·57	5,750		0·67	5,300	
G_5	0·65	5,400		0·92	4,500	
K_0	0·78	4,900		1·12	4,000	
K_5	0·98	4,300	Main sequence	1·57	3,200	Giants
M_0	1·45	3,400		1·73	3,000	
M_2	—	2,870		—	2,810	
M_8	—	—		—	1,780	

PERCENTAGE OF STARS OF THE DIFFERENT SPECTRAL TYPES (*see* Table on p. 159)

These are from the 225,000 stars listed in the Henry Draper Catalogue of Harvard.

Types 'A' and 'K' are therefore the most numerous, together accounting for ⅝ of all the stars; B contains a large proportion of the brightest, with percentage decreasing among the fainter. There are about 6000 stars brighter than magnitude 6·25 (the extreme limit of unaided sight) and of these only 20 are of the 'O' type. This Table can be compared with the more spectacular but less exact way of exhibiting the facts in Fig. 5 of Part I.

Statistics on Luminosity and Spectral Type

PERCENTAGE OF STARS OF THE DIFFERENT SPECTRAL TYPES

Visual magnitude	B_0 *to* B_5	B_8 *to* A_3	A_5 *to* F_2	F_5 *to* G_0	G_5 *to* K_2	K_5 *to* M_8
exceeding 2·24	28	28	7	10	15	12
2·25–3·24	25	19	10	12	22	12
3·25–4·24	16	22	7	12	35	8
4·25–5·24	9	27	12	12	30	10
5·25–6·24	5	38	13	10	28	6
6·25–7·24	4	30	12	14	33	7
7·25–8·24	2	26	11	16	37	8
8·25–9·24	1	27	10	21	34	7
below 9·25	1	33	8	25	29	4
Total of all brightnesses	2	29	9	21	33	6

NUMBERS OF STARS OF GIVEN APPARENT BRIGHTNESS

Seares and Van Rhijn have evaluated the following estimate, from counts on photographs of selected areas of the sky, excluding Globular Clusters and the external Galaxies. The figures below mag. 18 represent extrapolation only.

Brighter than mag.	*Photographic*	*Visual*
4	360	530
5	1,030	1,630
6	2,940	4,850
7	8,200	14,300
8	22,800	41,000
9	62,000	117,000
10	166,000	324,000
11	431,000	870,000
12	1,100,000	2,270,000
14	6½ million	13·8 million
16	33 "	71 "
18	143 "	296 "
20	505 "	1,000 "

In connection with this Table, the size of the telescope required for seeing these stars can usefully be estimated. The faintest magnitudes to be seen with a telescope of given aperture in inches are as follows:

Mag.	9	12	14	15	16	17	18	19
Aperture	1	4	10	16	25	40	63	100

Comparing these figures with the previous Table, it appears that a one-inch telescope could reveal about 100,000 stars compared with the 6000 visible to the naked eye, and a 40-inch telescope could reveal 100 million. These figures are considerably lowered in practice by absorption of light in the lenses, etc. The photographic limits extend themselves by the use of very long exposures.

RELATIVE NUMBERS OF STARS OF GIVEN ABSOLUTE MAGNITUDE

The above apparent brightnesses can be turned into an estimate of how bright the stars are intrinsically, apart from the accident of distance which decides their apparent luminosity. The figures of Van Rhijn may be quoted.

Abs. Mag.	*Brightness relative to Sun*	*Relative number in each category*
−5·0	10,000	1
−2·5	1,000	50
0	100	2,000
2·5	10	10,000
5·0	1	40,000
7·5	0·1	50,000
10·0	0·01	100,000
12·5	0·001	200,000

SIZE, DENSITY, MASS, TEMPERATURE, AND SPECTRAL TYPE

As a final tabulation, the following set of data, extracted by Baker's treatise, can serve to exhibit the several physical characteristics of a star obtained by the various methods which we have described in the early chapters of this book. It serves also as indicating the status of certain well known individual stars. 'Main sequence' and 'Giant' categories again refer to the branches of the diagram Fig. 5 of Part I. Two White Dwarfs are added for contrast in density.

Star	*Spectrum*	*Temperature*	*Diameter* (*sun* = 1)	*Mass* (*sun* = 1)	*Density* (*sun* = 1)
(Giants:)					
Antares	M_0	3,000	390	10	0·0000002
Aldebaran	K_5	3,200	72	4	0·00001
Arcturus	K_0	4,000	30	4	0·0001
Capella 'A'	G_0	5,300	16	4	0·001
(Main sequence:)					
β Centauri	B_0	19,000	6	4	0·02
Vega	A_0	10,600	2·6	3	0·2
Sirius 'A'	A_0	10,600	1·9	2·4	0·3
Altair	A_5	8,200	1·6	2	0·5
Procyon	F_5	6,300	2·3	1·1	0·1
α Centauri 'A'	G_0	5,750	1·3	1·1	0·5
Sun	G_0	5,750	1·0	1·0	1·0
70 Oph. 'A'	K_0	4,900	1·0	0·9	0·9
61 Cyg. 'A'	K_7	3,900	0·7	0·5	1·4
Krueger 60 'A'	M_3	3,300	0·3	0·3	9
(White Dwarf:)					
Sirius 'B'	A	7,800	0·034	0·96	30,000
o Eridani 'B'	A	11,000	0·019	0·44	60,000

Letters 'A' or 'B' denote members of a binary pair.

It may be noticed that among the giants Antares is a very extreme case of low density. Others somewhat similar are the stars Rigel, Deneb, Canopus, and Betelgeuse. These extraordinarily luminous and distended objects are often referred to as Super-giants, and the letter *c* written in front of their spectral type instead of *g*. Their spectrum lines are very sharp and narrow, owing to the extreme rarefaction of their atmospheres, lacking the 'pressure broadening' which spreads the wave-length of most laboratory lines and even some stellar lines.

The White Dwarf Stars

In Part I we gave a brief outline of the argument which led to the discovery of some stars that fail to conform to the usual relation between spectrum and luminosity. Not only does their abnormality show in the last of our Tables, but they fall quite outside the picture Fig. 5 of Part I, neither belonging to the main sequence nor to the diverging branch of giants. They are not over-luminous like the Super-giants, but under-luminous: they are hot and therefore white, but are faint because they are so small, and they are appropriately the White Dwarfs. In Part I a qualitative statement was made of the density of Sirius 'B', and of how the facts were discovered; but a density of tons to the cubic inch is so grossly at variance with the properties of terrestrial materials, that it will be important here to trace the detailed steps of the argument. The case also serves as an excellent example of an elementary but rigorous deduction from difficult but reliable observational data, typifying the convergence of evidences upon which the best of modern Astrophysics is based.

SEQUENCE OF THE DISCOVERIES ABOUT SIRIUS B

We will adapt the tale of Sirius B from Milne's Halley Lecture of 1932, using his marshalled data, although several numerical adjustments have been made by other workers which do not modify the main conclusion. For instance, temperatures of the several subdivisions of the 'A' stars are variously put around the figures we quote, and the final density comes out slightly differently in different treatments: the main point is that it always exceeds any figure possibly allowable without the ionisation theory which explains the anomaly. The steps in observation were as follows.

(*a*) In 1844 Bessel reported a 50-year periodicity in the slight drift or 'proper motion' of Sirius, suggesting that the star was a binary pair.

(*b*) In 1862 the components of the pair were detected, Sirius 'A' and Sirius 'B'.

(*c*) In 1915 Adams at Mount Wilson succeeded in photographing the spectrum of Sirius 'B', isolating its light from the 10,000-fold brighter light of the close neighbour 'A'. This provided at once the crucial discovery; the faint 'B' would not have raised any novel problem if it had turned out to be a cool star, but Adams' spectra proved that it is not much less hot than the far more brilliant 'A'.

(*d*) In 1925 Adams isolated the shift of spectral lines in 'B' as 0·29 units; this, if it were a Doppler displacement, would imply a velocity of 19 km. per sec. not shared by its companion. On Eddington's relativity calculation, a shift very close to this and equivalent to 20 km. per sec. must be expected from the Einstein gravitational effect upon spectral lines where a large mass is compressed into so small a radius.

The corresponding theoretical steps were as follows.

(*a*) In 1924 Eddington demonstrated that enormous densities such as that found for Sirius 'B' could be regarded, not as fantastic, but as normal in certain circumstances, in the light of what we knew about atomic structure.

(*b*) In 1926–9 Fowler and Stoner calculated the densities allowable to the White Dwarf state of a star.

(*c*) In 1930–6 Milne and Chandrasekhar worked out the circumstances in which a normal star might collapse into the White Dwarf state.

CALCULATION OF THE MEAN DENSITY OF SIRIUS B

Sirius 'A', the brightest star in our sky, is also one of the half-dozen nearest. Its parallax is $0''{\cdot}371$, giving a distance of 2·7 parsecs or 9 light-years, so that the visual Apparent Magnitude of 'A' which is $-1{\cdot}58$ and of 'B' which is $+8{\cdot}44$, mean Absolute Mag. of 'A' is 1·3 (visual), 0·97 (bolometric), and of 'B' 11·3 (visual), 11·25 (bolometric).

In other words, on our definitions of Magnitude in Chapter

II, 'B' emits about $\frac{1}{10000}$ of the radiation of 'A', and $\frac{1}{380}$ of our sun's radiation.

The masses are obtained from the binary orbit, according to the methods we described before, and in multiples of the sun's mass are 2·44 for 'A' and 0·95 for 'B'.

The surface temperature of 'A' was estimated as 11,300°, so that on Stefan's law its radiation *for a given area*, compared with the sun's, is,

$$(11{,}300/5{,}740)^4 = 15{\cdot}03.$$

Whereas, comparing the bolometric magnitude 0·97 with the solar 4·85, the *total* radiation of 'A' compared with the sun's is,

$$0{\cdot}4 \times (4{\cdot}85 - 0{\cdot}97) = 35{\cdot}6.$$

The surface area of Sirius 'A', compared with the sun's, is the quotient of total radiation divided by radiation per unit area,

$$35{\cdot}6/15{\cdot}03 = 2{\cdot}38.$$

The square root of this gives the radius, so that Sirius 'A' is 1·54 times the sun in radius.

Apply the same reasoning to Sirius 'B'. Adams' startling discovery of its unexpectedly hot spectrum assessed the temperature at about 8000°, so that its *total* radiation compared with the sun's is,

$$0{\cdot}4 \times (4{\cdot}85 - 11{\cdot}25) = 1/363$$

Its radiation *per unit area* is,

$$(8000/5740)^4 = 3{\cdot}79$$

The area of Sirius 'B' is therefore, in terms of the sun's,

$$\frac{1/363}{3{\cdot}79} = 1/1380$$

and the radius $\frac{1}{37}$ of the sun's.

Dividing now the masses which we quoted for 'A' and 'B' by $\frac{4}{3}\pi$ (radius)^3 to obtain the densities,

Sun: 1·41 gm. per cm.^{3}
Sirius 'A': 0·94.
Sirius 'B': 68,000 or 1·1 tons per cubic inch.

The temperature of Sirius 'B' is not, of course, accurately obtainable from so faint a star, and the estimate of 8000° may be wrong by hundreds; but it cannot be wrong by thousands. Indeed to make Sirius 'B' exhibit a density at all ordinary and not so grossly exceeding our entire acquaintance with terrestrial matter, it would have to be as cool as 2500 degrees and dull red —a complete reversal of the facts.

DENSITIES ALLOWED BY THE STAGES OF IONISATION

We pointed out in Part I that once we rid ourselves of the prejudice due to our terrestrial experiences, limited to temperatures at which atoms retain most of their electrons, the White Dwarf densities become understandable features of Nature, which we might even have predicted. All atomic theories based upon experiment, which means all since Rutherford's first suggestions of 1910, are forced to accept the facts which assign extremely small dimensions to the electron and to the positive nucleus around which electrons occupy 'energy levels' somewhat remotely like planets around the sun. Just as more than 99·9 per cent of the solar system is empty space, traversed by widely separated planets, so the vast majority of 'space' in the densest atom must be empty—though the early picture of a miniature solar system has given way to the wave-model. We do not now claim to know quite what could be meant by the 'boundary' of electron or nucleus, but whatever properties they each possess are certainly confined to extremely small dimensions. If the radius of a simple atom is about 10^{-8} cm., the radius (however difficult to define) assigned to electron or nucleus is more like 10^{-13} cm. or a hundred thousand times smaller. The atom is an 'open-work' structure, from which other atoms are excluded from entry except at high energies; this exclusion is due to the barrier of electric fields, not the barrier of encroaching on the actual electronic particle which is small—whatever that means today when we talk of waves more often than of particles. The mass of an atom is essentially some small multiple of $1{\cdot}65 \times 10^{-24}$ gm. (mass of hydrogen atom); so in 'occupying' its usual volume, or space over which the electrons exert their influence, density is limited by the impenetrability of electronic fields to 10 or 20 gms. per cm.^{3} But once those electrons are mostly removed by the multiple ionisation at the temperatures of millions of degrees at a star's centre, the volume could conceivably be compressed from its terrestrial $(10^{-8})^3$ to $(10^{-12})^3$, and the density accordingly could rise from about 10 to a million million. Various further considerations limit the likely possibilities to the million range. But Sirius 'B' at its density of less than a hundred thousand is obviously not at the extreme limit, and indeed denser White Dwarfs have since been discovered.

Students of mathematical physics are intrigued that here is a condition of matter in which the gas laws are more likely to be those of Fermi and Dirac's quantum statistics, than the familiar laws of Maxwell obeyed by gases in the laboratory.

We have been concerned with setting out the facts, and the inescapable deductions therefrom; so we are not at the moment going to enter upon the widespread speculations as to whether the White Dwarf state is the final fate to which all stars are condemned. We shall see in the discussions of Chapter VI, that a normal star is in equilibrium under gas pressure, radiation pressure, and gravitation: it has been suggested that at certain stages in evolution, when the hydrogen supplying energy (Chapter III) is exhausted, radiation pressure will fail and there be nothing to prevent collapse on to a dense central core. Some such possibility has been keenly canvassed as a conceivable origin of White Dwarfs; but about the detail of the collapse, or even its certainty, we are at present very ignorant. Mathematical Astrophysics has exploited conditions of equilibrium, and is not as yet very adaptable to the calculation of catastrophe.

The Cepheid Problem

SCHWARZSCHILD'S WAVES AND THE DELAY BETWEEN LIGHT AND MOTION

In Part I, in the Chapter V on unsteady stars, we brought together some evidence for the need to consider all 'unquiet' stars on a comparable basis. The precisely repeated pulsation of Cepheids, the less exact fluctuation of some red giants, the continuous ejection of gases by Wolf-Rayet stars, with the Planetary Nebulae which may represent accumulated ejecta, and the single catastrophe of an exploding star, may all imply differing degrees of an instability whose origin we discussed in Chapter VI, and which may be incipient in the spasmodic irregular variable stars. But before here raising a more exact treatment of stellar stability, some particular aspects of the Cepheid phenomenon and the Nova phenomenon must be described. They offer to the physicist some of the most striking instances in Nature, of harmonic oscillation and of rapid transport of material and energy on a huge scale; the principal clue to a star's stability may well lie in these individual hesitations between steady and catastrophic evolution.

We mentioned in Part I two main features of the periodic brightening and dimming of Cepheid variable stars, (*a*) the Period-Luminosity law enabling Absolute Magnitude at the middle of pulsation to be inferred from the time taken for the cycle from maximum to maximum brightness, (*b*) the phase lag. By the latter we mean that the fastest outward rush of gas occurs at the star's brightest moment, and the fastest inward fall of gas occurs at the dimmest moment. This was the fact which disposes of the earlier suggestion that the Cepheid is really a binary pair with periodic mutual eclipsing of its two members: but it raises one of the most difficult puzzles of stellar physics, for the following reason.

If the change in brightness were merely a swelling and contraction, the brightest and dimmest instants would be at the turning points, brightest when expansion gives way to contraction and dimmest when contraction gives way to re-expansion—a quarter-period *later* than actually occurs. If, on the other simple explanation, the changes in brightness were purely a consequence of temperature, contraction to smallest size would bring highest temperature and maximum light, and expansion would proceed to the point of greatest cooling and therefore dimming, a quarter-period *before* the dimming is actually to be seen, but again at a turning point of no velocity.

One hopeful attack on the problem has been that of Martin Schwarzschild in 1938; instead of regarding the whole star as pulsating, in analogy to the elastic vibrations or sound waves of common experiment, for instance the 'stationary' pattern of water beating against a reflecting wall, he investigated how such stationary waves in a star's interior could give way to progressively advancing waves as the outer layers are reached. Rosseland of Oslo, whose Darwin memorial lecture to the Royal Astronomical Society in 1943 is as brilliant and clear a dissection of the Cepheid problem as any ever published, classified the ways by which luminosity could have come so nearly into phase with velocity of gases instead of with radius; the work fits on to Schwarzschild's, because it is also a classification of how waves in the stellar interior could develop the character of standing patterns altered into progressive sequences of wave motion. The former character is usually due to reflection at a boundary without loss of energy or absorption; the standing waves on a vibrating rope fixed at both ends, or on a vessel of water agitated by some periodic impulse, are all due to forward wave trains which combine with those returning reflected from the boundary so that the pattern is a rising and falling *in situ*, instead of any travel of waves from end to end. A star's boundary might be expected to act as a reflecting wall; so the existence of travelling instead of stationary waves in its atmosphere must imply that somewhere the waves are deprived of their energy. Rosseland employs the analogy of energy dissipation by a shelving beach, which prevents the reflected wave from building the stationary pattern common in front of a steep wall; he shows that the phase lag between luminosity and expansion in a Cepheid star must imply some such dissipation, if it is to be associated with travelling waves according to Schwarzschild.

The most acute problem is therefore the source of energy dissipation. To this, Eddington made a contribution which has been historic, but may not be final.

EDDINGTON'S 'VALVE' FOR THE PHASING OF CEPHEID PULSATION

In 1941 Eddington, who had made a precise mathematical argument, a generation earlier, out of Shapley's first suggestion that Cepheids are pulsating bodies, attacked the problem of trapping energy in a Cepheid. He brought reasons for peculiar behaviour of the quantity γ, the ratio of specific heats, at some depth below a Cepheid's surface. γ had been the criterion of stability (Chapter VI of Part I), but in 1924 Eddington's closest approach was the statement 'low value of γ favours the setting up of pulsation'. Based upon a classical research of Fowler and Guggenheim in 1925, which showed γ lowered in a stellar region where the degree of ionisation varies greatly over a small depth, Eddington considered hydrogen whose fractional ionisation (on the Saha theory, Chapter II) will rise from 0·04 to 0·94 as we descend from a level at 9000° to one at 15,000° under a star's surface. 90 per cent of the ionisation of hydrogen occurs between these levels. Unsöld has exploited this partial ionisation zone, showing it to be the seat of vast upward and downward convection currents in the stellar gases; but Eddington in 1941 found employment for the expenditure of energy used up in converting ionisation from 0·04 to 0·94, suggesting this as a kind of 'valve' which may periodically trap an accumulation of the star's energy. He considers the hydrogen zone as a 'sink' into which energy disappears, and it is possible that he correctly claimed this was sufficient to retard the change of luminosity to fit the motions of the gases rather than their position. Eddington even finds grounds in this theory for the Period-Luminosity law, otherwise an intractable feature of Cepheids.

Neither Period-Luminosity law nor Phase-Lag can be regarded as solved problems, in particular when we recognise that Cepheids are only one class of pulsating star. For example, the red giants which pulsate with much longer periods, up to hundreds of days, exhibit a quite different phase law from that of the faster Cepheids, being brightest at fastest fall of the gases instead of at fastest up-rush of expansion as measured by the dark lines, although their occasional bright lines are nearer to the Cepheid habit. The importance of Schwarzschild's analysis, into travelling and standing waves, is that Scott has successfully applied much of the same idea to the long-period pulsations of the cool red star Mira Ceti.

ROSSELAND AND MULTIPLE PULSATIONS IN A STAR

Since Eddington's pioneer work of 1917, Cepheids have generally been treated as simple harmonic oscillators, and one of the greatest services of Rosseland's lecture in 1943, to which we have referred, was to call attention to multiple periods which may distort the actual motion and the light-curve into something more like actuality. It had been an over-simplification to regard the star as a distending and shrinking balloon, whose radial motions were not much more complex than the movements of a pendulum in its plane. Rosseland points out that even the simplest of pendulums, if permitted to swing over large amplitudes, increases its periodic time, and its vibrations cease to be representable by simple sine curves. Now the velocity of a Cepheid's gases, plotted against time (Fig. 7 of Part I) is certainly not a simple sine curve, alternately steep and gradual in its rate of change; Rosseland shows how length of periodic time can be correlated with distortion of a velocity-curve, and that the latter may explain some outstanding discrepancies between observed pulsation times and those predicted from the star's density.

A distorted curve such as suggested by Rosseland would be analysable into multiple periods, somewhat as a sound wave is analysed into its harmonics which control the tone quality of voice or musical instrument; it is therefore of considerable importance that 'overtones' in pulsating stars have in fact been discovered. The Dutch workers Woltjer and Kluyver have both studied the possibilities of harmonics or overtones in theory, and these have even been found in practice; the pulsating star R R Lyrae shows two distinct periods, a recurrence after every 0·57 days and a secondary recurrence after every 38·2 days. The latter seems analogous to the way the 'beat' between two tuning forks recurs as a slow waxing and waning of the combined note from two adjacent frequencies. Fath in 1940 detected in the star δ Scuti three distinct periods superposed, 0·19 days, 0·157 days, and 0·095 days.

GAMOW AND GAPOSCHKIN AND THE ATOMIC REACTIONS MAINTAINING A CEPHEID

None of the researches we have mentioned, for explaining the shape and phase of the luminosity and velocity curves of Cepheids, gives any suggestion as to the atomic mechanism at the star's centre; might this differ in Cepheids from that which maintains the *steady* stars? A pulsating star might be actuated by fluctuations of its central energy generator; whereas the researches we have been considering are not incompatible with a *steady* central source of energy, broken up by absorptions and reflections during its travel to the outer boundary of the star so that the ultimate light emission waxes and wanes periodically. It must be remembered that only in the *steady* stars is the fundamental law of p. 52 obeyed, namely that at any instant the energy emitted as light and heat exactly equals the energy generated internally; though obviously unless all irregularities here are speedily counteracted the star will either die into darkness or burst into excessive and perhaps explosive luminosity.

Whether the atomic sources of energy within a star can themselves fluctuate so rapidly as to maintain pulsation, is an unsolved problem, some preliminaries to which have been worked out by the present author in a paper of 1947. But that the atomic liberation, whether steady or fluctuating, does differ from that of the sun or hotter stars, has been discussed by Gamow, Greenfield, and Dr and Mrs Gaposchkin at Harvard. Of the nuclear reactions which we listed in Chapter III, those of the Bethe Carbon-Nitrogen cycle were most probable at temperatures between 15 and 25 million degrees, while those involving Lithium, Boron, Beryllium as 'fuel' could occur at temperatures not greatly exceeding one million degrees. Now *if* the Cepheids and the red giants are reliably representable by the mathematical device of polytrope models, mentioned in Chapter VI of Part I and to be discussed below, the central cores of many pulsating stars are far too cool for Bethe's cycle, and can only maintain stellar 'life', i.e. luminosity, by the Li, B, Be reactions, or alternatively by gravitational contraction. The latter offers only a very brief life of any considerable brightness, as seen in our argument about the sun: have we chanced to live and study astronomy just in the short life-time of these stars? Would our remote ancestors or descendants be robbed by Nature of that remarkable mystery of Mira the 'wonderful'? Even Lithium and the other light elements will not last the star long, and there is some evidence in Greenfield's work that the cooler stars must be 'living upon' that fuel. The possibility may account for why those elements seem so rare in the sun, which may have 'burnt them up' in a giant youth.

An alternative is to regard some of the cooler stars as not correctly representable by the polytrope models; however shrouded in a vast cool blanket of surrounding gas, whose luminosity is all that we see, they may possess intensely hot centres in which the Bethe reactions proceed as merrily as in the sun. We refer to this problem in discussing the stellar structure known as the 'shell-source', at the end of the book: the topic was briefly outlined in Chapter VI of Part I.

In the meantime the Gaposchkins at Harvard even went so far as to associate each of those specific reactions with a definite group of Cepheids, making contact with Gamow's suggestions assigning the Deuterium reaction to the long-period variable stars, with others as follows.

${}^{9}_{4}\mathrm{Be} + {}^{1}_{1}\mathrm{H} = {}^{6}_{3}\mathrm{Li} + {}^{4}_{2}\mathrm{He}$ (classical Cepheids of medium period)

${}^{10}_{5}\mathrm{B} + {}^{1}_{1}\mathrm{H} = {}^{11}_{6}\mathrm{C}$ (short period Cepheids)

${}^{2}_{1}\mathrm{H} + {}^{1}_{1}\mathrm{H} = {}^{3}_{2}\mathrm{He}$

${}^{7}_{3}\mathrm{Li} + {}^{1}_{1}\mathrm{H} = 2\,{}^{4}_{2}\mathrm{He}$

${}^{11}_{5}\mathrm{B} + {}^{1}_{1}\mathrm{H} = 3\,{}^{4}_{2}\mathrm{He}$

${}^{6}_{3}\mathrm{Li} + {}^{1}_{1}\mathrm{H} = {}^{4}_{2}\mathrm{He} + {}^{3}_{2}\mathrm{He}$

These may be regarded in the light of our list from Bethe, Chapter III. It is conceivable that stars evolve by burning up each element at the appropriate temperature, and then shrinking until the next reaction in turn becomes possible.

Until the question is solved of whether pulsating stars conform to the polytrope model, and thus have central temperatures which we can know with confidence, the success of these important researches is in abeyance.

AMOUNTS OF ENERGY LOST IN NOVA AND SUPERNOVA OUTBREAK

We have in Part I described the photometric sequence of a Nova, and of the far fewer and more distant explosions beyond our own range of stars which have been called Supernovae. The sudden brightening even of an 'ordinary' Nova may be ten-thousand-fold, and there occurs within the compass of its few weeks of violent life a fantastic sequence of spectra, by which in rapid succession the lines of extremely differing stellar types are imitated and mingled. Recollecting our brief survey of a few of the explanations which have been offered for these catastrophes, some can at once be dismissed, because we are now not quite ignorant of the quantities in Mass and Energy which must be involved. A stellar collision might well dissipate most of the participants' energy and material, but now that the densities of a star's outer layers are recognised to lie within a fairly narrow range, and so are the velocities with which those layers are shot off during the outbreak, it becomes feasible to state not only the total loss in excess radiated energy but also the total amount of material blown away; it turns out that the phenomena would be adequately accounted for by a star losing less than a thousandth of its 'capital' and reverting to a status still stellar and possibly normal again. We have in support of this, one or two instances of a recurrence of outbreak in the same star.

Gaposchkin of Harvard, by scrutinising the dozen most recent and therefore most accurately measured of the 120 Novae known to have occurred in our Milky Way, has established that the total radiation thrown off during an outbreak is within remarkably restricted range, between 3×10^{44} and 4×10^{45} ergs. The amount in the first 54 days of the outbreak ranges from 2 to 10×10^{44} ergs. The sun radiates at $3{\cdot}8 \times 10^{33}$ ergs per sec., or in the same 54 days of a Nova's acute stage a total of 2×10^{40} ergs. So the mean brightness of a Nova during two months of greatest activity is from 10,000 times to 50,000 times that of the sun—and much brighter at its brief maximum.

It is notable that some pulsating stars, at the maximum phase of their brightness, are emitting at a rate far exceeding the sun's and in fact nearer to the rate of a Nova. R R Lyrae, a 'hot' Cepheid, at greatest luminosity emits $0{\cdot}5 \times 10^{36}$ ergs per sec. in excess of its minimum light; η Aquilae, a 'cool' Cepheid, at $1{\cdot}7 \times 10^{36}$ in excess of its minimum, while Mira Ceti the famous red variable emits at 18×10^{36} compared with a Nova's 100×10^{36}: it is obvious that size of surface is a dominant feature, and the vast expansion of the Nova at its maximum certainly contributes as much as temperature to the colossal brilliance. Comparing these figures, it becomes evident that periodic instability goes with a brightness implying emissions of energy very high on the stellar scale, and the Nova only concentrates into its brief single fever a heat which a pulsating star might emit in the course of some dozen oscillations under restriction and the control of some surviving stability. The most striking feature of the Nova is not so much its output but its extreme faintness before and after, whereas the pulsating stars are very bright even at the minimum. Emphasising again our policy of connecting all unsteady stars, catastrophic or periodic, it is interesting to quote Gaposchkin that the light-curve of the irregular variable star S S Cygni (shown below, Fig. 19) implies a loss of 6×10^{38} ergs in one cycle of outbreak and recovery.

The material ejected during a Nova outbreak is probably only a millionth or hundred-thousandth of the star's original mass, a quantity obviously not exactly calculable because we know so little of the pre-Nova state in its extreme faintness. But it is certain that the outbreak is *not* the disruption of an entire star, as first thought. On the other hand, the supernovae are objects whose temporary luminosity seems comparable with that of an entire Galaxy, and which in Part I we stated as occurring exclusively in the external systems seen as the most distant Spiral Nebulae. We mentioned as possible exception the Crab Nebula, if indeed that curious patch of spreading light is the wreck of the great Nova seen by the Chinese in A.D. 1054. The luminosity of a Supernova must be about a hundred thousand times that of the ordinary Nova of our Galaxy, and in these rarest of catastrophes may indeed be realised the primitive notion of a star totally disrupted. Scaling up the loss of material, comparably with the luminosity scale, the factor of 10^5 would imply that the Supernova scatters to the sky a tenth or a half of its original mass. The process and the nature of the remnants have been discussed by Hoyle and others, after the very patient and thorough photographic search over many thousand plates by Baade and other Pacific Coast observers; but with such distant objects it is certain that the precision with which we study our Galactic Novae will be inevitably lacking.

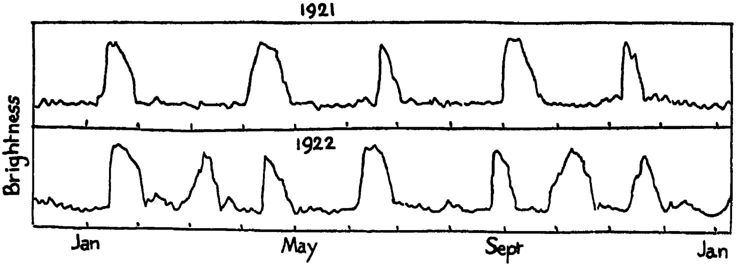

FIG. 19.

Peculiar stars typified by S S Cygni develop a mild form of Nova outbreak several times a year, too irregularly to be classified among the periodically repeating variable stars. The clue to catastrophic Novae may lie in these less violent and recurrent outbursts. (Diagram from Campbell of Lick Observatory, reproduced in the *Handbuch der Astrophysik.*)

SLOW AND FAST NOVAE AND THEIR EJECTED SHELLS

One useful attempt at classification of Novae has described them as (*a*) Fast, or (*b*) Slow. The total energy of each comes within the range we quoted, but type (*a*) fall from their peak maximum in a few days or a week or so, whereas type (*b*) remain near maximum for a month or two. The subsequent behaviour shows an even clearer distinction, the Fast following the course which we illustrated in Part I (Fig. 9), a roughly exponential decay curve upon which are superposed a number of nearly periodic minor oscillations: it almost looks as if a pulsation with periodicity of few days is an accompanying symptom of the Fast Nova's recovery. By contrast, the Slow Novae exhibit less regular minor oscillation, or even none at all; but a few months after maximum their far slower decline suddenly breaks down into a deep depression of the light-curve (Fig. 20). After this almost total dimming, there is another recovery of luminosity, in which earlier observers suspected a second outbreak like the original explosion; but examination of the curves of the few clear examples (1891, 1934, 1942) reveals recovery only towards a line which appears to be a continuation

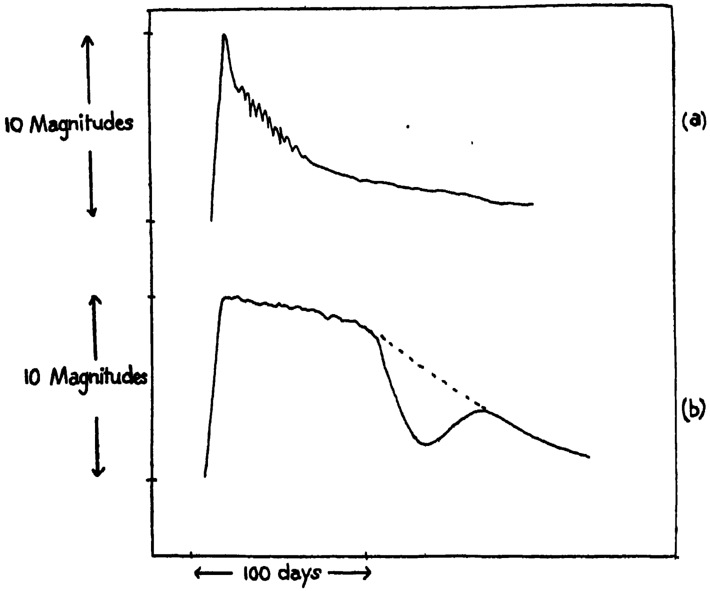

FIG. 20.

Besides the 'Fast Novae' which we illustrated previously, a few (*b*) tend to remain near their very high luminosity for weeks or months instead of only a few days as in (*a*). These 'Slow Novae' have also exhibited a unique sudden fall and recovery, breaking the gradual descent of their later stages. It seems possible that a cloud of unstable chemical composition forms and disintegrates between the shells of ejected gases, its absorbing properties causing the temporary dimming.

of the original fall. This has led Stratton and others to suggest that the 'deep gap' in the light-curve of Slow Novae (especially Nova Herculis of 1934) must imply the transient growth and dissipation of an obscuring compound in the ejected gases.

If we could indeed decide what is the composition of such an obscuration, as a cloud forming between the successive ejected shells of gas, we would probably make acquaintance with a unique state of matter—extremely tenuous gases shot through by beams of atomic particles at extremely high speeds, and the seat of unstable ionic compounds startling to our terrestrial chemistry.

The sequence of spectra is in every case bewildering in these brief weeks, not merely the successive imitating of spectra from one steady species of star to another adjoining it in the usual classes, but often a juxtaposition or superposition of extreme incompatibilities. For example, soon after the Nova maximum, the spectra of the highest temperature types appear close to the band spectra of Carbon compounds only otherwise seen in the coolest of steady stars: how can a star's atmosphere be at 30,000 degrees one day and 3,000 the next? But of course the 'atmosphere' pictured in the detailed researches upon steady stars is an unsafe guide to Novae: the vast Doppler effects, changing rapidly from 500 to 1500 kilometres per second, imply successive jets of erupting gases. These are driven from the star during the few days of its explosive phase, and settle into expanding concentric shells during the succeeding months. They may accumulate into the halo which becomes visible as a Planetary Nebula, and in the intervening years over which modern science has been able to follow a few post-novae there have actually become detectable detached patches of gaseous luminosity. Photographs of the recent changes in brightness *around* the sites of Nova Persei (of 1901), Nova Aquilae (of 1918), and Nova Pictoris (of 1925) are very convincing; we mentioned before the possibility that the Crab Nebula is an example of these ejected gases, traceable back to about 900 years ago from the present rate of motion, and conceivably originating in the explosion of A.D. 1054.

If we can elucidate these complex spectra, we may discover much that the chemist can only guess, the primitive association of atom with atom in fast-moving and immensely rarefied gases, to which our laboratory experiments are the clumsiest of approaches; CH, C_2, CN, are radicals difficult to isolate on earth, but they occur in stable equilibrium in the steady red stars, and we may discover how they originate if we can learn the secrets of the regions inside a Nova's expanding shells.

In the Chapter II on spectrophotometry, some account was given of the 'line-contours' of a complex spectral line broadened by the spreading speeds in a Doppler effect. When a Nova's gases are being ejected at 1000 km. per sec. the spread of the spectral line is very broad, quite unlike the same line of the same atoms in a steady star. Superposed on the broad *bright* line fluorescing under the ultra-violet rays of the exploding star, there is often a dark line bordering the bright at its violet edge. The correlation of this with what we have described as the Nova's environment, may be seen from Fig. 21, which is of a kind of

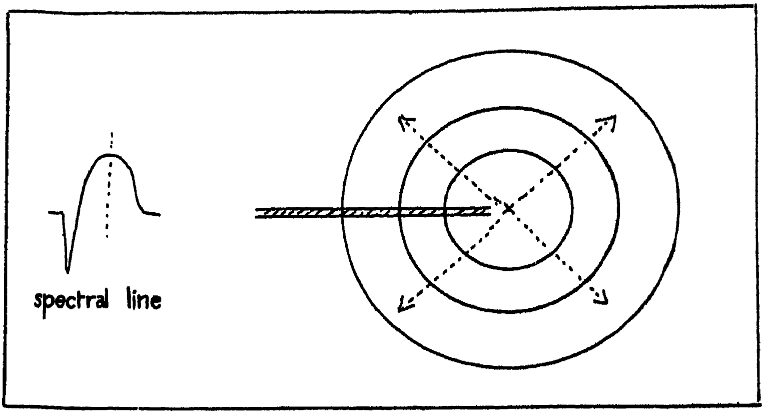

FIG. 21.

Several workers have pointed out that when a Nova or erupting star ejects sufficient gas to spread over a sphere large compared with the star's own size, the fluorescence of the expanding shell will give rise to an emission spectrum broadened by the Doppler effect as the material rushes in all directions away from and towards the observer, (dotted lines). At the violet edge of this broadened band is a sharp cut in the spectrum where gases moving only along the observer's line of sight (shaded) absorb the star's own radiation. The complex contour shown in the accompanying spectral line shows both these features.

diagram utilised by many writers. Here bright emission spectra from the further and receding portions of the ejected gas shells will be displaced to the red by the Doppler shift, while similar bright lines from the nearer and approaching portions of the shells are displaced to the violet, thus giving rise to the very broadened bright 'line contour', seen in the later development of a Nova spectrum. Meanwhile the shaded portion of the picture may show dark absorption spectra of the same materials, as it lies in our line of sight to the original star itself. This absorption line will have the full Doppler shift to violet, as it has no component in any other orientation, and will cut a narrow channel in the spread of the bright line—the whole contour referring to one substance only, which in the laboratory or in a quiet star would provide the single wave-length of the original undisplaced spectral line.

It is significant that the Wolf-Rayet spectra also exhibit these very broadened bright lines; this is the evidence that, although not undergoing the violent brief adventure of the Nova, they are extruding their atmospheres with steadier but persistent vigour. It is significant that some Wolf-Rayet stars lie at the centre of Planetary Nebulae—the latter may be the product of quiet accumulation of extruded gas, as well as sometimes the repository of a Nova's sudden ejecta. But Novae, Wolf-Rayet, and Planetaries confront us with too great a complexity of facts for anyone today to be sure of the exact nature of their interconnection which undoubtedly exists.

'FORBIDDEN' SPECTRA AT THE NEBULAR STAGE

The final spectrum of a Nova, months after the initial outburst, commonly contains the set of lines associated with any interstellar region occupied by nebulosity. Those great tracts of gas, fluorescing under the impact of radiations from stars embedded in them, shine with a few distinct wave-lengths only, lacking the colour background which underlies a stellar spectrum. The typical Nebular lines, seen over widespread gas clouds often with little obvious stellar connection, are the two green lines at 4959 and 5007, and the ultra-violet 3726 and 3729;

these defied the efforts of the earlier spectroscopists to reproduce them in any laboratory experiment, and their chemical anonymity was recognised by the label 'Nebulium'. But since 1910 atomic physics has been aware that there is no missing light element, and that the lines must denote not a new substance with no terrestrial counterpart but some unfamiliar condition of one of our familiar elements. Bowen of California, in 1927, worked out a convincing argument proving that the violet Nebular pair of lines are emitted by singly-ionised oxygen atoms, and the green lines by doubly ionised oxygen, the remaining unidentified lines denoting oxygen and nitrogen in even more highly ionised states. The mysterious lines in the spectrum of the Aurora Borealis, the earth's outer atmosphere stimulated by impact of ions and radiation from solar eruption, have likewise been identified as ionised constituents of the familiar air, and in the last few years even 'Coronium', as name given to the unknown spectrum of the sun's outermost envelope, has been identified by Edlén of Stockholm as being due to multiple ionisation of Iron and other metals. Some of these lines, notably the nebular spectrum characterising the late stage of a Nova, are referred to as 'forbidden' spectra—not merely because they do not occur in the usual laboratory experiments but because the reason for that failure is now evident, as follows.

All spectral lines denote the movement of a certain electron in the structure of energy levels which constitute atom or ion or molecule. Some of the transitions between one energy level and another have small intrinsic probability, but can occur if the atom rests long enough free from collisions; they only happen if their natural delay is not cut short by encountering a disturbing neighbour. In the laboratory, the residual gas pressure even in our best vacuum 'forbids' those particular transitions, because collisions are too frequent, and the spectrum omits those lines. Even if we were to succeed in obtaining better vacua, the line would be faint, and indeed the millions of miles of over-rarefied gas in the cloud surrounding the star is the only situation in Nature fulfilling both requirements: long freedom from collision with neighbouring atoms, and yet a great length of gas in the observer's line of sight, together permit us to see the otherwise 'forbidden' spectra.

Forbidden lines therefore reinforce in particular detail the conclusions stated in Chapter III of Part I, that stellar gases are very hot but very rarefied; the Novae and Planetary Nebulae show that luminosity can actually come from a state of matter more rarefied than the best vacuum of our laboratory technique, in the halo of ejected material surrounding at great distance the star's original structure.

THE LIGHT-CURVES OF R CORONAE BOREALIS STARS AND S S CYGNI STARS

Our argument has linked Novae, Planetary Nebulae, and Wolf-Rayet stars, and we have mentioned in Part I the work of Struve, Swings, Merrill, and others on the quieter 'enveloped' stars whose atmospheres extend into a halo of moving gases. The nature of the link between all these instances of vast eruption is as yet very uncertain; we do not know just how stability breaks down into pulsation of Cepheids, steady

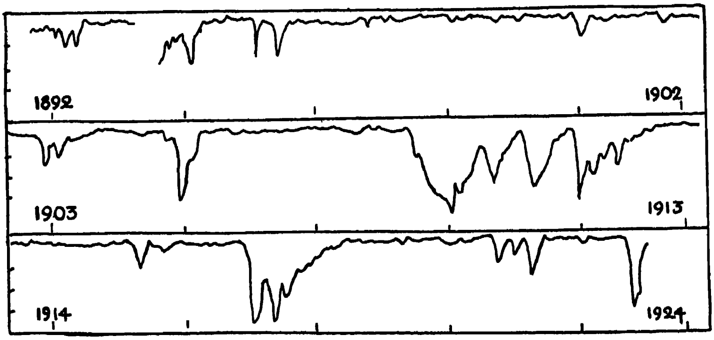

FIG. 22.

A class of peculiar variable stars inverting the habit of Fig. 19. R Coronae Borealis is steady for most of its time, with sudden brief dimmings at irregular intervals, down to a hundredth or a thousandth of its normal luminosity, with considerable changes in spectrum. The irregular time of repeat, as well as the extreme amount of the fading, shows that no second or eclipsing body is involved. This diagram, covering thirty years, is from Harvard Observatory.

evaporation of Wolf-Rayet type, or violent explosion of the Nova. But there are clues in intermediate phenomena. Among many groupings of irregularly fluctuating stars, those typified by S S Cygni, U Geminorum, and T Pyxidis, almost deserve the name of 'mild novae' through their repeated but irregular flashes into high luminosity which never lasts. On the other hand, stars like R Coronae Borealis appear as if inverted Novae; after a steady life for years the star suddenly dims by many hundreds-fold or thousands-fold in a few weeks, recovering more slowly in months (Fig. 22). If this enormous *fall* in luminosity, comparable with an inverted picture of a Nova's *rise*, has to be explained, like the dip in the recovery curve of 'slow' Novae, as a transient chemical association of highly opaque properties, the problem of what goes on in circumstellar gases becomes more acute. These intermediate freak instances may in the end be the clue to the simpler and more extreme.

LIMITS TO THE POSSIBLE INDEX

The problem of steady or catastrophic evolution, or the conditions under which a star's structure has stability, arises inevitably and insistently out of the observational data of the preceding chapters, where stars which are steady or which pulsate or erupt or even explode have been discussed as collected and analysed fact. For interpretation of those facts, attempting to settle what is the life history of a star, theory has to begin from 'models' or sets of supposed properties and laws which may be tested. The testing of the models implies finding whether known physical processes exemplified in any particular model would allow us to deduce phenomena which do actually occur. In Part I Chapter VI the particular type of model called 'polytrope' was introduced, without any quantitative treatment; it represents the idealised set of concentric spheres into which might be dissected the successive temperatures, densities, gas pressures, etc., which must obviously vary greatly along a star's radius from boundary to centre.

We proceed now to develop the notion of 'polytrope index', briefly mentioned in Part I as a means of discriminating between these models, and we shall relate to this index n the quantity 'gamma' (γ) or ratio of specific heats, which is a criterion of a star's stability. We shall be adapting, and amplifying, by filling in and explaining the steps, the standard treatments by Eddington, Milne, Chandrasekhar, and others of the great pioneers, Eddington's *Internal Constitution of the Stars* (1926) being the most readable, although superseded in modern developments by Chandrasekhar's *Stellar Structure* (1939) and more recent papers in the *Astrophysical Journal*, etc., from which we shall summarise the later modifications.

At any point in a steady star, the material is in equilibrium under (*a*) gravitational pull towards the star's centre, (*b*) pressure, which may include the mechanical pressure of radiation as well as the pressure due to motion of the atoms, molecules, or ions, of the gas. The 'adiabatic' law, according to which the quantities defining the state of the gas may vary without loss or gain of heat, expresses a particular form of the interdependence of pressure p, volume v, density ρ, and temperature T, and may be written in any of the following ways:

$$pv^{\gamma}=\text{constant};\ Tv^{\gamma-1}=\text{constant};\ p=\text{constant}\times\rho^{\gamma};$$
$$T\rho^{1-\gamma}=\text{constant}.$$

γ is the ratio between specific heat at constant pressure and at constant volume, though the separate heats lose their significance in the applications.

Polytrope models, of the distribution of physical properties along the star's radius, are usefully classified by stating for each a way in which pressure varies with density, and a more convenient variable than γ is the 'index n', such that,

$$p=\kappa\rho^{\frac{n+1}{n}}$$

The two kinds of specification are connected by the relation,

$$n=\frac{1}{\gamma-1},$$

γ for ordinary gases can vary from 1·333 to 1·667, which implies n from 3 to 1·5. Leaving until later the significance of these limits, it can be seen from our gravitational argument of Chapter III, Part II p. 146, that since,

$$q=\frac{3}{5-n},$$

n must always be less than 5, since otherwise the potential energy of the star would become infinite: that extreme would be reached with a depression of γ to 1·2, and it may be noticed that large values of n would imply reduction of γ towards unity. The temperature equation quoted above shows that the star would then tend to have uniform temperature throughout its body, in contrast to all evidence and to the necessity that any star can only continue to shine by reason of an internal energy source.

The greatest use of the index *n* is that it allows the density at the centre of a polytrope model—and hence of many stars—to be stated quite definitely in terms of the mean density obtainable from observational data, if once the index *n* is decided. The calculations began with Lane in 1870 and were perfected by Emden in 1907, a history which has been recorded in detail by their great successor Chandrasekhar. We quote some instances from the latter's complete table.

n:	Ratio of density at star's centre to mean density:
0	1·0
1	3·29
1·5	6·00
2	11·40
3	54·36
4	622·
5	Infinity.

The first value, zero, would imply a star of uniform density, impossible to any mobile gas under gravity; the table also shows we can reject values exceeding 5, as well as for the previous reason. The range could be further narrowed to between 3 and 1½ if we demand 'ordinary' materials of normal specific heats; this is doubtful in certain stages of ionisation discussed below, but we shall find that the need for a star's stability introduces separate restriction upon the value of *n*. Within the range mentioned, the table shows that a star's central density is between 6 and 54 times its mean density. Within the range, and around its limits, an effective value of γ can be depressed by radiation pressure or incomplete ionisation, below the values fixed by molecular structure in terrestrial gases; such values will be superseded by dissociation into atoms for most stellar material. For example, Chandrasekhar develops detailed discussion in which the outer part of a star may obey $n=3{\cdot}25$ and an inner core obey $n=1{\cdot}5$. Marginal values of γ and *n* may accompany the breakdown of stellar stability into convective flow of gases, and the facts recorded in the earlier chapters all insist that such breakdown is very common.

EXAMPLE OF PRESSURE AT THE CENTRE OF A STAR

We shall here trace, by simplifying a classical exposition by Eddington, and inserting some explanatory steps, the argument underlying the estimate of the central pressure in the star Capella, quoted without support in Part I. The gravitational acceleration g at any point r from the centre of a star, if M is the mass within r, is,

$$g = GM/r^2 = -d\phi/dr$$

where G is the universal gravitational constant and ϕ is the gravitational potential whose rate of change with r is thus an expression for g. If P is the pressure, including that of gas and of radiation, the condition of equilibrium conferring increase of pressure with depth in the star can also make use of the potential, so that,

$$dP = -g\rho\, dr = \rho\, d\phi$$

The relation between pressure and density in a polytrope model has the adiabatic form which we have already described,

$$P = \kappa\rho^{\gamma}$$

Differentiating this,

$$dP = \gamma\kappa\rho^{\gamma-1}d\rho$$

Our expression above, for dP in terms of potential enables this to give,

$$d\phi = \gamma\kappa\rho^{\gamma-2}d\rho$$

which, integrated, gives,

$$\frac{\gamma}{\gamma - 1}\kappa\rho^{\gamma-1} = \phi + \text{constant}$$

The constant is zero at the boundary of the star. If we use the index n instead of γ and,

$$\gamma = 1 + 1/n \quad n = 1/\gamma - 1$$

we can restate the pressure and density law,

$$P = \kappa\rho^{\gamma} = \frac{\rho\phi}{n+1}$$

and the integral giving ϕ,

$$\rho^{\gamma-1}=\frac{\phi}{\kappa\left(\frac{\gamma}{\gamma-1}\right)}=\frac{\phi}{(n+1)\kappa}$$

$$\rho=\left\{\frac{\phi}{(n+1)\kappa}\right\}^{n}$$

This gives the density in terms of the gravitational potential, for any model star of index n.

The mathematical problem of the polytropes is then to make and solve a differential equation to find ϕ at any value of r, i.e. at any point within the star's interior. The method by which Emden was able to tabulate the properties as they alter throughout any star's body, in terms of n, has been utilised and extended by all workers since, notably Eddington, Chandrasekhar and their pupils; it is best to convert the argument from a relation of ϕ to r, into a relation of u to z; these new quantities are proportional to ϕ and r but allow use of a standard mathematical form with known solutions. The new quantities are such that,

$$\phi=\phi_0\,u \quad \text{and} \quad r=z/\alpha\phi_0{}^{\frac{1}{2}(n-1)}$$

ϕ_0 is the potential at the star's centre, and α comes from Poisson's gravitational equation, so that,

$$\alpha=\sqrt{\frac{4\pi G}{\{(n+1)\kappa\}^{n}}}$$

Any radius, and the mass contained within any given radius, must next be written in terms of the new quantities u and z. The boundary of the star is where $u=0$, so if,

$$R=(r)_{u=0} \qquad GM=\left(-r^{2}\frac{d\phi}{dr}\right)_{u=0}$$

Let,

$$R'=(z)_{u=0} \qquad M'=\left(-z^{2}\frac{du}{dz}\right)_{u=0}$$

The values of these R' and M' are to be found in the Tables based on Emden's calculations. Our definitions connecting ϕ with u and r with z then give,

$$\frac{R}{R'}=\frac{1}{\alpha\phi_0^{\frac{1}{2}(n-1)}} \quad\text{and}\quad \frac{GM}{M'}=\frac{1}{\alpha\phi_0^{\frac{1}{2}(n-3)}}$$

$$\frac{GM}{M'}\times\frac{R'}{R}=\phi_0 \quad\text{and}\quad \left(\frac{GM}{M'}\right)^{n-1}\times\left(\frac{R'}{R}\right)^{n-3}=\frac{1}{\alpha^2}=\frac{\{(n+1)\kappa\}^n}{4\pi G}$$

We can apply these equations to work out the example given without proof in Part I, the star Capella; *if* this star is representable as a polytrope of index $n=3$, then,

$$n+1=4 \quad\text{and}\quad P_0=\frac{\rho_0\,\phi_0}{n+1}=\tfrac{1}{4}\rho_0\,\phi_0$$

the pressure and density and potential all having the particular values they possess at the star's centre. $n=3$ implies a γ just sufficient to be stable; values below 3 imply a safer star, but radiation pressure tends to keep γ low. From the astrophysical data, obtained by observation in the ways described in the earlier chapters, the mass and radius of this star are,

$$M=8\cdot3\times10^{33}\text{ grams,}\qquad R=9\cdot55\times10^{11}\text{ cm.}$$

Therefore the mean density, or mass divided by volume, is,

$$\rho_m=0\cdot00227\text{ grams per cm.}^3$$

From Emden's Table for $n=3$ which we quoted,

$$\rho_0/\rho_m=54\cdot36 \quad\text{Hence}\quad \rho_0=0\cdot123\text{ gm. per cm.}^3$$

From Emden's Tables also,

$$M'=2\cdot015 \quad R'=6\cdot901$$

so that,

$$\phi_0=\frac{GM}{M'}\times\frac{R'}{R}=1\cdot98\times10^{15}$$

$P_0=\frac{1}{4}\rho_0\phi_0=6\cdot11\times10^{13}$ dynes per cm.^{2}, or more than 10 million atmospheres.

TEMPERATURE AT THE CENTRE OF A STAR, ACCORDING TO MOLECULAR WEIGHT

We pointed out in Part I that molecular weight is not so indeterminate a quantity in a star as might be expected: we need not know very precisely the distribution of chemical elements, as these will mostly be very highly ionised. At the extreme, near the star's centre, all the electrons are removed from an atom, and there will be $z+1$ independent particles for each atom, where z is the atomic number or order of the element in the chemical Table, since z physically means the number of satellite electrons in any atom. For an element of atomic weight A, the molecular weight μ in such extreme stage of ionisation will therefore be,

$$\mu = A/(z+1).$$

Taking a few scattered examples:

	$z=$	$A=$	$\mu=$
Hydrogen	1	1	0·5
Helium	2	4	1·33
Oxygen	8	16	1·78
Carbon	20	40	1·91
Iron	26	56	2·07
Gold	79	197	2·46

It is evident that an assumed molecular weight $\mu=2$ will not be far in error for most material at the centre of a star, even though the heavier elements will retain a few of their electrons of which such a Table takes no account: actually μ will vary most with the fraction of hydrogen transmuted into helium by the reactions of Chapter III. Using this μ, the well-known 'law of perfect gases' to which terrestrial gases approximate,

$$PV = \text{gas constant} \times \text{Temperature}$$

becomes,

$$P = \frac{8{\cdot}26 \times 10^{7}}{\mu}\,\rho T$$

If μ is stated as multiple of the value for hydrogen,

$$P = \frac{1{\cdot}37 \times 10^{-16}}{H\mu}\,\rho T$$

where H is the mass of a hydrogen atom in grams.

Using now our previous expression for pressure in terms of potential and polytrope index,

$$P = \rho\phi/n+1$$

and combining this with the above gas law,

$$T = \mu\phi/\{(n+1) \times \text{gas constant}\}$$

This allows us to use the potential at the star's centre, already worked out for Capella, to find the temperature T_0 at the centre: inserting the values,

$$T_0 = \frac{2 \times 1{\cdot}98 \times 10^{15}}{4 \times 8{\cdot}26 \times 10^{7}} = 1{\cdot}20 \times 10^{7} \text{ degrees}$$

Although this is based on $\mu = 2$, the above Table indicates that the central temperature must still be several million degrees, whatever the chemical composition, even if the centre is occupied by differing proportions of hydrogen and helium.

These calculations afford reliable grounds for conviction that the temperatures and pressures at the centre of a star are essentially to be reckoned in millions of degrees and millions of atmospheres; they will vary (*a*) chiefly according to which polytrope model is nearest to actual fact, and (*b*) secondarily according to the extent of hydrogen conversion into helium, and (*c*) only slightly according to what other elements are present. It must be recollected that n, which we took as 3, can scarcely fall below $1\frac{1}{2}$, and we shall see later that only at cost of setting the star's gases into violent motion can it rise above 3. The restriction of the temperature argument to the 'ideal' gas, in the law $PV = KT$, occasioned the most serious of the early doubts, but in any highly ionised gas the reduced atomic size by liberation of electrons keeps the material a very 'perfect' gas even at these pressures. Of course in a White Dwarf star the law is not likely to hold—but neither is a White Dwarf a simple polytrope at all.

EDDINGTON'S STANDARD MODEL

In Part I, Chapter VI, (2) (*c*), we mentioned that the exceedingly useful connection between Mass of a star and its Absolute Luminosity has rational theoretical reasons enabling it to be predicted, as well as being an observational law discovered by plotting on a diagram those few instances where both M and L are known. To use the observational graph, inferring Mass when it happened that nearness of a star confers reliable knowledge of intrinsic brightness, or inferring L if M is known from a binary orbit, would be a gamble on the hypothesis that the very large number of unknown instances would behave like the few known cases, unless there were reason for expecting these to be exemplifying a general principle. This principle, worked out first by Eddington at the beginning of the 1920's, we will now argue step by step. We begin from the law of pressure increase inward along the radius of a gravitating sphere, already used here,

$$dP/dr = -g\rho$$

We add to this the relation describing the obstruction offered to radiation in emerging from the star's centre; this defines an absorption coefficient k such that along any distance x the intensity of radiation I_0 entering is reduced to I_1 emerging from a region,

$$I_1 = I_0 e^{-k\rho x}$$

where e is the usual base of natural logarithms. In becoming absorbed, the radiation H ergs per sec. gives up its momentum H/c to the absorbing material, where c is the velocity of light; this inevitable process of exchange of momentum between light and material constitutes the pressure of radiation, p_R, which although detectable with difficulty at laboratory temperatures becomes important at stellar temperatures. The reason is that radiation pressure increases (Stefan's law) with the rate T^4, whereas gas pressure p_G rises only proportionally to T alone. If gas pressure and radiation pressure contribute to the total opposing gravitation,

$$P = p_G + p_R$$

then since the second term arises from losses of momenta from radiation to the absorbing material,

$$\frac{dp_R}{dr} = -\frac{H}{c} k\rho$$

Combining this with the above relation of pressure to gravity,

$$dp_R = \frac{kH}{cg} dP$$

But H in any layer of the star comes from the internal energy sources, in a *steady* star equivalent to the luminosity L which it emits, so that,

$$H = L/4\pi r^2$$

The gravitational acceleration g, opposing the pressure, arises from Mass inside r and the gravitation constant G,

$$g = GM/r^2$$

so that,

$$\frac{H}{g} = \frac{1}{4\pi G} \frac{L}{M}$$

At any particular distance from the star's centre, say r,

$$L_r/M_r = \eta(L/M)$$

where the letters with no subscripts refer to the star's whole mass and the surface emission of light. η is evidently a way of expressing how energy sources are distributed within the star, and will increase from unity at the boundary, towards the centre. The previous expression for dp_R then becomes,

$$dp_R = \frac{L}{4\pi c GM} \eta k \, dP$$

Eddington's famous 'standard model', which simplified the vast complexity of the stellar structure problem, and made an approximation manageable and yet reliable, is the case in which,

$$\eta k = \text{constant throughout the star.}$$

In this idealised case, absorption coefficients may alter towards the star's centre, to balance increased energy generation, but the ratio of radiation pressure to gas pressure remains the same throughout the whole star. Modern research since Eddington, especially by Chandrasekhar and his school, has largely been occupied in finding what deviation from this convenient approximation is needed, now that we know something about how the star's energy is generated. In fact some of the most intriguing topics have become queries as to the relaxation of Eddington's restriction: but it must be remembered that without his original simplification of the 'standard model' no progress at all would have been possible.

RADIATION PRESSURE AND MASS IN THE STANDARD MODEL

A vital criterion for the existence of a star is the ratio of radiation pressure to gas pressure. We stated briefly in Part I how Eddington's treatment of this gave the first clue to why stellar masses do not vary as widely as their temperatures or densities or sizes. Let β be the fraction of total pressure contributed by gas pressure towards opposing gravitational contraction, and $(1 - \beta)$ the fraction contributed by radiation, so that,

$$p_R = (1 - \beta)\, P$$
$$p_G = \beta P$$

Then the integration of the previous equation connecting luminosity and radiation pressure gives,

$$L = \frac{4\pi c GM(1 - \beta)}{k_0}$$

if k_0 be written to cover the two factors in the constant product ηk. This equation exhibits the dependence of Luminosity upon

Mass, with also a proportionality taking account of the radiation pressure. In cases where the ideal gas law is obeyed, our previous statement of the usual gas property gives,

$$p_G = \frac{\text{gas constant}}{\mu}\,\rho T$$

and by Stefan's law for radiation intensities,

$$p_R = \tfrac{1}{3}\,aT^4$$

so that,

$$P = \frac{K\rho T}{\beta\mu} = \frac{1}{3}\,\frac{aT^4}{(1-\beta)}$$

using K for the gas constant.

We can eliminate T by dividing the second by the first of these expressions for P, and having thereby obtained T^3 and T, substituting this in the expression for P which involved T^4, and we discover that P is of the form,

$$P = \text{constant} \times \rho^{4/3}$$

where the constant is equal to,

$$\left\{\frac{3K^4(1-\beta)}{a\mu^4\beta^4}\right\}^{1/3}$$

Comparing these with the law which connects P with ρ in the polytrope model, it is seen that radiation itself is behaving like a gas whose value of γ is $\frac{4}{3}$, corresponding to a polytrope index $n = 3$. This allows us to return to the polytrope equation given previously, and treat the radiation as if it were one of the star's 'gases'. Quoting an earlier statement we made,

$$\left(\frac{GM}{M'}\right)^{n-1} \times \left(\frac{R'}{R}\right)^{n-3} = \frac{\{(n+1)\kappa\}^n}{4\pi G}$$

In the case of the radiation 'gas', since $n = 3$, $n - 3 = 0$ and $n + 1 = 4$, so that,

$$\left(\frac{GM}{M'}\right)^2 = \frac{(4\kappa)^3}{4\pi G} = \frac{4^3}{4\pi G} \times \frac{3K^4(1-\beta)}{a\mu^4\beta^4}$$

using the above value we have assigned to the constant κ.

From this we extract the quantities upon which radiation pressure depends, and discover that,

$$1-\beta=C\,M^2\mu^4\beta^4$$

where all the quantities in C are known, including the gravitation constant and Stefan's constant a, the gas constant K, and M' was 2·015 for a polytrope of $n=3$; in fact,

$$C=\frac{G^3\,4\pi a}{M'^2\times 192\times K^4}=7{\cdot}83\times 10^{-70}$$

It is convenient to write this in terms of the sun's mass (1·985 × 10^{33} gm.), so that any star's mass M is only some particular multiple of the sun's which is denoted in astronomical literature by the symbol ⊙. This makes,

$$1-\beta=0{\cdot}00309\left(\frac{M}{\odot}\right)^2\mu^4\beta^4$$

The important result is that the share demanded by radiation pressure in a star's life is calculable, and is seen to depend *only* upon the star's mass and mean molecular weight; the latter we have shown can vary only slightly.

This important equation stands with another which we derived before and quote in this connection,

$$L=4\pi c\,GM\,(1-\beta)/k_0$$

and the two together are the basis of much of our understanding of a star. It appears likely that the quantity k, the absorption coefficient included in k_0, will vary with temperature and with molecular weight so as to be proportional to,

$$\frac{\rho}{\mu T^{7/2}}$$

The growth of $(1-\beta)$ with M, the rising responsibility of radiation pressure for the more massive stars, can now be associated with observed bolometric magnitude m of a star through its luminosity which is fixed by the interdependence of $(1-\beta)$ and L and M. Eddington's pioneer Table, in which the data from Capella enable the scale of m to be adjusted so that one point is anchored in an actual case, shows how for a given temperature (here 5200°) at the star's surface, with an assumed molecular weight 2·11, stars of different mass would have different degrees of radiation pressure and exhibit different luminosities. The mass in the accompanying Table is shown as fraction or multiple of the sun's mass, so that at $M=1$, $m=4\cdot8$ which is the sun's Absolute magnitude.

$(1-\beta)$:	*Mass*	*Luminosity* (*magnitude*)
0·001	0·128	14·1
0·005	0·289	10·3
0·01	0·413	8·6
0·05	1·004	4·6
0·1	1·58	2·82
0·2	2·83	0·81
0·5	11·46	2·80
0·7	37·67	5·16
0·8	90·6	6·71

These figures may be compared with the Table in Part I, Chapter VI, where Eddington's picturesque and vivid account of the gas spheres which can actually *be* stars limited the mass to between 10^{32} and 10^{35}, or in the notation of this Table $\frac{1}{20}$ and 50; over this range, radiation pressure rises from a minute fraction to domination of the whole situation—a domination liable to disrupt the star if mass were higher still. The theoretical argument, which we have now carried through, brings this fundamental limitation into line with the observed Mass-Luminosity law, and the brightness which the heaviest stars do actually exhibit: the mass, and therefore the brightness of the star, is set between two limiting necessities—the star must be massive enough to shine visibly, but it must not be so massive that radiation pressure disrupts it. To Eddington is due the credit of first calculating rigorously why these limits arise from definitely known physics.

OTHER MODELS: POINT SOURCE AND SHELL SOURCE OF ENERGY

The first place at which even Eddington expected this argument to break down, was its dependence upon the law of ideal gases. This law was used in our linking of temperature, pressure, and molecular weight, above. At high pressures, even those readily attained in the laboratory, the ideal law does demonstrably begin to fail, and at a million atmospheres it seemed incredible that such a law could represent the facts. Jeans and others, indeed, began to talk of stellar interiors as more resembling the liquid than the gaseous state. That even dense stars could be fitted on to the Mass-Luminosity curve for ideal gases suggested that stellar material was unexpectedly simple and had not been unduly idealised. The reason, as we have hinted before, lies in the high degree of ionisation; so large a proportion of the electrons of each atomic structure have been detached at these temperatures, that there is a host of free particles for each atom, and the remnant or core is so reduced in size, that the material has lost much of its normal resistance to compression and possesses far more inter-collision space than even a moderately compressed terrestrial sample of gas. Giants and even ordinary Dwarfs possess nearly 'perfect' gas bodies, though the White Dwarf state cannot be expected to come under any such generalisation. In fact, it is the under-luminous (White Dwarfs) as well as the over-luminous (super-giants) and the extremely massive stars studied by Trumpler, which evade the simple relations between Mass, Luminosity, and radiation pressure, which seem to be the guiding principles in the Natural History of most ordinary stars in their maintenance of stable existence.

A breakdown of the other simplifying assumption is more serious: the 'standard model' comes under suspicion as soon as we know anything about the energy source at the star's centre. It is perhaps fortunate that everyone was very ignorant in that direction when Eddington first worked, or he might never have dared that simplifying assumption, and the subject might never have advanced as rapidly as it did. But it is high time to remove the undue simplification. Actually, though we used the standard model (ηk constant) of Eddington in our calculation, the results are of greater generality than the means adopted in their first proof; Chandrasekhar and others have shown that an M/L relation fitting the facts can be also derived for cases where ηk is no longer constant.

In all modern knowledge of the atomic transformations (Chapter III) which keep a star shining, it becomes very certain that these 'reactions' are extremely dependent upon temperature. We stated earlier that fears expecting stellar disruption from such temperature-dependence have been reassured by Cowling's calculations of stability, to which we return below. But even if not dangerous to a star's continued existence, these atomic reactions near the star's centre must increase enormously their onset around some temperature of a million or so degrees; the very steep gradient of temperature at such a transition zone makes it hard to believe in any constancy of the ratio between gas and radiation pressure, implied in the standard ηk constancy.

A first possibility of divergence from the standard model is to take account of this steep change in properties towards the centre, and as an extreme to suppose the whole energy-creating region located in a very small fraction of the star's volume near the centre; this is obviously to be called the 'point-source' model. We described, in Part I, Chapter VI, (3), how the attempt to build an evolutionary sequence on the gradual replacement of hydrogen by helium in the Bethe cycle must imply that a star's hydrogen is progressively 'burnt-up': this progressive exhaustion will occur in the central 'point-source' region more quickly than anywhere else in the star, so that the star's centre will become dead through lack of fuel. Since the surrounding regions, where helium production has been slow or negligible, will still possess plenty of hydrogen when the centre is exhausted, the seat of energy output must migrate outward to some zone still rich in hydrogen; such outwardly moving zone is called the 'shell-source' of energy, drifting slowly through the star as it in turn exhausts its fuel, and hollow inside. The deserted centre becomes isothermal or uniform in temperature, imitating the fantastic 'polytrope of infinite index' which was impossible to a purely gravitating star. We mentioned before the strange situation which has made this shell-source model worth investigating for Red Giants, and has induced Gamow to link the emergence of the shell at the star's surface with eruption of its outer layers. This particular attempt to recognise the incompleteness of any single polytrope to describe the course of evolution, is another aspect of the problem raised by the modern supposition that hydrogen supplies a star's energy: when the fuel is exhausted at one point, does the rest of the star replenish the supply before instability sets in?

Stability Criteria for a Star's Life

'GAMMA' FOR NON-DISSIPATION OF THE STAR

In Part I, Chapter VI, (2), it was stated that a star cannot remain stable unless,

$$\gamma > \tfrac{4}{3}$$

Eddington's and Biermann's theories were mentioned, in which they attempted to show that a zone of incomplete ionisation must lower the value of γ and may cause Cepheid pulsation or Nova eruption for that reason. We can now join that aspect of stellar stability on to the arguments for a star's energy which we developed in Part II. The relevant Sections in Part II are (*a*) in the notes for Chapter III on gravitation, and (*b*) for Chapter VI on polytrope index.

(*a*) Consider the fraction,

$$(3\gamma - 4) / 3(\gamma - 1)$$

which appeared in the expression for a star's energy of gravitational shrinkage. If γ exceeds $\tfrac{4}{3}$, 3γ exceeds 4, but if γ is less than $\tfrac{4}{3}$, 3γ is less than 4, the numerator in the above fraction becoming positive and negative respectively in the two instances. This reversal of sign affects the whole energy expression, so that if γ falls below $\tfrac{4}{3}$ we have a negative multiplier of the gravitational energy Ω in,

$$E = \frac{3\gamma - 4}{3(\gamma - 1)}\Omega \quad \text{where} \quad \Omega = -q\frac{GM^2}{R}$$

This double negative turns E the total energy into becoming positive. Only if γ exceeds $\tfrac{4}{3}$, so that E remains negative, can the star's shrinkage tend towards the natural end of minimum energy and therefore continue. If the above fraction becomes negative through fall of γ, the energy in the shrunken stage is no longer less than if diffused to infinity, and there is no agency for preventing the star from re-expanding and scattering itself over the heavens.

(*b*) Recollecting that γ reappears as an equivalent, for many purposes, of the polytrope index n such that,

$$\gamma = 1 + 1/n \qquad n = 1/\gamma - 1$$

a new light can be thrown upon the necessity that γ must exceed $\tfrac{4}{3}$; for this limiting value of γ means that n cannot exceed 3 for the star to be stable. This is the consideration which narrows the range which we had previously limited by saying that,

$$q = 3 \,/\, (5 - n)$$

must not become infinite or the star would possess infinite potential energy. This had demanded values of n less than 5, and an index 5 would imply $\gamma = 1{\cdot}2$ which is too low for stability, but we now see the other fraction connecting total energy with gravitation will further restrict n because γ itself must exceed $\tfrac{4}{3}$.

These limitations leave a narrow range within which to devise stellar models, and impose a warning that something will change in a star if n or γ become modified by any process within the star's structure; they also permit the use of polytropes for predicting internal temperatures and pressures which cannot depart widely from those which we tabulated.

We recall, however, that *locally* in some part of any star, γ may fall; indeed it is such occurrence that must set up the motions which lead to the unsteady luminosities we have described.

LOWERING OF GAMMA BY RADIATION PRESSURE

In the section on radiation pressure, we derived the intriguing fact that radiation itself behaves in a star like a gas of atoms, molecules, or electrons, but a gas whose γ is precisely $\tfrac{4}{3}$. Since in a very hot or massive star the ratio of radiation pressure to total pressure is no longer small, as denoted by $(1 - \beta)$ in our discussions, such a star will tend to assume the γ appropriate to its dominant constituent; so the rise in $(1-\beta)$ implies an effect as if the value of γ for the gases of the star were being lowered towards the critical danger limit $\frac{4}{3}$. This topic has been very thoroughly explored by Chandrasekhar, from whose Tables we take the following:

$1-\beta=$	0	0·2	0·4	0·6	0·8	1·0
$\Gamma_1=$	1·667	1·511	1·449	1·405	1·368	1·333

Γ_1 is defined as the 'effective' value to which a γ originally $\frac{5}{3}$ is lowered, in any star where there is so intense a field of radiation passing through the material that $(1-\beta)$ reaches the amounts tabulated. The calculation is made from the equation,

$$\Gamma_1=\beta+\frac{(4-3\beta)^2(\gamma-1)}{\beta+12\ \ (\gamma-1)(1-\beta)}$$

From the list, in our last Section, of the rise of $(1-\beta)$ for massive stars, we see here another aspect of the evolutionary dangers to which such enormous aggregates of matter are subject: the radiation pressure reduces the effective ratio of specific heats to near the danger level for dissipation of the star.

It may be noticed from the mass Table that in our own Sun the amount of matter is small enough to keep $(1-\beta)$ quite low. We may surmise what must be the eruptive life of an 'O-type' star, blowing off its atmosphere continuously through breakdown of stability, traceable to large M through reduced Γ due to enlarged $(1-\beta)$.

LOWERING OF GAMMA AT INCOMPLETE IONISATION

This source of instability was referred to in Part I, Chapter VI, (2) (*b*). If between any two zones in a star there is so steep a variation in the degree to which any electronic level in any atomic species is ionised, that in adjacent layers at not very different temperature electrons are free and then bound, then the outflowing energy is abnormally exploited. Eddington described it as expended on altering the ionisation instead of keeping the proper temperature gradient. The abnormality can usefully be exhibited as a local fall in γ. The pioneer calculations were by Fowler and Guggenheim in 1925, and Unsöld has developed much understanding of the outside portions of stars thereby. Undoubtedly the net result is to set a star's gases rushing upward and circulating in large-scale convection. Eddington regarded such instability, in a zone of incomplete ionisation of hydrogen, as likely cause of the pulsation of Cepheids, and Biermann tried to see the cause of Nova outbursts in similar instability at deeper levels.

GRADIENTS TOO STEEP FOR STABILITY: CONVECTIVE STARS

(1) The most useful discussions of stellar stability, involving the argument of both the preceding sections on γ as affected by radiation pressure and by incomplete ionisation, have in recent years tended to be rewritten in terms of 'gradients': in particular there is great significance in the gradient of temperature as it varies with depth in the star, the differential coefficient dT/dh. We will outline the following explanation of the situation, adapting from the writings of Plaskett and of Chandrasekhar, though the first notions go back to Schwarzschild (the elder) in 1906, and have been developed especially by Unsöld.

The gaseous structure of a star could be described as stable if, when a portion is displaced upward and its density and temperature are altering adiabatically, i.e. without heat exchange, it reaches a region of smaller density or higher temperature which forces it back to the original location. Gases changing only adiabatically will be in neutral equilibrium for such displacements, so that the stability can be decided by comparing the actual temperature gradient with that of a perfectly adiabatic atmosphere, the criterion being expressible by the following inequality:

$$\left.\begin{matrix}\text{stable}\\ \text{neutral}\\ \text{unstable}\end{matrix}\right\}\quad dT/dh \quad \begin{matrix}<\\ =\\ >\end{matrix}\left\{(dT/dh)_{\text{adiabatic}}\right.$$

If the actual gradient ever becomes too steep, in the third of these eventualities, small displacements due to temperature changes will not die out naturally, and the gas sets itself into convective motion, which cannot terminate until secondary effects counteract the instability. This instability is probably occurring in many stars at some layer beneath the surface, and may underlie the dissipation of atmospheres seen in unsteady stars.

(2) This criterion for stellar material to remain quiet or to rush upward, can be related to the γ which in the previous paragraphs controlled the stability. The law of ideal gases, used before, can be written,

$$\frac{dp}{p}=\left(\frac{\mu g}{KT}\right)dh$$

whereas for adiabatic changes p and T are related through γ so that,

$$\left(\frac{dp}{p}\right)_{\text{adiab.}}=\frac{\gamma}{\gamma-1}\ \frac{dT}{T}$$

and hence the adiabatic temperature gradient becomes,

$$\left(\frac{dT}{dh}\right)_{\text{adiab.}}=\frac{\mu g}{K}\ \frac{\gamma-1}{\gamma}$$

(3) The actual temperature gradient in a star largely depends on the flow of radiation and its absorption, and so we introduce the fraction of total pressure P contributed by radiation pressure p_R, and as previously,

$$p_R=(1-\beta)\,P=\tfrac{1}{3}\,aT^4$$

It will be convenient to write,

$$dp_R/dP=(1-\beta)\,\phi$$

where ϕ expresses the dependence upon local and mean absorption coefficients and rates of energy liberation, discussed under Eddington's standard model.

$$\frac{dp_R}{p_R}=\frac{dP}{P}\phi$$

$$\frac{dT}{T}=\tfrac{1}{4}\,\phi\,\frac{dP}{P}$$

(4) We now recall the use of a quantity η in discussing that standard model: η is the ratio of average energy generation within any radial distance r, relative to the average rate throughout the whole star. $k\eta$ was assumed constant over the entire star in the 'standard model' by which Eddington simplified stellar physics. If this product is no longer constant but of mean value $\overline{k\eta}$, then the function which we wrote as ϕ for expressing local circumstances in any part of a star can be written,

$$\phi = \frac{k\eta}{\overline{k\eta}}$$

The gradient affected by radiation pressure, when γ is depressed to the value Γ by the considerations we described, becomes,

$$\frac{dT}{T} = \frac{\Gamma - 1}{\Gamma}\,\frac{dP}{P}$$

But we also have,

$$\frac{dT}{T} = \frac{1}{4}\,\frac{k\eta}{\overline{k\eta}}\,\frac{dP}{P}$$

So the criterion for the gradient to allow the star to be stable becomes

$$\left.\begin{array}{r}\textbf{stable}\\ \textbf{neutral}\\ \textbf{unstable}\end{array}\right\} 4\,\frac{\Gamma - 1}{\Gamma}\begin{array}{c}>\\ =\\ <\end{array}\frac{k\eta}{\overline{k\eta}}$$

Chandrasekhar distinguished between several values of Γ, all beginning and ending at the same extreme but slightly differing in their middle course, and in this case Γ was defined by,

$$1 + \frac{(4 - 3\beta)\,(\gamma - 1)}{\beta^2 + 3\,(\gamma - 1)\,(1 - \beta)\,(4 + \beta)}$$

The portion k of the product $k\eta$ will vary at different atomic energy levels—diagrams have been plotted by Chandrasekhar and others—and the most likely law is that of Kramers, already quoted,

$$k \propto \frac{\rho}{T^{7/2}}$$

which seems likely to represent an average over a wide range of fluctuating circumstances. A study of stellar stability, such as will be needed for the future astronomer's assessment of his stars, will have to trace these effects of local radiation pressure and ionisation upon Γ and k while η is altering at the seat of atomic reactions violently dependent on any rise of temperature. The material of the star will only remain steady so long as the above inequality is maintained.

COWLING'S TREATMENT OF TEMPERATURE INDEX OF ATOMIC REACTION

The most serious danger to stability was anticipated, as we have mentioned, at the point where nuclear atomic transformations begin; as they rise so steeply with temperature, the liberation of heat accompanying their first onset might reinforce their rate and initiate a wave of explosive violence, comparable to the acceleration of chain reactions in an atomic bomb. If, as Eddington and Jeans thought, this was bound to disrupt a star, no stellar history could survive the attainment of a few million degrees at the body's centre. But not all stars become Novae, and it is tolerably certain that even our steady Sun long ago began to be the seat of Bethe's reactions and continues safely in that state of life.

It was Cowling, especially, among other workers, who showed that a star may safely contain convective motion of its gases, and indeed must do so, and yet not be disrupted or even expanded beyond small oscillations, while atomic reactions extremely temperature-dependent are proceeding within it. The 'point-source' model, as we have described, is enforced by the fact that atomic reactions proceed according to laws of the form,

$$\text{Reaction rate} \propto T^n,$$

where n is not the familiar square or cube but a power as high as 10 or 20. In the presence of reactions so temperature-dependent, it becomes inevitable that,

$$(1-\beta)_{\text{centre}} > (1-\beta)_{\text{mean}}$$

and the radiation gradient may vary greatly, becoming locally more steep than the gradient of total pressure in proportion to its magnitude. The gases will be set in motion by the excess temperature gradient which causes instability, and the star will develop what has come to be called 'Cowling's convective stellar core'. Cowling proved that if the law of energy generation (ϵ) and the law governing absorption coefficients k are such that,

$$\epsilon = \epsilon_0 \rho T^n \quad \text{and} \quad k = k_0 \rho^m T^s$$

then for fixed k, i.e. $m = s = 0$, convective flow of gases arises if $n > 4$. But if k follows Kramers' law which we quoted, i.e. $m = 1$, $s = -\frac{7}{2}$, convection is set up when $n > 8$. These numbers exceed the powers of temperature which the earlier workers thought dangerous. But the most important of Cowling's findings was that for either law of k any oscillations can stabilise themselves at small amplitude without 'getting out of hand' and damaging the star, if n lies within those limits, while if oscillation and convection both occur the pulsations stabilise themselves unless either n is very high or 'gamma' is already very low. The lower Γ is, the smaller is the value of n at which vibrations cannot remain within bounds. Cowling quotes three cases, assuming the law of Kramers: if $\Gamma = \frac{5}{3}$, oscillations stabilise with n as high as 400, an inconceivable figure for temperature index. If $\Gamma = \frac{10}{7}$, there is stability with n up to 18. If $\Gamma = \frac{40}{29}$, there is stability with n up to 4·98. Since most of Bethe's reactions which are capable of heating a star (Chapter III) have n between 10 and 20, a star changing its fuel owing to exhaustion of one constituent may be disrupted if its mass is high enough for Γ to be already near to $\frac{4}{3}$ by reason of radiation pressure.

Hydrogen Exhaustion and the Shell Source of Energy

COLLAPSE OF RADIATION PRESSURE

The conditions under which a star can maintain steady brightness, and the other conditions under which instability causes its internal gases to drift, or its envelope to pulsate, or the whole structure to disrupt, may now be summarised; these conditions govern the Natural History of a star, and when astronomy learns to apply them we shall know why the observed facts are an ordered evolutionary sequence of steady or unsteady shining of the stellar universe.

Maintenance of star during steady evolution:

The masses, temperatures, sizes, of stars, inferred from the luminosities and spectra by the methods of our earlier chapters, demand a definite and calculable rate of internal energy generation for any star to continue to shine. We have shown that three statements can summarise the knowledge and the query of recent stellar physics: (*a*) Gravitational shrinkage can only supply the right amount of energy for a brief fraction of stellar life-time, though this agency may become important in maintaining the star's brightness during transition stages after the exhaustion of each of the atomic species whose transmutations provide the energy in turn. (*b*) These atomic reactions, particularly those of the Bethe cycle where hydrogen forms helium with the aid of carbon and nitrogen, are adequate explanation of the energy of the Sun and most Stars of the main sequence. (*c*) But in the most luminous stars these reactions would proceed too rapidly, and in the cool red giants the reactions cannot occur at all unless such stars possess hot centres not calculable as polytrope models. Chemical composition may be non-uniform in such objects, and many of them may be vast nebulous spheres whose centres obey no one of our models and in which the usual atomic fuels are no longer available.

Convection or pulsation of material in a star:

The very high temperature dependence of atomic reaction causes the centre of many stars to be regions of violent circulatory motion. In the most massive stars, radiation pressure has lowered 'gamma' to the verge of evaporation or disruption of the whole body. As there are several modes of helium production as well as Bethe's cycle, with differing temperature dependence, and different effects upon the star's stability if gamma is low, a star may readily be set into radial pulsation during transition from one phase of its maintenance to another; it is possible that Cepheid and red giant oscillation arises thus, and is phased in the curious way observed, by the partial ionisation of hydrogen, in Eddington's theory.

Danger of evaporation or collapse of the star:

Exhaustion of any one element whose transmutation had supplied the star's energy, forces the 'point-source' at the star's centre to migrate outwards and become a 'shell-source' moving through the star unlike any polytrope model. When this shell nears the surface it may cause continuous ejection of the whole atmosphere, as seen in Wolf-Rayet stars and many other unquiet and 'enveloped' bodies surrounded by their own ejected gases.

If, however, exhaustion of the star's hydrogen were completed suddenly, the withdrawal of the principal source of radiation pressure may cause catastrophic collapse of the star on to a very small core at the centre. Milne and Chandrasekhar regarded this as the meaning of a Nova outburst, the excess gravitational energy of the collapse providing a brief flare-up before the star settled to the White Dwarf stage. Chandrasekhar calculated the onset of this hazard for stars of a small mass range, and Hoyle and others have followed up the implications with possible detectable wreckage from a single instance to be seen in the Crab Nebula. Whether hydrogen exhaustion, admittedly the crisis of a star's life, does occur gradually or with catastrophic suddenness, is a major problem with which we leave this essay:

we know almost nothing about how fast the rest of the star's gases rush to the support of any depleted energy source. The fact that the Nova catastrophe has been known to recur in the same star, suggests that not all ex-Novae are White Dwarfs, unless some means can be imagined of bringing the latter to unexpected revival of luminosity. The clue to both periodic and catastrophic fluctuation of a star's brightness will probably be found in the irregular variable stars, lying between quiet and unquiet evolution.

It must be clear that any such summary of the problems of stellar Natural History must preface many of its notions with 'IF'; we may have done service to general reader and to scientific worker by separating the most doubtful of the queries from the definite and unquestionable assertions. It is the juxtaposition of the unsolved interpretations with the triumphs of observation and inference, which makes stellar physics the most adventurous and most intriguing offshoot from modern atomic science.

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1. Stellar physics is discussed as part of any large account of Astronomy, of which those by:

SIR H. SPENCER JONES (Arnold, 1923 and later editions)
R. H. BAKER (Van Nostrand, 1930 and later)
RUSSELL, DUGAN, AND STEWART (Ginn, 1927 and later)

are specially recommended.

2. Among popular introductions requiring no previous acquaintance, a recent small one of outstanding merit is:

Frontiers of Astronomy: D. S. EVANS (Sigma Books, 1946)

On the topic of energy,

Birth and Death of the Sun: G. GAMOW (Viking Press, 1940)

A wide-ranging older book,

Modern Astrophysics: H. DINGLE (Collins, 1924).

3. Meant for students but not requiring expert knowledge are:

Stellar Astronomy: PETER DOIG (Hutchinson, 1947)

and the older standard course,

Astronomical Physics: F. J. M. STRATTON (Methuen, 1925).

A thorough historical treatment is:

A Hundred Years of Astronomy: R. L. WATERFIELD (Duckworth, 1938).

4. The three following mathematical treatises are an essential prelude to serious study at the graduate stage:

Internal Constitution of the Stars: SIR A. S. EDDINGTON (Cambridge, 1926)
Theoretical Astrophysics: S. ROSSELAND (Oxford, 1936)
Stellar Structure: S. CHANDRASEKHAR (Chicago, 1939).

The Eddington book, containing much mathematical argument, is nevertheless very readable, as we have shown in quotation; though old now, it is an irreplaceable masterpiece.

5. Special topics; we have stressed the importance of fluctuating stars, and their divergence from equilibrium conditions, so readers may like to consult the following, which are available in some libraries.

Variable Stars: C. P. AND S. GAPOSCHKIN (Harvard, 1938)
Thermodynamics of the Stars: E. A. MILNE
Novae: F. J. M. STRATTON.

The last two, with other important monographs, are in the German 'Handbuch der Astrophysik' and appear in English.

6. A series of recent books, readable without previous knowledge and yet carrying the student on into the heart of recent research, are the 'Harvard Books on Astronomy' J. and A. Churchill (London), of which the following are important to the present topic:

Atoms, Stars, and Nebulae: GOLDBERG AND ALLER (1946)
The Story of Variable Stars: CAMPBELL AND JACCHIA (1947)
Our Sun: D. H. MENZEL (1949).

Much of the current research in these subjects appears in the 'Astrophysical Journal' (Chicago), and the 'Monthly Notices of the Royal Astronomical Society' (London); while 'The Observatory' (c/o Greenwich Observatory and available to the public) publishes regular summaries of the discussion at astronomical meetings and articles surveying the trends in investigation all over the world.

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