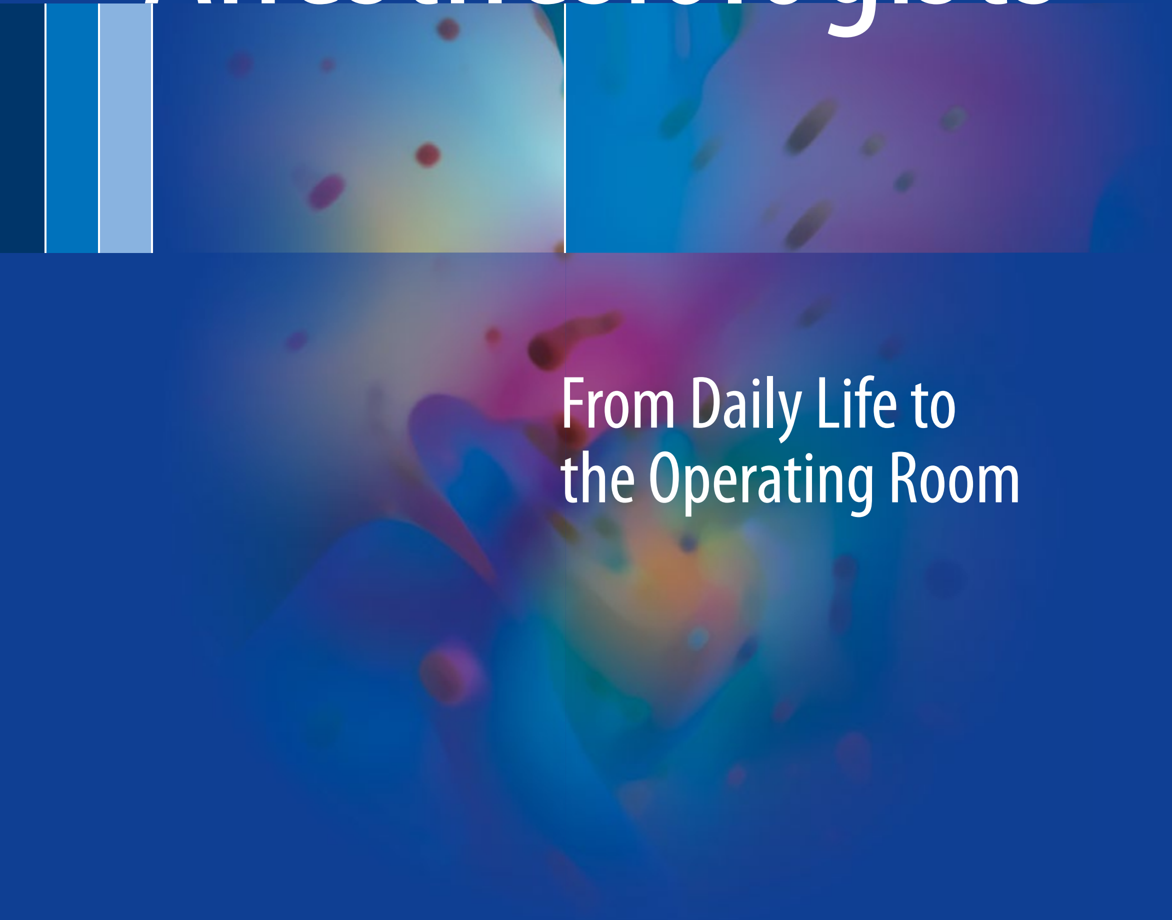
Antonio Pisano

Physics for Anesthesiologists



From Daily Life to the Operating Room

Springer

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Antonio Pisano
Cardiac Anesthesia and ICU
AORN Dei Colli, Monaldi Hospital
Naples
Italy

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To Marcella and Matteo

Preface

Physics is everywhere. This is not surprising, really, given that its field of interest is the description of how the matter around us behaves, from the "little" world of atoms and subatomic particles to the entire Universe, passing for the objects and situations of daily life.

Moreover, it is not a mystery that there are specific laws of physics at the basis of a lot of things we everyday do as anesthesiologists: most university training courses in anesthesiology, indeed, include the study of physics, and it is not uncommon to find more or less complex physics equations strewn among the pages of landmark anesthesia textbooks. These equations, however, are often skipped or soon forgotten.

Unfortunately, in fact, most people consider physics as an abstract and difficult matter (sometimes incomprehensible), if not even of little practical use. Conversely, there is no modern technological device (including smartphones) which does not rely on some conquest of physics. Furthermore, if you are an inquisitive person, physics can answer many interesting questions, sometimes in a surprisingly clear and illuminating way. For example: the ideal gas law tells us how to make a good coffee (with the "moka" pot); Henry's law explains the behavior of carbonated drinks and Champagne corks; Laplace's law reveals some secrets of bubbles; Bernoulli's theorem teaches us how an airplane can fly; thermodynamics shows the direction in which time flows; the study of heat and state changes of matter has many implications in everyday life, from our kitchens to the building of bridges. Moreover, there is a law of physics which accounts for the blue color of oceans, and a different one which explains why also the sky is blue; also, there is the physics that allows you to enjoy a concert, bats to avoid obstacles, and police to make you a speeding fine.

If, apart from being an inquisitive person, you are an anesthesiologist or an intensive care practitioner, you might take advantage (and, probably, some fun) in discovering, or rediscovering, the important implications (and applications) of physics in your daily clinical practice: for example, the abovementioned laws have something to do, approximately in the same order, with oxygen cylinders, blood gas analysis, airway management, hemodynamic monitoring, anesthetic vaporizers, pulse oximeters, near-infrared spectroscopy, and ultrasounds. And there is much more.

This book is intended for all who are interested in anesthesiology and intensive care medicine, from medical students to experienced clinicians, and can be read at different levels: to fully understand, in the training stage, the basic science that lies behind many aspects of anesthesia and intensive care medicine, and which is often only hinted elsewhere; to have a higher awareness about why we do what we do, as anesthesiologists, how the devices we commonly use work, and why we should not always blindly trust them; finally, just for curiosity, culture, or fun (maybe skipping some equation).

Although of varying difficulty, all topics are discussed through many examples from daily life and are accompanied by a lot of color illustrations for extra clarity. Moreover, the topic selection reflects both the interests and the basic mathematical knowledge of a clinician (such as the author himself is): accordingly, each aspect of physics discussed is strictly related to the clinical practice of anesthesia (and/or intensive care medicine), can be easily understood according to the recollections of high school (often refreshed within the text or in practical "boxes"), and allows to make interesting and enlightening comparisons with everyday life.

I would like to thank my wife Marcella and my son Matteo, to whom this book is dedicated, for their patience during the exciting but heavy commitment of writing a book. I would also like to thank Dr. Viviana Carillo and Dr. Pietro Castellone, two brilliant medical physicists (and friends), for their advices and their support.

Naples, Italy Antonio Pisano

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Part I

Gases, Bubbles and Surroundings

Perfect Coffee and Oxygen Cylinders: The Ideal Gas Law

1

Where's Physics

Daily life	How does a "moka" coffee pot work Troublesome passengers on the plane
Physics involved	Ideal gas law (and its special cases)
Clinical practice	Calculating the duration of an oxygen cylinder Decompression illness/Hyperbaric therapy

1.1 A Delicious Aroma… and a Nauseating Stench

The Italian "espresso" coffee pot, also called *moka*, is known around the world. In Italy, there is at least one in every kitchen (mine is the one in Fig. 1.1). However, very few people know how it works. All those to whom I asked answered that water flows through the funnel (which contains the ground coffee) after *boiling*. Indeed, this seems a widely held belief [1]. What happens in reality is that, *as temperature rises*, the *pressure* of the mixture of air and water vapor trapped between water and funnel increases. When this pressure overcomes the sum of the atmospheric pressure (at sea level, 760 mmHg or 101.3 kPa) and the pressure of filtration (which depends on physical properties of both water and ground coffee and on the funnel dimensions, according to the *Darcy's law of linear filtration*), the hot water is pushed into the funnel and through the ground coffee, from which it extracts aromatic compounds [2]. The coffee then comes out from the nozzle in the middle of the pot (Fig. 1.2). Only the small amount of water remaining at the end will boil (if the pot is not readily removed from the heat) due to the sudden decrease of pressure after the coffee came out (see Box 1.1).

A. Pisano, *Physics for Anesthesiologists*, DOI 10.1007/978-3-319-57330-4_1

Fig. 1.1 A *moka* coffee pot (from my kitchen)

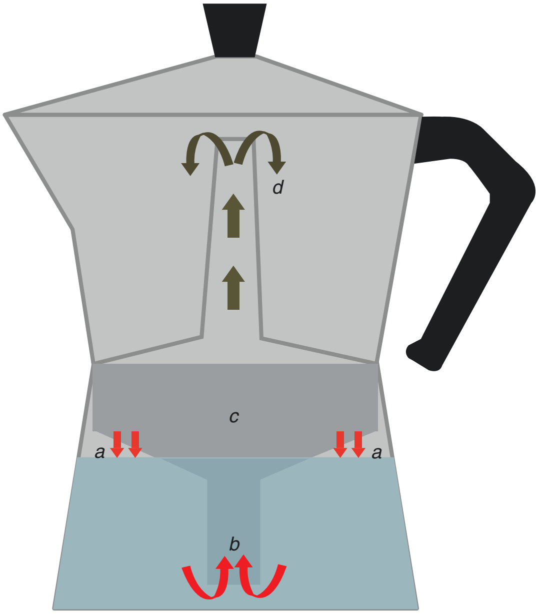

Fig. 1.2 Schematic representation of how the *moka* pot works. According to the Gay-Lussac's law (see Sect. 1.2.3), as temperature rises the pressure of the mixture of air and water vapor trapped between water and funnel increases (**a**), pushing water into the funnel (**b**) and through the ground coffee which is contained in the top of it (**c**). The coffee then comes out from the nozzle in the middle of the pot (**d**) after aromatic compounds have been dissolved in hot water

Box 1.1: Atmospheric Pressure and Boiling (See Also Chap. 12)
A liquid boils when its *saturated vapor pressure* (see Sect. 12.1.1), which depends only on temperature (for a given substance), equals the external atmospheric pressure. Saturated vapor pressure (SVP) of water at 20 °C (or 293 K) is 17.5 mmHg (or 2.3 kPa). When you put water on the stove to cook pasta, its vapor pressure increases with temperature. At 100 °C (373 K), water vapor pressure reaches 760 mmHg (or 101.3 kPa), that is the atmospheric pressure at sea level. Accordingly, water boils at 100 °C at sea level. In the mountains, where atmospheric pressure is lower, water needs to reach a lower SVP (and, accordingly, a lower temperature) to boil. This explains the statement "water will boil due to the sudden decrease of pressure" (in fact, as long as the coffee does not come out, pressure above the water in the pot is higher than atmospheric pressure due to the accumulating steam). Moreover, it explains why pasta is cooked badly in the high mountains (cooking temperature is lower)!

Now let's move from the sweet aroma of coffee to a much less pleasant smell. It can be very annoying to be seated on the plane next to a passenger who suffers from flatulence. This not only because the space on the airplane is limited, but also because his/her disorder might be accentuated during the flight due to the increase of intestinal gas volume caused by the reduction of atmospheric pressure with altitude. Indeed, among the many gastrointestinal problems which can occur in the high mountains [3] and, although to a lesser extent (due to cabin pressurization), on a plane, high altitude flatus expulsion (HAFE) syndrome seems to be related to the effect of pressure on gas volume [4]. Similarly, as mentioned, the *moka* pot uses the effect of temperature on gas pressure. Both are special cases of the so-called *ideal gas law*.

1.2 Ideal Gas Law

Pressure, volume, and temperature of an *ideal* gas are related by the following equation:

$$pV = nRT \tag{1.1}$$

where p is absolute pressure, V is volume, n is the number of moles of gas, R is the so-called *universal gas constant* (which has the same value for all gases, i.e., 8.31 J/mol K), and T is the temperature in kelvins (K).

One mole (1 mol) of a given substance contains 6.02×10^{23} atoms or molecules of that substance. This number is called the *Avogadro's number* (N_A). Accordingly, the number of molecules N present in a sample of a given substance is the product of N_A and the number of moles n:

$$N = n \cdot N_A \tag{1.2}$$

Equation 1.1 (*ideal gas law* or *universal gas law*) can therefore be rewritten for the number of molecules N (instead of for the number of moles n) as follows:

$$pV = \frac{N}{N_A} \cdot RT \tag{1.3}$$

$$pV = NkT \tag{1.4}$$

where $k = R/N_A$ is the so-called Boltzmann constant (1.38×10^{-23} J/K).

All *real* gases such as oxygen, as well as any mixture of different gases (not interacting each other) such as air follow the ideal gas law provided that the *density* (see Chap. 8) is sufficiently low (so that the volume of molecules is negligible compared to the volume which is occupied by the gas).

All other gas laws can be easily derived from Eq. 1.1 (or 1.4) as special cases.

1.2.1 Boyle's Law

If temperature remains constant, volume and pressure of a sample of any gas are inversely proportional.

In fact, since R (or k) is a constant and the quantity of gas (i.e., the number of moles n or the number of molecules N) is the same, if also temperature does not change the product nRT (or NkT) is a constant and Eq. 1.1 (or 1.4) can be written as follows:

$$pV = a\,\text{constant} \tag{1.5}$$

This is what happens to intestinal gases when altitude increases: as pressure decreases, volume must increase.

1.2.2 First Law of Gay-Lussac (or Charles's Law)

If pressure remains constant, the volume of a given amount of gas (e.g., contained in a deformable container such as a balloon) is directly proportional to its temperature:

$$V / T = a\,\text{constant or } V \propto T \tag{1.6}$$

This law, together with *Archimedes' principle*, explains why a hot air balloon flies (see Chap. 2).

1.2.3 Second Law of Gay-Lussac (or, Simply, Gay-Lussac's Law)

If volume remains constant, the pressure of a given amount of gas is directly proportional to its temperature:

$$p / T = a\,\text{constant or } p \propto T \tag{1.7}$$

This is the case of the coffee pot (see Fig. 1.2). The mixture of air and water vapor is trapped in a closed and nondeformable space (i.e., the volume is constant): according to Gay-Lussac's law, its pressure increases with temperature.

From a thermodynamic standpoint, the pressure of a gas in a container reflects the number of collisions of the gas molecules with the container walls (*kinetic theory of gases*). In particular, pressure is proportional to the so-called root-mean-square speed (v_{RMS}) of the molecules, which in turn depends on temperature, according to the following relationship:

$$v_{\mathrm{RMS}} = \sqrt{\frac{3RT}{M}} \tag{1.8}$$

where T is temperature, M is molar mass (i.e., the mass of 1 mole), and R the universal gas constant. So, higher temperature means faster molecules and, accordingly, more collisions (i.e., higher pressure). It can be said, indeed, that temperature of a given substance is a "measure" of the *kinetic energy* (E_{K}) of its molecules (see Box 1.2).

Box 1.2: Temperature and *Kinetic Energy*: A Look at the Kinetic Theory of Gases

According to the "kinetic theory of gases," volume, pressure, and temperature of a gas are all related to the motion of its molecules. A gas can be considered as a whole of tiny balls (whose volume is negligible with respect to the volume occupied by the gas) which continuously move, with different speeds, bumping with each other and with the container walls. The "average speed" of the molecules is expressed as the square root of the mean of the squares of each speed, i.e., the so-called root-mean-square speed (v_{RMS}).

In general, any object (whose mass is m) which moves with speed v has a *kinetic energy* (E_{K}) that is equal to:

$$E_{\mathrm{K}} = \frac{1}{2} m v^2$$

Similarly, the average kinetic energy of a gas molecule with molecular mass m is:

$$\overline{E}_{\mathrm{K}} = \frac{1}{2} \cdot m \left(v_{\mathrm{RMS}} \right)^2$$

After substituting for v_{RMS} from Eq. 1.8, the above equation becomes:

$$\overline{E}_{\mathrm{K}} = \frac{1}{2} \cdot m \cdot \frac{3RT}{M}$$

where T is temperature, M is molar mass, and R is the universal gas constant. Remembering that M/m is the Avogadro's number N_{A} (i.e., the mass of a mole divided by the mass of a molecule is equal to the number of molecules which are contained in a mole), we can write:

$$\overline{E}_{\mathrm{K}} = \frac{1}{2} \cdot \frac{3RT}{N_A}$$

and:

$$\overline{E}_{\mathrm{K}} = \frac{3}{2} \cdot KT$$

where $k = \mathrm{R}/N_{\mathrm{A}}$ is the Boltzmann constant.

It clearly results from the last equation that the temperature of a gas is nothing different from the average kinetic energy of its molecules.

1.2.4 Avogadro's Law

Two equal volumes of any gas at the same pressure and temperature hold the same number of molecules. Just look Eq. 1.4 (if p, V, and T do not change and k is a constant then N can assume only one value).

1.2.5 Dalton's Law

The total pressure exerted by a mixture of nonreactive gases is equal to the sum of the partial pressures of the individual gases. This law is discussed in detail in Chap. 3.

1.3 Calculating the Duration of an Oxygen Cylinder

Some anesthesiologists may have happened to transfer a patient on mechanical ventilation (or oxygen therapy) and to be worried that the oxygen cylinder could be emptied too early. As an example, a patient breathing through a Venturi mask (see Chap. 6) delivering an inspiratory oxygen fraction ($\mathrm{FiO_2}$) of 0.6 (or 60%) needs an oxygen flow of 15 liters per minute ($\mathrm{L\ min^{-1}}$). If the transfer takes 1 h, will a full cylinder with 10 L capacity be enough? And the same cylinder half full? And what about a 5 L cylinder?

Common cylinders for hospital or home use contain oxygen in the gas phase. In fact, oxygen cannot be liquefied at room temperature because its *critical temperature* (namely, the temperature above which a substance cannot exist in the liquid phase, regardless of the pressure applied) is very low (−118.57 °C or 154.58 K) [5]. As we will see, this is a fortune: in fact, it is not possible to know the content of a cylinder which contains a liquefied gas without a weighing scale!

For 1 h, the abovementioned patient needs:

$$15\,\mathrm{L\,min^{-1}} \times 60\,\mathrm{min} = 900\,\mathrm{L} \tag{1.9}$$

of oxygen. How many liters of oxygen are contained in a full 10 L cylinder? The answer is very simple: 10 L. And if it is half full, or almost empty? Again 10 L! Unlike solids and liquids, which have a *definite volume* (the former, but not the latter, also have a defined shape), gases always occupy all the volume that they have available. Accordingly, it is not enough to know the volume of a gas to know its "quantity."

It is clear from Eq. 1.1 (or 1.4) that, at a given temperature, the number of moles (or molecules, respectively) which are contained in a certain volume of gas depends on its pressure. Indeed, the "quantity" of oxygen which is contained in a cylinder is usually indicated by a pressure gauge and expressed in *bar* (1 bar ≈ 1 atm = 760 mmHg = 101.3 kPa). Oxygen cylinders are commonly filled up to 200 bar.

According to Boyle's law (Eq. 1.5), if a given amount of gas occupies the volume V_1 at a pressure p_1, it will occupy the volume V_2 ($>V_1$) at a pressure $p_1 < p_2$ (provided that temperature does not change), so that the product of its volume and pressure remains constant (see Fig. 1.3):

$$p_1 \cdot V_1 = p_2 \cdot V_2 \tag{1.10}$$

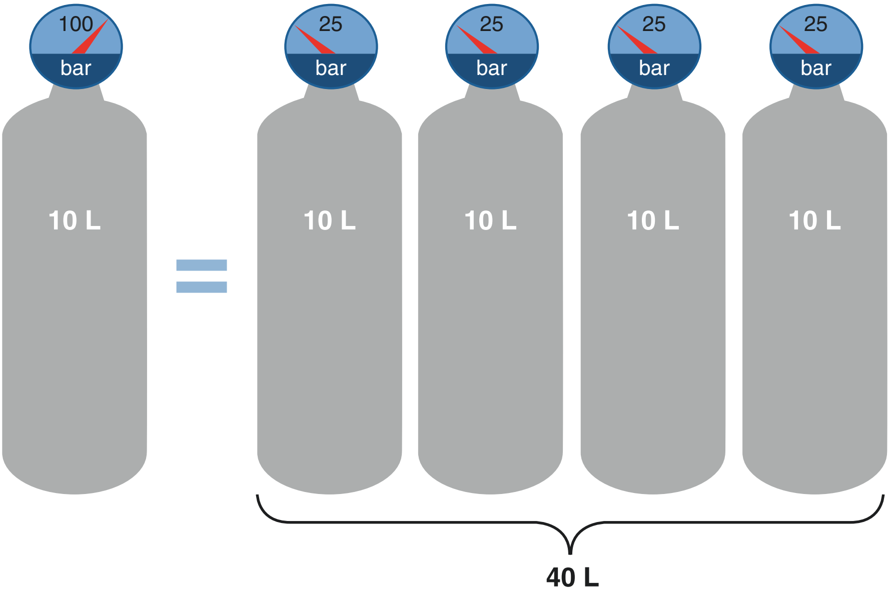

Fig. 1.3 Boyle's law. At a fixed temperature, the same amount of any gas which is contained in a 10 L cylinder at a pressure of 100 bar will occupy four cylinders identical to the first (or 40 L) at a pressure of 25 bar, so that the product of pressure and volume remains constant (10 × 100 = 25 × 40)

The volume V_2 can be easily calculated as follows:

$$V_2 = \frac{p_1 V_1}{p_2} \tag{1.11}$$

In the case of the oxygen cylinder, the volume V_2 which can be delivered to the patient *at atmospheric pressure* ($p_2 = 1$ atm ≈ 1 bar) is the product of the cylinder capacity (V_1) and the pressure p_1 (in bar) displayed on the gauge:

$$\text{Oxygen available}(L) = \text{Pressure in the cylinder}(\text{bar}) \times \text{Capacity of the cylinder}(L) \tag{1.12}$$

that is Eq. 1.11 for $p_2 = 1$ bar.

In the example above, a full 10 L cylinder (which has a pressure of 200 bar) will release 200 × 10 = 2000 L of oxygen, which will be certainly sufficient to meet the needs of our patient (i.e., 900 L for 1 h). Conversely, a half-full 10 L cylinder (pressure = 100 bar) will release 100 × 10 = 1000 L, as well as a full 5 L cylinder (200 × 5 = 1000): in both cases, I would be quite concerned! First, because the patient would have an oxygen reserve for only a few minutes more than expected (100 L/15 L min^{-1} ≈ 6 min), and most importantly, because it is advisable to never fully rely on the accuracy of these pressure gauges and, accordingly, to approximate by (large) defect the amount of available oxygen.

However, oxygen cylinders with a digital display which indicates the remaining "liters" by incorporating the calculation of the product of pressure and capacity are now available in order to help physicians… to forget physics!

In the case of cylinders containing a liquefied gas, e.g., carbon dioxide (CO_2), the pressure measured by the gauge does not correlate with the amount of the substance inside the cylinder. In fact, the liquid CO_2 is in equilibrium with its gaseous phase, whose pressure in these circumstances is always the *saturated vapor pressure* of CO_2 at the current temperature (see Chap. 12), whatever the amount of liquid contained in the cylinder. Accordingly, as mentioned, you should weigh the cylinder (and know its weight when full) in order to estimate the remaining amount of CO_2. The same applies to nitrous oxide.

1.4 Decompression Illness and Hyperbaric Therapy

Together with *Henry's law* (which is discussed in Chap. 4), Boyle's law has a key role in both the pathophysiology of decompression illness (DCI) and its recompression therapy with hyperbaric oxygen (HBO). A detailed discussion of these topics is beyond the scope of the present book and can be found in Vann et al. [6] In a few words, DCI includes two different diseases, both as a consequence of a reduction in environmental pressure (such as during the ascent of a diver): arterial gas embolism (AGE) and decompression sickness (DCS). While AGE occurs as a direct consequence of Boyle's law, since the reduced ambient pressure leads to expansion of alveolar gas causing pulmonary barotrauma and allowing gas to enter the arterial circulation, DCS involves the formation of bubbles in both tissues and blood due to Henry's law (see Chap. 4), followed by the increase in the size of such bubbles with decompression, according to Boyle's law. HBO therapy is the standard treatment for DCI [7], in which it acts primarily by reducing bubbles volume according, again, to Boyle's law. Moreover, a couple of practical issues in the care of patients with DCI or who need HBO result from Boyle's law. The first is that a patient with DCI who needs urgent recompression can be transferred to a hyperbaric chamber by helicopter provided to fly at low altitude (in order to avoid a further increase in the size of bubbles) [6]. Finally, in an intubated patient requiring HBO therapy the endotracheal tube cuff balloon should be filled with saline instead of air in order to avoid the reduction of its volume during compression [8].

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Boats, Balloons, and Air Bubbles: Archimedes' Principle

2

Where's Physics

Daily life	Boats (and ships), buoys, lifejackets, balloons for kids, ice cubes, hot air balloons, play dead afloat, swimming in fresh or salt water
Physics involved	Archimedes' principle
Clinical practice	Protect the brain from air embolism Facilitate de-airing (open-heart surgery) Handle a syringe

2.1 Archimedes' Principle: Gravity Not Always Makes You Fall

Everyone knows that boats, ships, lifejackets, buoys, and even the human body float in the sea. We also know that ice cubes float in our soft drink, that a balloon filled with helium flies upward (making children happy), as well as one filled with hot air (see Fig. 2.1). Moreover, many people are aware that swimming (or at least stay afloat) in the pool is a bit more exhausting than in the sea (provided that it is calm, of course). All of this is explained by the same physical law: *Archimedes' principle*.

It states that when a body is fully or partially submerged in a *fluid* (that is in a liquid, in a gas or in a gas mixture such as air), the fluid pushes it upward with a buoyant force that equals the weight of the fluid that has been displaced (Fig. 2.2).

According to *Newton's second law*, the weight of a body is the product of its mass and free-fall acceleration (see Box 2.1 and Chap. 11). Therefore, the buoyant force B on a body in a fluid has magnitude:

$$B = m_{\mathrm{f}} \cdot g \tag{2.1}$$

where m_{f} is the mass of the fluid that has been pushed out of the way by the body and g is free-fall acceleration.

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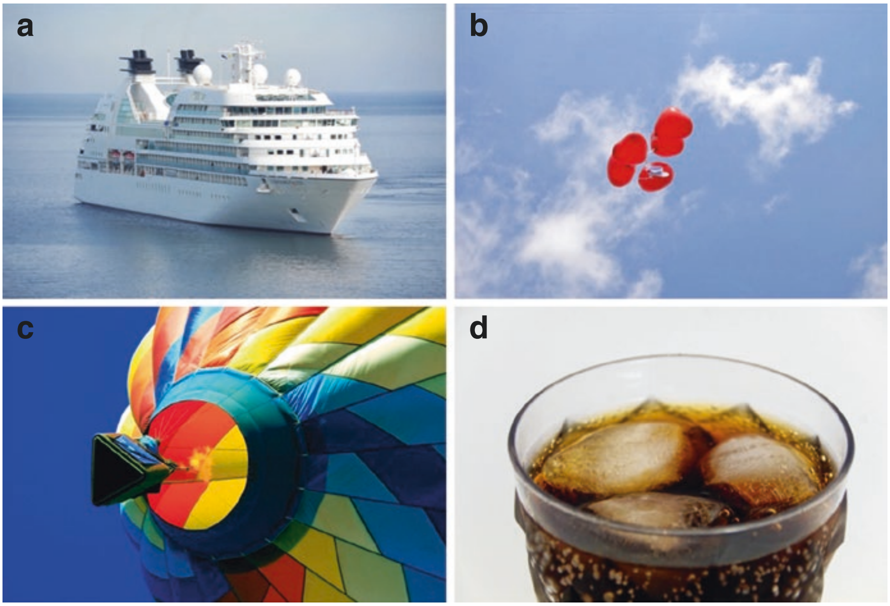

Fig. 2.1 Some examples of the Archimedes' principle from daily life. All these objects have a lower density than the fluid (**a**, **d**: water; **b**, **c**: air) in which they are immersed (see text)

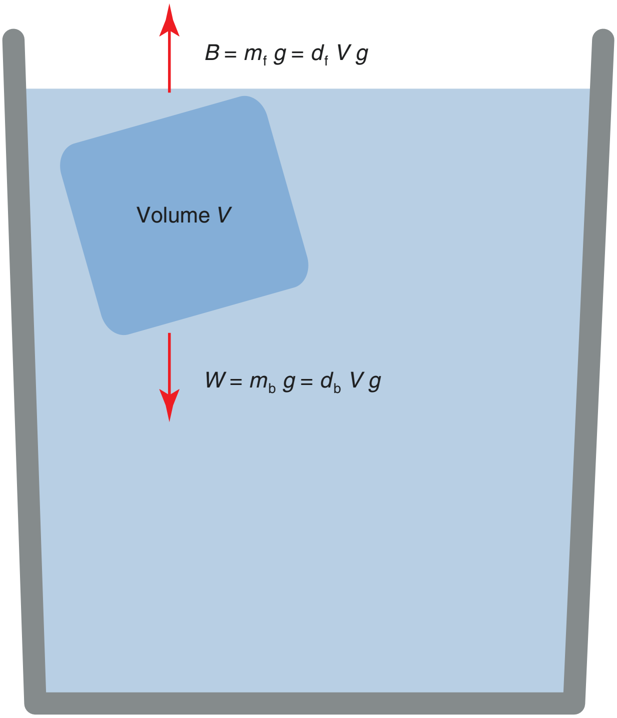

Fig. 2.2 The forces acting on a body of volume V and weight W that is submerged in a fluid. B buoyant force (Archimedes' principle), m_f mass of the volume V of fluid which is displaced by the body, d_f fluid density, g free-fall acceleration, m_b mass of the body, d_b density of the body

Box 2.1: Mass and Weight: Newton's Second Law (See Also Chap. 11)
Newton's second law states that the net force F acting on a body is the product of its mass m and its acceleration a (namely, the variation in speed over time):

$$F = m \cdot a$$

The mass of an object is generally described as the "quantity of matter" of which it is composed. However, Newton's second law provides a stricter definition of mass as the force that is needed to impart to a body an acceleration of unit value ($a = 1$). In other words, mass can be regarded as the "resistance" that a body opposes to a force which tries to make it accelerate.

The weight W of a body (which is often confused with its mass m) is the force that is exerted on it by effect of gravity. According to Newton's second law, it is:

$$W = m \cdot g$$

where g is the free-fall acceleration (i.e., the acceleration due to gravity), that is about 9.8 m/s^2 (or 32 ft/s^2).

Remembering that density d of a body is the mass per unit volume, i.e., the ratio between the mass m and the volume V of that body ($d = m/V$), m can be written as the product of d and V ($m = dV$) and Eq. 2.1 becomes:

$$B = d_{\mathrm{f}} \cdot V \cdot g \tag{2.2}$$

where d_{f} is the fluid density and V is the volume of the fluid displaced, *which is clearly equivalent to the volume of the body itself (or of its submerged portion).*

While the buoyant force pushes the body upwards, its weight pulls it down. According again to Newton's second law, the magnitude of the body weight W is:

$$W = m_{\mathrm{b}} \cdot g = d_{\mathrm{b}} \cdot V \cdot g \tag{2.3}$$

where m_{b} is the mass of the body and d_{b} is its density.

As you can see, the only difference in the calculation of the two forces (the buoyant force B and the weight W) is *density*: B is proportional to the density of the fluid, while W is proportional to the density of the body (see Fig. 2.2). The net force acting on the body is the difference between B and W (since the two forces have the same direction and opposite sign):

$$W - B = d_{\mathrm{b}} \cdot V \cdot g - d_{\mathrm{f}} \cdot V \cdot g = \left(d_{\mathrm{b}} - d_{\mathrm{f}}\right) \cdot V \cdot g \tag{2.4}$$

Accordingly, if the density of the body is lower than that of the fluid, the buoyant force will be greater than the body weight and the body will be pushed upward. Conversely, it will go down. If a body floats (i.e., it remains only partially submerged) it means that $B = W$.

Interestingly, the "fight" between the two forces W and B is not, as one might think, that between gravity and a different kind of force, but it is rather a "fight" of gravity against itself (due to the presence of the surrounding fluid). In fact, if we consider the example of an object submerged in a liquid, the buoyant force is due to the increase in *hydrostatic pressure* with depth (Stevin's law) (see Chap. 8). Since hydrostatic pressure is the pressure exerted by a column of liquid due to its weight, and weight is in turn an effect of gravity (see Box 2.1), it appears clear that, ultimately, also the buoyant force is due to gravity.

All the above mentioned phenomena of everyday life are easily explained with differences in density. All objects filled with air (which is less dense than water) float in the sea: even the very heavy ships float thanks to their large air-filled spaces (Fig. 2.1a). Helium has a lower density as compared with air [1]: therefore, balloons filled with this gas go upward in the air (Fig. 2.1b). According to Charles's law (see Chap. 1), the volume of a gas increases as temperature rises: a hot air balloon is initially inflated by heating the air (Fig. 2.1c), which becomes less dense (due to the increase in the balloon volume); when the balloon reaches its maximum volume, some air escapes from the opening at the base as temperature continues to rise, thus leading to a further decrease in density (the same volume, less molecules). Unlike almost all other substances, water expands (and, accordingly, its density decreases) when it becomes solid: this is why ice cubes float in water (and even more in soft drinks which contain sugar and other substances) (Fig. 2.1d). Finally, salt water is more dense than fresh water: that's why you float more easily in the sea than in a swimming pool.

2.2 The Anesthesiologist and Archimedes' Principle

The case which is usually encountered in the operating room is that where the "body" is represented by air bubbles. According to Archimedes' principle (and to common experience), air bubbles always tend to go upward in any liquid, including saline, drugs, and blood. A very trivial practical implication of this phenomenon pertains to how to handle a syringe if you realize that there is an air bubble inside and don't want to waste time, for example, to disconnect it from the tap along a venous line (Fig. 2.3): unless you do not need to inject the entire content of the syringe, just keep down the cone of the syringe!

However, Archimedes' principle has more strictly clinical applications in cardiac surgery. During cardiac interventions which imply the opening of a main heart chamber, de-airing before aortic declamping is needed in order to prevent air bubbles from reaching the brain and possibly causing ischemia. If de-airing is performed through a ventriculotomy, the anesthesiologist is asked to place the patient in the Trendelenburg position, so that the venting site is located above and air expulsion is favored [2]. Most noteworthy, Archimedes' principle helps cardiac anesthesiologists to prevent (or reduce) cerebral air embolism when air accidentally enters the circuits during cardiopulmonary bypass (CPB) or selective cerebral perfusion (SCP) in hypothermic circulatory arrest [3]. In these cases, in fact, the patient should

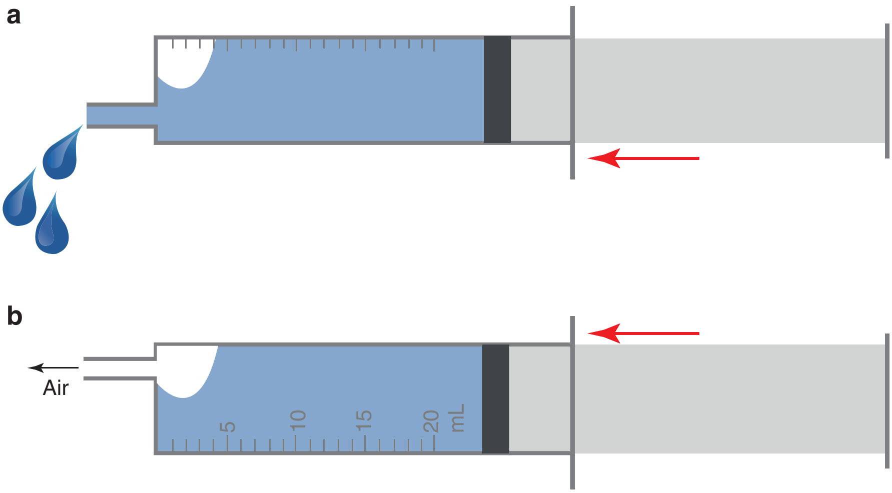

Fig. 2.3 Air bubble in a syringe. You can take advantage of Archimedes' principle in order to avoid injecting air to the patient. (**a**) cone down; (**b**) cone up

be immediately placed in steep Trendelenburg position in order to protect the brain from air embolism.

We will come back on Trendelenburg position in Chap. 11, where we will see how physics (and trigonometry) can help anesthesiologists to use it safely.

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Air Bubbles in the Blood Sample: Better or Worse Oxygenation? Dalton's Law and Fick's Law

3

Where's Physics

Daily life	The air we breathe Gas leaks Moisture on the walls of your home
Physics involved	Dalton's law Fick's law
Clinical practice	How an air bubble in the syringe affects blood gas analysis Physiology of respiration (not in this book!)

3.1 Dalton's Law: When You Do the Math, It All Adds Up!

Dalton's law was enunciated by the English physicist of the nineteenth century from which also comes the name of the unified atomic mass unit (Da), as well as the term "daltonism," the color blindness of which he was suffering and that he described for the first time [1]. As mentioned in Chap. 1, it states that *the total pressure exerted by a mixture of nonreactive gases is equal to the sum of the partial pressures of individual gases*:

$$P_{\mathrm{t}} = \sum_{i=1}^{m} P_i \text{ or } P_t = P_1 + P_2 + \ldots + P_m \tag{3.1}$$

where P_1, P_2, …, P_m are the partial pressures of each component, P_{t} is the total pressure, and m is the number of gases in the mixture.

In other words, *each component of the mixture exerts a partial pressure equal to the pressure which would exert if it occupied, alone, all the volume available* (see Fig. 3.1).

Thanks to this law it can be easily demonstrated that the ideal gas law discussed in Chap. 1 (Eq. 1.1) also applies to a mixture of nonreactive gases, provided that the

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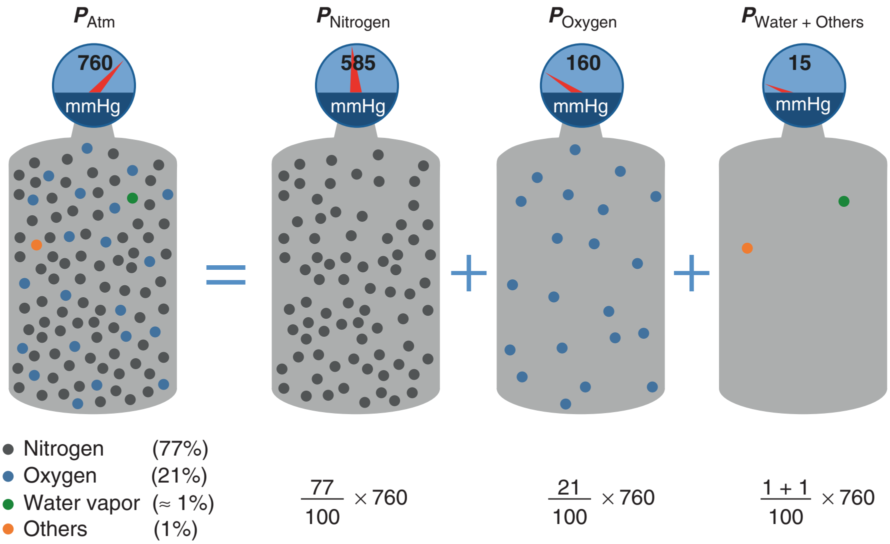

Fig. 3.1 Composition of atmospheric air and partial pressures of its components at sea level. It is clearly shown that the partial pressure of each component is equal to the pressure which it would exert if it occupied, alone, all the volume available (Dalton's law). P_{Atm} total atmospheric pressure

number of moles n is replaced with the sum of the numbers of mole of each component which are present in the mixture:

$$pV = \left(n_1 + n_2 + n_3 + \ldots\right) \cdot RT \tag{3.2}$$

where p is pressure, V is volume, R is the universal gas constant, T is the temperature (K), and n_1, n_2, n_3, etc. are the numbers of moles of each gas in the mixture.

Consider the example of a mixture of two gases in which there are n_1 moles of one gas and n_2 moles of the other one. According to the universal gas law, we can write:

$$p \cdot V_1 = n_1 \cdot RT \tag{3.3}$$

and

$$p \cdot V_2 = n_2 \cdot RT \tag{3.4}$$

If the first gas occupied all the available volume ($V_1 + V_2$), its ideal gas law could be written as follows:

$$p_1 \cdot \left(V_1 + V_2\right) = n_1 \cdot RT \tag{3.5}$$

and, after replacing n_1 RT with pV_1 (according to Eq. 3.3), we can calculate that its pressure p_1 would become:

$$p_1 = p \cdot \frac{V_1}{V_1 + V_2} \tag{3.6}$$

Similarly, if the second gas occupied, alone, all the available volume, its pressure would be:

$$p_2 = p \cdot \frac{V_2}{V_1 + V_2} \tag{3.7}$$

According to Dalton's law, the total pressure p of the mixture must be equal to $p_1 + p_2$. Therefore (using Eqs. 3.6 and 3.7):

$$p = p_1 + p_2 = \frac{pV_1}{V_1 + V_2} + \frac{pV_2}{V_1 + V_2} \tag{3.8}$$

and, after replacing pV_1 and pV_2 with $n_1 \cdot RT$ and $n_2 \cdot RT$, respectively (Eqs. 3.3 and 3.4), and $V_1 + V_2$ with the total volume V:

$$p = \frac{n_1\, RT}{V} + \frac{n_2\, RT}{V} \tag{3.9}$$

or

$$p \cdot V = \left(n_1 + n_2\right) \cdot RT \tag{3.10}$$

that is, indeed, what we wanted to prove.

Figure 3.1 shows the composition of atmospheric air and the calculation of the partial pressures of its main components according to Dalton's law. The air we breathe contains approximately 77% nitrogen, 21% oxygen, and a variable amount (around 1%) of water vapor (which is called *humidity* and depends largely on temperature, as we will see in Chap. 12 when talking about "saturated vapor pressure"), while other gases such as carbon dioxide (CO_2), noble gases (mainly Argon), and very small quantities of other gases represent, overall, the remaining 1%.

As mentioned in Chap. 1, atmospheric pressure (P_{Atm}) at sea level is about 760 mmHg (or 101.3 kPa). According to Dalton's law, it must be:

$$P_{Atm} = p_{\text{Nitrogen}} + p_{\text{Oxygen}} + p_{\text{Water}} + p_{\text{Others}} = 760\,\text{mmHg} = 101.3\,KPa \tag{3.11}$$

It is therefore easy to calculate, for example, the partial pressure of nitrogen as:

$$P_{\text{Nitrogen}} = \frac{77}{100} \times 760\,\text{mmHg} \approx 585\,\text{mmHg} \approx 78\,kPa \tag{3.12}$$

To further clarify the meaning of Dalton's law, just think what happens when you stay in a room (with poor ventilation) where there is a gas leak. Even if the gas which is filling the room is not strictly "toxic," you will probably die if you cannot get out quickly. In fact, as the gas accumulates, its partial pressure increases and that of all the components of air, *including oxygen*, must decrease so that the total pressure remains (approximately) the same. Therefore, you would die (due to hypoxia) just to make everything fit according to Dalton's law! (However, a window and Fick's law could save your life, as discussed in the next section).

A similar (but less dramatic) example is the humidification and heating of air as it passes through the airways. When air reaches the alveoli, its partial pressure of water vapor becomes equal to the *saturated vapor pressure* of water at 37 °C (i.e., the highest partial pressure of water vapor allowable at this temperature, as we will see in Chap. 12), which is 5–6 times higher than the usual water vapor pressure in the atmosphere. As a consequence, according to Dalton's law, the partial pressures of other gases decrease. It is known, indeed, that alveolar partial pressure of oxygen (pO_2) is slightly lower than that of the "dry" inhaled air [2].

3.2 Down the Slope: Fick's Law

Two bodies (whatever their state of aggregation), which stay in contact for a long enough time, diffuse into one another until they form a homogeneous mixture. This even occurs between two solids, although "a long enough time" could be really too long! Conversely, the delicious smell coming from the kitchen (i.e., a vapor which spreads through air, or a "gas" which diffuses into another gas), the cocoa powder which mixes with my son's milk (i.e., a solid that diffuses into a liquid in which it is soluble), and oxygen which moves from inhaled air to blood through the alveoli (i.e., a gas that diffuses into a liquid) are common examples of *diffusion* which occurs rather quickly.

A substance always diffuses from where its concentration is higher to where it is lower (see Fig. 3.2). More precisely, diffusion occurs along a concentration gradient (G_C), that is the difference in concentration per unit distance or, more correctly, the slope of the graph of concentration against distance:

$$G_C = \frac{c_1 - c_2}{\Delta x} \text{ or } G_C = \frac{dC}{dx} \tag{3.13}$$

where c_1 and c_2 are the concentrations of the substance at the point x and $x + \Delta x$, respectively, and dC/dx is the derivative of concentration with respect to distance

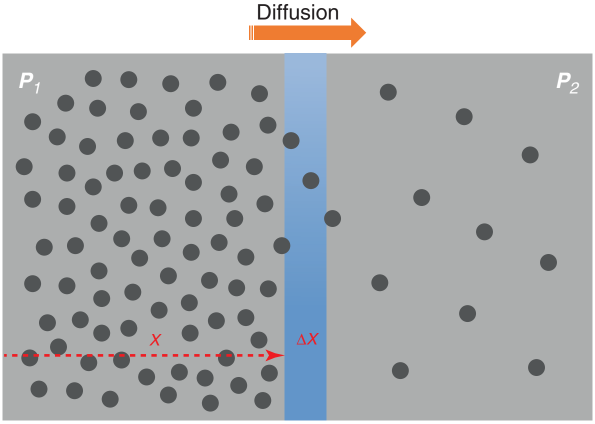

Fig. 3.2 According to Fick's law, a gas (whose molecules are represented by the little *gray balls*) diffuses through air (whose molecules are not shown) from the *left side* of a box where it has the partial pressure p_1 to the right side of it where it has the partial pressure $p_2 < p_1$

(that is, for those who remember a bit more mathematics, the slope of the curve of concentration against distance).

In particular, *Fick's law* states that the *diffusion flux*, i.e., the amount q of substance (expressed in moles) which flows through a unit area A during a unit time interval t, is proportional to the concentration gradient G_C:

$$J = -D \cdot G_C \tag{3.14}$$

where J $(=q/At)$ is the diffusion flux (mol·m^{-2} s^{-1}), D is the so-called *diffusion coefficient* (or *diffusivity*) (see Box 3.1), and the minus sign means that, as mentioned, the flow is directed towards the areas with lower concentration (i.e., in the opposite direction to the gradient, or in the direction of a "negative" gradient).

Box 3.1: A Closer Look at "Diffusivity" of Gases

When talking about alveolar gas exchanges, everyone knows that CO_2 is much more *diffusible* than oxygen.

According to the kinetic theory of gases (see Chap. 1), the *average kinetic energy* $\bar{E}_K$ (see Box 1.2) is the same for all the molecules in a mixture of gases. Accordingly, if we consider a mixture of the gases 1 and 2, it must be:

$$\bar{E}_K = \frac{1}{2} \cdot m_1 \cdot \left(v_{RMS1}\right)^2 = \frac{1}{2} m_2 \left(v_{RMS2}\right)^2$$

where m_1 and m_2 are the molecular masses of the gases 1 and 2, respectively, and v_{RMS1} and v_{RMS2} are their root-mean-square speeds. The equation above can easily be rearranged as follows:

$$\frac{vRMS2}{vRMS1} = \sqrt{\frac{m1}{m2}}$$

or, remembering that the average velocity v is the distance Δx traveled during the time t (as everybody knows):

$$\frac{t1}{t2} = \sqrt{\frac{m1}{m2}}$$

that is, the time that each component of the mixture needs to diffuse along a distance Δx is proportional to the square root of its molecular mass (*Graham's law of diffusion*, also known in some countries as Bunsen's law). According to this law, a gas with a higher molecular mass is less diffusible than one with a lower molecular mass.

If we remember the value of the molecular masses of oxygen (32 Da) and of CO_2 (44 Da), it is clear that *oxygen is more diffusible than* CO_2… and that there is something wrong! Really, there is no contradiction between Graham's law and the first sentence of this box, provided that you specify "when talking about alveolar gas exchanges."

Indeed, the *diffusion coefficient D* of Fick's law depends not only on the "solute" (i.e., the substance that diffuses) but also on the "solvent" (i.e., the substance in which the "solute" diffuses). When a gas diffuses into a liquid, the rate of diffusion is also directly proportional to the *solubility* of the gas in that liquid. Of course, CO_2 is much more soluble in blood than oxygen, so everything is explained! Finally, the solubility of a substance into another one also depends on temperature: in particular, the solubility of a gas into a liquid decreases with temperature (see Chap. 4), while the solubility of a solid into a liquid increases with temperature. (That's why the cocoa powder mixes more easily with my son's milk if it is hot. Unfortunately, he wants it always cold!).

In the case where the substance which diffuses is a gas, the concentrations c_1 and c_2 can be replaced with the partial pressures p_1 and p_2, and it can be said that *diffusion occurs along a partial pressure gradient*. Figure 3.2 shows a shoe box-shaped container inside which a gas diffuses, through air, from the point x where it has the partial pressure p_1 to the point $x + \Delta x$ where it has the partial pressure p_2 (which is lower than p_1 by magic or because a partition between the two areas was just removed). In this example, Eq. 3.14 (Fick's law) can be written as follows:

$$J = \frac{q}{At} = D\frac{p_1 - p_2}{\Delta x} \tag{3.15}$$

or

$$q = D \cdot A \cdot t \cdot \frac{p_1 - p_2}{\Delta x} \tag{3.16}$$

that is, the amount q of gas which passes through the thickness Δx over the time t is directly proportional to the section of the container A, to the time t and to the difference in partial pressures of the gas across Δx, and inversely proportional to Δx itself.

The gas that has accumulated in a room due to a leaking pipe will rapidly flow out thanks to the wide difference in its partial pressure between the room and the outside environment. Moreover, the higher the window size or the number of windows opened (i.e., the greater the area A through which the gas can pass) the faster the gas will diffuse away, just as described by Eq. 3.16. Of course, air (and therefore oxygen) will follow the reverse path.

As another example, the moisture (i.e., the partial pressure of water vapor) on the walls of your home also diffuses according to Fick's law [3].

3.3 Air Bubbles and Blood Gas Analysis

We all know that we should not leave air bubbles in the syringe containing a blood sample for gas analysis because it can affect the results, especially pO_2 and partial pressure of carbon dioxide (pCO_2). Actually, even a gas bubble whose relative volume is 0.5–1% that of the blood in the collection device is a potential source of significant error [4, 5].

One might think, instinctively, that air bubbles reduce the pO_2, since we are accustomed to think of air as the most oxygen-poor mixture as compared to the inspiratory oxygen fractions (FiO_2) we administer usually. Instead, the literature reports an increased (i.e., overestimated) pO_2 due to air bubbles [5–7]. In order to understand why (and why this is not always true!), we have just to remember the two laws discussed above (particularly, Dalton's law).

Given a barometric pressure at sea level of 760 mmHg (101.3 kPa) and a concentration of oxygen of about 21%, the pO_2 of room air is, according to Dalton's law, approximately:

$$pO_2 = \frac{21}{100} \times 760\,\text{mmHg} = 159.6\,\text{mmHg} \approx 21.3\,kPa \tag{3.17}$$

Since a gas, according to Fick's law, diffuses from where its partial pressure is higher to where it is lower, oxygen will diffuse from bubbles to blood if patient's arterial pO_2 is lower than the pO_2 of air, and from blood to bubbles if patient's arterial pO_2 is higher than that of air. Accordingly, air bubbles will tend to increase (i.e., overestimate) the pO_2 value on arterial gas analysis when patient's arterial pO_2 is below 159.6 mmHg (or 21.3 kPa), as usually. However, if the patient has a higher arterial pO_2 (e.g., around 200 mmHg or 26.7 kPa), as may occur during oxygen supplementation, mechanical ventilation, cardiopulmonary bypass (CPB), or extracorporeal membrane oxygenation (ECMO), with high FiO_2, then an air bubble in the syringe may lead to underestimation of pO_2 [8]. Moreover, it appears clear from Eq. 3.16 that the risk of a significant error in the determination of pO_2 is proportional to both the bubble size (the wider the "exchange surface" A, the higher the "rate of diffusion") and the delay between sampling and either analysis or bubble removal (the longer the contact time t between the bubble and blood, the higher the amount q of gas which diffuses from side to side) [6].

All this also applies to CO_2 (leaving out its different solubility in the blood as compared to oxygen). However, since the concentration of CO_2 in the air is very low (about 0.05%), the pCO_2 of air (approximately 0.38 mmHg or 0.05 kPa) is always much lower than that of the patient's arterial blood: accordingly, pCO_2 can only decrease (that is, be underestimated) due to the presence of an air bubble.

In conclusion, do not let air into the syringe, or at least remove it as soon as possible, when taking a blood sample for a gas analysis.

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4 Cold, Sparkling Drinks, and Blood Gas Analysis: Henry's Law

Where's Physics

Daily life	Sparkling water, carbonated drinks Flying corks (of Champagne) Bubbles in water which is not yet boiling
Physics involved	Henry's law
Clinical practice	Acid-base management during cardiopulmonary bypass (α-stat vs. pH-stat) Pathophysiology and treatment of decompression sickness

4.1 The Physics in a Soda Bottle: Henry's Law

When you open a can of soda or a bottle of sparkling water and you see a lot of bubbles appear, you are observing the effects of *Henry's law*: the degree a gas dissolves into a fluid (i.e., the solubility of the gas in that fluid) increases proportionally to the increase in partial pressure above the fluid, according to the equation:

$$C_x = k_H \bullet P_x \tag{4.1}$$

where C_x and P_x are, respectively, the concentration and the partial pressure of the gas x, and k_H is the so-called Henry's law constant.

Carbonated water and fizzy soft drinks are produced by adding carbon dioxide (CO_2) to water under high pressure. Accordingly, before the can or the bottle is opened the gas above the drink (almost pure CO_2) is at higher pressure than atmospheric (Fig. 4.1). When the container is opened, some of this gas flees (producing the typical hiss) and its partial pressure above the liquid decreases: therefore, according to Henry's law, the solubility of CO_2 decreases, and part of the dissolved CO_2 comes out of solution as bubbles.

Moreover, have you ever noticed that warmer is the drink, larger are the bubbles? Or also, that when it's hot, drinks left in a glass become "flat" more quickly? This too

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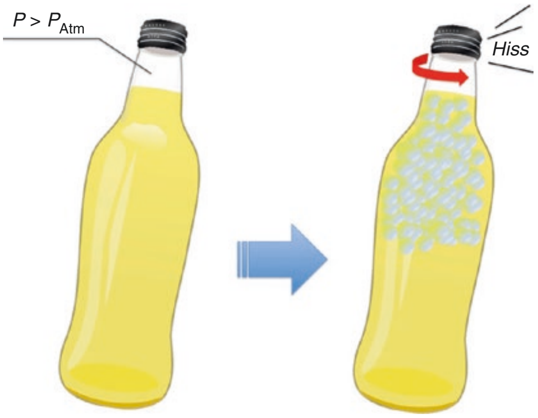

Fig. 4.1 The pressure of the gas (almost all CO_2) which is contained in a closed bottle of fizzy drink is slightly higher than atmospheric pressure (P_{Atm}). When the bottle is opened, this gas is released (producing the characteristic hiss) and replaced by air, whose partial pressure of CO_2 is very low (see Chap. 3, Sect. 3.3). According to Henry's law, the solubility of CO_2 in the liquid decreases and some CO_2 leaves the solution in the form of bubbles

is due to Henry's law. In fact, the Henry's constant is not at all constant (indeed, it should be called "Henry's coefficient"), but it depends on the gas, the solvent, and the *temperature*, according to complex "Van't Hoff-like" equations that is not the case to report here [1–5]. In particular, the higher the temperature, the lower the value of k_H (and, accordingly, the solubility of the gas). Accordingly, when the drink is left in a glass, its characteristic acidulous taste (which is due to the CO_2 *dissolved* in the form of carbonic acid) tends to fade more quickly in a hot day since the drink warms faster.

Thermodynamically, it can be said that an increase in temperature translates into an increase in the kinetic energy of the gas molecules (see Chap. 1), resulting in a faster movement of them and in an easier breaking of intermolecular bonds, which enables molecules to escape from the solution and increases pressure. In other words, the partial pressure of a gas dissolved in a liquid is a measure of the kinetic energy of the gas molecules.

All this, applied to a bottle of Champagne, leads to another interesting revelation: the higher the temperature of the Champagne, the faster the cork will fly (and the greater will be the distance reached or the damage to breakable objects). In Champagne, CO_2 forms spontaneously due to fermentation of sugar. Accordingly, pressure is much higher than that in a fizzy soft drink (up to six times that of the atmosphere) [6] and a lot of energy is released when the bottle is opened. Most of this energy generates a sound shock wave (see Chap. 15) which produces the characteristic laud "pop," while a small part makes flying the cork [7]. Due to the temperature dependence of the Henry's coefficient, the higher the temperature, the lower the amount of CO_2 dissolved: this means a higher pressure in the bottle and, accordingly, a greater velocity (i.e., a higher kinetic energy) of the cork.

Fig. 4.2 A pot of water on the stove. (**a**) The water is not yet boiling but you can see many tiny bubbles. These bubbles are made of gases (mainly oxygen and nitrogen) which leave the solution due to their reduced solubility into water as temperature rises (dependence on temperature of Henry's law constant). (**b**) The water is now boiling (the bubbles are made of water)

The relationship between Henry's coefficient and temperature also explains an event that can be observed by putting some water to boil: the little bubbles that appear before water starts to boil (Fig. 4.2). Someone might think that it is simply the slow beginning of boiling, but it is not. In fact, ebullition is, by definition, a tumultuous event (so, never slow) and the bubbles are made of water vapor. Instead, the "early" bubbles are formed by gases (such as oxygen) dissolved in water that come out of solution due to their reduced solubility with increasing temperature.

4.2 Acid-Base Management During Cardiopulmonary Bypass

One of the several things that alter the acid-base balance in a hypothermic patient [8–11] is what we have discussed above: the increase in the solubility of CO_2 into blood due to the lower temperature increases the concentration of dissolved CO_2 and, accordingly, reduces the partial pressure of CO_2 (pCO_2). This leads to respiratory alkalosis, which is masked by automated blood gas analysis (since it is performed at 37 °C), unless to correct values for the patient's temperature. Hence, there are two possible approaches to acid-base management during (hypothermic) cardiopulmonary bypass (CPB) or deep hypothermic circulatory arrest (DHCA) with selective cerebral perfusion (SCP): maintain normal values of pH and pCO_2 by adding CO_2 to the fresh oxygenator gas flow (pH-stat), or tolerate the respiratory alkalosis (α-stat) [10, 12–14]. Although pH-stat theoretically improves cerebral perfusion (due to the higher pCO_2), this may lead to an increased risk of air embolism during CPB (we will see in Chap. 5 that it is again physics which explains why an air bubble can occlude a blood vessel). Indeed, literature seems now in favor of α-stat in adult cardiac surgery, while pH-stat could be preferable during congenital procedures due to a more homogeneous brain cooling prior to circulatory arrest [14].

Beyond the choices and the different clinical settings, when addressing the "debate" on the best acid-base management strategy during CPB/DHCA, you may also think about boiling pots, carbonated drinks, flying corks, and Henry's law.

4.3 Pathophysiology and Treatment of Decompression Sickness

As mentioned in Chap. 1, decompression sickness (one of the two pathophysiological syndromes which are included in the more general term "decompression illness") occurs as a consequence of Henry's law [15, 16]. In fact, undersea the pressure increases by 1 atm (≈760 mmHg or 101.3 kPa) for every 10 m of depth (according to Stevin's law, as discussed in Chap. 8). As depth increases, the amount of inert gases (mainly nitrogen) dissolved in the diver's blood increases according to Henry's law (the higher the ambient pressure and, according to Dalton's law, the partial pressure of the gas, the higher its solubility). If the diver ascents too fast, the amount of gas which leaves the solution due to decompression (according again to Henry's law) exceeds the rate of washout from tissues: this leads to supersaturation (and bubble formation). Recompression treatment in a hyperbaric chamber promotes the resolution of bubbles through several mechanisms (see Chap. 1) including, of course, Henry's law.

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Bubbles, Tracheal Tube Cuffs, and Reservoir Bags: Surface Tension and Laplace's Law

5

Where's Physics

Daily life	Inflate a party balloon; turn it into a sword or a cute bunny Soap bubbles and raindrops The strange behavior of party balloons, soap bubbles, and beer foam
Physics involved	Surface tension, Laplace's law
Clinical practice	Reservoir bag in anesthesia breathing systems Tracheal tube cuff Pathophysiology of air embolism, dilated cardiomyopathy, aortic aneurysm, and alveolar collapse (with due cautions)

5.1 Physics in a Soap Bubble: Surface Tension and Laplace's Law

Laplace's law correlates the pressure difference between the inside and the outside of a bubble (or of a liquid drop) with its radius. Approximately, this relationship also applies to hollow objects with an elastic wall (including human organs) which contain a fluid (i.e., a liquid or a gas), provided that the wall is thin enough. Accordingly, examples abound. How does a soap bubble form? Why is inflating a balloon from your mouth more easy as it becomes bigger? How can you turn a party balloon into a cute bunny? When the contents of two balloons, or of two bubbles (e.g., in the beer foam), come into contact, why does the smaller one become even smaller and the bigger one further enlarge? How does an aortic aneurysm grow? (And when to undergo surgery?) How can an air bubble stop the flow in an arterial vessel? Why do we need surfactant in pulmonary alveoli? How a (good) reservoir bag can prevent barotrauma during anesthesia? Laplace's law answers, at least in part (and in certain cases with some approximation), to all these questions. To deal with it, however, we have first to remind the concept of *surface tension*.

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5.1.1 Also Liquids Care About their "Appearance": Surface Tension

All molecules of which a liquid is composed attract one another due to (rather weak) *cohesive forces* acting within a distance of a few nanometers (1 nm = 10^{-9} m). A molecule within the liquid is surrounded by other molecules from each side. Accordingly, for reasons of symmetry, the *vector sum* (or *resultant*) (see Chap. 10) of all the attractive forces which act on it is equal to zero (Fig. 5.1). Conversely, on the molecules that are located near the free surface of the liquid acts a net force which is directed toward the inside of the liquid, since such symmetry is lost. These forces are responsible for the so-called *surface tension,* which causes a liquid to always try to minimize its free surface and, accordingly, accounts for the roundish shape of raindrops, tears, and bubbles (as a sphere has the smallest possible surface area for a given volume). It is not entirely true, hence, that liquids have not a definite shape (as stated in Chap. 1), although the weak cohesive forces are easily exceeded by other forces (e.g., adhesive forces to the container) which often deprive liquids of their preferred shape!

In physics textbooks, surface tension is traditionally defined using the simple experimental device shown in Fig. 5.2. A thin liquid sheet is let form inside the space delimited by a U-shaped metallic frame and a thin metal wire which can slide along the frame without significant friction (see Chap. 11). Since the liquid surface tends to contract due to surface tension, a force F must be applied to keep the system in balance. Using a weighing scale, it can be shown that this force depends only on

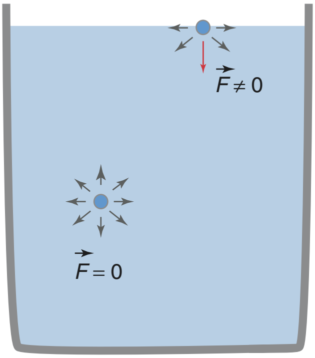
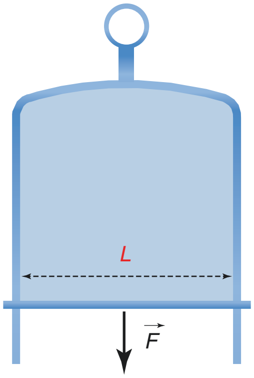


Fig. 5.1 Molecular basis of surface tension. Unlike a molecule within the liquid, which is equally attracted by other molecules from all directions, a molecule at the surface is mainly attracted toward the inside of the liquid, since the kind (and the density) of molecules which are present above the surface is different. Indeed, the values of surface tension due to the contact between two substances (e.g., liquid-to-liquid or liquid-to-gas) depend, in general, on the characteristics of both the substances. F net attractive force from the surrounding molecules

Fig. 5.2 Measuring surface tension (see text)



the frame width (i.e., on the length L of the edge of the liquid sheet). Considering that the liquid sheet has two sides, the surface tension γ can be defined as follows:

$$\gamma = \frac{F}{2L} \tag{5.1}$$

that is the *force per unit length of the edge*. It is evident that such a force is tangent to the liquid surface and perpendicular to the edge. In other words, due to surface tension, a liquid surface behaves approximately as a stretched elastic membrane.

An even more rigorous definition of surface tension relies on the concept of *work of a force* (Box 5.1). While it contracts, the liquid causes a shift of the metal wire and, accordingly, performs a work. Conversely, a work against surface tension must be performed in order to extend the liquid surface. In particular, if the metal wire moves (downward in Fig. 5.2) by a distance h, then the liquid surface will extend by an area:

$$S = 2Lh \tag{5.2}$$

where Lh is the area of a rectangle of base L and height h, multiplied by the two faces of the liquid sheet. The work needed to get the shift h (and, accordingly, the extension $2S$ of the liquid surface) is:

$$W = F \bullet h \tag{5.3}$$

The work which is needed to increase by 1 m^2 the liquid surface is W/S, and it is easy to show that this work is equal to the surface tension itself (after substituting from Eqs. 5.2 and 5.3):

$$\frac{W}{S} = \frac{Fh}{2Lh} = \frac{F}{2L} = \gamma \tag{5.4}$$

Box 5.1: Work of a Force (Mechanical Work)

By definition, the work W of a force, whose point of application moves, is the *scalar product* (or *dot product*) of the force F and the displacement d caused by the force itself, that is:

$$W = F \cdot d \cdot \cos\theta$$

where θ is the angle between the directions of the force and the displacement. In other words, W is the product of the displacement and the magnitude of the component of the force along the displacement itself (see Chap. 10). In the example of Fig. 5.2, the force F and the displacement h have the same direction ($\cos 0° = 1$), so W is simply $F \times h$ (Eq. 5.3).

Practically, work is the *energy* which is transferred to a body by a force acting on it. When you apply a force to an object, it accelerates (see Chap. 2, Box 2.1): this means that its kinetic energy increases. Similarly, surface tension can be also defined as the *surface potential energy* of the liquid per unit area (in fact, due to surface tension, the liquid has "the potential" to do a work, which means that some energy is "stored" in it).

Hence, it can be said that *surface tension is the work which is needed to extend the free surface of a liquid by a unit area.*

5.1.2 Laplace's Law

As stated above, liquids tend to assume a spherical shape due to surface tension. In fact, if the boundary surface of the liquid is a sphere (as in a raindrop or in a soap bubble), the forces due to surface tension (which are tangent to the surface) have a *resultant* which is perpendicular to the surface itself and, accordingly, lead to a pressure which "pushes" toward the center of the sphere. In order to understand how this pressure depends on surface tension and on the radius of the sphere, let's consider the soap bubble (i.e., a liquid "foil" of spherical shape) in Fig. 5.3. Imagine to inflate it a bit, so that its radius r becomes $r + \Delta r$. Remembering a little of mathematics (Box 5.2), its external surface will increase by:

Box 5.2: Two Little Memories of Mathematics (to Make Sense to Eq. 5.5)

Surface area of a sphere

The surface area of a sphere of radius R is equal to $4\pi R^2$

Square of a sum

$$(\mathrm{A}+\mathrm{B})^2 = \mathrm{A}^2 + 2\,AB + \mathrm{B}^2$$

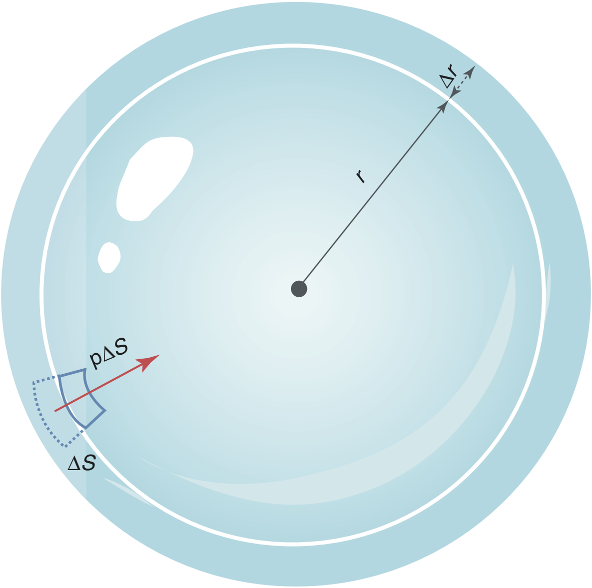


Fig. 5.3 Pressure (*p*) exerted perpendicularly to a portion Δ*S* of the (expanding) external surface of a soap bubble whose radius (*r*) increases by the value Δ*r*

$$S = 4\pi\left(r+\Delta r\right)^2 - 4\pi r^2 = 4\pi\left(r^2+\Delta r^2+2r\Delta r\right) - 4\pi r^2$$
$$= 8\pi r\Delta r + 4\pi\left(\Delta r\right)^2 \approx 8\pi r\Delta r \tag{5.5}$$

where $4\pi(\Delta r)^2$ can be neglected if Δr is very small as compared with r.

According to Eq. 5.4, the work to be done in order to obtain this size increase is:

$$W = S\gamma = 8\pi r\Delta r \bullet \gamma \tag{5.6}$$

At the same time, if p is the pressure due to surface tension and directed toward the center of the bubble, on each portion ΔS of the surface acts a force $p\Delta S$ (since pressure p is, by definition, force F per unit surface S, i.e., F/S), also directed toward the center (see Fig. 5.3). As the bubble size increases, the point of application of this force will move by a distance Δr. According to Eq. 5.3 (and to what described in Box 5.1), the work needed to extend each portion of the entire surface is the product of force (i.e., $p\Delta S$) and displacement Δr:

$$W_{\text{partial}} = p \bullet \Delta S \bullet \Delta r \tag{5.7}$$

To obtain the total work, we have to consider the sum of all the portions which form the surface, which is the surface area $4\pi r^2$ itself (see Box 5.2):

$$W_{\text{total}} = p \bullet 4\pi r^2 \bullet \Delta r \tag{5.8}$$

From Eqs. 5.6 and 5.8, is:

$$W = 8\pi r \Delta r \bullet \gamma = p 4\pi r^2 \Delta r \tag{5.9}$$

or:

$$p = \frac{2\gamma}{r} \tag{5.10}$$

This is a particular case of the so-called *Laplace's law*. Hence, the higher the radius, the lower the pressure. This is why, in the beer foam, the smaller bubbles become even smaller and the bigger ones further enlarge [1]. In fact, according to Fick's law (as discussed in Chap. 3), carbon dioxide diffuses from the smaller bubbles (where pressure is higher, according to Laplace's law) to the larger ones.

5.2 Reservoir Bags and Tracheal Tube Cuffs

If a liquid bubble behaves as if it were surrounded by an elastic membrane, an hollow object which really has an elastic wall behaves (approximately) in the same way. When you inflate a balloon from your mouth, you make less effort as it becomes bigger: in fact, the pressure within the balloon decreases as its radius increases

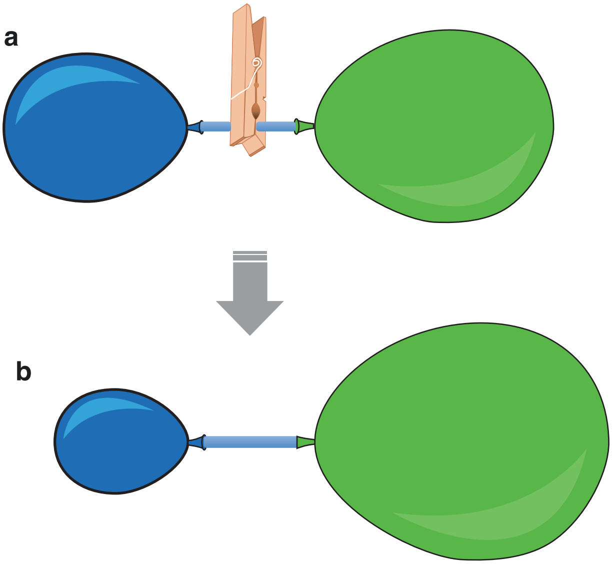


Fig. 5.4 In order to verify the effects of Laplace's law, you just need a couple of colored balloons, a straw, some adhesive tape, and a clothespin (or something like that). First put the clothespin on the straw, so as to prevent air flow between the two ends. Then, insert each end of the straw in a partially inflated balloon (and secure them with a bit of adhesive tape), making sure that one of the two balloons has been inflated more than the other. So you built the system shown in (**a**). Now just remove the clothespin and see the smaller balloon becoming even smaller and the bigger one further enlarging (**b**). This occurs because the pressure is higher in the balloon with the smallest radius, according to Laplace's law

(actually, this happens only after the balloon has reached a certain size, since, unlike what occurs in liquid bubbles due to surface tension, the work needed to extend a "real" elastic membrane depends on its previous extension. At the beginning, the resistance to stretching leads to an increase in pressure [2]). If you have any doubts, you can easily perform, at your home, the "experiment" shown in Fig. 5.4.

5.2.1 Some Unexpected Help from the Reservoir Bag

The reservoir bag in an anesthesia machine allows manual ventilation as well as a "visual" monitoring of spontaneous breathing. Moreover, thanks to Laplace's law, it can prevent barotrauma in case of malfunction or unintentional closing of the APL (adjustable pressure limiting) valve [3]. In fact, in the presence of an overflow or a flow obstruction in the breathing system, the radius of the reservoir bag increases (Fig. 5.5) and, according to Laplace's law, the pressure inside it decreases, thus preventing a dangerous rise in pressure in the entire breathing system and, consequently, in lungs. Accordingly, a reservoir bag which feels stiff should be replaced, since its *wall tension* (which we can define, similarly to surface tension γ, as the work required to extend the surface of an elastic membrane by a unit area) will be higher, for the same radius (or its radius will increase by a lesser extent for the same value of wall tension), thus providing a lower "pressure relief" (just have a look to Eq. 5.10).

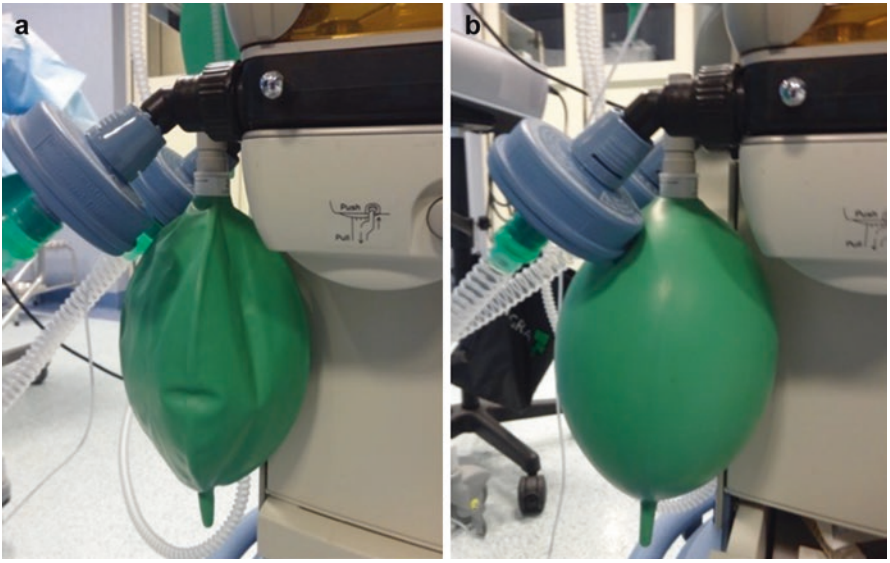


Fig. 5.5 A reservoir bag in an anesthesia machine. (**a**) With APL valve fully open, its radius and wall tension are small. (**b**) With APL valve closed, both wall tension and radius increase, so that the pressure inside remains constant (or increases slightly)

5.2.2 Monitoring Cuff Pressure: Can We Trust Our Fingers?

We all know that the inflation pressure of the cuff of an endotracheal (or tracheotomy) tube should be around 20–25 mmHg, and that an excessive pressure may cause mucosal ischemia and, consequently, may lead to tracheomalacia or tracheal stenosis [4]. Although pilot balloon palpation (the so-called finger-pressure technique) is a widespread method to estimate the cuff pressure (especially in the operating room), it should not be used due to its poor accuracy [5]. In particular, it has been shown that this technique often leads to significantly higher cuff pressures than recommended [6, 7], and that anesthesia practitioners, regardless of their experience, fail to identify even very high (about 88 mmHg!) cuff pressures [8]. This too can be explained, in part, by Laplace's law [9]. In fact, although the pressure p inside the pilot balloon is practically the same that in the cuff (according to *Pascal's principle*, as we will see in Chap. 8), what we really feel with our fingers is the *wall tension* T of the pilot balloon, that is (rearranging Eq. 5.10 and replacing surface tension γ with T):

$$T = \frac{p\,r}{2} \tag{5.11}$$

Since the radius of the pilot balloon is smaller than that of the cuff, its wall tension is lower as compared with the cuff: accordingly, the pilot balloon will always seem more "flabby" than it is the cuff indeed. If we could touch the cuff directly, we would be certainly more "accurate" in estimating its pressure!

5.3 Heart, Vessels, and Lungs

Laplace's law (written in various ways and adapted to different situations) is often invoked as part of cardiovascular and pulmonary physiology, as well as of pathophysiology of several diseases such as dilated cardiomyopathy, aortic stenosis, and aortic aneurysms. Actually, things are often much more complex, although the use of Laplace's law maintains its "educational" value. Finally, Laplace's law explains how an air bubble can counteract blood pressure and, consequently, stop circulation in an arterial vessel (in this case, it is exactly what happens).

In cardiovascular physiology, Laplace's law is often formulated in the following way:

$$\sigma = \frac{p\,r}{2h} \tag{5.12}$$

where σ is *wall stress* (i.e., tension acting over a cross-sectional area) and h is wall thickness (which appears due to the different definitions of wall "tension" and "stress"). Equation 5.12 applies to a spherical chamber (e.g., left ventricle, "sphere-shaped" aneurysms), while it becomes $\sigma = pr/h$ for a cylindrical vessel (that is, wall stress in a sphere for a given pressure is one half as compared with a cylinder with the same radius).

5.3.1 Left Ventricular Hypertrophy and Dilated Cardiomyopathy

A persistent increase in left ventricle (LV) afterload (such as in aortic stenosis) leads to an increased LV pressure and, according to Laplace's law (Eq. 5.12), to a higher wall stress. Consequently, the myocardial oxygen consumption increases due to the greater energy which is needed in order to develop such increased tension. As a compensatory mechanism, LV hypertrophy develops (that is, wall thickness increases) to reduce wall stress despite the increased LV pressure [10, 11].

In dilated cardiomyopathy, LV radius increases and, accordingly, a greater wall tension is needed to develop the same LV pressure. Heart failure occurs when wall tension to be developed becomes "too much." Of course, LV *is not* a sphere and, above all, wall stress is not homogeneously distributed along its thickness [10].

5.3.2 Aortic Aneurysm

An aortic aneurysm grows just like a party balloon (those of elongated shape, which are turned into cute bunnies by exploiting different radii of curvature, according to Laplace's law). When a weak spot develops in the aortic wall, the resulting expansion increases the diameter and, accordingly (Eq. 5.12), wall stress in that spot. As a consequence, the (weakened) vessel further enlarges [12]. Aneurysms often tend to assume a spherical shape in order to minimize wall stress (as stated, wall stress in a sphere is one half compared with a cylinder with the same radius). This, unfortunately, does not prevent wall rupture, which occurs when wall stress exceeds wall strength.

Surgeons mainly rely on diameter (and its progression) in order to estimate, according to Laplace's law, aortic wall stress and, hence, the risk of rupture and the indication for surgery [13, 14]. However, such an approach has been recently criticized, primarily because wall stress is not uniformly distributed within aortic wall, and other factors such as longitudinal stretch and vessel curvature should be taken into account [13–15].

5.3.3 Pulmonary Alveoli Are Not a House of Cards

Also the belief that the smaller pulmonary alveoli can exist only thanks to the presence of surfactant (which reduces wall tension and, accordingly, internal pressure for a given radius) and that, in the absence of it, they would inexorably converge in a unique big alveolus (because of Laplace's law) should be reassessed [16]. Of course, surfactant plays a key role in preventing the collapse of small airways. However, alveoli also have a solid anatomy that supports them.

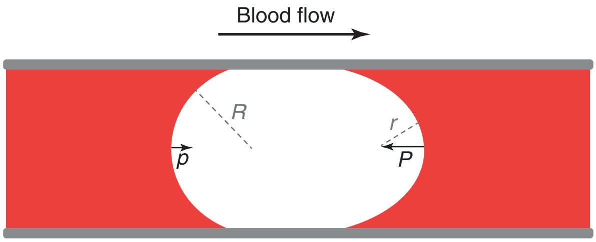


Fig. 5.6 An air bubble occluding a blood vessel. According to Laplace's law, the pressure *P* directed toward the center of the bubble (in the opposite direction to the blood flow) and acting on the meniscus with the lower radius of curvature (*r*) is higher than the pressure *p* (also directed toward the center of the bubble, but in the same direction of the blood flow) acting on the meniscus with the greater radius of curvature (R). Note that surface tension is the same, since it is due to the air-to-blood contact in both cases

5.3.4 Air Embolism

In Chaps. 1 and 2, we worried about air bubbles which could enter blood vessels during diving or cardiac surgery, respectively. Indeed, a gas bubble can develop a pressure which is sufficient to counteract the pressure difference that allows blood to flow (according to Hagen–Poiseuille equation, see Chap. 7). Due to the "force" exerted by blood, the two menisci of the bubble will deform as shown in Fig. 5.6 (in order to demonstrate this, we should rely again on surface tension, but with the help of some trigonometry). According to Laplace's law, the pressure due to surface tension in the meniscus which has the smaller radius of curvature is higher than that acting on the meniscus at the other side of the bubble. This pressure difference can be enough to counteract blood flow, causing ischemia.

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Part II

Fluids in Motion: Masks, Tubes, and Hemodynamics

The Venturi Mask Works (In Part) Like an Airplane: Continuity Equation and Bernoulli's Theorem

6

Where's Physics

Daily life	How an airplane flies Games with the hairdryer Watering the more distant plants
Physics involved	Bernoulli's theorem Continuity equation
Clinical practice	Venturi mask (Bernoulli's theorem and continuity equation) Echocardiographic determination of valve area in aortic or mitral stenosis (continuity equation) Echocardiographic estimation of pressure gradients (Bernoulli's theorem)

6.1 Garden Hoses and Heart Valve Stenosis: Continuity Equation

When you are watering a garden, you can reach the most distant areas by simply partially closing the hose with a thumb. In fact, the water speed increases when it flows through a decreased cross-sectional area. This occurs according to the so-called *continuity equation*, also known as *Leonardo's law* (after Leonardo da Vinci).

Figure 6.1 shows an ideal fluid flowing from the left to the right end of a tube segment with varying cross section. As an ideal fluid, we assume that it is incompressible (i.e., its density is constant), its *viscosity coefficient* is equal to zero (i.e., there are no internal forces which oppose the motion), and its flow is *laminar* (or *steady*), i.e., the speed of fluid at any point does not vary with time (see Chap. 7). Since the fluid is incompressible, the same volume ΔV which enters the tube at the left end during the time interval t must come out from the right end during the same time interval.

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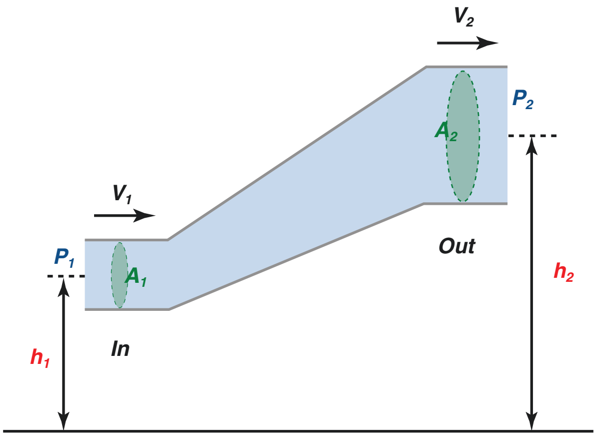

Fig. 6.1 A tube segment through which an ideal fluid flows from the input end (*In*) to the outlet end (*Out*). h_1, A_1, p_1, and v_1 and h_2, A_2, p_2, and v_2 are, respectively, the elevation, cross-sectional area, fluid pressure, and fluid speed at the two ends

If a "particle" of the fluid moves by the distance Δx during the time interval t, then the volume ΔV which flows through the cross-sectional area A during the same time interval is:

$$\Delta V = A\Delta x \tag{6.1}$$

(remembering that the volume of a cylinder with base area B and height h is the product of B and h). Since, as recalled in Chap. 3 (Box 3.1), the average velocity v of a body is the distance Δx which the body travels during the time t ($v = \Delta x/t$ and, accordingly, $\Delta x = vt$), Eq. 6.1 can be written as follows:

$$\Delta V = A\,v\,t \tag{6.2}$$

where v is fluid speed.

If we apply Eq. 6.2 to the two ends of the tube segment of Fig. 6.1, we have:

$$\Delta V = A_1\,v_1\,t = A_2\,v_2\,t \tag{6.3}$$

that is:

$$\frac{\Delta V}{t} = A_1\,v_1 = A_2\,v_2 = \text{a constant} \tag{6.4}$$

where v_1, A_1, v_2, and A_2 are, respectively, the fluid speeds and the cross-sectional areas at the left end and at the right end of the tube, and $\Delta V/t$ is the so-called *volume flow rate* (that is, in a blood vessel, the cardiac output).

Hence, *the volume flow rate of an ideal fluid which flows in a tube is the same all along the tube*. Since the volume flow rate is the product of fluid speed and cross-sectional area, *the flow speed must increase where the cross-sectional area decreases* (continuity equation).

This law is used sometimes by Formula 1 or motorbike racers (who won the race) to drench with Champagne much people as possible (including those more distant, who can be reached by partially closing the mouth of the bottle with the thumb, as with a garden hose). Most importantly, it allows echocardiographers to calculate valve area as part of the evaluation of the severity of mitral or aortic stenosis [1, 2], particularly when planimetry is not possible (e.g., heavily calcified valve) or poorly reliable (e.g., associated valve regurgitation). According to continuity equation, as an example, the product of the aortic valve area (AVA) and the blood flow velocity through it, e.g., measured as aortic velocity-time integral $\mathrm{VTI}_{\mathrm{Ao}}$ (see Chap. 15), must be equal to the product of the cross-sectional area of the left ventricle outflow tract ($\mathrm{CSA}_{\mathrm{LVOT}}$) and the VTI at this level ($\mathrm{VTI}_{\mathrm{LVOT}}$) [3]. Accordingly, AVA can be calculated as follows:

$$AVA = \frac{CSA_{\mathrm{LVOT}} \times VTI_{\mathrm{LVOT}}}{VTI_{Ao}} \tag{6.5}$$

Leaving aside this stuff for echocardiographers (and for anesthesiologists who are interested in echocardiography), we needed to know continuity equation in order to understand Bernoulli's theorem and how a Venturi mask works.

6.2 How Does an Airplane Fly? Bernoulli's Theorem

(Notice: if you really have never "digested" physics and/or mathematics, you can skip directly to Eq. 6.12. Conversely, the brave who want to fully understand can continue reading below). Let's consider again the tube segment shown in Fig. 6.1, in which an ideal fluid flows from the section A_1, which lies at a height h_1, to the section A_2 at a height h_2. As stated above, since the volume flow rate is constant, the fluid speed is inversely proportional to the cross-sectional area: accordingly, the speed v_2 of the fluid as it passes through the larger area A_2 is lower than the speed v_1 as it passes through the smaller area A_1. Finally, p_1 and p_2 are the pressures at the input and at the outlet, respectively.

Since we assumed that the fluid is nonviscous, its motion occurs without energy loss. In other words, the total mechanical energy of the system does not change (*principle of conservation of mechanical energy*). One aspect of this principle is the so-called *work-kinetic energy theorem*, which states that the work W that a force does on a body (see Chap. 5, Box 5.1) is equal to the change in kinetic energy (ΔE_{k}) of the body to which that force is applied:

$$W = \Delta E_{\mathrm{k}} \tag{6.6}$$

Remembering the definition of kinetic energy (see Chap. 1, Box 1.2), for a "particle" of fluid with mass m whose speed changes from v_1 to v_2, is:

$$\Delta E_{\mathrm{k}} = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2 \tag{6.7}$$

and, accordingly, the work-kinetic energy theorem can be written as follows:

$$W = \frac{1}{2} m v_2^2 - \frac{1}{2} m v_1^2 \tag{6.8}$$

where W is the work done by all the forces acting on the particle, which are, on the one hand, *gravity* (namely, the weight of the fluid particle) and, on the other hand, "surface" forces which push the fluid at the input level and are responsible for the pressure difference between p_1 and p_2.

The work done on the fluid particle by gravitational forces during the ascent from the height h_1 to the height h_2 is negative because the upward displacement h_2-h_1 has an opposite direction to that of gravity. As discussed in Chap. 5 (Box 5.1), work is the dot product of force F and displacement Δx. In this case, the displacement is $\Delta x = h_2-h_1$, the force is the weight of the particle, i.e., the product of its mass m and free-fall acceleration g (see Chap. 2, Box 2.1), and the angle between force and displacement is zero (see again Box 5.1). Accordingly, the "gravitational" work W_g is:

$$W_\mathrm{g} = -m\,g\left(h_2 - h_1\right) = m\,g\,h_1 - m\,g\,h_2 \tag{6.9}$$

Regarding the "surface" forces, imagine that the pressure difference p_1-p_2 makes the fluid particle move by the tract Δx. Then, the work W_p done due to this pressure difference is:

$$W_\mathrm{p} = F\Delta x = \left(p_1 - p_2\right) A\Delta x = \left(p_1 - p_2\right) A \frac{V}{A} = \left(p_1 - p_2\right) V = \left(p_1 - p_2\right)\frac{m}{d} \tag{6.10}$$

where: F has been replaced with $(p_1 - p_2)A$ remembering that, by definition, pressure is force per unit area ($p = F/A$ and, accordingly, $F = pA$); Δx has been replaced with V/A remembering that the volume V of a cylinder of base area A and height Δx is $V = A\Delta x$ (and, accordingly, is $\Delta x = V/A$); and, finally, V has been replaced with m/d remembering that, by definition, density is mass per unit volume ($d = m/V$ and, hence, $V = m/d$).

Substituting by Eqs. (6.8, 6.9, and 6.10), the work-kinetic energy theorem (Eq. 6.6) can then be written as follows:

$$W = W_\mathrm{g} + W_\mathrm{p} = m\,g\,h_1 - m\,g\,h_2 + \left(p_1 - p_2\right)\frac{m}{d} = \frac{1}{2} m v_2^2 - \frac{1}{2} m v_1^2 \tag{6.11}$$

or, after some simple rearrangements (divide by m and multiply by d all the terms):

$$d\,g\,h_1 + p_1 + \frac{1}{2} d v_1^2 = d\,g\,h_2 + p_2 + \frac{1}{2} d v_2^2 = \text{a constant} \tag{6.12}$$

that is the so-called *Bernoulli's equation* (or Bernoulli's theorem), where h_1, p_1, and v_1 are the elevation, pressure, and speed of a fluid entering a tube at one end, h_2, p_2, and v_2 the corresponding variables for the fluid emerging at the other end (Fig. 6.1), d is the fluid density, and g is the free-fall acceleration.

Of note, a simplified version of Bernoulli's equation allows echocardiographers to estimate maximum pressure gradients (e.g., across a stenotic aortic valve) from the maximum jet velocity measured by Doppler analysis [4] (but this, together with Eq. 6.5 mentioned above, is "physics for echocardiographers" which is discussed in depth in echocardiography textbooks. However, you can find some other note on these topics in Chap. 15, when we will talk about the Doppler effect).

6.2.1 And Now…Let This Plane Fly!

Bernoulli's theorem tells us something very important when considering the flow of a fluid that travels along a horizontal streamline, so that its elevation does not change. In this case, we have $h_1 = h_2$ and the previous equation becomes:

$$p_1 + \frac{1}{2} d v_1^2 = p_2 + \frac{1}{2} d v_2^2 = \text{a constant} \tag{6.13}$$

This means that *where the speed is higher the pressure is lower, and vice versa*. Bernoulli's theorem remains valid even for a fictitious *tube of flow*, i.e., in the absence of a real conduit. You can verify this at your home with a hairdryer and a sheet of paper. Place the sheet on a table, turn on the hairdryer, and approach it to the middle of the sheet, maintaining the air jet parallel to the sheet. At this point, air speed above the sheet will be much higher than below it and, according to Eq. 6.13, the pressure below the sheet will be higher than the pressure above it. Hence, you should see the sheet that rises up and "adheres" to the hairdryer. This is not very easy to perform: the sheet will fly away many times until you don't find the right "technique." However, if you succeed, you will see with your own eyes the Bernoulli's theorem at work and will discover how a plane flies. In fact, with an extreme simplification, it can be said that the particular airfoil shape of the wings (the top more convex than the bottom) yields a higher speed of air above wings and a lower speed below them, due to the greater distance which, during the same time, is traveled by air near the upper surface of the wings as compared to the lower surface (Fig. 6.2). According to Bernoulli's theorem, this implies a lower pressure on the top surface of the wings and a higher pressure on the bottom surface, hence generating the lift in the upward direction [5]. The airplane takes off when its speed is sufficient so that such upward lift overcomes its weight.

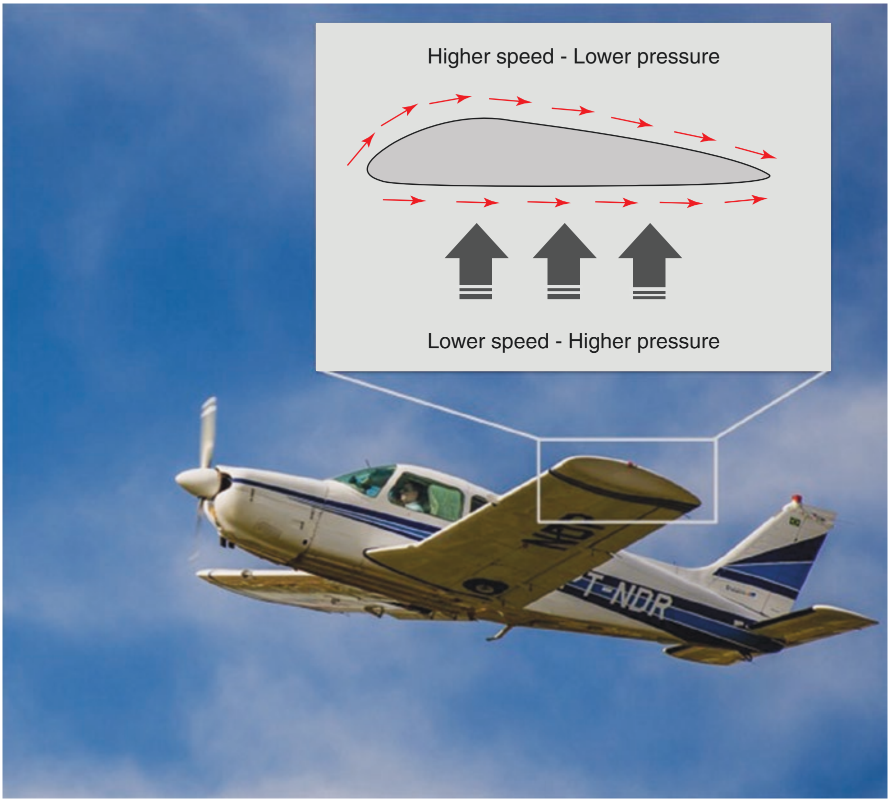

Fig. 6.2 How an airplane flies. Since the top of the wing is more curved than its bottom, the air flow (*red arrows*) has a higher speed near the upper surface as compared to the lower surface. This accounts, according to Bernoulli's theorem, for the pressure difference which generates the so-called "lift" (*upward arrows*)

6.3 Continuity and Bernoulli's Equations Work Together in a Venturi Mask

Even if someone in the past didn't agree [6], it is now generally accepted that also the way the Venturi mask works relies on Bernoulli's theorem [7, 8]. In a Venturi mask (Fig. 6.3a), 100% oxygen flows through tubing at a specified flow rate. At the level of the so-called Venturi valve (Fig. 6.3b), the pipe narrows (i.e., its cross-sectional area decreases) and there is a side port. According to continuity equation, the speed of oxygen increases as it flows through such narrowing and, according to Bernoulli's theorem, this leads to a drop in pressure (at subatmospheric values) that allows entrainment of atmospheric air from the side port. Different preformed Venturi valves, of different colors (Fig. 6.3c), allow the administration of *predictable* inspiratory oxygen fractions (FiO_2) (the greater the narrowing, the lower the FiO_2), provided that the oxygen flow rate indicated on each Venturi valve is used.

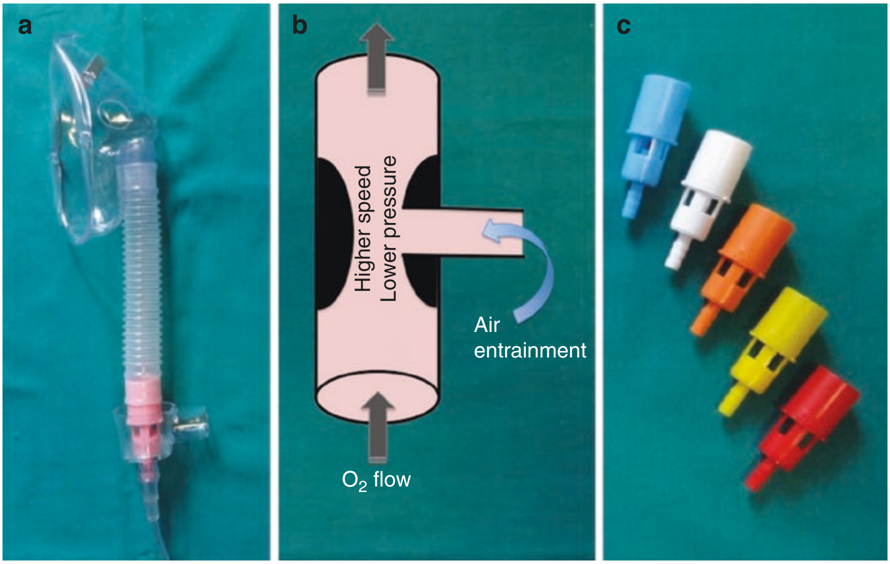

Fig. 6.3 (**a**) A Venturi mask assembled with a 50% (*pink*) Venturi valve. (**b**) Scheme and principle of operation of the Venturi valve (see text). (**c**) Different Venturi valves allow the administration of different inspired oxygen fractions: *blue*, 24%; *white*, 28%; *orange*, 31%; *yellow*, 35%; *red*, 40%. O_2 Oxygen

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From Tubes and Catheters to the Basis of Hemodynamics: The Hagen–Poiseuille Equation

7

Where's Physics

Daily life	"True" liquids and their motion (why we really need a heart) Pour ketchup and mayonnaise
Physics involved	Viscosity, Hagen–Poiseuille equation
Clinical practice	Endotracheal/tracheotomy tube and weaning from mechanical ventilation Work of breathing Peripheral vs. central venous catheter "Resistance" of an epidural or pulmonary artery catheter Understand hemodynamic monitoring and management: the relationship among blood pressure, cardiac output, and vascular resistances Apply ultrasound gel on the probe

7.1 Real Fluids Flow in a Different Way: Viscosity and Hagen–Poiseuille Equation

According to Bernoulli's theorem (see Chap. 6, Eq. 6.12), the pressure of an ideal fluid which flows through a horizontal tube with constant cross-sectional area is the same all along the tube: in fact, the elevation does not change and, due to continuity equation (Chap. 6, Eq. 6.4), also the fluid speed is constant (since the cross-sectional area is constant). Conversely, if we measure pressure along a tube in which a *real* fluid flows, we will find that it gradually decreases down the direction of flow. This occurs because, unlike ideal fluids (which we assumed to be nonviscous), real fluids are characterized by *viscosity*, i.e., "internal friction" forces which oppose to motion and, accordingly, cause energy loss during it.

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Let's briefly define viscosity, so as to be able to discuss the law which describes the flow of a real fluid through a cylindrical tube and its many implications in anesthesiology practice.

7.1.1 Viscosity

Figure 7.1 shows a plate which moves on the free surface of a liquid. As we will see in Chap. 11, according to *Newton's first law* (also referred as "law of inertia"), no (net) forces act on a body which is moving with *constant* speed on a frictionless surface. Accordingly, if a force F is needed in order to maintain the plate in uniform rectilinear motion (i.e., at a constant speed), it means that the liquid surface is not frictionless, and that such a force F must be equal to the friction forces: in fact, if it were lower, the plate could not move (since friction forces are those which oppose to motion), while if it were higher, its motion would be accelerated according to Newton's second law (see Chap. 2). Practically, the liquid layer which is in contact with the surface S of the plate will move with the same speed v of the plate itself, while the layers below will slide on each other with a gradually decreasing speed (due to friction), up to the layer near the bottom of the container, which is motionless. Experimentally, it is found that the friction force F_f is directly proportional to the surface S and the speed v of the plate, and inversely proportional to the height h of the liquid in the container:

$$F_f = \mu S \frac{v}{h} \tag{7.1}$$

where μ is the so-called *dynamic viscosity coefficient*, which depends on the fluid and varies (exponentially) with temperature (the lower the temperature, the higher the viscosity). As an example, at 0 °C, the viscosity coefficient of mercury is about 1.5 times that of water.

It can be said that viscosity is a measure of how much a fluid resists to flow (or to deformation by *shear stress*). While in liquids it is due, as mentioned, to the sliding of superimposed layers, viscosity of gases is due to the collisions among the gas molecules.

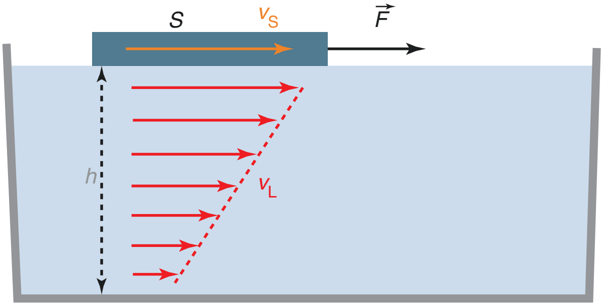

Fig. 7.1 Measuring the dynamic viscosity coefficient of a liquid (see text). S contact surface between the plate and the liquid; F force; h height; v_S plate speed; v_L speed of liquid layers

7.1.2 Hagen–Poiseuille Equation

Because of viscosity, a real fluid flows through a tube only if there is a pressure difference between its two ends. In fact, it is necessary to do a work (and, hence, apply a net force) against the friction forces in order to maintain the fluid in motion (that's why we need a heart!). Moreover, if the fluid speed is not too high (see Box 7.1), the flow of a real fluid is *laminar* (Fig. 7.2), that is, it occurs in parallel concentric layers which slide on one another with a parabolic velocity profile: the fluid speed is maximum in the middle and decreases gradually in the outer layers (something that is easy to understand considering what we said in Sect. 7.1.1).

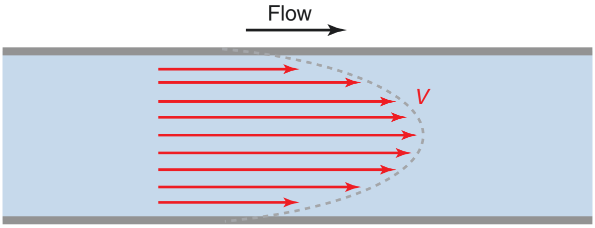

Fig. 7.2 Speed (v) profile of laminar flow. To better understand this figure, look again at Fig. 7.1 (in which the free surface of the liquid has the maximum speed, while the bottom of the container is motionless), and consider that here the whole pipe wall and, accordingly, the upper and lower ends in the two-dimensional representation, are motionless

Box 7.1: Laminar Vs. Turbulent Flow

It can be predicted if the flow of a fluid through a conduit would be laminar or *turbulent* (i.e., with chaotic changes in pressure and speed) using the so-called *Reynolds number* (Re):

$$\mathrm{Re} = \frac{2r\mathrm{V}d}{\mu} = \frac{2r\mathrm{V}}{\nu}$$

where r is the radius of the conduit, V is the mean speed of the fluid, d is fluid density, μ is its dynamic viscosity coefficient, and ν is the so-called *kinematic viscosity coefficient*, i.e., the ratio between dynamic viscosity coefficient and density ($\nu = \mu/d$).

The flow in a cylindrical conduit will be mostly laminar if Re is below a "critical" value of about 2000, while it will be mostly turbulent if it is $\mathrm{Re} > 4000$ (for intermediate values, the two types of flow coexist).

Accordingly, the flow of a given fluid (with its own viscosity and density) in a pipe will become turbulent when its speed will be higher than a certain value. The "sounds" coming from the heart, lungs, and arteries, which we listen to with our stethoscopes, are always caused by turbulent flow. As an example, where the carotid artery diameter is significantly reduced due to an atherosclerotic plaque, blood velocity increases (according to continuity equation) and the flow becomes turbulent, so that the typical “bruit” can be heard. Note that the “critical” Re value is lower than 2000 in blood vessels, due to complex vascular geometry and pulsatile flow patterns [1].

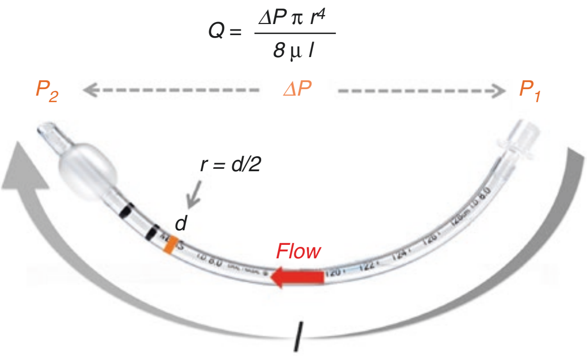

Fig. 7.3 Hagen–Poiseuille equation (for an endotracheal tube). Q volume flow rate; $\Delta P = P_1 - P_2$ pressure difference between the two ends; r radius; d diameter; μ dynamic viscosity coefficient (of air); l length

Hagen–Poiseuille equation states that, for a viscous fluid which moves with laminar flow through a cylindrical conduit of radius r and length l (such as the endotracheal tube in Fig. 7.3), the volume flow rate Q (as defined in Chap. 6) is directly proportional to the pressure difference ΔP between the two ends of the conduit and to the fourth power of its radius, and inversely proportional to the viscosity μ of the fluid and to the length of the conduit:

$$Q = \Delta P \frac{\pi r^4}{8 \mu l} \tag{7.2}$$

Accustomed as we are, as anesthesiologists and/or intensivists, to use each type of “conduit” (tracheal tubes, venous catheters, and so on), Hagen–Poiseuille equation is probably the physical law to which we are faced more often in our daily clinical practice. Some examples are discussed in the next section.

7.2 Tubes and Catheters: Some Implications of Hagen–Poiseuille Equation

When we administer intravenous fluids to a patient, the flow rate through the venous catheter is, according to Hagen–Poiseuille equation, directly proportional to the pressure difference between the solution bag and the vein, which is, in turn, proportional to the height of the bag with respect to the patient, according to *Stevin's law* (see Chap. 8). If rapid fluid resuscitation is required, a shorter intravenous catheter is preferable to a longer one of the same gauge [2, 3]. In fact, according again to Hagen–Poiseuille equation, the volume flow rate produced by the same pressure difference will be higher through a lower length catheter. Hence, although a triple-lumen central venous catheter usually includes a 16-gauge lumen, a 16-gauge peripheral venous catheter will allow a faster fluid administration since it is much shorter than an 8-in. (about 20 cm) central venous catheter. Moreover, we should not be surprised if packed red blood cells, as well as high-concentration glucose solutions, drip more slowly than saline, which has a lower viscosity.

One of the reasons why tracheotomy reduces the *work of breathing* (*WOB*) in mechanically ventilated patients is that tracheotomy cannulas are usually shorter than endotracheal tubes [4]. In fact, remembering the "physical" definition of work (see Chap. 5, Box 5.1), we can define WOB, roughly, as follows:

$$\mathrm{WOB} = F\,\Delta x = \Delta P\,A\,\Delta x = \Delta P\,A\frac{V}{A} = \Delta P\,V \tag{7.3}$$

where, similarly to what we did in Chap. 6 (Eq. 6.10), F has been replaced with $\Delta P\,A$ remembering that, by definition, pressure is force per unit area ($p = F/A$ and, accordingly, $F = pA$) and Δx (i.e., the "displacement") has been replaced with V/A remembering that the volume V of a cylinder of base area A and height Δx is $V = A\Delta x$ (and, accordingly, is $\Delta x = V/A$).

Hence, WOB is proportional to the pressure difference ΔP which is needed to develop the tidal volume (the so-called *driving pressure*). According to Hagen–Poiseuille equation, the patient will need to develop a lower ΔP to receive the same gas flow if the length of the tube is lower and, thus, he/she will do less work.

Furthermore, a larger internal diameter tracheotomy or endotracheal tube (ETT) may facilitate (and accelerate) the weaning from mechanical ventilation. In fact, for a given pressure difference, the patient will receive a much greater gas flow through a slightly larger tracheotomy cannula or ETT (since flow depends on the fourth power of radius) and, accordingly, the same flow will be obtained with a much lower driving pressure, again reducing WOB [5].

Finally, it is clear why a so high pressure of your thumb is needed to inject a liquid through a (tight and very long) epidural or pulmonary artery catheter.

7.3 Hagen–Poiseuille Equation and Hemodynamics

If we call *resistance* (R) the quantity $8\mu l/\pi r^4$ (flow, in fact, is hindered by the increase in length and viscosity, and favored by the increase in radius), Eq. 7.2 becomes:

$$Q = \frac{\Delta P}{R} \tag{7.4}$$

or

$$\Delta P = Q\,R \tag{7.5}$$

In the latter form, Hagen–Poiseuille equation is sometimes referred as the *Ohm's law of hydrodynamics*, due to its analogy to *Ohm's law for electric current*:

$$\text{P.D.} = i\,R_{\text{e}} \tag{7.6}$$

where P.D. is the potential difference (i.e., the voltage, a concept similar to the pressure difference), i the current intensity (defined similarly to the flow rate), and R_{e} the electric resistance [1, 6]. When applied to the cardiovascular system, these equations express the well-known relationship between arterial pressure and cardiac output (CO) and define systemic (or pulmonary) vascular resistance:

$$\text{MAP} - \text{CVP} = \text{CO} \times \text{SVR} \;\text{ or }\; \text{SVR} = \frac{\text{MAP} - \text{PVC}}{\text{CO}} \tag{7.7}$$

$$\text{MPAP} - \text{PAOP} = \text{CO} \times \text{PVR} \;\text{ or }\; \text{PVR} = \frac{\text{MPAP} - \text{PAOP}}{\text{CO}} \tag{7.8}$$

where MAP and MPAP are, respectively, mean arterial pressure and mean pulmonary artery pressure, CVP is central venous pressure, PAOP is pulmonary artery occlusion pressure, and SVR and PVR are, respectively, systemic and pulmonary vascular resistances. Cardiac output monitors use Eqs. 7.7 and 7.8 to display a "calculated value" of SVR and PVR, respectively.

We usually identify vascular resistance with the vascular "tone," namely the degree of vasodilation/vasoconstriction (which is, of course, its main determinant to be considered in clinical practice). However, this is a simplified way to define vascular resistance. In particular, from Eq. 7.2 it is clear that vascular resistance may also vary with blood viscosity, which is not constant [7]. Indeed, it is inversely related to *temperature* (as mentioned in Sect. 7.1.1) and increases with *hematocrit* [1]: both these effects can become significant during (hypothermic) cardiopulmonary bypass. Moreover, due to the presence of red blood cells, blood behaves as a *non-Newtonian fluid* (such as toothpaste, ketchup, mayonnaise, and ultrasound gel, to name but a few). Unlike *Newtonian fluids*, which have a characteristic viscosity (at a given temperature), the viscosity of non-Newtonian fluids depends on the "intensity" of the shear stress which is applied on them (expressed as *shear rate*, i.e., the rate at which a shearing deformation is applied to a body, or the speed difference between two points of the body divided by the distance between them) [8]. For example, it is easier to pour ketchup on a hamburger, or ultrasound gel on the probe (see Chap. 15), if you shake the bottle before: in fact, doing so, you apply a high shear rate to the fluid and, accordingly, its viscosity decreases (so that it comes out from the tight mouth of the bottle more easily) [9]. Similarly, blood viscosity increases at lower flow rates, affecting especially microcirculation [1, 8].

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Part III

Hemodynamic Monitoring

Toothpaste, Sea Deeps, and Invasive Pressure Monitoring: Stevin's Law and Pascal's Principle

8

Where's Physics

Daily life	"Under the sea" (or "Up in the sky") Squeezing toothpaste Lift a car (in a mechanical workshop) Save the life of someone who is choking (or who is in cardiac arrest)
Physics involved	Stevin's law and Pascal's principle
Clinical practice	Verify the inflation of a tracheal tube cuff Heimlich maneuver External cardiac massage Invasive pressure monitoring Measuring pulmonary artery occlusion pressure Height of the pressure transducer relative to the patient (leveling) Pressure transducer zeroing Phleboclysis

8.1 Fluids at Rest: Stevin's Law and Pascal's Principle

Chapters 6 and 7 have dealt with the motion of fluids, i.e., that part of *fluid mechanics* which is known as *fluid dynamics*. The so-called *statics of fluids*, on the contrary, deals with fluids which are "at rest." Below we will discuss two simple laws of fluid statics which are the basis of invasive pressure monitoring.

First, though, it is useful to recall the definitions of pressure and density, although both have been already mentioned in previous chapters.

In general, a force *F* which acts perpendicularly (and *uniformly*) on a flat surface *A* exerts a *pressure p* which can be defined as:

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$$p = \frac{F}{A} \tag{8.1}$$

that is the force per unit area. In particular, the pressure which is exerted by a fluid at rest due to its weight (which, as mentioned in Box 2.1, is a force) is called *hydrostatic pressure* and, as we will see shortly, depends on the depth within the fluid.

The *density d* of a fluid sample of mass *m* and volume *V*, *which has a uniform density*, is:

$$d = \frac{m}{V} \tag{8.2}$$

or the mass per unit volume. Of course, according to what discussed in Chap. 1, while the density of a liquid is (substantially) constant, that of a gas depends largely on pressure and temperature (in fact, the volume of a given mass of any gas varies with these two properties, according to the ideal gas law).

Note that in both definitions (Eqs. 8.1 and 8.2) we assumed, respectively, that the force is "uniformly applied" and, similarly, the density is "uniform." Otherwise, we should resort to the mathematical concept of *limit* for a correct (and more general) definition of these quantities. For example, the density of a fluid is, more rigorously, the limit of the ratio between its mass and volume for an infinitesimal volume of that fluid (however, "mass divided by volume" is enough for our purposes!).

In practice, the density of a substance is often reported as its *specific gravity*, i.e., the ratio between its density and that of water (which is around 1000 kg/m^3).

8.1.1 Under the Sea: Stevin's Law

Consider a given volume of liquid in a cylindrical container, e.g., a glass of water (Fig. 8.1). The force that this liquid exerts on the container base (which has an area *A*) is nothing but its weight *W*, that is, according to Newton's second law (see Chap. 2, Box 2.1):

$$W = mg \tag{8.3}$$

where *m* is the mass of fluid which is contained in the glass and *g* is free-fall acceleration.

The hydrostatic pressure p_i which is exerted on the base area *A* is, hence (Eq. 8.1):

$$p_\mathrm{i} = \frac{W}{A} = \frac{mg}{A} \tag{8.4}$$

According to Eq. 8.2, mass is the product of volume and density. Moreover, the volume *V* of a cylinder with base area *A* and height *h* is the product of *A* and *h*. After these substitutions, Eq. 8.4 becomes:

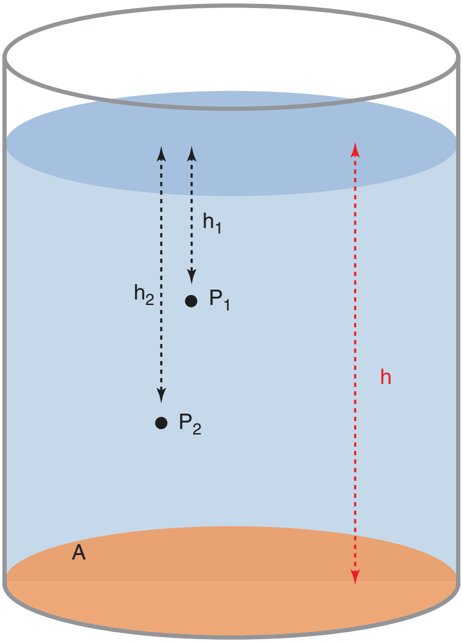

Fig. 8.1 The hydrostatic pressure that a certain volume of a given liquid exerts on the base area *A* of its container depends only on the height *h* of the liquid "column" (Stevin's law). Accordingly, the pressure P_1 at any point which lies at a depth h_1 is lower than the pressure P_2 at any point which is located at a greater depth h_2: in fact, a fluid column of greater height "pushes" above the latter

$$p_{\mathrm{i}} = \frac{dVg}{A} = \frac{dAhg}{A} = dgh \tag{8.5}$$

where *d* is the density of the liquid.

Equation 8.5 is known as *Stevin's law*. It states that the pressure which a column of a given fluid exerts on its base depends on its height but not on any horizontal dimension (and, hence, on the shape of the container). It goes without saying that, accordingly, the pressure at any point within a fluid at rest depends only on the depth *h* of that point (see Fig. 8.1).

It is interesting to note that Stevin's law is no more than a particular case of Bernoulli's theorem, namely the one which explains how an airplane flies and, at the same time, how the Venturi mask works (see Chap. 6). Considering again Fig. 8.1, Eq. 6.12 for two points at height h_1 and h_2 (where pressures are, respectively, p_1 and p_2) can be written as follows:

$$d\,g\,h_1 + p_1 = d\,g\,h_2 + p_2 \tag{8.6}$$

In fact, since the liquid is at rest, its speed is zero and, accordingly, the quantities $\frac{1}{2}\,d\,v_1^2$ and $\frac{1}{2}\,d\,v_2^2$ are both equal to zero.

After a simple rearrangement, Eq. 8.6 becomes:

$$p_1 - p_2 = d\,g\,h_2 - d\,g\,h_1 = d\,g\left(h_2 - h_1\right) = d\,g\,\Delta h \tag{8.7}$$

that is, the pressure difference between the two points is proportional to the difference in their depth (which is exactly what Stevin's law states).

Equation 8.7 explains how a phleboclysis works. According to Hagen–Poiseuille equation (see Chap. 7), a fluid flows through a venous catheter only if there is a pressure difference between its two ends. Such a difference is achieved due to the height difference between the bag and the vein (the height of the fluid column must be sufficient to exert a pressure higher than the venous pressure).

Stevin's law also explains why, as mentioned in Chap. 4, undersea the pressure increases by about 1 atm (≈760 mmHg or 101.3 kPa) for every 10 m of depth. In fact, the pressure exerted by 10 m of sea water is:

$$p = d g \Delta h = 1024\, Kg\, \mathrm{m}^{-3} \times 9.8\, ms^{-2} \times 10\, \mathrm{m} \approx 100{,}000\, Pa = 100\, kPa \approx 1\, atm \tag{8.8}$$

where $d = 1024$ Kg/m^3 is the density of sea water (which is a bit higher than that of pure water due to the presence of salt) and g, as stated in Chap. 2 (Box 2.1), is equal to 9.8 m/s^2.

The pressure at a depth h within a glass of water, or undersea, calculated according to Eq. 8.5, represents the so-called *gauge pressure*, i.e., the difference between the "absolute" (or total) pressure and atmospheric pressure. In fact, the pressure exerted by air (i.e., the atmospheric pressure p_{Atm}) must be added to that exerted by water. Accordingly, the *absolute pressure* (p_{tot}) at any depth h within a liquid is:

$$p_{tot} = p_{Atm} + d\, g\, h \tag{8.9}$$

As mentioned in Chap. 1, atmospheric pressure (which is 760 mmHg or 101.3 kPa at sea level) decreases as altitude increases: this, however, occurs only in part according to Stevin's law (see below).

8.1.2 Push, Squeeze, and Lift: Pascal's Principle

A variation in the pressure applied to an enclosed fluid is transmitted unchanged to each portion of the fluid and to the walls of its container, where it acts perpendicularly. This is the *Pascal's principle,* so called after Blaise Pascal, from whom also derives the name of the pressure unit of the International System of Units (SI), i.e., Newton/m^2 or pascal (Pa).

Thanks to Pascal's principle, it is possible to push out toothpaste by squeezing the tube at its bottom. Another useful application of this principle can be found in a mechanical workshop: in a hydraulic car lift (or, in general, *hydraulic lever*), for example, the Pascal's principle is used to magnify a force so much to allow lifting a car. Figure 8.2 shows a schematic representation of such a machinery. According to Pascal's principle, a pressure applied to the surface area A_0 on the right is transmitted unchanged to the surface area A_1 on the left. Remembering the definition of pressure (Eq. 8.1), it is therefore:

$$\frac{F_{\mathrm{in}}}{A_0} = \frac{F_{\mathrm{out}}}{A_1} \tag{8.10}$$

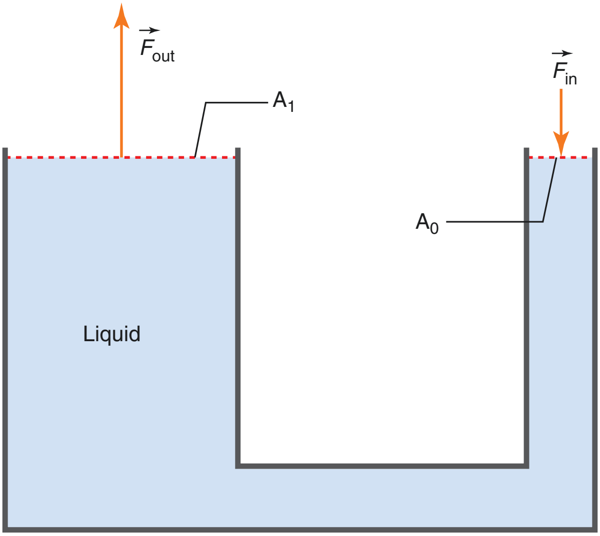

Fig. 8.2 Scheme of a hydraulic lever (see text). F_{in} Input force, which is applied to the surface area A_0; F_{out} Output force (exerted by the surface A_1)

Since A_0 is smaller than A_1, the output force F_{out} will be higher than the input force F_{in}. The "trick" is that the input force must be applied for a greater distance than that along which the output force applies. Accordingly, the *work* done by the two forces is the same (see Chap. 5, Box 5.1). However, the weight which we can lift is greater than that which we could lift with our own "force" (and this is what matters).

Furthermore, the Pascal's principle may save a life. Heimlich maneuver relies precisely on it: a pressure properly applied to the abdomen is transmitted undiminished to the thorax and then to the trachea, pushing out the foreign body [1]. Similarly, the abdomen gets up rhythmically during *external cardiac massage* (thanks to which the blood is pushed into the circulation, again according to Pascal's principle).

For a fluid whose weight is negligible, e.g., the air contained in an empty bottle (which, evidently, is not at all "empty" but full of air!), Pascal's principle tells us that *the pressure is anywhere the same*. In fact, this law can be also enunciated as follows: in the absence of *volume forces* (i.e., forces which act on all particles of a body, and are accordingly proportional to its volume: a typical example is exactly the weight), the pressure is constant at any point within the fluid.

Since the density of air (at 20 °C and 1 atm) is about 1.2 kg/m^3 (i.e., more than 800 times lower than that of water), we can certainly neglect the weight of the small amount of air that we inject into the cuff of an endotracheal tube (or of a supraglottic airway device). Accordingly, the pressure in the cuff is the same as that in the pilot balloon. Moreover, the pressure exerted by the tracheal wall on the cuff, which according to *Newton's third law* (see Chap. 11) is equal and opposite to that exerted by the cuff on the tracheal wall, is transmitted unchanged to the pilot balloon [2]. Hence, we can check the correct inflation of the cuff through the pilot balloon (we have already seen in Chap. 5 that it would be better to do it with a pressure gauge rather than "manually") [3–7].

Finally, it is again Pascal's principle which makes invasive pressure monitoring possible, as discussed below.

8.2 Invasive Pressure Monitoring

When we insert a catheter into an artery (e.g., the radial artery), the "system" comprising the catheter, the tubing, and the *pressure transducer* behaves as a closed container which holds a liquid (usually normal saline). Accordingly, any change in the arterial blood pressure is transmitted unaltered into the transducer (Pascal's principle). Moreover, if the artery and the transducer are at the same height, the pressure in the transducer is just the same that in the artery (Stevin's law). The pressure transducer consists of a membrane through which runs an electric current: the pressure and its changes within the transducer distort the membrane, leading to changes in its *electric resistance* [8]. These (very small) changes in electric resistance can be measured with high precision by means of a special electronic circuit (the so-called *Wheatstone bridge*) [9, 10], and are then displayed as a waveform which accurately reproduces the "original" arterial pressure waveform through a mathematical method known as *Fourier analysis*, according to which any complex waveform can be represented as the sum of simpler sine waves (called *harmonics*) whose frequencies are multiples of the fundamental frequency (all will be clearer in Chap. 15, which deals with mechanical waves and where, moreover, you can find another little consideration about the systems for invasive pressure monitoring) [9–12].

Of course, all this also applies to other pressures such as *central venous pressure* (CVP) or pulmonary artery pressure. Moreover, thanks again to Pascal's principle, we can measure left atrial pressure with a catheter whose tip is very far from the left atrium. In fact, after occluding a pulmonary artery branch by inflating the balloon of a pulmonary artery catheter, the pressure that we measure downstream, i.e., the *pulmonary artery occlusion pressure* (PAOP), is (approximately) the same that in the left atrium, since there are no valves in the pulmonary vasculature between pulmonary artery and left atrium.

8.2.1 Leveling: How Important Is the Difference?

Everyone of us knows very well that the correct *leveling* of the transducer (usually at the mid-thoracic position) is pivotal when measuring pressures with a fluid-filled system, such as invasive arterial pressure or CVP [11, 13, 14]. As an example, if the transducer lies 10 cm below the point at which the measurement should be made, according to Stevin's law the (hydrostatic) pressure exerted by a 10 cm high "column" of normal saline is added to the patient's true pressure, which will be accordingly overestimated. Conversely, the pressure will be underestimated if the transducer is located too high. In particular, every centimeter of height difference leads to an error of 0.74 mmHg: that is, if the transducer lies 10 cm above the correct level, the pressure will be underestimated by 7.4 mmHg. This is due to the different densities of mercury and normal saline [8, 14], as it can be easily shown by applying, again, Stevin's law.

According to Eq. 8.5, in fact, we can write:

$$d_{\mathrm{w}}\, h_{\mathrm{w}}\, g = d_{Hg}\, h_{Hg}\, g \tag{8.11}$$

or

$$\frac{h_{\mathrm{w}}}{h_{Hg}} = \frac{d_{Hg}}{d_{\mathrm{w}}} \tag{8.12}$$

where d_{w} and d_{Hg} are, respectively, the density of water (which approximates that of normal saline) and the density of mercury, and h_{w} is the height of a column of water that exerts the same hydrostatic pressure which is exerted by a column of mercury of height h_{Hg}. Hence, the conversion factor between cmH$_2$O and cmHg (beware, not mmHg) is equal to the ratio between the density of mercury (which is about 13,600 kg/m^3) and that of water, i.e., 13,600/1000 = 13.6 (that is the *specific gravity* of mercury). Finally, to convert cmHg to mmHg you have to multiply h_{Hg} by 10, then we have 1 mmHg = 13.6/10 = 1.36 cmH$_2$O, and 1 cmH$_2$O = 1/1.36 ≈ 0.74 mmHg. That's what we said above.

It is clear that a correct transducer leveling is particularly important when measuring filling pressures (CVP and PAOP). In fact, an error of 5–10 mmHg (due to a height difference of 7–14 cm, that are not so many!) is usually of little clinical relevance when measuring arterial blood pressure. Conversely, it may lead to big "blunders" when dealing, for example, with CVP, whose values (and changes) are most often in that order [11, 13, 14].

8.2.2 When the Difference Really Does Not Matter: Zeroing

All pressures to which we usually refer are, actually, *gauge pressures* (see Sect. 8.1.1). When you say: "Mr. Smith, your systolic pressure is 140 mmHg," the absolute systolic pressure in Mr. Smith's artery is, indeed, 900 mmHg (at sea level), i.e., 140 mmHg higher than the atmospheric pressure (760 mmHg) [8, 11]. Zeroing the transducer means "tell" the monitoring system how much atmospheric pressure is, so that it can subtract that value to the measured pressure in order to display the gauge pressure (i.e., the extra pressure as compared to the atmospheric pressure).

Does it matter where the transducer is located (relative to the patient) when zeroing it? Absolutely not. Also this according to Stevin's law.

In reality, atmospheric pressure does not vary with height (i.e., with altitude) according to Eq. 8.7, but according to an exponential equation which is known as *law of atmospheres*:

$$p = p_0 \, \mathrm{e}^{-Ah} \tag{8.13}$$

where p is the atmospheric pressure at an altitude h, p_0 is the atmospheric pressure for $h = 0$, e is the base of natural logarithm, and A is an appropriate constant (which includes *density*).

This is due to the decrease in air density with pressure (see above) and, hence, with altitude. However, for small altitude changes (so that the density variation can be neglected), Stevin's law also applies to atmospheric pressure. Of course, we can consider that air density variation is negligible within the same room. Therefore, we can calculate the change in atmospheric pressure for each meter of height change according to Eq. 8.7*:*

$$p = d\,g\Delta h = 1.2\,Kg\,\mathrm{m}^{-3} \times 9.8\,ms^{-2} \times 1\,\mathrm{m} \approx 11.8\,Pa = 0.012\,kPa \approx 0.09\,\mathrm{mmHg} \tag{8.14}$$

where $d = 1.2$ Kg/m^3 is air density at 20 °C and sea level. That is, an error of 0.0009 mmHg for every variation by 1 cm of the transducer height: it really matters very little where the transducer was during zeroing!

If you are really finicky, though, you'd better redo zeroing when, from the Intensive Care Unit on the ground floor, you bring the patient to Radiology which is, for example, at the fourth floor (a Δh of 12 m translates into an error of about 1 mmHg).

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Heat, Cardiac Output, and What Is the Future: The Laws of Thermodynamics

9

Where's Physics

Daily life	Past and future Temperature and thermometers Warm your hands Pans (and handles) Car radiators Fuel = motion + heat (and noise) Fridges and power plants
Physics involved	Laws of thermodynamics Specific heat
Clinical practice	Measuring cardiac output by thermodilution (derivation of the Stewart–Hamilton equation) Heat exchangers Vaporizers (see Chap. 12)

9.1 Temperature, Heat, and Energy: The Laws of Thermodynamics

Figure 9.1 shows a number of common objects and situations: (a) what is temperature and how it can be measured; (b) the conversion of the fuel chemical energy into kinetic energy (i.e., movement) and heat in a car engine; (c) warm your hands while keeping a cup of tea; (d) what happens in a fridge is impossible unless you also consider what happens, at the same time, in a power plant; (e) the direction in which time flows (i.e., what distinguishes past from future); (f) the choice of the materials with which a pot is constructed; (g) coolers (e.g., car radiators) and heaters, including heat exchanger of heart–lung machine; (h) measuring cardiac output by thermodilution

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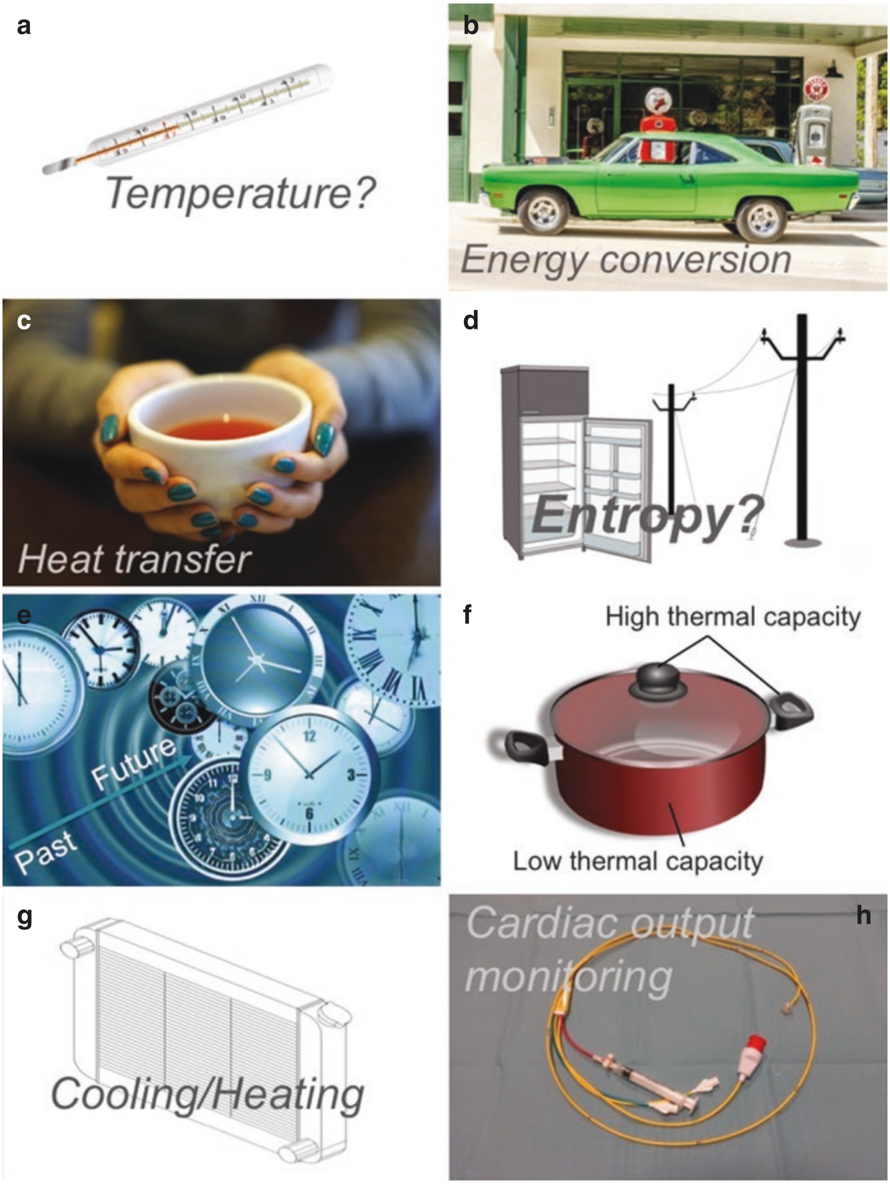

Fig. 9.1 Thermodynamics and its laws in everyday life, including that of the operating room/intensive care unit (**a**-**h**: see text)

(e.g., through a pulmonary artery catheter). All this, in a way or another, has something to do with *thermodynamics* and its laws. Hence, the laws of thermodynamics are maybe the best example of physical principles that we face continuously, "from daily life to the operating room" (as the subtitle of this book). Unfortunately, thermodynamics is a quite complex matter, which is not the case to discuss in detail here. Accordingly, below I've only tried to provide some hints about the meaning of the three laws of thermodynamics, as well as of the concept of specific heat capacity, which also are the principles on which the measurement of cardiac output by thermodilution relies. A thorough and systematic discussion on this topic, which is at the same time (wonderfully) easily understandable for non-physicists, can be found in the book *Four Laws That Drive the Universe* by Peter Atkins [1].

9.1.1 Temperature and Thermometers: The Zeroth Law of Thermodynamics

If a body X is put in close contact with another body Y, and no *heat* is transferred from X to Y or vice versa, it is said that the two bodies are in *thermal equilibrium* with each other. Heat (symbolized Q) is the *thermal energy* of a body, i.e., the "internal energy" due to the random motion of atoms and molecules within it.

If the body Y is, in turn, in thermal equilibrium with another body Z, then also X and Z are in thermal equilibrium with each other. This according to the so-called *zeroth law of thermodynamics*, which states that if two bodies are each in thermal equilibrium with some third body, then they are also in thermal equilibrium with each other. Evidently, these bodies must share a property that we call *temperature*. In other words, *two objects that are in thermal equilibrium have the same temperature*.

Now we can then define the heat Q as the energy which is transferred from one body to another (or between a "system" and its environment) due to a temperature difference between them. Simply, a cup of hot tea left in the kitchen will cool as some heat is transferred from it to the surrounding environment (whose temperature is lower). The energy transfer between the cup of tea and the environment will continue until both are at the same (room) temperature.

Thanks to the zeroth law of thermodynamics, we can say that the objects X and Z have an equal temperature without putting them into contact, using the object Y as a *thermoscope* (i.e., something with a measurable property which changes with temperature). A thermoscope can be turned into a *thermometer* after calibration, i.e., assigning an arbitrary "value" of temperature to certain reproducible thermal phenomena, e.g., 0 °C (32 °F) to the freezing point of water and (about) 100 °C (212 °F) to its boiling point at standard atmospheric pressure.

9.1.2 The First Law of Thermodynamics: It All Adds Up!

The *first law of thermodynamics* expresses the principle of energy conservation for a system which is not isolated (i.e., can exchange heat and *work* with the surrounding environment). It is usually formulated as follows:

$$\Delta U = Q - W \tag{9.1}$$

that is, the change in the *internal energy* ΔU of a system is equal to the difference between the net heat Q supplied to the system by its surrounding environment and the net work W (see Chap. 5, Box 5.1) done by the system itself. Alternatively, the work done *on* the system can be considered. In this case, the first law of thermodynamics can be written as follows:

$$\Delta U = Q + W \tag{9.2}$$

In other words, energy (e.g., heat) can be transferred from one body to another, or from a system to its surroundings and vice versa. It can also be converted into other forms of energy, such as mechanical work (which, as mentioned in Box 5.1, is "energy transfer" by means of a force). Indeed, mechanical work and heat are fully equivalent to each other (with an adequate proportionality coefficient): that is, the same change in the internal energy of a system can be obtained by either providing a certain quantity of heat to the system or doing some external work on it.

The only thing that energy cannot do is to disappear into nothing (or be created from nowhere).

Thus, if a car engine has a *thermal efficiency* of, say, 30% (i.e., only the 30% of the *potential energy* which is stored in the chemical bonds of the fuel is converted into useful mechanical work), the remaining 70% of the energy that is provided to it does not disappear, but is converted into other forms of energy (mostly heat, but also the work which is needed to overcome the friction among the gears, and the *sound energy* which produces the engine noise).

9.1.3 In Which Direction, Please? The Second Law of Thermodynamics

The second law of thermodynamics can be formulated in many ways. In its earliest form, which is known as *Kelvin statement*, it states that it is not possible to realize a process whose only result is to completely convert the heat absorbed from a thermal reservoir into work. In other words, a perfect engine (with a 100% thermal efficiency) cannot exist. Another way of expressing the second law of thermodynamics is the so-called *Clausius statement*: no process is possible whose only result is the transfer of heat from a colder body to a hotter one.

In order to describe mathematically the second law of thermodynamics (and to show that the two above mentioned statements say exactly the same thing), we need to introduce a new thermodynamic quantity: *entropy*.

Entropy is a measure of the "disorder" of a system and, at the same time, of the ability of that system to do work. In other words, entropy is the thermal energy per unit temperature which is *not* available for doing work: accordingly, it represents a measure of the "quality" of energy that is stored in a system. Entropy is, as physicists say, a *state function*, i.e., its value for a given state of the system is independent of how that state has been reached.

A just bought pack of playing cards (in which the cards are sorted in ascending order and separated by suit) has low entropy. When the cards are shuffled, entropy increases. The entropy of a charged battery is low, but it is much higher when the battery is exhausted (and can no longer do work, e.g., move the hands of a clock). These and other revealing (and funny) examples can be found in the beautiful books by the physicist Jim Al-Khalili [2, 3].

When some heat is absorbed from the surrounding environment, or released to it, by a system, the *change in entropy* ΔS of such a system can be calculated, roughly (i.e., without resorting to the integral calculus), as follows:

$$\Delta S = \frac{\Delta Q}{T} \tag{9.3}$$

where ΔQ is the energy transferred (as heat) between the system and the environment and T is the average temperature (in kelvins) at which the transfer occurred.

Hence, the same quantity of heat provided to a system will produce a greater increase in its entropy if temperature is lower, and a smaller increase in its entropy if temperature is higher (i.e., the system is already rather "chaotic"). Using a really enlightening analogy, Peter Atkins says that a sneeze causes a much higher increase of "entropy" in a silent library than in a busy road [1].

Now we are ready to formulate the second law of thermodynamics in equation form:

$$\Delta S \geq 0 \tag{9.4}$$

that is, entropy of a closed system can only increase (as a result of an *irreversible* process) or remain constant (following a *reversible* process), but *it can never decrease*.

If we consider the Universe as a closed system, we could say that the entropy of the Universe always increases during a spontaneous transformation. However, the entropy of any system which is located within the Universe (and is not isolated from it) can certainly decrease, provided that entropy increases somewhere else. If you hold a cup of hot tea between hands (as in Fig. 9.1c), heat will be transferred from the tea cup to your hands because this is the "direction" that the increase in entropy imposes (i.e., from the hotter body to the colder one), and *nothing else happens* (as the Clausius statement specifies). The fridge in your kitchen (in which total entropy decreases, since heat is continuously removed from warm food and beverages and transferred to the warmer environment) does not violate the second law of thermodynamics: in fact, entropy is increasing, at the same time, in a power plant, so that the entropy of the system "fridge + power plant" increases (Fig. 9.1d).

Ultimately, the second law of thermodynamics sets the direction in which time flows, namely that of the increase in the entropy of Universe, i.e., of the reduction in the "quality" of its total energy. Accordingly, it is also known as the "arrow of time." At the moment, this is the best way to distinguish past from future!

9.1.4 There Is Also a Third Law of Thermodynamics (Just to Know)

The third law of thermodynamics (also referred as *Nernst's theorem*) states something like this: it is not possible to cool a body up to the *absolute zero* in a finite number of operations. Most physicists say it is not even a "law" of thermodynamics, but simply something which follows from the second law. Furthermore, it deals with extremely low temperatures (absolute zero is the zero on the kelvin scale, i.e., −273.16 °C!): hence, we will not care about it here.

9.2 More or Less "Greedy": Specific Heat

When some heat is absorbed or released by an object, its temperature varies (unless a state change is ongoing, as discussed in Chap. 12). However, the extent of temperature change ΔT depends on the so-called *heat capacity* (or *thermal capacity*) C of the object, defined as:

$$C = \frac{Q}{T_1 - T_0} = \frac{Q}{\Delta T} \text{ or } Q = C\Delta T \tag{9.5}$$

where T_0 and T_1 are, respectively, the initial and the final temperature of the object. In other words, heat capacity of an object can be defined as the heat which needs to be provided to it in order to increase its temperature by 1 °C.

Heat capacity depends on the substance of which the object is made and on its mass. Accordingly, it is useful to refer to the *specific heat c* (also known as *specific heat capacity* or *specific thermal capacity*), i.e., the heat capacity per unit mass:

$$c = \frac{C}{m} \tag{9.6}$$

where C is heat capacity and m the mass of the object. After substituting from Eq. 9.6, Eq. 9.5 becomes:

$$Q = cm\Delta T \tag{9.7}$$

that is, specific heat c is the heat which is needed in order to raise 1 unit mass of a substance by 1 unit temperature.

For example, pans and pots are usually made of materials with low specific heat, so that a small amount of heat is needed to increase their temperature enough to cook your meals. Conversely, their handles should be preferably made of materials with high specific heat, as to not get burned when you touch them (Fig. 9.1f).

Water has a very high specific heat (e.g., about 5 times that of aluminum and 30 times that of mercury), i.e., it absorbs a great amount of heat with little increases in its temperature: thanks to this property, it is used in car radiators as a cooling fluid. For the same reason, water can "carry" a lot of heat, and it is therefore used in home central heating systems as well as in the heat exchanger of the heart–lung machine used for cardiopulmonary bypass (CPB) [4].

We will encounter again specific heat when discussing about vaporizers for volatile anesthetic agents (see Chap. 12). Here, its knowledge was essential in order to be able to move to the next sections.

9.3 Measuring Cardiac Output by Thermodilution

We all know that we can measure cardiac output (CO) with a pulmonary artery catheter (PAC) by means of the so-called *thermodilution* method [5–7]. A given volume of iced or room-temperature fluid (e.g., saline) is injected into the right atrium through the proximal lumen of the PAC, and the resulting change in blood temperature is measured in the pulmonary artery by a thermistor which is located at the tip of the PAC. Cardiac output is inversely proportional to the area under the curve of pulmonary artery blood temperature change over time (the so-called *thermodilution curve*), according to the modified *Stewart–Hamilton equation*:

$$CO = V_{\mathrm{i}} K_1 \frac{\left(T_{\mathrm{b}} - T_{\mathrm{i}}\right)}{\int_0^{\infty} \Delta T_{\mathrm{b}}\left(t\right) \mathrm{d}t} \frac{d_{\mathrm{i}} c_{\mathrm{i}}}{d_{\mathrm{b}} c_{\mathrm{b}}} \tag{9.8}$$

where V_{i} is the volume of injectate; T_{b} and T_{i} are baseline blood temperature and injectate temperature, respectively; d_{i} and c_{i} are, respectively, the density (or specific gravity, see Chap. 8) and the specific heat of the injectate; d_{b} and c_{b} are, respectively, the density and the specific heat of blood; K_1 is a computation constant which accounts for the heat exchange in transit, the catheter dead space, the injection time, and the unit conversion; and $\int_0^{\infty} \Delta T_{\mathrm{b}}(t)\, \mathrm{d}t$ is the integral of temperature change over time, i.e., the area under the thermodilution curve.

To make it less “frightening” (anyhow, it is the monitor software that does the calculation!), Eq. 9.8 is usually reported in the following short form:

$$CO = \frac{\left(T_{\mathrm{b}} - T_{\mathrm{i}}\right) K}{\int_0^{\infty} \Delta T_{\mathrm{b}}\left(t\right) \mathrm{d}t} \tag{9.9}$$

where K is a “comprehensive” constant defined as:

$$K = V_{\mathrm{i}} K_1 \frac{d_{\mathrm{i}} c_{\mathrm{i}}}{d_{\mathrm{b}} c_{\mathrm{b}}} = V_{\mathrm{i}} K_1 K_2 \tag{9.10}$$

with $K_2 = (d_{\mathrm{i}}\, c_{\mathrm{i}})/(d_{\mathrm{b}}\, c_{\mathrm{b}})$.

While Stewart–Hamilton equation, in any of its forms, can be found in every anesthesia textbook, its “origin” is rather shrouded in mystery (except for a few vague references to the fact that it follows from the laws of thermodynamics or from a “thermal energy balance” [5, 7]). Let’s try, then, to give a sense (for anesthesiologists) to what we discussed so far.

9.3.1 It's Just Thermodynamics, Beauty!

When a fluid bolus is injected into the right atrium, some heat is transferred from blood to the injectate (according to the second law of thermodynamics), leading to a reduction in temperature. According to Eq. 9.7, the amount of heat transferred can be calculated as follows:

$$Q_1 = c_i m_i \Delta T = c_i m_i \left(T_b - T_i\right) \tag{9.11}$$

where c_i, m_i, and T_i are, respectively, the specific heat, mass, and temperature of the injectate, and T_b is the baseline blood temperature.

Remembering the definition of density (see Chap. 8) and substituting by Eq. 8.2, Eq. 9.11 becomes:

$$Q_1 = c_i d_i V_i \left(T_b - T_i\right) \tag{9.12}$$

where d_i and V_i are the density and the volume of injectate, respectively.

Afterwards, *due to blood flow*, further heat exchange occurs, leading to the observed change in the pulmonary artery blood temperature over time. As above, we can apply Eq. 9.7 to calculate such heat transfer (whose "rate" evidently depends on cardiac output):

$$Q_2 = \sum c_b m_b \left(T_b - T_{(t)}\right) = \sum c_b d_b V_b \left(T_b - T_{(t)}\right) = \sum c_b d_b V_b \Delta T_{(t)} \tag{9.13}$$

where c_b, d_b, V_b, and T_b are, respectively, specific heat, density, volume, and (baseline) temperature of blood, $T_{(t)}$ is pulmonary artery blood temperature "as it changes over time," $\Delta T_{(t)}$ is the change in pulmonary artery blood temperature over time (t), and the symbol $\sum$ means that the total heat transfer is the sum of heats transferred for any value of pulmonary artery blood temperature over time (this is a not too rigorous notation from a mathematical standpoint, but I hope the concept is clear).

As mentioned in Chap. 6, cardiac output (CO) is the *volume flow rate* V_b/t (volume of blood per unit time t). Hence, after substituting by $V_b = \text{CO}\ t$, Eq. 9.13 becomes:

$$Q_2 = \sum c_b d_b COt\Delta T_{(t)} \tag{9.14}$$

According to the first law of thermodynamics (Eq. 9.2), the change in the internal energy of the system (in this case, the right heart and the pulmonary artery or, more generally, the human body) is equal to the heat transferred (considering the work done on the system as negligible). Hence, remembering that "energy is conserved," and assuming that blood and the injectate are mixed completely and instantaneously and that no other heat exchanges occur (and, accordingly, that pulmonary artery blood temperature remains constant except for the changes induced by the fluid bolus), it must be $Q_1 = Q_2$. From Eqs. 9.12 and 9.14, it is therefore:

$$c_i d_i V_i \left(T_b - T_i\right) = \sum c_b d_b COt\Delta T_{(t)} \tag{9.15}$$

or, rearranging a bit:

$$CO = V_{\mathrm{i}} \frac{(T_{\mathrm{b}} - T_{\mathrm{i}})}{\sum \Delta T_{(t)} t} \frac{c_{\mathrm{i}} d_{\mathrm{i}}}{c_{\mathrm{b}} d_{\mathrm{b}}} \tag{9.16}$$

As you can see, Eq. 9.16 closely resembles Eq. 9.8 (modified Stewart–Hamilton equation) and becomes identical to it remembering that the quantity roughly indicated as $\sum \Delta T_{(t)}\, t$ is more correctly described by the integral of the thermodilution curve $\int_0^\infty \Delta\, T_{\mathrm{b}}(t)\, \mathrm{d}t$, and adding an adequate computational constant K_1.

A thorough knowledge of the modified Stewart–Hamilton equation and of its "thermodynamic background" may help us to not blindly trust our thermodilution CO measurements: the number displayed on the monitor, although clinically useful, is not necessarily the "true" cardiac output. In fact, given its large number of assumptions, the thermodilution method has several sources of error [5–8]. For example, we assumed that there are no other heat exchanges apart from the injectate bolus. However, pulmonary artery blood temperature may vary due to physiological fluctuations with respiration, rapid intravenous fluid administration, and the so-called "baseline drift" soon after hypothermic CPB [5–7]. Moreover, the accuracy of the volume of injectate and, most importantly, of the measurement of its temperature is pivotal. Finally, as shown, the thermodilution CO equation includes the specific heat of blood (c_{b}) as a constant: however, this blood property is not indeed constant, but it varies slightly with the hematocrit [7, 9].

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Part IV

Forces in Action

Doors, Steering Wheels, Laryngoscopes, and Central Venous Catheters: The Moment of a Force

10

Where's Physics

Daily life	Doors and doorknobs Steering wheels Pliers, nutcrackers, and tweezers
Physics involved	Components of vectors Moment of a force (torque) and levers
Clinical practice	Inserting a central venous catheter Proper use of the laryngoscope

10.1 Vectors, Vector Sum, and Components of a Force

Physical quantities such as *force*, but also displacement, speed, and acceleration, to name but a few, cannot be defined by a single value: in addition to their "magnitude," in fact, we need to specify their *direction*. Quantities of this kind are called *vectors* (and are usually indicated by an arrow over their symbol).

The *projection* of a vector (e.g., a force) along an axis is called the *component* of the vector on that axis. To find the component of a vector (or, as it is said, to "resolve" the vector) we can place it on a rectangular (Cartesian) coordinate system and use *trigonometry*.

Figure 10.1 shows the components of a force $\vec{F}$ along the x and y axes. Called ϑ the angle made by the vector with the x axis, and remembering the definitions of *sine* and *cosine* (see Box 10.1), is:

$$F_x = F\ \cos\ \vartheta \tag{10.1}$$

and

$$F_y = F\ \sin\ \vartheta \tag{10.2}$$

where F_x and F_y are the x component and the y component of $\vec{F}$, respectively.

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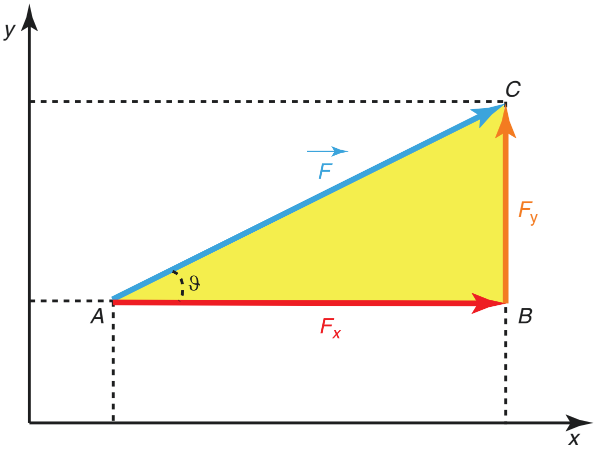

Fig. 10.1 Components of a force F along the x and y axes of a rectangular coordinate system

Box 10.1: Remembrances of Trigonometry (to Explain Eqs. 10.1 and 10.2) *Sine* and *cosine* are two trigonometric functions of an angle which are classically defined in the context of a right triangle. In particular, the sine of an angle ϑ is the ratio between the length of the side of the triangle which is opposite to ϑ and that of the hypotenuse, while the cosine of ϑ is the ratio between the length of the side which is adjacent to ϑ and that of the hypotenuse. Hence, in Fig. 10.1, is:

$$\sin\ \vartheta = BC / AC \quad \textit{and} \quad \cos\ \vartheta = AB / AC$$

or

$$BC = AC \sin\ \vartheta \quad \textit{and} \quad AB = AC\ \cos\ \vartheta$$

which are analogous to Eqs. 10.1 and 10.2, respectively.

For example, in Chap. 5 (Box 5.1) we defined the work W (of a force F that causes a displacement) as the product of the displacement d and the magnitude of the component of the force along the displacement itself (F_d). According to Eq. 10.1, such a component is:

$$F_\mathrm{d} = F\ \cos\ \vartheta \tag{10.3}$$

where here ϑ is, evidently, the angle between the directions of the force and the displacement. The work is, therefore:

$$W = d\ F_\mathrm{d} = d\ F\ \cos\ \vartheta \tag{10.4}$$

that is, as mentioned in Box 5.1, the *scalar* (or *dot*) *product* of force and displacement (in general, the scalar product of two vectors is the product of the magnitudes of the two vectors and of the cosine of the angle between their directions, or the product of the magnitude of one of the two vectors and the component of the second vector along the direction of the first one).

Two vectors cannot be summed algebraically. If we imagine that F_x and F_y (Fig. 10.1) are two different forces which act, at the same time, on an object (e.g., both applied to the point A), their *vector sum* (or *resultant*) can be obtained using the so-called *parallelogram method*: the two vectors are drawn head to tail (as in Fig. 10.1), and the resultant vector is that which goes from the tail of the one to the head of the other. It is easy to see that, in this case, the resultant vector is $\vec{F}$. In this particular case, in which the two forces are perpendicular to each other, the magnitude of their vector sum can be calculated by means of the *Pythagorean theorem* (see Chap. 16).

10.2 Pliers, Nutcrackers, Tweezers (and so on): Moment of a Force and the Levers

Why can we unscrew a bolt with the help of a wrench but not with bare hands? After all, which help do we get from an inanimate tool in terms of strength? Have you ever tried to open a heavy door by pushing it from a point very close to its hinges? It takes a lot more force than if you use the doorknob (it is not a coincidence, indeed, that doorknobs are where they are). Have you ever wondered why old trucks or buses had much larger steering wheels than cars? And why truckers are no longer the strong and muscled men than once?

Consider a body, such as a metal bar, that can rotate about an axis A (Fig. 10.2). A force F which is applied at a point P will cause rotation of such a body according to its direction and magnitude, but also depending on the position of its point of application P with respect to the rotation axis A, which is defined by the position vector r. As shown in Fig. 10.2, the force F can be resolved into two components: the *radial* component F_r, pointing along r, and the *tangential* (or *normal*) component F_n, perpendicular to r. Evidently, only the latter will rotate the metal bar. According to what discussed above, the magnitude of this normal component is:

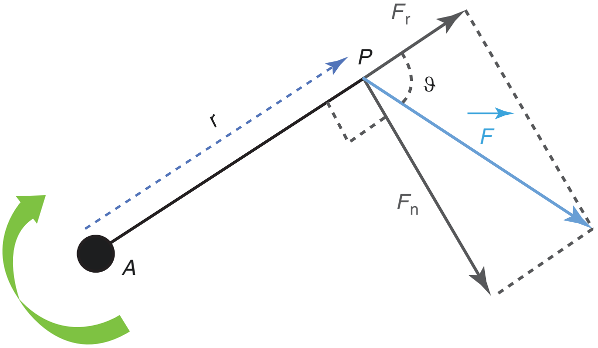

Fig. 10.2 A body such as a metal bar rotates about an axis A under the effect of a force F applied at a point P (which is located at a distance r from A). F_r radial component of F; F_n tangential (or normal) component of F

$$F_n = F \sin \vartheta \tag{10.5}$$

where ϑ is the angle that the directions of r and F make with each other.

In order to consider all factors involved in the rotation caused by F (magnitude, direction, and distance from the rotation axis), we define a vector called *moment M* (or *torque*) as the product of the distance r and the normal component F_n:

$$M = r\ F_n = r\ F \sin \vartheta \tag{10.6}$$

Therefore, the "actual" force causing rotation (i.e., the moment or torque of F) is as much more magnified the greater is the distance from the rotation axis.

This is the reason why the steering wheel of the old trucks or buses (that had no power steering) was much larger: a greater distance from the rotation axis allowed a greater "effective" force (i.e., a greater moment) with the same effort by the driver. With the introduction of power steering, however, all this is no longer necessary (and truckers can even be weakling!). Moreover, the kind of help provided by a wrench and the advantage of having doorknobs away from the hinge line are certainly clear now.

This is also why Archimedes said that it would raise the World with a *lever* long enough. On the principle of torque, in fact, is based the functioning of *levers*, which are classified according to the position of the fulcrum (i.e., the rotation axis) relative to the point of application of a force F which acts in order to win a "resistance" R, and that of R itself (Fig. 10.3). In a *first class lever* (such as a pliers), the fulcrum is located between the force and the resistance. This kind of lever is (mechanically) advantageous only if the fulcrum is closer to R than to F (and, accordingly, the moment of F is higher than that of R). In a *second class lever* (e.g., a nutcracker), the resistance is in the middle: this lever is always advantageous as F is always more distant from the fulcrum as compared to R. Finally, a *third class lever* (such as tweezers) is always disadvantageous since F is in the middle (closer to the fulcrum than R). Note that the terms "advantageous" and "disadvantageous" refer to the fact that the force applied is, respectively, "magnified" or "weakened" (as compared to the resistance to be overcome), not to the usefulness of the tool: in the case of the tweezers, for example, the "advantage" lies in a greater delicacy and precision.

10.3 Bend a Guidewire or Blow Up a Tooth: Matter of a Moment!

We all know that, when placing a central venous catheter, the advancement of the dilator is a delicate step. Apart from the potentially serious complications which can be associated with this maneuver [1–3], it is common experience that if the dilator (and, consequently, the guidewire inside) bends, at best we have to start all over again.

The force applied to the dilator should be perfectly parallel to it (so that the angle ϑ between the direction of the force and the long axis of the dilator is equal to zero

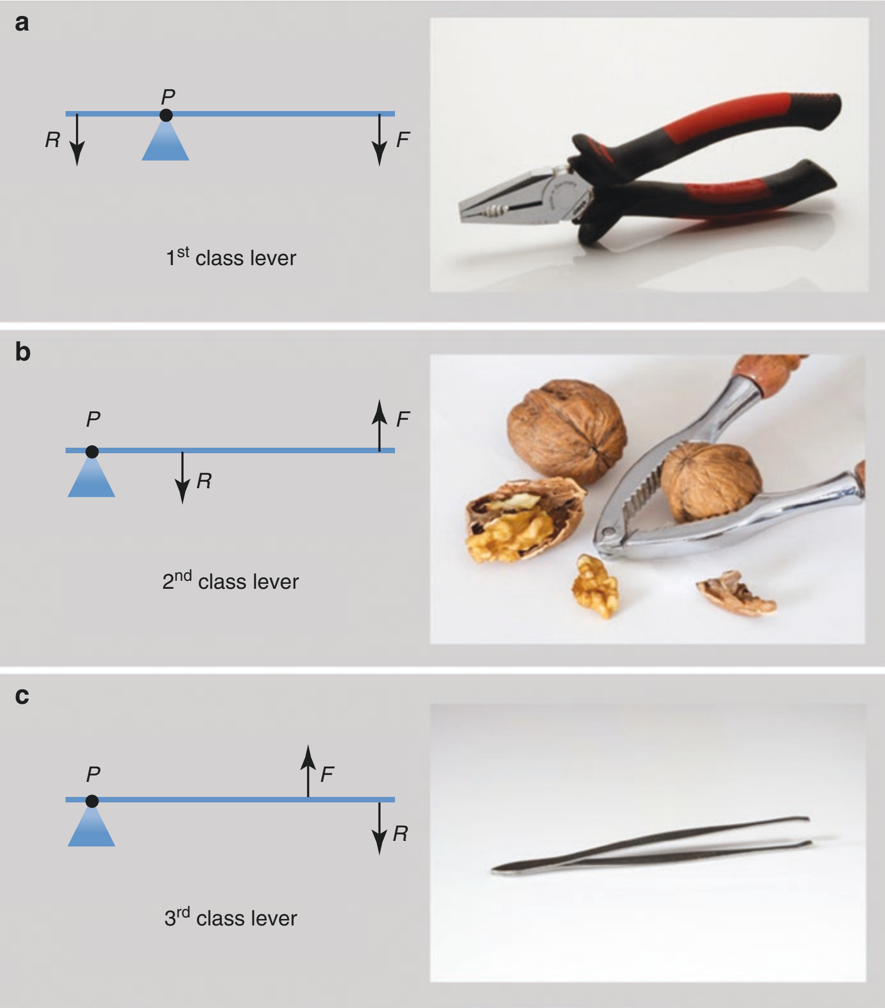

Fig. 10.3 Schematic representation and examples of the three basic types of levers: (**a**) first class lever; (**b**) second class lever; (**c**) third class lever. *P* fulcrum; *F* force; *R* resistance

and, accordingly, the moment is null): in this way, in fact, we will not cause rotation about the insertion site (which acts as a fulcrum) and, accordingly, the dilator will be not subjected to a torque tending to bend it. However, our thrust may not be perfectly parallel, so that a small tangential component appears (see Fig. 10.4). Although small, this component (perpendicular to the long axis of the dilator) could be enough to bend the dilator if its distance from the insertion site is large, since its moment will be high (Fig. 10.4a). Here's how physics explains why the proper way to hold the dilator is as close as possible to the insertion site, as we all know (Fig. 10.4b).

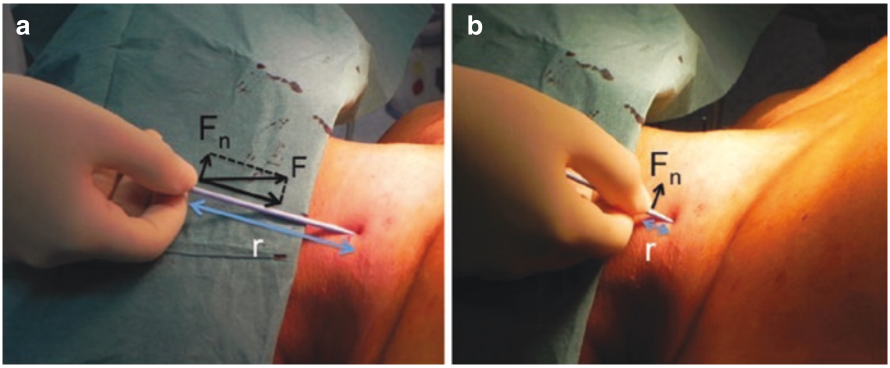

Fig. 10.4 Insertion of a central venous catheter. (**a**) Gripping the dilator distally, it acts as a second class lever; therefore the probability of winning its resistance (and bend it and the guidewire) increases. (**b**) Gripped more proximally to the insertion point, the dilator is more similar to a third class lever or, in any case, the moment of the force *F* we apply is much less and, consequently, the risk of bending the dilator is much lower. F_n normal component of *F*

Many authors warn clinicians not to use the *laryngoscope* as a lever in order to avoid *tooth injury* (including avulsion) during classic direct laryngoscopy [4–8]. Now that we know all about levers and the moment of a force, it should be clear that the major concern is not, as suggested [4, 5], to use the (upper) teeth as a fulcrum, but rather to use the laryngoscope as a second class lever, with the *epiglottic vallecula* as a fulcrum, so as to magnify the force acting against the "resistance" opposed by the teeth (just to be accurate from a physical standpoint). Anyway, stay away from the teeth during laryngoscopy!

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Friction, Trigonometry, and Newton's Laws: All About Trendelenburg Position

11

Where's Physics

Daily life	Motion or motionlessness (of everything) Cars and engines Weight Jets, rockets, rifles, and cannons
Physics involved	Newton's laws Gravity (gravitational force) Normal force Friction
Clinical practice	Safety in the Trendelenburg position Again something about the endotracheal tube cuff

11.1 Forces and Motion: Newton's Laws

The *Newton's laws*, also known as *laws of dynamics* (that is the branch of classical mechanics which deals with forces and their effect on motion), have been all already mentioned in previous chapters (see Chaps. 2, 7, and 8). Also the concept of *friction* has been faced several times (Chaps. 5 and 7). Moreover, in Chap. 10 we discussed how to resolve a force in its components and how to find the sum (or resultant) of different forces acting on a body. It is now time to put all together (and add some other little notion): this will allow us to take an (enjoyable, I hope) journey to discover how physics can help us to make *Trendelenburg position* safe (see Fig. 11.1).

Let's start from the Newton's laws.

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Fig. 11.1 A patient in the Trendelenburg position. The "degree" of tilt refers to the width of the angle ϑ between the operating table and a line parallel to the floor

11.1.1 Newton's First Law

Also referred to as *law of inertia*, *Newton's first law* states that if no *net* force acts on a body, such a body remains at rest if it was at rest, while it continues to move *at a constant speed* (i.e., with uniform rectilinear motion) if it was moving. Alternatively, Newton's first law can be enunciated as follows: a body cannot *accelerate* (i.e., its speed cannot change) if no *net* force acts on it. In reality, the first law can be derived (as a special case) from the *Newton's second law*, which establishes a quantitative relationship between the force applied to a body and the acceleration caused by this force.

11.1.2 Newton's Second Law

The *net* force acting on a body is the product of its *mass* and *acceleration*:

$$\vec{F} = m\,\vec{a} \tag{11.1}$$

where the vectors $\vec{F}$ and $\vec{a}$ are, respectively, the net force acting on the body and the acceleration caused by the force itself, and m is the mass of the body.

Equation 11.1 expresses the second law of dynamics (*Newton's second law*), according to which a given force applied to a body, *whatever its speed*, produce an acceleration of such a body (along the same axis of the force) whose magnitude is directly proportional to the force itself. Moreover, Eq. 11.1 allows to define mass as a characteristic of a body which determines "how much" the body accelerates due to a given force applied on it: the higher the mass, the lower the acceleration (see also Chap. 2, Box 2.1).

Evidently, Newton's second law applies regardless of the speed of a body (as stated above) only if we can consider the mass of such a body as independent from its speed. According to (Einstein's) *special relativity* (see Chap. 16), the mass of a body *increases* with its speed [1, 2]. However, this becomes evident only for speeds which are not negligibly low as compared to the *speed of light*, that is about 300,000 km/s (or 186,000 mi/s): at "ordinary" speeds to which we are accustomed, we can peacefully rely on Newton's law.

As mentioned in Chap. 2 (Box 2.1), Eq. 11.1 also allows to define the *weight* W of a body and its relationship with the mass m of the body itself. In fact, if the only force which is applied to the body is its weight (i.e., the magnitude of the *gravitational force,* that is, in our everyday life, the force which pulls everything towards the center of the Earth), then the acceleration produced is the *free-fall acceleration* (namely the acceleration due to *gravity*), and Eq. 11.1 becomes:

$$W = m\ g \tag{11.2}$$

where g is the free-fall acceleration that is equal to 9.8 m/s^2 or 32 ft/s^2 (as mentioned in Chap. 2).

Just to know, it is nowadays generally accepted that, according to *general relativity* (the wonderful "theory" by Albert Einstein, which I suggest to those who are intrigued to read from the author's own words [3], although it could be easier to rely on some clear and enjoyable divulgative books [1, 2, 4]), gravity is not properly a force acting at a distance between two bodies, but rather the result of a *curvature of spacetime* due to the presence of such bodies. Anyway, the knowledge of Eq. 11.2 (and simply refer to gravity as an attractive force exerted by the Earth) is all we need for our purposes: accordingly, feel free to ignore the last sentence.

11.1.3 Newton's Third Law

In its simplest formulation (the so-called *law of action and reaction*), the third law of dynamics states that if a body A exerts a force (which we can call "action") on a body B, then the body B exerts, on the body A, a force (reaction) that is equal in magnitude and opposite in direction to that exerted by A on B. The two bodies are said to *interact* with one another, while the opposite forces due to such interaction are often referred as an *action-reaction force pair* or (*Newton's*) *third-law force pair*. When you push with your hand on a wall, you are exerting a force against it, but also the wall is exerting, at the same time, an (equal and opposite) force against you: in fact, if you are wearing roller skates, or if the floor is wet, you could slip in the opposite direction to the wall (and it was the wall that pushed you!). Other common examples of action-reaction force pair are a bouncing ball, the propulsion of jets and rockets, and the recoil of firearms. When you shoot with a rifle, the rifle exerts a force on the bullet, but also the bullet exerts a force on the rifle. As a consequence, *both* objects move (in the opposite direction, since the two forces have opposite directions). One might wonder why the bullet moves much faster (and, accordingly, travels much farther) than the rifle despite the forces that the two objects exert on one another are equal in magnitude: this is due to the much lower mass of the bullet. In fact, according to the so-called *law of conservation of linear momentum* (another aspect of Newton's third law), the total linear momentum (i.e., the product of mass and speed) of a closed and isolated system remains constant. Accordingly, the linear momentum of the rifle must be equal to that of the bullet: hence, the lower the mass, the higher the speed.

We have already mentioned Newton's third law in Chap. 8, when discussing about the endotracheal tube cuff. With the endotracheal tube in the trachea, the pilot balloon expands soon after you have filled the system cuff-pilot balloon with a few milliliters of air. In fact, as the cuff pushes on the trachea, also the trachea pushes against the cuff and, according to Pascal's principle (see Chap. 8), against the wall of the pilot balloon. You can see with your own eyes the much greater amount of air which is needed indeed to inflate the pilot balloon when the tube is in your hands or placed on a table: in this case, the push due to the *interaction* between the cuff and the tracheal wall is lacking.

11.2 Forces Against Motion: Normal Force and Friction

Now we have to deal with two types of forces that, in our everyday experience, oppose to the motion of objects. Both of them are involved, together with Newton's second law and gravity, in determining the balance of an object on an inclined plane and, in our "anesthesiologic" example, that of the patient's body in the Trendelenburg position.

11.2.1 Normal Force: Physics … of the English Course

I don't know if the same occurs in other countries, but in Italy one of the first exercises of an entry level English course is, typically, "the pen is on the table." Physically speaking, if the pen is (motionless) on the table, it means that some kind of force must counterbalance the gravitational force which pulls down every object towards the center of the Earth (otherwise, the exercise would be "the pen is somewhere in the center of the Earth").

Actually, when a body lies on a surface, such as the pen on the table or a dresser on the floor, the surface exerts on the body a so-called *normal force* F_N, which completely *balances* (in the case of a horizontal surface such as the table or the floor) the gravitational force. This normal force is due to the deformation of the support surface and is always *perpendicular* to it (whence the term "normal," i.e., perpendicular).

If you are wondering whether, in the case of an object lying on a table, gravitational force and normal force are another example of action-reaction force pair, the answer is *no* (although this is a rather widespread belief). First, the action and reaction forces involved in Newton's third law never balance each other, since they are applied to two different bodies interacting with one another (remember that *both* the rifle and the bullet move). Conversely, gravitational force and the normal force exerted by the table are both applied to the pen, which consequently remains in balance (i.e., motionless, which according to Newton's first and second laws means that no *net* force acts on it). Moreover, while action and reaction forces are parallel to each other, normal force is always perpendicular to the supporting surface: in the case of the pen on the table, gravitational force and normal force happen to be parallel since the table is horizontal, but it would not be so on an inclined plane, as we will see below.

11.2.2 Why Your Car Needs an Engine: Friction

According to Newton's first law (or to second law, if you prefer), if an object is moving at a constant speed, we can be sure that no net force acts on it. Why do we need to press on the gas pedal (and, accordingly, to consume fuel so that a driving force is applied to our car) in order to maintain, say, 120 km/h (about 75 mph) on the highway, though? Are Newton's laws valid only in physics textbooks?

Really, Newtonian mechanics does not apply to all phenomena of the real world. As mentioned, it does not apply to objects moving at very high speed (i.e., not negligible as compared with that of light), whose behavior is described by *relativity*. Moreover, Newton's laws lose their validity for very small objects such as atoms and subatomic particles: in this case, we must rely on *quantum physics*. However, 120 km/h (or 75 mph) is absolutely negligible compared with the speed of light, and a car is definitely much bigger than an atom: accordingly, a car traveling on the highway is certainly comprised among those phenomena which are adequately described by Newton's laws.

The reason why our car needs a force to maintain a constant speed is that there is a force, which is called *friction*, that opposes to motion. Simply, when the car travels at a constant speed, the force generated by the engine exactly equals friction, so that no *net* force acts on the car (in full agreement with Newton's laws).

Generally speaking, whenever a force $\vec{F}$ tends to put in motion an object along a surface *S*, the surface itself exerts on the object a *frictional force* $\vec{F}_\mathrm{f}$ which is *parallel* to the surface and has an opposite direction to the (possible) motion, i.e., to the component of $\vec{F}$ which (possibly) causes the motion. The terms "possible" and "possibly" refer to the fact that, if the object is initially motionless, the force $\vec{F}$ will make it move only if the magnitude of the component of $\vec{F}$ parallel to the surface is *greater* than that of the frictional force (in fact, according to Newton's second law, you need a *net* force acting on the body in order to *accelerate* it). Consider, as an example, a heavy book lying on a table. If you push gently with a finger against it (parallel to the table), the book doesn't move due to the frictional force from the table. This kind of friction is known as *static friction*, which is the frictional force between two bodies which are not moving relative to each other. If you push harder (so that the force you apply is *greater* than the static frictional force), the book starts to slide along the table, and you need to keep pushing in order to *equal* the so-called *kinetic friction* (i.e., the frictional force acting between moving objects) and maintain the book in uniform rectilinear motion, as in the case of the car on the highway. Again according to Newton's second law, if the force you apply is greater than the kinetic frictional force, the book accelerates.

The maximum magnitude of *static* friction for an object lying on a surface S is:

$$F_{\mathrm{f}} = \mu F_{\mathrm{N}} \tag{11.3}$$

where F_{N} is the magnitude of the normal force exerted by the surface S on the object (see Sect. 11.2.1) and μ is the so-called *coefficient of static friction*, which depends on the characteristics of both the object and the surface: for example, μ is 1.0 for rubber on concrete (i.e., the maximum static frictional force is equal to the normal force exerted by the supporting surface), 0.2–0.6 for wood on metals, and is very low (about 0.03) for steel on ice, as ice skaters know very well!

11.3 Gravity vs. Friction: Safety in the Trendelenburg Position

A steep (up to 35–45°) *Trendelenburg position*, i.e., head down tilt, is often required in procedures such as laparoscopic (especially robotic) urologic and gynecologic surgery [5–7]. Apart from the well-known respiratory, hemodynamic, and cerebrovascular effects and the other possible complications (including soft tissue edema and ocular injury) [6, 8–10], a major concern of this position is the slipping of the patient along the operating table (under the action of gravity): this may even cause the patient to fall off the table or, if some restraining method such as shoulder blocks, wristlets, braces, or flexed knees is used, may lead to nerve injury (e.g., to the brachial plexus) due to compression or stretch [5, 6, 8].

Evidently, the patient does not slip and, accordingly, does not risk falling or having peripheral nerve injury (if restrained) as long as the frictional force exerted by the table is sufficient to balance the effects of gravity. This depends, intuitively, on the "degree" of tilt, expressed as the width of the angle ϑ between the table and a line parallel to the floor (Fig. 11.1). Which is, therefore, the maximum allowable tilt in order to prevent sliding?

An old article stated that some restraining method is needed to keep the patient in a 40° Trendelenburg position [11]. Let's give a closer look to the forces involved (and to their relationship with the tilt angle) in order to provide a "physical" explanation for this statement and to make some other interesting considerations.

As shown in Fig. 11.2, three forces (all of them applied *to* the patient) must balance each other to keep the patient in the Trendelenburg position without using blocks or other fasteners: the patient's weight W (i.e., the gravitational force), the normal force F_{N} exerted by the operating table as a supporting surface

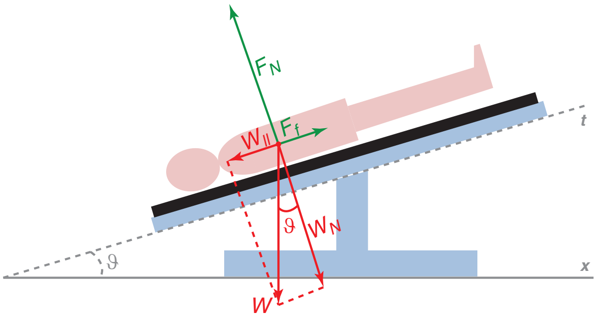

Fig. 11.2 The forces involved in the balance of a patient in the Trendelenburg position. *W* patient's weight, resolved in its components perpendicular (W_N) and parallel ($W_{//}$) to the operating table; F_N normal force; F_f frictional force

(see Sect. 11.2.1), and the frictional force F_f due to the tendency of the patient to slip down the table under the effect of gravitational force. The patient's weight can be resolved into two components (see Chap. 10): the component W_N, perpendicular (or "normal") to the table surface (t), is counterbalanced by the normal force F_N, while the component $W_{//}$, parallel to t, tends to slide the patient down the table.

Remembering the simple notions of trigonometry discussed in Chap. 10 (see Box 10.1 for quick reference) and noting that the angle ϑ between the table and the floor (x) is equal to the angle between the vectors W and W_N (since W is perpendicular to x and W_N is perpendicular to t), the magnitude of the parallel component is:

$$W_{//} = W \sin \vartheta \tag{11.4}$$

or, after substituting for Eq. 11.2:

$$W_{//} = mg \sin \vartheta \tag{11.5}$$

where m is the mass of the patient (what we usually call "weight") and g is the free-fall acceleration.

Similarly, the magnitude of the maximum static frictional force which opposes to sliding can be calculated as follows (keep in your mind Eq. 11.3 and, again, Eq. 11.2 and what discussed in Box 10.1):

$$F_f = \mu F_N = \mu W \cos \vartheta = \mu mg \cos \vartheta \tag{11.6}$$

where μ is the coefficient of static friction for the patient's skin to the table surface and F_N is the normal force.

Note that, for a ϑ value between 0 and 90°, sin ϑ increases with the angle width, while cos ϑ decreases. In particular, sin ϑ is lower than cos ϑ for $\vartheta < 45°$, sin ϑ is higher than cos ϑ for $\vartheta > 45°$, and sin ϑ = cos ϑ for $\vartheta = 45°$. Accordingly, $W_{//}$ increases, and F_f decreases, with the width of ϑ.

As mentioned, the patient remains in balance (i.e., he/she doesn't slip) or, if restrained, doesn't push or pull against restraints, as long as it is:

$$F_f = W_{//} \tag{11.7}$$

According to Eqs. 11.6 and 11.5 it must be, therefore:

$$\mu \; m g \; \cos \vartheta = m g \; \sin \vartheta \tag{11.8}$$

or

$$\mu \; \cos \vartheta = \sin \vartheta \tag{11.9}$$

This applies, of course, to any object lying on an inclined plane (or *ramp*). Consider, as an example, a wooden crate (perhaps full of bottles of champagne) on a downhill concrete road. Assuming a coefficient of static friction μ for wood on concrete of about 0.6, the crate will not slide down (if no one pushes against it!) on a road with a slope up to 30°: in fact, $\cos 30° = 0.866$ and $\sin 30° = 0.5 \cong 0.6 \times 0.866$, that is Eq. 11.9 for $\mu = 0.6$ and $\vartheta = 30°$). The precious bottles will be certainly safe on a 10° descent, unless the road is, say, snowy: in this case, a slope greater than 8° would be enough to make the crate slip (μ is just 0.14 for wood on wet snow, $\cos 8° = 0.99$, $\sin 8° = 0.139 \cong 0.14 \times 0.99$).

It should be noted that the coefficient of friction between two substances is not exactly constant (furthermore, Eq. 11.3 itself is only an approximation, as clearly explained by the famous Nobel laureate Richard Feynman in his landmark textbook *The Feynman Lectures on Physics* [12]).

In the case of the patient on the operating table, the issue is much more complex. The claim that some restraining method is needed to keep the patient at a 40° tilt [11] might make sense provided you do not rely too much on this value as a safety limit. Assuming as coefficient of static friction the median value ($\mu = 0.73$) found between in vivo human skin and an aluminum sample in a recent investigation [13], the patient would slip (along an aluminum table) with $\vartheta > 36°$ ($\cos 36° = 0.81$; $\sin 36° = 0.59 \cong 0.73 \times 0.81$). However, the top surface of operating tables is usually not made of aluminum. Moreover, different operating tables may exert different frictional forces, and the placement of a drape (e.g., a disposable bedsheet) between the table and the patient may result in a significant reduction in the coefficient of friction [14]. Most remarkably, a lot of factors may affect the frictional properties of human skin, including age, anatomical site, presence of hair, hydration, sweating, temperature, and ambient humidity, to name but a few [13, 15, 16]. Thus, it is very difficult (if not impossible) to exactly predict the maximum tilt angle allowable to prevent sliding: it might be much lower than 35–40°, depending on several factors.

A possible way to increase safety in the Trendelenburg position, allowing a higher tilt angle without resorting to restraining methods (or at least preventing nerve injury due to compression or stretch caused by them), is the use of anti-slip devices, such as pads or mattresses, which increase the coefficient of friction [5, 17, 18]. However, further studies are needed to better understand their usefulness and, in particular, to what extent they safely increase the maximum tilt allowed.

Finally, one might think that obese patients (simply because they weigh more) are at higher risk of downward slipping in the Trendelenburg position, as some author stated [18]. Conversely, it is clear from Eqs. 11.8 and 11.9 that mass (which in fact disappears from Eq. 11.8 since it is present in both its sides) equally contributes to both the force pushing down (i.e., the parallel component of the patient's weight) and the frictional force which opposes to it: accordingly, patient's body mass should not affect the risk of slipping, unless we consider, for some reason, the coefficient of static friction as a function of mass. Indeed, since the patient is not a wooden crate but a (complex) biological system, it cannot be excluded that an increased body mass (as well as a possible condition of morbid obesity) may affect the frictional properties of skin or of the whole patient's body (remember that the skin's coefficient of friction may vary among different anatomical sites). However, this may not necessarily reduce the coefficient of static friction, but even *increase* it, leading to a reduced risk of sliding. Consistently, although Klauschie et al. [18] found that body mass did not significantly affect the extent of patient's shift along an anti-slip device (pink egg-crate foam) in the Trendelenburg position, a trend towards a reduced shift can be noted in obese patients.

Future well-designed studies about these topics are desirable and would be very interesting, but are rather difficult to perform rigorously. Meanwhile, it is advisable not to exceed with tilt, especially since recent findings suggest that most robotic procedures can be performed safely (from a surgical standpoint) with a Trendelenburg angle of less than 30° [19].

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Part V

Inhalational Anesthesia

Physics in a Vaporizer: Saturated Vapor Pressure, Heat of Vaporization, and Thermal Expansion

12

Where's Physics

Daily life	Water boiling in your kitchen Pressure cooker Absolute and relative humidity A drink left in a glass in the garden (on a windy day) Clothes drying in the wind Train tracks, bridges, and thermostats
Physics involved	State changes of matter (particularly vaporization) Saturated vapor pressure Heat of transformation and evaporative cooling Thermal conductivity Specific heat Dalton's law Thermal expansion
Clinical practice	Anesthetic vaporizers (how they work, why are they so heavy, what makes them quite reliable, how they adapt to the ambient conditions) Humidification of inspired gases in the airways Cylinders containing liquefied gases

12.1 Why a Vaporizer Is Not Exactly a "Vaporizer": Saturated Vapor Pressure and Volatility

A thorough coverage of the different types of vaporizers, the way they work, and their advantages and drawbacks is beyond the scope of the present book and can be found elsewhere, including most anesthesia textbooks [1–4]. This chapter is limited to pointing out and discussing the many physical principles (some of which have already been mentioned in previous chapters) which are the basis of the operation and design of vaporizers, with special reference to the so-called *variable bypass*

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vaporizers, i.e., those most commonly used for the administration of halogenated anesthetics such as isoflurane and sevoflurane (but not desflurane). Let's start with the concept of saturated vapor pressure.

12.1.1 Saturated Vapor Pressure and Boiling Point

Any liquid, in an enclosed container, evaporates (i.e., vaporizes *from its surface*) until it reaches thermodynamic equilibrium, that is, until the number of molecules which have a kinetic energy high enough to escape from the liquid phase (to form vapor) equals that of molecules returning to the liquid. The pressure of the vapor in these circumstances is called *saturated vapor pressure* (SVP) and represents, evidently, the maximum vapor pressure possible under these conditions. Since, as discussed in Chap. 1 (see Box 1.2), the kinetic energy of molecules increases with temperature (or better, a higher temperature precisely *means* a higher average kinetic energy of molecules), it goes without saying that the saturated vapor pressure of any substance *increases with temperature* (see Fig. 12.1). For example, the SVP of water is 17.5 mmHg (or 2.3 kPa) at 20 °C (or 68 °F). When you put some water, whose initial temperature is 20 °C (68 °F), to boil in a pot, the *partial pressure* of water vapor just above the liquid surface, which is initially about 17.5 mmHg (i.e., equal to the SVP of water at that temperature), increases progressively as temperature rises. As mentioned in Chap. 1 (see Box 1.1), water will boil when its vapor pressure will equal the external atmospheric pressure. In these conditions, in fact,

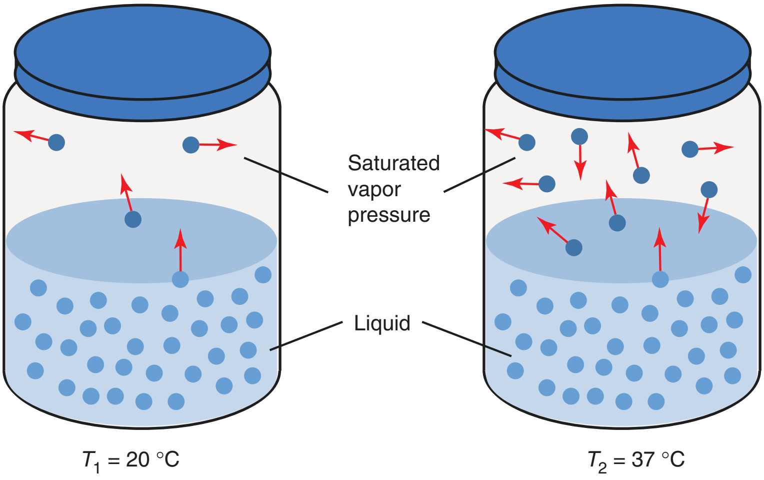

Fig. 12.1 Saturated vapor pressure (SVP) can be defined as the pressure exerted by a vapor which is in thermodynamic equilibrium with its condensed phase in an enclosed space. SVP is a function of temperature: in fact, a larger number of molecules has sufficient kinetic energy to leave the liquid phase at any temperature $T_2 > T_1$. *Red arrows* represent the speed vectors of each vapor molecule

water vapor has a sufficient pressure to form bubbles within the bulk of liquid, and vaporization turns from a "surface phenomenon" to a "volume phenomenon." The temperature at which this occurs is called *boiling point*. Evidently, the boiling point of a given substance varies according to the surrounding environment pressure. For example, the SVP of water is equal to 760 mmHg (or 101.3 kPa), that is, the atmospheric pressure at sea level, at a temperature of 100 °C (212 °F): this is, accordingly, the temperature at which water boils at sea level (or its *normal boiling point*). In a pressure cooker, the "surrounding environment pressure" becomes higher and higher as water vapor accumulates in an enclosed space: thus, the boiling point of water increases (up to 120 °C or 248 °F), and foods are cooked faster. As discussed in Chap. 1 (see Box 1.1), the opposite occurs in the high mountains.

The notions discussed above provide an explanation for what is stated in Chap. 3 about air humidity and the humidification of inhaled air in the airways (see Box 12.1). Also the issue, discussed in Chap. 1, of a cylinder containing a liquefied gas, whose content cannot be "measured" through a pressure gauge (which will always measure nothing but the SVP, whatever the remaining volume of liquid), is now certainly clearer.

Box 12.1: Saturated Vapor Pressure and (Air) Humidity

Atmospheric humidity can be defined roughly as the "amount" of water vapor in the air (*absolute humidity*). As in the example of the pot of water (Sect. 12.1.1), the higher the temperature, the higher the partial pressure of water vapor in the air (consider the pot of water as the sea, lakes, and so on in a certain region, and your kitchen as the surrounding atmosphere). Accordingly, *absolute humidity increases with temperature*. However, humidity is usually expressed as the ratio between the actual amount of water vapor and the maximum amount of water vapor which air can "hold" at a given temperature (*relative humidity*). This "maximum amount" is, evidently, the saturated vapor pressure of water at that temperature (similarly, the maximum humidification of inhaled air in the airways corresponds to the SVP of water at 37 °C, or 98.6 °F, as mentioned in Chap. 3). Since the saturated vapor pressure increases with temperature, relative humidity *decreases* with temperature (for a fixed "absolute amount" of water vapor in the air, i.e., if absolute humidity does not change): that's why when you turn on the heating at your home the air gets "dry" (whereas humidifiers, providing a source of water, increase absolute humidity and, thus, prevent an excessive reduction in relative humidity).

12.1.2 Volatility of Halogenated Anesthetics and … The "Trick" of the Variable-Bypass Vaporizer

The saturated vapor pressure of a liquid at a given temperature (usually 20 °C, or 68 °F) is a measure of the *volatility* of such liquid, i.e., of its tendency to vaporize: the higher the SVP at 20 °C (68 °F) of the liquid, the more quickly it vaporizes (at that temperature).

The term "vaporizer" suggests that these devices *cause* the vaporization of halogenated anesthetics. Instead, these drugs vaporize, *spontaneously*, very easily (for no other reason they are also called "volatile anesthetic agents"). As shown in Table 12.1, in fact, all halogenated anesthetics have a SVP at 20 °C much higher than water [5]. Sevoflurane, for example, which is about nine times more volatile than water (and nevertheless is the less volatile among the halogenated anesthetics), has a SVP at 20 °C of 157.5 mmHg (or 21 kPa). This would be, therefore, the pressure of sevoflurane vapor in an enclosed space containing the anesthetic in liquid form, in equilibrium with its vapor, at the temperature of 20 °C. If a fresh gas mixture flowed through such a *vaporizing chamber*, coming into contact with sevoflurane vapor, the concentration of sevoflurane in the outlet mixture would be, according to Dalton's law (see Chap. 3) and remembering that the atmospheric pressure at sea level is 760 mmHg,

$$\frac{157.5\,\text{mmHg}}{760\,\text{mmHg}} \times 100 = 20.7\% \tag{12.1}$$

It is evident that sevoflurane does not need at all a "vaporizer" to vaporize: in fact, it vaporizes even too much by itself! The issue is how to reduce this very high concentration to the clinically adequate (and safe) values of around 1–2% [6], and how to set the desired concentration with a simple action (that is, by turning the concentration dial).

In a common *variable bypass vaporizer* (see Fig. 12.2), the concentration of anesthetic delivered to the patient depends on how much fresh gas enters the vaporizing chamber (i.e., the space containing the liquid agent in equilibrium with its vapor) as compared with how much is diverted into a "bypass": the concentration dial simply sets this *splitting ratio*. Thus, the maximum concentration possible (i.e., about 21%) is more or less diluted by the fresh gas that flows into the bypass, in order to obtain the desired concentration. These vaporizers are, obviously, agent specific, since the splitting ratio needed to get a certain concentration depends on the SVP. For example, isoflurane is much more volatile than sevoflurane (see Table 12.1): accordingly, as compared to sevoflurane, a smaller fraction of the fresh gas flow needs to enter the vaporizing chamber to obtain the same concentration.

Table 12.1 Approximated values of saturated vapor pressure (SVP) at 20 °C of halogenated anesthetics

	SVP at 20 °C (68 °F)
Water	17.5 mmHg (2.3 kPa)
Sevoflurane	157.5 mmHg (21 kPa)
Enflurane	172 mmHg (23 kPa)
Isoflurane	240 mmHg (32 kPa)
Halotane	244 mmHg (32.5 kPa)
Desflurane	669 mmHg (89 kPa)

SVP of water is also reported for comparison

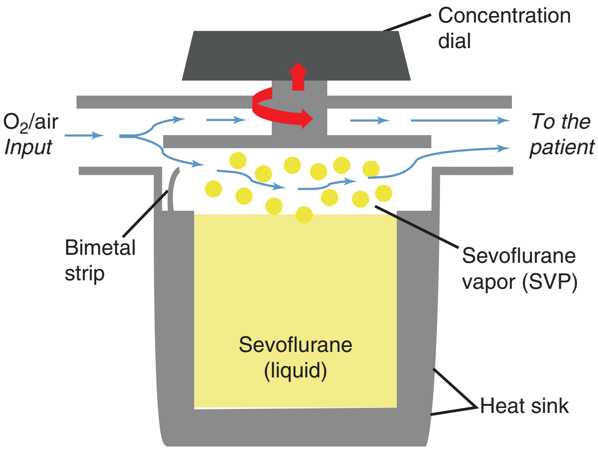

Fig. 12.2 Schematic representation of a variable bypass vaporizer for sevoflurane (see text). The *heat sink* (see Sect. 12.2.3) and the *bimetal strip* (see Sect. 12.3) provide temperature stabilization and compensation, respectively. *SVP* Saturated vapor pressure

12.1.3 Why Desflurane Needs a Special Kind of Vaporizer

As shown in Table 12.1, desflurane is the most volatile among halogenated anesthetics. Clearly, the higher the SVP of a liquid, the lower its boiling point. Since the SVP at 20 °C of desflurane (669 mmHg or 89 kPa) is already very close to the atmospheric pressure at sea level (760 mmHg or 101.3 kPa), its normal boiling point is only slightly higher than 20 °C (i.e., around 23 °C or 73 °F). Accordingly, desflurane could boil at room temperature (including that of an operating room), especially at higher altitudes (see Chap. 1), leading to hazardous doses delivered to the patient. Moreover, a very high bypass flow would be needed in order to dilute the concentration of desflurane at the desired values, since the concentration leaving the vaporizing chamber at 20 °C would be $100 \times (669/760) = 88\%$! Mainly for these reasons, the common variable bypass vaporizers are unsuitable for the administration of desflurane, that needs a special kind of vaporizer in which the anesthetic is heated at 39 °C (about 102 °F), reaching a vapor pressure of about 1500 mmHg (or 2 atm), and its inlet in the fresh gas flow is controlled through an electronically operated system consisting of a pressure-regulating valve and a pressure transducer [1, 3, 4, 7–9].

12.2 Why Vaporizers Are so Heavy: Heat of Transformation and the Need for Temperature Stabilization

If you have ever replaced the (variable bypass) vaporizer in your anesthesia machine, you well know how much these devices weigh (more or less like a 2-year-old child). As discussed below, also this has to do with physics, in particular with the *state change* which occurs in the vaporizer and the consequent need to keep its temperature constant in order to maintain its accuracy.

12.2.1 Some Notes About the State Changes of Matter

In Sect. 12.1.1 we reminded why the boiling point of a liquid depends on the surrounding pressure (as already mentioned in Chap. 1). This applies, actually, to every *state* (or *phase*) *change* of matter: the temperature at which any substance melts, vaporizes, etc. is characteristic of that substance, but varies with pressure. Accordingly, it makes no sense to say that the *melting point* of lead is 328 °C (or 622 °F) without specifying "at normal (i.e., sea-level) atmospheric pressure" (although in the current language, also among physicists, the term "melting point" conventionally refers to this pressure). Note that the temperature at which a state change occurs (under a given pressure) is always the same of the reverse transformation: for example, the boiling point of water at a given pressure is equal to its "condensing point" at that pressure, as well as its melting point is equal to its "freezing point." In fact, these are the temperatures at which two states (e.g., liquid and vapor, solid and liquid) *coexist*. Accordingly, they remain constant until *all* the amount of substance has been converted into another state. Consider again some water that is boiling in a pot (at sea level). We had put the water on the stove in order to transfer some *heat* to it and, hence, increase its temperature (see Chap. 9) up to its boiling point, i.e., 100 °C: as long as some liquid water remains in the pot, we can be sure that its temperature still remains 100 °C, *although we continue to provide heat.* Contrary to the heat supplied initially, which causes an increase in temperature, this heat is needed to free the water molecules from their clusters, i.e., to overcome the *cohesive forces* between them (see Chap. 5). In general, every state change of matter is accompanied by an *energy transfer* (as absorbed or released heat): the amount of energy which needs to be transferred so that a unit mass of a given substance completely undergoes a state change is called *heat of transformation* (or *latent heat*). In particular, the *heat of vaporization* (also referred as *latent heat of vaporization*) is the amount of energy which must be supplied to a unit mass of liquid in order to completely convert it into vapor.

12.2.2 Evaporative Cooling and Accuracy of Vaporizers

There is no doubt that, during inhalational anesthesia, the liquid anesthetic agent (say, sevoflurane) contained in the vaporizer is progressively converted into vapor. In fact, as the fresh gas mixture flows through the vaporizing chamber, it carries away some amount of vapor. Consequently, other sevoflurane molecules leave the liquid phase in order to restore the equilibrium (i.e., to maintain the vapor pressure at the value of the SVP). According to what discussed above, this process requires energy (heat of vaporization). Since, unlike the example of the pot of water on the stove, no one supplies heat from the outside, the anesthetic itself must provide this heat and, hence, it tends to cool (*evaporative cooling*) [1]. Accordingly, its SVP, which depends on temperature (see Sect. 12.1.1), decreases. Evaporative cooling of halogenated anesthetics is most pronounced when high fresh gas flows are used. In fact, the higher the flow of fresh gas through the vaporizing chamber, the faster the vapor has to be replaced from the liquid reservoir and, therefore, the more rapidly the temperature (and, as a consequence, the SVP) will drop. For the same reason, a liquid in an open container exposed to the wind (such as a soft drink left in a glass in the garden) is always at a temperature lower than the ambient temperature. Similarly, clothes will dry quickly (i.e., water will evaporate quickly) on a windy day, but they will appear cold.

Since the splitting ratio which must correspond to every mark on the concentration dial in order to obtain the desired anesthetic concentration is calibrated on a certain value of SVP of the anesthetic agent (e.g., that at 20 °C), the accuracy of variable bypass vaporizers decreases progressively as the temperature (and, accordingly, the SVP) of the anesthetic agent falls, leading to actual concentrations lower than those you set. For this reason, temperature must be kept as constant as possible in variable bypass vaporizers.

12.2.3 Temperature Stabilization (Heat Sink): Specific Heat and Thermal Conductivity

In modern variable bypass vaporizers, *temperature stabilization* is achieved thanks to the presence of a *heat sink*, namely a large mass of a metal alloy with high *specific heat* (see Chap. 9) and high *thermal conductivity* [4]. The latter is the constant k of the following equation:

$$\frac{Q}{t} = k\,A\,\frac{T_1 - T_2}{l} \tag{12.2}$$

where Q/t is the heat Q which is transferred by *conduction* over the time t (i.e., the so-called *heat conduction rate*) between the two faces of a sheet with thickness l and face area A whose temperatures are T_1 and T_2, respectively.

In other words, due to its high *heat capacity* (as discussed in Chap. 9, a large mass of a substance with high specific heat has a high heat capacity, that is the heat which such a substance must release in order to reduce its temperature by 1 °C), the heat sink is capable of releasing a lot of heat with a small decrease in its temperature. Moreover, due to its high thermal conductivity, it readily transfers heat to the anesthetic. Accordingly, the heat sink (to which the considerable weight of vaporizers is largely due) minimizes the fall in temperature (and, accordingly, in SVP) of the anesthetic due to evaporative cooling and, hence, preserves the accuracy of the vaporizer.

12.3 Thermal Expansion: Train Tracks, Thermostats … and Temperature Compensation in Vaporizers

In addition to temperature stabilization, modern variable bypass vaporizers are also *temperature compensated*; that is, the splitting ratio varies with temperature in order to keep the anesthetic concentration delivered constant [1–4]. In fact, ambient temperature in the operating room affects the SVP of the anesthetic agent and, accordingly, the share of fresh gas which needs to be directed into the vaporizing chamber in order to get a certain concentration. For example, if the operating room temperature is kept at around 17 °C (such as during interventions requiring deep hypothermic circulatory arrest), the SVP of sevoflurane is lower than that at 20 °C and, accordingly, a *higher* proportion of fresh gas needs to enter the vaporizing chamber as compared to that needed to get the same concentration at 20 °C.

Also temperature compensation relies on a physical phenomenon: *thermal expansion*. You surely know that, until a few years ago, between two consecutive train tracks there was always a little space that allowed them to lengthen, during the warmer seasons, without clashing each other: otherwise, they could be dangerously deformed (the same applies to the different sections of a bridge). In fact, when the temperature of a metal rod of length l is raised by an amount ΔT, its length will increase by an amount Δl, according to the equation

$$\Delta l = l\alpha \ \Delta T \tag{12.3}$$

where α is the so-called *coefficient of linear expansion*, a constant depending on the material.

The common liquid-in-glass clinic thermometers are based on the fact that, as temperature rises, liquids such as mercury (now banned in many countries due to toxicity and environmental hazard) or gallium expand to a greater extent than their glass container.

Another way to take advantage of the different coefficients of linear expansion of two different materials is to weld together two metals, for example copper and steel. Such a *bimetal strip* (Fig. 12.3) will bend as temperature varies, due to the

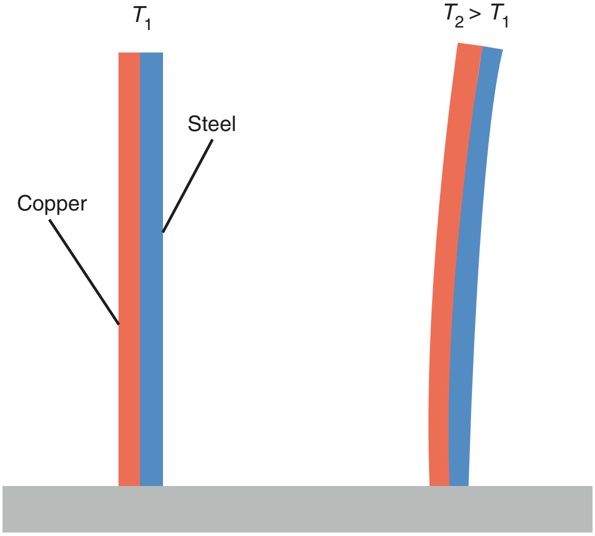

Fig. 12.3 A bimetal strip bends (evidently towards the side of the metal with the lower coefficient of linear expansion α) as its temperature increases. α is 17×10^{-6}/°C for copper and 11×10^{-6}/°C for steel

different elongation of the two juxtaposed strips. This is the way many thermostats work: as the bimetal strip bends due to temperature changes, it can make or break an electrical contact that operates the heating. A similar bimetal strip is the most commonly used method for temperature compensation in modern variable bypass vaporizers (see Fig. 12.2) [4, 10]: as temperature decreases, the bimetal strip bends so that an increased flow of fresh gas is allowed to enter the vaporizing chamber, while the opposite occurs if temperature increases. Other models have simple metal bellows or rods which expand or contract as temperature changes.

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Part VI

Waves

Light, Air Pollution, and Pulse Oximetry: The Beer-Lambert Law

13

Where's Physics

Daily life	Attenuation of sunlight by the atmosphere The colors of the sea The depth at which divers need a flashlight Air pollution and traffic ban
Physics involved	Electromagnetic waves and light Light absorption (Beer-Lambert law)
Clinical practice	Pulse oximetry Capnography Inhaled anesthetic analyzers The different colors of arterial and venous blood

13.1 Light, Waves, and the Anesthesiologist

This chapter and the next one deal with *light* and, more generally, with *electromagnetic waves*, while *sound waves* are discussed in Chap. 15. Waves are everywhere in our lives: the colors and the sounds of nature, the different sounds of musical instruments, surfing and earthquakes, most of the technological tools we use everyday (including DVDs, Blu-ray discs, remote controllers, smartphones, and microwave ovens, to name but a few), many medical devices (e.g., all the diagnostic imaging, practically), speeding fines, and even the strange way in which spaghetti break (see below) rely on waves and their behavior.

Why talk about light in a book for anesthesiologists and intensivists, though? A possible answer is that there is a light bulb in the laryngoscope or that, sometimes, we have to run faster than light! In reality, at least two physical phenomena involving electromagnetic waves, and the laws which describe them, have many important

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applications (and implications) in daily practice of anesthesia and intensive care medicine and, consistent with the spirit of this book, allow to provide a lot of interesting examples from everyday life.

In particular, this chapter addresses the so-called *Beer-Lambert law*, which describes the attenuation of light as it passes through absorbing substances: *pulse oximetry* (and some of its limitations) and certain methods of respiratory gas monitoring (such as *capnography*) are based on it. Other gas analyzers rely, instead, on light *scattering*, which is primarily involved in *near-infrared spectroscopy (NIRS) regional oximetry* (being, moreover, the cause of its main limitation), as well as in some aspects of *radioprotection*, as discussed in Chap. 14.

We will discuss all this, of course, not before to have briefly remembered some of the main features of light.

13.2 What Is Light

The description of light that is closest to our everyday experience is provided by the so-called *ray model*, according to which light travels in straight lines. Within certain limits, this model works well. However, a set of “rays” (not interacting with one another) is not what light *really* is. In fact, light undergoes *interference*, which is a typical behavior of waves: the *superposition* of two or more waves produces a resulting wave, or “interference pattern,” with a different *amplitude* (see below). Light can be described, therefore, as a wave (in particular, an electromagnetic wave), although it behaves, sometimes, as a (massless) *particle*: the *photon* (according to the so-called *photon model of light*). In reality, in the world of molecules, atoms, and subatomic particles (including the photon!), *all* particles *are* also waves, and vice versa. This is a fundamental point of *quantum physics* (or *quantum mechanics*), which is definitely beyond the scope of this book: to know more, I suggest you some books that I found particularly enlightening (excuse the pun) [1–3].

Rather we need to know something more about electromagnetic waves.

13.2.1 Light as a Wave

Generally speaking, a wave is a disturbance (i.e., a change in some physical property or dimension) which travels through a “medium,” carrying *energy*. Specifically, waves are produced by *oscillations* of such “dimension,” i.e., repetitive (or periodic) displacements, both over space and time, around an equilibrium value. Accordingly, they can be described by *sinusoidal* (i.e., sine or cosine) *equations*, or as the sum of sinusoidal waves (see Chap. 15). Waves are usually classified according to the medium in which the cyclic disturbance propagates:

1. *Mechanical waves* need a material medium. Sea waves, for example, travel through water (which is, accordingly, “the medium”), while the oscillating “dimension” is the height of the ruffle. This is almost always the first example

that comes to mind when talking about waves and can be found, practically, in all physics textbooks. I would add, therefore, a more intriguing one. Have you ever noticed that spaghetti almost never break in half, but at least in three or four fragments, when bended while holding them at their ends? As suggested recently, this may be due to *flexural waves* (i.e., changes in the curvature) traveling along the two fragments which are formed after the first breaking [4]. Also sound waves (see Chap. 15) are mechanical waves: in this case, the medium can be air, water, or something else, while what oscillates is density (and, hence, pressure).

2. *Electromagnetic waves* (Fig. 13.1) do not need a material medium and, accordingly, can travel through a vacuum. What oscillates is the *strength* of the electric field and of the magnetic field, which are perpendicular to each other (in physics, a “field” refers to something which has a defined value at every point in the space: for example, an electric field, which forms around a charged particle *A*, exerts on a second charged particle *B* a force which depends on the position of *B* in the space surrounding *A*). As shown in Fig. 13.1, electromagnetic waves travel perpendicularly to the planes in which the electric and magnetic fields oscillate: accordingly, they are *transverse waves*.

A third type of waves, called *matter waves*, has to do with that strange affair of particles which, according to quantum physics, behave also as waves (and, of course, does not concern us).

All waves are characterized by the following properties: *amplitude*, *period, frequency,* and *wavelength* (see Fig. 13.2).

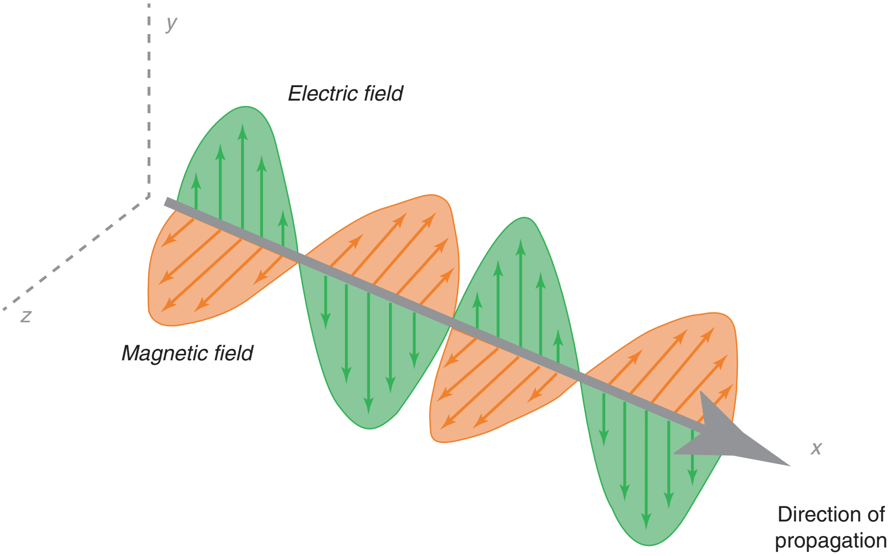
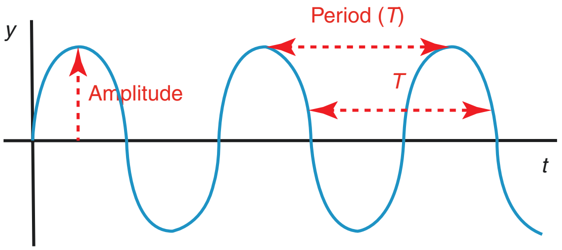
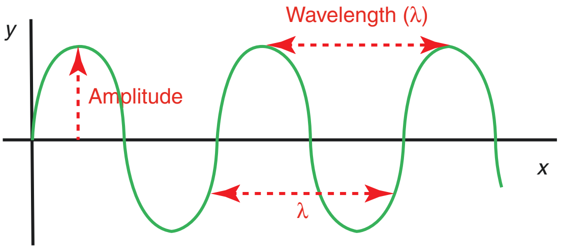


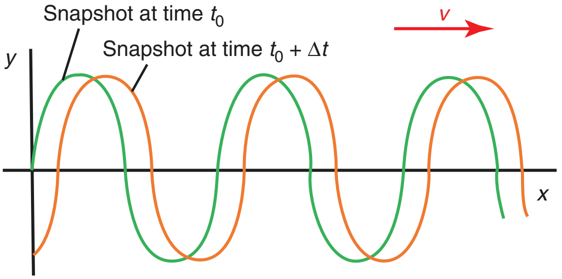
Fig. 13.1 In an electromagnetic wave, the strength of the electric field and that of the magnetic field oscillate perpendicularly to each other and to the direction of propagation



a **Graph** (displacement at one point over time)



b **Snapshot** of the wave at a time t_0



c The wave propagates *through the space* with speed v

Fig. 13.2 Some of the main properties of sinusoidal waves (see text)

The *amplitude* is the maximum "displacement" produced by the wave (Fig. 13.2a, b). The amplitude of an electromagnetic wave represents its *intensity* (symbolized I), that is, the mean *energy* transported by the wave over one cycle of oscillation, or the rate at which the wave transfers energy to a certain surface, i.e., energy per unit area and unit time (see also Chap. 14).

The *period* T of oscillation of a wave is the time interval in which a complete cycle occurs. It corresponds to the time interval between two wave crests (or between two any repetitions) in a *graph* displaying the "displacement" which occurs at one point in space *over time* (Fig. 13.2a). The frequency f is simply the number of oscillations per second. The unit of frequency is s^{-1} or *Hertz* (Hz). Evidently, it is

$$f = \frac{1}{T} \tag{13.1}$$

The *wavelength* λ is the distance between repetitions of the wave shape (e.g., two crests), as can be identified in a *snapshot* of the wave which shows the "displacement" *as a function of distance* (i.e., of the position in space) at a certain time instant (Fig. 13.2b). In other words, the wavelength arises from the propagation of the wave through the space with a certain speed (see Fig. 13.2c), and represents *the distance traveled by the wave in a time interval of one period*.

Remembering that speed (or velocity) is, roughly, distance divided by time (see Chaps. 3 and 6), the *wave speed* v (i.e., the distance traveled by the wave during a time interval Δt) is

$$v = \frac{\lambda}{T} \tag{13.2}$$

or, substituting for Eq. 13.1,

$$v = \lambda f \tag{13.3}$$

Note that the wave speed is a property of the medium in which the wave travels. Since electromagnetic waves don't need a material medium, they have their own speed *in vacuum* (although it decreases in materials such as water or glass), which is the same regardless of the type of electromagnetic wave (see below): this is the *speed of light* (symbolized c from Latin "celeritas," i.e., speed), which is about 300,000 km/s (or 186,000 mi/s), as mentioned in Chap. 11. We'll find out more about the speed of light in Chap. 16.

For electromagnetic waves traveling in a vacuum, therefore, Eq. 13.3 becomes

$$\lambda f = c \tag{13.4}$$

that is, wavelength and frequency are inversely proportional (since the speed of light c is a constant).

13.2.2 The Electromagnetic Spectrum

Visible light is an electromagnetic wave with a wavelength of about 400–700 nanometers (nm). As shown in Fig. 13.3, this is only a very small portion of the so-called *electromagnetic spectrum*, i.e., the whole range of the possible wavelengths of electromagnetic waves.

In order of decreasing wavelength (or, according to Eq. 13.4, increasing frequency), the electromagnetic spectrum includes, among others, radio (and TV) waves, microwaves, infrared (IR) radiation, visible light, ultraviolet (UV) radiation, X-rays, and gamma rays. Within the range of visible light (400–700 nm), we perceive each wavelength as a different color: for example, the shorter wavelengths (higher frequencies) correspond to blue/violet light, while the longer wavelengths (lower frequencies) are perceived as orange/red.

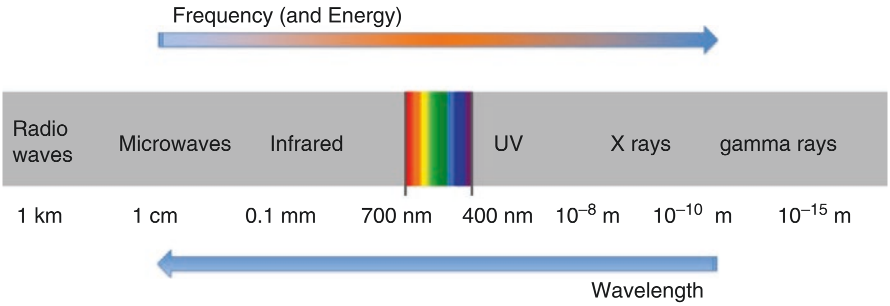


Fig. 13.3 A "qualitative" representation of the electromagnetic spectrum (the distances are not to scale). It can be seen, however, by reading the values of the wavelengths that visible light is only a very narrow part of the entire range of electromagnetic waves

The higher the frequency (or the shorter the wavelength), the greater the energy of the electromagnetic wave (for a given wave intensity). According to quantum physics, many quantities in the world of atoms and subatomic particles only exist as integer multiples of a minimum (or elementary) amount, called *quantum*. This also applies to electromagnetic waves: the above-mentioned photon represents, in fact, the quantum of light (or, in general, of electromagnetic radiation). For an electromagnetic wave of frequency f, the energy of a single photon (*photon energy*) is

$$E = h\,f \tag{13.5}$$

where h is a very small quantity called *Planck constant*.

Hence, one photon of violet light has a higher energy than one photon of red light, as well as the energy carried by an X-ray photon is much greater than that transported by a microwave photon (as one can easily guess). The photon energy, however, must not be confused with the energy carried by a certain electromagnetic wave, i.e., with the wave intensity: for a light of a given wavelength, a higher intensity simply means that the wave is made of more photons (and, hence, carries more energy).

13.3 Blue Oceans and Sea Deeps: Beer-Lambert Law

The *Beer-Lambert law*, also known as *Beer's law* (or Lambert-Beer law), correlates the absorption of an electromagnetic wave, as it passes through an object, with the thickness of the object and the amount and absorbing properties of the substances contained therein.

In particular, when a beam of *monochromatic light* (i.e., an electromagnetic radiation of a single wavelength) of intensity I_0 crosses a *solution* by a length L (see Fig. 13.4), the intensity I of the transmitted light decreases according to the following equation:

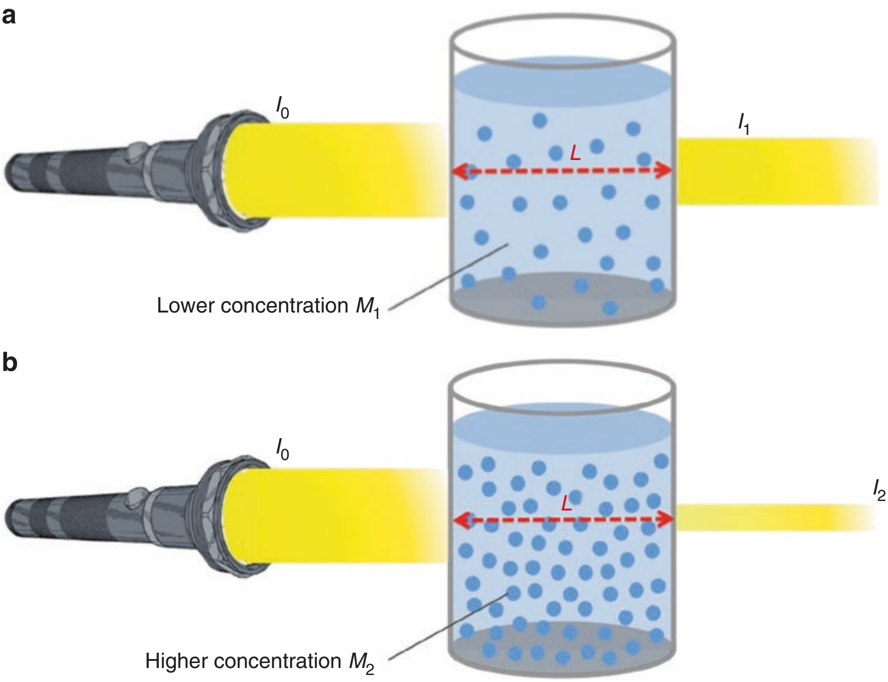


Fig. 13.4 Beer-Lambert law. When a monochromatic light beam of intensity I_0 crosses (by a length L) a solution containing a substance which absorbs light of that wavelength, the intensity of the transmitted light decreases exponentially with the increase in the concentration of the absorbing substance. Since the concentration in **a** is lower than in **b**, it is $I_1 > I_2$. Note that a flashlight like that drawn here does not emit monochromatic light (but, in this way, the figure is prettier!)

$$I = I_0 \ e^{-\varepsilon_\lambda ML} \tag{13.6}$$

where e is the base of natural logarithm, L is the path length, M is the molar concentration (or molarity) of the solution, and ε_λ is the so-called *molar attenuation coefficient* (or *molar extinction coefficient*), which is a characteristic of the solute and depends on the wavelength (in fact, it is here symbolized ε_λ to indicate that it is a function of wavelength). In practice, ε_λ is a measure of how much a chemical species attenuates light at a given wavelength.

If we define the *absorbance* A of the solution as log I_0/I, Eq. 13.6 can be written in the following more simple form:

$$A = \varepsilon_\lambda M \, L \tag{13.7}$$

Hence, the higher the concentration of the solution (and the greater the path length), the higher the absorbance, i.e., the lower the intensity of the transmitted light (Fig. 13.4).

A more complex version of this law allows to describe the attenuation of sun radiation as it travels through the atmosphere. Moreover, Beer's law accounts for the color of the sea (due to the dependence of the attenuation coefficient on the wavelength). In fact, the absorbance of seawater is higher for the longer wavelengths (those of red and orange), while shorter wavelengths (such as those of green, blue, and violet) are absorbed to a lesser extent and, accordingly, are more available for scattering, i.e., deviation in all directions (see Chap. 14). Since sunlight contains a poor amount of violet light, and our retina is poorly sensitive to violet [5], the sea looks blue (or green in some wonderful places!). Again according to Beer's law, the view around a diver will appear more and more blue as depth increases, until a flashlight will be needed (in fact, the absorbance for any wavelength increases with the path length).

Beer-Lambert law can be used to determine the concentration of a substance in a solution or in a gas mixture, according to the absorption of a (visible, IR, or UV) light of an appropriately chosen wavelength (*absorption spectroscopy*). At its simplest, if I have a solution which contains only one absorbing species (of which I know the molar attenuation coefficient for a given wavelength), I can irradiate a sample (of a certain thickness) of such a solution with a beam of light (of known intensity) of that wavelength and *measure* (thanks to a light detector) the intensity of the transmitted light: according to Eq. 13.6, the only unknown is the concentration of the absorbing species that can be therefore calculated. Some systems for the analysis of polluting substances in the air rely on this technique: also thanks to Beer's law, therefore, we can realize that our cities are polluted (and sometimes the authorities are compelled, for example, to ban the car traffic). What interests us, however, is that many devices we routinely use in the operating room and intensive care unit are also based on this physical law.

13.4 The Beer-Lambert Law in the Operating Room

As mentioned, pulse oximetry, capnography, and some other respiratory gas monitoring systems such as anesthetic (vapor) analyzers rely on absorption spectroscopy and, accordingly, on Beer-Lambert law. The last part of this chapter mainly addresses the physical and technological bases (and some of their clinical implications) of pulse oximetry. Also regional (e.g., cerebral) oximetry is based on Beer's law, but it also involves another physical phenomenon, i.e., the scattering of electromagnetic radiation, and will therefore be discussed in Chap. 14.

13.4.1 Pulse Oximetry

As we all know, pulse oximetry is the measurement of *arterial oxygen saturation* (S_aO_2), that is, roughly (i.e., considering the blood content of methemoglobin and carboxyhemoglobin as negligible), the ratio of the concentration of *oxyhemoglobin* (O_2Hb) to the sum of the concentrations of O_2Hb and *reduced hemoglobin* (Hb):

$$S_aO_2 = \frac{[O_2Hb]}{[O_2Hb]+[Hb]} \tag{13.8}$$

Such measurement is possible, by means of absorption spectroscopy, thanks to the different molar attenuation coefficients (ε_λ) of oxyhemoglobin and reduced hemoglobin for most wavelengths (see Fig. 13.5) [6–9].

In particular, a common pulse oximeter (Fig. 13.6) uses two monochromatic *light-emitting diodes* (LEDs) to alternately irradiate a finger (or an earlobe) with two wavelengths for which the difference between the molar attenuation coefficients of O_2Hb and Hb is particularly wide: 660 nm (red visible light) and 940 nm (infrared radiation) [6–8]. Even more importantly, as shown in Fig. 13.5, the molar

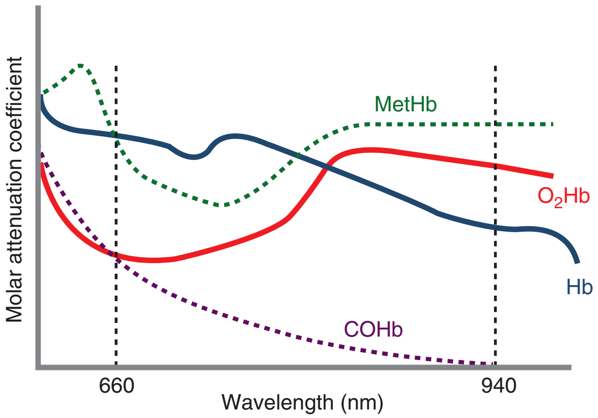


Fig. 13.5 Values of the molar attenuation coefficient of the different forms of hemoglobin as a function of wavelength. Note the differences between oxyhemoglobin (O_2Hb) and reduced hemoglobin (Hb) at the two wavelengths (660 and 940 nm) used in common pulse oximeters. The greater absorption of red light (660 nm) by reduced hemoglobin also accounts for the darker appearance of venous blood as compared to the "more red" arterial blood. *MetHb* methemoglobin; *COHb* carboxyhemoglobin

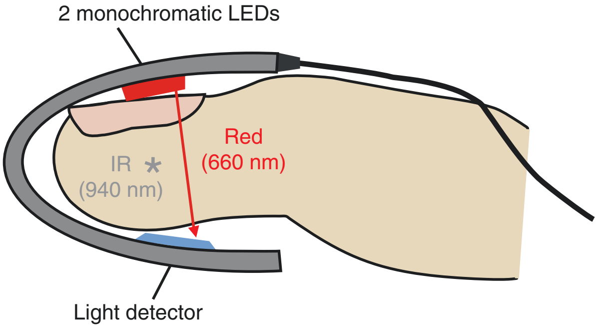


Fig. 13.6 Schematic representation of a pulse oximeter. The asterisk indicates the infrared (IR) beam (which I have not drawn as an arrow because it is not visible!). *LEDs* light-emitting diodes

attenuation coefficient of Hb is higher than that of O_2Hb at 660 nm, while the opposite occurs at 940 nm: accordingly, Hb absorbs more red (visible) light, but less infrared radiation, as compared with O_2Hb [6, 8]. This enables the pulse oximeter to "understand" if a variation in light absorption is due to a change in the concentration of reduced hemoglobin relative to that of O_2Hb (i.e., to a change in oxygen saturation) or rather, for example, to a change in the total hemoglobin concentration [8].

In practice, one of the two LEDs emits light (say, at 660 nm) while the other one is off: the absorption of this light is measured (as absorbance) by means of a light detector on the other side, which measures the intensity of the transmitted light (see Fig. 13.6). Subsequently, the absorbance for the second wavelength (940 nm) is measured while the first LED is turned off. The *ratio* between the two absorbances is then measured and compared with those found in healthy volunteers for empiric calibration [6, 9]. If the total hemoglobin concentration decreases (e.g., due to blood loss or hemodilution), the absorbance will be similarly reduced for both wavelengths, and the ratio will not change. Conversely, a reduction in oxygen saturation means that O_2Hb decreases as much as Hb increases (but the total hemoglobin concentration does not change): this leads to an increased absorbance for the 660 nm (red) wavelength (since it is increased, in proportion, the concentration of the species with the higher ε_λ for that wavelength) and to a reduced absorbance for the 940 nm (IR) wavelength (where the situation is reversed). Accordingly, the ratio between the two absorbances (red-to-infrared) increases, and this is translated by the oximeter software into a reduction in saturation.

As you can see, the situation is much more complex than the ideal case, discussed in the previous section, of a solution containing only one species which absorbs light. Since both Hb and O_2Hb, as well as other light absorbers which are present along the path length, absorb light at both wavelengths, in fact, pulse oximeters are not able to measure the absolute concentrations of the two forms of hemoglobin. Accordingly, the red-to-IR absorbance ratio must be matched with the ratios measured in healthy volunteers (in whom S_aO_2 is measured, at the same time, on blood samples) in order to get a value of oxygen saturation. This is one of the reasons why the accuracy of pulse oximeters significantly decreases for S_aO_2 values below 80% (it is not "nice" to go too down with arterial oxygen saturation in healthy volunteers!) [6, 8].

The accuracy of pulse oximeters may also be affected by high levels of carboxyhemoglobin (COHb) or methemoglobin (MetHb) [6–8]. For example, since the molar attenuation coefficient of COHb at 660 nm is very close to that of O_2Hb (see Fig. 13.5), pulse oximeters significantly overestimate S_aO_2 values in the presence of nonnegligible amounts of COHb.

Of course, the entire above-mentioned process must be only applied to what "pulsates," in order to eliminate the "noise" due to light absorption by tissues and nonarterial blood. Finally, the power cycle of the two LEDs (repeated many times a second) also includes a phase in which both LEDs are switched off, in order to reduce the interference from ambient light. The artifacts related to pulse detection and ambient light, as well as other well-known sources of error, are fully described elsewhere [6–9].

13.4.2 Capnography and Anesthetic Analyzers

Both *mainstream* and *sidestream capnometers* rely on absorption spectroscopy and, accordingly, on Beer-Lambert law [9, 10]. A monochromatic beam of infrared radiation with a wavelength of 4.28 μm, for which carbon dioxide (CO_2) exhibits an absorption peak [11], is used to measure the partial pressure of CO_2 (pCO_2) in the exhaled gas. In mainstream capnometers (nowadays rarely used in the operating room), the absorption-detecting device (including both the light-emitting element and the light-intensity detector) is positioned at the patient end of the respiratory circuit, while in sidestream capnometers a sample of exhaled gas is aspirated within the measurement device through a small-bore tube. The latter type of capnometers is often part of (sidestream) infrared absorption spectroscopy *respiratory gas analyzers*, which are usually integrated in the anesthesia machine. These devices use different wavelengths appropriately chosen to measure, in addition to pCO_2, the concentration of other respiratory gases such as nitrous oxide, and different volatile anesthetics (anesthetic analyzers).

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14 Scattering of Electromagnetic Waves: Blue Skies, Cerebral Oximetry, and Some Reassurance About X-Rays

Where's Physics

Daily life	Why the sky is blue Wet sand vs. dry sand How far car headlights illuminate
Physics involved	Scattering of electromagnetic waves Compton scattering The inverse square law (about light intensity) Raman scattering
Clinical practice	Near-infrared spectroscopy (NIRS) cerebral oximetry A few elements of radioprotection Some types of respiratory gas analyzers

14.1 Electromagnetic Waves Encounter Matter: Scattering

The trajectory of an electromagnetic wave which travels through a (substantially) transparent medium may be deviated in any direction as a result of the *interaction* between the wave and some obstacle or "inhomogeneity," such as the particles (including atoms or molecules) that are suspended in the medium. This phenomenon, which is referred to as *scattering* of the electromagnetic wave, should not be imagined as a "mechanical collision" with a body on which the wave "bounces". What happens indeed is that the oscillating electric field of the electromagnetic wave (see Chap. 13) causes the electrons within the atoms or molecules of which the body is made to move "up and down" [1]. As a consequence, the negative charges (i.e., the center of the electron cloud) *periodically* approach and move away from the positive charges (i.e., the center of the nucleus), producing what is called an oscillating *induced dipole moment* (see Box 14.1 if you are curious, otherwise read on). In practice, the atoms or molecules of the "obstacle" which is encountered by

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the electromagnetic radiation acquire part of the energy of the incident wave and become themselves oscillating electric fields, which act as a source of "*new*" electromagnetic radiation that propagates in different directions, always perpendicular to the direction of vibration of the charges (since, as mentioned in Chap. 13, electromagnetic waves are transverse waves; that is, they travel perpendicularly to the direction in which the oscillations of the electric field occur, namely, the same in which the electrons vibrate) [1].

Box 14.1: Induced Dipole Moment

Two equal and opposite (i.e., positive and negative) electric charges, separated by a small distance, form an *electric dipole*. Many molecules, such as that of water, have a permanent dipole; that is, positive and negative charges maintain a slight separation with one another. However, a neutral atom can be *polarized* by means of an external electric field, in order to obtain an *induced dipole*: since the electrons are located peripherally, the external electric field attracts them a bit more than it repulses the nucleus, thus leading to a small separation of charges. An electric dipole, both permanent or induced, has an electric field whose strength is determined by the magnitude of the so-called *dipole moment* p:

$$p = q\,d$$

where $\pm q$ is the value of the positive and negative electric charges and d is the distance between them. The dipole moment is a vector (see Chap. 10) which points from the negative to the positive charge: accordingly, it describes both the orientation and the strength of the dipole's electric field.

If an electric field induces a dipole moment within the context of an atom, an *oscillating* electric field like that of an electromagnetic wave will produce an *oscillating induced dipole moment*, i.e., a periodical widening and narrowing of the distance between the positive and negative charges of the atom and, accordingly, a second oscillating electromagnetic field associated to the oscillating dipole moment. That's how an incident electromagnetic wave produces a scattering wave.

For some aspects, however, the scattering of electromagnetic waves can be roughly likened to the collisions which occur between two (macroscopic) bodies. According to classical mechanics, a collision is said to be "elastic" if the total kinetic energy (see Chap. 1, Box 1.2) of the system made up of the two colliding objects is conserved, while it is called "inelastic" if the total kinetic energy of the system changes after the collision. For example, the collisions of the molecules of a gas against the walls of its container (see Chap. 1) are essentially elastic, those between two billiard balls are something quite similar to (although they are not, really) an elastic collision, while a car crash is definitely an inelastic collision.

Similarly, in *elastic scattering* the photon energy (see Chap. 13) doesn't change: according to Eq. 13.5, therefore, the scattered wave has the same frequency and, hence, wavelength (see Eq. 13.4) of the incident wave. Conversely, *inelastic scattering* involves a change in the energy and, accordingly, in the wavelength of the radiation.

As discussed below, both types of scattering of electromagnetic radiation may have a role in the daily practice of anesthesia and intensive care medicine. For example, the elastic scattering of infrared radiation is one of the principles on which *cerebral oximetry* is based and, at the same time, represents a major technological challenge which accounts for some limitations of this technique. Moreover, the special kind of (inelastic) scattering which ionizing radiations undergo (the so-called *Compton scattering*) must be considered in order to keep an appropriate distance, for example, during a chest X-ray in the intensive care unit (ICU). Finally, although nonionizing electromagnetic waves mainly undergo elastic scattering, inelastic scattering may also occur: this phenomenon, which is known as *Raman scattering*, is used in some types of respiratory gas analyzers.

14.2 Electromagnetic Scattering, Cerebral Oximetry, and Why the Sky is Blue

As mentioned, nonionizing electromagnetic waves such as visible light or infrared radiation mainly undergo elastic scattering. Hence, if we imagine a photon of a given wavelength (say, green visible light) which “strikes” an atom, as shown in Fig. 14.1a, such a photon may be diverted into a *random* direction, or better it has a certain *probability* to be deviated by a certain angle, but it will maintain the same wavelength (i.e., exactly the same color). Practically, the electrons of the atom will oscillate with the same frequency of the incident light (see Chap. 13).

In daily life, however, we don’t deal with one photon and one atom, but usually with a “beam” of light of a certain *intensity*, i.e., consisting of a certain amount of photons, which passes through a body containing many *scatterers* (i.e., particles able to deviate the electromagnetic waves). Accordingly, multiple scattering occurs and light is diverted in *all directions*, with different intensity depending on the scattering angle (Fig. 14.1b). In fact, since the intensity of light can be regarded as the “number of photons” (as discussed in Chap. 13), the intensity of scattered light will be higher in those directions along which the photons have a higher probability to be deviated. In other words, the “probability distribution” of a single scattering event translates into an “angular distribution” in the case of multiple scattering.

Now let’s see why the sky is blue [1, 2].

The elastic scattering of visible light by tiny particles such as atoms and molecules is described by the so-called Rayleigh theory for electromagnetic scattering (after lord J.W. Rayleigh). This model applies, in general, to the scattering of electromagnetic waves by particles whose diameter is lower than one-tenth of their wavelength. In the case of *Rayleigh scattering* by *molecules*, in particular, the intensity of scattered radiation I_{sctd} at any angle θ can be calculated as follows:

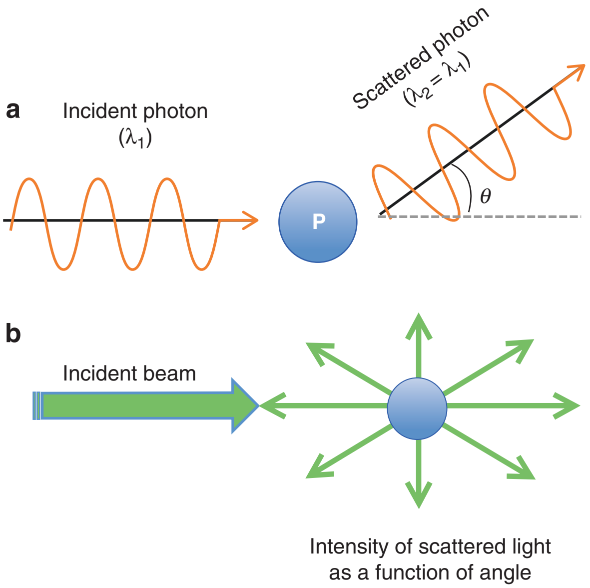

Fig. 14.1 Rayleigh scattering. (**a**) A photon (of nonionizing radiation) scattered at a certain angle θ from a small particle (P) such as an atom or a molecule has usually the same wavelength of the incident photon (elastic scattering). (**b**) A beam of light (i.e., a huge number of photons) passing through a "scattering medium" is scattered in all directions according to an angular distribution of intensities. Note that the intensity at 90° is one-half the backward and forward intensities (see text). λ_1 wavelength of the incident photon; λ_2 wavelength of the scattered photon

$$I_{\text{sctd}} = I_0 \frac{8\pi^4 N\alpha^2}{\lambda^4 R^2}\left(1+\cos^2\theta\right) \tag{14.1}$$

where I_0 is the intensity of the incident light; N the number of molecules (scatterers); α their *polarizability* (which is the tendency of the molecules to be polarized, i.e., as discussed in Box 14.1, the tendency of their opposite charges to be separated from each other by means of an external electric field; accordingly, it expresses how easily their electron cloud vibrates under the effect of the oscillating electric field of an electromagnetic wave); R the distance from scatterers; and λ the wavelength.

Note that, according to Eq. 14.1, the intensity of Rayleigh scattering at 90° is one-half of the forward and the backward intensities (see also Fig. 14.1): in fact, the coefficient $(1 + \cos^2\theta)$ is equal to 2 for both $\theta = 0°$ and $\theta = 180°$ ($\cos 0° = 1$, $\cos 180° = -1$) and to 1 for $\theta = 90°$ ($\cos 90° = 0$).

However, we have to focus on just one aspect of Eq. 14.1: the intensity of scattered radiation depends inversely on the fourth power of the wavelength:

$$I_{\text{sctd}} \propto \frac{1}{\lambda^4} \tag{14.2}$$

Therefore, the electromagnetic waves with a shorter wavelength are much more scattered than those with a longer wavelength. As sunlight (which contains practically all wavelengths of visible light and is, hence, substantially white) passes through the atmosphere, it is scattered by the air molecules (mainly nitrogen and oxygen). However, according to the Rayleigh's model, the blue and violet components (wavelengths around 400–450 nm) are scattered to a greater extent as compared with red-orange light (650–700 nm). Moreover, as mentioned in Chap. 13, sunlight does not contain large amounts of violet light and our retina is poorly sensitive to violet. Accordingly, when you look at the sky (in all directions except that of the sun), you see mainly blue light.

Light scattering also explains why wet sand is much darker than dry sand [3]. In fact, light is scattered more times in wet sand than in dry sand. For example, it may be entrapped into a water layer, where it is absorbed progressively as it is repeatedly scattered. Moreover, when the sand grains are wet (and, hence, bigger), light is more likely to be scattered in the forward direction, i.e., deeper in the sand (according to the Mie theory of scattering, which usually applies when the scatterers are "particles" larger than molecules). In both cases, a lesser amount of light "comes out" from the sand, which looks darker accordingly.

It's now time to see what all this has to do with cerebral oximetry.

14.2.1 The Unknown of Cerebral Near-Infrared Spectroscopy

In the last two decades, the monitoring of regional (especially cerebral) oxygen saturation of hemoglobin by means of *near-infrared spectroscopy* (NIRS) has been increasingly used, as a measure of oxygen supply/consumption balance, in several clinical settings, including cardiovascular surgery [4–9], pediatric anesthesia and ICU [10, 11], interventional neuroradiology procedures [12], and cardiac arrest [6, 13, 14]. In particular, some evidences suggest that the use of intraoperative cerebral oximetry, as well as the implementation of strategies aimed at preventing clinically significant cerebral desaturations, may help to improve important outcomes in patients undergoing cardiac surgery [6, 8, 15–18]. Moreover, a recent multicenter investigation showed an association between higher cerebral oximetry values during cardiopulmonary resuscitation and both the return of spontaneous circulation (ROSC) and a more favorable neurological outcome after cardiac arrest [14]. However, there are still not sufficient data which support the routine use of NIRS oximetry, whose clinical usefulness in anesthesia is indeed hotly debated [4–6, 10, 19, 20]. This is also due, maybe, to some limitations of this technique, including the lack of a "reference" value and of reliable absolute measurements [5, 21], the poor agreement among different devices [7, 22, 23], and accordingly, the lack of definite thresholds for either "normal values" or clinically important desaturations [6, 7, 19].

As discussed below, the scattering of electromagnetic radiation, on which NIRS partly relies, is also largely responsible for these limitations.

Like a pulse oximeter, a NIRS cerebral oximeter uses the Beer-Lambert law to estimate oxygen saturation thanks to the different molar attenuation coefficients of oxyhemoglobin and reduced hemoglobin for appropriately chosen wavelengths (see Chap. 13). In common commercially available NIRS devices, an electromagnetic radiation of two or more different wavelengths within the near-infrared (NIR) range (700–950 nm, i.e., the part of the infrared spectrum closest to the visible wavelengths) is emitted by as many light-emitting diodes (LEDs) or other types of emitting "optodes." The intensity of the radiation that, after having crossed a small area of the brain, reaches a couple of detecting optodes (positioned at a certain distance from one another) is then measured. NIR wavelengths have the advantage to be preferentially absorbed, with distinct absorption spectra, by a few substances (including hemoglobins), while the largest part of brain and other tissues is relatively transparent to them [5]. Nevertheless, since our head is much bigger than a finger, the emitting and detecting optodes cannot lie on opposite sides of the brain (as in the case of a finger in the pulse oximeter) because no radiation would reach the light detector due to the excessive path length [6].

However, infrared radiation undergoes scattering as it passes across brain: as in the case of sunlight passing through sand, some radiation comes back (or, as it is said, is "back-scattered") after a certain number of scattering events, thus reaching the detecting optodes which are located a few centimeters away from the emitting optodes on a single probe, usually an adhesive patch which is placed on the forehead (Fig. 14.2). Two patches are usually applied in adults (one on the left and the right).

Hence, cerebral oximetry is only possible thanks to scattering. Unfortunately, it's here that problems begin! In particular, the issues are two:

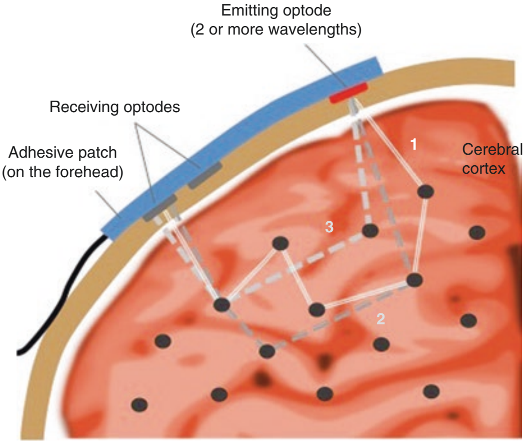

Fig. 14.2 Schematic representation of a NIRS cerebral oximetry probe. The simplified illustration shows that an infrared "ray" undergoes an unknown number of scattering events and, accordingly, may have followed different "true" path lengths (such as number 1, 2, or 3) before reaching the detectors

1. Unlike pulse oximetry, the exact path length of the electromagnetic radiation, which you need to know in order to calculate the absorbance according to Beer-Lambert law (see Chap. 13), is unknown. For example, we don't know precisely how many times the infrared radiation has been scattered before coming out from the brain and reaching the detecting optode (see Fig. 14.2).
2. The electromagnetic radiation that reaches the detecting optodes has not been attenuated only (or mostly) due to absorption, as in the case of pulse oximetry, but mainly due to scattering. In other words, *regardless of absorption by hemoglobins*, the intensity of the electromagnetic wave measured by the detecting optodes will be reduced because most of the radiation takes other directions (just take another quick look at Eq. 14.1 and Fig. 14.1).

Accordingly, Eq. 13.7 (Beer-Lambert law) assumes the following form [5, 24]:

$$A = \log \frac{I_0}{I} = \varepsilon_\lambda M \, L \;\; DPF + G \tag{14.3}$$

where A, I_0, I, ε_λ, M, and L are, respectively, the absorbance, the intensities of provided and measured radiation, the molar attenuation coefficient, the concentration (molarity) of the absorbing substance, and the distance between the emitting and the detecting optodes (as in Eq. 13.7), while DPF is the so-called *differential pathlength factor*, a coefficient of proportionality which accounts for the actual path length, and G is the attenuation due to scattering.

In practice, the actual path length of NIR radiation (i.e., the real optical distance between the emitting and the detecting optodes), which is referred to as the *differential pathlength* (DP), is the product of the geometrical distance L between the emitting and the detecting optodes and the differential path length factor DPF:

$$DP = L \times DPF \tag{14.4}$$

The scattering loss G depends on both the geometry and the scattering properties of the tissue, and is accordingly unknown [24]. However, once the probe is placed on the patient's forehead, it can be assumed that G (as well as the DPF) remains constant, so that the changes in the measured attenuation are only due to changes in the *absorption* by absorbing species such as hemoglobins: accordingly, the changes in the concentration of oxyhemoglobin and reduced hemoglobin (and, hence, the *changes* in cerebral oxygen saturation) can be measured by using a differential equation between the two species, i.e., measuring the difference in the attenuation at (a minimum of) two different wavelengths [5, 24]. In this way, the "factor" G is (mathematically) eliminated.

In order to provide *absolute* values of oxygen saturation (and not only a *trend*), an estimation of the DPF is needed. Most commercially available NIRS devices use a "continuous wave" technology and rely on *spatially resolved* (e.g., *multidistance*) *spectroscopy* to estimate the DPF. Briefly, the attenuation of infrared radiation (of different wavelengths) is measured by at least two detectors which are located at a different distance from the emitting optodes: in order to estimate the DPF, the increase in attenuation with the source-detector distance is assessed and matched with the dependency of scattering on the wavelength (which, as discussed, also explains why the sky is blue). Other NIRS devices use noncontinuous technologies such as radio-controlled intensity modulation (*frequency-resolved* or *frequency-domain* spectroscopy) or ultrashort pulsed laser (*time-resolved* or *time-of-flight* spectroscopy), whose description in details is beyond the scope of the present book and can be found elsewhere [5, 24].

The several "assumptions" which are made in the (different) algorithms of the different NIRS oximeters, as well as their relative technological complexity, may partly explain the above-mentioned limitations which hinder the widespread adoption of these monitoring tools. Moreover, the "accuracy" of NIRS oximeters is something very difficult to evaluate. Unlike pulse oximetry, in fact, the measurement is not made into an artery, but in a restricted region of brain which contains veins, arteries, and capillaries in a widely variable ratio (while the proprietary algorithms usually assume a fixed ratio of 25:75 or 30:70 between arteries and veins) [22, 25]. It is evident, though, that the expected value (that the device attempts to measure) does not exist "physiologically," but "is decided" at the same moment in which the probe is placed on the forehead of the patient. Accordingly, there is not a "true" value that can be measured by a "gold standard" technique [21].

14.3 Catch Me if You Can: X-Rays, Compton Scattering, and the Inverse Square Law

When a patient has a chest X-ray in the ICU, usually all staff flees as far away as possible for fear of being "hit" by radiation. In fact, although the X-ray beam is directed only towards the patient, some radiation undergoes scattering as it interacts with the patient's body. However, as discussed below, you don't need a huge distance to be safe.

14.3.1 X-Rays … from a Different Angle: Compton Scattering

As mentioned, unlike visible light or infrared radiation, X-rays (as well as gamma rays) undergo *inelastic scattering* when they encounter a material object. That is, the wavelength of scattered radiation is *longer* than that of the incident radiation (Fig. 14.3). This phenomenon, which is referred to as *Compton scattering* (or *Compton effect*), can be explained by interpreting the interaction between X-ray photons and electrons (in particular, the weakly bound electrons of the atoms within the object) as an *elastic collision*, similar to that between two billiard balls. In fact, the discovery of Compton effect was one of the best proofs of the quantum (or "photon") model of electromagnetic radiation (see Chap. 13), i.e., of the fact that light is a wave that sometimes behaves as a particle (and, accordingly, it can collide with another particle approximately like two billiard balls do with one another). Actually, an increase in wavelength of scattered radiation occurs for all wavelengths, although it is negligible for longer wavelengths, while it becomes significant for the shorter ones (X-rays and gamma rays). In practice, as shown in Fig. 14.3, when an X-ray (or a gamma ray) photon is scattered at a certain angle θ from an electron, also the electron is "sent away" (at an angle ϕ); that is, its kinetic energy increases. Since the total energy is conserved in an elastic collision, the energy of the photon must decrease. As discussed in Chap. 13, a lower photon energy means a lower frequency (Eq. 13.5) and, according to Eq. 13.4, a longer wavelength.

The *wavelength shift* $\Delta\lambda$ depends on the scattering angle θ according to the equation

$$\Delta\lambda = \frac{h}{mc}\left(1 - \cos\,\theta\right) \tag{14.5}$$

where h is the Planck constant (which has been already mentioned in Chap. 13), m is the mass of the particle from which scattering occurs (e.g., the electron), c is the speed of light, and the coefficient h/mc is known as *Compton wavelength.*

The maximum increase in wavelength (and, according to Eq. 13.5, the maximum decrease in photon energy) of scattered X-ray radiation occurs at 180°, while there is no wavelength shift for X-rays scattered forward ($\theta = 0°$). In fact, $\cos\,180° = -1$, so $1 - \cos\,\theta = 1-(-1) = 2$, and $\cos\,0° = 1$, that is, $1 - \cos\,\theta = 1-1 = 0$. In other words, the energy of scattered photons decreases as the scattering angle increases: accordingly, backscattered X-ray photons are those with the lowest energy, while X-ray photons which are scattered at 90° have an intermediate energy as compared with those scattered backward and forward ($\cos\,90° = 0$, so $1 - \cos\,\theta = 1-0 = 1$).

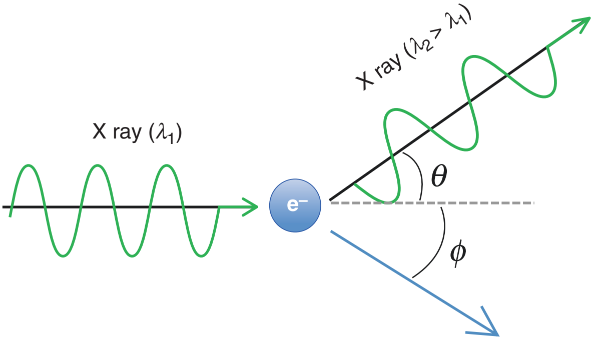

Fig. 14.3 Compton scattering. An X-ray photon of wavelength λ_1 which collides with an electron (e⁻) is scattered at an angle θ with *increased* wavelength λ_2, while the electron is "sent" at an angle ϕ

Of course, the number of photons which are scattered at a certain angle (as a result of the interaction with the patient) is only a small fraction of the amount of photons contained in the so-called *primary X-ray beam* emitted (only towards the patient) by the X-ray machine. In practice, the scattered radiation at any angle will have a much lower *intensity* (which is, as mentioned in Chap. 13, the *energy* carried by an electromagnetic wave) than that of the incident radiation for two reasons: (a) it consists of a much lower number of photons, (b) each of which has a lower energy.

On the basis of these considerations, it is usually estimated that the intensity of X-rays at an angle of 90° to the primary X-ray beam, *at a distance of 1 meter*, is about 1/1000 that of the primary beam [26]. In other words, the *absorbed dose* (expressed in joule/Kg or *gray*, symbolized *Gy*, that is, the energy of ionizing radiation absorbed by a unit mass of tissue) by someone who lies in this position is approximately 1/000 that absorbed by the patient. And what about, say, two meters?

14.3.2 Far Enough Away: The Inverse Square Law

As discussed in Chap. 13, the intensity I of an electromagnetic wave is the energy E per unit time t (which is referred to as the *power* $P = E/t$) which is transferred by the wave per unit area A (measured on a plane perpendicular to the direction of propagation of the wave):

$$I = \frac{E}{tA} = \frac{P}{A} \tag{14.6}$$

Considering a *point source* of electromagnetic radiation with power P_s which propagates uniformly (i.e., with the same intensity) in all directions, it can be said that, at a distance r from such a source, the power of the electromagnetic wave is distributed over the surface area of a sphere of radius r, that is, $4\pi r^2$ (see Chap. 5, Box 5.2). Accordingly, the intensity I of the electromagnetic wave at the distance r is

$$I = \frac{P_s}{4\pi r^2} \tag{14.7}$$

Hence, the intensity of an electromagnetic radiation generated by a point source decreases with the inverse square of the distance from the source (*inverse square law*). This simply comes from the fact that, as the distance from the source increases, the energy carried by the wave is distributed over a larger surface area: evidently, this area increases with the square of the distance (Fig. 14.4). The inverse square law applies, with good approximation, to X-rays scattered from the patient as well as, for example, to the headlights of your car (although they are not exactly point sources). Accordingly, if the intensity of scattered X-rays at 90° is 1/1000 that of the primary beam at a distance of 1 m from the patient, it will be roughly 1/4000 at 2 m, 1/9000 at 3 m, 1/16000 at 4 m, and so on. Hence, you don't need to go too far (unless you take the opportunity for a short break!).

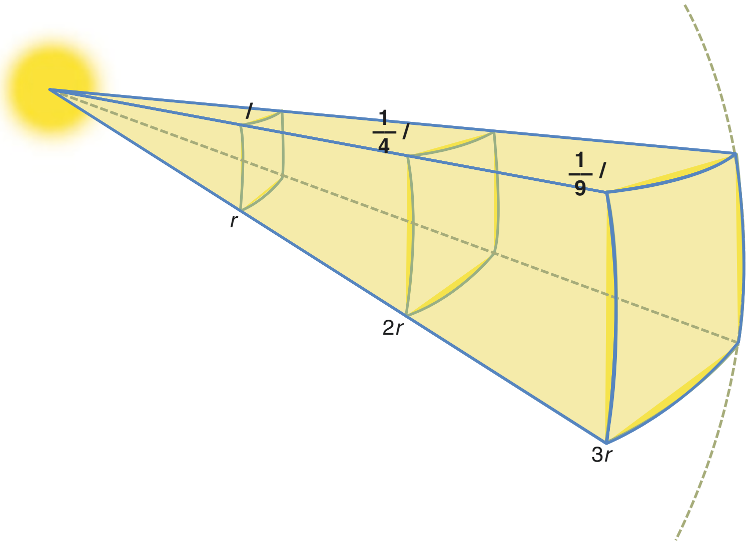

Fig. 14.4 The intensity *I* of an electromagnetic radiation decreases with the inverse square of the distance *r* from its (point) source. For example, the headlights of a car which is two times more distant than another from an observer will appear four times fainter. As shown, this is due to the larger surface (that increases with the square of the distance) over which the wave energy is distributed

14.4 Electromagnetic Scattering and Gas Analyzers: Raman Spectroscopy

Although, as discussed in Sect. 14.2, nonionizing electromagnetic waves such as visible light, infrared (IR) radiation, and ultraviolet (UV) radiation mainly undergo elastic scattering from molecules, about one every ten million scattering events may be *inelastic*, i.e., involve a change in the wavelength of the scattered radiation. This phenomenon is known as *Raman scattering* or "Raman effect" (so named after the Indian physicist C.V. Raman). Since the wavelength shift depends on the molecular species from which scattering occurs, the concentration of a specific component within a gas mixture can be measured according to the intensity of the scattered radiation of a certain wavelength (*Raman spectroscopy*). Some respiratory gas analyzers used in anesthesia rely on this technique to measure the concentration of gases such as oxygen, carbon dioxide, and nitrous oxide [27–29]. However, due to the extreme rarity of the phenomenon, a very high-intensity radiation source (an argon laser) is needed in order to generate a detectable "Raman signal." For this reason, these devices are much more expensive than those based on infrared absorption (see Chap. 13) and, accordingly, are rarely used in clinical practice.

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Origin and Propagation of Sound, Doppler Effect: Notes on Ultrasonography, Again Hemodynamic Monitoring, and … the Voice of Xenon

15

Where's Physics

Daily life	Flying balloons and funny voice Musical instruments Ambulance sirens, Formula 1 races, speeding fines, and bats The Universe is expanding
Physics involved	Origin and propagation of sound Doppler effect Fourier theorem Resonance
Clinical practice	A monster in the operating room (xenon anesthesia) Ultrasonography/echocardiography Measuring blood velocity and cardiac output (CO) Esophageal Doppler cardiac output monitoring Transcranial Doppler Invasive arterial pressure monitoring

15.1 The Nature of Sound

As mentioned in Chap. 13, sound is a mechanical wave, i.e., a periodic disturbance (or oscillation) which needs a material medium such as a gas, a liquid, or a solid in order to propagate (unlike electromagnetic waves, which can travel through a vacuum). In fact, what oscillates is the *density* (and, hence, the *pressure*) within the medium: accordingly, there cannot be a *sound wave* if there are not "particles" (e.g., air molecules) which can periodically come closer and move away with one another, i.e., undergo periodical *compression* and *rarefaction* (Fig. 15.1). Moreover, the medium must be *elastic*, i.e., its molecules must be able to change their reciprocal positions (which means a variation in density) when subjected to a "perturbation,"

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but there must be some force that makes them return to the initial positions once the perturbation is removed. Another difference with respect to electromagnetic waves is that, as shown in Fig. 15.1, sound waves are *longitudinal waves*; that is, they travel parallel to the direction in which the oscillation occurs. All this is usually summarized by saying that *a sound wave is a longitudinal wave* (*of compression and rarefaction,* or a "*pressure wave*") *which travels through an elastic medium*.

Sounds are produced by something, such as a guitar string, a drumskin, or our vocal cords, that vibrates periodically (conversely, what we call "noise" is an acoustic disturbance which propagates through an elastic medium *without* a precise periodicity). Although periodic, sounds we hear commonly, including our voice, do not have a *sinusoidal* (i.e., *sine* or *cosine*) waveform, but are generally more complex waves. However, according to the so-called *Fourier theorem*, any periodic wave can be described as the sum of sinusoidal waves whose frequencies (see Chap. 13) are integer multiples of its frequency. Accordingly, a sound can be always considered as the sum of a certain number of *pure tones*, called *harmonics*, each described by a sinusoidal wave. The wave with the lowest frequency is referred to as the *fundamental mode* or *first harmonic*, while the other sounds, whose frequencies are integer multiples of the first harmonic (see below), are called second harmonic, third harmonic, and so on (Fig. 15.2). This "set" of waves forms a *Fourier series* (or *harmonic series*). The study of Fourier series is known as *Fourier analysis*: as mentioned in Chap. 8, this mathematical tool is used for signal processing during invasive pressure monitoring and allows monitors to display an accurate representation of the arterial pressure waveform [1–4].

The main properties of waves (amplitude, frequency, wavelength, and speed of propagation) have already been discussed in Chap. 13.

Similarly to electromagnetic waves, the amplitude of a sound wave expresses the amount of *energy* carried by the wave, i.e., its *intensity*: a higher amplitude corresponds to a louder sound.

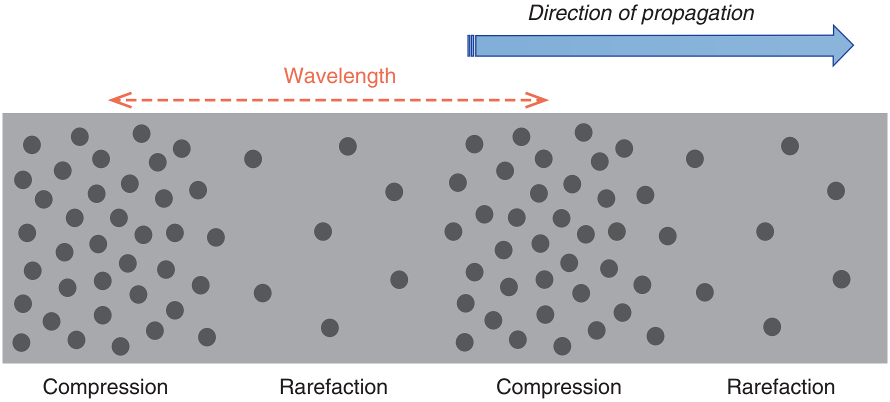

Fig. 15.1 The molecules of a material medium which is crossed by a sound wave periodically approach and move away from each other (compression and rarefaction) along the direction of wave propagation

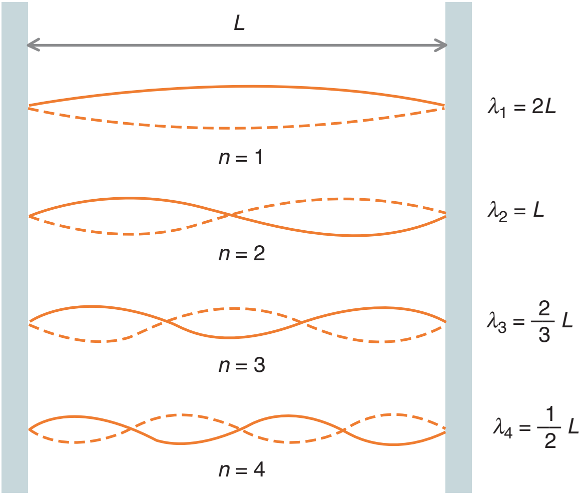

Fig. 15.2 The first four harmonics (or modes) on a string of length *L* (see text). The wavelength λ of each n^{th} harmonic is $2\,L/n$. *n* harmonic number

The frequency of the first harmonic (*fundamental frequency*) indicates the “pitch” of the sound: higher frequencies are heard as more acute sounds. We can hear sounds with a frequency between 20 and 20,000 Hertz (1 Hz = 1 “oscillation per second”), while “sound” waves with higher frequency (>20 kHz) are called *ultrasounds*, whose clinical use is known to everyone of us.

As discussed in Chap. 13, frequency f and wavelength λ are correlated with each other according to the following equation (which is analogous to Eq. 13.3):

$$f = \frac{v}{\lambda} \tag{15.1}$$

where v is the wave speed, namely, in this case, the *speed of sound*, which depends on both the density and the elastic properties of the medium in which sound propagates (see below).

Finally, the *timbre* (or “quality”) of a sound, i.e., what distinguishes my voice from that of someone else, or a musical note played on a clarinet from the same note played on a piano, depends on the percentage of the different harmonics which are present, i.e., on the waveform.

The next section is between serious and humorous: we will see something more about the origin and propagation of sound in order to explain something “strange” which might (theoretically) occur during a xenon anesthesia. However, we will also take the opportunity to make a further little consideration about invasive pressure monitoring.

The last part of this chapter briefly addresses another clinically relevant topic (to anesthesiologists and intensivists) of wave physics: the *Doppler effect* and its application to both ultrasonography (e.g., echocardiography) and noninvasive cardiac output monitoring.

15.2 A Monster in the Operating Room

If you should happen to induce *xenon anesthesia* [4–8] through a face mask (something quite unusual, however) and hear spooky sounds coming from the patient, you must not think the patient is turning into a monster!

Everyone knows that if you inhale the helium contained in a “flying balloon” (see Chap. 2), your voice becomes very similar, both in its rapidity and in its unusually high tone, to that of the Disney’s character Donald Duck [9, 10]. Let’s see why.

The speed of sound v through any medium depends on the *density* d of that medium, according to the following equation:

$$v = \sqrt{\frac{B}{d}} \tag{15.2}$$

where B is the so-called *bulk modulus*, which represents the elastic properties of the medium. In practice, B indicates how much the volume of a material varies as the pressure exerted on it changes: the lower the bulk modulus of a material, the higher its *compressibility*. Accordingly, the speed of sound is much lower in gases, which are highly compressible, than in (almost incompressible) liquids.

Since the density of helium is lower than that of air [7, 11], the speed of sound through helium (965 m/s) is much higher than through air (343 m/s at 20 °C): this explains why words seem to come out faster [10], but not why the voice looks higher.

As mentioned, our voice, as well as most sounds we hear in everyday life, is not formed by a single (sinusoidal) sound wave, but by a “mixture” of harmonics. Consider the example of a guitar string fixed at its two ends, as shown in Fig. 15.2. Imagine a sinusoidal wave traveling along the string from the left to the right. Once the wave has reached the right end, it undergoes *reflection* and starts to travel in the opposite direction, until it reflects again at the left end. In a short time, there will be many waves that *interfere* with one another. While most of these waves cancel each other out, only those with certain frequencies add up to form a *standing wave* (or *stationary wave*), in which the points of minimum and maximum amplitude are fixed in the space. These frequencies are called *resonant frequencies*, and the standing wave is said to form at *resonance* (something that must be avoided during invasive pressure monitoring, as discussed in Box 15.1).

Box 15.1: Natural Frequency and Resonance of Invasive Pressure-Monitoring Systems

The phenomenon of *resonance* does not concern sound waves only, but mechanical waves in general. Any object may undergo a *driven oscillation* when a periodic external force is applied to it (as when you push a child on a swing). When the frequency of such “imposed” oscillation matches the *natural frequency* of the object, i.e., the frequency at which any object tends to oscillate when left to itself (which depends on several factors including its length and stiffness), the object resonates, i.e., responds to such a solicitation by oscillating with large amplitude. For example, a tree may sway exaggeratedly (and, accordingly, be broken) due to a rather weak wind if the mean frequency of the wind gusts approaches its natural frequency [12].

While resonance is very useful for musical instruments, in which the sound produced by something that vibrates (e.g., a string or a reed) is amplified and enriched with harmonics by *resonators* (such as a pipe or a sound box), it can be a problem for rise buildings, bridges, airplanes … and invasive pressure-monitoring systems [2, 4, 13, 14].

As an example, the natural frequency of the wings of an airplane must be very far from the frequency of the vibrations produced by air turbulence; otherwise they could intensely oscillate and even (dramatically) break. Similarly, if one of the harmonics which form the arterial pressure waveform (see Chap. 8 and Sect. 15.1 in this chapter) has a frequency close to the natural frequency of the transducer-tubing-catheter system, the system may resonate; that is, its own oscillation will significantly add to the oscillation of blood pressure, leading to overestimation of systolic pressure and to underestimation of diastolic pressure. In order to avoid resonance, the natural frequency of an invasive blood pressure monitoring system must be at least 8–10 times higher than the measured heart rate (that is, the frequency of the first harmonic). For example, if heart rate is 240 beats per minute (i.e., 4 beats per second or 4 Hz), the natural frequency of the system must be at least 32–40 Hz. Although commonly used pressure-transducing systems have much higher natural frequencies (>200 Hz), adding too long tubing and three-way stopcocks, using small-diameter arterial cannulas, and the presence of air bubbles may considerably decrease the natural frequency of the system, leading to resonance.

In order to produce a standing wave on a string of length L, a wave must have one of the following wavelengths:

$$\lambda = \frac{2L}{n} \tag{15.3}$$

where $n = 1, 2, 3, \ldots$ is the so-called *harmonic number*, i.e., $n = 1$ for the first harmonic (fundamental mode), $n = 2$ for the second harmonic, $n = 3$ for the third harmonic, and so on. In other words, Eq. 15.3 provides the wavelengths of the different waves (harmonics) that form an harmonic series: the wavelength of the first harmonic ($n = 1$) will be equal to twice the length of the string, that of the second harmonic ($n = 2$) will be equal to the length of the string, that of the third harmonic ($n = 3$) will be equal to 2/3 of the length of the string, an so on (see Fig. 15.2).

Equation 15.3 also applies to a pipe of length L (with two open ends) filled with air: in this case, a standing wave is produced by sound waves of appropriate wavelengths (i.e., equal to 2 L, L, 2/3 L, etc.) which propagate through the air within the pipe. The corresponding frequencies f at which the pipe *resonates* (resonant frequencies) can be found according to Eq. 15.1 (after substituting for Eq. 15.3):

$$f = \frac{v}{\lambda} = n\frac{v}{2L} \tag{15.4}$$

where v is the speed of sound *through air*. Hence, the frequency of each harmonic n depends on the length L of the pipe and on the speed of sound through the gaseous medium contained in it.

Substituting for Eq. 15.2 (which provides the speed of sound through any medium), Eq. 15.4 becomes

$$f = \frac{n}{2L}\sqrt{\frac{B}{d}} \tag{15.5}$$

where B and d are, respectively, the bulk modulus and the density of the gas that fills the pipe: hence, *the lower the density, the higher the frequency* of each harmonic (including the first one, on which depends, as mentioned, the perceived “pitch” of the sound).

When our vocal cords vibrate to produce a sound, the oropharynx acts as a resonating cavity, as in a pipe organ, in a clarinet, and so on. Therefore, the vibration of vocal cords will produce a number n of harmonics, whose frequencies depend both on the size and shape of the oropharynx and on the speed of sound through the gas contained in it (which, in turn, depends on the density of that gas). When you inhale helium, the shape of your oropharynx does not change, but it is filled with helium instead of air: accordingly, the speed of sound v in it will be higher and, consequently, the harmonic frequency will be higher and the voice will look more acute [11].

Xenon has, conversely, a density about three times higher than that of air [7], so that the speed of sound through it is lower as compared with that through air: phonation while breathing xenon will therefore result in a slow and low-pitch voice, like that of a monster.

15.3 Bats, Speeding Fines, Echocardiography, and Cardiac Output: Ultrasounds and Doppler Effect

In the last years, anesthesiologists and intensivists are relying more and more often, in their clinical practice, on *ultrasonography* (e.g., perioperative transesophageal echocardiography [15, 16], ultrasound-guided peripheral nerve blocks [17] or vascular access [18, 19], pleural and lung ultrasound [20–22], and so on). It is common knowledge that an ultrasound image is formed from *reflection* of ultrasound waves which occurs at boundaries between tissues with different *acoustic impedance* [23, 24]. The latter, usually symbolized Z, is roughly the “resistance” that a tissue opposes to the transmission of sound waves through it, and is defined as the product of the tissue density d and the speed of sound v through it:

$$Z = d\,v \tag{15.6}$$

In particular, the higher the difference in the acoustic impedance between two neighboring tissues, the more intensely ultrasounds are reflected. In practice, since the speed of sound in soft tissues is relatively constant (about 1500 m/s, on average), reflection primarily occurs due to *density* changes.

The dependency of sound wave reflection on the difference in acoustic impedance at boundaries between different media is the reason why *ultrasound gel* (which has already been mentioned in Chap. 7 as an example of “non-Newtonian fluid”) is needed to get an ultrasound image. In fact, the acoustic impedance of air is much lower than that of our body, since both air density and speed of sound through it (as mentioned, about 340 m/s) are much lower than those of any tissue. Accordingly, at the air/body boundary, almost all ultrasounds would be reflected before entering the body, unless you replace the air “entrapped” between the probe and skin with a substance (the ultrasound gel) with an acoustic impedance similar to that of tissues.

In diagnostic ultrasound, a “beam” of ultrasounds with frequency between 1 and 20 million Hertz (or megahertz, symbolized MHz) is emitted by a set of *piezoelectric crystals*, which are made of a material which vibrates under the effect of an alternating electric current and, conversely, undergoes a change in its electric polarization in response to an applied *pressure* (*piezoelectric effect*). Accordingly, the reflected ultrasound waves (which, as discussed above, are “pressure waves”) are turned into an oscillating electric signal (and, then, into an image) thanks to the interaction with the same piezoelectric crystals which had previously produced them. The image is built according to the *time* which is needed for the waves to return to the probe: the waves reflected from more distant structures take more time to come back. This is exactly how bats identify obstacles and assess their distance [25].

Further “technological” details about the production and interpretation of ultrasound images are beyond the scope of this book and can be found in all ultrasound textbooks (such as [23]) as well as, nowadays, in many anesthesia textbooks (e.g., [24]).

In the context of ultrasonography, the so-called *Doppler analysis* allows, as known, to measure blood velocity within blood vessels or cardiac chambers (as mentioned in Chap. 6). This technique relies on a behavior which is common to *all* traveling waves, known as *Doppler effect*. The remaining part of this chapter is dedicated to the general description of this phenomenon, that we often encounter in daily life, as well as to some examples of its important applications in perioperative and intensive care medicine, such as echocardiography, noninvasive cardiac output monitoring (esophageal Doppler), and transcranial Doppler.

15.3.1 The Doppler Effect

If you have ever been overtaken by a police car or an ambulance with their sirens turned on, or if you have ever watched a Formula 1 race, then you should have heard in person the Doppler effect. It consists in a change in the perceived *frequency* (or wavelength) of a wave whose source is in motion relative to the observer. More generally, this phenomenon occurs when either the wave source or the observer (or both) are moving relative to the medium through which the wave travels. In particular, *for a stationary observer and a moving source*, the wave frequency increases if the source is approaching to the observer and decreases if it is moving away from it, according to the following equation:

$$f = f_0 \frac{v}{v \pm v_S} \tag{15.7}$$

where f is the frequency detected by the observer, f_0 is the frequency emitted by the source, v is the *wave speed* (e.g., through air), v_s is the speed of the source relative to the observer, and the sign ± is *negative* for an approaching source and *positive* for a receding one.

Since, as discussed above, a lower (fundamental) frequency of a sound wave means a less acute sound, the pitch of an ambulance siren which is receding from you is characteristically "waning." This effect is particularly evident during a Formula 1 race, due to the high speed of the wave source (i.e., the cars): the typical "wroom" made by the cars whizzing in front of you is rising (in its pitch) during "wro" and waning during "om."

As mentioned, Doppler effect does not concern only sound waves but also electromagnetic waves. For example, the wavelength of light coming from a source which is moving away from an observer increases (which, as discussed in Chap. 13, is the same as saying that its frequency decreases). Since the longer wavelengths (within the visible range) are those of red, it is said that light emitted by receding objects is "red shifted," i.e., shifted *towards* (but not necessarily to) the red. The observation of the *red shift* of light coming from galaxies led to the conclusion that the Universe is expanding, and is one of the proofs of the so-called Big Bang Theory. Similarly, the police can get the proof of your speeding thanks to the Doppler effect: a "speed radar gun," one of the several methods for speed detection, uses *radio waves* (see Chap. 13) "fired" against a moving car to measure its speed according to the variation in wavelength (which is often referred to as *Doppler shift*) of the reflected waves.

Also bats rely on the Doppler shift of ultrasounds they emit in order to estimate their own speed and to distinguish moving objects (e.g., insects) [25]. As discussed below, many echocardiographic measures, as well as some techniques commonly used for perioperative monitoring (such as esophageal Doppler and transcranial Doppler), use the Doppler effect in a similar way.

15.3.2 Cardiac Doppler Ultrasound

In echocardiography textbooks, Eq. 15.7 is usually written as follows (and referred to as the *Doppler equation*) [23, 24]:

$$v_{\mathrm{blood}} = \frac{v\left(f_{\mathrm{r}} - f_{\mathrm{e}}\right)}{2 f_{\mathrm{e}}\left(\cos\theta\right)} = v\,\Delta f / 2 f_{\mathrm{e}}\ \cos\ \theta \tag{15.8}$$

where v_{blood} is the blood flow velocity; v is the speed of sound in blood (1540 m/s); Δf is the Doppler shift, i.e., the difference between the frequency emitted by the probe (f_{e}) and that reflected to it (f_{r}); 2 accounts for the double travel of the ultrasound beam (from the probe to the reflecting source and then back to the probe); and θ is the intercept angle between the blood flow direction and the ultrasound beam.

The term cos θ "appears" since velocity is a vector (see Chap. 10): evidently, it is the component of the blood flow velocity parallel to the direction of the ultrasound beam which causes the Doppler shift. Accordingly, a close alignment between the ultrasound beam and the direction of blood flow is pivotal in cardiac Doppler measurements ($\theta < 20°$ is usually acceptable), while no Doppler shift can be recorded at 90° ($\cos 90° = 0$) [23].

Two echocardiographic applications of Doppler analysis have been already mentioned in Chap. 6: the measurement of blood flow velocity at both the aortic valve and the left ventricle outflow tract (LVOT) level allows to estimate, according to the continuity equation, the "actual" aortic valve area (AVA), while the pressure gradients across heart valves can be calculated from blood flow velocity thanks to the simplified Bernoulli's equation. Both techniques are used, for example, for the grading of valve stenosis.

Another example is the estimation of systolic pulmonary artery pressure as the sum of right atrial pressure and systolic pressure gradient across the tricuspid valve (calculated from the maximum regurgitant jet velocity, which in turn is measured according to Eq. 15.8) [23].

Let's see, finally, how cardiac output (CO) can be measured. As known, CO is the product of heart rate and stroke volume (SV), that is, the volume of blood ejected from the ventricle at each systole. Left ventricle SV can be calculated as the volume of blood ejected into the aorta at each systole, which can be approximated to the volume of a cylinder whose base area is the systolic cross-sectional area (CSA) of the left ventricle outflow tract and whose height is the mean distance traveled by blood cells during a systole. Since velocity is the first derivative (see Chap. 3) of distance over time, distance is the integral of velocity over time. Accordingly, the mean distance traveled by blood cells during a systole can be calculated as the integral of the Doppler systolic velocity-time curve (i.e., the area under the curve of blood flow velocity over time), which is called velocity-time integral (VTI). Remembering that the volume of a cylinder is the product of its base area and its height, it is

$$\mathrm{SV} = \mathrm{CSA} \times \mathrm{VTI} \tag{15.9}$$

The CSA is usually estimated by measuring (sonographically) the diameter of LVOT either at the aortic valve leaflet tips or in the ascending aorta, while the VTI is obtained from the measurement of aortic flow velocity by Doppler analysis at the same level [23].

15.3.3 Esophageal Doppler Cardiac Output Monitoring and Transcranial Doppler

Esophageal Doppler cardiac output (CO) monitors are commonly used as hemodynamic monitoring tools (for example to guide fluid therapy) in both perioperative and critical care settings [26, 27]. The way these devices measure the stroke volume is very similar to the echocardiographic method described above: the blood flow velocity in the *descending aorta* is measured, according to Eq. 15.8, by means of a Doppler ultrasound probe placed in the esophagus. The curve of flow velocity over time is then integrated to obtain the so-called *stroke distance* (i.e., the distance traveled by blood at each systole: in practice, the analogous of the "echocardiographic" VTI). The stroke distance, multiplied by the cross-sectional area (CSA) of the descending aorta, provides the *descending aortic stroke volume* (which is about 70% of the stroke volume). However, the most commonly used Doppler CO monitor (CardioQ-ODM, Deltex Medical, Chichester, UK) [28] neither measures nor estimates the CSA. Conversely, it translates the stroke distance into an absolute value of stroke volume using an algorithm developed by comparison with a thermodilution technique (in practice, it uses a nomogram based on patient's age, weight, and height to convert the stroke distance measured in a certain patient in a value of stroke volume). Of course, these devices have their best usefulness as trend monitors.

Transcranial Doppler (TCD) is another monitoring tool which relies on the Doppler equation. It uses a beam of ultrasounds to measure blood flow velocity within cerebral vessels such as the middle cerebral artery. TCD is useful, for example, for detection of cerebral emboli and as a trend monitoring of cerebral blood flow during cardiac surgery [29].

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Part VII

And Finally…

Activated Clotting Time and … a Brief Look at Relativity

16

Where's Physics

Daily life	So works the world (really!) GPS navigation systems
Physics involved	Special relativity
Clinical practice	Getting the result of activated clotting time faster (just for fun)

16.1 Surgeons Are Always in a Hurry

Activated clotting time (*ACT*), that is, the time (in seconds) needed for clot formation in a sample of whole blood after the addition of an activator, is still the most common method used to monitor the effectiveness of heparin anticoagulation in cardiac surgery [1]. Since an ACT above 480 s is usually required for cardiopulmonary bypass (CPB), it takes at least 8 min (480/60) from the time of sampling to get a useful result. Moreover, although the peak ACT prolongation has been reported to occur in less than 1 min, it is usually advisable to wait for 2–5 min after heparin administration before taking a blood sample for ACT [2]. Accordingly, even making a little discount (i.e., allowing surgeons to start CPB with an ACT above 400 s), it takes about 10 min from heparin administration to the onset of CPB.

These 10 min, however, seem to last an eternity, with surgeons who repeatedly ask the following: "How much is the ACT?" "Can I go on CPB?" It would be very nice to be able, say, to halve this time interval…

As discussed below, this is indeed *possible* … according to (Einstein's) *relativity*! [3–6].

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16.2 How to Get the Result of ACT Faster

One of the principles of relativity is *time dilation*: for someone or something who is moving, *time flows slower* (as compared with the reference frame in which that person/object is moving). In other words, *a clock ticks slowly when it moves* [3]. If, after the administration of heparin, the surgeon went out for a run around the hospital (*at an appropriate speed*), he/she might find the ACT ready in 5 min! In fact, for the surgeon, time would flow slower than for those who remained in the operating room (i.e., the reference frame with respect to which the surgeon runs). This is an "intraoperative" version of the famous *twin paradox*.

Before proceeding with the quantitative analysis of this phenomenon (and with its proof!), let's see the two fundamental ideas on which relativity relies.

16.2.1 The "Relativity" and the "Speed of Light" Postulates

Relativity is based on two basic postulates:

1. *All* the laws of physics apply in the same way whatever the *inertial reference frame* from which they are observed (*relativity postulate*). The term "inertial" refers to the fact that the *law of inertia,* or *Newton's first law* (see Chap. 11), must be valid. In other words, inertial reference frames are those reference frames in which a body moves *with constant speed* (or is motionless), which is the same as saying that inertial reference frames move relative to the body (and to each other) with constant speed. For example, if you walk with a friend at a speed of 3 km/h (in the travel direction) along a railroad car which is entering the station at a speed of 40 km/h, you are motionless relative to your friend, while you are moving at 3 km/h relative to the car and at 43 (i.e., 40 + 3) km/h relative to the station (this is a good approximation at these low speeds, although velocities do not add up in this way according to relativity). Your friend, the railroad car, and the station are *all* inertial reference frames (for you) which move relative to each other with different constant speeds. The part of relativity which deals with inertial reference frames is called *special relativity* (which includes the principle of time dilation further discussed below), while *general relativity* describes the effects of acceleration (and, hence, of *gravity*) on space-time and matter.
2. The *speed of light* c (in vacuum) is the same in all inertial reference frames (*speed of light postulate*). This is a fundamental point: it means that, if you walk at a speed of 3 km/h along the above-mentioned railroad car, holding a flashlight (turned on) in your hand, the speed of its light is not c relative to you, c + 3 km/h for someone sitting in the railroad car, and c + 43 km/h for the stationmaster (which is motionless relative to the station), but is exactly c for *all* three.

16.2.2 How Fast Should the Cardiac Surgeon Run

Figure 16.1 shows a classical representation of the *light clock*, which is used in all physics textbooks to prove Einstein's equation of *time dilation*. This equation will tell us to what extent time will slow for the surgeon, depending on his/her speed, as compared to the inertial reference frame in which he/she moves (e.g., the operating room).

A light clock consists in a light source and a light detector surmounted by a mirror at a distance h. The light source emits a pulse of light towards the mirror, where it is reflected and comes back to the detector. The clock "ticks" (i.e., it advances by a time interval Δt_0) whenever the detector receives a light pulse, leading, at the same time, to the emission of another light pulse from the source. Hence, when the clock is observed from its own reference frame, every "tick" corresponds to a time interval Δt_0. During this time interval, light travels, approximately, a distance 2 h (assuming that the light source and the light detector are very close).

Now imagine that the light clock is moving at a constant speed v (for example, we have put it in our checked luggage during a flight). Figure 16.2 shows what happens

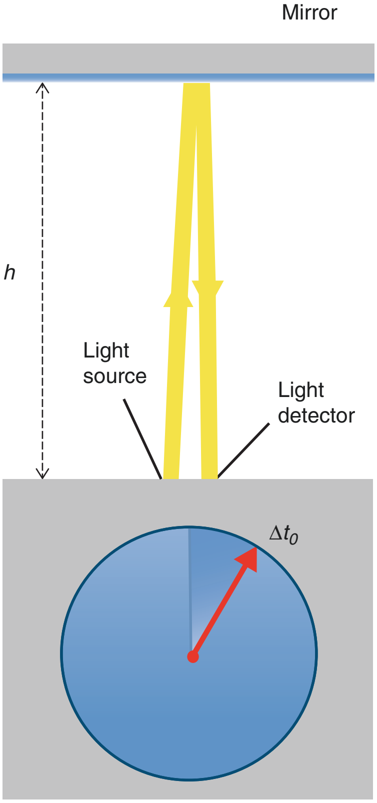

Fig. 16.1 The "light clock" (see text)

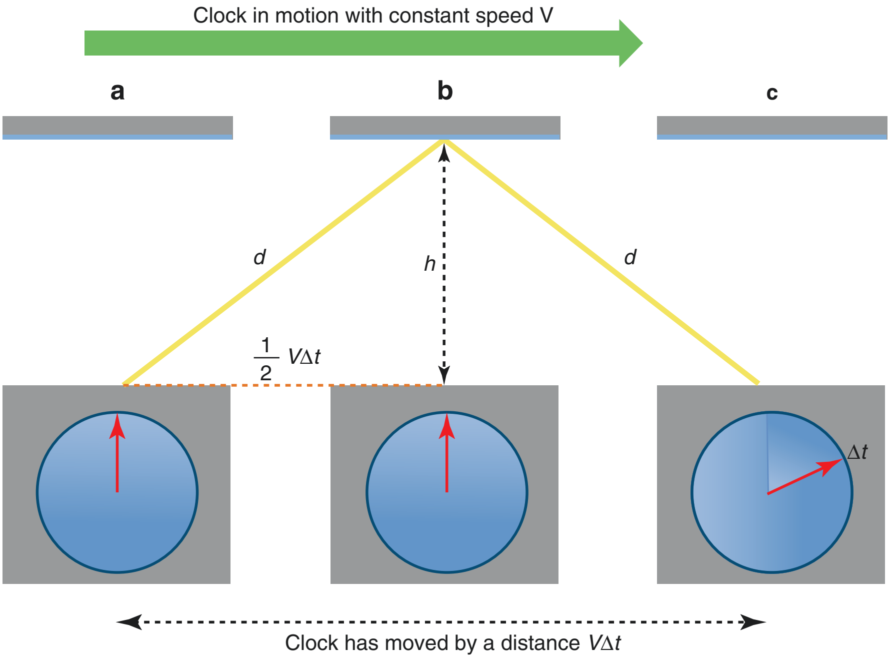

Fig. 16.2 A light clock observed from the inertial reference frame in which it moves with constant speed v. The three drawings (**a**), (**b**), and (**c**) represent three different positions in the space which the clock occupies during the time interval Δt

when one observes the clock from the reference frame in which the clock moves (therefore, not from the airplane but, for example, from the airport). As above, one "tick" of the clock corresponds to the time interval Δt which is needed for light to travel from the source to the mirror and then to the detector. In the same time interval, the clock moves by a distance $v\Delta t$. In fact, as recalled in Chaps. 3 and 6, the speed v of a body is the distance Δx which the body travels during the time Δt:

$$v = \frac{\Delta x}{\Delta t} \tag{16.1}$$

Accordingly, it is $\Delta x = v\Delta t$.

Evidently, during the time needed for the light to travel from the source to the mirror, the clock moves half of this distance, i.e., $v\Delta t/2$.

It appears clear from Fig. 16.2 that, when the clock is observed from the reference frame in which it moves, light covers a distance d from the source to the mirror (or from the mirror to the detector) which is longer than the distance h which light travels when the clock is observed in its own reference frame (in which it is at rest).

This distance d can be easily calculated according to the Pythagorean theorem (see Fig. 16.3):

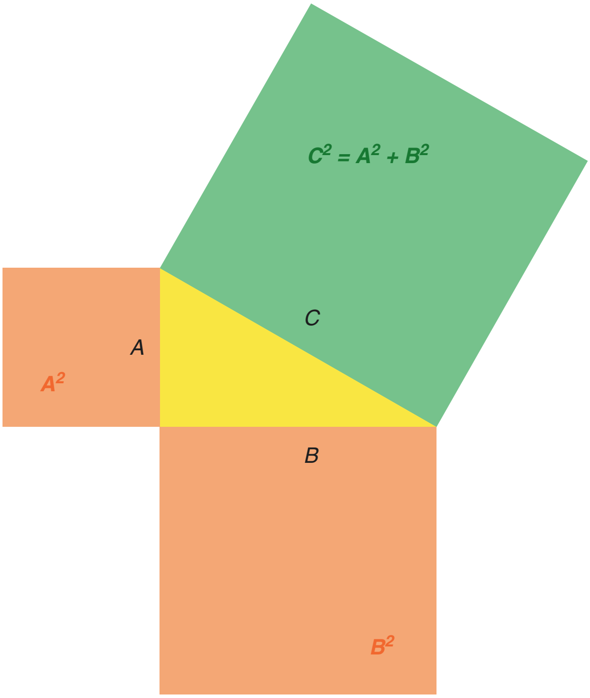

Fig. 16.3 The Pythagorean theorem states that, in a right triangle, the square of the hypotenuse *C* (i.e., the side opposite to the right angle) is equal to the sum of the squares of the other two sides *A* and *B*

$$d^2 = h^2 + \left(\frac{1}{2}v\Delta t\right)^2 \tag{16.2}$$

or

$$d = \sqrt{h^2 + \left(\frac{1}{2}v\Delta t\right)^2} \tag{16.3}$$

Hence, during the time interval Δt (i.e., between two "ticks"), light travels a distance:

$$2d = 2\sqrt{h^2 + \left(\frac{1}{2}v\Delta t\right)^2} \tag{16.4}$$

According to Eq. 16.1, and remembering that the speed of light is c, it is

$$c = \frac{2d}{\Delta t} \tag{16.5}$$

and, therefore

$$2d = c\Delta t \tag{16.6}$$

Putting Eqs. 16.4 and 16.6 together, we have

$$2\sqrt{h^2+\left(\frac{1}{2}v\Delta t\right)^2}=c\ \Delta t \tag{16.7}$$

On the other hand, in the clock's own reference frame (i.e., that of the airplane), light travels, as stated, the distance $2h$ in the time interval Δt_0. According again to Eq. 16.1, it is

$$2h=c\ \Delta t_0 \tag{16.8}$$

or

$$h=\frac{1}{2}c\ \Delta t_0 \tag{16.9}$$

After substituting for Eq. 16.9, Eq. 16.7 becomes

$$2\sqrt{\left(\frac{1}{2}c\,\Delta t_0\right)^2+\left(\frac{1}{2}v\Delta t\right)^2}=c\ \Delta t \tag{16.10}$$

and, after some algebra,

$$\Delta t=\frac{\Delta t_0}{\sqrt{1-(v/c)^2}} \tag{16.11}$$

Equation 16.11 is the equation of special relativity for time dilation, i.e., the quantitative relationship between the time Δt (which corresponds to the time interval between two clock "ticks" in a frame reference in which the clock moves with speed v) and the time Δt_0 (i.e., the time interval between two clock "ticks" in the clock's own frame) [3]. This means that, for those who are on the airplane, *life* itself flows more slowly as compared with those who are at the airport. Therefore, if there were two twins, one at the airport and one on the airplane, the traveler, on his/her return, would be younger than the brother remained at the airport.

Let's now calculate the speed at which the surgeon should travel in order to halve his/her waiting for the ACT, i.e., the speed for which it is $\Delta t = 2\Delta t_0$ (in other words, the speed for which 1 min in the surgeon's own reference frame lasts 2 min in that of the operating room). In order to do this, we just need to replace Δt with $2\Delta t_0$ in Eq. 16.11:

$$2\Delta t_0=\frac{\Delta t_0}{\sqrt{1-(v/c)^2}} \tag{16.12}$$

Accordingly, Δt_0 is cleared, and the equation becomes

$$\sqrt{1-(v/c)^2}=\frac{1}{2}\quad\text{or}\quad 1-(v/c)^2=\frac{1}{4} \tag{16.13}$$

and, after a few simple steps,

$$v = \frac{\sqrt{3}}{2} c \cong 0.87c \qquad (16.14)$$

Remembering that, as mentioned in Chaps. 11 and 13, the speed of light c is 300,000 km/s (or 186,000 mi/s), i.e., *one billion km/h* (or *670 million miles/h*), the surgeon should travel at a speed of 870 million km/h (or about 583 million miles/h), that is, about 700,000 times higher than that of a supersonic jet.

As you can easily realize, it is better to be resigned to wait for 10 min!

16.3 After All, It Was Just for Fun

The fact that the effects of relativity become evident only for incredibly high speeds, which probably we will never reach, does not mean that relativity doesn't concern our everyday life. For example, global positioning system (GPS) navigation devices would not be so precise without taking into account the effects of relativity on the satellites' clocks (which include both the "special relativity" effects, related to the high speed of satellites, and the "general relativity" effects, related to gravity) [3]. Anyhow, it is quite incorrect (and limiting) to talk only about a single aspect of relativity, which should be always considered as a (beautiful and complex) whole. In addition to time dilation, indeed, other relativistic phenomena which occur in moving objects (such as *length contraction*, *mass increase*, and the so-called *relativity of simultaneity* [3–6]), taken together, give a sense to some apparent paradoxes. However, the aim of this final chapter was just to have some fun with (harder) physics taking inspiration from an "anesthesiological" scenario (it is evident that the ACT was just a pretext). This is also to remember that, today, the term "physics" includes much more than what was dealt with in the previous 15 chapters of this book. Although we have sometimes fleetingly peeked into the world of *quantum mechanics*, in fact, this book deals almost entirely with topics of *classical physics* (i.e., what comes before relativity and quantum physics), in particular those with interesting applications in the field of anesthesia and intensive care medicine. The most curious of you will appreciate (I hope) the hints contained in this chapter (together with those about quantum physics in Chaps. 13 and 14) to broaden their horizons about physics.

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