PLANT DESIGN AND ECONOMICS FOR CHEMICAL ENGINEERS

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Chemical And Petroleum Engineering Series

FOURTH EDITION

MAX S. PETERS
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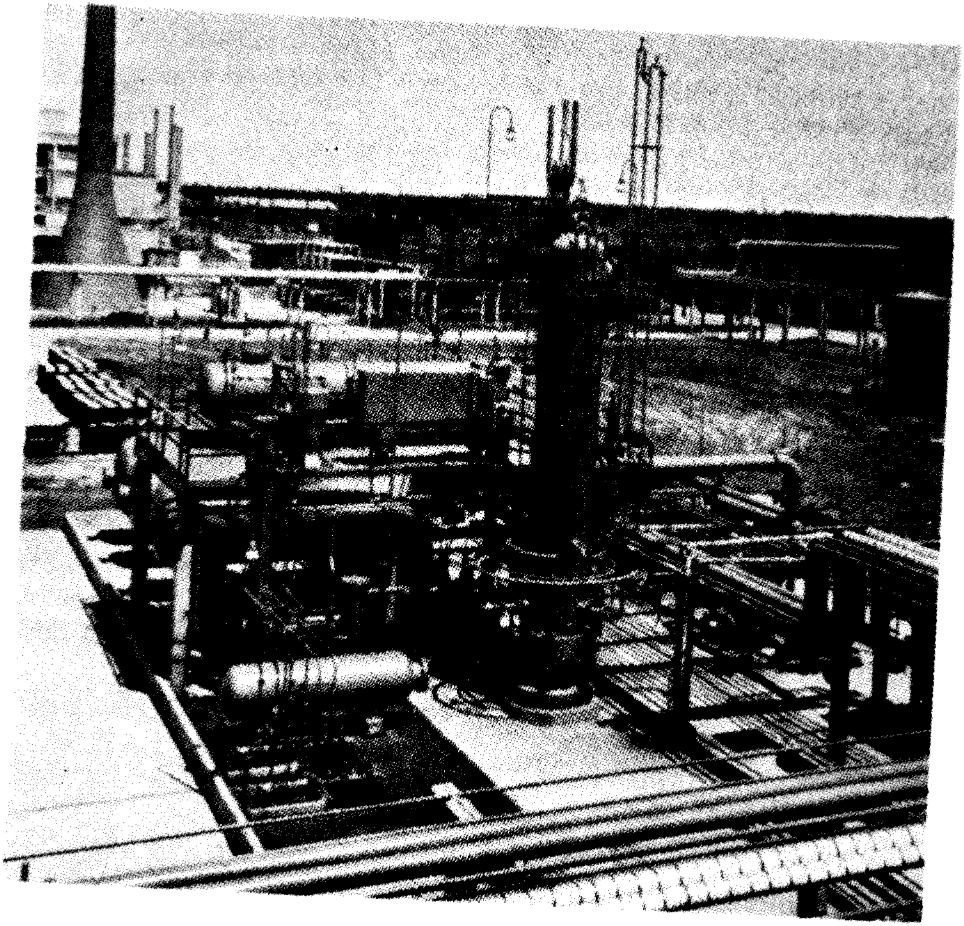

PLANT DESIGN AND ECONOMICS FOR CHEMICAL ENGINEERS

Fourth Edition

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Dr. Peters has served as President of the American Institute of Chemical Engineers, as a member of the Board of Directors for the Commission on Engineering Education, as Chairman of the President's Committee on the National Medal of Science, and as Chairman of the Colorado Environmental Commission. A Fellow of the American Institute of Chemical Engineers. Dr. Peters is the recipient of the George Westinghouse Award of the American Society for Engineering Education, the Lamme Award of the ASEE, the Award of Merit of the American Association of Cost Engineers, the Founders Award of the American Institute of Chemical Engineers, and the W. K. Lewis Award of the AIChE. He is a member of the National Academy of Engineering.

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He is past President of the American Institute of Chemical Engineers, past President of Sigma Xi, current President of the International Institute of Refrigeration, and has held offices in the Cryogenic Engineering Conference, the Society of Sigma Xi, the American Astronautical Society, the American Association for the Advancement of Science, the American Society for Engineering Education-Engineering Research Council, the Accreditation Board for Engineering and Technology, and the National Academy of Engineering.

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PREFACE

Advances in the level of understanding of chemical engineering principles, combined with the availability of new tools and new techniques, have led to an increased degree of sophistication which can now be applied to the design of industrial chemical operations. This fourth edition takes advantage of the widened spectrum of chemical engineering knowledge by the inclusion of considerable material on profitabilty evaluation, optimum design methods, continuous interest compounding, statistical analyses, cost estimation, and methods for problem solution including use of computers. Special emphasis is placed on the economic and engineering principles involved in the design of chemical plants and equipment. An understanding of these principles is a prerequisite for any successful chemical engineer, no matter whether the final position is in direct design work or in production, administration, sales, research, development, or any other related field.

The expression ***plant*** design immediately connotes industrial applications; consequently, the dollar sign must always be kept in mind when carrying out the design of a plant. The theoretical and practical aspects are important, of course; but, in the final analysis, the answer to the question "Will we realize a profit from this venture?" almost always determines the true value of the design. The chemical engineer, therefore, should consider plant design and applied economics as one combined subject.

The purpose of this book is to present economic and design principles as applied in chemical engineering processes and operations. No attempt is made to train the reader as a skilled economist, and, obviously, it would be impossible to present all the possible ramifications involved in the multitude of different plant designs. Instead, the goal has been to give a clear concept of the important principles and general methods. The subject matter and manner of presentation are such that the book should be of value to advanced chemical engineering undergraduates, graduate students, and practicing engineers. The information should also be of interest to administrators, operation supervisors, and research or development workers in the process industries.

The first part of the text presents an overall analysis of the major factors involved in process design, with particular emphasis on economics in the process industries and in design work. Computer-aided design is discussed early in the book as a separate chapter to introduce the reader to this important topic with the understanding that this tool will be useful throughout the text. The various costs involved in industrial processes, capital investments and investment returns, cost estimation, cost accounting, optimum economic design methods, and other subjects dealing with economics are covered both qualitatively and quantitatively. The remainder of the text deals with methods and important factors in the design of plants and equipment. Generalized subjects, such as waste disposal, structural design, and equipment fabrication, are included along with design methods for different types of process equipment. Basic cost data and cost correlations are also presented for use in making cost estimates.

Illustrative examples and sample problems are used extensively in the text to illustrate the applications of the principles to practical situations. Problems are included at the ends of most of the chapters to give the reader a chance to test the understanding of the material. Practice-session problems, as well as longer design problems of varying degrees of complexity, are included in Appendix C. Suggested recent references are presented as footnotes to show the reader where additional information can be obtained. Earlier references are listed in the first, second, and third editions of this book.

A large amount of cost data is presented in tabular and graphical form. The table of contents for the book lists chapters where equipment cost data are presented, and additional cost information on specific items of equipment or operating factors can be located by reference to the subject index. To simplify use of the extensive cost data given in this book, all cost figures are referenced to the all-industry Marshall and Swift cost index of 904 applicable for January 1, 1990. Because exact prices can be obtained only by direct quotations from manufacturers, caution should be exercised in the use of the data for other than approximate cost-estimation purposes.

The book would be suitable for use in a one- or two-semester course for advanced undergraduate or graduate chemical engineers. It is assumed that the reader has a background in stoichiometry, thermodynamics, and chemical engineering principles as taught in normal first-degree programs in chemical engineering. Detailed explanations of the development of various design equations and methods are presented. The book provides a background of design and economic information with a large amount of quantitative interpretation so that it can serve as a basis for further study to develop complete understanding of the general strategy of process engineering design.

Although nomographs, simplified equations, and shortcut methods are included, every effort has been made to indicate the theoretical background and assumptions for these relationships. The true value of plant design and economics for the chemical engineer is not found merely in the ability to put numbers ‘in an equation and solve for a final answer. The true value is found in obtaining an understanding of the reasons why a given calculation method gives a satisfactory result. This understanding gives the engineer the confidence and ability necessary to proceed when new problems are encountered for which there are no predetermined methods of solution. Thus, throughout the study of plant design and economics, the engineer should always attempt to understand the assumptions and theoretical factors involved in the various calculation procedures and never fall into the habit of robot-like number plugging.

Because applied economics and plant design deal with practical applications of chemical engineering principles, a study of these subjects offers an ideal way for tying together the entire field of chemical engineering. The final result of a plant design may be expressed in dollars and cents, but this result can only be achieved through the application of various theoretical principles combined with industrial and practical knowledge. Both theory and practice are emphasized in this book, and aspects of all phases of chemical engineering are included.

The authors are indebted to the many industrial firms and individuals who have supplied information and comments on the material presented in this edition. The authors also express their appreciation to the following reviewers who have supplied constructive criticism and helpful suggestions on the presentation for this edition: David C. Drown, University of Idaho; Leo J. Hirth, Auburn University; Robert L. Kabel, Pennsylvania State University; J. D. Seader, University of Utah; and Arthur W. Westerberg, Carnegie Mellon University. Acknowledgement is made of the contribution by Ronald E. West, Professor of Chemical Engineering at the University of Colorado, for the new Chapter 4 in this edition covering computer-aided design.

Max S. Peters
Klaus D. Timmerhaus

PROLOGUE

THE INTERNATIONAL SYSTEM OF UNITS (SI)

As the United States moves toward acceptance of the International System of Units, or the so-called SI units, it is particularly important for the design engineer to be able to think in both the SI units and the U.S. customary units. From an international viewpoint, the United States is the last major country to accept SI, but it will be many years before the U.S. conversion will be sufficiently complete for the design engineer, who must deal with the general public, to think and write solely in SI units. For this reason, a mixture of SI and U.S. customary units will be found in this text.

For those readers who are not familiar with all the rules and conversions for SI units, Appendix A of this text presents the necessary information. This appendix gives descriptive and background information for the SI units along with a detailed set of rules for SI usage and lists of conversion factors presented in various forms which should be of special value for chemical engineering usage.

Chemical engineers in design must be totally familiar with SI and its rules. Reading of Appendix A is recommended for those readers who have not worked closely and extensively with SI.

CHAPTER
1

INTRODUCTION

In this modern age of industrial competition, a successful chemical engineer needs more than a knowledge and understanding of the fundamental sciences and the related engineering subjects such as thermodynamics, reaction kinetics, and computer technology. The engineer must also have the ability to apply this knowledge to practical situations for the purpose of accomplishing something that will be beneficial to society. However, in making these applications, the chemical engineer must recognize the economic implications which are involved and proceed accordingly.

Chemical engineering design of new chemical plants and the expansion or revision of existing ones require the use of engineering principles and theories combined with a practical realization of the limits imposed by industrial conditions. Development of a new plant or process from concept evaluation to profitable reality is often an enormously complex problem. A plant-design project moves to completion through a series of stages such as is shown in the following:

1. Inception
2. Preliminary evaluation of economics and market
3. Development of data necessary for final design
4. Final economic evaluation
5. Detailed engineering design
6. Procurement
7. Erection
8. Startup and trial runs
9. Production

This brief outline suggests that the plant-design project involves a wide variety of skills. Among these are research, market analysis, design of individual pieces of equipment, cost estimation, computer programming, and plant-location surveys. In fact, the services of a chemical engineer are needed in each step of the outline, either in a central creative role, or as a key advisor.

CHEMICAL ENGINEERING PLANT DESIGN

As used in this text, the general term *plant design* includes all engineering aspects involved in the development of either a new, modified, or expanded industrial plant. In this development, the chemical engineer will be making economic evaluations of new processes, designing individual pieces of equipment for the proposed new venture, or developing a plant layout for coordination of the overall operation. Because of these many design duties, the chemical engineer is many times referred to here as a *design engineer.* On the other hand, a chemical engineer specializing in the economic aspects of the design is often referred to as a *cost engineer.* In many instances, the term *process engineering* is used in connection with economic evaluation and general economic analyses of industrial processes, while *process design* refers to the actual design of the equipment and facilities necessary for carrying out the process. Similarly, the meaning of plant design is limited by some engineers to items related directly to the complete plant, such as plant layout, general service facilities, and plant location.

The purpose of this book is to present the major aspects of plant design as related to the overall design project. Although one person cannot be an expert in *all* the phases involved in plant design, it is necessary to be acquainted with the general problems and approach in each of the phases. The process engineer may not be connected directly with the final detailed design of the equipment, and the designer of the equipment may have little influence on a decision by management as to whether or not a given return on an investment is adequate to justify construction of a complete plant. Nevertheless, if the overall design project is to be successful, close teamwork is necessary among the various groups of engineers working on the different phases of the project. The most effective teamwork and coordination of efforts are obtained when each of the engineers in the specialized groups is aware of the many functions in the *overall* design project.

PROCESS DESIGN DEVELOPMENT

The development of a process design, as outlined in Chap. 2, involves many different steps. The first, of course, must be the inception of the basic idea. This idea may originate in the sales department, as a result of a customer request, or to meet a competing product. It may occur spontaneously to someone who is acquainted with the aims and needs of a particular company, or it may be the result of an orderly research program or an offshoot of such a program. The operating division of the company may develop a new or modified chemical, generally as an intermediate in the final product. The engineering department of the company may originate a new process or modify an existing process to create new products. In all these possibilities, if the initial analysis indicates that the idea may have possibilities of developing into a worthwhile project, a preliminary research or investigation program is initiated. Here, a general survey of the possibilities for a successful process is made considering the physical and chemical operations involved as well as the economic aspects. Next comes the process-research phase including preliminary market surveys, laboratory-scale experiments, and production of research samples of the final product. When the potentialities of the process are fairly well established, the project is ready for the development phase. At this point, a pilot plant or a commercial-development plant may be constructed. A pilot plant is a small-scale replica of the full-scale final plant, while a commercial-development plant is usually made from odd pieces of equipment which are already available and is not meant to duplicate the exact setup to be used in the full-scale plant.

Design data and other process information are obtained during the development stage. This information is used as the basis for carrying out the additional phases of the design project. A complete market analysis is made, and samples of the final product are sent to prospective customers to determine if the product is satisfactory and if there is a reasonable sales potential. Capital-cost estimates for the proposed plant are made. Probable returns on the required investment are determined, and a complete cost-and-profit analysis of the process is developed.

Before the final process design starts, company management normally becomes involved to decide if significant capital funds will be committed to the project. It is at this point that the engineers' preliminary design work along with the oral and written reports which are presented become particularly important because they will provide the primary basis on which management will decide if further funds should be provided for the project. When management has made a firm decision to proceed with provision of significant capital funds for a project, the engineering then involved in further work on the project is known as ***capitalized*** engineering while that which has gone on before while the consideration of the project was in the development stage is often referred to as ***expensed*** engineering. This distinction is used for tax purposes to allow capitalized engineering costs to be amortized over a period of several years.

If the economic picture is still satisfactory, the final process-design phase is ready to begin. All the design details are worked out in this phase including controls, services; piping layouts, firm price quotations, specifications and designs for individual pieces of equipment, and all the other design information necessary for the construction of the final plant. A complete construction design is then made with elevation drawings, plant-layout arrangements, and other information required for the actual construction of the plant. The final stage consists of procurement of the equipment, construction of the plant, startup of the plant, overall improvements in the operation, and development of standard operating procedures to give the best possible results.

The development of a design project proceeds in a logical, organized sequence requiring more and more time, effort, and expenditure as one phase leads into the next. It is extremely important, therefore, to stop and analyze the situation carefully before proceeding with each subsequent phase. Many projects are discarded as soon as the preliminary investigation or research on the original idea is completed. The engineer working on the project must maintain a realistic and practical attitude in advancing through the various stages of a design project and not be swayed by personal interests and desires when deciding if further work on a particular project is justifiable. Remember, if the engineer's work is continued on through the various phases of a design project, it will eventually end up in a proposal that money be invested in the process. If no tangible return can be realized from the investment, the proposal will be turned down. Therefore, the engineer should have the ability to eliminate unprofitable ventures before the design project approaches a final-proposal stage.

GENERAL OVERALL DESIGN CONSIDERATIONS

The development of the overall design project involves many different design considerations. Failure to include these considerations in the overall design project may, in many instances, alter the entire economic situation so drastically as to make the venture unprofitable. Some of the factors involved in the development of a complete plant design include plant location, plant layout, materials of construction, structural design, utilities, buildings, storage, materials handling, safety, waste disposal, federal, state, and local laws or codes, and patents. Because of their importance, these general overall design considerations are considered in detail in Chap. 3.

Various types of computer programs and techniques are used to carry out the design of individual pieces of equipment or to develop the strategy for a full plant design. This application of computer usage in design is designated as ***computer-aided design*** and is the subject of Chap. 4.

Record keeping and accounting procedures are also important factors in general design considerations, and it is necessary that the design engineer be familiar with the general terminology and approach used by accountants for cost and asset accounting. This subject is covered in Chap. 5.

COST ESTIMATION

As soon as the final process-design stage is completed, it, becomes possible to make accurate cost estimations because detailed equipmept specifications and definite plant-facility information are available. Direct price quotations based on detailed specifications can then be obtained from various manufacturers. However, as mentioned earlier, no design project should proceed to the final stages before costs are considered, and cost estimates should be made throughout all the early stages of the design when complete specifications are not available. Evaluation of costs in the preliminary design phases is sometimes called "guesstimation" but the appropriate designation is ***predesign cost estimation.*** Such estimates should be capable of providing a basis for company management to decide if further capital should be invested in the project.

The chemical engineer (or cost engineer) must be certain to consider all possible factors when making a cost analysis. Fixed costs, direct production costs for raw materials, labor, maintenance, power, and utilities must all be included along with costs for plant and administrative overhead, distribution of the final products, and other miscellaneous items.

Chapter 6 presents many of the special techniques that have been developed for making predesign cost estimations. Labor and material indexes, standard cost ratios, and special multiplication factors are examples of information used when making design estimates of costs. The final test as to the validity of any cost estimation can come only when the completed plant has been put into operation. However, if the design engineer is well acquainted with the various estimation methods and their accuracy, it is possible to make remarkably close cost estimations even before the final process design gives detailed specifications.

FACTORS AFFECTING PROFITABILITY OF INVESTMENTS

A major function of the directors of a manufacturing firm is to maximize the long-term profit to the owners or the stockholders. A decision to invest in fixed facilities carries with it the burden of continuing interest, insurance, taxes, depreciation, manufacturing costs, etc., and also reduces the fluidity of the company's future actions. Capital-investment decisions, therefore, must be made with great care. Chapters 7 and 10 present guidelines for making these capital-investment decisions.

Money, or any other negotiable type of capital, has a time value. When a manufacturing enterprise invests money, it expects to receive a return during the time the money is being used. The amount of return demanded usually depends on the degree of risk that is assumed. Risks differ between projects which might otherwise seem equal on the basis of the best estimates of an overall plant design. The risk may depend upon the process used, whether it is well established or a complete innovation; on the product to be made, whether it is a staple item or a completely new product; on the sales forecasts, whether all sales will be outside the company or whether a significant fraction is internal, etc. Since means for incorporating different levels of risk into ***profitability*** forecasts are not too well established, the most common methods are to raise the minimum acceptable ***rate*** of ***return*** for the riskier projects.

Time value of money has been integrated into investment-evaluation systems by means of ***compound-interest*** relationships. Dollars, at different times, are given different degrees of importance by means of compounding or discounting at some preselected compound-interest rate. For any assumed interest value of money, a known amount at any one time can be converted to an equivalent but different amount at a different time. As time passes, money can be invested to increase at the interest rate. If the time when money is needed for investment is in the future, the present value of that investment can be calculated by discounting from the time of investment back to the present at the assumed interest rate.

Expenses, as outlined in Chap. 8, for various types of taxes and insurance can materially affect the economic situation for any industrial process. Because modern taxes may amount to a major portion of a manufacturing firm's net earnings, it is essential that the chemical engineer be conversant with the fundamentals of taxation. For example, income taxes apply differently to projects with different proportions of fixed and working capital. Profitability, therefore, should be based on income after taxes. Insurance costs, on the other hand, are normally only a small part of the total operational expenditure of an industrial enterprise; however, before any operation can be carried out on a sound economic basis, it is necessary to determine the insurance requirements to provide adequate coverage against unpredictable emergencies or developments.

Since all physical assets of an industrial facility decrease in value with age, it is normal practice to make periodic charges against earnings so as to distribute the first cost of the facility over its expected service life. This ***depreciation*** expense as detailed in Chap. 9, unlike most other expenses, entails no current outlay of cash. Thus, in a given accounting period, a firm has available, in addition to the net profit, additional funds corresponding to the depreciation expense. This cash is ***capital recovery,*** a partial regeneration of the first cost of the physical assets.

Income-tax laws permit recovery of funds by two accelerated depreciation schedules as well as by straight-line methods. Since cash-flow timing is affected, choice of depreciation method affects profitability significantly. Depending on the ratio of depreciable to nondepreciable assets involved, two projects which look equivalent before taxes, or rank in one order, may rank entirely differently when considered after taxes. Though cash costs and sales values may be equal on two projects, their reported net incomes for tax purposes may be different, and one will show a greater net profit than the other.

OPTIMUM DESIGN

In almost every case encountered by a chemical engineer, there are several alternative methods which can be used for any given process or operation. For example, formaldehyde can be produced by catalytic dehydrogenation of methanol, by controlled oxidation of natural gas, or by direct reaction between CO and H_2 under special conditions of catalyst, temperature, and pressure. Each of these processes contains many possible alternatives involving variables such as gas-mixture composition, temperature, pressure, and choice of catalyst. It is the responsibility of the chemical engineer, in this case, to choose the best process and to incorporate into the design the equipment and methods which will give the best results. To meet this need, various aspects of chemical engineering plant-design optimization are described in Chap. 11 including presentation of design strategies which can be used to establish the desired results in the most efficient manner.

Optimum Economic Design

If there are two or more methods for obtaining exactly equivalent final results, the preferred method would be the one involving the least total cost. This is the basis of an optimum economic *design*. One typical example of an optimum economic design is determining the pipe diameter to use when pumping a given amount of fluid from one point to another. Here the same final result (i.e., a set amount of fluid pumped between two given points) can be accomplished by using an infinite number of different pipe diameters. However, an economic balance will show that one particular pipe diameter gives the least total cost. The total cost includes the cost for pumping the liquid and the cost (i.e., fixed charges) for the installed piping system.

A graphical representation showing the meaning of an optimum economic pipe diameter is presented in Fig. l-l. As shown in this figure, the pumping cost increases with decreased size of pipe diameter because of frictional effects, while the fixed charges for the pipeline become lower when smaller pipe diameters are used because of the reduced capital investment. The optimum economic diameter is located where the sum of the pumping costs and fixed costs for the pipeline becomes a minimum, since this represents the point of least total cost. In Fig. l-l, this point is represented by *E*.

The chemical engineer often selects a final design on the basis of conditions giving the least total cost. In many cases, however, alternative designs do not give final products or results that are exactly equivalent. It then becomes necessary to consider the quality of the product or the operation as well as the total cost. When the engineer speaks of an optimum economic design, it ordinarily means the cheapest one selected from a number of equivalent designs. Cost data, to assist in making these decisions, are presented in Chaps. 14 through 16.

Various types of optimum economic requirements may be encountered in design work. For example, it may be desirable to choose a design which gives the maximum profit per unit of time or the minimum total cost per unit of production.

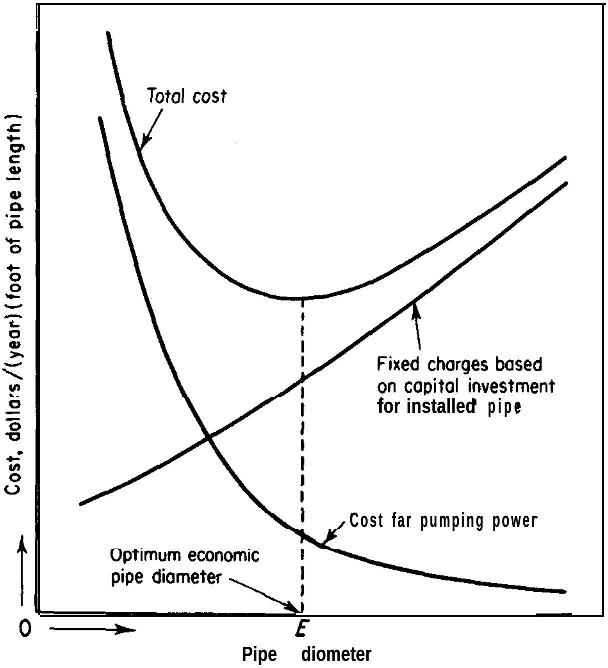

FIGURE 1-1
Determination of optimum economic pipe diameter for constant mass-throughput rate.

Optimum Operation Design

Many processes require definite conditions of temperature, pressure, contact time, or other variables if the best results are to be obtained. It is often possible to make a partial separation of these optimum conditions from direct economic considerations. In cases of this type, the best design is designated as the ***optimum operation design.*** The chemical engineer should remember, however, that economic considerations ultimately determine most quantitative decisions. Thus, the optimum operation design is usually merely a tool or step in the development of an optimum economic design.

An excellent example of an optimum operation design is the determination of operating conditions for the catalytic oxidation of sulfur dioxide to sulfur trioxide. Suppose that all the variables, such as converter size, gas rate, catalyst activity, and entering-gas concentration, are fixed and the only possible variable is the temperature at which the oxidation occurs. If the temperature is too high, the yield of SO, will be low because the equilibrium between SO,, SO,, and 0, is shifted in the direction of SO, and 0,. On the other hand, if the temperature is too low, the yield will be poor because the reaction rate between SO, and 0, will be low. Thus, there must be one temperature where the amount of sulfur trioxide formed will be a maximum. This particular temperature would give the

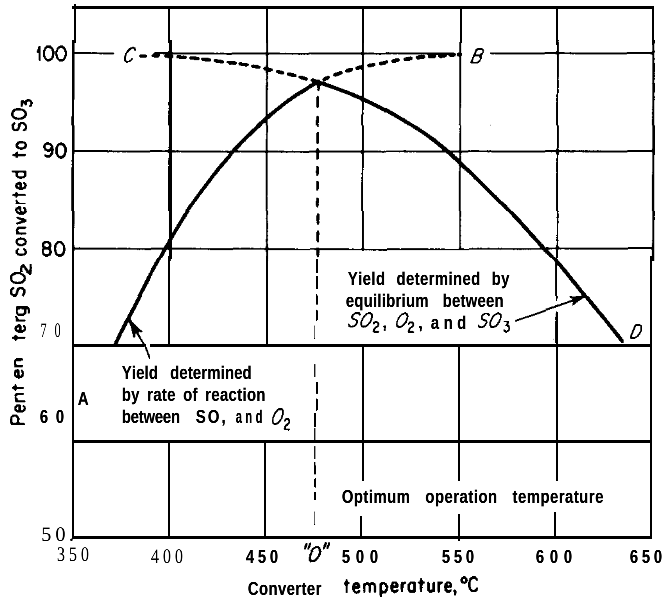

FIGURE 1-2
Determination of optimum operation temperature in sulfur dioxide converter.

optimum operation design. Figure 1-2 presents a graphical method for determining the optimum operation temperature for the sulfur dioxide converter in this example. Line AB represents the maximum yields obtainable when the reaction rate is controlling, while line *CD* indicates the maximum yields on the basis of equilibrium conditions controlling. Point 0 represents the optimum operation temperature where the maximum yield is obtained.

The preceding example is a simplified case of what an engineer might encounter in a design. In reality, it would usually be necessary to consider various converter sizes and operation with a series of different temperatures in order to arrive at the optimum operation design. Under these conditions, several equivalent designs would apply, and the final decision would be based on the optimum economic conditions for the equivalent designs.

PRACTICAL CONSIDERATIONS IN DESIGN

The chemical engineer must never lose sight of the practical limitations involved in a design. It may be possible to determine an exact pipe diameter for an optimum economic design, but this does not mean that this exact size must be used in the final design. Suppose the optimum diameter were 3.43 in. (8.71 cm). It would be impractical to have a special pipe fabricated with an inside diameter of 3.43 in. Instead, the engineer would choose a standard pipe size which could be purchased at regular market prices. In this case, the recommended pipe size would probably be a standard $3\frac{1}{2}$-in.-diameter pipe having an inside diameter of 3.55 in. (9.02 cm).

If the engineer happened to be very conscientious about getting an adequate return on all investments, he or she might say, "A standard 3-in.-diameter pipe would require less investment and would probably only increase the total cost slightly; therefore, I think we should compare the costs with a 3-in. pipe to the costs with the $3\frac{1}{2}$-in. pipe before making a final decision." Theoretically, the conscientious engineer is correct in this case. Suppose the total cost of the installed $3\frac{1}{2}$-in. pipe is \$5000 and the total cost of the installed 3-in. pipe is \$4500. If the total yearly savings on power and fixed charges, using the $3\frac{1}{2}$-in. pipe instead of the 3-in. pipe, were \$25, the yearly percent return on the extra \$500 investment would be only 5 percent. Since it should be possible to invest the extra \$500 elsewhere to give more than a 5 percent return, it would appear that the 3-in.-diameter pipe would be preferred over the $3\frac{1}{2}$-in.-diameter pipe.

The logic presented in the preceding example is perfectly sound. It is a typical example of investment comparison and should be understood by all chemical engineers. Even though the optimum economic diameter was 3.43 in., the good engineer knows that this diameter is only an exact mathematical number and may vary from month to month as prices or operating conditions change. Therefore, all one expects to obtain from this particular optimum economic calculation is a good estimation as to the best diameter, and investment comparisons may not be necessary.

The practical engineer understands the physical problems which are involved in the final operation and maintenance of the designed equipment. In developing the plant layout, crucial control valves must be placed where they are easily accessible to the operators. Sufficient space must be available for maintenance personnel to check, take apart, and repair equipment. The engineer should realize that cleaning operations are simplified if a scale-forming fluid is passed through the inside of the tubes rather than on the shell side of a tube-and-shell heat exchanger. Obviously, then, sufficient plant-layout space should be made available so that the maintenance workers can remove the head of the installed exchanger and force cleaning worms or brushes through the inside of the tubes or remove the entire tube bundle when necessary.

The theoretical design of a distillation unit may indicate that the feed should be introduced on one particular tray in the tower. Instead of specifying a tower with only one feed inlet on the calculated tray, the practical engineer will include inlets on several trays above and below the calculated feed point since the actual operating conditions for the tower will vary and the assumptions included in the calculations make it impossible to guarantee absolute accuracy.

The preceding examples typify the type of practical problems the chemical engineer encounters. In design work, theoretical and economic principles must be combined with an understanding of the common practical problems that will arise when the process finally comes to life in the form of a complete plant or a complete unit.

THE DESIGN APPROACH

The chemical engineer has many tools to choose from in the development of a profitable plant design. None, when properly utilized, will probably contribute as much to the optimization of the design as the use of high-speed computers. Many problems encountered in the process development and design can be solved rapidly with a higher degree of completeness with high-speed computers and at less cost than with ordinary hand or desk calculators. Generally overdesign and safety factors can be reduced with a substantial savings in capital investment.

At no time, however, should the engineer be led to believe that plants are designed around computers. They are used to determine design data and are used as models for optimization once a design is established. They are also used to maintain operating plants on the desired operating conditions. The latter function is a part of design and supplements and follows process design.

The general approach in any plant design involves a carefully balanced combination of theory, practice, originality, and plain common sense. In original design work, the engineer must deal with many different types of experimental and empirical data. The engineer may be able to obtain accurate values of heat capacity, density, vapor-liquid equilibrium data, or other information on physical properties from the literature. In many cases, however, exact values for necessary physical properties are not available, and the engineer is forced to make approximate estimates of these values. Many approximations also must be made in carrying out theoretical design calculations. For example, even though the engineer knows that the ideal-gas law applies exactly only to simple gases at very low pressures, this law is used in many of the calculations when the gas pressure is as high as 5 or more atmospheres (507 kPa). With common gases, such as air or simple hydrocarbons, the error introduced by using the ideal gas law at ordinary pressures and temperatures is usually negligible in comparison with other uncertainties involved in design calculations. The engineer prefers to accept this error rather than to spend time determining virial coefficients or other factors to correct for ideal gas deviations.

In the engineer's approach to any design problem, it is necessary to be prepared to make many assumptions. Sometimes these assumptions are made because no absolutely accurate values or methods of calculation are available. At other times, methods involving close approximations are used because exact treatments would require long and laborious calculations giving little gain in accuracy. The good chemical engineer recognizes the need for making certain assumptions but also knows that this type of approach introduces some uncertainties into the final results. Therefore, assumptions are made only when they are necessary and essentially correct.

Another important factor in the approach to any design problem involves economic conditions and limitations. The engineer must consider costs and probable profits constantly throughout all the work. It is almost always better to sell many units of a product at a low profit per unit than a few units at a high profit per unit. Consequently, the engineer must take into account the volume of production when determining costs and total profits for various types of designs. This obviously leads to considerations of customer needs and demands. These factors may appear to be distantly removed from the development of a plant design, but they are extremely important in determining its ultimate success.

CHAPTER
2

PROCESS DESIGN DEVELOPMENT

A principle responsibility of the chemical engineer is the design, construction, and operation of chemical plants. In this responsibility, the engineer must continuously search for additional information to assist in these functions. Such information is available from numerous sources, including recent publications, operation of existing process plants, and laboratory and pilot-plant data. This collection and analysis of all pertinent information is of such importance that chemical engineers are often members, consultants, or advisors of even the basic research team which is developing a new process or improving and revising an existing one. In this capacity, the chemical engineer can frequently advise the research group on how to provide considerable amounts of valuable design data.

Subjective decisions are and must be made many times during the design of any process. What are the best methods of securing sufficient and usable data? What is sufficient and what is reliable? Can better correlations of the data be devised, particularly ones that permit more valid extrapolation?

The chemical engineer should always be willing to consider completely new designs. An attempt to understand the controlling factors of the process, whether chemical or physical, helps to suggest new or improved techniques. For example, consider the commercial processes of aromatic nitration and alkylation of isobutane with olefins to produce high-octane gasolines. Both reactions involve two immiscible liquid phases and the mass-transfer steps are essentially rate controlling. Nitro-aromatics are often produced in high yields (up to 99 percent); however, the alkylation of isobutane involves numerous side reactions and highly complex chemistry that is less well understood. Several types of reactors have been used for each reaction. Then radically new and simplified reactors were developed based on a better understanding of the chemical and physical steps involved.

DESIGN-PROJECT PROCEDURE

The development of a design project always starts with an initial idea or plan. This initial idea must be stated as clearly and concisely as possible in order to define the scope of the project. General specifications and pertinent laboratory or chemical engineering data should be presented along with the initial idea.

Types of Designs

The methods for carrying out a design project may be divided into the following classifications, depending on the accuracy and detail required:

1. Preliminary or quick-estimate designs
2. Detailed-estimate designs
3. Firm process designs or detailed designs

Preliminary designs are ordinarily used as a basis for determining whether further work should be done on the proposed process. The design is based on approximate process methods, and rough cost estimates are prepared. Few details are included, and the time spent on calculations is kept at a minimum.

If the results of the preliminary design show that further work is justified, a ***detailed-estimate design*** may be developed. In this type of design, the cost-and-profit potential of an established process is determined by detailed analyses and calculations. However, exact specifications are not given for the equipment, and drafting-room work is minimized.

When the detailed-estimate design indicates that the proposed project should be a commercial success, the final step before developing construction plans for the plant is the preparation of a firm ***process design.*** Complete specifications are presented for all components of the plant, and accurate costs based on quoted prices are obtained. The firm process design includes blueprints and sufficient information to permit immediate development of the final plans for constructing the plant.

Feasibility Survey

Before any detailed work is done on the design, the technical and economic factors of the proposed process should be examined. The various reactions and physical processes involved must be considered, along with the existing and potential market conditions for the particular product. A preliminary survey of this type gives an indication of the probable success of’the project and also shows what additional information is necessary to make a complete evaluation. Following is a list of items that should be considered in making a feasibility survey:

1. Raw materials (availability, quantity, quality, cost)
2. Thermodynamics and kinetics of chemical reactions involved (equilibrium, yields, rates, optimum conditions)
3. Facilities and equipment available at present
4. Facilities and equipment which must be purchased
5. Estimation of production costs and total investment
6. Profits (probable and optimum, per pound of product and per year, return on investment)
7. Materials of construction
8. Safety considerations
9. Markets (present and future supply and demand, present uses, new uses, present buying habits, price range for products and by-products, character, location, and number of possible customers)
10. Competition (overall production statistics, comparison of various manufacturing processes, product specifications of competitors)
11. Properties of products (chemical and physical properties, specifications, impurities, effects of storage)
12. Sales and sales service (method of selling and distributing, advertising required, technical services required)
13. Shipping restrictions and containers
14. Plant location
15. Patent situation and legal restrictions

When detailed data on the process and firm product specifications are available, a complete market analysis combined with a consideration of all sales factors should be made. This analysis can be based on a breakdown of items 9 through 15 as indicated in the preceding list.

Process Development

In many cases, the preliminary feasibility survey indicates that additional research, laboratory, or pilot-plant data are necessary, and a program to obtain this information may be initiated. Process development on a pilot-plant or semiworks scale is usually desirable in order to obtain accurate design data.

Valuable information on material and energy balances can be obtained, and process conditions can be examined to supply data on temperature and pressure variation, yields, rates, grades of raw materials and products, batch versus continuous operation, material of construction, operating characteristics, and other pertinent design variables.

Design

If sufficient information is available, a preliminary design may be developed in conjunction with the preliminary feasibility survey. In developing the preliminary design the chemical engineer must first establish a workable manufacturing process for producing the desired product. Quite often a number of alternative processes or methods may be available to manufacture the same product. Except for those processes obviously undesirable, each method should be given consideration.

The first step in preparing the preliminary design is to establish the bases for ***design.*** In addition to the known specifications for the product and availability of raw materials, the design can be controlled by such items as the expected annual operating factor (fraction of the year that the plant will be in operation), temperature of the cooling water, available steam pressures, fuel used, value of by-products, etc. The next step consists of preparing a simplified flow diagram showing the processes that are involved and deciding upon the unit operations which will be required. A preliminary material balance at this point may very quickly eliminate some the alternative cases. Flow rates and stream conditions for the remaining cases are now evaluated by complete material balances, energy balances, and a knowledge of raw-material and product specifications, yields, reaction rates, and time cycles. The temperature, pressure, and composition of every process stream is determined. Stream enthalpies, percent vapor, liquid, and solid, heat duties, etc., are included where pertinent to the process.

Unit process principles are used in the design of specific pieces of equipment. (Assistance with the design and selection of various types of process equipment is given in Chaps. 14 through 16.) Equipment specifications are generally summarized in the form of tables and included with the final design report. These tables usually include the following:

1. ***Columns (distillation).*** In addition to the number of plates and operating conditions it is also necessary to specify the column diameter, materials of construction, plate layout, etc.
2. Vessels. In addition to size, which is often dictated by the holdup time desired, materials of construction and any packing or baffling should be specified.
3. ***Reactors.*** Catalyst type and size, bed diameter and thickness, heat-interchange facilities, cycle and regeneration arrangements, materials of construction, etc., must be specified.

4. ***Heat exchangers and furnaces.*** Manufacturers are usually supplied with the duty, corrected log mean-temperature difference, percent vaporized, pressure drop desired, and materials of construction.
5. ***Pumps and compressors.*** Specify type, power requirement, pressure difference, gravities, viscosities, and working pressures.
6. ***Instruments.*** Designate the function and any particular requirement.
7. ***Special equipment.*** Specifications for mechanical separators, mixers, driers, etc.

The foregoing is not intended as a complete checklist, but rather as an illustration of the type of summary that is required. (The headings used are particularly suited for the petrochemical industry; others may be desirable for different industries.) As noted in the summary, the selection of materials is intimately connected with the design and selection of the proper equipment.

As soon as the equipment needs have been firmed up, the utilities and labor requirements can be determined and tabulated. Estimates of the capital investment and the total product cost (as outlined in Chap. 6) complete the preliminary-design calculations. Economic evaluation plays an important part in any process design. This is particularly true not only in the selection for a specific process, choice of raw materials used, operating conditions chosen, but also in the specification of equipment. No design of a piece of equipment or a process is complete without an economical evaluation. In fact, as mentioned in Chap. 1, no design project should ever proceed beyond the preliminary stages without a consideration of costs. Evaluation of costs in the preliminary-design phases greatly assists the engineer in further eliminating many of the alternative cases.

The final step, and an important one in preparing a typical process design, involves writing the report which will present the results of the design work. Unfortunately this phase of the design work quite often receives very little attention by the chemical engineer. As a consequence, untold quantities of excellent engineering calculations and ideas are sometimes discarded because of poor communications between the engineer and management.?

Finally, it is important that the preliminary design be carried out as soon as sufficient data are available from the feasibility survey or the process-development step. In this way, the preliminary design can serve its main function of eliminating an undesirable project before large amounts of money and time are expended.

The preliminary design and the process-development work gives the results necessary for a detailed-estimate design. The following factors should be

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†See Chap. 13 for assistance in preparing more concise and clearer design reports.

established within narrow limits before a detailed-estimate design is developed:

1. Manufacturing process
2. Material and energy balances
3. Temperature and pressure ranges
4. Raw-material and product specifications
5. Yields, reaction rates, and time cycles
6. Materials of construction
7. Utilities requirements
8. Plant site

When the preceding information is included in the design, the result permits accurate estimation of required capital investment, manufacturing costs, and potential profits. Consideration should be given to the types of buildings, heating, ventilating, lighting, power, drainage, waste disposal, safety facilities, instrumentation, etc.

Firm process designs (or detailed designs) can be prepared for purchasing and construction from a detailed-estimate design. Detailed drawings are made for the fabrication of special equipment, and specifications are prepared for purchasing standard types of equipment and materials. A complete plant layout is prepared, and blueprints and instructions for construction are developed. Piping diagrams and other construction details are included. Specifications are given for warehouses, laboratories, guard-houses, fencing, change houses, transportation facilities, and similar items. The final firm process design must be developed with the assistance of persons skilled in various engineering fields, such as architectural, ventilating, electrical, and civil. Safety conditions and environmental-impact factors must also always be taken into account.

Construction and Operation

When a definite decision to proceed with the construction of a plant is made, there is usually an immediate demand for a quick plant startup. Timing, therefore, is particularly important in plant construction. Long delays may be encountered in the fabrication of major pieces of equipment, and deliveries often lag far behind the date of ordering. These factors must be taken into consideration when developing the final plans and may warrant the use of the Project Evaluation and Review Technique (PERT) or the Critical Path Method (CPM).† The chemical engineer should always work closely with construction personnel during the final stages of construction and purchasing designs. In this way, the design sequence can be arranged to make certain important factors

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†For further discussion of these methods consult Chap. 11.

that might delay construction are given first consideration. Construction of the plant may be started long before the final design is 100 percent complete. Correct design sequence is then essential in order to avoid construction delays.

During construction of the plant, the chemical engineer should visit the plant site to assist in interpretation of the plans and learn methods for improving future designs. The engineer should also be available during the initial startup of the plant and the early phases of operation. Thus, by close teamwork between design, construction, and operations personnel, the final plant can develop from the drawing-board stage to an operating unit that can function both efficiently and effectively.

DESIGN INFORMATION FROM THE LITERATURE

A survey of the literature will often reveal general information and specific data pertinent to the development of a design project. One good method for starting a literature survey is to obtain a recent publication dealing with the subject under investigation. This publication will give additional references, and each of these references will, in turn, indicate other sources of information. This approach permits a rapid survey of the important literature.

Chemical Abstracts, published semimonthly by the American Chemical Society, can be used for comprehensive literature surveys on chemical processes and operations.? This publication presents a brief outline and the original reference of the published articles dealing with chemistry and related fields. Yearly and decennial indexes of subjects and authors permit location of articles concerning specific topics.

A primary source of information on all aspects of chemical engineering principles, design, costs, and applications is "The Chemical Engineers' Handbook" published by McGraw-Hill Book Company with R. H. Perry and D. W. Green as editors for the 6th edition as published in 1984. This reference should be in the personal library of all chemical engineers involved in the field.

Regular features on design-related aspects of equipment, costs, materials of construction, and unit processes are published in *Chemical Engineering.* In addition to this publication, there are many other periodicals that publish articles of direct interest to the design engineer. The following periodicals are suggested as valuable sources of information for the chemical engineer who wishes to keep abreast of the latest developments in the field: *American Institute of Chemical Engineers' Journal, Chemical Engineering Progress, Chemical and Engineering News, Chemical Week, Chemical Engineering Science, Industrial and Engineering Chemistry Fundamentals, Industrial and Engineering Chemistry Process Design and Development, Journal of the American Chemical Society, Journal*

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†Abstracts of general engineering articles are available in the *Engineering Index*.

of Physical Chemistry, Hydrocarbon Processing, Engineering News-Record, Oil and Gas Journal, and Canadian Journal of Chemical Engineering.

A large number of textbooks covering the various aspects of chemical engineering principles and design are available.† In addition, many handbooks have been published giving physical properties and other basic data which are very useful to the design engineer.

Trade bulletins are published regularly by most manufacturing concerns, and these bulletins give much information of direct interest to the chemical engineer preparing a design. Some of the trade-bulletin information is condensed in an excellent reference book on chemical engineering equipment, products, and manufacturers. This book is known as the "Chemical Engineering Catalog,"‡ and contains a large amount of valuable descriptive material.

New information is constantly becoming available through publication in periodicals, books, trade bulletins, government reports, university bulletins, and many other sources. Many of the publications are devoted to shortcut methods for estimating physical properties or making design calculations, while others present compilations of essential data in the form of nomographs or tables.

The effective design engineer must make every attempt to keep an up-to-date knowledge of the advances in the field. Personal experience and contacts, attendance at meetings of technical societies and industrial expositions, and reference to the published literature are very helpful in giving the engineer the background information necessary for a successful design.

FLOW DIAGRAMS

The chemical engineer uses flow diagrams to show the sequence of equipment and unit operations in the overall process, to simplify visualization of the manufacturing procedures, and to indicate the quantities of materials and energy transfer. These diagrams may be divided into three general types: (1) qualitative, (2) quantitative, and (3) combined-detail.

A qualitative flow diagram indicates the flow of materials, unit operations involved, equipment necessary, and special information on operating temperatures and pressures. A quantitative flow diagram shows the quantities of materials required for the process operation. An example of a qualitative flow diagram for the production of nitric acid is shown in Fig. 2-1. Figure 2-2 presents a quantitative flow diagram for the same process.

Preliminary flow diagrams are made during the early stages of a design project. As the design proceeds toward completion, detailed information on flow quantities and equipment specifications becomes available, and combined-detail flow diagrams can be prepared. This type of diagram shows the

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†For example, see the *Chemical* Engineering Series listing at the front of this text.

‡Published annually by Reinhold Publishing, Stamford, CT.

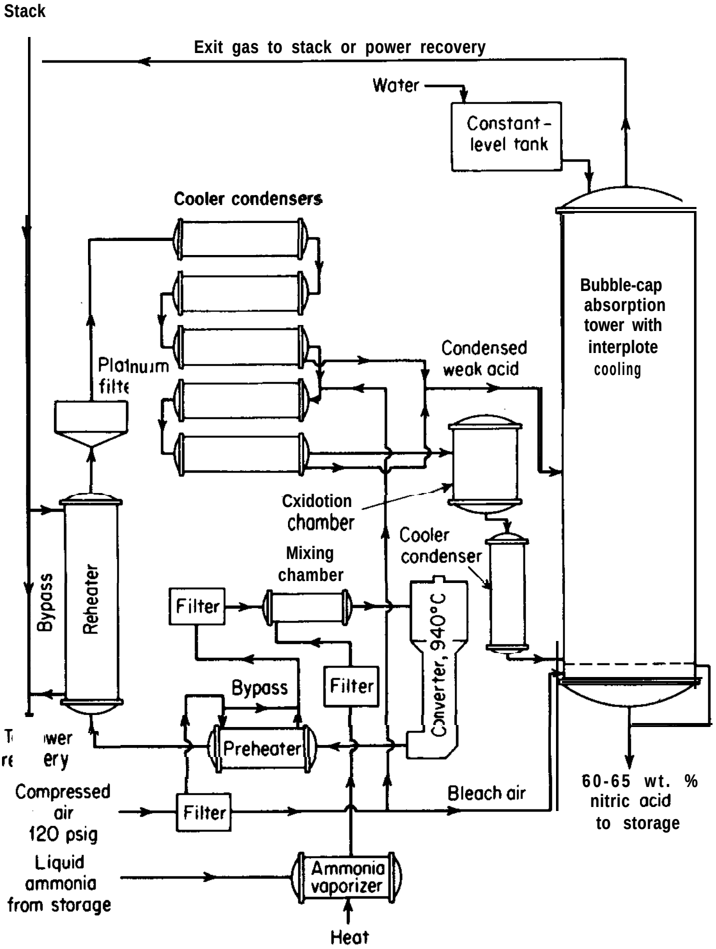

FIGURE 2-1
Qualitative flow diagram for the manufacture of nitric acid by the ammonia-oxidation process.

qualitative flow pattern and serves as a base reference for giving equipment specifications, quantitative data, and sample calculations. Tables presenting pertinent data on the process and the equipment are cross-referenced to the drawing. In this way, qualitative information and quantitative data are combined on the basis of one flow diagram. The drawing does not lose its effectiveness by presenting too much information; yet the necessary data are readily available by direct reference to the accompanying tables.

A typical combined-detail flow diagram shows the location of temperature and pressure regulators and indicators, as well as the location of critical control valves and special instruments. Each piece of equipment is shown and is designated by a defined code number. For each piece of equipment, accompany-

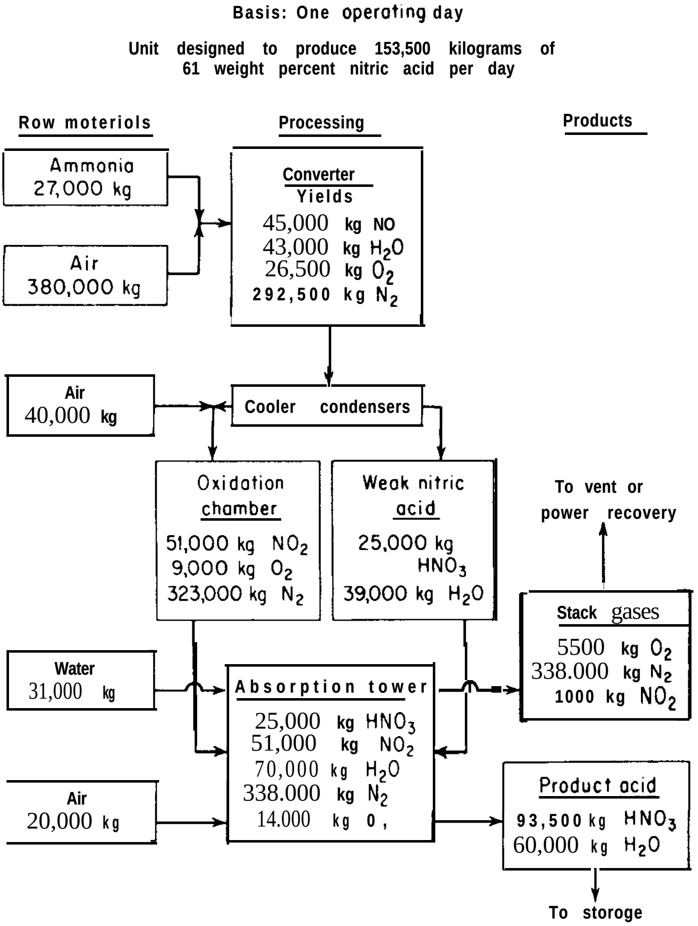

FIGURE 2-2
Quantitative flow diagram for the manufacture of nitric acid by the ammonia-oxidation process.

ing tables give essential information, such as specifications for purchasing, specifications for construction, type of fabrication, quantities and types of chemicals involved, and sample calculations.

Equipment symbols and flow-sheet symbols, particularly for detailed equipment flow sheets, are given in the Appendix.

THE PRELIMINARY DESIGN

In order to amplify the remarks made earlier in this chapter concerning the design-project procedure, it is appropriate at this time to look more closely at a specific preliminary design. Because of space limitations, only a brief presentation of the design will be attempted at this point.? However, sufficient detail will be given to outline the important steps which are necessary to prepare such a preliminary design. The problem presented is a practical one of a type frequently encountered in the chemical industry; it involves both process design and economic considerations.

Problem Statement

A conservative petroleum company has recently been reorganized and the new management has decided that the company must diversify its operations into the petrochemical field if it wishes to remain competitive. The research division of the company has suggested that a very promising area in the petrochemical field would be in the development and manufacture of biodegradable synthetic detergents using some of the hydrocarbon intermediates presently available in the refinery. A survey by the market division has indicated that the company could hope to attain 2.5 percent of the detergent market if a plant with an annual production of 15 million pounds were to be built. To provide management with an investment comparison, the design group has been instructed to proceed first with a preliminary design and an updated cost estimate for a nonbiodegradable detergent producing facility similar to ones supplanted by recent biodegradable facilities.

Literature Survey

A survey of the literature reveals that the majority of the nonbiodegradable detergents are alkylbenzene sulfonates (ABS). Theoretically, there are over 80,000 isomeric alkylbenzenes in the range of C_{10} to C_{15} for the alkyl side chain. Costs, however, generally favor the use of dodecene (propylene tetramer) as the starting material for ABS.

There are many different schemes in the manufacture of ABS. Most of the schemes are variations of the one shown in Fig. 2-3 for the production of sodium dodecylbenzene sulfonate. A brief description of the process is as follows:

This process involves reaction of dodecene with benzene in the presence of aluminum chloride catalyst; fractionation of the resulting crude mixture to recover the desired boiling range of dodecylbenzene; sulfonation of the dodecylbenzene and subsequent neutralization of the sulfonic acid with caustic soda; blending the resulting slurry with chemical “builders”; and drying.

Dodecene is charged into a reaction vessel containing benzene and aluminum chloride. The reaction mixture is agitated and cooled to maintain the reaction temperature of about 115°F maximum. An excess of benzene is used to suppress the formation of by-products. Aluminum chloride requirement is 5 to 10 wt% of dodecene.

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†Completion of the design is left as an exercise for the reader.

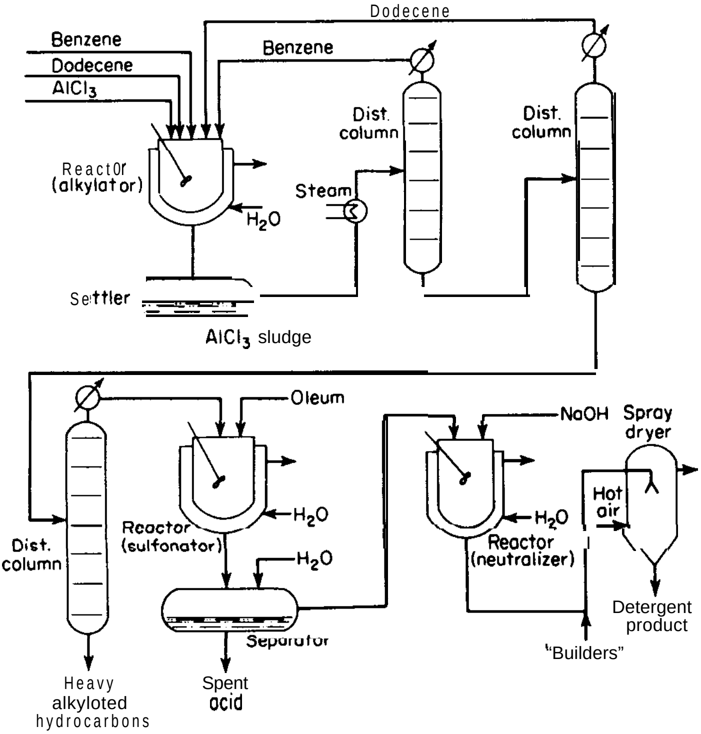

FIGURE 2-3
Qualitative flow diagram for the manufacture of sodium dodecylbenzene sulfonate.

After removal of aluminum chloride sludge, the reaction mixture is fractionated to recover excess benzene (which is recycled to the reaction vessel), a light alkylaryl hydrocarbon, dodecylbenzene, and a heavy alkylaryl hydrocarbon.

Sulfonation of the dodecylbenzene may be carried out continuously or batch-wise under a variety of operating conditions using sulfuric acid (100 percent), oleum (usually 20 percent SO,), or anhydrous sulfur trioxide. The optimum sulfonation temperature is usually in the range of 100 to 140°F depending on the strength of acid employed, mechanical design of the equipment, etc. Removal of the spent sulfuric acid from the sulfonic acid is facilitated by adding water to reduce the sulfuric acid strength to about 78 percent. This dilution prior to neutralization results in a final neutralized slurry having approximately 85 percent active agent based on the solids. The inert material in the final product is essentially Na_2SO_4.

The sulfonic acid is neutralized with 20 to 50 percent caustic soda solution to a pH of 8 at a temperature of about 125°F. Chemical "builders" such as trisodium phosphate, tetrasodium pyrophosphate, sodium silicate, sodium chloride, sodium sulfate, carboxymethyl cellulose, etc., are added to enhance the detersive, wetting, or other desired properties in the finished product. A flaked, dried product is obtained by drum drying or a bead product is obtained by spray drying.

The basic reactions which occur in the process are the following.

Alkylation:

$$C_6H_6 + C_{12}H_{24} \xrightarrow{AlCl_3} C_6H_5 \cdot C_{12}H_{25}$$

Sulfonation:

$$C_6H_5 \cdot C_{12}H_{25} + H_2SO_4 \longrightarrow C_{12}H_{25} \cdot C_6H_4 \cdot SO_3H + H_2O$$

Neutralization:

$$C_{12}H_{25} \cdot C_6H_4 \cdot SO_3H + NaOH \longrightarrow C_{12}H_{25} \cdot C_6H_4 \cdot SO_3Na + H_2O$$

A literature search indicates that yields of 85 to 95 percent have been obtained in the alkylation step, while yields for the sulfonation process are substantially 100 percent, and yields for the neutralization step are always 95 percent or greater. All three steps are exothermic and require some form of jacketed cooling around the stirred reactor to maintain isothermal reaction temperatures.

Laboratory data for the sulfonation of dodecylbenzene, described in the literature, provide additional information useful for a rapid material balance. This is summarized as follows:

1. Sulfonation is essentially complete if the ratio of 20 percent oleum to dodecylbenzene is maintained at 1.25.
2. Spent sulfuric acid removal is optimized with the addition of 0.244 lb of water to the settler for each 1.25 lb of 20 percent oleum added in the sulfonation step.
3. A 25 percent excess of 20 percent NaOH is suggested for the neutralization step.

Operating conditions for this process, as reported in the literature, vary somewhat depending upon the particular processing procedure chosen.

Material and Energy Balance

The process selected for the manufacture of the nonbiodegradable detergent is essentially continuous even though the alkylation, sulfonation, and neutralization steps are semicontinuous steps. Provisions for possible shutdowns for repairs and maintenance are incorporated into the design of the process by

specifying plant operation for 300 calendar days per year. Assuming 90 percent yield in the alkylator and a sodium dodecylbenzene sulfonate product to be 85 percent active with 15 percent sodium sulfate as inert, the overall material balance is as follows:
Input components:

$$\text{Product (85\% active)} = \frac{(15 \times 10^6)(0.85)}{(300)(348.5)} = 122 \text{ lb mol/day}$$

$$C_6H_6 \text{ feed} = (122)\left(\frac{1}{0.95}\right)\left(\frac{1}{0.90}\right) = 142.7 \text{ lb mol/day}$$

$$= (142.7X78.1) = 11{,}145 \text{ lb/day}$$

$$C_{12}H_{24} \text{ feed} = 142.7 \text{ lb mol/day}$$

$$= (142.7X168.3) = 24{,}016 \text{ lb/day}$$

20% oleum in = (1.25X11,145 + 24,016) = 43,951 lb/day

Dilution H_2O in = (0.244/1.25X43,951) = 8579 lb/day

20% NaOH in = (1.25X43,951) = 55,085 lb/day

$AlCl_3$ catalyst in = (0.05X11,145 + 24,016) = 1758 lb/day

Alkylation process:

Alkylate yield = (0.9X142.7X246.4) = 31,645 lb/day
Unreacted C_6H_6 = (0.1X11,145) = 1114 lb/day
Unreacted $C_{12}H_{24}$ = (0.1X24,016) = 2402 lb/day

Sulfur balance:

Sulfur in = (43,951X1.045X32.1/98.1) = 15,029 lb/day
Sulfur out = sulfur in detergent + sulfur in spent acid

$$\text{Sulfur in detergent} = \frac{(50{,}000)(0.85)(32.1)}{(348.5)} + \frac{(50{,}000)(0.15)(32.1)}{(142)}$$

$$= 3915 + 1695 = 5610 \text{ lb/day}$$

Sulfur out in acid = 15,029 − 5610 = 9419 lb/day

$$\text{Weight of 78\% } H_2SO_4 = (9419)\left(\frac{98.1}{32.1}\right)\left(\frac{1}{0.78}\right) = 36{,}861 \text{ lb/day}$$

The weight of the heavy alkylaryl hydrocarbon is obtained by difference as 3516 lb/day.

The material balance summary made by the design group for the process shown in Fig. 2-3 is given on a daily basis in Fig. 2-4. After a complete material balance is made, the mass quantities are used to compute energy balances

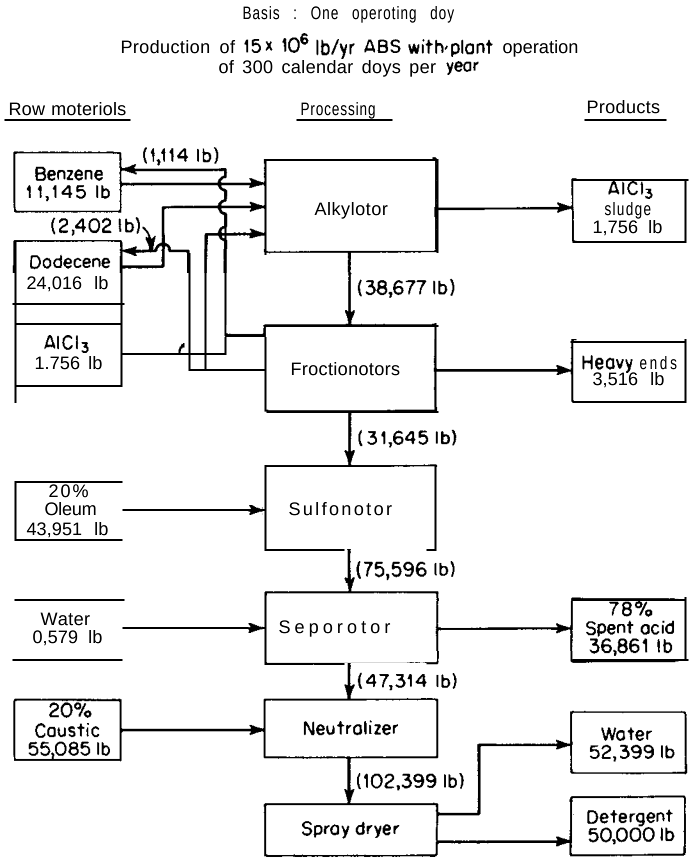

FIGURE 2-4
Quantitative flow diagram for the manufacture of sodium dodecylbenzene sulfonate.

around each piece of equipment. Temperature and pressure levels at various key points in the process, particularly at the reactors, serve as guides in making these heat balances. The complete calculations for the material and energy balances for each piece of equipment, because of their length, are not presented in this discussion.

Equipment Design and Selection

Equipment design for this preliminary process evaluation involves determining the size of the equipment in terms of the volume, flow per unit time, or surface area. Some of the calculations associated with the alkylation unit are presented in the following to indicate the extent of the calculations which are sometimes adequate for a preliminary design.

ALKYLATION UNIT EQUIPMENT DESIGN AND SELECTION

Reactor Volume

Assume a **4-h** cycle and operation of the alkylator at constant temperature and pressure of 115°F and 1 atm, respectively. The volume of reactants per day (with a 10% safety factor) is

$$V_r = \frac{(12,259)}{(8.34)(0.88)} + \frac{(26,418)}{(8.34)(0.7533)} + \frac{(1758)(7.48)}{(2.44)(62.4)}$$

$$= 1670 + 4160 + 86 = 5916 \text{ gal/day}$$

$$= \frac{5916}{6} = 986 \text{ gal/cycle}$$

If the reactor is 75 percent full on each cycle, the volume of reactor needed is

$$V_R = \frac{986}{0.75} = 1315 \text{ gal}$$

Select a **1300-gal,** glass-lined, stirred reactor.

HEAT OF REACTION CALCULATION

$$C_6H_{6(l)} + C_{12}H_{24(l)} \rightarrow C_6H_5 \,.\, C_{12}H_{25(l)}$$

$$\Delta H_r = \Delta H_{f(C_6H_5 \cdot C_{12}H_{25})l} - \Delta H_{f(C_6H_6)l} - \Delta H_{f(C_{12}H_{24})l}$$

The heats of formation ΔH_f of dodecylbenzene and dodecene are evaluated using standard thermochemistry techniques outlined in most chemical engineering thermodynamic texts, The heat formation of benzene is available in the literature.

$$\Delta H_{f(C_6H_5 \cdot C_{12}H_{25})l} = -54,348 \text{ cal/g mol}$$

$$\Delta H_{f(C_{12}H_{24})l} = -51,239 \text{ cal/g mol}$$

$$\Delta H_{f(C_6H_6)l} = 11,717 \text{ cal/g mol}$$

Thus,

$$\text{AH,} = -54,348 - 11,717 + 51,239 = -14,826 \text{ cal/g mol}$$

$$= -26,687 \text{ Btu/lb mol}$$

Assume heat of reaction is liberated in 3 h of the **4-h** cycle ($\frac{1}{6}$ of an operating day):

$$Q_r = \left(\frac{11,145}{78.1}\right)(-26,687)\left(\frac{1}{6}\right)\left(\frac{1}{3}\right) = -211,500 \text{ Btu/h}$$

Use a 10°F temperature difference for the cooling water to find the mass of cooling water required to remove the heat of reaction.

$$m_{H_2O} = \frac{Q_r}{C_p\ \Delta T} = \frac{211{,}500}{(1)(10)} = 21{,}150\ \text{lb/h}$$

$$q_{f(H_2O)} = \frac{21{,}150}{(60)(8.33)} = 42.3\ \text{gpm}$$

The volumetric flow rate is, therefore, 42.3 gpm. Select a 45-gpm centrifugal pump, carbon steel construction.

HEAT TRANSFER AREA NEEDED TO COOL REACTOR Assume water inlet of 80°F with a 10°F temperature rise. A reasonable overall heat transfer coefficient for this type of heat transfer may be calculated as 45 Btu/(h)(ft²)(°F).

$$\Delta T_{lm} = \frac{(115 - 80) - (115 - 90)}{2.303 \log \frac{35}{25}} = 29\ 7°\text{F}$$

$$A = \frac{0}{U \Delta T_{lm}} = \frac{211{,}500}{(45)(29.7)} = 158\ \text{ft}^2$$

A 1300-gal stirred reactor has approximately 160 ft² of jacket area. Therefore, the surface area available is sufficient to maintain isothermal conditions in the reactor.

SIZING OF STORAGE TANKS. Provide benzene and dodecene storage for six days:

$$V_{\text{benzene}} = (1670)(6) = 10{,}020\ \text{gal}$$

$$V_{\text{dodecene}} = (4160)(6) = 24{,}960\ \text{gal}$$

Select a 10,000-gal carbon steel tank for benzene storage and a 25,000-gal carbon steel tank for dodecene storage.

Provide holding tank storage for one day:

$$V_{\text{holding}} = 5918\ \text{gal}$$

Select a 6000-gal carbon steel tank for holding tank.

SIZING OTHER PUMPS. Provide benzene and dodecene filling of reactor in 10 min:

$$q_{f(\text{benzene})} = \frac{1670}{(6)(10)} = 27.8\ \text{gpm}$$

Select a 30-gpm centrifugal pump, carbon steel construction.

$$q_{f(\text{dodecene})} = \frac{4160}{(6)(10)} = 69.3\ \text{gpm}$$

TABLE 1
Equipment specifications for alkylation unit†

No. req'd.	Item and description	Size	Mat'l. const.
1	T-l, storage tank for benzene	10,000 gal	Carbon steel
1	T-Z, storage tank for dodecene	25,000 gal	Carbon steel
1	T-3, holding tank for alkylate	6,000 gal	Carbon steel
1	P-l, pump (centrifugal) for benzene transfer from T-l to R-l	30 gpm (up to 50 psi)	Carbon steel
1	P-2, pump (centrifugal) for dodecene transfer from T-2 to R-l	70 gpm (up to 50 psi)	Carbon steel
1	P-3, pump (centrifugal) for pumping cooling water to jacket of R-l	45 gpm (up to 50 psi)	Carbon steel
1	P-4, pump (positive displacement) for alkylate transfer from T-3 to C-l	10 gpm (150 psi)	Cast iron
1	R-l, reactor (stirred) alkylator	1,300 gal	Glass-lined

†See Fig. 2-5.

Select a 70-gpm centrifugal pump, carbon steel construction. The alkylate pump used to transfer alkylate from the holding tank to the benzene fractionator must operate continuously. Thus,

$$q_{f(\text{alkylate})} = \frac{(1670 + 4160)}{(24)(60)} = 4 \text{ gpm}$$

Select a 10-gpm positive displacement pump, carbon steel construction. A summary of the equipment needs for the alkylation unit in this preliminary process design is presented in Table 1. The preparation of similar equipment lists for the other process units completes the equipment selection and design phase of the preliminary design. Figure 2-5 shows a simplified equipment diagram for the proposed process and includes the specified size or capacity of each piece of process equipment.

Economics

The purchased cost of each piece of process equipment may now be estimated from published cost data or from appropriate manufacturers' bulletins. Regardless of the source, the published purchased-cost data must always be corrected to the current cost index. This procedure is described in detail in Chap. 6.

For the alkylation unit, purchased-equipment costs may be estimated using the equipment-specification information of Table 1 and the cost data presented in Chaps. 14 through 16 of this text. Table 2 presents these costs updated to January 1, 1990. The required fixed-capital investment for the nonbiodegradable detergent manufacturing process may be estimated from the total purchased-equipment cost using the equipment-cost ratio method outlined in Table 17 of Chap. 6. The total purchased-equipment cost is, presented in

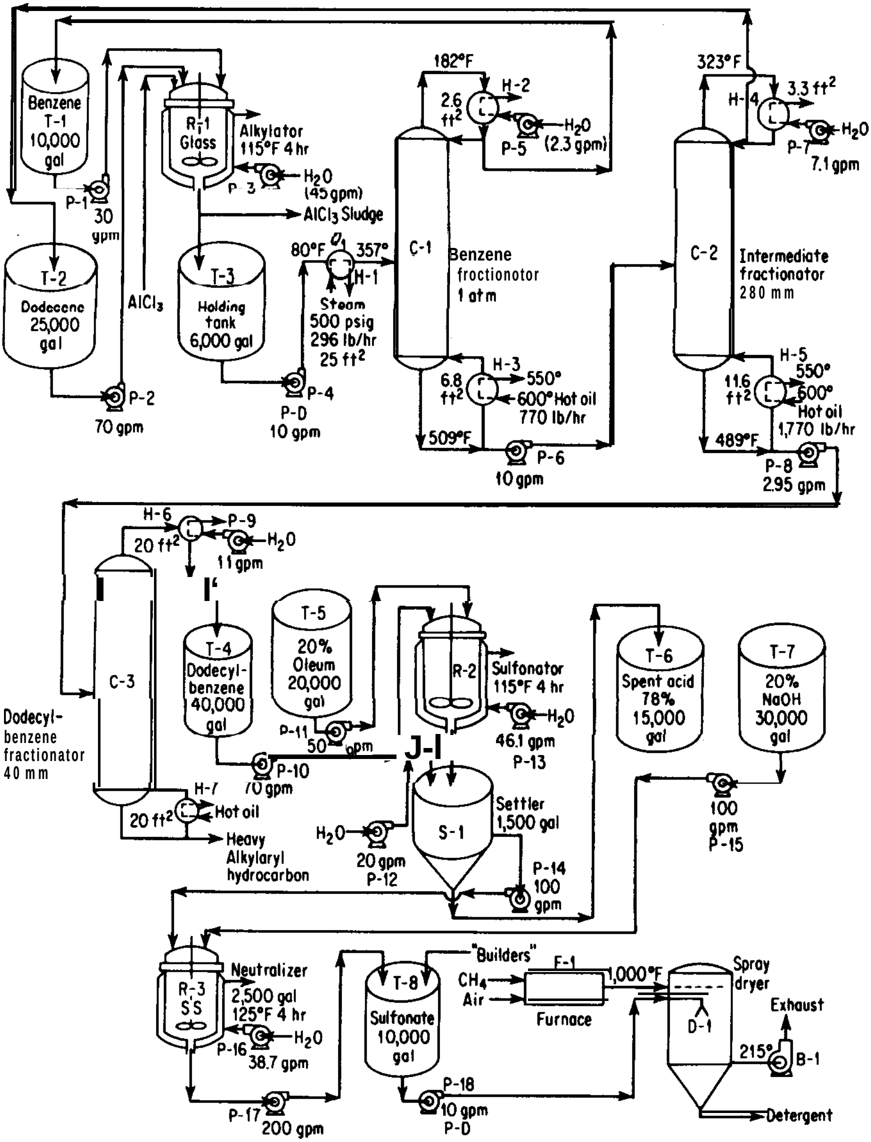

FIGURE 2-5

Simplified equipment diagram for the manufacture of sodium dodecylbenzene sulfonate.

TABLE 2
Estimated purchased-equipment cost for alkylation unit†

Designation	Item	Purchased cost
T-l	Storage tank	$ 21,800
T-2	Storage tank	36,700
T-3	Holding tank	16,200
P-l	Centrifugal pump (with motor)	1,500
P-2	Centrifugal pump (with motor)	1,700
P-3	Centrifugal pump (with motor)	1,600
P-4	Positive-displacement pump	6,200
R-l	Jacketed (stirred) reactor	58,400
		$144,100

†January 1, 1990 costs. See Fig. 2-5.

TABLE 3
Summary of purchased-equipment cost for complete process unit

Process unit	Purchased cost
Alkylation	$ 144,100
Fractionators	175,800
Sulfonation	245,100
Neutralization	163,700
Spray dryer	393,500
Auxiliary units	142,800
Total	$1,165,000

Table 3 and is the basis for the estimated fixed-capital cost tabulation given in Table 4. The probable error in this method of estimating the fixed-capital investment is as much as ±30 percent.

An evaluation of the operating labor and utilities requirements of the process must be made before the total product cost can be estimated. Details for evaluating these direct production costs are given in Chap. 6 and Appendix B. The estimate of the total product cost for the manufacture of 15 million lb per year detergent, based on methods outlined in Chap. 6, is presented in Table 5.

Once the total product cost has been estimated, the design group is in a position to evaluate for management the attractiveness of the proposed process using such measures of profitability as rate of return, payout time, or present worth. These methods are fully outlined in Chap. 10. The design report, as mentioned previously, completes the preliminary design.

TABLE 4
Fixed-capital investment estimate†

Items	Cost
Purchased equipment	**$1,165,000**
Purchased-equipment installation	547,600
Instrumentation and controls	209,700
Piping (installed)	768,900
Electrical (installed)	128,200
Buildings (including services)	209,700
Yard improvements	116,500
Service facilities (installed)	815,500
Land (purchase not required)	
Engineering and supervision	384,500
Construction expenses	477,700
Contractor's fee	244,700
Contingency	489,300
Fixed-capital investment	**$5,557,300**
Working capital	**1,001,900**
Total capital investment	**$6,559,200**

†**Equipment-cost** ratio percentages used in Table 4 are factors applicable to a fluid-processing plant as outlined in Chap. 6.

TABLE 5
Total product cost estimate

Items	cost
Direct production costs	
Raw materials	**$2,512,200**
Operating labor	963,500
Direct supervisory and clerical labor	192,700
Utilities	567,700
Maintenance and repairs	111,100
Operating supplies	16,700
Fixed charges	
Depreciation	555,700
Local taxes	111,100
Insurance	55,600
Plant-overhead costs	760,400
General expenses	
Administration	190,100
Distribution and selling	771,900
Research and development	385,900
Financing (interest)	524,600
Annual total product cost	**$7,719,200**
Total product cost per pound	$0.515

Summary

The preliminary design presented in this section was developed to show the logical step-by-step approach which is quite often followed for each new process design. The exact procedure may vary from company to company and from one design engineer to another. Likewise, the assumptions and rule-of-thumb factors used may vary from one company to the next depending to a large extent on design experience and company policy. Nevertheless, the basic steps for a process design are those outlined in this preliminary design covering the manufacture of a common household item.

No attempt has been made to present a complete design. In fact, to minimize the length, many assumptions were made which would have been verified or justified in a normal process design. Neither were any alternative solutions considered even though some were suggested by the literature survey. The investigation of these various alternatives is left to the reader.

COMPARISON OF DIFFERENT PROCESSES

In a course of a design project it is necessary to determine the most suitable process for obtaining a desired product. Several different manufacturing methods may be available for making the same material, and various processes must be compared in order to select the one best suited to the existing conditions.

The comparison can be accomplished through the development of complete designs. In many cases, however, all but one or two of the possible processes can be eliminated by a weighted comparison of the essential variable items, and detailed design calculations for each process may not be required. The following items should be considered in a comparison of this type:

1. Technical factors
    - a. Process flexibility
    - *b*. Continuous operation
    - c. Special controls involved
    - *d*. Commercial yields
    - e. Technical difficulties involved
    - *f*. Energy requirements
    - g. Special auxiliaries required
    - *h*. Possibility of future developments
    - *i*. Health and safety hazards involved
2. Raw materials
    - a. Present and future availability
    - *b*. Processing required
    - c. Storage requirements
    - *d*. Materials handling problems
3. Waste products and by-products
    - *a*. Amount produced
    - *b*. Value

  - c. Potential markets and uses
  - *d.* Manner of discard
  - e. Environmental aspects
- 4. Equipment
  - a. Availability
  - *b.* Materials of construction
  - c. Initial costs
  - *d.* Maintenance and installation costs
  - e. Replacement requirements
  - *f.* Special designs
- 5. Plant location
  - *a.* Amount of land required
  - *b.* Transportation facilities
  - c. Proximity to markets and raw-material sources
  - *d.* Availability of service and power facilities
  - e. Availability of labor
  - *f.* Climate
  - g. Legal restrictions and taxes
- 6. Costs
  - *a.* Raw materials
  - *b.* Energy
  - c. Depreciation
  - *d.* Other fixed charges
  - e. Processing and overhead
  - *f.* Special labor requirements
  - g. Real estate
  - *h.* Patent rights
  - *i.* Environmental controls
- 7. Time factor
  - a. Project completion deadline
  - *b.* Process development required
  - c. Market timeliness
  - *d.* Value of money
- 8. Process considerations
  - a. Technology availability
  - *b.* Raw materials common with other processes
  - c. Consistency of product within company
  - *d.* General company objectives

Batch Versus Continuous Operation

When comparing different processes, consideration should always be given to the advantages of continuous operation over batch operation. In many cases, costs can be reduced by using continuous instead of batch processes. Less labor is required, and control of the equipment and grade of final product is simplified. Whereas batch operation was common in the early days of the chemical industry, most processes have been switched completely or partially to continuous operation. The advent of many new types of control instruments has made this transition possible, and the design engineer should be aware of the advantages inherent in any type of continuous operation.

EQUIPMENT DESIGN AND SPECIFICATIONS

The goal of a “plant design” is to develop and present a complete plant that can operate on an effective industrial basis. To achieve this goal, the chemical engineer must be able to combine many separate units or pieces of equipment into one smoothly operating plant. If the final plant is to be successful, each piece of equipment must be capable of performing its necessary function. The design of equipment, therefore, is an essential part of a plant design.

The engineer developing a process design must accept the responsibility of preparing the specifications for individual pieces of equipment and should be acquainted with methods for fabricating different types of equipment. The importance of choosing appropriate materials of construction in this fabrication must be recognized. Design data must be developed, giving sixes, operating conditions, number and location of openings, types of flanges and heads, codes, variation allowances, and other information. Many of the machine-design details are handled by the fabricators, but the chemical engineer must supply the basic design information.

SCALE-UP IN DESIGN

When accurate data are not available in the literature or when past experience does not give an adequate design basis, pilot-plant tests may be necessary in order to design effective plant equipment. The results of these tests must be scaled up to the plant capacity. A chemical engineer, therefore, should be acquainted with the limitations of scale-up methods and should know how to select the essential design variables.

Pilot-plant data are almost always required for the design of filters unless specific information is already available for the type of materials and conditions involved. Heat exchangers, distillation columns, pumps, and many other types of conventional equipment can usually be designed adequately without using pilot-plant data.

Table 6 presents an analysis of important factors in the design of different types of equipment.? This table shows the major variables that characterize the

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†Adapted from Johnstone, R. E., and M. W. Thring, “Pilot Plants, Models, and Scale-up Methods,” McGraw-Hill Book Company, New York, 1957. See also Bisio, A., and R. L Kabel, “Scaleup of Chemical Processes,” J. Wiley & Sons, New York, 1985.

TABLE 6
Factors in equipment scale-up and design

Type of equipment	Is pilot plant usually necessary?	Major variables for operational design (other than flow rate)	Major variables characterizing size or capacity	Maximum scale-up ratio based on indicated characterizing variable	Approximate recommended safety or over-design factor, %
Agitated batch crystallizers	Yes	Solubility-temperature relationship	Flow rate Heat transfer area	>100:1	20
Batch reactors	Yes	Reaction rate Equilibrium state	Volume Residence time	>100:1	20
Centrifugal pumps	No	Discharge head	Flow rate Power input Impeller diameter	>100:1 >100:1 10:1	10
Continuous reactors	Yes	Reaction rate Equilibrium state	Flow rate Residence time	>100:1	20
Cooling towers	No	Air humidity Temperature decrease	Flow rate Volume	>100:1 10:1	15
Cyclones	No	Particle size	Flow rate Diameter of body	10:1 3:1	10
Evaporators	No	Latent heat of vaporization Temperatures	Flow rate Heat-transfer area	>100:1 >100:1	15
Hammer mills	Yes	Size reduction	Flow rate Power input	60:1 60:1	20
Mixers	No	Mechanism of operation System geometry	Flow rate Power input	>100:1 20:1	20
Nozzle-discharge centrifuges	Yes	Discharge method	Flow rate Power input	10:1 10:1	20 20

(continued)

TABLE 6
Factors in equipment scale-up and design (Continued)

Type of equipment	Is pilot plant usually necessary?	Major variables for operational design (other than flow rate)	Major variables characterizing size or capacity	Maximum scale-up ratio based on indicated characterizing variable	Approximate recommended safety or overdesign factor, %
Packed columns	No	Equilibrium data Superficial vapor velocity	Flow rate Diameter Height to diameter ratio	>100:1 10:1	15
Plate columns	No	Equilibrium data Superficial vapor velocity	Flow rate Diameter	>100:1 10:1	15
Plate-and-frame filters	Yes	Cake resistance or permeability	Flow rate Filtration area	>100:1 >100:1	20
Reboilers	No	Temperatures Viscosities	Flow rate Heat-transfer area	>100:1 >100:1	15
Reciprocating compressors	No	Compression ratio	Flow rate Power input Piston displacement	>100:1 >100:1 >100:1	10
Rotary filters	Yes	Cake resistance or permeability	Flow rate Filtration area	>100:1 25:1	20
Screw conveyors	No	Bulk density	Flow rate Diameter Drive horsepower	90:1 8:1	20
Screw extruders	No	Shear rate	Flow rate Power input	100: 1 100: 1	20 10
Sedimentation centrifuges	No	Discharge method	Flow rate Power input	10:1 10:1	20 20
Settlers	No	Settling velocity	Volume Residence time	>100:1	15
Spray columns	No	Gas solubilities	Flow rate Power input	10:1	20

TABLE 6
Factors in equipment scale-up and design (*Continued*)

Type of equipment	Is pilot plant usually neces-sary?	Major variables for operational design (other than flow rate)	Major variables characterizing size or capacity	Maximum scale-up ratio based on indi-cated char-acterizing variable	Approxi-mate recom-mended safety or over-design factor, %
Spray condensers	No	Latent heat of vaporization Temperatures	Flow rate Height to diameter ratio	70:1 12:1	20
Tube-and-shell heat exchangers	No	Temperatures Viscosities Thermal conductivities	Flow rate Heat-transfer area	>100:1 >100:1	15

size or capacity of the equipment and the maximum scale-up ratios for these variables. Information on the need for pilot-plant data, safety factors, and essential operational data for the design is included in Table 6.

SAFETY FACTORS

Some examples of recommended safety factors for equipment design are shown in Table 6. These factors represent the amount of overdesign that would be used to account for the changes in the operating performance with time.

The indiscriminate application of safety factors can be very detrimental to a design. Each piece of equipment should be designed to carry out its necessary function. Then, if uncertainties are involved, a reasonable safety factor can be applied. The role of the particular piece of equipment in the overall operation must be considered along with the consequences of underdesign. Fouling, which may occur during operation, should never be overlooked when a design safety factor is determined. Potential increases in capacity requirements are sometimes used as an excuse for applying large safety factors. This practice, however, can result in so much overdesign that the process or equipment never has an opportunity to prove its economic value.

In general design work, the magnitudes of safety factors are dictated by economic or market considerations, the accuracy of the design data and calculations, potential changes in the operating performance, background information available on the overall process, and the amount of conservatism used in developing the individual components of the design. Each safety factor must be chosen on basis of the existing conditions, and the chemical engineer should not hesitate to use a safety factor of zero if the situation warrants it.

SPECIFICATIONS

A generalization for equipment design is that standard equipment should be selected whenever possible. If the equipment is standard, the manufacturer may have the desired size in stock. In any case, the manufacturer can usually quote a lower price and give better guarantees for standard equipment than for special equipment.

The chemical engineer cannot be an expert on all the types of equipment used in industrial plants and, therefore, should make good use of the experience of others. Much valuable information can be obtained from equipment manufacturers who specialize in particular types of equipment.

Before a manufacturer is contacted, the engineer should evaluate the design needs and prepare a preliminary specification sheet for the equipment. This preliminary specification sheet can be used by the engineer as a basis for the preparation of the final specifications, or it can be sent to a manufacturer with a request for suggestions and fabrication information. Preliminary specifications for equipment should show the following:

1. Identification
2. Function
3. Operation
4. Materials handled
5. Basic design data
6. Essential controls
7. Insulation requirements
8. Allowable tolerances
9. Special information and details pertinent to the particular equipment, such as materials of construction including gaskets, installation, necessary delivery date, supports, and special design details or comments

Final specifications can be prepared by the engineer; however, care must be exercised to avoid unnecessary restrictions. The engineer should allow the potential manufacturers or fabricators to make suggestions before preparing detailed specifications. In this way, the final design can include small changes that reduce the first cost with no decrease in the effectiveness of the equipment. For example, the tubes in standard heat exchangers are usually 8, 12, 16, or 20 ft long, and these lengths are ordinarily kept in stock by manufacturers and maintenance departments. If a design specification called for tubes 15 ft long, the manufacturer would probably use 16-ft tubes cut off to the specified length.

HEAT EXCHANGER

Identification: Item Condenser Item No. *H-S* No. required I	Date I-I-90 By *JRL*

Function: Condense overhead *vapors from methanol fractionation column*

Operation: *Continuous*

Type: *Horizontal*
Fixed *tube sheet*
Expansion ring in shell
Duty *3,400,000* Btu/h Outside area *470* sq ft

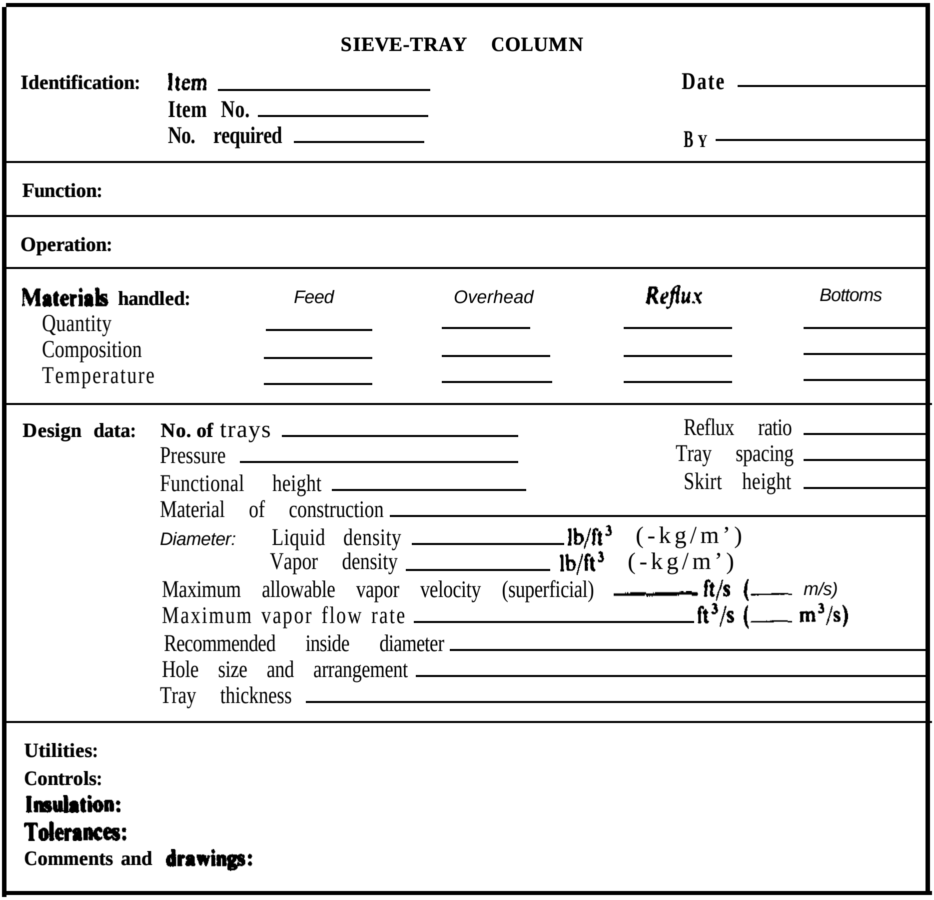
Tube side: Fluid handled *Cooling water* Flow rate 380 *gpm* Pressure 20 psig Temperature *15°C* to 25°C Head material Carbon steel	**Tubes:** *1* in. diam. *14* BWG *1.25″* Centers A Pattern *225* Tubes each 8 ft long *2* Passes Tube material *Carbon steel*
Shell aide: Fluid handled Methanol vapor Flow rate *7000 lb/h* Pressure 0 psig Temperature 65°C to *(constant temp.)*	**Shell:** *22* in. diam. I Passes (Transverse baffles Tube *support* Req’d) (Longitudinal baffles 0 Req’d) Shell material *Carbon steel*

Utilities: *Untreated cooling water*
Controls: *Cooling-water* rare *controlled by vapor temperature in vent* line
Insulation: *2-in. rock cork or equivalent; weatherproofed*
Tolerances: *Tubular Exchangers Manufacturers* Association *(TEMA) standards*
Comments **and drawings:** Location and sizes *of inlets* and *outlets are shown* on drawing

FIGURE 2-6
Specification sheet for heat exchangers using U.S. customary units.

Thus, an increase from 15 to 16 ft for the specified tube length could cause a reduction in the total cost for the unit, because the labor charge for cutting the standard-length tubes would be eliminated. In addition, replacement of tubes might become necessary after the heat exchanger has been in use, and the replacement costs with 16-ft tubes would probably be less than with 15-ft tubes.

Figures 2-6 and 2-7 show typical types of specification sheets for equipment. These sheets apply for the normal type of equipment encountered by a chemical engineer in design work. The details of mechanical design, such as shell or head thicknesses, are not included, since they do not have a direct effect on the performance of the equipment. However, for certain types of of equipment involving unusual or extreme operating conditions, the engineer may need



SIEVE-TRAY COLUMN

Identification: Item ____________
Item No. ____________
No. required ____________
Date ____________
By ____________

Function:

Operation:

Materials handled:	*Feed*	*Overhead*	*Reflux*	*Bottoms*
Quantity	____	____	____	____
Composition	____	____	____	____
Temperature	____	____	____	____

Design data: **No. of** trays ____________
Pressure ____________
Functional height ____________
Reflux ratio ____________
Tray spacing ____________
Skirt height ____________
Material of construction ____________
Diameter: Liquid density ____________ $\mathrm{lb/ft^3}$ (-kg/m’)
Vapor density ____________ $\mathrm{lb/ft^3}$ (-kg/m’)
Maximum allowable vapor velocity (superficial) ______ ft/s (____ *m/s*)
Maximum vapor flow rate ____________ $\mathrm{ft^3/s}$ (____ $\mathrm{m^3/s}$)
Recommended inside diameter ____________
Hole size and arrangement ____________
Tray thickness ____________

Utilities:
Controls:
Insulation:
Tolerances:
Comments and drawings:

FIGURE 2-7
Specification sheet for sieve-tray distillation column.

to extend the specifications to include additional details of the mechanical design. Locations and sizes of outlets, supports, and other essential fabrication information can be presented with the specifications in the form of comments or drawings.

MATERIALS OF CONSTRUCTION

The effects of corrosion and erosion must be considered in the design of chemical plants and equipment. Chemical resistance and physical properties of constructional materials, therefore, are important factors in the choice and design of equipment. The materials of construction may be resistant to the corrosive action of any chemicals that may contact the exposed surfaces. Possible erosion caused by flowing fluids or other types of moving substances must be considered, even though the materials of construction may have adequate chemical resistance. Structural strength, resistance to physical or thermal shock, cost, ease of fabrication, necessary maintenance, and general type of service required, including operating temperatures and pressures, are additional factors that influence the final choice of constructional materials.

If there is any doubt concerning suitable materials for construction of equipment, reference should be made to the literature,† or laboratory tests should be carried out under conditions similar to the final operating conditions. The results from the laboratory tests indicate the corrosion resistance of the material and also the effects on the product caused by contact with the particular material. Further tests on a pilot-plant scale may be desirable in order to determine the amount of erosion resistance or the effects of other operational factors.

PROBLEMS

1. Using *Chemical Abstracts* as a basis, list the original source, title, author, and brief abstract of three published articles dealing with three different processes for producing formaldehyde.
2. Prepare, in the form of a flow sheet, an outline showing the sequence of steps in the complete development of a plant for producing formaldehyde. A detailed analysis of the points to be considered at each step should be included. The outline should take the project from the initial idea to the stage where the plant is in efficient operation.
3. A process for making a single product involves reacting two liquids in a continuously agitated reactor and distilling the resulting mixture. Unused reactants are recovered as overhead and are recycled. The product is obtained in sufficiently pure form as bottoms from the distillation tower.
   (*a*) Prepare a qualitative flow sheet for the process, showing all pieces of equipment.
   (*b*) With cross reference to the qualitative flow sheet, list each piece of equipment and tabulate for each the information needed concerning chemicals and the process in order to design the equipment.
4. Figure 2-1 presents a qualitative flow diagram for the manufacture of nitric acid by the ammonia-oxidation process. Figure 2-2 presents a quantitative flow diagram for the same process. With the information from these two figures, prepare a quantitative energy balance for the process and size the equipment in sufficient detail for a preliminary cost estimate.
5. A search of the literature reveals many different processes for the production of acetylene. Select four different processes, prepare qualitative flow sheets for each, and discuss the essential differences between each process. When would one process be more desirable than the others? What are the main design problems which would

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†Detailed information on materials of construction is presented in Chap. 12.

require additional information? What approximations would be necessary if data are not available to resolve these questions?

6. Ethylene is produced commercially in a variety of different processes. Feed stocks for these various processes range from refinery gas, ethane, propane, butane, natural gasoline, light and heavy naphthas to gas and oil and heavier fractions. Prepare three different qualitative flow sheets to handle a majority of these feed stocks. What are the advantages and disadvantages of each selected process?
7. Gather all the available information on one of the ethylene processes for which a flow sheet was prepared in the preceding problem and make a preliminary material balance for the production of 50 million lb/yr of ethylene. Assume an operating factor of 90 percent.
8. One method of preparing acetaldehyde is by the direct oxidation of ethylene. The process employs a catalytic solution of copper chloride containing small quantities of palladium chloride. The reactions may be summarized as follows:

$$C_2H_4 + 2CuCl_2 + H_2O \xrightarrow{PdCl_2} CH_3CHO + 2HCl + 2CuCl$$

$$2CuCl + 2HCl + \tfrac{1}{2}O_2 \longrightarrow 2CuCl_2 + H_2O$$

In the reaction, $PdCl_2$ is reduced to elemental palladium and HCl, and is reoxidized by $CuCl_2$. During catalyst regeneration the CuCl is reoxidized with oxygen. The reaction and regeneration steps can be conducted separately or together.

In the process, 99.8 percent ethylene, 99.5 percent oxygen, and recycle gas are directed to a vertical reactor and are contacted with the catalyst solution under slight pressure. The water evaporated during the reaction absorbs the exothermic heat evolved, and make-up water is fed as necessary to maintain the catalytic solution concentration. The reacted gases are water-scrubbed and the resulting acetaldehyde solution is fed to a distillation column. The tail gas from the scrubber is recycled to the reactor. Inerts are eliminated from the recycle gas in a bleed stream which flows to an auxiliary reactor for additional ethylene conversion.

Prepare, in the form of a flow sheet, the sequence of steps in the development of a plant to produce acetaldehyde by this process. An analysis of the points to be considered at each step should be included. List the additional information that will be needed to complete the preliminary design evaluation.

9. Prepare a simplified equipment flow sheet for the acetaldehyde process outlined in Prob. 8. Identify temperature, pressure, and composition, wherever possible, at each piece of equipment.
10. Prepare a material balance and a qualitative flow sheet for the production of 7800 kg/h of acetaldehyde using the process described in the previous problem. Assume an operating factor of 90 percent and a 95 percent yield on the ethylene feed. Both ethylene and oxygen enter the process at 930 kPa.
11. Using the information developed in Prob. 10, make a basic energy balance around each piece of equipment and for the entire process. Prepare, a quantitative flow sheet to outline the results of the basic energy balance.
12. Prepare a material balance for the production of 7800 kg/h of acetaldehyde using the process described in Prob. 8. However, because 99.5 percent oxygen is unavailable, it will be necessary to use 830 kPa air as one of the raw materials. What steps of the process will be affected by this substitution in feed stocks? Assume an operating factor of 90 percent and a 95 percent yield on the ethylene feed.

13. Synthesis gas may be prepared by a continuous, noncatalytic conversion of any hydrocarbon by means of controlled partial combustion in a fire-brick lined reactor. In the basic form of this process, the hydrocarbon and oxidant (oxygen or air) are separately preheated and charged to the reactor. Before entering the reaction zone, the two feed stocks are intimately mixed in a combustion chamber. The heat produced by combustion of part. of the hydrocarbon pyrolyzes the remaining hydrocarbons into gas and a small amount of carbon in the reaction zone. The reactor effluent then passes through a waste-heat boiler, a water-wash carbon-removal unit, and a water cooler-scrubber. Carbon is recovered in equipment of simple design in a form which can be used as fuel or in ordinary carbon products.

    Prepare a simplified equipment flow sheet for the process, with temperatures and pressure conditions at each piece of equipment.

14. Make a material balance and a qualitative flow sheet for the synthesis gas process described in Prob. 13. Assume an operating factor of 95 percent and a feed stock with an analysis of 84.6 percent C, 11.3 percent H_2, 3.5 percent S, 0.13 percent O_2, 0.4 percent N_2, and 0.07 percent ash (all on a weight basis). The oxidant in this process will be oxygen having a purity of 95 percent. Production is to be 8.2 m^3/s.

15. Prepare an energy balance and a suitable flow sheet for the synthesis gas production requested in Prob. 14.

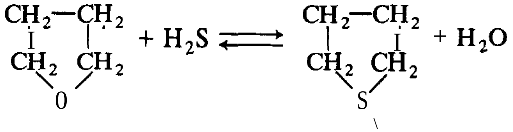
16. Size the equipment that is necessary for the synthesis gas production outlined in Probs. 13 and 14.

17. Estimate the required utilities for the synthesis gas plant described in the previous four problems.

18. Repeat the calculations of Probs. 14 to 17 by substituting air as the oxidant in place of the 95 percent purity oxygen.

19. In the face of world food shortages accompanying an exploding world population, many engineers have suggested that the world look to crude oil as a new source of food. Explore this possibility and prepare a flow sheet which utilizes the conversion of petroleum to food by organic microorganisms. What are the problems that must be overcome to make this possibility an economic reality?

20. A chemical engineering consultant for a large refinery complex has been asked to investigate the feasibility of manufacturing 1.44 x 10^{-2} kg/s of thiophane, an **odorant** made from a combination of tetrahydrofuran (THF) and hydragen sulfide. The essential reaction is given below:



The process consists essentially of the following steps:

(*a*) THF is vaporized and mixed with H_2S in a ratio of 1.5 moles H_2S to one mole of THF and reacted over an alumina catalyst at an average temperature of 672 K and 207 kPa.

(*b*) Reactor vapors are cooled to 300 K and phase separated.

(*c*) The noncondensable gases are removed and burned in a fume furnace while the crude thiophane is caustic washed in a batch operation.

(*d*) The caustic treated thiophane is then batch distilled in a packed tower and sent to storage before eventual shipment to commerical use.

(*e*) Recoverable THF is recycled back to the reactors from the batch column.

(*f*) The aqueous bottoms stream is stored for further processing in the plant.

(*g*) Carbon deposition on the catalyst is heavy (4 percent of THF feed) and therefore provision for regeneration of the catalyst must be made.

Assist the consultant in analyzing this process with a complete flow sheet and material balance, assuming 85 percent operating factor, 80 percent conversion in the reactor, and 90 percent recovery after the reactor. Outline the types of equipment necessary for the process. Determine approximate duties of heat exchangers and list overall heat balances *on* the plant. It is known that the heat of formation of THF is -59.4 kcal/g mol, H_2S is -4.77 kcal/g mol, and thiophane is -- 17.1 kcal/g mol.

What additional information would be required in order to complete the project analysis?

Physical properties:

THF MW = 72 sp gr = 0.887 Boiling pt. = 65°C

Vap. press. at 25°C = 176 mm Hg

Thiophane, MW = 88 Boiling pt. = 121°C

CHAPTER
3

GENERAL DESIGN CONSIDERATIONS

The development of a complete plant design involves consideration of many different topics. Quite understandably, the overall economic picture generally dictates whether or not the proposed facility will receive management approval. However, the application of engineering principles in the design of such a facility in a safe and environmentally acceptable fashion, along with some general design considerations, will ultimately determine whether these earlier economic goals can be met.

Before proceeding any further with the development of a process design and its associated economics, it will be desirable to consider an overall view of the various functions involved in a complete plant design. Particular emphasis in this discussion will be placed on important health, safety, loss prevention, and environmental considerations. Other items that will be noted briefly include plant location, plant layout, plant operation and control, utilities, structural design, storage, materials handling, patents, and legal restrictions.

HEALTH AND SAFETY HAZARDS

The potential health hazard to an individual by a material used in any chemical process is a function of the inherent toxicity of the material and the frequency and duration of exposure. It is common practice to distinguish between the short-term and long-term effects of a materiaf. A highly toxic material that’ causes immediate injury is classified as a safety hazard while a material whose effect is only apparent after long exposure at low concentrations is considered as an industrial health and hygiene hazard. The permissible limits and the precautions to be taken to ensure that such limits will not be exceeded are quite different for these two classes of toxic materials. Information on the effects of many chemicals and physical agents is accessible through computer databases such as MEDLAR and TOXLINE. A number of health effects noted in these sources besides that of cancer are dermatitis, neuropathy, irritation, reproductive damage, and acute poisoning.

The inherent toxicity of a material is measured by tests on animals. The short-term effect is expressed as LD_{50}, the lethal dose at which 50 percent of the test animals do not survive. Estimates of the LD,, value for humans are extrapolated from the animal tests. On the other hand, the permissible limits of concentration for the long-term exposure of humans to toxic materials is set by the ***threshold limit value*** (TLV). The latter is defined as the upper permissible concentration limit of the material believed to be safe for humans even with an exposure of 8 hr per day, 5 days per week over a period of many years. The handbook prepared by Sax† provides a comprehensive source of data as well as guidance on the interpretation and use of the data. Recommended TLV values are published in bulletins by the Occupational Safety and Health Agency (OSHA), the American Conference of Governmental Industrial Hygienists (ACGIH), the American Industrial Hygiene Association (AIHA), the National Institute for Occupational Safety and Health (NIOSH), and the United Ringdom Health and Safety Executive (HSE).

With the uncertainties involved in the designation of occupational exposure standards and the variability of the occupational environment, it would be unreasonable to interpret occupational limits as rigidly as one might interpret an engineering standard or specification. Fortunately, there has been a recent effort to make these rather subjective judgements more scientific and uniform by the application of statistics. The latter makes it possible to develop decision-making strategies that can prescribe how many samples to take, where and when to take them in the workplace, and how to interpret the results.

Sources of Exposure

The main objective of health-hazard control is to limit the chemical dosage of a chemical by minimizing or preventing exposure. It is not practical to measure or control the chemical dosage directly; rather, exposure is measured and limits are set for the control of such exposure.

The most common and most significant source of workplace exposure to chemicals and also the most difficult to control is inhalation. Workers become exposed when the contaminant is picked up by the air they breathe. Thus, an

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†N. T. Sax, “Dangerous Properties of Industrial Materials,;’ 6th ed., Van Nostrand Reinhold, New York, 1984.

understanding of the sources of contaminants to which workers are exposed is important for the recognition, evaluation, and control of occupational health hazards. For example, mechanical abrasions of solid materials by cutting, grinding, or drilling can produce small particles which can form an airborne dust cloud or solid aerosol. Liquid aerosols, on the other hand, may be produced by any process that supplies sufficient energy to overcome the surface tension of the liquid. This process occurs intentionally in spray coating and unintentionally when oil mist is generated from lubricants or coolants used on high-speed machinery. Liquid aerosols can also be produced by condensation.

Contaminant vapors are normally formed by allowing the liquid to evaporate into the air. A significant source of mercury poisoning is from worker exposure in laboratories where mercury has been spilled, trapped in cracks, and then evaporates at room temperature to exceed the TLV of 0.05 mg/m^3.

Gases are usually stored or processed in closed systems. Contamination of air with such gas occurs from fugitive emissions (leaks) or from venting. Essentially all closed systems leak to some degree. [The Environmental Protection Agency (EPA) through various studies has determined that emissions from just the synthetic organic chemical manufacturing industry in the United States are greater than 80,000 Mg/yr before emission controls are applied.] Obviously, the tightness of a system is directly related to the engineering and leak monitoring effort expended. This, in turn, depends on the consequences resulting from these emissions. High-value and very toxic materials are usually very tightly controlled. Contaminants that are neither valuable nor toxic but that create an undesirable atmosphere in neighboring communities are also controlled to maintain good public relations. Flammable materials likewise are carefully controlled because a leak may lead to a fire and a possible major loss in life and facility. Table 1 lists potential sources of air contamination in the chemical process industry, noting whether these are intermittent or continuous sources, whether workers are directly involved in the emission operation, the relative importance of the emission source, and the most probable control of the emission.

In typical well-maintained plants, pumps and valves are probably the major source of fugitive emissions. Monitoring and maintenance efforts are therefore generally focused on these sources. Taken as a whole, fugitive emissions, even without major seal failure, are the origin of the continuous background exposure of workers. This source of exposure may not, by itself, result in overexposure; but its presence reduces the margin within which other emissions may vary while still remaining under the acceptable TLV.

The continuous movement of materials through a process unit generally does not involve any situations for emission release and consequent exposure. However, some material-handling steps are difficult to accomplish with total containment. For example, whenever quantities of materials are allowed to accumulate in storage and then are removed for further processing, the possibility of release needs to be considered; for example, liquids entering fixed tankage generally displace air that must be vented to avoid overpressuring the tank. Control of such liquid-transfer operations can be achieved by using variable

TABLE 1
Potential sources of air contamination in the chemical process industry?

	Intermittent or continuous	Worker activity	Importance	Control‡
Fugitive emissions				
Pump seal leaks	Either	No	High	M
Flange leaks	Cont.	No	LOW	M
Agitator seal leaks	Either	No	Med	M
Valve stem leaks	Cont.	No	High	M
Process operations				
Sampling	Int.	Yes	Med	W, E
Filter change	Int.	Yes	LOW	W, P
Gauging	Int.	Maybe	LOW	E, W
Venting and flaring	Either	No	Med	E
Extruding	Either	Yes	Med	E
Material handling				
Solid addition	Int.	Yes	Med	E, P
Liquid transfer	Either	No	High	E
Bagging	Cont.	Yes	High	E
Drumming	Cont.	Yes	High	E, W
Bag dumping	Int.	Yes	High	E, W
Screening	Cont.	No	Med	E
Open mixing	Int.	No	Med	E, P
Milling	Either	No	Med	E, P
Maintenance				
Equipment opening	Int.	Yes	High	W, P
Instrument line draining	Int.	Yes	Med	W, P
Welding	Int.	Yes	High	E, W, P
Painting	Int.	Yes	Med	W, P
Sandblasting	Int.	Yes	High	E, P
Insulating	Int.	Yes	High	W, P§
Insulating removal	Int.	Yes	High	W, P
Chemical cleaning	Int.	Yes	Med	W, P
Degreasing	Int.	Yes	LOW	E
Cutting and burning	Int.	Yes	Med	W, P
Catalyst handling	Yes	Yes	High	W, P
Waste handling				
Bag house cleaning	Int.	Yes	High	P
Drain and sewer venting	Either	No	High	E
Spill clean up	Int.	Yes	Med	P
Sweeping	Int.	Yes	Low	W
Incineration	Either	Maybe	Med	E
Waste-water treatment	Cont.	No	Med	E
Sludge handling	Int.	Yes	Med	W, P

† Modified from S. Lipton and J. Lynch, "Health Hazard Control in the Chemical Process Industry," Wiley, New York, 1987.

‡ Legend: M = proper maintenance procedures, W = proper work practices, E = appropriate engineering design or modification, P = use of suitable personal protection.

§ Substitution of less toxic materials for asbestos is the most common control.

volume tanks, particularly those with floating roofs, or by scrubbing, flaring, or recovering the vented gas stream.

Solids handling can provide considerable exposure to contaminants whenever the operation is performed in an open atmosphere. Where possible, such operations should be retrofitted with a closed system. Even then, potential release problems exist, particularly during maintenance and repair of the system.

It should be recognized that the maintenance of any closed system can pose a hazardous exposure problem since most maintenance is performed while the plant is in operation and requires that workers be in close proximity to the operating equipment for long periods of time. Under such conditions, it is necessary to consider not only local contaminant releases but also physical hazards that may be present, such as noise and thermal radiation.

In a closed system, equipment that must be repaired should first be cleaned to reduce exposure before the system is opened. Where highly toxic process materials are present, it may be necessary to flush equipment with a low-toxicity stream, strip with steam, and then purge with nitrogen. In such situations, the equipment design should include special fittings necessary for the flushing and purging procedures.

Turnarounds, or major periodic overhauls of chemical plant units, are a special case of plant maintenance. Since the units are shut down, some exposure risks are avoided. However, since the unit is not in production, there is a time pressure to complete the turnaround and resume production. In such an environment, there is the potential for disorganization and misunderstanding on the part of workers with the unanticipated release of contaminants. To conduct a safe turnaround requires careful planning. Contingencies need to be anticipated to the greatest extent possible and plans made to deal with them.

It should be noted that the materials and operations used in a plant maintenance effort may involve a new set of hazards quite separate from the exposure hazards encountered with feedstocks, intermediates, and products for the process plant. For example, proper maintenance often involves such operations as welding, sandblasting, painting, chemical cleaning, catalyst handling, and insulation replacement. The maintenance of safe conditions requires extensive worker training in each one of these operations.

In the same vein, certain waste-handling procedures, even those performed intermittently, can result in very serious contaminant exposure without proper precautions. Workers need to be instructed in the proper procedures for cleaning up spills and accumulated debris. Spilled materials can become airborne and pose an inhalation hazard. Spills and chemical process wastes may end up in the waste-water treatment facilities where they again can be volatilized into the air and result in unexpected worker exposure.

Exposure Evaluation

If health hazards are to be controlled, they must be recognized and evaluated. A logical place to initiate the process of health-hazard recognition is with a total inventory of all materials present in the various stages of the process. Even when materials are present in only trace amounts, there is more than enough present to produce a potentially hazardous situation in a localized work area. Generally, feedstocks and products of a process are well known. Intermediates, by-products, and waste materials may be less conspicuous and may not even have been identified. Other materials, such as catalysts, additives, cleaning agents, and maintenance materials need to be identified to complete the inventory.

An estimate of the toxicity or intrinsic hazard is needed for each material identified in the inventory. Such information for many chemicals in the form of a Material Safety Data Sheet (MSDS) are required by the OSHA Hazard Communication Standard. (Other countries have similar requirements.) Standard hazard-data sources may need to be consulted for those chemical compounds for which no MSDSs are presently available. Adequate hazard data may be lacking for various mixtures that are unique to the plant. For such mixtures, it may be necessary to analyze the contents and then estimate the overall hazard based on the individual components.

To perform a risk assessment and then prioritize the exposure measurement effort requires an approximate initial exposure potential assessment. For each chemical present and for each source of exposure for that chemical, an estimate of exposure can be made. These exposure estimates combined with a toxicity estimate from the hazard data can then be combined to yield a risk estimate which can be used as a basis for prioritization of the measurement and monitoring effort.

It is generally not necessary to make an exposure estimate for every chemical/exposure source combination since many will be of such low significance that they can be neglected. For those chemical/exposure source combinations that could be near the top of the priority list, the exposure estimate is probably not needed beyond an order of magnitude. Methods for making this type of estimate have been developed by the EPA for the purpose of evaluating Premanufacturing Notifications (PMNs).

Contaminant concentrations in a typical plant environment are highly variable. The background level of exposure in a chemical plant is generally the result of a large number of small fugitive emissions, each varying with time. These variability aspects in the contaminant concentrations and the exposure of workers require that a sufficient number of samples be taken to permit characterization of the statistical distribution and permit estimation of exposure over the appropriate averaging time. In mathematical terms, the averaging time should be no longer than the biological half-time of a substance acting in the body. Although the range of biological half-times is continuous, for simplicity only a few discrete averaging times are commonly used. For fast-acting substances 15 and 30 min are used, while 8 h is most often used for substances with biological half-times longer than 8 h. The latter is generally labeled as the 8-h time weighted average (TWA).

The most commonly used methods for the analysis of airborne contaminants are listed in Table 2. Any method used for a particular contaminant must

TABLE 2
Air analysis methods

Method	Substance analyzed
Atomic absorption spectroscopy	Metals
Gas chromatography	Volatile organic compounds
Gravimetric	Nuisance dust, coal dust
Particle count	Asbestos
Ion-specific electrode	Halogens, HCN, NH_3
X-ray diffraction	Silica
Colorimetry	Miscellaneous

be appropriate for the sampling media, have sufficient sensitivity, and be reasonably free from interference. The ultimate confidence that can be placed on an analytical result depends in part on the accuracy of the method, but to a greater extent on how well the method has been validated for the particular purpose and on the reliability of the laboratory performing the test.

As noted earlier, the EPA has determined that fugitive emissions from process equipment are a large source of volatile organic compounds (VOC). The latter are defined by the EPA as organic compounds that participate in photochemical reactions. These reactions are of significance since the ozone level in the atmosphere is affected by the concentration of volatile organic compounds. Standards for ozone concentration in nonplant areas were originally one of the major concentration targets in the Clean Air Act.

In addition to the volatile organic compounds, EPA has added other regulations controlling a number of compounds which are neither carcinogenic agents or cause serious health problems to the public. These hazardous pollutants, controlled under the National Exposure Standards for Hazardous Pollutants (NESHAP), include benzene, vinyl chloride, mercury, asbestos, arsenic, beryllium, and radionuclides. The NESHAP regulations in combination with VOC emission-control regulations reduce exposures in the plant environment through equipment emission-control systems. This is in contrast to the specific objective of the Occupational Safety and Health Act (OSHA), which is the control of occupational exposures in the workplace. This is considered in the next section.

Exposure-Hazard Control

When it is concluded that an exposure problem exists, decisions need to be made regarding the implementation of hazard-control measures for the purpose of reducing exposure and correspondingly reducing the risks. However, a given set of exposure conditions does not lead to a fixed set of control strategies. There are many options. Since zero risk is not attainable, a decision must be made relative to the degree of risk reduction that is to be attained. Then a series of choices must be made from a wide range of options available to achieve the desired risk reduction. This choice of options is a judgemental decision since the precise degree of risk assessment achievable by a specific strategy is usually not known in advance. Furthermore, the strategy selected must meet company safety standards, comply with regulatory requirements, receive worker acceptability, and not adversely impact production and operability.

These are three general control principles utilized in reducing the exposure of workers to occupational health hazards. These involve source controls, transmission barriers, and personal protection. In the first strategy, measures are taken to prevent the release of the toxic contaminant to the air. The second strategy provides means for capturing or blocking the contaminant before it reaches the worker. The final strategy assumes the first two were unsuccessful and requires workers to wear some protective device to prevent contact with the toxic contaminant.

Containment eliminates most opportunities for exposure and is the preferred method of control in chemical manufacturing. Actually, containment in many chemical plants is dictated by pressure, temperature, fire, or product-loss requirements and is really not a health-hazard control option. However, it must be recognized that containment is never perfect, releases and exposure opportunities will still occur, and additional control will probably be required.

Basic or detailed changes in the way the process is permitted to operate can eliminate or reduce exposure. For example, rather than handling a material as a dry powder, it might be handled as a slurry or in solution. A special case of process change involves the substitution of a less hazardous material in the process for a more hazardous one. If such a substitution is not possible, then it may be necessary to completely isolate the process from the worker, as has been done in the manufacture of HCN (prussic acid).

The primary purpose of local exhaust ventilation is to control contaminant exposure by establishing a control surface or barrier between the emission source and the worker so that the contaminant is captured and does not reach the worker's breathing zone. Local exhaust ventilation is cumbersome, inconvenient, and requires considerable maintenance. It is an effective form of control that can be retrofitted to an existing plant and thus minimize a problem that was not anticipated in the original design. However, local exhaust ventilation is rarely completely effective since capture is not complete and not all release points are adequately covered.

Dilution ventilation, on the other hand, removes air containing a contaminant from the workplace after it has become mixed and been inhaled by the workers. The objective of dilution ventilation is not to prevent any exposure, but to keep the exposure to acceptable levels by dilution. This strategy should only be used in low release rate, low toxicity (low hazard) situations.

There are also procedures and precautions that can be taken by workers themselves to minimize exposure while on the job. Such practices do not generally eliminate a hazard by themselves but are necessary to prevent overexposure by emission sources not controlled by engineering design. Personal protection against exposure by inhalation can be accomplished by respirators. Such devices are capable of providing considerable protection when selected and used properly.

Various control options or combinations of options need to be selected to reduce the evaluated exposure level to an acceptable one. The best option or combination of options is then selected by means of a cost analysis. The latter is most useful when comparing two or more options that have approximately an equal probability of reducing the exposure below an appropriate occupational exposure limit. Costs, including capital and expense, of the various options may then be compared using such economic parameters as present net worth or annualized cost.

An engineering system or work procedure that is utilized to eliminate a health effect should be evaluated to determine the degree to which it reduces the occurrence of the health effect. Measurements of exposure, for use in comparison with occupational-exposure limits, need to be made over the averaging time appropriate to the standard.

Fire and Explosion Hazards

Besides toxic emissions, fire and explosion are the two most dangerous events likely to occur in a chemical plant. Considerable resources are expended to prevent both of these hazards or control them when they do occur because of an accident. These two hazards account for the major loss of life and property in the chemical and petroleum industry.

For a fire to occur, there must be a fuel, an oxidizer, and an ignition source. In addition, the combustion reaction must be self-sustaining. If air is the oxidizer, a certain minimum concentration of fuel is necessary for the flame to be ignited. While the minimum concentration required depends on the temperature of the mixture and to a lesser extent on the pressure, most interest generally is focused on the ignition conditions necessary at ambient temperature. The minimum concentration of fuel in air required for ignition at ambient temperature is known as the *lower flammable limit* (LFL). Any mixture of fuel and air below the LFL is too lean to burn. Conversely, the concentration above which ignition will not occur is labeled as the *upper flammable limit* (UFL). Both limits of flammability are published in various literature sources? for many hydrocarbons and chemicals. It should be noted that there is also a concentration of oxidizer that must be present for ignition, called the *limiting oxygen index* (LOI) with a meaning analogous to the LFL.

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†N. I. Sax and R. J. Lewis, Sr., "Hazardous Chemicals Desk Reference," Van Nostrand Reinhold, New York, 1987; C. E. Grant and P. J. Pagni, "Fire Safety Science," Hemisphere Publishing Corporation, New York, 1986; N. I. Sax and R. J. Lewis, Sr., "Dangerous Properties of Industrial Materials," 7th ed., Van Nostrand Reinhold, New York, 1989.

The flammability limits of mixtures can be estimated from the data for individual fuels by using le Chatelier's principle

$$\sum (y_i/\text{LFL}_i) = 1.0 \tag{1}$$

where y_i is the mole fraction of each component of the fuel in the air and the LFL_i is the corresponding LFL value for each component. A similar relationship can be used to estimate the UFL for a gas mixture. If the concentration of a mixture of fuel gases is known, the LFL for the mixture can be approximated from

$$(\text{LFL})_{\text{mix}} = 100 \Big/ \sum (p_i/\text{LFL}_i) \tag{2}$$

where p_i is the percentage of fuel in the original mixture, free from air and inert gases. The two preceding relationships provide reasonably good LFL and UFL values for mixtures of hydrocarbon gases and mixtures of hydrogen, carbon monoxide, and methane. The relationships provide poorer results for other gas mixtures.

If the concentration of fuel is within the flammability limits and the temperature of the mixture is high enough, the mixture will ignite. The temperature at which ignition will occur without the presence of a spark or flame is designated as the *autoignition* temperature (AIT). If the temperature is less than the AIT, a minimum amount of energy (as low as a few millijoules for hydrocarbons) is required for ignition of flammable mixtures.

When the fuel is a gas, the concentration required for flammability is reached by allowing more fuel to mix with a given quantity of air. However, if the fuel is a liquid, it must first be vaporized before it will burn. When the vapor concentration reaches the LFL, the vapor will ignite if an ignition source is present. The liquid temperature at which the concentration of the fuel in the air becomes large enough to ignite is labeled the *flash-point.* The latter is a measure of the ease of ignition of a liquid fuel.

Prevention of fires is best accomplished by keeping all flammable materials under close control. In most industrial operations, once the confined materials are released, it becomes very difficult to keep air from mixing with the materials to form a flammable mixture. It is then essential to eliminate as many ignition sources as possible. In fact, a' number of codes, like the National Electrical Code promulgated by the National Fire Protection Association (NFPA), specify in NFPA Standard 70(1) the elimination of all ignition sources or the use of protective devices to prevent potential ignition in areas where flammable mixtures are apt to occur. However, damage from the release may make this difficult. Thus, most designers of fire-protection systems assume that ignition generally will occur when a flammable material is released.

The heat-transfer rate in a fire depends on two mechanisms: convection and radiation. Calculation of the heat-transfer rate must be made by considering each of the mechanisms separately and then combining the result. If the fire is large, it will radiate at a constant flux; for most hydrocarbons and combustible chemicals, the radiant flux averages close to 30,000 Btu/h-ft^2.

Fires are classified into four groups: Class A fires are those burning ordinary solids; Class B fires are those burning liquids or gases; Class C fires are those that burn either Class A or Class B fuels in the presence of live electrical circuits; and Class D fires consume metals. Fire-protection systems can be divided into two large categories: passive and active. Active systems include such agents as water sprays, foam, and dry chemicals; these require that some action be taken, either by plant personnel or as a response by an automatic fire-protection system. Passive fire-protection systems do not require any action at the time of the fire. They are designed and installed at the time the plant is built and remain passively in place until needed.

One example of passive fire protection is insulating material (called fireproofing) that is applied to steel structural members and equipment supports in the plant. The time required for unprotected steel supports to fail during a fire is rather short. Fireproofing can significantly extend the failure time and provide additional time for fire fighters to reach the scene, apply cooling water to the supports, and bring the fire under control.

An explosion is a sudden and generally catastrophic release of energy, causing a pressure wave. An explosion can occur without a fire, such as the failure through overpressure of a steam boiler. It is necessary to distinguish between detonation and deflagration when describing the explosion of a flammable mixture. In a detonation, the chemical reaction propagates at supersonic velocity and the principal heating mechanism is shock compression. In a deflagration, the combustion process is the same as in the normal burning of a flammable mixture with the reaction propagating at subsonic velocity and experiencing a slow pressure buildup. Whether detonation or deflagration occurs in a flammable mixture depends on such factors as the concentration of the mixture and the source of ignition. Unless confined or ignited by a high-intensity source, most materials will not detonate. However, the pressure wave caused by a deflagration can still cause considerable damage.

An explosion can result from a purely physical reaction, from a chemical reaction, or from a nuclear reaction. A physical explosion is one in which a container fails, releasing its contents to the surroundings. The damage to the surroundings from the sudden expansion of the confined gas can be approximated by determining the maximum energy released from an isentropic expansion of the gas and converting this energy quantity to a TNT equivalent. (The energy released by an explosion of TNT is 4.52 MJ/kg or about 2000 Btu/lb.) A useful relation for this estimation is given by

$$E = \frac{p_b V}{(\gamma - 1)}\left[1 - \left(\frac{p_a}{p_b}\right)^{(\gamma-1)/\gamma}\right] \tag{3}$$

where E is the maximum energy release, V is the volume of the gas in the container, p_b is the burst pressure of the container, p_a is the pressure of the surrounding air, and γ is the ratio of the specific heats.

The amount of energy that is released from a chemical reaction involving a flammable fuel and oxidizer can be estimated from the heat of combustion of the fuel. The damage expected from the resulting explosion may be approximated by comparison with a similar energy release from a known charge of TNT.

There are two special kinds of explosions of particular importance to the chemical industry, namely, the boiling-liquid-expanding-vapor explosion (BLEVE) and the unconfined-vapor-cloud explosion (UVCE). In the former, heat leak into a container filled with a boiling liquid results in an excessive vaporization accompanied with a steady pressure buildup that ruptures the tank. The sudden depressurization causes very rapid vaporization with a substantial explosive force. An unconfined-vapor-cloud explosion, on the other hand, can result when a large cloud of gas or vapor forms following release of a flammable material. If ignition occurs, the cloud may either deflagrate, burning with a relatively low burning speed, or the burning speed may accelerate until the flame front reaches detonation velocities. Substantial destruction will occur if the flame front reaches high velocities. A method for approximating the potential for probable loss caused by a vapor-cloud explosion consists of estimating the quantity of combustible that can be released during an accident and then estimating the fraction of the material that is vaporized immediately after the spill. The explosive load is then considered to be 2 percent of the heat of combustion of the material vaporized.?

It is important to recognize that dusts and mists may also explode when ignited. A large number of solids can form explosive mixtures in air if they are sufficiently pulverized to remain well dispersed and suspended over a period of time. Some dusts are more sensitive than others to ignition whereas some dusts cause more severe explosions than others when ignited. The ignition sensitivity depends on the ignition temperature, the minimum ignition energy, and the minimum explosion concentration. The explosion severity, on the other hand, is a function of the maximum pressure measured during a test explosion and the maximum rate of pressure rise during the test. Since small dust particles are usually easier to ignite and burn more rapidly than larger particles, both the ignition sensitivity and explosion severity appear to be a function of particle size. Extensive data on the explosion characteristics of dusts can be found in the *Fire Protection Handbook*.‡

If an explosion occurs, whether it is from a physical reaction or a chemical reaction, an overpressure will be generated. Data are available to estimate the

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†J. A, Davenport, *Loss Prevention*, **11**:39 (1977).

‡G. P. McKinnon and K. Tower, "Fire Protection Handbook," National Fire Protection Association, Boston, MA, 1986.

effects of overpressure on personnel and equipment. To use the available information, it is necessary to equate the energy of the explosion in terms of equivalent quantities of TNT as discussed earlier. The explosive yield data are usually scaled in terms of $L/M_{\mathrm{TNT}}^{1/3}$, where L is the distance from the blast center and M is the equivalent yield in terms of mass of TNT. Even though present attempts in using this scaling parameter are rather crude, they do provide reasonable guidelines for locating process equipment and control facilities.?

It becomes clear that the chances a single fire or explosion will spread to adjoining units can be reduced by careful plant layout and judicious choice of construction materials. Hazardous operations should be isolated by location in separate buildings or by the use of brick fire walls. Brick or reinforced concrete walls can serve to limit the effects of an explosion, particularly if the roof is designed to lift easily under an explosive force.

Equipment should be designed to meet the specifications and codes of recognized authorities, such as the American Standards Association, American Petroleum Institute, American Society for Testing Materials, Factory Mutual Laboratories, National Fire Protection Association, and Underwriters' Laboratories. The design and construction of pressure vessels and storage tanks should follow API and ASME codes, and the vessel should be tested at 1.5 to 2 or more times the design pressure. Adequate venting is necessary, and it is advisable to provide protection by using both spring-loaded valves and rupture disks.

Possible sources of fire are reduced by eliminating all unnecessary ignition sources, such as flames, sparks, or heated materials. Matches, smoking, welding and cutting, static electricity, spontaneous combustion, and non-explosion-proof electrical equipment are all potential ignition sources. The installation of sufficient fire alarms, temperature alarms, fire-fighting equipment, and sprinkler systems must be specified in the design.

Personnel Safety

Every attempt should be made to incorporate facilities for health and safety protection of plant personnel in the original design. This includes, but is not limited to, protected walkways, platforms, stairs, and work areas. Physical hazards, if unavoidable, must be clearly defined. In such areas, means for egress must be unmistakable. All machinery must be guarded with protective devices4 In all cases, medical services and first-aid must be readily available for all workers.

[†] K. Gugan, "Unconfined Vapor Cloud Explosions," Gulf Publishing Company, Houston, TX, 1979.

$A general requirement for safeguarding all machinery is provided in Section 212 of the Occupational Safety Standard for General Industry (OSHA Standards, 29 CFR 1910):

Safety Regulations

The expressed intent of the Occupational Safety and Health Act (OSHA) originally enacted in 1970 is "to assure so far as possible every working man and woman in the Nation safe and healthful working conditions and to preserve our human resources. . . ." The act presently affects approximately 6 million workplaces and 70 million employees. Over 500 amendments to the Act have been introduced since the original legislation. A recent printing of the OSHA standards can be found in Title 29, Chapter XVII, Part 1910 of the *Code of Federal Regulations.*

Two of the standards directly related to worker health and important in design work are *Toxic Hazardous Substances* and *Occupational Noise Exposure.* The first of these two concerns the normal release of toxic and carcinogenic substances, carried via vapors, fumes, dust fibers, or other media. Compliance with the Act requires the designer to make calculations of concentrations and exposure time of plant personnel to toxic substances during normal operation of a process or plant. These releases could emanate from various types of seals and from control-valve packings or other similar sources. Normally, the designer can meet the limits set for exposure to toxic substances by specifying special valves, seals, vapor-recovery systems, and appropriate ventilation systems.

The list of materials declared hazardous is being updated at a rapid rate. Acceptable material exposure times and concentrations, likewise, are undergoing continuous revision. Thus, it is important that the *Federal Register* be examined closely before beginning the detailed design of a project. A useful publication, *Chemical Regulation Reporter,?* detailing these proposed and new regulations is now available to the design engineer. This weekly information service includes information concerning the *Toxic Substances Control Act* (a law administered by EPA rather than OSHA).

The *Occupational Noise Exposure* standard requires a well-planned, timely execution of steps to conform to the 90-dBA rule in the design stages of a project. Since many cities have adopted EPA's recommended noise-level criteria, or have stringent regulations of their own, design-stage noise control must also consider noise leaving the plant. It is a good idea, during plant design, to prepare two noise specifications: one to define the designer's own scope of work and the other to set vendor noise-level requirements for various pieces of equipment.

Other standards in the safety area that are most often citied by OSHA and which must be considered in detailed designs are the. *National Electric Code* and *Machinery and Machinery Guarding.* A cursory investigation by a designer of these and other OSHA standards quickly points out several problems, particularly in interpretation. The standards frequently do not allow for alternate

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†*Chemical Regulation Reporter,* Bureau of National Affairs, Inc., 1231 25th Street, NW, Washington DC 20037.

TABLE 3
Federal repositories of federal regulations

1. *Federal* Register (FR)-Published daily, Monday through Friday, excepting federal holidays. Provides regulations and legal notices issued by federal agencies. The *Federal Register* is arranged in the same manner as the CFR (see below), as follows:
   a. Title-Each title represents a broad area that is subject to federal regulations. There are a total of 50 titles. For example, Title 29 involves labor, and Title 40 is about protection of the environment.
   b. Chapter-Each chapter is usually assigned to a single issuing agency. For example, Title 29, Chapter XVII, covers the Occupational Safety and Health Administration; Title 40, Chapter I, covers the Environmental Protection Agency.
   c. Part-Chapters or subchapters are divided into parts, each consisting of a unified body of regulations devoted to a specific subject. For example, Title 40, Chapter I, Subchapter C, Part 50, is National Primary and Secondary Ambient Air Quality Standards. Title 29, Chapter XVII, Part 1910, is Occupational Safety and Health Standards. Parts can further be divided into subparts, relating sections within a part.
   d. Section-The section is the basic unit of the CFR (see below), and ideally consists of a short, simple presentation of one proposition.
   e. Paragraph-When internal division of a section is necessary, sections are divided into paragraphs (which may even be further subdivided).
2. *FR* Index-Published monthly, quarterly, and annually. The index is based on a consolidation of contents entries appearing in the month's issues of the *Federal Register* together with broad subject references. The quarterly and annual index consolidates the previous three months' and 12 months' issues, respectively.
3. *Code of Federal Regulations* **(CFR)–Published** quarterly and revised annually. A codification in book form of the general and permanent rules published in the *Federal Register* by the executive departments and agencies of the federal government.
4. *CFR* ***General Index***–**Revised** annually. July 1. Contains broad subject and title references.
5. *Cumulative List of CFR Sections* Affected-Published monthly and revised annually according to the following schedule: Titles **1–16** as of Jan. 1; 17-27 as of April 1; 28-41 as of July 1; 42-50 as of Oct. 1. The CFR is also revised according to these dates. Provides users of the CFR with amendatory actions published in the *Federal Register.*

designs that provide equivalent protection. Some sections are very specific, while others are rather vague. Additionally, some sections refer to other sets of codes such as ASME and ASNI. As a result, when a designer cannot obtain a satisfactory interpretation of a regulation from the standards, the regional or area OSHA should be contacted and an interpretation requested. Since many states also have approved plans comparable to that of the federal government, the designer must also be aware of these regulations.

It should be noted that maintaining an awareness of federal regulations is not an end in itself, but a necessary component for legally acceptable plant design. To aid the design engineer, Table 3 presents a listing of federal repositories for environmental and safety regulations.

LOSS PREVENTION

The phrase ***loss prevention*** in the chemical industry is an insurance term where the loss represents the financial loss associated with an accident. This loss not only represents the cost of repairing or replacing the damaged facility and taking care of all damage claims, but also includes the loss of earnings from lost production during the repair period and any associated lost sales opportunities.

As noted in the previous section, there are numerous hazards associated with chemical processing. The process designer must be aware of these hazards and ensure that the risks involved with these hazards are reduced to acceptable levels through the application of engineering principles and proven engineering practice. In its simplest terms, loss prevention in process design can be summarized under the following broad headings:

1. Identification and assessment of the major hazards.
2. Control of the hazards by the most appropriate means; for example, containment, substitution, improved maintenance, etc.
3. Control of the process, i.e., prevention of hazardous conditions in process operating variables by utilizing automatic control and relief systems, interlocks, alarms, etc.
4. Limitation of the loss when an incident occurs.

Identification can be as simple as asking "what-if' questions at design reviews. It can also involve the use of a checklist outlining the normal process hazards associated with a specific piece of equipment. The major weakness of the latter approach is that items not on the checklist can easily be overlooked. The more formalized hazard-assessment techniques include, but are not limited to, hazard and operability study (HAZOP), fault-tree analysis (FTA), failure mode-and-effect analysis (FMEA), safety indexes, and safety audits.

HAZOPS Study

The hazard and operability study, commonly referred to as. the HAZOP study, is a systematic technique for identifying all plant or equipment hazards and operability problems. In this technique, each segment (pipeline, piece of equipment, instrument, etc.) is carefully examined and all possible deviations from normal operating conditions are identified. This is accomplished by fully defining the intent of each segment and then applying guide words to each segment as follows:

No or not-no part of the intent is achieved and nothing else occurs (e.g., no flow)

More-quantitative increase (e.g., higher temperature)

Less-quantitative decrease (e.g., lower pressure)

As well as-qualitative increase (e.g., an impurity)

Part of-qualitative decrease (e.g., only one of two components in mixture)

Reverse-opposite (e.g., backflow)

Other than-no part of the intent is achieved and something completely different occurs (e.g., flow of wrong material)

These guide words are applied to flow, temperature, pressure, liquid level, composition, and any other variable affecting the process.

The consequences of these deviations on the process are then assessed, and the measures needed to detect and correct the deviations are established.

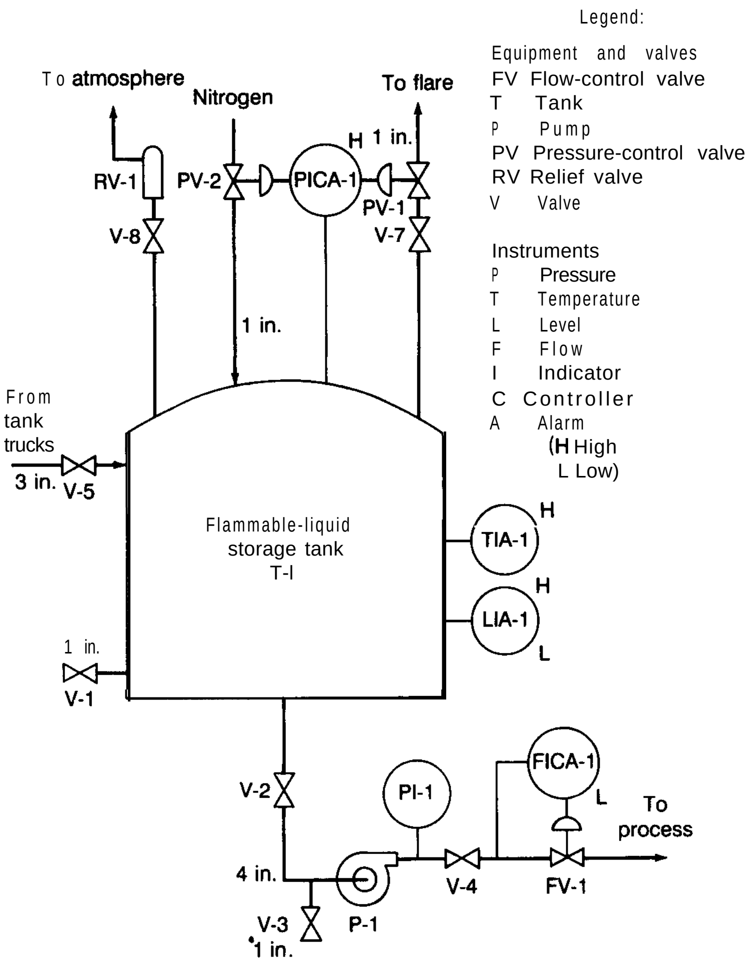

FIGURE 3-1
Piping and instrumentation diagram used in HAZOP example.

TABLE 4
HAZOP study results for process in Fig. 3-1

Equipment reference and operating conditions	Deviations from operating conditions	What event could cause this deviation?	Consequences of this deviation on item of equipment under consideration	Additional implications of this consequence	Process indications	Notes and questions
Storage tank T-l	Level Less	1. Tank runs dry	Pump **cavitates**	Damage to pump	LIA-1, FICA-l	Can reagent react/ explode if over-heated in pump?
		2. Rupture 4-in. discharge	Reagent released	Potential fire	LIA-1, FICA-l	Estimate release quantity. Consider second LAL shutdown on pump.
		3. V-3 open or broken	Reagent released	Potential fire	LIA-1	Estimate release quantity.
		4. V-l open or broken	Reagent released	Potential fire	LIA-1	Consider V-l protection.
		5. Tank rupture	Reagent released	Potential fire	LIA-1	What external events can cause rupture?
	More	6. Unload too much from tank truck	Tank overfills	Reagent released via RV-1	LIA-1	Is RV-1 designed to relieve liquid at loading rate? Consider second high level shutoff.
		7. Reverse flow from process	Tank overfills	Reagent released via RV-1	LIA-1	Consider check valve in pump discharge line.
	No	Same as less				Consider second LAH shutdown on feedlines.
	Composition Other than	8. Wrong reagent	Possible reaction	Possible tank rupture		Consider sampling before unloading.
	As well as	9. Impurity in reagent	If volatile, possible overpressure			Are other materials delivered in trucks?

		Possible problem in reactor			Are unloading connections different? What are possible impurities?
Pressure Less	10. Break 1-in. line to flare or 1-in. nitrogen line	Reagent released	Potential fire	PICA-1	Consider PAL to PICA-1 Consider independent PAL.
	11. Lose nitrogen	Tank implodes	Reagent released	PICA-1	Consider vacuum-break valve.
	12. PV-2 fails closed	Tank implodes	Reagent released	PICA-1	Consider PAL on PICA-1.
	13. PICA-1 fails, closing PV-2	Tank implodes	Reagent released	PICA-1	Tank not designed for vacuum.
More	14. PICA-1 fails closing PV-1	Reagent released via RV-1	Tank rupture if RV-1 fails	PICA-1	What is capacity of PV-1? RV-1? Consider independent PAH.
	15. PV-1 fails closed	Reagent released via RV-1	Tank rupture if RV-1 fails	PICA-1	Consider independent PAH.
	16. V-7 closed	Reagent released via RV-1	Tank rupture if RV-1 fails	PICA-1	Is V-7 locked open? Is **V-8** locked open? Consider independent PAH.
	17. **Overfill** tank	See Event 6	Tank rupture if RV-1 fails	PICA-1	Consider second high-level shutoff.
	18. Temperature of inlet is hotter than normal	Reagent released via RV-1	Tank rupture if RV-1 fails	PICA-1	What prevents high temperature of inlet? Consider independent PAH.
	19. High pressure in flare header	Reagent released via RV-1	Tank rupture if RV-1 fails	PICA-1	Can pressure in flare header exceed tank design? Consider alternative venting.

TABLE 4
HAZOP study results for process in Fig. 3-1 (*Continued*)

Equipment reference and operating conditions	Deviations fmm operating conditions	What event could cause this deviation?	Consequences of this deviation on item of equipment under consideration	Additional implications of this consequence	Process indications	Notes and questions
		20. Volatile impurity in feed	Reagent released via RV-1	Tank rupture if RV-1 fails	PICA-1	Consider independent PAH. Consider sampling before unloading.
	No	Same as less				
	Temperature					
	Less	21. Temperature of inlet is colder than normal	Possible vacuum (see less pressure)	Thermal stress on tank		What are temperature limits of tank%
		22. Low tank pressure	See Events 10–13	Thermal stress on tank		What are pressure limits of tank?
	More	23. Temperature of inlet is hotter than normal	See Event 18	Thermal stress on tank		What are temperature limits of tank?
		24. External fire	Tank fails	Reagent released		What could cause an external fire? What are fire-protection capabilities? Is fire protection adequate?
Feed pump P-l	Flow					
	Less	25. V-2 closed	Pump cavitates	Damage to pump	FICA-l	See Event 1.
		26. V-4 closed	Deadhead pump	Damage to pump	FICA-l	Any other problem with deadhead?
		27. Line plugs	Pump cavitates	Damage to pump	FICA-l	See Event 1.
		28. FV-1 fails closed	Deadhead pump	Damage to pump	FICA-1	See Event 26.

	29. FICA-l fails closing FV-1	Deadhead pump	Damage to pump	None	See Event 26.
	30. V-3 open	Reagent released		FICA-1	Estimate release quantity.
More	31. FV-1 fails open	Upset in reactor	Reagent released	FICA-1	Possible problem in reactor.
	32. FICA-l fails, opening FV-1	Upset in reactor	Reagent released	None	See Event 31.
Pressure More	33. V-4 closed	Deadhead pump	Damage to pump	PI-l, FICA-l	See Event 26.
	34. FV-1 fails closed	Deadhead pump	Damage to pump	PI-l, FICA-l	See Event 26.
	35. FICA-1 fails, closing FV-1	Deadhead pump	Damage to pump	PI-1	See Event 26.
	36. V-2 and V-4 closed	Deadhead pump	Overpressure in pump or line	PI-l, FICA-l	Evaluate need for hydraulic relief.
Less	37. V-2 closed	Pump **cavitates**	Damage to pump	PI-l, FICA-1	See Event 1.
	38. V-3 open	Reagent released		PI-1	See Event 3.
Temperature More	39. V-4 closed	Deadhead pump	Damage to pump	None	See Event 26.
	40. FV-1 fails closed	Deadhead pump	Damage to pump	None	See Event 26.
	41. FICA-l fails, closing FV-1	Deadhead pump	Damage to pump	None	See Event 26.

Since a majority of the chemical process industry now uses some version of HAZOP for all new facilities and selectively uses it on existing ones, an example of this technique, as originally described by Ozog,† is given in the following paragraphs.

Assume that a HAZOP study is to be conducted on a new flammable-reagent storage tank and feed pump as presented by the piping and instrument diagram shown in Fig. 3-1. In this scheme, the reagent is unloaded from tank trucks into a storage tank maintained under a slight positive pressure until it is transferred to the reactor in the process. For simplification, the system is divided into two elements-the tank T-l and the pump P-l and the feedline. Application of the guide words to these two elements is shown in Table 4 along with a listing of the consequences that result from the process deviations. Note that not all guide words are applicable to the process deviations listed. Also, some of the consequences identified with these process deviations have raised additional questions that need resolution to determine whether or not a hazard exists. This will require either more detailed process information or an estimation of release rates. For example, similar release rates could be the consequence of either Event 3 (V-3 open or broken) or Event 4 (V-l open or broken); however, the total quantity released through V-3 could be substantially reduced over that with V-l open or broken by closing V-2. Of the 41 events listed in Table 4, Event 5 (tank rupture) and Event 24 (external fire) would provide the worst consequences since both would result in instantaneous spills of the entire tank contents.

Hazard assessment is a vital tool in loss prevention throughout the life of the facility. Ideally, the assessment should be conducted during the conceptual-design phase, final design stage, and prestartup period as well as when the plant is in full operation. In the conceptual-design phase many potential hazards can be identified and significant changes or corrections made at minimal cost. Results of these assessments are key inputs to both site-selection and plant-layout decisions. The major hazards usually include toxicity, fire, and explosions; however, thermal radiation, noise, asphyxiation, and various environmental concerns also need to be considered.

A thorough hazard and risk assessment of a new facility is essential during the final design stage. At this stage, the piping and instrument diagrams, equipment details, and maintenance procedures are finalized. However, since equipment often has not been ordered, it is still possible to make changes without incurring major penalties or delays.

A hazard assessment during the prestartup period should be a final check rather than an initial assessment. This review should include the status of recommended changes from previous hazard studies and any significant design changes made after the final design. If serious hazards are identified at this time, it is unlikely that they can be eliminated without significant cost or startup delay.

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†H. Ozog, *Chem. Eng.*, **92**(4):161 (Feb. 18, 1985).

Since process and operating procedure changes are often made during or shortly after plant startup, it is strongly advised that hazard assessment not stop after startup. Rather, periodic hazard-assessment studies should be used to define the hazard potential of such changes throughout the life of the facility. The average time between reviews is about three years; more hazardous facilities are reviewed more frequently.

Fault-tree Analysis

The fault-tree analysis (FTA) is primarily a means of analyzing hazardous events after they have been identified by other techniques such as HAZOP. The FTA is used to estimate the likelihood of an accident by breaking it down into its contributing sequences, each of which is separated into all of its necessary events. The use of a logic diagram or fault tree then provides a graphical representation between certain possible events and an undesired consequence. The sequence of events forms pathways on the fault tree, provided with logical And and Or gates. The And symbol is used where coincident lower-order events are necessary before a more serious higher-order event occurs. By multiplying the probabilities of each event in this set, the probability of the next higher-order event is obtained. Correspondingly, when the occurrence of any one of a set of lower-order events is sufficient to cause a more serious higher-order event, the events in the set are joined by an Or gate' and the probabilities are added to obtain the probability of the higher-order event. Probabilities of the various events are expressed as a yearly rate. For example, a 1×10^{-3} chance occurrence per year would represent an event that average-wise would occur only once every 1000 years. Estimation of failure rates with any precision is generally difficult because of the limited prior data. In such cases, information from various sources is used and then revised to incorporate information that is site-specific.

Once a fault-tree analysis has been completed, it becomes rather easy to investigate the impact of alternative preventive measures. For example, in the developmenf of a FTA for Fig. 3-1 and its associated HAZOP study presented in Table 4, Ozog† has determined that the most probable event is a liquid release from the storage tank (Event 6) due to overfilling. However, by adding an independent high-level shutoff to the tank-truck unloading pump, the probability of a liquid release by this event is significantly reduced and Events 12 or 13 (PV-2 closed) become the most probable events. The probability of these events, in turn, could be reduced by the installation of an independent low-pressure alarm to the tank. This process of reducing the probability of the most probable event could be continued until an overall acceptable risk level is eventually achieved.

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†H. Ozog, *Chem. Eng.*, 92(4):161 (Feb. 18, 1985).

Magnitudes of events are typically expressed in terms of the amount of flammable or toxic material released during an event. Release rates are estimated by using appropriate single-phase and two-phase flow models. Since release duration is directly related to the cause and context of the release, its estimation is generally quite subjective.

The severity of the hazard usually cannot be related directly to the magnitude of a release since this is often a function of both the proximity and number of on- and off-site ignition sources. To determine the hazard severity requires quantifying, with the aid of state-of-the-art hazard models, the likely extent of toxic or flammable vapor-cloud travel under different atmospheric conditions, the thermal-radiation fields around vapor and liquid pool-fires, the overpressure from any anticipated explosions, and any missile or fragmentation activity that may result from a confined explosion. These hazard events can then be translated into hazard-zone estimates by incorporating criteria for human injury and property damage. Finally, the results of various loss scenarios can be combined and presented in risk profiles listing injuries, fatalities, and/or property damage. These results can be compared with data for other risks to the public and to workers in various related areas, and these serve as the basis for an assessment of whether or not the risks of the facility as designed are acceptable.

Failure Mode and Effect Analysis

The failure mode and effect analysis (FMEA) is generally applied to a specific piece of equipment in a process or a particularly hazardous part of a larger process. Its primary purpose is to evaluate the frequency and consequences of component failures on the process and surroundings. Its major shortcoming is that it focuses only on component failure and does not consider errors in operating procedures or those committed by operators. As a result, it has limited use in the chemical process industry.

Safety Indexes

The safety and loss prevention guide developed by the Dow Chemical Company? provides a method for evaluating the potential hazards of a process and assessing the safety and loss-prevention measures needed. In this procedure, a numerical “Fire and Explosion Index” is calculated, based on the nature of the process and the properties of the materials. The index can be used two different ways. In the preliminary design, the Dow index will indicate whether alternative,

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†*Dow’s Fire and Explosion Index Hazard Classification Guide,* 5th ed. AIChE, New York, 1981.

less hazardous processes should be considered in the manufacture of a specific chemical product. In the final design, after the piping and instrumentation diagrams and equipment layout have been prepared, the calculated index is used as a guide to the selection and design of the preventive and protective equipment needed for safe plant operation.

The Dow index applies only to main process units and does not cover process auxiliaries. Also, only fire and explosion hazards are considered. Recently the index has been expanded to include business-interruption losses. The principles and general approach used in the Dow method of hazard evaluation have been further developed by Mond in the United Kingdom to include toxicity hazards. This revised Mond index is described in a paper by Lewis.[†]

Safety Audits

The principal function of most safety audits in the past has been to verify the adequacy of safety equipment and safety rules. The former includes equipment for fire protection, personnel protection, and on-site emergency responses. In addition to reviewing the general safety rules, the audit has provided explicit safety rules for new process areas and associated emergency response procedures. However, with the greatly increased concerns for environmental health, safety, community relations, and loss prevention, safety audits have become significant, as well as continuous activities for all chemical process companies. Detailed checklists have been developed that cover every aspect of health, safety, and loss prevention. An example of such a checklist has been prepared by Whitehead[‡] and is shown in greatly condensed form in Table 5. (For complete details, the original table should be consulted.) A critical analysis of all the items on this checklist will generally identify the major hazards in a proposed or existing facility and assist in prescribing preventative actions. Because of their importance, several of the items on the checklist are amplified in later sections of this chapter.

A typical example of the steps involved in the development of a process plant is shown in Table 6. The enormity of the task confronting the design engineer is illustrated by the fact that each of the items in Table 5 must be considered at each of the stages in Table 6. It becomes apparent that considering these items only at the end of the design is unwise because decisions have been made that foreclose what might have been the optimum control option for occupational health reasons. Experience has shown that continuous integration

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[†] D. J. Lewis, *AIChE Loss Prevention Series, No.* **13**:29 (1979).

[‡] L. W. Whitehead, *Appl. Ind. Hyg.*, **2**:79 (1987); the unabridged table is also reproduced by L. Lipton and J. Lynch, "Health Hazard Control in the Chemical Process Industry," Wiley, New York, 1987, pp. 85-96.

TABLE 5
General safety checklist for identifying process hazards

1. External-plant considerations: location, site qualities and use, availability of services
   - A. Site
     1. Location variables-worker, materials and product access, emergency services, population exposure, security, rights of way, size
     2. Isolation, separation of on-site and adjacent hazard areas; considerations of quantities of materials, separation, and barriers
     3. Protection of materials, storage, plant access, and other critical areas against earthquakes, hurricanes, tornadoes, vandalism, air crashes, sabotage, etc., to practical limits
     4. Ice, snow, and water removal and control or storage, grounds management, site drainage
     5. Air quality aspects of site
   - B. People, materials, and energy flow into/out of site and within site-optimization of patterns and reduction of interferences
     1. Raw materials-inflow, handling, storage, distribution
     2. Products-outflow, handling, storage, packaging
     3. Workers
     4. Power services-electric, gas, cogeneration, backup power
     5. Water-supply, quality, treatment, storage
     6. Liquid wastes-wastewater, chemicals
     7. Solid wastes-collection, management, and disposition
     8. Material and energy losses to the surroundings

II. Internal plant, structure, and services considerations to provide maximum flexibility in a fully serviced area of controlled space
   - A. Flexible layout of space
     1. Power available throughout, without circuit overloads; local power takeoffs equipped with lockout provisions
     2. Water available throughout with adequate drainage; for cleaning
     3. Adequate roof support for weather extremes including snow, water, and wind
     4. Foundation load capacity
     5. Vertical space use and access
     6. Horizontal spatial layout
     7. Steam available as needed
     8. Compressed air and/or vacuum available as needed; including different breathing air system if needed (breathing air intake point not contaminated); also inerting gas systems, as needed
     9. Central coolant systems provided, as needed
     10. Drainage in plant-adequate for cleaning, does not leave standing water-size to include sprinkler flow
     11. Adequate preplanning for expansion
   - B. Materials maintenance and selection
     1. Fewest feasible dust-collecting surfaces and spaces; access for cleaning of surfaces, around and under machines
     2. Durable, noncoroding, cleanable structure and surfaces, resistant to chemicals used
     3. Fewest possible materials or fixtures requiring routine hand maintenance or cleaning
     4. Provision for cleaning outside and inside of structure
     5. Central and dispersed space for maintenance equipment and supplies, and repair of maintenance equipment
   - C. General environmental control system
     1. Thermal environment control
     2. General lighting levels-intensity, color, glare, contrast, etc.

TABLE 5
General safety checklist for identifying process hazards (*Continued*)

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3. General ventilation for odor, humidity, temperature control
4. Adequate makeup air to prevent indoor air-quality problems
5. Where feasible, general ventilation design for low-level control of toxic air contaminants
6. Buildings, processes, utility lines, process connections, and controls all have appropriate degree of environmental protection (e.g., against freezing or corrosion)

D. Minimum construction requirements (usually building codes)
  1. Construction types
  2. Allowable areas-those in codes are minimums, usually not optimum
  3. Allowable heights-are minimums, usually not optimum
  4. Fire separations and susceptibility to ignition from adjacent fires
  5. Exterior wall protection-meets adequate over- or underpressure requirements, explosion-relieving sash, walls, roof, as appropriate
  6. Fire-fighting requirements-water
  7. Fire-resistant materials or insulation, absence of openings to transmit flames
  8. Interior finishes-cleanable with reasonable methods
  9. Means of egress
  10. Fire protection systems-sprinklers, adequate water, extinguishers (halocarbons, CO_2, or special fire-fighting procedures for processes used), fire trucks, carts, etc.
  11. Vertical openings-controlled or minimized to reduce fire spread
  12. Hazardous areas
  13. Light and ventilation-minimum and optimum
  14. Sanitation, including drainage, water, sewerage
  15. Electrical wiring and grounding; lightning arrestors
  16. Provisions for handicapped
  17. Energy conservation
  18. High-hazard provisions

E. Rodent control

III. In-plant physical and organizational considerations

A. Design of work-implications for workplace design
  1. Teams versus assembly lines
  2. Individual team facilities
  3. Underlying work organization versus space organization
  4. Materials handling system to coincide with designed work patterns
  5. Worker participation in work and workplace design; worker participation in planning phase and ongoing hazard anticipation and recognition

B. Process health and safety review: preconstruction written occupational health assessment of each worksite or process or situation, prepared prior to final design
  1. Consider material or process substitutions possible to reduce hazard and risk
  2. Review of toxicity of feedstocks and products and their typical impurities, by-products, and intermediates, and effluents, catalysts, and solvents, additives of all types, unexpected products generated under abnormal process conditions
  3. Normal exposure potential
  4. 'Startup, shutdown, turnaround emergency, exposure potential
  5. Immediate control of occasional peak exposure
  6. Occupational health of maintenance staff considered, as well as operators
  7. Health and safety issues reviewed in major modification or automation planning
  8. Avoid process overcrowding; maintain adequate headroom, under equipment
  9. Failure modes and repairs considered in industrial hygiene evaluations
  10. Positive or negative pressure areas to control flow of contaminated air
  11. Does toxicity or flammability, stability, etc., justify extreme engineering controls, e.g., much higher standards, special facilities for dump, blowdown, quench, or deluge?

(Continued)

TABLE 5
General safety checklist for identifying process hazards (*Continued*)

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12. Microscale dispersion processes versus concentration
13. Needed automatic monitoring/sampling systems included?
14. Hierarchical emission control strategy
15. Select design and maintenance standards adequate for risk category; equipment designed to appropriate recognized standards
16. Detailed considerations of specific process conditions and safety (do not treat as all-inclusive)
17. Insure written operating procedures include safety checks, actions, maintenance
18. Insure operator training, formal and informal, includes safety and health training, including emergency procedures, protective equipment use, and hazard communication
19. Are normal staff personnel and distribution adequate for fire fighting and emergencies, particularly in small plants?

C. Isolation and separation of risks to minimize exposure
  1. Identification of toxic or radiation areas, minimize number of people exposed, degree of exposure
  2. Noise protection
  3. Radiation protection
  4. Fire-explosion risks separated from ignition hazards, other such risk areas-fire walls, curbs, dikes, barriers, etc.
  5. Boilers, other major pressure vessels
  6. Storage areas for hazardous materials, also compressed gas storage and restraint systems
  7. Carcinogen or biohazard areas
  8. Emergency chemical or other exposure refuge points
  9. Stored amounts of materials less than acceptable hazard amounts and areas affected if explosion or fire occur

D. Ergonomics considerations
  1. Job analysis needed for repetitive motion, biochemical stresses, etc.; can machines take over repetitive tasks?
  2. Acceptable information flow and control design at human-machine interfaces
  3. Workstations and materials handling evaluated for above, redesigned as needed

E. Material flow and handling systems and organization
  1. Horizontal transport systems-mechanical and pneumatic conveyors, carts, robot tugs, piping, augers
  2. Vertical transport-elevators, piping, etc.; gravity flow where possible
  3. Review transfer, measurement, and packaging points for exposure potential
  4. Storage-long term and temporary ready-to-use
  5. Proper location of controls, valves, etc., including access during emergencies
  6. Review materials packaging with respect to high-risk quantities and risk reduction
  7. Temperature and moisture control systems for dry bulk materials

F. People flow
  1. Worker access to worksites and facilities (carts, bicycles, scooters, foot)
  2. Visitors
  3. Conflicts between worker flows/locations and material flows/storage

G. Occupational health input into automation and mechanization
  1. Programming of machines to include safe movements, software interlocks, etc., as well as hardware interlocks
  2. Robots require two to three times more space

H. Industrial sanitation and services
  1. Water-quantity and quality as needed

TABLES
General safety checklist for identifying process hazards (Continued)

2. Food handling, eating, lounge/rest space away from worksite; vending machine sanitation
3. Solid waste collection and handling in-plant
4. Bacterial and insect control
5. Air cleaning where required
6. Sanitary facilities: toilets, washrooms-adequate number, size, and distribution; internal circular traffic patterns, not in busy areas
7. Personnel services, where required or useful
8. First aid and medical services space, and access
9. Facilities for industrial hygiene staff, laboratories, information handling
10. Space, facilities, fixtures planned throughout plant for health and safety equipment

I. Hazard communication within plant-consistent system of signage, placarding, content labeling, etc.; planned facilities for MSDS access

of environmental, safety, and occupational health issues into all design stages leads to the most cost-effective design. Examples of the kinds of interactions and hazard-control choices that need to be made at the various design stages are highlighted in the text by Lipton and Lynch.?

ENVIRONMENTAL PROTECTION

Because of the greater concern for the continued degradation of the environment, the Environmental Protection Agency (EPA) has systematically been rewriting and tightening many policies and regulations. The EPA has also been encouraging state and local governments, as well as industry, to take a more active role in environmental issues. Some of the important issues include the disposal of wastes, both hazardous and nonhazardous, effluent controls on wastewater and storm water runoff, and hydrocarbon emissions to the atmosphere. The EPA is also encouraging companies to perform environmental audits.

Waste disposal is a serious problem for many chemical plants. The EPA initiative that has greatly curtailed land disposal has had a great effect on waste disposal. The 1984 Resource Conservation and Recovery Act (RCRA) amendments have also made it more difficult to dispose of solid wastes. In addition, RCRA required all interim status hazardous-waste facilities to meet groundwater monitoring and insurance requirements by late 1985. This included hazardous-waste surface impoundments. Since 1988, a double liner and leachate collection system have also been required.

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†S. Lipton and J. Lynch, “Health Hazard Control in the Chemical Process Industry,” Wiley, New York, 1987.

TABLE 6
Sequence of steps in developing a project?

	Time sequence
Project steps	Process identification
	Laboratory verification
	‘Reaction flow schematic
	Preliminary flow process
	Preliminary economic evaluation
	Process development
	Mass and energy balance
	Process flow scheme
	Site selection
	Refined economic evaluation
	Design fixed
	Detailed economic evaluation
	Engineering flow scheme
	Basic design
	Detailed construction plan
	Detail design
	Procurement
	Construction
	Startup

† Modified from Ann. *Occup. Hyg.,* **30**:232 (1986).

The disposal of waste streams that contain large amounts of water is another challenge which faces a process engineer. Deep-well injection has been used in the past, but this method has been constrained by regulatory agencies. Recently, refinery wastewater and storm water runoff has been subject to more stringent Best Available Technology (BAT) effluent controls. The agreement covers nonconventional pollutants such as phenolic compounds, ammonia sulfide, and toxic pollutants such as chromium and hexavalent chromium.

Hydrocarbon emissions is an environmental problem which is prominent in some areas of the country. In these areas, pollutant levels occasionally exceed the national ambient air quality standard. One source of these pollutants involves emissions from refineries where about 85 percent of the hydrocarbons emitted to the atmosphere are from fugitive emissions. The principal contributors generally are valves, flanges, pumps, and compressors. Since 1981, valves and flanges must be repaired if they have emission concentrations greater than 10,000 ppm, when measured at the source with a hydrocarbon analyzer. They must be inspected every six months to a year. Pumps and compressors, however, need to be checked every shift to verify that they meet current emission standards.

It is becoming increasingly clear that chemical engineers must be versed in the latest federal and state regulations involving environmental protection, worker safety, and health. This need is especially great for engineers in design-related functions, such as capital-cost estimating, process and equipment design, and plant layout. It is particularly important to learn what is legally required by the Environmental Protection Agency (EPA), the Occupational Safety and Health Administration (OSHA), and corresponding regulatory groups at the state and local levels. As a minimum, every design engineer should understand how the federal regulatory system issues and updates its standards.

Every design engineer must be certain that a standard being used has not been revised or deleted. To be sure that a regulation is up-to-date, it must first be located in the most recent edition of the ***Code of Federal Regulations*** (CFR). ***Next,*** the ***Cumulative List of CFR Sections Affected*** must be checked to see if actions have been taken since the CFR was published. If action has been taken, the ***Cumulative List*** will indicate where the changes can be found in the ***Federal Register.*** The latter provides the latest regulations and legal notices issued by various federal agencies.

Environmental Regulations

Several key aspects of the U.S. Federal environmental regulation as spelled out in legislation entitled ***Protection of the Environment*** (Title 40, Chapter 1 of the CFR) are listed in Table 7. This checklist must also consider applicable state and local codes. Often these may be more stringent than the federal codes or may single out and regulate specific industries.

Note that Part 6 of Title 40, Chapter 1, in Table 7 requires the preparation of an Environmental Impact Statement (EIS). The National Environment Policy Act (NEPA) requires that federal agencies prepare such a statement in advance of any major “action” that may significantly alter the quality of the environment. To prepare the EIS, the federal agencies require the preparation of an Environmental Impact Assessment (EIA). The latter is required to be a full-disclosure statement. This includes project parameters that will have a positive environmental effect, negative impact, or no impact whatsoever. Generally, design engineers will only be involved with a small portion of the EIA preparation, in accordance with their expertise. However, each individual should be aware of the total scope of work necessary to prepare the EIA as well as the division of work. This will minimize costly duplication, as well as provide the opportunity for developing feasible design alternatives.

The preparation of an Environmental Impact Assessment requires determining what environmental standards require compliance by the project, obtaining baseline data, examining existing data to determine environmental safety of the project, preparing an effluent and emission summary with possible alternatives to meet acceptable standards, and finally preparing the environmental statement or report. Since it may require a full year to obtain baseline data such as air quality, water quality, ambient noise levels, ecological studies, and social surveys, emissions and effluents, studies should take place concurrently to avoid delay in preparing the EIA. The emissions and effluents studies must include all “significant” sources of pollution. Omission of data could cause inconsistencies

TABLE 7
Key aspects of U.S. federal environmental regulation
Based on Title 40 of the CFR

	Title 40-Protection of Environment
Chapter	I-Environmental Protection Agency
Part	Subchapter A-General
6	Preparation of Environmental Impact Statements (Information for the designer in preparing an EIA.)
	Subchapter C-Air Programs
50	National primary and secondary ambient air quality standards
53	Ambient air monitoring reference and equivalent methods
60	Standards of performance for new stationary sources
61	National emission standards for hazardous air pollutants
81	Air quality control regions, criteria, and control techniques
	Subchapter D-Water Programs
112	Oil pollution prevention
120	Water quality standards
122	Thermal discharges
128	Pretreatment standards
129	Toxic pollutant effluent standards
133	Secondary treatment information
	Subchapter E-Pesticide Programs
	Subchapter H-Ocean Dumping
	Subchapter N-Effluent Guidelines and Standards
Chapter	IV-Low-Emission Vehicle Certification Board
Chapter	V-Council on Environmental Quality
Part	
1500	Preparation of Environmental Impact Statements: guidelines

that could result in further time delays when negotiating with the regulatory agencies issuing the many required construction permits.

It becomes clear that environmental considerations not only can play a major factor in the choice of selecting a plant site but can also be quite costly. The American Petroleum Institute[†] has estimated that the preparation of an EIA for each site considered may range from $50,000 for small projects to $1.5 million for a large petroleum refinery. On the other hand, a detailed environmental assessment may quickly eliminate possible sites because of their highly restrictive standards.

[†] The Economic Impact of Environmental Regulations on the Petroleum Industry-Phase II Study, American Petroleum Institute, June 11, 1976.

Development of a Pollution Control System

Developing a pollution control system involves an engineering evaluation of several factors which encompass a complete system. These include investigation of the pollution source, determining the properties of the pollution emissions, design of the collection and transfer systems, selection of the control device, and dispersion of the exhaust to meet applicable regulations.

A key responsibility of the design engineer is to investigate the pollutants and the total volume dispersed. It is axiomatic that the size of equipment is directly related to the volume being treated and thus equipment costs can be reduced by decreasing the exhaust volume. Similarly, stages of treatment are related to the quantity of pollutants that must be removed. Any process change that favorably alters the concentrations will result in savings. Additionally, consideration should be given to changing raw materials used and even process operations if a significant reduction in pollution source can be attained. The extent to which source correction is justified depends on the cost of the proposed treatment plant.

For example, the characteristics of equipment for air pollution control, as specified in Table 8, often limit the temperature and humidity of inlet streams to these devices. Three methods generally considered for cooling gases below 500°F are dilution with cool air, quenching with a water spray, and the use of cooling columns. Each approach has advantages and disadvantages. The method selected will be dependent on cost and limitation imposed by the control device.

Selection of the most appropriate control device requires consideration of the pollutant being handled and the features of the control device. Often, poor system performance can be attributed to the selection of a control device that is not suited to the -pollutant characteristics. An understanding of the equipment operating principles will enable the design engineer to avoid this problem.

Air Pollution Abatement

The most recent changes in the U.S. Clean Air Act Amendments have changed the regulatory ground rules so that almost any air-pollutant-emitting new facility or modification is subject to the provisions of the law. For most situations, a New Source Review (NSR) application will have to be filed before construction is allowed. Source categories covered at this time include petroleum refineries, sulfur recovery plants, carbon black plants, fuel conversion plants, chemical process plants, fossil-fuel boilers (greater than 250 MM Btu/h heat input), and petroleum storage and transfer facilities (greater than 300,000-barrel capacity).

To obtain a construction permit, a new or modified source governed by the Clean Air Act must meet certain requirements. These include a demonstration that “best available control technology” (BACT) is to be used at the source. In addition, an air quality review must demonstrate that the source will not cause

TABLE 8

Air pollution control equipment characteristics

Control equipment	Optimum? size particle, microns	Optimum concentration, grains / ft^3	Temperature limitations, °F	Pressure drop, in. H_2O	Efficiency	Space requirements‡	Collected pollutant	Remarks
				Particulates pollutant				
Mechanical collectors								
Settling chamber	> 50	> 5	700	< 0.1	< 50	L	Dry dust	Good as precleaner
Cyclone	5-25	> 1	700	1-5	50-90	M	Dry dust	Low initial cost
Dynamic precipitator	> 10	> 1	700	Fan	< 80	M	Dry dust	
Impingement separator	> 10	> 1	700	< 4	< 80	S	Dry dust	
Bag filter	< 1	> 0.1	500	> 4	> 99	L	Dry dust	Bags sensitive to humidity, filter velocity, and temperature
Wet collector								
Spray tower	25	> 1	40-700	0.5	< 80	L	Liquid	Waste treatment required
Cyclone	> 5	> 1	40-700	> 2	< 80	L	Liquid	Visible plume possible
Impingement	> 5	> 1	40-700	> 2	< 80	L	Liquid	Corrosion
Venturi	< 1	> 0.1	40-700	1-60	< 99	S	Liquid	High temperature operation possible
Electrostatic precipitator	< 1	> 0.1	850	< 1	95-99	L	Dry or wet dust	Sensitive to varying condition and particle properties
				Gaseous pollutant				
Gas scrubber		> 1%	40-100	< 10	> 90	M-L	Liquid	Same as wet collector
Gas adsorber		§	40-100	< 10	> 90	L	Solid or liquid	Adsorbent life critical High initial and operating cost
Direct incinerator		Combustible vapors	2000	< 1	< 95	M	None	High operating costs
Catalytic combustion		Combustible vapors	1000	> 1	< 95	L	None	Contaminants could poison catalyst

† Minimum particle size (collected at approximately 90% efficiency under usual operating conditions)

‡ Space requirements: S = small, M = moderate, L = large

§ Adsorber (concentrations less than 2 ppm **non-regenerative system; greater than 2 ppm regenerative system)** or contribute to a violation of the Ambient Air Quality Standard (AAQS) or maximum allowable increase over the baseline concentration of sulfur dioxide and particulates in any area. (Three different clean air areas have been designated, with class I the most pristine encompassing national parks and forests.) Only when these steps indicate that the ambient air will not be significantly impacted by the source may a construction permit be issued.

Air pollution control equipment can essentially be classified into two major categories, those suitable for removing particulates and those associated with removing gaseous pollutants,. Particulates are generally removed by mechanical forces, while gaseous pollutants are removed by chemical and physical means.

PARTICULATE REMOVAL. The separating forces in a cyclone are the centrifugal and impact forces imparted on the particulate matter. Similar forces account for the particulate capture in mechanical collectors such as impingement and dynamic separators. In settling chambers, the separation is primarily the result of gravitational forces on the particulates. The mechanism in a wet collector involves contact between a water spray and the gaseous pollutant stream. Separation results primarily from a collision between the particulates and the water droplets. Separation also occurs because of gravitational forces on the large particles, or electrostatic and thermal forces on the small particles. The main separating forces in a bag filter are similar to those described in the wet collector, i.e., collision or attraction between the particle and the filter of the bag. Finally, the principal components in an electrostatic precipitator are a discharge plate and a collecting surface. The separation is effected by charging the particles with a high voltage and allowing the charged particles to be attracted to the oppositely charged collection plates.

To obtain the greatest efficiency in particulate removal, additional attention must be given to particle diameter and the air velocity. The particle size determines the separating force required, while the effectiveness of the control equipment is related to the stream velocity. Generally, the greater the relative velocity between the air stream and the collision obstacle for the particulates, the more effective the separating mechanism. The electrostatic precipitator is an exception to this generality, since here the particle diameter influences the migratory velocity and the power required to maintain the electrical field influences the equipment performance. Figure 3-2 illustrates the characteristics of various pollution particulates and the range of application for several control devices as related to particle size.

A review of Table 8 and Fig. 3-2 indicates that large-diameter particles can be removed with low-energy devices such as settling chambers, cyclones, and spray chambers. Submicron particles must be removed with high-energy units such as bag filters, electrostatic precipitators, and venturi scrubbers. Intermediate particles can be removed with impingement separators or low-energy wet collectors. Obviously, other equipment performance characteristics as noted in Table 8 will also have their influence on the final equipment

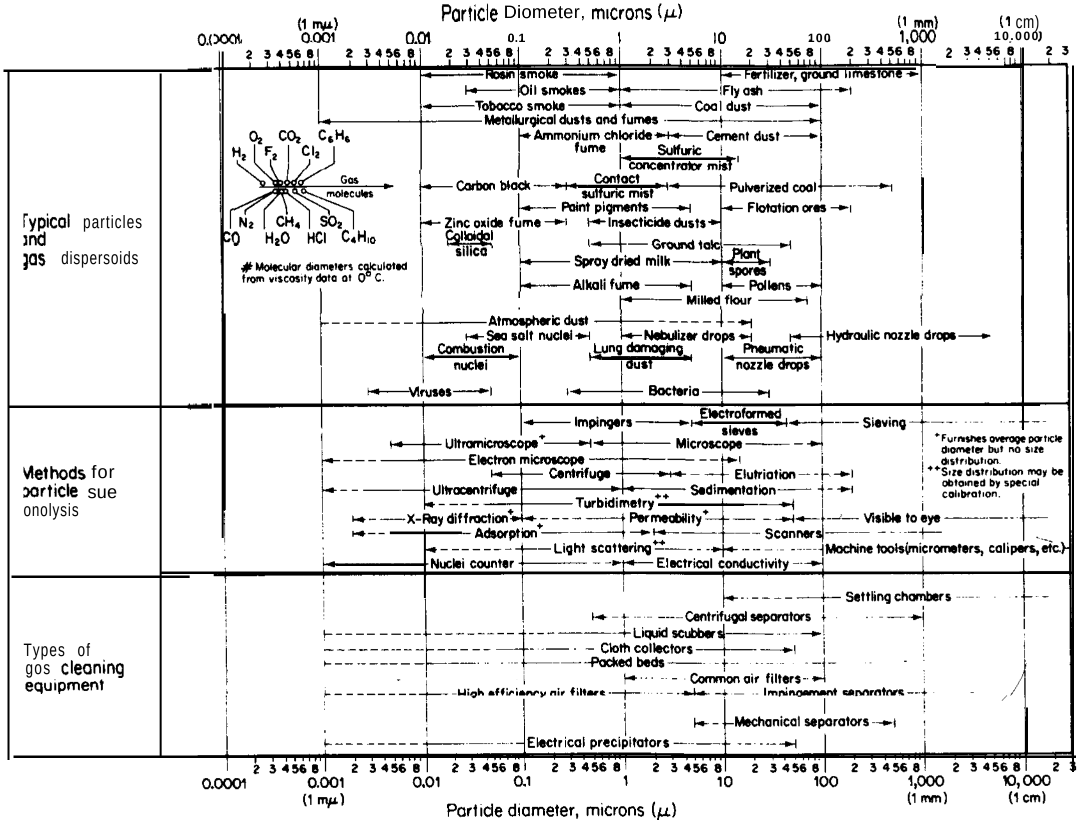

FIGURE 3-2
Characteristics of pollution particulates and control equipment for removal.

selection. Costs for much of the equipment considered in this section are given in Chap. 14 (Materials Transfer, Handling, and Treatment Equipment).

NOXIOUS GAS REMOVAL. Gaseous pollutants can be removed from air streams either by absorption, adsorption, condensation, or incineration. A list of typical gaseous pollutants that can be treated with these four methods is given in Table 9. Generally, condensation is not utilized as a method for removing a solvent vapor from air or other carrier gas unless the concentration of the solvent in the gas is high and the solvent is worth recovery. Since condensation cannot remove all of the solvent, it can only be used to reduce the solvent concentration in the carrier gas.

TABLE 9
Typical gaseous pollutants and their sources

Key element	Pollutant	Source
S	SO_2	Boiler, flue gas
	SO_3	Sulfuric acid manufacture
	H_2S	Natural gas processing, sewage treatment, paper and pulp industry
	R-SH (mercaptans)	Petroleum refining, pulp and paper
N	NO, NO,	Nitric acid manufacturing, high-temperature oxidation processes, nitration processes
	NH_3	Ammonia manufacturing
	Other basic N compounds, pyridines, amines	Sewage, rendering, pyridine base, solvent processes
Halogen:		
F	HF	Phosphate fertilizer, aluminum
	SiF_4	Ceramics, fertilizers
	CFC	Cleaning operations, refrigeration and air conditioning systems, insulation foams
Cl	HCl	HCl mfg., PVC combustion, organic chlorination processes
	Cl_2	Chlorine manufacturing
C	Inorganic	
	CO	Incomplete combustion processes
	CO_2	Combustion processes (not generally considered a pollutant)
	Organic	
	Hydrocarbons-paraffins, olefins, and aromatics	Solvent operations, gasoline, petrochemical operations, solvents
	Oxygenated hydrocarbons-aldehydes ketones, alcohols, phenols, and oxides	Partial oxidation processes, surface coating operations, petroleum processing, plastics, ethylene oxide
	Chlorinated solvents	Dry-cleaning, degreasing operations

Gas-liquid absorption processes are normally carried out in vertical, countercurrent flow through packed, plate, or spray towers. For absorption of gaseous streams, good liquid-gas contact is essential and is partly a function of proper equipment selection. Optimization of absorbers or scrubbers (as applied to noxious gas removal) is also important. The power consumption of a modem, high-energy scrubber at its peak can be considerable because of the high pressure drop involved. The latter difficulty has been alleviated in the spray scrubber with its low pressure drop even when handling large volumes of flue gases. In addition, sealing and plugging which can be a problem in certain scrubbing processes (e.g., using a limestone-slurry removal of sulfur dioxide from a flue gas) do not present difficulties when a spray is used in a chemically balanced system.

The use of dry adsorbents like activated carbon and molecular sieves has received considerable attention in removing final traces of objectional gaseous pollutants. Adsorption is generally carried out in large, horizontal fixed beds often equipped with blowers, condensers, separators, and controls. A typical installation usually consists of two beds; one is onstream while the other is being regenerated.

For those processes producing contaminated gas streams that have no recovery value, incineration may be the most acceptable route when the gas streams are combustible. There are presently two methods in common use: direct flame and catalytic oxidation. The former usually has lower capital-cost requirements, but higher operating costs, particularly if an auxiliary fuel is required. Either method provides a clean, odorless effluent if the exit-gas temperature is sufficiently high.

Each technique for removing pollutants from process gas streams is economically feasible under certain conditions. Each specific instance must be carefully analyzed before a commitment is made to any type of approach.

Water Pollution Abatement

Better removal of pollutants from wastewater effluents was originally mandated by the federal government in the Water Pollution Control Act of 1970 (P.L. 92-500). Since then the performance requirements for the various treatment technologies have been raised to new and higher standards with additional legislation aimed at regulating the amounts of toxic and hazardous substances discharged as effluents. Increased legal and enforcement efforts by various governmental agencies to define toxic and hazardous substances give evidence of the demands that will be placed on pollution technology in the future. The trend in effluent standards is definitely away from the broad, nonspecific parameters (such as chemical oxygen demand or biochemical oxygen demand) and towards limits on specific chemical compounds.

The problems of handling a liquid waste effluent are considerably more complex than those of handling a waste gas effluent. The waste liquid may contain dissolved gases or dissolved solids, or it may be a slurry in either concentrated or dilute form. Because of this complexity, priority should first be given to the possibility of recovering part or all of the waste products for reuse or sale. Frequently, money can be saved by installing recovery facilities rather than more expensive waste-treatment equipment. If product recovery is not capable of solving a given waste-disposal problem, waste treatment must be used. One of the functions of the design engineer then is to decide which treatment process, or combination of processes, will best perform the necessary task of cleaning up the wastewater effluent involved. This treatment can be either physical, chemical, or biological in nature, depending upon the type of waste involved and the amount of removal necessary.

PHYSICAL TREATMENT. The first step in any wastewater treatment process is to remove large floating or suspended particles by sources. This is usually followed by sedimentation or gravity settling. Where sufficient land area is available, earthen basins are sometimes used to remove settleable solids from dilute wastewater. Otherwise, circular clarifiers with rotating sludge scrapers or rectangular clarifiers with continuous chain sludge scrapers are used. These units permit removal of settled sludge from the floor of the clarifiers and scum removal from the surface. Numerous options for improving the operation of clarifiers are presently available.

Sludge from primary or secondary treatment that has been initially concentrated in a clarifier or thickener can be further concentrated by vacuum filtration or centrifugation. The importance of first concentrating a thin slurry by clarifier or thickener action needs to be recognized. For example, concentrating the sludge from 5 to 10 percent solids before centrifuging can result in a 250 percent increase in solids recovery for the same power input to the centrifuge.

Solid-liquid separation by flotation may be achieved by gravity alone or induced by dissolved-air or vacuum techniques. The mechanisms and driving forces are similar to those found in sedimentation, but the separation rate and solids concentration can be greater in some cases.

Adsorption processes, and in particular those using activated carbon, are also finding increased use in wastewater treatment for removal of refractory organics, toxic substances, and color. The primary driving forces for adsorption are a combination of (1) the hydrophobic nature of the dissolved organics in the wastewater and (2) the affinity of the organics to the solid adsorbent. The latter is due to a combination of electrostatic attraction, physical adsorption, and chemical adsorption. Operational arrangements of the adsorption beds are similar to those described for gaseous adsorption.

Three different membrane processes, ultrafiltration, reverse osmosis, and electrodialysis are receiving increased interest in pollution-control applications as end-of-pipe treatment and for inplant recovery systems. There is no sharp distinction between ultrafiltration and reverse osmosis. In the former, the separation is based primarily on the size of the solute molecule which, depending upon the particular membrane porosity, can range from about 2 to 10,000 millimicrons. In the reverse-osmosis process, the size of the solute molecule is not the sole basis for the degree of removal, since other characteristics of the solute such as hydrogen-bonding and valency affect the membrane selectivity. In contrast to these two membrane processes, electrodialysis employs the removal of the solute (with some small amount of accompanying water) from solution rather than the removal of the solvent. The other major distinction is that only ionic species are removed. The advantages due to electrodialysis are primarily due to these distinctions.

CHEMICAL TREATMENT. In wastewater treatment, chemical methods are generally used to remove colloidal matter, color, odors, acids, alkalies, heavy metals, and oil. Such treatment is considered as a means of stream upgrading by coagulation, emulsion breaking, precipitation, or neutralization.

Coagulation is a process that removes colloids from water by the addition of chemicals. The chemicals upset the stability of the system by neutralizing the colloid charge. Additives commonly used introduce a large multivalent cation such as Al^{+3} (from alum) or Fe^{+3} (from ferric chloride). Emulsion breaking is similar to coagulation. The emulsions are generally broken with combination of acidic reagents and polyelectrolytes. The common ion effect can also be useful in wastewater treatment. In this case an unwanted salt is removed from solution by adding a second soluble salt to increase one of the ion concentrations. Coagulant aids may then be needed to remove the precipitate.

One method for treating acid and alkaline waste products is by neutralization with lime or sulfuric acid (other available materials may also be suitable). Even though this treatment method may change the pH of the waste stream to the desired level, it does not remove the sulfate, chloride, or other ions. Therefore, the possibility of recovering the acid or alkali by distillation, concentration, or in the form of a useful salt should always be considered before neutralization or dilution methods are adopted.

Chemical oxidation is frequently another tool in wastewater treatment. Chemical oxidants in wide use today are chlorine, ozone, and hydrogen peroxide. The historical use of chlorine and ozone has been in the disinfection of water and wastewater. All three oxidizers are, however, receiving increased attention for removing organic materials from wastewaters that are resistant to biological or other treatment processes. The destruction of cyanide and phenols by chlorine oxidation is well known in waste-treatment technology. However, the use of chlorine for such applications has come under intense scrutiny because of the uncertainty in establishing and predicting the products of the chlorine oxidation reactions and their relative toxicity. Ozone, on the other hand, with only a short half-life is found to be effective in many applications for color removal, disinfection, taste and odor removal, iron and manganese removal, and in the oxidation of many complex inorganics, including lindane, aldrin, surfactants, cyanides, phenols, and organo-metal complexes. With the latter, the metal ion is released and can be removed by precipitation.

The need for chemical reduction of wastewaters occurs less often. The most common reducing agents are ferrous chloride or sulfate which may be obtained from a variety of sources.

BIOLOGICAL TREATMENT. In the presence of the ordinary bacteria found in water, many organic materials will oxidize to form carbon dioxide, water, sulfates, and similar materials. This process consumes the oxygen dissolved in the water and may cause a depletion of dissolved oxygen. A measure of the ability of a waste component to consume the oxygen dissolved in water is known as the *biochemical* oxygen *demand*. The biochemical oxygen demand (BOD) of a waste stream is often the primary factor that determines its importance as a pollutant. The biochemical oxygen demand of sewage, sewage effluents, polluted waters, or industrial wastes is the oxygen, reported as parts per million, consumed during a set period of time by bacterial action on the decomposable organic matter.

One of the more common biological wastewater treatment procedures today involves the use of concentrated masses of microorganisms to break down organic matter, resulting in the stabilization of organic wastes. These organisms are broadly classified as aerobic, anaerobic, and aerobic-anaerobic facultative. Aerobic organisms. require molecular oxygen for metabolism, anaerobic organisms derive energy from organic compounds and function in the absence of oxygen, while facultative organisms may function in either an aerobic or anaerobic environment.

Basically, the aerobic biological processes involve either the activated sludge process or the fixed-film process. The activated sludge process is a continuous system in which aerobic biological growths are mixed with wastewater and the resulting excess flocculated suspension separated by gravity clarification or air flotation. The predominant fixed-film process has, in the past, been the conventional trickling filter. In this process, wastewater trickles over a biological film fixed to an inert medium. Bacterial action in the presence of oxygen breaks down the organic pollutants in the wastewater. (An attempt to improve on the biological efficiency of the fixed-film process has resulted in the development of a useful rotating disk biological contactor.) A comparison of biological loading and area requirements for various aerobic biological processes is shown in Table 10.

Many organic industrial wastes (including those from food processing, meat packing, pulp and paper, refining, leather tanning, textiles, organic chemicals, and petrochemicals) are amenable to biological treatment; however, a fair number may prove to be refractory, i.e., nonbiodegradable. Thus, although the BOD removal may be excellent, the removal of the *chemical oxygen demand* (COD) may be low. Process evaluation prior to system design should center on characterization of the waste stream, particularly to determine the presence of inhibitory or toxic components relative to biological treatment, and the establishment of pollutant removal rates, oxygen requirements, nutrient requirements (nitrogen and phosphorus), sludge production, and solids settleability.

Anaerobic treatment is important in the disposal of municipal wastes but has not been widely used on industrial wastes. It has found some use in reducing highly concentrated BOD wastes, particularly in the food and beverage industries as a pretreatment or roughing technique. However, compared to

TABLE 10
Comparative requirements for processing 1700 lb BOD / day by various aerobic systems

System	Area, acres	Biological loading lb BOD/1000 ft^3	BOD removal, %
Stabilization pond	57†	0.09-0.23	70-90
Aerated lagoon	5.75‡	1.15-1.60	80-90
Activated sludge			
Extended	0.23	11.0-30.0	95‡
Conventional	0.08	33.0-400	90
High rate	0.046	57.0-150	70
Trickling filter			
Rock	0.2-0.5	0.7-50	40-70
Plastic disks	0.02-0.05	20-200	50-70

† 5 ft deep
‡ 10 ft deep

aerobic systems, anaerobic treatment is more sensitive to toxic materials and is more difficult to control.

Solid Waste Disposal

Solid wastes differ from air and water pollutants since these wastes remain at the point of origin until a decision is made to collect and dispose of them. There are several means of disposal available including recycling, chemical conversion, incineration, pyrolysis, and landfill. Federal regulations, local conditions, and overall economics generally determine which method is the most acceptable.

RECYCLING AND CHEMICAL CONVERSION. Resource recovery is a factor often overlooked in waste disposal. For example, specific chemicals may often be recovered by stripping, distillation, leaching, or extraction. Valuable solids such as metals and plastics can be recovered by magnets, electrical conductivity, jigging, flotation, or hand picking. Process wastes may at times also be converted to saleable products or innocuous materials that can be disposed of safely. The former would include hydrogenation of organics to produce fuels, acetylation of waste cellulose to form cellulose acetate, or nitrogen and phosphorus enrichment of wastes to produce fertilizer.

INCINERATION. The controlled oxidation of solid, liquid, or gaseous combustible wastes to final products of carbon dioxide, water, and ash is known as incineration. Since sulfur and nitrogen-containing waste materials will produce. their corresponding oxides, they should not be incinerated without considering their effect on air quality. A variety of incinerator designs are available.

Multiple-chamber incinerators, rotary kilns, and multiple-hearth furnaces are most widely used in industrial waste disposal.

Incineration has in the past provided certain advantages, particularly where land disposal sites are not available or are too remote for economic hauling. A properly designed and carefully operated incinerator can be located adjacent to a process plant and can be adjusted to handle a variety and quantity of wastes. Not only can heat recovery through steam generation reduce operating costs, but it can also save on pollution-control equipment. Additionally, the residue is a small fraction of the original weight and volume of the waste and may be acceptable for landfill.

PYROLYSIS. The most acceptable route to recycling wastes in the future may be through pyrolytic techniques in which wastes are heated in an air-free chamber at temperatures as high as 3000°F (1650°C). Pyrolysis seems to provide several advantages over incineration. These systems encounter far fewer air-pollution problems, handle larger throughputs resulting in lower capital costs, provide their own fuel, degrade marginally burnable materials, and have the added potential for recovering chemicals or synthesis gas.

LANDFILL. Sanitary landfill is basically a simple technique that involves spreading and compacting solid wastes into cells that are covered each day with soil. Care needs to be exercised that wastes disposed of in this fashion are either inert to begin with or are capable of being degraded by microbial attack to harmless compounds. The principal problems encountered in landfill operation are the production of leachates that may contaminate the surrounding groundwater and the potential hazards associated with the accumulation of flammable gases produced during the degradation of the waste material.

Thermal Pollution Control

Temperature affects nearly every physical property of concern in water quality management including density, viscosity, vapor pressure, surface tension, gas solubility, and gas diffusion. The solubility of oxygen is probably the most important of these parameters, inasmuch as dissolved oxygen in water is necessary to maintain many forms of aquatic life. This potential damage to the aquatic environment by changes in temperature, the reduction in the assimilative capacity of organic wastes due to increased temperature, and the federal enactment of more stringent water-temperature standards has led design engineers to investigate various offstream cooling systems to handle thermal discharges from processes and plants. Cooling towers are most often considered for this service, followed by cooling ponds and spray ponds in that order.

Cooling towers may be classified on the basis of the fluid used for heat transfer and on the basis of the power supplied to the unit. In wet cooling towers, the condenser cooling water and ambient air are intimately mixed. Cooling results from the evaporation of a portion of the water and to a lesser extent from the loss of sensible heat to the air. In dry cooling towers, the temperature reduction of the condenser water depends upon conduction and convection for the transfer of heat from the water to the air.

Mechanical draft cooling towers either force or induce the air which serves as the heat-transfer medium through the tower. For their driving force, natural draft cooling towers depend upon the density difference between the air leaving the tower and the air entering the tower.

Cooling ponds are generally only considered for heat removal when suitable land is available at a reasonable price, since such systems are simple, cheap, and frequently less water-intensive. It is normally assumed that all heat discharged to a cooling pond is lost through the air-water interface. With low heat-transfer rates, large surface areas are required.

When land costs are too high, spray ponds often provide a viable alternative to cooling ponds. It is estimated that a spray pond requires only about 5 to 10 percent of the area of a cooling pond due to the more intimate air-water contact. In addition, drift losses and corrosion problems are less severe than in cooling towers.

NOISE CONTROL. The design engineer should include noise studies in the design stage of any industrial facility. Generally, acoustical problems left for field resolution cost roughly twice as much. Unnecessary costs incurred in post-construction noise work may include the replacement of insulation, re-

TABLE 11
Equipment noise sources, levels, and potential control solutions

Equipment	Sound level in dBA† at 3 ft	Possible noise control treatments
Air coolers	87-94	Aerodynamic fan blades, decrease rev./min and increase pitch, tip and hub seals, decrease pressure drop.
Compressors	90-120	Install mufflers on intake and exhaust, enclosure of machine casing, vibration isolation and lagging of piping systems.
Electric motors	90-110	Acoustically lined fan covers, enclosures, and motor mutes.
Heaters and furnaces	95-110	Acoustic plenums, intake mufflers, ducts lined and damped.
Valves	< 80 to 108	Avoid sonic velocities, limit pressure drop, and mass flow, replace with special low noise valves, vibration isolation and lagging.
Piping	90-105	Inline silencers, vibration isolation and lagging.

† Defined as the sound intensity measured in units equal to ten times the logarithm of the square of the relative pressure associated with the sound wave.

design of piping configuration to accommodate silencers, modification of equipment, additional labor costs, and possible downtime to make necessary changes. Considerable judgment, therefore, must be exercised by the designer to establish final design-stage noise recommendations. These should not only consider the results of the equipment data-analysis procedure, but should also recognize additional factors such as administrative controls, feasibility of redesign, economic alternatives, intrusion of noise into the community, and the basic limitations of the equations employed in the applicable computer programs.

To attain efficient, effective, and practical noise control, it is necessary to understand the individual equipment or process noise sources, their acoustic properties and characteristics, and how they interact to create the overall noise situation. Table 11 presents typical process design equipment providing high noise levels and potential solutions to this problem.

PLANT LOCATION

The geographical location of the final plant can have strong influence on the success of an industrial venture. Considerable care must be exercised in selecting the plant site, and many different factors must be considered. Primarily, the plant should be located where the minimum cost of production and distribution can be obtained, but other factors, such as room for expansion and safe living conditions for plant operation as well as the surrounding community, are also important.

A general consensus as to the plant location should be obtained before a design project reaches the detailed estimate stage, and a firm location should be established upon completion of the detailed-estimate design. The choice of the final site should first be based on a complete survey of the advantages and disadvantages of various geographical areas and, ultimately, on the advantages and disadvantages of available real estate. The following factors should be considered in selecting a plant site:

1. Raw materials availability
2. Markets
3. Energy availability
4. Climate
5. Transportation facilities
6. Water supply
7. Waste disposal
8. Labor supply
9. Taxation and legal restrictions
10. Site characteristics
11. Flood and fire protection
12. Community factors

The factors that must be evaluated in a plant-location study indicate the need for a vast amount of information, both quantitative (statistical) and qualitative. Fortunately, a large number of agencies, public and private, publish useful information of this type greatly reducing the actual original gathering of the data.

Raw materials availability. The source of raw materials is one of the most important factors influencing the selection of a plant site. This is particularly true if large volumes of raw materials are consumed, because location near the raw-materials source permits considerable reduction in transportation and storage charges. Attention should be given to the purchased price of the raw materials, distance from the source of supply, freight or transportation expenses, availability and reliability of supply, purity of the raw materials, and storage requirements.

Markets. The location of markets or intermediate distribution centers affects the cost of product distribution and the time required for shipping. Proximity to the major markets is an important consideration in the selection of a plant site, because the buyer usually finds it advantageous to purchase from nearby sources. It should be noted that markets are needed for by-products as well as for major final products.

Energy availability. Power and steam requirements are high in most industrial plants, and fuel is ordinarily required to supply these utilities. Consequently, power and fuel can be combined as one major factor in the choice of a plant site. Electrolytic processes require a cheap source of electricity, and plants using electrolytic processes are often located near large hydroelectric installations. If the plant requires large quantities of coal or oil, location near a source of fuel supply may be essential for economic operation. The local cost of power can help determine whether power should be purchased or self-generated.

Climate. If the plant is located in a cold climate, costs may be increased by the necessity for construction of protective shelters around the process equipment, and special cooling towers or air-conditioning equipment may be required if the prevailing temperatures are high. Excessive humidity or extremes of hot or cold weather can have a serious effect on the economic operation of a plant, and these factors should be examined when selecting a plant site.

Transportation facilities. Water, railroads, and highways are the common means of transportation used by major industrial concerns. The kind and amount of products and raw materials determine the most suitable type of transportation facilities. In any case, careful attention should be given to local freight rates and existing railroad lines. The proximity to railroad centers and the possibility of canal, river, lake, or ocean transport must be considered: Motor trucking facilities are widely used and can serve as a useful supplement to rail and water facilities. If possible, the plant site should have access to all three types of transportation, and, certainly, at least two types should be available. There is usually need for convenient air and rail transportation facilities between the plant and the main company headquarters, and effective transportation facilities for the plant personnel are necessary.

Water supply. The process industries use large quantities of water for cooling, washing, steam generation, and as a raw material. The plant, therefore, must be located where a dependable supply of water is available. A large river or lake is preferable, although deep wells or artesian wells may be satisfactory if the amount of water required is not too great. The level of the existing water table can be checked by consulting the state geological survey, and information on the constancy of the water table and the year-round capacity of local rivers or lakes should be obtained. If the water supply shows seasonal fluctuations, it may be desirable to construct a reservoir or to drill several standby wells. The temperature, mineral content, silt or sand content, bacteriological content, and cost for supply and purification treatment must also be considered when choosing a water supply.

Waste disposal. In recent years, many legal restrictions have been placed on the methods for disposing of waste materials from the process industries. The site selected for a plant should have adequate capacity and facilities for correct waste disposal. Even though a given area has minimal restrictions on pollution, it should not be assumed that this condition will continue to exist. In choosing a plant site, the permissible tolerance levels for various methods of waste disposal should be considered carefully, and attention should be given to potential requirements for additional waste-treatment facilities.

Labor supply. The type and supply of labor available in the vicinity of a proposed plant site must be examined. Consideration should be given to prevailing pay scales, restrictions on number of hours worked per week, competing industries that can cause dissatisfaction or high turnover rates among the workers, and variations in the skill and productivity of the workers.

Taxation and legal restrictions. State and local tax rates on property income, unemployment insurance, and similar items vary from one location to another. Similarly, local regulations on zoning, building codes, nuisance aspects, and transportation facilities can have a major influence on the final choice of a plant site. In fact, zoning difficulties and obtaining the many required permits can often be much more important in terms of cost and time delays than many of the factors discussed in the preceding sections.

Site characteristics. The characteristics of the land at a proposed plant site should be examined carefully. The topography of the tract of land and’ the soil structure must be considered, since either or both may have a pronounced effect on construction costs. The cost of the land is important, as well as local building costs and living conditions. Future changes may make it desirable or necessary to expand the plant facilities. Therefore, even though no immediate expansion is planned, a new plant should be constructed at a location where additional space is available.

Flood and fire protection. Many industrial plants are located along rivers or near large bodies of water, and there are risks of flood or hurricane damage. Before selecting a plant site, the regional history of natural events of this type should be examined and the consequences of such occurrences considered. Protection from losses by fire is another important factor in selecting a plant location. In case of a major fire, assistance from outside fire departments should be available. Fire hazards in the immediate area surrounding the plant site must not be overlooked.

Community factors. The character and facilities of a community can have quite an effect on the location of the plant. If a certain minimum number of facilities for satisfactory living of plant personnel do not exist, it often becomes a burden for the plant to subsidize such facilities. Cultural facilities of the community are important to sound growth. Churches, libraries, schools, civic theaters, concert associations, and other similar groups, if active and dynamic, do much to make a community progressive. The problem of recreation deserves special consideration. The efficiency, character, and history of both state and local government should be evaluated. The existence of low taxes is not in itself a favorable situation unless the community is already well developed and relatively free of debt.

Selection of the Plant Site

The major factors in the selection of most plant sites are (1) raw materials, (2) markets, (3) energy supply, (4) climate, (5) transportation facilities, and (6) water supply. For a preliminary survey, the first four factors should be considered. Thus, on the basis of raw materials, markets, energy supply, and climate, acceptable locations can usually be reduced to one or two general geographical regions. For example, a preliminary survey might indicate that the best location for a particular plant would be in the south-central or south-eastern part of the United States.

In the next step, the effects of transportation facilities and water supply are taken into account. This permits reduction of the possible plant location to several general target areas. These areas can then be reduced further by considering all the factors that have an influence on plant location.

As a final step, a detailed analysis of the remaining sites can be made. Exact data on items such as freight rates, labor conditions, tax rates, price of land, and general local conditions can be obtained. The various sites can be inspected and appraised on the basis of all the factors influencing the final decision. Many times, the advantages of locating a new plant on land or near other facilities already owned by the concern that is building the new plant outweigh the disadvantages of the particular location. In any case, however, the final decision on selecting the plant site should take into consideration all the factors that can affect the ultimate success of the overall operation.

PLANT LAYOUT

After the process flow diagrams are completed and before detailed piping, structural, and electrical design can begin, the layout of process units in a plant and the equipment within these process units must be planned. This layout can play an important part in determining construction and manufacturing costs, and thus must be planned carefully with attention being given to future problems that may arise. Since each plant differs in many ways and no two plant sites are exactly alike, there is no one ideal plant layout. However, proper layout in each case will include arrangement of processing areas, storage areas, and handling areas in efficient coordination and with regard to such factors as:

1. New site development or addition to previously developed site
2. Type and quantity of products to be produced
3. Type of process and product control
4. Operational convenience and accessibility
5. Economic distribution of utilities and services
6. Type of buildings and building-code requirements
7. Health and safety considerations
8. Waste-disposal requirements
9. Auxiliary equipment
10. Space available and space required
11. Roads and railroads
12. Possible future expansion

Preparation of the Layout

Scale drawings, complete with elevation indications can be used for determining the best location for equipment and facilities. Elementary layouts are developed first. These show the fundamental relationships between storage space and operating equipment. The next step requires consideration of the safe operational sequence and gives a primary layout based on the flow of materials, unit operations, storage, and future expansion. By analyzing all the factors that are involved in plant layout, a detailed recommendation can be presented, and drawings and elevations, including isometric drawings of the piping systems, can be prepared.

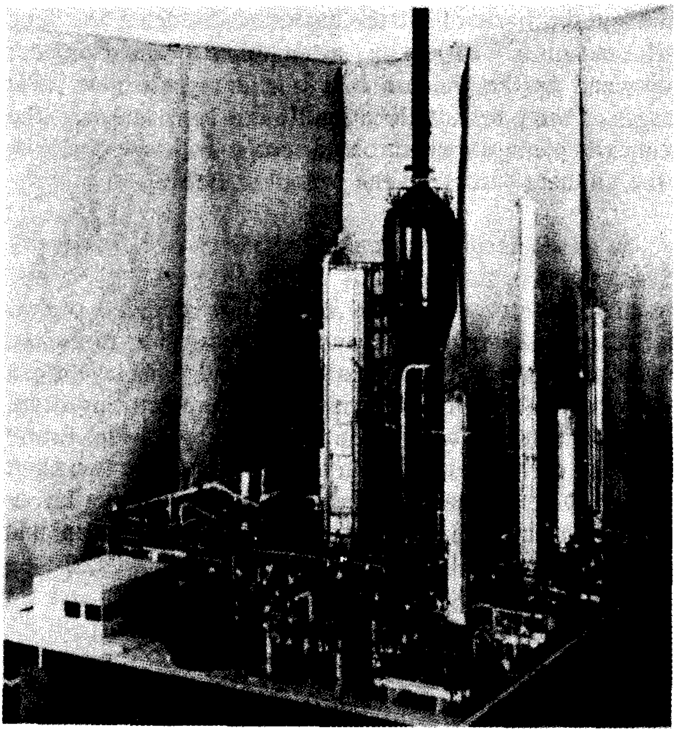

FIGURE 33
Scale model showing details of plant layout. (*Courtesy* of the *M. W. Kellogg Company.)*

Templates, or small cutouts constructed to a selected scale, are useful for making rapid and accurate layouts, and three-dimensional models are often made. The use of such models for making certain a proposed plant layout is correct has found increasing favor in recent years.

Figure 3-3 shows a view of a typical model for an industrial plant. Errors in a plant layout are easily located when three-dimensional models are used, since the operations and construction engineers can immediately see errors which might have escaped notice on two-dimensional templates or blueprints. In addition to increasing the efficiency of a plant layout, models are very useful during plant construction and for instruction and orientation purposes after the plant is completed.

PLANT OPERATION AND CONTROL

In the design of an industrial plant, the methods which will be used for plant operation and control help determine many of the design variables. For example, the extent of instrumentation can be a factor in choosing the type of process and setting the labor requirements. It should be remembered that maintenance work will be necessary to keep the installed equipment and facilities in good operating condition. The engineer must recognize the importance of such factors which are directly related to plant operation and control and must take them into proper account during the development of a design project.

Instrumentation

Instruments are used in the chemical industry to measure process variables, such as temperature, pressure, density, viscosity, specific heat, conductivity, pH, humidity, dew point, liquid level, flow rate, chemical composition, and moisture content. By use of instruments having varying degrees of complexity, the values of these variables can be recorded continuously and controlled within narrow limits.

Automatic control is the norm throughout the chemical industry, and the resultant savings in labor combined with improved ease and efficiency of operations has more than offset the added expense for instrumentation. (In most cases, control is achieved through the use of high-speed computers. In this capacity, the computer serves as a vital tool in the operation of the plant.) Effective utilization of the many instruments employed in a chemical process is achieved through centralized control, whereby one centrally located control room is used for the indication, recording, and regulation of the process variables. Panel boards have been developed which present a graphical representation of the process and have the instrument controls and indicators mounted at the appropriate locations in the overall process. This helps a new operator to quickly become familiar with the significance of the instrument readings, and rapid location of any operational variance is possible. An example of a graphic panel in a modern industrial plant is shown in Fig. 3-4.

Because of the many variables found in processing and the wide range over which these variables must be determined and controlled, the assistance of a skilled instrumentation engineer is essential in setting up a control system. Instrumentation problems caused by transmission lags, cycling due to slow or uncompensated response, radiation errors, or similar factors are commonly encountered in plant operation, but most of these problems can be eliminated if the control system is correctly designed.

Maintenance

Many of the problems involved in maintenance are often caused by a lack of thoroughly evaluating the original design and layout of plant and equipment. Sufficient space for maintenance work on equipment and facilities must be provided in the plant layout, and the engineer needs to consider maintenance and its safety requirements when making decisions on equipment.

Too often, the design engineer is conscious only of first costs and fails to recognize that maintenance costs can easily nullify the advantages of a cheap

FIGURE 3-4
Example of a graphic panel for a modern industrial plant with a computer-controlled system. *(Courtesy of C. F. Braun and Company.)*

initial installation. For example, a close-coupled motor pump utilizing a high-speed motor may require less space and lower initial cost than a standard motor combined with a coupled pump. However, if replacement of the impeller and shaft becomes necessary, the repair cost with a close-coupled motor pump is much greater than with a regular coupled pump. The use of a high-speed motor reduces the life of the impeller and shaft, particularly if corrosive liquids are involved. If the engineer fails to consider the excessive maintenance costs that may result, an error in recommending the cheaper and smaller unit can be made. Similarly, a compact system of piping, valves, and equipment may have a lower initial cost and be more convenient for the operators' use, but maintenance of the system may require costly and time-consuming dismantling operations.

UTILITIES

The primary sources of raw energy for the supply of power are found in the heat of combustion of fuels and in elevated water supplies. Fuel-burning plants are of greater industrial significance than hydroelectric installations because the physical location of fuel-burning plants is not restricted. At the present time, the most common sources of energy are oil, gas, coal, and nuclear energy. The decreasing availability of the first two sources of energy will necessitate the use of alternate forms of energy in the not-too-distant future.

In the chemical industries, power is supplied primarily in the form of electrical energy. Agitators, pumps, hoists, blowers, compressors, and similar equipment are usually operated by electric motors, although other prime movers, such as steam engines, internal-combustion engines, and hydraulic turbines are sometimes employed.

When a design engineer is setting up the specifications for a new plant, a decision must be made on whether to use purchased power or have the plant set up its own power unit. It may be possible to obtain steam for processing and heating as a by-product from the self-generation of electricity, and this factor may influence the final decision. In some cases , it may be justified by means of a HAZOP study to provide power to the plant from two independent sources to permit continued operation of the plant facilities if one of the power sources fails.

Power can be transmitted in various forms, such as mechanical energy, electrical energy, heat energy, and pressure energy. The engineer should recognize the different methods for transmitting power and must choose the ones best suited to the particular process under development.

Steam is generated from whatever fuel is the cheapest, usually at pressures of 450 psig (3100 kPa) or more, expanded through turbines or other prime movers to generate the necessary plant power, and the exhaust steam is used in the process as heat. The quantity of steam used in a process depends upon the thermal requirements, plus the mechanical power needs, if such a power is generated in the plant.

Water for industrial purposes can be obtained from one of two general sources: the plant's own source or a municipal supply. If the demands for water are large, it is more economical for the plant to provide its own water source. Such a supply may be obtained from drilled wells, rivers, lakes, damned streams, or other impounded supplies. Before a company agrees to go ahead with any new project, it must ensure itself of a sufficient supply of water for all industrial, sanitary, and safety demands, both present and future.

The value of an abundance of good water supplies is reflected in the selling price of plant locations that have such supplies. Any engineering techniques which are required to procure, conserve, and treat water significantly increase the operational cost for a plant or process. Increased costs of water processing have made maximum use of the processed water essential. In fact, the high costs of constructing and operating a waste treatment plant have led to concentration of industrial wastes with the smallest amount of water, except where treatment processes require dilution.

STRUCTURAL DESIGN

One of the most important aspects in structural design for the process industries is a correct foundation design with allowances for heavy equipment and vibrating machinery used. The purpose of the foundation is to distribute the load so that excessive or damaging settling will not occur. The type of foundation depends on the load involved and the material on which the foundation acts. It is necessary, therefore, to know the characteristics of the soil at a given plant site before the structural design can be started.

The allowable bearing pressure varies for different types of soils, and the soil should be checked at the surface and at various depths to determine the bearing characteristics. The allowable bearing pressure for rock is 30 or more ton/ft^2 (30×10^4 kg/m^2), while that for soft clay may be as low as 1 ton/ft^2 (1×10^4 kg/m^2). Intermediate values of 4 to 10 ton/ft^2 (4×10^4 to 10×10^4 kg/m^2) apply for mixtures of gravel with sand, hard clay, and hardpan.

A foundation may simply be a wall founded on rock or hardpan, or it may be necessary to increase the bearing area by the addition of a footing. Plain concrete is usually employed for making footings, while reinforced concrete, containing steel rods or bars, is commonly used for foundation walls. If possible, a foundation should extend below the frost line, and it should always be designed to handle the maximum load. Pilings are commonly used for supporting heavy equipment or for other special loads.

Maintenance difficulties encountered with floors and roofs should be given particular attention in a structural design. Concrete floors are used extensively in the process industries, and special cements and coatings are available which make the floors resistant to heat or chemical attack. Flat roofs are often specified for industrial structures. Felt saturated with coal-tar pitch combined with a coal-tar pitch-gravel finish is satisfactory for roofs of this type. Asphalt-saturated felt may be used if the roof has a slope of more than $\frac{1}{2}$in./ft (4.17 cm/m).

Corrosive effects of the process, cost of construction, and climatic effects must be considered when choosing structural materials. Steel and concrete are the materials of construction most commonly used, although wood, aluminum, glass blocks, cinder blocks, glazed tile, bricks, and other materials are also of importance. Allowances must be made for the type of lighting and drainage, and sufficient structural strength must be provided to resist normal loads as well as extreme loads due to high winds or other natural causes.

In any type of structural design for the process industries, the function of the structure is more important than the form. The style of architecture should be subordinated to the need for supplying a structure which is adapted to the proposed process and has sufficient flexibility to permit changes in the future. Although cost is certainly important, the engineer preparing the design should never forget the fact that the quality of a structure remains apparent long after the initial cost is forgotten.

STORAGE

Adequate storage facilities for raw materials, intermediate products, final products, recycle materials, off-grade materials, and fuels are essential to the operation of a process plant. A supply of raw materials permits operation of the process plant regardless of temporary procurement or delivery difficulties. Storage of intermediate products may be necessary during plant shutdown for emergency repairs while storage of final products makes it possible to supply the customer even during a plant difficulty or unforeseen shutdown. An additional need for adequate storage is often encountered when it is necessary to meet seasonal demands from steady production.

Bulk storage of liquids is generally handled by closed spherical or cylindrical tanks to prevent the escape of volatiles and minimize contamination. Since safety is an important consideration in storage-tank design, the American Petroleum Institute? and the National Fire Protection Association* publish rules for safe design and operation. Floating roof tanks are used to conserve valuable products with vapor pressures which are below atmospheric pressure at the storage temperature. Liquids with vapor pressures above atmospheric must be stored in vapor-tight tanks capable of withstanding internal pressure. If flammable liquids are stored in vented tanks, flame arresters must be installed in all openings except connections made below the liquid level.

Gases are stored at atmospheric pressure in wet- or dry-seal gas holders. The wet-gas holder maintains a liquid seal of water or oil between the top movable inside tank and the stationary outside tank. In the dry-seal holder the seal between the two tanks is made by means of a flexible rubber or plastic curtain. Recent developments in bulk natural gas or gas-product storage show that pumping the gas into underground strata is the cheapest method available. High-pressure gas is stored in spherical or horizontal cylindrical pressure vessels.

Solid products and raw materials are either stored in weather-tight tanks with sloping floors or in outdoor bins and mounds. Solid products are often packed directly in bags, sacks, or drums.

MATERIALS HANDLING

Materials-handling equipment is logically divided into continuous and batch types, and into classes for the handling of liquids, solids, and gases. Liquids and gases are handled by means of pumps and blowers; in pipes, flumes, and ducts; and in containers such as drums, cylinders, and tank cars. Solids may be handled by conveyors, bucket elevators, chutes, lift trucks, and pneumatic systems. The selection of materials-handling equipment depends upon the cost and the work to be done. Factors that‘must be considered in selecting such equipment include:

1. Chemical and physical nature of material being handled
2. Type and distance of movement of material
3. Quantity of material moved per unit time
4. Nature of feed and discharge from materials-handling equipment
5. Continuous or intermittent nature of materials handling

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†American Petroleum Institute, 50 W. 50th St., New York, NY.

‡National Fire Protection Association, 60 Batterymarch St., Boston, MA.

The major movement of liquid and gaseous raw materials and products within a plant to and from the point of shipment is done by pipeline. Many petroleum plants also transport raw materials and products by pipeline. When this is done, local and federal regulations must be strictly followed in the design and specification of the pipeline.

Movement of raw materials and products outside of the plant is usually handled either by rail, ship, truck, or air transportation. Some type of receiving or shipping facilities, depending on the nature of the raw materials and products, must be provided in the design of the plant. Information for the preparation of such specifications can usually be obtained from the transportation companies serving the area.

In general, the materials-handling problems in the chemical engineering industries do not differ widely from those in other industries except that the existence of special hazards, including corrosion, fire, heat damage, explosion, pollution, and toxicity, together with special service requirements, will frequently influence the design. The most difficult of these hazards often is corrosion. This is generally overcome by the use of a high-first-cost, corrosion-resistant material in the best type of handling equipment or by the use of containers which adequately protect the equipment.

PATENT CONSIDERATIONS

A patent is essentially a contract between an inventor and the public. In consideration of full disclosure of the invention to the public, the patentee is given exclusive rights to control the use and practice of the invention. A patent gives the holder the power to prevent others from using or practicing the invention for a period of 17 years from the date of granting. In contrast, trade secrets and certain types of confidential disclosures can receive protection under common-law rights only as long as the secret information is not public knowledge.

A new design should be examined to make certain no patent infringements are involved. If the investigation can uncover even one legally expired patent covering the details of the proposed process, the method can be used with no fear of patent difficulties. Although most large corporations have patent attorneys to handle investigations of this type, the design engineer can be of considerable assistance in determining if infringements are involved. An engineer, therefore, should have a working knowledge of the basic practices and principles of patent law.†

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†Statutes and general rules applying to United States patents are presented in the two pamphlets "Patent Laws" and "Rules of Practice of the United States Patent Office in Patent Cases," U.S. Government Printing Office.

PROBLEMS?

1. Identify some of the principal events leading to spillage and leakage in petroleum refineries and/or chemical plants. Provide specific examples, if possible, to support the selections.
2. Approximately 25 kg/s of vapor are released to the atmosphere. The lower explosive limit C_L for the vapor is 0.010 m^3 vapor/m' air. The vapor density of the vapor is 2 kg/m^3. If the wind speed U_v is 3 m/s, what is the distance d required for safe disposal of the vapor in the air as evaluated from the relation

$$d = \left(\frac{37Q}{U_v C_L}\right)^{0.552}$$

where Q is the escape rate in m^3/s?

3. A mercury spill occurs in an unventilated stockroom with some of the mercury trapped in several floor cracks. What is the maximum concentration of mercury that can be attained in the storeroom when the temperature is 23°C? If the Federal standard permissible exposure limit for mercury in the air is 0.1 mg/m^3, is the maximum concentration of mercury in the air acceptable?
4. Benzene has a PEL of 1 ppm for an 8-h exposure. If liquid benzene is evaporating at a rate of 2 ml/min in air whose temperature and pressure are 72°F and 12.8 psia, respectively, what must the ventilation rate of the air be to keep the benzene concentration below the PEL value?
5. Oxygen-deficient atmospheres can develop in closed containers by a number of seemingly harmless mechanisms. Corrosion inside of a closed container is such a mechanism. The typical corrosion rate for carbon steel in very moist air is about 0.005 in./yr and is about first order with respect to the oxygen concentration. If the corrosion reaction can be approximated by the reaction,

$$2Fe + \tfrac{3}{2}O_2 = Fe_2O_3$$

how long will it take to reduce the oxygen content in a closed 25-ft-diameter spherical container from 21 to 19.5 mol%?

6. Vinyl chloride, a suspected carcinogen, evaporates into the air at a rate of 7.5 g/min. If the permissible exposure limit (PEL) is 1 ppm, what flow rate of air will be necessary to achieve the PEL utilizing dilution ventilation? Assuming there is insufficient mixing of the vinyl chloride and the dilution air, a safety factor of 5 or more needs to be used to assure proper dilution. Under these conditions what flow rate is necessary?
7. Chemical cartridge respirators are personal protective devices used to adsorb harmful vapors and gases. A performance analysis of such respirators is similar to that used in a fixed bed adsorber. The generalized correlation of adsorption potential

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†A number of the problems are adapted from Safety, Health, and Loss Prevention in Chemical Processes: Problems for Undergraduate Engineering Curricula, © 1990 by the American Institute of Chemical Engineers, with permission from the Center for Chemical Process Safety/AIChE.

shows that the logarithm of the amount adsorbed is linear with the function $T/V [\log_{10}(f_s/f)]$ where T is the temperature in K, V is the molar volume of liquid at the normal boiling point in cm^3/g mole, f_s is the saturated fugacity (approximate as the vapor pressure), and f is the fugacity of the vapor (approximate as the partial pressure). Data for the adsorption of dichloropropane on a selected charcoal are as follows:

Amount adsorbed ($cm^3_{liq}/100\ g_{charcoal}$)	$T/V[\log_{10}(f_s/f)]$ (units as given)
1	21
10	11

If a respirator contains 100 g of this charcoal and breakthrough occurs when 82 percent of the adsorbent is saturated, how long can the respirator be used in a dichloropropane concentration of 750 ppm when the temperature is 80°F? Assume that the flow rate of contaminated air through the adsorbent is 45 l/min. The molar volume of the dichloropropane may be assumed to be 100 cm^3/g mole.

8. Determine which of the following liquids used in the laboratory could form flammable air-vapor mixtures if spilled in a storage cabinet and allowed to reach equilibrium.

acetone
benzene
carbon disulfide
ethyl ether
methyl alcohol
n-pentane

What type of fire extinguishers would be appropriate if a small spill is ignited? Note that if equilibrium is not attained, the vapor concentrations will be lower than calculated and a flammable air-vapor mixture could be present.

9. Estimate the flash-point of acetone and compare it with the experimental value given in the literature. *Hint:* Start with the basic principle that the fugacity in the vapor phase must equal that in the liquid phase. The lower flammable limit for acetone is 2.55 percent by volume.
10. The flash-point of a liquid mixture can be estimated by determining the temperature at which the equilibrium concentration of the flammable vapors in the air reach a concentration such that $\sum(y_i/LFL_i) = 1.0$ where y_i is the vapor phase mole percent of component i and LFL_i is the lower flammability limit concentration of component i expressed in mole percent. Estimate the flash-point of a liquid mixture containing 50 mol% n-octane and 50 mol% n-nonane. The LFL values for n-octane and n-nonane are 1.0 and 0.8 percent by volume.
11. A number of accidents have occurred as a result of the discharge of static electricity. The latter can be generated even by the flow of fluids. To provide additional protection against explosion or fire due to the discharge of static electricity, determine what temperature an acetone drum filling operation would have to maintain so that flammable mixtures would not be produced by vapor-air mixtures in equilibrium with the liquids. The lower flammable limit (LFL) for acetone vapor in air is 2.55 percent by volume.

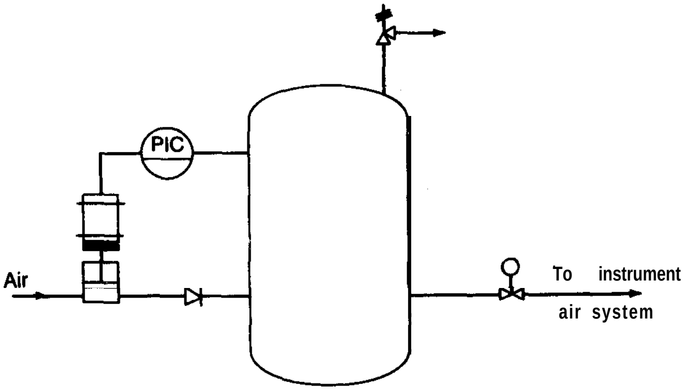
12. When a flammable material is burned, there will be an increase in either the volume of the gas produced (provided the pressure is constant) or the pressure in the container (provided the volume is constant). Calculate the volume of gas formed during the adiabatic combustion of 100 lb moles of gaseous propane at a constant pressure of 1 atm. Assume that the 200 percent theoretical air and the propane involved in the combustion are at 25°C and that the combustion goes to completion.
13. A mixture of ethylene and air with a C_p/C_v ratio of 1.25 is to be compressed from an initial temperature and pressure of 70°F and 14.7 psia, respectively, to 1000 psia. If the compression is performed essentially isentropically, what is the final temperature? If the autoignition temperature (the temperature at which a fuel-air mixture will ignite without the need for an ignition source) is 913°F, will an explosion occur if the mixture being compressed is flammable?
14. Many commercial chemical reactions are exothermic. If the heat that is generated is not appropriately removed, the reaction rate can get out of control and the reactor will be damaged if it is not adequately vented or protected. Consider a reaction in a continuously stirred reactor that has an activation energy of 28,000 cal/g mole. The water-cooled jacket surrounding the reactor utilizes cooling water with an inlet temperature of 15°C. What is the maximum temperature at which the reaction can operate without having the reaction run away? What actions might be taken to maintain safe operation of the reactor?
15. Materials like acetylene may decompose under the appropriate conditions with a fairly large liberation of energy. Determine the temperature and pressure attained in a closed spherical container when acetylene, initially at 25°C and 10 atm, rapidly decomposes to carbon and hydrogen after accidentally being subjected to an electrical spark. What thickness of carbon steel would have been required to contain the decomposition reaction if no relief valve had been available to relieve the pressure buildup? (Assume that a safety factor of 4 has been used in the thickness calculation.)
16. One pound of TNT explosive releases about 2000 Btu of energy when it detonates. Estimate the equivalent energy release rate in pounds of TNT when a 150-ft^3-high pressure cylinder filled with gaseous nitrogen fails at 2200 psia.
17. A 70-gal hot water tank in a worker change room has a working pressure of 75 psia and a burst pressure of 300 psia. During installation the relief valve becomes inoperable. After filling the tank, the fill and exit valves are shut and the heater energized. The pressure in the tank increases until the tank ruptures at 300 psia. Estimate the blast energy in terms of the TNT equivalent (2000 Btu/lb explosive). To simplify the calculation, assume that the explosion is adiabatic with the saturated water flashing isentropically to steam and water at 14.7 psia pressure. Since the explosion is rapid, very little air mixes with the steam.
18. Liquid chlorine may be transferred from a chlorine storage container by pressurizing with dry chlorine gas. Develop two systems
    (a) using recompressed chlorine vapor
    (b) using vaporized chlorine
    and provide the appropriate control and alarm system for each system.
19. A distillation unit has been designed to handle a very hazardous material. The unit utilizes a reflux drum and buffer storage. List several ways in which the inventory of the hazardous material can be reduced or eliminated. Sketch and instrument the system that is recommended.

20. Review the benzene storage system shown on page 107 in which benzene enters tank **T-101** through a 4-in. line **L-101** and is subsequently pumped out through pumps **P-101A** and B at 35 atm into a 3-in. line L-102. The accompanying table shows that part of a HAZOP study of line G102 has been completed. Provide the missing information to complete the study.

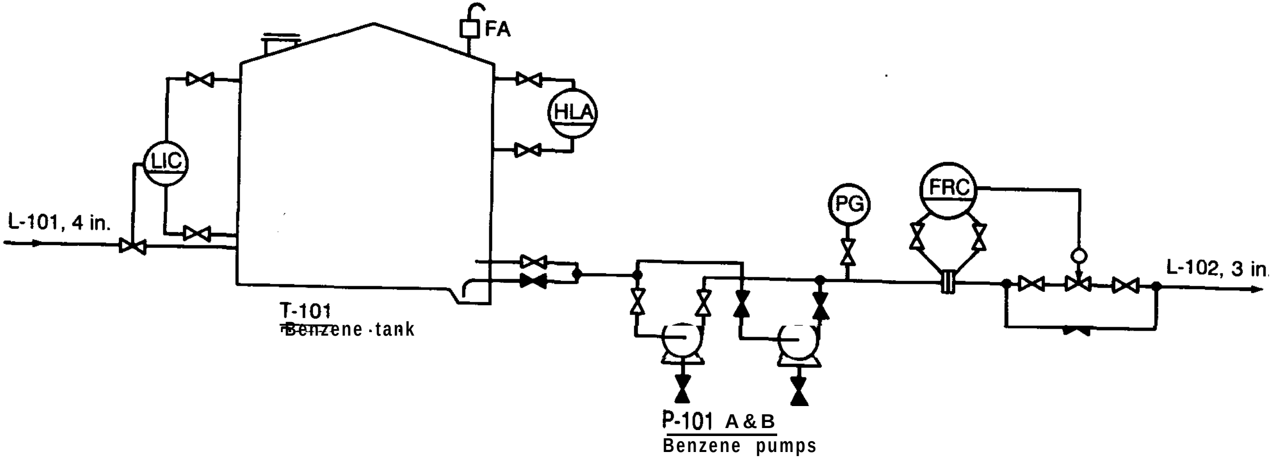
HAZOP for line L-102

Guide word	Property	Possible cause	Possible consequence	Action required
More	Temperature	—	Pump seal failure, vapor lock	Install a feedback line
Less	Temperature	Low ambient temperature	—	Steam tracing
More	Flow	Line fracture	Spillage (possible large explosion)	—
		Pump seal failure	Spillage (possible small explosion)	—
		Control fault		Consider bypass
No	Flow	—	Shutdown	Low level alarm
			Shutdown	Automatic startup of standby pump
Reverse	Flow	Pump failure	Backpressure on storage vessel	—
As well as	Impurities	—	Possible small detonation	Priming line

21. Produce a fault tree for the following system where the highest-order event is overpressure of the vessel. The possible causes of various failures should be suggested.



22. List all the areas in the sodium dodecylbenzene sulfonate process described in Chap. 2 that could be potential danger areas with respect to the health and safety of


FA
HLA
LIC
L-101, 4 in.
PG
FRC
L-102, 3 in.
T-101
Benzene tank
P-101 A & B
Benzene pumps

operating personnel. What precautions in each instance should be taken to minimize the hazard?

23. The following information has been obtained during a test for the BOD of a given industrial waste: 15 ml of the waste sample was added to a 500-ml BOD bottle, and the bottle was then filled with standard dilution water seeded with sewage organisms. A second 500-ml BOD bottle was filled with standard dilution water. After 5 days at 20°C, the blank and the diluted waste samples were analyzed for dissolved oxygen content by the sodium azide modification of the Winkler method. The blank results indicated a dissolved-oxygen content of 9.0 ppm. Results for the diluted sample showed a dissolved-oxygen content of 4.0 ppm. On the basis of the following assumptions, determine the BOD for the waste. The specific gravities of the liquids are 1.0; the waste sample contains no dissolved oxygen.
24. Outline and present solutions to the materials handling, waste disposal, and safety problems that are encountered in the nitric acid unit shown in Figs. 2-1 and 2-2.
25. Provide a design for handling the spent acid which is removed from the settler after the sulfonation process in the sodium dodecylbenzene sulfonate process described in Chap. 2. The final waste effluent must meet the waste-disposal standards of EPA.
26. A process for preparing acetaldehyde is by direct oxidation of ethylene. (This process is described in Prob. 8 of Chap. 2.) Completely analyze the various factor which should be considered in choosing a plant site for this process. With this information, outline possible geographical locations for the plant, noting the advantages and disadvantages of each site.
27. The trend in the fertilizer industry during the past few years has been toward larger and larger fertilizer plants. In terms of plant location, what are the more important factors that should be considered and which factors become even more important as the size of the plant is increased? Are these factors of equal importance regardless of the type of fertilizer produced? Analyze this situation for ammonia, urea, and phosphate fertilizer process plants.
28. Make an analysis of various means of transportation available at Chicago, IL, Houston, TX, and Denver, CO. Use this analysis to determine what form of transportation should be recommended from these three locations for the shipment of the following finished chemical products:

Quantity	Frequency	Distance of shipment
100 lb solid	Once a month	100 miles
50,000 lb solid	Once a month	100 miles
50,000 gal liquid	Once a week	100 miles
50,000 gal liquid	Once a week	500 miles
50,000 gal liquid	Daily	500-mile radius

Since destination of the chemical product can greatly affect the mode of transportation recommended, consider various destinations in this analysis, e.g., coastal location, river location, mountain location, etc. What approximate cost would have to be added to each shipment of finished chemical products as shipping charges in the above analysis?

29. Develop a complete plant layout for the sodium dodecylbenzene sulfonate plant that is described in Chap. 2. Indicate location of instruments, control valves, and panel boards. Provide possibilities for future expansion or revision of the plant.

30. A synthesis gas process is described in Probs. 13 through 18 of Chap. 2. Prepare a plant layout for a production of 25 MM scf/day which can use either air or 95 percent purity oxygen as the oxidant in this process.

31. Chlorine is transferred from a storage tank to a reactor via a vaporizer. The vaporizer may be heated by hot water, by steam, or by closed circuit heating using a heat transfer fluid that is inert to chlorine. The vaporizer may be a coil immersed in a heating bath, a vertical tube bundle, a concentric tube unit, or an evaporator. Indicate the relative merits of these different systems and select one method. Develop a piping and instrument diagram for the system selected making certain that all temperature, pressure, liquid level and flow control systems, and alarms are included for safe operation.

32. Determine the utilities requirement for the nitric acid unit shown in Figs. 2-1 and 2-2. Note that the exit gas from the absorption tower is to be used for power recovery.

33. Thrust forces caused by high-velocity, high-rate flow of fluids can cause forces of such a nature as to damage or rupture piping and associated equipment. Consider a S-in. schedule 40 steel pipe that rises vertically from an elbow off of a horizontal pipe. The pipe serves as the discharge end of a pressure relief valve and must discharge a flashing liquid/vapor mixture with an average density of 15 lb/ft^3 at a velocity of 150 ft/s. Determine the lateral force that will be exerted on the end of the pipe at the elbow.

34. A cylindrical tank 50 ft in diameter is used to store 10,000 barrels of liquid benzene. To provide adequate protection during an external fire, determine the venting rate and the vent area if the maximum tank pressure is fixed at 10 in. of water gauge. Is the calculated vent area realistic or should the tank specifications call for a weak seam roof? Heat transfer to the tank is by radiation from the hot soot particles and gases in the flame and convection from the hot gases. The radiant flux emitted from many burning hydrocarbon fuels is approximately 30,000 $Btu/hr\ ft^2$. The average accounts for fluctuations in temperature, emissivity, composition, and other variables in the flame. Note that the tank when heated from the fire will reradiate some of the energy it receives from the fire.

35. Obtain a copy of a patent dealing with a chemical process and outline the claims presented in this patent. Indicate the method used for presenting the specifications, and explain how the inventor presented proof of his or her claims.

CHAPTER 4

COMPUTER-AIDED DESIGN?

INTRODUCTION

Computing hardware and software are tools-of-the-trade for engineers. The capabilities provided by computers for fast calculation, large storage, and logical decisions plus the available technical and mathematical software permit engineers to solve larger problems and to do it much more rapidly than ever before possible. The engineers' emphasis can therefore shift from problem solving to planning, conceiving, interpreting, and implementing with the information made available. Design is one of the engineering functions that has been impacted by computers.

Chemical engineering stresses the processes for manufacturing chemicals and chemical-based products. In this effort the emphasis of the chemical engineer tends to be on the process rather than on the product, and that emphasis is reflected in the contents of this book. The computer aids most useful to chemical engineers in design are process- rather than product-oriented. The more widely known CAD/CAM (computer-aided design/computer-aided manufacturing) software is usually concerned with product-as-object and is highly graphical and spatial. Chemical process computer-aided design, on the other hand, is much less graphical. It is more concerned with the performance of process units (such as the classical unit operations of chemical

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†This chapter was prepared by Ronald E. West, Professor of Chemical Engineering, University of Colorado, Boulder, Colorado.

engineering) and the integration of these units into complete, consistent, efficient processes to produce chemical products. This chapter gives a brief introduction to computer software useful for process and plant design and to approaches in design that take advantage of the capabilities of computers and software.

An engineer may write a programming language code to solve a particular design problem. Before the explosive growth of software, this commonly was done. There is still a place for it when appropriate software is not available or as a learning technique. An example of code written specifically to solve a chemical-reactor-design problem is shown, in three different programming languages, in Chap. 16. (Chemical reactors, because of the great variety of reactor types, reaction kinetics, and specific design requirements, constitute a process-unit type for which available software may not be appropriate.) Because program writing and debugging is usually quite time consuming, it is recommended that available software be used and that a specific design program be written only after determining that appropriate software is not available. Many programs useful for chemical engineers have been published.?

Spread-sheet software has become an indispensible tool to engineers because of the availability on personal computers, ease of use, and adaptability to many types of problems. An example of spread-sheet application to a problem typical of chemical process design is presented later in this chapter.

Hundreds, even thousands, of programs have been written to solve problems commonly encountered by chemical engineers. Many of these have become available for purchase. In turn, many of these programs are for the design of individual units of chemical process equipment. Design programs are available for virtually every unit operation and every type of common process equipment. *Chemical Engineering* provides a feature, "Chemputer," in each issue reporting on computer hardware and software developments and periodically publishes a comprehensive list of programs useful to chemical engineers. The "Software" department of *Chemical Engineering Progress* likewise reports on software of interest to chemical engineers.

The Computer Aids for Chemical Education (CACHE) Corporation‡ makes available several programs intended mainly for educational use, but provides them at modest cost to anyone. CACHE is a not-for-profit organization whose purpose is to promote computer- and technology-based aids for chemical engineering education.

The critical need for chemical and physical property data is apparent to anyone who has worked on process-design projects. Property storage or predic-

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†G. Ross, "Computer Programming Examples for Chemical Engineers," Elsevier, Amsterdam, 1987; D. J. Deutsch, ed., "Microcomputer Programs for Chemical Engineers," Vol. 1, 1984 and Vol. 2, 1987, McGraw-Hill Book Co., New York; D. Wright, "Basic Programs for Chemical Engineers," Van Nostrand Reinhold Co., Inc., New York, 1986.

‡CACHE Corporation, P.O. Box 7939, Austin, TX 78713-7939.

tion is an enormous task because of the large number of substances and mixtures of possible interest and the virtually unlimited conditions (of temperature, pressure, and composition) at which they may be needed. Software to supply these properties is one of the most useful of all computer aids to chemical engineers. There are many computer programs available for supplying or predicting properties, primarily for pure components.

Process synthesis and *flow-sheeting* programs are those most specifically intended for design use. Process synthesis (or invention) involves generating a flow sheet for a process to produce a particular product or slate of products from specified raw materials. The process flow sheet identifies the chemical reactors and unit operations required and their sequence, the material and energy streams in the process, those streams to be recycled, and some characteristics of the equipment. This flow sheet should be a reasonable approximation of the best economic flow sheet for the product slate. Process synthesis software, a type of expert system program, has begun to appear in the late 1980s. Process synthesis is introduced in this chapter, but rapid changes are to be expected in this field.

"Flow-sheeting," as used in computer-aided-design, means performing on a specified flow sheet the calculations necessary to simulate the behavior of the process or to design the equipment and to determine values for key operating conditions. These calculations include mass and energy balances, process-equipment parameters, and cost estimation for the equipment and plant as well as an economic analysis of the process. This is chemical process design; it is what this textbook is all about. The potential of computerized flow-sheeting was recognized early by chemical engineers. Development of these programs began in the late 1950s and still continues. Flow-sheeting methods are emphasized in this chapter.

SPREAD-SHEETING

Spread-sheet software has achieved great popularity because of its availability for microcomputers at reasonable cost, the ease of learning and using the software, and its flexible application to many problems. Solutions for many engineering problems can be obtained more rapidly by spread sheet than by writing a language code program, such as FORTRAN or BASIC.

The term spread *sheet* refers to the row and column organizational form of the input and output of the software. Key features of spread-sheet software that are necessary or desirable for engineering problem solving include:

Mathematical functions (arithmetic, logical, relational operators, logarithms, exponents, etc.)

User definable functions (to use functions not in the library)

Formula and data replication (for ease of copying entries)

ASCII? data files (for easy transportability of data into and out of the spread sheet)

Iteration (repeat of the calculations automatically or under user control)

Multiple buffers (accommodation of several spread sheets at once)

Virtual memory (allows a microcomputer to act as though it were a much larger machine)

Macro capability (permits a user to write and use a procedure)

Functions (e.g., statistical, financial, data regression, matrix operations, and database functions)

Graphical output.

Spread-sheeting has been found to be especially useful for the mass and energy balancing, approximate sizing of equipment, cost estimating, and economic analysis steps of process design. It is less useful for more detailed equipment design, because the complex algorithms usually necessary for this step can be difficult to incorporate into a spread sheet. Examples of chemical engineering design applications of spread-sheeting software may be found in the literature.[‡] Julian illustrates how to draw a process flow sheet using Lotus 1-2-3™ as well as showing several examples of process mass balances.0 A spread-sheet calculation applied to a chemical-process mass balance, along with some tips for spread sheet use, is shown in the following ‘example.

Example 1 Reactor mass balances. The synthesis of methanol from carbon monoxide and hydrogen by the reversible reaction

$$CO + 2H_2 \rightleftharpoons CH_3OH$$

is to be conducted continuously. The reactor is followed by a condenser and drum that separate the methanol product from the unreacted carbon monoxide and hydrogen. A process flow sheet is shown in Fig. 4-1.

Mass balances are to be solved for this process. Perfect separation between methanol and the reactants is assumed. Unreacted reactants are recycled to the reactor to improve their utilization. The recycle stream within the process complicates solving the mass balances, for there is a circularity in the logic of the solution. The mass balance equations must be solved simultaneously rather than singly, or solved iteratively, as is done with a spread sheet.

It is recommended that the spread-sheet solution be built up by increments rather than trying to solve the complete problem in a single step. As an illustration, this example will be solved first without the complication of the recycle, then the recycle will be added.

[†] American Standard Code for Information Interchange.

SE. A. Grulke, "Using Spreadsheets for Teaching Design," *Chem. Eng. Ed.* **XX(3):128-131** + 153 (summer, 1986); W. P. Schmidt and R. S. Upadhye, Material Balances on a Spreadsheet, *Chem. Eng.*, **91(26):67-70** (1984).

[§] F. M. Julian, Flowsheets and Spreadsheets, *Chem. Eng. Prog.*, **81(9):35-39** (1985).

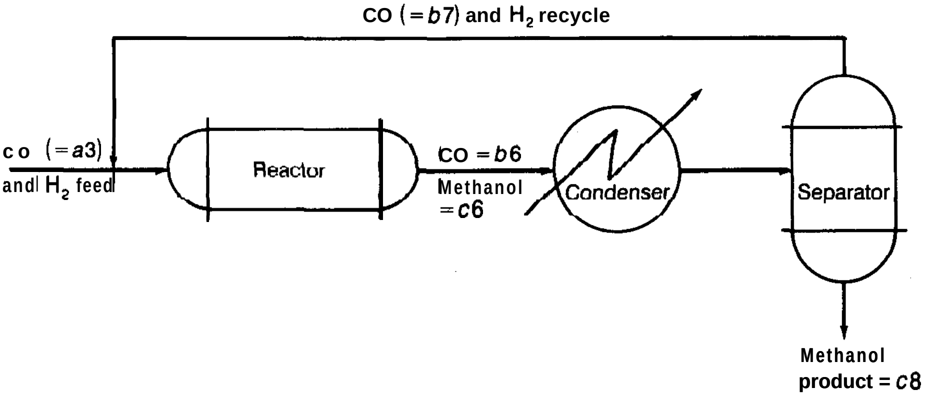

FIGURE 4-1
Methanol process for Example 1.

Solution. The feed consists of a 2 : 1 molar ratio of hydrogen-to-carbon monoxide, the same ratio in which they react; therefore the hydrogen-carbon monoxide ratio remains constant throughout the process. Thus, it is necessary to calculate the flow rates of only one of the reactants; carbon monoxide flow rates are calculated here. The hydrogen flow rate in any stream is twice that of carbon monoxide. Perfect separation between reactants and products is assumed.

Spread-sheet columns are labeled with lowercase letters and rows by numbers. A combination of a lowercase letter and a number identifies a cell in the spread sheet and represents the number stored in that cell; e.g., ***b3*** indicates the cell at the intersection of column ***b*** and row 3, and when used in an equation, ***b3*** stands for the numerical value stored in that cell. The spread-sheet location is used here to represent unknown quantities.

Mol balances for the reactor are (the symbol $*$ is used for multiplication),

$$b6 = (1 - b3) * a3$$

where a3 is the carbon monoxide feed flow rate (100 mol/h in this example), ***b2*** is the fractional conversion of carbon monoxide through the reactor (0.3), and ***b6*** is the carbon monoxide flow rate out of the reactor. Thus,

$$c6 = b3 * a3$$

where $c6$ is the methanol flow rate out of the reactor. Mol balances for the separation unit are

$$b7 = b6$$

where ***b7*** is the carbon monoxide flow rate out of the separator and

$$c8 = c6$$

where $c8$ is the methanol product flow rate.

The data and equations are entered into locations in the spread sheet as follows: The first five rows are used for labels and constants; the first column is used for labels and stream numbers. Each location that will contain a number is initialized to zero. The value 100, the carbon monoxide feed rate in mol/h, is entered into location $a3$, and the value 0.3, the fractional conversion, is entered into location $b3$. The right-hand side of each equation is entered into the cell representing the variable on the left-hand side of the equation. The entries into the spread sheet are indicated by the following table.

	a	*b*	*c*	*d*
1	Mass balances			
2	Reaction:	CO + 2H2	→	CH3OH
3	100	0.3		
4		Flow rates, MOL/H		
5	Stream no.	CO	CH3OH	
6	2	$(1 - b3) * a3$	$b3 * a3$	
7	3	$b6$	0.0	
8	4	0.0	$c6$	

The results of computation by the spread sheet are (omitting titles)

	a	*b*	*c*
3	100	0.3	
5	Stream no.	CO	CH3OH
6	2	70	30
7	3	70	0.0
8	4	0.0	30

The solution for the process without recycle has been obtained.

Now the recycle of the unreacted products to the reactor is added, as shown in Fig. 4-1. The reactor balance equations become

$$b6 = (1 - b3) * (a3 + b7)$$

and

$$c6 = b3 * (a3 + b7)$$

These new equations are substituted in place of the previous ones in the spread sheet. The separation-step equations are unchanged. The calculations are now circular, because $b6$ depends upon $b7$, which in turn depends upon $b6$. These equations will be solved iteratively; that is, the calculations will be repeated until the values in the table stop changing (within limits). The calculations can be performed automatically by the spread-sheet software. It is recommended, however, that the calculations be made utilizing the manual option of the software, at least initially until it is clear that the calculations are converging. The manual option has another advantage: It keeps the software from attempting to calculate as the equations are being entered (such calculation would result in errors because of the circularity).

If the case without the recycle was solved first, calculations for the case with recycle can utilize the values from the preceding table for the same feed rate and conversion. The result of the first iteration is then

	b	*c*
6	119	51
7	119	0.0
8	0.0	51

Clearly the values have changed since the previous calculation; convergence has not been attained. The user requests another calculation with the result

	b	c
6	153.3	65.7
7	153.3	0.0
8	0.0	65.7

Again the values have changed, so the calculations are repeated. Calculations are continued until the user is satisfied that no further "significant" change has occurred (the definition of significant is up to the user; a maximum change in values between iterations of 0.01 percent is a reasonable criterion).

The final results (to 0.01 percent) for this example are

	b	c
6	233.31	99.99
7	333.31	0.0
8	0.0	99.99

The exact values for this problem (by algebraic solution) are $233\frac{1}{3}$, 100, and $333\frac{1}{3}$, respectively.

If, after several manual trials, the calculations appear to be converging, it is reasonable to switch from manual to automatic iteration. It is strongly recommended that the calculations always be checked by a simple hand calculation. In this example, for instance, the user may add the feed and recycle carbon monoxide flow rates, multiply the sum by the conversion, and compare the result with the tabulated value for the methanol product rate $c8$.

A "scratch pad" can be very useful for a spread-sheet user. A scratch pad is an area (e.g., columns to the right of the main spread sheet) set aside for doing side calculations of parameters or constants for the main spread sheet, writing comments, locating macros, and other uses. A scratch pad would be useful in this example if the user needed mass balances for a particular product flow rate rather than a specified feed rate as just illustrated. The problem has been solved for a carbon dioxide feed rate of 100 mol/h as shown above, but suppose it is desired to know the feed rate required to produce 880 mol/h of methanol product. The problem might intentionally be solved first for a specified feed rate, because it is often more convenient to write the mass-balance equations for known process feed rates rather than known product rates.

The preceding results may be converted to the basis of the new product flow rate. Mass-balance equations are linear, so that if the product flow rate is doubled, for example, all flow rates in the process are doubled. In this example then, the new flow rates are obtained by multiplying the previously calculated values by the ratio of the new-to-previous product-stream flow rates $(880/99.99) = 8.801$. This ratio can be calculated in the scratch pad (say location $e3$), the new CO feed rate, 880.1, entered into $a3$, and the iterative calculations repeated until convergence is attained. The iteration could be avoided, however, if each mass balance had been entered as shown above, but multiplied by a coefficient e3 (the scratch-pad location). Initially, e3 would have been set equal to 1 and the calculations would have proceeded just as shown above. The user then enters $880/c8$ for location e3, repeats the calculation, and obtains the final flow rates without iteration.

PROCESS-EQUIPMENT-DESIGN PROGRAMS

A few examples of the many programs available for the design of chemical-process equipment are given here. Dozens of programs have been published.? Piping network design programs have been **reviewed.**‡

CACHE has PC programs available for the following operations: slurry flow in pipes, supercriticial fluid extraction, gas absorption with chemical reaction, design of flash vessels and distillation towers, heterogeneous reaction kinetics, and CSTR dynamics and stability. Also available from CACHE are instructional modules for binary distillation, material balances, and flow-sheet drawing. CACHE has the programs THEN and TARGET II that are useful in the design of heat-exchange networks.

PROPERTY DATA PROGRAMS

The property program developed by the Design Institute for Physical Properties of the American Institute of Chemical Engineers, known as DIPPR, is widely known and used. The PC version of DIPPR contains 26 constant and 13 temperature-dependent, pure-component properties for 766 common industrial chemicals.

There are at least 20 physical property databases commercially available as on-line data services.§ Five of the databases listed include vapor-liquid-equilibrium calculations for mixtures.

COADE/Chemstations, Inc. provides PREDICT™ and IS1 Software offers ChemSmart™ for property estimation on personal computers.

Chemical process flow-sheeting programs include property data and prediction methods for pure components and mixtures. Property data can be retrieved from these programs for use in flow-sheeting, or such a program can be used as a property database without using the flow-sheeting capability. Flow-sheeting programs are discussed later in this chapter.

PROCESS SYNTHESIS

The first step in plant design is to define a process, namely the chemical and physical processing steps to convert raw materials into desired products. The number of choices and combinations of unit operations, reactor designs, and ancillary processes that might be used for even a simple process can be quite

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†D. J. Deutsch, ed., "Microcomputer Programs for Chemical Engineers," Vols. 1 and 2, McGraw-Hill Book Company, New York, 1984 and 1987.

‡Piping Software Relieves Headaches, *Chem. Eng.*, 95(9):145–146 (June 20, 1988).

§A Wealth of Information Online, *Chem. Eng.*, 96(6):112–127 (June, 1989), describes these property databases as well as databases on environmental protection, safety and health, and patents.

large. For instance, Perry's Handbook shows 15 different ways just to separate a solid from a liquid.?

Chemical engineers have traditionally synthesized process flow sheets using experience, insight, invention, and sequential evaluation of alternatives. Recently there has been some analysis and computerization of this process. Douglas and co-workers have prepared a program, PIP-Process Invention Program-available through CACHE, that performs process synthesis.‡ The core of this approach is to build up a process in steps of increasing complexity, applying scientific principles and heuristics (rules of thumb) at each step. It is anticipated that computer-aided process synthesis will see a rapid growth in the future. Computer-aided process synthesis is illustrated by the following example using a process that is available on the PIP program.

Example 2 Synthesis of a styrene process. Styrene, the monomer of polystyrene, has enjoyed strong market growth over the past two decades. It is prepared starting with benzene and ethylene which react to form ethylbenzene; the ethylbenzene is dehydrogenated to yield styrene. Further information about styrene manufacture, properties, and uses is available.3 In this example, the steps involved in synthesizing a process to produce styrene from ethylbenzene will be illustrated. The procedure followed is analogous to that followed by the PIP program.

Experience will be cited frequently in this example for guidance of the synthesis procedure. In a computer process-synthesis program the guidance is provided by a set of heuristics summarizing the experience. Douglas and co-workers provide information about such heuristics.¶ The PIP program displays, at the request of the user, the heuristics used in selecting processing steps.

Solution

Step 1. Inputs and outputs. Establish whether the value of the reaction products exceeds the value of the reactants. The chemical reaction is written and balanced, as

$$\underset{\text{ethylbenzene}}{C_6H_5\text{—}C_2H_5} \rightleftharpoons \underset{\text{styrene}}{C_6H_5\text{—}C_2H_3} + \underset{\text{hydrogen}}{H_2} \qquad (1)$$

Values of \$0.42/lb for styrene, \$0.25/lb for ethylbenzene, and \$0.30/lb for hydrogen are used (the hydrogen value is based on its heat of combustion and a heat value of \$5/million Btu). 1 lb-mol each of styrene (104 lb) and hydrogen

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†R. H. Perry and D. Green, eds., "Perry's Chemical Engineers' Handbook," 6th ed., Fig. 19-133, pp. 19-108, McGraw-Hill Book Company, New York, 1984.

‡J. M. Douglas and R. L. Kirkwood, Design Education in Chemical Engineering, Part 2: Using Design Tools, *Chem. Eng. Ed.*, 23(2):120–127 (Spring, 1989); R. L. Kirkwood, J. M. Douglas, and M. H. Locke, A Prototype Expert System for Synthesizing Chemical Process Flowsheets, *Comput. and Chem. Eng.*, 12(4):329–343 (1988).

§M. Grayson and D. Eckroth, eds., "Kirk-Othmer Encyclopedia of Chemical Technology," 3rd ed., Vol. 21, pp. 770-801, 1983.

¶J. M. Douglas and R. L. Kirkwood, Design Education in Chemical Engineering, Part 1: Deriving Conceptual Design Tools, *Chem. Eng. Ed.*, 23(1):22–25 (winter, 1989).

(2 lb) requires 1 lb-mol of ethylbenzene (106 lb); Thus, on a basis of styrene, the value of the products is

$$104*0.42 + 2*0.30 = \$44.28$$

while the value of the ethylbenzene is

$$106*0.25 = \$26.50$$

The product value exceeds that of the raw materials, so it is possible for the process to be profitable and the process synthesis may continue. The cost of carrying out the process-the equipment, utilities, labor, overhead, etc.-has not been accounted for, nor has the fact that the conversion of reactant to product necessarily will be less than 100 percent.

The chemistry of the process is considered more thoroughly. Reaction (1) is an equilibrium reaction; it does not go to completion. The reaction is conducted in the gas phase over a solid, ferric-oxide based, catalyst. It is carried out at about 1-atm total pressure (low pressure favors the products of this equilibrium) and a temperature around 600°C (1112°F). Steam is added to lower the partial pressure of the products, thereby further favoring their formation. The reaction is endothermic, with the heat required being supplied by the steam. A steam-to-ethylbenzene molar ratio of 14 to 1 is representative. At typical process temperatures other reactions occur, including

$$C_6H_5—C_2H_5 \rightleftharpoons \underset{\text{benzene}}{C_6H_6} + \underset{\text{ethylene}}{C_2H_4} \tag{2}$$

and

$$C_6H_5—C_2H_5 + H_2 \rightleftharpoons \underset{\text{toluene}}{C_6H_5—CH_3} + \underset{\text{methane}}{CH,} \; . \tag{3}$$

Reactions (2) and (3) consume ethylbenzene without producing the desired styrene; they also produce by-products that must be separated from the product and disposed of or utilized.

The process feeds are ethylbenzene and steam and the products are condensed steam, styrene, benzene, toluene, hydrogen, methane, and ethylene. Including the steam, valued at \$0.01/lb, adds \$2.52 to the total feed cost, raising it to \$29.02 per 104 lb of styrene product. The value of the products still exceeds that of the inputs.

Our knowledge of the process at this point is shown schematically in Fig. 4-2.

Step 2. Recycle structure. It is necessary to have information on the extent of each of the three reactions at the reactor conditions. These extents could be calculated by using complete reaction-kinetic information, and this probably would be done in a more complete design of the process. Fogler presents examples of such models.? Process-invention programs, however, avoid lengthy calculations by simplifying the models used. A chemical reactor might be modeled by expressing the extent of each reaction as a function of the conversion-per-pass of a key reactant or, even more simply, as a constant. Simplified models are derived from detailed data or models of process operations and reasonable assumptions

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†H. S. Fogler, "Elements of Chemical Reaction Engineering," Chapter 9, Prentice-Hall, Englewood Cliffs, NJ, 1986.

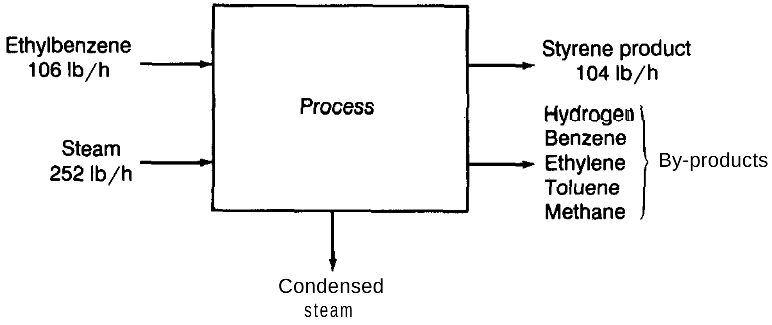

FIGURE 4-2
Input-output of styrene process for Example 2.

regarding the behavior of those operations. Douglas and Kirkwood discuss the development of simplified models.? Here the calculations are illustrated assuming a constant extent for each reaction. The values used for mass balance calculations are,

Fractional extent, reaction (1) = 0.47
reaction (2) = 0.025
reaction (3) = 0.005

The fractional extent-of-reaction is the fraction of the key reactant, ethylbenzene, in the reactor feed that reacts while passing through the reactor. In this example, 0.5 mol of ethylbenzene are consumed per 0.47 mol of styrene produced.

A total fractional extent-of-reaction less than 1.0 (0.5 in this example) means that not all of the reactant in the feed reacts, so unreacted reactant is present in the reactor-product stream. The latter stream contains ethylbenzene, all the reaction products, by-products, and steam. It is essential to separate the main product, styrene, from the rest of this mixture. Economically it is desirable and generally it is necessary to recover unreacted reactants and recycle them to the reactor. By-products must be separated and, if possible, effectively utilized. The condensed steam must be separated from the product stream and removed.

Normal boiling point values (°C) for the components of this system are:

hydrogen	− 252.5	water	100
methane	− 161.5	toluene	110.6
ethylene	-104	ethylbeniene	136
benzene	80.1	styrene	145

These values show that, with ambient cooling at 1 atm, hydrogen, methane, and ethylene are difficult to condense, but that steam, benzene, toluene, ethylbenzene, and styrene are easily condensed. Condensation will separate the latter five

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†J. M. Douglas and R. L. Kirkwood, Design Education in Chemical Engineering, Part 1: Deriving Conceptual Design Tools, *Chem. Eng. Ed.,* **23(1)**:22–25 (winter, 1989).

components as liquids and leave the first three in the gaseous state. Gaseous mixtures are very difficult to separate; when they are combustible, as here, experience indicates that it is usually best to use them as a fuel. Property data also show that water and the liquid hydrocarbons are highly insoluble in each other and thus will split into two liquid phases that can be separated easily. The reactor effluent should be condensed and then separated into the following streams: water (reuse if possible, or dispose of if not), benzene and toluene (these two do not need to be separated unless they are to be used separately), ethylbenzene (to be recycled to the reactor), styrene (the desired product), and gaseous fuel. It is assumed at this stage that the separations between components are complete.

Mass balances for the process are recalculated using the extent-of-reaction values from above and assuming that all unreacted ethylbenzene (EB) is recycled and converted to products. On the basis of 1 lb-mol (104 lb) of styrene product, the calculations are:

$$\begin{aligned}\text{Feed EB flowrate} &= (1\ \text{lb-mol styrene/h})\\ &\quad * (0.5\ \text{mol EB}/0.47\ \text{mol styrene})\\ &\quad * (106\ \text{lb EB/lb mol})\\ &= 112.8\ \text{lb/h}\end{aligned}$$

The EB feed rate to the reactor must be twice this, because only one-half of the feed reacts per pass and the EB recycle rate equals the feed rate; Thus,

$$\text{Reactor EB feed rate} = 225.6\ \text{lb/h}$$

$$\begin{aligned}\text{The steam feed rate is then} &= (225.6/106\ \text{mol EB/h})\\ &\quad * (14\ \text{mol steam/mol EB})\\ &\quad * (18\ \text{lb steam/mol steam})\\ &= 536\ \text{lb/h}\end{aligned}$$

The by-product benzene plus toluene and the fuel gas rates are calculated, from Eqs. (2) and (3), to be 5.2 and 3.6 lb/h, respectively. The raw materials and product values can be compared again. The benzene-toluene mixture is valued at \$0.10/lb and the fuel gas at \$0.18/lb (both based on heating value); condensed water has no value. The value of the feeds, ethylbenzene and steam, is \$33.54/104 lb of styrene. The value of the product plus the by-products and fuel gas is \$44.84/104 lb of styrene. The value of the outputs still exceeds that of the inputs, but the margin has narrowed.

The process, as identified so far, is shown in Fig. 4-3.

Step 3. Separation processes. The separation after condensation into a water phase, a hydrocarbon phase, and a gas phase is accomplished by gravity in one or two holding tanks (other phase-separation methods are available and could be considered in a more detailed design). The hydrocarbon phase needs to be separated into three fractions: (1) benzene plus toluene, (2) ethylbenzene, and (3) styrene. Experience shows that components with boiling points near ambient temperature (between, say, 0 and 200°C) and with adequate differences between their boiling points (at least 5°C) are usually most economically separated by distillation. An ordinary fractional distillation column yields two products; two distillations are required to produce the three hydrocarbon products in this process. Experience again provides guidance in planning this separation: First,

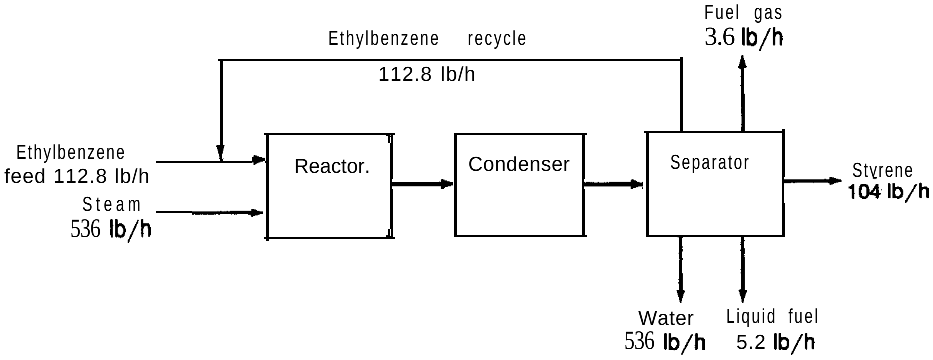

FIGURE 4-3
Styrene-process recycle structure for Example 2.

remove the lower boiling component (benzene and toluene) and, last, make the most difficult separation (closest boiling points-ethylbenzene and styrene). Another lesson of experience is to remove a desired product (styrene) finally as a distillate (lower boiling) product. That is not possible with this mixture as described; however, experience also shows that organic reactions almost always generate higher boiling "tars" that need to be removed. Thus, a final distillation step, with styrene as the distillate product and tar (not included in the material balances) as the bottom product, is recommended. Another complicating factor here is that styrene polymerizes when heated. In order to avoid significant polymerization, styrene distillation temperatures are lowered by operating under vacuum.

Separation processes in reality do not achieve perfect separation as we have assumed in the mass balances. Actual separation-product stream compositions must be specified if the separations are to be designed in more detail. Here again experience, as well as product and by-product specifications, would be used to establish these specifications. Equipment design is not included in this example, so these specifications are not discussed.

The process as it has been synthesized so far is shown in Fig. 4-4. The separation process devised here is the same as that found in industrial styrene production.?

Step 4. Heat integration. Both heating and cooling are needed in this process. Since supplying and removing heat is expensive, it is desirable to heat and cool using heat exchange between process streams. The purpose of heat integration is to satisfy the process heating and cooling requirements as economically as possible.

Required temperatures are specified; for example, a reactor outlet temperature of 600°C (1112°F) is needed. The temperature and energy

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†M. Grayson and D. Eckroth, eds., "Kirk-Othmer Encyclopedia of Chemical Technology," 3rd ed., Vol. 21, pp. 770-781, 1983.

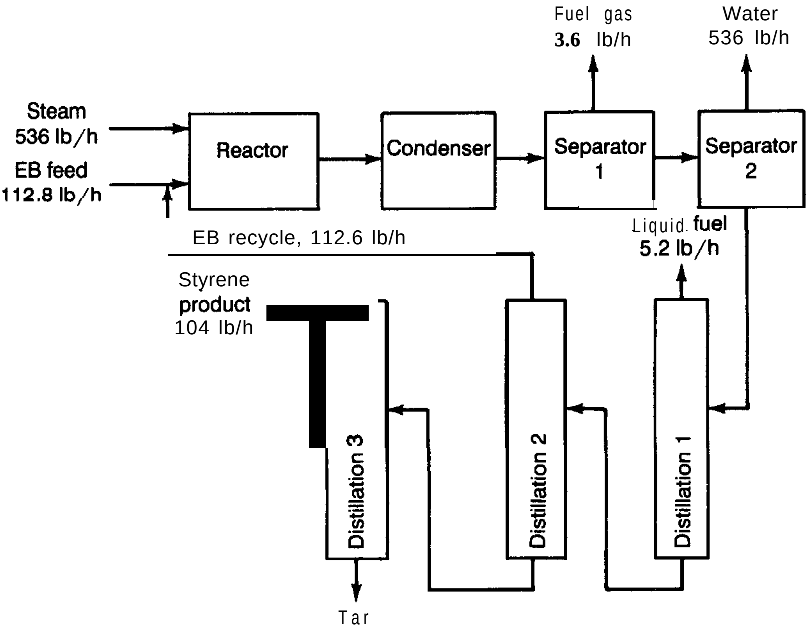

FIGURE 4-4
Styrene-process separation sequence for Example 2.

requirements for the heating and cooling loads are matched as closely as possible. Styrene reactors typically operate adiabatically (no heat is added), causing the temperature to drop as the endothermic reaction proceeds. The reactor-inlet temperature required to achieve the specified outlet temperature is calculated from a reactor energy balance,

$$\text{Enthalpy of products} = \text{enthalpy of reactants} - \text{heat of reaction}$$

Enthalpies of reactants and products are expressed with respect to a reference temperature of 25°C (77°F), and the heat of reaction is available at 25°C:

$$[m * C * (1112 - 77)]_p = [m * C * (T - 77)]_r - m * \Delta H_R$$

where m is the mass flow rate (lb/h), C is the heat capacity (Btu/lb-°F), $m * \Delta H_R$ is the total heat of reaction (Btu/h), and subscripts p and r denote products and reactants, respectively. The reactant and product mass flow rates are both 761.6 lb/h from the earlier mass balances. Using 0.53 Btu/lb-°F for the heat capacity of both products and reactants, 50,600 Btu/h as the total heat of reaction (styrene molar flow rate * heat of reaction for styrene), and solving this equation for T gives a required inlet temperature of 1237°F for the reactants.

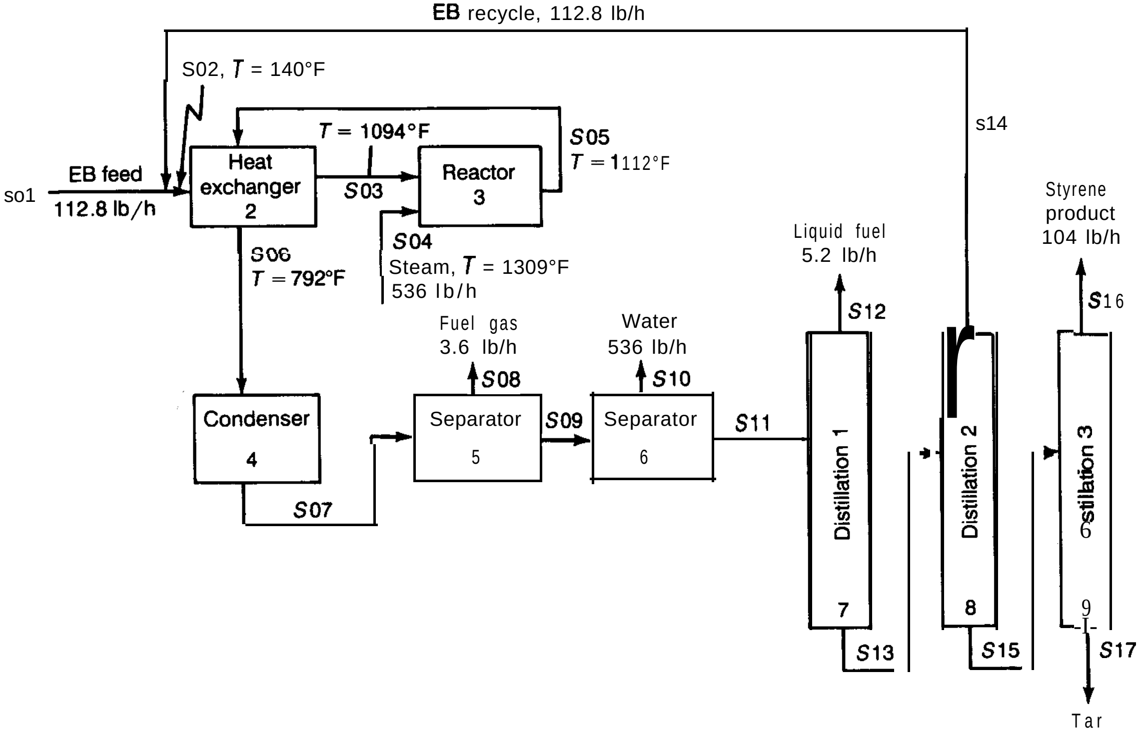

FIGURE 4-5
Styrene-process flow sheet for Example 2.

The ethylbenzene feed needs to be heated and the reactor effluent cooled. Heat exchange between them is feasible and desirable. Setting an approach temperature for a countercurrent exchanger (temperature difference between heated feed and reactor effluent) at 10°C, fixes the heated feed temperature at 590°C (1094°F). An energy balance for the heated feed and steam mixture establishes the required steam temperature:

$$[m * C * (T - 1237)]_s + [m * C * (1094 - 1237)]_{EB} = 0$$

where the subscripts s and EB signify steam and ethylbenzene, respectively. Using $C_s = 0.5$, $C_{EB} = 0.6$ Btu/lb-"F, and the flow rates calculated above gives

$$T(\text{of superheated steam}) = 1309°\text{F } (709°\text{C})$$

An energy balance around the feed-effluent heat exchanger,

$$[m * C * (1112 - T)]_{\text{effluent}} = [m * C * (1094 - 140)]_{\text{feed}}$$

gives

$$T = 792°\text{F as the outlet temperature of the effluent}$$

The reactor effluent must reach about 40°C (104°F) for condensation. Thus, further cooling is required, by cooling water or other process loads. Heating and cooling for the distillation columns represent such possible loads.

Figure 4.5 shows the styrene process that has been devised as a result of this analysis.

Step 5. Economic evaluation. Not shown in this example, but included in a process synthesis program such as PIP, are algorithms for costing and economic evaluation of the process. Process equipment is sized and priced and total plant investment is estimated. Requirements and costs for utilities and raw materials, other operating costs, and product values are estimated. These values are used to evaluate the profitability of the proposed process and can provide a sound basis for a more detailed design.

FLOW-SHEETING SOFTWARE

Flow-sheeting has been defined as "The use of computer aids to perform steady-state heat and mass balancing, sizing, and costing calculations for a chemical process."? Flow-sheeting and process design are analogous activities. An objective is specified, typically the desired annual production rate of the principal products, as is the process flow sheet. The process flow sheet defines the reactors, unit operations, and utilities needed, along with the streams of material and energy into and out of the process and between all the process units. Sufficient information is specified so that a unique solution exists to the posed problem. Feed and product compositions, temperatures, and pressures typically are specified, as are performance requirements and some operating characteristics of the process equipment.

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†A. W. Westerberg, H. P. Hutcbison, R. L. Motard, and P. Winter, "Process Flowsheeting," Cambridge University Press, Cambridge, England, 1979

Flow-sheeting is applied to steady-state conditions. The unsteady-state features of normally steady-state processes-start up, shutdown, disturbances, control actions-are usually analyzed separately from the process-design task. This results in a major simplification of the design problem, because time variations and integration with respect to time need not be performed (although integration with respect to space may still be required in models for particular operations, such as that of a plug-flow chemical reactor). The flow-sheeting language SPEED-UP is an exception in that it includes unsteady-state capability in its code.?

The solution to the problem is obtained by solving mass and energy balances to yield the quantity and state (i.e., composition, temperature, pressure) of all the streams and the utility requirements. Additional parameters for the process equipment, sufficient so that stream specifications are met and the cost of the equipment can be estimated, are calculated. The cost of equipment, raw materials, and utilities is estimated and an economic analysis is carried out. Methods of cost estimation and economic analysis are presented later in this text. This entire procedure may be repeated many times to examine modifications of the process flow sheet or to find optimal values of key process variables. Computer software can greatly simplify these repetitive calculations for the engineer. But even without the need for repetition, the software may simplify the calculations and provide detail and accuracy that would have been impossible otherwise.

Certain features of chemical process calculations contribute to their difficulty, complexity, and challenge. These include: large sets of nonlinear, algebraic equations, the need for large amounts of physical and chemical property data, the presence of operations that require very complex models, and the occurrence of recycle streams. The property issue has been discussed and the other matters are discussed in the following sections.

The flow-sheeting program FLOWTRAN is available to universities through CACHE. This program was developed by the Monsanto Company in the 1960s and made available for university use in 1973. FLOWTRAN is well documented with a manual and examples.+ It can be installed on many mainframe computers, but not on personal computers. FLOWTRAN is available through Chemical Engineering Departments at most universities. CHEMCAD II™ is a flow-sheeting program for personal computers offered by COADE/Chemstations, Inc. PROCESS™, the product of Simulation Sciences,

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[†] J. D. Perkins and R. W. H. Sargent, SPEED-UP: A Computer Program for Steady-State and Dynamic Simulation and Design of Chemical Processes, "Selected Topics on Computer-Aided Process Design and Analysis," R. S. H. Mah and G. V. Reklaitis, eds., AIChE Symposium Series, Vol. 78, No, 214, American Institute of Chemical Engineers, New York, 1982.

[‡] J. D. Seader, W. D. Seider, and A. C. Pauls, "FLOWTRAN Simulation--An Introduction," 3rd ed., CACHE, 1987; J. P. Clark, T. P. Koehler, and J. T. Sommerfeld, "Exercises in Process Simulation Using FLOWTRAN," 2nd ed., CACHE, 1980.

Inc., is available in main-frame and personal computer versions; many universities utilize PROCESS. ASPEN PLUS™, by Aspen Technology, Inc., is available for main-frame and personal computers. The ASPEN system was developed to handle solid-liquid-gas processes (such as coal conversions) and is probably the most versatile of the generally available flow-sheeting programs. ChemShare Corporation offers DESIGN II™ in a personal computer version. HYSIM™ is a personal computer flow-sheeting program from Hyprotech Ltd. In addition to

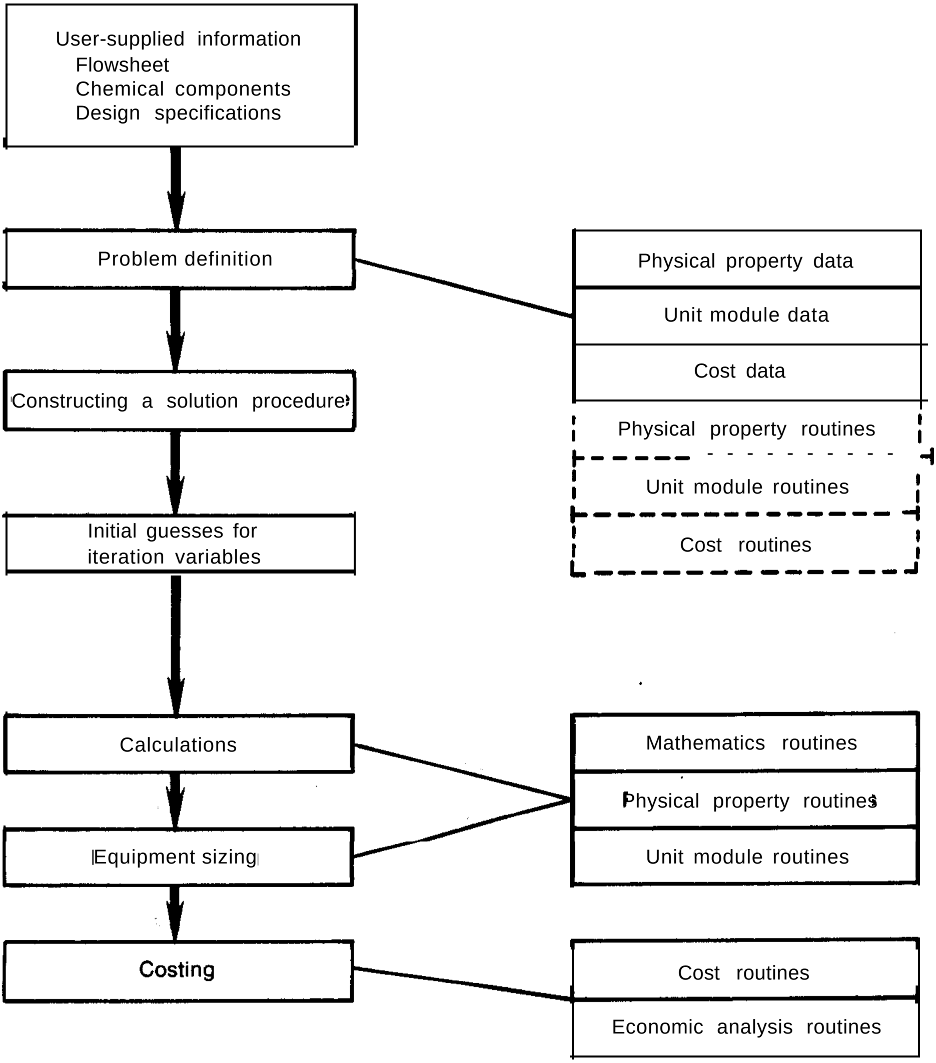

FIGURE 4-6
Development of a simulation program for a process system.

the foregoing commercially available programs, there are many proprietary flow-sheeting programs such as Exxon's COPE, DuPont's CPES, and Union Carbide's IPES.

An overall structure similar to that of many of the flow-sheeting programs is shown in Fig. 4-6.†

The unit operations, reactors, and other process features are represented by unit-module routines. These routines include operations associated with the equipment of the process (e.g., distillation columns and compressors) as well as changes occurring due to flow arrangements (e.g., composition and temperature changes that can occur when two or more streams are combined into one stream by connecting pipelines or are added simultaneously to a vessel). The description of a process consists of selecting the appropriate modules that represent the process and identifying the streams that flow into and out of each module. From a list of modules and stream connections, the process arrangement can be interpreted by a flow-sheeting program. Each unit-module routine consists of a program containing a mathematical model for the performance of one process unit. The model consists of equations relating the input and output stream conditions and the equipment parameters (specifications that determine the behavior of the equipment, such as reflux ratio and number of plates for a distillation column).

Degrees of Freedom

The number of unknowns and the number of equations relating these unknowns can become very large in a process-design problem. The number of unknowns and independent equations must be equal in order that a unique solution to a problem exists. Therefore, it is necessary to have a systematic method for enumerating them. The total number of independent extensive and intensive variables associated with each stream in a process is $C' + 2$, where C' is the number of independent chemical components in the stream. The quantity and the condition of the stream are completely determined by fixing the flow rate of each component in the stream (or, equivalently, the total flow rate and the mole or mass fractions of $C' - 1$ components) and two additional variables, usually the temperature and pressure, although other choices are possible. This number includes situations where physical and chemical equilibrium exist.‡

If all the inlet stream conditions to an operation are known, $(C' + 2)$ equations are required in order to calculate all the conditions of each outlet stream. For S outlet streams, a total of $S*(C' + 2)$ equations is necessary to relate the outputs to the inputs of the operation. The model of an operation

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† A. W. Westerberg, H. P. Hutchison, R. L. Motard, and P. Winter, "Process Flowsheeting," p. 12, Cambridge University Press, Cambridge, England, 1979.

‡ A. W. Westerberg, H. P. Hutchison, R. L. Motard, and P. Winter, "Process Flowsheeting," pp. 115-120, Cambridge University Press, Cambridge, England, 1979.

must have this number of equations, but most models have more, because the changes occurring in the operation are complex and more equations are needed to represent these changes. Distillation is an operation for which many equations must be solved within the module in order to determine the output stream conditions. The model equations introduce new unknowns as well. These new unknowns are parameters of the operations that are necessary to relate the outputs to the inputs. A simple example involves splitting one stream into two streams with no changes in temperature, pressure, or composition: the split between the two output streams is a variable in the overall mass-balance equation, and must be specified, e.g., by fixing the fraction of the input that goes into one of the outputs.

The total number of stream variables, the number of equations, and the number of equipment parameters can be summed. and the total degrees of freedom (unknowns minus equations) then determined. **A** unique solution to a problem exists only when the numbers of unknowns and equations are equal. Therefore, a number of variables equal to the number of degrees of freedom must be given values so that there will be a unique solution.

Example 3 Degrees of freedom for styrene process. Determine the degrees of freedom for the styrene process using the process flow diagram, Fig. 4-5.

Solution. There are 8 components (= C’) and 17 streams in the process. Thus there are

$$(8 + 2) * 17 = 170 \text{ total stream variables}$$

The process has 9 units and there are 15 output streams from these units:

$$\underset{\text{streams}}{17} - \underset{\text{external feeds}}{2} = \underset{\substack{\text{outputs from}\\ \text{process units}}}{15}$$

Therefore, there are

$$\underset{\text{outputs}}{15} * \underset{C'+2}{10} = \underset{\substack{\text{mass balance}\\ \text{equations}}}{150}$$

which results in

$$\underset{\text{variables}}{\mathbf{170}} - \underset{\text{equations}}{\mathbf{150}} = \underset{\substack{\text{more stream variables}\\ \text{than equations}}}{20}$$

Twenty stream variables need to be specified in order for a unique solution to exist. In principle, any 20 stream variables could be supplied; however, the usual solution strategy requires that the process feed streams be specified. Specifying the flow rate of each of the eight components, plus the temperature and pressure of the two feed streams (ethylbenzene and steam) reduces the number of variables to 150. Hence, a unique solution is available.

Equation Solution

The complete model of a chemical process can consist of hundreds, even thousands, of equations. The very simple process represented by the styrene

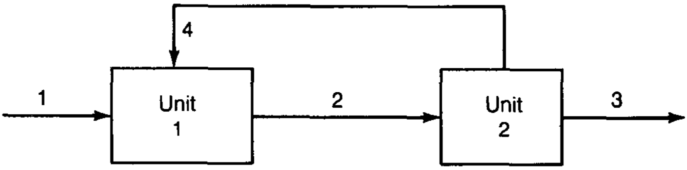

FIGURE 4-7
Illustration of circularity in recycle calculations.

example has 150 mass- and energy-balance equations and more than 200 total equations to solve when physical properties are included. Many of the equations in a chemical process model are nonlinear, in particular those representing performance and physical properties. The entire act is therefore nonlinear (although linear subsets are possible) and the well developed techniques of linear algebra are not applicable. The algorithm (method) for the solution of large sets of nonlinear equations is a key feature of a flow-sheeting program. Three types of algorithms have been developed: the sequential approach, the equation-oriented approach, and the two-tiered approach.? Since a sequential approach is used in most flow-sheeting programs, it is the only one discussed here. The sequential approach is based on calculating the outputs of a process module from a knowledge of the inputs and the necessary equipment parameters and proceeding sequentially through the modules of a process. Sequential calculation is problem-free until a recycle stream is encountered.

Recycle

Chemical processes more often than not contain recycle, a feature that complicates their analysis. Recycle often occurs, as in the styrene process where unreacted ethylbenzene is recovered and recycled back to the reactor as a physical mass flow. Recycle also occurs in the form of heat exchange (again in the styrene process) and sometimes as information, e.g., a specification that two variable temperatures must equal each other. The sequential-modular solution strategy is based upon knowing all inputs to a module and using these to calculate all outputs. When an input stream to a module is the output of a downstream module (i.e., there is recycle), calculations cannot be performed for the upstream module because one of its inputs is not yet known. This is illustrated in Fig. 4.7; unit 1 cannot be calculated because input stream 4 is the output of unit 2; nor can unit 2 be calculated because input stream 2 is an output of unit 1. This same problem of circular reasoning was encountered in Example 1. This dilemma in the sequential modular solution scheme can be

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†L. B. Evans, Computer-Aided Design (CAD): Advances in Process Flowsheeting Systems, "Recent Developments in Chemical Process and Plant Design," Y. A. Liu, H. A. McGee, Jr., and W. R. Epperly, eds., pp. 261-288, Wiley, New York, 1987.

resolved in the same manner as in the spread-sheet example; namely, calculate iteratively until the change in the calculated values is acceptable.

The identification of recycle loops within a process is termed ***partitioning***. This is done by the software of most flow-sheeting programs; however, in FLOWTRAN these loops must be identified by the user. Consequently, a procedure for performing this task is described here. Recycle loops usually can be identified fairly easily from a process flow diagram. If one can trace a closed path from an output of a process unit back to the input of that unit, then there is a recycle. In processes with many units it can be difficult to identify all of the independent recycle loops by tracing the flow diagram, but it can be done by hand in the same manner as it is done by some flow-sheeting software. In this procedure, start with an external feed stream to a process unit and follow one outlet from that unit. List each process unit encountered until either (a) an already listed process unit is encountered again or (b) the only remaining output stream of a unit is not a feed to any process unit. In the first case, a recycle loop has been identified; all the units in that loop are combined into a single block (they must be solved together iteratively) and that block is subsequently treated as is any other single unit and the procedure is continued. In the second case the identified unit is transferred from the process list to the beginning of a calculation list. This procedure is continued until all units have been shifted to the calculation list; this list, from the top down, indicates the order of calculation. Each block of units obtained in the first step is a recycle loop. This procedure will be illustrated in Example 4. The final result is one calculation order among many possible orders. There are algorithms that provide a calculation order with particular characteristics, such as the fewest possible tears of recycle loops. Such algorithms are intended to find an efficient calculation order, that is, one with fewer calculations or faster convergence. However, none of them can guarantee to be the most effective. Since FLOWTRAN does not employ such an algorithm, such algorithms will not be reyiewed here.

The iterative calculations of recycle are approached by "tearing" the recycle loop, that is, by selecting a stream in the loop to be the trial or assumed stream. Calculations are initiated by assuming values for the conditions of this stream, and the calculations then continue around the loop back to this assumed stream. The most recently calculated values for this stream are compared with the previous values and this procedure is continued (i.e., iterated) until consecutive values agree within a preset margin. This method is called direct or successive substitution. Various mathematical methods are used to accelerate the convergence of the calculations by guiding the selection of successive assumed values for the tear stream. These methods include those of Wegstein and Broyden and modifications thereof.? Partitioning, tearing, and solution by direct substitution without acceleration are illustrated in Example 4.

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†A. W. Westerberg. H. P. Hutchison, R. L. Motard, and P. Winter, "Process Flowsheeting," Chapter 3, Cambridge University Press, Cambridge, England, 1979.

Example 4 Computational order for styrene process. A styrene process flow diagram is shown in Fig. 4-5. From this flow sheet it is apparent that there are two recycle streams in this simple process: the unreacted ethylberizene is recycled and mixed with the fresh feed and the reactor effluent is recycled back to the heat exchanger.

Solution. The use of the calculation-ordering algorithm described above is demonstrated here. Starting with the styrene feed stream, follow the arrows and list the operations in the order encountered. The equipment numbers in order are,

1 2 3 2

Observe that the number 2 recurs, establishing that there is a recycle and that it includes operations 2 and 3. These operations are combined into one unit, label it 10. All operations within a loop must be solved together (usually by iteration). While following the streams, if an output stream is followed that is not a feed to another operation, e.g., SO8 from operation 5, check for another output from that operation (there is one—S09) and follow it. Continue listing the operations encountered:

1 10 4 5 6 7 8 1

Here the number 1 reappears, so another recycle loop has been found. This loop includes all the operations in this list. Note that the first loop, unit 10, is "nested" within this new loop. All the listed operations can now be combined into one new unit, label it 11. At operation 8, stream S14 was followed, it returned to operation 1; if stream S15 had been followed instead, operation 9 would have been transferred to the calculation list-the same result as obtained later. The choice of which output to follow first has no fundamental effect on the result except to perhaps change the calculation sequence. Continue following the outputs (there is only one from unit 11, S15) and listing the operations encountered:

11 9

The outputs from operation 9 are not feeds to any process dperation, so 9 is removed from this list and placed in a calculation list. This leaves only unit 11 in the process list and its output now is not a feed to any other operation (9 has been removed). Thus, operation 11 is now removed from the process list and added to the calculation list ahead (to the left) of what is already there. The calculation list is now

11 9

and nothing remains in the process list. The computation sequence is from left to right, so unit 11 is computed first and then operation 9 is computed.

One stream in each recycle loop must be chosen as the tear stream for that loop. It is the one to be assumed, checked for convergence, and iterated. There is not an unambiguous guideline for this choice, nor is it usually critical which stream is chosen. If there is one for which it is easier to make a reasonable initial estimate, probably it should be selected. The actual recycle stream is often selected as the tear stream.

A decision must be made whether the two recycle loops, units 10 and 11, are to be converged simultaneously or separately. Simultaneous convergence, as illustrated by Fig. 4-8*a*, is approached by checking and reestimating both tear streams once in each iteration. Separate convergence proceeds by converging the

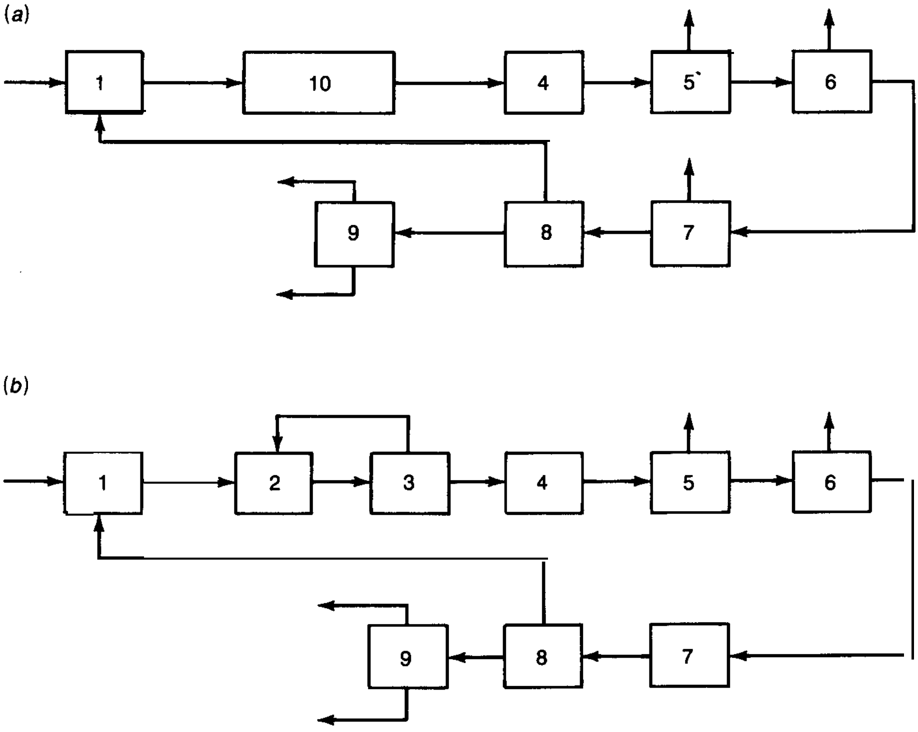


FIGURE 4-8
(*a*) Simultaneous and (*b*) separate convergence of nested recycle loops for Example 4.

inner loop (unit 10) during each iteration of the outer loop as suggested by Fig. 4-8*b*. The more appropriate choice, in terms of minimum computation time and convergence, is not readily apparent. It is often assumed that simultaneous convergence will be more efficient and that approach is illustrated here. All flow rates are in lb/h; stream numbers are from Fig. 4-5.

Start with feed stream SO1 = 112.8 lb EB/h

Assume that recycle stream S14 = 0

SO2 and SO3 are then 112.8

The steam feed SO4 = 112.8 * 14 * 18/106 = 268

The reactor effluent plus recycle SO5 = 380.8 and includes 112.8 * 0.5 = 56.4 lb EB/h. It has changed and not converged yet

SO6 is 380.8. Continuing through the separations, the EB recycle stream S14 = 56.4

The just-calculated values of the recycles, SO5 and S14, are now used and the calculations are repeated in the same sequence:

SO2 and SO3 = 169.2
SO4 = 402.2
S05 = so6 = 571.4
S14 = 169.2*0.5 = 84.6

Again, SO5 and S14 have changed. These new values are now used as the calculations are again repeated:

so2 = SO3 = 197.4
SO4 = 469.3
SO5 = SO6 = 666.7
s14 = 98.7

This sequence of calculations is repeated until SO.5 and S14 do not change significantly. The final results are, as obtained in Example 3:

SO2 = SO3 = 225.6
SO4 = 536
SO5 = 761.6
S14 = 112.8

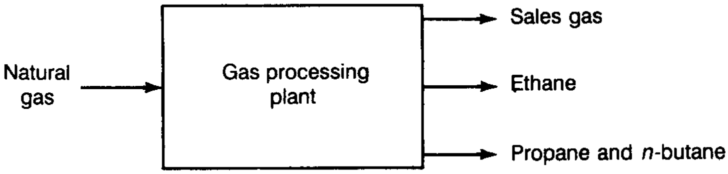
The styrene feed was used as the starting point in this example; the stream feed could have been chosen just as well. If it is, the same recycle loops will be identified, but a somewhat different calculation sequence will be identified. This is recommended as an exercise for the reader.

Many flow-sheeting programs perform the partitioning, solution ordering, and tearing functions discussed above and present the user with one or more choices of solution sequence and tear variables. FLOWTRAN, however, does not do this. The user must identify the recycle loops, the calculation sequence, and the tear streams. The preceding example illustrated their identification and selection.

PROBLEMS

1. A block diagram for a gas processing plant is shown below. The purpose of this process is to recover *ethane* and propane (+ butane) from a natural-gas stream because these components have more value as chemicals than as fuel. This problem is based upon the 1987 AIChE Student Contest Problem.

Natural gas is to be fed to the process at a rate of 2.5 x 10^6 g-mol/h. The feed-gas composition and product-gas specifications are given on page 135.



(a) Find the flow rates for each of the product streams for each of these values of the ethane mole fraction in the sales gas stream: 0.00, 0.01, 0.05.

(b) Find the value of the feed gas, the value of each of the product streams, and the difference between product and feed values ($/hr) for each of the ethane mole fractions in part (a).

Data:

Natural gas feed = $2.85/GJ of lower heating value (LHV)

Natural gas feed	Component	Mol%	LHV, kcal/gmol
	Methane	83.0	191.8
	Ethane	12.0	341.3
	Propane	4.0	488.5
	n-Butane	1.0	635.4

Products sales gas contains negligible propane and butane; value = $2.85/GJ (LHV).
Ethane product contains 1.0 mol% methane, 2.0 mol% propane, negligible butane; value = $0.18/kg.
Propane product contains negligible methane, all the n-butane and 2 mol% ethane; value = $0.20/kg.

2. A process has four streams with the characteristics given below. Devise a heat-exchange network to maximize the annual savings as compared to no heat exchange. Use a minimum approach temperature $\Delta T_{\min}$ = WC.

Process stream	T_{in}, °C	T_{out}, °C	$M * C_p$, MJ/(h) (°C)
"Cold" to be heated			
A	60	160	27.4
C	116	260	22.0
"Hot" to be cooled			
B	160	93	31.7
D	250	138	38.0

Cost data: Heat-exchanger costs are to be taken from Fig. 15-13 of this text. Use a design pressure of 150 psig for all exchangers except those using steam, for which a design pressure above the operating (steam) pressure should be used. Neglect pump and pumping costs.

Utilities: Cooling water, $0.15/1000 gal. (enters at 25°C, leaves at 50°C). Steam (saturated) at 6200 kPa, $15/Mg; at 2850 kPa, $13.2/Mg; at 790 kPa, $11/Mg; at 205 kPa, $7/Mg.

Annual cost of equipment, $/yr = (purchased cost, $)*(4.0)*(0.25)

Additional ***information:*** These estimated overall heat-transfer coefficients may be used (values are in J/s m^2K):

process stream to process stream-200
process stream to water-400
process stream to steam-550

Assume that all process streams have a heat capacity of 3.3 kJ/kg "C. Operation is 330 days per year, 24 h/day.

3. The academic version of the process invention program **(PIP)** contains several examples, as follows:

    HDA1—hydrodealkylation of toluene to produce benzene
    CYHEX1—cyclohexane production by hydrogenation of benzene
    STYR1—styrene production from ethylbenzene
    XYL1—production of m-xylene from toluene
    ANHYD1—reaction of acetone and acetic acid to form acetic anhydride
    BUTAL1—alkylation reaction of butene-1 and isobutane to produce **iso-octane**

    Select the "new plant design" mode and the "look at existing flow sheet" option, and follow the synthesis steps for one of these processes. 'Obtain the current flow sheet as output at each level of the synthesis procedure. Also list the heuristics used at each step.

4. Generate an alternative design for the process selected in Problem 3. Select the "new plant design" mode and the option "generate alternative to an existing flow sheet." There are many inputs that may be changed to produce an alternative flow sheet. A simple one is to input a new product rate, the process flow diagram will not change, but the mass and energy balances and equipment sizes and costs will. For example, double the product flow rate. More interesting changes may be introduced by changing process constraints such as reaction temperature and pressure, or even changing the choice of constraints. For example, replacing a specified constraint such as reactant molar ratio with another, such as the composition of the purge stream.

5. For the methanol synthesis process illustrated in Fig. 4-1, Example 1, assume that there are algorithms for calculating the outputs of each process unit from the inputs. Determine how many stream variables must be specified and decide what these should be so that a unique solution exists for the mass and energy balances. Identify all recycle loops, tear streams for these loops, and a calculation sequence.

6. Repeat Example 4 for the styrene synthesis process with a different selection for the tear streams to determine a new calculation sequence.

7. For a process flow sheet obtained in Problem 3, assume that algorithms are available to calculate the outputs from each process unit from known inputs. Determine the number of stream variables that must be specified, decide what they should be, identify all recycle loops, select tear streams for these loops, and establish a calculation sequence.

8. Toluene is converted to benzene catalytically according to the reaction

    $$C_7H_8 + H_2 \rightarrow C_6H_6 + \text{CH, } + 20{,}513 \text{ Btu/lb mol}$$

    For the following reactor feed, use the AREAC block of the CACHE FLOWTRAN simulation to determine the exit temperature for adiabatic operation at 90 percent conversion of toluene.

Species	*lb mol/h*
H_2	2000
CH_4	1000
C_6H_6	50
C_7H_8	400

Temperature = 800°F. Pressure = 500 psia

For solution, see Exercise 12 on page 81 of "Exercises in Process Simulation Using FLOWTRAN" by J. Peter Clark published by CACHE Corp., 77 Massachusetts Ave., Cambridge, Massachusetts in 1977. For information on the AREAC block, see pages A-146, A-147, and 15 of "FLOWTRAN Simulation-An Introduction" by J. D. Seader, W. D. Seider, and A. C. Pauls, 2d ed., published by CACHE Corp., in 1977.

CHAPTER
5

COST AND ASSET ACCOUNTING

The design engineer, by analyses of costs and profits, attempts to predict whether capital should be invested in a particular project. After the investment is made, records must be maintained to check on the actual financial results. These records are kept and interpreted by accountants. The design engineer, of course, hopes that the original predictions will agree with the facts reported by the accountant. There is little chance for agreement, however, if both parties do not consider the same cost factors, and comparison of the results is simplified if the same terminology is used by the engineer and the accountant.

The purpose of accounting is to record and analyze any financial transactions that have an influence on the utility of capital. Accounts of expenses, income, assets, liabilities, and similar item are maintained. These records can be of considerable value to the engineer, since they indicate where errors were made in past estimates and give information that can be used in future evaluations. Thus, the reason why the design engineer should be acquainted with accounting procedures is obvious. Although it is not necessary to know all the details involved in accounting, a knowledge of the basic principles as applied in economic evaluations is an invaluable aid to the engineer.

This chapter presents a survey of the accounting procedures usually encountered in industrial operations. Its purpose is to give an understanding of the terminology, basic methods, and manner of recording and presenting information as employed by industrial accountants.

OUTLINE OF ACCOUNTING PROCEDURE

The diagram in Fig. 5-1 shows the standard accounting procedure, starting with the recording of the original business transactions and proceeding to the final preparation of summarizing balance sheets and income statements. As the day-by-day business transactions occur, they are recorded in the *journal*. A single journal may be used for all entries in small businesses, but large concerns ordinarily use several types of journals, such as cash, sales, purchase, and general journals.

The next step is to assemble the journal entries under appropriate account headings in the *ledger*. The process of transferring the daily journal entries to the ledger is called *posting*.

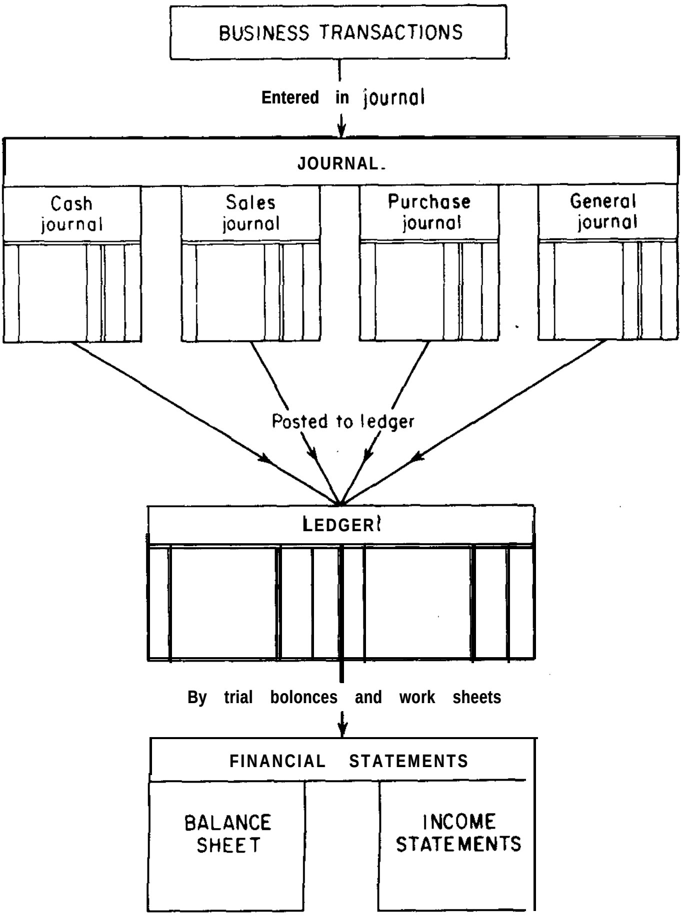


FIGURE 5-1
Diagram of accounting procedure.

Statements showing the financial condition of the business concern are prepared periodically from the ledger accounts. These statements are presented in the form of balance sheets and *income statements.* The balance sheet shows the financial condition of the business at a particular time, while the income statement is a record of the financial gain or loss of the organization over a given period of time.

BASIC RELATIONSHIPS IN ACCOUNTING

In the broadest sense, an *asset* may be defined as anything of value, such as cash, land, equipment, raw materials, finished products, or any type of property. At any given instant, a business concern has a certain monetary value because of its *assets.* At the same instant, many different persons may have a just claim, or *equity,* to ownership of the concern's assets. Certainly, any creditors would have a just claim to partial ownership, and the owners of the business would have some claim to ownership. Under these conditions, a fundamental relationship in accounting can be written as

$$\text{Assets} = \text{equities} \tag{1}$$

Equities can be divided into two general classes as follows: (1) *Proprietorship*-the claims of the concern or person who owns the asset; and (2) *liabilities* -the claims of anyone other than the owner. The term *proprietorship* is often referred to as *net worth* or simply as *ownership* or *capital.* Thus, Eq. (1) can be written as†

$$\text{Assets} = \text{liabilities} + \text{proprietorship} \tag{2}$$

The meaning of this basic equation can be illustrated by the following simple example. Five students have gone together and purchased a secondhand automobile worth $1000. Because they did not have the necessary $1000 they borrowed $400 from one of their parents. Therefore, as far as the students are concerned, the value of their asset is $1000, their proprietorship is $600, and their liability is $400.

Equation (2) is the basis for balancing assets against equities at any given instant. A similar equation can be presented for balancing costs and profits over any given time period. The total income must be equal to the sum of all costs and profits, or

$$\text{Total income} = \text{costs} + \text{profits} \tag{3}$$

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†Equation (2) is sometimes expressed as "Assets = liabilities," where "liabilities" is synonymous with "equities."

Any engineering accounting study can ultimately be reduced to one of the forms represented by Eq. (2) or (3).

THE BALANCE SHEET

A balance sheet for an industrial concern is based on Eq. (1) or (2) and shows the financial condition at any given date. The amount of detail included varies depending on the purpose. Consolidated balance sheets based on the last day of the fiscal year are included in the annual report of a corporation. These reports are intended for distribution to stockholders, and the balance sheets present the pertinent information without listing each individual asset and equity in detail.

Assets are commonly divided into the classifications of current, fixed, and miscellaneous. *Current assets,* in principle, represent capital which can readily be converted into cash. Examples would be accounts receivable, inventories, cash, and marketable securities. These are liquid *assets.* On the other hand, *fixed assets*, such as land, buildings, and equipment, cannot be converted into immediate cash. Deferred charges, other investments, notes and accounts due after 1 year, and similar items are ordinarily listed as miscellaneous assets under separate headings.

Modern balance sheets often use the general term *liabilities* in place of *equities. Current liabilities* are grouped together and include all liabilities such as accounts payable, debts, and tax accruals due within 12 months of the balance-sheet date. The *net working capital* of a company can be obtained directly from the balance sheet as the difference between current assets and current liabilities. Other liabilities, such as long-term debts, deferred credits, and reserves are listed under separate headings. Proprietorship, stockholders' equity, or capital stock and surplus complete the record on the equity (or liability) side of the balance sheet.

Consolidated balance sheets are ordinarily presented with assets listed on the left and liabilities, including proprietorship, listed on the right. As indicated in Eq. (1), the total value of the assets must equal the total value of the equities. A typical balance sheet of this type is presented in Fig. 5-2.

The value of property items, such as land, buildings, and equipment, is usually reported as the value of the asset at the time of purchase. Depreciation reserves are also indicated, and the difference between the original property cost and the depreciation reserve represents the *book* value of the property. Thus, in depreciation accounting, separate records showing accumulation in the depreciation reserve must be maintained. In the customary account, reserve for *depreciation* is not actually a separate fund but is merely a bookkeeping method for recording the decline in property value.

The ratio of total current assets to total current liabilities is called the *current ratio.* The ratio of immediately available cash (i.e., cash plus U.S. Government and other marketable securities) to total current liabilities is known as the cash *ratio.* The current and cash ratios are valuable for determining the ability to meet the financial obligations, and these ratios are examined

ABC PETROLEUM COMPANY
CONSOLIDATED BALANCE SHEET, DECEMBER 31, 1989

Assets	
Current assets	
Cash	$ 93.879000
Notes and accounts receivable	219,374,000
Inventories:	
Crude oil. petroleum products, and merchandise	152,021,000
Materials and supplies (at cost and condition value)	25,524,000
Total current assets	$490,798,000
Investments and long-term receivables (at cost)	$309.249.000
Properties, plants, and equipment (al cost)	
Production	$1.390.688.000
Manufacturing	669,915,000
Transportation	195.255.000
Marketing	188,405,000
Other	79,823,000
	$2,524,086,000
Less reserves for depreciation, depletion, and amortization	1,315,416,000
Total	$1.208.670.000
Prepaid and deferred charges	$ 20,347,000
TOTAL ASSETS	$2,029,064,000

Liabilities and stockholders' equity	
Current liabilities	
Accounts payable	$160,307,000
Long-term debt (due within one year)	3.514,000
Accrued taxes	58,938,000
Other accruals	20.796.000
Total current liabilities	$243.555.000
Long-term debt	$333,738,000
Deferred credits	
Federal income taxes	$ 86,845,000
Other	45,579,000
Total deferred credits	$132.424.000
Reserve for contingencies	$ 24.197.000
Stockholders' equity	
Common stock. $5 par value Shares authorized, 50.000.000 Shares issued. 34.465.956	$ 172,330,000
Capital in excess of par value of common stock	264,238,000
Earnings employed in the business	918,165,000
	$1,354,733,000
Less treasury stock (at cost), 1,096,627 shares	59,583,000
Total stockholders' equity	$1,295,150,000
TOTAL LIABILITIES AND STOCKHOLDE'RS EQUITY	$2,029,064,000

FIGURE 5-2
A consolidated balance sheet.

by banks or other loan concerns before credit is extended. From the data presented in Fig. 5-2, the current ratio for the company on December 31, 1989 was

$$\frac{\$490{,}798{,}000}{\$243{,}555{,}000} = 2.02$$

and the cash ratio was $93,879,000/$243,555,000 = 0.386.

THE INCOME STATEMENT

A balance sheet applies only at one specific time, and any additional transactions cause it to become obsolete. Most of the changes that occur in the balance sheet are due to revenue received from the sale of goods or services and costs incurred in the production and sale of the goods or services. Income-sheet accounts of all income and expense items, such as sales, purchases, depreciation, wages, salaries, taxes, and insurance, are maintained, and these accounts are summarized periodically in *income statements.*

A consolidated income statement is based on a given time period. It indicates surplus capital and shows the relationship among total income, costs, and profits over the time interval. The transactions presented in income-sheet accounts and income statements, therefore, are of particular interest to the engineer, since they represent the facts which were originally predicted through cost and profit analyses.

The terms gross *income* or gross revenue used by accountants refer to the total amount of capital received as a result of the sale of goods or service. *Net income* or *net* revenue is the total profit remaining after deducting all costs, including taxes.

Figure 5-3 is a typical example of a consolidated income statement based on a time period of 1 year. As indicated by Eq. (3), the total income ($345,155,242) equals the total cost ($302,600,732 + $18,854,000) plus the net income or profit ($23,700,510).

The role of interest on borrowed capital is clearly indicated in Fig. 5-3. Since the accountant considers interest as an expense arising from the particular method of financing, the cost due to interest is listed as a separate expense.

MAINTAINING ACCOUNTING RECORDS

Balance sheets and income statements are summarizing records showing the important relationships among assets, liabilities, income, and costs at one particular time or over a period of time. Some method must be used for recording the day-to-day events. This is accomplished by the use of *journals* and *ledgers.*

A journal can be a book, group of vouchers, or some other convenient computer printout in which the original record of a transaction is listed, while a ledger is a group of accounts giving condensed and classified information from the journal.

XYZ CHEMICAL COMPANY AND CONSOLIDATED SUBSIDIARIES
CONSOLIDATED INCOME STATEMENT FOR THE YEAR ENDING DECEMBER 31, 1989

Income	
Net sales	$341.822.557
Dividends from subsidiary and associated companies	798.483
Other	2.534.202
Total (or gross) income	$345,155,242
Deductions	
Cost of goods sold	$243,057.056
Selling and administrative expenses	42,167,634
Research expenses	10.651.217
Provision for employees' bonus	649.319
Interest expenses	3.323.372
Net income applicable to minority interests	143.440
Other	2.608.694
Total deductions	$302,600,732
Income before provision for income taxes	$ 42.554.510
Less provision for income taxes	18,854,000
Net income (net profit)	$ 23.700.510
Earned surplus at beginning of year	90.436.909
Total surplus	$114,137,419
Surplus deductions	
Preferred dividends ($3.85 per share)	$ 721,875
Common dividends ($2.50 per share)	13.148.300
Total surplus deductions	$ 13,870,175
EARNED SURPLUS AT END OF YEAR	$100,267,244

FIGURE 5-3
A consolidated income statement.

Debits and Credits

When recording business transactions, a *debit* entry represents an addition to an account, while a *credit* entry represents a deduction from an account. In more precise terms, a debit entry is one which increases the assets or decreases the equities, and a credit entry is one which decreases the assets or increases the equities.

Since accounting records must always show a balance between assets and equities, any single transaction must affect both assets and equities. Each debit entry, therefore, requires an equal and offsetting credit entry. For example, if a company purchased a piece of equipment by a cash payment, the assets of the company would be increased by the value of the equipment. This represents an addition to the account, and would therefore, be listed as a debit. However, the company had to pay out cash to obtain the equipment. This payment must be recorded as a credit entry, since it represents a deduction from the account. At least one debit entry and one credit entry must be made for each business

JOURNAL

Page 1

Date 19__		*Analysis*	*F*	*Debit*	*Credit*
April	3	Salaries	403	$ 758 00	$
		Cash	112		758 00
		Payment of salaries for week ending April 3			
	4	Rent	314	300 00	
		Cash	112		300 00
		Building rental for month of April			
	5	Cash	112	1041 00	
		Sales	201		1041 00
		Product *A* to X Company as per invoice No. 6839			
	8	Equipment	104	1800 00	
		Notes payable	521		1800 00
		Equipment for plant—6%, 90day note to Y Company			
	9	Purchases	608	881 00	
		Z Company	842		881 00
		Tools on open account-Terms-30 days			

FIGURE 5-4
A typical journal page.

transaction in order to maintain the correct balance between assets and equities. This is known as *double-entry bookkeeping.*

The Journal

A typical example of a journal page is shown in Fig. 5-4. The date is indicated in the first two columns. An analysis of the account affected by the particular transaction is listed in the third column, with debits listed first and credits listed below and offset to the right. A brief description of the item is included if necessary. The amounts of the individual debit and credit entries are indicated in the last two columns, with debits always shown on the left. When the journal entry is posted in the ledger, the number of the ledger account or the page number is entered in the fourth column of the journal page. This column is usually designated by "F" for "Folio."

The Ledger

Separate ledger accounts may be kept for various items, such as cash, equipment accounts receivable, inventory, accounts payable, and manufacturing expense. A typical ledger sheet is shown in Fig. 5-5. The ledger sheets serve as a secondary record of business transactions and are used as the intermediates

LEDGER

Cash 112

Date 19__		Analysis	F	Debit		Dare 19__		Analysis	F	Credit	
April	1	Balance forward	J-1	$ 945	00	April	3	Salaries	J-1	$ 758	00
							4	Rent	J-1	300	00
	5	Sales	J-1	1041	00		10	Purchases	J-2	175	00
	10	Sales	J-2	86	00		12	Insurance	J-2	455	00
	11	R Company	J-2	700	00		13	Taxes	J-2	875	00
	11	Sales	J-2	550	00		17	Salaries	J-3	821	00
	12	Sales	J-2	94	00		21	Purchases	J-3	985	00
	18	S Company	J-3	1200	00		29	Office supplies	J-3	158	00
	22	Sales	J-3	175	00						
	28	Sales	J-3	548	00		30	Purchases	J-3	154	00
	29	Sales	J-3	630	00						
	30	Sales	J-3	74	00			Balance		1362	00
		Total		$6043	00			Total		$6043	00
		Balance forward		$1362	00						

FIGURE **5-5**
Typical ledger sheet that has been closed and balanced.

between journal records and balance sheets, income statements, and general cost records.

COST ACCOUNTING METHODS

In the simplest form, cost accounting is the determination and analysis of the cost of producing a product or rendering a service. This is exactly what the designer engineer does when estimating costs for a particular plant or process, and cost estimation is one type of cost accounting.

Accountants in industrial plants maintain records on actual expenditures for labor, materials, power, etc., and the maintenance and interpretation of these records is known as *actual* or *post-mortem* cost accounting. From these data, it is possible to make accurate predictions of the future cost of the particular plant or process. These predictions are very valuable for determining future capital requirements and income, and represent an important type of cost accounting known as *standard* cost accounting. Deviations of standard costs from actual costs are designated as *variances.*

There are many different types of systems used for reporting costs, but all the systems employ some method for classifying the various expenses. One common type of classification corresponds to that presented in Chap. 6 (Cost Estimation). The total cost is divided into the basic groups of manufacturing costs and general expenses. These are further subdivided, with administrative, distribution, selling, financing, and research and development costs included under general expenses. Manufacturing costs include direct production costs, fixed charges, and plant overhead.

Each of the subdivided groups can be classified further as indicated in Chap. 6. For example, direct production costs can be broken down into costs for raw materials, labor, supervision, maintenance, supplies, power, utilities, laboratory charges, and royalties.

Each business corporation has its own method for distributing the costs on its accounts. In any case, all costs are entered in the appropriate journal account, posted in the ledger, and ultimately reported in a final cost sheet or cost statement.

Accumulation, Inventory, and Cost-of-Sales Accounts

In general, basic cost-accounting methods require posting of all costs in so-called *accumulation* accounts. There may be a series of such accounts to handle the various costs for each product. At the end of a given period, such as one month, the accumulated costs are transferred to *inventory* accounts, which give a summary of all expenditures during the particular time interval. The amounts of all materials produced or consumed are also shown in the inventory accounts., The information in the inventory account is combined with data on the amount of product sales and transferred to the cost-of-sales account. The cost-of-sales accounts give the information necessary for determining the profit or loss for each product sold during the given time interval. One type of inventory account is shown in Fig. 5-6, and a sample cost-of-sales account is presented in Fig. 5-7.

When several products or by-products are produced by the same plant, allocation of the cost to each product must be made on some predetermined basis. Although the allocation of raw-material and direct labor costs can be determined directly, the exact distribution of overhead costs may become quite complex, and the final method depends on the policies of the particular concern involved.

Materials Costs

The variation in costs due to price fluctuations can cause considerable difficulty in making the transfer from accumulation accounts to inventory and cost-of-sales accounts. For example, suppose an accumulation account showed the following:

ACCUMULATION ACCOUNT

Item: Chemical *A* **for use in producing product** *X*

Date 19__	*Received*	*cost*	*Balance on hand*	*Delivered for use in process*
May 2	**5,000 lb**	**\$0.0360/lb**	**5,000 lb**	
May 15	**10,000 lb**	**\$0.0390/lb**	**15,000 lb**	
May 17			**9,000 lb**	**6,000 lb**

MANUFACTURING COST WORKS INVENTORY

Refining of crude product D **For month of January. 19__**

Cost element		*Units on hand*						
Name	*Unit*	*Start of month*	*End of month*	*Units used per unit pro-duced*	*Total units used or pro-duced*	*Price per unit*	*Total cost*	*Cost per unit pro-duced*
Crude product D	**Gallons**	13,000	11,000	1.5000	150,000	so 2187	$41,800	$0.4180
Operating wages	Hours			0.0150	1,500	1.2500	1,975	0.0198
Operating supplies							890	0.0089
Maintenance wages	Hours			0.0250	2.500	2.0000	5,000	0.0500
Maintenance materials							10,500	0.1050
Utilities							8,000	0.0800
Depreciation	$ investment			5.0000	500,000	0.0100	5,000	0.0500
Overhead							3,800	0.0380
Total cost and production	**Gallons**	5,000	4,000		100,000	$0.7697	116.965	S0 1691

FIGURE 5-6
Example of one type of inventory account.

COST-OF-SALES ACCOUNT

Product E *For month of June, 19__*

Item	*This month*	*Last month*	*Year to date*
Sales, lb	475,000	590,000	3,220,000
	$/unit	S/unit	$/unit
Selling price	$0.200	$0.200	$0.200
Cost of sales:			
Manufacturing cost	0.120	0.100	0.105
Freight and delivery	0.007	0.008	0.007
Selling expense	0.018	0.020	0.016
Administrative expense	0.025	0.020	0.022
Research expense	0.010	0.008	0.008
Total cost of sales	S0.180	$0.156	$0.158
Profits before taxes	$0.020	S0.044	$0.042

FIGURE 5-7
Example of cost-of-sales account.

In transferring the cost of chemical *A* to the inventory and cost-of-sales accounts, there is a question as to what price applies for chemical ***A.*** There are three basic methods for handling problems of this type.

1. ***The current-average method.*** The average price of all the inventory on hand at the time of delivery or use is employed in this method. In the preceding example, the current-average price for chemical *A* would be $0.0380 per pound.
2. ***The first-in-first-out*** (or ***fifo***) ***method.*** This method assumes the oldest material is always used first. The price for the 6000 lb of chemical *A* would be $0.0360 per pound for the first 5000 lb and $0.0390 for the remaining 1000 lb.
3. ***The lust-in first-out*** (or ***lifo***) ***method.*** With this method, the most recent prices are always used. The price for the 6000 lb of chemical A would be transferred as $0.0390 per pound.

Any of these methods can be used. The current-average method presents the best picture of the true cost during the given time interval, but it may be misleading if used for predicting future costs.

The information presented in this chapter shows the general principles and fundamentals of accounting which are of direct interest to the engineer. However, the many aspects of accounting make it impossible to present a complete coverage of all details and systems in one chapter or even in one book.? The exact methods used in different businesses may vary widely depending on the purpose and the policies of the organization, but the basic principles are the same in all cases.

PROBLEMS

1. Prepare a balance sheet applicable at the date when the X Corporation had the following assets and equities:

Cash	$20,000	Common stock sold	$50,000
Accounts payable:		Machinery and equipment (at present value)	18,000
B Company	2,000		
C Corporation	8,000	Furniture and fixtures (at present value)	5,000
Accounts receivable	6,000		
Inventories	15,000	Government bonds	3,000
Mortgage payable	5,000	Surplus	2,000

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†C. T. Horngren, "Introduction to Financial Accounting," 2nd ed., Prentice-Hall, Inc., Englewood Cliffs, NJ, 1984; J. W. Wilkinson, "Accounting and lnformation Systems," Wiley, New York, 1982.

2. During the month of October, the following information was obtained in an antifreeze retailing concern:

Salaries	$3,000
Delivery expenses	700
Rent	400
Sales	15,100
Antifreeze available for sale during October (at cost)	20,200
Antifreeze inventory on Oct. 31 (at cost)	11,600
Other expenses	1,200
Earned surplus before income taxes as of Sept. 30	800

Prepare an income statement for the month of October giving as much detail as possible.

3. The following information applies to E Company on a given date:

Long-term debts	1,600
Debts due within 1 year	1,000
Accounts payable	2,300
Machinery and equipment (at cost)	10,000
Cash in bank	3,100
Prepaid rent	300
Government bonds	3,000
Social security taxes payable	240
Reserve for depreciation	600
Reserve for expansion	1,200
Inventory	1,600
Accounts receivable	1,700

Determine the current ratio, cash ratio, and working capital for Company E at the given date.

4. On Aug. 1, a concern had 10,000 lb of raw material on hand which was purchased at a cost of $0.030 per pound. In order to build up the reserve, 8000 lb of additional raw material was purchased on Aug. 15 at a cost of $0.028 per pound, and 2 days later 6000 lb was purchased from another supplier at $0.031 per pound. If none of the raw material was used until after the last purchase determine the total cost of 12,000 lb of the raw material on an inventory or cost-of-sales account for the month of August by (a) the current-average method, (*b*) the "fifo" method, and (*c*) the "lifo" method.

5. Prepare a complete list, with sample form sheets, of all cost-accounting records which should be maintained in a large plant producing ammonia, nitric acid, and ammonium nitrate. Explain how these records are used in recording, summarizing and interpreting costs and profits.

CHAPTER
6

COST ESTIMATION

An acceptable plant design must present a process that is capable of operating under conditions which will yield a profit. Since net profit equals total income minus all expenses, it is essential that the chemical engineer be aware of the many different types of costs involved in manufacturing processes. Capital must be allocated for direct plant expenses, such as those for raw materials, labor, and equipment. Besides direct expenses, many other indirect expenses are incurred, and these must be included if a complete analysis of the total cost is to be obtained. Some examples of these indirect expenses are administrative salaries, product-distribution costs, and costs for interplant communications.

A capital investment is required for any industrial process, and determination of the necessary investment is an important part of a plant-design project. The total investment for any process consists of fixed-capital investment for physical equipment and facilities in the plant plus working capital which must be available to pay salaries, keep raw materials and products on hand, and handle other special items requiring a direct cash outlay. Thus, in an analysis of costs in industrial processes, capital-investment costs, manufacturing costs, and general expenses including income taxes must be taken into consideration.

CASH FLOW FOR INDUSTRIAL OPERATIONS

Figure 6-1 shows the concept of cash flow for an overall industrial operation based on a support system serving as the source of capital or the sink for capital* receipts. Input to the capital sink can be in the form of loans, stock issues, bond releases, and other funding sources including the net cash flow returned to the

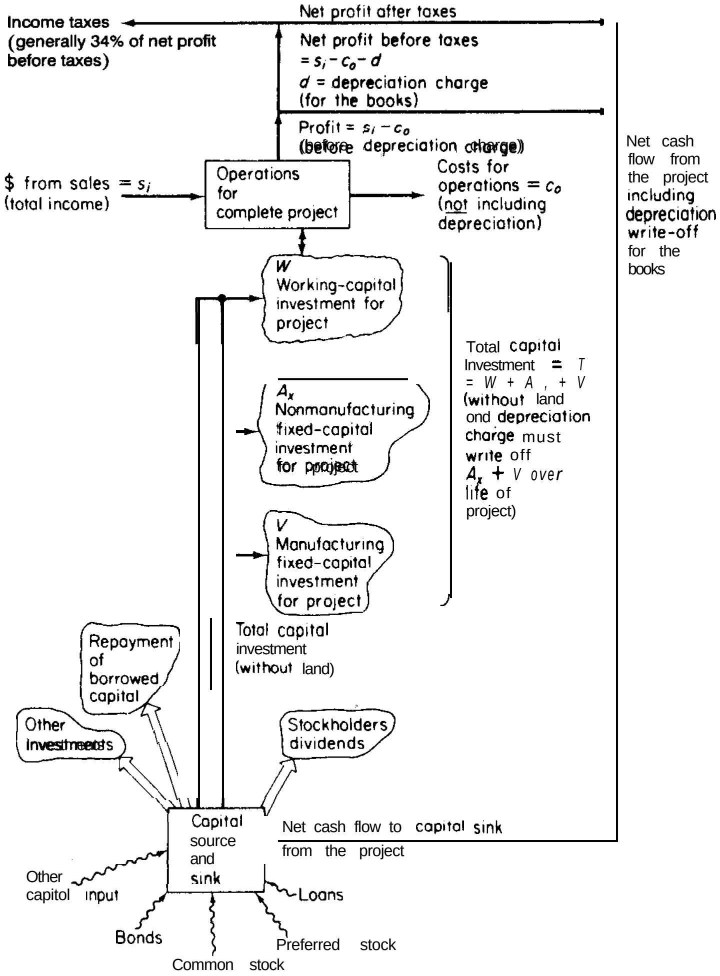

FIGURE 6-1
Tree diagram showing cash flow for industrial operations.

capital sink from each project. Output from the capital source is in the form of total capital investments for each of the company's industrial operations, dividends to stockholders, repayment of debts, and other investments.

The tree-growth concept, as shown in Fig. 6-1, depicts a trunk output to start the particular industrial operation designated as the total capital investment. This total capital investment includes all the funds necessary to get the project underway. This encompasses the regular manufacturing fixed-capital investment and the working-capital investment along with the investment required for all necessary auxiliaries and nonmanufacturing facilities. The cash flow for the capital investments can usually be considered as in a lump sum or in-an-instant such as for the purchase of land with a lump-sum payment or the provision of working capital as one lump sum at the start of the operation of the completed plant. Fixed capital for equipment ideally can be considered as in-an-instant for each piece of equipment although the payments, of course, can be spread over the entire construction period when considering the fixed-capital investment for a complete plant. Because income from sales and necessary operating costs can occur on an irregular time basis, a constant reservoir of working capital must be kept on hand continuously to draw from or add to as needed.

The rectangular box in Fig. 6-1 represents the overall operations for the complete project with working-capital funds moving in and out as needed but maintaining a constant fund as available working capital. Cash flows into the operations box as total dollars of income (s_i) from all sales while actual costs for the operations, such as for raw materials and labor, are shown as outflow costs (c_o). These cash flows for income and operating expenses can be considered as continuous and represent rates of flow at a given point in time using the same time basis, such as dollars per day or dollars per year. Because depreciation charges to allow eventual replacement of the equipment are in effect costs which are paid back to the company capital sink, these charges are not included in the costs for operations shown in Fig. 6-1. The difference between the income (s_i) and operating costs (c_o) represents gross profits before depreciation or income-tax charges $(s_i - c_o)$ and is represented by the vertical line rising out of the operations box.

Depreciation, of course, must be recognized as a cost before income-tax charges are made and before net profits are reported to the stockholders. Consequently, removal of depreciation in the cash-flow diagram as a charge against profit is accomplished at the top of the tree diagram in Fig. 6-1 with the depreciation charge (d) entering the cash-flow stream for return to the capital sink. The resulting new profit of $s_i - c_o - d$ is taxable, and the income-tax charge is shown as a cash-flow stream deducted at the top of the diagram. The remainder, or net profit after taxes, is now clear profit which can be returned to the capital sink, along with the depreciation charge, to be used for new investments, dividends, or repayment of present investment as indicated by the various trunks emanating from the capital source in Fig. 6-1.

Cumulative Cash Position

The cash flow diagram shown in Fig. 6-1 represents the steady-state situation for cash flow with s_i, c_o, and d all based on the same time increment. Figure 6-2 is for the same type of cash flow for an industrial operation except that it depicts the situation over a given period of time as the ***cumulative cash position.*** The time period chosen is the estimated life period of the project, and the time value of money is neglected.

In the situation depicted in Fig. 6-2, land value is included as part of the total capital investment to show clearly the complete sequence of steps in the

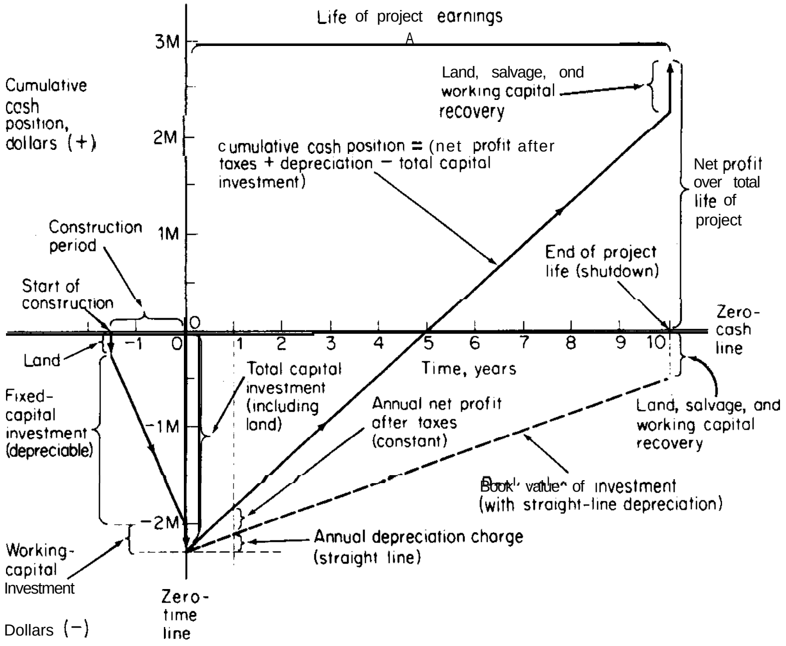

FIGURE 6-2
Graph of cumulative cash position showing effects of cash flow with time for an industrial operation neglecting time value of money.

full life cycle for an industrial process. The zero point on the abscissa represents that time at which the plant has been completely constructed and is ready for operation. The total capital investment at the zero point in time includes land value, fixed-capital and auxiliaries investment, and working capital. The cash position is negative by an amount equivalent to the total capital investment at zero time, but profits in the ideal situation come in from the operation as soon as time is positive. Cash flow to the company, in the form of net profits after taxes and depreciation charges, starts to accumulate and gradually pays off the full capital investment. For the conditions shown in Fig. 6-2, the full capital investment is paid off in five years. After that time, profits accumulate on the positive side of the cumulative cash position until the end of the project life at which time the project theoretically is shut down and the operation ceases. At that time, the working capital is still available, and it is assumed that the land can still be sold at its original value. Thus, the final result of the cumulative cash position is a net profit over the total life of the project, or a cash flow into the company capital sink (in addition to the depreciation cash flow for investment payoff) over the ten-year period, as shown in the upper right-hand bracket in Fig. 6-2.

The relationships presented in Fig. 6-2 are very important for the understanding of the factors to be considered in cost estimation. To put emphasis on the basic nature of the role of cash flow, including depreciation charges, Fig. 6-2 has been simplified considerably by neglecting the time value of money and using straight-line relationships of constant annual profit and constant annual depreciation. In the chapters to follow, more complex cases will be considered in detail.

FACTORS AFFECTING INVESTMENT AND PRODUCTION COSTS

When a chemical engineer determines costs for any type of commercial process, these costs should be of sufficient accuracy to provide reliable decisions. To accomplish this, the engineer must have a complete understanding of the many factors that can affect costs. For example, many companies have reciprocal arrangements with other concerns whereby certain raw materials or types of equipment may be purchased at prices lower than the prevailing market prices. Therefore, if the chemical engineer bases the cost of the raw materials for the process on regular market prices, the result may be that the process is uneconomical. If the engineer had based the estimate on the actual prices the company would have to pay for the raw materials, the economic picture might have been altered completely. Thus the engineer must keep up-to-date on price fluctuations, company policies, governmental regulations, and other factors affecting costs.

Sources of Equipment

One of the major costs involved in any chemical process is for the equipment. In many cases, standard types of tanks, reactors, or other equipment are used, and a substantial reduction in cost can be made by employing idle equipment or by purchasing second-hand equipment. If new equipment must be bought, several independent quotations should be obtained from different manufacturers. When the specifications are given to the manufacturers, the chances for a low cost estimate are increased if the engineer does not place overly strict limitations on the design.

Price Fluctuations

In our modern economic society, prices may vary widely from one period to another, and this factor must be considered when the costs for an industrial process are determined. It would obviously be ridiculous to assume that plant operators or supervisors could be hired today at the same wage rate as in 1975. The same statement applies to comparing prices of equipment purchased at different times. The chemical engineer, therefore, must keep up-to-date on price and wage fluctuations. One of the most complete sources of information on existing price conditions is the *Monthly Labor Review* published by the U.S. Bureau of Labor Statistics. This publication gives up-to-date information on present prices and wages for different types of industries.

Company Policies

Policies of individual companies have a direct effect on costs. For example, some concerns have particularly strict safety regulations and these must be met in every detail. Accounting procedures and methods for determining depreciation costs vary among different companies. The company policies with reference to labor unions should be considered, because these will affect overtime labor charges and the type of work the operators or other employees can do. Labor-union policies may even dictate the amount of wiring and piping that can be done on a piece of equipment before it is brought into the plant, and, thus, have a direct effect on the total cost of installed equipment.

Operating Time and Rate of Production

One of the factors that has an important effect on the costs is the fraction of the total available time during which the process is in operation. When equipment stands idle for an extended period of time, the labor costs are usually low; however, other costs, such as those for maintenance, protection, and depreciation, continue even though the equipment is not in active use.

Operating time, rate of production, and sales demand are closely interrelated. The ideal plant should operate under a time schedule which gives the maximum production rate while maintaining economic operating methods. In this way, the total cost per unit of production is kept near a minimum because the fixed costs are utilized to the fullest extent. This ideal method of operation is based on the assumption that the sales demand is sufficient to absorb all the material produced. If the production capacity of the process is greater than the sales demand, the operation can be carried on at reduced capacity or periodically at full capacity.

Figure 6-3 gives a graphical analysis of the effect on costs and profits when the rate of production varies. As indicated in this figure, the fixed costs remain constant and the total product cost increases as the rate of production increases. The point where the total product cost equals the total income is known as the *break-even point*. Under the conditions shown in Fig. 6-3, an ideal production rate for this chemical processing plant would be approximately 450,000 kg/month, because this represents the point of maximum net earnings.

The effects of production rate and operating time on costs should be recognized. By considering sales demand along with the capacity and operating characteristics of the equipment, the engineer can recommend the production rate and operating schedules that will give the best economic results.

Governmental Policies

The national government has many regulations and restrictions which have a direct effect on industrial costs. Some examples of these are import and export tariff regulations, restrictions on permissible depreciation rates, income-tax rules, and environmental regulations.

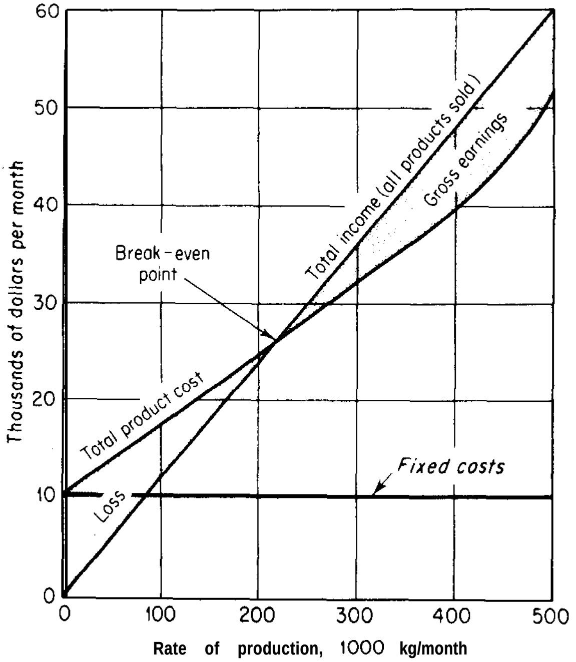

FIGURE 6-3
Break-even chart for chemical processing plant.

Prior to 1951, the United States had strict governmental regulations against rapid write-offs for industrial equipment. These restrictions increased the income-tax load for new companies during their first few years of existence and tended to discourage new enterprises. Therefore, during the Korean war, a fast amortization policy for certain defense installations was authorized. This policy permitted at least part of the value of the installation to be written off in 5 years as compared to an average of 10 to 15 years under the old laws. In 1954, a new law was passed permitting approximately two-thirds of the total investment for any process to be written off as depreciation during the first half of the useful life. A rapid write-off of this type can be very desirable for some concerns because it may reduce income taxes during the early years of the plant life.

In 1971 and again in 1981 and 1986, there were major changes in Federal income-tax regulations relative to acceptable methods for determining depreciation write-offs. Other changes have from time to time been adopted by Congress, and the cost engineer must keep up-to-date on these changes.?

Governmental policies with reference to capital gains and gross-earnings taxes should be understood when costs are determined. Suppose a concern decides to sell some valuable equipment before its useful life is over. The equipment has a certain *asset* or unamortized value, but the offered price may

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†For a discussion of depreciation write-off methods, see Chap. 9 (Depreciation).

be more than the unamortized value. This profit over the unamortized value would have been taxable as long-term capital gain at 28 to 30 percent if it had been held for nine months in 1977, one year from 1978 to mid 1984, and six months from mid 1984 through 1987. Starting in 1988, the period for long-term capital gain was one year and the tax rate on both short-term and long-term capital gains was generally the same as that for ordinary income. Therefore, in the example referred to above where a long-term capital gain would be realized by selling equipment, the capital gain would have been fairly large if a fast depreciation method had been used by the company. Prior to 1988, this gain would probably have been taxed at a low rate (perhaps as low as 28 percent) while the amount saved through fast depreciation allowance could have been at an income-tax rate of nearly 50 percent. However, after 1987, new Federal tax rules have been enacted which could make the capital-gains tax the same as the income tax on ordinary income of about 34 percent.?

The preceding examples illustrate why the chemical engineer should understand the effects of governmental regulations on costs. Each company has its own methods for meeting these regulations, but changes in the laws and alterations in the national and company economic situation require constant surveillance if optimum cost conditions are to be maintained.

CAPITAL INVESTMENTS

Before an industrial plant can be put into operation, a large sum of money must be supplied to purchase and install the necessary machinery and equipment. Land and service facilities must be obtained, and the plant must be erected complete with all piping, controls, and services. In addition, it is necessary to have money available for the payment of expenses involved in the plant operation.

The capital needed to supply the necessary manufacturing and plant facilities is called the *fixed-capital investment,* while that necessary for the operation of the plant is termed the *working capital.* The sum of the fixed-capital investment and the working capital is known as the *total capital investment.* The fixed-capital portion may be further subdivided into *manufacturing fixed-capital investment* and *nonmanufacturing fixed-capital investment.*

Fixed-Capital Investment

Manufacturing fixed-capital investment represents the capital necessary for the installed process equipment with all auxiliaries that are needed for complete process operation. #Expenses for piping, instruments, insulation, foundations, and site preparation are typical examples of costs included in the manufacturing fixed-capital investment.

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†For a discussion of income-tax rates, see Chap. 8 (Taxes and Insurance).

Fixed capital required for construction overhead and for all plant components that are not directly related to the process operation is designated as the nonmanufacturing fixed-capital investment. These plant components include the land, processing buildings, administrative, and other offices, warehouses, laboratories, transportation, shipping, and receiving facilities, utility and waste-disposal facilities, shops, and other permanent parts of the plant. The construction overhead cost consists of field-office and supervision expenses, home-office expenses, engineering expenses, miscellaneous construction costs, contractor's fees, and contingencies. In some cases, construction overhead is proportioned between manufacturing and nonmanufacturing fixed-capital investment.

Working Capital

The working capital for an industrial plant consists of the total amount of money invested in (1) raw materials and supplies carried in stock, (2) finished products in stock and semifinished products in the process of being manufactured, (3) accounts receivable, (4) cash kept on hand for monthly payment of operating expenses, such as salaries, wages, and raw-material purchases, (5) accounts payable, and (6) taxes payable.

The raw-materials inventory included in working capital usually amounts to a l-month supply of the raw materials valued at delivered prices. Finished products in stock and semifinished products have a value approximately equal to the total manufacturing cost for 1 month's production. Because credit terms extended to customers are usually based on an allowable 30-day payment period, the working capital required for accounts receivable ordinarily amounts to the production cost for 1 month of operation.

The ratio of working capital to total capital investment varies with different companies, but most chemical plants use an initial working capital amounting to 10 to 20 percent of the total capital investment. This percentage may increase to as much as 50 percent or more for companies producing products of seasonal demand because of the large inventories which must be maintained for appreciable periods of time.

ESTIMATION OF CAPITAL INVESTMENT

Of the many factors which contribute to poor estimates of capital investments, the most significant one is usually traceable to sizable omissions of equipment, services, or auxiliary facilities rather than to gross errors in costing. A check list of items covering a new facility is an invaluable aid in making a complete estimation of the fixed-capital investment. Table 1 gives a typical list of these items.

TABLE 1

Breakdown of fixed-capital investment items for a chemical process

---

Direct Costs

1. *Purchased equipment*
    - All equipment listed on a complete flow sheet
    - Spare parts and noninstalled equipment spares
    - Surplus equipment, supplies, and equipment allowance
    - Inflation cost allowance
    - Freight charges
    - Taxes, insurance, duties
    - Allowance for modifications during startup
2. *Purchased-equipment installation*
    - Installation of all equipment listed on complete flow sheet
    - Structural supports, insulation, paint
3. *Instrumentation and controls*
    - Purchase, installation, calibration, computer tie-in
4. *Piping*
    - Process piping-carbon steel, alloy, cast iron, lead, lined, aluminum, copper, ceramic, plastic, rubber, reinforced concrete
    - Pipe hangers, fittings, valves
    - Insulation-piping, equipment
5. *Electrical equipment and materials*
    - Electrical equipment -switches, motors, conduit, wire, fittings, feeders, grounding, instrument and control wiring, lighting, panels
    - Electrical materials and labor
6. *Buildings (including services)*
    - Process buildings-substructures, superstructures, platforms, supports, stairways, ladders, access ways, cranes, monorails, hoists, elevators
    - Auxiliary buildings-administration and office, medical or dispensary, cafeteria, garage, product warehouse, parts warehouse, guard and safety, fire station, change house, personnel building, shipping office and platform, research laboratory, control laboratory
    - Maintenance shops-electric, piping, sheet metal, machine, welding, carpentry, instrument
    - Building services-plumbing, heating, ventilation, dust collection, air conditioning, building lighting, elevators, escalators, telephones, intercommunication systems, painting, sprinkler systems, fire alarm
7. *Yard improvements*
    - Site development-site clearing, grading, roads, walkways, railroads, fences, parking areas, wharves and piers, recreational facilities, landscaping
8. *Service facilities*
    - Utilities-steam, water, power, refrigeration, compressed air, fuel, waste disposal
    - Facilities-boiler plant incinerator, wells, river intake, water treatment, cooling towers, water storage, electric substation, refrigeration plant, air plant, fuel storage, waste disposal plant, environmental controls, fire protection
    - Nonprocess equipment-office furniture and equipment, cafeteria equipment, safety and medical equipment, shop equipment, automotive equipment, yard material-handling equipment, laboratory equipment, locker-room equipment, garage equipment, shelves, bins, pallets, hand trucks, housekeeping equipment, fire extinguishers, hoses, fire engines, loading stations
    - Distribution and packaging-raw-material and product storage and handling equipment, product packaging equipment, blending facilities, loading stations

(Continued)

TABLE 1
Breakdown of fixed-capital investment items for a chemical process *(Continued)*

Direct Costs

9. *Land*
   - Surveys and fees
   - Property cost

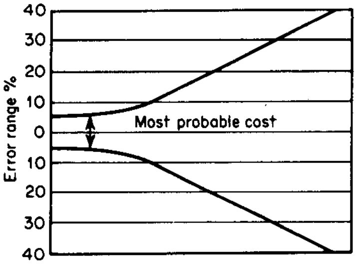
Indirect costs

1. *Engineering and supervision*
   - Engineering costs-administrative, process, design and general engineering, drafting, cost engineering, procuring, expediting, reproduction, communications, scale models, consultant fees, travel
   - Engineering supervision and inspection
2. *Construction expenses*
   - Construction, operation and maintenance of temporary facilities, offices, roads, parking lots, railroads, electrical, piping, communications, fencing
   - Construction tools and equipment
   - Construction supervision, accounting, timekeeping, purchasing, expediting
   - Warehouse personnel and expense, guards
   - Safety, medical, fringe benefits
   - Permits, field tests, special licenses
   - Taxes, insurance, interest
3. *Contractor's fee*
4. *Contingency*

Types of Capital Cost Estimates

An estimate of the capital investment for a process may vary from a predesign estimate based on little information except the size of the proposed project to a detailed estimate prepared from complete drawings and specifications. Between these two extremes of capital-investment estimates, there can be numerous other estimates which vary in accuracy depending upon the stage of development of the project. These estimates are called by a variety of names, but the following five categories represent the accuracy range and designation normally used for design purposes:

1. Order-of-magnitude estimate (ratio estimate) based on similar previous cost data; probable accuracy of estimate over ± 30 percent.
2. Study estimate (factored estimate) based on knowledge of major items of equipment; probable accuracy of estimate up to ±30 percent.
3. Preliminary estimate (budget authorization estimate; scope estimate) based on sufficient data to permit the estimate to be budgeted; probable accuracy of estimate within ±20 percent.



	Required information	Detailed estimate ±5% range	Definitive estimate ±10% range	Preliminary estimate ±20% range	Study estimate ±30% range	Order-of-magnitude estimate >±30% range
Site	Location	●	●	●	●	
	General description	●	●	●		
	Soil bearing	●	●	●		
	Location & dimensions R.R., roads, impounds, fences	●	●	●		
	Well-developed site plot plan & topographical map	●	●			
	Well-developed site facilities					
Process flow sheet	Rough sketches				●	
	Preliminary			●		
	Engineered	●	●			
Equipment list	Preliminary sizing & material specifications			●	●	
	Engineered specifications	●	●			
	Vessel sheets	●	●			
	General arrangement					
	(a) Preliminary		●	●		
	(b) Engineered	●				
Building and structures	Approximate sizes & type of construction			●	●	
	Foundation sketches		●	●		
	Architectural & construction	●	●	●		
	Preliminary structural design			●		
	General arrangements & elevations	●	●			
	Detailed drawings	●				
Utility requirements	Rough quantities (steam, water, electricity, etc.)				●	
	Preliminary heat balance			●		
	Preliminary flow sheets			●		
	Engineered heat balance	●	●			
	Engineered flow sheets	●	●			
	Well-developed drawings	●				
Piping	Preliminary flow sheet & specifications			●	●	
	Engineered flow sheet	●	●			
	Piping layouts & schedules	●				
Insulation	Rough specifications			●		
	Preliminary list of equipment & piping to be insulated		●			
	Insulation specifications & schedules	●				
	Well-developed drawings or specifications					
Instrumentation	Preliminary instrument list					
	Engineered list & flow sheet	●	●			
	Well-developed drawings	●				
Electrical	Preliminary motor list–approximate sizes			●	●	
	Engineered list & sizes	●	●			
	Substations, number & sizes, specifications	●	●	●		
	Distribution specifications	●				
	Preliminary lighting specifications			●		
	Preliminary interlock, control, & instrument wiring specs.		●			
	Engineered single-line diagrams (power & light)	●	●			
	Well-developed drawings	●				
Man-hours	Engineering & drafting	●	●	●	●	
	Labor by craft	●				
	Supervision	●				
Project scope standard processes	Product, capacity, location & site requirements. Utility & service requirements. Building & auxiliary requirements. Raw materials & finished product handling & storage requirements					●

FIGURE 6-4
Cost-estimating information guide.

TABLE 2
Typical average costs for making estimates (1990)†

Cost of project	Less than $2,000,000	$2,000,000 to $10,000,000	$10,000,000 to $100,000,000
Order-of-magnitude estimate	$ 3,000	$ 6,000	$ 13,000
Study estimate	20,000	40,000	60,000
Preliminary estimate	50,000	80,000	130,000
Definitive estimate	80,000	160,000	320,000
Detailed estimate	200,000	520,000	1,000,000

† Adapted from A. Pikulik and H. E. Diaz, Cost Estimating for Major Process Equipment, *Chem. Eng.*, 84(21):106 (Oct. 10, 1977).

4. Definitive estimate (project control estimate) based on almost complete data but before completion of drawings and specifications; probable accuracy of estimate within ± 10 percent.
5. Detailed estimate (contractor's estimate) based on complete engineering drawings, specifications, and site surveys; probable accuracy of estimate within ±5 percent.

Figure 6-4 shows the relationship between probable accuracy and quantity and quality of information available for the preparation of these five levels of estimates.? The approximate limits of error in this listing are plotted and show an envelope of variability. There is a large probability that the actual cost will be more than the estimated cost where information is incomplete or in time of rising-cost trends. For such estimates, the positive spread is likely to be wider than the negative, e.g., +40 and − 20 percent for a study estimate. Table 2 illustrates the wide variation that can occur in the cost of making a capital-investment estimate depending on the type of estimate.

Predesign cost estimates (defined here as order-of-magnitude, study, and preliminary estimates) require much less detail than firm estimates such as the definitive or detailed estimate. However, the predesign estimates are extremely important for determining if a proposed project should be given further consideration and to compare alternative designs. For this reason, most of the information presented in this chapter is devoted to predesign estimates, although it should be understood that the distinction between predesign and firm estimates gradually disappears as more and more detail is included.

It should be noted that the predesign estimates may be used to provide a basis for requesting and obtaining a capital appropriation from company management. Later estimates, made during the progress of the job, may indicate that the project will cost more or less than the amount appropriated. Management is then asked to approve a ***variance*** which may be positive or negative.

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†Adapted from a method presented by W. T. Nichols, *Znd. Eng. Chem.*, 43(10):2295 (1951).

TABLE 3

Cost indexes as annual averages

Year	Marshall and Swift installed-equipment indexes, 1926 = 100		*Eng. News-Record* construction index			Nelson-Farrar refinery construction index, 1946 = 100	Chemical engineering plant cost index 1957-1959 = 100
	All-industry	Process-industry	1913 = 100	1949 = 100	1967 = 100		
1975	444	452	2412	464	207	576	182
1976	472	479	2401	503	224	616	192
1977	505	514	2576	540	241	653	204
1978	545	552	2776	582	259	701	219
1979	599	607	3003	630	281	757	239
1980	560	675	3237	679	303	823	261
1981	721	745	3535	741	330	904	297
1982	746	774	3825	802	357	977	314
1983	761	786	4066	852	380	1026	317
1984	780	806	4146	869	387	1061	323
1985	790	813	4195	879	392	1074	325
1986	798	817	4295	900	401	1090	318
1987	814	830	4406	924	412	1122	324
1988	852	870	4519	947	422	1165	343
1989	895	914	4606	965	429	1194	355
1990 (Jan.)	904[†]	924	4673	979	435	1203	356

[†] All costs presented in this text are based on this value of the Marshall and Swift index unless otherwise indicated.

COST INDEXES

Most cost data which are available for immediate use in a preliminary or predesign estimate are based on conditions at some time in the past. Because prices may change considerably with time due to changes in economic conditions, some method must be used for updating cost data applicable at a past date to costs that are representative of conditions at a later time.[†] This can be done by the use of cost indexes.

A cost index is merely an index value for a given point in time showing the cost at that time relative to a certain base time. If the cost at some time in the past is known, the equivalent cost at the present time can be determined by multiplying the original cost by the ratio of the present index value to the index value applicable when the original cost was obtained.

[†] See Chap. 11 for a discussion of the strategy to use in design estimates to consider the effects of inflation or deflation on costs and profits in the future.

$$\text{Present cost} = \text{original cost}\left(\frac{\text{index value at present time}}{\text{index value at time original cost was obtained}}\right)$$

Cost indexes can be used to give a general estimate, but no index can take into account all factors, such as special technological advancements or local conditions. The common indexes permit fairly accurate estimates if the time period involved is less than 10 years.

Many different types of cost indexes are published regularly. Some of these can be used for estimating equipment costs; others apply specifically to labor, construction, materials, or other specialized fields. The most common of these indexes are the *Marshall and Swift all-industry and process-industry equipment indexes,* the *Engineering News-Record construction index,* the *Nelson-Farrar refinery construction index,* and the *Chemical Engineering plant cost index.* Table 3 presents a list of values for various types of indexes over the past 15 years.

Marshall and Swift Equipment Cost Indexes†

The Marshall and Swift (formerly known as Marshall and Stevens) equipment indexes are normally divided into two categories. The all-industry equipment index is simply the arithmetic average of individual indexes for 47 different types of industrial, commercial, and housing equipment. The process-industry equipment index is a weighted average of eight of these, with the weighting based on the total product value of the various process industries. The percentages used for the weighting in a typical year are as follows: cement 2; chemicals, 48; clay products, 2; glass, 3; paint, 5; paper, 10; petroleum, 22; and rubber, 8.

The Marshall and Swift indexes are based on an index value of 100 for the year 1926. These indexes take into consideration the cost of machinery and major equipment plus costs for installation, fixtures, tools, office furniture, and other minor equipment. All costs reported in this text are based on a Marshall and Swift all-industry index of 904 as reported for January 1, 1990 unless indicated otherwise.

Engineering News-Record Construction Cost Index‡

Relative construction costs at various dates can be estimated by the use of the *Engineering News-Record* construction index. This index shows the variation in

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†Values for the Marshall and Swift equipment cost indexes are published each month in *Chemical Engineering.* For a complete description of these indexes, see R. W. Stevens, *Chem. Eng.,* **54**(11):124 (Nov., 1947). See also *Chem. Eng.,* **85**(11):189 (May 8, 1978) and **92**(9):75 (April 29, 1985).

‡The *Engineering News-Record* construction cost index appears weekly in the *Engineering News-Record.* For a complete description of this index and the revised basis, see *Eng. News-Record,* **143**(9):398 (1949); **178**(11):87 (1967). History is in March issue each year; for example, see *Eng. News-Record,* **220**(11):54 (March 15, 1988).

labor rates and materials costs for industrial construction. It employs a composite cost for 2500 lb of structural steel, 1088 fbm of lumber, 2256 lb of concrete, and 200 h of common labor. The index is usually reported on one of three bases: an index value of 100 in 1913, 100 in 1949, or 100 in 1967.

Nelson-Farrar Refinery Construction Cost Index[†]

Construction costs in the petroleum industry are the basis of the Nelson-Farrar construction index. The total index percentages are weighted as follows: skilled labor, 30; common labor, 30; iron and steel, 20; building materials, 8; and miscellaneous equipment, 12. An index value of 100 is used for the base year of 1946.

Chemical Engineering Plant Cost Index[‡]

Construction costs for chemical plants form the basis of the *Chemical Engineering* plant cost index. The four major components of this index are weighted by percentage in the following manner: equipment, machinery, and supports, 61; erection and installation labor, 22; buildings, materials, and labor, 7; and engineering and supervision, 10. The major component, equipment, is further subdivided and weighted as follows: fabricated equipment, 37; process machinery, 14; pipe, valves, and fittings, 20; process instruments and controls, 7; pumps and compressors, 7; electrical equipment and materials, 5; and structural supports, insulation, and paint, 10. All index components are based on 1957-1959 = 100.

Other Indexes and Analysis

There are numerous other indexes presented in the literature which can be used for specialized purposes. For example, cost indexes for materials and labor for various types of industries are published monthly by the U.S. Bureau of Labor Statistics in the *Monthly Labor Review*. These indexes can be useful for special kinds of estimates involving particular materials or unusual labor conditions. Another example of a cost index which is useful for world-wide comparison of cost charges with time is published periodically in *Engineering Costs and*

[†] The Nelson-Farrar refinery construction index is published the first week of each month in the *Oil and Gus Journal*. For a complete description of this index, see *Oil Gas J.*, **63**(14):185 (1965); **74**(48):68 (1976); and **83**(52):145 (1985).

[‡] The *Chemical Engineering* plant cost index is published each month in *Chemical Engineering*. A complete description of this index is in *Chem. Eng.*, **70**(4):143 (Feb. 18, 1963) with recapping and updating in issues of **73**(9):184 (April 25, 1966); **76**(10):134 (May 5, 1969); **79**(25):168 (Nov. 13, 1972); **82**(9):117 (April 28, 1975); **85**(11):189 (May 8, 1978); **89**(8):153 (April 19, 1982); and **92**(9):75 (April 29, 1985).

Production Economics. This presents cost indexes for plant costs for various countries in the world including Australia, Belgium, Canada, Denmark, France, Germany, Italy, Netherlands, Norway, Japan, Sweden, the United Kingdom, and the United States.?

Unfortunately, all cost indexes are rather artificial; two indexes covering the same types of projects may give results that differ considerably. The most that any index can hope to do is to reflect average changes. The latter may at times have little meaning when applied to a specific case. For example, a contractor may, during a slack period, accept a construction job with little profit just to keep his construction crew together. On the other hand, if there are current local labor shortages, a project may cost considerably more than a similar project in another geographical location.

For use with process-equipment estimates and chemical-plant investment estimates, the *Marshall and Swift* equipment cost indexes and the *Chemical Engineering* plant cost indexes are recommended. These two cost indexes give very similar results, while the *Engineering News-Record* construction cost index, relative with time, has increased much more rapidly than the other two because it does not include a productivity improvement factor. Similarly, the *Nelson-Farrar* refinery construction index has shown a very large increase with time and should be used with caution and only for refinery construction.

COST FACTORS IN CAPITAL INVESTMENT

Capital investment, as defined earlier, is the total amount of money needed to supply the necessary plant and manufacturing facilities plus the amount of money required as working capital for operation of the facilities. Let us now consider the proportional costs of each major component of fixed-capital investment as outlined previously in Table 1 of this chapter. The cost factors presented here are based on a careful study by Bauman and associates plus additional data and interpretations from other more recent sources‡ with input based on modern industrial experience.

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†For methods used, see *Eng.* Costs *Prod. Econ.*, 6(1):267 (1982).

‡H. C. Bauman, "Fundamentals of Cost Engineering in the Chemical Industry," Reinhold Publishing Corporation, New York, 1964; K. M. Guthrie, "Process Plant Estimating, Evaluation, and Control," Craftsman Book Company of America, Solana Beach, CA, 1974; D. H. Allen and R. C. Page, Revised Techniques for Predesign Cost Estimating. *Chem. Eng.*, 82(5):142 (Mar. 3, 1975); W. D. Baasel, "Preliminary Chemical Engineering Plant Design," American Elsevier Publishing Company, Inc., New York, 1976; R. H. Perry and D. W. Green, "Chemical Engineers' Handbook," 6th ed., McGraw-Hi11 Book Company, Inc., New York, 1984; G. D. Ulrich, "A Guide to Chemical Engineering Process Design and Economics," Wiley, New York, 1984; R. K. Sinnott, "An Introduction to Chemical Engineering Design," Pergamon Press, Oxford, England, 1983; P. F. Ostwald, "AM Cost Estimator," McGraw-Hill Book Company, Inc., New York, 1988.

TABLE 4
Typical percentages of fixed-capital investment values for direct and indirect cost segments for multipurpose plants or large additions to existing facilities

Component	Range, %
Direct costs	
Purchased equipment	15-40
Purchased equipment installation	6-14
Instrumentation and controls (installed)	2-8
Piping (installed)	3-20
Electrical (installed)	2-10
Buildings (including services)	3-18
Yard improvements	2-5
Service facilities (installed)	8-20
Land	1-2
Total direct costs	
Indirect costs	
Engineering and supervision	4-21
Construction expense	4-16
Contractor's fee	2-6
Contingency	5-15
Total fixed-capital investment	

Table 4 summarizes this typical variation in component costs as percentages of fixed-capital investment for multiprocess grass-roots plants or large *battery-limit* additions. A *grass-roots* plant is defined as a complete plant erected on a new site. Investment includes all costs of land, site development, battery-limit facilities, and auxiliary facilities. A geographical boundary defining the coverage of a specific project is a *battery limit.* Usually this encompasses the manufacturing area of a proposed plant or addition, including all process equipment but excluding provision of storage, utilities, administrative buildings, or auxiliary facilities unless so specified. Normally this excludes site preparation and therefore, may be applied to the extension of an existing plant.

Example 1 Estimation of fixed-capital investment using ranges of process-plant component costs. Make a study estimate of the fixed-capital investment for a process plant if the purchased-equipment cost is $100,000. Use the ranges of process-plant component cost outlined in Table 4 for a process plant handling both solids and fluids with a high degree of automatic controls and essentially outdoor operation.

Solution

Components	Assumed % of total	Cost	Ratioed % of total
Purchased equipment	25	$100,000	23.0
Purchased-equipment installation	9	36,000	8.3
Instrumentation (installed)	7	28,000	6.4
Piping (installed)	8	32,000	7.3
Electrical (installed)	5	20,000	4.6
Buildings (including services)	5	20,000	4.6
Yard improvements	2	8,000	1.8
Service facilities (installed)	15	60,000	13.8
Land	1	4,000	0.9
Engineering and supervision	10	40,000	9.2
Construction expense	12	48,000	11.0
Contractor's fee	2	8,000	1.8
Contingency	8	32,000	7.3
		$436,000	100.0

Range will vary from $371,000 to $501,000 for normal conditions; if economy is inflationary, it may vary from $436,000–$566,000.

Purchased Equipment

The cost of purchased equipment is the basis of several predesign methods for estimating capital investment. Sources of equipment prices, methods of adjusting equipment prices for capacity, and methods of estimating auxiliary process equipment are therefore essential to the estimator in making reliable cost estimates.

The various types of equipment can often be divided conveniently into (1) processing equipment, (2) raw-materials handling and storage equipment, and (3) finished-products handling and storage equipment. The cost of auxiliary equipment and materials, such as insulation and ducts, should also be included.

The most accurate method for determining process equipment costs is to obtain firm bids from fabricators or suppliers. Often, fabricators can supply quick estimates which will be very close to the bid price but will not involve too much time. Second best in reliability are cost values from the file of past purchase orders. When used for pricing new equipment, purchase-order prices must be corrected to the current cost index. Limited information on process-equipment costs has also been published in various engineering journals. Costs, based on January 1, 1990 prices, for a large number of different types and capacities of equipment are presented in Chaps. 14 through 16. A convenient reference to these various cost figures is given in the Table of Contents and in the subject index.

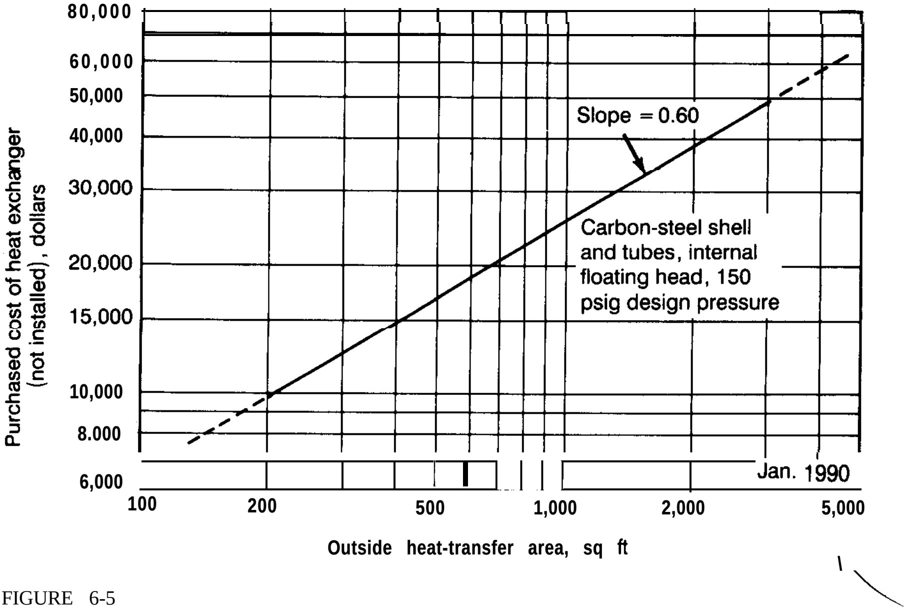

FIGURE 6-5
Application of "six-tenth-factor" rule to costs for shell-and-tube heat exchangers.

Estimating Equipment Costs by Scaling

It is often necessary to estimate the cost of a piece of equipment when no cost data are available for the particular size of operational capacity involved. Good results can be obtained by using the logarithmic relationship known as the ***six-tenths-factor*** rule, if the new piece of equipment is similar to one of another capacity for which cost data are available. According to this rule, if the cost of a given unit at one capacity is known, the cost of a similar unit with X times the capacity of the first is approximately $(X)^{0.6}$ times the cost of the initial unit.

$$\text{Cost of equip. } a = \text{cost of equip. } b \left(\frac{\text{capac. equip. } a}{\text{capac. equip. } b} \right)^{0.6} \qquad (1)$$

The preceding equation indicates that a log-log plot of capacity versus equipment cost for a given type of equipment should be a straight line with a slope equal to 0.6. Figure 6-5 presents a plot of this sort for shell-and-tube heat exchangers. However, the application of the 0.6 rule of thumb for most purchased equipment is an oversimplification of a valuable cost concept since the actual values of the cost capacity factor vary from less than 0.2 to greater than 1.0 as shown in Table 5. Because of this, the 0.6 factor should only be used in the absence of other information. In general, the cost-capacity concept should not be used beyond a tenfold range of capacity, and care must be taken to make certain the two pieces of equipment are similar with regard to type of construction, materials of construction, temperature and pressure operating range, and other pertinent variables.

TABLE 5
Typical exponents for equipment cost vs. capacity

Equipment	Size range	Exponent
Blender, double cone rotary, c.s.	50-250 ft^3	0.49
Blower, centrifugal	10^3–10^4 ft^3/min	0.59
Centrifuge, solid bowl, c.s.	10–10^2 hp drive	0.67
Crystallizer, vacuum batch, c.s.	500-7000 ft^3	0.37
Compressor, reciprocating, air cooled, two-stage, 150 psi discharge	10–400 ft^3/min	0.69
Compressor, rotary, single-stage, sliding vane, 150 psi discharge	10^2–10^3 ft^3/min	0.79
Dryer, drum, single vacuum	10–10^2 ft^2	0.76
Dryer, drum, single atmospheric	10–10^2 ft^2	0.40
Evaporator (installed), horizontal tank	10^2–10^4 ft^2	0.54
Fan, centrifugal	10^3–10^4 ft^3/min	0.44
Fan, centrifugal	2×10^4–7×10^4 ft^3/min	1.17
Heat exchanger, shell and tube, floating head, c.s.	100-400 ft^2	0.60
Heat exchanger, shell and tube, fixed sheet, c.s.	100-400 ft^2	0.44
Kettle, cast iron, jacketed	250-800 gal	0.27
Kettle, glass lined, jacketed	200-800 gal	0.31
Motor, squirrel cage, induction, 440 volts, explosion proof	5-20 hp	0.69
Motor, squirrel cage, induction, 440 volts, explosion proof	20-200 hp	0.99
Pump, reciprocating, horizontal cast iron (includes motor)	2–100 gpm	0.34
Pump, centrifugal, horizontal, cast steel (includes motor)	10^4–10^5 gpm × psi	0.33
Reactor, glass lined, jacketed (without drive)	50–600 gal	0.54
Reactor, s.s, 300 psi	10^2–10^3 gal	0.56
Separator, centrifugal, c.s.	50-250 ft^3	0.49
Tank, flat head, c.s.	10^2–10^4 gal	0.57
Tank, c.s., glass lined	10^2–10^3 gal	0.49
Tower, c.s.	10^3–2×10^6 lb	0.62
Tray, bubble cup, c.s.	3-10 ft diameter	1.20
Tray, sieve, c.s.	3-10 ft diameter	0.86

Example 2 Estimating cost of equipment using scaling factors and cost index. The purchased cost of a 50-gal glass-lined, jacketed reactor (without drive) was \$8350 in 1981. Estimate the purchased cost of a similar 300-gal, glass-lined, jacketed reactor (without drive) in 1986. Use the annual average Marshall and Swift equipment-cost index (all industry) to update the purchase cost of the reactor.

Solution. Marshall and Swift equipment-cost index (all industry)

(From Table 3) For 1981 721

(From Table 3) For 1986 798

From Table 5, the equipment vs. capacity exponent is given as 0.54:

$$\text{In 1986, cost of reactor} = (\$8350)\left(\frac{798}{721}\right)\left(\frac{300}{50}\right)^{0.54}$$

$$= \$24{,}300$$

Purchased-equipment costs for vessels, tanks, and process- and materials-handling equipment can often be estimated on the basis of weight. The fact that a wide variety of types of equipment have about the same cost per unit weight is quite useful, particularly when other cost data are not available. Generally, the cost data generated by this method are sufficiently reliable to permit order-of-magnitude estimates.

Purchased-Equipment Installation

The installation of equipment involves costs for labor, foundations, supports, platforms, construction expenses, and other factors directly related to the erection of purchased equipment. Table 6 presents the general range of installation cost as a percentage of the purchased-equipment cost for various types of equipment.

Installation labor cost as a function of equipment size shows wide variations when scaled from previous installation estimates. Table 7 shows exponents varying from 0.0 to 1.56 for a few selected pieces of equipment.

TABLE 6

Installation cost for equipment as a percentage of the purchased-equipment cost†

Type of equipment	Installation cost, %
Centrifugal separators	20-60
Compressors	30-60
Dryers	25-60
Evaporators	25-90
Filters	65-80
Heat exchangers	30-60
Mechanical crystallizers	30-60
Metal tanks	30-60
Mixers	20-40
Pumps	25-60
Towers	60-90
Vacuum crystallizers	40-70
Wood tanks	30-60

† Adapted from K. M. Guthrie, "Process Plant Estimating, Evaluation, and Control," Craftsman Book Company of America, Solana Beach, California, 1974.

TABLE 7

Typical exponents for equipment installation labor vs. size

Equipment	Size range	Exponent
Conduit, aluminum	0.5–2-in. diam.	0.49
Conduit, aluminum	2–4-in. diam.	1.11
Motor, squirrel cage, induction, 440 volts	1-10 hp	0.19
Motor, squirrel cage, induction, 440 volts	10–50 hp	0.50
Pump, centrifugal, horizontal	0.5–1.5 hp	0.63
Pump, centrifugal, horizontal	1.5–40 hp	0.09
Tower, c.s.	Constant diam.	0.88
Tower, c.s.	Constant height,	1.56
Transformer, single phase, dry	9-225 kva	0.58
Transformer, single phase, oil, class A	15-225 kva	0.34
Tubular heat exchanger	Any size	0.00

Tubular heat exchangers appear to have zero exponents, implying that direct labor cost is independent of size. This reflects the fact that such equipment is set with cranes and hoists, which, when adequately sized for the task, recognize no appreciable difference in size of weight of the equipment. The higher labor exponent for installing carbon-steel towers indicates the increasing complexity of tower internals (trays, downcomers, etc.) as tower diameter increases.

Analyses of the total installed costs of equipment in a number of typical chemical plants indicate that the cost of the purchased equipment varies from 65 to 80 percent of the installed cost depending upon the complexity of the equipment and the type of plant in which the equipment is installed. Installation costs for equipment, therefore, are estimated to vary from 25 to 55 percent of the purchased-equipment cost.

Insulation Costs

When very high or very low temperatures are involved, insulation factors can become important, and it may be necessary to estimate insulation costs with a great deal of care. Expenses for equipment insulation and piping insulation are often included under the respective headings of equipment-installation costs and piping costs.

The total cost for the labor and materials required for insulating equipment and piping in ordinary chemical plants is approximately 8 to 9 percent of the purchased-equipment cost. This is equivalent to approximately 2 percent of the total capital investment.

Instrumentation and Controls

Instrument costs, installation-labor costs, and expenses for auxiliary equipment and materials constitute the major portion of the capital investment required for instrumentation. This part of the capital investment is sometimes combined with the general equipment groups. Total instrumentation cost depends on the amount of control required and may amount to 6 to 30 percent of the purchased cost for all equipment. Computers are commonly used with controls and have the effect of increasing the cost associated with controls.

For the normal solid-fluid chemical processing plant, a value of 13 percent of the purchased equipment is normally used to estimate the total instrumentation cost. This cost represents approximately 3 percent of the total capital investment. Depending on the complexity of the instruments and the service, additional charges for installation and accessories may amount to 50 to 70 percent of the purchased cost, with the installation charges being approximately equal to the cost for accessories.

Piping

The cost for piping covers labor, valves, fittings, pipe, supports, and other items involved in the complete erection of all piping used directly in the process. This includes raw-material, intermediate-product, finished-product, steam, water, air, sewer, and other process piping. Since process-plant piping can run as high as 80 percent of purchased-equipment cost or 20 percent of tied-capital investment, it is understandable that accuracy of the entire estimate can be seriously affected by the improper application of estimation techniques to this one component.

Piping estimation methods involve either some degree of piping take-off from detailed drawings and flow sheets or using a factor technique when neither detailed drawings nor flow sheets are available. Factoring by percent of purchased-equipment cost and percent of fixed-capital investment is based strictly on experience gained from piping costs for similar previously installed chemical-process plants. Table 8 presents a rough estimate of the piping costs for various types of chemical processes. Additional information for estimating

TABLE 8
Estimated cost of piping

Type of process plant	Percent of purchased-equipment			Percent of fixed-capital investment
	Material	Labor	Total	Total
Solid †	9	7	16	4
Solid-fluid ‡	17	14	31	7
Fluid §	36	30	66	13

† A coal briquetting plant would be a typical solid-processing plant.
‡ A shale oil plant with crushing, grinding, retorting, and extraction would be a typical solid-fluid processing plant.
§ A distillation unit would be a typical fluid-processing plant.

TABLE 9
Component electrical costs as percent of total electrical cost

Component	Range, %	Typical value, %
Power wiring	25-50	40
Lighting	I-25	12
Transformation and service	9-65	40
Instrument control wiring	3-8	5

The lower range is generally applicable to grass-roots single-product plants; the higher percentages apply to complex chemical plants and expansions to major chemical plants.

piping costs is presented in Chap. 14. Labor for installation is estimated as approximately 40 to 50 percent of the total installed cost of piping. Material and labor for pipe insulation is estimated to vary from 15 to 25 percent of the total installed cost of the piping and is influenced greatly by the extremes in temperature which are encountered in the process streams.

Electrical Installations

The cost for electrical installations consists primarily of installation labor and materials for power and lighting, with building-service lighting usually included under the heading of building-and-services costs. In ordinary chemical plants, electrical-installations cost amounts to 10 to 15 percent of the value of all purchased equipment. However, this may range to as high as 40 percent of purchased-equipment cost for a specific process plant. There appears to be little relationship between percent of total cost and percent of equipment cost, but there is a better relationship to fixed-capital investment. Thus, the electrical installation cost is generally estimated between 3 and 10 percent of the fixed-capital investment.

The electrical installation consists of four major components, namely, power wiring, lighting, transformation and service, and instrument and control wiring. Table 9 shows these component costs as ratios of the total electrical cost.

Buildings Including Services

The cost for buildings including services consists of expenses for labor, materials, and supplies involved in the erection of all buildings connected with the plant. Costs for plumbing, heating, lighting, ventilation, and similar building services are included. The cost of buildings, including services for different types of process plants, is shown in Tables 10 and 11 as a percentage of purchased-equipment cost and tied-capital investment.

TABLE 10
Cost of buildings including services based on purchased-equipment cost

	Percentage of purchased-equipment cost		
Type of process plant†	New plant at new site (Grass roots)	New unit at existing site (Battery limit)	Expansion at an existing site
Solid	68	25	15
Solid-fluid	47	29	7
Fluid	45	5–18‡	6

† See Table 8 for definition of types of process plants.
‡ The lower figure is applicable to petroleum refining and related industries.

TABLE 11
Cost of buildings and services as a percentage of fixed-capital investment for various types of process plants

Type of process plant†	New plant at new site	New unit at existing site	Expansion at an existing site
Solid	18	1	4
Solid-fluid	12	7	2
Fluid	10	2–4‡	2

† See Table 8 for definition of types of process plants.
‡ The lower figure is applicable to petroleum refining and related industries.

Yard Improvements

Costs for fencing, grading, roads, sidewalks, railroad sidings, landscaping, and similar items constitute the portion of the capital investment included in yard improvements. Yard-improvements cost for chemical plants approximates 10 to 20 percent of the purchased-equipment cost. This is equivalent to approximately 2 to 5 percent of the fixed-capital investment. Table 12 shows the range in variation for various components of yard improvements in terms of the fixed-capital investment.

Service Facilities

Utilities for supplying steam, water, power, compressed air, and fuel are part of the service facilities of an industrial plant. Waste disposal, fire protection, and miscellaneous service items, such as shop, first aid, and cafeteria equipment and facilities, require capital investments which are included under the general heading of service-facilities cost.

The total cost for service facilities in chemical plants generally ranges from 30 to 80 percent of the purchased-equipment cost with 55 percent representing

TABLE 12
Typical variation in percent of fixed-capital investment for yard improvements

Yard improvement	Range, %	Typical value, %
Site clearing	0.4-1.2	0.8
Roads and walks	0.2-1.2	0.6
Railroads	0.3-0.9	0.6
Fences	0.1-0.3	0.2
Yard and fence lighting	0.1-0.3	0.2
Parking areas	0.1-0.3	0.2
Landscaping	0.1-0.2	0.1
Other improvements	0.2-0.6	0.3

an average for a normal solid-fluid processing plant. For a single-product, small, continuous-process plant, the cost is likely to be in the lower part of the range. For a large, new, multiprocess plant at a new location, the costs are apt to be near the upper limit of the range. The cost of service facilities, in terms of capital investment, generally ranges from 8 to 20 percent with 13 percent considered as an average value. Table 13 lists the typical variations in percentages of fixed-capital investment that can be encountered for various components of service facilities. Except for entirely new facilities, it is unlikely that all service facilities will be required in all process plants. This accounts to a large degree for the wide variation range assigned to each component in Table 13. The range also reflects the degree to which utilities which depend on heat balance are used in the process. Service facilities largely are functions of plant physical size and will be present to some degree in most plants. However, not always will there be a need for each service-facility component. The omission of these utilities would tend to increase the relative percentages of the other service facilities actually used in the plant. Recognition of this fact, coupled with a careful appraisal as to the extent that service facilities are used in the plant, should result in selecting from Table 13 a reasonable cost ratio applicable to a specific process design.

Land

The cost for land and the accompanying surveys and fees depends on the location of the property and may vary by a cost factor per acre as high as thirty to fifty between a rural district and a highly industrialized area. As a rough average, land costs for industrial plants amount to 4 to 8 percent of the purchased-equipment cost or 1 to 2 percent of the total capital investment. Because the value of land usually does not decrease with time, this cost should not be included in the fixed-capital investment when estimating certain annual operating costs, such as depreciation.

TABLE 13
Typical variation in percent of fixed-capital investment for service facilities

Service facilities	Range, %	Typical value, %
Steam generation	2.6-6.0	3.0
Steam distribution	0.2-2.0	1.0
Water supply, cooling, and pumping	0.4-3.7	1.8
Water treatment	0.5-2.1	1.3
Water distribution	0.1-2.0	0.8
Electric substation	0.9-2.6	1.3
Electric distribution	0.4-2.1	1.0
Gas supply and distribution	0.2-0.4	0.3
Air compression and distribution	0.2-3.0	1.0
Refrigeration including distribution	1.0-3.0	2.0
Process waste disposal	0.6-2.4	1.5
Sanitary waste disposal	0.2-0.6	0.4
Communications	0.1-0.3	0.2
Raw-material storage	0.3-3.2	0.5
Finished-product storage	0.7-2.4	1.5
Fire-protection system	0.3-1.0	0.5
Safety installations	0.2-0.6	0.4

Engineering and Supervision

The costs for construction design and engineering, drafting, purchasing, accounting, construction and cost engineering, travel, reproductions, communications, and home office expense including overhead constitute the capital investment for engineering and supervision. This cost, since it cannot be directly charged to equipment, materials, or labor, is normally considered an indirect cost in fixed-capital investment and is approximately 30 percent of the purchased-equipment cost or 8 percent of the total direct costs of the process plant. Typical percentage variations of tied-capital investment for various components of engineering and supervision are given in Table 14.

Construction Expense

Another expense which is included under indirect plant cost is the item of construction or field expense and includes temporary construction and operation, construction tools and rentals, home office personnel located at the construction site, construction payroll, travel and living, taxes and insurance, and other construction overhead. This expense item is occasionally included under equipment installation, or more often under engineering, supervision, and construction. If construction or field expenses are to be estimated separately, then Table 15 will be useful in establishing the variation in percent of fixed-capital investment for this indirect cost. For ordinary chemical-process

TABLE 14
Typical variation in percent of fixed-capital investment for engineering and services

Component	Range, %	Typical value, %
Engineering	1.5-6.0	2.2
Drafting	2.0-12.0	4.8
Purchasing	0.2-0.5	0.3
Accounting, construction. and cost engineering	0.2-1.0	0.3
Travel and living	0.1-1.0	0.3
Reproductions and communications	0.2-0.5	0.2
Total engineering and supervision (including overhead)	4.0-21.0	8.1

plants the construction expenses average roughly 10 percent of the total direct costs for the plant.

Contractor's Fee

The contractor's fee varies for different situations, but it can be estimated to be about 2 to 8 percent of the direct plant cost or 1.5 to 6 percent of the fixed-capital investment.

Contingencies

A contingency factor is usually included in an estimate of capital investment to compensate for unpredictable events, such as storms, floods, strikes, price

TABLE 15
Typical variation in percent of fixed-capital investment for construction expenses

Component	Range, %	Typical value, %
Temporary construction and operations	1.0-3.0	1.7
Construction tools and rental	1.0-3.0	1.5
Home office personnel in field	0.2-2.0	0.4
Field payroll	0.4-4.0	1.0
Travel and living	0.1-0.8	0.3
Taxes and insurance	1.0-2.0	1.2
Startup materials and labor	0.2-1.0	0.4
overhead	0.3-0.8	0.5
Total construction expanses	4.2-16.6	7.0

changes, small design changes, errors in estimation, and other unforeseen expenses, which previous estimates have statistically shown to be of a recurring nature. This factor may or may not include allowance for escalation. Contingency factors ranging from 5 to 15 percent of the direct and indirect plant costs are commonly used, with 8 percent being considered a fair average value.

Startup Expense

After plant construction has been completed, there are quite frequently changes that have to be made before the plant can operate at maximum design conditions. These changes involve expenditures for materials and equipment and result in loss of income while the plant is shut down or is operating at only partial capacity. Capital for these startup changes should be part of any capital appropriation because they are essential to the success of the venture. These expenses may be as high as 12 percent of the fixed-capital investment. In general, however, an allowance of 8 to 10 percent of the fixed-capital investment for this item is satisfactory.

Startup expense is not necessarily included as part of the required investment; so it is not presented as a component in the summarizing Table 26 for capital investment at the end of this chapter. In the overall cost analysis, startup expense may be represented as a one-time-only expenditure in the first year of the plant operation or as part of the total capital investment depending on the company policies.

Methods for estimating capital investment

Various methods can be employed for estimating capital investment. The choice of any one method depends upon the amount of detailed information available and the accuracy desired. Seven methods are outlined in thischapter, with each method requiring progressively less detailed information and less preparation time. Consequently, the degree of accuracy decreases with each succeeding method. A maximum accuracy within approximately ±5 percent of the actual capital investment can be obtained with method A.

METHOD A DETAILED-ITEM ESTIMATE. A detailed-item estimate requires careful determination of each individual item shown in Table 1. Equipment and material needs are determined from completed drawings and specifications and are priced either from current cost data or preferably from firm delivered quotations. Estimates of installation costs are determined from accurate labor rates, efficiencies, and employee-hour calculations. Accurate estimates of engineering, drafting, field supervision employee-hours, and field-expenses must be detailed in the same manner. Complete site surveys and soil data must be available to minimize errors in site development and construction cost estimates. In fact, in this type of estimate, an attempt is made to firm up as much of the estimate as possible by obtaining quotations from vendors and suppliers. Because of the extensive data necessary and the large amounts of engineering time required to prepare such a detailed-item estimate, this type of estimate is almost exclusively only prepared by contractors bidding on lump-sum work from finished drawings and specifications.

METHOD B UNIT-COST ESTIMATE. The unit-cost method results in good estimating accuracies for fixed-capital investment provided accurate records have been kept of previous cost experience. This method, which is frequently used for preparing definitive and preliminary estimates, also requires detailed estimates of purchased price obtained either from quotations or index-corrected cost records and published data. Equipment installation labor is evaluated as a fraction of the delivered-equipment cost. Costs for concrete, steel, pipe, electricals, instrumentation, insulation, etc., are obtained by take-offs from the drawings and applying unit costs to the material and labor needs. A unit cost is also applied to engineering employee-hours, number of drawings, and specifications. A factor for construction expense, contractor's fee, and contingency is estimated from previously completed projects and is used to complete this type of estimate. A cost equation summarizing this method can be given as†

$$C_n = \left[\Sigma(E + E_L) + \Sigma(f_x M_x + f_y M_L') + \Sigma f_e H_e + \Sigma f_d d_n\right](f_F) \qquad (2)$$

where C_n = new capital investment
E = purchased-equipment cost
E_L = purchased-equipment labor cost
f_x = specific material unit cost, e.g., f_p = unit cost of pipe
M_x = specific material quantity in compatible units
f_y = specific material labor unit cost per employee-hour
M_L' = labor employee-hours for specific material
f_e = unit cost-for engineering
H_e = engineering employee-hours
f_d = unit cost per drawing or specification
d_n = number of drawings or specifications
f_F = construction or field expense factor always greater than 1

Approximate corrections to the base equipment cost of complete, main-plant items for specific materials of construction or extremes of operating pressure and temperature can be applied in the form of factors as shown in Table 16.

METHOD C PERCENTAGE OF DELIVERED-EQUIPMENT COST. This method for estimating the fixed or total-capital investment requires determination of the delivered-equipment cost. The other items included in the total direct plant cost are then estimated as percentages of the delivered-equipment cost. The additional components of the capital investment are based on average percentages of the total direct plant cost, total direct and indirect plant costs, or total capital

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†H. C. Bauman, "Fundamentals of Cost Engineering in the Chemical Industry," Reinhold Publishing Corporation, New York, 1964.

TABLE 16
Correction factors for operating pressure, operating temperature, and material of construction to apply for fixed-capital investment of major plant items†‡

Operating pressure, psia (atm)	Correction factor
0.08 (0.005)	1.3
0.2 (0.014)	1.2
0.7 (0.048)	1.1
700.54) (48) 100 (6.8)	1.0 (base)
3000 (204)	1.21.1
6000 (408)	1.3

Operating temperature, °C	Correction factor
-80	1.3
0	1 .0 (base)
100	1.05
600	1.1
5,000	1.2
10,000	1.4

Material of construction	Correction factor
Carbon steel-mild	1 .0 (base)
Bronze	1.05
Carbon/molybdenum steel	1.065
Aluminum	1.075
Cast steel	1.11
Stainless steel	1.28 to 1.5
Worthite alloy	1.41
Hastelloy C alloy	1.54
Monel alloy	1.65
Nickel/inconel alloy	1.71
Titanium	2.0

† Adapted from D. H. Allen and R. C. Page, Revised Techniques for Predesign Cost Estimating, *Chem. Eng.*, **82(5)**: 142 (March 3, 1975).

‡ It should be noted that these factors are to be used *only* for complete, main-plant items and serve to correct from the base case to the indicated conditions based on pressure or temperature extremes that may be involved or special materials of construction that may be required. For the case of small or single pieces of equipment which are completely dedicated to the extreme conditions, the factors given in this table may be far too low and factors or methods given in other parts of this book must be used.

investment. This is summarized in the following cost equation:

$$C_n = [\Sigma E + \Sigma(f_1 E + f_2 E + f_3 E + \ldots)](f_I) \tag{3}$$

where $f_1, f_2 \ldots$ = multiplying factors for piping, electrical, instrumentation, etc.
f_I = indirect cost factor always greater than 1.

The percentages used in making an estimation of this type should be determined on the basis of the type of process involved, design complexity, required materials of construction, location of the plant, past experience, and other items dependent on the particular unit under consideration. Average values of the various percentages have been determined for typical chemical plants, and these values are presented in Table 17.

Estimating by percentage of delivered-equipment cost is commonly used for preliminary and study estimates. It yields most accurate results when applied to projects similar in configuration to recently constructed plants. For comparable plants of different capacity, this method has sometimes been reported to yield definitive estimate accuracies.

Example 3 Estimation of fixed-capital investment by percentage of delivered-equipment cost. Prepare a study estimate of the tied-capital investment for the process plant described in Example 1 if the delivered-equipment cost is $100,000.

Solution. Use the ratio factors outlined in Table 17 with modifications for instrumentation and outdoor operation.

Components	*cost*
Purchased equipment (delivered), *E*	$100,000
Purchased equipment installation, 39% *E*	39,000
Instrumentation (installed), 28% *E*	28,000
Piping (installed), 31% *E*	31,000
Electrical (installed), 10% *E*	10,000
Buildings (including services), 22% *E*	22,000
Yard improvements, 10% *E*	10,000
Service facilities (installed), 55% *E*	55,000
Land, 6% *E*	6,000
Total direct plant cost *D*	301,000
Engineering and supervision, 32% *E*	32,000
Construction expenses, 34% *E*	34,000
Total direct and indirect cost $(D + I)$	367,000
Contractor's fee, 5% $(D + I)$	18,000
Contingency, 10% $(D + I)$	37,000
Fixed-capital investment	$422,000

METHOD D "LANG" FACTORS FOR APPROXIMATION OF CAPITAL INVESTMENT. This technique, proposed originally by Lang† and used quite frequently to obtain order-of-magnitude cost estimates, recognizes that the cost of a

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†H. J. Lang, *Chem. Eng.*, **54**(10):117 (1947); H. J. Lang, *Chem. Eng.*, **55**(6):112 (1948).

TABLE 17
Ratio factors for estimating capital-investment items based on delivered-equipment cost

Values presented are applicable for major process plant additions to an existing site where the necessary land is available through purchase or present ownership.? The values are based on fixed-capital investments ranging from under $1 million to over $20 million.

Item	Percent of deliveredequipment cost for		
	Solid- processing **plant‡**	**Solid-fluid-** processing plant ‡	**Fluid-** processing plant ‡
Direct costs			
Purchased equipment-delivered (including fabricated equipment and process machinery) §	100	100	100
Purchased-equipment installation	45	39	47
Instrumentation and controls (installed)	9	13	18
Piping (installed)	16	31	66
Electrical (installed)	10	10	11
Buildings (including services)	25	29	18
Yard improvements	13	10	10
Service facilities (installed)	40	55	70
Land (if purchase is required)	6	6	6
Total direct plant cost	264	293	346
Indirect costs			
Engineering and supervision	33	32	33
Construction expenses	39	34	41
Total direct and indirect plant costs	336	359	420
Contractor's fee (about 5% of direct and indirect plant costs)	17	18	21
Contingency (about 10% of direct and indirect plant costs)	34	36	42
Fixed-capital investment	387	413	483
Working capital (about 15% of total capital investment)	68	74	86
Total capital investment	455	487	569

† Because of the extra expense involved in supplying service facilities, storage facilities, loading terminals, transportation facilities, and other necessary utilities at a completely undeveloped site, the fixed-capital investment for a new plant located at an undeveloped site may be as much as 100 percent greater than for an equivalent plant constructed as an addition to an existing plant.

‡ See Table 8 for definition of types of process plants.

§ Includes pumps and compressors.

TABLE 18

Lang multiplication factors for estimation of fixed-capital investment or total capital investment

Factor x delivered-equipment cost = fixed-capital investment or total capital investment for major additions to an existing plant.

Type of plant	Factor for Fixed-capital investment	Factor for Total capital investment
Solid-processing plant	3.9	4.6
Solid-fluid-processing plant	4.1	4.9
Fluid-processing plant	4.8	5.7

process plant may be obtained by multiplying the basic equipment cost by some factor to approximate the capital investment. These factors vary depending upon the type of process plant being considered. The percentages given in Table 17 are rough approximations which hold for the types of process plants indicated. These values, therefore, may be combined to give Lang multiplication factors that can be used for estimating the total direct plant cost, the fixed-capital investment, or the total capital investment. Factors for estimating the fixed-capital investment or the total capital investment are given in Table 18. It should be noted that these factors include costs for land and contractor's fees.

Greater accuracy of capital investment estimates can be achieved in this method by using not one but a number of factors. One approach is to use different factors for different types of equipment. Another approach is to use separate factors for erection of equipment, foundations, utilities, piping, etc., or even to break up each item of cost into material and labor factors.? With this approach, each factor has a range of values and the chemical engineer must rely on past experience to decide, in each case, whether to use a high, average, or low figure.

Since tables are not convenient for computer calculations it is better to combine the separate factors into an equation similar to the one proposed by Hirsch and Glazier‡

$$C_n = f_I\left[E(1 + f_F + f_p + f_m) + E_i + A\right] \tag{4}$$

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†Further discussions on these methods may be found in W. D. Baasel, "Preliminary Chemical Engineering Plant Design," American Elsevier Publishing Company, Inc., New York, 1976; S. G. Kirkham, Preparation and Application of Refined Lang Factor Costing Techniques, *AACE Bul.*, **15**(5):137 (Oct., 1972); C. A. Miller, Capital Cost Estimating-A Science Rather Than an Art, Cost *Engineers' Notebook,* ASCE A-1666 (June, 1978).

‡J. H. Hirsch and E. M. Glazier, *Chem. Eng. Progr.*, **56**(12):37 (1960).

where the three installation-cost factors are, in turn, defined by the following three equations:

$$\log f_F = 0.635 - 0.154 \log 0.001E - 0.992; + 0.506 \frac{f_v}{E} \tag{5}$$

$$\log f_p = -0.266 - 0.014 \log 0.001E - 0.156; + 0.556 \frac{p}{E} \tag{6}$$

$$\log f_m = 0.344 + 0.033 \log 0.001 E + 1.194; \tag{7}$$

and the various parameters are defined accordingly:

E = purchased-equipment on an f.o.b. basis
f, = indirect cost factor always greater than 1 (normally taken as 1.4)
f_F = cost factor for field labor
f_p = cost factor for piping materials
f_m = cost factor for miscellaneous items, including the materials cost for insulation, instruments, foundations, structural steel, building, wiring, painting, and the cost of freight and field supervision
E_i = cost of equipment already installed at site
A = incremental cost of corrosion-resistant alloy materials
e = total heat exchanger cost (less incremental cost of alloy)
f_v = total cost of field-fabricated vessels (less incremental cost of alloy)
p = total pump plus driver cost (less incremental cost of alloy)
t = total cost of tower shells (less incremental cost of alloy)

Note that Eq. (4) is designed to handle both purchased equipment on an f.o.b. basis and completely installed equipment.

METHOD E POWER FACTOR APPLIED TO PLANT-CAPACITY RATIO. This method for study or order-of-magnitude estimates relates the fixed-capital investment of a new process plant to the fixed-capital investment of similar previously constructed plants by an exponential power ratio. That is, for certain similar process plant configurations, the fixed-capital investment of the new facility is equal to the fixed-capital investment of the constructed facility C multiplied by the ratio $R,$ defined as the capacity of the new facility divided by the capacity of the old, raised to a power x. This power has been found to average between 0.6 and 0.7 for many process facilities. Table 19 gives the capacity power factor (x) for various kinds of processing plants.

$$C_n = C(R)'' \tag{8}$$

A closer approximation for this relationship which involves the direct and indirect plant costs has been proposed as

$$C_n = f\left[D(R)^x + I\right] \tag{9}$$

TABLE 19

Capital-cost data for processing plants (1990)†

Product or process	Process remarks	Typical plant size, 1000 tons / yr	Fixed-capital investment, million $	$ of fixed-capital investment per annual ton of product	Power factor (x)‡ for plant-capacity ratio
Chemical plants					
Acetic acid	CH_3OH and CO-catalytic	10	6	650	0.68
Acetone	Propylene-copper chloride catalyst	100	32	320	0.45
Ammonia	Steam reforming	100	24	240	0.53
Ammonium nitrate	Ammonia and nitric acid	100	5	50	0.65
Butanol	Propylene, CO, and H_2O—catalytic	50	40	800	0.40
Chlorine	Electrolysis of NaCl	50	28	550	0.45
Ethylene	Refinery gases	50	13	260	0.83
Ethylene oxide	Ethylene-catalytic	50	50	1000	0.78
Formaldehyde (37%)	Methanol-catalytic	10	16	1600	0.55
Glycol	Ethylene and chlorine	5	15	2900	0.75
Hydrofluoric acid	Hydrogen fluoride and H_2O	10	8	800	0.68
Methanol	CO,, natural gas, and steam	60	13	200	0.60
Nitric acid (high strength)	Ammonia-catalytic	100	6	65	0.60
Phosphoric acid	Calcium phosphate and H_2SO_4	5	3	650	0.60
Polyethylene (high density)	Ethylene-catalytic	5	16	3200	0.65
Propylene	Refinery gases	10	3	320	0.70
Sulfuric acid	Sulfur-catalytic	100	3	32	0.65
Urea	Ammonia and CO,	60	8	130	0.70

TABLE 19
Capital-cost data for processing plants **(1990)** (*Continued*)

Product or **process**	**Process** remarks	Typical plant **size, 1000 bbl / day**	**Fixed-capital investment, million $**	**$ of fixed-capital** investment per **bbl** / day	Power factor (x)‡ for **plant-capacity** ratio
		Refinery units			
Alkylation (H_2SO_4)	Catalytic	10	19	1900	0.60
Coking (delayed)	Thermal	10	26	2600	0.38
Coking (fluid)	Thermal	10	16	1600	0.42
Cracking (fluid)	Catalytic	10	16	1600	0.70
Cracking	Thermal	10	5	500	0.70
Distillation (atm.)	65% vaporized	100	32	3m	0.90
Distillation (vac.)	65% vaporized	100	19	200	0.70
Hydrotreating	Catalytic desulfurization	10	3	320	0.65
Reforming	Catalytic	10	29	2900	0.60
Polymerization	Catalytic	10	5	500	0.58

† Adapted from K. M. Guthrie, Capital and Operating Costs for 54 Chemical Processes, *Chem. Eng.*, **77**(13):140 (June 15, 1970) and K. M. Guthrie, "Process Plant Estimating, Evaluation, and Control," Craftsman Book Company of America, Solana Beach, California. 1974. See also J. E. Haselbarth, Updated Investment Costs for 60 Chemical Plants, *Chem. Eng.*, **74**(25):214 (Dec. 4, 1967) and D. Drayer, How to Estimate Plant Cost-Capacity Relationship, *Petro/Chem Engr.*, **42**(5):10 (1970).

‡ These power factors apply within roughly a three-fold ratio extending either way from the plant size as given.

TABLE 20
Relative labor rate and productivity indexes in the chemical and allied products industries for the United States (1989)†

Geographical area	Relative labor rate	Relative productivity factor
New England	1.14	0.95
Middle Atlantic	1.06	0.96
South Atlantic	0.84	0.91
Midwest	1.03	1.06
Gulf	0.95	1.22
Southwest	0.88	1.04
Mountain	0.88	0.97
Pacific Coast	1.22	0.89

† Adapted from J. M. Winton, Plant Sites, *Chem. Week*, 121(24):49 (Dec. 14, 1977), and updated with data from M. Kiley, ed., "National Construction Estimator," 37th ed., Craftsman Book Company of America, Carlsbad, CA, 1989. Productivity, as considered here, is an economic term that gives the value added (products minus raw materials) per dollar of total payroll cost. Relative values were determined by taking the average of Kiley's weighted state values in each region divided by the weighted average value of all the regions. See also Tables 23 and 24 of this chapter; H. Popper and G. E. Weismantel, Costs and Productivity in the Inflationary 1970's, *Chem. Eng.*, 77(1):132 (Jan. 12, 1970); and C. H. Edmondson, *Hydrocarbon Process.*, 53(7):167 (1974).

where f is a lumped cost-index factor relative to the original installation cost. D is the direct cost and Z is the total indirect cost for the previously installed facility of a similar unit on an equivalent site. The value of x approaches unity when the capacity of a process facility is increased by adding identical process units instead of increasing the size of the process equipment. The lumped cost-index factor f is the product of a geographical labor cost index, the corresponding area labor productivity index, and a material and equipment cost index. Table 20 presents the relative median labor rate and productivity factor for various geographical areas in the United States.

Example 4 Estimating relative costs of construction labor as a function of geographical area. If a given chemical process plant is erected near Dallas (Southwest area) with a construction labor cost of $100,000 what would be the construction labor cost of an identical plant if it were to be erected at the same time near Los Angeles (Pacific Coast Area) for the time when the factors given in Table 20 apply?

Solution

Relative median labor rate-Southwest 0.88 from Table 20
Relative median labor rate-Pacific Coast 1.22 from Table 20

$$\text{Relative labor rate ratio} = \frac{1.22}{0.88} = 1.3864$$

Relative productivity factor-Southwest 1.04 from Table 20
Relative productivity factor-Pacific Coast 0.89 from Table 20

$$\text{Relative productivity factor ratio} = \frac{0.89}{1.04} = 0.8558$$

Construction labor cost of Southwest to Pacific Coast = (1.3864)/(0.8558) = 1.620
Construction labor cost at Los Angeles = (1.620)($100,000) = $162,000

To determine the fixed-capital investment required for a new similar single-process plant at a new location with a different capacity and with the same number of process units, the following relationship has given good results:

$$C_n = R^x\left[f_E E + f_M M + f_L f_F e_L\left(E_L + f_y M_L'\right)\right](f_I)\frac{C}{C - I} \qquad (10)$$

where f_E = current equipment cost index relative to cost of the purchased equipment
f_M = current material cost index relative to cost of material
M = material cost
f_L = current labor cost index in new location relative to E_L and M_L' at old location
e_L = labor efficiency index in new location relative to E_L and M_L' at old location
E_L = purchased-equipment labor cost
M_L' = labor employee-hours for specific material
f_y = specific material labor cost per employee-hour
C = original capital investment

In those situations where estimates of fixed-capital investment are desired for a similar plant at a new location and with a different capacity, but with multiples of the original process units, Eq. (11) often gives results with somewhat better than study-estimate accuracy.

$$C_n = \left[R f_E E + R^x f_M M + R^x f_L f_F e_L\left(E_L + f_y M_L\right)\right](f_I)\frac{C}{C - I} \qquad (11)$$

More accurate estimates by this method are obtained by subdividing the process plant into various process units, such as crude distillation units, reformers, alkylation units, etc., and applying the best available data from similar previously installed process units separately to each subdivision. Table 19 lists some typical process unit capacity-cost data and exponents useful for making this type of estimate.

Example 5 Estimation of fixed-capital investment with power factor applied to plant-capacity ratio. If the process plant, described in Example 1, was erected in the Dallas area for a fixed-capital investment of $436,000 in 1975, determine what the estimated fixed-capital investment would have been in 1980 for a similar process plant located near Los Angeles with twice the process capacity but with an equal number of process units? Use the power-factor method to evaluate the new fixed-capital investment and assume the factors given in Table 20 apply.

Solution. If Eq. (8) is used with a 0.6 power factor and the Marshall and Swift all-industry index (Table 3), the fixed-capital investment is

$$C_n = Cf_E(R)^x$$

$$C_n = (436{,}000)\left(\frac{660}{444}\right)(2)^{0.6} = \$982{,}000$$

If Eq. (8) is used with a 0.7 power factor and the Marshall and Swift all-industry index (Table 3), the fixed-capital investment is

$$C_n = (436{,}000)\left(\frac{660}{444}\right)(2)^{0.7} = \$1{,}053{,}000$$

If Eq. (9) is used with a 0.6 power factor, the Marshall and Swift all-industry index (Table 3), and the relative labor and productivity indexes (Table 20), the fixed-capital investment is

$$C_n = f\left[D(R)^x + I\right]$$

where $f = f_E f_L e_L$, and D and Z are obtained from Example 1,

$$C_n = \left(\frac{660}{444}\right)\left(\frac{1.22}{0.88}\right)\left(\frac{1.04}{0.89}\right)\left[(308{,}000)(2)^{0.6} + 128{,}000\right]$$

$$C_n = (1.486)(1.620)(467.000 + 128{,}000)$$

$$C_n = \$1{,}432{,}000$$

If Eq. (9) is used with a 0.7 power factor, the Marshall and Swift all-industry index (Table 3), and the relative labor and productivity indexes (Table 20), the fixed-capital investment is

$$C_n = \$1{,}513{,}000$$

Results obtained using this procedure have shown high correlation with fixed-capital investment estimates that have been obtained with more detailed techniques. Properly used, these factoring methods can yield quick fixed-capital investment requirements with accuracies sufficient for most economic-evaluation purposes.

METHOD F INVESTMENT COST PER UNIT OF CAPACITY. Many data have been published giving the fixed-capital investment required for various processes per unit of annual production capacity such as those shown in Table 19. Although these values depend to some extent on the capacity of the individual plants, it is possible to determine the unit investment costs which apply for average conditions. An order-of-magnitude estimate.of the fixed-capital investment for a given process can then be obtained by multiplying the appropriate investment cost per unit of capacity by the annual production capacity of the proposed plant. The necessary correction for change of costs with time can be made with the use of cost indexes.

METHOD G TURNOVER RATIOS. A rapid evaluation method suitable for order-of-magnitude estimates is known as the "turnover ratio" method. Turnover ratio is defined as the ratio of gross annual sales to the fixed-capital investment,

$$\text{Turnover ratio} = \frac{\text{gross annual sales}}{\text{fixed-capital investment}} \tag{12}$$

where the product of the annual production rate and the average selling price of the commodities is the gross annual sales figures. The reciprocal of the turnover ratio is sometimes defined as the *capital ratio* or the *investment ratio*.† Turnover ratios of up to 5 are common for some business establishments and some are as low as 0.2. For the chemical industry, as a very rough rule of thumb, the ratio can be approximated as 1.

ORGANIZATION FOR PRESENTING CAPITAL INVESTMENT ESTIMATES BY COMPARTMENTALIZATION

The methods for estimating capital investment presented in the preceding sections represent the fundamental approaches that can be used. However, the direct application of these methods can often be accomplished with considerable improvement by considering the fixed-capital investment requirement by parts. With this approach, each identified part is treated as a separate unit to obtain the total investment cost directly related to it. Various forms of compartmentalization for this type of treatment have been proposed. Included in these are (1) the *modular estimate*,‡ (2) the *unit-operations estimate*,§ (3) the *functional-unit estimate*,¶ and (4) the *average-unit-cost estimate*.††

The same principle of breakdown into individual components is used for each of the four approaches. For the *modular estimate*, the basis is to consider individual modules in the total system with each consisting of a group of similar items. For example, all heat exchangers might be included in one module, all furnaces in another, all vertical process vessels in another, etc. The total cost estimate is considered under six general groupings including chemical processing, solids handling, site development, industrial buildings, offsite facilities, and

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†When the term *investment ratio* is used, the investment is usually considered to be the total capital investment which includes working capital as well as other capitalized costs.

‡W. J. Dodge et al., Metropolitan New York Section of AACE, The Module Estimating Technique as an Aid in Developing Plant Capital Costs, *Trans. AACE* (1962); K. M. Guthrie, Capital Cost Estimating, *Chem. Eng.*, **76**(6):114 (March 24, 1969); K. M. Guthrie, "Process Plant Estimating, Evaluation, and Control," Craftsman Book Company of America, Solana Beach, CA, 1974; A. Pikulik and H. E. Diaz, *Chem. Eng.*, **84**(21):106 (Oct. 10, 1977); R. H. Perry and D. H. Green, "Chemical Engineers' Handbook," 6th ed., McGraw-Hill Book Company, Inc., New York, 1984.

§E. F. Hensley, "The Unit-Operations Approach," American Association of Cost Engineers, Paper presented at Annual Meeting, 1967; E. W. Merrow, K. E. Phillips, and C. W. Meyers, "Understanding Cost Growth and Performance Shortfalls in Pioneer Process Plants," Rand Corporation, Santa Monica, CA, 1981; see also *Chem. Eng.*, **88**(3):41 (Feb. 9, 1981).

¶A. V. Bridgewater, The Functional-Unit Approach to Rapid Cost Estimation, *AACE Bull.*, **18**(5):153 (1976).

††C. A. Miller, New Cost Factors Give Quick Accurate Estimates, *Chem. Eng.*, **72**(19):226 (Sept. 13, 1965); C. A. Miller, Current Concepts in Capital Cost Forecasting, *Chem. Eng. Progr.*, **69**(5):77 (1973); 0. P. Charbanda, "Process Plant and Equipment Cost Estimation," Craftsman Book Company of America, Solana Beach, CA, 1979; S. Cran, Improved Factored Method Gives Better Preliminary Cost Estimates," *Chem. Eng.*, **88**(7):79 (Apr. 6, 1981).

project indirects. As an example of an equipment cost module for heat exchangers, the module would include the basic delivered cost of the piece of equipment with factors similar to Lang factors being presented for supplemental items needed to get the equipment ready for use such as piping, insulation, paint,- labor, auxiliaries, indirect costs, and contingencies.

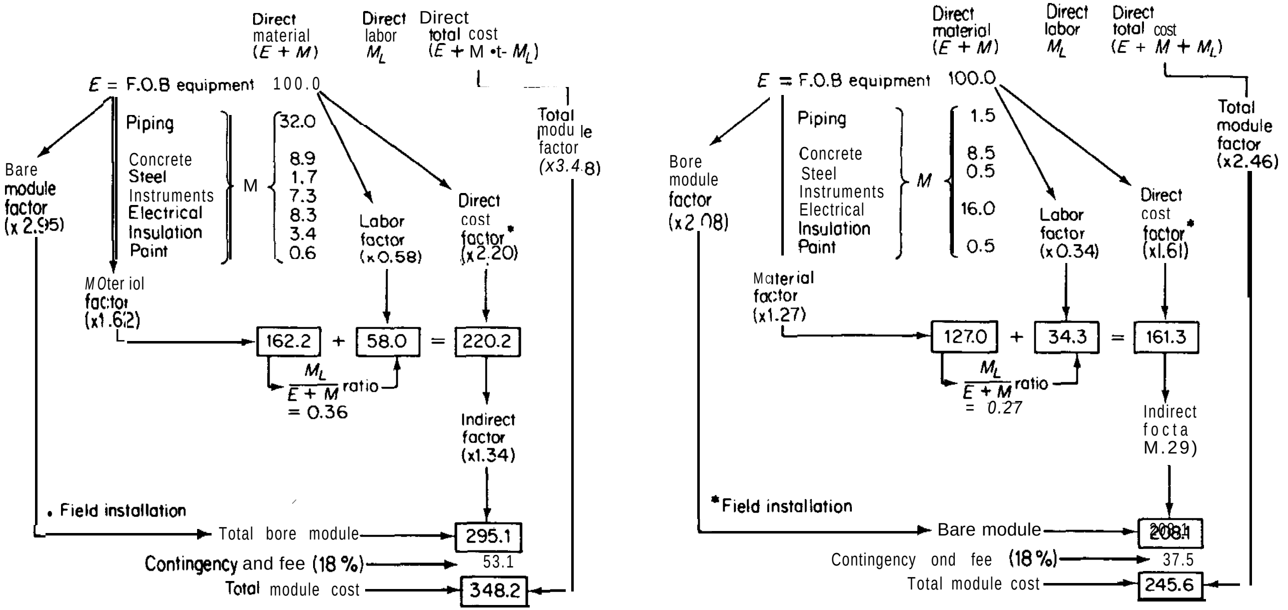
In presenting the basic data for the module factors, the three critical variables are size or capacity of the equipment, materials of construction, and operating pressure with temperature often being given as a fourth critical variable. It is convenient to establish the base cost of all equipment as that constructed of carbon steel and operated at atmospheric pressure. Factors, such as are presented in Table 16, are then used to change the estimated costs of the equipment to account for variation in the preceding critical variables. Once the equipment cost for the module is determined, various factors are applied to obtain the final fixed-capital investment estimate for the item completely installed and ready for operation. Figure 6-6 shows two typical module approaches with Fig. 6-6*a* representing a module that applies to a "normal" chemical process where the overall Lang factor for application to the f.o.b. cost of the original equipment is 3.482 and Fig. 6-6*b* representing a "normal" module for a piece of mechanical equipment where the Lang factor has been determined to be 2.456.

The modules referred to in the preceding can be based on combinations of equipment that involve similar types of operations requiring related types of auxiliaries. An example would be a distillation operation requiring the distillation column with the necessary auxiliaries of reboiler, condenser, pumps, holdup tanks, and structural supports. This type of compartmentalization for estimating purposes can be considered as resulting in a so-called *unit-operations estimate.* Similarly, the *functional-unit estimate* is based on the grouping of equipment by function such as distillation or filtration and including the fundamental pieces of equipment as the initial basis with factors applied to give the final estimate of the capital investment.

The *average-unit-cost* method puts special emphasis on the three variables of size of equipment, materials of construction, and operating pressure as well as on the type of process involved. In its simplest form, all of these variables and the types of process can be accounted for by one number so that a given factor to convert the process equipment cost to total fixed-capital investment can apply for each "average unit cost." The latter is defined as the total cost of the process equipment divided by the number of equipment items in that particular process. As the "average unit cost" increases, the size of the factor for converting equipment cost to total fixed-capital investment decreases with a range of factor values applicable for each "average unit cost" depending on the particular type of process, operating conditions, and materials of construction.

ESTIMATION OF TOTAL PRODUCT COST

Methods for estimating the total capital investment required for a given plant are presented in the first part of this chapter. Determination of the necessary



(a) "Normal" module for a chemical process unit with resultant Long factor of 3.482

(b) "Normal" module for a mechanical equipment unit with resultant Long factor of 2.456

FIGURE **6-6**
Example of a "normal" module as applied for estimating capital investment for a chemical process and a mechanical equipment unit. [Adapted from K. M. Guthrie, Capital Cost Estimating, *Chem. Eng.*, **76**(6):114 (March 24, 1969).]

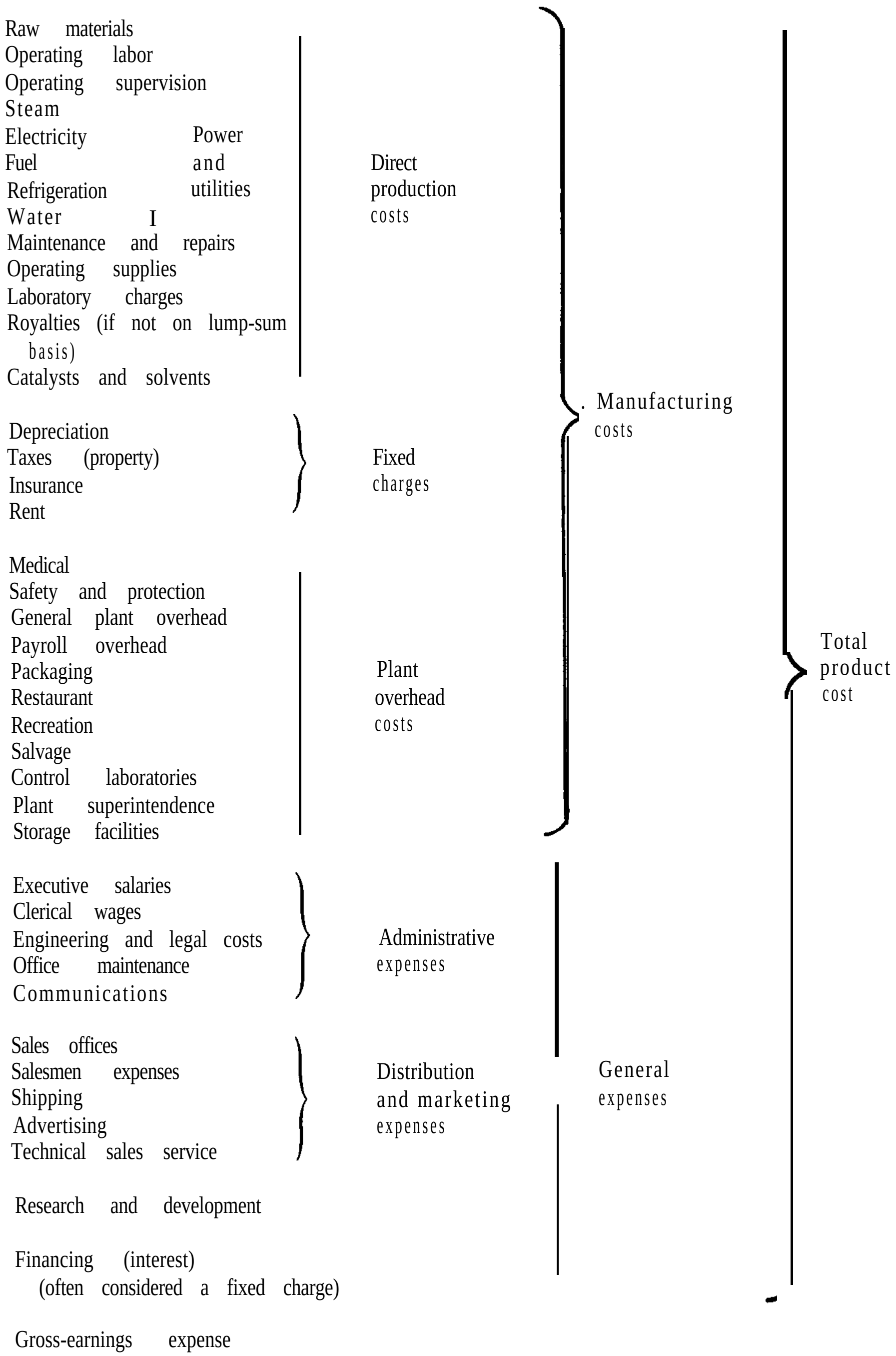


FIGURE 6-7
Costs involved in total product cost for a typical chemical process plant.

capital investment is only one part of a complete cost estimate. Another equally important part is the estimation of costs for operating the plant and selling the products. These costs can be grouped under the general heading of ***totalproduct cost.*** The latter, in turn, is generally divided into the categories of ***manufacturing costs*** and ***general expenses.*** Manufacturing costs are also known as ***operating*** or ***production costs.*** Further subdivision of the manufacturing costs is somewhat dependent upon the interpretation of direct and indirect costs.

Accuracy is as important in estimating total product cost as it is in estimating capital investment costs. The largest sources of error in total-product-cost estimation are overlooking elements of cost. A tabular form is very useful for estimating total product cost and constitutes a valuable checklist to preclude omissions. Figure 6-7 provides a suggested checklist which is typical of the costs involved in chemical processing operations.

Total product costs are commonly calculated on one of three bases: namely, daily basis, unit-of-product basis, or annual basis, The annual cost basis is probably the best choice for estimation of total cost because (1) the effect of seasonal variations is smoothed out, (2) plant on-stream time or equipment-operating factor is considered, (3) it permits more-rapid calculation of operating costs at less than full capacity, and (4) it provides a convenient way of considering infrequently occurring but large expenses such as annual turnaround costs in a refinery.

The best source of information for use in total-product-cost estimates is data from similar or identical projects. Most companies have extensive records of their operations, so that quick, reliable estimates of manufacturing costs and general expenses can be obtained from existing records. Adjustments for increased costs as a result of inflation must be made, and differences in plant site and geographical location must be considered.

Methods for estimating total product cost in the absence of specific information are discussed in the following paragraphs. The various cost elements are presented in the order shown in Fig. 6-7.

Manufacturing Costs

All expenses directly connected with the manufacturing operation or the physical equipment of a process plant itself are included in the manufacturing costs. These expenses, as considered here, are divided into three classifications as follows: (1) direct production costs, (2) fixed charges, and (3) plant-overhead costs.

Direct production costs include expenses directly associated with the manufacturing operation. This type of cost involves expenditures for raw materials (including transportation, unloading, etc.,); direct operating labor; supervisory and clerical labor directly connected with the manufacturing operation; plant maintenance and repairs; operating supplies; power; utilities; royalties; and catalysts.

It should be recognized that some of the variable costs listed here as part of the direct production costs have an element of fixed cost in them. For instance, maintenance and repair decreases, but not directly, with production level because a maintenance and repair cost still occurs when the process plant is shut down.

Fixed charges are expenses which remain practically constant from year to year and do not vary widely with changes in production rate. Depreciation, property taxes, insurance, and rent require expenditures that can be classified as fixed charges.

Plant-overhead costs are for hospital and medical services; general plant maintenance and overhead; safety services; payroll overhead including pensions, vacation allowances, social security, and life insurance; packaging, restaurant and recreation facilities, salvage services, control laboratories, property protection, plant superintendence, warehouse and storage facilities, and special employee benefits. These costs are similar to the basic fixed charges in that they do not vary widely with changes in production rate.

General Expenses

In addition to the manufacturing costs, other general expenses are involved in any company's operations. These general expenses may be classified as (1) administrative expenses, (2) distribution and marketing expenses, (3) research and development expenses, (4) financing expenses, and (5) gross-earnings expenses.

Administrative expenses include costs for executive and clerical wages, office supplies, engineering and legal expenses, upkeep on office buildings, and general communications.

Distribution and marketing expenses are costs incurred in the process of selling and distributing the various products. These costs include expenditures for materials handling, containers, shipping, sales offices, salesmen, technical sales service, and advertising.

Research and development expenses are incurred by any progressive concern which wishes to remain in a competitive industrial position. These costs are for salaries, wages, special equipment, research facilities, and consultant fees related to developing new ideas or improved processes.

Financing expenses include the extra costs involved in procuring the money necessary for the capital investment. Financing expense is usually limited to interest on borrowed money, and this expense is sometimes listed as a fixed charge.

Gross-earnings expenses are based on income-tax laws. These expenses are a direct function of the gross earnings made by all the various interests held by the particular company. Because these costs depend on the company-wide picture, they are often not included in predesign or preliminary cost-estimation figures for a single plant, and the probable returns are reported as the gross earnings obtainable with the given plant design. However, when considering net profits, the expenses due to income taxes are extremely important, and this cost must be included as a special type of general expense.

DIRECT PRODUCTION COSTS

Raw Materials

In the chemical industry, one of the major costs in a production operation is for the raw materials involved in the process. The amount of the raw materials which must be supplied per unit of time or per unit of product can be determined from process material balances. In many cases, certain materials act only as an agent of production and may be recoverable to some extent. Therefore, the cost should be based on the amount of raw materials actually consumed as determined from the overall material balances.

Direct price quotations from prospective suppliers are preferable to published market prices. For preliminary cost analyses, market prices are often used for estimating raw-material costs. These values are published regularly in journals such as the ***Chemical Marketing Reporter*** (formerly the ***Oil, Paint, and Drug Reporter***).

Freight or transportation charges should be included in the raw-material costs, and these charges should be based on the form in which the raw materials are to be purchased for use in the final plant. Although bulk shipments are cheaper than smaller-container shipments, they require greater storage facilities and inventory. Consequently, the demands to be met in the final plant should be considered when deciding on the cost of raw materials.

The ratio of the cost of raw materials to total plant cost obviously will vary considerably for different types of plants. In chemical plants, raw-material costs are usually in the range of 10 to 50 percent of the total product cost.

Operating Labor

In general, operating labor may be divided into skilled and unskilled labor. Hourly wage rates for operating labor in different industries at various locations can be obtained from the U.S. Bureau of Labor ***Monthly Labor Review.*** For chemical processes, operating labor usually amounts to about 15 percent of the total product cost.

In preliminary costs analyses, the quantity of operating labor can often be estimated either from company experience with similar processes or from published information on similar processes. Because the relationship between labor requirements and production rate is not always a linear one, a 0.2 to 0.25 power of the capacity ratio when plant capacities are scaled up or down is often used.

If a flow sheet and drawings of the process are available, the operating labor may be estimated from an analysis of the work to be done. Consideration

TABLE 21
Typical labor requirements for process equipment

Type of equipment	Workers/unit/shift
Dryer, rotary	$\frac{1}{2}$
Dryer, spray	1
Dryer, tray	$\frac{1}{2}$
Centrifugal separator	$\frac{1}{4}$–$\frac{1}{2}$
Crystallizer, mechanical	$\frac{1}{6}$
Filter, vacuum	$\frac{1}{8}$–$\frac{1}{4}$
Evaporator	$\frac{1}{4}$
Reactor, batch	1
Reactor, continuous	$\frac{1}{2}$
Steam plant (100,000 lb/h)	3

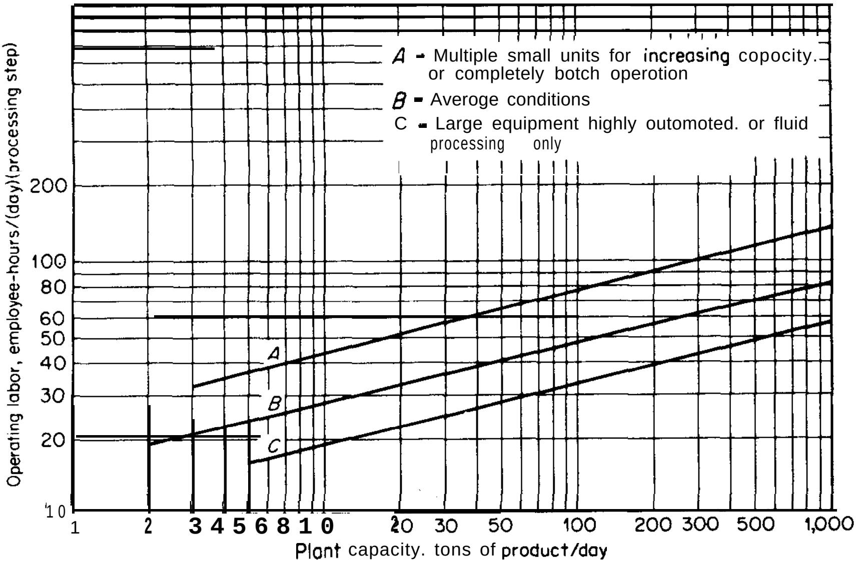


FIGURE 6-8
Operating labor requirements for chemical process industries.

must be given to such items as the type and arrangement of equipment, multiplicity of units, amount of instrumentation and control for the process, and company policy in establishing labor requirements. Table 21 indicates some typical labor requirements for various types of process equipment.

Another method of estimating labor requirements as a function of plant capacity is based on adding up the various principal processing steps on the flow

TABLE 22
Operating labor, fuel, steam, power, and water requirements for various processes†

	Capacity thousand ton/yr	Operating labor and supervision workhours/ ton	Maintenance labor and supervision workhours/ ton	Power and utilities, per ton/yr or bbl/day capacity			
				Fuel MM Btu/h	Steam lb/h	Power kWh	Water gph
		Chemical plants					
Acetone	100	0.518	0.315	.. ..	1.73	310	5.18
Acetic acid	10	1.483	0.984	. ..	......	**180**	0.58
Butadiene	100	0.345	0.285	.. .	0.012	130	0.73
Ethylene oxide	100	0.232	0.104	.. .	4.88	140	0.148
Formaldehyde	100	0.259	0.328	....	34.6	200	0.029
Hydrogen peroxide	100	0.288	0.352	.. ..	2.62	160	0.186
Isoprene	100	0.230	0.325	.....	0.81	710	0.001
Phosphoric acid	10	1.85	0.442	.	0.18	40	0.03
Polyethylene	100	0.259	0.295	...	0.23	450	0.0004
Urea	100	0.238	0.215	.. ..	0.33	135	0.0002
Vinyl acetate	100	0.432	0.528	.. ..	1.34	275	0.27
		Refinery units					
	Thousand bbl/day	Workhours/ bbl	Workhours/ bbl				
Alkylation	10	0.007	0.0895	.....	10.83	0.07	1.48
Coking (delayed)	10	0.011‡	0.0096	0.007	1.85	0.07	....
Coking (fluid)	10	0.0096	0.0058	0.012	2.55	0.06	0.64
Cracking (fluid)	10	0.0122	0.0115	. . .	(4.73)§	0.02	0.33
Cracking (thermal)	10	0.0096	0.0025	0.012	(2.55)§	0.06	0.64
Distillation (atm)	10	0.0048	0.0042	0.004	0.25	0.03	0.16
Distillation (MC)	10	0.0024	0.0154	0.003	0.95	0.04	0.18
Hydrotreating	10	0.0048	0.0028	0.006	0.92	0.01	0.14
Reforming, catalyt.	10	0.0048	0.0078	0.002	1.38	0.23	0.28
Polymerization	10	0.0024	0.0158	. . .	4.85	0.07	0.43

† Based on information from K. M. Guthrie, Capital and Operating Costs for 54 Chemical Processes, *Chem.* Eng., **77(13)**: 140 (June 15, 1970).

‡ Includes two coke cutters (1 shift/day).

§ Net steam generated.

TABLE 23

Cost tabulation for selected utilities and labor†‡

1989 costs based on U.S. Gulf Coast location

	cost
Steam costs	
Exhaust, \$/1000 lb	1.10
Pressure of 100 psig, \$/1000 lb	2.40
Pressure of 500 psig, \$/1000 lb	3.60
Fuel costs	
Gas at well head including gathering-system costs:	
Existing contracts, \$/million Btu	2.40
New contracts, \$/million Btu	3.00
Fuel oil in \$/million Btu with 6.25 million Btu/bbl	3.00
Gas transmission costs in ¢/100 miles	7.30
Plant fuel gas in \$/million Btu	3.20
Purchased power for midcontinent USA in ¢/kWh	7.00
Water costs	
Process water (treated) in ¢/1000 gal	80
Cooling water in ¢/1000 gal (tower or river)	10
Labor rates	
Supervisor, \$/h	28.00
Operators, \$/h	21.00
Helpers, \$/h	17.40
Chemists, \$/h	20.00
Labor burden as % of direct labors	25
Plant general overhead as % of total labor + burden	40

† Based on information updated from C. C. Johnnie and D. K. Aggarwal, Calculating Plant Utility Costs, *Chem. Eng. Progr.*, **73(11):84 (1977)** and M. Kiley, "National Construction Estimator," 37th ed., Craftsman Book Company of America, Carlsbad, CA, 1989.

‡ See Appendix B for a more detailed listing of utility and related costs.

§ Labor burden refers to costs the company must pay associated with and above the base labor rate, such as for Social Security, insurance, and other benefits.

sheets.¶ In this method, a process step is defined as any unit operation, unit process, or combination thereof, which takes place in one or more units of integrated equipment on a repetitive cycle or continuously, e.g., reaction, distillation, evaporation, drying, filtration, etc. Once the plant capacity is fixed, the number of employee-hours per ton of product per step is obtained from Fig. 6-8 and multiplied by the number of process steps to give the total employee-hours per ton of production. Variations in labor requirements from highly automated processing steps to batch operations are provided by selection of the appropriate curve on Fig. 6-8.

¶Method originally proposed by H. E. Wessel, New Graph Correlates Operating Labor for Chemical Processes, *Chem. Eng.*, **59(7):209** (July, 1952).

TABLE 24
Engineering ***News-Record*** **labor indexes to permit estimation of prevailing wage rates by location†**

(See table 23 for values of labor rates as \$/h)

	ENB Skilled Labor Index (December values). (Based on 1967 = 100)									
Location	**1980**	**1981**	**1982**	**1983**	**1984**	**1985**	**1986**	**1987**	**1988**	**1989**
Atlanta	256	304	330	312	312	312	330	335	348	392
Baltimore	281	304	333	337	329	339	355	377	395	478
Birmingham	289	309	320	332	343	343	338	355	368	422
Boston	265	2%	353	378	396	413	433	462	490	542
Chicago	289	314	349	355	358	361	381	401	417	442
Cincinnati	314	342	359	378	377	378	378	386	397	481
Cleveland	294	315	349	382	395	406	419	419	426	439
Dallas	302	352	386	409	385	364	357	353	335	434
Denver	281	324	366	406	406	359	373	373	347	386
Detroit	314	350	356	369	369	377	3%	412	433	441
Kansas City	307	340	372	394	397	404	410	427	436	498
Los Angeles	336	375	375	452	445	433	457	465	472	541
Minneapolis	276	314	351	388	372	397	407	417	426	461
New Orleans	297	325	350	376	376	311	376	3%	372	469
New York	250	274	303	334	361	381	408	427	456	470
Philadelphia	267	307	324	354	374	393	418	431	454	559
Pittsburgh	282	304	324	342	370	370	372	373	383	428
St Louis	262	297	306	318	350	362	378	382	400	448
San Francisco	307	330	381	400	407	411	442	455	464	508
Seattle	327	363	386	387	389	397	401	405	414	450

† Published in *Engineering* News Record monthly in the second issue of the month with summaries in the third issue of the March and December issues.

Example 6 Estimation of labor requirements. Consider a highly automated processing plant having a capacity of 100 tons/day of product and requiring principal processing steps of heat transfer, reaction, and distillation. What are the average operating labor requirements for an annual operation of 300 days?

Solution. The process plant is considered to require three process steps. From Fig. 6-8, for a capacity of 100 tons product/day, the highly automated process plant requires 33 employee-hours/day/processing step. Thus, for 300 days annual operation, operating labor required = (3)(33)(300) = 29,700 employee-hours/year.

Because of new technological developments including computerized controls and long-distance control arrangements, the practice of relating employee-hour requirements directly to production quantities for a given product can give inaccurate results unless very recent data are used. As a general rule of thumb,? the labor requirements for a fluids-processing plant, such as an ethylene oxide plant or others as shown in Table 22, would be in the low range of $\frac{1}{3}$ to 2 employee-hours per ton of product; for a solid-fluids plant, such as a polyethylene plant, the labor requirement would be in the intermediate range of 2 to 4 employee-hours per ton of product; for plants primarily engaged in solids processing such as a coal briquetting plant, the large amount of materials handling would make the labor requirements considerably higher than for other types of plants with a range of 4 to 8 employee-hours per ton of product being reasonable. The data shown in Fig. 6-8 and Table 22, where plant capacity and specific type of process are taken into account, are much more accurate than the preceding rule of thumb if the added necessary information is available.

In determining costs for labor, account must be taken of the type of worker required, the geographical location of the plant, the prevailing wage rates, and worker productivity. Table 20 presents data that can be used as a guide for relative median labor rates and productivity factors for workers in various geographical areas of the United States. Tables 23 and 24 provide data on labor rates in dollars per hour for the U.S. Gulf Coast region and average labor indexes to permit estimation of prevailing wage rates.

Direct Supervisory and Clerical Labor

A certain amount of direct supervisory and clerical labor is always required for a manufacturing operation. The necessary amount of this type of labor is closely related to the total amount of operating labor, complexity of the operation, and product quality standards. The cost for direct supervisory and clerical labor averages about 15 percent of the cost for operating labor. For reduced capacities, supervision usually remains fixed at the MO-percent-capacity rate.

Utilities

The cost for utilities, such as steam, electricity, process and cooling water, compressed air, natural gas, and fuel oil, varies widely depending on the amount of consumption, plant location, and source. For example, costs for a few selected utilities in the U.S. Gulf Coast region are given in Table 23. A more detailed list of average rates for various utilities is presented in Appendix B. The required utilities can sometimes be estimated in preliminary cost analyses from available information about similar operations as shown in Table 22. If such information is unavailable, the utilities must be estimated from a preliminary design. The utility may be purchased at predetermined rates from an outside source, or the service may be available from within the company. If the company supplied its own service and this is utilized for just one process, the entire cost of the service installation is usually charged to the manufacturing process. If the service is utilized for the production of several different products,

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†J. E. Haselbarth, Updated Investment Costs for 60 Chemical Plants, *Chem. Eng.*, 74(25):214 (Dec. 4, 1967).

the service cost is apportioned among the different products at a rate based on the amount of individual consumption.

Steam requirements include the amount consumed in the manufacturing process plus that necessary for auxiliary needs. An allowance for radiation and line losses must also be made.

Electrical power must be supplied for lighting, motors, and various process-equipment demands. These direct-power requirements should be increased by a factor of 1.1 to 1.25 to allow for line losses and contingencies. As a rough approximation, utility costs for ordinary chemical processes amount to 10 to 20 percent of the total product cost.

Maintenance and Repairs

A considerable amount of expense is necessary for maintenance and repairs if a plant is to be kept in efficient operating condition. These expenses include the cost for labor, materials, and supervision.

Annual costs for equipment maintenance and repairs may range from as low as 2 percent of the equipment cost if service demands are light to 20 percent for cases in which there are severe operating demands. Charges of this type for buildings average 3 to 4 percent of the building cost. In the process industries, the total plant cost per year for maintenance and repairs is roughly equal to an average of 6 percent of the fixed-capital investment. Table 25 provides a guide for estimation of maintenance and repair costs as a function of process conditions.

For operating rates less than plant capacity, the maintenance and repair cost is generally estimated as 85 percent of that at 100 percent capacity for a 75 percent operating rate, and 75 percent of that at 100 percent capacity for a 50 percent operating rate.

TABLE 25
Estimation of costs for maintenance and repairs

	Maintenance cost as percentage of fixed-capital investment (on annual basis)		
Type of operation	Wages	Materials	Total
Simple chemical processes	1-3	1-3	2-6
Average processes with normal operating conditions	2-4	3-5	5-9
Complicated processes, severe corrosion operating conditions, or extensive instrumentation	3-5	4-6	7-11

Operating Supplies

In any manufacturing operation, many miscellaneous supplies are needed to keep the process functioning efficiently. Items such as charts, lubricants, test chemicals, custodial supplies, and similar supplies cannot be considered as raw materials or maintenance and repair materials, and are classified as operating supplies. The annual cost for this type of supplies is about 15 percent of the total cost for maintenance and repairs.

Laboratory Charges

The cost of laboratory tests for control of operations and for product-quality control is covered in this manufacturing cost. This expense is generally calculated by estimating the employee-hours involved and multiplying this by the appropriate rate. For quick estimates, this cost may be taken as 10 to 20 percent of the operating labor.

Patents and Royalties

Many manufacturing processes are covered by patents, and it may be necessary to pay a set amount for patent rights or a royalty based on the amount of material produced. Even though the company involved in the operation obtained the original patent, a certain amount of the total expense involved in the development and procurement of the patent rights should be borne by the plant as an operating expense. In cases of this type, these costs are usually amortized over the legally protected life of the patent. Although a rough approximation of patent and royalty costs for patented processes is 0 to 6 percent of the total product cost, the engineer must use judgement because royalties vary with such factors as the type of product and the industry.

Catalysts and Solvents

Costs for catalysts and solvents can be significant and depend upon the specific manufacturing processes chosen.

FIXEDCHARGES

Certain expenses are always present in an industrial plant whether or not the manufacturing process is in operation. Costs that are invariant with the amount of production are designated as *fixed* costs or *fixed charges*. These include costs for depreciation, local property taxes, insurance, and rent. Expenses of this type are a direct function of the capital investment. As a rough approximation, these charges amount to about 10 to 20 percent of the total product cost.

Depreciation

Equipment, buildings, and other material objects comprising a manufacturing plant require an initial investment which must be written off as a manufacturing expense. In order to write off this cost, a decrease in value is assumed to occur throughout the usual life of the material possessions. This decrease in value is designated as ***depreciation.***

Since depreciation rates are very important in determining the amount of income tax, the Internal Revenue Service has established allowable depreciation rates based on the probable useful life of various types of equipment and other fixed items involved in manufacturing operations. While several alternative methods may be used for determining the rate of depreciation, a straight-line method is usually assumed for engineering projects. In applying this method, a useful-life period and a salvage value at the end of the useful life are assumed, with due consideration being given to possibilities of obsolescence and economic changes. The difference between initial cost and the salvage value divided by the total years of useful life gives the annual cost due to depreciation.

The annual depreciation rate for machinery and equipment ordinarily is about 10 percent of the tied-capital investment, while buildings are usually depreciated at an annual rate of about 3 percent of the initial cost.

Local Taxes

The magnitude of local property taxes depends on the particular locality of the plant and the regional laws. Annual property taxes for plants in highly populated areas are ordinarily in the range of 2 to 4 percent of the fixed-capital investment. In less populated areas, local property taxes are about 1 to 2 percent of the tied-capital investment.

Insurance

Insurance rates depend on the type of process being carried out in the manufacturing operation and on the extent of available protection facilities. On an annual basis, these rates amount to about 1 percent of the fixed-capital investment.

Rent

Annual costs for rented land and buildings amount to about 8 to 12 percent of the value of the rented property.

PLANT OVERHEAD COSTS

The costs considered in the preceding sections are directly related with the production operation. In addition, however, many other expenses are always involved if the complete plant is to function as an efficient unit. The expenditures required for routine plant services are included in ***plant-overhead*** costs. Nonmanufacturing machinery, equipment, and buildings are necessary for many of the general plant services, and the fixed charges and direct costs for these items are part of the plant-overhead costs.

Expenses connected with the following comprise the bulk of the charges for plant overhead:

Hospital and medical services
General engineering
Safety services
Cafeteria and recreation facilities
General plant maintenance and overhead
Payroll overhead including employee benefits
Control laboratories
Packaging
Plant protection
Janitor and similar services
Employment offices
Distribution of utilities
Shops
Lighting
Interplant communications and transportation
Warehouses
Shipping and receiving facilities

These charges are closely related to the costs for all labor ‘directly connected with the production operation. The plant-overhead cost for chemical plants is about 50 to 70 percent of the total expense for operating labor, supervision, and maintenance.

ADMINISTRATIVE COSTS

The expenses connected with top-management or administrative activities cannot be charged directly to manufacturing costs; however, it is necessary to include the ***administrative costs*** if the economic analysis is to be complete. Salaries and wages for administrators, secretaries, accountants, stenographers, typists, and similar workers are part of the administrative expenses, along with costs for office supplies and equipment, outside communications, administrative buildings, and other overhead items related with administrative activities. These costs may vary markedly from plant to plant and depend somewhat on whether the plant under consideration is a new one or an addition to an old plant. In the absence of more-accurate cost figures from company records, or for a quick estimate, the administrative costs may be approximated as 20 to 30 percent of the operating labor.

DISTRIBUTION AND MARKETING COSTS

From a practical viewpoint, no manufacturing operation can be considered a success until the products have been sold or put to some profitable use. It is necessary, therefore, to consider the expenses involved in selling the products. Included in this category are salaries, wages, supplies, and other expenses for sales offices; salaries, commissions, and traveling expenses for salesmen; shipping expenses; cost of containers; advertising expenses; and technical sales service.

Distribution and marketing costs vary widely for different types of plants depending on the particular material being produced, other products sold by the company, plant location, and company policies. These costs for most chemical plants are in the range of 2 to 20 percent of the total product cost. The higher figure usually applies to a new product or to one sold in small quantities to a large number of customers. The lower figure applies to large-volume products, such as bulk chemicals.

RESEARCH AND DEVELOPMENT COSTS

New methods and products are constantly being developed in the chemical industries. These accomplishments are brought about by emphasis on research and development. ***Research and development costs*** include salaries and wages for all personnel directly connected with this type of work, fixed and operating expenses for all machinery and equipment involved, costs for materials and supplies, direct overhead expenses, and miscellaneous costs. In the chemical industry, these costs amount to about 2 to 5 percent of every sales dollar.

FINANCING

Interest

Interest is considered to be the compensation paid for the use of borrowed capital. A fixed rate of interest is established at the time the capital is borrowed; therefore, interest is a definite cost if it is necessary to borrow the capital used to make the investment for a plant. Although the interest on borrowed capital is a fixed charge, there are many persons who claim that interest should not be considered as a manufacturing cost. It is preferable to separate interest from the other fixed charges and list it as a separate expense under the general heading of management or financing cost. Annual interest rates amount to 5 to 10 percent of the total value of the borrowed capital.

When the capital investment is supplied directly from the existing funds of a company, it is a debatable point whether interest should be charged as a cost.

For income-tax calculations, interest on owned money cannot be charged as a cost. In design calculations, however, interest can be included as a cost unless there is assurance that the total capital investment will be supplied from the company's funds and the company policies permit exclusion of interest as a cost.

GROSS-EARNINGS COSTS

The total income minus the total production cost gives the *gross earnings* made by the particular production operation, which can then be treated mathematically by any of several methods to measure the profitably of the proposed venture or project. These methods will be discussed later in Chaps. 7 and 10.

Because of income-tax demands, the final ***net profit*** is often much less than the gross earnings. Income-tax rates are based on the gross earnings received from all the company interests. Consequently, the magnitude of these costs varies widely from one company to another.

On an annual basis, the corporate income-tax laws for the United States in 1979 required payment of a 17, 20, 30, and 40 percent normal tax on the lst, 2nd, 3rd, and 4th $25,000, respectively, of the annual gross earnings of a corporation plus 46 percent of all annual gross earnings above $100,000. In addition, if other levies, such as state income taxes, were included, the overall tax rate could have been even higher. By 1988, the corporate income-tax laws had been changed to 15 percent on the first $50,000 of annual gross earnings, 25 percent on annual gross earnings of $50,000 to $75,000, and 34 percent on annual gross earnings above $75,000 plus a special graduated-tax phase-out of 5 percent on the gross earnings from $100,000 to $335,000. Tax rates vary from year to year depending on Federal and state regulations as is shown in the following example where 1988 Federal-tax rates are considered.

Example 7 Break-even point, gross earnings, and net profit for a process plant. The annual direct production costs for a plant operating at 70 percent capacity are $280,000 while the sum of the annual fixed charges, overhead costs, and general expenses is $200,000. What is the break-even point in units of production per year if total annual sales are $560,000 and the product sells at $40 per unit? What were the annual gross earnings and net profit for this plant at 100 percent capacity in 1988 when corporate income taxes required a 15 percent tax on the first $50,000 of annual gross earnings, 25 percent on annual gross earnings of $50,000 to $75,000, 34 percent on annual gross earnings above $75,000, and 5 percent on gross earnings from $100,000 to $335,000?

Solution. The break-even point (Fig. 6-3) occurs when the total annual product cost equals the total annual sales. The total annual product cost is the sum of the fixed costs (including fixed charges, overhead, and general expenses) and the direct production costs for n units per year. The total annual sales is the product of the number of units and the selling price per unit. Thus

$$\text{Direct production cost/unit} = \frac{280{,}000}{(560{,}000/40)} = \$20/\text{unit}$$

and the number of units needed for a break-even point is given by

$$200{,}000 + 20n = 40n$$

$$n = \frac{200{,}000}{20} = 10{,}000 \text{ units/year}$$

This is $[(10{,}000)/(14{,}000/0.7)]100 = 50\%$ of the present plant operating capacity.

$$\begin{aligned}
\text{Gross annual earnings} &= \text{total annual sales} - \text{total annual product cost} \\
&= \frac{14{,}000}{0.7} \text{ units } (\text{MO/unit}) \\
&\quad - \left[200{,}000 + \frac{14{,}000}{0.7} \text{ units } (\$20/\text{unit})\right] \\
&= 800{,}000 - 600{,}000 \\
&= \$200{,}000 \\
\text{Net annual earnings} &= \text{gross annual earnings} - \text{income taxes} \\
&= 200{,}000 - [(0.15)(50{,}000) + (0.25)(25{,}000) \\
&\qquad + (0.34)(200{,}000 - 75{,}000) \\
&\qquad + (0.05)(200{,}000 - 100{,}000)] \\
&= 200{,}000 - 61{,}250 \\
&= \$138{,}750
\end{aligned}$$

CONTINGENCIES

Unforeseen events, such as strikes, storms, floods, price variations, and other contingencies, may have an effect on the costs for a manufacturing operation. When the chemical engineer predicts total costs, it is advisable to take these factors into account. This can be accomplished by including a contingency factor equivalent to 1 to 5 percent of the total product cost.

SUMMARY

This chapter has outlined the economic considerations which are necessary when a chemical engineer prepares estimates of capital investment cost or total product cost for a new venture or project. Methods for obtaining predesign cost estimates have purposely been emphasized because the latter are extremely important for determining the feasibility of a proposed investment and to compare alternative designs. It should be remembered, however, that predesign estimates are often based partially on approximate percentages or factors that are applicable to a particular plant or process under consideration. Tables 26 and 27 summarize the predesign estimates for capital investment costs and total product costs, respectively. The percentages indicated in both tables give the ranges encountered in typical chemical plants. Because of the wide variations in different types of plants, the factors presented should be used only when more accurate data are not available.

TABLE 26

Estimation of capital investment cost (showing individual components)

The percentages indicated in the following summary of the various costs constituting the capital investment are approximations applicable to ordinary chemical processing plants. It should be realized that the values given can vary depending on many factors, such as plant location, type of process, complexity of instrumentation, etc.

- I. Direct costs = material and labor involved in actual installation of complete facility (70–85% of fixed-capital investment)
  - A. Equipment + installation + instrumentation + piping + electrical + insulation + painting (50–60% of fixed-capital investment)
    - 1. Purchased equipment (15–40% of fixed-capital investment)
    - 2. Installation, including insulation and painting (25–55% of purchased-equipment cost)
    - 3. Instrumentation and controls, installed (6-30% of purchased-equipment cost)
    - 4. Piping, installed (10–80% of purchased-equipment cost)
    - 5. Electrical, installed (10–40% of purchased-equipment cost)
  - B. Buildings, process and auxiliary (10–70% of purchasedequipment cost)
  - C. Service facilities and yard improvements (40–100% of purchased-equipment cost)
  - D. Land (I-2% of fixed-capital investment or 4-8% of purchased-equipment cost)
- II. Indict costs = expenses which are not directly involved with material and labor of actual installation of complete facility (15–30% of fixed-capital investment)
  - A. Engineering and supervision (5–30% of direct costs)
  - B. Construction expense and contractor's fee (6-30% of direct costs)
  - C. Contingency (5–15% of fixed-capital investment)
- Ill. Fixed-capital investment = direct costs + indirect costs
- IV. Working capital (10–20% of total capital investment)
- V. Total capital investment = fixed-capital investment + working capital

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TABLE 27

Estimation of total product cost (showing individual components)

The percentages indicated in the following summary of the various costs involved in the complete operation of manufacturing plants are approximations applicable to ordinary chemical processing plants. It should be realized that the values given can vary depending on many factors, such as plant location, type of process, and company policies.

Percentages are expressed on an annual basis.

- I. Manufacturing cost = direct production costs + fixed charges + plant overhead costs
  - A. Direct production costs (about 60% of total product cost)
    - 1. Raw materials (10–50% of total product cost)
    - 2. Operating labor (10–20% of total product cost)
    - 3. Direct supervisory and clerical labor (10–25% of operating labor)
    - 4. *Utilities (10–20% of* total *product* cost)
    - 5. *Maintenance and repairs (2–10% of fixed-capital investment)*
    - 6. Operating supplies (10–20% of cost for maintenance and repairs, or 0.5–1% of fixed-capital investment)
    - 7. Laboratory charges (IO-20% of operating labor)
    - 8. Patents and royalties (0–6% of total product cost)
  - B. Fixed charges (10–20% of total product cost)
    - 1. Depreciation (depends on life period, salvage value, and method of calculation-about 10% of fixed-capital investment for machinery and equipment and 2–3% of building value for buildings)
    - 2. Local taxes (1–4% of fixed-capital investment)
    - 3. Insurance (0.4–1% of fixed-capital investment)
    - 4. Rent (8–12% of value of rented land and buildings)

TABLE 27

Estimation of total product cost (showing individual components) (Continued)

- C. Plant-overhead costs (SO-70% of cost for operating labor, supervision, and maintenance, or **5-15%** of total product cost); includes costs for the following: general plant upkeep and overhead, payroll overhead, packaging, medical services, safety and protection, restaurants, recreation, salvage, laboratories, and storage facilities.

II. General expenses = administrative costs + distribution and selling costs + research and development costs

- A. Administrative costs (about 15% of costs for operating labor, supervision, and maintenance, or **2-6%** of total product cost); includes costs for executive salaries, clerical wages, legal fees, office supplies, and communications
- *B.* Distribution and selling costs **(2-20%** of total product cost); includes costs for sales offices, salesmen, shipping, and advertising
- C. Research and development costs **(2-5%** of every sales **dollar** or about 5% of total product cost)
- *D.* Financing (interest)? **(0-10%** of total capital investment)

III. Total product **cost‡ =** manufacturing cost + general expenses

IV. **Gross-earnings cost** (gross earnings = total income total product cost; amount of **gross-**earnings cost depends on amount of gross earnings for entire company and income-tax regulations; a general range for gross-earnings cost is **30-40%** of gross earnings)

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† Interest on borrowed money is often considered as a fiied charge.

‡ If desired, a contingency factor can be included by increasing the total product cost by l-5%.

NOMENCLATURE FOR CHAPTER 6

A = incremental cost of corrosion-resistant alloy materials
A, = nonmanufacturing fixed-capital investment
c_o = costs for operations (*not* including depreciation)
C = original capital investment
C_n = new capital investment
d = depreciation charge
d_n = number of drawings and specifications
D = total direct cost of plant
e = total heat exchanger cost (less incremental cost of alloy)
e_L = labor efficiency index in new location relative to cost of E_L and M_L'
E = purchased-equipment cost (installation cost not included) on f.o.b. basis
E_i = installed-equipment cost (purchased and installation cost included)
E_L = purchased-equipment labor cost (base)
f = lumped cost index relative to original installation cost
f_1, f_2 = multiplying factors for piping, electrical, instrumentation, etc.
f_d = unit cost per drawing and specification
f_e = unit cost for engineering
f_E = current equipment cost index relative to cost of E
f_F = construction or field-labor expense factor always greater than 1
f, = indirect cost factor always greater than 1

f_L = current labor cost index in new location relative to cost of E_L and M_L'
f_M = current material cost index relative to cost of M
f_m = cost factor for miscellaneous items
f_p = cost factor for piping materials
f_v = total cost of field-fabricated vessels (less incremental cost of alloy)
f_x = specific material unit cost, e.g., f_p = unit cost of pipe
f_y = specific material labor unit cost per employee-hour
H_e = engineering employee-hours
I = total indirect cost of plant
M = material cost
M_L' = labor employee-hours for specific material
M_L = direct labor cost for equipment installation and material handling
M_x = specific material quantity in compatible units
p = total pump plus driver cost (less incremental cost of alloy)
R = ratio of new to original capacity
s_i = total income from sales
t = total cost of tower shells (less incremental cost of alloy)
T = total capital investment
V = manufacturing fixed-capital investment
W = working-capital investment
x = exponential power for cost-capacity relationships

PROBLEMS

1. The purchased cost of a shell-and-tube heat exchanger (floating head and carbon-steel tubes) with 100 ft^2 of heating surface was $3000 in 1980. What will be the purchased cost of a similar heat exchanger with 200 ft^2 of heating surface in 1980 if the purchased-cost-capacity exponent is 0.60 for surface area ranging from 100 to 400 ft^2? If the purchased-cost-capacity exponent for this type of exchanger is 0.81 for surface areas ranging from 400 to 2000 ft^2, what will be the purchased cost of a heat exchanger with 1000 ft^2 of heating surface in 1985?
2. Plot the 1985 purchased cost of the shell-and-tube heat exchanger outlined in the previous problem as a function of the surface area from 100 to 2000 ft^2. Note that the purchased-cost-capacity exponent is not constant over the range of surface area requested.
3. The purchased and installation costs of some pieces of equipment are given as a function of weight rather than capacity. An example of this is the installed costs of large tanks. The 1980 cost for an installed aluminum tank weighing 100,000 lb was $390,000. For a size range from 200,000 to 1,000,000 lb, the installed cost-weight exponent for aluminum tanks is 0.93. If an aluminum tank weighing 700,000 lb is required, what is the present capital investment needed?
4. What weight of installed stainless-steel tank could have been obtained for the same capital investment as in the previous problem? The 1980 cost for an installed 304 stainless-steel tank weighing 300,000 lb was $670,000. The installed cost-weight exponent for stainless tanks is 0.88 for a size range from 300,000 to 700,000 lb.
5. The purchased cost of a 1400-gal stainless-steel tank in 1980 was $7500. The tank is cylindrical with flat top and bottom, and the diameter is 6 ft. If the entire outer

surface of the tank is to be covered with 2 in. thickness of magnesia block, estimate the present total cost for the installed and insulated tank. The Jan. 1, 1980 cost for the 2-in. magnesia block was \$2.20 per ft^2 while the labor for installing the insulation was \$5.00 per ft^2.

6. A one-story warehouse 120 by 60 ft is to be added to an existing plant. An asphalt-pavement service area 60 by 30 ft will be added adjacent to the warehouse. It will also be necessary to put in 500 lin ft of railroad siding to service the warehouse. Utility service lines are already available at the warehouse site. The proposed warehouse has a concrete floor and steel frame, walls, and roof. No heat is necessary, but lighting and sprinklers must be installed. Estimate the total cost of the proposed addition. Consult App. B for necessary cost data.
7. The purchased cost of equipment for a solid-processing plant is \$500,000. The plant is to be constructed as an addition to an existing plant. Estimate the total capital investment and the tied-capital investment for the plant. What percentage and amount of the fixed-capital investment is due to wst for land and contractor's fee?
8. The purchased-equipment cost for a plant which produces pentaerythritol (solid-fuel-processing plant) is \$300,000. The plant is to be an addition to an existing formaldehyde plant. The major part of the building cost will be for indoor construction, and the contractor's fee will be 7 percent of the direct plant cost. All other costs are close to the average values found for typical chemical plants. On the basis of this information, estimate the following:
   (a) The total direct plant cost.
   (*b*) The fixed-capital investment.
   (*c*) The total capital investment.
9. Estimate by the turnover-ratio method the fixed-capital investment required for a proposed sulfuric acid plant (battery limit) which has a capacity of 140,000 tons of 100 percent sulfuric acid per year (contact-catalytic process) using the data from Table 19 for 1990 with sulfuric acid cost at \$72 per ton. The plant may be considered as operating full time. Repeat using the cost-capacity-exponent method with data from Table 19.
10. The total capital investment for a chemical plant is \$1 million, and the working capital is \$100,000. If the plant can produce an average of 8000 kg of final product per day during a 365-day year, what selling price in dollars per kilogram of product would be necessary to give a turnover ratio of 1.0?
11. A process plant was constructed in the Philadelphia area (Middle Atlantic) at a labor cost of \$200,000 in 1980. What would the average costs for the same plant to be in the Miami, Florida area (South Atlantic) if it were constructed in late 1988? Assume, for simplicity, that the relative labor rate and relative productivity factor remain essentially constant.
12. A company has been selling a soap containing 30 percent by weight water at a price of \$10 per 100 lb f.o.b. (i.e., freight on board, which means the laundry pays the freight charges). The company offers an equally effective soap containing only 5 percent water. The water content is of no importance to the laundry, and it is willing to accept the soap containing 5 percent water if the delivered costs are equivalent. If the freight rate is 70 cents per 100 lb, how much should the company charge the laundry per 100 lb f.o.b. for the soap containing 5 percent water?
13. The total capital investment for a conventional chemical plant is \$1,500,000, and the plant produces 3 million kg of product annually. The selling price of the product is

\$0.82/kg. Working capital amounts to 15 percent of the total capital investment. The investment is from company funds, and no interest is charged. Raw-materials costs for the product are \$0.09/kg, labor \$0.08/kg, utilities \$0.05/kg, and packaging \$0.008/kg. Distribution costs are 5 percent of the total product cost. Estimate the following:

(*a*) Manufacturing cost per kilogram of product.
(*b*) Total product cost per year.
(*c*) Profit per kilogram of product before taxes.
(*d*) Profit per kilogram of product after taxes (use current rate).

14. Estimate the manufacturing cost per 100 lb of product under the following conditions:

Fixed-capital investment = \$2 million
Annual production output = 10 million lb of product
Raw materials cost = \$0.12/lb of product
Utilities
  100 psig steam = 50 lb/lb of product
  Purchased electrical power = 0.4 kWh/lb of product
  Filtered and softened water = 10 gal/lb of product
Operating labor = 20 men per shift at \$12.00 per employee-hour
Plant operates three hundred 24-h days per year
Corrosive liquids are involved
Shipments are in bulk carload lots
A large amount of direct supervision is required
There are no patent, royalty, interest, or rent charges
Plant-overhead costs amount to 50 percent of the cost for operating labor, supervision, and maintenance

15. A company has direct production costs equal to 50 percent of total annual sales and fixed charges, overhead, and general expenses equal to \$200,000. If management proposes to increase present annual sales of \$800,000 by 30 percent with a 20 percent increase in fixed charges, overhead, and general expenses, what annual sales dollar is required to provide the same gross earnings as the present plant operation? What would be the net profit if the expanded plant were operated at full capacity with an income tax on gross earnings fixed at 34 percent? what would be the net profit for the enlarged plant if total annual sales remained the same as at present? What would be the net profit for the enlarged plant if the total annual sales actually decreased to \$700,000?

16. A process plant making 2000 tons per year of a product selling for \$0.80 per lb has annual direct production costs of \$2 million at 100 percent capacity and other fixed costs of \$700,000. What is the fixed cost per pound at the break-even point? If the selling price of the product is increased by 10 percent, what is the dollar increase in net profit at full capacity if the income tax rate is 34 percent of gross earnings?

17. A rough rule of thumb for the chemical industry is that \$1 of annual sales requires \$1 of fixed-capital investment. In a chemical processing plant where this rule applies, the total capital investment is \$2,500,000 and the working capital is 20 percent of the total capital investment. The annual total product cost amounts to \$1,500,000. If the national and regional income-tax rates on gross earnings total 36 percent, determine the following:

(a) Percent of total capital investment returned annually as gross earnings.
(*b*) Percent of total capital investment returned annually as net profit.

18. The total capital investment for a proposed chemical plant which will produce $1,500,000 worth of goods per year is estimated to be $1 million. It will be necessary to do a considerable amount of research and development work on the project before the final plant can be constructed, and management wishes to estimate the permissible research and development costs. It has been decided that the net profits from the plant should be sufficient to pay off the total capital investment plus all research and development costs in 7 years. A return after taxes of at least 12 percent of sales must be obtained, and 34 percent of the research and development cost is tax-free (i.e., income-tax rate for the company is 34 percent of the gross earnings). Under these conditions, what is the total amount the company can afford to pay for research and development?
19. A chemical processing unit has a capacity for producing 1 million kg of a product per year. After the unit has been put into operation, it is found that only 500,000 kg of the product can be disposed of per year. An analysis of the existing situation shows that all fixed and other invariant charges, which must be paid whether or not the unit is operating, amount to 35 percent of the total product cost when operating at full capacity. Raw-material costs and other production costs that are directly proportional to the quantity of production (i.e., constant per kilogram of product at any production rate) amount to 40 percent of the total product cost at full capacity. The remaining 25 percent of the total product cost is for variable overhead and miscellaneous expenses, and the analysis indicates that these costs are directly proportional to the production rate during operation raised to the 1.5 power. What will be the percent change in total cost per kilogram of product if the unit is switched from the 1-million-kg-per-year rate to a time and rate schedule which will produce 500,000 kg of product per year at the least total cost? All costs referred to above are on a per-kilogram basis.
20. Estimate the total operating cost per day for labor, power, steam, and water in a plant producing 100 tons of acetone per day from the data given in Table 22 and using utility costs from Table 23. Consider all water as treated city water. The steam pressure may be assumed to be 100 psig. Labor costs average $20 per employee-hour. Electricity must be purchased. Plant operates 365 days per year.

CHAPTER
7

INTEREST AND INVESTMENT COSTS

A considerable amount of confusion exists among engineers over the role of interest in determining costs for a manufacturing operation. The confusion is caused by the attempt to apply the classical economist's definition of interest. According to the classical definition, interest is the money returned to the owners of capital for use of their capital. This would mean that any profit obtained through the uses of capital could be considered as interest. Modern economists seldom adhere to the classical definition. Instead, they prefer to substitute the term ***return on capital*** or ***return on investment*** for the classical ***interest.***

Engineers define interest as the ***compensation paid for the use of borrowed capital.*** This definition permits distinction between profit and interest. The rate at which interest will be paid is usually fixed at the time the capital is borrowed, and a guarantee is made to return the capital at some set time in the future or on an agreed-upon pay-off schedule.

TYPES OF INTEREST

Simple Interest

In economic terminology, the amount of capital on which interest is paid is designated as the ***principal,*** and ***rate of interest*** is defined as the amount of interest earned by a unit of principal in a unit of time. The time unit is usually taken as one year. For example, if $100 were the compensation demanded for giving someone the use of \$1000 for a period of one year, the principal would be \$1000, and the rate of interest would be 100/1000 = 0.1 or 10 percent/year.

The simplest form of interest requires compensation payment at a constant interest rate based only on the original principal. Thus, if \$1000 were loaned for a total time of 4 years at a constant interest rate of 10 percent/year, the simple interest earned would be

$$\$1000 \times 0.1 \times 4 = \$400$$

If P represents the principal, n the number of time units or interest periods, and i the interest rate based on the length of one interest period, the amount of simple interest Z during n interest periods is

$$Z = Pin \tag{1}$$

The principal must be repaid eventually; therefore, the entire amount S of principal plus simple interest due after n interest periods is

$$S = P + Z = P(1 + in) \tag{2}$$

Ordinary and Exact Simple Interest

The time unit used to determine the number of interest periods is usually 1 year, and the interest rate is expressed on a yearly basis. When an interest period of less than 1 year is involved, the *ordinary* way to determine simple interest is to assume the year consists of twelve 30-day months, or 360 days. The *exact* method accounts for the fact that there are 365 days in a normal year. Thus, if the interest rate is expressed on the regular yearly basis and d represents the number of days in an interest period, the following relationships apply:

$$\text{Ordinary simple interest} = Pi\frac{d}{360} \tag{3}$$

$$\text{Exact simple interest} = Pi\frac{d}{365} \tag{4}$$

Ordinary interest is commonly accepted in business practices unless there is a particular reason to use the exact value.

Compound Interest

In the payment of simple interest, it makes no difference whether the interest is paid at the end of each time unit or after any number of time units. The same total amount of money is paid during a given length of time, no matter which method is used. Under these conditions, there is no incentive to pay the interest until the end of the total loan period.

Interest, like all negotiable capital, has a time value. If the interest were paid at the end of each time unit, the receiver could put this money to use for earning additional returns. Compound *interest* takes this factor into account by stipulating that interest is due regularly at the end of each interest period. If payment is not made, the amount due is added to the principal, and interest is charged on this converted principal during the following time unit. Thus, an initial loan of $1000 at an annual interest rate of 10 percent would require payment of $100 as interest at the end of the first year. If this payment were not made, the interest for the second year would be ($1000 + $100)(0.10) = $110, and the total ***compound amount*** due after 2 years would be

$$\$1000 + \$100 + \$110 = \$1210$$

The compound amount due after any discrete number of interest periods can be determined as follows:

Period	Principal at start of period	Interest earned during period (i = interest rate based on length of one period)	Compound amount S at end of period
1	P	Pi	$P + Pi = P(1 + i)$
2	$P(1 + i)$	$P(1 + i)(i)$	$P(1 + i) + P(1 + i)(i) = P(1 + i)^2$
3	$P(1 + i)^2$	$P(1 + i)^2(i)$	$P(1 + i)^2 + P(1 + i)^2(i) = P(1 + i)^3$
n	$P(1 + i)^{n-1}$	$P(1 + i)^{n-1}(i)$	$P(1 + i)^n$

Therefore, the total amount of principal plus compounded interest due after n interest periods and designated as S is†

$$S = P(1 + i)^n \tag{5}$$

The term $(1 + i)^n$ is commonly referred to as the ***discrete single-payment compound-amount factor.*** Values for this factor at various interest rates and numbers of interest periods are given in Table 1.

Figure 7-1 shows a comparison among the total amounts due at different times for the cases where simple interest, discrete compound interest, and continuous interest are used.

NOMINAL AND EFFECTIVE INTEREST RATES

In common industrial practice, the length of the discrete interest period is assumed to be 1 year and the fixed interest rate i is based on 1 year. However, there are cases where other time units are employed. Even though the actual interest period is not 1 year, the interest rate is often expressed on an annual basis. Consider an example in which the interest rate is 3 percent per period

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†For the analogous equation for continuous interest compounding, see Eq. (12).

TABLE 1

Discrete compound-interest factor $(1 + i)^n$ at various values of i and n†

Number of interest periods, n	Value of $(1 + i)^n$ at indicated percent interest													
	1%	2%	3%	4 %	5%	6 %	7 %	8%	10%	12%	14%	16%	18%	20%
1	1.0100	1.0200	1.0300	1.0400	1.0500	1.0600	1.0709	1.0800	1.100	1.1209	1.1409	1.160	1.180	1.2000
2	1.0201	1.0404	1.0609	1.0816	1.1025	1.1236	1.1449	1.1664	1.210	1.2544	1.2996	1.345	1.392’	1.4400
3	1.0363	1.0612	1.0927	1.1249	1.1576	1.1910	1.2250	1.2597	1.331	1.4040	1.4815	1.560	1.643	1.7280
4	1.0406	1.0824	1.1256	1.1699	1.2155	1.2625	1.3108	1.3605	1.4641	1.5735	1.6890	1.810	1.938	2.0736
5	1.0510	1.1041	1.1593	1.2167	1.2763	1.3382	1.4026	1.4693	1.6105	1.7623	1.9264	2.100:	2.2871	2.4883
6	1.0615	1.1262	1.1041	1.2653	1.3401	1.4185	1.5007	1.5869	1.7716	1.9738	2.1950	2.4364	2.699	2.9860
7	1.0721	1.1487	1.2299	1.3159	1.4071	1.5036	1.6058	1.7138	1.9487	2.2107	2.5023	2.826:	3.185	3.6832
8	1.0829	1.1717	1.2668	1.3686	1.4775	1.6838	1.7182	1.8509	2.1438	2.4766	2.8592	3.278’	3.758	4.2998
9	1.0937	1.1961	1.3048	1.4233	1.5513	1.6895	1.8386	1.9990	2.3579	2.7731	3.2620	3.803	4.4351	5.1598
10	1.1046	1.2190	1.3439	1.4802	1.6289	1.7908	1.9672	2.1589	2.5931	3.1058	3.7072	4.4114	5.2331	6.1917
11	1.1157	1.2434	1.3842	1.6395	1.7103	1.8983	2.1049	2.3316	2.8531	3.4785	4.2262	5.117	6.1751	7.4301
12	1.1268	1.2662	1.4258	1.6010	1.7959	2.0122	2.2522	2.5182	3.1384	3.8960	4.8179	5.936	7.287	8.9161
13	1.1381	1.2936	1.4685	1.6651	1.8856	2.1329	2.4098	2.7196	3.4623	4.3635	6.4924	6.885	8.599’	10.699
14	1.1495	1.3195	1.5126	1.7317	1.9799	2.2609	2.5785	2.9372	3.7975	4.8871	6.2614	7.9875	10.147	12.839
15	1.1610	1.3459	1.5580	1.8009	2.0789	2.3966	2.7590	3.1722	4.1771	5.4736	7.1380	9.265	11.974	15.407
16	1.1726	1.3728	1.6047	1.8730	2.1829	2.5404	2.9522	3.4259	4.595	6.1304	8.1373	10.748	14.129	18.488
17	1.1843	1.4002	1.6528	1.9479	2.2920	2.6928	3.1588	3.7000	5.0546	6.8660	9.2765	12.468	16.672	22.186
18	1.1961	1.4282	1.7024	2.0258	2.4066	2.8543	3.3799	3.9960	5.5599	7.6900	10.575	14.462	19.673	26.623
19	1.2081	1.4568	1.7535	2.1068	2.5270	3.0256	3.6166	4.3157	6.115	8.6128	12.056	16.777	23.214	31.948
20	1.2202	1.4859	1.8061	2.1911	2.6533	3.2071	3.8697	4.6610	6.727	9.6463	13.744	19.461	27.393	38.338
21	1.2324	1.5157	1.8603	2.2788	2.7800	3.3996	4.1406	5.0338	7.400	10.304	15.668	22.574	32.324	46.005
22	1.2447	1.5460	1.9161	2.3699	2.9263	3.6035	4.4304	5.4365	8.1402	12.100	17.861	26.186	38.142	55.206
23	1.2522	1.5769	1.9736	2.4647	3.0715	3.8197	4.7408	5.8715	8.954	13.552	20.362	30.376	45.008	66.247
24	1.2697	1.6084	2.0328	2.5633	3.2251	4.0489	5.0724	6.3412	9.849	15.179	23.212	35.236	53.109	79.497
25	1.2824	1.6406	2.0938	2.6658	3.3864	4.2919	5.4274	6.8485	10.835	17.000	26.462	40.874	62.669	95.396
30	1.3478	1.8114	2.4273	3.2434	4.3219	5.7435	7.6123	10.063	17.449	29.960	50.950	87.044	143.37	237.38
35	1.4166	1.9999	2.8139	3.9461	6.5160	7.6861	10.677	14.785	28.102	52.800	98.101	180.31	328.00	590.67
40	1.4889	2.2080	3.2620	4.8010	7.0400	10.286	14.974	21.725	45.259	93.051	188.88	378.72	750.38	469.8
46	1.5648	2.4379	3.7816	5.8412	8.9850	13.765	21.002	31.920	72.891	163.99	163.68	795.44	1716.7	657.2
so	1.6446	2.6916	4.3839	7.1067	11.467	18.420	29.457	46.902	17.39	289.09	100.24	670.7	3927.3	1100.4

† Percent interest = (i)(100).

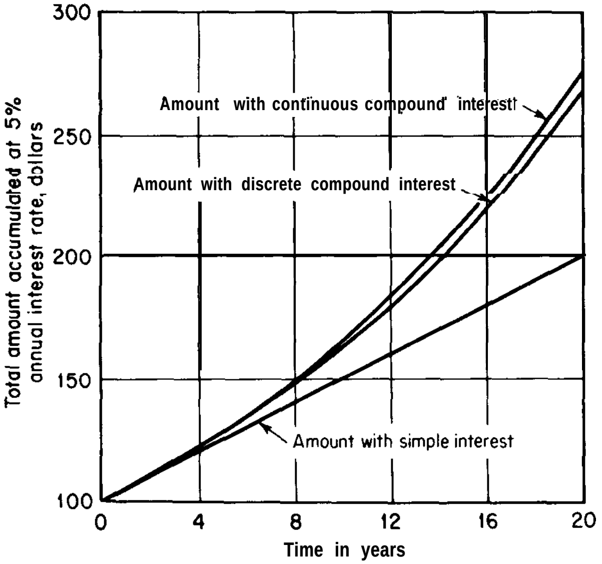

FIGURE 7-1
Comparison among total amounts accumulated with simple interest, discrete compound interest, and continuous compound nominal interest.

and the interest is compounded at half-year periods. A rate of this type would be referred to as "6 percent compounded semiannually." Interest rates stated in this form are known as nominal interest *rates*. The actual annual return on the principal would not be exactly 6 percent but would be somewhat larger because of the compounding effect at the end of the semiannual period.

It is desirable to express the exact interest rate based on the original principal and the convenient time unit of 1 year. A rate of this type is known as the *effective interest rate.* In common engineering practice, it is usually preferable to deal with effective interest rates rather than with nominal interest rates. The only time that nominal and effective interest rates are equal is when the interest is compounded annually.

Nominal interest rates should always include a qualifying statement indicating the compounding period. For example, using the common annual basis, $100 invested at a nominal interest rate of 20 percent compounded annually would amount to $120.00 after 1 year; if compounded semiannually, the amount would be $121.00; and, if compounded continuously, the amount would be $122.14. The corresponding effective interest rates are 20.00 percent, 21.00 percent, and 22.14 percent, respectively.

If nominal interest rates are quoted, it is possible to determine the effective interest rate by proceeding from Eq. (5).

$$S = P(1 + i)^n \tag{5}$$

In this equation, S represents the total amount of principal plus interest due after n periods at the periodic interest rate i. Let r be the nominal interest rate under conditions where there are m conversions or interest periods per year.

Then the interest rate based on the length of one interest period is r/m, and the amount S after 1 year is

$$S_{\text{after 1 year}} = P\left(1 + \frac{r}{m}\right)^m \tag{6}$$

Designating the effective interest rate as i_{eff}, the amount S after 1 year can be expressed in an alternate form as

$$S_{\text{after 1 year}} = P(1 + i_{\text{eff}}) \tag{7}$$

By equating Eqs. (6) and (7), the following equation can be obtained for the effective interest rate in terms of the nominal interest rate and the number of periods per year:

$$\text{Effective annual interest rate} = i_{\text{eff}} = \left(1 + \frac{r}{m}\right)^m - 1 \tag{8}$$

Similarly, by definition,

$$\textbf{Nominal annual interest rate} = m\left(\frac{r}{m}\right) = r \tag{9}$$

Example 1 Applications of different types of interest. It is desired to borrow \$1000 to meet a financial obligation. This money can be borrowed from a loan agency at a monthly interest rate of 2 percent. Determine the following:

(a) The total amount of principal plus simple interest due after 2 years if no intermediate payments are made.

(*b*) The total amount of principal plus compounded interest due after 2 years if no intermediate payments are made.

(*c*) The nominal interest rate when the interest is compounded monthly.

(*d*) The effective interest rate when the interest is compounded monthly.

Solution

(a) Length of one interest period = 1 month

Number of interest periods in 2 years = 24

For simple interest, the total amount due after n periods at periodic interest rate i is

$$S = P(1 + in) \tag{2}$$

P = initial principal = \$1000
i = 0.02 on a monthly basis
n = 24 interest periods in 2 years

$$S = \$1000(1 + 0.02 \times 24) = \$1480$$

(*b*) For compound interest, the total amount due after n periods at periodic interest rate i is

$$S = P(1 + i)^n \tag{5}$$

$$S = \$1000(1 + 0.02)^{24} = \$1608$$

(*c*) Nominal interest rate = 2 x 12 = 24% per year compounded monthly

(*d*) Number of interest periods per year = $m = 12$
Nominal interest rate = $r = 0.24$

$$\text{Effective interest rate} = \left(1 + \frac{r}{m}\right)^m - 1 \tag{8}$$

$$\text{Effective interest rate} = \left(1 + \frac{0.24}{12}\right)^{12} - 1 = 0.268 = 26.8\%$$

CONTINUOUS INTEREST

The preceding discussion of types of interest has considered only the common form of interest in which the payments are charged at periodic and discrete intervals, where the intervals represent a finite length of time with interest accumulating in a discrete amount at the end of each interest period. Although in practice the basic time interval for interest accumulation is usually taken as one year, shorter time periods can be used as, for example, one month, one day, one hour, or one second. The extreme case, of course, is when the time interval becomes infinitesimally small so that the *interest is compounded continuously.*

The concept of continuous interest is that the cost or income due to interest flows regularly, and this is just as reasonable an assumption for most cases as the concept of interest accumulating only at discrete intervals. The reason why continuous interest has not been used widely is that most industrial and financial practices are based on methods which executives and the public are used to and can understand. Because normal interest comprehension is based on the discrete-interval approach, little attention has been paid to the concept of continuous interest even though this may represent a more realistic and idealized situation.

The Basic Equations for Continuous Interest Compounding

Equations (6), (7), and (8) represent the basic expressions from which continuous-interest relationships can be developed. The symbol r represents the nominal interest rate with m interest periods per year. If the interest is compounded continuously, m approaches infinity, and Eq. (6) can be written as

$$S_{\text{after } n \text{ years}} = P \lim_{m \to \infty} \left(1 + \frac{r}{m}\right)^{mn} = P \lim_{m \to \infty} \left(1 + \frac{r}{m}\right)^{(m/r)(rn)} \tag{10}$$

The fundamental definition for the base of the natural system of logarithms ($e = 2.71828$) is[†]

$$\lim_{m \to \infty} \left(1 + \frac{r}{m}\right)^{m/r} = e = 2.71828\ldots \tag{11}$$

[†] See any book on advanced calculus. For example, W. Fulks, "Advanced Calculus," 3d ed., pp. 55-56, John Wiley & Sons, Inc., New York, 1978.

Thus, with continuous interest compounding at a nominal annual interest rate of r, the *amount* S an *initial principal* P will compound to in n years is†‡

$$S = Pe’” \tag{12}$$

Similarly, from Eq. (8), the effective annual interest rate i_{eff}, which is the conventional interest rate that most executives comprehend, is expressed in terms of the nominal interest rate r compounded continuously as

$$i_{\text{eff}} = e^r - 1 \tag{13}$$

$$r = \ln(i_{\text{eff}} + 1) \tag{14}$$

Therefore,

$$e^{rn} = (1 + i_{\text{eff}})^n \tag{15}$$

and

$$S = Pe’” = P(1 + i_{\text{eff}})^n \tag{16}$$

As is illustrated in the following example, a conventional interest rate (i.e., effective annual interest rate) of 22.14 percent is equivalent to a 20.00 percent nominal interest rate compounded continuously. Note, also, that a nominal interest rate compounded daily gives results very close to those obtained with continuous compounding.

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†The same result can be obtained from calculus by noting that, for the case of continuous compounding, the differential change of S with time must equal the nominal continuous interest rate times S, or $dS/dn = rS$. This expression can be integrated as follows to give Eq. (12):

$$\int_P^S \frac{dS}{S} = r\int_0^n dn$$

$$\ln\frac{S}{P} = rn \quad \text{or} \quad S = Pe^{rn} \tag{12}$$

‡A generalized way to express both Eq. (12) and Eq. (5), with direct relationship to the other interest equations in this chapter, is as follows:

Future worth = present worth x compound interest factor

$$S = PC$$

or

Future worth x discount factor = present worth

$$SF = P$$

$$\text{Discount factor} = F = \frac{1}{\text{compound interest factor}} = \frac{1}{C}$$

Although the various factors for different forms of interest expressions are derived in terms of interest rate in this chapter, the overall concept of interest evaluations is simplified by the use of the less-complicated nomenclature where designated factors are applied. Thus, expressing both Eqs. (12) and (5) as $SF = P$ would mean that F is e^{-rn} for the continuous interest case of Eq. (12) and $(1 + i)^{-n}$ for the discrete interest case of Eq. (5). See Table 4 for further information on this subject.

Example 2 Calculations with continuous interest compounding. For the case of a nominal annual interest rate of 20.00 percent, determine:

(*a*) The total amount to which one dollar of initial principal would accumulate after one 365-day year with daily compounding.

(*b*) The total amount to which one dollar of initial principal would accumulate after one year with continuous compounding.

(*c*) The effective annual interest rate if compounding is continuous.

Solution

(*a*) Using Eq. (6). $P = \$1.0$, $r = 0.20$, $m = 365$,

$$S_{\text{after 1 year}} = P\left(1 + \frac{r}{m}\right)^m = (1.0)\left(1 + \frac{0.20}{365}\right)^{365} = \$1.2213$$

(*b*) Using Eq. (12),

$$S = Pe^{rn} = (1\ .0)(e)^{(0.20)(1)} = \$1.2214$$

(*c*) Using Eq. (13),

$$i_{\text{eff}} = e^r \quad 1 = 1.2214 \quad 1 = 0.2214 \text{ or } 22.14\%$$

Tabulated values of i_{eff} and the corresponding r with continuous interest compounding are shown in Table 2.

TABLE 2
Effective annual interest rates compared to equivalent nominal interest rates with continuous interest

Effective annual rate of return, %	Nominal continuous rate of return, %	Effective annual rate of return, %	Nominal continuous rate of return, %
1	0.99504	35	30.010
2	1.9803	40	33.647
3	2.9559	45	37.156
4	3.9221	50	40.547
5	4.8790	60	47.000
6	5 8269	70	53.063
7	6.7659	80	58.779
8	7.6961	90	64.185
9	8.6178	100	69.315
10	9.5310	110	74.194
15	13.976	120	78.846
20	18.232	130	83.291
25	22.314	140	87.547
30	26.236	150	91.629

Example 3 Use of digital computer to give tabulated values of amount accumulated with continuous interest compounding. Present the digital computer program and the tabulated printout to six significant figures giving the amount to which an initial principal of $100 will accumulate year by year from 1 to 20 years with continuous interest compounding based on a nominal interest rate of 20 percent.

Solution. The equation to be solved on the digital computer is

$$S = Pe^{rn}$$

where S will be evaluated to six significant figures for

$$n = 1, 2, 3, \ldots, 20$$
$$P = \$100$$
$$r = 0.20$$

The Fortran IV program and the computer print-out follow:

```
$IBJOB   MAP
$IBFTC DECK 1
      DO 1 N = 1,20
      AN=N
        S = 100.*EXP(.20*AN)
  1   WRITE(6,2)N,S
  2   FORMAT(I4,F12.3)
      END
$ENTRY
```

Printout

```
   1     122.140     11      902.501
   2     149.182     12     1102.318
   3     182.212     13     1346.374
   4     222.554     14     1644.465
   5     271.828     15     2008.554
   6     332.012     16     2453.253
   7     405.520     17     2996.410
   8     495.303     18     3659.823
   9     604.965     19     4470.118
  10     738.906     20     5459.815
```

(NOTE: *The preceding illustrates the method used to prepare tabulated results of factors and emphasizes the simplicity of the procedure with a digital computer. This set of results represents a standard exponential function available in tabulated form in standard mathematical tables. See Prob. 8 at the end of this chapter for a requested computer solution for a more complicated continuous-interest case.)*

PRESENT WORTH AND DISCOUNT

It is often necessary to determine the amount of money which must be available at the present time in order to have a certain amount accumulated at some definite time in the future. Because the element of time is involved, interest must be taken into consideration. ***The present worth*** (or ***present value)*** of a future amount is the present principal which must be deposited at a given interest rate to yield the desired amount at some future date.?

In Eq. (5), S represents the amount available after n interest periods if the initial principal is P and the discrete compound-interest rate is i. Therefore, the present worth can be determined by merely rearranging Eq. (5).

$$\text{Present worth} = P = S\frac{1}{(1+i)^n} \tag{17}$$

The factor $1/(1+i)^n$ is commonly referred to as the *discrete single-payment present-worth factor.*

Similarly, for the case of continuous interest compounding, Eq. (12) gives

$$\text{Present worth} = P = S\frac{1}{e^{rn}} \tag{18}$$

Some types of capital are in the form of bonds having an indicated value at a future date. In business terminology, the difference between the indicated future value and the present worth (or present value) is known as the *discount.*

Example 4 Determination of present worth and discount. A bond has a maturity value of \$1000 and is paying discrete compound interest at an effective annual rate of 3 percent. Determine the following at a time four years before the bond reaches maturity value:

(a) Present worth.
(*b*) Discount.
(*c*) Discrete compound rate of effective interest which will be received by a purchaser if the bond were obtained for \$700.
(*d*) Repeat part (*a*) for the case where the nominal bond interest is 3 percent compounded continuously.

solution

(*a*) By Eq. (17), present worth = $S/(1+i)^n = \$1000/(1+0.03)^4$ = \$888
(*b*) Discount = future value − present worth = \$1000 \$888 = \$112
(*c*) Principal = \$700 = $S/(1+i)^n = \$1000/(1+i)^4$

$$i = \left(\frac{1000}{700}\right)^{1/4} - 1 = 0.0935 \text{ or } 9.35\%$$

(*d*) By Eq. (18), present worth = $S/e^{rn} = \$1000/e^{(0.03)(4)}$ = \$869

ANNUITIES

An *annuity* is a series of equal payments occurring at equal time intervals. Payments of this type can be used to pay off a debt, accumulate a desired

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†In the analyses presented in this chapter, effects of inflation or deflation on future worth are not considered. See Chap. 11 (Optimum Design and Design Strategy) for information on the strategy for dealing with inflation or deflation in design economic evaluations.

amount of capital, or receive a lump sum of capital that is due in periodic installments as in some life-insurance plans. Engineers often encounter annuities in depreciation calculations, where the decrease in value of equipment with time is accounted for by an annuity plan.

The common type of annuity involves payments which occur ***at the end of each interest period.*** This is ***known*** as an ***ordinary annuity.*** Interest is paid on all accumulated amounts, and the interest is compounded each payment period. ***An annuity term*** is the time from the beginning of the first payment period to the end of the last payment period. The ***amount of an annuity*** is the sum of all the payments plus interest if allowed to accumulate at a definite rate of interest from the time of initial payment to the end of the annuity term.

Relation between Amount of Ordinary Annuity and the Periodic Payments

Let R represent the uniform periodic payment made during n discrete periods in an ordinary annuity. The interest rate based on the payment period is i, and S is the amount of the annuity. The first payment of R is made at the end of the first period and will bear interest for $n - 1$ periods. Thus, at the end of the annuity term, this first payment will have accumulated to an amount of $R(1 + i)^{n-1}$. The second payment of R is made at the end of the second period and will bear interest for $n - 2$ periods giving an accumulated amount of $R(1 + i)^{n-2}$. Similarly, each periodic payment will give an additional accumulated amount until the last payment of R is made at the end of the annuity term.

By definition, the amount of the annuity is the sum of all the accumulated amounts from each payment; therefore,

$$S = R(1 + i)^{n-1} + R(1 + i)^{n-2} + R(1 + i)^{n-3} + \cdots + R(1 + i) + R \tag{19}$$

To simplify Eq. (19), multiply each side by $(1 + i)$ and subtract Eq. (19) from the result. This gives

$$Si = R(1 + i)^n - R \tag{20}$$

or

$$S = R\frac{(1 + i)^n - 1}{i} \tag{21}$$

The term $[(1 + i)^n - 1]/i$ is commonly designated as the ***discrete uniform-series compound-amount factor*** or the ***series compound-amount factor.***

Continuous Cash Flow and Interest Compounding

The expression for the case of continuous cash flow and interest compounding, equivalent to Eq. (21) for discrete cash flow and interest compounding, is developed as follows:

As before, let r represent the nominal interest rate with m conversions or interest periods per year so that $i = r/m$ and the total number of interest periods in n years is mn. With m annuity payments per year, let $\bar{R}$ represent the total of all ordinary annuity payments occurring regularly and uniformly throughout the year so that $\bar{R}/m$ is the uniform annuity payment at the end of each period. Under these conditions, Eq. (21) becomes

$$S = \frac{\bar{R}}{m}\,\frac{[1 + (r/m)]^{(m/r)(rn)} - 1}{r/m} \tag{22}$$

For the case of continuous cash flow and interest compounding, m approaches infinity, and Eq. (22), by use of Eq. (11), becomes[†]

$$S = \bar{R}\left(\frac{e^{rn} - 1}{r}\right) \tag{23}$$

Present Worth of an Annuity

The present worth of an annuity is defined as the principal which would have to be invested at the present time at compound interest rate i to yield a total amount at the end of the annuity term equal to the amount of the annuity. Let P represent the present worth of an ordinary annuity. Combining Eq. (5) with Eq. (21) gives, for the case of discrete interest compounding,

$$P = R\frac{(1 + i)^n - 1}{i(1 + i)^n} \tag{24}$$

The expression $[(1 + i)^n - 1]/[i(1 + i)^n]$ is referred to as the ***discrete uniform-series present-worth factor*** or the ***series present-worth factor,*** while the reciprocal $[i(1 + i)^n]/[(1 + i)^n - 1]$ is often called the ***capital-recovery factor.***

For the case of continuous cash flow and interest compounding, combination of Eqs. (12) and (23) gives the following equation which is analogous to Eq. (24):

$$P = \bar{R}\frac{e^{rn} - 1}{re^{rn}} \tag{25}$$

Example 5 Application of annuities in determining amount of depreciation with discrete interest compounding. A piece of equipment has an initial installed value of $12,000. It is estimated that its useful life period will be 10 years and its scrap

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[†] The same result is obtained from calculus by noting that the definition of $\bar{R}$ is such that the differential change in S with n is equal to $\bar{R}$, which is the constant gradient during the year, plus the contribution due to interest, or $dS/dn = \bar{R} + rS$. This expression can be integrated as follows to give Eq. (23):

$$\int_0^S \frac{dS}{\bar{R} + rS} = \int_0^n dn$$

$$\ln\frac{\bar{R} + rS}{\bar{R}} = rn \quad \text{or} \quad S = \bar{R}\left(\frac{e^{rn} - 1}{r}\right) \tag{23}$$

value at the end of the useful life will be \$2000. The depreciation will be charged as a cost by making equal charges each year, the first payment being made at the end of the first year. The depreciation fund will be accumulated at an annual interest rate of 6 percent. At the end of the life period, enough money must have been accumulated to account for the decrease in equipment value. Determine the yearly cost due to depreciation under these conditions.

(NOTE: This method for determining depreciation is based on an ordinary annuity and is known as the sinking-fund method.)

Solution. This problem is a typical case of an ordinary annuity. Over a period of 10 years, equal payments must be made each year at an interest rate of 6 percent. After 10 years, the amount of the annuity must be equal to the total amount of depreciation.

Amount of annuity = S
Total amount of depreciation = \$12,000 − \$2000 = \$10,000 = S
Equal payments per year = R = yearly cost due to depreciation
Number of payments = $n = 10$
Annual interest rate = $i = 0.06$.

From Eq. (21),

$$R = S\frac{i}{(1+i)^n - 1} = \$10{,}000\,\frac{0.06}{(1.06)^{10} - 1} = \$759/\text{year}$$

Yearly cost due to depreciation = \$759

Example 6 Application of annuities in determining amount of depreciation with continuous cash flow and interest compounding. Repeat Example 5 with continuous cash flow and nominal annual interest of 6 percent compounded continuously.

Solution. This problem is solved in exactly the same manner as Example 5, except the appropriate Eq. (23) for the continuous-interest case is used in place of the discrete-interest equation.

Amount of annuity = S
Total amount of depreciation = \$12,000 − \$2000 = S
Equal payments per year based on continuous cash flow and interest compounding = $\bar{R}$ = yearly cost due to depreciation
Number of years = $n = 10$
Nominal interest rate with continuous compounding = $r = 0.06$

From Eq. (23),

$$\bar{R} = S\frac{r}{e^{rn} - 1} = \$10{,}000\,\frac{0.06}{e^{(0.06)(10)} - 1} = \$730/\text{year}$$

Yearly cost due to depreciation = \$730

Special Types of Annuities

One special form of an annuity requires that payments be made at the beginning of each period instead of at the end of each period. This is known as an annuity due. An annuity in which the first payment is due after a definite number of years is called a *deferred annuity*. Determination of the periodic payments, amount of annuity, or present value for these two types of annuities can be accomplished by methods analogous to those used in the case of ordinary annuities.

PERPETUITIES AND CAPITALIZED COSTS

A *perpetuity* is an annuity in which the periodic payments continue indefinitely. This type of annuity is of particular interest to engineers, for in some cases they may desire to determine a total cost for a piece of equipment or other asset under conditions which permit the asset to be replaced perpetually without considering inflation or deflation.

Consider the example in which the original cost of a certain piece of equipment is \$12,000. The useful-life period is 10 years, and the scrap value at the end of the useful life is \$2000. The engineer reasons that this piece of equipment, or its replacement, will be in use for an indefinitely long period of time, and it will be necessary to supply \$10,000 every 10 years in order to replace the equipment. He therefore wishes to provide a fund of sufficient size so that it will earn enough interest to pay for the periodic replacement. If the discrete annual interest rate is 6 percent, this fund would need to be \$12,650. At 6 percent interest compounded annually, the fund would amount to $(\$12{,}650)(1 + 0.06)^{10} = \$22{,}650$ after 10 years. Thus, at the end of 10 years, the equipment can be replaced for \$10,000 and \$12,650 will remain in the fund. This cycle could now be repeated indefinitely. If the equipment is to perpetuate itself, the theoretical amount of total capital necessary at the start would be \$12,000 for the equipment plus \$12,650 for the replacement fund. The total capital determined in this manner is called the *capitalized cost*. Engineers use capitalized costs principally for comparing alternative choices.[†]

In a perpetuity, such as in the preceding example, the amount required for the replacement must be earned as compounded interest over a given length of time. Let P be the amount of present principal (i.e., the present worth) which can accumulate to an amount of S during n interest periods at periodic interest rate i. Then, by Eq. (5),

$$S = P(1 + i)^n \tag{5}$$

[†] For further discussion of capitalized costs used in engineering, see Chap. 10 and F. C. Jelen and M. S. Cole. Methods for Economic Analysis, Part I, *Hydrocarbon Proc.*, **53**(7):133 (1974); Part II, *Hydrocarbon Proc.*, **53**(9):227 (1974).

If perpetuation is to occur, the amount S accumulated after n periods minus the cost for the replacement must equal the present worth P. Therefore, letting C_R represent the replacement cost,

$$P - S - C, \tag{26}$$

Combining Eqs. (5) and (26),

$$P = \frac{C_R}{(1 + i)^n - 1} \tag{27}$$

The capitalized cost is defined as the original cost of the equipment plus the present value of the renewable perpetuity. Designating K as the capitalized cost and C_V as the original cost of the equipment,?

$$K = C_V + \frac{C_R}{(1 + i)^n - 1} \tag{28}$$

Example 7 Determination of capitalized cost. A new piece of completely installed equipment costs \$12,000 and will have a scrap value of \$2000 at the end of its useful life. If the useful-life period is 10 years and the interest is compounded at 6 percent per year, what is the capitalized cost of the equipment?

Solution. The cost for replacement of the equipment at the end of its useful life (assuming costs unchanged) = \$12,000 − \$2000 = \$10,000.

By Eq. (28)

$$\text{Capitalized cost} = C_V + \frac{C_R}{(1 + i)^n - 1}$$

where C_V = \$12,000
C_R = \$10,000
$i = 0.06$
$n = 10$

$$\text{Capitalized cost} = \$12{,}000 + \frac{\$10{,}000}{(1 + 0.06)^{10} - 1}$$

$$= \$12{,}000 + \$12{,}650 = \$24{,}650$$

Example 8 Comparison of alternative investments using capitalized costs. A reactor, which will contain corrosive liquids, has been designed. If the reactor is made of mild steel, the initial installed cost will be \$5000, and the useful-life period will be 3 years. Since stainless steel is highly resistant to the corrosive action of the liquids, stainless steel, as the material of construction, has been proposed as an alternative to mild steel. The stainless-steel reactor would have an initial installed cost of \$15,000. The scrap value at the end of the useful life would be

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†For the continuous-interest-compounding expression equivalent to the discrete-interest-compounding case given in Eq. (28), see Prob. 13 at the end of the chapter.

zero for either type of reactor, and both could be replaced at a cost equal to the original price. On the basis of equal capitalized costs for both types of reactors, what should be the useful-life period for the stainless-steel reactor if money is worth 6 percent compounded annually?

Solution. By Eq. (28), the capitalized cost for the mild-steel reactor is

$$K = C_V + \frac{C_R}{(1+i)^n - 1} = \$5000 + \frac{\$5000}{(1+0.06)^3 - 1}$$

$$K = \$5000 + \$26{,}180 = \$31{,}180$$

Therefore, the capitalized cost for the stainless-steel reactor must also be $31,180.

For the stainless-steel reactor,

$$\$31{,}180 = C_V + \frac{C_R}{(1+i)^n - 1} = \$15{,}000 + \frac{\$15{,}000}{(1+0.06)^n - 1}$$

Solving algebraically for n,

$$n = 11.3 \text{ years}$$

Thus, the useful-life period of the stainless-steel reactor should be 11.3 years for the two types of reactors to have equal capitalized costs. If the stainless-steel reactor would have a useful life of more than 11.3 years, it would be the recommended choice, while the mild-steel reactor would be recommended if the useful life using stainless steel were less than 11.3 years.

RELATIONSHIPS FOR CONTINUOUS CASH FLOW AND CONTINUOUS INTEREST OF IMPORTANCE FOR PROFITABILITY ANALYSES

The fundamental relationships dealing with continuous interest compounding can be divided into two general categories: (1) those that involve instantaneous or lump-sum payments, such as a required initial investment or a future payment that must be made at a given time, and (2) those that involve continuous payments or continuous cash flow, such as construction costs distributed evenly over a construction period or regular income that flows constantly into an overall operation. Equation (12) is a typical example of a lump-sum formula, while Eqs. (23) and (25) are typical of continuous-cash-flow formulas.

The symbols S, P, and R represent discrete lump-sum payments as future worth, present principal (or present worth), and end-of-period (or end-of-year) payments, respectively. A bar above the symbol, such as $\bar{S}$, $\bar{P}$, or $\bar{R}$, means that the payments are made *continuously* throughout the time period under consideration.† For example, consider the case where construction of a plant requires a continuous flow of cash to the project for one year, with the plant ready for operation at the end of the year of construction. The symbol $\bar{P}$ represents the total amount of cash put into the project on the basis of one year with a continuous flow of cash. At the end of the year, the compound amount of this $\bar{P}$ is

$$S_{\text{at end of one individual year}} = \bar{P}\frac{e^r - 1}{r} = P_{\text{at startup}} \tag{29}$$

If the plant is ready for operation after one year of construction time and the startup of the plant is designated as zero time, the future worth of the plant construction cost after n years with continuous interest compounding is

$$S_{\text{after } n \text{ years in operation}} = (P_{\text{at startup}})e^{rn} = \bar{P}\frac{e^r - 1}{r}e^{rn} \tag{30}$$

For profitability analyses, certain discounting or compounding factors based on continuous interest compounding are of sufficient importance that tables have been prepared which give values of the factors for various interest rates and time periods. Table 3 gives examples of tabulated factors for the following cases:‡§

(a) *Discount factors to give present worths for cash flows which occur in an instant at a point in time after the reference point.* These factors are used to convert one dollar of money, which must be available in an instant after time n (such as scrap value, working capital, or land value), to the present worth of this one dollar with continuous interest compounding. The appropriate equation for calculating the factor, therefore, is based on Eq. (12), and

$$\text{Factor} = 1.0\frac{1}{e^{rn}} = F_a \tag{31}$$

For example, if the nominal continuous interest rate is 20 percent and the

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† It should be noted that $\bar{S}$, $\bar{P}$ and $\bar{R}$ represent sums accumulated by continuous payment over an indicated time period without any interest accumulation. $\bar{R}$ represents a periodic accumulation normally based on one year, while $\bar{S}$ and $\bar{P}$ represent accumulations during a given period of time. Thus, $\bar{S}$ and $\bar{P}$ are interchangeable depending on the basic form of equation being used, and $\bar{S}$, $\bar{P}$, and $\bar{R}$ are interchangeable if the time period under consideration is limited to one year or one basic interest period.

‡ See Table 4 for a summary of the significance and meaning of the factors presented in Table 3. Extended values of the factors for parts (a) to (d) of Table 3 are given in Tables 5 to 8.

§ For illustrations of the applications of continuous interest compounding and continuous cash flow to cases of profitability evaluation, see Examples 2 and 3 in Chap. 10.

TABLE3

Discount and compounding factors for continuous interest and cash flows?

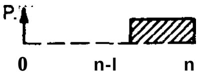
Discount factors to give present worths for cash flows which

(a) Occur in an instant at a point in time after the reference point

$$1.0\left(\frac{1}{e^{rn}}\right) = F_a$$

r as percent	1%	5%	10%	15%	20%	25%	30%	40%	50%	60%	80%	100%
$n =$ 1	0.990	0.951	0.905	0.861	0.819	0.779	0.741	0.670	0.606	0.549	0.449	0.368
2	0.980	0.905	0.819	0.741	0.670	0.606	0.549	0.449	0.368	0.301	0.202	0.135
3	0.970	0.861	0.741	0.638	0.549	0.472	0.407	0.301	0.223	0.165	0.091	0.050
4	0.961	0.819	0.670	0.549	0.449	0.368	0.301	0.202	0.135	0.091	0.041	0.018
5	0.951	0.779	0.606	0.472	0.368	0.286	0.223	0.135	0.082	0.050	0.018	0.007
10	0.905	0.606	0.368	0.223	0.135	0.082	0.050	0.018	0.007	0.002		
15	0.861	0.472	0.223	0.105	0.050	0.024	0.011	0.002	0.001			
20	0.819	0.368	0.135	0.050	0.018	0.007	0.002					
25	0.779	0.286	0.082	0.024	0.007	0.002	0.001					

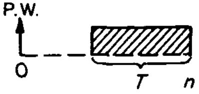
(b) Occur uniformly over one-year periods after the reference point



$$1.0\left(\frac{e^{r}-1}{r}\right)e^{-rn} = F_b$$

r as percent		1%	5%	10%	15%	20%	25%	30%	40%	50%	60%	80%	100%
$n =$ 1.	1st year	0.995	0.975	0.952	0.929	0.906	0.685	0.864	0.824	0.787	0.752	0.688	0.632
2.	2nd year	0.985	0.928	0.861	0.799	0.742	0.689	0.640	0.552	0.477	0.413	0.309	0.232
3.	3rd year	0.975	0.883	0.779	0.688	0.608	0.537	0.474	0.370	0.290	0.226	0.139	0.086
4.	4th year	0.966	0.840	0.705	0.592	0.497	0.418	0.351	0.248	0.176	0.124	0.062	0.032
5.	5th year	0.956	0.799	0.638	0.510	0.407	0.326	0.260	0.166	0.106	0.068	0.028	0.012

(c) Occur uniformly over a period of years-For period of years = T = 5 years



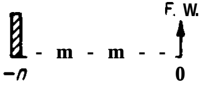
$$\frac{1}{T}\left(\frac{e^{rT}-1}{r}\right)e^{-rn} = F_c$$

r as percent		1%	5%	10%	15%	20%	25%	30%	40%	50%	60%	80%	100%
$n =$ 5.	1st 5 years	0.975	0.885	0.787	0.704	0.632	0.571	0.518	0.432	0.367	0.317	0.245	0.199
10.	2nd 5 years	0.928	0.689	0.477	0.332	0.232	0.164	0.116	0.058	0.030	0.016	0.004	0.001
15.	3rd 5 years	0.883	0.537	0.290	0.157	0.086	0.047	0.026	0.008	0.002	0.001		
20.	4th 5 years	0.840	0.418	0.176	0.074	0.032	0.013	0.006	0.001				
25.	5th 5 years	0.799	0.326	0.106	0.035	0.012	0.004	0.001					

(Continued)

TABLE 3

Discount and compounding factors for continuous interest and cash flows† *(Continued)*

r as percent	1%	5%	10%	15%	20%	25%	30%	40%	50%	60%	80%	100%
(d) Decline to zero at a constant rate over a period of years starting with the reference point												
 1st 5 years	0.983	0.922	0.852	0.791	0.736	0.687	0.643	0.568	0.506	0.456	0.377	0.320
1st 10 years	0.968	0.852	0.736	0.643	0.568	0.506	0.456	0.377	0.320	0.278	0.219	0.180
1st 15 years	0.952	0.791	0.643	0.536	0.456	0.394	0.347	0.278	0.231	0.198	0.153	0.124
1st 20 years	0.936	0.736	0.568	0.456	0.377	0.320	0.278	0.219	0.180	0.153	0.117	0.095
$\frac{2}{rn_T}[1-(1-e^{-rn_T})/rn_T] = F_d$ 1st 25 years	0.922	0.687	0.506	0.394	0.320	0.269	0.231	0.180	0.147	0.124	0.095	0.077
Compounding factors to give future worths for cash flows which												
(*e*) Occur in an instant at a point in time before the reference point												
 ½ year before	1.005	1.025	1.051	1.078	1.105	1.133	1.162	1.221	1.284	1.350	1.492	1.649
1 year before	1.010	1.051	1.105	1.162	1.221	1.284	1.350	1.492	1.649	1.822	2.226	2.718
1½ years before	1.015	1.078	1.162	1.252	1.350	1.455	1.568	1.822	2.117	2.460	3.320	4.482
2 years before	1.020	1.105	1.221	1.350	1.492	1.649	1.822	2.226	2.718	3.320	4.953	7.389
$1.0(e^{rn}) = C_e$ 3 years before	1.030	1.162	1.350	1.568	1.822	2.117	2.460	3.320	4.482	6.050	1.023	20.086
(f) Occur uniformly before the reference point												
 ½ year before	1.002	1.013	1.025	1.038	1.052	1.065	1.079	1.107	1.136	1.166	1.230	1.297
1 year before	1.005	1.025	1.052	1.079	1.107	1.136	1.166	1.230	1.297	1.370	1.532	1.718
1½ years before	1.008	1.038	1.079	1.121	1.166	1.213	1.263	1.370	1.489	1.622	1.933	2.321
2 years before	1.010	1.052	1.107	1.166	1.230	1.297	1.370	1.532	1.718	1.933	2.471	3.194
$\frac{1.0}{T}\left(\frac{e^{rT}-1}{r}\right) = C_f$ 3 years before	1.015	1.079	1.166	1.263	1.370	1.489	1.622	1.933	2.321	2.805	4.176	6.362

† r = nominal interest compounded continuously, percent/100; n = number of years; T and n_T = number of years in a time period. See Table 4 for significance and meaning of compounding factors. Extended values of factors for parts (a), (b), and (d) are given in Tables 5, 6, and 8, and Table 7 gives extended values for part (c) with $n = T$.

TABLE 4

Summary of significance and meaning of discount and compounding factors presented in Tables 3, 5, 6, 7, and 8†

As indicated in the footnote for Eq. (12), the common interest expressions can be written in simplified form by using discount-factor and compound-interest-factor notation. Following is a summary showing the significance and meaning of the compounding factors presented in Table 3. Derivations of the factors are presented in the text.

For part (a) in Table 3 and for Table 5

F_a = **Discount factor to give present worth for cash flows which occur in an instant at a point in time after the reference point.**

$$P = F_a S \qquad F_a = e^{-rn}$$

For part (b) **in** *Table 3 and for Table 6*

F_b = **Discount factor to give present worth for cash flows which occur uniformly over one-year periods after the reference point. ($\bar{S}$ is the total cash flow for the nth year.)**

$$P = F_b S_{(\text{nth year})} \qquad F_b = \left(\frac{e^r - 1}{r}\right) e^{-rn}$$

For part (c) in Table 3 and for Table 7 with n = T

F_c = **Discount factor to give present worth for cash flows which occur uniformly over a period of years** *T.* **(8 is the total cash flow for the T-year period.)**

$$P = F_c S_{(T\text{-year period})} \qquad F_c = \frac{1}{T}\left(\frac{e^{rT} - 1}{r}\right) e^{-rn}$$

NOTE: *For the case when the period of years T is based on the period immediately after the* **reference point,** $n = T$, *and* $F_c = (1 - e^{-rT})/rT$. *This is the factor presented in Table 7.*

For part (d) in Table 3 and for Table 8

F_d = **Discount factor to give present worth for cash flows which decline to zero at a constant rate over a period of years n_T starting with the reference point. ($\bar{S}$ is the total cash flow for the n_T-year period.)**

$$P = F_d \bar{S}_{(\text{declining to zero at constant rate in } n_T \text{ years})}$$

$$F_d = \frac{2}{rn_T}\left(1 - \frac{1 - e^{-rn_T}}{rn_T}\right)$$

For part (e) in Table 3

C_e = **Compounding factor to give future worth for cash flows which occur in an instant at a point in time before the reference point.**

$$s = C_e P \qquad C_e = e^{rn} = \frac{1}{F_a}$$

NOTE: *Table 6* **gives** *reciprocal values of* **C..**

(Continued)

TABLE 4

Summary of significance and meaning of discount and compounding factors presented in Tables 3, 5, 6, 7, and 8† *(Continued)*

For part (f) in Table 8

C_f = **Compounding factor to give future worth for cash flows which occur uniformly over a period of years** T **before the reference point. ($\bar{P}$ is the total cash flow for the T-year period.)**

$$S = C_f\bar{P} \qquad c, = \frac{e^{rT} - 1}{rT}$$

For n = T, $C_f = F_c/F_a = F_cC_e$.

NOTE: *On basis of* **above relationship,** C_f *can be generated from* F_a *and* F_c *values* **given in** *Tables 5 and Y.*

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† r = **nominal interest compounded continuously,** percent/100; n = **number of years;** T **and** n_T = **number of years in a time period.**

time period is 5 years, the appropriate factor, as shown in Table 3, is

$$\text{Factor} = \frac{1}{e^{(0.2)(5)}} = \frac{1}{2.7183} = 0.368$$

(*b*) *Discount factors to give present worths for cash flows which occur uniformly over one-year periods after the reference point.* For this situation, the factor would convert one dollar of money, as the total yearly amount flowing continuously and uniformly during the year (such as cash receipts for one year), to the present worth of this one dollar at zero time with continuous interest compounding. Thus, $\bar{R}$ (or $\bar{S}$) for the year in question is 1.0, and the appropriate equation for calculating the factor, based on Eqs. (23) and (12), is

$$\text{Factor} = 1.0\,\frac{e^{r} - 1}{r}e^{-rn} = F_b \tag{32}$$

As an example, if r represents 20 percent and n is the fifth year, the appropriate factor, as shown in Table 3, is

$$\text{Factor} = \frac{e^{(0.2)} - 1}{0.2}\,\frac{1}{e^{(0.2)(5)}} = \frac{1.2214 - 1}{0.2}\,\frac{1}{2.7183} = 0.407$$

(*c*) *Discount factors to give present worths for cash flows which occur uniformly over a period of years.* For this situation, a total amount of one dollar over a given time period is used as the basis. The cash flows continuously and uniformly during the entire period, and the factor converts the total of one dollar put in over the given time period to the present worth at zero time. This condition would be applicable to a case where cash receipts are steady over a given period of time, such as for five years. Designating T as the time period involved, the total amount put in each year is $\$1/T$, and the factor, based on Eqs. (23) and (12), is

$$\text{Factor} = \frac{1}{T}\,\frac{e^{rT} - 1}{r}\,e^{-rn} = F_c \tag{33}$$

For example, if the time period involved is the second five years (i.e., the 6th through the 10th years) and r represents 20 percent, the appropriate factor, as shown in Table 3, is

$$\text{Factor} = \frac{1}{5}\left(\frac{e^{(0.2)(5)} - 1}{0.2}\right)\left(\frac{1}{e^{(0.2)(10)}}\right) = \frac{1}{5}\left(\frac{2.7183 - 1}{0.2}\right)\frac{1}{7.3891} = 0.232$$

(*d*) *Discount factors to give present worths for cash flows declining to zero at a constant rate over a period* of *years starting with the reference point.* For this case, the assumption is made that the continuous cash flow declines linearly with time from the initial flow at time zero to zero flow at time n_T. A situation similar to this exists when the sum-of-the years-digits method is used for calculating depreciation in that depreciation allowances decline linearly with time from a set value in the first year to zero at the end of the life.†‡ For the case of continuous cash flow declining to zero at a constant rate over a time period of n_T, the linear equation for $\bar{R}$ is

$$\bar{R} = a - gn \tag{34}$$

where g = the constant declining rate or the gradient
$\bar{R}$ = instantaneous value of the cash flow
a = a constant

The discount factor is based on a total amount of one dollar of cash flow over the time period n_T and converts this total of one dollar to the present worth at time zero. Under these conditions, g equals $2/(n_T)^2$,§ and the

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† See Chap. 9 (Depreciation) for information on the sum-of-the-years-digits method for calculating depreciation.

‡ Equation (35) does not represent a true sum-of-the-years-digits factor. Normally, the constant declining rate or gradient for the sum-of-the-years-digits method. of depreciation is $1/\sum_1^{n_T} n = 2/n_T(n_T + 1)$. For the true case of continuous cash flow declining to zero at a constant rate, n_T is replaced by $n_T m$ as $m \to \infty$, and the constant gradient becomes $2/(n_T)^2$.

§ By definition of terms and conditions. $\bar{R}$ is zero when $n = n_T$ and $\bar{R}$ is a when $n = 0$. Also, if a total of one dollar is the cash flow during n_T

$$\int_0^{n_T} \bar{R}\,dn = an_T - \frac{g(n_T)^2}{2} = \$10$$

Because $\bar{R}$ is zero when $n = n_T$, a = gn_T. Therefore,

$$\$1.0 = g(n_T)^2 - \frac{g(n_T)^2}{2} = \frac{g(n_T)^2}{2} \quad \text{and} \quad g = \frac{2}{(n_T)^2}$$

factor is†

$$\text{Factor} = \frac{2}{m_T}\left(1 - \frac{1 - e^{-rn_T}}{m_T}\right) = F_d$$

As an example, if the cash flow declines at a constant rate to zero in 5 years and r is equivalent to 20 percent, the appropriate factor, as shown in Table 3, is

$$\text{Factor} = \frac{2}{(0.2)(5)}\left[1 - \frac{1}{(0.2)(5)}\left(1 - \frac{1}{e^{(0.2)(5)}}\right)\right]$$

$$= 2\left(1 - 1 + \frac{1}{2.7183}\right) = 0.736$$

(*e*) *Compounding factors to give future worths for cash flows which occur in an instant at a point in time before the reference point.* These factors merely show the future worth to which one dollar of principal, such as that for land purchase, will compound at continuous interest. Based on Eq. (12), the factor is

$$\text{Factor} = 1.0e^{rn} = C_e \tag{36}$$

For example, with r equivalent to 20 percent and a purchase made $1\frac{1}{2}$ years before the reference point, the appropriate factor, as shown in Table 3, is

$$\text{Factor} = e^{(0.2)(1.5)} = 1.350$$

(*f*) *Compounding factors to give future worths for cash flows which occur uniformly before the reference point.* The basis for these factors is a uniform and continuous flow of cash amounting to a total of one dollar during the given time period of T years, such as for construction of a plant. The factor converts this one dollar to the future worth at the reference time and is based on Eq. (23).

$$\text{Factor} = \frac{1.0}{T}\,\frac{e^{rT} - 1}{r} = C_f \tag{37}$$

As an example, for the case of continuous compounding at r equivalent to 20 percent for a period from 3 years before the reference time, the appropriate factor, as shown in Table 3, is

$$\text{Factor} = \frac{1}{3}\,\frac{e^{(0.2)(3)} - 1}{0.2} = \frac{1.8221 - 1}{(3)(0.2)} = 1.370$$

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†This can be derived by assuming an ordinary annuity with $\$gn_T$ for the first year, $\$g(n_T - 1)$ for the second year, etc., to $\$g$ for year n_T. The result with discrete interest compounding is

$$\text{Factor} = g\left[\frac{in_T - 1 + (1+i)^{-n_T}}{i^2}\right]$$

Letting $g = 2/(n_T)^2$ and replacing i by r/m and n_T by mn_T gives Eq. (35) for $m \to \infty$.

TABLE 5

Discount factors (F_a) with continuous interest to give present worths for cash flows which occur in an instant at a point in time after the reference point†‡

$100rn$	0	1	2	3	4	5	6	7	8	9
0	1.0000	0.99~1	0.9802	0.9704	0.9608	0.9512	0.9418	0.9324	0.9231	0.9139
10	0.9048	0.8958	0.8869	0.8781	0.8694	0.8607	0.8521	0.8437	0.8353	0.8270
20	0.8187	0.8106	0.8025	0.7945	0.7866	0.7788	0.7711	0.7634	0.7558	0.7483
30	0.7408	0.7334	0.7261	0.7189	0.7118	0.5047	0.6977	0.6907	0.6839	0.6771
40	0.6703	0.6637	0.6570	0.6505	0.6440	0.6376	0.6313	0.6250	0.6188	0.6126
50	0.6065	0.6005	0.5945	0.5886	0.5827	0.5770	0.5712	0.5655	0.5599	0.5543
60	0.5488	0.5434	0.5379	0.5326	0.5273	0.5220	0.5169	0.5117	0.5066	0.5016
70	0.4966	0.4916	0.4868	0.4819	0.4771	0.4724	0.4677	0.4630	0.4584	0.4538
80	0.4493	0.4449	0.4404	0.4360	0.4317	0.4274	0.4232	0.4190	0.4148	0.4107
90	0.4066	0.4025	0.3985	0.3946	0.3906	0.3867	0.3829	0.3791	0.3753	0.3716
100	0.3679	0.3642	0.3606	0.3570	0.3535	0.3499	0.3465	0.3430	0.3396	0.3362
110	0.3329	0.3296	0.3263	0.3230	0.3198	0.3166	0.3135	0.3104	0.3073	0.3042
120	0.3012	0.2982	0.2952	0.2923	0.2894	0.2865	0.2837	0.2808	0.2780	0.2753
130	0.2725	0.2698	0.2671	0.2645	0.2618	0.2592	0.2567	0.2541	0.2516	0.2491
140	0.2466	0.2441	0.2417	0.2393	0.2369	0.2346	0.2322	0.2299	0.2276	0.2254
150	0.2231	0.2209	0.2187	0.2165	0.2144	0.2122	0.2101	0.2080	0.2060	0.2039
160	0.2019	0.1999	0.1979	0.1959	0.1940	0.1921	0.1901	0.1882	0.1864	0.1845
170	0.1827	0.1809	0.1791	0.1773	0.1755	0.1738	0.1720	0.1703	0.1686	0.1670
180	0.1653	0.1637	0.1620	0.1604	0.1588	0.1572	0.1557	0.1541	0.1526	0.1511
190	0.1496	0.1481	0.1466	0.1451	0.1437	0.1423	0.1409	0.1395	0.1381	0.1367
200	0.1353	0.1340	0.1327	0.1313	0.1300	0.1287	0.1275	0.1262	0.1249	0.1237
210	0.1225	0.1212	0.1200	0.1188	0.1177	0.1165	0.1153	0.1142	0.1130	0.1119
220	0.1108	0.1097	0.1086	0.1075	0.1065	0.1054	0.1044	0.1033	0.1023	0.1013
230	0.1003	0.0993	0.0983	0.0973	0.0963	0.0954	0.0944	0.0935	0.0926	0.0916
240	0.0907	0.0898	0.0889	0.0880	0.0872	0.0863	0.0854	0.0846	0.0837	0.0829
250	0.0821	0.0813	0.0805	0.0797	0.0789	0.0781	0.0773	0.0765	0.0758	0.0750
260	0.0743	0.0735	0.0728	0.0721	0.0714	0.0707	0.0699	0.0693	0.0686	0.0679
270	0.0672	0.0665	0.0659	0.0652	0.0646	0.0639	0.0633	0.0627	0.0620	0.0614
280	0.0608	0.0602	0.0596	0.0590	0.0584	0.0578	0.0573	0.0567	0.0561	0.0556
290	0.0550	0.0545	0.0539	0.0534	0.0529	0.0523	0.0518	0.0513	0.0508	0.0503
300	0.0498	0.0493	0.0488	0.0483	0.0478	0.0474	0.0469	0.0464	0.0460	0.0455
310	0.0450	0.0446	0.0442	0.0437	0.0433	0.0429	0.0424	0.0420	0.0416	0.0412
320	0.0408	0.0404	0.0400	0.0396	0.0392	0.0388	0.0384	0.0380	0.0376	0.0373
330	0.0369	0.0365	0.0362	0.0358	0.0354	0.0351	0.0347	0.0344	0.0340	0.0337
340	0.0334	0.0330	0.0327	0.0324	0.0321	0.031i	0.0314	0.0311	0.0308	0.0305
350	0.0302	0.0299	0.0296	0.0293	0.0290	0.0287	0.0284	0.0282	0.0279	0.0276
360	0.0273	0.0271	0.0268	0.0265	0.0263	0.0260	0.0257	0.0255	0.0252	0.0250
3i0	0.0247	0.0245	0.0242	0.0240	0.0238	0.0235	0.0233	0.0231	0.0228	0.0226
380	0.0224	0.0221	0.0219	0.0217	0.0215	0.0213	0.0211	0.0209	0.0207	0.0204
390	0.0202	0.0200	0.0198	0.0196	0.0194	0.0193	0.0191	0.0189	0.0187	0.0185

(Continued)

TABLE 5

Discount factors (F_a) with continuous interest to give present worths for cash flows which occur in an instant at a point in time after the reference point†‡ (Continued)

100*rn*	0	1	2	3	4	5	6	7	8	9
400	0.0183	0.0181	0.01~0	0.01i~	0.0176	0.0174	0.0172	0.01i1	0.0169	0.0167
410	0.0166	0.0164	0.0162	0.0161	0.0159	0.0158	0.0156	0.0155	0.0153	0.0151
420	0.0150	0.0148	0.0147	0.0146	0.0144	0.0143	0.0141	0.0140	0.0138	0.0137
430	0.0136	0.0134	0.0133	0.0132	0.0130	0.0129	0.0128	0.0127	0.0125	0.0124
440	0.0123	0.0122	0.0120	0.0119	0.0118	0.0117	0.0116	0.0114	0.0113	0.0112
450	0.0111	0.0110	0.0109	0.0108	0.0107	0.0106	0.0105	0.0104	0.0103	0.0102
460	0.0101	0.0100	0.0099	0.009X	0.0097	0.0096	0.0095	0.0094	0.0093	0.0092
470	0.0091	0.0090	0.0089	0.0088	0.0087	0.0087	0.0086	0.0085	0.0084	0.0083
480	0.0082	0.0081	0.0081	0.0080	0.0079	0.0078	0.0078	0.00i7	0.0076	0.0075
490	0.0074	0.0074	0.0073	0 0072	0.0072	0.0071	0 0070	0.0069	0.0069	0.0068
100*rn*	0	10	20	30	40	50	60	70	80	90
500	0.006i	0.0061	0.0055	0.0050	0.0045	0.0041	0.003i	0.0033	0.0030	0.0027
600	0.0025	0.0022	0.0020	0.0018	0.001i	0.0015	0.0014	0.0012	0.0011	0.0010
700	0.0009	0.0008	0.0007	0.0007	0.0006	0.0006	0.0005	0.0005	0.0004	0.0004
800	0.0003	0.0003	0.0003	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0001
900	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
1000	0.0000									

† r = nominal interest compounded continuously, percent/100; n = number of years. See Tables 3 and 4 for information on F_a.

‡ The columns represent the unit increments from 1 to 9 for the lo-digit intervals of 100*rn* shown in the left column.

TABLES FOR INTEREST AND CASH-FLOW FACTORS

Tables of interest and cash-flow factors, such as are illustrated in Tables 1, 5, 6, 7, and 8 of this chapter, are presented in all standard interest handbooks and textbooks on the mathematics of finance as well as in appendices of most textbooks on engineering economy. Exponential functions for continuous compounding are available in the standard mathematical tables. The development of tables for any of the specialized factors is a relatively simple matter with the ready availability of digital computers, as is illustrated in Example 3 of this chapter.

The end-of-year convention is normally adopted for discrete interest factors (or for lump-sum payments) wherein the time unit of one interest period is assumed to be one year with interest compounding (or with lump-sum payments being made) at the end of each period. Thus, the *effective interest rate* is the form of interest most commonly understood and used by management and business executives.

In the tabulation of factors for continuous interest compounding and continuous cash flow, the nominal interest rate r is used for calculating the

TABLE 6

Discount factors (F_b) with continuous interest to give present worths for cash flows which occur uniformly over one-year periods after the reference point†

Year	1%	2%	3%	4%	5%	6%	7%	8%	9%	10%	11%	12%	13%	14%	15%
0-1	0.9950	0.9901	0.9851	0.9803	0.9754	0.9706	0.9658	0.9610	0.9563	0.9518	0.9470	0.9423	0.9377	0.9332	0.9286
1-2	0.9851	0.9705	0.9560	0.9418	0.9278	0.9141	0.9005	0.8872	0.8740	0.8611	0.8483	0.8358	0.8234	0.8112	0.7993
2-3	0.9753	0.9512	0.9278	[illegible]	0.8826	0.8808	0.8386	0.8189	[illegible]	0.7791	0.7600	0.7413	0.7230	0.7053	0.6879
3-4	0.9656	0.9324	0.9004	0.8694	0.8395	0.8107	0.7829	0.7560	0.7300	0.7050	0.6808	0.6574	0.6349	0.6131	0.5921
4-5	0.9560	0.9140	0.8737	0.8353	0.7986	0.7635	0.7299	0.6979	0.6672	0.6379	0.6099	0.5831	0.5575	0.5330	0.5096
5-6	0.9465	0.8959	0.8479	0.8026	0.7596	0.7190	0.6806	0.6442	0.6098	0.5772	0.5463	0.5172	0.4895	0.4634	0.4386
6-7	0.9371	0.8781	0.8229	0.7711	0.7226	0.6772	0.6346	0.5947	0.5573	0.5223	0.4894	0.4588	0.4299	0.4029	0.3775
7-8	0.9278	0.8607	0.7985	0.7409	0.6874	0.6377	0.5917	0.5490	0.5093	0.4726	0.4385	0.4069	0.3775	0.3502	0.3250
8-9	0.9185	0.8437	0.7749	0.7118	0.6538	0.6006	0.5517	0.5068	0.4655	0.4276	0.3928	0.3609	0.3314	0.3045	0.2797
9-10	0.9094	0.8270	0.7520	0.6839	0.6219	0.5656	0.5144	0.4678	0.4254	0.3369	0.3519	0.3201	0.2910	0.2647	0.2407
10-11	0.9003	0.8106	0.7298	0.6571	0.5916	0.5327	0.4796	0.4318	0.3888	0.3501	0.3152	0.2839	0.2556	0.2301	0.2072
11-12	0.8914	0.7946	0.7082	0.6312	0.5628	0.5016	0.4472	0.3986	0.3553	0.3168	0.2824	0.2518	0.2244	0.2000	0.1783
12-13	0.8825	0.7788	0.6873	0.6065	0.5353	0.4724	0.4169	0.3680	0.3248	0.2866	0.2530	0.2233	0.1970	0.1739	0.1535
13-14	0.8737	0.7634	0.6670	0.5827	0.5092	0.4449	0.3888	0.3397	0.2968	0.2593	0.2266	0.1981	0.1730	0.1512	0.1321
14-15	0.8650	0.7483	0.6473	0.5599	0.4844	0.4190	0.3625	0.3136	0.2713	0.2347	0.2030	0.1757	0.1519	0.1314	0.1137
15-16	0.8564	0.7335	0.6282	0.5380	0.4608	0.3946	0.3380	0.2895	0.2479	0.2123	0.1819	0.1558	0.1334	0.1143	0.0979
16-17	0.8479	0.7189	0.6096	0.5169	0.4383	0.3716	0.3151	0.2672	0.2266	0.1921	0.1629	0.1382	0.1172	0.0993	0.0842
17-18	0.8395	0.7047	0.5916	0.4966	0.4169	0.3500	0.2938	0.2467	0.2071	0.1739	0.1460	0.1225	0.1029	0.0864	0.0725
18-19	0.8311	0.6908	0.5741	0.4772	0.3966	0.3298	0.271u	0.2277	0.1893	0.1573	0.1308	0.1087	0.0903	0.0751	0.0824
19-20	0.8228	0.6771	0.5571	0.4584	0.3772	0.3104	0.2554	0.2102	0.1730	0.1423	0.1171	0.0964	0.0793	0.0653	0.0537
20-21	0.8147	0.6637	0.5407	0.4405	0.3588	0.2923	0.2382	0.1940	0.1581	0.1288	0.1049	0.0855	0.0697	0.0568	0.0462
21-22	0.8065	0.6505	0.5247	0.4232	0.3413	0.2753	0.2221	0.1791	0.1445	0.1165	0.0940	0.0758	0.0612	0.0493	0.0398
22-23	0.7985	0.6376	0.5092	0.4066	0.3247	0.2593	0.2071	0.1653	0.1320	0.1054	0.0842	0.0673	0.0537	0.0429	0.0343
23-24	0.7906	0.6250	0.4941	0.3907	0.3089	0.2442	0.1931	0.1526	0.1207	0.0954	0.0754	0.0596	0.0472	0.0373	0.0295
24-25	0.7827	0.6126	0.4795	0.3753	0.2938	0.2300	0.1800	0.1409	0.1103	0.0863	0.0676	0.0529	0.0414	0.0324	0.0254
25-30	0.7596	0.5772	0.4386	0.3334	0.2535	0.1928	0.1466	0.1115	0.0849	0.0646	0.0492	0.0374	0.0285	0.0217	0.0165
30-35	0.7226	0.5223	0.3775	0.2730	0.1974	0.1428	0.1033	0.0748	0.0541	0.0392	0.0284	0.0205	0.0149	0.0108	0.0078
35-40	0.6874	0.4726	0.3250	0.2235	0.1538	0.1058	0.0728	0.0501	0.0345	0.0238	0.0164	0.0113	0.0078	0.0054	0.0037
40-45	0.5538	0.4276	0.2797	0.1830	0.1197	0.0784	0.0513	0.0336	0.0220	0.0144	0.0094	0.0062	0.0041	0.0027	0.0017
45-50	0.6219	0.3869	0.2407	0.1498	0.0933	0.0581	0.0362	0.0225	0.0140	0.0087	0.0054	0.0034	0.0021	0.0013	0.0008

Year	16%	17%	18%	19%	20%	21%	22%	23%	24%	25%	26%	27%	28%	29%	30%
0-1	0.9241	0.9196	0.9152	0.9107	0.9063	0.9020	0.8976	0.8933	0.8890	0.8848	0.8806	0.8764	0.8722	0.8681	0.8640
1-2	0.7875	0.7759	0.7644	0.7531	0.7421	0.7311	0.7204	0.7098	0.6993	0.6891	0.6790	0.6690	0.6592	0.6495	0.6400
2-3	0.6710	0.6546	0.6385	0.6228	0.6075	0.5926	0.5781	0.5639	0.5501	0.5367	0.5235	0.5107	0.4982	0.4860	0.4741
3-4	0.5718	0.5522	0.5333	0.5150	0.4974	0.4804	0.4639	0.4481	0.4327	0.4179	0.4037	0.3899	0.3765	0.3637	0.3513
4-5	0.4873	0.4659	0.4455	0.4259	0.4072	0.3894	0.3723	0.3560	0.3404	0.3255	0.3112	0.2976	0.2846	0.2721	0.2802
5-6	0.4152	0.3931	0.3721	0.3522	0.3334	0.3156	0.2988	0.2829	0.2678	0.2535	0.2400	0.2272	0.2151	0.2036	0.1928
6-7	0.3538	0.3316	0.3108	0.2913	0.2730	0.2558	0.2398	0.2247	0.2106	0.1974	0.1850	0.1734	0.1626	0.1524	0.1428
7-8	0.3015	0.2798	0.2596	0.2409	0.2235	0.2074	0.1924	0.1786	0.1657	0.1538	0.1427	0.1324	0.1229	0.1140	0.1058
8-9	0.2569	0.2360	0.2168	0.1992	0.1830	0.1681	0.1544	0.1419	0.1303	0.1197	0.1100	0.1011	0.0929	0.0853	0.0784
9-10	0.2189	0.1991	0.1811	0.1647	0.1498	0.1363	0.1239	0.1127	0.1025	0.0933	0.0848	0.0772	0.0702	0.0638	0.0581
10-11	0.1866	0.1680	0.1513	0.1362	0.1227	0.1105	0.0995	0.0896	0.0807	0.0726	0.0654	0.0589	0.0530	0.0478	0.0430
11-12	0.1590	0.1417	0.1264	0.1126	0.1004	0.0895	0.0798	0.0711	0.0634	0.0566	0.0504	0.0450	0.0401	0.0357	0.0319
12-13	0.1355	0.1196	0.1055	0.0932	0.0822	0.0726	0.0641	0.0565	0.0499	0.0441	0.0389	0.0343	0.0303	0.0267	0.0236
13-14	0.1154	0.1009	0.0882	0.0770	0.0673	0.0588	0.0514	0.0449	0.0393	0.0343	0.0300	0.0262	0.0229	0.0200	0.0175
14-15	0.0984	0.0851	0.0736	0.0637	0.0551	0.0477	0.0113	0.0357	0.0309	0.0267	0.0231	0.0200	0.0173	0.0150	0.0130
15-16	0.0838	0.0718	0.0615	0.0527	0.0451	0.0387	0.0331	0.0284	0.0243	0.0208	0.0178	0.0153	0.0131	0.0112	0.0096
16-17	0.0714	0.0606	0.0514	0.0436	0.0369	0.0313	0.0266	0.0225	0.0191	0.0162	0.0137	0.0117	0.0099	0.0084	0.0071
17-18	0.0609	0.0511	0.0429	0.0360	0.0303	0.0254	0.0213	0.0179	0.0150	0.0126	0.0108	0.0089	0.0075	0.0063	0.0053
18-19	0.0519	0.0431	0.0358	0.0298	0.0248	0.0206	0.0171	0.0142	0.0118	0.0098	0.0082	0.0068	0.0057	0.0047	0.0039
19-20	[illegible]	[illegible]	0.0299	0.0246	0.0203	0.0167	0.0137	0.0113	0.0083	0.0077	0.0063	0.0052	0.0043	0.0035	0.0029
20-21	0.0377	0.0307	0.0250	0.0204	0.0166										
21-22	0.0321	0.0259	0.0209	0.0169	0.0136										
22-23	0.0274	0.0218	0.0175	0.0139	0.0111										
23-24	0.0233	0.0184	0.0146	0.0115	0.0091										
24-25	0.0199	0.0156	0.0122	0.0095	0.0075										
25-30	0.0126	0.0096	0.0073	0.0056	0.0043										
30-35	0.0057	0.0041	0.0030	0.0022	0.0016										
35-40	0.0025	0.0018	0.0012	0.0008	0.0006										
40-45	0.0011	0.0008	0.0005	0.0003	0.0002										
45-50	0.0005	0.0003	0.0002	0.0001	0.0001										

† Percent is nominal interest compounded continuously = 100r. Year indicates one-year period in which cash flow occurs. See Tables 3 and 4 for information on F_b.

TABLE 7

Discount factors (F_c) with continuous interest to give present worths for cash flows which occur uniformly over a period of T years after the reference point†‡

$100rT$	0	1	2	3	4	5	6	7	8	9
0	1.0000	0.9950	0.9901	0.9851	0.9803	0.9754	0.9706	0.9658	0.9610	0.9563
10	0.9516	0.9470	0.9423	0.9377	0.9332	0.9286	0.9241	0.9196	0.9152	0.9107
20	0.9063	0.9020	0.8976	0.8933	0.8891	0.8848	0.8806	0.8764	0.8722	0.8681
30	0.8639	0.8598	0.8558	0.8517	0.8477	0.8438	0.8398	0.8359	0.8319	0.8281
40	0.8242	0.8204	0.8166	0.8128	0.8090	0.8053	0.8016	0.7979	0.7942	0.7906
50	0.7869	0.7833	0.7798	0.7762	0.7727	0.7692	0.7657	0.7622	0.7588	0.7554
60	0.7520	0.7486	0.7452	0.7419	0.7386	0.7353	0.7320	0.7288	0.7256	0.7224
70	0.7192	0.7160	0.7128	0.7097	0.7066	0.7035	0.7004	0.6974	0.6944	0.6913
80	0.6883	0.6854	0.6824	0.6795	0.6765	0.6736	0.6707	0.6679	0.6650	0.6622
90	0.6594	0.6566	0.6537	0.6510	0.6483	0.6455	0.6428	0.6401	0.6374	0.6348
100	0.6321	0.6295	0.6269	0.6243	0.6217	0.6191	0.6166	0.6140	0.6115	0.6090
110	0.6065	0.6040	0.6016	0.5991	0.5967	0.5942	0.5918	0.5894	0.5871	0.5847
120	0.5823	0.5800	0.5777	0.5754	0.5731	0.5708	0.5685	0.5663	0.5641	0.5618
130	0.5596	0.5574	0.5552	0.5530	0.5509	0.5487	0.5466	0.5444	0.5424	0.5402
140	0.5381	0.5361	0.5340	0.5320	0.5299	0.5279	0.5259	0.5239	0.5219	0.5199
150	0.5179	0.5160	0.5140	0.5121	0.5102	0.5082	0.5064	0.5044	0.5026	0.5007
160	0.4988	0.4970	0.4952	0.4933	0.4915	0.4897	0.4879	0.4861	0.4843	0.4825
170	0.4808	0.4790	0.4773	0.4756	0.4739	0.4721	0.4704	0.4687	0.4671	0.4654
180	0.4637	0.4621	0.4605	0.4588	0.4571	0.4555	0.4540	0.4523	0.4508	0.4491
190	0.4476	0.4460	0.4445	0.4429	0.4414	0.4399	0.4383	0.4368	0.4354	0.4338
200	0.4323	0.4308	0.4294	0.4279	0.4265	0.4250	0.4236	0.4221	0.4207	0.4193
210	0.4179	0.4165	0.4151	0.4137	0.4123	0.4109	0.4096	0.4082	0.4069	0.4055
220	0.4042	0.4029	0.4015	0.4002	0.3989	0.3976	0.3963	0.3950	0.3937	0.3925
230	0.3912	0.3899	0.3887	0.3874	0.3862	0.3849	0.3837	0.3825	0.3813	0.3801
240	0.3789	0.3777	0.3765	0.3753	0.3741	0.3729	0.3718	0.3706	0.3695	0.3683
250	0.3672	0.3660	0.3649	0.3638	0.3627	0.3615	0.3604	0.3593	0.3582	0.3571
260	0.3560	0.3550	0.3539	0.3528	0.3517	0.3507	0.3496	0.3486	0.3476	0.3465
270	0.3455	0.3445	0.3434	0.3424	0.3414	0.3404	0.3393	0.3384	0.3374	0.3364
280	0.3354	0.3344	0.3335	0.3325	0.3315	0.3306	0.3296	0.3287	0.3277	0.3268
290	0.3259	0.3249	0.3240	0.3231	0.3221	0.3212	0.3203	0.3194	0.3185	0.3176
300	0.3167	0.3158	0.3150	0.3141	0.3132	0.3123	0.3115	0.3106	0.3098	0.3089
310	0.3080	0.3072	0.3064	0.3055	0.3047	0.3039	0.3030	0.3022	0.3014	0.3006
320	0.2998	0.2990	0.2982	0.2974	0.2966	0.2958	0.2950	0.2942	0.2934	0.2926
330	0.2919	0.2911	0.2903	0.2896	0.2888	0.2880	0.2873	0.2865	0.2858	0.2850
340	0.2843	0.2836	0.2828	0.2821	0.2814	0.2807	0.2799	0.2792	0.2785	0.2778
350	0.2771	0.2764	0.2757	0.2750	0.2743	0.2736	0.2729	0.2722	0.2715	0.2709
360	0.2702	0.2695	0.2688	0.2682	0.2675	0.2669	0.2662	0.2655	0.2649	0.2642
370	0.2636	0.2629	0.2623	0.2617	0.2610	0.2604	0.2598	0.2591	0.2585	0.2579
380	0.2573	0.2567	0.2560	0.2554	0.2548	0.2542	0.2536	0.2530	0.2524	0.2518
390	0.2512	0.2506	0.2500	0.2495	0.2489	0.2483	0.2477	0.2471	0.2466	0.2460

(Continued)

TABLE 7

Discount factors (F_c) with continuous interest to give present worths for cash flows which occur uniformly over a period of T years after the reference point†‡ (Continued)

100rT	0	1	2	3	4	5	6	7	8	9
400	0.2454	0.2449	0.2443	0.2437	0.2432	0.2426	0.2421	0.2415	0.2410	0.2404
410	0.2399	0.2393	0.2388	0.2382	0.2377	0.2372	0.2366	0.2361	0.2356	0.2350
420	0.2345	0.2340	0.2335	0.2330	0.2325	0.2319	0.2314	0.2309	0.2304	0.2299
430	0.2294	0.2289	0.2284	0.2279	0.2274	0.2269	0.2264	0.2259	0.2255	0.2250
440	0.2245	0.2240	0.2235	0.2230	0.2226	0.2221	0.2216	0.2212	0.2207	0.2202
450	0.2198	0.2193	0.2188	0.2184	0.2179	0.2175	0.2170	0.2166	0.2161	0.2157
460	0.2152	0.2148	0.2143	0.2139	0.2134	0.2130	0.2126	0.2121	0.2117	0.2113
470	0.2108	0.2104	0.21oC	0.2096	0.2091	0.2087	0.2083	0.2079	0.2074	0.2070
480	0.2066	0.2062	0.2058	0.2054	0.2050	0.2046	0.2042	0.2038	0.2034	0.2030
490	0.2026	0.2022	0.2018	0.2014	0.2010	0.2006	0.2002	0.1998	0.1994	0.1990
100rT	0	10	20	30	40	50	60	70	80	90
500	0.1987	0.1949	0.1912	0.1877	0.1843	0.1811	0.1779	0.1749	0.1719	0.1690
600	0.1663	0.1636	0.1610	0.1584	0.1560	0.1536	0.1513	0.1491	0.1469	0.1448
700	0.1427	0.1407	0.1388	0.1369	0.1351	0.1333	0.1315	0.1298	0.1282	0.1265
800	0.1250	0.1234	0.1219	0.1206	0.1190	0.1176	0.1163	0.1149	0.1136	0.1123
900	0.1111	0.1099	0.1087	0.1075	0.1064	0.1053	0.1043	0.1031	0.1020	0.1010
1000	0.1000	0.0990	0.0980	0.0971	0.0962	0.0952	0.0943	0.0935	0.0926	0.0917
1100	0.0909	0.0901	0.0893	0.0885	0.0877	0.0869	0.0862	0.0855	0.0847	0.0840
1200	0.0833	0.0826	0.0820	0.0813	0.0806	0.0800	0.0794	0.0787	0.0781	0.0775
1300	0.0769	0.0763	0.0758	0.0752	0.0746	0.0741	0.0735	0.0730	0.0725	0.0719
1400	0.0714	0.0709	0.0704	0.0699	0.0694	0.0690	0.0685	0.0680	0.0676	0.0671
1500	0.0667	0.0662	0.0658	0.0654	0.0649	0.0645	0.0641	0.0637	0.0633	0.0629
1600	0.0625	0.0621	0.0617	0.0613	0.0610	0.0606	0.0602	0.0599	0.0595	0.0592
1700	0.0588	0.0585	0.0581	0.0578	0.0575	0.0571	0.0568	0.0565	0.0562	0.0559
1800	0.0556	0.0552	0.0549	0.0546	0.0543	0.0541	0.0538	0.0535	0.0532	0.0529
1900	0.0526	0.0524	0.0521	0.0518	0.0515	0.0513	0.0510	0.0508	0.0505	0.0502
2000	0.0500	0.0497	0.0495	0.0492	0.049o	0.0487	0.0485	0.0483	0.0481	0.0478

† r = nominal interest compounded continuously, percent/100; T = n = number of years in time period. See Tables 3 and 4 for information on F_c.

‡ The columns represent the unit increments from 1 to 9 or 10 to 90 for the intervals of $100rT$ shown in the left column.

factors, but the tables are sometimes based on the effective interest rate. To avoid confusion between effective and nominal interest rates, the tables should always present a clear statement in the heading as to the type of interest basis used if there is any possibility for misunderstanding. In case such a necessary statement is not included with the continuous-interest table, the interest figures quoted are probably nominal, but it is advisable to check several of the factors by use of exponential tables to make certain that nominal, rather than effective, interest rates are quoted.

TABLE 8

Discount factors (F_d) with continuous interest to give present worths for cash flows which decline to zero at a constant rate over a period of years n_T starting with the reference point†‡

$100rn_T$	0	1	2	3	4	5	6	7	8	9
0	1.0000	0.9967	0.9934	0.9901	0.9868	0.9835	0.9803	0.9771	0.9739	0.9707
10	0.9675	0.9643	0.9612	0.9580	0.9549	0.9518	0.9487	0.9457	0.9426	0.9396
20	0.9365	0.9335	0.9305	0.9275	0.9246	0.9216	0.9187	0.9158	0.9129	0.9100
30	0.9071	0.9042	0.9013	0.8985	0.8957	0.8929	0.8901	0.8873	0.8845	0.8818
40	0.8790	0.8763	0.8736	0.8708	0.8682	0.8655	0.8628	0.8602	0.8575	0.8549
50	0.8523	0.8497	0.8471	0.8445	0.8419	0.8394	0.8368	0.8343	0.8317	0.8292
60	0.8267	0.8242	0.8218	0.8193	0.8169	0.8144	0.8120	0.8096	0.8072	0.8048
70	0.8024	0.8000	0.7976	0.7953	0.7930	0.7906	0.7883	0.7860	0.7837	0.7814
80	0.7791	0.7769	0.7746	0.7724	0.7701	0.7679	0.7657	0.7635	0.7613	0.7591
90	0.7570	0.7548	0.7527	0.7505	0.7484	0.7462	0.7441	0.7420	0.7399	0.7378
100	0.7358	0.7337	0.7316	0.7295	0.7275	0.7255	0.7235	0.7215	0.7195	0.7175
110	0.7155	0.7135	0.7115	0.7095	0.7076	0.7057	0.7037	0.7018	0.6999	0.6980
120	0.6961	0.6942	0.6923	0.6904	0.6885	0.6867	0.6848	0.6830	0.6812	0.6794
130	0.6776	0.6758	0.6740	0.6722	0.6704	0.6686	0.6668	0.6650	0.6632	0.6615
140	0.6598	0.6580	0.6563	0.6546	0.6529	0.6512	0.6495	0.6478	0.6461	0.6444
150	0.6428	0.6411	0.6394	0.6377	0.6361	0.6345	0.6329	0.6313	0.6297	0.6281
160	0.6265	0.6249	0.6233	0.6217	0.6201	0.6186	0.6170	0.6154	0.6139	0.6124
170	0.6109	0.6093	0.6078	0.6063	0.6048	0.6033	0.6018	0.6003	0.5988	0.5973
180	0.5959	0.5944	0.5929	0.5914	0.5900	0.5886	0.5871	0.5856	0.5842	0.5828
190	0.5814	0.5800	0.5786	0.5772	0.5758	0.5745	0.5731	0.5717	0.5703	0.5690
200	0.5677	0.5663	0.5649	0.5636	0.5623	0.5610	0.5596	0.5583	0.5570	0.5557
210	0.5544	0.5531	0.5518	0.5505	0.5492	0.5480	0.5467	0.5454	0.5441	0.5429
220	0.5417	0.5408	0.5391	0.5379	0.5367	0.5355	0.5342	0.5330	0.5318	0.5306
230	0.5294	0.5282	0.5270	0.5258	0.5246	0.5234	0.5222	0.5210	0.5198	0.5187
240	0.5176	0.5164	0.5152	0.5141	0.5130	0.5119	0.5107	0.5096	0.5085	0.5074
250	0.5063	0.5052	0.5041	0.5030	0.5019	0.5008	0.4997	0.4986	0.4975	0.4964
260	0.4953	0.4942	0.4931	0.4920	0.4910	0.4900	0.4889	0.4878	0.4868	0.4858
270	0.4848	0.4837	0.4827	0.4817	0.4807	0.4797	0.4787'	0.4777	0.4767	0.4757
280	0.4747	0.4737	0.4727	0.4717	0.4707	0.4698	0.4688	0.4678	0.4668	0.4658
290	0.4649	0.4639	0.4629	0.4620	0.4611	0.4602	0.4592	0.4582	0.4573	0.4564
300	0.4555	0.4545	0.4536	0.4527	0.4518	0.4509	0.4500	0.4491	0.4482	0.4473
310	0.4464	0.4455	0.4446	0.4437	0.4428	0.4419	0.4410	0.4401	0.4392	0.4384
320	0.4376	0.4367	0.4358	0.4349	0.4340	0.4332	0.4324	0.4316	0.4308	0.4300
330	0.4292	0.4283	0.4274	0.4266	0.4258	0.4250	0.4242	0.4234	0.4226	0.4218
340	0.4210	0.4202	0.4194	0.4186	0.4178	0.4170	0.4162	0.4154	0.4146	0.4138
350	0.4131	0.4123	0.4115	0.4107	0.4099	0.4091	0.4083	0.4076	0.4069	0.4062
360	0.4055	0.4047	0.4039	0.4031	0.4023	0.4016	0.4009	0.4002	0.3995	0.3988
370	0.3981	0.3973	0.3965	0.3958	0.3951	0.3944	0.3937	0.3930	0.3923	0.3916
380	0.3909	0.3902	0.3895	0.3888	0.3881	0.3874	0.3867	0.3860	0.3853	0.3846
390	0.3840	0.3833	0.3826	0.3819	0.3812	0.3805	0.3798	0.3791	0.3785	0.3779

(Continued)

TABLE 8

Discount factors (F_d) with continuous interest to give present wortbs for cash flows which decline to zero at a constant rate over a period of years n_T starting with the reference point†‡ (Continued)

$100rn_T$	0	1	2	3	4	5	6	7	8	9
400	0.3773	0.3766	0.3759	0.3752	0.3745	0.3738	0.3732	0.3726	0.3720	.37~4
410	0.3708	0.3701	0.3694	0.3687	0.3681	0.3675	0.3669	0.3663	0.3657	.3651
420	0.3645	0.3638	0.3632	0.3626	0.3620	0.3614	0.3608	0.3602	0.3596	0.3590
430	0.3584	0.3578	0.3572	0.3566	0.3560	0.3554	0.3548	0.3542	0.3536	.3530
440	0.3525	0.3519	0.3513	0.3507	0.3501	0.3495	0.3489	0.3483	0.3478	.3473
450	0.3468	0.3462	0.3456	0.3450	0.3444	0.3438	0.3432	0.3427	0.3422	.3417
460	0.3412	0.3406	0.3400	0.3394	0.3388	0.3383	0.3378	0.3373	0.3368	.3363
470	0.3358	0.3352	0.3346	0.3341	0.3336	0.3331	0.3326	0.3321	0.3316	.3311
480	0.3306	0.3300	0.3294	0.3289	0.3284	0.3279	0.3274	0.3269	0.3264	.3259
490	0.3254	0.3249	0.3244	0.3239	0.3234	0.3229	0.3224	0.3219	0.3214	.3209
$100rn_T$	0	10	20	30	40	50	60	70	80	90
500	0.3205	0.3157	0.3111	0.3065	0.3020	0.2978	0.2936	0.2895	0.2856	.2817
600	0.2779	0.2742	0.2707	0.2672	0.2637	0.2604	0.2572	0.2540	0.2509	.2479
700	0.2449	0.2420	0.2392	0.2364	0.2337	0.2311	0.2285	0.2260	0.2235	.2212
800	0.2188	0.2165	0.2143	0.2121	0.2098	0.2076	0.2056	0.2036	0.2016	.1995
900	0.1975	0.1957	0.1939	0.1920	0.1902	0.1884	0.1867	0.1850	0.1834	.1817
1000	0.1800	0.1784	0.1769	0.1753	0.1738	0.1723	0.1709	0.1694	0.1680	.1667
1100	0.1653	0.1639	0.1626	0.1613	0.1601	0.1588	0.1576	0.1563	0.1551	.1539
1200	0.1528	0.1516	0.1505	0.1494	0.1483	0.1472	0.1462	0.1451	0.1441	.1430
1300	0.1420	0.1410	0.1401	0.1390	0.1382	0.1372	0.1363	0.1354	0.1345	.1336
1400	0.1327	0.1318	0.1309	0.1301	0.1292	0.1284	0.1276	0.1268	0.1260	.1252
1500	0.1244	0.1236	0.1229	0.1221	0.1214	0.1207	0.1200	0.1193	0.1186	.1179
1600	0.1172	0.1165	0.1158	0.1151	0.1145	0.1139	0.1133	0.1126	0.1120	.1113
1700	0.1107	0.1101	0.1095	0.1089	0.1084	0.1078	0.1072	0.1066	0.1060	0.1054
1800	0.1049	0.1044	0.1039	0.1033	0.1028	0.1023	0.1018	0.1012	0.1007	.1002
1900	0.0997	0.0992	0.0987	0.0982	0.0978	0.0973	0.0968	0.0963	0.0959	.0955
2000	0.0950	0.0946	0.0942	0.0938	0.0933	0.0928	0.0923	0.0919	0.0915	0.0911

† r = nominal interest compounded continuously, percent/100; n_T = number of years in time period for cash flow to decrease to zero. See Tables 3 and 4 for information on F_d.

‡ The columns represent the unit increments from 1 to 9 or 10 to 90 for the intervals of $100rn_T$ shown in the left column.

COSTS DUE TO INTEREST ON INVESTMENT

Money, or any other negotiable type of capital, has a time value. When a business concern invests money, it expects to receive a return during the time the money is tied up in the investment. The amount of return demanded usually is related to the degree of risk that the entire investment might be lost.

One of the duties of a design engineer is to determine the net return or profit which can be obtained by making an investment. It is necessary, therefore, to find the total cost involved. Too often, the engineer fails to recognize the time value of money and neglects the effects of interest on cost. According to the modern definition of interest, the cost due to time value of an investment should be included as interest for that portion of the total capital investment which comes from outside sources.

Borrowed Capital versus Owned Capital

The question sometimes arises as to whether interest on owned capital can be charged as a true cost. The modern definition of interest permits a definite answer of “no” to this question. Court decisions and income-tax regulations verify this answer.

Interest Effects in a Small Business

In small business establishments, it is usually quite easy to determine the exact source of all capital. Therefore, the interest costs can be obtained with little difficulty. For example, suppose that a young chemical engineer has \$20,000 and decides to set up a small plant for producing antifreeze from available raw materials. For a working-capital plus fixed-capital investment of \$20,000, the chemical engineer determines that the proposed plant can provide a total yearly profit of \$8000 before income taxes. Since the investment was a personal one, interest obviously could not be included as a cost. If it has been necessary to borrow the \$20,000 at an annual interest rate of 10 percent, interest would have been a cost, and the total profit would have been $\$8000 - (0.10) \times (\$20{,}000) = \$6000$ per year.

Interest Effects in a Large Business

In large business establishments, new capital may come from issue of stocks and bonds, borrowing from banks or insurance companies, funds set aside for

TABLE 9
Source of new capital for corporations

Source of capital	Approximate amount of total new capital, %
External financing (loans from banks or other concerns, issue of stocks and bonds)	25
Profits earned but not distributed to stockholders as dividends	30
Depreciation funds set aside	25
Miscellaneous	20

replacement of worn-out or obsolete equipment, profits received but not distributed to the stockholders, and other sources. Therefore, it is often difficult to designate the exact source of new capital, and the particular basis used for determining interest costs should be indicated when the results of a cost analysis are reported.

An approximate breakdown showing the various sources of new capital for large corporations typical of the chemical industry is presented in Table 9.

SOURCE OF CAPITAL

One source of new capital is outside loans. Interest on such loans is usually at a fixed rate, and the annual cost can be determined directly.

New capital may also be obtained from the issue of bonds, preferred stock, or common stock. Interest on bonds and preferred-stock dividends must be paid at fixed rates. A relatively low interest rate is paid on bonds because the bond-holder has first claim on earnings, while higher rates are paid on preferred stock because the holder has a greater chance to lose the entire investment. The holder of common stock accepts all the risks involved in owning a business. The return on common stock, therefore, is not at a fixed rate but varies depending on the success of the company which issued the stock. To compensate for this greater risk, the return on common stock may be much higher than that on bonds or preferred stock.

Income-Tax Effects

The effect of high income-tax rates on the cost of capital is very important. In determining income taxes, interest on loans and bonds can be considered as a cost, while the return on both preferred and common stock cannot be included as a cost. Since corporate income taxes can amount to more than half of the gross earnings, the source of new capital may have a considerable influence on the net profits.

If the annual income-tax rate for a company is 34 percent, every dollar spent for interest on loans or bonds would have a true cost after taxes of only 66 cents. Thus, after income taxes are taken into consideration, a bond issued at an annual interest rate of 6 percent would actually have an interest rate of only $6 \times \frac{66}{100} = 4.0$ percent. On the other hand, the dividends on preferred stock must be paid from net profits after taxes. If preferred stock has an annual dividend rate of 7 percent, the equivalent rate before taxes would be $7 \times \frac{100}{66} = 10.6$ percent.

Despite the fact that it may be cheaper to use borrowed capital in place of other types of capital, it is unrealistic to finance each new venture by using borrowed capital. Every corporation needs to maintain a balanced capital structure and is therefore hesitant about placing itself under a heavy burden of debt.

A comparison of interest or dividend rates for different types of externally financed capital is presented in Table 10.

TABLE 10
Typical costs for externally financed capital

Income tax rate = 34% of (total income − total pretax cost)

Source of capital	Indicated interest or dividend rate, % / year	Actual interest or dividend rate before taxes, % / year	Actual interest or dividend rate after taxes, % / year
Bonds	6	6	4.0
Bank or other loans	7	7	4.6
Preferred stock	7	10.6	7
Common stock	0	13.6	9

METHODS FOR INCLUDING COST OF CAPITAL IN ECONOMIC ANALYSES

The cost of new capital obtained from bonds, loans, or preferred stock can be determined directly from the stated interest or dividend rate, adjusted for income taxes. However, the cost of new capital obtained from the issue of common stock is not so obvious, and some basis must be set for determining this cost. Probably the fairest basis is to consider the viewpoint of existing holders of common stock. If new common stock is issued, its percent return should be at least as much as that obtained from the old common stock; otherwise, the existing stockholders would receive a lower return after the issue of the new stock. Therefore, from the viewpoint of the existing stockholders, the cost of new common stock is the present rate of common-stock earnings.

A major source of new capital is from internal capital, including, primarily, undistributed profits and depreciation funds. Since this definitely is owned capital, it is not necessary to consider interest as a cost. However, some concerns prefer to assign a cost to this type of capital, particularly if comparisons of alternative investments are to be made. The reasoning here is that the owned capital could be loaned out or put into other ventures to give a definite return.

Two methods are commonly used for determining the cost of owned capital. In the first method, the capital is charged at a low interest rate on the assumption that it could be used to pay off funded debts or invest in risk-free loans. The second method requires interest to be paid on the owned capital at a rate equal to the present return on all the company's capital.

Design-Engineering Practice for Interest and Investment Costs

Many alternative methods are used by engineers when determining interest costs in an economic analysis of a design project. In preliminary designs, one of the following two methods is usually employed:

1. No interest costs are included. This assumes that all the necessary capital comes from owned capital, and any comparisons to alternative investments must be on the same basis.

2. Interest is charged on the total capital investment at a set interest rate. Rates equivalent to those charged for bank loans or bonds are usually employed. Under these conditions, the total profit represents the increase over the return that would be obtained if the company could invest the same amount of money in an outside loan at the given interest rate.

As the design proceeds to the final stages, the actual source of new capital should be considered in detail, and more-refined methods for determining interest costs can be used.

When interest is included as a cost, there is some question as to whether the interest costs should be based on the initial investment or on the average investment over the life of the project. Although this is a debatable point, the accepted design practice is to base the interest costs on the initial investment.

Because of the different methods used for treating interest as a cost, a definite statement should be made concerning the particular method employed in any given economic analysis. Interest costs become especially important when making comparisons among alternative investments. These comparisons, as well as the overall cost picture, are simplified if the role of interest in the economic analysis is clearly defined.

NOMENCLATURE FOR CHAPTER 7

a = a constant
C = compound interest factor
C_R = cost for replacement or other asset, dollars
C_V = original cost of equipment or other asset, dollars
d = number of days in an interest period, days, or derivative
e = base of the natural logarithm = 2.71828.. .
F = discount factor
g = constant declining rate or gradient
i = interest rate based on the length of one interest period, percent/100
i_{eff} = effective interest rate-exact interest rate based on an interest period of one year, percent/100
Z = total amount of interest during n interest periods, dollars
K = capitalized cost, dollars
m = number of interest periods per year
n = number of time units or interest periods
n_T = number of time units necessary for cash flow to decrease to zero
P = principal or present worth of capital on which interest is paid, dollars
$\bar{P}$ = principal or present worth considered as occurring regularly throughout the time period, dollars
r = nominal interest rate-approximate interest rate based on an interest period of one year or continuous interest rate, percent/100

R = uniform periodic payments made during n periods in an ordinary annuity, dollars/period

$\bar{R}$ = total of all ordinary annuity payments occurring regularly throughout the time period, dollars/period

S = future worth-amount of principal or present worth plus interest due after n interest periods, dollars

$\bar{S}$ = future worth considered as occurring continuously throughout the time period, dollars

T = time period, years

PROBLEMS

1. It is desired to have \$9000 available 12 years from now. If \$5000 is available for investment at the present time, what discrete annual rate of compound interest on the investment would be necessary to give the desired amount?
2. What will be the total amount available 10 years from now if \$2000 is deposited at the present time with nominal interest at the rate of 6 percent compounded semiannually?
3. An original loan of \$2000 was made at 6 percent simple interest per year for 4 years. At the end of this time, no interest had been paid and the loan was extended for 6 more years at a new, effective, compound-interest rate of 8 percent per year. What is the total amount owed at the end of the 10 years if no intermediate payments are made?
4. A concern borrows \$50,000 at an annual, effective, compound-interest rate of 10 percent. The concern wishes to pay off the debt in 5 years by making equal payments at the end of each year. How much will each payment have to be?
5. The original cost for a distillation tower is \$24,000 and the useful life of the tower is estimated to be 8 years. The sinking-fund method for determining the rate of depreciation is used (see Example **5**), and the effective annual interest rate for the depreciation fund is 6 percent. If the scrap value of the distillation tower is \$4000, determine the asset value (i.e., total book value of equipment) at the end of 5 years.
6. An annuity due is being used to accumulate money. Interest is compounded at an effective annual rate of 8 percent, and \$1000 is deposited at the beginning of each year. What will the total amount of the annuity due be after 5 years?
7. By use of a digital computer, develop and present a printout of the data of effective interest versus nominal interest compounded continuously as given in Table 2.
8. By use of a digital computer, develop and present a printout of the first five lines of Table 7.
9. For total yearly payments of \$5000 for 10 years, compare the compound amount accumulated at the end of 10 years if the payments are (*a*) end-of-year, (*b*) weekly, and (*c*) continuous. The effective (annual) interest is 20 percent and payments are uniform.
10. For the conditions of Prob. 9, determine the present worth at time zero for each of the three types of payments.
11. A heat exchanger has been designed for use in a chemical process. A standard type of heat exchanger with a negligible scrap value costs \$4000 and will have a useful life

of 6 years. Another proposed heat exchanger of equivalent design capacity costs $6800 but will have a useful life of 10 years and a scrap value of $800. Assuming an effective compound interest rate of 8 percent per year, determine which heat exchanger is cheaper by comparing the capitalized costs.

12. A new storage tank can be purchased and installed for $10,000. This tank would last for 10 years. A worn-out storage tank of capacity equivalent to the new tank is available, and it has been proposed to repair the old tank instead of buying the new tank. If the tank were repaired, it would have a useful life of 3 years before the same type of repairs would be needed again. Neither tank has any scrap value. Money is worth 9 percent compounded annually. On the basis of equal capitalized costs for the two tanks, how much can be spent for repairing the existing tank?
13. Equation (28) is the expression for capitalized cost based on discrete interest compounding. For continuous interest compounding, the expression becomes

$$K = C_V + \frac{C_R}{e^{rn} - 1}$$

Present a detailed derivation of this continuous-interest relationship going through each of the equivalent steps used in deriving Eq. (28).

14. The total investment required for a new chemical plant is estimated at $2 million. Fifty percent of the investment will be supplied from the company's own capital. Of the remaining investment, half will come from a loan at an effective interest rate of 8 percent and the other half will come from an issue of preferred stock paying dividends at a stated effective rate of 8 percent. The income-tax rate for the company is 30 percent of pre-tax earnings. Under these conditions, how many dollars per year does the company actually lose (i.e., after taxes) by issuing preferred stock at 8 percent dividends instead of bonds at an effective interest rate of 6 percent?
15. It has been proposed that a company invest $1 million in a venture which will yield a gross income of $1 million per year. The total annual costs will be $800,000 per year including interest on the total investment at an annual rate of 8 percent. In an alternate proposal, the company can invest a total of $600,000 and receive annual net earnings (before income taxes) of $220,000 from the venture. In this case, the net earnings were determined on the basis of no interest costs. The company has $1 million of its own which it wishes to invest, and it can always obtain an effective 6 percent annual interest rate by loaning out the money. What would be the most profitable way for the company to invest its $1 million?

CHAPTER
8

TAXES AND INSURANCE

Expenses for taxes and insurance play an important part in determining the economic situation for any industrial process. Because Federal, state, and local taxes may amount to a major portion of a concern's earnings, it is essential for the chemical engineer to understand the basic principles and factors underlying taxation. Insurance costs ordinarily are only a small part of the total expenditure involved in an industrial operation; however, adequate insurance coverage is necessary before any operation can be carried out on a sound economic basis.

Taxes are levied to supply funds to meet the public needs of a government, while insurance is required for protection against certain types of emergencies or catastrophic occurrences. Insurance rates and tax rates can vary considerably for business concerns as compared to the rates for individual persons. The information presented in this chapter applies generally to large business establishments.

TYPES OF TAXES

Taxes may be classified into three types: (1) property taxes, (2) excise taxes, and (3) income taxes. These taxes may be levied by the Federal government, state governments, or local governments.

Property Taxes

Local governments usually have jurisdiction over ***property taxes***, which are commonly charged on a county basis. In addition to these, individual cities and towns may have special property taxes for industrial concerns located within the city limits.

Property taxes vary widely from one locality to another, but the average annual amount of these charges is 1 to 4 percent of the assessed valuation. Taxes of this type are referred to as ***direct*** since they must be paid directly by the particular concern and cannot be passed on as such to the consumer.

Excise Taxes

Excise ***tares*** are levied by Federal and state governments. Federal excise taxes include charges for import customs duties, transfer of stocks and bonds, and a large number of other similar items. Manufacturers' and retailers' excise taxes are levied by Federal and state governments on the sale of many products such as gasoline and alcoholic beverages. Taxes of this type are often referred to as ***indirect*** since they can be passed on to the consumer. Many business concerns must also pay excise taxes for the privilege of carrying on a business or manufacturing enterprise in their particular localities.

Income Taxes

In general, ***income taxes*** are based on gross earnings, which are defined as the difference between total income and total product cost. Revenue from income taxes is an important source of capital for both Federal and state governments. National and state laws are the basis for these levies, and the laws change from year to year. State income taxes vary from one state to another and are a function of the gross earnings for individual concerns. Depending on the particular state and the existing laws, state income taxes may range from 0 to 5 percent or more of gross earnings.

FEDERAL INCOME TAXES

The Federal government has set up an extremely complex system for determining income taxes for business establishments. New laws are added and old laws are changed each year, and it would be impossible to present all the rules and interpretations in a few pages.? Accordingly, this section will deal only with the basic pattern of Federal income-tax regulations and give the methods generally used for determining Federal income taxes. It should be emphasized strongly that the final determination of income-tax payments should be made with the aid of legal and accounting tax experts.

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†Complete details are available in "Income Tax Regulations" and periodic "Income Tax Bulletins" issued by the U.S. Treasury Department, Superintendent of Documents, Internal Revenue Service and in services published by private concerns, such as the multivolume guide entitled "Prentice-Hall Federal Taxes," giving the latest tax laws with explanations and examples, which is published annually by Prentice-Hall Information Services, Paramus, NJ 07652.

The corporate income-tax rate in the United States has varied widely during the past 60 years. During the period from 1913 to 1935 the tax rate based on gross earnings increased from 1 to 13.75 percent. In 1938, the rate was increased to 19 percent, and, during the Second World War, it was 40 percent plus an excess-profits tax. In 1946, the standard income-tax rate for corporations was reduced to 38 percent, but the rate was increased to 42 percent in 1950 plus an excess-profits tax. During the Korean War the rate was 52 percent plus an excess-profits tax which could result in an overall tax rate of 70 percent on gross earnings. From 1954 through 1963, the corporation income-tax rate was 52 percent with reductions to 50 percent in 1964, 48 percent in 1965, and 46 percent in 1979 on gross earnings above certain base limits. Revision of the income-tax laws in 1986 resulted in a corporation income-tax rate of 34 percent on gross earnings above $75,000 beginning with taxable years starting after June 30, 1987. Table 1 summarizes the standard tax rates for corporations during the period from 1929 to 1988 and Table 2 presents a summary of Federal income taxes on corporations as applicable from 1965 to 1988 based on annual gross earnings.

The figures in Table 1 indicate the wide variations in income-tax rates caused by national emergencies, the prevailing economic situation, and the desires of lawmakers in office at any particular time. Figure 8-1 presents a

TABLE 1
United States national taxes on corporation profits

Year	Regular tax rate, %	Effective limit with wartime excess-profits tax, %
United States:		
1929	11	
1930-1931	12	
1932-1935	13.75	
1936-1937	15	
1938-1939	19	
1940	24	
1941	31	
1942-1943	40	80
1944-1945	40	72
1946-1949	38	
1950	42	52
1951	50.75	68
1952-1953	52	70
1954-1963	52	
1964	50	
1965-1978	48	
1979-1987	46	
1988-	34	

TABLE 2
Federal income taxes on corporations (1965 to 1990)

Year	Taxes	Limitations	Percent of gross earnings
1965-1974	Normal tax	On gross earnings	22
	Surtax	On gross earnings above $25,000	26
	Combined rate	On gross earnings above $25,000	48
	Capital-gains tax	Varies depending on accounting methods	25-30
1975-1978	Normal tax	On first $25,000 of gross earnings	20
		On gross earnings over $25,000	22
	surtax	On gross earnings over $50,000	26
	Combined rate	On gross earnings over $50,000	48
	Capital-gains tax	Varies depending on accounting methods	30
1979-1981	Normal tax	On first $25,000 of gross earnings	17
		On second $25,000 of gross earnings	20
		On third $25,000 of gross earnings	30
		On fourth $25,000 of gross earnings	40
	surtax	Graduated as shown to reach a combined rate of 46 percent on gross earnings above $100,000	
	Combined rate	On gross earnings above $100,000	46
	Capital-gains tax	Varies depending on accounting methods	28
1982-1987	Same as for 1979 to 1981 except the graduated tax-rate percentages were 16%, 19%, 30%, 40% and 46% for 1982 and 15%, 18%, 30%, 40%, and 46% for years from 1983 to 1987. Starting in 1983, the graduated tax rate was phased out for gross earnings above $1,000,000 by increasing taxes for those cases by the lesser of 5% of gross earnings in excess of $1,000,000 or $20,250.		
1988-	Normal tax	On first $50,000 of gross earnings	15
		On gross earnings of $50,000 to $75,000	25
	surtax	Graduated as shown to reach a combined rate of 34 percent on gross earnings above $75,000	
	Combined rate	On gross earnings above $75,000	34
	The graduated tax rate was phased out for gross earnings above $100,000 by increasing taxes for those cases by the lesser of 5% of gross earnings or $11,750.		
	Capital-gains tax	Varies depending on accounting methods but is generally the same as tax rate on all gross earnings for company	34

graphical representation showing the changes in income-tax rates for a typical chemical company for 1943 to 1988.

Many industries have special tax exemptions because of the type of product, market, or service involved in their business, or because the government wishes to offer particular support and inducement to concerns producing essential materials. Even for large concerns with high profits, the overall tax rate can vary widely from year to year depending on the. size of available tax

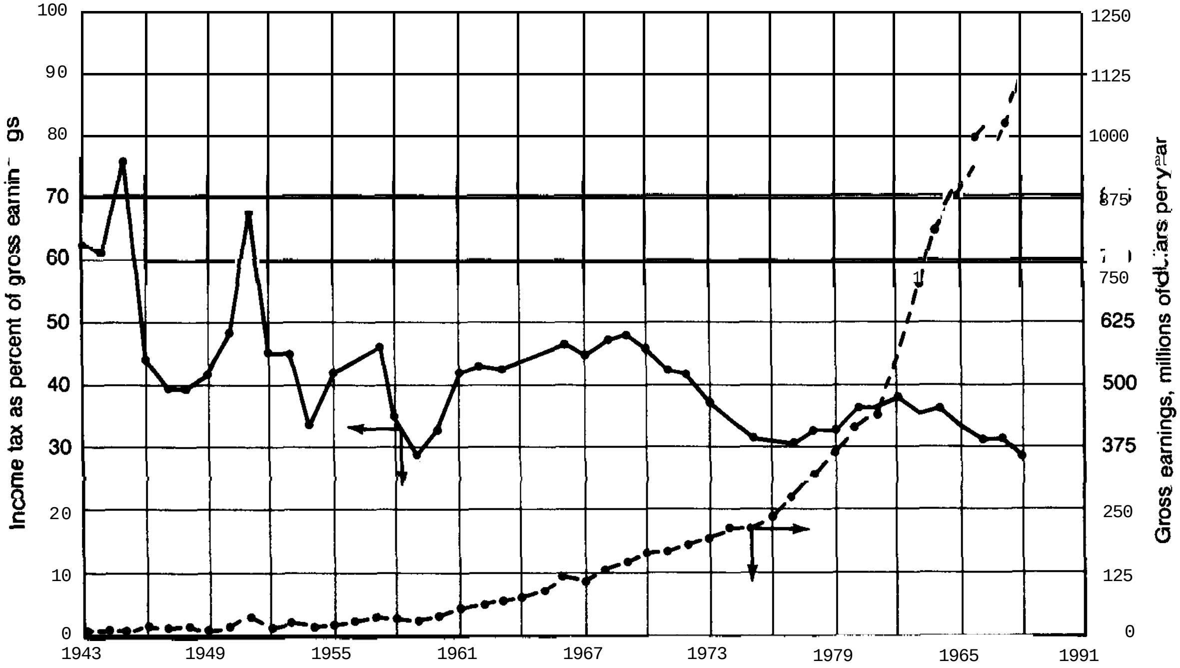

FIGURE 8-1
Example of variation in income-tax rate with time for a chemical company. (Based on *annual reports by Pfizer* Inc.)

deductions or because of carry-back or carry-forward provisions. These possible variations in income-tax effects can have an important influence on the optimum timing for expenditures or other financial transactions.

Normal Tax

A so-called *normal tax* has been levied by the Federal government on the earnings of corporations. This tax was at a rate set by the national lawmakers. For taxable years of 1965 to 1974, the normal tax in the United States was 22 percent of gross earnings. For the years 1975 through 1978, the normal tax was 20 percent of the first $25,000 of gross earnings and 22 percent of gross earnings above $25,000. For 1979, the normal tax was 17 percent of the first $25,000 of gross earnings, 20 percent of the second $25,000, 30 percent of the third $25,000, and 40 percent of the fourth $25,000. These percentages for 1979 were changed to 16, 19, 30, and 40 for 1982 and to 15, 18, 30, and 40 for 1982 to 1987. In 1988, the normal tax was changed to 15 percent of the first $50,000 of gross earnings and 25 percent of gross earnings of $50,000 to $75,000.

Surtax

In addition to the normal tax, corporations have had to pay a second Federal income tax on gross earnings above a certain base limit. This additional tax is known as a *surtax*. The base limit was $25,000 per year for taxable years of 1965 to 1974 and $50,000 per year for taxable years of 1975 through 1978. In this period, the surtax on gross earnings above the limit was at a rate of 26 percent, resulting in a tax rate of 48 percent for gross earnings above $25,000 in 1965 to 1974 and $50,000 in 1975 to 1978. For 1979 to 1987, a graduated tax rate was in effect for the normal tax with the base limit being $100,000 per year and a tax rate of 46 percent applying to gross earnings above this figure. As of 1988, the base limit was changed to $75,000 with a tax rate of 34 percent applying to annual gross earnings above this figure.

Example 1 Determination of annual income taxes. With the Federal income-tax regulations in effect in 1988, the graduated tax rate was phased out for gross earnings above $100,000 by increasing taxes for gross earnings above $100,000 by the lesser of 5 percent of the gross earnings in excess of $100,000 per year or $11,750 (see Table 2). Show that this results in a flat, overall tax rate of 34 percent for all cases of annual gross earnings exceeding $335,000.

Solution. From Table 1, annual tax on gross earnings of $335,000 is

15 percent of first $50,000 = 0.15 × $50,000 = $7500

plus 25 percent of $50,000 to $75,000 = 0.25 × $25,000 = $6250

plus 34 percent of $335,000 − $75,000 = 0.34 × $260,000 = $88,400

plus either 5 percent of $335,000 − $100,000 = 0.05 × $235,000 = $11,750 or $11,750, whichever is smaller.

Thus, total tax is \$7500 + \$6250 + \$88,400 + \$11,750 = **\$113,900** per year, Flat, overall tax rate is **(\$113,900/\$335,000)(100)** = 34 percent on gross earnings of \$335,000 or on any gross earnings greater than this because the limit of \$11,750 for use in adjusting the tax will **always** be less than 5 percent of gross earnings greater than \$335,000 - **\$100,000.**

Capital-Gains Tax

A ***capital-gains tax*** is levied on profits made from the sale of capital assets, such as land, buildings, or equipment. The profit is known as long-term capital gain if the asset was held for more than one year if it was acquired after December 31, 1987 or between December 31, 1977 and June 23, 1984 (six months for property acquired between June 23, 1984 and December 31, 1987, nine months in 1977, and six months before 1977). The profit is known as short-term capital gain if the time held is less than those indicated for long-term capital gain.

The net capital gain is the total of short-term and long-term capital gains, and this total is generally taxed for corporations at the same rate as ordinary income in the year the gain occurred. With this simplified definition of taxes on capital gains, the tax rate for corporations capital gains would be 34 percent starting in 1988. However, there are many special tax regulations for capital gains for corporations relative to items such as carry-back and carry-forward of net capital losses, alternative tax calculation methods, and minimum tax requirements. The tax regulations change regularly and are relatively complicated for corporation capital gains.?

Tax Exemption for Dividends Received

Corporations are given a partial tax exemption for dividends received. In general, only 15 percent of such dividends are considered as taxable income for corporations, with the remaining 85 percent being tax exempt.

Contributions

Corporate contributions to appropriate organizations, as defined by the income-tax laws, can be deducted as an expense up to 10 percent of the taxable income figured without regard to the contribution deduction and other special deductions. Thus, for a corporation which is paying income tax at a 34 percent rate, a contribution of \$10,000 would represent an actual cost to the corporation after taxes of only \$6600.

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†For a complete and up-to-date description, see the most recent annual issue of "Prentice-Hall Federal Tax Handbook," Prentice-Hall Information Services, Paramus, NJ 07652.

Carry-back and Carry-forward of Losses

The preceding analyses of taxes have been based on the assumption that the corporations involved were operating at a profit. In case the situation was one in which a loss resulted, some method of tax accounting needs to be available for this case of negative taxable income. To handle this possible situation, tax regulations permit the corporation to use the loss to offset profits in other years by *carry-back* or *carry-forward* of losses. Tax laws permit a corporation to carry its losses back as charges against profits for as many as three years before the loss or, if necessary, to carry the losses forward as charges against profits for as many as five years after the loss.

Investment Credit

The 1971 Revenue Act of the United States provided for a special first-year tax deduction on new investments for machinery, equipment, and certain other assets used in production processes, in the form of a 7 percent "investment *credit*" for the first year of the life for assets with over 7 years of estimated service life. The investment credit rate was increased to 10 percent for the years 1975 and later, with the possibility of a higher rate. The investment credit amount was limited to the first $25,000 of the corporation's tax liability for the year plus 50 to 90 percent (depending on the year) of the corporation's tax liability above $25,000. This investment credit was repealed in 1986 for properties placed in service after December 31, 1985 with special transition rules applying to carry-forwards, carry-backs, and certain types of property.

Tax revisions, such as those referred to in the preceding sections, are often made for the primary purpose of stimulating or controlling investments and the national economy. Accordingly, one can expect regular changes in the tax regulations, and the assistance of responsible tax experts who keep up with the latest developments is recommended for final evaluation of economic effects.

Taxes and Depreciation

Because Federal income taxes are based on gross earnings, which means that all costs have been deducted, the U.S. Treasury Department has devoted considerable effort to controlling one of the major costs in industrial operations, i.e., the cost for depreciation. The subject of depreciation costs is considered in Chap. 9, where some of the tax regulations by the U.S. Treasury Department are discussed.

In determining the influence of depreciation costs on income taxes, it should be clear that depreciation costs represent a deduction from taxable gross earnings. Thus, if d is the depreciation cost for the year and ϕ is the fractional tax rate,

$$\text{Tax "credit" for depreciation} = \phi d \tag{1}$$

Funds set aside for depreciation, although they represent a cost, normally go directly into the corporation treasury. Therefore, if S represents the total annual income or revenue and C represents the total annual costs with the exceptions of depreciation and taxes,

$$\text{Net annual cash flow to company after taxes}$$
$$= (S - C - d)(1 - \phi) + d = (S - C)(1 - \phi) + \phi d \qquad (2)$$

The preceding equation is applied to various situations of cash flow in Table 3 of Chap. 10.

Excess-Profits Tax

During times of national emergency, certain types of business concerns can realize extremely high income and profit. This is true in particular for concerns producing military necessities during wartime. An ***excess-profits tax*** may be levied, therefore, to supply the national government with part of these profits.

The system for determining excess-profits taxes is extremely complex. In general, the amount of the tax is based on the normal past earnings of a concern or on the total capital investment. Special provisions are made for new corporations or for concerns which do not have a normal past history to use as a basis. The excess-profits taxes are very unpopular with businessmen, and there is always considerable opposition to the levying of these taxes.

Tax Returns

Income-tax returns may be reported on a cash basis or on an accrual basis. When the cash basis is used, only money actually received or paid out during the year is reported. With the accrual method, income and expenses are included as of the time they were incurred, even though final payment has not yet been made.

Returns may be based on a standard calendar year or on a fiscal year. Any date may be chosen as the end of the fiscal year, and it is usually advisable to choose a time when the work of assembly and determination of the tax will be the most convenient. The tax payment itself is usually made in installments.

OTHER TAXES

The Federal Insurance Contribution Act levies a social security tax on most employers and also requires a certain percentage of employees' wages to be withheld. Special local assessments for tax purposes are often encountered, and concerns doing business in foreign countries must pay taxes based on the laws of the foreign countries involved.

The question sometimes arises in cost accounting whether certain service charges and license fees can be considered as taxes. If the charge can be regarded as part of a public duty to support government, it is legally correct to designate the charge as a tax. When the exaction is for a service and the amount charged is a reasonable fee for the service actually received, the cost cannot be considered as a tax. Fees for building permits, government inspections, formation of corporations, bridge and road tolls, and certain types of licenses cannot be charged as taxes because the primary purpose of these fees is to serve for control and regulation rather than to support government.

If a corporation is organized by an individual for the purpose of avoiding high personal-income taxes, the organization is classed as a personal-holding company, and special tax rates apply. The amount of income tax which must be paid by a private business exceeds that required of an equivalent corporation at surprisingly small gross earnings. Consequently, a private business should make a periodic analysis of the advantages and disadvantages of becoming incorporated.

INSURANCE

The annual insurance cost for ordinary industrial concerns is approximately 1 percent of the capital investment. Despite the fact that insurance costs may represent only a small fraction of total costs, it is necessary to consider insurance requirements carefully to make certain the economic operation of a plant is protected against emergencies or unforeseen developments.

The design engineer can aid in reducing insurance requirements if he or she understands the factors which must be considered in obtaining adequate insurance. In particular, the engineer should be aware of the different types of insurance available and the legal responsibilities of a concern with regard to accidents or other unpredictable emergencies.

LEGAL RESPONSIBILITY

A concern can obtain insurance to protect itself against loss of property owing to any of a number of different causes. In case a property loss occurs and the loss is covered by insurance, payment will be made for the damage even though the loss was caused by the owner's negligence.

Protection against unforeseen emergencies, other than direct property loss, can also be obtained through insurance. For example, injuries to employees or persons near the danger area may occur due to a fire or explosion, and the concern involved should have insurance adequate to handle claims made in these cases. It is, of course, impossible to insure against every possible emergency, but it is necessary to consider the results of a potential occurrence, and the legal responsibility for various types of events should be understood. The payments required for settling a case in which legal responsibility has been proved may be much greater than any costs due to direct property damage.

An assumed liability is one which the concern accepts in the form of a written contract or statement, while a ***legal liability*** is always in effect whether or not it is stated in writing. Legal liabilities include civil responsibility for events occurring because of damage or injuries due to negligence. A stronger type of legal liability is known as criminal liability. This is involved in cases where gross negligence or reckless disregard for the life and property of others is claimed.

The design engineer should be familiar with the legal aspects of any laws or regulations governing the type of plant or process involved in a design. In case of an accident, failure to comply with the definite laws involved is a major factor in fixing legal responsibility. Compliance with all existing laws, however, is not a sufficient basis for disallowance of legal liability. Every known safety feature should be included and extraordinary care in the complete operation must be proved before a good case can be presented for disallowing legal liability.

Many contracts include *hold-harmless agreements* wherein the legal responsibility for an accident or other type of event is indicated as part of a written agreement. Any new lease or contract should be examined by an expert to make certain all hold-harmless agreements are clearly stated and understood by both parties.

Any concern producing a product which may be dangerous to life or property has a legal responsibility to indicate the potential hazard by use of warning labels or other protective methods. The manufacturer must supply safe shipping containers and make certain that any hazards involved in their handling or use are clearly indicated. Legal liability also holds for defective or misrepresented products.

A manufacturing establishment may have on its property some object which would be highly attractive as a place for children to play. Two examples would be a quarry pit and a sand pile. An object of this type is known as an *attractive nuisance*, and the concern may be liable for injuries to children if the injuries are a result of their playing around or in the object. The liability would apply even though the children were obviously trespassing. High fences or some other effective safety measure should be used to keep children from gaining admittance to an attractive nuisance.

An industrial concern has a legal responsibility for property belonging to others as long as the property is on the concern's premises. This responsibility is known as a *bailee's liability*. The property may be stored equipment or materials, finished products, or products in process. If the property is damaged or destroyed, the bailee's liability is roughly a function of the degree of care used in safeguarding the property. In case the damaged or destroyed property is insured by the owner, the insurance company will pay the claim; however, the insurance company can then exercise its *subrogation rights* and attempt to force the bailee to pay the full amount received by the owner.

TYPES OF INSURANCE

Many different types of insurance are available for protection against property loss or charges based on legal liability. Despite every precaution, there is always the possibility of an unforeseen event causing a sudden drain on a company's finances, and an efficient management protects itself against such potential emergencies by taking out insurance. In order to make an intelligent analysis of insurance requirements for any kind of operation, it is necessary to understand the physical factors involved in carrying out the process and to be aware of the types of insurance available.

The major insurance requirements for manufacturing concerns can be classified as follows:

1. Fire insurance and similar emergency coverage on buildings, equipment, and all other owned, used, or stored property. Included in this category would be losses caused by lightning, wind- or hailstorms, floods, automobile accidents, explosions, earthquakes, and similar occurrences.
2. Public-liability insurance, including bodily injury and property loss or damage, on all operations such as those involving automobiles, elevators, attractive nuisances, bailee's charges, aviation products, or any company function carried on at a location away from the plant premises.
3. Business-interruption insurance. The loss of income due to a business interruption caused by a fire or other emergency may far exceed any loss in property. Consequently, insurance against a business interruption of this type should be given careful consideration.
4. Power-plant, machinery, and special-operations hazards.
5. Workmen's-compensation insurance.
6. Marine and transportation insurance on all property in transit.
7. Comprehensive crime coverage.
8. Employee-benefit insurance, including life, hospitalization, accident, health, personal property, and pension plans.

Self-Insurance

On an average basis, insurance companies pay out loss claims amounting to 55 to 60 cents for each dollar received. The balance is used for income taxes, salaries, commissions, administrative costs, inspection costs, and various overhead costs. Theoretically, a saving of 40 to 45 cents per dollar paid for insurance could be achieved by self-insurance. If insurance requirements are great, this saving could amount to a very large sum, and it would be worthwhile to consider the possibilities of self-insurance.

A careful analysis of all risks involved is necessary when considering self-insurance on possible losses or emergencies. The final decision should not be based on whether or not the insurable event will occur, because this is impossible to predict. Instead, the decision should be based on the total loss involved if the event or a series of such events were to occur. If an industrial concern has a number of widespread interests and sufficient funds available to handle simultaneous major losses in several of these interests, it might be reasonable to consider self-insurance on some of the potential hazards. On the other hand, if a single potential event could ruin the economic standing of the company, it would be very inadvisable to assume the risk involved in self-insurance.

There are several different ways of applying self-insurance. One method involves depositing money equivalent to an insurance premium into a special company fund. This fund can then be used to handle any losses or emergencies which may occur. At the outset, this fund would be small and would be inadequate to handle any major losses. Consequently, if this method is used, it may be necessary to supply an original base fund or else assume a disproportionate amount of risk until the fund has built up to a practical value. Under ordinary conditions, the premiums paid into a self-insurance reserve are not tax-deductible.

A second method may be used in which the company assumes all the risk and no payments are made into a reserve fund. This method is designated as "self-assumption of risk." Partial self-insurance may be obtained through the purchase of deductible insurance from regular agencies. The purchaser assumes the risk up to a certain amount and the insurance company agrees to pay for any additional losses.

The effects of income taxes should be considered when making a final decision regarding insurance. Because the premiums for standard insurance are tax-deductible, the actual cost after taxes for the protection may be much less than the direct premium charge. Another advantage of standard insurance is the inspection services supplied by the insurance companies. These companies require periodic inspections by specialists to make certain that the insurance rates are adequate, and the reports of these inspectors often indicate new ideas or methods for increasing the safety of the operation.

The overall policies of the particular manufacturing concern dictate the type and amount of insurance which will be held. It should be realized, however, that a well-designed insurance plan requires a great deal of skilled and informed investigation by persons who understand all the aspects of insurance as well as the problems involved in the manufacturing operation.

PROBLEMS

1. The fixed-capital investment for an existing chemical plant is $20 million. Annual property taxes amount to 1 percent of the fixed-capital investment, and state income taxes are 5 percent of the gross earnings. The net income per year after all taxes is $2 million, and the Federal income taxes amount to 34 percent of gross earnings. If the same plant had been constructed at a location where property taxes were 4 percent of the fixed-capital investment and state income taxes were 2 percent of the gross earnings, what would be the net income per year after taxes, assuming all other cost factors were unchanged?
2. The gross earnings for a small corporation were $54,000 in 1970. What would have been the percent reduction in Federal income taxes paid by the company if the tax

rates in effect in 1988 had been in effect in 1970? (See Table 2 for Federal income-tax rules in effect.)

3. During the period of one taxable year at a manufacturing plant, the total income on a cash basis was $21 million. Five million dollars of immediate debts due the company was unpaid at the end of the year. The company paid out $15 million on a cash basis during the year, and all of this amount was tax-deductible as a product cost. The company still owed $3 million of tax-deductible bills at the end of the year. If the total Federal income tax for the company amounts to 34 percent of the gross earnings, determine the amount of Federal income tax due for the year on a cash basis and also on an accrual basis.
4. Complete fire and allied-coverage insurance for one unit of a plant requires an annual payment of $700 based on an investment value of $100,000. If income taxes over a IO-year period average 30 percent of gross earnings, by how much is the net income, after taxes, reduced during this lo-year period owing to the cost of the insurance?
5. Self-insurance is being considered for one portion of a chemical company. The fixed-capital investment involved is $50,000, and insurance costs for complete protection would amount to $400 per year. If self-insurance is used, a reserve fund will be set up under the company's jurisdiction, and annual insurance premiums of $300 will be deposited in this fund under an ordinary annuity plan. All money in the fund can be assumed to earn interest at a compound annual rate of 5 percent. Neglecting any charges connected with administration of the fund, how much money should be deposited in the fund at the beginning of the program in order to have enough money accumulated to replace a complete $50,000 loss after 10 years?
6. A corporation shows a gross earnings or net profit before Federal income taxes of $200,000 in the taxable years of 1978, 1981, and 1988. The taxable income for the corporation in all 3 years is the $200,000 given, and there are no special tax exemptions for the corporation so that the situation described in this chapter under the headings of normal *tax* and *surtax* applies (see Table 2). Determine the Federal income tax paid by the corporation in each of the 3 years and find the amount of tax saved in 1988 because of the tax regulations in effect that year as compared to 1981 and 1978. Repeat for the case of a gross earnings of $50,000 instead of $200,000.
7. Locate the most recent issues of the Annual Report for Pfizer Inc. in your local business library past 1988 and extend the data for gross earnings and income tax as percent of gross earnings presented in Fig. 8-1 of this chapter.

CHAPTER
9

DEPRECIATION

An analysis of costs and profits for any business operation requires recognition of the fact that physical assets decrease in value with age. This decrease in value may be due to physical deterioration, technological advances, economic changes, or other factors which ultimately will cause retirement of the property. The reduction in value due to any of these causes is a measure of the ***depreciation.?*** The economic function of depreciation, therefore, can be employed as a means of distributing the original expense for a physical asset over the period during which the asset is in use.

Because the engineer thinks of depreciation as a measure of the decrease in value of property with time, depreciation can immediately be considered from a cost viewpoint. For example, suppose a piece of equipment had been put into use 10 years ago at a total cost of $31,000. The equipment is now worn out and is worth only $1000 as scrap material. The decrease in value during the lo-year period is $30,000; however, the engineer recognizes that this $30,000 is in reality a cost incurred for the use of the equipment. This depreciation cost was spread over a period of 10 years, and sound economic procedure would require part of this cost to be charged during each of the years. The application

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[†] According to the Internal Revenue Service, depreciation is defined as "A reasonable allowance for the exhaustion, wear, and tear of property used in the trade or business including a reasonable allowance for obsolescence." The terms ***amortization*** and ***depreciation*** are often used interchangeably. Amortization is usually associated with a definite period of cost distribution, while depreciation usually deals with an unknown or estimated period over which the asset costs are distributed. Depreciation and amortization are of particular significance as an accounting concept which serves to reduce taxes.

of depreciation in engineering design, accounting, and tax studies is almost always based on costs prorated throughout the life of the property.

Meaning of Value

From the viewpoint of the design engineer, the total cost due to depreciation is the original or new value of a property minus the value of the same property at the end of the depreciation period. The original value is usually taken as the total cost of the property at the time it is ready for initial use. In engineering design practice, the total depreciation period is ordinarily assumed to be the length of the property's useful life, and the value at the end of the useful life is assumed to be the probable scrap or salvage value of the components making up the particular property.

It should be noted here that the engineer cannot wait until the end of the depreciation period to determine the depreciation costs. These costs must be prorated throughout the entire life of the property, and they must be included as an operating charge incurred during each year. The property value at the end of the depreciation period and the total length of the depreciation period cannot be known with certainty when the initial yearly costs are determined. Consequently, it is necessary to estimate the final value of the property as well as its useful life. In estimating property life, the various factors which may affect the useful-life period, such as wear and tear, economic changes, or possible technological advances, should be taken into consideration.

When depreciation is not used in a prorated-cost sense, various meanings can be attached to the word value. One of these meanings involves appraisal of both initial and final values on the basis of conditions at a certain time. The difference between the estimated cost of new equivalent property and the appraised value of the present asset is ***known*** as the ***appraised depreciation.*** This concept involves determination of the values of two assets at one date as compared with the engineering-cost concept, which requires determination of the value of one asset at two different times.

Purpose of Depreciation as a Cost

Consideration of depreciation as a cost permits realistic evaluation of profits earned by a company and, therefore, provides a basis for determination of Federal income taxes. Simultaneously, the consideration of depreciation as a cost provides a means whereby funds are set aside regularly to provide recovery of the invested capital. When accountants deal with depreciation, they must follow certain rules which are established by the U.S. Bureau of Internal Revenue for determination of income taxes. These rules deal with allowable life for the depreciable equipment and acceptable mathematical procedures for allocating the depreciation cost over the life of the asset.

Although any procedure for depreciation accounting can be adopted for internal company evaluations, it is highly desirable to keep away from the necessity of maintaining two sets of accounting books. Therefore, the engineer should be familiar with Federal regulations relative to depreciation and should follow these regulations as closely as possible in evaluating depreciation as a cost.

TYPES OF DEPRECIATION

The causes of depreciation may be physical or functional. ***Physical depreciation*** is the term given to the measure of the decrease in value due to changes in the physical aspects of the property. Wear and tear, corrosion, accidents, and deterioration due to age or the elements are all causes of physical depreciation. With this type of depreciation, the serviceability of the property is reduced because of physical changes. Depreciation due to all other causes is known as ***functional depreciation.***

One common type of functional depreciation is ***obsolescence.*** This is caused by technological advances or developments which make an existing property obsolete. Even though the property has suffered no physical change, its economic serviceability is reduced because it is inferior to improved types of similar assets that have been made available through advancements in technology.

Other causes of functional depreciation could be (1) change in demand for the service rendered by the property, such as a decrease in the demand for the product involved because of saturation of the market, (2) shift of population center, (3) changes in requirements of public authority, (4) inadequacy or insufficient capacity for the service required, (5) termination of the need for the type of service rendered, and (6) abandonment of the enterprise. Although some of these situations may be completely unrelated to the property itself, it is convenient to group them all under the heading of functional depreciation.

Because depreciation is measured by decrease in value, it is necessary to consider all possible causes when determining depreciation. Physical losses are easier to evaluate than functional losses, but both of these must be taken into account in order to make fair allowances for depreciation.

Depletion

Capacity loss due to materials actually consumed is measured as ***depletion.*** Depletion cost equals the initial cost times the ratio of amount of material used to original amount of material purchased. This type of depreciation is particularly applicable to natural resources, such as stands of timber or mineral and oil deposits.

Costs for Maintenance and Repairs

The term ***maintenance*** conveys the idea of constantly keeping a property in good condition; ***repairs*** connotes the replacing or mending of broken or worn parts of a property. The costs for maintenance and repairs are direct operating expenses which must be paid from income, and these costs should not be confused with depreciation costs.

The extent of maintenance and repairs may have an effect on depreciation cost, because the useful life of any property ought to be increased if it is kept in good condition. However, a definite distinction should always be made between costs for depreciation and costs for maintenance and repairs.

SERVICE LIFE

The period during which the use of a property is economically feasible is known as the service *life* of the property. Roth physical and functional depreciation are taken into consideration in determining service life, and, as used in this book, the term is synonymous with ***economic*** or ***useful*** life. In estimating the probable service life, it is assumed that a reasonable amount of maintenance and repairs will be carried out at the expense of the property owner.

Many data are available concerning the probable life of various types of property. Manufacturing concerns, engineers, and the U.S. Internal Revenue Service (IRS) have compiled much information of this sort. All of these data are based on past records, and there is no certainty that future conditions will be unchanged. Nevertheless, by statistical analysis of the various data, it is possible to make fairly reliable estimates of service lives.

The U.S. Internal Revenue Service recognizes the importance of depreciation as a legitimate expense, and the IRS has issued formal statements which list recommended service lives for many types of properties.? Prior to July 12, 1962, Federal regulations for service lives and depreciation rates were based on the so-called ***Bulletin "F," "Income Tax Depreciation and Obsolescence-Estimated Useful Lives and Depreciation Rates"*** as originally published by the U.S. Internal Revenue Service in 1942. In July, 1962, the ***Bulletin "F"*** regulations were replaced by a set of new guidelines based on four groups of depreciable assets. The 1971 Revenue Act of the United States provided more flexibility in choosing depreciation life by allowing a choice of depreciation life of 20 percent longer or shorter than the guideline lives called for by earlier tax laws for machinery, equipment, or other assets put in service after December 31, 1970. This is known as the ***Class Life Asset Depreciation Range System*** (ADR).

Table 1 presents estimated service lives for equipment based on the four group guidelines as recommended by the Internal Revenue Service in the 1962 Federal regulations. These values, along with similar values as presented in ***Bulletin "F,"*** can serve as an indication of acceptable and useful lives to those not using other procedures.

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†For an up-to-date presentation of Federal ***income-tax*** regulations as related to depreciation, including estimation of service lives, see the latest annual issue of "Prentice-Hall Federal Taxes," Prentice-Hall Information Services, Paramus, NJ 07652.

TABLE 1

Estimated life of equipment

The following tabulation for estimating the life of equipment in years is an abridgement of information from "Depreciation-Guidelines and Rules" (Rev. Proc. 62-21) issued by the Internal Revenue Service of the U.S. Treasury Department as Publication No. 456 (7-62) in July, 1962. See Table 2 for an extended and more flexible interpretation including repair allowance as approved by the Federal regulations in 1971.

	Life, years
Group I: General business assets	
1. Office furniture, fixtures, machines, equipment	10
2. Transportation	
a. Aircraft	6
b. Automobile	3
c. Buses	9
d. General-purpose trucks	4-6
e. Railroad cars (except for railroad companies)	15
f. Tractor units	4
g. Trailers	6
h. Water transportation equipment	18
3. Land and site improvements (not otherwise covered)	20
4. Buildings (apartments, banks, factories, hotels, stores, warehouses)	40-60
Group II: Nonmanufacturing activities (excluding transportation, communications, and public utilities)	
1. Agriculture	
a. Machinery and equipment	10
b. Animals	3-10
c. Trees and vines	variable
d. Farm buildings	25
2. Contract construction	
a. General	5
b. Marine	12
3. Fishing	variable
4. Logging and sawmilling	6-10
5. Mining (excluding petroleum refining and smelting and refining of minerals)	10
6. Recreation and amusement	10
7. Services to general public	10
8. Wholesale and retail trade	10
Group III: Manufacturing	
1. Aerospace industry	8
2. Apparel and textile products	9
3. Cement (excluding concrete products)	20
4. Chemicals and allied products	11
5. Electrical equipment	
a. Electrical equipment in general	12
b. Electronic equipment	8
6. Fabricated metal products	12
7. Food products, except grains, sugar and vegetable oil products	12
8. Glass products	14
9. Grain and grain-mill products	17

(Continued)

TABLE 1
Estimated life of equipment (Continued)

	Life, years
Group III: Manufacturing (continued)	
10. Knitwear and knit products	9
11. Leather products	11
12. Lumber, wood products, and furniture	10
13. Machinery unless otherwise listed	12
14. Metalworking machinery	12
15. Motor vehicles and parts	12
16. Paper and allied products	
a. Pulp and paper	16
b. Paper conversion	12
17. Petroleum and natural gas	
a. Contract drilling and field service	6
b. Company exploration, drilling, and production	14
c. Petroleum refining	16
d. Marketing	16
18. Plastic products	11
19. Primary metals	
a. Ferrous metals	18
b. Nonferrous metals	14
20. Printing and publishing	11
21. Scientific instruments, optical and clock manufacturing	12
22. Railroad transportation equipment	12
23. Rubber products	14
24. Ship and boat building	12
25. Stone and clay products	15
26. Sugar products	18
27. Textile mill products	12-14
28. Tobacco products	15
29. Vegetable oil products	18
30. Other manufacturing in general	12
Group IV: Transportation, communications, and public utilities	
1. Air transport	6
2. Central steam production and distribution	28
3. Electric utilities	
a. Hydraulic	50
b. Nuclear	20
c. Steam	28
d. Transmission and distribution	30
4. Gas utilities	
a. Distribution	35
b. Manufacture	30
c. Natural-gas production	14
d. Trunk pipelines and storage	22
5. Motor transport (freight)	8
6. Motor transport (passengers)	8
7. Pipeline transportation	22
8. Radio and television broadcasting	6

(Continued)

TABLE 1
Estimated life of equipment *(Continued)*

	Life, years
Group IV: Transportation, communications, and public utilities (continued)	
9. Railroads	
a. Machinery and equipment	14
b. Structures and similar improvements	30
c. Grading and other right of way improvements	variable
d. Wharves and docks	20
10. Telephone and telegraph communications	variable
11. Water transportation	20
12. Water utilities	50

Table 2 gives a partial listing of the *Class Life Asset Depreciation Range System* (CLADR) as recommended for use by Federal regulations in 1971. The table shows the basic guideline life period as recommended in the earlier regulations along with the 20 percent variation allowed plus recommended guidelines for repair and maintenance allowance. Although these values are recommended by the Internal Revenue Service, the IRS does not require taxpayers to use the indicated lives. However, if other life periods are used, the taxpayer must be prepared to support the claim.

Tax-law changes put into effect with the 1981 Economic Recovery Act and modified in 1986 have instituted a new system of depreciation known as the *Accelerated Cost Recovery System* (ACRS). The latter has replaced the former ADR system for most tangible depreciable property used in a trade or business placed in service on or after January 1, 1981. In the ACRS [or *Modified Accelerated Cost Recovery System* (MACRS) which went into effect for property put into service on or after January 1, 19871, the recovery of capital costs as depreciation was determined over statutory periods of time using statutory percentages depending on the *class life* of the property and the number of years since the property was placed in service. The statutory periods of time were generally shorter than the useful life of the asset or the period for which it was used to produce income.

The statutory class lives for the Modified Accelerated Cost Recovery System are as follows where the key factor is the ADR (Asset Depreciation Range) midpoint designation, which corresponds in general to the asset guideline period shown in Table 2:

Three-year class-ADR midpoint of 4 years and less. This includes items such as machinery and equipment used in research, some automobiles, and certain types of trailers.

Five-year class-ADR midpoint of 4 to 10 years. This includes most production machinery, heavy trucks, and some automobiles and light trucks.

TABLE 2
Class life asset depreciation range†

	Asset depreciation range (ADR) (in years)			Annual asset guideline repair allowance, percentage of cost
Description of class life asset	Lower limit	Asset guideline period (Midpoint)	Upper limit	
Assets used in business activities:				
Office furniture, fixtures, and equipment	8	10	12	2
Information systems, computers, peripheral equipment	5	6	7	7.5
Data handling equipment, except computers	5	6	7	15
Airplanes, except commercial	5	6	7	14
Automobiles, taxis	2.5	3	3.5	16.5
Buses	7	9	11	11.5
Light general-purpose trucks	3	4	5	16.5
Heavy general-purpose trucks	5	6	7	10
Railroad cars and locomotives, except owned by railroad transportation companies	12	15	18	8
Tractor units for use over-the-road	3	4	5	16.5
Trailers and trailer-mounted containers	5	6	7	10
Vessels, barges, tugs and similar water-transportation equipment	14.5	18	21.5	6
Land improvements		20		
Industrial steam and electricity generation and/or distribution systems	22.5	33.5	33.5	2.5
Assets used in agriculture	8	10	12	11
Assets used in mining	8	10	12	6.5
Assets used in drilling of oil and gas wells	5	6	7	10
Assets used in exploration for and production of petroleum and natural gas deposits	5	6	7	10
Assets used in petroleum refining	13	16	19	7
Assets used in marketing of petroleum and petroleum products	13	16	19	4
Assets used in contract construction other than marine	4	5	6	12.5
Assets used in marine contract construction	9.5	12	14.5	5

† Values were excerpted from the listing given with full description of each category in the 1988 "Prentice Hall Federal Taxes" guide as updated from the original Federal regulation to the March 21, 1977 Revenue Procedure in the Internal Revenue Bulletin. The official documents originally setting up the ADR system were U.S. Treasury Decision 7128 in 1971 and Revenue Procedure 71-25 in 1971.

TABLE 2
Class life asset depreciation range† (*Continued*)

Description of class life asset	Asset depreciation range (ADR) (in years) Lower limit	Asset guideline period (Midpoint)	Upper limit	Annual asset guideline repair allowance, percentage of cost
Assets used in the manufacture of:				
Grain and grain-mill products	9.5	12	14.5	5
Sugar and sugar products	13.5	17	20.5	6
Vegetable oils and vegetable-oil products	14.5	18	21.5	4.5
Other food and kindred products	14.5	18	21.5	3.5
Tobacco and tobacco products	12	15	18	5
Knitted goods	6	7.5	9	7
Nonwoven fabrics	8	10	12	15
Wood products and furniture	5	6	7	10
Pulp and paper	8	10	12	6.5
Chemicals and allied products	9	11	13	5.5
Rubber products	11	14	17	5
Finished plastic products	9	11	13	5.5
Glass products	11	14	17	12
Cement	16	20	24	3
Machinery	8	10	12	11
Electrical equipment	9.5	12	14.5	5.5
Motor vehicles	9.5	12	14.5	9.5
Assets used in electric, gas, water, and steam utility services:				
Electric utility nuclear production plant	16	20	24	3
Electric utility steam production plant	22.5	28	33.5	5
Electric utility transmission and distribution plant	24	30	36	4.5
Gas utility distribution facilities	28	35	42	2
Gas utility manufactured gas production plant	24	30	36	2
Substitute natural gas-coal gasification (Lurgi process with advanced methanation)	14.5	18	21.5	15
Natural gas production plant	11	14	17	4.5
Liquefied natural gas plant	17.5	22	26.5	4.5
Water utilities	40	50	60	1.5
Central steam utility production and distribution	22.5	28	33.5	2.5

†Values were excerpted from the listing given with full description of each category in the 1988 "Prentice Hall Federal Taxes" guide as updated from the original Federal regulation to the March 21, 1977 Revenue Procedure in the Internal Revenue Bulletin. The official documents originally setting up the ADR system were U.S. Treasury Decision 7128 in 1971 and Revenue Procedure 71-25 in 1971.

Seven-year class-ADR midpoint of 10 to 16 years. Included here are items such as office furniture and equipment.

Ten-year class-ADR midpoint of 16 to 20 years. This includes properties such as tank cars and assets used in petroleum refining and food manufacturing.

Fifteen-year class-ADR midpoint of 20 to 25 years. Included here are items related to certain chemical production processes and some utilities.

Twenty-year class-ADR midpoint of 25 years or more. This includes many utilities and electrical distribution systems.

For the Accelerated Cost Recovery System in effect after 1980 and before 1987, the statutory classes were 3 year, 5 year, 10 year, and 15 year. The two classes of 7 year and 20 year were added in the Modified Accelerated Cost Recovery System for properties put into service on January 1, 1987 or later.

There has been considerable demand for a wider choice of service lives for properties, and the widespread revision and reinterpretation of the national income-tax laws in 1954, 1962, 1971, 1981, and 1986 met part of this demand. During times of national emergencies, the United States Congress may approve rapid-amortization policies to make it more attractive for concerns to invest in additional plants and equipment needed for the national welfare. Certificates of necessity can be obtained for certain types of industries, and these certificates permit writing off various percentages of the value of new equipment over selected periods of time.

SALVAGE VALUE

Salvage value is the net amount of money obtainable from the sale of used property over and above any charges involved in removal and sale. If a property is capable of further service, its salvage value may be high. This is not necessarily true, however, because other factors, such as location of the property, existing price levels, market supply and demand, and difficulty of dismantling, may have an effect. The term salvage *value* implies that the asset can give some type of further service and is worth more than merely its scrap or junk value.

If the property cannot be disposed of as a useful unit, it can often be dismantled and sold as junk to be used again as a manufacturing raw material. The profit obtainable from this type of disposal is known as the *scrap,* or *junk, value.*

Salvage value, scrap value, and service life are usually estimated on the basis of conditions at the time the property is put in use. These factors cannot be predicted with absolute accuracy, but improved estimates can be made as the property increases in age. It is advisable, therefore, to make new estimates from time to time during the service life and make any necessary adjustments in the depreciation costs. Because of the difficulties involved in making reliable estimates of salvage and scrap values, engineers often neglect the small error involved and designate these values as zero. Federal tax regulations generally limit salvage or scrap values to 10 percent or less of the initial value of the property.

PRESENT VALUE

The *present value* of an asset may be defined as the value of the asset in its condition at the time of valuation. There are several different types of present values, and the standard meanings of the various types should be distinguished.

Book Value, or Unamortized Cost

The difference between the original cost of a property, and all the depreciation charges made to date is defined as the *book value* (sometimes called *unamortized cost*). It represents the worth of the property as shown on the owner's accounting records.

Market Value

The price which could be obtained for an asset if it were placed on sale in the open market is designated as the *market value*. The use of this term conveys the idea that the asset is in good condition and that a buyer is readily available.

Replacement Value

The cost necessary to replace an existing property at any given time with one at least equally capable of rendering the same service is known as the *replacement value*.

It is difficult to predict future market values or replacement values with a high degree of accuracy because of fluctuations in market demand and price conditions.? On the other hand, a future book value can be predicted with absolute accuracy as long as a constant method for determining depreciation costs is used. It is quite possible for the market value, replacement value, and book value of a property to be widely different from one another because of unrealistic depreciation allowances or changes in economic and technological factors.

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†See Chap. 11 (Optimum Design and Design Strategy) for a discussion on inflation and the strategy for considering it.

METHODS FOR DETERMINING DEPRECIATION

Depreciation costs can be determined by a number of different methods, and the design engineer should understand the bases for the various methods. The Federal government has definite rules and regulations concerning the manner in which depreciation costs may be determined. These regulations must be followed for income-tax purposes as well as to obtain most types of governmental support. Since the methods approved by the government are based on sound economic procedures, most industrial concerns use one of the government-sanctioned methods for determining depreciation costs, both for income-tax calculations and for reporting the concern's costs and profits.[†] It is necessary, therefore, that the design engineer keep abreast of current changes in governmental regulations regarding depreciation allowances.

In general, depreciation accounting methods may be divided into two classes: (1) arbitrary methods giving no consideration to interest costs, and (2) methods taking into account interest on the investment. Straight-line, declining-balance, and sum-of-the-years-digits methods are included in the first class, while the second class includes the sinking-fund and the present-worth methods.

Straight-Line Method

In the straight-line *method* for determining depreciation, it is assumed that the value of the property decreases linearly with time. Equal amounts are charged for depreciation each year throughout the entire service life of the property. The annual depreciation cost may be expressed in equation form as follows:

$$d = \frac{V - V_s}{n} \tag{1}$$

where d = annual depreciation, \$/year
V = original value of the property at start of the service-life period, completely installed and ready for use, dollars
V_s = salvage value of property at end of service life, dollars
n = service life, years

The asset value (or book value) of the equipment at any time during the service life may be determined from the following equation:

$$V_a = V - a\,d \tag{2}$$

[†] An alternate procedure often used by industrial concerns is to use straight-line depreciation for reporting profits and one of the accelerated-depreciation methods as approved by Federal regulations for income-tax calculations.

where V_a = asset or book value, dollars, and a = the number of years in actual use.

Because of its simplicity, the straight-line method is widely used for determining depreciation costs. In general, design engineers report economic evaluations on the basis of straight-line depreciation unless there is some specific reason for using one of the other methods.

Because it is impossible to estimate exact service lives and salvage values when a property is first put into use, it is sometimes desirable to reestimate these factors from time to time during the life period of the property. If this is done, straight-line depreciation can be assumed during each of the periods, and the overall method is known as *multiple straight-line depreciation.* Figure 9-1 shows how the asset value of a property varies with time using the straight-line and the multiple straight-line methods for determining depreciation.

The straight-line method may be applied on the basis of units of production or predicted amount of service output, instead of life years. The depreciation may be based on miles, gallons, tons, number of unit pieces produced, or other measures of service output. This so-called *unit-of-production* or *service-output* method is particularly applicable when depletion occurs, as in the exploitation of natural resources. It should also be considered for properties having useful lives that are more dependent on the number of operations performed than on calendar time.

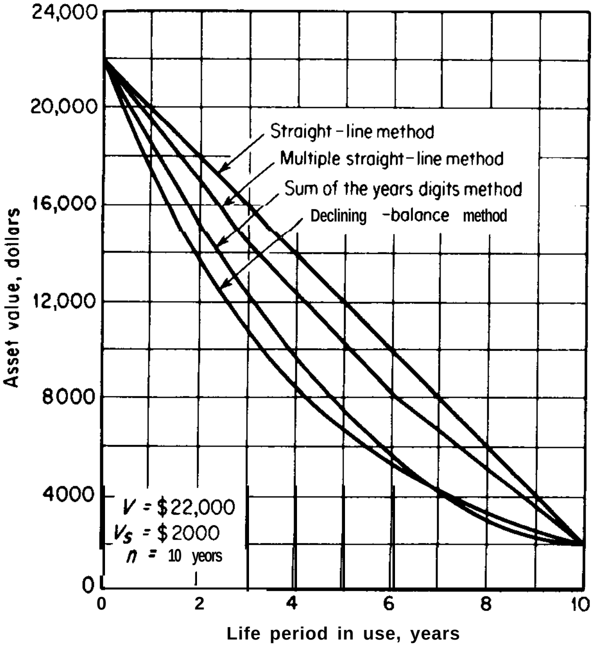

FIGURE 9-1
Comparison of straight-line, multiple straight-line, sum-of-the-years-digits, and declining-balance methods for determining depreciation.

Declining-Balance (or Fixed Percentage) Method

When the declining-balance method is used, the annual depreciation cost is a fixed percentage of the property value at the beginning of the particular year. The fixed-percentage (or declining-balance) factor remains constant throughout the entire service life of the property, while the annual cost for depreciation is different each year. Under these conditions, the depreciation cost for the first year of the property's life is $V\mathfrak{f}$, where $\mathfrak{f}$ represents the fixed-percentage factor.

At the end of the first year

$$\text{Asset value} = V_a = V(1 - \mathfrak{f}) \tag{3}$$

At the end of the second year

$$V_a = V(1 - \mathfrak{f})^2 \tag{4}$$

At the end of a years

$$V_a = V(1 - \mathfrak{f})^a \tag{5}$$

At the end of n years (i.e., at the end of service life)

$$V_a = V(1 - \mathfrak{f})^n = V_s \tag{6}$$

Therefore,

$$\mathfrak{f} = 1 - \left(\frac{V_s}{V}\right)^{1/n} \tag{7}$$

Equation (7) represents the textbook method for determining the fixed-percentage factor, and the equation is sometimes designated as the *Matheson formula.* A plot showing the change of asset value with time using this declining-balance depreciation method is presented in Fig. 9-1. Comparison with the straight-line method shows that declining-balance depreciation permits the investment to be paid off more rapidly during the early years of life. The increased depreciation costs in the early years are very attractive to concerns just starting in business, because the income-tax load is reduced at the time when it is most necessary to keep all pay-out costs at a minimum.

The textbook relationship presented in Eq. (7) is seldom used in actual practice, because it places too much emphasis on the salvage value of the property and is certainly not applicable if the salvage value is zero. To overcome this disadvantage, the value of the fixed-percentage factor is often chosen arbitrarily using a sound economic basis.

Prior to 1954, the United States government would not accept any depreciation method which permitted depreciation rates more than 50 percent greater than those involved in the straight-line method. In 1954, the laws were changed to allow rates up to twice (200 percent) those for the straight-line method. Under these conditions, one arbitrary method for choosing the value of

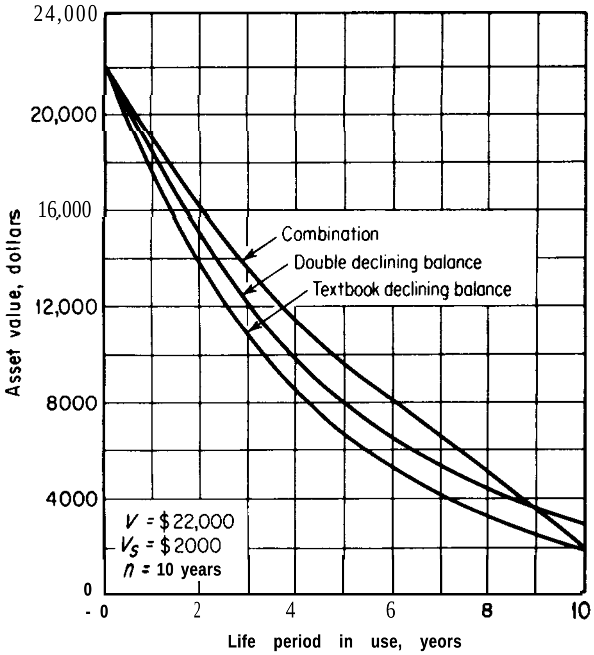

FIGURE 9-2
Types of declining-balance methods for determining depreciation.

f is to fix it at two times the reciprocal of the service life *n*.[†] This permits approximately two-thirds of the depreciable value to be written off in the first half of the useful life.[‡]

Figure 9-2 shows the effect of time on asset value when the declining-balance method of depreciation is used with an arbitrarily chosen value of *f*. It should be noted that the value of the asset cannot decrease to zero at the end of the service life and may possibly be greater than the salvage or scrap value. To handle this difficulty, it is sometimes desirable to switch from the declining-balance to the straight-line method after a portion of the service life has expired. This is known as the ***combination method.*** It permits the property to be fully depreciated during the service life, yet also gives the advantage of faster early-life write-offs. A curve showing this type of depreciation is presented in Fig. 9-2.

The main advantage of the declining-balance and the combination methods is that they permit greater depreciation allowances in the early life of the

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[†] The salvage value is considered to be zero, and the fixed-percentage factor is based on the straight-line rate of depreciation during the first year.

[‡] Based on the 1954 tax revision for depreciation accounting, any method could be used if the depreciation for the first two-thirds of the useful life of the property did not exceed the total of such allowances if they had been computed by the double declining-balance method.

property than in the later life. They are particularly applicable for units in which the greater proportion of the production occurs in the early part of the useful life or when operating costs increase markedly with age.

Example 1 Determination of depreciation by straight-line and declining-balance methods. The original value of a piece of equipment is $22,000, completely installed and ready for use. Its salvage value is estimated to be $2000 at the end of a service life estimated to be 10 years. Determine the asset (or book) value of the equipment at the end of 5 years using:

(a) Straight-line method.
(*b*) Textbook declining-balance method.
(*c*) Double declining-balance (200 percent) method (i.e., the declining-balance method using a fixed-percentage factor giving a depreciation rate equivalent to twice the minimum rate with the straight-line method).

Solution

(a) Straight-line method:

$$v = \$22{,}000$$

$$V_s = \$2000$$

$$n = 10 \text{ years}$$

$$d = \frac{V - V_s}{n} = \frac{20{,}000}{10} = \$2000/\text{year}$$

Asset value after 5 years = V_a, where a = 5, or

$$V_a = V - ad = 22{,}000 - (5)(2000) = \$12{,}000$$

(*b*) Textbook declining-balance method:

$$\mathfrak{f} = 1 - \left(\frac{V_s}{V}\right)^{1/n} = 1 - \left(\frac{2000}{22{,}000}\right)^{1/10} = 0.2131$$

Asset value after 5 years is

$$V_a = V(1 - \mathfrak{f})^a = (22{,}000)(1 - 0.2131)^5 = \$6650$$

(*c*) Double declining-balance (200 percent) method:

Using the straight-line method, the minimum depreciation rate occurs in the first year when $V = \$22{,}000$ and *the* depreciation = $2000. This depreciation rate is 2000/22,000, and the double declining-balance (or double fixed-percentage) factor is $(2)(2000/22{,}000) = 0.1818 = \mathfrak{f}$. (It should be noted that the double declining-balance method is often applied to cases where the salvage value is considered to be zero. Under this condition, the double fixed-percentage factor for this example would be 0.2000.)

Asset value after 5 years is

$$V_a = V(1 - \mathfrak{f})^a = (22{,}000)(1 - 0.1818)' = \$8060$$

Sum-of-the-Years-Digits Method

The sum-of-the-yearsdigits method is an arbitrary process for determining depreciation which gives results similar to those obtained by the declining-balance method. Larger costs for depreciation are allotted during the early-life years than during the later years. This method has the advantage of permitting the asset value to decrease to zero or a given salvage value at the end of the service life.

In the application of the sum-of-the-years-digits method, the annual depreciation is based on the number of service-life years remaining and the sum of the arithmetic series of numbers from 1 to n, where n represents the total service life. The yearly depreciation factor is the number of useful service-life years remaining divided by the sum of the arithmetic series. This factor times the total depreciable value at the start of the service life gives the annual depreciation cost.

As an example, consider the case of a piece of equipment costing $20,000 when new. The service life is estimated to be 5 years and the scrap value $2000. The sum of the arithmetic series of numbers from 1 to n is 1 + 2 + 3 + 4 + 5 = 15. The total depreciable value at the start of the service life is $20,000 − $2000 = $18,000. Therefore, the depreciation cost for the first year is $(\$18{,}000)(\frac{5}{15}) = \$6000$, and the asset value at the end of the first year is $14,000. The depreciation cost for the second year is $(\$18{,}000)(\frac{4}{15}) = \$4800$. Similarly, the depreciation costs for the third, fourth, and fifth years, respectively, would be $3600, $2400, and $1200.† Figure 9-1 presents a curve showing the change with time in asset value when the sum-of-the-years-digits method is used for determining depreciation.

Sinking-Fund Method

The use of compound interest is involved in the ***sinking-find method.*** It is assumed that the basic purpose of depreciation allowances is to accumulate a sufficient fund to provide for the recovery of the original capital invested in the property. An ordinary annuity plan is set up wherein a constant amount of money should theoretically be set aside each year. At the end of the service life,

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†Equations which apply for determining annual depreciation by the sum-of-the-years-digits method are

$$d_a = \text{depreciation for year } a = \frac{(n-a+1)}{\sum_{1}^{n} a}(V - V_s)$$

$$= \frac{2(n-a+1)}{n(n+1)}(V - V_s)$$

the sum of all the deposits plus accrued interest must equal the total amount of depreciation.

Derivation of the formulas for the sinking-fund method can be accomplished by use of the following notations in addition to those already given:

i = annual interest rate expressed as a fraction
R = uniform annual payments made at end of each year (this is the annual depreciation cost), dollars
$V - V_s$ = total amount of the annuity accumulated in an estimated service life of n years (original value of property minus salvage value at end of service life), dollars

According to the equations developed for an ordinary annuity in Chap. 7 (Interest and Investment Costs),

$$R = (V - V_s)\frac{i}{(1 + i)^n - 1} \tag{8}$$

The amount accumulated in the fund after a years of useful life must be equal to the total amount of depreciation up to that time. This is the same as the difference between the original value of the property V at the start of the service life and the asset value V_a at the end of a years. Therefore,

$$\text{Total amount of depreciation after } a \text{ years} = V - V_a \tag{9}$$

$$V - V_a = R\frac{(1 + i)^a - 1}{i} \tag{10}$$

Combining Eqs. (8) and (10),

$$V - V_a = (V - V_s)\frac{(1 + i)^{”} - 1}{(1 + i)^{”} - 1} \tag{11}$$

Asset (or book) value after a years = V_a

$$V_a = V - (V - V_s)\frac{(1 + i)^{”} - 1}{(1 + i)^{”} - 1} \tag{12†}$$

Since the value of R represents the annual depreciation cost, the yearly cost for depreciation is constant when the sinking-fund method is used. As shown in Fig. 9-3, this method results in book values which are always greater

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[†] Exactly the same result for asset value after a years is obtained if an annuity due (i.e., equal periodic payments at beginning of each year) is used in place of an ordinary annuity. The periodic payment with an annuity due would be $R/(1 + i)$. In accepted engineering practice, the sinking-fund method is based on an ordinary annuity plan.

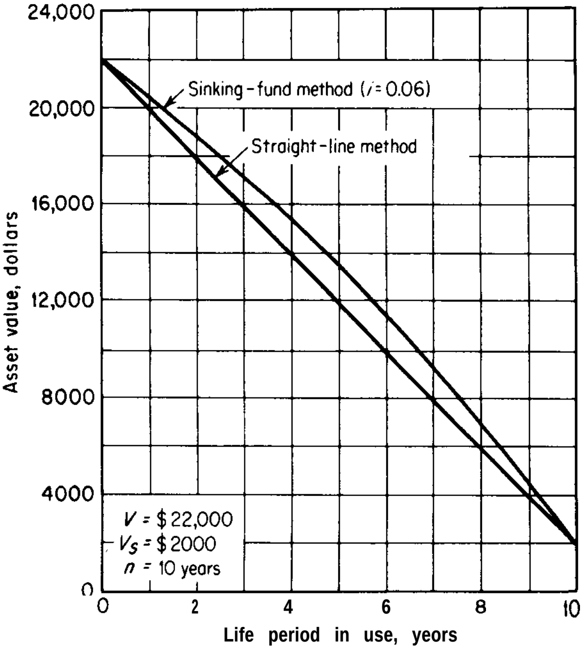

FIGURE 9-3
Asset values of property when depreciated by interest (sinking-fund) and no-interest (straight-line) methods.

than those obtained with the straight-line method. Because of the effects of interest in the sinking-fund method, the annual decrease in asset value of the property is less in the early-life years than in the later years.

Although the sinking-fund viewpoint assumes the existence of a fund into which regular deposits are made, an actual fund is seldom maintained. Instead, the money accumulated from the depreciation charges is put to work in other interests, and the existence of the hypothetical fund merely serves as a basis for this method of depreciation accounting.

The sinking-fund theory of cost accounting is now used by few concerns, although it has seen considerable service in the public-utilities field. Theoretically, the method would be applicable for depreciating any property that did not undergo heavy service demands during its early life and stood little chance of becoming obsolete or losing service value due to other functional causes.

The same approach used in the sinking-fund method may be applied by analyzing depreciation on the basis of reduction with time of future profits obtainable with a property. When this is done, it is necessary to use an interest rate equivalent to the annual rate of return expected from the use of the property. This method is known as the *present-worth method* and gives results similar to those obtained with the conventional sinking-fund approach. The sinking-fund and the present-worth methods are seldom used for depreciation cost accounting but are occasionally applied for purposes of comparing alternative investments.

Accelerated Cost Recovery System

The Accelerated Cost *Recovery* System (ACRS) is a system for determining depreciation allowances based on statutory annual percentages and class life periods established for the United States by Federal income-tax regulations. The basis for the statutory percentage factors is the declining-balance method of depreciation combined with the straight-line method. The original ACRS was in effect by Federal tax laws from 1981 through 1986 with a *Modified Accelerated Cost Recovery* System (MACRS) going into effect in 1987.

In general, ACRS allowed one-half of a full year's deduction for property in the year it was placed in service and no deduction in the year when the property was anticipated to be disposed of, although special month-by-month rules could be applied for some cases. Similarly, this so-called "half-year *convention"* applied for MACRS as one-half of a full year's deduction for property in the year it was placed in service, but it also allowed one-half of a full year's deduction during the year of disposal. Thus, the years of depreciable values equaled the class years for ACRS and equaled one more than the class years for MACRS.

The bases of calculation for the statutory percentage factors which were applied to the values of the original property to determine the yearly deductions are as follows:

For ACRS, the statutory percentages were based on a HO-percent declining balance with a switch to straight-line depreciation at the time appropriate to

TABLE 3
Statutory percentages for use in the Accelerated Cost Recovery System (ACRS). Property put in service after 1980 and before 1987.

Applicable recovery year	Applicable recovery percentage to give annual depreciation for class life of			
	3 years	5 years	10 years	15 years
1	25	15	8	5
2	38	22	14	10
3	37	21	12	9
4		21	10	8
5		21	10	7
6			10	7
7			9	6
8			9	6
9			9	6
10			9	6
11				6
12				6
13				6
14				6
15				6

maximize the deduction. The half-year convention applied for the first year when property was placed in service. Salvage value was taken as zero.

For MACRS, the statutory percentages were based on a 200-percent declining balance for class lives of 3, 5, 7, and 10 years and a 150-percent declining balance for class lives of 15 and 20 years with a switch to straight-line depreciation at the time appropriate to maximize the deduction. The half-year convention applied for the first year when property was placed in service and also for the year of disposal. Salvage value was taken as zero.

For both ACRS and MACRS, the statutory percentages have been calculated for each of a group of class years, and these, in turn, have been related to values of the *Class Life Accelerated Depreciation Range* (CLADR) as noted earlier. Results are given in Table 3 for ACRS and Table 4 for MACRS.

Details, such as shown in Tables 3 and 4, are presented annually as part of the United States Federal Income Tax Regulations. Tables are also given with conventions other than the half-year conventions for MACRS, such as mid-

TABLE 4
Statutory percentages for use in the Modified Accelerated Cost Recovery System (MACRS). Property put in service after 1986.

Applicable recovery year	Applicable recovery percentage to give annual depreciation for class life of					
	3 years	5 years	7 years	10 years	15 years	20 years
1	33.33	20.00	14.29	10.00	5.00	3.750
2	44.45	32.00	24.49	18.00	9.50	7.219
3	14.81	19.20	17.49	14.40	8.55	6.677
4	7.41	11.52	12.49	11.52	7.70	6.177
5		11.52	8.93	9.22	6.93	5.713
6		5.76	8.92	7.37	6.23	5.285
7			8.93	6.55	5.90	4.888
8			4.46	6.55	5.90	4.522
9				6.56	5.91	4.462
10				6.55	5.90	4.461
11				3.28	5.91	4.462
12					5.90	4.461
13					5.91	4.462
14					5.90	4.461
15					5.91	4.462
16					2.95	4.461
17						4.462
18						4.461
19						4.462
20						4.461
21						2.231

quarter with property placed in service in the first, second, third, or fourth quarter.?

During the period from 1981 through 1986, instead of using the applicable ACRS percentages to determine annual depreciation deductions, corporations were allowed to use straight-line depreciation over the recovery period under the following conditions:

For	Use recovery period of
3-year class-life property	3, 5, or 12 years
5-year class-life property	5, 12, or 25 years
lo-year class-life property	10, 25, or 35 years
15-year class-life property	15, 35, or 45 years

Businesses were expected, in general, to conform with the Modified Accelerated Cost Recovery System to determine depreciation deductions for real and tangible property put into service after 1986 when such deductions were involved in income-tax determinations. Figure 9-4 gives a comparison of depreciation results using methods of ACRS, MACRS, and double declining balance (200 percent) with no salvage value combined with straight line.

Example 2 Determination of percentage factors as given for Modified Accelerated Cost Recovery System. Calculate the percentage factors for a class life of 10 years as presented in Table 4 of this chapter for the Modified Accelerated Cost Recovery System (MACRS). Note that MACRS is based on a 200 percent declining balance for this class life with a switch to straight-line depreciation at the time appropriate to maximize the deduction. It is also based on salvage value being zero. The half-year convention in the first and last years applies. Use an initial property value of $22,000 to permit comparison of results to Fig. 9-4.

Solution. The declining-balance equation to use is (value of property at start of year)$(1 - f)$ = value of property at end of year with f being the declining-balance factor. The 200 percent declining-balance factor is based on two times the minimum depreciation rate which occurs in the first year when V = $22,000 and depreciation is $22,000/10 = $2200. Thus, the 200-percent declining-balance factor is $(2)(2200/22{,}000) = 0.20 = f$ which applies to each full year being considered.

For the first year, with the half-year convention, the investment is considered as being made at the midpoint of the year. Thus, the f which applies for the first year is $(2)(1100/22{,}000) = 0.10$.

Value at start of 1st year or at midpoint of 1st year = $22,000.

Value at end of 1st year = $22{,}000(1 - 0.10)$ = $19,800.

Percentage factor for 1st year = $[(22{,}000 - 19{,}800)/22{,}000](100) = 10.00\%$.

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†See Prentice-Hall Federal Taxes for the current year, Prentice-Hall Information Services, Paramus, NJ 07652.

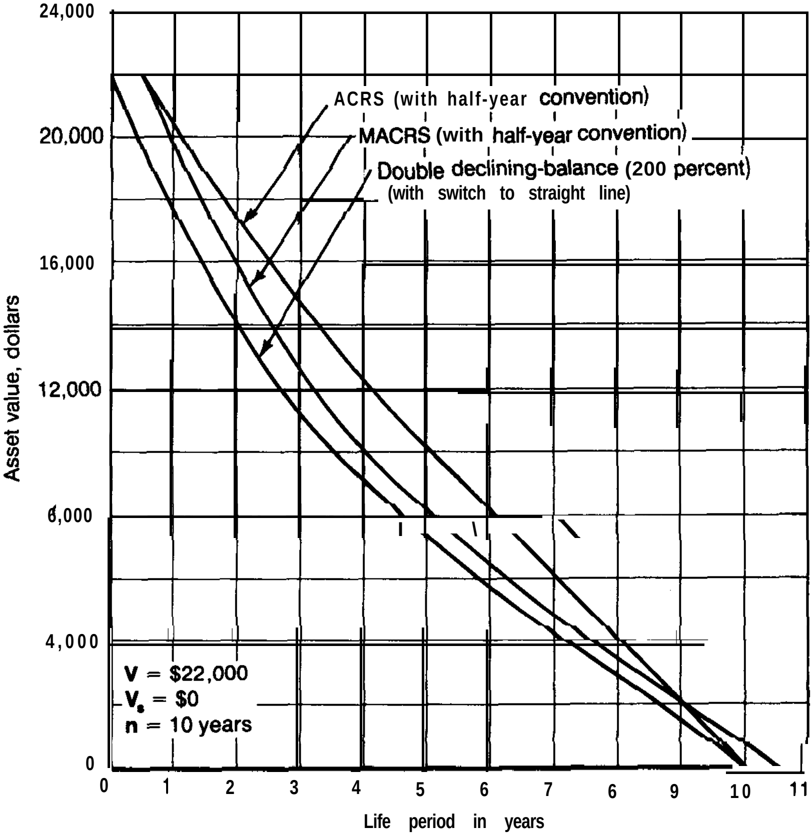

FIGURE 9-4
Asset values of property when depreciated by Accelerated Cost Recovery System (ACRS), Modified Accelerated Cost Recovery System (MACRS), and double declining-balance (200-percent) method with switch to straight-line.

Value at end of 2nd year = 19,800(1 − 0.20) = $15,840.

Percentage factor for 2nd year = [(19,800 − 15,840)/22,000](100) = 18.00%.

Value at end of 3rd year = 15,840(1 − 0.20) = $12,672.

Percentage factor for 3rd year = [(15,840 − 12,672)/22,000](100) = 14.40%.

Value at end of 4th year = 12,672(1 − 0.20) = $10,138.

Percentage factor for 4th year = [(12,672 − 10,138)/22,000](100) = 11.52%.

Value at end of 5th year = 10,138(1 − 0.20) = $8110.

Percentage factor for 5th year = [(10,138 − 8110)/22,000](100) = 9.22%.

Value at end of 6th year = 8110(1 − 0.20) = $6488.

Percentage factor for 6th year = [(8110 − 6488)/22,000](100) = 7.37%.

At this point, any further use of the declining-balance factor for the remaining 4.5 years of property life results in a deduction less than that obtained with the straight-line depreciation method. If one stays with the **200-percent** reducing-balance for the 7th year, the amount of depreciation in the 7th year would be 6488 - 6488(1 - 0.20) = \$1298 compared to 6488/4.5 = \$1442 if the switch is made to straight-line depreciation. Therefore, switch to straight-line method for the remaining 4.5 years of life with the annual depreciation to reduce the property value to zero after 4.5 years being **\$1442/year.**

Value at end of 7th year = 6488 - 1442 = \$5046.

Percentage factor for 7th year = (1442/22,000)(100) = 6.55% or 6.56%.

Value at end of 8th year = 5046 - 1442 = \$3604.

Percentage factor for 8th, 9th, and 10th years = (1442/22,000)(100) = 6.55%.

Value at end of 10th year = 3604 - (2)(1442) = \$720.

Percentage factor for 11th year up to half year = (720/22,000)(100) = 6.55/2 = 3.28%.

These percentages agree with those presented in Table 4 of this Chapter, and the year-end values agree with those shown in Fig. 9-4.

SINGLE-UNIT AND GROUP DEPRECIATION

In depreciation accounting procedures, assets may be depreciated on the basis of individual units or on the basis of various types of property groups or classifications. *The single-unit method* requires keeping records on each individual asset. Although the application of this method is simple, the large number of detailed records required makes the accounting expenses very high.

To simplify the accounting procedures, many concerns combine their various assets into groups for depreciation purposes. There are several types of group accounts employed, and the most common among these are composite accounts, classified accounts, and vintage-group accounts.

A *composite* account includes all depreciable assets in one single group, and an overall depreciation rate is applied to the entire account. With this method, the composite depreciation rate must be redetermined when important changes occur in the relative distribution of the service lives of the individual assets.

Instead of including all assets in a single depreciation account, it is possible to classify properties into general types, such as machinery and equipment, office furniture and fixtures, buildings, and transportation equipment. The records for these groups are known as *classified accounts.* A classified account is similar to a composite account because many items are included in the same group, regardless of life characteristics.

Another approach to group depreciation is to include in each account all similar assets having approximately the same service lives. These accounts are known as ***vintage-group accounts.*** A separate record is kept for each group and the same depreciation rate is applied to all the items included in each account. With this method, the advantages of single-unit depreciation are obtained since life characteristics serve as the basis. If a large number of items are contained in a vintage-group account, the overall depreciation results can be quite accurate because the law of averages will apply to the true service lives as compared with the estimated service lives.

ADJUSTMENT OF DEPRECIATION ACCOUNTS

The estimated service life and salvage value of a property are seldom exactly equal to the actual service life and salvage value. It is, therefore, advisable to adjust depreciation accounts by making periodic reestimations of the important variables. When a property is retired under conditions which do not permit exact agreement between estimated and actual values, the difference between the book depreciation and the actual depreciation may be handled in one of the following ways: (1) The gain or loss may be credited or charged on the financial record for the current period; (2) the difference may be credited or charged to a special depreciation reserve; or (3) the difference may be carried on the books for amortization during a reasonable future period.

According to the Federal income-tax laws, any gain on the retirement of a property is taxed as a capital gain. However, losses cannot be subtracted from the taxable income unless the maximum expected life was used. Because of the losses involved when a property must be retired before the end of its estimated service life, some concerns prefer to use a combination of methods 2 and 3 indicated in the preceding paragraph. A special depreciation reserve is built up by continuing the book depreciation of properties whose actual service lives exceed the estimated service lives. This fund is then used to handle losses due to early retirement of assets. The final choice of method for adjusting depreciation accounts depends on the accounting policies of the individual concern and on income-tax regulations.

EVALUATION OF DEPRECIATION METHODS

Comparison of the various depreciation methods shows that the declining-balance and the sum-of-the-years-digits methods give similar results. In both cases, the depreciation costs are greater in the early-life years of the property than in the later years. Annual depreciation costs are constant when the straight-line, sinking-fund, or present-worth method is used. Because interest effects are included in the sinking-fund and present-worth methods, the annual decrease in asset value with these two methods is lower in the early-life years than in the later years. The straight-line method is widely used for depreciation cost accounting because it is very simple to apply, both to groups and single units, and it is acceptable for cost-accounting purposes and for some income-tax determinations.

From the viewpoint of financial protection, it is desirable to make a greater charge for property depreciation during early life than during later life. This can be accomplished by use of the declining-balance or sum-of-the-years-digits method. The difficulties of accurate application to group accounts and income-tax restrictions have served to suppress the widespread usage of these methods. However, in recent years, a large number of industrial concerns have started using declining-balance and sum-of-the-years-digits depreciation, with many companies finding it desirable to use the combination method approved by Federal income-tax regulations of declining balance plus straight-line with statutory percentages for each year based on property life class.

The liberalized tax laws passed in 1954 first permitted use of double declining-balance depreciation as well as sum-of-the-years-digits depreciation for income-tax calculations. In general, these laws gave approval to any depreciation method which did not give faster write-offs during the first two-thirds of an asset's useful life than the double declining-balance method. These regulations were not applicable to assets with service lives of less than 3 years.

The final choice of the best depreciation method depends on a number of different factors. The type and function of the property involved is, of course, one important factor. Also, it is desirable to use a simple formula giving results as accurate as the estimated values involved. The advisability of keeping two separate sets of books, one for income-tax purposes and one for company purposes, should be considered. The final decision involves application of good judgment and an analysis of the existing circumstances.

NOMENCLATURE FOR CHAPTER 9

a = length of time in actual use, years
d = annual depreciation, \$/year
f = fixed-percentage or declining-balance factor
i = annual interest rate expressed as a fraction, percent/100
n = service life, years
R = uniform annual payments made in an ordinary annuity, dollars
V = original value of a property at start of service-life period, completely installed and ready for use, dollars
V_a = asset or book value, dollars
V_s = salvage value of property at end of service life, dollars

PROBLEMS

1. A reactor of special design is the major item of equipment in a small chemical plant. The initial cost of the completely installed reactor is \$60,000, and the salvage value at the end of the useful life is estimated to be \$10,000. Excluding depreciation costs

for the reactor, the total annual expenses for the plant are $100,000. How many years of useful life should be estimated for the reactor if 12 percent of the total annual expenses for the plant are due to the cost for reactor depreciation? The straight-line method for determining depreciation should be used.

2. The initial installed cost for a new piece of equipment is $10,000, and its scrap value at the end of its useful life is estimated to be $2000. The useful life is estimated to be 10 years. After the equipment has been in use for 4 years, it is sold for $7000. The company which originally owned the equipment employs the straight-line method for determining depreciation costs. If the company had used an alternative method for determining depreciation costs, the asset (or book) value for the piece of equipment at the end of 4 years would have been $5240. The total income-tax rate for the company is 34 percent of all gross earnings. Capital-gains taxes amount to 34 percent of the gain. How much net saving after taxes would the company have achieved by using the alternative (in this case, reducing-balance) depreciation method instead of the straight-line depreciation method?
3. A piece of equipment originally costing $40,000 was put into use 12 years ago. At the time the equipment was put into use, the service life was estimated to be 20 years and the salvage and scrap value at the end of the service life were assumed to be zero. On this basis, a straight-line depreciation fund was set up. The equipment can now be sold for $10,000, and a more advanced model can be installed for $55,000. Assuming the depreciation fund is available for use, how much new capital must be supplied to make the purchase?
4. The original investment for an asset was $10,000, and the asset was assumed to have a service life of 12 years with $2000 salvage value at the end of the service life. After the asset has been in use for 5 years, the remaining service life and final salvage value are reestimated at 10 years and $1000, respectively. Under these conditions, what is the depreciation cost during the sixth year of the total life if straight-line depreciation is used?
5. A property has an initial value of $50,000, service life of 20 years, and final salvage value of $4000. It has been proposed to depreciate the property by the text-book declining-balance method. Would this method be acceptable for income-tax purposes if the income-tax laws do not permit annual depreciation rates greater than twice the minimum annual rate with the straight-line method?
6. A piece of equipment having a negligible salvage and scrap value is estimated to have a service life of 10 years. The original cost of the equipment was $40,000. Determine the following:
   (a) The depreciation charge for the fifth year if double declining-balance depreciation is used.
   (*b*) The depreciation charge for the fifth year if sum-of-the-years-digits depreciation is used.
   (*c*) The percent of the original investment paid off in the first half of the service life using the double declining-balance method.
   (*d*) The percent of the original investment paid off in the first half of the service life using the sum-of-the-years-digits method.
7. The original cost of a property is $30,000, and it is depreciated by a 6 percent sinking-fund method. What is the annual depreciation charge if the book value of the property after 10 years is the same as if it had been depreciated at **$2500/year** by the straight-line method?

8. A concern has a total income of \$1 million/year, and all expenses except depreciation amount to \$600,000/year. At the start of the first year of the concern's operation, a composite account of all depreciable items shows a value of \$850,000, and the overall service life is estimated to be 20 years. The total salvage value at the end of the service life is estimated to be \$50,000. Thirty percent of all profits before taxes must be paid out as income taxes. What would be the reduction in income-tax charges for the first year of operation if the sum-of-the-years-digits method were used for depreciation accounting instead of the straight-line method?
9. The total value of a new plant is \$2 million. A certificate of necessity has been obtained permitting a write-off of 60 percent of the initial value in 5 years. The balance of the plant requires a write-off period of 15 years. Using the straight-line method and assuming negligible salvage and scrap value, determine the total depreciation cost during the first year.
10. A profit-producing property has an initial value of \$50,000, a service life of 10 years, and zero salvage and scrap value. By how much would annual profits before taxes be increased if a 5 percent sinking-fund method *were* used to determine depreciation costs instead of the straight-line method?
11. In order to make it worthwhile to purchase a new piece of equipment, the annual depreciation costs for the equipment cannot exceed \$3000 at any time. The original cost of the equipment is \$30,000, and it has zero salvage and scrap value. Determine the length of service life necessary if the equipment is depreciated (*a*) by the sum-of-the-years-digits method, and (*b*) by the straight-line method.
12. The owner of a property is using the unit-of-production method for determining depreciation costs. The original value of the property is \$55,000. It is estimated that this property can produce 5500 units before its value is reduced to zero: i.e., the depreciation cost per unit produced is \$10. The property produces 100 units during the first year, and the production rate is doubled each year for the first 4 years. The production rate obtained in the fourth year is then held constant until the value of the property is paid off. What would have been the annual depreciation cost if the straight-line method based on time had been used?
13. Calculate the percentage factors for a class life of 10 years as presented in Table 3 of this chapter for the Accelerated Cost Recovery System (ACRS). Note that ACRS is based on a HO-percent declining balance with switch to straight-line depreciation at the time appropriate to maximize the deduction. It is also based on salvage value being zero. The half-year convention in the first year applies, but the last half-year deduction cannot be claimed as such. Use an initial property value of \$22,000 to permit comparison to Fig. 9-4 and Example 2.
14. A materials-testing machine was purchased for \$20,000 and was to be used for 5 years with an expected residual salvage value of \$5000. Graph the annual depreciation charges and year-end book values obtained by using:
    (*a*) Straight-line depreciation.
    (*b*) Sum-of-digits depreciation.
    (*c*) Double-declining-balance depreciation.
    (*d*) ACRS with 5-year property recovery.
15. An asset with an original cost of \$10,000 and no salvage value has a depreciation charge of \$2381 during its second year of *service* when depreciated by the sum-of-digits method. What is its expected useful life?

CHAPTER
10

PROFITABILITY, ALTERNATIVE INVESTMENTS, AND REPLACEMENTS

The word *profitability* is used as the general term for the measure of the amount of profit that can be obtained from a given situation. Profitability, therefore, is the common denominator for all business activities.

Before capital is invested in a project or enterprise, it is necessary to know how much profit can be obtained and whether or not it might be more advantageous to invest the capital in another form of enterprise. Thus, the determination and analysis of profits obtainable from the investment of capital and the choice of the best investment among various alternatives are major goals of an economic analysis.

There are many reasons why capital investments are made. Sometimes, the purpose is merely to supply a service which cannot possibly yield a monetary profit, such as the provision of recreation facilities for free use of employees. The profitability for this type of venture cannot be expressed on a positive numerical basis. The design engineer, however, usually deals with investments which are expected to yield a tangible profit.

Because profits and costs are considered which will occur in the future, the possibilities of inflation or deflation affecting future profits and costs must be recognized. The strategy for handling effects of inflation or deflation is discussed in Chap. 11.

Investments may be made for replacing or improving an existing property, for developing a completely new enterprise, or for other purposes wherein a profit is expected from the outlay of capital. For cases of this sort, it is extremely important to make a careful analysis of the capital utilization.

PROFITABILITY STANDARDS

In the process of making an investment decision, the profits anticipated from the investment of funds should be considered in terms of a minimum profitability standard.† This profitability standard, which can normally be expressed on a direct numerical basis, must be weighed against the overall judgment evaluation for the project in making the final decision as to whether or not the project should be undertaken.

The judgment evaluation must be based on the recognition that a quantified profitability standard can serve only as a guide. Thus, it must be recognized that the profit evaluation is based on a prediction of future results so that assumptions are necessarily included. Many intangible factors, such as future changes in demand or prices, possibility of operational failure, or premature obsolescence, cannot be quantitized. It is in areas of this type that judgment becomes critical in making a final investment decision.

A primary factor in the judgment decision is the consideration of possible alternatives. For example, the alternatives to continuing the operation of an existing plant may be to replace it with a more efficient plant, to discontinue the operation entirely, or to make modifications in the existing plant. In reaching the final decision, the alternatives should be considered two at a time on a mutually exclusive basis.

An obvious set of alternatives involves either making the capital investment in a project or investing the capital in a safe venture for which there is essentially no risk and a guaranteed return. In other words, the second alternative involves the company's decision as to the cost of capital.

Cost of Capital

Methods for including the cost of capital in economic analyses have been discussed in Chap. 7. Although the management and stockholders of each company must establish the company's characteristic cost of capital, the simplest approach is to assume that investment of capital is made at a hypothetical cost or rate of return equivalent to the total profit or rate of return over the full expected life of the particular project. This method has the advantage of putting the profitability analysis of all alternative investments on an equal basis, thereby permitting a clear comparison of risk factors. This method is particularly useful for preliminary estimates, but it may need to be refined further to take care of income-tax effects for final evaluation.

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†One often-used basis for the minimum profitability standard is the value of money to the company, expressed as a rate, based on earnings after taxes.

BASES FOR EVALUATING PROJECT PROFITABILITY

Total profit alone cannot be used as the deciding profitability factor in determining if an investment should be made. The profit goal of a company is to maximize income above the cost of the capital which must be invested to generate the income. If the goal were merely to maximize profits, any investment would be accepted which would give a profit, no matter how low the return or how great the cost. For example, suppose that two equally sound investments can be made. One of these requires $100,000 of capital and will yield a profit of **$10,000/year**, and the second requires $1 million of capital and will yield **$25,000/year.** The second investment gives a greater yearly profit than the first, but the annual *rate of return* on the second investment is only

$$(\$25{,}000/\$1{,}000{,}000) \times (100) = 2.5 \text{ percent}$$

while the annual rate of return on the $100,000 investment is 10 percent. Because reliable bonds and other conservative investments will yield annual rates of return in the range of 6 to 9 percent, the $1 million investment in this example would not be very attractive; however, the 10 percent return on the $100,000 capital would make this investment worthy of careful consideration. Thus, for this example, the rate of return, rather than the total amount of profit, is the important profitability factor in determining if the investment should be made.

The basic aim of a profitability analysis is to give a measure of the attractiveness of the project for comparison to other possible courses of action. It is, therefore, very important to consider the exact purpose of a profitability analysis before the standard reference or base case is chosen. If the purpose is merely to present the total profitability of a given project, a simple statement of total profit per year or annual rate of return may be satisfactory. On the other hand, if the purpose is to permit comparison of several different projects in which capital might be invested, the method of analysis should be such that all cases are on the same basis so that direct comparison can be made among the appropriate alternatives.

Mathematical Methods for Profitability Evaluation

The most commonly used methods for profitability evaluation, as illustrated in Fig. 10-1, can be categorized under the following headings:

1. Rate of return on investment
2. Discounted cash flow based on full-life performance
3. Net present worth
4. Capitalized costs
5. Payout period

Each of these methods has its advantages and disadvantages, and much has been written on the virtues of the various methods. Because no single method is best for all situations, the engineer should understand the basic ideas involved in each method and be able to choose the one best suited to the needs of the particular situation.

RATE OF RETURN ON INVESTMENT. In engineering economic studies, rate of return on investment is ordinarily expressed on an annual percentage basis. The yearly profit divided by the total initial investment necessary represents the fractional return, and this fraction times 100 is the standard percent *return* on *investment*.[†][‡]

Profit is defined as the difference between income and expense. Therefore, profit is a function of the quantity of goods or services produced and the selling price. The amount of profit is also affected by the economic efficiency of the operation, and increased profits can be obtained by use of effective methods which reduce operating expenses.

To obtain reliable estimates of investment returns, it is necessary to make accurate predictions of profits and the required investment. To determine the profit, estimates must be made of direct production costs, fixed charges including depreciation, plant overhead costs, and general expenses. Profits may be expressed on a before-tax or after-tax basis, but the conditions should be indicated. Both working capital and fixed capital should be considered in determining the total investment.[§]

Returns Incorporating Minimum Profits as an Expense

The standard method for reporting rate of return on investment has been outlined in the preceding paragraphs. Another method which is sometimes used for reporting rate of return is based on the assumption that it must be possible to obtain a certain minimum profit or return from an investment before the necessary capital outlay will be desirable. This minimum profit is included as a

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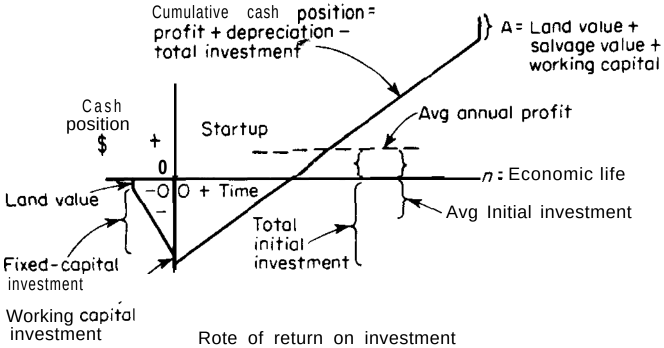
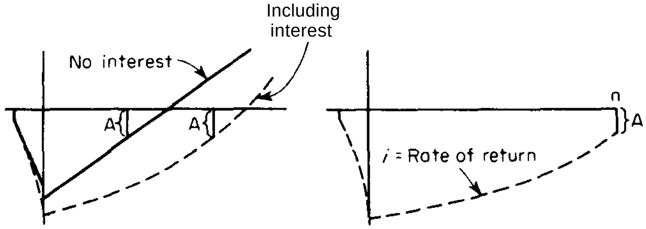
[†] The normal procedure is to base the percent return on investment on the total initial investment. However, because equipment depreciates during its useful life, it is sometimes convenient to base the rate of return on the average estimated investment during the life of the project. With this method, the rate of return is determined by dividing the average annual profit or saving by one-half the initial fixed-capital investment (or initial fixed-capital investment minus the estimated salvage value at the end of the useful life) plus the working-capital investment.

[‡] An article by J. Linsley, Return on Investment: Discounted and Undiscounted, *Chem. Eng.*, 86(11):201 (May 21, 1979), suggests that "return on investment" can be defined as net, after-tax profit plus depreciation divided by capital investment. This definition of return on investment where depreciation cash flow is included as part of the return is not used in this book. Instead, this method of handling cash flow is included in the profitability methods reported for discounted-cash-flow Profitability Index and Net Present Worth.

[§] Under some conditions, such as a profitability analysis based on a small component of an overall operation, the return on investment can be based on the fixed-capital investment instead of the total investment.

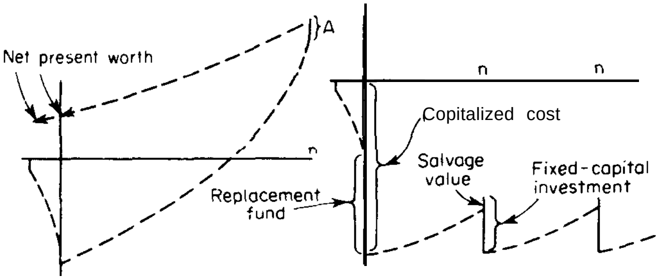


Rote of return on investment



Poyout period

Discounted cash flow



Net present worth

Capitalized cost

FIGURE 10-1
Graphical description of various profitability measures.

fictitious expense along with the other standard expenses. When return on investment is determined in this manner, the result shows the *risk earning rate*, and represents the return over and above that necessary to make the capital expenditure advisable. If the return is zero or larger, the investment will be attractive. This method is sometimes designated as *return* based *on capital recovery with minimum profit.*

The inclusion of minimum profit as an expense is rather unrealistic, especially when it is impossible to designate the exact return which would make a given investment worthwhile. One difficulty with this method is the tendency to use a minimum rate of return equal to that obtained from present investments. This, of course, gives no consideration to the element of risk involved in a new venture. Despite these objections, the use of returns incorporating minimum profits as an expense is acceptable providing the base, or minimum return and the general method employed are clearly indicated.

Example 1 Determination of rate of return on investment-consideration of income-tax effects. A proposed manufacturing plant requires an initial fixed-capital investment of \$900,000 and \$100,000 of working capital. It is estimated that the annual income will be \$800,000 and the annual expenses including depreciation will be \$520,000 before income taxes. A minimum annual return of 15 percent before income taxes is required before the investment will be worthwhile. Income taxes amount to 34 percent of all pre-tax profits.

Determine the following:

(a) The annual percent return on the total initial investment before income taxes.
(*b*) The annual percent return on the total initial investment after income taxes.
(*c*) The annual percent return on the total initial investment before income taxes based on capital recovery with minimum profit.
(*d*) The annual percent return on the average investment before income taxes assuming straight-line depreciation and zero salvage value.

Solution

(a) Annual profit before income taxes = \$800,000 − \$520,000 = \$280,000.

Annual percent return on the total initial investment before income taxes = [280,000/(900,000 + 100,000)](100) = 28 percent.

(*b*) Annual profit after income taxes = (\$280,000)(0.66) = \$184,800.

Annual percent return on the total initial investment after income taxes = [184,800/(900,000 + 100,000)](100) = 18.5 percent.

(*c*) Minimum profit required per year before income taxes = (\$900,000 + \$100,000)(0.15) = \$150,000.

Fictitious expenses based on capital recovery with minimum profit = \$520,000 + \$150,000 = \$670,000/year. Annual percent return on the total investment based on capital recovery with minimum annual rate of return of 15 percent before income taxes = [(800,000 − 670,000)/(900,000 + 100,000)](100) = 13 percent.

(*d*) Average investment assuming straight-line depreciation and zero salvage value = \$900,000/2 + \$100,000 = \$550,000.

Annual percent return on average investment before income taxes = (280,000/550,000)(100) = 51 percent.

The methods for determining rate of return, as presented in the preceding sections, give “point values” which are either applicable for one particular year or for some sort of “average” year. They do not consider the time value of money, and they do not account for the fact that profits and costs may vary significantly over the life of the project.

One example of a cost that can vary during the life of a project is depreciation cost. If straight-line depreciation is used, this cost will remain constant; however, it may be advantageous to employ a declining-balance or sum-of-the-years-digits method to determine depreciation costs, which will immediately result in variations in costs and profits from one year to another. Other predictable factors, such as increasing maintenance costs or changing sales volume, may also make it necessary to estimate year-by-year profits with variation during the life of the project. For these situations, analyses of project profitability cannot be made on the basis of one point on a flat time-versus-earning curve, and profitability analyses based on discounted cash flow may be appropriate. Similarly, time-value-of-money considerations may make the discounted-cash-flow approach desirable when annual profits are constant.

DISCOUNTED CASH FLOW

Rate of Return Based on Discounted Cash Flow†

The method of approach for a profitability evaluation by discounted cash flow takes into account the time value of money and is based on the amount of the investment that is unreturned at the end of each year during the estimated life of the project. A trial-and-error procedure is used to establish a rate of return which can be applied to yearly cash flow so that the original investment is reduced to zero (or to salvage and land value plus working-capital investment) during the project life. Thus, the rate of return by this method is equivalent to the maximum interest rate (normally, after taxes) at which money could be borrowed to finance the project under conditions where the net cash flow to the project over its life would be just sufficient to pay all principal and interest accumulated on the outstanding principal.

To illustrate the basic principles involved in discounted-cash-flow calculations and the meaning of rate of return based on discounted cash flow, consider the case of a proposed project for which the following data apply:

Initial fixed-capital investment = $100,000
Working-capital investment = $10,000
Service life = 5 years
Salvage value at end of service life = $10,000

Year	Predicted after-tax cash flow to project based on total income minus all costs except depreciation, $ (expressed as end-of-year situation)
0	(110,000)
1	30,000
2	31,000
3	36,000
4	40,000
5	43,000

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†Common names of methods of return calculations related to the discounted-cash-flow approach are *profitability index, interest rate of return, true rate of return,* and *investor's rate of return.*

TABLE 1
Computation of discounted-cash-flow rate of return

Year (n')	Estimated cash flow to project, $	Trial for $i = 0.15$		Trial for $i = 0.20$		Trial for $i = 0.25$		Trial for $i = 0.207$†	
		Discount factor, $\frac{1}{(1+i)^n}$	Present value, $	Discount factor, $\frac{1}{(1+i)^n}$	Present value, $	Discount factor, $\frac{1}{(1+i)^n}$	Present value, $	Discount factor, $\frac{1}{(1+i)^n}$	Present value, $
0	(110,000)								
1	30,000	0.8696	26.100	0.8333	25,000	0.8000	24,000	0.829	24,900
2	31,000	0.7561	23,400	0.6944	21,500	0.6400	19,800	0.687	21,200
3	36,000	0.6575	23,300	0.5787	20.700	0.5120	18.400	0.570	20,500
4	40,000	0.5718	22,900	0.4623	19.300	0.4096	16,400	0.472	18.800
5	43.000 +20,000	0.4971	31.300	0.4019	25.300	0.3277	20,600	0.391	24,600
	Total		127,000		111.800		99,200		110,000
Ratio = $\frac{\text{total present value}}{\text{initial investment}}$			1.155		1.016		0.902		1.000
								Trial is satisfactory	

†As illustrated in Fig. 10-2, interpolation to determine the correct rate of return can be accomplished by plotting the ratio (total present value/initial investment) versus the trial interest rate for three bracketing values and reading the correct rate from the curve where the ratio = 1.0.

NOTE: In this example, interest was compounded annually on an end-of-year basis and continuous interest compounding was ignored. Also, construction period and land value were not considered. The preceding effects could have been included in the analysis for a more sophisticated treatment using the methods presented in Examples 2 and 3 of this chapter.

Designate the discounted-cash-flow rate of return as i. This rate of return represents the after-tax interest rate at which the investment is repaid by proceeds from the project. It is also the maximum after-tax interest rate at which funds could be borrowed for the investment and just break *even* at the end of the service life.

At the end of five years, the cash flow to the project, compounded on the basis of end-of-year income, will be

$$(\$30,000)(1+i)^4 + (\$31,000)(1+i)^3 + (\$36,000)(1+i)^2 + (\$40,000)(1+i) + \$43,000 = S \quad (1)$$

The symbol S represents the future worth of the proceeds to the project and must just equal the future worth of the initial investment compounded at an interest rate i corrected for salvage value and working capital. Thus,

$$S = (\$110,000)(1+i)^5 - \$10,000 - \$10,000 \quad (2)$$

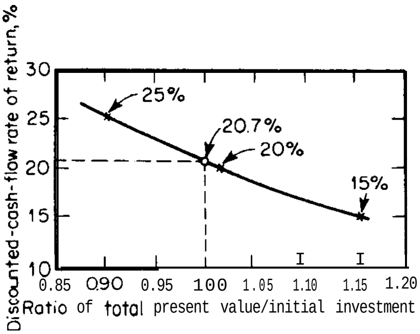

FIGURE 10-2
Graphical analysis for trial-and-error determination of discounted-cash-flow rate of return (see Table 1).

Setting Eq. (1) equal to Eq. (2) and solving by trial and error for i gives $i = 0.207$, or the discounted-cash-flow rate of return is 20.7 percent.

Some of the tedious and time-consuming calculations can be eliminated by applying a *discount factor* to the annual cash flows and summing to get a present value equal to the required investment. The discount factor for end-of-year payments and annual compounding is

$$d_{n'} = \frac{1}{(1 + i)”} = \text{discount factor} \qquad (3)$$

where i = rate of return

$n’$ = year of project life to which cash flow applies

This discount factor, $d_{n'}$, is the amount that would yield one dollar after n' years if invested at an interest rate of i. The discounted-cash-flow rate of return can be determined by the trial-and-error method illustrated in Table 1, where the annual cash flows are discounted by the appropriate discount factor to a total present value equal to the necessary initial investment.†

Example 2 Discounted-cash-flow calculations based on continuous interest compounding and continuous cash Row. Using the discount factors for continuous interest and continuous cash flow presented in Tables 5 to 8 of Chapter 7, determine the continuous discounted-cash-flow rate of return r for the example presented in the preceding section where yearly cash flow is continuous. The data follow.

Initial tied-capital investment = \$100,000
Working-capital investment = \$10,000
Service life = 5 years
Salvage value at end of service life = \$10,000

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†The significance of the use of discount factors, as illustrated in Table 1 and Example 2, can be seen by dividing both sides of Eq. (1) and Eq. (2) by $(1 + i)^5$, or by $(1 + i)^n$ for the general case where n is the estimated service life in years.

Year	Predicted after-tax cash flow to project based on total income minus all costs except depreciation with cash flow occurring continuously, $ (total of continuous cash flow for year indicated)
1	30,000
2	31,000
3	36,000
4	40,000
5	43,000

Solution. The following tabulation shows the final result of the trial-and-error solution using the factors F_a and F_b from Tables 5 and 6 in Chap. 7:

Year	Estimated continuous cash flow to project, $	Trial for r = 0.225		Present value, $
		Discount factor		
		F_b (from Table 6, Chap. 7)	F_a (from Table 5, Chap. 7)	
0	(110,000) In an instant			
0-1	30,000	0.8954		26,850
1-2	31,000	0.7151		22,200
2-3	36,000	0.5710		20,550
34	40,000	0.4560		18,250
4-5	43,000	0.3648		15,650
5	+20,000 In an instant		0.3246	6,500
				Total 110,000

Trial is satisfactory

Because the assumed trial value of $r = 0.225$ discounted all the cash flows to the present worth of $110,000, the continuous interest rate of 22.5 percent represents the discounted-cash-flow rate of return for this example which can be compared to the value of 20.7 percent shown in Table 1 for the case of discrete interest compounding and instantaneous cash flow.

NET PRESENT WORTH

In the preceding treatment of discounted cash flow, the procedure has involved the determination of an index or interest rate which discounts the annual cash flows to a zero present value when properly compared to the initial investment. This index gives the rate of return which includes the profit on the project, payoff of the investment, and normal interest on the investment. A related approach, known as the method of ***net present worth*** (or ***net present value*** or ***venture worth),*** substitutes the cost of capital at an interest rate i for the discounted-cash-flow rate of return. The cost of capital can be taken as the average rate of return the company earns on its capital, or it can be designated as the minimum acceptable return for the project. The net present worth of the project is then the difference between the present value of the annual cash flows and the initial required investment.

To illustrate the method for determining net present worth, consider the example presented in Table 1 for the case where the value of capital to the company is at an interest rate of 15 percent. Under these conditions, the present value of the cash flows is \$127,000 and the initial investment is \$110,000. Thus, the net present worth of the project is

$$\$127{,}000 \qquad \$110{,}000 = \$17{,}000$$

Work Sheet for Calculating Present Value and Net Present Worth

An example of a work sheet that can be used for handling discounted-cash-flow presentations to determine present value and net present worth is given in Table 2. The definitions as given in lines 16 and 17 of this table clearly show the preferred distinction between the terms net present *worth* and present value as used in this text. The table is particularly useful because it makes certain the user handles depreciation cash flow correctly by subtracting depreciation costs to determine tax costs (see lines 10 and 11) and including depreciation cash flow to determine the annual cash income (see lines 9 and 12). Line 14 shows four values of discount factors for 15 percent interest based on (*a*) continuous uniform cash flow and continuous interest compounding, (*b*) continuous uniform cash flow and finite (year-end) interest compounding, (*c*) finite (year-end) cash flow and continuous interest compounding, and (*d*) finite (year-end) cash flow and finite (year-end) interest compounding.

Lines 1, 2, and 3 (investments) in Table 2 would normally only be filled in for the first column (discount factor of 1.000) which is designated as the zero year for the operation, with the unit actually going into operation at the start of the so-called first year. It is assumed that working capital and salvage value will be recovered in a lump sum at the end of the estimated service life, so these values are listed on lines 1, 2, and 13 as positive (incoming funds) numbers in the end-of-life column. Since these are lump-sum instantaneous values, the discount factor to apply to them is the finite (year-end) cash flow factor as shown in line 14 in the end-of-life column.

Line 13 gives the annual cash flows for each of the operating years with the zero-year column giving only the total capital investment. In line 16, the present value of the annual cash flows to the project is obtained by summing the individual present values for each year of operation including the present value of the working-capital and salvage-value recovery at the end of the service life. Line 17 merely applies the definition of net present worth as used in this text as the difference between the total present value of the annual cash flows to the project and the initial required investment.

TABLE 2

Work sheet for presenting discounted-cash flow, present-value, and net-present-worth determinations

Project Title: ______________________________

Notes: 1. Dollar values can be in thousands of dollars and rounded to the nearest $1,000.
2. For lines 11 and 14, company policies will dictate which tax rate, interest, and discount factors to use.
3. The estimated service life for this example is taken as 5 years.
4. For lines 5, 6, and 7, see Table 27 of Chapter 6 for estimating information and basis.

Line	Item Numbers in ( ) designate line	Year						End-of-life working capital and salvage value
		1986 0	1987 1st	1988 2nd	1989 3rd	1990 4th	1991 5th	
1.	Fixed-capital investment							
2.	Working capital							
3.	Total capital investment (1 + 2)							
4.	Annual income (sales)							
5.	Annual manufacturing cost							
	(a) Raw materials							
	(b) Labor							
	(c) Utilities							
	(d) Maintenance and repairs							
	(e) Operating supplies							
	(f) Laboratory charges							
	(g) Patents and royalties							
	(h) Local taxes and insurance							
	(i) Plant overhead							
	(j) Other (explain in Notes)							
5-T.	Total of line 5							

6.	Annual general expenses							
	(a) Administrative							
	(b) Distribution and selling							
	(c) Research and development							
	(d) Interest							
	(e) Other (explain in Notes)							
6-T.	Total of line 6							
7.	Total product cost (5-T + 6-T)							
8.	Annual operating income (4 - 7)							
9.	Annual depreciation							
10.	Income before tax (8 - 9)							
11.	Income after 34% tax (0.66 × 10)							
12.	Annual cash income (9 + 11)							
13.	Annual cash flow (3 + 12)							(see heading above)
14.	Discount factors for 15% interest							
	(a) See footnote †	1.000	0.929	0.799	0.688	0.592	0.510	0.472
	(b) See footnote ‡	1.000	0.933	0.812	0.706	0.614	0.534	0.947
	(c) See footnote §	1.000	0.861	0.741	0.638	0.549	0.472	0.472
	(d) See footnote 11	1.000	0.870	0.756	0.658	0.572	0.497	0.497
15.	Annual present value (13 x 14)							

16. TOTAL present value of annual cash flows (sum of line 15 *not* including 0 year) = _______ in dollars or thousands of dollars

17. Net present worth = total present value of annual cash flows - total capital investment = line 16 - line 3 = _______ in dollars or thousands of dollars

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† Continuous uniform cash flow and continuous nominal interest (r) of 15%.

‡ Continuous uniform cash flow and finite effective interest (i) of 15%.

§ Finite (year-end) cash flow and continuous nominal interest (r) of 15%.

¶ Finite (year-end) cash flow and finite effective interest (i) of 15%.

Notes should be included with the table to explain the basis for special factors used, such as escalation factors, startup costs, and depreciation method. The notes can also be used to explain the methods used for estimating the various items as, for example, note 4 in Table 2 showing the methods used for estimating lines 5, 6, and 7.

The format shown in Table 2 is intended as an example, and a real case would undoubtedly include more columns to represent a life of more than five years. Similarly, capital is normally spent during the period of one or two years before operations begin and sales are made. Thus, the factors in the zero-year column could be changed to values other than 1.000 using methods presented in Chap. 7 (Interest and Investment Costs) as illustrated in Example 3 of this chapter.

CAPITALIZED COSTS†

The ***capitalized-cost*** profitability concept is useful for comparing alternatives which exist as possible investment choices within a single overall project. For example, if a decision based on profitability analysis were to be made as to whether stainless steel or mild steel should be used in a chemical reactor as one part of a chemical plant, capitalized-cost comparison would be a useful and appropriate approach. This particular case is illustrated in Example 9 of Chap. 7.

Capitalized cost related to investment represents the amount of money that must be available initially to purchase the equipment and simultaneously provide sufficient funds for interest accumulation to permit perpetual replacement of the equipment. If only one portion of an overall process to accomplish a set objective is involved and operating costs do not vary, then the alternative giving the least capitalized cost would be the desirable economic choice.

The basic equation for capitalized cost for equipment was developed in Chap. 7 as Eq. (28), which can be written as follows:

$$K = C_V + \frac{C_R}{(1+i)^n - 1} = \frac{C_R(1+i)^n}{(1+i)^n - 1} + V_s \tag{4}$$

where K = capitalized cost
C_V = original cost of equipment
C_R = replacement cost
V_s = salvage value at end of estimated useful life
n = estimated useful life of equipment
i = interest rate

$\frac{(1+i)^n}{(1+i)^n - 1}$ = capitalized-cost factor

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†For an analysis of the meaning of capitalized costs, development of related equations, and references, see Chap. 7 (Interest and Investment Costs).

Inclusion of Operating Costs in Capitalized-Costs Profitability Evaluation

The capitalized-costs concept can be extended to include operating costs by adding an additional capitalized cost to cover operating costs during the life of the project. Each annual operating cost is considered as equivalent to a necessary piece of equipment that will last one year.?

The procedure is to determine the present or discounted value of each year's cost by the method illustrated in Table 1. The sum of these present values is then capitalized by multiplying by the capitalized-cost factor given with Eq. (4). The total capitalized cost is the sum of the capitalized cost for the initial investment and that for the operating costs plus the working capital. This procedure is illustrated as part of Example 5 in this chapter.

PAYOUT PERIOD

Payout period, or ***payout time***,‡ is defined as the minimum length of time theoretically necessary to recover the original capital investment in the form of cash flow to the project based on total income minus all costs except depreciation. Generally, for this method, original capital investment means only the original, depreciable, fixed-capital investment, and interest effects are neglected. Thus,

$$\begin{matrix}\text{Payout period in years}\\ \text{(no interest charge)}\end{matrix} = \frac{\text{depreciable fixed-capital investment}}{\text{avg profit/yr} + \text{avg depreciation/yr}} \qquad (5)$$

Another approach to payout period takes the time value of money into consideration and is designated as ***payout period including interest.*** With this method, an appropriate interest rate is chosen representing the minimum acceptable rate of return. The annual cash flows to the project during the estimated life are discounted at the designated interest rate to permit computation of an average annual figure for profit plus depreciation which reflects the time value of money.0 The time to recover the tied-capital investment plus compounded interest on the total capital investment during the estimated life by

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†If annual operating cost is constant and the cost is considered as an end-of-year cost, the capitalized cost of operation is equal to the annual operating cost divided by i. Continuous interest compounding can be used to resolve the problem of whether an operating cost is an end-of-year or start-of-year cost. The effects of depreciation methods and taxes may be very important when capitalized costs are used to compare design alternatives involving operating costs.

‡Other equivalent names are ***payback period, payback time, payoff period, payoff time,*** and ***cash-recovery period.***

§This discounting procedure is similar to that illustrated in the footnote to part (*a*) of Example 5 in this chapter. Continuous interest tables, such as Tables 5 to 8 in Chap. 7, can also be used.

means of the average annual cash flow is the payout period including interest, or

Payout period including interest

$$= \frac{\text{depreciable fixed-capital investment} + \text{interest on total capital investment during estimated service life}}{(\text{avg profit/yr} + \text{avg depreciation/yr})_{\text{as constant annuity}}} \tag{6}$$

This method tends to increase the payout period above that found with no interest charge and reflects advantages for projects that earn most of their profits during the early years of the service life.

USE OF CONTINUOUS INTEREST COMPOUNDING

In the preceding presentation of methods for profitability evaluation, where interest was considered, it was generally treated as finite-period interest compounded annually. By use of the relationships developed in Chap. 7 (Interest and Investment Costs), it is a simple matter to convert to the case of continuous interest compounding in place of finite interest compounding.

For example, the discount factor $d_{n'} = 1/(1+i)^{n'}$, given as Eq. (3), becomes

$$d_{n'} = \frac{1}{e^{rn'}} \tag{7}$$

for the case of continuous interest compounding with r representing the nominal continuous interest. The preceding equation follows directly from Eq. (18) of Chap. 7.

The application of continuous interest compounding, along with a method of profitability evaluation which includes construction costs and other prestartup costs, is illustrated in the following example.

Example 3 Determination of profitability index with continuous interest compounding and prestartup costs. Determine the discounted-cash-flow rate of return (i.e., the profitability *index*) for the overall plant project described in the following, and present a plot of cash position versus time to illustrate the solution.

One year prior to startup of the plant, the necessary land is purchased at a cost of $200,000.

During the year prior to the startup, the plant is under construction with money for the construction and related activities flowing out uniformly during the entire year starting at zero dollars and totaling $600,000 for the year.

A working-capital investment of $200,000 is needed at the time the plant starts operation and must be retained indefinitely.

Salvage value for the plant at the end of the estimated useful life is $100,000.

The estimated useful life is 10 years.

Estimations of operating costs, income, and taxes indicate that the annual cash flow to the project (i.e., net profit plus depreciation per year) will be $310,000 flowing uniformly throughout the estimated life. This is an after-tax figure.

The concept of continuous interest compounding and continuous cash flow will be used. Neglect any effects due to inflation or deflation.

Solution. The procedure for this problem is similar to that illustrated in Table 1 in that a trial-and-error method is used with various interest rates until a rate is found which decreases the net cash position to zero at the end of the useful life. Let r represent the profitability index or discounted-cash-flow rate of return with continuous cash flow and continuous interest compounding.

1. Determination of cash position at zero time (i.e., at time of plant startup) in terms of unknown profitability index r.

Land value. The in-an-instant value of the land is \$200,000 one year before the zero reference point of plant startup time. The land value at zero time, therefore, is the future worth of this \$200,000 after one year with continuous interest compounding. Thus, by Eq. (36) of Chap. 7 or part (*e*) of Table 3 in Chap. 7.

$$\text{Compounded land value at zero time} = \$200{,}000(e^r)$$

Construction cost. The total construction cost of the plant during the one year prior to startup is \$600,000 occurring uniformly during the year. The compounded construction cost at zero time, therefore, is the future worth of this \$600,000 after one year flowing uniformly throughout the year with continuous compounding. Thus, by Eq. (37) of Chap. 7 or part (*f*) of Table 3 in Chap. 7.

$$\text{Compounded construction cost at zero time} = \$600{,}000\frac{e^r - 1}{r}$$

Working-capital investment. The working-capital investment of \$200,000 must be supplied at the time of plant startup or at the reference point of zero time.

Summary of cash position at zero time.

Total cash position at zero time

$$= CP_{\text{zero time}} = \$200{,}000(e^r) + \$600{,}000\frac{e^r - 1}{r} + \$200{,}000$$

2. Determination of cash position at end of estimated useful life (i.e., ten years from zero time) in terms of profitability index r. At the end of the useful life with the correct value of r, the total cash position, taking into account the working-capital investment, the salvage value, and the land value, must be zero.

After plant startup, the annual cash flow to the project (i.e., net profit plus depreciation) is \$310,000 flowing continuously and uniformly, and this annual figure is constant throughout the estimated useful life.

The following procedure for evaluating the total cash position at the end of the estimated useful life is analogous to the procedure used in establishing Eqs. (1), (2), (3), and (7) of this chapter.

At the end of each year, the compounded cash flow to the project, with continuous uniform flow and continuous compounding, gives, by Eq. (37) of

Chap. 7 or part (*f*) of Table 3 in Chap. 7, a future worth ($S_{\text{each year}}$) oft

$$S_{\text{each year}} = \$310{,}000\frac{e^r - 1}{r} \qquad (A)$$

At the end of 10 years, the total future worth (S) of the cash flow to the project, by Eq. (36) of Chap. 7, or part (*e*) of Table 3 in Chap. 7 becomes

$$S = (\$310{,}000)\frac{e^r - 1}{r}(e^{9r} + e^{8r} + e^{7r} + \cdots + e^r + 1) \qquad (B)$$

The future worth of the total flow to the project after 10 years must be equal to the future worth of the total cash position at zero time ($CP_{\text{zero time}}$) compounded continuously for 10 years minus salvage value, land value, and working-capital investment. Therefore, by Eq. (36) of Chap. 7 or part (*e*) of Table 3 in Chap. 7,

$$S = (CP_{\text{zero time}})(e^{10r}) - \$100{,}000 - \$200{,}000 - \$200{,}000 \qquad (C)$$

3. Determination of profitability index r. Equating Eq. (B) to Eq. (C) gives the following result with r as the only unknown, and a trial-and-error solution will give the profitability index r.

$$(\$310{,}000)\frac{e^r - 1}{r}(e^{9r} + e^{8r} + e^{7r} + \cdots + e^r + 1) - (CP_{\text{zero time}})(e^{10r})$$
$$+ \$100{,}000 + \$200{,}000 + \$200{,}000 = 0 \qquad (D)$$

The trial-and-error approach can be simplified by dividing Eq. (D) by e^{10r} and substituting the expression for $CP_{\text{zero time}}$ to give the present-value or discounted-cash-flow equation as follows:

$$(\$310{,}000)\frac{e^r - 1}{r}\sum_{n'=1}^{n'=10}\frac{1}{e^{rn'}} - \$200{,}000(e^r) - \$600{,}000\frac{e^r - 1}{r}$$
$$- \$200{,}000 + (\$100{,}000 + \$200{,}000 + \$200{,}000)\frac{1}{e^{10r}} = 0 \qquad (E)$$

where $1/e^{rn'}$ represents the discount factor for continuous cash flow and continuous compounding as given in Eq. (7) of this chapter.

Because the compounded annual flow to the project is constant for each year at $(\$310{,}000)[(e^r - 1)/r]$, the year-by-year use of the discount factor, as illustrated in Table 1 of this chapter, can be replaced by a one-step process wherein the equivalent present value is determined from Eq. (25) of Chap. 7, $P = \bar{R}[(e^{rn} - 1)/re^{rn}]$, with $\bar{R}$ = \$310,000, so that

$$\$310{,}000\frac{e^r - 1}{r}\sum_{n'=1}^{n'=10}\frac{1}{e^{rn'}} = 310{,}000\frac{\sim}{re^{rn}}$$

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†The concept of continuous and uniform cash flow with continuous interest compounding is obviously an assumption which is reasonable for some cash flow, such as costs for raw material and labor. However, it is clear that some major portions of the cash flow may not approximate continuous flow. For this reason, the annual cash flow is often estimated as an end-of-year-figure, and the interest factor in Eq. (A) is eliminated.

Similarly, in Eq. (*D*), the future-worth expression for the cash flow, **\$310,000**$[(e^r - 1)/r](e^{9r} + e^{8r} + e^{7r} + \cdots + e^r + 1)$, can be replaced by **\$310,000**$[(e^{rn} - 1)/r]$, as shown by **Eq. (23)** of Chap. 7.

With these simplifications, either Eq. (*D*) or (*E*) can be used for the trial-and-error solution for r. Table 3 shows the method of solution using the present-value Eq. (*E*) to give the correct value of $r = 0.26$.

Thus, the profitability index or discounted-cash-flow rate of return for this example is 26%.

TABLE 3

Computation of profitability index for Example 3†

n = 10 years

Basis: Eq. (*E*) and zero (present-value) time at plant startup

Trial for	$r = 0.20$	r = 0.25	$r = 0.40$	$r = 0.26$‡
a. Present value of cash flow to project	$\frac{e^r - 1}{r} = 1.107$	1.136	1.230	1.142
(\$310,000) $\frac{e^r - 1}{r} \sum_{n'=1}^{n'=10} \frac{1}{e^{rn'}}$ or	$\frac{e^{rn} - 1}{re^{rn}} = 4.31$	3.68	2.45	3.565
(\$310,000 $\frac{e^{rn} - 1}{re^{rn}}$	**\$1,335,000**	**\$1,140,000**	\$760,000	**\$1,107,000**
b. Present value of land	$e^r = 1.221$	1.284	1.492	1.297
(\$200,000) e^r	\$244,000	\$257,000	\$298,000	\$259,000
c. Present value of construction cost	$\frac{e^r - 1}{r} = 1.107$	1.136	1.230	1.142
(\$600,000) $\frac{e^r - 1}{r}$	\$664,000	\$682,000	\$738,000	\$685,000
d. Present value of **working**-capital investment	\$200,000	\$200,000	\$200,000	\$200,000
e. Present value of terminal land, working capital, and salvage value based on interest compounded continuously for n years	$\frac{1}{e^{nr}} = 0.135$	0.0822	0.0183	0.744
(\$500,000) $\frac{1}{e^{nr}}$	\$68,000	\$41,000	\$9,000	\$37,000
f. Total of all present values with interest of r. Should be zero at correct value of r. $f = a - b - c - d + e$	\$295,000 Rate too low	\$42,000 Rate too low	**−\$467,000** Rate too high	**\$0** Trial is satisfactory

† Graphical methods, special tables for particular cases, MAP1 worksheets and terminology, computer solutions, and rules of thumb are available to simplify the type of calculations illustrated in this table. For example, see G. A. Taylor, "**Managerial** and Engineering Economy: Economic Decision-Making, 2d ed., D. Van Nostrand Company, Inc.. Princeton, New Jersey, 1975.

‡ See Fig. 10-2 for example of graphical interpolation procedure.

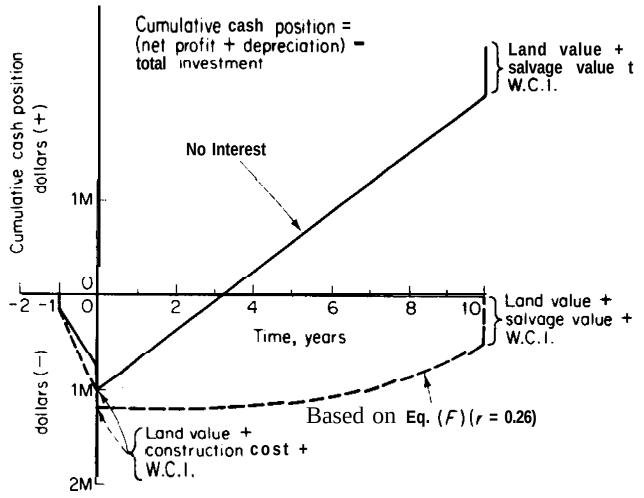

FIGURE 10-3
Illustrative plot showing cash position versus time to explain graphically the solution to Example 3. Dashed line is with interest or profitability index of 26 percent. Solid line is with no interest charge. (Note that method for calculating depreciation is not important except for income taxes.)

4. Graphical representation of problem solution. Equation (*D*) can be generalized, with the simplifications indicated in the preceding section of this problem, to give Table 3 (note that direct land value, salvage value, and working-capital investment are now included in the cash composition).

$$\begin{aligned}\text{Cash position at time } n &= (\text{annual constant cash flow to project})\,\frac{e^{rn}-1}{r} \\ &\quad - (\text{land value})e^{r(n+Z)} - (\text{construction cost})\frac{e^{rY}-1}{Yr}e^{nr} \\ &\quad - (\text{working-capital investment})(e^{nr}) \qquad (F)\end{aligned}$$

where Z is the time period in years the land is owned before startup and Y is the time period in years required for construction, In this example, Z and Y are both 1.0.

Figure 10-3 is the requested plot of cash position versus time for the case of $r = 0.26$ based on Eq. (*F*) and illustrates the concepts involved in the solution of this problem showing the cases with interest (or profitability index) and without interest.

DETERMINING ACCEPTABLE RETURNS

It is often possible to make a profit by the investment of capital, but it is not always easy to determine if a given return is sufficient to justify an investment. Many factors must be considered when deciding if a return is acceptable, and it is not possible to give one figure which will apply for all cases.

When dealing with ordinary industrial operations, profits cannot be predicted with absolute accuracy. Risk factors, therefore, must be given careful consideration, and the degree of uncertainty involved in predicted returns on investments plays an important role in determining what returns are acceptable.

A certain amount of risk is involved in any type of investment, but the degree of risk varies widely for different types of enterprises. For example, there is very little uncertainty in predicting returns on capital invested in government bonds, and the chances of losing the original capital are very small. However, money invested in a wildcat mining enterprise would stand a good chance of being lost completely with no return whatsoever.

If capital is available for investment in a proposed enterprise, it would also be available for use in other ventures. Therefore, a good basis for determining an acceptable return is to compare the predicted return and the risks involved with returns and risks in other types of investments.

Very conservative investments, such as government bonds, pay low returns in the range of 5 to 7 percent, but the risk involved is practically negligible. Preferred stocks yield returns of about 7 to 9 percent. There is some risk involved in preferred-stock investments since a business depression or catastrophe could cause reduction in returns or even a loss of the major portion of the capital investment. Common stocks may yield very high returns; however, the returns fluctuate considerably with varying economic conditions, and there is always the possibility of losing much or all of the original investment.

It can be stated that moderate risks are involved in common-stock investments. Certainly, at least moderate risks are involved in most industrial projects. In general, a 20 percent return before income taxes would be the minimum acceptable return for any type of business proposition, even if the economics appeared to be completely sound and reliable. Many industrial concerns demand a predicted pretax return of at least 30 percent based on reliable economic estimates before they will consider investing capital in projects that are known to be well engineered and well designed.

The final decision as to an acceptable return depends on the probable accuracy of the predicted return and on the amount of risk the investor wishes to take. Availability of capital, personal opinions, and intangible factors, such as the response of the public to changes or appearances, may also have an important effect on the final decision.

ALTERNATIVE INVESTMENTS

In industrial operations, it is often possible to produce equivalent products in different ways. Although the physical results may be approximately the same, the capital required and the expenses involved can vary considerably depending on the particular method chosen. Similarly, alternative methods involving varying capital and expenses can often be used to carry out other types of business ventures. It may be necessary, therefore, not only to decide if a given business venture would be profitable, but also to decide which of several possible methods would be the most desirable.

The final decision as to the best among alternative investments is simplified if it is recognized that each dollar of additional investment should yield an adequate rate of return. In practical situations, there are usually a limited number of choices, and the alternatives must be compared on the basis of incremental increases in the necessary capital investment.

The following simple example illustrates the principle of investment comparison. A chemical company is considering adding a new production unit which will require a total investment of $1,200,000 and will yield an annual profit of $240,000. An alternative addition has been proposed requiring an investment of $2 million and yielding an annual profit of $300,000. Although both of these proposals are based on reliable estimates, the company executives feel that other equally sound investments can be made with at least a 14 percent annual rate of return. Therefore, the minimum rate of return required for the new investment is 14 percent.

The rate of return on the $1,200,000 unit is 20 percent, and that for the alternative addition is 15 percent. Both of these returns exceed the minimum required value, and it might appear that the $2 million investment should be recommended because it yields the greater amount of profit per year. However, a comparison of the incremental investment between the two proposals shows that the extra investment of $800,000 gives a profit of only $60,000, or an incremental return of 7.5 percent. Therefore, if the company has $2 million to invest, it would be more profitable to accept the $1,200,000 proposal and put the other $800,000 in another investment at the indicated 14 percent return.

A general rule for making comparisons of alternative investments can be stated as follows: ***The minimum investment which will give the necessary functional results and the required rate of return should always be accepted unless there is a specific reason for accepting an alternative investment requiring more initial capital.*** When alternatives are available, therefore, the base plan would be that requiring the minimum acceptable investment. The alternatives should be compared with the base plan, and additional capital would not be invested unless an acceptable incremental return or some other distinct advantage could be shown.

Alternatives When an Investment Must Be Made

The design engineer often encounters situations where it is absolutely necessary to make an investment and the only choice available is among various alternatives. An example of this might be found in the design of a plant requiring an evaporation operation. The evaporator units must have a given capacity based on the plant requirements, but there are several alternative methods for carrying out the operation. A single-effect evaporator would be satisfactory. However, the operating expenses would be lower if a multiple-effect evaporator were used, because of the reduction in steam consumption. Under these conditions, the best number of effects could be determined by comparing the increased savings with the investment required for each additional effect. A

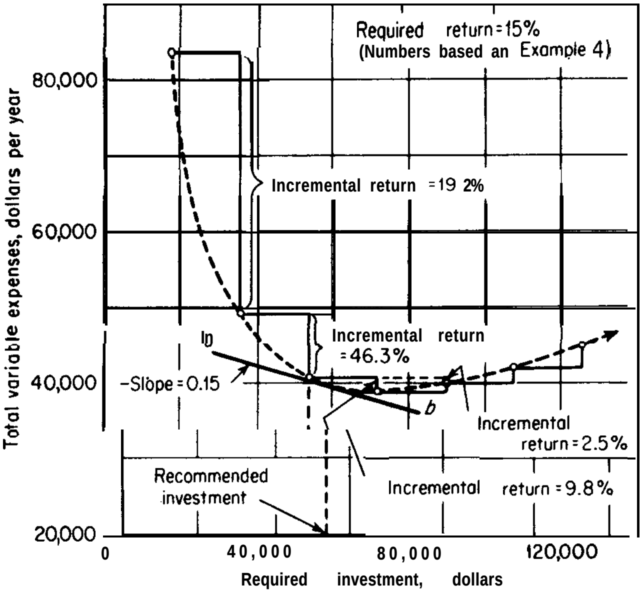

FIGURE 10-4
Comparison of alternative investments when one investment must be made for a given service and there are a limited number of choices.

graphical representation showing this kind of investment comparison is presented in Fig. 10-4.

The base plan for an alternative comparison of the type discussed in the preceding paragraph would be the minimum investment which gives the necessary functional results. The alternatives should then be compared with the base plan, and an additional investment would be recommended only if it would give a definite advantage.

When investment comparisons are made, alternatives requiring more initial capital are compared only with lower investments *which have been found to be acceptable.* Consider an example in which an investment of $50,000 will give a desired result, while alternative investments of $60,000 and $70,000 will give the same result with less annual expense. Suppose that comparison between the $60,000 and the $50,000 cases shows that $60,000 investment to be unacceptable. Certainly, there would be no reason to give further consideration to the $60,000 investment, and the next comparison should be between the $70,000 and the $50,000 cases. This type of reasoning, in which alternatives are compared in pairs on a mutually exclusive basis, is illustrated in the following simplified example.

AN EXAMPLE TO ILLUSTRATE PRINCIPLES OF ALTERNATIVE INVESTMENT ANALYSIS. In making a choice among various alternative investments, it is necessary to recognize the need to compare one investment to another on a mutually exclusive basis in such a manner that the ***return on the incremental investment*** is satisfactory. The following example illustrates this principle.

An existing plant has been operating in such a way that a large amount of heat is being lost in the waste gases. It has been proposed to save money by recovering the heat that is now being lost. Four different heat exchangers have been designed to recover the heat, and all prices, costs, and savings have been calculated for each of the designs. The results of these calculations are presented in the following:

Design	No. 1	No. 2	No. 3	No. 4
Total initial installed cost, $	10,000	16,000	20,000	26,000
Operating costs, $/yr	100	100	100	100
Fixed charges, % of initial cost/yr	20	20	20	20
Value of heat saved, $/yr	4,100	6,000	6,900	8,850

The company in charge of the plant demands at least a 10 percent annual return based on the initial investment for any unnecessary investment. Only one of the four designs can be accepted. Neglecting effects due to income taxes and the time value of money, which (if any) of the four designs should be recommended?

The first step in the solution of this example problem is to determine the amount of money saved per year for each design, from which the annual percent return on the initial investment can be determined. The net annual savings equals the value of heat saved minus the sum of the operating costs and fixed charges; thus,

For design No. 1,

$$\text{Annual savings} = 4100 - (0.2)(10{,}000) - 100 = \$2000$$

$$\text{Annual percent return} = \frac{2000}{10{,}000}(100) = 20\%$$

For design No. 2,

$$\text{Annual savings} = 6000 - (0.2)(16{,}000) - 100 = \$2700$$

$$\text{Annual percent return} = \frac{2700}{16{,}000}(100) = 16.9\%$$

For design No. 3,

$$\text{Annual savings} = 6900 - (0.2)(20{,}000) - 100 = \$2800$$

$$\text{Annual percent return} = \frac{2800}{20{,}000}(100) = 14\%$$

For design No. 4,

$$\text{Annual savings} = 8850 - (0.2)(26{,}000) - 100 = \$3550$$

$$\text{Annual percent return} = \frac{3550}{26{,}000}(100) = 13.6\%$$

Because the indicated percent return for each of the four designs is above the minimum of 10 percent required by the company, any one of the four designs would be acceptable, and it now becomes necessary to choose one of the four alternatives.

ALTERNATIVE ANALYSIS BY METHOD OF RETURN ON INCREMENTAL INVESTMENT. Analysis by means of return on incremental investment is accomplished by a logical step-by-step comparison of an acceptable investment to another which might be better. If design No. 1 is taken as the starting basis, comparison of design No. 2 to design No. 1 shows that the annual saving of \$2700 − \$2000 = \$700 results by making an additional investment of \$16,000 − \$10,000 = \$6,000. Thus, the percent return on the incremental investment is $\frac{700}{6000}(100)$ = 11.7 percent, and design No. 2 is acceptable by company policy in preference to design No. 1. This logical procedure results in the following tabulation and the choice of design No. 2 as the final recommendation:

Design No. 1 is acceptable.

Comparing design No. 1 to design No. 2, annual percent return = $\frac{700}{6000}(100)$ = 11.7 percent. Thus, design No. 2 is acceptable and is preferred over design No. 1.

Comparing design No. 2 to design No. 3, annual percent return = $\frac{100}{4000}(100)$ = 2.5 percent. Thus, design No. 3 compared to design No. 2 shows that the return is unacceptable and design No. 2 is preferred.

Comparing design No. 2 to design No. 4, annual percent return = $\frac{850}{10000}(100)$ = 8.5 percent. Thus, design No. 4 is not acceptable when compared to design No. 2, and design No. 2 is the alternative that should be recommended.

ALTERNATIVE ANALYSIS INCORPORATING MINIMUM RETURN AS A COST. Identical results to the preceding are obtained by choosing the alternative giving the greatest annual profit or saving if the required return is included as a cost for each case. For the heat-exchanger example, the annual cost for the required return would be 10 percent of the total initial investment; thus,

For design No. 1, annual savings above required return = $2000 - (0.1) \times (10{,}000) = \$1000$.

For design No. 2, annual savings above required return = $2700 - (0.1) \times (16{,}000) = \$1100$.

For design No. 3, annual savings above required return = $2800 - (0.1) \times (20{,}000) = \$800$.

For design No. 4, annual savings above required return = $3550 - (0.1) \times (26{,}000) = \$950$.

Because annual saving is greatest for design No. 2, this would be the recommended alternative which is the same result as was obtained by the direct analysis based on return on incremental investment.

This simplified example has been used to illustrate the basic concepts involved in making comparisons of alternative investments. The approach was based on using the simple return on initial investment in which time value of money is neglected. Although this method may be satisfactory for preliminary and rough estimations, for final evaluations a more sophisticated approach is needed in which the time value of money is considered along with other practical factors to assure the best possible chance for future success. Typical more advanced approaches of this type are presented in the following sections.

ANALYSIS WITH SMALL INVESTMENT INCREMENTS

The design engineer often encounters the situation in which the addition of small investment increments is possible. For example, in the design of a heat exchanger for recovering waste heat, each square foot of additional heat-transfer area can cause a reduction in the amount of heat lost, but the amount of heat recovered per square foot of heat-transfer area decreases as the area is increased. Since the investment for the heat exchanger is a function of the heat-transfer area, a plot of net savings (or net profit due to the heat exchanger) versus total investment can be made. A smooth curve of the type shown in Fig. 10-5 results.

The point of maximum net savings, as indicated by 0 in Fig. 10-5, represents a classical optimum condition. However, the last incremental investment before this maximum point is attained is at essentially a zero percent return. On the basis of alternative investment comparisons, therefore, some investment less than that for maximum net savings should be recommended.

The exact investment where the incremental return is a given value occurs at the point where the slope of the curve of net savings versus investment equals the required return. Thus, a straight line with a slope equal to the necessary return is tangent to the net-savings-versus-investment curve at the point representing the recommended investment. Such a line for an annual return on incremental investment of 20 percent is shown in Fig. 10-5, and the recommended investment for this case is indicated by ***RI***. If an analytical expression

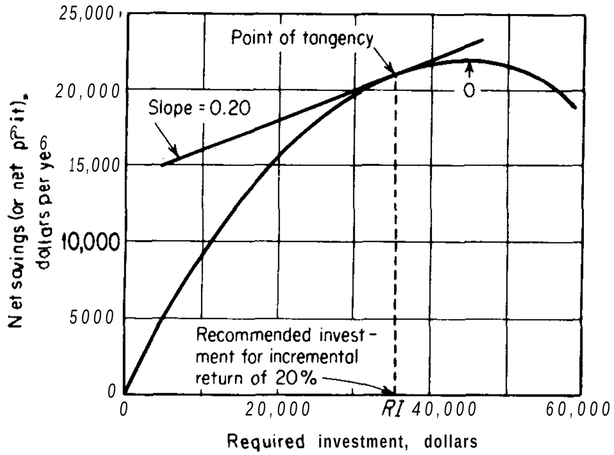

FIGURE 10-5
Graphical method for determining investment for a given incremental return when investment increments can approach zero.

relating net savings and investment is available, it is obvious that the recommended investment can be determined directly by merely setting the derivative of the net savings with respect to the investment equal to the required incremental return.

The method described in the preceding paragraph can also be used for continuous curves of the type represented by the dashed curve in Fig. 10-4. Thus, the line *ab* in Fig. 10-4 is tangent to the dashed curve at the point representing the recommended investment for the case of a 15 percent incremental return.

Example 4 Investment comparison for required operation with limited number of choices. A plant is being designed in which 450,000 lb per 24-h day of a water–caustic soda liquor containing 5 percent by weight caustic soda must be concentrated to 40 percent by weight. A single-effect or multiple-effect evaporator will be used, and a single-effect evaporator of the required capacity requires an initial investment of $18,000. This same investment is required for each additional effect. The service life is estimated to be 10 years, and the salvage value of each effect at the end of the service life is estimated to be $6000. Fixed charges minus depreciation amount to 20 percent yearly, based on the initial investment. Steam costs $0.60 per 1000 lb, and administration, labor, and miscellaneous costs are $40 per day, no matter how many evaporator effects are used.

Where X is the number of evaporator effects, $0.9X$ equals the number of pounds of water evaporated per pound of steam. There are 300 operating days per year. If the minimum acceptable return on any investment is 15 percent, how many effects should be used?

Solution. *Basis:* 1 operating day

X = total number of evaporator effects

$$\text{Depreciation per operating day (straight-line method)} = \frac{X(18{,}000 - 6000)}{(10)(300)}$$

$$= \$4.00X/\text{day}$$

$$\text{Fixed charges} - \text{depreciation} = \frac{X(18{,}000)(0.20)}{300} = \$12.00X/\text{day}$$

$$\text{Pounds of water evaporated per day} = (450{,}000)(0.05)\left(\tfrac{95}{5}\right) - (450{,}000)(0.05)\left(\tfrac{60}{40}\right)$$

$$= 393{,}800 \text{ lb/day}$$

$$\text{Steam costs} = \frac{(393{,}800)(0.60)}{X(0.9)(1000)} = \frac{\$262.5}{X} \text{ per day}$$

X = no. of effects	Steam costs per day	Fixed charges minus depreciation per day	Depreciation per day	Labor, etc., per day	Total cost per day
1	$262.5	$12	$ 4	$40	$318.5
2	131.3	24	8	40	203.3
3	87.5	36	12	40	175.5
4	65.6	48	16	40	169.6
5	52.5	60	20	40	172.5
6	43.8	72	24	40	179.8

Comparing two effects with one effect,

$$\text{Percent return} = \frac{(318.5 - 203.3)(300)(100)}{36{,}000 - 18{,}000} = 192\%$$

Therefore, two effects are better than one effect.

Comparing three effects with two effects,

$$\text{Percent return} = \frac{(203.3 - 175.5)(300)(100)}{54{,}000 - 36{,}000} = 46.3\%$$

Therefore, three effects are better than two effects.

Comparing four effects with three effects,

$$\text{Percent return} = \frac{(175.5 - 169.6)(300)(100)}{72{,}000 - 54{,}000} = 9.8\%$$

Since a return of at least 15 percent is required on any investment, three effects are better than four effects, and the four-effect evaporator should receive no further consideration.

Comparing five effects with three effects,

$$\text{Percent return} = \frac{(175.5 - 172.5)(300)(100)}{90{,}000 - 54{,}000} = 2.5\%$$

Therefore, three effects are better than five effects.

Since the total daily costs continue to increase as the number of effects is increased above five, no further comparisons need to be made.

A three-effect evaporator should be used.

ANALYSIS OF ADVANTAGES AND DISADVANTAGES OF VARIOUS PROFITABILITY MEASURES FOR COMPARING ALTERNATIVES

Of the methods presented for profitability evaluation and the economic comparison of alternatives, net present worth and discounted cash flow are the most generally acceptable, and these methods are recommended. Capitalized costs have limited utility but can serve to give useful and correct results when applied to appropriate situations. Payout period does not adequately consider the later years of the project life, does not consider working capital, and is generally useful only for rough and preliminary analyses. Rates of return on original investment and average investment do not include the time value of money, require approximations for estimating average income, and can give distorted results of methods used for depreciation allowances.

It is quite possible to compare a series of alternative investments by each of the profitability measures outlined in the early part of this chapter and find that different alternatives would be recommended depending on the evaluation technique used.[†] If there is any question as to which method should be used for a final determination, net ***present worth*** should be chosen, as this will be the most likely to maximize the future worth of the company.

Investment costs due to land can be accounted for in all the methods except payout period. Costs incurred during the construction period prior to startup can be considered correctly in both the net-present-worth and the discounted-cash-flow methods, while they are ignored in the return-on-investment methods and are seldom taken into account in determining payout period. None of the methods gives a direct indication of the magnitude of the project, although net present worth does give a general idea of the magnitude if interpreted correctly. In certain cases, such as for alternatives of different economic lives, the discounted-cash-flow rate-of-return method is very difficult to use for comparing investments correctly. The discounted-cash-flow rate-of-return method may give multiple or impossible answers for unusual cash-flow situations, such as a case where no cash investment is needed at the start and for certain replacement situations.

[†] This situation is illustrated in Example 5 of this chapter.

TABLE 4
Definitions to clarify income-tax situation for profitability evaluation

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Revenue = total income (or total savings)
Net profits = revenue − all expenses − income tax
All expenses = cash expenses + depreciation
Income tax = (revenue − all expenses)(tax rate)
Cash flow = net profits + depreciation
Cash flow = (revenue)(1 − tax rate) − (cash expenses)(1 − tax rate)
+ (depreciation)(tax rate)
Cash flow = (revenue)(1 − tax rate) − (all expenses)(1 − tax rate) + depreciation

For the case of a 34% tax rate

$1.00 of revenue (either as sales income or savings) yields a cash flow of $0.66.
$1.00 of cash expenses (as raw materials, labor, etc.) yields a cash outflow of $0.66.
$1.00 of depreciation yields a cash inflow of $0.34.

Consideration of Income Taxes

Income-tax effects can be included properly in all the profitability methods discussed in this chapter by using appropriate definitions of terms, such as those presented in Table 4. The methods of discounted-cash-flow rate of return and present worth are limited to consideration of cash income and cash outgo over the life of the project. Thus, depreciation, as a cost, does not enter directly into the calculations except as it may affect income taxes.

Net cash flow represents the difference between all cash revenues and all cash expenses with taxes included as a cash expense. Thus, discounted-cash-flow rate of return and present worth should be calculated on an after-tax basis, unless there is some particular reason for a pretax basis, such as comparison to a special alternate which is presented on a pre-tax basis.

Example 5 Comparison of alternative investments by different profitability methods. A company has three alternative investments which are being considered. Because all three investments are for the same type of unit and yield the same service, only one of the investments can be accepted. The risk factors are the same for all three cases. Company policies, based on the current economic situation, dictate that a minimum annual return on the original investment of 15 percent after taxes must be predicted for any unnecessary investment with interest on investment not included as a cost. (This may be assumed to mean that other equally sound investments yielding a 15 percent return after taxes are available.) Company policies also dictate that, where applicable, straight-line depreciation is used and, for time-value of money interpretations, end-of-year cost and profit analysis is used. Land value and prestartup costs can be ignored.

Given the following data, determine which investment, if any, should be made by alternative-analysis profitability-evaluation methods of

(*a*) Rate of return on initial investment
(*b*) Minimum payout period with no interest charge
(*c*) Discounted cash flow
(*d*) Net present worth
(*e*) Capitalized costs

Investment number	Total initial fixed-capital investment, $	Working-capital investment, $	Salvage value at end of service life, $	Service life, years	Annual cash flow to project after taxes,† $	Annual cash expenses‡ (constant for each year), $
1	100,000	10,000	10,000	5	See yearly tabulation §	44,000
2	170,000	10,000	15,000	7	52,000 (constant)	28,000
3	210,000	15,000	20,000	8	59,000 (constant)	21,000

† This is total annual income or revenue minus all costs except depreciation and interest cost for investment.

‡ This is annual coat for operation, maintenance, taxes, and insurance. Equals total annual income minus annual cash flow.

§ For investment number 1, variable annual cash flow to project is: year 1 = $30,000, year 2 = $31,000, year 3 = $36,000, year 4 = $40,000, year 5 = $43,000.

Solution

(a) Method of rate of return on initial investment.

Average annual profit = annual cash flow − annual depreciation cost

The average annual profits for investment No. 1, using straight-line depreciation are as follows:

Year	Average annual profit, dollars
1	$30{,}000 - \dfrac{(100{,}000 - 10{,}000)}{5} = 30{,}000 - 18{,}000 = 12{,}000$
2	31,000 − 18,000 = 13,000
3	36,000 − 18,000 = 18,000
4	40,000 − 18,000 = 22,000
5	43,000 − 18,000 = 25,000
	Total 90,000

For investment No. 1, the arithmetic average of the annual profits is 90,000/5 = $18,000.[¶]

The annual average rate of return on the first investment is

$$\frac{18{,}000}{100{,}000 + 10{,}000}(100) = 16.4\% \text{ after taxes}$$

[¶] An alternate method to obtain the average of the annual profits would be to determine the amount of the annuity R based on the end-of-year payments that would compound to the same future worth as the individual profits using an interest rate i of 0.15. With this approach, the average of the annual profits for investment No. 1 would be $17,100.

The method for determining this $17,100 is to apply the series compound-amount factor $[(1 + i)^n - 1]/i$ [see Eq. (21) in Chap. 71 to the annuity to give the future worth S of the annual incomes. The expression is $(12{,}000)(1 + i)^4 + (13{,}000)(1 + i)^3 + (18{,}000)(1 + i)^2 + (22{,}000)(1 + i) + 25{,}000 = R[(1 + i)^5 - 1]/i$. Solving for the case of i = 0.15 gives R = $17,100.

Because this return is greater than 15 percent, one of the three investments will be recommended, and it is only necessary to compare the three investments.

For investment number	Total initial investment	Average annual profit, dollars
1	\$110.000	\$18,000
2	\$180,000	$52{,}000 - \frac{170{,}000 - 15{,}000}{7} = 52{,}000 - 22{,}100 = \$29{,}900$
3	\$225,000	$59{,}000 - \frac{210{,}000 - 20{,}000}{8} = 59{,}000 - 23{,}800 = 135{,}200$

Comparing investment No. 2 with investment No. 1,

$$\text{Percent return} = \frac{29{,}900 - 18{,}000}{180{,}000 - 110{,}000}(100) = 17.0\%$$

Therefore, investment No. 2 is preferred over investment No. 1. Comparing investment No. 3 with investment No. 2,

$$\text{Percent return} = \frac{35{,}200 - 29{,}900}{225{,}000 - 180{,}000}(100) = 11.8\%$$

This return is not acceptable, and ***investment No. 2 should be recommended.***

The same result would have been obtained if a minimum return of 15 percent had been incorporated as an expense.

(*b*) Method of minimum payout period with no interest charge.

$$\text{Payout period (with no interest charge)} = \frac{\text{depreciable fixed-capital investment}}{\text{avg profit/yr + avg depreciation/yr}}$$

For investment number	*Payout period, years*
1	$\frac{90{,}000}{18{,}000 + 18{,}000} = 2.50$
2	$\frac{155{,}000}{29{,}900 + 22{,}100} = 2.98$
3	$\frac{190{,}000}{35{,}200 + 23{,}800} = 3.22$

The payout period for investment No. 1 is least; therefore, by this method, ***investment No. 1 should be recommended.***

(*c*) Method of discounted cash flow. For investment No. 1, as illustrated in Table 1, the rate of return based on discounted cash flow is 20.7 percent.

For investment No. 2, the discounted-cash-flow equation is

$$(52{,}000)\left[\frac{1}{1+i} + \frac{1}{(1+i)^2} + \cdots + \frac{1}{(1+i)^7}\right] + (10{,}000 + 15{,}000)\frac{1}{(1+i)}$$
$$= \$180{,}000$$

By trial-and-error solution, the discounted-cash-flow rate of return is 22.9 percent

Similarly, for investment No. 3,

$$(59{,}000)\left[\frac{1}{1+i} + \frac{1}{(1+i)^2} + \cdots + \frac{1}{(1+i)^8}\right] + (15{,}000 + 20{,}000)\frac{1}{(1+i)^8}$$

$$= \$225{,}000$$

By trial-and-error solution, the discounted-cash-flow rate of return is 21.5 percent.

To make a choice among the three alternatives, it is necessary to make a comparison among the three possible choices. This comparison can be made in a relatively straightforward manner using discounted-cash-flow rates of return by comparing pairs of investments on a mutually exclusive basis if the various alternatives have the same economic service lives. When different lengths of service life are involved, as in this problem, the best approach is to avoid the calculated rates of return and make the investment comparison by the net present-worth method as shown in part (*d*) of this problem. It would be possible to use discounted-cash-flow rates of return for comparison between investments with different service lives by assuming that each investment could be repeated at the end of its service life until a common end point was obtained for the investments being compared; however, this method becomes very involved mathematically and is not realistic.

If the service lives of the investments being compared are not widely different, the following ***approximate*** method using discounted-cash-flow rate of return can be employed for the comparison.†

In comparing a pair of alternatives, the base time is chosen as the longer of the two service lines. For the case of the investment with the shorter life, it is assumed that the accumulated amount at the end of its life can be invested at the minimum acceptable rate for the remaining time to equalize the two lives. The rate of return on the incremental investment can then be determined.

Comparison of investment No. 2 to investment No. 1. At the end of its 7-year service life, the net value of investment No. 2 is

$$(180{,}000)(1 + 0.229)' + 10{,}000 + 15{,}000 = \$785{,}000$$

With investment No. 1, the net value after 7 years is the amount accumulated in 5 years times the factor to let this accumulated amount be invested at 15 percent for 2 more years, or

$$\left[(110{,}000)(1 + 0.207)^5 + 10{,}000 + 10{,}000\right](1 + 0.15)' = \$398{,}000$$

Therefore, a gain of \$785,000 − \$398,000 = \$387,000 is made in 7 years by an added investment of \$70,000 if investment No. 2 is made instead of investment No, 1. The discounted-cash-flow rate of return for this incremental investment is found by

$$(70{,}000)(1 + i)^7 = 387{,}000$$

$$i = 0.277 \quad \text{or } 27.7\%$$

This return is greater than 15 percent; so investment No. 2 is preferred over investment No. 1.

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†The method is shown to illustrate the use of discounted-cash-flow rates of return for investment comparisons. It is correct only for comparisons involving equal service lives. If service lives are different, this method tends to favor the investment with the longest service life.

Comparison of investment **No. 3** ***to investment No. 2.*** At the end of its **8-year** service life, the net value of investment No. 3 is

$$(225{,}000)(1 + 0.215)^8 + 15{,}000 + 20{,}000 = \$1{,}105{,}000$$

For comparison, $180,000 invested in investment No. 2 would, with the last year at a 15 percent return, accumulate in 8 years to

$$\left[(180{,}000)(1 + 0.229)' + 10{,}000 + 15{,}000\right](1 + 0.15) = \$903{,}060$$

Therefore, a gain of **$1,105,000** − $903,000 = $202,000 is made *in* **8** years by an added investment of $45,000 by making investment No. 3 instead of investment No. 2. The discounted-cash-flow rate of return for this incremental investment is found by

$$(45{,}000)(1 + i)^8 = 202{,}000$$

$$i = 0.208 \text{ or } 20.8\%$$

This return is greater than **15 percent;** so investment No. 3 is preferred over investment No. 2. Therefore, ***investment No. 3 should be recommended.***

(*d*) **Method of net present** worth. For investment No. 1, as illustrated in Table 1, the present value of the cash flow to the project, discounted at an interest rate of 15 percent, is $127,000. Therefore, the net present worth of investment No. 1 is $127,000 − $110,000 = $17,000.

For investments 2 and 3, the present values of the cash flows to the projects are determined from the first two equations under part (*c*) of this problem, with $i = 0.15$. The resulting net present worth are:

For investment No. 2, net present worth = **$226,000** − $180,000 = $46,000

For investment No. 3, net present worth = **$278,000 − $225,000 = $53,000**

The greatest net present worth is found for investment No. 3; therefore, ***investment No. 3 should be recommended.***

(*e*) **Method of capitalized costs.** Capitalized costs for each investment situation must include the capitalized cost for the original investment to permit an indefinite number of replacements plus the capitalized present value of the cash expenses plus working capital.

Capitalized present value of cash expenses is determined as follows:

Let $C_{n'}$ be the annual cash expense in year n' of the project life. The present value of the annual cash expenses is then

$$\sum_{d-1}^{n'=n} C_{n'} \frac{1}{(1 + i)^{n'}}$$

and the capitalized present value is

$$\frac{(1 + i)^n}{(1 + i)'' - 1} \sum_{n'=1}^{n'=n} C``\frac{1}{(1 + i)^{`'}}$$

If $C_{n'}$ is constant, as is the case for this example, the capitalized present value becomes (annual cash expenses)/i. Therefore,

$$\text{Capitalized cost} = \frac{C_R(1+i)''}{(1+i)'' - 1} + V_s + \frac{\text{annual cash expenses}}{i} + \text{working capital}$$

where n = service life
i = annual rate of return
C_R = replacement cost
V_s = salvage value

For investment No. 1,

$$\text{Capitalized cost} = \frac{(90{,}000)(1+0.15)^5}{(1+0.15)^5 - 1} + 10{,}000 + \frac{44{,}000}{0.15} + 10\,000 = \$492\,000$$

For investment No. 2,

$$\text{Capitalized cost} = \frac{(155{,}000)(1+0.15)^7}{(1+0.15)' - 1} + 15{,}000 + \frac{28{,}000}{0.15} + 10{,}000$$

$$= \$460{,}000$$

For investment No. 3,

$$\text{Capitalized cost} = \frac{(190{,}000)(1+0.15)^8}{(1+0.15)^8 - 1} + 20{,}000 + \frac{21{,}000}{0.15} + 15{,}000$$

$$= \$457{,}000$$

The capitalized cost based on a minimum rate of return of 15 percent is least for investment No. 3; therefore, ***investment No. 3 should be recommended.***

Note: Methods (a) and (*b*) in this problem give incorrect results because the time value of money has not been included. Although investment No. 3 is recommended by methods (*c*), (*d*), and (*e*), it is a relatively narrow choice over investment No. 2. Consequently, for a more accurate evaluation, it would appear that the company management should be informed that certain of their policies relative to profitability evaluation are somewhat old fashioned and do not permit the presentation of a totally realistic situation. For example, the straight-line depreciation method may not be the best choice, and a more realistic depreciation method may be appropriate. The policy of basing time-value-of-money interpretations on end-of-year costs and profits is a simplification, and it may be better to permit the use of continuous interest compounding and continuous cash flow where appropriate. For a final detailed analysis involving a complete plant, variations in prestartup costs among alternatives may be important, and this factor should not be ignored.

REPLACEMENTS

The term “replacement,” as used in this chapter, refers to a special type of alternative in which facilities are currently in existence and it may be desirable to replace these facilities with different ones. Although intangible factors may have a strong influence on decisions relative to replacements, the design engineer must understand the tangible economic implications when a recommendation is made as to whether or not existing equipment or facilities should be replaced.

The reasons for making replacements can be divided into two general classes, as follows:

1. An existing property *must* be replaced or changed in order to continue operation and meet the required demands for service or production. Some examples of reasons for this type of necessary replacement are:
    a. The property is worn out and can give no further useful service.
    b. The property does not have sufficient capacity to meet the demand placed upon it.
    c. Operation of the property is no longer economically feasible because changes in design or product requirements have caused the property to become obsolete.
2. An existing property is capable of yielding the necessary product or service, but more efficient equipment or property is available which can operate with lower expenses.

When the reason for a replacement falls in the first general type, the only alternatives are to make the necessary changes or else go out of business. Under these conditions, the final economic analysis is usually reduced to a comparison of alternative investments.

The correct decision as the the desirability of replacing an existing property which is capable of yielding the necessary product or service depends on a clear understanding of theoretical replacement policies plus a careful consideration of many practical factors. In order to determine whether or not a change is advisable, the operating expenses with the present equipment must be compared with those that would exist if the change were made. Practical considerations, such as amount of capital available or benefits to be gained by meeting a competitor's standards, may also have an important effect on the final decision.

Methods of Profitability Evaluation for Replacements

The same methods that were explained and applied earlier in this chapter are applicable for replacement analyses. Net-present-worth and discounted-cash-flow methods give the soundest results for maximizing the overall future worth of a concern. However, for the purpose of explaining the *basic principles* of replacement economic analyses, the simple rate-of-return-on-investment method of analysis is just as effective as those methods involving the time value of money. Thus, to permit the use of direct illustrations which will not detract from the principles under consideration, the following analysis of methods for economic decisions on replacements uses the annual rate of return on initial investment as the profitability measure. The identical principles can be treated by more complex rate-of-return and net-present-worth solutions by merely applying the methods described earlier in this chapter.

Typical Example of Replacement Policy

The following example illustrates the type of economic analysis involved in determining if a replacement should be made: A company is using a piece of equipment which originally cost $30,000. The equipment has been in use for 5 years, and it now has a net realizable value of $6000. At the time of installation, the service life was estimated to be 10 years and the salvage value at the end of the service life was estimated to be zero. Operating costs amount to $22,000/year. At the present time, the remaining service life of the equipment is estimated to be 3 years.

A proposal has been made to replace the present piece of property by one of more advanced design. The proposed equipment would cost $40,000, and the operating costs would be $15,000/year. The service life is estimated to be 10 years with a nonzero salvage value. Each piece of equipment will perform the same service, and all costs other than those for operation and depreciation will remain constant. Depreciation costs are determined by the straight-line method.? The company will not make any unnecessary investments in equipment unless it can obtain an annual return on the necessary capital of at least 10 percent.

The two alternatives in this example are to continue the use of the present equipment or to make the suggested replacement. In order to choose the better alternative, it is necessary to consider both the reduction in expenses obtainable by making the change and the amount of new capital necessary. The only variable expenses are those for operation and depreciation. Therefore, the annual variable expenses for the proposed equipment would be $15,000 + $40,000/10 = $19,000.

The net realizable value of the existing equipment is $6000. In order to make a fair comparison between the two alternatives, all costs must be based on conditions at the present time. Therefore, the annual depreciation cost for the existing equipment during the remaining three years of service life would be $6000/3 = $2000. For purposes of comparison, the annual variable expenses if the equipment is retained in service would be $22,000 + $2000 = $24,000.

An annual saving of $24,000 - $19,000 = $5000 can be realized by making the replacement. The cost of the new equipment is $40,000, but the sale of the existing property would provide $6000; therefore, it would be necessary to invest

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†The use of the sinking-fund method for determining depreciation is sometimes advocated for replacement studies. However, for most practical situations, the straight-line method is satisfactory.

only $34,000 to bring about an annual saving of $5000. Since this represents a return greater than 10 percent, the existing equipment should be replaced.

Book Values and Unamortized Values

In the preceding example, the book value of the existing property was $15,000 at the time of the proposed replacement. However, this fact was given no consideration in determining if the replacement should be made. The book value is based on past conditions, and the correct decision as to the desirability of making a replacement must be based on present conditions.

The difference between the book value and the net realizable value at any time is commonly designated as the ***unamortized*** value. In the example, the unamortized value was $15,000 − $6000 = $9000. This means that a $9000 loss was incurred because of incorrect estimation of depreciation allowances.[†]

Much of the confusion existing in replacement studies is caused by unamortized values. Some individuals feel that a positive unamortized value represents a loss which would be caused by making a replacement. This is not correct because the loss is a result of past mistakes, and the fact that the error was not apparent until a replacement was considered can have no bearing on the conditions existing at the present time. ***When making theoretical replacement studies, unamortized values must be considered as due to past errors, and these values are of no significance in the present decision as to whether or not a replacement should be made.***

Although unamortized values have no part in a replacement study, they must be accounted for in some manner. One method for handling these capital losses or gains is to charge them directly to the profit-and-loss account for the current operating period. When a considerable loss is involved, this method may have an unfavorable and unbalanced effect on the profits for the current period. Many concerns protect themselves against such unfavorable effects by building up a surplus reserve fund to handle unamortized values. This fund is accumulated by setting aside a certain sum each accounting period. When losses due to unamortized values occur, they are charged against the accumulated fund. In this manner, unamortized values can be handled with no serious effects on the periodic profits.

Investment on which Replacement Comparison is Based

As indicated in the preceding section, the unamortized value of an existing property is based on past conditions and plays no part in a replacement study.

[†] As explained in Chap. 8 (Taxes and Insurance) and Chap. 9 (Depreciation) and as illustrated in Example 6 of this Chapter, part of the loss may be recovered by income-tax write-offs if the Internal Revenue Service will agree that the maximum expected life was used.

The advisability of making a replacement is usually determined by the rate of return which can be realized from the necessary investment. It is, therefore, important to consider the amount of the investment. ***The difference between the total cost of the replacement property and the net realizable value of the misting property equals the necessary investment.*** Thus, the correct determination of the investment involves only consideration of the present capital outlay required if the replacement is made.

Net Realizable Value

In replacement studies, the ***net realizable value*** of an existing property should be assumed to be the market value. Although this may be less than the actual value of the property as far as the owner is concerned, it still represents the amount of capital which can be obtained from the old equipment if the replacement is made. Any attempt to assign an existing property a value greater than the net realizable value tends to favor replacements which are uneconomical.

Analysis of Common Errors Made in Replacement Studies

Most of the errors in replacement studies are caused by failure to realize that a replacement analysis must be based on conditions existing at the present time. Some persons insist on trying to compensate for past mistakes by forcing new ventures to pay off losses incurred in the past. Instead, these losses should be accepted, and the new venture should be considered on its own merit.

INCLUDING UNAMORTIZED VALUE AS AN ADDITION TO THE REPLACEMENT INVESTMENT. This is one of the most common errors. It increases the apparent cost for the replacement and tends to prevent replacements which are really economical. Some persons incorporate this error into the determination of depreciation cost for the replacement equipment, while others include it only in finding the investment on which the rate of return is based. In any case, this method of attempting to account for unamortized values is incorrect. The unamortized value must be considered as a dead loss (or gain) due to incorrect depreciation accounting in the past.

USE OF BOOK VALUE FOR OLD EQUIPMENT IN REPLACEMENT STUDIES. This error is caused by refusal to admit that the depreciation accounting methods used in the past were wrong. Persons who make this error attempt to justify their actions by claiming that continued operation of the present equipment would eventually permit complete depreciation. This viewpoint is completely unrealistic because it gives no consideration to the competitive situation existing in modern business. The concern which can operate with good profit and still offer a given product or service at the lowest price can remain in business and force competitors either to reduce their profits or to cease operation.

When book values are used in replacement studies, the apparent costs for the existing equipment are usually greater than they should be, while the apparent capital outlay for the replacement is reduced. Therefore, this error tends to favor replacements which are really uneconomical.

Example 6 An extreme situation to illustrate result of replacement economic analysis. A new manufacturing unit has just been constructed and put into operation by your company. The basis of the manufacturing process is a special computer for control (designated as OVT computer) as developed by your research department. The plant has now been in operation for less than one week and is performing according to expectations. A new computer (designated as NTR computer) has just become available on the market. This new computer can easily be installed at once in place of your present computer and will do the identical job at far less annual cash expense because of reduced maintenance and personnel costs. However, if the new computer is installed, your present computer is essentially worthless because you have no other use for it.

Following is pertinent economic information relative to the two computers:

	OVT computer	NTR computer
Capital investment	$2,000,000	$1,000,000
Estimated economic life	10 years	10 years
Salvage value at end of economic life	0	0
Annual cash expenses	$250,000	$50,000

What recommendation would you make relative to replacing the present $2,000,000 computer with the new computer?

Solution. Assuming straight-line depreciation, the annual total expenses with the NTR computer = $50,000 + $1,000,000/10 = $150,000.

For replacement economic comparison, the OVT computer is worth nothing at the present time; therefore, annual total expenses with the OVT computer = $250,000 (i.e., no depreciation charge).

The $2,000,000 investment for the OVT computer is completely lost if the NTR computer is installed; so the total necessary investment to make an annual saving of $250,000 – $150,000 is $1,000,000. Therefore, the return on investment would be (100,000/1,000,000)(100) = 10 percent.

If your company is willing to accept a return on investment of 10 percent before taxes [or (0.66)(10) = 6.6 percent after taxes, assuming a 34 percent tax rate], the replacement should be made.

If income taxes are taken completely into consideration, the result can tend to favor the replacement. For example, if your company can write off the $2,000,000 loss in one lump sum against other capital gains which would normally be taxed at 30 percent, the net saving will be 30 percent of $2,000,000 or $600,000. Assuming a 34 percent tax rate on annual profits and dividing the $600,000 capital-gains tax saving over the ten years of the new computer life, the percent return after taxes =

$$\frac{600{,}000/10 + (0.66)(100{,}000)}{1{,}000{,}000}(100) = 12.6 \text{ percent}$$

Because of the reduced costs for the NTR computer, profitability evaluation including time value of money will tend to favor the replacement more than does the method of rate of return on investment as used for the solution of this example.

PRACTICAL FACTORS IN ALTERNATIVE-INVESTMENT AND REPLACEMENT STUDIES

The previous discussion has presented the theoretical viewpoint of alternative-investment and replacement studies; however, certain practical considerations also influence the final decision. In many cases, the amount of available capital is limited. From a practical viewpoint, therefore, it may be desirable to accept the smallest investment which will give the necessary service and permit the required return. Although a greater investment might be better on a theoretical basis, the additional return would not be worth the extra risks involved when capital must be borrowed or obtained from some other outside source.

A second practical factor which should be considered is the accuracy of the estimations used in determining the rates of return. A theoretically sound investment might not be accepted because the service life used in determining depreciation costs appear to be too long. All risk factors should be given careful consideration before making any investment, and the risk factors should receive particular attention before accepting an investment greater than that absolutely necessary.

The economic conditions existing at the particular time have an important practical effect on the final decision. In depression periods or in times when economic conditions are very uncertain, it may be advisable to refrain from investing any more capital than is absolutely necessary. The tax situation for the corporation can also have an effect on the decision.

Many intangible factors enter into the final decision on a proposed investment. Sometimes it may be desirable to impress the general public by the external appearance of a property or by some unnecessary treatment of the final product. These advertising benefits would probably receive no consideration in a theoretical economic analysis, but they certainly would influence management's final decision in choosing the best investment.

Personal whims or prejudices, desire to better a competitor's rate of production or standards, the availability of excess capital, and the urge to expand an existing plant are other practical factors which may be involved in determining whether or not a particular investment will be made.

Theoretical analyses of alternative investments and replacements can be used to obtain a dollar-and-cents indication of what should be done about a proposed investment. The final decision depends on these theoretical results plus practical factors determined by the existing circumstances.

NOMENCLATURE FOR CHAPTER 10

$C_{n'}$ = annual cash expenses in year II’, dollars
$CP_{\text{zero time}}$ = total cash position at zero time, dollars
C_R = replacement cost, dollars
C_V = original cost, dollars
$d_{n'}$ = discount factor for determining present value
e = base of the natural logarithm = 2.71828.. .
i = annual interest rate of return, percent/100
K = capitalized cost, dollars
n = estimated service life, years
n' = year of project life to which cash flow applies
P = present value, dollars
R = end-of-year (or ordinary) annuity amount, dollars/year
$\bar{R}$ = total of all ordinary annuity payments occurring regularly throughout the time period of one year, dollars/year
r = nominal continuous interest rate, percent/100
S = future worth, dollars
V_s = estimated salvage value at end of service life, dollars
X = number of evaporator units
Y = time period for construction, years
Z = time period land is owned before plant startup, years

PROBLEMS

1. A proposed chemical plant will require a fixed-capital investment of $10 million. It is estimated that the working capital will amount to 25 percent of the total investment, and annual depreciation costs are estimated to be 10 percent of the fixed-capital investment. If the annual profit will be $3 million, determine the standard percent return on the total investment and the minimum payout period.
2. An investigation of a proposed investment has been made. The following result has been presented to management: The minimum payout period based on capital recovery using a minimum annual return of 10 percent as a fictitious expense is 10 years; annual depreciation costs amount to 8 percent of the total investment. Using this information, determine the standard rate of return on the investment.
3. The information given in Prob. 2 applies to conditions before income taxes. If 34 percent of all profits must be paid out for income taxes, determine the standard rate of return after taxes using the figures given in Prob. 2.
4. A heat exchanger has been designed and insulation is being considered for the unit. The insulation can be obtained in thickness of 1, 2, 3, or 4 in. The following data

have been determined for the different insulation thicknesses:

	1 in.	2 in.	3 in.	4 in.
Btu/h saved	300,000	350,000	370,000	380,000
Cost for installed insulation	$1200	$1600	$1800	$1870
Annual fixed charges	10%	10%	10%	10%

What thickness of insulation should be used? The value of heat is 30 cents/1,000,000 Btu. An annual return of 15 percent on the fixed-capital investment is required for any capital put into this type of investment. The exchanger operates 300 days per year.

5. A company must purchase one reactor to be used in an overall operation. Four reactors have been designed, all of which are equally capable of giving the required service. The following data apply to the four designs:

	Design 1	Design 2	Design 3	Design 4
Fixed-capital investment	$10,000	$12,000	$14,000	$16,000
Sum of operating and fixed costs per year (all other costs are constant)	3,000	2,800	2,350	2,100

If the company demands a 15 percent return on any unnecessary investment, which of the four designs should be accepted?

6. The capitalized cost for a piece of equipment has been found to be $55,000. This cost is based on the original cost plus the present value of an indefinite number of renewals. An annual interest rate of 12 percent was used in determining the capitalized cost. The salvage value of the equipment at the end of the service life was taken to be zero, and the service life was estimated to be 10 years. Under these conditions, what would be the original cost of the equipment?

7. An existing warehouse is worth $500,000, and the average value of the goods stored in it is $400,000. The annual insurance rate on the warehouse is 1.1 percent, and the insurance rate on the stored goods is 0.95 percent. If a proposed sprinkling system is installed in the warehouse, both insurance rates would be reduced to three-quarters of the original rate. The installed sprinkler system would cost $20,000, and the additional annual cost of maintenance, inspection, and taxes would be $300. The required write-off period for the entire investment in the sprinkler system is 20 years. The capital necessary to make the investment is available. The operation of the warehouse is now giving an 8 percent return on the original investment. Give reasons why you would or would not recommend installing the sprinkler system.

8. A power plant for generating electricity is one part of a plant-design proposal. Two alternative power plants with the necessary capacity have been suggested. One uses a boiler and steam turbine while the other uses a gas turbine. The following informa-

tion applies to the two proposals:

	Boiler and steam turbine	Gas turbine
Initial investment	$600,000	$400,000
Fuel costs, per year	16,000	23,000
Maintenance and repairs, per year	12,000	15,000
Insurance and taxes, per year	18,000	12,000
Service life, years	20	10
Salvage value at end of service life	0	0

All other costs are the same for either type of power plant. A 12 percent return is required on any investment. If one of these power plants must be accepted, which one should be recommended?

9. The facilities of an existing chemical company must be increased if the company is to continue in operation. There are two alternatives. One of the alternatives is to expand the present plant. If this is done, the expansion would cost $130,000. Additional labor costs would be $150,000 per year, while additional costs for overhead, depreciation, taxes, and insurance would be $60,000 per year.

   A second alternative requires construction and operation of new facilities at a location about 50 miles from the present plant. This alternative is attractive because cheaper labor is available at this location. The new facilities would cost $200,000. Labor costs would be $120,000 per year. Overhead costs would be $70,000 per year. Annual insurance and taxes would amount to 2 percent of the initial cost. All other costs except depreciation would be the same at each location. If the minimum return on any acceptable investment is 9 percent, determine the minimum service life allowable for the facilities at the distant location for this alternative to meet the required incremental return. The salvage value should be assumed to be zero, and straight-line depreciation accounting may be used.

10. A chemical company is considering replacing a batch-wise reactor with a modernized continuous reactor. The old unit cost $40,000 when new 5 years ago, and depreciation has been charged on a straight-line basis using an estimated service life of 15 years and final salvage value of $1000. It is now estimated that the unit has a remaining service life of 10 years and a final salvage value of $1000.

    The new unit would cost $70,000 and would result in an increase of $5000 in the gross annual income. It would permit a labor saving of $7000 per year. Additional costs for taxes and insurance would be $1000 per year. The service life is estimated to be 12 years with a final salvage value of $1000. All costs other than those for labor, insurance, taxes, and depreciation may be assumed to be the same for both units. The old unit can now be sold for $5000. If the minimum required return on any investment is 15 percent, should the replacement be made?

11. The owner of a small antifreeze plant has a small canning unit which cost him $5000 when he purchased it 10 years ago. The unit has completely depreciated, but the owner estimates that it will still give him good service for 5 more years. At the end of 5 years the unit will be worth a junk value of $100. The owner now has an

opportunity to buy a more efficient canning unit for $6000 having an estimated service life of 10 years and zero salvage or junk value. This new unit would reduce annual labor and maintenance costs by $1000 **and increase annual expenses for taxes** and insurance by $100. All other expenses except depreciation would be unchanged. If the old canning unit can be sold for $600, what replacement return on his capital investment will the owner receive if he decides to make the replacement?

12. An engineer in charge of the design of a plant must choose either a batch or a continuous system. The batch system offers a lower initial outlay but, owing to higher labor requirements, exhibits a higher operating cost. The cash flows relevant to this problem have been estimated as follows:

	Year		Discounted-cash-flow rate of return	Net present worth at 10%
	0	1-10		
Batch system	−$20,000	$5600	25%	$14,400
Continuous system	−$30,000	$7650	22%	$17,000

Check the values given for the discounted-cash-flow rate of return and net present worth. If the company requires a minimum rate of return of 10 percent, which system **should be chosen?**

13. A company is considering the purchase and installation of a pump which will deliver oil at a faster rate than the pump already in use. The purchase and installation of the larger pump will require an immediate layout of $1600, but it will recover all the cost by the end of one year. The relevant cash flows have been established as follows:

	Year			Discounted-cash-flow rate of return	Net present worth at 10%
	0	1	2		
Install larger pump	−$1600	$20,000	0	1250%	$16,580
Operate existing pump	0	$10,000	$10,000	?	$17,355

Explain the values given for the discounted-cash-flow rate of return and net present worth. If the company requires a minimum rate of return of 10 percent, which alternative should be chosen?

14. An oil company is offered a lease of a group of oil wells on which the primary reserves are close to exhaustion. The major condition of the purchase is that the oil company must agree to undertake a water-flood project at the end of five years to make possible secondary recovery. No immediate payment by the oil company is

required. The relevant cash flows have been estimated as follows:

Year				Discounted-cash-flow rate of return	Net present worth at 10%
0	1-4	5	6–20		
0	$50,000	−$650,000	$100,000	?	$227,000

Should the lease-and-flood arrangement be accepted? How should this proposal be presented to the company board of directors who understand and make it a policy to evaluate by discounted-cash-flow rate of return?

15. For Example 3 in this chapter, determine the profitability index using the simplified Eq. (D) in the example instead of Eq. (E) as was used for Table 3. As a first approximation, assume the profitability index is 30 percent.

CHAPTER 11

OPTIMUM DESIGN AND DESIGN STRATEGY

An optimum design is based on the best or most favorable conditions. In almost every case, these optimum conditions can ultimately be reduced to a consideration of costs or profits. Thus, an optimum economic design could be based on conditions giving the least cost per unit of time or the maximum profit per unit of production. When one design variable is changed, it is often found that some costs increase and others decrease. Under these conditions, the total cost may go through a minimum at one value of the particular design variable, and this value would be considered as an optimum.

An example illustrating the principles of an optimum economic design is presented in Fig. 11-1. In this simple case, the problem is to determine the optimum thickness of insulation for a given steam-pipe installation. As the insulation thickness is increased, the annual fixed costs increase, the cost of heat loss decreases, and all other costs remain constant. Therefore, as shown in Fig. 11-1, the sum of the costs must go through a minimum at the optimum insulation thickness.

Although cost considerations and economic balances are the basis of most optimum designs, there are times when factors other than cost can determine the most favorable conditions. For example, in the operation of a catalytic reactor, an optimum operation temperature may exist for each reactor size because of equilibrium and reaction-rate limitations. This particular temperature could be based on the maximum percentage conversion or on the maximum amount of final product per unit of time. Ultimately, however, cost variables

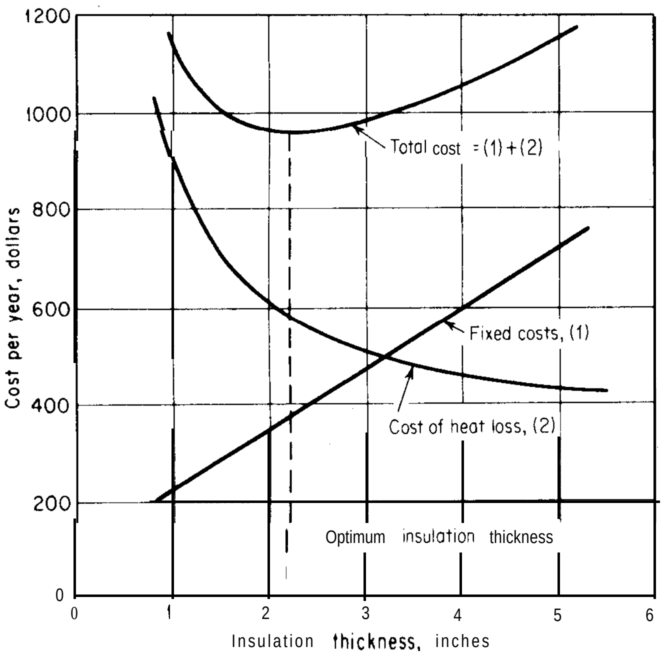

FIGURE 11-1
Illustration of the basic principle of an optimum design.

need to be considered, and the development of an optimum operation design is usually merely one step in the determination of an optimum economic design.

INCREMENTAL COSTS

The subject of incremental costs is covered in detail in Chap. 10 (Profitability, Alternative Investments, and Replacements). Consideration of incremental costs shows that a final recommended design does not need to correspond to the optimum economic design, because the incremental return on the added investment may become unacceptable before the optimum point is reached.† However, the optimum values can be used as a basis for starting the incremental-cost analyses.

This chapter deals with methods for determining optimum conditions, and it is assumed that the reader understands the role of incremental costs in establishing a final recommended design.

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†See Fig. 10-5 and the related discussion in Chap. 10. The material presented in Chap. 11 considers optimum designs based on maximum or minimum values of a specified variable. The same type of approach could be used if the term *optimum* (referring to an investment) were defined on the basis of a stipulated incremental return.

INTANGIBLE AND PRACTICAL CONSIDERATIONS

The various mathematical methods for determining optimum conditions, as presented in this chapter, represent on a theoretical basis the conditions that best meet the requirements. However, factors that cannot easily be quantitized or practical considerations may change the final recommendation to other than the theoretically correct optimum condition. Thus, a determination of an "optimum condition," as described in this chapter, serves as a base point for a cost or design analysis, and it can often be quantitized in specific mathematical form. From this point, the engineer must apply judgment to take into account other important practical factors, such as return on investment or the fact that commercial equipment is often available in discrete intervals of size.

As an example, consider the case where an engineer has made an estimation of the optimum pipe diameter necessary to handle a given flow stream based on minimizing the costs due to fixed charges and frictional pumping costs. The mathematical result shows that the optimum inside pipe diameter is 2.54 in. based on costs for standard (schedule 40) steel pipe. Nominal pipe diameters available commercially in this range are $2\frac{1}{2}$ in. (ID of 2.469 in.) and 3 in. (ID of 3.069 in.). The practical engineer would probably immediately recommend a nominal pipe diameter of $2\frac{1}{2}$ in. without going to the extra effort of calculating return on investment for the various sizes available. This approach would normally be acceptable because of the estimations necessarily involved in the optimum calculation and because of the fact that an investment for pipe represents only a small portion of the total investment.

Intangible factors may have an effect on the degree of faith that can be placed on calculated results for optimum conditions. Perhaps the optimum is based on an assumed selling price for the product from the process, or it might be that a preliminary evaluation is involved in which the location of the plant is not final Obviously, for cases of this type, an analysis for optimum conditions can give only a general idea of the actual results that will be obtained in the final plant, and it is not reasonable to go to extreme limits of precision and accuracy in making recommendations. Even for the case of a detailed and firm design, intangibles, such as the final bid from various contractors for the construction, may make it impractical to waste a large amount of effort in bringing too many refinements into the estimation of optimum conditions.

GENERAL PROCEDURE FOR DETERMINING OPTIMUM CONDITIONS

The first step in the development of an optimum design is to determine what factor is to be optimized. Typical factors would be total cost per unit of production or per unit of time, profit, amount of final product per unit of time, and percent conversion. Once the basis is determined, it is necessary to develop relationships showing how the different variables involved affect the chosen factor. Finally, these relationships are combined graphically or analytically to give the desired optimum conditions.

PROCEDURE WITH ONE VARIABLE

There are many cases in which the factor being optimized is a function of a single variable. The procedure then becomes very simple. Consider the example presented in Fig. 11-1, where it is necessary to obtain the insulation thickness which gives the least total cost. The primary variable involved is the thickness of the insulation, and relationships can be developed showing how this variable affects all costs.

Cost data for the purchase and installation of the insulation are available, and the length of service life can be estimated. Therefore, a relationship giving the effect of insulation thickness on fixed charges can be developed. Similarly, a relationship showing the cost of heat lost as a function of insulation thickness can be obtained from data on the value of steam, properties of the insulation, and heat-transfer considerations. All other costs, such as maintenance and plant expenses, can be assumed to be independent of the insulation thickness.

The two cost relationships obtained might be expressed in a simplified form similar to the following:

$$\text{Fixed charges} = \phi(x) = ax + b \tag{1}$$

$$\text{Cost of heat loss} = \phi^{i}(x) = \frac{c}{x} + d \tag{2}$$

$$\text{Total variable cost} = C_T = \phi(x) + \phi^{i}(x) = \phi^{ii}(x) = ax + b + \frac{c}{x} + d \tag{3}$$

where a, b, c, and d are constants and x is the common variable (insulation thickness).

The graphical method for determining the optimum insulation thickness is shown in Fig. 11-1. The optimum thickness of insulation is found at the minimum point on the curve obtained by plotting total variable cost versus insulation thickness.

The slope of the total-variable-cost curve is zero at the point of optimum insulation thickness. Therefore, if Eq. (3) applies, the optimum value can be found *analytically* by merely setting the derivative of C_T with respect to x equal to zero and solving for x.

$$\frac{dC_T}{dx} = a - \frac{c}{x^2} = 0 \tag{4}$$

$$x = \left(\frac{c}{a}\right)^{1/2} \tag{5}$$

If the factor being optimized (C_T) does not attain a usable maximum or minimum value, the solution for the dependent variable will indicate this condition by giving an impossible result, such as infinity, zero, or the square root of a negative number.

The value of x shown in Eq. (5) occurs at an optimum point or a point of inflection. The second derivative of Eq. (3), evaluated at the given point, indicates if the value occurs at a minimum (second derivative greater than zero), maximum (second derivative less than zero), or point of inflection (second derivative equal to zero). An alternative method for determining the type of point involved is to calculate values of the factor being optimized at points slightly greater and slightly smaller than the optimum value of the dependent variable.

The second derivative of Eq. (3) is

$$\frac{d^2C_T}{dx^2} = +\frac{2c}{x^3} \tag{6}$$

If x represents a variable such as insulation thickness, its value must be positive; therefore, if c is positive, the second derivative at the optimum point must be greater than zero, and $(c/a)^{1/2}$ represents the value of x at the point where the total variable cost is a minimum.

PROCEDURE WITH TWO OR MORE VARIABLES

When two or more independent variables affect the factor being optimized, the procedure for determining the optimum conditions may become rather tedious; however, the general approach is the same as when only one variable is involved.

Consider the case in which the total cost for a given operation is a function of the two independent variables x and y, or

$$C_T = \phi^{iii}(x, y) \tag{7}$$

By analyzing all the costs involved and reducing the resulting relationships to a simple form, the following function might be found for Eq. (7):

$$C_T = ax + \frac{b}{xy} + cy + d \tag{8}$$

where a, b, c, and d are positive constants.

GRAPHICAL PROCEDURE. The relationship among C,, x, and y could be shown as a curved surface in a three-dimensional plot, with a minimum value of C_T occurring at the optimum values of x and y. However, the use of a three-dimensional plot is not practical for most engineering determinations.

The optimum values of x and y in Eq. (8) can be found graphically on a two-dimensional plot by using the method indicated in Fig. 11-2. In this figure, the factor being optimized is plotted against one of the independent variables (x), with the second variable (y) held at a constant value. A series of such plots is made with each dashed curve representing a different constant value of the

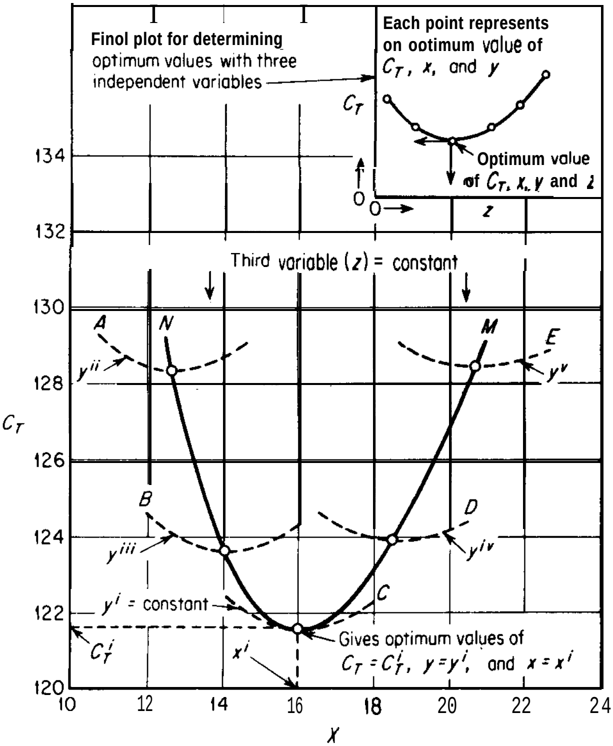

FIGURE 11-2
Graphical determination of optimum conditions with two or more independent variables.

second variable. As shown in Fig. 11-2, each of the curves (A, B, C, D, and E) gives one value of the first variable x at the point where the total cost is a minimum. The curve *NM* represents the locus of all these minimum points, and the optimum value of x and y occurs at the minimum point on curve *NM.*

Similar graphical procedures can be used when there are more than two independent variables. For example, if a third variable z were included in Eq. (8), the first step would be to make a plot similar to Fig. 11-2 at one constant value of z. Similar plots would then be made at other constant values of z. Each plot would give an optimum value of x, y, and C_T for a particular z. Finally, as shown in the insert in Fig. 11-2, the overall optimum value of x, y, z, and C_T could be obtained by plotting z versus the individual optimum values of C_T.

ANALYTICAL PROCEDURE. In Fig. 11-2, the optimum value of x is found at the point where $(\partial C_T/\partial x)_{y=y^i}$ is equal to zero. Similarly, the same results would be obtained if y were used as the abscissa instead of x. If this were done, the optimum value of y (i.e., y^i) would be found at the point where $(\partial C_T/\partial y)_{x=x^i}$ is equal to zero. This immediately indicates an analytical procedure for determining optimum values.

Using Eq. (8) as a basis,

$$\frac{\partial C_T}{ax} = a - \frac{b}{x^2 y} \tag{9}$$

$$\frac{\partial C_T}{\partial y} = c - \frac{b}{xy^2} \tag{10}$$

At the optimum conditions, both of these partial derivatives must be equal to zero; thus, Eqs. (9) and (10) can be set equal to zero and the optimum values of $x = (cb/a^2)^{1/3}$ and $y = (ab/c^2)^{1/3}$ can be obtained by solving the two simultaneous equations. If more than two independent variables were involved, the same procedure would be followed, with the number of simultaneous equations being equal to the number of independent variables.

Example 1 Determination of optimum values with two independent variables. The following equation shows the effect of the variables x and y on the total cost for a particular operation:

$$C_T = 2.33x + \frac{11{,}900}{xy} + 1.86y + 10$$

Determine the values of x and y which will give the least total cost.

Solution

Analytical method.

$$\frac{\partial C_T}{\partial x} = 2.33 - \frac{11{,}900}{x^2 y}$$

$$\frac{\partial C_T}{\partial y} = 1.86 - \frac{11{,}900}{xy^2}$$

At the optimum point,

$$2.33 - \frac{11{,}900}{x^2 y} = 0$$

$$1.86 - \frac{11{,}900}{xy^2} = 0$$

Solving simultaneously for the optimum values of x and y,

$$x = 16$$

$$y = 20$$

$$C_T = 121.6$$

A check should be made to make certain the preceding values represent conditions of minimum cost.

$$\frac{\partial^2 C_T}{\partial x^2} = \frac{(2)(11{,}900)}{x^3 y} = \frac{(2)(11{,}900)}{(16)^3(20)} = + \text{at optimum point}$$

$$\frac{\partial^2 C_T}{\partial y^2} = \frac{(2)(11{,}900)}{x y^3} = \frac{(2)(11{,}900)}{(16)(20)^3} = + \text{at optimum point}$$

Since the second derivatives are positive, the optimum conditions must occur at a point of minimum cost.

Graphical method. The following constant values of y are chosen arbitrarily:

$$y^{ii} = 32 \qquad Y^{iii} = 26 \qquad y^{i} = 20 \qquad Y^{iv} = 15 \qquad y^{v} = 12$$

At each constant value of y, a plot is made of C_T versus x. These plots are presented in Fig. 11-2 as curves *A, B,* C, *D,* and *E.* A summary of the results is presented in the following table:

y	Optimum x	Optimum C_T
$y^{ii} = 32$	12.7	12s.3
$y^{iii} = 26$	14.1	123.6
$y^{i} = 20$	16.0	121.6
$y^{iv} = 15$	18.5	123.9
$y^{v} = 12$	20.i	128.5

One curve (*NM* in Fig. 11-2) through the various optimum points shows that the overall optimum occurs at

$$x = 16$$
$$y = 20$$
$$C_T = 121.6$$

Note: In this case, a value of y was chosen which corresponded to the optimum value. Usually, it is necessary to interpolate or make further calculations in order to determine the final optimum conditions.

COMPARISON OF GRAPHICAL AND ANALYTICAL METHODS

In the determination of optimum conditions, the same final results are obtained with either graphical or analytical methods. Sometimes it is impossible to set up one analytical function for differentiation, and the graphical method must be used. If the development and simplification of the total analytical function require complicated mathematics, it may be simpler to resort to the direct graphical solution; however, each individual problem should be analyzed on the basis of the existing circumstances. For example, if numerous repeated trials are necessary, the extra time required to develop an analytical solution may be well spent.

The graphical method has one distinct advantage over the analytical method. The shape of the curve indicates the importance of operating at or very close to the optimum conditions. If the maximum or minimum occurs at a point where the curve is flat with only a gradual change in slope, there will be a considerable spread in the choice of final conditions, and incremental cost analyses may be necessary. On the other hand, if the maximum or minimum is sharp, it may be essential to operate at the exact optimum conditions.

THE BREAK-EVEN CHART FOR PRODUCTION SCHEDULE AND ITS SIGNIFICANCE FOR OPTIMUM ANALYSIS

In considering the overall costs or profits in a plant operation, one of the factors that has an important effect on the economic results is the fraction of total available time during which the plant is in operation. If the plant stands idle or operates at low capacity, certain costs, such as those for raw materials and labor, are reduced, but costs for depreciation and maintenance continue at essentially the same rate even though the plant is not in full use.

There is a close relationship among operating time, rate of production, and selling price. It is desirable to operate at a schedule which will permit maximum utilization of fixed costs while simultaneously meeting market sales demand and using the capacity of the plant production to give the best economic results. Figure 11-3 shows graphically how production rate affects costs and profits. The fixed costs remain constant while the total product cost, as

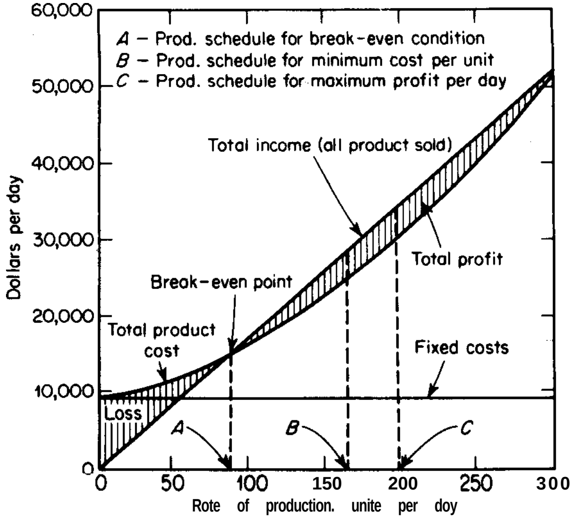

FIGURE 11-3
Break-even chart for operating production plant (based on situation presented in Example 2).

well as the profit, increases with increased rate of production. The point where total product cost equals total income represents the break-even point, and the optimum production schedule must be at a production rate higher than that corresponding to the break-even point.

OPTIMUM PRODUCTION RATES IN PLANT OPERATION

The same principles used for developing an optimum design can be applied when determining the most favorable conditions in the operation of a manufacturing plant. One of the most important variables in any plant operation is the amount of product produced per unit of time. The production rate depends on many factors, such as the number of hours in operation per day, per week, or per month; the load placed on the equipment; and the sales market available. From an analysis of the costs involved under different situations and consideration of other factors affecting the particular plant, it is possible to determine an *optimum rate of production* or a so-called *economic lot size.*

When a design engineer submits a complete plant design, the study ordinarily is based on a given production capacity for the plant. After the plant is put into operation, however, some of the original design factors will have changed, and the optimum rate of production may vary considerably from the "designed capacity." For example, suppose a plant had been designed originally for the batchwise production of an organic chemical on the basis of one batch every 8 hours. After the plant has been put into operation, cost data on the actual process are obtained, and tests with various operating procedures are conducted. It is found that more total production per month can be obtained if the time per batch is reduced. However, when the shorter batch time is used, more labor is required, the percent conversion of raw materials is reduced, and steam and power costs increase. Here is an obvious case in which an economic balance can be used to find the optimum production rate. Although the design engineer may have based the original recommendations on a similar type of economic balance, price and market conditions do not remain constant, and the operations engineer now has actual results on which to base an economic balance. The following analysis indicates the general method for determining economic production rates or lot sizes.

The total product cost per unit of time may be divided into the two classifications of *operating costs* and *organization costs.* Operating costs depend on the rate of production and include expenses for direct labor, raw materials, power, heat, supplies and similar items which are a function of the amount of material produced. Organization costs are due to expenses for directive personnel, physical equipment, and other services or facilities which must be maintained irrespective of the amount of material produced. Organization costs are independent of the rate of production.

It is convenient to consider operating costs on the basis of one unit of production. When this is done, the operating costs can be divided into two types of expenses as follows: (1) minimum expenses for raw materials, labor, power, etc., that remain constant and must be paid for each unit of production as long as any amount of material is produced; and (2) extra expenses due to increasing the rate of production. These extra expenses are known as ***superproduction costs.*** They become particularly important at high rates of production. Examples of superproduction costs are extra expenses caused by overload on power facilities, additional labor requirements, or decreased efficiency of conversion. Superproduction costs can often be represented as follows:

$$\text{Superproduction costs per unit of production} = mP^n \tag{11}$$

where P = rate of production as total units of production per unit of time
m = a constant
n = a constant

Designating h as the operating costs which remain constant per unit of production and O_c as the organization costs per unit of time, the total product cost c_T per unit of production is

$$c_T = h + mP^n + \frac{O_c}{P} \tag{12}$$

The following equations for various types of costs or profits are based on Eq. (12):

$$C_T = c_T P = \left(h + mP^n + \frac{O_c}{P}\right)P \tag{13}$$

$$r = s - c_T = s - h - mP^n - \frac{O_c}{P} \tag{14}$$

$$R' = rP = \left(s - h - mP^n - \frac{O_c}{P}\right)P$$

where C_T = total product cost per unit of time
r = profit per unit of production
R' = profit per unit of time
s = selling price per unit of production

OPTIMUM PRODUCTION RATE FOR MINIMUM COST PER UNIT OF PRODUCTION

It is often necessary to know the rate of production which will give the least cost on the basis of one unit of material produced. This information shows the selling price at which the company would be forced to cease operation or else operate at a loss. At this particular optimum rate, a plot of the total product cost per unit of production versus the production rate shows a minimum product cost; therefore, the optimum production rate must occur where $dc_T/dP = 0$. An analytical solution for this case may be obtained from Eq. (12), and the optimum rate P_o giving the minimum cost per unit of production is found as follows:

$$\frac{dc_T}{dP} = 0 = nmP_o^{n-1} - \frac{O_c}{P_o^2} \tag{16}$$

$$P_o = \left(\frac{O_c}{nm}\right)^{1/(n+1)} \tag{17}$$

The optimum rate shown in Eq. (17) would, of course, give the maximum profit per unit of production if the selling price remains constant.

OPTIMUM PRODUCTION RATE FOR MAXIMUM TOTAL PROFIT PER UNIT OF TIME

In most business concerns, the amount of money earned over a given time period is much more important than the amount of money earned for each unit of product sold. Therefore, it is necessary to recognize that the production rate for maximum profit per unit of time may differ considerably from the production rate for minimum cost per unit of production.

Equation (15) presents the basic relationship between costs and profits. A plot of profit per unit of time versus production rate goes through a maximum. Equation (15), therefore, can be used to find an analytical value of the optimum production rate. When the selling price remains constant, the optimum rate giving the maximum profit per unit of time is

$$P_o = \left[\frac{s-h}{(n+1)m}\right]^{1/n} \tag{18}$$

The following example illustrates the preceding principles and shows the analytical solution for the situation presented in Fig. 11-3.

Example 2 Determination of profits at optimum production rates. A plant produces refrigerators at the rate of P units per day. The variable costs per refrigerator have been found to be \$47.73 + $0.1P^{1.2}$. The total daily fixed charges are \$1750, and all other expenses are constant at \$7325 per day. If the selling price per refrigerator is \$173, determine:

(*a*) The daily profit at a production schedule giving the minimum cost per refrigerator.

(*b*) The daily profit at a production schedule giving the maximum daily profit.

(*c*) The production schedule at the break-even point.

Solution

(a) Total cost per refrigerator = $c_T = 47.73 + 0.1P^{1.2} + (1750 + 7325)/P$. At a production schedule for minimum cost per refrigerator,

$$\frac{dc_T}{dP} = 0 = 0.12P_o^{0.2} - \frac{9075}{P_o^2}$$

P_o = 165 units per day for minimum cost per unit

Daily profit at a production schedule for minimum cost per refrigerator

$$= \left[173 - 47.73 - 0.1(165)^{1.2} - \frac{9075}{165}\right]165$$

$$= \$4040$$

(b) Daily profit is

$$R' = \left(173 - 47.73 - 0.1P^{1.2} - \frac{1750 + 7325}{P}\right)P$$

At a production schedule for maximum profit per day,

$$\frac{dR'}{dP} = 0 = 125.27 - 0.22P_o^{1.2}$$

P_o = 198 units per day for maximum daily profit

Daily profit at a production schedule for maximum daily profit

$$= \left[173 - 47.73 - 0.1(198)^{1.2} - \frac{9075}{198}\right]198$$

$$= \$4400$$

(c) Total profit per day =

$$\left\{173 - \left[47.73 + 0.1P^{1.2} + \frac{(1750 + 7325)}{P}\right]\right\}P = 0$$

at the break-even point.

Solving the preceding equation for P,

$$P_{\text{at break-even point}} = 88 \text{ units/day}$$

OPTIMUM CONDITIONS IN CYCLIC OPERATIONS

Many processes are carried out by the use of cyclic operations which involve periodic shutdowns for discharging, cleanout, or reactivation. This type of operation occurs when the product is produced by a batch process or when the rate of production decreases with time, as in the operation of a plate-and-frame filtration unit. In a true batch operation, no product is obtained until the unit is shut down for discharging. In semicontinuous cyclic operations, product is delivered continuously while the unit is in operation, but the rate of delivery decreases with time. Thus, in batch or semicontinuous cyclic operations, the variable of total time required per cycle must be considered when determining optimum conditions.

Analyses of cyclic operations can be carried out conveniently by using the time for one cycle as a basis. When this is done, relationships similar to the following can be developed to express overall factors, such as total annual cost or annual rate of production:

$$\text{Total annual cost} = \text{operating and shutdown costs/cycle} \times \text{cycles/year} + \text{annual fixed costs} \tag{19}$$

$$\text{Annual production} = (\text{production/cycle})\,(\text{cycles/year}) \tag{20}$$

$$\text{Cycles/year} = \frac{\text{operating + shutdown time used/year}}{\text{operating + shutdown time/cycle}} \tag{21}$$

The following example illustrates the general method for determining optimum conditions in a batch operation.

Example 3 Determination of conditions for minimum total cost in a batch operation. An organic chemical is being produced by a batch operation in which no product is obtained until the batch is finished. Each cycle consists of the operating time necessary to complete the reaction plus a total time of 1.4 h for discharging and charging. The operating time per cycle is equal to $1.5P_b^{0.25}$ h, where P_b is the kilograms of product produced per batch. The operating costs during the operating period are \$20 per hour, and the costs during the discharge-charge period are \$15 per hour. The annual fixed costs for the equipment vary with the size of the batch as follows:

$$C_F = 340P_b^{0.8} \text{ dollars per batch}$$

Inventory and storage charges may be neglected. If necessary, the plant can be operated 24 h per day for 300 days per year. The annual production is 1 million kg of product. At this capacity, raw-material and miscellaneous costs, other than those already mentioned, amount to \$260,000 per year. Determine the cycle time for conditions of minimum total cost per year.

Solution

$$\text{Cycles/year} = \frac{\text{annual production}}{\text{production/cycle}} = \frac{1{,}000{,}000}{P_b}$$

$$\text{Cycle time} = \text{operating + shutdown time} = 1.5P_b^{0.25} + 1.4 \text{ h}$$

$$\text{Operating + shutdown costs/cycle} = (20)(1.5P_b^{0.25}) + (15)(1.4) \text{ dollars}$$

$$\text{Annual fixed costs} = 340P_b^{0.8} + 260{,}000 \text{ dollars}$$

$$\text{Total annual costs} = (30P_b^{0.25} + 21)(1{,}000{,}000/P_b) + 340P_b^{0.8} + 260{,}000 \text{ dollars}$$

The total annual cost is a minimum where $d(\text{total annual cost})/dP_b = 0$.

Performing the differentiation, setting the result equal to zero, and solving for P_b gives

$$P_{b,\text{ optimum cost}} = 1630 \text{ kg per batch}$$

This same result could have been obtained by plotting total annual cost versus P_b and determining the value of P_b at the point of minimum annual cost.

For conditions of minimum annual cost and 1 million kg/year production,

$$\text{Cycle time} = (1.5)(1630)^{0.25} + 1.4 = 11 \text{ h}$$

$$\text{Total time used per year} = (11)\left(\frac{1{,}000{,}000}{1630}\right) = 6750 \text{ h}$$

$$\text{Total time available per year} = (300)(24) = 7200 \text{ h}$$

Thus, for conditions of minimum annual cost and a production of 1 million kg/year, not all the available operating and shutdown time would be used.

SEMICONTINUOUS CYCLIC OPERATIONS

Semicontinuous cyclic operations are often encountered in the chemical industry, and the design engineer should understand the methods for determining optimum cycle times in this type of operation. Although product is delivered continuously, the rate of delivery decreases with time owing to scaling, collection of side product, reduction in conversion efficiency, or similar causes. It becomes necessary, therefore, to shut down the operation periodically in order to restore the original conditions for high production rates. The optimum cycle time can be determined for conditions such as maximum amount of production per unit of time or minimum cost per unit of production.

Scale Formation in Evaporation

During the time an evaporator is in operation, solids often deposit on the heat-transfer surfaces, forming a scale. The continuous formation of the scale causes a gradual increase in the resistance to the flow of heat and, consequently, a reduction in the rate of heat transfer and rate of evaporation if the same temperature-difference driving forces are maintained. Under these conditions, the evaporation unit must be shut down and cleaned after an optimum operation time, and the cycle is then repeated.

Scale formation occurs to some extent in all types of evaporators, but it is of particular importance when the feed mixture contains a dissolved material that has an inverted solubility. The expression ***inverted solubility*** means the solubility decreases as the temperature of the solution is increased. For a material of this type, the solubility is least near the heat-transfer surface where the temperature is the greatest. Thus, any solid crystallizing out of the solution does so near the heat-transfer surface and is quite likely to form a scale on this surface. The most common scale-forming substances are calcium sulfate, calcium hydroxide, sodium carbonate, sodium sulfate, and calcium salts of certain organic acids.

When true scale formation occurs, the overall heat-transfer coefficient may be related to the time the evaporator has been in operation by the straight-line equation?

$$\frac{1}{U^2} = a\theta_b + d \tag{22}$$

where a and d are constants for any given operation and U is the overall heat-transfer coefficient at any operating time θ_b since the beginning of the operation.

If it is not convenient to determine the heat-transfer coefficients and the related constants as shown in Eq. (22), any quantity that is proportional to the heat-transfer coefficient may be used. Thus, if all conditions except scale formation are constant, feed rate, production rate, and evaporation rate can each be represented in a form similar to Eq. (22). Any of these equations can be used as a basis for finding the optimum conditions. The general method is illustrated by the following treatment, which employs Eq. (22) as a basis.

If Q represents the total amount of heat transferred in the operating time θ_b, and A and At represent, respectively, the heat-transfer area and temperature-difference driving force, the rate of heat transfer at any instant is

$$\frac{dQ}{d\theta_b} = UA\,\Delta t = \frac{A\ At}{(a\theta_b + d)^{1/2}} \tag{23}$$

The instantaneous rate of heat transfer varies during the time of operation, but the heat-transfer area and the temperature-difference driving force remain essentially constant. Therefore, the total amount of heat transferred during an operating time of θ_b can be determined by integrating Eq. (23) as follows:

$$\int_0^Q dQ = A\,\Delta t \int_0^{\theta_b} \left(\frac{1}{a\theta_b + d} \right)^{1/2} d\theta_b \tag{24}$$

$$Q = \frac{2A\ At}{a} \left[(a\theta_b + d)^{1/2} - d^{1/2} \right] \tag{25}$$

CYCLE TIME FOR MAXIMUM AMOUNT OF HEAT TRANSFER. Equation (25) can be used as a basis for finding the cycle time which will permit the maximum amount of heat transfer during a given period. Each cycle consists of an operating (or boiling) time of θ_b h. If the time per cycle for emptying, cleaning, and recharging is θ_c, the total time in hours per cycle is $\theta_t = \theta_b + \theta_c$. Therefore, designating the total time *used* for actual operation, emptying, cleaning, and refilling as H, the number of cycles during H h = $H/(\theta_b + \theta_c)$.

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†W. McCabe and C. Robinson, *Ind. Eng. Chem.*, **16**:478 (1924).

The total amount of heat transferred during

$$H \text{ h} = Q_H = (Q/\text{cycle}) \times (\text{cycles}/H \text{ h})$$

Therefore,

$$Q_H = \frac{2A\,\mathrm{At}}{a}\left[(a\theta_b + d)^{1/2} - d^{1/2}\right]\frac{H}{\theta_b + \theta_c} \tag{26}$$

Under ordinary conditions, the only variable in Eq. (26) is the operating time θ_b. A plot of the total amount of heat transferred versus θ_b shows a maximum at the optimum value of θ_b. Figure 11-4 presents a plot of this type. The optimum cycle time can also be obtained by setting the derivative of Eq. (26) with respect to θ_b equal to zero and solving for θ_b. The result is

$$\theta_{b,\text{ per cycle for maximum amount of heat transfer}} = \theta_c + \frac{2}{a}\sqrt{a\,d\theta_c} \tag{27}$$

The optimum boiling time given by Eq. (27) shows the operating schedule necessary to permit the maximum amount of heat transfer. All the time available for operation, emptying, cleaning, and refilling should be used. For

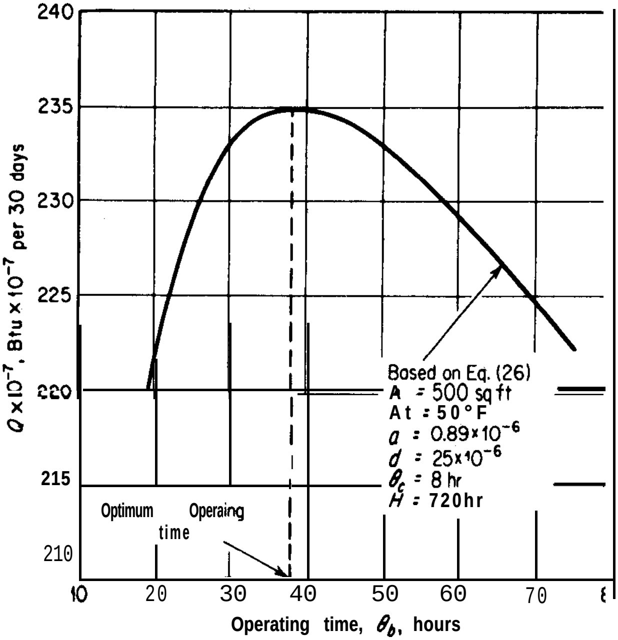

FIGURE 11-4
Determination of optimum operating time for maximum amount of heat transfer in evaporator with scale formation.

constant operating conditions, this same schedule would also give the maximum amount of feed consumed, product obtained, and liquid evaporated.

A third method for determining the optimum cycle time is known as the ***tangential method for finding optimum conditions,*** and it is applicable to many types of cyclic operations. This method is illustrated for conditions of constant cleaning time (θ_c) in Fig. 11-5, where a plot of amount of heat transferred versus boiling time is presented. Curve ***OB*** is based on Eq. (25). The ***average*** amount of heat transferred per unit of time during one complete cycle is $Q/(\theta_b + \theta_c)$. When the total amount of heat transferred during a number of repeated cycles is a maximum, the average amount of heat transferred per unit of time must also be a maximum. The optimum cycle time, therefore, occurs when $Q/(\theta_b + \theta_c)$ is a maximum.

The straight line ***CD'*** in Fig. 11-5 starts at a distance equivalent to θ_c on the left of the plot origin. The slope of this straight line is $Q/(\theta_b + \theta_c)$, with the values of Q and θ_b determined by the point of intersection between line ***CD'*** and curve ***OB.*** The maximum value of $Q/(\theta_b + \theta_c)$ occurs when line ***CD*** is tangent to the curve ***OB,*** and the point of tangency indicates the optimum value of the boiling time per cycle for conditions of maximum amount of heat transfer.

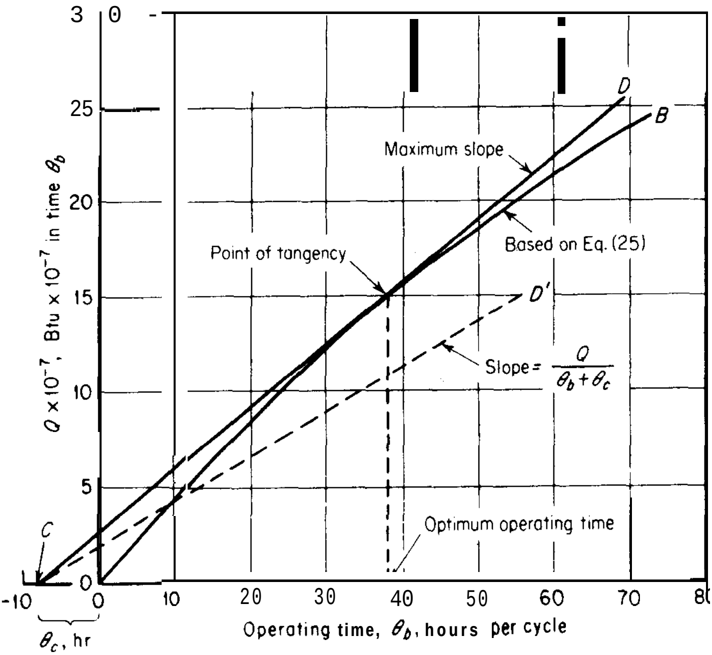

FIGURE 11-5
Tangential method for finding optimum time for maximum amount of heat transfer in evaporator with scale formation.

CYCLE TIME FOR MINIMUM COST 'PER UNIT OF HEAT TRANSFER There are many different circumstances which may affect the minimum cost per unit of heat transferred in an evaporation operation. One simple and commonly occurring case will be considered. It may be assumed that an evaporation unit of fixed capacity is available, and a definite amount of feed and evaporation must be handled each day. The total cost for one cleaning and inventory charge is assumed to be constant no matter how much boiling time is used. The problem is to determine the cycle time which will permit operation at the least total cost.

The total cost includes (1) fixed charges on the equipment and fixed overhead expenses, (2) steam, materials, and storage costs which are proportional to the amount of feed and evaporation, (3) expenses for direct labor during the actual evaporation operation, and (4) cost of cleaning. Since the size of the equipment and the amounts of feed and evaporation are fixed, the costs included in (1) and (2) are independent of the cycle time. The optimum cycle time, therefore, can be found by minimizing the sum of the costs for cleaning and for direct labor during the evaporation.

If C_c represents the cost for one cleaning and S_b is the direct labor cost per hour during operation, the total variable costs during H h of operating and cleaning time must be

$$C_{T,\,\text{for } H \text{ h}} = (C_c + S_b\theta_b)\frac{H}{\theta_b + \theta_c} \tag{28}$$

Equations (26) and (28) may be combined to give

$$C_{T,\,\text{for } H \text{ h}} = \frac{aQ_H(C_c + S_b\theta_b)}{2A\,\Delta t\left[(a\theta_b + d)^{1/2} - d^{1/2}\right]} \tag{29}$$

The optimum value of θ_b for minimum total cost may be obtained by plotting C_T versus θ_b or by setting the derivative of Eq. (29) with respect to θ_b equal to zero and solving for θ_b. The result is

$$\theta_{b,\,\text{per cycle for minimum total cost}} = \frac{C_c}{S_b} + \frac{2}{aS_b}\sqrt{a\,dC_c\,S_b} \tag{30}$$

Equation (30) shows that the optimum cycle time is independent of the required amount of heat transfer Q_H. Therefore, a check must be made to make certain the optimum cycle time for minimum cost permits the required amount of heat transfer. This can be done easily by using the following equation, which is based on Eq. (26):

$$\theta_t = \theta_b + \theta_c = \frac{2AH'\Delta t}{aQ_H}\left[(a\theta_{b,\text{opt}} + d)^{1/2} - d^{1/2}\right] \tag{31}$$

where H' is the total time *available* for operation, emptying, cleaning, and recharging. If θ_t is equal to or greater than $\theta_{b,\text{opt}} + \theta_c$, the optimum boiling time indicated by Eq. (30) can be used, and the required production can be obtained at conditions of minimum cost.

The optimum cycle time determined by the preceding methods may not fit into convenient operating schedules. Fortunately, as shown in Figs. 11-4 and 11-5, the optimum points usually occur where a considerable variation in the cycle time has little effect on the factor that is being optimized. It is possible, therefore, to adjust the cycle time to fit a convenient operating schedule without causing much change in the final results.

The approach described in the preceding sections can be applied to many different types of semicontinuous cyclic operations. An illustration showing how the same reasoning is used for determining optimum cycle times in filter-press operations is presented in Example 4.

Example 4 Cycle time for maximum amount of production from a plate-and-frame filter press. Tests with a plate-and-frame filter press, operated at constant pressure, have shown that the relation between the volume of filtrate delivered and the time in operation can be represented as follows:

$$P_f^2 = 2.25 \times 10^4(\theta_f + 0.11)$$

where P_f = cubic feet of filtrate delivered in filtering time θ_f h.

The cake formed in each cycle must be washed with an amount of water equal to one-sixteenth times the volume of filtrate delivered per cycle. The washing rate remains constant and is equal to one-fourth of the filtrate delivery rate at the end of the filtration. The time required per cycle for dismantling, dumping, and reassembling is 6 h. Under the conditions where the preceding information applies, determine the total cycle time necessary to permit the maximum output of filtrate during each 24 h.

Solution. Let θ_f = hours of filtering time per cycle.

Filtrate delivered per cycle = $P_{f,\text{cycle}} = 150(\theta_f + 0.11)^{1/2}$ ft³. Rate of filtrate delivery at end of cycle is

$$\text{Washing rate} \times 4 = \frac{dP_f}{d\theta_f} = \frac{150}{2}(\theta_f + 0.11)^{-1/2} \text{ ft}^3/\text{h}$$

$$\text{Time for washing} = \frac{\text{volume of wash water}}{\text{washing rate}}$$

$$= \frac{(4)(2)(150)(\theta_f + 0.11)^{1/2}}{(16)(150)(\theta_f + 0.11)^{-1/2}} = \frac{\theta_f + 0.11}{2} \text{ h}$$

$$\text{Total time per cycle} = \theta_f + \frac{\theta_f + 0.11}{2} + 6 = 1.5\theta_f + 6.06 \text{ h}$$

$$\text{Cycles per 24 h} = \frac{24}{1.5\theta_f + 6.06}$$

Filtrate in ft³ delivered/24 h is

$$P_{f,\text{cycle}}(\text{cycles per 24 h}) = 150(\theta_f + 0.11)^{1/2} \frac{24}{1.5\theta_f + 6.06}$$

At the optimum cycle time giving the maximum output of filtrate per 24 h,

$$\frac{d(\text{ft}^3 \text{ filtrate delivered/24 h})}{d\theta_f} = 0$$

Performing the differentiation and solving for θ_f,

$$\theta_{f,\text{opt}} = 3.8 \text{ h}$$

Total cycle time necessary to permit the maximum output of filtrate = $(1.5)(3.8) + 6.06 = 11.8$ h.

ACCURACY AND SENSITIVITY OF RESULTS

The purpose of the discussion and examples presented in the preceding sections of this chapter has been to give a basis for understanding the significance of optimum *conditions* plus simplified examples to illustrate the general concepts. Costs due to taxes, time value of money, capital, efficiency or inefficiency of operation, and special maintenance are examples of factors that have not been emphasized in the preceding. Such factors may have a sufficiently important influence on an optimum condition that they need to be taken into account for a final analysis. The engineer must have the practical understanding to recognize when such factors are important and when the added accuracy obtained by including them is not worth the difficulty they cause in the analysis.

A classic example showing how added refinements can come into an analysis for optimum conditions is involved in the development of methods for determining optimum economic pipe diameter for transportation of fluids. The following analysis, dealing with economic pipe diameters, gives a detailed derivation to illustrate how simplified expressions for optimum conditions can be developed. Further discussion showing the effects of other variables on the sensitivity is also presented.

FLUID DYNAMICS (OPTIMUM ECONOMIC PIPE DIAMETER)

The investment for piping and pipe fittings can amount to an important part of the total investment for a chemical plant. It is necessary, therefore, to choose pipe sizes which give close to a minimum total cost for pumping and fixed charges. For any given set of flow conditions, the use of an increased pipe diameter will cause an increase in the fixed charges for the piping system and a decrease in the pumping or blowing charges. Therefore, an optimum economic pipe diameter must exist. The value of this optimum diameter can be determined by combining the principles of fluid dynamics with cost considerations. The optimum economic pipe diameter is found at the point at which the sum of pumping or blowing costs and fixed charges based on the cost of the piping system is a minimum.

Pumping or Blowing Costs

For any given operating conditions involving the flow of *a* noncompressible fluid through a pipe of constant diameter, the total mechanical-energy balance can be reduced to the following form:

$$\text{Work}' = \frac{2\mathfrak{f}V^2L(1+J)}{g_cD} + B \tag{32}$$

where Work' = mechanical work added to system from an external mechanical source, ft · lbf/lbm

$\mathfrak{f}$ = Fanning friction factor, dimensionless?

V = average linear velocity of fluid, ft/s

L = length of pipe, ft

J = frictional loss due to fittings and bends, expressed as equivalent fractional loss in a straight pipe

g_c = conversion factor in Newton's law of motion, 32.17 ft · lbm/(s)(s)(lbf)

D = inside diameter of pipe, ft, subscript i means in.

B = a constant taking all other factors of the mechanical-energy balance into consideration

In the region of turbulent flow (Reynolds number greater than 2100), $\mathfrak{f}$ may be approximated for new steel pipes by the following equation:

$$\mathfrak{f} = \frac{0.04}{(N_{\text{Re}})^{0.16}} \tag{33}$$

where N_{Re} is the Reynolds number or $DV\rho/\mu$.

If the flow is viscous (Reynolds number less than 2100),

$$\mathfrak{f} = \frac{16}{N_{\text{Re}}} \tag{34}$$

By combining Eqs. (32) and (33) and applying the necessary conversion factors, the following equation can be obtained representing the annual pumping cost when the flow is turbulent:

$$C_{\text{pumping}} = \frac{0.273q_f^{2.84}\rho^{0.84}\mu_c^{0.16}K(1+J)H_y}{D_i^{4.84}E} + B' \tag{35}$$

where C_{pumping} = pumping costs as dollars per year per foot of pipe length when flow is turbulent

q_f = fluid-flow rate, ft^3/s

ρ = fluid density, lb/ft^3

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†Based on Fanning equation written as $\Sigma(\text{friction}) = 2\mathfrak{f}V^2L/g_cD$.

μ_c = fluid viscosity, centipoises
K = cost of electrical energy, \$/kWh
H_y = hours of operation per year
E = efficiency of motor and pump expressed as a fraction
B' = a constant independent of D_i

Similarly, Eqs. (32) and (34) and the necessary conversion factors can be combined to give the annual pumping costs when the flow is viscous:

$$C'_{\text{pumping}} = \frac{0.024 q_f^2 \mu_c K(1 + J)H_y}{D_i^4 E} + B' \tag{36}$$

where C'_{pumping} = pumping cost as dollars per year per foot of pipe length when flow is viscous.

Equations (35) and (36) apply to noncompressible fluids. In engineering calculations, these equations are also generally accepted for gases if the total pressure drop is less than 10 percent of the initial pressure.

Fixed Charges for Piping System

For most types of pipe, a plot of the logarithm of the pipe diameter versus the logarithm of the purchase cost per foot of pipe is essentially a straight line. Therefore, the purchase cost for pipe may be represented by the following equation:

$$c_{\text{pipe}} = XD_i^n \tag{37}$$

where c_{pipe} = purchase cost of new pipe per foot of pipe length, \$/ft
X = purchase cost of new pipe per foot of pipe length if pipe diameter is 1 in., \$/ft
n = a constant with value dependent on type of pipe

The annual cost for the installed piping system may be expressed as follows:

$$C_{\text{pipe}} = (1 + F)\, XD_i^n K_F \tag{38}$$

where C_{pipe} = cost for installed piping system as dollars per year per foot of pipe length?
F = ratio of total costs for fittings and installation to purchase cost for new pipe
K_F = annual fixed charges including maintenance, expressed as a fraction of initial cost for completely installed pipe

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†Pump cost could be included if desired; however, in this analysis, the cost of the pump is considered as essentially invariant with pipe diameter.

Optimum Economic Pipe Diameter

The total annual cost for the piping system and pumping can be obtained by adding Eqs. (35) and (38) or Eqs. (36) and (38). The only variable in the resulting total-cost expressions is the pipe diameter. The optimum economic pipe diameter can be found by taking the derivative of the total annual cost with respect to pipe diameter, setting the result equal to zero, and solving for D_i. This procedure gives the following results:

For turbulent flow,

$$D_{i,\mathrm{opt}} = \left[\frac{1.32 q_f^{2.84} \rho^{0.84} \mu_c^{0.16} K(1+J)H_y}{n(1+F)XEK_F}\right]^{1/(4.84+n)} \tag{39}$$

For viscous flow,

$$D_{i,\mathrm{opt}} = \left[\frac{0.096 q_f^{2} \mu_c K(1+J)H_y}{n(1+F)XEK_F}\right]^{1/(4.0+n)} \tag{40}$$

The value of n for steel pipes is approximately 1.5 if the pipe diameter is 1 in. or larger and 1.0 if the diameter is less than 1 in. Substituting these values in Eqs. (39) and (40) gives:

For turbulent flow in steel pipes,

$D_i \geq 1$ in.:

$$D_{i,\mathrm{opt}} = q_f^{0.448} \rho^{0.132} \mu_c^{0.025} \left[\frac{0.88K(1+J)H_y}{(1+F)XEK_F}\right]^{0.158} \tag{41}$$

$D_i < 1$ in.:

$$D_{i,\mathrm{opt}} = q_f^{0.487} \rho^{0.144} \mu_c^{0.027} \left[\frac{1.32K(1+J)H_y}{(1+F)XEK_F}\right]^{0.171} \tag{42}$$

For viscous flow in steel pipes,

$D_i \geq 1$ in.:

$$D_{i,\mathrm{opt}} = q_f^{0.364} \mu_c^{0.182} \left[\frac{0.064K(1+J)H_y}{(1+F)XEK_F}\right]^{0.182} \tag{43}$$

$D_i < 1$ in.:

$$D_{i,\mathrm{opt}} = q_f^{0.40} \mu_c^{0.20} \left[\frac{0.096K(1+J)H_y}{(1+F)XEK_F}\right]^{0.20} \tag{44}$$

The exponents involved in Eqs. (41) through (44) indicate that the optimum diameter is relatively insensitive to most of the terms involved. Since the exponent of the viscosity term in Eqs. (41) and (42) is very small, the value of $\mu_c^{0.025}$ and $\mu_c^{0.027}$ may be taken as unity over a viscosity range of 0.02 to 20 centipoises. It is possible to simplify the equations further by substituting average numerical values for some of the less critical terms. The following values are applicable under ordinary industrial conditions:

K = $0.09/kWh

J = 0.35 or 35 percent

H_y = 8760 h/year

E = 0.50 or 50 percent

F = 1.4

K_F = 0.20 or 20 percent

X = $0.74 per foot for 1-in.-diameter steel pipe

Substituting these values into Eqs. (41) through (44) gives the following simplified results:

For turbulent flow in steel pipes,

$D_i \geq 1$ in.:

$$D_{i,\mathrm{opt}} = 3.9 q_f^{0.45} \rho^{0.13} = \frac{2.2 w_m^{0.45}}{\rho^{0.32}} \tag{45}$$

where w_m = thousands of pounds mass flowing per hour.

$D_i < 1$ in.:

$$D_{i,\mathrm{opt}} = 4.7 q_f^{0.49} \rho^{0.14} \tag{46}$$

For viscous flow in steel pipes,

$D_i \geq 1$ in.:

$$D_{i,\mathrm{opt}} = 3.0 q_f^{0.36} \mu_c^{0.18} \tag{47}$$

$D_i < 1$ in.:

$$D_{i,\mathrm{opt}} = 3.6 q_f^{0.40} \mu_c^{0.20} \tag{48}$$

Depending on the accuracy desired and the type of flow, Eqs. (39) through (48) may be used to estimate optimum economic pipe diameters.[7] The simplified Eqs. (45) through (48) are sufficiently accurate for design estimates under ordinary plant conditions, and, as shown in Table 2, the diameter estimates obtained are usually on the safe side in that added refinements in the calculation methods generally tend to result in smaller diameters. A nomograph based on these equations is presented in Chap. 14 (Materials Transfer, Handling, and Treatment Equipment-Design and Costs).

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†This type of approach was first proposed by R. P. Genereaux, *Ind. Eng. Chem.*, **29**:385 (1937); *Chem. Met. Eng.*, **44**(5):281 (1937) and B. R. Sarchet and A. P. Colburn, *Ind. Eng. Chem.*, **32**:1249 (1940).

ANALYSIS INCLUDING TAX EFFECTS AND COST OF CAPITAL

The preceding analysis clearly neglects a number of factors that may have an influence on the optimum economic pipe diameter, such as cost of capital or return on investment, cost of pumping equipment, taxes, and the time value of money. If the preceding development of Eq. (39) for turbulent flow is refined to include the effects of taxes and the cost of capital (or return on investment) plus a more accurate expression for the frictional loss due to fittings and bends, the result is:†

For turbulent flow,

$$\frac{D_{\text{opt}}^{4.84+n}}{1 + 0.794L'_eD_{\text{opt}}} = \frac{0.000189YKw_s^{2.84}\mu_c^{0.16}\{[1 + (a' + b')M](1 - \phi) + ZM\}}{n(1 + F)X'[Z + (a + b)(1 - \phi)]E\rho^2} \tag{49}$$

where D_{opt} = optimum economic inside diameter, ft

X' = purchase cost of new pipe per foot of pipe length if pipe diameter is 1 ft, based on $c_{\text{pipe}} = X'D_{\text{opt}}^n$, \$/ft

L'_e = frictional loss due to fittings and bends, expressed as equivalent pipe length in pipe diameters per unit length of pipe, $L'_e = J/D_{\text{opt}}$

w_s = pounds mass flowing per second

M = ratio of total cost for pumping installation to yearly cost of pumping power required

Y = days of operation per year

a = fraction of initial cost of installed piping system for annual depreciation

a' = fraction of initial cost of pumping installation for annual depreciation

b = fraction of initial cost of installed piping system for annual maintenance

b' = fraction of initial cost of pumping installation for annual maintenance

ϕ = fractional factor for rate of taxation

Z = fractional rate of return (or cost of capital before taxes) on incremental investment

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†A similar equation was presented by J. H. Perry and C. H. Chilton, ed., “Chemical Engineers’ Handbook,” 5th ed., p. 5-32, McGraw-Hill Book Company, New York, 1973. See Prob. 15 at end of this chapter for derivation of Eq. (49) and comparison to form of equation given in above-mentioned reference.

TABLE 1

Values of variables used to obtain Eq. (50)

Turbulent flow--steel pipe-diameter 1 in. or larger

Variable	Value used	Variable	Value used
n			
L'	2.351.5	ϕ	0.200.32
$-_e$			
Y	328	X	31.0
K	0.09	$a + b$	0.18
μ_c	1.0	$a' + b'$	0.40
M	0.8	E	0.50

The variable F is a function of diameter and can be approximated by $F \cong 0.75/D_{opt} + 3.$

By using the values given in Table 1, for turbulent flow in steel pipes, Eq. (49) simplifies to:

$$D_i \geq 1 \text{ in.}$$

$$D_{\text{opt}} = \frac{(1 + 1.865 D_{\text{opt}})^{0.158} (0.32)\, w_s^{0.45}}{(1 + F)^{0.158} \rho^{0.32}} \tag{50}$$

SENSITIVITY OF RESULTS

The simplifications made in obtaining Eqs. (45) to (48) and Eq. (50) illustrate an approach that can be used for approximate results when certain variables appear in a form where relatively large changes in them have little effect on the final results. The variables appearing in Table 1 and following Eq. (44) are relatively independent of pipe diameter, and they are raised to a small power for the final determination of diameter. Thus, the final results are not particu-

TABLE 2

Comparison of optimum economic pipe diameter estimated from Eqs. (50) and (45)

Turbulent flow-schedule 40 steel pipe-approximate 15-ft spacings of fittings

$D_{i,\text{opt}}$ in.		w_s,	ρ
By Eq. (50)	By Eq. (45)	lb / s	lb / ft^3
10.0	11.2	4.50	200
5.0	6.1	4.5	2
3.0	3.9	12.5	35
1.5	2.2	0.45	2

larly sensitive to the variables listed in Table 1, and the practical engineer may decide that the simplification obtained by using the approximate equations is worth the slight loss in absolute accuracy.

Table 2 shows the extent **of** change in optimum economic diameter obtained by using Eq. (50) versus Eq. (45) and illustrates the effect of bringing in added refinements as well as changes in values of some of the variables.

HEAT TRANSFER (OPTIMUM FLOW RATE OF COOLING WATER IN CONDENSER)

If a condenser, with water as the cooling medium, is designed to carry out a given duty, the cooling water may be circulated at a high rate with a small change in water temperature or at a low rate with a large change in water temperature. The temperature of the water affects the temperature-difference driving force for heat transfer. Use of an increased amount of water, therefore, will cause a reduction in the necessary amount of heat-transfer area and a resultant decrease in the original investment and fixed charges. On the other hand, the cost for the water will increase if more water is used. An economic balance between conditions of high water rate-low surface area and low water rate-high surface area indicates that the optimum flow rate of cooling water occurs at the point of minimum total cost for cooling water and equipment fixed charges.

Consider the general case in which heat must be removed from a condensing vapor at a given rate designated by q Btu/h. The vapor condenses at a constant temperature of t'°F, and cooling water is supplied at a temperature of t_1°F. The following additional notation applies:

w = flow rate of cooling water, lb/h
c_p = heat capacity of cooling water, Btu/(lb)(°F)
t_2 = temperature of cooling water leaving condenser, °F
U = constant overall coefficient of heat transfer determined at optimum conditions, Btu/(h)(ft^2)(°F)
A = area of heat transfer, ft^2
At,,,, = log-mean temperature-difference driving force over condenser, °F
H_y = hours the condenser is operated per year, h/year
C_w = cooling-water cost assumed as directly proportional to amount of water supplied,? $/lb
C_A = installed cost of heat exchanger per square foot of heat-transfer area, \$/ft^2
K_F = annual fixed charges including maintenance, expressed as a fraction of initial cost for completely installed equipment

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†Cooling water is assumed to be available at a pressure sufficient to handle any pressure drop in the condenser; therefore, any cost due to pumping the water is included in C,.

The rate of heat transfer as Btu per hour can be expressed as

$$q = wc_p(t_2 - t_1) = UA\,\Delta t_{lm} = \frac{UA(t_2 - t_1)}{\ln[(t' - t_1)/(t' - t_2)]} \tag{51}$$

Solving for w,

$$w = \frac{q}{c_p(t_2 - t_1)} \tag{52}$$

The design conditions set the values of q and t_1, and the heat capacity of water may ordinarily be approximated as 1 Btu/(lb)(°F). Therefore, Eq. (52) shows that the flow rate of the cooling water is fixed if the temperature of the water leaving the condenser (t_2) is fixed. Under these conditions, the optimum flow rate of cooling water can be found directly from the optimum value of t_2.

The annual cost for cooling water is wH_yC_w. From Eq. (52),

$$wH_yC_w = \frac{qH_yC_w}{c_p(t_2 - t_1)} \tag{53}$$

The annual fixed charges for the condenser are AK_FC_A, and the total annual cost for cooling water plus fixed charges is

$$\text{Total annual variable cost} = \frac{qH_yC_w}{c_p(t_2 - t_1)} + AK_FC_A \tag{54}$$

Substituting for A from Eq. (51),

$$\text{Total annual variable cost} = \frac{qH_yC_w}{c_p(t_2 - t_1)} + \frac{qK_FC_A \ln[(t' - t_1)/(t' - t_2)]}{U(t_2 - t_1)} \tag{55}$$

The only variable in Eq. (55) is the temperature of the cooling water leaving the condenser. The optimum cooling-water rate occurs when the total annual cost is a minimum. Thus, the corresponding optimum exit temperature can be found by differentiating Eq. (55) with respect to t_2 (or, more simply, with respect to $t' - t_2$) and setting the result equal to zero. When this is done, the following equation is obtained:

$$\frac{t' - t_1}{t' - t_{2,\text{opt}}} - 1 + \ln\frac{t' - t_{2,\text{opt}}}{t' - t_1} = \frac{UH_yC_w}{K_Fc_pC_A} \tag{56}$$

The optimum value of t_2 can be found from Eq. (56) by a trial-and-error solution, and Eq. (52) can then be used to determine the optimum flow rate of cooling water. The trial-and-error solution can be eliminated by use of Fig. 11-6, which is a plot of Eq. (56).†

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†See Fig. 15-32 for a similar plot for counterflow coolers.

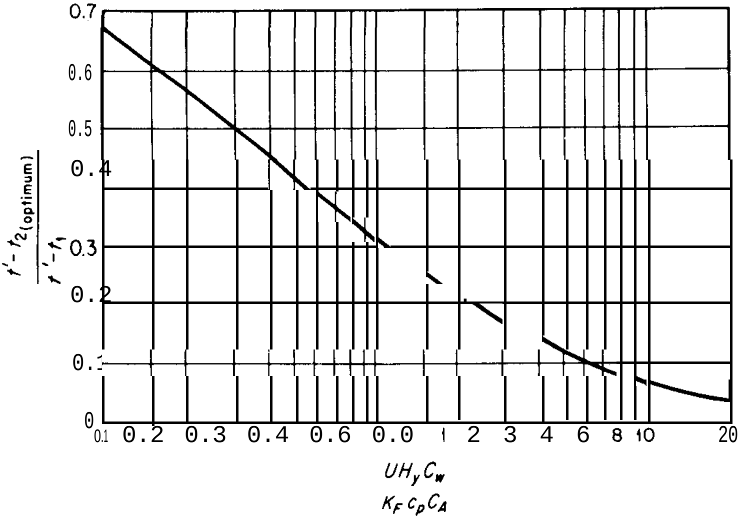

FIGURE 11-6
Solution of Eq. (56) for use in evaluating optimum flow rate of cooling medium in condenser.

Example 5 Optimum cooling-water flow rate in condenser. A condenser for a distillation unit must be designed to condense 5000 lb (2268 kg) of vapor per hour. The effective condensation temperature for the vapor is 170°F (350 K). The heat of condensation for the vapor is 200 Btu/lb (4.65×10^5 J/kg). Cooling water is available at 70°F (294 K). The cost of the cooling water is \$0.097 per 1000 gal (\$25.60 per 1000 m^3). The overall heat-transfer coefficient at the optimum conditions may be taken as 50 Btu/(h)(ft^2)(°F) (284 $J/m^2 \cdot s \cdot K$). The cost for the installed heat exchanger is \$34 per square foot of heat-transfer area (\$366 per square meter of heat-transfer area) and annual fixed charges including maintenance are 20 percent of the initial investment. The heat capacity of the water may be assumed to be constant at 1.0 Btu/(lb)(°F) (4.2 kJ/kg · K). If the condenser is to operate 6000 h/yr, determine the cooling-water flow rate in pounds per hour and in kilograms per hour for optimum economic conditions.

Solution

$$U = 50 \text{ Btu/(h)(ft}^2\text{)(°F)}$$

$$H_y = 6000 \text{ h/year}$$

$$K_F = 0.20$$

$$c_p = 1.0 \text{ Btu/(lb)(°F)}$$

$$C_A = \$34/\text{ft}^2$$

$$C_w = \frac{0.097}{(1000)(8.33)} = \$0.0000116/\text{lb}$$

$$\frac{UH_yC_w}{K_Fc_pC_A} = \frac{(50)(6000)(0.0000116)}{(0.20)(1.0)(34)} = 0.512$$

The optimum exit temperature may be obtained by a trial-and-error solution of Eq. (56) or by use of Fig. 11-6.

From Fig. 11-6, when the abscissa is 0.512,

$$\frac{t' - t_{2,\text{opt}}}{t' - t_1} = 0.42$$

where $t' = 170°\text{F}$
$t_1 = 70°\text{F}$
$t_{2,\text{opt}} = 128°\text{F}$

By Eq. (52), at the optimum economic conditions,

$$w = \frac{q}{c_p(t_2 - t_1)} = \frac{(5000)(200)}{(1.0)(128 - 70)} = 17{,}200 \text{ lb water/h } (7800 \text{ kg water/h})$$

MASS TRANSFER (OPTIMUM REFLUX RATIO)

The design of a distillation unit is ordinarily based on specifications giving the degree of separation required for a feed supplied to the unit at a known composition, temperature, and flow rate. The design engineer must determine the size of column and reflux ratio necessary to meet the specifications. As the reflux ratio is increased, the number of theoretical stages required for the given separation decreases. An increase in reflux ratio, therefore, may result in lower fixed charges for the distillation column and greater costs for the reboiler heat supply and condenser coolant.

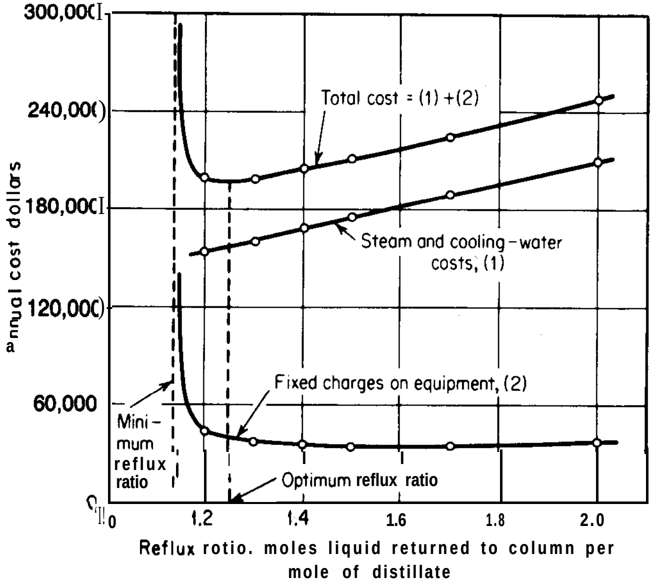

FIGURE 11-7
Optimum reflux ratio in distillation operation.

As indicated in Fig. 11-7, the optimum reflux ratio occurs at the point where the sum'of fixed charges and operating costs is a minimum. As a rough approximation, the optimum reflux ratio usually falls in the range of 1.1 to 1.3 times the minimum reflux ratio. The following example illustrates the general method for determining the optimum reflux ratio in distillation operations.

Example 6 Determination of optimum reflux ratio. A sieve-plate distillation column is being designed to handle 700 lb mol (318 kg mol) of feed per hour. The unit is to operate continuously at a total pressure of 1 atm. The feed contains 45 mol% benzene and 55 mol% toluene, and the feed enters at its boiling temperature. The overhead product from the distillation tower must contain 92 mol% benzene, and the bottoms must contain 95 mol% toluene. Determine the following:

(*a*) The optimum reflux ratio as moles liquid returned to tower per mole of distillate product withdrawn.

(*b*) The ratio of the optimum reflux ratio to the minimum reflux ratio.

(*c*) The percent of the total variable cost due to steam consumption at the optimum conditions.

The following data apply:

Vapor-liquid equilibrium data for benzene-toluene mixtures at atmospheric pressure are presented in Fig. 11-8.

The molal heat capacity for liquid mixtures of benzene and toluene in all proportions may be assumed to be 40 Btu/(lb mol)(°F) (1.67×10^5 J/kg mol · K).

The molal heat of vaporization of benzene and toluene may be taken as 13,700 Btu/lb mol (3.19×10^7 J/kg mol).

Effects of change in temperature on heat capacity and heats of vaporization are negligible. Heat losses from the column are negligible. Effects of pressure drop over the column may be neglected.

The overall coefficient of heat transfer is 80 Btu/(h)(ft^2)(°F) (454 J/m^2 · s · K) in the reboiler and 100 Btu/(h)(ft^2)(°F) (568 J/m^2 · s · K) in the condenser.

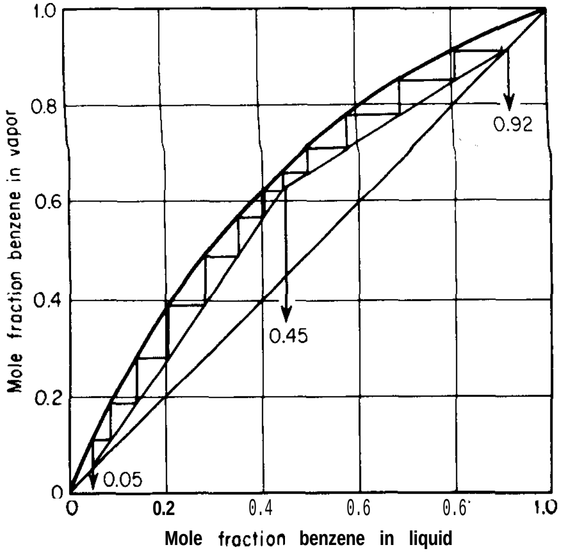

FIGURE 11-8
Equilibrium diagram for benzene-toluene mixtures at total pressure of 760 mm Hg (McCabe-Thiele method for deteimining number of theoretical plates).

The boiling temperature is 201°F (367 K) for the feed, 179°F (367 K) for the distillate, and 227°F (381 K) for the bottoms. The temperature-difference driving force in the reflux condenser may be based on an average cooling-water temperature of 90°F (305 K), and the change in cooling-water temperature is 50°F (27.8 K) for all cases. Saturated steam at 60 psi (413.6 kPa) is used in the reboiler. At this pressure, the temperature of the condensing steam is 292.7°F (418 K) and the heat of condensation is 915.5 Btu/lb (2.13 X 10^6 J/kg). No heat-savings devices are used.

The column diameter is to be based on a maximum allowable vapor velocity of 2.5 ft/s (0.76 m/s) at the top of the column. The overall plate efficiency may be assumed to be 70 percent. The unit is to operate 8500 h per year.

Cost data.

Steam = $1.50/1000 lb ($3.31/1000 kg).
Cooling water = $0.090/1000 gal or $0.108/10,000 lb ($0.238/10,000 kg).

The sum of costs for piping, insulation, and instrumentation can be estimated to be 60 percent of the cost for the installed equipment. Annual fixed charges amount to 15 percent of the total cost for installed equipment, piping, instrumentation, and insulation.

The following costs are for the installed equipment and include delivery and erection costs:

Sieve-plate distillation column
Values may be interpolated

Diameter in.	(m)	%/plate
60	(1.52)	2400
70	(1.78)	3000
80	(2.03)	3700
90	(2.29)	4500
100	(2.54)	5400

Condenser-tube-and-shell heat exchanger
Values may be interpolated

Heat-transfer area ft^2	(m^2)	$
800	(74.3)	19,500
1000	(92.9)	22,500
1200	(111.5)	25,200
1400	(130.1)	27,600
1600	(148.6)	29,700

Reboiler-tube-and-shell heat exchanger
Values may be interpolated

Heat-transfer area ft^2	(m^2)	$
1000	(92.9)	34,500
1400	(130.1)	42,300
1800	(167.2)	49,200
2200	(204.4)	55,500
2600	(241.5)	60,600

Solution. The variable costs involved are cost of column, cost of reboiler, cost of condenser, cost of steam, and cost of cooling water. Each of these costs is a function of the reflux ratio, and the optimum reflux ratio occurs at the point where the sum of the annual variable costs is a minimum. The total variable cost will be determined at various reflux ratios, and the optimum reflux ratio will be found by the graphical method.

Sample calculation for reflux ratio = 1.5:

Annual cost for distillation column. The McCabe-Thiele simplifying assumptions apply for this case, and the number of theoretical plates can be determined by the standard graphical method shown in Fig. 11-8. The slope of the enriching operating line is $1.5/(1.5+1) = 0.6$. From Fig. 11-8, the total number of theoretical stages required for the given separation is 12.1.

The actual number of plates = $(12.1 - 1)/0.70 = 16$.

The moles of distillate per hour (M_D) and the moles of bottoms per hour (M_B) may be determined by a benzene material balance as follows:

$$(700)(0.45) = (M_D)(0.92) + (700 - M_D)(0.05)$$

$$M_D = 322 \text{ moles distillate/h}$$

$$M_B = 700 - 322 = 378 \text{ moles bottoms/h}$$

Moles vapor per hour at top of column = $322(1 + 1.5) = 805$.

Applying the perfect-gas law,

$$\text{Vapor velocity at top of tower} = 2.5 \text{ ft/s} = \frac{(805)(359)(460 + 179)(4)}{(3600)(492)(\pi)(\text{diameter})^2}$$

$$\text{Diameter} = 7.3 \text{ ft}$$

Cost per plate for plate and vessel = \$4290

$$\text{Annual cost for distillation column} = (4290)(16)(1 + 0.60)(0.15) = \$16{,}470$$

Annual cost for condenser. Rate of heat transfer per hour in condenser = (moles vapor condensed per hour)(molal latent heat of condensation) = (805)(13,700) = 11,000,000 Btu/h.

From the basic heat-transfer-rate equation $q = UA\,\Delta t$,

$$A = \text{heat-transfer area} = \frac{(11{,}000{,}000)}{(100)(179 - 90)} = 1240 \text{ sq ft}$$

$$\text{Cost per square foot} = \frac{\$25{,}650}{1240}$$

$$\text{Annual cost for condenser} = \frac{25{,}650}{1240}(1240)(1 + 0.60)(0.15) = \$6150$$

Annual cost for reboiler. The rate of heat transfer in the reboiler (q_r) can be determined by a total energy balance around the distillation unit.

Base energy level on liquid at 179°F.

$$\text{Heat input} = \text{heat output}$$

$$q_r + (700)(201 - 179)(40) = 11{,}000{,}000 + (378)(227 - 179)(40)$$

$$q_r = 11{,}110{,}000 \text{ Btu/h} = UA \text{ At}$$

$$A = \text{heat-transfer area} = \frac{11{,}110{,}000}{(80)(292.7 \quad 227)} = 2120 \text{ ft}^2$$

$$\text{Cost per square foot} = \frac{\$54{,}300}{2120}$$

$$\text{Annual cost for reboiler} = \frac{54{,}300}{2120}(2120)(1 + 0.60)(0.15)$$

$$= \$13{,}020$$

Annual cost for cooling water. The rate of heat transfer in the condenser = 11,000,000 Btu/h. The heat capacity of water may be taken as 1.0 Btu/(lb)(°F).

$$\text{Annual cost for cooling water} = \frac{(11{,}000{,}000)(0.108)(8500)}{(1.0)(50)(10{,}000)}$$

$$= \$20{,}220$$

Annual cost for steam. The rate of heat transfer in the reboiler = 11,110,000 Btu/h.

$$\text{Annual cost for steam} = \frac{(11{,}110{,}000)(1.50)(8500)}{(915.5)(1000)}$$

$$= \$155{,}100$$

Total annual variable cost at reflux ratio of 1.5

$$\$16{,}470 + \$6150 + \$13{,}020 + \$20{,}220 + \$155{,}100 = \$210{,}960$$

By repeating the preceding calculations for different reflux ratios, the following table can be prepared:

Reflux ratio	Number of actual plates required	Column diameter, ft	Annual cost, dollars, for					Total annual cost, dollars
			Column	Condenser	Reboiler	Cooling water	Steam	
1.14	∞	6.7	∞	5610	11,880	17,340	132,900	∞
1.2	29	6.8	26,790	5730	12,120	17,820	136,500	198,960
1.3	21	7.0	19,860	58.50	12,390	18,600	142,500	199,200
1.4	18	7.1	17,760	6000	12,720	19,410	148,800	204,690
1.5	16	7.3	16,470	6150	13,020	20,220	155,100	210,960
1.7	14	7.7	15,870	6450	13,620	21,870	167,100	224,910
2.0	13	8.0	15,630	6840	14,400	24,300	185,400	246,570

(a) The data presented in the preceding table are plotted in Fig. 11-7. The minimum total cost per year occurs at a reflux ratio of 1.25.

$$\text{Optimum reflux ratio} = 1.25$$

(*b*) For conditions of minimum reflux ratio, the slope of the enriching line in Fig. 11-8 is 0.532

$$\frac{\text{Minimum reflux ratio}}{\text{Minimum reflux ratio} + 1} = 0.532$$

$$\text{Minimum reflux ratio} = 1.14$$

$$\frac{\text{Optimum reflux ratio}}{\text{Minimum reflw ratio}} = \frac{1.25}{1.14} = 1.1$$

(*c*) At the optimum conditions,

$$\text{Annual steam cost} = \$139{,}500$$

$$\text{Total annual variable cost} = \$198{,}000$$

$$\text{Percent of variable cost due to steam consumption} = \frac{139{,}500}{198{,}000}(\text{loo}) = 70\%$$

THE STRATEGY OF LINEARIZATION FOR OPTIMIZATION ANALYSIS

In the preceding analyses for optimum conditions, the general strategy has been to establish a partial derivative of the dependent variable from which the absolute optimum conditions are determined. This procedure assumes that an absolute maximum or minimum occurs within attainable operating limits and is restricted to relatively simple conditions in which limiting constraints are not exceeded. However, practical industrial problems often involve establishing the best possible program to satisfy existing conditions under circumstances where the optimum may be at a boundary or limiting condition rather than at a true maximum or minimum point. A typical example is that of a manufacturer who must determine how to blend various raw materials into a final mix that will meet basic specifications while simultaneously giving maximum profit or least cost. In this case, the basic limitations or constraints are available raw materials, product specifications, and production schedule, while the overall objective (or *objective function)* is to maximize profit.

LINEAR PROGRAMMING FOR OBTAINING OPTIMUM CONDITIONS

One strategy for simplifying the approach to a programming problem is based on expressing the constraints and the objective in a linear mathematical form. The "straight-line" or linear expressions are stated mathematically as

$$ax_1 + bx_2 + \cdots + jx_j + \cdots + nx_n = z \qquad (57)$$

where the coefficients $a \cdots n$ and z are known values and $x_1 \cdots x_n$ are unknown variables. With two variables, the result is a straight line on a two-dimensional plot, while a plane in a three-dimensional plot results for the case of three variables. Similarly, for more than three variables, the geometric result is a hyperplane.

The general procedure mentioned in the preceding paragraph is designated as *linear programming.* It is a mathematical technique for determining optimum conditions for allocation of resources and operating capabilities to attain a definite objective. It is also useful for analysis of alternative uses of resources or alternative objectives.

EXAMPLE OF APPROACH IN LINEAR PROGRAMMING

As an example to illustrate the basic methods involved in linear programming for determining optimum conditions, consider the following simplified problem. A brewery has received an order for 100 gal of beer with the special constraints that the beer must contain 4 percent alcohol by volume and it must be supplied immediately. The brewery wishes to fill the order, but no 4 percent beer is now in stock. It is decided to mix two beers now in stock to give the desired final product. One of the beers in stock (Beer A) contains 4.5 percent alcohol by volume and is valued at $0.32 per gallon. The other beer in stock (Beer B) contains 3.7 percent alcohol by volume and is valued at $0.25 per gallon. Water (W) can be added to the blend at no cost. What volume combination of the two beers in stock with water, including at least 10 gal of Beer A, will give the minimum ingredient cost for the 100 gal of 4 percent beer?

This example is greatly simplified because only a few constraints are involved and there are only three independent variables, i.e., amount of Beer A (V_A), amount of Beer B (V_B), and amount of water (V_W). When a large number of possible choices is involved, the optimum set of choices may be far from obvious, and a solution by linear programming may be the best way to approach the problem. A step-by-step rational approach is needed for linear programming. This general rational approach is outlined in the following with application to the blending example cited.

RATIONAL APPROACH TO PROBLEMS INVOLVING LINEAR PROGRAMMING

A systematic rationalization of a problem being solved by linear programming can be broken down into the following steps:

1. *A systematic description **of** the limitations or constraints.* For the brewery example, the constraints are as follows:
   a. Total volume of product is 100 gallons, or

$$V_W + V_A + V_B = 100 \tag{58}$$

   b. Product must contain 4 percent alcohol, or

$$0.0V_W + 4.5V_A + 3.7V_B = (4.0)(100) \tag{59}$$

   c. Volume of water and Beer B must be zero or greater than zero, while volume of Beer A must be 10 gal or greater; i.e.,

$$V_W \geq 0 \qquad V_B \geq 0 \qquad V_A \geq 10 \text{ or } V_A - S = 10 \tag{60}$$

   where S is the so-called "slack variable."

2. ***A systematic description of the objective.*** In the brewery example, the objective is to minimize the cost of the ingredients; i.e., the objective function is

$$C = \text{cost} = \text{a minimum} = 0.0V_W + 0.32V_A + 0.25V_B \tag{61}$$

3. ***Combination of the constraint conditions and the objective function to choose the best result out of many possibilities.*** One way to do this would be to use an intuitive approach whereby every reasonable possibility would be considered to give ultimately, by trial and error, the best result. This approach would be effective for the brewery example because of its simplicity. However, the intuitive approach is not satisfactory for most practical situations, and linear programming can be used. The computations commonly become so involved that a computer is required for the final solution. If a solution is so simple that a computer is not needed, linear programming would probably not be needed. To illustrate the basic principles, the brewery example is solved in the following by linear programming including intuitive solution, graphical solution, and computer solution.

From Eqs. (58) to (61), the following linearized basic equations can be written:

$$V_W + V_A + V_B = 100 \tag{58}$$

$$4.5V_A + 3.7V_B = 400 \tag{59}$$

$$0.32V_A + 0.25V_B = C = \text{minimum} \tag{61}$$

where C is designated as the objective function.

Combination of Eqs. (58) and (59) gives

$$V_A = 37.5 + 4.625V_W \tag{62}$$

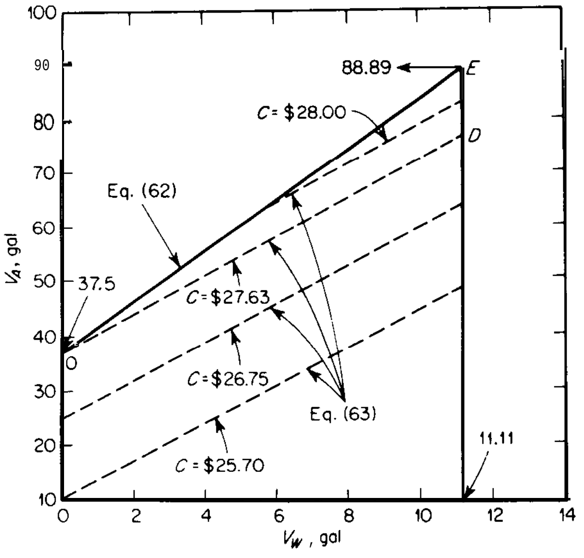

FIGURE **11-9**
Graphical representation of linear-programming solution based on brewery example.

Equation (62) is plotted as line *OE* in Fig. 11-9, and the optimum must fall on this line.

Equation (61) combined with Eq. (58) gives

$$V_A = \frac{C - 25}{0.07} + 3.57V_W \tag{63}$$

INTUITIVE SOLUTION. It can be seen intuitively, from Eqs. (63) and (62), that the minimum value of the objective function C occurs when V_W is zero. Therefore, the optimum value of V_A, from Eq. (62), is 37.5 gal and the optimum value of V_B, from Eq. (58), is 62.5 gal.

LINEAR PROGRAMMING GRAPHICAL SOLUTION. Figure 11-9 is the graphical representation of this problem. Line *OE* represents the overall constraint placed on the problem by Eqs. (58), (59), and (60). The parallel dashed lines represent possible conditions of cost. The goal of the program is to minimize cost (that is, C) while still remaining within the constraints of the problem. The minimum value of C that still meets the constraints occurs for the line *OD*, and the optimum must be at point 0. Thus, the recommended blend is no water, 37.5 gal of A, 62.5 gal of *B*, and a total cost C of $27.63 for 100 gal of blend.

LINEAR PROGRAMMING COMPUTER SOLUTION. Although the simplicity of this problem makes it trivial to use a computer for solution, the following is presented to illustrate the basic type of reasoning that is involved in developing a computer program for the linearized system.

An iterative procedure must be used for the computer solution to permit the computer to make calculations for repeated possibilities until the minimum objective function C is attained. In this case, there are four variables (V_A, V_B, V_W, and S) and three nonzero constraints (total volume, final alcohol content, and $V_A = 10 + S$). Because the number of real variables cannot exceed the number of nonzero constraints, one of the four variables must be zero.? Thus, one approach for a computer solution merely involves solving a four-by-three matrix with each variable alternatively being set equal to zero, followed by instruction that the desired combination is the one giving the least total cost.

The computer logic, from which the computer diagram, program, and solution can be developed directly, is presented in Tables 3 and 4.[‡]

[†] For proof of this statement, see any book on linear programming. For example, S. I. Gass, "Linear Programming: Methods and Applications," 3rd ed., p. 54, McGraw-Hill Book Company, New York, 1969.

[‡] In Prob. 18 at the end of this chapter, the student is requested to develop the full computer program and solve this problem on a computer.

TABLE 3

Computer logic for "linear programming" solution to brewery example

The computer must solve a linearized situation in which there are four variables and three nonzero constraints to meet a specified objective function of minimum cost. Under these conditions, one of the variables *must* be zero. Thus, one method for computer approach is to set each variable in turn equal to zero, solve the resulting three-by-three matrix, and determine the final solution to make cost a minimum.† Instead of one total computer solution with instructions for handling a three-by-four matrix, the approach will be simplified by repeating four times the solution by determinants of a standard case of three equations and three unknowns with each of the four variables alternatingly set equal to zero.

Basis: $E(I, J)$ and X(J); where $E(I, J)$ designates the appropriate coefficient, X(J) designates the appropriate variable, I designates the proper row, and J designates the proper column. Thus,

Equivalent to

Constraints

$$\begin{cases} E(1,1)X(1) + E(1,2)X(2) + E(1,3)X(3) = E(1,4) & \text{Eq. (58)} \\ E(2,1)X(1) + E(2,2)X(2) + E(2,3)X(3) = E(2,4) & \text{Eq. (59)} \\ E(3,1)X(1) + E(3,2)X(2) + E(3,3)X(3) = E(3,4) & \text{Eq. (60)} \end{cases}$$

Objective function

$$C(1)X(1) + C(2)X(2) + C(3)X(3) = \text{F(C)} \qquad \text{Eq. (61)}$$

C(J) designates the appropriate coefficient and F(C) is the objective function.

Logic and procedure based on arbitrary choice of one variable set equal to zero

Step 1: Read in to the computer the constant coefficients for the three variables being retained based on Eqs. (58), (59), and (60). Read in to computer the constant coefficients for the objective function, i.e., Eq. (61).

Step 2: Solve the resulting three equations simultaneously by determinants.

A. Evaluate the determinant of the coefficients of the system = F(E) = *D:*

B. Evaluate $DX(J) = F(W)$:

$$D \equiv \begin{vmatrix} E(1,1) & E(1,2) & E(1,3) \\ E(2,1) & E(2,2) & E(2,3) \\ E(3,1) & E(3,2) & E(3,3) \end{vmatrix} \qquad F(W) = \begin{vmatrix} W(1,1) & W(1,2) & W(1,3) \\ W(2,1) & W(2,2) & W(2,3) \\ W(3,1) & W(3,2) & W(3,3) \end{vmatrix}$$

where $W(I, J)$ designates the appropriate coefficient for the *DX* working matrix in which the column of equality constraints is substituted in the appropriate determinant column.

Step 3: Evaluate the objective function.

Step 4: Print out the values of the three variables and the value of the objective function.

Step 5: Repeat steps 1 to 4 three more times with each variable set equal to zero.

Step 6: After all results have been printed out, choose the set of results giving the minimum value for the objective function with all variables meeting the requirements.

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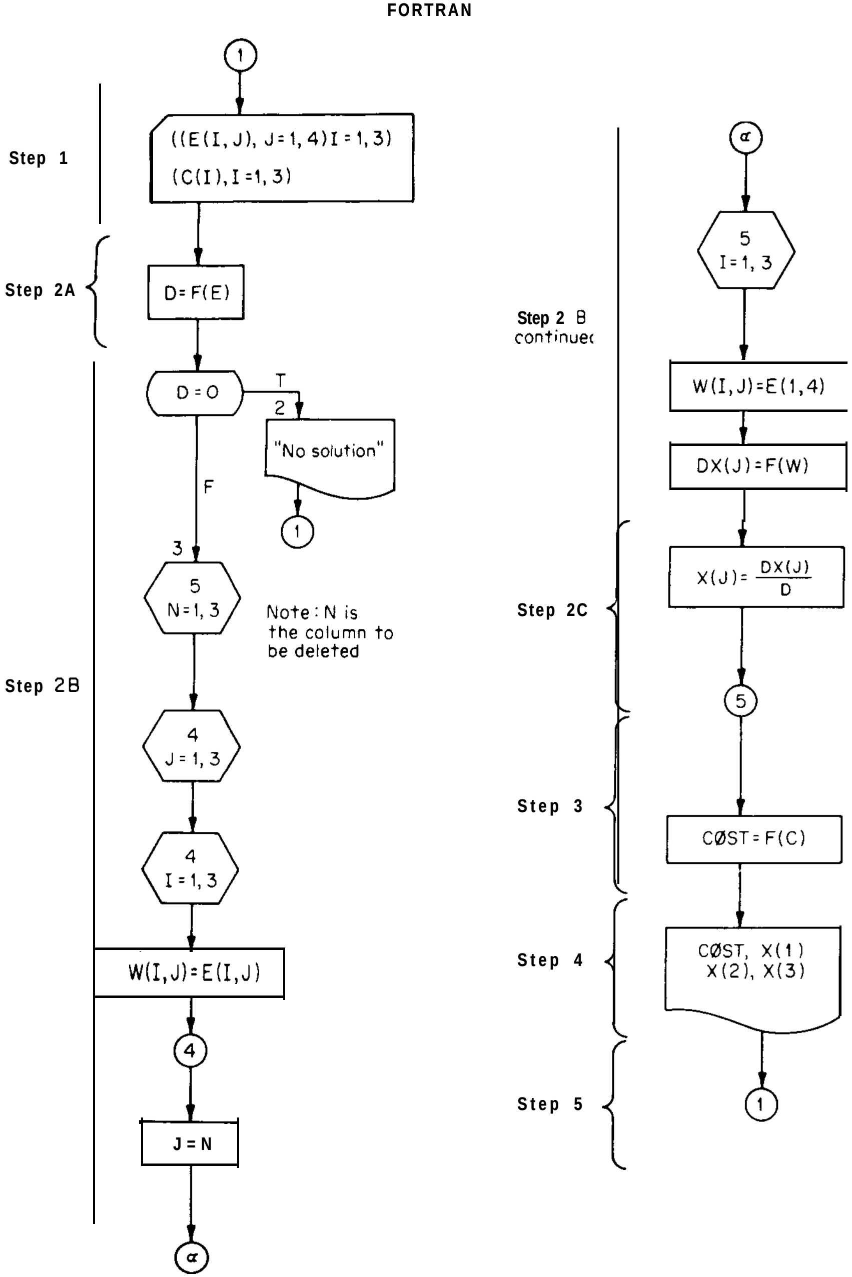
† For normal methods used for linear-programming solutions, there are various rules which can be used to determine which variable should be set equal to zero so that not all possible combinations must be tried. See later discussion of the *Simplex Algorithm*.

TABLE 4
Computer diagram based on logic of Table 3 for linear-programmed brewery example



GENERALIZATION OF STRATEGY FOR LINEAR PROGRAMMING

The basic problem in linear programming is to maximize or minimize a linear function of a form as shown in Eq. (57). There are various ***strategies*** that can be developed to simplify the methods of solution, some of which can lead to algorithms which allow rote or pure number-plugging methods of solution that are well adapted for machine solution.

In linear programming, the variables $x_1 \cdots x_n$ are usually restricted (or can be transformed) to values of zero or greater. This is known as a ***nonnegativity restriction*** on x_j; that is,

$$x_j \geq 0 \qquad j = 1, 2, \ldots, n$$

Consider a simple two-dimensional problem such as the following: The objective function is to maximize

$$3x_1 + 4x_2 \tag{64}$$

subject to the linear constraints of

$$2x_1 + 5x_2 \leq 10 \tag{65}$$

$$4x_1 + 3x_2 \leq 12 \tag{66}$$

$$x_1 \geq 0 \tag{67}$$

$$x_2 \geq 0 \tag{68}$$

This problem and its solution is pictured graphically in Fig. 11-10, which shows that the answer is $3x_1 + 4x_2 = 11$. From Fig. 11-10 it can be seen that the linear constraints, in the form of inequalities, restrict the solution region to the cross-hatched area. This solution region is a polygon designated as convex because all points on the line between any two points in the cross-hatched region are in the set of points that satisfy the constraints. The set of the objective function is a family of lines with slope of $-\frac{3}{4}$. The maximum value of the objective function occurs for the line passing through the polygon vertex D. Thus, the maximum value of the objective function occurs for the case of $3x_1 + 4x_2 = 11$ at $x_1 = \frac{15}{7}$ and $x_2 = \frac{8}{7}$.

For the two-dimensional case considered in the preceding, one linear condition defines a line which divides the plane into two half-planes. For a three-dimensional case, one linear condition defines a set plane which divides the volume into two half-volumes. Similarly, for an n-dimensional case, one linear condition defines a hyperplane which divides the space into two half-spaces.

For the n-dimensional case, the region that is defined by the set of hyperplanes resulting from the linear constraints represents a convex ***set*** of all points which satisfy the constraints of the problem. If this is a bounded set, the enclosed space is a convex polyhedron, and, for the case of monotonically increasing or decreasing values of the objective function, the maximum or minimum value of the objective function will always be associated with a vertex

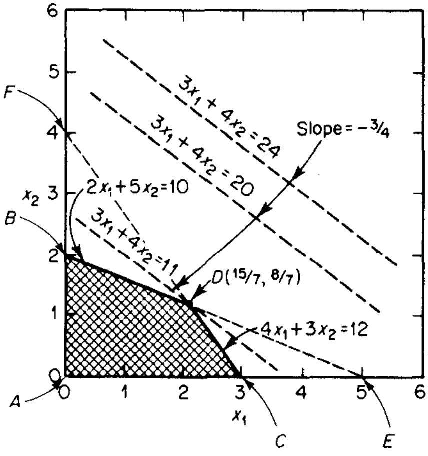


FIGURE 11-10
Graphical illustration of two-dimensional linear-programming solution.

or ***extreme point*** of the convex polyhedron. This indicates that the linear-programming solution for the model of inequality or equality constraints combined with the requested value for the objective function will involve determination of the value of the objective function at the extreme points of the set of all points that satisfy the constraints of the problem. The desired objective function can then be established by comparing the values found at the extreme points. If two extremes give the same result, then an infinite number of solutions exist as defined by all points on the line connecting the two extreme points.

SIMULTANEOUS EQUATIONS†

Linear programming is concerned with solutions to simultaneous linear equations where the equations are developed on the basis of restrictions on the variables. Because these restrictions are often expressed as inequalities, it is necessary to convert these inequalities to equalities. This can be accomplished by the inclusion of a new variable designated as a ***slack variable.***

For a restriction of the form

$$a_1x_1 + a_2x_2 + a_3x_3 \leq b \tag{69}$$

the inequality is converted to a linear equation by adding a slack variable S_4 to

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†Cases are often encountered in design calculations where a large number of design equations and variables are involved with long and complex simultaneous solution of the equations being called for. The amount of effort involved for the simultaneous solutions can be reduced by using the so-called ***structural-array algorithm*** which is a purely mechanical operation involving crossing out rows for equations and columns for variables to give the most efficient order in which the equations should be solved. For details, see D. F. Rudd and C. C. Watson, “Strategy of Process Engineering,” pp. 45-49, John Wiley & Sons, Inc., New York, 1968.

the left-hand side:

$$a_1x_1 + a_2x_2 + a_3x_3 + S_4 = b \tag{70}$$

The slack variable takes on whatever value is necessary to satisfy the equation and normally is considered as having a nonnegativity restriction. Therefore, the slack variable would be subtracted from the left-hand side for an inequality of the form

$$a_1x_1 + a_2x_2 + a_3x_3 \geq b \tag{71}$$

to give

$$a_1x_1 + a_2x_2 + a_3x_3 - S_4 = b \tag{72}$$

After the inequality constraints have been converted to equalities, the complete set of restrictions becomes a set of linear equations with n unknowns. The linear-programming problem then will involve, in general, maximizing or minimizing a linear objective function for which the variables must satisfy the set of simultaneous restrictive equations with the variables constrained to be nonnegative. Because there will be more unknowns in the set of simultaneous equations than there are equations, there will be a large number of possible solutions, and the final solution must be chosen from the set of possible solutions.

If there are m independent equations and n unknowns with $m < n$, one approach is to choose arbitrarily n m variables and set them equal to zero. This gives m equations with m unknowns so that a solution for the m variables can be obtained. Various combinations of this type can be obtained so that the total possible number of solutions by this process becomes

$$\binom{n}{m} = n!/m!(n - m)! \tag{73}$$

representing the total number of possible combinations obtainable by taking n variables m at a time. Another approach is to let $n - m$ combinations of variables assume any zero or nonzero value which results in an infinite number of possible solutions. Linear programming deals only with the situation where the excess variables are set equal to zero.

TWO EXAMPLES TO SHOW APPROACH BY SIMULTANEOUS EQUATIONS

To illustrate the introductory ideas presented for a linear-programming problem, consider the following example which is solved by using a step-by-step simultaneous-equation approach:

A production facility is being used to produce three different products, x_1, x_2, and x_3. Each of these products requires a known number of employee-hours and machine-hours for production such that

product x_1 requires 10 employee-hours and 15 machine-hours per unit,
product x_2 requires 25 employee-hours and 10 machine-hours per unit,
product x_3 requires 20 employee-hours and 10 machine-hours per unit.

The profit per unit is \$5 for x_1, \$10 for x_2, and \$12 for x_3. Over the base production period under consideration, a total of 300 employee-hours and a total of 200 machine-hours are available. With the special restriction that all employee-hours are to be used, what mix of products will maximize profits?

For this problem, the linear constraints are:

for machine hours,

$$15x_1 + 10x_2 + 10x_3 \leq 200 \tag{74}$$

for employee-hours,

$$10x_1 + 25x_2 + 20x_3 = 300 \tag{75}$$

The objective function is to maximize profits, or

$$\text{Maximize } 5x_1 + 10x_2 + 12x_3 \tag{76}$$

By including a slack variable for Eq. (74), the constraining equalities become

$$15x_1 + 10x_2 + 10x_3 + S_4 = 200 \tag{77}$$

$$10x_1 + 25x_2 + 20x_3 = 300 \tag{78}$$

For this case, $n = 4$ and $m = 2$. Setting any two of the variables equal to zero and solving the result gives

$$\binom{n}{m} = \frac{n!}{m!(n-m)!} = \frac{4!}{2!2!} = 6$$

possible solutions. These six solutions are shown in Table 5.

Solutions 3 and 4 are infeasible because the nonnegativity restrictionhas been violated, while Solution 6 is a feasible solution which maximizes the objective function. Thus, Solution 6 is the desired solution for this example and represents the optimal solution.

TABLE 5
Six solutions by simultaneous equations for example problem

Solution number	x_1	x_2	x_3	S_4	Objective function $5x_1 + 10x_2 + 12x_3$
1	1.2121	9.0909	0	0	121.27
2	5	0	12.5	0	175
3	30	0	0	-250	Infeasible (negative)
4	0	-20	40	0	Infeasible (negative)
5	0	12	0	80	120
6	0	0	15	50	180

For the example presented graphically in Fig. 11-10, the two linear constraining equations made into equalities by the slack variables S_3 and S_4 are

$$2x_1 + 5x_2 + S_3 + \text{OS}, = 10 \tag{79}$$

$$4x_1 + 3x_2 + \text{OS}, + S_4 = 12 \tag{80}$$

with the objective function being

$$3x_1 + 4x_2 + \text{OS}, + \text{OS}, = z = \text{a maximum} \tag{81}$$

In this case, there are two equations ($m = 2$) and four variables ($n = 4$). Thus, the approach with $n - m = 4 - 2 = 2$ of the variables being zero for each solution will involve having

$$\binom{n}{m} = \frac{n!}{m!(n \quad m)!} = \frac{4!}{2!\,2!} = 6$$

possible solutions. These solutions are shown in Fig. 11-10 as points A to F and in Table 6 as obtained by simultaneous-equation solution with the optimum result being the D solution.

TABLE 6
Six solutions by simultaneous equations for Fig. 11-10 example

Solution designation	x_1	x_2	S_3	S_4	Objective function $3x_1 + 4x_2$
A	0	0	10	12	0
B	0	2	0	6	a
C	3	0	4	0	9
D	15/7	8/7	0	0	11†
E	5	0	0	- a	Infeasible (negative)
F	0	4	-10	0	Infeasible (negative)

† Maximum feasible value; so the optimum solution is D.

The preceding examples, although they represent an approach for solving linear-programming problems, are very inefficient because of the large number of useless solutions that may be generated if many variables are involved. More efficient procedures are available, and these are discussed in the following sections.

GENERALIZATION OF LINEAR PROGRAMMING APPROACH FOR ALGORITHM SOLUTION

To permit efficient solutions for linear-programming problems, an algorithm can be developed. An algorithm, basically, is simply an objective mathematical method for solving a problem and is purely mechanical so that it can be taught to a nonprofessional or programmed for a computer. The algorithm may consist of a series of repeated steps or ***iterations.*** To develop this form of approach for linear-programming solutions, the set of linear inequalities which form the constraints, written in the form of "equal to or less than" equations is

$$\begin{aligned} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n &\le b_1 \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n &\le b_2 \\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots & \\ a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n &\le b_m \end{aligned} \tag{82}$$

or, in general summation form,

$$\sum_{j=1}^{n} a_{ij}x_j \le b_i \qquad i = 1,2,\ldots,m \tag{82a}$$

for

$$x_j \ge 0 \qquad j = 1,2,\ldots,n$$

where i refers to columns (or number of equations, m) in the set of inequalities and j refers to rows (or number of variables, n).

As has been indicated earlier, these inequalities can be changed to equalities by adding a set of slack variables, $x_{n+1} \cdots x_{n+m}$ (here x is used in place of S to simplify the generalized expressions), so that

$$\begin{aligned} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n + x_{n+1} &= b_1 \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n + x_{n+2} &= b_2 \\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots & \\ a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n + x_{n+m} &= b_m \end{aligned} \tag{83†}$$

---

†**This** can be written in standard matrix form and notation as

$$A\,X = B$$

where

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} & 1 & 0 & 0 & \cdots\cdots & 0 \\ a_{21} & a_{22} & \cdots & a_{2n} & 0 & 1 & 0 & \cdots\ \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & 0 & 0 & \cdots & 1\ \ \cdots & 0 \\ a_{m1} & a_{m2} & \cdots & a_{mn} & 0 & 0 & \cdots & \cdot\ \cdot\ \cdot\ 0 & 1 \end{bmatrix}$$

$$\mathbf{X} = \begin{bmatrix} x_1 \\ x_2 \\ \\ x_n \\ x_{n+1} \\ \\ x_{n+m} \end{bmatrix} \qquad \mathbf{B} = \begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{pmatrix}$$

$$x_j \ge 0 \qquad j = 1,2,\ldots,n+m$$

and standard matrix operations of multiplying, addition, etc., can be applied.

or, in general summation form,

$$\sum_{j=1}^{n} (a_{ij}x_j + x_{n+i}) = b_i \qquad i = 1,2,\ldots,m \tag{83a}$$

for

$$x_j \geq 0 \qquad j = 1,2,\ldots,n + m$$

In addition to the constraining equations, there is an objective function for the linear program which is expressed in the form of

$$z = \text{maximum (or minimum) of } c_1x_1 + c_2x_2 + \cdots + c_jx_j + \cdots + c_nx_n \quad (84)†$$

where the variables x_j are subject to $x_j \geq 0$ $(j = 1,2,\ldots, n + m)$. Note that, in this case, all variables above x_n are slack variables and make no direct contribution to the value of the objective function.

Within the constraints as indicated by Eqs. (82) and (83), a solution for values of the variables, x_j, must be found which meets the maximum or minimum requirement of the objective function, Eq. (84). As has been demonstrated in the preceding examples, the solution to a problem of this sort must lie on an extreme point of the set of possible feasible solutions. For any given solution, the number of equations to be solved simultaneously must be set equal to the number of variables, and this is accomplished by setting n (number of variables) minus m (number of equations) equal to zero and then proceeding to obtain a solution.

While the preceding generalization is sufficient to allow for reaching a final solution ultimately, it can be very inefficient unless some sort of special method is used to permit generation of extreme-point solutions in an efficient manner to allow rapid and effective approach to the optimum condition. This is what the ***simplex method*** does.‡

THE SIMPLEX ALGORITHM

The basis for the simplex method is the generation of extreme-point solutions by starting at any one extreme point for which a feasible solution is known and then proceeding to a neighboring extreme point. Special rules are followed which cause the generation of each new extreme point to be an improvement toward the desired objective function. When the extreme point is reached where no further improvement is possible, this will represent the desired optimum feasible solution. Thus, the simplex algorithm is an iterative process that starts at one extreme-point feasible solution, tests this point for optimality, and

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†In a more compact form using matrix notation, the problem is to find the solution to $AX = B$ which maximizes or minimizes $z = \mathbf{c}X$ where $X \geq 0$.

‡The simplex method and algorithm were first made generally available when published by G. B. Dantzig in "Activity Analysis of Production and Allocations," edited by T. C. Koopmans, Chap. XXI, Wiley, New York, 1951.

proceeds toward an improved solution. If an optimal solution exists, this algorithm can be shown to lead ultimately and efficiently to the optimal solution.

The stepwise procedure for the simplex algorithm is as follows (based on the optimum being a maximum):

1. State the linear-programming problem in standard equality form.
2. Establish the initial feasible solution from which further iterations can proceed. A common method to establish this initial solution is to base it on the values of the slack variables where all other variables are assumed to be zero. With this assumption, the initial matrix for the simplex algorithm can be set up with a column showing those variables which will be involved in the first solution. The coefficient for these variables appearing in the matrix table should be 1 with the rest of the column being 0.
3. Test the initial feasible solution for optimality. The optimality test is accomplished by the addition of rows to the matrix which give (a) a value of z_j for each column where z_j is defined as the sum of the objective-function coefficient for each solution variable (c_i corresponding to solution x_i in that row) times the coefficient of the constraining-equation variable for that column [a_{ij} in Eq. (83*a*)]: (that is, $z_j = \sum_{i=1}^{m} c_i a_{ij}$ $(j = 1, 2, \ldots, n)$), (*b*) c_j [see Eq. (84)], and (*c*) $c_j - z_j$. If $c_j - z_j$ is positive for at least one column, then a better program is possible.
4. Iteration toward the optimal program is accomplished as follows: Assuming that the optimality test indicates that the optimal program has not been found, the following iteration procedure can be used:
   a. Find the column in the matrix with the maximum value of $c_j - z_j$ and designate this column as k. The incoming variable for the new test will be the variable at the head of this column.
   b. For the matrix applying to the initial feasible solution, add a column showing the ratio of b_i/a_{ik}. Find the minimum ***positive*** value of this ratio and designate the variable in the corresponding row as the outgoing variable.
   c. Set up a new matrix with the incoming variable, as determined under (*a*), substituted for the outgoing variable, as determined under (*b*). The modification of the table is accomplished by matrix operations so that the entering variable will have a 1 in the row of the departing variable and zeros in the rest of that column. The matrix operations involve row manipulations of multiplying rows by constants and subtracting from or adding to other rows until the necessary 1 and 0 values are reached. This new matrix should have added to it the additional rows and column as explained under parts 3, 4*a*, and *4b*.
   d. Apply the optimality test to the new matrix.
   e. Continue the iterations until the optimality test indicates that the optimum objective function has been attained.

5. Special cases:
   a. If the initial solution obtained by use of the method given in the preceding is not feasible, a feasible solution can be obtained by adding more artificial variables which must then be forced out of the final solution.
   b. Degeneracy may occur in the simplex method when the outgoing variable is selected. If there are two or more minimal values of the same size, the problem is degenerate, and a poor choice of the outgoing variable may result in cycling, although cycling almost never occurs in real problems. This can be eliminated by a method of ratioing each element in the rows in question by the positive coefficients of the kth column and choosing the row for the outgoing variable as the one first containing the smallest algebraic ratio.
6. The preceding method for obtaining a maximum as the objective function can be applied for the case when the objective function is a minimum by recognizing that maximizing the negative of a function is equivalent to minimizing the function.

THE SIMPLEX ALGORITHM APPLIED TO THE EXAMPLE SHOWN IN FIGURE 11-10

In the example used previously, and whose graphical solution is shown in Fig. 11-10, the problem in standard linear-programming form is: Find the values of the variables which represent a solution to

$$2x_1 + 5x_2 + x_3 \quad = 10 \tag{85}$$

$$4x_1 + 3x_2 + \quad x_4 = 12 \tag{86}$$

which maximizes

$$3x_1 + 4x_2 \quad \text{(express as } 3x_1 + 4x_2 + \text{Ox, + Ox, } = z\text{)} \tag{87}$$

where $x_1 \geq 0$, $x_2 \geq 0$, $x_3 \geq 0$, and $x_4 \geq 0$.

The next step after the appropriate statement of the linear-programming problem is to establish an initial feasible solution from which further iterations can proceed. For this case, let $x_1 = x_2 = 0$, and $x_3 = 10$, $x_4 = 12$, $z = 0$. (Solution A in Fig. 11-10 or Table 6.) The corresponding matrix in a standard tableau form is shown in Table 7.

The top row, c_j, in the tableau permits a convenient recording of the coefficients on the variables in the objective function, with these values listed at the head of the appropriate columns.

The first column, c_i, gives the coefficients of the variables in the objective function for this first solution. In this case, both are zero because x_3 and x_4 do not appear in the objective function.

The second column, *Solution,* gives the variables involved in the current solution and shows the row for which the variables involved apply.

TABLE 7

Tableau form of matrix for initial feasible solution ($x_1 = x_2 = 0$)

c_j c_i	Solution	b	0 x_3	0 x_4	3 x_1	4 x_2	b_i/a_{ik}
0	x_3	10	1	0	2	⑤	--2 10_
0	x_4	12	0	1	4	3	--4 12_ 3
	z_j	0	0	0	0	0	
	$c_j - z_j$	0	0	0	3	4 (k)	

The third column, ***b***, gives the list of condition constants for the limiting equations.

The columns following b have x headings and represent variables. The ***slack*** variables are x_3 and x_4, designated as unity for the appropriate row, while the ***structural*** variables are x_1 and x_2 with normal matrix form based on the coefficients for x_1 and x_2 in the limiting equations.

The final column on the right, b_i/a_{ik}, is used to record the indicated ratios for each row during the iteration process.

The bottom two rows are included to give a convenient method for recording the objective-function row component z_j and the values of $c_j - z_j$ for each column. By definition of z_j as $\sum_{i=1}^{m} c_i a_{ij}$ for $j = 1, 2, \ldots, n$, the value of z_j is 0 for all columns because both c_i's are 0.

Because row $c_j - z_j$ in Table '7 has at least one positive value in it, a better optimal program is available. The variable at the head of the column (k) with the maximum value of $c_j - z_j$ is x_2. Therefore, x_2 will be the incoming variable. The minimum value of b_i/a_{ik} occurs for the x_3 row; so x_3 will be the outgoing variable and the encircled 5 becomes the so-called ***pivotal point.***

To eliminate x_3 from the basis, the use of the indicated pivotal point gives, as a first step, the matrix tableau shown in the top part of Table 8 where the corresponding element for the pivotal point has been reduced to 1 by dividing the x_3 row by 5. The bottom portion of Table 8 is the matrix tableau for the next iteration with $x_3 = x_1 = 0$. This is established by a matrix row operation to reduce the other elements in the (k) column to zero (i.e., for this case, the multiplying factor for the x_3 row is -3, and the x_3 result is added to the x_4 row). The values of z_j are $(4)(2) + (0)(6) = 8$, $(4)(\frac{1}{5})$ + (OX- $\frac{3}{5}$) = $\frac{4}{5}$, $(4)(0) + (0)(1) = 0$, $(4)(\frac{2}{5}) + (0)(\frac{14}{5}) = \frac{8}{5}$, and $(4)(1) + (0)(0) = 4$ for the five columns from left to right, respectively.

Therefore, from Table 8, another extreme-point solution is $x_2 = 2$, $x_4 = 6$, $x_1 = x_3 = 0$. (Point B in Fig. 11-10.) This is still not the optimal solution because row $c_j - z_j$ has a positive value in it. The encircled $\frac{14}{5}$ is the pivotal point for the next iteration which will have x_4 as the outgoing variable and x_1 as the incoming variable. The same procedure is followed for this second iteration as for the first iteration. The steps are shown in Table 9, where the pivotal-point

TABLE 8

Tableau form of matrix for first iteration $(x_1 = x_3 = 0)$

c_j			0	0	3	4	
c_i	Solution	b	x_3	x_4	x_1	x_2	b_i/a_{ik}
0	x_3	2	$\frac{1}{5}$	0	$\frac{2}{5}$	1	$\frac{2}{1} = 2$
0	x_4	12	0	1	4	3	$\frac{12}{3} = 4$
	z_j	0	0	0	0	0	
	$c_j - z_j$	0	0	0	3	4	
						(k)	

c_j			0	0	3	4	
c_i	Solution	b	x_3	x_4	x_1	x_2	b_i/a_{ik}
4	x_2	2	$\frac{}{5}$	0	$\frac{2}{5}$	1	$2/\frac{2}{5} = 5$
0	x_4	6	$-\frac{3}{5}$	1	0 $\frac{14}{5}$	0	$6/\frac{14}{5} = \frac{30}{14}$
	z_j	8	$\frac{4}{5}$	0	$\frac{8}{5}$	4	
	$c_j - z_j$	-8	$-\frac{4}{5}$	0	$\frac{7}{5}$	0	
					(k)		

TABLE 9

Tableau form of matrix for second iteration $(x_3 = x_4 = 0)$

c_j			0	0	3	4	
c_i	**Solution**	b	x_3	x_4	x_1	x_2	b_i / a_{ik}
4	x_2	2	$\frac{1}{5}$	0	3	1	$2/\frac{2}{5} = 5$
0	x_4	$\frac{30}{14}$	$-\frac{3}{14}$	$\frac{5}{14}$	1	0	$\frac{30}{14}/1 = \frac{30}{14}$
	z_j	8	$\frac{4}{5}$	0	$\frac{8}{5}$	4	
	$c_j - z_j$	-8	$-\frac{4}{5}$	0	$\frac{7}{5}$	0	
					(k)		

c_j			0	0	3	4	
c_i	**Solution**	b	x_3	x_4	x_1	x_2	b_i / a_{ik}
4	x_2	$\frac{8}{7}$	3	$-\frac{1}{5}$	0	1	
3	x_1	$\frac{15}{7}$	$-\frac{3}{14}$	$\frac{5}{14}$	1	0	
	z_j	11	$\frac{1}{2}$	$\frac{1}{2}$	3	4	
	$c_j - z_j$	-11	$-\frac{1}{2}$	$-\frac{1}{2}$	0	0	

element is first reduced to 1 by dividing the x_4 row by $\frac{14}{5}$, and the other elements in the (k) column are then reduced to zero by a matrix row operation involving a multiplication factor of $-\frac{2}{5}$ for the x_4 row and adding the x_4 row to the x_2 row.

The results shown in Table 9 give another extreme point of $x_1 = \frac{15}{7}$, $x_2 = \frac{8}{7}$, $x_3 = x_4 = 0$. The z_j value for the b column is $(4)(\frac{8}{7}) + (3)(\frac{15}{7}) = 11$, and the values for the other four columns from left to right are obtained by a similar addition as $(4)(\frac{2}{7}) + (3)(-\frac{3}{14}) = \frac{1}{2}$, $(4)(-\frac{1}{7}) + (3)(\frac{5}{14}) = \frac{1}{2}$, $(4)(0) + (3)(1) = 3$, and $(4)(1) + (3)(0) = 4$.

Because row $c_j - z_j$ has only negative or zero values in it, this is the optimal solution, and the objective function is a maximum of $z = (3)(\frac{15}{7}) + (4)(\frac{8}{7}) = 11$ at $x_1 = \frac{15}{7}$ and $x_2 = \frac{8}{7}$, which is the same solution (point D in Fig. 11-10) that was obtained by the graphical analysis in Fig. 11-10. Note that, in each basic, initial, table matrix where the column for that variable has all zeros except for the variable row which is 1, the b column gives the values of the variables and the objective function for that solution.

The preceding information can serve as an introduction to the methods of linear programming including the step-by-step rule approach used for a simplex algorithm. The reader is referred to any of the many standard texts on linear programming for proof of the theorems and rules used in this treatment and further extensions of the methods of linear programming.[†]

THE STRATEGY OF DYNAMIC PROGRAMMING FOR OPTIMIZATION ANALYSIS

The concept of dynamic programming is based on converting an overall decision situation involving many variables into a series of simpler individual problems with each of these involving a small number of total variables. In its extreme, an optimization problem involving a large number of variables, all of which may be subject to constraints, is broken down into a sequence of problems with each of these involving only one variable. A characteristic of the process is that the determination of one variable leaves a problem remaining with one less variable. The computational approach is based on the principle of optimality, which states that *an optimal policy has the property that, no matter what the initial state*

[†] For example, see W. J. Adams, A. Gewirtz, and L. V. Quintas, "Elements of Linear Programming," Van Nostrand Reinhold Company, New York, 1969; G. B. Dantzig, "Linear Programming and Extensions," 5th Pr., Princeton Univ. Press, Princeton, N.J., 1968; S. I. Gass, "Linear Programming: Methods and Applications," 3d ed., McGraw-Hill Book Company, New York, 1969; G. E. Thompson, "Linear Programming: An Elementary Introduction," Macmillan Book Company, New York, 1971; and T. F. Edgar and D. M. Himmelblau, "Optimization of Chemical Processes," McGraw-Hill Book Company, New York, 1988.

and initial decision, the remaining decisions must constitute an optimal policy with regard to the state resulting from the first decision.

The use of dynamic programming is pertinent for design in the chemical industry where the objective function for a complicated system can often be obtained by dividing the overall system into a series of stages. Optimizing the resulting simple stages can lead to the optimal solution for the original complex problem.

The general formulation for a dynamic-programming problem, presented in a simplified form, is shown in Fig. 11-11. On the basis of the definitions of terms given in Fig. 11-lla, each of the variables, x_{i+1}, x_i, and d_i, may be replaced by vectors because there may be several components or streams involved in the input and output, and several decision variables may be involved. The profit or return P_i is a scalar which gives a measure of contribution of stage i to the objective function.

For the operation of a single stage, the output is a function of the input and the decisions, or

$$x_i = h_i(x_{i+1}, d_i) \tag{88}$$

Similarly, for the individual-stage objective function P_i

$$P_i = g_i(x_{i+1}, x_i, d_i) \tag{89}$$

or, on the basis of the relation shown as Eq. (88),

$$P_i = g_i(x_{i+1}, d_i) \tag{90}$$

For the simple multistage process shown in Fig. 11-llb, the process design strategy to optimize the overall operation can be expressed as

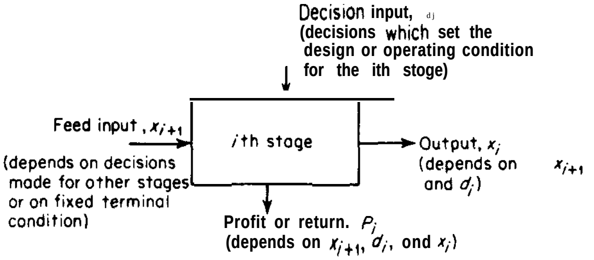
$$\mathfrak{f}_i(x_{i+1}) = \max_{d_i}\left[g_i(x_{i+1}, d_i) + \mathfrak{f}_{i-1}(x_i)\right] = \max_{d_i}\left[Q_i(x_{i+1}, d_i)\right] \tag{91}$$

for

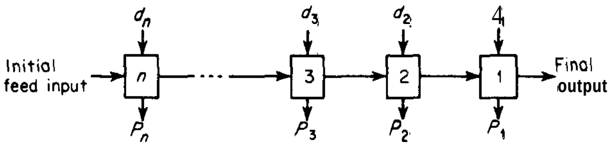
$$x_i = h_i(x_{i+1}, d_i) \qquad i = 1, 2, \ldots, n\text{-subject to } \mathfrak{f}_0 = 0$$

The symbolism $\mathfrak{f}_i(x_{i+1})$ indicates that the maximum (or optimum) return or profit from a process depends on the input to that process, and the terms in the square brackets of Eq. (91) refer to the function that is being optimized. Thus, the expression $Q_i(x_{i+1}, d_i)$ represents the combined return from all stages and must equal the return from stage i, or $g_i(x_{i+1}, d_i)$, plus the maximum return from the preceding stages 1 through $i - 1$, or $\mathfrak{f}_{i-1}(x_i)$.

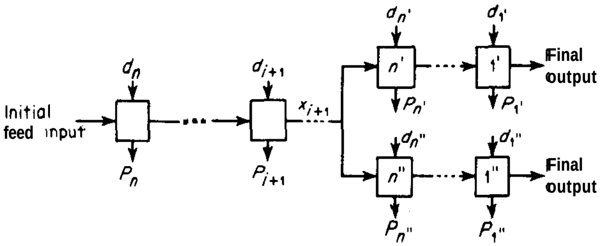
In carrying out the procedure for applying dynamic programming for the solution of appropriate plant-design problems, each input x_{i+1} is considered as a parameter. Thus, at each stage, the problem is to find the optimum value of the decision variable d_i for all feasible values of the input variable. By using the dynamic-programming approach involving n stages, a total of n optimizations must be carried out. This approach can be compared to the conventional approach in which optimum values of all the stages and decisions would be



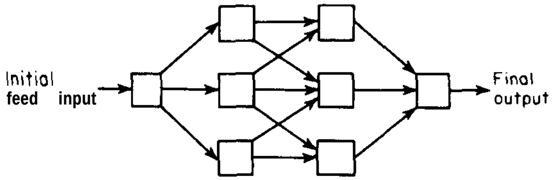
a. General formulation for one stage in a dynamic programming model



b. A simple multistage process with n stages



c. A multistage process with separating branches



d. A network illustration of dynamic programming showing various paths

FIGURE **11-11**
Illustration of stages involved in dynamic programming.

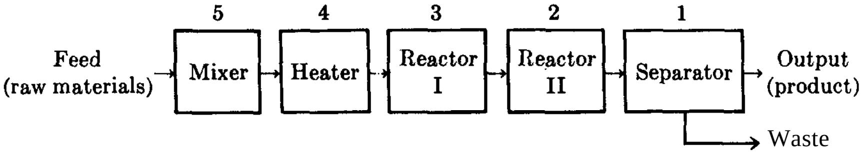
made by a basic probability combination analysis. Thus, the conventional method would have a computational effort that would increase approximately exponentially with the number of stages, while the dynamic-programming approach can give a great reduction in necessary computational effort because this effort would only increase about linearly with the number of stages. However, this advantage of dynamic programming is based on a low number of components in the input vector x_{i+1}, and dynamic programming rapidly loses its effectiveness for practical computational feasibility if the number of these components increases above two.

A SIMPLIFIED EXAMPLE OF DYNAMIC PROGRAMMING?

As an illustration of the general procedure and analytical methods used in dynamic programming, consider the example design problem presented in Table 10. The general procedure consists of the following steps:

1. Establish the sequence of single stages into which the process will be divided. These are shown as stages 1 to 5 in Table 10.
2. Decide on the units to be used for expressing the profit function for each individual stage and the overall process. For this case, the problem statement makes it clear that an appropriate unit for this purpose is the profit over a

TABLE 10
A dynamic-programming model for production of a new chemical with no recycle including specific data for an example



Feed: 50,000 lb/yr of raw material are fed to stage 5 of the above model of the process at a cost of $1 per pound.

The output from the 5-stage process must be at least 15,000 lb of product per year.

The overall objective function of the entire process is to optimize for a maximum profit over a five-year period. Assume equipment life period is 3 years.

a. Anticipated selling price of the product vs. annual production

Production, 1000 lb/yr	47.5	45.0	42.5	40.0	37.5	30.0	25.0	22.5	20.0	15.0
Expected selling price, $/lb	3.2	3.3	3.4	3.6	3.8	4.6	5.0	5.2	5.3	5.5

b. Operating costs for the mixing operation, $1000/yr

Mixing efficiency	1.0	0.8	0.6	0.5
Mixer *A*	12.0	6.0	3.0	2.0
B	8.0	4.0	2.5	1.5
C	5.0	3.0	2.0	1.0

c. Heater operating costs, $1000/yr

Mixing efficiency	Temperature, °F			
	650	700	750	800
1.0	0.5	1.0	6.0	10.0
0.8	1.0	1.5	8.0	12.0
0.6	1.5	2.5	10.0	16.0
0.5	2.0	3.0	12.0	20.0

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†Based on L. G. Mitten and G. L. Nemhauser, *Chem. Eng. Progr.*, 59(1):52 (1963).

TABLE 10

A dynamic-programming model for production of a new chemical with no recycle including specific data for an example (Continued)

d. Reactor, I and catalyst costs

Reactor	Initial cost, $1000	Operating cost, $1000/yr
I_A	40.0	4.0
I_B	20.0	2.0
I_C	5.0	1.0
Catalyst		
1	....	10.0
2	....	4.0

e. Percent conversion in reactor I

Temp., °F	650		700		750		800	
Catalyst	1	2	1	2	1	2	1	2
Reactor I_A	30	25	40	30	50	45	60	50
I_B	25	20	30	25	45	40	50	45
I_C	20	15	25	20	40	30	45	40

f. Total conversion from reactor I plus reactor II

Conversion in reactor	I	15	20	25	30	40	45	50	60
Second reactor	II_A	30	40	50	60	80	85	90	95
	II_B	45	60	75	85	90	95	95	95

g. Reactor II costs

Reactor	Initial cost, $1000	Operating cost, $1000/yr
II_A	60.0	10.0
II_B	80.0	20.0

h. Costs for the separation unit

	One large separator		Two small separators (cost per separator)	
% Conversion	Initial cost, $1000	Operating cost, $1000/yr	Initial cost. $1000	Operating cost, $1000/yr
30	12	2.5	7.5	1.5
40	12	3.0	7.5	1.5
45	12	4.0	7.5	1.5
50	15	4.0	9.0	1.5
60	15	5.0	9.0	2.0
75	20	6.0	12.0	2.0
80	20	6.5	12.0	2.0
85	20	7.0	12.0	2.0
90	20	7.5	12.0	2.5
95	20	8.0	12.0	2.5

i. Initial investment (4)

Mixer A.	10,000
Mixer B.	15,000
Mixer C.	25,000
Heater:	
700°F or less.	5,000
More than 700°F.	20,000

TABLE 11
Possible decisions, inputs, and outputs for the example presented in Table 10

For stage number	Decisions	Output from stage = input to next stage (expressed as relevant variable)
5	Type of mixer (that is, A, B, or C) Mixing efficiency (that is, η = 1.0, 0.8, 0.6, or 0.5)	Mixing efficiency
4	Temperature level at which heater operates (that is, 650, 700,750, or 800°F)	Temperature
3	Type of reactor (that is, I_A, I_B, or I_C) Type of catalyst (that is, 1 or 2)	Percent conversion
2	Type of reactor (that is, II_A, II_B, or none)	Percent conversion
1	Choice of separators (that is, one large or two small)	

five-year period of operation, and this unit will be used in the solution of the problem.

3. For each stage, determine the possible inputs, decisions, and outputs. These are shown in Table 11.
4. For each stage and for each combination of input decisions, establish the stage output.
5. Establish the optimal return from the overall process and from each stage by the application of the principle shown in Eq. (91).

Steps (1), (2), and (3) are completed, for the indicated example, in Tables 10 and 11. To carry out steps (4) and (5), it is necessary to assume a number of discrete levels for each of the decision variables. The size of the subdivisions for each decision variable, of course, represents an imposed constraint on the system solution, but these constraints are very useful for narrowing down the region which must receive the most careful attention for optimization.

The stage outputs are established in sequence for the subprocesses of stage 1, stage l-stage 2, stage l-stage 2-stage 3, stage l-stage 2-stage 3-stage 4, and finally stage l-stage 2-stage 3-stage 4-stage 5. The optimum is determined for each subprocess employing all the discrete levels chosen for the variables involved in that subprocess.

TABLE 12
One stage profits, $Q_1(x_2, d_1)$, \$1000

Stage 1 input, x_2, % conversion	Stage 1 decision, d_1	
	One separator	Two separators
95	700.0	**711.0***
90	685.0	**693.5***
85	667.5	**678.5***
80	667.5	**676.0***
75	662.5	**668.5***
60	650.0	652.0”
50	590.0	**592.0***
45	553.0	**555.0***
40	**503.0***	500.0
30	**388.0***	382.5

TABLE 13
Two-stage profits, $Q_2(x_3, d_2)$, \$1000

	Stage 2 decisions, d_2		
Stage 2 inpuf x_3, % conversion	Reactor II_A	Reactor II_B	No reactor II
60	601.0	531.0	**652.0***
50	583.5	531.0	**592.0***
45	**568.5***	531.0	555.0
40	**566.0***	513.5	503.0
30	**542.0***	498.5	388.0
25	482.0	**488.5***	
20	393.0	**472.0***	
15	278.0	**375.0***	

TABLE 14
Three-stage profits, $Q_3(x_4, d_3)$, \$1000

	Stage 3 decisions, d_3			
Stage 3 input, x_4, temperature	Reactor I_A	Reactor I_B	Reactor I_C	Catalyst
800	**542.0***	512.0	508.5	1
800	512.0	518.5	536.0	2
750	482.0	488.5	506.0	1
750	488.5	**516.0***	512.0	2
700	456.0	**462.0***	428.5	1
700	**462.0***	438.5	442.0	2
650	**432.0***	408.5	412.0	1
650	408.5	422.0	345.0	2

TABLE 15
Four-stage profits, $Q_4(x_5, d_4)$, \$1000

Stage 4 input, x_5, mixing efficiency	Stage 4 decisions, d_4, temperature			
	800	750	700	650
1.0	**472.0***	466.0	452.0	424.5
0.8	**462.0***	456.0	449.5	422.0
0.6	442.0	**446.0***	444.5	419.5
0.5	422.0	436.0	**442.0***	417.0

TABLE 16
Five-stage profits, $Q_5(x_F, d_5)$, \$1000

	Stage 5 decisions, d_5			
	Mixing efficiency			
Mixer	1.0	0.8	0.6	0.5
C	422.0	422.0	411.0	412.0
B	417.0	**427.0***	418.5	419.5
A	402.0	422.0	421.0	422.0

The subdivisions for the possible decisions in this example are shown by the data given in Table 10 and are summarized in Table 11. Thus, in stage 5, there are three possible decisions on the choice of mixer, and each of these has four possible efficiency decisions. In stage 4, there are four possible decisions on temperature level for the heater. In stage 3, there are three possible reactors and two possible catalysts. Stage 2 has three possible decisions of reactor II,, II,, or no reactor. In stage 1, there are two possible decisions of one large separator or two small separators. On an overall basis, therefore, the total possible modes of operation by a completely random approach would be

$$3 \times 4 \times 4 \times 3 \times 2 \times 3 \times 2 = 1728$$

By applying the technique of dynamic programming, the final optimum condition can be established by a stage-by-stage operation so that only about 15 modes of operation must be considered.

SUBPROCESS OF STAGE 1

For the dynamic-programming procedure involving only stage 1, it is desirable to base the analysis on final product sales with consideration of only the first stage. With this basis, all possible conversions of the entering stream must be considered. The data given in Table 10 show that at least 30 percent of the feed must be converted. This immediately indicates that the possibility of not including reactor II at stage 2 can only be considered if the conversion leaving reactor I is 30 percent or higher. Therefore, only those conversions of 30 percent or higher, as shown in Table 10*f*, need to be considered.

For each conversion (for example, for 50 percent conversion), the five-year profit can be evaluated for the cases of one large separator and two small separators. Therefore, using the data given in Table 10 and neglecting the cost of feed which is a constant,

Five-year profit using one large separator

$$= (5)(50{,}000)(0.5)(\$5.0) - \$15{,}000 - (5)(\$4{,}000) = \$590{,}000$$

Five-year profit using two small separators

$$= (5)(50{,}000)(0.5)(\$5.0) - (2)(\$9{,}000) - (2)(5)(\$1{,}500) = \$592{,}000$$

This indicates that the optimal operation of stage 1 with a 50 percent conversion requires the use of two small separators. These calculations are repeated for all feasible conversions, and the results [i.e., the one-stage profits $Q_1(x_2, d_1)$] are presented in Table 12 with the optimum condition for each conversion indicated by an asterisk.

SUBPROCESS OF STAGE 1-STAGE 2

This subprocess involves making a decision on the type of reactor II (II,, II,, or none). Possible conversions for the feed entering stage 2 can be established from Table 10*f* or Table 10*e* as 15, 20, 25, 30, 40, 45, 50, or 60 percent. All of these possibilities, including the decisions on conversion and reactor type, must be evaluated. Each result will give an exit conversion which represents the feed to stage 1, but the optimum condition for stage 1 has already been generated for the various feeds. Therefore, the sum of the optimum cost for stage 1 and the developed cost for stage 2 can be tabulated so that the optimum system for stage 1-stage 2 can be chosen for any appropriate feed to stage 2.

For example, if the stage-2 input conversion is chosen as 40 percent, the following data and calculations apply (neglecting cost of feed which remains constant):

Five-year profit using reactor II,

$$= \underset{\substack{\text{Optimum for}\\ \text{stage 1 with}\\ \text{80\% conversion}\\ \text{(Table 12)}}}{\$676{,}000} - \$60{,}000 - (5)(\$10{,}000) = \$566{,}000$$

Five-year profit using reactor II,

$$= \underset{\substack{\text{Optimum for}\\ \text{stage 1 with}\\ \text{90\% conversion}\\ \text{(Table 12)}}}{\$693{,}500} - \$80{,}000 - (5)(\$20{,}000) = \$513{,}500$$

Five-year profit using no reactor II = \$503,000

The preceding procedure can be repeated for all feasible combinations for the stage 1-stage 2 process, and the results [i.e., the one-stage-two-stage profits $Q_2(x_3, d_2)$] are tabulated in Table 13.

REMAINING SUBPROCESSES AND FINAL SOLUTION

The same type of optimizing procedure can now be followed for each of the remaining three subsystems, and the results are presented in Tables 14, 15, and 16 with asterisks being used to indicate the optimum sets. The final optimum for the full process can now be established directly from Table 16 as giving a five-year profit of \$427,000. The stage-wise operations should be as follows:

Stage 5: From Table 16, a type B mixer with an efficiency of 80 percent should be used.

Stage 4: From Table 15, the heater should be operated at 800°F.

Stage 3: From Tables 14 and 10, reactor I, with catalyst 1 should be used giving a 60 percent conversion.

Stage 2: From Table 13, no reactor II should be used.

Stage 1: From Table 12, two small separators should be used.

The preceding example illustrates the technique used in dynamic programming. This technique permits a great saving in the amount of computational effort involved as is illustrated by the fact that the stage-by-stage optimization approach used in the example involved consideration of only about 15 possible modes of operation. This can be compared to the total possible modes of operation of 1728 which would have had to be considered by a totally random approach.?

OTHER MATHEMATICAL TECHNIQUES AND STRATEGIES FOR ESTABLISHING OPTIMUM CONDITIONS

Many mathematical techniques, in addition to the basic approaches already discussed, have been developed for application in various situations that require determination of optimum conditions. A summary of some of the other common and more advanced mathematical techniques, along with selected references for additional information, is presented in the following:

APPLICATION OF LAGRANGE MULTIPLIERS+

When equality constraints or restrictions on certain variables exist in an optimization situation, a powerful analytical technique is the use of Lagrange multipliers. In many cases, the normal optimization procedure of setting the partial of the objective function with respect to each variable equal to zero and solving the resulting equations simultaneously becomes difficult or impossible mathematically. It may be much simpler to optimize by developing a ***Lagrange expression,*** which is then optimized in place of the real objective function.

In applying this technique, the Lagrange expression is defined as the real function to be optimized (i.e., the objective function) plus the product of the Lagrangian multiplier (λ) and the constraint. The number of Lagrangian multipliers must equal the number of constraints, and the constraint is in the form of an equation set equal to zero. To illustrate the application, consider the situation in which the aim is to find the positive value of variables x and y which make the product xy a maximum under the constraint that $x^2 + y^2 = 10$. For this simple case, the objective function is xy and the constraining equation, set equal to zero, is $x^2 + y^2 - 10 = 0$. Thus, the Lagrange expression is

$$\text{L.E.}\,(x, y) = xy + \lambda(x^2 + y^2 - 10) \tag{92}$$

[†] For additional basic information on dynamic programming, see R. E. Bellman and S. E. Dreyfus, "Applied Dynamic Programming," Princeton University Press, Princeton, N.J., 1962; R. Aris, "Discrete Dynamic Programming," Blaisdell Press, New York, 1964, S. E. Dreyfus and A. M. Law, "The Art and Theory of Dynamic Programming," Academic Press, New York, 1977; and T. F. Edgar and D. M. Himmelblau, "Optimization of Chemical Processes," McGraw-Hill Book Company, New York, 1988.

[‡] A. H. Boas, How to Use Lagrange Multipliers, *Chem. Eng.*, **70**(1):95 (1963); and T. F. Edgar and D. M. Himmelblau, "Optimization of Chemical Processes," McGraw-Hill Book Company, New York. 1988.

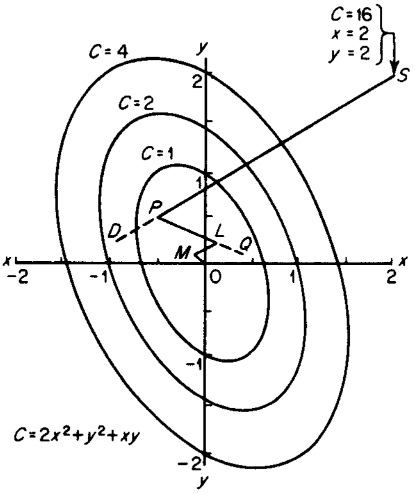

FIGURE 11-12
Method of steepest descent applied to unimodal surface.

Taking the partial of Eq. (92) with respect to x, y, and λ, and setting each result equal to zero gives

$$y + 2\lambda x = 0 \tag{93}$$

$$x + 2Ay = 0 \tag{94}$$

$$x^2 + y^2 - 10 = 0 \tag{95}$$

Simultaneous solution of the preceding three equations for x, y, and A gives, for the case where both x and y are positive, the optimum values of x equal to 2.24 and y equal to 2.24.

METHOD OF STEEPEST ASCENT OR DESCENT?

For the optimization situation in which two or more independent variables are involved, response surfaces can often be prepared to show the relationship among the variables. Figure 11-12 is an example of a unimodal response surface with a single minimum point. Many methods have been proposed for exploring such response surfaces to determine optimum conditions.

One of the early methods proposed for establishing optimum conditions from response surfaces is known as the method of steepest ascent or descent. The basis of this method is the establishment of a straight line or a two-dimensional plane which represents a restricted region of the curved surface. The gradient at the restricted region is then determined from the linearized approximation,

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†G. E. P. Box and K. B. Wilson, *J. Royal Star. Soc.*, **B13:1** (1951); W. D. Baasel, *Chem. Eng.*, **72(22):147** (1965); D. J. Wilde and C. S. Beightler, "Foundations of Optimization," Prentice-Hall, Inc., Englewood Cliffs, NJ, 1967.

and the desired direction of the gradient is established as that linear direction giving the greatest change in the function being optimized relative to the change in one or more of the independent variables. If the objective function is to be maximized, the line of steepest ascent toward the maximum is sought. For the case of a minimum as the desired objective, the approach would be by means of the steepest descent.

To illustrate the basic ideas involved, consider the case where the objective function to be minimized (C) is represented by

$$C = 2x^2 + y^2 + xy \tag{96}$$

where x and y are the independent variables. Equation (96) is plotted as a contour surface in Fig. 11-12, and the objective is to determine, by the method of steepest descent, the values of x and y which make C a minimum. Arbitrarily, a starting point of $x = 2$, $y = 2$, and $C = 16$ is chosen and designated as point S in Fig. 11-12. The gradient at point S is determined by taking the partial of C with respect to each of the independent variables to give

$$\frac{\partial C}{\partial x} = 4x + y = (4)(2) + 2 = 10 \tag{97}$$

$$\frac{\partial C}{\partial y} = 2y + x = (2)(2) + 2 = 6 \tag{98}$$

Both of these partials are positive which means that both x and y must change in the negative direction to head toward a minimum for C. The direction to be taken is established by recognizing that C must change more rapidly in the x direction than in the y direction in direct ratio to the partial derivatives. Thus, x should decrease faster than y in the ratio of (decrease in x)/(decrease in y) $= \frac{10}{6}$. Assume, arbitrarily, to decrease x linearly from point S in increments of 0.5. Then y must decrease in increments of $(0.5)\,\frac{6}{10} = 0.3$. Under these conditions, the first line of steepest descent is found as follows and is shown as line ***SD*** in Fig. 11-12.

$$\begin{array}{lll}
x_0 = 2.00 & y_0 = 2.00 & C_0 = 16.00 \\
x_1 = 1.50 & y_1 = 1.70 & C_1 = 9.94 \\
x_2 = 1.00 & y_2 = 1.40 & C_2 = 5.36 \\
x_3 = 0.50 & y_3 = 1.10 & C_3 = 2.26 \\
x_4 = 0.00 & y_4 = 0.80 & C_4 = 0.64 \\
x_5 = -0.50 & y_5 = 0.50 & C_5 = 0.50 \\
x_6 = -1.00 & y_6 = 0.20 & C_6 = 1.84
\end{array}$$

The minimum for line ***SD*** occurs at x_5, y_5; so a new line is now established using point x_5, y_5 as the starting point. Using the same procedure as was followed for finding line ***SD***, the line PQ is found with a minimum at ***L***. Thus, point ***L*** now becomes the new starting point. This same linearization procedure is repeated with each line getting closer to the true minimum of $C = 0$, $x = 0$, $y = 0$.

The method outlined in the preceding obviously can become very tedious mathematically, and a computer solution is normally necessary. The method also has limitations based on choice of scale and incremental steps for the variables, extrapolation past the region where the straight line approximates the surface, and inability to handle surfaces that are not unimodal.

EXPLORATION OF RESPONSE SURFACES BY GROUP EXPERIMENTS†

In addition to the method of steepest ascent and descent, many other strategies for exploring response surfaces which represent objective functions have been proposed. Many of these are based on making group experiments or calculations in such a way that the results allow a planned search of the surface to approach quickly a unimodal optimum point.

A typical example of an efficient search technique by group experiments is known as the Five-Point Method and is explained in the following. The basis of this method is first to select the overall range of the surface to be examined and then to determine the values of the objective function at both extremes of the surface and at three other points at equally spaced intervals across the surface. Figure 11-13 shows a typical result for these initial five points for a simplified two-dimensional case in which only one maximum or minimum is involved.

From these first five calculations, it can be seen that, by keeping the optimum point and the point on each side of it, the search area can be cut in half with assurance that the remaining area still contains the optimum value. In Fig. 11-13, the optimum is represented by the maximum profit, so the middle half of the search area is retained.

Two more calculations or experiments are then made in the remaining search area with these points again being equally spaced so that the remaining search area is again divided into four equal portions. As before, the optimum (highest profit) point is kept along with the points on each side of it, so the search area is again cut in half.

This procedure can be repeated to reduce the search area by a large amount with a relatively few calculations. For example, as shown in the following, 99.9 percent of the search area can be eliminated by a total of only 23

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†W. D. Baasel, Exploring Response Surfaces to Establish Optimum Conditions, *Chem. Eng.*, **72(22):147** (1965); D. J. Wilde and C. S. Beightler, "Foundations of Optimization," Prentice-Hall, Inc., Englewood Cliffs, NJ, 1967.

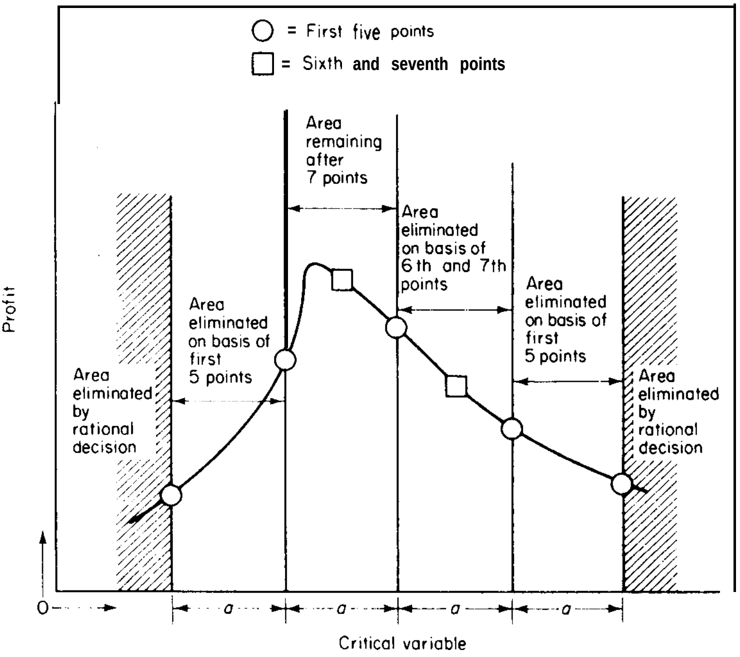

FIGURE 11-13
Illustration of Five-Point Method for group-experiment exploration of response surface.

calculations or experiments:

For step number	Number of new calculations	Total calculations	Fraction of region isolated = A
1	5	5	$\frac{1}{2}$
2	2	7	$(\frac{1}{2})^2$
3	2	9	$(\frac{1}{2})^3$
	·		·
m_s	2	$n_c = 3 + 2m_s$	$(\frac{1}{2})^{m_s} = \Delta$

$$\text{If}\quad \left(\tfrac{1}{2}\right)^{m_s} = \Delta, \quad m_s = \frac{\log \Delta}{-\log 2} = \frac{n_c - 3}{2}, \quad \text{and} \quad n_c = 3 - \frac{2 \log \Delta}{\log 2} \tag{99}$$

For the case where A is 0.001, or 99.9 percent of the surface has been eliminated, Eq. (99) gives the number of calculations needed (n_c) as 23.

A similar approach is used in the *Golden Section Search Technique* which uses as its basis a symmetrical placement of search points located at an arbitrary distance from each side of the search area.† This method can eliminate 99.9

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†See D. F. Rudd and C. C. Watson, "Strategy of Process Engineering," John Wiley & Sons, Inc., New York, 1968.

percent of the search area by a total of 17 search points as compared to 23 search points for the simple Five-Point Method.

A so-called dichotomous *search* for the optimum on a surface representing an objective function is conducted by performing the experiments or calculations in pairs. By locating the pairs at appropriate intervals over the surface, inappropriate regions can be eliminated quickly, and a sequential technique can be developed to permit rapid elimination of major portions of the surface. Similarly, the *simplex method,* based on a triangulation of experimental or calculated points, can be used to indicate the desired direction of a search. A highly effective sequential search technique, known as the *Fibonacci search* because the search sequence is based on Fibonacci numbers, can be employed when the objective function has only one optimum and is based on a single independent variable. Experimental errors involved in analyzing response surfaces can be eliminated partially by a so-called *evolutionary operations* (*EVOP*) technique based on measuring the response to the operating conditions a sufficient number of times so that the mean of the sample response approaches the true mean.

GEOMETRIC PROGRAMMING†

A technique for optimization, based on the inequality relating the arithmetic mean to the geometric mean for a set of numbers, has been called *geometric programming.* With this method, the basic idea is to start by finding the optimum way to distribute the total cost among the various factors of the objective function. This is then followed by an analysis of the optimal distribution to establish the final optimum for the objective function. Although this approach can become very involved mathematically and may involve nonlinear equations, it can handle equality and inequality constraints and can often be simpler than a direct nonlinear-programming approach.

OPTIMUM CONDITIONS FOR PRODUCTION, PLANNING, SCHEDULING, AND CONTROLS

A number of special numerical techniques have been developed for effective planning, scheduling, and control of projects. Two of these methods, critical

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†D. J. Wilde, *Ind. Eng. Chem.*, **57**(8):31 (1965); R. L. Zahradnik, "Theory and Techniques of Optimization for Practicing Engineers," Barnes & Noble, Inc., New York, 1971.

‡J. J. Moder and C. R. Phillips, "Project Management with CPM and PERT," 2d ed., Reinhold Publishing Corporation, New York, 1970; L. R. Shaffer, L. B. Ritter, and W. L. Meyer, "The Critical Path Method," McGraw-Hill Book Company, New York, 1965; W. P. Scarrah, Improve Production Efficiency via EVOP, *Chem. Eng.*, **94**(18):131 (1987).

path method (CPM) and program evaluation and review technique (PERT) have received particular attention and have shown the desirability of applying mathematical and graphical analyses to the planning and control of production processes.

The basis of both the critical path method and program evaluation and review technique is a graphical portrait, or network, showing the interdependencies of the various activities in the program leading from the initial input, or startup, to the end objective. PERT is of primary use for organizing and planning projects that involve research and development wherein the activities are usually being attempted for the first time. As a result, estimates of time, cost, and results cannot be made with accuracy, and probability and statistical concepts must be used to develop the predictions. In comparison, CPM is usually applied to projects for which relatively accurate estimations of time, cost, and results can be made, such as for construction projects.

For both CPM and PERT, the overall project is viewed as a series of activities or operations performed in an optimum sequence to reach a desired objective. Each activity is considered as having a beginning and an end so that the overall project consists of a series of these "events." The general technique, then, is to develop a mathematical model to give the best program or interrelated series of events to achieve a desired goal. The major difference in concept between CPM and PERT is that involved in estimating the time duration of activities. Thus, CPM may be relatively specific on time items, while PERT includes measures of the uncertainties involved.

When the series of activities is diagrammed, it can be seen that many possible paths exist between the "start" and the "end." The "critical path" is defined as that path involving the desired (usually shortest) duration for completion of the project. The mathematical concepts of both PERT and CPM are normally of sufficient complexity that a digital computer must be used for the solution. By the appropriate network computations, a final sequential procedure is developed which gives the "critical path" that must be followed from the "start" to the "end" to complete the job in the most efficient manner in a given duration of time.

THE STRATEGY OF ACCOUNTING FOR INFLATION IN DESIGN ESTIMATES

The method of correcting for price changes that have occurred in the past when estimating costs for design purposes has been discussed in Chap. 6 (Cost Estimation). As this discussion showed, the history of cost changes in the United States in the recent past has been strongly inflationary. For example, the Marshall and Swift All-Industry Installed-Equipment Cost Index doubled from 273 in 1968 to 545 in 1978. In the ten-year period from 1978 to 1988, the index increased by about 60 percent to 852. Other price indexes showed about the same factors of increase over these time intervals.

An effective interest rate of 7.18 percent will cause a doubling of value when compounded for 10 years while a 5 percent rate will give a 63 percent increase in 10 years and a 4 percent rate will give a 48 percent increase in 10 years. Consequently, past history of price changes in the United States would indicate that a rate of inflation of at least 3 percent and perhaps as high as 7 percent can be expected for at least the near future, and this factor should be taken into account in presenting design estimates of cost.

The critical element of the strategy for accounting for inflation in design estimates is to present the results in the form of present worth (present *value, profitability index, discounted cash flow*) with all future dollars discounted to the value of the present dollar at zero time. The discount factor must include both the interest required by the company as minimum return and the estimated interest rate of inflation. If profits on which income taxes are charged are involved, then the present worth based on the after-tax situation should be used.

In order to understand which form of discount factor to use with inflation (or with deflation), the two specific cases for constant annual income in the future and constant annual productivity in the future will be considered. In all cases, effective interest and instantaneous end-of-year cash flow will be assumed.

CASE OF CONSTANT ANNUAL INCOME IN THE FUTURE

Assume that a firm wishes to make an investment now to provide \$100,000 in cash at the end of each year for the next ten years. The firm expects to receive a 10 percent return ($i = 0.10$) on its investment irrespective of inflation effects. However, the firm also wishes to account for an assumed annual inflation of 7 percent ($i_{\text{inflation}} = 0.07$) so that the dollars its invests now are corrected for the fact that these dollars will be worth less in the future. Under these conditions, the question is how to establish the correct discount factor to determine the investment the firm needs to make at this time. In other words, what is the total present value of the future annual incomes of \$100,000 for 10 years discounted for both return on investment and inflation?

Consider the case of the first \$100,000 coming in at the end of the first year. The present value at zero time of this \$100,000 based only on the need to keep the purchasing power of the dollar constant by correcting for inflation is $(\$100{,}000)(1 + 0.07)^{-1}$ or, in general, $(\$100{,}000)(1 + i_{\text{inflation}})^{-n'}$ where n' is the year referred to. In addition, the firm demands a 10 percent direct return on the investment; so an additional discount factor of $(1 + 0.10)^{-1}$ or, in general, $(1 + i)^{-n'}$ must be applied to the annual income value to give its present value at zero time. Thus, the zero-time present value of the first \$100,000 is $(\$100{,}000) \times (1 + 0.07)^{-1}(1 + 0.10)^{-1}$. The total present value at zero time of all the annual incomes is merely the following sum:

$$\begin{array}{ll}
\text{For first year} & (\$100{,}000)(1+0.07)^{-1}(1+0.10)^{-1} \\
\quad + & \\
\text{For second year} & (\$100{,}000)(1+0.07)^{-2}(1+0.10)^{-2} \\
\quad + & \\
\text{For third year} & (\$100{,}000)(1+0.07)^{-3}(1+0.10)^{-3} \\
\quad + & \quad\quad + \\
\quad \cdot & \quad\quad \cdot \\
\quad + & \quad\quad + \\
\text{For tenth year} & (\$100{,}000)(1+0.07)^{-10}(1+0.10)^{-10}
\end{array}$$

or, in general

$$\text{The total present value} = \sum^{10} (\$100{,}000)\,(1+i_{\text{inflation}})^{-n'}(1+i)^{-n'}$$

The effective discount factor including both inflation and required return on investment is $[(1+i_{\text{inflation}})(1+i)]^{-n'}$ or $[1+i+i_{\text{inflation}}+(i_{\text{inflation}})(i)]^{-n'}$. Consequently, the effective combined interest (i_{comb}) including both inflation interest and required return on investment is

$$i_{\text{comb}} = i + i_{\text{inflation}} + (i_{\text{inflation}})(i) \tag{100}$$

The preceding situation, of course, is merely a case of an ordinary annuity (R = \$100,000 each year) at an interest rate of i_{comb} so that Eq. (24) of Chap. 7 (Interest and Investment Costs) applies as follows:

$$\text{Present value} = R\,\frac{(1+i_{\text{comb}})^n - 1}{i_{\text{comb}}(1+i_{\text{comb}})^n} \tag{101}$$

For the example under consideration, R = annual periodic payment = \$100,000, n = total life period = 10 years, i_{comb} = 0.10 + 0.07 + (0.07)(0.10) = 0.177, and present value (or necessary investment now) = \$100,000[(1 + 0.177)” − 1]/ 0.1770 + 0.177)” = \$452,240.

CASE OF CONSTANT ANNUAL PRODUCTIVITY IN THE FUTURE

For the typical situation of an industrial operation which has been designed to produce a set number of units per year which will be sold at the prevailing price, there would be no special problem with handling inflation except for the influence of income taxes. If the inflationary costs are considered as having the same effects on the selling price of the product as on the costs for the operation, then return on investment before taxes is the same whether or not inflation is taken into account. However, as illustrated by the following example, when income taxes are included in the analysis, the return on investment changes if inflation is taken into account. This is due to the fact that depreciation costs are not changed by inflation in normal accounting procedures.

Example 7 Return on investment before and after taxes with and without inflation. An investment of $1,000,000 will give annual returns as shown in the following over a life of five years. Assume straight-line depreciation, negligible salvage value, and 34 percent income taxes. What is the discounted-cash-flow rate of return on the investment (Profitability Index) before and after taxes with

(*a*) No inflation and annual returns of $300,000 each year (i.e., cash flow to the company of $300,000) before taxes?

(*b*) Inflation rate of 7 percent ($i_{\text{inflation}} = 0.07$) and a situation where the increase in profits due to inflation is also at an annual rate of 7 percent so the annual returns remain at the equivalent of $300,000 in zero-time dollars before taxes?

Solution

(*a*) For the case of no inflation, Eq. (24) of Chap. 7 [or Eq. (101) of this chapter with $i_{\text{comb}} = i$] applies as follows:

$$\text{Present value} = R\frac{(1+i)^n - 1}{i(1+i)^n}$$

For return on investment (i) before taxes,

$$\$1{,}000{,}000 = \$300{,}000\frac{(1+i)^5 - 1}{i(1+i)^5}$$

By trial and error, or by use of tables of $[(1+i)^n - 1]/i(1+i)^n$,

$$i = 0.152 \text{ or } 15.2\% \text{ return}$$

For return on investment (i) after taxes,

$$\text{Depreciation} = \frac{\$1{,}000{,}000 - 0}{5} = \$200{,}000 \text{ per year}$$

Taxable income = $300,000 − $200,000 = $100,000 per year

Taxes at 34% rate = $34,000 per year

Annual cash flow = $300,000 − $34,000 = $266,000

$$\$1{,}000{,}000 = \$266{,}000\frac{(1+i)^5 - 1}{i(1+i)^5}$$

$$i = 0.1033 \text{ or } 10.33\% \text{ return}$$

(*b*) For the case of 7% inflation

Year n'	Before-tax annual return based on zero-time dollars $	Actual dollars received at 7% inflation = 300,000 $(1 + 0.07)^{n'}$ $	Depreciation $	Taxable income $	Income tax at 34% $	After-tax annual cash flow $
1	300,000	321,000	200,000	121,000	41,140	279,860
2	300,000	343,470	200,000	143,470	48,780	294,690
3	300,000	367,513	200,000	167,513	56,954	310,559
4	300,000	393,239	200,000	193,239	65,701	327,538
5	300,000	420,766	200,000	220,766	75,060	345,706

For return on investment (i) before taxes, the actual annual return based on zero-time dollars is $300,000; so the return on the investment is exactly the same as for the case of no inflation, and i = 0.152 or 15.2% return.

For return on investment (i) after taxes, the annual cash flows based on zero-time dollars must have a total present value of **$1,000,000.** As is shown in the following tabulation, this occurs for a value of i = 0.0859:

Year n'	After-tax annual cash flow *A* in $	Inflation adjustment. After-tax annual cash flow based on zero-time dollars $= A\,(1 + 0.07)^{-n'}$ *B* in $	Inflation plus return adjustment. Present value at i = 0.0859 (8.5%) $= B\,(1 + 0.0859)^{-n'}$ $
1	279,860	261,551	240,853
2	294,690	257,394	218,268
3	310,559	253,518	197,971
4	327,538	249,876	179,687
5	345,706	246,475	163,221
		Total present value =	$1,000,000

Under these conditions, the return on investment after taxes with a 7% inflation rate is i = 0.0859 or 8.59% return.

Thus, as would be expected because profits for the inflation case increased at the same rate as the inflation, the before-tax return on the investment was the same for the cases with or without inflation at 15.2%. However, due to the depreciation costs remaining constant in the case of inflation, the after-tax return on the investment was different for the no-inflation case (10.33%) and the inflation case (8.59%).

The preceding example clearly shows that inflation effects can be important in determining returns on investment. The best strategy for handling such effects is to use the discounted-cash-flow or present-worth method for reporting returns on investment with the results based on the after-tax situation. This method of reporting can be handled easily and effectively by use of an appropriately arranged table for the presentation such as Table 2 in Chap. 10 (Profitability, Alternative Investments, and Replacements).?

NOMENCLATURE FOR CHAPTER 11

a = constant, or depreciation factor for installed piping system [See Eq. (82) for definition of a_i]

a' = depreciation factor for pumping installation

A = heat-transfer area, ft^2

b = constant, or maintenance factor for installed piping system [See Eq. (82) for definition of b_i]

b' = maintenance factor for pumping installation

B = constant

B' = constant

c = constant

c_i = objective-function coefficient for solution variable

c_j = objective-function coefficient for row in simplex algorithm matrix

c_p = heat capacity, Btu/(lb)(°F)

c_{pipe} = purchase cost of new pipe per foot of pipe length, \$/ft

c_T = total cost per unit of production, \$/unit of production

C = cost, or objective function

C_A = installed cost of heat exchanger per square foot of heat-transfer area, $\$/\text{ft}^2$

C_c = cost for one cleaning, dollars

C_F = fixed costs, \$/year

C_{pipe} = installed cost for piping system expressed as dollars per year per foot of pipe length, \$/(year)(ft)

C_{pumping} = pumping cost as dollars per year per foot of pipe length when flow is turbulent, \$/(year)(ft)

C'_{pumping} = pumping cost as dollars per year per foot of pipe length when flow is viscous, \$/(year)(ft)

C_T = total cost for a given unit of time, dollars

C_w = cost of cooling water, \$/lb

d = constant, or derivative, or design decision for dynamic programming

D = inside diameter of pipe, ft, or determinant

D_i = inside diameter of pipe, in.

E = efficiency of motor and pump expressed as a fraction

f = Fanning friction factor, dimensionless, or function for dynamic programming in Eq. (91) indicating optimum return depends on that input

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†For further discussion on this topic, see W. H. Griest, Jr., Making Decisions in an Inflationary Environment, *Chem. Eng. Progr.*, **75**(6):13 (1979).

F = ratio of total cost for fittings and installation to purchase cost for new pipe

g = function

g_c = conversion factor in Newton's law of motion, 32.17 ft lbm/(s)(s)lbf

h = operating costs which remain constant per unit of production, \$/unit of production, or function

i = annual effective interest rate of return, percent/100

i_{comb} = annual effective interest rate of change combining regular return and inflation estimate, percent/100

$i_{\text{inflation}}$ = annual effective interest rate of change based on inflation estimate, percent/100

H = total time used for actual operation, emptying, cleaning, and recharging, h

H' = total time available for operation, emptying, cleaning, and recharging, h

H_y = total time of operation per year, h/year

I = row

j = constant

J = frictional loss due to fittings and bends, expressed as equivalent fractional loss in a straight pipe, or column

k = designation for column in simplex algorithm matrix with maximum value of $c_j - z_j$

K = cost of electrical energy, \$/kWh

K_F = annual fixed charges including maintenance, expressed as a fraction of the initial cost for the completely installed equipment

L = length of pipe, ft

L'_e = frictional loss due to fittings and bends, expressed as equivalent pipe length in pipe diameters per unit length of pipe

m = constant, or number of independent equations

m_s = number of steps

it4 = ratio of total cost for pumping installation to yearly cost of pumping power required, \$/\$

M_B = bottoms flow rate, mol/h

M_D = distillate flow rate, mol/h

n = constant, estimated service life, or number of unknowns or stages

n' = year of project life to which cash flow applies

n_c = number of calculations

N_{Re} = Reynolds number = $DV\rho/\mu$, dimensionless

O_c = organization costs per unit of time, \$/day

P = rate of production, units of production/day, or return as objective function in dynamic programming

P_b = amount of production per batch, lb/batch

P_f = filtrate delivered in, filtering time θ_f h, ft^3

P_o = optimum rate of production, units of production/day

q = rate of heat transfer, Btu/h
q_f = rate of fluid flow, ft^3/s
q_r = rate of heat transfer in reboiler, Btu/h
Q = total amount of heat transferred in a given time, Btu
Q_H = total amount of heat transferred in H h, Btu
Q_i = function for dynamic programming indicating combined return from all stages
r = profit per unit of production, \$/unit of production
R = annual periodic payment in ordinary annuity, \$/year
R' = profit per unit of time, \$/day
s = selling price per unit of production, \$/unit of production
S = slack variable
S_b = direct labor cost per hour during operation, \$/h
t = temperature, °F
t_1 = temperature of cooling water entering condenser, °F
t_2 = temperature of cooling water leaving condenser, °F
t' = condensation temperature, °F
U = overall coefficient of heat transfer, Btu/(h)(ft^2)(°F)
V = average linear velocity, ft/s
V_A = volume of A, gal
V_B = volume of B, gal
V_W = volume of water, gal
w = flow rate, lb/h
w_m = thousands of pounds mass flowing per hour, 1000 lb/h
w_s = pounds mass flowing per second, lb/s
x = a variable
X = purchase cost for new pipe per foot of pipe length if pipe diameter is 1 in., \$/ft
X' = purchase cost for new pipe per foot of pipe length if pipe diameter is 1 ft, \$/ft
y = a variable
Y = days of operation per year, days/yr
z = a variable
z_j = objective-function row coefficient component for simplex algorithm matrix defined as $\sum_{j=1}^{m} c_i a_{ij}$ $(j = 1, 2, \ldots, n)$
Z = fractional rate of return on incremental investment

Greek symbols

α = symbol meaning go to next starting point
A = fraction of search area eliminated
At = temperature-difference driving force (subscript lm designates log mean), °F
θ_b = time in operation, h or h/cycle
θ_c = time for emptying, cleaning, and recharging per cycle, h/cycle
θ_f = filtering time, h or h/cycle

θ_t = total time per complete cycle, h/cycle
A = Lagrangian multiplier
μ = absolute viscosity, lbm/(s)(ft)
μ_c = absolute viscosity, cP
ρ = density, $\mathrm{lbm/ft^3}$
$\phi, \phi^i, \phi^{ii}, \phi^{iii}$ = function of the indicated variables, or fractional factor for rate of taxation

PROBLEMS

1. A multiple-effect evaporator is to be used for evaporating 400,000 lb of water per day from a salt solution. The total initial cost for the first effect is $18,000, and each additional effect costs $15,000. The life period is estimated to be 10 years, and the salvage or scrap value at the end of the life period may be assumed to be zero. The straight-line depreciation method is used. Fixed charges minus depreciation are 15 percent yearly based on the first cost of the equipment. Steam costs $1.50 per 1000 lb. Annual maintenance charges are 5 percent of the initial equipment cost. All other costs are independent of the number of effects. The unit will operate 300 days per year. If the pounds of water evaporated per pound of steam equals 0.85 x number of effects, determine the optimum number of effects for minimum annual cost.
2. Determine the optimum economic thickness of insulation that should be used under the following conditions: Saturated steam is being passed continuously through a steel pipe with an outside diameter of 10.75 in. The temperature of the steam is 400°F, and the steam is valued at $1.80 per 1000 lb. The pipe is to be insulated with a material that has a thermal conductivity of 0.03 $\mathrm{Btu/(h)(ft^2)(°F/ft)}$. The cost of the installed insulation per foot of pipe length is $4.5 x I_t, where I_t is the thickness of the insulation in inches. Annual fixed charges including maintenance amount to 20 percent of the initial installed cost. The total length of the pipe is 1000 ft, and the average temperature of the surroundings may be taken as 70°F. Heat-transfer resistances due to the steam film, scale, and pipe wall are negligible. The air-film coefficient at the outside of the insulation may be assumed constant at 2.0 $\mathrm{Btu/(h)(ft^2)(°F)}$ for all insulation thicknesses.
3. An absorption tower containing wooden grids is to be used for absorbing SO, in a sodium sulfite solution. A mixture of air and SO, will enter the tower at a rate of 70,000 $\mathrm{ft^3/min}$, temperature of 250°F, and pressure of 1.1 atm. The concentration of SO_2 in the entering gas is specified, and a given fraction of the entering SO, must be removed in the absorption tower. The molecular weight of the entering gas mixture may be assumed to be 29.1. Under the specified design conditions, the number of transfer units necessary varies with the superficial gas velocity as follows:

$$\text{Number of transfer units} = 0.32G_s^{0.18}$$

where G_s is the entering gas velocity as $\mathrm{lb/(h)(ft^2)}$ based on the cross-sectional area of the empty tower. The height of a transfer unit is constant at 15 ft. The cost for the installed tower is $1 per cubic foot of inside volume, and annual tied charges amount to 20 percent of the initial cost. Variable operating charges for the absorbent, blower power, and pumping power are represented by the following equation:

$$\text{Total variable operating costs as } \$/\text{h} = 1.8G_s^2 \times 10^{-8} + \frac{81}{G_s} + \frac{4.8}{G_s^{0.8}}$$

The unit is to operate 8000 h/year. Determine the height and diameter of the absorption tower at conditions of minimum annual cost.

4. Derive an expression for the optimum economic thickness of insulation to put on a flat surface if the annual fixed charges per square foot of insulation are directly proportional to the thickness, (a) neglecting the air film, (*b*) including the air film. The air-film coefficient of heat transfer may be assumed as constant for all insulation thicknesses.

5. A continuous evaporator is operated with a given feed material under conditions in which the concentration of the product remains constant. The feed rate at the start of a cycle after the tubes have been cleaned has been found to be 5000 kg/h. After 48 h of continuous operation, tests have shown that the feed rate decreases to 2500 kg/h. The reduction in capacity is due to true scale formation. If the down time per cycle for emptying, cleaning, and recharging is 6 h, how long should the evaporator be operated between cleanings in order to obtain the maximum amount of product per 30 days?

6. A solvent-extraction operation is carried out continuously in a plate column with gravity flow. The unit is operated 24 h/day. A feed rate of 1500 ft^3/day must be handled 300 days per year. The allowable velocity per square foot of cross-sectional tower area is 40 ft^3 of combined solvent and charge per hour. The annual fixed costs for the installation can be predicted from the following equation:

$$C_F = 8800F_{sf}^2 - 51{,}000F_{sf} + 110{,}000 \ \$/\text{year}$$

where F_{sf} = cubic feet of solvent per cubic foot of feed. Operating and other variable costs depend on the amount of solvent that must be recovered, and these costs are \$0.04 for each cubic foot of solvent passing through the tower. What tower diameter should be used for optimum conditions of minimum total cost per year?

7. Prepare a plot of optimum economic pipe diameter versus the flow rate of fluid in the pipe under the following conditions:
Costs and operating conditions ordinarily applicable in industry may be used.
The flow of the fluid may be considered as in the turbulent range.
The viscosity of the fluid may range from 0.1 to 20 centipoises.
The plot is to apply for steel pipe.
Express the diameters in inches and use inside diameters.
The plot should cover a diameter range of 0.1 to 100 in.
Express the flow rate in 1000 lb/h.
The plot should cover a flow-rate range of 10 to 100,000 lb/h.
The plot should be presented on log-log coordinates.
One line on the plot should be presented for each of the following fluid densities: 100, 50, 10, 1, 0.1, 0.01, and 0.001 lb/ft^3.

8. For the conditions indicated in Prob. 7, prepare a log-log plot of fluid velocity in feet per second versus optimum economic pipe diameter in inches. The plot should cover a fluid-velocity range of 1 to 100 ft/s and a pipe-diameter range of 1 to 10 in.

9. A continuous evaporator is being used to concentrate a scale-forming solution of sodium sulfate in water. The overall coefficient of heat transfer decreases according to the following expression:

$$\frac{1}{U^2} = 8 \times 10^{-6}\theta_b + 6 \times 10^{-6}$$

where U = overall coefficient of heat transfer, Btu/(h)(ft^2)(°F), and θ_b = time in operation, h.

The only factor which affects the overall coefficient is the scale formation. The liquid enters the evaporator at the boiling point, and the temperature and heat of vaporization are constant. At the operating conditions, 990 Btu are required to vaporize 1 lb of water, the heat-transfer area is 400 ft^2, and the temperature-difference driving force is 70°F. The time required to shut down, clean, and get back on stream is 4 h for each shutdown, and the total cost for this cleaning operation is $100 per cycle. The labor costs during operation of the evaporator are $20 per hour. Determine the total time per cycle for minimum total cost under the following conditions:

(a) An overall average of 65,000 lb of water per 24-h day must be evaporated during each 30-day period.

(*b*) An overall average of 81,000 lb of water per 24-h day must be evaporated during each 30-day period.

1LO. An organic chemical is produced by a batch process. In this process, chemicals *X* and Y react to form chemical Z. Since the reaction rate is very high, the total time required per batch has been found to be independent of the amounts of the materials, and each batch requires 2 h, including time for charging, heating, and dumping. The following equation shows the relation between the pounds of Z produced (lb,) and the pounds of X (lb,) and Y (lb,) supplied:

$$\text{lb}, = 1.5(1.1 \text{ lb}, \text{ lbz} + 1.3 \text{ lb}, \text{ lb}, - \text{lb}_X \text{ lb}_Y)^{0.5}$$

Chemical X costs $0.09 per pound. Chemical Y costs $0.04 per pound. Chemical Z sells for $0.80 per pound. If one-half of the selling price for chemical Z is due to costs other than for raw materials, what is the maximum profit obtainable per pound of chemical Z?

11. Derive an expression similar to Eq. (56) for finding the optimum exit temperature of cooling water from a heat exchanger when the temperature of the material being cooled is not constant. Designate the true temperature-difference driving force by $F_G \Delta t_{lm}$, where F_G is a correction factor with value dependent on the geometrical arrangement of the passes in the exchanger. Use primes to designate the temperature of the material that is being cooled.

12. Under the following conditions, determine the optimum economic thickness of insulation for a $1\frac{1}{2}$-in. standard pipe carrying saturated steam at 100 psig. The line is in use continuously. The covering specified is light carbonate magnesia, which is marketed in whole-number thicknesses only (i.e., 1 in., 2 in., 3 in., etc.). The cost of the installed insulation may be approximated as $20 per cubic foot of insulation. Annual fixed charges are 20 percent of the initial investment, and the heat of the steam is valued at $1.50 per 1 million Btu. The temperature of the surroundings may be assumed to be 80°F.

L. B. **McMillan,** *Trans. ASME,* **48:1269 (1926),** has presented approximate values of optimum economic insulation thickness versus the group $(kb_c H_y\ \Delta t/a_c)^{0.5}$, with pipe size as a parameter.

k = thermal conductivity of insulation, Btu/(h)(ft^2)(°F/ft)

b_c = cost of heat, \$/Btu

H_y = hours of operation per year, h/year

At = overall temperature-difference driving force, °F

a, = cost of insulation, \$/(ft^3)(year)

The following data are based on the results of **McMillan,** and these data are applicable to the conditions of this problem:

$\left(\frac{kb_c H_y \Delta t}{a_c}\right)^{0.5}$	Optimum economic thickness of insulation, in., for nominal pipe diameter of			
	$\frac{1}{2}$ in.	1 in.	2 in.	4 in.
0.1	....	0.40	0.5	0.6
0.2	0.80	0.95	1.1	1.3
0.3	1.20	1.4	1.6	1.9
0.5	1.85	2.1	2.45	2.9
0.8	2.75	3.1	3.6	4.3
1.2	3.80	4.3	4.9	

13. A catalytic process uses a catalyst which must be regenerated periodically because of reduction in conversion efficiency. The cost for one regeneration is constant at \$800. This figure includes all shutdown and startup costs, as well as the cost for the actual regeneration. The feed rate to the reactor is maintained constant at 150 lb/day, and the cost for the feed material is \$2.50 per pound. The daily costs for operation are \$300, and fixed charges plus general overhead costs are \$100,090 per year. Tests on the catalyst show that the yield of product as pounds of product per pound of feed during the first day of operation with the regenerated catalyst is 0.87, and the yield decreases as $0.87/(\theta_D)^{0.25}$, where θ_D is the time in operation expressed in days. The time necessary to shut down the unit, replace the catalyst, and start up the unit is negligible. The value of the product is \$14.08 per pound, and the plant operates 300 days per year. Assuming no costs are involved other than those mentioned, what is the maximum annual profit that can be obtained under these conditions?

14. Derive the following equation for the optimum outside diameter of insulation on a wire for maximum heat loss:

$$D_{\text{opt}} = \frac{2k_m}{(h_c + h,),}$$

where k_m is the mean thermal conductivity of the insulation and $(h_c + h_r)_c$ is the combined and constant surface heat-transfer coefficient. The values of k_m and $(h_c + h_r)_c$ can be considered as constants independent of temperature level and insulation thickness.

15. Derive Eq. (49) for the optimum economic pipe diameter and compare this to the equivalent expression presented as Eq. (5-90) in J. H. Perry and C. H. Chilton, ed., “Chemical Engineers’ Handbook,” 5th ed., p. 5-32, McGraw-Hill Book Company, New York, 1973.

16. Using a direct partial derivative approach for the objective function, instead of the Lagrangian multiplier as was used in Eqs. (92) to (95), determine the optimum values of x and y involved in Eqs. (92) to (95).

1 I. Find the values of x, y, and z that minimize the function $x + 2y^2 + z^2$ subject to the constraint that $x + y + z = 1$, making use of the Lagrangian multiplier.

18. For the mixing problem referred to in Tables 3 and 4 of this chapter, present the computer solution as
    (*a*) The computer diagram (similar to Table 4) based on the logic given in Table 3.
    (*b*) The computer program (Fortran language preferred).
    (*c*) The printout of the computer solution giving the minimum value of the objective function and the corresponding values of the variables.
    (*d*) The interpretation of the computer solution.

19. For the linear-programming example problem presented in this chapter where the simultaneous-equation solution is presented in Table 5, solve the problem using the simplex algorithm as was done in the text for the example solved in Fig. 11-10. Use as the initial feasible starting solution the case of solution 2 in Table 5 where $x_2 = S_4 = 0$. Note that this starting point should send the solution directly to the optimum point (solution 6) for the second trial.

20. From the data given for the dynamic-programming problem in Table 10 and the appropriate data from Table 13, show how the value of 462 was obtained in Table 14 for 700°F, Reactor I_B, and Catalyst 1.

21. Using the method outlined for steepest descent in Eqs. (96) to (98) and presented in Fig. 11-12, what would be the minimum value of C along the first line of steepest descent if the initial point had been chosen arbitrarily as x = 2 and y = 3 with x decreasing in increments of 0.5?

22. In order to continue the operation of a small chemical plant at the same capacity, it will be necessary to make some changes on one of the reactors in the system. The decision has been made by management that the unit must continue in service for the next 12 years and the company policy is that no unnecessary investments are made unless at least an 8 percent rate of return (end-of-year compounding) can be obtained. Two possible ways for making a satisfactory change in the reactor are as follows:
    (1) Make all the critical changes now at a cost of \$5800 so the reactor will be satisfactory to use for 12 years.
    (2) Make some of the changes now at a cost of \$5000 which will permit operation for 8 years and then make changes costing \$2500 to permit operation for the last 4 years.
    (*a*) Which alternative should be selected if no inflation is anticipated over the next 12 years?
    (*b*) Which alternative should be selected if inflation at a rate of 7 percent (end-of-year compounding) is assumed for all future costs?

CHAPTER 12

MATERIALS AND FABRICATION SELECTION

Any engineering design, particularly for a chemical process plant, is only useful when it can be translated into reality by using available materials and fabrication methods. Thus, selection of materials of construction combined with the appropriate techniques of fabrication can play a vital role in the success or failure of a new chemical plant or in the improvement of an existing facility.

MATERIALS OF CONSTRUCTION

As chemical process plants turn to higher temperatures and flow rates to boost yields and throughputs, selection of construction materials takes on added importance. This trend to more severe operating conditions forces the chemical engineer to search for more dependable, more corrosion-resistant materials of construction for these process plants, because all these severe conditions intensify corrosive action. Fortunately, a broad range of materials is now available for corrosive service. However, this apparent abundance of materials also complicates the task of choosing the "best" material because, in many cases, a number of alloys and plastics will have sufficient corrosion resistance for a particular application. Final choice cannot be based simply on choosing a suitable material from a corrosion table but must be based on a sound economic analysis of competing materials.

The chemical engineer would hardly expect a metallurgist to handle the design and operation of a complex chemical plant. Similarly, the chemical engineer cannot become a materials specialist overnight. But a good metallur-

TABLE 1
Comparison of purchased cost for metal plate

Material	Ratio $= \dfrac{\text{cost per pound for metal}}{\text{cost per pound for steel}}$
Flange quality steel†	1
304 stainless-steel-clad steel	5
316 stainless-steel-clad steel	6
Aluminum (99 plus)	6
304 stainless steel	7
Copper (99.9 plus)	7
Nickel-clad steel	8
Monel-clad steel	8
Inconel-clad steel	9
316 stainless steel	10
Monel	10
Nickel	12
Inconel	13
Hastelloy C	15

†Purchased cost for steel plate (January, 1990) can be approximated as 36 to 70 cents per pound, depending on the amount purchased.

gist must have a working knowledge of the chemical plant environment in which the recommendations will be applied. In the same manner, the chemical engineer should also understand something of the materials that make the equipment and processes possible.

The purpose of this chapter is to provide the process designer with a working knowledge of some of the major forms and types of materials available, what they offer, and how they are specified. With this background, the engineer can consult a materials specialist at the beginning of the design, not when the mistakes already have been made.

METALS

Materials of construction may be divided into the two general classifications of metals and *nonmetals*. Pure metals and metallic alloys are included under the first classification. Table 1 presents data showing the comparison of purchased cost for various types of metals in plate form.

Iron and Steel

Although many materials have greater corrosion resistance than iron and steel, cost aspects favor the use of iron and steel. As a result, they are often used as materials of construction when it is known that some corrosion will occur. If this is done, the presence of iron salts and discoloration in the product can be expected, and periodic replacement of the equipment should be anticipated.

In general, cast iron and carbon steel exhibit about the same corrosion resistance. They are not suitable for use with dilute acids, but can be used with many strong acids, since a protective coating composed of corrosion products forms on the metal surface.

Because of the many types of rolled and forged steel products used in industry, basic specifications are needed to designate the various types. The American Iron and Steel Institute (AISI) has set up a series of standards for steel products.? However, even the relatively simple product descriptions provided by AISI and shown in Table 2 must be used carefully. For instance, the AISI 1020 carbon steel does not refer to all 0.20 percent carbon steels. AISI 1020 is part of the numerical designation system defining the chemical composition of certain "standard steels" used primarily in bar, wire, and some tubular steel products. The system almost never applies to sheets, strip, plates, or structural material. One reason is that the chemical composition ranges of standard steels are unnecessarily restrictive for many applications.

Carbon steel plates for reactor vessels are a good example. This application generally requires a minimum level of mechanical properties, weldability, formability, and toughness as well as some assurance that these properties will be uniform throughout. A knowledge of the detailed composition of the steel alone will not assure that these requirements are met. Even welding requirements for plate can be met with far less restrictive chemical compositions than would be needed for the same type of steel used in bar stock suitable for heat treating to a minimum hardness or tensile strength.

Stainless Steels

There are more than 100 different types of stainless steels. These materials are high chromium or high nickel-chromium alloys of iron containing small amounts of other essential constituents. They have excellent corrosion-resistance and heat-resistance properties. The most common stainless steels, such as type 302 or type 304, contain approximately 18 percent chromium and 8 percent nickel, and are designated as 18-8 stainless steels.

The addition of molybdenum to the alloy, as in type 316, increases the corrosion resistance and high-temperature strength. If nickel is not included, the low-temperature brittleness of the material is increased and the ductility and pit-type corrosion resistance are reduced. The presence of chromium in the alloy gives resistance to oxidizing agents. Thus, type 430, which contains chromium but no nickel or molybdenum, exhibits excellent corrosion resistance to nitric acid and other oxidizing agents.

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†Specifications and codes on materials have also been established by the Society of Automotive Engineers (SAE), the American Society of Mechanical Engineers (ASME), and the American Society for Testing Materials (ASTM).

TABLE 2
AISI standard steels?
(XX's indicate nominal carbon content within range)

Carbon steels AISI series designations	Nominal composition or range‡
10XX	Non-resulfurized carbon steels with 44 compositions ranging from 1008 to 1095. Manganese ranges from 0.30 to 1.65%; if specified, silicon is 0.10 max. to 0.30 max., each depending on grade. Phosphorus is 0.040 max., sulfur is 0.050 max.
11XX	Resulfurized carbon steels with 15 standard compositions. Sulfur may range up to 0.33%, depending on grade.
B11XX	Acid Bessemer resulfurized carbon steels with 3 compositions. Phosphorus generally is higher than 11XX series.
12xx	Rephosphorized and resulfurized carbon steels with 5 standard compositions. Phosphorus may range up to 0.12% and sulfur up to 0.35%, depending on grade.
13xx	Manganese, 1.75%. Four compositions from 1330 to 1345.
4oxx	Molybdenum, 0.20 or 0.25%. Seven compositions from 4012 to 4047.
41XX	Chromium, to 0.95%, molybdenum to 0.30%. Nine compositions from 4118 to 4161.
43xx	Nickel, 1.83%, chromium to 0.80%, molybdenum, 0.25%. Three compositions from 4320 to E4340.
44xx	Molybdenum, 0.53%. One composition, 4419
46XX	Nickel to 1.83%, molybdenum to 0.25%. Four compositions from 4615 to 4626.
47xx	Nickel, 1.05%, chromium, 0.45%, molybdenum to 0.35%. Two compositions, 4718 and 4720
48XX	Nickel, 3.50%, molybdenum, 0.25%. Three compositions from 4815 to 4820.
50XX	Chromium, 0.40%. One composition, 5015
51xx	Chromium to 1.00%. Ten compositions from 5120 to 5160.
sxxxx	Carbon, 1.04%, chromium to 1.45%. Two compositions, 51100 and 52100.
61XX	Chromium to 0.95%, vanadium to 0.15% min. Two compositions, 6118 and 6150.
86XX	Nickel, 0.55%, chromium, 0.50%, molybdenum, 0.20%. Twelve compositions from 8615 to 8655.
87XX	Nickel, 0.55%, chromium, 0.50%, molybdenum, 0.25%. Two compositions, 8720 and 8740
88XX	Nickel, 0.55%, chromium, 0.50%, molybdehum, 0.35%. One composition, 8822
92XX	Silicon, 2.00%. Two compositions, 9255 and 9260.
50BXX	Chromium to 0.50%, also containing boron. Four compositions from 50B44 to 50B60.
51BXX	Chromium to 0.80%, also containing boron. One composition, 51B60.
81BXX	Nickel, 0.30%, chromium, 0.45%, molybdenum, 0.12%, also containing boron. One composition, 81B45.
94BXX	Nickel, 0.45%, chromium, 0.40%, molybdenum, 0.12%, also containing boron. Two compositions, 94B17 and 94B30.

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†When a carbon or alloy steel also contains the letter L in the code, it contains from 0.15 to 0.35 percent lead as a free-machining additive, i.e., 12L14 or 41L40. The prefix E before an alloy steel, such as E4340, indicates the steel is made only by electric furnace. The suffix H indicates an alloy steel made to more restrictive chemical composition than that of standard steels and produced to a measured and known hardenability requirement, e.g., 86308 or 94B30H.

‡For a detailed listing of nominal composition or range, see "Chemical Engineers' Handbook," 6th ed., McGraw-Hill Book Company, New York, 1984.

TABLE 3
Classification of stainless steels by alloy content and microstructure

Stainless steels	Chromium types	Martensitic	Hardenable (Types 403, 410, 414, 416, 416Se, 420, 431, 440A, 440B, 440C)
		Ferritic	Nonhardenable (Types 405, 430, 430F, 430Se, 442, 446)
	Chromium-nickel types	Austenitic	Nonhardenable, except by cold working (Types 201, 202, 301, 302, 302B, 303, 303Se, 304, 304L, 305, 308, 309, 309S, 310, 310S, 314, 316, 316L, 317, 321, 347, and 348)
			Strengthened by aging or precipitation hardening (Types 17-14 CuMo, 17-10P, HNM)
		Semiaustenitic	Precipitation-hardening (PH 15-7 Mo, 17-7 PH, AM 355)
		Martensitic	Precipitation-hardening (17-4 PH, 15-5 PH, Stainless W)

Although fabricating operations on stainless steels are more difficult than on standard carbon steels, all types of stainless steel can be fabricated successfully.? The properties of types 430F, 416, 410, 310, 309, and 303 make these materials particularly well suited for machining or other fabricating operations. In general, machinability is improved if small quantities of phosphorus, selenium, or sulfur are present in the alloy.

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†For a detailed discussion of machining and fabrication of stainless steels, see Selection of Stainless Steels, *Bulletin* OLE 11366, Armco Steel Corporation, Middeltown, Ohio 45042; and Fabrication of Stainless Steel, *Bulletin* 031478, United States Steel Corporation, Pittsburgh, Pa. 15230.

TABLE 4

Stainless steels most commonly used in the chemical process industries?

	Composition, %						
Type§	Cr	Ni	C max	Other significant **elements‡**	Major characteristics	Properties	Applications
301	**16.00–** 18.00	**6.00–** 8.00	0.15		High work-hardening rate combines cold-worked high strength with good ductility.	Good structural qualities.	Structural applications, bins and containers
302	**17.00–** 19.00	8.00- 10.00	0.15		Basic, general purpose austenitic type with good corrosion resistance and mechanical properties.	General purpose.	Heat exchangers, towers, tanks, pipes, heaters, general chemical equipment
303	17.00- 19.00	8.00- 10.00	0.15	s 0.15 min	Free machining modification of type 302; contains extra sulfur.	Type 303Se is **also** available for **parts** involving extensive machining.	Pumps, **valves,** instruments, fittings
304	18.00- 20.00	8.00- 12.00	0.08		**Low** carbon variation of type 302, minimizes carbide precipitation during welding.	General purpose. Also available **as 304L** with 0.03% carbon to minimize carbide precipitation during welding	Perforated blow-pit screens, heat-exchanger tubing, preheater tubes
305	17.00- 19.00	**10.00–** 13.00	0.12		Higher heat and corrosion resistance than type 304.	Good corrosion resistance.	Funnels, utensils, hoods
308	19.00- 21.90	**10.00–** 12.00	0.08		High Cr and Ni produce good heat and corrosion resistance. **Used** widely for welding rod.	In order of their numbers, these alloys show **increased** resistance to high temperature corrosion. Types 3085, **309S** and 3105 are **also** available for welded construction.	Welding rod, more ductile welds for type 430
309	22.00- 24.00	12.00- 15.06	0.20		High strength and **resistance** to scaling at high temperatures.		Welding rod for type 304, heat exchangers, pump parts
310	24.00- 26.00	19.00- 22.90	0.25		Higher alloy content improves basic characteristics of type 309.		Jacketed high-temperature, high-pressure reactors, **oil-refining-still tubes**

314	23.00-26.00	19.00-22.00	0.25	Si 1.5-3.0	High silicon content.	Resistant to oxidation in air to 2000°F.	Radiant tubes, carburizing boxes, annealing boxes
316	16.00-18.00	10.00-14.00	0.08	Mo 2.00-3.00	Mo improves general corrosion and pitting resistance and high temperature strength over that of type 302.	Resistant to high pitting corrosion. Also available as 316L for welded construction.	Distillation equipment for producing fatty acids, sulfite paper processing equipment
317	18.00-20.00	11.00-15.00	0.08	Mo 3.00-4.00	Higher alloy content improves basic advantages of type 316.	Type 317 has the highest aqueous corrosion resistance of all AISI stainless steels.	Process equipment involving strong acids or chlorinated solvents
321	17.00-19.00	9.00-12.00	0.08	Ti 5 x c, min	Stabilized to permit use in 420°- 870" C range without harmful carbide precipitation.	Stabilized with titanium and columbium-tantalum, respectively, to permit their use for large welded structures which cannot be annealed after welding.	Furnace parts in presence of corrosive fumes
347	17.00-19.00	9.00-13.00	0.08	Cb-Ta 10 x c, min.	Characteristics similar to type 321. Stabilized by Cb and Ta.		Like 302 but used where carbide precipitation during fabrication or service may be harmful, welding rod for type 321
403	11.50-13.50		0.15	Si 0.50 max.	Version of type 410 with limited hardenability but improved fabricability.	Not highly resistant to high temperature oxidation in air.	Steam turbine blades
405	11.50-14.50		0.08	Al 0.10-0.30	Version of type 410 with limited hardenability but improved weldability.	Good weldability and cladding properties.	Tower linings, baffles, separator towers, heat exchanger tubing
410	11.50-13.50		0.15		Lowest cost general purpose stainless steel.	Wide use where corrosion is not severe.	Bubble-tower parts for petroleum refining, pump rods and valves, machine parts, turbine blades

(continued)

TABLE 4
Stainless steels most commonly used in the chemical process industries? (Continued)

Type§	Composition, %				Major characteristics	Properties	Applications
	Cr	Ni	C max	Other significant elements‡			
416	12.00–14.00		0.15	S 0.15 min	Sulfur added for free machining version of type 410. Type 416Se also available.	The freest machining type of martensitic stainless.	Valve stems, plugs, gates, useful for screws, bolts, nuts, and other parts requiring considerable machining during fabrication
420	12.00-14.00		0.15 min		Similar to type 410 but higher carbon produces higher strength and hardness.	High-spring temper.	Utensils, bushings, valve stems and wear-resisting parts
430	14.00–18.00		0.12		Most popular of nonhardening chromium types. Combines good corrosion resistance (to nitric acid and other oxidizing media).	Good heat resistance and good mechanical properties. Also available in type 430F.	Chemical and processing towers, condensers. Furnace parts such as retorts and low stressed parts subject to temperatures up to 800 "C. Type 430 nitric-acid storage tanks, furnace parts, fan scrolls. Type 430F-pump shafts, instrument parts, valve parts
431	15.00–17.00	1.25–2.50	1.20		High yield point	Very resistant to shock.	Products requiring high yield point and resistance to shock
442	18.00–23.00		3.25		High chromium nonhardenable type	High temperature uses where high sulfur atmospheres make presence of nickel undesirable.	Fume furnaces, flare stacks, materials in contact with high sulfur atmospheres

446	23.00– *27.00*		*0.20*		**Similar to type 442 but Cr increased to provide maximum resistance to scaling. Especially suited to intermittent high temperatures.**	**Excellent corrosion resistance to many liquid solutions, fabrication difficulties limit its use primarily to high temperature applications. Useful in high sulfur atmospheres.**	**Burner nozzles, stack dampers, boiler baffles, furnace linings, glass molds**

† Adapted from Biennial Materials of Construction Reports published regularly in ***Chemical*** Engineering and from tabulations in Chemical Engineers' Handbook," 6th ed., McGraw-Hilt Book Company, New York, 1984.

$ For a detailed listing of nominal composition or range, see the latest issue of "Data on Physical and Mechanical Properties of Stainless and Heat-Resisting Steels," Carpenter Steel Company, Reading, PA 19603.

§ In general, stainless steels in the 300 series contain large amounts of chromium and nickel; those in the 400 series contain large amounts of chromium and little or no nickel; those in the 500 series contain low amounts of chromium and little or no nickel; in the 300 series, except for type 309, the nickel content can be 10 percent or less if the second number is zero and greater than 10 percent if the second number is one; in the 400 series, an increase in the number represented by the last two digits indicates an increase in the chromium content.

The types of stainless steel included in the 300 series are hardenable only by cold-working; those included in the 400 series are either nonhardenable or hardenable by heat-treating. As an example, type 410, containing 12 percent chromium and no nickel, can be heat-treated for hardening and has good mechanical properties when heat-treated. It is often used as a material of construction for bubble caps, turbine blades, or other items that require special fabrication.

Stainless steels exhibit the best resistance to corrosion when the surface is oxidized to a passive state. This condition can be obtained, at least temporarily, by a so-called "passivation" operation in which the surface is treated with nitric acid and then rinsed with water. Localized corrosion can occur at places where foreign material collects, such as in scratches, crevices, or corners. Consequently, mars or scratches should be avoided, and the equipment design should specify a minimum of sharp comers, seams, and joints. Stainless steels show great susceptibility to stress corrosion cracking. As one example, stress plus contact with small concentrations of halides can result in failure of the metal wall.

The high temperatures involved in welding stainless steel may cause precipitation of chromium carbide at the grain boundary, resulting in decreased corrosion resistance along the weld. The chances of this occurring can be minimized by using low-carbon stainless steels or by controlled annealing.

A preliminary approach to the selection of the stainless steel for a specific application is to classify the various types according to the alloy content, microstructure, and major characteristic. Table 3 outlines the information according to the classes of stainless steels-austenitic, martensitic, and ferritic. Table 4 presents characteristics and typical applications of various types of stainless steel while Table 5 indicates resistance of stainless steels to oxidation in air.

TABLE 5
Resistance of stainless steels to oxidation in air

Maximum temperature, °C	Stainless steel type
650	416
700	403, 405, 410, 414
800	430F
850	430,431
900	302, 303,304, 316, 317, 321, 347, 348, 17-14 CuMo
1000	302B, 308,442
1100	309, 310, 314, 329, 446

Hastelloy

The beneficial effects of nickel, chromium, and molybdenum are combined in Hastelloy C to give an expensive but highly corrosion-resistant material. A typical analysis of this alloy shows 56 percent nickel, 17 percent molybdenum, 16 percent chromium, 5 percent iron, and 4 percent tungsten, with manganese, silicon, carbon, phosphorus, and sulfur making up the balance. Hastelloy C is used where structural strength and good corrosion resistance are necessary under conditions of high temperatures. The material can be machined and is readily fabricated. It is used in the form of valves, piping, heat exchangers, and various types of vessels. Other types of Hastelloys are also available for use under special corrosive conditions.

Copper and its Alloys

Copper is relatively inexpensive, possesses fair mechanical strength, and can be fabricated easily into a wide variety of shapes. Although it shows little tendency to dissolve in nonoxidizing acids, it is readily susceptible to oxidation. Copper is resistant to atmospheric moisture or oxygen because a protective coating composed primarily of copper oxide is formed on the surface. The oxide, however, is soluble in most acids, and thus copper is not a suitable material of construction when it must contact any acid in the presence of oxygen or oxidizing agents. Copper exhibits good corrosion resistance to strong alkalies, with the exception of ammonium hydroxide. At room temperature it can handle sodium and potassium hydroxide of all concentrations. It resists most organic solvents and aqueous solutions of organic acids.

Copper alloys, such as brass, bronze, admiralty, and Muntz metals, can exhibit better corrosion resistance and better mechanical properties than pure copper. In general, high-zinc alloys should not be used with acids or alkalies owing to the possibility of dezincification. Most of the low-zinc alloys are resistant to hot dilute alkalies.

Nickel and its Alloys

Nickel exhibits high corrosion resistance to most alkalies. Nickel-clad steel is used extensively for equipment in the production of caustic soda and alkalies. The strength and hardness of nickel is almost as great as that of carbon steel, and the metal can be fabricated easily. In general, oxidizing conditions promote the corrosion of nickel, and reducing conditions retard it.

Monel, an alloy of nickel containing 67 percent nickel and 30 percent copper, is often used in the food industries. This alloy is stronger than nickel and has better corrosion-resistance properties than either copper or nickel. Another important nickel alloy is Inconel (77 percent nickel and 15 percent chromium). The presence of chromium in this alloy increases its resistance to oxidizing conditions.

Aluminum

The lightness and relative ease of fabrication of aluminum and its alloys are factors favoring the use of these materials. Aluminum resists attack by acids because a surface film of inert hydrated aluminum oxide is formed. This film adheres to the surface and offers good protection unless materials which can remove the oxide, such as halogen acids or alkalies, are present.

Lead

Pure lead has low creep and fatigue resistance, but its physical properties can be improved by the addition of small amounts of silver, copper, antimony, or tellurium. Lead-clad equipment is in common use in many chemical plants. The excellent corrosion-resistance properties of lead are caused by the formation of protective surface coatings. If the coating is one of the highly insoluble lead salts, such as sulfate, carbonate, or phosphate, good corrosion resistance is obtained. Little protection is offered, however, if the coating is a soluble salt, such as nitrate, acetate, or chloride. As a result, lead shows good resistance to sufuric acid and phosphoric acid, but it is susceptible to attack by acetic acid and nitric acid.

Tantalum

The physical properties of tantalum are similar to those of mild steel, with the exception that its melting point (2996°C) is much higher. It is ordinarily used in the pure form, and it is readily fabricated into many different shapes. The corrosion-resistance properties of tantalum resemble those of glass. The metal is attacked by hydrofluoric acid, by hot concentrated alkalies, and by materials containing free sulfur trioxide. It is resistant to all other acids and is often used for equipment involving contact with hydrochloric acid.

Silver

Because of its low mechanical strength and high cost, silver is generally used only in the form of linings. Silver is resistant to alkalies and many hot organic acids. It also shows fair resistance to aqueous solutions of the halogen acids.

Galvanic Action between Two Dissimilar Metals

When two dissimilar metals are used in the construction of equipment containing a conducting fluid in contact with both metals, an electric potential may be set up between the two metals. The resulting galvanic action can cause one of the metals to dissolve into the conducting fluid and deposit on the other metal. As an example, if a piece of copper equipment containing a solution of sodium chloride in water is connected to an iron pipe, electrolysis can occur between

TABLE 6

Electromotive series of metals

List of metals arranged in decreasing order of their tendencies to pass into ionic form by losing electrons

Metal	Ion	Standard electrode potential 1125°C
Lithium	Li^+	+2.96
Potassium	Ii^+	2.92
Calcium	Ca^{++}	2.87
Sodium	Na^+	2.71
Magnesium	Mg^{++}	2.40
Aluminum	Al^{3+}	1.70
Manganese	Mn^{++}	1.10
Zinc	Zn^{++}	0.76
Chromium	Cr^{++}	0.56
Gallium	Ga^{3+}	0.50
Iron	Fe^{++}	0.44
Cadmium	Cd^{++}	0.40
Cobalt	co^{++}	0.28
Nickel	Ni^{++}	0.23
Tin	Sn^{++}	0.14
Lead	Pb^{++}	0.12
Iron	Fe^{3+}	0.045
Hydrogen	H^+	0.0000
Antimony	Sb^{3+}	-0.10
Bismuth	Bi^{3+}	-0.23
Arsenic	As”+	-0.30
Copper	cu^{++}	-0.34
Copper	Cu^+	-0.47
Silver	Ag^+	-0.80
Lead	Pb^{4+}	-0.80
Platinum	Pt^{4+}	-0.86
Gold	Au^{3+}	-1.36
Gold	Au^+	-1.50

the iron and copper, causing high rates of corrosion. As indicated in Table 6, iron is higher in the electromotive series than copper, and the iron pipe will gradually dissolve and deposit on the copper. The farther apart the two metals are in the electromotive series, the greater is the possible extent of corrosion due to electrolysis.

NONMETALS

Glass, carbon, stoneware, brick, rubber, plastics, and wood are common examples of nonmetals used as materials of construction. Many of the nonmetals have low structural strength. Consequently, they are often used in the form of linings or coatings bonded to metal supports. For example, glass-lined or rubber-lined equipment has many applications in the chemical industries.

Glass and Glassed Steel

Glass has excellent resistance and is subject to attack only by hydrofluoric acid and hot alkaline solutions. It is particularly suitable for processes which have critical contamination levels. A chief drawback is its brittleness and damage by thermal shock. On the other hand, glassed steel combines the corrosion resistance of glass with the working strength of steel. Nucerite is a ceramic-metal composite made in a similar manner to glassed steel and resists corrosive hydrogen-chloride gas, chlorine, or sulfur dioxide at 650°C. Its impact strength is 18 times that of safety glass and the abrasion resistance is superior to porcelain enamel.

Carbon and Graphite

Generally, impervious graphite is completely inert to all but the most severe oxidizing conditions. This property, combined with excellent heat transfer, has made impervious carbon and graphite very popular in heat exchangers, as brick lining, and in pipe and pumps. One limitation of these materials is low tensile strength. Threshold oxidation temperatures are 350°C for carbon and 400°C for graphite.

Stoneware and Porcelain

Materials of stoneware and porcelain are about as resistant to acids and chemicals as glass, but with the advantage of greater strength. This is offset somewhat by poor thermal conductivity and susceptibility to damage by thermal shock. Porcelain enamels are used to coat steel, but the enamel has slightly inferior chemical resistance.

Brick and Cement Materials

Brick-lined construction can be used for many severely corrosive conditions, where high alloys would fail. Acidproof refractories can be used up to 900°C.

A number of cement materials are used with brick. Standard are phenolic and furane resins, polyesters, sulfur, silicate, and epoxy-based materials. Carbon-filled polyesters and furanes are good against nonoxidizing acids, salts, and solvents. Silica-filled resins should not be used against hydrofluoric or fluorosilicic acids. Sulfur-based cements are limited to 95°C, while resins can be used to about 175°C. The sodium silicate based cements are good against acids to 400°C.

Rubber and Elastomers

Natural and synthetic rubbers are used as linings or as structural components for equipment in the chemical industries. By adding the proper ingredients, natural rubbers with varying degrees of hardness and chemical resistance can be produced. Hard rubbers are chemically saturated with sulfur. The vulcanized products are rigid and exhibit excellent resistance to chemical attack by dilute sulfuric acid and dilute hydrochloric acid.

Natural rubber is resistant to dilute mineral acids, alkalies, and salts, but oxidizing media, oils, benzene, and ketones will attack it. Chloroprene or neoprene rubber is resistant to attack by ozone, sunlight, oils, gasoline, and aromatic or halogenated solvents. Styrene rubber has chemical resistance similar to natural. Nitrile rubber is known for resistance to oils and solvents. Butyl rubber's resistance to dilute mineral acids and alkalies is exceptional; resistance to concentrated acids, except nitric and sulfuric, is good. Silicone rubbers, also known as polysiloxanes, have outstanding resistance to high and low temperatures as well as against aliphatic solvents, oils, and greases. Chlorosulfonated polyethylene, known as hypalon, has outstanding resistance to ozone and oxidizing agents except fuming nitric and sulfuric acids. Oil resistance is good. Fluoroelastomers (Viton A, Kel-F) combine excellent chemical and high-temperature resistance. Polyvinyl chloride elastomer (Koroseal) was developed to overcome some of the limitations of natural and synthetic rubbers. It has excellent resistance to mineral acids and petroleum oils.

Plastics

In comparison with metallic materials, the use of plastics is limited to relatively moderate temperatures and pressures (230°C is considered high for plastics). Plastics are also less resistant to mechanical abuse and have high expansion rates, low strengths (thermoplastics), and only fair resistance to solvents. However, they are lightweight, are good thermal and electrical insulators, are easy to fabricate and install, and have low friction factors.

Generally, plastics have excellent resistance to weak mineral acids and are unaffected by inorganic salt solutions-areas where metals are not entirely suitable. Since plastics do not corrode in the electrochemical sense, they offer another advantage over metals: most metals are affected by slight changes in pH, or minor impurities, or oxygen content, while plastics will remain resistant to these same changes.

One of the most chemical-resistant plastics commercially available today is tetrafluoroethylene or TFE (Teflon). This thermoplastic is practically unaffected by all alkalies and acids except fluorine and chlorine gas at elevated temperatures and molten metals. It retains its properties up to 260°C. Chlorotrifluoroethylene or CFE (Kel-F) also possesses excellent corrosion resistance to almost all acids and alkalies up to 175°C. FEP, a copolymer of tetrafluoroethylene and hexafluoropropylene, has similar properties to TFE except that it is not recommended for continuous exposures at temperatures above 200°C. Also, FEP can be extruded on conventional extrusion equipment, while TFE parts must be made by complicated "powdered-metallurgy" techniques.

Polyethylene is the lowest-cost plastic commercially available. Mechanical properties are generally poor, particularly above 50°C, and pipe must be fully supported. Carbon-filled grades are resistant to sunlight and weathering.

Unplasticized polyvinyl chlorides (type I) have excellent resistance to oxidizing acids other than concentrated, and to most nonoxidizing acids. Resistance is good to weak and strong alkaline materials. Resistance to chlorinated hydrocarbons is not good.

Acrylonitrile butadiene styrene polymers (ABS) have good resistance to nonoxidizing and weak acids but are not satisfactory with oxidizing acids. Upper temperature limit is about 65°C. Resistance to weak alkaline solutions is excellent. They are not satisfactory with aromatic or chlorinated hydrocarbons but have good resistance to aliphatic hydrocarbons.

Chlorinated polyether can be used continuously up to 125°C, intermittently up to 150°C. Chemical resistance is between polyvinyl chloride and the fluorocarbons. Dilute acids, alkalies, and salts have no effect. Hydrochloric, hydrofluoric, and phosphoric acids can be handled at all concentrations up to 105°C. Sulfuric acid over 60 percent and nitric over 25 percent cause degradation, as do aromatics and ketones.

Acetals have excellent resistance to most organic solvents but are not satisfactory for use with strong acids and alkalies.

Cellulose acetate butyrate is not affected by dilute acids and alkalies or gasoline but chlorinated solvents cause some swelling. Nylons resist many organic solvents but are attacked by phenols, strong oxidizing agents, and mineral acids.

Polypropylene's chemical resistance is about the same as polyethylene, but it can be used at 120°C. Polycarbonate is a relatively high-temperature plastic. It can be used up to 150°C. Resistance to mineral acids is good. Strong alkalies slowly decompose it, but mild alkalies do not. It is partially soluble in aromatic solvents and soluble in chlorinated hydrocarbons.

Among the thermosetting materials are phenolic plastics filled with asbestos, carbon or graphite, and silica. Relatively low cost, good mechanical properties, and chemical resistance (except against strong alkalies) make phenolics popular for chemical equipment. Furane plastics, filled with asbestos, have much better alkali resistance than phenolic asbestos. They are more expensive than the phenolics but also offer somewhat higher strengths.

General-purpose polyester resins, reinforced with fiberglass, have good strength and good chemical resistance, except to alkalies. Some special materials in this class, based on bisphenol, are more alkali resistant. Temperature limit for polyesters is about 95°C. The general area of fiberglass reinforced plastics (FRP) represents a rapidly expanding application of plastics for processing equipment, and it is necessary to solve the problem of development of fabrication standards.

Epoxies reinforced with fiberglass have very high strengths and resistance to heat. Chemical resistance of the epoxy resin is excellent in nonoxidizing and weak acids but not good against strong acids. Alkaline resistance is excellent in weak solutions. Chemical resistance of epoxy-glass laminates may be affected by any exposed glass in the laminate.

Phenolic asbestos, general-purpose polyester glass, saran, and CAB (cellulose acetate butyrate) are adversely affected by alkalies, while thermoplastics generally show poor resistance to organics.

Wood

This material of construction, while fairly inert chemically, is readily dehydrated by concentrated solutions and consequently shrinks badly when subjected to the action of such solutions. It also has a tendency to slowly hydrolyze when in contact with hot acids and alkalies.

LOW- AND HIGH-TEMPERATURE MATERIALS

The extremes of low and high temperatures used in many recent chemical processes has created some unusual problems in fabrication of equipment. For example, some metals lose their ductility and impact strength at low temperatures, although in many cases yield and tensile strengths increase as the temperature is decreased. It is important in low temperature applications to

TABLE 7
Metals and alloys for low-temperature process use[†]

ASTM specification and grade	Recommended minimum service temp, °C
Carbon and alloy steels:	
T-1	-45
A 201, A 212, flange or firebox quality	
A 203, grades A and B ($2\frac{1}{4}$ Ni)	-60
A 203, grades D and E ($3\frac{1}{2}$ Ni)	-100
A 353 (9% Ni)	-195
Copper alloys, silicon bronze, 70-30 brass, copper	
Stainless steels type 302, 304L, 304, 310, 347	-255
Aluminum alloys 5052, 5083, 5086, 5154, 5356, 5454, 5456	

[†] See R. M. McClintock and H. P. Gibbons, "Mechanical Properties of Structural Materials at Low Temperatures," National Bureau of Standards, June 1960; Vol. 1-25, K. D. Timmerhaus (ed.), and Vol. 26, 28, 30, 32, 34, 36, R. P. Reed and A. F. Clark (ed.), "Advances in Cryogenic Engineering," Plenum Press, New York.

TABLE 8

Alloys for high-temperature process use†

	Nominal composition, %			
	Cr	Ni	Fe	Other
Ferritic steels:				
Carbon steel	. . . .		bal.	
2¼ chrome	2¼		bal.	Mo
Type 502	5		bal.	Mo
Type 410	12		bal.	
Type 430	16		bal.	
Type 446	27		bal.	
Austenitic steels:				
Type 304	18	8	bal.	
Type 321	18	10	bal.	Ti
Type 347	18	11	bal.	Cb
Type 316	18	12	bal.	Mo
Type 309	24	12	bal.	
Type 310	25	20	bal.	
Type 330	15	35	bal.	
Nickel-base alloys:				
Nickel	. . . .	bal.		
Incoloy	21	34	bal.	
Hastelloy B		bal.	6	Mo
Hastelloy C	16	bal.	6	W, Mo
60/15	15	bal.	25	
Inconel	15	bal.	7	
80/20	20	bal.		
Hastelloy X	22	bal.	19	Co, Mo
Multimet	21	20	bal.	Co
Rene 41	19	bal.	5	Co, Mo, Ti
Cast irons:				
Ductile iron	. . .		bal.	C, Si, Mg
Ni-Resist, D-2	2	20	bal.	Si, C
Ni-Resist, D-4	5	30	bal.	Si, C
Cast stainless (ACI types):				
HC	28	4	bal.	
HF	21	11	bal.	
HH	26	12	bal.	
HK	26	20	bal.	
HT	15	35	bal.	
HW	12	bal.	28	
Super alloys:				
Inconel X	15	bal.	7	Ti, Al, Cb
A 286	15	25	bal.	Mo, Ti
Stellite 25	20	10	Co-base	W
Stellite 21 (cast)	27.3	2.8	Co-base	R I O
Stellite 31 (cast)	25.2	10.5	Co-base	W

†"Chemical Engineers' Handbook," 6th ed., McGraw-Hill Book Company, New York, 1984

choose materials resistant to shock. Usually a minimum Charpy value of 15 ft . lbf (keyhole notch) is specified at the operating temperature. For severe loading, a value of 20 ft . lbf is recommended. Ductility tests are performed on notched specimens since smooth specimens usually show amazing ductility. Table 7 provides a brief summary of metals and alloys recommended for low-temperature use.

Among the most important properties of materials at the other end of the temperature spectrum are creep, rupture, and short-time strengths. Stress rupture is another important consideration at high temperatures since it relates stress and time to produce rupture. Ferritic alloys are weaker than austenitic compositions, and in both groups molybdenum increases strength. Higher strengths are available in Inconel, cobalt-based Stellite 25, and iron-base A286. Other properties which become important at high temperatures include thermal conductivity, thermal expansion, ductility, alloy composition, and stability.

Actually, in many cases strength and mechanical properties become of secondary importance in process applications, compared with resistance to the corrosive surroundings. All common heat-resistant alloys form oxides when exposed to hot oxidizing environments. Whether the alloy is resistant depends upon whether the oxide is stable and forms a protective film. Thus, mild steel is seldom used above 500°C because of excessive scaling rates. Higher temperatures require chromium. This is evident, not only from Table 5, but also from Table 8 which lists the important commercial alloys for high-temperature use.

GASKET MATERIALS

Metallic and nonmetallic gaskets of many different forms and compositions are used in industrial equipment. The choice of a gasket material depends on the corrosive action of the chemicals that may contact the gasket, the location of the gasket, and the type of gasket construction. Other factors of importance are the cost of the materials, pressure and temperature involved, and frequency of opening the joint.

TABULATED DATA FOR SELECTING MATERIALS OF CONSTRUCTION

Table 9 presents information on the corrosion resistance of some common metals, nonmetals, and gasket materials. Table 10 presents similar information for various types of plastics. These tables can be used as an aid in choosing materials of construction, but no single table can take into account all the factors that can affect corrosion. Temperature level, concentration of the corrosive agent, presence of impurities, physical methods of operation, and slight alterations in the composition of the constructional material can affect the degree of corrosion resistance. The final selection of a material of construction,

TABLE 9

Corrosion resistance of constructional materials?

Code designation for corrosion resistance

A = acceptable, can be used successfully
C = caution, resistance varies widely depending on conditions; used when some corrosion is permissible
X = unsuitable
Blank = information lacking

Code designation for gasket materials

a = asbestos, white (compressed or woven)
b = asbestos, blue (compressed or woven)
c = asbestos (compressed and rubber-bonded)
d = asbestos (woven and rubber-frictioned)
e = CR-S or natural rubber
f = Teflon

	Metals								Nonmetals					
			Stainless steel											
Chemical	**Iron and steel**	cast iron (Ni-resist)	**18-8**	**18-8 Mo**	Nickel	**Monel**	Red brass	**Aluminum**	**Industrial glass**	Carbon (Karbate)	**Phenolic** resins (**Haveg**)	**Acrylic resins (Lucite)**	**Vinylidene chloride** [Saran]	**Acceptable nonmetallic** gasket materials
Acetic acid. crude	C	C	C	C	C	C	C	A	A	A	A	A	C	b, c, d, f
Acetic acid, pure	X	X	C	A	C	A	X	A	A	A	A	X	X	b, c, d, f
Acetic anhydride	C	C	A	A	A	A	x	A	A	A	A	x	C	b, c, d, f
Acetone	A	A	A	A	A	A	A	A	A	A	C	X	C	a, e, f
Aluminum chloride	X	C	X	X	C	C	A	A	A	A	A	...	A	a, c, e, f
Aluminum sulfate	X	C	C	A	C	C	X	A	A	A	A	A	A	a, c, d, e, f
Alums	X	C	C	A	C	A	X	A	A	A	A	A	A	a, c, d, e, f
Ammonia **(gas)**	A	A	C	A	A	A	x	C	A	.	A		C	a, f
Ammonium chloride	C	A	C	C	A	A	C	C	A	A	A	A	A	b, c, d, e, f
Ammonium hydroxide	A	A	A	A	C	C	X	C	A		A	A	C	a, c, d, f
Ammonium phosphate **(monobasic)**	X	C	A	A		C	X	X	A	A	A	...		b, c, d, e, f
Ammonium phosphate **(dibasic)**	C	A	A	A		A	C	C	A	A	A			a, c, d, e, f
Ammonium phosphate **(tribasic)**	A	A	A	A	A	A	X	C	A	A	A	.	.	a, c, d, e, f
Ammonium sulfate	C	A	C	C	A	A	C	A	A	A	A	A	A	b, c, d, e, f
Aniline	A	A	A	A	. .	A	X	...	A	A	C	.	C	a, f

Benzene. **benzol**	A	A	A	A	A	A	A	A	A	A	A	, ,	**C**	**a, f**
Boric acid	X	C	A	A	A	A	C	A	A	A	A		A	**a, c, d, e, f**
Bromine	X	C	C	C	C	C	C	.	A	C	X		X	b, f
Calcium **chloride**	C	A	C	C	A	A	C	C	A	A	A	A	A	**b, c,** d, **e,** f
Calcium hydroxide	A	A	A	A		A	C						.	**a, c, d, e, f**
Calcium **hypochlorite**	x	C	C	A	C	C	C	C	A	A	C	. . .	C	**b, c, d,** f
Carbon **tetrachloride**	C	C	C	A	A	A	C	C	A	A	A	A	C	**a, f**
Carbonic acid	C	C	A	A	A	A	C	A	A	A	A	A	A	a. **e, f**
Chloracetic acid	x		x	**X**	C	C	**X**	C	A		A		. .	**b, f**
Chlorine, dry	A	A	C	A	A	A	A	A	A	A	A	.	X	**b, e, f**
Chlorine. wet	x	x	**X**	**X**	**X**	**X**	x	**X**	A	C	A		X	**b, e, f**
Chromic acid	C	C	C	C	C	C	**X**	C	A	X	X	X	A	b. f
Citric acid	x	C	C	A	C	A	C	A	A	A	A	A	A	b. **c,** d. **e,** f
Copper sulfate	**X**	C	A	A	C	C	x	x	A	A	A	X	.	**b, c, d, e, f**
Ethanol	A	A	A	A	A	A	A	A	A	A	A	.	A	**a, c, e, f**
Ethylene **glycol**	A	A	A	A	A	A	A	A	A	A	A	A	C	**a, c, e, f**
Fatty acids	C	C	A	A	A	A	C	A	A	A	A	.	A	**a, e, f**
Ferric chloride	x	x	x	C	**X**	x	X	x	A	C	A		A	**b, e,** f
Ferric sulfate	X	x	C	A	C	C	**X**	C	A	C	A		A	**b, c, e,** f
Ferrous sulfate	C	A	A	A	A	A	C	C	A	A	'A		C	
Formaldehyde	C	C	A	A	A	A	C	A	A	A	A		A	**a, c, e, f**
Formic acid	**X**		C	C	C	C	**X**	**X**	A	A	A		A	**b, c, e,** f
Glycerol	A	A		A	A	A	A	4	A	A	C	A	C	**a, c, e, f**
Hydrocarbons **(aliphatic)**	A	A	A	A	A	A	A	A	A	A	A	A	C	**a, c, d, f**
Hydrochloric acid	X	**X**	**X**	x	C	C	X	**X**	A	A	A	A	C	**b, c, d, f**
Hydrofluoric acid	C	x	**X**	**X**	C	C	X	x	x	A	C		C	**b, f**
Hydrogen peroxide	C	.	C	C	C	C	C	A	A	A	A	A	C	a. **e, f**
Lactic acid	x	C	C	A	C	C	A	C	A	A	A			**a, b, c, d, e,** f
Magnesium chloride	C	C	C	C	A	A	C	C	A	A	A		A	**b, c, e,** f
Magnesium sulfate	A	A	A	A	A	A	A	A	A		A		A	**b, c, e,** f
Methanol	A	A	A	A	A	A	A	A	A	A	A		A	**a, c, e, f**
Nitric acid	**X**	C	C	C	**X**	x	**X**	C	A	C	C		C	**b, f**
Oleic acid	C	C	A	A	A	A	C	A	A	A	A		A	**a, e, f**
Oxalic acid	C	C	C	C	C	A	C	C	A	A	A			**b, c, d, e, f**
Phenol (carbolic acid)	C	4	C	A	A	A	C	A	A	A	C	A	C	**a, f**
Phosphoric acid	C	C	C	A	C	C	**X**	**X**	C	A	A		A	**b, c, f**
Potassium hydroxide	C	C	A	A	A	A	**X**	x					C	**a, e, f**
Sodium **bisulfate**	X	C	A	A	A	A	C	C	A	A	A		A	**b, c, d, e,** f

†**From** miscellaneous sources. For additional details, see "Chemical Engineers' Handbook," 6th ed., McGraw-Hill Book Company, New York, 1984.

(Continued)

TABLE 9
Corrosion resistance of constructional materials† (Continued)

Chemical	Metals								Nonmetals						Acceptable nonmetallic gasket materials
			Stainless steel												
	Iron and steel	Cast iron (Ni-resist)	18-8	18-8 Mo	Nickel	Monel	Red brass	Aluminum	Industrial glass	Carbon (Karbate)	Phenolic resins (Haveg)	Acrylic resins ([Lucite	Vinylidene chloride (Saran)	
Sodium carbonate	A	A	A	A	A	A	C	C	C	A	A	X	.	a, c, d, e, f
Sodium chloride	A	A	C	C	A	A	C	C	A	A	A			a, c, d, e, f
Sodium hydroxide	A	A	A	A	A	A	C	X	C	A	A	A	C	a, c, d, f
Sodium hypochlorite	X	C	C	A	C	C	C	X	A	C	X		A	b, c, d, f
Sodium nitrate	A	A	A	A	A	A	C	A	A	A	A		.	b, c, d, e, f
Sodium sulfate	A	A	A	C	A	A	A	A	A	A	A	...	A	a, c, d, e, f
Sodium sulfide	A	A	C	A	A	A	X	X	C	A	A	.	.	a, e, f
Sodium sulfite	A	A	A	A	A	A	C	C	A	A	A			
Sodium thiosulfate	C		A	A	A	A	C	C	A	A	A	...	A	a, c, d, e, f
Stearic acid	C	A	A	A	A	A	C	A	A	A	A			a, e, f
Sulfur	A	C	C	C	C	C	C	C	A	A	A		.	a, e, f
Sulfur dioxide	C	C	C	C	C	C	C	A	A	A	A		A	a, f
Sulfuric acid (98 % to fuming)	A	C	x	C	x	X	X	C	A	X	X	X	C	b, f
Sulfuric acid (75-95 %)	A	C	x	X	X	C	X	X	A	C	X	X	C	b, f
Sulfuric acid (10–75 %)	X	C	x	X	C	C	X	X	A	A	C	C	A	b, f
Sulfuric acid (<10 %)	X	C	X	C	C	C	C	C	A	A	C	A	A	a. b, c, e, f
Sulfurous acid	X	.	C	A	X	X	C	C	A	A	A	...	C	b, c, d, e, f
Trichloroethylene	C	A	C	A	A	A	C	C	A	A	A	...	C	a, f
Zinc chloride	C	C	C	X	A	A	X	C	.	...	A	A	. .	b. c, d, e, f
Zinc sulfate	C	A	A	A	A	A	C	C	.	. .	. .	...	. .	b, c, d, e, f

†From miscellaneous sources. For additional details, see "Chemical Engineers' Handbook," 6th ed., McGraw-Hill Book Company, New York, 1984.

TABLE 10

Chemical resistance of plastics in various solvents?

Code designation for chemical resistance

S = good to 25°C
S_1 = good to 60°C
S_2 = good above 60°C
F = fair
U = unsatisfactory

	PVC rigid	PVC-plasticized	Polyethylene	Polypropylene	Methacrylates	Polyesters	Epoxies	Fluorocarbon	Polystyrene	ABS Polymers	Acetal polymers	henol-rmalehyde	olyaronate	Cl-polyether	Furan	Saran
Acetone	U	U	S	S	U	U	F	S?	U	U	S_1	U	S	S	F	S
Alcohols, methyl	S	St	S_1	S_1	S	S_1	S	S_2	S	S_1	S_1	F	S_1	S_2	S	S_1
ethyl	S	S_1	S_1	S_1	S	S_1	S	S_2	S	S_1	S_1	F	S_1	S_2	S	S_1
butyl	S	S_1	S_1	S_1	S	S_1	S	S_2	S	S	S_1	F	S_1	S_2	S	S_1
Aniline	U	U	S	S			S	S_2	S	U		U		F	U	U
Benzene	U	U	U	U	U	S_1	S	S_2	U	U	S	S	U	S	S	F
Carbon tetrachloride	U	U	U	U	U	S_1	S	S_2	U	U	S_1	S	U	S_2	S	S
Cyclohexanone	U	U						S_2	. . .	U						U
Ethyl acetate	U	U	U	U	U	U	F	S_2	U	U		U	U	S	S	F
Ethylene dichloride	U	U	U	U	U	U	F	S_2	U	U	S	S	U	S_2		S
Ethyl ether	U	U	S	S	U		S	S_2	U	U	S_1	S	S_1	S_2	S	U
Hexane	S	U	F	F	S	S	S	S_2	U	F	S	S	S_1	S	S	S_1
Kerosene	S	S_1	S	S	S	S	S	S?	U	F	S	S	S_1	S	S	S_1
Lubricating oils	S_1	S	S	S	S	S	S_2	S_2	F	S	S	S	S_1	S_2	S	S_1
Naphthalene	U	U	S	S	U	S	S	S?	U	U	S	S	F	S_2	S	S_1
Triethanolamine	S_1	S	S	S				S_2	. . .	U				S_2		
Xylene	U	U	U	U	U	S_2	S	S_2	U	U	S_1	S	U	S	S	F

†From Biennial Materials of Construction Reports published by *Chemical Engineering.*

therefore, may require reference to manufacturers' bulletins and consultation with persons who are experts in the particular field of application.?

SELECTION OF MATERIALS

The chemical engineer responsible for the selection of materials of construction must have a thorough understanding of all the basic process information available. This knowledge of the process can then be used to select materials of construction in a logical manner. A brief plan for studying materials of construction is as follows:

1. Preliminary selection
   Experience, manufacturer's data, special literature, general literature, availability, safety aspects, preliminary laboratory tests
2. Laboratory testing
   Reevaluation of apparently suitable materials under process conditions
3. Interpretation of laboratory results and other data
   Effect of possible impurities, excess temperature, excess pressure, agitation, and presence of air in equipment
   Avoidance of electrolysis
   Fabrication method
4. Economic comparison of apparently suitable materials
   Material and maintenance cost, probable life, cost of product degradation, liability to special hazards
5. Final selection

In making an economic comparison, the engineer is often faced with the question of where to use high-cost claddings or coatings over relatively cheap base materials such as steel or wood. For example, a column requiring an expensive alloy-steel surface in contact with the process fluid may be constructed of the alloy itself or with a cladding of the alloy on the inside of carbon-steel structural material. Other examples of commercial coatings for chemical process equipment include baked ceramic or glass coatings, flame-sprayed metal, hard rubber, and many organic plastics. The durability of coatings is sometimes questionable, particularly where abrasion and mechanical-wear conditions exist. As a general rule, if there is little economic incentive between a coated type versus a completely homogeneous material, a

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†Up-to-date information on various aspects of materials of construction is presented in the Biennial Materials of Construction Reports published by Chemical *Engineering.* A detailed treatment of this subject is given in "Chemical Engineers' Handbook," 6th ed., McGraw-Hill Book Company, New York, 1984. See also Current Literature on Materials of Construction, *Chem. Eng.*, **95**(15):69 (October 24, 1988).

selection should favor the latter material, mainly on the basis of better mechanical stability.

ECONOMICS IN SELECTION OF MATERIALS

First cost of equipment or material often is not a good economic criterion when comparing alternate materials of construction for chemical process equipment. Any cost estimation should include the following items:

1. Total equipment or materials costs
2. Installation costs
3. Maintenance costs
4. Estimated life
5. Replacement costs

When these factors are considered, cost comparisons bear little resemblance to first costs. Table 11 presents a typical analysis of comparative costs for alternative materials when based on return on investment. One difficulty with such a comparison is the uncertainty associated with "estimated life." Well-designed laboratory and plant tests can at least give order-of-magnitude estimates. Another difficulty arises in estimating the annual maintenance cost. This can only be predicted from previous experience with the specific materials.

Table 11 could be extended by the use of continuous compounding interest methods as outlined in Chaps. 7 and 10 to show the value of money to a company above which (or below which) material A would be selected over *B*, *B*

TABLE 11
Alternative investment comparison

	Material *A*	**Material** *B*	**Material** *c*
Purchased cost	$25,000	$30,000	$35,000
Installation cost	15,000	20,000	25,000
Total installed cost	40,000	50,000	60,000
Additional cost over *A*		10,000	20,000
Estimated life, years	4	10	10
Estimated maintenance cost/year	5,000	4,500	3,000
Annual replacement cost (installed cost/estimated life)	10,000	5,000	6,000
Total annual cost	15,000	9,500	9,000
Annual savings vs. cost for *A*		5,500	6,000
Tax on savings, 34%		1,870	2,040
Net annual savings		3,630	3,960
Return on investment over *A* (net savings/additional cost over *A*) 100		36.3%	19.8%

over C, etc. Table 11 indicates that material B is always better than material A (this, of course, is inherent in the yearly return on investment method used). However, depending on the value of money to a company, this may not always be true.

FABRICATION OF EQUIPMENT

Fabrication expenses account for a large fraction of the purchased cost for equipment. A chemical engineer, therefore, should be acquainted with the methods for fabricating equipment, and the problems involved in the fabrication should be considered when equipment specifications are prepared.

Many of the design and fabrication details for equipment are governed by various codes, such as the ASME Codes. These codes can be used to indicate definite specifications or tolerance limits without including a large amount of descriptive restrictions. For example, fastening requirements can often be indicated satisfactorily by merely stating that all welding should be in accordance with the ASME Code.

The exact methods used for fabrication depend on the complexity and type of equipment being prepared. In general, however, the following steps are involved in the complete fabrication of major pieces of chemical equipment, such as tanks, autoclaves, reactors, towers, and heat exchangers:

1. Layout of materials
2. Cutting to correct dimensions
3. Forming into desired shape
4. Fastening
5. Testing
6. Heat-treating
7. Finishing

Layout

The first step in the fabrication is to establish the layout of the various components on the basis of detailed instructions prepared by the fabricator. Flat pieces of the metal or other constructional material involved are marked to indicate where cutting and forming are required. Allowances must be made for losses caused by cutting, shrinkage due to welding, or deformation caused by the various forming operations.

After the equipment starts to take shape, the location of various outlets and attachments will become necessary. Thus, the layout operation can continue throughout the entire fabrication. If tolerances are critical, an exact layout, with adequate allowances for deformation, shrinkage, and losses, is absolutely essential.

Cutting

Several methods can be used for cutting the laid-out materials to the correct size. *Shearing* is the cheapest method and is satisfactory for relatively thin sheets. The edge resulting from a shearing operation may not be usable for welding, and the sheared edges may require an additional grinding or machining treatment.

Burning is often used for cutting metals. This method can be employed to cut and, simultaneously, prepare a beveled edge suitable for welding. Carbon steel is easily cut by an oxyacetylene flame. The heat effects on the metal are less than those involved in welding. Stainless steels and nonferrous metals that do not oxidize readily can be cut by a method known as *powder* or *flux* burning. An oxyacetylene flame is used, and powdered iron is introduced into the cut to increase the amount of heat and improve the cutting characteristics. The high temperatures involved may affect the materials, resulting in the need for a final heat-treatment to restore corrosion resistance or removal of the heat-affected edges.

Sawing can be used to cut metals that are in the form of flat sheets. However, sawing is expensive, and it is used only when the heat effects from burning would be detrimental.

Forming

After the constructional materials have been cut, the next step is to form them into the desired shape. This can be accomplished by various methods, such as by rolling, bending, pressing, bumping (i.e., pounding), or spinning on a die. In some cases, heating may be necessary in order to carry out the forming operation. Because of work hardening of the material, annealing may be required before forming and between stages during the forming.

When the shaping operations are finished, the different parts are assembled and fitted for fastening. The fitting is accomplished by use of jacks, hoists, wedges, and other means. When the fitting is complete and all edges are correctly aligned, the main seams can be tack-welded in preparation for the final fastening.

Fastening

Riveting can be used for fastening operations, but electric welding is far more common and gives superior results. The quality of a weld is very important, because the ability of equipment to withstand pressure or corrosive conditions is often limited by the conditions along the welds. Although good welds may be stronger than the metal that is fastened together, design engineers usually assume a weld is not perfect and employ weld efficiencies of 80 to 95 percent in the design of pressure vessels.

The most common type of welding is the *manual shielded-arc* process in which an electrode approximately 14 to 16 in. long is used and an electric arc is maintained manually between the electrode and the material being welded. The electrode melts and forms a filler metal, while, at the same time, the work material fuses together. A special coating is provided on the electrode. This coating supplies a flux to float out impurities from the molten metal and also serves to protect the metal from surrounding air until the metal has solidified and cooled below red heat. The type of electrode and coating is determined by the particular materials and conditions that are involved in the welding operation.

A *submerged-arc* process is commonly used for welding stainless steels and carbon steels when an automatic operation is acceptable. The electrode is a continuous roll of wire fed at an automatically controlled rate. The arc is submerged in a granulated flux which serves the same purpose as the coating on the rods in the shielded-arc process. The appearance and quality of the submerged-arc weld is better than that obtained by an ordinary shielded-arc manual process; however, the automatic process is limited in its applications to main seams or similar long-run operations.

Heliarc welding is used for stainless steels and most of the nonferrous materials. This process can be carried out manually, automatically, or semiautomatically. A stream of helium or argon gas is passed from a nozzle in the electrode holder onto the weld, where the inert gas acts as a shielding blanket to protect the molten metal. As in the shielded-arc and submerged-arc processes, a filler rod is fed into the weld, but the arc in the heliarc process is formed between a tungsten electrode and the base metal.

In some cases, fastening can be accomplished by use of various solders, such as brazing solder (mp, 840 to 905°C) containing about 50 percent each of copper and zinc; silver solders (mp, 650 to 870°C) containing silver, copper, and zinc; or ordinary solder (mp, 220°C) containing 50 percent each of tin and lead. Screw threads, packings, gaskets, and other mechanical methods are also used for fastening various parts of equipment.

Testing

All welded joints can be tested for concealed imperfections by X rays, and code specifications usually require X-ray examination of main seams. Hydrostatic tests can be conducted to locate leaks. Sometimes, delicate tests, such as a helium probe test, are used to check for very small leaks.

Heat-treating

After the preliminary testing and necessary repairs are completed, it may be necessary to heat-treat the equipment to remove forming and welding stresses, restore corrosion-resistance properties to heat-affected materials, and prevent stress-corrosion conditions. A low-temperature treatment may be adequate, or the particular conditions may require a full anneal followed by a rapid quench.

Finishing

The finishing operation involves preparing the equipment for final shipment. Sandblasting, polishing, and painting may be necessary. Final pressure tests at $1\frac{1}{2}$ to 2 or more times the design pressure are conducted together with other tests as demanded by the specified code or requested by the inspector.

PROBLEMS

1. A new plant requires a large rotary vacuum filter for the filtration of zinc sulfite from a slurry containing 1 kg of zinc sulfite solid per 20 kg of liquid. The liquid contains water, sodium sulfite, and sodium bisulfite. The filter must handle 8000 kg of slurry per hour. What additional information is necessary to design the rotary vacuum filter? How much of this information could be obtained from laboratory or pilot-plant tests? Outline the method for converting the test results to the conditions applicable in the final design.
2. For each of the following materials of construction, prepare an approximate plot of temperature versus concentration in water for sulfuric acid and for nitric acid, showing conditions of generally acceptable corrosion resistance:
   (*a*) Stainless steel type 302.
   (*b*) Stainless steel type 316.
   (*c*) Karbate
   (*d*) Haveg
   (See "Chemical Engineers' Handbook," 6th ed., McGraw-Hill Book Company, New York, 1984).
3. A process for sulfonation of phenol requires the use of a 3000-gal storage vessel. It is desired to determine the most suitable material of construction for this vessel. The time value of money is to be taken into account by use of an interest rate of 10 percent.

   The life of the storage vessel is calculated by dividing the corrosion allowance of $\frac{1}{8}$ in. by the estimated corrosion rate. The equipment is assumed to have a salvage value of 10 percent of its original cost at the end of its useful life.

   For the case in question, corrosion data indicate that only a few corrosion-resistant alloys will be suitable:

Vessel Type	Installed cost	Average corrosion rate, in./yr
Nickel clad	$80,000	0.020
Monel clad	$95,000	0.010
Hastelloy B	$180,000	0.0045

Determine which material of construction would be used with appropriate justification for the selection.

4. What materials of construction should be specified for the thiophane process described in Prob. 20 of Chap. 2? Note the extremes of temperatures and corrosion which are encountered in this process because of the regeneration step and the presence of H_2S and caustic.
5. A manhole plate for a reactor is to be 2 in. thick and 18 in. in diameter. It has been proposed that the entire plate be made of stainless steel type 316. The plate will have 18 bolt holes, and part of the face will need to be machined for close gasket contact. If the base price for stainless steel type 316 in the form of industrial plates is $4.00 per pound, estimate the purchased cost for the manhole plate.
6. Six tanks of different constructional materials and six different materials to be stored in these tanks are listed in the following columns:

Tanks	*Materials*
Brass-lined	20% hydrochloric acid
Carbon steel	10% caustic soda
Concrete	75% phosphoric acid for food products
Nickel-lined	98% sulfuric acid
Stainless steel type 316	Vinegar
Wood	Water

All tanks must be used, and all materials must be stored without using more than one tank for any one material. Indicate the material that should be stored in each tank.

7. For the design of internal-pressure cylindrical vessels, the **API-ASME** Code for Unified Pressure Vessels recommends the following equations for determining the minimum wall thickness when extreme operating pressures are not involved:

$$t = \frac{PD_m}{2SE} + C \quad \text{applies when} \quad \frac{D_o}{D_i} < 1.2$$

or

$$t = \frac{D_i}{2}\left(\sqrt{\frac{SE+P}{SE-P}} - 1\right) + C \quad \text{applies when} \quad \frac{D_o}{D_i} > 1.2$$

where t = wall thickness, in.

P = internal pressure, psig (this assumes atmospheric pressure surrounding the vessel)

D_m = mean diameter, in.

D_i = ID, in.

D_o = OD, in.

E = fractional efficiency of welded or other joints

C = allowances, for corrosion, threading, and machining, in.

S = design stress, lb/in.* (for the purpose of this problem, S may be taken as one-fourth of the ultimate tensile strength)

A cylindrical storage tank is to have an ID of 12 ft and a length of 36 ft. The seams will be welded, and the material of construction will be plain carbon steel (0.15 percent C). The maximum working pressure in the tank will be 100 psig, and the maximum temperature will be 25°C. No corrosion problems are anticipated. On the basis of the preceding equations, estimate the necessary wall thickness.

8. A proposal has been made to use stainless-steel tubing as part of the heat-transfer system in a nuclear reactor. High temperatures and extremely high rates of heat transfer will be involved. Under these conditions, temperature stresses across the tube walls will be high, and the design engineer must choose a safe wall thickness and tube diameter for the proposed unit. List in detail all information and data necessary to determine if a proposed tube diameter and gauge number would be satisfactory.

CHAPTER 13

THE DESIGN REPORT

A successful engineer must be able to apply theoretical and practical principles in the development of ideas and methods and also have the ability to express the results clearly and convincingly. During the course of a design project, the engineer must prepare many written reports which explain what has been done and present conclusions and recommendations. The decision on the advisability of continuing the project may be made on the basis of the material presented in the reports. The value of the engineer's work is measured to a large extent by the results given in the written reports covering the study and the manner in which these results are presented.

The essential purpose of any report is to pass on information to others. A good report writer never forgets the words "to others." The abilities, the functions, and the needs of the reader should be kept in mind constantly during the preparation of any type of report. Here are some questions the writer should ask before starting, while writing, and after finishing a report:

What is the purpose of this report?
Who will read it?
Why will they read it?
What is their function?
What technical level will they understand?
What background information do they have now?

ESTIMATED MANUFACTURING-COST STATEMENT

Product____________________

Basis: Capacity____________________ Operating rate____________________

M & S index____________________ Labor rate____________________

Raw materials ....

Operating labor .... ________

Operating supervision .... _ _ _

Maintenance and repairs ....

Operating supplies .... _ _ _

Power and utilities ....

Royalties .... ________

Direct-production cost .... ________

Depreciation .... _ _ _

Rent .... _ _ _

Taxes (property). ....

Insurance ....

Fixed charges. .... ________

Safety and protection ....

Payroll overhead ....

Packaging ....

Salvage ....

Control laboratories .... ________

Plant superintendence ....

General plant overhead ....

Plant-overhead cost .... ________

Factory-manufacturing costs .... ________

BY____________________ Date____________

FIGURE 13-1
Example of form for an informal summarizing report on factory manufacturing cost.

The answers to these questions indicate the type of information that should be presented, the amount of detail required, and the most satisfactory method of presentation.

Types of Reports

Reports can be designated as formal and *informal.* Formal reports are often encountered as research, development, or design reports. They present the results in considerable detail, and the writer is allowed much leeway in choosing the type of presentation. Informal reports include memorandums, letters, progress notes, survey-type results, and similar items in which the major purpose is to present a result without including detailed information. Stereotyped forms are often used for informal reports, such as those for sales, production, calculations, progress, analyses, or summarizing economic evaluations. Figures 13-1 through 13-3 present examples of stereotyped forms that can be used for presenting the summarized results of economic evaluations.

Although many general rules can be applied to the preparation of reports, it should be realized that each industrial concern has its own specifications and

ESTIMATED CAPITAL-INVESTMENT STATEMENT

Product ____________________
Basis: Capacity ____________________ M & S index ____________

Purchased equipment (delivered) .... ________
Installation of equipment .... ________
Insulation .... ________
Instrumentation. .... ________
Piping .... ________
Electrical installations .... ________
Buildings including services .... ________
Yard improvements .... ________
Service facilities ....
Land ....
Total physical cost .... ________
Engineering and construction .... ________
Direct plant cast .... ________
Contractor's fee .... ________
Contingency ....
Fixed-capital investment .... ________
Raw-materials inventory .... ________
Product and in-process inventory,, ... ________
Accounts receivable .... ________
Cash ....
Working capital .... ________
Total capital investment .... ________
BY ____________________ Date ____________________

FIGURE 13-2
Example of form for an informal summarizing report on capital investment.

regulations. A stereotyped form shows exactly what information is wanted, and detailed instructions are often given for preparing other types of informal reports. Many companies have standard outlines that must be followed for formal reports. For convenience, certain arbitrary rules of rhetoric and form may be established by a particular concern. For example, periods may be required after all abbreviations, titles of articles may be required for all references, or the use of a set system of units or nomenclature may be specified.

ORGANIZATION OF REPORTS

The organization of a formal report requires careful sectioning and the use of subheadings in order to maintain a clear and effective presentation.[†] To a lesser

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[†] Many books and articles have been written on effective technical writing. For example, see H. F. Ebel, C. Bliefert, and W. E. Russey, "The Art of Scientific Writing," VCH Publishers, 220 East 23rd St., New York 10010, 1987 and the review in *Chem. & Eng.* News, **66**(48):34 (Nov. 28, 1988).

ESTIMATED INCOME AND RETURN STATEMENT

Product ______ Sales price ______
Basis: Capacity ______ Operating rate ______
M & S index ______ Labor rate ______

Direct production costs .... ___
Fixed charges ...... ___
Plant overhead costs ___
Factory-manufacturing costs .. ______
Administrative costs ___
Distribution and selling costs ______
Research and development ______
Financing . . ______
General expenses . ______
Total product cost . . . . ______
Total income .. , ______
Fixed-capital investment ______ Working capital ______
Total-capital investment ______ Probable accuracy of estimate-
Gross earnings before taxes ______ Net profit after __% taxes ______
Annual return on capital before taxes ______%
Annual return on capital after __% taxes ______%

By ______ Date ______

FIGURE 13.3
Example of form for an informal summarizing report on income and return.

degree, the same type of sectioning is valuable for informal reports. The following discussion applies to formal reports, but, by deleting or combining appropriate sections, the same principles can be applied to the organization of any type of report.

A complete design report consists of several independent parts, with each succeeding part giving greater detail on the design and its development. A covering *Letter of Transmittal* is usually the first item in any report. After this come the *Title Page,* the *Table of Contents,* and an *Abstract* or *Summary* of the report. The Body of the report is next and includes essential information, presented in the form of discussion, graphs, tables, and figures. The *Appendix,* at the end of the report, gives detailed information which permits complete verification of the results shown in the body. Tables of data, sample calculations, and other supplementary material are included in the Appendix. A typical outline for a design report is as follows:

ORGANIZATION OF A DESIGN REPORT

I. Letter of transmittal
  A. Indicates why report has been prepared
  B. Gives essential results that have been *specifically requested*

II. Title page
  A. Includes title of report, name of person to whom report is submitted, writer's name and organization, and date
III. Table of contents
  A. Indicates location and title of figures, tables, and all major sections
IV. Summary
  A. Briefly presents essential results and conclusions in a clear and precise manner
V. Body of report
  A. Introduction
    1. Presents a brief discussion to explain what the report is about and the reason for the report; no results are included
  B. Previous work
    1. Discusses important results obtained from literature surveys and other previous work
  C. Discussion
    1. Outlines method of attack on project and gives design basis
    2. Includes graphs, tables, and figures that are essential for understanding the discussion
    3. Discusses technical matters of importance
    4. Indicates assumptions made and their justification
    5. Indicates possible sources of error
    6. Gives a general discussion of results and proposed design
  D. Final recommended design with appropriate data
    1. Drawings of proposed design
      a. Qualitative flow sheets
      b. Quantitative flow sheets
      c. Combined-detail flow sheets
    2. Tables listing equipment and specifications
    3. Tables giving material and energy balances
    4. Process economics including costs, profits, and return on investment
  E. Conclusions and recommendations
    1. Presented in more detail than in Summary
  F. Acknowledgment
    1. Acknowledges important assistance of others who are not listed as preparing the report
  G. Table of nomenclature
    1. Sample units should be shown
  H. References to literature (bibliography)
    1. Gives complete identification of literature sources referred to in the report
VI. Appendix
  A. Sample calculations
    1. One example should be presented and explained clearly for each type of calculation

- B. Derivation of equations essential to understanding the report but not presented in detail in the main body of the report
- C. Tables of data employed with reference to source
- D. Results of laboratory tests
  1. If laboratory tests were used to obtain design data, the experimental data, apparatus and procedure description, and interpretation of the results may be included as a special appendix to the design report.

Letter of Transmittal

The purpose of a letter of transmittal is to refer to the original instructions or developments that have made the report necessary. The letter should be brief, but it can call the reader's attention to certain pertinent sections of the report or give definite results which are particularly important. The writer should express any personal opinions in the letter of transmittal rather than in the report itself. Personal pronouns and an informal business style of writing may be used.

Title Page and Table of Contents

In addition to the title of the report, a title page usually indicates other basic information, such as the name and organization of the person (or persons) submitting the report and the date of submittal. A table of contents may not be necessary for a short report of only six or eight pages, but, for longer reports, it is a convenient guide for the reader and indicates the scope of the report. The titles and subheadings in the written text should be shown, as well as the appropriate page numbers. Indentations can be used to indicate the relationships of the various subheadings. A list of tables, figures, and graphs should be presented separately at the end of the table of contents.

Summary

The summary is probably the most important part of a report, since it is referred to most frequently and is often the only part of the report that is read. Its purpose is to give the reader the entire contents of the report in one or two pages. It covers all phases of the design project, but it does not go into detail on any particular phase. All statements must be concise and give a minimum of general qualitative information. The aim of the summary is to present precise quantitative information and final conclusions with no unnecessary details.

The following outline shows what should be included in a summary:

1. A statement introducing the reader to the subject matter
2. What was done and what the report covers
3. How the final results were obtained

4. The important results including quantitative information, major conclusions, and recommendations

An ideal summary can be completed on one typewritten page. If the summary must be longer than two pages, it may be advisable to precede the summary by an *abstract,* which merely indicates the subject matter, what was done, and a brief statement of the major results.

Body of the Report

The first section in the body of the report is the *introduction.* It states the purpose and scope of the report and indicates why the design project originally appeared to be feasible or necessary. The relationship of the information presented in the report to other phases of the company's operations can be covered, and the effects of future developments may be worthy of mention. References to *previous work* can be discussed in the introduction, or a separate section can be presented dealing with literature-survey results and other previous work.

A description of the methods used for developing the proposed design is presented in the next section under the heading of *discussion.* Here the writer shows the reader the methods used in reaching the final conclusions. The validity of the methods must be made apparent, but the writer should not present an annoying or distracting amount of detail. Any assumptions or limitations on the results should be discussed in this section.

The next section presents the *recommended design,* complete with figures and tables giving all necessary qualitative and quantitative data. An analysis of the cost and profit potential of the proposed process should accompany the description of the recommended design.

The body of a design report often includes a section giving a detailed discussion of all *conclusions* and *recommendations.* When applicable, sections covering *acknowledgment, table of nomenclature,* and *literature references* may be added.

Appendix

In order to make the written part of a report more readable, the details of calculation methods, experimental data, reference data, certain types of derivations, and similar items are often included as separate appendixes to the report. This information is thus available to anyone who wishes to make a complete check on the work, yet the descriptive part of the report is not made ineffective because of excess information.

PREPARING THE REPORT

The physical process of preparing a report can be divided into the following steps:

1. Define the subject matter, scope, and intended audience
2. Prepare a skeleton outline and then a detailed outline
3. Write the first draft
4. Polish and improve the first draft and prepare the final form
5. Check the written draft carefully, have the report typed, and proofread the final report

In order to accomplish each of these steps successfully, the writer must make certain the initial work on the report is started soon enough to allow a thorough job and still meet any predetermined deadline date. Many of the figures, graphs, and tables, as well as some sections of the report, can be prepared while the design work is in progress.

PRESENTING THE RESULTS

Accuracy and logic must be maintained throughout any report. The writer has a moral responsibility to present the facts accurately and not mislead the reader with incorrect or dubious statements. If approximations or assumptions are made, their effect on the accuracy of the results should, be indicated. For example, a preliminary plant design might show that the total investment for a proposed plant is $5,500,000. This is not necessarily misleading as to the accuracy of the result, since only two significant figures are indicated. On the other hand, a proposed investment of $5554,328 is ridiculous, and the reader knows at once that the writer did not use any type of logical reasoning in determining the accuracy of the results.

The style of writing in technical reports should be simple and straightforward. Although short sentences are preferred, variation in the sentence length is necessary in order to avoid a disjointed staccato effect. The presentation must be convincing, but it must also be devoid of distracting and unnecessary details. Flowery expressions and technical jargon are often misused by technical writers in an attempt to make their writing more interesting. Certainly, an elegant or forceful style is sometimes desirable, but the technical writer must never forget that the major purpose is to present information clearly and understandably.

Subheadings and Paragraphs

The use of effective and well-placed subheadings can improve the readability of a report. The sections and subheadings follow the logical sequence of the report outline and permit the reader to become oriented and prepared for a new subject.

Paragraphs are used to cover one general thought. A paragraph break, however, is not nearly as definite as a subheading. The length of paragraphs can vary over a wide range, but any thought worthy of a separate paragraph should require at least two sentences. Long paragraphs are a strain on the reader, and the writer who consistently uses paragraphs longer than 10 to 12 typed lines will have difficulty in holding the reader's attention.

Tables

The effective use of tables can save many words, especially if quantitative results are involved. Tables are included in the body of the report only if they are essential to the understanding of the written text. Any type of tabulated data that is not directly related to the discussion should be located in the appendix.

Every table requires a title, and the headings for each column should be self-explanatory. If numbers are used, the correct units must be shown in the column heading or with the first number in the column. A table should never be presented on two pages unless the amount of data makes a break absolutely necessary.

Graphs

In comparison with tables, which present definite numerical values, graphs serve to show trends or comparisons. The interpretation of results is often simplified for the reader if the tabulated information is presented in graphical form.

If possible, the experimental or calculated points on which a curve is based should be shown on the plot. These points can be represented by large dots, small circles, squares, triangles, or some other identifying symbol. The most probable smooth curve can be drawn on the basis of the plotted points, or a broken line connecting each point may be more appropriate. In any case, the curve should not extend through the open symbols representing the data points. If extrapolation or interpolation of the curve is doubtful, the uncertain region can be designated by a dotted or dashed line.

The ordinate and the abscissa must be labeled clearly, and any nomenclature used should be defined on the graph or in the body of the report. If numerical values are presented, the appropriate units are shown immediately after the labels on the ordinate and abscissa. Restrictions on the plotted information should be indicated on the graph itself or with the title.

The title of the graph must be explicit but not obvious. For example, a log-log plot of temperature versus the vapor pressure of pure glycerol should not be entitled "Log-Log Plot of Temperature versus Vapor Pressure for Pure Glycerol." A much better title, although still somewhat obvious, would be "Effect of Temperature on Vapor Pressure of Pure Glycerol."

Some additional suggestions for the preparation of graphs follow:

1. The independent or controlled variable should be plotted as the abscissa, and the variable that is being determined should be plotted as the ordinate.
2. Permit sufficient space between grid elements to prevent a cluttered appearance (ordinarily, two to four grid lines per inch are adequate).
3. Use coordinate scales that give good proportionment of the curve over the entire plot, but do not distort the apparent accuracy of the results.
4. The values assigned to the grids should permit easy and convenient interpolation.
5. If possible, the label on the vertical axis should be placed in a horizontal position to permit easier reading.
6. Unless families of curves are involved, it is advisable to limit the number of curves on any one plot to three or less.
7. The curve should be drawn as the heaviest line on the plot, and the coordinate axes should be heavier than the grid lines.

Illustrations

Flow diagrams, photographs, line drawings of equipment, and other types of illustrations may be a necessary part of a report. They can be inserted in the body of the text or included in the appendix. Complete flow diagrams, prepared on oversize paper, and other large drawings are often folded and inserted in an envelope at the end of the report.

References to Literature

The original sources of any literature referred to in the report should be listed at the end of the body of the report. References are usually tabulated and numbered in alphabetical order on the basis of the first author's surname, although the listing is occasionally based on the order of appearance in the report.

When a literature reference is cited in the written text, the last name of the author is mentioned and the bibliographical identification is shown by a raised number after the author's name or at the end of the sentence. An underlined number in parentheses may be used in place of the raised number, if desired.

The bibliography should give the following information:

1. For journal articles: (a) authors' names, followed by initials, (*b*) journal, abbreviated to conform to the "List of Periodicals" as established by Chemical Abstracts, (*c*) volume number, (*d*) issue number, if necessary, (*e*) page number, and (*f*) year (in parentheses). The title of the article is usually omitted. Issue number is omitted if paging is on a yearly basis. The date is sometimes included with the year in place of the issue number.

McCormick, J. E., *Chem. Eng.*, **95**(13):75–76 (1988).

McCormick, J. E., *Chem. Eng.*, **95**:75–76 (Sept. 26, 1988).

Gregg, D. W., and T. F. Edgar, *AIChE J.*, **24**:753–781 (1978).

2. For single publications, as books, theses, or pamphlets: (*a*) authors' names, followed by initials, (*b*) title (in quotation marks), (*c*) edition (if more than one has appeared), (*d*) volume (if there is more than one), (*e*) publisher, (*f*) place of publication, and (*g*) year of publication. The chapter or page number is often listed just before the publisher's name. Titles of theses are often omitted.

   Peters, M. S., "Elementary Chemical Engineering," 2d ed., p. 280, McGraw-Hill Book Company, New York, 1984.

   Heaney, M., PhD. Thesis in Chem. Eng., Univ. of Colorado, Boulder, CO. 1988.

3. For unknown or unnamed authors: (a) alphabetize by the journal or organization publishing the information.

   Chem. Eng., **95**(13):26 (1988).

4. For patents: (a) patentees' names, followed by initials, and assignee (if any) in parentheses, (*b*) country granting patent and number, and (*c*) date issued (in parentheses).

   Fenske, E. R. (to Universal Oil Products Co.), U.S. Patent 3,249,650 (May 3, 1986).

5. For unpublished information: (*a*) "in press" means formally accepted for publication by 'the indicated journal or publisher; (*b*) the use of "private communication" and "unpublished data" is not recommended unless absolutely necessary, because the reader may find it impossible to locate the original material.

   Morari, M., *Chem. Eng. Progr.*, in press (1988).

Sample Calculations

The general method used in developing the proposed design is discussed in the body of the report, but detailed calculation methods are not presented in this section. Instead, sample calculations are given in the appendix. One example should be shown for each type of calculation, and sufficient detail must be included to permit the reader to follow each step. The particular conditions chosen for the sample calculations must be designated. The data on which the calculations are based should be listed in detail at the beginning of the section, even though these same data may be available through reference to one of the tables presented with the report.

Mechanical Details

The final report should be submitted in a neat and businesslike form. Formal reports are usually bound with a heavy cover, and the information shown in the title page is repeated on the cover. If paper fasteners are used for binding in a folder, the pages should be attached only to the back cover.

The report should be typed on a good grade paper with a margin of at least 1 in. on all sides. Normally, only one side of the page is used and all material, except the letter of transmittal, footnotes, and long quotations, is double-spaced. Starting with the summary, all pages including graphs, illustrations, and tables should be numbered in sequence.

Written material on graphs and illustrations may be typed or lettered neatly in ink. If hand lettering is required, best results are obtained with an instrument such as a LeRoy or Wrico guide.

Short equations can sometimes be included directly in the written text if the equation is not numbered. In general, however, equations are centered on the page and given a separate line, with the equation number appearing at the right-hand margin of the page. Explanation of the symbols used can be presented immediately following the equation.

Proofreading and Checking

Before final submittal, the completed report should be read carefully and checked for typographical errors, consistency of data quoted in the text with those presented in tables and graphs, grammatical errors, spelling errors, and similar obvious mistakes. If excessive corrections or changes are necessary, the appearance of the report must be considered and some sections may need to be retyped.

Nomenclature

If many different symbols are used repeatedly throughout a report, a table of nomenclature, showing the symbols, meanings, and sample units, should be included in the report. Each symbol can be defined when it first appears in the written text. If this is not done, a reference to the table of nomenclature should be given with the first equation.

Ordinarily, the same symbol is used for a given physical quantity regardless of its units. Subscripts, superscripts, and lower- and upper-case letters can be employed to give special meanings. The nomenclature should be consistent with common usage (a list of recommended symbols for chemical engineering quantities is presented in Table 1).†

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†The nomenclature presented in Table 1 is consistent with the recommendations of the American Standards Association presented as American Standard ASA Y10.12. See also Appendix A for rules and recommendations relative to the use of the SI system of units.

TABLE 1

Letter symbols for chemical engineering

Listing is alphabetical by concept within each category. Illustrative units or definitions are supplied where appropriate?

Concept	Symbol	Unit of definition (U.S. customary system)	Concept	Symbol	Unit of definition (U.S. customary system)
1. General concepts					
Acceleration	a	(ft/s)/s	Moment of inertia	I	$(ft)^4$
Of gravity	g	(ft/s)/s	Newton law of motion, conversion factor in	g_c	$g_c = ma/F$, (lb)(ft)/$(s)^2$(lbf)
Base of natural logarithms	e				
Coefficient	C		Number		
Difference, finite	Δ		In general	N	
Differential operator	d		Of moles	n	
partial	a		Pressure	p	lbf/ft^2 ; atm
Efficiency	η		Quantity, in general	Q	
Energy, dimension of	E	Btu (ft)(lbf)	Ratio, in general	R	
Enthalpy	H	Btu	Resistance	R	
Entropy	S	Btu/°R	Shear stress	τ	lbf/ft^2
Force	F	lbf	Temperature		
Function	φ, ψ, χ		Dimension of	θ	
Gas constant, universal	R	To distinguish, use R_0	Absolute	T	K (Kelvin); °R (Rankine)
Gibbs free energy	G, F	$G = H - TS$, Btu	In general	T, t	°C; °F
Heat	Q	Btu	Temperature difference, logarithmic mean	$\bar{\theta}$	°F
Helmholtz free energy	A	$A = U - TS$, Btu			
Internal energy	U	Btu	Time		
Mass, dimension of	m	lb	Dimension of	T	s
			In general	t, τ	s;h
Mechanical equivalent of heat	J	(ft) (lbf)/Btu	Work	W	Btu

2. Geometrical concepts

Linear dimension		
Breadth	b	ft
Diameter	D	ft
Distance along path	s, x	ft
Height above datum plane	Z	ft
Height equivalent	H	ft (Use subscript p for equilibrium stage and t for transfer unit)
Hydraulic radius	r_H	ft; ft^2/ft
Lateral distance from datum plane	Y	ft
Length, distance or dimension of	L	ft
Longitudinal distance from datum plane	X	ft
Mean free path	λ	cm; ft
Radius	r	ft
Thickness		
In general	B	ft
Of film	B_f	ft

Wavelength	λ	cm; ft
Area		
In general	A	ft^2
Cross section	S	ft^2
Fraction free cross-section	σ	
Projected	A_P	ft^2
Surface		
Per unit mass	A_w, S	ft^2/lb
Per unit volume	A_v, a	ft^2/ft^3
Volume		
In general	V	ft^3
Fraction voids	ϵ	
Humid volume	v_H	ft^3/lb dry air
Angle	α, θ, ϕ	
In x, y plane	α	
In y, z plane	ϕ	
in z, x plane	θ	
Solid angle	ω	
Other		
Particle-shape. factor	ϕ_s	

3. Intensive properties

Absorptivity for radiation	α	
Activity	a	
Activity coefficient, molal basis	γ	
Coefficient of expansion		
Linear	α	(ft/ft)/°F
Volumetric	β	(ft^3/ft^3)/°F

Diffusivity		
Molecular, volumetric	D_v, δ	ft” /(h) (ft); ft^2/h
Thermal	α	$\alpha = k/c\rho$, ft^2/h
Emissivity ratio for radiation	ϵ	
Enthalpy, per mole	H	Btu/lb mol
Entropy, per mole	S	Btu/(lb mol) (°R)

(continued)

TABLE 1

Letter symbols for chemical engineering (Continued)

Listing is alphabetical by concept within each category. Illustrative units or definitions are supplied where **appropriate†**

Concept	Symbol	Unit of definition (U.S. customary system)
Compressibility factor	z	$z = pV/RT$
Density	P	lb/ft^3
Helmholtz free energy, per mole	A	Btu/lb mol
Humid heat	c_s	Btu(lb dry air) (° F)
Internal energy, per mole	U	Btu/lb mol
Latent heat, phase change	λ	Btu/lb
Molecular weight	M	lb
Reflectivity for radiation	P	
Specific heat	c	Btu/(lb) (° F)
At constant pressure	c_p	Btu/(lb) (° F)
At constant volume	c_v	Btu/(lb) (° F)

Concept	Symbol	Unit of definition (U.S. customary system)
Fugacity	f	lbf/ft^2 ; atm
Gibbs free energy, per mole	G, F	Btu/lb mol
Specific heats, ratio of	γ	
Surface tension	σ	lbf/ft
Thermal conductivity	k	Btu/(h) (ft^2) (° F/ft)
Transmissivity of radiation	τ	
Vapor pressure	p^*	lbf/ft^2 ; atm
Viscosity		
Absolute or coefficient of	μ	lb/(s)(ft)
Kinematic	ν	ft^2/s
Volume, per mole	V	ft^3 /lb mol

4. Symbols for concentrations

Concept	Symbol	Unit of definition (U.S. customary system)
Absorption factor	A	$A = L/K^* V$
Concentration, mass or moles per unit volume	c	lb/ft^3 ; lb mol/ft^3
Fraction		
Cumulative beyond a given size	ϕ	
By volume	x_v	
By weight	x_w	
Humidity	H, Y_H	lb/lb dry air
At saturation	H_s, Y^*	lb/lb dry ah
At wet-bulb temperature	H_w, Y_w	lb/lb dry air

Concept	Symbol	Unit of definition (U.S. customary system)
Mole or mass fraction		
In heavy or extract phase	x	
In light or raftinate phase	Y	
Mole or mass ratio		
In heavy or extract phase	X	
In light or raffinate phase	Y	
Number concentration of particles	n_p	number/ft^3

At adiabatic saturation temperature	H_a, Y_a	lb/lb dry air
Mass concentration of particles	c_p	lb/ft^3
Moisture content		
Total water to bone-dry stock	X_T	lb/lb dry stock
Equilibrium water to bone-dry stock	X^*	lb/lb dry stock
Free water to **bone-dry** stock	X	lb/lb dry stock

Phase equilibrium ratio	K^*	$K^* = y^*/x$
Relative distribution of two components		
Between two phases in equilibrium	α	$\alpha = K_i^*/K_j^*$
Between successive stages	β	$\beta_n = (y_i/y_j)_n/(x_j/x_i)_{n+1}$
Relative humidity	H_R, R_H	
Slope of equilibrium curve	m	$m = dy^*/dx$
Stripping factor	S	$S = K^*V/L$

5. Symbols for rate concepts

Quantity per unit time, in general	4	
Angular velocity	ω	
Feed rate	F	lb/h; lb mol/h
Frequency	f, N_f	
Friction velocity	u^*	$u^* = (g_c \tau_w \rho)^{1/2}$, ft/s
Heat transfer rate	4	Btu/h
Heavy or extract phase rate	L	lb/h; lb mol/h
Heavy or extract product rate	B	lb/h; lb mol/h
Light or raffinate phase rate	V	lb/h; lb mol/h
Light or raffinate product rate	D	lb/h; lb mol/h
Mass rate of flow	w	lb/s; lb/h
Molal rate of transfer	N	lb mol/h
Power	P	(ft) (lbf)/(s)
Revolutions per unit time	n	

In general	u	ft/s
Instantaneous local		
Longitudinal (x) component of	u	ft/s
Lateral (y) component of	v	ft/s
Normal (z) component of	w	ft/s
Volumetric rate of flow	q	ft^3/s; ft^3/h
Quantity per unit time, unit area		
Emissive power, total	W	Btu/(h) (ft^2)
Mass velocity, average	G	$G = w/S$, lb/(s) (ft^2)
Vapor or light phase	G, $\bar{G}$	lb/(h) (ft^2)
Liquid or heavy phase	L, L	lb/(h) (ft^2)
Radiation, intensity of	I	Btu/(h) (ft^2)
Velocity		
Nominal, basis total cross section of packed vessel	v_S	ft/s

(continued)

TABLE 1

Letter symbols for chemical engineering (Continued)

Listing is alphabetical by concept within each category. Illustrative units or definitions are supplied where appropriate?

Concept	Symbol	Unit of definition (U.S. customary system)
Velocity		
Quantity per unit time, unit volume		
Quantity reacted per unit time, reactor volume	N_R	(mol/s)/ft³
Space velocity, volumetric	Λ	(ft³/s)/ft³
Quantity per unit time, unit area, unit driving force, in general	k	
Eddy diffusivity	δ_E	ft²/h
Eddy viscosity	ν_E	ft²/h
Eddy thermal diffusivity	α_E	ft²/h
Volumetric average	$V, \bar{V}$	(ft²/s)/ft²; ft/s
Heat transfer coefficient		
Individual	h	Btu/(h)(ft²)(°F)
Overall	U	Btu/(h)(ft²)(“F)
Mass transfer coefficient		
Individual	k	lb mol/(h)(ft²)(driving force) To define driving force, use subscript: c for lb mol/ft³ *p* for atm *x* for mole fraction
Gas film	k_G	
Liquid film	k_L	
Overall	K	
Gas film basis	K_G	
Liquid film basis	K_L	
Stefan-Boltzmann constant	σ	0.173 x 10^{-8} Btu/(h)(ft²)(°R)⁴

6. Dimensionless numbers used in chemical engineering

Condensation number	N_{Co}	$\frac{h}{k}\left(\frac{\nu^2}{a}\right)^{1/3}$; $\frac{h}{k}\left(\frac{\nu^2}{g}\right)^{1/3}$
Euler number	N_{Eu}	$\frac{g_c p}{\rho u^2}$; $\frac{g_c \rho p}{G^2}$
Fanning friction factor	f	$\frac{g_c \rho D(\Delta p_f)}{2G^2(\Delta L)}$
Fourier number	N_{Fo}	$\frac{kt}{c\rho L^2}$ or $\frac{\alpha t}{L^2}$
Nusselt number	N_{Nu}	$\frac{hL}{k}$; $\frac{hD}{k}$
Peclet number	N_{Pe}	$\frac{Luc\rho}{k}$ or $\frac{Lu}{\alpha}$. $\frac{DV}{\alpha}$
Prandtl number	N_{Pr}	$\frac{c\mu}{k}$ or $\frac{\nu}{\alpha}$
Prandtl velocity ratio	u^+	$\frac{\bar{u}}{u^*}$

6. Dimensionless numbers used in chemical engineering (continued)

Froude number	N_{Fr}	$\frac{u^2}{aL}$; $\frac{u^2}{gL}$
Graetz number	N_{Gz}	$\frac{cLG}{k}$ or $\frac{LV}{\alpha}$
Grashof number	N_{Gr}	$\frac{L^3\rho^2\beta g\Delta t}{\mu^2}$ or $\frac{L^3\beta g\Delta t}{\nu^2}$
Heat transfer factor	j_H	$\frac{h}{cG}\left(\frac{c\mu}{k}\right)^{2/3}$ or $(N_{St})(N_{Pr})^{2/3}$
Lewis number	N_{Le}	$\frac{k}{c\rho D_v}$ or $\frac{\alpha}{D_v}$
Mass transfer factor	j_M	$\frac{k_c}{u}\left(\frac{\mu}{\rho D_v}\right)^{2/3}$
Reynolds number	N_{Re}	$\frac{Lu\rho}{\mu}$; $\frac{DG}{\mu}$
Reynolds number, local	y^+	$\frac{ru^*\rho}{\mu}$
Schmidt number	N_{Sc}	$\frac{\mu}{\rho D_v}$
Sherwood number	N_{Sh}	$\frac{k_cL}{D_v}$ or $j_M(N_{Re})(N_{Sc})^{1/3}$
Stanton number	N_{St}	$\frac{h}{c\rho u}$, $\frac{h}{cG}$
Vapor condensation number	N_{Cv}	$\frac{L^3\rho^2g\lambda}{k\mu\Delta t}$
Weber number	N_{We}	$\frac{Lu^2\rho}{g_c\sigma}$; $\frac{DG^2}{g_c\rho\sigma}$

(continued)

TABLE 1
Letter symbols for chemical engineering (Continued)

Listing is alphabetical by concept within each category. Illustrative units or definitions are supplied where appropriate7

Concept	Symbol	Unit of definition (U.S. customary system)		Concept	Symbol	Unit of definition (U.S. customary system)	
7. Modifying signs for principal symbols							
Concept	Remarks	Superscript	Subscript	Concept	Remarks	Superscript	Subscript
Average value	Written over symbol	‾ (Bar)		Partial molal quantity	Written over small capitals	‾ (Bar)	
Dimensionless form	Follows symbol	+ (Plus)		Sequence in time or space	Follows symbol	′ (Prime) ″ (Double prime)	1, 2, 3, etc.
Equilibrium value	Follows symbol	* (Asterisk)		Standard state	Follows symbol	° (Degree)	
Fluctuating component	Usually applied to local velocity	′ (Prime)		First derivative with respect to time	Written over symbol	. (Dot)	
Initial or reference value	Follows symbol		0 (zero)	Second derivative with respect to time	Written over symbol	.. (Double dot)	
Modified form	Follows symbol	′ (Prime) ″ (Double prime)					

†Units shown are for the U.S. Customary system. See Appendix A for units in the SI system. For a revised set of recommendations for symbols for use in chemical engineering based on the SI system, see E. Buck, Letter Symbols for Chemical Engineering, *Chem. Eng. Progr.*, 74(10):73 (1978) and AIChE Publication X-95, "SI Guide for AIChE," Latest Edition, 345 E. 47th St., New York, NY 10017.

Abbreviations

Time and space can be saved by the use of abbreviations, but the writer must be certain the reader knows what is meant. Unless the abbreviation is standard, the meaning should be explained the first time it is used. The following rules are generally applicable for the use of abbreviations:

1. Abbreviations are acceptable in tables, graphs, and illustrations when space limitations make them desirable
2. Abbreviations are normally acceptable in the text only when preceded by a number [3 cm/s (three centimeters per second)]
3. Periods may be omitted after abbreviations for common scientific and engineering terms, except when the abbreviation forms another word (e.g., in. for inch)
4. The plural of an abbreviation is the same as the singular (pounds--lb) (kilograms-kg)
5. The abbreviation for a noun derived from a verb is formed by adding *n* (concentration-concn)
6. The abbreviation for the past tense is formed by adding *d* (concentrated-coned)
7. The abbreviation for the participle is formed by adding *g* (concentrating—concg)

Examples of accepted abbreviations are shown in Table 2.

RHETORIC

Correct grammar, punctuation, and style of writing are obvious requirements for any report. Many engineers, however, submit unimpressive reports because they do not concern themselves with the formal style of writing required in technical reports. This section deals with some of the restrictions placed on formal writing and presents a discussion of common errors.

Personal Pronouns

The use of personal pronouns should be avoided in technical writing. Many writers eliminate the use of personal pronouns by resorting to the passive voice. This is certainly acceptable, but, when applicable, the active voice gives the writing a less stilted style. For example, instead of saying “We designed the absorption tower on the basis of . . . ,” a more acceptable form would be “The absorption tower was designed on the basis of . . . ” or “The basis for the absorption-tower design was . , . .”

The pronoun “one” is sometimes used in technical writing. In formal writing, however, it should be avoided or, at most, employed only occasionally.

TABLE 2
Accepted abbreviations

American Chemical Society	ACS
American Institute of Chemical Engineers	AIChE
American Iron and Steel Institute	AISI
American Petroleum Institute	API
American Society of Mechanical Engineers	ASME
American Society for Testing Materials	ASTM
American wire gauge	AWG
Ampere	A
Angstrom	Å
Atmosphere	atm
Average	avg
Barrel	bbl
Baumé	Bé
Biochemical oxygen demand	BOD
Boiling point	bp
Bottoms	btms
British thermal unit	Btu
Brown and Sharpe gauge number	B&S
Calorie	cal
Capacity	cap.
Catalytic	cat.
Centigrade	C
Centigram	cg
Centimeter	cm
Centipoise	cP
Centistoke	cs
Chemically pure	CP
Concentrate	conc
Critical	crit
Cubic	cu
Cubic centimeter	cc
Cubic foot	cu ft or ft^3
Cubic foot per minute	cfm or ft^3/min
Cubic foot per second	cfs or ft^3/s
Cubic inch	cu in. or in.^{3}
Degree	deg or °
Diameter	diam
Dilute	dil
Distill or distillate	dist

Efficiency	eff
Electromotive force	emf
Equivalent	equiv
Ethyl	Et
Evaporate	evap
Experiment	expt
Experimental	exptl
Extract	ext
Feet per minute	fpm or ft/min
Figure	fig.
Foot	ft
Foot pound	ft.lb
Gallon	gal
Gallons per minute	gpm or gal/min
Grain	spell out
Gram	gm (sometimes g) or spell out
Height equivalent to a theoretical plate	HETP
Height of a transfer unit	HTU
Horsepower	hp
Hour	h
Hundredweight (100 lb)	cwt
Inch	in.
Inside diameter	ID or i.d.
Insoluble	insol
Kilogram	kg
Kilometer	km
Kilovolt	kV
Kilowatt	kW
Kilowatt-hour	kWh
Liquid	liq
Liter	1 or spell out
Logarithm (base 10)	log
Logarithm (base e)	ln
Maximum	max
Melting point	mp

TABLE 2
Accepted abbreviations *(Continued)*

Meter	m	Saybolt Universal seconds	sus
Methyl	Me	Second	s
Micron	p or mu	Society of Automotive Engineers	SAE
Mile	mi	Soluble	sol
Miles per hour	mph	Solution	soln
Milliampere	mA	Specific gravity	sp gr
Million electron volts	meV	Specific heat	sp ht
Millivolt	mV	Square	sq
Minute	min	Square foot	sq ft or ft^2
Molecular	mol	Standard	std
Ounce	oz	Standard temperature and pressure	STP
Outside diameter	OD or o.d.		
Overhead	ovhd	Tank	tk
		Technical	tech
Page	p.	Temperature	temp
Pages	pp.	Tetraethyl lead	TEL
Parts per million	ppm	Thousand	M
Pint	pt	Ton	spell out
Pound	lb		
Pound centigrade unit	Pcu	Tubular Exchangers Manufacturers Association	TEMA
Pounds per cubic foot	lb/cu ft or lb/ft^3		
Pounds per square foot	lb/sq ft or lb/ft^2	Volt	V
Pounds per square inch	psi or $lb/in.^2$	Volt-ampere	VA
Pounds per square inch absolute	psia	Volume	vol
Pounds per square inch gauge	psig		
		watt	W
Quart	qt	Watthour	Wh
		Weight	wt
Refractive index	RI or n		
Revolutions per minute	rpm or r/min	Yard	yd

Tenses

Both past and present tenses are commonly used in report writing; however, tenses should not be switched in one paragraph or in one section unless the meaning of the written material requires the change. General truths that are not limited by time are stated in the present tense, while references to a particular event in the past are reported in the past tense (e.g., “The specific gravity of mercury is 13.6.” “The experiment was performed . . . ”).

Diction

Contractions such as “don’t” and “can’t” are seldom used in technical writing, and informal or colloquial words should be avoided. Humorous or witty statements are out of place in a technical report, even though the writer may feel they are justified because they can stimulate interest. Too often, the reader will be devoid of a sense of humor, particularly when engrossed in the serious business of digesting the contents of a technical report. A good report is made interesting by clarity of expression, skillful organization, and the significance of its contents.

Singular and Plural

Many writers have difficulty in determining if a verb should be singular or plural. This is especially true when a qualifying phrase separates the subject and its verb. For example, "A complete list of the results ***is*** (not ***are***) given in the appendix."

Certain nouns, such as "number," and "series," can be either singular or plural. As a general rule, the'verb should be singular if the subject is viewed as a unit ("The number of engineers in the United States is increasing") and plural if the things involved are considered separately ("A number of the workers are dissatisfied"). Similarly, the following sentences are correct: "***Thirty thousand gallons was*** produced in two hours." "The tests show that 18 batches were run at the wrong temperature."

Dangling Modifiers

The technical writer should avoid dangling modifiers that cannot be associated directly with the words they modify. For example, the sentence "Finding the results were inconclusive, the project was abandoned" could be rewritten correctly as "Finding the results were inconclusive, the investigators abandoned the project."

Poor construction caused by dangling modifiers often arises from retention of the personal viewpoint, even though personal pronouns are eliminated. The writer should analyze the work carefully and make certain the association between a modifying phrase and the words referred to is clear.

Compound Adjectives

Nouns are often used as adjectives in scientific writing. This practice is acceptable; however, the writer must use it in moderation. A sentence including "a centrally located natural gas production plant site is . , . " should certainly be revised. Prepositional clauses are often used to eliminate a series of compound adjectives.

Hyphens are employed to connect words that are compounded into adjectives-for example, "a hot-wire anemometer," "a high-pressure line"-but no hyphen appears in "a highly sensitive element."

Split Infinitives

Split infinitives are acceptable in some types of writing, but they should be avoided in technical reports. A split infinitive bothers many readers, and it frequently results in misplaced emphasis. Instead of "The supervisor intended to carefully check the data," the sentence should be "The supervisor intended to check the data carefully."

That-Which

Many technical writers tend to overwork the word *that*. Substitution of the word *which* for *that* is often acceptable, even though a strict grammatical interpretation would require repetition of *that*. The general distinction between the pronouns "that" and "which" can be stated as follows: *That* is used when the clause it introduces is necessary to define the meaning of its antecedent; *which* introduces some additional or incidental information.

COMMENTS ON COMMON ERRORS

1. The word ***data*** is usually plural. Say "data are," not "data is."
2. "Balance" should not be used when "remainder" is meant.
3. Use "different from" instead of "different than."
4. The word "farther" refers to distance, and "further" indicates "in addition to."
5. "Affect," as a verb, means "to influence." It should never be confused with the noun "effect," which means "result."
6. "Due to" should be avoided when "because of' can be used.
7. Use "fewer" when referring to numbers and "less" when referring to quantity or degree.

CHECK LIST FOR THE FINAL REPORT

Before submitting the final draft, the writer should make a critical analysis of the report. Following is a list of questions the writer should ask when evaluating the report. These questions cover the important considerations in report writing and can serve as a guide for both experienced and inexperienced writers.

1. Does the report fulfill its purpose?
2. Will it be understandable to the principal readers?
3. Does the report attempt to cover too broad a subject?
4. Is sufficient information presented?
5. Is too much detail included in the body of the report?
6. Are the objectives stated clearly?

7. Is the reason for the report indicated?
8. Is the summary concise? Is it clear? Does it give the important results, conclusions, and recommendations? Is it a true summary of the entire report?
9. Is there an adequate description of the work done?
10. Are the important assumptions and the degree of accuracy indicated?
11. Are the conclusions and recommendations valid?
12. Are sufficient data included to support the conclusions and recommendations?
13. Have previous data and earlier studies in the field been considered?
14. Is the report well organized?
15. Is the style of writing readable and interesting?
16. Has the manuscript been rewritten and edited ruthlessly?
17. Is the appendix complete?
18. Are tables, graphs, and illustrations presented in a neat, readable, and organized form? Is all necessary information shown?
19. Has the report been proofread? Are pages, tables, and figures numbered correctly?
20. Is the report ready for submittal on time?

PROBLEMS

1. Prepare a skeleton outline and a detailed outline for a final report on the detailed-estimate design of a distillation unit. The unit is to be used for recovering methanol from a by-product containing water and methanol. In the past, this by-product has been sold to another concern, but the head of the engineering-development group feels that the recovery should be accomplished by your company. The report will be examined by the head of the engineering-development group and will then be submitted to the plant management for final approval.
2. List ten words you often misspell and five grammatical errors you occasionally make in formal writing.

Correct the following sentences:

3. "Using the mass transfer coefficients and other physical data, Schmidt and Nusselt numbers were calculated for each experiment."
4. "The excellent agreement between the experimental and theoretical values substantiate the validity of the assumptions."
5. "This property makes the packing more efficient than any packing."
6. "He is an engineer with good theoretical training and acquainted with industrial problems."
7. "Wrought iron is equally as good as stainless steel because no temperatures will be used at above 25°C."
8. "The pressure has got to be maintained constant or the tank will not empty out at a constant rate."

Interpret, rewrite, and improve the following:

9. "The purchasing division has contracted with the X Chemical Company to supply 20,000 kg of chemical ***A*** which corresponds closely to the specifications presented and 10,000 kg of chemical ***E.***"
10. "A more rigorous derivation would be extremely complicated and would hardly be justified in view of the uncertainties existing with respect to basic information necessary for practical applications of the results."
11. "An important factor in relation to safety precautions is first and foremost giving to workmen some kind of a clear and definite instruction along the line of not coming into the radioactive areas in connection with their work."

CHAPTER

14

MATERIALS TRANSFER, HANDLING, AND TREATMENT EQUIPMENT— DESIGN AND COSTS

The design and cost estimation for equipment items and systems used for the transfer, handling, or treatment of materials is vitally involved in almost every type of plant design. The most common means for transferring materials is by pumps and pipes. Conveyors, chutes, gates, hoists, fans, and blowers are examples of other kinds of equipment used extensively to handle and transfer various materials. Many forms of special equipment are used for the treatment of materials, as, for example, filters, blenders, mixers, kneaders, centrifugal separators, crystallizers, crushers, grinders, dust collectors, kettles, reactors, and screens. The design engineer must decide which type of equipment is best suited for the purposes and be able to prepare equipment specifications that will satisfy the operational demands of the process under reasonable cost conditions. Consequently, theoretical design principles, practical problems of operation, and cost considerations are all involved in the final choice of materials transfer, handling, and treatment equipment.

PUMPS AND PIPING

POWER REQUIREMENTS

A major factor involved in the design of pumping and piping systems is the amount of power that is required for the particular operation. Mechanical power must be supplied by the pump to overcome frictional resistance, changes in elevation, changes in internal energy, and other resistances set up in the flow system.

The various forms of energy can be related by the total energy balance or the total mechanical-energy balance. On the basis of 1 lbm[†] of fluid flowing under steady conditions, the total energy balance may be written in differential form as

$$\frac{g}{g_c}\, dZ + d(pv) + \frac{V_i\, dV_i}{g_c} + du = \delta Q + \delta W \tag{1}$$

The total mechanical-energy balance in differential form is

$$\frac{g}{g_c}\, dZ + v\, dp + \frac{V_i\, dV_i}{g_c} = S W, - \delta F \tag{2}$$

where g = local gravitational acceleration, usually taken as 32.17 ft/(s)(s)

g_c = conversion factor in Newton's law of motion, 32.17 ft · lbm mass/(s)(s)(lbf)

Z = vertical distance above an arbitrarily chosen datum plane, ft

v = specific volume of the fluid ft^3/lbm

p = absolute pressure, lbf/ft^2

V_i = instantaneous or point velocity, ft/s

u = internal energy, ft · lbf/lbm

Q = heat energy imparted as such to the fluid system from an outside source, ft · lbf/lbm

W = shaft work, gross work input to the fluid system from an outside source, ft · lbf/lbm

W_o = mechanical work imparted to the fluid system from an outside source,[‡] ft · lbf/lbm

F = mechanical-energy loss due to friction, ft · lbf/lbm

[†] The U.S. customary system of units is used here to clarify the role of g and g_c and to show the common usage by design engineers in the U.S.

[‡] The mechanical work W_o is equal to the total shaft work W minus the amount of energy transmitted to the fluid as a result of pump friction or pump inefficiency. When W_o is used in the total mechanical-energy balance, pump friction is not included in the term for the mechanical-energy loss due to friction F.

Integration of these energy balances between "point 1" where the fluid enters the system and "point 2" where the fluid leaves the system gives
Total energy balance:

$$Z_1\frac{g}{g_c} + p_1v_1 + \frac{V_1^2}{2\alpha g_c} + u_1 + Q + W = Z_2\frac{g}{g_c} + p_2v_2 + \frac{V_2^2}{2\alpha g_c} + u_2 \qquad (3)$$

Total mechanical-energy balance:

$$Z_1\frac{g}{g_c} - \int_1^2 v\,dp + \frac{V_1^2}{2\alpha g_c} + W_o = Z_2\frac{g}{g_c} + \frac{V_2^2}{2\alpha g_c} + \Sigma F \qquad (4)$$

where V is the average linear velocity, ft/s, and α is the correction coefficient to account for use of average velocity, usually taken as 1.0 if flow is turbulent and 0.5 if flow is viscous. Equations (1) through (4) are sufficiently general for treatment of almost any flow problem and are the basis for many design equations that apply for particular simplified conditions.

Evaluation of the term $\int_1^2 v\,dp$ in Eq. (4) may be difficult if a compressible fluid is flowing through the system, because the exact path of the compression or expansion is often unknown. For noncompressible fluids, however, the specific volume v remains essentially constant and the integral term reduces simply to $v(p_2 - p_1)$. Consequently, the total mechanical-energy balance is especially useful and easy to apply when the flowing fluid can be considered as noncompressible.

Friction

Frictional effects are extremely important in flow processes. In many cases, friction may be the main cause for resistance to the flow of a fluid through a given system. Consider the common example of water passing through a pipe. If no frictional effects were present, pipes of very small diameters could be used for all flow rates. Under these conditions, the pumping-power costs for forcing 100,000 gal of water per hour through a $\frac{1}{8}$-in.-diameter pipe would be the same as the power costs for forcing water at the same mass rate through a pipe of equal length having a diameter of 2 ft. In any real flow process, however, frictional effects are present, and they must be taken into consideration.

When a fluid flows through a conduit, the amount of energy lost due to friction depends on the properties of the flowing fluid and the extent of the conduit system. For the case of steady flow through long straight pipes of uniform diameter, the variables that affect the amount of frictional losses are the velocity at which the fluid is flowing (V), the density of the fluid (ρ), the viscosity of the fluid (μ), the diameter of the pipe (D), the length of the pipe (L), and the equivalent roughness of the pipe (ε). By applying the method of dimensional analysis to these variables, the following expression, known as the

Funning equation, can be obtained for the frictional effects in the system:

$$dF = \frac{-dp_f}{\rho} = \frac{2fV^2\,dL}{g_cD} \tag{5}$$

The friction factor f is based on experimental data and has been found to be a function of the Reynolds number and the relative roughness of the pipe (ε/D). Figure 14-1 presents a plot of the friction factor versus the Reynolds number in straight pipes. In the viscous-flow region, the friction factor is not affected by the relative roughness of the pipe; therefore, only one line is shown in Fig. 14-1 for Reynolds numbers up to about 2100. In the turbulent-flow region, the relative roughness of the pipe has a large effect on the friction factor. Curves with different parameters of the dimensionless ratio e/D are presented in Fig. 14-1 for values of Reynolds numbers greater than 2100. A table on the plot indicates values for ε for various pipe-construction materials. As the methods for determining f do not permit high accuracy (± 10 percent), the value of the friction factor should not be read to more than two significant figures, and Fig. 14-1 gives adequate accuracy for determining the numerical size of the friction factor.

The values for equivalent pipe roughness given in Fig. 14-1 are only approximations, even for new pipe, and the values may increase because of surface pitting and corrosion after the pipe is in service. The design engineer, therefore, should recognize the inherent inaccuracies in estimating the effects of pipe roughness, and this matter should be taken into consideration when the final design is prepared.

Curves similar to Fig. 14-1 are sometimes presented in the literature with a different defining value of f. For example, mechanical engineers usually define the friction factor so that it is exactly four times the friction factor given in Eq. (5).

The Reynolds number range between 2100 and 4000 is commonly designated as the *critical* region. In this range, there is considerable doubt as to whether the flow is viscous or turbulent. For design purposes, the safest practice requires the assumption that turbulent flow exists at all Reynolds numbers grater than 2100.

A mathematical expression for the friction factor can be obtained from the equation for the straight line in the viscous-flow region of Fig. 14-1. Thus, at Reynolds numbers below 2100

$$f = \frac{16}{N_{\mathrm{Re}}} = \frac{16\mu}{DV\rho} \tag{6}$$ †

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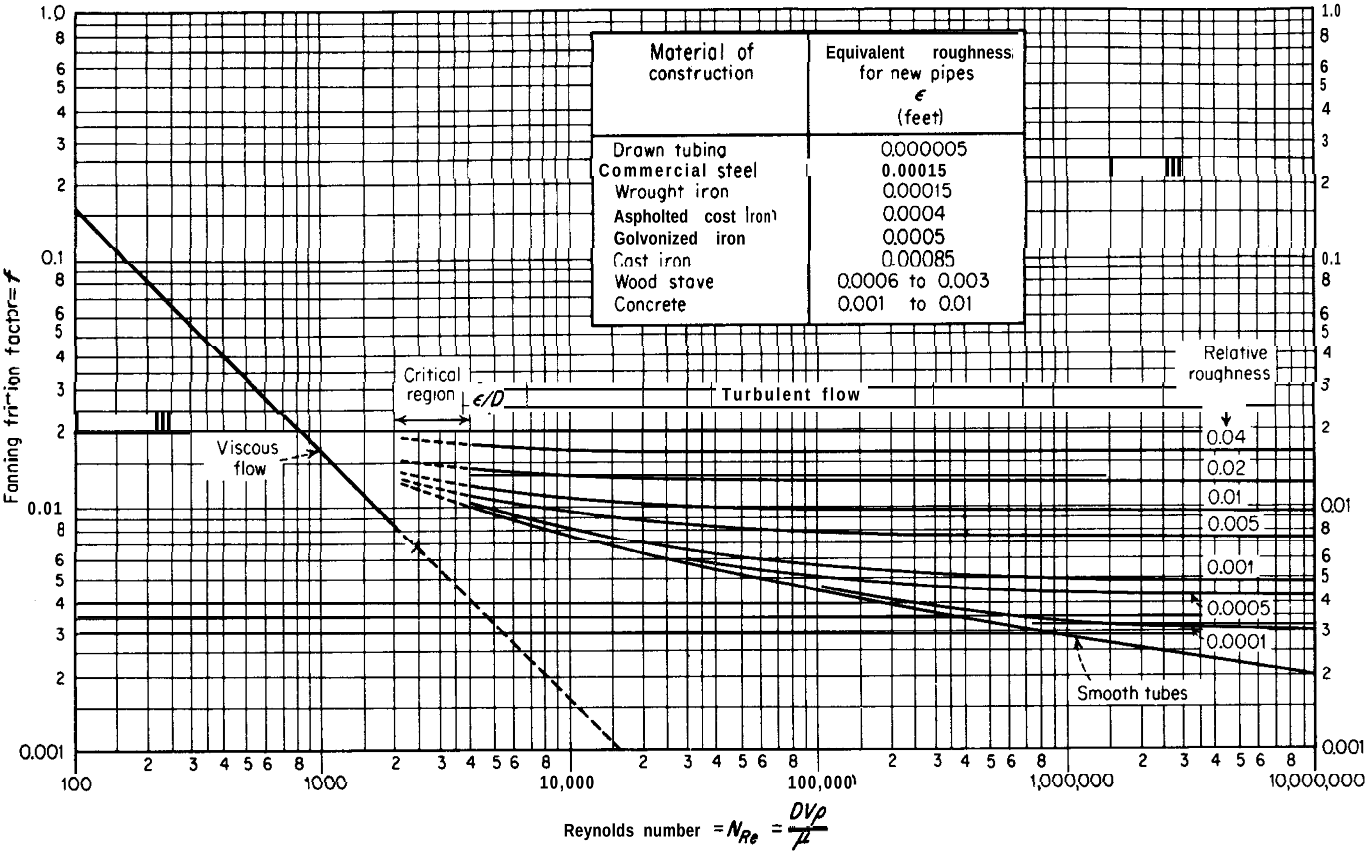
†**Substitution** of this expression for f into Eq. (5) results in the well-known **Hagen-Poiseuille** law for viscous flow.



Material of construction	Equivalent roughness for new pipes ϵ (feet)
Drawn tubing	0.000005
Commercial steel	0.00015
Wrought iron	0.00015
Asphalted cast iron	0.0004
Galvanized iron	0.0005
Cast iron	0.00085
Wood stave	0.0006 to 0.003
Concrete	0.001 to 0.01

FIGURE 14-1
Fanning friction factors for long straight pipes. [Based on L. F. Moody, *Trans. ASME*, **66**:671–684 (1944).]

Approximate equations showing the relationship between the friction factor and the Reynolds number in the turbulent-flow region have been developed. Two of these equations follow:

For smooth pipe or tubes,

$$f = \frac{0.046}{(N_{\mathrm{Re}})^{0.2}} \tag{7}$$

For new iron or steel pipe,

$$f = \frac{0.04}{(N_{\mathrm{Re}})^{0.16}} \tag{8}$$

INTEGRATED FORM OF THE FANNING EQUATION. If the linear velocity, density, and viscosity of the flowing fluid remain constant and the pipe diameter is uniform over a total pipe length L, Eq. (5) can be integrated to give the following result:

$$F = \frac{-\Delta p_f}{\rho} = \frac{2fV^2L}{g_cD} \tag{9}$$

In a strict sense, Eq. (9) is limited to conditions in which the flowing fluid is noncompressible and the temperature of the fluid is constant. When dealing with compressible fluids, such as air, steam, or any gas, it is good engineering practice to use Eq. (9) only if the pressure drop over the system is less than 10 percent of the initial pressure. If a change in the fluid temperature occurs, Eq. (9) should not be used in the form indicated unless the total change in the fluid viscosity is less than approximately 50 percent based on the maximum viscosity.†‡ If Eq. (9) is used when pressure changes or temperature changes are involved, the best accuracy is obtained by using the linear velocity, density, and viscosity of the fluid as determined at the average temperature and pressure. Exact results for compressible fluids or nonisothermal flow can be obtained from the Fanning equation by integrating the differential expression, taking all changes into consideration.

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†Overall effects of temperature on the friction factor are more important in the streamline-flow range where f is directly proportional to the viscosity than in the turbulent-flow range where f is approximately proportional to $\mu^{0.16}$.

‡For heating or cooling of fluids, a temperature gradient must exist from the pipe wall across the flowing fluid. A simplified design procedure for this case is as follows: When temperature and viscosity changes must be taken into consideration, the friction factor for use in Eq. (9) should be taken as the isothermal friction factor (Fig. 14-1) based on the arithmetic-average temperature of the fluid divided by a correction factor ϕ, where $\phi = 1.1(\mu_a/\mu_w)^{0.25}$ when DG/μ_a is less than 2100 and $\phi = 1.02(\mu_a/\mu_w)^{0.14}$ when DG/μ_a is greater than 2100. [G = mass velocity, lb/(h)(ft^2 of cross-sectional area); μ_a = viscosity of fluid at average hulk temperature, lb/(s)(ft); μ_w = viscosity of fluid at temperature of wall, lb/(s)(ft).]

TABLE 1

Expressions for evaluating frictional losses in the flow of fluids through conduits

For noncircular, cross-sectional area and turbulent flow, replace D by $4R_H = 4$ (cross-sectional flow area/wetted perimeter).

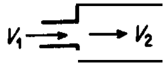
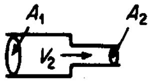
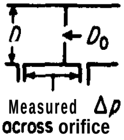
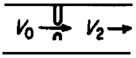
Friction caused by	General expression for frictional loss	Limited expression and remarks
Flow through long straight pipe of constant cross-sectional area	$dF = \frac{2f V^2 dL}{g_c D}$	For case in which fluid is essentially noncompressible and temperature is constant $F = \frac{2f V^2 L}{g_c D}$
Sudden enlargement	$F_e = \frac{(V_1 - V_2)^2}{2\alpha g_c}$	The following values for α may be used in design calculations: turbulent flow, $\alpha = 1$; streamline flow, $\alpha = 0.5$ 
Sudden contraction	$F_c = \frac{K_c V_2^2}{2\alpha g_c}$	The following values for α may be used in design calculations: turbulent flow, $\alpha = 1$; streamline flow, $\alpha = 0.5$  For $\frac{A_2}{A_1} < 0.715$, $K_c = 0.4\left(1.25 - \frac{A_2}{A_1}\right)$ For $\frac{A_2}{A_1} > 0.715$, $K_c = 0.75\left(1 - \frac{A_2}{A_1}\right)$ For conical or rounded shape, $K_c = 0.05$
Fittings, valves, etc.	$F = \frac{2f V^2 L_e}{g_c D}$	L_e/D per fitting (dimensionless) 45° elbows: 15 90° elbows, std. radius: 32 90° elbows, medium radius: 26 90° elbows, long sweep: 20 90° square elbows: 60 180° close-return bends: 75

TABLE 1
Expressions for evaluating frictional losses in the flow of fluids through conduits (*Continued*)

Friction caused by	General expression for rictional loss	Limited expression and remarks	
Fittings, valves, etc.	$F = \frac{2fV^2L_e}{g_cD}$		L_e/D per fitting (dimensionless)
		80° medium-radius return bends	50
		Tee (used as elbow, entering run)	60
		Tee (used as elbow, entering branch)	90
		Couplings	Negligible
		Unions	Negligible
		Gate valves, open	7
		Globe valves, open	300
		Angle valves, open	170
		Water meters, disk	400
		Water meters, piston	600
		Water meters, impulse wheel	300
Sharp-edged orifice	$-\Delta p_f = F\rho$	$\frac{D_0}{D}$	$\frac{\Delta p_f(100)}{\Delta p \text{ across orifice}} = \%$
		0.8	40
		0.7	52
		0.8	63
		0.5	73
		0.4	81
		0.3	89
		0.2	95
			
Rounded orifice	$F = \frac{(V_0 - V_2)^2}{2\alpha g_c}$	The following values for α may be used in design calculations: turbulent flow, $\alpha = 1$; streamline flow, $\alpha = 0.5$ 	
Venturi	$-\Delta p_f = F\rho$	$-\Delta p_f = \frac{1}{8}$ to $\frac{1}{10}$ of total pressure drop from upstream section to venturi throat	

For turbulent flow in a conduit of noncircular cross section, an equivalent diameter can be substituted for the circular-section diameter, and the equations for circular pipes can then be applied without introducing a large error. This equivalent diameter is defined as four times the hydraulic radius R_H, where the hydraulic radius is the ratio of the cross-sectional flow area to the wetted perimeter. When the flow is viscous, substitution of $4R_H$ for D does not give accurate results, and exact expressions relating frictional pressure drop and velocity can be obtained only for certain conduit shapes.

FRICTIONAL EFFECTS DUE TO END LOSSES, FITTINGS, ORIFICES, AND OTHER INSTALLATIONS. If the cross-sectional area of a pipe changes gradually to a new cross-sectional area, the disturbances to the flow pattern can be so small that the amount of mechanical energy lost as friction due to the change in cross section is negligible. If the change is sudden, however, an appreciable amount of mechanical energy can be lost as friction. Similarly, the presence of bends, fittings, valves, orifices, or other installations that disturb the flow pattern can cause frictional losses. All of these effects must be included in the friction term appearing in the total mechanical-energy balance. Recommended expressions for evaluating the important types of frictional losses are presented in Table 1.

Design Calculations of Power Requirements

LIQUIDS. For noncompressible fluids, the integrated form of the total mechanical-energy balance reduces to

$$W_o = \Delta Z + \Delta\left(\frac{V^2}{2\alpha g_c}\right) + \Delta(pv) + \sum F \qquad (10)$$

where g is assumed to be numerically equal to g_c. Because the individual terms in Eq. (10) can be evaluated from the physical properties of the system and the flow conditions, the design engineer can apply this equation to many liquid-flow systems without making any major assumptions. The following example illustrates the application of Eq. (10) for a design calculation of the size of motor necessary to carry out a given pumping operation.?

Example 1 Application of the total mechanical-energy balance to noncompressible-flow systems. Water at 61°F is pumped from a large reservoir into the top of an overhead tank using standard 2-in.-diameter steel pipe (ID = 2.067 in.). The reservoir and the overhead tank are open to the atmosphere, and the difference in vertical elevation between the water surface in the reservoir and the discharge point at the top of the overhead tank is 70 ft. The length of the pipeline

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†The examples in the Chapter are presented using the U.S. customary system of units. For similar examples using SI units, see M. S. Peters, “Elementary Chemical Engineering,” 2d ed., pp. 89-118, McGraw-Hill Book Company, New York, 1984.

is 1000 ft. Two gate valves and three standard 90° elbows are included in the system. The efficiency of the pump is 40 percent. This includes losses at the entrance and exit of the pump housing. If the flow rate of water is to be maintained at 50 gpm and the water temperature remains constant at 61°F, estimate the horsepower of the motor required to drive the pump.

Solution

Basis: 1 lb of flowing water

Total mechanical-energy balance between point 1 (surface of water in reservoir) and point 2 (just outside of pipe at discharge point):

$$W_o = Z_2 - Z_1 + \frac{V_2^2}{2\alpha g_c} - \frac{V_1^2}{2\alpha g_c} + p_2v_2 - p_1v_1 + \Sigma F$$

Points 1 and 2 are taken where the linear velocity of the fluid is negligible; therefore

$$\frac{V_2^2}{2\alpha g_c} = 0 \quad \text{and} \quad \frac{V_1^2}{2\alpha g_c} = 0$$

$p_1 = p_2$ = atmospheric pressure, $v_1 = v_2$, since liquid water can be considered as a noncompressible fluid. $p_2v_2 - p_1r_1 = 0$. Z_2 − Z, = 70 ft . lbf/lbm (assuming $g = g_c$).

Determination of friction:

$$\text{Average velocity in 2-in. pipe} = \frac{(50)(144)}{(60)(7.48)(2.067)^2(0.785)} = 4.78 \text{ ft/s}$$

Viscosity of water at 61°F = 1.12 cp = (1.12)(0.000672) lb/(ft)(s)

Density of water at 61°F = 62.3 lb/ft^3

$$\text{Reynolds number in 2-in. pipe} = \frac{(2.067)(4.78)(62.3)}{(12)(1.12)(0.000672)} = 68{,}000$$

$$\frac{\varepsilon}{D} = \frac{(0.00015)(12)}{2.067.} = 0.00087$$

Friction factor = $\mathfrak{f}$ = 0.0057 (from Fig. 14-1)

$$\text{Total } L_e \text{ for fittings and valves} = \frac{(2)(7)(2.067)}{12} + \frac{(3)(32)(2.067)}{12}$$

$$= 19 \text{ ft}$$

Friction due to flow through pipe and all fittings

$$= \frac{2\mathfrak{f}V^2(L + L,)}{g_cD}$$

$$= \frac{(2)(0.0057)(4.78)^2(1000 + 19)(12)}{(32.17)(2.067)}$$

$$= 47.9 \text{ ft . lbf/lbm}$$

Friction due to contraction and enlargement (from Table 1)

$$= \frac{(0.5)(4.78)^2}{(2)(1)(32.17)} + \frac{(4.78 - 0)^2}{(2)(1)(32.17)}$$

$$= 0.53 \text{ ft} \cdot \text{lbf/lbm}$$

$$\Sigma F = 47.9 + 0.53 = 48.4 \text{ ft} \cdot \text{lbf/lbm}$$

From the total mechanical-energy balance, W_0 = theoretical mechanical energy necessary from pump = 70 + 48.4 = 118.4 ft · lbf/lbm.

$$\text{hp of motor required to drive pump} = \frac{(118.4)(50)(62.3)}{(0.40)(60)(7.48)(550)} = 3.74 \text{ hp}$$

$$\left.\frac{\text{ft}\cdot\text{lbf}}{\text{lbm}}\right|\left.\frac{\text{gal}}{\text{min}}\right|\left.\frac{\text{min}}{\text{s}}\right|\left.\frac{\text{ft}^3}{\text{gal}}\right|\left.\frac{\text{lbm}}{\text{ft}^3}\right|\left.\frac{\text{s (hp)}}{\text{ft}\cdot\text{lbf}}\right| = \text{hp}$$

A 4.0-hp motor would be adequate for a design estimate.

GASES. Because of the difficulty that may be encountered in evaluating the exact integral of v *dp* and *dF* for compressible fluids, use of the total mechanical-energy balance is not recommended for compressible fluids when large pressure drops are involved. Instead, the total energy balance should be used if the necessary data are available.

If g is assumed to be numerically equal to g_c, the integrated form of the total energy balance [Eq. (3)] can be written as

$$W = \Delta Z + \Delta h + \Delta\left(\frac{V^2}{2\alpha g_c}\right) - Q \qquad (11)$$

where h = enthalpy = $u + pv$, ft · lbf/lbm.

When Eq. (11) is applied in design calculations, information must be available for determining the change in enthalpy over the range of temperature and pressure involved. The following illustrative examples show how Eq. (11) can be used to calculate pumping power when compressible fluids are involved.

Example 2 Application of total energy balance for the flow of an ideal gas. Nitrogen is flowing under turbulent conditions at a constant mass rate through a long, straight, horizontal pipe. The pipe has a constant inside diameter of 2.067 in. At an upstream point (point 1), the temperature of the nitrogen is 70°F, the pressure is 15 psia, and the average linear velocity of the gas is 60 ft/s. At a given downstream point (point 2), the temperature of the gas is 140°F and the pressure is 50 psia. An external heater is located between points 1 and 2, and 10 Btu is transferred from the heater to each pound of the flowing gas. Except at the heater, no heat is transferred as such between the gas and the surroundings. Under these conditions, nitrogen may be considered to be an ideal gas, and the mean heat capacity C_p of the gas is 7.0 Btu/(lb mol)(°F). Estimate the total amount of energy (as foot-pounds force per pound of the flowing gas) supplied by the compressor located between points 1 and 2.

Solution

Basis: 1 lb of flowing nitrogen

Total energy balance between points 1 and 2 for horizontal system and turbulent flow:

$$W = h_2 - h_1 + \frac{V_2^2}{2g_c} - \frac{V_1^2}{2g_c} - Q$$

where $V_1 = 50$ ft/s and $V_2 = (50)(600)(15)/(530)(50) = 17$ ft/s. Since nitrogen is to be considered an ideal gas,

$$h_2 - h_1 = \frac{C_p}{M}(T_2 - T_1) = \frac{(7)(140 - 70)(778)}{28} = 13{,}600 \text{ ft} \cdot \text{lbf/lbm}$$

$$Q = (10)(778) \text{ ft} \cdot \text{lbf/lbm}$$

W = total energy supplied by compressor

$$= 13{,}600 + \frac{(17)^2}{(2)(32.17)} - \frac{(50)^2}{(2)(32.17)} - 7780$$

$$= 5790 \text{ ft} \cdot \text{lbf/lbm}$$

Example 3 Application of total energy balance for the Row of a nonideal gas (steam turbine). Superheated steam enters a turbine under such conditions that the enthalpy of the entering steam is 1340 Btu/lb. On the same basis, the enthalpy of the steam leaving the turbine is 990 Btu/lb. If the turbine operates under adiabatic conditions and changes in kinetic energy and elevation potential energy are negligible, estimate the maximum amount of energy obtainable from the turbine per pound of entering steam.

Solution

Bask 1 lb of entering steam

For an adiabatic system and negligible change in potential and kinetic energies, the total energy balance becomes

$$W = h_2 - h, = 990 - 1340 = -350 \text{ Btu/lb of steam}$$

Maximum energy obtainable from the turbine = 350 Btu/lb of steam.

When the data necessary for application of the total energy balance are not available, the engineer may be forced to use the total mechanical-energy balance for design calculations, even though compressible fluids and large pressure drops are involved. The following example illustrates the general method for applying the total mechanical-energy balance under these conditions.†

[†] For additional discussion and methods for integrating the total mechanical-energy balance when the flow of a compressible fluid and high pressure drop are involved, see R. H. Perry and D. Green, "Chemical Engineers' Handbook," 6th ed., McGraw-Hill Book Company, New York, 1984.

Example 4 Application of the total mechanical-energy balance for the flow of a compressible fluid with high pressure drop. Air is forced at a rate of 15 lb/min through a straight, horizontal, steel pipe having an inside diameter of 2.067 in. The pipe is 3000 ft long, and the pump is located at the upstream end of the pipe. The air enters the pump through a 2.067-in.-ID pipe. The pressure in the pipe at the downstream end of the system is 5 psig, and the temperature is 70°F. If the air pressure in the pipe at the entrance to the pump is 10 psig and the temperature is 80°F, determine the following:

(a) The pressure in the pipe at the exit from the pump.

(*b*) The mechanical energy as foot-pounds force per minute added to the air by the pump, assuming the pump operation is isothermal.

Solution

Basis: 1 lb mass of flowing air

Because the amount of heat exchanged between the surroundings and the system is unknown, the total energy balance cannot be used to solve this problem. However, an approximate result can be obtained from the total mechanical-energy balance.

Designate point 1 as the entrance to the pump, point 2 as the exit from the pump, and point 3 as the downstream end of the pipe. Under these conditions, the total mechanical-energy balance for the system between points 2 and 3 may be written as follows:

$$\int_2^3 v\,dp + \int_2^3 \frac{V\,dV}{\alpha g_c} = -\int_2^3 \delta F = -\int_2^3 \frac{2\mathfrak{f} V^2\,dL}{g_c D} \qquad (A)$$

The mass velocity G [as lbm/(s)(ft^2)] is constant, and

$$V = Gv$$
$$dV = G\,dv$$

Eliminating V and dV from Eq. (A) and dividing by v^2 gives

$$-\int_2^3 \frac{dp}{v} = \int_2^3 \frac{G^2\,dv}{\alpha g_c v} + \int_2^3 \frac{2\mathfrak{f} G^2\,dL}{g_c D} \qquad (B)$$

Assume that air acts as an ideal gas at the pressures involved, or

$$v = \frac{RT}{Mp} \qquad (C)$$

where M = molecular weight of air ≈ 29 lb/lb mol
R = ideal-gas-law constant = 1545 (lbf/ft^2)(ft^3)/(lb mol)(°R)
T = temperature, °R

Substituting Eq. (C) into Eq. (B) and integrating gives

$$\frac{M}{2RT_{\text{avg}}}\left(p_2^2 - p_3^2\right) = \frac{G^2}{\alpha g_c}\ln\frac{v_3}{v_2} + \frac{2\mathfrak{f}_{\text{avg}} G^2 L}{g_c D} \qquad (D)$$

T_{avg} represents the average absolute temperature between points 2 and 3, and temperature variations up to 20 percent from the average absolute value will introduce only a small error in the final result. The error introduced by using a constant $\mathfrak{f}_{\text{avg}}$ (based on average temperature and pressure) instead of the exact integrated value is not important unless pressure variations are considerably greater than those involved in this problem.

If the pump operation is isothermal, $T_2 = 80°\text{F}$ and $T_{\text{avg}} = 75 + 460 = 535°\text{R}$. NOTE: If the pump operation were assumed to be adiabatic, a different value for T_{avg} would be obtained.

At 535°R,

$$\mu_{\text{air}} = 0.018 \text{ cp} = (0.018)(0.000672) \text{ lb/(s)(ft)}$$

$$G = \frac{(15)(144)(4)}{(60)(2.067)^2(\pi)} = 10.77 \text{ lb/(s)(ft}^2)$$

$$N_{\text{Re}} = \frac{DG}{\mu} = \frac{(2.067)(10.77)}{(12)(0.018)(0.000672)} = 153{,}000$$

$$\alpha = 1$$

$$\frac{\varepsilon}{D} = \frac{(0.00015)(12)}{2.067} = 0.00087$$

From Fig. 14-1,

$$f_{\text{avg}} = 0.0052$$

$$p_3 = (5 + 14.7)(144) = 2840 \text{ lbf/ft}^2$$

Since air is assumed to act as an ideal gas,

$$\frac{v_3}{v_2} = \frac{T_3 p_2}{T_2 p_3} = \frac{(530)p_2}{(540)(2840)} = \frac{p_2}{2890}$$

Substituting into Eq. (D),

$$\frac{29}{(2)(1545)(535)}\left[p_2^2 - (2840)^2\right] = \frac{(10.77)^2}{32.17}\ln\frac{p_2}{2890} + \frac{(2)(0.0052)(10.77)^2(3000)(12)}{(32.17)(2.067)}$$

By trial-and-error solution,

(*a*)
$$p_2 = \text{pressure in pipe at exit from pump}$$
$$= 6750 \text{ psf} = \frac{6750}{144} \text{ psia} = 47 \text{ psia}$$

(*b*) The mechanical energy added by the pump can be determined by making a total mechanical-energy balance between points 1 and 3:

$$\int_1^3 v\,dp + \int_1^3 \frac{V\,dV}{\alpha g_c} = \int_1^3 \delta W_o - \int_1^3 \delta F \qquad (E)$$

The friction term, by definition, includes all friction except that occurring at the pump. Therefore,

$$\int_1^3 \delta F = \int_2^3 \delta F$$

and

$$\int_1^2 v\,dp + \int_2^3 v\,dp + \int_1^2 \frac{V\,dV}{\alpha g_c} + \int_2^3 \frac{V\,dV}{\alpha g_c} = W_o - \int_2^3 \delta F \qquad (F)$$

Subtracting Eq. (A) from Eq. (F) gives

$$\int_1^2 v\,dp + \int_1^2 \frac{V\,dV}{\alpha g_c} = W_o \qquad (G)$$

The value of $\int_1^2 v\,dp$ depends on the conditions or path followed in the pump, and the integral can be evaluated if the necessary p-v relationships are known. Although many pumps and compressors operate near adiabatic conditions, the pump operation will be assumed as isothermal in this example.

For an ideal gas and isothermal compression,

$$\int_1^2 v\,dp = \frac{RT}{M}\ln\frac{p_2}{p_1}$$

$$V_1 = Gv_1 = \frac{(10.77)(359)(540)(14.7)}{(29)(492)(24.7)} = 87\ \text{ft/s}$$

$$V_2 = V_1\frac{p_1}{p_2} = \frac{(87)(24.7)}{47} = 46\ \text{ft/s}$$

$$W_o = \frac{RT}{M}\ln\frac{p_2}{p_1} + \frac{V_2^2}{2g_c} - \frac{V_1^2}{2g_c}$$

$$W_o = \frac{(1545)(540)}{29}\ln\frac{47}{24.7} + \frac{(46)^2}{(2)(32.17)} - \frac{(87)^2}{(2)(32.17)}$$

$$= 18{,}400\ \text{ft}\cdot\text{lbf/lbm}$$

Mechanical energy added to the air by the pump, assuming the pump operation is isothermal = (18,400)(15) = 276,000 ft · lbf/min.

The total power supplied to the pump could be determined if the isothermal efficiency of the pump (including any end effects caused by the pump housing) were known.

PIPING STANDARDS

Pipe Strength

Iron and steel pipes were originally classified on the basis of wall thickness as standard, extra-strong, and double-extra-strong. Modern industrial demands for more exact specifications have made these three classifications obsolete. Pipes are now specified according to wall thickness by a standard formula for *schedule number* as designated by the American Standards Association.

The bursting pressure of a thin-walled cylinder may be estimated from the following equation:

$$P_b = \frac{2S_T t_m}{D_m} \tag{12}$$

where P_b = bursting pressures (difference between internal and external pressures), psi
S_T = tensile strength, psi
t_m = minimum wall thickness, in.
D_m = mean diameter, in.

A safe working pressure P_s can be evaluated from Eq. (12) if the tensile strength is replaced by a safe working stress S,.

$$P_s = \frac{2S_s t_m}{D_m} \tag{13}$$

Schedule number is defined by the American Standards Association as the approximate value of

$$1000\frac{P_s}{S_s} = \text{schedule number} \tag{14}$$

For temperatures up to 250°F, the recommended safe working stress is 9000 psi for lap-welded steel pipe and 6500 psi for butt-welded steel pipe.? If the schedule number is known, the safe working pressure can be estimated directly from Eq. (14).

Ten schedule numbers are in use at the present time. These are 10, 20, 30, 40, 60, 80, 100, 120, 140, and 160. For pipe diameters up to 10 in., schedule 40 corresponds to the former “standard” pipe and schedule 80 corresponds to the former “extra-strong” pipe. The original “double-extra-strong” pipe is not represented by a definite schedule number.

Nominal Pipe Diameter

Pipe sizes are based on the approximate diameter and are reported as nominal pipe sizes. Although the wall thickness varies depending on the schedule number, the outside diameter of any pipe having a given nominal size is constant and independent of the schedule number. This permits the use of standard fittings and threading tools on pipes of different schedule numbers. A table showing outside diameters, inside diameters, and other dimensions for pipes of different diameters and schedule numbers is presented in the Appendix.

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†For allowable stresses at other temperatures and for other materials of construction, see R. H. Perry and D. Green, “Chemical Engineers’ Handbook,” 6th ed., p. 6-80, McGraw-Hill Book Company, New York, 1984.

Tubing

Copper tubing and brass tubing are used extensively in industrial operations. Other metals, such as nickel and stainless steel, are also available in the form of tubing. Although pipe specifications are based on standard nominal sizes, tubing specifications are based on the actual outside diameter with a designated wall thickness. Conventional systems, such as the Birmingham wire gauge (BWG), are used to indicate the wall thickness. Common designations of tubing dimensions are given in the Appendix.

Fittings and Other Piping Auxiliaries

Threaded fittings, flanges, valves, flow meters, steam traps, and many other auxiliaries are used in piping systems to connect pieces of pipe together, change the direction of flow, regulate the flow, or obtain desired conditions in a flow system. Flanges are usually employed for piping connections when the pipe diameter is 3 in. or larger, while screwed fittings are commonly used for smaller sizes. In the case of cast-iron pipe used as underground water lines, bell-and-spigot joints are ordinarily employed rather than flanges.

The auxiliaries in piping systems must have sufficient structural strength to resist the pressure or other strains encountered in the operation, and the design engineer should provide a wide safety margin when specifying the ratings of these auxiliaries. Fittings, valves, steam traps, and similar items are often rated on the basis of the safe operating pressure as (a) low pressure (25 psi), (*b*) standard (125 psi), (*c*) extra-heavy (250 psi), or (*d*) hydraulic (300 to 10,000 psi). Figures D-5 and D-6 in the Appendix show examples of standard designations used to indicate various types of fittings and auxiliaries in sketches of piping systems.

DESIGN OF PIPING SYSTEMS

The following items should be considered by the engineer when developing the design for a piping system:

1. Choice of materials and sizes
2. Effects of temperature level and temperature changes
    a. Insulation
    b. Thermal expansion
    c. Freezing
3. Flexibility of the system for physical or thermal shocks
4. Adequate support and anchorage
5. Alterations in the system and the service

6. Maintenance and inspection
7. Ease of installation
8. Auxiliary or stand-by pumps and lines
9. Safety
   a. Design factors
   b. Relief valves and flare systems

In the early years of industrial development in the United States, many plants buried their outside pipelines. The initial cost for this type of installation is low because no supports are required and the earth provides insulation. However, location and repair of leaks are difficult, and other pipes buried in the same trench may make repairs impossible. Above-ground piping systems in industrial plants have proven to be more economical than buried systems, and, except for major water and gas lines, most in-plant piping systems in new plants are now located above ground or in crawl-space tunnels.

Thermal expansion and the resultant pipe stresses must be considered in any piping system design. For example, if the temperature changes from 50 to 600°F, the increase in length would be 4.9 in. per 100 ft for steel pipe and 7.3 in. per 100 ft for brass pipe. This amount of thermal expansion could easily cause a pipe or wall to buckle if the pipe were fastened firmly at each end with no allowances for expansion. The necessary flexibility for the piping system can be provided by the use of expansion loops, changes in direction, bellows joints, slip joints, and other devices.

The possibility of solidification of the fluid should not be overlooked in the design of a piping system. Insulation, steam tracing, and sloping the line to drain valves are methods for handling this type of problem.

Water hammer may cause extreme stresses at bends in pipelines. Consequently, liquid pockets should be avoided in steam lines through the use of steam traps and sloping of the line in the direction of flow. Quick-opening or quick-closing valves may cause damaging water hammer, and valves of this type may require protection by use of expansion or surge chambers.

A piping system should be designed so that maintenance and inspection can be accomplished easily, and the possibility of future changes in the system should not be overlooked. Personal-safety considerations in the design depend to a large extent on the fluids, pressures, and temperatures involved. For example, an overhead line containing a corrosive acid should be shielded from open walkways, and under no conditions should an unprotected flange in this type of piping system be located immediately over a walkway.

Pipe Sizing

The design engineer must specify the diameter of pipe that will be used in a given piping system, and economic factors must be considered in determining the optimum pipe diameter. Theoretically, the optimum pipe diameter is the

TABLE 2
"Rule-of thumb" economic velocities for sizing steel pipelines

Turbulent flow

Type of fluid	Reasonable velocity, ft/s
Water or fluid similar to water	3-10
Low-pressure steam (25 psig)	50-100
High-pressure steam (100 psig and up)	100-200
Air at ordinary pressures (25-50 psig)	50-100

The preceding values apply for motor drives. Multiply indicated velocities by 0.6 to give reasonable velocities when steam turbine drives are used.

Viscous flow (liquids)

Nominal pipe diameter, in.	Reasonable velocity, ft/s		
	μ_c† = 50	μ_c = 100	μ_c = 1000
1	1.5-3	1-2	0.3-0.6
2	2.5-3.5	1.5-2.5	0.5-0.8
4	3.5-5.0	2.5-3.5	0.8-1.2
8		4.0-5.0	1.3-1.8

† μ_c = viscosity, centipoises.

one that gives the least total cost for annual pumping power and fixed charges with the particular piping system. Many short-cut methods have been proposed for estimating optimum pipe diameters, and some general "rules of thumb" for use in design estimates of pipe diameters are presented in Table 2.

The derivation of equations for determining optimum economic pipe diameters is presented in Chap. 11 (Optimum Design and Design Strategy). The following simplified equations [Eqs. (45) and (47) from Chap. 11] can be used for making design estimates:

For turbulent flow ($N_{\mathrm{Re}} > 2100$) in steel pipes

$$D_{i,\mathrm{opt}} = 3.9 q_f^{0.45} \rho^{0.13} \tag{15}$$

For viscous flow ($N_{\mathrm{Re}} < 2100$) in steel pipes

$$D_{i,\mathrm{opt}} = 3.0 q_f^{0.36} \mu_c^{0.18} \tag{16}$$

where $D_{i\,\mathrm{opt}}$ = optimum inside pipe diameter, in.
q_f = fluid flow rate, ft^3/s
ρ = fluid density, $\mathrm{lb/ft}^3$
μ_c = fluid viscosity, centipoises

The preceding equations are the basis for the nomograph presented in Fig. 14-2, and this figure can be used for estimating the optimum diameter of steel pipe under ordinary plant conditions. Equations (15) and (16) should not be applied when the flowing fluid is steam, because the derivation makes no allowance for the effects of pressure drop on the value of the flowing material. Equation (15) is limited to conditions in which the viscosity of the fluid is between 0.02 and 20 centipoises.

As discussed in Chap. 11, the constants in Eqs. (15) and (16) are based on average cost and operating conditions. When unusual conditions are involved or when a more accurate determination of the optimum diameter is desired, other equations given in Chap. 11 can be used.

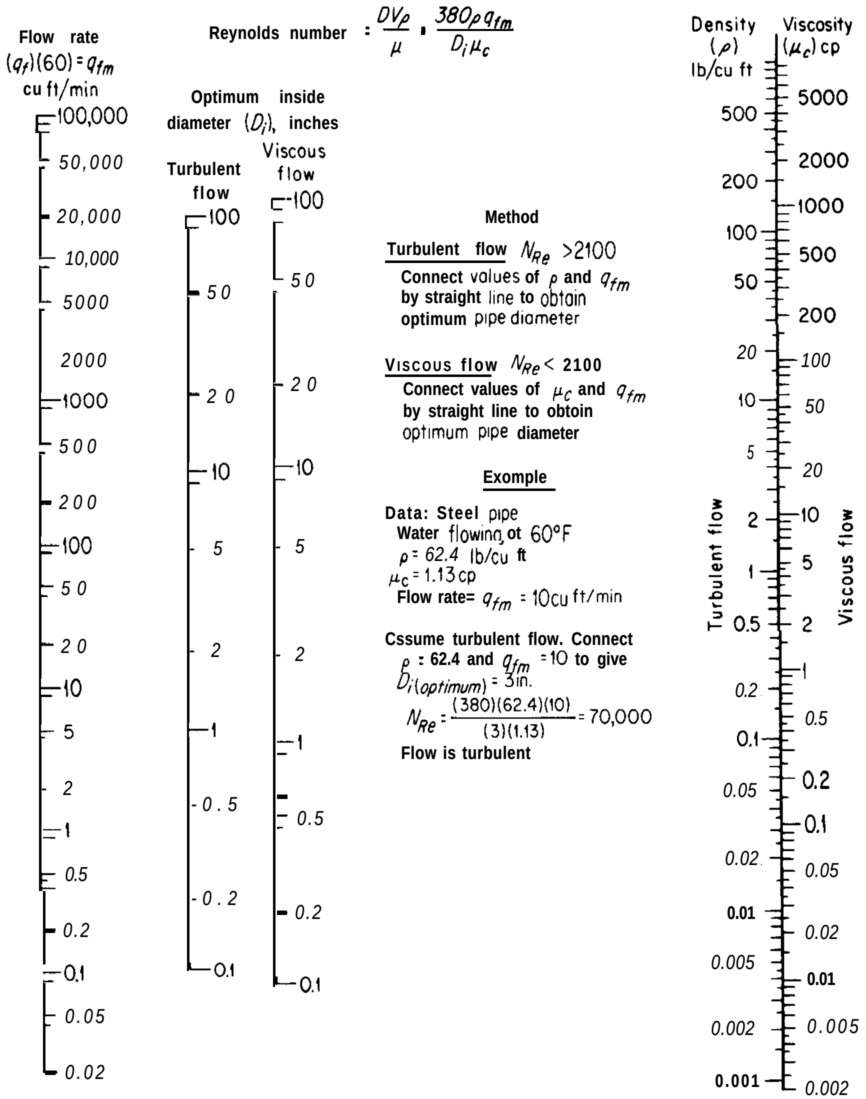
COSTS FOR PIPING AND PIPING-SYSTEM AUXILIARIES

Piping is a major item in the cost of chemical process plants. These costs in a fluid-process plant can run as high as 80 percent of the purchased equipment cost or 20 percent of the fixed-capital investment. There are essentially two basic methods for preparing piping-cost estimates-the percentage of installed equipment method and the material and labor take-off method. Several variations of each method have appeared in the literature.

The percentage of installed equipment method as described in Chap. 6 is a quick procedure for preliminary or order-of-magnitude type of cost estimates. In the hands of experienced estimators it can be a reasonably accurate method, particularly on repetitive type units. It is not recommended on alteration jobs or on projects where the total installed equipment is less than $100,000.

The material and labor take-off method is the recommended method for definitive estimates where accuracy within 10 percent is required. To prepare a cost estimate by this method usually requires piping drawings and specifications, material costs, fabrication and erection labor costs, testing costs, auxiliaries, supports, and painting requirements. The take off from the drawings must be made with the greatest possible accuracy because it is the basis for determining material and labor costs. In the case of revisions to existing facilities, thorough field study is necessary to determine job conditions and their possible effects.

Although accurate costs for pipes, valves, and piping system auxiliaries can be obtained only by direct quotations from manufacturers, the design engineer can often make satisfactory estimates from data such as those presented in Figs. 14-3 through 14-34. The cost of materials and installation time presented in these figures covers the types of equipment most commonly encountered in industrial operations.



Method

Turbulent flow $N_{Re} > 2100$

Connect values of ρ and q_{fm} by straight line to obtain optimum pipe diameter

Viscous flow $N_{Re} < 2100$

Connect values of μ_c and q_{fm} by straight line to obtain optimum pipe diameter

Example

Data: Steel pipe
Water flowing at 60°F
$\rho = 62.4$ lb/cu ft
$\mu_c = 1.13$ cp
Flow rate $= q_{fm} = 10$ cu ft/min

Assume turbulent flow. Connect $\rho = 62.4$ and $q_{fm} = 10$ to give $D_i(\text{optimum}) = 3$ in.

$$N_{Re} = \frac{(380)(62.4)(10)}{(3)(1.13)} = 70{,}000$$

Flow is turbulent

FIGURE 14-2
Nomograph for estimation of optimum economic pipe diameters with turbulent or viscous flow based on Eqs. (15) and (16).

Piping labor consists of cutting, fitting, welding and/or threading, and field assembly. It frequently may be as high as 200 percent of materials cost. This labor is generally calculated on either the "diameter-inch" method or the "lineal-foot" method. In the diameter-inch method, all connections (threaded or welded) are counted and multiplied by the nominal pipe diameter. This diameter-inch factor is then multiplied by labor factors of employee-hour/diam-

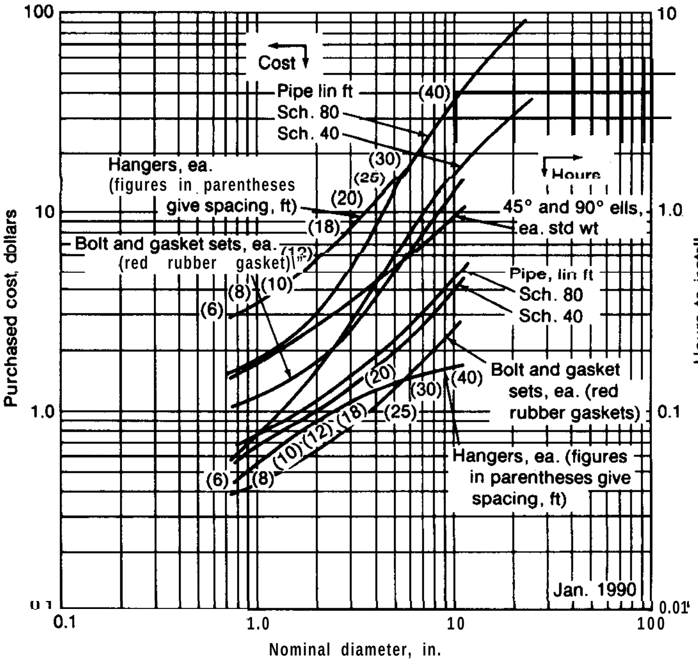


FIGURE 14-3
Cost and installation time for carbon-steel welded pipe and fittings.

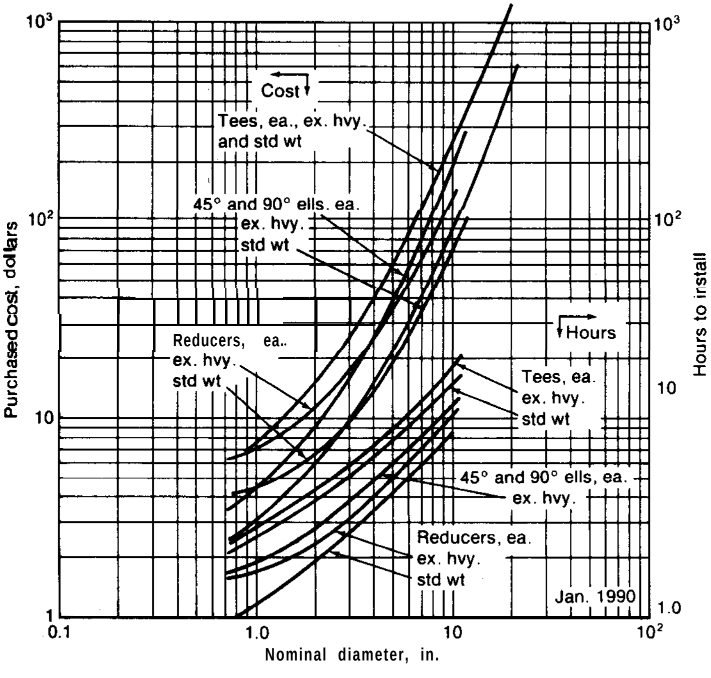


FIGURE 14-4
Cost and installation time for carbon-steel welded pipe fittings.

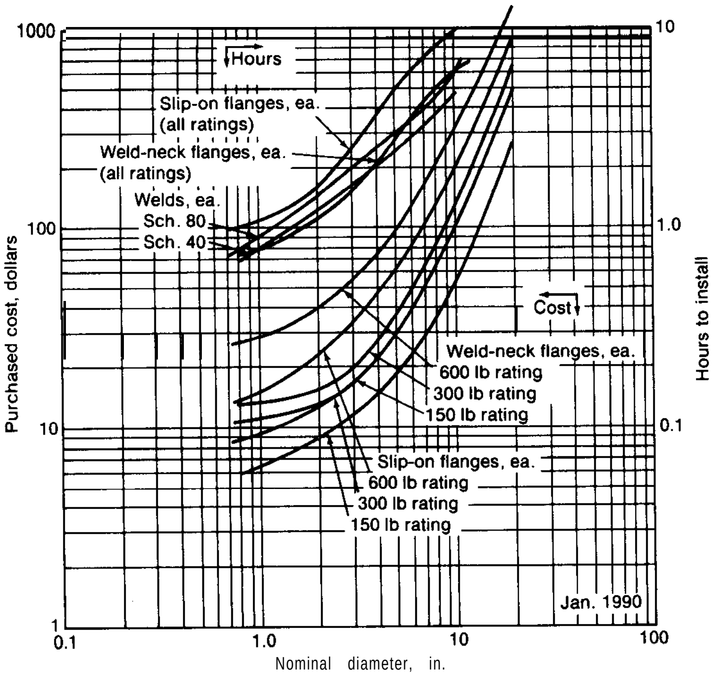


FIGURE 14-5
Cost and installation time for carbon-steel welded pipe fittings.

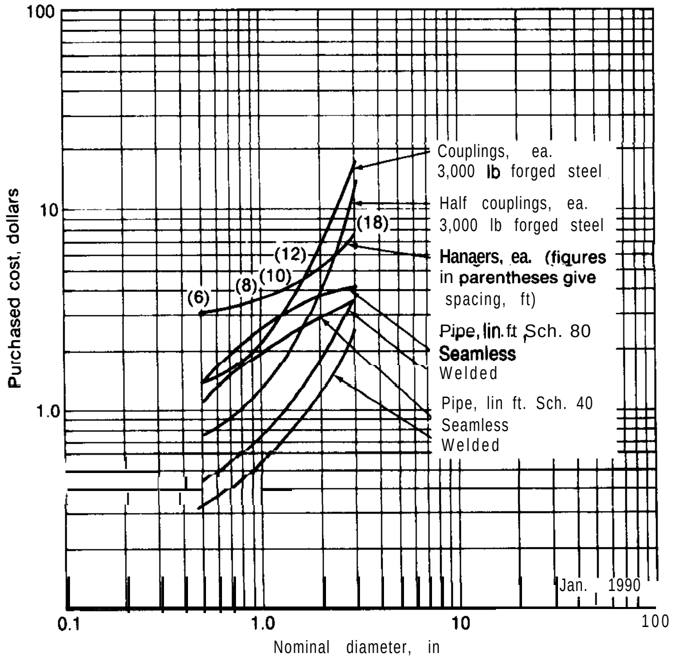


FIGURE 14-6
Cost of carbon-steel screwed pipe and fittings.

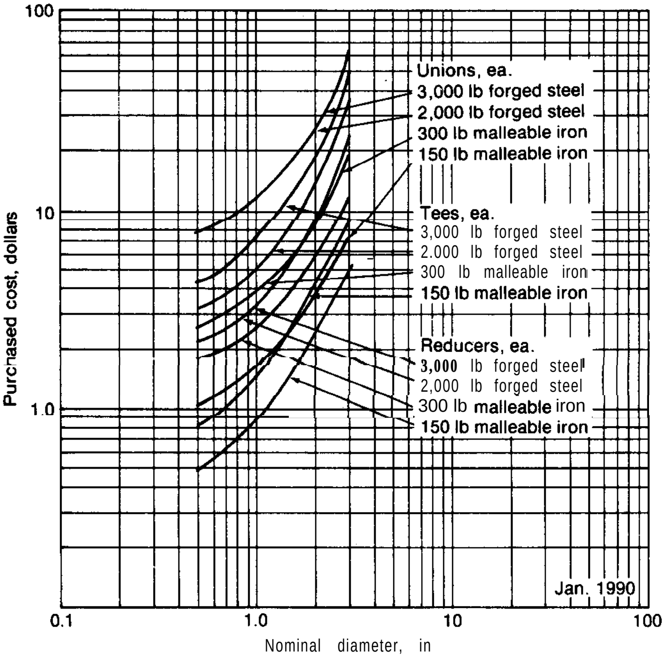


FIGURE 14-7
Cost of carbon-steel screwed pipe fittings.

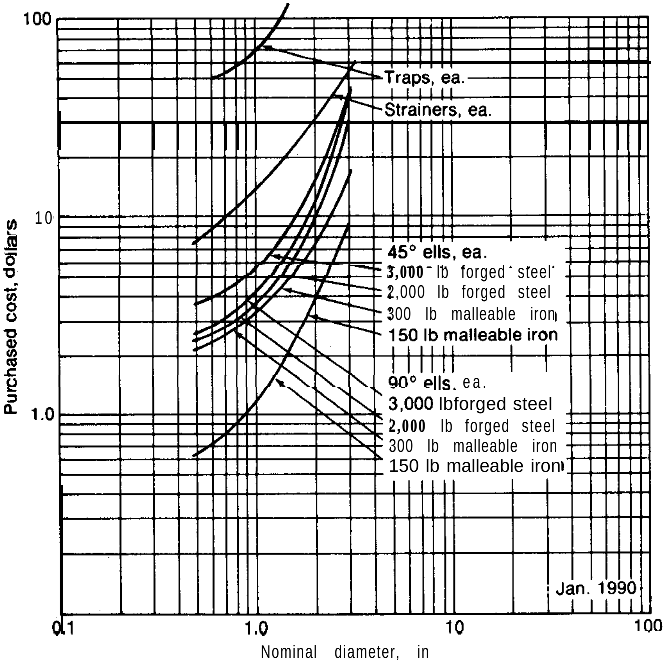


FIGURE 14-8
Cost of carbon-steel screwed pipe fittings.

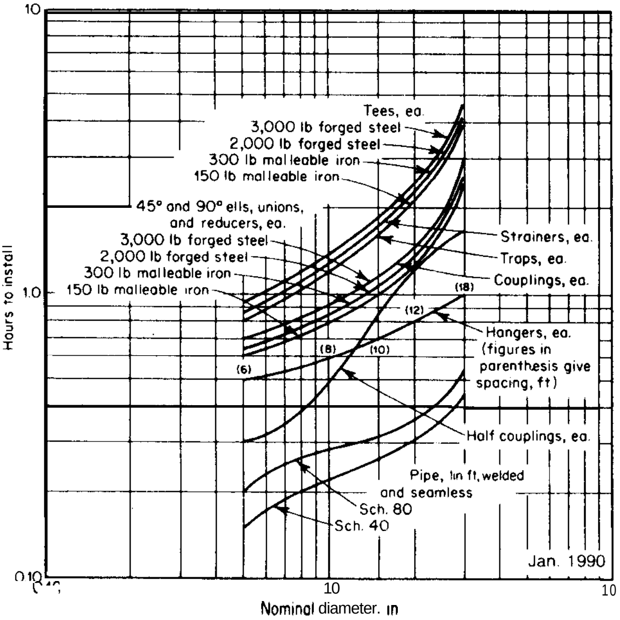


FIGURE 14-9
Installation time for carbon-steel screwed pipe fittings.

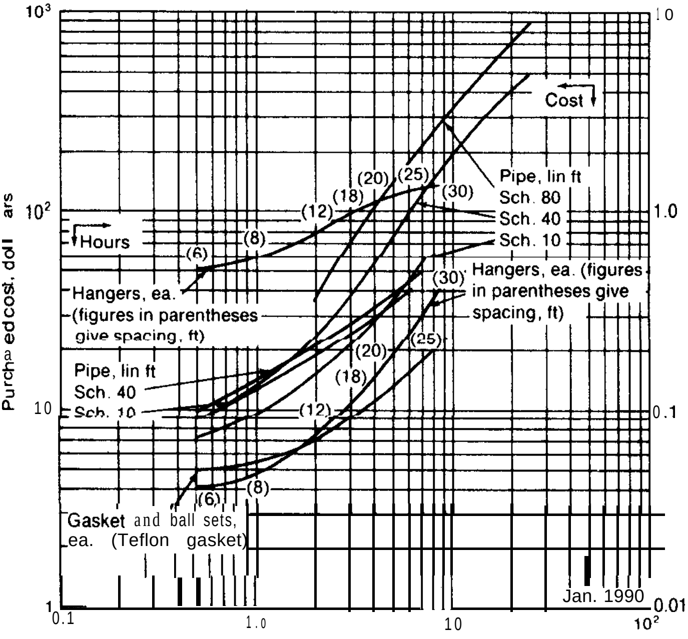


FIGURE 14-10
Cost and installation time for stainless-steel welded pipe and fittings. Prices are for types 304 and 304L. For types 316 and 347, multiply by 1.25; for type 316L, multiply by 1.45.

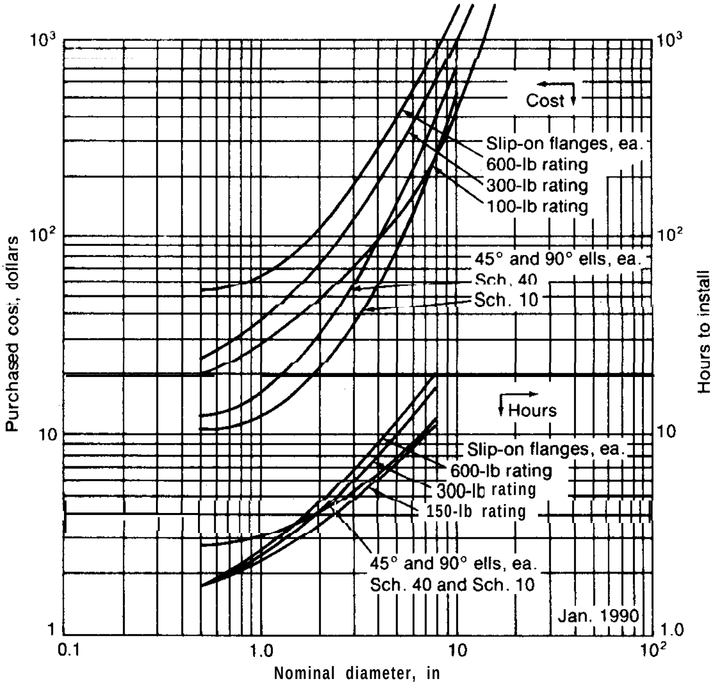


FIGURE 14.11
Cost and installation time for stainless-steel pipe fittings. Prices are for types 304 and 304L. For types 316 and 347, multiply by 1.25; for type 316L, multiply by 1.45.

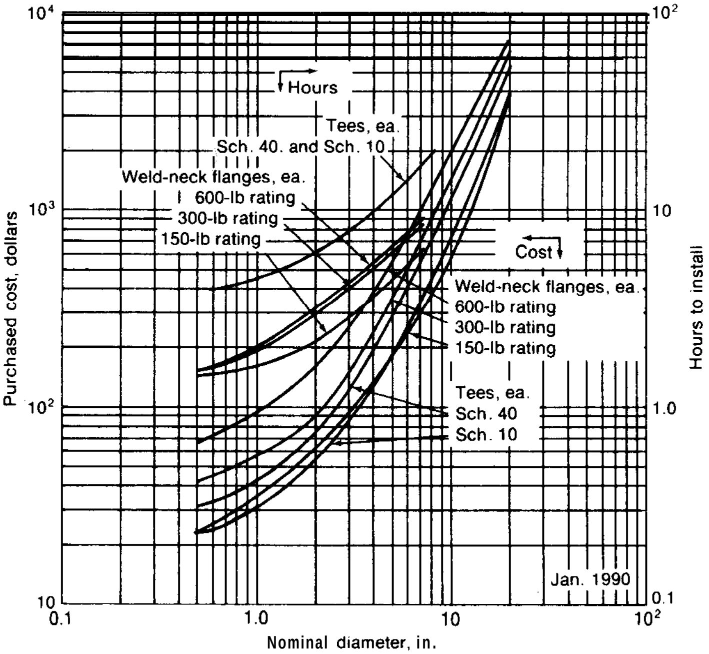


FIGURE 14-12
Cost and installation time for stainless-steel pipe fittings. Prices are for types 304 and 304L. For types 316 and 346, multiply by 1.25; for type 316L, multiply by 1.45.

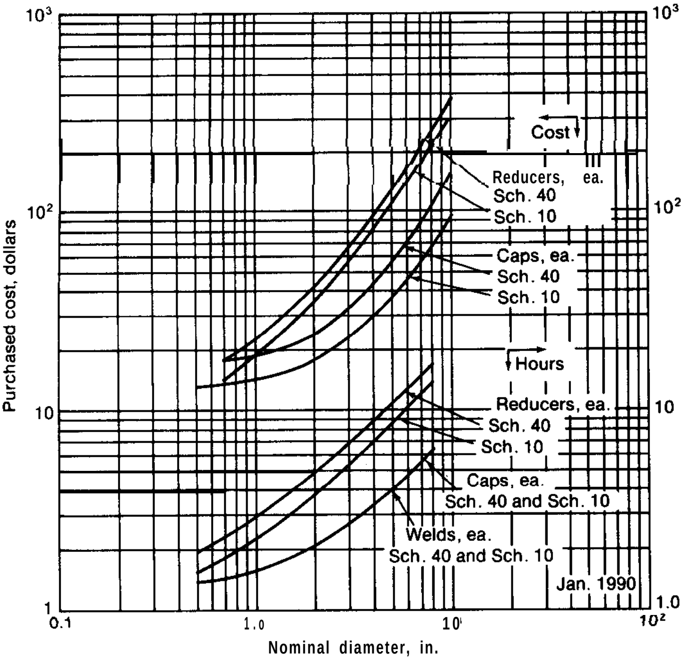


FIGURE 14-13
Cost and installation time for stainless-steel pipe fittings. Prices are for types 304 and **304L.** For types 316 and 347, multiply by 1.25; for type **316L,** multiply by 1.45.

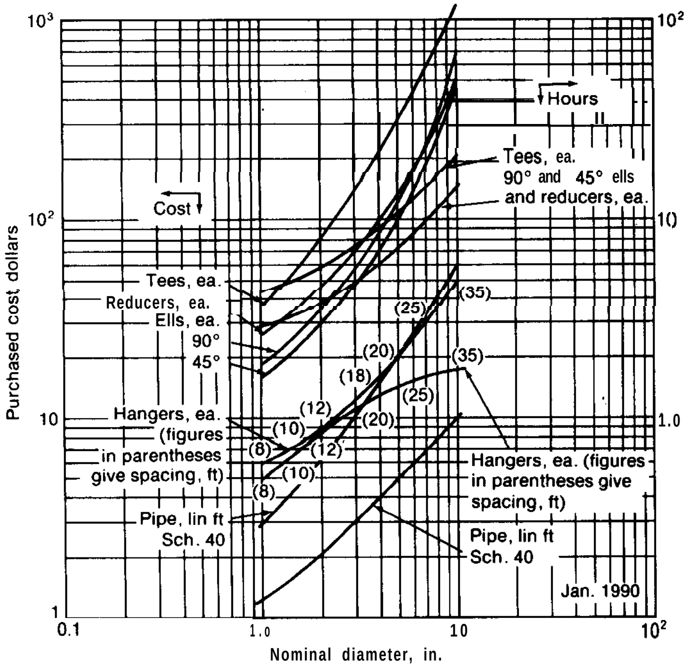


FIGURE 14-14
Cost and installation time for aluminum welded pipe and fittings.

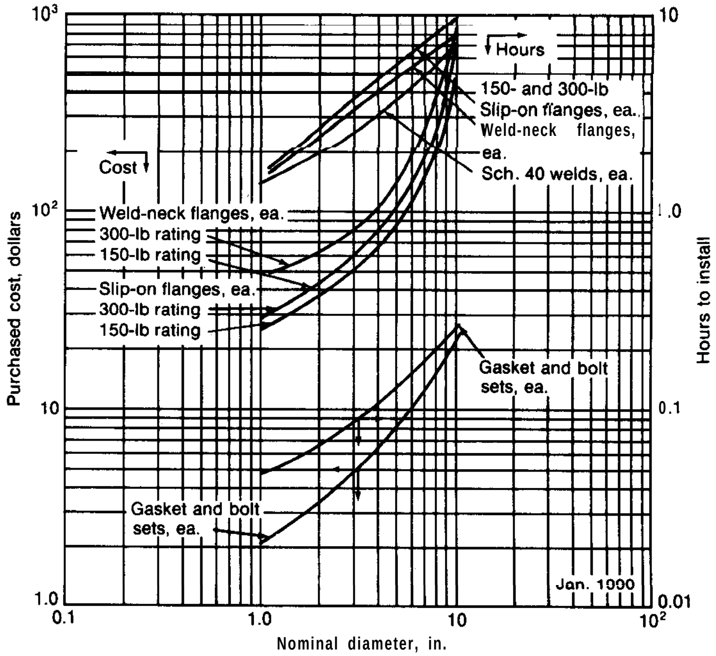


FIGURE **14-15**
Cost and installation time for aluminum welding and pipe fittings.

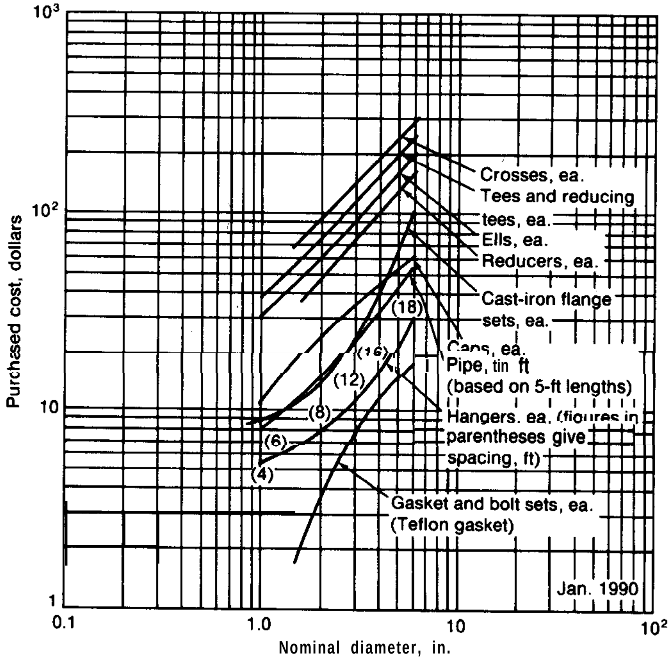


FIGURE 14-16
Cost of heat-resistant glass pipe and fittings.

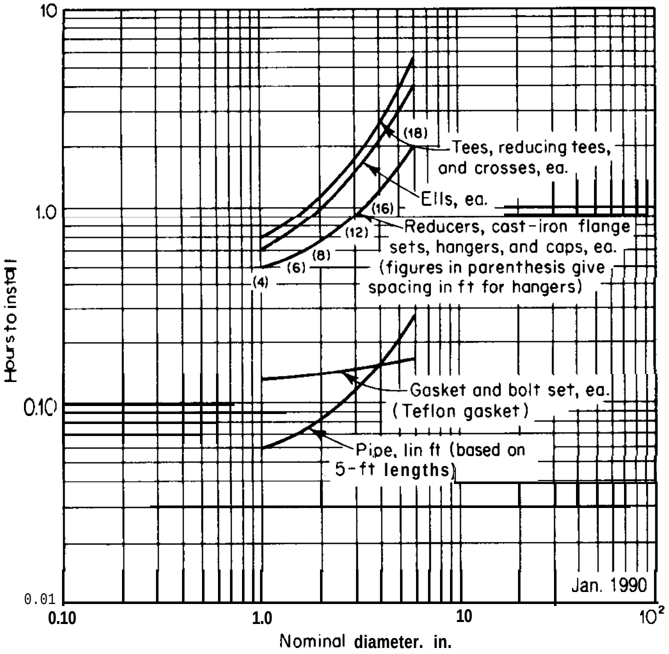


FIGURE 14-17
Installation time for heat-resistant glass pipe and fittings.

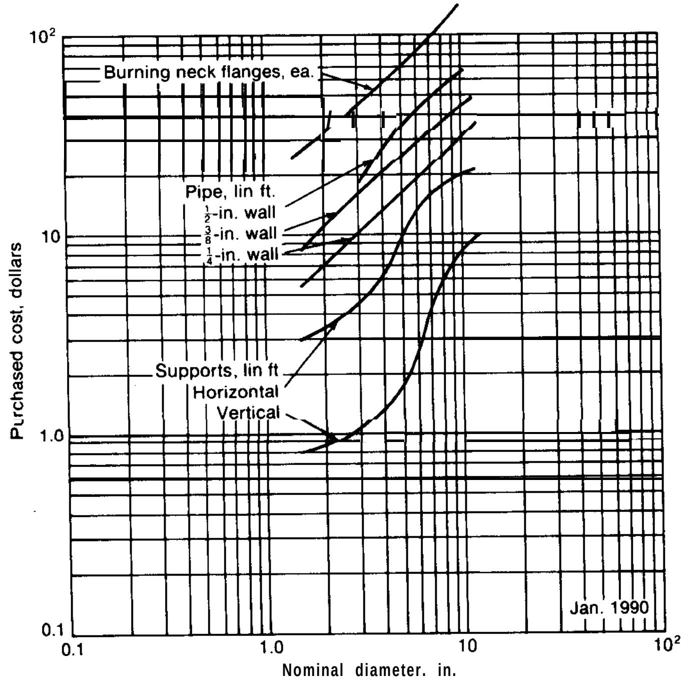


FIGURE **14-18**
Cost of chemical-lead pipe.

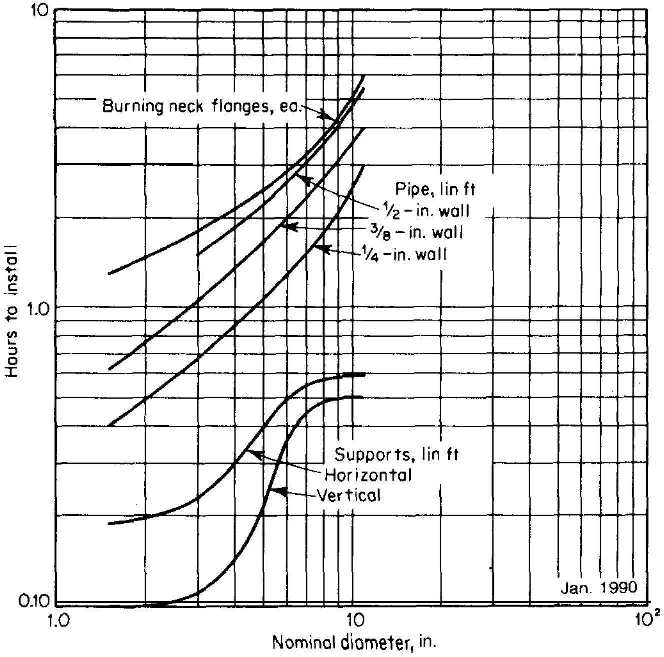


FIGURE 14-19
Installation time for chemical-lead pipe.

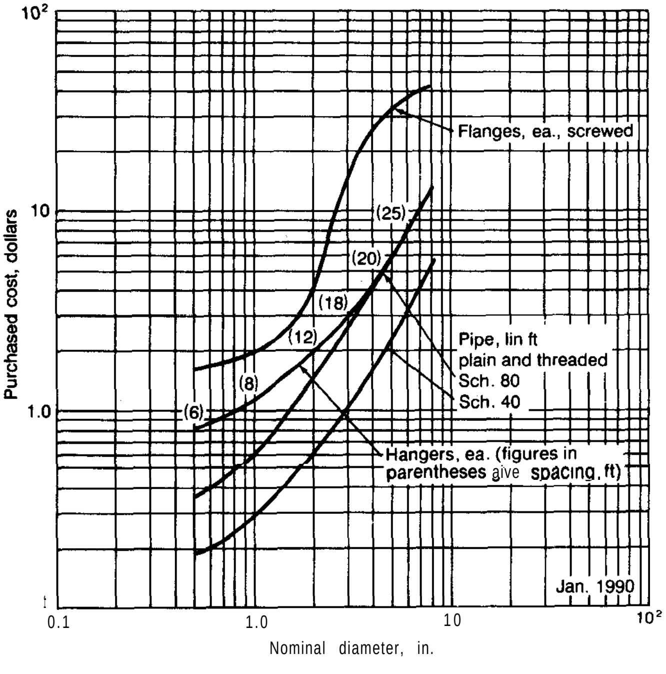


FIGURE 14-20
Cost of PVC plastic pipe and fittings.

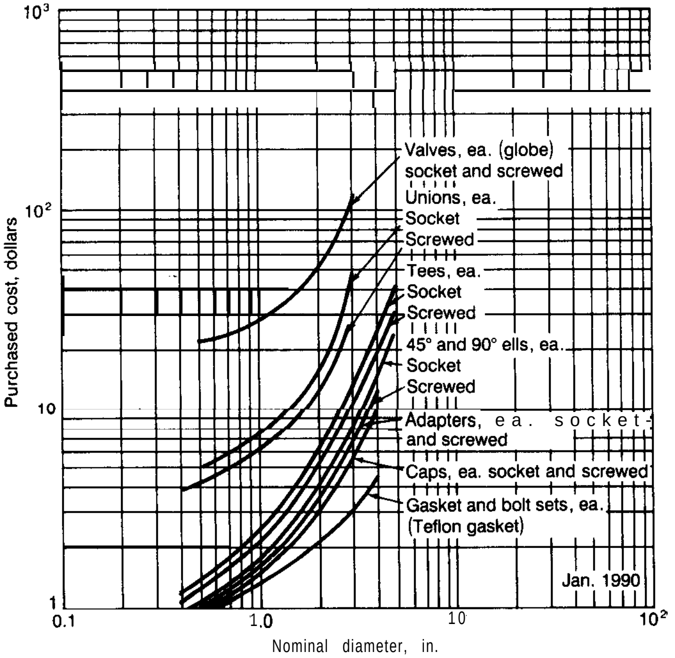


FIGURE 14-21
Cost of PVC plastic pipe fittings.

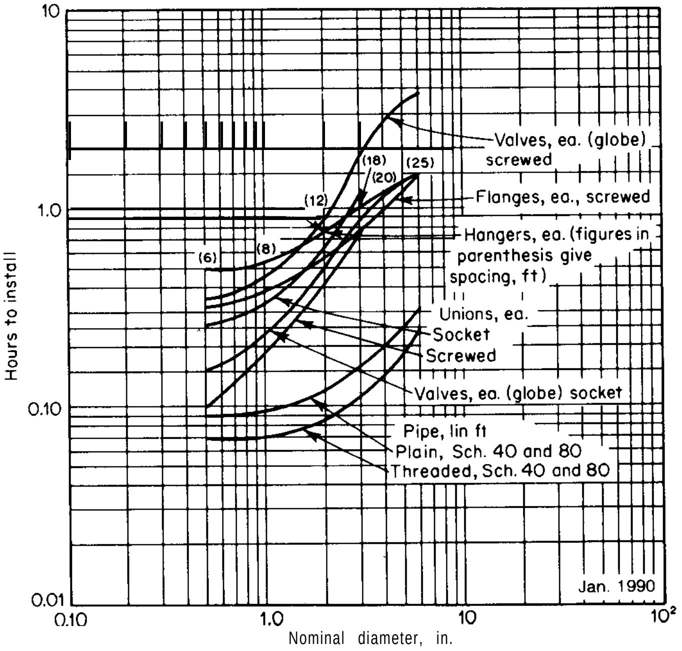


FIGURE 14-22
Installation time for PVC plastic pipe and fittings.

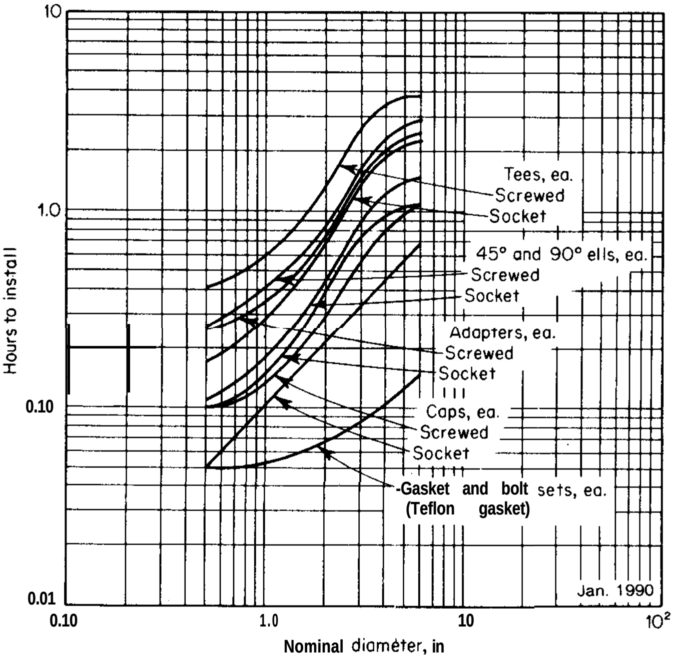


FIGURE 14-23
Installation time for PVC plastic pipe fittings.

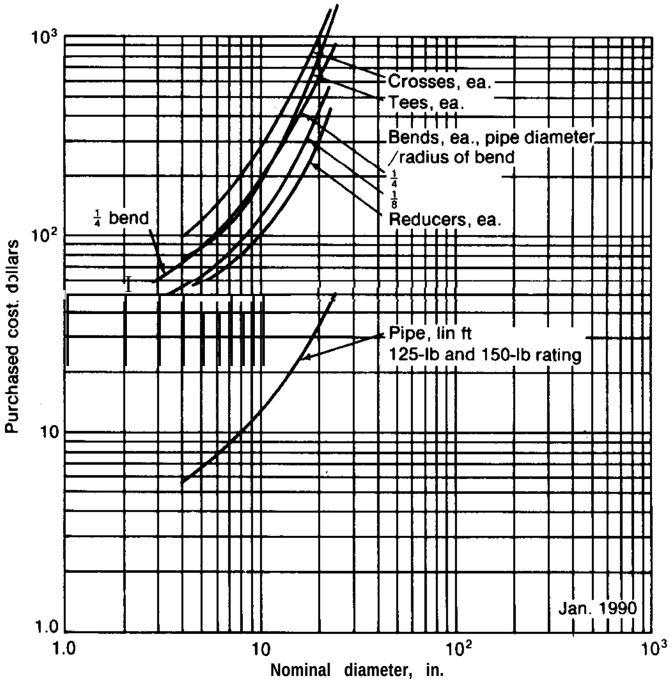


FIGURE 14-24
Cost for cast-iron mechanical-joint pipe and fittings.

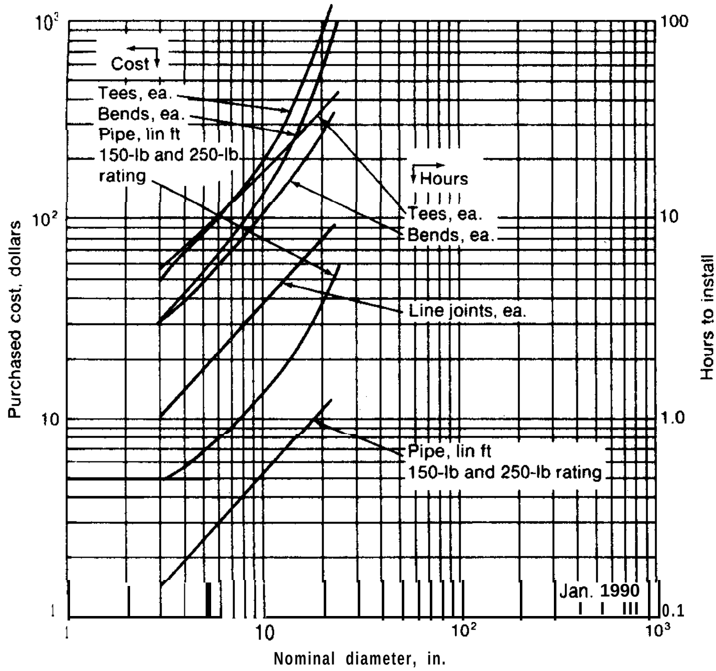


FIGURE 14-25
Cost and installation time for cast-iron bell-and-spigot pipe and fittings.

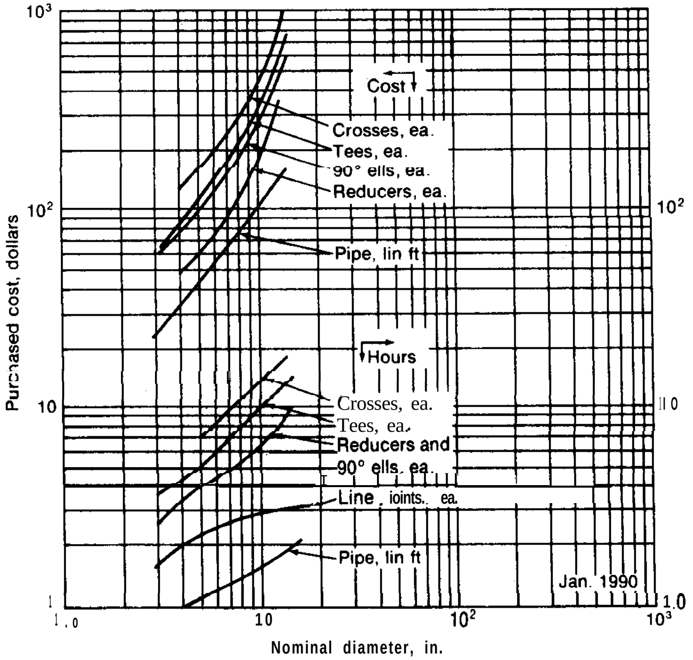


FIGURE 14-26
Cost and installation time for cast-iron flanged pipe and fittings. Cost of the pipe includes joint material.

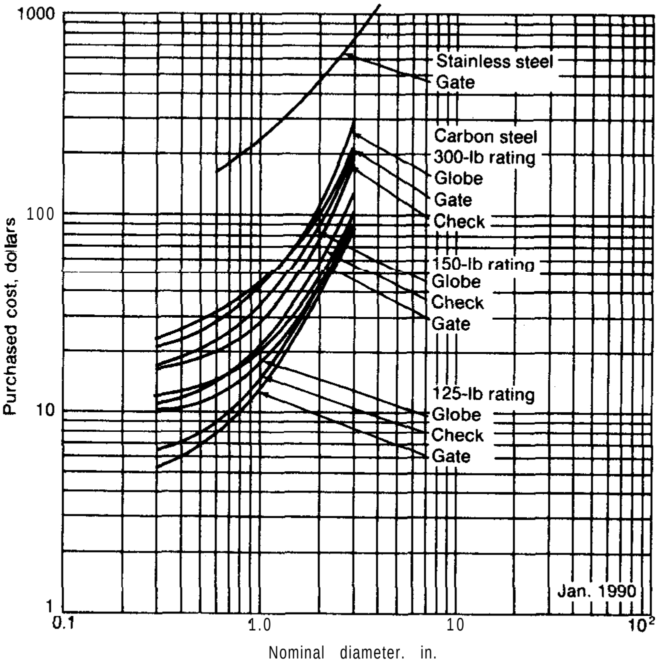


FIGURE 14-27
Screwed valves. For water, oil, **and** gas.

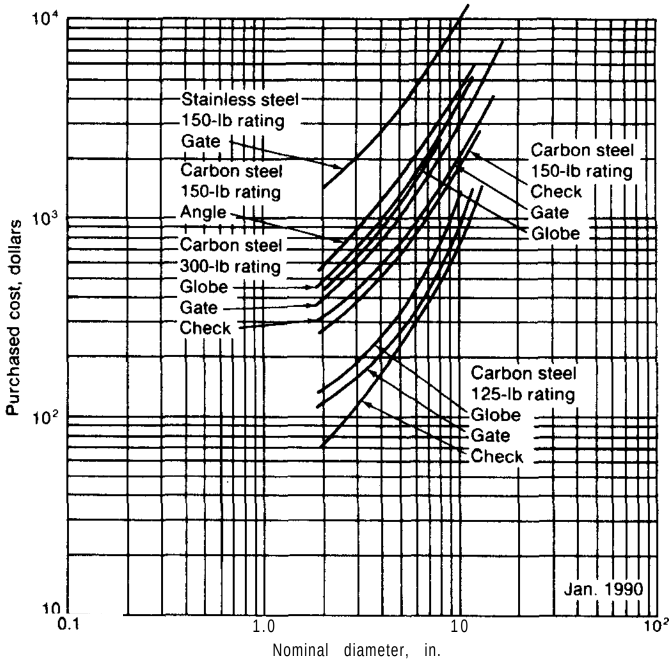


FIGURE 14-28
Flanged valves. For water, oil, and gas.

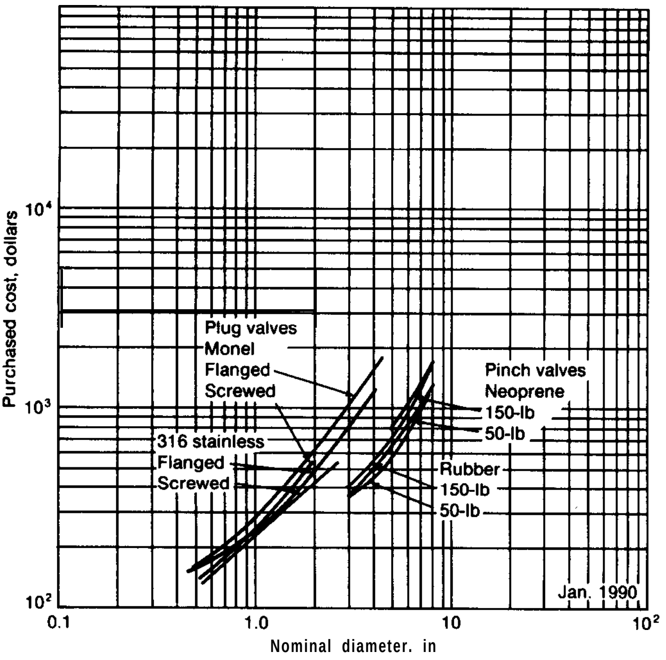


FIGURE 14-29
Cost of plug and pinch valves.

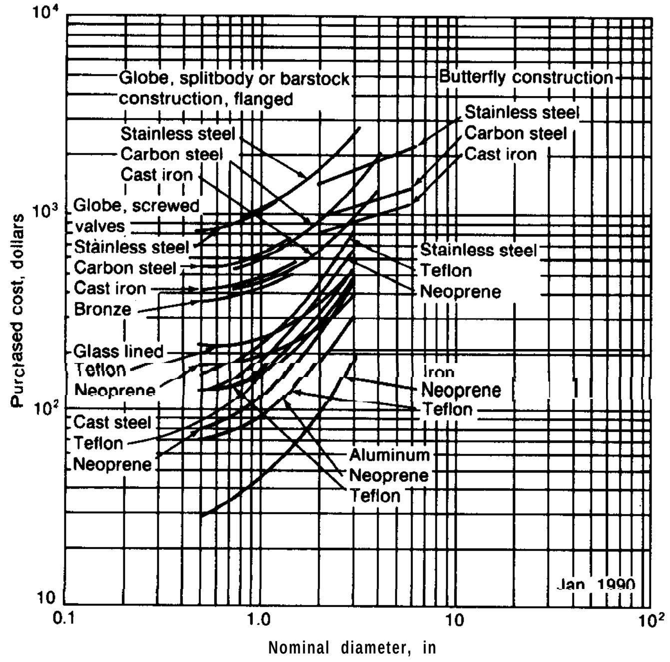


FIGURE 14-30
Cost of diaphragm valves.

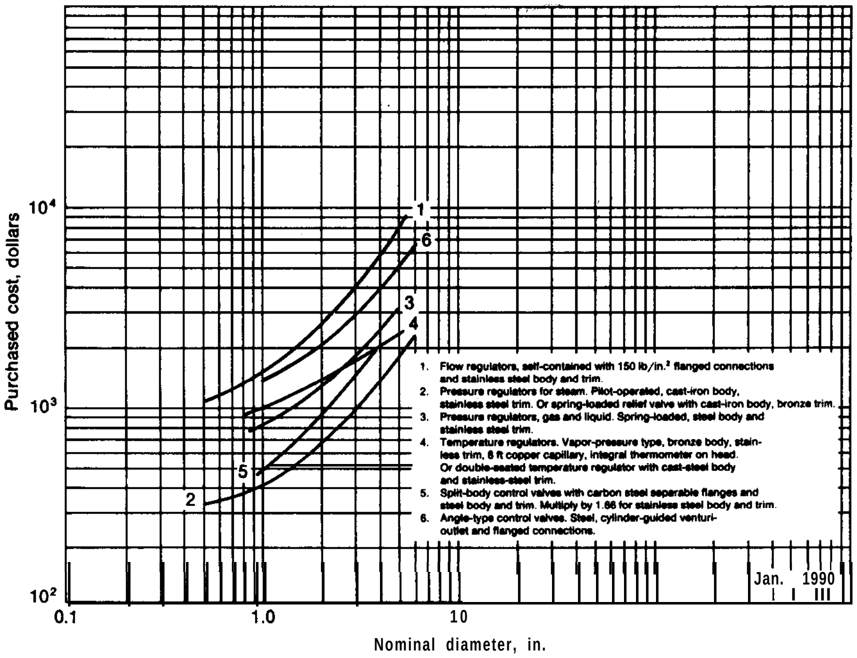


FIGURE 14-31
Costs for control and relief valves.

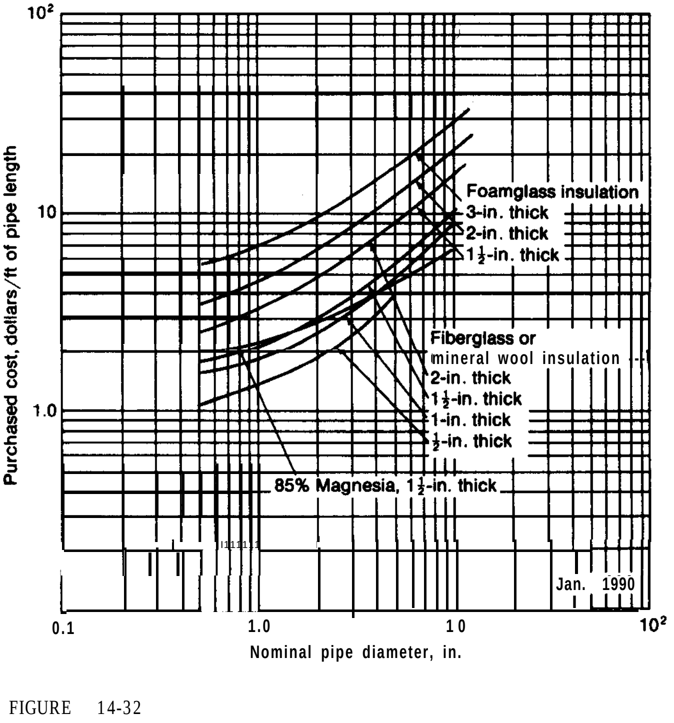


FIGURE 14-32
Cost of pipe insulation. Price includes cost of standard covering.

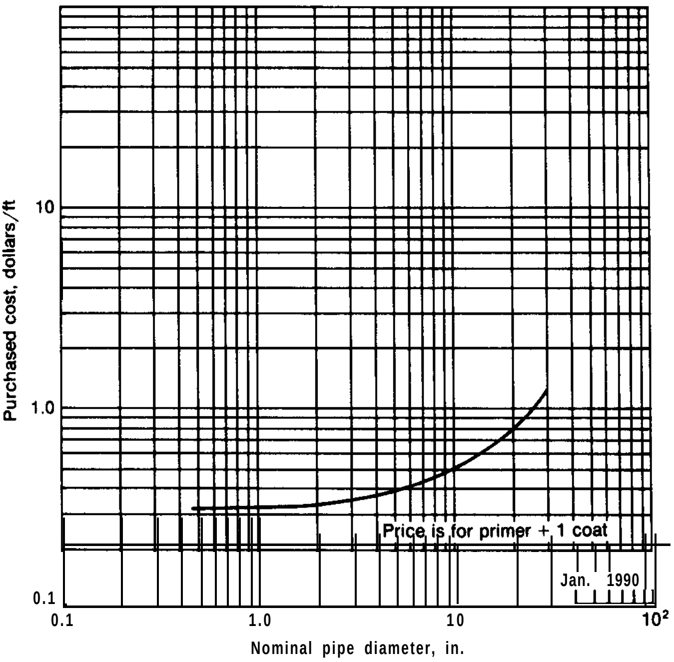


FIGURE 14-33
Cost of pipe painting. Cost includes material and labor, no overhead.

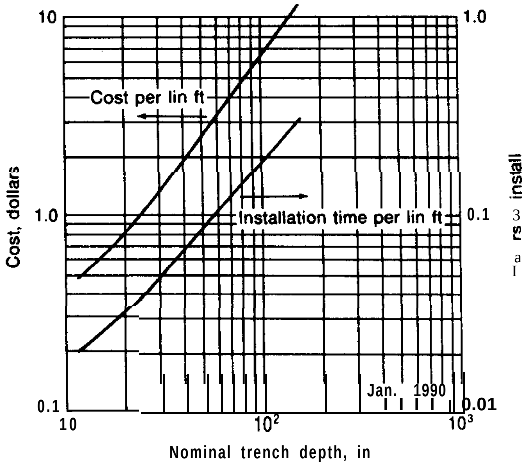


FIGURE **14-34**
Cost and installation time for 2-ft-wide trench in damp sandy loam sloped $\frac{1}{2}$ to 1.

eter-inch to yield employee-hours to fabricate and erect a piping system. Such a technique requires less data for each line size and for varying conditions of complexity; however, it also has certain limitations. Many estimators use 1.0 employee-hour/diameter-inch for welding carbon-steel pipe. This factor can give labor cost estimates that are up to 25 to 30 percent too high, particularly if an efficiently operated field-fabrication shop is available. The lineal-foot method, on the other hand, estimates the piping-installation costs by applying employee-hour units to the erection of the pipe (considering the length of the piping system), the installation of valves, fittings, and auxiliaries, and the welding or threading of piping components. Accurate estimation by this method requires that the engineering and design of the system be well along so that piping flowsheets, elevations, isometrics, etc., are available for a material take-off.

PUMPS

Pumps are used to transfer fluids from one location to another. The pump accomplishes this transfer by increasing the pressure of the fluid and, thereby, supplying the driving force necessary for flow. Power must be delivered to the pump from some outside source. Thus, electrical or steam energy may be transformed into mechanical energy which is used to drive the pump. Part of this mechanical energy is added to the fluid as work energy, and the rest is lost as friction due to inefficiency of the pump and drive.

Although the basic operating principles of gas pumps and liquid pumps are similar, the mechanical details are different because of the dissimilarity in physical properties of gases and liquids. In general, pumps used for transferring gases operate at higher speeds than those for liquids, and smaller clearances between moving parts are necessary for gas pumps because of the lower viscosity of gases and the greater tendency for the occurrence of leaks.

The different types of pumps commonly employed in industrial operations can be classified as follows:†

1. Reciprocating or positive-displacement pumps with valve action: piston pumps, diaphragm pumps, plunger pumps
2. Rotary positive-displacement pumps with no valve action: gear pumps, lobe pumps, screw pumps, eccentric-cam pumps, metering pumps
3. Rotary centrifugal pumps with no valve action: open impeller, closed impeller, volute pumps, turbine pumps
4. Air-displacement systems: air lifts, acid eggs or blow cases, jet pumps, barometric legs

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†For a detailed discussion on different types of pumps, see R. H. Perry and D. Green, "Chemical Engineers' Handbook," 6tb ed., McGraw-Hill Book Company, New York, 1984.

Many different factors can influence the final choice of a pump for a particular operation. The following list indicates the major factors that govern pump selection:

1. The amount of fluid that must be pumped. This factor determines the size of pump (or pumps) necessary.
2. The properties of the fluid. The density and the viscosity of the fluid influence the power requirement for a given set of operating conditions; corrosive properties of the fluid determine the acceptable materials of construction. If solid particles are suspended in the fluid, this factor dictates the amount of clearance necessary and may eliminate the possibility of using certain types of pumps.
3. The increase in pressure of the fluid due to the work input of the pumps. The head change across the pump is influenced by the inlet and downstream-reservoir pressures, the change in vertical height of the delivery line, and frictional effects. This factor is a major item in determining the power requirements.
4. Type of flow distribution. If nonpulsating flow is required, certain types of pumps, such as simplex reciprocating pumps, may be unsatisfactory. Similarly, if operation is intermittent, a self-priming pump may be desirable, and corrosion difficulties may be increased.
5. Type of power supply. Rotary positive-displacement pumps and centrifugal pumps are readily adaptable for use with electric-motor or internal-combustion-engine drives; reciprocating pumps can be used with steam or gas drives.
6. Cost and mechanical efficiency of the pump.

Reciprocating Pumps

A ***reciprocating piston pump*** delivers energy to a flowing fluid by means of a piston acting through a cylinder. Although steam is often employed as the source of power for this type of pump, the piston can be activated by other means, such as a rotating crankshaft operated by an electric motor. Thus, reciprocating piston pumps can be classified as ***steam pumps*** or ***power pumps.*** They can also be classified as ***single-acting*** or ***double-acting*** depending on whether energy is delivered to the fluid on both the forward and backward strokes of the piston.

Specifications for reciprocating steam pumps are expressed in abbreviated form as diameter of the steam cylinder, diameter of the water cylinder, and length of the piston stroke in inches. For example, a $7 \times 4\frac{1}{2} \times 12$ pump has a steam-cylinder diameter of 7 in., a water-cylinder diameter of $4\frac{1}{2}$ in., and a stroke length of 12 in. In general, reciprocating steam pumps having stroke lengths less than 10 in. should not operate at more than 100 strokes per minute because of excessive wear. For longer stroke lengths, reasonable piston speeds are in the range of 50 to 90 ft/min.

In reciprocating steam pumps, the steam is not used expansively as in the common types of steam engines. Instead, essentially the full pressure of the steam is maintained throughout the entire stroke by keeping the steam-inlet ports fully open during the stroke. As the piston moves forward through the cylinder on the delivery side of the pump, the fluid is compressed and forced out of the cylinder. By a system of opening and closing valves, the piston can deliver energy to the fluid with every stroke.

The rate of fluid discharge from the cylinder is zero at the beginning of a piston stroke and increases to a maximum value when the piston reaches full speed. If only one discharge cylinder is used, the flow rate will pulsate. These pulsations can be reduced by placing an air chamber on the discharge line or by using a number of delivery cylinders compounded. Simplex pumps have only one delivery cylinder, *duplex* pumps have two cylinders, and *triplex* pumps have three cylinders.

The theoretical fluid displacement of a piston pump equals the total volume swept by the piston on each delivery stroke. Because of leakage past the piston and the valves and failure of the valves to close instantly, this theoretical displacement is not attained in actual practice. The volumetric efficiency, defined as the ratio of the actual displacement to the theoretical displacement, is usually in the range of 70 to 95 percent.

When a steam pump is used, the pressure of the steam in pounds per unit area times the area of the piston would be the maximum force that could be exerted on the work-delivery piston if the machine were perfect and no friction were involved. However, friction is involved and work must be done on the liquid (or work-receiving fluid) under conditions in which the steam pressure is a finite amount greater than the liquid pressure. The ratio of the pressure theoretically required on the steam piston to the pressure actually exerted by the steam is known as the *pressure efficiency* or *steam-end efficiency*. It includes the effects of piston and rod friction, momentum changes in acceleration of the piston and fluid, and leakage of fluid past the piston. The pressure efficiency varies from about 50 percent for small pumps up to 80 percent for large pumps.

Another so-called efficiency, known as *hydraulic efficiency*, is sometimes given for reciprocating pumps. This efficiency indicates losses due to velocity changes in the inlet and outlet of the pump, friction, and valves. It is defined as the ratio of the actual head across the pump to the sum of the actual head pumped and the losses in the suction and discharge lines.

For pumps driven by electric motors, overall efficiency is usually defined as the work done on the fluid divided by the electrical energy supplied to the motor. Attempts to apply this type of definition to the overall efficiency of steam pumps can give misleading results because the term “energy supplied by the steam” can have many meanings. For design estimates, an overall efficiency for steam pumps is sometimes defined as the work done on the fluid divided by the ideal work that could have been obtained by the isentropic expansion of the steam from its initial temperature and pressure to its exhaust pressure. The

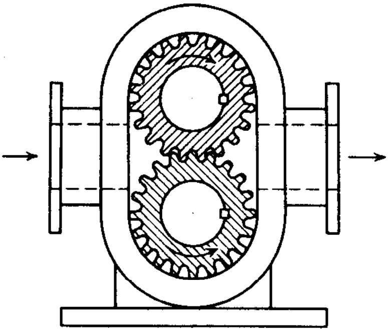

FIGURE 14-35
Cutaway view of external gear (rotary) pump.

numerical value of this type of overall efficiency is significant only when the operating conditions are specified for the particular pump.

Reciprocating pumps, in general, have the advantage of being able to deliver fluids against high pressures and operate with good efficiency over a wide range of operating conditions. A major disadvantage of piston and plunger pumps is that they cannot be used with fluids which contain abrasive solids. Reciprocating diaphragm pumps, however, are satisfactory for handling fluids with large amounts of suspended solids at low heads.

Rotary Positive-Displacement Pumps

Pumps of this type combine a rotary motion with a positive displacement of the fluid. A common type of rotary gear pump is illustrated in Fig. 14-35. Two intermeshing gears are fitted into a casing with a sufficiently close spacing to seal off effectively each separate tooth space. As the gear rotate in opposite directions, fluid is trapped in each tooth space and is delivered to the exit side of the pump. Similar results can be obtained by using a rotating eccentric cam or separately driven impellers having several lobes.

No priming is required with rotary positive pumps, and they are well adapted for pumping highly viscous fluids. A constant rate of delivery is obtained, and the fluid may be delivered at high pressures. Because of the small clearance that must be maintained, this type of pump should not be used with nonlubricating fluids or with fluids containing solid particles.

Rotary Centrifugal Pumps

In a centrifugal pump the fluid is fed into the pump at the center of a rotating impeller and is thrown outward by centrifugal force. The fluid at the outer periphery of the impeller attains a high velocity and, consequently, high kinetic energy. The conversion of this kinetic energy into pressure energy supplies the pressure difference between the suction side and the delivery side of the pump.

Different forms of impellers are used in centrifugal pumps. One common type, known as a *closed impeller,* consists of a series of curved vanes attached to a central hub and extending outward between two enclosing plates. An open impeller is similar, except that there are no enclosing plates. Impellers of this type are used in *volute pumps*, which are the simplest form of centrifugal pumps.

Energy losses caused by turbulence at the point where the liquid path changes from radial flow to tangential flow in the pump casing can be decreased by using so-called *turbine pumps*. With this type of centrifugal pump, the liquid flows from the impeller through a series of fixed vanes forming a diffusion ring. The change in direction of the fluid is more gradual than in a volute pump, and a more efficient conversion of kinetic energy into pressure energy is obtained.

For an ideal centrifugal pump, the speed of the impeller (N r/min) should be directly proportional to the rate of fluid discharge (q gpm), or

$$\frac{N_1}{N_2} = \frac{q_1}{q_2} \tag{17}$$

The head (or pressure difference) produced by the pump is a function of the kinetic energy developed at the point of release from the impeller. The head developed by an ideal pump, therefore, should be directly proportional to the square of the impeller speed:

$$\frac{\text{Head}_1}{\text{Head}_2} = \frac{q_1^2}{q_2^2} = \frac{N_1^2}{N_2^2} \tag{18}$$

The power required for a perfect pump is directly proportional to the product of the head and the flow rate; therefore,

$$\frac{\text{Power}_1}{\text{Power}_2} = \frac{q_1^3}{q_2^3} = \frac{N_1^3}{N_2^3} \tag{19}$$

The preceding equations apply for the ideal case in which there are no frictional, leakage, or recirculation losses. In any real pump, however, these losses do occur, and their magnitudes can be determined only by actual tests. As a result, characteristic curves are usually supplied by pump manufacturers to indicate the performance of any particular centrifugal pump. Figure 14-36 shows a typical set of characteristic curves for a centrifugal pump.

The ratings of centrifugal pumps are ordinarily based on the head and capacity at the point of maximum efficiency. The size of the pump is usually specified as the diameter of the discharge opening. The rating for the pump referred to in Fig. 14-36 would be 140 gpm and a head of 40 ft if water (viscosity of 1 centistoke) is the fluid involved. From the data shown in Fig. 14-36 , if the head is increased to 50 ft, the capacity will decrease to 80 gpm and the efficiency of the pump will decrease. The capacity could be decreased to 80 gpm at a head of 40 ft by throttling the discharge so that a head of 50 ft is actually generated within the pump, but this would result in a reduction in the pump efficiency. Consequently, the design engineer should always attempt to give the necessary

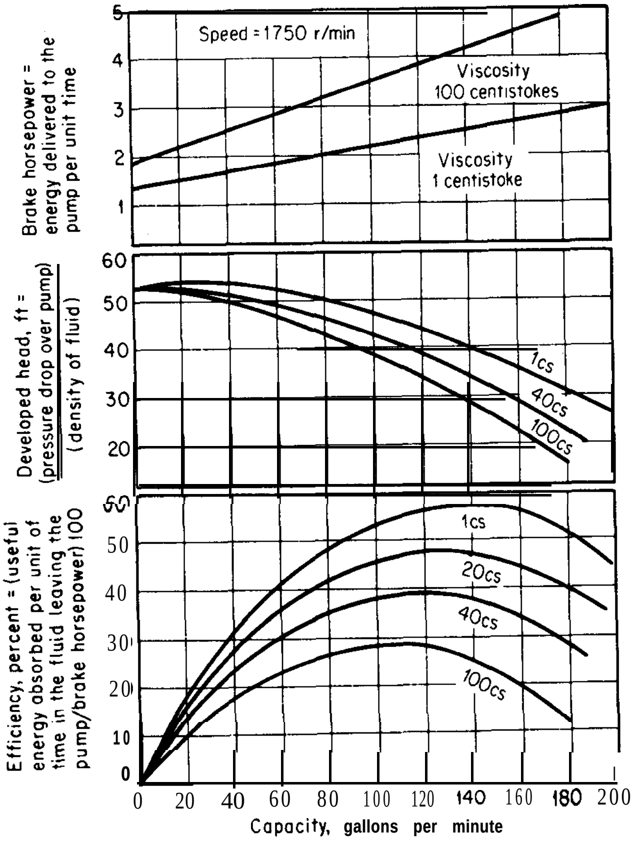

FIGURE 1436
Characteristic curves for a typical centrifugal pump showing effect of viscosity.

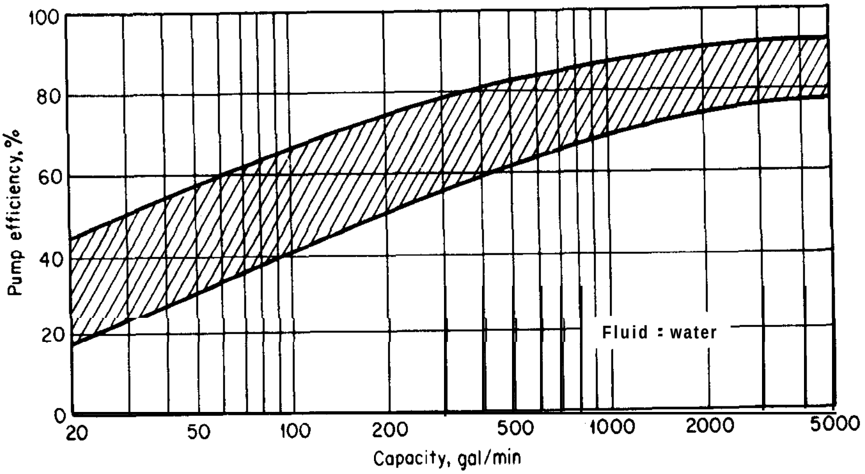

FIGURE 1437
Efficiencies of centrifugal pumps.

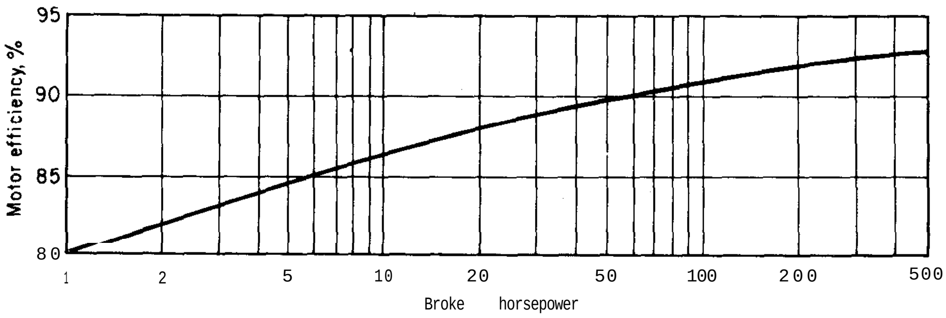

FIGURE 1438
Efficiencies of three-phase motors.

pump specifications as accurately as possible in order to obtain the correct pump which will operate at maximum efficiency.

Figure 14-37 gives values that are suitable for design estimates of centrifugal-pump efficiencies. Because pump and driver efficiencies must both be considered when total power costs are determined, necessary design data on the efficiency of electric motors are presented in Fig. 14-38.

The following list gives the major advantages and disadvantages of centrifugal pumps:

Advantages

1. They are simple in construction and cheap.
2. Fluid is delivered at uniform pressure without shocks or pulsations.
3. They can be coupled directly to motor drives. In general, the higher the speed, the smaller the pump and motor required for a given duty.
4. The discharge line may be partly shut off or completely closed off without damaging the pump.
5. They can handle liquid with large amounts of solids.
6. There are no close metal-to-metal fits.
7. There are no valves involved in the pump operation.
8. Maintenance costs are lower than for other types of pumps.

Disadvantages

1. They cannot be operated at high heads.
2. They are subject to air binding and usually must be primed.
3. The maximum efficiency for a given pump holds over a fairly narrow range of conditions.
4. They cannot handle highly viscous fluids efficiently.

Air-Displacement Systems

The pumps discussed in the preceding sections depend on the mechanical action of pistons, impellers, plungers, or other devices to move the fluid. Movement of fluids can also be accomplished by the use of air pressure, and many types of air-displacement systems have been developed for this purpose. The most common of these systems are air lifts, acid eggs, and jet pumps.

In the simple *air lift*, compressed air is introduced into the submerged end of the discharge pipe at a distance of H_s ft below the liquid surface. Because the air-and-liquid mixture is lighter than the liquid alone, the mixture rises through the discharge pipe and is expelled into an overhead receiver at a distance of H_t ft above the liquid surface. Although equations for the operation of an air lift can be developed theoretically, the frictional effects are so complex that the following empirical equation is often assumed for the design basis:?

$$V_{\text{air}} = 0.8 \frac{H_t}{C_a \log\left[(H_s + 34)/34\right]} \tag{20}$$

where V_{air} is the cubic feet of free air (i.e., at normal atmospheric conditions) required to raise 1 gal of water, and C_a has the following values:

H_t, ft	Recommended value of $H_s/(H_s + H_t)$	C_a (outside air line)
20-125	0.65	348
126-250	0.60	335
251-400	0.50	296
401-650	0.40	246
651-700	0.35	216

Acid eggs or *blow cases* are simply closed vessels with inlet and outlet lines and an air connection. Air is admitted to the vessel and forces the liquid out through the discharge line. Operation of acid eggs is intermittent, and the elevation attained depends on the air pressure. Although these systems are inexpensive and easy to operate, they are inefficient. Their use is limited primarily to batchwise operations with corrosive fluids.

Jet pumps, employing water, steam, or gas, as the operating medium, are often used for transferring fluids. The operating medium flows rapidly through an expanding nozzle and discharges into the throat of a venturi. As the operating medium issues from the nozzle, the high velocity head causes a decrease in the pressure head. If the resulting pressure is less than that of the

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†F. W. O'Neil, "Compressed Air Data," 5th ed., pp. 188-191, Ingersoll-Rand Company, New York, 1954.

TABLE 3
Steam-jet average consumption of steam at 100 psig in pounds per hour†

For larger or smaller capacities, steam consumption is approximately proportional to the capacity.

Capacity		Suction pressure. in. Hg absolute										
Wt of gas-vapor mixture handled, lb/h	Wt % net dry air (non-condensables) in gas-vapor mixture	0.5		1.0		1.5	2.0	3.0	4.0		6.0
		3-stage	2-stage	3-stage	2-stage	2-stage	2-stage	2-stage	2-stage	I-stage	I-stage
10	100	73	QQ	59	70	58	50	42	38	58	36
10	70	59	84	47	60	49	42	35	31	63	39
10	40	45	68	33	47	38	32	26	23	68	41
10	10	24	45	16	28	21	17	14	12	74	42

† J. H. Perry, "Chemical Engineers' Handbook," 4th ed., McGraw-Hill Book Company, New York, 1963.

second fluid at that point, the fluid will be sucked into the venturi throat along with the operating medium and discharged from the venturi outlet.

Jet pumps are used to remove air, gases, or vapors from condensers and vacuum equipment, and the steam jets can be connected in series or parallel to handle larger amounts of gas or to develop a greater vacuum. The capacity of steam-jet ejectors is usually reported as pounds per hour instead of on a volume basis. For design purposes, it is often necessary to make a rough estimate of the steam requirements for various ejector capacities and conditions. The data given in Table 3 can be used for this purpose.

Barometric-leg pumps are used for assisting in maintaining a vacuum when condensable vapors are involved. Auxiliary pumps are necessary to remove any fixed gases that may accumulate in the leg. For water condensation, the leg usually empties into an open well, and the vertical length of the leg must be longer than 34 ft.

Gas Compressors

Movement of gases can be accomplished by use of fans, blowers, vacuum pumps, and compressors. Fans are useful for moving gases when pressure differences less than about 0.5 psi are involved. Centrifugal blowers can handle large volumes of gases, but the delivery pressure is limited to approximately 50 psig. Reciprocating compressors can be employed over a wide range of capacities and pressures, and they are used extensively in industrial operations. Sizes of reciprocating compressors ranging from less than 1 to 3000 hp are available, and some types can give delivery pressures as high as 4000 atm.

Compressor efficiencies are usually expressed as isentropic efficiencies, i.e., on the basis of an adiabatic reversible process. Isothermal efficiencies are sometimes quoted, and design calculations are simplified when isothermal efficiencies are used. In either case, the efficiency is defined as the ratio of the power required for the ideal process to the power actually consumed.

Because the energy necessary for an isentropic compression is greater than that required for an equivalent isothermal compression, the isentropic efficiency is always greater than the isothermal efficiency. For reciprocating compressors, isentropic efficiencies are generally in the range of 70 to 90 percent and isothermal efficiencies are about 50 to 70 percent. Multistage compression is necessary for high efficiency in most large compressors if the ratio of the delivery pressure to the intake pressure exceeds approximately 5 : 1.

Expressions for the theoretical power requirements of gas compressors can be obtained from the basic equations of thermodynamics. For an ideal gas undergoing an isothermal compression (pv = constant), the theoretical power requirement for any number of stages can be expressed as follows:

$$\text{Power} = p_1 v_1 \ln\frac{p_2}{p_1} \tag{21}$$

or

$$\text{hp} = 3.03 \times 10^{-5} p_1 q_{fm_1} \ln\frac{p_2}{p_1} \tag{22}$$

where power = power requirement, ft . lbf/lbm
p_1 = intake pressure, lbf/ft^2
v_1 = specific volume of gas at intake conditions, ft^3/lbm
p_2 = final delivery pressure, lbf/ft^2
q_{fm_1} = cubic feet of gas per minute at intake conditions

Similarly, for an ideal gas undergoing an isentropic compression (pv^k = constant), the following equations apply:

For single-stage compressor

$$\text{Power} = \frac{k}{k-1} p_1 v_1 \left[\left(\frac{p_2}{p_1} \right)^{(k-1)/k} - 1 \right] \tag{23}$$

$$\text{hp} = \frac{3.03 \times 10^{-5} k}{k-1} p_1 q_{fm_1} \left[\left(\frac{p_2}{p_1} \right)^{(k-1)/k} - 1 \right] \tag{24}$$

$$p_2 = p_1 \left(\frac{v_1}{v_2} \right)^k = p_1 \left(\frac{T_2}{T_1} \right)^{k/(k-1)} \tag{25}$$

$$T_2 = T_1 \left(\frac{v_1}{v_2} \right)^{k-1} = T_1 \left(\frac{p_2}{p_1} \right)^{(k-1)/k} \tag{26}$$

For *multistage compressor,* assuming equal division of work between cylinders and intercooling of gas to original intake temperature,

$$\text{hp} = \frac{3.03 \times 10^{-5} k_s}{k - 1} p_1 q_{fm_1} \left[\left(\frac{p_2}{p_1} \right)^{(k-1)/kN_s} - 1 \right] \tag{27}$$

$$T_2 = T_1 \left(\frac{p_2}{p_1} \right)^{(k-1)/kN_s} \tag{28}$$

where k = ratio of specific heat of gas at constant pressure to specific heat of gas at constant volume

v_2 = specific volume of gas at final delivery conditions, ft^3/lbm

T_1 = absolute temperature of gas at intake conditions, °R

T_2 = absolute temperature of gas at final delivery conditions, °R

N_s = number of stages of compression

Cost of Pumping Machinery

Figures 14-39 through 14-54 give approximate costs for different types of pumps, compressors, blowers, fans, and motors. Although the data from these figures

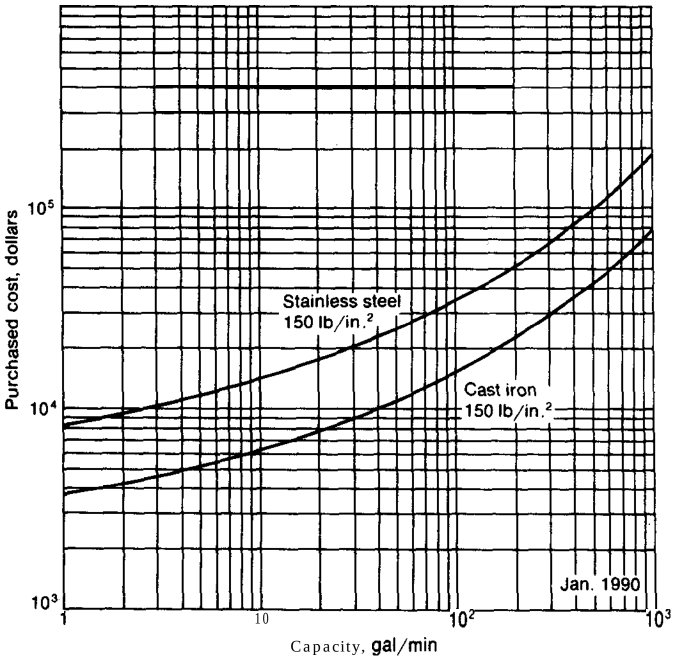


FIGURE 14-39
Cost of reciprocating pumps. Price includes pump and motor.

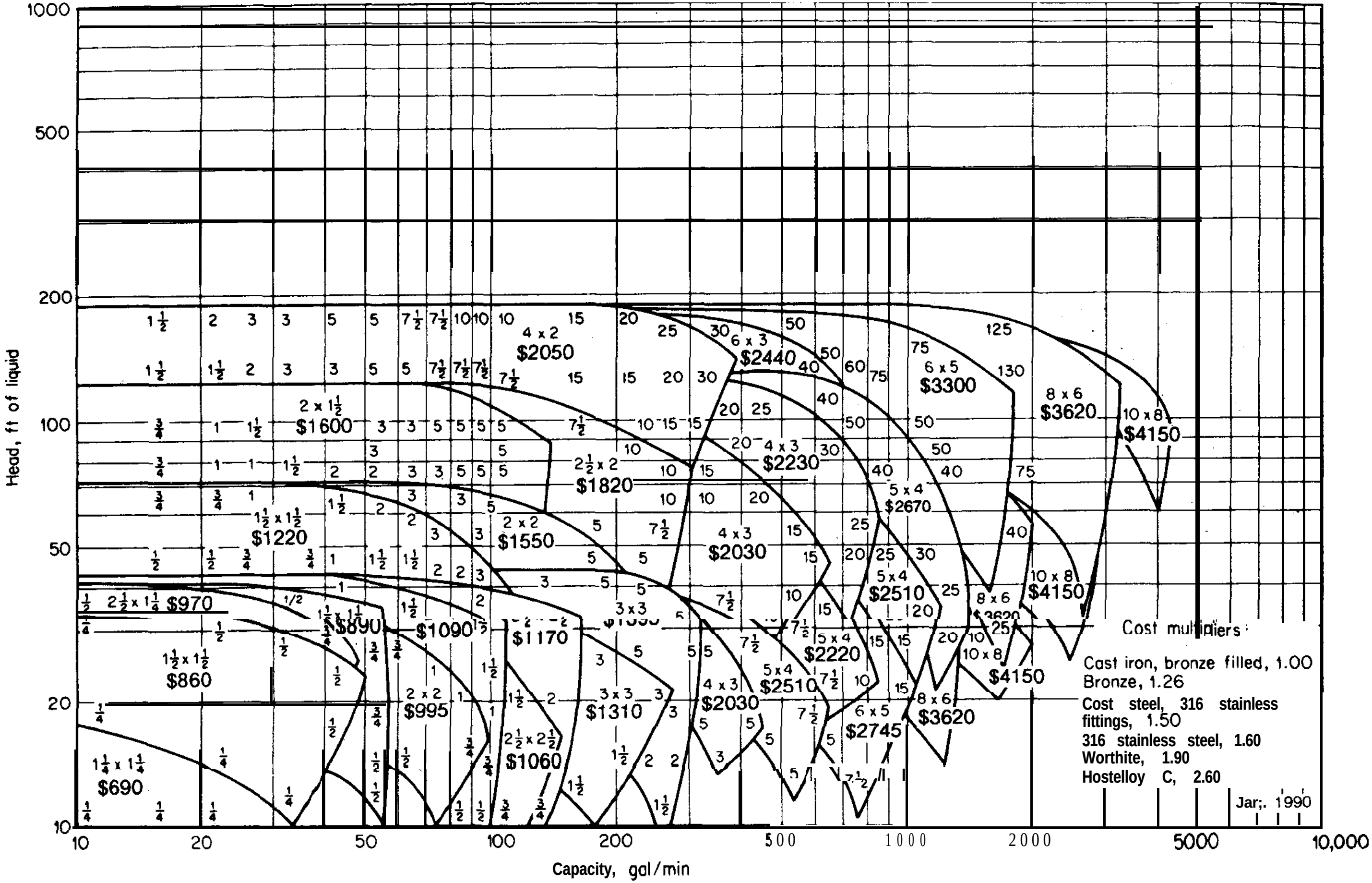


FIGURE 14-40

Pumps: general-purpose centrifugal (single- and two-stage, single-suction). Price includes pump, steel base, and coupling, but no motor. Small numbers

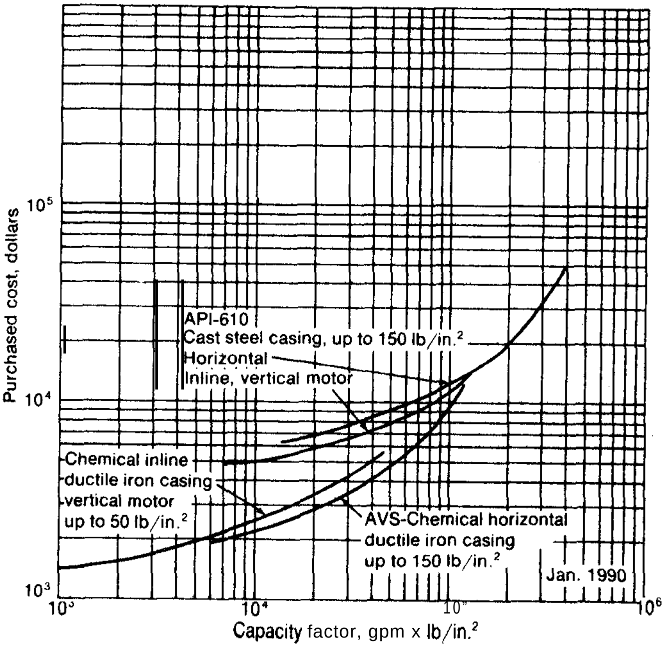


FIGURE 14-41
Cost of centrifugal pumps. Price includes motor.

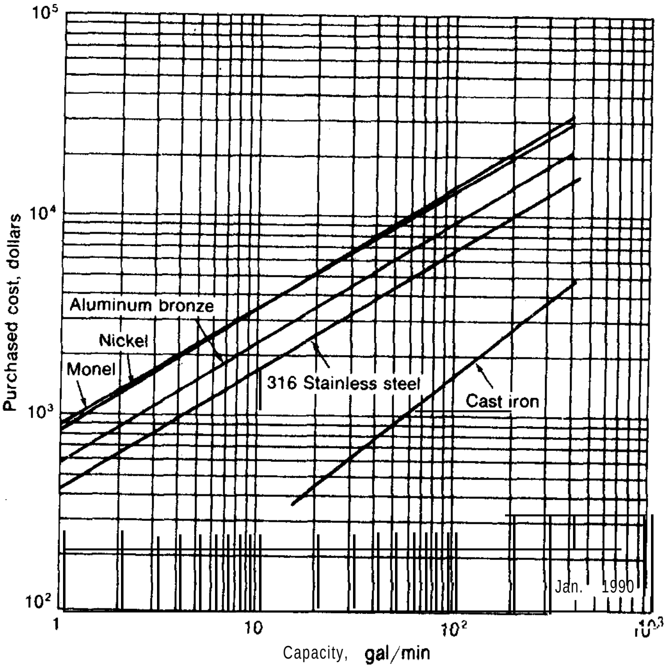


FIGURE 14-42
Cost of gear pumps, 100 psig discharge pressure. Cost includes pump, base plate, and V-belt drive, but no motor.

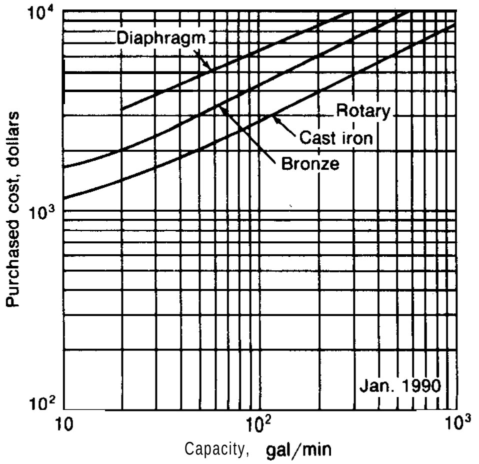


FIGURE 14.43
Cost of diaphragm and rotary pumps.

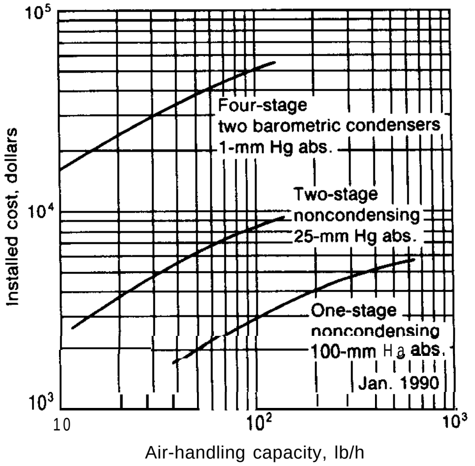


FIGURE 14-44
Cost of steam-jet ejectors. Carbon-steel construction, 1000 lb/h steam consumption.

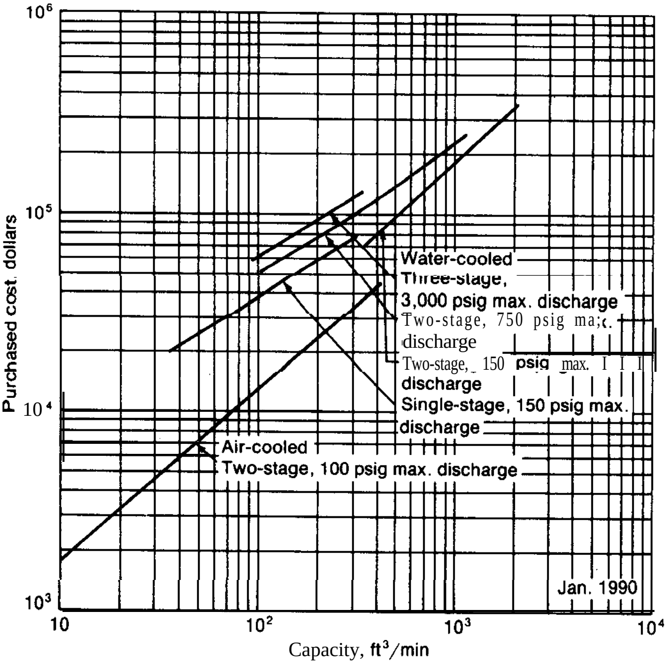
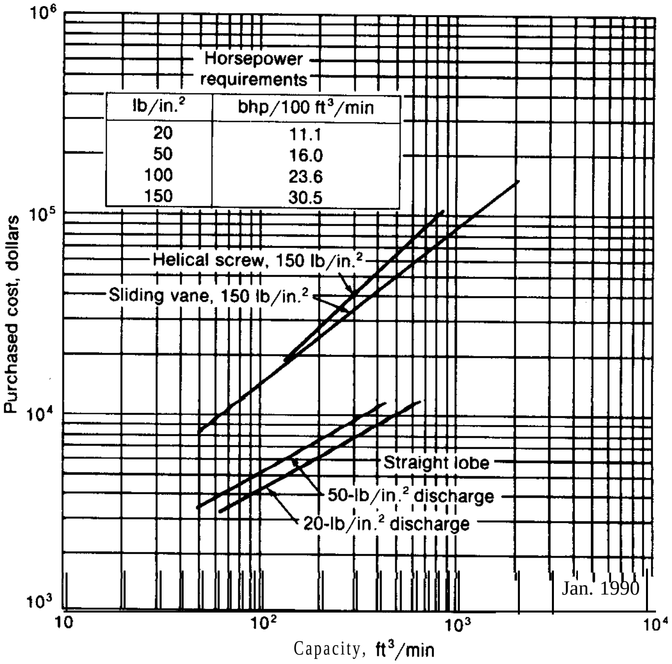


FIGURE 14-45
Cost of reciprocating compressors.



lb/in.^{2}	bhp/100 ft^3/min
20	11.1
50	16.0
100	23.6
150	30.5

FIGURE 14-46
Single-stage rotary compressors. Prices are for completely packaged compressor units (freight and installation costs excluded). The straight lobe prices also exclude aftercooler, trap, and controls.

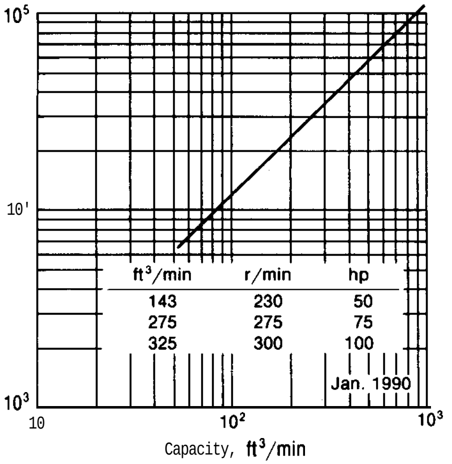


FIGURE 14-47
Cost of air compressors.

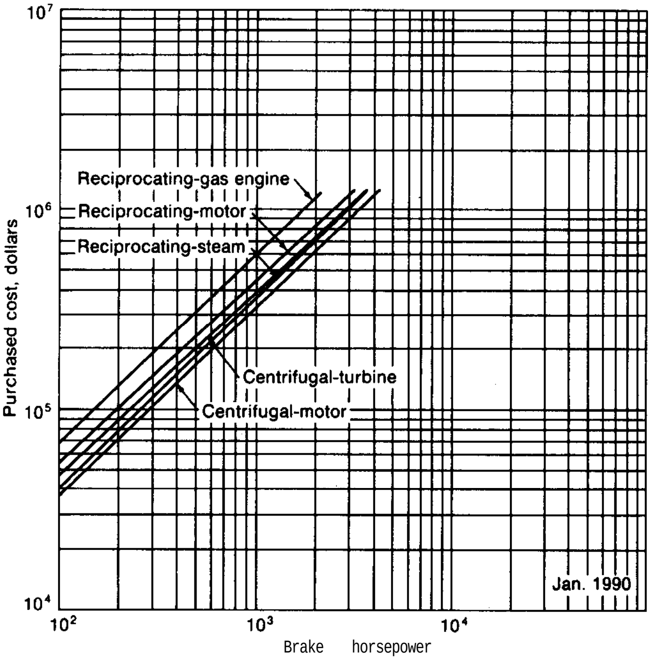


FIGURE **14-48**
Compressor costs. Prices includes drive, gear mounting, base plate, and normal auxiliary **equipment;** operating pressure to 1000 psig.

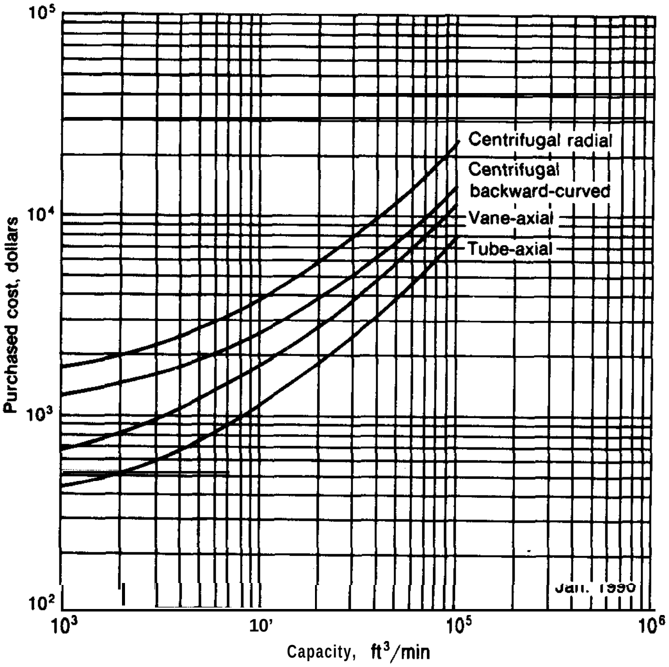


FIGURE **14-49**
Cost of centrifugal fans.

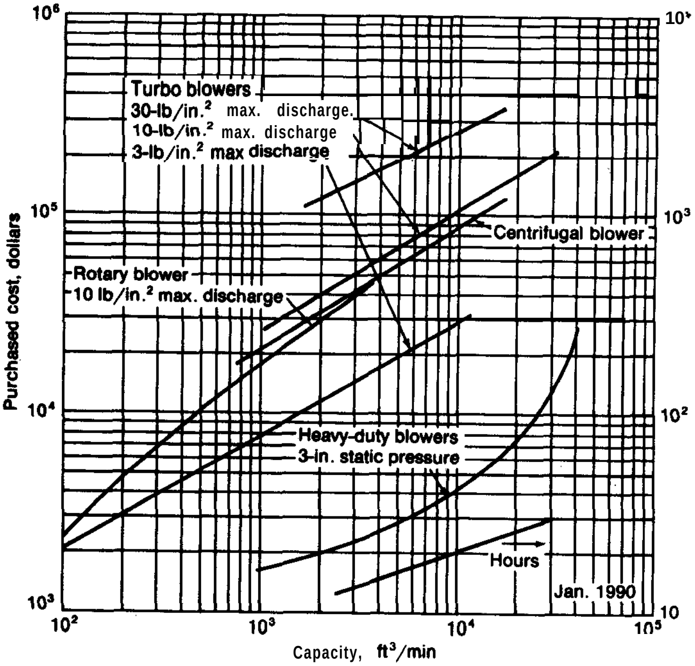


FIGURE **14-50**
Blowers (heavy-duty, industrial type).

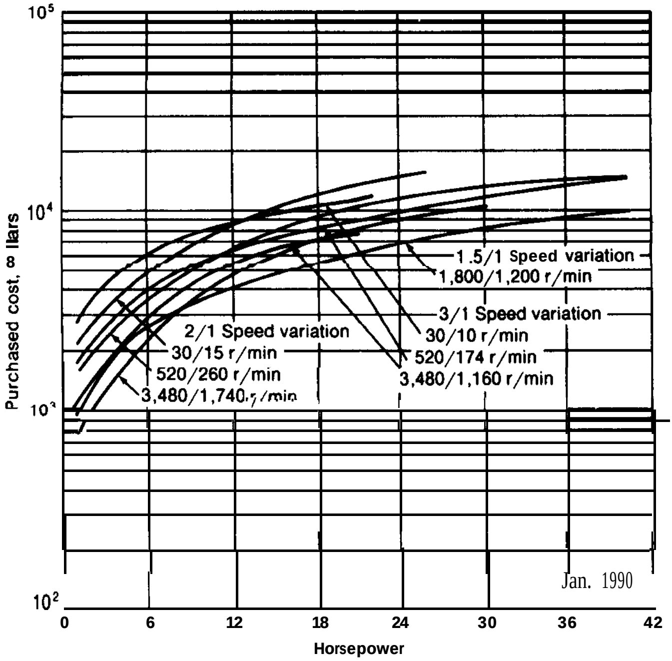


FIGURE **14-51**
Variable-speed drives. Price includes handwheel control with a built-in indicator and TEFC motors as an integral part of the unit.

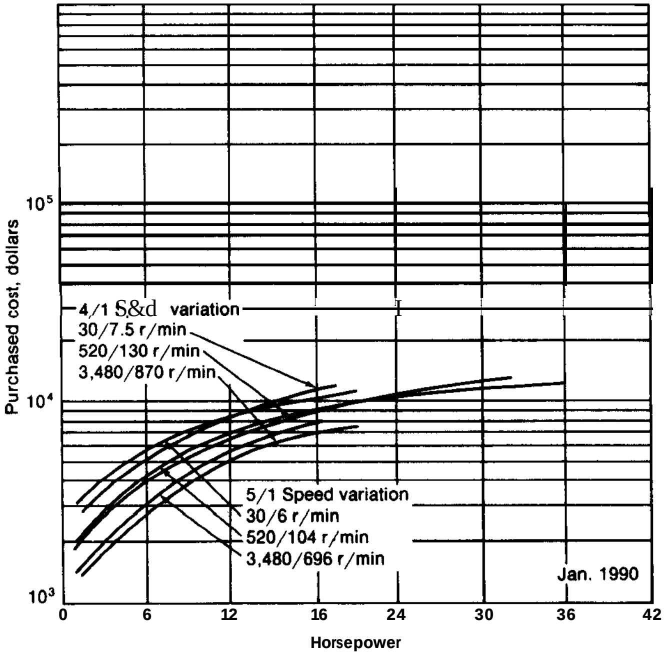


FIGURE **14-52**
Variable-speed drives. Price includes handwheel control with a built-in indicator and TEFC motors as an integral part of the unit.

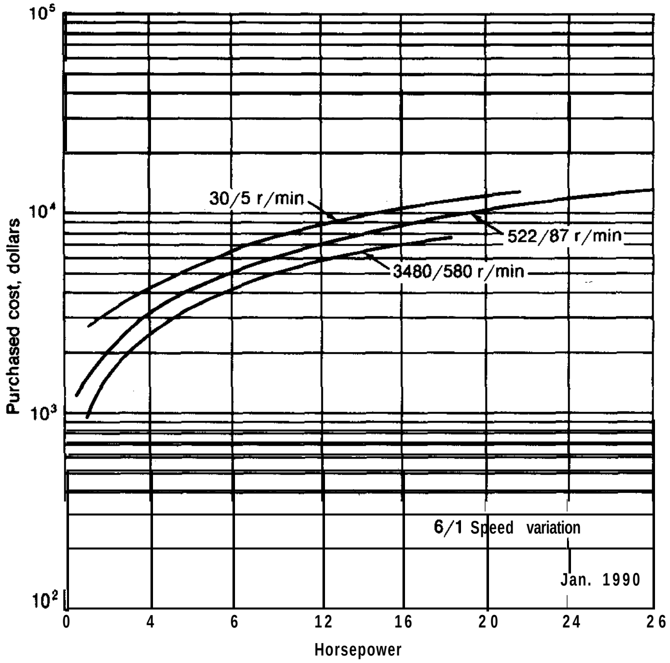


FIGURE **14-53**
Variable-speed drives. Price includes handwheel control with a built-in indicator and TEFC motors as an integral part of the unit.

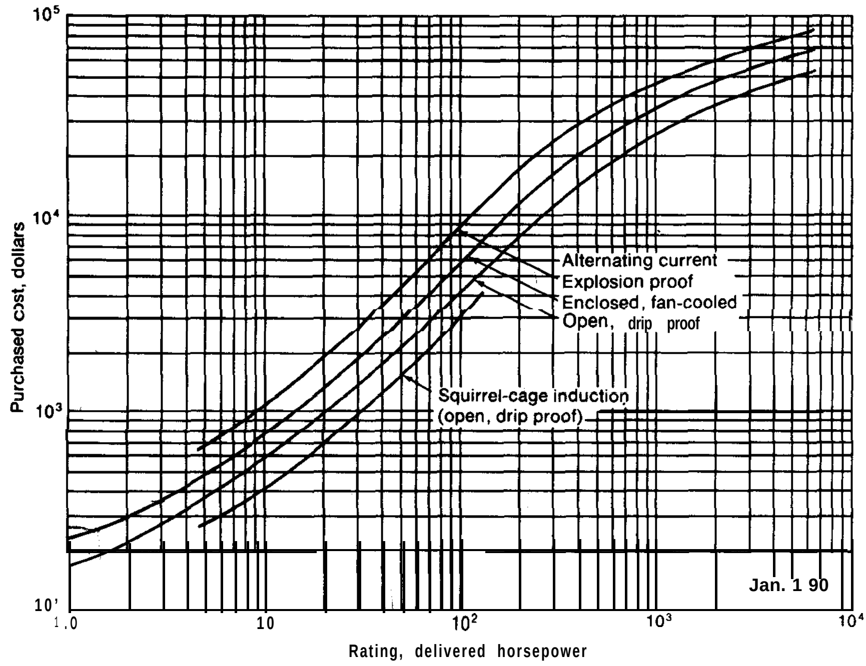


FIGURE 14-54
Cost of electric motors.

can be used for preliminary design estimates, firm estimates should be based on manufacturers' quotations.

FLOW MEASURING EQUIPMENT

Orifice meters, venturi meters, rotameters, and displacement meters are used extensively in industrial operations for measuring the rate of fluid flow. Other flow-measuring devices, such as weirs, pitot tubes, anemometers, and wet-test meters, are also useful in industrial operations.? In general, orifice meters are the cheapest and most flexible of the various types of equipment for measuring flow rates. Despite the inherent disadvantage of large permanent pressure drops with orifice installations, they are one of the most common types of flow-measuring equipment in industrial operations. Venturi meters are expensive and must be carefully proportioned and fabricated. However, they do not cause a large permanent pressure drop and are, therefore, very useful when power cost is an important factor.

Basic equations for the design and operation of orifice meters, venturi meters, and rotameters can be derived from the total energy balances presented at the beginning of this chapter. The following equations apply when the flowing fluid is a liquid, and they also give accurate results for the flow of gases if the pressure drop caused by the constriction is less than 5 percent of the upstream pressure:

For orifice meters and venturi meters

$$q_f = C_d S_c \sqrt{\frac{2g_c v(p_1 - p_c)}{1 - (S_c/S_1)^2}} \tag{29}$$

For rotameters

$$q_f = C_d S_c \sqrt{\frac{V_p 2g(\rho_p - 1/v)v}{S_p\left[1 - (S_c/S_1)^2\right]}} \tag{30}$$

where q_f = flow rate, ft^3/s
C_d = coefficient of discharge
S_c = cross-sectional flow area at point of minimum cross-sectional flow area, ft^2
S_1 = cross-sectional flow area in upstream section of duct before constriction, ft^2

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†Detailed descriptions of various types of flow-measuring equipment and derivations of related equations are presented in essentially all books dealing with chemical engineering principles. See R. H. Perry and D. Green, "Chemical Engineers' Handbook," 6th ed., McGraw-Hill Book Company, New York, 1984.

g_c = gravitational constant in Newton's law of motion, 32.17 ft · lbm/(s)(s)(lbf)
g = local gravitational acceleration, ft/(s)(s)
v = average specific volume of fluid, ft^3/lb
p_1 = static pressure in upstream section of duct before constriction, psf
p_c = static pressure at point of minimum cross-sectional flow area, psf
V_p = volume of plummet, ft^3
S_p = maximum cross-sectional area of plummet, ft^2
ρ_p = density of plummet, lb/ft^3

The value of the coefficient of discharge C_d for orifice meters depends on the properties of the flow system, the ratio of the orifice diameter to the upstream diameter, and the location of the pressure-measuring taps. Values of C_d for sharp-edged orifice meters are presented in Fig. 14-55. These values apply strictly for pipe orifices with ***throat taps,*** in which the downstream pressure tap is located one-third of one pipe diameter from the downstream side of the orifice plate and the upstream tap is located one pipe diameter from the upstream side. However, within an error of about 5 percent, the values of C_d indicated in Fig. 14-55 may be used for manometer taps located anywhere between the orifice plate and the hypothetical throat taps.

Venturi meters usually have a tapered entrance with an interior total angle of 25 to 30” and a tapered exit with an interior angle of 7”. Under these conditions, the value of the coefficient of discharge may be assumed to be 0.98 if

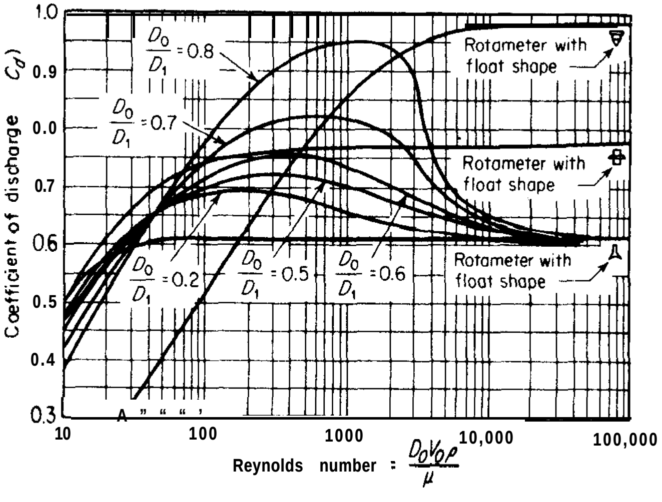

FIGURE 14-55

Coefficients of discharge for square-edged orifices with centered circular openings and for rotameters. (Subscript 0 indicates "at orifice or at constriction" and subscript 1 indicates "at upstream section.")

the Reynolds number based on conditions in the upstream section is greater than 5000.

Values of C_d for various plummet shapes in rotameters are presented in Fig. 14-55. The Reynolds number applicable to the rotameter coefficient of discharge is based on the flow conditions through the annular opening between the plummet and the containing tube. The equivalent diameter for use in the Reynolds number consists of the difference between the diameter of the rotameter tube at the plummet location and the maximum diameter of the plummet.

TANKS, PRESSURE VESSELS, AND STORAGE EQUIPMENT

Storage of liquid materials is commonly accomplished in industrial plants by use of cylindrical, spherical, or rectangular tanks. These tanks may be constructed of wood, concrete, fiber reinforced plastic (FRP), or metal. Metal is the most common material of construction, although use of FRP is becoming increasingly important. The design of storage vessels involves consideration of details such as wall thickness, size and number of openings, shape of heads, necessary temperature and pressure controls, and corrosive action of the contents.

The same principles of design apply for other types of tanks, including pressure vessels such as those used for chemical reactors, mixers, and distillation columns. Fur these cases, the shell is often designed and its cost estimated separately with the other components, such as tray assemblies, agitators, linings, and packing units, being handled separately. Process pressure vessels are normally designed in accordance with the *ASME Boiler and Pressure Vessel Code*.[†] They are usually cylindrical shells capped with an elliptical or hemispherical head at each end with installation in either a vertical or horizontal position. A major concern in the design is to make certain the walls of the vessel are sufficiently thick to permit safe usage under all operating conditions.

The necessary wall thickness for metal vessels is a function of (1) the ultimate tensile strength or the yield point of the metal at the operating temperature, (2) the operating pressure, (3) the diameter of the tank, and (4) the joint or welding efficiencies.[*] Table 4 presents a summary of design equations and data for use in the design of tanks and pressure vessels based on the ASME Boiler and Pressure Vessel Code as specified in Section VIII of Division 1.

[†] The ASME Boiler and Pressure Vessel Code is published by the ASME Boiler and Pressure Vessel Committee, American Society of Mechanical Engineers, New York City, with a new edition coming out every three years. Section VIII of the Code deals specifically with pressure vessels with the basic rules being given in Division 1 and alternative rules being presented in Division 2.

[‡] In the design of vacuum vessels, the ratio of length to diameter must also be taken into consideration.

TABLE 4
Design equations and data for pressure vessels

a = 2 for thicknesses < 1 in. and 3 for thicknesses ≥ 1 in.
C_c = allowance for corrosion, in.
D_a = the major axis of an ellipsoidal head, before corrosion allowance is added, in.
E_J = efficiency of joints expressed as a fraction
IDD = inside depth of dish, in.
L_a = inside radius of hemispherical head or inside crown radius of torispherical head, before corrosion allowance is added, in.
n = 1.2 for $D \leq 60$ in., 1.21 for $D = 61\text{-}79$ in., 1.22 for $D = 80\text{-}106$ in., and 1.23 for $D > 106$ in.
OD = outside diameter, in.
P = maximum allowable internal pressure, psig
r = knuckle radius, in.
r_i = inside radius of the shell, before corrosion allowance is added, in.
S = maximum allowable working stress, psi
t = minimum wall thickness, in.
ρ_m = density of metal, lbm/in.^{3}

Recommended design equations for vessels under internal pressure	Limiting conditions
For cylindrical shells	
$t = \dfrac{Pr_i}{SE_J - 0.6P} + C_c$	$t \leq r_i/2$ or $P \leq 0.385SE_J$
$t = r_i\left(\dfrac{SE_J + P}{SE_J - P}\right)^{1/2} - r_i + C_c$	$t > r_i/2$ or $P > 0.385E_J$
For spherical shells	
$t = \dfrac{Pr_i}{2SE_J - 0.2P} + C_c$	$t \leq 0.356r_i$ or $P \leq 0.665SE_j$
$t = r_i\left(\dfrac{2SE_J + 2P}{2SE_J - P}\right)^{1/3} - r_i + C_c$	$t > 0.356r_i$ or $P > 0.665SE_J$
For ellipsoidal head	
$t = \dfrac{PD_a}{2SE_J - 0.2P} + C_c$	0.5 (minor axis) = 0.250,
For torispherical (spherically dished) head	
$t = \dfrac{0.885\,PL_a}{SE_j - 0.1P} + C_c$	r = knuckle radius = 6% of inside crown radius and is not less than $3t$
For hemispherical head Same as for spherical shells with $r_i = L_a$	

(Continued

TABLE 4
Design equations and data for pressure vessels (Continued)

Properties of vessel heads (Include corrosion allowance in variables)	2: 1 Ellipsoidal	**Hemi-spherical**	Standard **ASME** torispherical
Capacity as volume in head, in?	$\frac{\pi D_a^3}{24}$	$\frac{2}{3}\pi L_a^3$	$0.9\left[\frac{2\pi L_a^2}{3}(IDD)\right]$
IDD = inside depth of dish, in.	$\frac{D_a}{4}$	L_a	$L_a - [(L_a - r)^2 - (L_a - t - r)^2]^{1/2}$
Approximate weight of **dished** portion of head, **lbm**	$\rho_m\left[\frac{\pi(nD_a + t)^2 t}{4}\right]$	$\rho_m[2\pi L_a^2 t]$	$\rho_m\left[\frac{\pi(OD + OD/24 + at)^2 t}{4}\right]$

Joint efficiencies

For double-welded butt joints
if fully radiographed = 1.0
if spot examined = 0.85
if not radiographed = 0.70

In general, for spot examined
if electric resistance weld = 0.85
if lap welded = 0.80
if single-butt welded = 0.60

Recommended stress values

Metal	Temp., °F	*S*, psi
Carbon steel (SA-285, Cr. C)	-20 to 650	13,700
	750	12,000
	850	8,300
Low-alloy steel for resistance to H_2 and H_2S (SA-387, **Gr.12C1.1**)	-20 to 800	13.700
	950	11,000
	1050	5,000
	1200	1,000
High-tensile steel for heavy-wall vessels (SA-302, **Gr.B**)	-20 to 750	20,000
	850	16,800
	950	10,000
	1000	6,200
High-alloy steel for cladding and corrosion resistance		
Stainless 304 (SA-240)	-20	18,700
	650	11,200
	800	10,500
	1000	9,700
Stainless 316 (SA-240)	-20	18,700
	650	11.500
	800	11,000
	1000	10,600
Nonferrous metals		
Copper (SB-11)	100	6,700
	400	3.000
Aluminum (SB-209, 1100-0)	100	2,300
	400	1,000

See the latest **ASME** Boiler and Pressure Vessel Code for further details.

COSTS FOR TANKS, PRESSURE VESSELS, AND STORAGE EQUIPMENT

Cost data for mixing tanks including agitators,? storage tanks, and pressure tanks are presented in Figs. 14-56 to 14-58 while Table 5 gives costs for selected containers. In determining the total cost for the vessel, allowances must be made for nozzles on the unit, supports and foundations, platforms, labor, and indirect costs as well as for all of the internals in the vessel.‡

Numerous articles have been published which give methods for obtaining vessel costs based on estimates of costs for the individual components, such as for materials, labor, nozzles, manholes, and overhead related to fabrication, to arrive at an estimated cost (f.o.b) at the fabricator's shop. Final installed cost can be obtained by applying factors to account for freight, labor, materials, engineering, and overhead related to getting the unit to the plant and installing

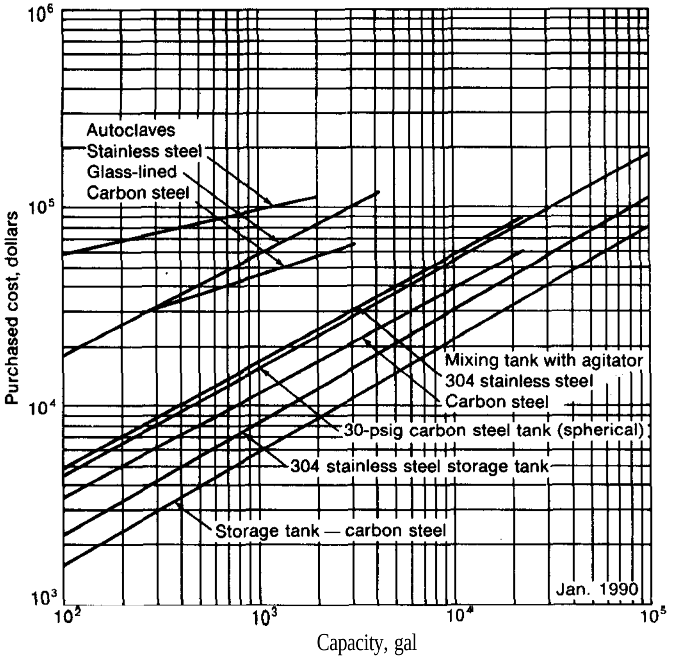

FIGURE 14-56
Cost of mixing, storage, and pressure tanks. Price for the mixing tank includes the cost of the driving unit.

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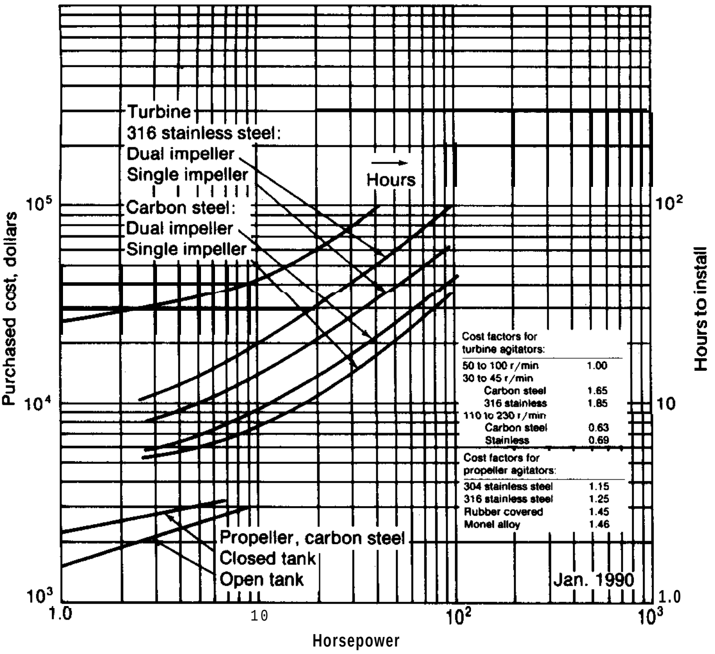
†For methods of calculating power requirements for agitators, see R. H. Perry and D. Green, "Chemical Engineers' Handbook," 6th ed., McGraw-Hill Book Company, New York, 1984.

‡See the information presented in Chap. 16 on costs for plate and packed towers where cost data are given for tower shells including manholes and nozzles, internals such as trays and packing, and auxiliaries such as ladders and insulation. Cost data for reactor vessels are also presented in Chap. 16 under the section on costs for reactor equipment components.



Cost factors for turbine agitators:	
50 to 100 r/min	1.00
30 to 45 r/min	
Carbon steel	1.65
316 stainless	1.85
110 to 230 r/min	
Carbon steel	0.63
Stainless	0.69

Cost factors for propeller agitators:	
304 stainless steel	1.15
316 stainless steel	1.25
Rubber covered	1.45
Monel alloy	1.46

FIGURE 14-57
Cost of turbine and propeller agitators.

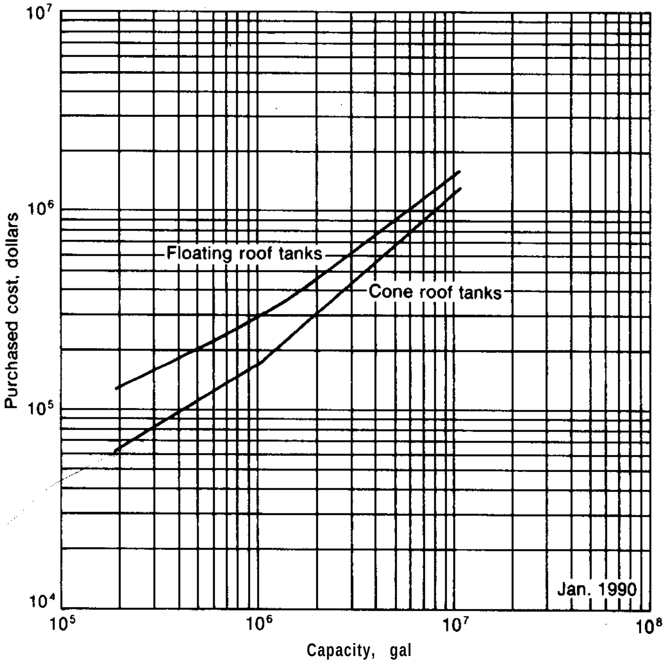


FIGURE 14-58
Cost of large-volume carbon-steel storage tanks.

TABLE 5
Approximate costs of small containers for chemical products (Jan., 1990)

Container size, description	Unit cost	Usable volume, ft^3
55-gal steel drum, new	$ 24.30	7.35
55-gal steel drum, used, cleaned	13.60	7.35
55-gal aluminum drum	112.00	7.35
55-gal type 304 stainless steel drum	297.00	7.35
30-gal steel drum	16.2	4.00
16-gal steel drum	3.3	2.14
61-gal fiber drum, dry products only	10.9	8.15
55-gal fiber drum, dry products only	9.9	7.35
47-gal fiber drum, dry products only	9.7	6.28
41-gal fiber drum, dry products only	9.2	5.48
30-gal fiber drum, dry products only	7.8	4.00
15-gal fiber drum, dry products only	3.4	2.00
Multiwall paper bags, polyethylene film	0.44-0.50	1.33
Corrugated cartons, 24 x 16 x 6 in.	0.89	1.33
l-gal glass jug, plastic cap	1.09	0.1335
l-gal polyethylene jar or bottle	0.52	0.1335
1-qt glass jar, plastic cap	0.45	0.034
1-qt polyethylene bottle	0.24	0.034
Pallets, expendable, 40 X 48 in. to 44 x 50 in.	7.46-13.60	
Pallets, warehouse type, 40 X 48 in. to 44 x 50 in.	15.00-20.90	

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it ready for use. These methods take into account the materials of construction to be used as well as operating temperature and pressure.

The most reliable method for estimating the costs for tanks and pressure vessels is to obtain the assistance of a representative of a vessel fabricator. In many cases, these representatives can give an estiamte based on a cost per unit weight for the particular vessel called for, or an actual delivered or installed price can be quoted. In addition, expert help with experience is needed to make good estimates of allowances to use for corrosion and to advise on the most appropriate materials of construction. Nevertheless, rough preliminary estimates of costs or tanks and pressure vessels can be made on the basis of gross methods such as are illustrated in Figs. 14-56 to 14-58. Some general rules of thumb for making cost estimates for pressure vessels are given in Table 6.†‡

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†Adapted from R. H. Perry and D. Green, "'Chemical Engineers' Handbook," 6th ed., McGraw-Hill Book Company, New York, 1984; ASME Boiler and Pressure Vessel Code, Section VIII, Div. 1, ASME, New York, 1977; H. Rase, "Chemical Reactor Design for Process Plants-Vol. II-Case Studies and Design Data," John Wiley and Sons, New York, 1977; and other references.

‡An example illustrating the use of Table 4 and Table 6 in the design and costing of a reactor vessel is given in Chap. 16 with the section on reactors.

TABLE 6

Rules of thumb for use in preliminary estimates of costs for pressure vessels

Costs for vessel (January, 1990—Including nozzles, manholes, and saddle or skirt but no special internals such as trays or agitators) as dollars per pound of weight of fabricated unit f.o.b. with carbon steel as the cost basis = $80(W_v)^{-0.34}$ where W_v is the total weight in pounds (applicable in W_v range of 800 lb to 100,000 lb).

To account for the extra weight due to nozzles, manholes, and skirts or saddles, increase the weight calculated for the smooth vessel including top and bottom by 1.5% for vessels to be installed in a horizontal position and by 20% for vessels to be installed in a vertical position.

Steel density can be taken as 489 lb/ft^3 or 0.283 lb/in^3.

Cost factors to convert from carbon steel as the material of construction for the fabricated unit follow:

	Shell-material cost factors
Carbon steel	1.0 (basis)
Stainless steel 304	2.0 to 3.5
Stainless steel 316	2.3 to 4.3
Monel	4.5 to 9.8
Titanium	4.9 to 10.6

Cost factors to convert from an internal pressure of up to 50 psig for carbon steel at temperatures below 800°F†

Pressure	Pressure factor	Pressure	Pressure factor
up to 50 psig	1.0 (basis)	800 psig	3.8
100	1.3	900	4.0
200	1.6	1000	4.2
300	2.0	1500	5.4
400	2.4	2000	6.5
500	2.8	3000	8.8
600	3.0	4000	11.3
700	3.3	5000	13.8

In general, the minimum wall thickness, not including allowances for corrosion, for any plate subject to pressure should not be less than $\frac{3}{32}$ in. for welded or brazed construction and not be less than $\frac{3}{16}$ in. for riveted construction except that the thickness of walls for unfired steam boilers should not be less than $\frac{1}{4}$ in.

A corrosion allowance of 0.010 to 0.015 in./yr, or about $\frac{1}{8}$ in. for a 10-year life is a reasonable value.

For high-pressure vessels, hemispherical heads are usually the most economical.

Lang factors to convert from the base cost of the delivered vessel (costed as if it were of carbon-steel material of construction so that weight becomes the primary measure of installation cost) to the cost of the vessel installed with all necessary auxiliaries except special internals such as trays or agitators are 3.0 for vessels installed in a horizontal position and 4.0 for vessels installed in a vertical position.

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†If the data are available, it is much better to use the design equations presented in Table 4 of this chapter to obtain necessary wall thickness based on the stress value at the operating temperature in place of using the given pressure factors since there is a critical interrelationship among material of construction, operating pressure, and operating temperature in establishing the design and cost of pressure vessels.

FILTERS

The primary factor in the design of filters is the cake resistance or cake permeability. Because the value of the cake resistance can be determined only on the basis of experimental data, laboratory or pilot-plant tests are almost always necessary to supply the information needed for a filter design. After the basic constants for the filter cake have been determined experimentally, the theoretical concepts of filtration can be used to establish the effects of changes in operating variables such as filtering area, slurry concentration, or pressure-difference driving force.

In recent years, there has been considerable advance in the development of filtration theory, but the development has not reached the stage where an engineer can design a filter directly from basic equations as with a fractionation tower or a heat exchanger. Instead, the final design should be carried out by the technical personnel in filtration-equipment manufacturing concerns or by someone who has access to the necessary testing equipment and has an extensive understanding of the limitations of the design equations.

Choice of a filter for a particular operation depends on many factors. Some of the more important of these are:

1. Fixed and operating costs
2. Quantities and value of materials to be handled
3. Properties of the fluid, such as viscosity, density, and corrosive nature
4. Whether the valuable product is to be the solid, the fluid, or both
5. Concentration, temperature, and pressure of slurry
6. Particle size and shape, surface characteristics of the particles, and compressibility of the solid material
7. Extent of washing necessary for the filter cake

Although a wide variety of filters is available on the market, the types can generally be classified as *batch* or *continuous.*

There are many times when the engineer wishes to make a preliminary design without asking for immediate assistance from a specialist in the field. The theoretical equations presented in the following sections are adequate for this purpose.?

DESIGN EQUATIONS

The rate at which filtrate is obtained in a filtering operation is governed by the materials making up the slurry and the physical conditions of the operation. The

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†For application of these equations to give optimum values of maximum production and minimum cost, see N. H. Chen, Liquid-Solid Filtration: Generalized Design and Optimization Equations, *Chem. Eng.*, **85**(17):97 (July 31, 1978).

major variables that affect the filtration rate are:

1. Pressure drop across the cake and the filter medium
2. Area of the filtering surface
3. Viscosity of the filtrate
4. Resistance of the cake
5. Resistance of the filter medium

The rate of filtrate delivery is inversely proportional to the combined resistance of the cake and filtering medium, inversely proportional to the viscosity of the filtrate, and directly proportional to the available filtering area and the pressure-difference driving force. This statement can be expressed in equation form as

$$\frac{dV}{d\theta} = \frac{AAP}{(R_K + R_F)\mu} \tag{31}$$

where V = volume of filtrate delivered in time θ
A = area of the filtering surface
AP = pressure drop across filter
R_K = resistance of the cake
R_F = resistance of the filter medium
μ = viscosity of the filtrate

Cake resistance R_K varies directly with the thickness of the cake, and the proportionality can be expressed as

$$R_K = Cl \tag{32}$$

where C is a proportionality constant and 1 is the cake thickness at time θ.

It is convenient to express R_F in terms of a fictitious cake thickness 1, with resistance equal to that of the filter medium. Thus,

$$R_F = Cl, \tag{33}$$

Designating w as the pounds of dry-cake solids per unit volume of filtrate, ρ_c as the cake density expressed as pounds of dry-cake solids per unit volume of wet filter cake, and V_F as the fictitious volume of filtrate per unit of filtering area necessary to lay down a cake thickness l_F, the actual cake thickness plus the fictitious cake thickness is

$$l + l_F = \frac{w(V + AV_F)}{\mathrm{PC}A} \tag{34}$$

Equations (31) to (34) can be combined to give

$$\frac{dV}{d\theta} = \frac{A^2\,\mathrm{AP}}{\alpha w(V + AV_F)\mu} \tag{35}$$

where α equals C/p, and is known as the *specific* cake resistance. In the usual range of operating conditions, the value of the specific cake resistance can be related to the pressure difference by the empirical equation

$$a = \alpha'(\Delta P)^s \tag{36}$$

where a' is a constant with value dependent on the properties of the solid material and s is a constant known as the *compressibility exponent of the cake.* The value of s would be zero for a perfectly noncompressible cake and unity for a completely compressible cake. For commercial slurries, the value of s is usually between 0.1 and 0.8.

The following general equation for rate of filtrate delivery is obtained by combining Eqs. (35) and (36):

$$\frac{dV}{D\theta} = \frac{A^2(\Delta P)^{1-s}}{a'w(v + AV_F)\mu} \tag{37}$$

This equation applies to the case of constant-rate filtration. For the more common case of constant-pressure-drop filtration, A, AP, s, a', w, V_F, and μ can all be assumed to be constant with change in V, and Eq. (37) can be integrated between the limits of zero and V to give

$$V^2 + 2AV_FV = \frac{2A^2(\Delta P)^{1-s}}{\alpha' w\mu}\theta \tag{38}$$

Batch Filters

Equations (37) and (38) are directly applicable for use in the design of batch filters. The constants a', s, and V_F must be evaluated experimentally, and the general equations can then be applied to conditions of varying A, AP, V, θ, w, and μ. One point of caution is necessary, however. In the usual situations, the constants are evaluated experimentally in a laboratory or pilot-plant filter. These constants may be used to scale up to a similar filter with perhaps 100 times the area of the experimental unit. To reduce scale-up errors, the constants should be obtained experimentally with the same slurry mixture, same filter aid, and approximately the same pressure drop as are to be used in the final designed filter. Under these conditions, the values of a' and s will apply adequately to the larger unit. Fortunately, V_F is usually small enough for changes in its value due to scale-up to have little effect on the final results.

The following example illustrates the methods for determining the constants and applying them in the design of a large plate-and-frame filter.

Example 5 Estimation of filtering area required for a plate-and-frame filtration operation. A plate-and-frame filter press is to be used for removing the solid material from a slurry containing 5 lb of dry solids per cubic foot of solid-free liquid. The viscosity of the liquid is 1 centipoise, and the filter must deliver at least 400 ft^3 of solid-free filtrate over a continuous operating time of 2 h when the pressure-difference driving force over the filter unit is constant at 25 psi. On the basis of the following data obtained in a small plate-and-frame filter press, estimate the total area of filtering surface required.

Experimental data. The following data were obtained in a plate-and-frame filter press with a total filtering area of 8 ft^2:

Total volume of filtrate (V), ft^3	Time from start of filtration (θ), h, at constant pressure difference of		
	AP = 20 psi	AP = 30 psi	AP = 40 psi
5	0.34	0.25	0.21
8	0.85	0.64	0.52
10	1.32	1.00	0.81
12	1.90	1.43	1.17

The slurry (with filter aid) was identical to that which is to be used in the large filter. The filtrate obtained was free of solid, and a negligible amount of liquid was retained in the cake.

Solution. An approximate solution could be obtained by interpolating for values of V at AP = 25 psi and then using two of these values to set up Eq. (38) in the form of two equations involving only the two unknowns V_F and $(\Delta P)^{1-s}/\alpha' w\mu$. By simultaneous solution, the values of V_F and $(\Delta P)^{1-s}/\alpha' w\mu$ could be obtained. The final required area could then be determined directly from Eq. (38). Because this method puts too much reliance on the precision of individual experimental measurements, a more involved procedure using all the experimental data is recommended.

The following method can be used to evaluate the constants V_F, s, and a’ in Eq. (38):

Rearrange Eq. (38) to give

$$\frac{\theta\,\Delta P}{V/A} = \frac{\alpha' w\mu(\,\Delta P)^s}{2}\frac{V}{A} + \alpha' w\mu V_F(\Delta P)^s$$

At constant AP a plot of $\theta\,\Delta P/(V/A)$ vs. V/A should give a straight line with a slope equal to $\alpha' w\mu(\Delta P)^s/2$ and an intercept at $V/A = 0$ of $\alpha' w\mu V_F(\Delta P)^s$. Figure 14-59 presents a plot of this type based on the experimental data for this problem. Any time the same variable appears in both the ordinate and abscissa of a straight-line plot, an analysis for possible misinterpretation should be made. In this case, the values of θ and AP change sufficiently to make a plot of this type acceptable.

The following slopes and intercepts are obtained from Fig. 14-59:

ΔP, psf	Slope = $\frac{\alpha' w\mu(\Delta P)^s}{2}$, (h)(lb)/($ft^4$)	Intercept = $\alpha' w\mu V_F(\Delta P)^s$, (h)(lb)/$ft^3$
20 X 144	2380	70
30 X 144	2680	80
40 X 144	2920	90

Values of α', s, and V_F could now be obtained by simultaneous solution with any three of the appropriate values presented in the preceding list. However, a

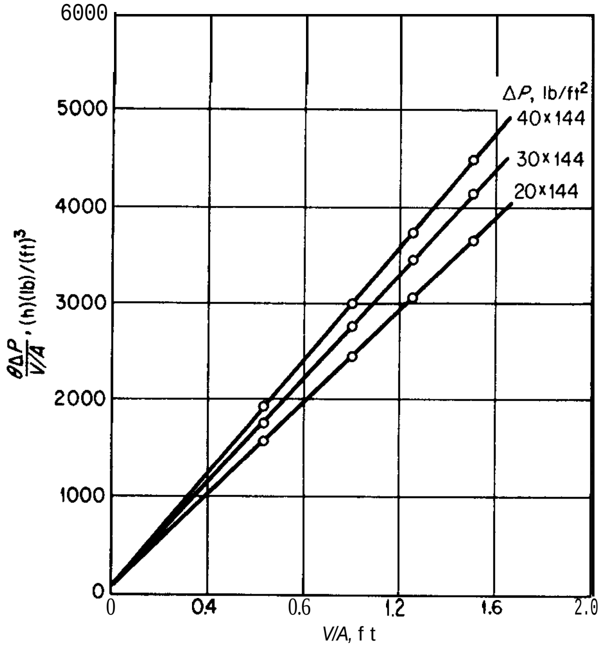


FIGURE **14-59**
Plot for evaluation of constants for filtrate-rate equation in **Ex**ample 5.

better idea as to the reliability of the design constants is obtained by using the following procedure:

Take the logarithm of the expressions for the slope and the intercept in Fig. 14-59. This gives

$$\log\ (\text{slope}) = s \log \text{A} P + \log \frac{\alpha' w u}{2}$$

$$\log\ (\text{intercept}) = s \log \text{AP} + \log \alpha' w \mu V_F$$

A log-log plot of the Fig. 14-59 slopes versus AP should give a straight line with a slope of s and an intercept at log $(\Delta P) = 0$ of log $(\alpha' \omega \mu / 2)$. In this way, s and α' can be evaluated, and the consistency of the data can be checked. This plot is ptesented in Fig. 14-60. From the slope and intercept,

$$s = 0.3$$

$$\frac{\alpha' w \mu}{2} = 220$$

Similarly, a log-log plot of the Fig. 14-59 intercepts versus AP should give a straight line with a slope of s and an intercept at log $(\Delta P) = 0$ of log $(\alpha' w \mu V_F)$, from which V_F could be evaluated. Because the value of V_F is relatively small, the intercepts read from Fig. 14-59 are not precise. The value of V_F, therefore, will be estimated from the combined results of Figs. 14-59 and 14-60.

$w = 5$ lb/ft^3

$\mu = 2.42$ lb/(h)(ft)

$\alpha' = \frac{(220)(2)}{(5)(2.42)} = 36$ with units equivalent to α units of h^2/lb

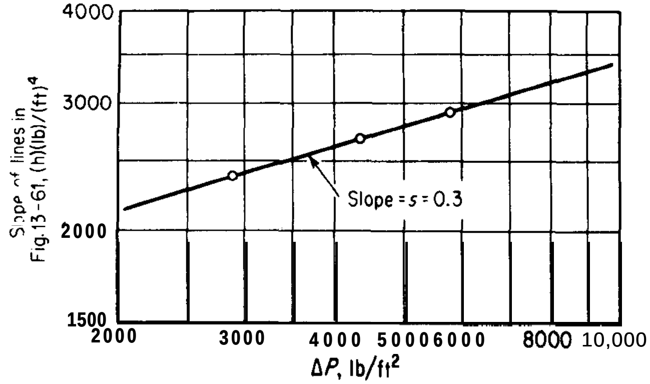


FIGURE **14-60**
Secondary plot for evaluation of constants for filtrate-rate equation in Example 5.

On the basis of the Fig. 14-59 intercept for the 30-psi line,

$$V_F = \frac{80}{\alpha' w\mu(\text{ AP})"} = \frac{80}{(36)(5)(2.42)(30 \times 144)^{0.3}}$$

$$\approx 0.015 \text{ ft}^3/\text{sq ft}$$

Substitution of the constants into Eq. (38) gives the final equation for use in evaluating the total filtering area needed for the large filter:

$$V^2 + 0.03AV = \frac{2A^2(\Delta P)^{1-0.3}}{36w\mu}\theta$$

For the conditions of this problem,

$V = 400 \text{ ft}^3$

$\text{A } P = 25 \text{ X } 144 \text{ psf}$

$w = 5 \text{ lb/ft}^3$

$\mu = 2.42 \text{ lb/(h)(ft)}$

$\theta = 2 \text{ h}$

Substituting the indicated values gives

$$(400)^2 + (0.03)(400)A = \frac{2A^2(25 \times 144)^{0.7}(2)}{(36)(5)(2\ 42)}$$

Solving for A,

$$A = 240 \text{ ft}^2$$

The total area of filtering surface required is approximately 240 ft^2.

Continuous Filters

Many types of continuous filters, such as rotary-drum or rotary-disk filters, are employed in industrial operations. Development of the general design equations for these units follows the same line of reasoning as that presented in the development of Eq. (38). The following analysis is based on the design variables for a typical rotary vacuum filter of the type shown in Fig. 14-61.

It is convenient to develop the design equations in terms of the total area available for filtering service, even though only a fraction of this area is in direct use at any instant. Designate the total available area as A, and the fraction of this area immersed in the slurry as ψ_f. The effective area of the filtering surface then becomes $A_D\psi_f$, and Eq. (31) can be expressed in the following form:

$$\frac{dV}{d\theta} = \frac{A_D\psi_f\Delta P}{(R_K + R_F)\mu} \tag{39}$$

According to Eqs. (32) and (33),

$$R_K + R_F = C(l + l_F) \tag{40}$$

With a continuous filter, the cake thickness at any given location on the submerged filtering surface does not vary with time. The thickness, however, does vary with location as the cake builds up on the filtering surface during passage through the slurry. The thickness of the cake leaving the filtering zone is a function of the slurry concentration, cake density, and volume of filtrate delivered per revolution. This thickness can be expressed by the following equation:

$$\text{‘leaving filtering zone} = \frac{wV_R}{\rho_c A_D} \tag{41}$$

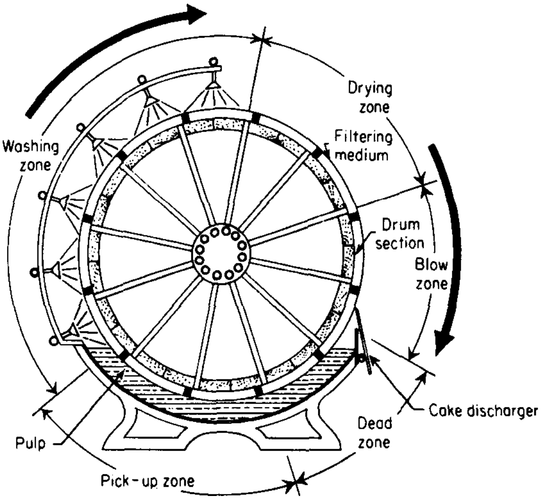


FIGURE **14-61**
Cross-sectional end view of rotary vacuum-drum filter. (*Eimco Corporation.*)

where V_R is the volume of filtrate delivered per revolution and ρ_c is the cake density as pounds of dry-cake solids per unit volume of wet filter cake leaving the filter zone.

An average cake thickness during the cake-deposition period can be assumed to be one-half the sum of the thicknesses at the entrance and exit of the filtering zone. Since no appreciable amount of cake should be present on the filter when it enters the filtering zone,

$$1_{avg} = \frac{wV_R}{2\rho_c A_D} \tag{42}$$

By using the same procedure as was followed in the development of Eq. (34),

$$l + 1, = 1, + l_F = \frac{w(V_R/2 + A_D\psi_f V_F)}{\rho_c A_D} \tag{43}$$

Combination of Eqs. (39), (40), and (43), with $\alpha = C/\rho_c$, gives

$$\frac{dV}{d\theta} = \frac{2A_D^2\psi_f \text{ AP}}{\alpha w(V_R + 2A_D\psi_f V_F)\mu} \tag{44}$$

Integration of Eq. (44) between the limits of $V = 0$ and $V = V_R$, and $\theta = 0$ and θ = l/N,,, where N_R is the number of revolutions per unit time, gives

$$V_R^2 + 2A_D\psi_f V_F V_R = \frac{2A_D^2\psi_f \Delta P}{\alpha w\mu N_R} \tag{45}$$

or, by including Eq. (36),

$$V_R^2 + 2A_D\psi_f V_F V_R = \frac{2A_D^2\psi_f(\Delta P)^{1-s}}{\alpha' w\mu N_R} \tag{46}$$

The constants in the preceding equations can be evaluated by a procedure similar to that described in Example 5. Equation (46) is often used in the following simplified forms, which are based on the assumptions that the resistance of the filter medium is negligible and the filter cake is noncompressible:

$$\text{Volume of filtrate per revolution} = V_R = A,\sqrt{\frac{2\psi_f \Delta P}{\alpha w\mu N_R}} \tag{47}$$

$$\text{Volume of filtrate per unit time} = V_R N_R = A,\sqrt{\frac{2\psi_f N_R \Delta P}{\alpha w\mu}} \tag{48}$$

$$\text{Weight of dry cake per unit time} = V_R N_R w = A,\sqrt{\frac{2\psi_f N_R w \Delta P}{\alpha\mu}} \tag{49}$$

Example 6 Effect of pressure difference on capacity of a rotary vacuum filter. A rotary vacuum filter with negligible filter-medium resistance delivers 100 ft^3 of filtrate per hour when a given $CaCO_3$-H_2O mixture is filtered under known conditions. How many cubic feet of filtrate will be delivered per hour if the pressure drop over the cake is doubled, all other conditions remaining constant? Assume the $CaCO_3$ filter cake is noncompressible.

Solution. Equation (48) applies for this case:

$$100 = A_D \sqrt{\frac{2\psi_f N_R}{\alpha w \mu}} \sqrt{\Delta P_1} \tag{A}$$

$$\text{Unknown filtrate rate} = A_D \sqrt{\frac{2\psi_f N_R}{\alpha w \mu}} \sqrt{2\,\Delta P_1} \tag{B}$$

Dividing Eq. (*B*) by Eq. (*A*),

$$\text{Unknown filtrate rate} = 100\sqrt{2} = 141 \text{ ft}^3/\text{h}$$

Air Suction Rate in Rotary Vacuum Filters

A vacuum pump must be supplied for the operation of a rotary vacuum filter, and the design engineer may need to estimate the size of pump and power requirement for a given filtration unit. Because air leakage into the vacuum system may supply a major amount of the air that passes through the pump, design methods for predicting air suction rates must be considered as approximate since they do not account for air leakage.

The rate at which air is drawn through the dewatering section of a rotary vacuum filter can be expressed in a form similar to Eq. (39) as

$$\frac{dV_a}{d\theta} = \frac{A_D \psi_a \Delta P}{(R'_F + R'_K)\mu_a} \tag{50}$$

where V_a = volume of air at temperature and pressure of surroundings drawn through cake in time θ

ψ_a = fraction of total surface available for air suction

μ_a = viscosity of air at temperature and pressure of surroundings

The cake resistance R'_K is directly proportional to the cake thickness l, and the filter-medium resistance R'_F can be assumed to be directly proportional to a fictitious cake thickness l'_F. Designating C' as the proportionality constant,

$$R'_K + R'_F = C'(l + l'_F) \tag{51}$$

If the cake is noncompressible, l must be equal to the thickness of the cake leaving the filtering zone. Therefore, by Eq. (41) and using the same procedure as was followed in the development of Eq. (34),

$$l + l'_F = \frac{w(V_R + A_D \psi_a V'_F)}{\rho_c A_D} \tag{52}$$

where V'_F is the fictitious volume of filtrate per unit of air-suction area necessary to lay down a cake of thickness l'_F.

Combination of Eqs. (50) to (52) gives

$$\frac{dV_a}{d\theta} = \frac{A_D^2 \psi_a \, \Delta P}{\beta w (V_R + A_D \psi_a V_F') \mu_a} \tag{53}$$

where β equals C'/ρ_c and is *known* as the *specific air-suction cake resistance.*

Integration of Eq. (53) between the limits corresponding to $V_a = 0$ and $V_a = V_{aR}$, where V_{aR} designates the volume of air per revolution, gives

$$V_{aR} = \frac{A_D^2 \psi_a \, \Delta P}{\beta w (V_R + A_D \psi_a V_F') \mu_a N_R} \tag{54}$$

If the cake is compressible, a rough correction for variation in β with change in *AP* can be made by use of the following empirical equation:

$$\beta = \beta' (\Delta P)^{s'} \tag{54a}$$

where β' and s' are constants.

By neglecting the resistance of the filter medium, Eq. (54) can be simplified to

$$\text{Volume of air per revolution} = V_{aR} = \frac{A_D^2 \psi_a \, \Delta P}{\beta w V_R \mu_a N_R} \tag{55}$$

$$\text{Volume of air per unit time} = V_{aR} N_R = \frac{A_D^2 \psi_a \, \Delta P}{\beta w V_R \mu_a} \tag{56}$$

Equations (47), (49), and (56) can be combined to give

$$\text{Volume of air per unit time} = \frac{A_D \psi_a}{\beta \mu_a} \sqrt{\frac{\alpha \mu N_R \, \Delta P}{2 w \psi_f}} \tag{57}$$

$$\frac{\text{Volume of air per unit time}}{\text{Weight of dry cake per unit time}} = \frac{\psi_a}{\psi_f} \frac{\mu}{\mu_a} \frac{\alpha}{2\beta w} \tag{58}$$

If the constants in the preceding equations are known for a given titer system and the assumption of no air leakage is adequate, the total amount of suction air can be estimated. This value, combined with a knowledge of the air temperature and the pressures at the intake and delivery sides of the vacuum pump, can be used to estimate the power requirements of the vacuum pump by methods described elsewhere in this chapter.

Example 7 Estimation of horsepower motor required for vacuum pump on a rotary vacuum filter. A rotary vacuum-drum filter is to handle a slurry containing 20 lb of water per 1 lb of solid material. Tests on the unit at the conditions to be used for the filtration have shown that the dimensionless ratio of α/β is 0.6 and 19 lb of filtrate (not including wash water) is obtained for each 21 lb of slurry. The temperature of the surroundings and of the slurry is 70°F, and the pressure of the surroundings is 1 atm. The pressure drop to be maintained by the vacuum pump is 5 psi. The fraction of the drum area submerged in the slurry is 0.3, and the fraction of the drum area available for air suction is 0.1. On the basis of the following assumptions, estimate the horsepower of the motor necessary for the vacuum pump if the unit handles 50,000 lb of slurry per hour.

Assumptions:

Resistance of filter medium is negligible.

Any effects caused by air leakage are taken into account in the value given for α/β.

For air at the temperature involved, heat capacity at constant pressure divided by heat capacity at constant volume is 1.4.

The vacuum pump and motor have an overall efficiency of 50 percent based on an isentropic compression.

The value of β is based on the temperature and pressure of the air surrounding the filter.

The filter removes all of the solid from the slurry.

Solution. Because the value given for α/β applies at the operating conditions for the filtration and the resistance of the filter medium is negligible, Eq. (58) can be used:

$$\frac{\text{Volume of air per unit time}}{\text{Weight of dry cake per unit time}} = \frac{\psi_a}{\psi_f}\frac{\mu}{\mu_a}\frac{\alpha}{2\beta w}$$

$\psi_a = 0.1$
$\psi_f = 0.3$
μ = viscosity of water at 70°F = 0.982 centipoise = 0.982 $\times$ 2.42 lb/(h)(ft)
μ_a = viscosity of air at 70°F = 0.018 centipoise = 0.018 $\times$ 2.42 lb/(h)(ft)

$$\frac{\alpha}{\beta} = 0.6$$

Density of water at 70°F = 62.3 lb/ft^3
Pounds filtrate per pound of dry-cake solids = 19

$$w = \frac{1}{19/62.3} = 3.28 \text{ lb dry-cake solids/ft}^3 \text{ filtrate}$$

$$\text{Weight of dry cake/h} = (50{,}000)\left(\tfrac{1}{21}\right) = 2380 \text{ lb/h}$$

$$\text{Volume of air/h} = \frac{(2380)(0.1)(0.982 \times 2.42)(0.6)}{(0.3)(0.018 \times 2.42)(2 \times 3.28)}$$

$$= 3960 \text{ ft}^3\text{/h at 70°F and 1 atm}$$

By Eq. (24)

Theoretical horsepower for isentropic single-stage compression

$$= \frac{3.03 \times 10^{-5}k}{k-1} P_1 q_{fm_1}\left[\left(\frac{P_2}{P_1}\right)^{(k-1)/k} - 1\right]$$

k = ratio of heat capacity of gas at constant pressure to heat capacity of gas at constant volume
= 1.4

P_1 = vacuum-pump intake pressure = (14.7 − 5)(144) psf

P_2 = vacuum-pump delivery pressure = $(14.7)(144)$ psf

q_{fm_1} = cubic feet of gas per minute at vacuum-pump intake conditions

$$= \frac{(3960)(14.7)(144)}{(60)(14.7 - 5)(144)} = 100 \text{ cfm at } 70°\text{F and } 9.7 \text{ psia}$$

Horsepower of motor required for vacuum pump

$$= \frac{(3.03 \times 10^{-5})(1.4)(14.7 - 5)(144)(100)}{(0.5)(1.4 - 1)} \left| \left(\frac{14.7}{9.7} \right)^{(1.4-1)/1.4} - 1 \right|$$

$$= 3.7 \text{ hp}$$

A 4-hp motor would be satisfactory.

Costs for Filters

Information to permit estimation of the cost for various types of filters is presented in Figs. 14-62 through 14-65.

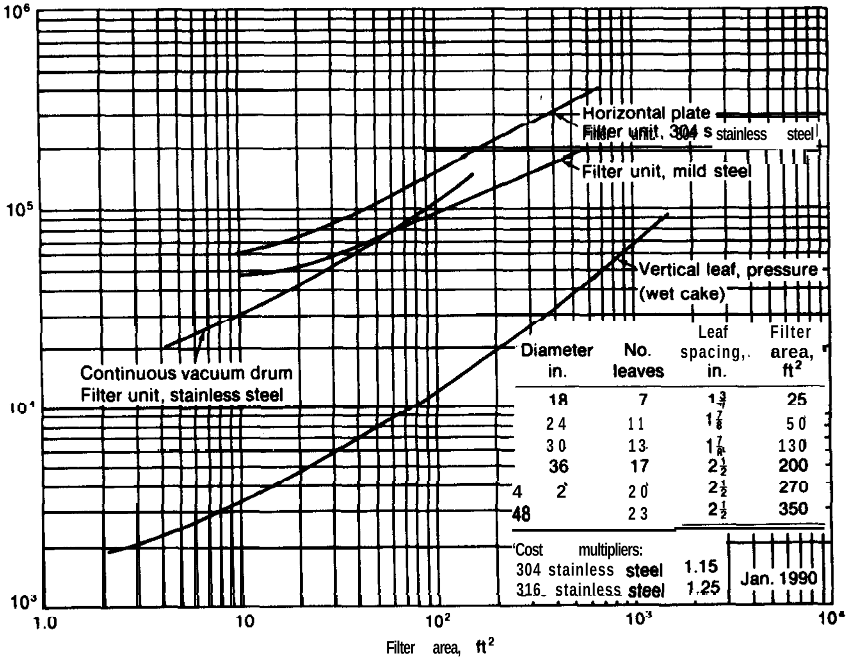


FIGURE 14-62
Filters.

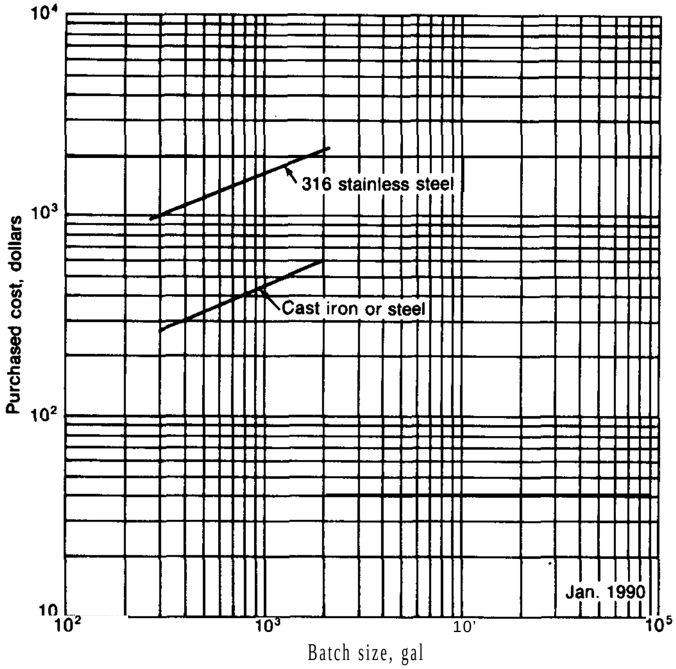


FIGURE 14.63
Cartridge-type filters.

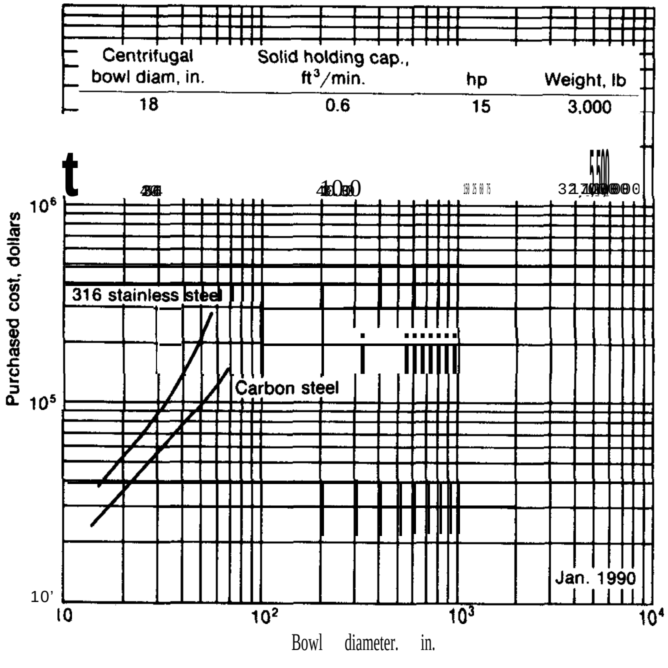


FIGURE **14-64**
Centrifugal filters. Continuous solid bowl. Price does not include motor and drive.

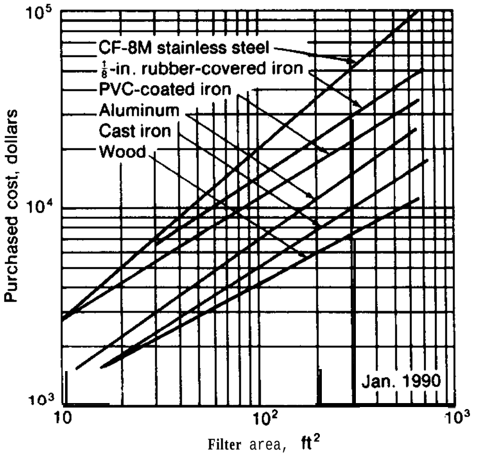
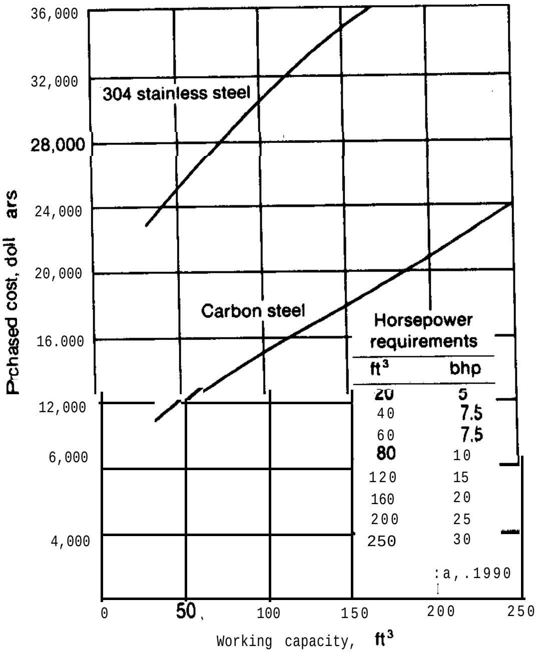


FIGURE 14-65
Plate-and-frame filters. Order-of-magnitude capital-cost estimating data.

MISCELLANEOUS PROCESSING EQUIPMENT COSTS

Cost data for blenders and mixers, kneaders, centrifugal separators, crystallizers, crushing and grinding equipment, dust collectors, electrostatic precipitators, and screens are presented in Figs. 14-66 through 14-88.



Horsepower requirements

ft³	bhp
20	5
40	7.5
60	7.5
80	10
120	15
160	20
200	25
250	30

FIGURE 14-66
Double-cone rotary blenders. Price does not include motor.

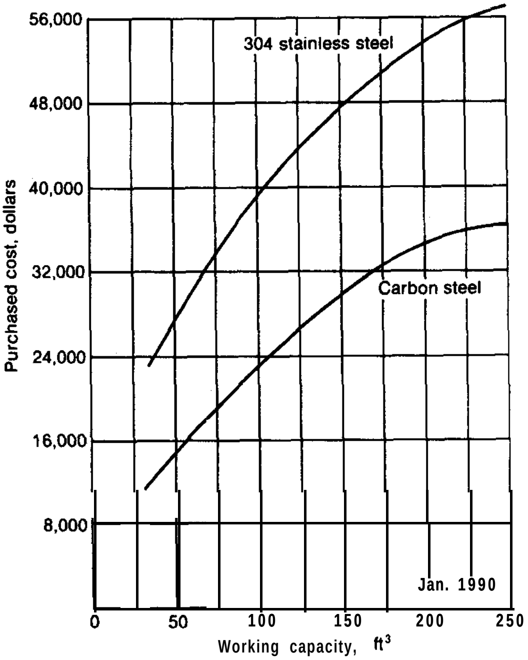


FIGURE 14-67
Ribbon blenders. Price includes standard floor-mounted support, baffled shell cover, antifriction pillow blocks mounted on outboard bearing shelves, stuffing box, flanged inlet opening, and lever-operated discharge gate.

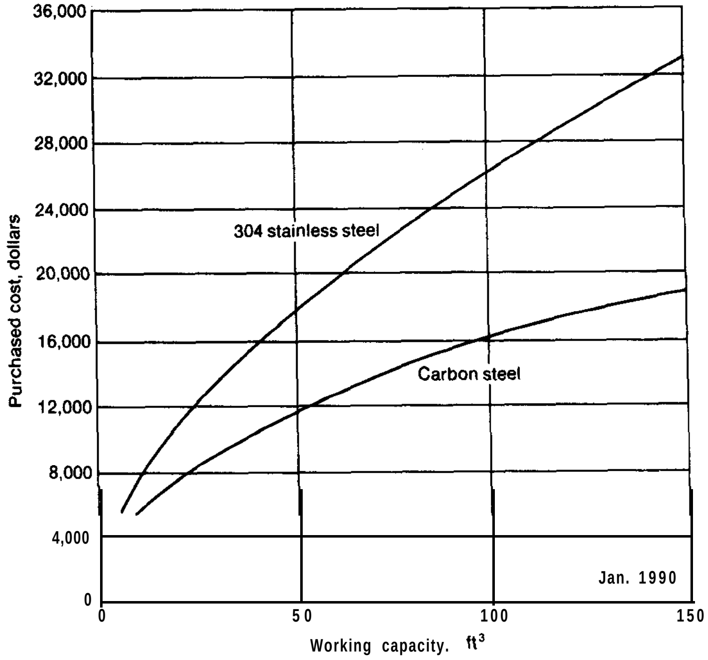


FIGURE 14-68
Twin-shell blenders. Price includes blender only.

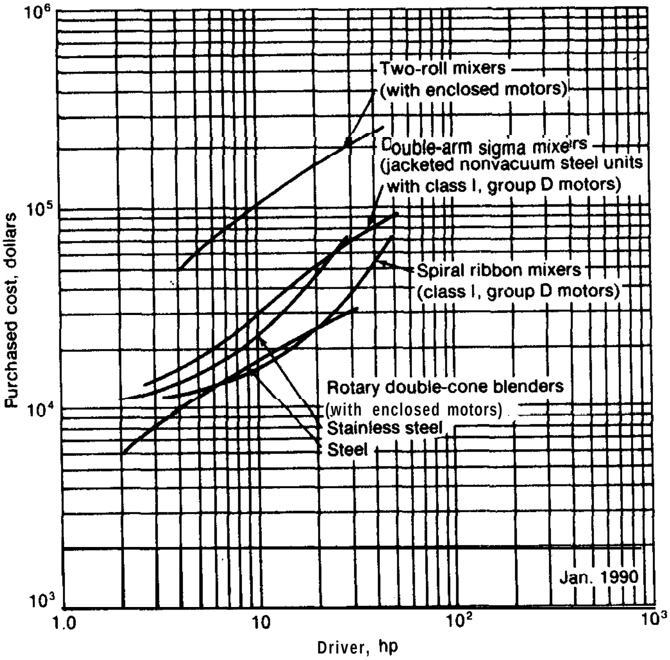


FIGURE 14-69
Cost of mixers and blenders.

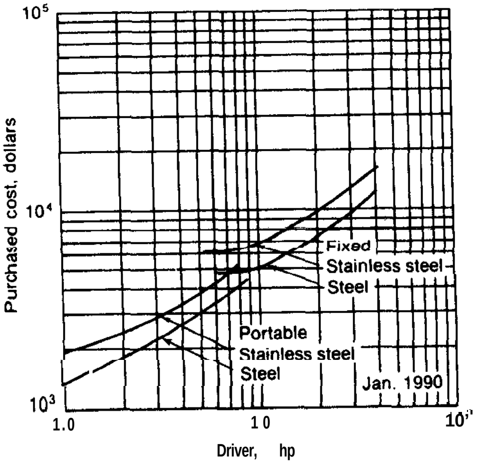


FIGURE 14-70
Cost of propeller mixers.

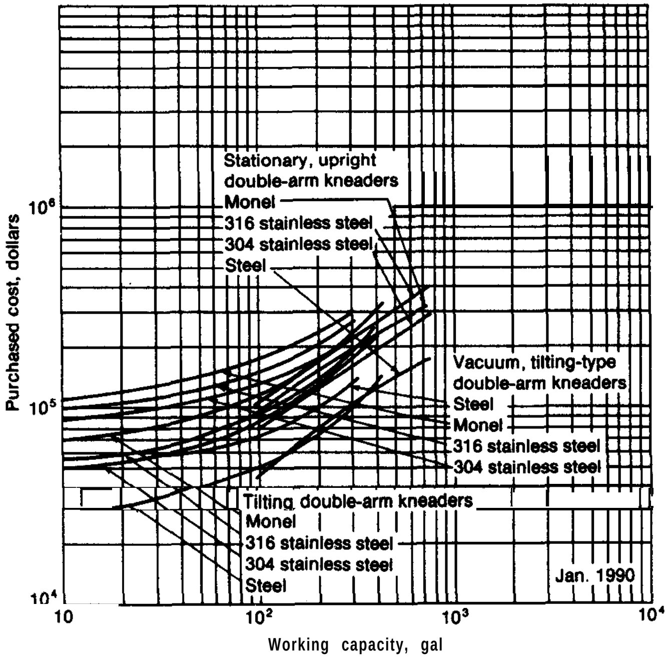


FIGURE 14-71
Cost of kneaders. Price includes machine, jacket, gear reducer and drive, cover, nozzles, and agitator. Motor and starter are not included in purchase price.

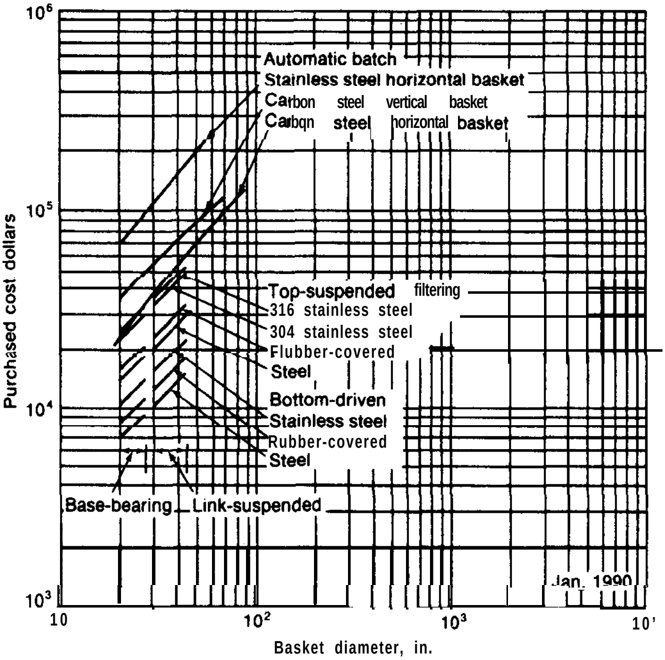


FIGURE 14.72
Centrifugal separators.

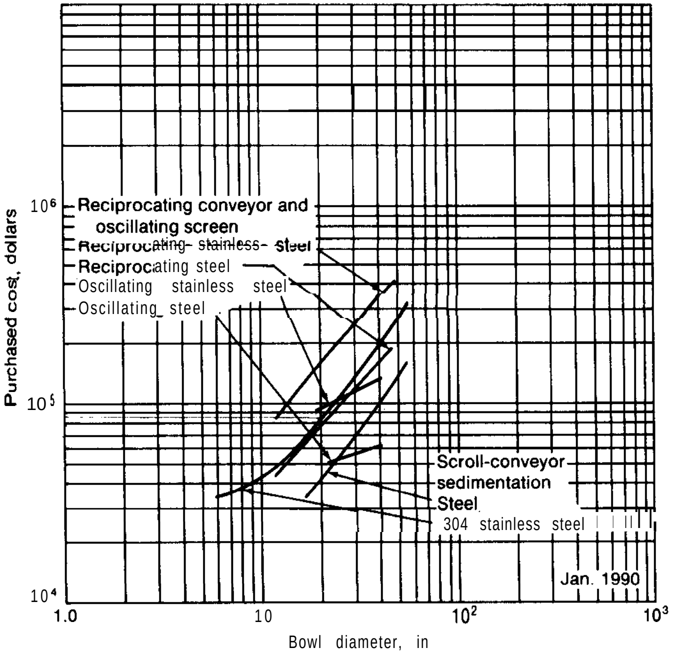


FIGURE 14.73
Centrifugal separators.

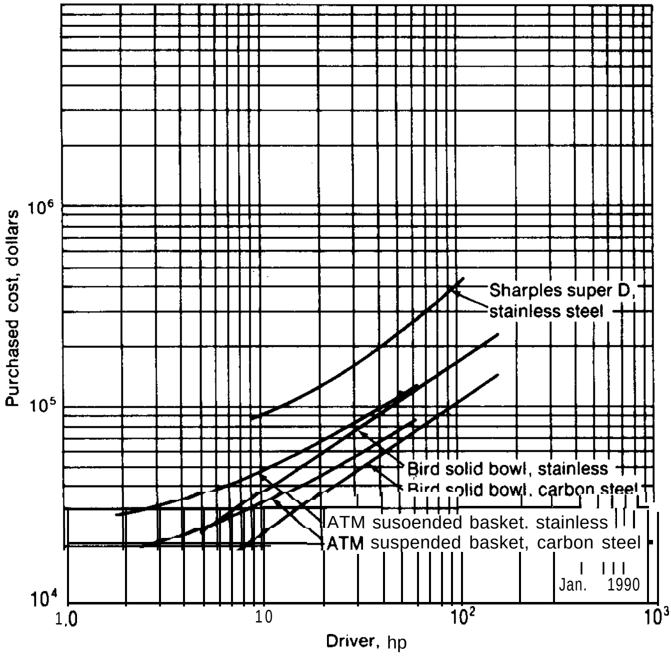


FIGURE 14-74
Centrifuges.

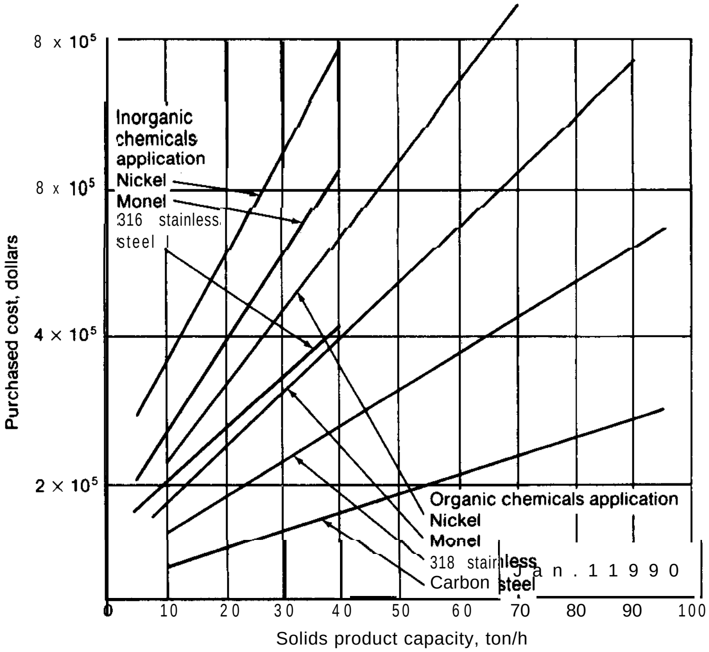


FIGURE 14-75
Centrifuges: solid-bowl, screen-bowl, and pusher types.

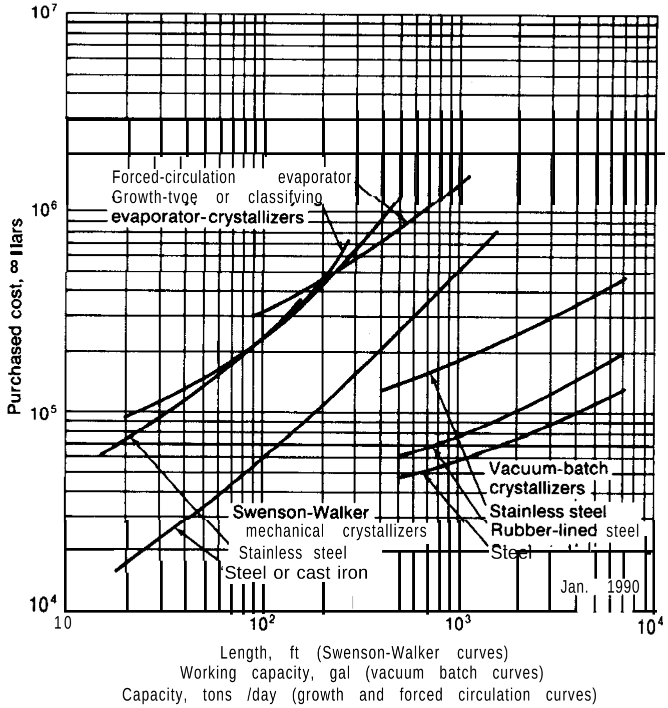
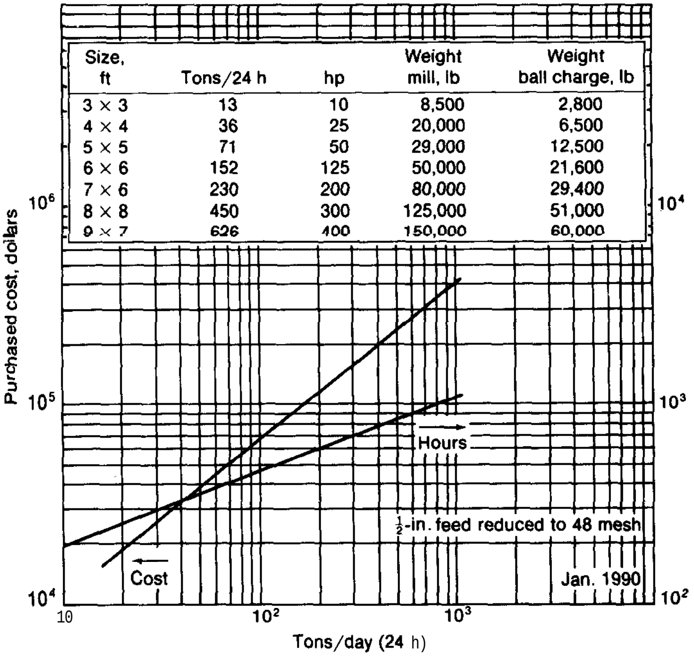


FIGURE 14-76
Crystallizers.



Size, ft	Tons/24 h	hp	Weight mill, lb	Weight ball charge, lb
3 × 3	13	10	8,500	2,800
4 × 4	36	25	20,000	6,500
5 × 5	71	50	29,000	12,500
6 × 6	152	125	50,000	21,600
7 × 6	230	200	80,000	29,400
8 × 8	450	300	125,000	51,000
9 × 7	626	400	150,000	60,000

FIGURE 14-77
Ball mills. Ball charge is $315/ton. Price includes liner, motor, drive, and **guard.**

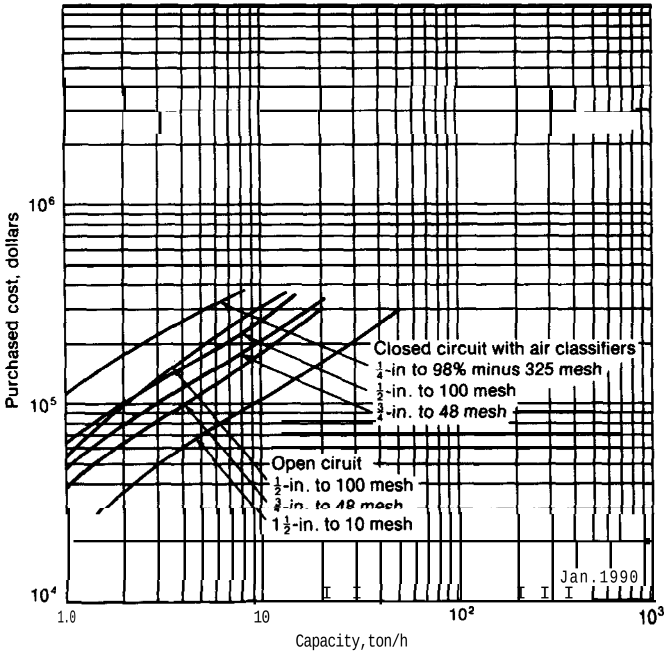


FIGURE 14-78
Ball mill dry grinding. Price includes installation, classifier, motors, drives, and an average allowance for foundations and erection. Does not include freight, auxiliary equipment, or equipment for handling the material.

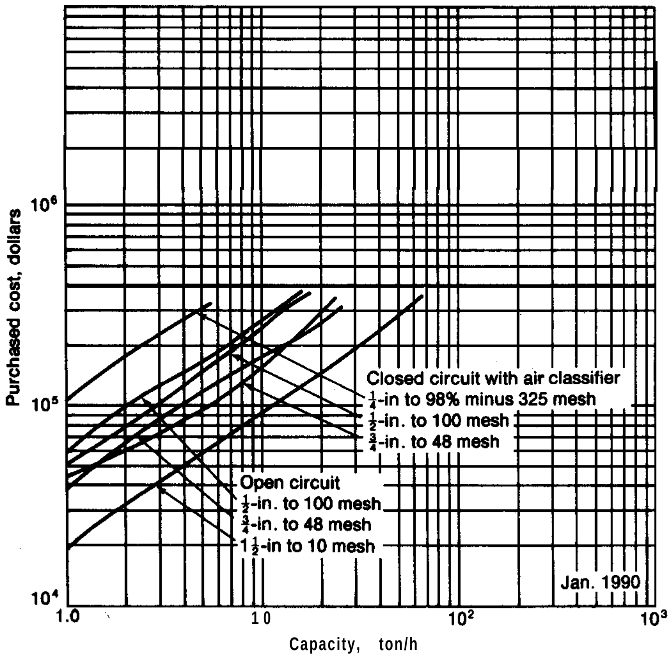
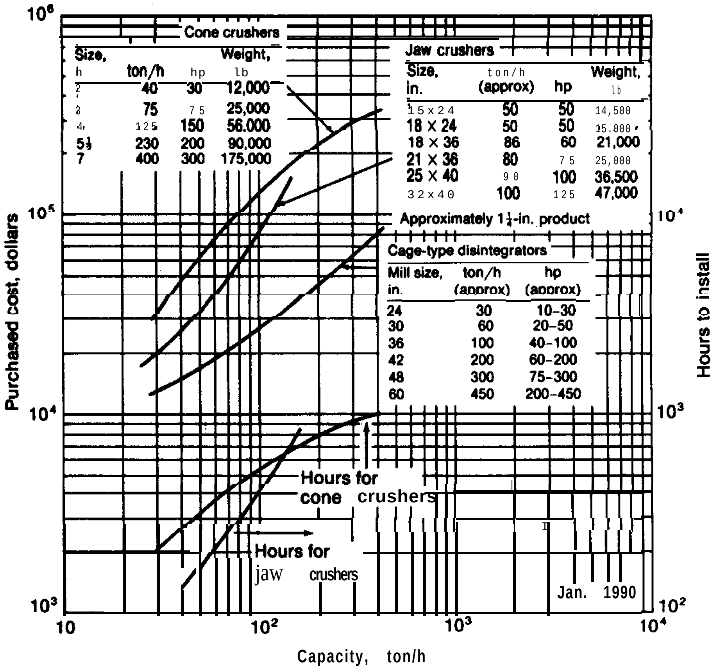


FIGURE **14-79**
Ball mill wet grinding. Price includes installation, classifier, motors, drives, and an average allowance for foundations and erection. Does not include freight, auxiliary equipment, or equipment for handling the material.



Cone crushers

Size, h	ton/h	hp	Weight, lb
2	40	30	12,000
3	75	75	25,000
4	125	150	56,000
5½	230	200	90,000
7	400	300	175,000

Jaw crushers

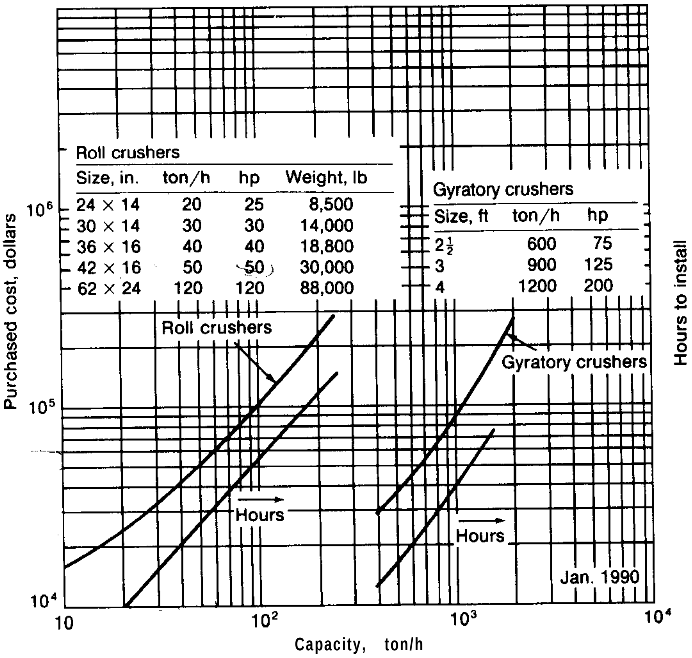
Size, in.	ton/h (approx)	hp	Weight, lb
15 x 24	50	50	14,500
18 X 24	50	50	15,000
18 × 36	86	60	21,000
21 X 36	80	75	25,000
25 X 40	90	100	36,500
32 x 40	100	125	47,000

Approximately $1\frac{1}{4}$-in. product

Cage-type disintegrators

Mill size, in.	ton/h (approx)	hp (approx)
24	30	10–30
30	60	20–50
36	100	40–100
42	200	60–200
48	300	75–300
60	450	200–450

FIGURE **14-80**
Crushers and disintegrators. Price includes motor, drive, and guard.



Roll crushers

Size, in.	ton/h	hp	Weight, lb
24 × 14	20	25	8,500
30 × 14	30	30	14,000
36 × 16	40	40	18,800
42 × 16	50	50	30,000
62 × 24	120	120	88,000

Gyratory crushers

Size, ft	ton/h	hp
$2\frac{1}{2}$	600	75
3	900	125
4	1200	200

FIGURE 14-81
Crushers. Price includes motor and drive.

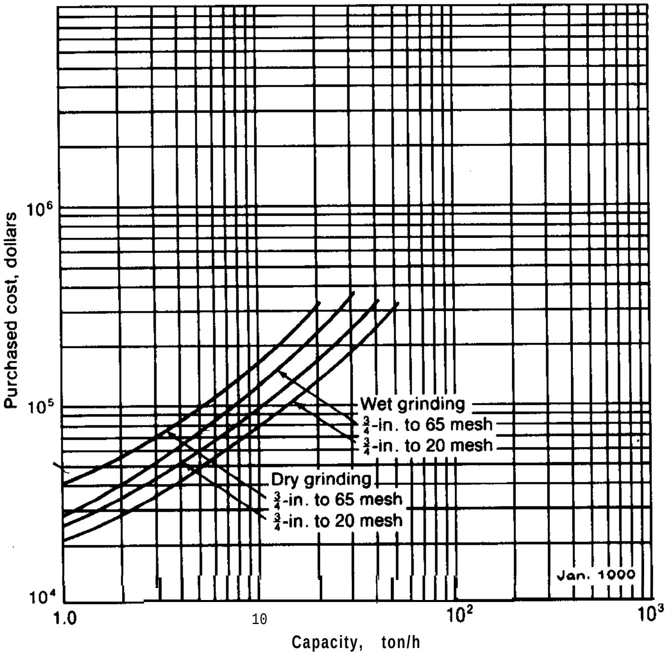
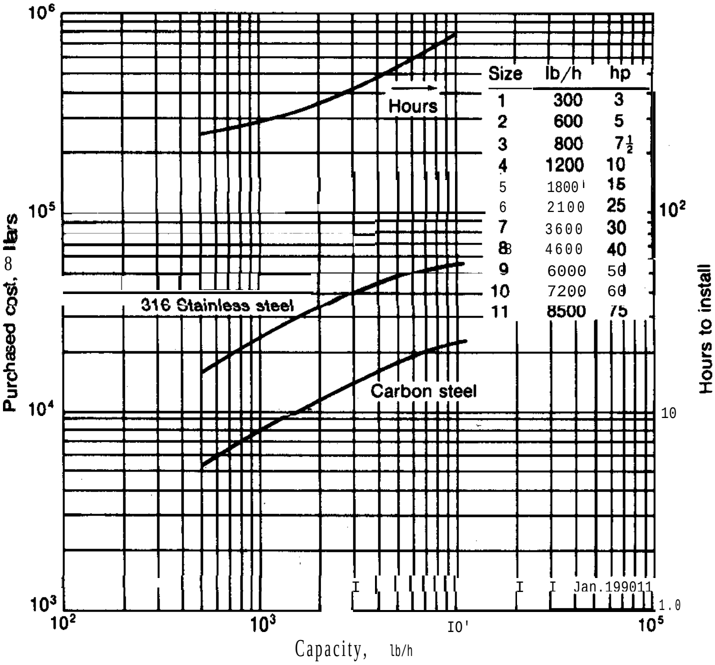


FIGURE 14-82
Rod mill in open circuit. Price includes installation, classifier, motors, drives, and an average allowance for foundations and erection, Does not include freight, auxiliary equipment, or equipment for handling material.



Size	lb/h	hp
1	300	3
2	600	5
3	800	7½
4	1200	10
5	1800	15
6	2100	25
7	3600	30
8	4600	40
9	6000	50
10	7200	60
11	8500	75

FIGURE 14-83
Pulverizers. Price does not include motor and drive. Cost of legs for units 1 through 5 is $180 and of stands for units 6 through 11 is $525. Add 15% for explosion-proof construction and $680 for flexible coupling.

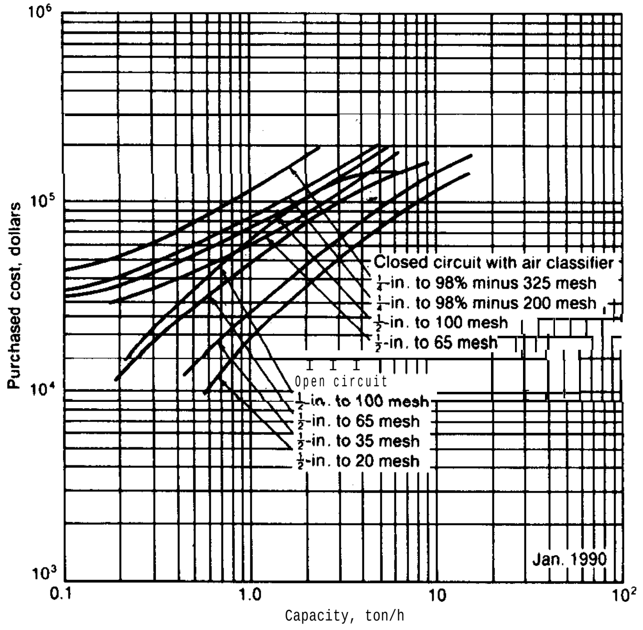


FIGURE 14-84
Pebble mill dry grinding. Price includes installation, classifier, motors, drives, and an average allowance for foundations and erection. Does not include freight, auxiliary equipment, or equipment for handling the material.

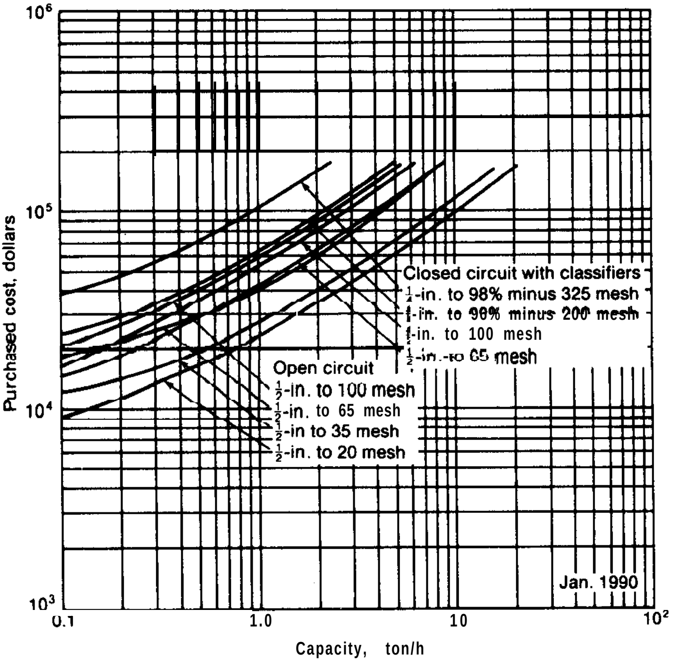


FIGURE **14-85**
Pebble mill wet grinding. Price includes installation, classifier, motors, drives, and an average allowance for installation and erection. Does not include freight, auxiliary equipment, or equipment for handling the material.

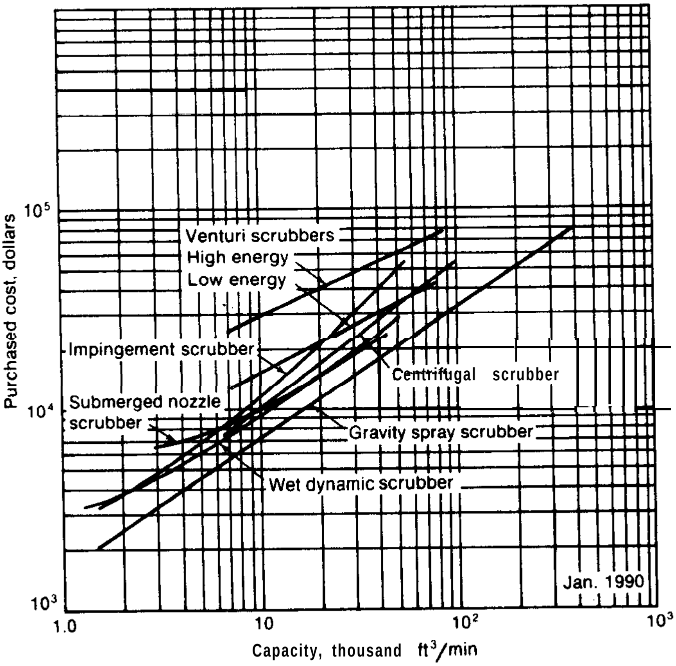


FIGURE **14-86**
Cost of wet dust collectors.

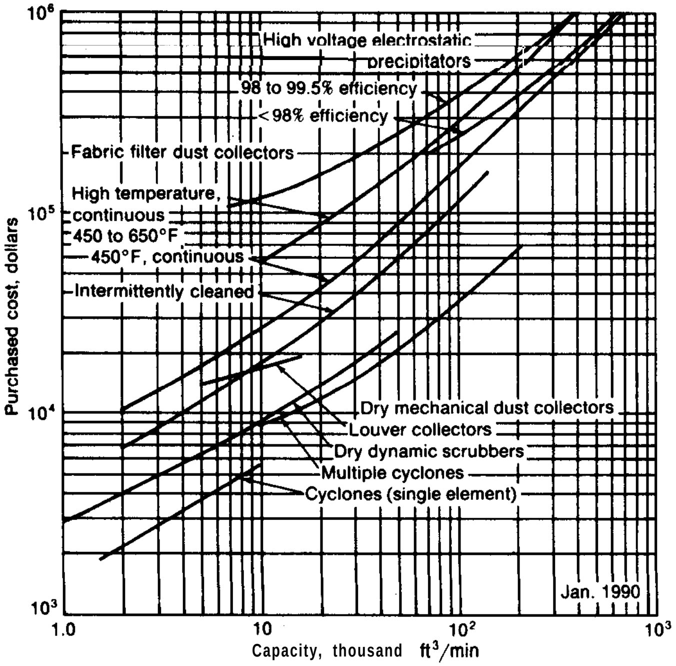
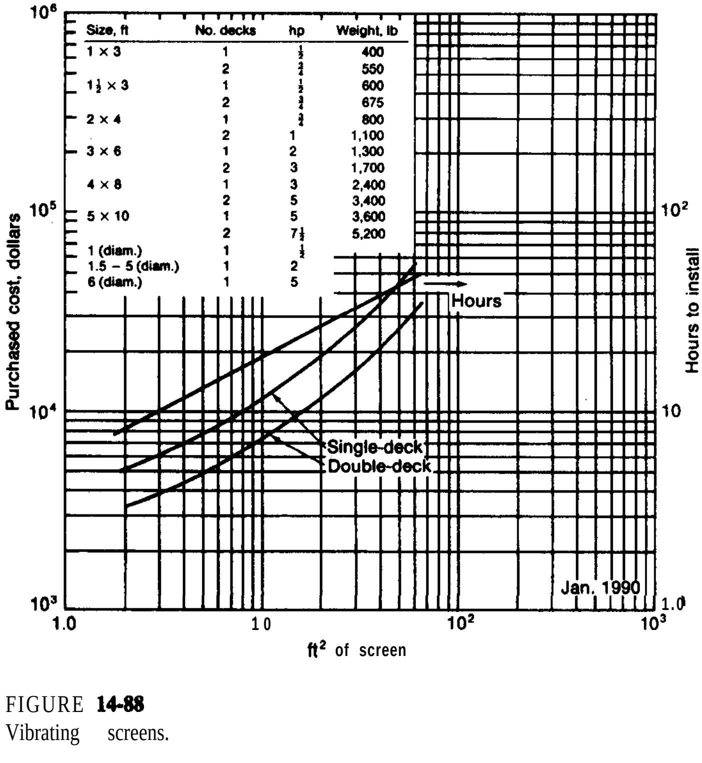


FIGURE **14-87**
Cost of dry, mechanical dust collectors, high-voltage electrostatic precipitators. and fabric-filter dust collectors.



Size, ft	No. decks	hp	Weight, lb
1 × 3	1	½	400
	2	¾	550
1½ × 3	1	½	600
	2	¾	675
2 × 4	1	¾	800
	2	1	1,100
3 × 6	1	2	1,300
	2	3	1,700
4 × 8	1	3	2,400
	2	5	3,400
5 × 10	1	5	3,600
	2	7½	5,200
1 (diam.)	1	½	
1.5 – 5 (diam.)	1	2	
6 (diam.)	1	5	

FIGURE **14-88**
Vibrating screens.

MATERIALS-HANDLING EQUIPMENT COSTS

Materials-handling cost for chutes, conveyors, gates, and hoists are given in Figs. 14-89 through 14-95. In addition, Table 7 lists costs for materials-handling equipment by automotive means.

TABLE 7
Costs for automotive materials-handling equipment (Jan. 1990)

Equipment	cost, $
Fork lift trucks	
3,000 lb 15-in. load center 72-in. lift	22,050
4,000 lb 15-in. load center 72-in. lift	24,150
5,000 lb 24-in. load center 72-in. lift	26,850
6,000 lb 24-in. load center 84-in. lift	30,600
8,000 lb 24-in. load center 84-in. lift	34,050
10,000 lb 24-in. load center 92-in. lift	39,450
Hand truck, heavy duty	188
Hydraulic pallet truck, 4000 lb	2,720
Jack lift electric pallet truck, 4000 lb	9,690
Payloaders	
18 ft^3, gas	18,000
20 ft^3, gas	25,800
1 yd^3, gas	29,550
1.75 yd^3, gas	53,550
1.75 yd^3, diesel	59,850
4.0 yd^3, gas	89,700
4.0 yd^3, diesel	96,900
Railway tank cars (8000 gal)	
Steel	39,450
Aluminum	57,450
Stainless steel	96,900
Tank trailers (4300 gal unlined)	
Carbon steel	35,700
Aluminum	46,500
Stainless steel	57,450
Tractors	
Gasoline	35,700
Diesel	71,400
Tractor shovel	
$2\frac{1}{4}$ yd^3 bucket 105 hp	74,708
$2\frac{1}{4}$ yd^3 bucket 125 hp	81,600
$3\frac{1}{4}$ yd^3 bucket 150 hp	102,008
Walkie pallet truck, 4000 lb	6,750
Battery	1,808
Charger	1,425

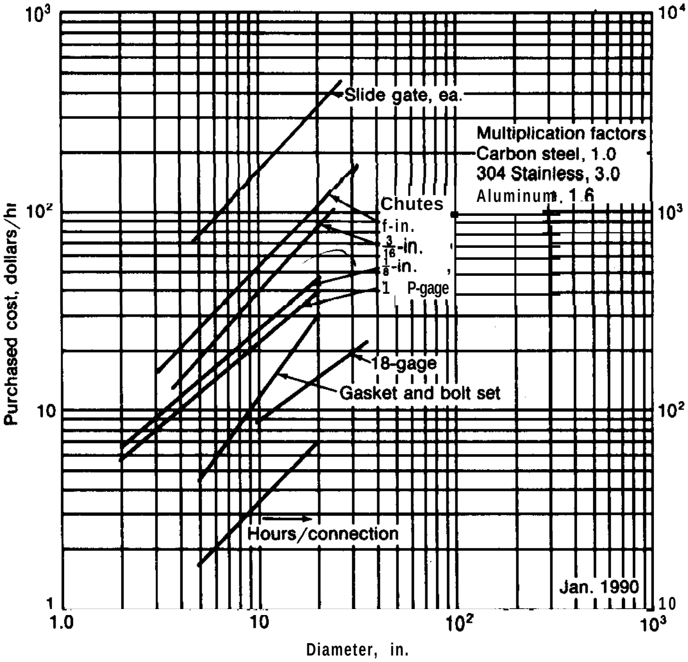
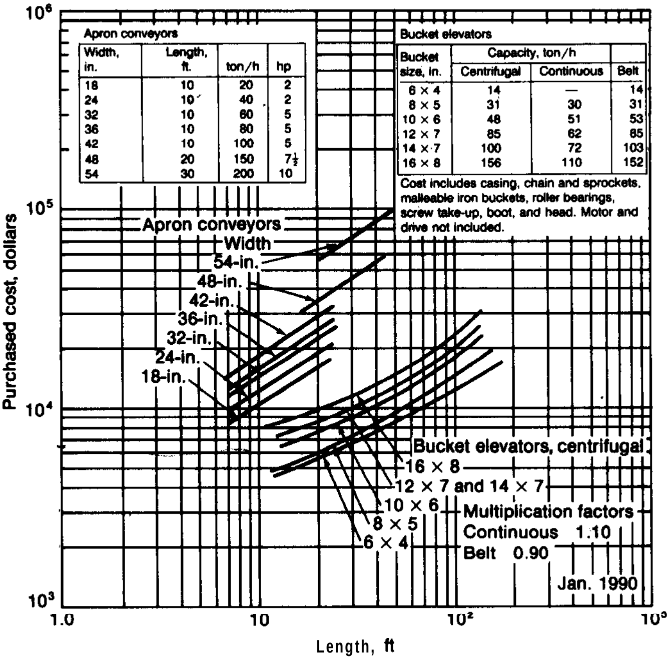


FIGURE 14-89
Chutes and gates. Price for flexible connections, \$70 to \$123 each. Transition piece, round to square, five times the per-foot price.



Apron conveyors

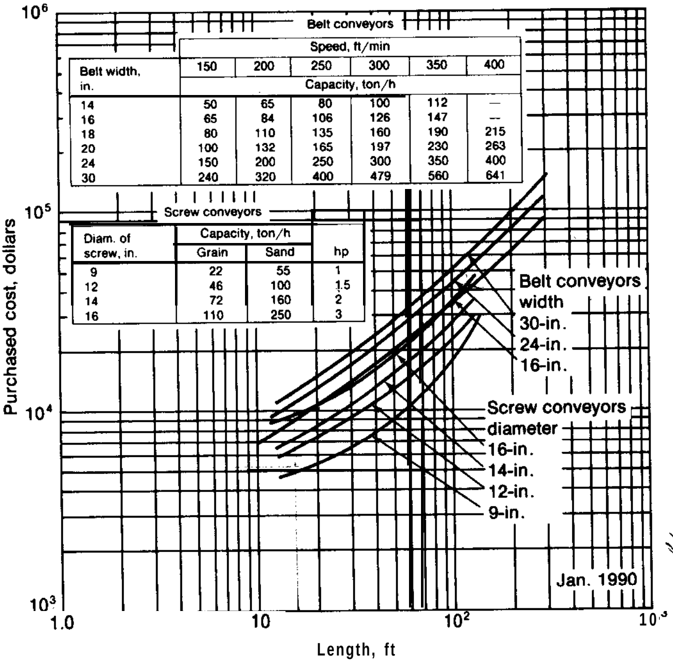
Width, in.	Length, ft.	ton/h	hp
18	10	20	2
24	10	40	2
32	10	60	5
36	10	80	5
42	10	100	5
48	20	150	7½
54	30	200	10

Bucket elevators

Bucket size, in.	Capacity, ton/h		
	Centrifugal	Continuous	Belt
6 × 4	14	—	14
8 × 5	31	30	31
10 × 6	48	51	53
12 × 7	85	62	85
14 × 7	100	72	103
16 × 8	156	110	152

Cost includes casing, chain and sprockets, malleable iron buckets, roller bearings, screw take-up, boot, and head. Motor and drive not included.

FIGURE 14-90
Conveying equipment. Cost of apron conveyors and bucket elevators.



Belt conveyors

Belt width, in.	Speed, ft/min 150	200	250	300	350	400
	Capacity, ton/h					
14	50	65	80	100	112	—
16	65	84	106	126	147	—
18	80	110	135	160	190	215
20	100	132	165	197	230	263
24	150	200	250	300	350	400
30	240	320	400	479	560	641

Screw conveyors

Diam. of screw, in.	Capacity, ton/h Grain	Sand	hp
9	22	55	1
12	46	100	1.5
14	72	160	2
16	110	250	3

HELICOIDE
BRASIL

FIGURE 14-91
Conveying equipment. Cost of belt and screw conveyors.

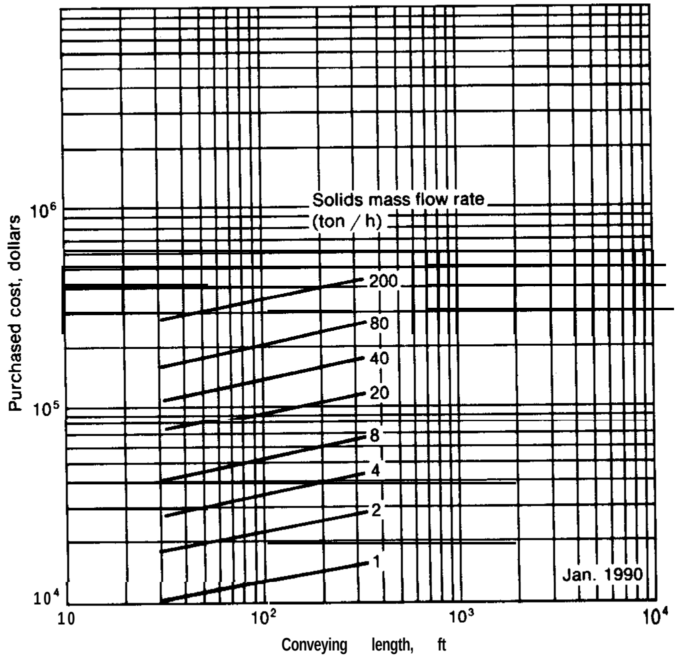
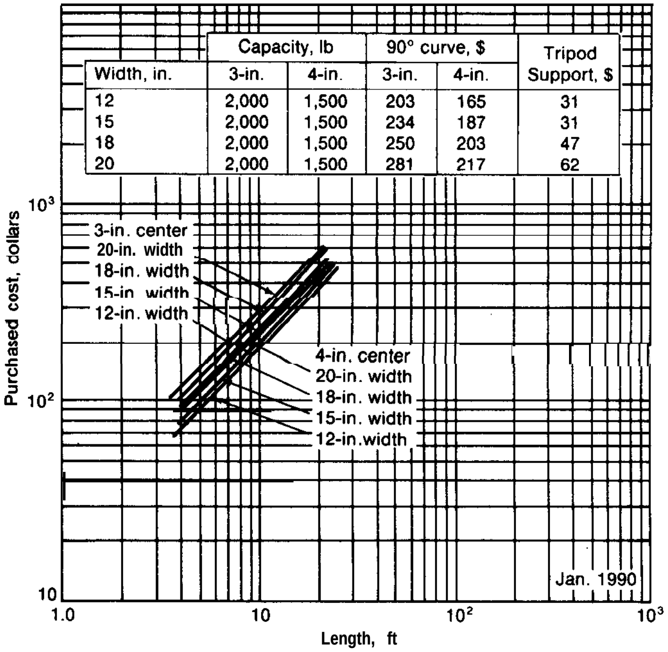
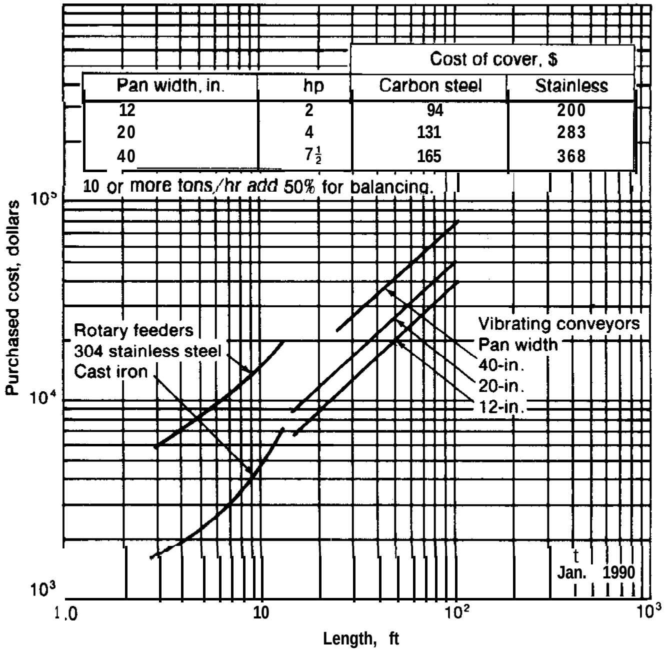


FIGURE 14-92
Purchased equipment costs for pneumatic solids-conveying equipment. Drives are included.



Width, in.	Capacity, lb 3-in.	Capacity, lb 4-in.	90° curve, $ 3-in.	90° curve, $ 4-in.	Tripod Support, $
12	2,000	1,500	203	165	31
15	2,000	1,500	234	187	31
18	2,000	1,500	250	203	47
20	2,000	1,500	281	217	62

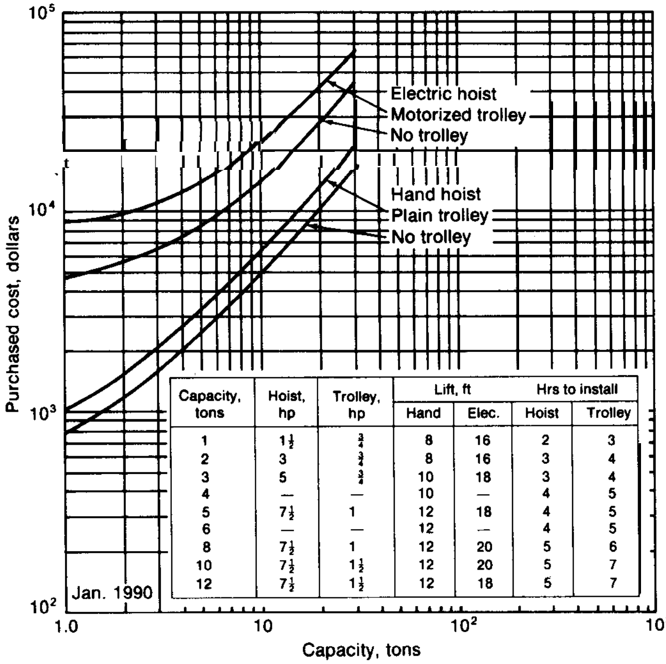
FIGURE 14-93
Roller conveyors.



Pan width, in.	hp	Cost of cover, $ Carbon steel	Cost of cover, $ Stainless
12	2	94	200
20	4	131	283
40	$7\frac{1}{2}$	165	368

10 or more tons/hr add 50% for balancing.

FIGURE 14-94
Conveying equipment. Cost of rotary feeders and vibrating conveyors.



Capacity, tons	Hoist, hp	Trolley, hp	Lift, ft		Hrs to install	
			Hand	Elec.	Hoist	Trolley
1	$1\frac{1}{2}$	$\frac{3}{4}$	8	16	2	3
2	3	$\frac{3}{4}$	8	16	3	4
3	5	$\frac{3}{4}$	10	18	3	4
4	—	—	10	—	4	5
5	$7\frac{1}{2}$	1	12	18	4	5
6	—	—	12	—	4	5
8	$7\frac{1}{2}$	1	12	20	5	6
10	$7\frac{1}{2}$	$1\frac{1}{2}$	12	20	5	7
12	$7\frac{1}{2}$	$1\frac{1}{2}$	12	18	5	7

FIGURE 14-95
Hoisting equipment. Extra costs include: acid-resistant construction, \$340; dust-tight construction, \$150; power reels, \$740; and chain container, \$140.

NOMENCLATURE FOR CHAPTER 14

a = special correction constant for pressure-vessel design, see Table 4

A = area of filtering surface, ft^2

A, = total available surface for filtration on continuous filter, ft^2

C = a proportionality constant relating cake thickness to resistance to liquid passage

C' = a proportionality constant relating cake thickness to resistance to air passage

C_a = coefficient in air-lift design equation

C_c = allowance for corrosion, in.

C_d = coefficient of discharge for orifice or rotameter

C_p = heat capacity at constant pressure, Btu/(lb mol)(°F)

D = diameter, ft

D_a = inside length of the major axis of an ellipsoidal head, in., see Table 4

D_i = inside diameter, in.

D_m = mean diameter, in.

D_o = orifice diameter, ft

E_J = efficiency of joints, expressed as a fraction

f = Fanning friction factor

F = mechanical-energy loss due to friction, ft . lbf/lbm

F_c = frictional loss due to sudden contraction, ft · lbf/lbm

F_e = frictional loss due to sudden expansion, ft · lbf/lbm

g = local gravitational acceleration, ft/(s)(s)

g_c = conversion factor in Newton's law of motion, 32.17 ft · lbm/(s)(s)(lbf)

G = mass velocity, lbm/(h)(ft^2 of cross-sectional area)

h = enthalpy, ft · lbf/lbm

H_s = submergence, ft

H_t = lift, ft

IDD = inside depth of dish, in., see Table 4

k = ratio of specific heat at constant pressure to specific heat at constant volume

K_c = coefficient in expression for frictional loss due to sudden contraction

1 = cake thickness at time θ, ft

I, = fictitious cake thickness for liquid flow with resistance equal to that of filter medium, ft

l'_F = fictitious cake thickness for air flow with resistance equal to that of filter medium, ft

L = length, ft

L_a = inside radius of hemispherical head or inside crown radius of torispherical head, in., see Table 4

L_e = fictitious length of straight pipe with the equivalent resistance of a pipe fitting of same nominal diameter as pipe, ft

M = molecular weight, lbm/lbmol

n = special correction constant for pressure-vessel design, see Table 4

N = speed of impeller, r/min

N_R = number of revolutions per unit time, revolutions/h

N_{Re} = Reynolds number, equals $DV\rho/\mu$, dimensionless

N_s = number of stages of compression

OD = outside diameter, in., see Table 4

p = absolute pressure, lbf/ft^2

p_c = static pressure at point of minimum cross-sectional flow area, lbf/ft^2

P = maximum allowable internal pressure, psig, or total pressure; AP means pressure-difference driving force across filter; P_1 refers to vacuum-pump intake pressure; P_2 refers to vacuum-pump delivery pressure, lbf/ft^2

P_b = bursting pressure, psig

P_s = safe working pressure, psig

q = rate of fluid discharge, gpm
q_f = fluid-flow rate, ft^3/s
q_{fm} = fluid-flow rate, ft^3/min
q_{fm_1} = cubic feet of gas flowing per minute at vacuum-pump intake conditions, ft^3/min
Q = heat energy provided as such to the fluid system from an outside source, ft · lbf/lbm
r = knuckle radius, in., see Table 4
r_i = inside radius of shell, in., see Table 4
R = ideal-gas-law constant, 1545 $(lbf/ft^2)(ft^3)/(lb\ mol)(°R)$
R_F = resistance of filter medium to passage of liquid
R'_F = resistance of filter medium to passage of air
R_H = hydraulic radius, ft
R_K = resistance of filter cake to passage of liquid
R'_K = resistance of filter cake to passage of air
s = compressibility exponent of filter cake, defined by Eq. (36)
s' = a constant, defined by Eq. (54*a*)
S = maximum allowable working stress, $lbf/in.^2$
S_1 = cross-sectional flow area in upstream section of duct before constriction, ft^2
S_c = cross-sectional flow area at point of minimum cross-sectional flow area, ft^2
S_p = maximum cross-sectional area of plummet, ft^2
S_s = safe working stress, $lbf/in.^2$
S_T = tensile strength, $lbf/in.^2$
t = minimum wall thickness, in., see Table 4
T = temperature, °R
u = internal energy, ft · lbf/lbm
v = specific volume of fluid, ft^3/lbm
V = average linear velocity, ft/s, or volume of filtrate delivered in time θ, ft^3
V_a = volume of air at temperature and pressure of surroundings drawn through filter cake in time θ, ft^3
V_{air} = volume of free air required in air lift to raise 1 gal of water, ft^3/gal
V_{aR} = volume of air drawn through filter cake per revolution, ft^3/revolution
V_F = fictitious volume of filtrate per unit of filtering area necessary to lay down a cake of thickness l_F, ft^3/ft^2
V'_F = fictitious volume of filtrate per unit of air-suction area necessary to lay down a cake of thickness l'_F, ft^3/ft^2
V_i = instantaneous or point linear velocity, ft/s
V_o = average linear velocity through orifice, ft/s
V_p = volume of plummet, ft^3
V_R = volume of filtrate delivered per revolution, ft^3/revolution

w = weight of dry-cake solids per unit volume of filtrate, lb/ft^3
W = shaft work, gross work input to the fluid system from an outside source, ft · lbf/lbm
W_o = mechanical work imparted to the fluid system from an outside source, ft · lbf/lbm
W_v = weight of vessel, lbm, see Table 6
Z = vertical distance above an arbitrarily chosen datum plane, ft

Greek Symbols

α = correction coefficient to account for use of average velocity or specific cake resistance in units of h^2/lb
α' = a constant, defined by Eq. (36)
β = specific air-suction cake resistance, h^2/lb
β' = a constant, defined by Eq. (54*a*)
A = difference
Δp_f = change in pressure due to friction, lb/ft^2
ε = equivalent roughness, ft
θ = time, h
μ = absolute viscosity of fluid or absolute viscosity of filtrate, lb/(s)(ft) or lb/(h)(ft)
μ_a = absolute viscosity of fluid at average bulk temperature or absolute viscosity of air at temperature and pressure of surroundings, lb/(s)(ft) or lb(h)(ft)
μ_c = absolute viscosity of fluid, centipoises
μ_w = absolute viscosity of fluid at average temperature of wall, lbm/(s)(ft)
ρ = density of fluid, lbm/ft^3
ρ_c = cake density as weight of dry-cake solids per unit volume of wet filter cake, lbm/ft^3
ρ_m = density of metal for pressure-vessel design, lbm/in.^{3}, see Table 4
ρ_p = density of plummet, lbm/ft^3
ϕ = correction factor for nonisothermal flow = $1.1(\mu_a/\mu_w)^{0.25}$ when DG/μ_a is less than 2100 and $1.02(\mu_a/\mu_w)^{0.14}$ when DG/μ_a is greater than 2100
ψ_a = fraction of total surface area available for air suction
ψ_f = fraction of total available surface for filtration immersed in slurry

PROBLEMS

1. A lean oil is to be used as the absorbing medium for removing a component of a gas. As part of the design for the absorption unit, it is necessary to estimate the size of the motor necessary to pump the oil to the top of the absorption tower. The oil must be pumped from an open tank with a liquid level 10 ft above the floor and forced

through 150 ft of schedule number 40 pipe of 3-in. nominal diameter. There are five 90° elbows in the line, and the top of the tower is 30 ft above the floor level. The operating pressure in the tower is to be 50 psig, and the oil requirement is estimated to be 50 gpm. The viscosity of the oil is 15 centipoises, and its density is 53.5 lb/ft^3. If the efficiency of the pumping assembly including the drive is 40 percent, what horsepower motor will be required?

2. Hydrogen at a temperature of 20°C and an absolute pressure of 1380 kPa enters a compressor where the absolute pressure is increased to 4140 kPa. If the mechanical efficiency of the compressor is 55 percent on the basis of an isothermal and reversible operation, calculate the pounds of hydrogen that can be handled per minute when the power supplied to the pump is 224 kW. Kinetic-energy effects can be neglected.
3. For the conditions indicated in Prob. 2, determine the mechanical efficiency of the pump on the basis of adiabatic and reversible operation. A single-stage compressor is used, and the ratio of heat capacity at constant pressure to the heat capacity at constant volume for the hydrogen may be assumed to be 1.4.
4. A steel pipe of 4-in. nominal diameter is to be used as a high-pressure steam line. The pipe is butt-welded, and its schedule number is 40. Estimate the maximum steam pressure that can be used safely in this pipe.
5. A preliminary estimate is to be made of the total cost for a completely installed pumping system. A pipeline is to be used for a steady delivery of 250 gpm of water at 60°F. The total length of the line will be 1000 ft, and it is estimated that the theoretical horsepower requirement (100 percent efficiency) of the pump is 10.0 hp. Using the following additional data, estimate the total installed cost for the pumping system.

   Materials of construction-black steel (standard weights are satisfactory)
   Number of fittings (equivalent to tees)—40
   Number of valves (gate)—4
   Insulation-85 percent magnesia, $1\frac{1}{2}$ in. thick
   Pump-centrifugal (no standby pump is needed)
   Motor-ac, enclosed, 3-phase, 1800 r/min

6. A centrifugal pump delivers 100 gpm of water at 60°F when the impeller speed is 1750 r/min and the pressure drop across the pump is 20 psi. If the speed is reduced to 1150 r/min, estimate the rate of water delivery and the developed head in feet if the pump operation is ideal.
7. A two-stage steam jet is to be used on a large vacuum system. It is estimated that 9 kg of air must be removed from the system per hour. The leaving vapors will contain water vapor at a pressure equivalent to the equilibrium vapor pressure of water at 15°C. If a suction pressure of 2.0 in. Hg absolute is to be maintained by the jet, estimate the pounds of steam required per hour to operate the jet.
8. The rate of flow of a liquid mixture is to be measured continuously. The flow rate will be approximately 40 gpm, and rates as low as 30 gpm or as high as 50 gpm can be expected. An orifice meter, a rotameter, and a venturi meter are available. On the basis of the following additional information, would you recommend installation of the orifice meter, the venturi meter, or the rotameter? Give reasons for your choice.

   Density of liquid = 58 lb/ft^3
   Viscosity of liquid = 1.2 centipoises

Diameter of venturi throat = 1 in.
Upstream diameter of venturi opening = 2 in.
Manometers connected across the venturi and the orifice contain a nonmiscible liquid (sp gr = 1.56) in contact with the liquid mixture
The maximum possible reading on these manometers is 15 in.
Orifice is square-edged with throat taps
Diameter of orifice opening = 1 in.
Diameter of upstream chamber for orifice meter = 3 in.

Calibration curve for rotameter is for water at 60°F and gives the following values:

Rotameter reading	*Flow rate, ft^3/min*
2.0	2.0
4.0	4.0
6.0	6.0
8.0	8.0

The density of the plummet in the rotameter is 497 lb/ft^3.

9. A spherical carbon-steel storage tank has an inside diameter of 30 ft. All joints are butt-welded with backing strip. If the tank is to be used at a working pressure of 30 psig and a temperature of 80°F, estimate the necessary wall thickness. No corrosion allowance is necessary.

10. Estimate the cost of the steel for the spherical storage tank in the preceding problem if the steel sheet costs $0.25 per pound. On the basis of the data presented in Fig. 14-56, estimate the fraction of the purchased cost of the tank that is due to the cost for the steel.

11. Filtration tests with a given slurry have indicated that the specific cake resistance α is 157 h^2/lb. The fluid viscosity is 2.5 lb/(h)(ft), and 3 lb of dry cake is formed per cubic foot of filtrate. The cake may be assumed to be noncompressible, and the resistance of the filter medium may be neglected. If the unit is operated at a constant pressure drop of 5 psi, what is the total filtering area required to deliver 30 ft^3 of filtrate in $\frac{1}{2}$ h?

12. A rotary filter with a total filtering area of 8 ft^2 has been found to deliver 10 ft^3 of filtrate per minute when operating at the following conditions:

    Fraction of filtering area submerged in the slurry = 0.2
    r/min = 2
    Pressure drop = 20 psi

    Another rotary filter is to be designed to handle the same slurry mixture. This unit will deliver 100 ft^3 of filtrate per minute and will operate at a pressure drop of 15 psi and a revolving speed of 1.5 r/min. If the fraction of filtering area submerged in the slurry is 0.2, estimate the total filtering area required for the new unit. In both cases, it may be assumed that no solids pass through the filter cloth, the cake is noncompressible, and the resistance of the filtering medium is negligible.

13. A plate-and-frame filter press is used to filter a known slurry mixture. At a constant pressure drop of 10 psi, 50 ft^3 of filtrate is delivered in 10 min, starting with a clean filter. In a second run with the same slurry and filter press, 40 ft^3 of filtrate is

obtained in 9 min when the pressure drop is 6 psi, starting with a clean filter. What is the compressibility exponent for the cake if the resistance of the filter medium is negligible?

14. A slurry containing 1 lb of filterable solids per 10 lb of liquid is being filtered with a plate-and-frame filter press having a total filtering area of 250 ft^2. This unit delivers 10,000 lb of filtrate during the first 2 h of filtration, starting with a clean unit and maintaining a constant pressure drop of 10 psi. The resistance of the filter medium is negligible. The time required for washing and dumping is 3 h/cycle. The pressure drop cannot exceed 10 psi, and the unit is always operated at constant pressure drop.

    The filter press is to be replaced by a rotary vacuum-drum filter with negligible filter-medium resistance. This rotary filter can deliver the filtrate at a rate of 1000 lb/h when the drum speed is 0.3 r/min. Assuming the fraction submerged and the pressure drop are unchanged, what drum speed in r/min is necessary to make the amount of filtrate delivered in 24 h from the rotary filter exactly equal to the maximum amount of filtrate obtainable per 24 h from the plate-and-frame filter?

15. A plate-and-frame filter press with negligible filter-medium resistance is being used to filter a water slurry of constant composition. Experimental tests show that, during continuous operation, 300 ft^3 of filtrate is delivered when the pressure drop is 20 psi and 150 ft^3 of filtrate is delivered when the pressure drop is 5 psi. The unit is to be operated at a constant pressure drop of 15 psi during filtration and washing. The cake will be washed with 10 ft^3 of water at the end of 2 h of continuous filtration. If reverse thorough washing (i.e., wash rate = $\frac{1}{4}$ of final filtrate delivery rate) is used, estimate the time required for washing.

16. A slurry is filtered, and the filter cake is washed by use of a plate-and-frame filter press operated at a constant pressure drop of 40 psi throughout the entire run. Experimental tests have been carried out on this equipment, and the results for the slurry mixture used can be expressed as follows for any one pressure drop:

$$\theta \Delta P = k'\left(\frac{V}{A}\right)^2 + k''\frac{V}{A}$$

    where k' and k'' are constants. At a pressure drop of 40 psi, 0.02 lb of filtrate is collected in 1.8 min for each square inch of cloth area, and 0.08 lb of filtrate per square inch of cloth area is collected in 22.2 min. Calculate the time required to filter and wash the cake formed when 0.11 lb of filtrate has been collected per square inch of cloth area if an amount of wash water equal to half the filtrate is used. The specific gravities of the filtrate and wash water are 1.0, and both are at the same temperature. Simple forward washing is used so that the washing rate is equal to the filtrate delivery rate at the end of the filtration.

CHAPTER

15

HEAT-TRANSFER EQUIPMENT—DESIGN AND COSTS

Equipment for transferring heat is used in essentially all the process industries, and the design engineer should be acquainted with the many different types of equipment employed for this operation. Although relatively few engineers are involved in the manufacture of heat exchangers, many engineers are directly concerned with specifying and purchasing heat-transfer equipment. Process-design considerations, therefore, are of particular importance to those persons who must decide which piece of equipment is suitable for a given process.

Modern heat exchangers range from simple concentric-pipe exchangers to complex surface condensers with thousands of square feet of heating area. Between these two extremes are found the conventional shell-and-tube exchangers, coil heaters, bayonet heaters, extended-surface finned exchangers, plate exchangers, furnaces, and many varieties of other equipment. Exchangers of the shell-and-tube type are used extensively in industry and are often identified by their characteristic design features. For example, U-tube, fin-tube, fixed-tubesheet, and floating-head exchangers are common types of shell-and-tube exchangers. Figure 15-1 shows design details of a conventional two-pass exchanger of the shell-and-tube type.

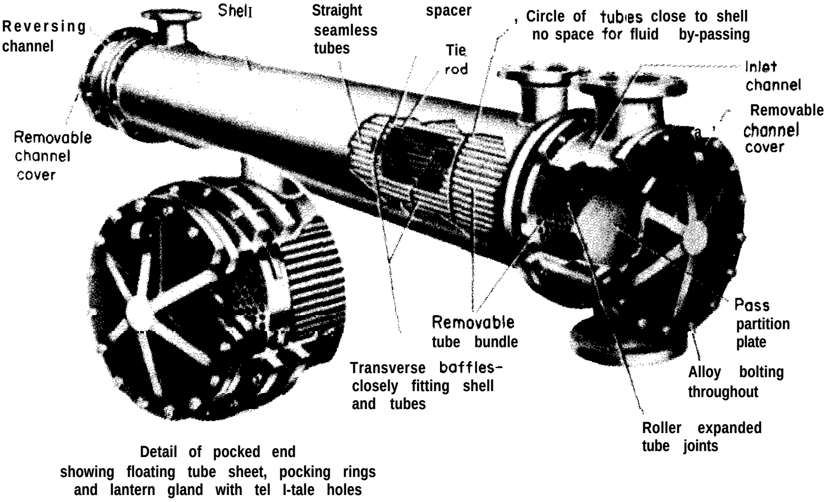


FIGURE 15-1
Two-pass shell-and-tube heat exchanger showing construction details. (*Ross Heat Exchanger Division of American-Standard.*)

Intelligent selection of heat-transfer equipment requires an understanding of the basic theories of heat transfer and the methods for design calculation. In addition, the problems connected with mechanical design, fabrication, and operation must not be overlooked. An outline of heat-transfer theory and design-calculation methods is presented in this chapter, together with an analysis of the general factors that must be considered in the selection of heat-transfer equipment.

Determination of appropriate coefficients of heat transfer is required for design calculations on heat-transfer operations. These coefficients can sometimes be estimated on the basis of past experience, or they can be calculated from empirical or theoretical equations developed by other workers in the field. Many semiempirical equations for the evaluation of heat-transfer coefficients have been published. Each of these equations has its limitations, and the engineer must recognize the fact that these limitations exist. A summary of useful and reliable design equations for estimating heat-transfer coefficients under various conditions is presented in this chapter. Additional relations and discussion of special types of heat-transfer equipment and calculation methods are presented in the numerous books and articles that have been published on the general subject of heat transfer.

BASIC THEORY OF HEAT TRANSFER

Heat can be transferred from a source to a receiver by conduction, convection, or radiation. In many cases, the exchange occurs by a combination of two or three of these mechanisms. When the rate of heat transfer remains constant and is unaffected by time, the flow of heat is designated as being in a steady state; an *unsteady state* exists when the rate of heat transfer at any point varies with time. Most industrial operations in which heat transfer is involved are carried out under steady-state conditions. However, unsteady-state conditions are encountered in batch processes, cooling and heating of materials such as metals or glass, and certain types of regeneration, curing, or activation processes.

Conduction

The transfer of heat through a fixed material is accomplished by the mechanism known as *conduction.* The rate of heat flow by conduction is proportional to the area available for the heat transfer and the temperature gradient in the direction of the heat-flow path. The rate of heat flow in a given direction, therefore, can be expressed as†

$$\frac{dQ}{d\theta} = -kA\frac{dt}{dx} \tag{1}$$

where Q = amount of heat transferred in time θ h, Btu

k = the proportionality constant, designated as thermal conductivity and defined by Eq. (1), Btu/(h)(ft^2)(°F/ft)

A = area of heat transfer perpendicular to direction of heat flow, ft^2

t = temperature, °F

x = length of conduction path in direction of heat flow, ft

The thermal conductivity is a property of any given material, and its value must be determined experimentally. For solids, the effect of temperature on thermal conductivity is relatively small at normal temperatures. Because the conductivity varies approximately linearly with temperature, adequate design accuracy can be obtained by employing an average value of thermal conductivity based on the arithmetic-average temperature of the given material. Values of thermal conductivities for common materials at various temperatures are listed in the Appendix.

For the common case of steady-state flow of heat, Eq. (1) can be expressed as

$$\frac{Q}{\theta} = q = -kA_m\frac{\Delta t}{x} \tag{2}$$

where A_m = mean area of heat transfer perpendicular to direction of heat flow, ft^2

q = rate of heat transfer, Btu/h

Δt = temperature-difference driving force, °F

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†In accord with accepted design practice in the United States, the U.S. customary system of units is used in this chapter. See Appendix A for conversion to SI units.

Convection

Transfer of heat by physical mixing of the hot and cold portions of a fluid is known as heat ***transfer*** by ***convection.*** The mixing can occur as a result of density differences alone, as in ***natural convection,*** or as a result of mechanically induced agitation, as in ***forced convection.***

The following equation is used as a basis for evaluating rates of heat transfer by convection:

$$\frac{dQ}{de} = hA\,\Delta t \tag{3}$$

The proportionality constant h is designated as the heat-transfer coefficient, and it is a function of the type of agitation and the nature of the fluid. The heat-transfer coefficient, like the thermal conductivity k, is often determined on the basis of experimental data. For steady-state conditions, Eq. (3) becomes

$$q = hA\,\Delta t \tag{4}$$

Radiation

When radiant heat energy is transferred from a source to a receiver, the method of heat transfer is designated as ***radiation. The*** rate at which radiant heat energy is emitted from a source is

$$\frac{dQ}{d\theta} = \sigma\varepsilon A T^4 \tag{5}$$

where a = Stefan-Boltzmann constant
= 0.1713×10^{-8} Btu/(h)(ft^2)(°R)^4
ε = emissivity of surface
A = exposed-surface area of heat transfer, ft^2
T = absolute temperature, “R.

The emissivity depends on the characteristics of the emitting surface and, like the thermal conductivity and the heat-transfer coefficient, must be determined experimentally. Part of the radiant energy intercepted by a receiver is absorbed, and part may be reflected. In addition, the receiver, as well as the source, can emit radiant energy.

The engineer is usually interested in the net rate of heat interchange between two bodies. Some of the radiated energy indicated by Eq. (5) may be returned to the source by reflection from the receiver, and the receiver, of course, emits radiant energy which can be partly or completely absorbed by the source. Equation (5), therefore, must be modified to obtain the net rate of radiant heat exchange between two bodies. The general steady-state equation is

$$q_{\text{from body 1 to body 2}} = 0.171A\left[\left(\frac{T_1}{100}\right)^4 F_{A_1}F_{E_1} - \left(\frac{T_2}{100}\right)^4 F_{A_2}F_{E_2}\right] \tag{6}$$

TABLE 1

Values of F_A and F_E for use in Eq. (7)

ϵ_1 = emissivity of surface 1

ϵ_2 = emissivity of surface 2

Assumptions: Base area is smooth and no direct self-radiation is intercepted by the base surface; separating medium behveen surfaces is nonabsorbing; emissivity is equal to absorptivity

Orientation of surfaces	Area, A	F_A	F_E
Surface A_1 small compared with the totally enclosing surface A_2 (for example, heat loss from equipment to surroundings)	A_1	1	ϵ_1
Two parallel planes of equal area when length and width are large compared with distance between planes	A_1 or A_2	1	$\dfrac{1}{\dfrac{1}{\epsilon_1}+\dfrac{1}{\epsilon_2}-1}$
Surface A_1 is a sphere with radius r_1 inside a concentric sphere with radius r_2	A_1	1	$\dfrac{1}{\dfrac{1}{\epsilon_1}+\left(\dfrac{r_1}{r_2}\right)^2\left(\dfrac{1}{\epsilon_2}-1\right)}$
Surface A_1 is a cylinder with radius r_1 inside a concentric cylinder with radius r_2 when length is large compared with diameter	A_1	1	$\dfrac{1}{\dfrac{1}{\epsilon_1}+\dfrac{r_1}{r_2}\left(\dfrac{1}{\epsilon_2}-1\right)}$

or, in an alternative form,

$$q_{\text{from body 1 to body 2}} = 0.171A\left[\left(\frac{T_1}{100}\right)^4 - \left(\frac{T_2}{100}\right)^4\right]F_A F_E \tag{7}$$

where A represents the area of one of the surfaces, F_A is a correction factor based on the relative orientation of the two surfaces and the surface chosen for the evaluation of A, and F_E is a correction factor based on the emissivities and absorptivities of the surfaces.

Table 1 indicates methods for evaluating F_A and F_E for several simple cases. Table 2 lists the emissivities of surfaces commonly encountered in industrial operations. Additional tables, plots, and methods for evaluating the correction factors are available in the various texts and handbooks on heat transfer.

The design engineer often encounters the situation in which a nonblack body is surrounded completely by a large enclosure containing a nonabsorbing gas. An example would be a steam line exposed to the atmosphere. Under these conditions, little error is introduced by assuming that none of the heat radiated from the source is reflected to it, and Eq. (6) or (7) can be simplified to

$$q_{\text{radiated from pipe}} = 0.171A_1\epsilon_1\left[\left(\frac{T_1}{100}\right)^4 - \left(\frac{T_2}{100}\right)^4\right] \tag{8}$$

TABLE 2
Normal emissivities of various surfaces

Surface	Temperature, °F	Emissivity
Aluminum, polished plate	73	0.040
Aluminum, rough plate	78	0.055
Aluminum paint	. . . .	0.3-0.6
Aluminum-surfaced roofing	100	0.22
Asbestos paper	100	0.93
Brass, dull	120-660	0.22
Brass, polished	100-600	0.096
Brick, building	. . . . .	0.80-0.95
Brick, refractory	. .	0.75-0.90
Copper, oxidized	77	0.78
Iron, cast plate, smooth	73	0.80
Iron, oxidized	212	0.74
Iron, polished	800	0.14
Lead, gray, oxidized	75	0.28
Nickel, pure, polished	440	0.07
	710	0.087
Oil paint	212	0.92-0.96
Roofing paper	69	0.91
Silver, polished	100	0.022
	700	0.031
Steel, polished	212	0.066
Steel sheet, oxidized	75	0.80
Water	32	0.95
	212	0.963
Zinc, galvanized sheet iron, gray oxidized	75	0.276

where subscript 1 refers to the pipe and subscript 2 refers to the surroundings. It is often convenient to use Eq. (8) in the following form, analogous to the equation for heat transfer by convection:

$$q_{\text{radiated from pipe}} = h_r A_1 \, \Delta t = h_r A_1 (T_1 - T_2) \tag{9}$$

where h_r is a fictitious heat-transfer coefficient based on the rate at which radiant heat leaves the surface of the pipe. Combination of Eqs. (8) and (9) gives

$$h_r = \frac{0.171\varepsilon_1\left[(T_1/100)^4 - (T_2/100)^4\right]}{T_1 - T_2} \tag{10}$$

Equations (8), (9), and (10) are based on the assumption that none of the heat radiated from the source is reflected to it. This, of course, implies that the air surrounding the pipe has no effect. In other words, air is assumed to be nonabsorbing and nonreflecting. This assumption is essentially correct for gases such as oxygen, nitrogen, hydrogen, and chlorine. Other gases, however, such as carbon monoxide, carbon dioxide, sulfur dioxide, ammonia, organic gases, and water vapor, exhibit considerable ability to absorb radiant energy in certain regions of the infrared spectrum. As a result, the design engineer dealing with heat transfer in furnaces or other equipment in which absorbing gases are present may find it necessary to consider radiation from gases, as well as ordinary surface radiation.

Example 1 Combined heat transfer by convection and radiation. The OD of an uninsulated steam pipe is 4.5 in. The outside-surface temperature of the pipe is constant at 300°F, and the pipe is located in a large room where the surrounding temperature is constant at 70°F. The heat content of the steam is valued at $1.60 per 10^6 Btu. The emissivity of the pipe surface is 0.7, and the heat-transfer coefficient for heat loss from the surface by convection is 1.4 Btu/(h)(ft^2)(°F). Under these conditions, determine the cost per year for heat losses from the uninsulated pipe if the length of the pipe is 100 ft.

Solution. From Eq. (4), heat loss by convection = $hA\,\Delta t_f$

$$A = \frac{(3.14)(4.5)(100)}{12} = 118 \text{ ft}^2$$

$$\Delta t_f = 300 - 70 = 230°\text{F}$$

$$q_{\text{convection}} = (1.4)(118)(230) = 38{,}000 \text{ Btu/h}$$

From Eq. (8),

$$\text{Heat lost by radiation} = 0.171 A_1 \varepsilon_1 \left[\left(\frac{T_1}{100}\right)^4 - \left(\frac{T_2}{100}\right)^4\right]$$

$$= (0.171)(118)(0.7)\left[\left(\frac{300 + 460}{100}\right)^4 - \left(\underline{\;,\;,\;,\;,\;,\;}\right)\right]$$

$$= 36{,}000 \text{ Btu/h}$$

Yearly cost for heat losses per 100 ft of pipe

$$= \frac{(38{,}000 + 36{,}000)(24)(365)(1.60)}{10^6} = \$1037$$

OVERALL COEFFICIENTS OF HEAT TRANSFER

Many of the important cases of heat transfer involve the flow of heat from one fluid through a solid retaining wall into another fluid. This heat must flow through several resistances in series. The net rate of heat transfer can be related to the total temperature-difference driving force by employing an overall coefficient of heat transfer U; thus, for steady-state conditions,

$$q = UA\,\Delta t_{oa} \qquad (11)$$

where At,, represents the total temperature-difference driving force. Because there are usually several possible areas on which the numerical value of the overall coefficient can be based, the units of U should include a designation of the base area A. For example, if an overall coefficient is based on the inside area of a pipe, the units of this U should be expressed as **Btu/(h)(ft^2** of inside **area)(°F).†**

The overall coefficient (including a dirt or fouling resistance) can be related to the individual coefficients or resistances by the following equation:

$$\frac{1}{U_d} = \frac{A}{h'A'_f} + \frac{A}{h''A''_f} + \frac{Ax,}{kA_{m_w}} + \frac{A}{h'_dA'_f} + \frac{A}{h''_dA''_f} \tag{12}$$

where A represents the base area chosen for the evaluation of U_d, and the primes refer to the different film resistances involved.

Fouling Factors‡

After heat-transfer equipment has been in service for some time, dirt or scale may form on the heat-transfer surfaces, causing additional resistance to the flow of heat. To compensate for this possibility, the design engineer can include a resistance, called a *dirt, scale,* or *fouling factor,* when determining an overall coefficient of heat transfer. Equation (12) illustrates the method for handling the fouling factor. In this case, the fouling coefficients h'_d and h''_d are used to account for scale or dirt formation on the heat-transfer surfaces.

Fouling factors, equivalent to **l/h,,** are often presented in the literature for various materials and conditions. Values of h_d for common process services are listed in Table 3. Because the scale or dirt resistance increases with the time the equipment is in service, some basis must be chosen for numerical values of fouling factors. The common basis is a time period of 1 year, and this condition applies to the values of h_d presented in Table 3. When the correct fouling factors are used, the equipment should be capable of transferring more than the required amount of heat when the equipment is clean. At the end of approximately 1 year of service, the capacity will have decreased to the design value, and a shutdown for cleaning will be necessary. With this approach, numerous shutdowns for cleaning individual units are not necessary. Instead, annual or periodic shutdowns of the entire plant can be scheduled, at which time all heat-transfer equipment can be cleaned and brought up to full capacity.

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†To change units for heat-transfer coefficients from **Btu/(h)(ft^2)(°F)** to **SI** units of **J/(s)(m^2)(K)** or **W/(m^2)(K)**, multiply by 5.678.

‡For a detailed review of fouling factors related to heat exchangers, see J. W. Suitor, W. J. Marner, and R. B. Ritter, The History and Status of Research in Fouling of Heat Exchangers in Cooling Water Service, *Can. J. Chem. Eng.*, **55(4):374 (1977).**

TABLE 3
Individual heat-transfer coefficients to account for fouling

	h_d for water, Btu/(h)(ft^2)(°F)			
Temperature of heating medium:	Up to 240°F		240–400°F	
Temperature of water:	125°F or less		Above 125°F	
Water velocity, ft/s:	3 and less	Over 3	3 and less	Over 3
Distilled	2000	2000	2000	2000
Sea water	2000	2000	1000	1000
Treated boiler feedwater	1000	2000	500	1000
Treated make-up for cooling tower	1000	1000	500	500
City, well, Great Lakes	1000	1000	500	500
Brackish, clean river water	500	1000	330	500
River water: muddy, silty†	330	500	250	330
Hard (over 15 grains/gal)	330	330	200	200
Chicago Sanitary Canal	130	170	100	130

	h_d for miscellaneous process services, Btu/(h)(ft^2)(°F)
Organic vapors, liquid gasoline	2000
Refined petroleum fractions (liquid), organic liquids, refrigerating liquids, brine, oil-bearing steam	1000
Distillate bottoms (above 25°API), gas oil or liquid naphtha below 500°F, scrubbing oil, refrigerant vapors, air (dust)	500
Gas oil above 500°F, vegetable oil	330
Liquid naphtha above 500°F, quenching oils	250
Topped crude (below 25°API), fuel oil	200
Cracked residuum, coke-oven gas, illuminating gas	100

†Mississippi, Schuylkill, Delaware, and East Rivers and New York Bay.

MEAN AREA OF HEAT TRANSFER

The cross-sectional area of heat transfer A in Eq. (1) can vary appreciably along the length of the heat-transfer path x. Therefore, the shape of the solid through which heat is flowing must be known before Eq. (1) can be integrated to give Eq. (2). The exact value for A,, based on the limiting areas A, and A,, follows for three cases commonly encountered in heat-transfer calculations:

1. Conduction of heat through a solid of constant cross section (example, a large flat plate)

$$A_m = A_{\text{arith.avg}} = \frac{A_1 + A_2}{2} \tag{13}$$

2. Conduction of heat through a solid when the cross-sectional area of heat transfer is proportional to the radius (example, a long hollow cylinder)

$$A_m = A_{\text{log. mean}} = \frac{A_1 - A_2}{\ln(A_1/A_2)} \tag{14}$$

3. Conduction of heat through a solid when the cross-sectional area of heat transfer is proportional to the square of the radius (example, a hollow sphere)

$$A_{,,,} = A_{\text{geom. mean}} = \sqrt{A_1A_2} \tag{15}$$

When the value of A_1/A_2 (or A_2/A_1 if *A,* is larger than *A,)* does not exceed 2.0, the mean area based on an arithmetic-average value is within 4 percent of the logarithmic-mean area and within 6 percent of the geometric-mean area. This accuracy is considered adequate for most design calculations. It should be noted that the arithmetic-average value is always greater than the logarithmic mean or the geometric mean.

MEANTEMPERATURE-DIFFERENCEDRMNGFORCE

When a heat exchanger is operated continuously, the temperature difference between the hot and cold fluids usually varies throughout the length of the exchanger. To account for this condition, Eqs. (4) and (11) can be expressed in a differential form as

$$dq = h\,dA\,\Delta t_f \tag{16}$$

$$dq = U\,dA\,\Delta t_{oa} \tag{17}$$

The variations in At and the heat-transfer coefficients must be taken into account when Eqs. (16) and (17) are integrated. Under some conditions, a graphical or stepwise integration may be necessary, but algebraic solutions are possible for many of the situations commonly encountered with heat-transfer equipment.

CONSTANT HEAT-TRANSFER COEFFICIENT. The integrated forms of Eqs. (16) and (17) are often expressed in the following simplified forms:

$$q = hA\,\Delta t_{f_m} \tag{18}$$

$$q = UA\ \text{At,,,} \tag{19}$$

where the subscript *m* refers to a mean value. Under the following conditions, the correct mean temperature difference is the logarithmic-mean value:

1. U (or h) is constant.
2. Mass flow rate is constant.
3. There is no partial phase change.

4. Specific heats of the fluids remain constant.
5. Heat losses are negligible.
6. Equipment is for counterfiow, parallel flow, or any type of flow if the temperature of one of the fluids remains constant (phase change can occur) over the entire heat-transfer area.

$$\Delta t_{oa_m} = \Delta t_{oa_{\text{log mean}}} = \frac{\Delta t_{oa_1} - \Delta t_{oa_2}}{\ln\left(\Delta t_{oa_1}/\Delta t_{oa_2}\right)} \tag{20}$$

where **At,,,** and Δt_{oa_2} represent the two terminal values of the overall temperature-difference driving force. For design calculations, an **arithmetic**-average temperature difference can be used in place of the logarithmic-mean value if the ratio $\Delta t_{oa_1}/\Delta t_{oa_2}$ (or $\Delta t_{oa_2}/\Delta t_{oa_1}$ if Δt_{oa_2} is greater than Δt_{oa_1}) does not exceed 2.0.

When multipass exchangers are involved and the first five conditions listed in the preceding paragraph apply, values for **At,,,** can be obtained by integrating Eq. (17) or, more simply, from plots that give the correct Δt_{oa_m} for various types of multipass exchangers as a function of the logarithmic-mean temperature difference for **counterflow**.†‡ Figure 15-2 is a plot of this type for the common case of an exchanger with one shell pass and two or more even-numbered tube passes.

VARIABLE HEAT-TRANSFER COEFFICIENT. If the heat-transfer coefficient varies with temperature, one can assume that the complete exchanger consists of a number of smaller exchangers in series and that the coefficient varies linearly with temperature in each of these sections. When the last five conditions listed in the preceding section hold and the overall coefficient varies linearly with temperature, integration of Eq. (17) gives

$$q = A\frac{U_1\,\Delta t_{oa_2} - U_2\,\Delta t_{oa_1}}{\ln\left(U_1\,\Delta t_{oa_2}/U_2\,\Delta t_{oa_1}\right)} \tag{21}$$

The values of q and A in Eq. (21) apply to the section of the equipment between the limits indicated by the subscripts 1 and 2. Consequently, the total value of q or A for the entire exchanger can be obtained by summing the quantities for each of the individual sections.

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†For other cases, see R. H. Perry and D. Green, "Chemical Engineers' Handbook," 6th ed., McGraw-Hill Book Company, New York, 1984.

‡These plots present graphically the results of integrating Eq. (17).

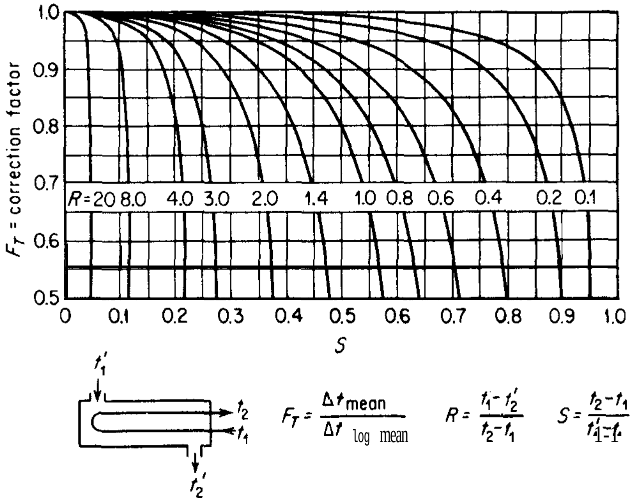

FIGURE 15-2
Chart for determining correct, mean temperature-difference driving force for an exchanger with one shell pass and two or more even-numbered tube passes. (Correction factor F_T is based on the $\Delta t_{\log\ \text{mean}}$ for counterflow. If F_T is below 0.7, operation of the exchanger may not be practical.)

UNSTEADY-STATE HEAT TRANSFER

When heat is conducted through a solid under unsteady-state conditions, the following general equation applies:

$$\frac{\partial t}{\partial \theta} = \frac{1}{\rho c_p}\left[\frac{\partial}{\partial x}\left(k_x\frac{\partial t}{\partial x}\right) + \frac{\partial}{\partial y}\left(k_y\frac{\partial t}{\partial y}\right) + \frac{\partial}{\partial z}\left(k_z\frac{\partial t}{\partial z}\right)\right] \tag{22}$$

where c_p is the heat capacity of the material through which heat is being conducted and x, y, and z represent the Cartesian coordinates. The solution of any problem involving unsteady-state conduction consists essentially of integrating Eq. (22) with the proper boundary conditions.

For a homogeneous and isotropic material, Eq. (22) becomes

$$\frac{\partial t}{\partial \theta} = \alpha\left(\frac{\partial^2 t}{\partial x^2} + \frac{\partial^2 t}{\partial y^2} + \frac{\partial^2 t}{\partial z^2}\right) \tag{23}$$

where α = thermal diffusivity = $k/\rho c_p$, ft^2/h.

Many cases of practical interest in unsteady-state heat transfer involve one-dimensional conduction. For one-dimensional conduction in the x direction, Eq. (23) reduces to

$$\frac{\partial t}{\partial \theta} = \alpha\frac{\partial^2 t}{\partial x^2} \tag{24}$$

Solutions of Eqs. (23) and (24) for various shapes and boundary conditions are available in the literature. The simplest types of problems are those in which the surface of a solid suddenly attains a new temperature and this temperature remains constant. Such a condition can exist only if the temperature of the surroundings remains constant and there is no resistance to heat transfer between the surface and the surroundings (i.e., surface film coefficient is infinite). Although there are few practical cases when ‘these conditions occur, the solutions of such problems are of interest to the design engineer because they indicate the results obtainable for the limiting condition of the maximum rate of unsteady--state heat transfer.

Figure 15-3 presents graphically the results of integrating the unsteady-state equations for a sudden change from a uniform surface and bulk temperature to a new constant surface temperature when the surface film coefficient is infinite. The reference temperature in this plot is at the center point, center

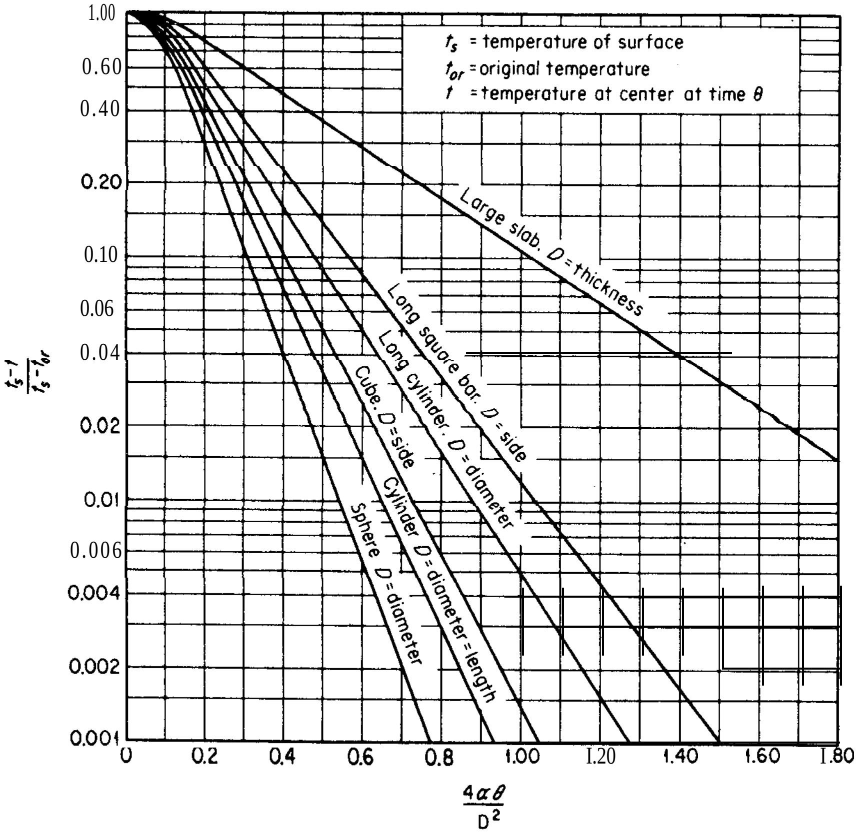

FIGURE **15-3**
Midpoint or midplane temperature for unsteady-state heating or cooling of solids having negligible surface resistance.

line, or center plane, and results are presented for a slab, a square bar, a cube, a cylinder, and a sphere.

Example 2 Estimation of minimum time required for unsteady-state cooling. A solid steel cylinder has a diameter of 3 ft and a length of 3 ft. The temperature of the cylinder is 1000°F. It is suddenly placed in a well-ventilated room where the temperature of the room remains constant at 90°F. Estimate the minimum time (i.e., surface film coefficient is infinite) the cylinder must remain in the room before the temperature at the center can drop to 150°F. Assume the following values for the steel: $k = 24.0$ Btu/(h)(ft^2)(°F/ft); $c_p = 0.12$ Btu/(lb)(°F); $\rho = 488$ lb/ft^3.

Solution

$$\alpha = k/\rho c_p = 24/(488)(0.12) = 0.41 \text{ ft}^2/\text{h}$$

$$t_s = 90°\text{F}$$

$$t = 150°\text{F}$$

$$t_{or} = 1000°\text{F}$$

$$\frac{t_s - t}{t_s - t_{or}} = \frac{-60}{-910} = 0.066$$

From Fig. 15-3,

$$\frac{4\alpha\theta}{D^2} = 0.42$$

$$\theta = \frac{(0.42)(3)^2}{(4)(0.41)} = 2.3 \text{ h}$$

The minimum time before temperature at center of cylinder can drop to 150°F = 2.3 h.

DETERMINATION OF HEAT-TRANSFER COEFFICIENTS

Exact values of convection heat-transfer coefficients for a given situation can be obtained only by experimental measurements under the particular operating conditions. Approximate values, however, can be obtained for use in design by employing correlations based on general experimental data. A number of correlations that are particularly useful in design work are presented in the following sections. In general, the relationships applicable to turbulent conditions are more accurate than those for viscous conditions. Film coefficients obtained from the correct use of equations in the turbulent-flow range will ordinarily be within ±20 percent of the true experimental value, but values determined for viscous-flow conditions or for condensation, boiling, natural convection, and shell sides of heat exchangers may be in error by more than 100 percent. Because of the inherent inaccuracies in the methods for estimating film coefficients, some design engineers prefer to use overall coefficients based on

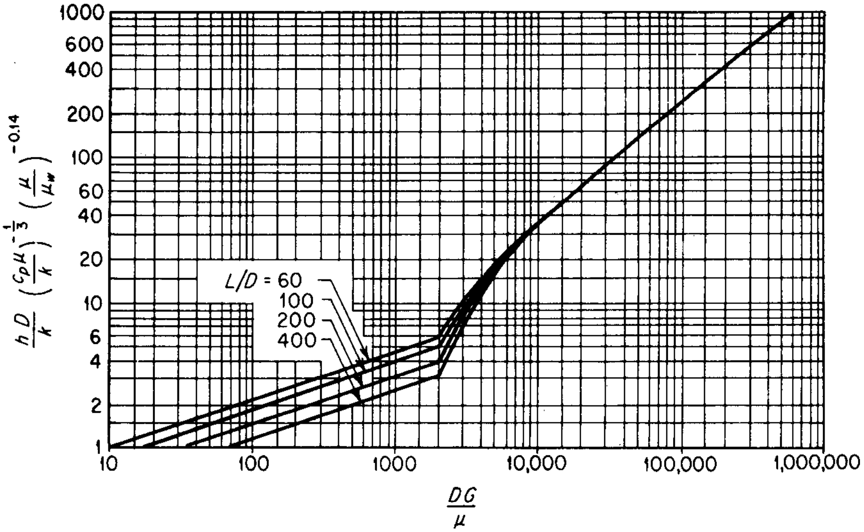

FIGURE 15-4
Plot for estimating film coefficients for fluids flowing in pipes and tubes. [Based on Eqs. (25) and (26).]

past experience, while others include a large safety factor in the form of fouling factors or fouling coefficients.

Film Coefficients for Fluids in Pipes and Tubes (No Change in Phase)

The following equations are based on the correlations presented by Sieder and Tate:[†]

For viscous flow $(DG/\mu < 2100)$,

$$\frac{h_iD}{k} = 1.86\left(\frac{DG}{\mu}\frac{c_p\mu}{k}\frac{D}{L}\right)^{1/3}\left(\frac{\mu}{\mu_w}\right)^{0.14} = 1.86\left(\frac{4wc_p}{\pi kL}\right)^{1/3}\left(\frac{\mu}{\mu_w}\right)^{0.14} \qquad (25)$$

For turbulent flow above the transition region $(DG/\mu > 10{,}000)$,

$$\frac{h_iD}{k} = 0.023\left(\frac{DG}{\mu}\right)^{0.8}\left(\frac{c_p\mu}{k}\right)^{1/3}\left(\frac{\mu}{\mu_w}\right)^{0.14} \qquad (26)^{‡}$$

For transition region $(DG/\mu = 2100$ to $10{,}000)$, see Fig. 15-4,

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[†] E. N. Sieder and G. E. Tate, *Ind. Eng. Chem.*, **28**:1429 (1936).

[‡] In some references, the constant 0.027 is used in place of 0.023. The constant 0.023 is recommended in order to make Eq. (26) generally applicable for water, organic fluids, and gases at moderate Δt.

where D = diameter of pipe or tube (inside), ft
G = mass velocity inside tube, lb/(h)(ft^2 of cross-sectional area)
c_p = heat capacity of fluid at constant pressure, Btu/(lb)(°F)
μ = viscosity of fluid (subscript w indicates evaluation at wall temperature), lb/(h)(ft)
L = heated length of straight tube, ft
w = weight rate of flow per tube, lb/h
k, μ, and c_p are evaluated at the average bulk temperature of the fluid.

Equations (25) and (26) are applicable for organic fluids, aqueous solutions, water, and gases. The two equations are plotted in 15-4 to facilitate their solution and to indicate the values for use in the transition region.

SIMPLIFIED EQUATIONS. For common gases, Eq. (26) can be simplified to give the following approximate equation:?

$$h_i = \frac{0.014 c_p G^{0.8}}{D^{0.2}} \tag{27}$$

Similarly, for water at ordinary temperatures and pressures,

$$h_i = \frac{150(1 + 0.011 t_b)(V')^{0.8}}{(D')^{0.2}} \tag{28}$$

Equations (27) and (28) are dimensional, and a value of h_i as Btu/(h)(ft^2)(°F) is obtained only if the following units are employed for the indicated variables:

c_p = heat capacity of fluid, Btu/(lb)(°F)
D = diameter, ft
D' = diameter, in.
G = mass velocity inside tube, lb/(h)(ft^2)
t_b = average (i.e., bulk) temperature of water, °F
V' = velocity of water, ft/s

Noncircular Cross Section-Equivalent Diameter

The situation is often encountered in which a fluid flows through a conduit having a noncircular cross section, such as an annulus. The heat-transfer coefficients for turbulent flow can be determined by using the same equations that apply to pipes and tubes if the pipe diameter D appearing in these equations is replaced by an equivalent diameter D_e. Best results are obtained if

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†Equations (27) and (28) are derived by neglecting the viscosity correction factor in Eq. (26) and substituting average values for the physical properties.

this equivalent diameter is taken as four times the hydraulic radius, where the hydraulic radius is defined as the cross-sectional flow area divided by the *heated* perimeter. For example, if heat is being transferred from a fluid in a center pipe to a fluid flowing through an annulus, the film coefficient around the inner pipe would be based on the following equivalent diameter:

$$D_e = 4 \text{ x hydraulic radius} = 4 \text{ x } \frac{\pi D_2^2/4 - \pi D_1^2/4}{\pi D_1} = \frac{D_2^2 - D_1^2}{D_1}$$

where D_1 and D_2 represent, respectively, the inner and outer diameters of the annulus.

The difference between the hydraulic radii for heat transfer and for fluid flow should be noted. In the preceding example, the correct equivalent diameter for evaluating friction due to the fluid flow in the annulus would be four times the cross-sectional flow area divided by the *wetted* perimeter, or 4 X$(\pi D_2^2/4 - \pi D_1^2/4)/(\pi D_2 + \pi D_1) = D_2 -$ D,.

Film Coefficients for Fluids Flowing Outside Pipes and Tubes (No change in Phase)

In the common types of baffled shell-and-tube exchangers, the shell-side fluid flows across the tubes. The equations for predicting heat-transfer coefficients under these conditions are not the same as those for flow of fluids inside pipes and tubes. An approximate value for shell-side coefficients in a cross-flow exchanger with segmental baffles and reasonable clearance between baffles, between tubes, and between baffles and shell can be obtained by using the following correlation:?*

$$\frac{h_o D_o}{k_f} = \frac{a_o}{F_s}\left(\frac{D_o G_s}{\mu_f}\right)^{0.6}\left(\frac{c_p \mu}{k}\right)_f^{1/3} \tag{29}$$

where a, = 0.33 if tubes in tube bank are staggered and 0.26 if tubes are in line
F_s = safety factor to account for bypassing effects§

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†A. P. Colburn, *Trans. AIChE*, **29**:174 (1933).

‡For an alternate approach which takes pressure drop into consideration, see D. A. Donohue, *Ind. Eng. Chem.*, **41**(11):2499 (1949).

§Amount of shell-side bypassing between the cross baffles and the shell, between tubes and tube holes in baffles, and between outermost tubes and shell depends on the manufacturing methods and tolerances for the exchanger. The amount of bypassing can have a large influence on the shell-side heat-transfer coefficient. The value of F_s is usually between 1.0 and 1.8, and a value of 1.6 is often recommended.

G_s = shell-side mass velocity across tubes, based on minimum free area between baffles at shell axis,† lb/(h)(ft^2)

Subscript f refers to properties at average film temperature

Equation (29) can be used to obtain approximate film coefficients for hydrocarbons, aqueous solutions, water, and gases when the Reynolds number $(D_o G_s/\mu_f)$ is in the common operating range of 2000 to 32,000.

Film Coefficients and Overall Coefficients for Miscellaneous Cases

Table 4 presents equations for use in the estimation of heat-transfer film coefficients for condensation, boiling, and natural convection. Values showing the general range of film coefficients for various situations are indicated in Table 5.

The design engineer often prefers to use overall coefficients directly without attempting to evaluate individual film coefficients. When this is the case, the engineer must predict an overall coefficient on the basis of past experience with equipment and materials similar to those involved in the present problem. Design values of overall heat-transfer coefficients for many of the situations commonly encountered by design engineers are listed in Table 6.

PRESSURE DROP IN HEAT EXCHANGERS

The major cause of pressure drop in heat exchangers is friction resulting from flow of fluids through the exchanger tubes and shell. Friction due to sudden expansion, sudden contraction, or reversal in direction of flow also causes a pressure drop. Changes in vertical head and kinetic energy can influence the pressure drop, but these effects are ordinarily relatively small and can be neglected in many design calculations.

Tube-side Pressure Drop

It is convenient to express the pressure drop for heat exchangers in a form similar to the Fanning equation as presented in Chap. 14. Because the transfer of heat is involved, a factor must be included for the effect of temperature

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†The free area for use in Eqs. (29) and (31) occurs where the total cross-sectional area of the shell (normal to direction of flow) is a maximum, and the free area at this axis plane is based on the transverse or diagonal openings that give the smallest free area. The free area S_o for use in evaluating G_s for the common case of a full-packed shell and transverse openings giving the smallest free area can be estimated to be

$$\frac{\text{(ID of shell)(clearance between adjacent tubes)(baffle spacing)}}{\text{Center-to-center distance between adjacent tubes}}$$

TABLE 4
Equations and methods for estimating film coefficients of heat transfer for common cases

Type of heat transfer	**Limitations**
Film-type condensation of vapors: Outside horizontal tubes $h = 0.725\left(\frac{k_f^3\rho_f^2 g\lambda_c}{N_V D_o \mu_f \Delta t_f}\right)^{1/4} = 0.95\left(\frac{k_f^3\rho_f^2 gL}{w\mu_f}\right)^{1/3}$ For steam, average h at 1 atm pressure $= \frac{3100}{(N_V D_o)^{1/4}(\Delta t_f)^{1/3}}$	**McAdams**† Pure saturated vapors $\frac{2w}{L\mu_f} < 2000$ Physical properties are for condensate.
Vertical tubes $h = 1.47\left(\frac{\pi D_o k_f^3\rho_f^2 g}{4w\mu_f}\right)^{1/3}$ For steam, average h at 1 atm pressure $= \frac{4000}{L^{1/4}(\Delta t_f)^{1/3}}$	**McAdams**† Pure saturated vapors $4w/\pi D_o\mu_f < 2000$ Physical properties are for condensate.
Boiling liquids outside horizontal tubes: Film boiling (above critical Δt_f) $h_{co} = 0.62\left[\frac{k_v^3\rho_v(\rho_L - \rho_v)g\lambda_c}{D_o\mu_v\,\Delta t_f}\right]^{1/4}$	Bromley ‡ For saturated liquids on submerged surfaces; film coefficient h_{co} is for conduction through the vapor; no radiation effect is included.
Nucleate boiling (below critical At,) Value of h depends on Δt_f, type of surface, and materials involved (critical Δt_f for water as temperature drop from heating surface to liquid is approximately 45°F).	
Natural convection: General equation $\frac{h_c L}{k_f} = c_c[(N_{Gr})_f(N_{Pr})_f]^n$	Perry § Values of c_c and n depend on N_{Gr} X N_{Pr} and the geometry of the surfaces.

(Continued)

change on the friction factor. Under these conditions, the pressure drop through the tube passes (i.e., tube side) of a heat exchanger may be expressed as follows (subscript i refers to inside of tubes at bulk temperature):

$$-\Delta P_i = \frac{B_i 2 f_i G^2 L n_p}{g_c \rho_i D_i \phi_i} \tag{30}$$

TABLE 4

Equations and methods for estimating film coefficients of heat transfer for common cases (Continued)

Type of heat transfer		Limitations
Simplified equations for air $h_c = K(\Delta t)^{0.25}$		**Kern¶** Ordinary air temperature, atmospheric pressure, normal conditions; for more exact equations, see Perry.5
Physical arrangement of equipment	K	
Horizontal plates facing upward	0.38	
Horizontal plates facing downward	0.20	
Vertical plates more than 2 ft high	0.30	
Vertical plates less than 2 ft high	$\frac{0.28}{(\text{vertical height, ft})^{0.25}}$	
Vertical pipes more than 1 ft high	$\frac{0.22}{(D_o, \text{ft})^{0.25}}$	
Horizontal pipes	$\frac{0.27}{(D_o, \text{ft})^{0.25}}$	

A, = latent heat of condensation, Btu/lb

g = local gravitational acceleration, ft/(h) (h)

N_V = number of rows of tubes in a vertical tier

D_o = outside diameter of tube, ft

Δt_f = temperature-difference driving force across film, °F

L = heated length of straight tube or length of heat-transfer surface, ft

w = weight rate of flow of condensate per tube from lowest point on condensing surface, lb/(h) (tube)

(No, $)_f$ = Grashof number $= \dfrac{L^2\rho f\,^2 g\beta_f\,\Delta t_f}{\mu_f^2}$

$(N_{Pr})_f$ = Prandtl number $= \left(\dfrac{c_p\mu}{k}\right)_f$

β = coefficient of volumetric expansion, 1/°R

c_c, K, n = constants

Subscript f designates "at average film temperature"

Subscript v designates vapor at average vapor temperature

Subscript L designates liquid at average liquid temperature

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†W. H. McAdams, "Heat Transmission," 3rd ed., McGraw-Hill Book Company, New York, 1954.

‡L. A. Bromley, *Chem. Eng. Progr.*, **46**:221 (1950).

§R. H. Perry and D. Green, "Chemical Engineer's Handbook," 6th ed., McGraw-Hill Book Company, New York, 1984.

¶D. Q. Kern, "Process Heat Transfer," McGraw-Hill Book Company, New York, 1950.

where f_i = Fanning friction factor for isothermal flow based on conditions at the arithmetic-average temperature of the fluid; f_i as function of D_iG/μ_i is shown in Fig. 14-1.

n_p = number of tube passes

g_c = conversion factor in Newton's law of motion, (32.17) x $(3600)^2$ ft · lbm/(h)(h)(lbf)

ϕ_i = correction factor for nonisothermal flow is equal to $1.1(\mu_i/\mu_w)^{0.25}$ when D_iG/μ_i is less than 2100 and $1.02(\mu_i/\mu_w)^{0.14}$ when D_iG/μ_i

TABLE 5
Order of magnitude of individual film coefficients

Condition	h, Btu/(h) (ft^2) (°F)
Dropwise condensation of steam	**10,000-20,000**
Film-type condensation of steam	1 ,ooo-3,000
Boiling water	**300-9,000**
Film-type condensation of organic vapors	200-400
Heating or cooling of water	so-3,000
Heating or cooling of organic solvents	30-500
Heating or cooling of oils	**10-120**
Superheated steam	5-30
Heating or cooling of air (Low value for free **convection**– High value for forced convection)	0.2-20

is greater than 2100; μ_i is viscosity at arithmetic-average (bulk) temperature of fluid, and μ_w is viscosity of fluid at average temperature of the inside-tube wall surface

B_i = correction factor to account for friction due to sudden contraction, sudden expansion, and reversal of flow direction

$= 1 + (F_e + F_e + F_r)/(2\mathfrak{f}_i G^2 L/g_c \rho_i^2 D_i \phi_i)$†

Shell-side Pressure Drop

The pressure drop due to friction when a fluid is flowing parallel to and outside of tubes can be calculated in the normal manner described in Chap. 14 by using a mean diameter equal to four times the hydraulic radius of the system and by including all frictional effects due to contraction and expansion. In heat exchangers, however, the fluid flow on the shell side is usually across the tubes, and many types and arrangements of baffles may be used. As a result, no single

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†F_e (friction due to sudden enlargement) and F_c (friction due to sudden contraction) can be estimated by the methods indicated in Chap. 14. $F_e = (V_i - V_{\text{header}})^2/2g_c$. $F_c = K_c V_i^2/2g_c$. F_r (friction due to reversal of flow direction) depends on the details of the exchanger construction, but a good estimate for design work is $(0.5V_i^2/2g_c)$ X $(n_p - 1)/n_p$. Designating S_i/S_H as the ratio of total inside-tube cross-sectional area per pass to header cross-sectional area per pass and K_1 as $[1-(S_i/S_H)]^2 + K_c + 0.5(n_p - 1)/n_p$,

$$B_i = 1 + \frac{K_1 D_i \phi_i}{4\mathfrak{f}_i L}$$

If the flow is highly turbulent and in smooth tubes, $\mathfrak{f}_i$ may be taken as $0.046(D_i G/\mu_i)^{-0.2}$. Combining the preceding relationships with Eqs. (4), (11), (26), and the rate equation for no phase change [Eq. (36)] gives, for smooth tubes and turbulent flow,

$$B_i = 1 + \frac{0.51 K_1 n_p \, \Delta t_{fi} (\mu_i/\mu_w)^{0.28}}{(t_2 - t_1)_i (c_p \mu/k)_i^{2/3}}$$

where $t_2 - t_1$ represents the total change in the temperature of the fluid as it flows through the exchanger tubes.

explicit equation can be given for evaluating pressure drop on the shell side of all heat exchangers.

For the case of flow across tubes, the following equation can be used to approximate the pressure drop due to friction (subscript *o* refers to outside of tubes at bulk temperature):

$$-\Delta P_o = \frac{B_o 2 f' N_r G_s^2}{g_c \rho_o} \tag{31}$$

where f' = special friction factor for shell-side flow

N_r = number of rows of tubes across which shell fluid flows

B_o = correction factor to account for friction due to reversal in direction of flow, recrossing of tubes, and variation in cross section; when the flow is across unbaffled tubes, B_o can be taken as 1.0; as a rough

TABLE 6

Approximate design values of overall heat-transfer coefficients

The following values of overall heat-transfer coefficients are based primarily on results obtained in ordinary engineering practice. The values are approximate because variation in fluid velocities, amount of noncondensable gases, viscosities, cleanliness of heat-transfer surfaces, type of baffles, operating pressure, and similar factors can have a significant effect on the overall heat-transfer coefficients. The values are useful for preliminary design estimates or for rough checks on heat-transfer calculations.

Upper range of overall coefficients given for coolers may also be used for condensers. Upper range of overall coefficients given for heaters may also be used for evaporators.

Units of coefficients are Btu/(h) (ft^2) (°F).

Hot fluid	Cold fluid	Fouling coefficient, h_d	Overall coefficient, U_d
	Coolers		
Water	Water	1000	250-500
Methanol	Water	1000	250-500
Ammonia	Water	1000	250-500
Aqueous solutions	Water	1000	250-500
Light organics—viscosities less than 0.5 cP (benzene, toluene, acetone, ethanol, gasoline, light kerosene, and naphtha)	Water	300	75-150
Medium organics—viscosities between 0.5 and 1.0 cP (kerosene, straw oil, hot gas oil, hot absorber oil, some crude oils)	Water	300	50-125
Heavy organics—viscosities greater than 1.0 cP (cold gas oil, lube oils, fuel oils, reduced crude oils, tars, and asphalts)	Water	300	5-75
Gases	Water	300	2-50
Water	Brine	300	100-200
Light organics	Brine	300	40-100

TABLE 6
Approximate design values of overall heat-transfer coefficients (Continued)

Hot fluid	Cold fluid	Fouling coefficient, h_d	Overall coefficient, U_d
Heaters			
Steam	Water	1000	200-700
Steam	Methanol	1000	200-700
Steam	Ammonia	1000	200-700
Steam	Aqueous solutions:		
	Less than 2.0 cP	1000	200-700
	More than 2.0 cP	1000	100-500
Steam	Light organics	300	100-200
Steam	Medium organics	300	50-100
Steam	Heavy organics	300	6-60
Steam	Gases	300	5-50
Dowtherm	Gases	300	4-40
Dowtherm	Heavy organics	300	6-60
Exchangers (no phase change)			
Water	Water	1000	250-500
Aqueous solutions	Aqueous solutions	1000	250-500
Light organics	Light organics	300	40-75
Medium organics	Medium organics	300	20-60
Heavy organics	Heavy organics	300	10-40
Heavy organics	Light organics	300	30-60
Light organics	Heavy organics	300	10-40

approximation, B_o can be estimated as equal to the number of tube crosses.

The friction factor f' is a function of the Reynolds number of the flowing fluid and the arrangement of the tubes. For the common case of D_oG_s/μ_f in the range of 2000 to 40,000, the friction factor in Eq. (31) may be represented as†

$$f' = b_o\left(\frac{D_oG_s}{\mu_f}\right)^{-0.15} \tag{32}$$

The following approximate equations for b_o are based on the data of Grimson:‡§

---

† For general cases with higher Reynolds numbers, see R. H. Perry and D. Green, "Chemical Engineer's Handbook," 6th ed., pages 5-50 to 5-53, McGraw-Hill Book Company, New York, 1984. For added pressure drop due to flow through baffle-pass area, see page 5-40 of "Chemical Engineers' Handbook," 6th ed.

‡ E. D. Grimson, *Trans. ASME*, **59**:583 (1937); **60**:381 (1938).

§ In Eqs. (33) and (34), best results are obtained if x_T is between 1.5 and 4.0. For design purposes, the range can be extended down to 1.25. When the Reynolds number is between 2000 and 10,600 with tubes in line, f' for a given tube spacing can be assumed to be constant at the value obtained when the Reynolds number is 10,060.

For staggered tubes,

$$b_o = 0.23 + \frac{0.11}{(x_T - 1)^{1.08}} \tag{33}$$

For tubes in line,

$$b_o = 0.044 + \frac{0.08x_L}{(x_T - 1)^{0.43+1.13/x_L}} \tag{34}$$

where x_T = ratio of pitch (i.e., tube center-to-center distance) transverse to flow to tube diameter, dimensionless; and x_L = ratio of pitch parallel to flow to tube diameter, dimensionless.

Example 3 Estimation of film coefficient and pressure drop inside tubes in a shell-and-tube exchanger. A horizontal shell-and-tube heat exchanger with two tube passes and one shell pass is being used to heat 70,000 lb/h of 100 percent ethanol from 60 to 140°F at atmospheric pressure. The ethanol passes through the inside of the tubes, and saturated steam at 230°F condenses on the shell side of the tubes. The tubes are steel with an OD of $\frac{3}{4}$ in. and a BWG of 14. The exchanger contains a total of 100 tubes (50 tubes per pass), and the average number of tubes in a vertical tier can be taken as 6. The ratio of total inside-tube cross-sectional area per pass to header cross-sectional area per pass can be assumed to be 0.5. Estimate the film coefficient for the ethanol and the pressure drop due to friction through the tube side of the exchanger.

Solution. From the Appendix,

Tube wall thickness = 0.083 in.

Tube ID = 0.584 in.

Tube OD = 0.75 in.

Flow area per tube = 0.268 in.^{2}

Inside surface area per lin ft = 0.1529ft^2

k for steel = 26 Btu/(h)(ft^2)(°F/ft)

At (60 + 140)/2 = 100°F,

μ for ethanol = 0.9 centipose

k for ethanol = 0.094 Btu/(h)(ft^2)(°F/ft)

c_p for ethanol = 0.62 Btu/(lb)(°F)

ρ for ethanol = 49 lb/ft^3

$$\text{Mass velocity} = G = \frac{(70{,}000)(144)}{(0.268)(50)} = 752{,}000 \text{ lb/(h)(ft}^2\text{)}$$

$$N_{\mathrm{Re}} = \frac{DG}{\mu} = \frac{(0.584)(752{,}000)}{(12)(0.9)(2.42)} = 16{,}800$$

$$\frac{c_p\mu}{k} = \frac{(0.62)(0.9)(2.42)}{0.094} = 14\ \ 4$$

Equation (26), for the evaluation of the average coefficient, applies under these conditions. The only remaining unknown in the equation is μ_w.

Evaluation of μ_w. As a first approximation, assume the following:

At over ethanol film = 70% of average total At = 0.70(230 − 100) = 91°F

Δt over steam film = 8% of average total At = 0.08(230 − 100) = 10°F

Wall temperature for evaluation of μ_w = 100 + 91 = 191°F.

$\mu_w = \mu$ of ethanol at 191°F = 0.4 centipose

$\mu/\mu_w = 0.9/0.4 = 2.25$

From Eq. (26),

$$h_i = \frac{k}{D}(0.023)\left(\frac{DG}{\mu}\right)^{0.8}\left(\frac{c_p\mu}{k}\right)^{1/3}\left(\frac{\mu}{\mu_w}\right)^{0.14}$$

$$= \frac{(0.094)(12)(0.023)(16{,}800)^{0.8}(14.4)^{1/3}(2.25)^{0.14}}{0.584}$$

$$= 290 \text{ Btu/(h)(ft}^2\text{)(°F)}$$

Check on At assumptions. From Table 3, a fouling coefficient of 1000 Btu/(h)(ft^2)(°F) is adequate for the ethanol. No fouling coefficient will be used for the steam.

From Table 4, the steam film coefficient can be approximated as

$$\frac{3100}{[(6)(0.75/12)]^{1/4}(10)^{1/3}} = 1800 \text{ Btu/(h)(ft}^2\text{)(°F)}$$

By Eq. (12) (basing U_d on the inside tube area),

$$\frac{1}{U_d} = \frac{A}{h'A'_f} + \frac{A}{h''A''_f} + \frac{Ax_w}{kA_w} + \frac{A}{h'_dA'_f}$$

$$= \frac{1}{290} + \frac{0.584}{(1800)(0.75)} + \frac{(0.584)(0.083)}{(26)(0.667)(12)} + \frac{1}{1000}$$

$$= 0.00345 + 0.00043 + 0.00023 + 0.001 = 0.00511$$

$$\text{Percent At over ethanol film} = \frac{0.00345}{0\ 00511}(100) = 68\% \text{ of total At}$$

$$\text{Percent At over steam film} = \frac{0.00043}{0.00511}(100) = 8.4\% \text{ of total At}$$

The At assumptions are adequate.

Determination of pressure drop due to friction. Assume U_d is constant over length of exchanger:

$$U_d = \frac{1}{0.00511} = 196 \text{ Btu/(h)(ft}^2 \text{ of inside area)(°F)}$$

$$\Delta t_{oa,m} = \frac{(230 - 60) + (230 - 140)}{2} = 130°\text{F} = \Delta t_{oa,\text{ arith avg}}$$

The arithmetic-average value is satisfactory because the temperature of the steam remains constant and the ratio $\Delta t_1/\Delta t_2$ is less than 2.0.

$$q = (70{,}000)(0.62)(140 - 60) = 3{,}470{,}000 \text{ Btu/h}$$

By Eq. (19),

$$A = \frac{4}{U_d \text{ At,,, }_m} = \frac{3{,}470{,}000}{(196)(130)} = 136 \text{ ft}^2 \text{ of inside tube area}$$

$$\text{Length per tube} = L = \frac{136}{(0.1529)(100)} = 8.9 \text{ ft}$$

By Eq. (30),

$$-\Delta P_i = \frac{B_i 2 \mathfrak{f}_i G_i^2 L n_p}{g_c \rho_i D_i \phi_i}$$

$$B_i = 1 + \frac{F_e + F_c + F_r}{2\mathfrak{f}_i G_i^2 L / g_c \rho_i^2 D_i \phi_i} = 1 + \frac{0.51 K_1 n_p \, \Delta t_{fi} (\mu_i/\mu_w)^{0.28}}{(t_2 - t_1)_i (c_p \mu/k)_i^{2/3}}$$

$$K_1 = \left(1 - \frac{S_i}{S_H}\right)^2 + K_c + \frac{0.5(n_p - 1)}{n_p}$$

$$= (1 - 0.5)^2 + 0.3 + (0.5)(1)(\tfrac{1}{2}) = 0.8$$

K_c (Chap. 14, Table 1) = $(0.4)(1.25 - 0.5) = 0.3$

$$B_i = 1 + \frac{(0.51)(0.8)(2)(0.68)(230 - 100)(2.25)^{0.28}}{(140 \quad 60)(14.4)^{2/3}} = 1\ 19$$

$\mathfrak{f}_i$ (from Fig. 14-1) = 0.0066

$n_p = 2$

$$\phi_i = 1.02\left(\frac{\mu_i}{\mu_w}\right)^{0.14} = 1.02(2.25)^{0.14} = 1.14$$

$$-\Delta P_i = \frac{(1.19)(2)(0.0066)(752{,}000)^2(8.9)(2)}{(3600)^2(32.17)(49)(0.584/12)(1.14)} = 140 \text{ psf}$$

The pressure drop due to friction through the tube side of the exchanger is approximately 140 psf or 1 psi.

If the cost for power is $0.08 per kilowatthour, the pumping cost due to frictional pressure drop on the tube side of the exchanger is

$$(140)(70{,}000)(0.08)/(49)(2.655 \times 10^6) = \$0.0060 \text{ per hour.}$$

The low power cost and low pressure drop indicate that a higher liquid velocity would be beneficial, since it would give a better heat-transfer coefficient and have little effect on the total cost.

Example 4 Estimation of film coefficient and pressure drop on shell side in a shell-and-tube exchanger. A shell-and-tube heat exchanger with one shell pass and one tube pass is being used as a cooler. The cooling medium is water on the shell side of the exchanger. Five segmental baffles with a 25 percent cut are used on the shell side, and the baffles are spaced equally 2 ft apart. The safety factor F_s for use in evaluating the shell-side film coefficient is 1.6. The inside diameter of the shell is 23 in. The OD of the tubes is 0.75 in., and the tubes are staggered. Clearance between tubes is 0.25 in., and the flow of water across the tubes is normal to this clearance. There is a total of 384 tubes in the exchanger, and the shell can be considered to be full-packed. Water flows through the shell side at a rate of 90,000 lb/h. The average temperatures of the water is 90°F, and the average temperature of the tube walls on the water side is 100°F. Under these conditions, estimate the heat-transfer coefficient for the water and the pressure drop due to friction on the shell side of the exchanger. Neglect pressure drop due to flow through baffle-pass area.

Solution. Average water-film temperature = 95°F. From the Appendix,

μ for water at 95°F = 0.724 centipose

k for water at 95°F = 0.361 Btu/(h)(ft^2)(°F/ft)

c_p for water at 95°F = 1 Btu/(lb)(°F)

ρ for water at 90°F = 62.1 lb/ft^3

μ for water at 100°F = 0.684 centipose

Calculation of Reynolds number = $D_o G_s/\mu_f$

$D_o = 0.75/12$ ft

$\mu_f = (0.724)(2.42)$ lb/(h)(ft)

Free area for evaluation of G_s occurs at center plane of shell where number of tube rows normal to flow direction = 23/(0.75 + 0.25) = 23. Free area is based on the smallest flow area at the center plane. In this case, the transverse openings give the smallest free area.

$$\text{Free area between baffles} = \frac{(23)(0.25)(2)}{12} = 0\ 96\ \text{ft}^2$$

$$G_s = \frac{90{,}000}{0.96} = 94{,}000\ \text{lb/(h)(ft}^2\text{)}$$

$$\frac{D_o G_s}{\mu_f} = \frac{(0.75)(94{,}000)}{(12)(0.724)(2.42)} = 3360$$

$$\left(\frac{c_p \mu}{k}\right)_f = \frac{(1)(0.724)(2.42)}{0.361} = 4\ 85$$

Equation (29) may be used under these conditions.

$$h_o = \frac{k_f}{D_o}\frac{a_o}{F_s}\left(\frac{D_oG_s}{\mu_f}\right)^{0.6}\left(\frac{c_p\mu}{k}\right)_f^{1/3}$$

$$a_o = 0.33 \qquad F_s = 1.6$$

$$h_o = \frac{(0.361)(12)(0.33)(3360)^{0.6}(4.85)^{1/3}}{(0.75)(1.6)} = 260 \text{ Btu/(h)(ft}^2\text{)(°F)}$$

Determination of pressure drop due to friction. Equations (31) to (33) apply for this case:

$$-\Delta P_o = \frac{B_o 2 f' N_r G_s^2}{g_c \rho_o}$$

$$f' = \left[0.23 + \frac{0.11}{(x_T - 1)^{1.08}}\right]\left(\frac{D_oG_s}{\mu_f}\right)^{-0.15}$$

$$f' = \left[0.23 + \frac{0.11}{(1.0/0.75 - 1)^{1.08}}\right](3360)^{-0.15} = 0.175$$

B_o = roughly the number of tube crosses = number of baffles + 1 = 6. N_r = number of tube rows across which shell fluid flows = 23 minus the tube rows that pass through the cut portions of the baffles. With 25 percent cut baffles, the fluid will flow across approximately one-half of the tubes. In this case, N_r will be taken as 14. This gives some allowance for neglecting friction due to reversal of flow direction and friction due to flow parallel to the tubes:

$$-\Delta P_o = \frac{(6)(2)(0.175)(14)(94{,}000)^2}{(32.17)(3600)^2(62.1)} = 10 \text{ psf}$$

Pressure drop due to friction on the shell side of the exchanger is approximately 10 psf.

CONSIDERATIONS IN SELECTION OF HEAT-TRANSFER EQUIPMENT

When the design engineer selects heat-transfer equipment, it is necessary to consider the basic process-design variables and also many other factors, such as temperature strains, thickness of tubes and shell, types of baffles, tube pitch, and standard tube lengths. Under ordinary conditions, the mechanical design of an exchanger should meet the requirements of the ASME or API-ASME Safety Codes. The Tubular Exchanger Manufacturers Association (TEMA) publishes standards on general design methods and fabrication materials for tubular heat exchangers.

Tube Size and Pitch

The standard length of tubes in a shell-and-tube heat exchanger is 8, 12, or 16 ft, and these standard-length tubes are available in a variety of different diameters and wall thickness. Exchangers with small-diameter tubes are less expensive per square foot of heat-transfer surface than those with large-diameter tubes, because a given surface can be fitted into a smaller shell diameter; however, the small-diameter tubes are more difficult to clean. A tube diameter of $\frac{3}{4}$ or 1 in. OD is the most common size, but outside diameters ranging from $\frac{5}{8}$ to $1\frac{1}{2}$ in. are found in many industrial installations.

Tube-wall thickness is usually specified by the Birmingham wire gauge, and variations from the nominal thickness may be $\pm$ 10 percent for "average-wall" tubes and + 22 percent for "minimum-wall" tubes. Pressure, temperature, corrosion, and allowances for expanding the individual tubes into the tube sheets must be taken into consideration when the thickness is determined.

Tube pitch is defined as the shortest center-to-center distance between adjacent tubes, while the shortest distance between two tubes is designated as the *clearance.* In most shell-and-tube exchangers, the pitch is in the range of 1.25 to 1.50 times the tube diameter. The clearance should not be less than one-fourth of the tube diameter, and $\frac{3}{16}$ in. is usually considered to be a minimum clearance.

Tubes are commonly laid out on a square pattern or on a triangular pattern, as shown in Fig. 15-5. Although a square pitch has the advantage of easier external cleaning, the triangular pitch is sometimes preferred because it permits the use of more tubes in a given shell diameter. Table 7 indicates the

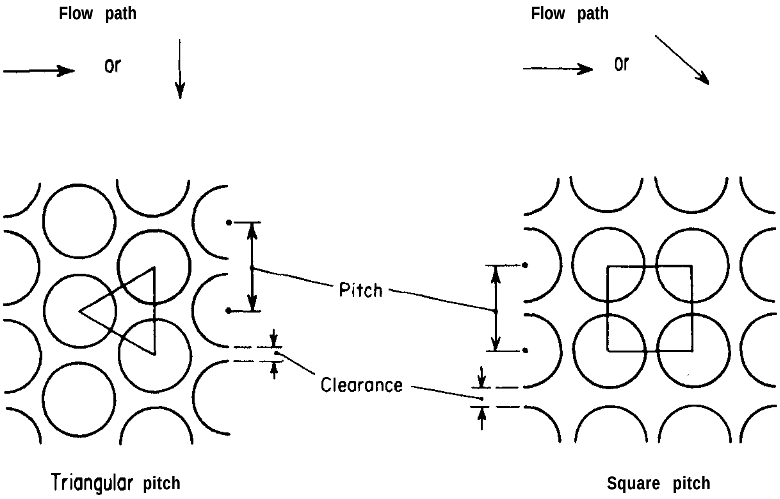

FIGURE 15-5
Conventional tube-plate layouts.

TABLE 7
Number of tubes in conventional tubesheet layouts

Shell ID, in.	One-pass		Two-pass		Four-pass	
	Square pitch	Triangular pitch	Square pitch	Triangular pitch	Square pitch	Triangular pitch
¾-in.-OD tubes on 1-in. pitch						
8	32	37	26	30	20	24
12	81	92	76	82	68	76
15¼	137	151	124	138	116	122
21¼	277	316	270	302	246	278
25	413	470	394	452	370	422
31	657	745	640	728	600	678
37	934	1074	914	1044	886	1012
1-in.-OD tubes on 1¼-in. pitch						
8	21	21	16	16	14	16
12	48	55	45	52	40	48
15¼	81	91	76	86	68	80
21¼	177	199	166	188	158	170
25	260	294	252	282	238	256
31	406	472	398	454	380	430
37	596	674	574	664	562	632

number of tubes that can be placed in an exchanger with conventional tube sizes and pitches.

Shell Size

For shell diameters up to 24 in., nominal pipe sizes apply to the shell. Inside diameters are usually indicated, and schedule number or wall thickness should also be designated. In general, a shell thickness of $\frac{3}{8}$ in. is used for shell diameters between 12 and 24 in. unless the fluids are extremely corrosive or the operating pressure on the shell side exceeds 300 psig.

Thermal Strains

Thermal expansion can occur when materials, such as the metal components of a heat exchanger, are heated. For example, in a shell-and-tube heat exchanger, thermal expansion can cause an elongation of both the tube bundle and the shell as the temperature of the unit is increased. Because the tube bundle and the shell may expand by different amounts, some arrangement may be necessary

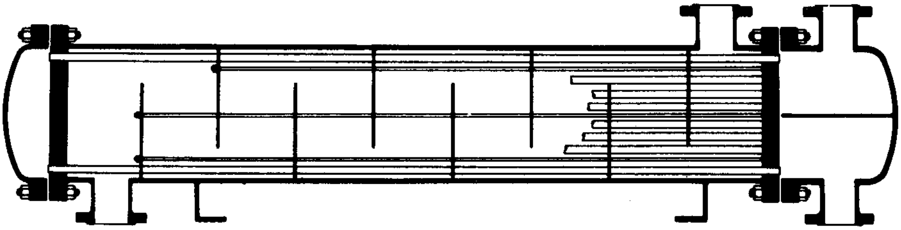

FIGURE 15-6
Heat exchanger with fixed tube sheets, two tube passes, and one shell pass. (*Struthers-Wells Corporation.*)

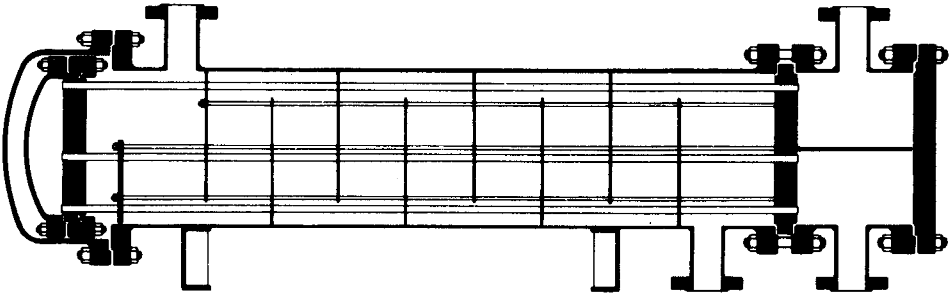

FIGURE 15-7
Heat exchanger with internal floating head, two tube passes, and one shell pass. (*Struthers-Wells Corporation*.)

to reduce thermal strains. Figure 15-6 shows a fixed-tube-sheet exchanger with no allowance for expansion, while Figs. 15-7 and 15-8 illustrate two conventional types of exchangers with a floating head or a slip joint to relieve stresses caused by thermal expansion. Temperature stresses due to tube elongation can also be avoided by using U-shaped tubes, and some exchangers have a U-type bellows loop or ring in the shell to handle thermal elongation of the shell.

Use of the fixed-head type of exchanger should be limited to exchangers with short tubes or to cases in which the maximum temperature difference between shell and tubes is less than 50°F. In general, floating-head exchangers with removable bundles are recommended for most services.

Cleaning and Maintenance

Heat exchangers require periodic cleaning, tube replacements, or other maintenance work. The inside of straight tubes can be cleaned easily by forcing a wire brush or worm through the tubes, but cleaning of the outside of the tubes usually requires removal of the entire tube bundle from the exchanger. Consequently, many exchangers are provided with removable tube bundles, and the

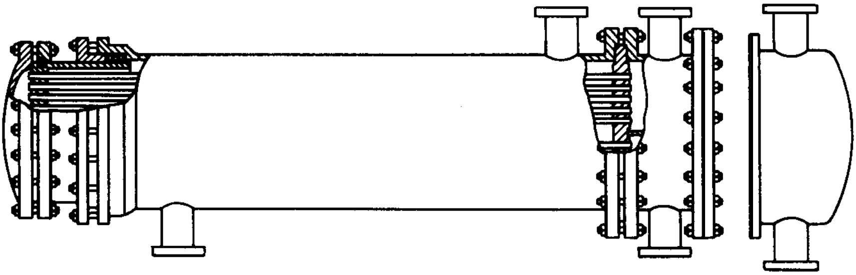

FIGURE 15-8
Heat exchanger with external floating head, two tube passes, and one shell pass. (*Struthers-Wells Corporation*.)

pitch and arrangement of the tubes are often dictated by the amount and type of cleaning that are required.

Baffles

Although the presence of baffles in the shell side of a shell-and-tube exchanger increases the pressure drop on the shell side, the advantage of better mixing of the fluid and increased turbulence more than offsets the pressure-drop disadvantage. The distance between baffles is known as the *baffle spacing.* In general, baffle spacing is not greater than a distance equal to the diameter of the shell or less than one-fifth of the shell diameter.

The most common type of baffle used in heat exchangers is the ***segmental baffle***, illustrated in Fig. 15-9. Many segmental baffles have a baffle height that is 75 percent of the inside diameter of the shell. This arrangement is designated as *25 percent cut segmental baffles.* Other types of baffles include the *disk-and-doughnut baffle* and the orifice *baffle,* shown in Figs. 15-10 and 15-11. Longitudinal baffles of the permanent type (i.e., welded to the shell) or removable type are also found in some exchangers.

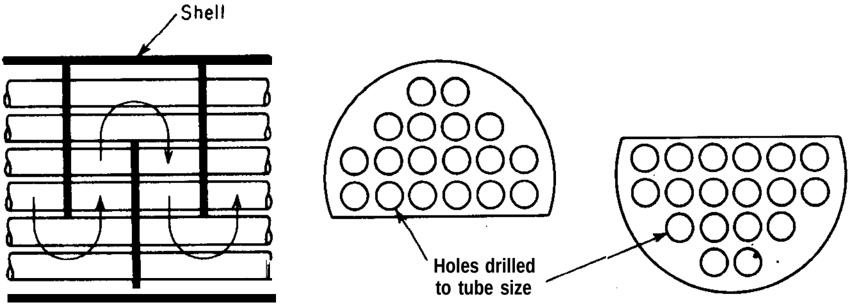

FIGURE 15-9
Segmental baffles.

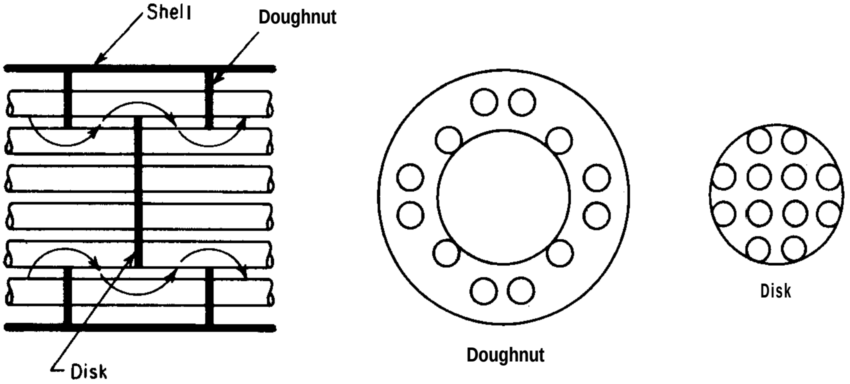

FIGURE **15-10**
Disk-and-doughnut baffles.

Segmental and disk-and-doughnut baffles contain tube-pass holes of size close to that of the diameter of the tubes. With these two types of baffles, the clearance between the tubes and the edge of the holes may range from 1 percent to as high as 3 percent of the tube diameter. As a result, some fluid passes through the clearance spaces, but the major portion of the fluid should flow between the baffles in a direction perpendicular to the tubes.

Orifice baffles should be spaced reasonably close together to produce frequent increases in fluid velocity through the orifice openings between the tubes and the baffles. This type of baffle should not be used for fluids with high fouling characteristics.

Straight tie rods are used to hold baffles in place. Usually, four to six rods of $\frac{1}{8}$- to $-in. diameter are necessary. These are fitted to the fixed tube sheet, and short lengths of pipe sleeves are placed around the tie rods to form shoulders between adjacent baffles. The thickness of baffle plates should be at

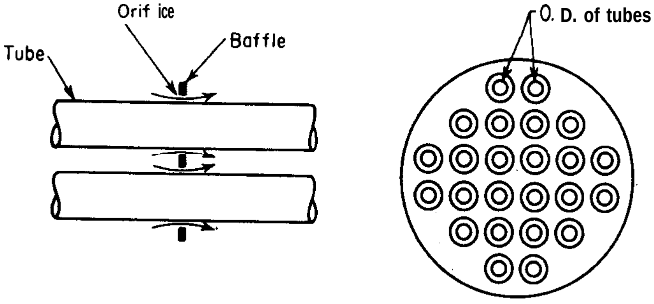

FIGURE 15-11
Orifice baffles.

least twice the thickness of the tube walls and is ordinarily in the range of $\frac{1}{8}$ to $\frac{1}{4}$ in. The tube sheets should have a thickness at least as great as the outside diameter of the tubes. In industrial exchangers, $\frac{7}{8}$ in. is ordinarily considered as a minimum thickness for tube sheets.

Fluid Velocities and Location of Fluids

The major factors involved in determining the best location for fluids in a heat exchanger are the fouling and corrosion characteristics of the fluids, pressure drop across the unit, materials costs, and general physical characteristics of the fluids and the exchanger. When one of the fluids is highly corrosive, it should flow inside the tubes to avoid the expense of corrosion-resistant materials of construction on the shell side. Because cleaning inside tubes is easier than external cleaning, consideration should always be given to locating the fluid with the greatest fouling tendencies inside the tubes. If the other factors are equal and one fluid is under high pressure, the expense of a high-pressure shell construction can be avoided by passing the high-pressure fluid through the tubes.

The velocities of the fluids passing through the shell side and the tube side of an exchanger can have a large influence on the heat-transfer coefficients and the pressure drop. At high velocities, the beneficial effects of large film coefficients can be counterbalanced by the detrimental effects of high pressure drop. If one of the fluids is much more viscous than the other, pressure drop on the tube side may be excessive when the viscous fluid is passed through the tubes at the velocity necessary for adequate rates of heat transfer. The effects of fluid velocities and viscosities, therefore, must be considered carefully before a final decision is made concerning the best routing of the fluids.

Ineffective Surface

In the course of heating a fluid, noncondensable gases, such as absorbed air, may be evolved. If these gases are not removed, they can collect in the exchanger and form an effective blanket around some of the heat-transfer surface. Adequate provision, therefore, should be made for venting noncondensables. The heat-transfer surface can also become ineffective because of build-up of condensate when condensing vapors are involved. Consequently, drains, steam traps with bypasses, and sight glasses to indicate condensate level are often necessary auxiliaries on heat exchangers. When high pressures are used, relief valves or rupture disks may be essential for protection. Vapor blanketing can occur in boilers if the critical temperature-difference driving force is exceeded, and the design engineer must take this factor into consideration. Inadequate baffling on the shell side of an exchanger can result in poor distribution of the shell-side fluid, with a resulting ineffective use of the available surface area.

Use of Water in Heat Exchangers

Because of the abundance of water and its high heat capacity, this material is used extensively as a heat-exchange medium. At high temperatures, water exerts considerable corrosive action on steel, particularly if the water contains dissolved oxygen. Nonferrous metals, therefore, are often employed in heat exchangers when water is one of the fluids. To reduce costs, the water may be passed through the more expensive tubes, and the shell side of the exchanger can be constructed of steel. When water is used in contact with steel, a large corrosion allowance should be included in the mechanical design.

Liquid mixtures that contain solids tend to foul heat exchangers very rapidly, because the solids can settle out and bake into a cake on the hot walls. This difficulty can be reduced by passing the fluid through the exchanger at a velocity which is sufficient to keep the solids in suspension. As a standard practice, a fluid velocity of at least 3 fps should be maintained in an exchanger when the fluid is water. This minimum velocity is particularly important if the water is known to contain any suspended solids. Since it is very difficult to eliminate low-velocity pockets on the shell side of an exchanger, water that contains suspended solids should be passed through the tube side of the exchanger.

As far as possible, suspended solids should be removed from water before it enters a heat exchanger. This can be accomplished by use of settling tanks or filters. Screen filters are commonly used on water lines to remove debris, such as sticks, pebbles, or pieces of algae, but a screen filter will not remove finely dispersed solids.

Another difficulty encountered when water is employed as a heat-transfer medium is the formation of mineral scale. At temperatures higher than 120°F, the formation of scale from water of average mineral and air content tends to become excessive. Consequently, an outlet water temperature above 120°F should be avoided. In many cases, it is advisable to soften the water by a chemical treatment before using it in a heat exchanger. Hot water leaving a heat exchanger may be reused by employing a cooling tower to lower the temperature of the water. This reduces both the softening costs and the amount of water that must be purchased.

Use of Steam in Heat Exchangers

Steam has a high latent heat of condensation per unit weight and is, therefore, very effective as a heating medium. Exhaust steam is often available as a by-product from power plants. Because exhaust steam usually has a saturation temperature in the range of 215 to 230°F, the material is useful for heating only in the delivery-temperature range below about 200°F. If a material must be heated to higher temperatures, process steam is necessary. At a pressure of 200 psig, the condensing temperature of steam is 382°F. Consequently, when delivery temperatures much higher than 300°F are required, the high steam pressure necessary may eliminate steam as a useful heating medium. Under these conditions, another heating medium, such as Dowtherm, which does not require high pressures, might be chosen.

The film coefficients of heat transfer for pure condensing steam are ordinarily in the range of 1000 to 3000 $\text{Btu}/(\text{h})(\text{ft}^2)(^\circ\text{F})$. The condensing-steam film, therefore, is seldom the controlling resistance in a heat exchanger, and approximate values of steam-film coefficients are usually adequate for design purposes.[†] If the steam is superheated, it is standard design practice to neglect the superheat and assume all the heat is delivered at the condensing temperature corresponding to the steam pressure. Similarly, subcooling of the condensate and pressure drop due to friction on the steam side of the exchanger are usually neglected. Under these conditions of assumed isothermal condensation, the true temperature-difference driving force is the same as the logarithmic-mean value.

Specifications for Heat Exchangers

The design engineer should consider both process design and mechanical design when preparing the specifications for a heat exchanger. The fabricator ordinarily has a plant that is tooled to produce standard parts at the least cost and in the shortest time. Consequently, attempts to specify every detail on an exchanger should not be made unless one is thoroughly familiar with the fabricator's equipment and methods. On the other hand, the specifications must be sufficiently complete to permit an adequate evaluation by the fabricator.

The following list presents the basic information that should be supplied to a fabricator in order to obtain a price estimate or firm quotation on a proposed heat exchanger:

Process information

1. Fluids to be used
    - *a.* Include fluid properties if they are not readily available to the fabricator
2. Flow rates or amounts of fluids
3. Entrance and exit temperatures
4. Amount of vaporization or condensation
5. Operating pressures and allowable pressure drops
6. Fouling factors
7. Rate of heat transfer

Mechanical information

1. Size of tubes
    - *a.* Diameter
    - *b.* Length
    - c. Wall thickness
2. Tube layout and pitch
    - *a.* Horizontal tubes
    - *b.* Vertical tubes
3. Maximum and minimum temperatures and pressures
4. Necessary corrosion allowances
5. Special codes involved
6. Recommended materials of construction

[†] For design estimates under ordinary conditions, a conservative value of 1500 $\text{Btu}/(\text{h})(\text{ft}^2)(^\circ\text{F})$ is often assumed for the condensing-steam film coefficient.

Some of the preceding information can be presented in the form of suggestions, with an indication of the reasons for the particular choice. This would apply, in particular, to such items as fouling factors, tube layout, codes, and materials of construction. An example of a typical specification table for heat exchangers is presented in Fig. 2-6.

HEAT-TRANSFER EQUIPMENT COSTS

The major factors that can influence the cost for heat-transfer equipment are indicated in the following list:

1. Heat-transfer area
2. Tube diameter and gauge
3. Tube length
4. Pressure
5. Materials of construction for tubes and shell
6. Degree and type of baffling
7. Supports, auxiliaries, and installation
8. Special features, such as floating heads; removable bundles; multipass, finned surfaces; and U bends

Certain manufacturers specialize in particular types of exchangers and can, therefore, give lower quotations on their specialities than other manufacturers. As a result, price quotations should be obtained from several manufacturers before a firm cost is listed.

Figure 15-12 presents average cost data for heat exchanger tubing constructed of steel. The average purchased cost of complete heat exchangers with materials of construction is given in Figs. 15-13 through 15-18, while the cost for

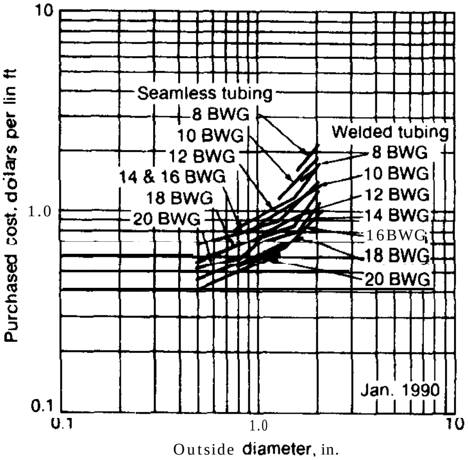

FIGURE 15-12
Welded and seamless heat-exchanger tubing. Basis of the cost data includes: quantity, 40,000 lb or ft; lengths, cut lengths within range of 10–24 ft; specifications, seamless, ASTM-A-129 and welded ASTM-A-214; wall thickness, minimum wall. Material of construction is low-carbon (less than 0.18 percent) steel.

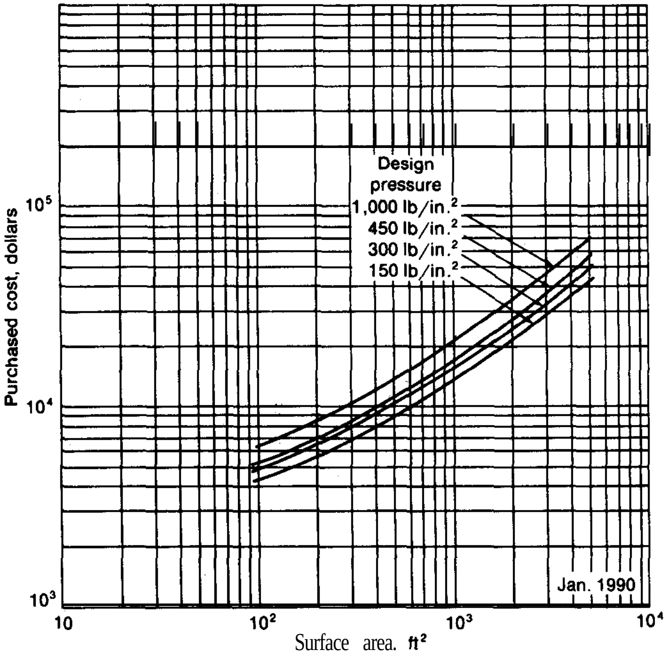

FIGURE **15-13**
Floating-head heat exchangers with $\frac{3}{4}$-in. OD x 1-in. square pitch and 16-ft bundles of carbon-steel construction.

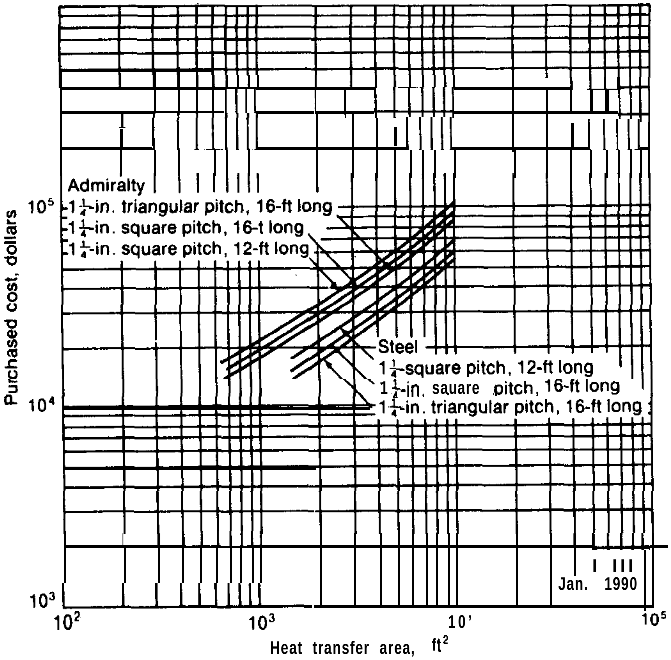

FIGURE 15-14
Finned-tube floating-head exchangers at 150 psi. Cost is for 1-in. OD Turfin tubes.

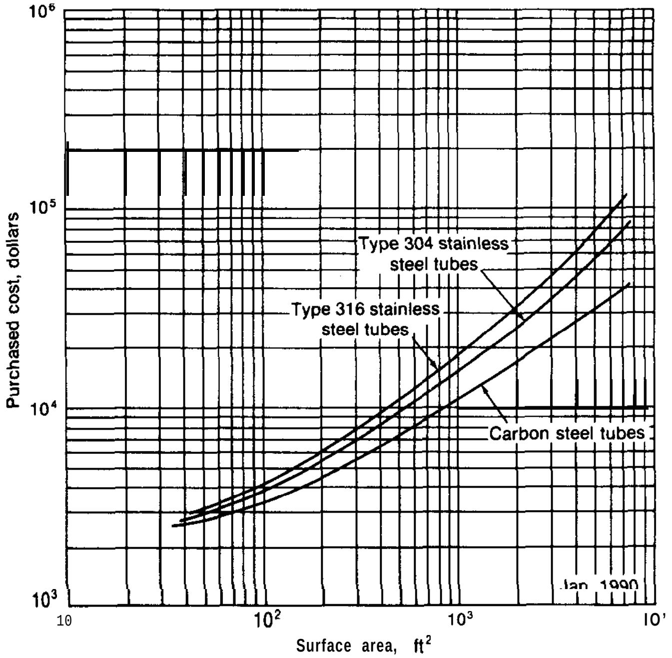

FIGURE 15-15
Fixed-tube-sheet exchangers with $\frac{3}{4}$-in. OD x l-in. square pitch and 16- or 20-ft bundles and carbon-steel shell operating at 150 psi.

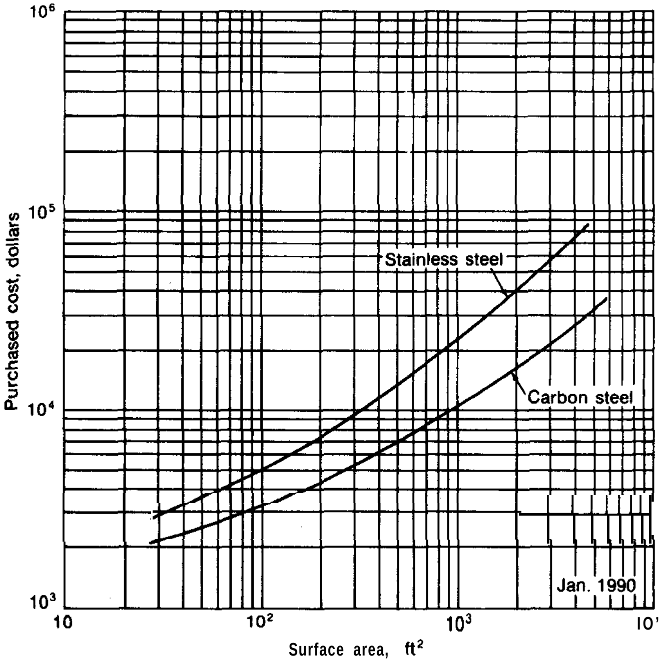

FIGURE 15-16
U-tube heat exchanger with l-in. tubes x l-in. square pitch and 16-ft bundles operating at 150 psi.

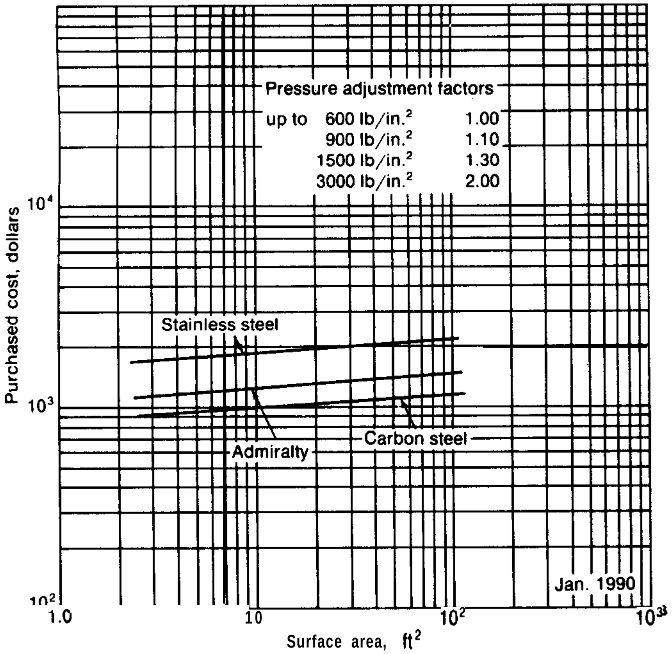

FIGURE 15-17
Cost of double-pipe heat exchangers.

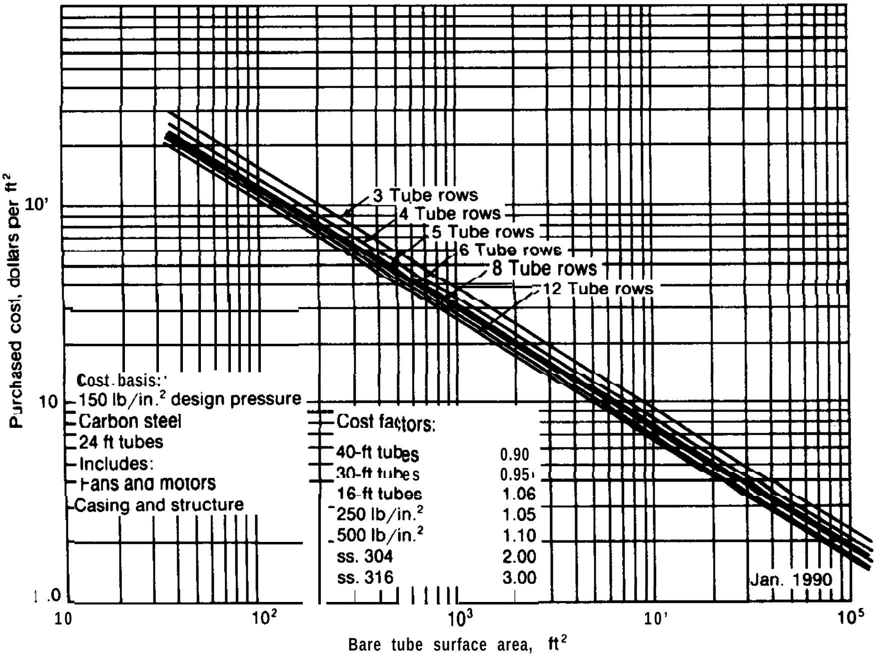

FIGURE 15-18
Cost of air-cooled heat exchangers.

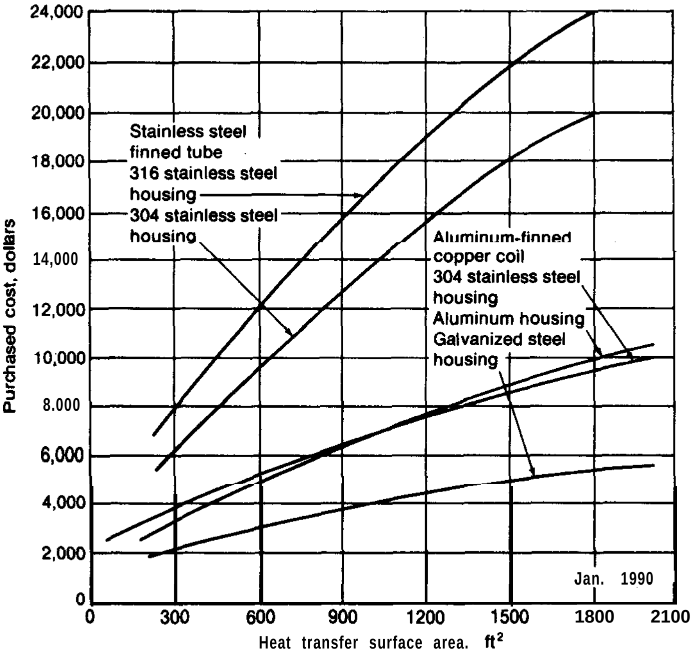

FIGURE **15-19**
Cost of tank vent condensers.

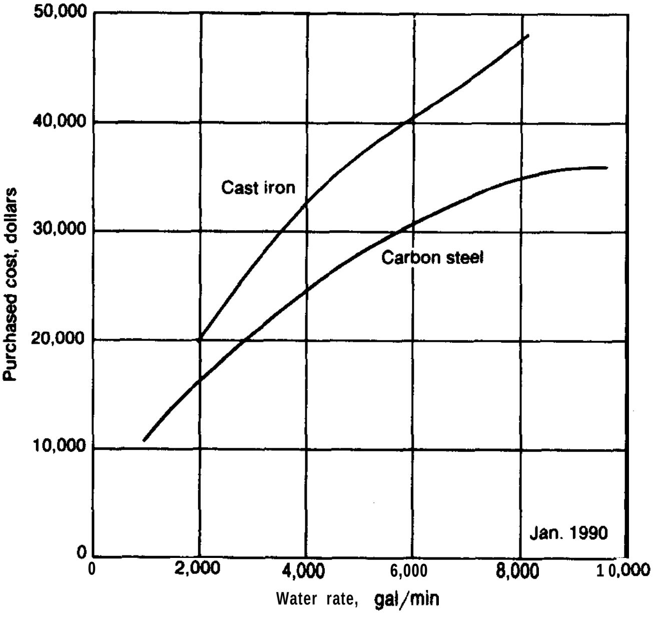

FIGURE 15-20
Cost of multijet spray-type condensers. Basic construction consists of a shell, water nozzle, case and plate, and spray-type nozzles.

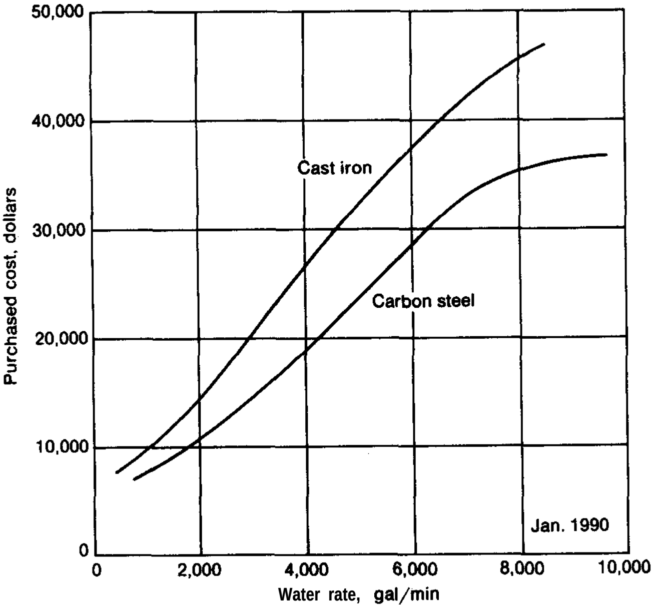

FIGURE 15-21
Cost of **multijet** barometric condensers.

coolers and barometric condensers is given in Figs. 15-19 through 15-21. Because of the many possible variations in heat exchangers, these data must be regarded as approximate and should be used only for preliminary rough estimates.

Relative Costs

The relative effect of tube diameter, tube length, and operating pressure on the purchased cost of shell-and-tube heat exchangers is shown in Figs. 15-22 through 15-24. If two floating heads are used in a single shell in place of one floating head, the exchanger cost will be increased by approximately 30 percent,

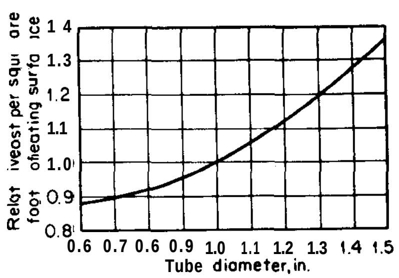

FIGURE 15-22
Effect of tube diameter on cost of conventional **shell-and-tube** heat exchangers.

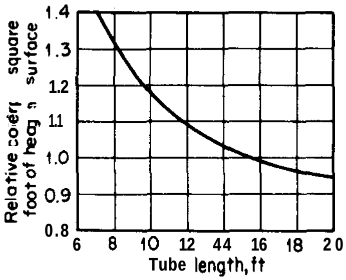

FIGURE 15-23
Effect of tube length on cost of conventional shell-and-tube heat exchangers.

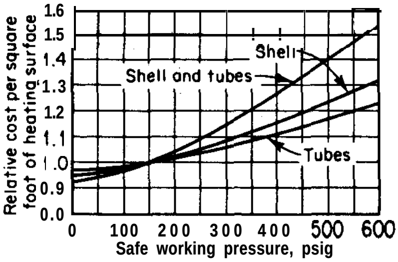

FIGURE 15-24
Effect of operating pressure on cost of conventional shell-and-tube heat exchangers.

and a decrease in cost of about 10 or 15 percent occurs if fixed tube sheets or U tubes are used instead of one floating head. Table 8 lists relative costs for tube-and-shell heat exchangers on the basis of the material of construction used for the tubes and the exchanger, and Fig 15-25 shows the costs of heat exchangers with two types of stainless-steel tubes relative to all-carbon-steel construction.

Costs for forced circulation evaporators, long tube evaporators, and agitated falling-film evaporators are presented in Figs. 15-26 and 15-27. Information on purchased cost of specialized heaters is given in Figs. 15-28 and 15-29, while Figs. 15-30 and 15-31 present costs for process furnaces and direct-fired heaters.

OPTIMUM DESIGN OF HEAT EXCHANGERS

Two types of quantitative problems are commonly encountered by the design engineer when dealing with heat-transfer calculations. The first type is illustrated by Examples 3 and 4 in this chapter. In these examples, all of the design variables are set, and the calculations involve only a determination of the indicated nonvariant quantities. The conditions specified for Examples 3 and 4 give low pressure drops through the exchangers. Alternative conditions could be specified which would give higher pressure drops and less heat-transfer area. By choosing various conditions, the engineer could ultimately arrive at a final design that would give the least total cost for fixed charges and operation. Thus,

TABLE 8

Relative costs of heat-exchanger tubing and heat exchangers with 1500 ft^2 of surface†

Material	Tubing	Heat exchanger
Zirconium-seamless	25.1	7.1
Hastelloy-C-276-welded	18.2	5.9
Zirconium-20 BWG-seamless	15.8	5.2
Inconel-625 welded	15.1	5.0
Carpenter-20 CB3-welded	8.6	3.3
Incoloy-825-welded	7.6	3.0
Monel-400-seamless	1.5	3.0
Titanium-welded	6.8	2.8
E-Brite-26-1-welded	5.2	2.4
Titanium-20 BWG-welded	3.6	1.9
316L stainless steel -welded	3.2	1.8
Cu/Ni-70/30-seamless	2.9	1.8
Cu/Ni-90/30-welded	2.4	1.6
304L stainless steel-welded	2.2	1.6
Carbon steel	1.0	1.0

Basis: Tubing is 1 in. OD X 16 BWG, except as noted. Heat exchangers are TEMA-type BEM with $\frac{3}{4}$ in. OD x 16 BWG X 20 ft welded tubing and mild-carbon steel shells.

† Estimates will be conservative for smaller exchangers and too low for larger exchangers.

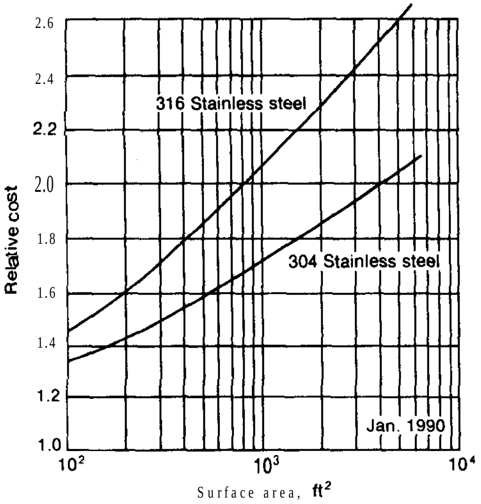

FIGURE 15-25
Cost of heat exchangers with stainless-steel tubes relative to all-carbon-steel construction.

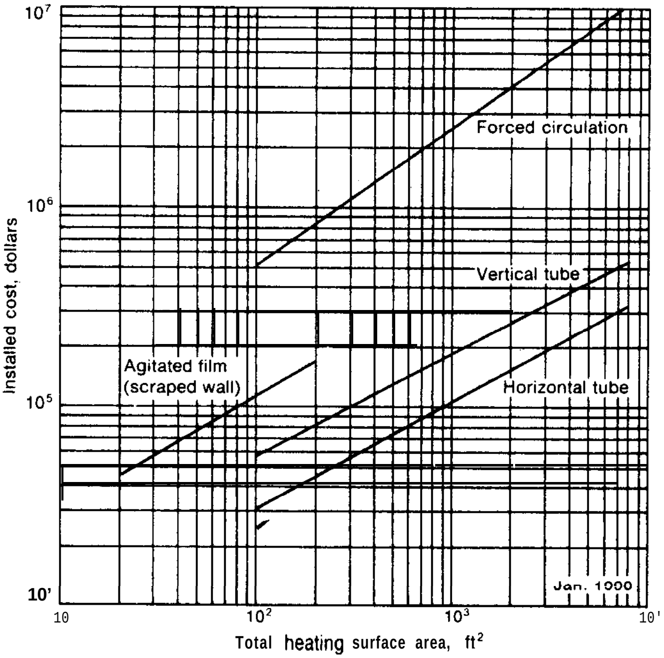

FIGURE 15-26
Installed cost of evaporators.

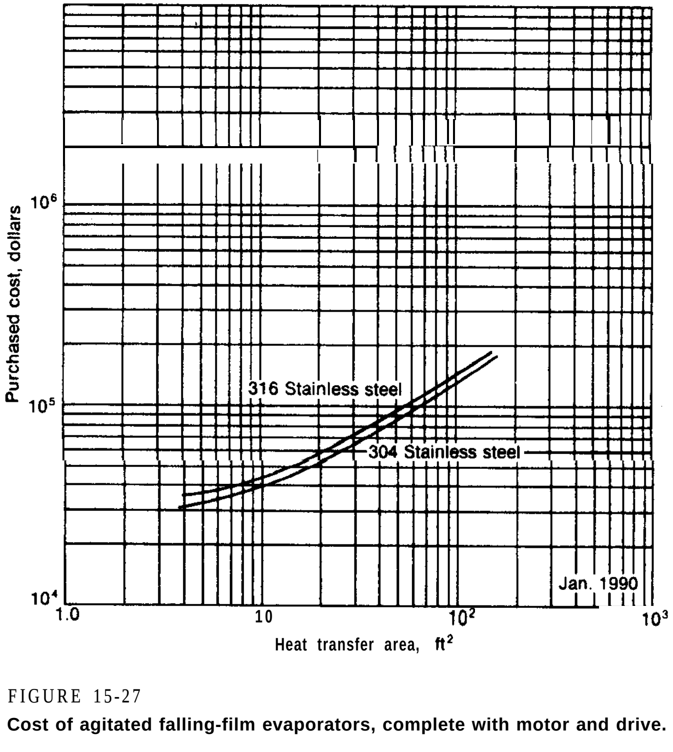

FIGURE 15-27
Cost of agitated falling-film evaporators, complete with motor and drive.

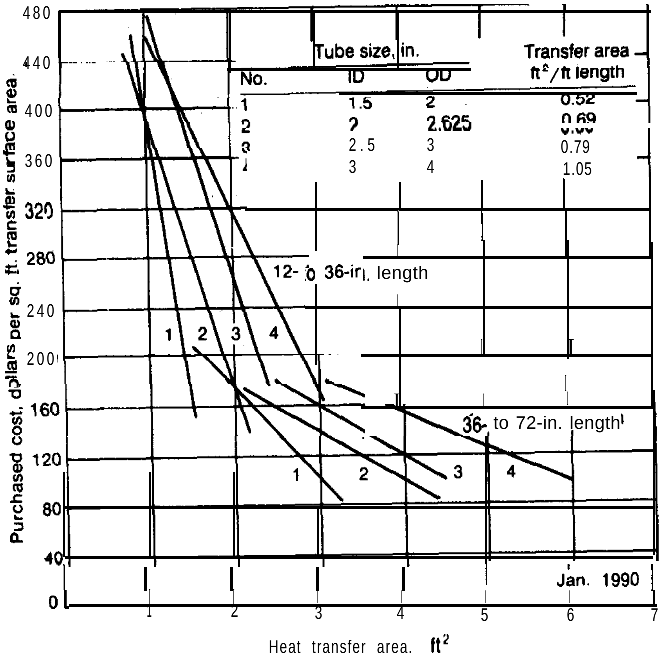

FIGURE 15-28
Cost of bayonet heaters.

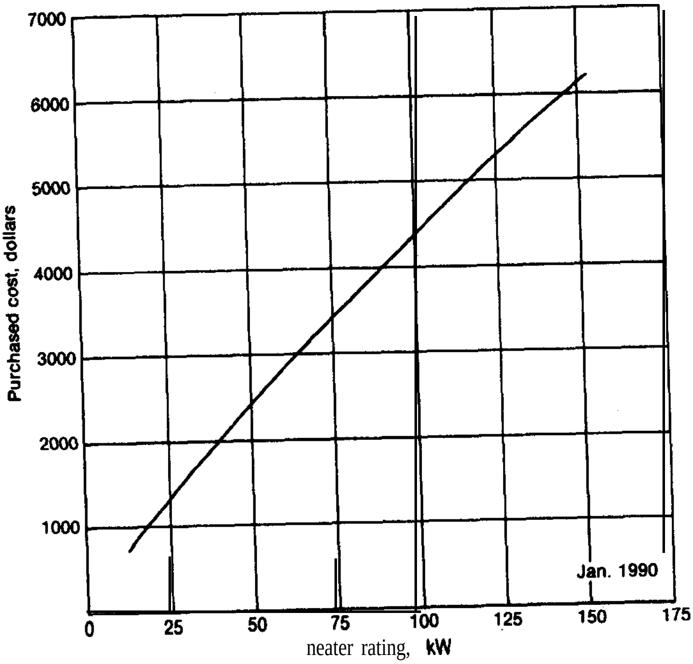

FIGURE 15-29
Cost of electric immersion heaters.

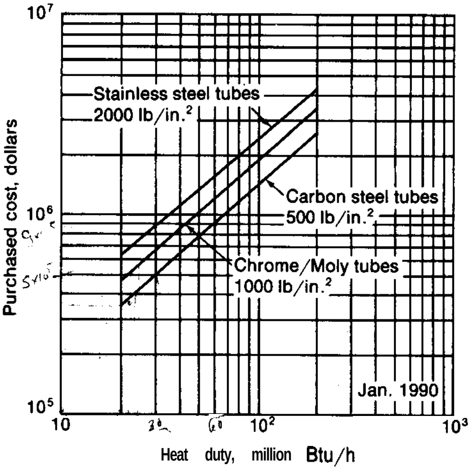

FIGURE **15-30**
Cost of process furnaces, box type with horizontal radiant tubes.

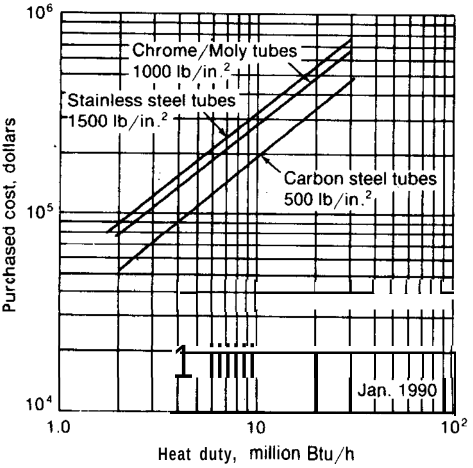

FIGURE **15-31**
Cost of direct-fired heaters, cylindrical type with vertical tubes.

the second type of quantitative problem involves conditions in which at least one variable is not fixed, and the goal is to obtain an optimum economic design.

In general, increased fluid velocities result in larger heat-transfer coefficients and, consequently, less heat-transfer area and exchanger cost for a given rate of heat transfer. On the other hand, the increased fluid velocities cause an increase in pressure drop and greater pumping costs. The optimum economic design occurs at the conditions where the total cost is a minimum. The basic problem, therefore, is to minimize the sum of the variable annual costs for the exchanger and its operation.

GENERAL CASE†

The design of most heat exchangers involves initial conditions in which the following variables are known:

1. Process-fluid rate of flow
2. Change in temperature of process fluid
3. Inlet temperature of utility fluid (for cooling or heating)

With this information, the engineer must prepare a design for the optimum exchanger that will meet the required process conditions. Ordinarily, the-following results must be determined:

1. Heat transfer area
2. Exit temperature and flow rate of utility fluid
3. Number, length, diameter, and arrangement of tubes
4. Tube-side and shell-side pressure drops

The variable annual costs of importance are the fixed charges on the equipment, the cost for the utility fluid, and the power cost for pumping the fluids through the exchanger. The total annual cost for optimization, therefore, can be represented by the following equation:

$$C_T = A_o K_F C_{A_o} + w_u H_y C_u + A_o E_i H_y C_i + A_o E_o H_y C_o \tag{35}$$

where C_T = total annual variable cost for heat exchanger and its operation, \$/year

C_{A_o} = installed cost of heat exchanger per unit of outside-tube heat-transfer area, \$/ft^2

C_u = cost of utility fluid, \$/lb

C_i = cost for supplying 1 ft . Ibf to pump fluid flowing through inside of tubes, \$/ft · lbf

C_o = cost for supplying 1 ft · Ibf to pump fluid flowing through shell side of unit, \$/ft · lbf

A, = area of heat transfer, ft^2; subscript o designates outside of tubes

K_F = annual fixed charges including maintenance, expressed as a fraction of initial cost for completely installed unit, dimensionless

w_u = flow rate of utility fluid, lb/h

H_y = hours of operation per year, h/year

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†The analysis presented in this section is similar to that of M. Cichelli and M. Brinn, *Chem. Eng.*, **65**(5):196 (1956). See also Chap. 11 and H. T. Bates, S. S. Patel, and K. G. Shaw, Design Optimization of 1-2 Pass and 2-4 Pass Shell and Tube Heat Exchangers, *Paper 47a*, 61st AIChE Nat. Mtg., Houston, Tex., Feb. 22, 1967

E_i = power loss inside tubes per unit of outside tube area, ft · lbf/(h)(ft^2)

E_o = power loss outside tubes per unit of outside tube area, ft · lbf/(h)(ft^2)

An optimum design could be developed from Eq. (35) by the laborious procedure of direct trial and error, taking all possible variables into consideration; however, the procedure can be simplified considerably by using the method of partial derivatives.

Under ordinary circumstances, the effect of tube diameter on total cost at the optimum operating conditions is not great, and a reasonable choice of tube diameter, wall thickness, and tube spacing can be specified at the start of the design. Similarly, the number of tube passes is usually specified. If a change in phase of one of the fluids occurs (for example, if the utility fluid is condensing steam), solution of Eq. (35) for optimum conditions can often be simplified. For the case of no change in phase, the solution can become complex, because the velocities and resulting power costs and heat-transfer coefficients can be varied independently over a wide range of values. In the following analysis, the general case of steady-state heat transfer in shell-and-tube exchangers with no change in fluid phase is considered, and a specified tube diameter, wall thickness, number of passes, and arrangement of baffles and tubes are assumed. Simplifications are indicated for the common conditions of turbulent flow.

Choice of Primary Independent Variables

The heat-transfer area A, can be related to the flow rates and the temperature changes by an overall heat balance and the rate equation. If heat losses are assumed as negligible and q is designated as the rate of heat transfer to the utility fluid,

$$4 = w_u c_{p_u}(t_2 - t_1) = w'c'_p(t'_1 - t'_2) = U_o A_o \text{ At,} \qquad (36)\dagger$$

where the primes refer to the process fluid. Subscript 1 refers to the entering temperature, and subscript 2 refers to the leaving temperature.

From Eq. (36),

$$w_u = \frac{4}{c_{p_u}(\Delta t_1 - \Delta t_2 + t'_1 - t'_2)} \qquad (37)$$

where At, = $t'_2 - t_1$ and At, = $t'_1 - t_2$. Since q, c_{p_u}, At,, t'_1, and t'_2 are constant, w_u is a function only of the independent variable Δt_2.‡

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†Under these conditions, q is negative if the exchanger is used for heating the process fluid, and q is positive if the process fluid is cooled.

‡The heat capacity, viscosity, and density of the fluids are taken as constant at an assumed average temperature. This assumption is adequate for most cases. If greater precision is necessary, the results of the initial calculation can be used to obtain new values for the physical constants, and the corresponding optimum conditions can be determined.

The area A, is known if U_o and At, are fixed. The overall coefficient U_o is known if the inside and outside film coefficients, h_i and h_o, are fixed, and, for a given number of tube passes, At,,, varies only with changes in *At,*. Therefore, A, is a function of h_i, ***h,,*** and At,, as shown by the following equations:

$$\frac{\Delta t_m}{q} = \frac{1}{U_o A_o} = \frac{1}{A_o}\left(\frac{D_o}{D_i h_i} + \frac{1}{h_o} + R_{dw}\right) \tag{38}$$

$$\frac{F_T(\Delta t_2 - \Delta t_1)}{q \ln(\Delta t_2/\Delta t_1)} = \frac{1}{U_o A_o} = \frac{1}{A_o}\left(\frac{D_o}{D_i h_i} + \frac{1}{h_o} + R_{dw}\right) \tag{39}$$

where U_o = overall coefficient of heat transfer based on outside tube area ***A,,***
F_T = correction factor on logarithmic-mean temperature difference for counterflow to account for number of passes; F_T = 1 if unit is counterflow and single-pass on shell and tube sides (see Fig. 15-2)
R_{dw} = combined resistance of tube wall and scaling or dirt factors,

$$\frac{D_o x_w}{k_w D_{w_{\text{log mean}}}} + \frac{D_o}{D_i h_{d_i}} + \frac{1}{h_{d_o}}$$

For a set diameter and tube arrangement, Eqs. (26) and (29) show that h_i is fixed by the mass velocity G inside the tubes and h_o is fixed by the mass velocity G_s outside the tubes. Similarly, since the heat-transfer area ***A,,*** mass velocities, and flow rates determine the length of the tubes ***L,*** Eqs. (30) and (31) show that E_i and E_o are functions of ***A,,*** flow rates, and, respectively, G and G_s.† Thus, E_i and E_o are functions of h_i, h_o, and At,.

The variables in Eq. (35) are ***A,,*** w_u, E_i, and E_o, and their values are set if h_i, h_o, and At, are known. Partial differentiation therefore, of Eq. (35) with respect to the three independent variables h_i, h_o, and At, would lead to a solution for the optimum conditions. However, the resulting equations are cumbersome, and the procedure is simplified by retaining the following four variables:

1. Tube-inside coefficient of heat transfer, h_i
2. Tube-outside coefficient of heat transfer, h_o
3. Temperature-difference driving force for counterflow based on temperature of utility fluid at exit from exchanger, At, = $t_1' - t_2$
4. Outside tube area of heat transfer, ***A,***

Optimization Procedure

The first step in the optimization procedure is to express Eq. (35) in terms of the fundamental variables. The following relationships for power loss inside tubes and power loss outside tubes are developed in Table 9 for conditions of

---

†See Eqs. (*B*) and (*H*) in Table 9.

TABLE 9

Development of equations for optimization of heat-exchanger design

***Conditions*:** Turbulent flow in shell-and-tube heat exchangers with cross flow on the shell side.

Power Loss inside Tubes

$$(A) \qquad -\Delta p_i = \frac{B_i 2 f_i G^2 L n_p}{g_c \rho_i D_i \phi_i} = \frac{2 f_i G_i^2 L n_p}{g_c \rho_i D_i \phi_i} + (F_e + F_c + F_r) n_p \rho_i \qquad (30)$$

$$(Al) \quad \phi_i = 1.02 \left(\frac{\mu_i}{\mu_{w_i}} \right)^{0.14}$$

$$(A2) \quad S_i = \frac{\pi D_i^2 N_t}{4 n_p}$$

$$(A3) \quad A. = N_t \pi D_o L$$

where S_i = cross-sectional flow area inside tubes per pass

N_t = total number of tubes in exchanger = (number of tubes per pass) n,

$$(B) \quad E_i = \frac{-\Delta p_i w_i}{\rho_i A_o} = \frac{-\Delta p_i G S_i}{\rho_i A_o} = \frac{-\Delta p_i G D_i^2}{4 \rho_i D_o L n_p}$$

For turbulent flow in tubes,

$$(C) \quad f_i = \frac{0.046}{(N_{Re})^{0.2}} = \frac{0.046}{(D_i G/\mu_i)^{0.2}} \qquad \text{[Chap. 14, Eq. (7)]}$$

Combining Eqs. *(A)*, *(B)*, and (C),

$$(D) \qquad E_i = \frac{0.023 B_i \mu_i^{0.2} D_i^{0.8} G^{2.8}}{g_c D_o \rho_i^2 \phi_i}$$

$$(E) \quad \frac{h_i D_i}{k_i} = 0.023 \left(\frac{D_i G}{\mu_i} \right)^{0.8} \left(\frac{c_{p_i} \mu_i}{k_i} \right)^{1/3} \left(\frac{\mu_i}{\mu_{w_i}} \right)^{0.14} \qquad (26)$$

$$(E1) \quad G = \left[\frac{h_i D_i^{0.2} \mu_i^{0.8}}{0.023 k_i} \left(\frac{k_i}{c_{p_i} \mu_i} \right)^{1/3} \left(\frac{\mu_{w_i}}{\mu_i} \right)^{0.14} \right]^{1.25}$$

Combining Eqs. (Al), *(D)*, and (El),

$$(F) \quad E_i = h_i^{3.5} B_i \frac{D_i^{1.5} \mu_i^{1.83} (\mu_{w_i}/\mu_i)^{0.63}}{(1.02)(0.023)^{2.5} g_c D_o \rho_i^2 k_i^{2.33} c_{p_i}^{1.17}}$$

$$\frac{1}{(1.02)(0.023)^{2.5}} = 12{,}200$$

Power Loss outside Tubes

$$(G) \quad -\Delta p_o = \frac{B_o 2 f' N_r G_s^2}{g_c \rho_o} \qquad (31)$$

$$(H) \quad E_o = \frac{-\Delta p_o w_o}{\rho_o A_o} = \frac{-\Delta p_o G_s S_o}{\rho_o A_o} = \frac{-\Delta p_o G_s S_o}{\rho_o N_t \pi D_o L}$$

$$(I) \quad S_o = \frac{N_c D_c L}{n_b}$$

(Continued)

TABLE 9

Development of equations for optimization of heat-exchanger design (Continued)

---

where S_o = shell-side free-flow area across shell axis
N_c = number of clearances between tubes for flow of shell-side fluid across shell axis
D_c = clearance between tubes to give smallest free area across shell axis
n_b = number of baffle spaces = number of baffles + 1

For turbulent flow across tubes,

$$(J) \quad f' = b_o\left(\frac{D_oG_s}{\mu_{f_o}}\right)^{-0.15} \qquad (32)$$

See Eqs. (33) and (34) for values of b_o in terms of tube size and arrangement. Combining Eqs. (G), (*H*), (I), and (*J*),

$$(K) \quad E_o = \frac{2B_ob_o\mu_{fo}^{0.15}D_cG_s^{2.85}N_rN_c}{\pi g_cD_o^{1.15}\rho_o^2n_bN_t}$$

$$(L) \quad \frac{h_oD_o}{k_{f_o}} = \frac{a_o}{F_s}\left(\frac{D_oG_s}{\mu_{f_o}}\right)^{0.6}\left(\frac{c_{p_o}\mu_o}{k_o}\right)_f^{1/3} \qquad (29)$$

a. = 0.33 for staggered tubes and 0.26 for in-line tubes

$$(L1) \quad G_s = \left[\frac{h_oD_o^{0.4}\mu_{fo}^{0.6}F_s}{k_{f_o}a_o}\left(\frac{k_o}{c_{p_o}\mu_o}\right)_f^{1/3}\right]^{1.67}$$

Combining Eqs. (*K*) and (*L*1),

$$(M) \quad E_o = h_o^{4.75}\frac{B_o}{n_b}\frac{N_rN_c}{N_t}\frac{2b_oD_cD_o^{0.75}F_s^{4.75}\mu_{fo}^{1.42}}{\pi a_o^{4.75}g_c\rho_o^2k_{fo}^{3.17}c_{p_{fo}}^{1.58}}$$

---

turbulent flow and shell-side fluid flowing in a direction normal to the tubes:

$$E_i = \psi_ih_i^{3.5} \qquad (40)$$

$$E_o = \psi_oh_o^{4.75} \qquad (41)$$

where

$$\psi_i = B_i\left[\frac{12{,}200D_i^{1.5}\mu_i^{1.83}(\mu_{w_i}/\mu_i)^{0.63}}{g_cD_o\rho_i^2k_i^{2.33}c_{p_i}^{1.17}}\right] \qquad (40a)$$

$$\psi_o = \frac{B_o}{n_b}\frac{N_rN_c}{N_t}\left(\frac{2b_oD_cD_o^{0.75}F_s^{4.75}\mu_{fo}^{1.42}}{\pi a_o^{4.75}g_c\rho_o^2k_{fo}^{3.17}c_{p_{fo}}^{1.58}}\right) \qquad (41a)$$

All the terms in the brackets are set by the design conditions or can be approximated with good accuracy on the first trial. The values of B_i and B_o/n_b are not completely independent of the film coefficients, but they do not vary enough to be critical. As a first approximation, B_i is usually close to 1, and B_o is often taken to be equal to or slightly greater than the number of baffle passes n_b. The value of the safety factor F_s depends on the amount of bypassing and is often taken as 1.6 for design estimates.

The ratio $N_r N_c / N_t$ depends on the tube layout and baffle arrangement. For rectangular tube bundles and no baffles, this ratio is equal to 1.0. For other tube layouts and segmental baffles, the ratio is usually in the range of 0.6 to 1.2.

Equation (35) can now be expressed in terms of the primary variables At,, h_i, h,, and A,:

$$C_T = A_o K_F C_{A_o} + \frac{q H_y C_u}{c_{p_u}(\Delta t_1 - \Delta t_2 + t_1' - t_2')} + A_o \psi_i h_i^{3.5} H_y C_i + A_o \psi_o h_o^{4.75} H_y C_o \tag{42}$$

Only three of the four variables in Eq. (42) are independent. Under these conditions, optimization can be accomplished by use of the Lagrange multiplier method.? The necessary relationship for applying the constant Lagrangian multiplier A is given by Eq. (43):

$$\lambda\left[\frac{F_T(\Delta t_2 - \Delta t_1)}{q \ln(\Delta t_2/\Delta t_1)} - \frac{1}{A_o}\left(\frac{D_o}{D_i h_i} + \frac{1}{h_o} + R_{dw}\right)\right] = 0 \tag{43}$$

Equations (42) and (43) can now be added to give the following equation for optimization by partial differentiation with respect to each of the four primary variables:

$$C_T = A_o K_F C_{A_o} + \frac{q H_y C_u}{c_{p_u}(\Delta t_1 - \text{At, } + t_1' - t_2')} + A_o \psi_i h_i^{3.5} H_y C_i + A_o \psi_o h_o^{4.75} H_y C_o + \lambda\left[\frac{F_T(\Delta t_2 - \Delta t_1)}{q \ln(\Delta t_2/\Delta t_1)} - \frac{1}{A_o}\left(\frac{D_o}{D_i h_i} + \frac{1}{h_o} + R_{dw}\right)\right] \tag{44}$$

OPTIMUM **VALUE** OF h_o. The following relationship between the optimum values of h; and h_o is obtained by taking the partial derivative of Eq. (44) with respect to h_i and then with respect to h_o, setting the results equal to zero, and

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†See Chap. 11.

eliminating A,, and A:

$$\frac{\partial C_T}{\partial h_i} = 3.5A_{o,\text{opt}}\psi_i h_{i,\text{opt}}^{2.5} H_y C_i + \frac{\lambda D_o}{A_{o,\text{opt}} D_i h_{i,\text{opt}}^2} = 0 \tag{45a}$$

$$\frac{\partial C_T}{\partial h_o} = 4.75A_{o,\text{opt}}\psi_o h_{o,\text{opt}}^{3.75} H_y C_o + \frac{A}{A_{o,\text{opt}} h_{o,\text{opt}}^2} = 0 \tag{45b}$$

$$h_{o,\text{opt}} = \left(\frac{0.74\psi_i C_i D_i}{\psi_o C_o D_o}\right)^{0.17} h_{i,\text{opt}}^{0.78} \tag{45c}$$

OPTIMUM VALUE OF h_i. The optimum value of h_i can be determined by setting the partial derivatives of Eq. (44) with respect to A, and with respect to h_i equal to zero and eliminating A, and A. This gives a result with $h_{i,\text{opt}}$ and $h_{o,\text{opt}}$ as the only unknowns, and simultaneous solution with Eq. (45*c*) yields Eq. (46*b*), where $h_{i,\text{opt}}$ is the only unknown.

$$\frac{\partial C_T}{\partial A_o} = K_F C_{A_o} + \psi_i h_{i,\text{opt}}^{3.5} H_y C_i + \psi_o h_{o,\text{opt}}^{4.75} H_y C_o$$

$$+ \frac{\lambda}{A_{o,\text{opt}}^2}\left(\frac{D_o}{D_i h_{i,\text{opt}}} + \frac{1}{h_{o,\text{opt}}} + R_{dw}\right) = 0 \tag{46a}$$

$$h_{i,\text{opt}}^{3.5}\Bigg|\, 2.5\psi_i H_y C_i + \frac{3.5\psi_i H_y C_i D_i R_{dw} h_{i,\text{opt}}}{D_o}$$

$$+ 2{,}9\left(\frac{\psi_i C_i D_i}{D_o}\right)^{0.83} (\psi_o C_o)^{0.17} H_y h_{i,\text{opt}}^{0.22} \underset{I}{} = K_F C_{A_o} \tag{46b}$$

OPTIMUM VALUE OF U_o. A trial-and-error or graphical method can be used to obtain $h_{i,\text{ opt}}$ from Eq. (46*b*). Then, by Eqs. (38) and (45*c*), the value of $U_{o,\text{opt}}$ can be determined as

$$U_{o,\text{opt}} = \left(\frac{D_o}{D_i h_{i,\text{opt}}} + \frac{1}{h_{o,\text{opt}}} + R_{dw}\right)^{-1} \tag{47}$$

OPTIMUM VALUE OF At,. The value of $U_{o,\text{opt}}$ is now known, and $\Delta t_{2,\text{opt}}$ can be determined by setting the partial derivatives of Eq. (44) with respect to At, and with respect to A, equal to zero and eliminating A. The result can be combined with Eqs. (39) to (41) to give

$$\frac{F_T U_{o,\text{opt}} H_y C_u}{c_{p_u}\left(K_F C_{A_o} + E_{i,\text{opt}} H_y C_i + E_{o,\text{opt}} H_y C_o\right)}$$

$$= \left(1 + \frac{t_1' - t_2'}{\Delta t_1 - \Delta t_{2,\text{opt}}}\right)^2 \left(\ln\frac{\Delta t_{2,\text{opt}}}{\Delta t_1} - 1 + \frac{\Delta t_1}{\Delta t_{2,\text{opt}}}\right) \tag{48}$$

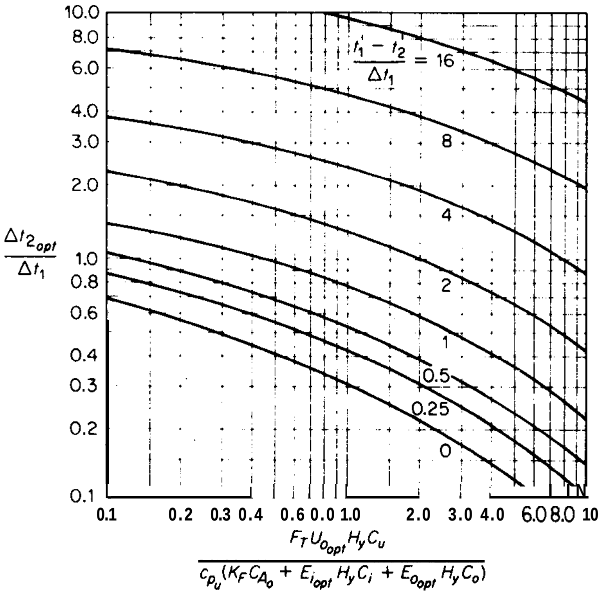

FIGURE **15-32**
Graphical solution of Eq. (48) for evaluation of optimum Δt_2 in heat exchangers.

Equation (48) can be solved for $\Delta t_{2,\text{opt}}$ by trial and error or by Fig. 15-32 which is a plot of Eq. (48).

OPTIMUM VALUE OF A,. Since $\Delta t_{2,\text{opt}}$ and, therefore, At,,, opt are now known, $A_{o,\text{opt}}$ can be determined directly from Eq. (38).

OPTIMUM VALUE OF G AND G_s. Equations (El) and ($L1$) in table 9 give G_{opt} and G_s, opt, respectively, in terms of $h_{i,\text{opt}}$ and $h_{o,\text{opt}}$.

OPTIMUM VALUE OF w_u. The flow rate of the utility fluid (w_u) is set by the value of At,. Therefore, when $\Delta t_{2,\text{opt}}$ is known, $w_{u,\text{opt}}$ can be calculated from Eq. (37).

OPTIMUM VALUES OF S_i AND N,. The optimum flow area inside the tubes per pass can be calculated from the following equation:

$$S_{i,\text{opt}} = \frac{w_i}{G_{\text{opt}}} \tag{49}$$

The optimum total number of tubes in the exchanger is

$$N_{t,\text{opt}} = \frac{4 n_p S_{i,\text{opt}}}{\pi D_i^2} \tag{50}$$

OPTIMUM VALUE OF L. The optimum length per tube is set by the optimum heat-transfer area and the total number of tubes. Thus, for a given tube diameter

$$L_{\text{opt}} = \frac{A_{o,\text{opt}}}{\pi D_o N_{t,\text{opt}}} \tag{51}$$

OPTIMUM VALUES OF S_o, N_c, AND n_b. The following equation gives the optimum shell-side free-flow area across the shell axis:

$$S_{o,\text{opt}} = \frac{w_o}{G_{s,\text{opt}}} \tag{52}$$

The number of clearances N_c for flow between tubes across the shell axis is determined by the number of tubes in the shell, the pitch of the tubes, and the arrangement of the tubes. For the common case of a cylindrical shell and transverse clearances giving the minimum free area, the following equations can be used to obtain an approximation of $N_{c,\text{opt}}$:[†][‡]

With square pitch and N_t greater than 25

$$N_{c,\text{opt}} = 1.37(N_{t,\text{opt}})^{0.475} \tag{53}$$

With equilateral triangular pitch and N_t greater than 20

$$N_{c,\text{opt}} = 0.94 + \left(\frac{N_{t,\text{opt}} - 3.7}{0.907}\right)^{1/2} \tag{54}$$

The optimum number of baffle spaces can be estimated by Eq. (I) in Table 9 as follows:

$$n_{b,\text{opt}} = \frac{N_{c,\text{opt}} D_c L_{\text{opt}}}{S_{o,\text{opt}}} \tag{55}$$

SUMMARY OF PROCEDURE FOR GENERAL CASE OF OPTIMUM DESIGN

In the preceding analysis, consideration has been given to the general case in heat-exchanger design in which the following conditions apply:

1. The flow rate and necessary temperature change of the process fluid are known.
2. The inlet temperature of the utility fluid is known.
3. The exchanger is a shell-and-tube type with crossflow baffling, and flow is in the turbulent range on both the tube side and the shell side.

[†] W. H. McAdams, "Heat Transmission," 3d ed., p. 434, McGraw-Hill Book Company, New York, 1954.

[‡] Equations (53) and (54) are based on the assumption that shell inside diameter/tube pitch = number of tubes in a row across the shell axis = number of clearances across the shell axis. No allowance is made for decrease in available tube space when more than one tube pass is used. The equations are general and are not necessarily limited to optimum conditions.

4. No partial phase changes occur.
5. Necessary safety factors are known.

The following information may be specified for the design or can be assumed as a reasonable approximation:

1. Tube diameter, wall thickness, pitch, and arrangement
2. Number of tube passes
3. Heat-transfer resistance caused by tube walls, dirt, and scale

The following information must be assumed for the first trial and then checked when the optimum conditions are obtained (usually, no more than two trials are necessary, and an experienced engineer can often make adequate assumptions on the first trial):

1. Average bulk and film temperatures
2. Values for B_i (usually 1.0), B_o/n_b (usually 1.0), and $N_r N_c/N_t$ (usually 1.0)

The calculation procedure is as follows:

1. Determine $h_{i,\mathrm{opt}}$ from Eq. (46*b*)
2. Determine $h_{o,\mathrm{opt}}$ from Eq. (45*c*)
3. Determine $U_{o,\mathrm{opt}}$ from Eq. (47)
4. Determine $\Delta t_{2,\mathrm{opt}}$ from Eq. (48) or Fig. 15-32
5. Determine $A_{o,\mathrm{opt}}$ from Eq. (39)
6. Determine G_{opt} and $G_{s,\mathrm{opt}}$ from Eqs. (El) and (*L*1) in Table 9
7. Determine $w_{u,\mathrm{opt}}$ from Eq. (37)
8. Determine $S_{i,\mathrm{opt}}$ and $N_{t,\mathrm{opt}}$ from Eqs. (49) and (50)
9. Determine L_{opt} from Eq. (51)
10. Determine $S_{o,\mathrm{opt}}$, $N_{c,\mathrm{opt}}$, and $n_{b,\mathrm{opt}}$ from Eqs. (52) through (55)
11. Check assumptions; if any are invalid, make new reasonable assumptions and repeat procedure.

Example 5 Development of the optimum design for a shell-and-tube heat exchanger. A gas under pressure with properties equivalent to air must be cooled from 150 to 100°F. Cooling water is available at a temperature of 70°F. Use of a Shell-and-tube floating-head heat exchanger with cooling water as the utility fluid has been proposed. On the basis of the following data and specifications, determine the tube length, number of tubes, and installed cost for the optimum exchanger which will handle 20,000 lb of the gas per hour.

1. Exchanger specifications
   - a. Steel shell-and-tube exchanger with cross-flow baffling.
   - *b.* Cooling water passes through shell side of unit.

c. One tube pass and countercurrent flow.
d. Tube OD = 1.0 in. Tube ID = 0.782 in.
e. $\frac{15}{16}$-in triangular pitch. Tubes are staggered.

2. costs
a. Data presented in Fig. 15-13 are applicable.
b. Installation cost equals 15 percent of purchased cost.
c. Annual **fixed** charges including maintenance equal 20 percent of installed cost.
d. Cost for cooling water (not including pumping cost) is \$0.009 per 1000 lb.
e. Cost for energy supplied to force the cooling water and the gas through the exchanger (including effect of pump-and-motor efficiency and cost) is \$0.04 per **kWh.**

3. General
a. Average absolute pressure of gas in exchanger is 10 atm.
b. Unit operates 7000 h/year.
c. In the friction relations, B_i can be taken as 1.2, and B_o can be taken as equal to the number of tube crosses, assuming the optimum number of baffles can be installed in the unit.
d. The safety factor F_s for the outside film coefficient is 1.3.
e. Fouling coefficient for cooling water is 1500 Btu/(h)(ft^2)(°F). Fouling coefficient for gas is 2000 Btu/(h)(ft^2)(°F).
f. At the optimum conditions, flow on tube side and shell side is turbulent.
g. The factor $N_r N_c / N_t$ can be taken as 1.0.

Solution. Assumptions:

Exit temperature of cooling water = t_2 = 110°F
Average At over cooling-water film = 10 percent of total At
Average At over air film = 80 percent of total **At**
Purchased cost per square foot outside heat-transfer area = \$34.00
Temperatures and physical properties:
$t_1' = 150°F$, $t_2' = 100°F$, $t_1 = 70°F$, $t_2 = 110°F$
At, = $t_2' - t_1$ = 30°F
Average bulk water temperature = 90°F
Average bulk gas temperature = 125°F
Average water-film temperature = 90 + (0.1/2)(35) = 91.8°F
Inside wall temperature = 125 − (0.8)(35) = 97°F

From Appendix: For water at 91.8°F, k = 0.359 Btu/(h)(ft^2)(°F/ft), μ = (0.748)(2.42) = 1.81 lb/(h)(ft), c_p = 1 Btu/(lb)(°F). For water at 90°F, ρ = 62.1 lb/ft^3. For air at 125°F, k = 0.0162 Btu/(h)(ft^2)(°F/ft), μ = (0.019) x (2.42) = 0.046 lb/(h)(ft), c_p = 0.25 Btu/(lb)(°F), ρ = (29)(492)(10)/(359) × (460 + 125) = 0.68 lb/ft^3. For air at 97°F, μ = (0.0185)(2.42) = 0.045 lb(h)(ft).

Determination of optimum U_o. From Eq. (40*a*)

$$\psi_i = B_i \frac{12{,}200 D_i^{1.5} \mu_i^{1.83} (\mu_{w_i}/\mu_i)^{0.63}}{g_c D_o \rho_i^2 k_i^{2.33} c_{pi}^{1.17}}$$

$$\psi_i = \frac{(1.2)(12{,}200)(0.782/12)^{1.5}(0.046)^{1.83}(0.045/0.046)^{0.63}}{(32.17)(3600)^2(\frac{1}{12})(0.68)^2(0.0162)^{2.33}(0.25)^{1.17}}$$

$$\psi_i = 4.1 \times 10^{-3}$$

From Eqs. (33) and (41*a*),

$$b_o = 0.23 + \frac{0.11}{(1.25 - 1)^{1.08}} = 0.72$$

$$\psi_o = \frac{B_o}{n_b}\frac{N_r N_c}{N_t}\frac{26_o D_c D_o^{0.75} F_s^{4.75} \mu_{f_o}^{1.42}}{\pi a_o^{4.75} g_c \rho_o^2 k_{f_o}^{3.17} c_{p_{fo}}^{1.58}}$$

$$\psi_o = \frac{(1)(1)(2)(0.72)(0.25/12)(\frac{1}{12})^{0.75}(1.3)^{4.75}(1.81)^{1.42}}{(3.14)(0.33)^{4.75}(32.17)(3600)^2(62.1)^2(0.359)^{3.17}(1)^{1.58}}$$

$$\psi_o = 3.8 \text{ x } 10^{-11}$$

$$R_{dw} = \frac{x_w D_o}{k_w D_{w_{\text{log mean}}}} + \frac{D_o}{D_i h_{d_i}} + \frac{1}{h_{d_o}}$$

$$= \frac{(0.109/12)(\frac{1}{12})}{(26)(0.891/12)} + \frac{6}{(0.782/12)(2000)} + \frac{1}{1500}$$

$$= 1.7 \text{ x } 10^{-3}$$

$$C_i = C_o = \frac{\$0.04}{2.655 \times 10^6} \text{ per ft} \cdot \text{lbf}$$

From Eq. (46*b*),

$$(h_{i,\text{opt}})^{3.5}(7000)\left(\frac{0.04}{2.655 \times 10^6}\right)$$

$$\text{x}\left[(2.5)(4.1 \text{ x } 10^{-3}) + \frac{(3.5)(4.1 \text{ x } 10^{-3})(0.782/12)(1.7 \text{ x } 10^{-3})h_{i,\text{opt}}}{\frac{1}{12}}\right.$$

$$\left. + 2.9\left(\frac{4.1 \text{ x } 10^{-3} \text{ x } 0.782/12}{\frac{1}{12}}\right)^{0.83}(3.8 \text{ x } 10^{-11})^{0.17}(h_{i,\text{opt}})^{0.22}\right]$$

$$= (0.2)(34)(1.15)$$

By trial and error,

$$h_{i,\text{opt}} = 85 \text{ Btu/(h)(ft}^2\text{)(°F)}$$

From Eq. (45*c*),

$$h_{o,\text{opt}} = \left[\frac{(0.74)(4.1 \text{ x } 10^{-3})(0.782/12)}{(3.8 \text{ x } 10^{-11})(\frac{1}{12})}\right]^{0.17}(85)^{0.78} = 670 \text{ Btu/(h)(ft}^2\text{)(°F)}$$

From Eq. (47),

$$U_{o,\text{opt}} = \left[\frac{1}{(0.782)(85)} + \frac{1}{670} + 0.0017\right]^{-1}$$

$$= (0.0150 + 0.00149 + 0.0017)^{-1} = (0.0182)^{-1}$$

$$= 55 \text{ Btu/(h)(ft}^2\text{)(°F)}$$

$$\text{Percent total At over air film} = \frac{0.0150}{0\ 0182}(100) = 82 \text{ percent}$$

$$\text{Percent total At over water film} = \frac{0.00149}{0\ 0182}(100) = 8.2 \text{ percent}$$

Therefore, the percent At assumptions are adequate.

Determination of optimum At,:

C_u = \$9 x 10^{-6} per lb of water

$F_T = 1$

$$\frac{F_T U_{o,\text{opt}} H_y C_u}{c_{p_u}\left(K_F C_{A_o} + E_{i,\text{opt}} H_y C_i + E_{o,\text{opt}} H_y C_o\right)}$$

$$= \frac{(1)(55)(7000)(9 \text{ x } 10^{-6})}{(1)\left[(0.2)(34.00)(1.15) + (4.1 \times 10^{-3})(85)^{3.5}(7000) \text{ x } \left(\frac{0.04}{2.655 \times 10^{-6}}\right) + (3.8 \text{ x } 10^{-11})(670)^{4.75}(7000)\left(\frac{0.04}{2.655 \times 10^{6}}\right)\right]}$$

$$= 0.31$$

$$\frac{t_1' - t_2'}{\Delta t_1} = \frac{150 - 100}{30} = 1.67$$

From Fig. 15-32

$$\frac{\text{A } t_{2,\text{opt}}}{\Delta t_1} = 1.44$$

$$\Delta t_{2,\text{opt}} = t_1' - t_{2,\text{opt}} = (30)(1.44) = 43°\text{F}$$

$$t_{2,\text{opt}} = 150 - 43 = 107°\text{F}$$

The assumption of t_2 = 110°F is adequate.

Determination of optimum A,:

$$\Delta t_{m,\text{opt}} = \frac{30 + 42}{2} = 36°\text{F}$$

$$A_{o,\text{opt}} = \frac{q}{U_{o,\text{opt}}\,\Delta t_{m,\text{opt}}} = \frac{(20{,}000)(0.25)(50)}{(55)(36)} = 126 \text{ ft}^2$$

From Fig. 15-13, the assumed purchased cost of \$34.00 per square foot of outside area is low, but it will be assumed as adequate and will be used to complete the example, leaving a repeat calculation up to the reader if so desired.

Cost of installed exchanger = (126)(34.00)(1.15) = \$4930

Determination of optimum number of tubes and tube length: From Eq. (El) in Table 9,

$$G_{i,\text{opt}} = \left[\frac{(85)(0.782/12)^{0.2}(0.046)^{0.8}}{(0.023)(0.0162)}\left(\frac{0.0162}{0.25 \times 0.046}\right)^{1/3}\left(\frac{0.045}{0.046}\right)^{0.14}\right]^{1.25}$$

$$= 133{,}000 \text{ lb/(h)(ft}^2\text{)}$$

$$S_{i,\text{opt}} = \frac{20{,}000}{133{,}000} = 0.150 \text{ ft}^2$$

$$\text{Number of tubes} = \frac{(4)(0.150)}{(3.14)(0.782/12)^2} = 45$$

$$\text{Tube length} = \frac{126}{(3.14)(\frac{1}{12})(45)} = 10.7 \text{ ft}$$

A check on the Reynolds numbers indicates that flow on tube side and shell side is turbulent.

For the optimum exchanger:

Installed cost = \$4930

Number of tubes = 45

Tube length = 10.7 ft

Note: An optimum design of this type can be used as a guide for the final selection of an exchanger. However, practical factors of operation must also be considered. In this case, a large number of baffles are required to give the optimum conditions. Because the water-film coefficient is large relative to the air-film coefficient, a reduction in the number of baffles would have little effect on the optimum design.

SIMPLIFIED CASES OF OPTIMUM DESIGN

Immaterial Power Costs on Shell Side or Tube Side

In the operation of heat exchangers, the power cost for one of the fluids is often immaterial. For example, process cooling water is ordinarily supplied under sufficient pressure so that no pumping-power costs are necessary for the water. The highest useful water velocity should be employed in order to minimize the heat-transfer resistance caused by the water film, and a reasonable water-film coefficient can be assumed. Thus, if power costs on one side of the exchanger are immaterial, the problem reduces to one in which the film coefficient and, therefore, the fluid velocity on one side of the exchanger are fixed. Under these conditions, the number of independent variables for the development of the type of optimum design discussed in the preceding sections is two. Possible choices of the basic independent variables could be h_i, At,; h_i, A,; or A,, Δt_2.†

SHELL.-SIDE POWER COST IMMATERIAL. In this case, the value of C_o in Eqs. (35) and (44) is zero and h_o is constant. The optimization can be accomplished by differentiating Eq. (44) under the given conditions. The following expression for the evaluation of $h_{i,\text{opt}}$ is obtained by setting the partial derivatives of Eq.

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†An alternative approach for this type of situation is described in Chap. 11 for the case of optimum flow rate of cooling water in a condenser. Equation (56) in Chap. 11 is merely a special case of Eq. (48) where $t_1' - t_2'$ is zero.

(44) with respect to h_i and with respect to A, equal to zero:

$$h_{i,\text{opt}}^{3.5}\left[2.5\psi_i H_y C_i + \frac{3.5\psi_i H_y C_i D}{D_o}\left(\frac{1}{h_o} + R_{dw}\right)h_{i,\text{opt}}\right] = K_F C_{A_o} \tag{56}$$

The value of $h_{i,\text{opt}}$ can be determined from Eq. (56), and $U_{o,\text{opt}}$ can then be calculated from Eq. (38). When $U_{o,\text{opt}}$ is known, the optimum value of At, can be obtained from Eq. (48) or Fig. 15-32, and the optimum design is fixed.

TUBE-SIDE POWER COST IMMATERIAL. Optimization of the heat-exchanger design for this situation is based on the assumption that C_i is zero and h_i is constant. The procedure is similar to that for the case of shell-side power immaterial as described in the preceding paragraph. The optimum value of h_o can be determined from the following equation, which is obtained by setting the partial derivatives of Eq. (44) with respect to h_o and with respect to A, equal to zero:

$$h_{o,\text{opt}}^{4.75}\left[3.75\psi_o H_y C_o + 4.75\psi_o H_y C_o\left(\frac{D_o}{D_i h_i} + R_{dw}\right)h_{o,\text{opt}}\right] = K_F C_{A_o} \tag{57}$$

If power costs on both the tube side and the shell side are to be considered, the ratio of these two costs can be evaluated from the following equation:

$$\frac{\text{Power cost on tube side}}{\text{Power cost on shell side}} = \frac{E_i C_i}{E_o C_o} = \frac{\psi_i h_i^{3.5} C_i}{\psi_o h_o^{4.75} C_o} \tag{58}$$

As in the previous cases, Eq. (58) applies to a crossflow shell-and-tube exchanger with flow in the turbulent range and no phase change. The equation is not limited to optimum conditions.

Flow Rate of Utility-Fluid Fixed

A situation sometimes occurs in which the maximum temperature of a utility coolant fluid is fixed, thereby setting the coolant flow rate. In other cases, the operating circumstances may arbitrarily set the utility-fluid flow rate as well as the process-fluid flow rate. Under these conditions, At, is constant, and only two independent variables are involved in the optimization procedure. Since At, was held constant in the development of Eqs. (45*c*), (46*b*), and (47), these equations can be used for determining the optimum overall heat-transfer coefficient. This coefficient, combined with the known values of At, and At,, is sufficient to permit calculation of the optimum heat-transfer area by Eq. (39), and the rest of the design variables are then established.

Change in Phase of One Fluid (Temperature of the Fluid Remains Constant)

The procedure for developing an optimum heat-exchanger design is simplified if the temperature of one of the fluids remains constant. This condition is often encountered when one of the fluids changes phase, as in a condenser or a steam heater.

When the process fluid is the one that changes phase, the three independent variables h_i, h_o, and At, are involved, and the general procedure for developing the optimum-design equations is similar to that outlined for the case of no phase change.

Under many conditions, however, determination of the optimum overall coefficient is simplified, because the pressure drop and the resulting power costs for a condensing fluid can be taken as zero and the heat-transfer resistance of the condensing-fluid film is relatively small, permitting the assumption of a reasonable constant value for the film coefficient. Thus, if the fluid that changes phase is the process fluid, the number of independent variables in the design is reduced to two, as in the case of immaterial power costs on the shell side or tube side. If the change in phase occurs on the shell side, Eq. (56) can be used to evaluate $h_{i,\text{opt}}$, and $U_{o\ \text{opt}}$ can then be obtained from Eq. (38). Similarly, if the phase change is on the tube side, Eq. (57) gives the optimum value of h_o. When $U_{o,\text{opt}}$ is known, Eq. (48) with $t_1' - t_2'$ equal to zero, or the bottom line in Fig. 15-32, can be used to evaluate the optimum value of At,. The procedure for this situation, as applied to a condenser with immaterial power costs for the utility fluid, is presented in detail in Chap. 11.

If the condensing fluid happens to be the utility fluid, as in a steam heater, the value of At, is set by the condensing temperature, and the independent variables are h_i and h,. For the common case of negligible power cost and constant value of the film coefficient on the phase-change side of the exchanger, only one independent variable remains. In this situation, the optimum design can be established by setting the partial derivatives of Eq. (44) with respect to A, and with respect to the independent film coefficient equal to zero. The resulting optimum film coefficient for change in shell-side phase can be calculated from Eq. (56), and that for change in tube-side phase is given by Eq. (57).

Velocity of One Fluid Fixed (h_i or h_o fixed)

Because of the fouling effects, there may be a limit on the velocity of one of the fluids in a heat exchanger. For example, the velocity of cooling water in tubes of a shell-and-tube exchanger is often specified as 3 ft/s. If the velocity of one fluid is specified, the coefficient for that fluid is set, and the independent variables become At, and the film coefficient of the other fluid.

INSIDE-TUBE VELOCITY **FIXED.** If the inside-tube velocity is fixed, h_i is constant, and the optimum value of h_o can be established by eliminating λ/A_o

from Eqs. (4%) and (46*a*) to give

$$h_{o,\text{opt}}^{4.75}\left[3.75\psi_o H_y C_o + 4.75\psi_o H_y C_o\left(\frac{D_o}{D_i h_i} + R_{dw}\right)h_{o,\text{opt}}\right] = K_F C_{A_o} + \psi_i h_i^{3.5} H_y C_i \tag{59}$$

Trial-and-error or graphical solution of Eq. (59) yields the value of $h_{o,\text{opt}}$, and $U_{o,\text{opt}}$ can be calculated from Eq. (39). Figure 15-32 or Eq. (48) can then be used to obtain $\Delta t_{2,\text{opt}}$, and the optimum design conditions are fixed.

OUTSIDE-TUBE VELOCITY FIXED. When h_o is constant, the procedure for evaluation of $h_{i,\text{opt}}$ and the other optimum conditions is similar to that described in the preceding paragraph. By eliminating *A/A,* from Eqs. (45*a*) and (46*a*), the following expression for evaluation of $h_{i,\text{opt}}$ is obtained:

$$h_{i,\text{opt}}^{3.5}\left[2.5\psi_i H_y C_i + \frac{3.5\psi_i H_y C_i D_i}{D_o}\left(\frac{1}{h_o} + R_{dw}\right)h_{i,\text{opt}}\right] = K_F C_{A_o} + \psi_o h_o^{4.75} H_y C_o \tag{60}$$

GENERAL METHODS FOR PROCESS DESIGN OF HEAT EXCHANGERS

The procedures used for developing the design of heat exchangers vary with the type of problem and the preference of the worker. Some engineers prefer to develop the design for a heat exchanger by a method known as *rating an exchanger.* In this method, the engineer assumes the existence of an exchanger and makes calculations to determine if the exchanger would handle the process requirements under reasonable conditions. If not, a different exchanger is assumed, and the calculations are repeated until a suitable design is developed. For example, with a given set of process requirements, the engineer could assume the existence of an exchanger with a designated tube size, tube spacing, baffle type, baffle spacing, and number of tubes and passes. The engineer might then proceed through the process-design calculations by computing an overall heat-transfer coefficient and evaluating all flow rates, areas, lengths, and pressure drops. Repeated trials may be necessary to obtain an accurate overall coefficient. If the results of the final design indicate that the assumed exchanger has reasonable dimensions, reasonable cost, and acceptable pressure drops, the unit is considered as adequate and the design is complete.

An alternative approach, of course, is to base the design on optimum economic conditions, using the methods described in the preceding sections. No matter which approach is used, the general method of attack for a given set of process conditions consists of the following steps:

1. Determine the rates of flow and rate of heat transfer necessary to meet the given conditions.

2. Decide on the type of heat exchanger to be used, and indicate the basic equipment specifications.
3. Evaluate the overall heat-transfer coefficient and also the film coefficients, if necessary. In many cases, fluid velocities must be determined in order to obtain accurate heat-transfer coefficients.
4. Evaluate the mean temperature-difference driving force.
5. Determine the necessary area of heat transfer and the exchanger dimensions.
6. Analyze the results to see if all dimensions, costs, pressure drops, and other design details are satisfactory.
7. If the results of (6) show that the exchanger is not satisfactory, the specifications given in (2) are inadequate. Choose new specifications and repeat steps 3 through 7 until a satisfactory design is obtained.

NOMENCLATURE FOR CHAPTER 15

a_o = constant in Eq. (29) for evaluating outside film coefficient of heat transfer, dimensionless

A = area of heat transfer, ft^2; subscripts m designates mean area; subscript o designates outside area; subscript f designates film area

b_o = constant in Eq. (32) for evaluating shell-side friction factor, dimensionless

B_i = correction factor in Eq. (30) to account for friction due to sudden contraction, sudden expansion, and reversal of flow direction, dimensionless

B_o = correction factor in Eq. (31) to account for friction due to reversal of flow direction, recrossing of tubes, and variation in cross section, dimensionless

c_c = constant, dimensionless; defined in Table 4

c_p = heat capacity, Btu/(lb)(°F); prime refers to process fluid

C_{A_o} = installed cost of heat exchanger per unit of outside-tube heat-transfer area, $\$/ft^2$

C_i = cost for supplying 1 ft · lbf to pump the fluid through the inside of the tubes, \$/ft · lbf

C_o = cost for supplying 1 ft · lbf to pump the fluid through the shell side of the exchanger, \$/ft · lbf

C_T = total annual variable cost for heat exchanger and its operation, \$/year

C_u = cost of utility fluid, \$/lb

D = diameter or distance, ft

D' = diameter, in.

D_c = clearance between tubes to give smallest free area across shell axis, ft

D_e = equivalent diameter = 4 x hydraulic radius, ft

E = power loss per unit of outside-tube heat-transfer area. ft · lbf/(h)(ft^2); subscript i designates inside tubes, and subscript o designates outside tubes

f_i = Fanning friction factor for isothermal flow, dimensionless
f' = special friction factor for shell-side flow, dimensionless
F_A = correction factor for radiant heat transfer based on relative orientation of surfaces, dimensionless; defined by Eqs. (6) and (7)
F_c = friction due to sudden contraction, ft · lbf/lbm
F_e = friction due to sudden enlargement, ft · lbf/lbm
F_E = correction factor for radiant heat transfer based on emissivities and absorptivities of surfaces, dimensionless; defined by Eqs. (6) and (7)
F_r = friction due to reversal of flow direction, ft · lbf/lbm
F_s = safety factor in Eq. (29) to account for bypassing on shell side of exchanger, dimensionless
F_T = correction factor on logarithmic-mean At for counterflow to give mean At, dimensionless; defined in Fig. 15-2
g = local gravitational acceleration, ft/(s)(s)
g_c = conversion factor in Newton's law of motion, 32.17 ft · lbm/(s)(s)(lbf)
G = mass velocity inside tubes, lb/(h)(ft^2)
G_s = shell-side mass velocity across tubes based on the minimum free area between baffles across the shell axis, lb/(h)(ft^2)
h = film coefficient of heat transfer, Btu/(h)(ft^2)(°F); subscript c indicates convection; subscript d represents dirt or fouling; subscript co indicates conduction
h_r = film coefficient for heat transfer by radiation, Btu/(h)(ft^2)(°F)
H_y = hours of operation per year, h/year
k = thermal conductivity, Btu/(h)(ft^2)(°F/ft); subscripts x, y, and z refer to direction of heat-flow path
K = dimensional constant; defined in Table 4
K_c = constant in expression for evaluating friction due to sudden contraction, dimensionless
K_F = annual fixed charges including maintenance, expressed as a fraction of the initial cost for the completely installed unit, dimensionless
K_1 = constant for evaluation of B_i, dimensionless; defined with Eq. (30)
L = heated length of straight tube or length of heat-transfer surface, ft; if tubes in parallel are involved, L is the length of one tube
n = constant, dimensionless
n_b = number of baffle spaces = number of baffles plus 1, dimensionless
n_p = number of tube passes, dimensionless
N_c = number of clearances between tubes for flow of shell-side fluid across shell axis, dimensionless
N_{Gr} = Grashof number = $L^3\rho^2 g\beta\,\Delta t/\mu^2$, dimensionless
N_{Pr} = Prandtl number = $c_p\mu/k$, dimensionless
N_r = number of rows of tubes across which shell fluid flows, dimensionless
N_{Re} = Reynolds number = DG/μ, dimensionless
N, = total number of tubes in exchanger = number of tubes per pass $X\, n_p$, dimensionless
N_V = number of rows of tubes in a vertical tier, dimensionless

P, p = pressure, $\mathrm{lbf/ft^2}$
4 = rate of heat transfer, **Btu/h**
Q = amount of heat transferred in time, θ, Btu
r = radius, ft
R = temperature ratio for evaluating F_T, dimensionless; defined in Fig. 15-2
R_{dw} = combined resistance of tube wall and scaling or dirt factors, $[\mathrm{Btu/(h)(ft^2)(^\circ F)}]^{-1}$; defined with **Eq.** (39).
S = temperature ratio for evaluating F_T, dimensionless; defined in Fig. 15-2
S_H = cross-sectional flow area of header per pass, $\mathrm{ft^2}$
S_i = cross-sectional flow area inside tubes per pass, $\mathrm{ft^2}$
S_o = shell-side free-flow area across the shell axis, $\mathrm{ft^2}$
t = temperature, °F; subscript b refers to average bulk temperature; subscript *or* refers to original temperature; subscript s refers to surface; in general, primes refer to the process fluid, subscript 1 refers to the entering temperature, and subscript 2 refers to the leaving temperature
t' = temperature of second fluid in a heat exchanger, °F; refers, in general, to process fluid
T = absolute temperature, °R
U = overall coefficient of heat transfer, $\mathrm{Btu/(h)(ft^2)(^\circ F)}$; subscript d indicates that a dirt or fouling factor is included; subscript o indicates based on outside area and fouling factor included
V = velocity, **ft/h**; subscript i indicates in tubes
V' = velocity, **ft/s**
w = weight rate of flow, lb/h; no subscript indicates per tube; subscript u indicates total flow rate of utility fluid; subscript i indicates total flow rate of inside-tube fluid; subscript o indicates total flow rate of outside-tube fluid
w' = total weight rate of flow of process fluid, lb/h
x = length of conduction path, ft
x_L = ratio of pitch parallel to flow to tube diameter, dimensionless
x_T = ratio of pitch transverse to flow to tube diameter, dimensionless
y = length of conduction path, ft
z = length of conduction path, ft

Greek symbols

α = thermal **diffusivity** = $k/\rho c_p$, $\mathrm{ft^2/h}$
β = coefficient of volumetric expansion, 1/°R
A = At designates temperature-difference driving force, °F; subscript f designates across film; subscript m designates mean At; subscript oa or no subscript designates overall At; At, = $t'_2 - t_1$; At, = $t'_1 - t_2$; AP and Ap designate pressure drop; Ap = – A P
ε = emissivity, dimensionless
θ = time, h

A, = latent heat of condensation, Btu/lb
λ = Lagrangian multiplier, dimensionless; defined by Eq. (43)
μ = absolute viscosity, lb/(h)(ft)
π = 3.1416 ···
ρ = density, lb/ft^3
σ = Stefan-Boltzmann dimensional constant for radiant heat transfer; defined by Eq. (5)
ϕ = correction factor for nonisothermal flow, dimensionless; defined with Eq. (30)
ψ_i, ψ_o = dimensional factors for evaluation of E_i and E_o; defined with Eqs. (40) and (41)

Subscripts

f = across film or at average film temperature
i = inside pipe or tube, based on average bulk temperature
L = liquid at average liquid temperature
m = mean
o = outside pipe or tube, based on average bulk temperature
oa = overall
opt = optimum conditions
u = utility fluid
v = vapor at average vapor temperature
w = tube or pipe wall, based on temperature at wall surface

PROBLEMS

1. A single-pass shell-and-tube heat exchanger contains 60 steel tubes. The ID of the tubes is 0.732 in., and the OD is 1.0 in. The shell side of the exchanger contains saturated steam at 290°F, and water passes through the tubes. The unit is designed with sufficient tube area to permit 15,000 gph of water to be heated from 70 to 150°F. In the course of this design, an h_d of 1500 $\text{Btu/(h)(ft}^2\text{)(°F)}$ was assumed to allow for scaling on the water side of the tube. The film coefficient for the steam is 2000 $\text{Btu/(h)(ft}^2\text{)(°F)}$. No safety factor other than the one scale value was used in carrying out the exchanger design. Estimate the temperature of saturated steam which must be used when the exchanger is new (i.e., no scale present) if the water enters the unit at a rate of 15,000 gph and is heated from 70 to 150°F.

2. A horizontal heat exchanger has seven steel tubes enclosed in a shell having an ID of 5.0 in. The OD of the tubes is 1.0 in., and the tube wall thickness is 0.10 in. Pure ethyl alcohol flows through the 1.0-in.-OD tubes. The ethyl alcohol enters the unit at 150°F and leaves at 100°F. Water at 70°F enters the shell side of the unit and flows countercurrent to the ethyl alcohol. It is necessary to cool 50,000 lb of ethyl alcohol per hour, and it has been decided to use 100,000 lb of water per hour.

   Under the following conditions, determine the total pumping cost for the two fluids in the exchanger as dollars per year:

   (*a*) There are no baffles, and flow on shell side can be considered to be parallel to the tubes.

(*b*) The outside of the shell is insulated, and there is no heat loss from the shell.
(*c*) The unit operates three hundred 24-h days per year.
(*d*) The efficiency of both pumps is 60 percent.
(*e*) Contraction, expansion, and fitting losses can be accounted for by increasing the straight-section frictional pressure drop by 20 percent.
(*f*) The specific heat of the ethyl alcohol may be assumed to be constant at 0.60. For water, the value may be assumed to be 1.0.
(*g*) The specific gravity of the ethyl alcohol may be assumed constant at 0.77. For water, the value may be assumed to be 1.0.
(*h*) No scale is present, and no safety factor is to be applied to the heat-transfer coefficients.
(*i*) Cost of power is \$0.08 per kilowatthour.

3. A heat exchanger is to be constructed by forming copper tubing into a coil and placing it inside an insulated steel shell. If the following data apply, what should be the length of the coil?
(*a*) Water will flow inside the tubing, and a hydrocarbon vapor will condense on the outside of the tubing.
(*b*) ID of tubing = 0.5 in.
(*c*) OD of tubing = 0.6 in,
(*d*) Condensate rate = 1000 lb/h.
(*e*) Temperature of condensation = 190°F.
(*f*) Heat of vaporization of hydrocarbon at 190°F = 144 Btu/lb.
(*g*) Heat-transfer coefficient for condensing vapor = 250 Btu/(h)(ft^2)(°F).
(*h*) Inlet water temperature = 50°F.
(*i*) Outlet water temperature = 90°F.
(*j*) Heat losses from the shell may be neglected.

4. A solid surface at 1100 K is radiating to a second surface at 330 K. What temperature of the hot surface would be required if it were desired to double the number of joules transmitted per hour, the sink temperature and both emissivities remaining constant? What will be the percentage increase of the radiation coefficient h_r under the changed conditions?

5. A plywood-manufacturing concern is using a binder that requires a temeprature of 180°F in order to obtain adequate holding strength. Large slabs of the plywood at an initial uniform temperature of 70°F are placed in a heater, and the heating unit is maintained at a constant temperature. The manufacturer wishes to pass the slabs through the heater continuously at such a rate that each slab remains in the heater for 15 min. If the slabs are 1 in. thick, determine the minimum constant temperature (i.e., with negligible surface resistance) required for the heater if the temperature at the center of each slab is to reach 180°F before leaving the heater. The following average data apply to the plywood: density = 35 lb/ft^3; thermal conductivity = 0.10 Btu/(h)(ft^2)(°F/ft); heat capacity = 0.50 Btu/(lb)(°F). Consider the plywood as homogeneous and isotropic.

6. A heat exchanger with two tube passes has been proposed for cooling distilled water from 93 to 85°F. The proposed unit contains 160 copper tubes, each $\frac{3}{4}$ in. OD, 18 BWG, and 16 ft long. The tubes are laid out on a $\frac{15}{16}$-in. triangular pitch, and the shell ID is $15\frac{1}{4}$ in. Twenty-five percent cut segmental baffles spaced 1 ft apart are located in the shell. The correction factor F_s, for use in evaluating the outside-tube film coefficient can be assumed to be 1.3. Cooling water at 75°F will be used to remove the

heat, and this fluid will flow through the tubes at a velocity of 6.7 fps. Under these conditions, the fouling coefficient is **2000 Btu/(h)(ft^2)(°F)** for the distilled water and 1000 **Btu/(h)(ft^2)(°F)** for the cooling water. The pressure drop on the tube side and on the shell side cannot exceed 10 psi. Would the proposed unit be satisfactory for cooling 175,000 lb of distilled water per hour?

7. Determine the installed cost, tube length, and number of tubes for the optimum exchanger that will meet the following operating conditions and specifications:
   (a) Twenty thousand pounds of air per hour is cooled from 150 to 100°F inside the tubes of a shell-and-tube exchanger.
   (*b*) Water is used as the cooling medium. The water enters the unit at 70°F and leaves at **100°F.**
   (*c*) All other conditions are the same as those specified in Example 5 of this chapter, except that B_i is not given.
8. Present a detailed derivation of Eq. **(48)** in this chapter, using, as a starting point, any of the other equations that are given.
9. Air, for use in a catalytic oxidation process, is to be heated from 200 to 520°F before entering the oxidation chamber. The heating is accomplished by the product gases, which cool from 720 to 400°F. A steel one-pass shell-and-tube exchanger with crossflow on the shell side will be used. The average absolute pressure on both the shell side and tube side can be assumed to be 10 atm, and the hot gases will pass through the tubes. The exchanger must handle 15,000 lb of the colder gas per hour and operates continuously for 8000 h/year. The properties of the product gases can be considered as identical to air. The cost for power delivered to either gas is $0.08 per kilowatthour. The OD of the tubes is 1.0 in.; the ID is 0.782 in. The tubes will be arranged in line with a square pitch of 1.5 in. All thermal resistances except those of the gas films may be neglected. The safety factor F_s for the outside film coefficient is 1.4. The terms B_o/n_b and N_rN_c/N_t can both be assumed to be equal to 1.0. The cost data presented in Fig. 15-13 apply. Installation costs are 15 percent of the purchased cost, and annual **fixed** charges including maintenance are 20 percent of the installed cost. Under these conditions, estimate the tube length and purchased cost for the optimum exchanger.

CHAPTER
16

MASS-TRANSFER AND REACTOR EQUIPMENT—DESIGN AND COSTS

The transfer of mass from one phase to another is involved in the operations of distillation, absorption, extraction, humidification, adsorption, drying, and crystallization. The principal function of the equipment used for these operations is to permit efficient contact between the phases. Many special types of equipment have been developed that are particularly applicable for use with a given operation, but finite-stage contactors and continuous contactors are the types most commonly encountered. A major part of this chapter, therefore, is devoted to the design aspects and costs of stagewise plate contactors and continuous packed contactors.

Chemical reactions and the equipment in which such reactors are carried out play an important role in chemical process analysis. This involves mass transfer as well as chemical kinetics, and a portion of this chapter deals with the major aspects of chemical reactor design and the costs of related equipment.

The initial cost for the operating equipment includes expenses for foundations, supports, installation, shell and shell internals, insulation, pumps, blowers, piping, heaters, coolers, and other auxiliaries, such as instruments, controls, heat exchangers, or special accessory equipment. Operating costs include power for circulating the fluids, maintenance, labor, cooling water, steam, and unrecovered materials. As illustrated in Chap. 11, a balance can be made among these various costs to yield an optimum economic design.

FINITE-STAGE AND CONTINUOUS CONTACTORS

Because the equipment for a finite-stage contactor consists of a series of interconnected individual units or stages, a study of the overall assembly is best made on the basis of the flow and mass-transfer characteristics in each individual stage. Thus, for a sieve-, valve-, or bubble-cap-tray contactor, each tray can be considered as a separate entity, and the total design requires an analysis of the stepwise operation from one tray to another. In a differential-stage contactor, such as a packed column, the contacting operation can be considered as occurring continuously throughout the unit, since there are no fixed locations where the equipment is divided physically into finite sections. The overall analysis for a differential-stage contactor, therefore, can be based on a differential length or height.

Net mass transfer between two phases can occur only when there is a driving force, such as a concentration difference, between the phases. When equilibrium conditions are attained, the driving force and, consequently, the net rate of mass transfer becomes zero. A state of equilibrium, therefore, represents a theoretical limit for mass-transfer operations. This theoretical limit is used extensively in mass-transfer calculations.

A theoretical stage is defined as a contacting stage in which equilibrium is attained between the various phases involved. Thus, in a sieve-tray column, a theoretical or perfect plate is one in which the liquid leaving the tray is in equilibrium with the gas leaving the tray. The same approach is often used for packed columns, where the HETP is defined as the height of the packed column necessary to give a separation equivalent to one theoretical plate. A more rigorous method for evaluating the performance of a continuous contactor requires a differential treatment of the separation process and gives results that can be expressed in terms of the mass-transfer coefficient or as the number of transfer units. The transfer unit is similar to the theoretical plate, but the transfer unit is based on a differential change in equilibrium conditions and actual concentrations, and the theoretical plate is based on finite changes.

The design of most mass-transfer equipment requires evaluation of the number of theoretical stages or transfer units. Methods for carrying out these calculations for various types of mass-transfer operations are presented in many general chemical engineering books, such as those indicated in the Chemical Engineering Series list of books given at the front of this text.

Because methods for determining theoretical stages and transfer units are covered extensively in the types of references mentioned in the preceding paragraph, the details will not be repeated here. Other important decisions, however, must be made in the design of mass-transfer equipment, and an error in these decisions can be just as detrimental as an error in evaluating the number of theoretical stages. The following sections deal with design factors for finite-stage and differential-stage contactors as related to the direct operational characteristics of the equipment.

FINITE-STAGE CONTACTORS

The most common types of finite-stage contactors are bubble-cap-tray, sieve-tray, or valve-tray units, although turbogrid trays and other speciality types of units are also used in industrial operations. Many of the units constructed in the past have used bubble caps for the contactor, but sieve and valve types of contactors are less expensive than bubble caps and are just as efficient; so bubble caps are now seldom used for new equipment. Nevertheless, it is still worthwhile to consider the aspects of bubble-cap-tray design in some detail, since many of the same principles apply to valve-tray, sieve-tray, and bubble-cap-tray design and large numbers of bubble-cap-tray columns remain in service for which the engineer needs to know design principles in order to understand the operation.

Plate or tray towers are particularly useful as compared to packed towers when fluctuations in the vapor or liquid rate may occur or where major changes in the overall capacity of the column are anticipated. The liquid held on each tray makes the finite-stage contactor useful for cases where time must be allowed for a chemical reaction, as in an absorption tower for producing nitric acid by absorption and aqueous reaction of nitrogen dioxide. The problems of poor liquid or gas distribution encountered at various loading capacities with packed towers can be avoided with plate towers, and plate towers are usually easier to clean when solid deposits are involved. On the other hand, packed towers often can operate with lower pressure drop than plate units which can be a major advantage for high-vacuum service, and the cost for a packed tower capable of doing the equivalent job is often considerably cheaper than that for a plate tower.

Critical factors in the design of finite-stage contactors, other than the determination of the number of stages theoretically necessary for the required operation, are (1) diameter of column so that flooding or excessive entrainment will not occur, (2) the operating efficiency of the trays expressing how close the operation comes to the theoretically perfect tray, and (3) pressure drop generated across each tray. Other factors of importance in the design are appropriate dimensions and form of the contactor assemblies, liquid flow patterns on the trays, entrainment, tray spacing, downcomers for carrying fluid from one tray to another, and tray stability. Detailed empirical equations and other relationships have been developed to aid in designing the finite-stage contacting equipment taking the various factors and types of contactors into account. Rules of thumb are also useful for application in developing preliminary designs. These methods are discussed in the following sections.†

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†For further treatment on design methods for finite-stage contactors used for fractionation including a tabulation of recommended limits on design variables for bubble-cap trays, perforated trays, and valve trays, see M. **VanWinkle,** "Distillation," Chap. 14, McGraw-Hill Book Company, New York, 1967. See also A. P. Economopoulos, Computer Design of Sieve Trays and Tray Columns, *Chem. Eng.,* **85(27):109** (Dec. 4, 1978) and Series of Articles on Distillation Design, *Chem. Eng.,* **95(13):71** (Sept. 26, 1988).

BUBBLE-CAP-TRAY, SIEVE-TRAY, AND VALVE-TRAY UNITS

Examples of the three common forms of finite-stage contactors (bubble cap, sieve, and valve) are shown in Fig. 16-1 which represents a tray tower in operation and illustrates the basic form of each of the three contactors. The tower consists of a series of individual trays, each equipped with a series of contacting units to achieve close contact and resultant mass transfer between a gas phase and a liquid phase. With the bubble-cap contactor, the gas passes upward through the risers into the bubble caps, where the liquid is depressed, permitting the gas to bubble through the slots or notches in the cap into the liquid. As the bubbles are dispersed into and rise through the liquid on the tray, a large amount of interfacial area exists between the gas and liquid phases, thereby permitting effective mass transfer. Liquid flows downward from tray to tray through downcomers, and the necessary gas-liquid contact is made as the liquid passes across each tray. Figures 16-2 to 16-4 show typical forms of bubble caps and operations including effects of excessive liquid gradient while Fig. 16-5 shows examples of liquid-flow patterns for finite-stage trays.

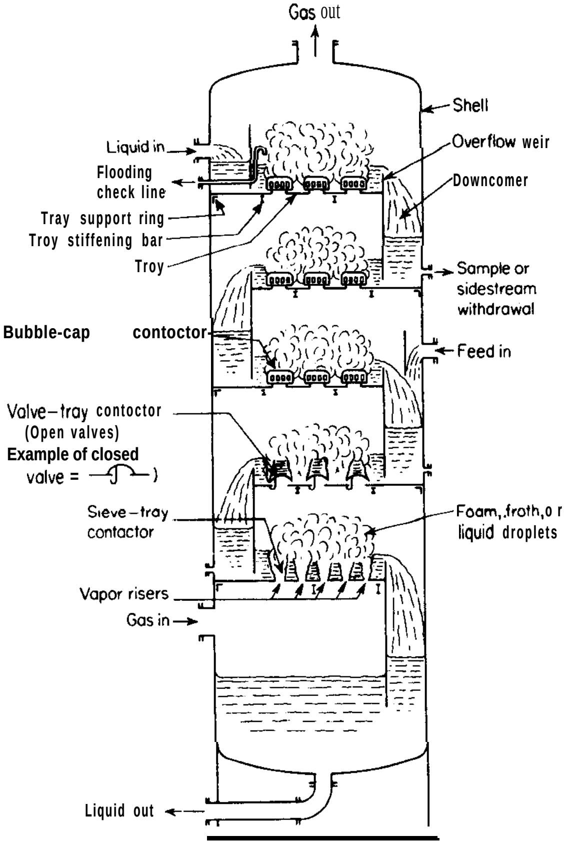


FIGURE 16-1
Cross-sectional view of finite-stage contactor tower in operation showing an example of a sieve tray, a valve tray, and a bubble-cap tray.

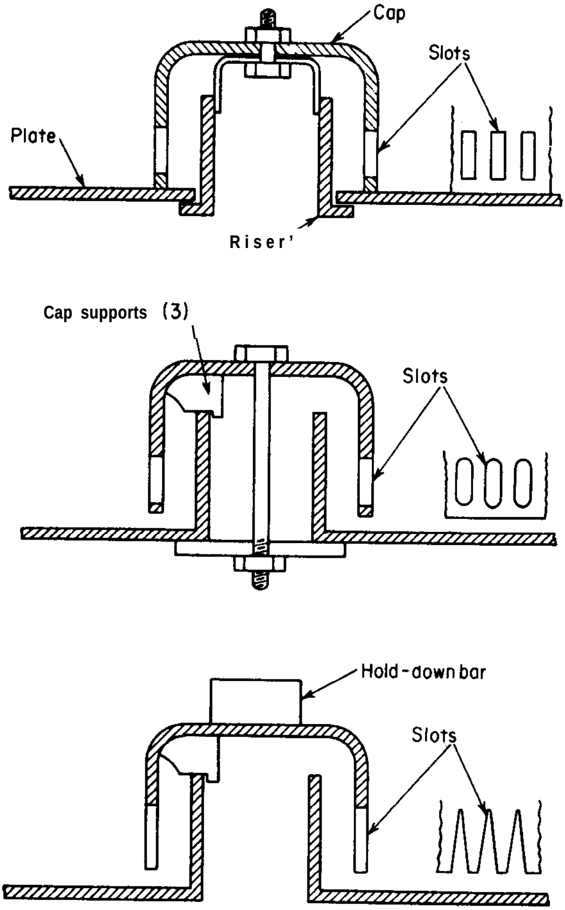


FIGURE **16-2**
Methods for holding bubble caps in place.

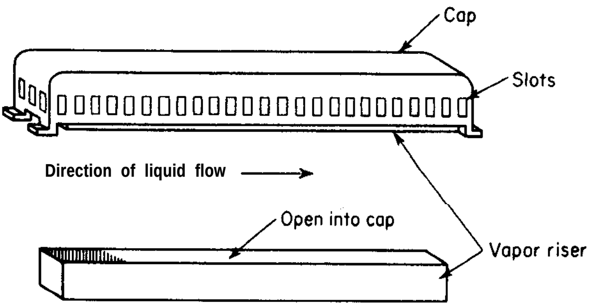


FIGURE 163
Tunnel cap.

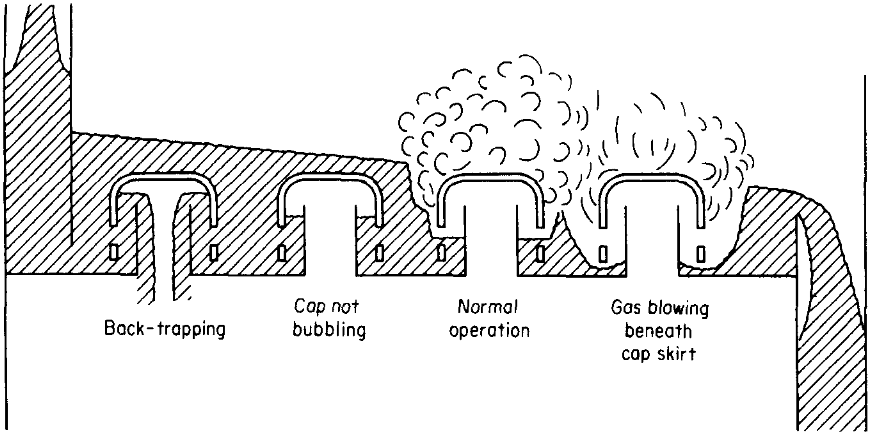
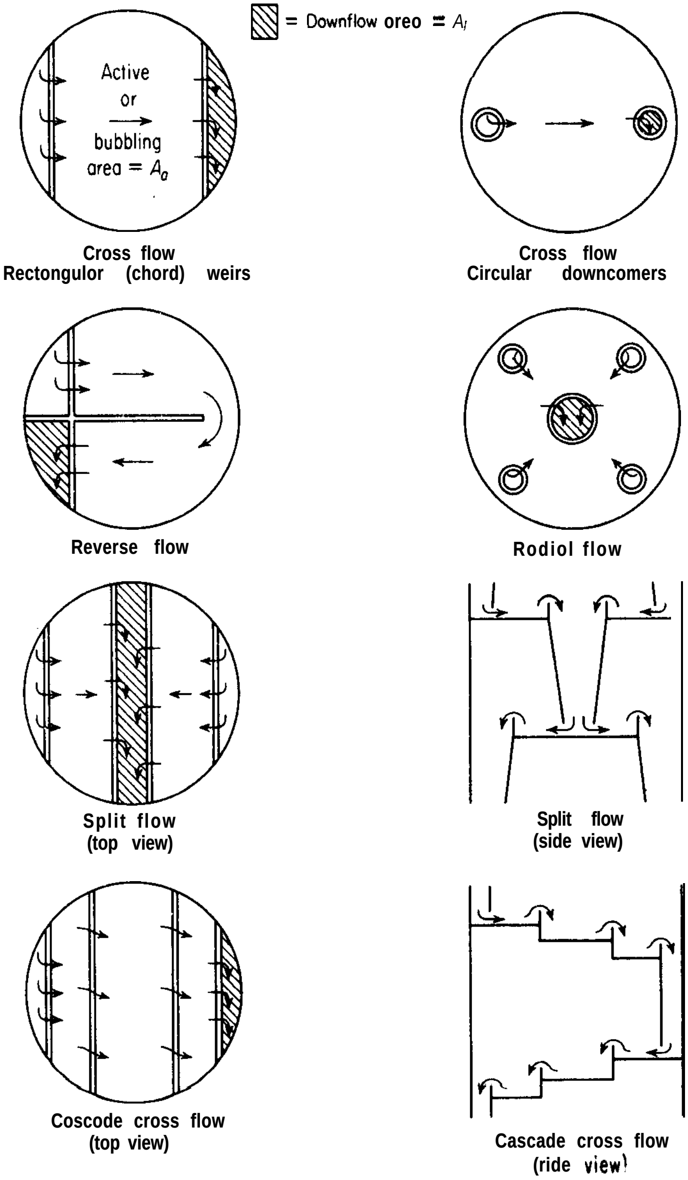


FIGURE 16-4
Cross-sectional view of bubble-cap tower showing effect of excessive liquid gradient.

The sieve-tray contactor shown in Fig. 16-1 is presented in a form known as a *crossflow plate contactor.* The tray consists of a flat plate perforated with many small holes that are drilled or punched in a size range of $\frac{1}{8}$ to $\frac{1}{2}$ in. diameter. Liquid flows across the plate as shown in Fig. 16-1, through the froth or spray which develops, and passes over a weir into the downcomer leading to the tray below. The upward flow of the vapor keeps the liquid from flowing through the holes, and the operation of the tray is basically the same as that of a bubble-cap tray. If the flow of gas is low, some or all of the liquid may drain down through the perforations so that some of the contacting areas may be bypassed. If the entire transfer of the liquid from one tray to another is by this so-called "weeping" action with no downcomer being used, the type of unit is designated as a *counterflow plate contactor.*

Because best results are normally obtained with full crossflow plate operation for a sieve tray, units may be designed with a lift valve over the hole in the plate or over a riser from the plate so that the rising gas lifts this valve to allow the vapors to be passed horizontally into the liquid as is illustrated in Fig. 16-1. This liquid cannot easily flow back down the holes in the plate when the gas flow is low because the valve tends to close with the reduced gas flow.

Many modifications of the three types of contactors just discussed have been developed in an effort to reduce costs, reduce pressure drop, equalize vapor flow through each contactor, increase plate efficiencies, or, in general, improve the operating performance of the tower. An example of this for modification of bubble-cap towers is the old *Uniflux tray* originally developed by Socony-Vacuum, which consisted of a series of interlocking S-shaped sections which were assembled in the form of tunnel caps with slot outlets on one side only. Segmental downcomers, similar to the downcomers in conventional bubble-cap columns, were provided. The vapors issued from the Uniflux caps in



Cross flow
Rectangular (chord) weirs

Cross flow
Circular downcomers

Reverse flow

Radial flow

Split flow
(top view)

Split flow
(side view)

Cascade cross flow
(top view)

Cascade cross flow
(side view)

FIGURE 16-5
Liquid flow patterns for different types of finite-stage trays.

only one direction in such a manner as to give equal vapor distribution among the caps. Even though the simplified construction resulted in costs 30 to 50 percent less than for equivalent bubble-cap trays of conventional design while providing large capacities and good plate efficiencies, the Uniflux tray has not been able to compete effectively with sieve trays and is no longer being fabricated.

Various types of valve arrangements are used for the valve-tray towers. Some of these are liftable disk-type valves which come as a cap with each riser. As the vapor rate increases, the valve lifts to provide a larger opening until it reaches a limit point which still directs the vapors horizontally into the liquid. Float-valve trays are designed to operate on the same principle as the liftable disk-type valves except that a floating pivotal valve is used in each cap.

A variation of the normal sieve tray is the *Turbogrid* tray developed by the Shell Development Company. The Turbogrid tray uses slots instead of circular holes and operates as a counterflow plate unit with no downcomers. A typical tray consists of a flat grid of parallel slots. The slots can be stamped perforations in a flat metal sheet or the open spaces between parallel bars.

MAXIMUM ALLOWABLE VAPOR VELOCITIES

The vapor velocity in a finite-stage contactor column can be limited by the liquid handling capacity of the downcomers or by entrainment of liquid droplets in the rising gases. In most cases, however, downcomer limitations do not set the allowable vapor velocity; instead, the common design basis for choosing allowable vapor velocities is a function of the amount of gas entrainment which can result in improper operation or flooding of the column.

A tower must have sufficient cross-sectional area to handle the rising gases without excessive carry-over of liquid from one tray to another. By assuming that the frictional drag of the vapor on suspended liquid droplets should not exceed the average weight of a droplet, Souders and Brown† derived the following equation applicable for any specific location in the column:

$$V_m = K_v\sqrt{\frac{\rho_L - \rho_G}{\rho_G}} \tag{1‡}$$

where V_m = maximum allowable superficial vapor velocity (based on cross-sectional area of empty tower), ft/s, and K_v = an empirical constant, ft/s. An alternative form of Eq. (1) in terms of mass velocity follows:

$$G_m = V_m\rho_G = K_v\sqrt{\rho_G(\rho_L - \rho_G)} \tag{2}$$

where G_m = maximum allowable mass velocity of vapor, lb/(s)(ft^2).

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†M. Souders and G. G. Brown, *Ind. Eng. Chem.*, **26**:98 (1934).

‡As a general rule of thumb, V_m should be near 4 ft/s for finite-stage fractionation towers operating at atmospheric pressure.

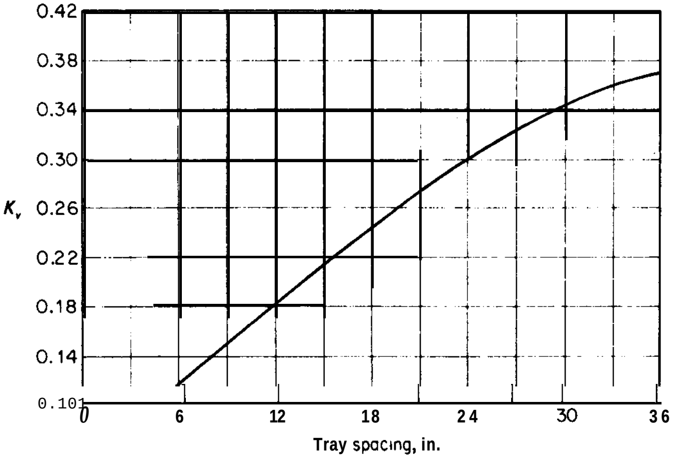

FIGURE 16-6
Rough estimate values (±25%) for K_v in Eq. (1) for maximum allowable velocities in finite-stage towers *(weir height less than 15% of plate spacing).*

Equation (1) or (2) can be used as an empirical guide for estimating the maximum vapor velocities in plate columns. The constant K_v is a major function of the plate spacing and also varies to a lesser extent with depth of liquid on the tray, ratio of liquid flow rate to gas flow rate, surface tension of the liquid, density of the gas and liquid, and physical arrangement of the tray components.

While it is always best to obtain values for the constant K_v based on data obtained with fluids, equipment, and operating pressures and temperatures similar to those involved in the particular design, a rough approximation of K_v can be obtained from Fig. 16-6.† This figure gives values of K_v as a function of tray spacing only and should not be relied on for better than ±25 percent for maximum allowable velocities.

An alternate approach for estimating maximum allowable velocities has been presented by Fair (see reference given in footnote for preceding paragraph) which is based on data obtained with sieve-tray and other types of finite-stage columns and takes into account the effect of surface tension of the liquid in the column, the ratio of the liquid flow rate to the gas flow rate, gas and liquid densities, and dimensions and arrangement of the contactor.‡ In this method, the basic equation for the maximum allowable vapor velocity, equiva-

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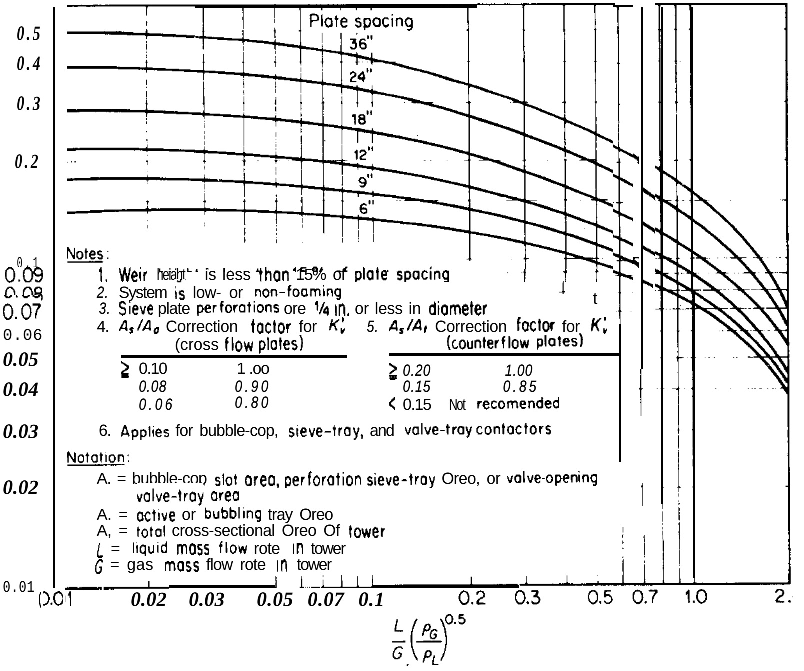
†Figure 16-6 is adapted from values of constants presented by J. R. Fair, *Petro/Chem. Eng.*, 33(10):45 (1961) and gives K_v values that are much less conservative than the original values recommended by Souders and Brown.

‡See also R. H. Perry and D. Green, "Chemical Engineers Handbook," 6th ed., Sec. 18, McGraw-Hill Book Company, New York, 1984.



Notes:

1. Weir height is less than 15% of plate spacing
2. System is low- or non-foaming
3. Sieve plate perforations are 1/4 in. or less in diameter
4. A_s/A_a Correction factor for K'_v (cross flow plates)

≥ 0.10	1.00
0.08	0.90
0.06	0.80

5. A_s/A_t Correction factor for K'_v (counterflow plates)

≥ 0.20	1.00
0.15	0.85
< 0.15	Not recomended

6. Applies for bubble-cap, sieve-tray, and valve-tray contactors

Notation:

A_s = bubble-cap slot area, perforation sieve-tray area, or valve-opening valve-tray area
A_a = active or bubbling tray area
A_t = total cross-sectional area of tower
L = liquid mass flow rate in tower
G = gas mass flow rate in tower

FIGURE 16-7
Chart for estimating values of K_v (± 10%) in Eq. (3). [J. R. Fair, *Petro/Chem. Eng.*, **33**(10):45 (1961). *With* permission.]

lent to Eq. (1), is

$$V'_m = K'_v\left(\frac{\sigma}{20}\right)^{0.2}\sqrt{\frac{\rho_L-\rho_G}{\rho_G}} \tag{3}$$

where V'_m = maximum allowable vapor velocity based on net area for vapor flow which is usually the active or bubbling cross-sectional area of the tower (A_a in Fig. 16-5) plus the area of one downcomer (A, in Fig. 16-5), ft/s
K'_v = an empirical constant, ft/s
σ = surface tension of the liquid in the tower, dyn/cm.

Figure 16-7 presents values of K'_v as a function of plate spacing and also presents information on limitation of the applicability of the results.

Although the allowable velocity given by Eq. (1) may be conservative for many types of operations while Eq. (3) is somewhat less conservative due to the use of the active or bubbling cross-sectional area of the tower along with the other corrections, many engineers employ vapor velocities in the range of 65 to 80 percent of V_m or V'_m to make certain their column will be operable. The actual velocity to be used is applied to set the diameter of the column. The additional dimensions of the trays are then established, and an investigation of other limiting factors, such as downcomer capacity, should be made.

In many cases, the vapor rate changes over the length of the tower, and the theoretical diameter based on the allowable vapor velocity varies. Occasionally, two different diameters are used for different sections of one tower. Cost considerations, however, usually make it impractical to vary the diameter, and the constant diameter should be based on the tower location where allowable velocity and throughput rates require the largest diameter.

Example 1 Determination of distillation-column diameter on basis of allowable vapor velocity. A sieve-tray distillation tower is to be operated under the following conditions:

	At top of tower	At bottom of tower
Liquid rate	245 lb mol/h	273 lb mol/h
Vapor rate	270 lb mol/h	310 lb mol/h
Vapor molecular weight	70	110
Temperature	220°F	260°F
Pressure	1 .0 atm	1.1 atm
Liquid density	44 lb/ft³	42 lb/ft³
Liquid surface tension	20 dyn/cm	20 dyn/cm

The tray spacing is 24 in. with a weir height of 3 in. or 12.5 percent of the tray spacing. The ideal-gas law can be assumed as applicable to the vapors, and the system is nonfoaming. Total area of sieve holes (A,) is 10 percent of the active or bubbling tray area (A,). A rough rule of thumb is that downcomer area (A, in Fig. 16-5) is 5 percent of total column cross-sectional area ($2A_d$ + A, in Fig. 16-5). The molecular weights of the liquid and the gas may be assumed as identical at any given point in the column. If the tower diameter is to remain constant over the entire length, compare the minimum diameter estimated using Fig. 16-6 with that obtained by using Fig. 16-7.

Solution. From Fig. 16-6, $K_v = 0.30$. At top of tower,

$$\rho_G = \frac{(70)(492)}{(359)(680)} = 0.141\ \text{lb/ft}^3$$

$$\rho_L = 44\ \text{lb/ft}^3$$

$$V_m = 0.30\sqrt{\frac{44 - 0.141}{0.141}} = 5.29\ \text{ft/s based on total cross-sectional area}$$

$$\text{Minimum diameter} = \left[\frac{(70)(270)(4)}{(0.141)(\pi)(5.29)(3600)}\right]^{1/2} = 3.0\ \text{ft}$$

At bottom of tower,

$$\rho_G = \frac{(110)(492)(1.1)}{(359)(720)(1.0)} = 0.23 \text{ lb/ft}^3$$

$$\rho_L = 42 \text{ lb/ft}^3$$

$$V_m = 0.30\sqrt{\frac{42 - 0.23}{0.23}} = 4.04 \text{ ft/s}$$

$$\text{Minimum diameter} = \left[\frac{(110)(310)(4)}{(0.23)(\pi)(4.04)(3600)}\right]^{1/2} = 3.6 \text{ ft}$$

The limiting diameter occurs at the bottom of the tower; therefore, the minimum diameter based on the maximum allowable vapor velocity is 3.6 ft obtained with the use of Fig. 16-6.

Figure 16-7 can be used to obtain K_v' directly since $A_s/A_a = 0.10$, weir height is 12.5 percent of tray spacing, and system is nonfoaming.

At top of tower,

$$\frac{L}{G}\left(\frac{\rho_G}{\rho_L}\right)^{0.5} = \frac{245}{270}\left(\frac{0.141}{44}\right)^{0.5} = 0.051$$

From Fig. 16-7, $K_v' = 0.36$

$$V_m' = (0.36)\left(\frac{20}{20}\right)^{0.2}\sqrt{\frac{44 - 0.141}{0.141}}$$

$= 6.35$ ft/s based on active area plus area of one downcomer

With rule of thumb that downcomer area $(A,)$ = 5 percent of total cross-sectional area,

$$V_m = (6.35)\left(\frac{100 - 5}{100}\right) = 6.0 \text{ ft/s based on total cross-sectional area}$$

$$\text{Minimum diameter} = \left[\frac{(70)(270)(4)}{(0.141)(\pi)(6.0)(3600)}\right]^{1/2} = 2.8 \text{ ft}$$

At bottom of tower,

$$\frac{L}{G}\left(\frac{\rho_G}{\rho_L}\right)^{0.5} = \frac{273}{310}\left(\frac{0.23}{42}\right)^{0.5} = 0.065$$

From Fig. 16-7, $K_v' = 0.35$

$$V_m' = (0.35)\left(\frac{20}{20}\right)^{0.2}\sqrt{\frac{42 - 0.23}{0.23}}$$

$= 4.72$ ft/s based on active area plus area of one downcomer

$$V_m = (4.72)\left(\frac{100-5}{100}\right)$$

$$= 4.5 \text{ ft/s based on total cross-sectional area}$$

$$\text{Minimum diameter} = \left[\frac{(110)(310)(4)}{(0.23)(\pi)(4.5)(3600)}\right]^{1/2} = 3.5 \text{ ft}$$

The limiting diameter occurs at the bottom of the tower; therefore, the minimum diameter based on the maximum allowable vapor velocity obtained with use of Fig. 16-7 is 3.5 ft compared to 3.6 ft obtained with use of Fig. 16-6.

PLATE AND COLUMN EFFICIENCIES

As discussed in the first part of this chapter, the design of mass-transfer equipment often requires evaluation of the number of theoretical stages necessary to accomplish a desired separation. To complete the design, information must be available that shows the relationship between these ideal values and the actual performance of the equipment. The translation of ideal stages into actual finite stages can be accomplished by the use of plate efficiencies.

Types of Plate Efficiency

Three hinds of plate efficiencies may be used for expressing the relationship between the performance of theoretical and actual stages. They are (1) overall column efficiency or overall plate efficiency, (2) Murphree plate efficiency, and (3) point efficiency or local efficiency.

The *overall column efficiency* applies to the total number of stages and is defined as the number of theoretical stages required to produce a given separation divided by the number of stages actually necessary to produce the same separation. The *Murphree plate efficiency* applies to a single plate. It is defined as the ratio of the actual change in average vapor composition accomplished by a given plate to the change in average vapor composition if the vapors leaving the plate were in equilibrium with the liquid leaving the plate. *The point efficiency* is similar to the Murphree plate efficiency, except that the point efficiency applies to a single location on a given tray.

Although the overall column efficiency has no fundamental mass-transfer basis, it is widely used because of its simplicity. The number of actual stages required for a given separation is simply equal to the number of theoretical stages divided by the overall column efficiency. The Murphree plate efficiency is more fundamental than the overall value but is less convenient to use because it must be applied to each individual plate. The point efficiency is of considerable theoretical interest but is seldom used in design practice because it requires knowledge of the variations in liquid composition across the tray and integration of the point efficiencies over the entire tray. Point efficiencies are always less than 100 percent, but Murphree plate efficiencies may be greater than 100

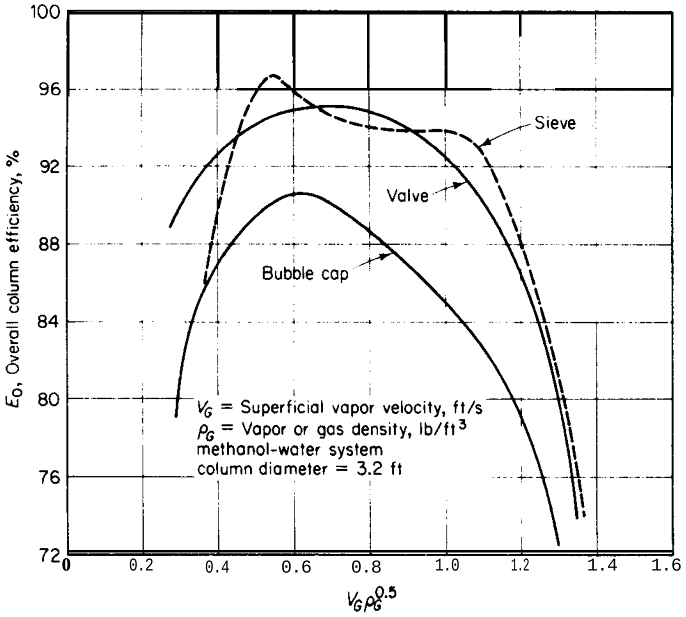


FIGURE 16-8
Comparison of column efficiency for bubble-cap, sieve, and valve finite-stage contactors.

percent and are usually greater than point efficiencies on the same tray because of variation in liquid composition across the tray.

Factors Influencing Plate and Column Efficiencies

A comparison of overall column efficiencies for bubble-cap, sieve, and valve finite-stage contactors is presented in Fig. 16-8 which also shows the effects of the superficial vapor velocity (based on cross-sectional area of empty tower) and the gas density.? In general, the three types of contactors give plate efficiencies in the range of 80 to 90 percent when the column is operated at appropriate conditions with sieve and valve contactors generally giving slightly higher efficiencies than bubble-cap contactors. The major factors that influence plate and column efficiencies are discussed in the following:

VAPOR VELOCITY. As shown in Fig. 16-8, efficiencies improve with increase in superficial vapor velocity when the velocity is low, reach a fairly constant value over the range of velocities corresponding to normal acceptable operation, and

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[†] **Kastenek** et *al.*, *Proc. Intern. Symp. Distillation 1969,* Institute of Chemical Engineers, London (1969).

then decrease at higher velocities. Vapor slot velocities appear to have no appreciable effect on efficiency if they are kept in the range indicated by Eqs. (20) and (21) as presented later in this chapter.

LIQUID DEPTH ABOVE VAPOR OPENINGS. Increase in depth of the liquid above the vapor openings tends to increase the efficiency, especially if this liquid depth is less than 1 in.

PLATE SPACING. Because of entrainment carry-over, the effect of plate spacing is related to the superficial vapor velocity. Too small a plate spacing can cause a low efficiency if the vapor velocity is greater than the allowable value.

LENGTH OF LIQUID PATH. The length of the liquid path across a tray is an important factor in determining the degree of liquid concentration gradient across the tray. Thus, if the length of the liquid path is long enough that an appreciable liquid concentration gradient is established, the Murphree plate efficiency is greater than the point efficiency. In general, as the length of the liquid path is increased, the overall column efficiency increases. The effect of liquid-path length is usually negligible if the length is less than 5 ft, but increasing the length to 10 to 15 ft may increase the overall column efficiency by 20 to 40 percent.

LIQUID RESISTANCE TO INTERPHASE MASS TRANSFER. Liquid viscosity, gas solubility in absorbers, and relative volatility in rectification columns are important factors in determining the liquid resistance to interphase mass transfer. Increase in liquid viscosity, decrease in gas solubility for absorbers, and increase in relative volatility for rectification columns cause an increase in the liquid resistance to interphase mass transfer and a resultant reduction in plate efficiency. The ratio of the liquid rate to the gas rate influences the relative importance of the liquid resistance to interphase mass transfer. An increase in the ratio of liquid rate to gas rate reduces the importance of the liquid resistance and can cause an increase in the plate efficiency.

OTHER FACTORS. Design details of the column, such as vapor-opening dimensions, plate layout, or the total number of trays can have an effect on the efficiencies. The exact influence of these factors is best determined by experimental tests.

In multicomponent mass-transfer operations, the assumption is usually made that the same plate efficiency applies to all components being separated. The overall column efficiency is then considered in terms of the key components, and the same efficiency is assumed for the lighter and heavier components. This assumption is not necessarily correct because of the different properties of the components. More exact results can be obtained by using Murphree plate efficiencies and accounting for the difference in efficiencies.

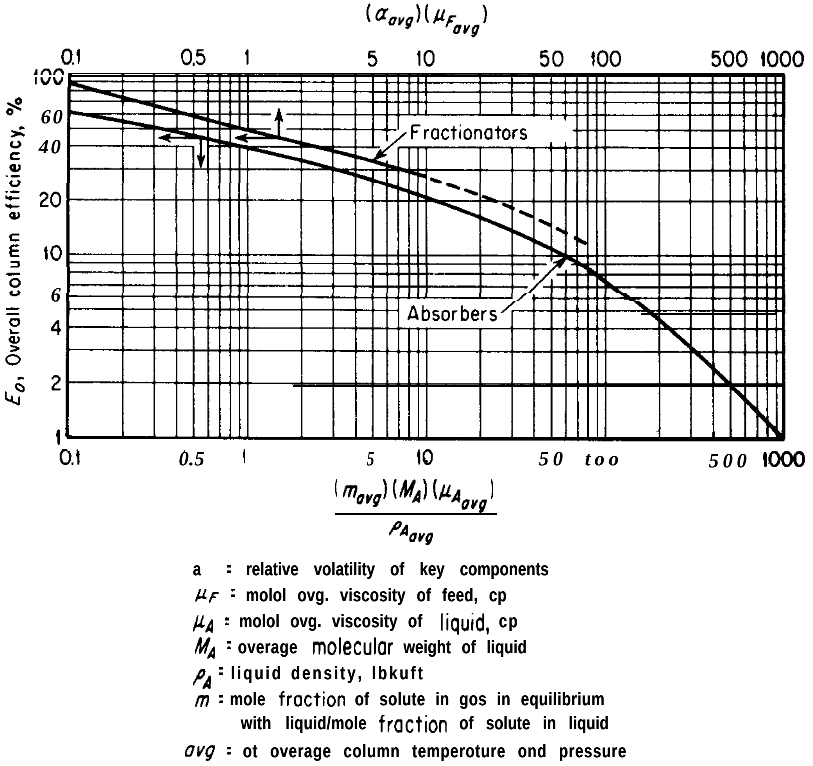

FIGURE 16-9
Overall column efficiencies for finite-stage contactor fractionators and absorbers.

Correlations for Estimation of Plate Efficiencies

Overall column efficiencies are based on performance data, and no exact correlation of the results obtained with various mixtures and types of columns can be presented. Many generalized correlations have been developed, however, and these are useful for making design estimates when no other data are available.

For standard types of finite-stage contactor columns operated in the range of allowable velocities where the overall column efficiencies are essentially constant, O'Connell has correlated efficiency data on the basis of liquid viscosity and relative volatility (or gas solubility).† The results for fractionators and absorbers are presented in Fig. 16-9. This correlation is based, primarily, on experimental data obtained with bubble-cap columns having a liquid path of less than 5 ft and operated at a reflux ratio near the minimum value. Figure 16-9 is adequate for design estimates with most types of commercial equipment and

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†H. E. O'Connell, *Trans. AIChE*, **42**:751 (1946).

mixtures, although efficiencies determined directly from equipment operating near the conditions involved in the design are always to be preferred.

The correlation shown by Fig. 16-9 can be extended to include the effects of liquid submergence and the ratio of liquid flow rate to vapor flow rate by use of the following simplified equations for estimating overall column efficiencies:†

For fractionators,

$$\log E_o = 1.67 - 0.25 \log (\mu_{F,\text{avg}} \alpha_{\text{avg}}) + 0.30 \log \frac{L'_M}{V'_M} + 0.09 \left(S_m + \frac{c}{2} \right) \qquad (4)$$

For absorbers,

$$\log E_o = 1.60 - 0.38 \log \frac{m_{\text{avg}} M_A \mu_{A,\text{avg}}}{\rho_{A,\text{avg}}} + 0.25 \log \frac{L'_M}{V'_M} + 0.09 \left(S_m + \frac{c}{2} \right) \qquad (5)$$

where E_o = overall column efficiency, percent
S_m = static submergence, ft
c = slot height, ft
L'_M = molal liquid flow rate, lb mol/h
V'_M = molal vapor flow rate, lb mol/h

$\mu_{F,\text{avg}}$, α_{avg}, m_{avg}, M_A, $\mu_{A,\text{avg}}$, $\rho_{A,\text{avg}}$ are defined in Fig. 16-9.

Equations (4) and (5) are based on the relationships shown by Fig. 16-9 plus additional experimental results on the effects of liquid submergence and the ratio L'_M/V'_M. Use of these equations should be limited to conditions in which the ratio L'_M/V'_M is in the range of 0.4 to 8 and the static submergence is less than 1.5 in.

A simplified empirical equation for finite-stage contactor columns operating on petroleum and similar hydrocarbons has been presented by Drickamer and Bradford.* Their results are based on plant tests with 54 refinery columns used for distillation or absorption of hydrocarbons. The columns were of the standard bubble-cap or perforated-plate type operated under typical refinery conditions. The results were correlated on the basis of the single variable, liquid viscosity, to give

$$E_o = 17 - 61.1 \log \mu_{F,\text{avg}} \qquad (6)$$

Use of Eq. (6) should be limited to commercial towers for which no other data are available. It gives adequate results for the fractionation of petroleum and similar hydrocarbons, but it is not recommended if the relative volatility of the key components is greater than 4.0 or if the value of $\mu_{F,\text{avg}}$ is outside the range of 0.07 to 1.4 centipoises.

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†J. C. Chu et al., *J. Appl. Chem.*, 1529 (1951). For sieve-tray units, replace $(S_m + c/2)$ by the weir height. For valve-tray units, replace $(S_m + c/2)$ by liquid height above the base of the valve opening.

‡H. G. Drickamer and J. R. Bradford, *Trans. AIChE*, **39**:319 (1943); *Petrol. Refiner*, **22**(10):105 (1943).

Example 2 Estimation of overall column efficiency. A continuous fractionation unit has been designed to operate on a liquid feed containing components, *A*, *B*, C, and *D*. Calculations have shown that 20 theoretical stages are necessary in the column, not including the reboiler. On the basis of the following data, estimate the overall column efficiency and the number of actual trays needed in the column by (a) Fig. 16-9, (*b*) Eq. (4), and (*c*) Eq. (6):

Component	Mole fraction in			Viscosity of liquid at 260°F, centipoises
	Feed	Overhead	Bottoms	
A	0.10	0.25		0.040
B	0.30	0.70	0.03	0.110
C	0.40	0.05	0.64	0.138
D	0.20		0.33	0.175

Materials *B* and *C* are considered as the key components.

Relative volatility of the key components is independent of concentration and equals 1.94 at 260°F and the average column pressure. Feed temperature = 85°F. Overhead temperature = 240°F. Bottoms temperature = 280°F.

The fractionating unit is a sieve-tray column with a standard type of tray design. The vapor velocity through the tower is about 90 percent of the maximum allowable value.

Tower diameter = 4.8 ft. Weir height = 2 in. The ratio L'_M/V'_M is 0.7 in the rectifying section of the column and 1.2 in the stripping section.

Solution. Average column temperature = (280 + 240)/2 = 260°F.

Molal average viscosity of feed at average column temperature = $\mu_{F,\text{avg}}$ = (0.040)(0.10) + (0.110)(0.30) + (0.138)(0.40) + (0.175)(0.20) = 0.127 centipoise.

Average relative volatility of key components at average column temperature = α_{avg} = 1.94.

Since L'_M/V'_m is different in the rectifying and stripping sections, an average value must be used, or else a separate efficiency for each section can be obtained. In this case an arithmetic-average value is adequate:

$$\frac{L'_M}{V'_M} = \frac{0.7 + 1.2}{2} = 0.95$$

$$\text{Weir height} = \frac{2}{12} = 0.167 \text{ ft.}$$

(a) Efficiency and total number of trays by Fig. 16-9:

$$(\alpha_{\text{avg}})(\mu_{F,\text{avg}}) = (1.94)(0.127) = 0.246$$

From Fig. 16-9,

$$E_o = 70\%$$

$$\text{Number of actual trays needed in column} = \frac{20}{0.7} = 29$$

(*b*) Efficiency and total number of trays by Eq. (4):

$$\log E_o = 1.67 - 0.25 \log 0.246 + 0.3 \log 0.95 + 0.09(0.167)$$

$$E_o = 68\%$$

$$\text{Number of actual trays needed in column} = \frac{20}{0.68} = 29$$

(*c*) Efficiency and total number of trays by Eq. (6):

$$E_o = 17 - 61.1 \log 0.127 = 72\%$$

$$\text{Number of actual trays needed in column} = \frac{20}{0.72} = 28$$

PRESSURE DROP OVER FINITE-STAGE CONTACTORS

As the gas passes through a finite-stage contactor, the pressure of the gas decreases because of the following causes:

1. Pressure drop through the contactor assembly
    a. Contraction as the gas enters the riser or sieve opening
    b. Friction in the riser or sieve opening and in the annular space if a bubble-cap unit is used
    c. Friction due to change in direction of gas flow for bubble-cap and valve units
    d. Passage of the gas through the slots for a bubble-cap unit
2. Pressure drop due to liquid head above slots, sieve openings, or valve openings.

Because the pressure drop is uniform throughout the gas space above or below a tray, the gas pressure drop across a given tray must be the same irrespective of the location on the tray. A design estimate of the pressure drop, therefore, can be obtained on the basis of an average contactor assembly, such as a bubble-cap, a sieve tray, or a valve unit. Because of the liquid gradient which may exist with bubble-cap or valve-tray units, the rate of gas flow through the individual caps or valves may vary from the liquid-inlet to the liquid-outlet location on the tray. For a stable plate, an average contactor assembly for use in pressure-drop calculations is one located at the point of average liquid gradient. The flow rate of gas through this assembly can then be assumed to be the flow rate per assembly assuming each assembly delivers the same amount of gas.

In making pressure-drop calculations for bubble-cap, sieve-tray, or valve contactor units, the same general principles are applied with the only differences being based on the geometrical arrangement of the individual contacting units. A rule of thumb for correctly designed bubble caps or valve trays is that the total pressure drop per tray will be about two times the pressure drop equivalent to the average head of liquid above the top of the bubble-cap slots or the top portion of the valve opening. The equivalent rule of thumb for total

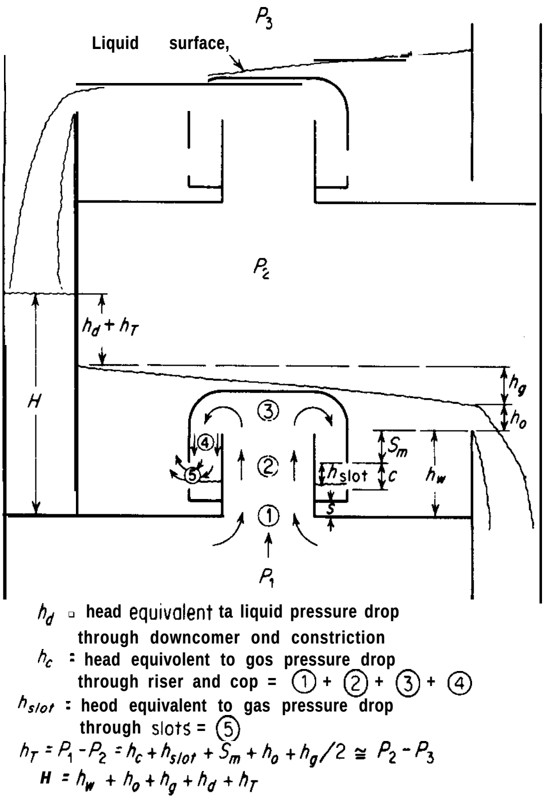


FIGURE 16.10
Cross-sectional view of bubble-cap tower showing flow and nomenclature for pressure-drop calculations. (Units of all symbols are feet of liquid.)

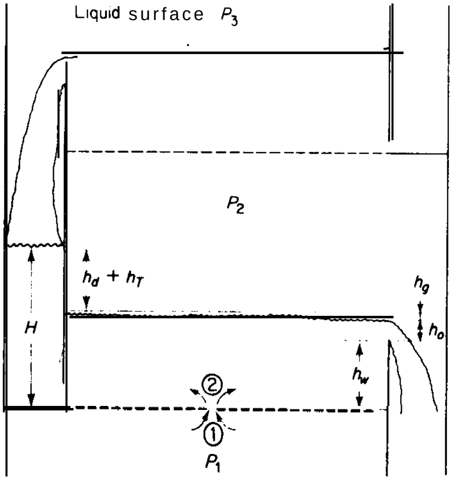
pressure drop over a correctly designed sieve tray is to multiply the total head of liquid on the tray by 2.0 to give the liquid head equivalent to the total pressure drop for that plate.

For a typical bubble-cap column, the following pressure drops per tray are considered reasonable, and they also would be order-of-magnitude values for sieve trays or valve trays:

Total pressure	Pressure drop per tray
30 mm Hg	3 mm Hg or less
1 atm	0.07-0.12 psi
300 psi	0.15 psi

Figures 16-10 and 16-11 present a cross-sectional view of an average bubble cap and a typical sieve tray showing the various causes of pressure drop expressed as liquid head equivalent to the pressure drop. The total pressure drop across the tray in U.S. conventional pressure units is related to the total liquid head h_T (expressed in feet) by the following equation:

$$\Delta p_T = \frac{h_T \rho_L g}{144 g_c} \tag{7}$$



h_d = head equivalent to liquid pressure drop through downcomer and constriction
h_c = head equivalent to gas pressure drop through the holes in the sieve tray = 1 + ②
$h_T = P_1 - P_2 = h_c + h_w\ t\ h_o + h_g/2 \cong P_2 - 8$ ③
$H = h_w + h_o + h_g + h_d + h_T$

FIGURE 16-11
Cross-sectional view of sieve-tray tower showing flow and nomenclature for pressure-drop calculations. (Units of all symbols are feet of liquid.)

where Ap, = total pressure drop of gas across tray, psi
ρ_L = density of the liquid, lb/ft^3
g = local acceleration due to gravity, ft/(s)(s)
g_c = conversion factor in Newton's law of motion, 32.17 ft · lbm/(s)(s)(lbf)

In most cases, g and g_c are considered to be numerically equal.

PRESSURE DROP THROUGH THE CONTACTOR ASSEMBLY. Causes for pressure drop through the contactor assembly are shown in Fig. 16-10 for bubble caps as (1) contraction, (2) friction in riser, (3) reversal of flow direction, and (4) friction in annular space. Similarly, Fig. 16-11 shows for sieve trays that this cause for pressure drop is (1) contraction and (2) friction in the sieve hole. The total pressure drop due to the preceding causes is primarily a function of the kinetic head. The pressure drop as feet of liquid equivalent to one kinetic head is

$$h_H = \frac{V_c^2}{2g}\frac{\rho_G}{\rho_L} \tag{8}$$

where V_c = maximum linear velocity of gas in riser, reversal area, annulus, or sieve hole, ft/s
ρ_G = gas density, lb/ft^3
ρ_L = density of liquid, lb/ft^3
g = local acceleration due to gravity, ft/(s)(s)

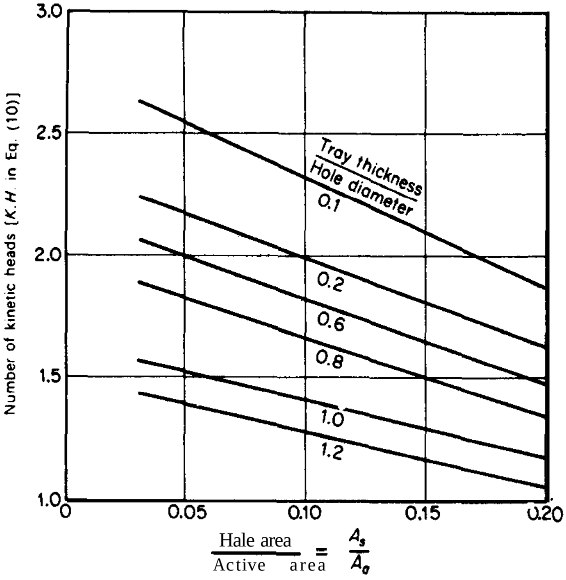


FIGURE **16-12**
Chart for estimation of number of kinetic heads for calculation of pressure drop due to gas flow through sieve tray holes, h_c in Eq. (10). ($\frac{3}{16}$-in. diameter holes.)

The total pressure drop through the riser and bubble cap is usually in the range of four to eight kinetic heads, depending on the cap design. A factor of six kinetic heads is a reasonable average figure. With this factor, the pressure drop due to gas flow through the riser and bubble cap expressed as liquid head is

$$h_c = \frac{3V_c^2}{g}\frac{\rho_G}{\rho_L} \tag{9}$$

For sieve trays, the number of kinetic heads equivalent to the total pressure drop through the plate itself is a function of the ratio of the sieve-hole diameter to the tray thickness and the ratio of the hole area per tray to the active area per tray as shown in Fig. 16-5. This pressure drop for a reasonable sieve-tray design is generally in the range of 1 to 3 kinetic heads, and Fig. 16-12 can be used to choose the most reasonable number to use in preliminary designs.[†] Designating the number of kinetic heads obtained from Fig. 16-12 as *K.H.*, the pressure drop due to gas flow through the holes for a sieve tray expressed as liquid head is

$$h_c = (K.H.)\frac{V_c^2\rho_G}{2g\rho_L} \tag{10}$$

It should be noted that this sieve-tray value for pressure drop through the contactor assembly is considerably less than for the equivalent case of bubble

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[†] Adapted from I. Liebson, R. Kelley, and L. Bullington, How to Design Perforated Trays, *Petrol. Refiner*, **36(2):127** (1957); **36(3):288** (19.57).

caps with the same gas velocity in the cap as in the sieve hole as shown by the difference in Eq. (9) and Eq. (10) where the *K.H.* range is from 1 to 3. Equivalent values for valve trays would normally be closer to those for bubble-cap trays, but the actual result depends so much on the design of the valve that it is recommended that vendors be contacted for appropriate values even in preliminary designs.?

Pressure drop through bubble-cap slots is a direct function of the vertical distance the liquid is depressed below the top of the slots. In Fig. 16-10, this distance is represented by h_{slot}. The difference in pressure between the inside of the cap and the liquid outside the cap at the top of the slots is defined as the pressure drop through the slots. Consequently, h_{slot} is the liquid head equivalent to the pressure drop through the slots.

At low gas velocities, intermittent bubbling through the slots is obtained because of liquid surface-tension effects, and the pressure drop is a function of the surface tension and the slot dimensions. When the velocity increases sufficiently, the gas issues from the slots in a steady stream, and the effect of surface tension becomes unimportant.

Under normal operating conditions, the slots remain partly open continuously and the gas is delivered in a steady stream. For this situation, the following equations, developed by Cross and Ryder,‡ can be used to estimate the average slot opening and the pressure drop through the slots:

For rectangular-shaped slots

$$h_{\text{slot}} = 1.5\left(\frac{Q_s}{b}\right)^{2/3}\left[\frac{\rho_G}{(\rho_L - \rho_G)g}\right]^{1/3} \tag{11}$$

For triangular-shaped slots

$$h_{\text{slot}} = 1.85\left(\frac{cQ_s}{b}\right)^{2/5}\left[\frac{\rho_G}{(\rho_L - \rho_G)g}\right]^{1/5} \tag{12}$$

where Q_s = volumetric flow rate of gas per slot, ft^3/s

b = width of slot at base, ft

c = height of slot, ft

Equations (11) and (12) apply when h_{slot} is less than c, and a design value for h_{slot} of $0.5c$ is often recommended.

PRESSURE DROP DUE TO LIQUID HEAD ABOVE SLOTS, SIEVE HOLES, OR VALVE OPENINGS. Reference to Fig. 16-10 shows that the total head above bubble-cap slots for an average cap is the sum of static submergence S_m, height of liquid crest above weir h_o, and average liquid gradient $0.5h_g$. The same

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†For further information on valve-tray design, see W. L. Bolles, Estimating Valve Tray Performance, *Chem. Eng. Progr.*, **72**(9):43 (1976).

‡C. A. Cross and H. Ryder, *J. Appl. Chem.*, **2**:51 (1952).

would hold for valve trays with static submergence being the distance to the top of the open valve. For sieve trays, liquid gradient is usually ignored and liquid head above the sieve holes is merely the weir height h_w plus the height of liquid crest above the weir h_o. The static submergence is determined directly by the construction details of the plate, and design methods are available for estimating the liquid crest over the weir and the liquid gradient.

The head of liquid over the weir, based on a straight segmental downcomer, can be estimated by the following modification of the Francis weir formula:

$$h_o = \left(\frac{1.7Q_L}{l_w\sqrt{g}} \right)^{2/3} \tag{13}$$

where Q_L = volumetric flow rate of liquid, ft^3/s, and l_w = weir length, ft.

The liquid gradient across the tray (h_g) can be approximated by use of the following equation developed by Davies.[†]

For bubble caps arranged on equilateral-triangle centers and caps covered with liquid to a depth not greater than 1 in.,

$$(h_gF_C)^{1.5}(1.5r - 1.4) + (h_gF_C)^{0.5}(3r)\left[h_w + h_o + \frac{s(l_r - l_c)}{l_c}\right] = \frac{0.42Q_Lr^{1.5}}{C_Dl_c} \tag{14}$$

where F_C = a correction factor to account for variations in gas rate and gas density; values of F_C are given in Fig. 16-13 as a function of $V\sqrt{\rho_G}$ and Q_L/W_T

r = number of rows of caps perpendicular to direction of liquid flow

s = skirt clearance, ft

l_r = total free space between risers perpendicular to direction of liquid flow, average of various rows, ft

l_c = total free space between caps perpendicular to direction of liquid flow, average of various rows, ft

C_D = liquid-gradient factor; value of C_D can be obtained from Fig. 16-13

V = superficial linear gas velocity (based on cross-sectional area of empty tower), ft/s

W_T = average width of tray normal to direction of liquid flow, ft, computed as average of total tray width at each row of caps

For caps arranged on square centers and for tunnel caps, replace the constants 0.42 and 1.4 in Eq. (14) by 0.37 and 1.0, respectively.

All the variables in Eq. (14) except the liquid gradient are fixed by the tray design and the operating conditions, and the value of h_g can be determined by a trial-and-error or graphical procedure. Equation (14) is based on the assump-

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[†] J. A. Davies, Petrol. *Refiner*, **29**(9):121 (1950); *Ind. Eng. Chem.*, **39**:774 (1947).

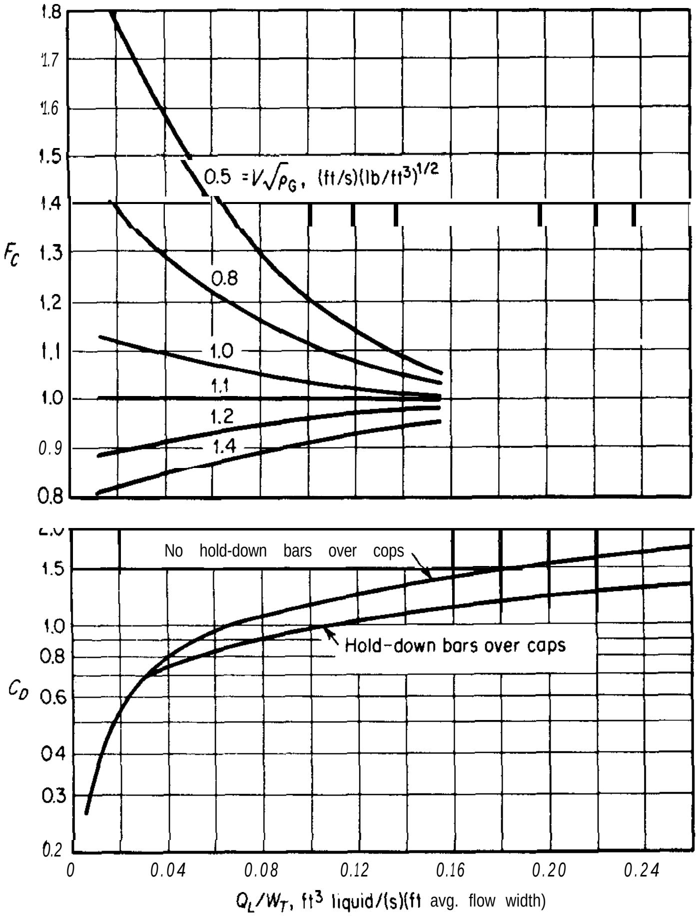

FIGURE 16-13
Plot for evaluation of liquid-gradient factor C_D and correction factor F_C in Eq. (14).

tion that the liquid level is below the top of the caps. In most bubble-cap towers, however, the liquid level is above the caps. Equation (14), therefore, is conservative for design estimates, since it tends to give high values for h_g.

EVALUATION OF TOTAL PRESSURE DROP PER TRAY. The total pressure drop of the gas across a bubble-cap tray as indicated by Fig. 16-10 equals the sum of the pressure drop through the cap assembly and the pressure drop due to liquid head above the slots, or

$$h_T = h_c + h_{\text{slot}} + S_m + h_o + 0.5h_g \tag{15}$$

where h_T = head of liquid equivalent to total pressure drop of gas across tray, ft.

Similarly, the total pressure drop of the gas across a sieve tray, as indicated in Fig. 16-11, becomes

$$h_T = h_c + h_w + h_o + 0.5h_g \tag{16}$$

Because liquid gradient is very small in most sieve tray towers, the last term in Eq. (16) is often dropped.

Because the liquid in most finite-stage contactors is usually aerated, the clear-liquid density is greater than the density of the aerated liquid on the tray. Therefore, if ρ_L is taken as the clear-liquid density, the values of Δp_T and h_T given by Eqs. (7) and (15) may be high, and these values are often multiplied by 0.7 to give an approximate correction for the liquid aeration.?

LIQUID HEAD IN DOWNCOMER. If the head of liquid in the downcomer is greater than the tray spacing plus the weir height, flooding will occur and liquid will build up on the trays. In design practice, the height of liquid in the downcomer (based on clear-liquid density) should be less than 50 percent of the tray spacing.

Liquid head in the downcomer is composed of five individual heads as follows:

1. Weir height h_w, which is established by the tray design.
2. Height of crest over weir h_o, which can be calculated by Eq. (13).
3. Liquid gradient h_g, which can be calculated by Eq. (14).
4. Head of liquid equivalent to the frictional flow resistance in the downcomer and in the passage of the fluid from the downcomer onto the plate. This head is usually relatively small, but it can be estimated as three. kinetic heads for the liquid, based on the linear liquid velocity at the minimum cross-sectional area for downcoming liquid flow, or

$$h_d = \frac{3}{2g}\left(\frac{Q_L}{A_d}\right)^2 \tag{17}$$

   where h_d = head of liquid equivalent to liquid pressure drop due to flow through downcomer and constriction, ft, and A, = minimum cross-sectional area for downcoming liquid flow, ft^2.
5. Head of liquid which the gas must overcome as the gas passes through the next tray above. Assuming the same gas pressure drop over adjacent trays, this liquid head is equal to $h,$.

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†For further treatment on aeration and additional methods for estimating pressure drop with finite-stage contactors, see R. H. Perry and D. Green, "Chemical Engineers' Handbook," 6th ed., Sect. 18, McGraw-Hill Book Company, New York, 1984.

The total head in the downcomer, H, as shown in Figs. 16-10 and 16-11, is equal to the sum of the preceding five heads, or

$$H = h_w + h_o + h_g + h_d + h_T \tag{18}$$

Combination of Eqs. (18) and (15) gives for bubble caps

$$H = 2h_o + 1.5h_g + h_w + h_d + h_c + h_{\text{slot}} + S_m \tag{19}$$

Examples 3 and 4 presented in the following illustrate methods for estimating pressure drop with bubble-cap contactors and with sieve-tray contactors. The examples also give information as to typical design conditions for the two types of contactors.

Example 3 Determination of pressure drop and liquid height in downcomer for bubble-cap plate. The following specifications apply to a bubble-cap plate:

Diameter = 10.0 ft
Tray spacing = 26 in.
Liquid crossflow
Weir length = 6.2 ft
Weir height = 3.0 in.
Skirt clearance = 0.5 in.
Static submergence = 0.5 in.
Rectangular slots
Height of slots = 1.5 in.
Width of slots = 0.3 in.
Total riser cross-sectional area = 9 ft^2
Caps are bolted to tray (no hold-down bars)
Clearance between bottom of downcomer and plate = 2.5 in.
Number of rows of caps perpendicular to direction of liquid flow = 11

For an average row of caps perpendicular to direction of liquid flow, total free space between risers = 4.4 ft, total free space between caps = 2.7 ft, width of tray = 9 ft

Cross-sectional areas for vapor flow through riser, direction-reversal space, annular cap space, and slots are equal.

Caps are arranged on equilateral-triangle centers staggered perpendicular to direction of liquid flow, and liquid depth above caps is less than 1 in.

This bubble-cap plate is to be used under the following conditions:

Vapor density = 0.15 lb/ft^3
Liquid density = 50 lb/ft^3
Superficial vapor velocity based on cross-sectional area of empty tower = 1.8 ft/s
Liquid flow rate = 1 ft^3/s
Surface tension of liquid is such that Eq. (11) is applicable.

Estimate the gas pressure drop across the tray, the percent of this pressure drop due to liquid head above the top of the bubble-cap slots, and the liquid head in the downcomer.

Solution. Liquid head equivalent to pressure drop through riser and cap (h_c): By Eq. (9),

$$h_c = \frac{3V_c^2}{g}\frac{\rho_G}{\rho_L}$$

$$\text{Cross-sectional area of empty tower} = (10)^2\frac{\pi}{4} = 78.5 \text{ ft}^2$$

V_c = linear velocity of gas in riser, reversal area, annular cap space, and slots

$$= \frac{(1.8)(78.5)}{9} = 15.7 \text{ ft/s}$$

$$h_c = \frac{(3)(15.7)^2(0.15)}{(32.17)(50)} = 0.069 \text{ ft}$$

Liquid head equivalent to pressure drop through slots (h_{slot}): By Eq. (11),

$$h_{\text{slot}} = 1.5\left(\frac{Q_s}{b}\right)^{2/3}\left[\frac{\rho_G}{(\rho_L - \rho_G)g}\right]^{1/3}$$

Q_s = volumetric flow rate of gas per slot

$$= \frac{(15.7)(1.5)(0.3)}{144} = 0.049 \text{ ft}^3/\text{s}$$

$$b = \text{slot width} = \frac{0.3}{12} \text{ ft}$$

$$h_{\text{slot}} = 1.5\left(\frac{0.049}{0.3/12}\right)^{2/3}\left[\frac{0.15}{(50 - 0.15)(32.17)}\right]^{1/3} = 0.107 \text{ ft}$$

Height of liquid crest over weir (h_o): By Eq. (13),

$$h_o = \left(\frac{1.7Q_L}{l_w\sqrt{g}}\right)^{2/3}$$

Q_L = liquid flow rate = 1.0 ft^3/s

l_w = weir length = 6.2 ft

$$h_o = \left[\frac{(1.7)(1.0)}{(6.2)(32.17)^{1/2}}\right]^{2/3} = 0.132 \text{ ft}$$

Liquid gradient (h_g): By Eq. (14),

$$(h_gF_C)^{1.5}(1.5r - 1.4) + (h_gF_C)^{0.5}(3r)\left[h_w + h_o + \frac{s(l_r - l_c)}{l_c}\right] = \frac{0.42Q_Lr^{1.5}}{C_D l_c}$$

where r = rows of caps perpendicular to direction of fluid flow = 11

h_w = weir height = 3.0/12 = 0.25 ft

h_o = 0.132 ft

s = skirt clearance = 0.5/12 = 0.0416 ft

l_r = average total free space between risers perpendicular to direction of liquid flow = 4.4 ft

1, = average total free space between bubble caps perpendicular to direction of liquid flow = 2.7 ft

V = superficial linear gas velocity based on empty tower = 1.8 ft/s

W_T = average tray width perpendicular to direction of fluid flow = 9 ft

$Q_L/W_T = \frac{1}{9} = 0.111$ ft^3/(s)(ft)

$V\sqrt{\rho_G} = (1.8)(0.15)^{1/2} = 0.7$ (ft/s)(lb/ft^3)$^{1/2}$

From Fig. 16-13,

$$F_C = 1.12$$

$$C_D = 1.2$$

$$(h_g)^{1.5}(1.12)^{1.5}[(1.5)(11) - 1.4] + (h_g)^{0.5}(1.12)^{0.5}(3)(11)$$

$$\times\left[0.25 + 0.132 + \frac{(0.0416)(4.4 - 2.7)}{2.7}\right] = \frac{(0.42)(1.0)(11)^{1.5}}{(1.2)(2.7)}$$

By trial and error,

$$h_g = 0.088 \text{ ft}$$

Pressure drop across tray based on clear-liquid density (Δp_T):

By Eq. (15), liquid head equivalent to the pressure drop across the tray is

$$h_T = h_c + h_{\text{slot}} + S_m + h_o + 0.5h_g$$

$$S_m = \text{static submergence} = \frac{0.5}{12} = 0.0416 \text{ ft}$$

$$h_T = 0.069 + 0.107 + 0.0416 + 0.132 + (0.5)(0.088) = 0.3936 \text{ ft}$$

By Eq. (7),

$$\Delta p_T = \frac{h_T \rho_L}{144}\frac{g}{g_c} = \frac{(0.3936)(50)(32.17)}{(144)(32.17)} = 0.137 \text{ psi}$$

Percent of Δp_T due to liquid head (average) above top of plate

$$= \frac{S_m + h_o + 0.5h_g}{h_T}(100) = \frac{0.0416 + 0.132 + (0.5)(0.088)}{0.3936}(100) = 55\%$$

Liquid head in downcomer (H): By Eq. (18),

$$H = h_w + h_o + h_g + h_d + h_T$$

By Eq. (17), head due to liquid flow through downcomer constriction of area A, is

$$h_d = \frac{3}{2g}\left(\frac{Q_L}{A_d}\right)^2$$

A_d = (weir length)(clearance between downcomer and plate)

$$= (6.2)\left(\frac{2.5}{12}\right) = 1.29\ \text{ft}^2$$

$$h_d = \frac{(3)}{(2)(32.17)}\left(\frac{1.0}{1.29}\right)^2 = 0\ 028\ \text{ft}$$

$$H = 0.25 + 0.132 + 0.088 + 0.028 + 0.3936 = 0.8916\ \text{ft}$$

Liquid head in downcomer based on clear-liquid density = (0.8916)(12) = 10.7 in.

Example 4 Determination of pressure drop and liquid height in dowucomer for sieve tray. The conditions of operation and design as given in Example 3 apply for a sieve-tray column with the exception that the design conditions for the bubble caps and weir height are replaced by the following design conditions for the sieve tray:

Sieve-tray holes are drilled on equilateral-triangle pattern with hole diameter = $\frac{3}{16}$ in.

Plate thickness = $\frac{3}{16}$ in.

Active area of plate is 88 percent of the total column cross-sectional area

The total area of sieve holes is 5 percent of the active area of the plate

Weir height = 2.0 in.

Liquid gradient (h_g) is negligible.

Estimate the gas pressure drop across the tray, the percent of the pressure drop due to liquid head above the sieve holes, and the liquid head in the downcomer.

Solution. Liquid head equivalent to pressure through the holes (h_c): By Eq. (10),

$$h_c = (K.H.)\frac{V_c^2\rho_G}{2g\rho_L}$$

$$V_c = \frac{1.8}{(0.88)(0.05)} = 40.9\ \text{ft/s}$$

K.H. from Fig. 16-12 (with area of sieve hole/active area of plate = 0.05 and tray thickness/hole diameter = 1.0) = 1.5

$$h_c = \frac{(1.5)(40.9)^2(0.15)}{(2)(32.17)(50)} = 0.117\ \text{ft}$$

Height of liquid crest over weir (h_o): Same as calculated in Example 3 by Eq. (13),

$$h_o = 0.132 \text{ ft}$$

$$\text{Height of weir} = h_w = \frac{2.0}{12} = 0.167 \text{ ft}$$

Pressure drop across tray based on clear-liquid density (Δp_T): By Eq. (16), liquid head equivalent to the pressure drop across the tray [assuming liquid gradient (h_g) is negligible] is

$$h_T = h_c + h_w + h_o = 0.117 + 0.167 + 0.132 = 0.416 \text{ ft}$$

By Eq. (7),

$$\Delta p_T = \frac{h_T \rho_L g}{144 g_c} = \frac{(0.416)(50)(32.17)}{(144)(32.17)} = 0.144 \text{ psi}$$

Percent of Δp_T due to liquid head above the sieve holes

$$= \frac{h_o + h_w}{h_T}(100) = \frac{0.132 + 0.167}{0.416}(100) = 72\%$$

Liquid head in downcomer (H): By Eq. (18), neglecting liquid gradient,

$$H = h_w + h_o + h_d + h_T$$

From Example 3, head due to liquid flow through downcomer constriction (h_d) by Eq. (17) is

$$h_d = 0.028 \text{ ft}$$

$$H = 0.167 + 0.132 + 0.028 + 0.416 = 0.743 \text{ ft}$$

Liquid head in downcomer based on clear-liquid density = (0.743)(12) = 8.9 in.

OTHER DESIGN FACTORS FOR FINITE-STAGE CONTACTORS

In addition to the critical design factors for finite-stage contactors of number of theoretical trays, maximum allowable vapor velocity, column efficiency, and pressure drop as discussed earlier, a number of other factors are of importance in the development of the design. These factors are discussed in the following sections.

Bubble Caps and Risers

Bubble-cap assemblies in the form of round bell caps are commonly used with diameters ranging from 4 to 7 in. A 6-in.-diameter cap with a 4-in.-diameter riser is a standard size used in many industrial operations. Cap diameters as large as 8 in. have been employed successfully in some operations, and 3-in. caps with 2-in. risers are used in many vacuum towers. Tunnel caps are ordinarily 3 to 6 in. wide and 12 or more in. long. Comparison of the two types shows that tunnel caps have the advantage of a smaller number of units for installation for a given slot and riser area, but the round caps are more adaptable to changes in tray layout and can be purchased and stocked in one standard size.

The caps are slotted around the lower periphery and can be anchored to the plate with teeth touching the plate or suspended to permit a so-called "skirt clearance" between the plate and the cap. Slots can be of a saw-tooth type or in the form of punched holes, usually rectangular or triangular. In common practice, a skirt clearance in the range of 0.5 to 1.5 in. is recommended to prevent plugging of the slots by residue buildup. Since the purpose of the slots is to disperse the gas into the liquid in the form of small bubbles, sufficient slot area should be provided in order that no gas may pass through the skirt clearance.

As illustrated in Fig. 16-2, the caps can be held in place by bolts or by a hold-down bar; however, the hold-down bar is seldom used because it interferes with the flow distribution. Since periodic removal or maintenance of the caps may be necessary, the preferred method for holding caps and trays in place permits removal of the caps by one person working in the crawl space above the plate.

SLOT VELOCITIES AND RELATIVE DIMENSIONS. After the design of the cap has been established, the next step is to determine the number of bubble caps to be used per tray. This number is set by the allowable gas velocity through the slots. If the velocity is too high, pressure drop may be excessive and the liquid may be blown away from the cap, thus resulting in poor efficiency as shown by Fig. 16-8. On the other hand, if the velocity is too low, the gas bubbles will have little opportunity to disperse through the liquid, and the efficiency of the tray will be low. Davies has recommended the following equations for use in preliminary estimates of slot velocities in rectification columns:?

$$\text{Maximum linear slot velocity, ft/s} = \frac{12}{\rho_G^{0.5}} \tag{20}$$

$$\text{Minimum linear slot velocity, ft/s} = \frac{3.4}{\rho_G^{0.5}} \tag{21}$$

where ρ_G = gas density, lb/ft^3.

In common design practice, the riser area, total slot area, and passage area in the annular space under the cap are approximately equal in order to reduce pressure losses caused by expansion and contraction. Gas bubbles issuing from the slots are seldom projected more than about 1 in. from the cap, and a clearance between caps in the range of 1 to 3 in. is usually sufficient to eliminate large amounts of undesirable bubble coalescence. The caps should be spaced evenly over the entire tray. A clearance between the shell and adjacent

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†J. A. Davies, *Petrol. Refiner,* **29**(9):121 (1950).

caps of less than 2 in. is recommended in order to reduce the possibility of liquid bypassing the bubble-contact regions. For towers 3 or more feet in diameter, a plate layout with the total riser area in the range of 10 to 20 percent of the tower cross-sectional area is common, with the greater riser area being more easily obtainable in larger-diameter towers.

Sieve Trays

The common sieve trays are constructed of flat metal sheets with holes drilled or punched to form the sieve plate. Hole sizes in the range of $\frac{1}{8}$ to $\frac{1}{2}$ in. diameter are used with $\frac{3}{16}$ in. being a common size. The design range for ratio of tray thickness to sieve hole diameter is usually 0.1 to 0.7. The liquid head on a sieve tray should be in the range of 2 in. to 4 in. The sieve holes should be spaced as equilateral triangles with a pitch-to-diameter ratio greater than 2.0 but less than 5.0 with an optimum of about 3.8. The downcomer inlet velocity should not exceed 0.4 feet per second to allow for adequate area for vapor disengaging. At the time of installation of new trays, they should be inspected to make certain that smooth and clean holes are provided with a minimum of surface irregularities on the side toward the flowing gas.

Valve Trays

The major advantage of valve trays over sieve trays is that high efficiencies can be maintained over a wider range of operating liquid and gas throughputs than with sieve trays. ***Turnover ratios,*** defined as the ratio of the maximum allowable throughput rate to the minimum allowable throughput rate, as high as 10 can be obtained with valve trays, while the ratio is much less with sieve trays. In general, good design information on the various devices used as the contactor for valve trays is proprietary in that they are manufactured by individual concerns which hold patent rights and related private knowledge on the design and operation of the units.?

Valve-tray perforations are larger than those for sieve trays being on the order of 1.5 in. for circular holes and 6 in. long for the case of rectangular-slot holes. There are many methods to control the movement of the valve lids, but the lids generally are allowed a vertical movement in the range of $\frac{1}{4}$ to $\frac{1}{2}$ in., and the weight of the lid varies depending on the intended use of the valves.

Basically, the overall model for valve trays is similar to that for sieve trays with the exceptions that the vapor flow area in the contactor can vary for valve units depending on the extent of the valve opening, and the number of kinetic

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†**Examples** of special information which is available on valve-tray design are Glitch, Inc., “Ballast Tray Design Manual,” Bulletin 4900, Dallas, Tex.; Koch Engineering Co., “Flexitray Design Manual,” Bulletin 960, Wichita, Kans.; and Nutter Engineering Co., “Float Valve Tray Design Manual,” Tulsa, Okla.

heads for calculation of the gas flow pressure drop through the contactor (h_c) is usually greater for valve trays than for sieve trays. Thus, many of the basic principles of fluid mechanics as used for sieve-tray design also apply for valve-tray design so that rough preliminary designs for valve-tray columns can often be made by engineers for use in considering alternatives before contacting a specific manufacturer. Because valve units are more complex mechanically than sieve trays, valve trays are more expensive to fabricate, and this extra cost must be recognized in pricing the units. On the other hand, proprietary knowhow will become available for the valve units if they are purchased, and this advantage to the user plus the advantage of wider range of turnover ratios may swing the balance in favor of the more expensive valve trays.

Shell and Trays

Cylindrical shells are commonly used for finite-stage contactor columns. The shell may be constructed of short sections that are bolted together or in the form of one long cylinder. Manholes should be provided to give access to the individual trays for cleaning, maintenance, and installation. Adequate foundations and tower supports must be provided. In general, the size of the foundations is set by the size necessary to resist the overturning stresses on the column, since this load usually exceeds the direct load caused by the weight of the column assembly.

Both the corrosion characteristics of the fluids involved and costs dictate the acceptable materials of construction for the shell and trays. Bubble caps, valves, and trays are usually made of a suitable metal to facilitate fabrication, but the shell material can be glass, plastic, impervious carbon, wood, glass-lined or resin-lined steel, or metal. Despite the additional weight involved, trays for bubble-cap units are often made of cast iron, usually 0.5 in. thick. With cast-iron trays, the risers may be fabricated as a permanent part of the tray. Lighter-gauge alloy metals may be cheaper than cast iron, and the final decision must be made on the basis of the situation for each individual case.

Supporting beams are used to stiffen the trays, and the trays must be fastened securely to prevent movement caused by gas surges. To allow for thermal expansion and to facilitate installation, slotted bolt holes should be provided in the supporting rings, and there should be adequate clearance between the tray and shell wall.

TRAY LEVEL. The calculations for finite-stage contactor operations are based on the assumption that the trays are perfectly level. A tolerance of $\pm\frac{1}{8}$ in. change in vertical tray level is usually specified for the location of tray supports and fastening of the trays to these supports. Variations from the horizontal level are also caused by foundation settling, tray deflections caused by operating conditions, tower deflection due to wind loads, and uneven corrosion of the tray or tray supports. For design purposes, a maximum change in tray level of $\frac{1}{2}$ in. is a safe assumption, and this figure can be used as a safety factor in setting the seal dimensions of the tray.

LEAKAGE AND WEEP HOLES. To ensure a minimum of liquid leakage from one tray to another, bubble-cap risers should be fitted firmly or sealed into the trays, and an effective seal is necessary around the tray supports. In large columns, the trays are often made in sections for ease of installation. Each of these sections must be installed carefully to minimize leakage.

Provision for draining the liquid from a bubble-cap tray when the unit is not in operation is made by the use of *weep holes*. These holes are usually located near the overflow weir so that any delivery during operation follows approximately the same path as the overflow fluid. The weep holes must be large enough to prevent plugging, but they should not deliver an excessive amount of fluid during operation. A size in the range of $\frac{1}{4}$- to $\frac{5}{8}$-in. diameter is usually adequate.

Liquid Flow

High tray efficiency is achieved when all the contactor units are delivering gas uniformly into liquid that is evenly distributed over the entire tray surface. The beneficial effects of a liquid concentration gradient are obtained if liquid crossflow is used, where the liquid enters on one side of the tray and makes one pass across the tray. For tower diameters larger than 4 ft, better liquid distribution and less change in liquid head can often be achieved by using split flow, radial flow, or cascade flow, as illustrated in Fig. 16-5.

Because of the flow resistance of the caps and risers in bubble-cap columns, there is a decrease in liquid depth as the liquid passes across the tray. Figure 16-4 shows an extreme example in which this liquid gradient is so great that only one out of the four bubble caps is operating normally. In general, the dimensionless ratio of total liquid gradient to pressure-drop head caused by the bubble-cap assembly should be less than 0.4 in order to ensure adequate vapor distribution, and, for single-pass crossflow trays, the rate of liquid flow across the tray should be less than 0.22 $ft^3/(s)(ft)$ diameter.

A total liquid gradient of 0.5 in. over one tray is usually acceptable, but a different flow pattern should be considered if the value approaches 1 in. The liquid gradient can be reduced by decreasing the number of rows of caps through which the liquid flows or by decreasing the velocity of liquid flow past the caps. A higher skirt clearance or a higher weir will sometimes be sufficient to reduce excessive liquid gradients to acceptable values.

Entrainment

As a gas passes through the contactor unit into the liquid, a large amount of turbulence is set up, and liquid particles can become entrained with the gas. Carry-over of these liquid particles from one tray to the tray above is known as *entrainment*. It is often defined as the weight of liquid entrained per unit weight of gas. Liquid can be entrained by the gas as a result of violent splashing of the liquid or because of extensive foaming or frothing.

TABLE 1
Recommended tray spacings for towers

Tower diameter, ft	Tray spacing, in.
	6 minimum
4 or less	18–20 (no manways in trays)
6-10	24
12-24	36

Entrainment has an adverse effect on the column operation in that it reduces the concentration change per tray and, consequently, decreases the efficiency. Nonvolatile impurities can be carried up a tower by entrainment, resulting in off-color or impure overhead products.

The major factors that determine the amount of entrainment are plate spacing, depth of liquid on the tray, and vapor velocity in the space between the plates. Slot or sieve-hole vapor velocity and liquid flow rate have some effect on the entrainment, but they are not of major importance.

Tray Spacing

Tray spacing in large columns is usually determined by the need for easy access for maintenance and inspection. With columns less than 3 ft in diameter, the minimum value of tray spacings is about 6 in., and greater values are used in most cases. In the petroleum industry, an 18-in. spacing is considered to be the minimum value below which entrainment or tendency toward flooding becomes excessive, and a tray spacing of 20 to 24 in. is often set as a minimum for reasons of accessibility. Table 1 presents recommended tray spacings for petroleum rectification columns of various diameters.

Downcomers and Weirs

Downcomers for conducting the liquid from one tray to the next tray below may be in the form of circular pipes or segments of the tower isolated from the rising gas by means of vertical or angled plates. Some vapor is entrained in the liquid as the liquid enters the downcomer, and sufficient residence time should be provided in the downcomer to permit escape of the entrained vapor. A residence time of 5 s, evaluated as the volume of the downcomer divided by the volumetric flow rate of the downcoming fluid, is enough to permit release of most of the entrained vapor. Vapor release is also accomplished by providing a calming section before the liquid flows into the downcomer. This is supplied by locating the bubble caps, valves, or sieve holes 3 to 5 in. from the downcomer weir or by blanking off the cap or valve slots that face toward the downcomer. The liquid head in the downcomer should not be greater than one-half the plate spacing.

The discharge end of the downcomer must project far enough into the tray liquid so that no gas bubbles can enter the open end and bypass the bubble caps. When the liquid contains no sediment, a seal pot or discharge weir is often placed around the discharge end of the downcomer to make certain that no free vapor can enter the open end. The distance between the liquid level on the loaded discharge plate and the bottom of the downcomer when no liquid is flowing is known as the *downcomer liquid seal.* A downcomer liquid seal, based on a perfectly level tray, in the range of $\frac{1}{2}$ to $1\frac{1}{2}$ in. is usually satisfactory.

An extension of the downcomer plate can be used as an overflow entrance weir, or a separate overflow weir may be provided. Since adequate vapor-disengaging space is necessary, extension of circular-pipe downcomers to form an overflow weir is not recommended if the column diameter is greater than 3 ft. Straight rectangular weirs are often used, and, on crossflow trays, these generally have a length in the range of 0.6 to 0.8 times the column diameter.

The height of the overflow weir is a major factor in determining the head of liquid on the tray. Although plate efficiency is increased slightly as the liquid head is increased, the beneficial effect is seldom enough to counterbalance the detrimental pressure-drop effects caused by high heads. This is particularly true when pressure drop is an important factor, as with towers operating under vacuum. The distance between the top of the bubble-cap slots and the liquid surface when the static liquid is just ready to flow over the overflow weir is known as the *static submergence.* Table 2 shows recommended values of static submergence for various operating pressures.

TABLE 2
Typical values of static submergence for bubble-cap plate columns

Operating pressure	Static submergence, in.
Vacuum (30 mm Hg abs.)	0
Atmospheric	$\frac{1}{2}$
100 psig	1
300 psig	$1\frac{1}{2}$
500 psig	$1\frac{1}{2}$

Example 5 Determination of holdup time in downcomer. A valve-tray tower with 24-in. plate spacing and liquid crossflow contains straight segmental downcomers. The overflow weir at the downcomer entrance is formed by an extension of the downcomer plate. The height of this weir is 3 in. The inside diameter D of the tower is 5 ft, and the weir length is 0.60. If liquid with a density of 55 lb/ft^3 flows across the plate at a rate of 30,000 lb/h, estimate the residence or holdup time in the downcomer from this plate.

Solution

Volumetric flow rate of downcoming fluid = 30,000/(55)(3600) = 0.152 ft^3/s
Weir length = $l_w = (0.6)(5) = 3$ ft
Perpendicular distance from weir to center of tower = $d_{wc} = [(D/2)^2 - (l_w/2)^2]^{1/2} = [(\frac{5}{2})^2 - (\frac{3}{2})^2]^{1/2} = 2$ ft
If θ designates the angle in degrees subtended from the center of the tower by the weir,

$$l_w = D \sin \frac{\theta}{2}$$

$$\sin \frac{\theta}{2} = 0.6$$

$$\theta = 73.7\text{”}$$

Downcomer cross-sectional area is

$$\frac{\pi D^2 \theta}{1440} - \frac{l_w d_{wc}}{2} = \frac{(3.14)(5)^2(73.7)}{1440} - \frac{(3)(2)}{2} = 1.02 \text{ ft}^2$$

Alternatively, downcomer cross-sectional area is

$$\frac{\pi D^2}{8} - \frac{l_w d_{wc}}{2} - \frac{D^2}{4}\left(\sin^{-1} \frac{2d_{wc}}{D}\right)_{\text{radians}} = 1.02 \text{ ft}^2$$

$$\text{Effective height of downcomer} = \frac{24 + 3}{12} = 2.25 \text{ ft}$$

$$\text{Residence time} = \frac{\text{volume of downcomer}}{\text{volumetric flow rate of downcoming fluid}}$$

$$= \frac{(2.25)(1.02)}{0.152} = 15 \text{ s}$$

NOTE: The preceding answer is based on the assumption that the descending liquid occupies all the available volume in the downcomer.

Plate Stability

The term *plate stability* refers to the ability of the plate to maintain satisfactory operating characteristics when flow rates change or when unsteady-state conditions exist. Vapor and liquid distribution on the plate are the primary factors that determine plate stability. A stable plate is one in which all the contacting units are handling appropriate amounts of vapor, and the best efficiencies are obtained when the vapor flow is evenly distributed among the contacting units. Thus, an ideal sieve plate would have all the vapor holes discharging uniformly even when fluctuations in flow rates occur. In an actual bubble-cap column, the vapor is not distributed uniformly among the caps primarily because of the gradient in liquid height across the tray.

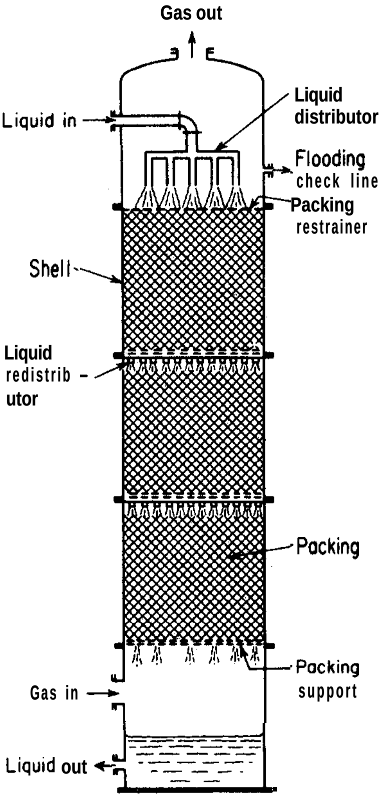


FIGURE 16-14
Cross-sectional view of packed tower in operation.

CONTINUOUS CONTACTORS—PACKED TOWERS

As illustrated in Fig. 16-14, the common type of packed tower consists of a cylindrical shell containing an inert packing material. These towers usually operate with countercurrent fluid flow. For the case in which liquid and vapor phases are involved, the liquid descends through the column in the form of films distributed over the packing surface, and the vapors rise through the spaces between the packing particles. Consequently, a large amount of vapor-liquid contact area becomes available, thus resulting in efficient mass-transfer operations.

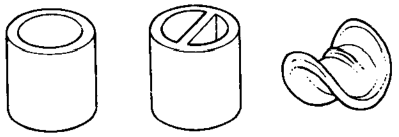
The design of a packed tower requires consideration of mechanical factors, such as pressure drop, flow capacities, and foundation load. In addition, consideration must be given to the factors that influence the effectiveness of contact between the fluid phases. A satisfactory packing should have the following properties:

1. *Low pressure drop.* Since the pressure drop through the packing is a direct function of the fluid velocities, a large free cross-sectional area should be available between the packing particles in order to give a low pressure drop.

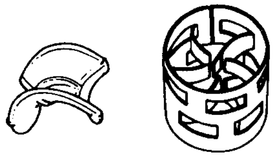
2. *High capacity.* The packing should permit high fluid rates without excessive pressure drop or build-up of liquid in the tower. Because flooding or carry-over of liquid out of the tower can occur above certain limiting fluid velocities, a large free cross-sectional area is desirable for high capacities.
3. *Low weight and low liquid holdup.* The total weight of the column and the resultant foundation load is low if the weight of the packing and the liquid holdup in the tower are low. The amount of liquid holdup, however, must be sufficiently great to retain an effective driving force for mass transfer.
4. *Large active surface area per unit volume.* To give high efficiencies, the packing must provide a large amount of contact area between the two fluid phases. This can be accomplished by using irregularly shaped packings that permit extensive distribution of liquid over surface area which can be contacted directly by the second fluid.
5. *Large free volume per unit total volume.* This property is particularly important if time must be available for a gas-phase chemical reaction, such as the oxidation of nitric oxide in the aqueous absorption of nitrogen dioxide.
6. *Miscellaneous.* High durability. High corrosion resistance. Low side thrust on tower walls. Low cost.

TYPES OF PACKING

Although many different types of packing are available for obtaining efficient contact between two fluid phases, the types can generally be classified as *random* or *stacked.* A random packing is one that is merely dumped into a containing shell, and the individual pieces are not arranged in any particular pattern. Pall rings, Intalox saddles, Raschig rings, and Berl saddles, as shown in Fig. 16-15, are the most common of the random packings used in industrial operations. Pall rings and Intalox saddles are generally replacing the older Raschig rings and Berl saddles because, in most cases, the Pall rings and Intalox



(*a*) Raschig ring (*b*) Lessing ring (*c*) **Berl** saddle



(*d*) **Intaloz** saddle (*e*) Pall ring

FIGURE 16-15
Single pieces of typical random packings.

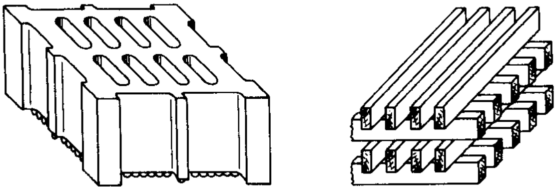
saddles permit a more economical tower design than the other packings. Pall rings, made of metal or plastic, have the same general form as Raschig rings being open cylinders with the height equal to the diameter. However, Pall rings are stamped during the forming operation so that part of the original cylinder wall is cut with the projections bent inward leaving holes in the wall. The projections nearly touch at the center and the result is an opening of the ring and utilization of the interior of the ring to give improved vapor-liquid contact. They are available in sizes from $\frac{5}{8}$ to 3 in. or more.

Saddle-shaped packings, such as Intalox saddles and Berl saddles, are available in sizes from $\frac{1}{4}$ to 2 in. These packings are formed from chemical stoneware, plastics, or any other material that can be shaped by punch dies. They form an interlocking structure which gives less side thrust and more active surface area per unit volume than Raschig rings.

Raschig rings, as illustrated in Fig. 16-15, are simply hollow cylinders with the outside diameter equal to the height. They are usually made of inert materials that are cheap and light, such as porcelain, chemical stoneware, or carbon. Other materials of construction, such as clay, plastic, steel, and metal alloys, are also used. Raschig rings are available in sizes ranging from $\frac{1}{4}$ to 3 in. or more. Because breakage of fragile packing may occur when the pieces are dropped into an open shell, the initial packing charge is sometimes made by filling the empty tower with water and then dumping the packing slowly into the water.

Additional active surface can be provided by adding a single web or cross web on the inside of a Raschig ring. When a single web is present, the packing is known as Lessing rings. With a solid cross web, the packing is known as cross-partition rings which are normally available in sizes ranging from 3 to 6 in. and are almost always used as a stacked packing.

Stacked packings, in general, give lower pressure drops for equivalent fluid capacities than random packings. However, this advantage is gained at the expense of higher initial costs due to the extra installation labor. The ring packings of nominal sizes 3 in. or larger are often used as a stacked packing. Other examples of stacked packings are shown in Fig. 16-16. A list of the important physical characteristics for common packings used in industrial operations is presented in Table 3.



Drip- point grid

Wood grids

FIGURE 16.16
Stacked grid packings.

TABLE 3

Physical characteristics of commercial tower packings

For use when direct experimental data are not available

Packing	Nominal size, in.	Approximate average weight per ft^3 of tower volume, lb	Approximate average total surface area of packing, ft^2/ft^3 of tower volume	Percent free-gas space = ϵ x 100	Packing factor, a_p/ϵ^3, effective surface area/(void fraction)', ft^2 per ft^3 of tower volume (dry-packed values for use with Fig. 16-20)
Random packings:					
Stoneware	¼	46	240	13	168
Raschig rings	⅜	51	134	68	494
	½	50	122	64	511
	¾	44	80	13	199
	1	40	58	13	150
	1½	42	35	68	108
	2	31	28	14	46
	3	40	19	14	
Carbon Raschig rings	½	27	114	14	313
	1	27	51	14	110
	1½	34	38	61	92
	2	27	29	14	56
Steel Raschig rings (wall thickness = $\frac{1}{16}$ in.)	½	132	118	13	
	1	13	51	85	
	2	38	31	92	
Lessing rings (porcelain)	1	50	69	66	
	½	58	40	60	
	2	49	32	68	
Intalox saddles (porcelain)	½	34	190	18	
	1	34	18	18	100
	1½	30	60	81	52
Berl saddles (porcelain)	3	56	214	58	4225
	½	54	155	60	514
	1	45	19	69	229
	1½	38	52	10	19
Pall rings (steel)	⅝	31	104	93	10
	1	30	63	94	48
	1½	26	39	95	28
	2	24	31	96	20
Pall rings (polypropylene)	⅝	7¼	104	81	91
	1	53	63	90	52
	1½	4¾	39	91	32
	2	4½	31	92	25
Stacked packings (stacked for maximum surface area per unit of tower volume):					
Raschig rings	2		32	80	
Cross-partition rings (porcelain)	4 × 4 × ⅜	81	32	53	
	6 × 6 × ⅝	10	21	48	

LIQUID DISTRIBUTION

In absorption and distillation towers, the packing should be wetted completely by the descending liquid in order to provide the maximum amount of active surface area. For this reason, the entering liquid should be distributed uniformly over the top of the packing at a flow rate sufficient to permit wetting of all the packing surface. The packing support plate should provide ample space for passage of both liquid and gas with good distribution of the gas. A cone-shaped packing support is often used in small columns to distribute the entering gas uniformly throughout the packing and reduce the possibility of liquid flooding at the support plate.

Even with distributors at the top and bottom of the tower, the descending liquid has a tendency to channel or flow toward the column wall, thus resulting in packing regions where the surface is dry. Stacked packings do not give as good liquid distribution as random packings, and, in general, distribution difficulties tend to become excessive if the ratio of tower diameter to packing diameter is less than 7.

Loss of active surface area due to improper liquid distribution can be reduced by using intermediate redistributor plates, as shown in Fig. 16-14.

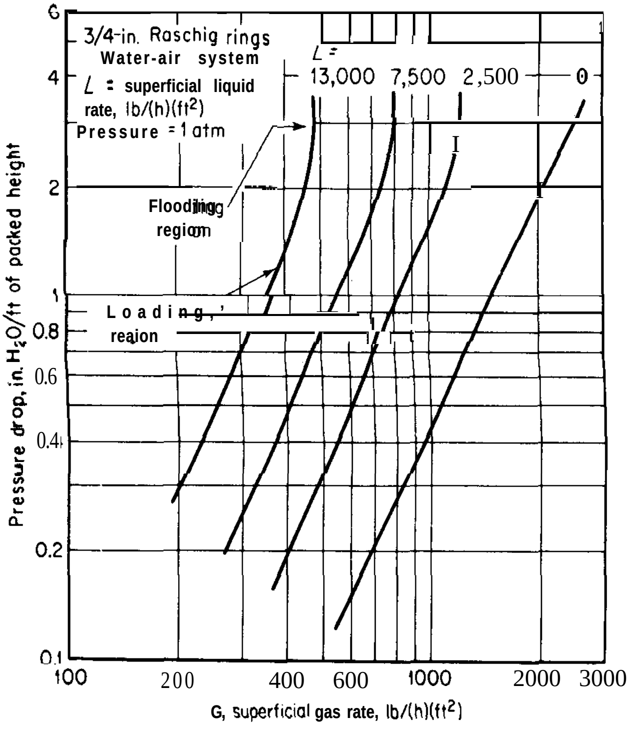


FIGURE **16-17**
Packed-tower pressure drop as function of gas rate and liquid rate.

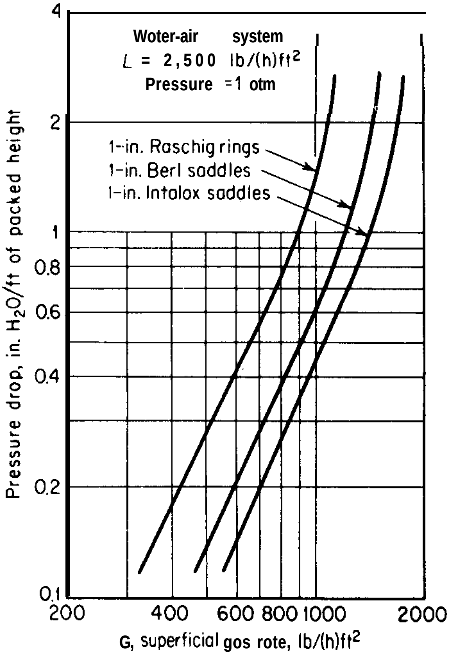


FIGURE **16-18**
Packed-tower pressure drop as function of **gas** rate and type of packing.

Complete wetting of the packing by preflooding the column at the start of the operation or use of a self-wetting packing, such as protruded packing, can decrease the detrimental effects of poor liquid distribution.

PRESSURE DROP

The primary factors that affect pressure drop in packed towers are (1) fluid-flow rates, (2) density and viscosity of the fluids, and (3) size, shape, orientation, and surface of the packing particles. Figure 16-17 illustrates the effects of fluid rates at constant operating pressure, and Fig. 16-19 shows how increased gas velocity due to reduction in operating pressure can affect the pressure drop.

As indicated by Figs. 16-17 through 16-19, a log-log plot of pressure drop per foot of packed height versus gas rate gives a straight-line relationship over the lower range of pressure drops. The point where the line first starts to curve upward is often designated as the *loadingpoint,* to indicate that liquid is starting to build up in the column and is reducing the effective free space for gas flow. At the *flooding point* the pressure-drop-versus-gas-rate curve becomes almost vertical, and a liquid layer starts to build up on top of the packing. The flooding point represents the upper limiting conditions of pressure drop and fluid rates for practical tower operation.

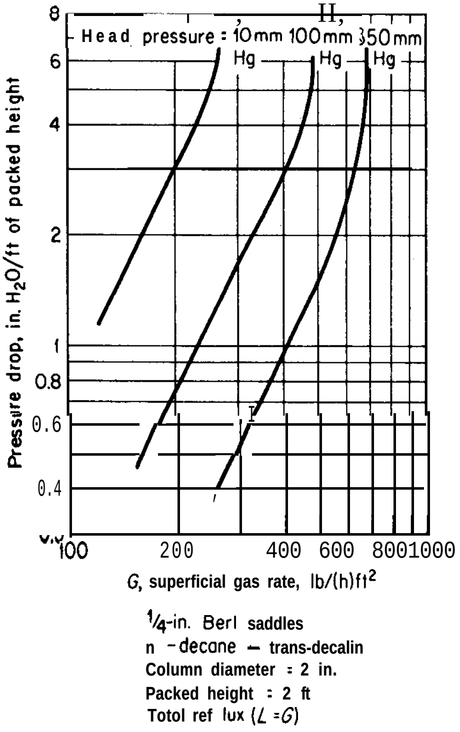


FIGURE 16-19
Pressure drop in packed distillation tower as function of gas rate and operating pressure.

Estimation of Pressure Drop in Packed Towers

Many methods have been proposed for estimating pressure drop in packed towers. Most of these methods are based primarily on experimental data obtained with countercurrent flow of water and air through various types of packed towers. Because of the empirical nature of these correlations and the fact that the effects of some of the variables are not included, it is always best to predict pressure drops in the design of packed towers on the basis of experimental data obtained with equipment operating under conditions equivalent to those involved in the design.? If such data are not available, approximations can be made by using the methods outlined in the following discussion.

IRRIGATED PACKINGS. Because the operation of packed towers in the range between the loading region and the flooding point may give erratic fluctuations, many packed towers are operated in the preloading range. In this range, the

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[†] For a compilation of various experimental data, see R. H. Perry and D. Green, "Chemical Engineers' Handbook," 6th ed., Sect. 18, McGraw-Hill Book Company, New York, 1984.

TABLE 4

Constants for estimating pressure drop in packed towers by Eq. (22)

Apply only to preloading range. Above this range, pressure drop predicted by Eq. (22) will be too low.

Packing	Nominal size, in.	γ x 10^8	Φ x 10’	Valid for range of L, lb/(h)(ft^2)
Raschig rings	½	139	7.2	300-9000
	¾	33	4.5	2000–11,000
	1	32	4.3	400–27,000
	1½	12	4.0	700–18,000
	2	11	2.3	700–22,000
Berl saddles	½	60	3.4	300–14,000
	¾	24	3.0	400–14,000
	1	16	3.0	700–29,000
	1½	8	2.3	700–22,000
Intalox saddles	1	12	2.8	2500–14,000
	1½	6	2.3	2500–14,000

slope of the straight lines in Figs. 16-17 through 16-19 is approximately 2.0, and the pressure drop can therefore be taken as directly proportional to the square of the gas mass velocity.

Leva† has correlated experimental data to obtain the following empirical equation for estimating pressure drop in packed beds under preloading conditions with simultaneous counterflow of liquid and gas:

$$\frac{\Delta p}{h} = \gamma(10)^{\Phi L/\rho_L}\frac{G^2}{\rho_G} \tag{22}$$

where Ap = pressure drop, psf

h = packed height, ft

γ, Φ = constants with value dependent on packing size and type (see Table 4)

L = superficial liquid mass velocity (based on cross-sectional area of empty tower), lb/(h)(ft^2)

ρ_L = density of liquid, lb/ft^3

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†M. Leva, *Chem. Eng. Progr. Symp. Ser.*, 50(10):51 (1954).

TABLE 5

Pressure drop at flooding point in packed towers

Values are based on countercurrent flow of liquid and air at atmospheric pressure. In the range between the loading region and the flooding point, an approximation of pressure drop Δp at gas mass velocity G can be obtained from Ap = $\Delta p_F(G/G_m)^{3.2}$. The pressure drop at the loading point is usually in the range of one-fourth to one-sixth of the pressure drop at the flooding point

Packing	Nominal size, in.	Pressure drop at flooding point, $\Delta p_F/h$, in. water/ft of packed height						
		Kinematic viscosity of liquid, centistokes = centipoises/specific gravity						
		0.6	1.0	3.0	6.0	10	20	40
Raschig rings	$\frac{1}{4}$	4.4	4.0	3.3	3.0	2.8	2.6	2.4
	$\frac{1}{2}$	3.9	3.5	2.9	2.7	2.5	2.3	2.1
	$\frac{3}{4}$	3.3	3.0	2.5	2.3	2.1	1.9	1.8
	1	3.3	3.0	2.5	2.3	2.1	1.9	1.8
	$1\frac{1}{2}$	2.8	2.5	2.1	1.9	1.8	1.6	1.5
	2	2.8	2.5	2.1	1.9	1.8	1.6	1.5
Berl saddles	$\frac{1}{4}$	1.4	1.3	1.2	1.1	1.0	1.0	0.9
	$\frac{1}{2}$	2.1	2.0	1.8	1.7	1.6	1.5	1.4
	$\frac{3}{4}$	2.7	2.5	2.3	2.1	2.0	1.9	1.8
	1	2.7	2.5	2.3	2.1	2.0	1.9	1.8
	$1\frac{1}{2}$	2.4	2.2	2.0	1.8	1.7	1.6	1.5

G = superficial gas mass velocity (based on cross-sectional area of empty tower), lb/(h)(ft^2)
ρ_G = density of gas, lb/ft^3

The effect of liquid viscosity is not included in Eq. (22). Although an increase in liquid viscosity tends to cause an increase in pressure drop if flow rates are held constant, the results of Eq. (22) can be accepted as a reasonable approximation if the liquid involved has a viscosity less than about 2 centipoises.

At the flooding point for a given packing and set of fluids, the pressure drop per foot of packed height remains approximately constant with variations in fluid rates and operating pressure. Table 5 presents typical pressure drops for various fluids and packings at the flooding point.? The table also indicates a method for estimating pressure drop when a packed column is operated in the range between the loading point and the flooding point.

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†F. A. Zenz, Chem. Eng., 60(8):176 (1953).

DRY PACKINGS. The following equation, by Ergun,[†] can be used to estimate pressure drop caused by the flow of a gas through dry packings:

$$\frac{\Delta p}{h} = \frac{1-\varepsilon}{\varepsilon^3}\frac{G^2}{d_p g_c \rho_G}\left[\frac{150(1-\varepsilon)\mu_G}{d_p G} + 1.75\right] \tag{23}$$

where ε = fractional void volume in bed, ft^3 void/ft^3 of packed-tower volume
μ_G = absolute viscosity of gas, lb/(ft)(h)
d_p = effective diameter of packing particle, ft, or diameter of a sphere with same surface-to-volume ratio as packing particle
$= 6(1-\varepsilon)/a_p$
a_p = surface area of packing per unit of packed-tower volume, ft^2/ft^3

Equation (23) accounts for simultaneous kinetic and viscous energy losses and is applicable to the single-phase flow of liquids as well as gases.[‡]

Example 6 Estimation of pressure drop in packed tower. A column 2 ft in diameter is packed with $\frac{3}{4}$-in. stoneware Raschig rings. Air, at an average pressure of 1 atm and an average temperature of 70°F, flows through the tower at a superficial mass velocity of 600 lb/(h)(ft^2). Estimate the pressure drop through the dry packing by Eq. (23). If water at 70°F flows countercurrent to the air at a rate of 8000 lb/h, estimate the pressure drop by Eq. (22). Express the answers as inches of water per foot of packed height, and compare the calculated results with the values shown in Fig. 16-17.

Solution. For dry packing,

$$\frac{\Delta p}{h} = \left(\frac{1-\varepsilon}{\varepsilon^3}\right)\frac{G^2}{d_p g_c \rho_G}\left[\frac{150(1-\varepsilon)\mu_G}{d_p G} + 1.75\right]$$

From Table 3,

$$\varepsilon = 0.73$$

$$a_p \text{ (based on total surface area)} = 80\ \text{ft}^2/\text{ft}^3$$

$$d_p = \frac{6(1-\varepsilon)}{a_p}$$

$$= \frac{6(1-0.73)}{80} = 0.0202\ \text{ft}$$

[†] S. Ergun, *Chem. Eng. Progr.*, **48**:89 (1952).

[‡] For additional information and methods for estimating pressure drop and flooding velocities in packed towers, see R. H. Perry and D. Green, “Chemical Engineers’ Handbook,” 6th ed., Sect. 18, McGraw-Hill Book Company, New York, 1984.

$$\mu_G \text{ (at } 70°\text{F)} = 0.018 \text{ centipoise}$$

$$= 0.018 \times 2.42 = 0.044 \text{ lb/(ft)(h)}$$

$$\rho_G = \frac{(29)(492)}{(359)(530)}$$

$$= 0.075 \text{ lb/ft}^3$$

$$\frac{AP}{h} = \frac{1 - 0.73}{(0.73)^3} \frac{(600)^2}{(0.0202)(32.17)(3600)^2(0.075)} \left[\frac{150(1 - 0.73)(0.044)}{(0.0202)(600)} + 1.75 \right|$$

$$= 0.67 \text{ lb/(ft}^2\text{)(ft)} = \frac{(0.67)(33.93)(12)}{(14.7)(144)} = 0.13 \text{ in. water/ft}$$

According to Fig. 16-17, $\Delta p/h$ = about 0.15 in. water/ft for $L = 0$ (dry packing). For irrigated packing, Eq. (22) applies.

$$\frac{\Delta p}{h} = \gamma(10)^{\Phi L/\rho_L} \frac{G^2}{\rho_G}$$

From Table 4,

$$\gamma = 33 \text{ x } 10^{-8} \qquad \Phi = 4.5 \text{ x } 10^{-3}$$

$$\rho_L = 62.3 \text{ lb/ft}^3$$

$$L = \frac{8000}{(\pi/4)(2)^2} = 2550 \text{ lb/(h)(ft}^2)$$

$$\frac{AP}{h} = 33 \times 10^{-8}(10)^{(0.0045X2550)/62.3} \frac{(600)^2}{(0.075)} = 2.42 \text{ lb/(ft}^2\text{)(ft)}$$

$$= \frac{(2.42)(33.93)(12)}{(14.7)(144)} = 0.47 \text{ in. water/ft}$$

According to Fig. 16-17, $\Delta p/h$ = about 0.48 in. water/ft.

ALLOWABLE VAPOR VELOCITY

The limiting vapor velocity for practical operation of a packed tower is set by the flooding point. As in the case of towers utilizing finite-stage contactors, the design engineer can determine the necessary diameter of a packed tower on the basis of the limiting vapor velocity. The maximum allowable vapor velocity is commonly designated as the superficial velocity at the flooding conditions. Because the tower operation may become unstable as the flooding point is approached, the design value for allowable vapor velocity is usually estimated to be 50 to 70 percent of the maximum allowable velocity, and this allowable velocity is used to establish the column diameter.

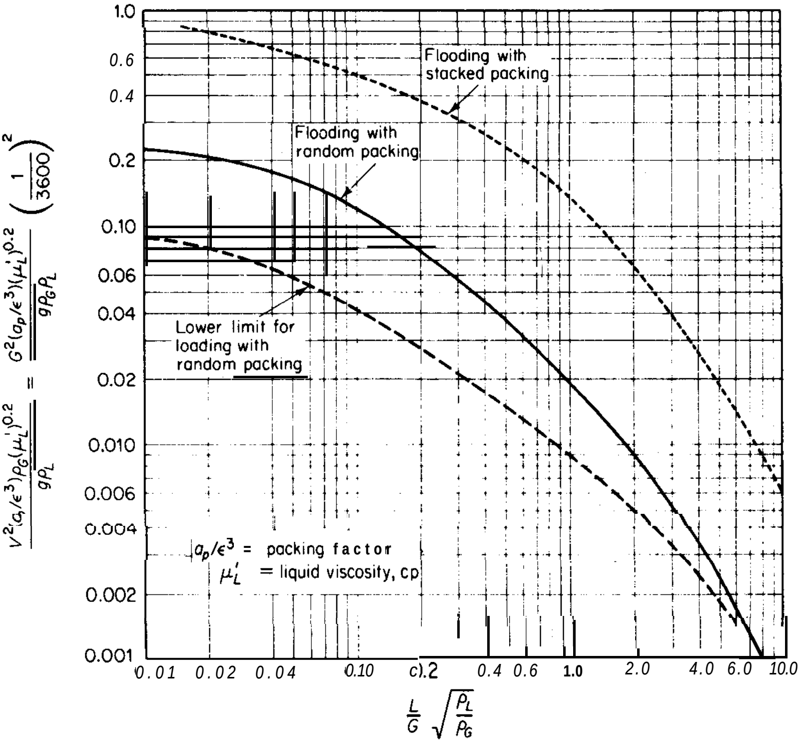

FIGURE 16-20
Correlation for estimating flooding rate in packed towers.

General Correlation for Flooding Rate

Figure 16-20 presents a correlation for flooding conditions in packed towers.[†][‡] The flooding line for random packings is based on experimental data obtained with 15 different liquids, 3 gases, and various random packings, such as Raschig rings, Berl saddles, spheres, and helices. Best results are obtained if the value of the packing factor (a_p/ε^3) is based on direct experimental measurements with

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[†] The correlation presented in Fig. 16-20 is based on the method proposed by T. K. Sherwood, G. H. Shipley, and F. A. L. Holloway, *Ind. Eng. Chem.*, **30**:765 (1938), with the modification of experimental determination of appropriate a_p/ε^3 values as given by Lobo, et al., *Trans. AIChE*, **41**:693 (1945).

[‡] Many other correlations for flooding rates in packed towers have been proposed. R. H. Perry and D. Green, "Chemical Engineers' Handbook," 6th ed., Sect. 18, McGraw-Hill Rook Company, New York, 1984, gives packing characteristics for flooding. F. G. Eichel, *Chem. Eng.*, **73**(19):197 (1966), discusses the effects of packing size on column capacity. H. X. Nguyen, *Chem. Eng.*, **85**(26):181 (Nov. 20, 1978) presents methods showing how computer programs can expedite packed-tower design.

the particular packed tower; however, in the absence of experimental values, the data given in Table 3 can be used to approximate a_p/ε^3.

The dashed line in Fig. 16-20 represents an approximate lower limit of the loading range for random packings and can be used as a rough check to determine what percent of the maximum allowable velocity will give operating conditions in the preloading range.? As indicated by the upper line in Fig. 16-20, stacked packings have higher flow capacities than random packings. This increased capacity can be attributed partly to the decreased holdup and lower pressure drop due to the regular arrangement of the packing.

Simplified Approximate Method for Estimating Flooding Velocities in Packed Distillation Towers‡

At the flooding velocity in a packed tower, liquid starts to build up over the packing. It can be assumed, therefore, that flooding occurs when the downward pressure of the descending films or droplets equal the upward velocity pressure of the rising vapor. With this assumption and considering the gas density as negligible in comparison to the liquid density, the pressure-balance equation can be written as

$$\left(\frac{V_m^2}{2g}\right)\rho_G = H_p\rho_L \tag{24}$$

where H_p is a constant for each packing and can be considered as the mean diameter or film length of the liquid droplets. If K_p is defined as $(2gH_p)^{0.5}$, Eq. (24) can be expressed in a form similar to Eq. (1) as

$$V_m = K_p\sqrt{\frac{\rho_L}{\rho_G}} \tag{25}$$

Since the superficial gas mass velocity equals $V\rho_G$, Eq. (25) can be written in the following alternative form:

$$G_m = V_m\rho_G = K_p\sqrt{\rho_L\rho_G} \tag{26}$$

Equations (24) through (26) do not include any effects due to liquid viscosity or liquid flow rate. Consequently, the equations are most useful when experimental data for the packing are available at known L/G ratios and liquid viscosities. For design purposes, liquid-viscosity effects can be assumed negligible if the viscosity is under 2 centipoises. Because of errors that may be introduced by neglecting the influence of the liquid rate, use of Eq. (25) or (26)

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†M. Leva, *Chem. Eng. Progr. Symp.* Ser., **50**(10):51 (1954). Leva states that the correlation presented in Fig. 16-20 is improved if the ordinate group is multiplied by $(\rho_{\text{water}}/\rho_L)^2$.

‡M. S. Peters and M. R. Cannon, *Ind. Eng. Chem.*, **44**:1452 (1952).

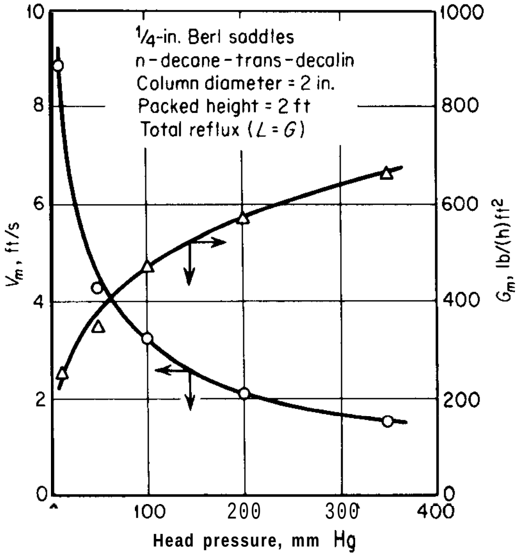

FIGURE 16-21
Effect of operating pressure in packed distillation column on maximum allowable gas rates as superficial linear velocity and mass velocity.

should be limited to cases in which the liquid rate is less than 1.5 times the gas rate.

Effect of Operating Pressure on Flooding Velocities

Because many packed towers are operated at reduced pressures, the effect of total operating pressure on flooding rates is of particular interest to both the design engineer and the operations engineer. As a rough rule of thumb, the maximum allowable vapor velocity V_m in packed distillation columns is from 1 to 3 ft/s at atmospheric pressure and increases with decreasing pressure in direct proportion to the square root of the pressure ratios. Similarly, the maximum allowable mass velocity G_m decreases with decreasing pressure in direct proportion to the square root of the pressure ratios. A typical example of the effect of operating pressure on maximum allowable vapor velocity and maximum allowable gas mass velocity is presented in Fig. 16-21.

The relationship between the maximum allowable rates in a packed distillation tower at two different pressures for a given packing, reflux ratio, and set of fluids can be estimated by writing Eq. (25) or Eq. (26) for each pressure and eliminating the constant K_p. This procedure gives

$$V_{m_1} = V_{m_2}\sqrt{\frac{\rho_{G_2}\rho_{L_1}}{\rho_{G_1}\rho_{L_2}}} \tag{27}$$

$$G_{m_1} = G_{m_2}\sqrt{\frac{\rho_{G_1}\rho_{L_1}}{\rho_{G_2}\rho_{L_2}}} \tag{28}$$

where subscript 1 designates average conditions at one operating pressure and subscript *2* designates average conditions at the second operating pressure.

If the vapor acts as a perfect gas, $\rho_G = M_G P/RT$, and Eqs. (27) and (28) can be expressed as

$$V_{m_1} = V_{m_2}\left(\frac{P_2}{P_1}\right)^{0.5}\left(\frac{\rho_{L_1}}{\rho_{L_2}}\right)^{0.5}\left(\frac{T_1}{T_2}\right)^{0.5}\left(\frac{M_{G_2}}{M_{G_1}}\right)^{0.5} \tag{29}$$

$$G_{m_1} = G_{m_2}\left(\frac{P_1}{P_2}\right)^{0.5}\left(\frac{\rho_{L_1}}{\rho_{L_2}}\right)^{0.5}\left(\frac{T_2}{T_1}\right)^{0.5}\left(\frac{M_{G_1}}{M_{G_2}}\right)^{0.5} \tag{30}$$

where R = ideal-gas-law constant
P = total pressure
T = absolute temperature
M_G = molecular weight of gas

Example 7 Estimation of maximum allowable gas rate in packed tower. A random-packed distillation tower is being operated at an average pressure equal to 1 atm. The reflux ratio is such that an average value of 1.0 may be assumed for *L/G.* The column is being operated at 60 percent of the maximum allowable gas rate. Under these conditions, the average gas rate through the tower is 100 lb/h and the average superficial gas velocity is 1.5 ft/s. If the operating pressure is reduced until the average pressure in the tower is 100 mm Hg and the ratio L/G is unchanged, estimate the maximum allowable gas rate as pounds per hour by (a) use of the simplified approximate Eq. (26), and (*b*) use of Fig. 16-20. The following data apply:

	At average temperature and atmospheric pressure	At average temperature and 100 mm Hg
ρ_G, lb/ft^3	0.20	0.031
ρ_L, lb/ft^3	50	52
μ'_L, centipoises	0.5	0.1

Solution

(*a*) By Eq. (26)

$$G_m = K_p\sqrt{\rho_L \rho_G}$$

At a pressure of 1 atm

$$G_m = \frac{100}{(0.60)(\text{cross-sectional area})}\ \text{lb/(h)(ft}^2)$$

$$K_p = \frac{100}{(0.60)(\text{cross-sectional area})[(50)(0.20)]^{1/2}}$$

At pressure = 100 mm Hg

$$G_m = K_p\sqrt{\rho_L \rho_G} = \frac{(100)[(52)(0.031)]^{1/2}}{(0.60)(\text{cross-sectional area})[(50)(0.20)]^{1/2}}$$

Maximum allowable gas rate at 100 mm Hg pressure by Eq. (26) is

$$G_m(\text{cross-sectional area}) = \frac{100}{0.60}\sqrt{\frac{(52)(0.031)}{(50)(0.20)}} = 67 \text{ lb/h}$$

The same result could be obtained directly from Eq. (28).

(*b*) At pressure = 1 atm

$$\frac{L}{G}\sqrt{\frac{\rho_G}{\rho_L}} = (1)\sqrt{\frac{0.20}{50}} = 0.063$$

From Fig. 16-20,

$$\frac{V_m^2(a_p/\varepsilon^3)\rho_G(\mu'_L)^{0.2}}{g\rho_L} = 0.15$$

$$V_m = \frac{1.5}{0.60} = 2.5 \text{ ft/s}$$

$$\frac{a_p}{\varepsilon^3} = \frac{(0.15)(32.17)(50)}{(2.5)^2(0.20)(0.5)^{0.2}} = 222$$

At a pressure of 100 mm Hg

$$\frac{L}{G}\sqrt{\frac{\rho_G}{\rho_L}} = (1)\sqrt{\frac{0.031}{52}} = 0.024$$

From Fig. 16-20,

$$\frac{V_m^2(a_p/\varepsilon^3)\rho_G(\mu'_L)^{0.2}}{g\rho_L} = 0.20$$

$$V_m = \left[\frac{(0.20)(32.17)(52)}{(222)(0.031)(0.7)^{0.2}}\right]^{1/2} = 7.2 \text{ ft/s}$$

$$\text{Cross-sectional area of empty tower} = \frac{100}{(1.5)(0.20)(3600)} = 0.0927 \text{ ft}^2$$

Maximum allowable gas rate at 100 mm Hg by Fig. 16-20 is

$$V_m\rho_G(\text{cross-sectional area}) = (7.2)(0.031)(0.0927)(3600) = 74 \text{ lb/h}$$

PACKING EFFICIENCIES

The ability of a given packing to achieve effective mass transfer between a gas phase and a liquid phase is commonly expressed in an empirical form as the height of packing equivalent to one transfer unit (HTU) or the height of packing equivalent to one theoretical plate (HETP). HTU can be expressed on the basis of the number of transfer units calculated from gas-phase driving-force data (NTU_G) or liquid-phase driving-force data (NTU_L) as shown in the following.†

$$NTU_G = \int_{y_1}^{y_2} \frac{dy}{y - y_L^*} \tag{31}$$

$$NTU_L = \int_{x_1}^{x_2} \frac{dx}{x_G^* - x} \tag{32}$$

$$HTU_G = \frac{Z}{NTU_G}; \qquad HTU_L = \frac{Z}{NTU_L} \tag{33}$$

where y, x = gas- and liquid-phase concentrations, respectively

y_L^*, x_G^* = gas- and liquid-phase concentrations, respectively, corresponding to values in equilibrium with the liquid- and gas-phase concentrations

Z = height of packing corresponding to distance from the lower to upper limits indicated by the subscripts on y and x

Numerous empirical methods and results giving data for HTU and HETP values for various packings have been presented in the literature.‡

Empirical Prediction of HTU

The following equations giving empirical correlations of data for HTU_G based on a large amount of published data for Raschig rings and Berl saddles have been developed by Cornell et *al.*:§

For Raschig rings:

$$HTU_G = \frac{\psi Sc_G^{0.5}}{(Lf_1 f_2 f_3)^{0.6}} \left(\frac{D'}{12}\right)^{1.24} \left(\frac{Z}{10}\right)^{1/3} \tag{34}$$

For Berl saddles:

$$HTU_G = \frac{\psi Sc_G^{0.5}}{(Lf_1 f_2 f_3)^{0.5}} \left(\frac{D'}{12}\right)^{1.11} \left(\frac{Z}{10}\right)^{1/3} \tag{35}$$

where HTU_G = height of a gas-phase transfer unit, ft

ψ = parameter for a given packing material, see Figs. 16-22 and 16-23 for values

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†HTU can also be based on overall gas or overall liquid values which combine HTU_G and HTU_L. See any standard chemical engineering text on mass transfer for details.

‡See R. H. Perry and D. Green, "Chemical Engineers' Handbook," 6th ed., Sect. 18, McGraw-Hill Rook Company, New York, 1984, for a detailed summary.

§D. Cornell, W. G. Knapp, and J. R. Pair, Mass Transfer Efficiency-Packed Columns-Part 1, *Chem. Eng. Progr.*, **56**(7):68 (1960).

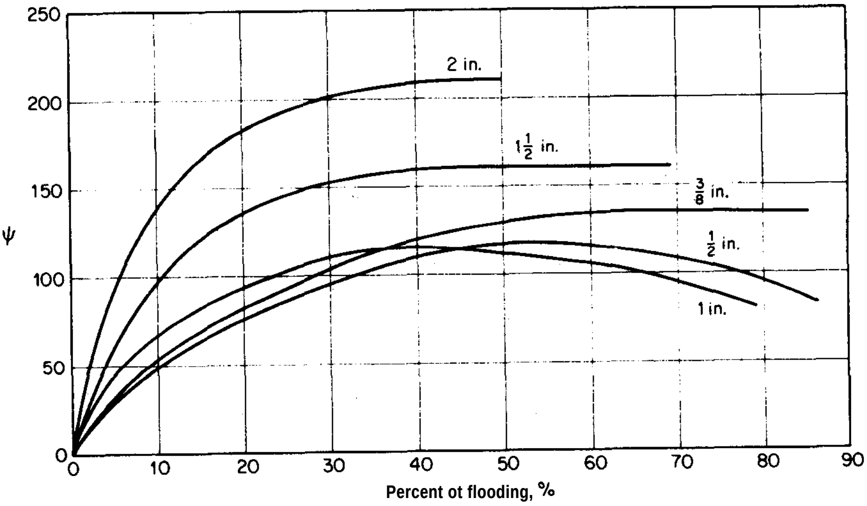

FIGURE 16-22
ψ parameter used in **Eq.** (34) to develop HTU_G values for several sizes of Raschig ring tower packing.

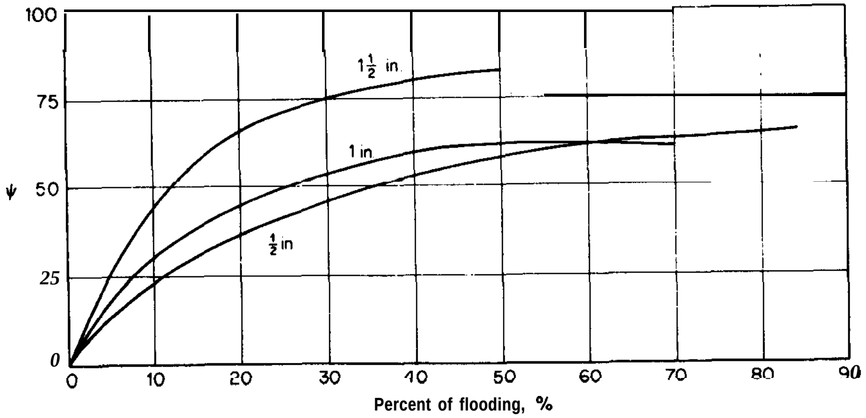

FIGURE 16-23
ψ parameter used in Eq. (35) to develop HTU_G values for several sizes of Berl saddle tower packing.

Sc_G = gas-phase Schmidt number = $\mu_G/\rho_G D_G$
L = superficial liquid mass flow rate, lb/(h)(ft^2)
$f_1 = (\mu_L/2.42)^{0.16}$
$f_2 = (62.4/\rho_L)^{1.25}$
$f_3 = (72.8/\sigma)^{0.8}$
D' = column diameter, in.
Z = packed height, ft
μ_G = gas viscosity, lb/(ft)(h)
μ_L = liquid viscosity, lb/(ft)(h)
ρ_G = gas density, lb/ft^3
ρ_L = liquid density, lb/ft^3
σ = surface tension, dyn/cm
D_G = gaseous-diffusion coefficient, ft^2/h

To apply Eqs. (34) and (35), values of ψ are first obtained from Fig. 16-22 or Fig. 16-23 depending on the size of Raschig rings or Berl saddles being used.

Empirical Prediction of HETP

On the basis of analysis of published data on distillation operations with packed towers, Murch has presented the following empirical equation as a correlation to be used for estimating HETP:†

$$HETP = K_1 G^{K_2} D'^{K_3} Z^{1/3} \frac{\alpha \mu'_L}{\rho'_L} \tag{36}$$

where $HETP$ = height equivalent to a theoretical plate, in.
K_1, K_2, and K_3 = empirical constants as a function of the packing, see Table 6 for values of these constants
G = superficial gas mass flow rate, lb/(h)(ft^2)
D' = column diameter, in.
Z = height of packing, ft
α = relative volatility
μ'_L = liquid viscosity, centipoise
ρ'_L = liquid density, g/cc.

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†D. Murch, Height of Equivalent Theoretical Plate in Packed Fractionation Columns-An Empirical Correlation, *Ind. Eng. Chem.*, **45**:2616 (1953). See also M. S. Kuk, Key Design Variables in Packed Towers, *Chem. Eng. Progr.*, **75**(5):68 (1979).

TABLE 6
Packing constants for use in Eq. (36)

	Summary of packing constant			
	Size, in.	K_1	K_2	K_3
Rings	$\frac{1}{4}$ 8 1	2.10	-0.37	1.24
	$\overline{2}$ 1.0	0.57 8.53	-0.10 -0.34	1.24 1.24
	2.0	0.42	0	1.24
Saddles	$\frac{1}{2}$	5.62	-0.45	1.11
	1.0	0.76	-0.14	1.11
McMahon	$\frac{1}{4}$	0.017	+0.50	1.00
	$\frac{3}{8}$	0.20	+0.25	1.00
	$\frac{1}{2}$	0.33	+0.20	1.00
Protruded packing	0.16	0.39	+0.25	0.30
	0.24	0.076	+0.50	0.30
	0.48	0.45	+0.30	0.30
	1.0	3.06	+0.12	0.30
Stedman	2	0.077	+0.48	0.24
	3	0.363	+0.26	0.24
	6	0.218	+0.32	0.24

RELATIVE MERITS OF PLATE AND PACKED TOWERS

The choice between use of a plate tower or a packed tower for a given mass-transfer operation should, theoretically, be based on a detailed cost analysis for the two types of contactors. Thus, the optimum economic design for each type would be developed in detail, and the final choice would be based on a consideration of costs and profits at the optimum conditions. In many cases, however, the decision can be made on the basis of a qualitative analysis of the relative advantages and disadvantages, and the need for a detailed cost comparison is eliminated. The following general advantages and disadvantages of plate and packed towers should be considered when a choice must be made between the two types of contactors:

1. Stage efficiencies for packed towers must be based on experimental tests with each type of packing. The efficiency varies, not only with the type and size of packing, but also with the fluid rates, the fluid properties, the column diameter, the operating pressure, and, in general, the extent of liquid dispersion over the available packing surface.

2. Because of liquid-dispersion difficulties in packed towers, the design of plate towers is considerably more reliable and requires less safety factor when the ratio of liquid mass velocity to gas mass velocity is low.
3. Plate towers can be designed to handle wide ranges of liquid rates without flooding.
4. If the operation involves liquids that contain dispersed solids, use of a plate tower is preferred because the plates are more accessible for cleaning.
5. Plate towers are preferred if interstage cooling is required to remove heats of reaction or solution, because cooling coils can be installed on the plates or the liquid-delivery line from plate to plate can be passed through an external cooler.
6. The total weight of a dry plate tower is usually less than that of a packed tower designed for the same duty. However, if liquid holdup during operation is taken into account, both types of towers have about the same weight.
7. When large temperature changes are involved, as in distillation operations, plate towers are often preferred because thermal expansion or contraction of the equipment components may crush the packing.
8. Design information for plate towers is generally more readily available and more reliable than that for packed towers.
9. Random-packed towers are seldom designed with diameters larger than 4 ft, and diameters of commercial plate towers are seldom less than 2 ft.
10. Packed towers prove to be cheaper and easier to construct than plate towers, if highly corrosive fluids must be handled.
11. Packed towers are usually preferred if the liquids have a large tendency to foam.
12. The amount of liquid holdup is considerably less in packed towers.
13. The pressure drop through packed towers may be less than the pressure drop through plate towers designed for the same duty. This advantage, plus the fact that the packing serves to lessen the possibility of tower-wall collapse, makes packed towers particularly desirable for vacuum operation.

MASS-TRANSFER EQUIPMENT COSTS

PLATE AND PACKED TOWERS

The purchased cost for plate and packed towers can be divided into the following components: (1) cost for shell, including heads, skirts, manholes, and nozzles; (2) cost for internals, including trays and accessories, packing, supports, and distributor plates; and (3) cost for auxiliaries, such as platforms, ladders, handrails, and insulation.

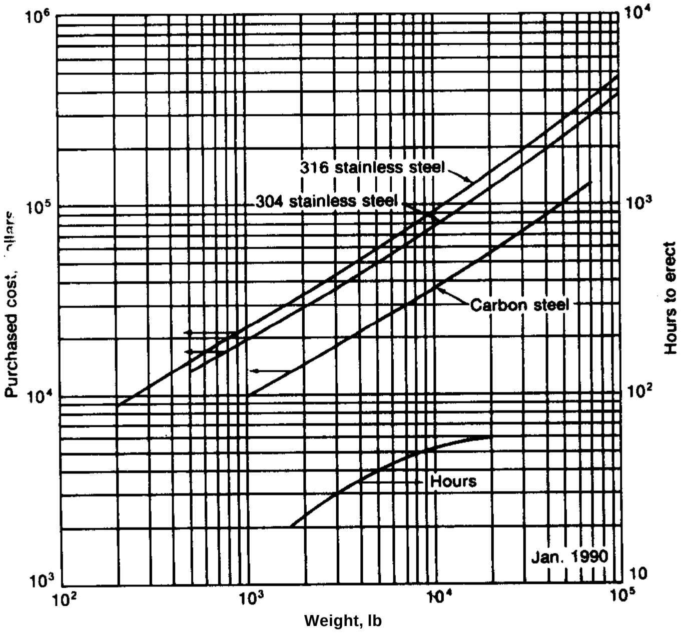

FIGURE 16-24
Fabricated costs and installation time of towers. Costs are for shell with two heads and skirt, but without trays, packing, or connections.

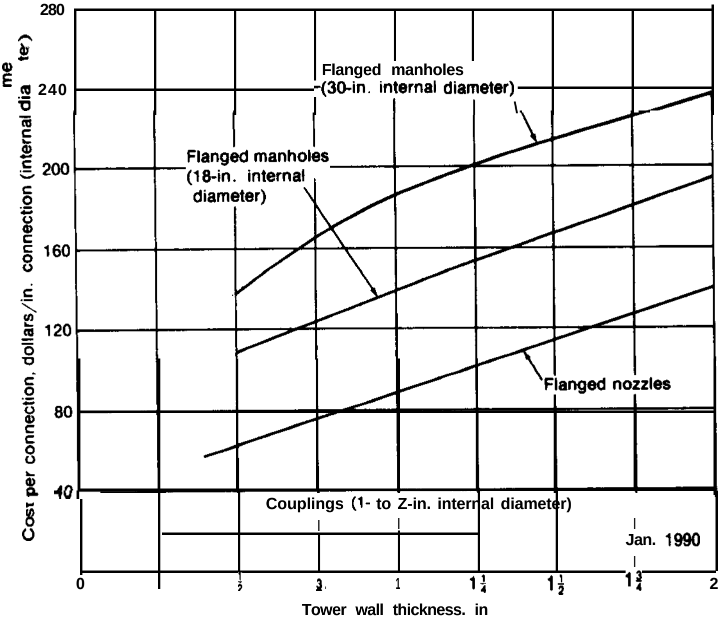

FIGURE 16-25
Approximate installed cost of steel-tower connections. Values apply to 300-lb connections. Multiply costs by 0.9 for 150-lb connections and by 1.2 for 600-lb connections.

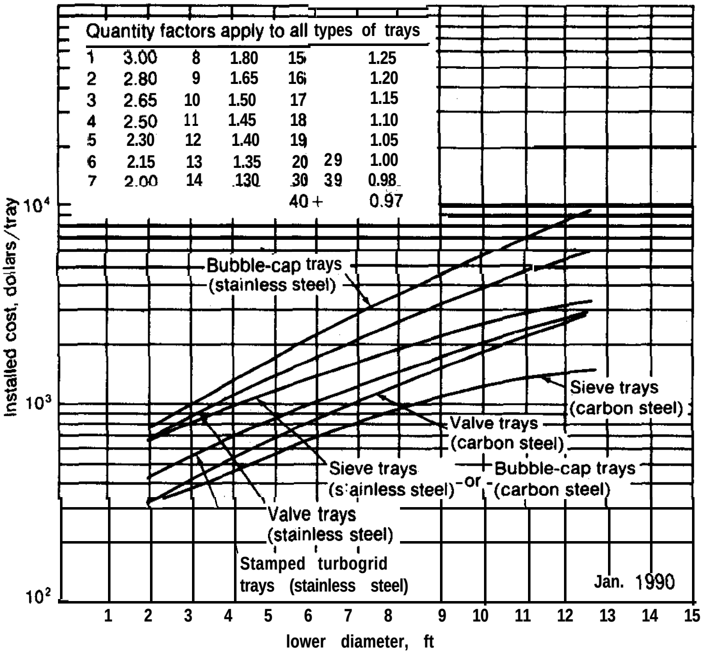

FIGURE 16.26
Cost of trays in plate towers. Price includes tray deck, bubble caps, risers, downcomers, and structural-steel parts.

The cost for fabricated tower shells is quite often estimated on the basis of weight. Figure 16-24 indicates the approximate cost for fabricated steel towers based on weight. This cost is for the tower shell only, without trays, packing, or connections.? The cost for installed connections, such as manholes and nozzles, may be approximated from the data in Fig. 16-25.

Tray costs are shown in Fig. 16-26 for conventional installations. The approximate prices for various types and sizes of the most common industrial packings are indicated in Table 7. Costs for other packing materials are listed in Table 8. For rough estimates, the cost for a distributor plate in a packed tower can be assumed to be the same as that for one bubble-cap tray.

Table 9 presents data that can be used for estimating the cost of such auxiliaries as ladders, platforms, and handrails. The installed cost of several industrial-type insulations, all with aluminum jackets, for towers and tanks may

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†See also the information presented in Chap. 14 on costs for tanks, pressure vessels, and storage equipment along with information on design methods for tanks and pressure vessels.

TABLE 7
Costs of tower packings, uninstalled, January, 1990
Prices in dollars per ft^3, 100 ft^3 orders, f.o.b. manufacturing plant

	Size, in., $ / ft^3			
	1	$1\frac{1}{2}$	2	3
Raschig rings:				
Chemical porcelain	12.8	10.3	9.4	7.8
Carbon steel	36.5	23.9	20.5	16.8
Stainless steel	155	117	87.8	—
Carbon	52	46.2	33.9	31.0
Intalox saddles:				
Chemical stoneware	17.6	13.0	11.8	10.7
Chemical porcelain	18.8	14.1	12.9	11.8
Polypropylene	21.2	—	13.1	7.0
Berl saddles:				
Chemical stoneware	27.0	21.0	—	
Chemical porcelain	33.5	21.5	15.6	—
Pall rings:				
Carbon steel	29.3	19.9	18.2	—
Stainless steel	131	99.0	86.2	—
Polypropylene	21.2	14.4	13.1	

TABLES
Costs of miscellaneous packing materials, January, 1990

Materials	cost, $ / ft^3
Activated carbon	45
Alumina	42
Calcium chloride	11
Coke	5
Crushed limestone	11
Resin	2.56
Silica gel	84

TABLE 9
Cost of tower auxiliaries

Item	Cost, January, 1990	Amount for typical tower	
Ladder	$0.68/lb	30 lb / ft of height	
Platforms and handrails	0.68/lb	Tower d&meter, ft	Weight, lb
		4	1700
		6	2300
		8	2800
		10	3300

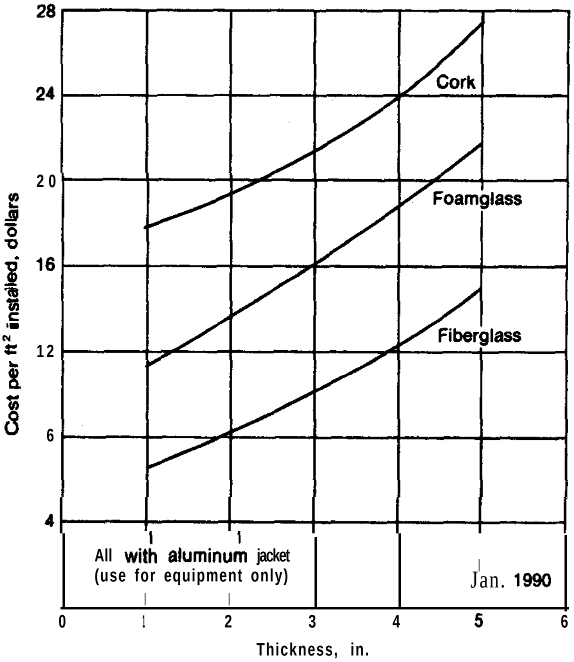

FIGURE 16-27
Installed cost of various industrial insulations for towers and tanks.

TABLE 10
Relative fabricated cost for metals used in tray-tower construction

Materials of construction	Relative cost per ft² of tray area (based on carbon steel = 1)
Sheet-metal trays:	
Steel	1
4–6% chrome- $\frac{1}{2}$ moly alloy steel	2.1
11-13% chrome type 410 alloy steel	2.6
Red brass	3
Stainless steel type 304	4.2
Stainless steel type 347	5.1
Monel	7.0
Stainless steel type 316	5.5
Inconel	8.2
Cast-iron trays	2.8

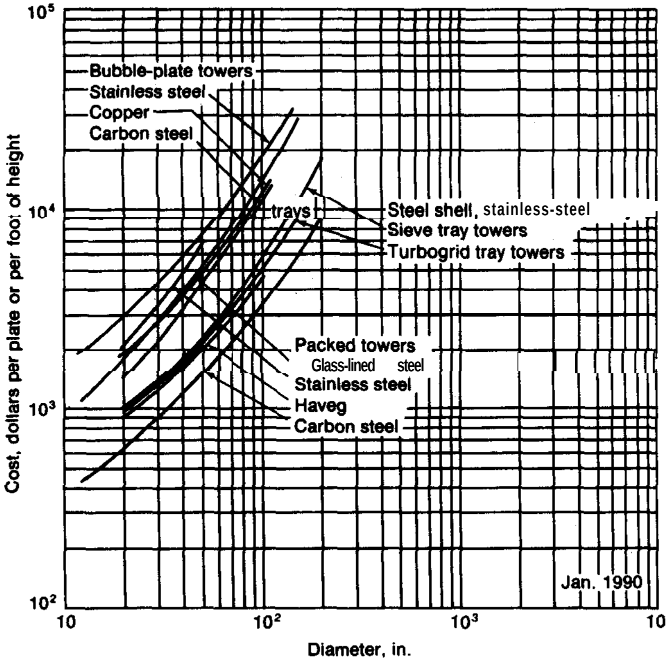

FIGURE 16-28
Cost of towers including installation and auxiliaries.

be approximated from Fig. 16-27. Relative fabricated costs for various materials used in tray-tower construction are listed in Table 10.

The complete installed cost of various types of towers is given in Fig. 16-28. This cost includes all the components normally associated with a tower as outlined above.

Example 8 Estimation of cost for bubble-cap tower. A distillation tower contains 18 steel bubble-cap trays. An 18-in. manhole is located above each tray, and one manhole is located below the bottom tray. The ID of the tower is 6 ft, and the total height including the skirt is 50 ft. The shell is steel (density = 490 lb/ft^3) with a $\frac{5}{8}$-in. wall thickness. Six l-in. couplings and the following flanged nozzles are attached to the tower: one 10-in. vapor-line nozzle; three 4-in. nozzles; and six Z-in. nozzles. On the basis of the data presented in Fig. 16-24, Fig. 16-25, and Fig. 16-26, estimate the cost of the tower with trays installed, but not including cost for auxiliaries or tower installation. The total weight of the shell, including heads and skirt, may be assumed to be 1.12 times the weight of the cylindrical shell. Material of construction is carbon steel.

Solution. Total weight of shell = $(6)(\frac{5}{8})(\frac{1}{12})(3.14)(50)(490)(1.12)$ = 27,000 lb:

Purchased cost of steel shell = (27,000 lb—From Fig. 16-24)	$68,100
Cost for 19 installed 18-in. manholes = (19)(116)(18)	39,672
Cost for 1 installed 10-in. nozzle = (1)(69)(10)	690
Cost for 3 installed 4-in. nozzles = (3)(69)(4)	828
Cost for 6 installed 2-in. nozzles = (6)(69)(2)	828
Cost for 6 installed l-in. couplings = (6)(19)(1)	114
Cost for 18 installed steel bubble-cap trays = (18)(1380)	24,840
Total	$135,072

Estimated total purchased cost for tower = $135,000.

DRYERS

Cost data for drum dryers, rotary dryers, tray dryers, and tumble dryers are presented in Figs. 16-29 through 16-32. The data available cover both atmospheric and pressure systems.

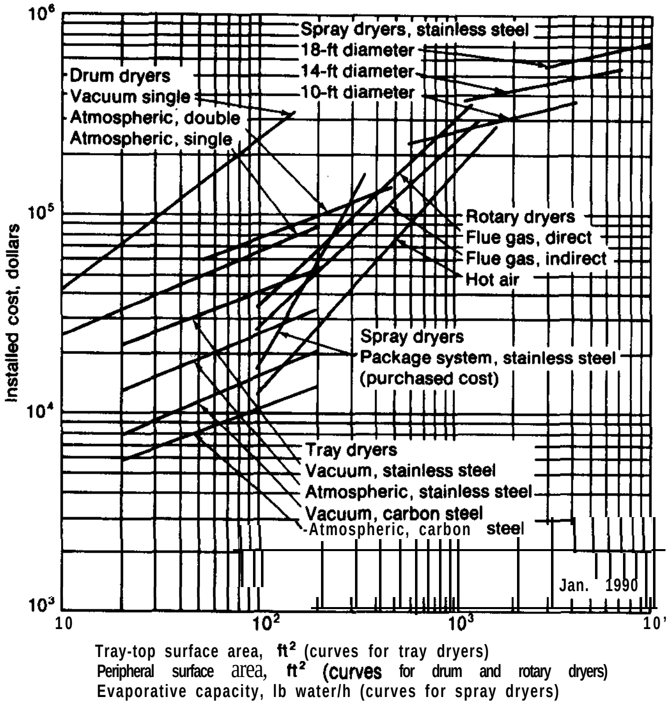

FIGURE **16-29**
Installed cost of dryers.

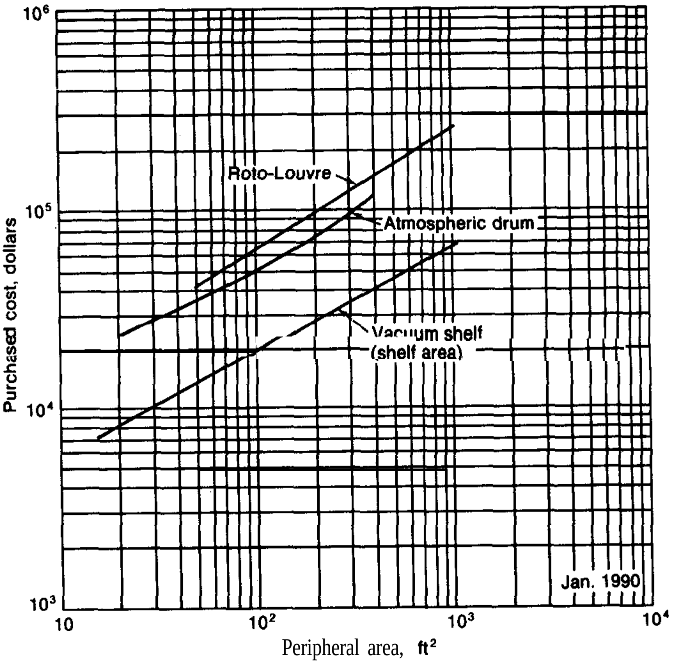

FIGURE **16-30**
Costs for dryers, steel construction. Price includes auxiliaries (motor, drive, fan, vacuum pump, condenser, and receiver).

REACTORS

Application of the principles of chemical reaction kinetics for the design of chemical reactors is an area of activity that is particularly appropriate for chemical engineers.† For reactor design, the fundamental ideas of process analysis, equipment design, economic evaluation, and optimization are all combined to give a mathematical model of the reactor system. Prior to the ready availability of large computers, mathematical models for application to overall reactor systems were generally restrained to a relatively simple form so that solution of the resulting equations could be accomplished by standard computational techniques.

Now that sophisticated computer equipment is widely available, the chemical engineer can concentrate much more heavily on analysis of the system and

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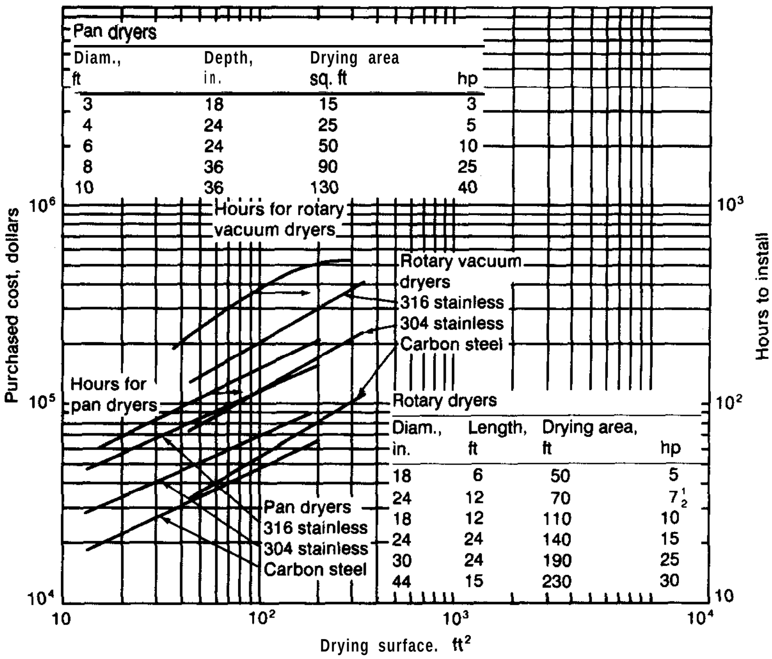
†For an overall treatment of principles and techniques as applied to reactor design, see H. F. **Rase,** “Chemical Reactor Design for Process Plants, Vol. I-Principles and Techniques; Vol. II-Case Studies and Design Data,” John Wiley & Sons, New York, 1977.



Pan dryers

Diam., ft	Depth, in.	Drying area sq. ft	hp
3	18	15	3
4	24	25	5
6	24	50	10
8	36	90	25
10	36	130	40

Rotary dryers

Diam., in.	Length, ft	Drying area, ft	hp
18	6	50	5
24	12	70	$7\frac{1}{2}$
18	12	110	10
24	24	140	15
30	24	190	25
44	15	230	30

FIGURE 16-31
Drying equipment. Price includes motor and drive.

development of the best mathematical model. Solution of complex mathematical models for reactor-design systems, which would have been impossible before the availability of computers, can now be accomplished with ease so that the actual solution techniques are no longer nearly so important as the basic analysis involved in developing the mathematical model.

The basic mathematical model for a reactor system is developed from (1) material balances including inflow, outflow, reaction rates, mixing effects, and diffusional effects; (2) energy balances including heats of reaction, heat transfer, and latent and sensible heat effects; (3) reaction-rate expressions incorporating mechanism definition and temperature functionality; (4) economic evaluations; and (5) special constraints on the design system. The material balances and the energy balances referred to in (1) and (2) are developed from the fundamental conservation equations based on a differential volume of the reacting system, or,

Material balance:

$$\begin{array}{c}\text{Rate of accumulation}\\ \text{of the ith species in}\\ \text{the volume element}\end{array} = \begin{array}{c}\text{net rate of inward}\\ \text{flow of the ith}\\ \text{species to the}\\ \text{volume element}\end{array} + \begin{array}{c}\text{rate of genera-}\\ \text{tion of the ith}\\ \text{species in the}\\ \text{volume element}\end{array} \qquad (37)$$

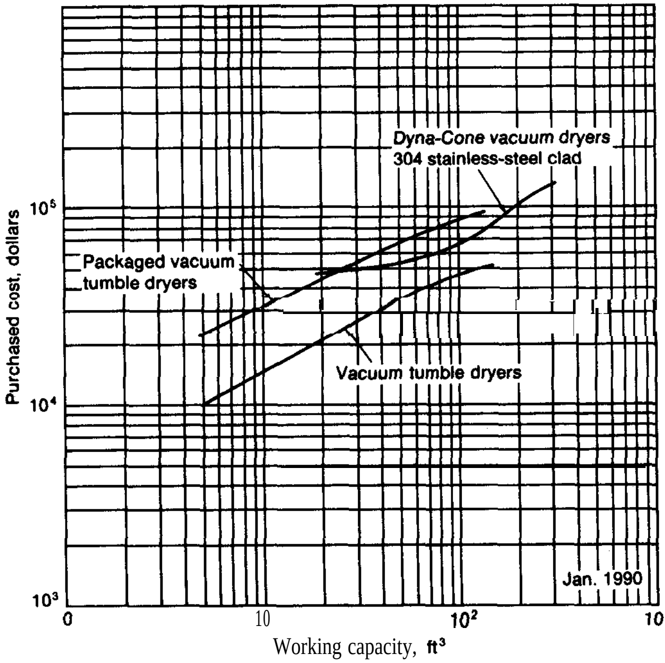


FIGURE 16-32
Cost of tumble and Dyna-Cone vacuum dryers. For packaged vacuum tumble dryers, price includes the following equipment: jacketed shell for steam or electric water heater, water pump, vacuum pump, condenser, receiver, piping, and temperature-pressure- and motor controls. For the vacuum tumble dryers, price includes frame and neoprene-lined butterfly outlet valve. For Dyna-Cone vacuum dryers, price includes power supply, butterfly discharge valve, fan-cooled motor, internal brake, gear, coupling, belt or chain guards, replaceable cartridge filter, and 304 stainless steel thermocouple well.

Energy balance:

$$\begin{matrix}\text{Rate of energy} & & \text{net rate of inward} & & \text{rate of generation}\\ \text{accumulation in} & = & \text{energy flow to the} & + & \text{of energy in the}\\ \text{the volume element} & & \text{volume element} & & \text{volume element}\end{matrix} \qquad (38)$$

There are many types of chemical reactors which operate under various conditions, such as batch, flow, homogeneous, heterogeneous, steady state, etc. Thus, one general mathematical description which would apply to all types of reactors would be extremely complex. The general approach for reactor design, therefore, is to develop the appropriate mathematical model which will describe the specific reaction system for that particular form of reactor under consideration. For example, if the reaction system is to be evaluated for steady-state operating conditions, the derivative terms on the left-hand sides of Eqs. (37) and (38) are zero which results in a simplified mathematical model.

It is convenient to use different forms for expressing the rate of generation of the ith species and the accompanying rate of energy generation due to the heat of reaction which are appropriate to the particular reaction system under consideration. For example, if a batch reactor of volume V_b involving a homogeneous reaction is under consideration, the convenient form for expressing the rate of reaction, based on material i as a reactant, is

$$r_i = -\frac{1}{V_b}\frac{dN_i}{dt} = k_i(C_a, C_b, \ldots, C_i, \ldots, C_{n-1}, C_n) \tag{39}$$

where N_i = amount of base-reactant material i present in the volume V_b at time t

t = time

k_i = function for species i related to the specific reaction-rate constant

$C_a, \ldots, C_n$ = concentration of species a,. . . , n in the reactor at time t

On the other hand, for a flow reactor where concentration varies throughout the reactor, it is usually more convenient to base the rate on a differential volume of the reactor, and a commonly used rate form is

$$r_i = -\operatorname{div}(\mathbf{U}C_i) \tag{40}$$

where U is the velocity vector with normal x, y, and z directed velocity elements using Cartesian coordinates. For the case of a plug-flow reactor and z representing the length of the reactor in the direction of flow, Eq. (40) reduces to the familiar form of

$$r_i = -\frac{d(U_z C_i)}{dz} \tag{41}$$

Similar forms of simplified expressions can be developed for back-mix tank reactors and other types of reactor systems.†‡ A typical example of the analysis used in developing the mathematical model for a reactor design is presented in the following section.

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†Design equations for use with various idealized types of reactor systems are presented later in this chapter.

‡For further development of standard mathematical forms of rate expressions and basic developments for design expressions, see any standard text on chemical engineering reaction kinetics.

AN EXAMPLE OF MATHEMATICAL MODELING FOR REACTOR DESIGN

Consider the case of a reactor design dealing with a unit that is to be used to chlorinate benzene with ferric chloride as the catalyst.† A semibatch reactor is to be used containing an initial charge of pure benzene into which dry chlorine gas is fed at a steady state. The reaction vessel is operated at a constant pressure of two atmospheres and is equipped with coils to maintain a constant temperature of 55°C. The liquid is agitated to give a homogeneous mixture. The unit contains a reflux condenser which returns all vaporized benzene and chlorobenzenes to the reactor while allowing the generated hydrogen chloride gas and excess chlorine gas to leave the system. Monochlorobenzene, dichlorobenzene, and trichlorobenzene are produced through successive irreversible reactions as follows, where the *k's* represent the forward rate constants for the equations as written:

$$C_6H_6 + Cl_2 \xrightarrow{k_1} C_6H_5Cl + HCl$$

$$C_6H_5Cl + Cl_2 \xrightarrow{k_2} C_6H_4Cl_2 + HCl$$

$$C_6H_4Cl_2 + Cl_2 \xrightarrow{k_3} C_6H_3Cl_3 + HCl$$

For this semibatch system, it is desired to know at what time the reaction should be stopped in order to obtain a maximum yield from the original benzene of any one of the three products, based on the data and assumptions given in Table 11.

Equation (37) forms the basis for the initial design analysis of this system. Equation (38) is not used for the kinetic analysis because the system is isothermal. Because there is no input or output flow of benzene or chlorobenzenes, the material balances, by Eq. (37), become

For benzene,

$$-\frac{1}{V_b}\frac{dN_B}{dt} = k_1\frac{N_B}{V_b}\frac{N_{Cl}}{V_b} \tag{42}$$

For monochlorobenzene,

$$\frac{1}{V_b}\frac{dN_M}{dt} = k_1\frac{N_B}{V_b}\frac{N_{Cl}}{V_b} - k_2\frac{N_M}{V_b}\frac{N_{Cl}}{V_b} \tag{43}$$

For dichlorobenzene,

$$\frac{1}{V_b}\frac{dN_D}{dt} = k_2\frac{N_M}{V_b}\frac{N_{Cl}}{V_b} - k_3\frac{N_D}{V_b}\frac{N_{Cl}}{V_b} \tag{44}$$

---

†Adapted from information and methods presented by R. B. MacMullin, *Chem. Eng. Progr.*, **44**:183 (1948) and A. Carlson, *Instr. and Control Systems*, **38**(4):147 (1965).

TABLE 11

Basic data and assumptions for reactor-design example dealing with optimization of yields for benzene chlorinator

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Data for chlorobenzene reactor-design example

The feed rate of the dry chlorine is 1.4 lb mole of chlorine/(h) (lb mole of initial benzene charge).

The following rate constants are estimated values for the catalyst used at 55°C (assumed for this problem):

$$k_1 = 510\ (\text{lb mol/ft}^3)^{-1}(\text{h})^{-1}$$
$$k_2 = 64\ (\text{lb mol/ft}^3)^{-1}(\text{h})^{-1}$$
$$k_3 = 2.1\ (\text{lb mol/ft}^3)^{-1}(\text{h})^{-1}$$

There is negligible liquid or vapor holdup in the reflux condenser.

Volume changes in the reacting mixture are negligible, and the volume of liquid in the reactor remains constant at 1.46 ft^3/lb mole of initial benzene charge.

Hydrogen chloride has a negligible solubility in the liquid mixture.

The chlorine gas fed to the system goes into the liquid solution immediately up to its solubility limit of 0.12 lb mole of chlorine/lb mole of original benzene, and this value then remains constant.

Each reaction is second order as written.

Nomenclature

F = feed rate of chlorine as (lb mol/h)/(lb mole of initial benzene charge) = 1.4
V_b = volume of reactor liquid contents = $1.46N_0$; (V_b/N_0 = 1.46)
N_0 = lb moles of benzene present at time 0
N_B = lb moles of benzene present at time t
N_{Cl} = lb moles of chlorine present in liquid at time t
N_D = lb moles of dichlorobenzene present at time t
N_M = lb moles of monochlorobenzene present at time t
N_T = lb moles of trichlorobenzene present at time t
t = time in hours

$$B = \frac{N_B}{N_0} \qquad D = \frac{N_D}{N_0} \qquad T = \frac{N_T}{N_0}$$

$$C = \frac{N_{Cl}}{N_0} \qquad M = \frac{N_M}{N_0}$$

For trichlorobenzene,

$$\frac{1}{V_b}\frac{dN_T}{dt} = k_3 \frac{N_D}{V_b}\frac{N_{Cl}}{V_b} \tag{45}$$

For chlorine, subject to the solubility limit of $N_{Cl}/N_0 = 0.12$, there is an input flow term in Eq. (37). Therefore, the rate expression for the consumption of chlorine can be written as

$$\frac{1}{V_b}\frac{dN_{Cl}}{dt} = \frac{N_0F}{V_b} - k_1\frac{N_B}{V_b}\frac{N_{Cl}}{V_b} - k_2\frac{N_M}{V_b}\frac{N_{Cl}}{V_b} - k_3\frac{N_D}{V_b}\frac{N_{Cl}}{V_b} \tag{46}$$

subject to a maximum value of

$$\frac{N_{Cl}}{N_0} = 0.12 \tag{47}$$

Equations (42) to (47) can be converted to a more convenient form for solution by rearranging with use of the nomenclature and data shown in Table 11 to bring in the primary variables being sought (namely, B, M, D, T, and C). The result is

$$\frac{dB}{dt} = -\frac{k_1 N_0}{V_b} BC \tag{48}$$

$$\frac{dM}{dt} = \frac{k_1 N_0}{V_b} BC - \frac{k_2 N_0}{V_b} MC \tag{49}$$

$$\frac{dD}{dt} = \frac{k_2 N_0}{V_b} MC - \frac{k_3 N_0}{V_b} DC \tag{50}$$

$$\frac{dT}{dt} = \frac{k_3 N_0}{V_b} DC \tag{51}$$

$$\frac{dC}{dt} = F - \frac{k_1 N_0}{V_b} BC - \frac{k_2 N_0}{V_b} MC - \frac{k_3 N_0}{V_b} DC \tag{52}$$

$$\text{subject to a maximum of } C = 0.12 \tag{53}$$

The solution can be checked at any point by an overall material balance which indicates that

$$1 = B + M + D + T \tag{54}$$

Thus, the result of the mathematical analysis is five differential equations [Eqs. (48) through (52)] with five unknowns (B, M, D, T, C). The solution to the problem is now completed by simultaneous solution of the above equations for the five variables as a function of time to give results from which the optimum times can be chosen.

Without the availability of a computer, the calculations for this solution would be very tedious and time consuming.? With computers available, the design engineer's task is essentially finished with the correct generation of the mathematical model as given in Eqs. (48) to (53) and summarized as an information flow diagram in Table 12.

For this example, the final result can be obtained by use of either an analog or digital computer. The details for obtaining the solution with a digital computer using the three computer languages of FORTRAN, BASIC, and PASCAL are presented in the following:+ Tables 13, 14, and 15 provide the

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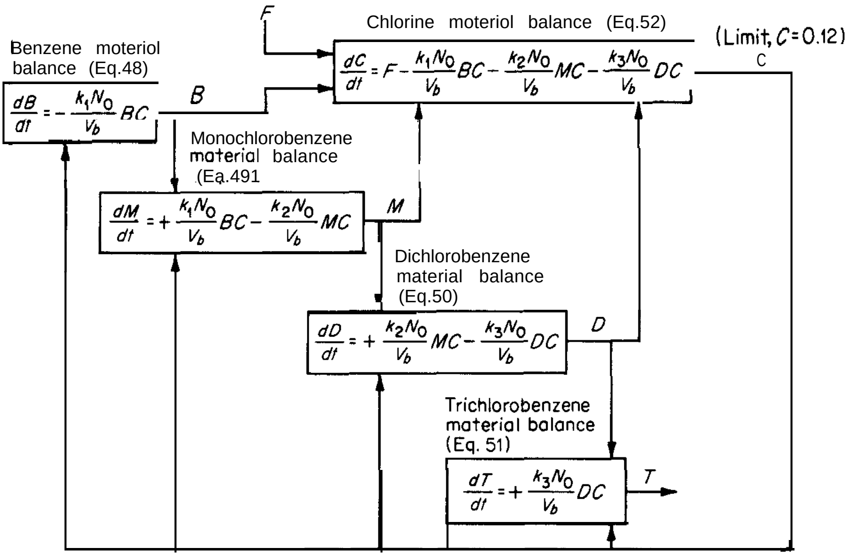
[†] For example, see the amount of labor involved in the hand solution presented by J. M. Smith, "Chemical Engineering Kinetics," 2d ed., pp. 79-82, McGraw-Hill Book Company, New York, 1970, in which the same problem is solved with the simplification that no excess chlorine passes through the system.

[‡] For a full solution using an analog computer, see M. S. Peters and K. D. Timmerhaus, "Plant Design and Economics for Chemical Engineers," 3d ed., p. 782, McGraw-Hill Book Company, New York, 1980.

TABLE12

Information flow diagram of mathematical model for chlorobenzene reactor-design example



source-language programs for the three solutions which give essentially the same results for each program. Table 16 along with Fig. 16-33 present the final results for the FORTRAN program.?

EQUATIONS FOR REACTOR-DESIGN APPLICATION

The common, idealized designations for types of reactors are batch, *plug-flow,* and *buck-mix.* In an idealized *batch* reactor, the reactants are mixed at the same time and no reaction mixture is removed during the reaction. Complete mixing is assumed during the entire reaction so that the entire reactor contents are at the same temperature and concentration at any one instant. The composition (and possibly the temperature) changes with time. The idealized *plug-flow* reactor is a tubular reactor with the reacting fluid moving through it with no back mixing or radial concentration gradients. Conditions are in a steady state so the concentrations and temperature profile along the length of the reactor do

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†Solution of the ordinary differential equations was accomplished using the elementary Euler method. Improvement could be made by using a smaller time step or by use of a different integration routine. Help in these solutions by E. C. Roche, Jr. of New Jersey Institute of Technology is acknowledged.

TABLE 13

Printout of FORTRAN program for solution of chlorobenzene reactor-design example

```
      PROGRAM REACT
C     DEFINITION OF VARIABLES
C      B = BZ, M = MONO, D = DI'S, T = TRI'S, C = CL2, & TAU = TIME
C      F = CL2 FEED, V=REACTOR VOLUME
C      ALL FLOWS ARE PER MOLE OF BENZENE CHARGED TO REACTOR
      REAL B(1000),M(1000),D(1000),T(1000),C(1000),TAU(1000)
C     PROCESS CONSTANTS: RATE CONSTANTS ARE K1, K2, & K3
C     EULER TIME STEP = DTAU
      REAL K1,K2,K3
      DATA K1 / 510./, K2 / 64./, K3 / 2.1 / , V / 1.46 /
      DATA F / 1.4 / , DTAU / 0.0020 /
C     INITIALIZE ALL PROCESS VARIABLES
      N=l
      B(1) = 1.0
      M(1) = 0.0
      D(1) = 0.0
      T(1) = 0.0
      C(1) = 0.0
      TAU(1) -0.0
      NN=N - 1
      WRITE (6,30) NN,TAU(N),B(N),M(N),D(N),T(N),C(N)
C     INTEGRATION OF ODE'S VIA EULER
10    N = N + 1
      TAU(N) = TAU(N - 1) + DTAU
      DB= - (K1 / V*B(N - 1)*C(N - 1))*DTAU
      DM = ( + K1 / V*B(N - 1)*C(N - 1) - K2 / V*M(N - 1)*C(N - 1))*DTAU
      DD = ( + K2 / V*M(N - 1)*C(N - 1) - K3 / V*D(N - 1)*C(N - 1))*DTAU
      DT= (K3 / V*D(N - 1)*C(N - 1))*DTAU
      DC = (F - K1 / V*B(N - 1)*C(N - 1) - K2 / V*M(N - 1)*C(N - 1) -
         K3 / V*D(N - 1)*C(N - 1))*DTAU
      B(N) = B(N - 1) + DB
      M(N) = M(N - 1) + DM
      D(N) = D(N - 1) + DD
      T(N) = T(N - 1) + DT
C     CHECK BZ MATERIAL BALANCE . . . BZ RINGS
      RING = B(N) + M(N) + D(N) + T(N)
      IF (ABS(RING - 1.0) .LT. 0.0001) GO TO 20
      B(N) = B(N) /RING
      M(N) = M(N) / RING
      D(N) = D(N) /RING
      T(N) ❑ TCN) /RING
C     CHLORINE SOLUBILITY CHECK
20    C(N) = C(N - 1) + DC
      IF (C(N) .GT. 0.12) C(N)=0.12
C     OUTPUT RESULTS AT TIME TAU(N) . . . BUT AT 0.1HR INCREMENTS
      NN=N - 1
      IF (MOD(NN,50) .NE. 0) GO TO 40
      WRITE (6,30) NN,TAU(N),B(N),M(N),D(N),T(N),C(N)
30    FORMAT (15,F10.4,5E12.4)
C     TIME CHECK
40    IF (TAU(N) .LT. 2.0) GO TO 10
      STOP
      END
```

TABLE 14
Printout of BASIC program for solution of chlorobenzene reactor-design example

```
5 CLS
6 KEY OFF
8 OPEN "O", #1, “RESULTS”
lo REM DEFINITION OF VARIABLES
20 REM B=BZ,M=MONO,D=DI'S,T=TRI'S,C=CL2,& TAU=TIME
30 REM ALL FLOWS ARE PER MOLE OF BENZENE CHARGED TO THE REACTOR
40 DIM B(1000),M(1000),D(1000),T(1000),C(1000),TAU(1000)
50 REM PROCESS CONSTANTS: RATE CONSTANTS ARE K1, K2, & K3
60 RER EULER TIME STEP = DTAU
70 REM
80 READ K1,K2,K3,V,F,DTAU
81 REN PRINT K1,K2,K3,V,F,DTAU:PRINT
90 REM
120 REM INITIALIZE ALL PROCESS VARIABLES
130 B(0)=1: M(0)=0: D(0)= 0: T(0)=0: C(0)= 0: TAU(0)=0
139 FORMS ="#### #.## #.###### #.###### #.###### #.###### #.######"
140 N=0
150 REM PRINT N,TAU(N),B(N),M(N),D(N),T(N),C(N)
151  PRINT #1, USING FORMS; N,TAU(N),B(N),M(N),D(N),T(N),C(N)
200 REM INTEGRATION OF ODE’S VIA EULER
210 N=N+1
211  TAU(N)=TAU(N-1)+DTAU
220   DB=-K1/V*B(N-1)*C(N-1)*DTAU
230    DM=(+K1/V*B(N-1)*C(N-1)-K2/V*M(N-1)*C(N-1))*DTAU
240    DD=(+K2/V*M(N-1)*C(N-1)-K3/V*D(N-1)*C(N-1))*DTAU
250  DT=K3/V*D(N-1)*C(N-1)*DTAU
260 DC= (F - K1/V*B(N - 1)*C(N - 1) - K2/V*M(N - 1)*C(N - 1) -
    K3/V*D(N - 1)*C(N - 1)*DTAU
280 REM PRINT “LINE 280",N,DTAU,TAU(N),DB,DM,DD,DT,DC
281 REM PRINT “LINE 281”,B(N - 1),M(N - 1),D(N - 1),T(N - 1),C(N - 1)
290 REM PRINT
299 REM IF N=5 THEN STOP
300 B(N)=B(N-1)+DB
310 M(N)=M(N-1)+DM
320 D(N)=D(N-1)+DD
330 T(N)=T(N-1)+DT
340 REM CHECK THE BZ MATERIAL BALANCE . . . BZ RINGS
350 RING=B(N)t M(N)t D(N)+T(N)
360 IF ABS(RING)<.0001THEN GOT0 420
370 B(N)=B(N)/RING
380 M(N)=M(N)/RING
390 D(N)=D(N)/RING
400 T(N)=T(N)/RING
410 REN CHECK CL2 SOLUBILITY
420 C(N)=C(N-1)+DC
430 IF C(N)>.12 THEN C(N)=.12
500 REM PRINT BATCH REACTOR CONTENTS AT 0.1 HR TIME INCREMENTS
510 IF INT(N/50)*50 <> N THEN GOT0 600
540 FORNS ="#### #.## #.###### #.###### #.###### #.###### #.######"
550  REN PRINT N,TAU(N),B(N),M(N),D(N),T(N),C(N)
551  PRINT #1, USING FORMS; N,TAU(N),B(N),M(N),D(N),T(N),C(N)
600 IF N = 1000 GOT0 999
610 GOT0 210
900 REM
910 REN . . . DATA . . .
920 DATA 510.0, 64.0, 2.1, 1.46, 1.4, 0.0020
999 END
```

TABLE 15
Printout of PASCAL program for solution of chlorobenzene reactor-design example

---

```
PROGRAM REACT(INPUT,OUTPUT):
 {*DEFINITION OF VARIABLES*)
 {* B=BZ, M=MONO, D=DI'S, T=TRI'S, C=CL2, & *}
 {* TAU=TIME, F=CL2 FEED, V= REACTOR VOLUME *}

TYPE VECTOR = ARRAY [0..1001] OF REAL;

VAR  B,M,D,T,C,TAU:VECTOR;
     N:INTEGER;
     DB,DM,DD,DT,DC,RING:REAL;
 {*PROCESS CONSTANTS: RATE CONSTANTS ARE K1,K2, & K3*}
 {*EULER TIME STEP =DTAU*}
{*}
CONST  K1=510.0;K2=64.0;K3=2.1;V=1.46;F=1.4;DTAU=0.0020;
       WW = 16; WD = 5;
{*}
BEGIN {PROGRAM REACT)
 {*INITIALIZE ALL PROCESS VARIABLES*)
 B[0]:=1.0;
 M[0]:=0.0;
 D[0]:= 0.0;
 T[0]:=0.0;
 C[0]:=0.0;
 TAU[0]:=0.0;
 N:=0;
  WRITELN (N:5,TAU[N]:WW:WD,B[N]:WW:WD,M[N]:WW:WD,D[N]:WW:WD,
           T[N]:WW:WD,C[N]:WW:WD);
  <*INTEGRATION OF ODE'S VIA EULER*)
  N:=N+1;
WHILE ((TAU[N]<=2.0 )AND (N<=1000))DO
 BEGIN
  TAU[N]:=TAU[N-11 t DTAU;
  DB:= - (K1 /V*B[N - 1]*C[N - 1])*DTAU;
  DM:= (K1 / V*B[N -  1]*C[N-I        I        -K2/V*M[N-1]*C[N-1])*DTAU;
  DD:=(K2/V*M[N-1]*C[N-1          1         -K3/V*D[N-1]*C[N-1])*DTAU;
  DT:= (K3 /V*D[N - 1]*C[N  - 1])*DTAU;
  DC:=(F-K1/V*B[N-1]*C[N-1              1              -K2/V*M[N-1]*C[N-1]-
       K3/V*D[N - 1]*C[N  - 1])*DTAU;
  B[N]:=B[N-1]+D B ;
  M[N]:=M[N -  1]+DM;
  D[N]:=D[N-1]+D D ;
  T[N]:=T[N-1]+D T ;
  {*CHECK BZ MATERIAL BALANCE . . . BZ RINGS*)
  RING:=B[N]+M[N]+D[N]+T[N];
  IF ABS(RING- 1.0) >= 0.0001
   THEN BEGIN
         B[N]:=B[N]/RING;
         M[N]:=M[N]/ RING;
         D[N]:=D[N]/ RING;
         T[N]:=T[N]/RING
        {*CHLORINE SOLUBILITY CHECK*>
        END; {IFTHEN}
```

TABLE 15

Printout of PASCAL program for solution of chlorobenzene reactor-design example (Continued)

```
  C[N]:=C[N - 1]+DC;
  IF(C[N] > 0.12)
   THEN C[N]:=0.12;
   {*OUTPUT RESULTS AT 0.1HR INCREMENTS*>
  IF N MOD 50 = 0
   THEN URITELN (N:5,TAU[N]:WW:WD,B[N]:WW:WD,M[N]:WW:WD,D[N]:WW:WD,
                 T[N]:WW:WD,C[N]:WW:WD);
   {*TIME CHECK*>
  N:=N+1
 END; {DO WHILE)
END. {PROGRAM REACT)
```

not change with time. An idealized *back-mix flow reactor* is equivalent to a *continuous stirred-tank reactor (CSTR)* where the contents of the reactor are completely mixed so that the complete contents of the reactor are at the same concentration and temperature as the product stream. The unit is operated at steady state so the flow rates of the inlet and outlet streams as well as the reactor conditions remain unchanged with time.

TABLE 16

Printed-out results from computer program of Table 13 for chlorobenzene reactor-design example

(Note: $.8011E-01 = .8011 \times 10^{-1}$)

```
       TINE
        HR        BENZ         MONO        DICHLO         TRI          CL2

  0    .0000   .1000E+01   .0000E+00   .0000E+00   .0000E+00   .0000E+00
 50    .1000   .8657E+00   .1331Et00   .1210E-02   .2324E-06   .4522E-02
100    .2000   .7307E+00   .2639E+00   .5443E-02   .2361E-05   .5217E-02
150    .3000   .5997E+00   .3868E+00   .1352E-01   .9786E-05   .6135E-02
200    .4000   .4740E+00   .4994E+00   .2659E-01   .2876E-04   .7387E-02
250    .5000   .3556E+00   .5979E+00   .4635E-01   .7120E-04   .9160E-02
300    .6000   .2474E+00   .6771E+00   .7531E-01   .1609E-03   .1177E-01
350    .7000   .1536E+00   .7289E+00   .1172E+00   .3475E-03   .1575E-01
400    .8000   .8011E-01   .7423E+00   .1768E+00   .7372E-03   .2182E-01
450    .9000   .3219E-01   .7077E+00   .2586E+00   .1545E-02   .3050E-01
500   1.0000   .9118E-02   .6262E+00   .3616E+00   .3136E-02   .4127E-01
550   1.1000   .1715E-02   .5158E+00   .4765E+00   .5976E-02   .5325E-01
600   1.2000   .2021E-03   .3976E+00   .5916E+00   .1059E-01   .6745E-01
650   1.3000   .1304E-04   .2844E+00   .6979E+00   .1769E-01   .8673E-01
700   1.4000   .3532E-06   .1834E+00   .7882E+00   .2839E-01   .1150E+00
750   1.5000   .4502E-08   .1083E+00   .8492E+00   .4252E-01   .1200E+00
800   1.6000   .5651E-10   .6381E-01   .8787E+00   .5746E-01   .1200E+00
850   1.7000   .7092E-12   .3760E-01   .8897E+00   .7274E-01   .1200E+00
900   1.8000   .8901E-14   .2216E-01   .8897E+00   .8811E-01   .1200E+00
950   1.9000   .1117E-15   .1306E-01   .8835E+00   .1034E+00   .1200E+00
```

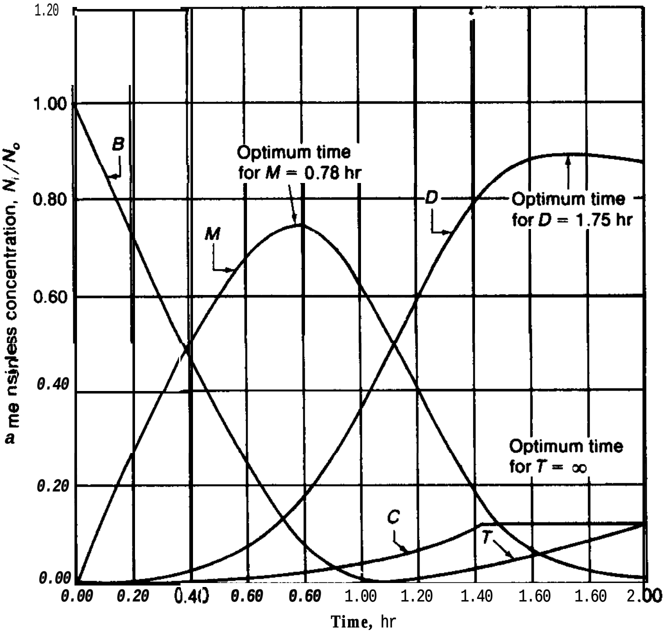

FIGURE 16-33
Plot of dimensionless concentration vs. time for chlorobenzene reactor-design example showing digital-computer results based on Table 13 program.

BATCH REACTORS

For batch reactors, Eq. (39) can be rearranged to give a basic design equation as†

$$\int_0^t dt = t = -\int_{N_{io}}^{N_{ie}} \frac{dN_i}{V_b r_i} \tag{55}$$

where N_{ie} = moles of material i left at the end of reaction time t
N_{io} = moles of material i at the start of the reaction

In terms of conversions (X_i), where X_{ie} represents the total fractional conversion of material i during the reaction time t with N_{io} being a constant,

$$N_i = N_{io} - N_{io}X_i = N_{io}(1 - X_i) \quad \text{and} \quad dN_i = -N_{io}\,dX_i \tag{56}$$

Substituting Eq. (56) into Eq. (55) gives

$$\int_0^t dt = t = N_{io}\int_0^{X_{ie}} \frac{dX_i}{V_b r_i} \tag{57}$$

---

†Note that r_i as used in this chapter represents the reaction rate expressed as the rate of *disappearance* of reactant i. The form $-r_i$ represents the rate of *generation* of the ith species.

Equation (57) represents the basic design equation for use with batch reactors. In case the reactor volume (V_b) remains constant during the entire reaction, integration of the equation can be simplified by recognizing that N_{io}/V_b is merely the concentration of the reactant at the start of the reaction and removing the V_b term from under the integral sign.

TUBULAR PLUG-FLOW REACTORS

Tubular reactors are often designed for steady-state operation on the basis of idealized plug flow so that Eq. (41) applies, or

$$\int_0^z dz = z = -\int_{(U_zC_i)_o}^{(U_zC_i)_e} \frac{d(U_zC_i)}{r_i} \tag{58}$$

To simplify this expression, designate the constant cross-sectional area of the flow reactor as A„ the total volume of the reactor to point z as V_R so that V_R = A„ and the constant feed rate of material i as F_i. Then multiply both sides of Eq. (58) by A, and note that $A_RU_zC_i$ equals the moles of material i flowing at length z in the reactor per unit of time which is also $F_i - F_iX_{iz}$ where X_{iz} represents the fractional conversion of material i in the entering feed at any distance z along the reactor length. Because $d(F_i - F_iX_{iz}) = -F_i\, dX_{iz}$, the result of the preceding definitions and operations can be written as

$$\int_0^{zA_R} d(zA_R) = \int_0^{V_R} dV_R = V_R = F_i\int_0^{X_{iz}} \frac{dX_{iz}}{r_i} \quad \text{or} \quad \frac{V_R}{F_i} = \int_0^{X_{iz}} \frac{dX_{iz}}{r_i} \tag{59}$$

As an alternate expression for plug-flow reactors, the more general case can be considered in which the reactant i enters the reactor already partly converted with this initial conversion, designated as X_{io}, taking the place of the zero conversion assumed initially in Eq. (59). The resulting design equation equivalent to Eq. (59) is

$$\frac{V_R}{F_i} = \int_{X_{io}}^{X_{iz}} \frac{dX_{iz}}{r_i} \tag{60}$$

Equations (59) and (60) are in a form often used for tubular, idealized, plug-flow reactors when the reaction rate is based on the volume of the reactor. When the reaction is heterogeneous, such as one occurring on the surface of a catalyst, it is common practice to base the reaction rate on the mass of the catalyst rather than on the volume of the reactor and substitute r_{ic} for r_i. The resulting design equation equivalent to Eq. (60) is

$$\frac{W_c}{F_i} = \int_{X_{io}}^{X_{iz}} \frac{dX_{iz}}{r_{ic}} \tag{61}$$

where W_c = mass of the catalyst in the reactor
r_{ic} = rate of the reaction as moles of material i converted per unit of time per unit of catalyst mass

SPACE VELOCITY. In design treatment for plug-flow reactors, the concept of space velocity is often used where space velocity is defined as the ratio of the volumetric feed rate to the volume of the reactor, or

$$\text{Space velocity} = \frac{F_i v_F}{V_{RP}} = \frac{F_i}{V_R C_{iF}} \tag{62}$$

where v_F = the volume of the feed per mole of material i in the feed
C_{iF} = the concentration of material i in the feed as moles of i per unit volume = l/v,

BACK-MIX REACTORS

For the steady-state operation of idealized back-mix reactors, Eq. (37) becomes

$$\begin{pmatrix}\text{Rate of accumulation}\\ \text{of } i\end{pmatrix} = \begin{pmatrix}\text{inflow rate of } i\\ \text{-outflow rate of } i\end{pmatrix} + \begin{pmatrix}\text{rate of generation}\\ \text{of } i\end{pmatrix}$$

$$0 = F_i - F_i(1 - X_{ie}) + (-r_i)V_B \tag{63}$$

Therefore,

$$F_i X_{ie} = r_i V_B \quad \text{or} \quad \frac{V_B}{F_i} = \frac{X_{ie}}{r_i} \tag{64}$$

where X_{ie} = final conversion based on fraction of entering material i converted
V_B = total volume of the back-mix reactor

In case the feed reactant material (i in F_i) on which the conversion is based enters the reactor with part of the material already converted, the initial or entering conversion becomes X_{io} instead of zero as was the case for ‘Eq. (64). Under these conditions, Eq. (64) assumes the more general form of

$$\frac{V_B}{F_i} = \frac{X_{ie} - X_{io}}{r_i} \tag{65}$$

EXPRESSIONS FOR r_i

It is normally necessary to use a simplified or empirical expression for the reaction rate r_i in terms of constants and reactant and product concentrations which can be assumed from the stoichiometry of a proposed reaction mechanism or developed purely empirically on the basis of experimental data. One of the key components of the rate expression is the specific rate constant k_i which must almost always be determined directly from laboratory or other rate data.

The most common way of presenting a rate constant is in the form of an Arrhenius equation as

$$k = Ae^{-E/RT} \tag{66}$$

where k = specific rate constant with appropriate units to fit the rate equation
A = frequency factor, same units as k
E = activation energy, units so that E/RT is dimensionless
R = perfect-gas-law constant, units so that E/RT is dimensionless
T = absolute temperature, units so that E/RT is dimensionless

It is important to be certain that the rate equation, as a function of concentrations, and the rate constant used in the design equations apply for the reaction conditions of temperature, pressure, and concentration to be used in the reactor.

MECHANICAL FEATURES OF DESIGN

Chemical reactors basically come in the form of tanks, such as for batch reactors or back-mix flow reactors, large cylinders, such as for fluidized-bed or plug-flow reactors, or multiple tubes inside a cylindrical container, such as for plug-flow reactors when special needs exist for temperature control. High pressure and extremes of temperature as well as corrosive action of the materials involved can introduce complications in the design which must be handled by the design engineer.

Tank reactors must be designed with closed ends which can withstand the operating conditions. Cooling or heating can be provided by internal coils or pipes or by jacketing. Vents, pressure releases, sampling inlets, and seals and auxiliaries for agitation must be provided, as well as adequate foundations, supports, and facilities for personnel access to handle operations, inspection, and maintenance.

Some flow reactors are designed as large cylinders closed at both ends through which the reactant flows while others are designed in the form of multiple tubes located in parallel with the reactant flowing through the inside of the tubes. This latter arrangement is the form usually used for thermal cracking units in the petroleum industry since it is well adapted to allow the necessary heating to attain the appropriate temperatures for reaction. In both of the forms discussed in the preceding, the simple assumption of ideal plug-flow operation is often assumed for the design even though it is clear that some back mixing will occur, especially in the large-cylinder flow units.

Many reactors fall in the classification of fluid-solid catalytic units where the catalyst may be retained in a *fixed-bed* position in the reactor with the reactant flowing through the catalyst bed, or the unit may be operated as a *fluidized-bed* reactor with the catalyst particles being suspended in the flowing fluid due to motion of the fluid. A third type of reactor is one in which the catalyst particles fall slowly through the fluid by gravity in the form of a so-called *moving bed.*

Fixed-bed reactors can be designed as units in large cylinders with jacketed or internal-coil heating or cooling as needed. They can also be designed as multiple units with heating or cooling between the separate beds to maintain the necessary temperature levels. The design for units of this type is often based on the assumption of ideal plug-flow although a series of back-mix reactors can also be used for the design basis.

The fluidized-bed reactor involves a rapid movement of the solid catalytic particles throughout the bed so that the operation can come close to one of uniform temperature throughout the reactor. The actual flow pattern for the operation of a fluidized bed is very complex and is between that for the ideal back-mix reactor and the ideal plug-flow reactor so that special methods for design may be required to approximate the real situation.

The fluidized-bed reactor allows catalyst to be regenerated while the unit is in operation by continuously removing a portion of the catalyst from the reactor for regeneration treatment and subsequent flow-back into the reactor. Because there is a tendency for the catalyst particles to be carried over in the product stream, auxiliary units, such as cyclone separators or dust collectors, must be provided for separating out the solid particles or catalyst fines.

ESTIMATION OF COSTS FOR REACTOR EQUIPMENT COMPONENTS

The design and costing of reactor vessels are handled in a manner similar to that for regular mixing and pressure vessels as described in Chap. 14 or for heat exchangers as described in Chap. 15. The most reliable results are obtained by direct fabricator quotations or by the assistance of a fabricator representative who is an expert in the design of reactor vessels.

One critical item in the design of reactor vessels, in addition to establishing the type and size of the reactor by methods mentioned previously, is to select the correct materials of construction and wall thicknesses to handle the given operating conditions. Table 4 in Chap. 14 gives design equations and stress values for various materials of construction which can be applied to the design of reactor vessels, while Table 6 of Chap. 14 presents some rules of thumb that can be helpful in the design and costing of reactor vessels.? Figures 14-56 and 14-57 give data which can be used for estimating costs of reactor vessels. Costs for tubular flow reactors with internal tubes can often be estimated by considering the reactor unit as equivalent to a heat exchanger so that the data presented in Chap. 15 can be used for the cost estimate. Examples of costs for kettle reactors are given in Fig. 16-34, while Fig. 16-35 gives data on cost and installation time for jacketed and agitated reactors.

Example of Design and Costing for Reactor Vessel

The following example illustrates how a pressure vessel used as a reactor can be designed and its cost estimated using the design equations and rules-of-thumb

---

†See also the information presented earlier in this chapter on costs for plate and packed vessels and the cost information given in Chap. 14 for tanks, pressure vessels, and storage equipment.

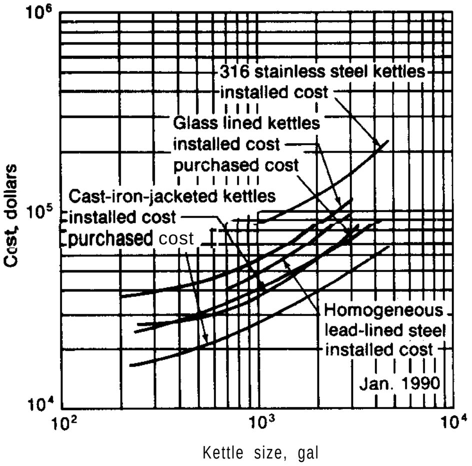

FIGURE 16-34
Cost of kettles. Purchased cost includes kettle, jacket, agitator, thermometer well, drive and support, manhole cover, stuffing box, and clamps where **necessary**. Installed cost includes above plus installation costs (labor, material, and overhead), design, and expense. (No piping, structural changes, instrumentation, or electrical charges are included.)

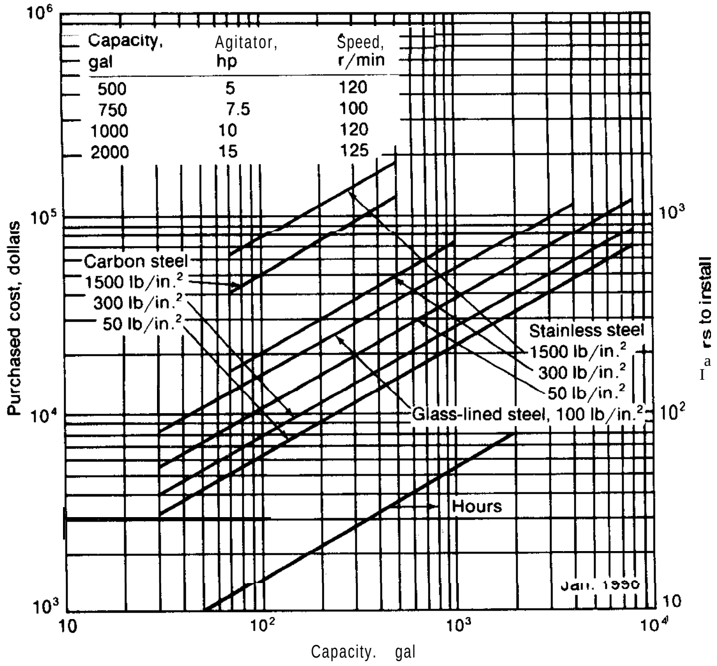

FIGURE **16-35**
Cost and installation time of jacketed and stirred reactors.

presented in Chap. 14. Consider the case where a reactor is to be designed and a preliminary cost estimate (January, 1990) is to be made for the installed reactor for the following conditions:

The reactor will be cylindrical with a 9 ft inside diameter and a length of 30 ft. It will be constructed of carbon steel (SA-285, Gr. C) and will operate at a temperature of 750°F and an internal pressure of 100 psig. It will require double-welded butt joints and will be spot-examined by the radiograph technique. It will be operated as a flow reactor in a horizontal position with the reacting materials flowing through it. There are no major heating or cooling needs, and, therefore, no special internal accessories are needed. Heads for the reactor will be hemispherical and the same thickness as the shell. The corrosion allowance for the wall thickness is $\frac{1}{8}$ in. Neglect freight and other delivery charges.

The solution for this case will be made by the use of Tables 4 and 6 in Chap. 14. For carbon steel (SA-285, Gr. C) at 750°F, the stress value is 12,000 psi and the joint efficiency is 0.85 for the given fabrication conditions as shown in Table 4 of Chap. 14.

The minimum wall thickness is (see Table 4 of Chap. 14 for equations and nomenclature)

$$\text{Thickness} = \frac{P\ \text{(inside radius)}}{SE_J - 0.6P} + C_c$$

$$= \frac{\overbrace{(100)}^{P}\left[\overbrace{(9)(12)(0.5) + 0.125}^{\text{inside radius}}\right]}{\underbrace{(12{,}000)}_{S}\underbrace{(0.85)}_{E_J} - (0.6)\underbrace{(100)}_{P}} + \overbrace{0.125}^{C_c}$$

$$= 0.66\ \text{in.}$$

The vessel weight, with density of steel = 489 lb/ft^3, is (see Tables 4 and 6 of Chap. 14)

$$\text{Weight of shell} = \pi(9)(30)\left(\frac{0.66}{12}\right)(489) = 22{,}800\ \text{lb}$$

$$\text{Weight of two heads} = (2\pi)\left[\frac{(9)(12)}{2}\right]^2(0.66)\left[\frac{489}{(12)^3}\right](2) = 6{,}840\ \text{lb}$$

Total weight including 15 percent increase (see Table 6 of Chap. 14) for nozzles, manholes, and saddles = (22,800 + 6,840)(1.15) = 34,100 lb.

F.o.b. cost for reactor based on Table 6 of Chap. 14 = (cost per lb) ~(34,100) = $(60)(34{,}100)^{-0.34}$ (34,100) = (2.30)(34,100) = \$78,000. (See Fig. 16-24 for an approximate check on this number.)

The cost for installation (Lang factor of 3.0 by Table 6 of Chap. 14) will be twice the cost of the f.o.b. unit if it is constructed of carbon steel. In this case, the material of construction is carbon steel, so it is not necessary to use a materials cost-conversion factor to obtain the f.o.b. cost; thus, the estimated total installed cost of the reactor is \$78,000 + (2)(\$78,000) = \$234,000.

NOMENCLATURE FOR CHAPTER 16

a_p = surface area of packing per unit of packed-tower volume, ft^2/ft^3

A = frequency factor, same units as k

A, = active or bubbling tray area, ft^2

A, = minimum cross-sectional area for downcoming liquid flow in finite-stage tower, ft^2

A_R = cross-sectional area of flow reactor, ft^2

A, = bubble-cap slot area, perforation sieve-tray area, or valve-opening valve-tray area, ft^2

A_t = total cross-sectional area of tower, ft^2

b = width of slot at base, ft

B = n_B/N_0 in Eqs. (48), (49), (52), and (54)

c = height of slot, ft

C = N_{Cl}/N_0 in Eqs. (48) to (53)

$C_a, C_b, C_i, C_{n-1}, C_n$ = concentrations of species a,. . . , C_n at time t, mol/ft^3

C_D = liquid gradient factor; see Fig. 16-13

C_{iF} = concentration of material i in feed, mol/ft^3

d_p = effective diameter of packing particle, ft; defined with Eq. (23)

d_{wc} = perpendicular distance from weir to center of tower, ft

D' = inside diameter of tower, in.

D = inside diameter of tower, ft, or N_D/N_0 in Table 11 and Eqs. (50), (51), (52), and (54)

D_G = gaseous-diffusion coefficient, ft^2/h

E = activation energy, units so that E/RT is dimensionless

E_o = overall column efficiency, percent

$f_1 = (\mu_L/2.42)^{0.16}$

$f_2 = (62.4/\rho_L)^{1.25}$

$f_3 = (72.8/\sigma)^{0.8}$

F = feed rate of chlorine as (lb mol/h)/(lb mol of initial benzene charge)

F_C = correction factor for evaluation of liquid gradient; see Fig. 16-13

F_i = feed rate of material i, mol/h

g = local acceleration due to gravity; usually taken as 32.17 ft/(s)(s)

g_c = conversion factor in Newton's law of motion, 32.17 ft . lbm/(s)(s)(lbf)

G = superficial mass velocity of gas (based on cross-sectional area of empty tower), $lb/(h)(ft^2)$

G_m = maximum allowable superficial mass velocity of gas (based on cross-sectional area of empty tower) $lb/(s)(ft^2)$ or $lb/(h)(ft^2)$

h = packed height, ft

h_c = head of liquid equivalent to pressure drop due to gas flow through riser and cap or through sieve holes, ft of liquid

h_d = head of liquid equivalent to pressure drop due to liquid flow through downcomer and constriction, ft of liquid

h_g = liquid gradient across tray, ft of liquid

h_H = head of liquid equivalent to pressure drop due to one kinetic head, ft of liquid

h_o = head of liquid over weir, ft

h_{slot} = distance liquid is depressed below top of slot = head of liquid equivalent to pressure drop due to gas flow through slots, ft of liquid

h_T = head of liquid equivalent to total pressure drop of gas across tray, ft of liquid

h_w = height of weir, ft

H = total head of liquid in downcomer, ft

H_p = a constant for evaluating maximum allowable gas rates in packed towers, ft

$HETP$ = height of packing equivalent to one theoretical plate, in.

HTU_G = height of a gas-phase transfer unit, ft

k = specific reaction rate constant with appropriate units to fit the rate equation

k_i = specific reaction rate constant for species i

k„ k_2, k_3 = specific reaction rate constants for species 1, 2, and 3

K_1, K_2, K_3 = empirical constants as a function of the packing, see Table 6 and Eq. (36)

K_p = a constant for evaluating maximum allowable gas rates in packed towers, ft/s

K'_v = a constant for evaluating maximum allowable gas rates in finite-stage towers, ft/s, see Eq. (3)

K_v = a constant for evaluating maximum allowable gas rates in finite-stage towers, ft/s, see Eq. (1)

$K.H.$ = number of kinetic heads equivalent to pressure drop through sieve-tray plate, see Fig. 16-12

l_c = total free space between bubble caps perpendicular to direction of liquid flow, average of various rows, ft

l_r = total free space between bubble-cap risers perpendicular to direction of liquid flow, average of various rows, ft

1, = weir length, ft

L = superficial mass velocity of liquid (based on cross-sectional area of empty tower), lb/(h)(ft^2)

L'_M = molal liquid flow rate, lb mol/h

m_{avg} = mole fraction of solute in gas in equilibrium with liquid/mole fraction of solute in liquid at average column temperature and pressure

A4 = N_M/N_0 in Eqs. (49), (50), (52), and (54)
M_A = average molecular weight of liquid absorbent, lb/lb mol
M_G = molecular weight of gas, lb/lb mol
N_B = lb mol of benzene present at time t
N_{Cl} = lb mol of chlorine present at time t
N_D = lb mol of dichlorobenzene present at time t
N_i = amount of base material i present in the volume V_b at time t
N_{ie} = moles of material i left at end of reaction time t
N_{io} = moles of material i at the start of the reaction
N_M = lb mol of monochlorobenzene present at time t
N_0 = lb mol of benzene present at start
N_T = lb mol of trichlorobenzene present at time t
NTU = number of transfer units, see Eqs. (31) and (32)
p = pressure; Ap is pressure drop in packed tower, psf or in. water
p_F = pressure at flooding; Ap, is pressure drop in packed column at flooding point, psf or in. water
p_T = pressure; Δp_T is total pressure drop of gas across finite-stage tray, psi
P = total pressure, expressed as equivalent ft of liquid or psf
Q_L = volumetric flow rate of liquid, ft^3/s
Q_s = volumetric flow rate of gas, ft^3/s
r = number of rows of caps perpendicular to direction of liquid flow
r_i = rate of reaction of material i expressed as rate of disappearance of reactant i, $\mathrm{mol}/(\mathrm{ft}^3)(\mathrm{h})$
r_{ic} = rate of reaction as moles of material i converted per unit of time per unit of catalyst mass
R = ideal-gas-law constant, 1545 psf $\cdot$ $\mathrm{ft}^3/(\mathrm{lb\ mol})(°\mathrm{R})$
s = skirt clearance, ft
SC, = gas-phase Schmidt number = $\mu_G/\rho_G D_G$
S_m = static submergence, distance from top of bubble-cap slot to liquid surface when liquid is just ready to flow over the weir, ft
t = time, h
T = absolute temperature, °R, or N_T/N_0 in Table 11 and Eqs. (51) and (54)
U = velocity vector, ft/s
U_z = velocity vector in z direction, ft/s
v_F = volume of feed per mole of material i in the feed, $\mathrm{ft}^3/\mathrm{mol}$
V = superficial linear velocity of gas (based on cross-sectional area of empty tower), ft/s
V_b = volume of batch reactor, ft^3
V_B = volume of back-mix reactor, ft^3

V_c = linear velocity of gas in riser, reversal area, or annulus of bubble cap (maximum value) or in sieve hole, ft/s
V_m' = maximum allowable superficial linear velocity of gas (based on "net" cross-sectional area of tower for vapor flow), ft/s, see Eq. (3)
V_m = maximum allowable superficial linear velocity of gas (based on cross-sectional area of empty tower), ft/s, see Eq. (1)
V_M' = molal gas flow rate, lb mol/h
V_R = volume of tubular reactor to point z, ft^3
W_c = mass of catalyst in reactor, lb
W_T = average width of tray normal to direction of liquid flow (computed as the average of the total tray width at each row of caps), ft
X_i = fractional conversion of material i
X_{ie} = fractional conversion of material i in reaction time t
X_{io} = fractional conversion of material i at start of reaction or at $t = 0$ or $z = 0$
X_{iz} = fractional conversion of material i at distance z along reactor length
y, x = gas- and liquid-phase concentrations, respectively
y_L^*, x_G^* = gas- and liquid-phase concentrations, respectively, corresponding to values in equilibrium with the liquid- and gas-phase concentrations
z = length of reactor in direction of flow, ft
Z = packed height, ft

Greek symbols

α = relative volatility
α_{avg} = relative volatility of key components at average column temperature and pressure
γ = constant, value dependent on packing size and type; see Table 4
ε = fractional void volume in packed bed, ft^3 void/ft^3 of packed-tower volume
θ = angle subtended from center of tower by chord weir, deg
$\mu_{A,avg}$ = molal average viscosity of liquid absorbent at average column temperature and pressure, centipoises
$\mu_{F,avg}$ = molal average viscosity of feed at average column temperature and pressure, centipoises
μ_G = absolute viscosity of gas, lb/(ft)(h)
μ_L = liquid viscosity, lb/(ft)(h)
μ_L' = liquid viscosity, centipoises
π = constant = 3.1416
$\rho_{A,avg}$ = density of liquid absorbent at average column temperature and pressure, lb/ft^3

ρ_G = gas density, lb/ft^3

ρ_L = liquid density, lb/ft^3

ρ'_L = liquid density, g/cc

σ = surface tension of liquid, dyn/cm

ψ = parameter for a given packing material, see Figs. 16-22 and 16-23

Φ = constant, value dependent on packing size and type; see Table 4

PROBLEMS†

1. A sieve-tray tower has an ID of 5 ft, and the combined cross-sectional area of the holes on one tray is 10 percent of the total cross-sectional area of the tower. The height of the weir is 1.5 in. The head of liquid over the top of the weir is 1 in. Liquid gradient is negligible. The diameter of the perforations is $\frac{3}{16}$ in., and the superficial vapor velocity (based on the cross-sectional area of the empty tower) is 3.4 ft/s. The pressure drop due to passage of gas through the holes may be assumed to be equivalent to 1.4 kinetic heads (based on gas velocity through holes). (Tray thickness = hole diameter and active area = 90 percent of total area-see Fig. 16-12). If the liquid density is 50 lb/ft^3 and the gas density is 0.10 lb/ft^3, estimate the pressure drop per tray as pounds force per square inch.
2. An existing bubble-cap distillation tower has an ID of 8 ft and a tray spacing of 30 in. The tower is operating under a pressure of 70 psig, and a slot liquid seal of 2 in. is maintained. At the point of maximum volumetric vapor flow, the molecular weight of the vapor is 100, the rate of vapor flow is 1500 lb mol/h, the liquid density is 55 lb/ft^3, and the temperature is 175°F. The pressure drop through the tower is negligible, and the ideal gas law is applicable to the rising vapors. Approximately what percent of the maximum allowable flow rate is being used in the tower?
3. A mixture of benzene and toluene containing 60 mole percent benzene is to be separated into an overhead product containing 96 mole percent benzene and a bottoms containing 25 mole percent benzene. A valve-tray tower has been designed which is intended to accomplish this separation at atmospheric pressure. At the operating conditions chosen, calculations have shown that 6.1 theoretical stages are necessary. The temperature is 181°F at the top of the column and 213°F in the reboiler. Assuming the reboiler acts as one theoretical stage, estimate the number of actual trays required. Benzene and toluene mixtures may be considered as ideal. At 197°F, the vapor pressure of pure benzene is 1070 mm Hg and the vapor pressure of pure toluene is 429 mm Hg.
4. For the conditions given in Example 3, what weir height would be necessary to reduce the dimensionless ratio of liquid gradient/pressure-drop head caused by bubble-cap assemblies to the recommended maximum value of 0.4?
5. Air is passing through a bubble-cap plate at a superficial vapor velocity of 2 ft/s. Water is flowing across the tray at a rate of 0.1 ft^3/(s)(ft diameter). The water and air are at a temperature of 70°F, and the pressure is 1 atm. Assuming stable

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†See Appendix C for reactor-design problems.

operation of the plate and a liquid gradient of 0.4 in., estimate the percent of the total pressure drop across the tray due to the head of liquid above the slots. The following specifications apply to the unit:

Inside diameter ≈ 6 ft
Minimum total cross-sectional area of riser, direction-reversal space in cap, or annular cap space = 15 percent of tower cross-sectional area
Total slot area = 10 percent of tower cross-sectional area
Length of chord weir = 0.7 x tower diameter
Slots in caps are triangular, and slot width at base = 0.4 in.
Slot height = 1.5 in.
Static submergence = 1.0 in.

6. A random-packed absorption tower is to be used for removing a hydrocarbon from a gas mixture by countercurrent scrubbing with an oil. Stoneware Raschig rings of 1-in. nominal size will be used as the packing. The tower must handle a gas rate of 2000 lb/h and a liquid rate of 6000 lb/h. A gas velocity equal to 70 percent of the maximum allowable velocity at the given liquid and gas rates will be used. The gas density is 0.075 lb/ft^3. The liquid density is 55 lb/ft^3. The viscosity of the oil is 20 centipoises. Under these conditions, estimate the necessary tower diameter and the pressure drop through the tower as inches of water per foot of packed height.

7. A blower is being used to force 200 lb of air per hour through a 12-in. inside-diameter tower countercurrent to water flowing at a rate of 6000 lb/h. The tower is packed to a height of 10 ft with 1-in. Berl saddles. The average pressure in the tower is 1 atm, and the temperature is 75°F. The blower-motor combination used at the gas inlet has an overall efficiency of 50 percent. Estimate the power cost per 8000 operating hours for forcing the air through the packing if the cost of power is $0.08 per kilowatthour.

8. A random-packed distillation tower with an inside diameter of 6 in. is being operated at a condenser pressure of 100 mm Hg. The following data are obtained during operation:

Packed height = 3 ft
Gas rate = 100 lb/h
Liquid rate = 90 lb/h
Average molecular weight of gas = 100
Average temperature = 110°F
Total pressure drop = 12 mm Hg
Liquid viscosity = 1.2 centipoises
Liquid density ≈ 55 lb/ft^3

A larger distillation tower is needed for the same mixture, and it has been decided to use a 12-in. inside-diameter tower with the same type and size of packing. The packed height will be 3 ft. The condenser pressure will be 50 mm Hg, and the average temperature can be assumed to be 95°F. At this temperature, the liquid viscosity is 1.4 centipoises and the liquid density is 57 lb/ft^3. The 12-in. tower will be operated at the same percent of the maximum allowable gas mass velocity as the 6-in. tower, and the same ratio of L/G will be used. The average molecular weight of the gas is unchanged. With the given packing and fluids, the pressure drop at the flooding point can be assumed to be 16 mm Hg/ft of packed height. Neglecting the effect of temperature on fluid densities at the flooding point, estimate the gas rate as pounds per hour at the indicated operating conditions in the 12-in. tower.

9. A special test on the 6-in. distillation tower described in Prob. 8 indicates that the total pressure drop is 2.9 mm Hg when the gas rate is 50 lb/h. For this test, the condenser pressure was maintained at 100 mm Hg, the *L/G* ratio was the same as shown in Prob. 8, and temperature change was negligible. Operation can be considered to be under preloading conditions. Estimate the total pressure drop at the operating conditions indicated in Prob. 8 for the 12-in. column.

10. A random-packed tower is to be used for contacting 3000 lb/h of air with 4000 lb/h of water. The tower will be packed with l-in. chemical porcelain Raschig rings. The operating pressure is 1 atm, and the temperature is 70°F. Assuming column operation is in the preloading range, estimate the optimum diameter per foot of packed height to give minimum annual cost for fixed charges and blower operating charges. The following additional data apply:

Annual fixed charges = 0.2 times cost of installed unit
Cost of installed unit, complete with distributor plates, supports, and all auxiliaries = 2.0 times purchased cost of shell and packing
Cost of delivered power = $0.08/kWh
Operating time = 8000 h/year
Purchased cost of shell = $\$40/\text{ft}^3$ of packed volume
Purchased cost of packing = $\$22.00/\text{ft}^3$

CHAPTER 17

STATISTICAL ANALYSIS IN DESIGN

Application of statistical analysis in process design has become so extensive that every chemical engineer must have a basic knowledge of this branch of mathematics. Statistical design and analysis help the engineer to make decisions by getting the most out of the data. In many cases, where too little is known about a chemical process to permit a mathematical model, a statistical approach may indicate the manner of the direction in which to proceed with the design.

Besides analyzing and correlating data by statistical means, the chemical engineer also uses statistics in the development of quality control to establish acceptable limits of process variables and in the design of laboratory, pilot plant, and process plant (evolutionary operation) experiments. In the latter application, statistical strategy in the design of experiments enables the engineer to set experimental variables at levels that will yield maximum information with a minimum amount of data.

The basis for any statistical analysis is the fact that all data are to some extent, one way or another, subject to chance errors. These chance errors may arise whether the problem involves an estimation, the development of a reliable model, or the testing of a hypothesis. For example, since no experimentally determined value in the laboratory is absolute, it is generally necessary to determine by statistical analysis the reliability of the newly obtained data, The eventual use of these data by the engineer in design work may often determine the necessary confidence level that will be required concerning the true value of the data.

The specific techniques of statistical analysis in design require extensive discussions and can only be mentioned here for lack of space. The objective of this presentation is primarily to describe and briefly illustrate the concepts of statistical analysis with applications in the chemical process industry.

BASIC CONCEPTS

Statistical methods are based on the single concept of variability. It is through this fundamental concept that a basis is determined for design of experiments and analysis of data. Full utilization of this concept makes it possible to derive maximum information from a given set of data and to minimize the amount of data necessary to derive specific information.

AVERAGES

One of the most useful quantities for providing a measure of the location of data, making inferences about the true mean, μ, and comparing sets of data by statistical methods is the sample mean, $\bar{x}$, defined as the sum of all the values of a set of data divided by the number of observations, n, making up the data sample. Mathematically, the mean can be symbolized by

$$\bar{x} = \frac{1}{n} \sum x_i \tag{1}$$

The value of the mean obtained in this manner is not necessarily the true mean μ, unless all the values being averaged represent the total population. Another measure of location that is frequently used is the sample median. The median is defined as that value of the data that has the same number of values below it as above it once the data are arranged in order of value. A median is not affected by extremely large or small values and is frequently used when little is known about the underlying probability laws. A third measure of location of data is the *mode.* This term is defined as the most frequently occurring value in a set of data and finds application when there is a large frequency of one value to be averaged with one or a few numerically large values.

Example 1 Evaluation of mean and median. Five analyses have been run on a natural gas sample to determine the methane content with the following results: 92.4, 92.8, 92.3, 93.0, and 92.5 percent. What are the sample mean and the sample median?

Solution

$$\bar{x} = \frac{1}{n} \sum x_i$$

$$\bar{x} = \$92.4 + 92.8 + 92.3 + 93.0 + 92.5)$$

$$\bar{x} = 92.6$$

Retabulating the data in order of magnitude results in:

$$92.3, 92.4, 92.5, 92.8, 93.0$$

The median value is underlined.

In addition to the sample mean, there is also a harmonic mean and a geometric mean. The harmonic *mean* H is the reciprocal of the arithmetic mean of the reciprocals, and is used whenever reciprocals are involved, e.g., rates. For example, to obtain the average flow rate in a pipe delivering A gal at R_1 gpm and B gal at R_2 gpm, it is necessary to use the harmonic mean.

$$\bar{H} = \frac{n}{\sum \frac{1}{x_i}} \tag{2}$$

Example 2 Evaluation of harmonic mean. What is the average rate of flow for a pump delivering 100 gal at 50 gpm and 100 gal at 10 gpm?

Solution

$$\bar{H} = \frac{n}{\sum \frac{1}{x_i}}$$

$$n = 2$$

$$\bar{H} = \frac{2}{\frac{1}{50} + \frac{1}{10}}$$

$$\bar{H} = 16.7 \text{ gpm}$$

This value could also have been obtained by determining the total time necessary to pump 100 gal at the two rates and dividing this into the total number of gallons pumped. Thus,

$$\tau_1 = \frac{100}{50} = 2 \text{ min}$$

$$\tau_2 = \frac{100}{10} = 10 \text{ min}$$

$$\text{Average rate} = \frac{\text{loo} + \text{loo}}{12} = 16.7 \text{ gpm}$$

Note that the mean would be 30 gpm, which is nearly twice as much as the harmonic mean.

The geometric *mean* $\bar{G}$ is the nth root of the product of n terms, and is used when averaging numbers involving growth data, e.g., population growth.

VARIABILITY OF DATA

There are a variety of ways in which to characterize the variability of data. One of the more useful quantities, although not the simplest, is the ***true standard deviation*** σ, which is defined as the square root of the sum of the squares of the deviations of the data points from the true mean divided by the number of observations n.

$$\sigma = \sqrt{\frac{\Sigma(x_i - \bar{x})^2}{n}} \tag{3}$$

Units for the standard deviation are the same as for the individual observation. Because the true mean is practically never known, the true standard deviation is generally a theoretical quantity. However, σ may be approximated by the estimated standard deviation, $s(x)$, where

$$s(x) = \sqrt{\frac{\Sigma(x_i - \bar{x})^2}{n - 1}} \tag{4}$$

In this relationship $(n - 1)$ appears in the denominator rather than n in order to make $s(x)$ an unbiased estimator of σ.

In statistical analysis it is necessary to use a quantity called degrees of *freedom*, designated henceforth as d.f. This quantity allows for a mathematical correction of the data for constraints placed upon the data. In this case, in the calculation of the estimated standard deviation, the number of observations n is fixed and the estimated standard deviation is calculated from the mean. Only $(n - 1)$ of the observations or sample terms can be varied and the last term is fixed by $\bar{x}$ and n. Thus, there are only $(n - 1)$ degrees of freedom in estimating the standard deviation from a sample of the population data.

Squaring the true standard deviation gives a term called the ***true variance***, σ^2. It can be shown that the standard deviation of means $\bar{\sigma}$, calculated for samples taken from the total population of data, will have a true standard deviation equal to $\sigma/\sqrt{n}$, where n is the sample size. In other words, the spread of these means is less than the spread of the overall data around the group mean.

In the same manner, squaring the estimated standard deviation results in an *estimated variance* $s^2(x)$. An estimated standard deviation of the mean has the same relation to the estimated standard deviation of the population as the true standard deviation of the mean, that is, $s(\bar{x})$ is given by $s(x)/\sqrt{n}$.

Example 3 Determining the estimated standard deviation. What is the estimated standard deviation for five weighings of a sample with the following values: 18.5, 18.4, 18.6, 18.3, and 18.7? What is the estimated standard deviation of the mean for this sample?

Solution

$$\bar{x} = \frac{18.5 + 18.4 + 18.6 + 18.3 + 18.7}{5} = 18.5$$

$$s(x) = \sqrt{\frac{\Sigma(x_i - \bar{x})^2}{n-1}} = \sqrt{\frac{\left[\Sigma x^2 - (\Sigma x)^2/n\right]}{n-1}}$$

$$= \left[\frac{(18.5)^2 + (18.4)^2 + (18.6)^2 + (18.3)^2 + (18.7)^2 - (92.5)^2/5}{5-1}\right]^{1/2}$$

$$= 0.158$$

$$s(\bar{x}) = \frac{s(x)}{\sqrt{n}}$$

$$= \frac{0.158}{\sqrt{5}}$$

$$= 0.071$$

Another measure of the spread of the data is the mean ***deviation,*** md. This is simply the arithmetic mean of the deviation from the mean without regard to sign.

$$\text{md} = \frac{\Sigma|\bar{x} - x_i|}{n} \tag{5}$$

For a normal distribution, the true standard deviation is approximately 1.25 times the mean deviation.

Example 4 Evaluation of mean deviation. Find the mean deviation of the following results found for five different samples: 0.57, 0.73, 0.69, 0.63, and 0.70.

Solution

$$\bar{x} = 0.66$$

$$\text{md} = \frac{\Sigma|\bar{x} - x_i|}{n}$$

$$= \frac{|0.57 - 0.66| + |0.73 - 0.66| + |0.69 - 0.66| + |0.63 - 0.66| + |0.70 - 0.66|}{5}$$

$$= 0.052$$

The spread of the data would be reported as ±0.05 since the third decimal point would have very little meaning.

The last measure of dispersion to be noted here is the range, which is the difference between the largest and the smallest value in any data that are reported. For small quantities of data (less than 10) the range is a useful number for comparison and is used in sampling and quality control. However, since this measure of variance is only affected by two of the data points, it obviously loses power as the size of the data sample or the number of observations increases.

THEORETICAL AND EMPIRICAL FREQUENCY DISTRIBUTION

When dealing with large amounts of data, it is convenient to form an array of the data in such a way that the frequencies of occurrence of given values or ranges of values can be tabulated and graphed. This grouping is accomplished by the designation of ranges which are called *class intervals.* The relative frequency of the class intervals is called an empirical distribution or an empirical frequency distribution and is used to estimate the theoretical frequency distribution,

Statistical formulas are based on various mathematical distribution functions representing these frequency distributions. The most widely used of all continuous frequency distributions is the *normal distribution,* the common bell-shaped curve. It has been found that the normal curve is the model of experimental errors for repeated measurements of the same thing. Assumption of a normal distribution is frequently and often indiscriminately made in experimental work because it is a convenient distribution on which many statistical procedures are based. However, some experimental situations subject to random error can yield data that are not adequately described by the normal distribution curve.

Usefulness of the normal distribution curve lies in the fact that from two parameters, the true mean μ and the true standard deviation σ, the distribution of all of the data can be established. The true mean determines the value on which the bell-shaped curve is centered, and σ determines the "spread" of the curve. A large σ gives a broad flat curve, while a small σ yields a tall narrow curve with most probability concentrated on values near the mean. It is impossible to find the exact value of the true mean from information provided by a sample. But an interval within which the true mean most likely lies can be found with a definite probability, for example, 0.95 or 0.99. The 95 *percent confidence* level indicates that while the true mean may or may not lie within the specified interval, the odds are 19 to 1 that it does.† Assuming a normal distribution, the 95 percent limits are $\bar{x} \pm 1.96\bar{\sigma}$ where $\bar{\sigma}$ is the true standard deviation of the sample mean. Thus, if a process gave results that were known to fit a normal distribution curve having a mean of 11.0 and a standard deviation of 0.1, it would be clear from Fig. 17-1 that there is only a 5 percent chance of a result falling outside the range of 10.804 and 11.196.

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†Note that the value of the true mean is always fixed, but that the interval specified by the confidence level because of its dependence upon the mean does not necessarily always include the true mean.

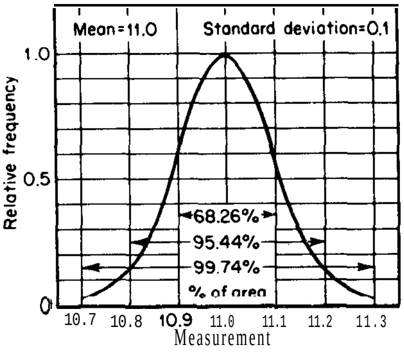

FIGURE 17-1
Probability curve.

As suggested above it is customary to work at the 95 percent or sometimes at the 99 percent probability level. The 95 percent probability level, which gives a 5 percent chance of a Type I error, represents the usual optimum for minimizing the two types of statistical error. A Type I error is a false rejection by a statistical test of the null hypothesis when it is true. Conversely, a Type II error is a false acceptance of the null hypothesis by a statistical test. The probability level at which statistical decisions are made will obviously depend on which type of error is more important.

STATISTICAL TESTS

There are a large number of statistical distributions which have for the most part been derived from the normal frequency distribution. The principles underlying only three of these distributions will be presented here. These distributions form the basis for the most frequently used statistical tests of significance.

There are two important statistical tests which can be used to determine whether the differences between two sets of data are real and significant or are just due to chance errors. Both assume that the experimental results are independently and normally distributed. One is the t-test and the other is the chi-squared test. The t-test applies only to continuous data-usually a measurement. The chi-squared test in many cases can approximate frequency or count type of data.

Statistics also provides a function for testing whether the scatter of two sets of data-as measured by the variance-is such as would be expected from two samples from the same population. This test is known as the F-test.

Tabular values for the above statistical tests are available in most statistical texts or engineering handbooks.[7]

[†] See, for example, R. E. Walpole and R. H. Myers, "Probability and Statistics for Engineers and Scientists," 2d ed., Macmillan Publishing Company, Inc., New York, 1978; and B. W. Lindgren, G. W. McElrath, and D. A. Berry, "Introduction to Probability and Statistics," 4th ed., Macmillan Publishing Company, Inc., New York, 1978.

THE r-TEST

The r-test compares the observed difference between averages with the inherent variability within the data-as measured by the standard deviation-to tell whether the difference is significant. Success of this test depends on the fact that the means of groups of samples from a distribution will tend to locate themselves around a normal distribution.?

The t-test consists of setting up a hypothesis for the value of the mean, calculating t from the observed mean and the estimated standard deviation, and comparing the calculated t with tabulated values of t. If the calculated t exceeds the tabulated value at the proper degree of freedom, then the original hypothesis may be rejected with the chance that the rejection is wrong corresponding to the probability level. Three common hypotheses and their corresponding t-equations are given below:

Hypothesis A: $\mu = \mu_0$ (one sample test)

$$\text{d.f.} = n - 1$$

$$t = \frac{\bar{x} - \mu_0}{s(\bar{x})}$$

Hypothesis B: $\mu_1 - \mu_2 = 0$ (two sample test-matched pairs)

$$\text{d.f.} = n - 1$$

$$t = \frac{X - O}{s(\bar{x})}$$

Hypothesis C: $\mu_1 = \mu_2$ (two sample test)

$$\text{d.f.} = n_1 + n_2 - 2$$

$$t = \frac{x_1 - x_2}{S(X)}$$

where

$$s(\bar{x}) = \frac{s(x)}{\sqrt{n}} \tag{6}$$

$$\bar{s}(\bar{x}) = \bar{s}(x)\sqrt{\frac{1}{n_1} + \frac{1}{n_2}} \tag{7}$$

$$\bar{s}(x) = \sqrt{\frac{\Sigma(\bar{x}_1 - x_1)^2 + \Sigma(\bar{x}_2 - x_2)^2}{n_1 + n_2 - 2}} \tag{8}$$

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†**Alternative** tests which are sometimes used in place of the f-tests are the Wilcoxon's tests. These tests are described by J. L. Hodges and E. L. Lehman, "Basic Concepts of Probability and Statistics," 2d ed., Holden-Day, Inc., San Francisco, 1970.

Tabular values for the t-test are normally presented for various degrees of freedom. The probability level is the chance of obtaining a value larger than the tabular values by chance alone. As the sample size (degrees of freedom) increases, the t-values approach the probability values for the normal distribution curve.

The following examples illustrate the use of the t-test.

Example 5 Example of t-test using Hypothesis A. Consider the analysis of a gas sample in Example 1. Is the mean significantly different from an assumed true mean of 92.5?

Solution. Before a statistical test is applied, the level of significance is generally selected. It is customary to prescribe a significance of 95 percent or more before rejecting a specific hypothesis. The 95 percent level will be chosen in this example. This means that, if the calculated absolute value of t is larger than that which would occur by chance alone 1 time in 19, the hypothesis would be assumed to be false. From Example 1

$$\bar{x} = 92.6$$

$$s(x) = 0.292$$

$$s(x) = 0.130$$

Assuming Hypothesis A

$$\mu = \mu_0 = 92.5$$

$$\text{d.f.} = n - 1 = 5 - 1 = 4$$

$$t = \frac{\bar{x} - \mu_0}{s(\bar{x})}$$

$$t = \frac{92.6 - 92.5}{0.130} = 0.77$$

The tabulated value of t (from a t-table for a “double-sided” or “two-tailed” test which effectively treats the observed difference between $\bar{x}$ and μ of 0.1 as ± 0.1) for a 95 percent significance level and four degrees of freedom is 2.78. Since the calculated value of t is less than 2.78, the hypothesis that $\mu = 92.5$ is not rejected.

Looking at it another way, the calculated value of t for four degrees of freedom corresponds to a probability level of approximately 0.49 (using the same r-table). This means that rejection of the hypothesis that $\mu = \mu_0$ can be expected to be in error approximately 49 percent of the time, or conversely, there is approximately a 51 percent chance that μ is not significantly different from μ_0.

To determine if there is any difference in the measurements for two sets of data made under similar conditions, it is useful to try to pair the measurements and use the r-test on the average differences between the measurements. The hypothesis to be tested under such conditions is whether $\mu_1 - \mu_2$ is different from zero.

Example 6 Example of t-test using Hypothesis B. Seven samples of catalyst are analyzed for carbon content by two technicians with the following results. Is there any difference between the two analyses?

Sample	Analysis 1	Analysis 2	Difference
1	15.1	14.6	-0.5
2	14.7	14.2	-0.5
3	15.2	15.0	-0.2
4	15.3	15.3	0.0
5	14.9	14.0	-0.9
6	14.7	14.6	-0.1
7	15.1	14.5	-0.6

solution

$$x_{\text{difference}} = -0.4$$

$$s(x) = 0.316$$

$$s(x) = 0.119$$

Assuming Hypothesis B

$$\mu_1 - \mu_2 = 0$$

$$\text{d.f.} = 6$$

$$t = \frac{0 - \bar{x}}{s(\bar{x})}$$

$$t = \frac{0 - (-0.4)}{0.119} = 3.36$$

The tabulated t-value ("double-sided" test) for a 95 percent significance level and six degrees of freedom is 2.45. Thus, there is greater than a 95 percent probability that the two analyses are not the same, i.e., the hypothesis may be rejected with a likelihood of less than 5 percent error. Note that this test has not indicated whether one analysis was better than the other, but rather whether or not there was any significant difference between the two.

Hypothesis C, useful for determining if differences exist between two materials, assumes that $\bar{x}_1 = \bar{x}_2$. In this case, rather than using the estimated standard deviation of the mean, $s(\bar{x})$, an overall better estimate is often obtained if a pooled estimate of the standard deviation of the mean, $s(\bar{x})$, is used.[†] A handy rule of thumb for samples of 5 to 10 items is not to pool the

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[†] To determine if pooling is permissible use the F-test. If it is not, use Welch's approximation. The latter is outlined by K. A. Brownlee, "Statistical Theory and Methodology in Science and Engineering," 2d ed., John Wiley and Sons, New York, 1965.

standard deviations if the ratio of the individual standard deviations is greater than 2.2.

Example 7 Example of t-test using Hypothesis C. Hydrogen gas from two sources of supply gives the following hydrogen content. Is there a significant difference between the hydrogen content of the two streams?

Source 1	*Source 2*
65.0	64.0
64.5	69.0
74.5	61.5
64.0	69.0
75.0	67.5
74.0	
67.0	

Solution

For source I	*For source 2*
$\bar{x}_1 = 69.1$	$\bar{x}_2 = 66.2$
$n = 7$	$n = 5$
$s(x_1) = 5.11$	$s(x_2) = 3.32$
$s^2(x_1) = 26.1$	$s^2(x_2) = 11.0$

Assuming Hypothesis C,

$$\mu_1 = \mu_2$$

$$\text{d.f.} = n, + n_2 - 2 = (7 + 5 - 2) = 10$$

$$\bar{s}(x) = \sqrt{\frac{\Sigma(\bar{x}_1 - x_1)^2 + \Sigma(\bar{x}_2 - x_2)^2}{n_1 + n_2 - 2}} = \sqrt{\frac{(n_1 - 1)s^2(x_1) + (n_2 - 1)s^2(x_2)}{n_1 + n_2 - 2}}$$

$$= \sqrt{\frac{(6)(26.1) + (4)(11.0)}{10}} = 4.47$$

$$\bar{s}(\bar{x}) = \bar{s}(x)\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$$

$$= 4.47\sqrt{\frac{1}{7} + \frac{1}{5}} = 2.62$$

$$t = \frac{\bar{x}_1 - \bar{x}_2}{\bar{s}(\bar{x})}$$

$$= \frac{(69.1 - 66.2)}{2.62} = 1.11$$

From the r-table ("double-sided" test) for ten degrees of freedom, the 1.11 corresponds to a probability level of between 0.20 and 0.30. This indicates that it would not be reasonable to reject the hypothesis. In other words, the hydrogen content of the two streams does not show a significant difference. A t-value of 2.23 or 3.17 would have been necessary to have rejected the hypothesis with a confidence level of 95 or 99 percent, respectively.

THE CHI-SQUARED TEST

The chi-squared test provides a test for determining whether the observed count (enumeration data with distinct integers which would produce a continuous distribution curve) differs significantly from an expected count. Fundamentally, if samples are withdrawn from a distribution of known frequency, the resulting distribution frequency of the samples will be different from the parent distribution. As the samples get larger, the frequency distribution will approach more closely the frequency distribution of the total population. Chi-squared is a mathematical function which gives the probability of getting differences between the two frequencies, for varying sample sizes, when expressed in the following form:

$$\chi^2 = \sum \frac{(f - f')^2}{f'} \tag{9}$$

where f is the observed frequency and f' is the expected frequency.

The procedure used in making the chi-squared test is quite similar to that described for the t-test. After making a hypothesis that the observed frequencies are equal to the expected frequencies, chi-squared is calculated by Eq. (9), and the result is compared at the proper level of degrees of freedom with the tabulated values. Any tabular value of chi-squared is the maximum value which could occur by chance alone, at a particular probability level. The degrees of freedom have the same meaning as for the r-test and represent the number of independent sums of squares.

There are two restrictions to the chi-squared test. One is that the test should not be used when expected frequencies are less than five. The other restriction is that when only one degree of freedom is involved, the difference $(f - f')$ is reduced by 0.5 before squaring. This is necessary to correct for a bias in the numerical answer when the chi-squared distribution, which is a continuous mathematical function, is applied to discrete numbers. Adjustment is not necessary when the degrees of freedom exceed one.

Example 8 Testing homogeneity of data. The feed to a reactor is handled by four pumps which have experienced a total of 40 failures during the past year. The distribution of failures per pump is as follows:

Pump 1: 16
Pump 2: 9
Pump 3: 6
Pump4: 9

Is the maintenance supervisor justified in saying that pump 1 has had too many failures during the past year compared to the other pumps?

Solution. The expected failures, assuming equal failure rates for all four pumps (null hypothesis), from a total of 40 failures divided among the four pumps would be 10

$$\chi^2 = \sum \frac{(f - f')^2}{f'}$$

$$= \frac{(16 - 10)^2 + (9 - 10)^2 + (6 - 10)^2 + (9 - 10)^2}{10} = \frac{54}{10} = 5.4$$

With four possible categories of numbers and with the total fixed, there are three degrees of freedom. This corresponds to a probability level of approximately 0.25 and indicates that, if all the pumps were operating under similar conditions, a chi-squared value as large as 5.4 would be expected one time in four by chance alone. Thus there is a 25 percent chance that the supervisor could be wrong in the analysis of the operation of pump 1.

Example 9 Effect of additional data on the homogeneity test. Suppose that the pump failures in Example 8 are analyzed for the past two years and the total pump failures are found to be 80 with the same relative distribution as previously.

Pump 1: 32
Pump 2: 18
Pump 3: 12
Pump 4: 18

Are the pump failures distributed in such a manner as to indicate that all pumps are operating essentially the same?

Solution. The expected failures would be $\frac{80}{4} = 20$

$$\chi^2 = \frac{(32 - 20)^2 + (18 - 20)^2 + (12 - 20)^2 + (18 - 20)^2}{20} = \frac{216}{20} = 10.8$$

This chi-squared value, at three degrees of freedom, is between the 0.02 and 0.01 level and there is approximately only 1 chance in 75 of obtaining this distribution if the pumps were in fact operating the same.

Example 9 illustrates that the same relative distribution of data produces a different statistical conclusion when there are different amounts of data available. Chi-squared varies directly with the sample size when the relative distribution within the sample is unchanged. Thus, it is possible to determine the sample necessary to give a significant test.

A chi-squared test applied to data which can be arranged in various categories is usually called a contingency *test*, and χ^2 is used to determine whether the data are homogeneous within themselves.

THE F-TEST

The analysis of variance as performed by the F-test makes it possible to separate the total variance of a process into its component parts. The procedure for this test is similar to that followed in the t-test. Assuming a normal distribution, a null hypothesis that $\sigma_1^2 = \sigma_2^2$ is set up and F is calculated from the two estimated variances, $F = s_1^2/s_2^2$. If the calculated F exceeds the expected value, then with the indicated probability of error, the null hypothesis may be rejected.

With the F-test the degrees of freedom corresponding to the two variances need not be identical. Most statistics texts will tabulate F values for the 0.05 and 0.01 probability level. The degrees of freedom for the variance in the numerator are normally listed across the top of the table while the degrees of freedom for the variance in the denominator are given in the left-hand column. The F tables contain the maximum values that occur by chance alone at a given probability level. For example, if the numerator variance had 10 degrees of freedom and the denominator had 12, the F tables would indicate that the hypothesis of equal variances could be rejected with a 5 percent chance of being wrong for F values exceeding 2.76. Likewise, for the same degrees of freedom, an F value exceeding 4.30 would indicate that there would only be a 1 percent chance of error by rejecting the hypothesis of equal variances.

Example 10 Comparison of two variances. A simplified analytical procedure is proposed for a routine laboratory test. It is necessary to determine not only whether the new procedure gives the same results as the old, i.e., whether the means of a duplicate set are the same, but also whether the precision of the new test is as good as the current test. The data for the two tests are as follows:

Current method	***Revised method***
79.7	79.2
79.5	79.7
79.6	79.5
79.5	79.4
79.7	80.0
	79.6
	79.8

Solution. For current method:

$$\bar{x}_1 = 79.6$$

$$\text{d.f.} = n - 1 = 4$$

$$s_1^2(x) = \frac{\Sigma(x - \bar{x})^2}{n - 1} = 0.01$$

For revised method:

$$\bar{x}_2 = 79.6$$

$$\text{d.f.} = n - 1 = 6$$

$$s_2^2(x) = 0.07$$

F-test hypothesis:

$$\frac{s_2^2(x)}{s_1^2(x)} = \frac{0.07}{0.01} = 7.0$$

The sample means of the two procedures are the same, but the difference in variance (or scatter) is significant at the 5 percent probability level. This is evident from the F table which indicates a value of 6.2 at the 5 percent probability level for the degrees of freedom present.

To determine whether the mean obtained in the current procedure is significantly different from the mean of the revised method, it would be necessary to proceed as in Example 7 and use the t-test. However, the t-test would not indicate which method is more precise.

The F-test provides a statistical measure of the variability for two sets of measurements. If there are more than two sets, the chi-squared test is used to determine whether the variability of the separate sets forms a homogeneous group. The relationship for chi-squared under these conditions is given as follows:

$$\chi^2 = 2.303\,\frac{\log \bar{s}^2(x)\Sigma(k-1) - \Sigma(k-1)\log s^2(x)}{C} \qquad (10)$$

where $\bar{s}^2(x)$ is the pooled variance $= \dfrac{\Sigma(k-1)s^2(x)}{\Sigma(k-1)}$

$$C = 1 + \frac{1}{3(n-1)}\left[\Sigma\frac{1}{(k-1)} - \frac{1}{\Sigma(k-1)}\right]$$

k = number of datum points to calculate $s^2(x)$

n = number of sets of data

d.f. = $n - 1$

The use of the chi-squared test in one case and the use of the F-test in another case to test variances is not surprising when it is realized that these two tests are interrelated. To find the confidence limits for the variance of a normal population, the following definition of the chi-squared is used:

$$\chi^2 = \frac{\text{d.f.}\ s^2}{\sigma^2} \qquad (11)$$

At infinite degrees of freedom $s^2 = \sigma^2$. Therefore χ^2 corresponds to F times the degrees of freedom for values at infinite degrees of freedom in the denominator.

The t-test provides a method for comparing two means while the F-test permits a comparison of two or more means. It can be shown that if a group of samples of size n from a total population has a sample variance $s^2(x)$, then the means of the samples have a variance

$$s^2(\bar{x}) = \frac{s^2(x)}{n}$$

Thus, to determine whether the means of several sets of measurements are similar, it is necessary to calculate the pooled variance of the sets of samples, $\bar{s}^2(x)$, and the variance of the means of the sets, $s^2(\bar{x})$. If the means have only a normal amount of variation, then $ns^2(\bar{x})/\bar{s}^2(x)$ is usually 1.0. If the means are different, then $s^2(\bar{x})$ will include not only the variance within the sets of samples, but will include the variance due to the difference of the means, and the ratio of the variances will be different from the tabulated F values. This test is the simplest form of the analysis of variance.

Example 11 Analysis of variance for three sets of data. Three different reactors used at different locations but using the same process give the following yields. It is desired to determine whether the means for the three reactors are similar.

Yields		
1	*2*	*3*
11.9	11.9	11.2
11.8	12.4	9.8
10.9	12.0	10.6
11.4	12.1	10.4

Solution. The analysis of variance is made in the following manner:

Source of variance	*Degrees of freedom*	Sum *of squares*
Between means	n - 1	$\sum \frac{(\text{set sums})^2}{k} - \frac{(\Sigma x)^2}{\Sigma k}$
Within sets of data	$\sum k - n$	$\sum x^2 - \sum \frac{(\text{set sums})^2}{k}$
Total	$\sum k - 1$	$\sum x^2 - \frac{(\Sigma x)^2}{\Sigma k}$

where n = number of sets of data

k = number of datum points in each set of data

For the three sets of data

Reactors	1	2	3
Set sums	46.0	48.4	42.0
Mean	11.5	12.1	10.5

$$\sum \frac{(\text{set sums})^2}{k} \frac{(42.0)^2.0)^2}{4} + \frac{(48.4)^2}{4} + \frac{}{4} = 1555.64$$

$$\frac{(\Sigma x)^2}{\Sigma k} = \frac{(46.0 + 48.4 + 42.0)^2}{4 + 4 + 4} = 1550\ 41$$

$$\Sigma x^2 = (11.9)^2 + (11.8)^2 + \cdots + (10.6)^2 + (10.4)^2 = 1557.40$$

$$s^2(x) = \frac{\text{sum of squares}}{\text{d.f.}}$$

Between means

$$s^2(x) = \frac{1555.64 - 1550.41}{3 - 1} = 2.62$$

Within sets of data

$$s^2(x) = \frac{1557.40 - 1555.64}{12 - 3} = 0.186$$

$$F = \frac{s^2(x) \text{ between means}}{s^2(x) \text{ within sets of data}}$$

$$= \frac{2.62}{0.186} = 41.1$$

F tables tabulated for 0.05 and 0.01 probability levels with d.f. (numerator) of 2 and d.f. (denominator) of 9 indicate values of 4.26 and 8.02, respectively. Since the calculated value is larger than either one of the tabulated values, it may be concluded that the means of the three reactors are significantly different at both the 0.05 and 0.01 probability levels.

The analysis-of-variance test has great utility in statistical analysis because it can be applied in suitable cases to any number of classifications. For example, if experiments are run at several temperatures and in several reactors, the yield data could be analyzed to see if the reactor mean results are similar and to determine the variance of the individual measurements. If the data are divided into groups according to the classification, the mean square of each category is the corresponding sum of squares divided by the proper degrees of freedom. An F-test applied to each classification, comparing it with the variance within sets, will establish whether the variance due to this classification exists, i.e., whether the means are significantly different. If the variance due to one classification does exist, it may be calculated from the mean squares in the following manner:

Mean squares for	*Estimates variances of*
Rows	s^2 + (no. in row) (s^2 rows)
Columns	s^2 + (no. in column) (s^2 columns)
Within sets of samples	s^2

In a set of experiments with two variables, the effect of the variables might be different at various levels of each other. This effect is called "interaction." The effect of this interaction is included in the analysis of variance in the "within-set" variance.

CONFIDENCE-INTERVAL ESTIMATION

Statistical analysis development involves the theoretical determination of the distributions of certain quantities—such as mean, standard deviation, and variance-which would be expected to occur by chance alone. Thus, in analyzing experimental data, statistical theory serves as a powerful tool for determining with a reasonable degree of certainty whether certain observed differences might have been due to chance.

In terms of the previously mentioned normal distribution, the probability that a randomly selected observation x from a total population of data will be within so many units of the true mean μ can be calculated. However, this leads to an integral which is difficult to evaluate. To overcome this difficulty, tables have been developed in terms of $\mu \pm Z\sigma$.† This means that, if the true standard deviation σ of a particular normal distribution under study is known and assuming that the difference between the sample $\bar{x}$ and the true mean μ is only the result of chance and that the individual observations are normally distributed, then a confidence interval in estimating μ can be determined. This measure was referred to previously as the confidence level.

Since by definition,

$$\pm Z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} \tag{12}$$

it follows by rearrangement, that

$$\mu = \bar{x} \pm \frac{Z\sigma}{\sqrt{n}} \tag{13}$$

Therefore, corresponding to a selected probability level which determines the value of Z, it can be stated that the confidence interval of μ will be given by

$$\bar{x} - \frac{Z\sigma}{\sqrt{n}} < \mu < \bar{x} + \frac{Z\sigma}{\sqrt{n}} \tag{14}$$

The *confidence interval* for μ is defined as the random interval in which μ is predicted to lie for a selected probability level. A high probability level will give a larger confidence interval for μ.

If the true standard deviation is not known, a corresponding confidence interval can still be determined. However, this estimate must utilize the t-distribution instead of the Z-distribution since the t-concept includes the additional variation introduced by the estimate of standard deviation. In this case the rearranged t-equation is used.

$$\mu = \bar{x} \pm \frac{ts(x)}{\sqrt{n}} \tag{15}$$

This indicates that the true mean lies within some range of the observed mean,

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†See for example Table 1-15; in R. H. Perry and D. Green, "Chemical Engineers' Handbook," 6th ed., McGraw-Hill Book Company, New York, 1984.

the range being set by the degrees of freedom used in getting s and the probability level of t.

$$\bar{x} - \frac{ts}{\sqrt{n}} < \mu < \bar{x} + \frac{ts(x)}{\sqrt{n}} \tag{16}$$

Example 12 Confidence interval for the true mean. Set the confidence interval for the true mean in Example 5 for probability levels of 95 and 99 percent.

Solution

From Example 5	*From t-table*
$\bar{x} = 92.6$	$t_{0.95} = 2.78$
d.f. = 4	$t_{0.99} = 4.60$
$s(x) = 0.292$	

$$\mu = \bar{x} \pm \frac{ts(x)}{\sqrt{n}}$$

Probability level	*Confidence interval*
95 percent	$92.6 \pm \frac{(2.78)(0.292)}{\sqrt{5}} = 92.2$ to 93.0
99 percent	$92.6 \pm \frac{(4.60)(0.292)}{\sqrt{5}} = 92.0$ to 93.2

This shows that for a 95 percent confidence level it would be more correct to report the gas analysis in Example 5 as 92.6 ± 0.4 percent rather than simply as 92.6 percent.

Assuming a normal distribution, a confidence interval for σ^2 can also be obtained. In this case the chi-squared distribution is used as the basis for evaluating the chance deviation of the estimated variance from the true variance. Rearranging Eq. (11) results in a confidence interval for the true variance in terms of the degrees of freedom, the estimated variance, and the chi-squared distribution as

$$\text{d.f.}\frac{s^2}{\chi_P^2} \le \sigma^2 \le \text{d.f.}\frac{s^2}{\chi_{1-P}^2} \tag{17}$$

where P is the probability listed in the χ^2 tables. Note that the χ^2 table lists values for low and high probability values. In other words, 90 percent of the chi-squared distribution lies in the range 0.95 to 0.05, and 95 percent of this distribution lies in the range 0.975 and 0.025, etc. Setting the 90 percent probability range of the true variance from the estimated variance means that σ^2 has only a 5 percent chance of being smaller than $\text{d.f.}(s^2)/\chi_{P=0.05}^2$ and a 5 percent chance of being larger than $\text{d.f.}(s^2)/\chi_{P=0.95}^2$.

Often it is of interest in dealing with a normal population to determine what percentage of the observations can be expected to fall within some specified interval when the mean and standard deviation are not known.

Estimates of this type are called *tolerance intervals.* Such intervals are in a form of $\bar{x} \pm Ks(x)$ similar to the previously described confidence intervals of $\bar{x} \pm ts(x)/\sqrt{n}$. The value of K (which is a function of n, α, and γ) is selected in such a manner that it can be said with a probability of $100-\alpha$ (corresponding to an error of α percent) that the interval will cover at least a proportion γ of the population.†

REGRESSION ANALYSIS

The statistical techniques which have been discussed to this point were primarily concerned with the testing of hypotheses. A more important and useful area of statistical analysis in engineering design is the development of mathematical models to represent physical situations. This type of analysis, called *regression analysis*, is concerned with the development of a specific mathematical relationship including the mathematical model and its statistical significance and reliability. It can be shown to be closely related to the Analysis of Variance model.

REGRESSION OF TWO VARIABLES

The usual procedure when dealing with one dependent and one independent variable, e.g., reaction rate and pressure, is to plot the data and draw what appears to be the best line through the points. The "best" line in a statistical sense is the line which minimizes the sum of the squares of deviations of observed points from the line. This is similar in principle to the way the mean of a group of data minimizes the scatter of the data when measured as the sum of the squares of deviation.

If the regression, however, is to provide the maximum amount of information on precision, differences between sets of data, and interaction between variables, it will be necessary to use the statistical technique of regression analysis.

The application of regression analysis involves four steps:

1. Selection of a mathematical model to represent the data.
2. Calculation of the coefficients in the mathematical model selected.
3. Statistical test of the mathematical model to represent the physical situation.
4. Evaluation of the mathematical model to determine the direction for further improvements.

The appropriate model to be used for a specific application depends upon what is theoretically known about the process, the number of variables and data points, and the relative reliability of the data. There are no specific rules for

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†Values of K can be found in A. H. Bowker and G. J. Lieberman, "Engineering Statistics," 2d ed., page 314, Prentice-Hall Inc., Englewood Cliffs, New Jersey, 1972.

selecting a model. Rather, only experience by the engineer will dictate the appropriate course of selection. In general, the selection of nonlinear models should be avoided unless there is a sound theoretical basis for selecting a particular type ("Nonlinear," in this case, refers to the unknown regression coefficients.)

Assuming, for the moment, that the relationship is best represented by a straight line, the "least-squares" line is the regression giving the minimum squared deviation of the data from the line. The theoretical least-squares line can be represented mathematically as

$$y = \alpha + \beta(x - \bar{x}) + e_i \tag{18}$$

where α, β are unknown constants and e_i is the error term which is normally and independently distributed with zero mean and constant unknown variance. Note that the independent variable x is assumed to be known without error. The least-squares solution yields

$$y = a + b(x - \bar{x}) \tag{19}$$

where a = $\bar{y}$, the mean value of y

$$b = \frac{\Sigma(\bar{x} - x)(\bar{y} - y)}{\Sigma(\bar{x} - x)^2}$$, the slope of the least-squares line

If the mathematical model indicates that the regression will go through the origin, e.g., in the case of the yield from a reactor as a function of time, the calculations can obviously be simplified over those shown in Eq. (19). The slope expression is obtained by differentiating the linear equation and setting the sum of squares of the derivative at a **minimum.** Terms in the slope expression may be simplified in the following manner:

$$\Sigma(\bar{x} - x)(\bar{y} - y) = \Sigma xy - \frac{\Sigma x \Sigma y}{n} \tag{20}$$

$$\Sigma(\bar{x} - x)^2 = \Sigma x^2 - \frac{(\Sigma x)^2}{n} \tag{21}$$

A least-squares line may be drawn in several simple ways once the calculation is completed. The line goes through the $\bar{x}$, $\bar{y}$ point and the y intercept. If the intercept is off the figure, the line may be drawn through the $\bar{x}$, $\bar{y}$ point with a slope of b.

Example 13 Linear regression of two variables. Determine the best linear regression to represent the following data.

Y	x
3.0	1
4.1	2
5.2	3
6.0	4
6.7	5

Solution

$$\Sigma x = 15 \qquad \bar{x} = \frac{15}{5} = 3$$

$$\Sigma x^2 = 55$$

$$\Sigma(\bar{x} - x)^2 = \Sigma x^2 - \frac{(\Sigma x)^2}{n} = 55 - \frac{(15)^2}{5} = 10$$

$$\Sigma y = 25 \qquad \bar{y} = \frac{25}{5} = 5$$

$$\Sigma y^2 = 133.74$$

$$\Sigma(\bar{y} - y)^2 = \Sigma y^2 - \frac{(\Sigma y)^2}{n} = 133.74 - \frac{(25)^2}{5} = 8.74$$

$$\Sigma xy = 84.3$$

$$\Sigma(\bar{x} - x)(\bar{y} - y) = \Sigma xy - \frac{\Sigma x \Sigma y}{n} = 84.3 \quad \frac{(15)(25)}{5} = 9.3$$

$$y = a + b(x - \bar{x})$$

$$a = \bar{y} = 5$$

$$b = \frac{\Sigma(\bar{x} - x)(\bar{y} - y)}{\Sigma(\bar{x} - x)^2} = \frac{9.3}{10} = 0.93$$

$$y = 5.0 + 0.93(x - 3)$$

$$y = 5.0 + 0.93x - 2.79$$

$$y = 2.21 + 0.93x$$

The linear regression representing the above data is shown on Fig. 17-2.

Quite often the regression will not be too good and the question arises as to when a regression is not significant, i.e., when is the amount of variance removed by the regression less than could be expected by chance. In such cases, one can test the hypothesis that the regression coefficient β is zero or alternatively find a confidence interval for β and see whether it includes zero.

Returning once again to an analysis of the least-squares line, it is now possible to make confidence statements concerning the slope, the intercept, and any value of y predicted by the equation. The statistical quantities required to make these statements are given in the next four equations and are the residual variances $s^2(Y)$, the variance of the slope $s^2(b)$, the variance of the mean $s^2(y)$,

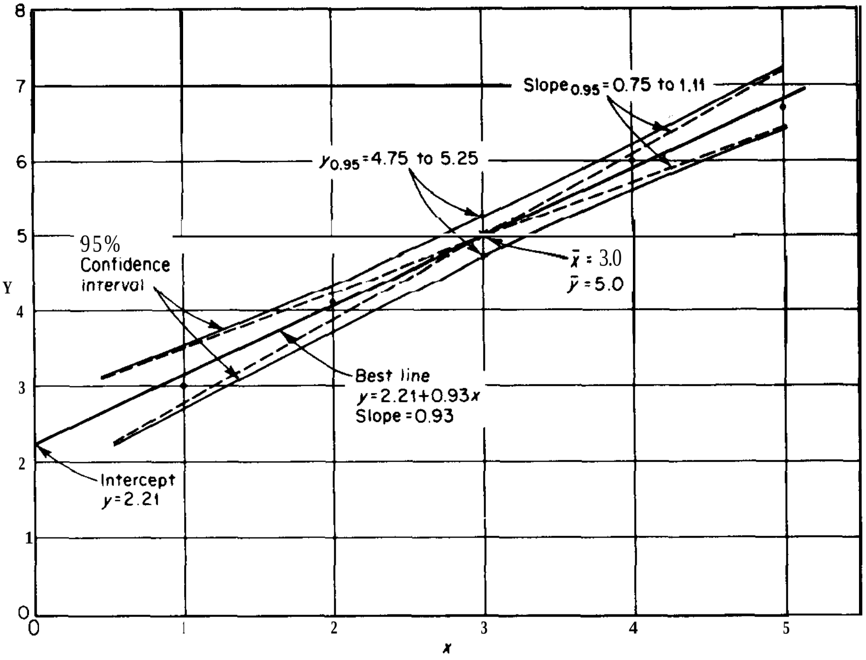

FIGURE 17-2
Correlation of data from Example 13 with "least-squares" line.

and the variance of a single point, $s^2(y_i)$, respectively.

$$s'(Y) = \frac{1}{\text{d.f.}\dagger}\left[\Sigma(\bar{y}-y)^2 - b\Sigma(\bar{x}-x)(\bar{y}-y)\right] \tag{22}$$

$$s^2(b) = \frac{s^2(Y)}{\Sigma(\bar{x}-x)^2} \tag{23}$$

$$s^2(\bar{y}) = \frac{s^2(Y)}{n} \tag{24}$$

$$s^2(y_i) = s^2(Y)\left[\frac{1}{n} + \frac{(\bar{x}-x_i)^2}{\Sigma(\bar{x}-x)^2}\right] \tag{25}$$

The confidence interval on the slope can be calculated from the variance of the slope in the same manner as was used to determine the confidence range for the mean. Thus for a 95 percent confidence interval on the slope

$$b - t_{0.95}s(b) < b_{0.95} < b + t_{0.95}s(b) \tag{26}$$

Similar expressions for the confidence interval of the mean $\bar{y}$, or with any point included in the data, y_i, can be formulated.

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†The degrees of freedom for a linear correlation with one dependent and one independent variable is $n-2$.

Example 14 Analysis of linear regression. What is the confidence range of the slope and the mean of the dependent variable for a 95 percent confidence level in Example 13?

Solution

$$s^2(Y) = \frac{1}{\text{d.f.}}\left[\Sigma(\bar{y} - y)^2 - b\Sigma(\bar{x} - x)(\bar{y} - y)\right]$$

$$= \frac{1}{3}[8.74 - (0.93)(9.3)] = 0.03$$

$$s^2(b) = \frac{s^2(Y)}{\Sigma(\bar{x} - x)^2}$$

$$= \frac{0.03}{10} = 0.003$$

$$s^2(\bar{y}) = \frac{s^2(Y)}{n}$$

$$= \frac{0.03}{5} = 0.006$$

The 95 percent confidence interval of the slope is then

$$b_{0.95} = b \pm t_{0.95}s(b)$$

$$= 0.93 \pm (3.18)\sqrt{0.003} = 0.93 \pm 0.175$$

and the 95 percent confidence interval of the mean $\bar{y}$ is

$$\bar{y}_{0.95} = \bar{y} \pm t_{0.95}s(\bar{y})$$

$$= 5.0 \pm (3.18)\sqrt{0.006} = 5.0 \pm 0.25$$

Thus,

$$b_{0.95} = 0.75 \text{ to } 1.11$$

$$\bar{y}_{0.95} = 4.75 \text{ to } 5.25$$

This confidence limit is also shown on Fig. 17-2 to provide a comparison with the best line developed previously in Example 13.

If the calculated regression is to be used to predict what will occur if another set of data points is to be obtained, it will be necessary to add the residual variance $s^2(Y)$ to Eq. (25) to obtain the variance of the predicted point. Thus,

$$s^2(y_p) = s^2(Y)\left[\frac{1}{n} + \frac{(\bar{x} - x_i)^2}{\Sigma(\bar{x} - x)^2}\right] + s^2(Y) \tag{27}$$

The significance of the regression can also be tested by determining whether the slope is significantly different from zero by means of

$$t = \frac{b - O}{s(b)} \tag{28}$$

in a manner similar to that used in testing the mean $\bar{x}$ difference.

The variance of the slope permits a comparison of two regression lines in the same way that two means were compared by the t-test. Thus,

$$t = \frac{b_1 - b_2}{\bar{s}(b)} \tag{29}$$

where $\bar{s}(b)$ is given by

$$i(b) = \sqrt{\left[\frac{(\text{d.f.})_1 s^2(Y_1) + (\text{d.f.})_2 s^2(Y_2)}{(\text{d.f.})_1 + (\text{d.f.})_2}\right]\left[\frac{1}{\Sigma(\bar{x}_1 - x_1)^2} + \frac{1}{\Sigma(\bar{x}_2 - x_2)^2}\right]}$$

REGRESSION OF MULTIPLE VARIABLES

A similar regression procedure may be followed for any number of independent variables. The method is fairly involved for more than two independent variables and under such circumstances it would be best to refer to a good text in statistical analysis. However, since regressions with two independent variables occur quite frequently in chemical engineering design, a brief description of the regression procedure is given in the next few paragraphs.

The expression relating two independent variables x_1 and x_2 to a dependent variable y with a linear model may be expressed as

$$y = \alpha + \beta_1(x_1 - \bar{x}_1) + \beta_2(x_2 - \bar{x}_2) + e_i \tag{30}$$

The least-squares solution yields

$$y = a + b_1(x_1 - \bar{x}_1) + b_2(x_2 - \bar{x}_2) \tag{31}$$

where a = $\bar{y}$, the mean value of y

$$b, = C_{11}\Sigma' x_1 y + C_{12}\Sigma' x_2 y$$

$$C_{11} = \frac{\Sigma' x_2^2}{K}$$

$$C_{12} = -\frac{\Sigma' x_1 x_2}{K}$$

$$b_2 = C_{12}\Sigma' x_1 y + C_{22}\Sigma' x_2 y$$

$$C_{22} = \frac{\Sigma' x_1^2}{K}$$

$$K = \Sigma' x_1^2 \Sigma' x_2^2 - (\Sigma' x_1 x_2)^2$$

and the summations are further defined as

$$\Sigma' y^2 = \Sigma y^2 - \bar{y}\Sigma y$$

$$\Sigma' x_1^2 = \Sigma x_1^2 - \bar{x}_1\Sigma x_1$$

$$\Sigma' x_2^2 = \Sigma x_2^2 - \bar{x}_2\Sigma x_2$$

$$\Sigma' x_1 y = \Sigma x_1 y - \bar{x}_1\Sigma y$$

$$\Sigma' x_2 y = \Sigma x_2 y - \bar{x}_2\Sigma y$$

$$\Sigma' x_1 x_2 = \Sigma x_1 x_2 - \bar{x}_1\Sigma x_2$$

while the residual variance is given by

$$s^2(Y) = \frac{\Sigma(y_i - \bar{y})^2 - (b_1\Sigma' x_1 y + b_2\Sigma' x_2 y)}{n - 3} \tag{32}$$

and the variance of the two coefficients b_1 and b_2 is

$$s^2(b_1) = C_{11}s^2(Y) \tag{33}$$

$$s^2(b_2) = C_{12}s^2(Y) \tag{34}$$

NONLINEAR REGRESSIONS OF VARIABLES

Regressions obviously do not have to be linear, although the linear ones are most frequently employed and definitely the most easily handled. In fact, it is often possible to modify nonlinear relationships to give linear correlations. The most common example is to transform an exponential equation into a linear logarithmic equation. If such a transformation is not possible, the regression may become quite involved either because of the stepwise solution which is necessary or the tedious iterative method which must be followed.

USE IN EVALUATION OF PROCESS DATA

The use and benefits of regression analysis can be appreciable, particularly in the evaluation of process data. In these applications, processes having as many as fifty variables, which are continuously changing over months of operation, can be evaluated by this technique. For these, the daily log records for say 400 to 500 data points are analyzed through the selected model (usually linear as a first approximation) to determine the relative effects of each variable on the response. This analysis in many cases has led to qualitative and often to quantitative determination of key operating variables whose effect had been masked on individual data point comparisons by the simultaneous changes in other less important, but unknown, variables.

STATISTICAL STRATEGY IN DESIGN OF EXPERIMENTS

In many exploratory experimental programs, the researcher is often confronted with the problem of determining the possible effect of a large number of factors. In such situations, it is necessary to establish an acceptable procedure for choosing the conditions of each experimental run. Statistical strategy in the design of experimental programs involves the systematic and controlled procedure for developing the correct combinations of variable conditions to determine a reliable analysis. Three basic types of statistically designed experiments are most often used in the chemical industry. These are:

1. Factorial design
2. Fractional factorial design
3. Box-Wilson design

Before investigating these methods briefly, it will be necessary to become familiar with the terminology which is used in this particular application of statistical analysis. (The terminology is not always very meaningful in industrial problems since the early work in this area was originally developed in agricultural experimentation.)

Experimental variables are usually called factors. The particular value of the variable is called the level of the factor. The combination of factors used in a particular experimental run is called a *treatment*. The result of the run is designated as the *effect*. If the material being processed is limited in quantity so that it may be necessary to use several batches of material that are similar but not identical in characteristics, then each batch is called a *block*. Repetitions of the same experiment at the same conditions are known as *replications*.

Applications of these terms to an experimental program can best be illustrated by considering the optimization of operating conditions for a reaction using small bench-scale reactors. If it is desired to determine the effect on the yield of varying temperature, pressure, and residence time by studying this at two temperatures, two pressures, and two contact times, the experimental program will be equivalent to one of three factors, each at two levels. The treatments will be various selected combinations of the levels of the factors. The effects are the yields of each run. If more than one reactor is used in the study and the reactors are suspect of small differences, then data obtained from each reactor may be established as a block. Choice of blocks is based on previous experience or suspected differences. The purpose of the blocks is to eliminate the effect of a variable that often cannot be measured accurately nor controlled by establishing groups that are similar in this variable.

FACTORIAL DESIGN

Factorial design is useful in making a preliminary survey of system factors (variables). It does not yield a quantitative relationship showing the effect of a factor; it merely determines whether there is any effect. The chemical engineer can therefore use factorial design as a first step to determine the influential factors and then use regression analysis on further experimental runs to develop a quantitative relationship among the important factors.

Factorial design can be applied to any number of factors and levels. Frequently, in exploratory work, only two levels of each factor are chosen for the factorial design. Two factors and two levels constitute a 2^2 factorial design and permit setting up a 2 x 2 analysis-of-variance table with one factor on columns and another on rows. By determining the row effect and the column effect, it is possible to determine, for example in a reactor, whether a variation in temperature or pressure affects the reaction yield. A choice of substantially different values of temperature and pressure would be desirable to ensure conclusions which would be based on a wide range of factors.

In the traditional approach to experimentation, involving a large number of factors, it is normal to vary one factor while holding all the others constant to determine the effect of that factor. The experiment is then repeated for all other factors, each time holding all others constant except the one being studied. In factorial design, rather than varying only one factor at a time, several factors are varied at once in a prearranged pattern to obtain more information with possibly fewer runs. Besides giving an estimate of the size of the error variation in the data, this method also permits evaluation of possible interactions between factors with a minimum of experimentation.

The advantages of factorial design can best be illustrated by considering a simple experiment involving the effect of three factors (A, B, C) each at two levels $(1, 2)$ on the yield of a reaction X. In the traditional design, the effect of varying C is determined by the difference in the yields while holding factors A and B constant. The effects of A and B on the yield are obtained similarly. This requires a minimum of four experimental runs. Four replicate runs must be made to obtain an estimate of the experimental error.

In the 2^3 factorial design, a total of eight experimental runs is also made as represented by the corners of the cube in Fig. 17-3. The effect of varying C is determined by comparing the average of the C, plane to that of the C_2 plane.

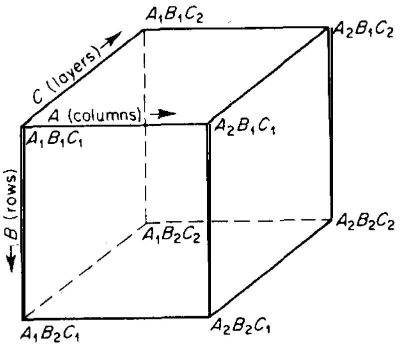

FIGURE 17-3
Cube representing a factorial design involving three factors at two levels for each.

That is, the average of $X(A_1B_1C_1)$, $X(A_2B_1C_1)$, $X(A_1B_2C_1)$, and $X(A_2B_2C_1)$ is compared with the average of $X(A_1B_1C_2)$, $X(A_2B_1C_2)$, $X(A_1B_2C_2)$, and $X(A_2B_2C_2)$. Although A and B also vary, their variation is the same in the two sets, C, and C_2, so that their effects cancel out. The effects of varying A and B are determined similarly from comparing the other two planes.

One advantage of the factorial design over the traditional design is readily apparent. In the traditional design, the difference between the means of sets of two observations was used to determine a main effect; in the factorial design, the difference in the means of sets of four observations was used. The greater number of observations gives a better estimate of the true difference in the means. Note that, in both cases, the same number of experimental runs was required.

Another advantage of the factorial design is that interaction between factors can be estimated. To determine the interaction between A and B, it is necessary to average the effect of C. That is, in Fig. 17-3, the top and bottom planes are averaged. This results in

$$X(A_1B_1\bar{C}) \quad \text{and} \quad X(A_2B_1\bar{C}) \qquad B_1 \text{ held constant}$$

$$X(A_1B_2\bar{C}) \quad \text{and} \quad X(A_2B_2\bar{C}) \qquad B_2 \text{ held constant}$$

The effect of varying A at B_1 is obtained from a difference of the first two values

$$X(A_1 - A_2)_{B_1} = X(A_1B_1\bar{C}) - X(A_2B_1\bar{C}) \tag{35}$$

while the effects of varying A at B_2 is evaluated from the difference of the last two values

$$X(A_1 - A_2)_{B_2} = X(A_1B_2\bar{C}) - X(A_2B_2\bar{C}) \tag{36}$$

If $X(A_1 -$ A,) at B, varies significantly from $X(A_1 - A_2)$ at B_2, there is an interaction between A and B. The interactions between A and C, and B and C, are evaluated in a similar manner.

The analysis of results from a 2^3 factorial experiment requires a three-way analysis of variance. That is, in addition to having row and column effects, there are also layer effects and interactions among the three.

FRACTIONAL FACTORIAL DESIGN

Procedures are available for 2” designs involving two levels of n factors where n can, in principle, be any large number. However, the required number of runs for large n may be prohibitive. For a five-factor experiment, 2^5 runs are required in a single block; but block size must be held to a minimum to control known sources of error. For this reason, fractional factorials which utilize some integer fraction (a multiple of the number of levels) of the total factorial experiments are used. The five-factor experiment at two levels would involve a total of 32 experiments in the factorial design whereas a $\frac{1}{2}$ fractional factorial design would involve only 16 experiments. Likewise a 2^{10} factorial design could be analyzed by a $\frac{1}{2}$ fractional factorial design with 32 tests. This simplification reduces the block size at the expense of the accuracy of some of the higher-order interactions.

As the number of factors increases, the number of possible interactions rapidly grows. Higher-order interactions are possible. For example, a 2^4 design requires a four-way analysis of the variance. There are four main effects, six second-order interactions (such as row X column), and four third-order interactions (such as row x column x layer). (Speaking of rows, columns, and layers in a four-way analysis of variance is not too meaningful because a fourth dimension is needed and the geometrical analogy breaks down.)

BOX-WILSON DESIGN

The Box-Wilson experimental designs are a general series of experiments that have been developed to efficiently serve as a basis for deriving the mathematical model of a physical process. Their usefulness is enhanced in the study of industrial applications because most physical situations can usually be approximated by a quadratic function over a reasonable range of the factors. For a two-factor system, the generally used form of this model is

$$y = b_0 + b_1x_1 + b_2x_2 + b_{11}x_1^2 + b_{12}x_1x_2 + b_{22}x_2^2 \tag{37}$$

Similar equations can be formulated for models with more than two factors but are obviously more complex.

A technique for designing which experimental tests should be carried out to evaluate the coefficients of the model is the Box-Wilson composite rotatable design. For the purpose of the experiment, the independent factors are each specified at five levels. The specific values of these five levels for each factor depend on the number of factors included in the model and the range over which they are to be studied. The design principle includes three types of combinations, the axial, factorial, and center points. Axial points include each factor at its extreme level with the other factors at their center-point level. The center point is a single test at the average level of each factor. For purposes of estimating experimental error, the center point is usually repeated three to five times during the experiment. Designs for any number of factors can be developed from these principles.

When the experimental tests have been completed, a regression analysis is carried out to determine the coefficients in the assumed model. An analysis of variance by means of the F-test is usually carried out after the model has been developed to determine its significance.

EVOLUTIONARY OPERATION

The technique of evolutionary operation (EVOP) involves systematic small changes in process variables during the operation of the process. The results of previous small changes are used to suggest further changes so as to approach optimum operating conditions by a series of small steps. None of the individual changes which are made, however, are sufficient to upset the process to the point of producing an unacceptable product.

With two independent variables and one response, a response surface may be visualized much like a contour map of a mountain peak. The two independent variables would be represented by the two horizontal dimensions, and the vertical distance would represent the response. (For more than two variables, this geometric analogy breaks down.) Evolutionary operation assumes that an optimum exists. To determine this optimum, experimental runs are made at four points which are removed from the original operating conditions on the response surface. The optimum or highest response of these tests is then used to select four new operating points, and the process is repeated until the optimum is reached. Since the response surface must be visualized, small changes are made to explore it gradually.

The basic assumption made in this simple-minded procedure above is that a smooth response surface exists which gradually rises to a single optimum. The procedure outlined would not necessarily locate the highest of several optimums. Here is where judgement of the experiments and possible past experience must be used to place the first runs on the response surface of the highest "peak." Actually, the optimum may be on a rather flat plateau of the surface, where several sets of near-optimum conditions may be preferable to the true optimum from other considerations.

OTHER STATISTICAL METHODS

Propagation of error, sequential analysis, and quality control are additional statistical techniques with which the chemical engineer in design should be acquainted. The intent of this section will only be to outline the value of these tools and leave the details to other references.

PROPAGATION OF ERROR

Cost estimation of a process design generally involves the summation of costs of individual pieces of equipment making up the process. If the precision of the cost estimate for the individual items is known, what is the precision of the total estimate?

There is no rigorous mathematical method for combining the individual precisions, but the variances of the individual measurements may be combined. The precision, at the 95 percent confidence level, is defined as approximately twice the true standard deviation. Since the latter is the square root of the true variance, this permits evaluation of the precision for the combined operation.

The variance of a sum or difference is the sum of the variances assuming independence. Thus, if

$$y_t = f(x_1)$$

the variance of the combined operation y, is given approximately by

$$\sigma^2(y_t) = \Sigma\left(\frac{\partial f}{\partial x_i}\right)^2 \sigma^2(x_i) \tag{38}$$

The use of this relation is best illustrated in the following example.

Example 15 Propagation of error. What is the precision? of a cost estimate including three items if the individual costs and **precisions** of these items are as follows:

Item	Cost	*Precision*
A_1	\$1000	±\$50
A_2	1000	±\$100
A_3	2000	±\$100

Solution

C, =A, +A, +A, **where** C_t istotalcost

$$\sigma^2(C_t) = \sigma^2(\text{ A,}) + \sigma^2(A_2) + \sigma^2(A_3)$$

$$C_t \pm c_t = (A_1 \pm a_1) +_{,}(A_2 \pm a_2) + (A_3 \pm a_3)$$

Assume that a, = $2\sigma(A_1)$, $a_2 = 2\sigma(A_2)$, $a_3 = 2\sigma(A_3)$, and c, = $2\sigma(C_t)$.

$$\sigma^2(C_t) = \frac{a_1^2}{4} + \frac{a_2^2}{4} + \frac{a_3^2}{4}$$

$$\sigma(C_t) = \sqrt{\frac{a_1^2}{4} + \frac{a_2^2}{4} + \frac{a_3^2}{4}}$$

$$c_t = 2\sigma(C_t) = \sqrt{a_1^2 + a_2^2 + a_3^2}$$

$$c_t = 150$$

therefore

$$C_t \pm \text{c,} = (1000 + 1000 + 2\text{ooo}) \pm 150$$

$$= \$4000 \pm \$150$$

In this case, the precision of the total cost estimate is better percent-wise than that of either of the three individual items of equipment.

SEQUENTIAL ANALYSIS

In the usual statistical test, a hypothesis is made about the population from which the sample is taken. A statistical quantity is calculated from the sample

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†**Precision** in this example refers to a confidence interval of 217, equivalent to approximately a 95 percent confidence level.

and the hypothesis is either accepted or rejected on the basis of the test. When the hypothesis is rejected, it is with a known probability, established by the significance level of the test, of a false rejection. If the hypothesis is accepted, it is normally on the basis of having insufficient evidence for its rejection, and not with an established probability of a false acceptance. In some cases the probability of false acceptance of the hypothesis, the Type II error, can be controlled by the size of the sample.

In sequential testing, a hypothesis is established and limits are set for both Type I and Type II errors. The data are then accumulated sequentially, and, as each new observation is obtained, the information from all the data is tested against the hypothesis with one of three actions possible: the hypothesis may be rejected, the hypothesis may be accepted, or no decision may be made and another datum point is obtained. Data are collected until the hypothesis is either rejected or accepted.

In general, the method involves plotting some cumulative function of the data as ordinate against the number of observations as abscissa. Two parallel guide lines are drawn, depending upon the particular test involved and the probability levels selected. When the plot of the cumulated function of the data crosses either of the parallel guide lines, the indicated decision is made. As long as the plot stays within the parallel lines, no decision is indicated without a greater probability of error than originally established. The slope of the guide line is a function of the hypothesis tested. The heights of the guide lines or the magnitude of the control values are established by both the hypothesis being tested and the probability of its false acceptance and false rejection,

QUALITY CONTROL

Statistical analysis, as applied to production or other processes in which quantities of materials are continuously being tested or measured, is known as quality control. In this statistical method, some measurable attribute of the processed material is used as a criterion of the quality of the product. Random samples are drawn from the production line in succeeding time intervals, and the means of small groups of these samples are compared with some standard. Statistical methods, particularly the t-test, provide a method of determining when the measured mean differs from the control value by an amount greater than would be expected by chance.

NOMENCLATURE FOR CHAPTER 17

a = estimate of α in regression equation

b = estimate of β in regression equation

b,, b_2 = estimates of β_1 and β_2 in multiple regression equation

C = constant defined by Eq. (10)
C_{11}, C_{12}, C_{22} = constants defined by Eq. (31)
d.f. = degrees of freedom
F = F-test
f = observed frequency
f' = expected frequency
$\overline{G}$ = geometric mean
$\overline{H}$ = harmonic mean
k = number of datum points in Eq. (10)
md = mean deviation
n = sample size
P = probability level
s(b) = estimated standard deviation of slope
$\bar{s}(b)$ = pooled estimate of standard deviation of slope
$s^2(b)$ = estimated variance of slope
$s(x)$ = estimated standard deviation
$s(\bar{x})$ = estimated standard deviation of mean
$\bar{s}(x)$ = pooled estimate of standard deviation of sample
$\bar{s}(\bar{x})$ = pooled estimate of standard deviation of mean
$s^2(x)$ = estimated variance
$\bar{s}^2(x)$ = pooled estimate of variance of sample
$s^2(Y)$ = estimated residual variance from correlation
$s^2(\bar{y})$ = estimated variance of mean
$s^2(y_i)$ = estimated variance of a datum point
t = t-test
$\bar{x}$ = mean
$\bar{y}$ = mean of dependent variable
Z = defined by Eq. (12)

Greek Symbols

α = percent of Type I error
β_1 = unknown parameter in multiple regression problem
β_2 = unknown parameter in multiple regression problem
γ = proportion of total statistical population
σ = true standard deviation
$\bar{\sigma}$ = true standard deviation of the sample mean
σ^2 = true variance
7 = time
μ = true mean
μ_0 = some fixed number
χ^2 = chi-squared test

PROBLEMS

1. Compute the mean, median, and mode for the following distribution of scores:

scows	*Frequency*
110-119	
100-109	0
90-99	2
80-89	5
70-79	10
60-69	13
50-59	9
40-49	4
30-39	5
20-29	0
10-19	1

Does this distribution fit a normal distribution curve?

2. If a normal distribution has a mean of 118 and an estimated standard deviation of 11, what percentage of the scores will be found between score limits of 100 and **118?** What percentage will be expected to lie below a score of **140?** What Z values correspond to scores of 100, 118, and **140?**

3. If material is received from two sources of supply and separate analysis on samples from the two shipments gives the results listed below, would there be a justification (0.95 probability level) for saying there was a difference between the two materials?

Shipment 1	*Shipment 2*
84.0	77.5
77.0	74.0
76.0	79.0
84.5	71.5
74.5	77.0
73.0	75.5
85.0	78.5

4. If the scatter in the individual samples of the previous problem is reduced to that listed below, would there be a justification (0.95 probability level) for saying there was a difference between the two materials? (Note that mean values of the two shipments have remained unchanged.)

Shipment 1	*Shipment 2*
80.0	74.5
77.5	76.5
79.0	77.0
79.5	75.0
78.0	76.5
79.5	77.0
80.5	76.5

5. The analysis of a flue gas gives the following CO, volume content: 23.8, 23.4, 24.1, 24.6, 23.9, and 23.1. What is the confidence interval for the true mean for a probability level of 0.95 and 0.99? How much could the confidence range be improved if the number of analyses were doubled assuming the original distribution remained unchanged?
6. The analysis of an acid gas stream from a gas treating plant is known to give an average deviation of 0.5, i.e., hundreds of analyses have verified this fact. In a series of analyses the following results were obtained: 22.5, 24.7, 23.2, 21.9, 22.7, and 22.9. Should the second analysis be discarded based on a 0.95 probability level?
7. The mean difference between duplicate determinations of water in a solvent was found to be 1.0 ppm. A single determination of a sample gave 15 ppm water. What is the confidence interval for probability levels of 0.95 and 0.99?
8. Two shifts give the following C_4 content from samples taken in the overhead of a butane splitter in a refinery. Is there a significant difference (0.99 probability level) between the analyses obtained during the two shifts?

	C_4 content, %	
Hour	*Shift 1*	*Shift 2*
1	27.7	27.8
2	28.3	27.0
3	28.0	27.6
4	28.4	27.4
5	27.9	28.0
6	28.1	27.8
7	28.4	27.2
8	27.8	27.7

9. Information on the effect of temperature on a specific reaction has been obtained from four different reactors as follows:

	Yields in reactor			
Temperatures, °F	*1*	*2*	*3*	4
300	10.4	10.8	10.6	10.8
325	10.9	11.6	11.7	12.1
350	12.1	12.9	12.8	13.5

By a two-way analysis of variance determine whether the variance between the reactors and between the temperatures is highly significant (0.01 probability level). Even though no replicates were made at any condition, estimate the precision of a result by the within set variance.

10. The total gas stream to a reactor unit is 5000 scfh of fresh feed, 12,500 scfh of recycle, and 100 scfh for an instrument bleed stream. If the standard deviation of each measurement is 5 percent of the individual flow, what are the 95 percent

precision limits of each stream, the 95 percent precision limits for total flow, and the 95 percent confidence interval for total flow?

11. The pressure drop in a process is the summation of the pressure drop in the lines, that contributed by two control valves, the pressure drop in the column, and the pressure drop in the condenser. What is the precision of the calculated total pressure drop in psia calculated at a 95 percent confidence level for the following data?

	Pressure drop	*Standard deviation*
Line	5.2 psia	0.05 psia
Control valves (each)	10.0 in. H_2O	0.10 in. H_2O
Column	6.8 psia	0.10 psia
Condenser	2.5 psia	0.05 psia

12. Five different batches of a commercial plastic are sampled to determine the variation of inert additives between batches. Since each batch consists of several thousand pounds of material, several samples are taken from each batch to determine the variation in the batch. With the following information, determine whether or not the variability of the separate batches forms a homogeneous test. Use a 95 percent probability level in the statistical analysis.

Batch	*Number of* samples	*Variance in* inert material, %
1	7	5.2
2	6	7.6
3	7	6.1
4	5	5.9
5	8	8.0
6	6	4.3
7	7	5.7

13. The tabulated data represent analyses of nitrogen in a crude nitrogen stream made by five different operators with three different types of analyzers. Establish if there is any significant difference (0.95 probability level) between operators and analysis equipment.

Operator	*Analyzers*		
	1	*2*	*3*
A	*96.6*	*97.0*	*97.6*
B	*96.4*	*96.0*	*97.2*
C	*97.0*	*95.0*	*96.4*
D	*96.2*	*95.6*	*97.4*
E	*96.8*	*97.0*	*97.8*

14. Find the best straight line through the following data points.

x	1.00	1.50	2.00	2.50	3.00	3.50	4.00	4.50
Y	2.21	3.07	4.19	5.51	6.20	7.13	8.92	9.49

What are the 95 percent confidence limits for the slope of this straight line? Is the predicted value of y at $x = 0$ significantly different from zero at the 95 percent confidence level?

15. Determine whether a linear correlation of y with x accounts for all but the error variation in y from the following replicate values of y at different levels of x.

x	1.00	1.50	2.00	2.50	3.00	3.50	4.00	4.50
Y	2.67	4.13	4.62	6.31	6.95	8.27	10.02	10.90
	2.43	3.98	5.43	6.07	7.20	8.41	10.46	10.76
	2.80	4.09	5.07	6.18	7.25	8.25	9.97	11.15

16. Experimental data for the heat capacity in cal/(g . mol)(°C) as a function of temperature are given as follows:

Temperature, K	*Heat capacity*
300	19.65
400	26.74
500	32.80
600	37.74
700	41.75
800	45.06
900	47.83
1000	50.16

Fit the data to an empirical heat-capacity equation of the form

$$C_p = a + b_1T + b_2T^2$$

by simple regression techniques.

APPENDIX A

THE INTERNATIONAL SYSTEM OF UNITS (SI)

As the International System of Units, or the so-called SI units, become accepted in the U.S., there will be a long transition period when both the U.S. customary system and SI will be in use simultaneously. The design engineer, in particular, will need to be able to think and work in both systems because of the wide variety of persons involved in design considerations. Accordingly, this text has used a mixture of the two systems.

The purpose of this Appendix is to provide a description of SI units along with rules for conversion and rules for usage in the written form. Conversion factors are given with one table presenting a full detailed list with extensive footnote explanations and another table giving conversion factors in a simplified form for units commonly encountered by chemical engineers.

The name SI is derived from Systbme International d'Unités and has evolved from an original basis of a given length (meter) and mass (kilogram) established by members of the Paris Academy of Science in the late eighteenth century. The original system was known as the metric system, but there are differences in the modern SI system and the old metric system based primarily on new names being added for derived terms.

The current International System of Units (SI) is a metric system of measurement which has been adopted internationally by the General Conference of Weights and Measures and is described in an International Standard

(ISO 1000)[†] and in numerous other **publications.**[‡] Usage differences among countries have been resolved by a series of international conferences resulting in a set of seven base units, two supplementary units, and derived units as given in the following:

SI BASE UNITS ON WHICH THE ENTIRE SYSTEM IS FOUNDED

Name, Symbol	Definition
meter,[§] m-(length)	The meter is the length equal to 1650763.73 wavelengths in vacuum of the radiation corresponding to the transition between the levels $2p_{10}$ and $5d_5$ of the krypton-86 atom.
kilogram, kg-(mass)	The kilogram is a unit of mass (not force). A prototype of the kilogram made of platinum-iridium is kept at the International Bureau of Weights and Measures, Skres, France. (The kilogram is the only base unit having a prefix and defined by an artifact.)
second, s-(time)	The second is the duration of 9 192631770 periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the cesium-133 atom.
ampere, A-(electric current)	The ampere is that constant current which, if maintained in two straight parallel conductors of infinite length and of negligible circular cross section, and placed 1 meter apart in a vacuum, would produce between these conductors a force equal to $2 \times 10^{-7}\ \mathrm{m \cdot kg/s^2}$ (newton) per meter of length.

(Continued)

[†] **International** Standard, “SI Units and Recommendations for the Use of Their Multiples and of Certain Other Units,” ISO **1000-1973(E),** American National Standards Institute, 1430 Broadway, New York City, NY 10018.

[‡] **The** basic English document for SI is the National Bureau of Standards Special Publication 330 which can be obtained from the Superintendent of Documents, U.S. Government Printing Office, Washington, D.C. 20402 as document SD Catalog No. **C13.10:330/3.** This is the authorized English translation of the official document of the international body. For guidance in U.S. usage, the most widely recognized document in use is the ASTM Standard for Metric Practice available from the American Society for Testing and Materials, 1916 Race St., Philadelphia, PA. 19103.

[§] **The** spelling of *metre* (and *litre*) is commonly accepted internationally and is recommended by the ASTM. However, the spelling as meter (and liter) is widely used in the United States and is the spelling used in this book.

SI BASE UNITS ON WHICH THE ENTIRE SYSTEM IS FOUNDED (Continued)

Name, Symbol	Definition
kelvin, K-(temperature)	The kelvin is the fraction 1/273.16 of the thermodynamic temperature of the triple point of water. The kelvin is a unit of thermodynamic temperature (T). The word (or symbol) "degree" is not used with kelvin. The Celsius (formerly centigrade) temperature is also used. Celsius temperature (symbol t) is defined by the equation $t = T - T_0$, where T_0 equals 273.15 K. A degree Celsius (°C) is thus equal to one kelvin. The term centigrade should not be used because of possible confusion with the French unit of angular measurement, the grade.
mole, mol-(amount of substance)	The mole is the amount of substance of a system which contains as many elementary entities as there are atoms in 0.012 kilograms of carbon 12. When the mole is used, the elementary entities must be specified and may be atoms, molecules, ions, electrons, other particles, or specified groups of such particles.
candela, cd-(luminous intensity)	The candela is the luminous intensity, in the perpendicular direction, of a surface of 1/600 000 square meter of a blackbody at the temperature of freezing platinum (2045 K) under a pressure of 101325 $\mathrm{m}^{-1} \cdot \mathrm{kg} \cdot \mathrm{s}^{-2}$.

SUPPLEMENTARY UNITS

Name, Symbol	Definition
radian, rad-(plane angle)	The radian is the plane angle between two radii of a circle which cuts off, on the circumference, an arc equal in length to the radius.

(Continued)

SUPPLEMENTARY UNITS *(Continued)*

Name, Symbol	Definition
steradian, sr-(solid angle)	The steradian is the solid angle which, having its vertex in the center of a sphere, cuts off an area of the surface of the sphere equal to that of a square with sides of length equal to the radius of the sphere.

DERIVED UNITS

Derived units are algebraic combinations of the seven base units or two supplementary units with some of the combinations being assigned special names and symbols. Examples are shown in Table 1.

For the chemical engineer, the seven base units and two supplementary units are no problem because they have been used regularly in technical work of a chemical nature. However, the SI units for some of the derived terms, such as for pressure, are not familiar or in common usage in the U.S. customary system

TABLE 1
Common derived units with special names and symbols acceptable in SI

Name	Symbol	Quantity	Expression in terms of SI base units	Expression in terms of other units
becquerel	**Bq**	radioactivity	s^{-1}	
coulomb	C	quantity of electricity or electric charge	$A \cdot s$	
farad	F	electric capacitance	$m^{-2} \cdot kg^{-1} \cdot s^4 \cdot A^2$	C/V
gray	**Gy**	absorbed radiation	$m^2 \cdot s^{-2}$	J/kg
henry	H	electric inductance	$m^2 \cdot kg \cdot s^{-2} \cdot A^{-2}$	Wb/A
hertz	Hz	frequency	s^{-1}	
joule	J	energy, work, or quantity of heat	$m^2 \cdot kg \cdot s^{-2}$	N·m
lumen	lm	luminous flux	$cd \cdot sr$	
lux	lx	illuminance	$m^{-2} \cdot cd \cdot sr$	lm/m^2
newton	N	force	$m \cdot kg \cdot s^{-2}$	$J \cdot m^{-1}$
ohm	Ω	electric resistance	$m^2 \cdot kg \cdot s^{-3} \cdot A^{-2}$	V/A
pascal	Pa	pressure or stress	$m^{-1} \cdot kg \cdot s^{-2}$	N/m^2
siemens	S	electric conductance	$m^{-2} \cdot kg^{-1} \cdot s^3 \cdot A^2$	A/V
tesla	T	magnetic flux density	$kg \cdot s^{-2} \cdot A^{-1}$	Wb/m^2
volt	V	electric potential, potential difference, or electromotive force	$m^2 \cdot kg \cdot s^{-3} \cdot A^{-1}$	W/A
watt	W	power or radiant flux	$m^2 \cdot kg \cdot s^{-3}$	J/s
weber	Wb	magnetic flux	$m^2 \cdot kg \cdot s^{-2} \cdot A^{-1}$	V·s

of units. The SI pressure unit is the pascal (Pa) (rhymes with *rascal*) which is a newton (N) per square meter, or $N \cdot m^{-2}$. Since a newton is an SI derived unit of force as mass (kg) times acceleration (m/s^2), the net expression of the pascal in terms of SI base units is

$$Pa = N \cdot m^{-2} = kg \cdot m \cdot s^{-2} \cdot m^{-2} = m^{-1} \cdot kg \cdot s^{-2}.$$

Chemical engineers have commonly used atmospheres as a unit for pressure. Although the unit of atmosphere (1 atm = 101.325 kPa) was internationally authorized as an SI derived unit, this authorization was granted for a limited time only and its use should be minimized.

Another common set of units used by chemical engineers is the calorie (or British thermal unit) for energy. The units of calorie [1 cal = 4.1868 J where J is the symbol for joule (rhymes with pool) which is a newton-meter with base units of $m^2 \cdot kg \cdot s^{-2}$] and British thermal unit (1 Btu = $1.055\,056 \times 10^3$ J) are not acceptable with SI units.

In the SI system, the kilogram is restricted to the unit of mass so that it is not acceptable to use a unit of force as kilogram-force which would be analogous to the U.S. customary unit of pound-force. The newton is the unit of force in the SI system and should be used in place of kilogram-force. Confusion can occur because the term weight is used to mean either force or *muss.* In common everyday use, the term weight normally means mass, but, in physics, weight usually means the force exerted by gravity. Because of the ambiguity involved in the dual use of the term *weight,* the term should be avoided in technical practice unless the conditions are such that the meaning is totally clear.

Table 1 lists common derived SI units with special names. The table also gives the approved SI symbol and the expression for the term in base units and in terms of other units. Table 2 gives examples of other derived units which are commonly used in chemical engineering including a description and SI units. Table 3 shows units which are not officially recognized as usable with SI but which are authorized for use to a certain extent, while Table 4 gives units which are not acceptable for use with SI.

ADVANTAGES AND GUIDELINES FOR THE SI SYSTEM

An advantage of the SI system is its total coherence in that all of the units are related by unity. Thus, as can be seen from Table 1, a force of one newton exerted over a length of one meter gives an energy of one joule, while one joule occurring over a time period of one second results in a power of one watt. Mass is always measured in kilograms and force in newtons when dealing with the SI system so that the confusion often found in the U.S. customary system of using both pounds force and pounds mass is eliminated.

A fundamental characteristic of the SI system is the fact that each defined quantity has only one unit. Thus, the fundamental SI unit of energy is the joule and the fundamental SI unit of power is the watt. While a joule is defined as a newton meter, it refers to a unit force moving through a unit distance. The

TABLE 2

Other derived units commonly used in chemical engineering with description in terms of acceptable SI units

Quantity	Description	Symbol	Expression in terms of **SI** base units
acceleration	meter per second squared	$\mathrm{m/s^2}$	$\mathrm{m \cdot s^{-2}}$
area	square meter	$\mathrm{m^2}$	$\mathrm{m^2}$
coefficient of heat transfer (U.S. symbol of h or U)	watt per square meter kelvin	$\mathrm{W/(m^2 \cdot K)}$ $\mathrm{J/(m^2 \cdot K \cdot s)}$	$\mathrm{kg \cdot s^{-3} \cdot K^{-1}}$
concentration (of amount of substance)	mole per cubic meter	$\mathrm{mol/m^3}$	$\mathrm{mol \cdot m^{-3}}$
current density	ampere per square meter	A/m’	$\mathrm{A \cdot m^{-2}}$
density (mass density) (U.S. symbol of ρ)	kilogram per cubic meter	$\mathrm{kg/m^3}$	$\mathrm{kg \cdot m^{-3}}$
electric charge density	coulomb per cubic meter	$\mathrm{C/m^3}$	$\mathrm{m^{-3} \cdot s \cdot A}$
electric field strength	volt per meter	V/m	$\mathrm{m \cdot kg \cdot s^{-3} \cdot A^{-1}}$
electric flux density	coulomb per square meter	$\mathrm{C/m^2}$	$\mathrm{m^{-2} \cdot s \cdot A}$
energy density	joule per cubic meter	$\mathrm{J/m^3}$	$\mathrm{m^{-1} \cdot kg \cdot s^{-2}}$
force	newton	N or J/m	$\mathrm{m \cdot kg \cdot s^{-2}}$
heat capacity or entropy	joule per kelvin	J/K	$\mathrm{m^2 \cdot kg \cdot s^{-2} \cdot K^{-1}}$
heat flow rate (U.S. symbol of Q or q)	watt	W or J/s	$\mathrm{m^2 \cdot kg \cdot s^{-3}}$
heat flux density or irradiance	watt per square meter	W/m’	$\mathrm{kg \cdot s^{-3}}$
luminance	**candella** per square meter	cd/m’	cd-m-’
magnetic field strength	ampere per meter	A/m	$\mathrm{A \cdot m^{-1}}$
modulus of elasticity or Young’s modulus	gigapascal	GPa	$\mathrm{10^{-9} \cdot m^{-1} \cdot kg \cdot s^{-1}}$
molar energy	joule per mole	J/mol	$\mathrm{m^{-2} \cdot kg \cdot s^{-2} \cdot mol^{-1}}$
molar entropy or molar heat capacity	joule per mole kelvin	J/(mol·K)	$\mathrm{m^2 \cdot kg \cdot s^{-2} \cdot K^{-1} \cdot mol^{-1}}$
moment of force or torque	newton meter	N·m	$\mathrm{m^2 \cdot kg \cdot s^{-2}}$
moment of inertia	kilogram meter squared	$\mathrm{kg \cdot m^2}$	$\mathrm{kg \cdot m^2}$
momentum	kilogram meter per second	kg·m/s	$\mathrm{kg \cdot m \cdot s^{-1}}$
permeability	henry per meter	H/m	$\mathrm{m \cdot kg \cdot s^{-2} \cdot A^{-2}}$
permittivity	farad per meter	F/m	$\mathrm{m^{-3} \cdot kg^{-1} \cdot s^4 \cdot A^2}$
power	kilowatt	kW	$\mathrm{10^{-3} \cdot m^2 \cdot kg \cdot s^{-3}}$
pressure (U.S. symbol of P or p)	kilopascal	kPa	$\mathrm{10^{-3} \cdot m^{-1} \cdot kg \cdot s^{-2}}$
specific energy	joule per kilogram	J/kg	$\mathrm{m^2 \cdot s^{-2}}$
specific heat capacity or specific entropy (U.S. symbol of c_p, c_v, or s)	joule per kilogram · kelvin	J/(kg·K)	$\mathrm{m^2 \cdot s^{-2} \cdot K^{-1}}$
specific volume	cubic meter per kilogram	$\mathrm{m^3/kg}$	$\mathrm{m^3 \cdot kg^{-1}}$
stress	megapascal	MPa	$\mathrm{10^{-6} \cdot m^{-1} \cdot kg \cdot s^{-2}}$
surface tension	newton per meter	N/m	$\mathrm{kg \cdot s^{-2}}$
thermal conductivity (U.S. symbol of k)	watt per meter kelvin	W/(m·K)	$\mathrm{m \cdot kg \cdot s^{-3} \cdot K^{-1}}$
torque	newton meter	N·m	$\mathrm{m^2 \cdot kg \cdot s^{-2}}$
velocity or speed	meter per second	m/s	$\mathrm{m \cdot s^{-1}}$
viscosity-absolute or dynamic (U.S. symbol of μ)	**pascal** second	Pa·s	$\mathrm{m^{-1} \cdot kg \cdot s^{-1}}$
viscosity-kinematic (U.S. symbol of ν)	square meter per second	$\mathrm{m^2/s}$	$\mathrm{m^2 \cdot s^{-1}}$
volume	cubic meter	$\mathrm{m^3}$	$\mathrm{m^3}$
wave number	1 per meter	1/m	$\mathrm{m^{-1}}$
work energy (U.S. symbol of Win foot pounds force)	Joule	J or N·m	$\mathrm{m^2 \cdot kg \cdot s^{-2}}$

TABLE 3

Non-SI units which are acceptable for use

The following common units have been authorized for use with SI to a certain extent and continue in use on an unofficially accepted basis.

Name	Symbol	Value in SI units
time-minute, hour, day, year	min, h, d, yr	60s, 3 600s, 86 400s, ≈ 365d
angle-degree, minute, second	°, ′, ″	$(\pi/180)$ rad, $(1/60)°$, $(1/60)′$
liter†	1 (or L)	1 dm^3
nautical mile	nautical mile	1 852 m
knot (1 nautical mile per hour)	knot	0.513 9 m/s
hectare	ha	10^4 m^2
ångström	A	0.1 *nm* = 10^{-10} *m*
are	a	10^2 m^2
atmosphere pressure	atm	101.325 kPa
bar pressure	bar	10^5 Pa
galileo or gal	Gal	10^{-2} m/s”
metric ton	t	10^3 kg

† The SI unit of volume is the cubic meter, and this unit or one of its regular multiples is preferred for all cases. However, the special name ***liter*** has been approved for the cubic decimeter, but the use of this unit is restricted to the measurements of liquids and gases. No prefix other than ***milli*** should be used with liter.

TABLE 4

Common units which are not acceptable with SI

Despite the fact that the following units have been used in the past, they are not acceptable with SI

Name	Symbol	Value in SI units
British thermal unit	Btu	1.055 056 X 10^3 J
calorie	Cd	4.186 8 J
dyne	dyn	10^{-5} N
erg	erg	10^{-7} J
fermi	Fm	10^{-15} m
gamma	γ	10^{-9} T
gauss	Gs, G	10^{-4} *T*
kilogram-force	kgf	9.806 65 N
lambda	λ	10^{-6} liter
maxwell	Mx	10^{-8} Wb
metric carat		200 mg
micron	μ	1 micrometer
oersted	Oe	$(1000/4\pi)A\cdot m^{-1}$
phot	ph	10’ lx
poise	P	0.1 Pa·s
stere	st	1 m^3
stilb	sb	1 cd/cm^2
stokes	St	1 cm^2/s
torr		101 325/760 Pa
X unit		1.002 X 10^{-4} nm (approximately)

expression “newton meter” is used in the SI system to refer to torque in which there is no indication of motion or movement. Thus, the SI system is very explicit that joule and newton meter are different units.

The SI system has a series of approved prefixes and symbols for decimal multiples as shown in Table 5.

The common usage of “psi” and “atmosphere” for units of pressure will be replaced by the pascal in the SI system. Because a pascal, as a force of one newton against an area of one square meter, is a very small unit, it is convenient to deal with kilopascals (kPa) rather than pascals in many cases.† The following conversion factors are useful for making the transition from the U.S. customary system for pressure designations:

To convert to kPa from	multiply by
psi	6.895
atmosphere	101.325
torr	0.133 3
bar	100.000

RULES FOR USE OF SI UNITS

1. **Periods.** A period is never used after a symbol of an SI unit unless it is used to designate the end of a sentence.

2. **Capitalization.** Capitals are not used to start units that are written out except at the beginning of a sentence. However, when the units are expressed as symbols, the first letter of the symbol is capitalized when the name of the unit was derived from the name of a person. For example, it is correct to write

5 pascals or 5 Pa
5 newtons or 5 N
5 meters or 5 m
300 kelvins or 300 K

But note that the following temperature forms are correct:

200 degrees Celsius or 300°C
100 degrees Fahrenheit or 100°F

In the SI system, it is very important to follow the precise, agreed-upon use of uppercase and lowercase letters. This importance is shown by the following examples taken from Tables 3 and 5 and base-unit definitions:

G for giga; g for gram
K for kelvin; k for kilo
M for mega; m for milli
N for newton; n for nano
T for tera; t for metric ton

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†To give an idea as to the approximate magnitude of a pressure of one pascal, it would be equivalent to the extra pressure exerted on the palm of an open hand when a person blows a sharp breath on the hand.

TABLE 5

SI unit prefixes

Multiplication factor	Prefix	Symbol	Pronunciation (USA) (1)	Meaning (in USA)	Meaning (in other countries)
1 000 000 000 000 000 000 = 10^{18}	exa (2)	E	ex'a (a as in about)	One quintillion times (3)	trillion
1 000 000 000 000 000 = 10^{15}	peta (2)	P	as in petal	One quadrillion times (3)	thousand billion
1 000 000 000 000 = 10^{12}	tera	T	as in terrace	One trillion times (3)	billion
1 000 000 000 = 10^{9}	giga	G	jig'a (a as in about)	One billion times (3)	milliard
1 000 000 = 10^{6}	mega	M	as in megaphone	One million times	
1000 = 10^{3}	kilo	k	as in kilowatt	One thousand times	
100 = 10^{2}	hecto	h (4)	heck'toe	One hundred times	
10 = 10	deka	da (4)	deck'a (a as in about)	Ten times	
0.1 = 10^{-1}	deci	d (4)	as in decimal	One tenth of	
0.01 = 10^{-2}	centi	c (4)	as in sentiment	One hundredth of	
0.001 = 10^{-3}	milli	m	as in military	One thousandth of	
0.000 001 = 10^{-6}	micro	μ (5)	as in microphone	One millionth of	
0.000 000 001 = 10^{-9}	nano	n	nan'oh (an as in ant)	One billionth of (3)	milliardth
0.000 000 000 001 = 10^{-12}	pico	p	peek'oh	One trillionth of (3)	billionth
0.000 000 000 000 001 = 10^{-15}	femto	f	fem'toe (fem as in feminine)	One quadrillionth of (3)	thousand billionth
0.000 000 000 000 000 001 = 10^{-18}	atto	a	as in anatomy	One quintillionth of (3)	trillionth

1. The first syllable of every prefix is accented to assure that the prefix will retain its identity. Therefore, the preferred pronunciation of kilometer places the accent on the first syllable, not the second.
2. Approved by the 15th General Conference of Weights and Measures (CGPM), May-June 1975.
3. These terms should be avoided in technical writing because the denominations above one million are different in most other countries, as indicated in the last column.
4. While hecto, deka, deci, and centi are SI prefixes, their use should generally be avoided except for the SI unit-multiples for area and volume and nontechnical use of centimeter, as for body and clothing measurement. The prefix hecto should be avoided also because the longhand symbol h may be confused with k.
5. Although SI rules prescribe vertical (roman) type, the sloping *(italics)* form is usually acceptable in the USA for the Greek letter μ because of the scarcity of the upright style.

3. **Plurals.** As indicated in some of the preceding examples, the plural is used in the normal grammatical sense when the units are written out as words, but plurals are never used with the unit symbols. For numerical values greater than 1, equal to 0, or less than − 1, the names of units are plural. All other values take the singular form for the unit names. For example, the following forms are correct:

200 kilograms	or	200 kg
1.05 meters	or	1.05 m
0 degrees Celsius	or	0°C
−2 degrees Celsius	or	− 2°C
3 kelvins	or	3K
0.9 meter	or	0.9 m
− 0.5 degree Celsius	or	−0.5°C
1 kelvin	or	1 K
- 1 degree Celsius	or	1°C

An "s" is added to form the plurals of unit names as illustrated in the preceding except that hertz, lux, and siemens remain unchanged and henry becomes henries.

4. **Groupings of numbers and decimal points.** The common U.S. practice of using commas to separate multiples of 1000 is not followed with SI which uses a space instead of a comma to separate the multiples of 1000. For decimals, the space is filled on both sides of the decimal point. The decimal point is placed on the line as a regular period for U.S. usage rather than at mid-line height or use of a comma as is frequent European practice. When writing numbers with values less than one, a zero should be placed ahead of the decimal.

Numbers with many digits should be set off in groups of three digits away from the decimal point on both the left and the right. For example, the following forms are correct:

57 321684.52169
0.431684 2

If there are only four digits to the left or right of the decimal point, the use of the space is optional unless there is a column of figures which is aligned on the decimal point with one or more numbers having more than four digits to the left or right of the decimal point. Thus, the following forms are correct:

3200	or	3 200
0.6854	or	0.685 4

$$\begin{array}{r} 13.6 \\ +\ 15\ 057 \\ +\ 3200 \\ \hline 18\ 270.6 \end{array}$$

5. Spacing, hyphens, and italics. When a unit symbol is given after a number, a space is always left between the number and the symbol with the exception of cases where the symbol appears in the superscript position, such as degree, minute, and second of plane angles. The symbol for degree Celsius may be written either with or without a space before the degree symbol. For example,

68 kHz
60 mm
10^6 N
plane angle of 20° 45’ 26”
20°C or 20 °C (20°C is preferred and 20” C is *not* acceptable)

For both symbols and names of units having prefixes, no space is left between letters making up the symbol or name. For example,

kA, kiloampere; mg, milligram

The symbols when printed are always given as roman (vertical) type. Sloping letters (or *italics)* are reserved for quantity symbols such as *m* for mass, 1 for length, or general algebraic quantities such as a, *b*, or c. When the algebraic quantity is used, there is no space used between the algebraic quantity and the numerical coefficient. For example,

5 m means a distance of 5 meters,
but $5m$ means 5 times the algebraic quantity m

When a quantity is used in an adjectival sense, a hyphen should be used between the number and the symbol except for symbols appearing in the superscript position. For example,

He bought a 35-mm film; but,
the width of the film is 35 mm.
He bought a 5-kg ham; but,
the mass of the ham is 5 kg.
However, it is correct to write—
He bought a 100°C thermometer which covers a temperature range of 100°C.

A space should be left on each side of signs for multiplication, division, addition, and subtraction except within a compound symbol. The product dot (as in N . m) is used for the derived unit symbol often with no space on either side. The product dot should not be used as a multiplier symbol for calculations. For example,

Write 6 m x 8 m (not 6 mx8 m or 6 m · 8 m)
kg/m^3 or $\text{kg} \cdot \text{m}^{-3}$
$\text{m}^2 \cdot \text{kg} \cdot \text{s}^{-2}$

6. **Prefixes.** In general, it is desirable to keep numerical values between 0.1 and 1000 by the use of appropriate prefixes shown prior to the unit symbol. Prefixes and symbols along with pronunications and meanings as acceptable in

the SI system are given in Table 5. Some typical examples are

$$5\,527 \text{ Pa} = 5.527 \times 10^{-3} \text{ Pa} = 5.527 \text{ kPa}$$
$$0.051 \text{ m} = 51 \times 10^{-3} \text{ m} = 51 \text{ mm}$$
$$0.235 \times 10^{-6} \text{ s} = 0.235\ \mu\text{s} = 235 \times 10^{-3}\ \mu\text{s} = 235 \text{ ns}$$

Two or more SI prefixes should not be used simultaneously for the designation of a unit. For example,

write 1 pF instead of 1 $\mu\mu$F

For cases that fall outside the range covered by single prefixes, the situation should be handled by expressing the value with powers of ten as applied to the base unit.

With reference to the spelling with prefixes, there are three cases where the final vowel in a prefix is omitted. These are megohm, kilohm, and hectare. In all other cases, both vowels are retained and both are pronounced. No space or hyphen should be used.

7. Combination of units. It is desirable to avoid the use of prefixes in the denominator of compound units with the one exception of the base unit kg. For example,

use kN/m instead of N/mm
use kg/s instead of g/ms

The single exception is to use J/kg instead of mJ/g.

Use a solidus (/) to indicate a division factor. Avoid the use of a double solidus. For example,

write $\text{J}/(\text{s} \cdot \text{m})^2$ or $\text{J} \cdot \text{s}^{-2} \cdot \text{m}^{-2}$ instead of $\text{J}/\text{s}^2/\text{m}^2$

When the denominator of a unit expression is a product, it should normally be shown in parentheses. For example,

$$\text{W}/(\text{m}^2 \cdot \text{K})$$

If an expression is given for units raised to a power, such as square millimeters, the power number refers to the entire unit and not just to the last symbol. For example,

mm^2 means (mm)’ instead of milli(square meters) or m(m^2)

Symbols and unit names should not be used together in the same expression. For example,

write joules per kilogram or J/kg instead of
joules/kilogram or joules/kg or joules $\cdot$ kg^{-1}

8. Guidelines for calculations. It is generally desirable to carry out calculations in base units and then convert the final answers to appropriate-size numbers by use of correct prefixes.

9. Confusion of meaning of billion in U.S.A. and other countries. In the United States, billion means a thousand million (prefix *giga*), but, in most other countries, it means a million million (prefix *tera*.) Because of possible confusion as to the meaning, the term billion should be avoided in technical writing. As shown in Table 5, the same possible confusion exists with *quintillion* (prefix *exa* in U.S.A.), *quadrillion* (prefix *peta* in U.S.A.) and *trillion* (prefix *tera* in U.S.A).

10. Round-offs in conversions. In making a conversion of a number to new units, the number of significant digits should not be increased or decreased. It is, therefore, necessary to use sufficient precision in the conversion factor to preserve the precision of the quantity converted.

11. **Conversions between SI and U.S. customary units.** SI and U.S. customary units can be presented with the U.S. customary units first followed by SI units in parentheses, as 2.45 in. (62.2 mm) or as preferred SI units first with the U.S. customary units in parentheses, such as 170 kPa (24.7 psi).

Table 6 presents a detailed list of conversion factors that can be used to convert between U.S.-British units and SI units, while Table 7 gives a simplified and abbreviated list of equivalences for converting unacceptable units commonly used by chemical engineers into acceptable SI units.

TABLE 6
Conversion factors for converting from U.S. customary units to SI units—alphabetical listing in detail?

To convert from	To	Multiply by
abampere	ampere (A)	1.000 000*E+01
abcoulomb	coulomb (C)	1.000 000*E+01
abfarad	farad (F)	1.000 000*E+09
abhenry	henry (H)	1 .000 000*E-09
abmho	siemens (S)	1.000 000*E+09
abohm	ohm (Ω)	1.000 000*E-09
abvolt	volt (V)	1.000 000*E-08
acre foot (U.S. survey)‡	meter^3 (m^3)	1.233 489 E+03
acre (U.S. survey)‡	meter* (m^2)	4.046 873 E+03
ampere hour	coulomb (C)	3.600 000*E+03
are	meter^2 (m^2)	1.000 000*E+02
ångstrom	meter (m)	1.000 000*E-10
astronomical unit	meter(m)	1.495 979 E+11
atmosphere (standard)	pascal (Pa)	1.013 250*E+05
atmosphere (technical = 1kgf/cm^2)	pascal (Pa)	9.806 650*E+04
bar	pascal (Pa)	1.000 000*E+05
barn	meter' (m^2)	1.000 000*E-28
barrel (for petroleum, 42 gal)	meter^3 (m^3)	1.589 873 E-01
board foot	meter' (m^3)	2.359 737 E-03
British thermal unit (International Table) §	joule (J)	1.055 056 E+03
British thermal unit (mean) §	joule (J)	1.055 87 E+03
British thermal unit (thermochemical) §	joule (J)	1.054 350 E+03

† See end of Table ‡ See end of Table § See end of Table

TABLE 6

Conversion factors for converting from U.S. customary units to SI units—alphabetical listing in detail? (Continued)

To convert from	To	Multiply by
British thermal unit (39” F)	joule (J)	1.059 6 7 E+03
British thermal unit (59°F)	joule (J)	1.054 8 0 E+03
British thermal unit (60°F)	joule (J)	1.054 6 8 E+03
Btu (International Table)·ft/h·ft^2·°F (*k*, thermal conductivity)	watt per meter kelvin (W/m·K)	1.730 735 E+OO
Btu (thermochemical)·ft/h·ft^2·°F (*k*, thermal conductivity	watt per meter kelvin (W/m·K)	1.729 577 E+OO
Btu (International Table). in./h·ft^2·°F (*k*, thermal conductivity)	watt per meter kelvin (W/m·K)	1.442 279 E–01
Btu (thermochemical)·in./h·ft^2·°F (*k*, thermal conductivity)	watt per meter kelvin (W/m·K)	1.441 314 E–01
Btu (International Table)·in./s·ft^2·°F (*k*, thermal conductivity)	watt per meter kelvin (W/m·K)	5.192 204 E+02
Btu (thermochemical)·in./s·ft^2·°F (*k*, thermal conductivity)	watt per meter kelvin (W/m·K)	5.188 732 E+02
Btu (International Table)/h	watt (W)	2.930 711 E–01
Btu (thermochemical)/h	watt (W)	2.928 75 1 E–01
Btu (thermochemical)/min	watt(W)	1.757 250 E+01
Btu (thermochemical)/s	watt (W)	1.054 350 E+03
Btu (International Table)/ft^2	joule per meter* (J/m^2)	1.135 653 E+04
Btu (thermochemical)/ft^2	joule per meter’ (J/m’)	1.134 893 E+04
Btu (International Table)/ft^2·h	watt per meter’ (W/m^2)	3.154 591 E+OO
Btu (thermochemical)/ft^2·h	watt per meter’ (W/m^2)	3.152 481 E+OO
Btu (thermochemical/ft^2·min	watt per meter’ (W/m^2)	1.891 489 E+02
Btu (thermochemical)/ft^2·s	watt per meter’ (W/m’)	1.134 893 E+04
Btu (thermochemical)/in.^{2}·s	watt per meter* (W/m*)	1.634 246 E+06
Btu (International Table)/h·ft^2·°F (C, thermal conductance)	watt per meter’ kelvin (W/m’ ·K)	5.678 263 E+OO
Btu (thermochemical)/h·ft^2·°F (*C*, thermal conductance)	watt per meter’ kelvin (W/m’ ·K)	5.674 466 E+OO
Btu (International Table)/s·ft^2·°F	watt per meters kelvin (W/m* ·K)	2.044 175 E+04
Btu (thermochemical)/s·ft^2·°F	watt per metre’ kelvin (W/m’ ·K)	2.042 808 E+04
Btu (International Table)/lb	joule per kilogram (J/kg)	2.326 000*E+03
Btu (thermochemical)/lb	joule per kilogram (J/kg)	2.324 444 E+03
Btu (International Table)/lb·°F (c, heat capacity)	joule per kilogram kelvin (J/kg·K)	4.186 800*E+03
Btu (thermochemical)/lb·°F (c, heat capacity)	joule per kilogram kelvin (J/kg·K)	4.184 000 E+03
bushel (U.S.)	meter” (m^3)	3.523 907 E--02
caliber (inch)	meter(m)	2.540 OOO*E-02
calorie (International Table)	joule (J)	4.186 800*E+00
calorie (mean)	joule (J)	4.190 02 E+OO
calorie (thermochemical)	joule (J)	4.184 000*E+00
calorie (15°C)	joule (J)	4.185 80 E+OO
calorie (20°C)	joule (J)	4.18190 E+OO
calorie (kilogram, International Table) §	joule (J)	4.186 800*E+03

† See end of Table   § See end of Table   (*Continued*)

TABLE6

Conversion factors for converting from U.S. customary units to SI units—alphabetical listing in detail? (*Continued*)

To convert from	To	Multiply by
calorie (kilogram, mean) §	joule (J)	4.190 02 E+03
calorie (kilogram, thermochemical) §	joule (J)	4.184 000*E+03
cal (thermochemical)/cm^2	joule per meter^2 (J/m’)	4.184 000*E+04
cal (International Table)/g	joule per kilogram (J/kg)	4.186 800*E+03
cal (thermochemical)/g	joule per kilogram (J/kg)	4.184 000*E+03
cal (International Table)/g·°C	joule per kilogram kelvin (J/kg·K)	4.186 800*E+03
cal (thermochemical)/g·°C	joule per kilogram kelvin (J/kg·K)	4.184 000*E+03
cal (thermochemical)/min	watt (W)	6.973 333 E–02
cal (thermochemical)/s	watt (W)	4.184 000*E+00
cal (thermochemical)/cm^2·min	watt per meter” (W/m^2)	6.973 333 E+02
cal (thermochemical)/cm^2·s	watt per meter’ (W/m^2)	4.184 000*E+04
cal (thermochemical)/cm·s·°C	watt per meter kelvin (W/m·K)	4.184 000*E+02
carat (metric)	kilogram (kg)	2.000 OOO*E-04
centimeter of mercury (0°C)	pascal (Pa)	1.333 22 E+03
centimeter of water (4°C)	pascal (Pa)	9.806 38 E+01
centipoise	pascal second (Pa·s)	1 .000 000*E-03
centistokes	meter’ per second (m^2/s)	1.000 000*E–06
circular mil	metre’ (m^2)	5.067 075 E–10
clo	kelvin meter’ per watt (K·m^2/W)	2.003 712 E–01
cup	meter^3 (m^3)	2.365 882 E–04
curie	becquerel (Bq)	3.700 000*E+ 10
day (mean solar)	second (s)	8.640 000 E+04
day (sidereal)	second (s)	8.616 409 E+04
degree (angle)	radian (rad)	1.745 329 E–02
degree Celsius	kelvin (K)	$t_K = t_{°C} + 273.15$
degree Fahrenheit	degree Celsius	$t_{°C} = (t_{°F} - 32)/1.8$
degree Fahrenheit	kelvin (K)	$t_K = (t_{°F} + 459.67)/1.8$
degree Rankine	kelvin (K)	$t_K = t_{°R}/1.8$
°F·h·ft^2/Btu (international Table) (*R*, thermal resistance)	kelvin meter’ per watt (K·m^2/W)	1.761 102 E–01
°F·h·ft^2/Btu (thermochemical) (*R*, thermal resistance)	kelvin meter’ per watt (K·m^2/W)	1.762 280 E–01
denier	kilogram per meter (kg/m)	1.111 111 E–07
dyne	newton (N)	1.000 000*E–05
dyne·cm	newton meter (N·m)	1.000 OOO*E-07
dyne/cm’	pascal (Pa)	1.000 OOO*E-01
electronvolt	joule (J)	1.602 19 E-19
EMU of capacitance	farad (F)	1.000 000*E+09
EMU of current	ampere (A)	1.000 000*E+01
EMU of electric potential	volt (V)	1.000 000*E–08
EMU of inductance	henry (H)	1 .000 000*E–09
EMU of resistance	ohm (Ω)	1.000 OOO*E-09
ESU of capacitance	farad (F)	1.112 650 E–12
ESU of current	ampere (A)	3.335 6 E–10
ESU of electric potential	volt(V)	2.997 9 E+02
ESU of inductance	henry (H)	8.987 554 E+11
ESU of resistance	ohm (Ω)	8.987 554 E+11

† See end of Table § See end of Table

TABLE 6
Conversion factors for converting from U.S. customary units to SI units—alphabetical listing in detail? (Continued)

To convert from	To	Multiply by
erg	joule (J)	1.000 000*E−07
erg/cm’ •s	watt per meter’ (W/m^2)	1.000 000*E-03
erg/s	watt (W)	1.000 000*E-07
faraday (based on carbon-12)	coulomb (C)	9.648 70 E+04
faraday (chemical)	coulomb (C)	9.649 57 E+04
faraday (physical)	coulomb (C)	9.652 19 E+04
fathom	meter (m)	1.828 8 E+00
fermi (femtometer)	meter (m)	1.000 000*E-15
fluid ounce (U.S.)	meters (m^3)	2.957 353 E−05
foot	meter (m)	3.048 000*E-01
foot (U.S. survey)‡	meter(m)	3.048 006 E−01
foot of water (39.2°F)	pascal (Pa)	2.988 98 E+03
ft^2	meters (m^2)	9.290 304*E−02
ft^2 /h (thermal diffusivity)	meters per second (m^2 /s)	2.580 640*E−05
ft^2/s	meter’ per second (m^2/s)	9.290 304*E−02
ft^3 (volume; section modulus)	meter^3 (m^3)	2.831 685 E−02
ft^3/min	meter^3 per second (m^3/s)	4.719 474 E−04
ft^3/s	meter’ per second (m^3/s)	2.831 685 E−02
ft’ (moment of section)	meter^4 (m^4)	8.630 975 E−03
ft/h	meter per second (m/s)	8.466 667 E−05
ft/min	meter per second (m/s)	5.080 000*E-03
ft/s	meter per second (m/s)	3.048 000*E-01
ft/s^2	meter per seconds (m/s”)	3.048 000*E-01
footcandle	lux (lx)	1.076 391 E+01
footlambert	candela per meters (cd/m’)	3.426 259 E+00
ft•lbf	joule (J)	1.355 818 E+00
ft•lbf/h	watt (W)	3.766 161 E−04
ft•lbf/min	watt (W)	2.259 697 E−02
ft•lbf/s	watt (W)	1.355 818 E+00
ft•poundal	joule (J)	4.214 011 E−02
free fall, standard (*g*)	meter per seconds (m/s^2)	9.806 650*E+00
gal	meter per second’ (m/s^2)	1.000 000*E−02
gallon (Canadian liquid)	meters (m^3)	4.546 090 E−03
gallon (U.K. liquid)	meter^3 (m^3)	4.546 092 E−03
gallon (U.S. dry)	meters (m^3)	4.404 884 E−03
gallon (U.S. liquid)	meter’ (m^3)	3.785 412 E−03
gal (U.S. liquid)/day	meters per second (m^3 /s)	4.381 264 E−08
gal (U.S. liquid)/min	meter^3 per second (m^3/s)	6.309 020 E−05
gal (U.S. liquid)/hp•h (SFC, specific fuel consumption)	kilogram per joule (kg/J)	1.410 089 E−09
gamma	tesla (T)	1.000 000*E−09
gauss	tesla (T)	1.000 000*E-04
gilbert	ampere	7.957 747 E−01
gill (U.K.)	meter^3 (m^3)	1.420 654 E−04
gill (U.S.)	meter^3 (m^3)	1.182 941 E−04
grad	degree (angular)	9.000 000*E-01
grad	radian (rad)	1.570 796 E−02

† See end of Table ‡ See end of Table

(Continued)

TABLE 6

Conversion factors for converting from U.S. customary units to SI units—alphabetical listing in detail? (Continued)

To convert from	To	Multiply by
grain (1/7000 lb avoirdupois)	kilogram (kg)	6.479 891*E−05
gram (lb avoirdupois/7000)/gal (U.S. liquid)	kilogram per meter’ (kg/m’)	1.711 806 E−02
gram	kilogram (kg)	1.000 OOO*E-03
g/cm^3	kilogram per $meter^3$ (kg/m^3)	1.OOO000*E+03
gram-force/cm*	pascal (Pa)	9.806 650*E+01
hectare	meter’ (m’)	1.000 000*E+04
horsepower (550 ft·lbf/s)	watt (W)	7.456 999 E+02
horsepower (boiler)	watt (W)	9.809 50 E+03
horsepower (electric)	watt (W)	7.460 000*E+02
horsepower (metric)	watt (W)	7.354 99 E+02
horsepower (water)	watt (W)	7.460 43 E+02
horsepower (U.K.)	watt(W)	7.457 0 E+02
hour (mean solar)	second (s)	3.600 000 E+03
hour (sidereal)	second (s)	3.590 170 E+03
hundredweight (long)	kilogram (kg)	5.080 235 E+01
hundredweight (short)	kilogram (kg)	4.535 924 E+01
inch	meter (m)	2.540 000*E−02
inch of mercury (32°F)	pascal (Pa)	3.386 38 E+03
inch of mercury (60°F)	pascal (Pa)	3.376 85 E+03
inch of water (39.2°F)	pascal (Pa)	2.490 82 E+02
inch of water (60” F)	pascal (Pa)	2.488 4 E+02
in^2	meter* (m^2)	6.45 1 600*E−04
in! (volume; section modulus)	meter’ (m^3)	1.638 706 E-OS
in? /min	meter” per second (m^3/s)	2.731 177 E−07
in? (moment of section)	meter’ (m^4)	4.162 314 E−07
in./s	meter per second (m/s)	2.540 OOO*E-02
$in./s^2$	meter per second’ (m/s’)	2.540 OOO*E-02
kayser	1 per meter (l/m)	1.000 000*E+02
kelvin	degree Celsius	$t_{°C} = t_K - 273.15$
kilocalorie (International Table)	joule (J)	4.186 800*E+03
kilocalorie (mean)	joule (J)	4.190 02 E+03
kilocalorie (thermochemical)	joule (J)	4.184 000*E+03
kilocalorie (thermochemical)/min	watt (W)	6.973 333 E+01
kilocalorie (thermochemical)/s	watt (W)	4.184 000*E+03
kilogram-force (kgf)	newton (N)	9.806 650*E+00
kgf·m	newton meter (N·m)	9.806 650*E+00
$kgf \cdot s^2/m$ (mass)	kilogram (kg)	9.806 650*E+00
kgf/cm^2	pascal (Pa)	9.806 650*E+04
kgf/m^2	pascal (Pa)	9.806 650*E+00
kgf/mm^2	pascal (Pa)	9.806 650*E+06
km/h	meter per second (m/s)	2.777 778 E−01
kilopond	newton (N)	9.806 650*E+00
kW·h	joule (J)	3.600 000*E+06
kip (1000 lbf)	newton (N)	4.448 222 E+03
kip/in^2 (ksi)	pascal (Pa)	6.894 757 E+06
knot (international)	meter per second (m/s)	5.144 444 E−01

†See end of Table

TABLE6
Conversion factors for converting from U.S. customary units to SI units—alphabetical listing in detail† (Continued)

To convert from	To	Multiply by
lambert	candela per meters (cd/m')	1/π *E+04
lambert	candela per meters (cd/m*)	3.183 099 E+03
langley	joule per meter' (J/m")	4.184 000*E+04
league	meter (m)	[see footnote ‡]
light year	meter (m)	9.460 55 E+ 15
liter	meter' (m^3)	1.000 OOO*E-03
maxwell	weber (Wb)	1.000 000*E–08
mho	siemens (S)	1.000 000*E+00
microinch	meter (m)	2.540 OOO*E-08
micron	meter (m)	1 .000 000*E -06
mil	meter (m)	2.540 OOO*E-05
mile (international)	meter (m)	1.609 344*E+03
mile (statute)	meter (m)	1.609 3 E+03
mile (U.S. survey)‡	meter (m)	1.609 347 E+03
mile (international nautical)	meter (m)	1.852 000*E+03
mile (U.K. nautical)	meter (m)	1.853 184*E+03
mile (U.S. nautical)	meter (m)	1.852 000*E+03
mi^2 (international)	meter' (m^2)	2.589 988 E+06
mi^2 (U.S. survey)‡	meter' (m^2)	2.589 998 E+06
mi/h (international)	meter per second (m/s)	4.410 400*E–01
mi/h (international)	kilometer per hour (km/h)	1.609 344*E+00
mi/min (international)	meter per second (m/s)	2.682 240*E+01
mi/s (international)	meter per second (m/s)	1.609 344*E+03
millibar	pascal (Pa)	1.000 000*E+02
millimeter of mercury (0°C)	pascal (Pa)	1.333 2 2 E+02
minute (angle)	radian (rad)	2.908 882 E–04
minute (mean solar)	second (s)	6.000 000 E+01
minute (sidereal)	second (s)	5.983 617 E+01
month (mean calendar)	second (s)	2.628 000 E+06
oersted	ampere per meter (A/m)	1.951 141 E+01
ohm centimeter	ohm meter (Ω·m)	1.000 OOO*E-02
ohm circular-mil per foot	ohm millimeter' per meter (Ω·mm^2/m)	1.662 426 E–03
ounce (avoirdupois)	kilogram (kg)	2.834 952 E–02
ounce (troy or apothecary)	kilogram (kg)	3.110 348 E -02
ounce (U.K. fluid)	meter^3 (m^3)	2.841 307 E–05
ounce (U.S. fluid)	meters (m^3)	2.951 353 E -05
ounce-force	newton (N)	2.780 139 E–01
ozf·in.	newton meter (N·m)	7.061 552 E–03
oz (avoirdupois)/gal (U.K. liquid)	kilogram per meter^3 (kg/m^3)	6.236 021 E+OO
oz (avoirdupois)/gal (U.S. liquid)	kilogram per meters (kg/m^3)	7.489 152 E+OO
oz (avoirdupois)/in.^{3}	kilogram per meter' (kg/m^3)	1.729 994 E+03
oz (avoirdupois)/ft^2	kilogram per meter" (kg/m')	3.051 517 E–01
oz (avoirdupois)/yd^2	kilogram per meter' (kg/m')	3.390 575 E–02
parsec	meter (m)	3.085 678 E+16
peck (U.S.)	meter^3 (m^3)	8.809 768 E–03
pennyweight	kilogram (kg)	1.555 174 E–03

† See end of Table ‡ See end of Table

(Continued)

TABLE 6
Conversion factors for converting from U.S. customary units to SI units—alphabetical listing in detail? (Continued)

To convert from	TO	Multiply by
perm (0°C)	kilogram per **pascal** second meter’ **(kg/Pa•s•m^2)**	5.721 35 E−11
perm (23°C)	kilogram per **pascal** second meter’ **(kg/Pa•s•m^2)**	5.745 25 E−11
perm•in. (0°C)	kilogram per **pascal** second meter **(kg/Pa•s•m)**	1.453 22 E−12
perm•in. (23°C)	kilogram per **pascal** second meter **(kg/Pa•s•m)**	1.459 29 E−12
phot	lumen per **meter^2 (lm/m^2)**	1.000 000*E+04
pica (printer’s)	meter **(m)**	4.217 518 E−03
pint (U.S. dry)	meter” **(m^3)**	5.506 105 E−04
pint (U.S. liquid)	meter’ **(m^3)**	4.731 765 E−04
point (printer’s)	meter (m)	3.514 598*E−04
poise (absolute viscosity)	**pascal** second **(Pa•s)**	1.000 OOO*E-01
pound (lb avoirdupois)	kilogram (kg)	4.535 924 E−01
pound (troy or apothecary)	kilogram (kg)	3.732 417 E−01
lb•ft^2 (moment of inertia)	kilogram meter* **(kg•m^2)**	4.214 011 E−02
lb•in^2 (moment of inertia)	kilogram meter’ **(kg• m^2)**	2.926 397 E−04
lb/ft.h	**pascal** second **(Pa•s)**	4.133 789 E−04
lb/ft•s	**pascal** second **(Pa•s)**	1.488 164 E+OO
lb/ft^2	kilogram per meter’ (kg/m’)	4.882 428 E+OO
lb/ft^3	kilogram per meters **(kg/m^3)**	1.601 846 E+01
lb/gal (U.K. liquid)	kilogram per **meter^3 (kg/m^3)**	9.977 633 E+01
lb/gal (U.S. liquid)	kilogram per mete? (kg/m”)	1.198 264 E+02
lb/h	kilogram per second (kg/s)	1.259 979 E−04
lb/hp•h (SFC, specific fuel consumption)	kilogram per joule (kg/J)	1.689 659 E−07
lb/in^3	kilogram per meter” **(kg/m^3)**	2.767 990 E+04
lb/min	kilogram per second (kg/s)	7.559 873 E−03
lb/s	kilogram per second (kg/s)	4.535 924 E−01
lb/yd^3	kilogram per **meter^3 (kg/m^3)**	5.932 764 E−01
poundal	newton **(N)**	1.382 550 E−01
poundal/ft^2	**pascal** (Pa)	1.488 164 E+OO
poundal•s/ft^2	**pascal** second **(Pa•s)**	1.488 164 E+OO
pound-force **(lbf)**	newton (N)	4.448 222 E+OO
lbf•ft	newton meter **(N•m)**	1.355 818 E+OO
lbf•ft/in.	newton meter per meter **(N•m/m)**	5.337 866 E+01
lbf•in.	newton meter **(N•m)**	1.129 848 E−01
lbf•in./in.	newton meter per meter **(N•m/m)**	4.448 222 E+00
lbf•s/ft^2	**pascal** second **(Pa•s)**	4.788 026 E+01
lbf/ft	newton per meter (N/m)	1.459 390. E+01
lbf/ft^2	**pascal** (Pa)	4.788 026 E+01
lbf/in.	newton per meter (N/m)	1.751 268 E+02
lbf/in^2 (psi)	**pascal** (Pa)	6.894 757 E+03
lbf/lb (thrust/weight [mass] ratio)	newton per kilogram (N/kg)	9.806 650 E+00
quart (U.S. dry)	**meter^3 (m^3)**	1.101 221 E−03
quart (U.S. liquid)	**meter^3 (m^3)**	9.463 529 E−04

†**See** end of Table

TABLE 6
Conversion factors for converting from U.S. customary units to SI units—alphabetical listing in detail[†] (Continued)

To convert from	To	Multiply by
rad (radiation dose absorbed)	gray (Gy)	1.000 OOO*E-02
rhe	1 per pascal second (1/Pa·s)	1.000 000*E+01
rod	meter (m)	[see footnote $]
roentgen	coulomb per kilogram (C/kg)	2.58 E-04
second (angle)	radian (rad)	4.848 137 E-06
second (sidereal)	second (s)	9.972 696 E-01
section	meter' (m^2)	[see footnote ‡]
shake	second (s)	1.000 000*E-08
slug	kilogram (kg)	1.459 390 E+01
slug/ft·s	pascal second (Pa·s)	4.788 026 E+01
slug/ft^3	kilogram per meter^3 (kg/m^3)	5.153 788 E+02
statampere	ampere (A)	3.335 640 E-10
statcoulomb	coulomb (C)	3.335 640 E-10
statfarad	farad (F)	1.112 650 E-12
stathenry	henry (H)	8.987 554 E+11
statmho	siemens (S)	1.112 650 E-12
statohm	ohm (Ω)	8.987 554 E+11
statvolt	volt(v)	2.997 925 E+02
stere	meter' (m^3)	1.000 000*E+00
stilb	candela per meter* (cd/m')	1.000 000*E+04
stokes (kinematic viscosity)	meters per second (m^2 /s)	1.000 OOO*E-04
tablespoon	meter^3 (m^3)	1.478 676 E-OS
teaspoon	meters (m^3)	4.928 922 E--O6
tex	kilogram per meter (kg/m)	1.000 000*E-06
therm	joule (J)	1.055 056 E+08
ton (assay)	kilogram (kg)	2.916 667 E-02
ton (long, 2240 lb)	kilogram (kg)	1.016 047 E+03
ton (metric)	kilogram (kg)	1.000 000*E+03
ton (nuclear equivalent of TNT)	joule (J)	4.184 E+09
ton (refrigeration)	watt (W)	3.516 800 E+03
ton (register)	meters (m^3)	2.831 685 E+OO
ton (short, 2000 lb)	kilogram (kg)	9.071 847 E+02
ton (long)/yd^3	kilogram per meter^3 (kg/m^3)	1.328 939 E+03
ton (short)/h	kilogram per second (kg/s)	2.519 958 E-01
ton-force (2000 lbf)	newton (N)	8.896 444 E+03
tonne	kilogram (kg)	1.000 000*E+03
torr (mm Hg, 0°C)	pascal (Pa)	1.333 2 2 E+02
township	meter* (m^2)	[see footnote ‡]
unit pole	weber (Wb)	1.256 637 E-07
W·h	joule (J)	3.600 000*E+03
W·s	joule (J)	1.000 000*E+00
W/Cm^2	watt per meter^2 (W/m')	1.000 000*E+04
W/in?	watt per meter^2 (W/m^2)	1.550 003 E+03
yard	meter (m)	9.144 OOO*E-01
yd^2	meter' (m^2)	8.361 274 E-01
yd^3	meter^3 (m^3)	7.645 549 E-01
yd^3 /min	meter^3 per second (m^3/s)	1.274 258 E-02
year (calendar)	second (s)	3.153 600 E+07
year (sidereal)	second (s)	3.155 815 E+07
year (tropical)	second (s)	3.155 693 E+07

See next page for footnotes. *(Continued)*

*Exact **equivalence.**

†Adapted from ASTM Standard for Metric Practice E 380-76. The conversion factors are listed in standard form for computer readout as a number greater than one or less than ten with six or less decimal points. The number is followed by the letter E (for exponent), a plus or minus symbol, and two digits which indicate the power of 10 by which the number must be multiplied. An asterisk (*) after the sixth decimal place indicates that the conversion factor is exact and that all subsequent digits are zero. All other conversion factors have been rounded to the figures given. Where less than six decimal places are shown, more precision is not warranted.

For example, 1.013 250*E + OS is exactly 1.013 250 x 10^5 or 101 325.0.

1.589 873 E − 01 has the last digit rounded off to 3 and is

1.589 873 x 10^{-1} or 0.158 987 3.

‡Since 1893, the U.S. basis of length measurement has been derived from metric standards. In 1959, a small refinement was made in the definition of the yard to resolve discrepancies both in this country and abroad which changed its length from 3600/3937 m to 0.9144 m exactly. This resulted in the new value being shorter by two parts in a million. At the same time, it was decided that any data in feet derived from and published as a result of geodetic surveys within the U.S. would remain with the old standard (1 ft = 1200/3937 m) until further decision. This foot is named the U.S. survey foot. As a result, all U.S. land measurements in U.S. customary units will relate to the meter by the old standard. All the conversion factors in this table for units referenced to this footnote are based on the U.S. survey foot rather than on the international foot.

Conversion factors for the land measures given below may be determined from the following relationships:

1 league = 3 miles (exactly)

1 rod = $16\frac{1}{2}$ feet (exactly)

1 section = 1 square mile (exactly)

1 township = 36 square miles (exactly)

§By definition, one calorie (International Table) is exactly 4.186 8 absolute joules which converts to 1.055 056 X 10” joules for one Btu (International Table). Also, by definition, one calorie (thermochemical) is exactly 4.184 absolute joules which converts to 1.054350 × 10^3 joules for one Btu (thermochemical). A mean calorie is $\frac{1}{100}$th of the heat required to raise the temperature of one gram of water at one atmosphere pressure from 0°C to 100°C and equals 4.19002 absolute joules. In all cases, the relationship between calorie and British thermal unit is established by 1 cal/(g . °C) = 1 Btu/(lb . “F). A mean Btu, therefore is $\frac{1}{180}$th of the heat required to raise the temperature of one pound of water at one atmosphere pressure from 32°F to 212°F and equals 1.055 87 X 10^3 joules. When values are given as Btu or calories, the type of unit (International Table, thermochemical, mean, or temperature of determination) should be given. In all cases for this table, conversions involving joules are based on the absolute joule.

TABLE 7

Abbreviated list of equivalences for converting units commonly used by chemical engineers to acceptable SI units

Unacceptable unit	Acceptable SI unit with unit conversion factor
ångström	0.1 nm*
atmosphere (standard)	101.325 kPa
Btu†	1.055 056 kJ
Btu/(lb$_m$·°F) (heat capacity)	4.186 8 kJ/(kg·K)*
Btu/h	0.293 971 1 W
Btu/ft^2	11.356 53 kJ/m^2
Btu/(ft^2 · h·°F) (heat transfer coefficient)	5.678 263 J/(m^2·s·K)
Btu/(ft^2·h) (heat flux)	3.154 591 J/(m^2·s)
Btu/(ft·h·°F) (thermal conductivity)	1.730 735 J/(m·s·K)
calorie†	4.186 8 J*
cal/(g·°C) (heat capacity)	4.186 8 kJ/(kg·K)*
centipoise (absolute viscosity)	1.0 mPa·s*
centistoke (kinematic viscosity)	1.0 $\times$ 10^{-6} m^2/s*
t (°F)	(t + 459.67)/(1.8) K
t (°R)	t/(1.8) K*
dyne	10.0 μN*
erg	100 pJ*
foot ‡	0.3048 m*
ft^2	9.290 304 $\times$ 10^{-2} m^2 *
ft^3	2.831685 $\times$ 10^{-2} m^3
gallon (U.S. liquid)	3.785 412 $\times$ 10^{-3} m^3
horsepower (550 ft·lb$_f$/s)	745.699 9 W
inch	2.54 $\times$ 10^{-2} m*
in. Hg (60°F) (inches mercury pressure)	3.376 85 kPa
in. H_2O (60° F) (inches water pressure)	0.248 84 kPa
kg$_f$ (kilogram force)	9.806 65 N*
mile	1609.344 m*
mm Hg (0°C) (millimeters mercury pressure)	0.133 322 kPa
poise (absolute viscosity)	0.1 Pa·s*
lb$_f$ (pounds force)	4.448 222 N
lbm (pounds mass-avoirdupois)	0.453 592 4 kg
psi (pounds per square inch pressure)	6.894 757 kPa
stoke (kinematic viscosity)	1.0 $\times$ 10^{-4} m^2/s*
yard	0.9144 m*

* Exact equivalence.

† British thermal unit and calorie are reported as the International Table values as adopted in 1956 for all cases in this table. The exact conversion factor for Btu (International Table) to kJ is 1.055 055 852 62. The Btu (thermochemical) is 1.054 350 kJ and the calorie (thermochemical) is exactly 4.184 J. (See footnote § for Table 6).

‡ The foot is reported as the International Table value and holds for all cases of length in this table.

APPENDIX B

AUXILIARY, UTILITY, AND CHEMICAL COST DATA[†]

CONTENTS

AUXILIARY COST DATA **Page**

[†] Costs reported are Jan. 1, 1990 values except as noted.

INSTRUMENTATION

UTILITIES

CHEMICALS

TABLE 1

Hourly wage rates of craft labor in selected U.S. cities*

Employer's burden includes state and federal taxes, insurance, etc. (Hourly wage including fringe benefits + employer's burden, July 1989)

City	Bricklayers	Carpenters	Electricians	General construction laborers	Structural iron workers	Painters	Plasterers	Plumbers or pipefitters	Sheet-metal workers
Atlanta, Ga.	18.79 + 4.93	20.20 + 5.94	23.83 + 4.93	13.96 + 4.10	21.74 + 9.08	21.12 + 5.42	18.61 + 5.42	22.49 + 4.91	21.50 + 5.14
Boston, Mass.	27.14 + 7.11	26.33 + 7.74	30.30 + 6.27	20.23 + 5.94	27.17 + 11.35	28.38 + 7.29	25.98 + 7.74	31.17 + 6.80	32.38 + 7.74
Chicago, Ill.	25.52 + 6.69	25.65 + 7.54	27.45 + 5.68	20.59 + 6.05	28.41 + 11.87	22.00 + 5.65	23.95 + 7.11	26.83 + 5.85	26.42 + 6.31
Dallas, Tex.	21.11 + 5.53	24.06 + 7.07	25.12 + 5.20	12.89 + 3.79	19.51 + 8.15	19.58 + 5.03	20.69 + 6.17	22.49 + 4.91	24.86 + 5.94
Denver, Colo.	24.82 + 6.51	25.20 + 7.40	25.64 + 5.31	15.04 + 4.42	24.21 + 10.11	25.08 + 6.44	22.51 + 6.70	27.91 + 6.09	25.64 + 6.13
Detroit, Mich.	27.84 + 7.30	26.56 + 7.80	28.75 + 5.95	20.59 + 6.05	28.65 + 11.97	25.74 + 6.61	26.19 + 7.77	28.46 + 6.21	29.79 + 7.12
Houston, Tex.	23.90 + 6.26	23.38 + 6.87	27.97 + 5.79	17.36 + 5.10	23.47 + 9.80	23.76 + 6.10	21.73 + 6.46	28.18 + 6.15	27.71 + 6.62
Los Angeles, Calif.	30.16 + 7.90	29.28 + 8.60	36.52 + 7.56	25.96 + 7.63	31.86 + 13.31	29.70 + 7.63	32.80 + 9.75	37.94 + 8.27	32.89 + 7.86
Newark, N.J.	29.23 + 7.66	29.96 + 8.80	29.53 + 6.11	20.41 + 6.00	33.10 + 13.83	not available	29.06 + 8.63	33.06 + 7.21	30.04 + 7.18
Philadelphia, Pa.	25.29 + 6.63	22.70 + 6.67	30.04 + 6.22	20.94 + 6.15	28.41 + 11.87	22.88 + 5.88	21.90 + 6.52	27.91 + 6.09	28.49 + 6.81
San Francisco, Calif.	35.03 + 9.18	34.05 + 10.00	43.77 + 9.06	26.13 + 7.68	33.10 + 13.83	35.42 + 9.10	35.16 + 10.47	50.41 + 10.99	40.92 + 9.78
Seattle, Wash.	27.61 + 7.24	17.48 + 5.14	not available	23.09 + 6.79	27.42 + 11.46	24.86 + 6.39	26.70 + 7.94	33.88 + 7.39	30.82 + 7.37

*Adapted from M. Kiley, Ed., "National Construction Estimator," 37th ed., Craftsman Book Company, Carlsbad, California, 1989. Units are dollars per hour.

TABLE 2
Building and construction costs (Jan. 1990)

Item	Unit	cost, $	Employee—hr to install
Floors			
Asphalt tile	ft^2	1.40	†
Concrete, prestressed, 4-in. thick	ft^2	10.70	†
Steel grating	ft^2	27.81	†
Wood deck, 2-in. thick	ft^2	3.70	†
Foundations, includes excavation, backfill, and forming: flat slab, 1 yd^3 concrete, 5.3 ft^2 forms, 100 lb reinforcing steel‡	yd^3	223	6
Pits and basins: 1 yd^3 concrete, 17.5 ft^2 forms, 115 lb reinforcing steel‡	yd^3	285	8
Walls and piers: 1 yd^3 concrete, 57.0 ft^2 forms, 160 lb reinforcing steel‡	yd^3	593	16
Lumber			
Structural, plain	MBF§	910	30
Structural, creosoted	MBF	1,135	30
Plywood (exterior unsanded):			
$\frac{3}{8}$-in.	ft^2	0.32	1
$\frac{1}{2}$in.	ft^2	0.43	1
$\frac{5}{8}$-in.	ft^2	0.46	1
f-in.	ft^2	0.54	1
Piling (20-25 ton load, 60 ft long)			
Wood, treated	Ea.	670	†
Wood, untreated	Ea.	570	†
Composite	Ea.	1,180	†
Concrete	Ea.	4,300	†
Test pile	Ea.	8,500	†
Load test	Ea.	10,200	†
Equipment "on and off site"	Per job	14,800	†
Roofs			
Aluminum, corrugated, 0.032-in. thick	ft^2	2.32	†
Built-up, 5-ply	ft^2	0.91	t
Reinforced concrete, 4-in. thick	ft^2	10.70	t
Steel, 26 gauge	ft^2	1.44	†
Transite, $\frac{3}{8}$-in. thick	ft^2	2.47	t
Sprinkler systems, exposed. Add 28% for concealed systems			
Wet system	ft^2	1.65	†
Dry system	ft^2	2.10	†
4-in. alarm valve, wet system	Ea.	1,080	40
4-in. alarm valve, dry system	Ea.	1,650	50
din. alarm valve, wet system	Ea.	1,290	40
6-in. alarm valve, dry system	Ea.	1,800	50
Compressor to operate 500 heads	Ea.	980	16
(Valves and compressor not included in ft^2 price)			
Structural steel			
Grating, If-in. standard	ft^2	10.18	0.33

†Labor cost is included in material cost.
‡To adjust: forms, \$3.25/ft^2; reinforcing steel, 44¢/lb.
§MBF refers to 1000 board feet.

(Continued)

TABLE 2
Building and construction costs (Jan. 1990) ***(Continued)***

Item	Unit	cost, $	Employee—hr to install
Grating, expanded metal	ft^2	4.80	0.17
Grating, checker plate, $-in.	ft^2	6.80	0.13
Handrail, standard If-in. pipe, 2 rails, welded	Lineal ft	22.60	t
Handrail, bar type	Lineal ft	7.60	0.70
Steel ladder, with safety cage (Add 25% for aluminum)	Lineal ft	76.00	t
Steel ladder, without safety cage (Add 25% for aluminum)	Lineal ft	48.30	t
Operating platforms, including stairs	ft^2	42.40	1.8
Stairway	Vert. ft	91.60	
Stair treads, **12-in.** wide, galvanized	Ea.	30.50	1
Building steel, shop fab	lb	0.64	0.01
Platform and support steel, shop flab	**lb**	0.80	0.02
Toeplate steel, 4 in. x $\frac{1}{4}$-in.	Lineal ft	5.40	t
Walls			
Siding			
Transite	ft^2	2.60	t
Aluminum	ft^2	2.20	t
Steel-coated	ft^2	1.80	t
Brick			
4-in.	ft^2	5.50	t
8-in.	ft^2	9.50	t
10-in.	ft^2	11.60	†
12-in.	ft^2	13.70	†
Concrete block, reinforced			
6-in.	ft^2	5.25	†
8-in.	ft^2	5.50	t
12-in.	ft^2	8.10	†
Windows, industrial			
Steel, tixed	ft^2	16.30	t
Aluminum	ft^2	12.40	t
Wood	ft^2	7.30	t
Total cost of erected buildings (median values)			
Laboratory: steel frame, masonry walls, floor and **roof;** heating, lighting, and plumbing	ft^2	91	t
Office: steel frame, masonry walls, floor, and **roof;** heating, lighting, and plumbing	ft^2	62	t
Process building: multilevel, **12-ft** clearance, steel platforms, heating, lighting, and plumbing			
Masonry construction	ft^2	41	t
Aluminum on steel	ft^2	43	t
Transite on steel	ft^2	32	†
Open structure: **3-level,** steel, with lighting and plumbing	ft^2	29	t
Warehouse: single story, **15-ft** clearance. Steel frame, masonry walls, floor, and roof; heating, lighting, and plumbing	ft^2	28	t

†Labor cost is included in material cost.

TABLE 2

Building and construction costs (Jan. 1990) (Continued)

Item	Unit	cost, $	Employee-hr to install
Doors			
Metal: Steel frame, 8 x 8 ft, automatic	Ea.	2,085	30
Steel rolling, 12 X 12 ft, manual	Ea.	1,320	24
Swing, with frame, 3 x 7 ft, $1\frac{3}{4}$-in. thick	Ea.	228	t
Wood: Sectional, overhead, 12 x 12 ft	Ea.	1,369	t
Swing exterior, with frame, 3 x 7 ft	Ea.	195	t
Excavation			
Machine	yd^3	4-7	t
Hand	yd^3	23-44	t

†Labor cost is included in material cost,

TABLE 3

Costs for yard improvements (Jan. 1990)

Item		Unit	cost, $	Employee-hr to install
Docks and wharfs				
All concrete, 100 lineal ft wide		Lineal ft	33,200	†
Timber, with wood deck, 100 lineal ft wide		Lineal ft	10,100	†
Dredging		yd^3	4.20	†
Pipe bridges (includes structural steel and foundations)				
Heavy duty		Lineal ft	65	2.5
Light duty		Lineal ft	41	1.8
Pipe column, steel, extra heavy				
16 ft-4 in. diameter		Ea.	126	1.2
16 ft-6 in. diameter		Ea.	260	1.4
22 ft-6 in. diameter		Ea.	350	1.9
Plant fence				
6 ft chain link (3-strand B.W.)		Lineal ft	7.78	t
3- to 4-ft wide person gate		Ea.	200	†
8-ft wide equipment gate		Ea.	330	†
30-ft wide double gate (manual)		Ea.	664	†
20-ft wide gate (automatic)		Ea.	4,500	t
Relocate plant fence		Lineal ft	5.40	†
Railroads				
Track (90 lb)		Lineal ft	63	t
Track (75 lb)		Lineal ft	48	†
Turnout		Ea.	17,030	†
Ties, creosoted (6 in. x 8 in. x 8 ft)		Ea.	38	†
Grade and ballast		Lineal ft	38.50	t
Cars, ore, 24-in. track				
Capacity, ft^3	Wt, lb			
12	660	Ea.	1,040	†
16	700	Ea.	1,110	t
20	930	Ea.	1,240	t

†Labor cost is included with material cost.

(Continued)

TABLE 3
Cost for yard improvements (Jan. 1990) (Continued)

Item	Unit	cost, $	Employee—hr to install
Locomotives, mine, battery type			
Size, tons Wt, lb			
9 19,000	Ea.	102,000	
12 26,000	Ea.	117,000	
Locomotive, mine, Diesel type			
$1\frac{1}{2}$ 3,000 2 cyl	Ea.	31,600	
3 7,000 4 cyl	Ea.	39,400	
Roads and walkways			
Concrete, slab, mesh reinforced			
Thickness, in. Sub-base, in.			
4 6	yd^2	21.50	†
6 6	yd^2	21.80	†
8 6	yd^2	26.50	†
Paving asphalt			
3 12	yd^2	12.90	†
Sewers			
Reinforced concrete, thickness, in. (installed)			
12	Lineal ft	20.90	†
15	Lineal ft	23.30	†
18	Lineal ft	26.10	†
24	Lineal ft	36.00	†
30	Lineal ft	47.40	†
Vitrified clay, thickness, in. (installed)			
4	Lineal ft	14.00	†
6	Lineal ft	15.90	†
8	Lineal ft	19.50	†
12	Lineal ft	30.25	†
15	Lineal ft	38.80	†
18	Lineal ft	48.70	†
24	Lineal ft	71.40	†
Site development			
Clearing and grubbing	yd^2	0.90	†
Grade out	yd^3	2.80	†
Cut, fill, and compact	yd^3	4.50	†
New fill compacted	yd^3	9.00	†
Topsoil	yd^3	14.40	†
Gravel till	yd^3	15.60	†
Crushed stone ($\frac{3}{4}$ in.)	yd^3	15.60	†
Seeding	yd^2	0.80	†
Sluiceway			
Open, piled, and sheathed	Lineal ft		†
Wells			
200 gpm, 400 ft deep, 15 hp	Ea.	26,000	†
500 gpm, 200 ft deep, 40 hp	Ea.	34,000	†
1200 gpm, 400 ft deep, 75 hp	Ea.	47,000	†

†Labor cost is included with material cost.

TABLE 4
Cost of electrical installations (Jan. 1990)

Main transformer stations, installed cost, $/kVa

Capacity, kVA	Primary voltage, 25 kV A	Primary voltage, 25 kV B	Primary voltage, 46 kV A	Primary voltage, 46 kV B	Primary voltage, 69 kV A	Primary voltage, 69 kV B
3,000	130	105	150	127	167	148
5,000	81	70	91	78	102	88
10,000	53	45	57	49	63	53
20,000	39	36	42	39	49	45

A: Two three-phase transformers, 60 cycles, two main incoming supply lines
B: Two three-phase transformers, 60 cycles, one main incoming supply line

Secondary transformer stations, installed cost, $/ kVa

Capacity, kVA	2400 / 240 volts	4200 / 600 volts	13,200 / 600 volts
600	34	89	112
1000	28	65	81
1500	24	50	63
2000	21	42	52

Distribution feeders, installed cost, $/ kVa main transformer capacity

Capacity, kVA	Primary voltage, 42 kV A	Primary voltage, 42 kV B	Primary voltage, 13.2 kV A	Primary voltage, 13.2 kV B
3,000	52-104	16-36		
5,000	36-71	13-52		
10,000			19-42	6-16
20,000			13-26	3-10

A: Underground
B: Overhead

Lighting, installed cost, $/ kW

Avg. watts per ft^2	Explosion proof	Vapor proof	Standard
1	7460	4740	3890
2	5060	3080	2430
3	4220	2370	1540
4	3890	1870	1265
5	3730	1700	1200

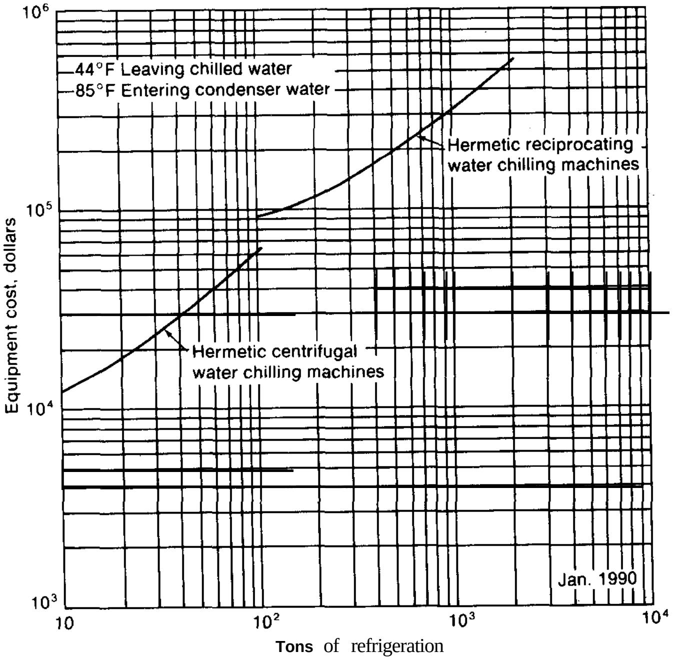


FIGURE B-1
Cost of air conditioning. Price includes compressor, motor, starter, controls, cooler, condenser, and refrigerant. Does not include cooling tower, pumps, foundations, ductwork, and installation costs.

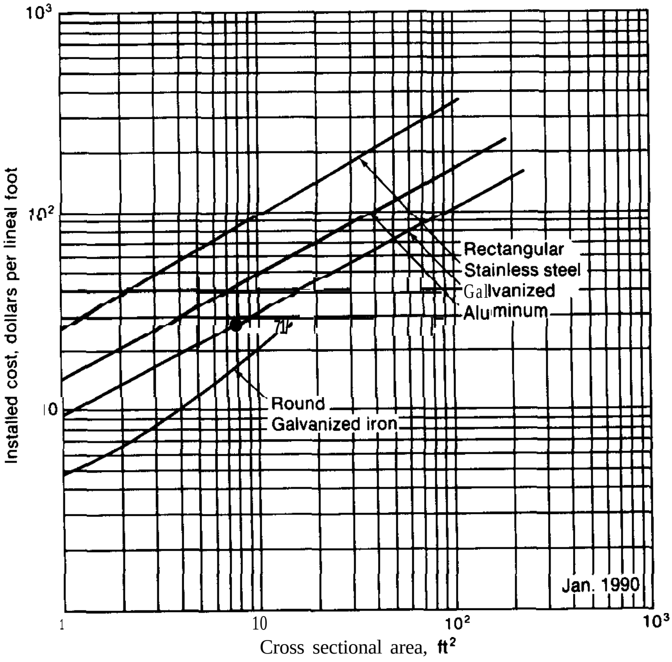


FIGURE B-2
Cost of ductwork. Price is for shop-fabricated ductwork with hangers and supports installed.

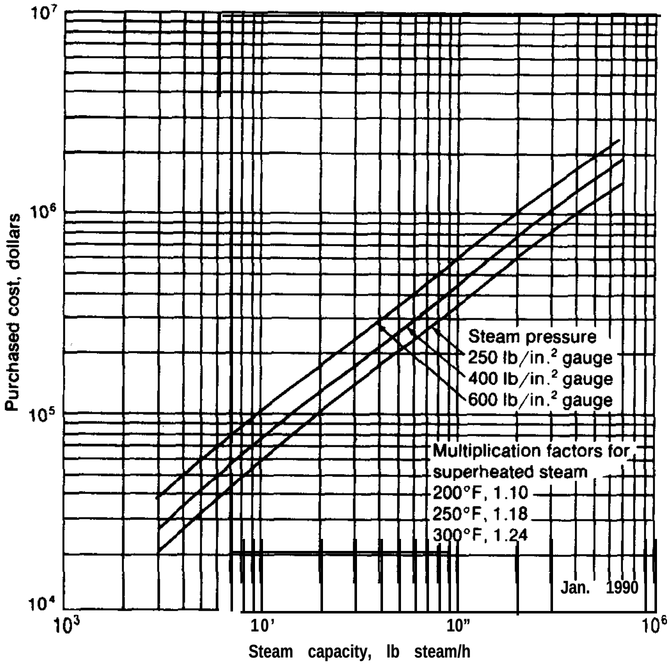


FIGURE B-3
Cost of package boiler plants. Price includes complete boiler, feed-water deaerator, boiler feed pumps, chemical injection system, stack, and shop assembly labor.

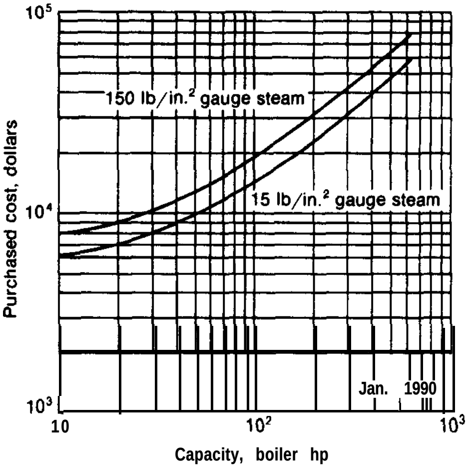


FIGURE B-4
Cost of steam generators. Price is for packaged unit with steel tubes, gas-fired. Multiply by 1.02 for oil-fired boiler. Multiply horsepower by 33,500 for number of Btu's.

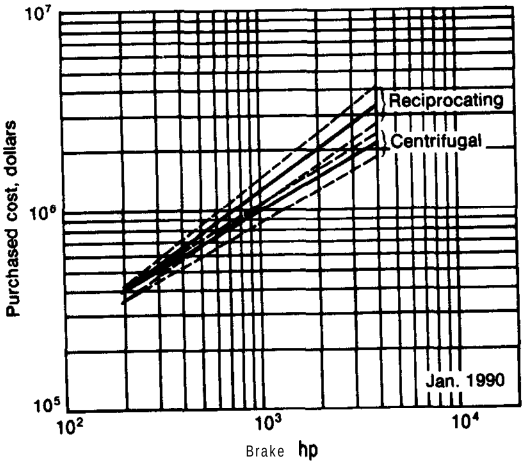
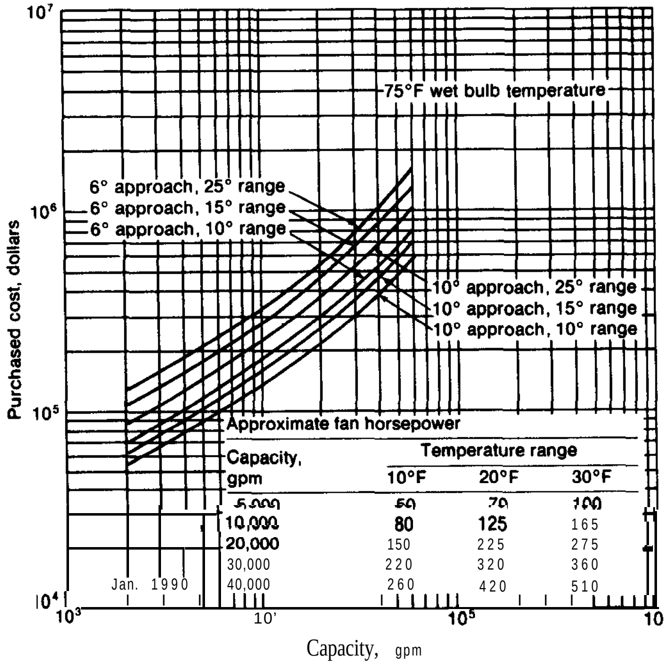


FIGURE B-5
Cost of compressor plants. (Dotted lines indicate range of costs for the two types of plants.)



Approximate fan horsepower

Capacity, gpm	Temperature range 10°F	20°F	30°F
5,000	50	70	100
10,000	80	125	165
20,000	150	225	275
30,000	220	320	360
40,000	260	420	510

FIGURE B-6
Cost of cooling towers. Prices are for conventional, wood-frame, induced-draft, cross-flow, cooling towers. Price does not include external piping, power wiring, special foundation work, or field labor.

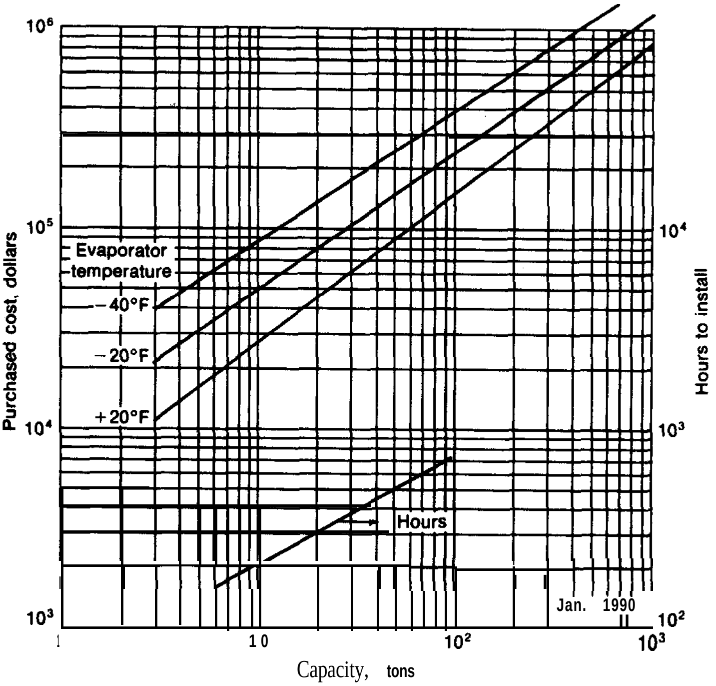


FIGURE B-7
Cost of industrial refrigeration,

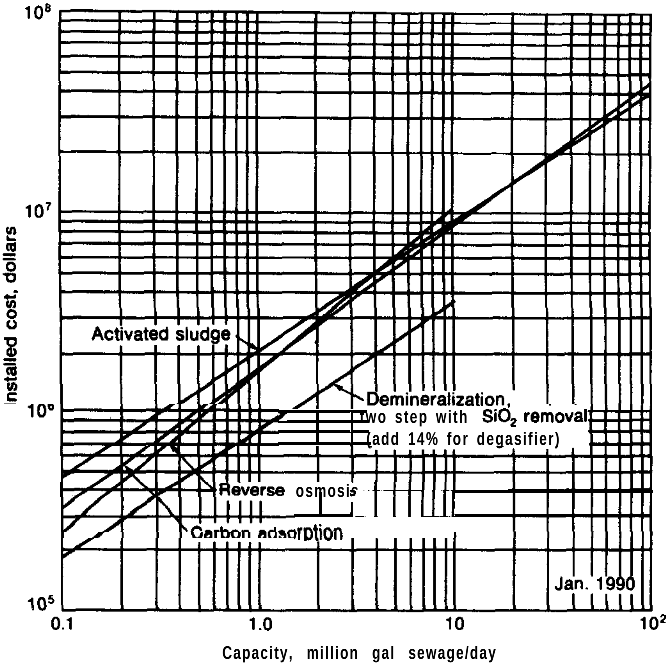


FIGURE B-8
Cost of wastewater-treatment plants.

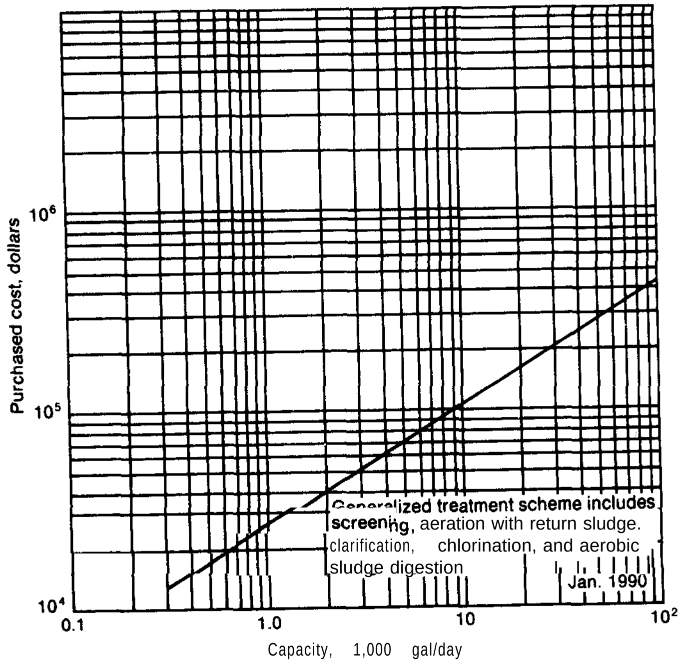


FIGURE B-9
Cost of small packaged wastewater-treatment plants.

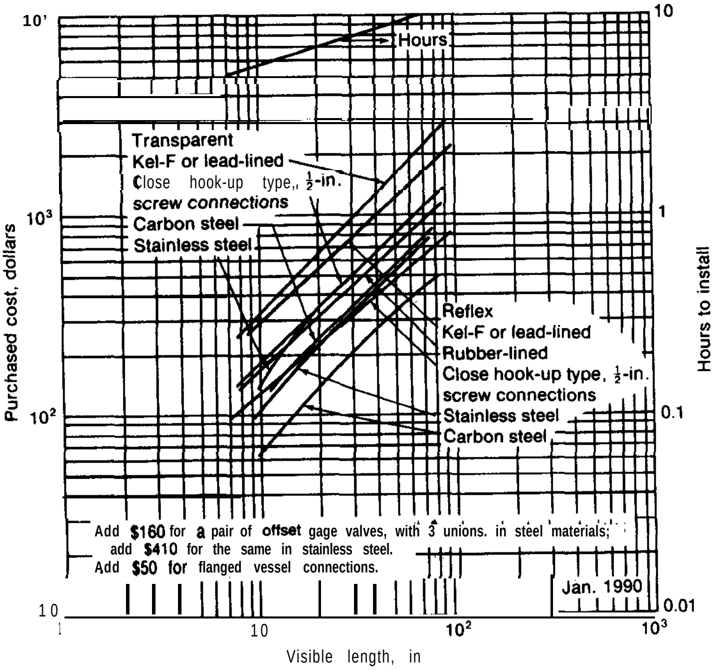


FIGURE B-10
Cost of liquid-level gages, flat-glass type.

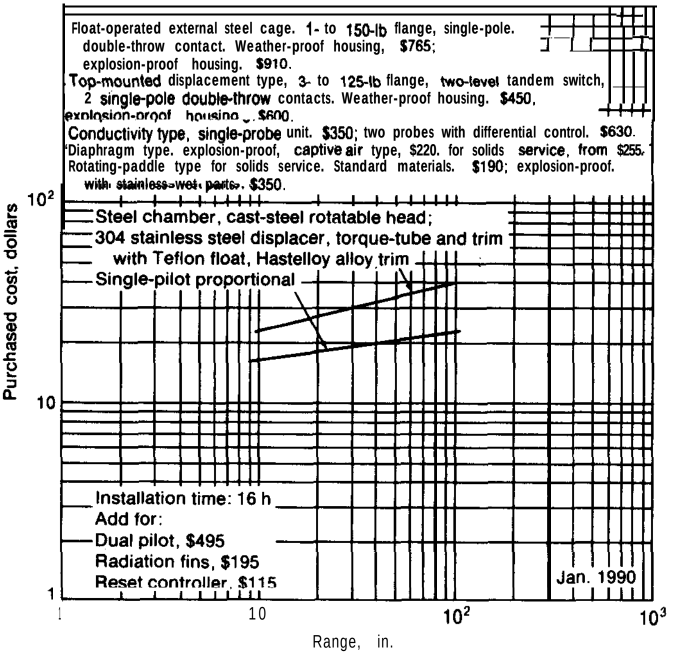


FIGURE B-11
Cost of level controllers.

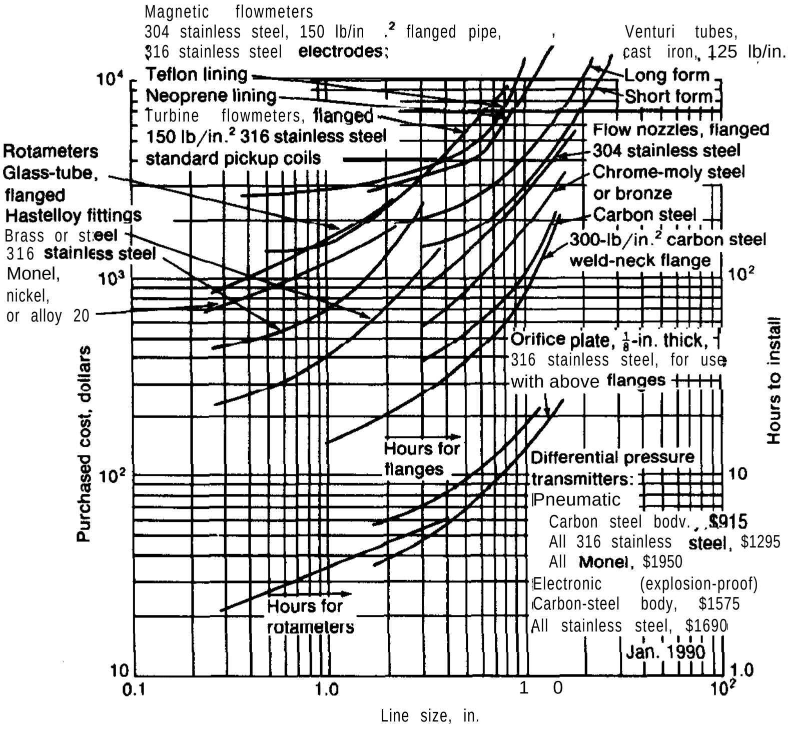


FIGURE B.12
Cost of flow indicators.

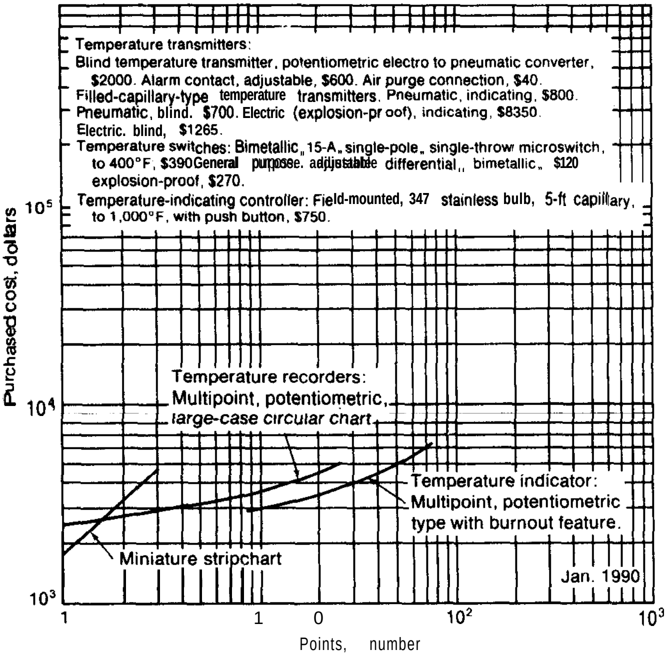
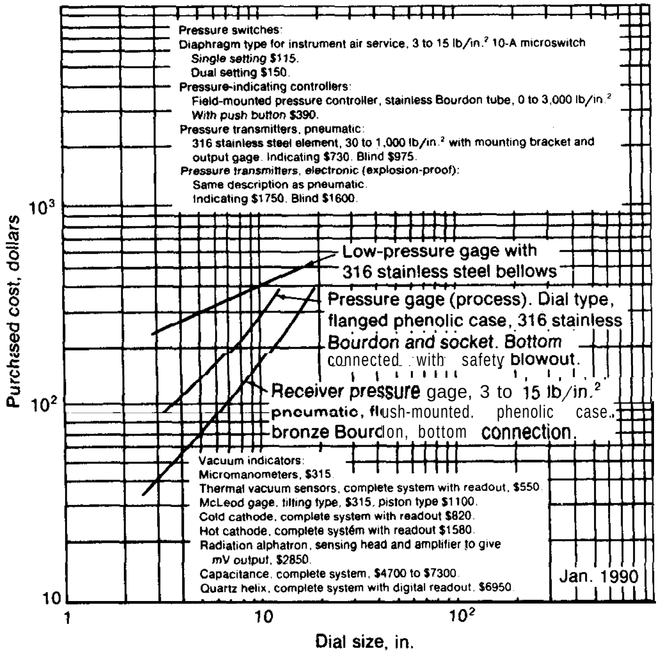

Temperature transmitters:
Blind temperature transmitter, potentiometric electro to pneumatic converter, $2000. Alarm contact, adjustable, $600. Air purge connection, $40.
Filled-capillary-type temperature transmitters. Pneumatic, indicating, $800. Pneumatic, blind. $700. Electric (explosion-proof), indicating, $8350. Electric. blind, $1265.
Temperature switches: Bimetallic, 15-A, single-pole, single-throw microswitch, to 400°F, $390. General purpose. adjustable differential, bimetallic, $120 explosion-proof, $270.
Temperature-indicating controller: Field-mounted, 347 stainless bulb, 5-ft capillary, to 1,000°F, with push button, $750.

FIGURE B-13
cost of temperature recorders and indicators.


Pressure switches:
Diaphragm type for instrument air service, 3 to 15 lb/in.^{2} 10-A microswitch
Single setting $115.
Dual setting $150.
Pressure-indicating controllers:
Field-mounted pressure controller, stainless Bourdon tube, 0 to 3,000 lb/in.^{2}
With push button $390.
Pressure transmitters, pneumatic:
316 stainless steel element, 30 to 1,000 lb/in.^{2} with mounting bracket and output gage. Indicating $730. Blind $975.
Pressure transmitters, electronic (explosion-proof):
Same description as pneumatic.
Indicating $1750. Blind $1600.

Vacuum indicators:
Micromanometers, $315.
Thermal vacuum sensors, complete system with readout, $550.
McLeod gage, tilting type, $315, piston type $1100.
Cold cathode, complete system with readout $820.
Hot cathode, complete system with readout $1580.
Radiation alphatron, sensing head and amplifier to give *mV* output, $2850.
Capacitance, complete system, $4700 to $7300.
Quartz helix, complete system with digital readout, $6950.

FIGURE B-14
Cost of pressure indicators.

TABLE 5
Rates for various industrial utilities

Utility	Cost (Jan. 1990)
Steam:	
500 psig	\$3.25–\$3.90/1000 lb
100 psig	1.50–3.20/1000 lb
Exhaust	0.80–1.50/1000 lb
Electricity:	
Purchased	0.035–0.13/kWh†
Self-generated	0.025–0.065/kWh
Cooling water:	
Well	0.10–0.50/1000 gal
River or salt	0.06–0.20/1000 gal
Tower	0.06–0.26/1000 gal
Process water:	
City	0.35–1.50/1000 gal
Filtered and softened	0.50–1.50/1000 gal
Distilled	2.25–4.00/1000 gal
Compressed air:	
Process air	0.06–0.20/1000 ft^3 (at SC)‡
Filtered and dried for instruments	0.12–0.36/1000 ft^3 (at SC)‡
Natural gas:	
Interstate (major pipeline companies)	2.40–3.15/1000 ft^3 (at SC)‡
Intrastate (new contracts)	2.40–3.50/1000 ft^3 (at SC)‡
Intrastate (renegotiated or amended contracts)	2.75–3.75/1000 ft^3 (at SC)‡
Manufactured gas	1.65–4.85/1000 ft^3 (at SC)‡
Fuel oil	17.00–24.00/bbl
Coal	30.00–55.00/ton
Refrigeration (ammonia), to 34°F	2.00/ton-day (288,000 Btu removed)

†Highly dependent upon location. Convert to mils by 1000 mils = one dollar.

‡For these cases, standard conditions are designated as a pressure of 29.92 in. Hg and a temperature of 60°F.

TABLE 6
Costs for selected industrial chemicals (Jan. 1990)†

Chemical and conditions	Cost quote unit	cost, $
Acetaldehyde, 99%, tanks	lb	0.47
Acetic acid, tech., tanks	lb	0.29
Acetic anhydride, tanks	lb	0.46
Acetone, tanks	lb	0.29
Acrylonitrile, tanks	lb	0.45
Allyl alcohol, tanks, Bayport, Texas	lb	1.00
Ammonia, anhyd., fertilizer, tanks, Midwest	ton	110
Aniline, tanks	lb	0.57
Benzaldehyde, tech., drums	lb	0.73
Benzene, indust., barges	gal	1.50
Benzoic acid, tech., bags	lb	0.55
n-Butyl alcohol, syn., ferment., tanks	lb	0.38
Butyric acid, tanks	lb	0.76
Calcium carbonate, nat., dry-ground, carlots	ton	49
Carbon tetrachloride, tech., drums, carlots	lb	0.31
Chlorine, tanks	ton	190
Chloroform, tech., tanks, delivered	lb	0.36
Copper chloride (cupric), anhyd., carlots	lb	2.37
Ethyl alcohol, 190 proof, USP tax free, tanks	gal	2.00
Ethyl ether, refined, tanks	lb	0.52
Ethylene, contract, delivered	lb	0.24
Ethylene oxide, tanks	lb	0.60
Formaldehyde, 37% by wt., inhibited 7% methanol, tanks, Gulf coast	lb	0.10
Glycerine, syn., 99.7%, tanks, delivered	lb	0.85
Hexane, indust., tanks	gal	0.74
Hydrochloric acid, 20°Be, tanks	ton	55
Isobutyl alcohol, tanks, delivered	lb	0.38
Lime, chemical, pebble (quicklime), bulk	ton	39
Methanol, syn., barges, Gulf coast	gal	0.32
Nitric acid, 36 Be, tanks	ton	175
Oxalic acid, bags, carlots	lb	0.60
Pentaerythritol, tech., bags, carlots	lb	0.71
Phenol, syn., tanks	lb	0.41
Propylene glycol, indust., tanks	lb	0.56
Soda ash, dense, 58%, paper bags, carlots	ton	146
Sodium hydroxide, tech. (caustic soda), bulk	ton	560
Sulfur, crude, 99.5%, 50-lb bags, mines	100 lb	13.60
Sulfuric acid, 100%, tanks, works	ton	75
Toluene, petroleum, indust., tanks	gal	0.76
Urea, 46% N, indust., bulk, Gulf coast	ton	210

†Obtained from "Chemical Marketing Reporter," published by Schnell Publishing Company, Inc., New York, N.Y. Unless otherwise indicated, prices are for large lots, f.o.b., New York City or eastern U.S.A.

APPENDIX C

DESIGN PROBLEMS

CONTENTS

MAJOR PROBLEMS

MINOR PROBLEMS

PRACTICE SESSION PROBLEMS

MAJOR PROBLEMS†

Problem 1. Plant for Solvent Rendering of Raw Tissues

Memorandum
To: Assistant Design Engineer
From : A. B. White, Head
Process Development Division

You are to prepare a complete preliminary-estimate design for a new plant for the solvent rendering of raw tissues by an extraction process. Some pieces of equipment are now on hand which we believe can be used in the new plant. Please submit a complete report on the design which you feel will be most favorable for our company. We are particularly interested in total investment, yearly profit before taxes, and the probable percent return on the investment. We shall be interested in the reason for your particular choice of solvent. The design report should also include the number of operators necessary and the approximate operating procedure.

We are considering expansion of our present plant to include an extraction process for treating 50 tons of raw fish per day. The average analysis of the

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†Time period of 30 days recommended for individual student solution.

fish as received by our plant is as follows:

	% by weight
Water	70
Soluble oils	10
Insolubles	20

The general process has been described in some detail in the literature and in patents (see references at end of problem); however, our proposed method is briefly presented in the following:

THE GENERAL PROCESS

The process is to be carried out at a sufficiently low temperature for the biological substances to remain essentially unchanged, except for the removal of oil and water. The top temperature limit for the insoluble materials and the oils is 90°C.

The raw fish as received are ground and delivered to a slurry tank where the pulped material is agitated. The slurry is then sent to cookers where the oil is extracted by a suitable solvent and the water is evaporated along with the solvent. The evolved vapors are condensed, and the solvent and the water are separated by decanting.

The liquid-and-solid mixture from the batch cookers is filtered, and the solid is desolventized to give a fish-meal product.

The filtrate, containing solvent and dissolved fish oil, is sent to a steam-jacketed kettle where the oil is recovered by distilling off the solvent. The oils are finally passed through a steam stripper where the final traces of solvent are removed and the oil is deodorized. The fish-oil product is obtained from the bottom of the steam stripper.

A general flow diagram is included with this problem (Fig. C-l), but this diagram does not show any of the details.

SOLVENTS. The following solvents should be considered for possible use: trichloroethylene, ethylene dichloride, perchloroethylene, and carbon tetrachloride. The solvents may be purchased in tank-car lots since storage space is available. Note that some of these are on the toxic list.

SPECIAL CONDITIONS. To ensure adequate water removal, the cooker must be operated for $\frac{1}{2}$ hr at the boiling temperature of the pure solvent.

The final dry fish meal may not contain more than 1 percent oil by weight.

The weight ratio of solvent to insolubles at the end of each cooker batch must be 3 : 1. The fish-meal filter cake must be washed with a weight of solvent at least equal to the weight of the dry fish meal in order to ensure adequate oil removal.

The mixture in the slurry tank contains 50 wt % solvent and 50 wt % fish. Cooling water is available at 15°C.

The solid material remaining after filtration contains 1.0 lb of solvent per pound of dry fish meal.

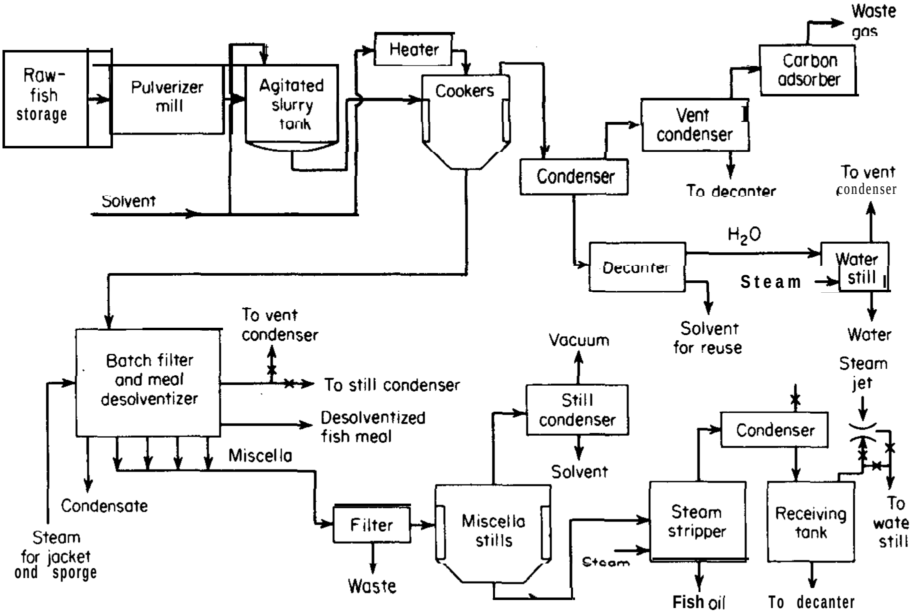

FIGURE C-1
Suggested rough flow diagram showing extraction process for fish-oil recovery. (Details are not included and changes may be necessary.)

Tanks larger than 9 ft in diameter or 11 ft high cannot be used under the plant conditions.

The steam pressure in the plant is 30 psig.

Superheated solvent vapor may be used to provide additional heat for the cooker.

ASSUMPTIONS AND DATA. The effective molecular weight of the oil may be assumed to be 150. The vapor pressure of the oil is negligible under the conditions of the process.

Heat capacities and specific volumes for individual components making up the mixtures may be considered as additive. The following values apply between 20 and 100°C.

c_p of insolubles = 0.2 Btu/(lb)(°F)
c_p of oil = 0.3 Btu/(lb)(°F)
sp gr of oil ≈ 0.7
Equivalent sp gr of insolubles = 0.8

The only source of solvent loss is from the waste water discarded from the water still.

The liquid leaving all condensers is 5°C below the boiling point.

The temperature of the slurry entering the cookers is 25°C.

The temperature of the liquid entering the Miscella stills is 30°C.

The liquid enters the steam stripper at the same temperature it had leaving the Miscella still.

The desolventized fish meal contains no solvent.

The total fixed-capital investment for the installed equipment may be obtained by multiplying the cost of all the primary and accessory equipment by 4. This factor takes care of construction costs and all other fixed costs such as piping, valves, instrumentation, new buildings, etc. (This, of course, does not include the original solvent cost or any raw materials.)

Overall heat-transfer coefficient in all condensers = 150 Btu/(h)(ft^2)(°F).

Overall heat-transfer coefficient in cookers and in Miscella stills = 100 Btu/(h)(ft^2)(°F) (applies for walls or tubes).

Coils may not be used in the cookers.

The liquid leaving the Miscella still may be assumed to contain 8 percent solvent by weight.

The steam stripper is steam-jacketed, and the oil must leave the bottom at a temperature less than 100°C. In order to ensure complete removal of the solvent from the oil and to deodorize the oil, use excess steam amounting to 30 percent of the weight of the oil.

No agitators are necessary in the cookers or Miscella tanks, because the boiling supplies sufficient agitation.

The total amount of steam theoretically necessary for the process should be increased by 20 percent to take care of unaccounted-for heat losses.

The plant operates 300 days per year.

EQUIPMENT. The following equipment is now available at the plant:

Fish-pulverizing mill
All necessary storage tanks and receiving tanks
The slurry tank complete with agitator
Batch filter and meal desolventizer
Decanter still
Water still
Vacuum ejectors (valued at $4800)
Waste filter
Steam stripper
Vent condenser and carbon adsorber

These pieces of equipment are not installed. Preliminary estimates have indicated that all of these pieces of equipment have adequate capacity for the present design. They are valued at a total of $300,000. For the purpose of your cost estimates, you may assume that $300,000 will cover the cost of the listed equipment. However, your design should include complete specifications for all the equipment necessary for the new plant. In this way, we will be able to have a final check to show us if the present equipment on hand is satisfactory.

It will be necessary to purchase the cookers, the Miscella stills, the condensers and the heaters for these pieces of equipment, and all pumps.

COST. The necessary working-capital investment which must be kept on hand for the new plant is estimated to be $145,000.

Annual fixed charges = 25 percent of total investment.

Annual direct production costs for this process (with the exception of raw material and steam costs) plus costs for plant overhead, administration, office help, distribution, and contingencies are $480,000 per year. This $480,000 includes the cost of operating any vacuum equipment and is unchanged even if vacuum is not used in the process.

Tanks. Suitable for cookers or Miscella stills (diameter less than 9 ft):

	Cost for one 1000-gal tank (installed)	
Material	**Without steam jacket**	**With steam jacket**
Carbon steel	$11,400	$20,000
Stainless steel	29,000	44,000

The tanks are available in standard sizes of (in gallons) 100, 200, 300, 500, 750, 1000, 1500, 2000, 5000, 10,000.

Assume that the cost of each tank varies as $(\text{volume})^{2/3}$.

If heating coils are to be inserted in a tank, increase the cost of the tank by 10 percent and by 10 times the cost of the pipe or tubing making up the coil. (This is an approximation to take care of extra costs for fabrication, special materials, etc. It applies only to a reasonable length of coil).

Heat exchangers. Shell-and-tube type (for condensers and heaters):

	Dollars per ft^2 of tube surface (installed) (pressures from 25 psig to 0 in. Hg)					
	Square feet					
Material	**50**	**100**	200	300	500	**1000**
Carbon steel	160	130	94	78	65	49
Stainless steel	224	182	132	109	91	69

Steam cost. $1.30/1000 lb.

Materials.

Cost of raw fish (delivered at plant) = $140.00/ton

Selling price for fish meal of plant-product grade = $0.40/lb

Selling price for fish oil of plant-product grade = $0.65/lb

References

1. E. Levin and R. K. Finn, A Process for Dehydrating and Defatting Tissues at Low Temperature, *Chem. Eng. Progr.*, **51**:223 (1955).

2. E. Levin and F. Lerman, An Azeotropic Extraction Process for Complete Solvent Rendering of Raw Tissues, *J. Am. Oil Chem. Soc.*, **28**:441 (1951).
3. E. M. Worsham and E. Levin, Simultaneous Defatting and Dehydrating of Fatty Substances, U.S. Patent No. 2503,312 (Apr. 11, 1950).
4. E. Levin, Production of Dried, Defatted Enzymatic Material, U.S. Patent No. 2,503,313 (Apr. 11, 1950).
5. E. Levin, Drying and Defatting Tissues, U.S. Patent No. 2,619,425 (Nov. 25, 1952).
6. Defatting Distillery, *Ind. Eng. Chem.*, **42**:14A (June, 1950).
7. E. W. McGovern, Chlorohydrocarbon Solvents, *Znd. Eng. Chem.*, **35**:1230 (1943).

Problem 2. Pentaerytbritol Plant

You are a design engineer with the ABC Chemical Company, located at Wilmington, Del. Mr. Charles B. Ring, Supervisor of the Process Development Division, has asked you to prepare a preliminary design of a new plant that will produce pentaerythritol.

The following memo has been given to you by Mr. Ring:

Dear Sir:

We are considering construction of a pentaerythritol plant at Louisiana, Missouri. At the present time, we have an anhydrous ammonia plant located at Louisiana, Missouri, and we own sufficient land at this site to permit a large expansion.

As yet, we have not decided on the final plant capacity, but we are now considering construction of a small plant with a capacity of 60 tons of technical grade pentaerythritol per month. Any work you do on the design of this plant should be based on this capacity.

Sufficient water, power, and steam facilities for the new plant are now available at the proposed plant site.. Some standby equipment is located at our existing plant in Louisiana, and we may be able to use some of this in the pentaerythritol plant. However, you are to assume that all equipment must be purchased new.

Please prepare a preliminary design for the proposed plant. This design will be surveyed by the Process Development Division and used as a basis for further decisions on the proposed plant. Make your report as complete as possible, including a detailed description of your recommended process, specifications and cost estimates for the different pieces of equipment, total capital investment, and estimated return on the investment assuming we can sell all our product at the prevailing market price. We shall also be interested in receiving an outline of the type and amount of labor required, the operating procedure, and analytical procedures necessary.

Present what you consider to be the best design. Although our legal department will conduct a patent survey to determine if any infringements are involved, it would be helpful if you would indicate if you know that any part of your design might involve patent infringements.

Enclosed you will find information on our utilities situation, amortization policy, labor standards, and other data.

C. B. King, Supervisor
Process Development Division

Enclosure: Information for Use in Design of Proposed Pentaerythritol Plant

Utilities. Water-available for pumping at 65°F; cost, \$0.14 per 1000 gal. Steam-available at 100 psig: cost, \$1.50 per 1000 lb. Electricity-cost, \$0.07 per kWh.

Storage. No extra storage space is now available.

Transportation facilities. A railroad spur is now available at the plant site.

Labor. Chief operators, \$16.00 per hour; all helpers and other labor, \$12.00 per hour.

Amortization. Amortize in 10 years.

Return on investment. For design calculations, we require at least a 20 percent return before income taxes on any unnecessary investment.

Income taxes. The income-tax load for our company amounts to 34 percent of all profits.

Raw-materials costs. All raw materials must be purchased at prevailing market prices.

Heats of reaction. With sodium hydroxide as the catalyst, 64,000 Btu is released per pound mole of acetaldehyde reacted. With calcium hydroxide as the catalyst, 60,000 Btu is released per pound mole of acetaldehyde reacted.

Heat-transfer coefficients. Steam to reactor liquid, 200 Btu/(h)(ft^2)(°F). Water to reactor liquid, 150 Btu/(h)(ft^2)(°F).

Problem 3. Formaldehyde Plant

Illinois Chemical Process Corporation
Plant Development Division
Urbana, Illinois

Gentlemen:

We are considering entering the field of production of formaldehyde, and we should like to have you submit a complete preliminary design for a 70-ton-per-day formaldehyde plant (based on 37 wt % formaldehyde) to us.

We have adequate land available for the construction of the plant at Centralia, Illinois, and sufficient water and steam are available for a plant of the desired capacity. Please make your report as complete as possible, including a detailed description of your recommended process, specifications and cost estimates for the different pieces of equipment, total capital investment, and estimated return on the investment assuming we can sell all our production at the prevailing market price. We shall also be interested in receiving an outline of the amount and type of labor required, the operating procedure, and analytical procedures necessary.

Enclosed you will find information concerning our utilities situation, amortization policy, labor standards, product specification, etc.

Very truly yours,

A. B. Blank
Technical Superintendent
Centralia Chemical Company
Centralia, Illinois

ABB:jf
Enclosure

Enclosure

Utilities. Water-available for pumping at 70°F; cost, $0.14 per 1000 gal. Steam-available at 200 psig; cost, $1.50 per 1000 lb. Electricity-cost, $0.07 per kWh.

Storage. No extra storage space is now available.

Transportation facilities. A railroad spur is now available at the plant site.

Land. The plant site is on land we own which is now of no use to us. Therefore, do not include the cost of land in your analysis.

Labor. Chief operators, $16.00 per hour; all helpers and other labor, $12.00 per hour.

Amortization. Amortize in 10 years.

Return on investment. For design comparison, we require at least a 15 percent return per year on any unnecessary investment.

Product specification. All formaldehyde will be sold as N.F. solution containing 37.0 wt % formaldehyde and 8 wt % methanol. All sales may be considered to be by tank car.

MINOR PROBLEMS†

Problem 1. Optimum Temperature for Sulfur Dioxide Reactor

The head of your design group has asked you to prepare a report dealing with the use of a special catalyst for oxidizing SO, to SO,. This catalyst has shown excellent activity at low temperatures, and it is possible that it may permit you

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† Time period of 14 days recommended for individual student solution.

to get good SO, yields by using only one standard-size reactor instead of the conventional two.

Your report will be circulated among the other members of your design group and will be discussed at the group meetings. This report is to be submitted to the head of your design group.

Some general remarks concerning your report follow:

A. The report should include the following:
  1. Letter of transmittal (the letter to the head of your group telling him you are submitting the report and giving any essential results if applicable)
  2. Title page
  3. Table of Contents
  4. Summary (a concise presentation of the results)
  5. Body of report
    a. Introduction (a brief discussion to explain what the report is about and reason for the report; no results should be included here)
    b. Discussion (outline of method of attack on the problem; do not include any detailed calculations; this should bring out technical matters not important enough to be included in the Summary; indicate assumptions and reasons; include any literature survey results of importance; indicate possible sources of error, etc.)
    c. Final recommended conditions (or design if applicable) with appropriate data (a drawing is not necessary in this case although one could be included if desired)
  6. Appendix
    a. Sample calculations (clearly presented and explained)
    b. Table of nomenclature (if necessary)
    c. Bibliography (if necessary)
    d. Data employed

B. The outline as presented above can be changed if desired (for example, a section on conclusions and recommendations might be included)

C. The report can be made more effective by appropriate subheadings under the major divisions

The Problem

A new reactor has recently been purchased as a part of a contact sulfuric acid unit. This reactor is used for oxidizing SO, to SO, employing a vanadium oxide catalyst.

Using the following information and data, determine the temperature at which the reactor should be operated to give the maximum conversion of SO, to SO,, and indicate the value (as percent) of this maximum conversion. Ten thousand pounds of SO, enters the reactor per day.

Air is used for the SO, oxidation, and it has been decided to use 300 percent excess air. Preheaters will permit the air and SO, to be heated to any desired temperature, and cooling coils in the reactor will maintain a constant temperature in the reactor. The reactor temperature and the entering air-and-SO, temperature will be the same. The operating pressure may be assumed to be 1 atm.

The inside dimensions of the reactor are 5 by 5 by 8 ft. One-half of the inside reactor volume is occupied by the catalyst.

Your laboratory has tested a special type of vanadium oxide catalyst, and, on their recommendation, you have decided to use it in the reactor. This catalyst has a void fraction of 60 percent (i.e., free space in catalyst/total volume of catalyst = 0.6).

The reaction $2SO_2$ + 0, → $2SO_3$ proceeds at a negligible rate except in the presence of a catalyst.

Your laboratory has run careful tests on the catalyst. The results indicate that the reaction is not third-order but is a complex function of the concentrations. Your laboratory reports that the reaction rate is proportional to the SO, concentration, inversely proportional to the square root of the SO, concentration, and independent of the oxygen concentration.?

This may be expressed as

$$\frac{dx}{dt} = k\frac{a - x}{x^{1/2}}$$

where a = SO, originally present as pound mol/ft^3

x = number of lb moles of SO, converted in t sec of catalyst contact time per cubic foot of initial gas

k = specific reaction-rate constant, (lb mol/ft^3)$^{1/2}$/s; this may be assumed to be constant at each temperature up to equilibrium conditions

The laboratory has obtained the data given in Table 1 for the reaction-rate constant. These data are applicable to your catalyst and your type of reactor.

TABLE I

$k \times 10^4$, (lb mol/ft^3)$^{1/2}$/s†	Temperature, °C
14	350
30	400
60	450
100	500
210	550

† Applicable only to the conditions of this problem.

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†Modern tests indicate that this may not be the case with some vanadium oxide reactors. However, the information given above should be used for the solution of this problem.

TABLE 2

ΔF^0, cal/g mol	Temperature, °C
-9120	350
-8000	400
-7000	450
-5900	500
-4900	550

The data of standard-state free-energy values at different temperatures given in Table 2 were obtained from the literature. These data apply to the reaction

$$\text{so,} + \tfrac{1}{2}O_2 \rightleftharpoons \text{so,}$$

The fugacities of the gases involved may be assumed to be equal to the partial pressures.

Problem 2. Heat-Exchanger Design

To: Assistant Process Engineer
From: Dr. A. B. Green, Chief Design Engineer
Mountain View Chemical Company
Boulder, Colorado

We are in the process of designing a catalytic cracker for our petroleum division. As part of this work, will you please submit a design for a single-pass heat exchanger based on the information given below?

It is estimated that 200,000 gph of oil *A* must be heated from 100 to 230°F. The heating agent will be saturated steam at 50 psig.

The engineering department has indicated that the exchanger will cost $60.00 per square foot of inside heating area. This cost includes all installation.

You can neglect any resistance due to the tube walls or steam film; so all the heat-flow resistance will be in the oil film.

The cost of power is 7 cents per kilowatthour, and the efficiency of the pump and motor installation is estimated to be 60 percent.

Do not consider the cost of steam or exchanger insulation in your analysis.

The oil will flow inside the tubes in the heat exchanger. Following are data on oil *A* which have been obtained from the Critical Tables and Perry's Chemical Engineers' Handbook:

Avg viscosity of oil *A* between 100 and 230°F = 6 centistokes
Avg density of oil *A* between 100 and 230°F = 0.85 g/cm^3
Avg sp ht of oil *A* between 100 and 230°F = 0.48
Avg thermal conductivity of oil *A* between 100 and 230°F = 0.08 $Btu/(h)(ft^2)(°F/ft)$

We recommend that the Reynolds number be kept above 5000 in this type of exchanger.

The tubes in the exchanger will be constructed from standard steel tubing. Tube sizes are available in $\frac{1}{2}$-in.-diameter steps. Tubing wall is 16 BWG.

We are particularly interested in the diameter of the tubes we should use, the length of the tubes, and the total cost of the installed unit. In case the exchanger length is unreasonable for one unit, what would you recommend?

Assume the unit will operate continuously for 300 days each year. Fixed charges are 16 percent.

Remember that our company demands a 20 percent return on all extra fixed-capital investments.

You may base your calculations on a total of 100 tubes in the exchanger.

Please submit this information as a complete formal report. Include a short section outlining what further calculations would have been necessary if the specification of 100 tubes in a one-pass exchanger had not been given.

Following are recommended assumptions:

You may assume that the Fanning friction factor can be represented by

$$f = \frac{0.04}{(N_{Re})^{0.16}}$$

You may assume that the oil-film heat-transfer coefficient is constant over the entire length of the exchanger.

For simplification, assume that the oil-film heat-transfer coefficient may be represented by (standard heat-transfer nomenclature)

$$h = 0.023;\ (N_{Re})^{0.8}(N_{Pr})^{0.4}$$

where all variable values are at the average value between 100 and 230°F.

Problem 3. Design of Sulfur Dioxide Absorber

You are a member of a group of design engineers designing equipment for the recovery of SO, from stack gases.

The group leader has asked you to determine the optimum size of the SO, absorption tower. Specifically, he has asked you to determine the height and cross-sectional area of the optimum absorption tower and to present your recommendations in the form of a formal report.

Your group has held several meetings to discuss the proposed overall design. Following is a list of conditions, assumptions, and data on which the group has decided to base the design:

100,000 ft^3 of gas per minute at 300°F and 1 atm are to be treated.

The entering gas contains 0.3 percent by volume SO, and 11.0 percent CO,, with the balance being N_2, O_2, and H_2O.

The average molecular weight of the entering gas = 29.4

The mole percent SO, in the exit gas is to be 0.01 percent.

The entering and exit pressures of the absorption column may be assumed to be 1 atm for purposes of calculating the SO, pressures.

The zinc oxide process will be used for recovering the SO,. In this process, a solution of H_2O, $NaHSO_3$, and Na_2SO_3 is circulated through the absorption tower to absorb the SO,. This mixture is then treated with ZnO, and the $ZnSO_3$ formed is filtered off, dried, and calcined to yield practically pure SO,. The ZnO from the calciner is reused, and the sulfite-bisulfite liquor from the filter is recycled.

The absorption tower will contain nonstaggered wood grids of the following dimensions:

Clearance = 1.5 in.

Height = 4 in.

Thickness = $\frac{1}{4}$ in.

Free cross-sectional area = 85.8%

Active absorption area per cubic foot of volume = a = 13.7 ft^2/ft^3

The average density of the gas at the tower entrance can be assumed to be 0.054 lb/ft^3. The sulfite-bisulfite liquid has a density of 70 lb/ft^3 and can be considered as having a zero equilibrium SO, vapor pressure at both the inlet and outlet of the tower.

The sulfite-bisulfite liquid must be supplied at a rate of 675 $\text{lb}/(\text{h})(\text{ft}^2$ of column cross-sectional area).

The optimum design can be assumed to be that corresponding to a minimum total power cost for circulating the liquid and forcing the gas through the tower. You may assume that this optimum corresponds to the optimum that would be obtained if fixed charges were also considered.

The following simplified equations are applicable for grids of the dimensions to be used:

$$K_g = 0.00222(G_0)^{0.8}$$

$$\frac{\Delta h_w}{L} = 0.23 \times 10^{-7}(G_0)^{1.8}$$

where K_g = molar absorption coefficient, lb mole of component absorbed/$(\text{h})(\text{ft}^2)(\text{atm})_{\text{log mean}}$

G_0 = superficial mass velocity of gas in tower, $\text{lb}/(\text{h})(\text{ft}^2)$

Ah,,, = pressure drop through tower, in. of water

L = height of tower, ft

The liquid is put into the absorption tower by means of a nozzle at the top of the tower. The pressure just before the nozzle is 35 psig. Assume the pump for the liquid must supply power to lift the liquid to the top of the tower and compress the liquid to 35 psig. Use a 10 percent safety factor on the above pumping-power requirements to take care of the friction in the lines and other minor losses.

The gas blower has an overall efficiency of 55 percent.

The pump has an overall efficiency of 65 percent.

Problem 4. Utilization of Liquid Methane Refrigeration for Liquefaction of Nitrogen and Oxygen

Management of a natural-gas transmission company is considering using liquid methane as a heat sink in producing 210 tons/day of liquid nitrogen and 64 tons/day of liquid oxygen for a neighboring customer. Accordingly, one of the company's engineers has outlined a scheme for doing this. The process description is as follows:

Air is compressed from atmospheric conditions to 20 atm and then purified. The clean dry gas is then chilled by counter-current heat exchange with liquid methane boil-off. The partially liquefied air stream serves as the reboiler stream for a dual-pressure air separation column. Before entering the 5-atm high-pressure lower section of the dual-pressure column, the high-pressure air stream is bled through a J-T valve. The bottoms of the lower column is enriched to approximately 40 mole % oxygen and is the feed for the 1-atm low-pressure upper column. The condensing vapors in the high-pressure column serve as a reboiler for the low-pressure column. High-purity N_2 is withdrawn from the top of the high-pressure column, reduced in pressure, and used as reflux for the low-pressure column. High-purity liquid oxygen is withdrawn as bottoms from the low-pressure column while high-purity nitrogen vapor is withdrawn from the top of this same column. The latter stream is warmed to room temperature, compressed to 20 atm, and then cooled in this same heat exchanger down to a low temperature. This high-pressure pure nitrogen stream is then condensed by counter-current exchange with the liquid methane. After liquefaction, it is expanded to atmospheric pressure, and the resulting vapor is recycled and combined with the overhead stream from the low-pressure column to complete the liquefaction cycle all over again.

As a chemical engineer, you have been asked to analyze the process and make recommendations. Start by making as complete a flow sheet and material balance as possible assuming an 85 percent operating factor. Outline the types of equipment necessary for the process. Determine approximate duties of heat exchangers and sizes of major pieces of equipment. Instrument the plant as completely as possible outlining special problems which might be encountered. List all of the additional information which would be needed in order to finish completely the design evaluation.

Problem 5. Production of High-Purity Anhydrous Ammonia

A chemical company is considering the production of 1000 tons/day of high-purity anhydrous ammonia. The method selected is a high-pressure steam-methane reforming process. The process description is as follows:

The steam-methane reforming process produces ammonia by steam reforming natural or refinery gas under pressure, followed by carbon monoxide shift, purification of raw synthesis gas, and ammonia synthesis. In the process, saturated and unsaturated hydrocarbons are decomposed by steam according to the basic equation:

$$CH_4 + H_2O \rightarrow CO + 3H_2$$

Feed streams high in olefins or sulfur require pretreatment. The primary reformer converts about 70 percent of a natural-gas feed into raw synthesis gas, in the presence of steam using a nickel catalyst. In the secondary reformer, air is introduced to supply the nitrogen required for ammonia. The heat of combustion of the partially reformed gas supplies the energy to reform the remainder of the gas after reacting with the oxygen in the air. High-pressure reforming conserves 30-40 percent in compressor horsepower over usual practices giving low-pressure synthesis gases.

Next, the mixture is quenched and sent to the shift converter. Here CO is converted to CO_2 and H_2. When heat is still available after satisfying the water requirement for the shift reaction, a waste-heat boiler may be installed. Shift reactor effluent, after heat recovery, is cooled and compressed, then goes to the gas-purification section. CO_2 is removed from the synthesis gas in a regenerative MEA (monoethanolamine) or other standard recovery system. After CO_2 removal, CO traces left in the gas stream are removed by methanation. The resulting pure synthesis gas passes to the oil separator, is mixed with a recycle stream, cooled with ammonia refrigeration, and goes to the secondary separator where anhydrous ammonia (contained in the recycle stream) is removed. Synthesis gas is then passed through heat exchange and charged to the catalytic ammonia converter. Product gas from the converter is cooled and exchanged against converter feed gas. Anhydrous liquid ammonia then separates out in the primary separator and, after further cooling, goes to the anhydrous ammonia product flash drum. The feed to the reforming section is normally in excess of 300 psig. The pressure, however, is not fixed and may be varied to provide optimum design for specific local conditions. Temperatures in primary and secondary reformers are 1400 to 1800°F, while shift reaction temperatures are 700 to 850°F in the first stage and 450 to 550°F in the second stage. Ammonia synthesis is normally performed at 3000 psig. Temperature in the quench-type ammonia converter is accurately and flexibly controlled inside the catalyst mass to allow a catalyst basket temperature gradient giving a maximum yield of ammonia per pass, regardless of production rate.

Analyze this process and make as complete a flow sheet and material balance as possible assuming a 90 percent operating factor. Outline the types of equipment necessary for the process. Determine approximate duties of heat exchangers and sizes of the major pieces of equipment. What additional information will be needed in order to finish completely the preliminary design evaluation?

PRACTICE-SESSION PROBLEMS?

Problem 1. Cost for Hydrogen Recovery by Activated-Carbon Adsorption Process+

What is the cost, as cents per 1000 ft^3 (at SC) of 95 mol % H_2, for recovering hydrogen of 95 mol % purity from 10 million ft^3 (at SC) of gaseous feed per day if the following conditions apply?

Feed composition by volume = 72.5 percent H_2 and 27.5 percent CH,.

A hydrogen-recovery method based on selective adsorption by activated carbon will be used.

For the carbon adsorption, three separate beds will be needed so that one bed can be in continuous use while the other two are being desorbed or reactivated. Base the recovery on a single pass and an absorption cycle of 1 hour per bed.

0.00838 mol of material is adsorbed per hour per pound of activated carbon.

The composition of the adsorbed phase is 96.8 mol % CH,, the balance being hydrogen.

The cost of activated carbon is $3.65 per pound.

The annual amount of additional carbon necessary is 15 percent of the initial charge.

The total plant cost (equipment, piping, instrumentation, etc.) equals $4,750,000.

[†] The following problems are recommended for solution by students working in groups of two or three during a three-hour practice session. Many of these problems have been adapted from old Annual AIChE Student Contest Problems as shown by footnotes for the individual problems. The original problems and winning solutions are available from AIChE Headquarters, 345 E. 47 St., New York, NY 10017. A useful procedure for using these Practice-Session Problems is to have students examine the original AIChE Student Contest Problem and Solution before the class period and then have someone (student or teacher) present a discussion on it in the first $\frac{1}{2}$ hour or 1 hour of the practice session. Then the students can break up into groups and carry out the solution to the problem assigned during the last 2 hours of the practice session. It is desirable for the students to have an opportunity to examine the correct solution to the problem immediately after they turn in their solution at the end of the 3-hour session.

[‡] Adapted from 1947 AIChE Student Contest Problem.

The capital investment equals the total plant cost plus process materials (process materials are considered as auxiliaries-i.e., process materials are only the initial charge of activated carbon).

The capital investment must be completely paid off (no scrap value) in 5 years.

The operating cost per year, including labor, fuel, water, feed, regeneration, fixed charges minus depreciation, and repairs and maintenance, equals $2,250,000 (operating costs not included in this value are the cost of the additional make-up carbon necessary and depreciation).

The plant operates 350 days/year.

Problem 2. Adsorption-Tower Design for Hydrogen Purification by Activated Carbon†

Your plant is producing 10 million ft^3 (measured at SC) per day of a gas containing 72.5 vol % H_2 and 27.5 vol % CH_4. It is proposed to pass this gas through activated carbon to obtain a product gas containing 95 vol % H_2. The activated carbon shows a preferential selective adsorption of the CH_4.

An adsorption-desorption-regeneration cycle using three fixed beds will be used. One bed will be regenerated and purged over an 8-h period. During this period, the other two beds will be on alternate 1-h adsorption and 1-h desorption cycles to permit a continuous operation. A single pass of the gas will be used.

Each individual bed may be designed to include a number of carbon-packed towers in parallel. The diameter of the individual towers must all be the same, and the diameter may be 6, 9, 12, or 15 ft.

Determine the following:

1. The number of individual units (or towers) in each bed to give the minimum total cost for the towers.
2. The height and diameter of the towers for the conditions in part 1.

Data and information previously obtained for the chosen conditions of the process (i.e., adsorption at 400 psia, desorption at 20 psia, and an average adsorption temperature of 110°F) *are as follows:*

0.0063 lb mol of material is adsorbed per hour per pound of activated carbon.

---

†Adapted from 1947 AIChE Student Contest Problem.

TABLE 3
Cost data

Column diameter, ft	6	9	12	15
Dollars per foot of length	1170	2140	3280	5840
Cost per tower for skirt or support, dollars	1950	3240	4540	5200

The composition of the adsorbed phase is 96.8 mol % CH,, the balance being H,.

The carbon has a bulk density of 0.30 g/cm^3.

The gas velocity in the adsorbers should not exceed 1 ft/s based on the cross-sectional area of the empty vessel. This applies to all the cycles including the adsorption and the regeneration and purge.

The feed gases enter the adsorbers at 400 psia and 80°F. The product gases leave the adsorbers at almost 400 psia and a temperature of 140°F. Regeneration includes 30 ft^3 (SC) of flue gas per pound of carbon and 80 standard cubic feet of purging air per pound of carbon. The flue gas is at 600°F (its maximum temperature) and 5 psig, and the air is at 90°F and 5 psig. The air may reach a maximum temperature of 600°F at the start of the purging.

For each head (either top or bottom), add equivalent cost of 5 ft additional length per vessel. Cost data are given in Table 3.

Problem 3. Design of Rotary Filter for Sulfur Dioxide Recovery System[†]

As a member of a design group working on the design of a recovery system for SO,, you have been asked to estimate the area necessary for a rotary vacuum filter to handle a zinc sulfite filtration. You are also to determine the horsepower of the motor necessary for the vacuum pump. Do not include any safety factors in your results.

The following conditions have been set by your group:

A slurry containing 20 lb of liquid per pound of dry $ZnSO_3 \cdot 2.5H_2O$ is to be filtered on a continuous rotary filter to give a cake containing 0.20 lb of H_2O per pound of dry hydrate.

One hundred pounds of $ZnSO_3 \cdot 2.5H_2O$ in the slurry mixture will be delivered to the filter per minute.

[†] See Chap. 14 of this text for design basis.

A drum speed of 0.33 rpm will be used, and a vacuum of 10.2 in. mercury below atmospheric pressure will be used.

The fraction of drum area submerged will be assumed to be 0.25.

The fraction of drum area available for air suction will be assumed to be 0.10.

The temperature of the slurry is 110°F, and the air into the vacuum pump can be considered to be at 110°F. C_p/C_v for air at 110°F = 1.4. The temperature and pressure of the air surrounding the filter are 70°F and 1 atm, respectively.

The specific cake resistance and the specific air-suction cake resistance can be assumed to be independent of pressure drop, drum speed, temperature, fraction of drum submerged, fraction of drum available for suction, and slurry concentration. However, to eliminate possible errors due to this assumption, a lab test should be run at conditions approximating the planned design. The results of these tests can be used as a basis for the design (i.e., cake and filtrate compositions and densities can be assumed to be the same for the design as those found in the lab).

The vacuum pump and motor have an efficiency (isentropic) of 85 percent.

It can be assumed that all of the $ZnSO_3 \cdot 2.5H_2O$ is filtered off and none remains in the filtrate.

The resistance of the filter medium can be assumed to be negligible.

Laboratory data compiled at the request of the SO,-recovery design group (results of filtration of zinc sulfite slurry on an Oliver rotary vacuum filter) are as follows:

Total area of filter	4.15 ft^2
Fraction of area submerged	0.20
Fraction of area available for air suction	0.10
Vacuum	9 in. Hg below atmospheric pressure
Slurry concentration	12 lb liquid/lb dry $ZnSO_3 \cdot 2.5H_2O$
Temperature	110°F
Drum speed	0.40 r/min
Density of wet cake leaving filtering zone	100 lb/ft^3
Pounds of liquid per pound of dry $ZnSO_3 \cdot 2.5H_2O$ in wet cake leaving filtering zone	0.6
Density of filtrate	68.8 lb,‘ft ^{3}
Viscosity of filtrate	0.6 centipoise
Pounds of water per pound of dry $ZnSO_3 \cdot 2.5H_2O$ in final cake	0.20
Time interval	5.0 min
Volume of filtrate	0.95 ft^3
Volume of air at SC	8.5 ft^3

Problem4 Return on Investment for Chlorine Recovery System-f

The off-gas from a chloral production unit contains 15 vol % Cl,, 75 vol % HCl, and 10 vol % $EtCl_2$. This gas is produced at a rate of 150 cfm based on 70°F and 2 psig. It has been proposed to recover part of the Cl, by absorption and reaction in a partially chlorinated alcohol (PCA). The off-gas is to pass continuously through a packed absorption tower counter-current to the PCA, where Cl, is absorbed and partially reacts with the PCA. The gas leaving the top of the tower passes through an alcohol condenser and thence to an existing HCl recovery unit.

The reaction between absorbed Cl, and the PCA is slow, and only part of the absorbed Cl, reacts in the tower. Part of this PCA from the bottom of the tower is sent to a retention system where the reaction is given time to approach equilibrium. The rest of the PCA is sent to the chloral production unit. Ethyl alcohol is added to the PCA going to the retention system. Then the PCA is recycled from the retention system to the top of the absorption column, and ethyl alcohol is added at a rate sufficient to keep the recycle rate and recycle concentration constant.

Your preliminary calculations have set the optimum conditions of operations, and all costs have been determined except the cost for the absorption tower. Using the following data and information, determine the yearly percent return on the capital expenditure:

The gases leaving the top of the tower contain 2 vol % Cl, (on the basis of no PCA vapors present in the gas).

The PCA entering the top of the tower contains 0.01 mol % free Cl,.

The PCA leaving the bottom of the tower contains 0.21 mol % free Cl,.

The recycle rate is 200 gpm (entering the top of the absorption tower).

The gas rate at the top of the column is 27.2 lb mol/h.

The PCA entering the top of the tower has a density of 68.5 lb/ft^3 and an average molecular weight of 70.

The column is operated at a temperature of 35°C and a pressure of 1 atm.

The gases enter the bottom of the tower at 70°F and 2 psig.

No Cl, is absorbed in the alcohol condenser.

The column is packed with l-in. porcelain Raschig rings, and a porcelain tower is used.

Laboratory tests with a small column packed with Raschig rings have been conducted. These tests were carried out at 35°C using PCA and off-gas having the same concentrations as those in your proposed design. These data have been scaled up to apply to l-in. Raschig rings and are applicable to your column. The results are given in Table 4.

---

†Adapted from 1949 AIChE Student Contest Problem.

TABLE 4

m_2G_2/L_2 based on conditions at dilute end of column	Height of a transfer unit (log-mean method), ft
0.178	3.6
0.571	4.8
1.04	6.2
1.53	7.6

The absorption of Cl_2 in the PCA follows Henry's law, and the following relation may be used for determining the number of transfer units:

$$N_t = \frac{y_1 - y_2}{\Delta y_m}$$

where Δy_m = log-mean driving force = $\dfrac{\Delta y_1 - \Delta y_2}{\ln(\Delta y_1/\Delta y_2)}$

$$\Delta y_1 = y_1 - y_1^*$$

$$\Delta y_2 = y_2 - y_2^*$$

Equilibrium data for Cl_2 and the PCA at 35°C and 1 atm are given in Table 5. These data apply over the entire length of the column.

The maximum allowable velocity (based on conditions of the gas at the inlet to the tower) is 1.5 ft/s. The column will operate at 60 percent of the maximum allowable velocity.

TABLE 5

Free Cl_2 in PCA, x, mole %	Free Cl_2 in gas (based on PCA vapor-free gas), y, mole %
0.1	2.0
0.2	4.0
0.3	6.0

Necessary cost data are given in the following:

1. Tower:

Diameter, in	12	18	24	30	36	42
Porcelain, $/ft of length	450	650	880	1170	1490	1850
Top or bottom heads, porcelain, each, $	390	550	750	970	1230	1520

2. Tower packing. One-inch Raschig rings, porcelain = $24.00 per cubic foot.
3. Capital expenditure minus cost of absorption tower = $80,000.
4. Net annual savings (taking alcohol loss and all other costs, such as interest, rent, taxes, insurance, depreciation, maintenance, and other overhead expenses into consideration) = $40,000.

NOTE: This $40,000 has been determined by developing an accurate estimate of the absorption-tower cost and can be taken as the actual net savings.

Nomenclature for Prob. 4

G = molar gas flow, mol/h
L = molar liquid flow, mol/h
m = slope of equilibrium curve, y/x
N_t = number of transfer units
y = mole fraction of chlorine in gas
y^* = equilibrium mole fraction of chlorine in gas

Subscripts

1 = bottom of tower
2 = top of tower

Problem 5. Economic Analysis of Chlorine Recovery System[†]

The data shown in Table 6 have been obtained for a chlorine recovery system in which all values have been determined at the optimum operating conditions of retention time and recycle rates.

Determine the following:

1. The percent Cl_2 in exit gas at break-even point.
2. The maximum net annual savings and percent Cl_2 in exit gas where it occurs.

[†] Adapted from 1949 AIChE Student Contest Problem.

TABLE 6

Mole % Cl, in exit gas	Total capital expenditure, dollars	Total annual operating costs (fixed costs, production costs, overhead, etc.), dollars	Gross annual savings by using process, dollars
0.2	368,000	336,000	456,000
1.0	304,000	316,000	444,000
2.0	272,000	296,000	420,000
5.0	212,000	268,000	332,000
10.0	136,000	236,000	188,000

3. The maximum percent return on the capital expenditure and percent Cl, in exit gas where it occurs.
4. Which investment would you recommend and why?

Problem 6. Optimum Thickness of Insulation

Insulation is to be purchased for 3 miles of 10-in.-OD pipe carrying saturated steam at 250°F. The average air temperature for the year for the surroundings is 45°F.

It is estimated that the life period of the installation will be 20 years with negligible scrap value. The sum of fixed charges excluding depreciation is 10 percent, and maintenance is estimated to be 2 percent annually of the F.C.I.

One company has submitted a bid which includes installation at a cost of $\$0.25D^{1.3}$ per lineal foot, where D is the outside diameter of the lagging in inches. Using the following data and equations, what thickness of insulation should be used for this job in order to give a 50 percent return on the full investment?

The line will be in continuous service 365 days/year. The steam is valued at \$1.30 per 1000 lb. The thermal conductivity of the lagging is 0.04 Btu/(h)(ft^2)(°F/ft). Steam-film and pipe resistance may be neglected.

Heat losses by conduction and convection from the surface may be calculated by the use of the equation

$$h_c = 0.42\left(\frac{\Delta t_s}{D_o'}\right)^{0.25}$$

where Δt_s = temperature difference between surface of lagging and air, °F
D_o' = OD, in.
h_c = Btu/(h)(ft^2)(°F)

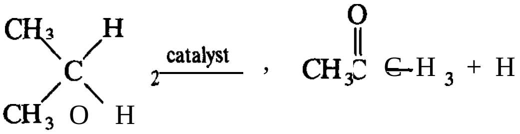
An average value of h_r = 1.2 Btu/(h)(ft^2)(°F) may be used to determine heat losses by radiation. This is an adjusted value such that total heat loss per hour may be calculated by the equation

$$Q = (h_c + h_r)A \text{ At,}$$

A mathematical setup with all necessary numbers and a description of the method for final solution will be satisfactory.

Problem 7. Capacity of Plant for Producing Acetone from Isopropanolf

Acetone is produced by the dehydrogenation of isopropanol according to the following reaction:



The reverse of the above reaction can be neglected.

The catalyst used for the process decreases in activity as the amount of isopropanol fed increases. This effect on the reaction rate is expressed in the following:

$$k = \frac{0.000254NT}{V}\left(2.46 \ln \frac{1}{1-\alpha} - \alpha\right)$$

where α = fraction of isopropanol converted to acetone and side products
= (moles isopropanol converted)/(moles isopropanol supplied)
k = reaction-rate constant, s^{-1}
N = lb mol of isopropanol fed to converter per hour
T = absolute temperature, “R.
V = catalyst volume, ft^3

The feed rate of isopropanol (N) is maintained constant throughout the entire process.

The fresh catalyst has an activity such that $k = 0.30\ \text{s}^{-1}$.

After 10,000 lb of isopropanol per cubic foot of catalyst has been fed, $k = 0.15\ \text{s}^{-1}$.

A plot of log k versus total *isopropanol fed as pound moles* is a straight line.

The maximum production of acetone is 76.1 lb mol/h. This maximum production can be considered as at zero time (i.e., when $k = 0.30\ \text{s}^{-1}$)

A constant temperature of 572°F and a constant pressure of 1 atm are maintained throughout the entire process.

---

†Adapted from 1948 AIChE Student Contest Problem.

The catalyst volume $V = 250\ \text{ft}^3$.

$$\text{Efficiency} = \frac{\text{moles acetone produced}}{\text{moles isopropanol consumed}}(100) = 98\% \text{ at } 572°\text{F}$$

The catalyst can be restored to its original activity with a 48-hour reactivation.

The catalyst will be thrown away after the last operating period each year.

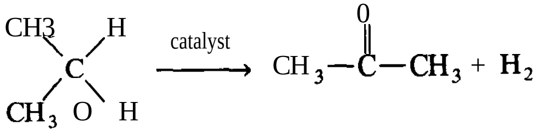
The unit will operate 350 days/year (this includes reactivation shutdowns). The other 15 days are used for repairs and replacing the old catalyst.

If nine catalyst reactivations are used per year, what will the production of acetone be in pounds per year? Assume all acetone produced is recovered.

Outline your method of solution. The actual mathematical calculations are not necessary.

Problem 8. Equipment Design for the Production of Acetone from Isopropanol†

You are designing a plant for the production of acetone from isopropanol. Acetone is produced according to the following reaction:



The reaction can be assumed as irreversible.

The catalyst used in this process decreases in activity as the process proceeds. This effect on the reaction rate can be expressed as follows:

$$k = \frac{0.000254}{V} NT\left(2.46 \ln \frac{1}{1-\alpha} - \alpha\right)$$

where α = fraction of isopropanol converted to acetone and side products
= (moles isopropanol converted)/(moles isopropanol supplied)
k = reaction-rate constant, s^{-1}
N = lb mole of isopropanol fed to converter per hour
T = absolute temperature, °R
V = catalyst volume, ft^3

The catalyst must be regenerated periodically throughout the operation.

The fresh catalyst has an activity such that $k = 0.30\ s^{-1}$.

The products from the reactor (unconverted isopropanol, acetone, side products, and some water from the impure isopropanol feed) are sent to a continuous distillation column where purified acetone is removed.

---

†Adapted from 1948 AIChE Student Contest Problem.

TABLE 7

Per 1000 lb/h of acetone distilled over and removed as product	
Column cross-sectional area, ft^2	6.4 (based on 12-in. plate spacing and 2-in. liquid depth)
Condenser area, ft^2	220
Steam, lb/h	1100

A distillation column, calandria, and condenser are now available in your plant, and you are to determine if these can be used for the purification step.

Using the following data, determine whether or not the column, calandria, and condenser are usable. If these are not satisfactory, determine the purchase cost of the necessary equipment. Do not buy any new equipment unless you need it. However, the present equipment may be used in another part of the plant at a later date.

Feed rate of isopropanol (N) is kept constant at 97 moles of isopropanol per hour during the entire operation.

A constant temperature of 572°F and a constant pressure of 1 atm are maintained throughout the entire operation.

The catalyst volume $V = 250\ \text{ft}^3$.

$$\text{Efficiency of conversion} = \frac{\text{moles acetone produced}}{\text{moles isopropanol consumed}}(100) = 98\% \text{ at } 572^\circ\text{F}$$

The present distillation tower contains 50 actual plates. Your calculations have indicated that a reflux ratio of 1: 1 will allow 98 percent of the acetone to be removed, assuming a 40 percent plate efficiency. The product material may be assumed as 100 percent acetone. These conditions are satisfactory, and you have made calculations at these conditions, giving the results shown in Table 7.

The overall heat-transfer coefficient in the calandria is 250 $\text{Btu}/(\text{h})(\text{ft}^2)(^\circ\text{F})$.

Saturated steam is available at 50 psig. Neglect any sensible heat transfer from steam condensate to boiling liquid. The temperature-difference driving force At in the calandria may be assumed to be 90°F.

Your calculations have indicated that the present condenser on the column is satisfactory and may be used for this process.

The necessary data on the present equipment are given in Table 8. Table 9 shows the installed-cost data for any new equipment which must be purchased.

TABLE 8

Number of plates in column	50
Column diameter	72 in.
Plate spacing	12 in.
Liquid depth on each plate	2 in.
Calandria heat-transfer area	104 ft^2
Calandria shell working pressure	60 psig

TABLE 9
Steel heat exchangers

Surface, ft^2	cost, \$ / ft^2
100	130
200	94
300	78
500	65

Steel Distillation Column

Estimate on the basis of \$160.00 per square foot of tray area. Column diameters should be even multiples of 6 in.

Problem 9. Quick-Estimate Design of Debutanizer†

You are the chief design engineer at a large petroleum refinery. The head projects engineer has asked you to make a preliminary design estimate for a proposed debutanizer. You are to present the following preliminary information to a group meeting 3 h from now:

1. Number of plates for proposed debutanizer column
2. Diameter of proposed column
3. Outside tube area required for heating coils in reboiler (coils to contain saturated steam at 250 psia, and average heat of vaporization of hydrocarbons in reboiler may be taken as 5000 cal/g mol)

The following information has been supplied you by the projects engineer:

Charge stock from catalytic cracker to debutanizer = 5620 BPSD (barrels per service day). (See Table 10).
Debutanizer to operate at 165 psia.
Two fractions are to be obtained-OVHD and BTMS.
OVHD is to contain 98.5 percent of the butanes and lighter components with a contamination of 1.5 mol % pentanes and pentenes.

The debutanizer must fractionate between NC, and IC,.

A search through your debutanizer-design card file gives the information shown in Cards A to C.

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†Adapted from L. J. Coulthurst, *Chem. Eng. Progr.*, **44**:257 (1948).

TABLE 10

Debutanizer charge stock from catalytic cracker

°API (60°F) = 100.3

Viscosity = 42 SSU at 120°F

Component	Mole %	Molecular weight
C_2'	0.1	28
C_2	1.2	30
H_2S	2.1	34
C_3''	16.3	42
C_3	6.9	44
IC_4''	6.5	56
NC:’	14.3	56
IC_4	10.8	58
NC_4	3.9	58
C_5''	11.9	70
IC_5	9.7	72
NC_5	2.3	72
C_6	11.8	86
C_7	2.1	96
C_8	0.1	112
	100.0	

Card A Debutanizer: 30 trays

Feed 574 BPSD	Reflux 635 BPSD	OVHD 354 BPSD	BTMS 220 BPSD
5897 lb/h 84.6 mol/h	5630 lb/h 100.6 mol/h	3114 lb/h 56.0 mol/h	2783 lb/h 28.6 mol/h
Mole %	Composition same as OVHD	Mole %	Mole %
C_3'' 12.4	Reflux ratio = 1.8 : 1	C_3'' 18.8	C_4’s 1.4
C_3 6.1		C_3 9.2	C_5—400°F 98.6
C_4’s 47.9		C_4’s 71.6	100.0
C_5—400°F 33.6		C_5—400°F 0.4	
100.0		100.0	
“API = 68.8	“API = 100.7	“API = 100.7	“API = 39.7
Basis: feed	119 mol%	66.2 mol%	33.8 mol%

Card B Values of K at 165 psia

Listings derived principally from the data of Scheibel and Jenny, *Ind. Eng. Chem.*, **37**:80 (1945).

Hydrocarbon	260°F	280°F	285°F	290°F	300°F
n-Butane	1.75	2.00	2.06	2.13	2.25
n-Pentene	1.15	1.30	1.34	1.39	1.47
Isopentane	1.03	1.19	1.23	1.28	1.37
n-Pentane	0.90	1.06	1.11	1.16	1.25
n-Hexane	0.46	0.58	0.61	0.64	0.70
n-Heptane	0.24	0.31	0.32	0.35	0.39
n-Octane	0.13	0.17	0.18	0.19	0.21

Card C Recommended overall heat-transfer coefficients (based on outside coil heating area)

	Debutanizer service	Transfer rate, Btu/(h)(ft^2)(°F)
Condenser	Butane to water	100–110
Reboiler	Gasoline to steam	120-140
Reboiler	Gasoline to hot oil	50-75
Exchanger	Gasoline to gasoline	80
Preheater	Gasoline to steam	110-120
Cooler	Gasoline to water	75–90

Problem 10. Economic Analysis of Formaldehyde-Pentaerythritol Plant†

You are a member of a firm doing consulting work on chemical engineering design. The G. I. Treyz Chemical Company of Cooks Falls, N.Y., has asked your firm to determine the advisability of adding a pentaerythritol production unit to its present formaldehyde plant. You have been sent to Cooks Falls to analyze the situation and obtain the necessary details.

A conference with G. Victor Treyz, owner of the plant, supplies you with the following information:

The present formaldehyde plant cost $1,140,000 several years ago and is in satisfactory operating condition. It produces formaldehyde by the oxidation of

---

†Adapted from a real-life situation for one of the co-authors of this text.

methanol. The yearly fixed charges (interest, rent, taxes, insurance, and depreciation) at the plant amount to 15 percent of the initial investment.

The plant capacity is 100,000 lb of formalin (37.2 percent HCHO, 8 percent CH_3OH as inhibitor, and 54.8 percent H_2O by weight) per day.

Miscellaneous costs (salaries, labor, office expenses, lab supplies, maintenance, repairs, communication, sales, silver catalyst replacement, etc.) amount to $360,000 per year.

The overall efficiency of conversion of methanol into formaldehyde equals 80 percent, i.e., 0.8 lb of CH_3OH is converted to HCHO per pound of CH_3OH decomposed.

The total cost of utilities (fuel, electricity, steam, water, etc.) equals 5 percent of the total cost of producing the formaldehyde.

Mr. Treyz has included all the smaller costs such as insurance benefits, etc., in the overhead cost so that the total cost of the present operation is the sum of fixed charges, overhead cost, utilities, and methanol.

The proposed PE plant is to produce 6000 lb of final pentaerythritol per day using the inhibited formalin produced at the plant. The basic reaction involving lime, acetaldehyde, and formaldehyde can be assumed to be going to completion. Only 70 percent of the PE produced in the reaction is obtained as the final product. Any costs due to the presence of excess $Ca(OH)_2$ can be neglected. The calcium formate formed must be discarded.

The initial installed cost of the proposed plant is $500,000. It can be assumed that the yearly cost of the new operation minus raw-materials costs will be 40 percent of the initial installed cost. This 40 percent includes fixed charges, overhead, utilities, and all other expenses except raw materials.

Both plants operate 350 days/year.

Mr. Treyz feels he can sell 6000 lb of PE per day at the present market price. He can also sell 100,000 lb of formalin per day, but he is willing to make the new investment if it will give him better than a 30 percent yearly return on the PE initial plant investment. Ignore income-tax effects and working capital.

Following is a list of prices supplied by Mr. Treyz. All these prices are f.o.b. Cooks Falls, N.Y., and they are to be used for the cost estimate.

Methanol, carload lots	\$ 0.60/gal = \$0.091/lb
Formaldehyde (or Formalin), 37.2% HCHO, 8% CH_3OH	0.10/lb
Acetaldehyde	0.46/lb
Lime as pure CaO	40.00/ton
Pentaerythritol	0.72/lb

From the preceding information, determine the following:

1. The present profit per year on the formaldehyde unit.
2. The total profit per year if the PE unit were in operation.
3. Yearly percent return on the PE initial plant investment.
4. Should the Treyz Company make the investment?

Problem 11. Operating Time for Catalytic Polymerization Reactor to Reach Minimum Allowable Conversion†

A catalytic polymerization plant is to operate continuously with a special catalyst with properties and results as given below. Under the following specified operating conditions, how many days could each reactor remain on stream from the time of fresh (catalyst age zero) catalyst charge until the percent propylene conversion through the catalyst bed drops to 93.75 percent. What will the pressure drop across the catalyst bed be at that time? How often should a new recharged unit come on stream if all units are to operate on identical staggered schedules (i.e., how many days will each unit be down for dumping and recharging)?

Total feed stream is 15,000 barrels per day and is 40 percent by volume propylene and 60 percent by volume propane.

Five reactors are available, each holding 20,000 lb of the catalyst.

All reactors will operate on identical staggered schedules.

The unit will be operated with the same flow rate in four of the reactors while the fifth one is down for dumping and catalyst recharging, with this dumping and recharging requiring at least 5 days but no more than 8 days.

The temperature in the reactors will be maintained constant at 430°F. 0.715 barrel of polymer is obtained for every barrel of propylene converted. One barrel equals 42 gallons.

$$\text{Catalyst age factor} = \text{A} = \frac{\text{gallons of polymer produced since catalyst charging}}{\text{pounds of catalyst charged}}$$

$$A = \frac{\left[\int_0^D F(t)\,dt\right]\left[\text{conversion factors to give gallons of polymer}\right]_{\text{arith. ave}}}{\text{pounds of catalyst}}$$

t = time, days

D = time in days at which A P/F^2 is to be calculated

$F = F(t)$ = reactor total feed rate as thousands of barrels/day

AP = pressure drop across reactors, psi

At the operating pressure and temperature of 430°F, the following data apply for the special catalyst used:

A plot of log A P/F^2 versus catalyst age, A, is linear with A $P/F^2 = 0.2$ at $A = 0$ and $\Delta P/F^2 = 100$ at $A = 84.90$.

(Resultant equation is log $\Delta P/F^2 = 0.03179A - 0.699$.)

The conversion of propylene to polymer at the time of fresh catalyst charge (catalyst age zero) is 97.66 percent.

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†Adapted from 1974 AIChE Student Contest Problem.

The following data apply for the catalyst and indicated temperature (linear interpolation is satisfactory):

A	$\dfrac{\%\text{ conversion of propylene at catalyst age } A}{\%\text{ conversion of propylene at catalyst age zero}}$
0	1.00
10	0.995
20	0.99
30	0.98
40	0.97
50	0.96
60	0.935
70	0.91
80	0.87
90	0.82
100	0.76

Problem 12. Sizing and Costing of Multicomponent Distillation Column for Biphenyl Recovery Unit†

The feed to a multicomponent distillation column is as follows (in order of decreasing volatility):

Component	Feed rate as lb mol/h
Toluene	0.488
Naphthalene	1.599
Biphenyl	16.835
O-Methyl biphenyl	0.208
P-Methyl biphenyl	0.333
M-Methyl biphenyl	0.121
Diphenylenemethane (Fluorene)	1.029
Phenanthrene	0.769
M-Terphenyl	1.919
	23.301

A carbon steel, bubble-cap distillation column with ten trays operated at an average pressure of 400 mm Hg, an average temperature of 280°C, and a reflux ratio of 8 to 1 will give an overhead product containing 98 percent of the entering biphenyl and a negligible amount of methyl biphenyl and heavier. Based on the following data and assumptions, estimate the cost of the distillation tower including installation and auxiliaries (not including reboiler and condenser) at the present time:

Pressure drop in column can be neglected and average temperature and pressure can be used for calculations.

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†Adapted from 1975 AIChE Student Contest Problem.

Ideal gas laws apply to the gas mixture under these conditions.

Liquid feed is at its boiling point.

Column operates adiabatically and constant molal overflow assumption is acceptable.

The average molecular weight of the gas can be taken as that of biphenyl = $C_6H_5 \cdot C_6H_5 = 154.2$.

The average specific gravity of the liquid in the column is 0.72.

The tray spacing is 24 inches with a 2-inch slot liquid seal.

Assume the surface tension of the liquid is 20 dyn/cm.

Size the column by using the K_v from both of the plots given in Chap. 16 for maximum allowable vapor velocity, and use an 85 percent safety factor on the maximum allowable vapor velocity. For comparison purposes, give an answer for each of the two K_v estimates.

Use Fig. 16-28 in Chap. 16 which gives cost data per plate for the final cost estimate.

Problem 13. Design of Reactor for Coal Conversion to Nonpolluting Fuel Oil (Plus Partial Solution)†

A plant is being designed to produce low-sulfur oil from coal under the conditions as outlined in the following. A major factor in the design is to minimize the volume of the reactor, and you are to carry out some preliminary studies for the reactor system. Specifically, you are to determine the total volume of the reactor if it is operated isothermally at 800°F for the case of a single, ideal, plug-flow reactor operation and for the case of a single, back-mix (continuous-stirred-tank reactor) reactor system with the conditions and assumptions as outlined in the following.

Operating Conditions

Plant is to produce 50,000 barrels (based on 60°F)/day of low-sulfur oil (0.4 wt % sulfur) from coal. Following are specifications for the coal feed and the product oil:

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†Adapted from 1976 AIChE Student Contest Problem.

Coal Feed

Bulk density, lb/ft^3 = 45.0

†**Proximate** Analysis, wt %	
Moisture	1.5
Ash	10.3
Volatile matter	35.5
Fixed Carbon	52.1
Total	100.0

†**Ultimate** Analysis, wt %	
Carbon	70.2
Hydrogen	4.6
Nitrogen	1.0
Sulfur	3.6
Oxygen	10.5
Ash	10.1
Total	loo.0

Oil Product

4.4” API = Density of 64.97 lb/ft^3 at 60°F
Density = 44.8 lb/ft^3 at 800°F

Boiling distribution: true boiling point cut, wt %	
C_5—400°F	8.1
400-650°F	32.1
650-975°F	22.1
975°F +	31.7
Total	**100.0**

Ultimate Analysis, wt %	
Carbon	90.2
Hydrogen	8.5
Nitrogen	0.8
Sulfur	0.4
Oxygen	
Ash	0.1
Total	**100.0**

Coal in the slurry is 35 percent by weight with the balance being recycled oil of the same composition as the product oil.

A nickel-molybdenum on alumina catalyst in the form of $\frac{1}{8}$-in. spheres is used with a desulfurization activity (A,) of 1.25 and a bulk density of 42.0 lb/ft^3.

The Following Assumptions Apply for the Reactor System:

Pressure = 2500 psia and negligible pressure drop across the reactor.

25,000 ft^3 of gas at SC (SC = 60°F and 1 atm) flow to reactor per barrel of slurry feed (based on 60°F).

One barrel = 42 gal.

85 percent of the gas to the reactor is hydrogen, and the other 15 percent is methane with negligible H_2S content.

Yield of product is 4.2 barrels of product oil (at 60°F) per ton of coal (as received).

Average molecular weight of fuel oil is 301.

No hydrogen or methane or hydrogen sulfide is dissolved in the slurry.

Necessary heating and cooling units are available so reactors can be assumed to operate isothermally at 800°F.

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†See Perry’s “Chemical Engineers’ Handbook’ for discussion of these and methods of analysis (6th ed., p. 9-4). Proximate and ultimate analyses in this case were carried out with air-dried coal samples; so the oxygen and hydrogen in the “moisture” reported in the proximate analysis are included in the ultimate analysis.

Partial pressure of hydrogen for plug flow reactor can be assumed as constant at the arithmetic average of entrance and exit pressures.

Assume the term $(1 + K_{HS}P_{HS})$ in the rate equation stays constant for the plug-flow reactor at the arithmetic average of the entering and exit values.

Assume negligible volume change during the reaction so that $C_s = C_{s_o}(1 - X_s)$.

15 wt % of the fuel oil passing through the reactor is vaporized in the reactor section, and this can be doubled to 30 percent on a molal basis considering different volatilities of the components.

The carbon in the coal which is lost to the gas stream is converted to CH,, C_2H_6, C_3H_8, and C_4H_{10} in equal volume amounts so that the average carbon to hydrogen ratio of the resultant gas is 0.35714 on a mole basis considering hydrogen as being 1.008 lb per mole of hydrogen.

All the nitrogen in the coal that is lost is converted to gaseous NH,.

All the sulfur in the coal that reacts goes to H_2S.

Reactor sizing will be based on the rate equation for the desulfurization reaction as follows:

$$-r_s = k_s A_s \frac{C_s^2 P_H}{C_{s_o}(1 + K_{HS}P_{HS})}$$

where $-r_s$ = rate of sulfur removal, lb · mol/h · lb catalyst
k_s = reaction rate constant, ft^3/h · lb cat ·(psia)
$k_s = \exp(14.76 - 55{,}000/RT) = 7.405 \times 10^{-4}$ at 800°F
K_{HS} = adsorption constant for H_2S inhibition, (psia)$^{-1}$
$K_{HS} = 0.10 \exp(1200/RT) = 0.162$ at 800°F
R = 1.987 Btu/(lb mol)(°R) = Gas-law constant
T = temperature, °R
A, = desulfurization activity
C_s = sulfur concentration in slurry, lb mol/ft^3
P_H = hydrogen partial pressure, psia
P_{HS} = hydrogen sulfide partial pressure, psia

The reactor performance equations are as follows:

For plug-flow For back-mix (CSTR)

$$\frac{W}{Q} = C_{s_o}\int_{X_{s_i}}^{X_{s_f}} \frac{dX_s}{-r_s} \qquad \frac{W}{Q} = C_{s_o}\frac{(X_{s_f} - X_{s_i})}{-r_s}$$

where W = catalyst charge, lb
X_s = fractional conversion of sulfur
C_{s_o} = concentration of sulfur in slurry feed to reactor, lb mol/ft^3
Q = volumetric feed rate of slurry, ft^3/h
i = inlet value
f = final value

Suggestions

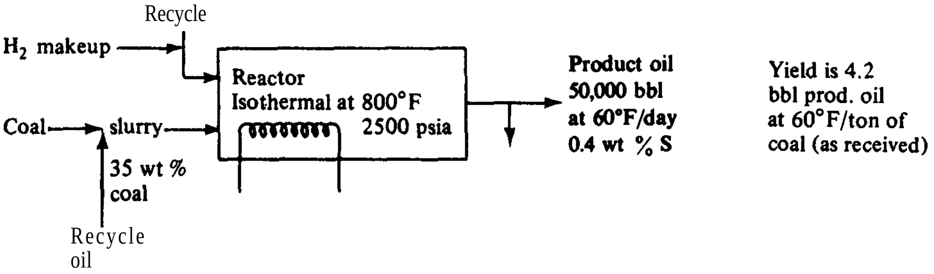
Base material balances on one ton (2000 lb) of coal as received.

Integrate rate expression for plug flow analytically (not graphically or by approximations).

See information as provided for initial part of solution presenting necessary material balances for the conditions given for this problem, and understand what was done.

Problem 13. Partial Solution

Following is the first part of the solution to Problem 13 dealing with the design of a reactor for coal conversion to nonpolluting fuel oil:



Material Balances

Choose as basis: 1 ton of coal as received = 2000 lb = 4.2 bbl of product oil at 60°F

$$\text{Fuel oil produced} = 4.2 \text{ bbl} \times 42 \frac{\text{gal}}{\text{bbl}} \times \frac{1}{7.48} \frac{\text{ft}^3}{\text{gal}} \times 64.97 \frac{\text{lb}}{\text{ft}^3} = 1532 \frac{\text{lb prod. oil}}{\text{ton coal}}$$

Material balance, overall-2000 lb coal

	Feed		Prod. Oil		Difference	
Mtl.	wt %	lb	wt %	lb	lb	
C	70.2	1404	90.2	1381.95	22.05	
H	4.6	92	8.5	130.23	-38.23	gain in H
N	1.0	20	0.8	12.26	7.74	
S	3.6	72	0.4	6.13	65.87	
O	10.5	210	—	—	—	
Ash	10.1	202	0.1	1.53	200.47	
Total	100	2000	100	1532.1		

Hydrogen material balance

22.05 lb of C in coal is burned to CH_4, C_2H_6, C_3H_8, or C_4H_{10} as 0.35714 mol C per mol H

$$\text{Hydrogen used to burn C} = \frac{22.05}{12.011}\text{mol} \times \frac{1 \text{ mol H}}{0.35714 \text{ mol C}} \times 1.008 \frac{\text{lb H}}{\text{mol H}}$$

$$= 5.19 \text{ lb H}$$

$$\text{Hydrogen used to make } NH_3 = \frac{7.74}{14.007}\text{mol N} \times 3 \frac{\text{mol H}}{\text{mol N}} \times 1.008 \frac{\text{lb H}}{\text{mol H}}$$

$$= 1.67 \text{ lb H}$$

$$\text{Hydrogen used to make } H_2S = \frac{65.87}{32.064}\text{ mol S} \times 2\,\frac{\text{mol H}}{\text{mol S}} \times 1.008\,\frac{\text{lb H}}{\text{mol H}}$$

$$= 4.14 \text{ lb H}$$

$$\text{Hydrogen used to make } H_2O = \frac{210}{16}\text{ mol O} \times 2\,\frac{\text{mol H}}{\text{mol O}} \times 1.008\,\frac{\text{lb H}}{\text{mol H}}$$

$$= 26.46 \text{ lb H}$$

Hydogen gain = 38.23 lb

Total H used = 75.69 lb H

$$\frac{75.69}{2.016} = 37.544 \text{ lb mol } H_2$$

Material balance, at reactor inlet, **for** *slurry concentration, basis-2000* lb coal

For slurry, 35 wt % is coal and 65 wt % is oil.

$$2000 \text{ lb coal} \times \frac{0.65 \text{ lb oil}}{0.35 \text{ lb coal}} = 3714 \text{ lb of recycle oil}$$

$$= \frac{3714 \text{ lb}}{44.8 \text{ lb/ft}^3 \text{ at } 800°\text{F}}$$

$$= 82.9 \text{ ft}^3 \text{ oil at } 800°\text{F}/2000 \text{ lb coal}$$

$$\text{Volume of coal} = \frac{2000 \text{ lb}}{45 \text{ lb/ft}^3} = 44.44 \text{ ft}^3$$

Total volume of slurry to reactor

$$= 82.9 + 44.44 = 127.34 \text{ ft}^3/2000 \text{ lb of coal fed}$$

Sulfur content of fuel oil in slurry

$$= 3714 \times 0.004 = 14.856 \text{ lb S}$$

Sulfur content of coal in slurry

$$= 2000 \times 0.036 = \underline{72.00 \text{ lb S}}$$

$$\text{Total} = 86.86$$

$$= \frac{86.86}{32.066} = 2.709 \text{ lb mol S}$$

Concentration of sulfur entering reactor in slurry

$$= C_{s_o} = \frac{2.709}{127.34} = 0.0213 \text{ lb mol/ft}^3$$

At reactor outlet

$$\text{Oil} = \underset{3714}{\text{Recycl.}} + \underset{1532}{\text{Prod.}} = 5246 \text{ lb} \quad \frac{5246 \text{ lb}}{44.8 \text{ lb/ft}^3} = 117.1 \text{ ft}^3 \text{ at } 800°\text{F}$$

$$\text{Sulfur in outlet oil} = 5246 \times 0.004 = 20.98 \text{ lb} = \frac{20.98}{32.066} = 0.65444 \text{ lb mol}$$

Concentration of sulfur in oil leaving reactor

$$= C_{S_f} = \frac{0.65444}{117.1} = 0.00559 \text{ lb mol/ft}^3$$

$$X_{S_f} = 1 - \frac{C_{S_f}}{C_{S_o}} = 1 - \frac{0.00559}{0.0213} = 0.738 \cong 0.74$$

$$X_{S_f} = \frac{C_{S_o} - C_S}{C_{S_o}}$$ ‘assuming constant fluid volumetric flow rate

Material balance for gas at entrance and exit **of** *reactor, basis-2000* lb coal

25,000 SCF of gas/bbl of slurry at 60°F is given as condition

$$\text{bbl of slurry at 60°F/2000 lb coal} = \left(\overset{\text{ft}^3\text{ coal}}{\frac{2000}{45}} + \overset{\text{ft}^3\text{ oil at 60°F}}{\frac{3714}{64.97}}\right)$$

$$\times\ 7.48 \text{ gal/ft}^3 \times \frac{1 \text{ bbl}}{42 \text{ gal}}$$

$$= 18.1 \text{ bbl/2000 lb coal}$$

Gas to reactor = 25,000 x 18.1 = 452,450 SCF of gas/2000 lb coal

Gas is 85% H_2; So, 0.85 X 452,450 = 384,583 SCF H_2/2000 lb coal

$$\frac{359 \times 520}{492} = 380 \text{ ft}^3\text{/mol at SC of 60°F and 1 atm}$$

$$\frac{384{,}583}{380} = 1014 \text{ lb mol } H_2\text{/2000 lb coal}$$

$$\text{15\% of the gas is CH,; so, } 452{,}450 \times \frac{0.15}{380} = \underline{179} \text{ lb mol } CH_4\text{/2000 lb coal}$$

$$\text{Total} = 1193$$

$$P_H \text{ at entrance to reactor} = \frac{1014}{1193}(2500) = 2125 \text{ psia}$$

$$P_{HS} \text{ at entrance} = 0 \text{ (given)}$$

Material balance, gas at exit of reactor, basis-2000 lb coal

Amount of fuel oil entering reactor is 3714 lb or

$$\frac{3714}{\text{avg mol wt } 301} = 12.34 \text{ lb mol/2000 lb coal}$$

Fuel oil is 15 wt % or 30 mol % vaporized; so 12.34 x 0.30 = 3.7 moles of fuel oil are vaporized in reactor leaving 8.64 moles of fuel oil in liquid and 3.7 moles of fuel oil in gas at reactor exit.

Assume no moles of H_2 or CH, or NH, or H_2S are dissolved in the liquid.

Total moles of H_2 in gas at exit $= 1014 - \underset{\text{used in reactor}}{37.544} = 976.5$ lbmol

Total moles of H_2O in gas at exit $= \dfrac{26.46}{2.016} = 13.1$ lbmol

Total moles of NH, in gas at exit $= \dfrac{7.7;}{14.007} = 0.55$ lbmol

Total moles of H_2S in gas at exit $= \dfrac{4.14}{2.016} = 2.06$ lb mol

Total moles of CH, in gas at exit $= 179.0$ lbmol

Total moles of fuel oil in gas at exit $= 3.7$ lbmol

Total exit gas moles = 1174.91 lb mol/2000 lb coal

Total entering gas moles = 1193 lb mol/2000 lb coal

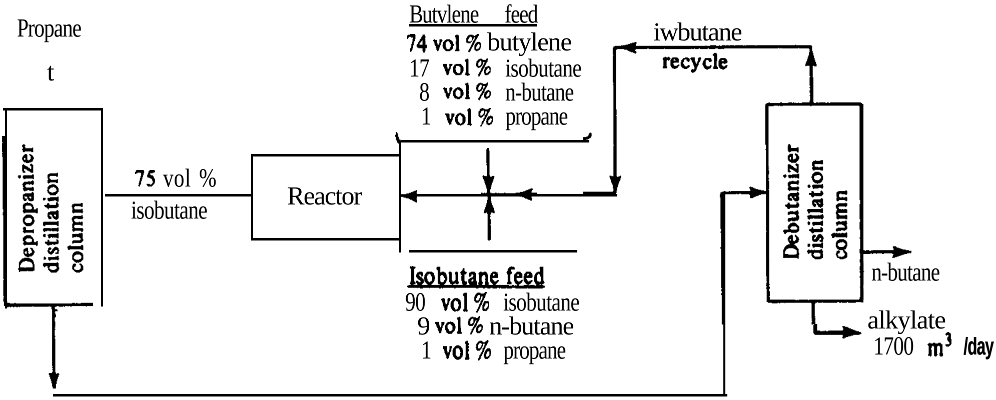
P_H at reactor exit $= \dfrac{976.5}{1174.91}(2500) \cong 2075$ psia

P_{HS} at reactor exit $= \dfrac{2.06}{1174.91}(2500) \cong 4.4$ psia

This completes the major work on material balances needed for solving this problem.

Now proceed, using these results and other information given in the problem, to complete the reactor design analysis requested.

Problem 14. Material Balance for Alkylation Plant Evaluation†



The simplified diagram of a catalytic alkylation unit is shown above.

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†Adapted from 1977 AIChE Student Contest Problem.

In the reactor, butylene and isobutane react to form C_8 "alkylate" according to the following reaction:

$$C_4H_8 + C_4H_{10} \rightarrow C_8H_{18}$$

The unit is to produce product alkylate at a rate of 1700 m^3/day (10,693 bbl/day).

The yield is 1.72 m^3 alkylate per m^3 butylene consumed; 1.10 m^3 of isobutane are consumed per m^3 butylene consumed.

The reactor effluent is to contain 75 vol % isobutane.

It may be assumed that the recycle is pure isobutane and that propane, alkylate, and n-butane are completely recovered as pure products in the columns. Propane and n-butane do not react.

Under these conditions, (*a*) How much of each of the feed streams is required in m^3/day and in bbl/day? (*b*) How much isobutane must be recycled in m^3/day and in bbl/day?

Problem 15. Cost For Producing Butadiene Sulfone†

A design has been completed for the production of butadiene sulfone from 1,3 butadiene and sulfur dioxide. Based on the following results from the completed design and the basic company requirements as listed, determine what the selling price of the butadiene sulfone product should be as dollars per pound based on:

(A) The case where the company demands a 20 percent continuous nominal interest rate of return after taxes on any investment [i.e., profitability index (*r*) is 20 percent].

(B) The case where the company demands a 20 percent finite effective end-of-year interest rate of return after taxes on any investment [i.e., profitability index (*i*) is 20 percent].

The total capital investment for the complete plant is $300,000, and it produces 10 million pounds of butadiene sulfone per year.

A patent royalty charge of 5 percent of the annual sales value before taxes must be paid.

Working capital is 10 percent of the fixed capital investment (F.C.I.) ($27,000).

Special startup costs for the first year *only* are 10 percent of the F.C.I. ($27,000).

The plant operates 330 days/yr (90 percent on-stream factor).

The income-tax rate for the company is 48 percent of the gross profits.

---

†Adapted from 1970 AIChE Student Contest Problem.

Straight-line method is used for calculating depreciation cost ($27,000/yr).

Calculation of costs per year gives a total of $758,000/yr with this including all costs except those for royalty, income taxes, and startup in the first year of operation. This $758,000 includes the annual depreciation cost of $27,000.

Theannual costs and the annual income are considered, by company policy, as end-of-year lump-sum transactions.

Life period of plant is 10 years.

Scrap value of plant at end of life is zero.

Neglect interest during construction period and value-of-land effects.

NOTE: Rate of return calculations for your company must be based on discounted-cash-flow procedures to account for the time value of money.

Problem 16. Cost of Reboiler for Alkylation Unit Heat-Pump Fractionator if Fractionation Column Operation is Assumed to be at Minimum Reflux Ratio†

An alkylation unit heat-pump fractionator is being designed for continuous operation to meet the following conditions:

Feed is 30,000 barrels/day (104,640 lb mole/day) of a liquid at its boiling temperature with a composition of 24 volume percent i-butane and 76 volume percent n-butane (23.3 mole percent i-butane and 76.7 mole percent n-butane). The product stream as i-butane-rich overhead product is to be 5000 barrels/day (17,000 lb mole/day) of which 400 barrels/day is n-butane with the rest being i-butane (composition is 91.7 mole percent i-butane and 8.3 mole percent n-butane).

The temperature of the materials at the bottom of the tower is 90°F.

At 90°F, the equilibrium vapor pressure of i-butane is 62 psia and that for n-butane is 44 psia.

Raoult's law and Dalton's law hold for the mixtures involved.

Relative volatility is constant for all compositions of the mixture at the value found for the bottom of the tower (90°F temperature).

An alkylation unit provides 30×10^6 Btu/h of heat for reboiling in the fractionation column.

The overall heat-transfer coefficient (U) in the reboiler is 120 Btu/(h)(ft^2 outside area)(°F).

The At (constant) in the reboiler for use in the equation $q = UA$ At is 20°F.

The heat of vaporization for the mixture in the reboiler can be taken as 152 Btu/lb.

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†Adapted from 1980 AIChE Student Contest Problem.

The cost for the reboiler is \$25/ft^2 of outside heat-transfer area.

Neglect pressure drop in the column.

Assume ideal liquids and ideal gases.

Mc Cabe-Thiele assumptions apply for the fractionation column

Physical properties =	i-butane	n-butane
Molecular formula	C_4H_{10}	C_4H_{10}
Molecular weight	58.12	58.12
Density, lb/gal (at 60°F)	4.69	4.87

One barrel is 42 gal measured at 60°F.

What would be the cost of the reboiler if the column operation is assumed to be at minimum reflux ratio?

Problem 17. Incremental Investment Comparison for Two Conversions for a Dicyanobutene Reactor System†

As one step in the process of making nylon, dichlorobutene (DCB) is reacted with sodium cyanide to form dicyanobutene (DNB). A CSTR is used to carry out the reaction and special controls and materials of construction are required.

You are asked, on the basis of the simplifying assumptions given in the following, to make a preliminary estimate of incremental return on investment when conditions for two conversions are compared. At a later time, using details as given in the 1981 AIChE student contest problem, it may be possible to extend the scope of this problem to include all effects including losses and major effects of working capital, but these items will not be considered at this time.

Following are the conditions for the problem:

For going from 80- to 85-percent conversions, where the rate of reaction as kg DCB converted per minute per kg of DCB charged is known as 0.0169 at 80 percent conversion and 0.0131 at 85 percent conversion, what is the incremental return on the extra capital investment required under the following conditions? A single continuous stirred tank (back mix) reactor is to be used in each case. The reaction is

$$C_4H_6Cl_2 + 2NaCN \xrightarrow{\text{catalyst}} C_4H_6(CN)_2$$

NaCN M.W. = 49

DCB	DNB
M.W. = 125	M.W. = 106

A total of 90 x10^6 kg of DNB is to be produced per year (8000 h). The catalyst is $NaCu(CN)_2$ with a molecular weight of 138.6, and 0.038 kg of copper (atomic

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†Adapted from 1981 AIChE Student Contest Problem.

weight of copper is 63.6) are needed per kg DCB charged to the system. The catalyst solution is as follows:

wt. %

$NaCu(CN)_2 \approx 6.5$

$NaCN \approx 17.3$

$H_2O = 76.2$

Liquid density = 1.15 x 10^3 kg/m^3

The DCB charge is 100 percent DCB and has a liquid density of 1.16 x 10^3 kg/m^3.

None of the cyanide in the catalyst is consumed in the reaction and there is no catalyst loss. Therefore, consider the catalyst initial cost as a component of the total fixed capital investment based on the amount of catalyst needed for a 2-h operating period.

The sodium cyanide solution added for the reaction is as follows (enough is always charged for 100 percent DCB conversion no matter what the actual conversion is):

Wt. %

$NaCN = 26.0$

$Na_2CO_3 = 1.0$

NH, = 0.3

$NaOH = 0.2$

$H_2O = 72.5$

Liquid density = 1.13 x 10^3 kg/m^3

The hydrogen cyanide solution added for pH control is only about 1 percent by weight of the NaCN stream, and the HCN stream effects can be neglected for this calculation.

Reactor purchased costs are as follows with straight-line interpolation or extrapolation to 30,000 gal acceptable:

Working capacity (1000 gal.)	cost ($1000)
5.0	108
10.0	204
12.5	230
15	260
24	390

The total fixed capital investment (FCI) for the entire system taking all costs for heat-exchanger equipment, pumps, piping, installation, etc., into account is equal to the initial cost of the catalyst solution plus 4.5 times the purchased cost of the reactor. Assume none of the unreacted materials can be recovered.

The density of the average reaction mixture is 1.14 x 10^3 kg/m^3
The cost of raw materials is as follows:

DCB solution = \$0.62/kg
Catalyst solution = \$0.30/kg
NaCN solution = \$0.082/kg

In making the investment comparison, ignore working capital and consider only the FCI with an annual charge for depreciation of 8 percent of FCI.

Assume all operating costs except those for raw materials and depreciation are constant for the two conversion cases under consideration. Do not consider effects of income taxes.

Problem 18. Recycle Compressor Power Cost for Methanation Unit of SNG Plant†

In the design of a substitute natural gas (SNG) plant for producing SNG from coal, you have been asked to determine the power cost, in dollars per year, for compressing the recycle gas for the bulk methanation portion of the plant with a single bulk methanator using electric-motor-driven compressors of conventional centrifugal type with adiabatic efficiency of 75 percent and maximum compression ratio per stage of 3.5. Following are the special conditions for your design in this preliminary evaluation:

The feed is a mixture of CH,, CO, CO,, H_2, and N_2 with poisoning H_2S having been removed, and only a small amount of CO, present. The gas will be fed to a single bulk methanator at a pressure of 360 psia and temperature of 450°F. The feed rate for the initial gas brought into the system (before recycle gas is added) is 390 x 10^6 standard cubic feet per day, where a standard cubic foot is defined as at 60°F and 1 atm (i.e., 1 lb mole = 379.5 standard cubic feet). The critical methanation reaction is CO + $3H_2$ = CH, + H_2O with an exothermic heat of reaction of AH = – 95,404 Btu/lb mole CO. Ignore any heat effects due to CO, methanation reaction or overall heat losses.

The initial entering gas (before recycle is added) contains 63.4 mole percent hydrogen, and 90 percent of this entering hydrogen reacts by the methanation reaction given in the preceding with a final methanator exit-gas temperature of 900°F, and there is no other reaction. The molecular weight of the gas mixture entering the methanator has been calculated to be 15.5 and for the exit gas is 16.5. Assume the molecular weight of the entering feed can be taken as 15.5.

The average heat capacity of the gas in the reactor can be taken as 0.47 Btu/(lb)(°F), and this average applies over a temperature range of 450 to 900°F. Enough gas must be recycled through the compressor to the single-bulk-

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†Adapted from 1982 AIChE Student Contest Problem.

methanator entry-gas stream so that the temperature rise from 450 to 900°F is maintained.

The inside diameter of the methanator to be used has been calculated to be 24.8 ft and the methanator height of the catalyst-packed bed is 12.4 ft.

Use Eq. (24) in Chap. 14 of this text to calculate the theoretical compressor power. The brake horsepower efficiency of the electric motor

$$\left[\frac{\text{power delivered to turbine}}{\text{power provided as electrical energy}}(100)\right]$$

is 80 percent. The value of k can be taken as 1.35, where k is the ratio of heat capacity of the gas at constant pressure to heat capacity at constant volume.

The gas is cooled before it enters the compressor to a temperature such that the exit gas from the compressor will be at 450°F and 360 psia assuming ideal adiabatic compression. To get the compressor entering-gas flow rate, assume that $p_1 v_1^k = p_2 v_2^k$. Neglect pressure drop due to the cooler.

The following equation, taken from Perry's "Chemical Engineers' Handbook," 6th ed., p. 5-53 [Eq. (5-196)], can be used to determine the pressure drop in the methanator:

$$AP = \frac{2fG^2L(1-\varepsilon)^{3-h}}{D_p g_c \rho \phi_S^{3-h} \varepsilon^3}$$

where AP = pressure drop, lb force/ft^2

f = friction factor = a function of Reynolds number = 1.0 for this case

G = fluid superficial mass velocity based on empty methanator cross-sectional area, lb/(s)(ft^2)

L = actual depth of methanator bed in ft plus 3 ft more added on to account for AP due to nozzles, distributor, and supports

ε = voidage (fractional free volume of packing) = 0.40 for this case

h = exponent = a function of the Reynolds number = 2.0 for this case

D_p = average catalyst particle diameter, ft = 0.0238 ft for this case

g_c = dimensional constant = 32.17 (lb mass) (ft)/(lb force)(s^2)

ρ = fluid density, lb/ft^3, based on average temperature and entering pressure = 0.464 lb/ft^3 for this case

ϕ_S = shape factor for the solid catalyst particles = 1.0 for this case

Under the given conditions, determine:

(a) The power cost for driving the recycle gas compressor as dollars per year if the purchase cost for electricity is \$0.06/kWh and the plant operates 330 days per year.

(b) The net present value of the initial fixed-capital costs plus the discounted operating cost of the recycle-gas compressor over a 20-year life period with the time value of money being 11 percent per year with interest compounding annually and expenses paid annually at end of year. Equation (24) in Chap.

7 of this text is applicable for this case with i = 0.11 since inflation effects cancel out by assuming that the purchase cost of power will inflate. Capital cost for the related installation can be taken as $5,000,000, which is the initial fixed capital investment for the related equipment to be used in determining the net present value.

Problem 19. Process Design for Wood-Pulp Production Plant†

In a process for producing bleached wood pulp from southern pine trees. chipped wood is charged to a digester where it is heated to 346°F and cooked with a water-alkali mixture. After cooking, the mixture is passed to a blow tank where the pressure is reduced to atmospheric, and part of the water evaporates. The mixture is then passed through a series of washers and filters (brown stock washers) where the pulp is removed and the remaining liquid (black liquor) is sent to a multiple-effect evaporator for concentration followed by chemical treatment to prepare the liquid for recycling to the digester.

For the following conditions, you are to determine (a) the flow rate in tons per day and the weight percent of dissolved solids for the black liquor leaving the brown-stock-washers system passing to the evaporators and (b) the rate of final bleached dry pulp production as tons per day:

Cycle time per batch is 185.75 min with eight digesters in continuous use so there are 62.02 digester charges per 24-h day.

Density of the solid wood before chipping is 40.6 $\mathrm{lb/ft^3}$ and contains 52 percent H_2O with the remaining 48 percent being cellulose, lignin, and various carbohydrates. After chipping, 1 $\mathrm{ft^3}$ of the chips contains 12.5 lb of dry wood and 13.5 = $12.5(\frac{52}{48})$ lb of water. (This corresponds to a void fraction of 0.36.)

The volume of each digester is 4150 $\mathrm{ft^3}$ with the chipped wood being added to fill the digester to the full 4150 $\mathrm{ft^3}$.

The cooking liquor added to the digester takes up the void space and contains 10.25 wt % of alkali as dissolved solids with the balance being water.

The liquid in the digester has a ratio of 3.5 lb of total liquid including the water in the wood per pound of dry wood.

In the digestion step, 45 wt % of the dry wood is left as unbleached pulp which goes on through the process and comes out of the brown stock washers as pulp product for further treatment. The final yield of bleached pulp is 90 percent of the unbleached pulp yield. The other 55 percent of the dry wood in the digester charge (except for a small amount of turpentine which is removed from the digester and has a negligible effect on the total amounts) is dissolved in the cooking liquor and passes through the system to the evaporators as dissolved solids. It is later burned in a recovery furnace for its heat value.

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†Adapted from 1983 AIChE Student Contest Problem.

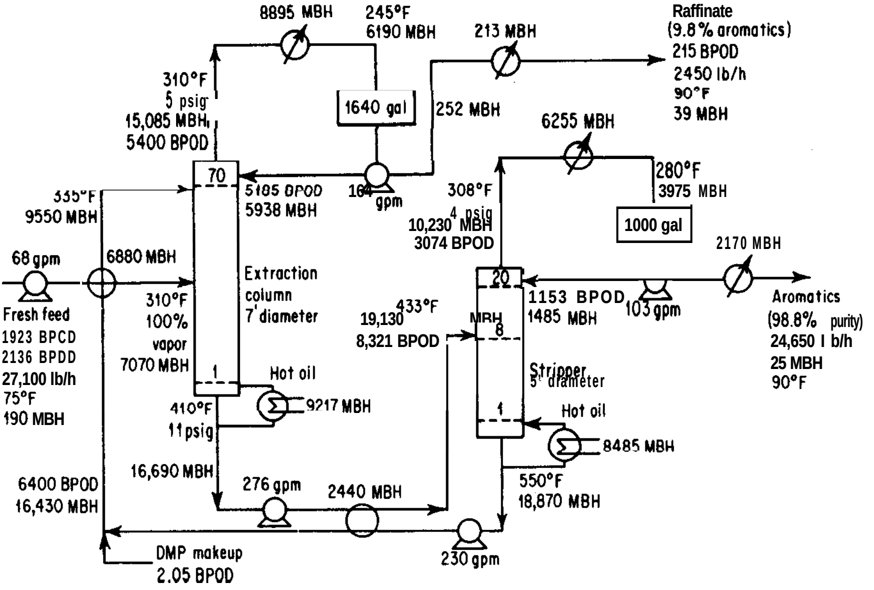

FIGURE C-2
Paraffin removal by extractive distillation with dimethylphthalate.

In the pressure reduction from the digester to the blow tank, 14 wt % of the stream leaving the digester is lost as steam released in the blow step.

In the brown stock washers, water is used to wash the final pulp in the last filter and the dilution factor (D) used is 4 where the pounds of wash water used per pound of dry pulp in the wet filter product is D plus the amount of water in the wet filter product per pound of dry pulp.

The pulp product from the last washer contains 82 wt % water and 18 wt % unbleached pulp. This is sent to the final bleaching unit where a 90 percent conversion of the unbleached pulp to bleached pulp product occurs.

Note that a total material balance around the brown-stock-washers system shows that the entering stream from the blow unit plus the wash water added at the final filter must equal the weight of wet-pulp product plus the black liquor product. This reduces to a material balance result of tons/day of black liquor to the evaporators = tons/day of stream from blow unit to brown stock washers + $(D - 1)$(tons/day of dry pulp in wet-pulp product).

Problem 20. Paraffin Removal by Extractive Distillation with Dimetbylphthalate

Figure C-2 represents a flow sheet prepared by a junior design engineer of part of an aromatics plant which provides paraffin removal by extractive distillation with dimethylphthalate (DMP). Check this design and make recommendations concerning the design conclusions of the junior design engineer who worked on the project. If there is an inconsistency in the results, indicate where the error is, what must be done to correct the error, and what would be the magnitude of the error if it were not corrected. The basis for the design is as follows:

1. 90 percent operating factor for the process
2. 60 percent tray efficiency of the columns
3. The fresh feed is of the following composition

Component		BPCD	Relative volatility to Ox
C_8 paraffin	C_8-Pn	11	1.52
C_8 naphthene	C_8-N	24	
C_9 paraffin	C_9-Pn	68	
C_9 naphthene	C_9-N	9	
ethylbenzene	EB	112	1.22
para-xylene	Px	358	1.155
281°F paraffin	281°F-Pn	49	1.15
meta-xylene	Mx	739	1.135
ortho-xylene	Ox	500	1.0
292°F paraffin	292°F-Pn	34	0.90
Heavies	300°F-C_9	19	0.70
		1923	

4. The addition of DMP enhances the relative volatility of the paraffins and naphthenes an average of 25 percent above their normal values
5. $(L/D)_{\text{min}}$ of the extraction column is 20
6. $(L/D)_{\text{min}}$ of the stripper is 0.5
7. $(L/D)_{\text{act}} = 1.2(L/D)_{\text{min}}$
8. Dimethylphthalate properties, formula $C_6H_4(CO_2CH_3)_2$

Temp, °F	Vapor press, psia	Liquid enthalpy, Btu/lb	Vapor enthalpy, Btu/lb
60	....	0	
100	....	12	
200	....	42	
300	0.2	73	
400	1.7	110	251
500	8.2	148	281
550	17.0	169	293
600	....	191	306

9. Since the DMP makeup is so small compared to the other streams, it may be neglected in the overall heat and material balance.
10. Pump sizes may be assumed correct even though each one includes a fixed safety factor.

Duties of condensers and reboilers are given in MBH (million Btu per hour). BPCD is the designation for barrels per calendar day and BPOD is that for barrels per operating day. Note that the number of plates in each column is indicated by the number at the top of the column. Significant temperatures are given wherever necessary.

Problem 21. Optimum Operating Range for Commercial Production of Styrene

The following laboratory runs reported in Tables 11, 12, and 13 covering styrene production by catalytic dehydrogenation are to be analyzed with respect to three variables. These are (1) product value, (2) equipment and operating costs, and (3) regeneration costs. With these three variables in mind, determine the optimum operating range which should be prescribed for commercial production of styrene.

Liquid hourly space velocity (L.H.S.V.) = volumes of liquid charged per hour per gross volume of catalyst. Liquid volume has been corrected to 60°F in all cases.

Process period = length of time charging stock is passed through the catalyst bed between successive regenerations.

Conversion = the percentage of ethylbenzene charged which is transformed into other products.

Laboratory data

TABLE 11

Effect of pressure at 550°C block temperature, 1.0 L.H.S.V., and 30-min process period

Run No. (arbitrary)	Pressure at tube outlet, mm of Hg, absolute	% conversion	Once-through yields, wt % on ethylbenzene		Styrene ultimate yield, wt %	100 C/ C_8H_8†
			Styrene	Carbon		
5	80	29.0	25.6	0.51	88	2.0
6	110	29.9	27.4	0.59	92	2.2
7	250	29.6	25.6	0.96	86	3.7
8‡	742	21.9	18.3	1.5	84	8.2
9	747	25.8	21.0	1.2	81	5.7
10%	1530§	16.9	11.4	2.0	67	18
11%	3320¶	12.7	6.3	2.0	50	41

† Pounds of catalyst carbon per 100 lb of styrene.

‡ The superatmospheric pressure unit was used in these runs (8, 10, 11), the quartz tube unit in the others.

§ Equivalent of 15 psig.

¶ Equivalent to 50 psig.

TABLE 12

Effect of pressure at 600°C block temperature and 30-min process period

Run No. (arbitrary)	Pressure at tube outlet, mm of Hg, absolute	L.H.S.V. at 60°F	% conversion	Once-through yields, wt % on ethylbenzene		Styrene ultimate yield, wt %	100 C/C_8H_8†
				Styrene	Carbon		
12	80	0.99	54.6	49.4	0.77	90	1.6
13	80	1.50	45.3	42.0	0.66	92	1.6
14	80	1.98	41.9	39.0	0.36	93	0.92
15	80	4.02	33.2	31.4	0.17	95	0.54
16	250	0.645	60.5	45.7	4.4	76	9.7
17	250	0.98	60.9	49.5	3.0	80	6.1
18	250	1.49	56.9	48.9	1.5	85	3.1
19	250	2.19	53.0	48.1	1.1	91	2.3
20	250	2.51	50.7	44.2	0.92	87	2.1
21	250	3.30	44.9	41.0	0.56	91	1.4
22	250	4.05	41.1	37.4	0.46	92	1.2
23	250	7.98	27.6	25.7	0.14	93	0.55
24	250	9.92	26.4	24.5	0.15	93	0.61
25	742	0.67	44.7	25.8	6.0	58	23
26	748	0.99	43.8	28.6	4.2	65	15
27	741	1.49	43.8	32.3	2.5	73	7.7
28	751	2.21	40.8	33.0	1.8	80	5.5
29	742	3.97	38.4	32.3	0.99	84	3.1
30	751	7.96	26.5	23.8	0.26	90	1.1

† Pounds of talyst carbon per 100 lb of styrene.

TABLE 13
Effect of temperature and space velocity at 250-mm outlet pressure and 30-min process period

Run No. (arbitrary)	Block temperature C	L.H.S.V. at 60°F	% conversion	Once-through yields, wt % on ethylbenzene		Styrene ultimate yield, wt %	106 C/ C_8H_8†
				Styrene	Carbon		
31	550	0.67	46.5	34.9	1.4	86	3.9
7	550	1.02	29.6	25.6	0.96	86	3.7
32	550	2.02	27.9	25.6	0.42	92	1.7
33	550	4.06	19.2	18.0	0.25	94	1.4
34	550	8.02	11.6	10.9	0.16	94	1.5
35	570	0.69	49.9	42.2	2.1	84	5.0
36	570	0.98	46.2	40.6	1.1	87	2.7
37	570	1.50	45.1	40.9	0.74	90	1.8
16-24	600	See Table 12.					
38	630	1.48	66.1	54.4	3.8	81	7.0
39	630	2.17	66.1	54.4	2.9	81	5.3
40	630	2.98	58.8	53.2	1.5	90	2.8

† Pounds of catalyst carbon per 100 lb of styrene.

ULTIMATE YIELDS. Having determined once-through styrene yield, the ultimate yield obtainable under the conditions employed in any run can be calculated providing the percentage of unreacted ethylbenzene is known. The latter could be approximated by considering the vacuum distillates as binary mixtures of styrene and ethylbenzene. Fractional distillation data, however, have shown that small percentages of lower-boiling materials, definitely identified by physical constants and through preparation of their nitro derivatives as benzene and toluene, are also present. Hence, in all ultimate-yield calculations, allowance has been made for the formation of these by-products.

APPENDIX D

TABLES OF PHYSICAL PROPERTIES AND CONSTANTS

CONTENTS

TABLE 1
General engineering conversion factors and constants?

Length	
1 inch	2.54 centimeters
1 foot	30.48 centimeters
1 yard	91.44 centimeters
1 meter	100.00 centimeters
1 meter	39.37 inches
1 micron	10^{-6} meter
1 mile	5280 ft
1 kilometer	0.6214 mile

Mass	
1 pound †	16.0 ounces
1 pound †	453.6 grams
1 pound †	7000 grains
1 ton (short)	2000 pounds †
1 kilogram	1000 grams
1 kilogram	2.205 pounds †

† Avoirdupois.

Volume	
1 cubic inch	16.39 cubic centimeters
1 liter	61.03 cubic inches
1 liter	1.057 quarts
1 cubic foot	28.32 liters
1 cubic foot	1728 cubic inches
1 cubic foot	7.481 U.S. gallons
1 U.S. gallon	4.0 quarts
1 U.S. gallon	3.785 liters
1 U.S. bushel	1.244 cubic feet

Density

1 gram per cubic centimeter . . . 62.43 pounds per cubic foot

1 gram per cubic centimeter . . . 8,345 pounds per U.S. gallon

1 gram mole of an ideal gas at 0°C and 760 mm Hg is equivalent to 22.414 liters

1 pound mole of an ideal gas at 0°C and 760 mm Hg is equivalent to 359.0 cubic feet

Density of dry air at 0°C and 760 mm Hg . . . 1.293 grams per liter = 0.0807 pound per cubic foot

Density of mercury . . . 13.6 grams per cubic centimeter (at +2°C)

Pressure	
1 pound per square inch	2.04 inches of mercury
1 pound per square inch	51.71 millimeters of mercury
1 pound per square inch	2.31 feet of water
1 atmosphere	760 millimeters of mercury
1 atmosphere	2116.2 pounds per square foot
1 atmosphere	33.93 feet of water
1 atmosphere	29.92 inches of mercury
1 atmosphere	14.7 pounds per square inch

Temperature scales	
Degrees Fahrenheit (F)	1.8 (degrees C) + 32
Degrees Celcius (C)	(degrees F − 32)/1.8
Degrees Kelvin (K)	degrees C + 273.15
Degrees Rankine (R)	degrees F + 459.7

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† See also Tables 6 and **7 in Appendix A for SI Conversion factors and more exact conversion** factors.

TABLE 1

General engineering conversion factors and constants? (Continued)

Power

1 kilowatt	137.56 foot-pounds force per second
1 kilowatt	56.87 Btu per minute
1 kilowatt	1.341 horsepower
1 horsepower	550 foot-pounds force per second
1 horsepower	0.707 Btu per second
1 horsepower	745.7 watts

Heat, energy, and work equivalents

	Cal	Btu	**ft·lb**	**kWh**	hp-hr	Joules	liter-atm
Cd	1	3.97×10^{-3}	3.086	1.162×10^{-6}	1.558×10^{-6}	4.1840	4.129×10^{-2}
Btu	252	1	778.16	2.930×10^{-4}	3.930×10^{-4}	1055	10.41
ft·lb	0.3241	1.285×10^{-3}	1	3.766×10^{-7}	5.0505×10^{-7}	1.356	1.338×10^{-2}
kWh	860,565	3412.8	2.655×10^{6}	1	1.341	3.60×10^{6}	**35,534.3**
hp-hr	641,615	2545.0	1.980×10^{6}	0.7455	1	2.685×10^{6}	26,494
Joules	0.239	9.478×10^{-4}	0.7376	2.773×10^{-7}	3.725×10^{-7}	1	9.869×10^{-3}
liter-atm	24.218	9.604×10^{-2}	74.73	2.815×10^{-5}	3.774×10^{-5}	101.33	1

Constants

e 2.7183

A 3.1416

Gas-law constants:

R . . . . . 1.987 (cal)/(g mol) (K)

R . . . . . 82.06 (cm^3) (atm)/(g mol) (K)

R . . . . . 10.73 (lb/in.‘) (ft^3)/(lb mol) (°R)

R . . . . . 0.730 (atm) (ft^3)/(lb mol) (°R)

R . . . . . 1545.0 (lb/ft^2) (ft^3)/(lb mol) (°R)

g_c . . . . 32.17 (ft) (lbm)/(s) (s) (lbf)

Analysis of air

By weight: oxygen, 23.2%; nitrogen, 76.8%

By volume: oxygen, 21.0%; nitrogen, 79.0%

Average molecular weight of air on above basis = 28.84 (usually rounded off to 29)

True molecular weight of dry air (including argon) = 28.96

Viscosity

1 centipoise . . . . . 0.01 g/(s) (cm)

1 centipoise , ... 0.000672 lb/(s) (R)

1 centipoise . . . . . 2.42 lb/(h) (ft)

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† See also Tables 6 and 7 in Appendix A for SI Conversion factors and more exact conversion factors.

TABLE 2
Viscosities of gases

Coordinates for use with Fig. D-1

No.	Gas	X	Y	No.	Gas	X	Y
1	Acetic acid	7.7	14.3	29	Freon 113	11.3	14.0
2	Acetone	8.9	13.0	30	Helium	10.9	20.5
3	Acetylene	9.8	14.9	31	Hexene	8.6	11.8
4	Air	11.0	20.0	32	Hydrogen	11.2	12.4
5	Ammonia	8.4	16.0	33	$3H_2 + 1N_2$	11.2	17.2
6	Argon	10.5	22.4	34	Hydrogen bromide	8.8	20.9
7	Benzene	8.5	13.2	35	Hydrogen chloride	8.8	18.7
8	Bromine	8.9	19.2	36	Hydrogen cyanide	9.8	14.9
9	Butene	9.2	13.7	37	Hydrogen iodide	9.0	21.3
10	Butylene	8.9	13.0	38	Hydrogen sulfide	8.6	18.0
11	Carbon dioxide	9.5	18.7	39	Iodine	9.0	18.4
12	Carbon disulfide	8.0	16.0	40	Mercury	5.3	22.9
13	Carbon monoxide	11.0	20.0	41	Methane	9.9	15.5
14	Chlorine	9.0	18.4	42	Methyl alcohol	8.5	15.6
15	Chloroform	8.9	15.7	43	Nitric oxide	10.9	20.5
16	Cyenogen	9.2	15.2	44	Nitrogen	10.6	20.0
17	Cyclohexene	9.2	12.0	45	Nitrosyl chloride	8.0	17.6
18	Ethene	9.1	14.5	46	Nitrous oxide	8.8	19.0
19	Ethyl acetate	8.5	13.2	47	Oxygen	11.0	21.3
20	Ethyl alcohol	9.2	14.2	48	Pentene	7.0	12.8
21	Ethyl chloride	8.5	15.6	49	Propane	9.7	12.9
22	Ethyl ether	8.9	13.0	50	Propyl alcohol	8.4	13.4
23	Ethylene	9.5	15.1	51	Propylene	9.0	13.8
24	Fluorine	7.3	23.8	52	Sulfur dioxide	9.6	17.0
25	Freon 11	10.0	15.1	53	Toluene	8.6	12.4
26	Freon 12	11.1	16.0	54	2,3,3-Trimethylbutane	9.5	10.5
27	Freon 21	10.8	15.3	55	Water	8.0	16.0
28	Freon 22	10.1	17.0	56	Xenon	9.3	23.0

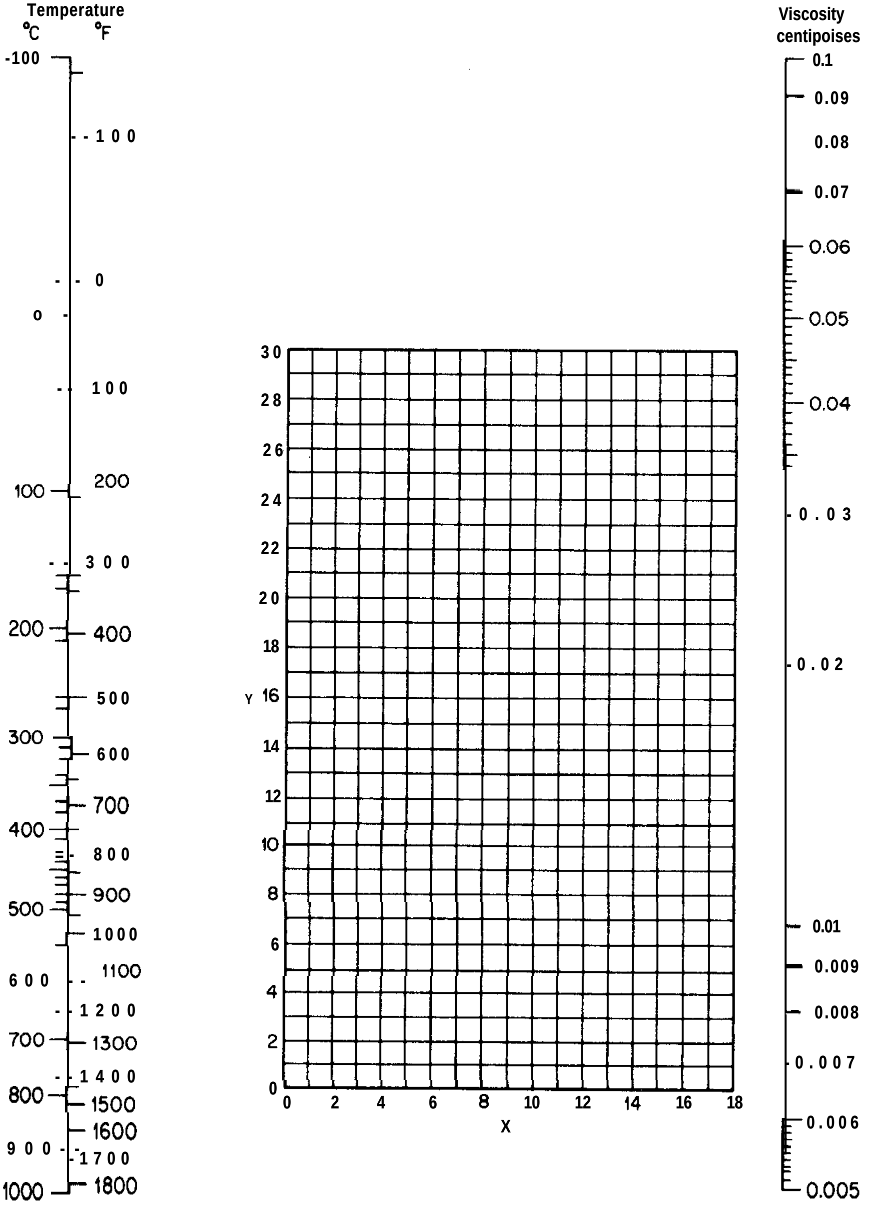


FIGURE D-1
Viscosities of gases at 1 atm. (For coordinates see Table 2.)

TABLE 3

Viscosities of liquids

Coordinates for use with Fig. D-2

No.	Liquid	X	Y	No.	Liquid	X	Y
1	Acetaldehyde	15.2	4.8	56	Freon 22	17.2	4.7
2	Acetic acid, 100%	12.1	14.2	57	Freon 113	12.5	11.4
3	Acetic acid, 70%	9.5	17.0	58	Glycerol, 100 %	2.0	30.0
4	Acetic anhydride	12.7	12.8	59	Glycerol, 50%	6.9	19.6
5	Acetone, 100 %	14.5	7.2	60	Heptene	14.1	8.4
6	Acetone, 35 %	7.9	15.0	61	Hexane	14.7	7.0
7	Allyl alcohol	10.2	14.3	62	Hydrochloric acid, 31.5 %	13.0	16.6
8	Ammonia, 100%	12.6	2.0	63	Isobutyl alcohol	7.1	18.0
9	Ammonia, 26 %	10.1	13.9	64	Isobutyric acid	12.2	14.4
10	Amyl acetate	11.8	12.5	65	Isopropyl alcohol	8.2	16.0
11	Amyl alcohol	7.5	18.4	66	Kerosene	10.2	16.9
12	Aniline	8.1	18.7	67	Linseed oil, raw	7.5	27.2
13	Anisole	12.3	13.5	68	Mercury	18.4	16.4
14	Arsenic trichloride	13.9	14.5	69	Methanol, 100 %	12.4	10.5
15	Benzene	12.5	10.9	70	Methanol, 90 %	12.3	11.8
16	Brine, $CaCl_2$, 25%	6.6	15.9	71	Methanol, 40%	7.8	15.5
17	Brine, NaCl, 25%	10.2	16.6	72	Methyl acetate	14.2	8.2
18	Bromine	14.2	13.2	73	Methyl chloride	15.0	3.8
19	Bromotoluene	20.0	15.9	74	Methyl ethyl ketone	13.9	8.6
20	Butyl acetate	12.3	11.0	75	Naphthalene	7.9	18.1
21	Butyl alcohol	8.6	17.2	76	Nitric acid, 95%	12.8	13.8
22	Butyric acid	12.1	15.3	77	Nitric acid. 60%	10.8	17.0
23	Carbon dioxide	11.6	0.3	78	Nitrobenzene	10.6	16.2
24	Carbon disulfide	16.1	7.5	79	Nitrotoluene	11.0	17.0
25	Carbon tetrachloride	12.7	13.1	80	Octane	13.7	10.0
26	Chlorobenzene	12.3	12.4	81	Octyl alcohol	6.6	21.1
27	Chloroform	14.4	10.2	82	Pentachloroethane	10.9	17.3
28	Chlorosulfonic acid	11.2	18.1	83	Pentane	14.9	5.2
29	Chlorotoluene, ortho	13.0	13.3	84	Phenol	6.9	20.8
30	Chlorotoluene, meta	13.3	12.5	85	Phosphorus tribromide	13.8	16.7
31	Chlorotoluene, para	13.3	12.5	86	Phosphorus trichloride	16.2	10.9
32	Cresol, meta	2.5	20.8	87	Propionic acid	12.8	13.8
33	Cyclohexanol	2.9	24.3	88	Propyl alcohol	9.1	16.5
34	Dibromoethane	12.7	15.8	89	Propyl bromide	14.5	9.6
35	Dichloroethane	13.2	12.2	90	Propyl chloride	14.4	7.5
36	Dichloromethane	14.6	8.9	91	Propyl iodide	14.1	11.6
37	Diethyl oxalate	11.0	16.4	92	Sodium	16.4	13.9
38	Dimethyl oxalate	12.3	15.8	93	Sodium hydroxide, 50 %	3.2	25.8
39	Diphenyl	12.0	18.3	94	Stannic chloride	13.5	12.8
40	Dipropyl oxalate	10.3	17.7	95	Sulfur dioxide	15.2	7.1
41	Ethyl acetate	13.7	9.1	96	Sulfuric acid, 110%	7.2	27.4
42	Ethyl alcohol, 100%	10.5	13.8	97	Sulfuric acid, 98 %	7.0	24.8
43	Ethyl alcohol, 95%	9.8	14.3	98	Sulfuric acid, 60%	10.2	21.3
44	Ethyl alcohol, 40%	6.5	16.6	99	Sulfuryl chloride	15.2	12.4
45	Ethyl benzene	13.2	11.5	100	Tetrachloroethane	11.9	15.7
46	Ethyl bromide	14.5	8.1	101	Tetrachloroethylene	14.2	12.7
47	Ethyl chloride	14.8	6.0	102	Titanium tetrachloride	14.4	12.3
48	Ethyl ether	14.5	5.3	103	Toluene	13.7	10.4
49	Ethyl formate	14.2	8.4	104	Trichloroethylene	14.8	10.5
50	Ethyl iodide	14.7	10.3	105	Turpentine	11.5	14.9
51	Ethylene glycol	6.0	23.6	106	Vinyl acetate	14.0	8.8
52	Formic acid	10.7	15.8	107	Water	10.2	13.0
53	Freon 11	14.4	9.0	108	Xylene, ortho	13.5	12.1
54	Freon 12	16.8	5.6	109	Xylene, meta	13.9	10.6
55	Freon 21	15.7	7.5	110	Xylene, para	13.9	10.9

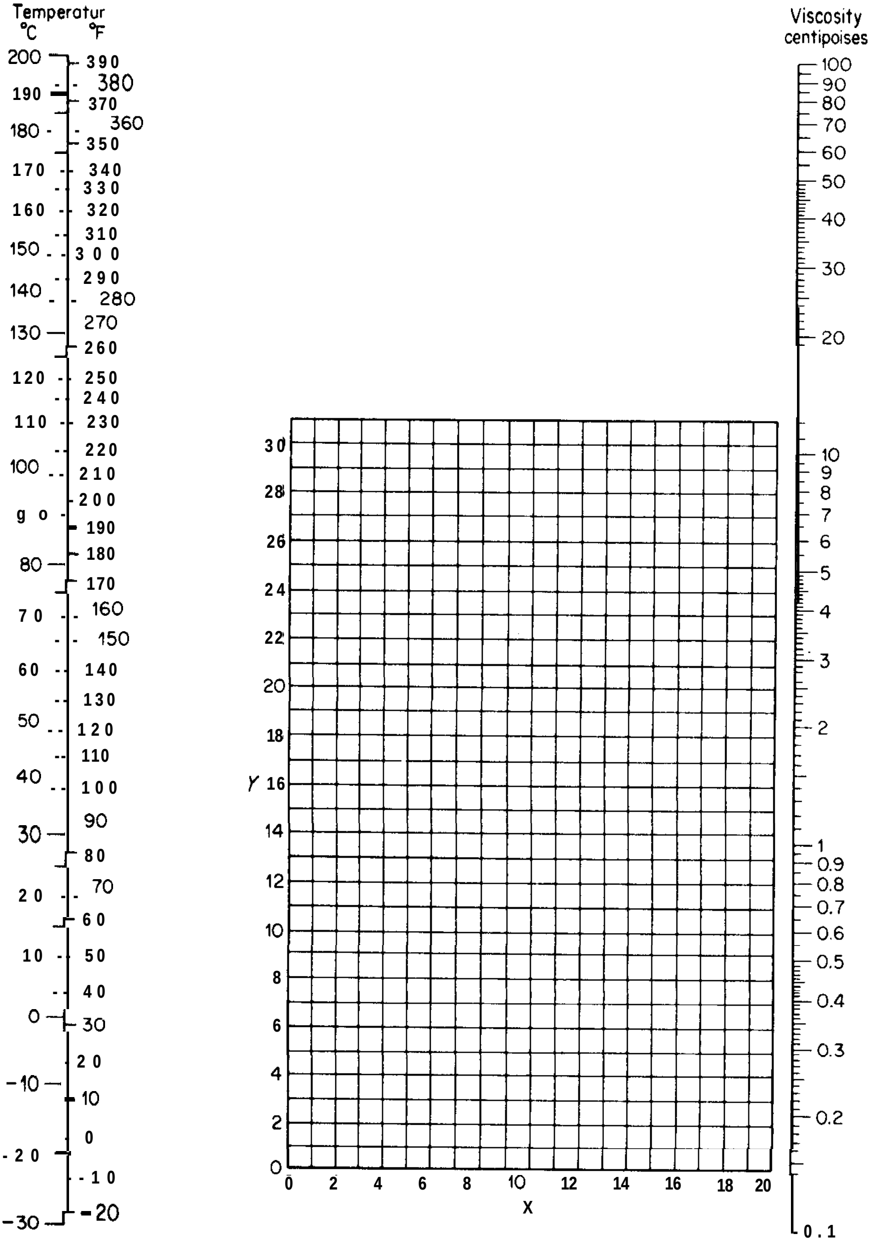


FIGURE D-2
Viscosities of liquids at 1 atm. (For coordinates see Table 3.)

TABLE 4
Density, viscosity, and thermal conductivity of water

Temperature, °F	Density of liquid water, lb/ft^3	Viscosity of water, centipoises	Thermal conductivity of water, Btu/(h)(ft^2)(°F/ft)
32	62.42	1.794	0.320
40	62.43	1.546	0.326
50	62.42	1.310	0.333
60	62.37	1.129	0.340
70	62.30	0.982	0.346
80	62.22	0.862	0.352
90	62.11	0.764	0.358
100	62.00	0.684	0.363
110	61.86	0.616	0.367
120	61.71	0.559	0.371
130	61.55	0.511	0.375
140	61.38	0.470	0.378
150	61.20	0.433	0.381
160	61.00	0.401	0.384
170	60.80	0.372	0.386
180	60.58	0.347	0.388
190	60.36	0.325	0.390
200	60.12	0.305	0.392
210	59.88	0.287	0.393
212	59.83	0.284	0.393

TABLE 5

Thermal conductivity of metals

Metal	k, Btu/(h)(ft^2)(°F/ft)		
	At 32°F	At 212°F	At 572°F
Aluminum	117	119	133
Brass (7030)	56	60	66
Cast iron	32	30	26
Copper	224	218	212
Lead	20	19	18
Nickel	36	34	32
Silver	242	238	. . .
Steel (mild)	. . .	26	25
Tin	36	34	
Wrought iron	. . .	32	28
Zinc	65	64	59

TABLE 6

Thermal conductivity of nonmetallic solids

Material	Temperature, °F	k, Btu/(h)(ft^2)(°F/ft)
Asbestos-cement boards	68	0.43
Bricks:		
Building	68	0.40
Fire clay	392	0.58
	1832	0.95
Sil-0-Cel	400	0.042
Calcium carbonate (natural)	86	1.3
Calcium sulfate (building plaster)	77	0.25
Celluloid	86	0.12
Concrete (stone)	. . . .	0.54
Cork board	86	0.025
Felt (wool)	86	0.03
Glass (window)	. . . .	0.3-0.61
Rubber (hard)	32	0.087
Wood (across grain) :		
Maple	122	0.11
Oak	59	0.12
Pine	59	0.087

TABLE 7

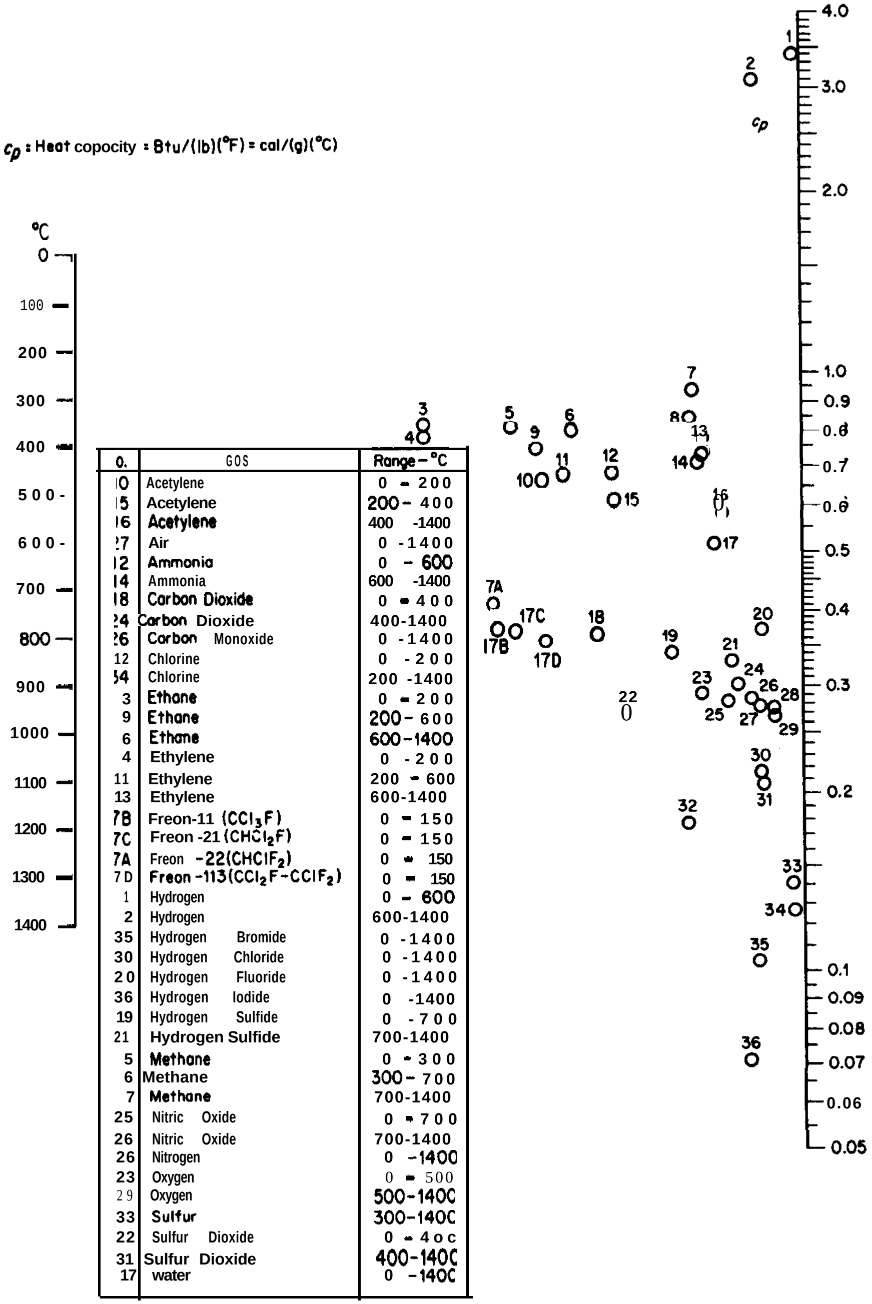
Thermal conductivity of liquids

Liquid	Temperature, °F	k, Btu/(h)(ft^2)(°F/ft)
Acetic acid:		
100%	68	0.099
50%	68	0.20
Acetone	86	0.102
	167	0.095
Benzene	86	0.092
	140	0.087
Ethyl alcohol ,		
100%	68	0.105
	122	0.087
40 %	68	0.224
Ethylene glycol	32	0.153
Glycerol:		
100%	68	0.164
	212	0.164
40%	68	0.259
n-Heptane	86	0.081
Kerosene	68	0.086
Methyl alcohol:		
100%	68	0.124
	122	0.114
40%	68	0.234
n-Octane	86	0.083
Sodium chloride brine, 25%	86	0.330
Sulfuric acid :		
90%	86	0.210
30%	86	0.300
Toluene	86	0.086
Water	32	0.320
	200	0.392

TABLE 8

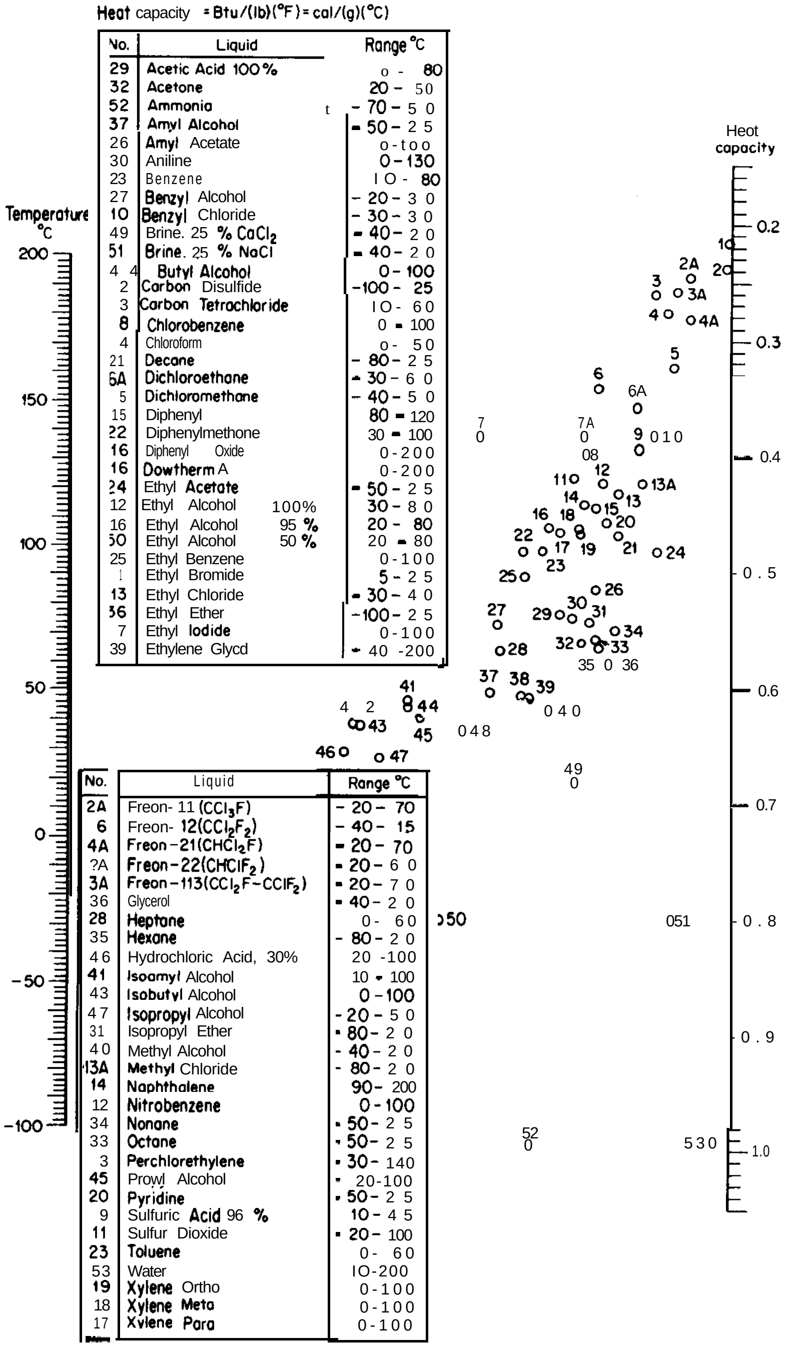
Thermal conductivity of gases

Gas	Temperature, °F	k, Btu/(h)(ft^2)(°F/ft)
Air	32	0.0140
	212	0.0183
	392	0.0226
Ammonia	32	0.0128
	122	0.0157
Carbon dioxide	32	0.0085
	212	0.0133
Chlorine	32	0.0043
Hydrogen	32	0.100
	212	0.129
Methane	32	0.0175
	122	0.0215
Nitrogen	32	0.0140
	212	0.0180
Oxygen	32	0.0142
	212	0.0185
Sulfur dioxide	32	0.0050
	212	0.0069
Water vapor	200	0.0159
	600	0.0256



0.	GOS	Range - °C
10	Acetylene	0 - 200
15	Acetylene	200 - 400
16	Acetylene	400 - 1400
27	Air	0 - 1400
12	Ammonia	0 - 600
14	Ammonia	600 - 1400
18	Carbon Dioxide	0 - 400
24	Carbon Dioxide	400-1400
26	Carbon Monoxide	0 - 1400
12	Chlorine	0 - 200
34	Chlorine	200 - 1400
3	Ethane	0 - 200
9	Ethane	200 - 600
6	Ethane	600-1400
4	Ethylene	0 - 200
11	Ethylene	200 - 600
13	Ethylene	600-1400
7B	Freon-11 (CCl_3F)	0 - 150
7C	Freon -21 ($CHCl_2F$)	0 - 150
7A	Freon -22 ($CHClF_2$)	0 - 150
7D	Freon -113 ($CCl_2F-CClF_2$)	0 - 150
1	Hydrogen	0 - 600
2	Hydrogen	600-1400
35	Hydrogen Bromide	0 - 1400
30	Hydrogen Chloride	0 - 1400
20	Hydrogen Fluoride	0 - 1400
36	Hydrogen Iodide	0 - 1400
19	Hydrogen Sulfide	0 - 700
21	Hydrogen Sulfide	700-1400
5	Methane	0 - 300
6	Methane	300 - 700
7	Methane	700-1400
25	Nitric Oxide	0 - 700
26	Nitric Oxide	700-1400
26	Nitrogen	0 - 1400
23	Oxygen	0 - 500
29	Oxygen	500-1400
33	Sulfur	300-1400
22	Sulfur Dioxide	0 - 400
31	Sulfur Dioxide	400-1400
17	water	0 - 1400

FIGURE D-3
Heat capacities c_p of gases at 1 atm pressure.



No.	Liquid	Range °C
29	Acetic Acid 100%	0 - 80
32	Acetone	20 - 50
52	Ammonia	-70 - 50
37	Amyl Alcohol	-50 - 25
26	Amyl Acetate	0-100
30	Aniline	0-130
23	Benzene	10 - 80
27	Benzyl Alcohol	-20 - 30
10	Benzyl Chloride	-30 - 30
49	Brine. 25 % $CaCl_2$	-40 - 20
51	Brine. 25 % NaCl	-40 - 20
44	Butyl Alcohol	0 - 100
2	Carbon Disulfide	-100 - 25
3	Carbon Tetrachloride	10 - 60
8	Chlorobenzene	0 - 100
4	Chloroform	0 - 50
21	Decane	-80 - 25
6A	Dichloroethane	-30 - 60
5	Dichloromethane	-40 - 50
15	Diphenyl	80 - 120
22	Diphenylmethone	30 - 100
16	Diphenyl Oxide	0-200
16	Dowtherm A	0-200
24	Ethyl Acetate	-50 - 25
12	Ethyl Alcohol 100%	30 - 80
16	Ethyl Alcohol 95 %	20 - 80
50	Ethyl Alcohol 50 %	20 - 80
25	Ethyl Benzene	0-100
1	Ethyl Bromide	5 - 25
13	Ethyl Chloride	-30 - 40
36	Ethyl Ether	-100 - 25
7	Ethyl Iodide	0-100
39	Ethylene Glycd	-40 -200

No.	Liquid	Range °C
2A	Freon- 11 (CCl_3F)	-20 - 70
6	Freon- 12(CCl_2F_2)	-40 - 15
4A	Freon-21($CHCl_2F$)	-20 - 70
?A	Freon-22($CHClF_2$)	-20 - 60
3A	Freon-113($CCl_2F-CClF_2$)	-20 - 70
36	Glycerol	-40 - 20
28	Heptane	0 - 60
35	Hexane	-80 - 20
46	Hydrochloric Acid, 30%	20 -100
41	Isoamyl Alcohol	10 - 100
43	Isobutyl Alcohol	0 -100
47	Isopropyl Alcohol	-20 - 50
31	Isopropyl Ether	-80 - 20
40	Methyl Alcohol	-40 - 20
13A	Methyl Chloride	-80 - 20
14	Naphthalene	90 - 200
12	Nitrobenzene	0 - 100
34	Nonane	-50 - 25
33	Octane	-50 - 25
3	Perchlorethylene	-30 - 140
45	Propyl Alcohol	-20-100
20	Pyridine	-50 - 25
9	Sulfuric Acid 96 %	10 - 45
11	Sulfur Dioxide	-20 - 100
23	Toluene	0 - 60
53	Water	10-200
19	Xylene Ortho	0-100
18	Xylene Meta	0-100
17	Xylene Para	0-100

FIGURE D-4
Heat capacities of liquids.

TABLE 9

Specific gravities of liquids

The values presented in this table are based on the density of water at 4°C and a total pressure of 1 attn.

$$\text{Specific gravity} = \frac{\text{density of material at indicated temperature}}{\text{density of liquid water at 4°C}}$$

Density of liquid water at 4°C = 1.000 g/cm^3 = 62.43 lb/ft^3

Pure liquid	Formula	Temperature, °C	Specific gravity
Acetaldehyde	CH_3CHO	18	0.783
Acetic acid	CH_3CO_2H	0	1.067
		30	1.038
Acetone	CH_3COCH_3	20	0.792
Benzene	C_6H_6	20	0.879
n-Butyl alcohol	$C_2H_5CH_2CH_2OH$	20	0.810
Carbon tetrachloride	CCl_4	20	1.595
Ethyl alcohol	CH_3CH_2OH	10	0.798
		30	0.791
Ethyl ether	$(CH_3CH_2)_2O$	25	0.708
Ethylene glycol	$CH_2OH{\cdot}CH_2OH$	19	1.113
Glycerol	$CH_2OH{\cdot}CHOH{\cdot}CH_2OH$	15	1.264
		30	1.255
Isobutyl alcohol	$(CH_3)_2CHCH_2OH$	18	0.805
Isopropyl alcohol	$(CH_3)_2CHOH$	0	0.802
		30	0.777
Methyl alcohol	CH_3OH	0	0.810
		20	0.792
Nitric acid	HNO_3	10	1.531
		30	1.495
Phenol	C_6H_5OH	25	1.071
n-Propyl alcohol	$CH_3CH_2CH_2OH$	20	0.804
Sulfuric acid	H_2SO_4	10	1.841
		30	1.821
Water	H_2O	4	1.000
		100	0.958

TABLE 10

Specific gravities of solids

The specific gravities as indicated in this table apply at ordinary atmospheric temperatures. The values are based on the density of water at 4°C.

$$\text{Specific gravity} = \frac{\text{density of material}}{\text{density of liquid water at } 4^\circ\text{C}}$$

Density of liquid water at 4 °C = 1.000 g/cm^3 = 62.43 lb/ft^3

Substance	Specific gravity
Aluminum, hard-drawn	2.55-2.80
Brass, cast-rolled	8.4-8.7
Copper, cast-rolled	8.8-8.95
Glass, common	2.4-2.8
Gold, cast-hammered	19.25-19.35
Iron :	
Gray cast	7.03-7.13
Wrought	7.6-7.9
Lead	11.34
Nickel	8.9
Platinum, cast-hammered	21.5
Silver, cast-hammered	10.4-10.6
Steel, cold-drawn	7.83
Tin, cast-hammered	7.2-7.5
White oak timber, air-dried	0.77
White pine timber, air-dried	0.43
Zinc, cast-rolled	6.9-7.2

TABLE 11

Properties of saturated steam†

Values in table based on zero enthalpy of liquid water at 32°F

Temperature, °F	Absolute pressure, psi	Volume of vapor, ft^3/lb	Enthalpy		Latent heat of evaporation, Btu/lb
			Liquid, Btu/lb	Vapor, Btu/lb	
32	0.0885	3306	0.00	1075.8	1075.8
35	0.0999	2947	3.02	1077.1	1074.1
40	0.1217	2444	8.05	1079.3	1071.3
45	0.1475	2036.4	13.06	1081.5	1068.4
50	0.1781	1703.2	18.07	1083.7	1065.6
55	0.2141	1430.7	23.07	1085.8	1062.7
60	0.2563	1206.7	28.06	1088.0	1059.9
65	0.3056	1021.4	33.05	1090.2	1057.1
70	0.3631	867.9	38.04	1092.3	1054.3
75	0.4298	740.0	43.03	1094.5	1051.5
80	0.5069	633.1	48.02	1096.6	1048.6
85	0.5959	543.5	53.00	1098.8	1045.8
90	0 6982	468.0	57.99	1100.9	1042.9
95	0.8153	404.3	62.98	1103.1	1040.1
100	0.9492	350.4	67.97	1105.2	1037.2
105	1.1016	304.5	72.95	1107.3	1034.3
110	1.2748	265.4	77.94	1109.5	1031.6
115	1.4709	231.9	82.93	1111.6	1028.7
120	1.6924	203.27	87.92	1113.7	1025.8
125	1.9420	178.61	92.91	1115.8	1022.9
130	2.2225	157.34	97.90	1117.9	1020.0
135	2.5370	138.95	102.90	1119.9	1017.0
140	2.8886	123.01	107.89	1122.0	1014.1
145	3.281	109.15	112.89	1124.1	1011.2
150	3.718	97.07	117.89	1126.1	1008.2
155	4.203	86.52	122.89	1128.1	1005.2
160	4.741	77.29	127.89	1130.2	1002.3
165	5.335	69.19	132.89	1132.2	999.3
170	5.992	62.06	137.90	1134.2	996.3
175	6.715	55.78	142.91	1136.2	993.3

TABLE 11

Properties of saturated steam† (Continued)

Temperature, °F	Absolute pressure, psi	Volume of vapor, ft³/lb	Enthalpy		Latent heat of evaporation, Btu/lb
			Liquid, Btu/lb	Vapor, Btu/lb	
180	7.510	50.23	147.92	1138.1	990.2
185	8.383	45.31	152.93	1140.1	987.2
190	9.339	40.96	157.95	1142.0	984.1
195	10.385	37.09	162.97	1144.0	981.0
200	11.526	33.64	167.99	1145.9	977.9
210	14.123	27.82	178.05	1149.7	971.6
212	14.696	26.80	180.07	1150.4	970.3
220	17.186	23.15	188.13	1153.4	965.2
230	20.780	19.382	198.23	1157.0	958.8
240	24.969	16.323	208.34	1160.5	952.2
250	29.825	13.821	218.48	1164.0	945.5
260	35.429	11.763	228.64	1167.3	938.7
270	41.858	10.061	238.84	1170.6	931.8
280	49.203	8.645	249.06	1173.8	924.7
290	57.556	7.461	259.31	1176.8	917.5
300	67.013	6.466	269.59	1179.7	910.1
310	77.68	5.626	279.92	1182.5	902.6
320	89.66	4.914	290.28	1185.2	894.9
330	103.06	4.307	300.68	1187.7	887.0
340	118.01	3.788	311.13	1190.1	879.0
350	134.63	3.342	321.63	1192.3	870.7
360	153.04	2.957	332.18	1194.4	862.2
370	173.37	2.625	342.79	1196.3	853.5
380	195.77	2.335	353.45	1198.1	844.6
390	220.37	2.0836	364.17	1199.6	835.4
400	247.31	1.8633	374.97	1201.0	826.0

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† Abridged from "Thermodynamic Properties of Steam," by J. H. Keenan and F. G. Keyes, copyright, 1937, by Joseph H. Keenan and Frederick G. Keyes. Published by John Wiley and Sons, Inc., New York. See also A.S.M.E. Steam Tables.

TABLE 12

Heat-exchanger and condenser-tube data

Tube OD, in.	BWG	Wall thick-ness, in.	ID, in.	Flow area per tube, in.^{2}	Surface per lin ft, ft^2		Weight per lin ft, lb steel
					Outside	Inside	
½	12	0.109	0.282	0.0625	0.1309	0.0748	0.493
	14	0.083	0.334	0.0876	0.1309	0.0874	0.403
	16	0.065	0.370	0.1076	0.1309	0.0969	0.329
	18	0.049	0.402	0.127	0.1309	0.1052	0.258
	20	0.035	0.430	0.145	0.1309	0.1125	0.190
¾	10	0.134	0.482	0.182	0.1963	0.1263	0.965
	11	0.120	0.510	0.204	0.1963	0.1335	0.884
	12	0.109	0.532	0.223	0.1963	0.1393	0.817
	13	0.095	0.560	0.247	0.1963	0.1466	0.727
	14	0.083	0.584	0.268	0.1963	0.1529	0.647
	15	0.072	0.606	0.289	0.1963	0.1587	0.571
	16	0.065	0.620	0.302	0.1963	0.1623	0.520
	17	0.058	0.634	0.314	0.1963	0.1660	0.469
	18	0.049	0.652	0.334	0.1963	0.1707	0.401
1	8	0.165	0.670	0.335	0.2618	0.1754	1.61
	9	0.148	0.704	0.389	0.2618	0.1843	1.47
	10	0.134	0.732	0.421	0.2618	0.1916	1.36
	11	0.120	0.760	0.455	0.2618	0.1990	1.23
	12	0.109	0.782	0.479	0.2618	0.2048	1.14
	13	0.095	0.810	0.515	0.2618	0.2121	1.00
	14	0.083	0.834	0.546	0.2618	0.2183	0.890
	15	0.072	0.856	0.576	0.2618	0.2241	0.781
	16	0.065	0.870	0.594	0.2618	0.2277	0.710
	17	0.058	0.884	0.613	0.2618	0.2314	0.639
	18	0.049	0.902	0.639	0.2618	0.2361	0.545
1¼	8	0.165	0.920	0.665	0.3271	0.2409	2.09
	9	0.148	0.954	0.714	0.3271	0.2498	1.91
	10	0.134	0.982	0.757	0.3271	0.2572	1.75
	11	0.120	1.01	0.800	0.3271	0.2644	1.58
	12	0.109	1.03	0.836	0.3271	0.2701	1.45
	13	0.095	1.06	0.884	0.3271	0.2775	1.28
	14	0.083	1.08	0.923	0.3271	0.2839	1.13
	15	0.072	1.11	0.960	0.3271	0.2896	0.991
	16	0.065	1.12	0.985	0.3271	0.2932	0.900
	17	0.058	1.13	1.01	0.3271	0.2969	0.808
	18	0.049	1.15	1.04	0.3271	0.3015	0.688

TABLE 12

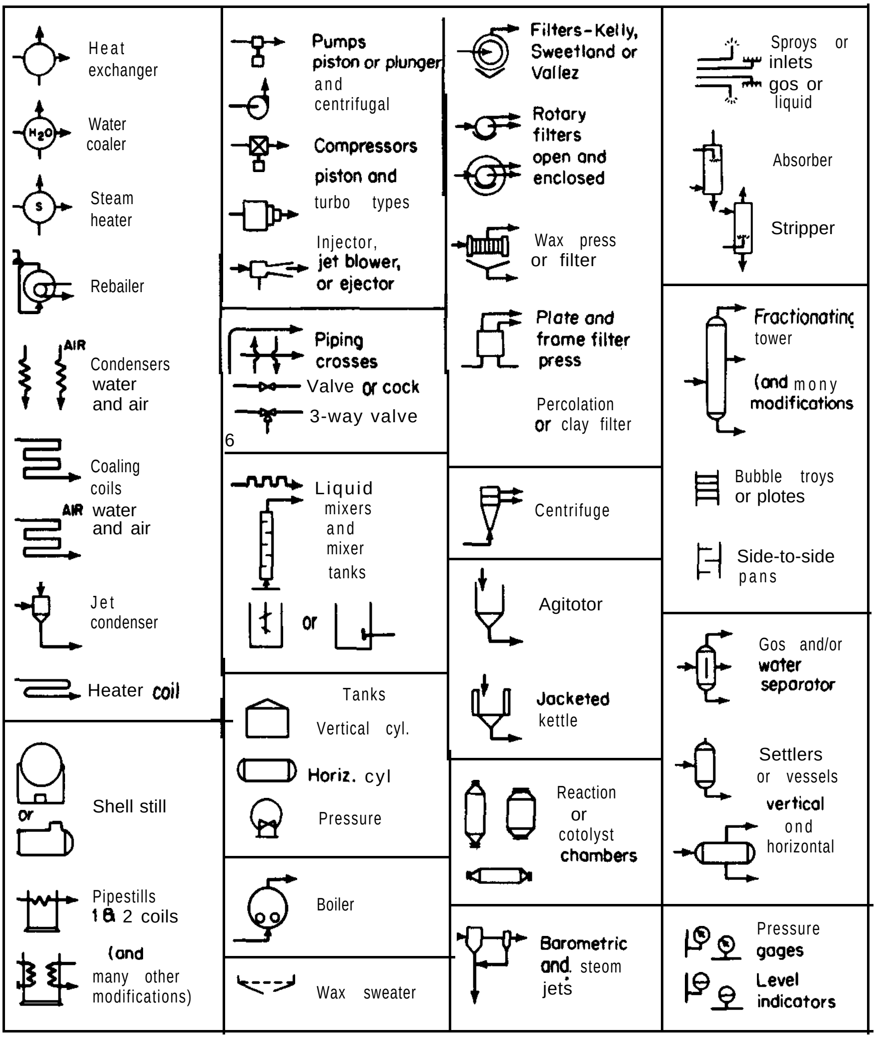
Heat-exchanger and condenser-tube data *(Continued)*

Tube OD, in.	BWG	Wall thickness, in.	ID, in.	Flow area per tube, in.^{2}	Surface per lin ft, ft^2		Weight per lin ft, lb steel
					Outside	Inside	
1½	8	0.165	1.17	1.075	0.3925	0.3063	2.57
	9	0.148	1.20	1.14	0.3925	0.3152	2.34
	10	0.134	1.23	1.19	0.3925	0.3225	2.14
	11	0.120	1.26	1.25	0.3925	0.3299	1.98
	12	0.109	1.28	1.29	0.3925	0.3356	1.77
	13	0.095	1.31	1.35	0.3925	0.3430	1.56
	14	0.083	1.33	1.40	0.3925	0.3492	1.37
	15	0.072	1.36	1.44	0.3925	0.3555	1.20
	16	0.065	1.37	1.47	0.3925	0.3587	1.09
	17	0.058	1.38	1.50	0.3925	0.3623	0.978
	18	0.049	1.40	1.54	0.3925	0.3670	0.831

TABLE 13
Steel-pipe dimensions

Nominal pipe size, in.	OD, in.	Schedule No.	ID, in.	Flow area per pipe, in.^{2}	Surface per lin ft, ft^2		Weight per lin ft, lb steel
					Outside	Inside	
⅛	0.405	40†	0.269	0.058	0.106	0.070	0.25
		80‡	0.215	0.036	0.106	0.056	0.32
¼	0.540	40	0.364	0.104	0.141	0.095	0.43
		80	0.302	0.072	0.141	0.079	0.54
⅜	0.675	40	0.493	0.192	0.177	0.129	0.57
		80	0.423	0.141	0.177	0.111	0.74
½	0.840	40	0.622	0.304	0.220	0.163	0.85
		80	0.546	0.235	0.220	0.143	1.09
¾	1.05	40	0.824	0.534	0.275	0.216	1.13
		80	0.742	0.432	0.275	0.194	1.48
1	1.32	40	1.049	0.864	0.344	0.274	1.68
		80	0.957	0.718	0.344	0.250	2.17
1¼	1.66	40	1.380	1.50	0.435	0.362	2.28
		80	1.278	1.28	0.435	0.335	3.00
1½	1.90	40	1.610	2.04	0.498	0.422	2.72
		80	1.500	1.76	0.498	0.393	3.64
2	2.38	40	2.067	3.35	0.622	0.542	3.66
		80	1.939	2.95	0.622	0.508	5.03
2½	2.88	40	2.469	4.79	0.753	0.647	5.80
		80	2.323	4.23	0.753	0.609	7.67
3	3.50	40	3.068	7.38	0.917	0.804	7.58
		80	2.900	6.61	0.917	0.760	10.3
4	4.50	40	4.026	12.7	1.178	1.055	10.8
		80	3.826	11.5	1.178	1.002	15.0
6	6.625	40	6.065	28.9	1.734	1.590	19.0
		80	5.761	26.1	1.734	1.510	28.6
8	8.625	40	7.981	50.0	2.258	2.090	28.6
		80	7.625	45.7	2.258	2.000	43.4
10	10.75	40	10.02	78.8	2.814	2.62	40.5
		60	9.75	74.6	2.814	2.55	54.8
12	12.75	30	12.09	115	3.338	3.17	43.8
16	16.0	30	15.25	183	4.189	4.00	62.6
20	20.0	20	19.25	291	5.236	5.05	78.6
24	24.0	20	23.25	425	6.283	6.09	94.7

† Schedule 40 designates former "standard" pipe.
‡ Schedule 80 designates former "extra-strong" pipe.



FIGURED-5
Equipment symbols.

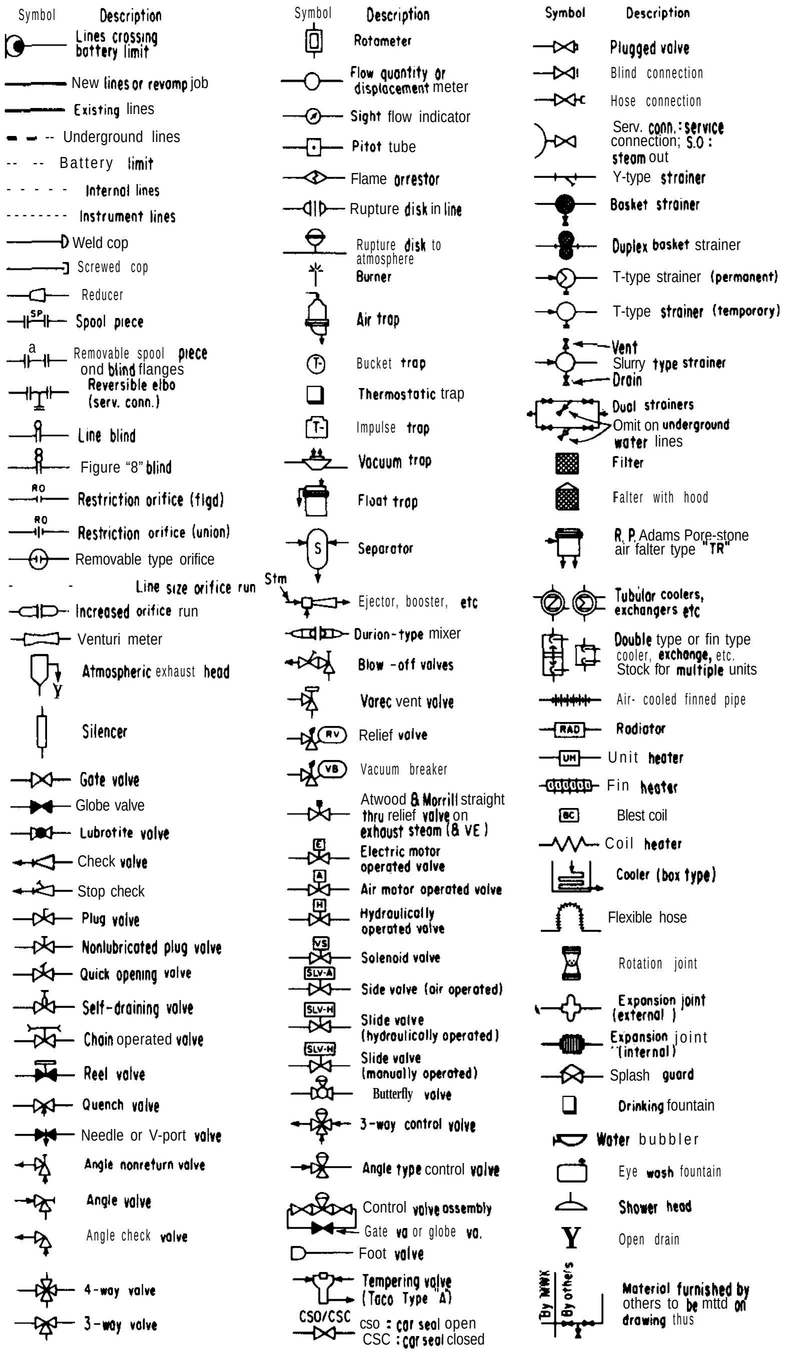


FIGURE D-6
Flow sheet symbols, particularly for detailed equipment flow sheets. (*Courtesy of the M. W. Kellogg Co.*)

TABLE 14

International atomic weights

Substance	Symbol	Atomic weight	At. No.
Actinium	Ac	221.03	89
Aluminum	Al	26.98	13
Americium	Am	243	95
Antimony	Sb	121.75	51
Argon	A	39.948	18
Arsenic	As	74.92	33
Astatine	At	210	85
Barium	Ba	137.33	56
Berkelium	Bk	247	97
Beryllium	Be	9.012	4
Bismuth	Bi	208.98	83
Boron	B	10.81	5
Bromine	Br	79.904	35
Cadmium	Cd	112.41	48
Calcium	Ca	40.08	20
Californium	Cf	251	98
Carbon	C	12.011	6
Cerium	Ce	140.12	58
Ceslum	cs	132.91	55
Chlorine	Cl	35.457	17
Chromium	Cr	52.00	24
Cobalt	co	58.93	27
Copper	cu	63.55	29
Curium	Cm	247	96
Dysprosium	Dy	162.50	66
Einsteinium	Es	254	99
Erbium	Er	167.26	68
Europium	Eu	152.0	63
Fermium	Fm	257	100
Fluorine	F	19.00	9
Francium	Fr	223	87
Gadollnium	Gd	157.25	64
Gallium	Ga	69.72	31
Germanium	Ge	12.59	32
Gold	Au	197.0	79
Hafnium	Hf	178.49	72
Helium	He	4.003	2
Holmium	Ho	164.93	67
Hydrogen	H	1.008	1
Indium	In	114.82	49
Iodine	I	126.90	53
Iridium	Ir	192.2	77
Iron	Fe	55.85	26
Krypton	Kr	83.80	36
Lanthanum	La	138.91	57
Lawrencium	Lr	260	103
Lead	Pb	201.2	82
Lithium	Li	6.941	3
Lutetium	Lu	174.97	71
Magnesium	Mg	24.31	12
Manganese	Mn	54.94	25
Mendelevium	Md	258	101
Mercury	Hg	200.59	80
Molybdenum	Mo	95.94	42
Neodymium	Nd	144.24	60
Neon	Ne	20.179	10
Neptunium	Np	237	93
Nickel	Ni	58.10	28
Niobium (Columbian)	Nb	92.91	41
Nitrogen	N	14.007	7
Nobelium	No	259	102
Osmium	Os	190.2	76
Oxygen	0	16	8
Palladium	Pd	106.4	46
Phosphorus	P	30.975	15
Platinum	Pt	195.09	78
Plutonium	Pu	244	94
Polonium	PO	209	84
Potassium	K	39.098	19
Praseodymium	Pr	140.91	59
Promethium	Pm	145	61
Protactinium	Pa	231	91
Radium	Ra	226.03	88
Radon	Rn	222	86
Rhenium	Re	186.2	75
Rhodium	Rh	102.91	45
Rubidium	Rb	85.47	31
Ruthenium	Ru	101.1	44
Samarium	Sm	150.4	62
Scandium	SC	44.96	21
Selenium	Se	78.96	34
Silicon	Si	28.09	14
Silver	Ag	107.868	47
Sodium	Na	22.990	11
Strontium	Sr	81.62	38
Sulfur	S	32.06	16
Tantalum	Ta	180.95	73
Technetium	Tc	97	43
Tellurium	Te	127.60	52
Terbium	Tb	158.93	65
Thallium	Tl	204.37	81
Thorium	Th	232.04	90
Thulium	Tm	168.93	69
Tin	Sn	118.69	50
Titanium	Ti	47.90	22
Tungsten (Wolfram)	W	183.85	74
Uranium	U	238.03	92
Vanadium	V	50.94	23
Xenon	Xe	131.30	54
Ytterbium	Yb	173.04	70
Yttrium	Y	88.91	39
Zinc	Zn	65.38	30
zirconium	Zr	91.22	40

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SUBJECTINDEX

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2-1

The vapor pressure of mercury is obtained from <u>Perry's Chemical Engineers' Handbook</u>, 6th ed., p. 3-201 as

T, °C	Vapor press, bar
10	7.101×10^{-7}
20	1.729×10^{-6}
30	3.968×10^{-6}

By interpolation, the vapor pressure of mercury at 23°C is 2.2×10^{-6} bar. For low pressures, at equilibrium, the vapor phase mol fraction of mercury can be obtained with the use of Dalton's law, or

$y = p_v^{\circ}/p$ where p = system pressure of 1.013 bar

$y = 2.2 \times 10^{-6}/1.013 = 2.17 \times 10^{-6}$ kg mols Hg / kg mols air

but $y =$ mols Hg / (mols air + mols Hg) $\cong$ mols Hg / mols air

The specific volume of air is obtained from the ideal gas relation $v = RT/p = (8314)(296.1)/101{,}300 = 24.305\ m^3/\text{kg mol}$

The concentration of mercury in air at equilibrium is then

$$c = (2.17 \times 10^{-6}/24.305)(200.6) = 17.9 \times 10^{-6}\ \text{kg Hg}/m^3$$
$$= 17.9\ \text{mg Hg}/m^3$$

Since the PEL (permissible exposure limit is 0.1 mg Hg/m^3, the concentration of mercury in the air greatly exceeds this limit and therefore is <u>not acceptable.</u>

Answer

2-2

This problem has been simplified considerably with a number of assumptions. Actually, the corrosion rate for the iron (carbon steel) over a year is not in a fixed oxygen level atmosphere. It is most likely that the corrosion rate of iron is limited by the diffusion rate of oxygen to the tank surface. Under the circumstances, the analysis would require a corrosion model, information on the initial rate before corrosion product buildup, and need some indication of the diffusivity of the oxygen in the corrosion layer. As indicated in the problem, the reaction rate is a function of the surface are inside the tank. For a 7.6-m (inside) diameter spherical container, the inside area is given by

$$A = 4\pi r_i^2 = \pi D_i^2 = \pi (7.6)^2 = 181.5 \text{ m}^2$$

Iron corrosion $= (181.5)(0.127 \times 10^{-3})(7850)$ $\qquad \rho_{Fe} = 7850 \text{ kg/m}^3$

$= 180.9$ kg/y

$= (180.9)/(365)(24) = 0.02065$ kg/h

$= 0.02065/55 = 3.7545 \times 10^{-4}$ kg mol/h

Oxygen rate $= (3.7545 \times 10^{-4})(1.5 \text{ kg mol } O_2 / 2.0 \text{ kg mol Fe})$

$= 2.816 \times 10^{-4}$ kg mol/h

Volume of sphere $= (4/3)\pi r_i^3 = \pi D_i^3/6 = \pi (7.6)^3/6 = 229.8 \text{ m}^3$

At room temperature (21.1°C) and 1 atm, the specific volume inside the tank, assuming an ideal gas, is

$$v = (22.4)(294.2/273.1) = 24.147 \text{ m}^3/\text{kg mol air}$$

The oxygen concentration under these conditions is $(0.21)(24.147) = 8.696$ kg mol O_2/m^3

For a first order reaction,

$$\frac{dc}{d\theta} = -kc$$ where k = reaction rate constant, θ = time, and c = concentration

2-2 (continued)

When $c = 8.696 \times 10^{-4}$ kg mol/m^3, the O_2 rate is 2.816×10^{-4} kgmol/h, and the tank volume is 229.8 m^3. At this condition

$$\frac{dc}{d\theta} = \frac{-2.816 \times 10^{-4}}{229.8} = -1.225 \times 10^{-6} \text{ kgmol/m}^3\cdot\text{h}$$

Solving for the reaction rate constant gives

$$k = -\left(\frac{dc}{d\theta}\right)/c = (1.225 \times 10^{-6})/8.696 \times 10^{-3} = 1.408 \times 10^{-4}\,\text{h}$$

Now solve for the time when O_2 concentration is reduced to 19.5 mol percent. Integrating the original rate equation results in

$$\ln(c_1/c_2) = k(\theta_2 - \theta_1) \qquad \text{where } \theta_1 = 0$$

$$\theta_2 = \ln(c_1/c_2)/k$$

$$= \ln(0.21/0.195)/1.408 \times 10^{-4}$$

$$= \underline{\underline{526 \text{ h or approx. } 22 \text{ days}}}$$

Answer

2-3

Assume that the dilution air is available at a temperature of 20°C and a pressure of 1 atm. The evaporation rate is 7.5 g/min or 0.0075 kg/min; Thus,

$$\dot{n}_{vc} = 0.0075/78 = 9.62 \times 10^{-5} \text{ kg mol vinyl chloride/min}$$

At 1 atm use the ideal gas to obtain the volumetric flow rate

$$\dot{m}_{v,vc}\ n_{cv}RT/p = (9.62 \times 10^{-5})(8314)(293/1.013 \times 10^{5}$$
$$= 2.31 \times 10^{-3} \text{ m}^3/\text{min}$$

Since the PEL for vinyl chloride is 1ppm,

$$\dot{m}_{v,air} = (\dot{m}_{v,vc})(10^{6}) = (2.31 \times 10^{-3})(10^{6})$$
$$= 2.31 \times 10^{3} \text{ m}^3/\text{min}$$

If a safety factor of 5 is required,

$$\dot{m}_{v,air} = (5)(2.31 \times 10^{3})/60 = \underline{\underline{192.5 \text{ m}^3/\text{s}}}$$

Answer

This is a fairly high flow rate and would need one or two large blowers.

2-4

The density of benzene at 22°C is 879 kg/m^3. The molecular weight is 78.1 kg/kg mol.

Since the pressure is low, assume the ideal gas law is applicable to the air stream that is used for ventilation.

$$\dot{n}_{Bz} = \left(2.0 \times 10^{-6}\ \frac{m^3}{min}\right)\left(879\ \frac{kg}{m^3}\right) / 78.1\ \frac{kg}{kg\ mol}$$

$$= 2.2506 \times 10^{-5}\ kg\ mol/min$$

$$\dot{m}_{v,Bz}\ \ nRT/p = \left(2.2506 \times 10^{-5}\ \frac{kg\ mol}{min}\right)\left(8314\ \frac{J}{kg\ mol \cdot K}\right)(295\ K)$$

$$6.27 \times 10^{-4}\ m^3/min$$

Assume complete mixing of the benzene with the dilution air

$$\dot{m}_{v,air} = \left(6.27 \times 10^{-4}\ \frac{m^3\ Bz}{min}\right)\left(10^6\ \frac{m^3\ air/min}{m^3\ Bz/min}\right)$$

$$= \underline{\underline{627\ m^3\ air/min}}$$

Answer

2-5

To determine the length of time the cannister will be usable, we will need to find a relationship between the amount adsorbed and $(T/V)\log(f_s/f)$. Let $x = (T/V)\log(f_s/f)$ and y the amount adsorbed in cm^3(liq)/100 g charcoal. From the data given,

when $y = 1$ $\quad x = 21$
$y = 10$ $\quad x = 11$

assuming a linear relation of $\log y = ax + b$

$$\log 1 = 21a + b$$
$$\log 10 = 11a + b$$

or

$$\log(1/10) = 10a \qquad a = -0.1$$
$$b = \log 10 - 11(-0.1) \qquad b = 2.1$$

The molecular weight of dichloropropane (DCP) is 113, density is 1160 kg/m^3, and molar volume = 0.1 m^3/kg mol. The vapor pressure from Perry's Chemical Engineers' Handbook, 6ed., p.354,

Temp., °C	Vapor press., mm Hg
19.4	40
28	60

Assuming the Antoine equation can be used to represent the vapor pressure of DCP

$$\log p = A + B/T$$
$$\log 40 = A + B/292.55$$
$$\log 60 = A + B/301.15$$

Solving the latter two equations simutaneously results in

$$B = -1802 \qquad A = 7.76$$

At 26.9°C or 300.05K

$$\log p = 7.76 + (-1802/300.05) = 1.755$$
$$p = 56.9 \text{ mm Hg}$$

Determine the fugacity of DCP by assuming that it is essentially equal to the partial pressure. Thus,

2-5 (continued)

$f \simeq p$

$p = y p_T$ where p_T = total pressure and $y = 750/10^6$

$p = (750/10^6)(760) = 0.57 \text{ mm Hg}$

therefore, $f_s/f = 56.9/0.57 = 99.82$

$x = (T/V)\log(f_s/f) = (300/100)\log(99.82) = 5.99$

Now determine how much gas has been adsorbed

$\log y = -0.1x + 2.1 = (-0.1)(5.99) + 2.1 = 1.503$

$y = 31.84 \text{ cm}^3/100 \text{ g}$ charcoal

For 100% saturation, amount adsorbed per 100g charcoal is

$m = (31.84 \text{ cm}^3/100 \text{g C})(1.16 \text{ g/cm}^3) = 36.93 \text{g}$

At 82% saturation, amount adsorbed before breakthrough is

$m_{82\%} = (36.93)(0.82) = 30.3 \text{ g}$

Now determine volume of air associated with 30.3 g DCP

$$n_{air} = \left(\frac{30.3 \text{ g DCP}}{\frac{113 \text{ g DCP}}{\text{g mol DCP}}}\right)\left(\frac{10^6 \text{ g mol air}}{750 \text{ g mol DCP}}\right) = 357.5 \text{ g mol air}$$

$$\text{Vol of air} = nRT/p = \frac{(357.5 \text{ g mol})}{1 \text{ atm}}\left(82.06 \frac{\text{atm}\cdot\text{cm}^3}{\text{g mol}\cdot\text{K}}\right)(300 \text{ K})$$

$$= 8.8 \times 10^6 \text{ cm}^3 = 8{,}800 \text{ ℓ} = 8.8 \text{ m}^3$$

At 45 ℓ/min

$$\theta = (8{,}800 \text{ ℓ})/(45 \text{ ℓ/min}) = \underline{\underline{195 \text{ min} = 3.25 \text{h}}}$$

Answer

Cannister should last 3.25h before break-through occurs.

2-6

If the mixture is at the flash point

$$\sum \frac{y_i}{LFL_i} = 1.0$$

Assuming Raoult's law applies

$$y_i = p_{v,i}\, x_i / p$$

where $p_{v,i}$ is the vapor pressure of component i, p the pressure, and x_i and y_i the mol fraction of component in the liquid and vapor, respectively. Vapor pressure of C_8 and C_9 are available from <u>Perry's Chemical Engineers' Handbook</u>, 6th ed., p. 3-58

Assume flash temperature of 18°C

Comp	x_i	$p_{v,i}$, atm	y_i	LFL_i
C_8	0.5	0.0123	0.00615	0.01
C_9	0.5	0.00365	0.00183	0.008

$$\sum \frac{y_i}{LFL_i} = \frac{0.00615}{0.01} + \frac{0.00183}{0.008} = 0.844 \text{ temp. too low}$$

Assume flash temperature of 21.1°C

Comp	x_i	$p_{v,i}$, atm	y_i	LFL_i
C_8	0.5	0.0145	0.00725	0.01
C_9	0.5	0.0044	0.0022	0.008

$$\sum \frac{y_i}{LFL_i} = \frac{0.00725}{0.01} + \frac{0.0022}{0.008} \cong 1.0$$

The flash point for this mixture is 21.1°C
Answer

2-7

At equilibrium, $f_{i,L} = f_{i,v}$

Since

$$f_{i,L} = \gamma_i f_{o,L} x_i$$
$$f_{i,v} = \varphi_i f_{o,v} y_i$$
$$\gamma_i f_{o,L} x_i = \varphi_i f_{o,v} y_i$$

At low pressures

$$f_{o,v} = p$$
$$f_{o,L} = p_v^\circ$$
$$\varphi_i = 1.0$$

Therefore $\gamma_i p_v^\circ x_i = p y_i$

For an ideal solution, $\gamma_i = 1$

and $x_i p_v^\circ = p y_i$

Definitions

$f_{i,L}$ = fugacity of liquid
$f_{i,v}$ = fugacity of vapor
γ_i = liquid phase activity coeff,
φ_i = vapor phase activity coeff.
$f_{o,L}$ = pure comp. liq. fugacity
$f_{o,v}$ = pure comp. vapor fugacity
p = total pressure
p_v° = vapor pressure

If the acetone has no air dissolved in it, $x_i = 1.0$ and y_i is the LFL or $y_i = 0.0255$.

At a total pressure of 1 atm,

$$p_v^\circ = y_i p / x_i$$
$$= (0.0255)(1.0)/1.0$$
$$= 0.0255 \text{ atm} = 19.38 \text{ mm Hg}$$

Acetone has a vapor pressure of 19.38 mm Hg at a temperature of -21.4 °C (<u>Perry's Chemical Engineers' Handbook</u>, 6th ed., p. 3-50.

$$T_{flash} \text{ (calculated)} = \underline{\underline{-21.4\,°C}}$$

Answer

The handbook <u>Dangerous Properties of Industrial Materials</u> provides a measured flash point for acetone of -17.8°C.

2-8

At equilibrium, acetone, benzene, ethyl ether, and n-pentane are too rich to ignite. Carbon disulfide and methyl alcohol will always be in the flammable range. It should be noted that all of the vaporizing liquids can be ignited if not at equilibrium. Some are also quite toxic.

Answer

The types of fire extinguishers recommended when a small spill is ignited are as follows:

Liquid	Extinguishers recommended
acetone	alcohol foam, dry chemical, CO_2
benzene	foam, dry chemical, CO_2
carbon disulfide	foam, water blanket
ethyl ether	alcohol foam, dry chemical, CO_2
methyl alcohol	alcohol foam, dry chemical, CO_2
n-pentane	foam, dry chemical, CO_2

Answer

2-9

The stoichiometric balance for the reaction is given by

$$C_3H_8 + 5O_2 + 5(3.76)N_2 \longrightarrow 3CO_2 + 4H_2O + 5(3.76)N_2$$

For 200% theoretical air, the balance becomes

$$C_3H_8 + 10O_2 + 10(3.76)N_2 \longrightarrow 3CO_2 + 4H_2O + 5O_2 + 10(3.76)N_2$$

Since all the energy generated from the combustion process is absorbed by the product gases, the final temperature of the products is determined by means of an adiabatic flame temperature calculation. The heat capacities for each component is a function of the temperature and requires a knowledge of the C_p-temperature relation of those components. Present software is available and can be used to rapidly converge with a heat balance on the appropriate flame temperature. When this is done, a temperature of **1527 K** is obtained.

Answer

The iterative calculation can also be done by using an enthalpy table as a function of temperature for each of the coponents. The latter is available in Holman's *Thermodynamics*, 4th ed., Table A-18, pp 755-6 and is used in the hand calculation of this problem. The energy balance is

$$(\bar{h}_f^\circ)_{C_3H_8} = 3(\bar{h}_f + \bar{h}_T - \bar{h}_{298})_{CO_2} + 4(\bar{h}_f^\circ + \bar{h}_T - \bar{h}_{298})_{H_2O} + 5(\bar{h}_f^\circ + \bar{h}_T - \bar{h}_{298})_{O_2}$$
$$+ 37.6(\bar{h}_f + \bar{h}_T - \bar{h}_{298})_{N_2}$$

where $\bar{h}_f^\circ$ is the molar heat of formation, $\bar{h}_T$ the molar enthalpy at temperature T, and $\bar{h}_{298}$ the molar enthalpy at 298 K for the components as designated by the subscripts. Molar heats of formation for these components are from Table A-6 of the same text.

Heat losses to the surroundings are assumed to be negligible.

2-9 (continued)

To obtain the final temperature, assume different values of temperature until the energy balance is consistent. Units in kJ/kg mol.

Assume T = 1555 K

$$-103{,}925 = 3(-393{,}782 + 74{,}392 - 9374) + 4(-241{,}997 + 60{,}680 - 9904) + 5(51{,}307 - 8664) + 37.6(49{,}034 - 8676)$$

$$-103{,}925 \neq -20{,}500$$

Temperature assumed is too high; assume T = 1500 K

$$-103{,}925 = 3(-393{,}782 + 71{,}131 - 9374) + 4(-241{,}997 + 58{,}050 - 9904) + 5(49{,}272 - 8664) + 37.6(47{,}092 - 8676)$$

$$-103{,}925 \neq -123{,}997$$

Temperature assumed is too low. By interpolation, the final temperature under adiabatic conditions is 1511 K

The ideal gas law is valid at low pressures and high temperatures. Thus,

$$V_f = n_f RT/p$$

$$n_f = 3 + 4 + 5 + 37.6 = 49.6 \text{ kg mol/kg mol fuel}$$
$$= 4960 \text{ kg mol}/100 \text{ kg mol fuel}$$

$$V_f = (4960)(8314)(1511)/101{,}300 = 615{,}000 \text{ m}^3$$
$$= \underline{\underline{6.15 \times 10^5 \text{ m}^3}}$$

Answer

Ratio of final volume to initial volume is given by

$$V_f/V_i = \frac{n_f T_f}{n_i T_i} = \frac{(4960)(1511)}{(4860)(298)} = 5.17$$

2-10

Since the lower flammability limit (LFL) for acetone vapor in air is 2.55%, the maximum vapor pressure the acetone could have would be

$$p_v^\circ = (0.0255)(760) = 19.38 \text{ mm Hg}$$

The temperature equivalent to this vapor pressure may be determined utilizing the experimental data provided in Perry's Chemical Engineers' Handbook, 6th ed., p. 3-50 and the use of the Antoine equation with the form

$$\ln p_v^\circ = a - b/(t + 273)$$

The vapor pressure of acetone in the range of interest is

t, °C	p_v°, mm Hg
-40.5	5
-20.8	20

Substituting these values in the expression

$$\ln 5 = a - b/(-40.5 + 273)$$
$$\ln 20 = a - b/(-20.8 + 273)$$

Solving the equations simultaneously gives a = 19.36 and b = 4126.3. Thus,

$$\ln 19.38 = 19.36 - 4126.3/(t + 273)$$
$$t = -21.3°C$$

The acetone would have to be cooled below -21.3°C to avoid an acetone-air mixture that could be ignited with a discharge of static electricity. Such cooling is not very practical and the use of an inert gas blanket should be considered during the filling operation.

Answer

2-11

The decomposition reaction is the following

$$C_2H_2 \rightleftharpoons 2C + H_2$$

Component	mols in	mols at temp. T	mol fraction
C_2H_2	1	$1-x$	$1-x$
C	0	$2x$	0*
H_2	0	x	x

*Assumes solid carbon product

Using the nomenclature and methodology from Sandler's <u>Chemical and Engineering Thermodynamics</u>

$$K_a(T) = \frac{a_C^2 a_{H_2}}{a_{C_2H_2}} \simeq \frac{y_{H_2}}{y_{C_2H_2}} = \frac{x}{1-x} \quad \text{Where } x = \text{extent of reac.}$$

$$K_a(T) = e^{-(\Delta G_{rx}(T)/RT)}$$

Assume for both carbon and hydrogen

$$\Delta G_f(25°C) = 0 \quad \text{and} \quad \Delta H_f(25°C) = 0$$

$$\Delta G_{rx}(25°C) = -\Delta G_f(25°C)_{C_2H_2} = -50{,}000 \text{ cal/g mol}$$

$$\Delta H_{rx}(25°C) = -\Delta H_f(25°C)_{C_2H_2} = -54{,}194 \text{ cal/g mol}$$

The heat capacities for C, H_2 and C_2H_2 fit the relation $Cp_i = a_i + b_iT + c_iT^2 + d_iT^3 + e_i/T^2$ cal/g mol·K where T is in kelvins.

Comp.	a	$b \times 10^2$	$c \times 10^5$	$d \times 10^9$	e
C	2.673	0.2617	–	–	116,900
H_2	6.952	−0.04576	0.09563	−0.2079	–
C_2H_2	5.21	2.2008	−1.559	4.349	–

Data are from Sadler and <u>Perry's Chemical Engineers' Handbook</u>

For heats of reaction at other temperatures besides 25°C or T_1

$$\Delta H^\circ_{rx}(T) = \Delta H^\circ_{rx}(T_1) + \Delta a(T-T_1) + (\Delta b/2)(T^2-T_1^2) + (\Delta c/3)(T^3-T_1^3) + (\Delta d/4)(T^4-T_1^4) - \Delta e\left(\frac{1}{T} - \frac{1}{T_1}\right)$$

Where $\Delta a = \sum \nu_i a_i$, etc. and ν_i = stoichiometric coefficient

2-11 (continued, 1)

$K_a(T)$ for other temperatures besides at T_1 or 25 °C

$$\frac{\ln K_a(T)}{\ln K_a(T_1)} = \frac{\Delta a}{R}\ln\frac{T}{T_1} + \frac{\Delta b}{2R}(T-T_1) + \frac{\Delta c}{6R}(T^2-T_1^2) + \frac{\Delta d}{12R}(T^3-T_1^3) + \frac{\Delta e}{2}\left(\frac{1}{T^2}-\frac{1}{T_1^2}\right) + \frac{1}{R}\left\{-\Delta H^\circ_{rx}(T_1) + \Delta a T_1 + \frac{\Delta b}{2}T_1^2 + \frac{\Delta c}{3}T_1^3 + \frac{\Delta d}{4}T_1^4 - \frac{\Delta e}{T_1}\right\}\left(\frac{1}{T}-\frac{1}{T_1}\right) \quad (1)$$

For this reaction $K_a(T) = x/(1-x)$ or

$$x = \frac{K_a(T)}{1+K_a(T)} \quad (2)$$

From an energy balance for an adiabatic constant volume reaction

$$x = \frac{-\sum \int n_{i,in} C_{v_i}\, dT}{\Delta H^\circ_{rx}(T)} \quad (3)$$

where $n_{i,in}$ are the mols that enter into the reaction. Using the heat capacity constants tabulated earlier

$\Delta a = 7.088$

$\Delta b = -1.7232 \times 10^{-2}$

$\Delta c = 1.6546 \times 10^{-5}$

$\Delta d = -4.5569 \times 10^{-9}$

$\Delta e = 2.338 \times 10^{5}$

$$K_a(25\,°C) = e^{-[(-50{,}000)/(1.987)(298.1)]} = 4.5 \times 10^{36}$$

Since the final temperature will be relatively high, the ideal gas law may be assumed to be valid. Thus,

$$C_{v,i} = C_{p,i} - R = a_i - R + b_iT + c_iT^2 + d_iT^3 + e_i/T^2$$

$\sum[n_i(a_i - R)]_{in} = 3.223$ $\quad\sum(n_ib_i)_{in} = 2.2008 \times 10^{-2}$ $\quad\sum(n_ic_i)_{in} = -1.559 \times 10^{-5}$

$\sum(n_id_i)_{in} = 9.34 \times 10^{-9}$ $\quad\sum(n_ie_i)_{in} = 0$

With this information, Eqs. (1), (2) and (3) can be solved simultaneously. The results are

$T = 2967$ K and $x = 0.984$

The pressure will then be obtained from

2-11 (continued, 2)

$$p(T) = p_1(T_2/T_1) = 10(2967/298) = 99.6 \text{ atm}$$

With a safety factor of 4, the tank should be able to withstand a pressure of essentially 400 atm.

For spherical containers, Table 12-10 provides a relation to obtain the minimum wall thickness t of the tank as

$$t = \frac{P r_i}{2SE_j - 0.2P}$$

Where P is the maximum allowable pressure, r_i the inside radius, S the maximum allowable working stress, E_j the efficiency of the welded joint. Assume that the carbon steel used in the tank is SA-442 Grade 55 with an S value of 94.7 MPa and that the welds are butt joints with complete penetration and spot examined resulting in a weld efficiency of 85%.

The diameter of the spherical tank was not given. Assume inside diameters of 1, 2, and 5 meters. For a 1-m diameter sphere

$$t = \frac{(400)(1.013 \times 10^5)(0.5)}{2(94.7 \times 10^6)(0.85) - (0.2)(400)(1.013 \times 10^5}$$

$$= 0.1324 \text{ m} \quad \text{or} \quad 5.21 \text{ in}$$

For 2 and 5 m inside diameters, the minimum thicknesses are $\underline{\underline{0.265 \text{ m and } 0.662 \text{ m}}}$.
Answer

The results show the advantage of having a properly designed relief valve and burst disk assembly closely tied in with the reactor containing the decomposition reaction of acetylene. It would be imprudent to recommend a spherical container with a wall thickness greater than 0.0125 m because of the reactor costs involved.

2-12

The rate at which energy is produced in a reactor is given by the relation

$$\frac{dq}{d\theta} = \Delta H_{rx}\, k_o C V e^{-(E/RT)}$$

where $dq/d\theta$ is the heat liberated over time, ΔH_{rx} the heat of reaction, C the concentration of the reactant, V the volume of the reactor, E the activation energy, R the gas constant, and T the temperature.

When the reaction is exothermic, heat must be removed from the reactor to keep the temperature from increasing beyond control. The heat transfer rate can be written as

$$\frac{dq}{d\theta} = UA(T - T_o)$$

where U is the overall heat transfer coefficient, A the heat transfer area, and T_o the coolant temperature. If heat can be removed as rapidly as heat is generated by the reaction, the reaction can be kept under control. Thus, at steady state

$$\Delta H_{rx} k_o C V e^{-(E/RT)} = UA(T - T_o)$$

To simplify the relation, let $B = \Delta H_{rr} k_o C V$ and $K = UA$. Thus,

$$B e^{-(E/RT)} = K(T - T_o)$$

or $$B e^{-(E/RT)} - K(T - T_o) = 0$$

The maximum temperature where the steady-state energy balance is valid can be found by setting its derivative with respect to temperature equal to zero. Doing this, resuts in

$$B e^{-(E/RT)} (E/RT_c^2) - K = 0$$

where T_c is the critical or maximum temperature. However,

$$B e^{-(E/RT)} = K(T_c - T_o)$$

at the maximum temperature. Substituting this into the previous equation and solving for K gives

2-12 (continued)

$$K(T_c - T_o)(E/RT_c^2) = K$$

or $(T_c - T_o) = RT_c^2/E$ which can be used to find T_c

Substituting values

$$(T_c - 288.1) = 1.987\,T_c^2/28,000$$

$$T_c = 294.3\text{ K}$$

This means that if the temperature in the reactor rises above 294.3 K (21.1°C), the reactor will begin to generate energy faster than it can be removed and the reaction will run away. Note that only a 6.2°C rise in temperature in the reactor can be tolerated.

Answer

For safe operation of the reactor some or all of the following actions can be taken:

1. Increase the size of the heat exchanger
2. Inject an inhibitor into the reactor or quench the reaction if it is ready to go out of control
3. Reduce the feed to the reactor
4. Provide adequate venting for the reactor for emergency use
5. Add diluents to the reactor to reduce the reaction rate. (These will probably have to be separated from the product after the reaction is completed)

2-13

A system for transferring liquid chlorine from a chlorine storage container with pressurized chlorine gas is shown below. A feature of this design is a relief system that utilizes a duplicate burst system to simplify replacement of the discs with a mechanical interlock between the two burst systems. Bottom outlets have been avoided so that joints are not exposed to liquid chlorine and the only leakage will be of a gaseos nature.

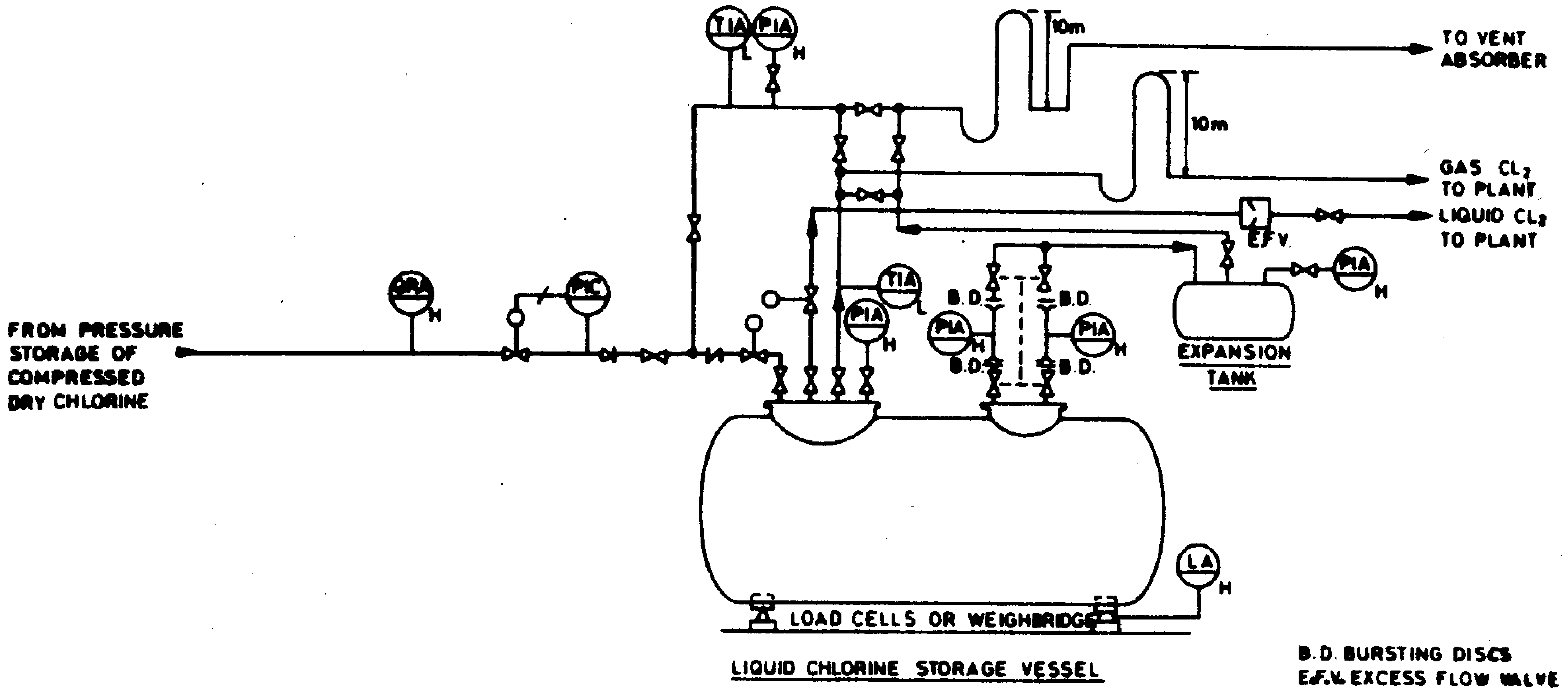
The severity of the leak is reduced by lowering the pressure within the storage container or system. A facility must be available to accomplish this pressure reduction during chlorine transfer as well. A waste Chlorine absorption unit that uses caustic or some simular chlorine consuming process is recommended. In addition, an expansion vessel is provided with an alarm to handle any pressure build-up in the container. The expansion tank is automatically vented to the absorption system (see figure).

The recompressed dry chlorine can be stored in a pressure storage receiver fitted with a relief valve set to discharge to the waste chlorine absorption unit at a predetermined pressure.

Pressurization with vaporized chlorine can function with a pressure storage tank that can utilize an external heater to vaporize the liquid chlorine. Control of the vaporization heater is tied to the pressure control system for the storage tank. Relief valves must permit over-pressurization to be relieved with excess chlorine vapor going to the waste chlorine absorption unit described above.

One possible instrumented arrangement for the chlorine vaporizer is shown in the second figure.

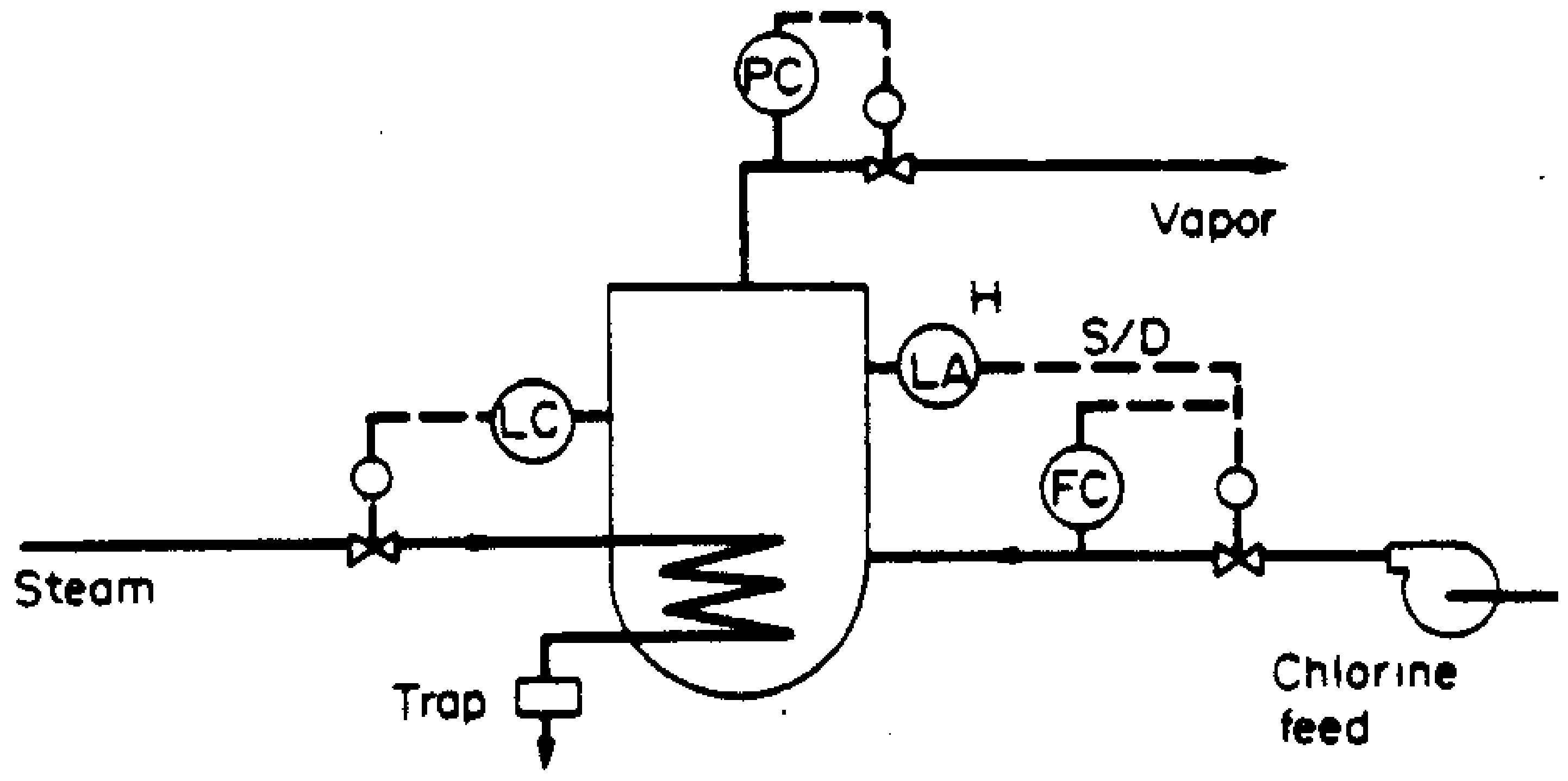
2-13 (continued, 1)



One possible arrangement with appropriate controls and alarm system for transferring liquid chlorine by using recompressed chlorine vapor.

Answer

2-13 (continued, 2)



Answer

2-14

Reducing the inventory of a hazardous material in a process includes, but is not limited to the following items:

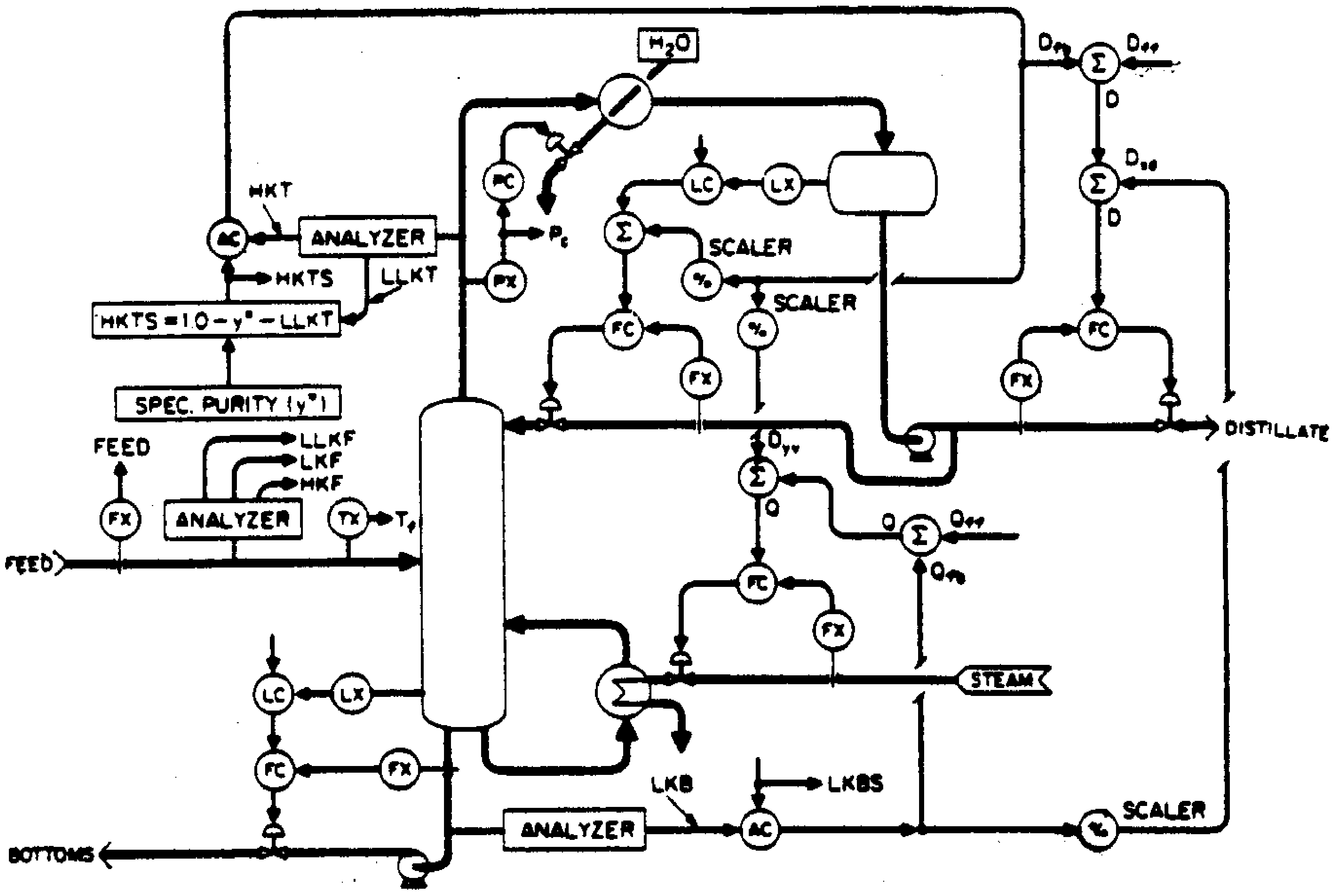
1. Investigate the possibility of producing the end product by another process that does not the use of a very hazardous material.
2. Determine whether the hazardous material may be reacted or combined before the distillation step to form a non-hazardous material. If the hazardous material is a reactant, recycle the stream back to the process to reduce the hazardous material content before the distillation step.
3. Depending upon how the hazardous material is used in the process, determine whether there are more efficient ways of absorbing, adsorbing, trapping, complexing, etc, the hazardous materials to avoid using a distillation operation that generally has considerably greater difficulty in attaining ppm concentrations in a product stream.
4. If elimination of the hazardous material is not possible, operate the distillation operation on a continuous basis. This will require less material being handled, smaller equipment used with possible elimination of the buffer storage, and improved chances of containment should there be a leak.
5. Improve the efficiencies of the column to reduce build-up of hazardous materials. Guidelines for such improvements are given by K.E. Nelson, Hydrocarbon Processing 69(3), 93-98 (1990).

Answer

A distillation column can be controlled in many ways. Generally, there is no one "best" way. Which one is to be prefered depends on the purpose of the separation, the types of controls available for each service, the variability of the feed, and the relative importance of the product recovery and product purity. Each case needs to be analyzed separately.

2-14 (continued)

A typical distillation column with appropriate instrumentation in which basic controls are assisted with predictive and analyzer loops is outlined in <u>Perry's</u> <u>Chemical</u> <u>Engineers'</u> <u>Handbook</u>, 6th ed., p. 22-118 and is reproduced below with the permission of McGraw-Hill.



Nomenclature Summary for Fig.

Symbol	Explanation
D_{fb}	Contribution of feedback control to distillate-flow set point
D_{ff}	Contribution of predictive control
D_{nd}	Contribution of noninteractive decoupler to distillate-flow set point
D_{yw}	Contribution of noninteractive decoupler to reboiler-heat set point
HKF	Heavy key feed-stream component
HKT	Heavy key overhead-product-stream-component concentration, weight percent
HKTS	Specified concentration of heavy key component in overhead product, weight percent
LKBS, x^*	Specified concentration of light key component in bottoms product, weight percent
LKF, z	Light-key-feed-stream-component concentration, weight percent
LLKF	Lighter-than-light-key-feed-stream-component concentration, weight percent
LLKT	Lighter-than-light-key-overhead-product-stream-component concentration, weight percent
Q	Reboiler heat, BTU/unit time
Q_{fb}	Contribution of feedback control to steam-flow set point
Q_{ff}	Contribution of predictive control to steam-flow set point
Q_r/F	Weight ratio, reboiler-heat-to-feed
x^*, *LKBS*	Specified concentration of light key component in bottoms product, weight percent
z, *LKF*	Light-key-feed-stream-component concentration, weight percent

2-15

HAZOP for Line L-102

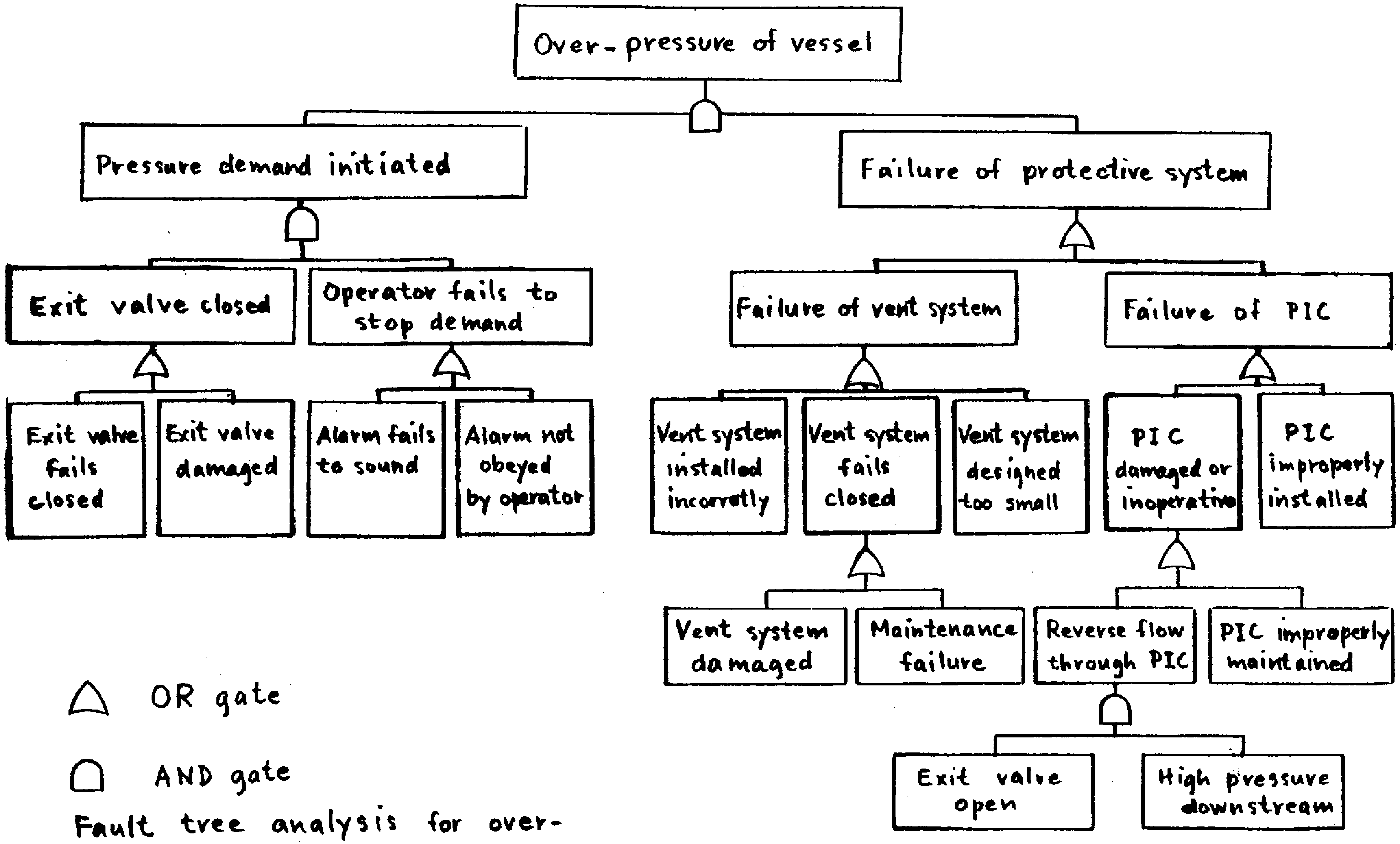
Guide word	Property	Possible cause	Possible consequence	Action required
More	Temperature	A	Pump seal failure, vapor lock	Install a feed-back line
Less	Temperature	Low ambient temperature	B	steam tracing
More	Flow	Line fracture	Spillage (possible large explosion)	C
		Pump seal failure	Spillage (possible small explosion)	D
		Control fault		Consider bypass
No	Flow	E	Shutdown	Low level alarm
		F	Shutdown	Automatic start-up of standby pump
Reverse	Flow	Pump failure	Backpressure on storage vessel	G
As well as	Impurities	H	Possible small detonation	Priming line

The missing items below complete the HAZOP study above:

A. Steam tracing operating incorrectly or downstream valve closed
B. Thermal stress on line
C. Install flow control with automatic shutoff
D. Flow diverted through standby pump capable of automatic startup
E. Tank empty
F. Pump failure
G. Relieve backpressure, divert flow through standby pump with automatic startup
H. Pump drawing in air or operator error

Answer

2-16



Fault tree analysis for over-pressurizing vessel

2.25

2-17

The important factors to be considered for plant location in the fertilizer industry are (1) raw materials, (2) markets, (3) transportation facilities, (4) power and fuel availability, (5) waste disposal, and (6) water supply. The order of importance varies with the fertilizer considered and to some extent to the size of plant contemplated. Analysis of this situation for ammonia, urea, and phosphate fertilizer requires a knowledge of the raw materials required and the most commonly used process for each fertilizer. A concise review of these requirements is given by G.T. Austin in Shreve's Chemical Process Industries, 5th ed. About 90% of US ammonia production uses natural gas as the feedstock. The reactions are

$$CH_{4(g)} + H_2O \xrightarrow[2860\ \text{kPa},\ 515^\circ C]{\text{Ni catalyst}} CO_{(g)} + 3\,H_{2(g)}$$

$$CH_{4(g)} + \text{air} \xrightarrow[670^\circ C]{\text{Ni catalyst}} CO_{(g)} + 2H_{2(g)} + N_{2(g)}$$

$$1/2\,N_{2(g)} + 3/2\,H_{2(g)} \xrightarrow[\text{pressure}]{\text{Fe catalyst}} NH_{3(g)}$$

The process requires large amounts of power and cooling water. Since natural gas is easily transported by pipeline, the plant need not be located near the gas well. Rather, it is more profitable to locate the plant close to the markets. Thus, the latter is the most important factor in plant location. Adequate power and water supply would also rank quite high. As the size of the plant increased, more consideration would have to be given to transportation facilities since the markets would be from the plant. Adequate supply of natural gas would also become important with plant size.

The commercial processes in current use for the manufacture of urea are based on two reactions, namely

$$CO_{2(g)} + 2NH_{3(l)} \xrightarrow[185^\circ C - 193^\circ C]{22\ \text{MPa}} NH_4CO_2NH_{2(l)}$$

2-17 (continued)

$$NH_4CO_2NH_2 \longrightarrow NH_2CONH_{2(l)} + H_2O_{(l)}$$

This process requires approximately 1/4 the power and cooling water of that required for the ammonia process per unit weight of fertilizer produced. Since ammonia is one of the principal raw materials required for the process, it is much more important to consider the source of the raw materials in a selection of the plant site for the production of urea. This factor will be the most important consideration even with an increase in plant size. However, since the fertilizer markets are not as large as for the raw material, careful consideration must be given to markets and the transportation necessar to reach those markets.

Phosphate fertilizer is obtained from phosphate rock ore. The accidulation of phosphate rock to produce superphosphate fertilizer can probably best be described by the following reaction:

$$CaF_2 \cdot 3Ca_3(PO_4)_2 + 7\ H_2SO_4 + 3H_2O \longrightarrow 3CaH_4(PO_4)_2 \cdot H_2O + 2HF + 7CaSO_4$$

Since the phosphate rock must be concentrated after it is mined, plant location is determined by the location of the phosphate rock ore regardless of plant size. The aspects of waste disposal of the non-phosphate rock is directly related to the mining operation. Markets directly affect plant size.

Answer

2-18

The diluted waste sample contains 15g of waste and 485g of dilution water. The total oxygen available for consumption by the waste is

$$\text{g of } O_2 = (485)(9)/10^6 = 4.365 \times 10^{-3}\,\text{g}$$

The final oxygen content of the diluted sample is

$$\text{g of } O_2 = (500)(4)/10^6 = 2.0 \times 10^{-3}\,\text{g}$$

The oxygen depletion due to the 15 g waste sample is

$$O_2 \text{ depletion} = 4.365 \times 10^{-3} - 2.0 \times 10^{-3} = 2.365 \times 10^{-3}\,\text{g}$$

The BOD, defined as the oxygen depletion in parts oxygen consumed per 10^6 parts of waste is then

$$\text{BOD} = (2.365 \times 10^{-3})(10^6/15) = \underline{\underline{158 \text{ parts } O_2/10^6 \text{ parts waste}}}$$

Answer

2-19

The solution to this problem involves some of the same safety aspects as considered in the solution of Problem 2-13.

The most common method to vaporize chlorine is to use steam in a heating coil immersed in the chlorine liquid in the vaporizer. A schematic with typical instrumentation is shown in the solution of Problem 2-13 and will not be repeated here.

The disadvantage of using hot water to vaporize the chlorine is the need for a larger coil and a greater potential for freezing of the coil (NBP of Cl_2 is $-34°C$). Economics is probably the major deterrent for not using a closed circuit heating with a heat transfer fluid that is inert to chlorine.

The use of a heating coil immersed below the liquid level in a chlorine vaporizer is the preffered arrangement to accomplish the vaporization. Not only is the coiled tube less expensive, it is also less susceptible to leaks and failures.

Answer

2-20

The net heat transfer flux to a storage tank during a fire comes from radiation and convection attributed to the fire that surrouds tank and can be approximated by

$$q_{net} = q_r + q_c + q_{rr}$$

where q_r is the radiation flux from the fire, q_c the convection flux from the fire, and q_{rr} the radiation flux back from the tank. The radiation fluxes are proportional to the differences in the fourth powers of the absolute temperatures for the radiating and absorbing surfaces. The convection flux can be estimated of natural convection heat transfer. Since the surface of interest in this problem is large and vertical, the relations for natural convection on vertical plane surfaces are the most appropriate ones to use. The flame can be assumed to have transport properties equal to that of hot air in the flame and the fact that most of the gas in the flame is nitrogen. Flame temperatures vary from fuel to fuel, but a temperature of about 1500K will provide a reasonable average value for most situations.

If the benzene tank, assumed to be a vertical cylinder, is to store 1600 m^3, its minimum height is given by

$$V = \pi D_i^2 Z/4$$
$$Z = 4V/\pi D_i^2 = (4)(1600)/\pi(15)^2 = 9.05 \text{ m}$$

Storage tanks generally have an ullage volume of 10% or more to take care of liquid expansion and provide additional volume for some vaporization to occur without appreciably increasing the tank pressure. Heat transfer to the surface area surrounding the ullage space will be minimal compared to the surface area wetted by the full tank contents. Thus, only the latter area will be used in the heat transfer calculation. Also assume that the temperature of the wetted wall to be at the boiling point temperature of the fluid being stored in the tank. There will be a temperature gradient through the tank wall, but this will be small and have minimal effect on the re-

2-20 (continued, 1)

quired venting rate during a fire.

$$q_r = 95{,}000 \text{ W/m}^2$$

is given in the problem statement and assumes that the external fire completely surrounds the storage tank. The convective flux from the fire is given by

$$q_c = h(T_f - T_w)$$

where T_f is the flame temperature and T_w is the wall temperature, assumed to be the same as the normal boiling temperature of the fluid in the tank. The heat transfer coefficient in US customary units for natural convection along a vertical surface may be obtained numerically from

$$Nu = \left\{ 0.825 + \frac{0.387\, R_a^{1/6}}{\left[1 + \left(\frac{0.492}{P_r}\right)^{9/16}\right]^{8/27}} \right\}^2$$

as given in <u>Perry's Chemical Engineers' Handbook</u>, 6th ed., where $Nu = hL/k$, $Pr = C_p\mu/k$, $Ra = Gr\,Pr$, and $Gr = L^3\rho^2 g \beta \Delta T/\mu^2$. Since this calculation is rather lengthy and natural convective heat transfer coefficients for air generally have values of 5 to 10 W/m²·K, we will assume an average value of 7.5 W/m²·K and only calculate a better value if analysis warrants it.

Using this assumption

$$q_c = (7.5)(1500 - 353) = 8600 \text{ W/m}^2\cdot\text{K}$$

The heat flux radiated back from the tank surface to space can be estimated from

$$q_{rr} = \epsilon \sigma T_w^4 \qquad \text{assume } \epsilon = 1.0$$

$$= (1.0)(0.972 \times 10^{-8})(353)^4 = 150 \text{ W/m}^2\cdot\text{K}$$

From these calculations

$$q_{net} = 950{,}000 + 8600 - 150 = 103{,}450 \text{ W/m}^2\cdot\text{K}$$

2-20 (continued, 2)

The vapor boil-off rate is then

$$\dot{m} = (q_{net}) A / \Delta H_v \qquad \Delta H_v = \text{heat of vaporization}$$

$$= (q_{(net)}) \pi D Z / \Delta H_v$$

$$= (103,450) \pi (15)(9.05) / 3.943 \times 10^5 = 111.9 \text{ kg/s}$$

At the NBP of benzene, the vapor density is 2.755 kg/m^3.
Thus

$$\dot{m}_v = \dot{m} / \rho = 111.9 / 2.755 = 40.65 \text{ m}^3/\text{s}$$

The vent area needed for this volumetric flow rate can be determined by use of an orifice equation of the form

$$A = \frac{\dot{m}}{0.61 [(2) \Delta p \rho]^{1/2}} \qquad \Delta p = 250 \text{ Pa}$$

$$= \frac{111.9}{(0.61)[(2)(250)(2.755)]^{1/2}} = 4.94 \text{ m}^2$$

The area for the vent is much too large for the tank. Therefore a weak seam roof <u>should be specified</u>

Answer

Several assumptions were made in the solution of this problem and need to be verified. First, the convective flux is only 9% of the total heat flux. Even if it were changed ± 100%, the overall result would only be changed slightly. Thus, an exact calculation of the heat transfer coefficient would not change the final conclusion.

There is some resistance to heat transfer through the walls of the tank. For a tank wall of 0.0125 m, the temperature gradient is

$$\Delta T = q_{(net)} t_w / k_w = (103,450)(0.0125)/45 = 28.7°\text{C}$$

This will have a negligible effect on the results, and thus this assumption is acceptable. However, there is also a thermal resistance with the film on the inside of the tank. Typical heat transfer coefficients are on the order of several thousand $W/m^2 \cdot K$. Assuming a value of

2-20 (continued,3)

2,000 $W/m^2 \cdot K$, the ΔT across the film is

$$\Delta T = q_{(net)}/h = 103{,}450/2000 = 51.7\,^{\circ}C$$

The combination of these two resistances only reduces the convective flux rate by approximately 6%. This change would have negligible effect on the final answer.

As a footnote, the NFPA Fire Protection Handbook and API 650 all reommend the following equation for the net heat flux to a tank subjected to an external fire:

$$q_{(net)} = 43{,}200\, A^{0.82}$$

where $q_{(net)}$ is in watts and A is the wetted area in m^2. For this problem

$$q_{(net)} = 43{,}200\,[\pi D_t Z]^{0.82} = 43{,}200\,[\pi(15)(9.05)]^{0.82}$$
$$= 6.19 \times 10^6\ W$$

from this

$$\dot{m}_v = q_{(net)}/\Delta H_v \rho$$
$$= 6.19 \times 10^6/(3.943 \times 10^5)(2.755) = 5.6\ m^3/s$$

The major difference in results is that the heat flux in the problem statement applies to a fire that completely surrounds the tank. Generally fires are seldom large enough to surround large tanks. In addition a soot layer builds up on the cool tank walls which further restricts the heat transfer. Thus, the total heat flux is reduced substantially. However, the venting rate even under these conditions will be large and a tank with a weak roof seam would be recommended.

3-1

The production of formaldehyde still relies on the chemical process developed nearly 50 years ago. Nearly 70% of new installations in the US utilize a metal oxide catalyst to form formaldehyde by the oxidation of methanol

$$CH_3OH + \frac{1}{2} O_2 \rightarrow HCHO$$

Process 1

In this process shown above, vaporized methanol is mixed with air and recycle tail gas and passed through catalyst filled tubes in a heat exchanger reactor. Heat released by the exothermic reaction is removed by vaporization of a high-boiling heat transfer fluid on the outside of the tubes. Steam is normally produced by condensing the heat transfer fluid. Product leaving the bottom of the reactor is cooled and then transferred to the base of an absorber. Formaldehyde concentration in the product is adjusted by controlling the amount of water added to the top of the absorber. A product of 1 to 55% formaldehyde can be produced. Byproducts in the reaction include CO, CO_2, dimethylether, and formic acid. The latter is removed by ion exchange. Plant yields between 88 – 92% are reported.

Reference: "Kirk-Othmer Encyclopedia of Chemical Technology", Vol. II, p. 929, J. Wiley, New York, 1994.

Process 2

A feed mixture is generated by sparging air into a pool of heated methanol and combining the vapor with steam. The mixture passes through a superheater to a catalyst bed of silver crystals or layers of silver gauze. The reactor product is rapidly cooled in a steam generator followed by a water-cooled heat exchange and then fed to the bottom of an absorption tower. Most of the methanol, water, and formaldehyde is condensed in the water-cooled bottom section of the absorber. Almost complete removal of the remaining methanol and formaldehyde from the tail gas occurs at the top of the tower by countercurrent contact with the process water. Absorber

3-1 (continued)

bottoms go to a distillation column where methanol is recovered for recycle to the reactor. The bottoms stream from the distillation tower is usually sent to an anion exchanger which reduces the formic acid to specifications. The product contains up to 55% formaldehyde and less than 1.5% methanol.

Reference: J.H. Martin and M.T. Butter, *Oil & Gas J.* **72**, 71(Mar. 11, 1974).

Process 3

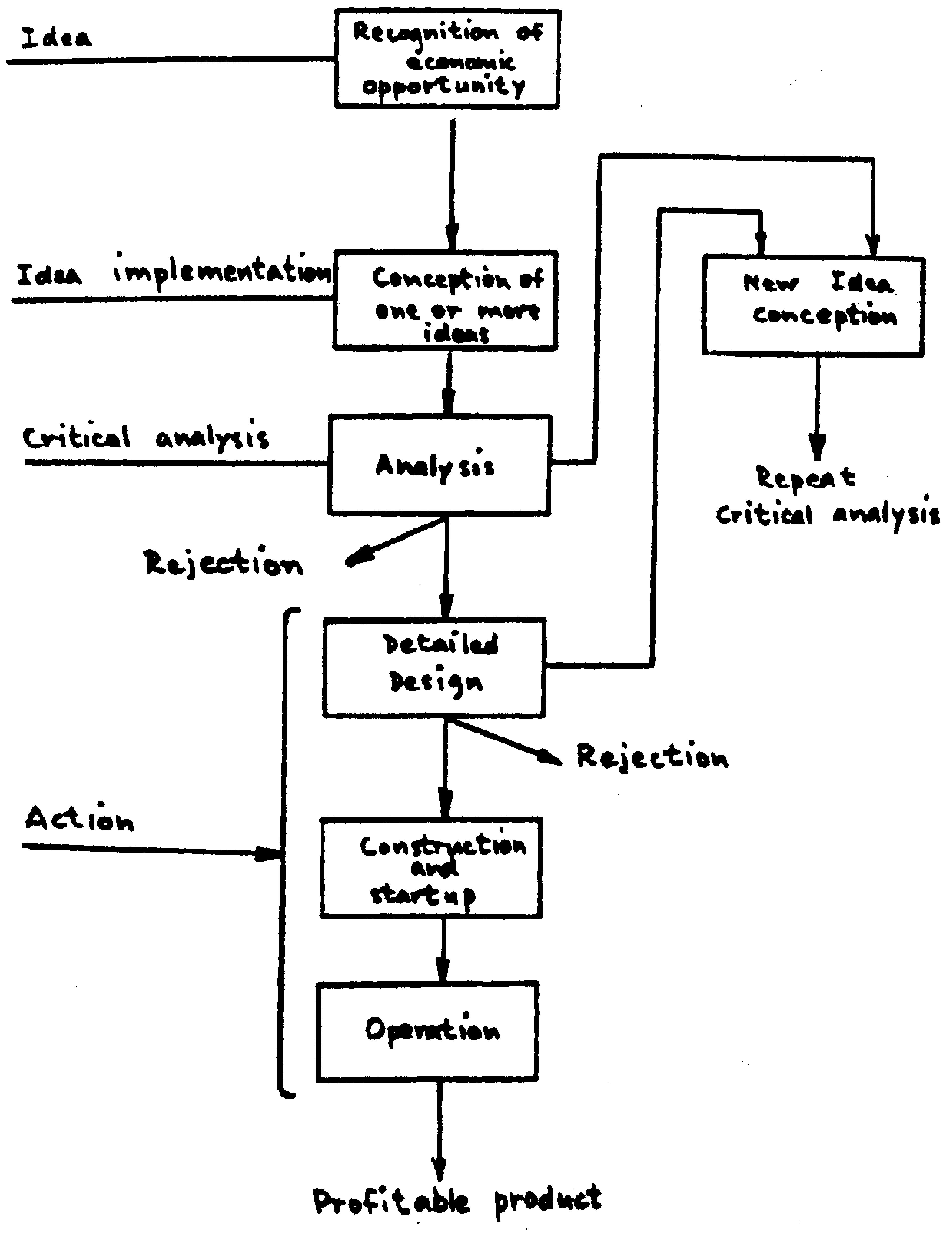
Another process to produce formaldehyde involves the partial oxidation of methane. The incentive for such a process is the reduction of raw material costs by avoiding the capital and operating expense of initially producing the methanol from methane. In this process methane gas is mixed with excess air, preheated to around 600°C and fed into a vertical, tubular reactor with a conical bottom, filled with ceramic packing saturated with a $K_2B_4O_2$ solution. The conversion of methane depends on the temperature of the feed mixture and its content of nitrogen oxides. Exit gases are rapidly cooled to 400°C in a tubular shell condenser with water. The gas is further cooled to approximately 33°C in a heat exchanger and transferred to a scrubber where formaldehyde is absorbed in a dilute solution of formaldehyde solution to provide formaldehyde of the desired concentration. Since energy requirements are quite high, the processing plant is best coupled with a cheap source of power.

Reference: M. Brown and N. Parkyns, *Catal. Today* **5**(3), 305 (1991).

J. Hargreaves, G.Hutchings, and R. Joyner, *Nature* **348**, 28 (Nov. 1990).

3-2

A general outline showing the sequence of steps in the development of a formaldehyde plant is shown below:



3-2 (continued)

Recognition of economic opportunity – this can come from a variety of sources. Technical services may recognize a customer's need in a resin or adhesive. Market research may uncover another use of formaldehyde as a feed product or intermediary requirement in a process.

Conception of one of more ideas – again this can come from a variety of sources, namely, the chemist, process engineer, plant operator, technical service personnel, maintenance crew, etc. The idea can be one of an entirely new process like the utilization of natural gas for a raw material (see Problem 3-1) rather than methanol. On the other hand, the idea can involve a change in operating conditions and arrangement of equipment while using the same raw materials as demonstrated by the Formox process developed by Reichhold Chemicals (see *Hydrocarbon Process.*, **44**(11), 215 (1965) or the Montecatini process (see *Hydrocarbon Process.*, **44**(11), 216 (1965).

Analysis – this step requires the greatest technical interaction on the part of the design engineer. For the production of formaldehyde this step includes the following considerations:

1. Raw materials (availability, quantity, quality, cost)
2. Thermodynamics and kinetics of the chemical reaction of, for example, the Formox process

   $$2CH_3OH+O_2 \rightarrow 2HCHO+2H_2O$$

   in terms of equilibrium, yields, rates, optimum conditions, etc.
3. Facilities and equipment available and what must be purchased
4. Estimation of production costs and total investment for various alternative designs
5. Profits (probable and optimum per kilogram of formaldehyde as well as annually, return on investment)
6. Materials of construction
7. Safety considerations

3-2 (continued)

8. Markets (present and future supply and demand, present uses, new uses, etc.)
9. Competition (overall production statistics, comparison of various manufacturing processes, product specification, e.g. 37% formaldehyde)
10. Properties of formaldehyde, properties of byproducts, disposal of waste products – this can be rather difficult in the case of the formaldehyde process.
11. Sales and sales service
12. Shipping restrictions, labeling, container selection
13. Plant location
14. Patent situation and legal restrictions

The analysis step may indicate that there is insufficient data which may require additional research. The analysis step involves the development of a preliminary design. This may lead to rejection of the entire project or it may initiate the development of additional new ideas. The latter should undergo further critical analysis to determine whether these, in turn, are feasible and should be pursued further or also rejected.

Detailed design – the following factors should be established within narrow limits before a detailed design is performed:

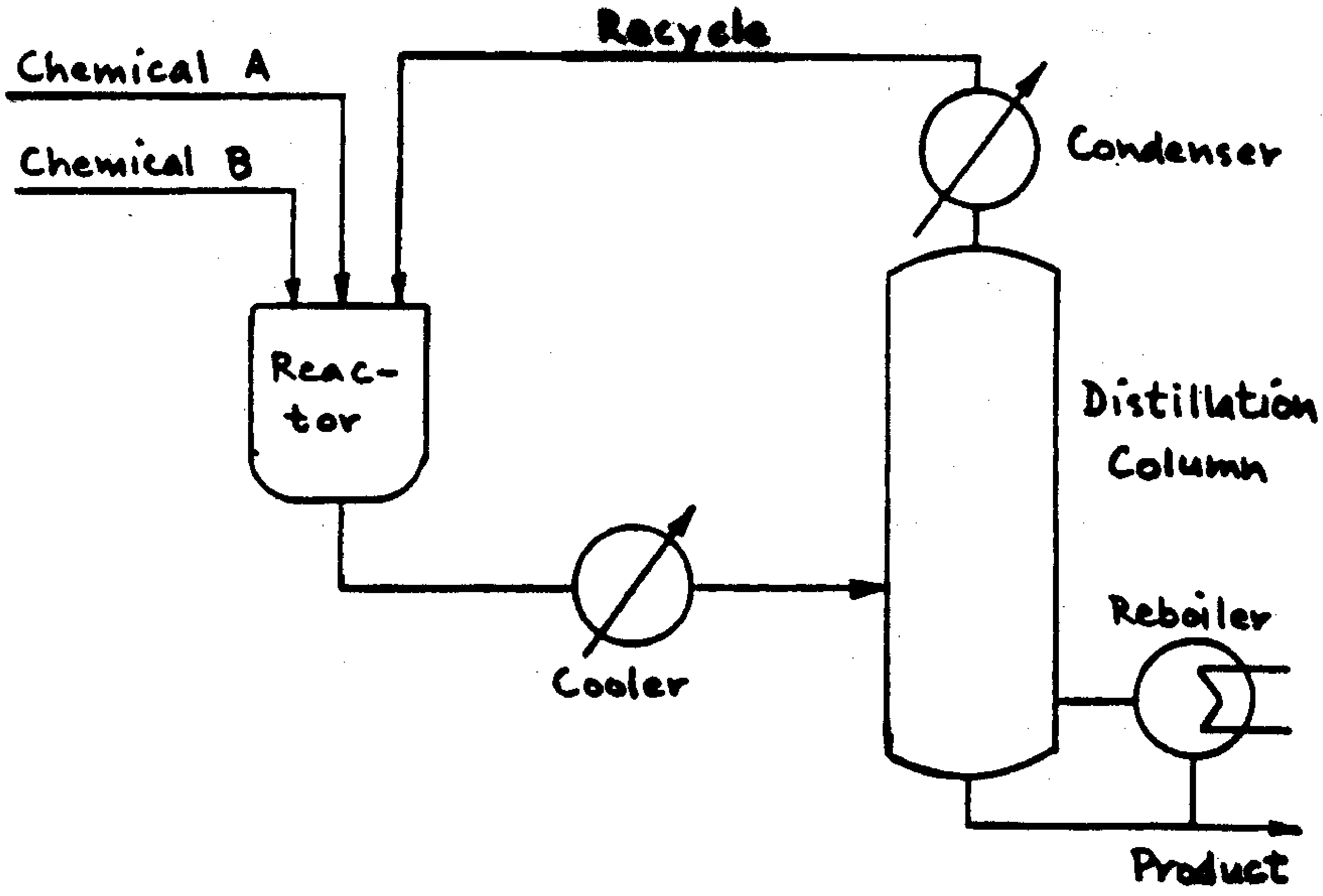
1. Selected process
2. Material and energy balances
3. Temperature and pressure ranges
4. Raw material and product specifications
5. Yields, reaction rates
6. Materials of construction
7. Utilities requirements
8. Plant site

If the detailed design indicates a profitable venture that meets the return criteria set forth by company managers, a recommendation will be made to go ahead with the construction if funds are available.

3-2 (continued)

Construction – the final design should be developed in such a manner that items requiring the longest time for completion can be started first. Careful coordination between the design engineer and the contractor will help minimize construction delays and assist with more orderly plant startup. Detailed operating procedures need to be developed not only for startup procedures, but also for eventual continuous operation. During operation, engineers and operating personnel should be encouraged to continuously monitor the process to see where the process can be improved or simplified.

3-3



Information needed for the design:

Raw materials – chemicals involved, stream capacities, thermal properties of materials, physical properties of materials, temperature, pressure, material concentrations, environmental hazards

Product – some considerations listed for the raw materials above, product purity, build-up of byproducts

Reactor – rate expression, conversion under various operating conditions, heats of reaction, thermal and physical properties of entering and exiting materials, temperature, pressure, stream capacities

Cooler – quenching temperature, vapor-liquid equilibria, heat load, thermal and physical properties

3-3 (continued)

Distillation column – vapor-liquid equilibria, heat of vaporization, thermal and physical properties, stream capacities, temperature, pressure, reflux ratio, plate efficiency, stream capacities

Condenser and reboiler – heat of condensation and vaporization, heat loads, thermal and physical properties, temperature, pressure, heat transfer coefficient, stream capacities

3-4

Acetylene is currently produced by the thermal cracking of hydrocarbon feeds. This thermal cracking requires considerable quantities of energy since the reactions are strongly endothermic. Further dissociation into carbon and hydrogen of all hydrocarbons begins at relatively low temperatures. Thus, the reactions must be allowed to proceed very rapidly and quenched very quickly to stable temperatures.

One method of distinguishing between processes for acetylene production is by the manner in which the energy for cracking is supplied.

1. Regenerative method

This method involves periodic cycles of heating a hot surface of ceramic materials and then cracking the hydrocarbons on these hot surfaces. An example of this method is the Wulff process. High purity acetylene is possible from a feedstock of ethane to gas oil. Byproducts are an aromatic distillate fraction and a hydrogen-rich off gas. The reaction is a vapor phase reaction in the presence of steam. Low conversion between 12 to 32 wt% is a major drawback of this method. Also, startup and shutdown are quite long compared to some of the other processes.

2. Partial oxidation

Acetylene is obtained from the incomplete combustion of methane. Oxygen and natural gas are reacted and about one-third of the entering methane is cracked and the rest is combusted to provide the necessary heat for the cracking process. The yield based on the carbon in the natural gas is usually about 31 wt%. The process generates considerable soot (carbon) that must be removed continuously for good operation. The process is licensed by BASF.

3. Two-step procedure

This process involves the generation of combustion gases at about 1500K and subsequent injection of hydrocarbons into these hot combustion gases. The feedstock normally is naptha cuts

3-4 (continued)

to middle distillates. High purity acetylene is obtained with a high yield of 50 to 54 wt% from this feedstock. The process generally is identified as the Hoechst HTP process.

4. Submerged-flame process

In this process the reaction between hydrocarbons and oxygen occurs in a submerged-flame burner where the burners remain submerged in oil during operation. The burner projects a flame into which oil and oxygen are fed. Combustion and cracking of the oil takes place at the boundaries of the flame. The gases are rapidly quenched by the surrounding oil which is continuously cooled. The feed is crude oil rather than natural gas. The process is designed to recycle the soot and combust it in the burner. Product yields are in the same range as those obtained for processes that use the light hydrocarbons for feed stock.

Since the flowsheets for all of these four processes are readily available in the literature along with useful information on the operating details and procedures, they will not be reproduced here. Some of the earlier references to the processes listed above provide much more useful information than is available from later references. Accordingly, these references are listed rather than the more recent references.

1. Regenerative method: *Hydrocarbon Process.* **46**(11), 139 (1967)
2. Partial oxidation: *Hydrocarbon Process.* **48**(11), 142 (1969)
3. Two-step procedure: *Hydrocarbon Process.* **46**(11), 138 (1969)
4. Submerged-flame process: *Chem. Eng.* **76**(3), 82 (1969)

The choice of one process over another is largely dependent upon the projected needs. The availability of feed stocks, the desired product purity, and byproduct marketability will be overriding considerations. As noted above, one basic difference is the form of energy input. The availability of one form of energy over another could make one process more attractive than another. The licensing and fees requirements could also provide advantages of one process over another. The principal design problems involve the chemical reactions and energy requirements.

3-4 (continued)

The latter will be a major part of the operating costs and will be reflected in the sales price. Material and energy balances around the reactor or burner will be necessary. However, complete kinetic data for these complex reactions will probably not be available. In the case of these well-known processes, the information needed would be available from the licensor.

The other area of concern in all of these processes is in the separation reaction. Most require the use of selective solvents. The mass transfer calculations for these solvents would be needed. Again, data would be available from the licensor, though approximations can be made with today's sophisticated computer software.

There are many similarities in the various separation techniques. Thus, comparison of the processes can provide economic information in some cases without making an actual detailed cost estimate.

3,12

3-5

Some of the most important and widely employed processes for ethylene production utilize thermal cracking of a hydrocarbon feed stock in the presence of a steam atmosphere. As noted in the preliminary design in Chapter 3, the ethylene process based on thermal cracking consists basically of four distinct process units, namely (1) thermal cracking and quenching, (2) compression and acid gas removal, (3) subcooling and product separation, and (4) refrigeration. Selection of a feed stock for an ethylene plant is primarily dependent on feed stock availability, relative feed price, yield structure, plant construction cost, as well as the sale price for the byproducts. The thermal cracking of ethane and propane by the fixed tubular heater is the most common process in the United States, and the thermal cracking of naptha by the fixed tubular heater is the most common process in Europe and Japan.

The capital cost of an ethylene plant increases as the molecular weight of the feed stock increases since the ethylene yield decreases with increasing molecular weight. In theory, an ethylene plant can be designed to process a full range of feed stocks from ethane to gas oil. However, in practice, complete feed stock flexibility to cover so broad a range is neither technically practical nor economically attractive because of various constraints in equipment design.

Distribution of product yields depends on the type of feed stock, hydrocarbon partial pressure, residence time, as well as cracking severity or coil outlet temperature. Typical yield distribution patterns for a variety of feedstocks is shown in the table below. The cracked gas recovery section in an ethylene plant is complex and allows a variety of possible flow schemes. In the separation section it is necessary to separate hydrogen and methane fractions, high-purity ethylene and propane, ethane and propane fractions, C_4s and gasoline. The design must provide for removal of acetylene and methyl acetylene/propadiene (MAPD). For the separation process, there are maximum temperature limits to prevent polymerization fouling, and minimum temperature limits to prevent hydrocarbon freezing or hydrate formation. Within the separation process, there are several possible variations. Acetylene can be removed from the cracked gas stream in the

Typical pyrolysis yields of ethylene plant feedstocks (in mass percent).

Feedstock	Ethane	Ethane	Propane	Propane	n-Butane	Isobutane	n-Pentane	Isopentane	Naphtha	Naphtha	Kerosene	Gas Oil	Gas Oil (Vac)
Conversion%	50	60	75	90	95	92	95	95					
Severity									Medium	High	High	High	High
Comp. Yield													
H_2	3.06	3.55	1.08	1.29	1.0	1.39	0.8	0.9	0.63	0.8	0.63	0.65	0.47
CH_4	2.6	4.2	18.83	24.7	21.8	23.6	17.52	20.6	11.7	15.3	11.4	10.6	7.9
C_2H_2	0.12	0.25	0.17	0.33	0.4	0.46	0.55	0.42	0.3	0.75	0.37	0.35	0.2
C_2H_4	41.65	48.2	25.0	34.5	35.8	10.3	36.5	17.95	24.0	29.8	23.2	24.0	19.5
C_2H_6	50.0	40.0	5.15	4.4	5.1	0.73	5.6	3.2	3.9	3.75	3.4	3.25	2.83
C_3H_4	0.1	0.02	0.22	0.34	0.55	2.46	0.95	1.7	0.76	1.1	0.85	1.0	0.38
C_3H_6	0.89	1.11	18.86	13.96	16.4	21.2	19.8	19.0	15.75	14.1	13.8	14.45	11.3
C_3H_8	0.14	0.17	25.0	10.0	0.15	0.33	0.55	0.55	0.41	0.25	0.45	0.45	0.36
C_4H_6	0.5	1.07	1.05	2.65	3.4	1.94	3.7	4.6	4.6	4.85	4.4	4.7	5.5
C_4H_8	0.25	0.21	1.28	1.0	1.7	16.7	2.6	14.38	6.85	4.2	4.0	4.5	4.88
C_4H_{10}	0.35	0.27	0.07	0.05	5.0	8.0	0.1	0.1	1.0	0.25	0.1	0.1	0.09
C_5s	0.2	0.27	1.43	1.81	1.65	2.12	6.1	7.5	5.0	2.3	3.0	3.3	5.0
C_6H_6	0.2	0.48	1.09	2.2	2.58	3.42	1.8	4.1	3.15	6.3	7.2	5.8	4.16
C_7H_8	0.03	0.06	0.33	0.48	1.12	1.51	0.6	1.25	4.4	4.9	4.0	3.15	2.5
C_8H_8					0.45	0.67	0.1	0.25	2.65	2.0	2.2	1.2	1.28
C_8H_{10}							0.06	0.15	0.52	0.7	0.75	0.7	0.5
C_6-C_8 PON*		0.14	0.14	1.44	1.2	3.18	0.1	0.1	8.83	2.0	1.9	1.8	2.0
C_9-200^0 C							1.48	1.75	3.75	2.8	4.85	2.4	2.0
Fuel oil			0.3	0.85	1.7	1.99	1.09	1.5	1.8	3.8	13.5	17.6	29.15

* PON (Paraffin, Olefin, Naphthene).

3-5 (continued)

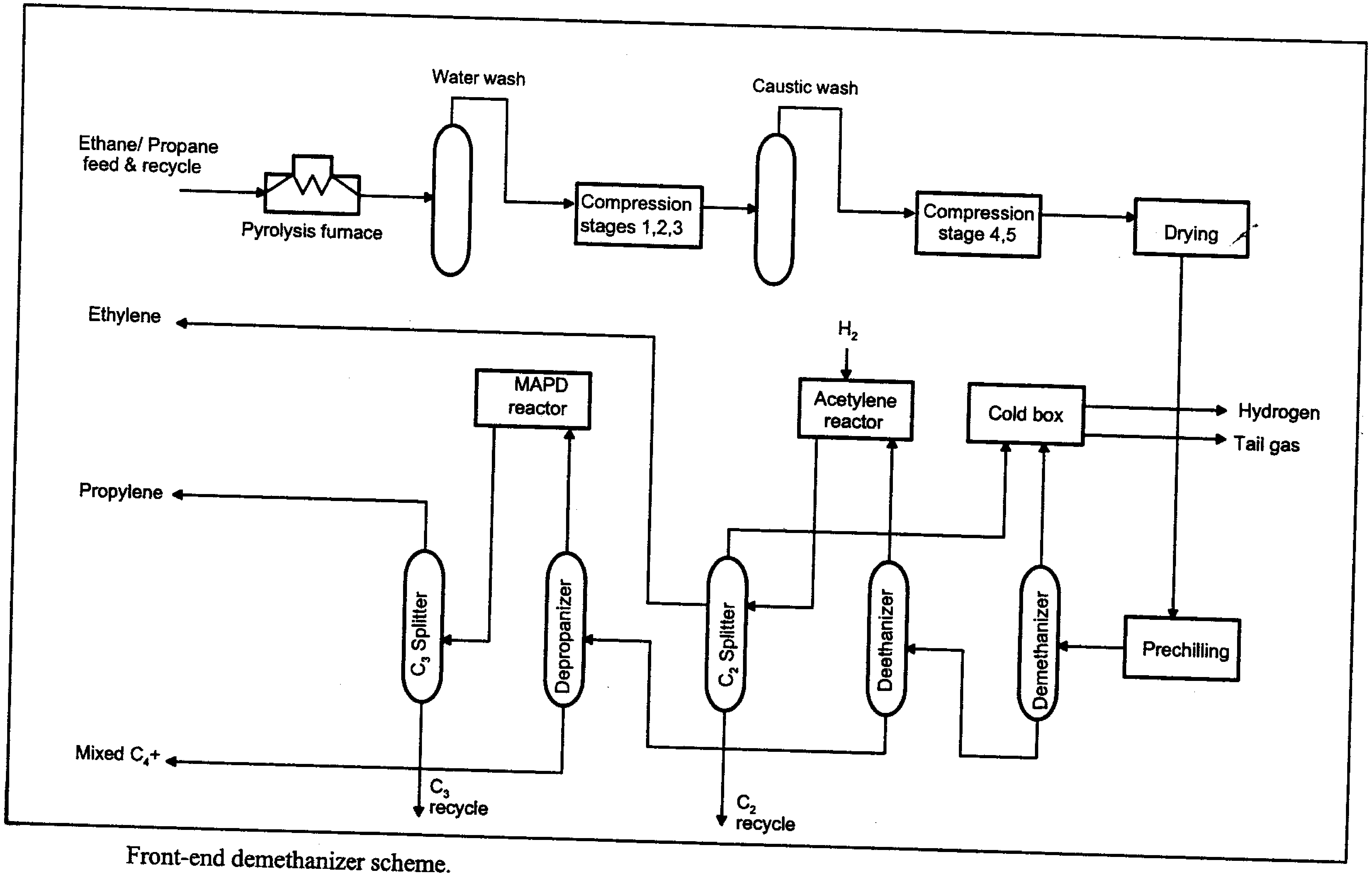
presence of the hydrogen produced in the furnaces or from a concentrated demethanized and depropanized C_2 cut. Such process arrangements are identified as the front-end demethanizer, the front-end deethanizer, and the front-end deproponizer scheme. They are briefly described below.

Front-end demethanizer

In this scheme the dried cracked gas is chilled and sent to the demethanizer/cold-box system as shown in the attached figure. The demethanizer bottoms are sent to the deethanizer. The overhead from the deethanizer is hydrogenated to remove the acetylene. From the acetylene converter, the ethylene-ethane stream passes to the C_2 splitter. The deethanizer bottoms are sent to a depropanizer which separates the C_3 components from the heavier C_4 components. The overhead from the depropanizer is fed to a methyl acetylene/propadiene hydrogenation reactor. A C_3 splitter separator separates the propylene from propane. The bottoms stream from the depropanizer passes to a debutanizer where the C_4 fraction is separated as an overhead from the C_5 and higher components.

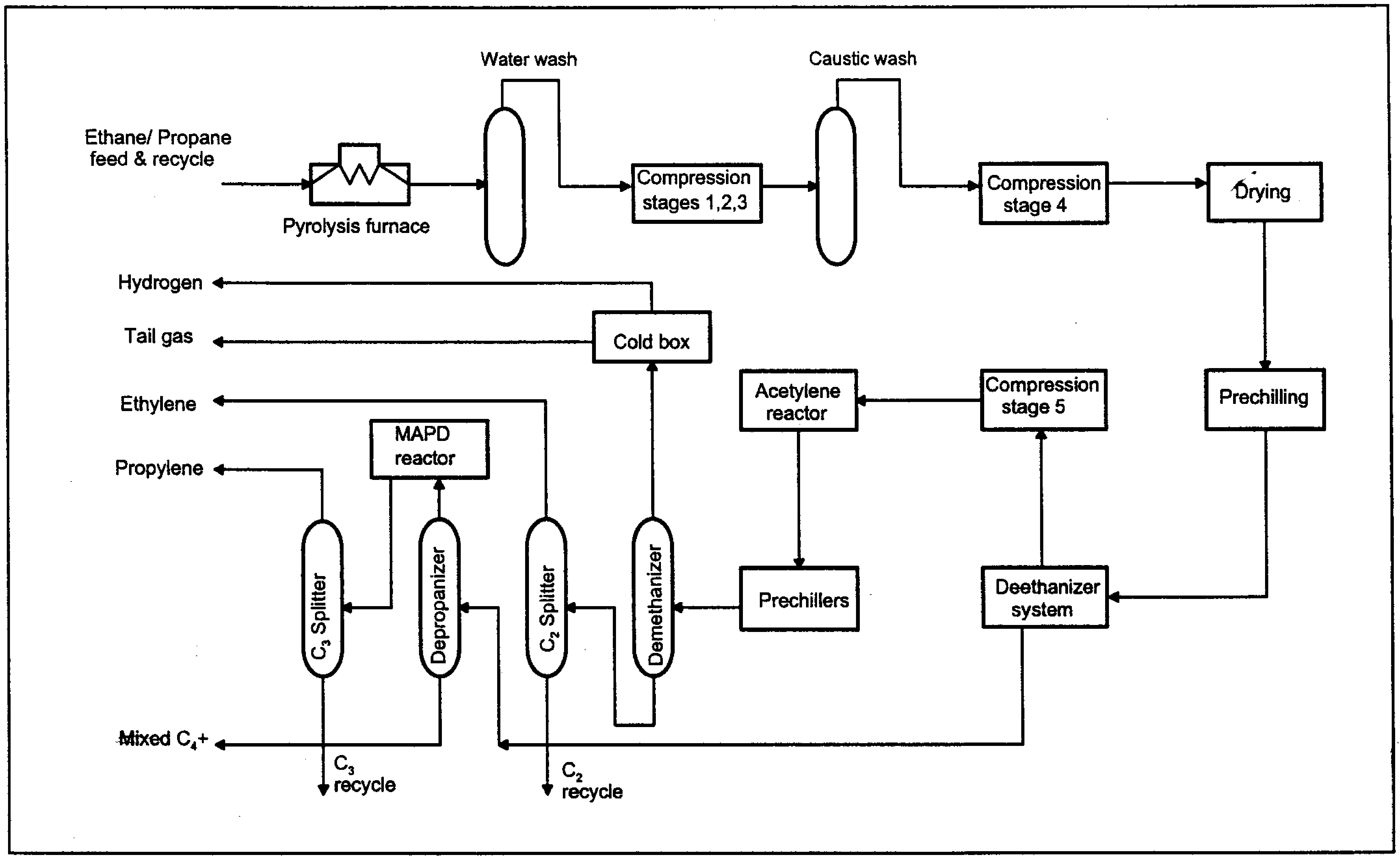
Front-end deethanizer

As noted in the figure showing the front-end deethanizer, the cracked gas is dried and chilled before it is sent to the deethanization section. Overhead from the deethanization section is compressed and sent to a front-end acetylene reactor where acetylene is hydrogenated to ethylene and ethane. Following the acetylene reactor, the C_2 and lighter components are chilled and sent to the demethanizer which separates methane and hydrogen from the heavier components. Hydrogen and methane are separated in the cold-box, while the demethanizer bottoms are fractionated in the C_2 splitter to produce ethylene product and ethane for recycle. The bottoms from the deethanizer are depropanized, and the mixed C_3s are sent to the methyl acetylene/propadiene reactor for hydrogenation. The C_3 reactor effluent goes to a C_3 splitter to produce polymer grade propylene and propane for recycle.



Front-end demethanizer scheme.

3.15



Front-end deethanizer scheme.

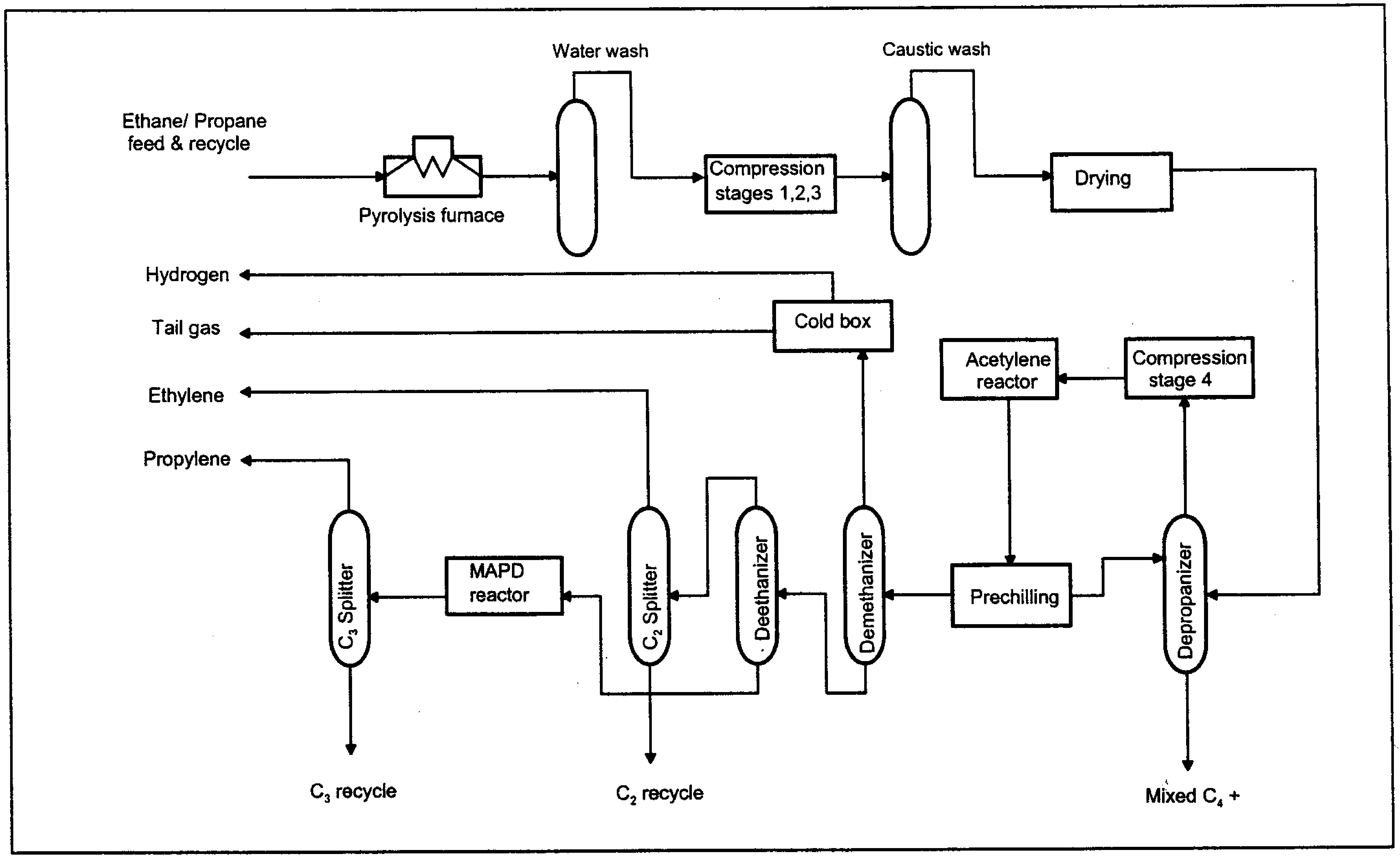
3-5 (continued)

Front-end depropanizer

The cracked gas in this scheme shown on the next figure is compressed before being caustic-washed and dried. The dried cracked gas is sent to the depropanizer where the overhead is compressed. It then goes to a front-end acetylene reactor where the acetylene is hydrogenated to ethylene and ethane. After hydrogenation, the gases are partially condensed to provide reflux for the depropanizer. The net overhead liquid and vapor, which contain the C_3 and lighter components, go forward to the demethanizer prechillers. The net C_3 and lighter stream from the depropanizer are fractionated in the demethanizer, with the resulting bottoms stream being further fractionated in a deethanizer to a methyl acetylene/propadiene hydrogenation reactor. The reactor effluent is sent to the C_3 splitter.

In addition to the location of the demethanizer, deethanizer, and depropanizer, the location of the reactor for hydrogenating the C_2 and C_3 acetylenes is dictated by the scheme selected. In both the front-end deethanization and front-end depropanizer separation schemes, the C_2 acetylene hydrogenation converters are located ahead of the cracked gas chilling train and demethanizer. By contrast, in the front-end demethanization scheme, the acetylene converter follows the demethanizer and deethanizer. In this scheme, the hydrogen in the cracked gas has already been removed in the gas chilling train and demethanizer and an external addition of hydrogen is necessary in the acetylene converter. However, hydrogenation of the C_3 acetylenes in all schemes requires an external addition of hydrogen.

Separation of the ethylene-ethane mixture also provides several choices. This separation is commonly achieved in a high-pressure column, utilizing close-cycle propylene refrigeration. This process can also be carried out by low-pressure fractionation, using open-cycle ethylene refrigeration. Alternatively, the selection can also be achieved with absorption using highly selective solvents. The latter separation technique is seldom used. However, the low-pressure fractionation has received some support since the relative volatility between ethylene and ethane is higher and the separation is easier, resulting in a lower reflux ratio and fewer trays than in the



Front-end depropanizer scheme.

3-5 (continued)

high-pressure column. However, this fractionation scheme requires the use of ethylene refrigeration instead of propylene refrigeration. Both fractionation schemes have advantages and disadvantages. For example, the lower design pressure reduces the wall thickness of the column with a resultant cost saving. However, this cost reduction is offset by the higher material cost of equipment and piping due to the lower design temperature.

A good example of the front-end demethanizer scheme is the licensed processes developed by the M.W. Kellogg Company, *Hydrocarbon Process.* **68**(11), 102(1989), and by the Stone and Webster Engineering Company, *Hydrocarbon Process.*, **64**(11), 137(1985). The process developed by Linde A.G., *Hydrocarbon Process.* **64**(11), 137(1985) for producing ethylene from hydrocarbon liquids is an example of the deethanizer scheme. The advantages and disadvantages of these processes are listed below. (Other advantages and disadvantages of these processing schemes were listed earlier.)

Kellogg scheme advantage:

High olefin yields due to the very small (less than 0.1 second) reaction times in the pyrolysis furnace unit. Process handles feeds ranging from ethane to vacuum gas oils. Provides ultra-purified hydrogen stream for catalytic conversion. Generates excess high pressure steam for use elsewhere in a multi-purpose plant.

Kellogg scheme disadvantages:

Steam pyrolysis is performed at temperatures higher than most other ethylene processes creating more coking potential. Process requires two separate water quench exchangers.

Stone and Webster scheme advantages:

Handles feeds ranging from gases to liquid hydrocarbons. Process is characterized by a highly selective pyrolysis coil. Steam pyrolysis temperature not as high as the Kellogg process reducing

3-5 (continued)

the coking potential. Operating history of quench exchanger shows it to be basically non-fouling. Removes acid gas with patented amine process.

Stone and Webster scheme disadvantages:

Requires an oil quench (with liquid feed stocks) and also a water quench. Ethylene yields slightly less than that obtained with the Kellogg process.

Linde AG advantages:

Feed is cracked in more conventional tubular furnaces which can be adapted to meet specific requirements of high capacity, high severity, or high selectivity. Recompression of cracked gases is to a lower pressure than other processes.

Linde AG disadvantages:

Operates only on hydrocarbon liquids. Requires an oil and a water quench.

3:21

3-6

To assist in preparing the material balance for the process shown in Fig. 3-13, the information provided below can simplify the calculations. The flow of ethylene in stream 533 is 62,008 kg/h. From the preliminary design, the C_4 – C_6 component listed in streams 413 and 416 includes 1-3 butadiene, i-butene, 1-butene, n-butane, C_{5s}PON, benzene, and toluene. The calculated component distribution in kg/h is

Component	Stream 413		Stream 416	
acetylene			0.005	
ethylene			0.005	
ethane			1.2	
MAPD	97.5		458	
propylene	2,220		20,572	
propane	1912		14,321	
1-3 butadine	1635	8734	2790.2	4776
i-butene	294.2		563	
1-butene	275.5		515	
n-butane	45.4		67.8	
C_{5s} PON	2250		715	
benzene	3476		122.5	
toluene	757.9		2.5	
heavy gasoline	1858		0.55	

Design information:

Distillation columns were simulated and designed with the CHEMCAD-SCDS method using the Soave-Redlich-Kwong equation of state. Reflux ratio for C-601 was set at 1.5 R_{min}. For C-602, C-603, C-604, and C-605 it was 1.2 R_{min}. Cooling water was available with an inlet temperature of 29°C and an outlet temperature of 35°C. Plate efficiency of the valve trays was assumed to be 85%. Compressor K-601 assumes a polytropic efficiency of 85%. Removal of MAPD (methylacetylene and propadiene) assumes a level of the C_3 acetylene at the exit of 2500 ppm in reactor R-60 and an exit level less than 50 ppm in reactor R-602. The Arrhenius equation parameters for the two C_3 acetylene removal and propylene hydrogenation reactions specified as

3-6 (continued)

$$C_3H_4 + H_2 \rightarrow C_3H_6 \qquad (1)$$

$$C_3H_6 + H_2 \rightarrow C_3H_8 \qquad (2)$$

are

	Activation energy, E cal/g mol	Temp range, °C	Frequency factor, A
(1)	1.18 x 10^4	100 – 130	2.3 x 10^6
(2)	1.17 x 10^4	100 - 130	5.25 x 10^3

The CC-Therm software was used for the thermal and mechanical design of all shell-and-tube heat exchangers including E-601 through E-612.

The overall mass balance was developed with the use of ASPEN PLUS and CHEMCAD-III software programs utilizing the Soave-Redlich-Kwong equation of state relation. This equation of state provides a good match between simulated properties and actual properties reported in the literature.

To complete the material balance, components in streams 215 and 325 are provided in kg/h as

Component	Stream 215 (77.3°C, 179 kPa)	Stream 325 (62.8°C, 234 kPa)
$C_{5,s}$ PON		1.8
benzene		25.2
toluene		25.2
heavy gasoline	9.1	483.1

Figure 3-15 provides a summary of the mass balances around various parts of the separation section and can be used to verify results.

The material balance result is summarized in the following table with each component flow provided in kg/h.

3-6 (continued)

Material balance results in kg/h for the product separation section shown in Fig. 3-13.

	Stream numbers							
Components	602, 621	603	604	605,605	609	610,611	612,613	614
hydrogen					27.8	27.8		16.3
methane	2.9							
acetylene	0.02	0.22	0.005			0.005	0.005	
ethylene	61,946	62.5	0.005			0.005	0.005	
ethane	59.9	11,893	1.2			1.2	1.2	
MAPD[†]			555	0.6		555	99.9	
propylene		2.1	22,791.5	0.06		22,791.5	23,166.7	
propane			16,232.3	0.5		16,232.3	16,340.2	
1,3 butadiene			1.3	4423.6		1.3	1.3	
i-butene			0.6	856.2		0.6	0.6	
1-butene			0.4	790.1		0.4	0.4	
n-butane				113.2				
$C_{5,s}$PON[‡]				2964.8				
benzene				3598.6				
boluene				760.4				
heavy gasoline				1859				

	Stream numbers					
Components	615	616,617,618	619	620	607	608
hydrogen	16.3	0.01	0.01			
methane						
acetylene	0.005	0.005	0.005			
ethylene	0.005	0.005	0.005			
ethane	1.2	1.2	1.2			
MAPD[†]	99.9	1.9		1.9	0.6	
propylene	23,166.7	23,031.6	23,004.8	26.8	0.07	
propane	16,340.2	16,589.4	94.8	16,494.6	0.5	
1,3 butadiene	1.3	1.3		1.3	4423.5	0.05
i-butene	0.6	0.6		0.6	856.2	
1-butene	0.4	0.4		0.4	790.1	0.005
n-butane					113.2	0.01
$C_{5,s}$PON[‡]					0.03	2964.8
benzene						3598.6
boluene						760.4
heavy gasoline						1859.0

[†] Methyl acetylene/propadiene

[‡] Paraffin, olefin, napthene

3-7

The material balance from Problem 3-6 and either ASPEN PLUS or CHEMCAD-III computer software is used to develop the energy balance around each piece of equipment in the ethylene separation section. For example, around distillation column, C-601, the computer program establishes the heat content of streams 533, 602, and 603 above a selected datum plane. The distillation calculation indicates the flow rates of the overhead and bottoms streams. The reflux and reboil then indicate the flow rates of the streams that are returned to the column and permits evaluation of the condenser and reboiler duties. In kW this can be expressed as

$$h_{533} + \dot{q}_{E-602} - (\dot{q}_{E-601} + h_{602} + h_{603}) = 0$$

$$15{,}208 + 19{,}479 - (15{,}639 + 29{,}674 - 10{,}626) = 0$$

Thus $\dot{q}_{E-601} = -15{,}639\,\text{kW}$

$\dot{q}_{E-602} = 19{,}479\,\text{kW}$

Similarly

$\dot{q}_{E-603} = -4422\,\text{kW}$

$\dot{q}_{E-604} = 8050\,\text{kW}$

$\dot{q}_{E-605} = 1245\,\text{kW}$

$\dot{q}_{E-606} = -636\,\text{kW}$

$\dot{q}_{E-607} = -325\,\text{kW}$

$\dot{q}_{E-608} = -30{,}752\,\text{kW}$

$\dot{q}_{E-609} = 29{,}288\,\text{kW}$

$\dot{q}_{E-610} = -1432\,\text{kW}$

$\dot{q}_{E-611} = 890\,\text{kW}$

$\dot{q}_{E-612} = -507\,\text{kW}$

Equipment dimensions have already been summarized in Table 3-7, p. 111. These dimensions are to be verified in the problem.

3-8

The material balance for Fig. P3-8 was developed with the use of ASPEN PLUS and CHEMCAD-III software programs utilizing the Soave-Redlich-Kwong (SRK) equation of state relation. This equation of state provides a good match between simulated properties and actual properties reported in the literature.

The distillation column was simulated and designed with the CHEMCAD-SCDS method, also using the SRK equation of state. Reflux ratio for the distillation column was set at 1.2 R_{min}. Plate efficiency of the valve trays was assumed to be 0.85. Cooling water was available with an inlet temperature of 29°C and an exit temperature of 35°C. The CC-Therm software was used for the thermal and mechanical design of all the shell-and-tube heat exchangers.

The material balance in kg/h for various locations in Fig. P3-8 are given below

	Stream no.			
	1, 2, 3, 4	5	6	7, 15, 16
Ethylene, kg/h	62,014	29.4	61,986	55,352
Ethane, kg/h	11,956	5776.6	6178	60
	73,970	5806	68,164	55,412
Ethylene, mol fract.	0.8475	0.0055	0.915	0.999

	Stream no.			
	8, 9	10	11, 12, 13, 14	17, 18
Ethylene, kg/h	6632.5	6597.2	35.3	61,949.2
Ethane, kg/h	6119.5	0.8	6118.7	60.8
	12752	6598	6154	62,010
Ethylene, mol fract.	0.5373	0.999	0.0055	0.999
		Answer		

3-8 (continued)

An energy balance around the pieces of equipment in Fig. P3-8 provides the heat duties for the seven heat exchangers, condenser and reboiler as:

E-S1, E-52 $\dot{q}_{2-3} = 4053\,\text{kW}$

E-S3 $\dot{q}_{3-4} = 2671\,\text{kW}$

E-S4 $\dot{q}_{8-9} = -80\,\text{kW}$

E-S4 $\dot{q}_{12-13} = 80\,\text{kW}$

E-S5 $\dot{q}_{\text{cond}} = -3005\,\text{kW}$

E-S6 $\dot{q}_{\text{reboil}} = 2000\,\text{kW}$

E-S7 $\dot{q}_{11-12} = 770\,\text{kW}$

E-S8 $\dot{q}_{15-16} = -2384\,\text{kW}$

E-S9 $\dot{q}_{17-18} = -2205\,\text{kW}$

Answer

The enthalpy content in kW of each stream based on a fixed datum plane is given below:

Stream no.	Enthalpy content	Stream no.	Enthalpy content
1, 2	15,207 kW	10	3139
3	19,260	11	-5671
4	21,931	12	-4901
5	-4542	13, 14	-4821
6	26,474	15	31,006
7	27,938	16	28,622
8	-1466	17	33,856
9	-1546	18	31,651

Answer

3-9

Without information on the process streams exchanging heat in E-S1, E-S2, E-S3 and E-S7, the areas required for these heat exchangers cannot be evaluated. CC-Therm results for these four exchangers and their cost using Fig. 14-18 are as follows:

Exchanger data	E-S1	E-S2	E-S3	E-S7
Heat duty, kW	3271	782	2671	770
$\Delta T_{\ln m}$, °C	12.2	26.2	9.8	18.6
U, $W/m^2 \cdot K$	756	310	636	312
A, m^2	355	96	429	133
Cost, $ (Jan. 2002)	27,500	11,800	31,000	14,500

For the other heat exchangers, condenser, and reboiler, the sizes of the heat exchangers can be evaluated to provide

Exchanger	Area, m^2	Cost, $ (Jan. 2002)
E-S4, $\dot{q}_{8\text{-}9}$	68	9,400
E-S5, $\dot{q}_{\text{cond}}$	449	32,800
E-S6, $\dot{q}_{\text{reboil}}$	98	11,900
E-S8, $\dot{q}_{15\text{-}16}$	443†	32,000
E-S9, $\dot{q}_{17\text{-}18}$	488†	34,000

† Utilizes cooling water with an inlet of 29°C and an outlet of 35°C. With the CHEMCAD-SCDS computer software, the information for the distillate column, assuming 85% tray efficiency, is

Diameter, 1.52 m
Number of actual trays, 57
Tray spacing, 0.61 m
Installed cost from Fig. 15-15 ($1250/tray), $71,250
(For purchased cost, approximately $57,000)

The power required by the two centrifugal rotary compressors and their cost from Fig. 12-28 are

Compressor (7 to 15)	3008 kW	$700,000
Compressor (10 + 16 to 17)	2722 kW	$650,000

3-9 (continued)

The area and cost[†] of the two membrane units are

Membrane (4 to 5 and 6)	8900 m^2	\$2,314,000
Membrane (6 to 7 and 8)	995 m^2	\$259,000

[†] Cost of membranes is \$260/m^2 based on vendor quotation.

Answer

3-10

A membrane cascade system with recycling and utilizing facilitated transport technology can be shown to recover ethylene from ethane at the required polymer-grade composition. The separation can be achieved without the aid of a distillation column. Two or more membrane units with associated compression equipment are sufficient for separating ethylene from ethane at high purity and recovery. However, the high recovery rate of ethylene as well as the high purity product dictates high recycle rates between membrane units and large membrane area requirements.

Figure P3-10 shows a two-stage membrane cascade with recycling for producing ethylene product with a polymer-grade composition of 0.999 mol percent. The fresh feed pressure of 2605 kPa and temperature of 28°C are identical to the conditions shown for stream 4 in the series configuration hybrid system presented in Fig. P3-8. The process conditions for this cascade configuration are shown in the accompanying table.

Process conditions for the two-stage membrane system in Fig. P3-10

	Stream no.					
	1	2	3	4	5	6
Temp., °C	28	33	31	31	10.9	43.2
Press., kPa	2605	2605	2605	2605	827	1344
Ethylene, kg/h	62,014	91,072	153,086	64.8	153,021.2	153,021.2
Ethane, kg/h	11,956	17,558	29,514	11,895.2	17,618.8	17,618.8
Ethylene mol fract.	0.8475	0.8475	0.8475	0.0056	0.9029	0.9029

	Stream no.					
	7	8	9	10	11	12
Temp., °C	33	33	80.4	20.2	237	33
Press., kPa	1337	1337	2619	103	1970	1944
Ethylene, kg/h	153,021.2	91,072	91,072	61,949.2	61,949.2	61,949.2
Ethane, kg/h	17,618.8	17,558	17,558	60.8	60.8	60.8
Ethylene mol fract.	0.9029	0.8475	0.8475	0.9990	0.9990	0.9990

3:30

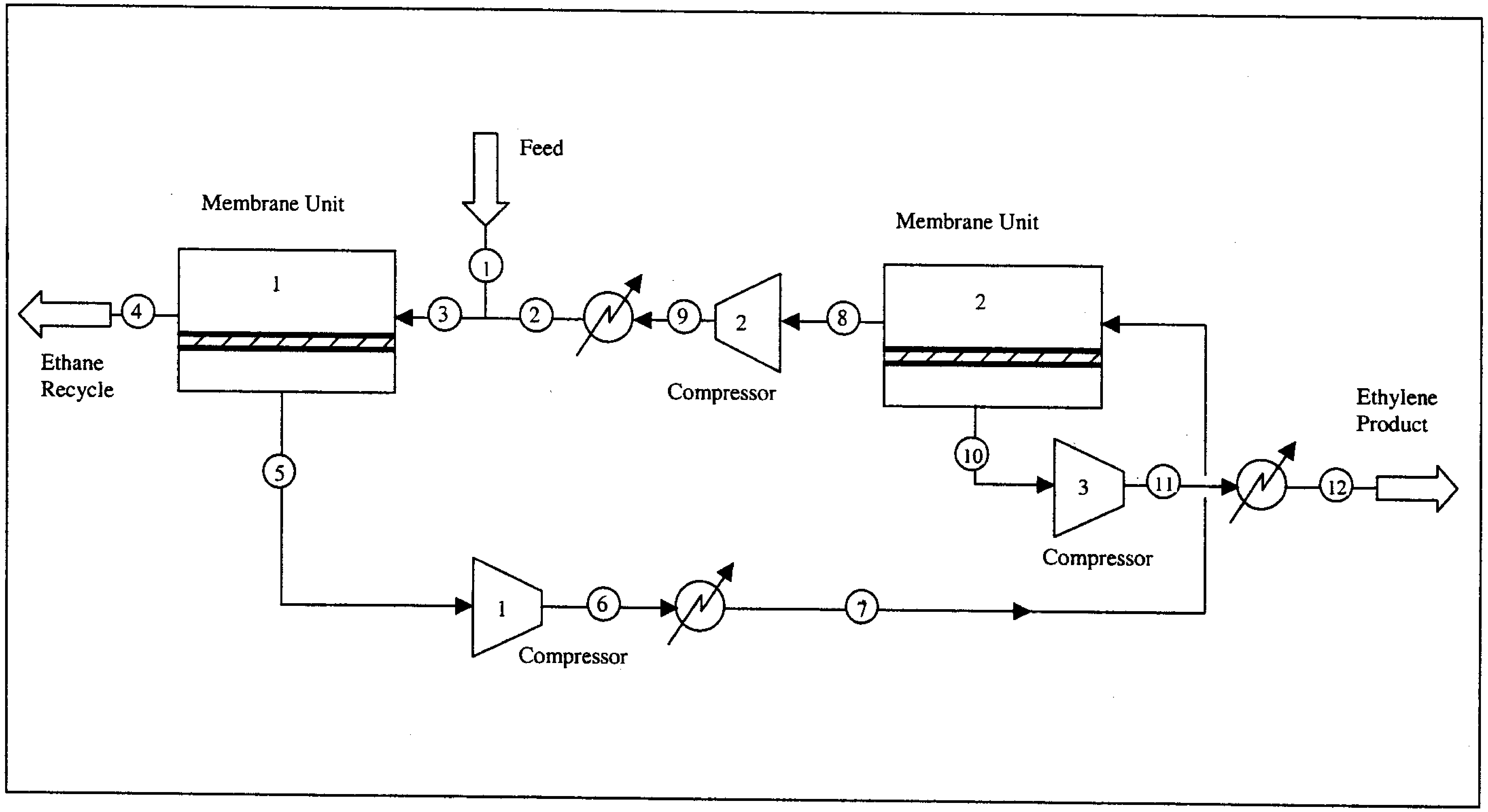


Figure P3-10 : Membrane cascade for ethane/ethylene separation.

3-10 (continued)

In this membrane cascade system, the fresh feed stream 1 is mixed with the recycle in stream 2 and then fed to the first membrane stage. In the first membrane unit, the recycled ethane is separated as the retentate and the recovered ethylene is concentrated as the permeate. There is a trade off between the membrane area requirement and the operating pressures. Analysis shows that the optimal permeate pressure for the first membrane is close to 827 kPa. This pressure balances the trade off between the compression costs and the capital costs. The overall performance of the first membrane stage is lower when compared to the second membrane stage. This low performance is because of the high percentage of ethylene recovered in the permeate side as well as the lower pressure ratio of the first stage which requires a very large membrane area.

The permeate stream after leaving the first membrane stage is compressed to 1337 kPa and cooled to 33°C before being sent to the second membrane stage. This stream could be compressed to the pressure of the fresh feed in a two-stage compressor before being sent to the second membrane stage. However, the reduction in compression load due to the reduction in flow rate to the second stage compressor can be shown to be more significant than theeffect the lower pressure would have on the membrane area requirement.

Polymer-grade ethylene is produced in the second membrane stage at a permeate pressure of 103 kPa. Permeate pressure can be increased to lower the ethylene product compression cost, but this can only be achieved when the membrane feed composition is above its minimum value. When the permeate pressure is increased, the required feed composition is increased. Also, the membrane area required increases as the permeate pressure is increased. Finally, an increase in permeate pressure requires an increase in the recycling rate with an increase in compression costs. The second-stage performance is more efficient than the first stage with respect to both the selectivity and flux rate. This performance is affected by the feed and permeate pressures as well as by the feed, retentate, and permeate compositions.

3-10 (continued)

Even though the pressure difference has a significant impact on the membrane separation process, the pressure ratio is more important than the pressure difference for this particular process. Higher pressure ratios increase both the selectivity and flux rate and these lead to improved membrane unit performance.

3-11

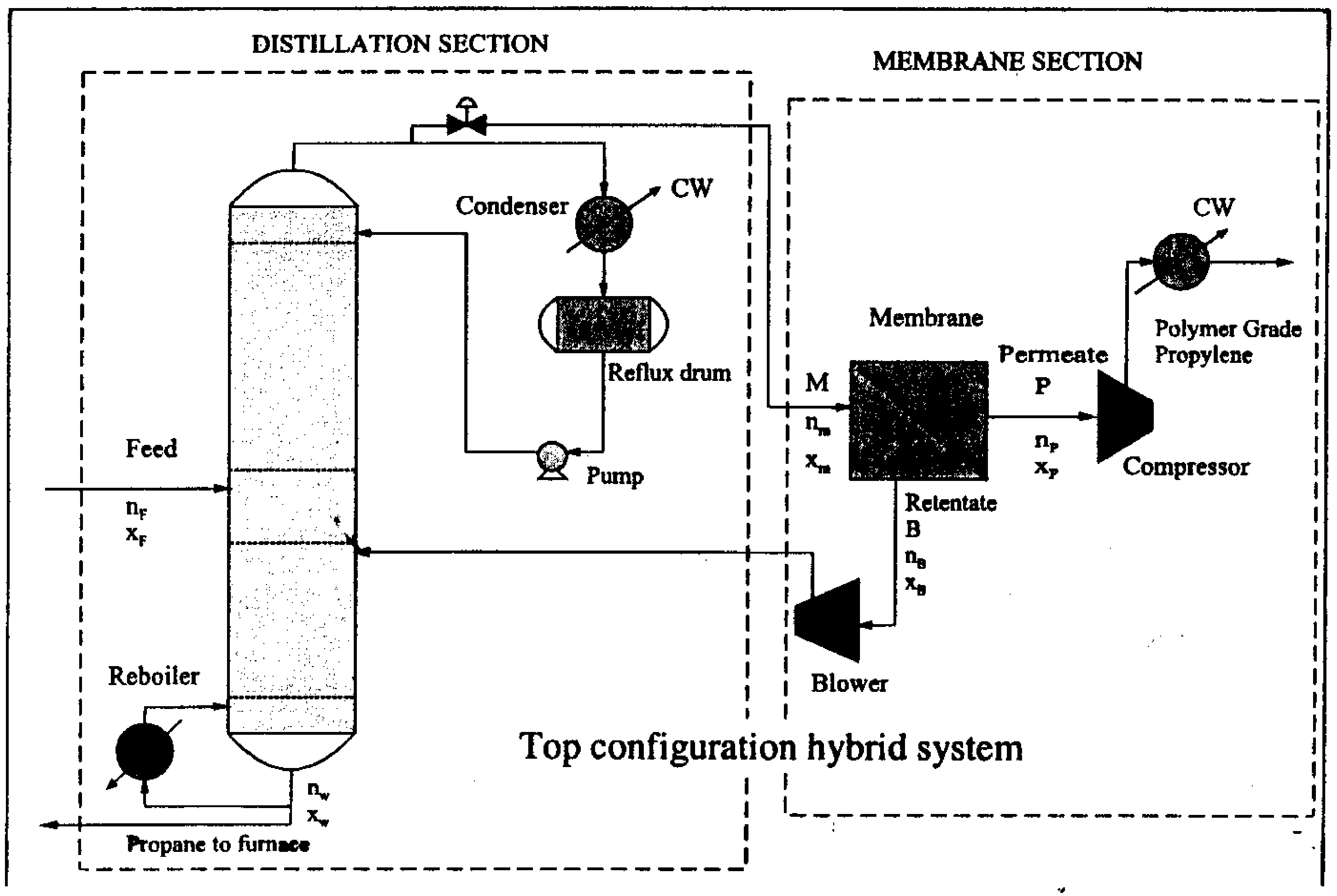
The separation of propylene from propane utilizing facilitated transport technology is similar to that used in the separation of ethylene from ethane. Figure 3-6 provides several possible configurations for the hybrid distillation membrane separation system. Another possible configuration is the combination of the separate top and bottom configurations into a hybrid system that utilizes a membrane separation system at both ends of the distillation column. Schematics of these different hybrid systems are provided in the figures below.

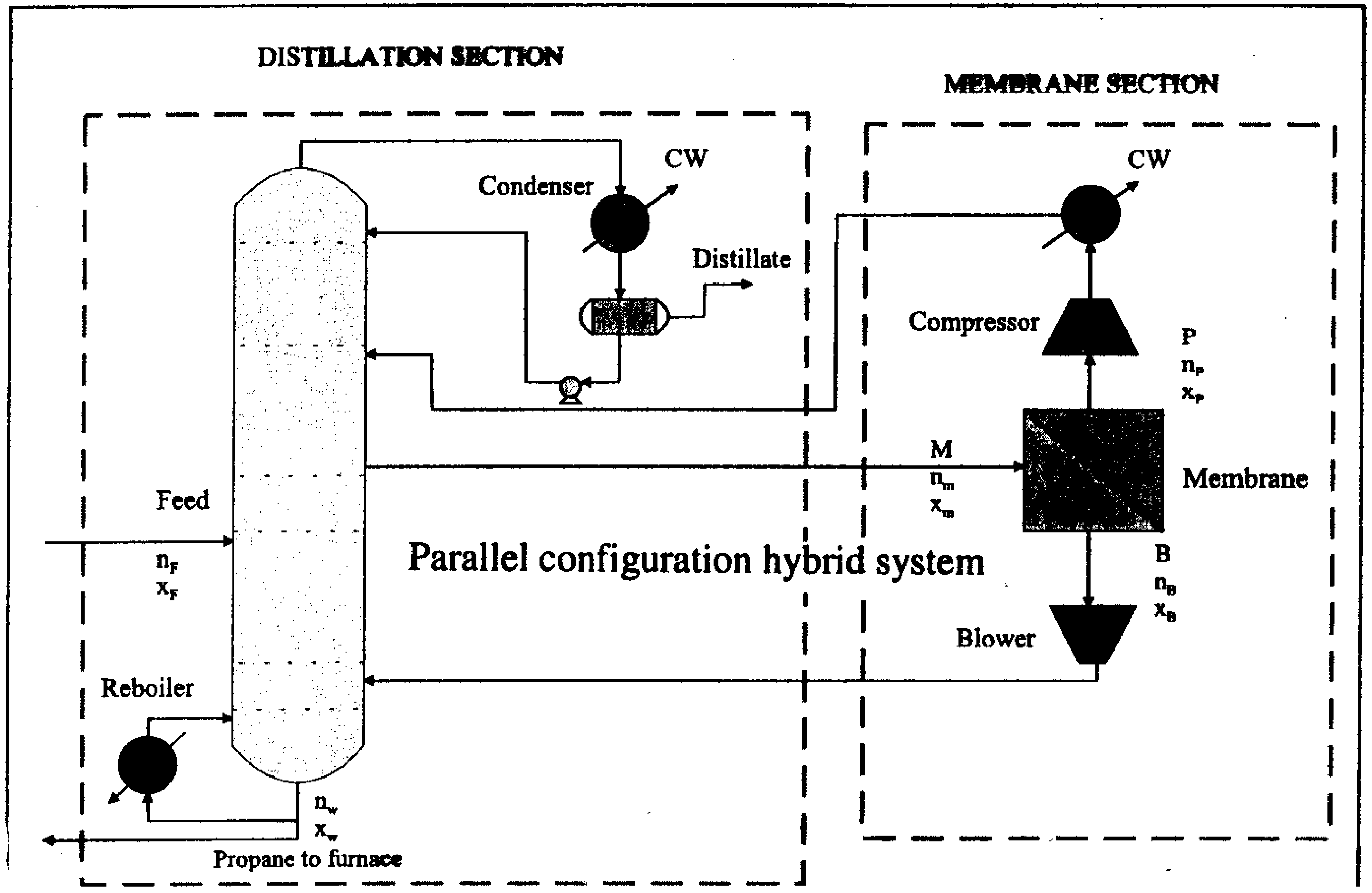
A review of these four different configurations indicates that the top configuration will provide a reduction of separation stages in the rectification section of the distillation column. This reduction in the number of stages will only have a small effect on process and energy savings over the conventional C_3 splitters. The parallel configuration will have less effect on the number of stages required, but should provide greater energy savings since both ends of the column receive benefit from this configuration. The bottom configuration can also provide a greater reduction in the stages required for separation and will have an energy savings in the reboiler. The top-bottom configuration offers the best opportunity to obtain both a capital savings and an energy savings. Thus, this configuration appears to offer the greater annual savings and warrants further evaluation.

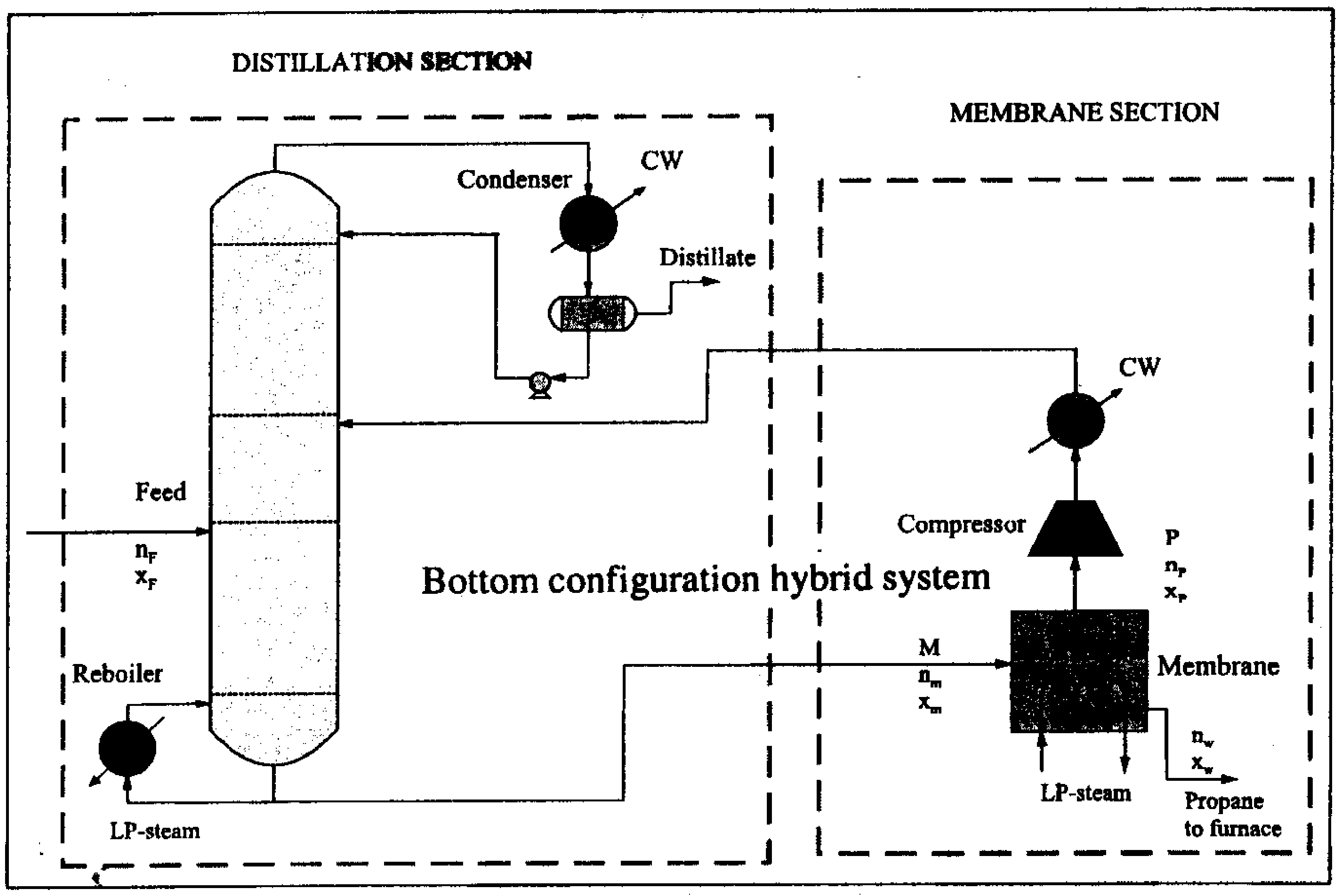
Answer

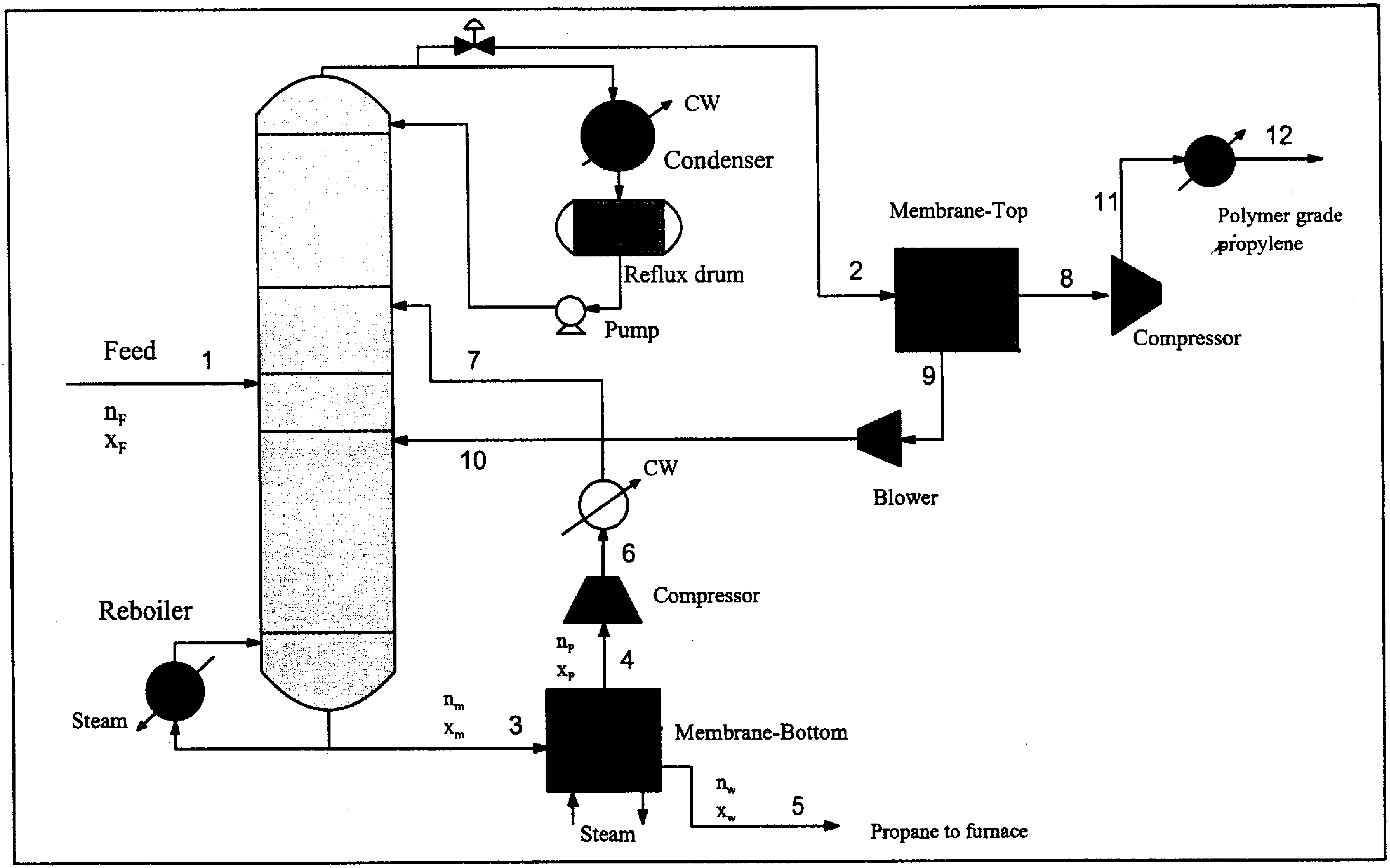
Note: the four configurations have been investigated and the top-bottom configuration provided the largest capital and processing savings among these configurations.

3:34


Top configuration hybrid system


Parallel configuration hybrid system


Bottom configuration hybrid system

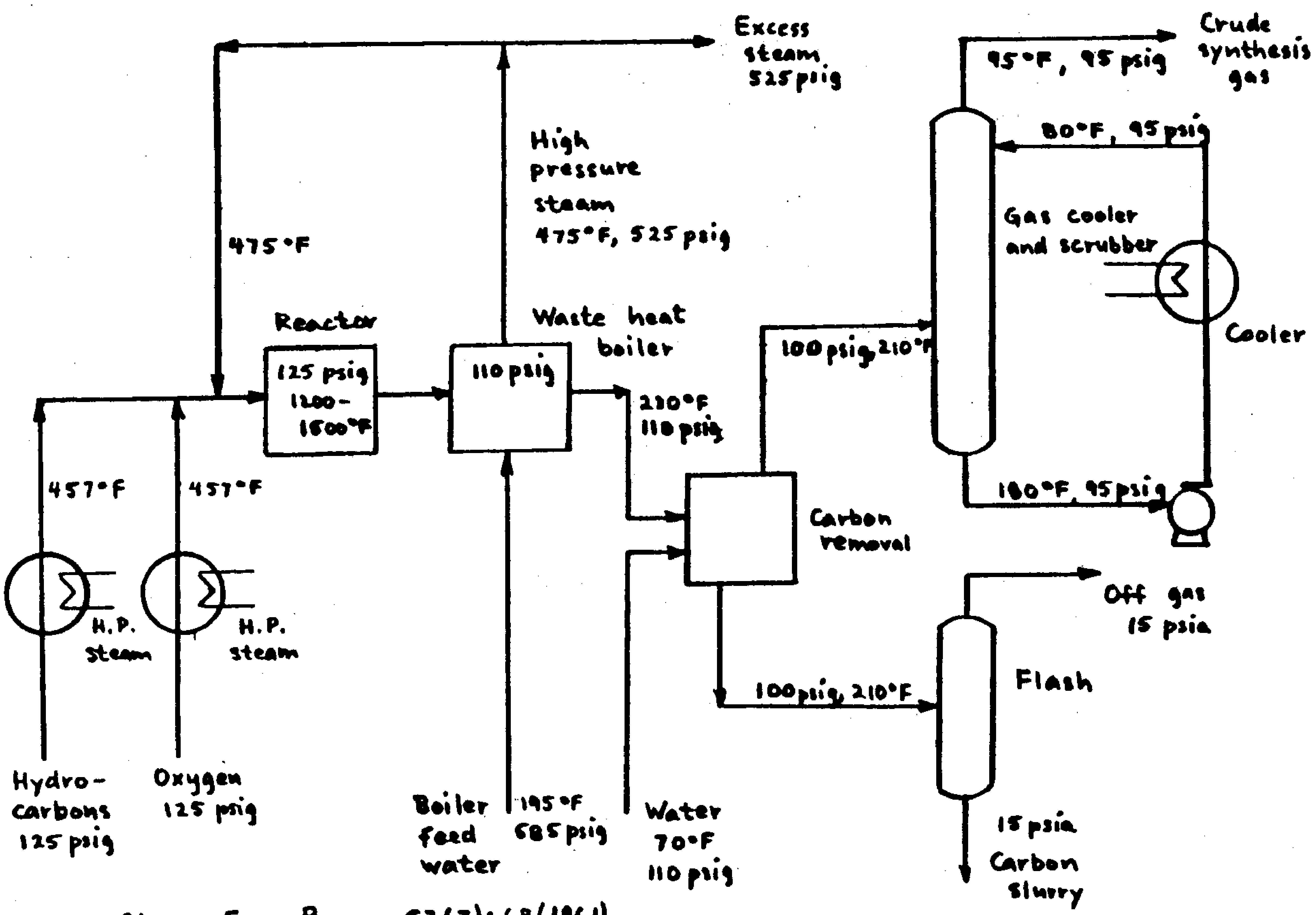


Top-bottom configuration hybrid system (*TBCHS*) for the propane/propylene separation.

3:35

3-12

A process flow sheet for preparing synthesis gas from any hydrocarbon is given below.

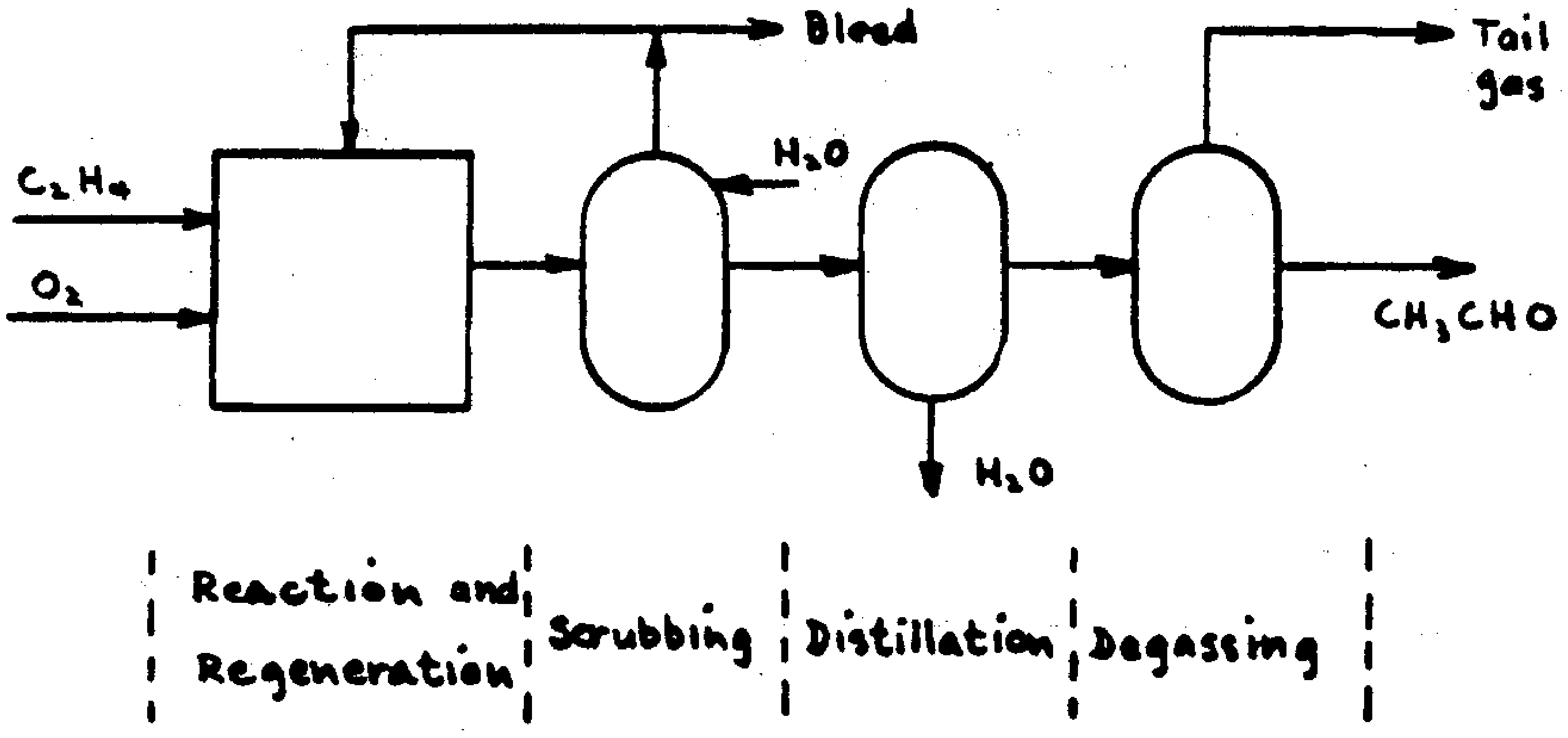


see Chem. Eng. Progr., 57(7): 68(1961)

The operating conditions for the synthesis gas process using the given feed stock, a heavy fuel oil, are presented in an article by S.C. Singer and L.W. ter Haar in *Chem. Eng. Progr.*, 57(7), 68(1961). The material balance may be made either with software programs by ASPEN PLUS or CHEMCAD and using these operating conditions and the final composition of the crude synthesis gas.

3-13

The simplified flow sequence for producing acetaldehyde from ethylene is given below:



Reaction and regeneration:

The main considerations are the reaction kinetics, equilibrium relations, and heat of reaction. The relationship of these factors will determine the reactor configuration and the cooling water requirements. Obviously, physical and thermal data for all the flow streams will be needed. Catalyst activity with information on its decline with time and reactivation time will be needed for regeneration design purposes.

Scrubbing:

The equilibrium relationship between the various reactor components and water will be the important design factor in this unit. Physical and thermal data for all the flow streams will be needed. Build up of inerts is controlled by a bleed stream. The composition of the latter must be known to determine the requirements for an auxiliary reactor designed to complete the oxidation of unconverted ethylene.

3-13 (continued)

Distillation:

The distillation unit requires equilibrium and vapor pressure data for the mass transfer calculations. Separation of key components must be specified and separation efficiencies need to be established. Physical and thermal data for all the flow streams will again be needed. Heat transfer coefficients will be required for the design of heat exchangers, condenser, and reboiler.

Degassing:

This unit will require much of the same information that is required for the distillation unit only with different compositions and flows.

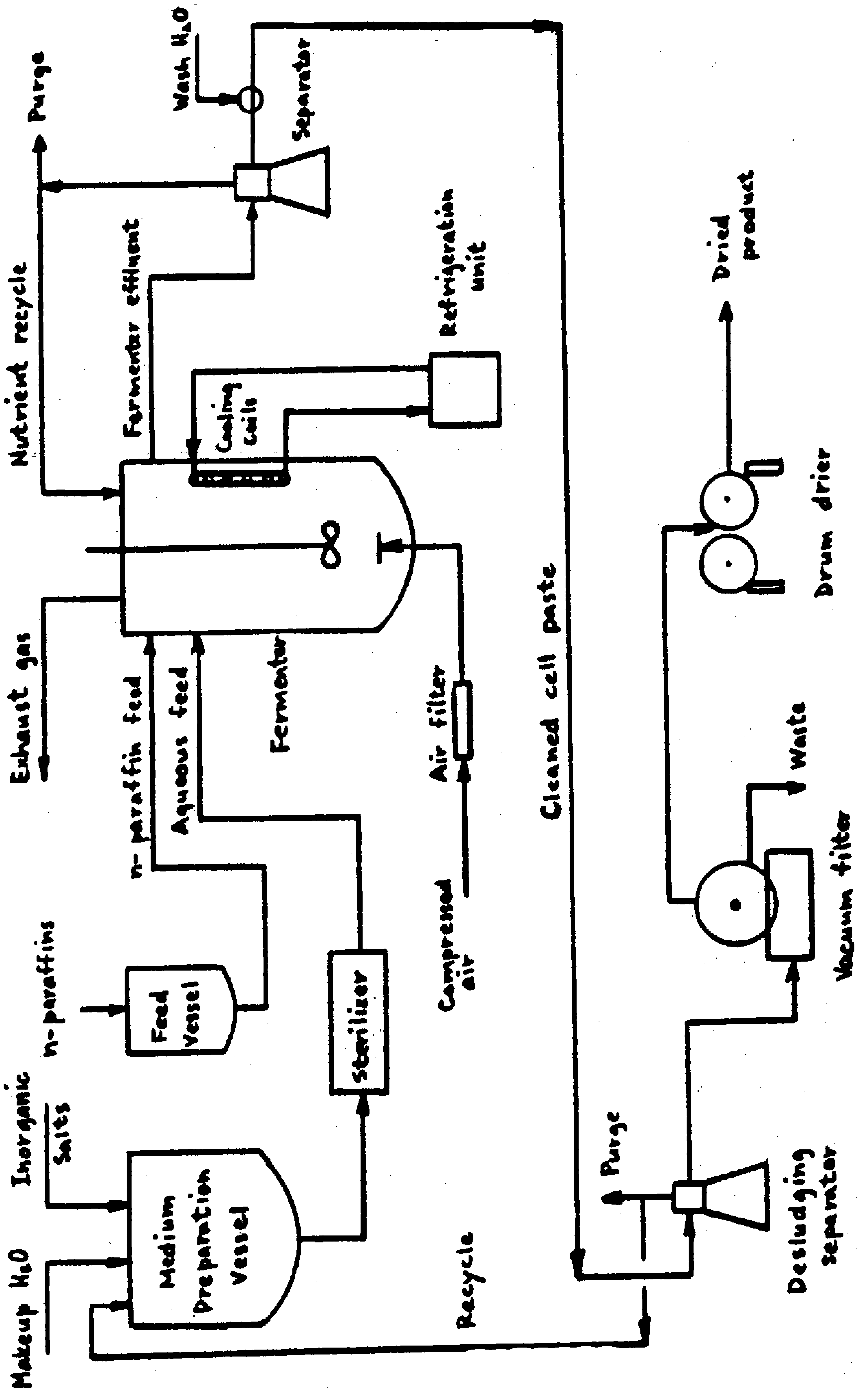
A simplified equipment flow sheet for the acetaldehyde process is given in *Hydrocarbon Process*. **44**(11), 159(1965). Since this is a proprietary process, only sketchy information is supplied with regards to temperature, pressure, and stream composition at each piece of equipment. Some of this information can be deduced from the information supplied for the raw materials and utilities requirements per short ton of acetaldehyde.

3-14

A simplified equipment flow sheet for the acetaldehyde process is given in *Hydrocarbon Process.* **46**(11), 135(1967). In this variation of the process, the spent catalyst is reoxidized with air in a separate regenerator. Ethylene is converted to acetaldehyde in a single-pass contact with the catalyst solution at a slightly higher pressure than used in a single-stage unit. In a separator following the reactor, the pressure is reduced and reaction heat vaporizes the acetaldehyde product from the catalyst solution. The spent catalyst from the separator is pumped to the regenerator where it is reoxidized by contact with air. A comparison of raw materials and utilities required for the two-stage acetaldehyde process may be made with that given for an actual 75,000 short ton/year plant discussed in the same reference noted above. Since the solution of this problem is quite long, it is not presented here.

3-15

A flow sheet outlining the conversion of petroleum to food supplements using organic microorganisms is presented below;



3-15 (continued)

In the process an aqueous feed containing the necessary inorganic nutrients is mixed in a vessel before being sent to the fermentor with a metered amount of paraffinie feed stock. Compressed and sterilized air serves as the oxygen supply for the growing cells in the fermentor.

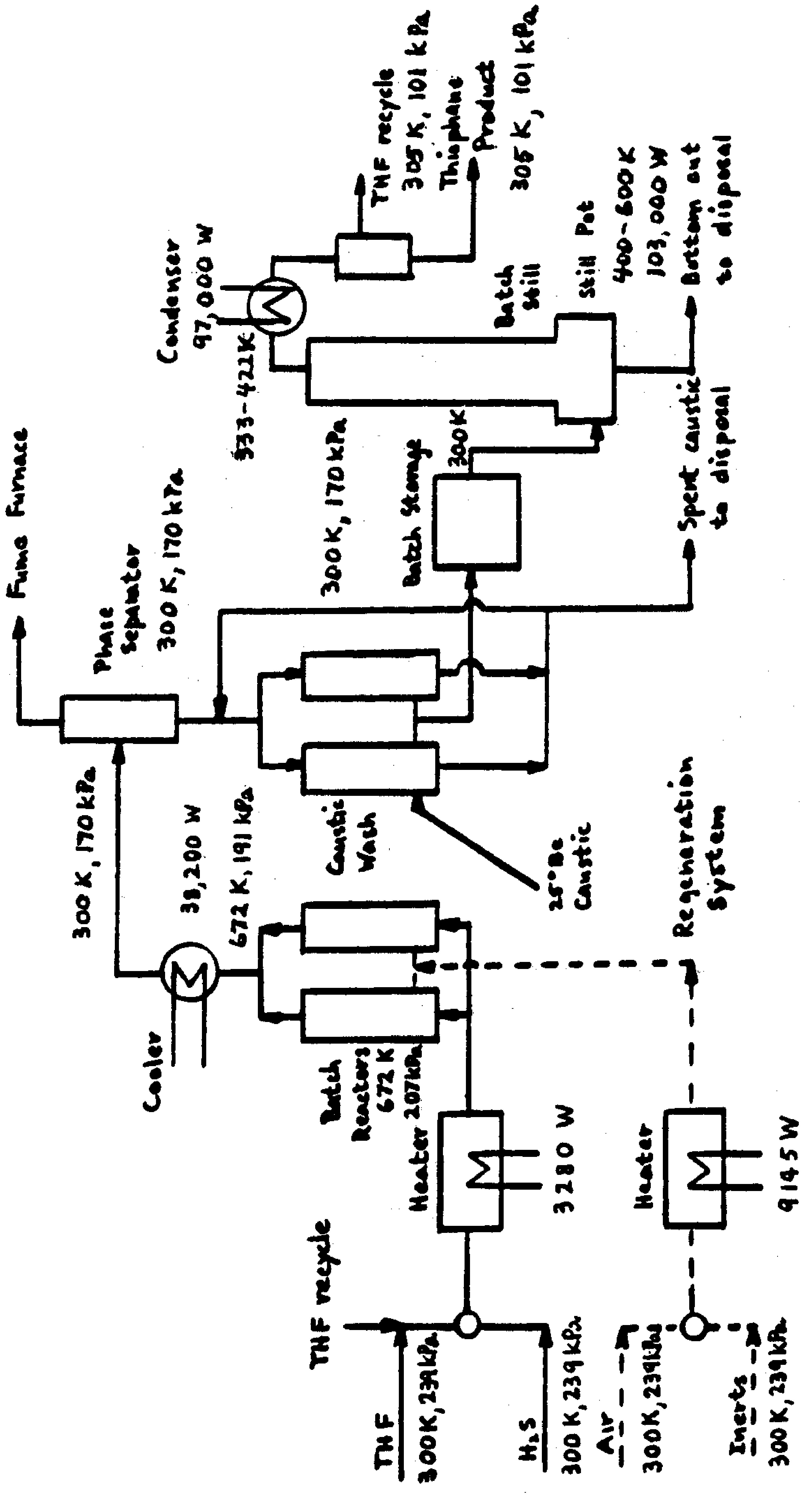
The growth of single cells is an autolytic process. The exponential growth rate can be maintained as long as the concentrations of all nutrients are kept above their respective critical values in the fermenting brew. The ratio of the paraffins to aqueous feedrates need to be maintained as high as possible. The temperature during fermentation must be kept constant by the careful control of heat removal.

At steady state, the product stream is continuously removed from the fermentor at a rate equal to feedrate. The product stream is concentrated in a desludging separator to a paste of 20 to 30% dry solids. After washing with warm water to remove adsorbed or absorbed hydrocarbons, a second desludging separator produces a cleaned paste that is further dehydrated in a rotary vacuum filter. A final drying step in an atmospheric drum dryer reduces the moisture content to less than 10%, and the powdered product is packaged for the customer.

The potential of using crude oil to produce human or animal-feed supplements has been recognized since the early 1960s. Many of the problems to accomplish this feat were listed by Wang in a *Chem. Eng.* 75(18), 99 (1968) publication. Several of these problems include separation of straight-chain hydrocarbons from isoparaffins, cyclic paraffins, and napthenes, development of a continuous non-aseptic processing technique, better understanding of the mass transfer characteristics of the process, control of the heat of fermentation, establishment of the required cell separation and purification technique, and improvement of the overall economics of the system. A number of these problems have been addressed over the past 35 years and suitable solutions have been formulated. However, the high demand for crude oil in meeting the world's transportation needs has increased the cost for crude oil to such a level that it is impractical to consider the use of this commodity to produce human or animal-feed supplements.

3-16

The process flow diagram for the production of 1.40 x 10^{-2} kg/s of thiophane is presented below with appropriate temperature, pressure, and heat exchanger duties. The material balance can be developed with the aid of suitable computer software based on the conditions listed on the flow sheet.

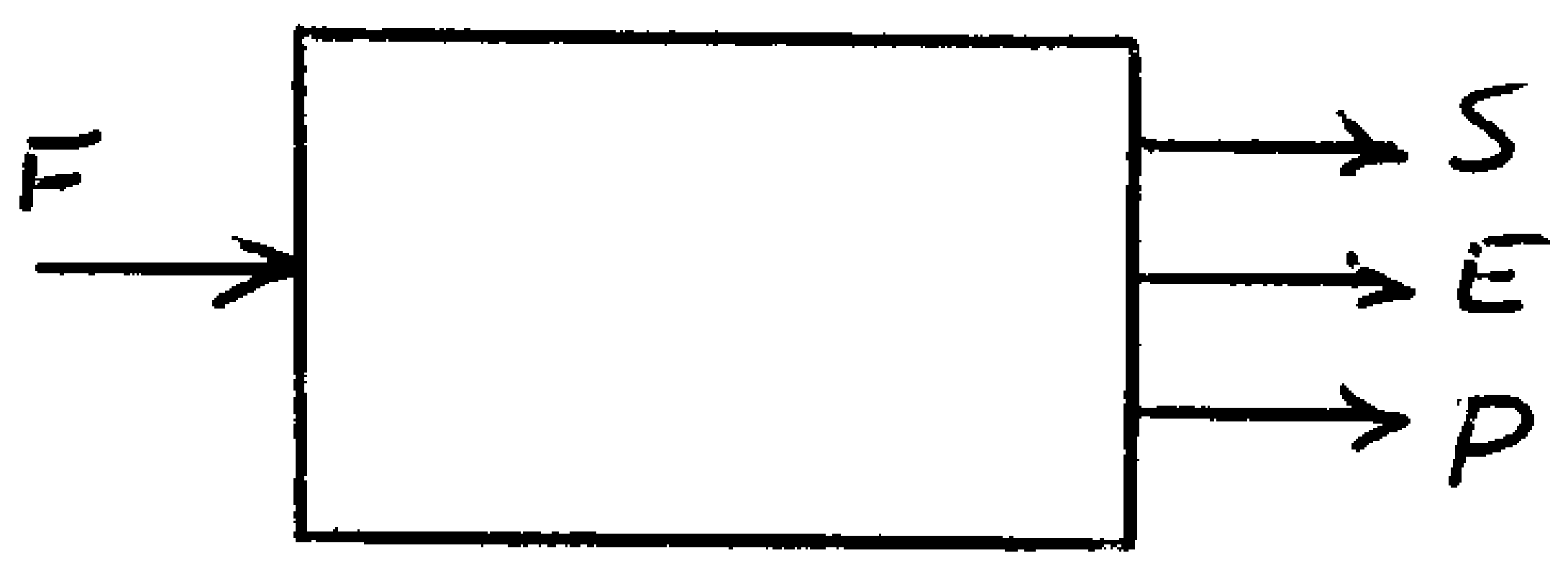


PROBLEM 4-1

Basis: 1 hour, 2.5×10^3 kgmol feed

Feed contains 83, 12, 4 and 1 mol % of methane, ethane, propane and n-butane, respectively. Let F, S, E and P represent the flowrates of feed, sales gas, ethane product and propane product, respectively.

a) I/O



b) Mole balances

Methane $(0.83)F = (0.995)S + (0.01)E$

Ethane $(0.12)F = (0.005)S + (0.97)E + (0.02)P$

Total $F = S + E + P$

Solving gives,

$\underline{S} = (0.8330)F = \underline{2.083\times10^3 \text{ kgmol/h}}$

$\underline{E} = (0.1184)F = \underline{0.296\times10^3 \text{ kgmol/h}}$

$\underline{P} = (0.0486)F = \underline{0.121\times10^3 \text{ kgmol/h}}$

c) $LHV = \sum (\text{mol fr.})_i (LHV)_i$

Feed LHV $= (0.83)(0.802) + (1.428)(0.12) + (2.044)(0.04) + (2.686)(0.01) = 0.946$ GJ/kg mol

Sales Gas LHV $= (0.995)(0.802) + (0.005)(1.428) = 0.805$ GJ/kg mol

$MW_{ave} = \sum (\text{mol fr.})_i (MW)_i$

Ethane Prod. $MW = (0.01)(16) + (0.97)(30) + (0.02)(44) = 30.14$

Feed value $= (2.5\times10^3)(4.00)(0.946) = \underline{\underline{\$9.46\times10^3/h}}$

ANS.

Problem 4-1 (continued-1)

Sales gas value $= (2.083)(3.25)(0.805)$

$= \underline{\underline{\$5,450/h}}$ ANS

Ethane product value $= (0.296\times10^3)(0.40)(30.14)$

$= \underline{\underline{\$3,568/h}}$ ANS.

Propane product contains all butane in the feed, 0.02 mol fr. ethane, no methane, and the propane not included in other streams.

mols butane/h $= (0.01)(2.5\times10^3) = 25$

mols propane/h $= (0.04)(2.5\times10^3) - (0.02)(0.296\times10^3) = 94.08\times10^3$

mols ethane $= (.02)((25+94.08)/0.98) = 2.43$

Propane product value $= (0.44)(2.43*30 + 94.08*44 + 25*58) = \underline{\underline{\$2459/h}}$ ANS.

d) $0 = (5,450 + 3,568 + 2,459) - 9,460 - C$

$C = 11,477 - 9,460 = \underline{\underline{\$2,017/h}}$ ANS.

C = Separation cost.

PROBLEM 4-2

NGL (natural gas liquids) consists of ethane, propane, iso- and normal butane, and C_5 and heavier hydrocarbons. See Chapter 3, Fig. 3-1 for information.

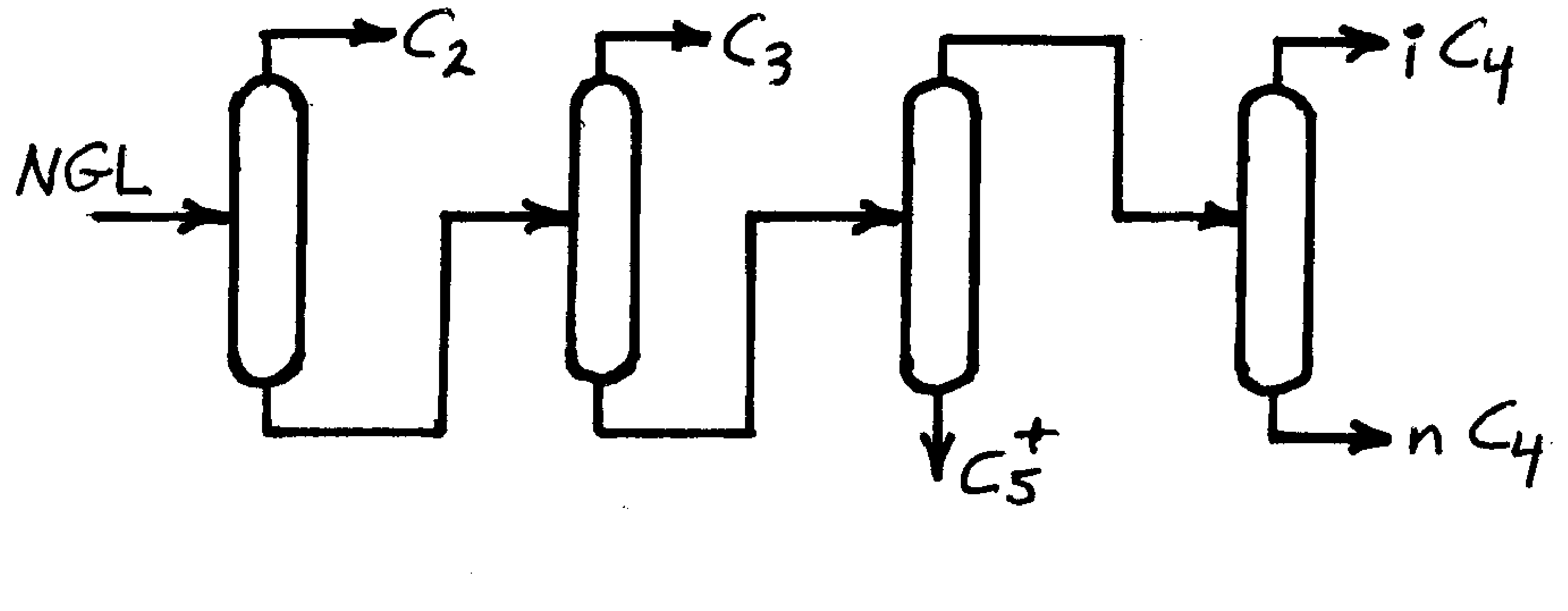
Pertinent properties are:

	Boiling pt. °C 101 kPa	506 kPa	1.01 MPa	Typical relative amt.
Ethane - C_2	-88.6	-52.8	- 32	Most
Propane - C_3	-42.1	-1.4	26.9	2nd most
i-butane - iC_4	-11.7	39.0	66.8	4th most
n-butane - nC_4	-0.5	50.0	79.5	3rd most
gasoline - C_5^+		greater		least

Hueristics:

1) Remove most volatile as overhead products one-by-one
2) Remove largest quantities early
3) Save most difficult separation until last.

1) and 2) lead to removing C_2 in first col., C_3 in second col. 3) leads to separating (iC_4 + nC_4) from C_5^+, then separating C_4's. So, suggested train is

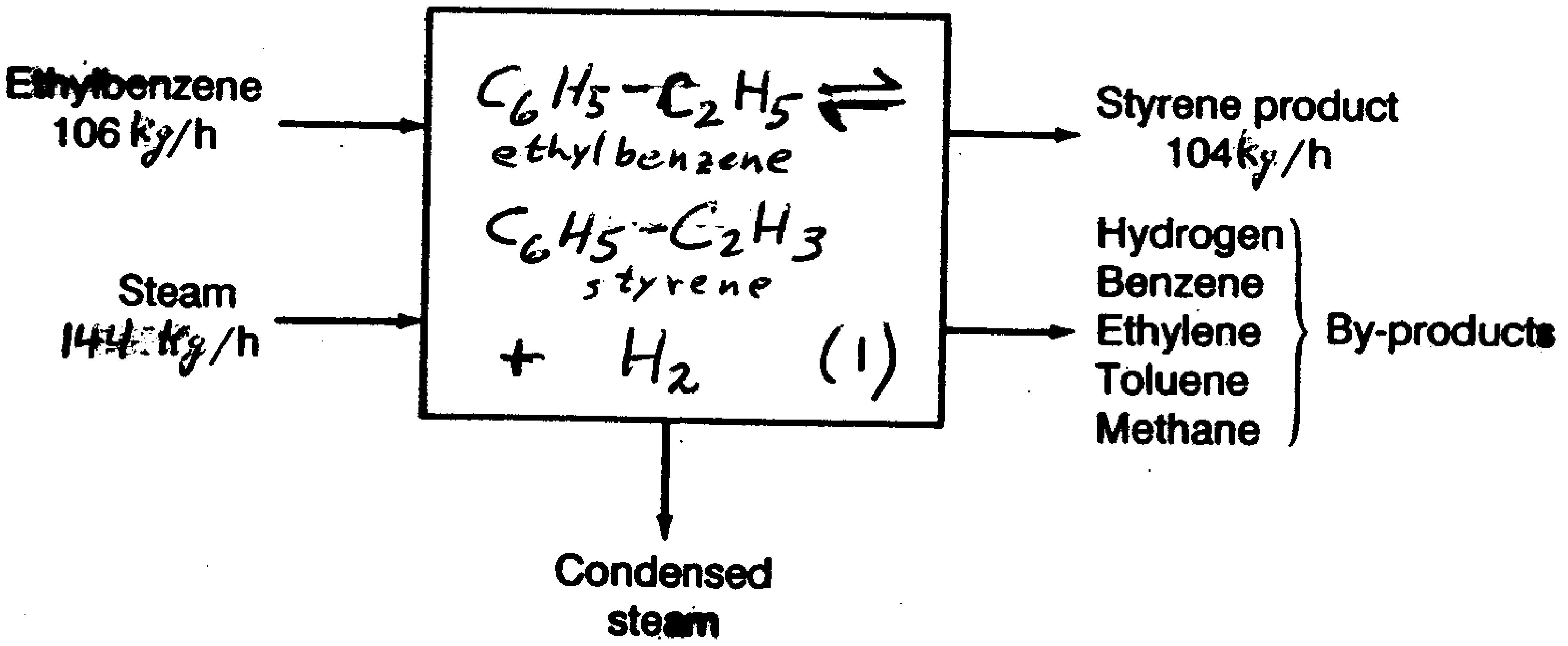


PROBLEM 4-3

Sources: Kirk-Othmer Encyclopedia of Chemical Technology, 4th ed., Vol. 22. Ullman's Encyclopedia of Industrial Chemistry, 5th ed., Vol. A25. Peters and Timmerhaus 4th ed., www.chemicalmarketreporter.com.

Styrene is made from ethylbenzene. The main process is catalytic dehydrogenation of ethylbenzene.

I/O



Values of \$0.57/kg for styrene, \$0.48/kg for ethylbenzene, and \$0.15/kg for hydrogen are used (the hydrogen value is based on its heat of combustion and a heat value of \$1.26/GJ). 1 kg-mol each of styrene (104 kg) and hydrogen (2 kg) requires 1 kg-mol of ethylbenzene (106 kg); Thus, on a basis of 1 kg-mol of styrene, the value of the products is

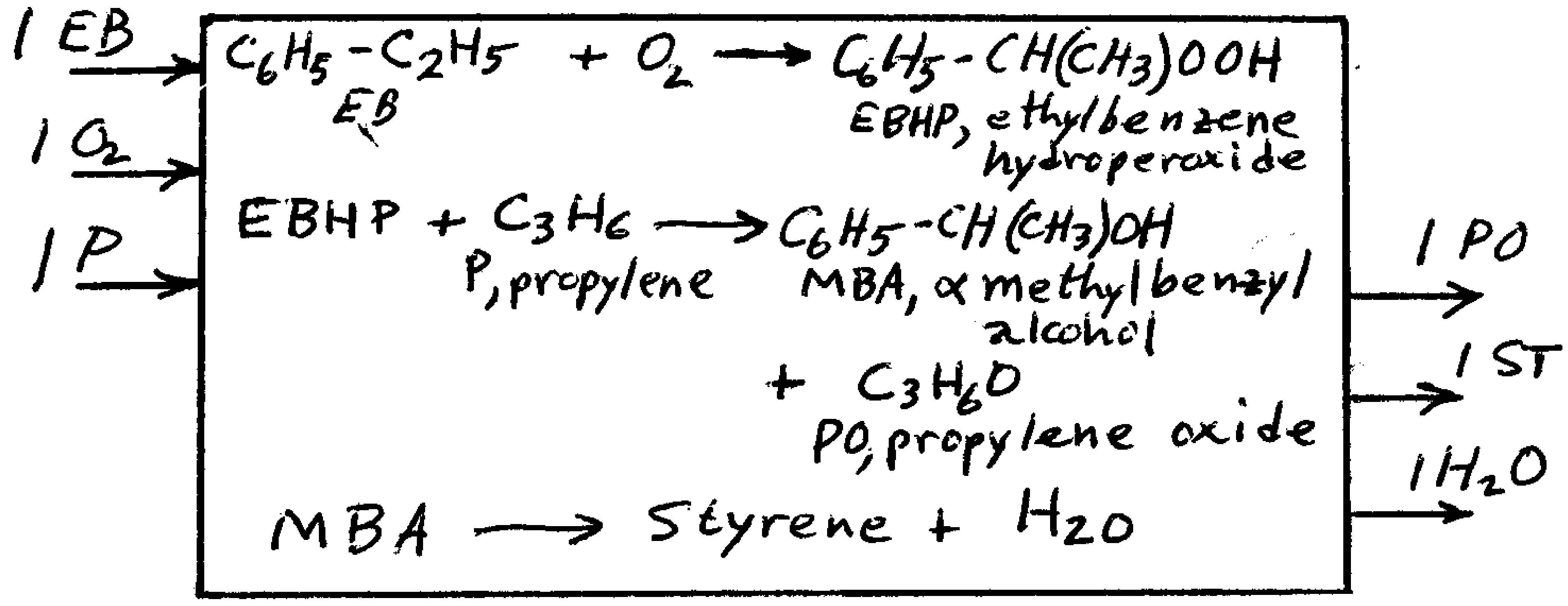
$$104 * 0.57 + 2 * 0.15 = \$59.6$$

while the value of the ethylbenzene is

$$106 * 0.48 = \$50.1$$

The product value exceeds that of the raw materials, so it is possible for the process to be profitable and the process synthesis may continue. The cost of carrying out the process—the equipment, utilities, labor, overhead, etc.—has not been accounted for, nor has the fact that the conversion of reactant to product necessarily will be less than 100 percent.

PROBLEM 4-3 (continued -1)
Another commercial but less used reaction path to styrene (ST) from ethylbenzene (EB) is:



With P @ $ 0,42/kg, PO @ $1,41/kg, H_2O valueless, and O_2 @ $0,04/kg, the preliminary economics are:

$$(104)(0,57) + (58)(1.41) - (106)(0.48) - (32)(0,04) - (42)(0.42)$$
$$= \$141 - 70 = \$\,71/\text{kg mol}$$

Clearly this process appears to be more attractive economically, due to the value of propylene oxide.
The current styrene price, $0,57/kg, is at the low end of the historical range, which has been $ 0,50 to 1,50/kg. This is undoubtedly due to excess capacity, suggesting this is not a good time to invest in styrene capacity. The propylene oxide route looks much more attractive, but should be considered only after market studies for styrene and propylene oxide.

PROBLEM 4-4

We select the ethylbenzene dehydrogenation path, because it is most widely used. For a similar analysis of the propylene oxide path, see problem 4-14.

The chemistry of the process is considered more thoroughly. Reaction (1) is an equilibrium reaction; it does not go to completion. The reaction is conducted in the gas phase over a solid, ferric-oxide based, catalyst. It is carried out at about 1-atm total pressure (low pressure favors the products of this equilibrium) and a temperature around 600°C (1112°F). Steam is added to lower the partial pressure of the products, thereby further favoring their formation. The reaction is endothermic, with the heat required being supplied by the steam. A steam-to-ethylbenzene molar ratio of 8 to 1 is representative. At typical process temperatures other reactions occur, including

$$\underset{}{C_6H_5{-}C_2H_5} \rightleftharpoons \underset{\text{benzene}}{C_6H_6} + \underset{\text{ethylene}}{C_2H_4} \tag{2}$$

and

$$C_6H_5{-}C_2H_5 + H_2 \rightleftharpoons \underset{\text{toluene}}{C_6H_5{-}CH_3} + \underset{\text{methane}}{CH_4} \tag{3}$$

Reactions (2) and (3) consume ethylbenzene without producing the desired styrene; they also produce by-products that must be separated from the product and disposed of or utilized.

The process feeds are ethylbenzene and steam and the products are condensed steam, styrene, benzene, toluene, hydrogen, methane, and ethylene. Including the steam, valued at $0.0[illegible]/kg, adds $[illegible] to the total feed cost, raising it to $30.7 per 104 kg of styrene product. The value of the products still exceeds that of the inputs, but barely. It is necessary to have information on the extent of each of the three reactions at the reactor conditions. These extents could be calculated by using complete reaction-kinetic information, and this probably would be done in a more complete design of the process.

Here the calculations are illustrated assuming a constant extent for each reaction. The values used for mass balance calculations are,

Fractional extent, reaction (1) = 0.47
reaction (2) = 0.025
reaction (3) = 0.005

Because it is a large volume product, run continously.

PROBLEM 4-4 (continued - 1)

The fractional extent-of-reaction is the fraction of the key reactant, ethylbenzene, in the reactor feed that reacts while passing through the reactor. In this example, 0.5 mol of ethylbenzene are consumed per 0.47 mol of styrene produced.

A total fractional extent-of-reaction less than 1.0 (0.5 in this example) means that not all of the reactant in the feed reacts, so unreacted reactant is present in the reactor-product stream. The latter stream contains ethylbenzene, all the reaction products, by-products, and steam. It is essential to separate the main product, styrene, from the rest of this mixture. Economically it is desirable and generally it is necessary to recover unreacted reactants and recycle them to the reactor. By-products must be separated and, if possible, effectively utilized. The condensed steam must be separated from the product stream and removed.

Mass balances for the process are recalculated using the extent-of-reaction values from above and assuming that all unreacted ethylbenzene (EB) is recycled and converted to products. On the basis of 1 kg-mol (104 kg) of styrene product, the calculations are:

$$\begin{aligned}\text{Feed EB flowrate} &= (1\ \text{kg-mol styrene/h})\\ &\quad *(0.5\ \text{mol EB}/0.47\ \text{mol styrene})\\ &\quad *(106\ \text{kg EB/kg mol})\\ &= 112.8\ \text{kg/h}\end{aligned}$$

The EB feed rate to the reactor must be twice this, because only one-half of the feed reacts per pass and the EB recycle rate equals the feed rate. Thus,

$$\text{Reactor EB feed rate} = 225.6\ \text{kg/h}$$

$$\begin{aligned}\text{The steam feed rate is then} &= (225.6/106\ \text{mol EB/h})\\ &\quad *(8\ \text{mol steam/mol EB})\\ &\quad *(18\ \text{kg steam/mol steam})\\ &= 306\ \text{kg/h}\end{aligned}$$

The by-product benzene plus toluene and the fuel gas rates are calculated, from Eqs. (2) and (3), to be 5.2 and 3.6 kg/h, respectively. The raw materials and product values can be compared again. The benzene-toluene mixture is valued at \$0.2/kg and the fuel gas at \$0.[illegible]/kg (both based on heating value); condensed water has no value. The value of the feeds, ethylbenzene and steam, is \$57.3/104 kg of styrene. The value of the product plus the by-products and fuel gas is \$60.[illegible]/104 kg of styrene. The value of the outputs still exceeds that of the inputs, but the margin has narrowed.

The process, as identified so far, is shown in Fig. 4-7.

PROBLEM 4-4 (Continued-2)

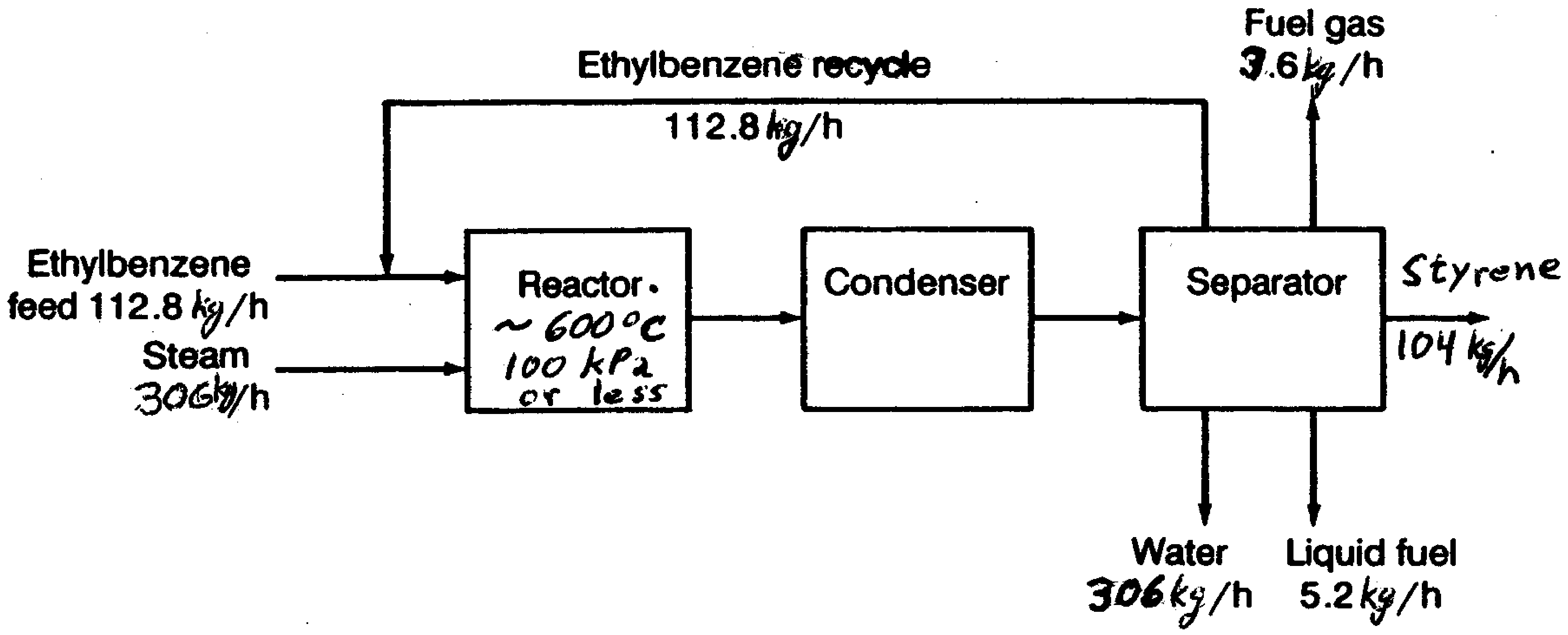


FIGURE 4-4 FUNCTIONS DIAGRAM

PROBLEM 4-5

One reactor is needed. The reaction is endothermic. The steam serves both as a diluent and a heat source. A packed bed of catalyst will serve as the reactor. If the reaction is adiabatic, temperature will decrease along the bed. An alternative is to have several beds in series with reheating between the beds.

Decision: Use adiabatic fixed bed reactor.

Separation processes. The **separation** after condensation into a water **phase, a** hydrocarbon phase, and a gas phase is accomplished by gravity in one or two holding tanks (other phase-separation methods are available and could be considered in a more detailed design). The liquid hydrocarbon phase needs to be separated into three fractions: (1) benzene plus toluene, (2) ethylbenzene, and (3) styrene. Experience shows that components with boiling points near ambient temperature (between, say, 0 and 200°C) and with adequate differences between their boiling points (at least 5°C) are usually most economically separated by distillation.

Normal boiling point values (°C) for the components of this system are:

hydrogen	−252.5	water	100
methane	−161.5	toluene	110.6
ethylene	−104	ethylbenzene	136
benzene	80.1	styrene	145

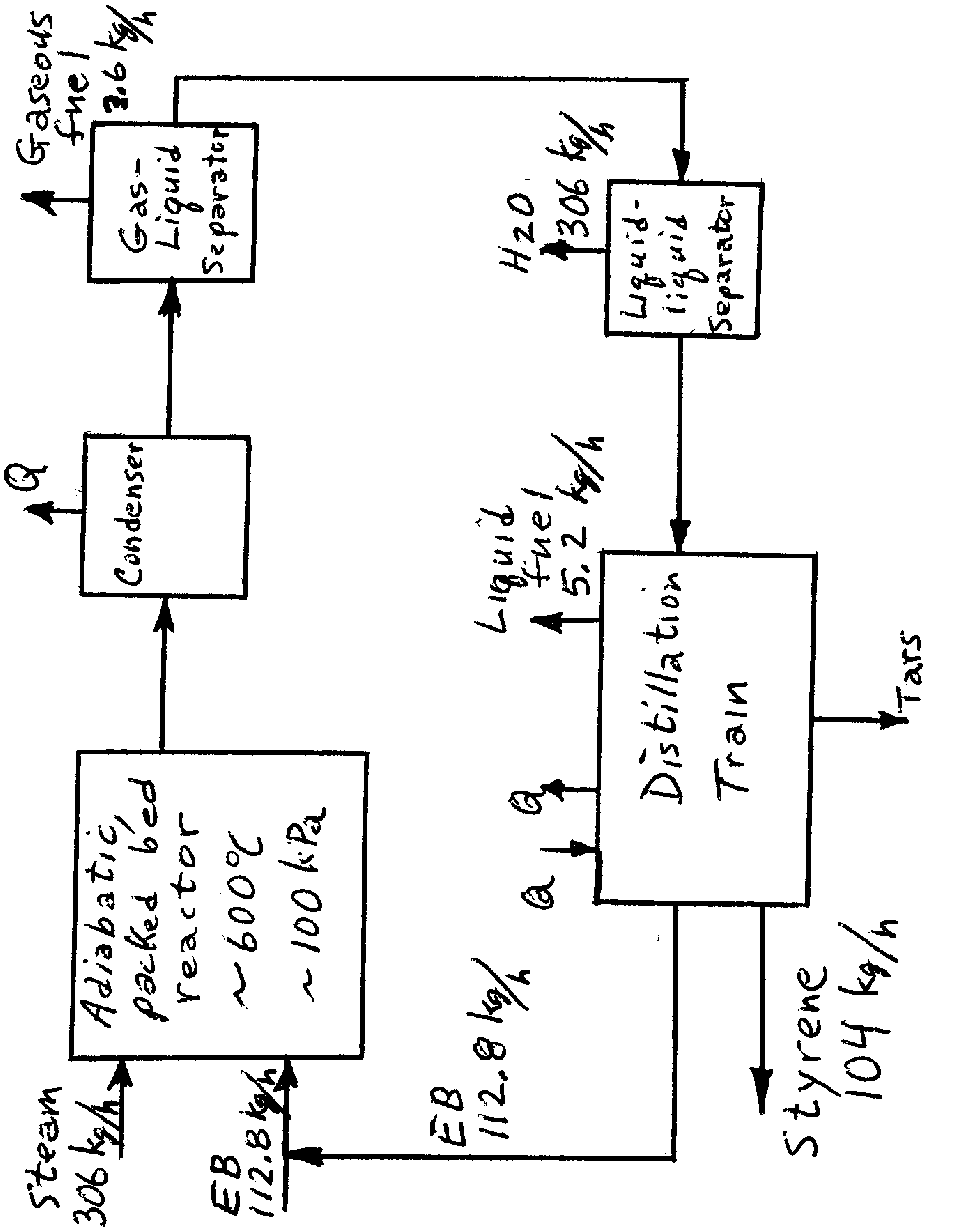
These values show that, with ambient cooling at 1 atm, hydrogen, methane, and ethylene are difficult to condense, but that steam, benzene, toluene, ethylbenzene, and styrene are easily condensed. **Condensation** will separate the latter **five** components as liquids and leave the **first** three in the gaseous state. Gaseous mixtures are very difficult to separate; when they are combustible, as here, experience indicates that it is usually best to use them as a fuel. Property data also show that water and the liquid hydrocarbons are highly insoluble in each other and thus will split into two liquid phases that can be separated **easily**.

Decision: Condense benzene and higher boiling components. Use holding tank to separate non-condensibles from liquids.

PROBLEM 4-5 (Continued-1)

<u>Decision:</u> Separate water from hydrocarbon liquids via holding tank (decanter).

<u>Decision:</u> Separate the hydrocarbon liquids by a distillation train.



OPERATIONS DIAGRAM

PROBLEM 4-6

An ordinary fractional distillation column yields two products; two distillations are required to produce the three hydrocarbon products in this process. Experience again provides guidance in planning this separation: First, remove the lower boiling component (benzene and toluene) and, last, make the most difficult separation (closest boiling points—ethylbenzene and styrene). Another lesson of experience is to remove a desired product (styrene) finally as a distillate (lower boiling) product. That is not possible with this mixture as described; however, experience also shows that organic reactions almost always generate higher boiling "tars" that need to be removed. Thus, a final distillation step, with styrene as the distillate product and tar (not included in the material balances) as the bottom product, is recommended. Another complicating factor here is that styrene polymerizes when heated. In order to avoid significant polymerization, styrene distillation temperatures are lowered by operating under vacuum.

Decision: Use 3 distillation columns.

The first column can be at or near atmospheric pressure, as the dilution of the styrene plus added polymerization inhibitors will limit styrene polymerization. The second column, EB-styrene separation needs to be under vacuum. For example at 13.3 kPa (100 mm Hg) absolute pressure, styrene boils at 82°C. This will be the reboiler temperature. The third column also needs to be under vacuum; at 5.3 kPa styrene boils at 60°C.

Packed distillation columns should be used to reduce pressure drops and residence times. A vacuum system, e.g. steam-jet ejectors, is needed.

Separation processes in reality do not achieve perfect separation as we have assumed in the mass balances. Actual separation-product stream compositions must be specified if the separations are to be designed in more detail. Here again experience, as well as product and by-product specifications, would be used to establish these specifications. Equipment design is not included in this example, so these specifications are not discussed.

PROBLEM 4-6 (continued-1)

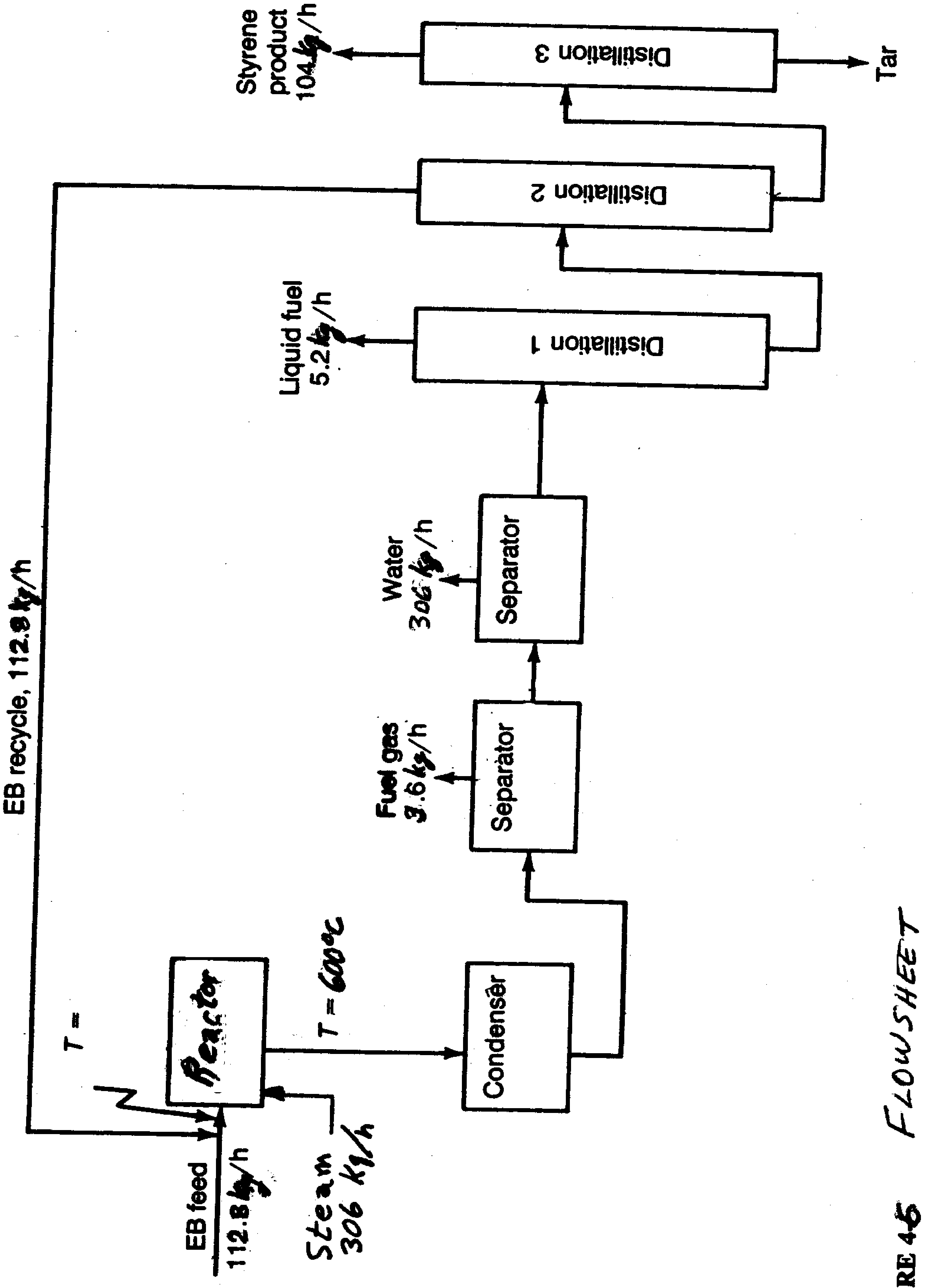


FIGURE 4.5 FLOWSHEET

PROBLEM 4-6 (continued - 2)

Heat integration. Both heating and cooling are needed in this process. Since supplying and removing heat is expensive, it is desirable to heat and cool using heat exchange between process streams. The purpose of heat integration is to satisfy the process heating and cooling requirements as economically as possible.

Required temperatures are specified; for example, a reactor outlet temperature of 600°C (1112°F) **is needed.** The temperature and energy requirements for the heating and **cooling loads** are matched as closely as possible. Styrene reactors typically operate **adiabatically** (no heat is added), causing the temperature to drop as the endothermic reaction proceeds. The reactor-inlet temperature required to achieve the specified outlet temperature is calculated from a reactor energy balance,

$$\text{Enthalpy of products} = \text{enthalpy of reactants} - \text{heat of reaction}$$

Enthalpies of reactants and products are expressed with respect to a reference temperature of 25°C (77°F), and the heat of reaction is available at 25°C:

$$[m * C * (600 - 25)]_p = [m * C * (T - 25)]_r - m * \Delta H_R$$

where m is the mass flow rate (kg/h), C is the heat capacity (kJ/kg-°C), $m * \Delta H_R$ is the total heat of reaction (kJ/h), and subscripts p and r denote products and reactants, respectively. The reactant and product mass flow rates are both 531.6 kg/h including steam from the earlier mass balances. Using 2.2 kJ/kg-°C for the heat capacity of both products and reactants, 117,500 kJ/h as the total heat of reaction (styrene molar flow rate * heat of reaction for styrene), and solving this equation for T gives a required inlet temperature of 700°C for the reactants.

PROBLEM 4-6 (continued - 3)

The ethylbenzene feed needs to be heated and the reactor effluent cooled. Heat exchange between them is feasible and desirable. Setting an approach temperature for a countercurrent exchanger (temperature difference between heated feed and reactor effluent) at 30°C, fixes the heated feed temperature at 570°C (1058°F). An energy balance for the heated feed and steam mixture establishes the required steam temperature:

$$[m * C * (T - 700)]_s + [m * C * (570 - 700)]_{EB} = 0$$

where the subscripts s and EB signify steam and ethylbenzene, respectively. Using $C_s = 2.5$, $C_{EB} = 2.5$ kJ/kg-°C, and the flow rates calculated above gives

$$T(\text{of superheated steam}) = 796°C$$

An energy balance around the feed-effluent heat exchanger,

$$[m * C * (600 - T)]_{\text{effluent}} = [m * C * (570 - 60)]_{\text{feed}}$$

where $m^*C_{feed} = (225.6)(2.5)$ and $m^*C_{effl.} = (2.2)(531.6)$

$T = 354°C$ as the outlet temperature of the effluent

The reactor effluent must reach about 40°C (104°F) for condensation. Thus, further cooling is required, by cooling water or other process loads. Heating ~~and cooling~~ for the distillation columns represent such possible loads.

Figure 4-6, on the next page, shows the heat integrated flowsheet.

The analysis evokes the question of utilizing the heat in the 354°C effluent. The only places where heat is needed are the distillation column reboilers. Analysis of the distillations to find reboiler heat loads is needed, but is not attempted here. Such application would result in some change in the flowsheet.

The earlier decisions should be revisited. Changes these would change the flowsheet.

PROBLEM 4-6 (continued - 4)

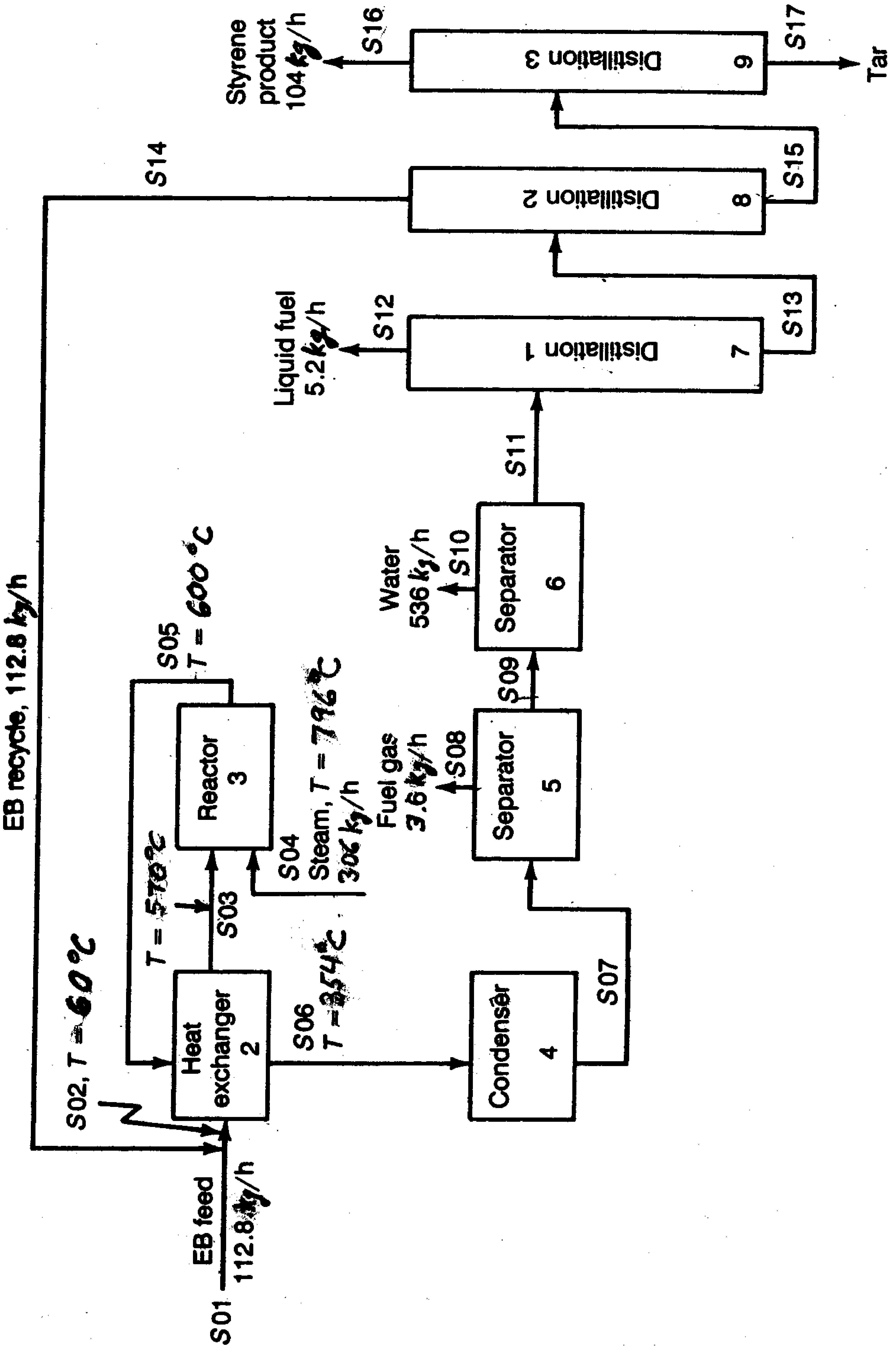


FIGURE 4. Heat Integrated Flowsheet

PROBLEM 4-7

Based on *Kirk-Othmer Encylcopedia,* assume that a reasonable catalyst lifetime is 18 months. Assume that in order to achieve a continuous operation, twice the amount of catalyst needed in the reactor is purchased.

Basis: 104 kg/h of styrene product. This requires 225.6 kg/h of reactor feed (see problem 4-6).

Given: 1 kg of catalyst is required in the reactor per 100 kg/h of reactor feed.

Thus,

(1/100)(225.6 kg /h of reactor feed) = 2.256 kg catalyst/104 kg styrene

Assume that twice the required amount of catalyst is purchased, so that there is sufficient catalyst to achieve continuous flow of product (e.g. by using 2 reactors).

The purchase cost of catalyst = (4.512 kg cat.)(5\$/kg cat.)/(104 kg styrene/h)

= \$0.217/(kg styrene/h)

Based on the assumed 18-month catalyst lifetime, the operating lifetime of the catalyst is

(18/12)(365)(24) = 13,140 h

Purchase cost of cat. = (\$0.217)/(kg styrene/h)(13,140 h) = \$1.65*10^{-5}/kg styrene

The regeneration cost = (2.256 kg cat.)(\$0.15/kg)/((104 kg styrene/h)(800h))

= \$4.1*10^{-6}/kg styrene

Comparing these costs with the styrene value of \$0.57/kg and ethylbenzene cost of \$0.48/kg it is clear that, for the stated and assumed conditions, the catalyst cost does not have a significant impact on the economics of the process.

PROBLEM 4-8

A simulation can be performed using one of the overall process simulators listed in Appendix F, including Aspen Plus, CHEMCAD, Hysis, or others.

In order to perform the simulation, the components and the physical property routine must be specified. The main components in the styrene process are ethylbenzene, benzene, ethylene, toluene, methane, styrene, hydrogen, and H_2O. For example, the Peng-Robinson method for properties could be used.

The operations needed include: mixing, a reactor, splitting, heat exchange, cooling and condensing, distillation, recycle, and steam jet ejectors. The ethylebenzene conversion per pass is specified, the styrene selectivity can be estimated along with the extent of the benzene and toluene reactions. These could be calculated with a kinetic based reactor model, and should be for more detailed design. A simplified distillation model could be used, such as one based on the FUG method. Pressures must be established for the distillation columns. Split and recovery estimates are needed for the components.

The simulator will calculate material and energy balances. These would be useful in looking for heat integration opportunities. Approximate equipment specifications would also be established.

PROBLEM 4-9

See problem 4-3 for the two significant processes for styrene, and for pertinent sources.

Ethylbenzene is the only raw material for the dominant dehydrogenation process discussed in problems 4-3 through 4-8. Ethylbenzene and propylene are the raw materials for the route of secondary importance, the propylene oxide process.

Ethylbenzene is made from benzene and ethylene by

$$C_6H_6 + C_2H_4 \longrightarrow C_6H_5\text{-}C_2H_5$$

See Chapter 3 for a detailed description of the ethylene-production process.

Preliminary economics, on the basis of one kgmol of ethylbenzene show:

Product value = (\$0.48/kg)(106 kg) = \$50.9
Reactant values, Benzene value = (\$0.40/kg)(78) = \$31.2
Ethylene value = (\$0.53/kg)(28) = <u>\$14.8</u>
Product value – Reactant values = \$ 4.9

This result illustrates that, if the producer must purchase the benzene and ethylene at these prices, the operation is only marginally promising.

For the propylene oxide process, the economics of ethylbenzene production are the same as above. Propylene is a co-product of the ethylene process described in detail in chapter 3, and is economically favorable as shown there. The propylene oxide process itself is economically promising as shown in Problem 4-3.

In fact, the standard in the industry is that styrene production is integrated from benzene and the ethylene raw material (e.g. propane) through styrene (and propylene oxide for the second process). Styrene production is part of an integrated petrochemical process.

PROBLEM 4-10

Based on problem 4-9, there is no strong incentive to integrate ethylbenzene production with styrene production, for either the dehydrogenation or the propylene oxide route. However, the reality of the situation is that they are always integrated. Styrene is by far the major market for ethylbenzene, and the economics are favorable when the styrene producer controls raw material supply. The price of ethylbenzene on the open market does not reflect its value to the integrated producer.

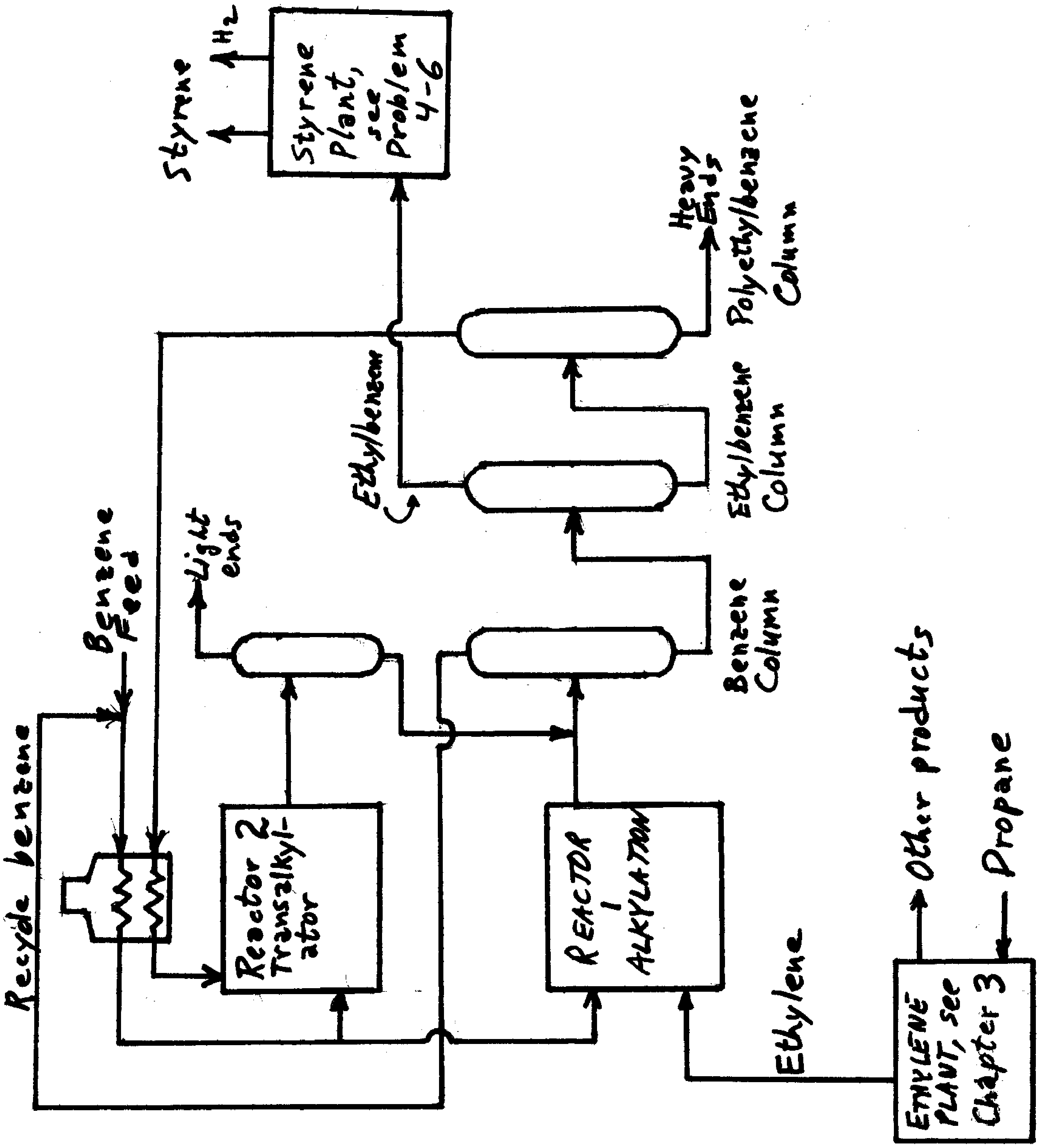
Ethylene and propylene are co-products of ethylene production (see Chapter 3), and this is an economically favorable process. So, it follows that the ethylene (and propylene), benzene, and ethylbenzene production would be integrated with the styrene production.

See problem 4-12 for preliminary economics.

PROBLEM 4-11

Reaction 1. Vapor-phase alkylation of Benzene + ethylene to ethylbenzene over zeolite catalyst.

Reaction 2. Vapor-phase transalkylation of benzene with polyethylbenzenes (by-products of 1.) over zeolite.



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PROBLEM 4-12

The preliminary economics of the overall process shown in Problem 4-11 are shown.

$$C_3H_8 + C_6H_6 \longrightarrow C_6H_5\text{-}C_2H_3 + H_2 + CH_4$$

Product value (neglect H_2 and CH_4) = (\$0.57/kg)(104 kg) = \$59.3

Reactant values: Propane = (\$0.27/kg)(44 kg) = \$11.9
Benzene = (\$0.40/kg)(78kg) = \$31.2

Value of products – reactants = \$16.2

Note: Benzene price is from www.icislor.com, a good source for chemical and petrochemical prices.

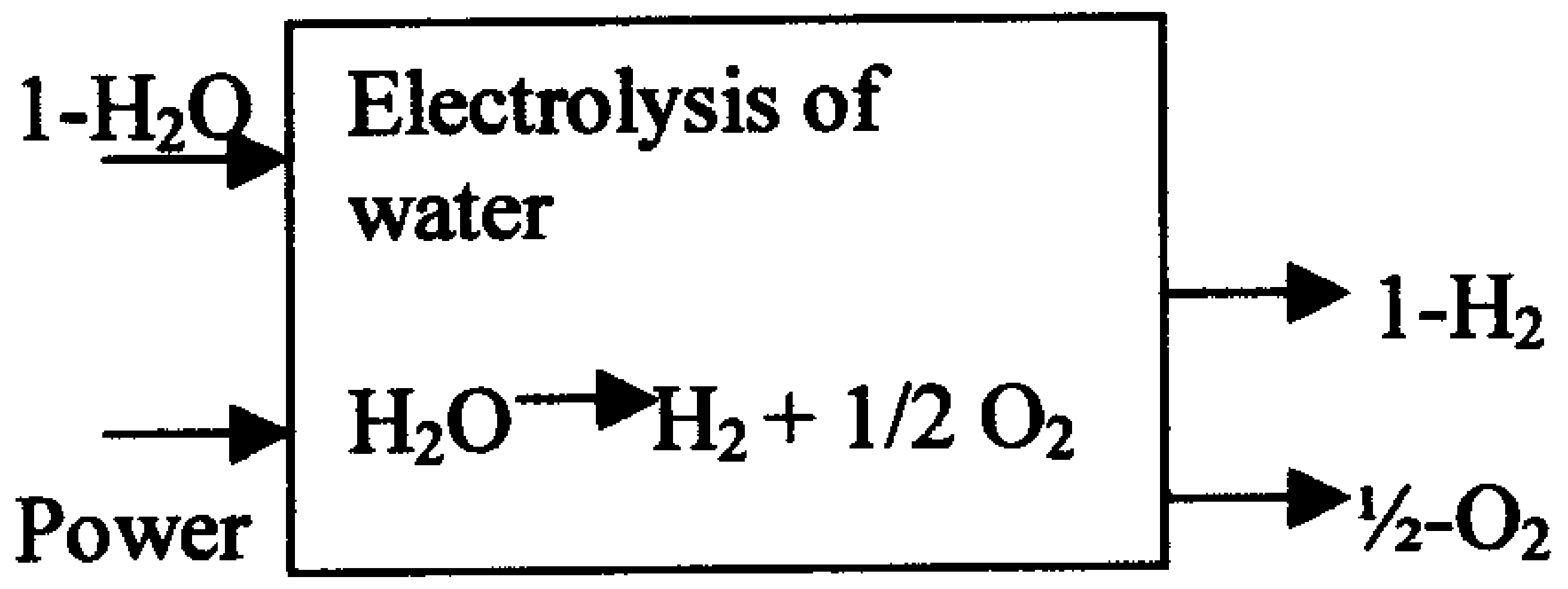
This result shows a significant economic improvement compared to the result in problem 4-9. Similarly, the propylene oxide pathway, shown in problem 4-3 to be favorable, also shows improvement.

PROBLEM 4-13

Sources: *Kirk-Othmer Encyclopedia of Chemical Technology*, 4th ed., and *Ullman's Encyclopedia of Industrial Chemistry,* 5th ed.

For either of the rates suggested, a continuous process should be used. The potential of hydrogen as a fuel for fuel cells would require very large production rates and large plants. An alternative that has been suggested is small electrolysis plants located at refueling stations.

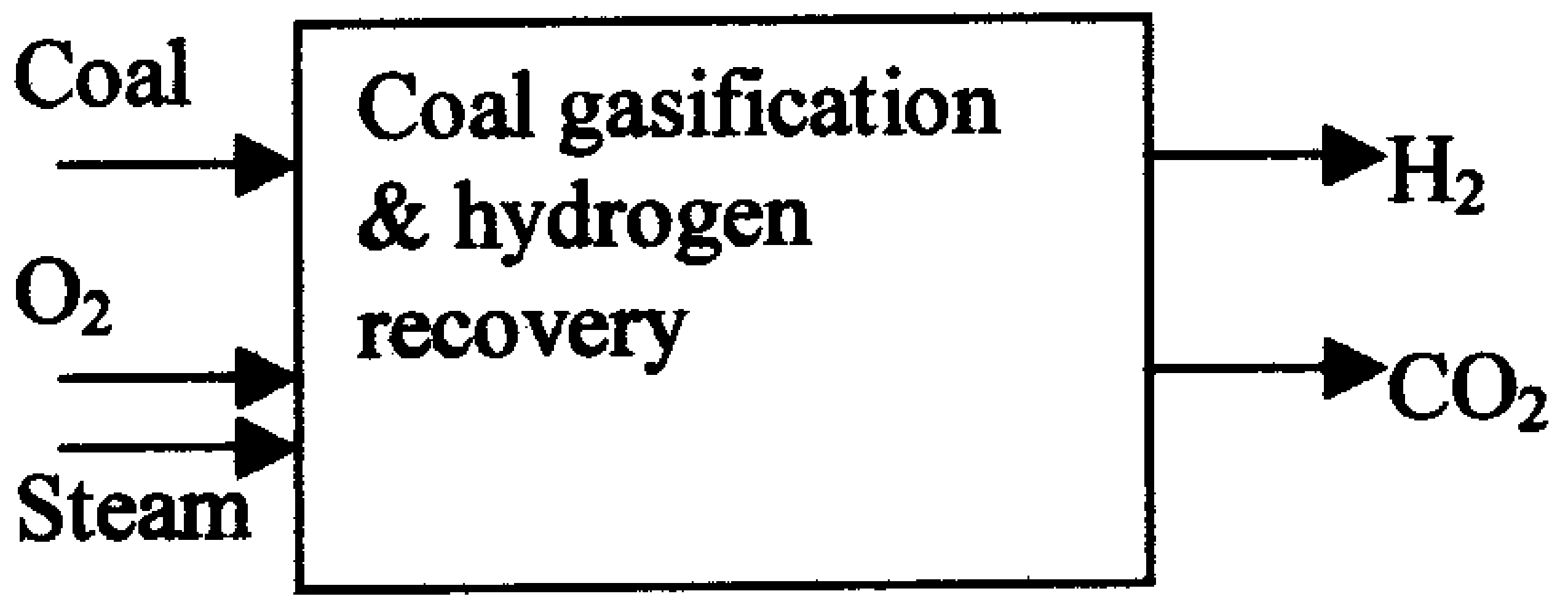
Water. Hydrogen can be obtained from water by any decomposition process, but the most likely candidate is electrolysis. An I/O diagram and preliminary economics are shown.



	Price	Amount/ kgmol H_2	Cost/ kgmol H_2
H_2O	negl.	18 kg	negl.
Elec.	\$0.05/kWh	50 kWh	\$-2.50
H_2	\$0.67/kg	2 kg	\$ 1.34
O_2	\$ 0.04/kg	16 kg	\$ 0.64
		Net	\$-0.52

A price for hydrogen difficult to come by, but \$0.67 is at the high end of the range (\$0.01 to 0.06 per standard m^3) for pipeline hydrogen from Kirk-Othmer. This pathway is not economically attractive for pipeline-scale quantities.

Coal. Coal may be gasified to produce a gas containing CO, CO_2, and H_2. Hydrogen may be recovered from this gas.

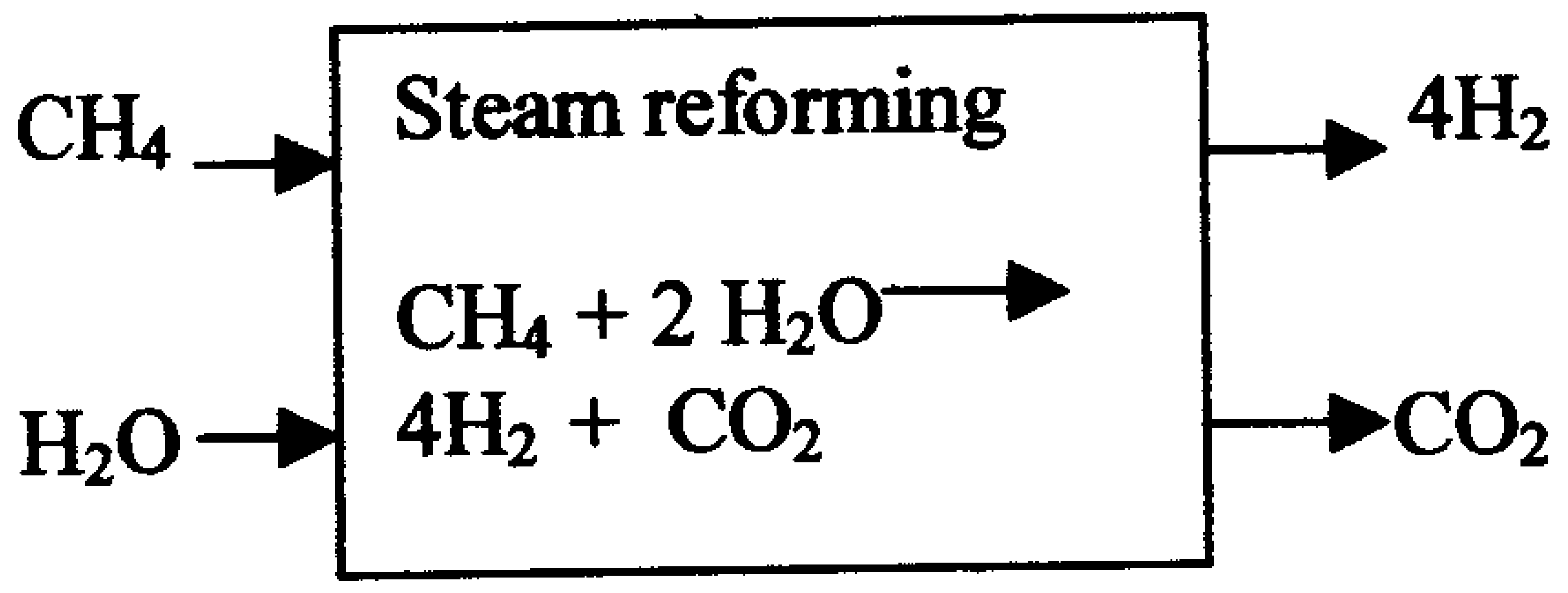


	Price	Amount/ kgmol/H_2	Cost/ kgmol H_2
Coal	\$0.055/kg	~17	\$-0.94
O_2	\$0.04/kg	~4	\$-0.16
Steam	\$0.008/kg	~20	\$-0.16
H_2	\$0.67/kg	2	\$ 1.34
		Net	\$ 0.08

This pathway is marginally economically attractive with no processing cost included.

PROBLEM 4-13 (continued-1)

Natural gas. Natural gas (NG, mainly methane) is converted to a gas containing primarily CO_2 and H_2 by reacting with steam, the steam-reforming process.



	Price	Amount/ kgmol H_2	Cost/ kgmol H_2
NG	\$0.13/kg	4 kg	\$-0.50
Steam	\$0.008/kg	9 kg	\$-0.07
H_2	\$0.67/kg	2 kg	\$ 1.34
		Net	\$0.77

This pathway looks potentially economical. Indeed, this well developed process is the way in which industrial hydrogen is produced.

PROBLEM 4-14

Note corrections: The first four words should be replaced by "For the second..." The sentence beginning, "100 percent separation of all ethylbenzene..." should read, "100 percent separation and recycle of all ethylbenzene from the reactor exit stream." The stated catalyst life of 1100 hrs is actually the period between regenerations. Assume a catalyst life of 1 year. Due to lack of thermodynamic data for several components, heat integration should not be required.

See problem 4-3 for I/O diagram and preliminary economics.

First consider the catalyst cost for the dehydration of α-methylbenzyl alcohol. Given that 1 kg of catalyst is required per 100 kg/hr of feed, the amount of catalyst required per 100 kg/h of styrene product is approximately (122/104). Three catalysts are required, thus,

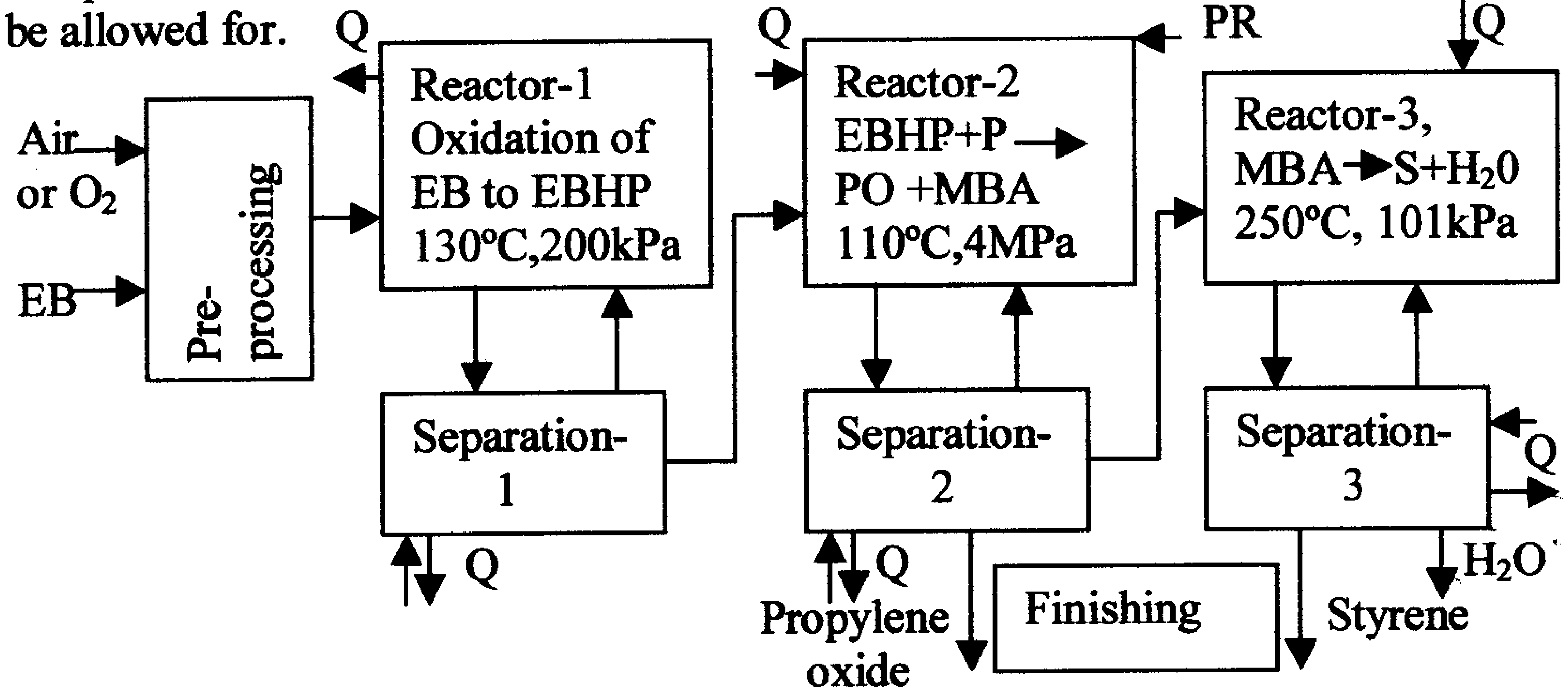
Catalyst required = (3)(122/104 kg feed/kg product)(1kg cat./100kg feed/h)
= 0.0352 kg catalyst/kg styrene product/h

At 1100 h per regeneration, there would be 7 regenerations over the assumed one year lifetime of the catalyst. Further assuming that twice the total amount required catalyst is purchased, the catalyst cost becomes

Catalyst cost = (2)(0.0352 kg cat./kg styrene/h)($6.6/kg cat.)(1/8760 h) +
(7)(0.0352 kg cat./kg styrene/h)($0.25/kg cat.)(1/8760 h)
= $$6*10^{-5}$/kg styrene

Comparison with problem 4-3 shows that this cost does not significantly affect the process economics.

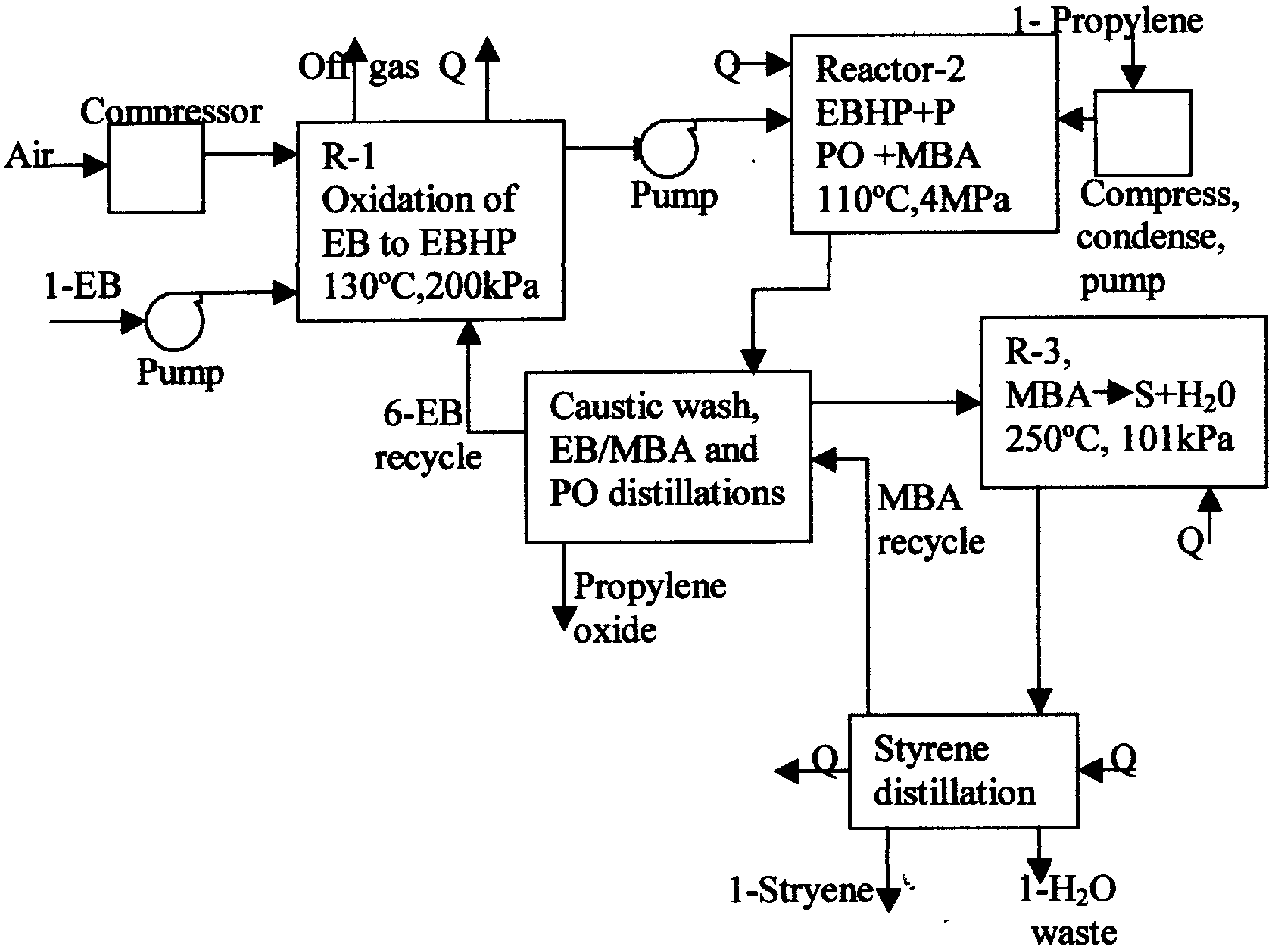
a) Functions. Three separate reactors are required for this process. Separation and purification is required after each reactor. Preprocessing and finishing should be allowed for.



PRpropylene, PO=propylene oxide, EB=ethylbenzene, EBHP= ethylbenzene hydroperoxide, and MBA=α-methylbenzyl alcohol, S=styrene

HOMEWORK 4-14 (continued-1)

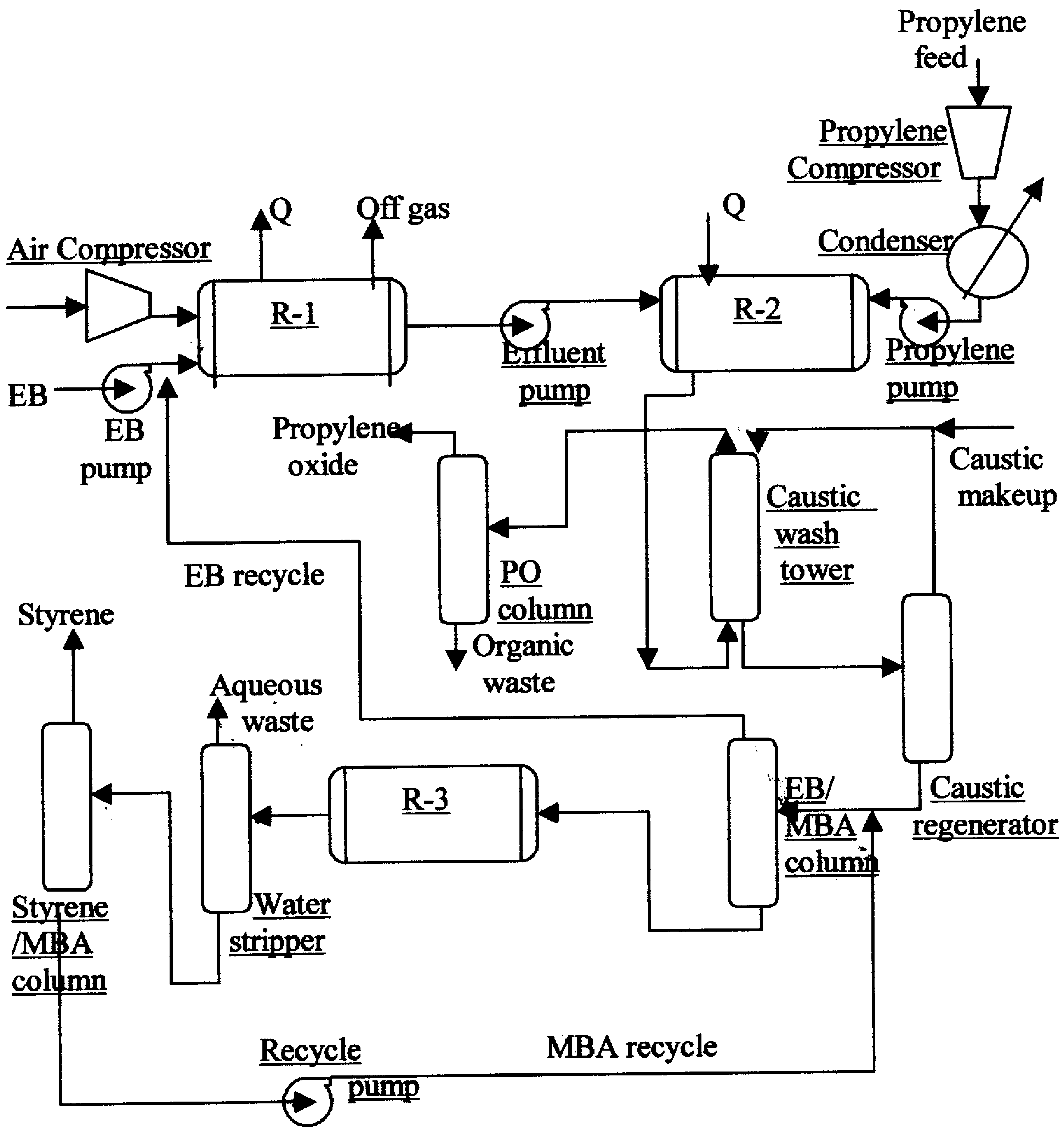
b) Operations. Air is used for the liquid-phase oxidation of EB, as the nitrogen moderates the temperature. Air needs to be compressed and EB pumped into reactor-1. EB conversion is only about 15 percent in reactor-1, to minimize byproducts. The products of reactor 1 are sent directly to reactor 2. Unreacted EB is recycled after reactor-2. Conversion of EBHP is essentially complete. Caustic solution is used to absorb all of the organics except PO, which is purified by distillation. EB and MBA are separated by distillation, the EB recycled and the MBA goes to R-3. From R-3, styrene is recovered and purified by distilation and MBA is recycled to the first separation process. Insufficient data available in the literature and on the web to obtain the heat duty values, Q.



c. Flowsheet. The operations diagram is coverted to a flowsheet by defining the recovery and separation steps. See the next page. Because of the lack of heat of reaction data, heat integration has not been accomplished in this study.

PROBLEM 4-14 (continued-2)

d. The economics has been examined in problem 4-3 and above.



PROBLEM 4-15

Sources: www.chemicalmarketreporter.com, *Kirk-Othmer Encyclopedia of Chemical Technology*.

a) The major process for acrylic acid production is from propylene and oxygen:

Reaction 1, $\underset{\text{propylene}}{CH_2{=}CHCH_3} + O_2(\text{air}) \rightarrow \underset{\text{acrolein}}{CH_2{=}CHCHO} + H_2O$ Exothermic

Reaction 2, $CH_2{=}CHCHO + \frac{1}{2}O_2\ (\text{air}) \rightarrow \underset{\text{acrylic acid}}{CH_2CHCOOH}$ Exothermic

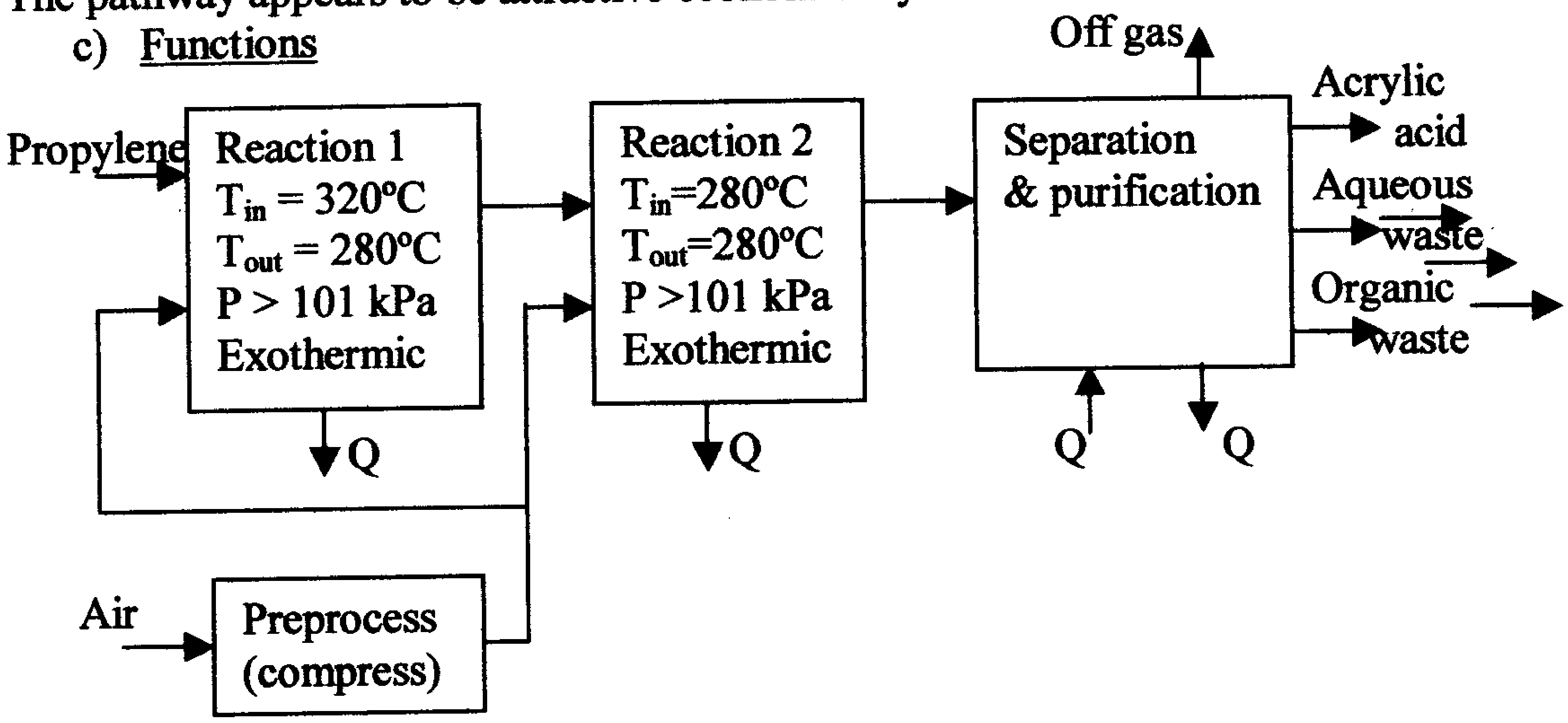
Acrylic acid is a high volume chemical, continuous processing is used. Similar catalysts are used for both reactions, but the catalysts and conditions are sufficiently different that the reactions are conducted separately. Bismuth molybdate and molybdenum vanadium oxides typically are the bases for the catalysts in the first and second reactions, respectively. Effluent from the first reactor can go directly to the second reactor without processing. Fixed bed, shell-and-tube, solid-catalyst reactors are used for the gas-phase reaction. Reactors are cooled with circulating molten salts. Additional process information is presented as needed.

b) Basis: one kgmol of acetic acid

	Price	Amount	Value
Propylene	\$0.42/kg	42 kg	\$17.64
O_2 (air)	negligible	48 kg	negl.
Acetic Acid	\$1.0/kg	72 kg	\$72.00
H_2O	negligible	18 kg	negl.

The pathway appears to be attractive economically.

c) Functions



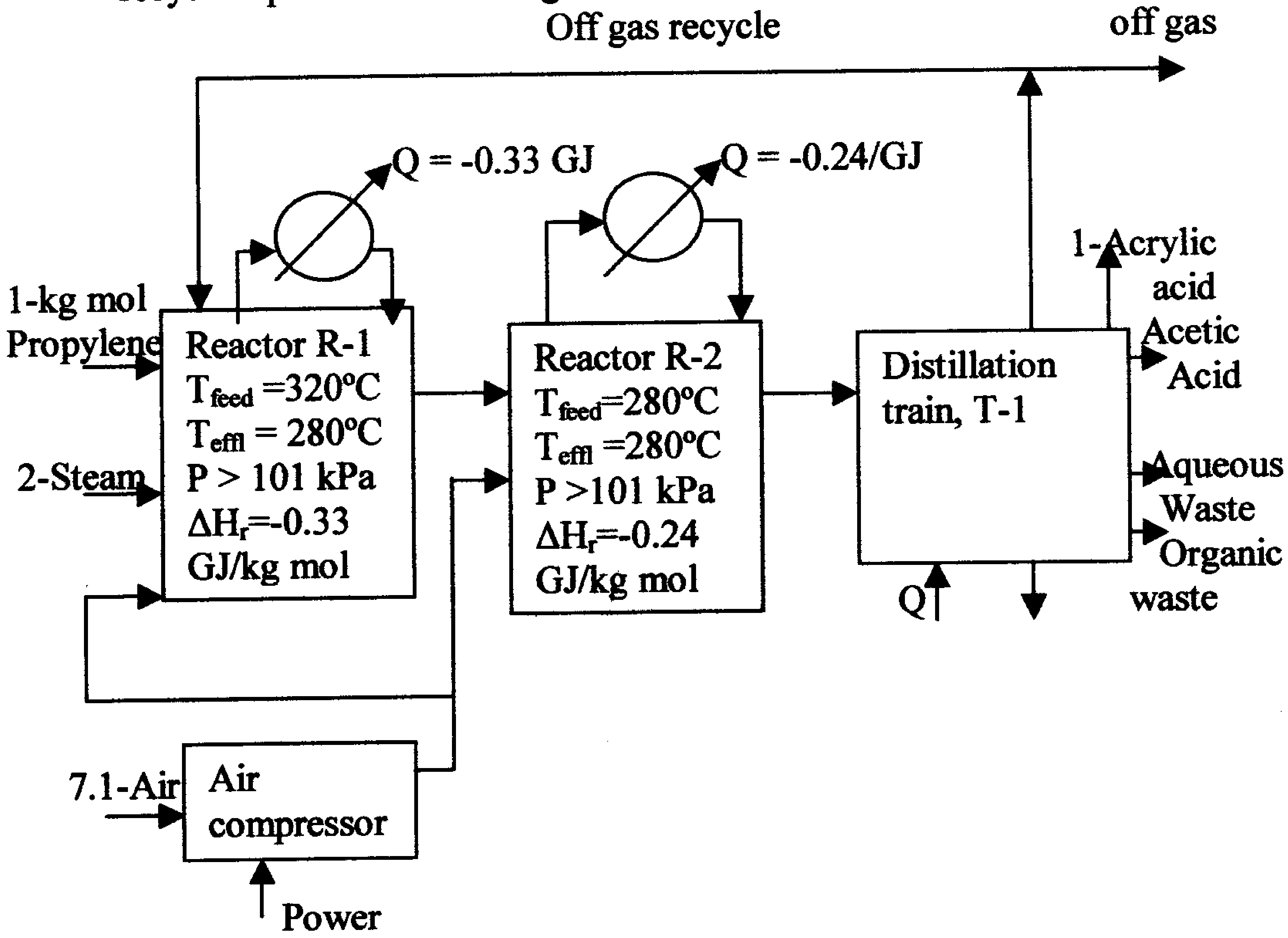
PROBLEM 4-15 (continued-1)

Operations. Steam is added to the reactor feed as a diluent. Molten salt is circulated through each reactor and cooled externally, usually by generating low-pressure steam. Conversion of ethylene and acrolein is nearly complete and 85 to 90 percent is converted to acrylic acid with the remainder being mostly acetic acid, a valuable byproduct. Depending on the conversions, recycle of off gases may or may not be necessary.

Decision: Design for 90% conversion to acrylic acid and the remainder acetic acid.

The separation of products is usually accomplished by first absorbing reactor-2 effluent into water, followed by extraction of the organics into a solvent such as butyl acetate, and distillation to recover the solvent, acrylic acid and acetic acid, and aqueous and organic wastes. However, if relatively high propylene and low steam concentrations in the feed are used, extraction can be dispensed with. In the interest of minimization of processing, this design will be employed, but it does require recycle.

Decision: Separate by distillation, do not use solvent extraction, but recycle a portion of the off gas.



PROBLEM 4-15 (continued-2)

Flowsheet. The distillation train must remove water, and recover and purify acetic and acrylic acids. Water and the organics form a heterogeneous azeotrope. The in order of increasing normal boiling points, in °C, is:

non-condensables, mainly nitrogen	
acrolein	52.5
water	100
acetic acid	118
acrylic acid	141

To prevent reactions, especially polymerization of the acrylic acid, the temperature in the distillation train should be kept below 100°C. At 100°C the vapor pressure of acrylic acid is 24 kPa.

Decision: Operate the distillation columns at about 16 kPa (90°C b.p. for acrylic acid). Remove water in the first column.

The overhead from the first column, C-1 contains the components more volatile than acetic acid. Following condensation, the two-phase liquid plus gas goes to a tank where sufficient holding time allows separation into a gas, an aqueous phase, and an organic phase. The gas is vented, part of it to be recycled and the rest sent to a flare. The organic phase is completely refluxed to the column. The aqueous phase is a waste stream, but it probably should be sent to a stripping column to reduce the organic content, and the organics sent back to column C-1. An aqueous stripping column is not shown in the flowsheet on the next page.

The remaining organic phase consists of acetic and acrylic acids, and less volatile components. Following the heuristic to remove largest quantities first, the second column, C-2, should separate into two streams, predominantly acetic and acrylic acids.

Decision: Next separate between acetic and acrylic acids.

Acetic acid is the distillate product, and should be obtainable in adequately pure form to meet specifications. The acrylic acid bottom product contains nearly all of the heavier components from the reactions. It needs to be distilled in order to meet specifications, as suggested by the heuristic to finally recover product streams as distillate products, and this is done in column C-3.

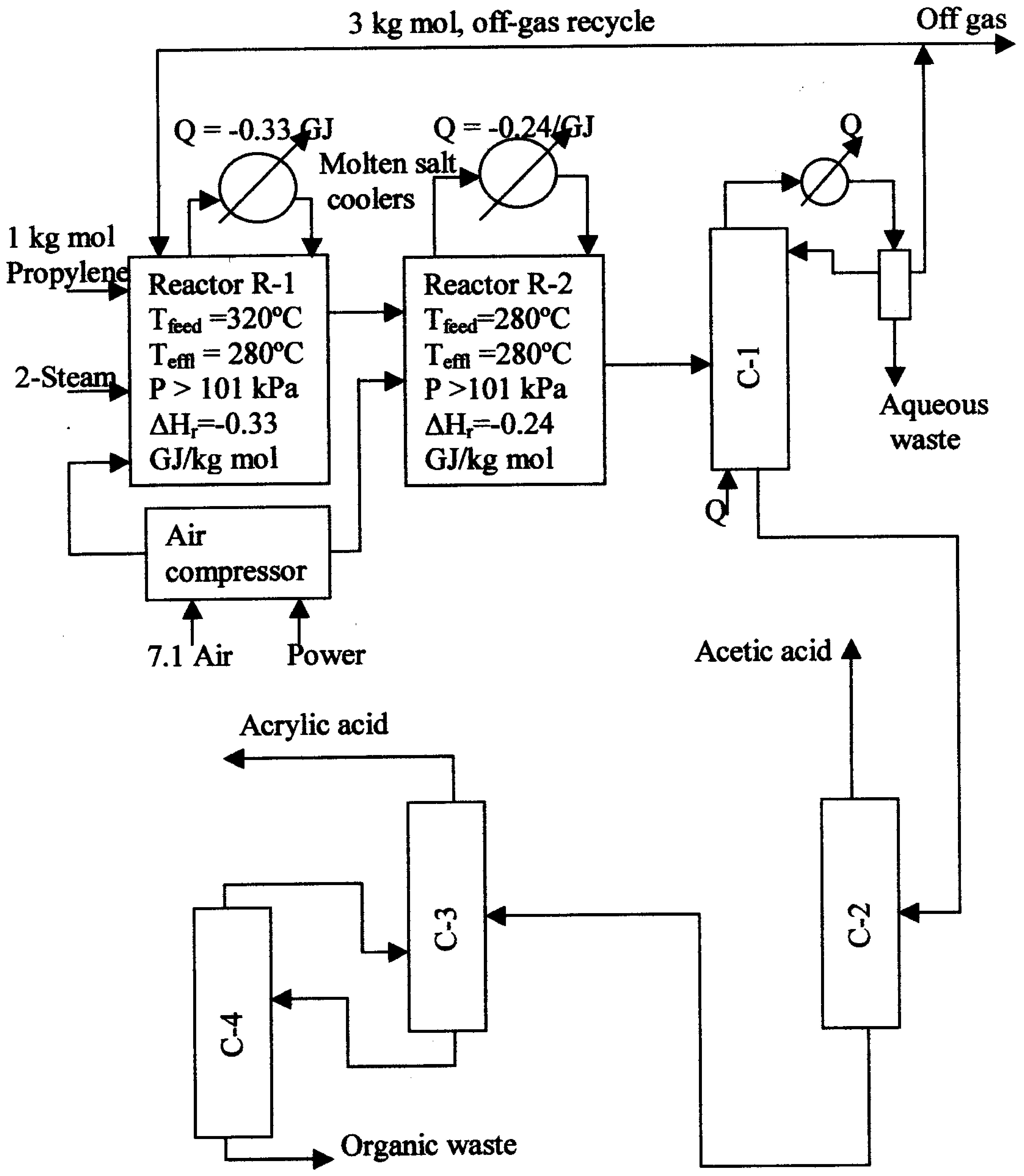
Decision: Distill the acrylic acid and recover it as a distillate product.

The bottom product from the acrylic acid column, C-3, is an organic liquid waste, but it is likely to still contain enough acrylic acid to make it worth while to send it to column C-4 to strip the acrylic acid. The acrylic acid is then recycled to the acrylic acid column.

Decision: Strip the C-3 bottom stream to recover acrylic acid values.

PROBLEM 4-15 (continued-3)

The flowsheet obtained is shown below.



PROBLEM 4-15 (continued-4)

d) The heat sources in the process are the two exothermic reactions. The column condensers are heat sources, but at temperature too low to be of much interest. The heat sinks are the distillation column reboilers. The temperatures and heat duties should be compared to see if some matching can be done, and surely some can. Generating steam in the coolers for the molten salts streams is one manner of heat integration, but it may be possible to use the molten salt streams directly as heat sources. Using the molten salt conserves some heat, because there is a temperature drop of the order of 25°C associated with every heat exchange. See Problem 4-6 for an illustration of heat integration with distillation.

e) A simulation can be performed using one of the overall process simulators listed in Appendix F, including Aspen Plus, CHEMCAD, Hysis and others.

In order to perform the simulation, components and the physical property package must be specified. The main components of this process are water, air, propylene, acrolein, acrylic acid, acetic acid, and acrylic acid dimer. The Peng-Robinson method could be used for example.

The operations needed include: two reactors, splitting, mixing, cooling, condensing, recycle and distillation. The propylene and acrolein conversions and product yields are to be specified. These could be calculated if sufficient kinetic data were available. A simplified distillation model, such as the FUG method, could be used. Split and recoveries need to be estimated.

The simulator will calculate material and energy balances. These would be useful in looking for heat integration opportunities.

PROBLEM 4-16

Sources: *Ullman's Encyclopedia of Industrial Chemistry*, 5th ed., *Chemical Market Reporter.*

a) The dominant chemical pathway for vinyl acetate production is from catalytic, gas-phase partial oxidation of ethylene and acetic acid together.

$$C_2H_4 + CH_3COOH + 1/2O_2 \longrightarrow CH_3COOCH{=}CH_2 + H_2O$$

This oxidation can also be done in the liquid phase. "Syngas", CO and H_2, usually from steam reforming of natural gas, can also be the basis for vinyl chloride. This is a multistep pathway through methanol or methyl acetate made from syngas reacting with syngas.

b) Gross economics. Ethylene and acetic acid oxidation. Basis: 1 kg mol of vinyl acetate.

	Price	Amount	Value
Ethylene	\$0.53/kg	28 kg	\$-14.84
Acetic acid	\$1.00/kg	60 kg	\$-60.00
Oxygen	\$0.04/kg	16 kg	\$- 0.64
Vinyl acetate	\$1.30/kg	86 kg	\$111.80
		Net	\$ 36.32

This result looks attractive, and is the same for both the gas- and liquid phase oxidation. This pathway would be even more favorable if the producer also made the acetic acid by the reaction

$$CH_3OH + CO \longrightarrow CH_3COOCH{=}CH_2$$

and made the CO from natural gas or other sources.

Natural gas route to syngas and vinyl acetate. The overall reaction is

$$4CH_4 + 2H_2O \longrightarrow CH_3COOCH{=}CH_2 + 7H_2$$

On a basis of 1 kg mol of vinyl acetate, the gross economics is

	Price	Amount	Value
Methane	\$0.13/kg	64 kg	\$ -8.32
H_2O	\$0.008/kg	36 kg	\$ -0.29
Vinyl acetate	\$1.30/kg	86 kg	\$111.80
Hydrogen	\$0.67/kg	14 kg	\$ 9.38
		Net	\$112.57

This pathway looks exceptionally promising. However, because of the numerous steps, and equilibrium and competing reactions it has not proven to be a commercial success as yet.

c) *Ullman's* describes this rather complex process in considerable detail, so the steps of process development will not be shown here. Rather, the reader is referred to *Ullman's*. The oxygen concentration in the ethylene-acetic acid

PROBLEM 4-16 (continued-1)
oxidation reactor feed must be carefully limited to less than 8 volume %. This concentration is only about half of the stoichiometric amount in the equation. This low concentration is required to stay below the lower combustion limit of the gas mixture, primarily for safety reasons. However, it also restricts the oxidation of ethylene. The ethylene conversion is typically on about 10% per pass through the reactor, as excess ethylene is used. As a result of very specific catalysts, the selectivity to of ethylene to vinyl acetate is as high as 94 percent, and 99% based on acetic acid, the more expensive reactant.

Vinyl acetate is a flammable, reactive liquid. It can be stored in steel, aluminum, or stainless steel containers under nitrogen. For shipping the same materials are used, and a stabilizer, such as hydroquinone, is added. The containers must have Symbol F flammable liquid labels.

d) Based on current industrial practice, the ethylene/acetic acid pathway is the most promising when all factors are considered. This is probably due to raw materials costs and, perhaps more importantly, the cost of processing. The combination of processes required for the syngas route is complicated, yields are not so high, and it does not appear to be economically competitive at the present time. The potential of the syngas route is high, as indicated by the preliminary economics, and it is worthy of further attention and development.

e. See problem 4-15 solution for a discussion of using a simulation program to produce a PFD for this problem.

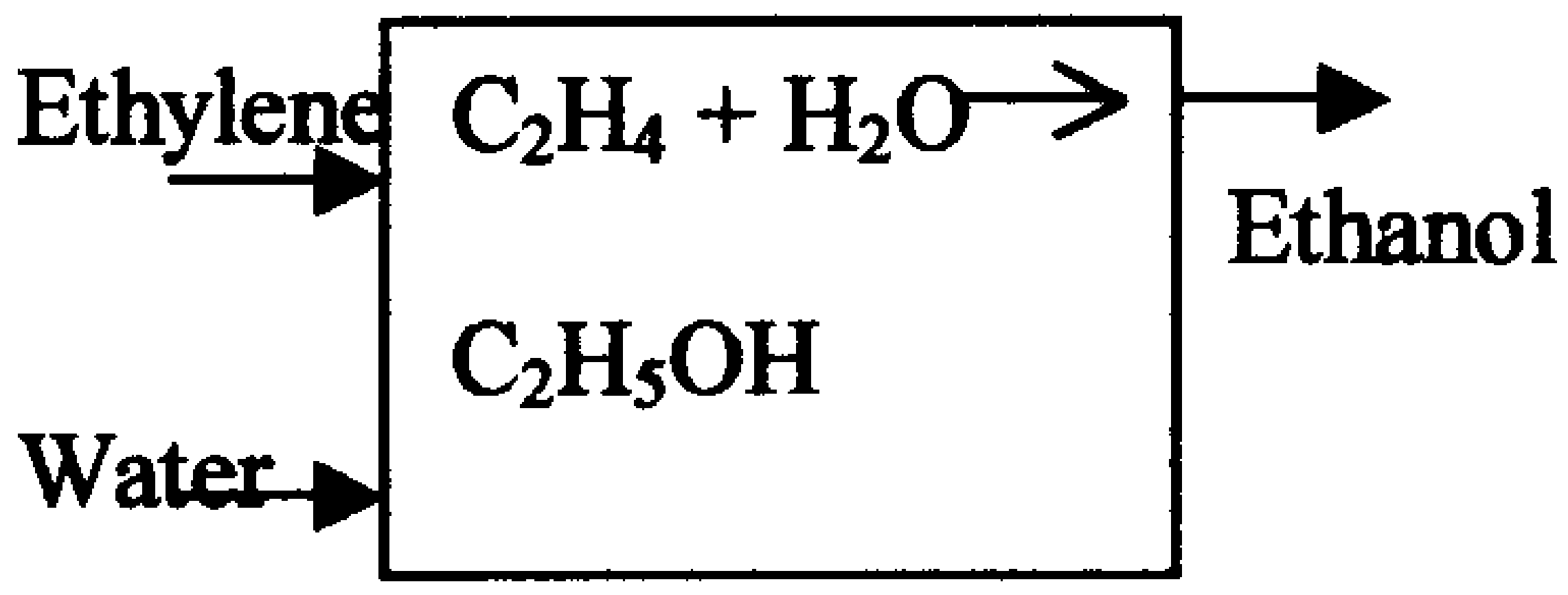
PROBLEM 4-17

Source: *Kirk-Othmer Encyclopedia of Chemical Technology,* 4th ed.

Industrial ethanol can be produced synthetically by reaction of ethylene and water, or by fermentation of a suitable feed. Ethanol and corn prices and data from www.icislor.com and www.ohiocorn.org.

Ethylene pathway. Ethylene and water can be reacted in the liquid or gas phase at elevated temperature and pressure with a catalyst. The gas-phase reaction is used most often.

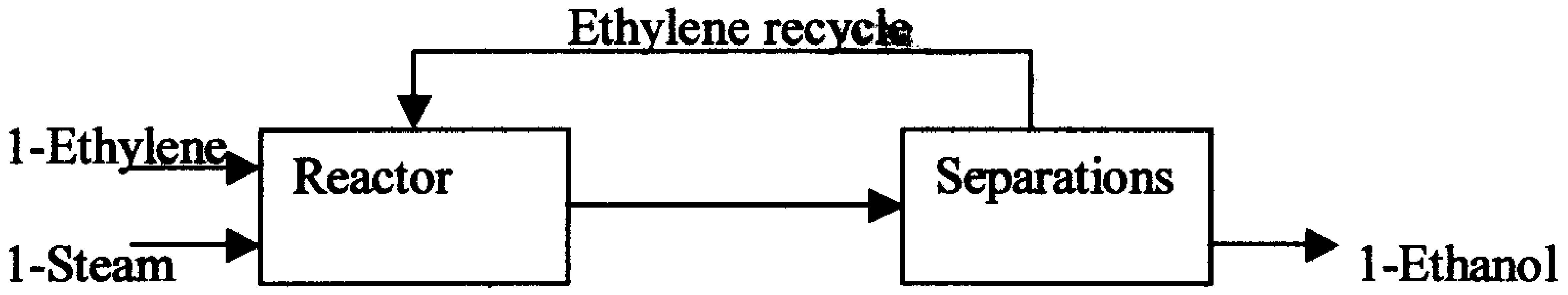
I/O Basis: 1 kg mol ethanol



	Price/kg	Amount	Cost
C_2H_4	\$0.53	28 kg	\$-14.84
H_2O	\$0.008	18 kg	\$ -0.15
C_2H_5OH	\$0.84	46 kg	\$38.64
		Net	\$23.65

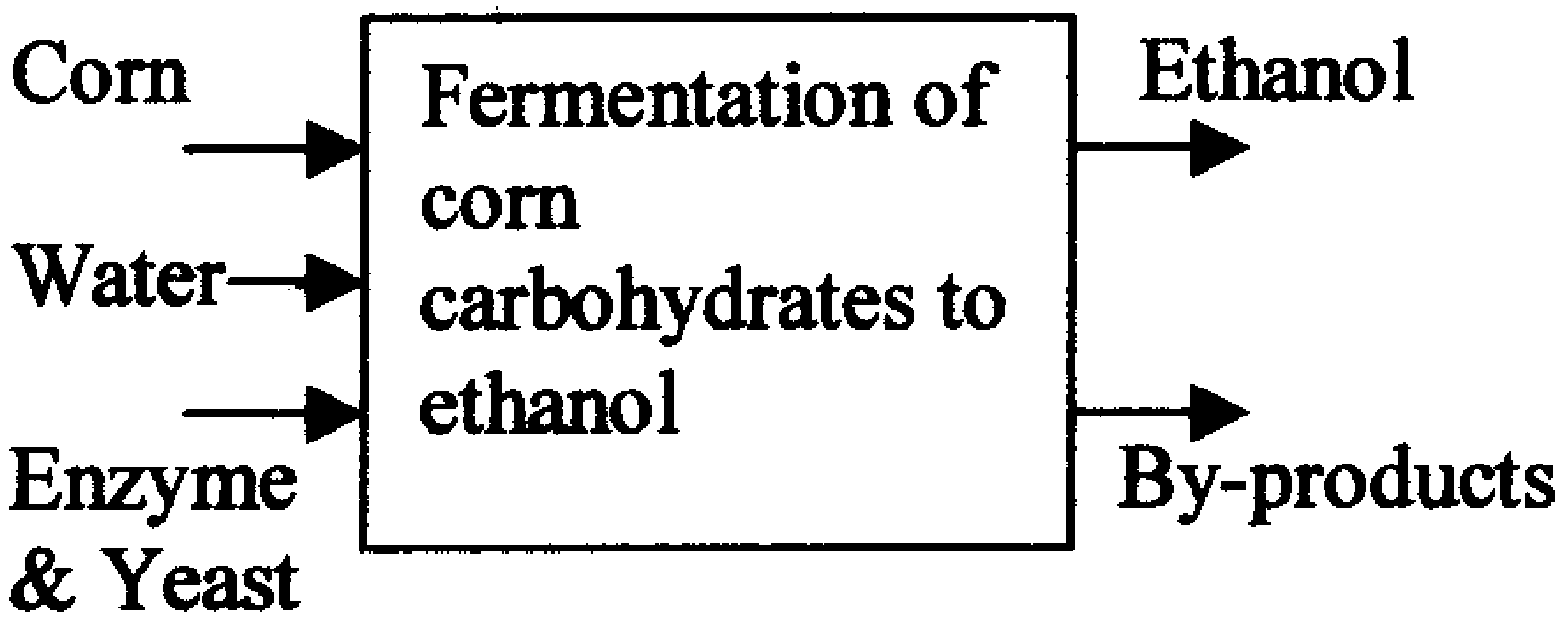
This process looks economically favorable.

Functions



Ethanol from carbohydrate. Ethanol can be produced by fermentation of carbohydrates from a wide variety of sources; fermentation is one of the oldest of human-mediated chemical reactions. In the U.S., corn is the most widely utilized carbohydrate source for this fermentation when industrial/fuel ethanol is to be the product, so this comparison will use corn as the raw material.

I/O Basis: 1 bushel (~56 lb, ~25 kg) corn yields ~2.5 gallons (~7.5 kg of ethanol)



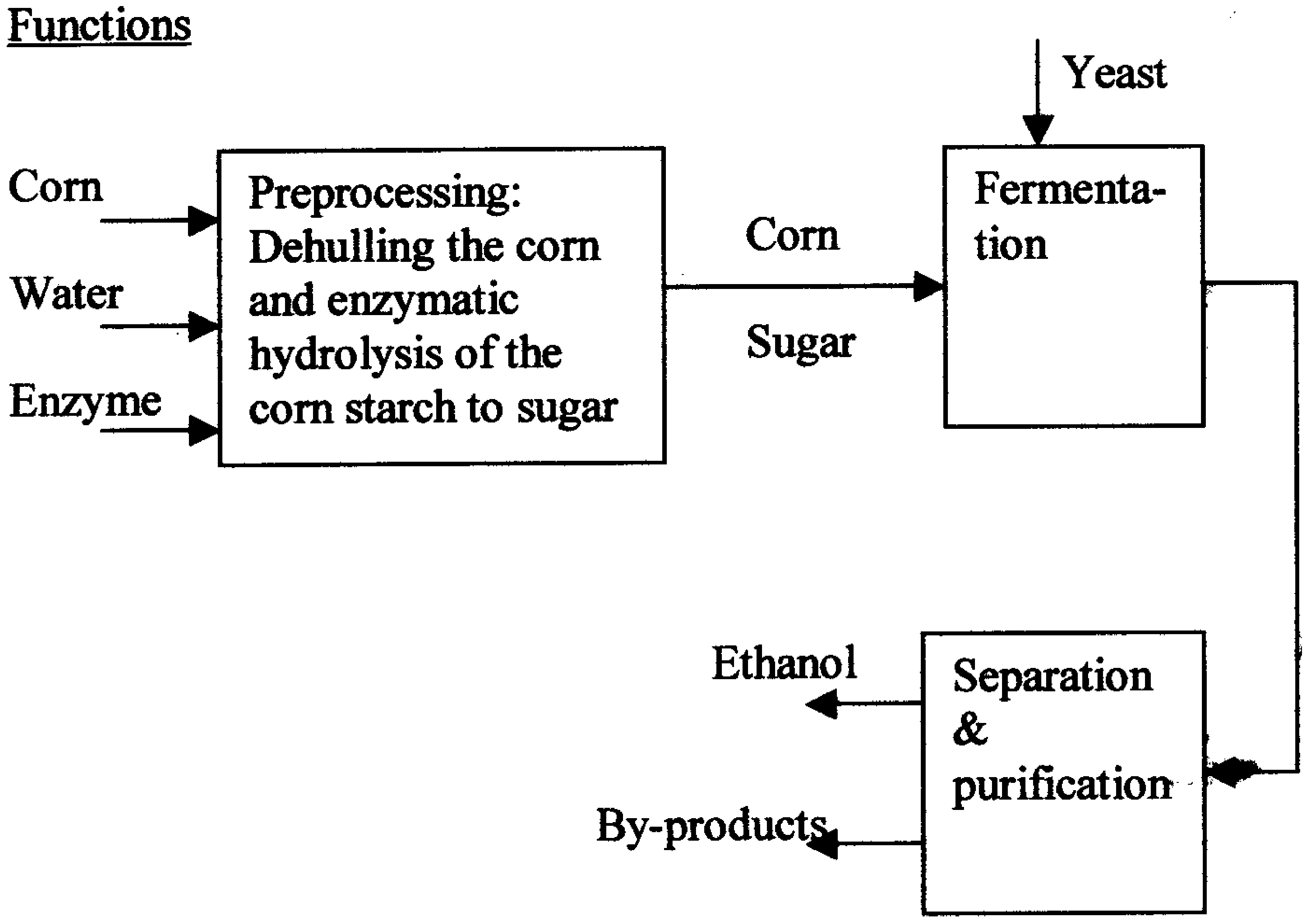
	Price/kg	Amount	Value
Corn	\$0.08	25 kg	\$-2.00
Ethanol	\$0.33	7.5 kg	\$ 2.48
		Net	\$ 0.48

(~\$2.95/kg mol ethanol)

PROBLEM 4-17 (continued-1)

This process looks economically favorable, though not nearly as favorable as the ethylene process. Moreover, the processing is more complex. Several by-products, including animal feed are produced. There is a currently a U.S. government subsidy which helps the operation be profitable. The fermentation step of the process is usually batch, but the rest can be continuous.

Functions

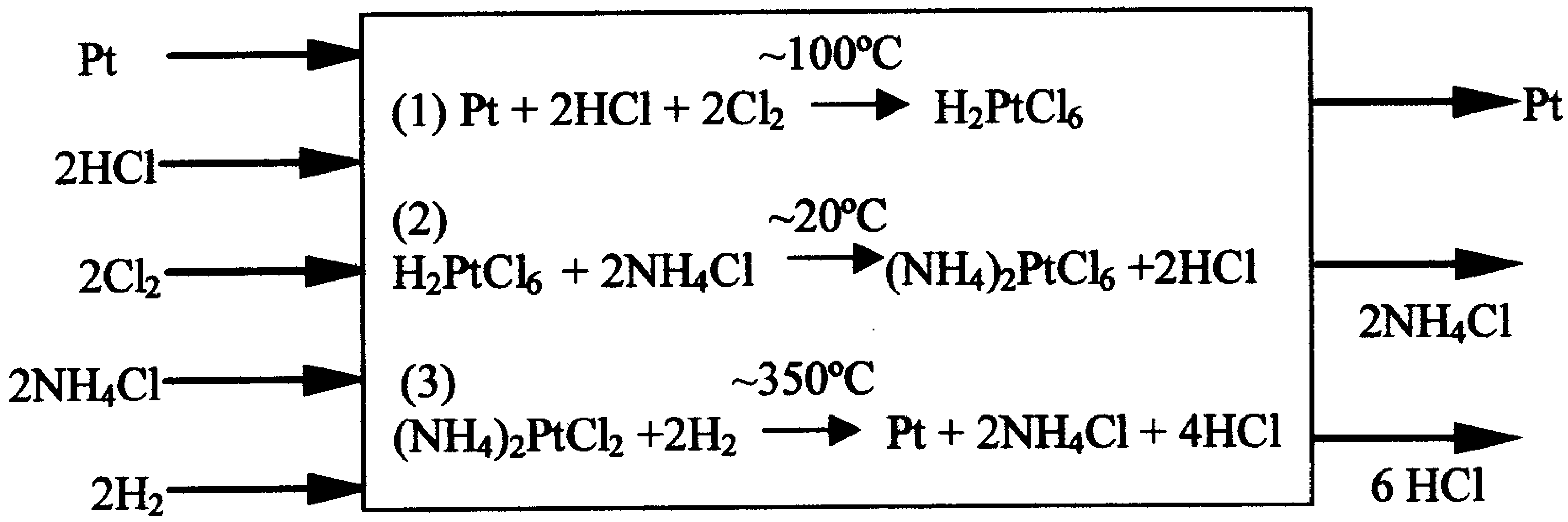


PROBLEM 4-18.

NOTE CORRECTION: There should be 2-g of metal per unit, not 0.5 kg.
Sources: *Kirk-Othmer Encyclopedia of Chemical Technology*, 4th ed., and *Ullman's Encyclopedia of Industrial Chemistry.*

Input-output and preliminary economics.

Platinum (and other Pt-group metals, primarily Palladium) can be dissolved from the ceramic, after crushing it, in strong oxidizing acids, such as 6M aqueous hydrochloric acid plus chlorine. The Pt can be precipitated by adding ammonium chloride. The ammonium chloride can be decomposed to Pt powder by heating in the presence of hydrogen.



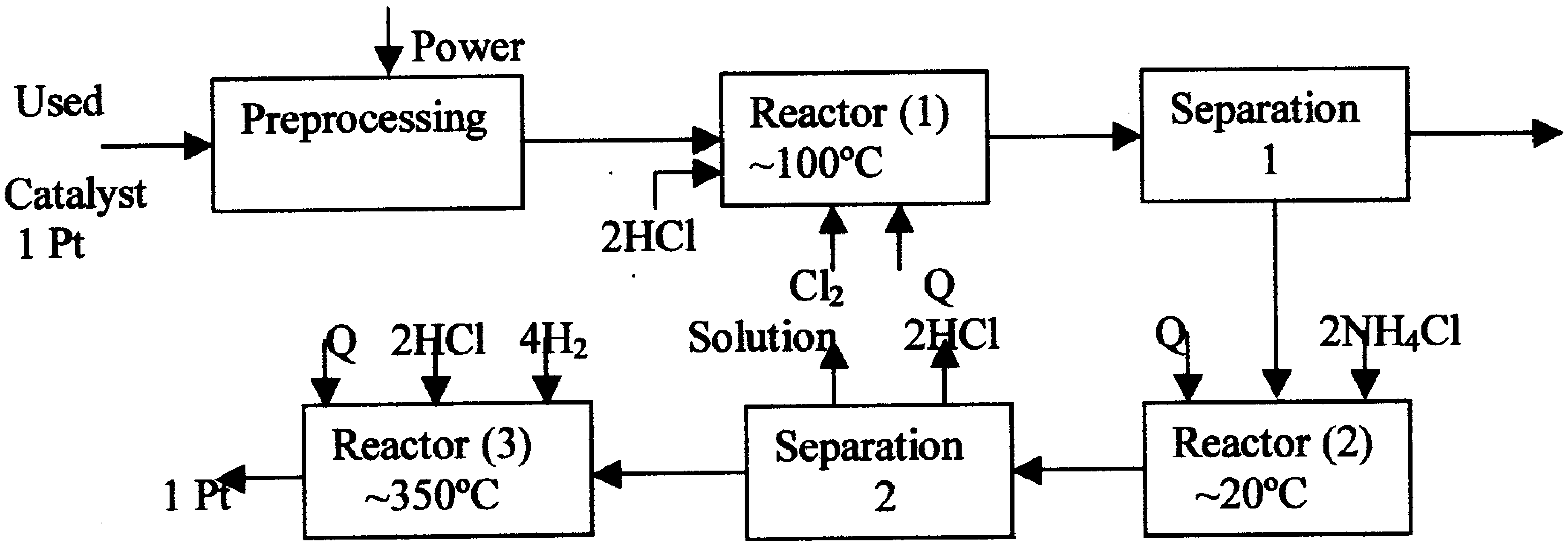
Prices are from *Chemical Market Reporter, Kirk-Othmer, Ullman's*, www.camfordinfo.com, www.chemexpo.com and www.kitco.com, and are time averaged.

Product values: Pt @ \$16/g = 195*16 = \$3,120. NH_4Cl is potentially recoverable, but ignore value. HCl potentially recoverable as HCl and Cl_2, but ignore value

Raw materials' values: Used Pt catalyst on ceramic monolithic is a waste product. However, collecting it for processing may be the most difficult and expensive step of recovery. Nonetheless, it will be considered valueless. For HCl @ \$0.27/kg, value= 0.27*2*36.5 = \$19.7. With Cl_2 @ \$0.21/kg, value = 0.21*2* 71 = \$29.8. Ammonium chloride @ \$0.8/kg has a value of 0.8*2*53.5 = \$85.6. The value of hydrogen @ \$5/kg = 5*2*2 = \$20.0. So the total value of the reactants is \$155.1, almost negligible compared to the potential value of the recovered Pt. However, reprocessing, collection, and other costs have hampered the development of recycling for this material.

PROBLEM 4-18 (continued-1)

Functions diagram. Because of collection difficulties, the capacity at any one site is likely to be small, and because of the difficult and hazardous nature of the processing, it is recommended that the process be batch. A preprocessing step including crushing is needed to prepare steel jacketed catalytic converters for Pt recovery. A reaction (dissolution) step is followed by separation of the crushed ceramic from the solution by filtration. A second reaction step precipitates the Pt as $(NH_4)_2PtCl_6$. The precipitate is filtered from the solution and then heated (calcined) to decompose the solid to Pt metal. The metal is washed. Finishing may be required to obtain powder of the right properties.



PROBLEM 6-1

CORRECTION: The 100 m² in the first sentence should be 10 m².

Use Eqn. 6-2 with the appropriate exponent.

$$\text{1990 Cost for } 20\,m^2 = \$4{,}200\left(\frac{20}{10}\right)^{0.6} = \underline{\underline{\$6{,}370}}$$

ANS.

$$\text{1990 Cost for } 100\,m^2 = \$6{,}370\left(\frac{40}{20}\right)^{0.6}\left(\frac{100}{40}\right)^{0.81}$$

$$= \$20{,}268.$$

Estimate the 2000 cost using an index from Table 6-2.

Using the Marshall & Swift process-industry installed equipment index,

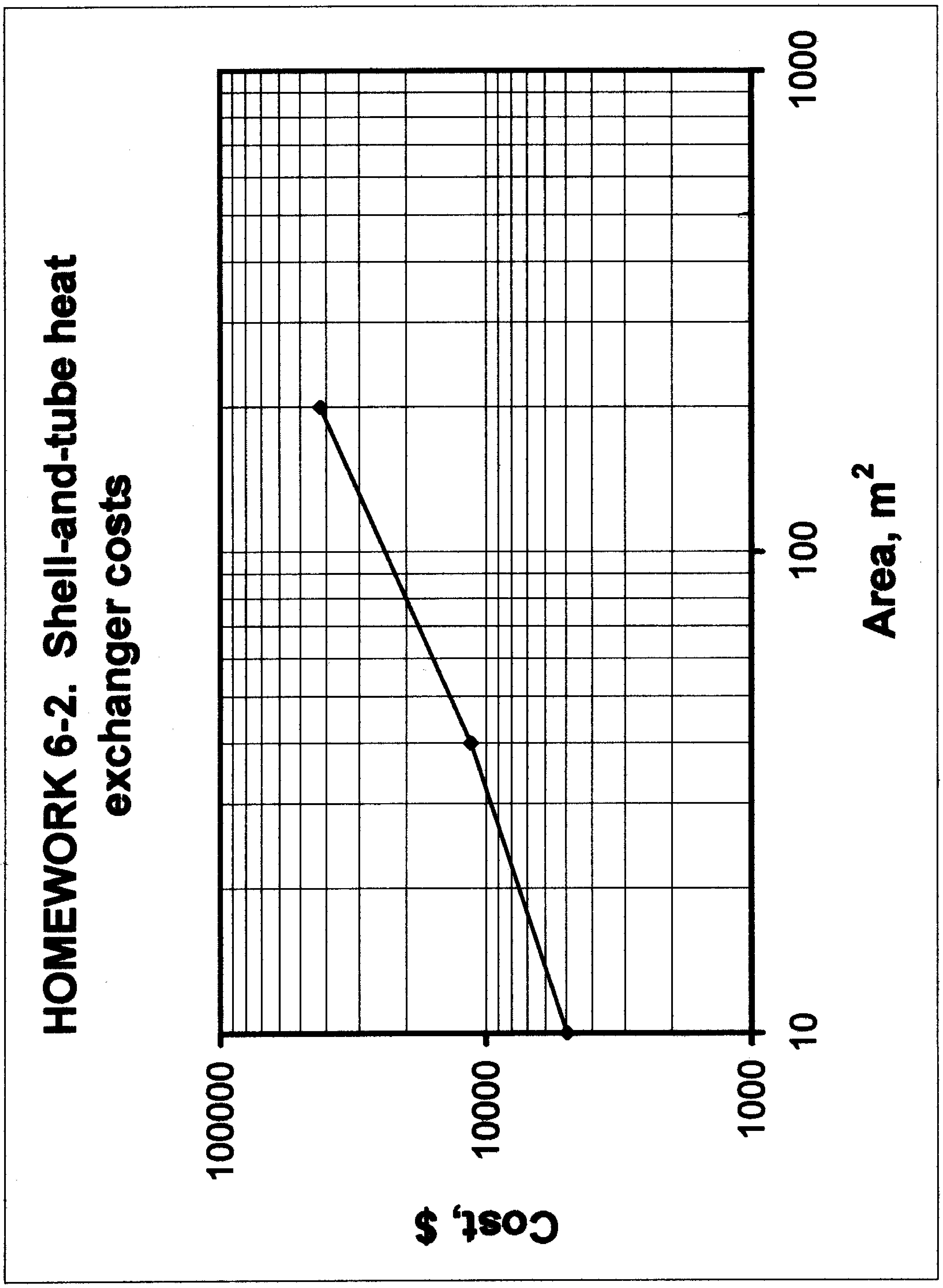
$$\text{2000 cost for } 100\,m^2 = (20{,}268)\left(\frac{1097.7}{929.3}\right) = \underline{\underline{\$23{,}900}}$$

ANS.

For comparison, using the Chemical Engineering Plant Cost index gives

$$\text{2000 cost for } 100\,m^2 = (20{,}268)\left(\frac{394.1}{357.6}\right) = \$22{,}300$$

PROBLEM 6-2


HOMEWORK 6-2. Shell-and-tube heat exchanger costs
Cost, $
100000
10000
1000
Area, m^2
10
100
1000

PROBLEM 6-3

Use Eq. 6-2 with appropriate exponent. Although it is out of range, in the absence of other information, assume that the 0.93 exponent can be used down to 45,000 kg. Then apply the Marshall & Swift installed equipment cost index from 1990 to 2000 (see Table 6-2 for index values).

Installed cost for 300,000 kg tank in 2000 $= \$640{,}000\left(\frac{300{,}000}{45{,}000}\right)^{0.93}\left(\frac{1097.7}{929.3}\right)$

$$= \$\underline{\underline{4.4 \times 10^6}}$$

ANS.

(Note, because of the uncertainties in the method, the value is rounded to 2 digits).

PROBLEM 6-4

Use Eq. 6-2 with appropriate exponent. A value of $4.4×10^6 was obtained in problem 6-3, but the unrounded value should be used in the calculations here and the final result rounded.

$$\$4,410276 = 1.1\times10^6\left(\frac{wt.}{135,000}\right)^{0.88}\left(\frac{1097.7}{929.3}\right)$$

$$wt. = (111,030)^{1/0.88} = \underline{\underline{540,000\ kg}}$$

Note that this value lies outside the stated range for the 0.88 exponent. In the absence of other information, assume the extrapolation is reasonable.

PROBLEM 6-5.

Find tank height and surface area.

$$\text{Vol.} = 5\,m^3 = \frac{\pi D^2}{4} H \;;\; D = 2\,m,\; H = \text{height}.$$

$$H = (4)(5)/(2^2\pi) = 1.59\,m$$

$$\text{Surface Area} = 2\pi D^2/4 + \pi D H$$

$$= 2\pi(4/4) + 2\pi\,(1.59) = 16.27\,m^2$$

The purchased cost of the tank, in 1995 = \$10,900. The installation cost for metal tanks is in the range 30 to 60 %, part of which is for insulation. Select a value, say 40%, without insulation.

1995 installed cost without insulation =

$$(1 + 0.4)\$10,900 = \$15,260$$

The 1995 insulation cost is

$$= \$(40 + 95)/_{m^2}\;(16.27)\,m^2 = \$2,200.$$

Take current = 2000, and use the M&S installed equipment index to update the costs.*

$$\text{2000 cost of tank} = (15,260 + 2,200)\left(\frac{1097.7}{1029.0}\right)$$

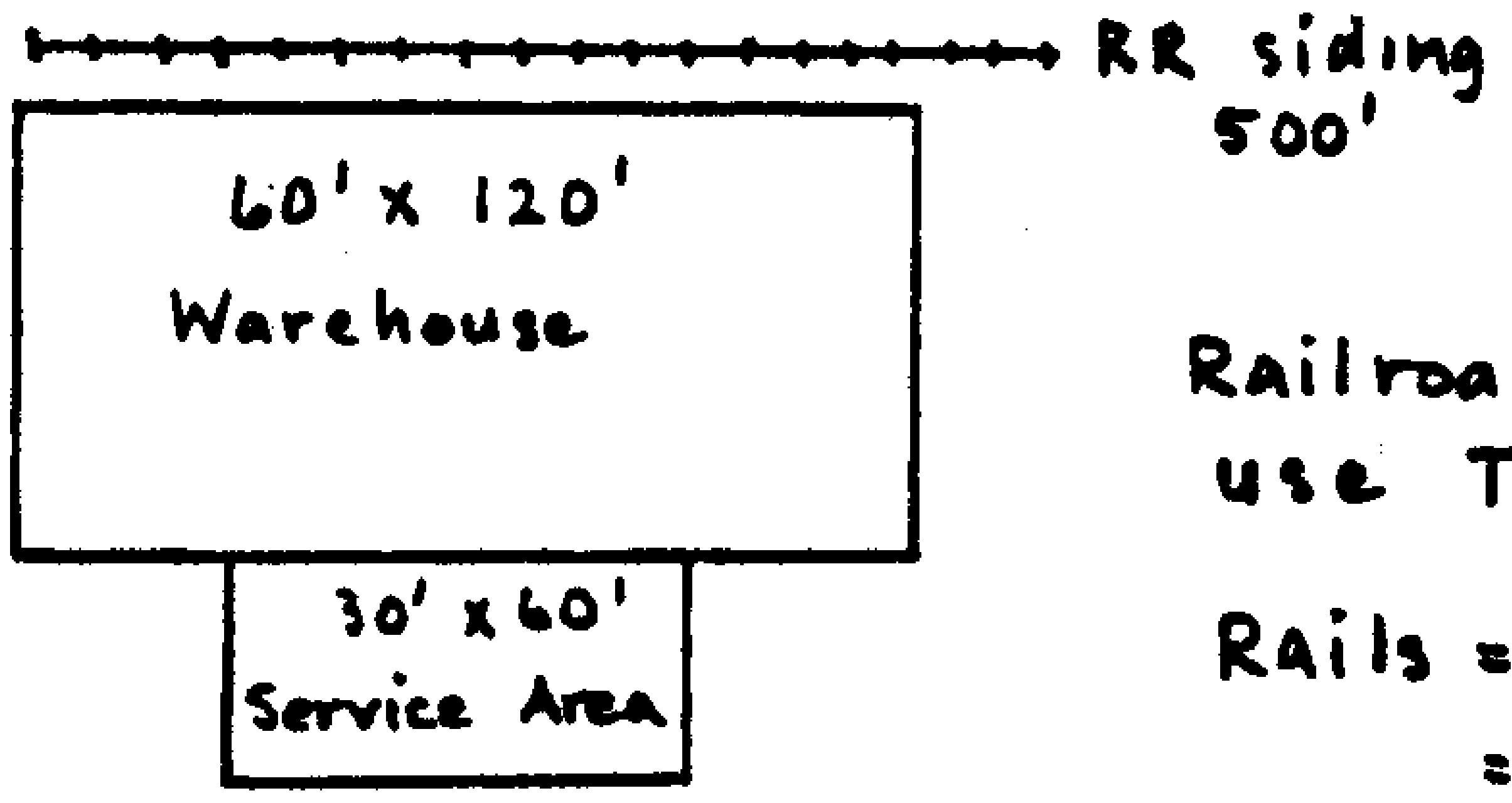
installed and insulated

$$= \$\,\underline{\underline{18,600}}\;\text{ANS.}$$

*It might be preferable to use materials and labor indexes for the insulation if available.

PROBLEM 6-6

The necessary costs can be found in the two tables following this solution (Tables 2 and 3, Appendix B, 4th edition of Peters and Timmerhaus.



Basis: Jan. 1990 cost figures

Railroad siding costs - use Table 3 - Appendix B

Rails = (500 ft)(\$63 / lin. ft)
= \$31,500 (90 lb Track)

Assume necessity of turnout cost

Turnout cost = \$17,030

Total railroad siding cost = \$ 48,530

Warehouse costs - use Table 2 - App. B

Steel frame, masonry walls, floor and roof, heating, lighting and plumbing - \$28/ft²

Cost = (\$28/ft²)(60 ft)(120 ft) = \$201,600

(heating was not required, but is included in cost)

Sprinkler costs - use Table 2 - App. B

Dry system, no heat - \$2.10/ft²

Cost = (\$2.10/ft²)(60ft)(120ft) = \$15,120

Asphalt for service area - use Table 3 - App. B, \$12.90/yd²

Cost = \$12.90/yd²)(60ft/3)(120ft/3) = \$10,320

Total cost of proposed addition

48,530 + 201,600 + 15,120 + 10,320 = \$ 275,500 <u>Answer</u>

Problem 6-6 (continued)

TABLE 2
Building and construction costs (Jan. 1990)

Item	Unit	Cost, $	Employee—hr to install
Floors			
Asphalt tile	ft^2	1.40	†
Concrete, prestressed, 4-in. thick	ft^2	10.70	†
Steel grating	ft^2	27.81	†
Wood deck, 2-in. thick	ft^2	3.70	†
Foundations, includes excavation, backfill, and forming: flat slab, 1 yd^3 concrete, 5.3 ft^2 forms, 100 lb reinforcing steel‡	yd^3	223	6
Pits and basins: 1 yd^3 concrete, 17.5 ft^2 forms, 115 lb reinforcing steel‡	yd^3	285	8
Walls and piers: 1 yd^3 concrete, 57.0 ft^2 forms, 160 lb reinforcing steel‡	yd^3	593	16
Lumber			
Structural, plain	MBF§	910	30
Structural, creosoted	MBF	1,135	30
Plywood (exterior unsanded):			
$\frac{3}{8}$-in.	ft^2	0.32	1
$\frac{1}{2}$in.	ft^2	0.43	1
$\frac{5}{8}$-in.	ft^2	0.46	1
$\frac{3}{4}$-in.	ft^2	0.54	1
Piling (20–25 ton load, 60 ft long)			
Wood, treated	Ea.	670	†
Wood, untreated	Ea.	570	†
Composite	Ea.	1,180	†
Concrete	Ea.	4,300	†
Test pile	Ea.	8,500	†
Load test	Ea.	10,200	†
Equipment "on and off site"	Per job	14,800	†
Roofs			
Aluminum, corrugated, 0.032-in. thick	ft^2	2.32	†
Built-up, 5-ply	ft^2	0.91	†
Reinforced concrete, 4-in. thick	ft^2	10.70	†
Steel, 26 gauge	ft^2	1.44	†
Transite, $\frac{3}{8}$-in. thick	ft^2	2.47	†
Sprinkler systems, exposed. Add 28% for concealed systems			
Wet system	ft^2	1.65	†
Dry system	ft^2	2.10	†
4-in. alarm valve, wet system	Ea.	1,080	40
4-in. alarm valve, dry system	Ea.	1,650	50
6-in. alarm valve, wet system	Ea.	1,290	40
6-in. alarm valve, dry system	Ea.	1,800	50
Compressor to operate 500 heads	Ea.	980	16
(Valves and compressor not included in ft^2 price)			
Structural steel			
Grating, $1\frac{1}{4}$-in. standard	ft^2	10.18	0.33

†Labor cost is included in material cost.
‡To adjust: forms, \$3.25/ft^2; reinforcing steel, 44¢/lb.
§MBF refers to 1000 board feet.

(Continued)

Problem 6-6 (Continued)

TABLE 2
Building and construction costs (Jan. 1990) (*Continued*)

Item	Unit	Cost, $	Employee—hr to install
Grating, expanded metal	ft^2	4.80	0.17
Grating, checker plate, $\frac{5}{16}$-in.	ft^2	6.80	0.13
Handrail, standard $1\frac{1}{2}$-in. pipe, 2 rails, welded	Lineal ft	22.60	†
Handrail, bar type	Lineal ft	7.60	0.70
Steel ladder, with safety cage (Add 25% for aluminum)	Lineal ft	76.00	†
Steel ladder, without safety cage (Add 25% for aluminum)	Lineal ft	48.30	†
Operating platforms, including stairs	ft^2	42.40	1.8
Stairway	Vert. ft	91.60	
Stair treads, 12-in. wide, galvanized	Ea.	30.50	1
Building steel, shop fab	lb	0.64	0.01
Platform and support steel, shop flab	lb	0.80	0.02
Toeplate steel, 4 in. × $\frac{1}{4}$-in.	Lineal ft	5.40	†
Walls			
Siding			
Transite	ft^2	2.60	†
Aluminum	ft^2	2.20	†
Steel-coated	ft^2	1.80	†
Brick			
4-in.	ft^2	5.50	†
8-in.	ft^2	9.50	†
10-in.	ft^2	11.60	†
12-in.	ft^2	13.70	†
Concrete block, reinforced			
6-in.	ft^2	5.25	†
8-in.	ft^2	5.50	†
12-in.	ft^2	8.10	†
Windows, industrial			
Steel, fixed	ft^2	16.30	†
Aluminum	ft^2	12.40	†
Wood	ft^2	7.30	†
Total cost of erected buildings (median values)			
Laboratory: steel frame, masonry walls, floor and roof; heating, lighting, and plumbing	ft^2	91	†
Office: steel frame, masonry walls, floor, and roof; heating, lighting, and plumbing	ft^2	62	†
Process building: multilevel, 12-ft clearance, steel platforms, heating, lighting, and plumbing			
Masonry construction	ft^2	41	†
Aluminum on steel	ft^2	43	†
Transite on steel	ft^2	32	†
Open structure: 3-level, steel, with lighting and plumbing	ft^2	29	†
Warehouse: single story, 15-ft clearance. Steel frame, masonry walls, floor, and roof; heating, lighting, and plumbing	ft^2	28	†

†Labor cost is included in material cost.

Problem 6-6 (Continued)

TABLE 2
Building and construction costs (Jan. 1990) (*Continued*)

Item	Unit	Cost, $	Employee—hr to install
Doors			
Metal: Steel frame, 8 × 8 ft, automatic	Ea.	2,085	30
Steel rolling, 12 × 12 ft, manual	Ea.	1,320	24
Swing, with frame, 3 × 7 ft, $1\frac{3}{4}$-in. thick	Ea.	228	†
Wood: Sectional, overhead, 12 × 12 ft	Ea.	1,369	†
Swing exterior, with frame, 3 × 7 ft	Ea.	195	†
Excavation			
Machine	yd^3	4–7	†
Hand	yd^3	23–44	†

†Labor cost is included in material cost.

TABLE 3
Costs for yard improvements (Jan. 1990)

Item	Unit	Cost, $	Employee—hr to install
Docks and wharfs			
All concrete, 100 lineal ft wide	Lineal ft	33,200	†
Timber, with wood deck, 100 lineal ft wide	Lineal ft	10,100	†
Dredging	yd^3	4.20	†
Pipe bridges (includes structural steel and foundations)			
Heavy duty	Lineal ft	65	2.5
Light duty	Lineal ft	41	1.8
Pipe column, steel, extra heavy			
16 ft-4 in. diameter	Ea.	126	1.2
16 ft-6 in. diameter	Ea.	260	1.4
22 ft-6 in. diameter	Ea.	350	1.9
Plant fence			
6 ft chain link (3-strand B.W.)	Lineal ft	7.78	†
3- to 4-ft wide person gate	Ea.	200	†
8-ft wide equipment gate	Ea.	330	†
30-ft wide double gate (manual)	Ea.	664	†
20-ft wide gate (automatic)	Ea.	4,500	†
Relocate plant fence	Lineal ft	5.40	†
Railroads			
Track (90 lb)	Lineal ft	63	†
Track (75 lb)	Lineal ft	48	†
Turnout	Ea.	17,030	†
Ties, creosoted (6 in. × 8 in. × 8 ft)	Ea.	38	†
Grade and ballast	Lineal ft	38.50	†
Cars, ore, 24-in. track			
Capacity, ft^3 — Wt, lb			
12 — 660	Ea.	1,040	†
16 — 700	Ea.	1,110	†
20 — 930	Ea.	1,240	†

†Labor cost is included with material cost.

(*Continued*)

Problem 6-6 (continued)

TABLE 3
Cost for yard improvements (Jan. 1990) (*Continued*)

Item	Unit	Cost, $	Employee—hr to install
Locomotives, mine, battery type			
Size, tons Wt, lb			
9 19,000	Ea.	102,000	
12 26,000	Ea.	117,000	
Locomotive, mine, Diesel type			
1½ 3,000 2 cyl	Ea.	31,600	
3 7,000 4 cyl	Ea.	39,400	
Roads and walkways			
Concrete, slab, mesh reinforced			
Thickness, in. Sub-base, in.			
4 6	yd²	21.50	†
6 6	yd²	21.80	†
8 6	yd²	26.50	†
Paving asphalt			
3 12	yd²	12.90	†
Sewers			
Reinforced concrete, thickness, in. (installed)			
12	Lineal ft	20.90	†
15	Lineal ft	23.30	†
18	Lineal ft	26.10	†
24	Lineal ft	36.00	†
30	Lineal ft	47.40	†
Vitrified clay, thickness, in. (installed)			
4	Lineal ft	14.00	†
6	Lineal ft	15.90	†
8	Lineal ft	19.50	†
12	Lineal ft	30.25	†
15	Lineal ft	38.80	†
18	Lineal ft	48.70	†
24	Lineal ft	71.40	†
Site development			
Clearing and grubbing	yd²	0.90	†
Grade out	yd³	2.80	†
Cut, fill, and compact	yd³	4.50	†
New fill compacted	yd³	9.00	†
Topsoil	yd³	14.40	†
Gravel fill	yd³	15.60	†
Crushed stone (¾ in.)	yd³	15.60	†
Seeding	yd²	0.80	†
Sluiceway			
Open, piled, and sheathed	Lineal ft		†
Wells			
200 gpm, 400 ft deep, 15 hp	Ea.	26,000	†
500 gpm, 200 ft deep, 40 hp	Ea.	34,000	†
1200 gpm, 400 ft deep, 75 hp	Ea.	47,000	†

†Labor cost is included with material cost.

PROBLEM 6-7

Use capital investment estimating method C - Percentage of Delivered Equipment Cost, with percentages given for solid processing in Table 6-9.

Purchased equipment cost = \$500,000.

Fixed capital investment $= (3.97)(5 \times 10^5)$
$= \underline{\underline{\$1.99 \times 10^6}}$ ANS.

Total capital investment $= (4.67)(5 \times 10^5)$
$= \underline{\underline{\$2.34 \times 10^6}}$ ANS.

Engineering and supervision = 33% of purchased equip $= (0.33)(5 \times 10^5)$
$= \underline{\underline{\$165,000}}$ ANS.

$= \left(\frac{1.65 \times 10^5}{1.99 \times 10^6}\right) \times 100 = \underline{\underline{8.3\%}}$ of fixed cap. inv.

Contractor's fee = 17% of purchased equip. $= (0.17)(5 \times 10^5) = \underline{\underline{\$85,000}}$ ANS.

$= \left(\frac{8.5 \times 10^4}{1.99 \times 10^6}\right) 100 = \underline{\underline{4.3\%}}$ of fixed cap. inv.

PROBLEM 6-8

Table 6-9 shows the cost components as typical percentages of purchased equipment cost, E. However, two of these, buildings and contractor's fee are to change. The problem says that building costs will be high. From Table 6-3 select from the high end of the building range, say 18% of fixed capital investment, FCI.

The total direct plant cost is the sum of the items above it. Thus

$$\text{Direct} = (273/100)E + (18/100)FCI$$

The Contractor's fee is to be 7% of direct costs. The total indirect cost is the sum of the items engineering through contingency. Thus

$$\text{Indirect} = (32+34+4+37)E/100 + (0.07)[(273/100)E + (18/100)FCI]$$

$$FCI = \text{Direct} + \text{Indirect}$$

$$= (2.73)E + (0.18)FCI + 1.07E + + (0.07)((2.73)E + (0.18)FCI)$$

From which

$$FCI - 0.1926\,FCI = 1.197 \times 10^6$$

and

$$FCI = \underline{\$1.483 \times 10^6} \quad \text{ANS.} \quad (= 4.94E)$$

$$\text{TOTAL DIRECT COST} = \underline{\underline{\$1.086 \times 10^6}} \quad \text{ANS.} \quad (= 3.62E)$$

$$\text{WORKING CAP.} = (.75)(E) = \$225{,}000$$

$$TCI = FCI + WC = \$\,\underline{\underline{1.708 \times 10^6}} \quad (= 5.69E)$$

ANS.

PROBLEM 6-9

Using cost and capacity values from Table 6-11 and Eq. 6-11, the turnover ratio, TOR, is

$$TOR = \frac{\text{sales revenue, \$/yr}}{\text{capital investment, \$}} =$$

$$= \frac{(9 \times 10^7\ \text{kg/yr})(86/10^3\ \text{\$/kg})}{\$\,4 \times 10^6} = 1.935\ \text{yr}^{-1}$$

$$\therefore\ FCT = \frac{(1.3 \times 10^8\ \text{kg/yr})(\$86/10^3\ \text{kg})}{1.935\ \text{yr}^{-1}}$$

$$= \underline{\underline{\$\,5.78 \times 10^6}}$$

ANS.

Using the plant capacity ratio Eq. 69, exponent from Table 6-11 = 0.65.

$$FCI = \$4 \times 10^6 \left(\frac{1.3 \times 10^8}{9 \times 10^7}\right)^{0.65}$$

$$= \underline{\underline{\$\,5.08 \times 10^6}}$$

ANS.

Chap. 6 - Prob. 10

Use method G in Chapter 5

F.C.I. = total capital investment - working capital

$= \$1,000,000 - 100,000 = \$900,000$

$$\text{Turnover ratio} = \frac{\text{gross annual sales}}{\text{fixed capital investment}} = 1.0$$

$$\$/\text{kg selling price} = \frac{\text{fixed capital investment}}{\text{annual production}}$$

$$= \frac{\$900,000}{(8000)(365)} = \$0.308$$ Answer

PROBLEM 6-11

Using rate and productivity factors from Table 6-12

$$\text{1990 labor cost in So. Atl.} = \$425{,}000\left(\frac{0.84}{1.06}\right)\left(\frac{0.96}{0.91}\right)$$

$$= \$355{,}300$$

To update this to 1998, use the Engineering News-Record construction index (1967=100 basis)

$$\text{1998 labor cost in So. Atl.} = \$355{,}300\left(\frac{551}{441}\right)$$

$$= \underline{\underline{\$444{,}000}}$$

ANS.

Note, it would be better to use the location and labor specific indexes. However, these are not readily available, so this construction index, of which labor is a major part, is used as a substitute.

PROBLEM 6-12

Basis: 50 kg of 70% soap solution.

Cost delivered to customer, 50kg,

$$= \$20 + (50)(\$1.50/50)$$

$$= \$21.50$$

The 95% solution must result in the same amount of soap, i.e. (50)(0.7) = 35 kg, and the total delivered cost.

Amt. of 95% solution $= 35/0.95$

$= 36.82$ kg

let $p =$ price per 50kg of 95% solution, then $p/50 =$ price per kg, and

$$21.5 = 36.82\left(p/50 + 1.5/50\right)$$

thus

$$p/50 = 21.5/36.82 - 1.5/50$$

$$= 0.554 \ \$/kg$$

and $p = (50)(0.554) = \underline{\underline{\$27.69/50kg}}$

ANS.

PROBLEM 6-13

The main calculations are performed using the spreadsheet for annual total product cost at 100% capacity, **Annual TPC**, from the book's web site.

Because some of the terms in the sum for TPC depend upon TPC, a key to the solution is expressing the total product cost in equation form as:

$$\text{TPC} = \Sigma(\text{all terms not depending on TPC}) + \Sigma(\text{all terms depending on TPC})$$

$$= \Sigma(\text{all terms not depending on TPC}) + \Sigma(\text{Factor}_i)(\text{TPC})$$

which gives,

$$\text{TPC}[1-\Sigma(\text{Factor}_i)] = \Sigma(\text{all terms not depending on TPC})$$

and,

$$\text{TPC} = [\Sigma(\text{all terms not depending on TPC})]/[1-\Sigma(\text{Factor}_i)]$$

The following calculations are made from the spreadsheet results shown on the next page.

a) Manufacturing cost per kg product = $(\$1.244*10^6/\text{y})/(3*10^6\text{kg/y}) = \$0.417/\text{kg}$

b) Total product cost per year = $1.427/y

c) Profit per kg of product before taxes = gross profit = selling price, $/kg - TPC/kg

$= \$0.82 - (\$1.427*10^6)/(3*10^6)$

$= \$0.344/\text{kg}$

d) After tax profit per kg product = net profit = (gross profit)(1-Φ)

$= (\$0.344/\text{kg})(1-0.35) = \$0.224/\text{kg}$

6.18

	A	B	C	D	E	F	G	H
1	**Title: Problem 6-13**							
2	**Annual total product cost at 100% capacity**							
3	See Figure 6-7 and 6-8					Date: Nov. 25, 2002		
4	Default, may be changed				Subtotal			
5	Input from problem statement				**RESULT**			
6	Required, may be calculated here, in linked worksheet, or entered manuall							
7	Product:			N/A				
8	Operating time			8000	hours per year			
9	Capacity			3	10^6 kg per year			
10	Fixed Capital Investment, FCI			1.275	million $			
11						User variables		Calculated values, million $
12				Suggested factor		Cost per amount, or factor	Amount, millions	
13	Raw materials					0.09	3	0.270
14	Operating labor					0.08	3	0.240
15	Operating supervision			0.15	of operating labo	0.15	0.240	0.036
16	Utilities					0.05	3	0.150
17	Maintenance and repairs			0.07	of FCI	0.07	1.275	0.089
18	Operating supplies			0.15	of maintenance &	0.15	0.089	0.013
19	Laboratory charges			0.15	of operating labo	0.15	0.240	0.036
20	Royalties (if not on lump-sum basis			0	of TPC	0	1.427	0.000
21	Catalysts and solvents			0		0		0.000
22					**Variable Production Costs**		=	**0.835**
23	Taxes (property)			0.02	of FCI	0.02	1.275	0.026
24	Financing (interest)			0	of FCI	0	1.275	0.000
25	Insurance			0.01	of FCI	0.01	1.275	0.013
26	Rent			0	of FCI	0	1.275	0.000
27	Depreciation			0.1	of FCI	0.1	1.275	0.128
28						**Fixed Charges**	=	**0.166**
29	Plant overhead, general			0.6	of labor, supervis	0.6	0.365	0.219
30	Plant overhead, packaging					0.008	3	0.024
31						**Plant Overhead**	=	**0.243**
32						**Manufacturing cost**	=	**1.244**
33	Administration			0.15	of labor, supervis	0.15	0.365	0.055
34	Distribution & selling			0.05	of TPC	0.05	1.427	0.071
35	Research & Development			0.04	of TPC	0.04	1.427	0.057
36						**General Expense**	=	**0.183**
37				**TOTAL PRODUCT COST**		=	**TPC** =	**1.427**
38								

PROBLEM 6-14

The main calculations are performed using the spreadsheet for annual total product cost at 100% capacity, **Annual TPC**, from the book's web site.

Because some of the terms in the sum for TPC depend upon TPC, a key to the solution is expressing the total product cost in equation form as:

$$\text{TPC} = \Sigma(\text{all terms not depending on TPC}) + \Sigma(\text{all terms depending on TPC})$$

$$= \Sigma(\text{all terms not depending on TPC}) + \Sigma(\text{Factor}_i)(\text{TPC})$$

which gives,

$$\text{TPC}[1-\Sigma(\text{Factor}_i)] = \Sigma(\text{all terms not depending on TPC})$$

and,

$$\text{TPC} = [\Sigma(\text{all terms not depending on TPC})]/[1-\Sigma(\text{Factor}_i)]$$

The following calculations are made from the spreadsheet results shown on the next page.

Manufacturing $= (\$11.122*10^6/\text{y})/(9*10^6\text{kg/y})(100) = \$123.58/100\text{kg}$
cost per 100 kg product

6:20

	A	B	C	D	E	F	G	H
1	**Title: Problem 6-14**							
2	**Annual total product cost at 100% capacity**							
3	See Figure 6-7 and 6-8					Date: Nov. 25, 2002		
4	Default, may be changed				Subtotal			
5	Input from problem statement				**RESULT**			
6	Required, may be calculated here, in linked worksheet, or entered manuall							
7	Product:			N/A				
8	Operating time			7200	hours per year			
9	Capacity			9	10^6 kg per year			
10	Fixed Capital Investment, FCI			4	million $			
11				Default factor, user may change		User variables		Calculated values, million $
12						Cost per amount, or factor	Amount, millions	
13	Raw materials					0.25	9	2.250
14	Operating labor					25	0.105	2.628
15	Operating supervision			0.2	of operating labo	0.2	2.628	0.526
16	Utilities							2.765
17	Maintenance and repairs			0.07	of FCI	0.07	4	0.280
18	Operating supplies			0.15	of maintenance &	0.15	0.280	0.042
19	Laboratory charges			0.15	of operating labo	0.15	2.628	0.394
20	Royalties (if not on lump-sum basis			0	of TPC	0	12.788	0.000
21	Catalysts and solvents			0		0		0.000
22					**Variable Production Costs =**			**8.885**
23	Taxes (property)			0.02	of FCI	0.02	4	0.080
24	Financing (interest)			0	of FCI	0	4	0.000
25	Insurance			0.01	of FCI	0.01	4	0.040
26	Rent			0	of FCI	0	4	0.000
27	Depreciation			0.1	of FCI	0.1	4	0.400
28						**Fixed Charges =**		**0.520**
29	Plant overhead, general			0.5	of labor, supervis	0.5	3.434	1.717
30						**Plant Overhead =**		**1.717**
31						**Manufacturing cost =**		**11.122**
32	Administration			0.15	of labor, supervis	0.15	3.434	0.515
33	Distribution & selling			0.05	of TPC	0.05	12.788	0.639
34	Research & Development			0.04	of TPC	0.04	12.788	0.512
35						**General Expense =**		**1.666**
36				**TOTAL PRODUCT COST**		=	**TPC** =	**12.788**
37								

Chap. 6 - Prob. 15

Gross earnings = total sales − direct and indirect production costs
= $800,000 − (.5)(800,000) − 200,000
= $200,000

Gross earnings under increased annual sales held constant = $200,000 = x − .5x − 1.2 (200,000)
x = $880,000 Answer

If plant operated at full capacity under increased annual sales,

Gross earnings = (1.3)(800,000) − (.5)(1.3)(800,000) − (1.2)(200,000)
= $280,000

Net profit = (1 − .35)(280,000) = $182,000 Answer

If total annual sales remained the same as at present,

Gross earnings = 800,000 − 400,000 − (1.2)(200,000)
= 160,000

Net profit = (1 − .35)(160,000) = $104,000 Answer

If total annual sales decreased to $700,000

Gross earnings = 700,000 − 350,000 − (1.2)(200,000)
= 110,000

Net profit = (1 − .35)(110,000) = $71,500 Answer

PROBLEM 6-16

Assume 8000 h/y operating time.

At breakeven, annual values of sales revenue, s, and total product cost, TPC, are equal. Let x = fraction of annual capacity produced at breakeven.

$$s = x\,(5000\ \text{kg/d})(8000/24\ \text{d/y})(1.75\ \$/\text{kg})$$

At breakeven,

$$s = (\$2\times10^6/\text{y})\,x + \$700{,}000/\text{y}$$

setting the two equal and solving gives

$$x = 0.764$$

The production rate is the

$$(0.764)(5000)\left(\frac{8000}{24}\right) = 1.273\times10^6\ \text{kg/y}$$

$$\text{Fixed cost/kg} = \frac{7\times10^5}{1.273\times10^6} = \underline{\underline{\$0.55/\text{kg}}}$$

ANS.

At 100% capacity and \$1.75/kg,

$$\text{Net profit} = (0.65)\big((5000)(8000/24)(1.75) - 2.7\times10^6\big)$$

$$= \$140{,}800/\text{y}$$

If the selling price increases 10%

$$\text{Net profit} = (0.65)\big(1.1(2.917\times10^6) - 2.7\times10^6\big)$$

$$= \$330{,}400/\text{y}$$

And the increase = $\underline{\underline{\$189{,}600/\text{y}}}$ ANS.

PROBLEM 6-17

Fixed capital inv. $= (0.8)(2.5\times10^6)$
$= \$2.0\times10^6$

Annual sales $\simeq (0.5)FCI = \$1.0\times10^6/y$

Annual total product cost
$= \$1.5\times10^6/y$

$\therefore$ There would be an annual loss of $\$0.5\times10^6/y$ ANS.

If a ratio of \$1 of sales per \$1 of fixed capital inv. was achieved, then

Annual sales $\simeq FCI = \$2\times10^6/y$

Gross profit $= 2\times10^6 - 1.5\times10^6$
$= \$0.5\times10^6/y$
$= \dfrac{0.5\times10^6}{2.5\times10^6}\times100 = 20\,\%/y$

and Net profit $= \dfrac{(0.65)(0.5\times10^6)}{2.5\times10^6}\times100$
$= 13\,\%/y$

Chap. 6 - Prob. 18

Total capital investment = $\$1 \times 10^6$

Total annual sales = $\$1.5 \times 10^6$

Net profits must pay off TCI and R+D in 7 years

Require 12% of sales after taxes = $(.12)(\$1.5\times10^6) = \$180{,}000$

34% of R+D is tax free; tax rate is 34% of gross earnings

Basis 1 year

Return after taxes = \$180,000

Total R+D cost = R

Net R+D cost = R - am't saved by being tax free and added to cost of product.

Amount saved by being tax free = R (0.34)

Net R+D cost = R − R(.34) = .66 R

thus

$$180{,}000 = \frac{1.0\times10^6 + .66R}{7}$$

$$R = \frac{260{,}000}{.66} = \$394{,}000$$ Answer

PROBLEM 6-19

Note that all costs referred to are on a kg basis

Choose as basis, first, 1 kg of product at full capacity

Let
x = total prod. cost/kg of product, full capacity
R = rate of production as kg/yr
T = total prod. cost/kg of product, half capacity

Fixed costs are constant $= (0.35)(10^6)(x)$ in \$/yr

or $= \frac{(0.35)(10^6)(x)}{R}$ in \$/kg

Raw materials are constant per kg of product or $= 0.4x$

Other variable costs are directly proportional to $(R)^{1.5}$ or variable costs $= C(R)^{1.5}$ where C is a proportionality constant which may be evaluated when $R = 10^6$ as

$$0.25x = C(10^6)^{1.5} \quad \text{or} \quad C = (0.25)(10^{-9})x$$

thus, the total cost/kg at $R = 0.5 \times 10^6$ kg/yr is

$$T = \frac{(0.35)(10^6)x}{(0.5 \times 10^6)} + 0.4x + (0.25)(10^{-9})(0.5\times10^6)^{1.5}x$$

$$= (0.7 + 0.4 + 0.088)x = 1.188x$$

$$\left(\frac{T-x}{x}\right)(100) = \left(\frac{T}{x} - 1\right)100 = (1.188-1)(100) = 18.8\%$$

Answer

Note: A common error in this problem, which often occurs, is the assumption that the other variable costs per year are directly proportional to $(R)^{1.5}$.

Chapter 7

7-1.

$$i = \frac{0.06}{2} = 0.03 \qquad N = 10 \cdot 2 = 20 \qquad P = \$10{,}000$$

$$F = P(1+i)^N$$

$$F = \$10{,}000(1.03)^{20}$$

$$\boxed{F = \$18{,}061}$$

7-2.

$$F = P(F/P, i, N) = \frac{A\left[(1+i)^N - 1\right]}{i}$$

$$\$50{,}000 = \frac{A\left[(1.06)^{10} - 1\right]}{.06}$$

$$\boxed{A = \$3{,}793}$$

$$V = \$50{,}000 - \$5000 = \$45{,}000$$

$$d = \frac{\$45{,}000}{n} = \$4{,}500$$

asset value after 5 years $= \$50{,}000 - 5(\$4{,}500)$

$$= \boxed{\$27{,}500}$$

7-3. (1) End of year

$$F = \frac{A\left[(1+i)^N - 1\right]}{i}$$

$$= \frac{\$10{,}000\left[(1.08)^{10} - 1\right]}{.08}$$

$$= \boxed{\$144{,}866}$$

$$P = F/(1+i)^N$$

$$= \frac{\$144,866}{(1.08)^{10}}$$

$$= \boxed{\$67,101}$$

(2) Weekly

$A = \$10,000/52 = \$192.3077$

$i = .08/52 = 1.538462 \times 10^{-3}$

$N = 520$

$$F = \frac{\$192.3077\left[(1.001538462)^{520} - 1\right]}{1.538462 \times 10^{-3}}$$

$$F = \boxed{\$153,022}$$

$$P = \boxed{\$68,799}$$

(3) Continuously

$$F = \bar{A}\,(F/\bar{P}, r, N) = \bar{A}\left(\frac{e^{rN} - 1}{r}\right)$$

$$= \$10,000\,\frac{e^{(0.08)(10)} - 1}{0.08}$$

$$= \boxed{\$153,193}$$

$$P = F/e^{rN}$$

$$= \boxed{\$68,834}$$

7-4 $CC = C_V + P$

(1) $$\left.\begin{aligned} F &= P(1+i)^N \\ F-P &= C_R \end{aligned}\right\} \text{Combine}$$

$$P\left[(1+i)^N - 1\right] = C_R$$

$$\boxed{CC = C_V + \frac{C_R}{(1+i)^N - 1}}$$

(2) $$\left.\begin{aligned} F &= Pe^{rN} \\ F-P &= C_R \end{aligned}\right\} \text{Combine}$$

$$P\left[e^{rN} - 1\right] = C_R$$

$$\boxed{CC = C_V + \frac{C_R}{e^{rN} - 1}}$$

7-5

(1) $C_V = \$20{,}000$ 6-year life $i = 0.06$

(2) $C_V = \$34{,}000$ $C_S = \$4000$ 10-year life

$$CC_1 = \$20{,}000 + \frac{\$20{,}000}{(1.06)^6 - 1}$$

$$= \underline{\$67{,}788}$$

$$CC_2 = \$34{,}000 + \frac{(\$34{,}000 - \$4{,}000)}{(1.06)^{10} - 1}$$

$$= \underline{\$71{,}934}$$

$\boxed{\text{Heat exchanger (1) is cheaper}}$

7-6. (1) $C_V = \$10{,}000$ 10-year life $i = 0.06$

(2) $C_V = ?$ (repair) 3-year life

$$CC_1 = \$10{,}000 + \frac{\$10{,}000}{(1.06)^{10}-1}$$

$$CC_2 = C_{repair} + \frac{C_{repair}}{(1.06)^3-1}$$

Set $CC_1 = CC_2$ & solve for C_{repair}

$$\boxed{C_{repair} = \$3{,}632}$$

7-7. Stock dividends are not deductible

\$5,000,000 @ 8% → \$400,000 / yr in dividends

Bond interest payments are deductible

\$5,000,000 @ 6% → \$300,000/ yr in interest

@ 35% → \$105,000 is deducted

actual cost \$195,000 / yr

$$\boxed{\text{Difference } \$205{,}000/\text{yr}}$$

7-8 (1) \$1,000,000 investment with annual cash flow of \$200,000

(2) \$600,000 investment with annual cash flow of \$220,000

+ \$400,000 @ 6% = \$24,000 in interest.

<u>Option (2) is better</u> just from the cash flow difference. The added interest income makes it even better.

7-9

$FCI = \$20 \times 10^6$

$Tax_{prop} = 1\%$ of FCI

$Tax_{state} = 5\%$ of Gross Earnings (GE)

$Tax_{Fed} = 35\%$ "

$$GE - 0.05GE - 0.35GE - 0.01 \cdot FCI = \$2{,}000{,}000$$

$$\underline{GE = \$3{,}000{,}000}$$

2nd plant

$Tax_{prop} = 0.04 \cdot FCI$

$Tax_{state} = 0.02 \cdot GE$

$$\text{Net Inc.} = (\$3{,}000{,}000)(1 - .35 - .02) - 0.04(20 \times 10^6)$$

$$\boxed{= \$1{,}090{,}000}$$

7-10.

$F = \$50{,}000$ $i = 0.06$

$N = 10$

$A = \$400/yr$

$$F = \frac{A[(1+i)^N - 1]}{i} + P(1+i)^N$$ solve for P

$$\$50{,}000 = \frac{\$400[1.06^{10} - 1]}{.06} + P(1.06)^{10}$$

$$\boxed{P = \$24{,}976}$$

7-11 MACRS - Cap. Gain = \$7000 - \$1728 = \$5272

Tax_{CG} = .20(\$5272) = \$1054.40

SL - Cap Gain = \$7000 - \$2000 = \$5,000

Tax_{CG} = \$1000

Income Tax: 0.35(\$8272 - 8000) = \$95.20

Tax paid w/ SL
over that paid w/ MACRS

DIFFERENCE: \$95.20 - 54.40 = $\boxed{\$40.80}$

7-12 (1) Depreciation = \$200,000/yr

Taxable income = \$200,000 - \$200,000 = 0

Cash flow = \$200,000/yr

(2) Depreciation = \$120,000/yr

Taxable income = \$220,000 - \$120,000 = \$100,000

Taxes = \$35,000

Cash Flow = \$220,000 - \$35,000 = \$185,000/yr

Interest income is \$24,000/yr (see 7-8)

Total cash flow = \$209,000/yr

Option (2) is better

7-13 SL: \$10,000/yr

\$20,000 paid in 2 yrs (40%)

MACRS: 5th year 0.1152(\$50,000) = \$5760

20% + 32% in 2 yrs = 52%

7-14. 1st year of MACRS - 20%

20% of \$850,000 = \$170,000

SL: $d = \frac{\$850{,}000}{9.5} = \$89{,}474\ /yr$

Difference = \$80,526 less deducted in SL

Taxes = .35 (\$80,526)

= $\boxed{\$28{,}184}$ less paid w/ MACRS

7-15

\$6,000,000 5yr (20%/yr) = \$1,200,000/yr

\$4,000,000 10yr (10%/yr) = \$ 400,000/yr

Total 1st year = \$1,600,000

MACRS (15yrs) → 5% in 1st year = \$500,000

7-16 unit of production method \$10/unit

year	units	deprec.	cum
1	200	\$2000	\$2000
2	400	\$4000	\$6000
3	800	\$8000	\$14,000
4	1600	\$16,000	\$30,000
5	1600	\$16,000	\$46,000
6	.	.	.
7	:	:	:
8	.	.	
9	1600	\$16,000	\$110,000

SL w/ 9 yrs $d = \frac{\$110{,}000}{9} = \boxed{\$12{,}222/yr}$

7-17

SL: $d = \dfrac{\$35,000 - \$5,000}{5} = \$6000$

MACRS: percentages from Table 7-9

sum-of-the-digits: $d = \dfrac{2(n-a+1)}{n(n+1)}(V - V_s)$

	method/Depr.		
Year	SL	MACRS	SOTD
1	\$6,000	\$6,000	\$10,000
2	"	\$9,600	\$8,000
3	"	\$5,760	\$6,000
4	"	\$3,456	\$4,000
5	"	\$3,456	\$2,000
6	0	\$1,728	0

7-18

$$d = \frac{2(n-a+1)}{n(n+1)}(V - V_s)$$

where n is the recovery period

a is the year for which the depreciation allowance, d, is being determined

V is the original equipment cost

V_s is the salvage value

$d = \$2381$

$a = 2$

$V = \$10,000$

$V_s = 0$

Solve for n

$$\boxed{n = 6 \text{ years}}$$

CHAPTER 8

Chap. 8 - Prob 1:

$$F = P\left(1+\frac{i}{m}\right)^{Nm} \qquad (\text{eq. } 7\text{-}6)$$

(a) From the problem statement

$P =$ \$10,000

$i = 0.06$

$N = 10$

$m = 12$

$$\therefore \quad F = 10{,}000\left(1+\frac{0.06}{12}\right)^{120}$$

$$= 18{,}193.97$$

Total amount of funds = \$18,193.97 Answer

(b) $P =$ \$10,000

$F =$ \$20,000

$i = 0.06$

$m = 2$

To calculate N

$$20{,}000 = 10{,}000\left(1+\frac{0.06}{2}\right)^{2N}$$

$\Rightarrow$ N = 11.7 years Answer

(c)

$$F = Pe^{rN} \qquad (\text{eq. } 7\text{-}18)$$

$F = 2P$ and $r = 0.06$

Substituting

$$2P = Pe^{0.06N}$$

$$\Rightarrow N = \frac{\ln 2}{0.06}$$

$\Rightarrow$ N = 11.55 years Answer

Chap. 8 – Prob 2:

Total investment = T.I. = Working capital + fixed capital

$$T.I. = 0.25\,(T.I.) + 10 \times 10^6$$

$$\Rightarrow \quad T.I. = \$\,13.33 \times 10^6$$

$$\text{Percent return on investment} = \frac{\text{Annual profit}}{\text{Total investment}} \times 100$$

$$= \frac{3 \times 10^6}{13.33 \times 10^6} \times 100$$

$$= 22.5$$

Answer

$$\text{Payout period} = \frac{\text{Fixed capital investment}}{\text{Annual profit} + \text{Annual depreciation}}$$

$$= \frac{10 \times 10^6}{3 \times 10^6 + 1 \times 10^6}$$

$$= 2.5 \text{ years}$$

Answer

Chap. 8 – Problem 3

Let the total capital investment be T

Then, fixed capital investment = 0.85 T

$$PBP = \frac{\text{Fixed capital investment}}{\text{Annual cash flow}}$$

$$\Rightarrow \text{Annual cash flow} = A_j = \frac{0.85T}{PBP}$$

$$= \frac{0.85T}{5}$$

$$= 0.17T$$

Net profit = Cash flow + depreciation

Depreciation = 10% of fixed capital investment

$$= (0.1)(0.85T)$$

$$= 0.085T$$

$$\therefore \text{Net Profit} = A_j + 0.085T$$

$$= (0.17 + 0.085)T$$

$$= 0.255T$$

$$ROI = \left(\frac{\text{Net profit}}{\text{Total Capital Investment}}\right) \times 100$$

$$= \frac{0.255T}{T} \times 100$$

$$= 25.5\ \%$$

Answer

Chap. 8 – Prob. 4

Cost of replacing Pump A after 4 years = C_R = \$20,000 – \$2,000

= \$18,000

Installed cost of Pump A = C_V = \$ 20,000

Annual effective interest rate = r = 0.15

Number of years of useful life = n = 4 years

Capitalized cost = K = $C_V + \dfrac{C_R}{(1+i)^n - 1}$

$= 20{,}000 + \dfrac{18{,}000}{(1+0.15)^4 - 1}$

= \$44,031.84

To be competitive, the capitalized cost of Pump B should be the same as Pump A, i.e., \$44,031.84

Capitalized cost of Pump B = K = \$44,031.84

Cost of replacing Pump B after n years = C_R = 25,000 – 4000

= \$ 21,000

Installed cost of Pump B = C_V = 25,000

Annual effective interest rate = r = 0.15

Using $K = C_V + \dfrac{C_R}{(1+i)^n - 1}$

we have

$$44{,}031.84 = 25{,}000 + \frac{21{,}000}{(1+0.15)^n - 1}$$

$\Rightarrow$ n = 5.3 years

Service life of Pump B = 5.3 years **Answer**

Chap. 8 — Prob. 5

Basis : 1 year

For 0.025 m insulation

$$\text{Money saved on heat} = \frac{(88)(3600)(24)(300)(1.50)}{1{,}000{,}000} = \$3{,}421.44$$

$$\text{Fixed charges} = (0.1)(8{,}000) = \$800$$

$$\text{Total savings} = 3421.44 - 800 = \$2621.44$$

For 0.051 m insulation

$$\text{Money saved on heat} = \frac{(102)(3600)(24)(300)(1.50)}{1{,}000{,}000} = \$3965.76$$

$$\text{Fixed charges} = (0.1)(10{,}100) = \$1{,}010$$

$$\text{Total savings} = 3{,}965.76 - 1{,}010 = \$2955.76$$

For 0.076 m insulation

$$\text{Money saved on heat} = \frac{(108)(3600)(24)(300)(1.50)}{1{,}000{,}000} = \$4199.04$$

$$\text{Fixed charges} = (0.1)(11{,}100) = \$1110$$

$$\text{Total savings} = 4199.04 - 1110 = \$3{,}089.04$$

For 0.102 m insulation

$$\text{Money saved on heat} = \frac{(111)(3600)(24)(300)(1.50)}{1{,}000{,}000} = \$4315.68$$

$$\text{Fixed charges} = (0.1)(11{,}500) = \$1150$$

$$\text{Total savings} = 4315.68 - 1150 = \$3165.68$$

Comparing 0.025 m insulation to 0.051 m insulation

$$\text{Return on investment} = \left(\frac{2955.76 - 2621.44}{10{,}100 - 8000}\right)(100) = 15.92\,\% \Rightarrow \underline{\text{Prefer } 0.051}$$

Comparing 0.051 m insulation with 0.076 m insulation

$$\text{Return on investment} = \frac{(3089.04 - 2955.76)}{(11{,}100 - 10{,}100)}(100) = 13.33\,\% \Rightarrow \underline{\text{Prefer } 0.051}$$

Comparing 0.051 m insulation with 0.102 m insulation

$$\text{Return on investment} = \frac{(3165.68 - 2955.76)}{(11500 - 10100)}(100) = 14.99\% \Rightarrow \underline{\text{Prefer } 0.051}$$

Answer

Chap. 8 – Prob. 6

Cost of replacing Pump A after 2 years = C_{RA} = \$15,000

Installed cost of Pump A = C_{VA} = \$15,000

Service life of Pump A = n_A = 2 years

Capitalized cost of Pump A $= K_A = C_{VA} + \dfrac{C_{RA}}{(1+i)^{n}-1}$

$$= 15{,}000 + \frac{15{,}000}{(1+i)^2-1}$$

Cost of replacing Pump B after 5 years = C_{RB} = \$22,000

Installed cost of Pump B = C_{VB} = \$22,000

Service life of Pump B = n_B = 5 years

Capitalized cost of Pump B $= K_B = 22{,}000 + \dfrac{22{,}000}{(1+i)^5-1}$

To be competitive, $K_A = K_B$

$$\Rightarrow \quad 15{,}000 + \frac{15{,}000}{(1+i)^2-1} = 22{,}000 + \frac{22{,}000}{(1+i)^5-1}$$

Solving by trial and error, $i = 0.63$

Annual interest rate for the two pumps to be competitive is 63%. <u>Answer</u>

The pumps are competitive at a very high interest rate. Suppose we assume a more reasonable rate of 10%.

Then, the capitalized cost of Pump A $= 15{,}000 + \dfrac{15{,}000}{(1+0.1)^2-1}$

$$= \$86{,}428.57$$

The capitalized cost of Pump B $= 22{,}000 + \dfrac{22{,}000}{(1+0.1)^2-1}$

$$= \$58{,}035.44$$

I would recommend Pump B as significantly less money needs to be invested to replace it. <u>Answer</u>

Chap 8 – Prob. 7

15% return is required for the design to be acceptable.

Comparing No. 2 to No. 1, incremental return is:

$$\left(\frac{3,000 - 2,800}{12,000 - 10,000}\right) \times (100) = 10\% \qquad \text{Reject design No. 2}$$

Comparing No. 3 to No. 1, incremental return is:

$$\left(\frac{3,000 - 2,350}{14,000 - 10,000}\right) \times (100) = 16.25\% \qquad \text{Reject design No. 1}$$

Comparing No. 3 to No. 4, incremental return is:

$$\left(\frac{2,350 - 2,100}{16,000 - 14,000}\right) \times (100) = 12.5\% \qquad \text{Reject design No. 4}$$

Based on the above analysis, Design No. 3 is recommended

Answer

Chap. 8 – Prob. 8

(a) From Table 8-3, we have:

All expenses = cash expenses + depreciation

Cash expenses = \$2,000,000

Depreciation = \$1,000,000 (assuming straight line depreciation of total capital over 10 years)

All expenses = 2,000,000 + 1,000,000
= \$3,000,000

Income tax = (revenue – all expenses) (tax rate)
= (\$8,000,000 – \$3000000) (0.35) (Assume a tax rate of 35%)

= \$1,750,000

Net profits = revenue – all expenses – income tax
= \$8,000,000 – \$3,000,000 – \$1,750,000
= \$3,250,000

Total capital investment = \$10,000,000

$$ROI = \frac{\text{Net profits}}{\text{Total capital investment}}$$

$$= \frac{3,250,000}{10,000,000}$$

= 0.325 or 32.5% Answer

b) Cash flow A_j = net profits + depreciation (Table 8-3)
= \$3,250,000 + \$1,000,000
= \$4,250,000

Fixed capital investment = Total capital investment – working capital
= \$10,000,000 – \$1,000,000
= \$9,000,000

$$PBP = \frac{\$9,000,000}{4,250,000} = 2.11 \text{ years}$$ Answer

Chap. 8 – Prob. 9

Insurance cost without sprinklers:

$(0.5\times10^6)(0.011)(20) = 0.11\times10^6$

$(0.4\times10^6)(0.0095)(20) = 0.076\times10^6$

Total = \$186,000

Insurance cost with sprinklers:

$(0.52\times10^6)(0.011)(20)(3/4) = 0.086\times10^6$

$(0.4\times10^6)(0.0095)(20)(3/4) = 0.057\times10^6$

Total = \$143,000

Installation + maintenance costs = 20,000 + (300)(20)

= \$26,000

Total cost with sprinklers = 143,000 + 26,000

= \$169,000

Savings in 20 years with sprinkler system = \$17,000

Annual % return $= \dfrac{17{,}000}{(20)(20{,}000)}\times 100 = 4.25\%$

This is less than the present 8% return; therefore, from the viewpoint of the stockholders, the sprinkler system <u>should not be purchased.</u> From the viewpoint of safety, the 4.25% return on investment might be considered acceptable.

Chap 8 - Prob 10

(a) Annual cash flow = Annual revenue − annual operating expense + annual depreciation

We first compute the amount depreciated using \$50 million as the basis and an MACRS depreciation schedule with a class life of 5 years (Table 7-9)

Year	Depreciation rate, %	Amount depreciated, in millions of \$
1	20.00	10.00
2	32.00	16.00
3	19.20	9.60
4	11.52	5.76
5	11.52	5.76
6	5.76	2.88
7	0.00	0.00

Based on the amount depreciated, the annual cash flow can be computed, as shown below (all figures, except year, in millions of dollars)

Year	Annual revenue	Annual operating exp.	Amount dep.	Cash flow
1	7.0	4.0	10.00	13.00
2	10.0	5.6	16.00	20.40
3	15.0	6.8	9.60	17.80
4	20.0	7.8	5.76	17.96
5	22.5	8.8	5.76	19.46
6	24.0	9.6	2.88	17.28
7	25.0	10.0	0.00	15.00

Answer

(b) Net present worth = $\sum_{j=1}^{N} PWF_{cf,j}\,[s_j - c_j - d_j](1-\phi) + rec_j + d_j] - \sum_{j=-b}^{N} PWF_{v,j} F_j$

Assume that the tax rate = $\phi = 0.35$

Assume that the recovery of working capital = $rec_j = 0$

$PWF_{cf,j} = (1+0.15)^{-j}$

s_j = annual revenue from the above table

c_j = annual operating expense from the above table

d_j = annual depreciation from the above table

Chap 8 - Prob 10 (Cont'd)

Substituting, we get:

$$
\begin{aligned}
\text{Net present worth} = {} & (1.15)^{-1}\{(7\times10^6 - 4\times10^6 - 10\times10^6)(1-0.35) + 10\times10^6\} \\
& + (1.15)^{-2}\{(10\times10^6 - 5.6\times10^6 - 16\times10^6)(1-0.35) + 16\times10^6\} \\
& + (1.15)^{-3}\{(15\times10^6 - 6.8\times10^6 - 9.6\times10^6)(1-0.35) + 9.6\times10^6\} \\
& + (1.15)^{-4}\{(20\times10^6 - 7.8\times10^6 - 5.76\times10^6)(1-0.35) + 5.76\times10^6\} \\
& + (1.15)^{-5}\{(22.5\times10^6 - 8.8\times10^6 - 5.76\times10^6)(1-0.35) + 5.76\times10^6\} \\
& + (1.15)^{-6}\{24\times10^6 - 9.6\times10^6 - 2.88\times10^6)(1-0.35) + 2.88\times10^6\} \\
& + (1.15)^{-7}\{25\times10^6 - 10\times10^6 - 0)(1-0.35) + 0\} \\
& - 50\times10^6 \\
= {} & \$ -13{,}885{,}994.52
\end{aligned}
$$

Answer

(c) DCFR

$$
\begin{aligned}
0 = {} & (1+r)^{-1}\{5{,}450{,}000\} + (1+r)^{-2}\{8{,}460{,}000\} + (1+r)^{-3}\{8{,}690{,}000\} \\
& + (1+r)^{-4}\{9{,}946{,}000\} + (1+r)^{-5}\{10{,}921{,}000\} + (1+r)^{-6}\{10{,}368{,}000\} \\
& + (1+r)^{-7}\{9{,}750{,}000\} - 50{,}000{,}000
\end{aligned}
$$

Calculating by trial and error

$r \approx 0.035$ or 3.5%

Answer

Chap 8 - Prob. 11

Annual savings by choice of steam unit

Fuel	\$70,000
Maintenance	\$ 3,000
Insurance and taxes	− \$ 6,000
Depreciation : $\frac{400{,}000}{10} - \frac{600{,}000}{20}$	= \$10,000
Total savings	= \$ 77,000

% incremental return on investment if steam unit chosen :

$$\left(\frac{77{,}000}{600{,}000 - 400{,}000}\right)(100) = 38.5\%$$

12% return on investment is required ; thus the steam turbine unit is an acceptable alternative. <u>Answer</u>

Chap 8 – Prob 12

Let x be the initial manufacturing cost at both locations

For old plant:

Capital investment $= \$130{,}000$

Manufacturing cost $= \$150{,}000 + 60{,}000 + x$

$= \$210{,}000 + x$

For new plant:

Capital investment $= \$200{,}000$

Manufacturing cost $= \$120{,}000 + 70{,}000 + (0.02)(200{,}000) + x + \text{depreciation}$

$= \$194{,}000 + x + \text{depreciation}$

A return of 9% after taxes of 35% are required

This is equivalent to $\frac{9}{1-0.35} = 13.8\%$ return before taxes

Rate of return $= 0.138 = \dfrac{-\Delta \text{ manufacturing costs}}{\Delta \text{ investment}}$

$= \dfrac{\text{New manufacturing cost} - \text{old manufacturing cost}}{\text{Old investment} - \text{new investment}}$

$= \dfrac{(194{,}000 + x + \text{depreciation}) - (210{,}000 + x)}{130{,}000 - 200{,}000}$

$= \dfrac{\text{depreciation} - 16000}{-70{,}000}$

$\Rightarrow \text{depreciation} = (0.138)(-70{,}000) + 16{,}000$

$= \$6340$ per year

Recovery period for new investment $= \dfrac{\text{Capital investment}}{\text{annual depreciation}}$

$= \dfrac{200{,}000}{6340}$

$= 31.5$ years   Answer

Chap 8 - Prob 13

Definitions from Table 8-3 are used in this solution

Old unit:

Let the revenue be = R

Let the cash expense be = E

Annual depreciation allowed = $\frac{40{,}000}{10}$ = \$4,000 (assuming straight line method)
(Note that in this method no salvage value may be taken as stated on page 311)

$\therefore$ All expenses = $E + 4000$

Tax = T = (revenue − all expenses) (tax rate)
= $(R - E - 4000)(0.35)$

Net Profit = P_{old} = Revenue − All expenses − Tax
= $R - (E + 4000) - (R - E - 4000)(0.35)$
= $0.65(R - E) - 2600$

New Unit:

Revenue = R (same as old unit)

Cash expenses = $E - 15{,}000$

Annual depreciation allowed = $\frac{70{,}000}{10}$ = \$7,000

All expenses = $E - 15{,}000 + 7000$ = $E - 8{,}000$

Tax = (revenue − all expenses) (tax rate)
= $(R - E + 8000)(0.35)$

Net profit = P_{new} = Revenue − All expenses − Tax
= $R - (E - 8000) - (R - E + 8000)(0.35)$
= $0.65(R - E) + 5200$

Increase in profit if one switches to new unit = $P_{new} - P_{old}$
= \$7,800

New investment = 70,000 − 5000 = \$65,000

% Return = $\frac{7800}{65000} \times 100 = 12\%$. Since this is less than 15% do NOT make change.

8:15

Chap 8 - Prob 14

Fixed capital investment = \$1,000,000

Depreciation = $\frac{1{,}000{,}000}{5} = \$\,200{,}000$

Cash flow = $(s_j - c_j - d_j)(1-\phi) + rec_j + d_j$ where $\phi = 0.35$

Present worth = $PWF_{ct,j}\ \{\text{cash flow}\}$

where $PWF_{ct,j} = \left(\frac{e^r - 1}{r}\right) e^{-rj}$ where $r = 0.10$

Using the above formulae, we can generate the following spreadsheet

Year	Revenue	Product Cost	Depreciation	Cashflow	Present Worth
1	500,000	100,000	200,000	330,000	314,036.52
2	500,000	100,000	200,000	330,000	284,151.99
3	500,000	100,000	200,000	330,000	257,111.36
4	500,000	100,000	200,000	330,000	232,643.98
5	500,000	100,000	200,000	330,000	210,504.97

If we assume an inflation rate of 5% for both revenues and expenses, we get the following:

Year	Revenue	Product Cost	Depreciation	Cash flow	Present Worth
1	500,000	100,000	200,000	330,000	314,036.52
2	525,000	105,000	200,000	343,000	295,345.86
3	551,250	110,250	200,000	356,650	277,875.05
4	578,812.50	115,762.50	200,000	370,982.50	261,535.89
5	607,753.13	121,550.63	200,000	386,031.63	246,247.21

Chap 8 - Prob. 15

Old unit: Depreciation costs $= \frac{600}{5} =$ \$120 per year

let $x =$ labor + maintenance costs

and $y =$ taxes + insurance costs

Manufacturing costs $= x + y$

New unit: Depreciation costs $= \frac{6000}{10} =$ \$600 per year

Manufacturing costs $= (x - 1000) + (y + 100)$

Savings with new unit $= (x + y) - [(x - 1000) + (y + 100)]$

$=$ \$900 per year

% replacement return $= \left[\frac{900 - (600 - 120)}{6000 - 600}\right](100)$

$= 7.78\%$ Answer

Chap. 8 – Prob. 16

The project cost compounded over 10 years must be equal to the cash flow to the project compounded over 10 years using the given discounted-cash-flow rate of return

Batch system:

Future worth of project cost

$$S = 20{,}000\,(1.25)^{10} = \$186{,}200$$

Cash flow to project (based on end-of-year income)

$$S = 5600\left[\sum_{n=0}^{y-1}(1.25)^{y}\right] = (5600)(33.28) = \$186{,}400$$

This gives a good check on discounted-cash-flow rate of return

Continuous system

Future worth of project cost

$$S = 30{,}000\,(1.22)^{10} = (30{,}000)(7.3) = \$219{,}000$$

Cash flow to project (based on end-of-year income)

$$S = 7650\left[\sum_{n=0}^{y-1}(1.22)^{y}\right] = (7650)(27.72) = \$212{,}000$$

This gives a good check on discounted-cash-flow rate of return.

$$\text{Present worth} = \underbrace{\underbrace{\left[\frac{1}{(1+i)^{n}}\right]}_{\text{Discount factor}} \times \text{annual cash flow}}_{\text{Present value}} - \begin{array}{c}\text{initial}\\ \text{cost}\end{array}$$

For batch system

Year, n	Cash flow to project, $	Discount factor, i=0.10	Present value, $
1	5600	0.909	
2	5600	0.827	
3	5600	0.751	
4	5600	0.683	
5	5600	0.621	
6	5600	0.564	
7	5600	0.514	
8	5600	0.467	
9	5600	0.424	
10	5600	0.386	
	Total	6.146	$34,400

Chap 8 - Prob. 16 (Cont'd)

Present value = (5600)(6.146) = \$34,400
Present worth = 34,400 − 20,000 = \$14,400

For continuous system:
Same discount factor applies
Present value = (7650)(6.146) = \$47,000
Present worth = 47,000 − 30,000 = \$17,000

The present worth values for both the batch and continuous systems check.

Rate of return:

A comparison of investments shows that an additional investment of \$10,000 yields incremental benefits of 7650 − 5600 or \$2050 in years 1 to 10.

This is a discounted-cash-flow rate of return of 16%.

$$S = 2050\left[\sum_{n=0}^{y-1}(1.16)^{y}\right] = 2050(21.4) = (10,000)(1.16)^{10} = \$44,000$$

Because the 16% incremental return exceeds the required return of 10%, the continuous system should be selected, as shown by present worth calculations.

Answer

Chap. 8 – Prob. 17

On the basis of the positive present worth at 10% the lease-and-water-flood arrangement should be undertaken. There is no rate of interest (less than infinity) which will yield a zero present worth. Therefore, if the board of directors understood only discounted-cash-flow rate of return, it would be necessary to reeducate the board to fully comprehend the limitations of the discounted-cash-flow rate-of-return method and demonstrate the effectiveness of the present-worth method before this proposal could be presented and fully evaluated.

Answer

Chap. 8 – Prob. 18

(a) A capital investment of $100 million is spent over three years. Assuming that the investment in each of the three years are equal to $33,333,333, we obtain the following depreciation every year:

Year	Amount depreciated from 1st year capital	Amount depreciated from 2nd year capital	Amount depreciated from 3rd year capital	Total
1	3,333,333	0	0	3,333,333
2	3,333,333	3,703,704	0	7,037,037
3	3,333,333	3,703,704	4,166,667	11,203,704
4	3,333,333	3,703,704	4,166,667	11,203,704
5	3,333,333	3,703,704	4,166,667	11,203,704
6	3,333,333	3,703,704	4,166,667	11,203,704
7	3,333,333	3,703,704	4,166,667	11,203,704
8	3,333,333	3,703,704	4,166,667	11,203,704
9	3,333,333	3,703,704	4,166,667	11,203,704
10	3,333,333	3,703,704	4,166,667	11,203,704
				Total = $100,000,000

Note that the investment of the 1st year is depreciated over 10 years, the investment of the second year is depreciated over 9 years and the investment of the third year is depreciated over 8 years.

Chap 8 – Prob 18 (Cont'd)

Net profit = (sales – operating expenses – depreciation) (1 – tax rate)

Tax rate = 0.35

Year	Sales, \$	Expenses, \$	Depreciation, \$	Net Profit, \$
1	75×10^6	100×10^6	3,333,333	−18,416,666
2	112.5×10^6	100×10^6	7,037,037	3,550,926
3	150×10^6	100×10^6	11,203,704	25,217,592
4	150×10^6	100×10^6	11,203,704	25,217,592
5	150×10^6	100×10^6	11,203,704	25,217,592
6	150×10^6	100×10^6	11,203,704	25,217,592
7	150×10^6	100×10^6	11,203,704	25,217,592
8	150×10^6	100×10^6	11,203,704	25,217,592
9	150×10^6	100×10^6	11,203,704	25,217,592
10	150×10^6	100×10^6	11,203,704	25,217,592
			100×10^6	186,874,996

$$ROI = \frac{\sum_{j=1}^{N} N_j / 10}{\text{Total investment}} \times 100$$

$$= \frac{186{,}874{,}996/10}{100\times10^6 + 20\times10^6} \times 100$$

$$= 15.5\ \%$$

Answer

(b) $$PBP = \frac{\text{Fixed capital investment}}{\text{Average cash flow}}$$

Cash flow = Net profits + depreciation

$$\text{Average cash flow} = \frac{186{,}874{,}996 + 100{,}000{,}000}{10} = 28{,}687{,}500$$

Fixed capital investment = \$100,000,000

$$PBP = \frac{100{,}000{,}000}{28{,}687{,}500} = 3.5 \text{ years}$$

Chap 8 – Prob 19

We first compute the depreciation for each year

Year	Amount depreciated from 1st year capital	Amount depreciated from 2nd year capital	Amount depreciated from 3rd year capital	Total
1	2×10^6	0	0	2×10^6
2	2×10^6	3.33×10^6	0	5.33×10^6
3	2×10^6	3.33×10^6	6.25×10^6	11.58×10^6
4	2×10^6	3.33×10^6	6.25×10^6	11.58×10^6
5	2×10^6	3.33×10^6	6.25×10^6	11.58×10^6
6	2×10^6	3.33×10^6	6.25×10^6	11.58×10^6
7	2×10^6	3.33×10^6	6.25×10^6	11.58×10^6
8	2×10^6	3.33×10^6	6.25×10^6	11.58×10^6
9	2×10^6	3.33×10^6	6.25×10^6	11.58×10^6
10	2×10^6	3.33×10^6	6.25×10^6	11.58×10^6

$$\text{Present worth} = PWF_{c\,f,j}\left[(s_j - c_j - d_j)(1-\phi) + rec_j + d_j\right]$$

From Table 7-5, $PWF_{c\,f,j} = \left(\frac{e^r-1}{r}\right)e^{-rj}$

Assume $r = 0.1$

Year, j	s_j	c_j	d_j	$\left(\frac{e^r-1}{r}\right)e^{-rj}$	Present Worth
1	75×10^6	100×10^6	2×10^6	0.951	−14,788,050
2	112.5×10^6	100×10^6	5.33×10^6	0.861	8,601,820
3	150×10^6	100×10^6	11.58×10^6	0.779	28,474,787
4	150×10^6	100×10^6	11.58×10^6	0.705	25,769,865
5	150×10^6	100×10^6	11.58×10^6	0.638	23,320,814
6	150×10^6	100×10^6	11.58×10^6	0.577	21,091,081
7	150×10^6	100×10^6	11.58×10^6	0.522	19,080,666
8	150×10^6	100×10^6	11.58×10^6	0.473	17,289,569
9	150×10^6	100×10^6	11.58×10^6	0.428	15,644,684
10	150×10^6	100×10^6	11.58×10^6	0.387	14,146,011

Total = \$158,631,247 $\approx$ \$$158.63\times10^6$

Chap 8 – Prob 19 (cont'd)

Present worth of all capital investments

Year, j	Capital investment, F_j	$PWF_{u,j} = \left(\frac{e^r-1}{r}\right)e^{-rj}$	$PWF_{u,j} \times F_j$
1	20×10^6	0.951	19.02×10^6
2	30×10^6	0.861	25.83×10^6
3	50×10^6	0.779	38.95×10^6
		Total	$\$83.80\times10^6$

Net present worth = Present worth of all cash flow – present worth of all capital investments

$$= 158.63\times10^6 - 83.80\times10^6$$

$$= \$74.83\times10^6$$

Answer

CHAPTER 9

PROB. 9-1

From prob. description,

Objective Function = Annual Cost Dependent Upon No. of Effects

Decision Variable(s) = No. of Effects $\triangleq X$

Investment Cost $= 18k\$ + 15k\$(X-1)$

$\therefore$ ① = Fixed Annual Charge ex. Depr. $= [18k\$ + 15k\$(X-1)]0.15$

② = Annual Maint. Charges $= [18k\$ + 15k\$(X-1)]0.05$

③ = Annual Depr. Charges $= [18k\$ + 15k\$(X-1)]0.10$

$$①+②+③ = [18k\$ + 15k\$(X-1)](0.30) = 4500X - 900\ \$$$

$$\left\{\begin{matrix}\text{Annual}\\ \text{Stm. Cost}\end{matrix}\right\} = \frac{2\times10^5\ \text{kg}\ H_2O\ \text{evap.}}{\text{oper. Day}} * \frac{\text{kg Stm}}{0.85X\ \text{kg}\ H_2O\ \text{evap}} * \frac{0.0033\$}{\text{kg. Stm}} * \frac{300\ \text{oper. Da.}}{\text{y.}}$$

$$\Rightarrow \left\{\begin{matrix}\text{Annual}\\ \text{Stm. Cost}\end{matrix}\right\} = \frac{232941\$}{X}$$

$$\therefore \left\{\begin{matrix}\text{Annual Cost}\\ \text{Dependent}\\ \text{Upon No. of Effects}\end{matrix}\right\} = C_A = 4500X - 900 + \frac{232941}{X}$$

OPTIMUM NO. OF EFFECTS SATISFIES

$$\left.\frac{dC_A}{dX}\right|_{X_{OPT}} = 0 = 4500 - \frac{232941}{X_{OPT}^2}$$

$$\therefore X_{OPT} = 7.19 \Rightarrow \boxed{X_{OPT} = 7\ \text{EFFECT}}$$

Problem 9.2 Optimum Insulation Thickness

MathCad File

Input Data

$k_{ins} := 5 \cdot 10^{-2} \quad \frac{W}{s \cdot m}$ $\quad h_o := 11.4 \quad \frac{W}{s \cdot m^2}$

$T_{stm} := 480 \quad K \qquad T_{surr} := 295 \quad K$

$\Delta H_v := 1.912 \cdot 10^6 \quad \frac{J}{kg}$ Heat of Vaporization of Sat.Stm. at 480 K

$L := 300 \quad m \qquad D_o := 0.273 \quad m$

$c_{stm} := 0.004 \quad \frac{dollars}{kg}$ $\qquad Install\_Cost(I_t) := 180 \cdot I_t \quad \frac{dollars}{m}$

Annual Fixed Costs

$AFC(I_t) := 300 \cdot Install\_Cost(I_t) \cdot 0.20$

Annual Steam Cost

Use heat resistances to determine overall h.t. coefficient as function of I_t. Neglect resistance except for insulation and outer film. First express the areas involved:

$A_o(I_t) := \pi \cdot (D_o + 2 \cdot I_t) \cdot L \quad m^2 \qquad A_m(I_t) := .5 \cdot [D_o + (D_o + 2 \cdot I_t)] \quad m^2$

$UA_o(I_t) := \frac{(h_o \cdot A_o(I_t) \cdot k_{ins} \cdot A_m(I_t))}{k_{ins} \cdot A_m(I_t) + I_t \cdot h_o \cdot A_o(I_t)} \qquad \frac{W}{K}$ from rearranging overall resistance from individual resistances.

$q(I_t) := UA_o(I_t) \cdot (T_{stm} - T_{surr}) \quad W$

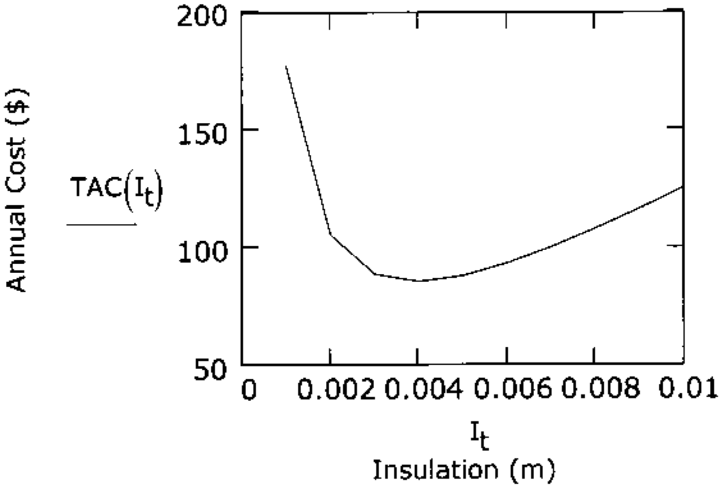
$m_{stm}(I_t) := \frac{q(I_t)}{\Delta H_v} \quad \frac{kg}{s}$

$ASC(I_t) := m_{stm}(I_t) \cdot c_{stm} \cdot 3600 \cdot 24 \cdot 365$ Annual Steam Cost

Total Annual Cost

$TAC(I_t) := AFC(I_t) + ASC(I_t) \quad \frac{dollars}{y}$

$I_t := 0.001, 0.002 .. 0.01$ Define a range of values for plotting



$j := 1, 2 .. 10$

$TABLE_{j,1} := 0.001 + (j-1) \cdot 0.001$ $TABLE_{j,2} := TAC(TABLE_{j,1})$

I_t TAC

TABLE =

	1	2
1	$1\cdot10^{-3}$	177.242
2	$2\cdot10^{-3}$	105.318
3	$3\cdot10^{-3}$	88.459
4	$4\cdot10^{-3}$	85.413
5	$5\cdot10^{-3}$	87.9
6	$6\cdot10^{-3}$	93.156
7	$7\cdot10^{-3}$	99.995
8	$8\cdot10^{-3}$	107.824
9	$9\cdot10^{-3}$	116.312
10	0.01	125.263

Problem 9.3: Absorption Tower

Math Cad File

Input Data:

$F_V := 33 \quad \frac{m^3}{s} \qquad M_W := 29.1 \quad \frac{kg}{kgmol}$

$T := 400 \quad K \qquad P := 111 \quad kPa$

$HTU := 4.5 \quad m \qquad NTU(G_s) := 1.05 \cdot G_s^{0.18} \qquad Op\_Hr := 8000 \quad \frac{h}{y}$

$c_{tower} := 40 \quad \frac{dollars}{m^3} \qquad f_{FAC} := 0.2$ factor for Fixed Annual Cost

$V\_OP\_Cost(G_s) := 1.22 \cdot 10^{-3} \cdot G_s^{2} + 0.1098 \cdot G_s^{-1} + 0.0244 \cdot G_s^{-0.8} \qquad \frac{dollars}{s}$

Solution:

$F_m := F_V \cdot \left(\frac{1}{22.4}\right) \cdot \left(\frac{P}{101.3}\right) \cdot \left(\frac{273}{T}\right) \cdot M_W \qquad F_m = 32.061 \quad \frac{kg}{s}$

$G_s(D) := \frac{F_m}{0.25 \cdot \pi \cdot D^2}$

$z(D) := HTU \cdot NTU(G_s(D)) \quad m$ Height of Packing

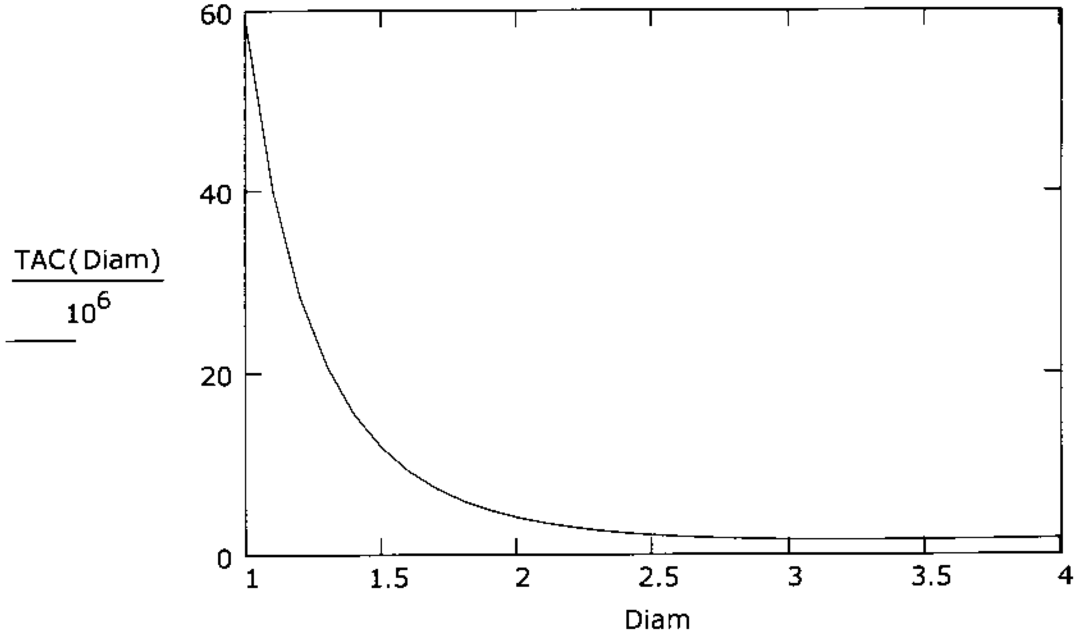
$Vol(D) := z(D) \cdot \left(0.25 \cdot \pi \cdot D^2\right) \quad m^3$ Volume of Packing

$Ann\_Fixed\_Cost(D) := Vol(D) \cdot c_{tower} \cdot f_{FAC} \qquad \frac{dollars}{y}$

$Ann\_Var\_Cost(D) := V\_OP\_Cost(G_s(D)) \cdot 3600 \cdot Op\_Hr \qquad \frac{dollars}{y}$

$TAC(D) := Ann\_Fixed\_Cost(D) + Ann\_Var\_Cost(D)$

Diam := 1 , 1.1 .. 4



$F(D) := \frac{d}{dD}TAC(D)$ Take first derivative of Objective Function and set = 0

x := 2.5 Initial Guess Solve for Optimum Diameter below

Given

$F(x) = 0$

$D_{opt} := Find(x)$

$\boxed{D_{opt} = 3.272}$ m Optimum Diameter

$H_{opt} := z(D_{opt})$ $\boxed{H_{opt} = 6.012}$ m Optimum Height

$G_{sOPT} := G_s(D_{opt})$ $G_{sOPT} = 3.814$ $\frac{kg}{s \cdot m^2}$ Mass Velocity at Opt. Diam.

Costs at Optimum

$TAC(D_{opt}) = 1.581 \times 10^6$ \$/y Total Annual Cost

$Ann\_Fixed\_Cost(D_{opt}) = 404.342$ \$/y Total Annual Fixed Cost

$Ann\_Var\_Cost(D_{opt}) = 1.581 \times 10^6$ \$/y Total Annual Variable Cost

Optimum primarily depends upon variable costs in this problem.

PROB. 9.4

Let H_{op} = hrs. operation / yr.

δ_{ins} = insulation thickness (m)

c_{FC} = annual fixed cost associated w/ insulation / m of ins

c_H = cost of heat as \$/ J

ΔT = Temp. Driving Force (°K)

k = thermal conductivity (W/m-°K)

h_{air} = Air film heat transf. coefficient W/m²-°K

ⓐ Neglect air film resistance

∴ at steady state

$$q = \frac{Q}{A} = \frac{k\,\Delta T}{\delta_{ins}} \Rightarrow \frac{\text{Cost Heat}}{m^2\text{-}y.} = \frac{k\Delta T\, H_{op}\, c_H}{\delta_{ins}}$$

$$\frac{\text{Yearly Fixed Charges}}{m^2} = c_{FC}\,\delta_{ins} \quad \text{(By Prob. Statement)}$$

$$\Rightarrow \frac{\text{Total Annual Cost}}{m^2} = \frac{k\Delta T\, H_{op}\, c_H}{\delta_{ins}} + c_{FC}\,\delta_{ins}$$

$$\frac{d}{d\delta_{ins}}\left[\frac{\text{Total Ann. Cost}}{m^2}\right]\Bigg|_{\delta_{opt}} = 0 = -\frac{k\Delta T\, H_{op}\, c_H}{\delta_{opt}^2} + c_{FC}$$

$$\therefore \boxed{\delta_{opt} = \left[\frac{k\,\Delta T\, H_{op}\, c_H}{c_{FC}}\right]^{1/2}}$$

ⓑ with air film resistance included

$$q = U\Delta T \quad \text{where } \frac{1}{U} = \frac{\delta_{ins}}{k} + \frac{1}{h_{AIR}}$$

$$\therefore U = \frac{k\,h_{AIR}}{k + \delta_{ins}\,h_{air}}$$

$$\Rightarrow q = \frac{k\,h_{air}}{k + \delta_{ins}\,h_{air}}\,\Delta T$$

PROB. 9.A (CON'T)

ⓑ CON'T

$$\frac{\text{Tot. Ann. Cost}}{m^2} = \frac{k h_{air} \Delta T H_{op}}{k + \delta_{ins} h_{air}} c_H + \delta_{ins} c_{FC}$$

Taking the Derivative with respect to δ_{ins}

$$\left.\frac{d(\text{Tot. Ann. Cost})}{d\,\delta_{ins}}\right|_{\delta_{OPT}} = 0 = -k h_{air} \Delta T c_H \frac{h_{air} H_{OP}}{(k + \delta_{OPT} h_{air})^2} + c_{FC}$$

$$\therefore \quad (k + \delta_{OPT} h_{air})^2 = \frac{k h_{air}^2 \Delta T c_H H_{OP}}{C_{FC}}$$

and $$\boxed{\delta_{OPT} = \left[\frac{k \Delta T c_H H_{OP}}{c_{FC}}\right]^{1/2} - \frac{k}{h_{air}}}$$

Problem 9.5 Evaporator

MathCad File

Objective Function = Production over 30 days
Decision Variable = Time onstream if cleaning time is 6 hours

Input Data

$R_1 := 5000\ \frac{kg}{h}$ at $\theta_1 := 0$ and $R_2 := 2500\ \frac{kg}{h}$ at $\theta_2 := 48$ h

$\theta_C := 6$ h

As noted on p.396, since the rate of production is proportional to U, $R = \frac{dP}{d\theta}$ will satisfy a relation of the form $\frac{1}{R^2} = a*\theta + b$

If we let $P(\theta_B)$ be the production up to θ_B, we can obtain the value by integrating the rate, R, from 0 to θ_B, namely,

$$P(\theta_B, a, b) := \int_0^{\theta_B} \frac{1}{(a \bullet \theta + b)^{0.5}}\, d\theta$$

Therefore

$$P(\theta_B, a, b) \rightarrow 2. \bullet \frac{(\theta_B \bullet a + b)^{\frac{1}{2}}}{a} - 2. \bullet \frac{b^{\frac{1}{2}}}{a}$$

Therefore

$$P_{30}(\theta_B, a, b, \theta_C) := P(\theta_B, a, b) \bullet \frac{(24 \bullet 30)}{\theta_B + \theta_C}$$

$$\frac{d}{d\theta_B} P_{30}(\theta_B, a, b, \theta_c) \rightarrow \frac{720.0}{(\theta_B \bullet a + b)^{\frac{1}{2}} \bullet (\theta_B + \theta_c)} - 720 \bullet \frac{\left[2. \bullet \frac{(\theta_B \bullet a + b)^{\frac{1}{2}}}{a} - 2. \bullet \frac{b^{\frac{1}{2}}}{a}\right]}{(\theta_B + \theta_c)^2}$$

The constants "a" and "b" may be evaluated from the given data

$$a := \frac{\left[\left(\frac{1}{R_2^{\,2}}\right) - \left(\frac{1}{R_1^{\,2}}\right)\right]}{\theta_2 - \theta_1} \qquad a = 2.5 \times 10^{-9}$$

$$b := \frac{1}{R_1^{\,2}} \qquad b = 40 \times 10^{-9}$$

Set the derivative P_{30} equal to zero and solve for the optimal θ_B

$$F(\theta_B) := \frac{d}{d\theta_B} P30(\theta_B, a, b, \theta_C)$$

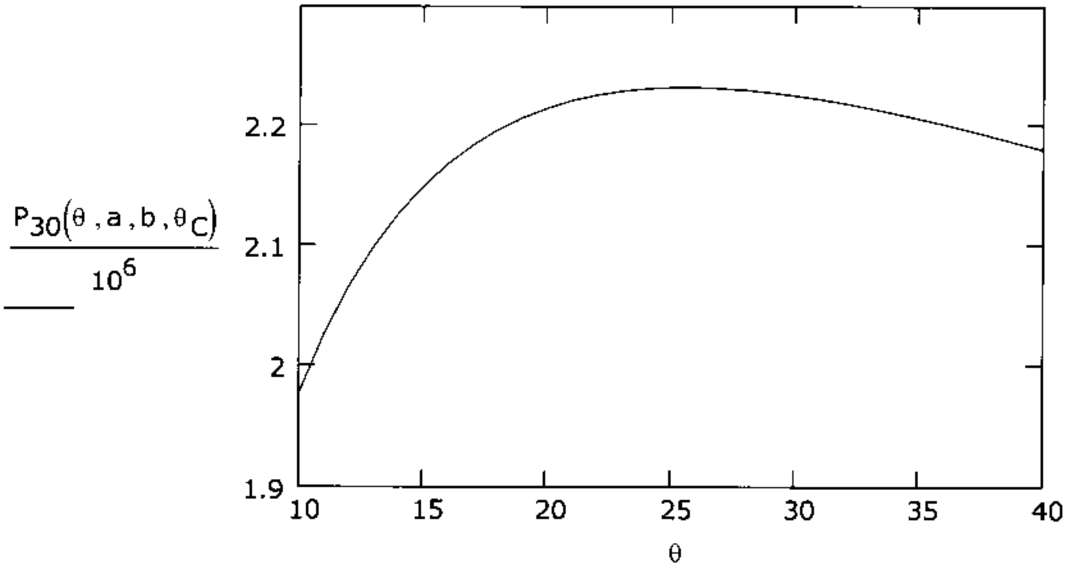
Initial Guess $\theta := 20$

Given

$F(\theta) = 0$

$\theta_{bOPT} := \text{Find}(\theta)$ $\boxed{\theta_{bOPT} = 25.596}$

$\theta := 10, 11 .. 40$ Define range for plotting



Problem 9.6 Solvent Extraction

MathCad File

Objective Function = Minimum Cost per Year
Decision Variable = Tower Diameter

Input Data:

$L_f := 40 \ \frac{m^3}{d}$ Feed Rate $\quad Op\_h := 24 \ \frac{h}{da} \quad Op\_d := 300 \ \frac{da}{y}$

$v_{allow} := 12.2 \ \frac{m}{h}$ recommended velocity of solvent and feed combined

Let F_{sf} = m^3 of solvent/m^3 of feed

$C_F(F_{sf}) := 8800 \cdot F_{sf}^2 - 51000 \cdot F_{sf} + 110000$ Fixed Charges (\$/y)

$C_{OP}(F_{sf}) := 1.41 \cdot L_f \cdot F_{sf} \cdot Op\_d$ Operating Costs (\$/y)

Solution

$C_{total}(F_{sf}) := C_F(F_{sf}) + C_{OP}(F_{sf})$ Total Cost (\$/y)

The optimum value of F_{sf} is found by setting the derivative of the total cost equal to zero and solving for the optimum F_{sf}

$g(F_{sf}) := \frac{d}{dF_{sf}} C_{total}(F_{sf}) \qquad \frac{d}{dF_{sf}} C_{total}(F_{sf}) \rightarrow 17600 \cdot F_{sf} - 34080.00$

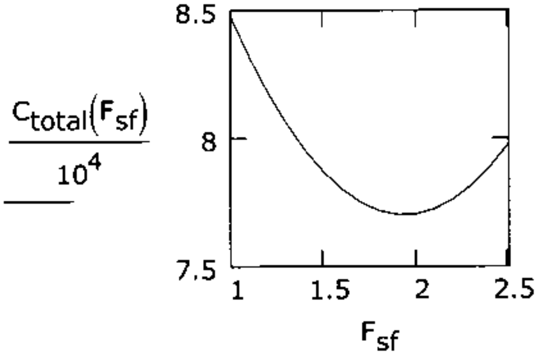
Setting this equal to zero, one obtains

$F_{sfOPT} := \frac{34080}{17600} \qquad F_{sfOPT} = 1.936$

Now we can determine the Tower diameter using the allowable total velocity

$$D_{opt} := \left[\left(\frac{4}{\pi}\right) \cdot \left(\frac{40}{24}\right) \cdot \left(1 + F_{sfOPT}\right)\right]^{0.5} \qquad \boxed{D_{opt} = 2.496} \ m$$

$F_{sf} := 1.0, 1.1 .. 2.5$



Problem 9.7 Optimum Pipe Diameter vs Flow Rate MathCad File

For the range of viscosity considered, the viscosity term will range from 1 to 1.08 in the equation for diameters less than 0.0254m and from 1 to 1.077 in the equation for diameters greater than 0.0254 for the stated viscosity range. Therefore, we will plot $y = D_{I,opt}/\mu_{CP}{}^{0.025}$ and $z = D_{I,opt}/\mu_{CP}{}^{0.025}$ in the two cases where μ_{CP} is in cP.

From the equations on page 404 modified to account for viscosity in cP,

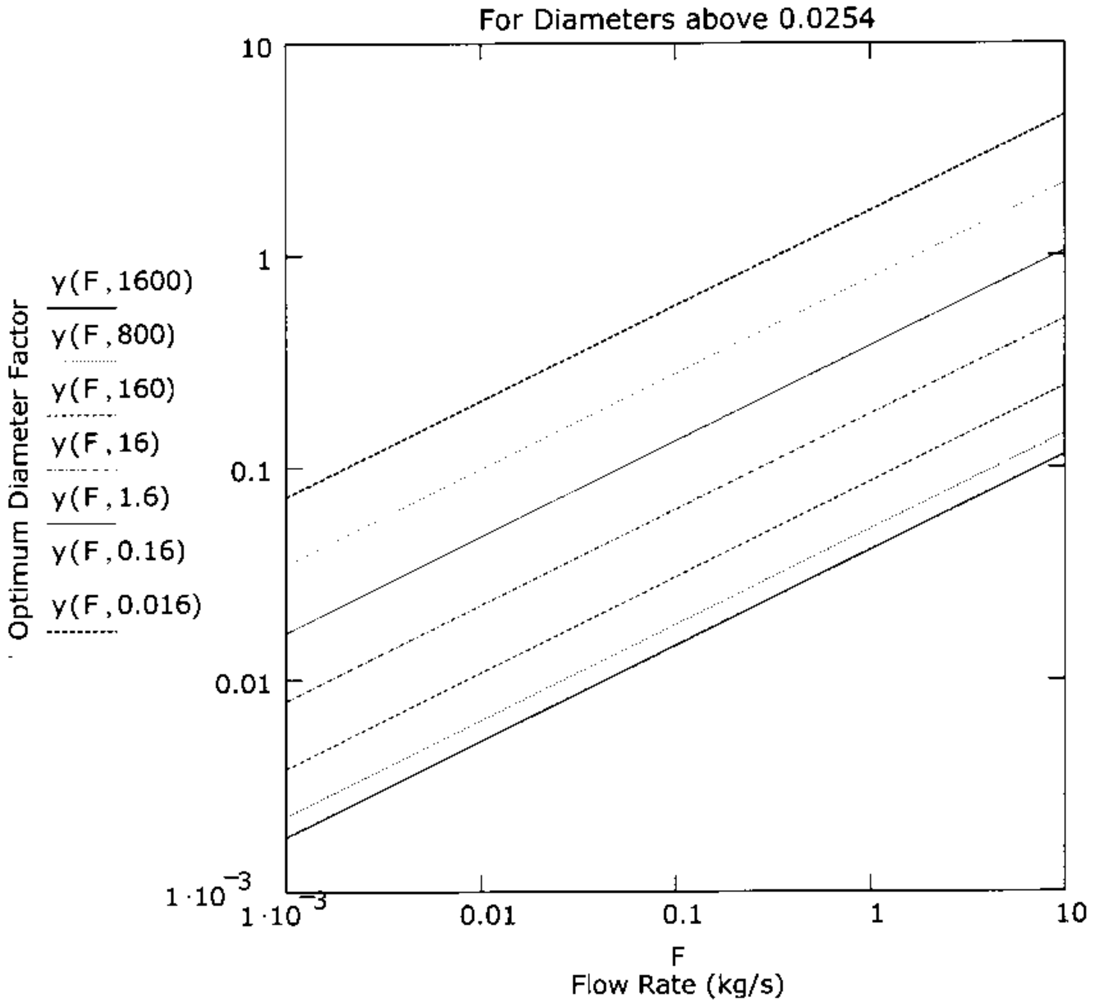
Turbulent Flow with Diameter greater than or equal to 0.0254

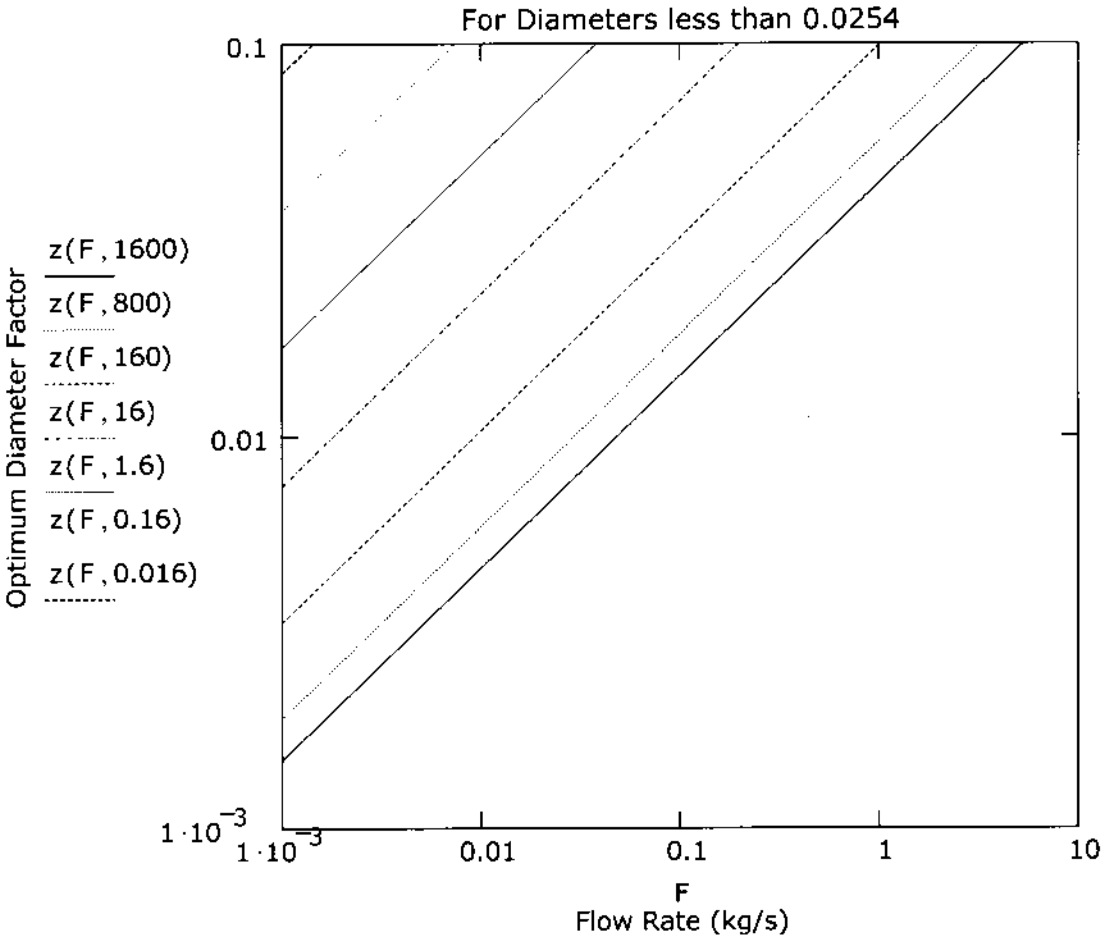
$$y(F_m, \rho) := 0.363 \cdot (1000)^{0.025} \cdot F_m{}^{0.45} \cdot \rho^{-0.32} \quad = D_{I,opt}/\mu_{CP}{}^{0.025}$$

Turbulent Flow with Diameter less than or equal to 0.0254

$$z(F_m, \rho) := 0.49 \cdot (1000)^{0.025} \cdot F_m{}^{0.49} \cdot \rho^{-0.35} \quad = D_{I,opt}/\mu_{CP}{}^{0.027}$$

$F := 0.001, 0.01 .. 10$ Define the range of flow rates to be plotted




For Diameters less than 0.0254
Optimum Diameter Factor
z(F, 1600)
z(F, 800)
z(F, 160)
z(F, 16)
z(F, 1.6)
z(F, 0.16)
z(F, 0.016)
0.1
0.01
1·10^-3
1·10^-3
0.01
0.1
1
10
F
Flow Rate (kg/s)

Problem 9.8 Optimum Pipe Diameter vs Velocity MathCad File

The basis will be eqs. 9-76 and 9-77 on p.404, namely,

TURBULENT FLOW and diameter greater than or equal to 0.0254m

(1) $D_{I,opt}=0.363*(q_f^{0.45})*(\rho^{0.13})*(\mu^{0.025})$ with q_f in m^3/s

ρ in kg/m^3

μ in Pa-s

TURBULENT FLOW and diameter less than or equal to 0.0254m

(2) $D_{I,opt}=0.49*(q_f^{0.49})*(\rho^{0.14})*(\mu^{0.02})$ with q_f , ρ, μ in same units

If we substitute $q_f = v*\pi*(D_{I,opt})^2$ and $\mu = \mu_{CP}$(to place viscosity in units of cP) into the above equations, we obtain, respectively,

(1') $(D_{I,opt})^{0.1}=(7.221x10^{-2})*(v^{0.45})*(\rho^{0.13})*(\mu_{CP}{}^{0.025})$

(2') $(D_{I,opt})^{0.02}=(1.0347)*(v^{0.49})*(\rho^{0.14})*(\mu_{CP}{}^{0.027})$

Solving these equations for $D_{I,opt}$ and, similar to Problem 9.7, defining $y = D_{I,opt}/\mu_{CP}{}^{0.25}$ and $z = D_{I,opt}/\mu_{CP}{}^{1.35}$ to keep only ρ and v on the right hand side, we obtain:

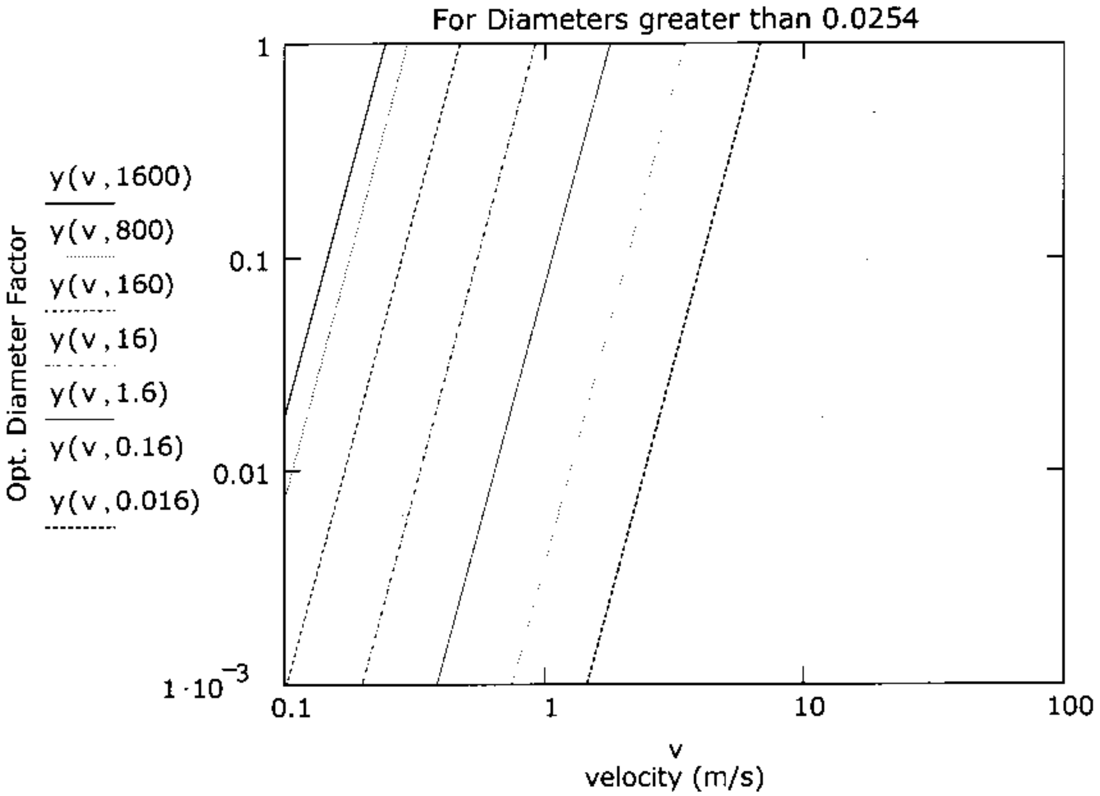
(1'') $y = D_{I,opt}/\mu_{CP}{}^{0.25} = (3.855x10^{-2})*(v^{4.5})*(\rho^{1.3})$

(2'') $z = D_{I,opt}/\mu_{CP}{}^{1.35} = (5.505)*(v^{24.5})*(\rho^{7})$

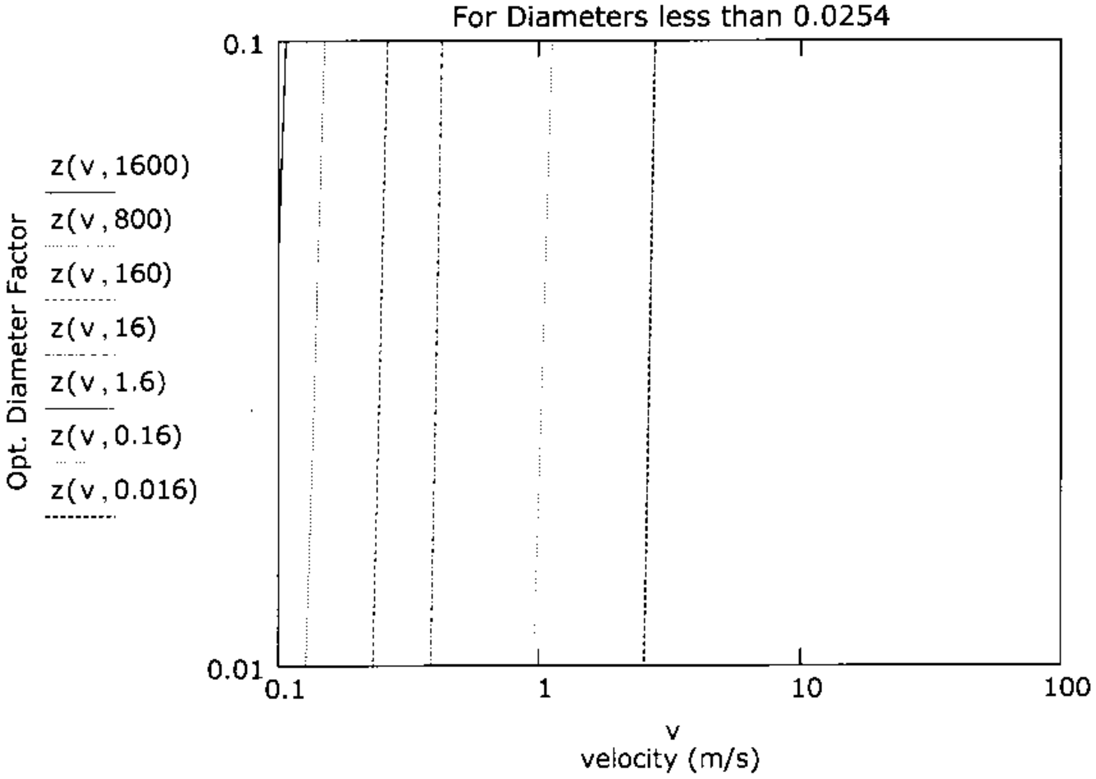
$v := 0.1, 2 .. 30$ Define range of values of v for plotting

$y(v,\rho) := 3.885 \cdot 10^{-2} \cdot v^{4.5} \cdot \rho^{1.3}$ for diameters greater than 0.0254m

$z(v,\rho) := 5.505 \cdot v^{24.5} \cdot \rho^{7}$ for diameters less than 0.0254m



For Diameters greater than 0.0254



For Diameters less than 0.0254

Problem 9.9 Evaporator - Minumum Total Cost MathCad File

Objective Function: Minimum Total Cost
Decision Variable: Cycle Time

This approach is discussed on pp.399-400.

The problem statement gives the correlation

$(1/U)^2=(6.88\text{x}10^{-5})*\theta_B + 0.186$ where U is in kJ/s-m^2-K and θ_B is in s.

If we want U in kJ/h-m^2-K and θ_B in hours, the correlation becomes:

$(1/U)^2=(6.88\text{x}10^{-5}/3600)*\theta_B + (0.186/3600^2)$

Therefore

$a := \frac{\left(6.88 \cdot 10^{-5}\right)}{(3600)}$ $\quad a = 1.911 \times 10^{-8}$ $\quad c := \frac{0.186}{(3600)^2}$ $\quad c = 1.435 \times 10^{-8}$

Other Input Data:

$\lambda := 2300 \ \frac{\text{kJ}}{\text{kg}}$ $\quad A := 37 \ \text{m}^2$ $\quad \Delta T := 40 \ \text{C}$

$\theta_C := 4$ h $\quad C_C := 100$ \$ per cycle $\quad S_B := 20$ \$ per hour

Part a

$F_m := 30000 \ \frac{\text{kgH2Oevaporated}}{\text{da}}$

From equation 9-61

$$\theta_{Bopt} := \left(\frac{C_C}{S_B}\right) + \left[\left(\frac{2}{a \cdot S_B}\right) \cdot \left(a \cdot c \cdot C_C \cdot S_B\right)^{0.5}\right] \qquad \boxed{\theta_{Bopt} = 8.875} \ \text{h}$$

$$\theta_T := \theta_{Bopt} + \theta_C \qquad \boxed{\theta_T = 12.875} \ \text{h}$$

Check using available time

$H_{avail} := 30 \cdot 24$ h $\qquad Q_H := F_m \cdot 30 \cdot \lambda$ kJ

$$\theta_t := \left[\frac{\left(2 \cdot A \cdot H_{avail} \cdot \Delta T\right)}{\left(a \cdot Q_H\right)}\right] \cdot \left[\left(a \cdot \theta_{Bopt} + c\right)^{0.5} - c^{0.5}\right] \qquad \boxed{\theta_t = 16.653} \ \text{h}$$

Since the min. cost opt. is less than the available, we may use θ_{Bopt}

Part b

$F_{m2} := 37000 \ \frac{\text{kgH2Oevaporated}}{\text{da}}$

Using the results of part (a), we have for the available cycle time

$\theta_{t2} := \theta_t \cdot \left(\frac{F_m}{F_{m2}}\right)$ $\qquad \theta_{t2} = 13.503$ h

Therefore, the available cycle time is still greater that the cycle time for min. cost.

✱ If, however, the new rate had been 40000

$F_{m2} := 40000 \; \frac{\text{kgH2Oevaporated}}{\text{da}}$ $\qquad \theta_{t2} := \theta_t \cdot \left(\frac{F_m}{F_{m2}}\right)$ $\qquad \theta_{t2} = 12.49$ h

Therefore, the available cycle time would be less than the cycle time for min. cost and one would have to determine the cycle time by solving the following equation:

$\theta_b := 8$ Initial Guess

Given

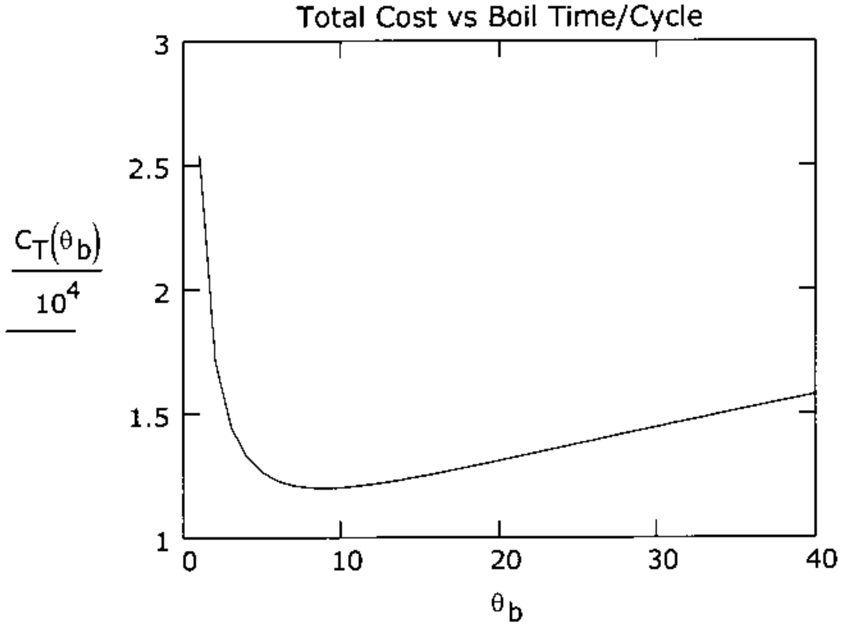
$$\theta_C + \theta_b = \left[\frac{(2 \cdot A \cdot H_{avail} \cdot \Delta T)}{(a \cdot Q_H)}\right] \cdot \left[(a \cdot \theta_b + c)^{0.5} - c^{0.5}\right]$$

$\theta_b := \text{Find}(\theta_b)$ $\qquad \boxed{\theta_b = 2.083}$ h

If the initial guess had been 14, the solution would have converged to 32.5 h

$$C_T(\theta_b) := \frac{\left[a \cdot Q_H \cdot (C_C + S_B \cdot \theta_b)\right]}{2 \cdot A \cdot \Delta T \cdot \left[(a \cdot \theta_b + c)^{0.5} - c^{0.5}\right]}$$

$\theta_b := 1, 2 .. 40$ range for plotting



Problem 9.10 Maximum Profit per kg Product Math Cad File

Problem Info:

$C\_X := 0.2$ \$/kg X $C\_Y := 0.1$ \$/kg Y $SP\_Z := 1.75$ \$/kg Z

$C\_Other := 0.5 \cdot SP\_Z$ $C\_Other = 0.875$

If we set $kg_Z = 1$ in the given relationship between kg_X, kg_Y, and kg_Z, we can solve for kgX_Z and kgY_Z which we define as the kg_X/kg_Z and kg_Y/kg_Z respectively. The given relationship, therefore becomes:

$$1 = (1.5)^2[1.1*kgX\_Z + 1.3*kgY\_Z - kgX\_Z*kgY\_Z]$$

Solving for kgY_Z as a funtion of kgX_Z, we obtain:

$$kgY\_Z = [1 - 2.475*kgX\_AZ] / [2.925 - 2.25*kgX\_Z]$$

Defining this relation as a function

$$kgY\_Z(kgX\_Z) := \frac{(1 - 2.475 \cdot kgX\_Z)}{(2.925 - 2.25 \cdot kgX\_Z)}$$ kgY_Z as a function of kgX_Z

$$C\_kgZ(kgX\_Z) := C\_X \cdot kgX\_Z + C\_Y \cdot kgY\_Z(kgX\_Z) + C\_Other$$ *Cost/kg Z*

$$P\_kgZ(kgX\_Z) := SP\_Z - C\_kgZ(kgX\_Z)$$ Profit/kg Z

Note: In this case maximizing the profit is the same as minimizing the cost. Also, the optimum is not a function of the selling price or the cost of items other than the raw material costs. Therefore, the optimum coulod be determined by minimizing the raw material costs.

To determine the optimum value of kgX_Z, we take the derivative of the Profit function (or, if one wishes the raw material cost function), set it equal to zero, and solve for kgX_Z

$$\frac{d}{dkgX\_Z}P\_kgZ(kgX\_Z) \rightarrow -.2 + \frac{.2475}{(2.925 - 2.25 \cdot kgX\_Z)} - .225 \cdot \frac{(1 - 2.475 \cdot kgX\_Z)}{(2.925 - 2.25 \cdot kgX\_Z)^2}$$

Defining the optimum value of kgX_Z as x_{opt}

$$F(kgX\_Z) := \frac{d}{dkgX\_Z}P\_kgZ(kgX\_Z)$$ Define the function corresponding to the derivative

Set up to solve for x_{opt}

$x_{opt} := 1.6$ Initial Guess

Given

$F(x_{opt}) = 0$

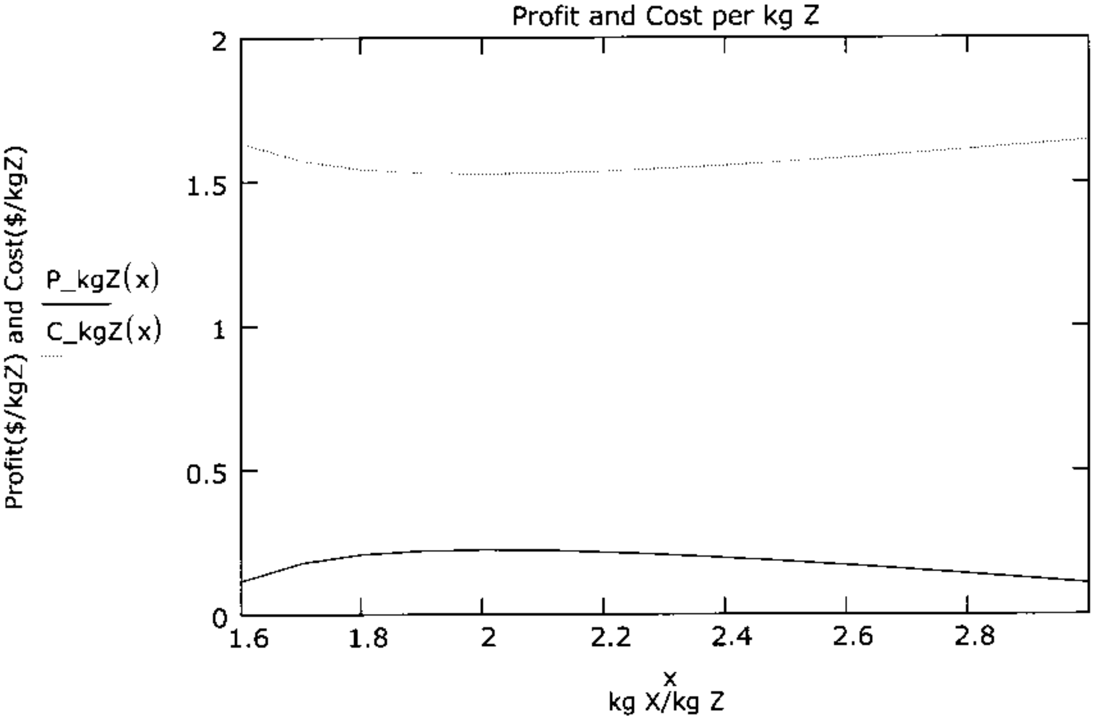
$x_{opt} := Find(x_{opt})$ $\boxed{x_{opt} = 2.002}$ Optimum kg_X/kg_Z

$y_{opt} := kgY\_Z(x_{opt})$ $\boxed{y_{opt} = 2.504}$ Optimum kg_Y/kg_Z

$\boxed{C\_kgZ(x_{opt}) = 1.526}$ Minimum Total Cost/kg_Z

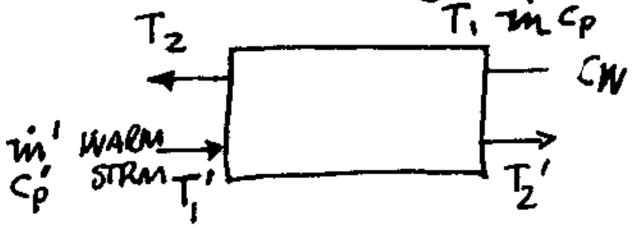
$\boxed{P\_kgZ(x_{opt}) = 0.224}$ Maximum Profit/kg_Z

$x := 1.6, 1.7 .. 3$ Range for plotting



PROB 9-11

Use nomenclature of p. 407 with $F_G \triangleq (\Delta T_{ACTUAL}/\Delta T_{lm})$ and primes to designate items from stream being cooled



$$\dot{q} = \dot{m} c_p (T_2 - T_1)$$
$$= \dot{m}' c_p' (T_1' - T_2')$$
$$= U A F_G \Delta T_{lm} \quad \text{where } \Delta T_{lm} \triangleq \frac{(T_1' - T_2) - (T_2' - T_1)}{\ln\left[\frac{T_1' - T_2}{T_2' - T_1}\right]}$$

$$\left\{\begin{matrix}\text{Total Annual} \\ \text{Cost}\end{matrix}\right\} = C_T = \left\{\begin{matrix}\text{Exchr. Fixed} \\ \text{Cost}\end{matrix}\right\} + \left\{\begin{matrix}\text{Cooling Water} \\ \text{Cost (note in text)}\end{matrix}\right\}$$

But $$\left\{\begin{matrix}\text{Exchr. Fixed} \\ \text{Cost}\end{matrix}\right\} = K_F C_A A = K_F C_A \frac{\dot{q}}{U F_G \Delta T_{lm}}$$

$$\left\{\begin{matrix}\text{Cooling Water} \\ \text{Cost}\end{matrix}\right\} = \dot{m} H_y C_W = H_y C_W \frac{\dot{q}}{c_p (T_2 - T_1)}$$

$$\therefore C_T = \dot{q}\left[\frac{H_y C_W}{c_p (T_2 - T_1)} + \frac{K_F C_A}{U F_G \Delta T_{lm}}\right] \qquad ①$$

To find the T_2 to minimize C_T, say T_{2opt}, we can solve the equation

$$\left.\frac{dC_T}{dT_2}\right|_{T_{2opt}} = 0 \qquad ②$$

From ① and ②

$$\left.\frac{dC_T}{dT_2}\right|_{T_{2opt}} = \dot{q}\left[\frac{H_y C_W}{c_p}\left\{\frac{-1}{(T_{2_{opt}} - T_1)^2}\right\} + \frac{K_F C_A}{U F_G}\left\{\frac{-1}{\Delta T_{lm}^2}\frac{d}{dT_2}(\Delta T_{lm})\right\}\right] = 0$$

PROB. 9-11 (con't)

$$\therefore -\frac{d}{dT_2}(\Delta T_{lm})\Big|_{T_{2,opt}} = \frac{H_y C_W U F_G}{c_p K_F C_A}\frac{(\Delta T_{lm})^2}{(T_{2opt}-T_1)^2}$$ ③

If we let $\Delta T_h = T_1' - T_2$ & $\Delta T_c = T_2' - T_1$

and then $u = \Delta T_h - \Delta T_c$ & $v = \ln\frac{\Delta T_h}{\Delta T_c} = \ln \Delta T_h - \ln \Delta T_c$

$$\frac{d}{dT_2}(\Delta T_{lm}) = \frac{d}{dT_2}\left(\frac{u}{v}\right) = \frac{1}{v}\frac{du}{dT_2} - \frac{u}{v^2}\frac{dv}{dT_2}$$

But $\frac{du}{dT_2} = -1$ & $\frac{dv}{dT_2} = \frac{-1}{\Delta T_h}$

and substituting into ③ with $\Delta T_{lm} = \frac{\Delta T_h - \Delta T_c}{\ln[\Delta T_h/\Delta T_c]}$

$$\frac{1}{\ln\left[\frac{\Delta T_h}{\Delta T_c}\right]} + \frac{1}{\left\{\ln\left[\frac{\Delta T_h}{\Delta T_c}\right]\right\}^2}\frac{1}{\Delta T_h} = \frac{H_y C_W U F_G}{c_p K_F C_A}\frac{1}{(T_{2opt}-T_1)^2}\left\{\frac{\Delta T_h - \Delta T_c}{\ln\left[\frac{\Delta T_h}{\Delta T_c}\right]}\right\}^2$$

Rearranging and cancelling like terms, we obtain

$$\frac{H_y C_W U F_G}{c_p K_F C_A} = (T_{2opt}-T_1)^2\left\{\frac{\ln(\Delta T_{hopt}/\Delta T_c)}{(\Delta T_{hopt}-\Delta T_c)^2} + \frac{1}{\Delta T_{hopt}(\Delta T_{hopt}-\Delta T_c)^2}\right\}$$

$$= \frac{(T_{2opt}-T_1)^2}{(\Delta T_{hopt}-\Delta T_c)^2}\left\{\frac{(\Delta T_{hopt}-\Delta T_c)}{\Delta T_{lm,opt}} + \frac{1}{\Delta T_{opt}}\right\}$$

where $\Delta T_{hopt} = T_1' - T_{2opt}$ & $\Delta T_{lmopt} = \frac{\Delta T_{hopt} - \Delta T_c}{\ln\left[\frac{\Delta T_{hopt}}{\Delta T_c}\right]}$

PROB. 9-12

Temp. of Sat. Stm at 800 kPa ≅ 170.4°C = 443.6°K

$k \cong 0.066 \frac{W}{m\text{-}K}$ (estimated from C. Geankoplis "Transport Proc. and Unit Ops., 3ed., Prentice-Hall) at 148°C

$H_y = 365 * 24 * 3600 = \quad 3.154 \times 10^7 \text{ s/y}$

$T_{surr} = 300\ ^\circ K$

$\therefore\ \Delta T = 443.6 - 300 = 143.6\ ^\circ K$

$C_{ins} = 700\ \$/m^3$

$C_{FIXED} = 0.2 * [\text{Initial Investment}]$

$C_{stm} = 1.42 \times 10^{-9}\ \$/J \Longrightarrow b_c = 1.42 \times 10^{-6}\ \$/kJ$

$a_c = (0.2)(700) = 140\ \$/m^3\text{-}y$

$$\therefore \left(\frac{k\, b_c\, H_y \Delta T}{a_c}\right)^{1/2} = \left[\frac{(0.066 \times 10^{-3})(1.42 \times 10^{-6})(3.154 \times 10^{7})(143.6)}{140}\right]^{1/2}$$

$$= 0.055 = \text{FACTOR}$$

FROM TABLE	0.025 m	0.051 m
@ FACTOR = 0.03	0.010	0.013
@ FACTOR = 0.06	0.024	0.028

BY LINEAR INTERPOLATION

Opt. Econ. Thickness of Insulation = 0.0236

∴ SELECT 0.025 m insulation available

Problem 9.13 Optimum Cycle Time for Maximum Profit MathCad File

Input Information:

$F := 70\ \frac{kg}{da}$ $Op\_da := 300\ \frac{Oper\_da}{y}$ $c_{feed} := 5.50$ \$/kg

$c_{oper} := 300$ \$/oper.da *Cost for Operations* $c_{regen} := 800$ \$/regen *Cost of Regeneration*

$A\_C\_Other := 100000$ \$/y Annual Fixed, O/H, etc.

$Y(\theta) := 0.87 \cdot \theta^{-0.25}\ \frac{kgProd}{kgFeed}$ Product Yield

$P_S := 31$ \$/kgProd *Product Value*

$Prod\_Rate(\theta) := F \cdot Y(\theta)$ *Production Rate*

Assuming the yield for the first day is 0.87

$$Prod(\theta_D) := \int_1^{\theta_D} Prod\_Rate(\theta)\,d\theta + F \cdot Y(1)$$

$$Prod(\theta_D) \rightarrow 81.20 \cdot \theta_D^{\frac{3}{4}} - 20.30$$

$A\_Inc(\theta_D) := Prod(\theta_D) \cdot \frac{Op\_da}{\theta_D} \cdot P_S$ Annual Income as Function of θ_D

$A\_C\_Feed := F \cdot c_{feed} \cdot Op\_da$ Annual Cost of Feed

$A\_C\_Oper := c_{oper} \cdot Op\_da$ Annual Cost for Operations

$A\_C\_Regen(\theta_D) := c_{regen} \cdot \frac{Op\_da}{\theta_D}$ Annual Cost for Regenerations

$A\_C\_Total(\theta_D) := A\_C\_Feed + A\_C\_Oper + A\_C\_Regen(\theta_D) + A\_C\_Other$

$Ann\_Profit(\theta_D) := A\_Inc(\theta_D) - A\_C\_Total(\theta_D)$

The optimum θ_D is found by setting the first derivative of the annual profit to zero and solving for $\theta_{D_{opt}}$

$$F(\theta_D) := \frac{d}{d\theta_D} Ann\_Profit(\theta_D)$$

$$\frac{d}{d\theta_D} Ann\_Profit(\theta_D) \rightarrow \frac{5.664 \cdot 10^5}{\theta_D^{\frac{5}{4}}} - 9300 \cdot \frac{\left(81.20 \cdot \theta_D^{\frac{3}{4}} - 20.30\right)}{\theta_D^2} + \frac{240000}{\theta_D^2}$$

Solve for θ_{Dopt}

Initial Guess $\theta_D := 4$

Given

$$F(\theta_D) = 0$$

$$\theta_{Dopt} := \text{Find}(\theta_D) \qquad \boxed{\theta_{Dopt} = 2.986} \qquad \text{da}$$

$\text{Ann\_Profit}(\theta_{Dopt}) = 1.254 \times 10^5$

$\text{A\_Inc}(\theta_{Dopt}) = 5.113 \times 10^5$

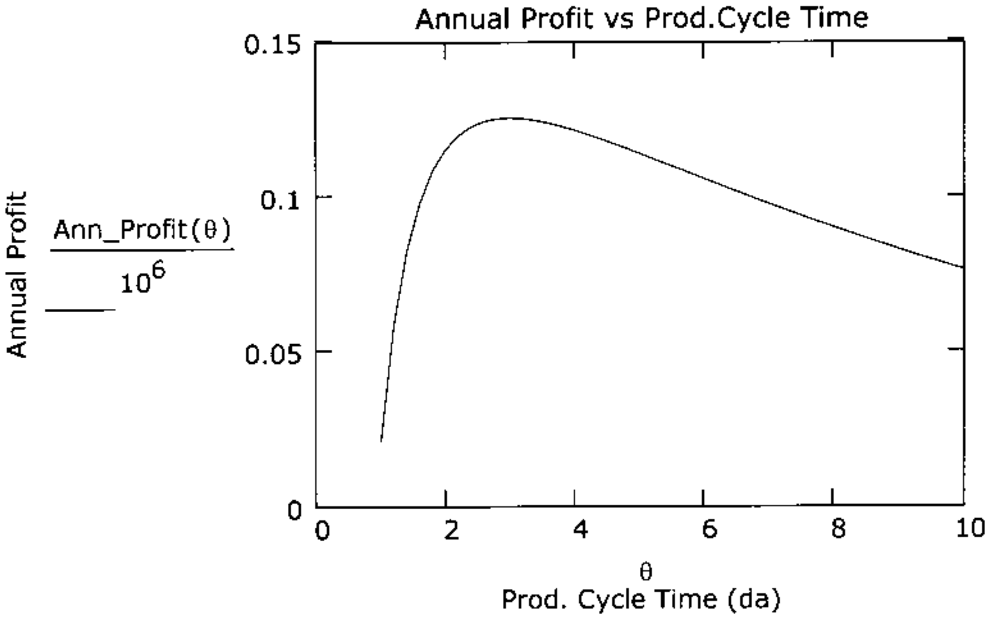
$\text{A\_C\_Feed} = 1.155 \times 10^5$

$\text{A\_C\_Regen}(\theta_{Dopt}) = 8.039 \times 10^4$

$\text{A\_C\_Oper} = 9 \times 10^4$

$\text{A\_C\_Total}(\theta_{Dopt}) = 3.859 \times 10^5$

$\theta := 1, 1.2 .. 10$ range for plotting



PROB. 9-14

$$\dot{Q} = \frac{\Delta T_{ov}}{R_{th,ov}}$$

$$R_{th,ins} = \frac{\ln(r_2/r_1)}{2\pi L k_m}$$ = thermal resist. of insulation

$$R_{th,out} = \frac{1}{(h_c+h_r)_c 2\pi L r_2}$$ = thermal resist. due to conv. and radiation from outer portion of insulation

$$\therefore R_{th,ov} = \left[\frac{\ln(r_2/r_1)}{k_m} + \frac{1}{(h_c+h_r)_c r_2}\right]\frac{1}{2\pi L}$$

$$\Rightarrow \dot{Q} = \frac{2\pi L\,\Delta T_{ov}}{\left[\frac{\ln(r_2/r_1)}{k_m} + \frac{1}{(h_c+h_r)_c r_2}\right]}$$

To find r_{2opt}

$$\left.\frac{d\dot{Q}}{dr_2}\right|_{r_{2opt}} = 0$$

But

$$\frac{d\dot{Q}}{dr_2} = 2\pi L\,\Delta T_{ov}(-1)\frac{1}{\left[\frac{\ln(r_2/r_1)}{k_m} + \frac{1}{(h_c+h_r)_c r_2}\right]^2}\left\{\qquad\qquad\right\}$$

$$\boxed{\frac{1}{r_2}\frac{1}{k_m} + \frac{(-1)}{(h_c+h_r)_c}\frac{1}{r_2^2}}$$

Setting this derivative to zero, we obtain

$$\frac{1}{k_m r_{2opt}} = \frac{1}{(h_c+h_r)_c r_{2opt}^2}$$

$$\therefore r_{2opt} = \frac{k_m}{(h_c+h_r)_c} \quad ; \quad \boxed{D_{opt} = 2r_{2opt} = \frac{2k_m}{(h_c+h_r)_c}}$$

PROB 9-15

Using the nomenclature of pp. 401-404, from eq. 9-66 for turbulent flow,

$$C_{pumping} = \frac{1.248\times10^{-4}\, q_f^{2.84}\, \rho^{0.84}\, \mu_c^{0.16}\, K(1+J)H_y}{D_i^{4.84}\, E} + B'$$

From eq. 9-69, if K_F is factor for fixed charges (incl. maintenance) expressed as fraction of initial cost of installed pipe,

$$C_{pipe} = (1+F)X\left(\frac{D_i}{0.0254}\right)^n K_F$$

In order to use the same variables as text in eq. 9-80, we substitute

$$q_f = \frac{\dot{m}}{\rho} \Rightarrow q_f^{2.84}\, \rho^{0.84} \text{ becomes } \frac{\dot{m}^{2.84}}{\rho^2}$$

$$J = L'_e D_i$$

$$H_y = 24Y \qquad X = X'$$

$$\therefore C_{pumping} = \frac{2.995\times10^{-3}\, \dot{m}^{2.84}\, \mu_c^{0.16}\, KY(1+L'_e D_i)}{\rho^2\, E\, D_i^{4.84}} + B'$$

Using the factor M we may express the initial cost of the pumping installation as:

$$C_{pump.install} = M\, C_{pumping}$$

$$\therefore \left\{\begin{matrix}\text{Total Annual}\\ \text{Cost}\end{matrix}\right\} = C_T = C_{pumping}(1-\Phi) \quad \text{(POWER COST)}$$

$$+\left[C_{pipe}(a+b) + MC_{pumping}(a'+b')\right](1-\Phi) \quad \text{(Depr + Maint. Cost)}$$

$$+\left[C_{pipe} + MC_{pumping}\right]Z \quad \text{(Cost of Capital)}$$

PROB 9-15 (con't.)

or, upon rearranging

$$C_T = C_{pipe}\underbrace{\left[(a+b)(1-\Phi)+Z\right]}_{A_1} + C_{pumping}\underbrace{\left[(1-\Phi)+M(a'+b')(1-\Phi)+MZ\right]}_{A_2}$$

or $C_T = A_1 C_{pipe} + A_2 C_{pumping}$

The optimal pipe diameter is determined from the equation

$$\left.\frac{dC_T}{dD_i}\right|_{D_{opt}} = A_1 \left.\frac{dC_{pipe}}{dD_i}\right|_{D_{opt}} + A_2 \left.\frac{dC_{pumping}}{dD_i}\right|_{D_{opt}} = 0 \quad ①$$

From the expressions for C_{pipe} and $C_{pumping}$

$$\left.\frac{dC_{pipe}}{dD_i}\right|_{D_{opt}} = n(1+F)X'(D_{opt})^{n-1}\frac{1}{(0.0254)^n}$$

$$\left.\frac{dC_{pumping}}{dD_i}\right|_{D_{opt}} = -\frac{2.995\times10^{-3}\,\dot{m}^{2.84}\,\mu_c^{0.16}\,KY}{\rho^2 E}\left[3.84 D_{opt}^{-4.84}L_e' + 4.84 D_{opt}^{-5.84}\right]$$

$$= -\frac{1.45\times10^{-2}\,\dot{m}^{2.84}\,\mu_c^{0.16}\,KY}{\rho^2 E}\left[D_{opt}^{-5.84} + 0.7934 L_e' D_{opt}^{-4.84}\right]$$

Substituting these expressions for the derivatives into ① above and multiplying through by $D_{opt}^{5.84}$, we obtain

$$\frac{D_{opt}^{4.84+n}}{1+0.7934 L_e' D_{opt}} = \frac{(1.45\times10^{-2})(0.0254)^n\,\dot{m}^{2.84}\,\mu_c KY}{\rho^2 E\, n(1+F)X'}\;\frac{A_2}{A_1}$$

where
$$\frac{A_2}{A_1} = \frac{\left[(1-\Phi)+ZM+(a'+b')M\right]}{\left[Z+(a+b)(1-\Phi)\right]}$$

PROB. 9-16 Const. Optimum

The objective function in this expression is xy and the constaint is $x^2 + y^2 = 10$ (this relates the values of x & y).

$$\therefore\ x^2 + y^2 = 10 \Rightarrow y = (10 - x^2)^{0.5}$$

Therefore, the objective function

$$\Phi(x, y) = xy = x(10 - x^2)^{0.5} = \phi(x)$$

$$\Rightarrow \left.\frac{d\phi}{dx}\right|_{x_{opt}} = 0 = \frac{x_{opt}}{\cancel{2}}(10 - x_{opt}^2)^{-0.5}(-\cancel{2}x_{opt}) + (10 - x_{opt}^2)^{0.5}$$

Solving for x_{opt}, we have $x_{opt} = 0\sqrt{5})^{0.5}$

& $y_{opt} = \sqrt{5}$

where only $x \geqq 0$ and $y \geqq$ are being sought.

PROB. 9-17 Const Optimum Using Lagrange Multiplier

Objective Function $u(x, y, z) = x + 2y^2 + z^2$

Constraint $v(x, y, z) = x + y + z - 1$

Forming the Lagrange expression

$$L(x, y, z, \lambda) = x + 2y^2 + z^2 + \lambda(x + y + z - 1)$$

To find the optimum, we set the partial derivative of L with respect to $x, y, z,$ & $\lambda = 0$ and solve

$$\therefore\ \frac{\partial L}{\partial x} = 1 + \lambda = 0 \Longrightarrow \lambda = -1$$

$$\frac{\partial L}{\partial y} = 4y + \lambda = 0 \Longrightarrow y = -\frac{1}{4}\lambda = 0.25$$

PROB 9-17 (CON'T)

$$\frac{\partial L}{\partial z} = 2z+\lambda=0 \Rightarrow z = -\frac{1}{2}\lambda = 0.5$$

$$\frac{\partial L}{\partial \lambda} = x+y+z-1=0 \Rightarrow x = 1-y-z = 0.25$$

$$\therefore \boxed{x_{opt} = 0.25;\ y_{opt} = 0.25;\ z_{opt} = 0.5}$$

PROB 9-18

CASE A: No inflation

ALT. 1: Present Value = $\$5800$

ALT. 2: Present Value = $5000 + 2500(1+0.08)^{-8} = \$6351$

∴ SELECT ALTERNATIVE 1 AS IT HAS LOWEST PRESENT VALUE

CASE B: 7% INFLATION ($\$2500$ IS COST 8 YEARS FROM NOW)

ALT. 1: Present Value = $\$5800$

ALT. 2: $PV = 5000 + \left[(2500)(1+0.07)^{-8}\right](1+0.08)^{-8}$

$= \$5786$

∴ SELECT ALTERNATIVE 2 AS IT HAS LOWEST PRESENT VALUE

(NOTE: INFLATION & COST OF CAPITAL SHOULD BE CONSIDERED SEPARATELY, I.E.

$(1+0.07)^{-8}(1+0.08)^{-8} \neq (1+0.15)^{-8}$!)

Problem 9-19 Optimum ΔT_{min}(Using Example 9-8) Network

Input Data: $\Phi := 0.35$ $\quad$ $MAR := 0.15$

Stream Data:

Stream S1

$CP\_S1 := 50\ \frac{kJ}{s \cdot C}$

$In\_S1\_spec := 20\ C$

$Out\_S1\_spec := 160\ C$

$x_{S1a} := 0.48$

$CP\_S1a := x_{S1a} \cdot CP\_S1$

$CP\_S1b := \left(1 - x_{S1a}\right) \cdot CP\_S1$

Stream S2

$CP\_S2 := 55\ \frac{kJ}{s \cdot C}$

$In\_S2\_spec := 120\ C$

$Out\_S2\_spec := 260\ C$

Stream S3

$CP\_S3 := 30\ \frac{kJ}{s \cdot C}$

$In\_S3\_spec := 280\ C$

$Out\_S3\_spec := 60\ C$

Stream S4

$CP\_S4 := 40\ \frac{kJ}{s \cdot C}$

$In\_S4\_spec := 180\ C$

$Out\_S4\_spec := 20\ C$

Hot Utility

$T\_HOS := 320\ C$

$T\_HOR := 310\ C$

$C_{HO} := 2.25 \cdot 10^{-6}$ \$/kJ

Cold Utility

$T\_CWS := 10\ C$

$T\_CWR := 20\ C$

$C_{CW} := 0.25 \cdot 10^{-6}$ \$/kJ

Exchanger Capital Cost Estimate

$Cost\_Exchr(A) := 1000 \cdot A^{0.6}$ where A is the surface area in m²

Exchanger Overall Heat Transfer Coefficients

$U_{E1} := 1\frac{W}{m^2 \cdot s}$ $\quad$ $U_{E2} := 1\frac{W}{m^2 \cdot s}$ $\quad$ $U_{E3} := 1\ \frac{W}{m^2 \cdot s}$ $\quad$ $U_{E4} := 1\ \frac{W}{m^2 \cdot s}$

$U_{E5} := 1\ \frac{W}{m^2 \cdot s}$ $\quad$ $U_{C1} := 1\ \frac{W}{m^2 \cdot s}$ $\quad$ $U_{H1} := 1\ \frac{W}{m^2 \cdot s}$

Set up calculations for sequence of ΔT_{min}

Pinch Information

$T_{pinch} := 120\ C$ $\quad$ $klast := 81$ $\quad$ $DT_{first} := 2$ $\quad$ $DT_{last} := 22$

$k := 1, 2 .. klast$

$\delta := \frac{\left(DT_{last} - DT_{first}\right)}{klast - 1}$ $\quad$ $\Delta T_{min_k} := DT_{first} + (k - 1) \cdot \delta$ *Sequence of ΔT_{min}*

Define Functions Useful In Calculations

$QDOT(CP, \Delta T) := CP \bullet \Delta T$ Heat Rate

$\Delta T_{LM}(\Delta T1, \Delta T2) := \dfrac{(\Delta T1 - \Delta T2)}{\ln\left(\dfrac{\Delta T1}{\Delta T2}\right)}$ log-mean temperature difference

$Area(QDOT, U, LMTD) := \dfrac{QDOT}{U \bullet LMTD}$ Exchanger Surface Area

For network on p.432 - starting at the pinch - calculations repeated for each k

Exchanger E-3

$In\_S4E3 := In\_S4\_spec$

$Out\_S4E3_k := T_{pinch} + \Delta T_{min_k}$

$In\_S2E3 := In\_S2\_spec$

$\Delta T_k := In\_S4E3 - Out\_S4E3_k$

$DQ_{E3_k} := QDOT(CP\_S4, \Delta T_k)$ $\dfrac{kJ}{s}$ Heat Rate

$Out\_S2E3_k := In\_S2E3 + \dfrac{DQ_{E3_k}}{CP\_S2}$

$\Delta T1_k := In\_S4E3 - Out\_S2E3_k$

$\Delta T2_k := Out\_S4E3_k - In\_S2E3$

$LMTD\_E3_k := \Delta T_{LM}(\Delta T1_k, \Delta T2_k)$

$A_{E3_k} := Area\left(DQ_{E3_k}, U_{E3}, LMTD\_E3_k\right)$

$C_{E3_k} := Cost\_Exchr\left(A_{E3_k}\right)$ \$

Exchanger E-2

$In\_S4E2_k := Out\_S4E3_k$

$In\_S1bE2 := In\_S1\_spec$

$Out\_S1bE2 := T_{pinch}$

$\Delta T := Out\_S1bE2 - In\_S1bE2$

$DQ_{E2} := QDOT(CP\_S1b, \Delta T)$ $\dfrac{kJ}{s}$ Heat Rate

$Out\_S4E2_k := In\_S4E2_k - \dfrac{DQ_{E2}}{CP\_S4}$

$\Delta T1_k := In\_S4E2_k - Out\_S1bE2$

$\Delta T2_k := Out\_S4E2_k - In\_S1bE2$

$LMTD\_E2_k := \Delta T_{LM}(\Delta T1_k, \Delta T2_k)$

$A_{E2_k} := Area(DQ_{E2}, U_{E2}, LMTD\_E2_k)$ m^2

$C_{E2_k} := Cost\_Exchr(A_{E2_k})$ $

Exchanger E-4

$Out\_S1E4 := Out\_S1\_spec$

$In\_S1E4 := T_{pinch}$

$Out\_S3E4_k := T_{pinch} + \Delta T_{min_k}$

$\Delta T := Out\_S1E4 - In\_S1E4$

$DQ_{E4} := QDOT(CP\_S1, \Delta T)$ $\frac{kJ}{s}$ Heat Rate

$In\_S3E4_k := Out\_S3E4_k + \frac{DQ_{E4}}{CP\_S3}$

$\Delta T1_k := In\_S3E4_k - Out\_S1E4$

$\Delta T2_k := Out\_S3E4_k - In\_S1E4$

$LMTD\_E4_k := \Delta T_{LM}(\Delta T1_k, \Delta T2_k)$

$A_{E4_k} := Area(DQ_{E4}, U_{E4}, LMTD\_E4_k)$ m^2

$C_{E4_k} := Cost\_Exchr(A_{E4_k})$ $

Exchanger E-1

$In\_S3E1_k := Out\_S3E4_k$

$Out\_S3E1 := Out\_S3\_spec$

$In\_S1aE1 := In\_S1\_spec$

$Out\_S1aE1 := Out\_S1bE2$

$\Delta T_k := In\_S3E1_k - Out\_S3E1$

$DQ_{E1_k} := QDOT(CP\_S3, \Delta T_k)$ $\frac{kJ}{s}$ Heat Rate

$\Delta T1_k := In\_S3E1_k - Out\_S1aE1$

$\Delta T2 := Out\_S3E1 - In\_S1aE1$

$LMTD\_E1_k := \Delta T_{LM}(\Delta T1_k, \Delta T2)$

$A_{E1_k} := Area(DQ_{E1_k}, U_{E1}, LMTD\_E1_k)$ m^2

$C_{E1_k} := Cost\_Exchr(A_{E1_k})$ \$

Exchanger E-5

$In\_S2E5_k := Out\_S2E3_k$

$In\_S3E5 := In\_S3\_spec$

$Out\_S3E5_k := In\_S3E4_k$

$\Delta T_k := In\_S3E5 - Out\_S3E5_k$

$DQ_{E5_k} := QDOT(CP\_S3, \Delta T_k)$ $\frac{kJ}{s}$ Heat Rate

$$Out\_S2E5_k := In\_S2E5_k + \frac{DQ_{E5_k}}{CP\_S2}$$

$\Delta T1_k := In\_S3E5 - Out\_S2E5_k$

$\Delta T2_k := Out\_S3E5_k - In\_S2E5_k$

$LMTD\_E5_k := \Delta T_{LM}(\Delta T1_k, \Delta T2_k)$

$A_{E5_k} := Area(DQ_{E5_k}, U_{E5}, LMTD\_E5_k)$ m^2

$C_{E5_k} := Cost\_Exchr(A_{E5_k})$ \$

Exchanger C-1

$In\_CWC1 := T\_CWS$

$Out\_CWC1 := T\_CWR$

$In\_S4C1_k := Out\_S4E2_k$

$Out\_S4C1 := Out\_S4\_spec$

$\Delta T_k := In\_S4C1_k - Out\_S4C1$

$DQ_{C1_k} := QDOT(CP\_S4, \Delta T_k)$ $\frac{kJ}{s}$ Heat Rate

$\Delta T1_k := In\_S4C1_k - Out\_CWC1$

$\Delta T2 := Out\_S4C1 - In\_CWC1$

$LMTD\_C1_k := \Delta T_{LM}(\Delta T1_k, \Delta T2)$

$A_{C1_k} := \mathrm{Area}\left(DQ_{C1_k}, U_{C1}, LMTD\_C1_k\right)$ m^2

$C_{C1_k} := \mathrm{Cost\_Exchr}\left(A_{C1_k}\right)$ \$

Exchanger H-1

$In\_S2H1_k := Out\_S2E5_k$

$Out\_S2H1 := Out\_S2\_spec$

$In\_HOH1 := T\_HOS$

$Out\_HOH1 := T\_HOR$

$\Delta T_k := Out\_S2H1 - In\_S2H1_k$

$DQ_{H1_k} := \mathrm{QDOT}\left(CP\_S2, \Delta T_k\right)$ $\frac{kJ}{s}$ Heat Rate

$\Delta T1_k := Out\_HOH1 - In\_S2H1_k$

$\Delta T2 := In\_HOH1 - Out\_S2H1$

$LMTD\_H1_k := \Delta T_{LM}\left(\Delta T1_k, \Delta T2\right)$

$A_{H1_k} := \mathrm{Area}\left(DQ_{H1_k}, U_{H1}, LMTD\_H1_k\right)$ m^2

$C_{H1_k} := \mathrm{Cost\_Exchr}\left(A_{H1_k}\right)$ \$

Cost Summary

$Cost\_Hot_k := DQ_{H1_k} \cdot C_{HO} \cdot 3600 \cdot 24 \cdot 365$ \$/y

$Cost\_Cold_k := DQ_{C1_k} \cdot C_{CW} \cdot 3600 \cdot 24 \cdot 365$ \$/y

$Cost\_Util_k := Cost\_Hot_k + Cost\_Cold_k$

$Cap\_Exchr_k := C_{E1_k} + C_{E2_k} + C_{E3_k} + C_{E4_k} + C_{E5_k} + C_{C1_k} + C_{H1_k}$ \$

$T_{hs_k} := 5 \cdot Cap\_Exchr_k$ \$ Estimated Total Investment Cost

$Ann\_Depr_k := \frac{Cap\_Exchr_k}{7}$ \$/y

$C_{T_k} := Cost\_Util_k \cdot (1 - \Phi) + Ann\_Depr_k \cdot (1 - \Phi) + MAR \cdot T_{hs_k}$ \$/y**Total Annual Cost**

Construct Table of Answers and Plot

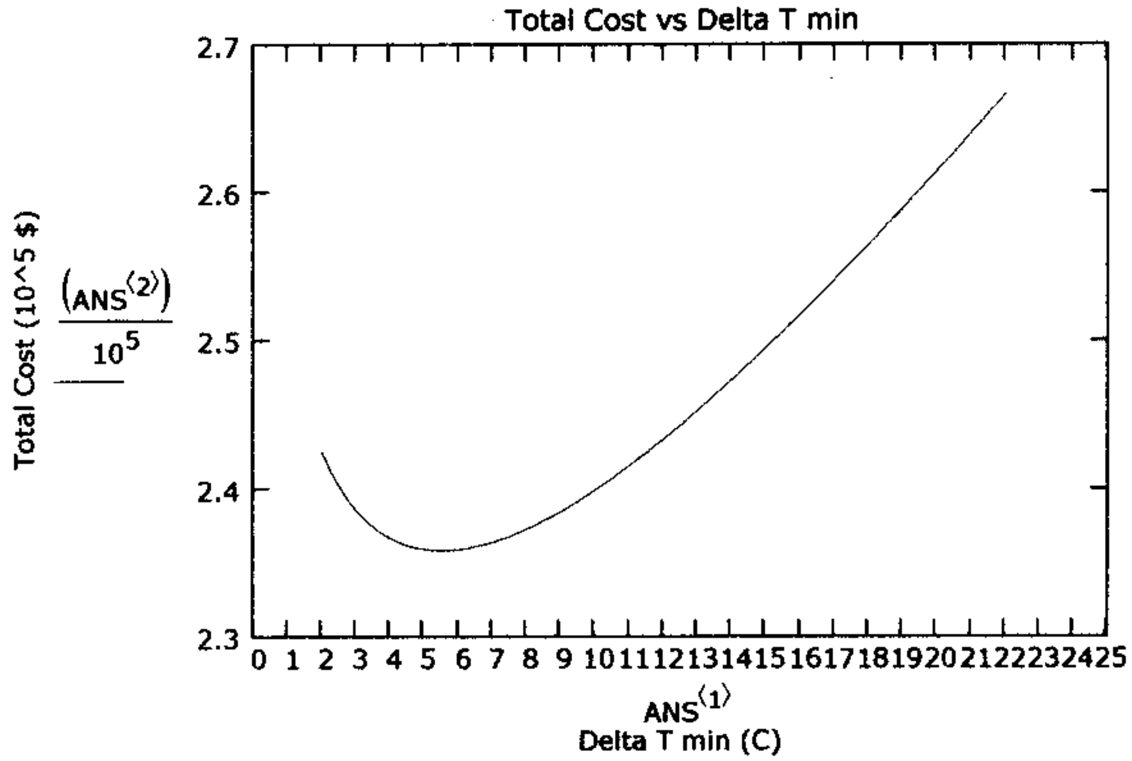
$ANS_{k,1} := \Delta Tmin_k$

$ANS_{k,2} := C_{T_k}$

$ANS_{k,3} := Cost\_Util_k$

ANS =

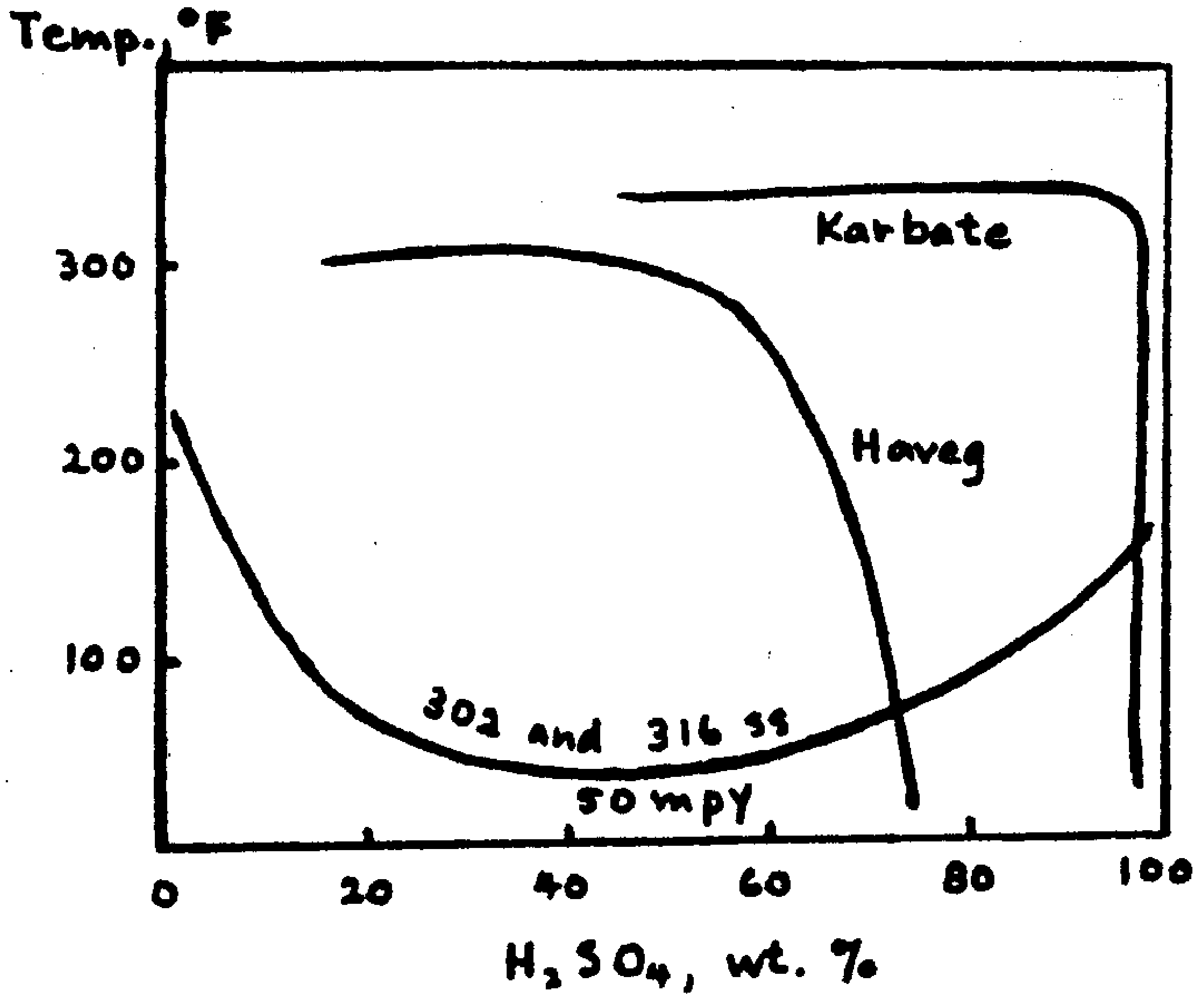
	ΔT_{min}	C_T	C_{UTIL}
	1	2	3
1	2	$2.425\cdot10^5$	$1.99\cdot10^5$
2	2.25	$2.412\cdot10^5$	$2.003\cdot10^5$
3	2.5	$2.402\cdot10^5$	$2.016\cdot10^5$
4	2.75	$2.393\cdot10^5$	$2.03\cdot10^5$
5	3	$2.386\cdot10^5$	$2.043\cdot10^5$
6	3.25	$2.379\cdot10^5$	$2.056\cdot10^5$
7	3.5	$2.374\cdot10^5$	$2.069\cdot10^5$
8	3.75	$2.37\cdot10^5$	$2.082\cdot10^5$
9	4	$2.366\cdot10^5$	$2.096\cdot10^5$
10	4.25	$2.364\cdot10^5$	$2.109\cdot10^5$
11	4.5	$2.362\cdot10^5$	$2.122\cdot10^5$
12	4.75	$2.36\cdot10^5$	$2.135\cdot10^5$
13	5	$2.359\cdot10^5$	$2.148\cdot10^5$
14	5.25	$2.358\cdot10^5$	$2.162\cdot10^5$
15	5.5	$2.358\cdot10^5$	$2.175\cdot10^5$
16	5.75	$2.358\cdot10^5$	$2.188\cdot10^5$

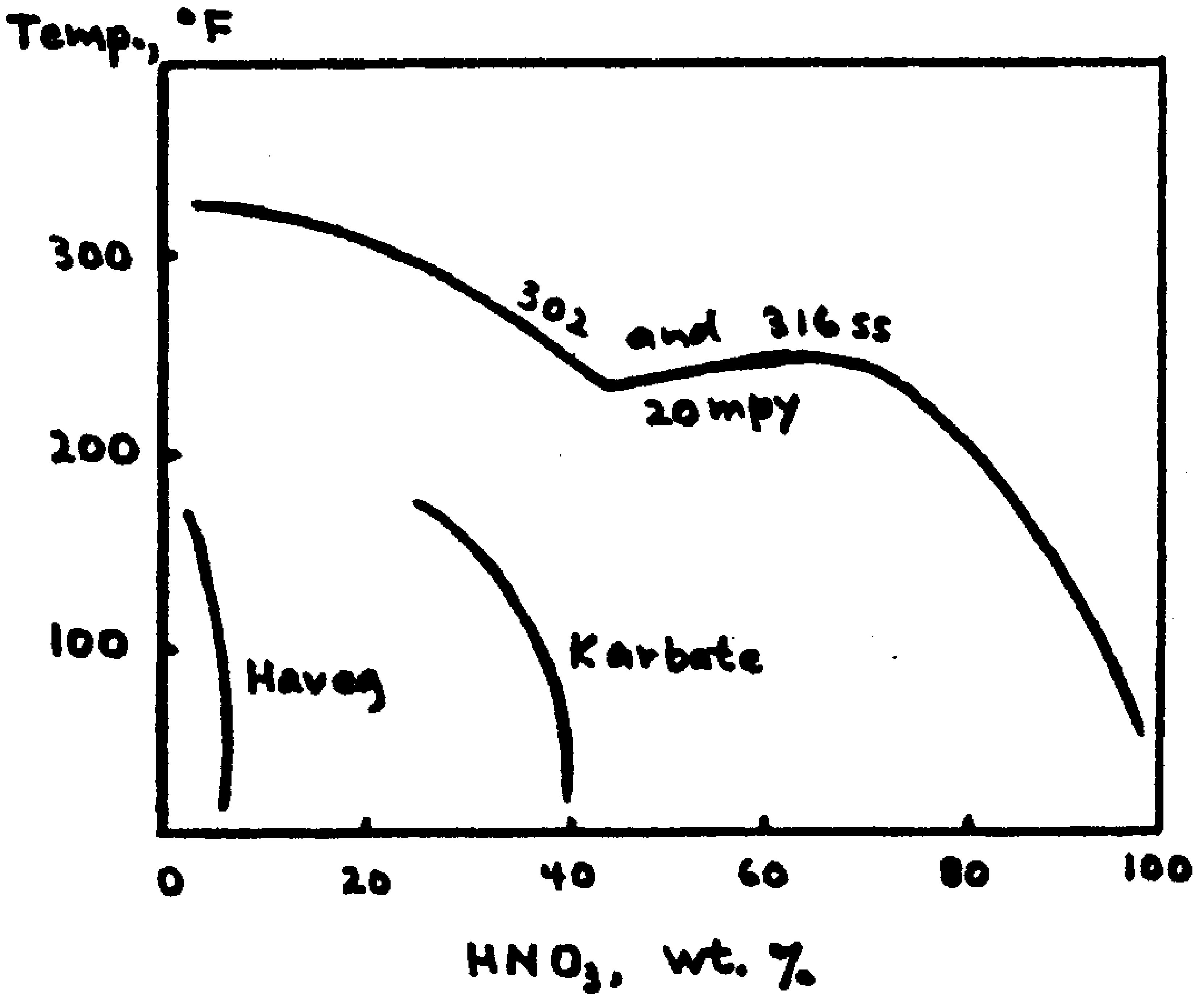


Very flat optimum between ΔT_{min} of 5 to 6 degrees C. Largest exchanger area is 181 m^2.

10-1

Approximate plots showing conditions of generally acceptable corrosion resistance can be constructed from information available in "Perry's Chemical Engineers' Handbook" and regular materials review including corrosion resistance published by Chemical Engineering.





mpy = mils per year

10-2

Allowable corrosion is 3.175 mm or 0.003175 m. Assume interest payable at end of year

	Investment comparison		
	Ni-clad	Monel-clad	Hastelloy
Total installed cost, \$	88,000	105,000	198,000
Additional cost over Ni-clad, \$		17,000	110,000
Estimated life, y	6.35	12.7	27.9
Replacement cost, \$	79,200	94,500	178,200
Annual replacement cost, \$ (replacement cost/est. life)	12,470	7,440	6,390
Annual interest charge, \$	8,800	10,500	19,800
Total annual cost, \$	21,270	17,940	26,190
Annual savings over Ni-clad	–	3,330	NA
Return on investment, % (annual savings/add. cos) 100	–	19.6	NA

The most suitable material of construction for the kettle is Monel-clad.

Answer

10-3

The additional information necessary in the design of the rotary vacuum filter includes values of α, β, μ, and w. (See Chapter 15 of text for nomenclature.) Added information needed to design rotary filters includes: T, Δp, ψ_a, ψ_f, safety factor, vacuum pump efficiency, and N_r.

From laboratory tests with the above data, we can calculate α and β (μ and w can be obtained from lab tests). Thus all the available information is available to calculate the area of the rotary filter under the final conditions using Eq. (15-101). Equations (12-22) to (12-25) and Eqs. (15-102) to (15-110) can be used to calculate the size of the vacuum pump that is required.

Answer

10-4

The density of 316ss is 7850 kg/m^3

The 316ss will only be available as a square sheet that will need to be cut into a circle with a diameter of 0.5 m.

$$m = (0.5)^2(0.05)(7850) = 98.1\ kg$$

Assume machined material is 50% of purchased cost

$$\text{total cost} = (98.1)(2)(\$10.00) = \underline{\$1960}$$

Answer

Alternate method involves estimation of fabrication time

cutting and trimming	6 h
drilling holes	8 h
finishing	8 h
Total hours	22 h

Labor at \$25/h plus 50% overhead added to material cost

$$\text{total cost} = (98.1)(\$10.00) + (22)(1.5)(\$25.00)$$
$$= \underline{\underline{\$1810}}$$

Alternate Answer

10-5

The selection of materials for the available tanks is shown below.

Tanks	Material
Brass-lined	Water
Carbon steel	98% sulfuric acid
Concrete	20% hydrochloric acid
Nickel-lined	10% caustic soda
316 stainless	75% phosphoric acid
Wood	Vinegar

Answer

10-6

Information on cylindrical tank is the following:

$D_i = 3.6\,m$ $L = 11\,m$ $E = 0.9$ $P = 790\,kPa = 690\,kPag$

$T = 25°C$

From Table 12-10 for a cylindrical shell

$$t = \frac{P r_i}{S E_J - 0.6P} + C_c \qquad \text{when } t \le r_i/2$$

S is the maximum allowable working stress in kPa. It is often assumed that this value is essentially one fourth of the ultimate tensile strength at the operating temperature. "Perry's Chemical Engineer's Handbook", 6th ed., p 23-41 provides an ultimate tensile strength of 62,400 psia or 430,000 kPa:

Assume that C_c is negligible relative to the first term

$$t = \frac{(690)(1.8)}{(430{,}000/4)(0.9) - 0.6(690)}$$

$= 0.0128$ or $\underline{\underline{0.013\,m}}$ to take care of C_c

Answer

10-7

This is a standard type of problem treated in any reference dealing with nuclear power reactor design. There are also numerous examples of this type of treatment in the literature including J.R. Envico, "Corrosion of Nuclear Materials," Materials 7(10), 464 (1967) and R.E. Skavdahl and E.L. Zebroski, "Finding Materials for Fast Reactors in the Future," Chem. Eng., 76 (17), 114 (1969). Other reference materials are available in handbook published on this subject.

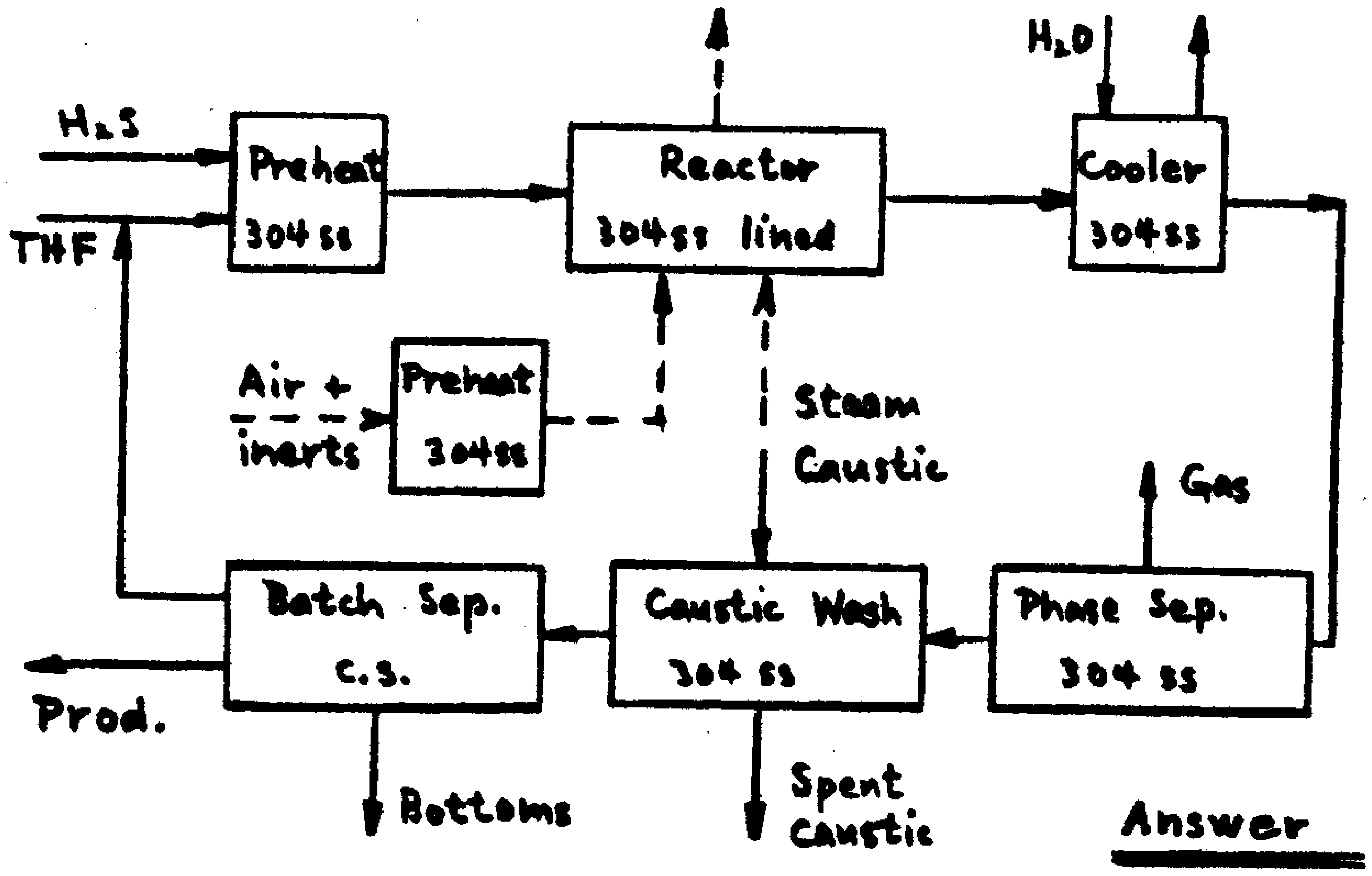
Typical data or information needed for determining whether a given tube diameter and gauge number is satisfactory are given below:

- Type of stainless steel
  - Poisson's ratio
  - Young's modulus
  - Coefficient of thermal expansion
  - Thermal conductivity
  - Creep stress limit
- Type of heating
- Rate of heat transfer
- Properties of fluids for heat transfer calculations
- Temperature at wall surfaces and wall ΔT
- Number of tubes
- Length of tubes
- Safety factors to be considered

Answer

10-8

The general flowsheet for thiophane production is given below. Materials of construction for each unit are included.



Answer

10-9

Two systems were developed in Problem 2-13 to transfer liquid chlorine from a liquid storage container; one using recompressed chlorine vapor and the other using vaporized chlorine. Flow diagrams describe these systems. Please refer back to Problem 2-13 for details. The information below applies to either system.

For dry chlorine gas, 317ss, nickel, Monel, and carbon steel have been the recommended materials of construction. In the event of water contamination of the chlorine gas, carbon steel should not be considered. If there is a potential for leakage of the chlorine, the most acceptable material of construction would be 317ss.

If wet chlorine is involved, other choices, besides 317ss, are Nucerite (a ceramic-metal composite), glass-clad steel or Haveg. The least expensive of these choices generally is 317ss because of its greater flexibity to meet special design conditions.

<u>Answer</u>

10-10

With reference to Fig. 2-1 in the 4th ed. of Plant Design and Economics for Chemical Engineers, each piece of equipment identified in the nitric acid plant is listed below with the most appropriate material of construction. Note that other choices are possible but have not been included because they are generally more expensive.

Piece of equiment	Material choice	
Ammonia vaporizer	cs	
Ammonia filter	cs	
Air filter	cs	
Oxidation chamber	430ss	850°C top temp.
Cooler condenser (lower)	430ss	
Cooler condenser (upper)	430ss	
Absorption tower	430ss	
Converter	316ss	
Mixing chamber	316ss	
Preheater	316ss	
Reheater	316ss	

<u>Answer</u>

11-1

Depending upon the type of report prepared, company guidelines for preparing written reports can vary significantly from the typical outline presented in the chapter. If the report is prepared for use within the design group, it may eliminate the letter of transmittal, title page, and table of contents (particularly if the report is not lengthy.

The body of the report also can be decreased when reporting to the design group who are familiar with the project. For example, the report could provide the recommended design and provide sufficient economic details that strongly support or reject the project. If the report is prepared for the decision makers in the company, greater emphasis will probably be required if investment in the project is recommended. Such a report will generally have a much shorter appendix since the decision makers will not be interested in the calculation details, but how investment in the project will affect the bottom line of the company. The decision makers will assume that the design group has carefully reviewed the design calculations and have made a sensitivity analysis of the consequences of their recommendation before its transmittal.

11-2

Chemical engineering department guidelines for preparing laboratory reports generally follow the typical outline for process design projects as presented in this chapter. Sometimes the departmental guidelines request more information in the body of the report to provide immediate access to the assumptions made and the justification for those assumptions. Also, often more information is required in the analysis of possible sources of error and their potential effect on the final result. Some reports may not require an economic evaluation particularly when only experimental results are being analyzed. In fact, there may be several different outlines depending on the purpose of the report.

11-3

Review of a design report prepared by a colleague will need to be done by the reviewer. Answering the questions posed in in the problem should provide the reviewer with positive suggestions that can be transmitted back to the colleague.

11-4

Every design report must be checked with a spell checker to avoid embarrestment on the part of the writer, but also to emphasize to the reader that the writer is a professional who has made every effort to prepare a technical study of the highest quality.

11-5

There are numerous software packages that prepare various types of visual material. These packages can cover the preparation of simple graphs to the delopment of highly automated visuals. The cost for the packages that prepare simple but instructive graphs are quite modest and can be programmed to provide various ways to display the material of interest using color to provide desired hightlights. The more complex packages are considerably more expensive, but the cost has decreased dramatically in the past decade. Such software packages are capable of displaying animated sequences, halftones, and videos. Because of the competition, these software packages are continuously being improved with greater flexibility for the user. With the constant updating of these software packages it is difficult to specify advantages and disadvantages of a specific package since these change even over a short time period.

Regardless of the software selected, there are numerous guidelines concerning the density of information that is to be used in visuals. The font size of the lettering needs to be adjusted to meet the requirements placed on it by the location of the presentation. Colors on the visual need to be selected that will enhance the images projected. Whatever material that is to be presented should be easily readable by all those attending the presentation.

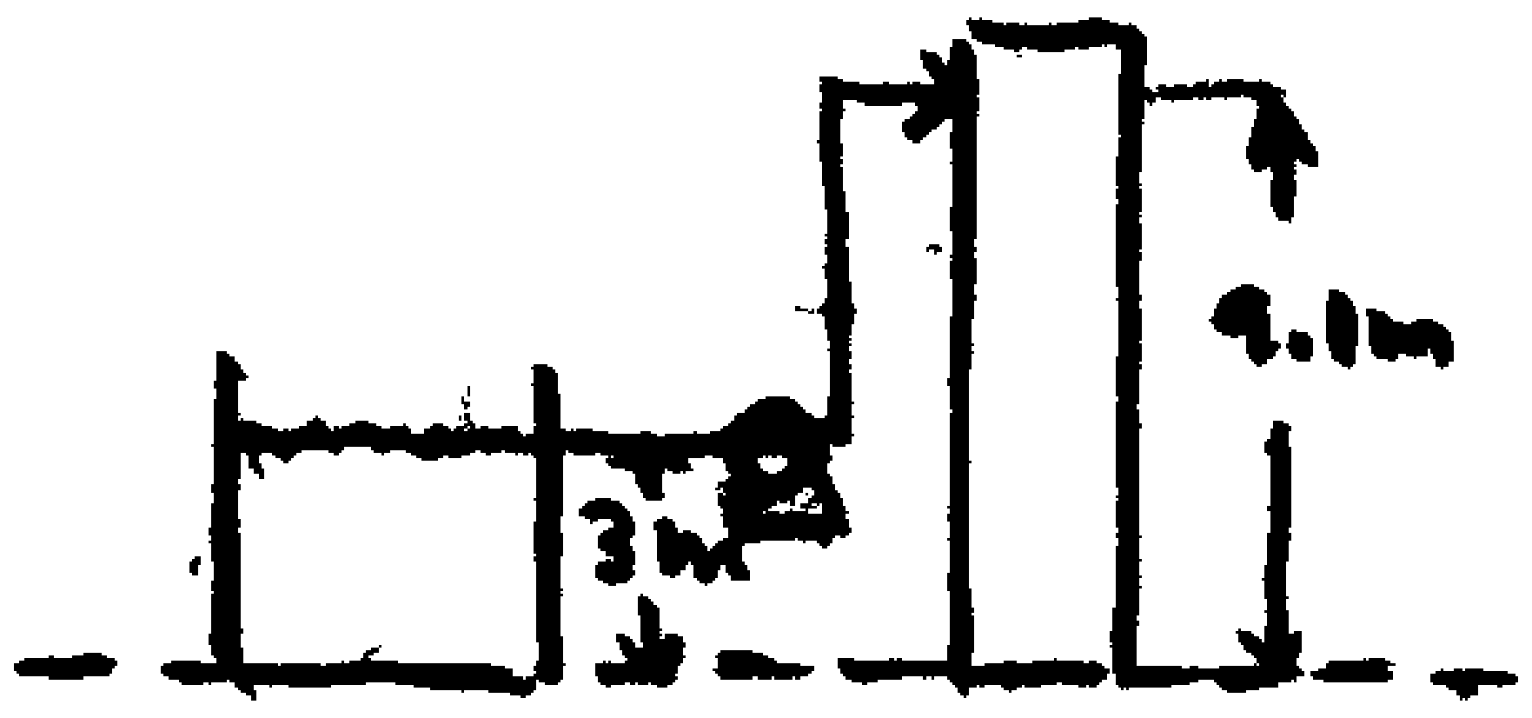
11-6

A skeleton outline and a detailed outline for a final report on the detailed-estimate design of a distillation unit can have many forms. The skeleton outline will probably include the items listed.

1. Introduction to the project
   - Listing of items specifically requested
   - Actions taken to meet those requests
2. Results from the project
3. Recommended design
   - Brief description of proposed design
   - Listing of economics supporting recommendation
4. Conclusions

The detailed outline will be very similar to the outline detailed in Table 11-1. However, in the Discussion section (V.C.) that covers the technical matters of importance there will be a detailed analysis of the steps involved in the deisign of a distillation as outlined in Fig. 15-1 of the text. This analysis will recognize the specific problems that are encountered in obtaining high purity methanol from a water-methanol mixture.

12-1



$D_i = 0.078\text{m}$ $\quad A_i = \pi D_i^2/4 = 4.778 \times 10^{-3}\text{m}^2$

5 - 90° elbows

$\dot{m}_i = 2.7\text{kg/s}$

$\mu_i = 15\text{cP}$

$\rho_i = 857\text{kg/m}^3$

$\eta_p = 0.4$

Find power requirement for pump

$$gz_1 + p_1v_1 + V_1^2/2\alpha + \int pdv + W = gz_2 + p_2v_2 + V_2^2/2\alpha + F$$

$$V = 2.7\frac{\text{kg}}{\text{s}}\ \frac{\text{m}^3}{857\text{kg}}\ \frac{1}{4.778\times10^{-3}\text{m}^2} = 0.659\text{m/s}$$

$$Re = DV\rho/\mu = (0.078)(0.659)(857)/0.015 = 2936$$

From Fig. 12-1, $\epsilon/D = 0.000046/0.078 = 0.00059$ based on commercial steel, $f = 0.011$

$$L_e = 46 + 5(32)(0.078) = 58.5\text{m}$$

$$F = 2fV^2L_e/D$$

$$= (2)(0.011)(0.659)^2(58.5)/0.078 = 7.17$$

Assume $v_1 = v_2$, $\int pdv = 0$ and $V_1 = 0$

$$W = g(z_2 - z_1) + (p_2 - p_1)/\rho + V_2/2 + F$$

$$= (9.806)(6.1) + (445 - 101)/857 + (0.659)^2/2 + 7.17$$

$$\doteq 468.6\ \text{Nm/kg}$$

$$P_{theo} = (468.6)(2.7) = 1265\text{ W} = 1.27\text{ kW}$$

$$P_{act} = (1.265)/0.4 = \underline{\underline{3.18\text{ kW}}} \text{ or } 3.2\text{ kW}$$

Answer

12-2

$\dot{m} = 2.14$ kgs

$L = 21$ m

$D_i = 0.0409$m

$\mu_L = 5\times10^{-4}$ Pa.s

$\rho_L = 849$ kg/m^3

The pressure drop is entirely from friction since there is no elevation change. Equivalent resistances of valves and fittings are obtained from Table 12-1.

Fittings and valves	Equivalent resistance in pipe diameters
6 90° elbows	192
1 tee (used as elbow)	60
1 gate valve (open)	7
1 globe valve (open)	300
	559

Equivalent length = (559)(0.0409) = 22.9 m

Total length of pipe = 21 + 22.9 = 43.9 m

$V_i = (2.14)/(849)(0.785)(0.0409)^2 = 1.92$ m/s

$Re = DV\rho/\mu = (0.0409)(1.92)(849)/0.0005 = 1.33\times10^5$

$\epsilon/D_i = 0.000046/0.0409 = 0.0011$

$f = 0.0053$ from Fig. 12-1

$F = 2fV^2L_e/D = (2)(0.0053)(1.92)^2(43.9)/0.0408$

$= 41.9$ Nm/kg

$\Delta p = F\rho = (41.9)(849) = 35{,}575 \text{ Pa} = \underline{\underline{35.6 \text{ kPa}}}$

Answer

12-3

Volumetric flow rate of cooling water = $0.0095\ m^3/s$

$\rho_L = 996\ kg/m^3 \quad \mu_L = 0.893\ cP$

$\dot{m}_{H_2O} = (0.0095)(996) = 9.46\ kg/s$

Assume that $Re > 2100$, use Eq.(12-15) to determine optimum pipe diameter

$$D_{i,opt} = 0.344\, \dot{m}_v^{0.45}\, \rho^{0.13} \quad \text{where } D_i \text{ is in meters}$$

$$= (0.344)(0.0095)^{0.45}(996)^{0.13} = 0.1038\ m = 4.087\ in.$$

Check Re,

$$Re = DG/\mu = (0.1038)(9.46/0.785(0.1038)^2/0.000893$$

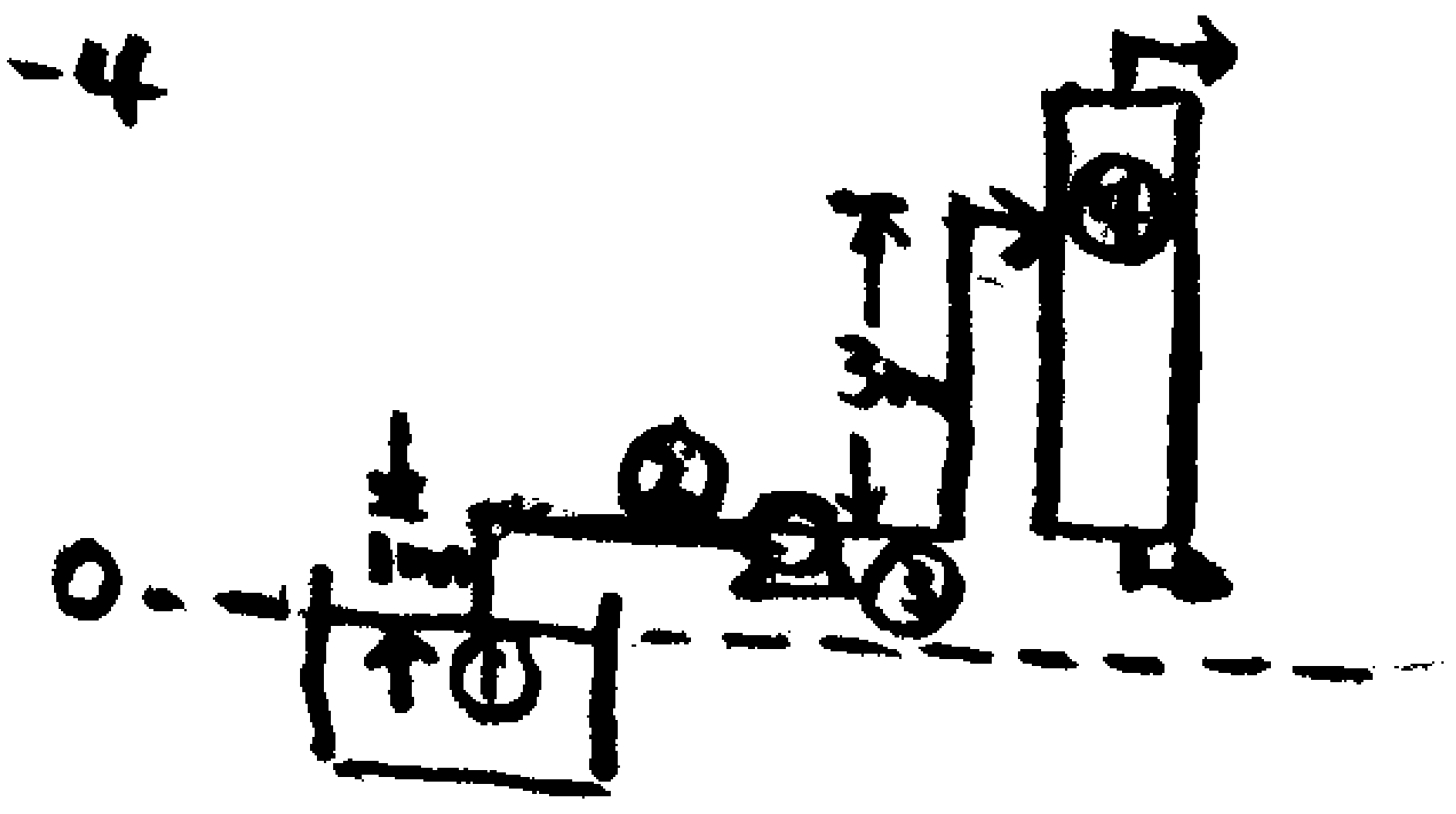
$$= 130{,}000 \quad \therefore Re > 2100$$

Thus, the use of Eq.(12-15) is valid.

The inside diameter of a Schedule 40, 4 in. steel pipe is 4.026 in. The next larger size is 5.047 in.. Therefore, a 4 in., Schedule 40 pipe should be selected even though it is slightly smaller than the computed value of 4.087 in.

Answer

12-4



$T_{82} = 38°C$

$p^{\circ}_{v82} = 26.4\ kN/m^2$

$\rho_{82} = 860\ kg/m^3$

$\dot{m}_v = 0.0025\ m^3/s$

$\eta_p = 0.60$

$p_1 = 101.3\ kPa$

$\Delta p_{1-2} = 3450\ N/m^2$

$\Delta p_{3-4} = 37,900\ N/m^2$

Assume $Re > 2100$, use Eq. (12-15) to obtain optimum pipe diam.

$$D_{i,opt} = 0.344\ \dot{m}_v^{0.45}\ \rho^{0.13} = (0.344)(0.0025)^{0.45}(860)^{0.13} = 0.0559\ m$$

Use a 2in., Schedule 40 (2.067 in or 0.0525 m) pipe for the system.

$$V_4 = (0.0025)/(0.785)(0.0525)^2 = 1.15\ m/s$$

Designate the level of the tank as the datum plane for the pumping system. Since $V_1 = 0$, the mechanical energy balance, Eq. (12-2) can be rearranged into

$$W\eta_p = p_4/\rho + gZ_4 + V_4^2/2\alpha - p_1/\rho + F \quad \text{where } \alpha = 0$$

$$= \frac{445,000}{860} + (9.806)(4) + \frac{(1.15)^2}{2} - \frac{101,300}{860} + \frac{3450 + 37,900}{860}$$

$$= 487.8\ Nm/kg$$

Since $W\eta_p$ also is the head produced, $\underline{\underline{H = 487.8\ Nm/kg}}$

$$\dot{m} = (0.0025)(860) = 2.15\ kg/s$$

Answer

Power input is thus

$$P = (2.15)(487.6)/0.6 = 1747\ kJ/s = \underline{\underline{1.75\ kW}}$$

Answer

It can be shown that $NSPH = (p_1 - p^{\circ}_v - F_s)/\rho g - Z_1$, where p_1 is the pressure at the surface of the tank, p°_v the vapor pressure of the fluid and F the friction in the suction line

$$NSPH = \frac{1}{9.806}\left[\frac{101,300 - 26,400 - 3450}{860}\right] - 1 = 8.47 - 1$$

$$= 7.47\ m$$

The available NSPH is larger than the net positive head of 3m required. The pump should be suitable.

Answer

12-5

$\dot{m}_v = 0.0063\ m^3/s$ at 15°C

$N_s = 1800$ rpm

$\Delta p = 138$ kPa

For an ideal centrifugal pump, the speed of the impeller is proportional to the rate of fluid discharge. The head developed by an ideal pump is proportional to the square of the impeller speed. Thus from Eq. (12-17)

$$m_{v,2} = m_{v,1}\left(\frac{N_{r,2}}{N_{r,1}}\right)$$

$$= (0.0063)(1200/1800) = 0.0042\ m^3/s$$

$$\text{or } \dot{m}_2 = \dot{m}_{v,2}\,\rho = (0.0042)(998) = \underline{\underline{4.19\ kg/s}}$$

Answer

The new head with the reduction in impeller speed is given by

$$H = \left(\frac{N_{r_2}}{N_{r_1}}\right)^2\left(\frac{\Delta p}{\rho}\right)$$

$$= \left(\frac{1200}{1800}\right)^2\left(\frac{138{,}000}{998}\right) = 61.6\ Nm/kg$$

This is equivalent to $(61.6)/(9.806) = \underline{\underline{6.28\ m}}$

Answer

12-6 $\dot{m}_{H_2O} = 15.75\ kg/s$, $\rho_L = 998\ kg/m^3$, $\mu_L = 1.120\ cP$

Assume turbulent flow. From Eq. (12-15)

$D_{i,opt} = 0.344\ m_v^{0.45} \rho^{0.13} = 0.344(15.75/998)^{0.45}(998)^{0.13} = 0.130m.$

Use 5 in nominal steel pipe, Schedule 40, $D_i = 5.047\ in = 0.128\ m$

From Fig. 12-3, $Re = 1280\ \rho \dot{m}_v / D_i \mu$

$= (1280)(998)(15.75/998)/(0.128)(1.120) = 141{,}000$

Flow is turbulent. To use Fig 12-20 to obtain cost of pump and motor need to develop equivalent pressure drop across pump.

$\Delta p = (7.5\ kNm/s)(998\ kg/s)/((15.75\ kg/s) = 475.2\ kPa$

$(m^3/s)(kPa) = (15.75/998)(475.2) = 7.5$

From Fig 12-20, cost of pump and motor is $3600

From Fig. 12-4, cost of 0.128m diameter carbon seamless pipe is $23/m

Pipe cost = (23)(305) = $7015

From Fig 14-4 (4th edition) the cost of fittings indexed to Jan. 1, 2002 is $70 /tee

Fittings cost = (70)(40) = $2800

From Fig. 12-8, the cost of a gate valve, carbon steel, flanged, with a 860 rating, is $310/ valve

Valve cost = (310)(4) = $1240

From Fig. 12-12, the cost of 0.038 m 85% magnesia is $16 /m

Insulation cost = (16)(305) = $4880

Information from Table 6-5 indicates that the cost of installation of pumps is from 25 to 60 % of the purchased equipment cost. Use an average value of 40%, or a factor of 1.4. Installed cost is then

Cost = (3600 + 7015 + 2800 + 1240 + 4880)(1.4) = $27,300

Answer

12-7

Two-step steam jet, suction pressure of 50 mm Hg

Vapor pressure of water at 15°C = 13.25 mm Hg

$$\% \text{ dry air} = \frac{(50-13.25)(29)}{(50-13.25)(29)+(13.25)(18)} = 0.817 \cong 82\%$$

Total weight of mixture/h = (10/0.817) = 12.25 kg/h

From Table 12-7 for 4.5 kg/h of mixture at 82% air, the steam requirement is

$$19 + (12/30)(4) = 20.6 \text{ kg/h}$$

Estimated total steam required for 12.25 kg/h is

$$(20.6)(12.25/(4.5) = \underline{\underline{56 \text{ kg/h}}}$$

12-8

Compress methane at 25°C and 100 kPa to 6200 kPa

Determine the number of stages required. Try two stages:

$(6200/100)^{1/2} = 7.87$ this is too high, try three stages:

$(6200/100)^{1/3} = 3.95$ this is adequate. $\underline{\underline{\text{3 stages required}}}$

The power required/stage is obtained with the use of Eq. (12-22b)

$$P = (2.78\times10^{-4})(0.085)(3600)(10^{5})\left(\frac{1.31}{1.31-1}\right)\left[(3.95)^{0.2366}-1\right](10^{-3})/0.8$$

$$= 17.25\text{ kW}$$

The power required by three stages is

$$p = (3)(17.25) = \underline{\underline{51.75\text{ kW}}}$$

Answer

The exit temperature of the gas leaving the first stage is

$$T_2 = T_1(p_2/p_1)^{(k-1)/k} = (298.1)(3.95)^{0.2366} = 412.6\text{K} = \underline{\underline{139.5\,°\text{C}}}$$

Answer

To obtain the cooling water required for the intercooler and aftercooler involves a heat balance. The heat load for one stage is

$$\dot{q} = \dot{m}C_p\,\Delta T = (0.085)(0.65)(2433)(139.5-25) = 15{,}400\text{ J/s}$$

for three stages this totals to 46,200 J/s

Cooling water (assuming an average temperature of the water of 32°C) required is

$$\dot{m}_{H_2O} = \dot{q}/C_p\,\Delta T = 46{,}200/(4178)(15) = \underline{\underline{0.737\text{ kg/s}}}\text{ or}$$

Answer

$$\underline{\underline{7.4\times10^{-4}\text{ m}^3/\text{s}}}$$

Answer

12-9

The use of a software package in the solution of Problem 12-8 provides results that are about 2% lower than those obtained for the power requirement by the hand calculation. The difference occurs essentially from the variation in the C_p/C_v ratio of the methane as it is compressed in each stage of compression since the process is not really adiabatic and reversible.

The software package calculation for the cooling water demand shows nearly a 6% deviation from the hand calculation. This deviation comes from the variation in the C_p/C_v ratio of the methane to obtain the exit gas temperature, the change in C_p of the methane affecting the heat load, and the use of the changing C_p value of water rather than using an average value of the water temperature. Obviously, the calculations with the computer software package are more accurate.

Answer

12-10

The mechanical energy balance, Eq.(2-2) in differential form is given as

$$dz + v\,dp + V\,dV = \int W - \int F \qquad \text{where } dz;\ V\,dV \text{ and } \int F = 0$$

$$W = \int v\,dp \qquad pv = RT/M$$

$$= \int_1^2 (RT/M)\,dp/dv = (RT/M)\ln p_2/p_1 = \frac{(8.314)(293)}{2.016}\ln\frac{4140}{1380}$$

$$= 1.327 \times 10^3 \text{ kJ/kg}$$

Actual power supplied is 224 kW

The kilograms of H_2 compressed is given by

$$(224 \text{ kJ/s})\left(\frac{0.55}{1.327 \times 10^3 \text{ kJ/kg}}\right) = \underline{\underline{0.093 \text{ kg/s}}}$$

Answer

For the isothermal and reversible case, the theoretical power is $p_1 v_1 \ln(p_2/p_1)$

$$\eta_{iso} = \frac{\text{theoretical power (isothermal)}}{\text{actual power}} = 0.55$$

For the adiabatic and reversible case, the theoretical power

$$\eta_{ad} = [k/(k-1)]\,p_1 v_1 \left[\left(p_2/p_1\right)^{(k-1)/k} - 1\right]$$

$$\eta_{ad} = \frac{\text{theoretical power (adiabatic)}}{\text{actual power}}$$

$$= \frac{\text{theoretical power (adiabatic)}}{\text{theoretical power (isothermal)}}(0.55)$$

$$= \frac{(1.4/0.4)\left[\left(\frac{4140}{1390}\right)^{0.4/1.4} - 1\right]}{\ln\left(\frac{4140}{1390}\right)}(0.55) = 0.646$$

$$= \underline{\underline{64.6\%}}$$

Answer

12-11

The actual suction density of the air is

$$\rho_1 = pM/RT = (10^5)(29)/(8314)(305) = 1.144 \text{ kg/m}^3$$

and the discharge density is

$$\rho_2 = (1.144)(103/101) = 1.178 \text{ kg/m}^3$$

Average density of the air removed is

$$\rho_{ave} = (1.144 + 1.178)/2 = 1.161 \text{ kg/m}^3$$

The mass flow rate is

$$\dot{m} = (4.7 \text{ std m}^3/\text{s})(1.144) = 5.38 \text{ kg/s}$$

In the mechanical energy balance, $Z_1 = Z_2$, $V_1 = 0$, $V_2 = 40$ m/s, $F = 0$. The developed pressure is

$$(p_2 - p_1)\,\rho_{ave} = (103{,}000 - 100{,}000)/1.161 = 2585 \text{ Nm/kg}$$

The velocity head is

$$V_2^2/2\alpha = (40)^2/2 = 800 \text{ Nm/kg} \qquad \alpha = 1.0$$

$$W = [(p_2 - p_1)\,\rho_{ave} + V_2^2/2]/\eta$$
$$= (2585 + 800)(0.65) = 5210 \text{ Nm/kg}$$

The power requirement therefore is

$$P = (5210)(5.38) \cong 28{,}000 \text{ J/s} = \underline{\underline{28 \text{ kW}}}$$

Answer

12-12

The mixer in Example 12-6 has the following shape factors:

$S_1 = 1.0$, $S_2 = 1.0$ and $S_6 = 1.0$. S_3 and S_4 are also the same.

$\mu = 120$ Pa·s $\rho = 1121$ kg/m^3

The Reynolds number for this mixing system is

$$Re = D_a^2 N_r \rho/\mu = (0.6)^2(90/60)(121)/120 = 5.04 \text{ (laminar)}$$

For laminar flow, the Froude number may be neglected. Since the shape factors match those used to develop Fig. 12-40, this figure may be used to obtain the power requirement for this mixing problem. From Fig. 12-40, the power function is approximately 12.5.

$$P = \phi N_r^3 D_a^5 \rho$$
$$= (12.5)(90/60)^3(0.6)^5(1121)(10^{-3}) = \underline{\underline{3.67 \text{ kW}}}$$

There should be no diffence in the power requirement if baffles were added since with laminar flow the power function does not change as oberved in Fig. 12-40.

Answer

12-13

$D_t = 2\,m$ $D_a = 0.67\,m$ $C = 2\,m$ $\mu = 0.012\,Pa \cdot s$

$\rho = 1500\,kg/m^3$

Determine the Reynolds number for the mixing of the solution

$$Re = D_a^2 N_r \rho/\mu = (0.67)^2(90/60)(1500)/0.012$$
$$= 84,200 \quad \text{(turbulent flow)}$$

For turbulent flow, $N_{P_0} = K_T$. Values for K_T are given in Table 12-9 for various types of impellers operating in tanks with four baffles ($S_5 = 0.1$)

$$K_T = 6.30$$

From Eq. (12-59)

$$P = K_T N_r^3 D_a^5 \rho$$
$$= (6.30)(90/60)^3(0.67)^5(1500) = 4306\,J/s = \underline{\underline{4.3\,kW}}$$

Answer

12-14

$D_i = 9\text{ m}$ $p_i = 310\text{ kPa}$ $T_i = 27°C$ no corrosion considered

Table 12-10 provides the relationship to determine the recommended wall thickness for a spherical tank under pressure as

$$t = \frac{Pr_i}{2SE_j - 0{,}2P} + C_c$$

where S is the maximum allowable stress and P the maximum allowable internal gauge pressure given in Table 12-10 for carbon steel

Assume lap weld, spot examined, $E_j = 0{,}80$

$$t = \frac{(209)(4.5)}{2(94{,}500)(0{,}8) - 0{,}2(209)} = \underline{\underline{0.0062\text{ m} = 0.244\text{ in.}}}$$

With the small wall thickness relative to r_i, it is satisfactory to obtain the volume occupied by the wall by multiplying the inside area with the minimum wall thickness.

$$A_{sphere} = 4\pi r_i^2 \qquad V_{sphere} = (4/3)\pi r_i^3$$

$$\rho_{steel} = 7826\text{ kg/m}^3 \text{ (from Table D-10)}$$

Cost of steel used in the tank based on \$1.10/kg is

$$\text{Cost} = 4\pi(4.5)^2(0.0062)(7826)(1.10 = \$13{,}600$$

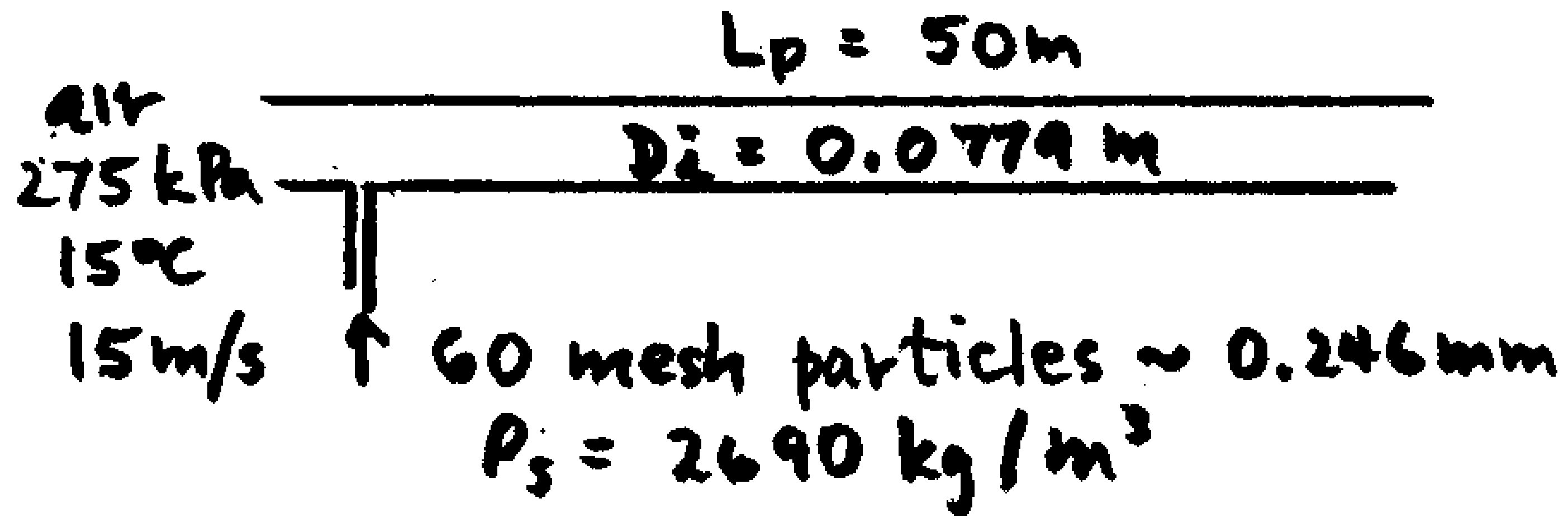
Cost of tank based on Fig. 12-52 is about \$205,000 for a tank capacity of 381 m^3

Fraction of tank cost due to the steel required is

$$13{,}600/205{,}000 = 0.066 = \underline{6.6\%}$$

Answer

12-15



Determine Δp
$m_p/m_{air} = 4$

Density of air at the inlet is

$\rho_{air} = (275)(29)/(8.314)(288) = 3.33 kg/m^3$

$G = \rho V = (3.33)(15) = 49.9 kg/m^2 \cdot s$

$Re = DG/\mu = (0.0779)(49.9)/(1.8 \times 10^{-5}) = 2.16 \times 10^5$

From Fig. 12-1, f = 0.0045 for commercial pipe

Assume that the average density of the air in the flow process is 3.30 kg/m³. Calculate the density of the air at the discharge to verify ρ_{ave} for pure air flow from

$\Delta p = p_1 - p_2 = (G^2/\rho_{ave})(2fL_p/D + (2.303/\alpha)\log(p_1/p_2)$

neglect the log term to simplify the calculation

$= (49.9)^2(2)(0.0045)(50)/(3.30)(0.0779) = 4359 Pa$

$p_2 = p_1 - \Delta p = 275{,}000 - 4359 = 270{,}641 Pa$

$\rho_2 = (270.6)(29)/(8.314)(288) = 3.27 kg/m^3$

Thus, $\rho_{ave} = (3.33 + 3.27)/2 = 3.30 kg/m^3$ ρ_{ave} verified

Energy is required to move the solid particles and this is obtained from the air through the action of drag forces between the air and the solid. This is represented by

$E = r\left[(p_2 - p_1)/\rho_s + (V_2^2 - V_1^2)_s/2 + g(z_2 - z_1)\right]$ where r is the solid/gas ratio

Assuming that the pressure drop is a small fraction of the absolute pressure, the air can be considered as an incompressible fluid of constant average density. Neglecting change in velocity head and assuming the kinetic energy factor to be unity, the mechanical energy balance provides the required energy. This is given by

$(p_2 - p_1)/\rho_{ave} + g(z_2 - z_1)_{air} = -E + \Sigma F$ (1)

12:16

where ΣF is the total friction in the stream

Eliminating E from both equations and solving for $p_1 - p_2$ gives

$$p_1 - p_2 = \frac{g(1+r)(z_2 - z_1) + (r/2)(V_2^2 - V_1^2)_s + \Sigma F}{1/\rho_{ave,air} + r/\rho_s}$$

The total friction term is the product of β and that from the flow of the air by itself and is given by

$\Sigma F = 2\beta f L V_{avg}^2 / g D$ where V_{ave} is the average superficial velocity of the air in the pipe

An empirical relation for the relative pressure of the solid is given by Vogt and White (Ind. Eng. Chem. 40:1731(1948)).

$$\beta = a(D/D_p)^2(\rho_{ave} r/\rho_s Re)^k + 1$$

According to Vogt and White, $a = 8$ and $k = 0.57$ for the conditions in this problem. This permits evaluation of the pressure drop for the combined air and solid flow.

$D_p = 60$ mesh $= 0.000246$ m

$$\beta = 8.0\left(\frac{0.0779}{2.46\times10^{-4}}\right)^2\left(\frac{(3.30)(4)}{(2690)(2.16\times10^{5})}\right)^{0.57} + 1 = 36.3$$

Substituting this into Eq. (1), noting that $z_1 = z_2$ and the velocity of the air and solid at the discharge is 15 m/s

$$p_1 - p_2 = \frac{(4/2)[(15)^2 - 0] + 2(36.3)(0.0045)(50)(15.12)^2/(9.806)(.0779)}{(1/3.30) + (4/2690)}$$

$\Delta p = 17{,}520$ Pa $= \underline{\underline{17.5 \text{ kPa}}}$ (6.4% of original press.)

Answer

12-16

Equation (12-60) can be used to obtain the power consumption under the new conditions. Since the only variables being considered are the feed rate and the volume-surface mean diameter, Eq.(12-60) can be used for the original operation and then for the new operation. It will not be necessary to convert units of Eq.(12-60) since the conversion factors can be included in an overall constant.

For the original operation

$$\frac{P_1}{\dot{m}_s} = C\left(\frac{1}{D_{pp}^{1/2}} - \frac{1}{D_{pf}^{1/2}}\right)$$

where C includes the work index as well as the unit conversions which remain the same for both conditions

$$\frac{6.35}{10{,}000} = C\left[\frac{1}{(0.005)^{1/2}} - \frac{1}{(0.069)^{1/2}}\right] = 10.335\,C$$

For the new operation

$$\frac{P_2}{9{,}000} = C\left[\frac{1}{(0.004)^{1/2}} - \frac{1}{(0.069)^{1/2}}\right] = 12.0\,C$$

Dividing the second equation by the first, permits evaluation of P_2

$$\frac{P_2\,(10{,}000)}{(6.35)(9{,}000)} = \frac{12.0}{10.335}$$

$$P_2 = (12)(6.35)(9{,}000)/((10.335)(10{,}000)) = \underline{\underline{6.64\text{ kW}}}$$

Answer

Chapter 13

13-1. (a) $\frac{V_R}{F_{io}} = \frac{X}{-r_i}$

8% drop in conversion

$X_{new} = .92\, X_{orig.}$

$\Longrightarrow \underline{V_{R_{new}} = .92\, V_{R_{orig}}}$

(b) Yes, it is possible that the decrease is due to the malfunctioning of one mixer; however, it is not possible to tell which one since no matter which one fails, the process goes from 4 reactors in series to 3.

13-2 (a) space time $= \dfrac{V_R\, c_{iF}}{F_i}$

$$\frac{V_R}{F_i} = \frac{X}{-r_i} \quad ; \quad X = 0.5$$

$$-r = k\,[CH_3OH]^2[(C_6H_5)_3CCl]$$

$$k = 4.48\times10^{-3}\left(\frac{m^3}{kgmol}\right)^2\!\Big/ s \quad @\ 298K\ ;\ 101kPa$$

feed conc: $0.11\ \frac{kgmol}{m^3}$ triphenyl methyl chloride

$0.055\ \frac{kgmol}{m^3}$ methanol

$$\frac{X}{-r_i} = \frac{0.5}{4.48\times10^{-3}\left(\frac{m^3}{kgmol}\right)^2\!\Big/ s\left[\frac{.055}{2}\right]^2\left[.11-\frac{.055}{2}\right]\left(\frac{kgmol}{m^3}\right)^3}$$

$$= 1.789\times10^{6}\ \frac{m^3\cdot s}{kgmol}$$

$$\frac{V_R}{F_i}\,c_{iF} = 1.789\times10^{6}\ \frac{m^3\cdot s}{kgmol}\cdot 0.055\ \frac{kgmol}{m^3} = \underline{\underline{9.839\times10^{4}\ s}}$$

(b) order of reaction is positive for both reactants ⇒ minimize volume with <u>PFR or batch</u>

13-3. (a) $BrCN + CH_3NH_2 \longrightarrow NH_2CN + CH_3Br$
$\quad(1)\qquad(2)$

$$\frac{V_R}{F_i} = \frac{X_i}{-r_i}$$

$V_R = 2\ m^3$

$F_1 = .01\ \frac{kg\,mol}{s}$; $F_2 = .02\ \frac{kg\,mol}{s}$

$k = 2.22\ \frac{m^3}{kg\,mol \cdot s}$

For (1) $$\frac{V_R}{F_1} = \frac{\frac{C_{o_1} - C_1}{C_{o_1}}}{k\, C_1 C_2}$$

C_o - is a combined feed conc.

$C_{o_1} = \frac{.05\ kg\,mol}{m^3}$ $\qquad C_{o_2} = \frac{.1\ kg\,mol}{m^3}$

Solve for C_1 $\qquad (C_2 = C_1 + .05)$

$C_1 = 0.0196\ \frac{kg\,mol}{m^3}$

$$X_1 = \frac{.05 - .0196}{.05}$$

$$\underline{\underline{X_1 = 0.607}}$$

Similarly, for C_2

$$\frac{V_R}{F_2} = \frac{\frac{C_{o_2} - C_2}{C_{o_2}}}{k\, C_1 C_2}$$

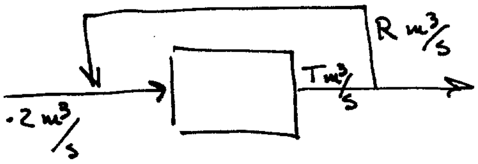
$$X_2 = \frac{C_{o_2} - C_2}{C_{o_2}} = \frac{.1 - .0696}{.1}$$

$$= \underline{\underline{0.304}}$$

(b) recycle of 90% of unreacted reactants
20% of water
20% of product

assume product recycle will not dilute reactants.

recycle flowrate:



$$T = .2 + R$$
$$R = .2T$$
$$T = 1.25(.2\ m^3/s) = .25\ m^3/s$$
$$R = .05\ m^3/s$$

New feed rates:

(1) $$.0196 \frac{kg\ mole}{m^3} \cdot \frac{.2\ m^3}{sec} = .00392 \frac{kg\ mole}{s} \text{ effluent}$$

90% of this is $.003528 \frac{kg\ mol}{s}$

$$F_{1_{new}} = .01 \frac{kg\ mol}{s} + .003528 \frac{kg\ mol}{s}$$

$$F_{1\,new} = .013528\ mol/sec$$

$$C_{o_1} = \frac{.013528\ mol/s}{.25\ m^3/s} = .05411\ kg\ mol/m^3$$

(2) $$.0696 \frac{kg\ mole}{m^3} \cdot \frac{.2\ m^3}{s} = .01392 \frac{kg\ mole}{s} \text{ effluent}$$

$$90\% \rightarrow .012528 \frac{kg\ mol}{s}$$

$$F_{2_{new}} = .02 + .012528 = .032528 \frac{kg\ mol}{s}$$

$$C_{o_2} = \frac{.032528 \frac{kg\ mol}{s}}{.25\ m^3/s} = 0.1301 \frac{kg\ mol}{m^3}$$

$$c_2 = c_1 + .076$$

(1) $$\frac{V_R}{F_1} = \frac{\dfrac{C_{01} - C_1}{C_{01}}}{k\, C_1\, (C_1 + .076)}$$

solve for C_1

$$C_1 = 0.0200 \ \frac{\text{kg mol}}{m^3}$$

$$X_1 = \frac{.05411 - .0200}{.05411}$$

$$= \underline{\underline{0.63}}$$

(2) $$X_2 = \underline{\underline{0.26}}$$

13-4

(1) $C_3H_6 + \frac{3}{2} O_2 \rightarrow C_3H_4O_2 + H_2O$

(2) $C_3H_6 + \frac{5}{2} O_2 \longrightarrow C_2H_4O_2 + CO_2 + H_2O$

(3) $C_3H_6 + \frac{9}{2} O_2 \longrightarrow 3CO_2 + 3H_2O$

$$-r_i = A_i \exp\left(\frac{-E_A}{RT}\right) P_{prop} P_{O_2}$$

Reaction	E_A, kJ/kgmol	A_i (kg mol/m³)(kPa)$^{-2}$(s)$^{-1}$
1	3590	4.42×10^1
2	5980	5.07×10^4
3	4780	2.45×10^2

Want to maximize r_1 relative to r_2 & r_3

$$\frac{r_1}{r_2} = \frac{A_1}{A_2} \exp\left[\frac{E_{A_2} - E_{A_1}}{RT}\right]$$

$$\frac{r_1}{r_3} = \frac{A_1}{A_3} \exp\left[\frac{E_{A_3} - E_{A_1}}{RT}\right]$$

since $E_{A_2}, E_{A_3} > E_{A_1}$, Low T's maximize ratio's & hence selectivity for rxn 1.

All 3 rxn's are 1st order in reactants, so altering concentrations will not change selectivity, but equal amounts of propylene & O_2 are required. Pressure changes will not affect selectivity.

13-5. $-r = kc^2$

$k = 5.4 \times 10^7 \exp\left(\frac{-100,250}{RT}\right) \quad m^3/kgmol \cdot s$

3 CSTR's

$V_1 = 8\ m^3 \quad V_2 = 6\ m^3 \quad V_3 = 4\ m^3$

$T = 500K$
$P = 101\ kPa$
$v = 7\ m^3/day$

(a) assume ideal gas

$$C_{AO} = .02437\ \frac{kgmol}{m^3}$$

$$\varepsilon = -\frac{1}{2}$$

$$X_A = \frac{1 - \frac{C_A}{C_{AO}}}{1 - \frac{C_A}{2C_{AO}}}$$

$$-r = kC^2 = \frac{v\, C_{AO} X_A}{V}$$

solve for X_A, C_A for each reactor, remembering to adjust v.

Reactor	X_A	C_A
1	.742	.00999
2	.224	.00873
3	.091	.008315

overall

$$\boxed{X_A = 0.794}$$

(b) batch reactor - 50% time

(see Levenspiel)

2nd order batch

$$\frac{(1+\varepsilon_A)X_A}{1-X_A} + \varepsilon_A \ln(1-X_A) = kC_{A0}t$$

$$\varepsilon_A = -\frac{1}{2} \; ; \; X_A = 0.794$$

$$k = 1.817\times10^{-3} \; \frac{m^3}{\text{kgmol s}}$$

$$C_{A0} = .02437 \; \frac{\text{kg mol}}{m^3}$$

$$\Rightarrow t = 61359 \text{ s}$$

$$v = 8.1\times10^{-5} \; \frac{m^3}{s}$$

to process that quantity & accounting for operating 50% of the time

$$V = 2vt$$

$$\boxed{V = 9.94 \text{ m}^3}$$

13-6 Slurry tank reactor

$2.6 \frac{kgmol}{m^3}$ glucose $10^{-3} \frac{m^3}{s}$

120% H_2 $T = 423 K$

$-r_{glucose} = k C_{H_2}^{0.6} C_{glu}$ $k = 3.76 \times 10^{-4} \left(\frac{kgmol}{m^3}\right)^{-.6} / s$

$P = 10{,}000 \text{ kPa}$ $S_{H_2} = 5 \frac{kgmol}{m^3}$

$V = 2 m^3$ CSTR

$$\frac{V}{F} = \frac{X_A}{-r_A}$$

$$F = \frac{2.6 kgmol}{m^3} \cdot 10^{-3} \frac{m^3}{s}$$

$$= 2.6 \times 10^{-3} \frac{kgmol}{s}$$

$$X_A = \frac{2.6 - C_{glu}}{2.6}$$

$$-r_A = k \left(5 \frac{kgmol}{s}\right)^{.6} C_{glu}$$

Solve for C_{glu}

$C_{glu} = 0.874$

$$\boxed{X_A = 0.66}$$

13-7 $-r_{PO} = k C_{PO}$

$k = 4.71 \times 10^9 \exp\left(\frac{-63{,}010}{RT}\right) / s$

$T = 300\ K$

Feed : 10 wt % PO (aqueous) .01 m^3/s

0.1 wt % H_2SO_4 (catalyst) .01 m^3/s

90% conversion

$$\frac{V}{v} = -\frac{1}{k} \ln \frac{C_A}{C_{A0}}$$

$k = 0.0503/s$

$\frac{C_A}{C_{A0}} = .1$

$v = .02\ m^3/s$

$$\boxed{V = 0.9155\ m^3}$$

PROBLEM 13-8

Correction: Add, the ethanol yield is 0.47 kg ethanol/kg of glucose consumed. The correct unit for the rate coefficient, *k*, is 1/s, not kg/m^3·s

a) The time for a batch reaction is given by Eq. 13-8. Since N_i/V_R is the reactant concentration, glucose in this case, the equation can be written as

$$\theta = \int_{c_{\text{glucose},i}}^{c_{\text{glucose},f}} d(c_{\text{glucose}})/(r_{\text{glucose}}) \approx \sum_{j=1}^{j=g} (\Delta c_{\text{glucose}})/(r_{\text{glucose}})_{j,\text{average}}$$

where $c_{\text{glucose},f}$ is the final glucose concentration = (0.05*10) kg/m^3, $c_{\text{glucose,i}}$ the initial glucose concentration=10 kg/m^3, $\Delta c_{\text{glucose}}$ the incremental change of glucose concentration in the numerical integration (a negative quantity), and $g=(c_{\text{glucose},f} - c_{\text{glucose},i})/\Delta c_{\text{glucose}}$. Because the rate is a complicated form in concentration, a numerical integration is recommended. Any acceptable numerical integration routine may be used. Here, integration was performed with a spreadsheet using the trapezoidal approximation. The Δ-increment size was decreased in successive integrations until the result does not change significantly with increment size. The term $(r_{\text{glucose}})_{j,\text{average}} = (r_{\text{glucose},j\text{-}1} + r_{\text{glucose},j})/2$ for integration by the trapezoidal method.

The reaction rate is given by

$$r_{\text{glucose}} = -(1.53*10^{-3})\Big(1 - (c_{\text{ethanol}}/93)\Big)^{0.52}\Big((c_{\text{glucose}})(c_{S.cerevisiae})/(c_{\text{glucose}} + 1.7)\Big)$$

The concentrations are related as follows:

$$c_{\text{glucose},j} = c_{\text{glucose},i} + \sum_{j=1}^{j} \Delta c_{\text{glucose}} \qquad c_{\text{ethanol},j} = -(0.47)\sum_{j=1}^{j} \Delta c_{\text{glucose},j}$$

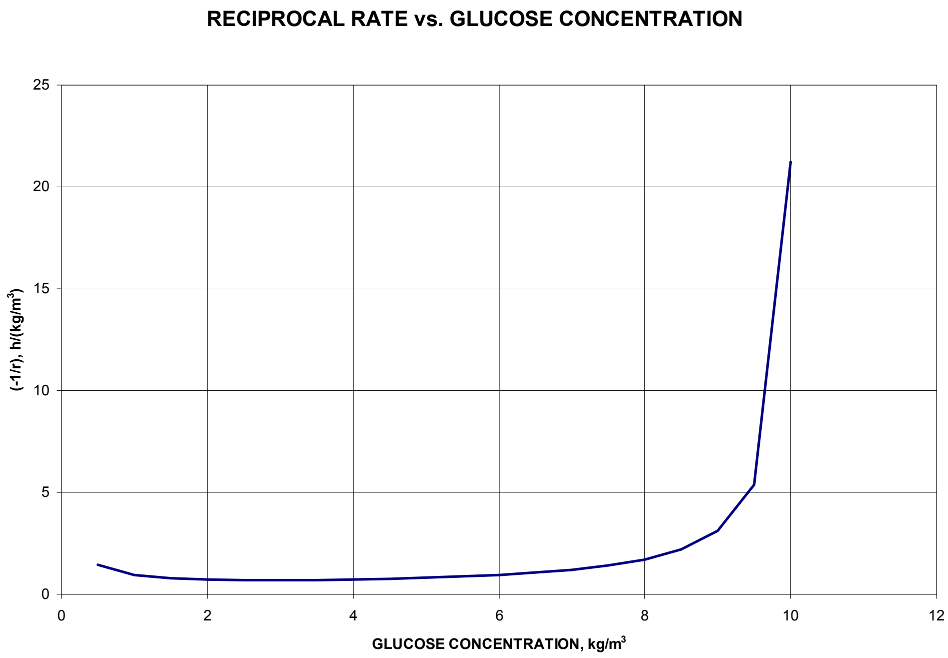
$$c_{S.cereisiaei,j} = 0.01 - (0.06)\sum_{j=1}^{j} \Delta c_{\text{glucose}}$$

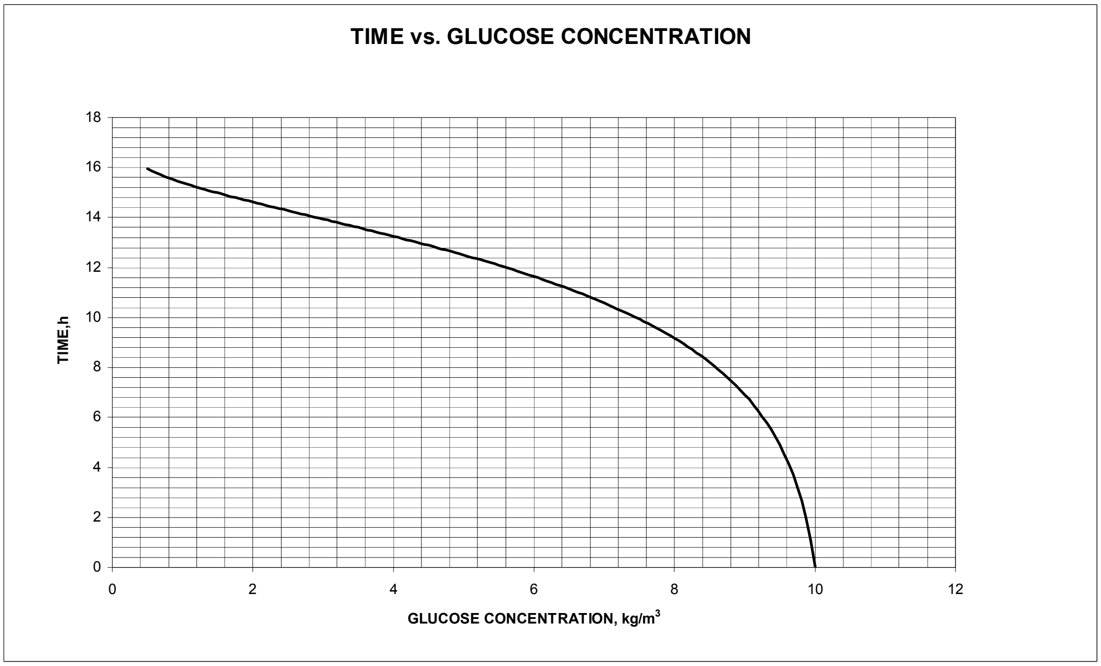
The reciprocal of rate as a function of c_{glucose} and the value of the integral (which is time) as a function of c_{glucose} are shown on the next page.

The result for these conditions is

θ = 16.0 h ANSWER

PROBLEM 13-8 (continued-1)


RECIPROCAL RATE vs. GLUCOSE CONCENTRATION


TIME vs. GLUCOSE CONCENTRATION

PROBLEM 13-8 (continued-2)

b) The PFR will have a volume of 7 m^3. Eq. (13-13a) can be written in terms of the total volumetric flowrate V, assumed constant, and glucose concentration as

$$V_R = (V)\int_{c_{\text{glucose},i}}^{c_{\text{glucose},f}} \mathrm{d}(c_{\text{glucose}})/(r_{\text{glucose}})$$

The integral must be reevaluated with the new initial concentration of $c_{S.cerevisiae}$ (=0.9 kg/m^3). The result is a value of 2.23 h.

Thus, the flow rate needed is

$$V = V_R/(2.23) = (7\ \text{m}^3)/(2.23\ \text{h}) = \underline{\underline{3.1\ \text{m}^3/\text{h}}}\ \ \underline{\underline{\text{ANSWER}}}$$

c) The volume of a single CSTR for this reaction, assuming that the feed concentration of $c_{S.cerevisiae}$ the same as in part b), is obtained from Eq. (13-18) in the volumetric flowrate/concentration form as

$$V_R = V(c_{\text{glucose},f} - c_{\text{glucose},i})/(r_{\text{glucose},f})$$

where $r_{\text{glucose},f}$ is the rate of glucose reaction at the conditions leaving the reactor.

$$V_R = (3.1\ \text{m}^3/\text{h})(0.5 - 10\ \text{kg/m}^3)/(-1.79\ \text{kg/m}^3\cdot\text{h}) = \underline{\underline{16.6\ \text{m}^3}}\ \ \underline{\underline{\text{ANSWER}}}$$

From Fig. 13-15, choosing a glass-lined, jacketed, agitated reactor, the cost is estimated to be

$$\underline{\underline{\$\ 120{,}000\quad \text{ANSWER}}}$$

For the case of three equal volume reactors, an iterative solution is required, wherein reactor sizes are represented as rectangular areas on a reciprocal rate vs. concentration graph. The outlet concentration is varied until the three reactor volumes (rectangular areas on the graph) are equal. See Ex. 13-7. The result, as shown on the graph on the following page, is that the value of the integral for each of the three reactors is 0.90 h. Thus, the volume of each reactor is

$$V_R = V^*(0.90) = (3.1)(.90) = 2.8\ \text{m}^3$$

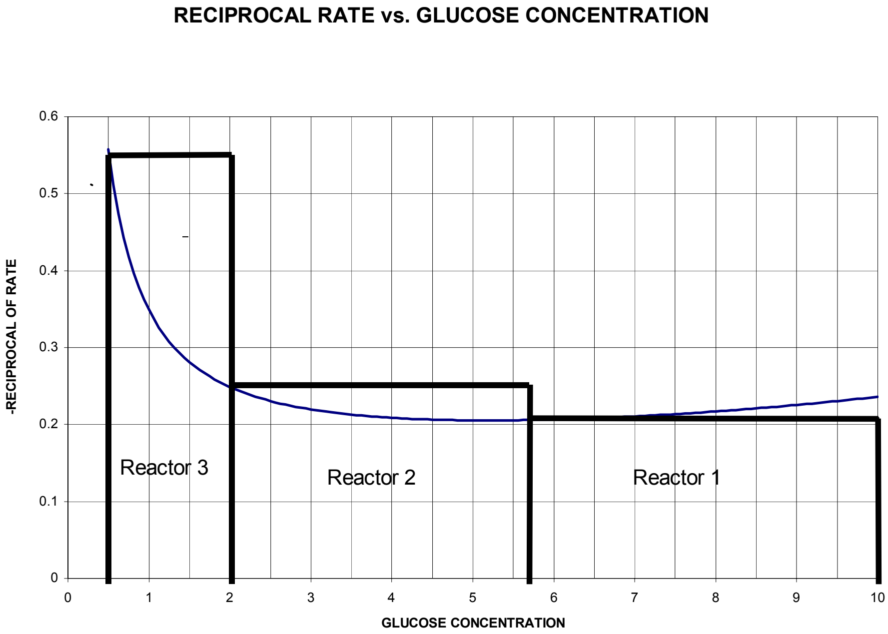
and the total volume of the 3 reactors = 3*2.8 = $\underline{\underline{8.4\ \text{m}^3\ \text{ANSWER}}}$

From Fig. 13-15, again choosing the glass-lined reactor, the cost is estimated to be

$$\underline{\underline{\$\ 50{,}000\ \text{each, or}\ \$150{,}000\ \text{for the three}\quad \text{ANSWER}}}$$

Even though the total volume is just half of that for a single CSTR, the cost is greater for the three reactors. This is a consequence of the economy of scale for such reactors. There are other factors, such as flexibility of multiple reactors that would influence such a selection. If a material other than glass-lined was chosen, the cost figures would be different, but the relative values would be very similar.

PROBLEM 13-8 (continued-3)



PROBLEM 13-9

This solution uses the reciprocal rate vs. concentration curve developed in part b) of problem 13-8 and shown below. The term "best arrangement" is used to mean the combination of the given reactors that gives the greatest degree of conversion for the given reactor sizes and types, and the flowrate and conditions of part b), problem 13-8.

Each of the reactor types and volumes suggested in parts a) through d), will be matched with the reciprocal rate vs. concentration curve to find the lowest concentration achievable.

This reaction is autocatalyzed, and the reaction rate initially increases as the glucose concentration decreases, because the biomass concentration increases. Eventually, the glucose concentration decreases sufficiently to cause the reaction rate to decrease; the increase of ethanol concentration contributes somewhat to the decrease also. Two guidelines are helpful. For PFR combinations, the order or sequence of PFR reactors makes no difference, because the reactor volume is proportional to the area under the reciprocal rate vs. concentration curve. So this area and the resulting outlet concentration are fixed, regardless of the order. The order of CSTRs is important, however, because the conditions at the reactor outlet determine the reaction rate in the reactor. Starting at the initial glucose concentration, as long as the reciprocal rate is decreasing (rate is increasing), the conversion achieved with a given volume of CSTRs will be greater than that for the same volume of PFRs, and the larger CSTR should be be used first. These guidelines are illustrated by the following solution.

The following solution is based on the conditions for part b) of problem 13-8. A graph of reciprocal rate vs. concentration is shown in the solution for problem 13-8, part b), and the value of the integral vs. concentration from that problem is shown on the following page.

a) Eq. (13-13a) for PFR, in concentration terms, is

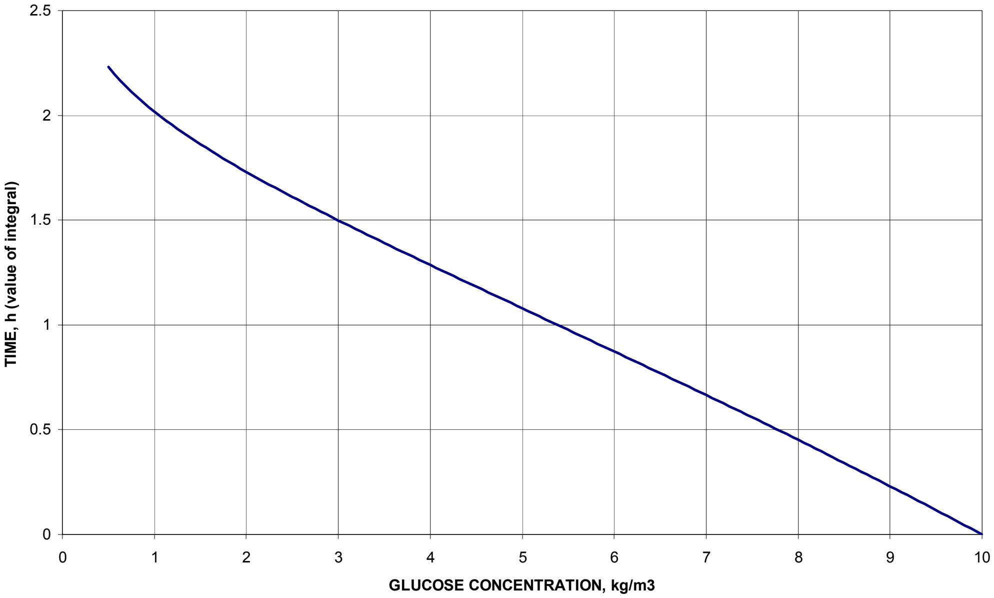
$$V_R = (V)\int_{c_{\text{glucose},i}}^{c_{\text{glucose},f}} \mathrm{d}(c_{\text{glucose}})/(r_{\text{glucose}})\text{; thus, Intergal} = V_R/\mathrm{V} = (2+1)/3.1 = 0.968$$

For two PFRs in series in either order, the value of the integral is fixed, so the result is independent of the arrangement. The only variable left is $c_{\text{glucose},f}$ and this is read from the abcissa of the graph on the following page for the ordinate equal to 0.968.

The result is, $\underline{\underline{c_{\text{glucose},f} = 5.6\ \text{kg/m}^3}}$ <u>ANSWER</u>

PROBLEM 13-9 (continued-1)

TIME vs. GLUCOSE CONCENTRATION FOR CONDITIONS OF 13-8-b



PROBLEM 13-9 (continued-2)

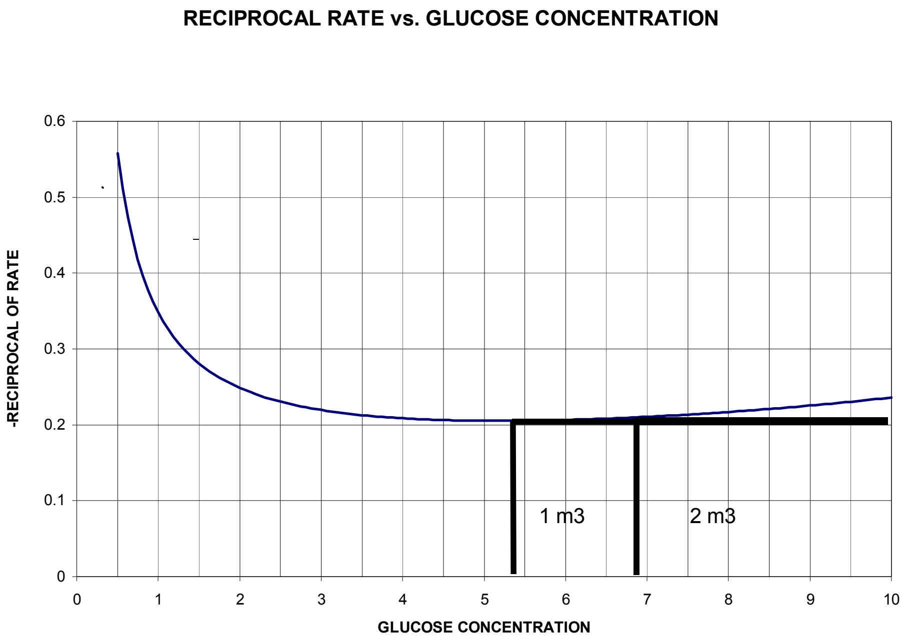
b) For CSTRs, Eq. 13-18 in concentration form, gives the volume is by

$$V_R = V(c_{\text{glucose},f} - c_{\text{glucose},i})/(r_{\text{glucose},f})$$

A value of $c_{\text{glucose},f}$ is assumed, the corresponding $(1/r_{\text{glucose},f})$ is found from the graph below, and iteration is continued until the foregoing equation is satisfied. Since sequence matters for the two unequal CSTR case, both combinations must be tried to find the one that gives the lowest $c_{\text{glucose},f}$. The better combination, the 2-m^3 CSTR followed by the 1-m^3, is shown on the graph. In this case, the first reactor reduces the glucose concentration to 6.9 kg/m^3 and the second to 5.3 kg/m^3. So the final result is, a bit lower than the value obtained with the 2 PFRs in part a).

$$\underline{\underline{c_{\text{glucose},f} = 5.3\ \text{kg/m}^3 \quad \text{ANSWER}}}$$

For comparison, if the 1-m^3 is placed first, $c_{\text{glucose},f}$ = 5.5 kg/m^3, so the result is better with the bigger reactor first.



c) If the choice is between the four reactors given, it is obvious that the 2-m^3 PFR and the 2-m^3 CSTR should be used. From the above graph and the results in parts a) and b), it is clear that putting the 2-m^3 CSTR first, followed by the the 2-m^3 PFR will achieve the lowest value of $c_{\text{glucose},f}$. The value is obtained by starting with the 6.9 kg/m^3 from the 2-m^3 CSTR, finding the time on the first graph corresponding to that concentration (0.7h), and then adding to that the space time of the 2-m^3 PFR (0.65 h) and then, from the same graph, finding the concentration corresponding to the total time (1.35 h). The result is

$$\underline{\underline{c_{\text{glucose},f} = 3.7\ \text{kg/m}^3 \quad \text{ANSWER}}}$$

Note that some might interpret the statement of this part to mean the use of one PFR and one CSTR of equal, but unspecified, volumes, to reach the original goal of 95 % conversion. The result for this case is a volume of 3.3 m^3 for each reactor.

d) If all four reactors available reactors are combined, it can be deduced from the results above that it is best to put the 2-m^3 CSTR first, the 1-m^3 CSTR second and to follow these with the 2 PFRs in either order. The result (obtained as in part c) is

$$\underline{\underline{c_{\text{glucose},f} = 1.2\ \text{kg/m}^3 \quad \text{ANSWER}}}$$

e) The glucose conversion = $[(c_{\text{glucose},i} - c_{\text{glucose},f})/\ c_{\text{glucose},i}](100)$, and the values are as follows:

a) 44%
b) 47%
c) 63%
d) 88%

PROBLEM 13-10

Corrections: The heat of reaction is 89.98 MJ/kg mol CH_3OH, not kJ/kg mol. The symbol $\bar{p}_i$ represents fugacity, in kPa, of component i, which equals the partial pressure, $p_i = Py_i$, in kPa, when the fugacity coefficient is 1.

a) At equilbrium, the numerator of the reaction rate expression is equal to zero, or,

$$K_{eq} = (p_{\mathrm{CH3OH}})/(p_{\mathrm{CO}})(p_{\mathrm{H2}})^2 = P(y_{\mathrm{CH3OH}})/(P^3(y_{\mathrm{CO}})(y_{\mathrm{H2}})^2)$$

The mol fractions can be expressed in terms of the conversion of CO, X, starting with 1 mol of CO and 2 moles of hydrogen, as follows:

$$CO + 2H_2 \rightleftharpoons CH_3OH$$

(1-X) (2-2X) (X) (moles). At any conversion, the total number of mols is the sum and equals (3-2X). So the mole fractions are given by,

$$y_{\mathrm{CO}} = (1-X)/(3-2X); \quad y_{\mathrm{H2}} = (2-2X)/(3-2X); \text{ and } y_{\mathrm{CH3OH}} = X/(3-2X)$$

Substituting these terms into the equilibrium expression gives,

$$K_{eq} = [X/(3-2x)]/[P^2(4(1-X)^3/(3-2X)^3)], \text{ or } 4K_{eq}P^2 = X(3-2X)^2/(1-X)^3$$

This latter equation can be solved by entering the known values and iterating on X to find the conversion at equilibrium.

The result is $\underline{X_{eq} = 0.610 \quad \text{ANSWER}}$

b) A packed-bed reactor is specified, and is best modeled as a plug-flow reactor. The catalyst-to-feed ratio is given by Eq. 13-15. In order to find the ratio, the integral of $(dX_{iz})/(-r_{ic})$ must be evaluated between the limits $X_{io} = 0$ and $X_{iz} = 90$ percent of the equilibrium value from part a (0.9*0.61=0.549).

The reaction rate for CO is written as

$$-r_{\mathrm{CO}} = \frac{\{[4P^3(1 - X)^3/(3-2X)^3] \;-\; (P)(X)/[(3 - 2X)(3.18*10^{-7})]\}}{[23400 + 126(P)(1-X)/(3-2X) + (47)(P)(2-2X)/(3-2X)]^2}$$

Because the rate rate expression is a complex function of X, analytical integration is impractical and a numerical evalution is easier. Any numerical integration method can be used. Here, the value of $-r_{CO}$ has been calculated for values of X incremented from 0 by a constant until X= 0.549 has been passed. The area under the curve of $1/(-r_{CO})$ vs. X was obtained by the trapezoid rule, and the calulation was repeated with a smaller increment size until the value of the integral does not change significantly. Graphs of $1/(-r_{CO})$ vs. X and the value of the integral vs. X are shown on the second following page. Interpolating for X= 0.549 gives a value for the integral equal to 7.2.

By Eq. 13-15, this is the value of W_c/F_{io}, i.e.

$$\underline{W_c/F_{io} = \text{7.2 kg catalyst per kg mol CO/min} \quad \text{ANSWER}}$$

PROBLEM 13-10 (continued-1)

c) For a methanol production rate of 50 metric tons/h, the CO feed rate to the reactor must be

(50,000 kg CH_3OH/h)(1 kg mol/32 kg)(1 kg mol CO/0.549 kg mol CH_3OH)(1h/60 min) = 47.43 kg mol CO/min. And therefore the reactor volume (filled with catalyst) is

(7.2 kg cat./kg mol CO/min)(47.43 kg mol CO/min)/(653 kg cat./m^3 reactor vol.) = <u>0.523 m^3 reactor volume</u> <u>ANSWER</u>

Because the large heat of reaction must be removed, it is expected that a shell-and-tube reactor will be needed, with the catalyst packed inside the tubes.

The heat duty for the reactor is

(0.43 kg mol CH_3OH/s)(89,980 kJ/kg mol) = 38,650 kJ/s

The tube number, length and diameter must be selected so as to provide sufficient volume for the reaction and sufficient area for the heat transfer. This is done as illustrated in Ex. 13-8. Using 0.0508-m diameter and 6.09-m tubes from the example, the number of tubes needed is

$(0.523\ m^3)/(0.00956\ m^3 \text{ per tube}) = 55$

The available surface area is then,

$(0.973\ m^2/\text{tube})(55 \text{ tubes}) = 53.5\ m^2$

The heat transfer capability must be checked. Assume a $\Delta T_{\text{log mean}}$ of 50ºC and a heat transfer coefficient of 0.25 kW/$m^2 \cdot K$. The heat transfer rate is then

$Q = (0.25)(53.5)(50) = 2140$ kW= 669 kJ/s

Clearly in this case, heat transfer dominates the sizing of the reactor. The area, A, required to handle the heat removal is

$A = (38,650)/(.25)(50) = 3090\ m^2$

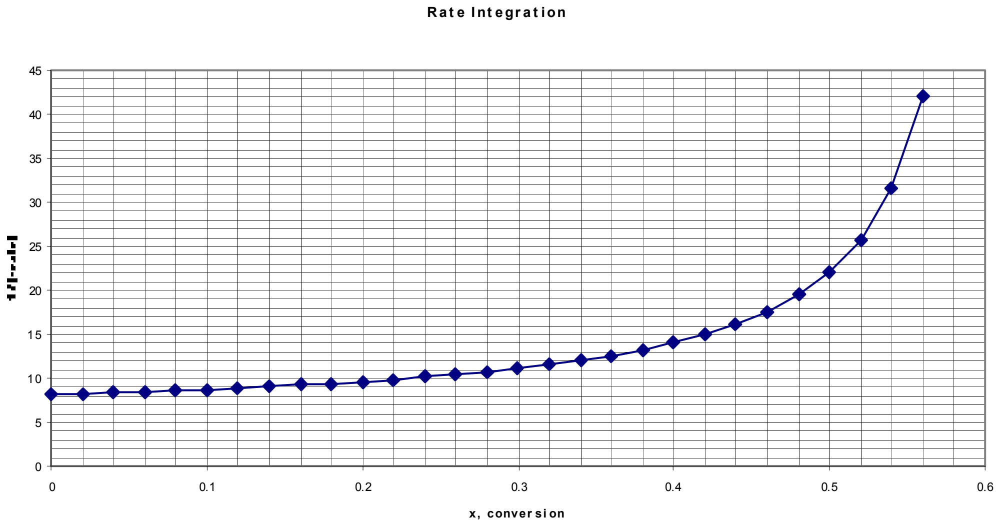
and the number of tubes required is 3090/(0.973) = 3176. From Fig. 14-17, it is found that the maximum size for a standard floating head shell-and-tube exchanger is 1000-m^2 area, with a base cost of $70,000. Therefore, three such exchangers would be reasonable. With carbon steel, 690 kPa design pressure, and an adjustment for tube diameter of ~1.75/0.91 (from Fig. 14-21) and 0.96 for tube length, the base cost is $70,000 for each exchanger.

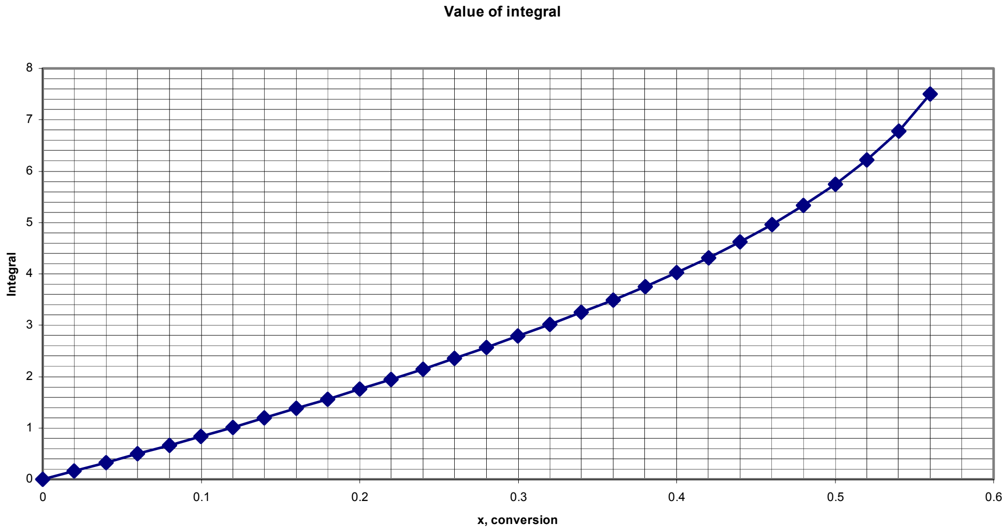
The total cost of the exchangers is estimated to be

(3)($70,000)(1.75/0.91)(0.96) = <u>$388,000</u> <u>ANSWER</u>

It would be expected that the same amount of catalyst as calculated in part b) would still be used. It would be mixed with sufficient inert catalyst support to pack the tubes full.

PROBLEM 13-10 (continued-2)





PROBLEM 13-11

The time for a batch reaction is given by Eq. 13-8. Since N_i/V_R is the reactant concentration, such as lactose in this case, the equation can be written as

$$\theta = \int_{c_{\text{lactose},i}}^{c_{\text{lactose},f}} d(c_{\text{lactose}})/(r_{\text{lactose}}) \approx \sum_{j=1}^{j=g} (\Delta c_{\text{lactose}})_j/(r_{\text{lactose}})_{j,\text{average}}$$

where $c_{\text{lactose},f}$ is the final lactose concentration = 20 kg/m^3, $c_{\text{lactose},i}$ the initial lactose concentration = 50 kg/m^3, and $g = (c_{\text{lactose},i} - c_{\text{lactose},f})/\Delta c_{\text{lactose}}$; $\Delta c_{\text{lactose}}$ is negative. Because the rate is a complicated form in concentration, a numerical integration is recommended. Any acceptable numerical integration routine may be used. Here, integration was performed with a spreadsheet using the trapezoidal approximation. The Δ-increment size was decreased in successive integrations until the result does not change significantly with increment size. The term $(r_{\text{lactose}})_{j,\text{average}} = (r_{\text{lactose},j\text{-}1} + r_{\text{lactose},j})/2$ for integration by the trapezoidal method.

The reaction rate is given by

$$r_{\text{lactose}} = -(0.901)\Big(1+(c_{\text{propionic acid}}/4.4214)\Big)^{-1}\Big((c_{\text{lactose}})(c_{P.acidipropionici})/(c_{\text{lactose}} + 32.5)\Big)$$

The concentrations are related as follows:

$$c_{\text{lactose},j} = c_{\text{lactose,i}} + \sum_{j=1}^{j} \Delta c_{\text{lactose}}; \quad c_{\text{propionic acid},j} = -(0.307)\sum_{j=1}^{j} \Delta c_{\text{lactose}};$$

$$c_{P.acidipropionici,j} = 0.1 - (0.145)\ )\sum_{j=1}^{j} \Delta c_{\text{lactose}}$$

The rate as a function of c_{lactose} and the value of the integral as a function of c_{lactose} are shown on the next page.

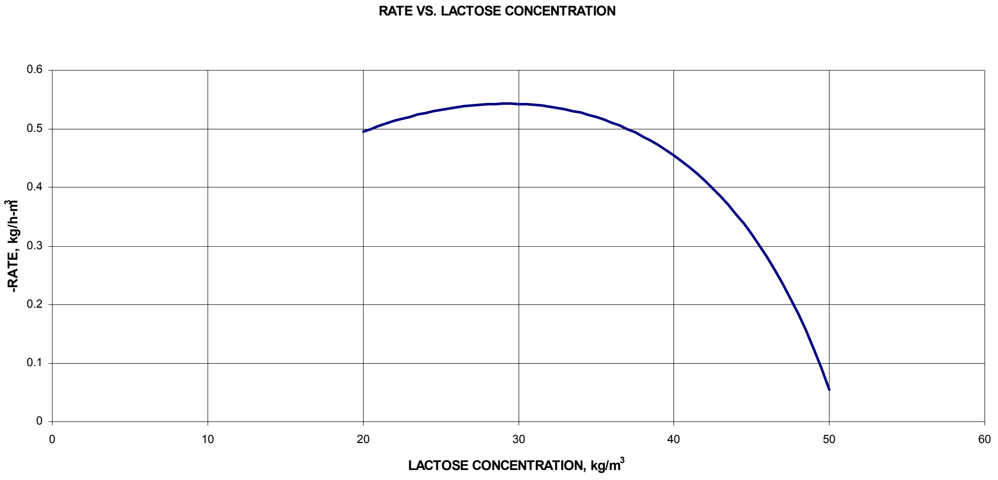
The result for these conditions is

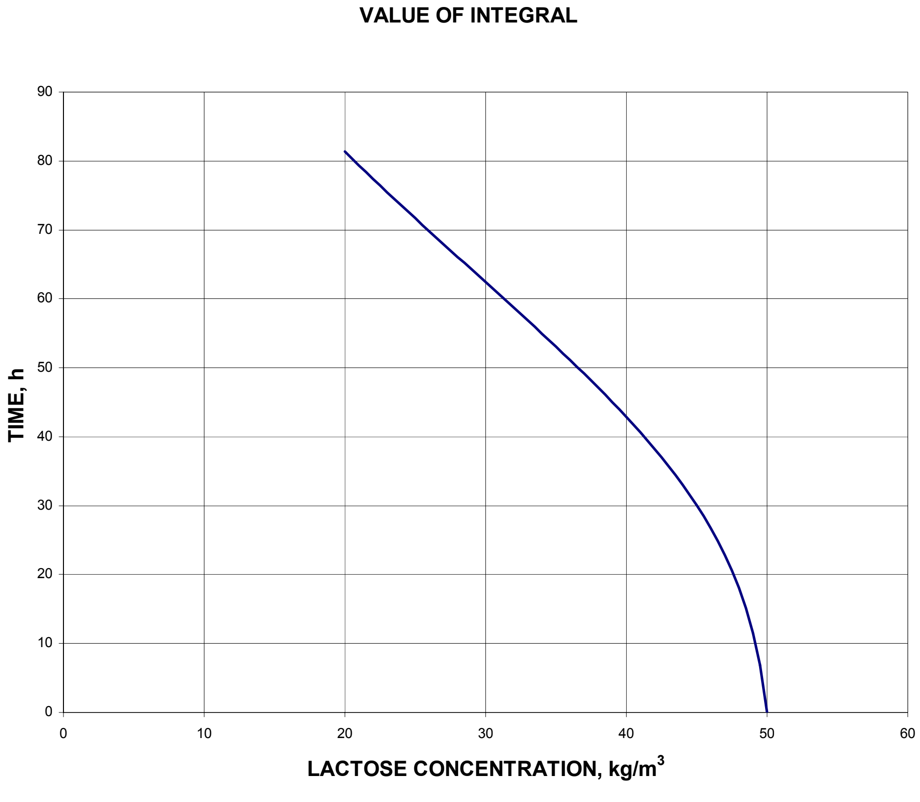
θ = 81.4 h ANSWER

The concentrations of lactose, *P. acidipropionici* and propionic acid are shown vs. time in the graph on the second next page.

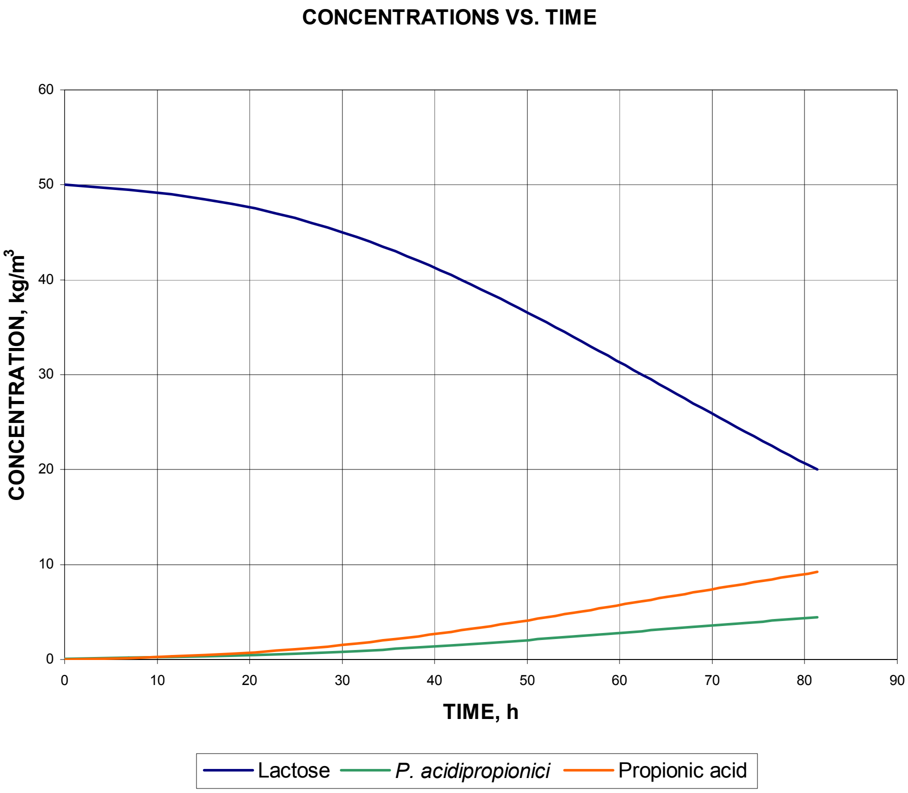
The required reaction time can be substantially reduced by increasing the initial concentration of biomass, *P. acidipropionici*. For example, if the initial concentration is 1 kg/m^3, rather than 0.1 kg/m^3, the time required is 44.4 h.

PROBLEM 13-11 (continued-1)





PROBLEM 13-11 (continued-2)



PROBLEM 13-12

Clarification: The third sentence should read, “In the proposed recovery process, each kg of catalyst-containing ceramic is mixed with two kg of aqueous sodium cyanide solution, and the Pt is dissolved by complexing with the cyanide.”

a) Assuming complete Pt recovery and a 90% operating factor, the annual production rate is

$$= (12\ \text{blocks/h})(0.05\ \text{kg-Pt/block})(24*365*0.9\ \text{h/y}) = 4{,}730\ \text{kg-Pt/y}$$

This small rate indicates use of a batch reactor. See Ch. 4.
ANSWER

b) There is no effect of pressure on the reaction, it is liquid phase. So, ambient pressure, or a pressure somewhat lower to prevent cyanide leakage, is recommended.

The temperature is selected to achieve the specified reactor space time of 4 h. Using Eq. 13-10, with θ in s.

$$\theta = (N_{io}/V_R)\int_0^{Xie}[\mathrm{d}X_i/(-r_i)] = c_{io}\int_0^{Xie}\{\mathrm{d}X_i/[(c_{io})(\mathrm{k}_2)(1\text{-}X_i)]\} = (1/\mathrm{k}_2)\ln[1/(1\text{-}X_{ie})]$$

$$= 4*3600 = \{\ln[1/(1\text{-}0.9999)]\}/[(5*10^7)\exp(-13720/\mathrm{T})]$$

from which,

$$\exp(-13720/\mathrm{T}) = \ln(1/0.0001)/[(5*10^7)(4*3600)] = 1.28*10^{-11}$$

$$-13720/\mathrm{T} = -25.08,\ \text{and}\ \mathrm{T} = 547\ \mathrm{K}\quad \text{ANSWER}$$

c) The reactor volume, for a space time of 4 h, is given by Eq. 13-7 as

$$V_R = (F_i{*}v_{iF})\theta$$

where $(F_i{*}v_{iF})$ is equal to the average volumetric flowrate of the reactor feed. This is the flowrate of catalyst blocks plus cyanide solution. Assuming the volume of this mixture is 0.003 $\mathrm{m^3}$/kg of ceramic catalyst (approximated as 1 kg of ceramic per 2 kg of solution, thus ~3 kg/kg of ceramic), the reactor volume is

$$V_R = (12\ \text{blocks/h})(5\ \text{kg ceramic/block})(0.003\ \mathrm{m^3}/\text{kg ceramic})(4\mathrm{h})$$

$$= 0.72\ \mathrm{m^3}\ \text{ANSWER}$$

PROBLEM 13-12 (continued-1)

d) The reactor must contain a hot solution of sodium cyanide/hydrocyanic acid, as well as the abrasive ceramic material of the catalyst block. Either a steel or glass-lined reactor might be used. Steel has the problem of being somewhat subject to corrosion by weak acids, while a glass-lined vessel has the problem of being subject to scratching and erosion from the ceramic catalyst support. Costs are estimated from Fig. 13-15 to be

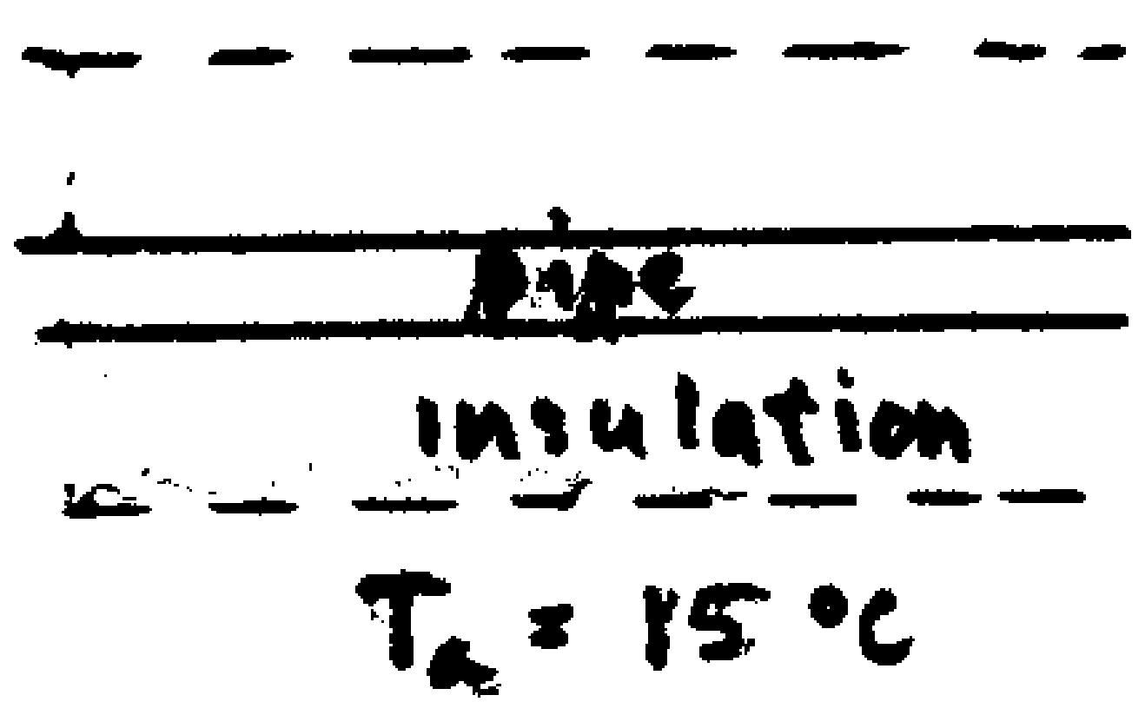
Glass-lined-reactor cost = $24,000
Steel reactor cost = $10,000 (TENTATIVE ANSWERS)

Further study, including more detailed corrosion and erosion information and an alternative investment comparison, would be needed to make a definitive recommendation.

14-1

T_p = temperature on outside of pipe

T_a = temperature of ambient air



T_p = 100°C

h_{air} = 10 W/m²·K

D_p = 0.01 m

Assume that the thermal resistance on the fluid side of the pipe can be neglected. First consider the heat transfer per meter of uninsulated pipe. For the latter, the thermal resistance is

$$R_{T_a} = \frac{T_p - T_a}{\dot{q}} = \frac{1}{2\pi r_p hL} = \frac{1}{2\pi(0.005)(10)(1)} = 3.183\text{(K/W)/m}$$

The rate of heat loss per meter of pipe is then

$$\dot{q} = \frac{T_p - T_a}{R_{T_a}} = \frac{100-15}{3.183} = \underline{\underline{26.70\text{W/m}}}$$

Answer

The thermal resistance of the first layer of insulation is given by

$$R_{T_1} = \frac{\ln r_1 / r_p}{2\pi Lk} = \frac{\ln(0.01/0.005)}{2\pi(1)(0.1)} = 1.1032\text{(K/W)/m}$$

where r_1 = radius of pipe plus one layer of insulation.

Because of a doubling of the surface area of the insulation, the thermal resistance becomes half of that for the uninsulated pipe as given by

$$R_{T_a} = \frac{1}{2\pi r_1 hL} = \frac{1}{2\pi(0.01)(10)(1)} = 1.592\text{(K/W)/m}$$

The total thermal resistance is

$$R_T = 1.1032 + 1.592 = 2.695\text{(K/W)/m}$$

and the heat flow is

$$\dot{q} = \frac{100-15}{2.695} = 31.54 \text{ W/m}$$

Note that adding 0.005m of insulation increased the heat loss.

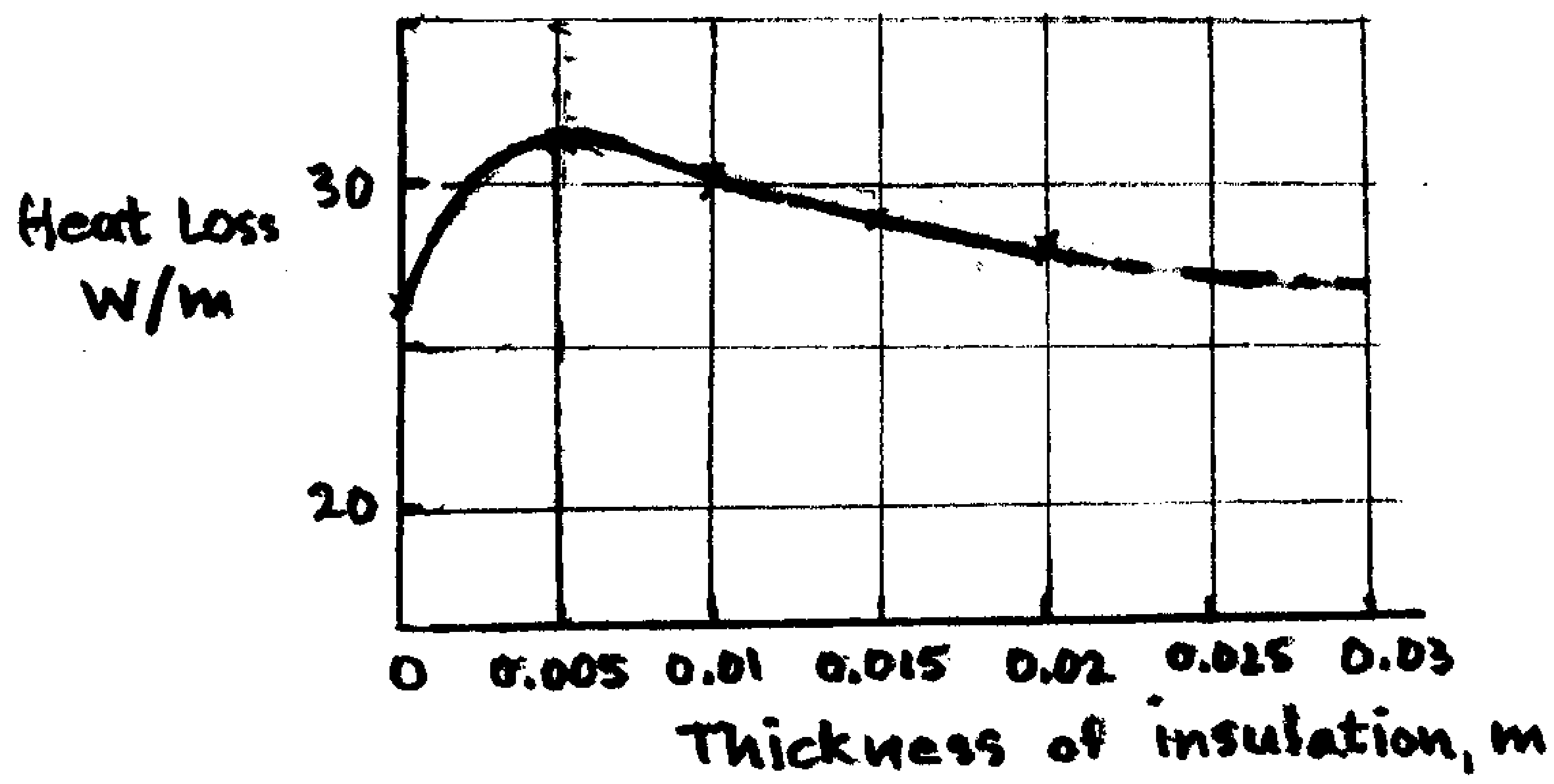
14-1 (continued)

The addition of another 0.005m of insulation on the pipe results in

$$R_{T_2} = 1.748\text{(K/W)/m} \quad R_{T_a} = 1.061\text{(K/W)/m}, \quad R_T = 2.809\text{(K/W)/m}$$

and the heat loss is 30.25W/m

These calculations are repeated with the addition of each layer of insulation. The results are shown below.



For small diameter pipes there is a critical thickness of insulation that produces the minimum thermal resistance. To be effective insulation thicknesses must be greater than this value. In this problem the critical thickness of insulation is 0.005m as shown on the figure. This can be verified by differentiating the sum of the conductive and convective resistance with respect to the outside radius and setting the result equal to zero. Thus

$$R_T = \frac{\ln r_2 / r_p}{2\pi L h} + \frac{1}{2\pi r^2 L k}$$

$$\frac{dR_T}{dr_2} = 0 \quad \text{and} \quad r_2 = k/h = 0.1/10 = 0.1 \text{ m}$$

In practice, the optimum thickness of insulation involves an economic balance between additional costs for the insulation and the energy savings attributed to the insulation. In this problem, it requires adding 0.02m of insulation to reduce the heat losses per meter of pipe to the heat losses shown for the uninsulated pipe. Since additional insulation only reduces the losses by small amounts, it is more economical to operate the pipe without the addition of any insulation. This is often the case for small pipes where the heat loss is relatively small. **Answer**

14-2

$T = 350\text{K} = 77^\circ\text{C} \quad \text{Re} = 5\times10^4 \quad \text{ID} = 0.0254\text{m}$

For the same Reynolds number, the average velocity will be different for each fluid

$$V = \frac{\text{Re}\,\mu}{D\rho} \quad \text{for air} \quad V = \frac{(5\times10^4)(2\times10^{-5})}{(0.0254)(0.955)} = 41.22\,\text{m/s}$$

$$\text{for H}_2\text{O} \quad V = \frac{(5\times10^4)(3.72\times10^{-4})}{(0.0254)(973)} = 0.75\,\text{m/s}$$

$$\text{for oil} \quad V = \frac{(5\times10^4)(3.56\times10^{-2})}{(0.0254)(854)} = 83\,\text{m/s}$$

Use Eq. (14-18) to obtain the individual heat transfer coefficient. Assume that $(\mu/\mu_w)^{0.14}$ is essentially unity.

$$h = (k/D)(0.023)\,\text{Re}^{0.8}\,\text{Pr}^{1/3}$$

$$h_{\text{air}} = (0.030/0.254)(0.023)(5\times10^4)^{0.8}[(1050)(2\times10^{-5})(0.030)]^{1/3} = 136\ \text{W/m}^2\cdot\text{K}$$

$$h_{H_2O} = (0.668/0.0254)(0.023)(5\times10^4)^{0.8}[(4190)(3.72\times10^{-4})(0.668)]^{1/3} = 4610\ \text{W/m}^2\cdot\text{K}$$

$$h_{\text{oil}} = (0.138/0.0254)(0.023)(5\times10^4)^{0.8}[(2116)(3.56\times10^{-2})(0.138)]^{1/3} = 5860\ \text{W/m}^2\cdot\text{K}$$

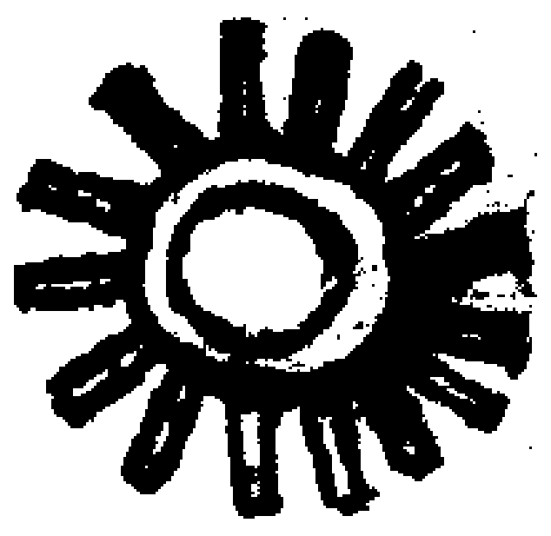
The term $\rho V^2/2$ for each fluid, because of the constant Reynolds number, provides an indication of the magnitude of the pressure loss in the pipe. The values for air, water, and oil are 811 N/m^2, 273 N/m^2, and 2.94×10^6 N/m^2, respectively.

Summary	Air	Water	Oil
Velocity, m/s	41.2	0.75	83
Heat transfer coeff, W/m$^2\cdot$K	136	4610	5860
Relative pressure drop, N/m^2	811	273	2.94×10^6

Obviously the pressure drop for the oil is too large and the velocity of the oil must be reduced to a reasonable value. However, in reducing the velocity, the flow may become laminar with a large decrease in the heat transfer coefficient. This indicates that heat transfer rates for oil that are equivalent to those for water cannot be achieved with pressure drops similar to those obtained with water flow.

14'.4

14-3



$D_2 = 0.022\text{m} \quad D_1 = 0.028\text{m} \quad L_p = 0.012\text{m} \quad t_s = 0.002\text{m}$

Inside surface areas/unit length of pipe, $A_2 = \pi D_2 = \pi(0.022) = 0.069\,\text{m}^2/\text{m}$

Outside surface area (without fins)/unit length of pipe, $A_1 = \pi(0.028) = 0.0879\,\text{m}^2/\text{m}$

Outside surface area not covered with fins/unit length of pipe,

$$A_p = 0.0879 - \pi(0.028)(0.002)/0.01 = 0.0703\,\text{m}^2/\text{m}$$

Surface area of fin/unit length of pipe, $A_f = \dfrac{2\pi[(0.04)^2 - (0.028)^2}{4(0.01)} + \dfrac{\pi[(0.04)(0.002)}{0.01}$

$$= 0.1533\,\text{m}^2/\text{m}$$

Total outside surface area, $A_T = 0.1533 + 0.0703 = 0.2236\,\text{m}^2/\text{m}$

We can use Eq. (14-26) to obtain the fin efficiency. For a quick approximation, use the approach suggested in Chap. 2.5.3 of the *Heat Exchanger Design Handbook.*

Effective fin height $L_e = (L_f + t_f)/2 = (0.012 + 0.002)/2 = 0.007\,\text{m}$

Fin parameter $M = (h/k_f t_f/2)^{1/2} = [200/(25)(0.001)^{1/2} = 89.44$

$$\eta_f = [1 + 1/3(ML_e)^2(D_f/D_o)^{1/2}]^{-1}$$
$$= \{1 + 1/3[(89.44)(0.007)]^2[(0.040/0.028)]^{1/2}\}^{-1} = 0.865$$

Since no fouling factors were given, assume the heat transfer occurs when the heat exchange process has just initiated operation. Calculate the effective outside heat transfer coefficient.

$$h_e = (h_o/A_o)(A_f\eta_f + A_p)$$
$$= (200/0.2236)[(0.1533)(0.865) + 0.0703] = 181.5\ \text{W/m}^2\cdot\text{K}$$

This permits calculation of the overall heat transfer coefficient based on <u>total</u> outside area (fins + outside area not covered by fins)

14-3 (continued)

$$\frac{1}{U_o}=\frac{A_o}{h_i A_1}+\frac{D_1 \ln D_1/D_2}{2k_w}\frac{A_o}{A_1}+\frac{1}{h_e'}$$

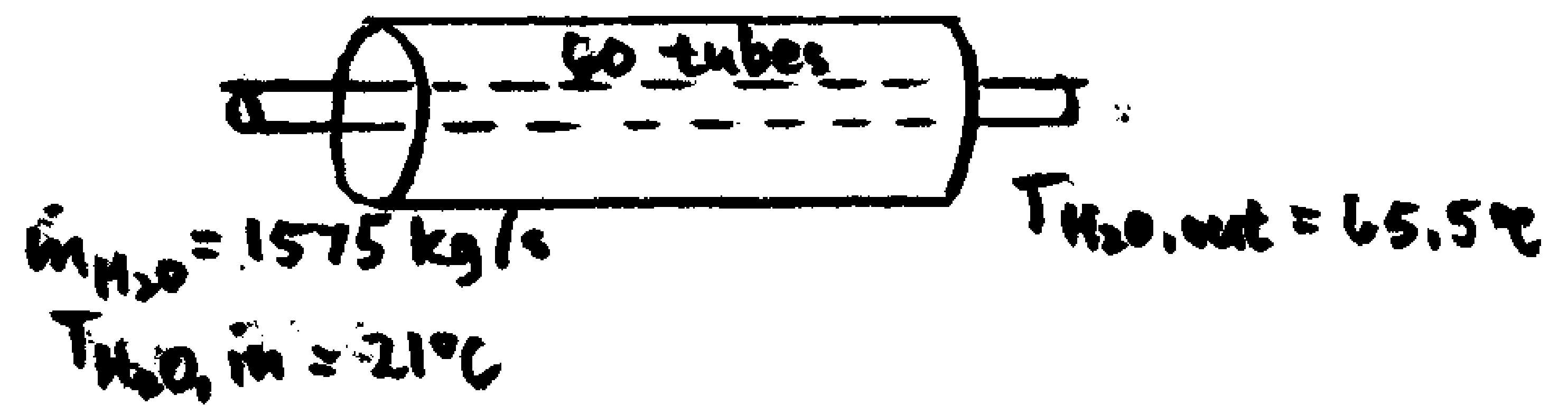
$$=\frac{0.2236}{(1500)(0.069)}+\frac{(0.028)\ln(0.028/0.022)}{(2)(25)}\frac{(0.2236)}{0.0879}+\frac{1}{181.5}$$

$$=2.160\times10^{-3}+3.434\times10^{-4}+5.5096\times10^{-3}=8.013\times10^{-3}$$

$$U_o=\underline{\underline{125\ \mathrm{W/m^2\cdot K}}}$$

If the heat transfer coefficient is based on the outside area of the pipe, A_1, as it is used in Eq. (14-29), the value for the coefficient would be $\underline{\underline{318\ \mathrm{W/m^2\cdot K}}}$. Thus, great care must be exercised to define what area is used as a reference point when fins are involved.

14-4 Tube and shell heat exchanger



$h_{H_2O} +$ with scale $= 8250\ \mathrm{W/m^2 \cdot K}$

$h_{\text{steam}} = 11{,}400\ \mathrm{W/m^2 \cdot K}$

$T_{1\ \text{steam}} = 143.3^\circ\mathrm{C}$

$\rho_{av,H_2O} = 988.5\ \mathrm{kg/m^3}$

$K_{\text{steel}} = 45\ \mathrm{W/m^2 \cdot K}$

Case 1 = Design with scale, $h_{o,H_2O} = 8250\ \mathrm{W/m^2 \cdot K}$

Case 2 = Design with no scale

$\dot{q} = \dot{q}_2 \quad \dot{q}_1 = U_1 A_1 \Delta T_1 \quad \dot{q} = U_2 A_2 \Delta T_2 \quad A_1 = A_2 \quad \Delta T_2 = (U_1 / U_2) \Delta T_1$

Base both U_1 and U_2 on the inside area

$$\Delta T_{1,\text{avg}} = \frac{(143.3-21)+(143.3-65.5)}{2} = 100^\circ C \quad \Delta T_{2,\text{avg}} = \frac{(T_{2,s}-21)+(T_{2,s}-65.5)}{2} = T_{2,s} - 43.3$$

Determine U based on average values for physical properties

$$V_{1,\text{avg}} \left(\frac{15.75}{60}\frac{\text{kg}}{\text{s}}\right)\left(\frac{\text{m}^3}{988.5\,\text{kg}}\right)\left(\frac{4}{\pi(0.0186)^2\,\text{m}^2}\right) = 0.977\ \text{m}^3/\text{s}$$

$$\text{Re}_1 = DV\rho/\mu_{\text{avg}} = (0.0186)(0.977)(988.5)/0.000616 = 29{,}160 \text{ turbulent flow}$$

$$h_i = (k/D_i)(0.023)\,\text{Re}^{0.8}\,\text{Pr}^{1/3} \text{ neglecting viscosity correction}$$

$$= (0.636/0.0186)(0.023)(29{,}160)^{0.8}\left[\frac{(4.179)(0.000616)}{0.636}\right]^{1/3} = 4678\ \mathrm{W/m^2 \cdot K}$$

From Eq. (14-4a)

$$\frac{1}{U_1} = \frac{1}{4678} + \frac{1}{8250} + \frac{\pi(0.0186)(0.0034)}{(45)(0.0686)} + \frac{(0.0186)}{(11400)(0.0254)} = 4.636\times10^{-4}\ \mathrm{m^2 \cdot K/W}$$

$$U_1 = 2157\ \mathrm{W/m^2 \cdot K} \qquad \text{where } A_{m,W} = 0.068\ \mathrm{m^2}$$

$$\frac{1}{U^2} = 4.636\times10^{-4} - \frac{1}{h_D} = 4.636\times10^{-4} - \frac{1}{8250} = 3.424\times10^{-4}\ \mathrm{m^2 \cdot K/W}$$

$$U_{2_1} = 2920\ \mathrm{W/m^2 \cdot K}$$

14-4 (continued)

From above

$\Delta T_2(U_1/U_2)\Delta T_1 = (2157/2980)(100) = 72.4^{\circ}\text{C}$

$\Delta T_{2,\text{avg}} = 72.4 = T_{2,\text{steam}} - 43.3$ if $\Delta T_1/\Delta T_2 < 2.0$ in case 2

$T_{2,s} = 72.4 + 43.3 = 115.7^{\circ}\text{C}$ $\Delta T_1 = 115.7 - 21 = 94.7^{\circ}\text{C}$

$\Delta T_2 = 115.7 - 65.5 = 50.2^{\circ}\text{C}$

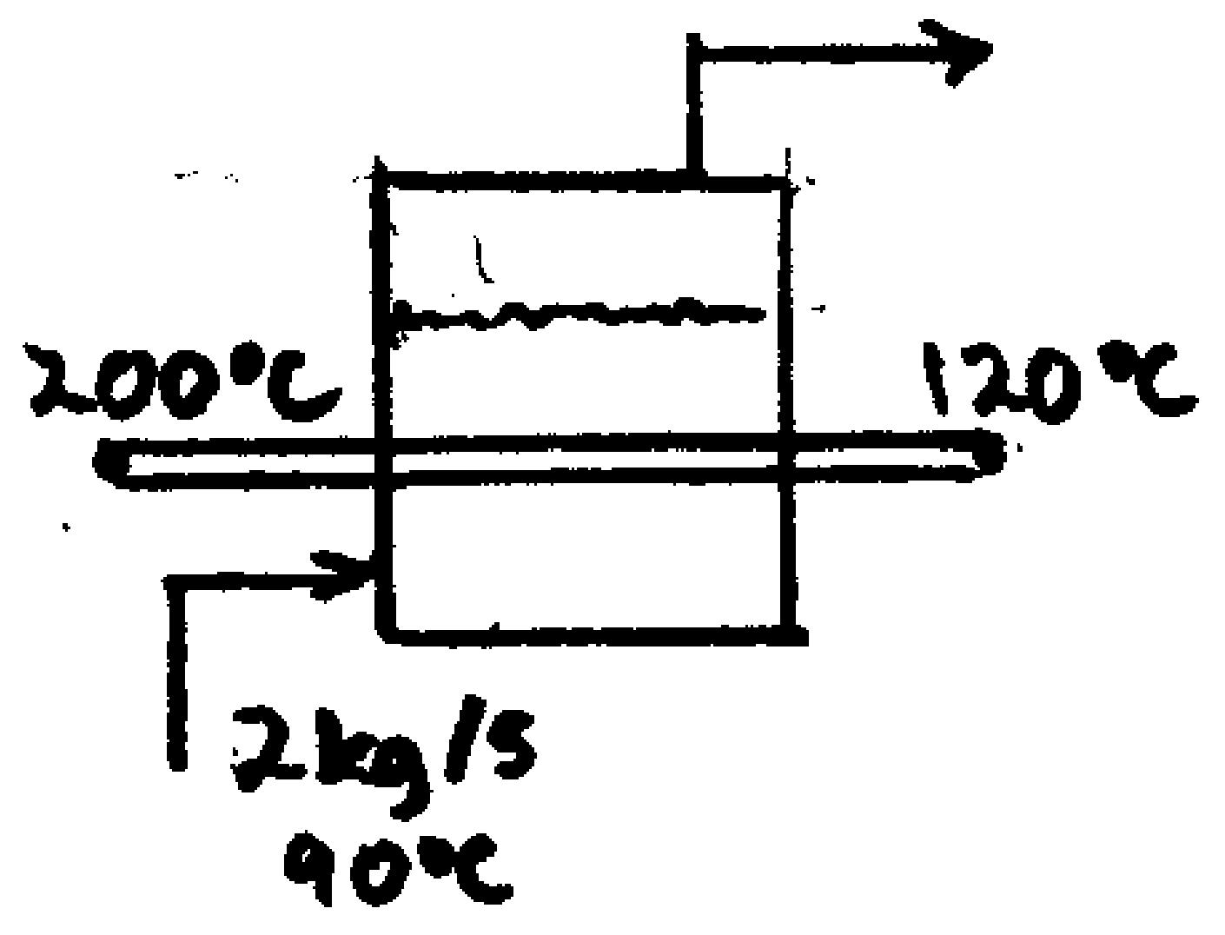
$\Delta T_1/\Delta T_2 = 94.7/50.2 < 2.0$

thus, using averages is acceptable

$\therefore$ Temperature of steam required is **115.7°C or 116°C**

Answer

14-5



$\Delta H_v = 200\,\text{kJ/kg} \;@\; 90^\circ\text{C}$

$\dot{m}_c = 2\,\text{kg/s}$

$T_{h,1} = 200^\circ\text{C} \quad C_{p,h} = 2.2\,\text{kW/kg}\cdot\text{K}$

$T_{h,2} = 120^\circ\text{C}$

$U_o = 400\,\text{W/m}^2\cdot\text{K} \quad \text{Find}\, \dot{m}_h, \Delta T_{\ln m}, A$

The evaporization process is represented by Fig. 14-2c. Since C_p is constant for the hot fluid, a $\Delta T_{\ln m}$ can be determined from

$$\Delta T_m = \frac{(200-90)-(120-90)}{\ln 110/30} = \frac{80}{1.299} = 61.6^\circ\text{C}$$

$$\dot{q}_c = \dot{m}_c \Delta H_V = (2)(200{,}000) = 400{,}000\,\text{J/s} \quad \dot{q}_c = \dot{q}_h$$

$$\dot{q}_h = \dot{m}_h Cp_h (T_{h,m} - T_{h,out})$$

$$\dot{m}_h = \dot{q}_h / Cp_h (T_{h,m} - T_{h,out}) = -400{,}000/(2{,}200)(120-200)$$

$$= \underline{\underline{2.27\ \text{kg/s}}}$$

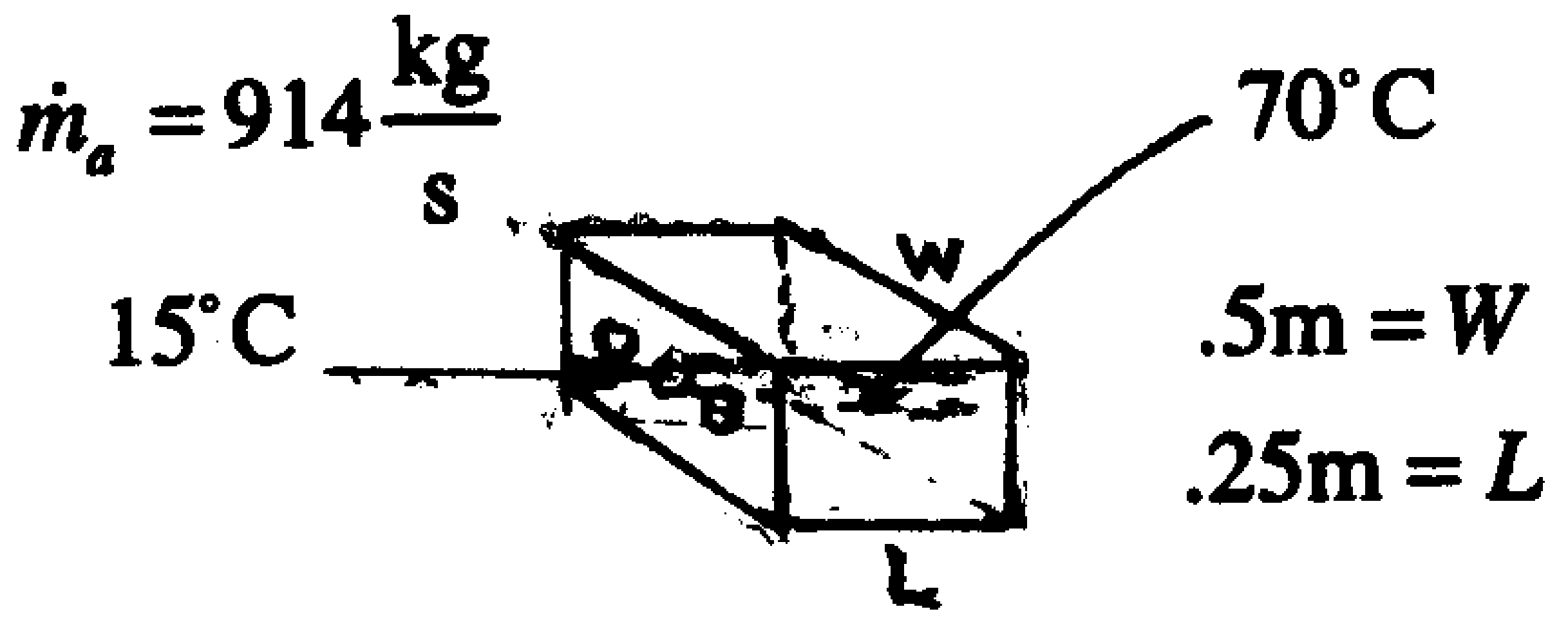
Answer

$$A = \dot{q}/U\Delta T_{em} = 400{,}000/(400)(61.6) = \underline{\underline{16.2\text{m}^2}}$$

Answer

14\9

14-6



$D_o = 0.0104\text{m}$

$L_p = 1.0\text{m}$

staggered array with 60° triangular pitch

56 tubes 7x8 with a pitch of 0.313m

Fluid properties of air at 15°C

$\mu = 1.79\times10^{-5}\,\text{Pa}\cdot\text{s} \quad k = 2.53\times10^{-2}\,\text{W/m}\cdot\text{k} \quad Cp = 1007\,\text{J/kgK}$

$\rho = 1.226\,\text{kg/m}^3$

Calculate the maximum air velocity from

$$V_{\max} = \frac{m}{\rho n(P-D)L}$$

where n = number of tubes in a row
P = pitch of 0.0313 m

$$= \frac{0.914}{(1.226)(8)(0.0313-0.0104)(0.5)} = 8.92\,\text{m/s}$$

The corresponding Reynolds number is

$$\text{Re} = DV_{\max}\rho/\mu = (0.0104)(8.92)(1.226)/1.79\times10^{-5} = 6350$$

Use Eq. (14-22) to obtain average heat transfer coefficient

$$\frac{\bar{h}_o D_o}{K} = a\,\text{Re}^m\,\text{Pr}^{0.34}\,F_1 F_2$$

Table 14-1 provides a = 0.273, m = 0.635. Since the difference between the bulk air temperature and the pipe surface temp is small, $F_1 \cong 1$. Table 14-2 indicates that $F_2 = 0.97$ for a staggered array of seven rows. Thus,

14-6 (continued)

$$\bar{h}_o = (2.53\times10^{-2}/0.0104)(0.273)(6350)^{0.635}\left[\frac{(1007)(1.79\times10^{-5})}{2.53\times10^{-2}}\right]^{0.34}(0.97)$$

$$=149.2\,\text{W/m}^2\cdot\text{K}$$

Now calculate the log mean temperature difference in terms of the average heat transfer coefficient

$$\Delta T = \frac{(T_p - T_1) - (T_p - T_2)}{\ln\left[(T_p - T_1) - (T_p - T_2)\right]}$$ this can also be written in terms of $\bar{h}_o$ as

$$= (T_p - T_1)\left[\frac{1 - e^{-A\bar{h}_o/\dot{m}C_p}}{A\bar{h}_o/\dot{m}C_p}\right]$$ $A = \pi DLN_T$ where N_T = number of tubes

$$A = \pi(0.0104(1)(56) = 1.829\,\text{m}^2$$

$$A\bar{h}_o/\dot{m}C_p = (1.829)(149.2)/(0.914)(1007) = 0.296$$

$$\Delta T = (70 - 15)\left(\frac{1 - e^{-0.296}}{0.296}\right) = 47.6^\circ\text{C}$$

The total heat transfer rate is then

$$\dot{q} = \bar{h}A\Delta T = (149.2)(1.829)(47.6) = 12{,}989\,\text{W} \cong 13{,}000\,\text{W}$$

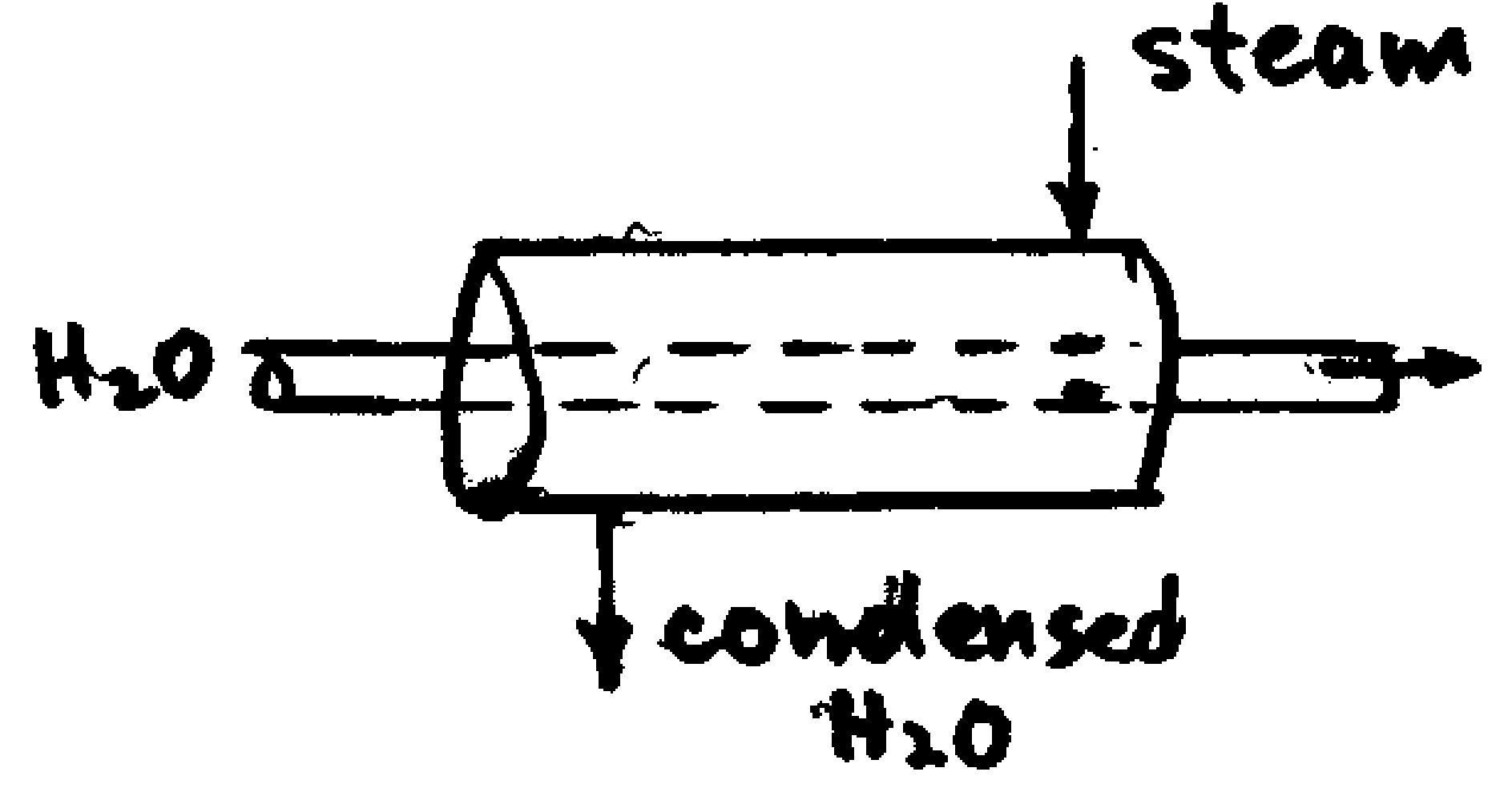
This corresponds to an increase in the air temperature of

$$T_{exit} - T_{inlet} = \dot{q}/\dot{m}C_p = 12{,}989/(0.914)(1007) = 14.1^\circ\text{C}$$

and an exit temperature of 14.1 + 15 = **29.1°C**

Answer

14-7



turbulent flow

$\dot{m}_{H_2O}, T_{H_2O,in}, T_{H_2O,out}, T_s, \Delta p_{tube}$ assumed to be constant

assume negligible thermal resistance of tube and for steam condensate film, tube diameter is available

$\dot{q} = U_i A_i \Delta T_{lm} = \text{constant} / U_i$, From Eq. (14-14a),

$$\frac{1}{U_i} = \frac{1}{h_i} + \frac{A_i x_w}{k_w A_{w,m}} + \frac{D_i}{h_o D_o}$$

assume last two terms are negligible

From Eq. (14-18)

$$h_i = 0.023\ \text{Re}_i^{0.8}\ \text{Pr}_i^{1/3} (\mu / \mu_w)^{0.14}$$

assume $(\mu / \mu_w)^{0.14} \sim 1.0$

$$h_i \cong \text{Re}_i^{0.8} \cong K(DV\rho / \mu)^{0.8} \qquad V = \dot{m} / (\rho \pi D_i^2 / 4) \qquad V \cong 1 / D_i^2 / 4$$

$$\cong (D_i / D_i^2)^{0.8} \cong 1 / D^{0.8} \cong D^{-0.8}$$

$$\frac{1}{U_1} \cong \frac{1}{D_i^{-0.8}}$$

$$A_i \cong D_1^{0.8}$$

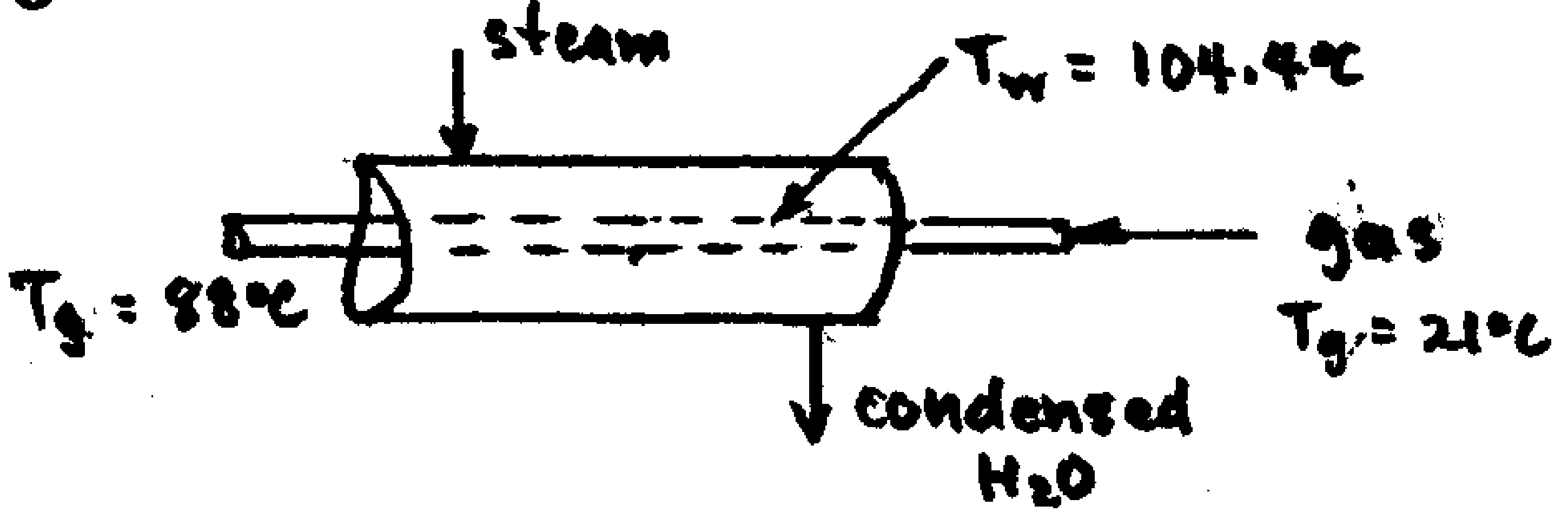
Answer

Assuming flow is turbulent

Relative D_i	Relative A_i
1	1
2	1.74
3	2.41
4	3.02
5	3.62
10	6.31

14', 12

14-8



Assume that the gas has the properties of air. These properties, particularly density, depend on the pressure of the gas. Since such information was not included, consider two cases: one with a pressure of 101 kPa and the other with a pressure of 202 kPa for the gas. At 101 kPa, the properties for the air are:

$$\rho = 1.078\,\text{kg/m}^3 \quad C_p = 1046\,\text{J/kg}\cdot\text{K} \quad k = 0.0282\,\text{W/m}\cdot\text{K} \quad \mu = 0.018\,\text{cP}$$

Determine the optimal velocity from a rearrangement of Eq. (13-15) assuming turbulent flow and $D_i = 0.023$.

For a pressure of 101 kPa

$$V^{0.45} = \left[D_i/(0.363)\right]A^{0.45}\rho^{0.13}$$

$$V^{0.45} = (0.023)/(0.363)[(0.785)(0.023)^2]^{0.45}(1.078)^{0.13} = 2.09$$

$$V \cong 5.2\,\text{m/s}$$

$$V = \dot{m}/\rho A_c$$

Since $\dot{m}_{tube} = V\rho A_c = (5.2)(1.078)0.785)(0.023)^2 = 2.3278\times10^{-3}\,(\text{kg/s})/\text{tube}$

$$N_T = \dot{m}/\dot{m}_{tube} = 0.075/2.3278\times10^{-3} = \underline{\underline{32.2 \text{ tubes} \sim 32 \text{ tubes}}}$$

Answer

The heat transferred to the gas is

$$\dot{q}_i = \dot{m}\,C_p\,\Delta T_{gas} \qquad \Delta T_{\ln m} = \frac{(104.4-21)-(104.4-88)}{\ln 83.4/16.4}$$

$$= (0.075)(1046)(88-21) \qquad = 41.2^\circ\text{C}$$

$$= 5260\,\text{J/s}$$

$$\dot{q}_i = A_i U_i\,\Delta T_{\ln m}$$

From Eq. (14-4a)

14-8 (continued)

$$\frac{1}{U_1}=\frac{1}{h_i}+\frac{A_i x_W}{k_w A_{w,m}}+\frac{D_i}{h_o D_o}$$

assume that the thermal resistance of the pipe and the steam film are negligible

From Eq. (14-18)

$$h_i=(k/D_i)(0.023)(\mathrm{Re})^{0.8}(\mathrm{Pr})^{1/3}(\mu/\mu_W)^{0.14}$$

$$=\left(\frac{0.0282}{0.023}\right)(0.023)\left[\frac{(0.023)(5.2)(1.078)}{0.000018}\right]^{0.8}\left[\frac{(1046)(0.000018)}{0.0282}\right]^{1/3}$$

$$=29.9\,W/\mathrm{m}^2\cdot\mathrm{K}\cong 30\ \mathrm{W/m^2\cdot K}$$

$$L=(\dot{q}/N_T)\pi D_i \mu_i \Delta T_{\ln m}$$

$$=(5260/32)(\pi(0.023)(30)(41.2)$$

$$=\underline{\underline{1.84\,\mathrm{m\ or\ }6.0\,\mathrm{ft}}}$$

Answer

If the gas pressure is 202 kPa, $\rho=2.1545\,\mathrm{kg/m^3}$ and using the same procedure as above

$V=4.25\,\mathrm{m/s}$

$N_T=19.7$ tubes $\cong$ **20 tubes**

Answer

$h_i=44.3\,\mathrm{W/m^2\cdot K}$

$L=\underline{\underline{2.13\ \mathrm{m}=7\ \mathrm{ft}}}$

Answer

This procedure can be followed to obtain appropriate values for other gas conditions.

14-8 (continued)

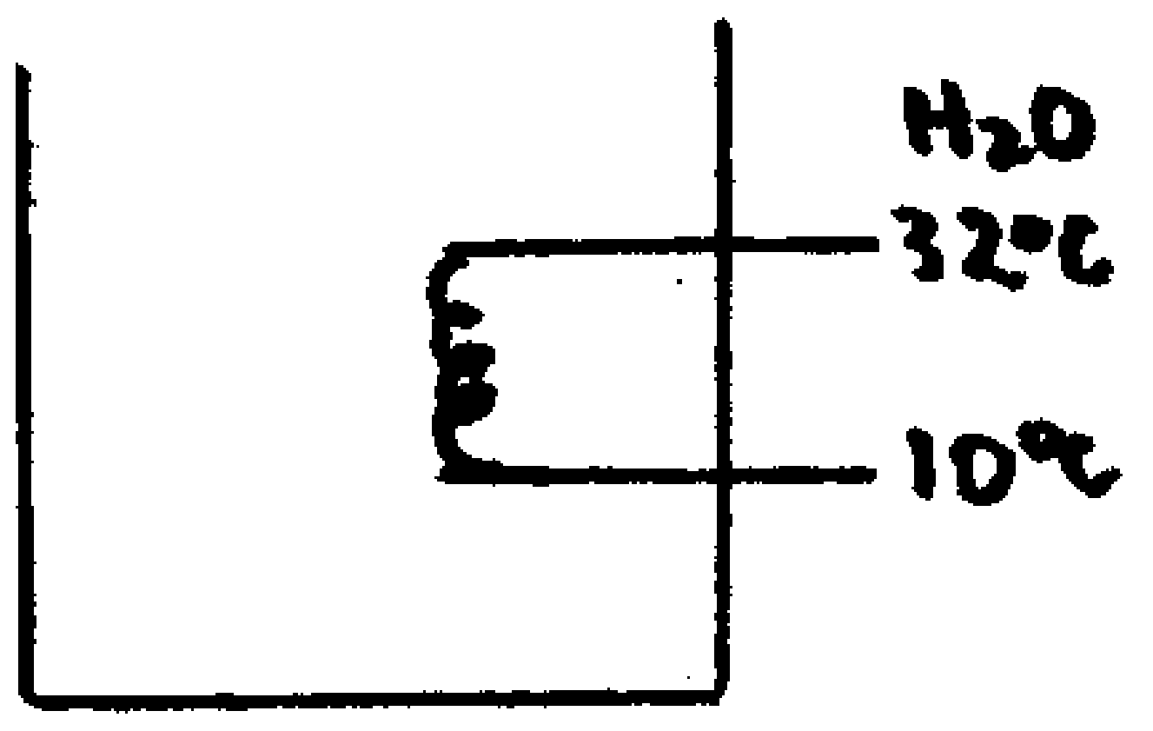
Consider the two cases for providing a 20 percent safety factor.

1. Adding 20% more tubes of the same diameter and length increases the heat transfer area by 20%, but the mass flow in each tube is now 20% less and the velocity in each tube is reduced by 20%. This reduces the heat transfer coefficient inside the tubes by approximately 16% which will require about a 20% increase in heat transfer area to obtain the desired heat transfer. Thus, the desired safety factor will not be achieved by this action.
2. Increasing the tube diameter by 20 percent while keeping the same number fo tubes and same tube length, increases the heat transfer area by 55% and reduces the velocity in each tube by 30.6% and the heat transfer coefficient by 25.3%. The increase in diameter of the tubes provides approximately a 7.5% safety factor. Again the desired safety factor will not be achieved.

A better choice to obtain a 20% safety factor is to increase the length of the tubes by 20%. This will maintain the same mass flow and velocity in each tube. The heat transfer coefficient will not change and 20% more heat can be transferred. The disadvantage of this approach is that the pressure drop across the exchanger will be increased by 20%. If this exceeds the allowable limit, then additional changes in tube size and number will need to be taken.

14'. 15

14-9



$\Delta H_{\text{cond}} = 335\,kJ/\text{kg at } 88^\circ\text{C} \quad \dot{m}_{\text{cond}} = 0.126\,\text{kg/s}$

$h_{\text{cond}} = 1420\,W/\text{m}^2\cdot\text{K} \qquad k_{\text{pipe}}\ @\ 21^\circ\text{C}{=}385\ \text{W/mK}$

$\text{D}_{\text{i}} = 0.0127\,\text{m}$

$D_o = 0.0152\,m$

Heat requirement is

$$\dot{q} = (0.126)(335) = 42.2\,\text{kJ/s} = \dot{m}_{\text{H}_2\text{O}} C_{p_{\text{H}_2\text{O}}}(32-10)$$

$$\dot{m}_{\text{H}_2\text{O}} = 42.2/(4.186)(22) = 0.458\,\text{kg/s}$$

$$V_{\text{H}_2\text{O}} = (0.485/997)/0.785(0.0127)^3 = 3.63\text{m/s}$$

$$\text{Re}_{H_2O} = \frac{DV\rho}{\mu} = \frac{(0.0127)(3.63)(997)}{0.000983} = 46{,}750 \quad \text{flow is turbulent}$$

Obtain heat transfer coefficient for the water using Eq. (14-18) with physical properties at the average temperature of the water, namely 21°C.

$$h_i = (0.023)\left(\frac{0.604}{0.0127}\right)(46{,}750)^{0.8}\left[\frac{(4186)(0.000983)}{0.604}\right]^{1/3} = 11{,}110\,\text{W/m}^2\cdot\text{K}$$

Base the overall heat transfer coefficient on the inside area using Eq. (14-4a)

$$\frac{1}{U_i} = \frac{1}{11{,}110} + \frac{(0.0127)\ln(0.0152/0.0127)(0.0025)}{(0.0152-0.0127)(385)} + \frac{0.0127}{(1420)(0.0152)}$$

$$= 6.843\times10^{-4}$$

$$U_i = 1461\ \text{W/m}^2\cdot\text{K}$$

$$\dot{q} = U_i A \Delta T_{\text{avg}} = 42.2\,\text{kJ/s}$$

$$\Delta T_{\text{avg}} = [(88-15)+(88-32)]/2 = 64.5^\circ\text{C}$$

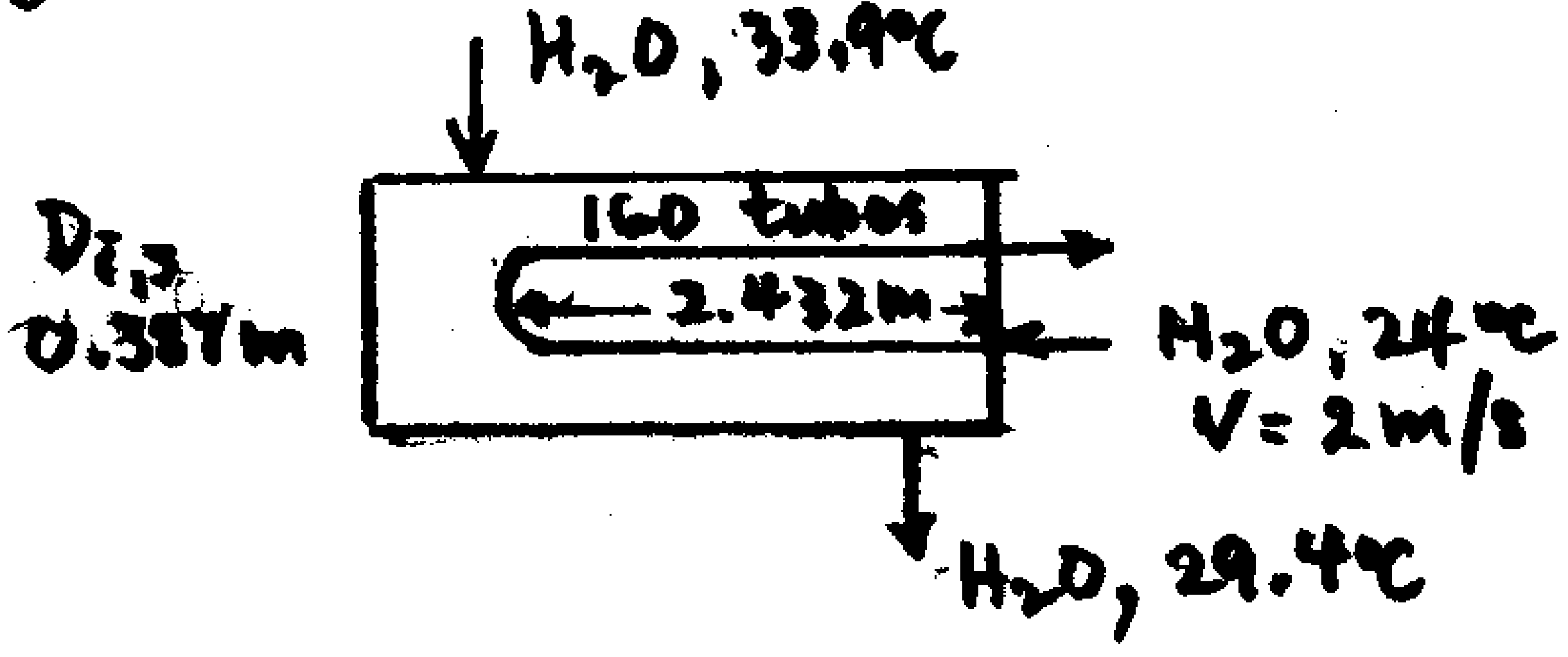
$$\text{A}{=}\pi\text{D}_{\text{i}}L = \dot{q}/U_i\Delta T_{\text{avg}}$$

$$L = \dot{q}/\pi D_i U_i \Delta T_{\text{avg}} = (42{,}200)(6.843\times10^{-4})/\pi(0.0127)(64.5)$$

$$L = \underline{\underline{11.2\,\text{m}}}$$

Answer

14-10



$D_o = 0.0191$ m, 18BWG $\quad t_W = 0.0025$m

$V = 2.0$m/s $\quad D_i = 0.0166$m

Δp_i and $\Delta p_o < 69$kPa

$h_{o,d}$(distilled water) = 11,360 W/m^2·K $\qquad h_{i,d}$(cooling water) = 5680 W/m^2·K

From a heat balance on distilled water side

$$\dot{q} = -(22)(4179)(33.9 - 29.4) = -414,400\,\text{J/s}$$

this equals heat gained by cooling water

$$\dot{m}_{H_2O} = \left(\frac{160}{2}\right)\left(\frac{\pi(0.0166)^2}{4}\right)(997)(20) = 34.52\,\text{kg/s}$$

$$\dot{q} = \dot{m}C_p(T_{exit} - T_{inlet})$$

$$T_{H_2O,exit} = \dot{q}/\dot{m}C_p + T_{inlet} = 414,400/(34.52)(4180) + 24$$

$$= 26.9^\circ\text{C}$$

$$\Delta T_{\ln mean} = \frac{(33.9 - 26.9) - (29.4 - 24.0)}{\ln(33.9 - 26.9)/(29.4 - 24.0)} = 6.2^\circ\text{C}$$

Correct ΔT for the two tube pass arrangement by obtaining correction factor

$$P = \frac{26.9 - 24}{33.9 - 24} = 0.29 \qquad R = \frac{33.9 - 29.4}{26.9 - 24} = 1.55 \quad \text{From Fig.14-4} \quad F = 0.94$$

$$\Delta T = (0.94)(6.2) = 5.8^\circ\text{C}$$

An approximate value for the shell-side heat transfer coefficient in a cross-flow exchanger with segmented baffles and reasonable clearance between baffles, between tubes, and between baffles and shell can be obtained using the relation developed by Colburn.[1]

$$h_o = \frac{k}{D_o}\frac{a_o}{F_s}\left(\frac{D_o G_s}{\mu_f}\right)^{0.6}\left(\frac{C_p\mu}{k}\right)_f^{1/3} \quad \text{Subscript } f \text{ at average film temperature.}$$

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[1] It can be estimated more accurately by using the Bell-Delaware method outlined in Example 14-6 or by using a computer software program.

14-10 (continued)

where $a_o = 0.33$ when tubes in a tube bank are staggered, F_s a correction factor to account for bypassing effects usually between 1.1 and 1.7, and G_s the shell-side mass velocity across the tube based on the minimum free area between baffles at the shell axis. The free area S for use in evaluating G_s for the case of a full-packed shell and transverse openings giving the smallest free area is estimated from

$$S = \frac{\text{(ID of shell)(clearance between adjacent tubes)(baffle spacing)}}{\text{center-to-center distance between adjacent tubes}}$$

$$= \frac{(0.387)(0.0238 - 0.0191)(0.3)}{0.0238} = 0.023\,\text{m}^2$$

$$G_s = 22/0.023 = 956.5 \text{ kg/m}^2\cdot\text{s}$$

$$\text{Pr} = (C_p\mu/k)_f = (4179)(0.000786)/0.618 = 5.32 \qquad \text{at 304K film temperature}$$

$$\text{Re} = D_o G_s/\mu_f = (0.0191)(956.5)/0.000786 = 23{,}240$$

Assume an F_s of 1.3 to account for the bypassing effects.

$$h_o = (0.618/0.0191)(0.33/1.3)(23{,}240)^{0/6}(5.32)^{1/3} = 5973 \text{ W/m}^2\cdot\text{K}$$

The heat transfer coefficient inside the tubes is given by Eq. (14-18). Assume negligible viscosity effects.

$$h_i = 0.023(k/D_i)\text{Re}^{0.8}\,\text{Pr}^{1/3}$$

$$\text{Re}_i = D_i V\rho/\mu = (0.0166)(2.0)(996)/0.000821 = 40{,}275$$

$$\text{Pr}_i = C_p\mu/k = (4179)(0.000821)/0.609 = 5.63$$

$$h_i = (0.023)(0.609/0.0166)(40{,}275)^{0.8}(5.63)^{1/3} = 7252 \text{ W/m}^2\cdot\text{K}$$

The overall heat transfer coefficient is then obtained from Eq. (14-4). Assume tube is copper $k_w = 383$ W/m·K.

14-10 (continued)

$$\frac{1}{U_o} = \frac{D_o}{h_i D_i} + \frac{D_o}{h_{i,d} D_i} + \frac{D_o \ln(D_o / D_i) x_w}{k_w (D_o - D_i)} + \frac{1}{h_o} + \frac{1}{h_{o,d}}$$

$$= \frac{0.0191}{(7252)(0.0166)} + \frac{0.0191}{(5680)(0.0166)} + \frac{0.0191 \ln(0.0191/0.0166)}{383} + \frac{1}{5973} + \frac{1}{11{,}360}$$

$$= 6.213 \times 10^{-4}$$

$U_o = 1610\ \mathrm{W/m^2 \cdot K}$

Area required is

$$A_o = \dot{q} / U_o \Delta T_{mean} = 414{,}400/(1610)(5.8) = 44.4\ \mathrm{m^2}$$

Area available is

$$A_o = (160)\pi(0.0191)(4.876) = 46.8\ \mathrm{m^2}$$

The area is adequate with copper tubes, but would not be adequate for steel tubes. †

Pressure drop calculation inside the tube using Eq. (14-23)

$$\Delta p_i = 2\beta_i + G_i^2 L n_p / \rho_i D_i \phi_i$$

Because of the small temperature change $\phi_i \cong 1$ and assuming few expansions and contractions because of the liquid flows, $\beta \cong 1$. For a Re of 40,275, $f = 0.0055$ from Fig.(12-1)

$$\Delta p_i = (2)(0.0055)[(2.0)(996)]^2 (4.876)(2)/(996)(0.0166)(10^{-3}) = 25.75\ \mathrm{kPa}$$

The inside pressure drop is adequate.

The shell-side pressure drop may be obtained with Eq.(14-24).

$$\Delta p_o = 2 B_o f' N_{tr} G_s^2 / \rho_o$$

$$N_{tr} = 0.387 / 0.0238 = 16.3 \quad \text{as a maximum}$$

Combining Eqs.(14-24a) and 14-24c results in

---

† If the tubes were of steel, the U_o would be reduced to 1483 $\mathrm{W/m^2 \cdot K}$ and the area required would be 48.2 $\mathrm{m^2}$, slightly above that available.

14-10 (continued)

$$f' = \left[0.23 + \frac{0.11}{(X_T^{-1})^{1.08}}\right]\left(\frac{D_o G_s}{\mu_f}\right)^{-0.15}$$

$$= \left[0.23 + \frac{0.11}{\left(\dfrac{0.0238}{0.0191} - 1\right)^{1.08}}\right]\left(\frac{1}{23,240}\right)^{0.15} = 0.161$$

Assume that $B_o = 16$ as a maximum

$$\Delta p_o = (2)(16)(0.161)(16.3)(956.5)^2 / 995 = 77,220 \text{ Pa} = \mathbf{77.2 kPa}$$

ANSWER

The use of $B_o = 16$ and $N_{tr} = 16.3$ provides a large double safety factor in the calculation. The allowable pressure drop is 69 kPa, but the safety factor should be sufficient to accept the heat exchanger to cool 22 kg/s of distilled water.

Addendum:

This problem can be calculated using appropriate software such as CC-Therm. When this done, the overall heat transfer coefficient is about 10% higher and the pressure drop for the shell side is nearly 20% lower. Thus. the computer solution also indicates that the proposed heat exchanger can handle the specified heat load.

14-11

First calculate the ideal cross-flow heat transfer coefficient

$$\mathrm{Re} = D_o V_{\max} \rho / \mu \qquad \text{where } V_{\max} \text{ is given by Eq. (14-39) using Eq. (14-40)}$$

$$S_m = L_B[(D_s - D_{OTL}) + \frac{D_{OTL} - D_o}{P_T}(P_T - D_o)] \qquad D_{OTL} = D_s - \Delta_b = 0.54 - 0.035 = 0.505$$

$$= (0.127)[0.035 + \frac{0.505 - 0.0254}{0.03175}(0.03175 - 0.0254)] = 0.01662\,\mathrm{m}^2$$

$$V_{\max} = \frac{5.5}{(730)(0.01662)} = 0.4533\,\mathrm{m/s}$$

Use Eq. (14-22) to obtain the ideal heat transfer coefficient for a Reynolds number of 21,010 with a $\mu = 4\times10^{-4}\,\mathrm{Pa\cdot s}$

$$h_o = \frac{k}{D_o} a\,\mathrm{Re}^m\,\mathrm{Pr}^{0.34}\,F_1F_2 \qquad \text{neglect the correction factors } F_1 \text{ and } F_2$$

$$= \left(\frac{0.13}{0.0254}\right)(0.211)\left[\left(2.1\times10^4\right)^{0.651}\right]\left[\frac{(2470(4\times10^{-4}}{0.13}\right]^{0.34} = 1402\,\mathrm{W/m^2\cdot K}$$

Now calculate correction factors J_c, J_L, and J_B in Eq. (14-42) and use Fig. 14-45

$$F_c = \frac{1}{\pi}\left[\pi + \frac{2(D_s - 2L_c)}{D_{OTL}}\sin\left(\cos^{-1}\frac{(D_s - 2L_c)}{D_{OTL}}\right) - 2\cos^{-1}\frac{(D_s - 2L_c)}{D_{OTL}}\right]$$

$$L_c = (0.25)D_s = (0.25)(0.54) = 0.135\,\mathrm{m}$$

$$\frac{D_s - 2L_c}{D_{OTL}} = \frac{0.54 - 2(0.135)}{0.505} = 0.5346$$

$$F_c = \frac{1}{\pi}\left[\pi + (2)(0.5346)\sin(\cos^{-1}0.5346) - 2\cos^{-1}0.5346\right]$$

$$= 0.647$$

$$J_c = 0.55 + (0.72)(0.647) = 1.016$$

To calculate J_C, determine the leakage areas from Eqs. (14-43a) and (14-43b)

14-11 (continued)

$$S_{sb} = D_s\left(\frac{\Delta_{sb}}{2}\right)\left[\pi - \cos^{-1}\left(1-\frac{2L_c}{D_s}\right)\right]$$

$$= (0.54)\left(\frac{0.005}{2}\right)\left[\pi - \cos^{-1}\left(1-\frac{2(0.135)}{0.54}\right)\right]$$

$$= 2.826\times10^{-3}\,\mathrm{m}^2$$

$$S_{tb} = \pi D_o\left(\frac{\Delta_{tb}}{2}\right)N_T\left(\frac{1+F_c}{2}\right)$$

$$= \pi(0.0254)\left(\frac{0.0008}{2}\right)(158)\left(\frac{1+0.647}{2}\right) = 4.153\times10^{-3}$$

$$\frac{S_{sb}+S_{tb}}{s_m} = \frac{2.826\times10^{-3}+4.153\times10^{-3}}{1.662\times10^{-2}} = 0.4199$$

$$\frac{S_{sb}}{S_{sb}+S_{tb}} = \frac{2.826\times10^{-3}}{2.826\times10^{-3}+4.153\times10^{-3}} = 0.4049$$

J_L is obtained from Fig. 14-46 as 0.57

To calculate J_B, obtain the bypass area fraction F_{bp} from Eq. (14-44)

$$F_{bp} = (L_B / S_m)(D_s - D_{OTL}) = (0.127/1.662\times10^{-2})(0.035) = 0.2674$$

For $N_{ss}/N_c = 0.2$, Fig. 14-47 provides a value for J_B of 0.92

The individual heat transfer coefficient on the shell side is then

$$h_s = h_o J_C J_L J_B = (1402)(1.016)(0.57)(0.92) = \underline{\underline{747\ \mathrm{W/m^2\cdot K}}}$$

Answer

The procedure to obtain the pressure drop on the shell side follows the steps outlined in Eqs. (14-46) through (14-51). The ideal cross-flow pressure drop is obtained from

14-11 (continued)

$$\Delta p_c = (K_a + N_c K_f)(\rho V_{\max}^2 / 2) \quad \text{neglect } K_a \text{, obtain } K_f \text{ from Table 14-10}$$

$$K_f = 0.267 + \frac{0.249\times10^4}{21{,}010} - \frac{0.927\times10^7}{(21{,}010)^2} + \frac{0.1\times10^4}{(21{,}010)^3} = 0.3625 \text{ for } 2\times10^4 < \text{Re} < 2\times10^6$$

$$N_c = D_s(1 - 2L_c / D_s) / P_{TD}$$

$$= (0.54)[1 - 2(0.135)/0.54]/(0.03175) = 8.5$$

The cross-flow pressure drop on one baffle space is obtained from

$$\Delta p_c = (8.5)(0.3625)(730)(0.4533)^2 / 2 = 231\,\text{Pa}$$

Now calculate the pressure drop for the window zone between baffles with the window flow area given by Eq. (14-49)

$$S_w = (D_s^2 / 4)\left[\cos^{-1} D_B - D_B(1 - D_B^2)^{1/2}\right] - (N_T / 8)(1 - F_c)\pi D_o^2$$

$$D_B = (D_s - 2L_s) / D_s = [0.54 - 2(0.135)]/0.54 = 0.5$$

$$S_w = (0.54)^2 / 4\left\{\cos^{-1} 0.5 - 0.5[1 - (0.5)^2]^{1/2}\right\} - (158/8)(1 - 0.647)\pi(0.0254)^2$$

$$= 0.04474 - 0.01414 = 0.0306\,\text{m}^2$$

Calculate the effective number of cross flow rows in the window zone

$$N_{cw} = 0.8L_c / P_{TP} = (0.8)(0.135)/0.03175 = 3.40$$

The window zone pressure drop is given by Eq. (14-47b)

$$\Delta p_w = (2 + 0.6N_{cw})\dot{m}_T^2 / 2S_m S_w \rho \quad \text{where } \dot{m}_T = 5.5\,\text{kg/s}$$

$$= [2 + 0.6(3.4)](5.5)^2 / (2)(0.01662)(0.0306)(730) = 164.6\,\text{Pa}$$

Now use Figs. 14-48 and 14-49 to obtain leakage and bypass correction factors R_B and R_L for use in Eq. (14-51).

From Fig. 14-48, $R_B = 0.76$

Fig. 14-49, $R_L = 0.33$

$$\Delta p_s = [(N_B - 1)\Delta p_c R_B + N_B \Delta p_w] R_L + 2\Delta p_c R_B (1fN_{cw} / N_c)$$

where N_B from Eq. (14-36) is $L_s / (L_B + t_b) - 1$

14-11 (continued)

$$N_B = \frac{4.267}{0.127 + 0.005} - 1 = 31.3 \text{ baffles}$$

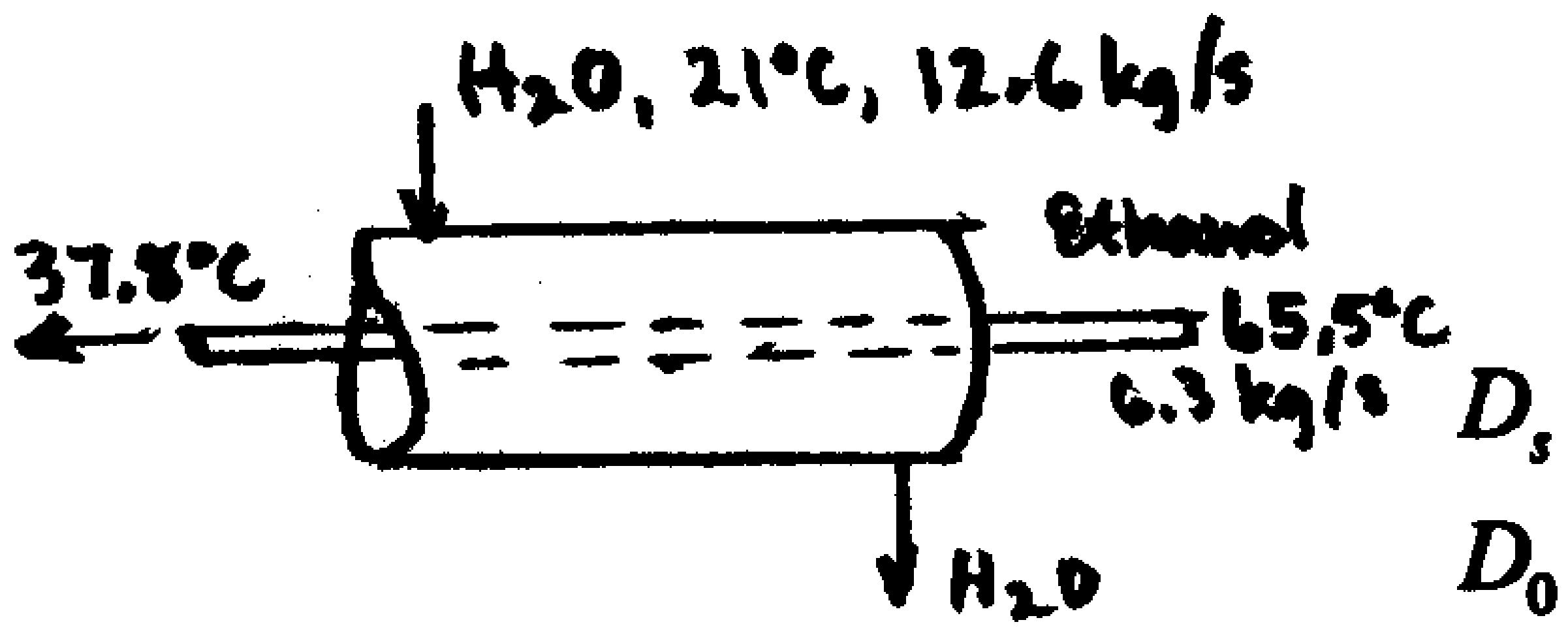
$$\text{or } 31$$

$$\Delta p_s = [(31-1)(231)(0.76) + (31)(165)](0.33) + 2(231)(0.76)(1 + 3.4/8.5)$$

$$= 3426 + 492 = \underline{\underline{\textbf{3918 Pa or 3.9 kPa}}}$$

Answer

14-12



$D_s = 0.127\text{m} \quad N_T = 7$

$D_0 = 0.0254\text{m}$

$D_i = 0.0203\text{m}$

For the alcohol,

$T_{avg} = 51.65^\circ\text{C}$

$C_p = 2512\text{J/kg}\,^\circ\text{C}$

$\rho = 770\,\text{kg/m}^3$

$k = 0.151\ \text{W/m}\cdot\text{K}$ (Appendix D)

$$\dot{q}_{alc} = \dot{m}C_p\Delta T = (6.3)(2512)(65.5-37.8)(10^{-3})$$
$$= 438.4\,\text{kJ/s}$$
$$\Delta T_{H_2O} = \dot{q}/\dot{m}C_p = (438{,}400)/(12.6)(4186)$$
$$= 8.3^\circ\text{C} \quad T_{ave,H_2O} = 25.1^\circ\text{C}$$

Velocity of ethyl alcohol

$$V_{alc} = (6.3\,\text{kg/s}/7)\left(\frac{1}{770\text{kg/m}^3}\right)\frac{4}{\pi(0.0203)^2\text{m}^2} = 3.61\text{m/s}$$
$$\text{Re} = DV\rho/\mu = (0.0203)(3.61)(770)/0.0007 = 80{,}600$$

Calculate h_{alc} with Eq.(14-18) assuming $(\mu/\mu_w)^{0.14} \cong 1$

$$h_{alc} = (0.151/0.0203)(0.0203)(80{,}600)^{0.8}\left[(2512)(0.0007)/0.151\right]^{1/3}$$
$$= 3263\,\text{W/m}^2\cdot\text{K}$$

For the water side, the available cross-sectional area available for flow is

$$A = \frac{\pi D_S^2}{4} - \frac{7\pi D_0^2}{4} = \frac{\pi(0.127)^2}{4} - \frac{7\pi(0.0254)^2}{4} = 0.00915\,\text{m}^2$$
$$V_{H_2O} = (12.6)/(996)(0.00915) = 1.38\,\text{m/s}$$

The combined diameter for the water side is estimated by assuming that the 7 tubes are combined into one tube or $7\pi D_0^2/4 = \pi D_{comb}^2/4$.

14-12 (continued)

$$D_{combined} = \left[7(0.0254)^2\right]^{1/2} = 0.0672\text{m}$$

as suggested by the <u>*Heat Exchanger Design Handbook*</u>, the effective diameter for the water side can be taken as 0.127-0.0672 = 0.0598m.

$$\text{Re} = D_e\rho V/\mu = (0.0598)(996)(1.38)/0.000899 = 91,400$$

$$\text{assume } h_{H_2O} = \left(\frac{0.610}{0.0598}\right)(0.023)(91,400)^{0.8}\left[(4180)(0.000899)/0.610\right]^{1/3}$$

$$= 4002\ \text{W/m}^2\cdot\text{K}$$

Base overall coefficient on outside area of tubes

$$\frac{1}{U_0} = \frac{1}{4002} + \frac{0.0254}{(3263)(0.0203)} + \frac{(0.0254)(0.00254)}{(45)(0.02286)}$$

$$= 2.4487\times10^{-4} + 3.8346\times10^{-4} + 6.272\times10^{-5}$$

$$= 6.9105\times10^{-4}$$

$$U_0 = 1447\ \text{W/m}^2\cdot\text{K}$$

$$\Delta T_{\ln mean} = \frac{(65.5-29.3)-(37.8-21)}{\ln 36.2/16.8} = \frac{19.4}{0.767} = 25.3^\circ\text{C}$$

Outside tube area is then

$$A = \dot{q}/U_0\Delta T_{\ln m} = 438,400/(1447)(25.3) = 12.0\,\text{m}^2$$

Tube length

$$L = (12.0/7)/(\pi)(0.0254) = 21.5\,\text{m}$$

From the mechanical energy balance, Eq.(12-2), the work that must be provided in the pumping process reduces to

$$W = \sum F = 2fV^2L/D$$

From Fig. 12-1 values for the friction factor for smooth pipes are

14-12 (continued)

$$\mathrm{Re} = 80{,}600 \quad f = 0.0061$$

$$\mathrm{Re} = 91{,}400 \quad f = 0.0059$$

$$\sum F_{alc} = (2)(0.0061)(3.61)^2(21.5)(1.2)/0.0203 = 202\ \mathrm{Nm/kg}$$

$$\sum F_{H_2O} = (2)(0.0059)(1.38)^2(21.5)(1.2)/0.0598 = 9.7\ \mathrm{Nm/kg}$$

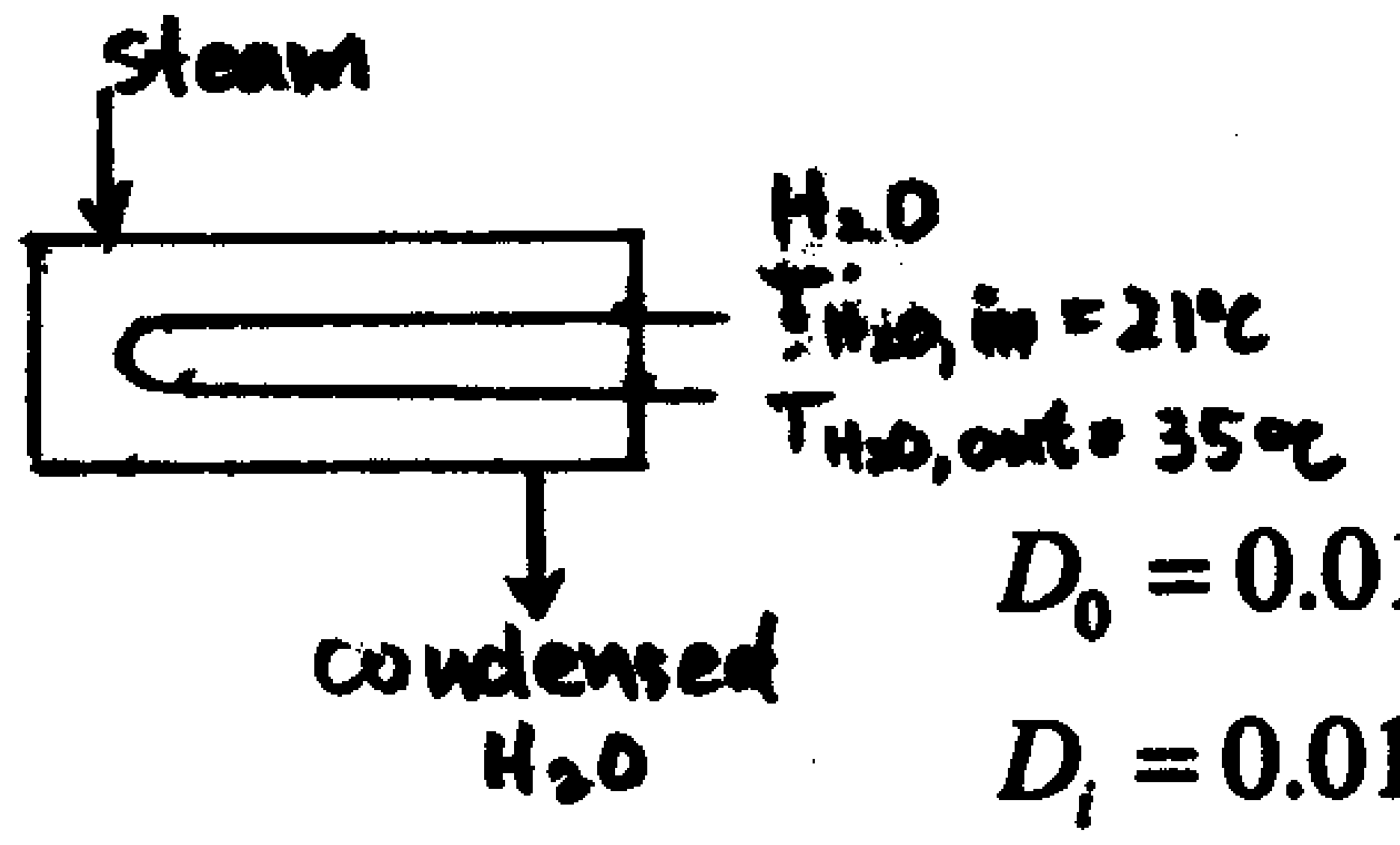
Power cost

$$\mathrm{Cost}_{alc} = \left(\frac{202}{0.6}\right)(6.3)\left(\frac{1}{1000}\right)(300)(24)(0.08) = \$1222/\mathrm{y}$$

$$\mathrm{Cost}_{H_2O} = \left(\frac{9.7}{0.6}\right)(12.6)\left(\frac{1}{1000}\right)(300)(24)(0.08) = \$117/\mathrm{y}$$

Total pumping cost = 1222 + 117 = **$ 1339/y**
Answer

14-13



$D_0 = 0.0159\text{m}$ 18 gauge $L_p = 1.83\text{m}$ $T_{sat} 51.7°\text{C}$

$D_i = 0.0134\text{m}$ 129 tubes/pass $V_{H_2O} = 1.22\text{m/s}$

Total area for heat transfer based on D_0

$$A = (\pi D_0) L_p N_T (2) = \pi(0.0159)(1.83)(129)(2) = 23.6\text{m}^2$$

Total cross-sectional area for water flow/tube pass

$$A_C = (\pi D_i^2 / 4) N_T = (\pi / 4)(0.0134)^2 (129) = 0.0182\text{m}^2$$

Although this is a tube-tube pass heat exchanger, the mean temperature difference is equal to the log mean temperature difference with the correction factor F = 1

$$\Delta T_{lnm} = \frac{(51.7-21)-(51.7-35)}{\ln 30.7/16.7} = 23°\text{C}$$

To simplify the problem evaluate the overall heat transfer coefficient at the location where the water temperature is (21+35)/2=28°C. To obtain the heat transfer coefficient by Eq.(14-78) requires a wall temperature. Since the thermal resistance in the water stream is at least twice that in the condensing steam, assume a wall temperature of 46°C. At a fluid temperature of 28°C and a film temperature of 37°C, the properties for water are

$\rho = 996\text{kg/m}^2$ $\mu = 0.838\text{cP}$ $C_p = 4179\text{J/kg} \cdot \text{K}$

$k = 0.614\text{W/mk}$ $\mu_w = 0.710\text{cP}$

$$h_i = (k / D)(0.023)(D_i V \rho / \mu)^{0.8} (C_p \mu / k)^{1/3} (\mu / \mu_w)^{0.14}$$

$$= \left(\frac{0.614}{0.0134}\right)(0.023)\left[\frac{(0.0134)(1.22)(996)}{8.38\times10^{-4}}\right]^{0.8}\left[\frac{(4179)(8.38\times10^{-4})}{0.614}\right]^{1/3}\left[\frac{8.38\times10^{-4}}{7.10\times10^{-4}}\right]^{0.14}$$

$$= 5195\text{W/m}^2 \cdot \text{K}$$

For the condensing steam use Eq.(14-78) for a film temperature of 48.9°C.

14-13 (continued)

$$\bar{h}_0 = 0.725\left[\frac{k_L^3\rho_L(\rho_L-\rho_V)g\lambda_c}{\mu_L D_0(T_{sat}-T_w)}\right]^{1/4} \qquad \lambda_c = 2388\,\text{kJ/kg}$$

$$\bar{h}_0 = 0.725\left[\frac{(0.642)^3(989)(989-0.07)(9.806)(2388\times10^3)}{(0.000567)(0.0159)(51.7-46)}\right]^{1/4}$$

$$= 13{,}435\,\text{W/m}^2\cdot\text{K}$$

$$\frac{1}{U_0} = \frac{0.0159}{(5195)(0.0134)} + \frac{(0.0159)\ln(0.0159/0.0134)}{(2)(99.7)} + \frac{1}{12{,}450}$$

$$U_0 = 3160\,\text{W/m}^2\cdot\text{K}$$

Check on the wall temperature

$$(3160)(51.6-28) = 13{,}435(51.7-T_w)$$

$T_w = 46.1^\circ C$ compared to an assumed wall temperature of 46°C; further iteration with a new wall temperature is not necessary. Thus the overall heat transfer coefficient is **3160W/m$^2\cdot$K**

Answer

The kilograms of steam condensed is

$$\dot{q}/\lambda = UA\Delta T_{lm}/\lambda$$

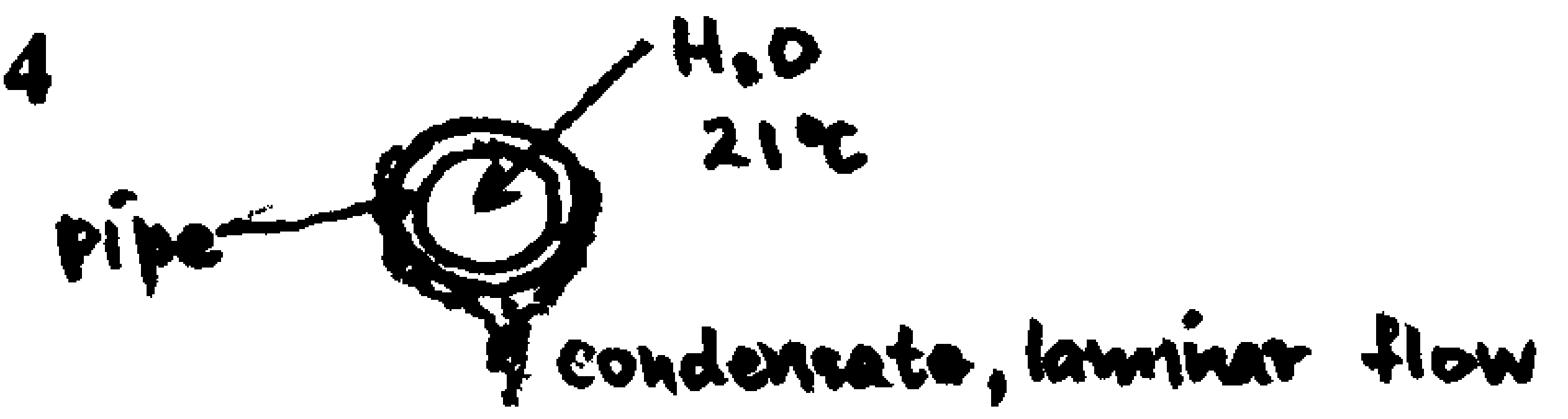
$$= (3160)(23.6)(23)/2388\times10^3$$

$$= \mathbf{0.72\,kg/s}$$

Answer

Note that the calculation has not included various operating conditions that could alter the condensation process such as fouling effects, presence of noncondensibles in the steam, subcooling of the condensate, etc.

14-14



$D_0 = 0.030\text{m}$

$D_i = 0.025\text{m}$

Determine h_i from Eq.(14-18). Assume a velocity of 1 m/s for the water flow

$$\mathrm{Re}_{H_2O} = (DV\rho/\mu)_i = (0.025)(1)(998)/0.000982 = 25,400$$

$$\mathrm{Pr}_{H_2O} = \left(\frac{C_p\mu}{k}\right) = (4186)(0.000982)/0.599 = 6.86$$

$$h_{H_2O} = \left(\frac{0.599}{0.025}\right)(0.023)(25,400)^{0.8}(6.86)^{1/3} \quad \text{assume}\left(\mu/\mu_w\right)^{0.14} \cong 1$$

$$h_{H_2O} = 3498\ \text{W/m}^2\cdot\text{K}$$

Calculations for the outside condensation coefficient uses Eq. (14-78). However, since the temperature of the wall T_w is unknown, this requires an iterative calculation that is best performed with a simple computer software program. The procedure is as follows:

> Assume a T_w and calculate the condensation coefficient. This permits evaluation of an overall coefficient and a heat transfer rate. From the latter T_w may be evaluated. If the value does not match the initial assumption, modify T_w and repeat the process. The first step is provided below.
>
> The average T_w would be $(127+21)/2 = 74^\circ\text{C}$ or 347K. Begin with a T_w of 330 K.

$$h_{i-C_s} = 0.725\left[\frac{k_L^3\rho_L(\rho_L-\rho_v)g\lambda_C}{\mu_L D_0(T_{sat}-T_w)}\right]^{1/4}$$

$$= 0.725\left[\frac{(0.082)^3 551(551-14.1)(9.806)(283,800)}{(0.0001285)(0.03)(400-330)}\right]^{1/4}$$

$$= 825\ \text{W/m}^2\cdot\text{K}$$

$$\frac{1}{U_0} = \frac{1}{825} + \frac{0.03}{(3498)(0.025)} = 1.5552\times10^{-3}$$

$$U_0 = 643$$

$$\dot{q} = -U_o(T_{sat}-T_w) = 643(400-294) = 68,160\ \text{W/m}^2$$

$$T_w = T_{sat} - \dot{q}/h_{i-C_s} = 400 - 68,160/825 = 317.4\text{K}$$

Temperature assumed is high and needs to be lowered for the next iterarion.

14-14 (continued)

For an assumed T_w of 317.4K

$$h_{i-C_5} = 792\,\mathrm{W/m^2 \cdot K}$$
$$U_0 = 622.6\,\mathrm{W/m^2 \cdot K}$$
$$T_w = 316.7\,\mathrm{K}$$

after several iterations the final values are

$$U_0 = \underset{\textbf{Answer}}{\underline{\underline{\mathbf{619\,W/m^2 \cdot K}}}} \qquad \dot{q} = \underset{\textbf{Answer}}{\underline{\underline{\mathbf{66.2\,kW/m^2}}}}$$

The rate of condensation per unit length of tube is

$$\begin{aligned}\dot{m}_{cond} &= (\dot{q}/\Delta H_c)\pi D \\ &= (66.2/283.8)\pi(0.03) = \underset{\textbf{Answer}}{\underline{\underline{\mathbf{0.022(kg/s)/m}}}}\end{aligned}$$

14-15

The CC-Therm software is used to design the partial condenser E-601. The tabulated data and results are shown below.

	Tube-side	Shell-side
Flow, kg/s	35.30	64.57
Non-condensed flow, kg/s	-	17.07
Operating pressure kPa	109	194.4
Press-drop allowed, kPa	34.5	34.5
Outside tube diameter, m	0.0127	-
Inside tube diameter, m	0.0094	-
Length of tubes, m	3.05	-
Temperature of fluids, °C	30.1	-46
Heat exchanged, kW	1565.3	-1565.3

Results:

	Tube-side	Shell-side
Re	8091	952
Velocity, m/s	2.05	2.89
h (clean), $\mathrm{W/m^2 \cdot K}$	3161	5838
Δp (calc), kPa	5.7	5.65

$\Delta T_{\mathrm{lnm}} = 16.2^\circ\mathrm{C}$

$U(\mathrm{calc})^* = \underline{\mathbf{807\,W/m^2 \cdot K}}$

Answer

$A_{required}(\mathrm{calc})^+ = \underline{\underline{\mathbf{1198\,m^2}}}$

Answer

$N_{T\,required} = \underline{\underline{\mathbf{2688\ Tubes}}}$

Answer

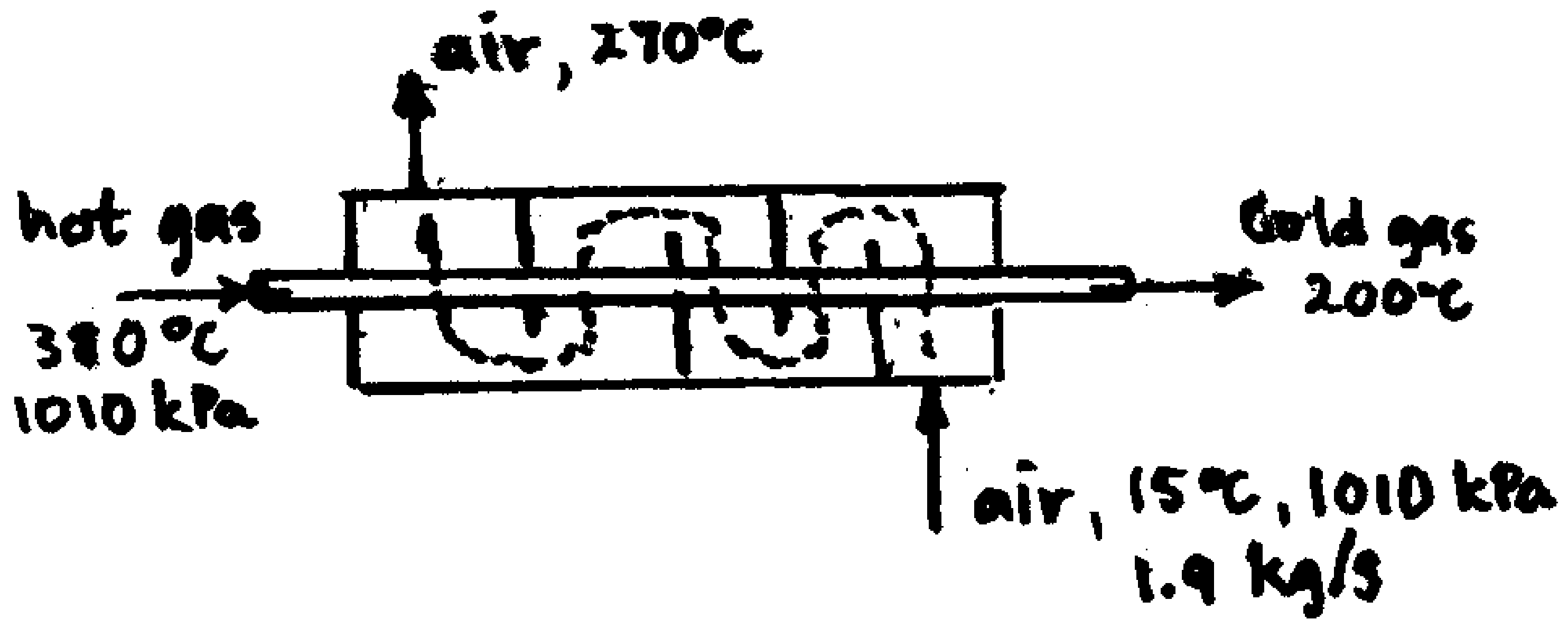
Inside shell diameter = $\underline{\underline{\mathbf{1.32\,m}}}$

Answer

* This includes a fouling factor of $1.76\times10^{-4}\,\mathrm{m^2 \cdot K/W}$ for both sides of the tube.

+ To meet the area requirement requires some ineffective area that is required to mechanically assemble the shell-and-tube heat exchanges. For example, for a class R, type AEL, the actual area needs to be increased to 1211 $\mathrm{m^2}$.

14-16



Heat exchanger, sq. pitch, 0.0381 m
$D_i = 0.0191$ m
$D_o = 0.0254$ m
5000 h/y
Cost data use Fig. 14-19; power cost is $0.12/kWh

The optimum length of the heat exchanger can be estimated with a hand calculation similar to the one that was used in the solution of Example 5 in Chapter 15 of the 4th edition of *Plant Design and Economics for Chemical Engineers*. The optimum length can also be estimated by using available computer software and rapidly examining a number of lengths, determining the pressure drop of the shell and tube side of the exchanger and the required area to establish an annual operating and fixed cost to identify the exchanger requiring the minimum annual cost. The latter approach has been taken to solve this problem.

The procedure is similar to that outlined in Example 14-9. Based on average temperatures on both sides of the exchanger, an estimate of the heat load can be obtained. With an estimated overall heat transfer coefficient, a preliminary area can be established to minimize the number of heat exchanger lengths that will need to be examined. With a heat load of approximately 530 kJ/s, it will require a heat exchanger of at least several meters to meet the heat load. Thus, the computer software was used to examine exchanger lengths above 4m. Near 6.1 m, a minimum in the annual operating and fixed cost was established. It should be noted that the cost curve was rather flat near the minimum.

Length of exchanger = **6.096 m**

Answer

Purchased cost for the optimum heater = **$11,200**

Answer

15:1

15-1

A material balance for the depropanizer follows the procedure outlined in Example 15-2. The results are shown below.

	Mol %			Mols/100 kg mol of feed	
	Feed	Distillate	Bottoms	Distillate	Bottoms
Methane	26.0	43.5		26.0	
Ethane	9.0	15.0		9.0	
Propane	25.0	41.0	1.0	24.6	0.4
n-Butane	17.0	0.5	41.7	0.3	16.7
n-Pentane	11.0		27.4		11.0
n-Hexane	12.0		29.9		12.0
	100.0	100.0	100.0	59.9	40.1

Determine the minimum stages with Eq. (15-1), assume 96°C as the average temperature for the distillation column

$$N_{\min} = \ln\left[(x_{LK}/x_{HK})_D(x_{HK}/x_{LK})_B\right]/\ln(\alpha_{LK/HK})$$

$$= \ln\left[(0.41/0.005)(0.417)/0.01)\right]/\ln(1.55/0.8)$$

$$= \mathbf{12.3\,stages}$$

Answer

Use the Underwood method to calculate $R_{\min}$ using Eqs. (15-2) and (15-3)

$$\sum_{i=1}^{n}\frac{\alpha_i x_{F,i}}{\alpha_i - \Theta} = 1 - q \qquad \text{where } q = 0.34$$

$$\frac{18.75(0.26)}{18.75-\Theta}+\frac{4.75(0.09)}{4.75-\Theta}+\frac{1.9375(0.25)}{1.9375-\Theta}+\frac{1(0.17)}{1-\Theta}+\frac{0.48(0.11)}{0.48-\Theta}+\frac{0.24(0.12)}{0.24-\Theta}=0.66$$

After several iterations, $\Theta = 1.351$

$$R_{\min}+1=\sum_{i=1}^{n}\alpha_i x_{D,i}/(\alpha_i-\Theta)$$

$$R_{\min}=\frac{18.75(0.435)}{18.75-1.351}+\frac{4.75(0.15)}{4.75-1.351}+\frac{1.9375(0.41)}{1.9375-1.351}+\frac{1(0.005)}{1-1.351}-1$$

$$= 0.4687+0.2096+1.3544-0.0142-1$$

15-1 (continued)

$= 1.0185$ or **1.02**

Answer

Determine number of theoretical stages with the use of Eq. (15-6) with $R = 1.5(1.02) = 1.53$

$$\frac{N - N_{\min}}{N+1} = 0.75\left[1 - \left(\frac{R - R_{\min}}{R+1}\right)^{0.566}\right]$$

$$\frac{N - 12.3}{N+1} = 0.75\left[1 - \left(\frac{1.53 - 1.02}{1.53+1}\right)^{0.566}\right]$$

Solving for N results in **N=23 theoretical stages**.

Answer

The location of the feed is obtained from Eq. (15-6)

$$\log\frac{N_D}{N_B} = 0.206\log\left\{\left(\frac{B}{D}\right)\left(\frac{x_{HK}}{x_{LK}}\right)_F\left[\frac{(x_{LK})_B}{(x_{HK})_B}\right]^2\right\}$$

$$= 0.206\log\left\{\left(\frac{.401}{.599}\right)\left(\frac{0.17}{0.25}\right)\left(\frac{0.01}{0.005}\right)^2\right\}$$

$$N = N_D + N_B$$

Solving these two equations shows that the feed is located **13 stages** above the reboiler.

Answer

Use of a computer simulation provides the following results:

$N_{\min}$ (minimum stages) = 13 stages

$R_{\min}$ (minimum reflux) = 1.068

N (theoretical stages) = 25 stages

N_B (feed location above reboiler) = 15 stages

15-2

A material balance around the C_2 splitter is similar to that developed in Problem 3-6. The distillate shows a concentration of 99.9 mol percent ethylene, 0.093 mol percent ethane, and 0.007 mol percent methane. Likewise, the bottoms then provide a 99.43 mol percent ethane, 0.56 mol percent ethylene, and 0.01 mol percent propylene. Essentially this is a two component distillation similar to that presented in Examples 15-2 and 15-3. The material balance in kg/h is

Component[†]	Feed (601)	Distillate (602)	Bottoms (603)
Methane	2.9	2.9	--
Ethylene	62,121	62,067	62.9
Ethane	11,974	62.1	11,903
Propylene	2.1	--	2.1
	74,100	62,132	11,968

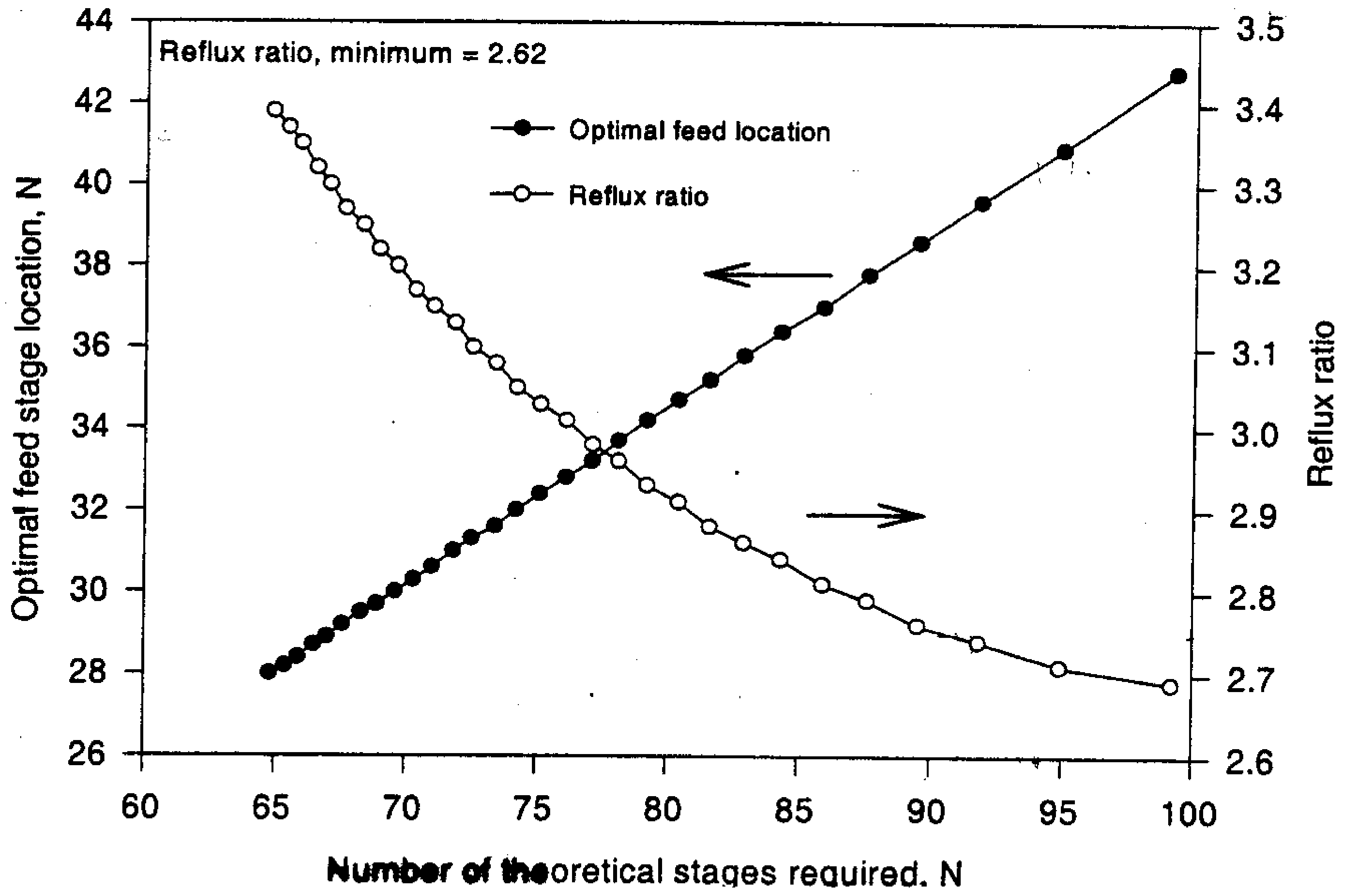
[†] Acetylene and propane have been neglected in the material balance.

The minimum reflux ratio can be evaluated for this two component distillation by using the Fenske-Underwood-Gilliland method and then determining what ratio factor to use to obtain the desired separation using 94 theoretical trays. This approach uses Eq. (15-1), (15-2), (15-3) and (15-4). If this approach is used, N_{min} = 21.2 stages and R_{min} = 2.62. A trial and error calculation with Eq. (15-4) where R is unknown, establishes that a value of 2.75 for R is required to obtain 94 theoretical trays. Thus R = (1.05)(2.62) or 2.75 for this column. This is reasonable since the ratio factor for low temperatures distillation columns is generally between 1.05 and 1.10.

Answer

With the aid of a suitable computer software program, it is possible to investigate the effect of a number of reflux ratios on the number of stages that will be required to obtain the desired separation between ethylene and ethane. This approach is outlined on the computer software output which plots the effect of decreased reflux on the number of theoretical plates required for the separation.

15-2 (continued)



Note that for 94 theoretical plates a reflux ratio of 2.75 is required to make the desired separation. Since R_{min} was also established as 2.62 by the computer software, the R_{min} was also increased by a factor of 1.05.

The computer software also established the feed location for a 94 theoretical plate column as the **41^{st} plate** above the reboiler.

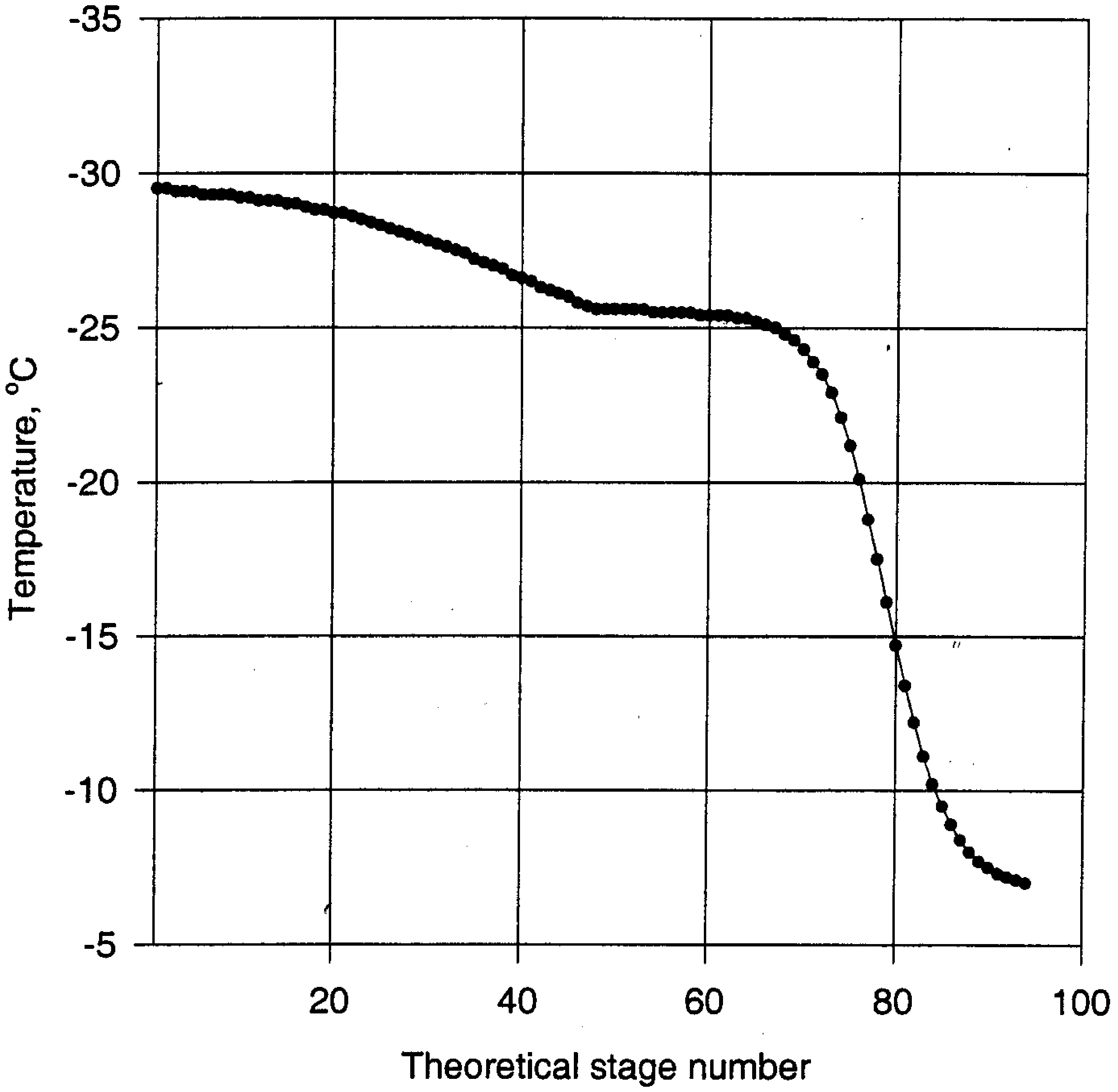
Answer

The temperature profile for the column examined in this problem is shown on the attached figure.

Answer

15:5

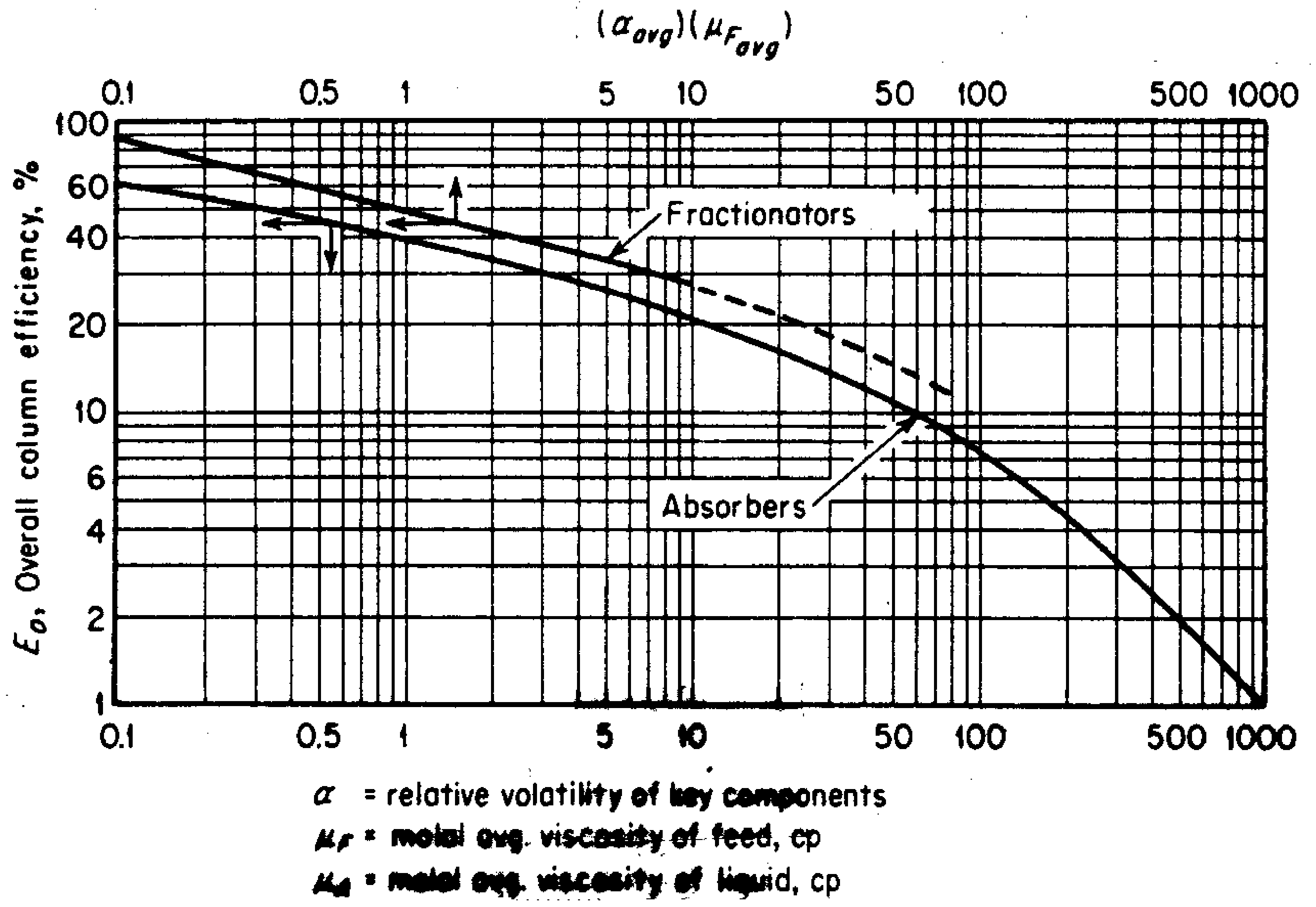
15- 2 (continued)



Temperature profile for the ethylene-ethane column as a function of theoretical stage number.

15-3

The overall column efficiency for finite-stage fractinators and absorbers has been correlated by O'Connell[†] and is shown in the figure below.



Overall column efficiencies for finite-stage contactor fractionators and absorbers.

To determine the overall column efficiency E_o, determine $\alpha_{avg}\mu_{avg}$

$T_{avg} = (82.8 + 100.5)/2 = 91.6^\circ C$ $\qquad$ $\alpha_{avg} = 1070/429 = 2.49$

At $91.6^\circ C$ $\mu_{Bz} = 0.28 cP$ $\qquad$ $\mu_{Tol} = 0.30 cP$ $\qquad$ from Appendix D

$\mu_{avg} = \mu_{molal\ avg} = (0.28)(0.6) + (0.30)(0.4) = 0.288\, cP$

$E_o = 52\%$ from above figure

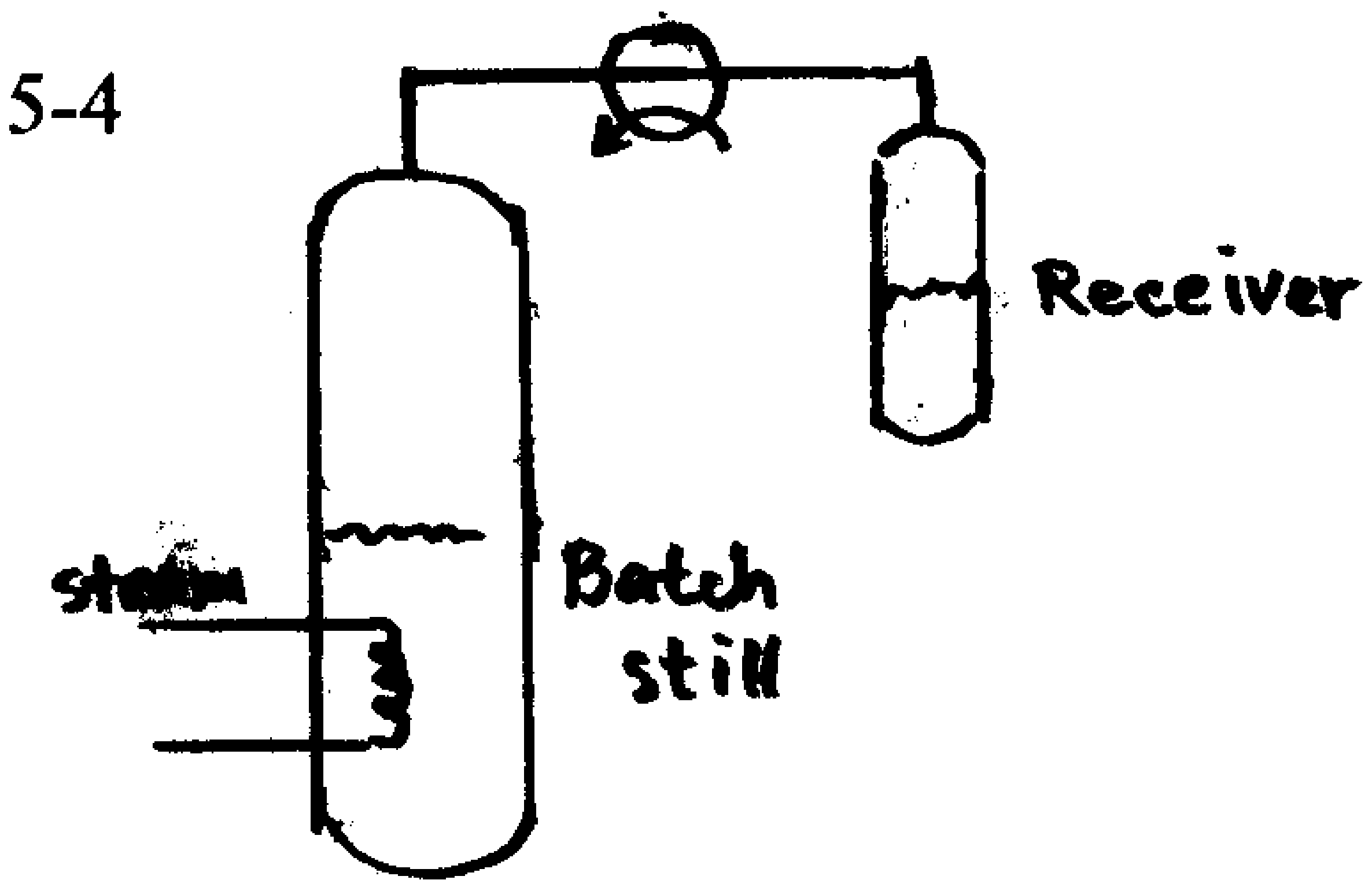
Assume reboiler equivalent to 1 theoretical stage

$N_{act} = (6.1 - 1)/0.52 = 9.8$ or **10 actual trays**

Answer

[†] H.E. O'Connell, *Trans. AIChE* **42**, 751 (1946).

15-4



Separate a mixture of 60 mol % benzene and 40 mol % toluene in a batch distillation to obtain a residual of 10 mol % benzene.

Assume α is constant at 2.9 and column holdup is negligible.

Under the above assumptions, the Rayleigh equation can be integrated, resulting in Eq. (15-29).

$$\ln\frac{L_2'}{L_2'} = \frac{1}{\alpha_{ij}-1}\ln\frac{x_{i2}(1-x_{i1})}{x_{i1}(1-x_{i2})} + \ln\frac{1-x_{i1}}{1-x_{i2}}$$

Substituting, assume $L_1' = 100\,\text{kg mols}$

$$= \frac{1}{1.9}\ln\frac{0.1(1-0.6)}{0.6(1-0.1)} + \ln\frac{(1-0.6)}{(1-0.1)}$$

$$= -1.3698 - 0.8109 = -2.1807$$

$$L_2' = (100)(0.113) = 11.3\,\text{kg mols}$$

By a material balance

	Batch Still		
	Start	End	Receiver
Benzene	60	1.13	58.87
Toluene	40	10.17	29.83
	100	11.3	88.7

Mol fraction of benzene in the receiver is

Mol fraction = 58.87 / 88.7 = **0.664 or 66.4%**

Answer

15-5

The required column diameter and pressure drop in a packed absorption column can be estimated quickly with the use of Fig. 15-7 or obtained quite accurately with one of several appropriate computer simulation programs. Both approaches will be used in the solution of this problem.

To use Fig. 15-7 requires developing a value for the abscissa and from this obtaining the superficial gas mass flux G under flooding conditions and from this the maximum allowable value of G. From this both the column diameter and pressure drop can be determined.

$V = 900/A_c$ $\quad L = 2700/A_c$ $\quad$ where A_c is the cross-sectional area

$$\frac{L}{V}[\rho_V/(\rho_L-\rho_V)]^{1/2} = (2700/900)[1.2(881-1.2)]^{1/2} = 0.1108$$

At an abscissa value of 0.1108 on Fig. 15-7

$$G^2F_p\mu_L^{0.1}/\rho_v(\rho_L-\rho_v) = 0.18 \qquad F_p = 50 \text{ from Fig. 15-7}$$

Thus

$$G_{\text{flood}} = \left[\frac{\rho_v(\rho_2-\rho_v)(0.18)(3600)^2}{F_p\mu_L^{0.1}}\right]^{1/2} = \left[\frac{(1.2)(881-1.2)(0.18)(3600)^2}{(50)(0.02)^{0.1}}\right]^{1/2}$$

$$= 8535 \text{ kg/h}\cdot\text{m}^2$$

$$G_{\text{allowable}} = (8535)(0.7) = 5975 \text{ kg/h}\cdot\text{m}^2$$

This is equivalent to an ordinate value of

$$\frac{(5975/3600)^2(50)(0.02)^{0.1}}{(1.2)(881-1.2)} = 0.088$$

For a value of 0.088 $\quad \underline{\Delta p \cong \mathbf{700\,Pa}/\text{meter}}$

Answer

$$\underline{D = (A_c/0.785)^{1/2} = [(900/5975)/0.785]^{1/2} = \mathbf{0.44\,m}}$$

Answer

With a computer simulation program

$D = 0.52$ m $\quad \Delta p = 615$ Pa/meter

15.9

15-6

There are several ways to estimate the pressure drop that occurs in a packed tower filled with 0.0254 m Raschig ring packing under preloading conditions. One way to do this is to use the empirical relationship developed by Leva[†]. The relation in US customary units is given by

$$\Delta p / Z = \gamma (10)^{\phi L/\rho_L} G^2 / \rho_V$$

where Δp is the pressure drop in $\mathrm{lb_f / ft^2}$, Z packed height in ft, γ, and ϕ constants dependent on the packing size and type of packing, L the superficial liquid mass velocity based on the cross-sectional area of the empty tower in $\mathrm{lb/h{\cdot}ft^2}$, and ρ_L and ρ_V the density of the liquid and vapor respectively. For 0.0254m Raschig rings, $\gamma = 32\times10^{-8}$ and $\phi = 4.3\times10^{-3}$.

Since the relationship is in US customary units, it will be easier to provide the solution in those units and convert the resulting answer to SI units. Accordingly,

$$L = (1820)(2.204)/0.785D^2 = 5110/D^2 \ \mathrm{lb/h\cdot ft^2}$$
$$G = (1365)(2.204)/0.785D^2 = 3832/D^2 \ \mathrm{lb/h\cdot ft^2}$$
$$\rho_L = 1000/16.018 = 62.4 \ \mathrm{lb/ft^3}$$
$$\rho_V = (29/359)(4.92/530) = 0.075 \ \mathrm{lb/ft^3}$$

$$\Delta p / Z = 32\times10^{-8}[10^{(4.3\times10^{-3})(5110/D^2)/62.4}][(3832)/D]^2/0.075]$$
$$= \left(\frac{62.3}{D^4}\right)(10)^{0.35/D^2}$$

Product cost to overcome the Δp per foot of packed height in \$/y

$$\text{Blower cost} = \left(\frac{\Delta p}{Z}\right)\left[\frac{(1365)(2.204)(8000)}{(0.075)(2.655\times10^6)}\right](0.08) = 9.667\frac{\Delta p}{Z}$$

Set $Z = 1$ ft

$$= (9.667)(62.3/D^4)(10)^{0.35/D^2} = (605/D^4)(10)^{0.35/D^2} (\$/\mathrm{y})/\mathrm{ft}$$

[†] M. Leva, *Chem. Eng. Prog. Symp. Ser.*, 50(10, 51(1954).

15-6 (continued)

Fixed charges in $(\$/\mathrm{y})/\mathrm{ft}$

$$\text{Packed volume} = Z(0.785D^2)$$

$$\text{Purchased tower cost} = (1413)(0.02831)(Z)(0.785D^2)$$

$$\text{Purchased packing cost} = (777)(0.02831)(Z)(0.785D^2)$$

When $Z = 1\,ft$

$$\text{Fixed charges} = (2)[(1413+777)(0.02831)(0.785D^2)(0.2)$$
$$= 1947D^2 (\$/\mathrm{y})/\mathrm{ft}$$

$$\text{Total annual cost/ft} \quad = 19.47D^2 + (605/D^4)(10)^{0.35/D^2}$$

$$\text{At optimum } D, \frac{d(\text{total annual cost}/\text{ft})}{dD} = 0$$

$$\frac{d(19.47D^2)}{dD} + \frac{d[(605/D^4)(10)^{0.35/D^2}]}{dD} = 0$$

Solving this equation shows that D_{opt} = 2.05 ft = **<u>0.625 m</u>**

Answer

This can be verified by using Mathcad as shown below:

Diameter, ft	Diameter, m	Annual cost $(\$/\mathrm{y})/\mathrm{m}$
1	0.305	4506
1.5	0.457	705
2.0	0.610	406.80
2.02	0.616	405.80
<u>2.05</u>	<u>0.625</u>	<u>404.50</u> min
2.06	0.628	407.50
2.1	0.640	408.50
2.5	0.762	457
3.0	0.914	602

15:11

15-7

Based on the concept of lowering the activity coefficient of the solute by the addition of a solvent, a table of interactions for solvent screening has been developed by Cusack, et. al.[†] which incorporate the earlier suggestions by Robbins.[‡] With the aid of this table, the following solvents are suggested (the underlined component is the solute),

Components	Solvent(s)
a. water and acetic acid	aromatic alcohols
b. water and acetone	trichloroethane, methyl isobutyl ketone, any multihalo-paraffin with an active H
c. water and aniline	methylcyclohexane
d. water and ethyl alcohol	phenol

Answer

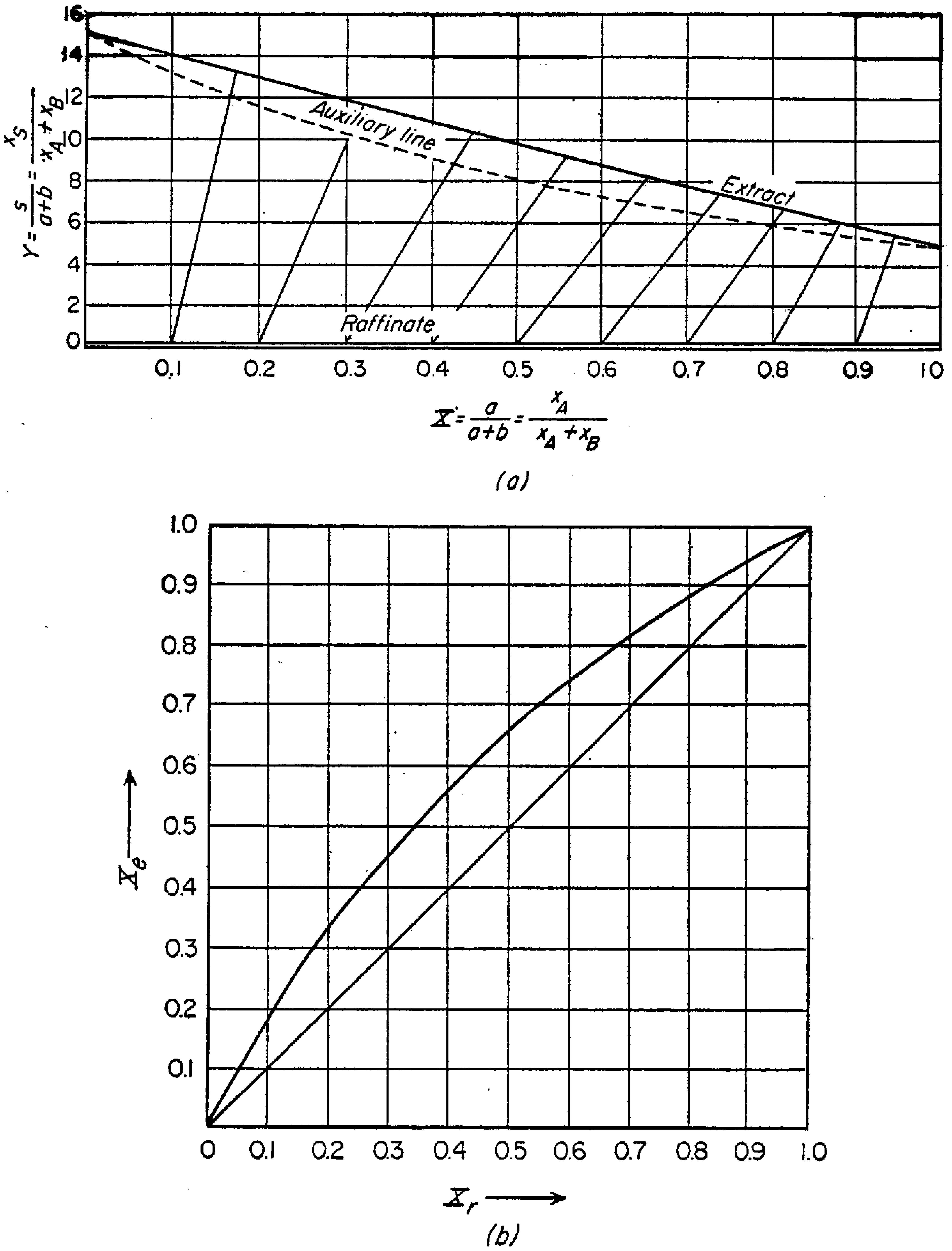
[†] R.W. Cusak, P. Fromeaux, and P. Glate, *Chem. Eng.* 98(2), 66(1991).

[‡] L.A. Robbins, *Chem. Eng. Progr.* 76(10), 58(1980).

15-8

Once the extraction data is obtained from the literature, this problem can be solved using either an available computer simulation software programs or developing a modified Ponchon-Savarit diagram. The latter will be used in this solution.

The extraction data at 25°C obtained by Varteressian and Fenske† are plotted below.



System aniline–*n*-heptane–MCH at 25°C: (*a*) *XY* diagram. *a*, solute, MCH. *b*, diluent, *n*-heptane. *s*, solvent, aniline. (*b*) Equilibrium, raffinate and extract, on solvent-free basis, X_e vs. X_r.

---

† K.A. Varteressian and M.R. Fenske, *Ind. Eng. Chem.*, 61(7), 184 (1954).

15-8 (continued)

In the first figure X is defined as the ratio of the mass of the solute to the summed mass of the solute and the diluent. Likewise Y is defined as the ratio of mass of the solvent to the summed mass of the solute and the diluent. The second figure provides the equilibrium between the raffinate X_r and extract X_e, on a solvent-free basis.

The extract-raffinate data from the first figure is used to develop the Ponchon-Savarit diagram. The solvent content of the raffinate is low and Y_r is considered zero. X_F, X_o, and X_B are located at values of 0.40, 0.98, and 0.01, respectively. The upper operating point P_D is located using the given reflux ratio of 7.7. Thus,

$$\frac{P_D - 5}{5} = 7.7 \quad P_D = 43.5$$

The line P_DFP_B drawn through P_D and F establishes P_B to satisfy the material balance around the extraction unit. Construction of the Ponchon-Savarit diagram can now be completed with the use of equilibrium tie lines obtained from the second figure on the previous page. The fully constructed diagram is shown below showing that **26 ideal stages** are required to effect the desired separation. Feed is located on the 11th stage. **Answer**

Per 100 kg of feed, the solvent-free raffinate product is

$$[(0.98-0.40)/(0.98-0.01)]100 = 59.8\,\text{kg}$$

The solvent-free extract product is 100 – 59.8 = 40.2 kg

The solvent required, per kg of product is given by

$$\underline{(P_B - Y_r)(59.8) = (29.2-0)(59.8) = \mathbf{1745\,kg}}$$

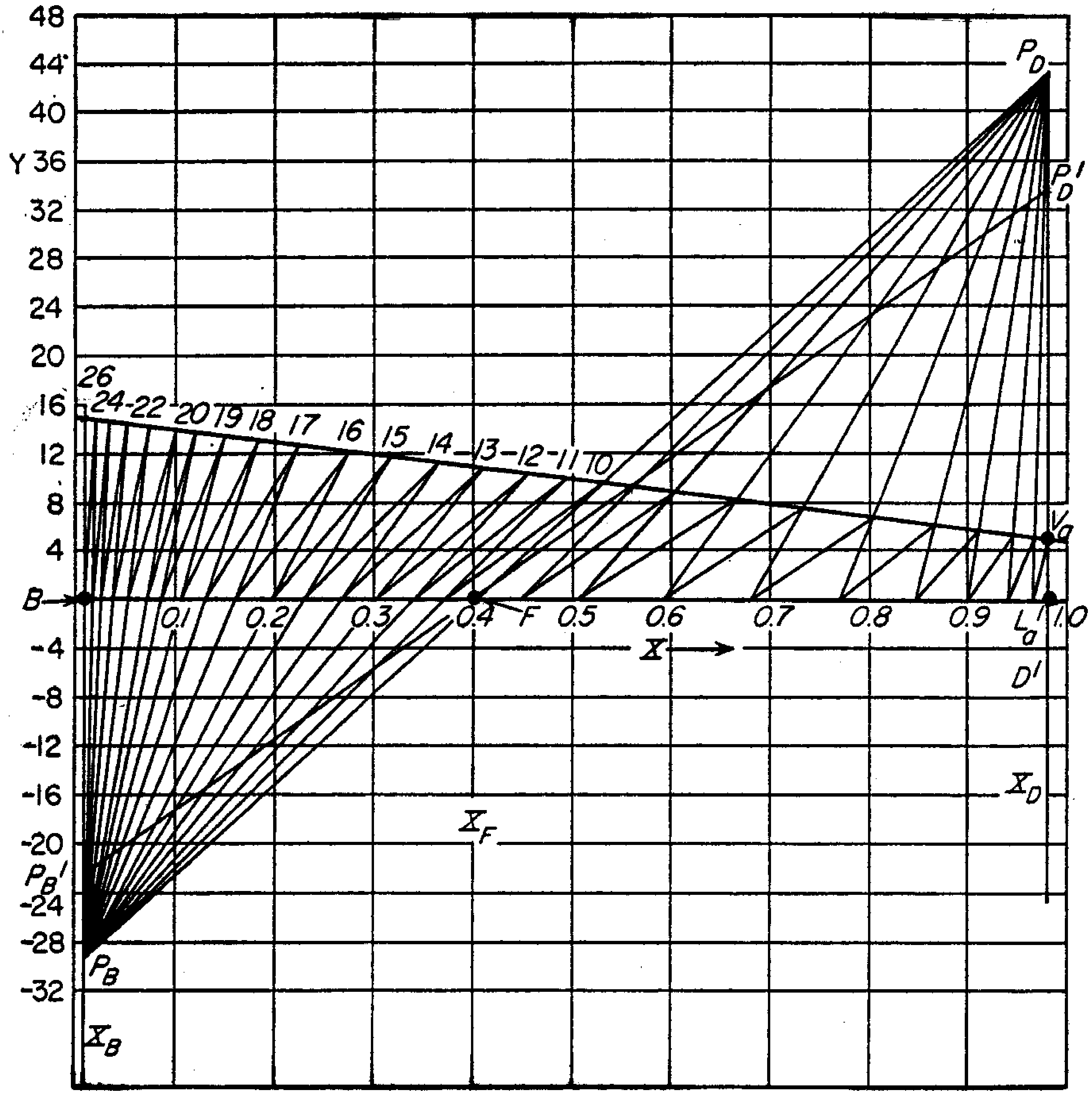
Answer

The solvent removed in the solvent separator is given by

$$\underline{(P_D - Y_r)(40.2) = (43.5-0)(40.2) = \mathbf{1745\,kg}}$$

Answer

15-8 (continued)



Ponchon-Savarit diagram

15-9

From Example 15-6, $\alpha^*_{O_2,N_2} = 2.99$, $r = p_R / p_P = 0.1$

$$n_F = 1513\,\text{kg mol/h},\ x_{F_1O_2} = 0.21$$

To obtain α_{O_2,N_2} use Eq. (15-60) after it has been simplified to Eq. (3) of Example 15-6, namely

$$\alpha_{O_2,N_2} = \alpha^*_{O_2,N_2}\left[\frac{x_{F_1O_2}(\alpha_{O_2,N_2}-1)+1-r\alpha_{O_2,N_2}}{x_{F_1O_2}(\alpha_{O_2,N_2}-1)+1-r}\right]$$

Setting, $x_{F_1O_2} = 1$, this relation can be solved with Mathcad to obtain a value of 2.603 for α_{O_2,N_2}.

An overall module material balance on O_2 results in

$$x_{F_1O_2} n_F = x_{R_1O_2}(1-\Theta)n_F + y_{P_1O_2}\Theta n_F$$

where $y_{P_1O_2}$ is defined in Eq. (15-66) as

$$y_{P_1O_2} = x_{R_1O_2}{}^{1/(1-\alpha)}\left(\frac{1-\Theta}{\Theta}\right)\left[\left(1-x_{R_1O_2}\right)^{\alpha/(x-1)}\left(\frac{x_{F_1O_2}}{1-x_{F_1O_2}}\right)^{\alpha/(\alpha-1)} - x_{R_1O_2}^{1/(1-\alpha)}\right]$$

Here α is α_{O_2,N_2} obtained above as 2.99.

For a stage cut of 0.4, the last two equations can be solved with Mathcad to give

$$x_{R_1O_2} = 0.122 \quad y_{P_1O_2} = 0.342$$

Use Eq. (15-59a) to obtain the required area

$$A_M = \frac{y_{P_1O_2} n_P}{P_{M_1O_2}(x_{R_1O_2}\underline{p_R} - y_{P_1O_2}\underline{p_P})}$$

$$= \frac{(0.342)(1513)(0.4)}{3.50\times10^{-8}\left[0.122(1.013\times10^{6}) - 0.342(1.013\times10^{3})\right]}$$

$$= \underline{\mathbf{66{,}500\ m^2}}$$

Answer

15-10

From the information given in the problem the interparticle void fraction ε_P, the extraparticle void fraction ε, and the pore radius r_p are obtained from

$$\varepsilon_P = \left(\frac{1}{777} - \frac{1}{2178}\right)^{-1} \Bigg/ \left(\frac{1}{777}\right)^{-1} = 0.643$$

$$\varepsilon = \left(\frac{1}{509} - \frac{1}{777}\right)^{-1} \Bigg/ \left(\frac{1}{509}\right)^{-1} = 0.345$$

$$r_P = 2\varepsilon_P / A\rho_P = (2)(0.643)/(1.1\times10^{9})(777) = 1.504\times10^{-9}\,\text{m}$$

The fluid phase diffusivity is $D_i = 7.42\times10^{-2}\,\text{cm}^2/\text{s}$

The pore diffusivity is obtained from (Satterfield, *Mass Transfer in Heterogeneous Catalysis*, p. 43, MIT, Cambridge, MA 1970)

$$D_p = \frac{1}{\tau}\left[\left(\frac{3}{4r_p}\right)\left(\frac{\pi M}{2RT}\right)^{1/2} + \frac{1}{D_i}\right]^{-1}$$

For a tortuosity factor of 4, the D_P is $1.45\times10^{-3}\,\text{cm}^2/\text{s}$

To determine whether pore diffusion is controlling, evaluate the Reynolds and Schmidt dimensionless numbers and use the figure below which relates the effect of the Re Sc product, the distribution ratio, and the diffusivity ratio on the height of a transfer unit.

$$\text{Re} = Vd_P/\upsilon = (0.3)(0.0017)/(1.08\times10^{-5}) = 47.2$$

$$\text{Sc} = \upsilon/D_i = (1.08\times10^{-5})/(7.42\times10^{-6}) = 1.455$$

$$\text{Re}\,\text{Sc} = (47.2)(1.455) = 68.7$$

The isotherm parameters based on the feed concentration are

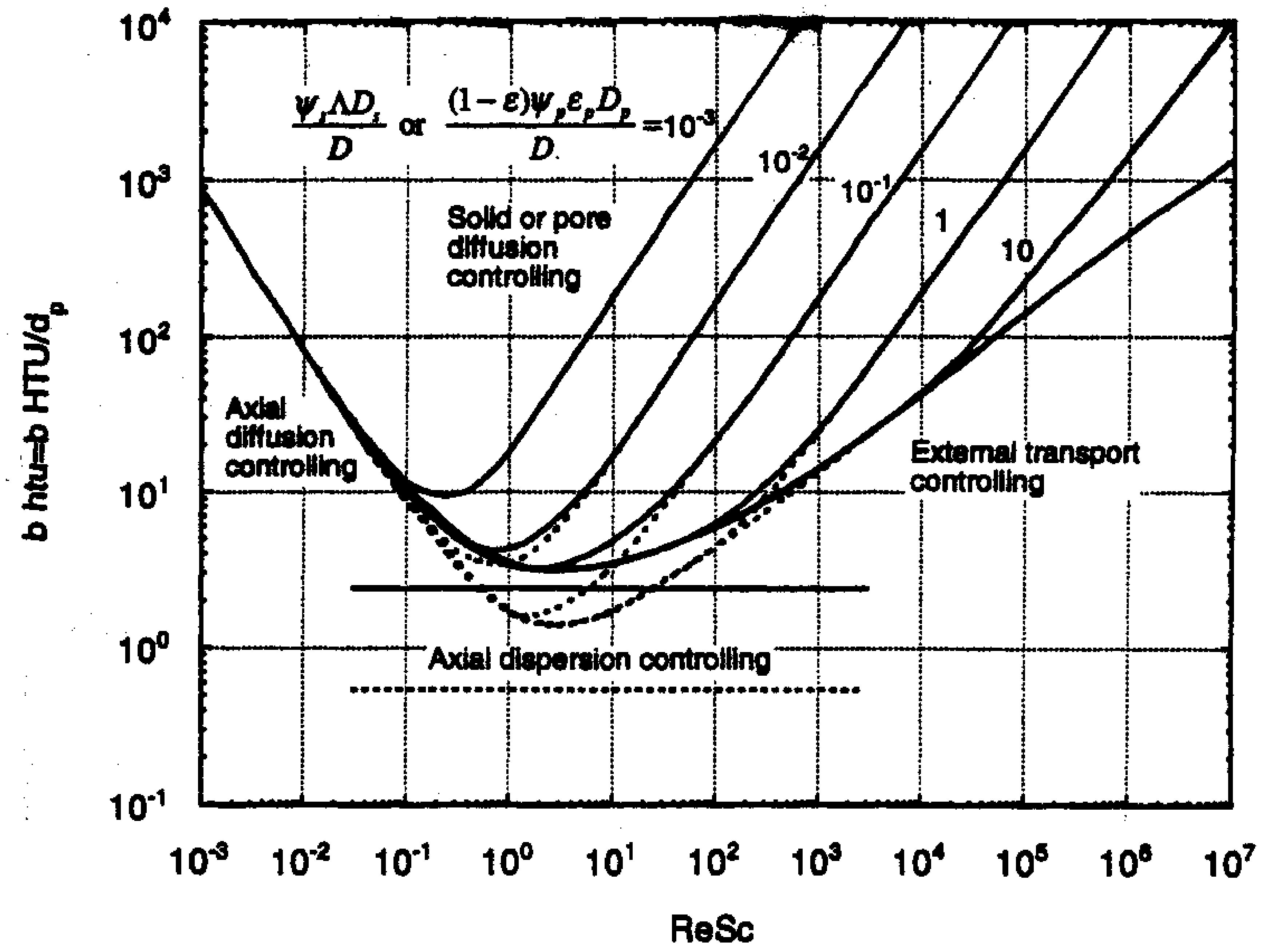
$$R = 1/(1 + K_a p_a) = 1/(1 + 0.346(10)(0.1)) = 0.7424$$

where a correction factor Ψ_p for constant R is given by

$$\Psi_p = 0.775/(1 - 0.225R^{0.5}) = 0.961$$

and $(1-\varepsilon)\Psi_p\varepsilon_p D_p / D_i) = (1-0.345)(0.961)(0.643)(1.45\times10^{-3})/0.0742 = 7.93\times10^{-3}$

15-10 (continued)



Effect of *ReSc* group, distribution ratio, and diffusivity ratio on height of a transfer unit. Dotted lines for gas and solid lines for liquid-phase systems.

Location of this point at an abscissa value of 68.7 for the ReSc product, indicates that **pore diffusion** is controlling.

Answer

When pore diffusion is controlling, the expression for the rate coefficient can be approximated from *Perry's Chemical Engineers' Handbook*, 7th ed., Table 16-2 as

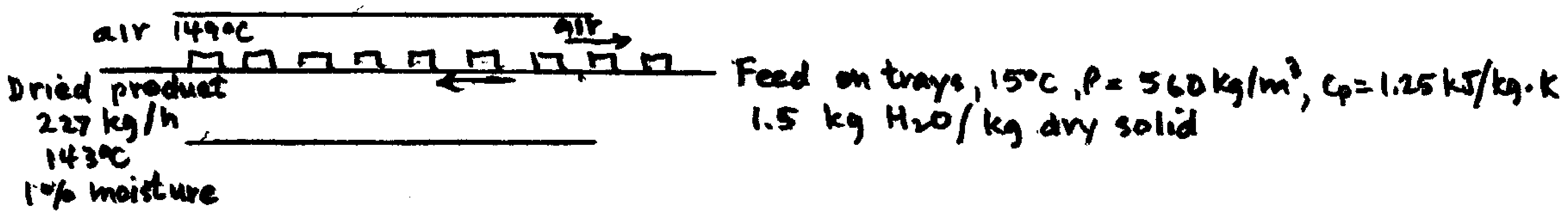
$$k = 15\Psi_p(1-\varepsilon)\varepsilon_p D_p / \Lambda r_p^2$$

Substituting appropriate values in this relation with the isotherm parameter $\Lambda = 9.56$ results in

k = 0.13 s^{-1}

Answer

15-11



In this drying process assume shrinkage during drying is negligible. Also assume operation at 1 atm.

Heat transfer requirement is approximated from the following heat balance for the water and the solid:

Heat to evaporate moisture (C_{p,H_2O} vapor $= 1.884\,\mathrm{kJ/kg\cdot K}$)

$$\dot{q}_{H_2O} = 227(1.5-0.01)[(100-15)(4.186)+2256+1.884(60-100)]$$
$$= 887{,}887 \text{ or } 857{,}900\,\mathrm{kJ/h}$$

Heat of solid

$$\dot{q}_{\text{solid}} = (1.25)(227)(143-15) = 36{,}200\,\mathrm{kJ/h}$$
$$\dot{q}r = 857{,}900 + 36{,}200 - 894{,}100\,\mathrm{kJ/h}$$

Relative humidity of entering air with 0.03 kg H_2O/kg dry air at 149°C is ~1%

Required mass of air is

$$\dot{m}_{air} = 894{,}100/(1.06)(149-60)$$
$$= 9478\ \mathrm{kg/h}$$

where 1.06 kJ/kg·K is the humid heat at 149°C and 1% RH

Rate of water removal

$$\dot{m}_{H_2O} = (227)(1.5-0.01) - 338.2\,\mathrm{kg/h}$$

Humidity of air leaving the dryer $= 0.03 + 338.2 / 9478$

$= 0.0656$ kg water/kg dry air

15-11 (continued)

This exit humidity corresponds to a relative humidity of 48% and 60°C. The wet bulb temperature at this condition is

$$T_{wb} = 46.7^{\circ}\mathrm{C}$$

Mass of air recirculated

$$\begin{aligned}\dot{m}_{air} &= 9478(0.03-0.01)/(0.0656-0.01)\\ &= 3410\,\mathrm{kg/h}\end{aligned}$$

The heat required for the constant rate section is 642,000 kJ/h

Air temperature where surface drying stops is

$$T = 60+(149-60)(642,000/894,100) = 123.9^{\circ}C$$

The number of transfer units (NTU) is defined by

$$NTU = \ln\frac{T_1 - T_{wb}}{T_c - T_{wb}}$$

where T_1 = air temperature entering the constant-rate zone

T_c = air temperature at the critical moisture content

$$NTU = \ln\frac{123.9-46.7}{60-46.7} - \underline{\mathbf{1.75}}$$

Answer

Under normal tray drying conditions, NTU in the falling rate section is obtained from

$$NTU = \frac{T_c - T_2}{\Delta T_m} \qquad \text{where} \qquad \Delta T_m = \frac{(123.9-46.7)-(149-143)}{\ln(123.9-46.7)/(149-143)}$$

$$= \frac{(149-123.9)}{27.9} \qquad\qquad =27.9^{\circ}\mathrm{C}$$

$$= \underline{\mathbf{0.90}}$$

Answer

$NTU_{total} = 1.75+0.90 = 2.65$

15-11 (continued)

The open cross-sectional area of the tunnel drier is given by

$$A = 9478/(2.71)(3600) = 0.97\,\text{m}^2$$

The length of the dryer is obtained with the empirical equation

$$L = (72.77)(0.0381)(2.71)^{0.2}(2.65) = 8.96\,\text{m or } \underline{\mathbf{9m}}$$

Answer

15.21

15-12

In this evaporation process assume a temperature driving force of 5°C between the wall temperature and the evaporating water, i.e., the T_W = 105°C. Since the Reynolds number of the liquid flow down the vertical will change, it will be necessary to evaluate values at both ends to arrive at an average heat transfer coefficient.

The mass flow of water per tube periphery is given by

$$\Gamma_o = \dot{m}/\pi D_i = 0.01/\pi(0.05) = 0.0637\,\text{kg/m}\cdot\text{s}$$

The Reynolds number is given by $4\Gamma_o/\mu_L$. At the periphery

$$\text{Re}_{in} = (4)(0.0637)/2.79\times10^{-4} = 913.3 \quad \mu_L = 2.79\times10^{-4}\text{Pa}\cdot\text{s}$$

The film is laminar, but waves are on the surface. If laminar flow in this wavy regime (30<Re<1600) continues down to the bottom of the tube, Kutateladze (*Fundamentals of Heat Transfer*, Academic Press, New York, 1963) developed the following relation to determine the Reynolds as a function of distance down the tube:

$$\text{Re}_{\text{out}}^{1.22} = \text{Re}_{\text{in}}^{1.22} - 3.69\frac{k_L(T_W - T_S)L}{\Delta H_V \mu_L}\left[\frac{\rho_L(\rho_L - \rho_V)g}{\mu_L^2}\right]^{1/3}$$

Properties required are: $k_L = 0.68$ W/m·K; $\Delta H_V = 2.03\times10^6$ J/kg

$\rho_L = 958\,\text{kg/m}^3$; $\rho_V = 0.6\,\text{kg/m}^3$

Substituting these values and using Mathcad to solve for Re at the lower end of the tube

$$\text{Re}_{\text{out}}^{1.22} = (913.3)^{1.22} - 3.69\frac{(0.68)(105-100)3}{(2.03\times10^6)(2.79\times10^{-4})}\left[\frac{958(958-0.6)9.8}{(2.79\times10^{-4})^2}\right]^{1/3} = 856$$

$\text{Re}_{\text{out}} = 253.3$; this indicates continued wavy laminar flow

The fraction of water evaporated is $(\text{Re}_{\text{min}} - \text{Re}_{\text{out}})/\text{Re}_{\text{in}} = 0.726$. Since the average heat transfer coefficient can be related to the inlet and exit Reynolds numbers as

$$\bar{h} = \frac{(\text{Re}_{\text{in}} - \text{Re}_{\text{out}})}{-\int_{\text{Re}_{\text{in}}}^{\text{Re}_{\text{out}}} d\,\text{Re}/h}$$

15-12 (continued)

An expression can be formulated for the average heat transfer assuming that T_W and T_S remain constant down the length of the tube. Kutateladze has developed such a relation

$$\bar{h} = 0.922 k_L \left[\frac{\rho_L(\rho_L - \rho_V) g}{\mu_L^2} \right]^{1/3} \frac{(\mathrm{Re}_{\mathrm{in}} - \mathrm{Re}_{\mathrm{out}})}{(\mathrm{Re}_{\mathrm{in}}^{1.22} - \mathrm{Re}_{\mathrm{out}}^{1.22})}$$

$$= (0.922)(0.68) \left[\frac{(958)(958 - 0.6)(9.8)}{(2.79 \times 10^{-4})} \right]^{1/3} \left[\frac{913.3 - 253.3}{(913.3)^{1.22} - (253.3)^{1.22}} \right]$$

$$= 6.23 \times 10^3 \ \mathrm{W/m^2 \cdot K}$$

The volumetric rate of steam can now be evaluated. The total rate of heat transfer is

$$\dot{q} = \bar{h}(T_W - T_S)\pi D L$$

$$= (6.23 \times 10^3)(105 - 100)\pi(0.05)(3) = 1.47 \times 10^4 \mathrm{W}$$

The volume of steam generated per second is

$$v_{\mathrm{steam}} = \dot{q} / \Delta H_V \rho_V = 1.47 \times 10^4 / (2.03 \times 10^6)(0.6)$$

$$= \underline{\mathbf{0.012\ m^3/s}}$$

Answer

The maximum velocity of the steam is at the top of the tube. Thus,

$$V_{\max} = 0.012 / (0.785)(0.05)^2 = 6.1 \ \mathrm{m/s}$$

This is about 22% of the flooding velocity of 27 m/s, but even this velocity can have some effect on the interfacial shear stress. **Answer**

15.23

15-13

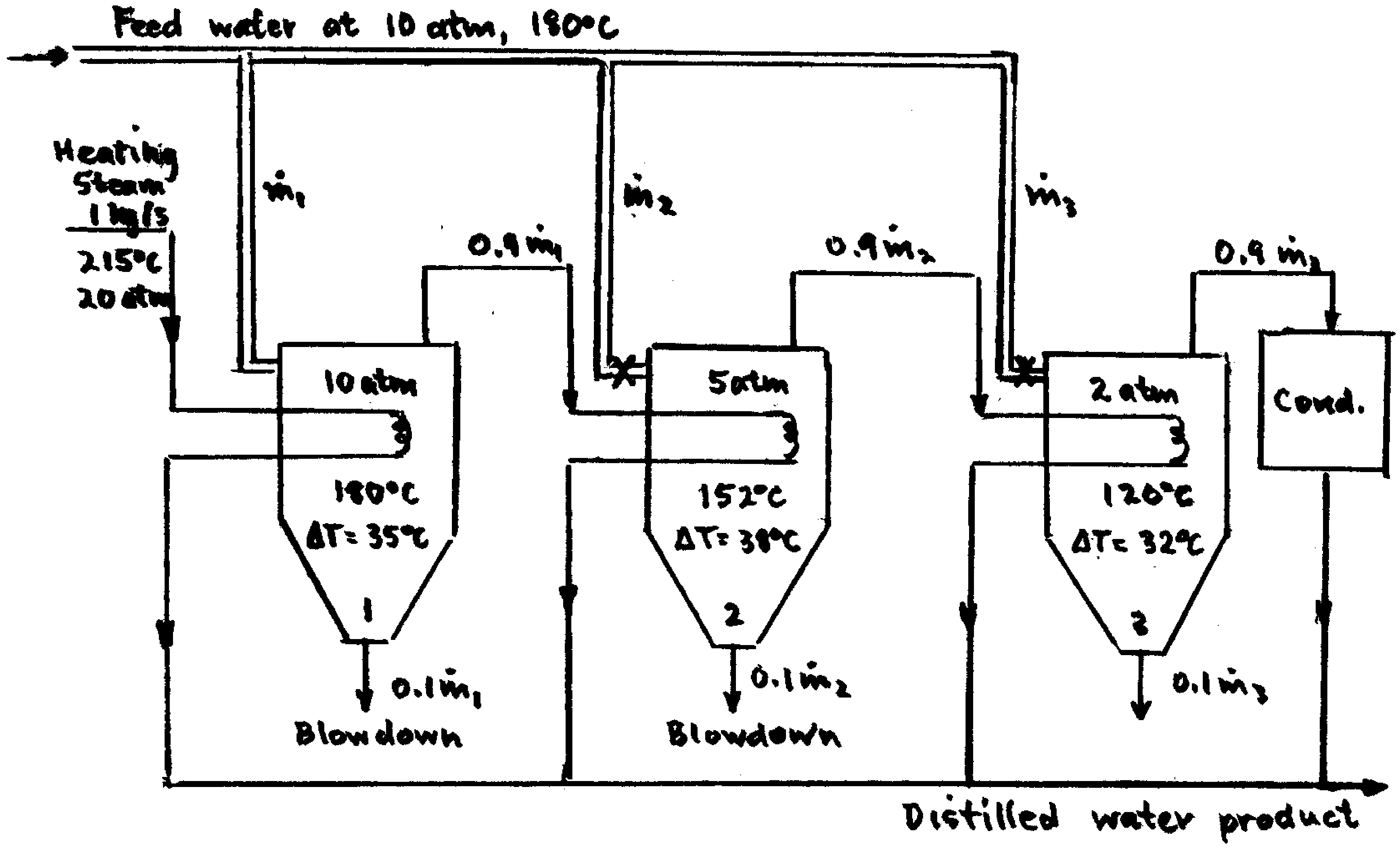


Diagram of triple-effect evaporator

To obtain essentially equal temperature-differences over the triple effect evaporator, operate the first evaporator at 10 atm, the second at 5 atm, and the third at 2 atm. (The latter choice was made to maintain a positive driving force from the exit from the evaporator.) The driving temperature differences are

Stage 1: $\Delta T_1 = 215 - 180 = 35^\circ\text{C}$

Stage 2: $\Delta T_2 = 180 - 152 = 28^\circ\text{C}$

Stage 3: $\Delta T_3 = 152 - 120 = 32^\circ\text{C}$

15-13 (continued)

Values of enthalpy for steam and water for the 3 stages used in the solution

	Steam	Stage 1	Stage 2	Stage 3
Pressure, atm	20	10	5	2
Temp., °C	215	180	152	120
h_V, kJ/kg$\times 10^{-3}$		2.778	2.749	2.706
h_L, kJ/kg$\times 10^{-3}$		0.763	0.641	0.504
ΔH_V, kJ/kg$\times 10^{-3}$	1.881	2.015	2.108	2.202

For stage 1, assume a steam feed of 1 kg/s is condensed at 20 atm and that the liquid being blown down has the same enthalpy $h_{1,L}$ as that entering. The heat balance for stage 1 is

$$\dot{q}_1 + \dot{m}_1 h_{1,L} = 0.9\dot{m}_1 h_{1,V} + 0.1\dot{m}_1 h_{1,L}$$

$$\dot{q}_1 = 0.9\dot{m}_1 \Delta H_{1,V}$$

$$\dot{m}_1 = \dot{q}_1 / 0.9\Delta H_{1,V} = 1.881\times 10^3 / (0.9)(2.015\times 10^3) = 1.037\,\text{kg/s}$$

For stage 2, $\dot{m}_{\text{in}} = (0.9)(1.037) = 0.9345\,\text{kg/s}$ and is a source of heat for evaporation. The 1 kg/s of condensed steam from stage 1 can be added to the distilled water product. Assume in the heat balance for stage 2, all steam coming from stage 1 to stage 2 is condensed. Thus,

$$\dot{q}_2 = (0.9335)(2.015\times 10^3) = 1.881\times 10^3\,\text{kJ/s}$$

which is identical to $\dot{q}_1$ since the liquid enters and leaves stage 1 with the same enthalpy.

Since the pressure in stage 2 is 5 atm, some of the feedwater entering is vaporized because of the isenthalpic pressure reduction. The overall energy balance for stage 2 is

$$\dot{q}_1 + \dot{m}_2 h_{1,L} = 0.9\dot{m}_2 h_{2,V} + 0.1\dot{m}_2 h_{2,L}$$

$$\dot{m}_2 = \dot{q}_2 / (0.9h_{2,V} + 0.1h_{2,L} - h_{1,L})$$

$$= 1.881\times 10^3 / [(0.9)(2.749) + (0.1)(0.641) - (0.763)]10^3$$

$$= 1.06\ \text{kg/s}$$

15-13 (continued)

For stage 3, $\dot{m}_{\text{in}} = (0.9)(1.06) = 0.954\,\text{kg/s}$ and the rate of condensate to the product water line is 0.9345 kg/s. The energy balance for this stage is

$$\dot{q}_3 + \dot{m}_3 h_{1,L} = 0.9\dot{m}_3 h_{3,V} + 0.1\dot{m}_3 h_{3,L}$$

$$\dot{m}_3 = \dot{q}_3 / 0.9h_{3,V} + 0.1h_{3,L} - h_{1,L}$$

$$\dot{m}_3 = (0.954)(2.202\times10^6)/[(0.9)(2.706)+(0.1)(0.504)-0.763]10^3$$

$$= 1.220\text{ kg/s}$$

Steam is produced at a rate of (0.9)(1.220) or 1.098 kg/s and this is condensed in the condenser before being added to the distilled water product.

The total amount of distilled water produced is the sum of the water obtained from the three stages plus the water from the condenser. This total is

Stage 1	1.000 kg/s
Stage 2	0.934
Stage 3	0.954
Condenser	1.098
	3.986 kg/s
	Answer

If the feedwater at 10 atm had been flashed down to the final product pressure of 2 atm in the third stage, the fraction of steam generated would have been

$$x = (h_{1,L} - h_{3,L}) / \Delta H_{3,V}$$

$$= (0.736 - 0.504)10^3 / 2.202\times10^3$$

$$= 0.105$$

The amount of additional distilled water product would be

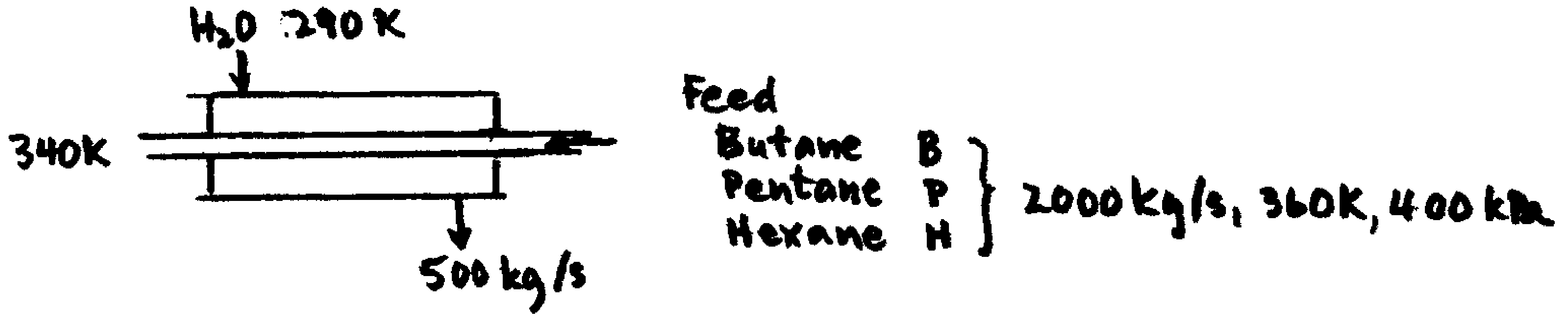
$$\dot{m} = (0.105((3.317) = \underline{\mathbf{0.349\text{ kg/s}}}$$

Answer

15-13 (continued)

where 3.317 kg/s is the sum of the entering feedwater along with the 1 kg/s of entering steam. Blowdown rate is 10% of the feedwater rate.

15-14



To determine the heat removed in the condensation process requires establishing the vapor-liquid relationship across the heat exchanger. The rate of heat removal changes because of a variation in the heat transfer coefficient. To approximate this change will require dividing the heat exchanger into several smaller sections. One convenient way of doing this is to use 5°C temperature intervals to divide the exchanger into four sections. Establishing the amounts of vapor and liquid of each component at these temperature intervals requires the use of equilibrium constants. The solution at 360K will be shown below.

Convert feed to mols (B refers to butane, P to pentane, H to hexane)

$\dot{n}_B$ = (200/3)(58.1) = 1.147 kg mol/h

$\dot{n}_P$ = (200/3)(72.1) = 0.924 kg mol/h

$\dot{n}_H$ = (200/3)(86.2)= 0.773 kg mol/h

Total 2.844

From equilibrium relations in mass units, the following expression relates the ratios of mols of gas to mols of liquid at a specific temperature.

$$\dot{n}_T = \left(1+\frac{\dot{n}_V}{\dot{n}_L}\right)\sum_{i=1}^{n}\frac{\dot{n}_T}{K(\dot{n}_V/\dot{n}_L)+1} \quad \text{and } \dot{n}_T = \dot{n}_V + \dot{n}_L$$

at 360 K

$K_B = 3.108$ $K_P = 1.105$ $K_H = 0.458$

$$2.844 = \left(1+\frac{\dot{n}_V}{\dot{n}_L}\right)\left[\frac{2.844}{3.108(\dot{n}_V/\dot{n}_L)+1}+\frac{2.844}{1.105(\dot{n}_V/\dot{n}_L)+1}+\frac{2.844}{0.458(\dot{n}_V/\dot{n}_L)+1}\right]$$

15-14 (continued)

Use of Mathcad quickly determines $\dot{n}_V$ and $\dot{n}_L$ for each component.

Results from the calculation for a temperature of 360K

	$\dot{n}_L$	$\dot{n}_V$	$\dot{n}_T$
Butane, kg mol/h	0.0117	1.1353	1.147
Pentane, kg mol/h	0.0261	0.8979	0.924
Hexane, kg mol/h	0.0507	0.7223	0.773
	0.0885	2.7555	2.844

To obtain the energy content of this mixture at 360K, requires use of molar enthalpies relative to a reference temperature (340K in this problem) obtained from Maddox, *Heat Exchanger Design Handbook*. Units are in kJ/kgmol.

	Butane		Pentane		Hexane	
Temp.	Liq.	Vap.	Liq.	Vap.	Liq.	Vap.
340	0	18,075	0	23,940	0	28,820
345	813	18,336	945	24,502	1,031	29,568
350	1,569	18,773	1,976	25,057	2,180	30,318
355	2,441	19,121	2,734	25,620	3,240	31,068
360	3,371	19,586	3,579	26,176	4,167	31,818

The energy content in kJ/kg at 360K is then

$$\begin{aligned} h_{\text{mean}} &= \frac{1}{\dot{m}_T}\sum_{i=1}^{n}(h_B\dot{n}_B + h_P\dot{n}_P + h_H\dot{n}_H)_V + (h_B\dot{n}_B + h_P\dot{n}_P + h_H\dot{n}_H)_L \\ &= \frac{1}{200}[19{,}586)(1.1353) + (26{,}176)(0.8979) + (31{,}818)(0.7223) + \\ &\quad (3{,}371)(0.0117) + (3{,}579)(0.0261) + (4{,}167)(0.0507)] \\ &= 345.4\,\text{kJ/kg} \end{aligned}$$

Repeating this same procedure at a temperature of 340K provides a h_{mean} of 27.5 kJ/kg. Thus, the heat removed in the condenser is

$$\dot{q}_r = 200(345.4 - 27.5) = 63{,}580\,\text{kJ/s}$$

15-14 (continued)

The calculations of 340K, 345K, 350K, 355K and 360K can be obtained by use of a spreadsheet. The results are tabulated below:

	340K	345K	350K	355K	360K
$\dot{n}_{B,L}$ kg mol/s	0.9532	0.5572	0.3070	0.1269	0.0117
$\dot{n}_{P,L}$ kg mol/s	0.8631	0.6739	0.4701	0.2373	0.0261
$\dot{n}_{H,L}$ kg mol/s	0.7549	0.6778	0.5562	0.3452	0.0507
$\dot{n}_{B,V}$ kg mol/s	0.1939	0.5898	0.8392	1.0201	1.135
$\dot{n}_{P,V}$ kg mol/s	0.0609	0.2500	0.4539	0.6867	0.898
$\dot{n}_{H,V}$ kg mol/s	0.0187	0.0957	0.2173	0.4193	0.722
h_V, kJ/kg	27.5	108.1	181.7	261.1	345.4
$\dot{q}$, kJ/s	-63,580	-47,450	-32,740	-16,850	0

The outlet temperature of the cooling water can now be determined.

$$\begin{aligned} T_{H_2O,\text{out}} &= T_{H_2O,\text{in}} + \dot{q}_T / \dot{m}_{H_2O} C_{pH_2O} \\ &= 290 + 63,850/(500)(4.2) = 290 + 30.3 \\ &= 320.3\,\text{K} \end{aligned}$$

The inlet vapor mass flow rate is given by

$$\dot{m}_V = (1.135)(58.1) + (0.989)(72.1) + (0.722)(86.2) = 193\,\text{kg/s}$$

Similarly the outlet vapor mass flow rate using information from the table is determined as 17.3 kg/s.

Since the vapor flows within the tube, assume that the vapor heat transfer coefficient varies with $\text{Re}^{0.8}$ or $\dot{m}_V^{0.8}$. Thus, at the outlet,

$$\begin{aligned} h_{V,\text{out}} &= h_{V,\text{in}} (\dot{m}_{V,\text{out}} / \dot{m}_{V,\text{in}})^{0.8} = (150)(17.3/193)^{0.8} \\ &= 21.8\,\text{W/m}^2 \cdot \text{K} \end{aligned}$$

This same calculation can be repeated for each temperature increment along the condensation curve occurring within the heat exchanger. The results of these calculations are tabulated below:

15-14 (continued)

Temp, K (mixture)	Temp, K (water)	$\dot{m}_V$, kg/s	h_V, W/m$^2\cdot$K
340	290	17.3	21.8
345	297.7	60.6	59.3
350	304.7	100.2	89
355	312.3	144.9	119
360	320.3	193	150

Additional corrections to h_V can be made because of condensation conditions, but will not be included in this solution. With the establishment of the four temperature sections in the heat exchanger, the Silver-Bell-Ghaly method can now be used to approximate the required areas in these four sections. Consider the increment from 360K to 355K. The $\Delta\dot{q}$ for this section is 16,850 kJ/s. This can be approximated in this section by

$$\Delta\dot{q}_V = \dot{m}_{V,\text{avg}} C_p \Delta T$$

where $\dot{m}_{V,\text{avg}} = (193 + 144.9)/2 = 168.95\,\text{kg/s}$

$$\Delta\dot{q}_V = (168.95)(2)(5) = 1689.5\,\text{kJ/s}$$

Thus, $\dfrac{\Delta\dot{q}_V}{\Delta\dot{q}} = \dfrac{1689.5}{16{,}850} = 0.1003$

This ratio is inserted in the equation used in the method to obtain the increment of area required in this section. This relation is

$$\Delta A = \Delta\dot{q}\left(1 + \frac{\Delta\dot{q}_V}{\Delta\dot{q}} U'/\bar{h}_V\right)/U'(\bar{T}_E - \bar{T}_C)$$

where $\bar{T}_E$ is the mean equilibrium temperature for the increment, $\bar{T}_C$ the mean temperature of the cooling liquid, U' the overall heat transfer coefficient from the coolant to the condensate film interface and $\bar{h}_V$ the mean vapor heat transfer coefficient. For this section

$$\bar{T}_E = (360 + 355)/2 = 357.5\text{K}$$

$$\bar{T}_C = (320.3 + 312.3)/2 = 316.3\text{K}$$

15-14 (continued)

$$U' = 800\ \mathrm{W/m^2 \cdot K}$$

$$\bar{h}_v = (150 + 119)/2 = 134.5\ \mathrm{W/m^2 \cdot K}$$

Substituting these values in the incremental area equation

$$\Delta A = 16.85\times10^6[1+(0.1003)(800)/134.5]/((800)(357.5-316.3)$$
$$= 816\,\mathrm{m^2}$$

Similar calculations for the three other sections shows required areas of 722, 633, and 611 $\mathrm{m^2}$ for a total of

$$A_T = 816 + 722 + 633 + 611 = \underline{\mathbf{2782\ m^2}}$$

Answer

Using one of the available computer simulation programs shows a requirement of 3090 $\mathrm{m^2}$. However, this program includes a safety factor to recognize fouling particularly by the water, use of commercial lengths and diameters, and inclusion of construction details.

15-15

For a rotary filter, the volume of filtrate per unit time is given by Eq. (15-101b)

$$\frac{\text{Volume of filtrate}}{\text{time}} = A_D\left(\frac{2\Psi_f N_r \Delta p}{\alpha w \mu}\right)^{1/2}$$

This equation can be used for both the small and large rotary filter.

Small filter: $$0.3 = 0.75\left(\frac{2(0.2)(2)(140)}{\alpha w \mu}\right)^{1/2} \quad (1)$$

Large filter: $$3 = A_D\left(\frac{2(0.2)(1.5)(100)}{\alpha w \mu}\right)^{1/2} \quad (2)$$

Since α, w, and μ remain constant in both filters, Eq. (2) can be divided by Eq. (1) to obtain the new area of the filter drum

$$\frac{3 = A_D\left[(2)(0.2)(1.5)(100)\right]^{1/2}}{0.3 = 0.75\left[(2)(0.2)(2)(140)\right]^{1/2}}$$

$A_D = 1\underline{0.25\ \text{m}^2}$

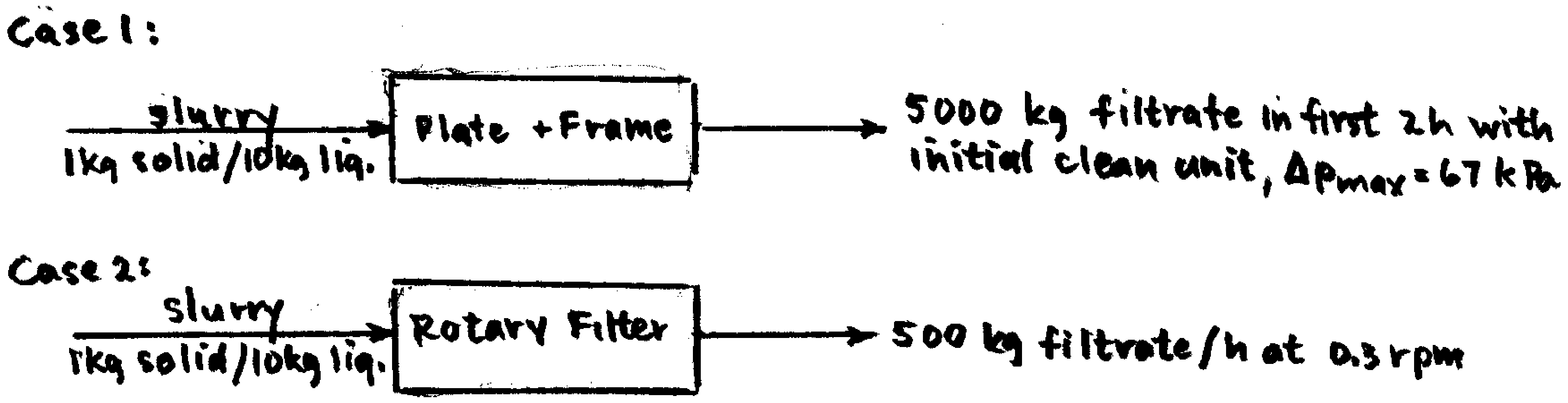
Answer

Purchased cost of filter (Jan. 2002) from Figure 15-39 is given as **<u>$120,000</u>**

Answer

15-16

Conditions for the two filtration systems:



In each case both Ψ and Δp are constant and provide negligible filter-unit resistance.

For case 1, let w_F = kg filtrate rate/h and ρ_F = kg/m^3

$$w_F / \rho_F = \upsilon, \text{m}^3/\text{h}$$

From Eq. (15-91) $\qquad\qquad\qquad\qquad \Delta p = 67\,\text{kpa},\ A = 25\text{m}^2$

$$\upsilon^2 = \frac{2\Delta p A^2}{\alpha' w \mu}\Theta = \left(\frac{w_F}{\rho_F}\right)^2$$

$$\frac{(5000)^2}{2} = 1.25\times10^7 = \frac{2\Delta p A^2 \rho_F^2}{\alpha' w \mu} \qquad \text{since } \Theta = 2\text{h}$$

$$w_F = (1.25\times10^7\Theta)^{1/2}$$

Product as kg of filtrate/24 h $= (12.5\times10^6\Theta)^{1/2}\left(\dfrac{24}{\Theta+3}\right)$

$$= \frac{8.485\times10^4\Theta^{1/2}}{\Theta+3}$$

At Θ_{opt} for maximum production per day, set

$$\frac{d(\text{product/24h})}{d\Theta} = 0 \text{ and solve for } \Theta_{opt} \text{ with a result that } \Theta_{opt} \cong 3\text{h}$$

15-16 (continued)

Maximum product/24 h $= \dfrac{8.485\times10^4(3)^{1/2}}{3+3} = 2.45\times10^4\,\mathrm{kg/24h}$

For equal product rate in the rotary filter per hour requires

$$\dot{m} = 2.45\times10^4/24 = 1020\,\mathrm{kg/h}$$

From Eq. (15-101b)

$$\text{Volume of filtrate/unit time} = A_D\left(\frac{2\Psi_f N_r \Delta\rho}{\alpha w \mu}\right)^{1/2}$$

$$\text{kg of filtrate/h} = A_D\rho_F\left(\frac{2\Psi_f \Delta p}{\alpha w \mu}\right)^{1/2}(N_r)^{1/2}$$

Where N_r is in revolutions /h

From the data for the operating rotary filter

$$500\,\mathrm{kg/h} = A_D\rho_F\left(\frac{2\Psi_f \Delta p}{\alpha w \mu}\right)^{1/2}[(0.3)(60)]^{1/2} \quad (1)$$

For the required rotary filter

$$1020 = A_D\rho_L\left(\frac{2\Psi_f \Delta p}{\alpha w \mu}\right)^{1/2}(N_r)^{1/2} \quad (2)$$

Divide Eq. (2) by Eq. (1), cancel common terms and solve for N_r

$$\frac{1020}{500} = \frac{(N_r)^{1/2}}{[(0.3)(60)]^{1/2}}$$

$N_r = 74.9$ r/h or **1.25 rpm**

Answer

APPENDIX C

Practice Session Problems

Pr. Ses. Prob. 1

Basis: 1 hr.

$$\text{Moles pure } H_2 \text{ in} = \begin{array}{c|c|c} 10{,}000{,}000 & 72.5 & \\ \hline 359 & 100 & 24 \end{array} = 842 \text{ moles}$$

$$\text{Moles pure } CH_4 \text{ in} = \begin{array}{c|c|c} 10{,}000{,}000 & 27.5 & \\ \hline 359 & 100 & 24 \end{array} = 319 \text{ ''}$$

$$\text{Total} \quad \overline{1161} \text{ ''}$$

X = lb act. C necessary per bed.

$$0.95 = \frac{842 - 0.00838\,(0.032)\,X}{842 - 0.00838\,(0.032)\,X + 319 - 0.00838\,(0.968)\,X}$$

X = 33,900 lb. charcoal per bed

$$\text{Amt. pure } H_2 \text{ prod} = 842 - 0.00838\,(0.032)(33{,}900) = 832.8 \text{ moles}$$

$$\text{Amt. pure } CH_4 \text{ prod.} = 319 - 0.00838\,(0.968)(33{,}900) = \underline{44} \text{ moles}$$

$$\text{Total} \quad 876.8$$

$$\text{Check} = \frac{832.8}{876.8}(100) = 95\%$$

Cu. ft. of 95% H_2 (at S.C.) produced per

$$\text{year} = \begin{array}{c|c|c|c} 876.8 & 359 & 24 & 350 \\ \hline & & & \end{array} = 2{,}650{,}000{,}000 \text{ ft}^3$$

Cost Accounting:

Charcoal initial cost = (33,900)(3)(\$3.65) = \$371,200

Charcoal makeup cost = (0.15)(\$371,200) = \$55,700

Capital investment = \$4,750,000 + \$371,200 = \$5,121,200.

Pr. Scs. Prob. 1 (Continued)

Cost Per Year

$$\text{Amortization} = \frac{1}{5}(\$5{,}121{,}200) = \$1{,}024{,}000$$

$$\text{Other operating costs} = \$2{,}250{,}000 + \$55{,}700 = \$2{,}305{,}700$$

$$\text{Total cost per year} = \$3{,}329{,}700$$

Cost per 1000 ft^3 (at s.c.) of 95% H_2 =

$$\frac{\$3{,}329{,}700}{2{,}650{,}000{,}000}\bigg|\frac{1000}{} = \$1.256 \approx 126\ ¢/1000\ ft^3\ (s.c.)$$

Answer

Pr. Ses. Prob. 2

Basis: 1 hr.

Moles pure H_2 in: $\begin{array}{c|c} 10{,}000{,}000 & 0.725 \\ \hline 359 & 24 \end{array} = 842$ moles

Moles pure CH_4 in = $\begin{array}{c|c} 10{,}000{,}000 & 0.275 \\ \hline 359 & 24 \end{array} = \underline{319}$ moles

X = lb. carbon necessary/bed   Total   1161

$$0.95 = \frac{842 - (0.00632)(0.032)X}{842 - (0.00632)(0.032)X + 319 - (0.00632)(0.968)X}$$

$X = 45{,}000$ lb. charcoal/bed

Moles H_2 produced = $842 - 0.000202X = 832.9$ moles

Moles CH_4 produced = $319 - 0.00612X = \underline{44}$ "

Total   876.9 "

Volume of ads. bed = $\begin{array}{c|c} 45{,}000 & \\ \hline 0.30 & 62.42 \end{array} = 2400$ ft^3

Find limiting gas flow - (based on max. vel. of 1 ft/sec)

H_2 Feed gas rate = $\begin{array}{c|c|c|c} 10{,}000{,}000 & & 14.7 & 540 \\ \hline 24 & 3600 & 400 & 492 \end{array} = 4.67$ act. ft^3/sec.

Regen. + Flue gas rate = $\begin{array}{c|c|c|c|c|c} 80+30 & 45{,}000 & 14.7 & 1060 & & \\ \hline & & 19.7 & 492 & 3600 & 8 \end{array}$

$= 277$ ft^3/sec.

Thus, the regeneration cycle puts the most gas through per second, and this rate will set the necessary column diameter.

At a max. gas velocity of 1 ft/sec, the cross-sectional area (total) per bed must be 277 ft^2.

Pr. Ses. Prob. 2 (Continued)

Determination of Individual No. of Towers per bed:

Dia., ft	X-Sect. area/tower ft²	No. of Towers to Give 277 ft² Area	Tower Ht. ft	$/ft. length
6	28.2	10	8.52(+10)	1170
9	63.6	5	7.55 (+10)	2140
12	113.1	3	7.07 (+10)	3280
15	176.5	2	6.81 (+10)	5840

Dia. ft	Skirt cost $	Cost, $/Tower	Total cost, $ (3 beds)
6	1950	21,670 + 1950	236,200 × 3
9	3240	37,560 + 3240	204,000 × 3
12	4540	56,020 + 4540	181,700 × 3
15	5200	98,100 + 5200	206,600 × 3

Sample Calc. $D = 6$ ft. $\text{Area} = (0.785)(6)^2 = 28.2 \text{ ft}^2$

$\text{No. of towers} = \dfrac{277}{28.2} = 10$ (rounded off)

$\text{Tower Ht.} = \dfrac{2400}{(28.2)(10)} = 8.52$ ft. $\quad \text{Cost/tower} = \$1950 + (18.52)(\$1170) = \$23,620$

$\text{Total cost (3 beds)} = (\$23,620)(10)(3) = \$236,200 \times 3$

Part a: 3 Towers per bed for minimum cost
Answer

Part b: Tower diameter = 12 ft.
Tower height = 7.1 ft. // Answer

Pr. Ses. Prob. 3

$$N_R V_R = \text{Vol. Filtrate/min} = A_D \sqrt{\frac{2\psi_f \, \Delta P \, N_R}{\alpha w \mu}} \ \text{ft}^3/\text{min}. \quad \text{(Eq. 48)}$$

(Chap. 14)
(4th ed.)

N_R = Rev./min.

V_R = vol. filt./rev. = ft^3/rev.

A_D = Total filter area, ft^2

ΔP = Pres. drop, psi

ψ_f = Fract. drum area submerged

w = lb. dry cake/ft^3 filtrate

μ = fluid viscosity

α = Sp. cake resistance

From lab. data

$\text{Vol. filt./min} = \frac{0.95}{5} = 0.19$ ft^3/min

$V_R = \frac{0.19}{0.4} = 0.475 \qquad N_R = 0.4$

$\psi_f = 0.20$

$\Delta P = \frac{9}{29.92}(14.7) = 4.42$ psi

let x = lb wet cake

$\frac{x}{1.6}$ = lb dry cake

$\frac{x}{1.6}(13) = x + 65.3$

$x = \frac{65.3 \,|\, 1.6}{|\, 11.4} = 9.17$ lb

Calc. of w: (Basis = 5 min)

Vol. filtrate = 0.95 ft^3

lb. of filtrate = (0.95)(68.3) = 653

lb dry cake = $\frac{x}{1.6} = \frac{9.17}{1.6} = 5.73$

w = lb dry cake/ft^3 filtrate

$= \frac{5.73}{0.95} = 6.03$

Basis: 1 min. + lab. data:

$$\text{(Eq. 48)} \quad 0.19 = 4.15 \sqrt{\frac{2 \,|\, 4.42 \,|\, 0.2 \,|\, 0.4}{|\, \alpha\mu \,|\, |\, 6.03}}$$

(Chap. 14)

$$\alpha\mu = \left(\frac{4.15}{0.19}\right)^2 \left(\frac{2 \,|\, 4.42 \,|\, 0.2 \,|\, 0.4}{| \quad | \quad |\, 6.03}\right)$$

Pr. Scs. Prob. 3 (Continued)

Basis: 1 min. + Design Data

100 lb. $ZnSO_3 \cdot 2.5H_2O$ in Cake

(100) (20) = 2000 lb. of liquid to filter

lb. of liquid in wet cake = (100) (0.6) = 60

Wt. of wet cake lug. filt. area = 160 lb.

Wt. of filtrate = 2000 − 60 = 1940 lb.

Vol. of filtrate = $\frac{1940}{68.8} = 28.2\ ft^3$

w = lb. dry cake/ft^3 filtrate = $\frac{100}{28.2} = 3.54$

$N_R = 0.33$ Rev/min.

$\Delta P = 5$ psi

$\psi_F = 0.25$.

(Eq. 48 4th ed.) (Chap. 14)
$$28.2 = A_D \sqrt{\begin{array}{c|c|c|c} 2 & 5 & 0.25 & 0.33 \\ \hline & \alpha\mu & & 3.54 \end{array}}$$

Substitute value of $\alpha\mu$ from preceding page and solve for A_D. $A_D = \underline{\underline{437\ ft^2}}$

Answer

(4th ed. (Chap. 14) (Eq. 56)

Vacuum Pump Power:

Lab. Data Vol. air/min. as ft^3 at s.c. = $\frac{8.5}{6} = \frac{A_D^2 \psi_a \Delta P}{\beta \mu_a w V_R}$

ψ_a = Fract. drum area for air suction

μ_a = air viscosity

β = sp. air cake res.

$$\frac{8.5}{5} = \begin{array}{c|c|c} \overline{4.15}^2 & 0.1 & 4.42 \\ \hline \beta\mu_a & 6.03 & 0.475 \end{array}$$

$$\beta\mu_a = \begin{array}{c|c|c|c} 5 & \overline{4.15}^2 & 0.1 & 4.42 \\ \hline 8.5 & & 6.03 & 0.475 \end{array}$$

Pr. Ses. Prob. 3 (Continued)

Design Data

Vol. filtrate/rev. = $V_R = \frac{28.2}{0.33} = 84.6$ ft³/rev.

Vol. air/min = $\frac{\overline{437}^{2}}{\beta \mu_a} \Big| \frac{0.1}{3.54} \Big| \frac{5}{84.6} = 202$ ft³ at sc./min.

$\psi_a = 0.1$

By. subst. $\beta \mu_a$ from preceding page.

V_1 actual or $q_{fm_1} = \frac{202}{} \Big| \frac{570}{492} \Big| \frac{14.7}{9.7}$

= 355 ft³/min at 9.7 psi & 110°F pump intake cond.

$$\text{Work} = \frac{k}{k-1}\, p_1 V_1 \left[\left(\frac{p_2}{p_1}\right)^{\frac{k-1}{k}} - 1\right] (3.03\times10^{-5}) \text{ hp.}$$

$k = \frac{c_p}{c_v} = 1.4$

p_1 = intake pres. = 9.7 (144) lb/ft.²

p_2 = exhaust pres. = 14.7 (144) lb/ft.²

V_1 = ft³/min into pump = q_{fm_1} in Eq. 24 (4th ed)

$$\text{Work (hp.)} = \frac{3.03\times10^{-5}}{} \Big| \frac{1.4}{1.4-1} \Big| \frac{(9.7)(144)}{\underset{\overset{\uparrow}{\text{Eff}}}{0.85}} \Big| 355 \left[\left(\frac{14.7}{9.7}\right)^{\frac{0.4}{1.4}} - 1\right]$$

Motor size = $\underline{\underline{7.8 \text{ hp}}}$ Answer

Round off to $\underline{\underline{8 \text{ hp}}}$ Answer.

Pr. Ses. Prob. No. 4

Determination of H.TU.

$G_2 = 27.2$ moles/hr.

$L_2 = \frac{200}{7.48} \Big| 60 \Big| \frac{68.5}{70} = 1570$ moles/hr

$m_2 = 20$ $\qquad \frac{m_2 G_2}{L_2} = \frac{20 \,|\, 27.2}{|\, 1570} = 0.347$

HTU = 4.1 ft by linear interpolation

Determination of N.T.U

y_1^* in equil. with 0.21% PCA

$N.T.U. = N_t = \frac{y_1 - y_2}{\Delta y_m}$ $\qquad \Delta y_1 = 0.15 - 0.042 = 0.108$

$\Delta y_2 = 0.02 - 0.002 = 0.018$

y_2^* in equil/ with 0.01% PCA

$$\Delta y_m = \frac{\Delta y_1 - \Delta y_2}{\ln \frac{\Delta y_1}{\Delta y_2}} = \frac{0.108 - 0.018}{\ln \frac{0.108}{0.018}} = 0.0503$$

$$N.T.U. = N_t = \frac{0.15 - 0.02}{0.0503} = 2.59$$

Column Ht = (2.59)(4.1) = 10.6 ft Use 11 feet

Determination of Column Diameter

Gas enters at $\frac{150}{60}$ ft^3/sec. (Max. possible value would be at 35°C and 14.7 psia or 178.5 ft^3/min

Gas inlet vel = $1.5 | 0.6$ = 0.9 ft/sec. To give D = 2.05 ft)

D = Diameter, ft: $\qquad 0.785D^2 = \frac{150}{60} \Big| \frac{}{0.9}$ $\quad$ D = 1.88 ft Use 2 ft

Column Cost

Tower jacket = (\$880)(11) = \$9680

Heads = (2)(\$750) = 1500

Packing = (0.785)(4)(11)(\$24) = 829

or 12,009

Total cap. expend. = \$12,009 + \$80,000 = \$92,009

% Return on investment = (\$40,000/\$92,009) 100 = 43.5 %

Answer

Pr. Ses. Prob. No. 5

% Cl_2	Net Annual Savings, \$	% Return on Capital Investment
0.2	120,000	32.6
1.0	128,000	42.0
2.0	124,000	45.6
5.0	64,000	30.2
10.0	-48,000	-35.4

For Part 4

% Cl_2	Inv	Net Svg.	Rel	%
2.5	260,000	116,000	8000	$\frac{8000}{120}$ Yes
2.0	272,000	124,000		
1.5	284,000	124,000	0	$\frac{0}{120}$ No
1.0	304,000	128,000	4000	$\frac{4000}{200}$ No
0.5		124,000	No	

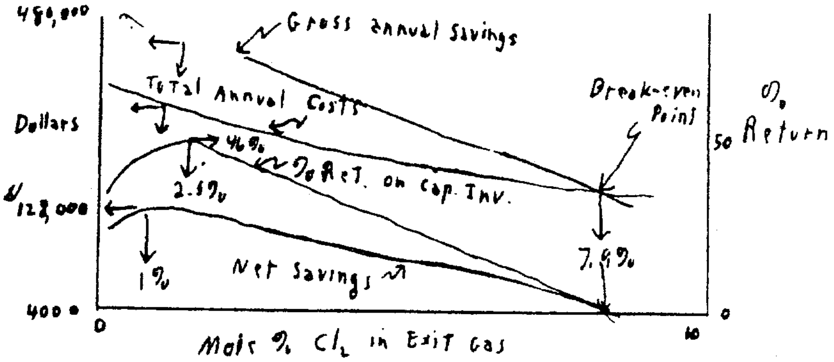
From Plot

1. % Cl_2 at break even point = 7.9%
Answer

2. % Cl_2 at Max. Net Annual Savings = 1% //Answer
Max. Net Annual Savings = \$128,000

3. % Cl_2 at Max. % Return = 2.5% //Answer
Max. % Return = 46%

4. See comparision above in right upper corner:
It is close in range of 1.5 to 2%
Recommend 2% Cl_2 with \$272,000 investment
Answer



Pr. Ses. Prob. 6

D = O.D. of lagging in inches
t_s = T° of outside lagging surface in °F

With insulation:

Initial inv. for insul. $= 0.25\ D^{1.3}(3)(5280) = 3960\,D^{1.3}$

Yearly Deprec. $= \frac{3960\,D^{1.3}}{20} = 198\,D^{1.3}$ \$/yr

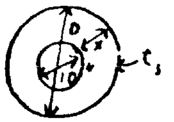
Total cost of equip/yr. $= 198\,D^{1.3} + 3960\,D^{1.3}\left(\frac{10+2}{100}\right) = 673.2\,D^{1.3}$ \$/yr.

$$Q = UA\Delta t = \frac{k\,A_{avg}\,\Delta t_L}{x} = (h_c + h_r)(A_o)\,\Delta t_s$$

x = ins. thickness $= \frac{D-10}{2}$ inches

$$A_{avg} = \frac{D-10}{\ln\frac{D}{10}}\ \frac{\pi}{12}(3)(5280)$$

or assume $\frac{A_o}{A_i} < 2.0$; then $A_{avg} = \left(\frac{D+10}{2}\right)\frac{\pi}{12}(3)(5280)$



$A_o = \frac{D}{12}\pi(3)(5280)$

$A_i = \frac{10}{12}\pi(3)(5280)$

$\Delta t_L = 250 - t_s$

$\Delta t_s = t_s - 45$

$x = \frac{D-10}{2}$ inches

$h_c + h_r = 0.42\left(\frac{\Delta t_s}{D}\right)^{0.25} + 1.2$

$Q_{\text{Conv. \& Rad.}}$ (as BTU/hr) =

$$\left[1.2 + 0.42\left(\frac{t_s-45}{D}\right)^{0.25}\right]\frac{D}{12}\pi(3)(5280)\,(t_s-45)$$

Eq. Ⓐ ↑

$Q_{\text{cond.}}$ (as BTU/hr) =

$$(0.04)\frac{\pi}{12}(3)(5280)\frac{D-10}{\ln\frac{D}{10}}\ \frac{(250-t_s)}{D-10}(2)(12)$$

Eq. Ⓑ ↑

Eq. Ⓒ ↓

Ⓐ = Ⓑ ∴

$$\left[1.2 + 0.42\left(\frac{t_s-45}{D}\right)^{0.25}\right]D(t_s-45) = (0.04)\frac{D-10}{\ln\frac{D}{10}}\ \frac{(250-t_s)}{(D-10)}(2)(12).$$

Pr. Ses Prob 6 (Continued)

Without Insulation $Q'(\text{BTU}/\text{hr}) = (h_c + h_r) A_o (\Delta t_s)$

$$A_o = \pi \frac{10}{12}(3)(5280) \qquad \Delta t_s = 250-45 \qquad h_c = 0.42\left(\frac{250-45}{10}\right)^{0.25}$$

$$h_c = 0.895 \ \frac{\text{BTU}}{\text{hr ft}^2\,°\text{F}}$$

$$Q' = (0.895 + 1.2)\pi \frac{10}{12}(3)(5280)(205) = 1.78 \times 10^7 \ \frac{\text{BTU}}{\text{hr}}$$

Saving on steam with ins. as \$/yr. =

$$\left[1.78\times10^7 - (0.04)\frac{\pi}{12}(3)(5280)\frac{D-10}{\ln\frac{D}{10}}\left(\frac{250-t_s}{D-10}\right)(2)(12)\right]\overbrace{(24)\left(\frac{365}{945.5}\right)\left(\frac{1.30}{1000}\right)}^{0.01201}$$

$\uparrow$ ht. of evap of H_2O at 250°F

Net savings with Ins. as \$/yr = Steam savings - Costs

$$\text{Return on Investment} = \frac{\text{Net savings/yr}}{\text{Total Inv.}} = 0.50 =$$

$$\frac{\left[1.78\times10^7 - (0.04)\frac{\pi}{12}(3)(5280)\frac{D-10}{\ln\frac{D}{10}}\left(\frac{250-t_s}{D-10}\right)(2)(12)\right](0.01208) - 673.2\,D^{1.3}}{3960\,D^{1.3}}$$

Eq. Ⓓ ↗

METHOD FOR COMPLETING SOLUTION

Assume a value of D (or t_s) and calculate the value of t_s (or D) from eq. C. Put these values of D and t_s into eq. D and check the assumption. Repeat until eq. D is satisfied

$$\text{Lagging thickness} = \frac{D-10}{2} \text{ inches}$$

Answer

Pr. Ses. Prob. No. 7

At start, moles isopropanol consumed/hr. $= \frac{76.1}{0.98}$

$N =$ Moles isoprop. fed to converter/hr $= \frac{76.1}{0.98\,\alpha} = \frac{77.6}{\alpha}$ (Keep constant)

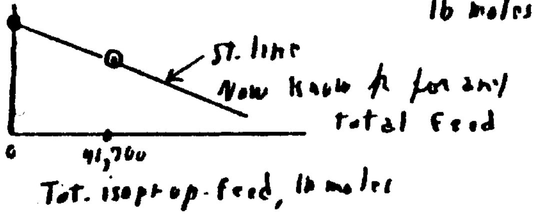
At start, $k = 0.3 = \frac{0.000254 \mid 77.6 \mid 572+460}{250 \mid \alpha \mid}\left[2.46 \ln\frac{1}{1-\alpha} - \alpha\right]$

Solve for α $(\alpha = 0.78)$

Therefore, can solve for $N = \frac{77.6}{\alpha}$ $(= 99.7$ moles/hr$)$

k is $0.15\ \text{sec}^{-1}$ when total feed $= \frac{10{,}000 \mid 250}{\mid 60.1} = 41{,}700$ lb moles

Fig. 1 plot log k



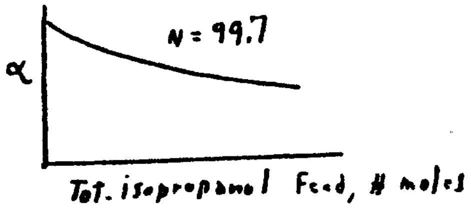
Need α at various total feeds

$k = \frac{0.000254 \mid \underbrace{99.7}_{N} \mid 1032}{\mid 250 \mid}\left[2.46 \ln\frac{1}{1-\alpha} - \alpha\right]$

Fig. 2 plot

From Fig. 1 and Fig. 2, plot

Fig. 3



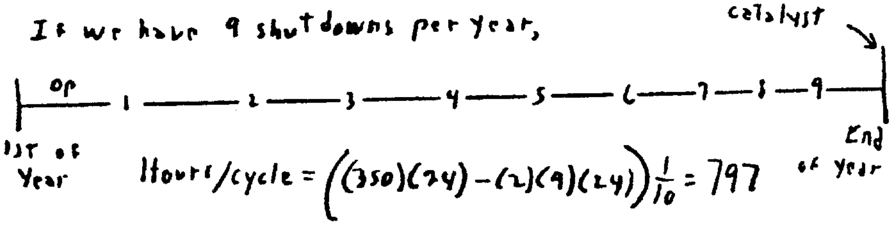
Pr. Ses. Prob. 7 (Continued)

Get moles of isopropanol feed per cycle

(Neglect down time)

If we have 9 shutdowns per year,

Throw away catalyst

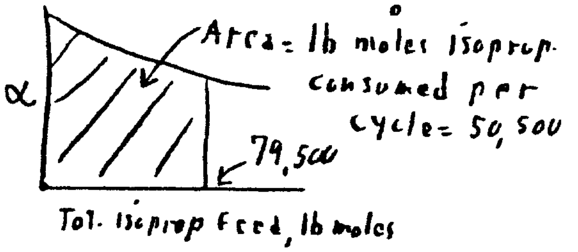


Hours/cycle $= \left((350)(24) - (2)(9)(24)\right)\frac{1}{10} = 797$

Total isopropanol fed/cycle $= 797(99.7) = 79{,}500$ lb moles

$$\alpha = \frac{d\,(\text{isoprop. cons.})}{d\,(\text{isoprop. sup.})} \quad \text{or} \quad \text{isoprop cons.} = \int_0^{79{,}500} \alpha\, d\,(\text{isoprop. sup})$$

From Fig. 3



Acetone produced in lb per year =

(moles isoprop cons./cycle)(efficiency)(cycles/yr)(M.W. of Acetone)

$= (50{,}500)(0.98)(10)(58.1) = \underline{\underline{28{,}800{,}000}}$ lb/yr

Pr. Ses. Prob. No. 8

Get amount of acetone prod. at max. prod rate ($k = 0.3\ sec^{-1}$)

$k = \frac{0.000254}{V} NT \left[2.46 \ln \frac{1}{1-\alpha} - \alpha\right]$ $N = 97$ $V = 250$ $T = 1032°R$

$k = 0.3$

Solve for α: $\alpha = \underline{0.78}$

lb acetone delivered from top of dist col. as product =

$$(97)(0.78)(0.98)(0.98)(58.1) = 4220\ lb/hr.$$

Necessary column cross sect. area = $\frac{4220}{1000}(6.4) = 27.0\ ft^2$

Actual column cross sect. area = $0.785(6)^2 = 28.3\ ft^2$

The present column is satisfactory

The present condenser is satisfactory

Calandria Calc.

Ht. of cond. of steam at 64.7 psia

$$q = \frac{4220}{1000}(1100)(911.8) = UA\Delta t\ Btu/hr$$

$U = 250\ BTU/hr\,ft^2\,°F$ $\Delta t = 90°F$

Calculate value of $A = 188\ ft^2$

Available area is $104\ ft^2$; so supply new calandria

If use 20% safety factor, Area $= 188/1.2 = 226\ ft^2$

Therefore, Area of at least $200\ ft^2$ should be provided.

Cost = ($160) (200) = $32,600 at least

Answer

Pr. Scs. Prob. No. 9

	CHARGE STOCK				OVHD		BTMS	
Comp.	Mol %	M.W.	Avg M.W	Mols/Hr	Mols/Hr	Mol %	Mols/Hr	Mol %
C_2''	0.1	28	0.03	0.8	0.8	0.2		
C_2	1.2	30	0.36	10.0	10.0	1.9		
H_2S	2.1	34	0.71	17.5	17.5	3.3		
C_3''	16.3	42	6.85	135.7	135.7	26.3		
C_3	6.9	44	3.03	57.5	57.5	11.1		
IC_4''	6.5	56	3.64	54.2	54.2	10.5		
NC_4''	14.3	56	8.00	119.2	119.2	23.0		
IC_4	10.8	58	6.26	90.0	90.0/484.9	17.4		
LK → NC_4	3.9	58	2.26	32.4	25.0	4.8	7.4	2.4
C_5''	11.9	70	8.33	99.1	4.8	0.9	94.3	29.9
HK → IC_5	9.7	72	6.98	80.8	3.0	0.6	77.8	24.7
NC_5	2.3	72	1.65	19.2			19.2	6.1
C_6	11.8	86	10.14	98.3			98.3	31.1
C_7	2.1	96	2.02	17.5			17.5	5.5
C_8	0.1	112	0.11	0.8			0.8/135.8	0.3
	100.0	60.4≃	60.35	833.0	517.7	100	315.3	100

% OVHD = $\frac{517.7}{833}(100) = 62.2\%$

Get Sp. Gr. 60/60

$$°API = \frac{141.5}{Sp. Gr. 60/60} - 131.5 = 100.3$$

Sp. Gr. = 0.613

$$\text{Total moles/hr} = \frac{5620 \mid 42 \mid 0.613 \mid 62.4}{\mid 7.48 \mid 24 \mid 604} = \underline{833}$$

Get No. of Trays & Amount of Reflux From Card A

About the same case: Card A has 66.2% OVHD. We have 62.2% OVHD and poorer C_5-C_4 separation. Fewer plates and lower reflux ratio could be used.

Use 27 Trays and return 115 mol% feed as Reflux

Pr. Ses. Prob. # 9 (Continued)

Distillate = 517.7 moles/hr   Reflux = $\frac{115}{100}(833) = 957$ moles/hr

Assume feed near B.P.; so $V = \bar{V}$ and diameter is set for $V = 517.7 + 957 = 1474.7$ moles/hr

At one Atm. pres. = assume MAV = 3 ft/sec.

At 165 psia & 85% of MAV, the MAV (superficial) =

$$3\sqrt{\frac{14.7}{165}}(.85) = 0.77 \text{ ft/sec.}$$

Determine Diameter at point of higest Temp.

Thus, get Temp. at reboiler. Use $y = kx$ and Card B.

With x values for BTMS as given and k values on card B, get y for each component at trial and error temperature until $\Sigma y = 1.0$.

Result is; Temp. = 282°F

Thus $$0.77 = \frac{1474.7 \;|\; 359 \;|\; 14.7 \;|\; 460+282 \;|\; \;|\; \;|\;}{\;|\; \;|\; 165 \;|\; 492 \;|\; 3600 \;|\; 0.785 \;|\; D^2}$$

D = 5.73

For Reboiler Area - Use Card C $U = 130 \ \text{BTU/hr ft}^2\,{}^\circ\text{F}$ is safe

$$q = (1474.7)(5000)(1.8) = (130)\,A\,(\underset{\uparrow}{401}-282)$$

t° of cond. of steam at 250 psia

A = 860 ft²

Final Results Use column with 27 trays ± 10%

Use column diameter of 5.73 ft ± 10%

Use Reboiler area of 860 ft² ± 10%

Answer

Pr. Ses. Prob. No. 10

Determine cost of Prod. HCHO

Basis: 1 operating day

$$F.C. = \frac{1{,}140{,}000}{350}\bigg|\frac{15}{100} = \$\,489$$

$$Misc. = \frac{360{,}000}{350} = \$\,1029$$

$$CH_3OH \text{ cost} = (57{,}600)(0.091) = \$\,5242$$

$$\text{Utilities cost} = x = \$\,356$$

$$\text{Total cost} \quad \$\,7116$$

$$CH_3OH + \tfrac{1}{2}O_2 \rightarrow HCHO + H_2O$$

$$\underset{32}{CH_3OH} \rightarrow \underset{30}{HCHO} + H_2$$

lb. Methanol necess =

$$\frac{100{,}000}{}\bigg|\frac{37.2}{100}\bigg|\frac{32}{30}\bigg|\frac{}{0.8} + \frac{100{,}000}{}\bigg|\frac{8}{100} = 57{,}600$$

If utilities cost $= x$

$$\frac{x}{x+489+1029+5242} = 0.05$$

$$\text{Total income from HCHO only} = (100{,}000)(0.100) = \$\,10{,}000$$

$$\text{Present profit/yr} = (10{,}000-7116)(350) = \$\,\underline{\underline{1{,}009{,}000}}$$

Answer 1

Situation with P.E. Plant in use

Basis: One operating day

$$\underset{56.1}{CaO} + H_2O \rightarrow \underset{74.1}{Ca(OH)_2}$$

$$\underset{74.1}{Ca(OH)_2} + 2\,\underset{88}{CH_3CHO} + \underset{240}{8\,HCHO} \rightarrow 2\,\underset{272}{C(CH_2OH)_4} + \underset{130}{Ca(COOH)_2}$$

Final P.E. Prod. = 6000 lb.

$$\text{P.E. Prod. in reaction} = \frac{6000}{0.7} = 8570 \text{ lb.}$$

$$\text{lb } Ca(OH)_2 \text{ necess.} = \frac{74.1}{272}\bigg|\frac{8570}{} = 2330 \text{ lb.}$$

$$\text{lb } CH_3CHO \text{ necess.} = \frac{88}{272}\bigg|\frac{8570}{} = 2780 \text{ lb.} \cong \frac{2780}{}\bigg|\frac{0.46}{} = \$\,1279$$

$$\text{lb HCHO necess.} = \frac{240}{272}\bigg|\frac{8570}{0.372} = 20{,}400 \text{ lb.} \cong \frac{20{,}400}{}\bigg|\frac{0.07116}{} = 1452$$

$$\text{lb CaO necess.} = \frac{2330}{}\bigg|\frac{56.1}{74.1} = 1770 \text{ lb} \cong \frac{1765}{}\bigg|\frac{40}{2000} = \underline{35}$$

$$\text{Total raw mtl. cost} = \$\,2766$$

Pr. Ses. Prob. No. 10 (Continued)

$$\text{All other costs} = \frac{0.4 \;\big|\; 500,000}{\;\big|\; 350} = \$571$$

$$\text{Total cost} = 571 + 2766 = \$3337 \quad \text{per day}$$

$$\text{Cost per lb.} = \frac{3337}{6000} = \$0.556 \quad \text{per lb}$$

Total profit per year with P.E. plant operating =

$$\frac{0.72 - 0.556 \;\big|\; 6000 \;\big|\; 350}{\;\big|\;\;\big|\;} +$$

$$+ \frac{0.100 - 0.07116 \;\big|\; 100,000 - 20,400 \;\big|\; 350}{\;\big|\;\;\big|\;}$$

$$= \underset{\text{Answer 2}}{\underline{\underline{\$1,147,500}}} \quad \text{Total Yearly Profit}$$

$$\text{Yearly return on P.E. Investment} = 1,147,500 - 1,009,600 = \$138,500$$

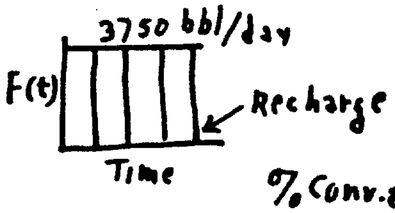
$$\%\ \text{return on P.E. Investment} \quad \frac{138,500}{500,000}(100) = \underset{\text{Answer 3}}{\underline{\underline{27.7\%}}}$$

The % return does not reach the required 30%.

Therefore, the investment should <u>not</u> be made in the P.E. plant.

$$\underset{\text{Answer 4}}{\underline{\underline{\text{NO}}}}$$

Pr. Ses. Prob. 11



$$F = \frac{15{,}000}{4}\Big|\frac{}{1000} = 3.75$$

or 3750 Barrels/day

% Conv. at cat. age A = 93.75% by problem statement

If % conversion at Time 0 (Cat. Age Factor = A = 0) is 97.66%, then $\frac{\%\text{ Conv. at Cat. Age } A}{\%\text{ Conv. at Cat. Age } 0} = \frac{93.75}{97.66} = 0.9600$

and, by table in Problem, A = Cat. age factor = 50

$$\log \frac{\Delta P}{F^2} = 0.03179A - 0.699 = 0.03179(50) - 0.699$$

$$\doteq 0.8905 = \log \frac{\Delta P}{(3.75)^2}$$

$\Delta P = 109.3$ psi at 93.75% Conv. Answer

$$A = \frac{\left[\int_0^D F(t)\,dt\right] \times \left[\text{conv. factor to give gal. of polymer}\right]_{a.a.}}{\text{Pounds of catalyst}}$$

Arith. Avg. (a.a.) conv. factor to give gal. of polymer

$$= 42\,\frac{\text{gal.}}{\text{bbl}} \times 0.4\,\frac{\text{bbl propylene in feed}}{\text{bbl of feed}} \times 0.715\,\frac{\text{bbl of polymer prod.}}{\text{bbl of propylene conv.}}$$

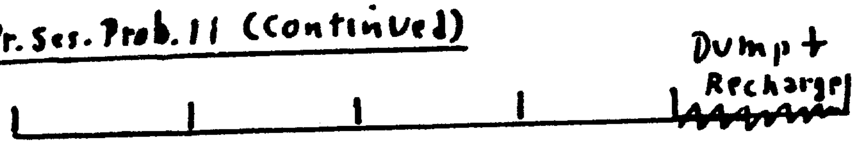
$$\times \frac{93.75 + 97.66}{2 \times 100}\left[\frac{\text{bbl of propylene conv.}}{\text{bbl of propylene in feed}}\right]_{a.a.} = 11.50$$

$$A = 50 = \left[\int_0^D F(t)\,dt\right] \times \frac{11.50}{20{,}000} = \frac{15{,}000}{4}\,D\,\frac{11.50}{20{,}000}$$

$$D = \frac{50}{}\Big|\frac{20{,}000}{15{,}000}\Big|\frac{4}{11.5} = 23.19$$ Answer

Days each reactor is on stream

Pr. Ses. Prob. 11 (Continued)

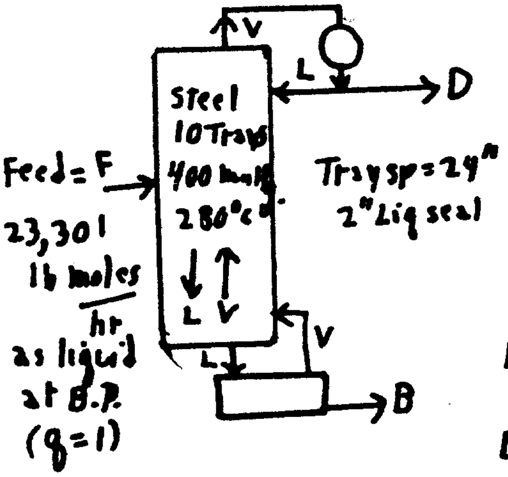


Dump + Recharge time = $\frac{23.19}{4}$ = 5.80 Days

Time each unit should come on stream = Days between each new recharged unit coming on stream = 5.8 Days

Answer

Pr. Ses. Prob. 12



Assume const. T° + Pres. + ideal gas + adiabatic + const. molal overflow + q = 1.

Gas m.w. = 154.2

Liq. dens = 0.72 × 62.4 = 44.9 lb/ft³

D contains 98% of the biphenyl entering

B contains 2% of the biphenyl entering and all below that

D contains all entering above biphenyl

F, D, B, V, + L = lb moles/hr

Pr. Ses. Prob. 12 (Continued)

Material Balances

$$\frac{L}{D} = R.R. = 8 \qquad L = 8D$$

$$V = L + D = 9D$$

$$D = 0.488 + 1.599 + 0.98(16.835) = 18.5853\ \frac{\text{lb moles}}{\text{hr}}$$

$$V = 9D = (9)(18.5853) = 167.2677\ \frac{\text{lb moles}}{\text{hr}}$$

$$\rho_G = \frac{m}{Vol} = \frac{PM}{RT} = \frac{154.2}{359}\Bigg|\frac{273}{280+273}\Bigg|\frac{400}{760} = 0.112\ \frac{\text{lb}}{\text{ft}^3}$$

$$\rho_L = 44.9\ \text{lb/ft}^3$$

Using Eq. 1 & Fig. 16-6; (Chap. 16) $K_V = 0.30$ for 24" plate spacing

$$V_m = K_V\sqrt{\frac{\rho_L - \rho_G}{\rho_G}} = 0.30\sqrt{\frac{44.9 - 0.112}{0.112}} = 6.0\ \text{ft/sec}$$

Actual superficial $V_m = (6.0)(0.85) = 5.1$ ft/sec

Let D_F = Column diameter in feet

$$\frac{167.2677}{}\Bigg|\frac{}{3600}\Bigg|\frac{359}{}\Bigg|\frac{760}{400}\Bigg|\frac{553}{273}\Bigg|\frac{}{0.785\,D_F^2} = 5.1$$

$$D_F = 4.0\ \text{FT}$$

By Fig. 16-28, cost per plate = \$4100 in Jan., 1990.
This includes installation and auxiliaries.

Cost of Tower installed with auxiliaries (no reboiler or condenser) = (\$4100)(10) = \$41,000
Answer

Pr. Ses. Prob. 12 (Continued)

Using Eq. 3 (Chap 16) + Fig. 16-7; (4th ed.)

$$V_m' = K_v' \left(\frac{\sigma}{20}\right)^{0.2} \sqrt{\frac{\rho_L - \rho_G}{\rho_G}}$$

$$\frac{L}{G}\left(\frac{\rho_G}{\rho_L}\right)^{0.5} = \frac{8 \times 18.5}{166.6}\left(\frac{0.112}{44.9}\right)^{0.5} = 0.044$$

By Fig. 16-7, $K_v' = 0.36$

Assume no correction factors needed and $V_m = 0.95 V_m'$ as in Example 1 of Chap. 16 (4th ed.)

$$V_m = (0.36)\left(\frac{20}{20}\right)^{0.2} \sqrt{\frac{44.9 - 0.112}{0.112}}\,(0.95) = 6.8 \text{ ft/sec}$$

Actual superficial $V_m = (6.8)(0.85) = 5.8$ ft/sec

$$\frac{167.2679}{} \bigg| \frac{}{3600} \bigg| \frac{359}{} \bigg| \frac{760}{400} \bigg| \frac{553}{273} \bigg| \frac{}{0.785 D_F^2} = 5.8$$

$D_F = 3.8$ ft

By Fig. 16-28, cost per plate = \$3900 in Jan., 1990.

Cost of Tower installed with auxiliaries (no reboiler or condenser) = (\$3900)(10) = \$39,000

Answer

Pr. Ses. Prob. 13

Case of Single back-mix reactor operated Isothermally at 800°F

$$\frac{W}{Q} = C_{S_0}\left(\frac{X_{SF} - X_{Si}}{-r_S}\right)$$

$$-r_S = k_S A_S \frac{C_S^2 P_H}{C_{S_0}(1 + K_{HS} P_{HS})} \text{ lb moles/hr-lb cat.}$$

$C_S = C_{S_0}(1 - X_{SF})$

$P_H = P_{H_F} = 2075$ psia

$P_{HS} = P_{HS_F} = 4.4$ psia

$K_{HS} = 0.162$ psia^{-1}

$C_{S_0} = 0.0213$ lb mole/ft^3

$X_{Si} = 0$

$X_{SF} = 0.74$

$k_S = 7.405 \times 10^{-4}$ ft^3/hr-lb cat-psia

$A_S = 1.25$

Cat. Density = 42 lb/ft^3

4.2 Bbl. of Prod oil/ton coal

At 50,000 Bbl of prod. oil/day, need 50,000/4.2 tons coal/day

There fore,

$$Q = \frac{50{,}000}{4.2} \times \frac{1}{24} \times 127.34 \frac{\text{ft}^3\text{ slurry}}{\text{Ton coal}} = 63{,}300 \text{ ft}^3 \text{ slurry/hr}$$

$$-r_S = k_S A_S \frac{C_{S_0}(1 - X_{S_F})^2 P_{HF}}{(1 + K_{HS} P_{HS_F})}$$

$$-r_S = \frac{7.405 \times 10^{-4} \,\big|\, 1.25 \,\big|\, 0.0213 \,\big|}{} \; \frac{(1-0.74)^2 \,\big|\, 2075}{1 + (0.162)(4.4) \,\big|} =$$

$$= 0.001625$$

$$W = \text{lb cat. charge} = Q C_{S_0} \frac{X_{S_F} - X_{Si}}{-r_S}$$

$$W = \frac{63{,}300 \,\big|\, 0.0213 \,\big|\, (0.74 - 0)}{\big|\, 0.001625 \,\big|}$$

$$= 613{,}991$$

Vol. of CSTR needed if no extra space other than for catalyst =

$$\frac{613{,}991}{42} = 14{,}600 \text{ ft}^3$$

Answer

Pr. Ses. Prob. 13 (Continued)

Case of Ideal, Single, Plug-flow Reactor Operated Isothermally at 800°F

$$\frac{W}{Q} = C_{S_0} \int_{x_{S_i}}^{x_{S_F}} \frac{dx_S}{-r_S} \qquad -r_S = k_S A_S \frac{C_S^2\, p_H}{C_{S_0}(1 + k_{HS} p_{HS})}$$

$$-r_S = \underbrace{\frac{k_S A_S\, p_{H_{avg}}\, C_{S_0}}{(1 + K_{HS}\, p_{HS})_{Avg.}}}_{\text{constant by assumption}} (1 - x_S)^2 \ \frac{\text{lb moles}}{\text{hr-lb cat.}}$$

$$= \frac{(1+0) + [1 + (0.162)(4.4)]}{2} = 1.3564$$

$C_{S_0} = 0.0213$ lb moles/ft^3

$$p_{H_{avg}} = \frac{2125 + 2075}{2} = 2100 \text{ psia}$$

$p_{HS_0} = 0$

$K_{HS} = 0.162$ psia^{-1}

$x_{S_i} = 0$

$x_{S_f} = 0.74$

$C_{S_0} = 0.0213$ lb mole/ft^3

$k_S = 7.405 \times 10^{-4}$ ft^3/hr-lb cat.-psia

$A_S = 1.25$

$Q = 63,300$ ft^3 slurry/hr

Cat. density $= 42$ lb/ft^3

Pr.Ser. Prob.13 ((Continued)

$$-r_s = \frac{7.405\times10^{-4}\,\big|\,1.25\,\big|\,2100\,\big|\,0.0213}{\big|\quad\big|\quad\big|\,1.3564}(1-x_s)^2$$

$$-r_s = 0.03057(1-x_s)^2$$

$$W = \frac{Q\,C_{s0}}{0.03057}\int_0^{x_{s_f}} \frac{dx_s}{(1-x_s)^2}$$

$$\int_0^{x_{s_f}} \frac{dx_s}{(1-x_s)^2} = \frac{1}{1-x_s}\Bigg]_0^{x_{s_f}=0.74} = 2.85$$

$$W = \frac{63{,}300\,\big|\,0.0213\,\big|\,2.85}{\big|\,0.03057\,\big|} = 125{,}699 \text{ lb. cat. needed}$$

Vol. of plug-flow reactor needed if no extra space other than for catalyst =

$$\frac{125{,}699}{42} = 3{,}000 \text{ ft}^3$$

Answer

Versus 14,600 ft³ needed for CSTR

Pr. Ses. Prob. 14

$$\frac{1700 \text{ m}^3 \text{ alkylate/day}}{1.72 \dfrac{\text{m}^3 \text{ alkylate}}{\text{m}^3 \text{ butylene consumed}}} = 988.37 \text{ m}^3 \text{ Butylene consumed/day}$$

$$\frac{988.37}{0.74} = 1336 \text{ m}^3 \text{ Butylene Feed/day}$$

$$918.37 \text{ m}^3 \text{ Butylene consumed/day} \times 1.10 \frac{\text{m}^3 \text{ isobutane cons.}}{\text{m}^3 \text{ butylene cons.}}$$

$$= 1087.2 \text{ m}^3 \text{ isobutane consumed/day}$$

$$\underbrace{(1336)(0.17)}_{\text{isobutane From butylene feed}} + \underbrace{(0.9)(\text{Isobutane Feed/day})}_{\text{isobutane From isobutane feed}} = 1087.2$$

Isobutane Feed/day = 956 m^3/day

Answer (a)

$$\text{Reactor Feed} = \underbrace{1336 + 956}_{2291.36} + \underbrace{\text{Recycle}}_{R \text{ as m}^3\text{/day}}$$

Volume change of flow stream through reactor =

$$\underbrace{(1.72)(988.37)}_{\text{Inc. in Vol. due to Butylene conv. to alkylate}} - \underbrace{(1.1)(988.37) - 988.37}_{\text{Loss in vol due to isobutane conv. to alkylate; Loss in vol due to Usage of butylene}} = -375.6 \text{ m}^3$$

Pr. Ses. Prob 14 (Continued)

P = Reactor product as m^3/day = $2291.36 + R - 375.6$

$= 1915.76 + R$

Reactor product is 75 vol-% isobutane, and, since all isobutane coming out of reactor must be in recycle stream R (all exit streams are assumed to be pure),

$$R = 0.75P \quad \text{and} \quad P = 1915.76 + 0.75P$$

$$P = \frac{1915.76}{0.25} = 7663.12 \text{ m}^3 \text{ prod./day}$$

R = amout of isobutane recycled =

$(0.75)(7663.12) = 5750 \text{ m}^3\text{/day}$

Answer

Pr. Ses. Prob. 15

See solution to 1970 AIChE Stud. Cont. Prob. in Fall, 1970 AIChE Stud. Members Bul. - P. 75 - Table

Part A: 20% Cont. nom. int. rate of return after taxes required on any investment

$T.C.I. \cong 300{,}000 = F.C.I. + W.C.I. = F.C.I. + 0.1\, F.C.I.$
$= 1.1\, F.C.I.$

$F.C.I. = 300{,}000/1.1 \cong 273{,}000.$ $\quad W.C.I. \cong 27{,}000$

round off

$$\text{Annual Depr. cost} = \frac{273{,}000 - 0}{10} \cong 27{,}000$$

round-off

Annual cost without depreciation before taxes $= 758{,}000 - 27{,}000 + 0.05y = 731{,}000 + 0.05y$
+27,000 startup in 1st yr. only

y = annual income before taxes

Incl. depr.

A. Taxable profit per normal year $= [y - (758{,}000 + 0.05y)]$

48% of this is tax cost

B. Income taxes per normal year $= 0.48[y - (758{,}000 + 0.05y)]$

In-an-instant at end of year,

Cash flow to project after taxes in normal year =
$0.52[y - (758{,}000 + 0.05y)] + 27{,}000$

Cash flow to project after taxes in 1st year =
$0.52[y - (758{,}000 + 27000 + 0.05y)] + 27{,}000$

Cash flow to project after taxes in 10th year =
$0.52[y - (758{,}000 + 0.05y)] + \underset{\text{deprec.}}{27{,}000} + \underset{\text{W.C.I.}}{27{,}000}$

Pr. Ses. Prob. 15 (Continued)

YEAR	Cost-Incl. Depr.	Inc. before Taxes	Cash Flow = A to Project after Taxes	r = 20% Discount factor (Fa). Table 5 Chap. 7 100rn	 Fa	Pres. Value = $F_a(A)$
0	(300,000)					
1	785,000 + 0.05Y	Y	$0.52[Y-(785{,}000)+0.05Y)]$ +27,000	20	0.8187	
2	758,000 + 0.05Y		$0.52[Y-(758{,}000+0.05Y)]$ +27,000	40	0.6703	
3	"		"	60	0.5488	
4	"		"	80	0.4493	
5	"		"	100	0.3679	
6	"		"	120	0.3012	
7	"		"	140	0.2466	
8	"		"	160	0.2019	
9	"		"	180	0.1653	
10	"	↓	$0.52[Y-(758{,}000+0.05Y)]$ +27,000 +27,000	200	0.1353	
					$\Sigma f_a = 3.9053$	ΣP.V.

Profits Present Value after Taxes = Σ P.V. =

$$(3.9053)(0.52)[0.95Y-758{,}000)-(0.52)(27{,}000)(0.8187)+(3.9053)(27{,}000)+(0.1353)(27{,}000)$$

Profits Present Value after Taxes = $300{,}000 = 1.929Y - 1{,}441{,}712$

$$Y = \frac{300{,}000 + 1{,}441{,}712}{1.929} = \$902{,}909$$

$$\text{Price} = \frac{902{,}909}{10\times10^6} = \underline{\underline{0.0903}} \text{ per lb. of prod.}$$

Answer A

Note: In 1970 AIChE Stud. Contest Solution, error was made in yr 2 on by including deprec as a cost and did not correct for income-tax effect correctly. Also had error in year 10 cash flow by taking 0.52 of WCI.

Pr. Ses. Prob. 15 (Continued)

Part B, 20% finite eff. end-of-year rate of return after Taxes required on any investment

Year	Cost-Incl. Depr.	Inc. before taxes	Cash flow = A to Project after Taxes	$i = 20\%$ Discount factor $= D.F. = \frac{1}{(1+i)^n}$ Table 1 Chap. 7	Pres. Value $= (D.F.)($
0	(300,000)				
1	(785,000 + 0.05Y)	Y	$0.52[Y-(785,000+0.05Y)]$ + 27,000	$\frac{1}{1.2} = 0.833$	
2	(758,000 + 0.05Y)	↓	$0.52[Y-(758,000+0.05Y)]$ + 27,000	$\frac{1}{1.44} = 0.6944$	
3	"		"	$\frac{1}{1.728} = 0.5787$	
4	"		"	$\frac{1}{2.0736} = 0.4823$	
5	"		"	$\frac{1}{2.4883} = 0.4019$	
6	"		"	$\frac{1}{[illegible].9860} = 0.3349$	
7	"		"	$\frac{1}{3.5832} = 0.2791$	
8	"		"	$\frac{1}{4.2998} = 0.2326$	
9	"		"	$\frac{1}{5.1598} = 0.1938$	
10	"	↓	$0.52[Y-(758,000+0.05Y)]$ + 27,000 + 27,000	$\frac{1}{6.1917} = 0.1615$	
				$\Sigma D.F. = 4.1922$	$\Sigma P.V.$

Profits Present Value after Taxes $= \Sigma P.V. = 300,000$

$$= (4.1922)(0.52)(0.95Y - 758,000) - (0.52)(27,000)(0.833) + (4.1922)(27,000) + (0.1615)(27,000)$$

$$= 2.0709Y - 1,546,534 = 300,000$$

$$Y = \frac{300,000 + 1,546,534}{2.0709} = \$891,662$$

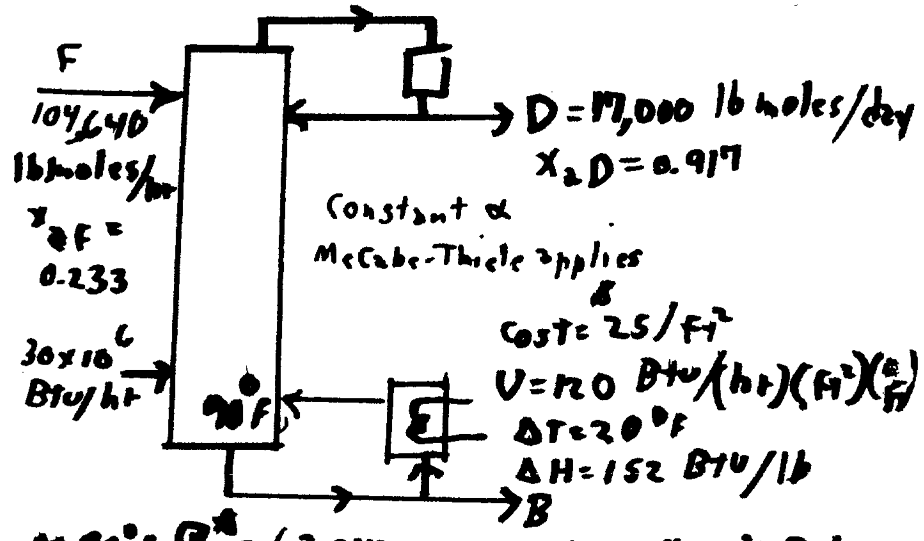
$$\text{Price} = \frac{891,662}{10 \times 10^6} = \$\underline{\underline{0.0892}} \text{ per lb. of Prod}$$

Answer B

Pr. Ses. Prob. 16

Basis, 1 day

a = i-butane
b = n-butane



At 90°F $P_a^* = 62$ psia
$P_b^* = 44$ psia

? = Cost of reboiler at min. R.R.

Overall Mt'l Balance

$F = D + B$

$B = 104,640 - 17,000$
$= 87,640$ lb moles

$x_{aF} F = x_{aD} D + x_{aB} B$

$0.233 \times 104,640 = 0.917 \times 17000 + x_{aB} \times 87,640$

$x_{aB} = 0.1003$

Get α at Bottom temp. of 90°F and constant α:

$p_a = P_a^* x_a = 62 \times 0.1003 = 6.2185$ psia

$p_b = P_b^* x_b = 44 \times 0.8997 = 39.5865$ psia

$P_T = 45.8053$ psia

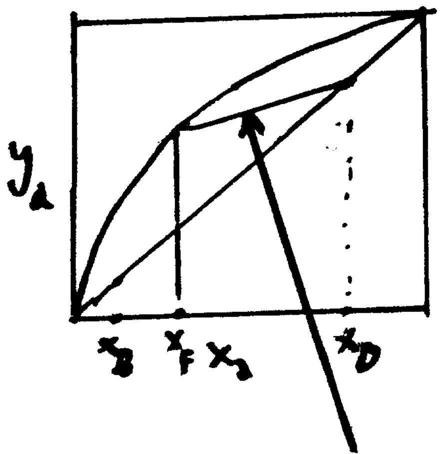
Pr. Sec. Prob. 16 (Continued)

$$y_a = \frac{P_a}{P_T} = \frac{6.2185}{45.8053} = 0.136$$

$$y_b = \frac{P_b}{P_T} = \frac{39.5865}{45.8053} = 0.864$$

$$\alpha = \frac{y_a}{y_b}\,\frac{x_b}{x_a} = \frac{0.136}{0.864}\,\frac{0.8997}{0.1003} = 1.41$$

McCabe Thiele Diagram



For Feed, $x_{aF} = 0.233$

$x_{bF} = 0.767$

$y_{bF} = 1 - y_{aF}$

$$\alpha = 1.41 = \frac{y_{aF}}{1-y_{aF}}\,\frac{0.767}{0.233}$$

$y_{aF} = 0.300$

Enriching line for min. R.R.

$$\text{Slope} = \frac{0.917-0.300}{0.917-0.233} = 0.902 = \frac{L}{V}$$

$V = L + D = 0.902V + 17{,}000$

$$V = \frac{17{,}000}{0.098} = 173{,}469 \text{ lb moles/day} \times \frac{1}{24} = 7228 \ \frac{\text{lb moles}}{\text{hr}}$$

$V = \bar{V}$

Pr. Ses. Prob 16 (continued)

Reboiler heat needed $= (\bar{V})(\Delta H)(M.W.) -$ $30\times10^6 \frac{BTU}{hr}$

$= (7228)(152)(58.12) - 30\times10^6$

$= 33.85\times10^6$ BTU/hr

$Q = UA\Delta T$

$$A = \frac{Q}{U\Delta T} = \frac{33.85\times10^6}{(120)(20)} = 14{,}104 \text{ ft}^2$$

Reboiler cost $= 14{,}104 \times 25 = \$\underline{\underline{350{,}000}}$

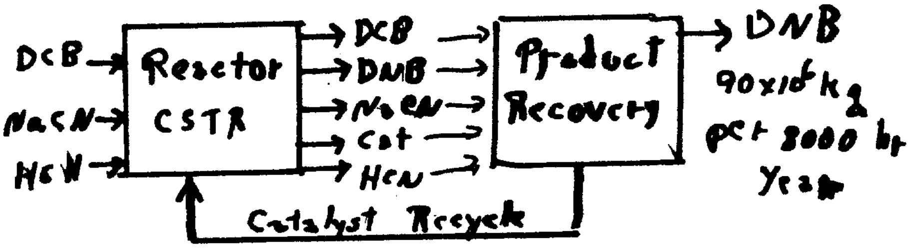
Answer

Ap C Pr:34

Pr. Ses. Prob. 17

$$\overset{DCB}{C_4H_6Cl_2} + 2NaCN \xrightarrow{Catalyst} \overset{DNB}{C_4H_6(CN)_2}$$

M.W. = 125 MW = 49 M.W. = 106



? = % return on extra investment for going from 80% to 85% conversion

Catalyst solution

	wt %
$Na_2Cu(CN)_2$ (M.W. = 138.6)	= 6.5
NaCN	= 17.3
H_2O	= 76.2

Density = 1.18×10^3 kg/m^3

0.038 kg of Cu (At. wt. = 63.6) needed per kg of DCB charge

Basis: 100 kg of cat. solution = 6.5 kg $Na_2Cu(CN)_2$

$$= \frac{6.5 \mid 63.6}{\mid 138.6} = 2.98 \text{ kg Cu}$$

$$= \frac{2.98}{0.038} = 78.42 \text{ kg DCB charge/100 kg of cat. solution}$$

So need $\frac{100}{78.42} = 1.275$ kg cat. solution per kg of DCB charge

Pr. Ses. Prob. 17 (continued)

Amount of DCB feed (No side losses)

for product rate of $\frac{90{,}000{,}000}{8000} = 11{,}250$ kg DNB/hr.

amount of DCB needed in feed if yield is 80%

$= \frac{11{,}250}{}\left|\frac{125}{106}\right|\frac{}{0.8} = 16{,}583$ kg DCB/hr

amount of DCB needed in feed if yield is 85%

$= \frac{11{,}250}{}\left|\frac{125}{106}\right|\frac{}{0.85} = 15{,}608$ kg DCB/hr

Total DCB feed stream is

$\frac{16{,}583}{1.16 \times 10^3} = 14.30$ m^3/hr at 80% conv.

$\frac{15{,}608}{1.16 \times 10^3} = 13.46$ m^3/hr at 85% conv.

Total Catalyst stream is

$(16{,}583)(1.275) = 21{,}143$ kg cat. str./hr at 80% conv.

$\times \frac{1}{1150} = 18.39$ m^3/hr

$(15{,}608)(1.295) = 19{,}900$ kg cat. str./hr at 85% conv.

$\times \frac{1}{1150} = 17.30$ m^3/hr.

NaCN stream is

$\frac{16{,}583}{}\left|\frac{(2)(49)}{125}\right|\frac{}{0.26} = 50{,}004$ kg/hr $\times \frac{1}{1130} = 44.25$ m^3/hr for 80% conv.

$\frac{15{,}608}{}\left|\frac{(2)(49)}{125}\right|\frac{}{0.26} = 47{,}064$ kg/hr $\times \frac{1}{1130} = 41.65$ m^3/hr for 85% conv.

Pr. Seg. Prob. 17 (Continued)

Total feed coming in (Ignoring HCN stream)

For 80% conversion

	Vol. as m^3/hr	Wt. as kg/hr	Value $/kg	Value $/hr
DCB Stream	14.30	16,583	0.62	10,281
Catalyst Stream	18.39	21,143	—	—
NaCN Stream	44.25	50,004	0.082	4,100
Total	76.94	87,730		$14,381

For 85% conversion

	Vol. as m^3/hr	Wt. as kg/hr	Value $/kg	Value $/hr
DCB Stream	13.46	15,608	0.62	9,677
Catalyst stream	17.30	19,900	-	-
NaCN stream	41.65	47,064	0.082	3,859
Total	72.41	82,572		$13,536

Thus: Raw material costs per year

For 80% conv. = 14,381 × 8000 = $115.0×$10^6$

for 85% conv. = 13,536 × 8000 = $108.3×$10^6$

Now get cost for reactors (CSTR)

V_R = vol. of reactor, m^3

X = fractional conv. of DCB as kg DCB in feed converted per kg of DCB feed

F = feed rate of DCB as kg DCB/hr

r = kg DCB conv. in reactor/(hr)(m^3 of reactor)

Pr. Seg. Prob. 17 (Continued)

Classic C.S.T.R. Equation No. 64 (4th ed.) in Chap. 16 is

$$V_R = F\frac{x}{r} \quad \text{based on} \quad 0 = F - F(1-x) + (-r)V_R$$

If rate is given as $\frac{\text{kg DCB conv.}}{(\text{kg DCB feed})(\text{hr})}$, change to

$\frac{\text{kg DCB conv.}}{(\text{m}^3 \text{ of reactor})(\text{hr})}$ by mult. by $\frac{\text{kg DCB feed}}{\text{m}^3 \text{ of Feed}}$.

$$r\,(\text{for } 80\%\text{ conv.}) = \frac{0.0169 \mid 60 \mid 16{,}583}{ \mid \mid 76.94} = 218.5\ \frac{\text{kg DCB}}{(\text{m}^3)(\text{hr})}$$

$$r\,(\text{for } 85\%\text{ conv.}) = \frac{0.0131 \mid 60 \mid 15{,}608}{ \mid \mid 72.41} = 169.4\ \frac{\text{kg DCB}}{(\text{m}^3)(\text{hr})}$$

$$V_R\,(\text{for } 80\%\text{ conv.}) = \frac{16{,}583 \mid 0.8}{\phantom{16{,}583} \mid 218.5} = 60.7\ \text{m}^3 \times 264\ \frac{\text{gal}}{\text{m}^3}$$

$= F\frac{x}{r}$

$= 16{,}037$ gal.

$$V_R\,(\text{for } 85\%\text{ conv.}) = \frac{15{,}608 \mid 0.85}{\phantom{15{,}608} \mid 169.4} = 78.3\ \text{m}^3 \times 264\ \frac{\text{gal}}{\text{m}^3}$$

$= 20{,}682$ gal.

Reactor cost for 80% conv. $= 260{,}000 + \frac{1.037}{9}(130{,}000)$

$= \$274{,}000 \times 4.5 = \$1{,}233{,}000.$

Reactor cost for 85% conv. $= 260{,}000 + \frac{3.682}{9}(130{,}000)$

$= \$342{,}000 \times 4.5 = \$1{,}539{,}300.$

Cost for Cat. Solution (80% conv.) $= 21{,}143 \times 2 \times 0.30$

$= \$12{,}685$

Cost for Cat. Solution (85% conv.) $= 19{,}900 \times 2 \times 0.30$

$= \$11{,}940$

Pr Ses. Prob. 17 (Continued)

Total F.C.I. (80% Conv.) $= 1{,}233{,}000 + 12{,}685$

$\overset{\$}{=} 1{,}245{,}685$

Total F.C.I. (85% Conv) $= 1{,}539{,}300 + 11{,}940$

$\overset{\$}{=} 1{,}551{,}240$

Total annual variable cost

For 80% conv. $= \underset{\text{Raw mtl.}}{115.0\times10^6} + \underset{\text{Depreciation}}{1{,}245{,}685 \times 0.08}$

$\overset{\$}{=} 115.1\times10^6$

for 85% conv. $= \underset{\text{Raw mtl.}}{108.3\times10^6} + \underset{\text{Depreciation}}{1{,}551{,}240 \times 0.08}$

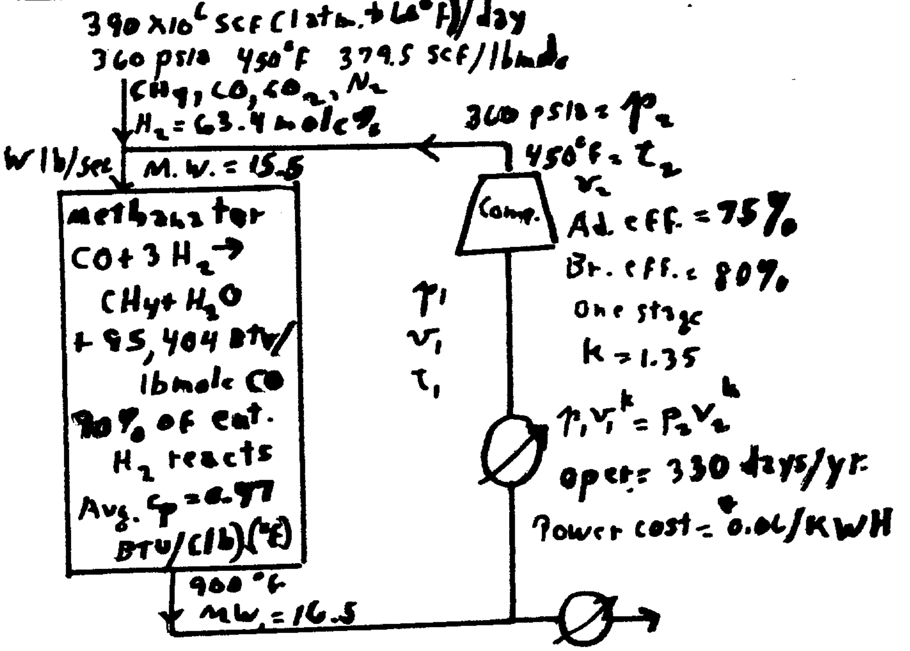
$\overset{\$}{=} 108.4\times10^6$

% return by going from 80% to 85% Conv

$$= \frac{(115.1 - 108.4)\times10^6}{1{,}551{,}240 - 1{,}245{,}685}(100) = \underline{\underline{22\%}}$$

Answer

Pr. Ses. Prob. 18



D of Methanator = 24.8 ft

L of Methanator = 12.4 + 3 = 15.4 ft

Theo. Hp (Eq 24-Ch. 14) (4th ed) $= \dfrac{3.03\times10^{-5}k}{k-1} P_1 q_{fm_1} \left[\left(\dfrac{P_2}{P_1}\right)^{\frac{k-1}{k}} - 1\right]$

$$\text{Act. Hp.} = \frac{\text{Theo. Hp}}{\underset{\text{Br. eff.}}{0.80}\;\Big|\;\underset{\text{Ad. eff.}}{0.75}}$$

ΔP (P. 5-53 of Perry's 6th) $= \dfrac{2 f G^2 L (1-\epsilon)^{3-n}}{D_p g_c \rho \phi_s^{3-n} \epsilon^3}$ lb/ft²

$f = 1$ $G = \text{lb}/(\text{sec})(\text{ft})^2$ $L = 15.4$ ft $\epsilon = 0.4$ $n = 2$

$D_p = 0.0238$ ft $g_c = 32.17\ (\text{lb})(\text{ft})/(\text{lb})(\text{sec})^2$

$\rho = 0.464\ \text{lb}/\text{ft}^3$ $\phi_s = 1.0$ $i = 0.11$ $n = 20$

P.V. (Eq 24-Ch. 7) $= R\dfrac{(1+i)^n - 1}{i(1+i)^n}$ FCI = $5,000,000

Pr. Sec. Prob. 18 (Continued)

? ⓐ = Power cost/yr for recycle-gas compression

? ⓑ = Net Present Value = F.C.I. + P.V. at 11% annual int. and 20-year life.

SOLUTION

Determine flow through reactor:

$$\text{Entering gas} = \frac{390\times10^6\ \text{scf}}{24\times3600\ \text{sec}}\bigg|\frac{\text{lb mole}}{379.5\ \text{scf}} = 11.9\ \frac{\text{lb mole}}{\text{sec}}$$

$11.9 \times 15.5 = 184$ lb/sec

$$\text{Lb Moles } H_2 \text{ reacting/sec} = (11.9)(0.634)(0.9) = 6.79\ \text{lb/sec}$$

$$\text{Heat released/sec} = \frac{6.79}{}\bigg|\frac{95,404}{3} = 215,936\ \frac{\text{BTU}}{\text{sec}}$$

Let W (lb/sec) = required flowrate to have temp. go from 450°F to 900°F. (No heat loss)

$$215,936 = W(0.47)(900-450)$$

W = 1021 lb/sec enter at 360 psia and 450°F

$$\text{Flow rate through compressor} = 1021 - 184 = 837\ \frac{\text{lb}}{\text{sec}}$$

$$G\ (\text{for } \Delta P \text{ Eq.}) = \frac{837\ \text{lb}}{\text{sec}}\bigg|\frac{4}{\pi(24.8^2)\ \text{ft}^2} = 1.734\ \frac{\text{lb}}{(\text{sec})(\text{ft})^2}$$

$$\Delta P = \frac{2}{0.0238}\bigg|\frac{1}{32.17}\bigg|\frac{(1.734)^2}{0.464}\bigg|\frac{15.4}{(1)^1}\bigg|\frac{(0.6)^1}{(0.4)^3} = 2444\ \frac{\text{lb}}{\text{ft}^2}$$

$$= \frac{2444}{144} = 17.0\ \text{psia}$$

$P_2 = 360$ psia $\qquad P_1 = 360 - 17 = 343$ psia

$$v_2\ (\text{at } 360\ \text{psia} + 450°\text{F}) = \frac{379.5}{}\bigg|\frac{910}{520}\bigg|\frac{14.7}{360}\bigg|\frac{}{16.5} = 1.643\ \frac{\text{ft}^3}{\text{lb}}$$

Pr. Ses. Prob. 18 (Continued)

$$v_1 = v_2\left(\frac{P_2}{P_1}\right)^{1/k} = 1.643\left(\frac{360}{343}\right)^{\frac{1}{1.35}} = 1.703\ \frac{ft^3}{lb}$$

$$= \frac{1.703\ ft^3}{lb}\left|\frac{837\ lb}{sec}\right|\frac{60\ sec}{min} = \underset{\underset{q_{fm}}{\parallel}}{85{,}522}\ \frac{ft^3}{min}$$

$$\text{Theo. Hp.} = \frac{3.03\times10^{-5}}{}\left|\frac{1.35}{0.35}\right|\frac{343\times144}{}\left|\frac{85{,}522}{}\right.$$

$$\times\left[\left(\frac{360}{343}\right)^{\frac{0.35}{1.35}} - 1\right] = 6228\ Hp$$

$$\text{Act. Hp} = \frac{6228}{0.80}\left|\frac{}{0.75}\right. = 10{,}380\ Hp$$

1 Hp = 0.746 kW. Plant operates 330 days/yr.

ⓐ Power cost/yr $= \frac{10{,}380\ Hp}{}\left|\frac{0.746\ kW}{Hp}\right|\frac{330\times24\ hr}{yr}\left|\frac{\$0.06}{kWhr}\right.$

$= \$3{,}680{,}000/yr$

Answer ⓐ

ⓑ Present Worth of power cost (20 years at 11%)

$$= 3{,}680{,}000\ \frac{(1+0.11)^{20}-1}{(0.11)(1+0.11)^{20}}$$

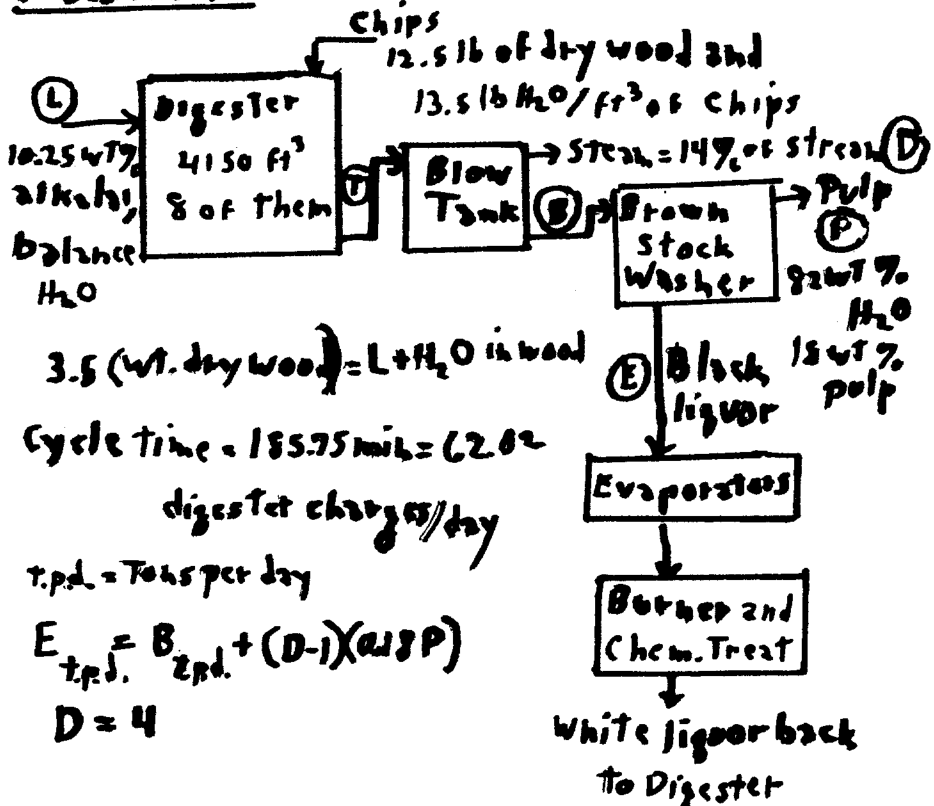
$= \$29{,}300{,}000$

Net Present Worth $= 29{,}300{,}000 + \overbrace{5{,}000{,}000}^{\text{F.C.I.}}$

$= \$34{,}300{,}000$

Answer ⓑ

Pr. Ses. Prob 19 Ap CPr=42



3.5 (wt. dry wood) = L + H_2O in wood

Cycle time = 185.75 min = 62.02 digester charges/day

t.p.d. = Tons per day

$$E_{t.p.d.} = B_{t.p.d.} + (D-1)(0.18P)$$

$D = 4$

Total charge of dry wood = $\dfrac{4150 \mid 12.5 \mid 62.02}{ \mid \mid 2000}$

= 1608.6 t.p.d. dry wood

Total charge of H_2O in wood = $\dfrac{4150 \mid 13.5 \mid 62.02}{ \mid \mid 2000}$

= $\underline{1737.3}$ t.p.d. H_2O in wood

Total wt. of wood charge = 3345.9 t.p.d. total wood

Ⓛ = wt. of liquid to Digester = 3.5 (1608.6) - 1737.3

= 3892.8 t.p.d. white liquor

Pr. Ses. Prob. 19 (Continued)

WT. of alkalai in Ⓛ = (0.1025)(3892.8) = 399 t.p.d.

WT. of H_2O in Ⓛ = (0.8975)(3892.8) = 3494 t.p.d.

WT. of stream T to Blow Tank = 3892.8 + 3345.9
= 7238.7 t.p.d.

WT. of H_2O off in Blow Tank = (7238.7)(0.14) = 1013.4 t.p.d.

H_2O in stream B to B.S. Washer = 1737.3 wood
+ 3494 in Ⓛ − 1013.4 in Blow
= 4217 t.p.d.

Alkalai in stream B to B.S. Washer = 399 t.p.d.

Pulp in stream T and in stream B = (1608.6)(0.45)
= 724 t.p.d. unbleached pulp.

Dissolved solids wood in streams T and B =
(1608.6)(0.55) = 885 t.p.d.

Ignore the 3 t.p.d. loss of turpentine.

Total wt. of stream B = 4217.9 H_2O + 399 alk.
+ 885 dis. wood + 724 pulp = 6226 t.p.d.
(above includes 399 + 885 = 1284 t.p.d dis. solids)

Check: Ⓑ = 3345.9 + 3892.8 − 1013.4 = 6225.3 t.p.d
Check is OK

Now procede to Brown Stock Washer:

Pr. Sec. Prob. 19 (Continued)

B.S. Washer $E_{t.p.d}$ to Evap. = $B_{t.p.d}$ to B.S.W. + $(D-1)(0.18P)$

$B_{t.p.d.}$ To B.S.W. = 6226 t.p.d.

$$D = \frac{(\text{Ton water wash} + \text{Ton water in prod})}{\text{ton dry pulp}} = 4$$

P = Ton wet pulp/day

Dry unbleached pulp = 724 t.p.d

$$P = \frac{724}{0.18} = 4022 \text{ t.p.d}$$

$E_{t.p.d}$ to Evap. = $6226 + (4-1)(724) = \underline{\underline{8398 \text{ t.p.d}}}$

Answer a

WT. of dissolved solids in stream B and in stream E = 399 alk + 885 diss. wood = 1284 t.p.d

WT. % of dissolved solids in stream E = $\frac{1284}{8398}(100)$

$$= \underline{\underline{15.3 \text{ WT.\%}}}$$

Answer a

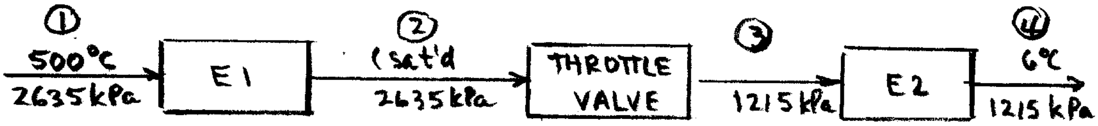
Rate of prod. of Final, bleached, dry pulp

$= (0.9)(724) = \underline{\underline{652 \text{ t.p.d.}}}$

Answer b

Pr. Ses. Prob. 21

This problem is similar to one outlined by Seider, Seader and Lewin in Process Design Principles. The authors suggest using Aspen Plus to complete the material and energy balance. The flow sheet for the process is



Using the Aspen Plus subroutine HEATER to model the two heat exchangers and VALVE to simulate the action of the throttling valve the following results are obtained (these calculations use the SRK equation of state to compute the thermodynamic properties):

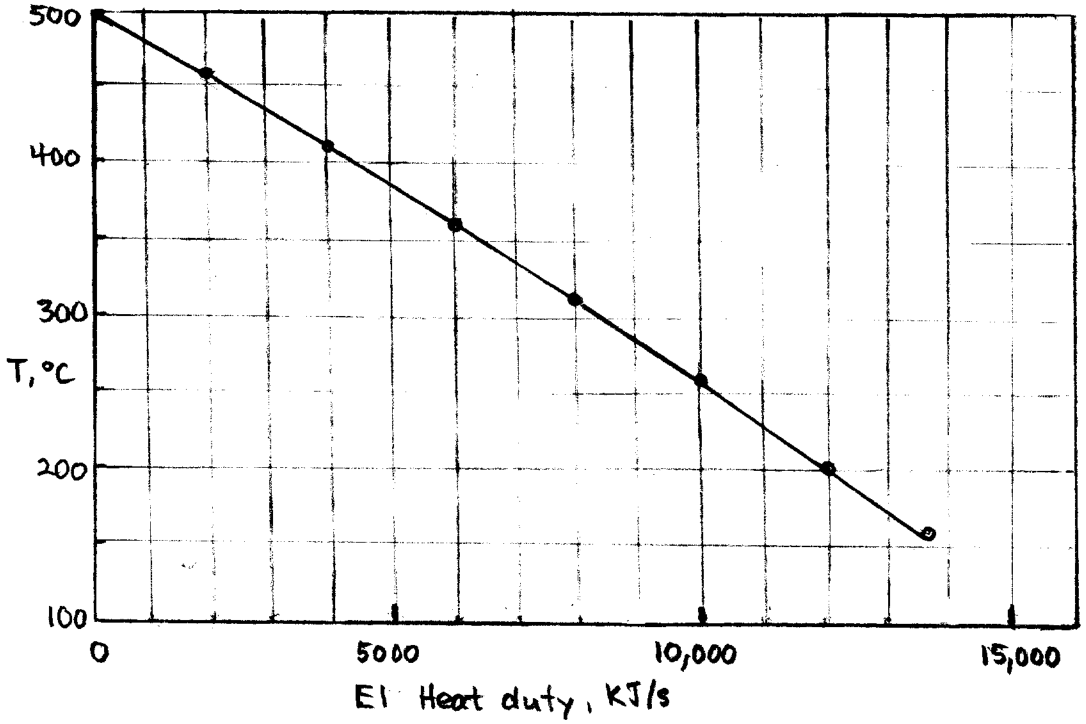
Stream	1	2	3	4
Temp., °C	500	158	140	6
Press., kPa	2635	2635	1215	1215
Mass flow, kg/s	33.37	33.37	33.37	33.37
Mol flow, kg mol/s	0.5388	0.5388	0.5388	0.5388
Enthalpy cont., kJ/s	−13,808	−27,516	−27,516	−43,045
Heat removed	E1 = 13,708 kJ/s		E_2 = 15,529 kJ/s	

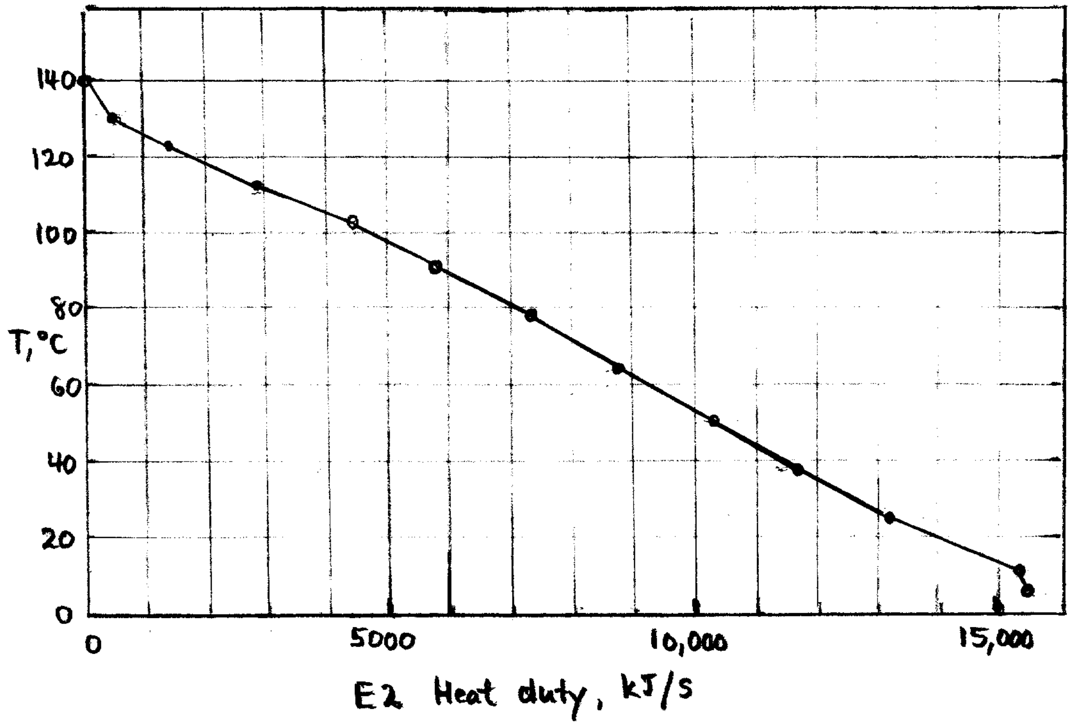
Note that the saturation temperature of the mixture at 2635 kPa is 158°C. Adiabatic expansion of this exit stream from E1 reduces the temperature to 140°C at a pressure of 1215 kPa. Thus, the exit stream upon adiabatic expansion is slightly superheated. This superheat must be removed in the entrance section of E2 as shown on the temperature diagram for this heat exchanger. Since the bubble point of the mixture is 10°C at 1215 kPa another small change occurs at this temperature to indicate subcooling of the mixture.

[illegible]

Pr. Ses. Prob. 21 (cont.)

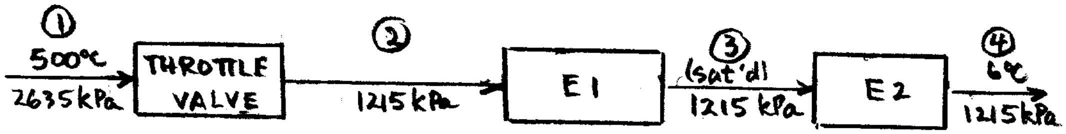
The temperature profiles in E1 and E2 are shown below:





Pr. Ses. Prob. 21 (cont.)

There are numerous arrangements that could be suggested to achieve the same product end point. One suggested arrangement below, involves reducing the pressure before entering the first exchanger. Thus,



In this alternative arrangement the inlet temperature to E1 will be more than 20°C below that for the initial arrangement. With a lower temperature driving force the area of E1 will be larger than for the initial arrangement. However, because of the lower pressure, the required thickness of the tubes in E1 may be somewhat less. The effect on E2 with this arrangement will be minimal. (Of course, E1 and E2 could be combined into one larger area exchanger.) Comparison of the two arrangements will need to be verified with an economic analysis. Without this analysis, the initial arrangement is preferred over the alternative arrangement.

Ap C Pr: 48

Pr. Session Problem 22

From an analysis of the cooling process, the hot flue gas enters at 149°C and a 37.7°C dew point (equivalent to a corresponding humidity of about 0.042 kg water/kg dry air with an enthalpy of 276 kJ/kg and an adiabatic saturation temperature of 50°C). During cooling the hot flue will proceed along the adiabatic saturation line until condensation occurs at 50°C. The cooling process then follows the saturation line down to the exit temperature of 32°C. At that point the water content is 0.031 kg water/kg dry air with an enthalpy of 116 kJ/kg. Thus, this cooling is one of dehumidification in which the mass and energy flows are in the same direction.

Calculate the total heat load

$$\dot{q}_T = \dot{m}(h_{in} - h_{out}) = (2.52\,kg/s)(276 - 116)kJ/kg$$
$$= 403\,kW$$

Latent heat total of the total heat load is the amount of water that must be condensed

$$\dot{q}_{cond} = (2.52)(0.042 - 0.031)(2402) = 66.5\,kW$$

Thus the sensible heat load is 336.5 kW or 83.5% of the total heat load.

Temperature rise of the water is 49 - 26.8 = 22.2°C, the water requirement is

$$\dot{m}_{H_2O} = 403/(4.186)(22.2) = 4.34\,kg/s$$

Assuming essentially linear temperature changes throughout the cooler, the LMTD = 32°C.

For this system the liquid-to-gas flow rate ratio is

4.34/2.52 = 1.72

For flow ratios of 1.5 to 3 in direct gas-liquid heat exchange, a packed tower is generally selected to effect the heat transfer and maintain a reasonable pressure drop. See Table 3-18 of D.R. Woods, *Process Design and Engineering Practice*, p. 3-73, Prentice Hall (1995).

To size the tower assume a gas velocity of 1 m/s. Since the density of the flue gas changes from about 0.8 kg/m^3 at the inlet to about 1.1 kg/m^3 at the exit, assume an average of about 1 kg/m^3.

Cross-sectional area required with these assumptions is

$$A = \dot{m}/\rho V = 2.52/(1)(1) = 2.52\,m^2$$

Pr. Ses. Prob. 22 (cont.)

ApC Pr: 49

or

$$D = (A/0.785)^{1/2} = (2.52/0.785)^{1/2} = \underline{1.79\,m}$$

ANSWER

The gas and liquid loadings for the cooler are

$$\text{gas}: 2.52\,kg/s\ /\ 2.52\,m^2 = 1\,kg/s\cdot m^2$$

$$\text{liquid}: 4.34/2.52 = 1.72\,kg/s\cdot m^2$$

The overall volumetric heat transfer coefficient is obtained by combining a gas and liquid coefficient. Woods provides a value of 3 kW/m^2•°C for a typical volumetric overall heat transfer coefficient for a packed tower under the above conditions (see Table 3-18). For greater accuracy values for the individual gas and liquid volumetric heat transfer coefficient may be obtained using the relations

$$h_V = 3(\dot{m}_V/A)(\dot{m}_L/A)^{0.2}$$

$$h_L = 42(\dot{m}_V/A)^{0.7}(\dot{m}_L/A)^{0.5}$$

These values can be used to obtain the overall volumetric heat transfer coefficient.

Using the value of 3 kW/m^2•°C for U, the volume can be determined from

$$\text{volume} = \dot{q}_T / \upsilon \Delta T_{\ln m}$$

$$= (403)/(3)(32) = 4.2\,m^3$$

$$L = 4.2/2.52 = \underline{1.67\,m}$$

ANSWER

This provides essentially a 1:1 ratio for the L/D ratio.

The pressure drop in the cooler is obtained directly from using Fig. 2-17 provided by Woods. For Pall rings this would be approximately 0.35kPa/m. Thus, the total pressure drop in the cooler with a length of 1.67 + 0.2 = 1.87 m. (the 0.2 m is added to provide sufficient clearance in the packed tower) is

$$\Delta p = (1.87)(0.35) = 0.65\,kPa$$

This is within the 5 kPa pressure drop allowed.

ApC Pr: 50

Pr. Ses. Prob. 22 (cont.)

The approximate cost of a carbon steel tower with these dimensions (requires an extrapolation of the cost curve of Fig. 15-11) without any packing is $12,000 on Jan. 2002.

ANSWER

Note: This problem was originally developed by H. Sawitstowski and W. Smith, *Mass Transfer Process Calculations*, p. 456, Interscience Publishers, New York (1963). In these calculations, they arrived at a total heat load of 405 kW, a calculated volumetric heat transfer coefficient of 2.4 $kW/m^2 \cdot {}^\circ C$, a diameter of 1.83 m and a height of 1.98 m.

Pr. Session Problem 23

ApC Pr :51

To provide a basis for devising alternative schemes for creating the streams required, list the components in increasing order of boiling temperature and note the various characteristics of the components. Such an analysis shows a very wide range in boiling temperatures, from −196 to 98°C. All components have similar solubility parameters, and are non-polar. Hydrogen sulfide is the only component that is fairly corrosive. Except for CO_2 and N_2 most of the components have similar flammability ratings.

To obtain the specific streams required, use a heuristic approach to suggest possible separation schemes. Note that some of the heuristics will provide conflicts that will require further analysis before a decision on the separation scheme is proposed. (See Appendix F).

1. Remove the most hazardous, corrosive or unstable component first. In this case this would be hydrogen sulfide.
2. Save the most difficult separation until last. The easiest to separate are those that exhibit the greatest difference in molar mass or between boiling temperatures, or between freezing temperatures. Several of the molar masses are within the 30 to 45 range. The largest difference in temperature occurs between the N_2 and C_1 and the rest of the components.
3. Separate the most plentiful components early in the process. Here CO_2 should be removed early because it sublimes that can result in blocked lines and because if forms an azeotrope with C_2.
4. Try to avoid adding an agent to achieve the separation. There appears to be a reasonable difference in boiling temperature except for the most plentiful substance, CO_2, which sublimes. According to heuristic#3, CO_2 should be separated early. Thus, distillation might be useful for all separations except the separation of CO_2.
5. Leave high specific recoveries until last.

6. Avoid extremes in operating conditions. Atmospheric pressure operation is acceptable for iC_3 and higher. For the other species we should require a higher pressure and this is available with the inlet pressure of 2 MPa.
7. When considering separation options, recognize safety, controllability, and materials of construction integration as well as energy conservation.

Note that some of the heuristics offer some conflicts. For example, heuristic#1 recommends removal of the H_2S first; heuristic#3 says separate the CO_2 early; heuristic#2 says separate C_1 from C_2 early, or separate CO_2 from C_2 last. However, removing the corrosive component first makes the most sense. This will probably involve absorbing H_2S preferentially in an absorbing liquid. Such H_2S removal is used in every one of the four proposed options available to the design engineer. In each separation option, assume the H_2S is removed first.

Option 1:

Separate the C_1 cut first: use distillation without reflux and without the temperatures that might cause CO_2 freezing. Do not use a solvent in the first column. This allows more CO_2 to go overhead and the latter is lost by scrubbing the methane to free it from the CO_2. The bottoms is sent to an extractive distillation column. Figure 23a shows the variation. This option is shown in Fig. P-23a.

Option 2:

Concentrate the CO_2 using membrane technology. Distill the retentate to remove the methane. Take the bottoms from the demethanizer and perform an azeotropic distillation to separate CO_2 from C_2 using a C_4 extraction fluid. Treat the azeotrope overhead with a membrane distillation hybrid to remove the CO_2. Now combine the various streams as shown in Fig. P-23b.

Option 3:

Separate the C_1 cut first using simple distillation with some C_4 solvent added near the top of the distillation column to prevent the CO_2 from freezing. This separates the C_1 from the rest of the feed. The bottoms goes to an extractive distillation column with the C_4 must used as the solvent to separate the CO_2 from the C_2. This option is shown in Fig. D-23c.

Option 4:

Partially separate the C_2 / CO_2 with an absorption process using a C_4 stream as the absorbent. With the overhead, a CO_2 / C_1 separation is pursued. The bottoms stream is separated in an absorption column using a C_4 stream to serve as the absorption stream. This configuration is outlined in Fig. P-23d.

Additional information on process design configurations is given by D.R. Woods, *Process Design and Engineering Practice: Selecting and Sizing Homogenous Separations,* McMaster University, Hamilton, ON, Canada (1993).

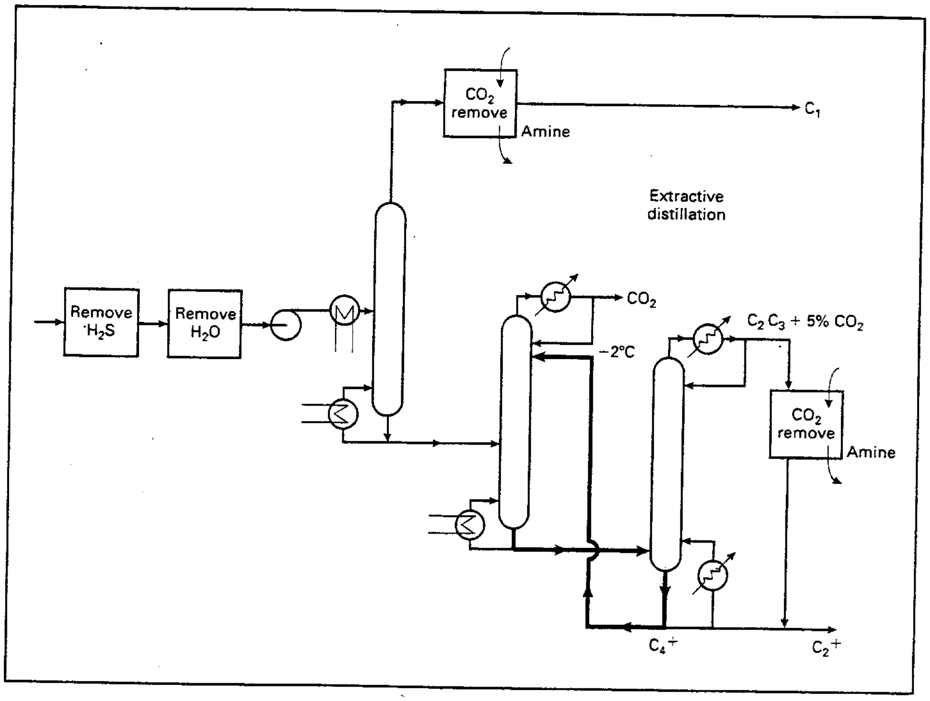


Figure P23a.

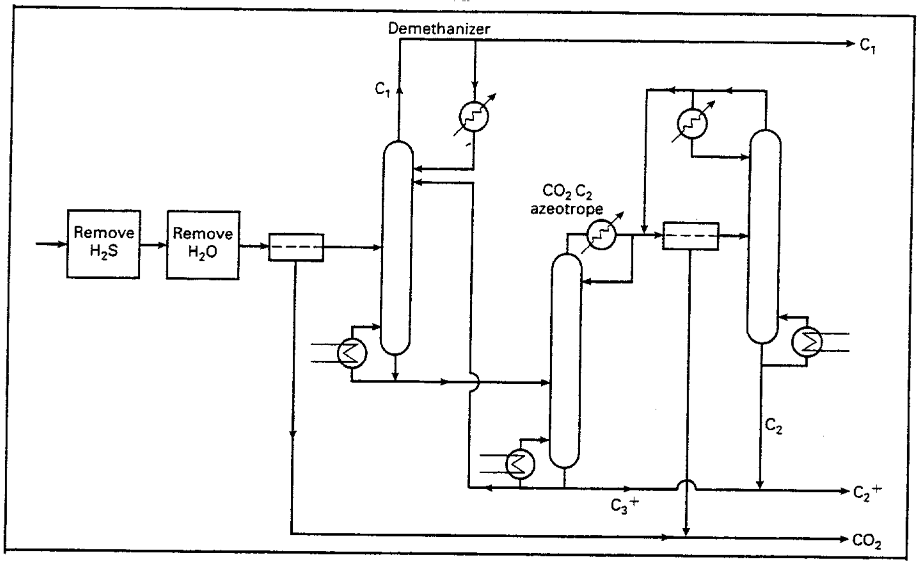


Figure P23b.

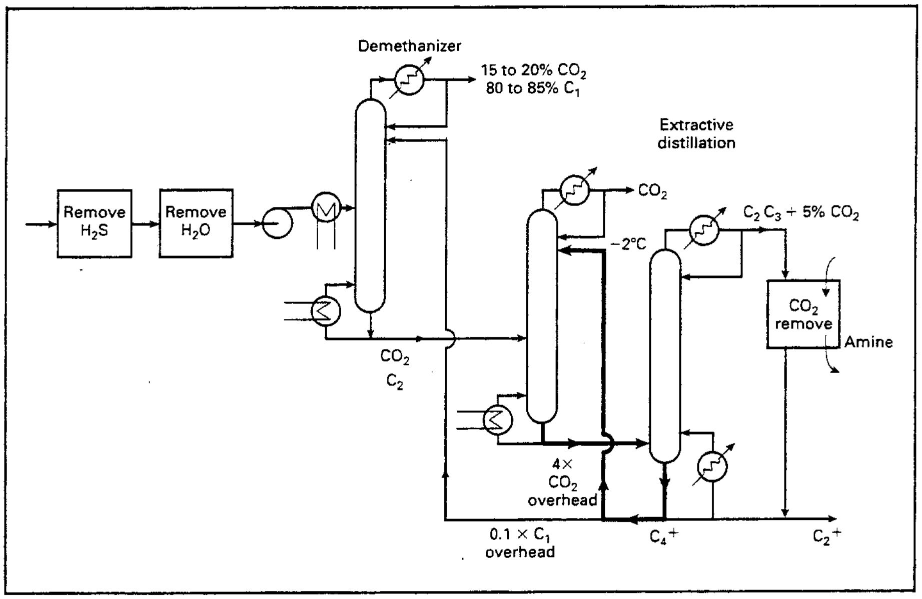


Figure P23c.

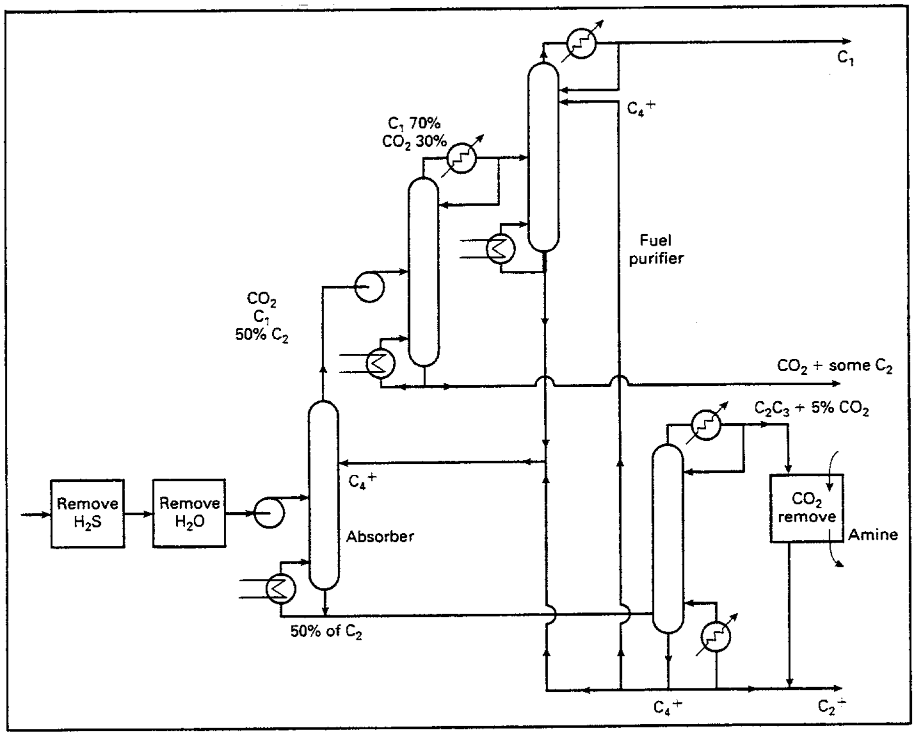


Figure P23d.

ApC Pr:56

Pr. Session Problem 24

(a) For the base case, the superficial permeate velocity is

$$v = \frac{40L}{m^2 h}\;\frac{h}{3600\,s}\;\frac{10^{-3}m^3}{L} = 1.11\times10^{-5}\,m/s$$

Assume essentially a water solution with a kinematic viscosity of $1\times10^{-6}\,m/s$, $D_i = 5\times10^{-11}\,m^2/s$

$$\mathrm{Sc} = \frac{\mu}{\rho D_i} = \frac{1\times10^{-6}}{5\times10^{-11}} = 20,000$$

From T.K. Sherwood, R.L. Pigford, and C.R. Wilkie, *Mass Transfer*, p. 169, McGraw-Hill, New York (1975) a relationship between Sc, Re, and Sh is given by

$$\begin{aligned}\mathrm{Sh} &= 0.096\,\mathrm{Re}^{0.913}\,\mathrm{Sc}^{0.346}\\ &= (0.0096)(25,000)^{0.913}(20,000)^{0.346}\\ &= 3060\end{aligned}$$

The mass transfer coefficient k_c based on concentration is given by

$$k_c = \frac{\mathrm{Sh}D_i}{D} = \frac{(3060)(5\times10^{-11})}{0.015} = 1.02\times10^{-5}\,m/s$$

The rejection rate R in ultrafiltration is related to the equivalent partition function K by the relation

$$\frac{1-R}{R} = \left(\frac{K}{1-K}\right)\exp\left(\frac{v}{k_c}\right)$$

Foro this relation as v/k_c approaches zero, the rejection approaches 1-K and is designated as R_{max}. Substituting in the previous equation

$$\frac{1-0.75}{0.75} = \frac{\mathrm{K}}{1-\mathrm{K}}\exp\frac{1.11\times10^{-5}}{1.02\times10^{-5}}$$

$$\mathrm{K} = 0.112/1.112 = 0.101$$

If the flux is reduced to $20\mathrm{L}/m^2 \cdot h$ (half of the original flux)

$$\mathrm{v} = 1.11\times10^{-5}/2 = 0.555\times10^{-5}\,m/s$$

$$\frac{1-\mathrm{R}}{\mathrm{R}} = \frac{0.101}{1-0.101}\exp\frac{0.555\times10^{-5}}{1.02\times10^{-5}}$$

Solving for R gives

$$\mathrm{R} = 0.84$$

$\mathrm{R}_{\mathrm{macx}}$ is obtained by allowing the flux to decrease to zero. Then

$$\mathrm{R}_{\max} = 1 - \mathrm{K} = 1 - 0.101 = \underline{0.90}$$

Answer (a)

(b) The diffusivity of large molecules varies with the $-1/3$ power of the size and the mass transfer coefficient k varies with $\mathrm{D}_i^{-0.65}$ or $\mathrm{M}^{-0.22}$.

Predict a new partition function K_2 from K_1 where

$$K_1 = 0.101 = (1-\lambda_1)^2$$

where λ is the ratio of molecular size to pore size. For the case in part (a), $\lambda_1 = 0.682 = D_1/D_{\mathrm{pore}}$

$$D_2 \cong D_1\left(\frac{10,000}{30,000}\right)^{1/3} = 0.693\,D_1$$

$$\lambda_2 = (\lambda_1/D_1)D_2 \qquad \lambda_2 = (0.682)(0.693) = 0.473$$

$$K_2 = (1-0.473)^2 = 0.278$$

From above

$$\mathrm{k}_{c,2} = \mathrm{k}_{c,1}\left(\frac{10,000}{30,000}\right)^{-0.22} = (1.02\times10^{-5})(1.27)$$

$$= 1.30\times10^{-5}\,m/s$$

At a $\mathrm{v} = 1.11\times10^{-5}\,m/s$

Pr. Ses. Prob. 24 (cont.)

$$\frac{1-R_2}{R_2} = \left(\frac{0.278}{1-0.278}\right)\exp\frac{1.11\times10^{-5}}{1.30\times10^{-5}} = 0.904$$

$$R_2 \cong \underline{0.525}$$

Answer

(c) Often the diffusion effect can be considerable if the selective layer thickness is very thin, i.e., between 0.1 to 0.2 μm as in this case. The basic equation for solute flux in the selective layer is similar to that for mass transfer in the boundary layer, but the diffusion term adds to the convective term.

$$vc_2 = vc - D_e\frac{dc}{dy}$$

where the effective diffusivity D_e is given by $D_{pore}\varepsilon/\tau$. Here D_{pore} is the diffusivity in the pores, ε the porosity fraction, and τ the tortuosity factor. Assuming the same parttion function at both boundaries of the selective layer, the differential equation can be integrated to

$$\frac{c_2}{c_s} = \frac{K\exp(vL/D_e)}{K-1+\exp(vL/D_e)} \qquad \text{L=thickness of the selective layer}$$

For a $D_{pore} = 1\times10^{-11} m^2/s,\ \varepsilon = 0.5,\ \tau = 2,\ M = 30{,}000, \text{and } D_i = 5\times10^{-11} m^2/s$

$$D_e = (1\times10^{-11})(0.5)/2 = 2.5\times10^{-12} m^2/s$$

$$\frac{vL}{D_e} = \frac{(0555\times10^{-5})(2\times10^{-7})}{2.5\times10^{-12}} = 0.444$$

Thus $$\frac{c_2}{c_3} = \frac{(0.101)\exp(0.444)}{(0.101-1+\exp(0.444))} = 0.238$$

Molecular diffusion has a significant effect on the rejection since without it

$$\frac{c_2}{c_3} = 0.101$$

Thus, the permeate concentration would be more than twice as high, indicating that the partition coefficient would be lower than that estimated in part (a).

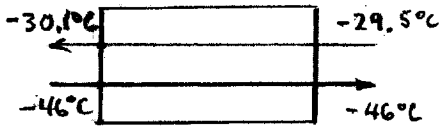
tion requirement. E603 and E604, as well as E608 and E609 can be eliminated because of the large heat exchange loads that could not be provided by the remaining heat exchangers. For example, E612 could provide less than 10% of the heat load required in E609 and necessitate two additional heat exchangers. The heal load available in E612 would most likely be better utilized in another section of the ethylene plant. E610 can also be eliminated because of the requirement of a cooling medium below 35°C.

This elimination of the above heat exchangers from those shown on Fig. 3-13 leaves exchangers E605, E606, E607, and E611 to be used in the heat integration. Using the procedure described in Chap. 9, a pinch technology plot can be developed as shown on the attached plot. This graphical summary of enthalpy rate change as a function of temperature indicates that the stream arrangement for E605 is adequate and should not be changed. The heat load from E606 can be partially used to heat up the stream in E611. This will reduce the cold water requirement in E606 and the low pressure steam requirement in E611. It will necessitate the addition of two heat exchangers to take care of the additional heat loads specified. There are no additional heat exchange opportunities to accept the heat load of E607. Just as with E612, part of the heat load could have been provided to E609. It would be better to find a more suitable heat exchange somewhere else.

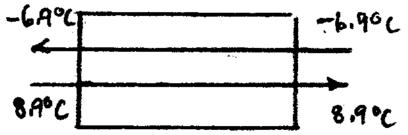
Pr. Ses. Prob. 25 (cont.)

The conditions for the 12 heat exchangers listed in Fig. 3-13 are listed below:

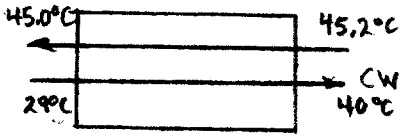
E601



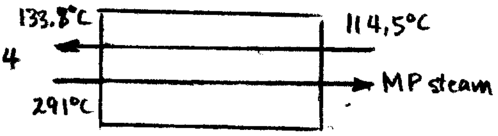
E602



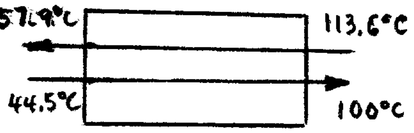
E603



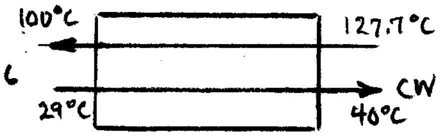
E604



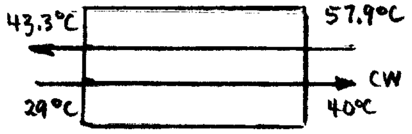
E605



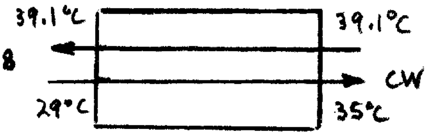
E606



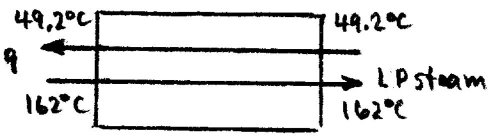
E607



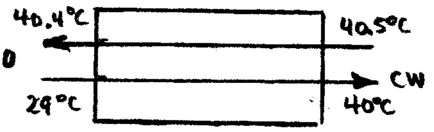
E608



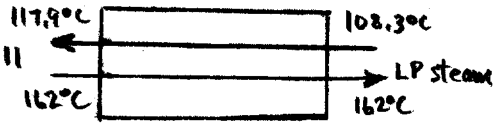
E609



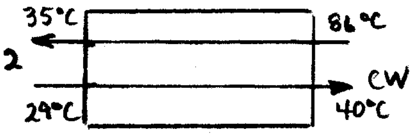
E610



E611

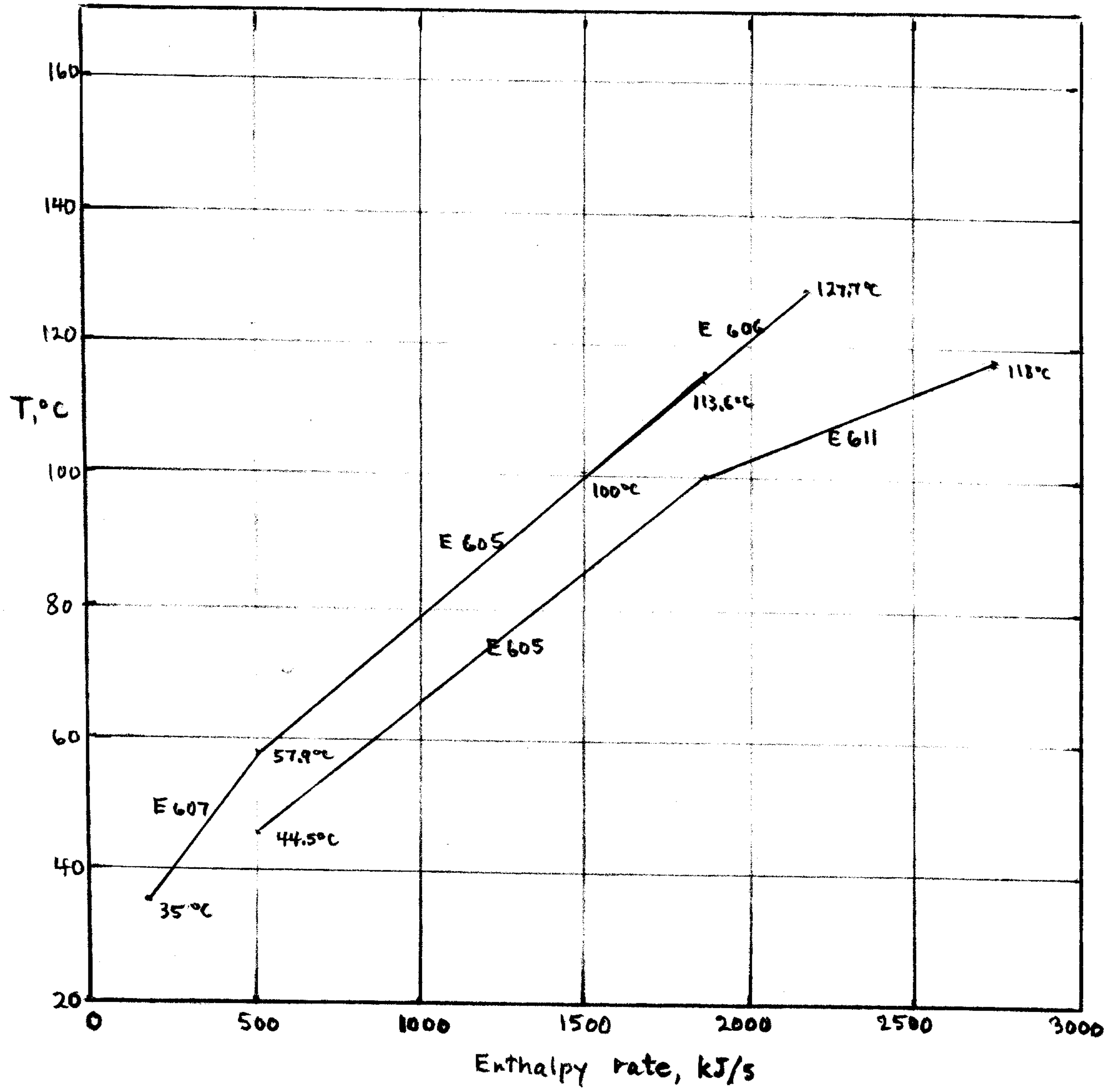


E612



A quick review of the temperature and load requirements for some of the heat exchangers makes it possible to eliminate several of the heat exchangers for the heat integration. E601 and E602 can be eliminated because of the C_3 refrigera-

Pr. Ses. Prob. 25 (cont.)



Pr. Ses. Prob. 26

ApC Pr: 62

Recognize that NPW can be expressed in terms of y_2, the $GeCl_4$ mol fraction in the scrubber effluent as,

$$NPW = (\$100/\text{kg})(0.633\ \text{kg mol/min.})(214.39\ \text{kg/kg mol})(60 \times 24 \times 335\ \text{operating min/y})(y_{2,present} - y_{2,new})(1-0.35) - 1.41\left[(18{,}590) + (7{,}160)(0.40)\left(\ln \frac{0.001475}{y_{2,new}}\right)\right]$$

To maximize NPW w.r.t. $y_{2,new}$, differentiate and set $= 0$. Recognize that although $y_{2,present}$ is not stated, it is a constant, and therefore drops out upon differentiation.

$$\frac{d(NPW)}{d(y_{2,new})} = (\$4.25 \times 10^{10})\left(\cancelto{0}{\frac{d(y_{2,present})}{d(y_{2,new})}} - \cancelto{-1}{\frac{d(y_{2,new})}{d(y_{2,new})}}\right)$$

$$- 4038 \frac{d}{dy_{2,new}}\left(\cancelto{0}{\ln(0.001475)} - \cancelto{-1/y_{2,new}}{\ln y_{2,new}}\right)$$

$$0 = -4.25 \times 10^{10} + 4038/(y_{2,new})$$

$$y_{2,new} = \underline{\underline{9.5 \times 10^{-8}}}$$

ANS.

Note, this gives $z = 3.86$ m