29 Engineering Acoustic Band Gaps in Phononic Crystals

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1. INTRODUCTION

In recent years, the study of acoustic and elastic waves propagation in periodic materials, known as "phononic crystals", has received increasing amount of attention. Analogous to photonic crystals, a large and robust band gap is essential to all applications of phononic crystals. In this work, a perturbative approach is applied to phononic crystals and two main results are obtained. Firstly, we show that a perturbation analysis can provide us an efficient method to enlarge an existing acoustic band gap. Secondly, by extending the perturbative analysis to disordered phononic crystals, we can quantitatively estimate the effect of the disorder on the size of an acoustic band gap. Due to the difference in the mathematical structures between Maxwell equations in a photonic crystal and acoustic wave equation in a phononic crystal, we find that it is much more efficient to enlarge an acoustic band gap than a photonic band gap. Numerical simulations using the Multiple Scattering Method verify all the conclusions above.

2. ENGINEERING ACOUSTIC BAND GAPS

An acoustic Bloch state with an eigenfrequency $\omega_{n\mathbf{k}}$ and an eigenfield $p_{n\mathbf{k}}(\mathbf{r})$ in a phononic crystal satisfy the following acoustic pressure wave equation,

$$-\frac{\omega_{n\mathbf{k}}^2 p_{n\mathbf{k}}(\mathbf{r})}{\rho(\mathbf{r})\, c_l(\mathbf{r})^2} = \nabla\cdot\left(\frac{1}{\rho(\mathbf{r})}\nabla p_{n\mathbf{k}}(\mathbf{r})\right). \tag{1}$$

Here $\rho(\mathbf{r})$ and $c_l(\mathbf{r})$ are the periodic mass density function and longitudinal wave speed function of the phononic crystal, respectively. If we alter the microstructure by two small periodic functions, $\delta\rho$ and δc_l, the new eigenfrequency, $\varpi_{n\mathbf{k}}$, can be estimated from a perturbative analysis on Eq. (1), which yields

$$\left(\frac{\varpi_{n\mathbf{k}}}{\omega_{n\mathbf{k}}}\right)^2 - 1 \approx \frac{\int \delta\left(\frac{1}{\rho}\right)\left|\nabla p_{n\mathbf{k}}\right|^2 d\mathbf{r}}{\omega_{n\mathbf{k}}^2 \int \frac{\left|p_{n\mathbf{k}}\right|^2}{\rho c_l^2} d\mathbf{r}} - \frac{\int \delta\left(\frac{1}{\rho c_l^2}\right)\left|p_{n\mathbf{k}}\right|^2 d\mathbf{r}}{\int \frac{\left|p_{n\mathbf{k}}\right|^2}{\rho c_l^2} d\mathbf{r}}. \qquad (2)$$

Here the integrations are taken over a unit cell. By applying Eq. (2) to the band edge states, we find an efficient way to enlarge or reduce an existing band gap. A similar engineering method (X. Zhang *et al.*, 2000) for photonic crystals has been developed before, but here exists an important difference. In acoustic case, there are two terms of opposite signs on the RHS of Eq. (2), thus the eigenfrequencies at upper and lower band edges can be shifted upward and downward, respectively, for a simple alteration in the microstructure. This is impossible in the photonic case, since there is only one term on the RHS of the corresponding equation for Eq. (2).

In order to illustrate the engineering method explicitly, we consider a case of a two-dimensional phononic crystal consisting of a square lattice of water cylinders in mercury background ($\rho_w/\rho_m = 0.076, c_w/c_m = 1.056$) (M. S. Kushwaha and P. Halevi, 1996). At cylinder radius $R = 0.29a$ (a is the lattice constant), the band structure is shown in Fig. 1(a).

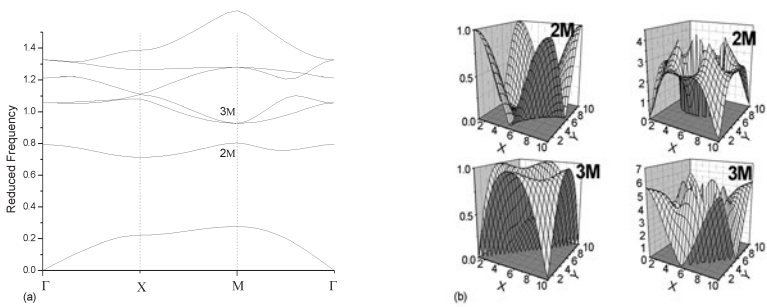

Fig. 1 (a) Band structure for a square lattice of water cylinder in mercury background. (b) Field distributions for 2M and 3M states in a unit cell. Left column is $|p|$ and right column is $|\nabla p|$.

The second band gap is from the 2M state ($f_{2\mathrm{M}} = 0.8047$, here we use dimensionless frequency $f = \omega a/2\pi c_m$) to the 3M state ($f_{3\mathrm{M}} = 0.9278$). We will enlarge this band gap by using our engineering method. The eigenfield distributions of the gap edge states as well as their derivatives are plotted in Fig. 1(b). If we insert small water cylinders, from the eigenfield distributions in Fig. 1(b), we find that the right positions of insertions are the corners of the unit cell. In this way, the eigenfrequency $f_{2\mathrm{M}}$ is reduced due to the large $|p_{2\mathrm{M}}|$ and small $|\nabla p_{2\mathrm{M}}|$ at insertion points, while $f_{3\mathrm{M}}$ is increased due to the small $|p_{3\mathrm{M}}|$ and large $|\nabla p_{3\mathrm{M}}|$ at insertion points. Thus the second band gap will be enlarged. For verification, we have calculated the band gap shift by using the Multiple Scattering Method. The estimated gap shift by Eq. (2) and the calculated gap shift are plotted, respectively, as dashed lines and solid lines in Fig. 2.

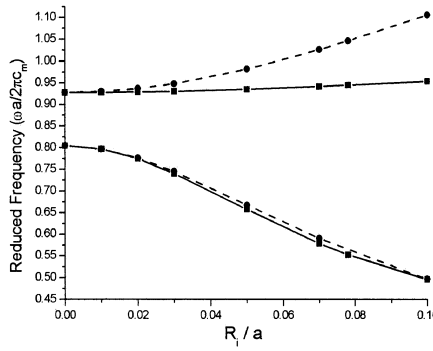

Fig. 2 Estimated (dashed lines) and calculated (solid lines) gap edge shifts.

Significant enhancement of gap size is obtained. When the inserted water cylinder radius $R_i = 0.1a$, the gap is enlarged 3.7 times. Although Eq. (2) fits calculation accurately only when $R_i < 0.03a$, it provides us an effective guide on how to engineer an existing band gap at our will.

3. DISORDER EFFECTS

Since the presence of various kinds of disorder is inevitable during the fabrication process of phononic crystals, it is thus important to study the effects due to disorder on the quality of a band gap. The structure of full band gaps of a finite-sized disordered phononic crystal can be obtained from the radiation power spectrum. Consider a circular sample of radius R_s, which is excited by a line source at a fixed frequency located near the centre. The radiation power can be calculated by using the Multiple Scattering Method. For frequencies inside a full gap, the density of states is zero, which in turn gives a divergent impedance and vanishing radiation power. The validity of this method has been established elsewhere (X. Zhang *et al.*, 2001).

The effects due to disorder on acoustic band gaps can also be estimated by applying the perturbation analysis to a disordered sample with two disordered functions $1/\rho + \delta(1/\rho)$ and $1/\rho c_l^2 + \delta(1/\rho c_l^2)$. The shifts at two gap edge states can be estimated from the perturbative analysis, which gives

$$\left(\frac{\varpi_{n\mathbf{k}}}{\omega_{n\mathbf{k}}}\right)^2 - 1 \approx \sum_i \left[\frac{\int_i \delta\left(\frac{1}{\rho}\right) \cdot \left|\nabla p_{n\mathbf{k}}\right|^2 d\mathbf{r}}{\omega_{n\mathbf{k}}^2 \int_c \frac{\left|p_{n\mathbf{k}}\right|^2}{\rho c_l^2} d\mathbf{r}} - \frac{\int_i \delta\left(\frac{1}{\rho c_l^2}\right) \cdot \left|p_{n\mathbf{k}}\right|^2 d\mathbf{r}}{\int_c \frac{\left|p_{n\mathbf{k}}\right|^2}{\rho c_l^2} d\mathbf{r}} \right]. \tag{3}$$

Here the summation sums all unit cells inside the sample.

As an example, we consider the case of a 2-dimensional disordered phononic crystal consisting of a square lattice of water cylinders (R = 0.29a) in mercury background. Here two kinds of disorder are considered, i.e. site randomness and size randomness with dr and dxy denoting the strengths of disorder, respectively. For site randomness, the cylinders are randomly displaced away from their respective lattice points with displacements uniformly distributed within a circle of radius dxy. While for size randomness, the cylinders' radii are uniformly distributed over $[R-dr, R+dr]$. The sample is in a circular shape enclosing 184 unit cells.

In Fig. 3 we plotted the change of gap size as a function of disorder strength. The solid lines are obtained from radiation power and the dashed lines are the results of Eq. (3). Excellent quantitative agreement between the two results clearly demonstrates the validity of the perturbative approach. The size randomness is more effective in reducing the size of a gap. The similar result has been found in the case of photonic crystal (Z. Y. Li *et al.*, 2000). Moreover, by comparing these results with the case of photonic crystals, we find that acoustic band gaps are more robust against size randomness than photonic band gaps are. This may be due to the cancellation of the two terms on the RHS of Eq. (3) in each unit cell.

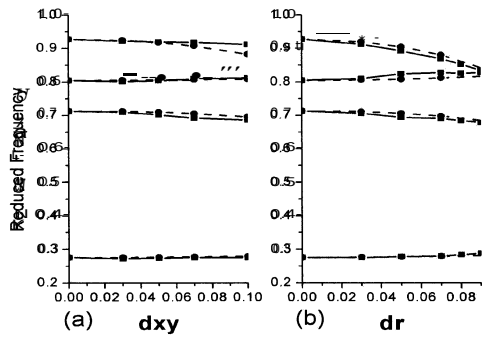

Fig. 3 (a) site randomness (b) size randomness.

4. SUMMARY

Based on a perturbative approach, a simple, systematic, and efficient method to engineer acoustic gaps is proposed. The effects due to disorder on the size of a band gap are estimated by the same perturbative analysis. Both results are verified by the "exact" numerical calculations using the Multiple Scattering Method.

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