30 Quantum Dynamics of Coupled Quantum-Dot Qubits and Dephasing Effects Induced by Detections

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1 INTRODUCTION

Recently, the investigations on the coherent tunnelling in a coupled quantum-dot (QD) system have carried out both experimentally (Blick *et al.*, 1998, and Oosterkamp *et al.*, 1998), and theoretically (Tsukada *et al.*, 1997, and Wu *et al.*, 2000). However, all of these studies did not take the influence of the measurement into account. It is well known that measurement itself will certainly induce dephasing. This effect was studied (Aleiner *et al.*, 1997, Levinson, 1997, and E. Buks *et al.*, 1998) in detail. Also, Gurvitz *et al.* (1996, 1997) derived modified rate equations and studied the dephasing effect induced by measuring the electron state in a coupled QD system via a quantum point contact.

Motivated by these studies, we derive the more generalized rate equation for the coupled QD system irradiated by a microwave field in the presence of a quantum point contact (detector). We investigate the quantum dynamics of the coupled QD system, and find the photon-assisted tunnelling in the coupled QD system when the frequency of the microwave field matches the energy difference between the ground states of the two dots. It is also shown that measurements enforce the coupled QD system to dephase.

2 COUPLED QUANTUM-DOT SYSTEM

The proposed coupled QD system and the detector are schematically shown in Figure 2.1. A quantum point contact is placed near dot 1 as the detector. Its resistance is very sensitive to the electrostatic potential, which may be influenced by electrons filled in dot 1 and 2. The detector is represented by a barrier (sandwiched between an emitter S and a collector D). The chemical potentials of the emitter and the collector are denoted as μ_S and μ_D. $V_d = \mu_S - \mu_D$ is the applied voltage between the emitter and the collector. The Hamiltonian of the entire system can be written as

$$H = H_{DD} + H_{PC} + H_{FD} + H_I \,, \tag{2.1}$$

where $$H_{DD} = E_1 c_1^+ c_1 + E_2 c_2^+ c_2 + \Omega_0 (c_2^+ c_1 + c_1^+ c_2) \,, \tag{2.2}$$

$$H_{PC} = \sum_L \varepsilon_L a_L^+ a_L + \sum_R \varepsilon_R a_R^+ a_R + \sum_{LR} \Omega_{LR} (a_L^+ a_R + a_R^+ a_L) \,, \tag{2.3}$$

$$H_{FD} = -\vec{P} \cdot \vec{E}(t)(c_1^+ c_2 + c_2^+ c_1) \,, \tag{2.4}$$

$$H_I = -\sum_{LR} \Omega'_{LR} c_1^+ c_1 (a_L^+ a_R + a_R^+ a_L) \,. \tag{2.5}$$

Here, H_{DD} and H_{PC} are the Hamiltonians describing the isolated QD system and the quantum point contact, respectively. The interaction between the QD system and the point contact (microwave field) is denoted by H_I (H_{FD}).

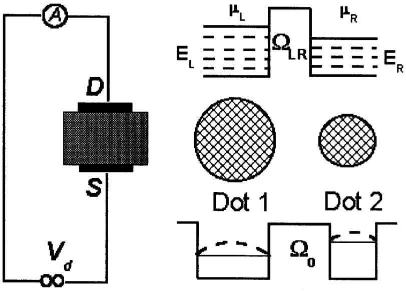

Figure 2.1 Schematic illustration of the coupled quantum-dot system and the quantum point contact detector (left part), whose energy levels are shown on right upper.

In the two-dimensional Fock space composed of the two states $|1\rangle$ and $|2\rangle$, we can obtain the current following through the detector

$$I_d(t) = D_2 \sigma_{11}(t) + D_1 \sigma_{22}(t) \,. \tag{2.6}$$

Here, $D_{1(2)} = T_{1(2)} V_d / 2\pi$ is the transition rate of an electron hoping from emitter to the collector as the electron stays at $|1\rangle$ ($|2\rangle$). After tracing out the point contact states, the reduced density matrixes can be expressed by

$$\dot{\sigma}_{11}(t) = i\Omega_0 (\sigma_{12} - \sigma_{21}) - i\vec{P} \cdot \vec{E}(\sigma_{12} - \sigma_{21}) \,, \tag{2.7}$$

$$\dot{\sigma}_{22}(t) = i\Omega_0 (\sigma_{21} - \sigma_{12}) - i\vec{P} \cdot \vec{E}(\sigma_{21} - \sigma_{12}) \,, \tag{2.8}$$

$$\dot{\sigma}_{12}(t) = i\varepsilon\sigma_{12} + i\Omega_0(\sigma_{11} - \sigma_{22}) - i\vec{P}\cdot\vec{E}(\sigma_{11} - \sigma_{22}) - \Gamma_d\sigma_{12}/2\,, \qquad (2.9)$$

where $\Gamma_d = (\sqrt{D_2} - \sqrt{D_1})^2$ is the dephasing rate.

3 NUMERICAL RESULTS AND DISCUSSIONS

The electron occupation probabilities for different magnitudes of the microwave field are studied in Figure 3.2. As shown by the dotted lines in Figures 3.2(a), 3.2(c), and 3.2(e), they are sinusoidal oscillations with periods 1.255, 0.62, 0.314 when $\vec{P}\cdot\vec{E}_\omega$ are equal to $5\Omega_0$ $10\Omega_0$ $20\Omega_0$, in agreement with the formula of the one-qubit logic gates

$$U = \begin{bmatrix} \cos\theta & -ie^{-i\varphi}\sin\theta \\ -ie^{i\varphi}\sin\theta & \cos\theta \end{bmatrix}, \qquad (3.1)$$

where $\theta = (\vec{P}\cdot\vec{E}_\omega \cdot t)/2$ and φ is the initial phase of the microwave field. This is just what the photon-assisted tunneling should be. It can be seen that this coupled QD system may be used as a qubit for quantum computing and information.

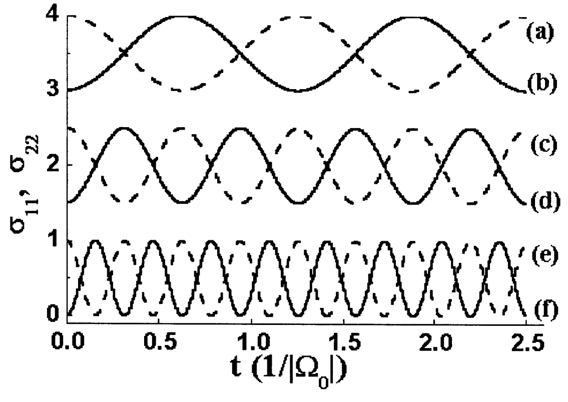

Figure 3.1 Evolution of the electron probability as a function of time for different magnitudes of the microwave field: (a) and (b) P · $E_\omega=5\Omega_0$; (c) and (d) P · $E_\omega=10\Omega_0$; (e) and (f) P · $E_\omega=20\Omega_0$. The solid (dotted) line is the probability of state |2>(|1>). The lines are offset vertically for clarity.

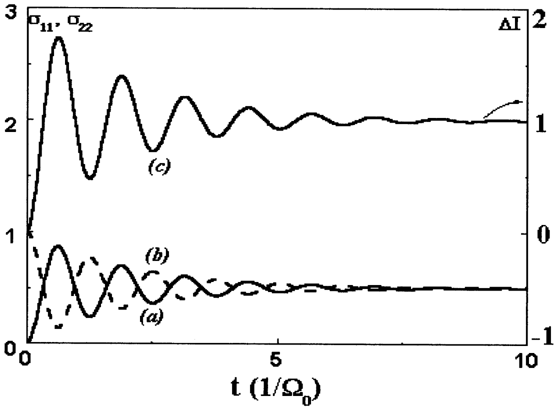

Figure 3.2 Curves (a) and (b) show evolutions of the electron probabilities for dephasing rates $\Gamma_d=3\Omega_0$ and curve (c) corresponding current through the detector.

The influence induced by detecting has been shown in Figure 3.2. It can be seen from curves (a) and (b) that the probabilities decay quickly and approach to 1/2 at sufficiently later time, revealing that the QD system lost its coherence. Comparing the curve (c) with the curve (a), we can find that when the electron probability approaches the maximum, the current also reaches the maximum in phase. This phenomena indicates that one can truly extract the information of the coupled QD system by means of measuring the current variation.

4 CONCLUSION

In conclusion, using rate equations we demonstrate that this coupled QD system may perform all the operations of single qubit. By measuring the current variation we can extract the information of the coupled QD system. Also, we show that in the presence of the dephasing, the oscillating current through the detector decays drastically. For the application of the coupled QD system in the quantum computing and information, keeping an appropriate dephasing rate is necessary.

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REFERENCES

Aleiner, I. L., Wingreen, N. S. and Meir, Y., 1997, Dephasing and the orthogonality catastrophe in tunneling through a quantum dot: the "Which Path?" interferometer, *Physical Review Letters*, **79**, pp. 3740-3743.

Blick, R. H. *et al.*, 1998, Formation of a coherent mode in a double quantum dot, *Physical Review Letters*, **80**, pp. 4032-4035.

Buks, E. *et al.*, 1998, Dephasing in electron interference by a 'which-path' detector, *Nature (London)*, **391**, pp. 871-874.

Gurvitz, S. A., 1997, Measurements with a noninvasive detector and dephasing mechanism, *Physical Review B*, **56**, pp. 15215-15223.

Gurvitz, S. A. and Prager. Ya. S., 1996, Microscopic derivation of rate equations for quantum transport, *Physical Review B*, **53**, pp. 15932-15943.

Levinson, Y., 1997, Dephasing in a quantum dot due to coupling in a quantum dot, *Europhysics Letters*, **39**, pp. 299-304.

Oosterkamp, T. H., *et al.*, 1998, Microwave spectroscopy of a quantum-dot molecule, *Nature(London)*, **395**, pp. 873-876.

Tsukada, N. *et al.*, 1997, Dynamical control of quantum tunneling due to ac stack shift in an asymmetric coupled quantum dot, *Physical Review B*, **56**, pp. 9231-9234.

Wu, N. J. *et al.*, 2000, Quantum computer using coupled-quantum-dot molecules, *Jpn. Journal of Applied Physics*, **39**, pp. 4642-4646.