# 31 Coherent Dynamics and Quantum Information Processing in Josephson Charge Devices

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## 1. INTRODUCTION

Quantum information technology focuses on the quantum-state engineering of a system, with which the quantum states of the system can be prepared, manipulated and readout quantum mechanically. Recently, the "macroscopic" quantum effects in low-capacitance Josephson-junction circuits have received renewed attention because suitable Josephson devices may be used as qubits for quantum information processing (QIP) (Makhlin *et al.*, 1999, 2001 and Mooij *et al.*, 1999) and are expected to be scalable to large-scale circuits using modern micro fabrication techniques.

Experimentally, the energy-level splitting and the related properties of state superpositions were observed in the Josephson charge (Nakamura *et al.*, 1997 and Bouchiate *et al.*, 1998) and phase devices (van der Wal *et al.*, 2000 and Friedman *et al.*, 2000). Moreover, coherent oscillations were demonstrated in the Josephson charge device prepared in a superposition of two charge states (Nakamura *et al.*, 1999). These experimental observations reveal that the Josephson charge and phase devices are suitable for solid-state qubits in QIP. To realize QIP devices of practical use, the next immediate challenge involved is to implement two-bit coupling and then to scale up the architectures to many qubits in a feasible fashion.

A subtle way of coupling Josephson charge qubits was designed in terms

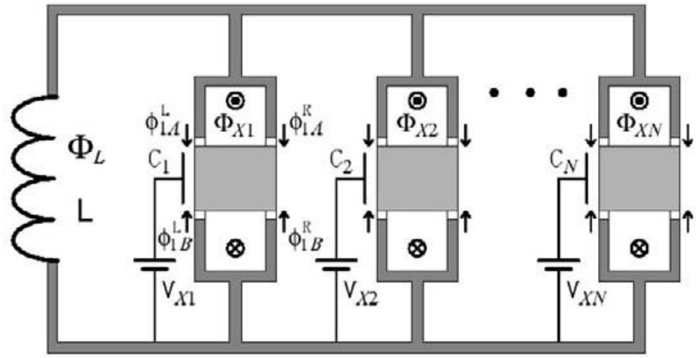


Figure 1: Schematic illustration of the proposed quantum device, where all Josephson charge-qubit structures are coupled by a common superconducting inductance.

of the oscillator modes in a *LC* circuit formed by an inductance and the qubit capacitors (Makhlin *et al.*, 1999, 2001). In that design, interbit coupling is switchable and any two charge qubits can be coupled. However, an appropriate quantum-computing (QC) scheme still lacks with this design and the interbit coupling terms calculated applies only to the case *both* when eigen-frequency of the *LC* circuit is much faster than the quantum manipulation times *and* when the phase conjugate to the total charge on the qubit capacitors fluctuates weakly. Here we propose a new QC scheme based on charge-qubit structures. In our proposal, a common inductance (but not *LC* circuit) is used to couple all Josephson charge qubits. Because the proposed QC architectures have appropriate Hamiltonians, we are able to formulate an efficient QC scheme by means of these Hamiltonians. Moreover, our QC scheme is also scalable because any two charge qubits (*not* necessarily neighbors) can be effectively coupled by an experimentally accessible inductance.

## 2. QUANTUM DEVICE

The proposed quantum device consists of $N$ Cooper-pair boxes coupled by a common superconducting inductance $L$ (see Fig. 1). For the $k$th Cooper-pair box, a superconducting island with charge $Q_k = 2en_k$ is weakly coupled by two symmetric dc SQUID's and biased by an applied voltage $V_{Xk}$ through a gate capacitance $C_k$. The two symmetric dc SQUID's are assumed to be identical and all junctions in them have Josephson coupling energy $E^0_{Jk}$ and capacitance $C_{Jk}$. Since the size of the loop is small (~ 1 μm), we ignore the self-inductance effects of each SQUID loop. The effective coupling energy produced by a SQUID (pierced with a magnetic flux $\Phi_{Xk}$) is given by $-E_{Jk}(\Phi_{Xk})\cos\Phi_{kA(B)}$ with $E_{Jk}(\Phi_{Xk}) = 2E_{Jk}^0\cos(\pi\Phi_{Xk}/\Phi_0)$, where $\Phi_0 = h/2e$ is the quantum flux. The effective phase drop $\Phi_{kA}(B)$, with subscript $A(B)$ labelling the SQUID above (below) the island, equals the average value, $[\Phi^L_{kA}(B) + \Phi^R_{kA(B)}]/2$, of the phase drops across the left and right Josephson junctions in the dc SQUID.

The quantum dynamics of the Josephson charge device is governed by the Hamiltonian

$$H = \sum_{k=1}^{N} H_k + \frac{1}{2}LI^2, \tag{1}$$

with

$$H_k = E_{ck}(n_k - C_k V_{Xk}/2e)^2 - E_{Jk}(\Phi_{Xk})(\cos\Phi_{kA} + \cos\Phi_{kB}). \tag{2}$$

Here $E_{ck} = 2e^2/(C_k + 4C_{Jk})$ is the charging energy of the superconducting island and $I = \sum_{k=1}^{N} I_k$ is the total supercurrent through the superconducting inductance, as contributed by all coupled Cooper-pair boxes. The phase drops $\Phi_{kA}^{L}$ and $\Phi_{kA}^{L}$ are related to the total flux $\Phi = \Phi_L + LI$ through the inductance $L$ by the constraint $\Phi_{kB}^{L} - \Phi_{kA}^{L} = 2\pi\Phi/\Phi_0$, where $\Phi_L$ is the applied magnetic flux threading $L$. In order to implement QC in a feasible way, the magnetic fluxes through the two SQUID loops of each Cooper-pair box are designed to have the *same* values but *opposite* directions. Because this pair of fluxes *cancel* each other in any loop enclosing them, there is $\Phi_{kB}^{L} - \Phi_{kA}^{L} = \Phi_{kB}^{R} - \Phi_{kA}^{R}$, which yields $\Phi_{kB} - \Phi_{kA} = 2\pi\Phi/\Phi_0$ for the average phase drops across the Josephson junctions in SQUID's. Here, each Cooper-pair box is operated both in the charging regime $E_{ck} >> E_{ck}^{0}$ and at low temperatures $k_B T << E_{ck}$. Moreover, we assume that the superconducting gap is larger than $E_{ck}$, so that quasiparticle tunneling is prohibited in the system.

## 3. ONE- AND TWO-BIT STRUCTURES

For any given Cooper-pair box, say $i$, we choose $\Phi_{Xk} = \Phi_0/2$ and $V_{Xk} = (2n_k + 1)e/C_k$ for all boxes except $k = i$. As shown in Fig. 2(a), the inductance $L$ only couples the $i$th Cooper-pair box to form a superconducting loop and the Hamiltonian of the system is $H = H_i + LI_i^2/2$, with $H_i = E_{ci}(n_i - C_i V_{Xi}/2e)^2 - 2E_{Ji}(\Phi_{Xi})\cos(\pi\Phi/\Phi_0)\cos\varphi_i$. Here, the phase $\varphi_i = (\varphi_{iA} + \varphi_{iB})/2$ is canonically conjugate with the number of the extra Cooper pairs on the island and the circulating supercurrent $I_i$ in the loop is given by $I_i = 2I_{ci}\cos\varphi_i \sin(\pi\Phi_L/\Phi_0 + \pi LI_i\Phi_0)$, where $I_{ci} = -\pi E_{Ji}(\Phi_{Xi})/\Phi_0$. Expanding each operator function into a power series, we can cast the Hamiltonian of the system to (You *et al.*, 2001)

$$H = \varepsilon_i(V_{Xi})\sigma_z^{(i)} - \overline{E}_{Ji}\,\sigma_x^{(i)}, \tag{3}$$

where $\varepsilon_i(V_{Xi}) = \overline{E}_{ci}\,[C_i V_{Xi}/e - (2n_i + 1)]$ and the spin $-1/2$ representation of the reduced Hamiltonian is based on charge states $|\uparrow\rangle_i = |n_i\rangle$ and $|\downarrow\rangle_i = |n_i+1\rangle$. Retained up to the terms second order in the expansion parameter, there is $\overline{E}_{Ji} = E_{Ji}(\Phi_{Xi})\cos(\pi\Phi_L/\Phi_0)\xi$, with $\xi = 1 - \frac{1}{2}(\pi LI_{ci}/\Phi_0)^2\sin^2(\pi\Phi_L/\Phi_0)$.

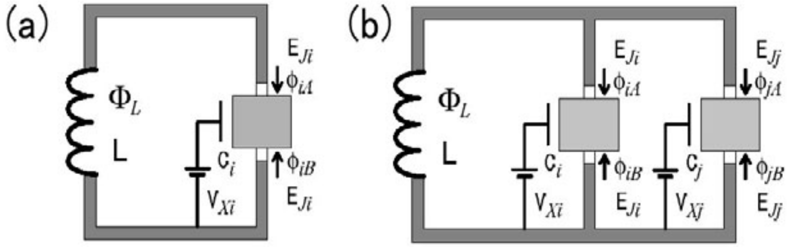


Figure 2: (a) One- and (b) two-bit structures.

To couple any two Cooper-pair boxes, say $i$ and $j$, we choose $\Phi_{Xk} = \Phi_0/2$ and $V_{Xk} = (2n_k + 1)e/C_k$ for all boxes except $k = i$ and $j$. The inductance $L$ is shared by the Cooper-pair boxes i and j to form superconducting loops [see Fig. 2(b)]. The Hamiltonian of the system is $H = H_i + H_j + L(I_i + I_j)^2/2$, where $I_i = 2I_{ci} \cos\varphi_i \sin[\pi\Phi_L/\Phi_0 + \pi L(I_i + I_j)/\Phi_0]$ is the circulating supercurrent contributed by the Cooper-pair box $i$. Interchanging $i$ and $j$ in $I_i$ gives the expression for circulating current $I_j$. In the spin –1/2 representation, the Hamiltonian of the system is reduced to

$$H = \sum_{k=i,j} [\varepsilon_k(V_{Xk})\sigma_z^{(k)} - \overline{E}_{Jk}\, \sigma_x^{(k)}] + \Pi_{ij}\sigma_x^{(i)}\, \sigma_x^{(j)}. \tag{4}$$

Retained up to the second-order terms in expansion parameters, $\overline{E}_{Ji}$ and $\Pi_{ij}$ are given by $\overline{E}_{Ji} = \overline{E}_{Ji}(\Phi_{Xi})\cos(\pi\Phi_L/\Phi_0)\xi$, with $\xi = 1 - \frac{1}{2}[(\pi LI_{ci}/\Phi_0)^2 + 3(\pi LI_{cj}/\Phi_0)^2] \sin^2(\pi\Phi_L/\Phi_0)$, and $\Pi_{ij} = -LI_{ci}I_{cj}\sin^2(\pi\Phi_L/\Phi_0)$.

## 4. COMPUTING WITH QUBITS

The quantum system evolves according to $U(t) = \exp(-i2\pi H_t/h)$. Initially, we choose $\Phi_{Xk} = \Phi_0/2$ and $V_{Xk} = (2n_k + 1)e/C_k$ for all boxes in Fig. 1, so that the Hamiltonian of the system is H = 0 and no evolution occurs to the system. Then, we *switch* fluxes $\Phi_{Xk}$ and/or gate voltages $V_{Xk}$ away from the above initial values for periods of times to implement operations requried for QC. For any two Cooper-pair boxes, say i and j, when fluxes $\Phi_{Xi}$ and $\Phi_{Xj}$ are switched away from the initial value $\Phi_0/2$ for a given period of time $\tau$, the Hamiltonian of the system becomes $H = -\overline{E}_{Ji}\, \sigma_x^{(i)} - \overline{E}_{Ji}\, \sigma_x^{(j)} + \Pi_{ij}\sigma_x^{(i)}\, \sigma_x^{(j)}$. This anisotropic Hamiltonian is Ising-like (Burkard *et al.*, 1999), with its anisotropic direction and the "magnetic" field along the $x$ axis. When the parameters are suitably chosen so that $\overline{E}_{Ji} = \overline{E}_{Ji} = \Pi_{ij} = -h/8\tau$ for the switching time τ, we obtain a two-bit gate:

$$U_{CPS}' = -e^{i\pi/4}U_{2b} = e^{i\pi/4}[1-\sigma_x^{(i)} - \sigma_x^{(j)} + \sigma_x^{(i)}\sigma_x^{(j)}], \tag{5}$$

which does not alter the two-bit states $|+\rangle_i|+\rangle_j$, $|+\rangle_i|-\rangle_j$ and $|-\rangle_i|+\rangle_j$, but transforms $|-\rangle_i|-\rangle_j$ to $-|-\rangle_i|-\rangle_j$ Here $|\pm\rangle$ are defined by $|\pm\rangle = (|\uparrow\rangle \pm |\downarrow\rangle)/\sqrt{2}$.

For any Cooper-pair box, say $i$, one can shift flux $\Phi_{Xi}$ and/or gate voltage $V_{Xi}$ for a given switching time $\tau$ to derive one-bit rotations. A universal set of one-bit gates $U_z^{(i)}(\alpha) = -\exp[i\alpha\sigma_z^{(i)}]$ and $U_x^{(i)}(\beta) = -\exp[i\beta\sigma_x^{(i)}]$, where $\alpha = 2\pi\varepsilon_i(V_{Xi})\tau/h$ and $\beta = 2\pi E_{Ji}\tau/h$, can be defined by choosing $E_{Ji} = 0$ and $\varepsilon_i(V_{Xi}) = 0$ in Hamiltonian (3), respectively. Combining $U_{CPS}'$ with one-bit rotations, we obtain the controlled-phase-shift gate $U_{CPS}$ for the basis states $|\uparrow>_i|\downarrow>_j$, $|\downarrow>_i|\uparrow>_j$, and $|\downarrow>_i|\downarrow>_j$:

$$U_{CPS} = H_j^+ H_i^+ U_{CPS}' H_i H_j, \qquad (6)$$

where $H$ is the Hadamard gate $H_i = e^{-i\pi/2} U_z^{(i)}(\frac{\pi}{4}) U_x^{(i)}(\frac{\pi}{4}) U_z^{(i)}(\frac{\pi}{4})$. The controlled-NOT gate is given by

$$U_{CNOT} = V_j^+ U_{CPS} V_j, \qquad (7)$$

where $V_j = U_z^{(j)}(-\frac{\pi}{4}) U_x^{(j)}(\frac{\pi}{4}) U_z^{(j)}(\frac{\pi}{4})$. This gate transforms the basis states as $|\uparrow>_i|\uparrow>_j \rightarrow |\uparrow>_i|\uparrow>_j$, $|\uparrow>_i|\downarrow>_j \rightarrow |\uparrow>_i|\downarrow>_j$ $|\downarrow>_i|\uparrow>_j \rightarrow |\downarrow>_i|\downarrow>_j$ and $|\downarrow>_i|\downarrow>_j \rightarrow |\downarrow>_i|\uparrow>_j$. This conditional two-bit gate and one-bit rotations provide a complete set of gates required for QC (Lloyd, 1995). Usually, a two-bit operation is much slower than a one-bit operation. Our designs for conditional gates UCPS and UCNOT are efficient since only one two-bit operation $U_{CPS}'$ is used.

The typical switching time $\tau^{(1)}$ during a one-bit operation is of the order $\hbar/E_j^0$. For the experimental value of $E_j^0$ ~ 100 mK, there is $\tau^{(1)}$ ~ 0:1 ns. The switching time $\tau^{(2)}$ for the two-bit operation is typically of the order $(\hbar/L)(\Phi_0/\pi E_j^0)^2$. Choosing of $E_j^0$ ~ 100 mK, there is $\tau^{(2)} \sim 10\tau^{(1)}$ (i.e., ten times slower than the one-bit rotation), we derive an inductance of experimentally accessible value, $L$ ~ 30 nH. As compared with our proposal, when the two-bit operation is also chosen ten times slower than the one-bit rotation, the inductance should be $(C_J = C_{qb})^2 L$ in the quantum computers designed by Makhlin *et al.* (1999, 2001). For their previous design (Makhlin *et al.*, 1999), $C_J \sim 11C_{qb}$ as $C_g/C_J$ ~ 0:1, requiring an inductance of value ~ 3:6 μH. This inductance is too large to fabricate in nanometer scales. In their improved design (Makhlin *et al.*, 2001), $C_J \sim 2C_{qb}$. The value of the inductance is greatly reduced to ~ 120 nH (four times the value of the inductance used in our scheme).

## 5. CONCLUSION

In conclusion, we propose a QIP device based on Josephson charge qubits. We employ a common inductance to couple all charge qubits and design switchable interbit couplings by using two dc SQUID's to connect the island in each Cooper-pair box. The proposed QC architectures are scalable since any two charge qubits can be effectively coupled by an experimentally accessible inductance. Using appropriate Hamiltonians of the QC architectures, we ormulate an efficient QC scheme in which only one two-bit operation is used in the controlled-phase-shift and controlled-NOT gates.

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