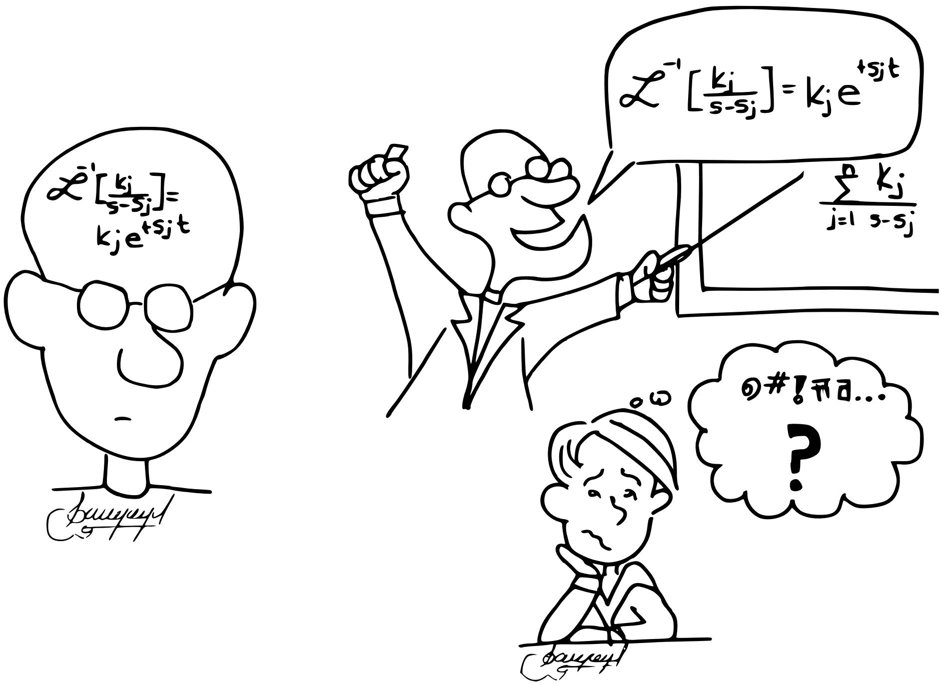
Part II

The Laplace Transform

Chapter 4

Introduction to the Laplace Transform

“Why do I have to use Laplace Transform to solve an Electric Circuit?”



An engineering student said this while he was solving an electric circuit problem in our study. The study of Laplace transform is considered an important topic in many university programs towards an engineering degree, for example electrical engineering. However the expression mentioned above, and other results in our study, show that it is important to study students’ and teachers’ views of how Laplace transforms are handled and made relevant in engineering education. In this study we will focus on the teachers’ views.

he Laplace transform could be understood as a process used in, for example, engineering to solve real situations and we describe this process in 3 stages (Fig.1):

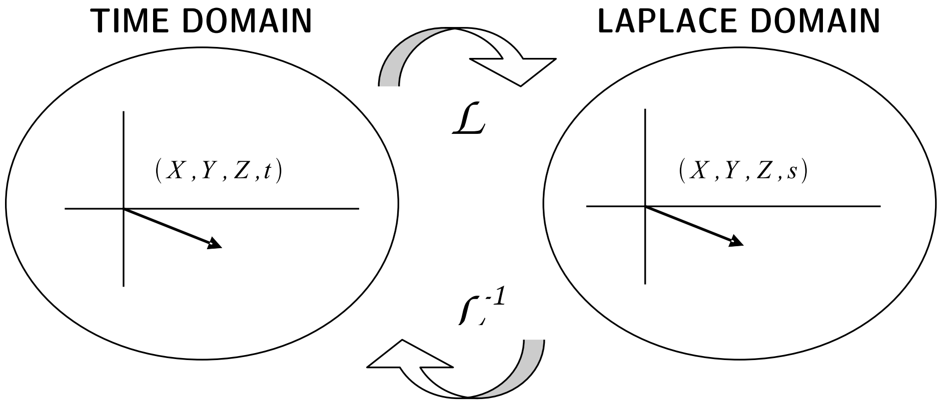
First stage To represent a system in the time domain.

Second stage To transform apply equivalence of that system in the Laplace domain making reduction of calculation and obtaining a solution in the Laplace domain.

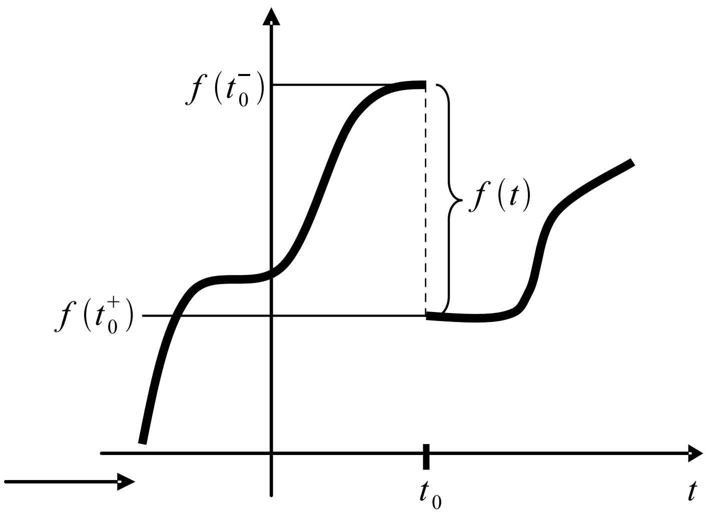
Third stage To transform the solution in the Laplace domain back to a solution in the original time domain applying the Laplace inverse.

$$s = \sigma + j\omega \tag{4.1}$$

ω [radians / second] = "angular frequency"



Definition : $f : \mathbb{R} \to \mathbb{R}$ is sectional continuous in $[a, b] \subset \mathbb{R}$ have only a finite number of finite discontinuities in $[a, b] \subset \mathbb{R}$.

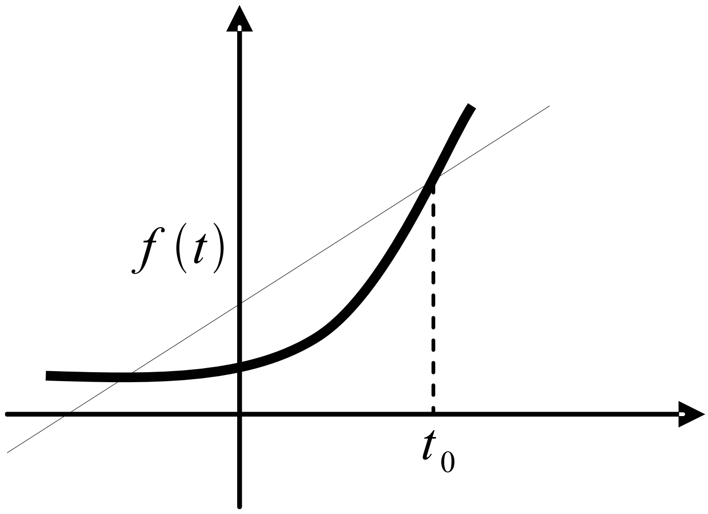


$f(t)$ is a finite discontinuity in t_0

$$f({t_0}^+) = \lim_{\epsilon \to 0} f(t_0 + \epsilon) \qquad \text{limit by right} \tag{4.2}$$
$$f({t_0}^-) = \lim_{\epsilon \to 0} f(t_0 - \epsilon) \qquad \text{limit by left} \tag{4.3}$$

Definition : $f : \mathbb{R} \to \mathbb{R}$
f is exponential order

$$\iff \exists_n \; k > 0, t_0, b \in R \colon |f(t)| < ke^{bt} \forall t > t_0 \tag{4.4}$$



Definition : a function $f : \mathbb{R} \to \mathbb{R}$ that is sectional continuous in $[a, b] \subset \mathbb{R}$ and is exponential order, that carry the Dirichlet conditions out.

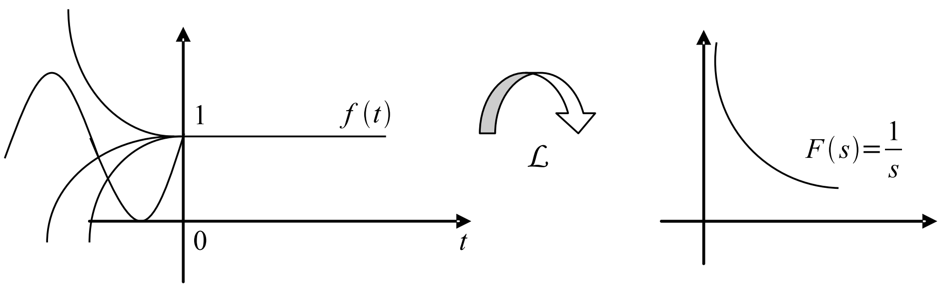
Definition : $f : \mathbb{R} \to \mathbb{R}$ that carry out the Dirichlet conditions. It is defined **the Laplace transform** like:

$$F(s) = \mathcal{L}\{f(t)\} = \int_0^\infty f(t)e^{-st}dt \tag{4.5}$$

Example 1. *To find the Laplace transform in the function $f(t) = 1$.*

$$\mathcal{L}\{1\} = \int_0^\infty (1)e^{-st}dt = \frac{1}{s}e^{-st}\bigg|_0^\infty = \frac{1}{s}[0-1] = \frac{1}{s} \tag{4.6}$$

$$\Rightarrow \mathcal{L}\{1\} = \frac{1}{s} \qquad \textit{(to } s > 0\textit{)} \tag{4.7}$$



And a more formal explanation is possible to find Körner (1988) about the Laplace transform:

> *"No one who has used a shower in a student residence can fail to become aware o the problems caused by the fact that the temperature of the water does not respond instantly to the controls. First the water comes out cold so you turn the hot tap on full, only to be scalded ten seconds later. The natural response produces icy cold water after ten seconds of agony. Shivering you then turn the hot tap full on."* Körner (1988)

The simplest mathematical analogue of such a system is the differential equation

$$F'(s) + KF(s - \eta) = G(s) \tag{4.8}$$

Where $\eta > 0$ and $F(s) = G(s) = 0$ for $s \leq 0$. We commence our treatment by making the substitution $s = t\eta$ and writing $f(t) = \eta^{-1}F(\eta t), g(t) = G(\eta t), k = K\eta$. Our equation then becomes

$$\left.\begin{aligned} f'(t) + kf(t-1) = g(t) \\ f'(t) = g(t) = 0 \text{ for } t < 0 \end{aligned}\right\} \tag{4.9}$$

It is easy to check that (if "g" is continuous) (4.9) has a unique solutions and that(if "g" is bounded) this solution lies in ϵ.

Some authors have written about engineering education concerning to learn and teach the Laplace transform. Carstensen and Bernhard (2002, 2004) have studied how engineering students solve electric circuits using the Laplace transform, Bernhard and Carstensen (2002) have discussed learning electric circuit theory for engineering students.

4.1 Application of the Laplace transform

It is common in engineering education to find the perspective that the Laplace transform is just a theoretical and mathematical concept (outside of the real world) without any application in others areas. The transforms are considered as a tool to make mathematical calculations easier. However, it is important to notice that "frequency domain" is possible appreciate also in the real world and applied in other areas like, for example, economics.

> *"The most popular application of the Laplace transform is in electronic engineering, but it has also been applied to the economic and managerial problems, and most recently, to Materials Requirement Planning (MRP)"*
> Yu and Grubbström (2001)

The article of Grubbström (1967) shows the application the Laplace transform to:

- Deterministic Economic Process
- Stochastic Economic Processes

It is pointed out that the metod of the Laplace transform has found an increasing number of applications in the fields of physics and technology. For example, the possibility of solving problems in the area of discounting with the aid of this method. Without any loss of general validity, it is shown that the discount factor can always be written in an exponential manner which implies that the present value of a "cash-flow" will obtain a very simple form in the Laplace transform terminology. This simplicity holds good for stochastic as well as for deterministic economic processes. Also it could be applied to all mathematical simplifications of reality. Grubbström (1996) consider a stochastic inventory process in which demand is generated by individuals separated by independent stochastic time intervals whereas production takes place in batches of possibily varying sizes at different points in time. The resulting processes are analyzed using the Laplace transform methodology. Then Grubbström and Molinder (1994) designed a generalized input-matrix to incorporate requirements as well as production lead times by means of z-transform methodology in a discrete time model. The theory is extended to continous time using the Laplace transform, wich enables it to incorporate the possibility of batch production at finite production rates. Also Grubbström and Molinder (1996) developed a basic method of how such safety master production plans can be determinated in simple cases using the Laplace transform.

Other application: A telephone or simple intercommuticator does not need to be modulated, it is only necessary to have a couple of machines that transform the pressure waves into electric energy, the electric energy is sending by a couple of cables of copper (Cu) and in the receptor side is used a similar machine that convert the variations of electricity in variations of pressure. A microphone and speaker are built in same way. We talk to the microphone by the diafragma and we make the current in cables variabe, by other hand, we put variable current in the cables and the diafragma produces sound. We can use the space instead of the cable to send signals. There is a lot of difference between send a signal trough the cable and to send it by the empthy. It is like to travel trough the flat road or to go trough rough road. The air is not as conductive as the copper, otherwise we would be electrocuted. The alternating current (amplitud variable) has the capacity to travel through the space. To transmitt a signal from point A to point B trough space we need power and a very high frequency (almost radiating). For lower frequencies we need more power and in the extreme case (continous current), it does not matter how much power is supplied, it radiates nothing. In certain special frequencies, the higher level of the atmosphere (ionosphere) that act as reflectors, and rebound in them is possible to get a signal in large distances. This is "the short wave". If the frequency is too high, it passes across of the large and is lost in the exterior space; if it is too low, does not arrive and it is absorbed by the earth.

The problem is that the frequncy of the sounds that we are able to hear is between 20Hz and 18 Khz (in people good hearing). If we directly convert the waves of the sound to electricity, they will be of a very low frequency that only can be transmitted by cable. The solution to transmit sounds by the air trough large distances, consist in to send a signal with high enough frequency so that it can radiate, but modifiying it in a proportional way to the variations of the sound that we want to send. The name of this frequency is "carrier" and the low is "modulation". Thera are different things that we can modify in a carrier and for it exist different methods of modulation: amplitude (AM), frequency (FM), phase (PM), etc.

The sounds are variations of pressure in the air and exist pure sounds and composed. The pure sounds consist in only frequency (for example the "beep" of a computer) and are not common in the nature. The majority of the "real" sounds consist in thousand of different fequencies emitted at the same time. This let us to distinguish between a natural sound (rich in resonance) from an artificial (with not many components). The natural sounds like our voice, the music, the noice, etc. they do not consist in a frequency but in a band of frequencies with many fundamentals and other harmonies that produce rebound and resonancy in the first.

Laplace proposed, before the radio exist, that the resultant is not only one frequency, but three different: the original of 7500000Hz, the superior side of 7500000+300 and the inferior side of 7500000-300. Among other things, Laplace discovered a general method demonstrating that any form of wave can be described as a serie of pure sin waves. The Laplace transform is a mathematical method to find this equivalence and it is the enough simple to be easy to progam. In the real life, natural sounds are transmitted that consist in thousands of simultaneous frequencies in tone and amplitude, it make the situation complicated, in the sense that the resultant modulated is not only three frequencies, but the carrier and two bands of frequencies, the quantites of those frequencies is enormous, but is taken just a range until the amplitudes are significant, the rest of them is disregarded.

Chapter 5

Study 3: Student Views of Using the Laplace Transform in Solving Electric Circuits Problems

5.1 Introduction

This research has its origins in a Study 1, focused on finding the difficulties that students have solving electric circuits. The results showed the special importance that the Laplace transform topic has for engineering students. For this reason we are interested in identifying the link between the mathematical models and the physical effect.

Most studies regarding students understanding of electric circuits are however in the domain of pre-university students understanding of simple DC-circuits. According to this body of research (see for example Duit and von Rhöneck (1997); Bernhard and Carstensen (2002), and references therein) students tend to mix concepts such as voltage, current, power and energy. This means that students do not clearly distinguish between these concepts and from this view follows misconceptions such as current consumption, battery as constant current supply, no current – no voltage and voltage as a part or property of current.

Few researches studying the understanding of electricity have written about higher level education and specifically about engineering education. Carstensen and Bernhard (2002) and Ryegård (2004) has written about promoting learning through interactive sessions and Carstensen and Bernhard (2004) have studied how engineering students solve electric circuits using the Laplace transform in labwork. They state:

> *"In many engineering programs at college level the application of the Laplace transform is nowadays considered too difficult for the students to understand."* Carstensen and Bernhard (2004)

To understand the more complex concepts, engineering students has to apply previous knowledge in physics and mathematics. In the case of, for example, analysis of electrical circuits it is necessary for the students to develop mathematical matrices and differential equations. They need to know the electric behavior of the different elements in an electric circuit.

For engineering students the motivation for learning electric circuits is different from that of children since the engineering student has chosen to learn electric circuits while the school pupil is mandated to learn it.

The aim of this study is to find these difficulties through the analysis of the students' answers, how the students link the theoretical/model world and the real/physical world. To find information about the interpretation of complex concepts and difficulties in analyzing and solving in electric circuits problems.

5.2 Method and Sample

This study is a survey of the answers from the same 109 engineering students as in Study 2.

The data analysis strategy for each stage was quantitative and qualitative analysis: Categorizing, comparing and summarizing information. Presenting and explaining results of previous studies and their interrelation and conclusions. In the last part of the questionnaire a solved problem is given and the students are asked to explain some parts of the solution; for example, to explain the meaning of the symbol "*s*" of the Laplace transform method applied to solve the electric circuit.

Figure 5.1: *Questionnaires to students from three different countries*

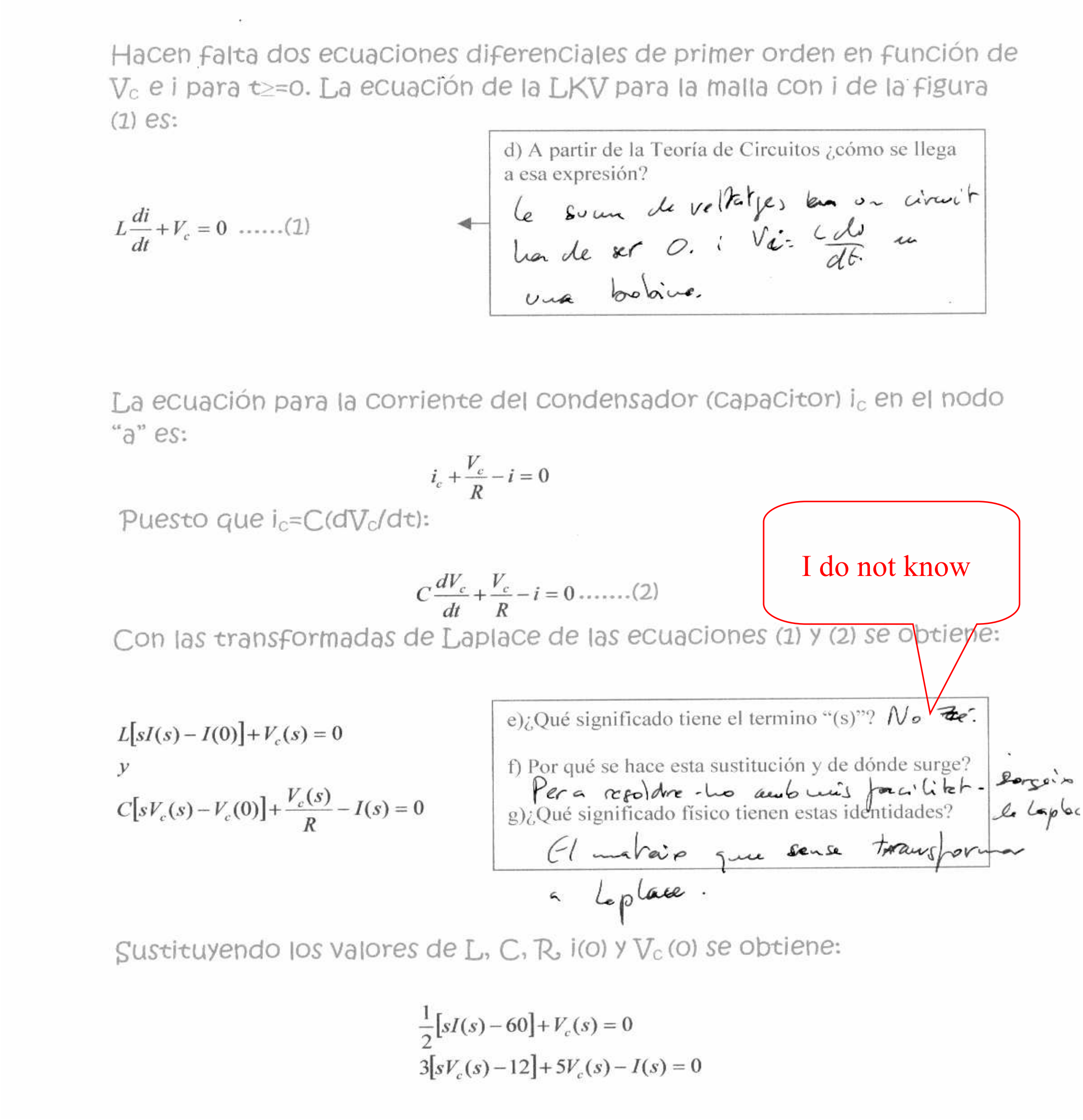
5.3 Results

This is the continuation of Study 2, but in this case we are focusing on knowing how the students interpret the Laplace transform models in a solved electric circuit problem. Normally the students has to solve electric circuits, but our strategy was to make the opposite; to let the students explain parts of the procedure of the solved the problem.

A particular observation we made was that most of the students were perplexed because the problem was already solved. It seemed strange to them to explain the process of solving the problem, since they usually do not do that.

General observations

It was interesting to notice that some students preferred not to answer those questions (for any reason that we are not concerned with) but others answered, like the next example: in the question about the meaning of symbol "s" in the mathematical model used, an student answer was "I don't know" as Figure 5.2 shows.



Hacen falta dos ecuaciones diferenciales de primer orden en función de V_C e i para t≥=0. La ecuación de la LKV para la malla con i de la figura (1) es:

$$L\frac{di}{dt}+V_c=0 \;......(1)$$

d) A partir de la Teoría de Circuitos ¿cómo se llega a esa expresión?

La suma de voltatges en un circuit ha de ser 0. i $V_i = L\frac{di}{dt}$ en una bobina.

La ecuación para la corriente del condensador (capacitor) i_C en el nodo "a" es:

$$i_c+\frac{V_c}{R}-i=0$$

Puesto que $i_C=C(dV_C/dt)$:

$$C\frac{dV_c}{dt}+\frac{V_c}{R}-i=0\;.......(2)$$

I do not know

Con las transformadas de Laplace de las ecuaciones (1) y (2) se obtiene:

$$L[sI(s)-I(0)]+V_c(s)=0$$

y

$$C[sV_c(s)-V_c(0)]+\frac{V_c(s)}{R}-I(s)=0$$

e)¿Qué significado tiene el termino "(s)"? No sé.

f) Por qué se hace esta sustitución y de dónde surge? Per a resoldre-ho amb més facilitat.

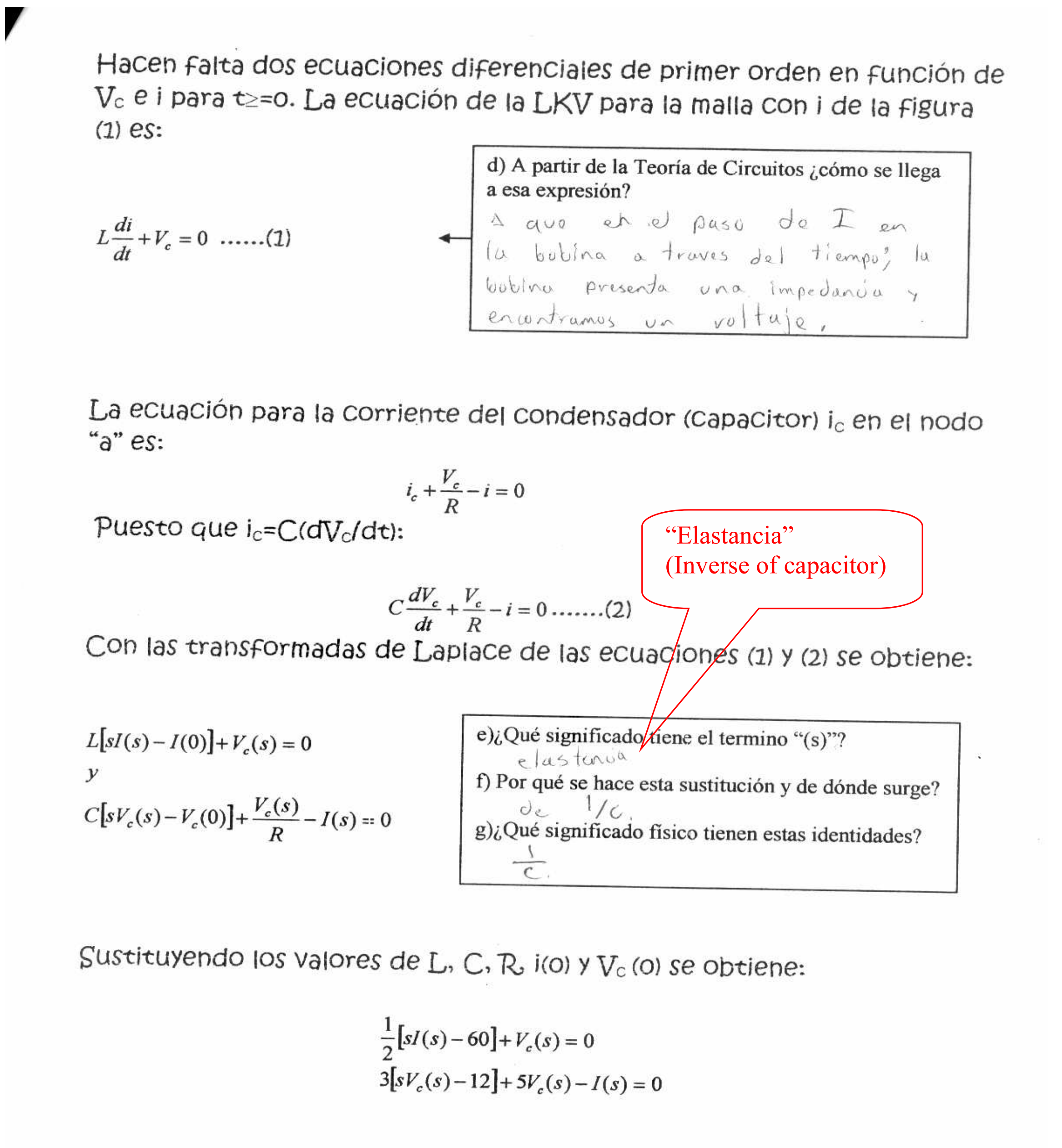
g)¿Qué significado físico tienen estas identidades? El mateix que sense transformar a Laplace.

Sustituyendo los valores de L, C, R, i(0) y V_C (0) se obtiene:

$$\frac{1}{2}[sI(s)-60]+V_c(s)=0$$
$$3[sV_c(s)-12]+5V_c(s)-I(s)=0$$

Figure 5.2: *Part of the questionnaire answered by student from Catalonia*

In other answers we find some confusion with other symbols assigned to electric elements. For electricity, in the Mexican context, the letter "s" is use also to represent the inverse of capacitance ($1/C$), called elastancia (Figure 5.3):



Hacen falta dos ecuaciones diferenciales de primer orden en función de V_C e i para t≥=0. La ecuación de la LKV para la malla con i de la figura (1) es:

$$L\frac{di}{dt}+V_c=0 \;......(1)$$

d) A partir de la Teoría de Circuitos ¿cómo se llega a esa expresión?

A que es el paso de I en la bobina a traves del tiempo; la bobina presenta una impedancia y encontramos un voltaje.

La ecuación para la corriente del condensador (capacitor) i_C en el nodo "a" es:

$$i_c+\frac{V_c}{R}-i=0$$

Puesto que $i_C=C(dV_C/dt)$:

$$C\frac{dV_c}{dt}+\frac{V_c}{R}-i=0\;.......(2)$$

Con las transformadas de Laplace de las ecuaciones (1) y (2) se obtiene:

$$L[sI(s)-I(0)]+V_c(s)=0$$

y

$$C[sV_c(s)-V_c(0)]+\frac{V_c(s)}{R}-I(s)=0$$

e) ¿Qué significado tiene el termino "(s)"?

elastancia

f) Por qué se hace esta sustitución y de dónde surge?

de 1/C.

g) ¿Qué significado físico tienen estas identidades?

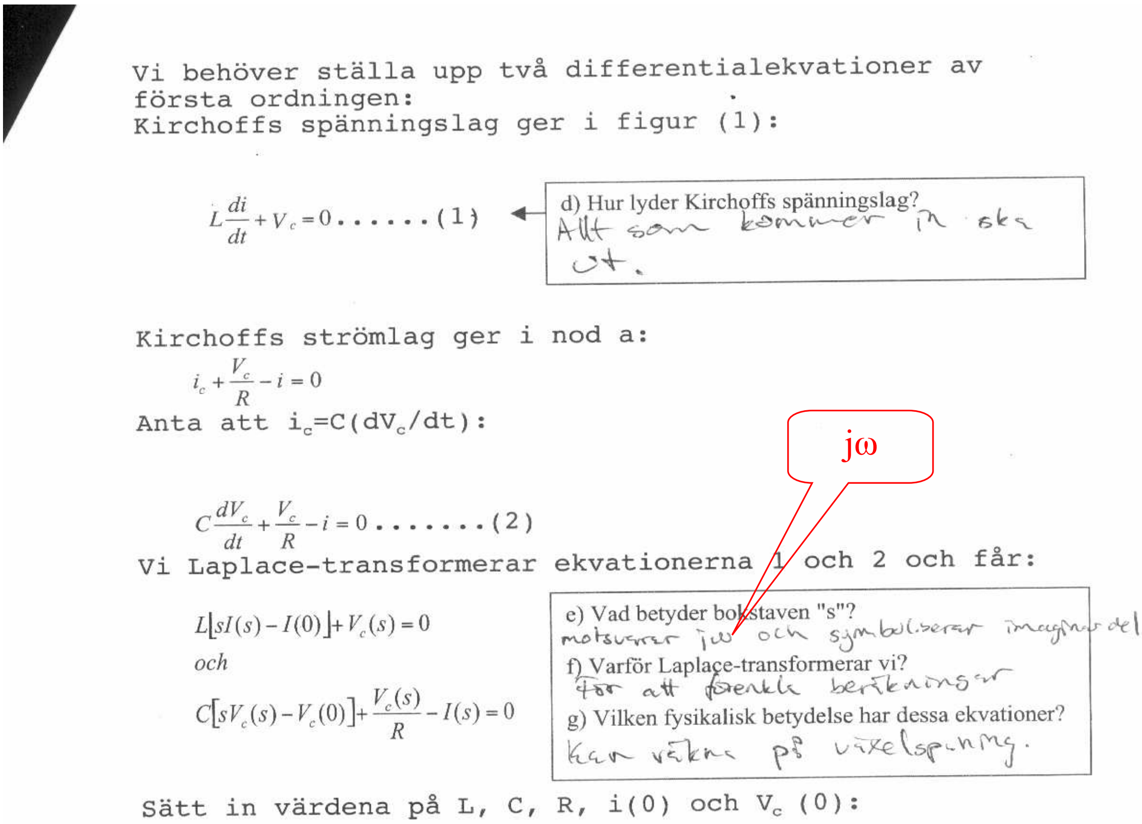
1/C.

Sustituyendo los valores de L, C, R, i(0) y V_C (0) se obtiene:

$$\frac{1}{2}[sI(s)-60]+V_c(s)=0$$
$$3[sV_c(s)-12]+5V_c(s)-I(s)=0$$

Figure 5.3: *Part of the questionnaire answered by student from Mexico*

Not all cases were wrong. In other answers we appreciated some interesting explanations that were not the answer expected but was other way very near of the expert's point of view (Figure 5.4):


Vi behöver ställa upp två differentialekvationer av första ordningen:
Kirchoffs spänningslag ger i figur (1):

$$L\frac{di}{dt}+V_c=0 \ldots\ldots (1)$$

d) Hur lyder Kirchoffs spänningslag?
Allt som kommer in ska ut.

Kirchoffs strömlag ger i nod a:

$$i_c+\frac{V_c}{R}-i=0$$

Anta att $i_c=C(dV_c/dt)$:

$$C\frac{dV_c}{dt}+\frac{V_c}{R}-i=0 \ldots\ldots (2)$$

Vi Laplace-transformerar ekvationerna 1 och 2 och får:

$$L[sI(s)-I(0)]+V_c(s)=0$$

och

$$C[sV_c(s)-V_c(0)]+\frac{V_c(s)}{R}-I(s)=0$$

e) Vad betyder bokstaven "s"?
motsvarar jω och symboliserar imaginär del
f) Varför Laplace-transformerar vi?
För att förenkla beräkningar
g) Vilken fysikalisk betydelse har dessa ekvationer?
Kan räkna på växelspänning.

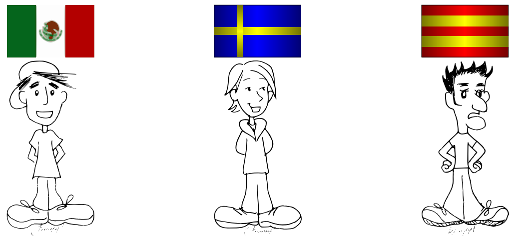
Sätt in värdena på L, C, R, i(0) och V_c (0):

Figure 5.4: *Part of the questionnaire answered by student from Sweden*

The meaning of the symbol "*s*"

Although the way to answer was different for every context, it was interesting to see some incidences on explanations about meanings. The next tables show a classification about the students' answers.

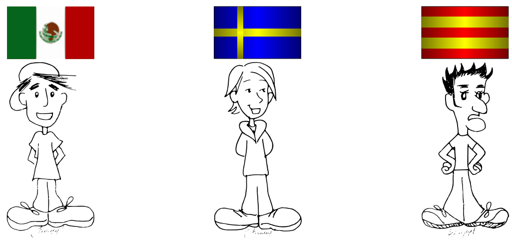
Table 5.1 shows the correct answer from the engineering students explaining the meaning of the symbol "*s*" in a solved problem of electric circuit. This was the only answer repeated in all of the three contexts.



	Number of Students		
	Mexico	Sweden	Catalonia
The Laplace transform	7	1	3

Table 5.1: *Correct answer to the question "What does the 's' mean?"*

Table 5.2 shows the arguments more near to the right answer explaining the meaning of the symbol "s" in a solved problem of electric circuit.

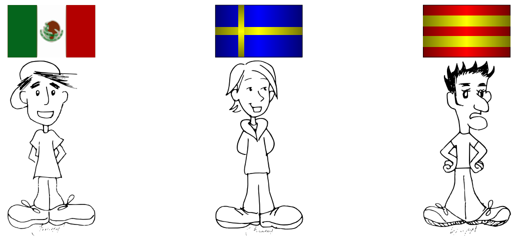


	Number of Students		
	Mexico	Sweden	Catalonia
Differential operator	1	2	
$j\omega$		4	
Symbolization of imaginary part		1	
Corresponds to "t" in the time domain, applied to the Laplace domain		1	
Laplace operator		1	3

Table 5.2: *Close to expert views in answers to the question "What does the 's' mean?"*

We can observe that the answer "Laplace operator" was repeated in Sweden and Catalonia while the answer "differential operator" was repeated in Sweden and Mexico.

Table 5.3 shows the arguments from the students who did not understand the meaning of the symbol "s" in the solved electric circuit problem.



	Number of Students		
	Mexico	Sweden	Catalonia
Confusion with "s ($1/C$)"	2		
A constant		1	
I do not know		2	1
Is equal to time			1

Table 5.3: *Misconceptions in answers to the question "What does the 's' mean?"*

In our data we can see that students often use the mathematical language to solve electric problems, but it is unusual that they have to know the meaning of the mathematical formulas. When learning about electric circuits, especially more complex concepts, like the Laplace transform, this is very typical.

Generally it assumes that the engineering students need intellectual and methodological maturity to understand and to integrate the new concepts, dedicating small or no attention to the contextualization of advanced knowledge. Often they do not understand the necessity of a solid scientific formation. And here the pragmatic and dominating question regarding the training for their future professional future arises: *"is this usable in real life?"*.

It is important to notice that students are able to manage complex concepts without a deeper understanding of the meaning of them, and this is a limitation for students when they have to confront other kind of problems, as Study 1 shows. Although for some opinions in the professional environment this aspect is not relevant, arguing that while the student give god result is not necessary to stop in details.

Transformation of differential equations

The students were asked why the substitutions in Expressions 5.1 and 5.2 is made, and what gains is made from the transformation.

$$L\frac{di}{dt} + V_c = 0 \qquad \text{to} \qquad L[sI(s) - I(0)] + V_c(s) = 0 \tag{5.1}$$

$$C\frac{dV_c}{dt} + \frac{V_c}{R} - i = 0 \qquad \text{to} \qquad C[sV_c(s) - V_c(0)] + \frac{V_c(s)}{R} - I(s) = 0 \tag{5.2}$$

Table 5.4. Concerns the reasoning where the differential equation is transformed into the Laplace domain by the Laplace transform.

	Number of Students		
			
	Mexico	Sweden	Catalonia
To solve differential equations more easy	2	6	2
To calculate V and I with impedance			1
To calculate the dynamics, the change with time			1
To easily interpret the circuit; example: when you apply a force it produces movement		1	
To make it easier to use computer simulations and find nice expressions		1	
To be able to calculate transients		1	
To be able to analyze what happens at t=0		1	
Alternating current is complex		1	
It is more simple than the $j\omega$-method		1	
We don't know anything better		1	
I don't know		1	

Table 5.4: *Reasoning from the engineering students explaining the transformation of the differential equation to the frequency domain by the Laplace transform*

From Table 5.4 we can observe the repeated argumentation in the three different countries was "to more easily solve differential equations" and it is true but not enough because as we can see in the rest of the answers they never talk about the benefit that the Laplace transform gives them. It implies that they do not have other perspective (for example in other areas) than mathematic ones.

From the interviews with teachers Study 4 we find that the students see the Laplace transform as an obstacle to understanding the electric circuits instead of the tool it was meant to be. One teacher explicitly states that this is due to the problems students have in connecting the abstract concept to the real circuit.

5.4 Conclusions and implications

To teach the Laplace transform as a separate mathematical topic seems to make it an obstacle for learning. From the survey we can conclude that it is important to teach simple concepts that ought to have been understood earlier, especially at extreme values, e.g. abstracting an open circuit to an infinite resistance or a short circuit to a zero resistance.

This study show the links between theoretical issues and the real circuits have to be made explicitly, something that is also shown in other studies and in the symposium Interaction in Labwork - linking the object/event world to the theory/model world.

It is common that students learn to make mathematical operations without understanding what they are doing. They just repeat procedures that they have learned to solve the problem.

The Laplace transform is one of the many fields that have a teaching contents where it is very easy to disassociate the form and the meaning; the application and understanding of mechanic rules. The idea of some students concerning the Laplace transform is that it is a knowledge of strict and unquestionable rules that are applied to problems with just one solution, problems very far of the reality.

The disconnection between the application and understanding of procedures in specific situations can be dreadful in engineering education because some students think: mathematics is not necessary understand but it is necessary to know the adequate procedure to solve the problem. For this reason some students use superficial techniques to solve specific circumstances and there is not estrange to notice not motivation and absurd to make just mathematic calculus to pass the subject.

It is necessary understand the process: "to go" and "come back" between the formal character, the strict mathematic language and it intuitive and contextual meaning.

We relate our results in this study with the work of Dubinsky (1996) who considers it necessary to develop a theory about mental process, to explain what happend in the mind of students. He says that an action is a transformation of objects that a person perceive as something external. A person who only can understand a transformation as an action can only make that action, reacting to external indications that give him exact detail about the steps that he has to do. For example, a student that is not able to interpret a situation like a function, with the exception that he has a formula to obtain values, he is restricted to a concept of action of a function. In this case the student cannot make many things with this function, except to evaluate it in specific points and manipulate the formula.

It is necessary to mark that mathematics is not disconnected of calculations but it is important do not do the routine calculations without understand the reality.

The mathematic is not just a description group of elements.

Chapter 6

Study 4: Teachers Perspective about Difficulties to for Engineering Students to Understand the Laplace Transform

6.1 Method and Sample

The study consists in individual interviews with 22 teachers from different Universities: Linköping Universitiet (Sweden), Escuela Superior de Ingeniería Mecánica y Eléctrica de Zacatenco del Instituto Politécnico Nacional (México), Universitat de Barcelona (Catalonia), Escuela Politécnica Superior de Mondragon (Basque Country) and Universitat Politècnica de Catalunya (Catalonia) who are teaching topics that could be related to the Laplace transform.

6.2 Results

For each interview a corresponding transcription was made, and subsequently analyzed.

The first part consisted in to know the relevance of the Laplace transform topic in Engineering Education.

6.2.1 Importance of the Laplace transform in Engineering Education

The first question to the teachers was

1: *"What is the importance of the Laplace transform in Engineering Education?"*

The following is a summary of the areas where the Laplace transform is considered important by the teachers.

1. Importance in specific areas as
    - Automatic Control
    - Circuit theory
    - Economics (from the statistical point of view)
2. Importance as a tool
    - To solve differential equations
    - For static analysis
    - For continuous systems
3. To facilitate calculation working in the Laplace domain
4. Is fundamental to understand the systems
5. To solve problems eliminating noise, perturbations, etc
6. As a way of describing development of processes

6.2.2 Difficulties to Learn the Laplace Transform

To know the difficulties to learn the Laplace transform, from the teachers' perspective, we found three different kind of answers, where a group of teachers considered it difficult to learn this topic, another group considered it easy to learn, and the third group considered both possibilities.

After the analysis of each answer we found "key-points" that express the views of the teachers in each group. In Table 6.1 the teachers are grouped according to their views (difficult, not difficult or both), and they are compared by country.

2: *Is the Laplace transform a difficult topic to learn for engineering students?*

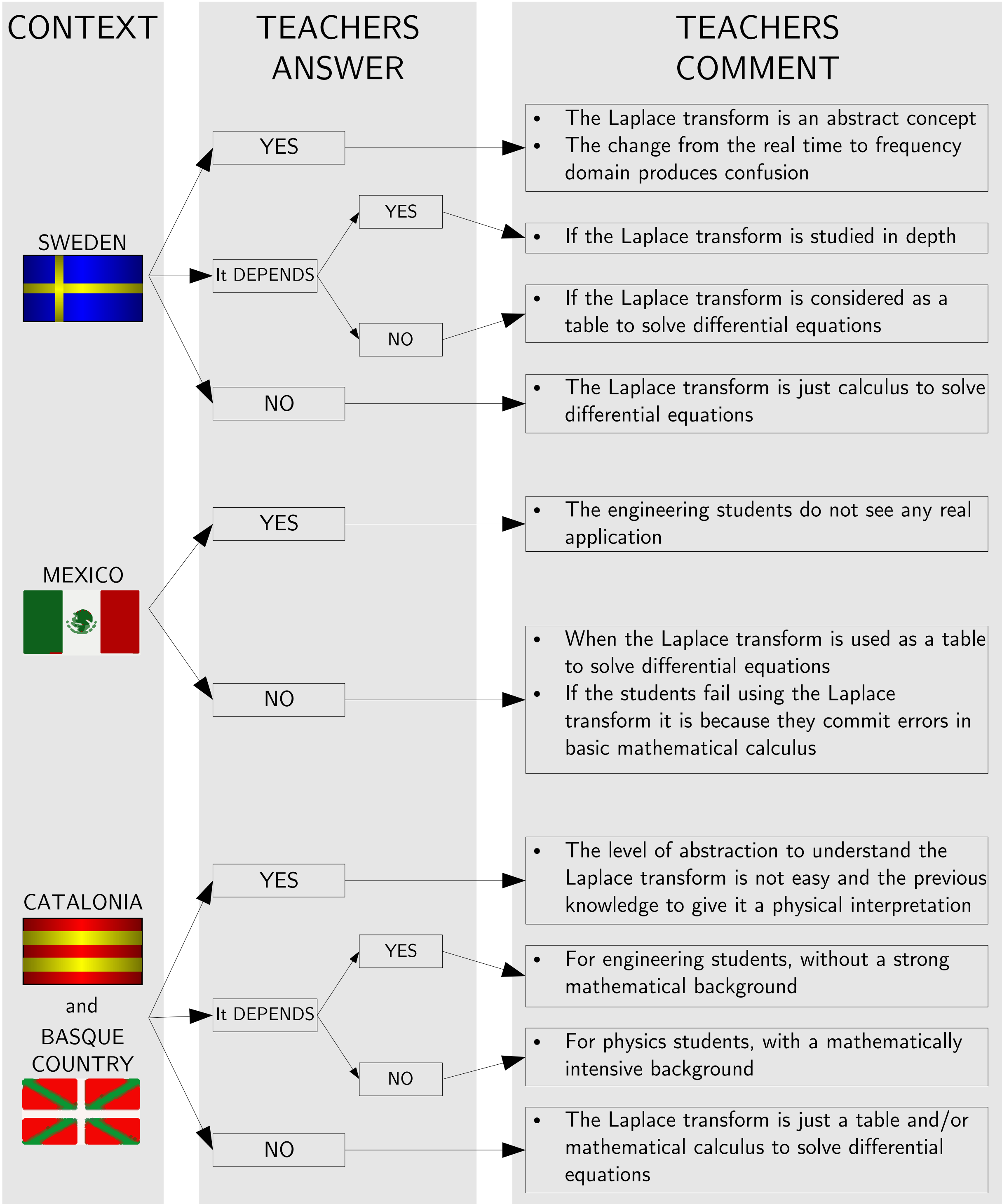


Table 6.1: *Teachers' answers about difficulties to learn the Laplace transform*

We can observe the same argument in the category "not difficult" for three different contexts where teachers said that the Laplace transform is a mathematical calculus to solve differential equations.

Table 6.2 corresponds to the explanations given by the first group of teachers (in Table 6.1), those who considered the Laplace transform topic as difficult to learn. Each sentence in the table is the key-point obtained through the analysis of their answers, and comparing the different countries.

	Mexico	Sweden	Basque Country / Catalonia
Mathematical background necessary to understand the Laplace transform		x	x
The Laplace transform is a rather abstract topic		x	x
Previous knowledge necessary to understand the Laplace transform			x
Mix with other transforms		x	
The change between the time domain and the Laplace domain		x	
To link the Laplace transform with a real event	x		
Disconnection with the focus of the study program	x		

Table 6.2: *Teachers' answers about the origin of difficulties to lean the Laplace transform*

From Table 6.2, we observe two first categories are common in Sweden and Catalonia–Basque Country, making reference to the necessary knowledge of mathematics to understand the subject and the level of abstraction.

6.2.3 Use of the Laplace transform to solve real problems

From the Table 6.1 three groups of teachers perspectives was identified:

A Teachers that consider the Laplace transform as a difficult topic to learn

B Teachers that consider the Laplace transform in both possibilities: difficult and not difficult to learn

C Teachers that consider the Laplace transform not difficult to learn

The teachers were asked to comment on the sentence make by an engineering student in Study 1:

"To use the Laplace transform formulas is not necessary to solve real problems, it is just a requirement."

Their comments are summarized below.

Perspectives from teachers in group A

The diagram in Figure 6.1 corresponds to the teachers perspective (group A).

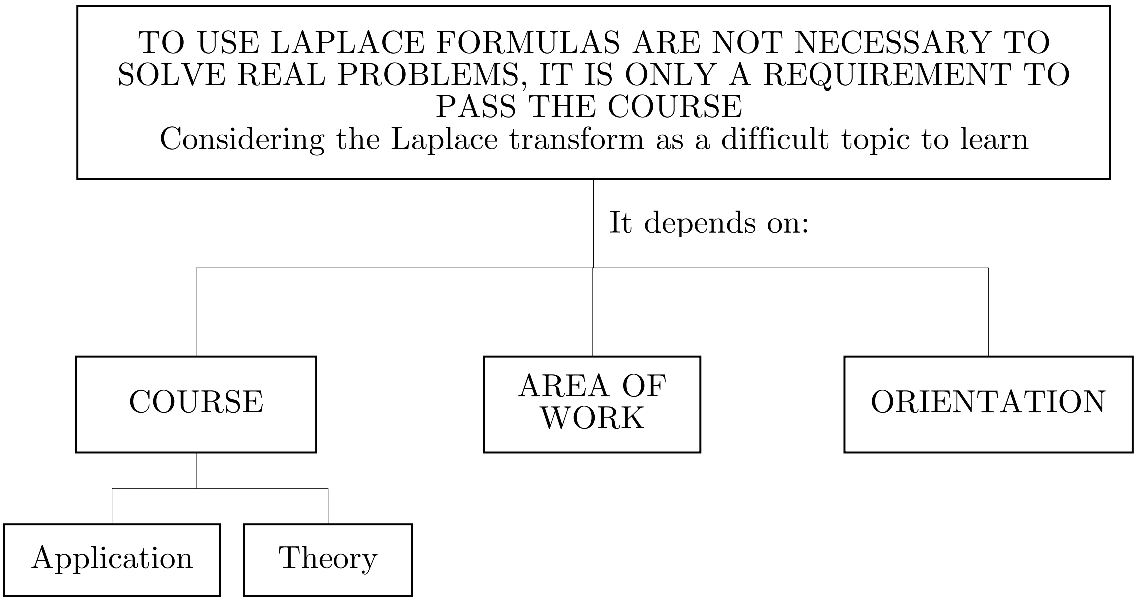


Figure 6.1: *Teachers perspective about the sentence made by engineering student seeing the Laplace transform just a requirement to pass the course*

1. Kind of Course:

    - For an **Applied** course, like control systems, the Laplace transform is used as a tool for solving real problems but it is not necessary care much about details of itself.
    - For a **Theoretical** course like transform theory, the focus is to know the fundamental of the subject in itself and not necessary its applications.

2. Area of work:

    - To solve problems working in Companies applying automatic control (for example, working in SAAB[1] with aircraft dynamics) you use the Laplace transform.
    - When solving problems using using computer technology the Fourier transform and the Z transform is used instead of the Laplace transform.

3. Orientation: It happens when student ignore the importance and application of the Laplace transform in the subject.

Perspectives from teachers in group B

All the teachers of this group disagreed with the sentence made by the student and classify the reason in two categories.

1. It depend on the case:

    - Conceptually is necessary. It's important to know the meaning of the complex variable "s", the meaning of the tables and to know how to use them and to know the difference between the time and frequency domain.
    - Not necessary of the job to do the Laplace transforms as an integral solution like mathematic procedure in the practice.
    - In the automatic control case, if you don't know anything about the Laplace Transform then the subject wouldn't work at all.

2. It depend on the job:

    - Definitely the Laplace transform is necessary for a university qualified engineer. For an electrical engineer, are necessary transforms to analyze frequency. They have to be able to solve circuit problems, looking frequency, understand questions of frequency and stability which are easily analyzed in transforms compared to time.
    - Technicians don't use it so much, they might suffice that they just use it superficially as methodology.

It is interesting notice that is making a difference between levels of study (engineers and technicians) while the first group made reference more about application and theory.

Perspectives from teachers in group C

In general the teachers from group C made the same points as the teachers in group B; all the teachers expressed disagreement.

- It is a mathematical tool but it is possible to use it directly for filters.

[1]Company that produces aircrafts in Sweden

- It is necessary for automatic control.
- The student ignores the importance or use of the Laplace transform.
- It depends:
  1. In Static System is not necessary.
  2. In Dynamic systems, the transitory part is more easy solve it using the Laplace transform than differential equations.

Though the teachers share the same views as the teachers from groub B, they also add that when the students ignore the utility of the tool (the Laplace transform) then they loose the focus and sometimes the interest.

6.2.4 Application of the Laplace transform

The previous statement by the student in Study 1 was focusing on the Laplace transform as only a requirement in the study plan. The following sentence from the same study focused in its application and we found different kind of answers.

"I do not see any application of the Laplace transform, they are just mathematical operations!."

Tables 6.3 and 6.4 has the relevant transcriptions of the interviews corresponding to teachers who consider the Laplace transform a difficult topic to learn (group A).

Interview	Teachers Answer	Comment
I4	...It seems very strange ... it's like to say: Laplace transform isn't very important... It depends on what program you are using, in applied physics and electrical engineering is kind of theory like electronic circuits;... then the subject is very important; but if you take Laplace transform courses, just like something you have to take as an engineer, but really you are in the mechanics or something else, then maybe you don't really see the need for it. But ...both in physics and electrical engineering, transforms are different in different ways, it's really important. So, for me it's insult... I guess in that kind of course, if you have a course in Laplace transform, in a program, but you don't use it very much often, for example mechanics or whatever, you have to introduce real problems, I mean, real life problems	*It is possible observe that this discourse remarks 2 points:* *1. The importance of the Laplace transform, over all in physics and electric engineering.* *2. To introduce real life problems in subjects where the Laplace transform is used more seldom.*
I2	I think... we as teachers are in some way bad prepared to point out the powerfulness of the transform in some sense for practical problems, I think we have too few practical problems to show the students...as a teacher is always easy to use the standard and schools examples...and that is of course for reason because the practical real problems for instance at SAAB[2] they are very complicated... if you have a lecture two hours I think you have problems to present the problem and then make any solution within those two hours... that is why is more used school examples and of course students have problems to see what you really used it for.	*The time that the programs are designed is not enough to solve other kind of problems (more practical application).*

[2]Company that produces aircrafts in Sweden

(continued)

Interview	Teachers Answer	Comment
I6	I don't think that he understood what the Laplace transform is because....it is difficult to understand what is behind they most to use, usually when they arrive to the late course is the understand why they need to study the Laplace transform, when they study in the transform theory course I don't think that they understand too much why they have to study it but it's the same when you study derivative in calculus you don't understand immediately why you need it but they should understand late why before finish. You don't need a course of transform theory if it is only a tool	*It suggests that to learn the Laplace transform is an implicit process not immediately to understand.*

Table 6.3: *Transcriptions from the interviews with the teachers, regarding the application of the Laplace transform*

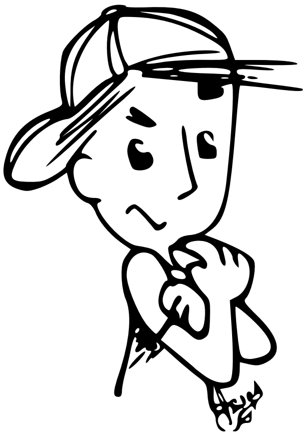
Table 6.4 shows a very important explanation for the way that the teacher explain how it is possible to physically apreciate the Lapalce transform, and replying the students statement. We decided to show it, respecting its original way, but also including a translation.

Interview	Teachers Comment	Translation
I9	Bueno, es que si tu lo analizas fisicamente no lo vas a ver ...pero si tu quieres saber ¿por qué se cae algun puente cuando entra en resonancia?, si tu quieres saber ¿por qué un motor se quema?,¿ por qué un controlador no puede llegar a la estabilidad? lo puedes determinar mediante los polos que son directamente obtenidos de la transformada de Laplace de un sistema; pero asi fisicamente, si tu me preguntas ¿qué es (1/s)? Fisicamente yo sé que es un integrador pero tu no lo vas a ver jamás	*Well, if you analyze it physically, you are not going to see ... but if you want to know why is a bridge fallen when it is in resonance? If you want to know why a motor burns?, why a controller cannot get stability? You can determine it by means of the poles that are directly obtained from the Laplace transform of a system; but this way if you ask me, what is (1/s) physically? Physically I know that it is an integrator but you are never going to see*

Table 6.4: *Transcriptions from the interviews with the teachers, regarding the application of the Laplace transform*

6.2.5 Importance of the Laplace transform for the future profession

The following statement was made by a student in Study 1:



"The Laplace transform is just a requirement to pass the course and unnecessary for his future job."

The following is the result of the analysis of the answers from the teachers where we show the most important idea that the teachers expressed. In this aspect we make a classification of the teachers according to the group that was according to their views (see Table 6.1), but we did not consider it necessary to make the comparisson between the countries.

Perspectives from teachers in group A

It depends of the job where the student will work.

Because:

As a designer of electronic products and/or mechanic machines, perhaps he will never solve anything using the Laplace transform "by hand" because there are tools able to do it like Maple, Matlab, Mathematic, etc. But he will use much theory of the Laplace transform.

Perspectives from teachers in group B

The expression is a wrong idea.

Because:

Perhaps the student will not have to do all the mathematical calculus in detail but he has to know conceptually the meaning of the Laplace transform to use it as a tool or an easier way to solve problems in other areas. But it is necessary to develop the all mathematical calculus related to the Laplace transform and have an understanding of it before it is possible to use it or apply it to solve problems.

Perspectives from teachers in group C

The expression is not true.

Because:

The Laplace transform is a system of solution very easy, it has application in areas as Automatic Control and to solve electric circuits using only differential equations is complicated, for example 3rd order systems (systems of triple integral or triple derivate or big systems of equations, etc.), then with the Laplace transform is possible to get the transfer function and to introduce in a simulation system and the problem is solved in a more easy way.

In the work profession, instead to do all these calculations, the engineer will consult a book of tables where he can tabulate the solutions; but he has to know what it is, like using integrals or differential equations that they learn in previous courses.

6.2.6 Suggestions from Teachers regarding Difficulties in Learning the Laplace Transform

Table 6.5 shows the suggestions to solve the difficulties and to help engineering students to understand the Laplace trnasform (from the experts' point of view).

	Mexico	Sweden	Basque Country / Catalonia
To explain the benefits of the Laplace transform and link with any real application		1	1
To make the students notice the necessity of using the Laplace transform to more easily solve an electric circuit problem than by doing it with differential equations	2	1	1
To solve electric circuits problems with some program of simulation	1		
To make the students to develop projects (like physical simulators) using the Laplace transform	1		1
Theory should be taught (to mature it) before any application		2	
In Automatic Control the students do not have any problems		1	

Table 6.5: *The Teachers' perspectives about solutions to solve difficulties to learn the Laplace transforms*

6.2.7 Teacher View on Interaction between Physics, Mathematics and Technology in the Laplace transform

Model of analysis

In the process of learning the Laplace transform many factors are involved, and three of them are very important:

Mathematics : all the elements that describe the Laplace transform.

Physics : the Laplace transform as a part in the nature.

Technology and/or application : the roll of the Laplace transform in different areas – like a tool in economics or automatic control.

We are interested in knowing the interaction or links among these concepts in the process of learning from the perspective of experts. Therefore, some of the questions in the interviews was focused on knowing how the teachers relate, or link, these aspects, in the context of the Laplace transform.

In the process of learning of the Laplace transform (students solving problems), how are these three different aspects related?

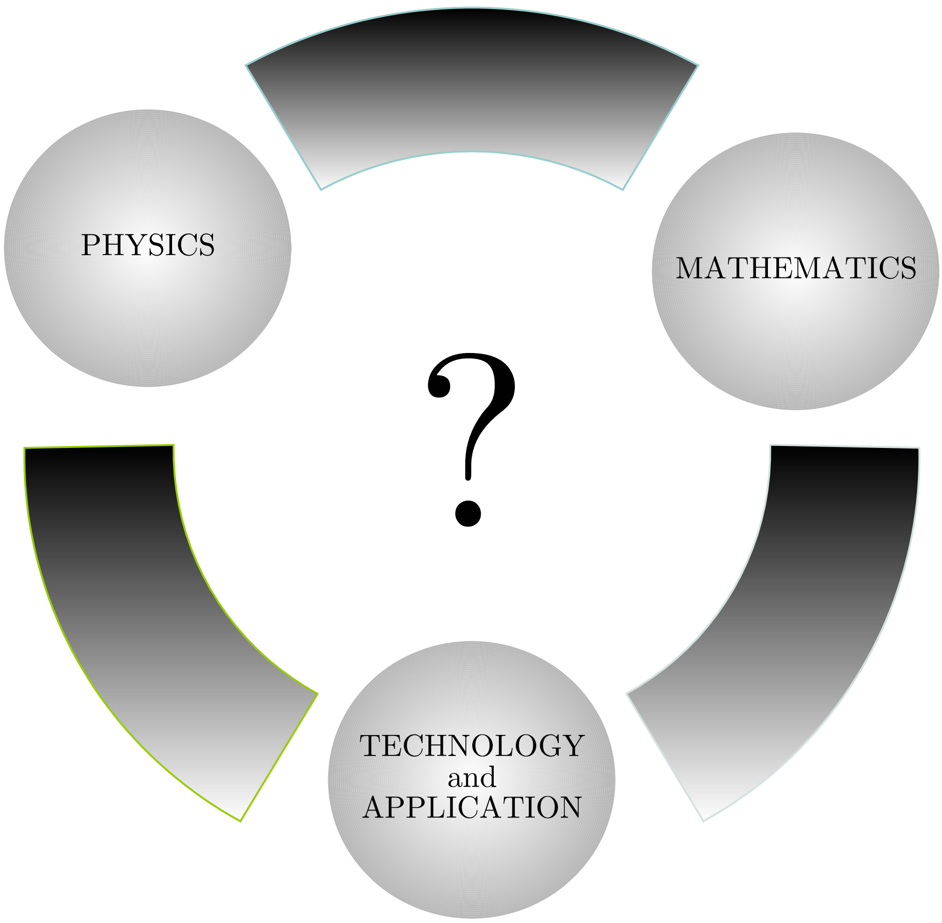


Figure 6.2: *The three aspects involved in learning the Laplace transform*

We show the analysis of five teachers from the interviews, that we consider relevant in the sense that they relate every aspect with the Laplace transform. The answers was completely different among them.

Teacher from Interview 5 (I5)

Mexico

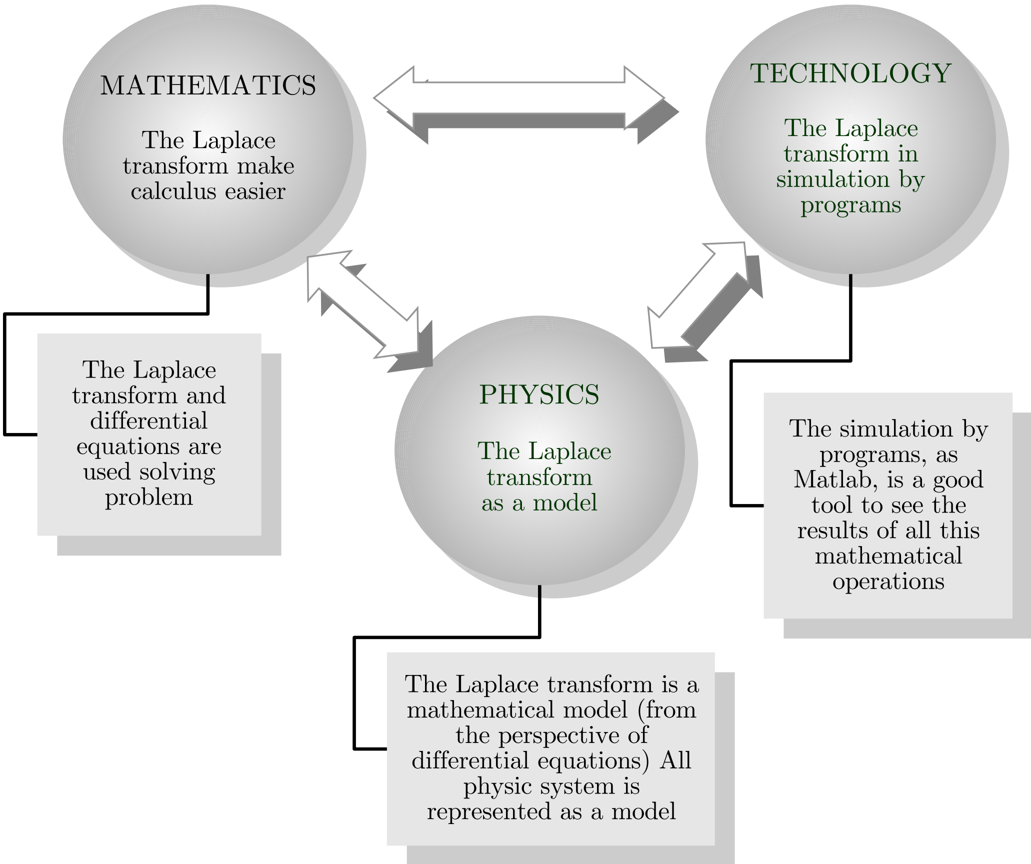


Figure 6.3: *Diagram of Relations for Teacher from Interview 5*

In automatic control students can use the Laplace transform to do more easy mathematical calculus but is not the only alternative, they can also use differential equations but the process of solve became more complicated. The Laplace transform is more use as a mathematical model.

In the physics aspect, the Laplace transform is a model and all physic system you have to represent as a model. Then from the perspective of differential equation the Laplace transform is a mathematical model.

The program of simulation correspond to technological an application part of the Laplace transform; for example students ask: *"where I can see the results of that mathematical operations?"* So, the simulation by programs is a good alternative. And talking about application, Matlab is the part of Bode diagrams in simulation.

Teacher from Interview 17 (I17)

Mexico

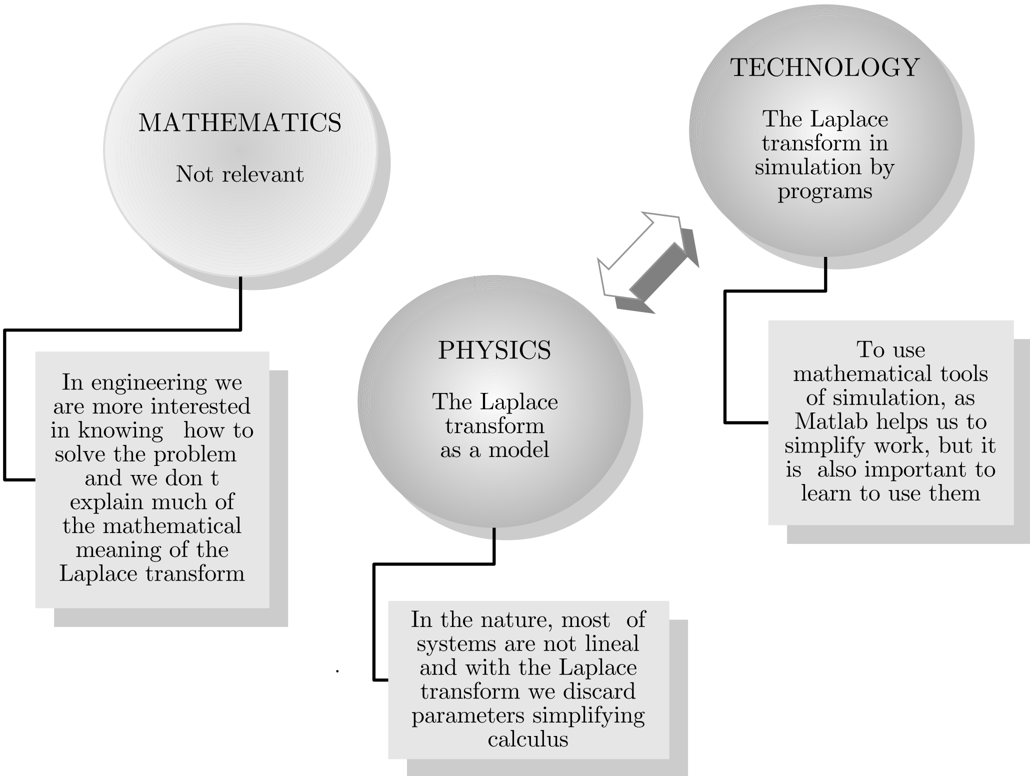


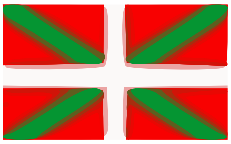
Figure 6.4: *Diagram of Relations for Teacher from Interview 17*

For engineering student, in high levels, it is a requirement to have a good mathematical background. They have to know what the Laplace transform is. In, for example, automatic control it is more relevant to get specific results and not the meaning of tool we use.

The application is important because in the nature, most systems work in a non-lineal way, and with the Laplace transform is possible to rule out parameters or characteristics, simplifying mathematical evaluations and calculus.

Not many tools of mathematical analysis simulate this process; but when we talk with students about zeros, poles, etc, then we do it in Laplace domain; when we talk about frequency (to increase or reduce frequency) then we do it in Laplace domain (not in time domain) and it simplifies and lets us relate to other tools, as oscilloscopes. For this case, the Matlab program is a good tool of simulation, but it is also important to learn to use it.

Teacher from Interview 8 (I8)



Basque Country

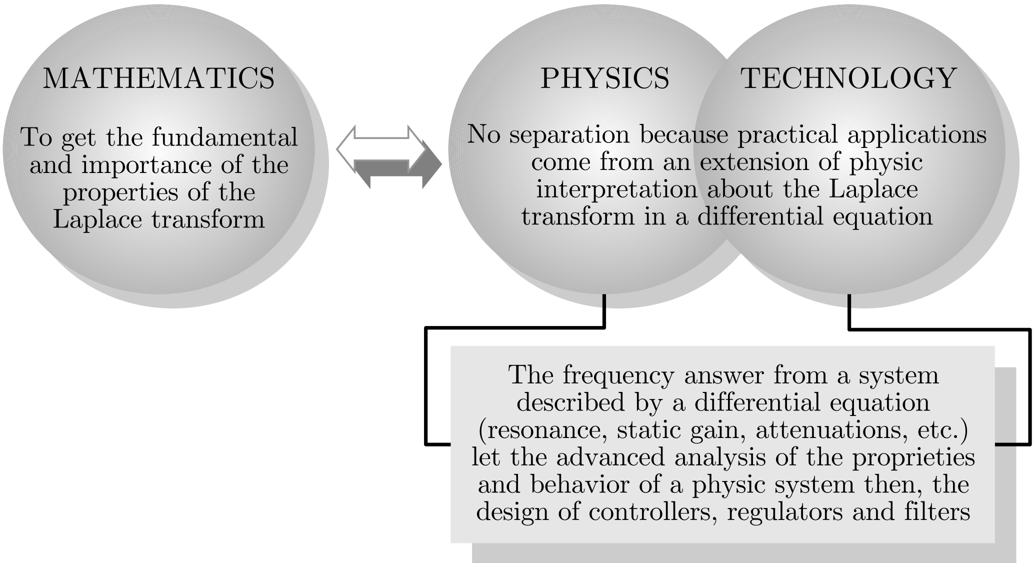
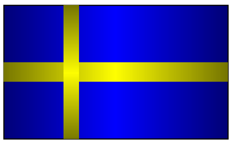


Figure 6.5: *Diagram of Relations for Teacher from Interview 8*

It is important to get the mathematical base (fundamentals) of the Laplace transform and to know its properties solving lineal differential equations (polynomial relation). It can be developed in detail by studying every property individually and by analysing advantages and disadvantages to solve differential equations.

It shouldn't be a complete separation between technological and physical point of view because practical applications come from an extension of physic interpretation about the Laplace transform in a differential equation. The Fourier transform is related with adding of sinusoidal signals and the Laplace transform with adding of "absorb" sinusoidal signals (exponentials). The relation among time domain, frequency and the Laplace transform.

Teacher from Interview 15 (I15)



Sweden

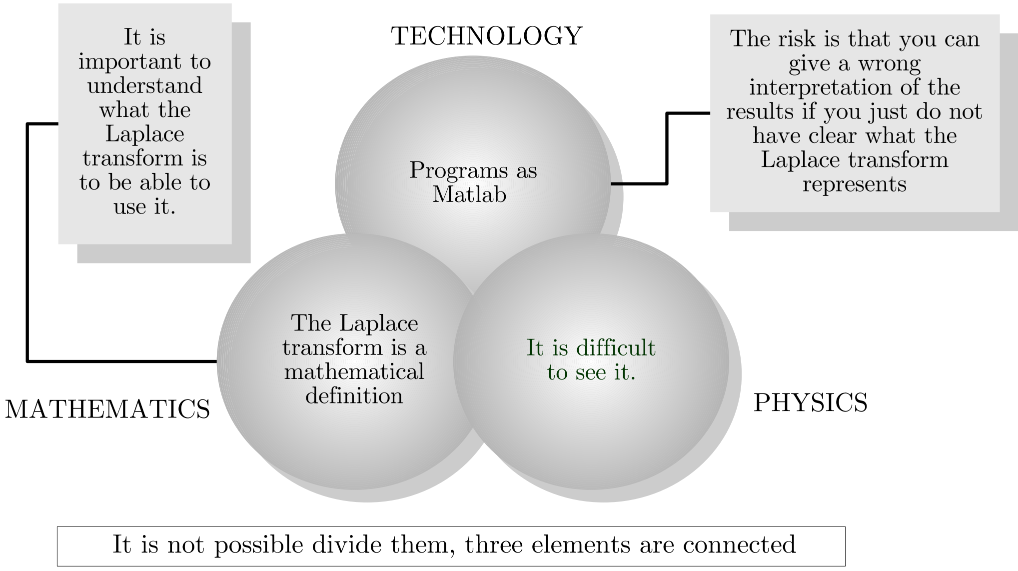


Figure 6.6: *Diagram of Relations for Teacher from Interview 15*

It is very difficult to divide in three aspects because are all connected and it is not possible to make a clear division.

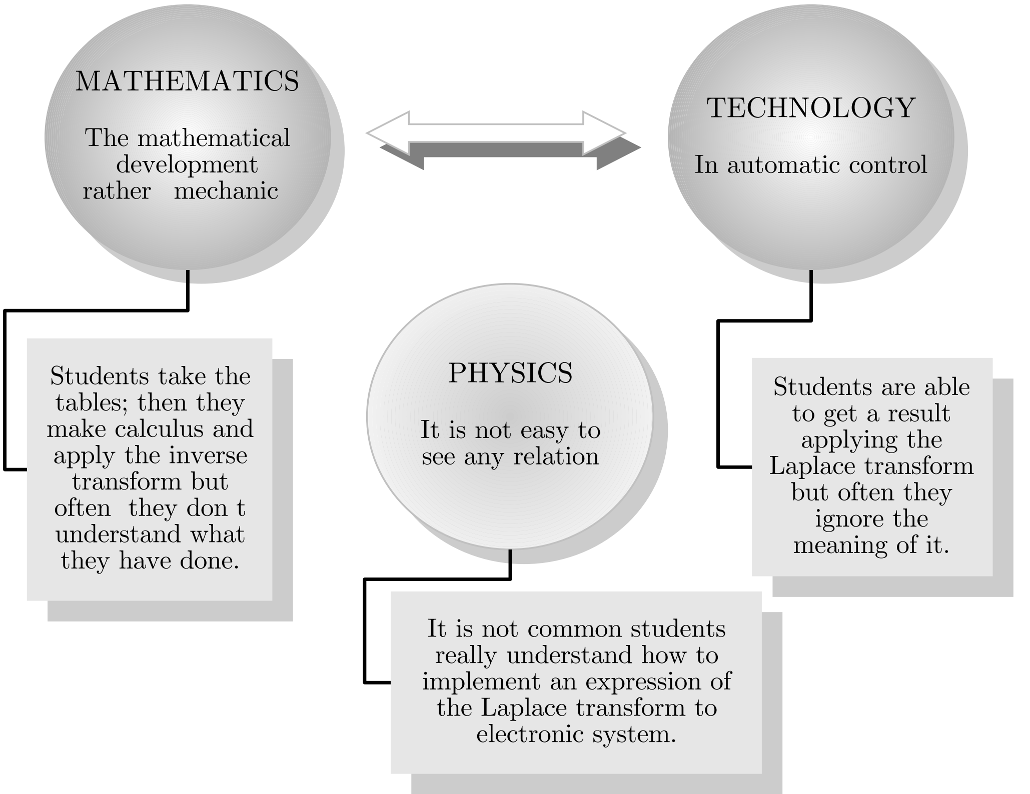
Firstly, it is important for students to understand what the Laplace transform is to be able to use it. The Laplace Transform is a mathematical definition with all the properties and then it makes it possible to change and to study some problems in Laplace domain instead of time domain that can be very much more difficult. Because of the transfer function it becomes much easier to study the properties of the system in terms of Laplace.

It is also possible to use some program, for example Matlab, but it is only way of computation. Matlab only makes computation faster, so that you don't have to do them by hand. But still one needs to understand what is behind. The risk is that the interpretation of the results will be wrong if you don not have a clear understanding of what the Laplace transform represents.

It is important to understand why and how to use the Laplace transform then it is possible to use a program.

Teacher from Interview 12 (I12)

Mexico



In automatic control we do not study the Laplace transform, we just apply it.

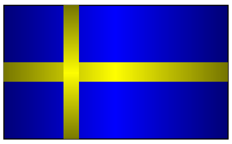
Figure 6.7: *Diagram of Relations for Teacher from Interview 12*

The mathematical development is rather mechanical, for example, students take the Laplace transform tables, do the transformation, then they realize the calculations an finally they do the inverse transformation but at the end they often do not know what they did.

In, at least, automatic control students notice that if they concern with a pole with one ”s” or with a zero they can modify the behavior of a controller, but often they ignore that on having affected this pole or this zero they are adding a system, not modifying what already they have made.

The Physical relation is because not all students really deal how to implement an expression of the Laplace transform to an electronic system.

Teacher from Interview 4 (I4)



SWEDEN

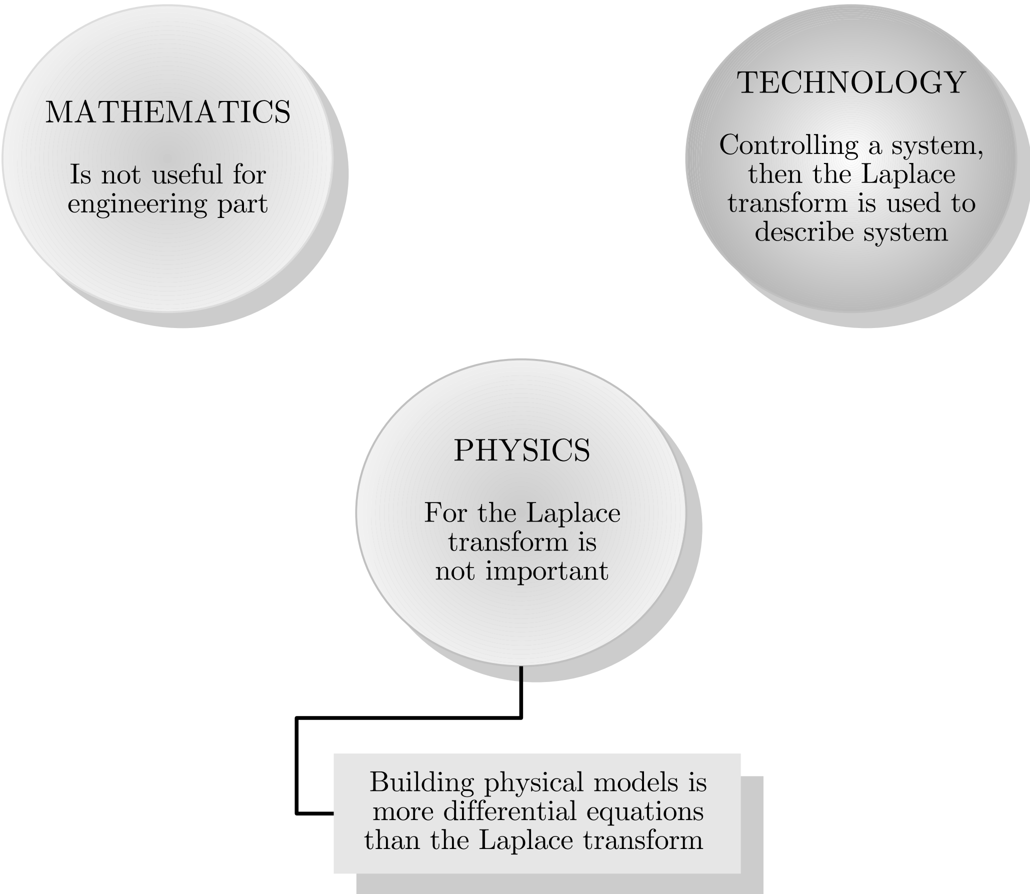


Figure 6.8: *Diagram of Relations for Teacher from Interview 4*

From the mathematical aspect, with the Laplace transform is possible do a lot of mathematical details and not useful for engineering.

Physics is more about differential equations than the Laplace transform. When building physical models, differential equation is used as the way of describing systems. In automatic control the Laplace transform can be used, but it is not necessary for all the automatic control. Differential equations can be used instead. So from this perspective the Laplace transform is not important.

An application can be trying to control something then the Laplace transform is fundamental for describing the system

6.3 Synthesis

To summarize some of the important points made by the teachers we present the following extracts, that we consider that they are this.

To teach the Laplace transform as a separate mathematical topic seems to make it an obstacle for learning.

As we can see from the results we can observe that:

1. It is important the focus where the Laplace transform is taught, because it could cause any kind of confusion for the students when they have to apply it in an specific field.

Interview	Teachers Comment	Translation
I8	Si el único objetivo es ser capaz de resolver ecuaciones diferenciales, estaría de acuerdo con el alumno, al decir que aprender la transformada de Laplace es sólo un requisito para pasar el curso, pero en la asignatura de circuitos, lo fundamental es hacer ver al alumno la utilidad que tiene la transformada de Laplace a la hora de resolver el circuito, pero sobre todo para interpretar el comportamiento del circuito para un caso general sin tener que resolverlo y dar bases para el diseño de circuitos, por ejemplo: filtros	*If the aim is just to be able to solve differential equations, I would agree with the student, to say that to learn the Laplace transform is only a requirement to pass the course, but in the subject of circuits is fundamental to make see the student the utility that the Laplace transform has at the moment of solving the circuit, but especially to interpret the behaviour of the circuit for a general case without solve it and to give bases for the design of circuits, for example: filters*

Table 6.6: *Comments from teacher interviews*

And:

Interview	Teachers Comment
I4	... the most important way or motivation in the students is to explain why they benefit from learning this staff that you use it in the following courses ...

Table 6.7: *Comments from teacher interviews*

Another important aspect that we need to know is:

Interview	Teachers Comment	Translation
I9	... Un ejemplo todavia mas sencillo: como las raices cuadaradas, es decir, nos enseñan a hacer raices cuadradas en el instituto o en la enseñanza basica pero al cabo de unos años practicamente a todos se nos ha olvidado como hacer la operacion de la raiz cuadrada, mira te aseguro que a mi se me ha olvidado tambien y eso que tambien he sido profesor de matematicas, pero sin embargo lo que no se nos puede olvidar es el concepto de que significado tiene la raiz cuadrada; como poder trabajar con areas y con longitudes. Entonces para poder conceptualmente llegar a entender el significado pues uno tiene que trabajar previamente y tiene que primero haber hecho y haber trabajado con las operaciones y entonces de esa manera uno llega mas facilmente al concepto y lo importante con el tiempo quede el concepto y uno sepa instrumentalmente utilizarlo	*An simple example is: the squared root, that is to say, we learned to do squared roots in the basic education but after a few years practically is forgotten to do the operation of the square root, I have forgotten too, and I have been a teacher of mathematics, but I cannot forget the meaning of the square root concept and to be able to apply it in areas and with lengths. Then, to understand conceptual meaning, it is necessary to work it before with mathematical operations and on this way is more easily to understand the concept and the important thing is that, by the time, the concept keeps and you can use it instrumentally*

Table 6.8: *Comments from teacher interviews*

2. The second point it is somewhat related to the first because when the Laplace transform is mixed with the other transforms without specific focus, it could cause a problem for engineering students when they have to apply it in on specific problem:

Interview	Teachers Comment
I4	... there are so many different transforms that they are almost the same but not completely the same and ... are easily mix them ... I think that is important to try to see the connections but also the separations between different transforms

Table 6.9: *Comments from teacher interviews*

and:

Interview	Teachers Comment	Translation
I19	... un alumno mío me dijo: -¡No profesor. Es que la transformada de Laplace es como una caja negra que nadie sabe que hace y nadamas sirve para torturar a los alumnos!- Cuando conoces la utilidad que tiene cualquier transformada ya sea Laplace, Fourier, la que sea, te das cuenta como te facilita la vida a la hora de resolver los problemas	*... One of my students told me: -No teacher. The Laplace transform is like a black box that nobody knows and it only serves to torture the students! When you know the usefulness that any transform has, as Laplace, Fourier, anyone else; you realize how it facilitates you the life at the moment to solve problems*

Table 6.10: *Comments from teacher interviews*

3. The Laplace transform is considered a concept not easy to understand without previous mathematical knowledge. It is not on a basic level of understanding, where the students have to use other resources of thinking. Other important aspect involved in the Laplace transform is that when the focus is not clear, it is not easy to understand (in the beginning) the change of "worlds", in this case to move from the time domain to the frequency domain. The experience and the results of this research show that it has not been emphasised in detail when the Laplace transform is taught.

Interview	Teachers Comment	Translation
I9	El dominio del tiempo a priori es mas sencillo de entender porque es la realidad, el dominio de la frecuencia compleja al fin y al cabo es una construccion matematica que ayuda conceptualmente a resolver los problemas pero que realmente no tiene una significacion directa; es decir, siempre tenemos que antitransformar al dominio del tiempo para realmente recuperar las señales que vamos a observar en la realidad.	*The time domain is more simply to understand because it is the reality, the complex frequency domain is a mathematical construction that helps to solve conceptual the problems but it does not have a direct meaning; is to say, always we have to apply the Laplace transform inverse to the time domain to recover the signals that we are going to observe in the reality.*

Table 6.11: *Comments from teacher interviews*

4. Working with simulation of the Laplace transform, Matlab is the program more used.

6.4 Conclusions

A last important aspect is the "application". In engineering education is important that students have a view of the relation between theory and the real world. It is demonstrated that not making this connection will weaken their interest and motivation, promoting the simulation or practical activities.

> *"The Laplace transform has also been applied to various problems: evaluation of payments, reliability and maintenance strategies, utility factions of analysis, choice of investments, assembly line and queuing system problems, theory of system and element behaviors, investigation of the dispatching aspect of job-shop scheduling, assessing econometric models and may others areas."* Yu and Grubbström (2001)

Also the Laplace transform has been applied to the evaluation of payments, to reliability and maintenance strategies, to utility function analysis, to the choice of investments, to assembly line and queuing system problems, to the theory of systems and elements behavior, to the investigation of the dispatching aspect of job/shop scheduling, for assessing econometric models, to study dynamical economic systems, Grubbström and Yinzhong (1990).

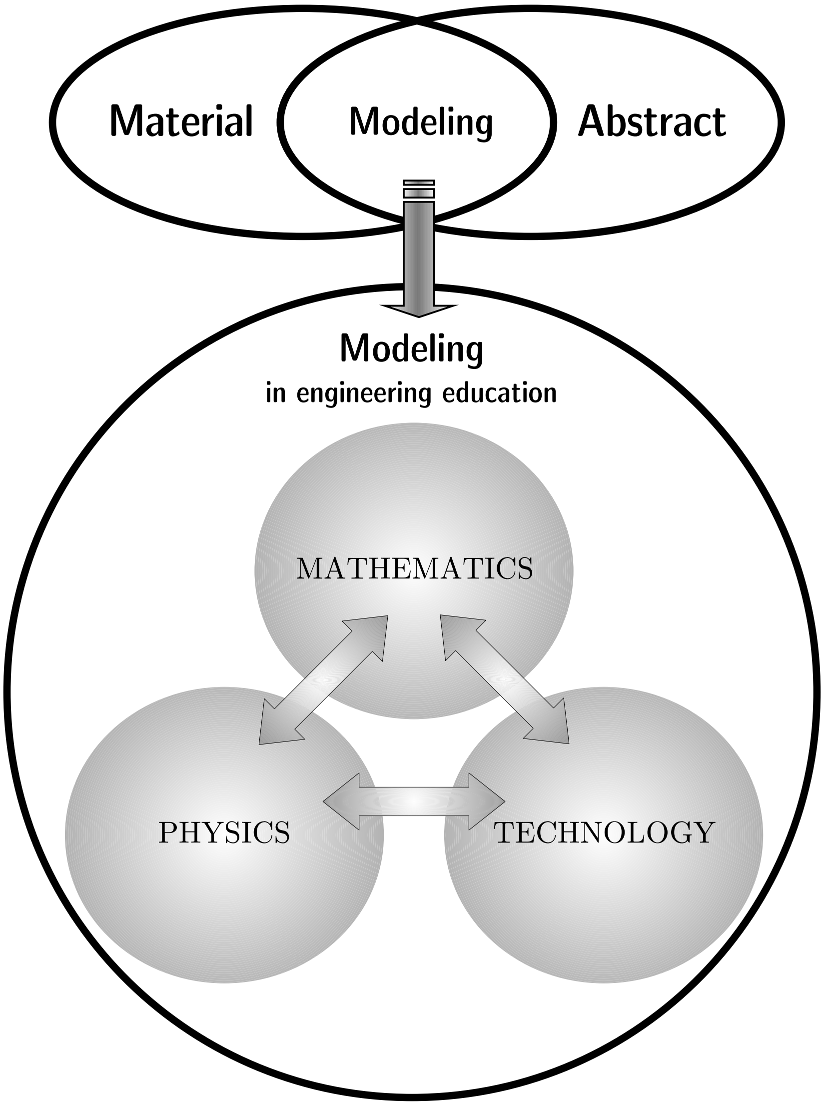


Figure 6.9: *Modeling in Engineering Education*

The Laplace transform is basically mathematics but mathematics can be interpreted and in the physical world, the experiments and empirical data, etc. are represent with a model to analyze it and that data became abstract. It is not the real world; it's a description of kind of process as it might take place. The theoretical process might take place in the real world but, is the solution that you get with the computing using the Laplace transform, is the solution in the process that we find in the real world.