CHAPTER 23
VIBRATION AND SHOCK

Wayne Tustin
Equipment Reliability Institute
Santa Barbara, California

23.1 VIBRATION

In any structure or assembly, certain whole-body motions and certain deformations are more common than others; the most likely (easiest to excite) motions will occur at certain natural frequencies. Certain exciting or forcing frequencies may coincide with the natural frequencies (resonance) and give relatively severe vibration responses.

We will now discuss the much-simplified system shown in Fig. 23.1. It includes a weight W (it is technically preferred to use mass M here, but weight W is what people tend to think about), a spring of stiffness K, and a viscous damper of damping constant C. K is usually called the spring rate; a static force of K newtons will statically deflect the spring by δ mm, so that spring length l becomes $\delta + l$. (In "English" units, a force of K lb will statically deflect the spring by 1 in.) This simplified system is constrained to just one motion—vertical translation of the mass. Such single-degree-of-freedom (SDF) systems are not found in the real world, but the dynamic behavior of many real systems approximate the behavior of SDF systems over small ranges of frequency.

Suppose that we pull weight W down a short distance further and then let it go. The system will oscillate with W moving up-and-down at natural frequency f_N, expressed in cycles per second (cps) or in hertz (Hz); this condition is called "free vibration." Let us here ignore the effect of the damper, which acts like the "shock absorbers" or dampers on your automobile's suspension—using up vibratory energy so that oscillations die out. f_N may be calculated by

$$f_N = \frac{1}{2\pi}\sqrt{\frac{Kg}{W}} \tag{23.1}$$

It is often convenient to relate f_N to the static deflection δ due to the force caused by earth's gravity, $F = W = Mg$, where g = 386 in./sec^2 = 9807 mm/sec^2, opposed by spring stiffness K expressed in either lb/in. or N/mm. On the moon, both g and W would be considerably less (about one-sixth as large as on earth). Yet f_N will be the same. Classical texts show Eq. (23.1) as

$$f_N = \frac{1}{2\pi}\sqrt{\frac{K}{M}}$$

In the "English" System:

$$\delta = \frac{F}{K} = \frac{W}{K}$$

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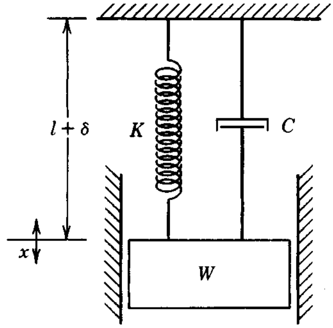

Fig. 23.1 Single-degree-of-freedom system.

Then

$$f_N = \frac{1}{2\pi}\sqrt{\frac{g}{\delta}} = \frac{1}{2\pi}\sqrt{\frac{386}{\delta}}$$
$$= \frac{19.7}{2\pi\sqrt{\delta}} = \frac{3.13}{\sqrt{\delta}} \tag{23.2a}$$

In the International System:

$$\delta = \frac{F}{K} = \frac{Mg}{K}$$

Then

$$f_N = \frac{1}{2\pi}\sqrt{\frac{g}{\delta}} = \frac{1}{2\pi}\sqrt{\frac{9807}{\delta}}$$
$$= \frac{99.1}{2\pi\sqrt{\delta}} = \frac{15.76}{\sqrt{\delta}} \tag{23.2b}$$

Relationships (23.2a) and (23.2b) often appear on specialized "vibration calculators." As increasingly large mass is supported by a spring; δ becomes larger and f_N drops.

Let $K = 1000$ lb/in. and vary W:

W (lb)	$\delta = \frac{W}{K}$ (in.)	f_N (Hz)
0.001	0.000 001	3 130
0.01	0.000 01	990
0.1	0.000 1	313
1.	0.001	99
10.	0.01	31.3
100.	0.1	9.9
1 000.	1.	3.13
10 000.	10.	0.99

Let $K = 1000$ N/mm and vary M:

M (kg)	$\delta = \frac{9.81M}{K}$	f_N (Hz)
0.00102	10 nm	4 980
0.0102	100 nm	1 576
0.102	1 μm	498
1.02	10 μm	157.6
10.2	100 μm	49.8
102.	1 mm	15.76
1 020.	10 mm	4.98
102 000.	1 m	0.498

Note, from Eqs. (23.2a) and (23.2b), that f_N depends on δ, and thus on both M and K (or W and K). As long as both load and stiffness change proportionately, f_N does not change.

The peak-to-peak or double displacement amplitude D will remain constant if there is no damping to use up energy. The potential energy we put into the spring becomes zero each time the mass passes through the original position and becomes maximum at each extreme. Kinetic energy becomes maximum as the mass passes through zero (greatest velocity) and becomes zero at each extreme (zero velocity). Without damping, energy is continually transferred back and forth from potential to kinetic energy. But with damping, motion gradually decreases; energy is converted to heat. A vibration pickup on the weight would give oscilloscope time history patterns like Fig. 23.2; more damping was present for the lower pattern and motion decreased more rapidly.

Assume that the "support" at the top of Fig. 23.1 is vibrating with a constant D of, say, 1 in. Its frequency may be varied. How much vibration will occur at weight W? The answer will depend on

1. The frequency of "input" vibration.
2. The natural frequency and damping of the system.

Let us assume that this system has an f_N of 1 Hz while the forcing frequency is 0.1 Hz, one-tenth the natural frequency, Fig. 23.3. We will find that weight W has about the same motion as does the input, around 1 in D. Find this at the left edge of Fig. 23.3; transmissibility, the ratio of response vibration divided by input vibration, is $1/1 = 1$. As we increase the forcing frequency, we find that the response increases. How much? It depends on the amount of damping in the system. Let us assume that our system is lightly damped, that it has a ratio C/C_c of 0.05 (ratio of actual damping to "critical" damping is 0.05). When our forcing frequency reaches 1 Hz (exactly f_N), weight W has a response D of about 10 in., 10 times as great as the input D. At this "maximum response" frequency, we have the condition of "resonance"; the exciting frequency is the same as the f_N of the load. As we further increase the forcing frequency (see Fig. 23.3), we find that response decreases. At 1,414 times f_N, the response has dropped so that D is again 1 in. As we further increase the forcing frequency, the response decreases further. At a forcing frequency of 2 Hz, the response D will be about 0.3 in. and at 3 Hz it will be about 0.1 in.

Note that the abscissa of Fig. 23.3 is "normalized"; that is, the transmissibility values of the preceding paragraph would be found for another system whose natural frequency is 10 Hz, when the forcing frequency is, respectively, 1, 10, 14.14, 20, and 30 Hz. Note also that the vertical scale of Fig. 23.3 can represent (in addition to ratios of motion) ratios of force, where force can be measured in pounds or newtons.

The region above 1.414 times f_N (where transmissibility is less than 1) is called the region of "isolation." That is, weight W has less vibration than the input; it is *isolated*. This illustrates the use of vibration isolators—rubber elements or springs that reduce the vibration input to delicate units in aircraft, missiles, ships, and other vehicles, and on certain machines. We normally try to set f_N (by selecting isolators) considerably below the expected forcing frequency of vibration. Thus, if our

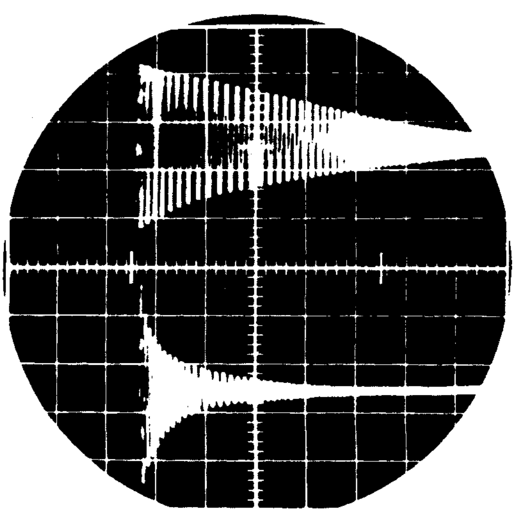

Fig. 23.2 Oscilloscope time history patterns of damped vibration.

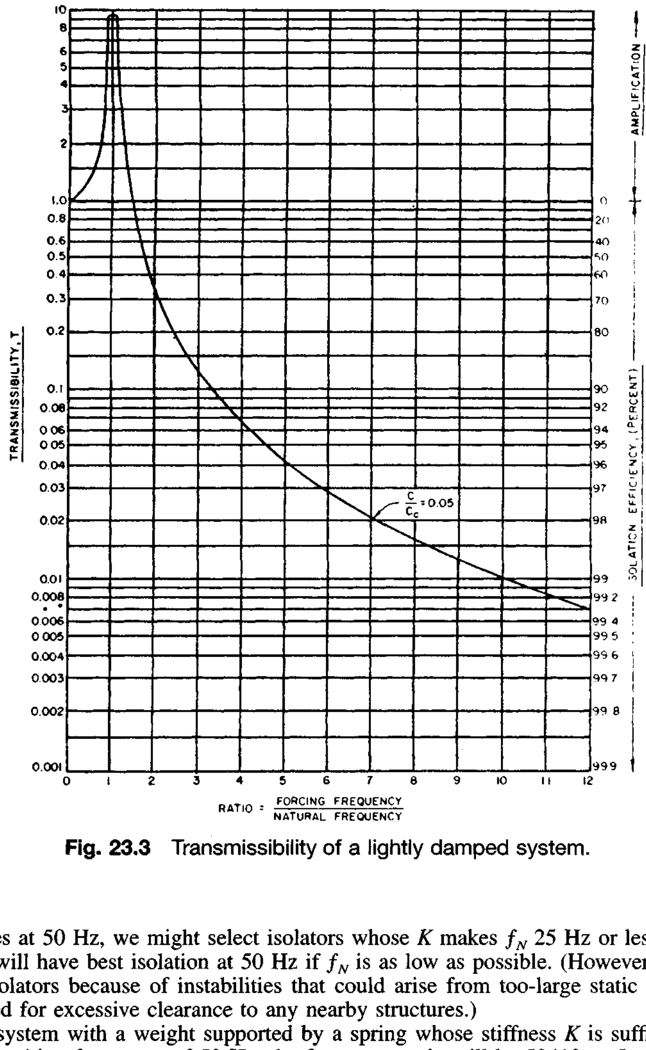

Fig. 23.3 Transmissibility of a lightly damped system.

support vibrates at 50 Hz, we might select isolators whose K makes f_N 25 Hz or less. According to Fig. 23.3, we will have best isolation at 50 Hz if f_N is as low as possible. (However, we should not use too-soft isolators because of instabilities that could arise from too-large static deflections, and because of need for excessive clearance to any nearby structures.)

Imagine a system with a weight supported by a spring whose stiffness K is sufficient that f_N = 10 Hz. At an exciting frequency of 50 Hz, the frequency ratio will be 50/10 or 5, and we can read transmissibility = 0.042 from Fig. 23.3. The weight would "feel" only 4.2% as much vibration as if it were rigidly mounted to the support. We might also read the "isolation efficiency" as being 96%. However, as the source of 50-Hz vibration comes up to speed (passing slowly through 10 Hz), the isolated item will "feel" about 10 times as much vibration as if it were rigidly attached, without any isolators. Here is where damping is helpful: to limit the "Q" or "mechanical buildup" at resonance. Observe Fig. 23.4, plotted for several different values of damping. With little damping present, there is much resonant magnification of the input vibration. With more damping, maximum transmissibility is not so high. For instance, when C/C_c is 0.01, "Q" is about 40. Even higher Q values are found with certain structures having little damping; Q's over 1000 are sometimes found.

Most structures (ships, aircraft, missiles, etc.) have Q's ranging from 10 to 40. Bonded rubber vibration isolating systems often have Q's around 10; if additional damping is needed (to keep Q lower), dashpots or rubbing elements may be used. Note that there is less buildup at resonance, but that isolation is not as effective when damping is present.

Figure 23.1 shows guides or constraints that restrict the motion to up-and-down translation. A single measurement on an SDF system will describe the arrangement of its parts at any instant. Another SDF system is a wheel attached to a shaft. If the wheel is given an initial twist, that system will also oscillate at a certain f_N, which is determined by shaft stiffness and wheel inertia. This imagined rotational system is an exact counterpart of the SDF system shown in Fig. 23.1.

If weight W in Fig. 23.1 did not have the guides shown, it would be possible for weight W to move in five other motions—five additional degrees of freedom. Visualize the six possibilities:

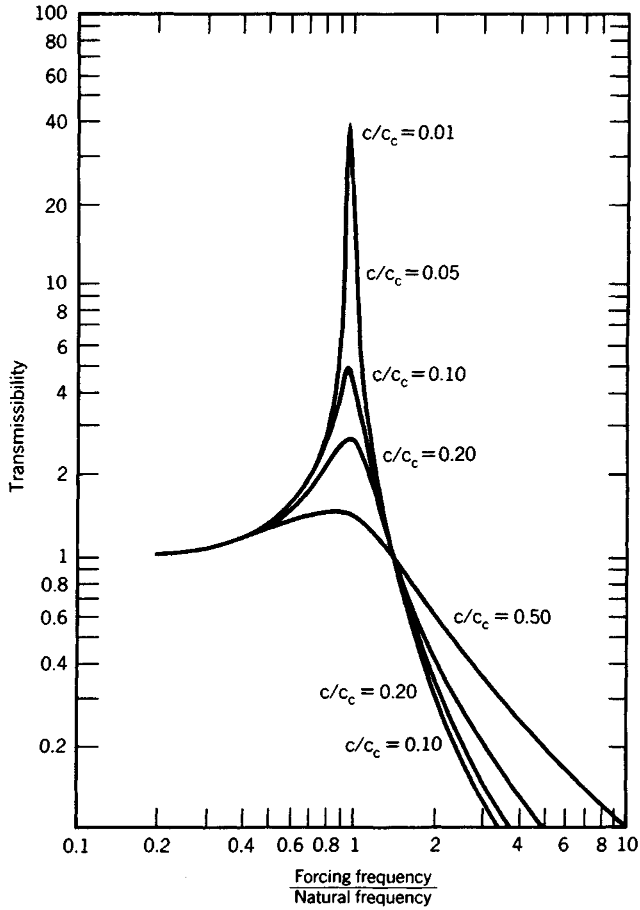

Fig. 23.4 Transmissibility for several different values of damping.

Vertical translation.
North–south translation.
East–west translation.
Rotation about the vertical axis.
Rotation about the north–south axis.
Rotation about the east–west axis.

Now this solid body has six degrees of freedom—six measurements would be required in order to describe the various whole-body motions that may be occurring.

Suppose now that the system of Fig. 23.1 were attached to another mass, which in turn is supported by another spring and damper, as shown in Fig. 23.5. The reader will recognize that this is more typical of many actual systems than is Fig. 23.1. Machine tools, for example, are seldom attached directly to bedrock, but rather to other structures that have their own vibration characteristics. Weight W_2 will introduce six additional degrees of freedom, making a total of 12 for the system of Fig. 23.5. That is, in order to describe all of the possible solid-body motions of W_1 and W_2, it would be necessary to consider all 12 motions and to describe the instantaneous positions of the two masses.

The reader can extend that reasoning to include additional masses, springs, and dampers, and additional degrees of freedom—possible motions. Finally, consider a continuous beam or plate, where mass, spring, and damping are distributed rather than being concentrated as in Fig. 23.1 and 23.5. Now we have an infinite number of possible motions, depending on the exciting frequency, the

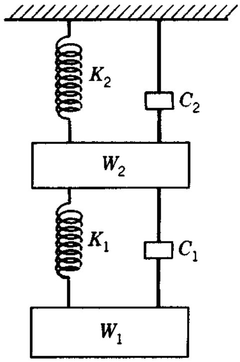

Fig. 23.5 System with 12 degrees of freedom.

distribution of mass and stiffness, the point or points at which vibration is applied, etc. Fortunately, only a few of these motions are likely to occur in any reasonable range of frequencies.

Figure 23.6 consists of three photographs, each taken at a different forcing frequency. Up-and-down shaker motion is coupled through a holding fixture to a pair of beams. (Let us only consider the left-hand beams for now.) Frequency is adjusted first to excite the fundamental shape (also called the "first mode" or "fundamental mode") of beam response. Then frequency is readjusted higher to excite the second and third modes. There will be an infinite number of such resonant frequencies, an infinite number of modes, if we go to an infinitely high test frequency. We can mentally extend this reasoning to various structures found in vehicles, machine tools, and appliances, etc.: an infinite number of resonances could exist in any structure. Fortunately, there are usually limits to the exciting frequencies that must be considered. Whenever the frequency coincides with one of the f_N's of a structure, we will have a resonance; high stresses, large forces, and large motions result.

All remarks about free vibration, f_N, and damping apply to continuous systems. We can "pluck" the beam of Fig. 23.6 and cause it to respond in one or more of the patterns shown. We know that vibration will gradually die out, as indicated by the upper trace of Fig. 23.2, because there is internal (or hysteresis) damping of the beam. Stress reversals create heat, which uses up vibratory energy. With more damping, vibration would die out faster, as in the lower trace of Fig. 23.2. The right-hand cantilever beam of Fig. 23.6 is made of thin metal layers joined by a layer of a viscoelastic damping material, as in Fig. 23.7. Shear forces between the layers use up vibratory energy faster and free vibration dies out more quickly.

Statements about forced vibration and resonance also apply to continuous systems. At very low frequencies, the motion is the same (transmissibility = 1) at all points along the beams. At certain f_N's, large motion results.

The points of minimum D are called "nodes" and the points having maximum D are called "antinodes." With a strobe light and/or vibration sensors we could show that (in a pure mode) all points along the beam are moving either in-phase or out-of-phase and that phase reverses from one side of a node to the other. Also bending occurs at the attachment point and at the antinodes; these are the locations where fatigue failures usually occur.

It is possible for several modes to occur at once, if several natural frequencies are present as in complex or in broad-band random vibration. It is also possible for several modes to simultaneously be excited by a shock pulse (and then to die out). Certain modes are most likely to cause failure in a particular installation; the frequencies causing such critical modes are called "critical frequencies."

Intense sounds can cause modes to be excited, especially in thin panels; stress levels in aircraft and missile skins may cause fatigue failures. Damping treatments on the skin are often very effective in reducing such vibration.

Figure 23.6 shows forced vibration of two beams whose length is adjusted until their f_N's are identical. The conventional solid beam responds more violently than the damped laminated beam. We say that the maximum transmissibility (often called mechanical "Q") of the solid beam is much greater than that of the damped beam. Assuming that the vibration continues indefinitely, which beam will probably fail in fatigue first? Here is one reason for using damping.

Resonance is sometimes helpful and desirable; at other times it is harmful. Some readers will be familiar with deliberate applications of vibration to move bulk materials, to compact materials, to remove entrapped gases, or to perform fatigue tests. Maximum vibration is achieved (assuming the vibratory force is limited) by operating the system at resonance.

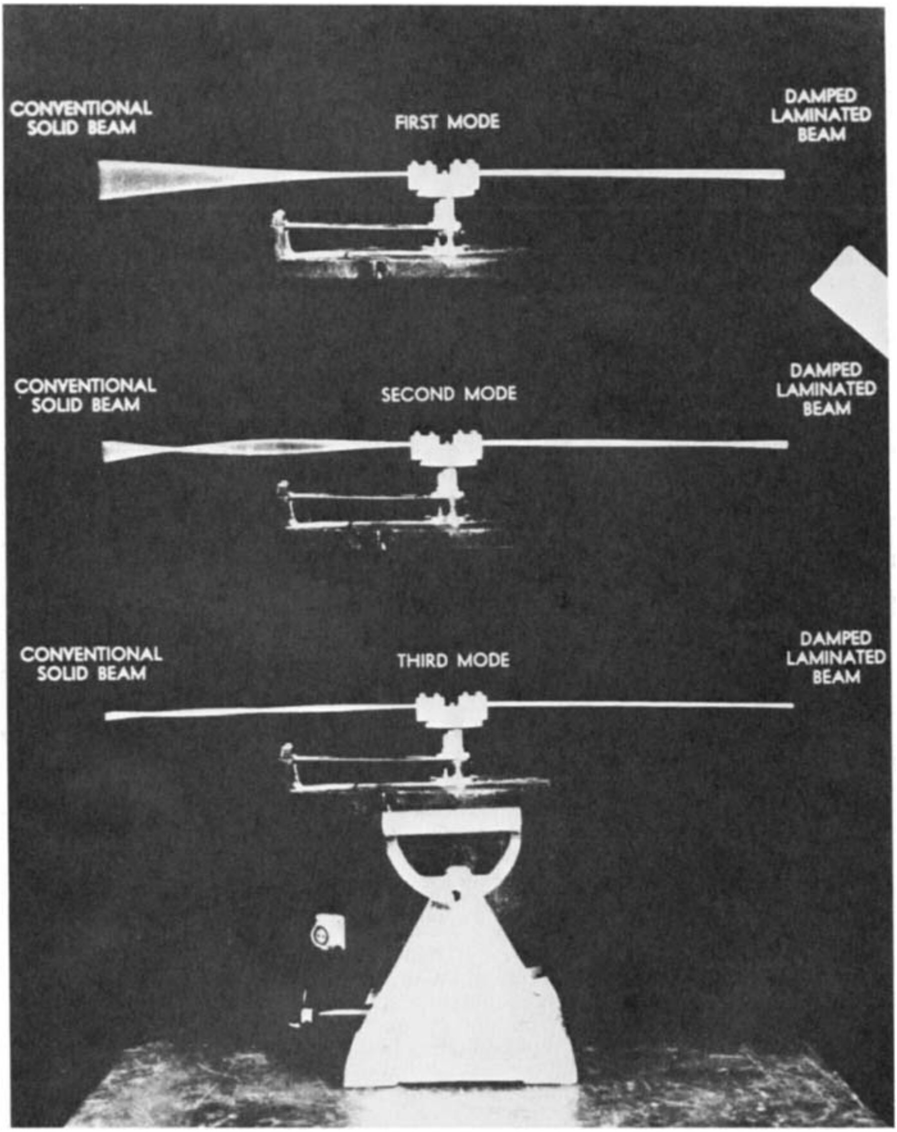

Fig. 23.6 Pair of beams excited at three different forcing frequencies.

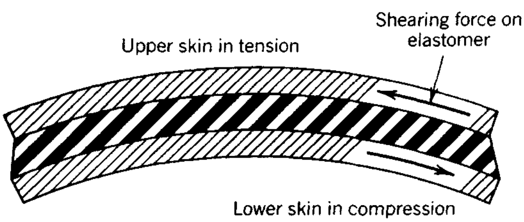

Fig. 23.7 Detail of laminated beam. (Courtesy of Lord Manufacturing Co.)

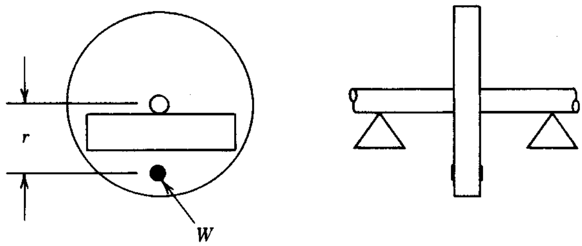

Fig. 23.8 Static balancing of a disk.

A resonant vibration absorber can sometimes reduce motion, if the vibration input to a structure is at a fixed frequency. Imagine, for example, weaving machines in a relatively soft, multistory factory building. They happen to excite an up-and-down resonant motion of their floor. (Some "old timers" claim they saw D's of several inches.) A remedy was to attach springs to the undersides of those floors, directly beneath the offending machines. Each spring supported a pail which was gradually filled with sand until the f_N of the spring/pail was equal to the exciting frequency of the weaving machine above it. A dramatic reduction in floor motion told maintenance people that the spring was correctly loaded. Similar methods (using tanks filled with water) have been used on ships. However, any change in exciting frequency necessitates a bothersome readjustment.

23.2 ROTATIONAL IMBALANCE

Where rotating engines are used in ships, automobiles, aircraft, or other vehicles; where turbines are used in vehicles or electrical power generating stations; where propellers are used in ships and aircraft—in all of these and many other varied applications, imbalance of the rotating members causes vibration.

Consider first the simple disk shown in Fig. 23.8. This disk has some extra material on one side so that the center of gravity is not at the rotational center. If we attach this disk to a shaft and allow the shaft to rotate on knife-edges, we observe that the system comes to rest with the heavy side of the disk downward. This type of imbalance is called *static* imbalance, since it can be detected statically. It can be measured statically, also, by determining some weight W at some radius r that must be attached to the side opposite the heavy side, in order to restore the center of gravity to the rotational center and thus to bring the system into static balance; that is, so that the disk will have no preferred position and will rest in any angular position. The product Wr is the value of the original imbalance. It is often expressed in units of ounce-inches, gram-millimeters, etc.

Static balancing is the simplest technique of balancing, and is often used for the wheels on automobiles, for instance. It locates the center of gravity at the center of the wheel. But we will show that this compensation is not completely satisfactory. Let us now support the disc and shaft of Fig. 23.8 by a bearing at each end and cause the disk and shaft to spin. A rotating vector force of $Mr\omega^2$ lb results, in phase with the center of gravity of the rotating system, as shown in Fig. 23.9. W is the

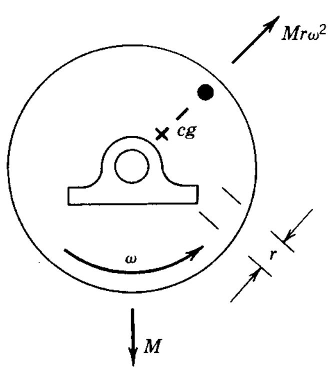

Fig. 23.9 Unbalanced disk in rotation.

total weight, ω is the angular velocity in radians per second, and r is the radial distance (inches or mm) from the shaft center to the center of gravity.

The shaft and bearings must absorb and transmit not only the weight of the rotor but also a new force, one which rotates; one which, at high rotational speeds, may be greater than the weight of the rotor.

Though your automotive mechanic may only statically balance the wheels of your car by adding wheel weights on the light side of each wheel, this static balancing results in noticeable improvement in car ride, in passenger comfort, and in tire wear, because the force $Mr\omega^2$ is greatly reduced.

A numerical example may interest the reader. Imbalance can be measured in ounce-inches (or, in metric units, gram-millimeters). One ounce-inch means that an excess or deficiency of weight of one ounce exists at a radius of 1 in. How big is an ounce-inch? It sounds quite small, but at high rotational speeds (since force is proportional to the square of rotational speed) this "small" imbalance can cause very high forces. You will recall that centrifugal force may be calculated by

$$F = \frac{Mv^2}{r}$$

where M is the mass in kilograms (or weight in pounds divided by 386 in./sec^2, the acceleration due to the earth's gravity); r is the radius in inches; and v is the tangential velocity in inches per second. We can calculate $v = 2\pi fr$, where f is the frequency of rotation in hertz and r is the radius in inches or mm. Then

$$F = \frac{M}{r}(2\pi fr)^2 = 4\pi^2 Mf^2 r = \frac{4\pi^2}{386} Wrf^2 = 0.1023Wrf^2$$

Let us calculate the force that results from 1 oz-in. of imbalance on a member rotating at 8000 rpm or 133 rps:

$$1 \text{ oz} = \tfrac{1}{16} \text{ lb}$$
$$r = 1 \text{ in.}$$

Then

$$F = 0.1023(\tfrac{1}{16})(1)(133)^2 = 114 \text{ lb}$$

If this centrifugal force of 114 lb occurred in an electrical motor, for instance, whose weight was less than 114 lb, the imbalance force acting through the bearings would lift the motor off its supposed once each revolution, or 8000 times a minute. If the motor were fastened to some framework, vibratory force would be apparent.

In most rotating elements, such as motor armatures or engine crankshafts, the mass of the rotor is distributed along the shaft rather than being concentrated in a disk as shown in Figs. 23.8 and 23.9. If we test such a rotor as we tested in Fig. 23.8, we may find that we have static balance, then the rotating element has no preferred angular position, and that the center of gravity coincides with the shaft center. But when we spin such a unit, we may find severe forces being transmitted by shaft and bearings. Obviously we are not truly balanced; since this new imbalance is apparent only when the system is rotated, we call it *dynamic imbalance.*

As a simplified example of such a system, consider Fig. 23.10. If the two imbalances P and Q are exactly equal, if they are exactly 180° apart, and if the two disks are otherwise uniform and identical, this system will be statically balanced. But if we rotate the shaft, each disk will have rotating centrifugal force similar to Fig. 23.9. These two forces are out-of-phase with each other. The result is dynamic imbalance forces in our simple two-disk system; they must be countered by two rotating forces rather than by one rotating force as before, in static balancing.

If we again consider one wheel of our automobile, having spent our money for static balancing only, we may still have unbalanced forces at high speeds. We may find it necessary to both statically and dynamically balance the wheel to reduce the forces to zero. Few automotive mechanics, doing this work every day, are aware of this. You will find it quite difficult to find repair shops that both statically and dynamically balance the wheels of your automobile, but the results, in increased comfort and tire wear, often repay one for the effort and expense.

Imagine that we have a perfectly homogeneous and balanced rotor. Now we add a weight on one side of the midpoint. If we now spin this rotor, but do not rigidly restrain its movement, the motion will resemble the left sketch in Fig. 23.11; the centerline of the shaft will trace out a cylinder. On the other hand, suppose that we had added two equal weights on opposite sides, equidistant from the center, so that we have static balance. If we spin the rotor, its centerline will trace out two cones,

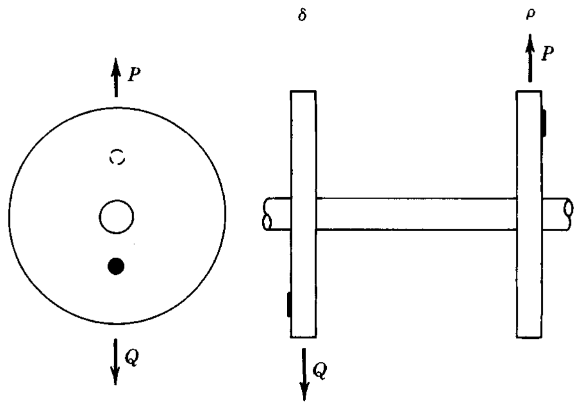

Fig. 23.10 Schematic of unbalanced shaft.

as shown in the right sketch; the apex of each cone will be at the center of gravity of the rotor. In practical unbalanced rotors, the motion will be some complex combination of these two movements.

Imbalance in machinery rotors can come from a number of sources. One is lack of symmetry; the configuration of the rotor may not be symmetrical in design, or a core may have shifted in casting, or a rough cast or forged area may not be machined. Another source is lack of homogeneity in the material due, perhaps, to blowholes in a casting or to some other variation in density. The rotor (a fan blade, for example) may distort at operating rpm. The bearings may not be aligned properly.

Generally, manufacturing processes are the major source of imbalance; this includes manufacturing tolerances and processes that permit any unmachined portions, any eccentricity or lack of squareness with the shaft, or any tolerances that permit parts of the rotor to shift during assembly. When possible, rotors should be designed for inherent balance. If operating speeds are low, balancing may not be necessary; today's trends are all toward higher rpm and toward lighter-weight assemblies; balancing is more required than it was formerly.

Figure 23.12 shows two unbalanced disks on a shaft; they represent the general case of any rotor, but the explanation is simpler if the weight is concentrated into two disks. The shaft is supported by two bearings, a distance l apart. At a given rotational speed, one disk generates a centrifugal force P, while the other disk generates a centrifugal force Q. In the plane of bearing 1, forces P and Q may be resolved into two forces by setting the sum of the moments about plane 2 equal to zero; the force diagram is shown in Fig. 23.12. Similarly, in the plane of bearing 2, forces P and Q may be resolved into two forces by setting the sum of the moments about plane 1 equal to zero; this force diagram is also shown in Fig. 23.12. Both force diagrams represent rotating vectors with angular velocity ω radians per second. Force P has been replaced by two forces, one at each bearing plane:

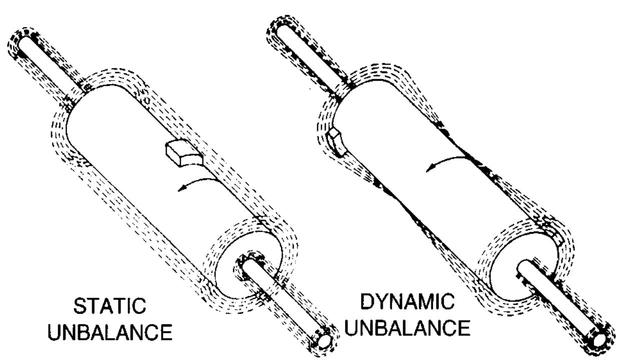

Fig. 23.11 Imbalance in a rotor; the rotor on the left has a weight on one side at the midpoint; the rotor on the right has two equal weights on opposite sides, equidistant from the center.

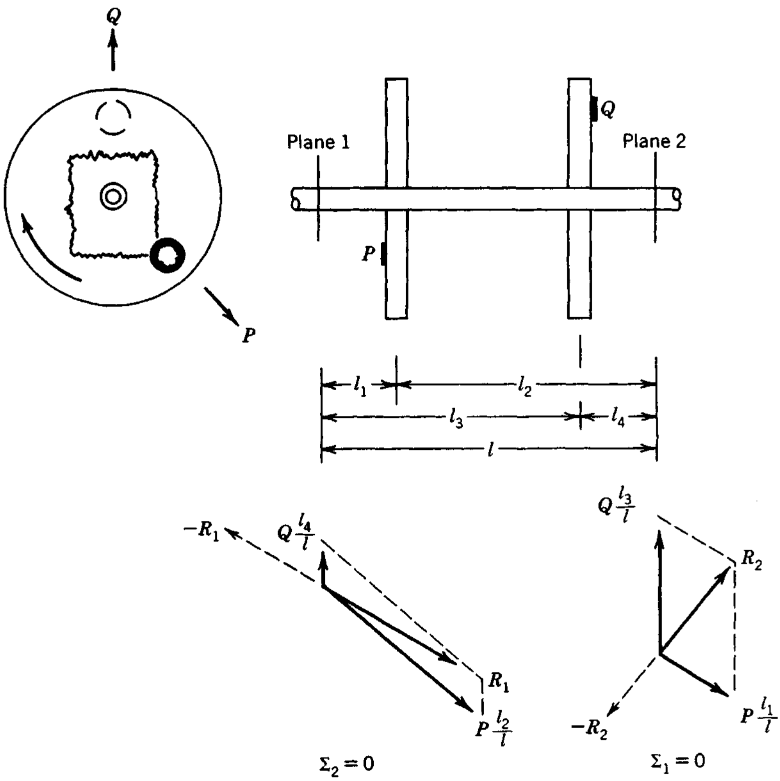

Fig. 23.12 Representation of the general case of an unbalanced rotor.

$P(l_1/l)$ and $P(l_2/l)$. Similarly, force Q may be replaced by two forces, one at each bearing plane: $Q(l_3/l)$ and $Q(l_4/l)$.

We may combine the two forces at bearing plane 1 into one resultant force R_1. We may combine the two forces at bearing plane 2 into one resultant force R_2. If we can somehow apply rotating counterforces at the bearings, namely, $-R_1$ and $-R_2$, we will achieve complete static and dynamic balance. No force will be transmitted from our rotor to its bearings.

In practice, of course, we do not actually use forces at the bearings. We add balancing weights at two locations along the shaft and at the proper angles around the shaft. These weights each generate a centrifugal force, which is also proportional to the square of shaft speed. Or we may remove weight. Either way, we must achieve forces $-R_1$ and $-R_2$. The amount of each weight added (or subtracted) depends on where along the shaft we can conveniently perform the physical operation.

When we have our automobile wheels balanced, to use this simple example for the last time, only the weight and angular position of the counterweights may be selected. The weights are always attached to either the inner or outer rim of the wheel. In early automobile wheels, with their large diameter/thickness ratio, static balancing was usually sufficient. Modern automobiles have smaller, thicker wheels (turning at higher speeds) and they approach Fig. 23.11. Dynamic balancing is definitely better.

In the case of a rotating armature we may subtract weight at a convenient point along the length of the armature (usually at the end) and at the proper angular position by simply drilling out a bit of the armature material. This operation is repeated at the other end. Then the armature is both statically and dynamically balanced.

As an example, consider the rotor shown in Fig. 23.13. This rotor has 3 oz-in. of imbalance at station 2, located 3 in. from the left end, and at an angular position of 90° from an arbitrary reference. Another imbalance of 2 oz-in. exists at station 3, located 5 in. from the right end, and at an angle of 180° from the same reference. We want to statically and dynamically balance the rotor, by means of corrections at the two ends, at stations 1 and 4.

Let us now draw a vector diagram of the forces at station 1, as shown in Fig. 23.13. First we take a summation of forces about station 4, just as we did in Fig. 23.12. The imbalance at station 2,

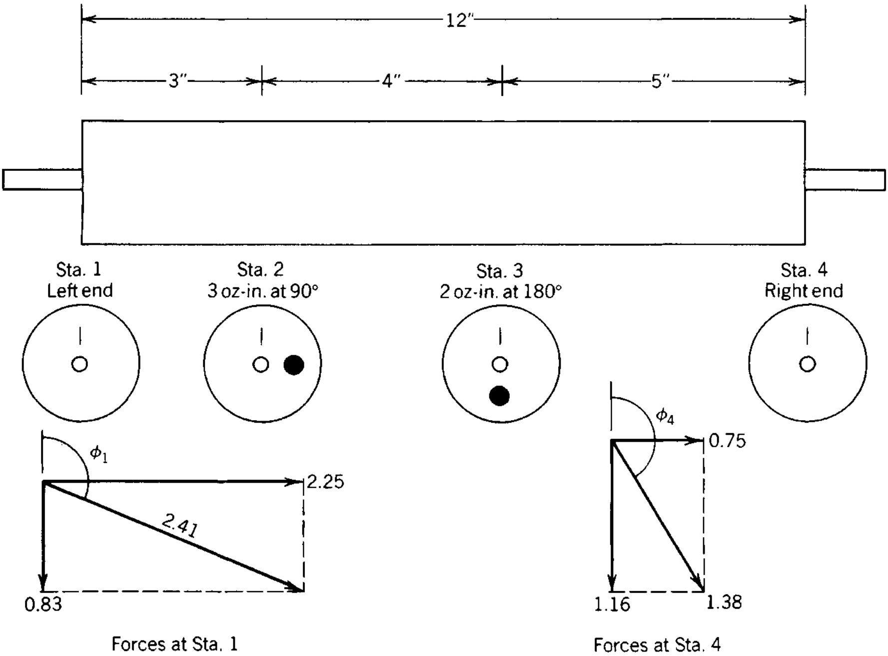

Fig. 23.13 Forces on an unbalanced rotor.

when sensed at station 1, is 3 × 9⁄12 = 9⁄4 = 2.25 or a vector 2.25 at 90°. The imbalance at station 3, when sensed at station 1, is 2 × 5⁄12 = 5⁄6 = 0.83 or a vector 0.83 at 180°. By taking the square root of 2.25^2 plus 0.83^2, we find that we must remove 2.41 oz-in. Now at what angle shall we remove weight?

$$\begin{aligned}\phi_1 &= 90° + \tan^{-1}\frac{0.83}{2.25}\\ &= 90° + 20.3° = 110.3°\end{aligned}$$

So we see that we must remove 2.41 oz-in. at an angle of 110.3° from our reference. We may do this by removing 2.41 oz at a radius of 1 in., or by removing 1 oz at a radius of 2.41 in. or any convenient combination of weight and radius.

Let us now draw a vector diagram of forces at station 4, summing forces about station 1. The imbalance at station 2, when sensed at station 4, is 3 × 3⁄12 = 3⁄4 = 0.75 or a vector 0.75 at 90°. The imbalance at station 3, when sensed at station 4, is 2 × 7⁄12 = 7⁄6 = 1.16 or a vector of 1.16 at 180°. By taking the square root of 0.75^2 plus 1.16^2, we get a resultant of 1.38 oz-in. Now to get the proper angle:

$$\begin{aligned}\phi_4 &= 180° - \tan^{-1}\frac{0.75}{1.16}\\ &= 180° - 32.8° = 147.2°\end{aligned}$$

So we see that we must remove 1.38 oz-in. at an angle of 147.2° from our reference. We may again select any convenient combination of weight and radius along this 147.2° radius line.

The reader must be cautioned that elastic bodies cannot be balanced in this manner; imbalance must be removed in the plane where it exists, rather than in some plane that happens to be convenient.

We have reviewed the meanings of several terms and shown how balancing may be accomplished. Now we will discuss the test equipment used in learning the amount and location of imbalance.

The simplest balancing machines make use of gravity. A rotor, on its shaft, may rest on hard knife-edges, which are straight and level. The heavy side of the disk will seek out its lowest level and thus automatically indicate the angle at which a balancing weight must be added. Various weights are added until the disk has no preferred position. Or the disk may be horizontal with its center pivoted on a point, or attached to a string. The heavy side will seek its lowest level, thus indicating the angle at which a correcting weight should be added. Weight is added until the disk is level. With either of these gravity machines, which accomplish static balancing only (the disk is stationary, not rotating), we recognize that balancing could have been accomplished by removing material from the heavy side, rather than by adding material to the light side.

Static balancing is occasionally helpful as a first step in dynamic balancing. A rotor may be so badly unbalanced that it cannot be brought up to speed in a dynamic balancing machine. It must be given a preliminary static balance.

There are many variations on the basic idea of centrifugal (dynamic) balancing machines. We will not attempt to describe any one unit; rather we will mention rather briefly some of the variations that a reader may encounter.

Bearings may be supported in a soft, flexible manner, to approximate the unrestricted rotor of Fig. 23.11. Bearings are supported by thin rods or wires which are soft in a single plane; the resulting motion is primarily in that plane. Motion of the flexible bearing supports is measured as an indication of imbalance force. The earliest machines measured motion mechanically. Subsequent machines measured motion electrically, which pickups that sensed either displacement or velocity. Newer machines employ force-sensing transducers to generate a signal for the electronic system. A readout informs the operator how much imbalance exists and where, and usually tells this in terms of how much material must be removed (or added) and where. Some machines require that the work-piece be removed to, say, a drill press. Others combine the balancing operation with drilling, welding, soldering, etc., so the part can be balanced and rechecked without removing it from the machine. On some machines the entire operation is automatic.

The reader may be interested to learn that transducers and meters of some balancing machines are capable of measuring peak-to-peak displacements of about 0.08 μin. This extreme sensitivity makes it possible to detect very small imbalances.

Table 23.1 indicates the balancing accuracy of some common commercial items. Gryscopes are held to even closer tolerances, on the order of 0.000 000 25 oz-in., since even slight vibration causes long-time drifting.

In-place dynamic balancing of rotating machinery is often needed when it is impossible or inconvenient to remove a rotor for balancing. A stroboscopic light is synchronized to the rpm of the rotor to be balanced; this is usually done with a vibration pickup on the frame of the machine. It may be necessary to "tune out" interfering vibrations from other machines by means of a filter between pickup and strobe light; when this is accomplished, the offending rotor will appear to be "frozen" and not moving. A reference mark on the rotor and the angle at which the mark appears indicate the angle of imbalance. Readout from the vibration pickup is proportional to the amount of imbalance.

The rotor is stopped, and a compensating weight is attached at some location. The rotor is again spun at operating rpm and any improvement (or degradation) is noted in the vibration level, along with the new reference angle. Different weights and radial and angular locations are tried until satisfactory balance is achieved. The "cut-and-try" process is greatly speeded by drawing vector diagrams representing imbalance forces or from use of a specially programmed calculator. Skill in this work comes from wide experience; a wide range of problems arises, as compared to production-line balancing work.

23.3 VIBRATION MEASUREMENT

Let us first discuss the measurement of sinusoidal vibratory *displacement,* normally the peak-to-peak displacement or double-amplitude D measured in inches or in millimeters. If a structure is steadily

Table 23.1 Selected Balancing Accuracies

Part	Accuracy of Balance (oz-in.)
Vacuum sweeper armature	0.00005–0.0001
Automobile crankshaft	0.0001–0.0002
Automobile flywheel	0.0002–0.0005
Automobile clutch	0.001–0.002
Aircraft crankshaft	0.0001–0.0004
Aircraft supercharger	0.00055–0.0001
Electric motor armatures, 1 hp	
1800 rpm	0.0001–0.0002
3600 rpm	0.00005–0.0001
Small instrument motor armatures	0.000005–0.00001

vibrating with a large enough D, we can estimate motion by merely holding a ruler alongside. The reader is requested to mentally estimate the smallest D which would be read with ±10% accuracy, ½ or ¼ in.? 10 or 5 mm? If we could hold the ruler steady, if the vibration remained constant, and if we took several readings and averaged them, we might get an accuracy of ±10% at ¼ in. or 10 mm D.

Optical techniques for measuring vibratory displacement D are limited in accuracy, especially at the small displacements we find in vibration measurement and testing. Let us calculate D under the test conditions of 10g (peak) acceleration at a frequency of 1000 Hz by using the following relationships:

English unit example:

$$A = 0.0511 f^2 D$$

or

$$D = \frac{A}{0.0511 f^2} = \frac{10}{.0511(1000)(1000)} = 0.0001957 \text{ in.}$$

International System example

$$A = 0.002\,02 f^2 D$$

or

$$D = \frac{A}{0.002\,02 f^2} = \frac{10}{0.002\,02(1000)(1000)} = 0.00495 \text{ mm}$$

This is about 200 μin., an example of the extremely small displacements we find in much vibration work. At higher frequencies, D is still smaller; for example, at 10g (peak), 2000 Hz, D would be about 50 μin. or 1.3 μm.

A very popular optical technique uses the "optical wedge" device, shown in Fig. 23.14a, which is cemented onto a structure that will be vibrating. This device depends on our eyes' persistence of vision; it works best above 15 Hz. Velocity is zero at the extremes of position. There we get a crisp image. In between, where the pattern is moving, we let a lighter, gray image. The result *appears* to be two images, overlapping each other as in Fig. 23.14*b*. The ends of the images will be separated by exactly D. D is read by noting where the two crisp images intersect, then by looking directly below this intersection on the ruled scale. In Fig. 23.17*b* D is 0.50 in. With great care, accuracies of ±10% or even ±5% are possible, particularly at larger amplitudes. This technique is most useful when an "unknown" D is to be measured.

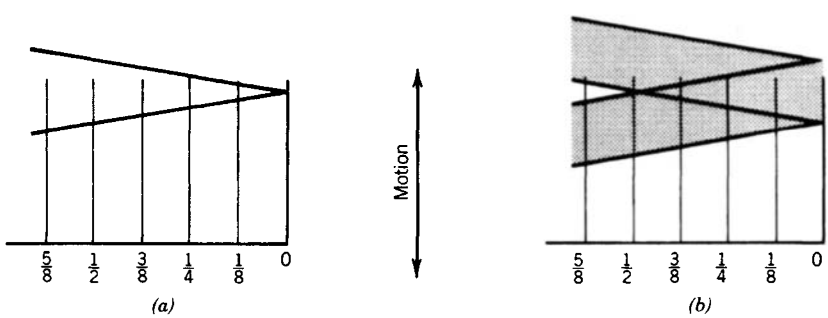

Fig. 23.14 Vibration measurement by "optical wedge" device.

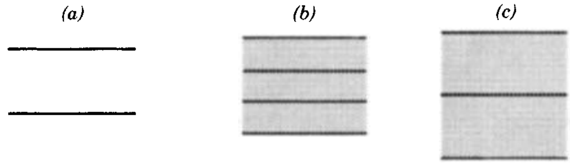

Fig. 23.15 Vibration calibration.

When one wants an optical indication that D has reached a particular value, the scheme of Fig. 23.15 is useful. Two parallel lines, perpendicular to the direction of motion, are drawn ½ in. apart. When the target is stationary, the lines appear as in Fig. 23.15*a*. To adjust the D of a shaker to, say, ¼ in., increase shaker power until two gray bands separated by a white band, all ¼ in. high, appear as in Fig. 23.15*b*. For a D of ½ in., increase the power to the shaker until the two gray bands just merge, as in Fig. 23.15*c*.

Suppose you wish to calibrate an accelerometer/cable/amplifier/meter system at 10*g* (peak) and 20 Hz. Carefully rule lines 0.49 in. apart, then adjust D until the pattern resembles Fig. 23.15*c*. Some laboratories use lines scribed through dull paint on the surface of a metal block bolted to the table of a shaker. Lines are scribed with spacings of 0.100, 0.200, 0.036, 0.49, etc., in. These D's, at known frequencies, give useful levels of velocity and acceleration to check velocity pickups and accelerometers. Use a magnifying glass or a low-power microscope to determine when areas merge.

Accuracy statements on these techniques are not conclusive. Factors include accuracy of original construction of the device used, accuracy and judgment of the user, the D being read (greatest accuracy at large D's), etc. Only with all factors aiding the operator do optical pattern techniques give $\pm 5\%$ accuracy.

For measuring D's smaller than 0.1 in., use optical magnification. Use a 40× or 50× microscope with internal "cross hairs" and with cross-feeds that permit moving the entire microscope body. A stroboscopic light is helpful. Focus the microscope on some well-defined point or edge on the vibrating structure (no motion); with the structure vibrating, move the microscope body until the cross hairs line up with the extreme excursion of the point or edge being observed. Read a dial wheel on the cross-feed. Move the microscope until the cross hairs line up with the other extreme position, and take another reading. D equals the distance the microscope body was moved—the difference in readings. In practice, this technique is difficult to use: the "neutral" position of the vibrating structure may shift and lead to large errors. "Filar" elements inside the microscope are helpful.

A better microscope technique, generally used with about a 40× or 50× instrument, requires a calibrated reticle or eyepiece, having regularly spaced lines, usually 0.001 in. apart. Before the structure commences to vibrate, focus the microscope on a bit of Scotchlite tape, fine garnet, or emery paper. You will see the grains of the target; with a strong light from the side, each grain will reflect a bright point of light, as in Fig. 23.16*a*. When the structure vibrates, each point of light "stretches" into a line as in Fig. 23.16*b*. The length of any line is estimated by comparing it with the rulings. In Fig. 23.16*b*, the rulings are 0.001 in. apart, and D is 0.005 in.

Optical (and mechanical) magnification is also used in largely obsolete hand-held instruments typified by Fig. 23.17. A probe maintains contact with a vibrating surface. Motion resulted in a bright pattern appearing on a scale: the pattern width represents D. A similar unit permanently recorded waveforms of instantaneous displacement versus time on a paper strip. As paper speed was known,

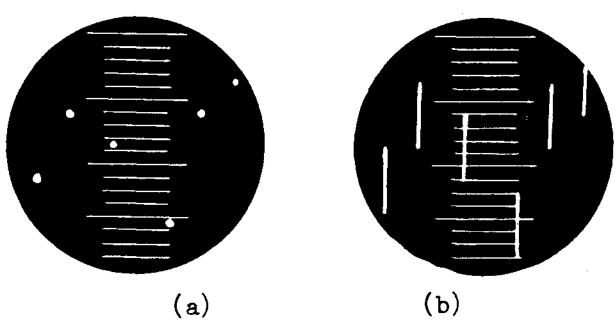

Fig. 23.16 Vibration measurement with a microscope.

Fig. 23.17 Hand-held vibration measurement instrument.

frequency could be calculated. These instruments were satisfactory when a vibrating machine was so heavy that its motion was not affected by the mass of the probe. The instrument case could not move. Thus the operator stood on a nonvibrating support and held the instrument steady against forces transmitted through it. These limitations were never fully met; consequently, these instruments were seldom used for measuring the tiny displacements found in most vibration situations, originating in faulty bearings, gears, etc. They were mainly used in measuring the result of rotational imbalance upon heavy machines such as engines, pumps, etc.

There are a number of displacement-sensing pickups that send an electrical signal to remote amplifiers, meters, oscilloscopes, oscillographs, tape recorders, analyzers, etc. Several different physical methods can convert changes in position into electrical signals: these include linkages from vibrating structures to the sliders of stationary variable resistances; also noncontacting variations in capacitance, inductance, eddy currents, etc., which are caused by changes in distance. Note that some portion of each instrument must be held stationary, if one is to gain accurate information about the *absolute* motion of a vibrating structure. This is very difficult to do. The "background noise" of vibration in many industrial buildings is 0.005 in. *D* or more; this will certainly prevent accurate measurement of, say, 0.001 in. *D*. And many times we want to read *D*'s of only a few microinches. The frame vibration of a gas turbine is normally very small (unless resonances in the support structure are excited). However, there may be large, low-frequency motion due to other machines nearby; this may "mask" the signal of interest.

On the other hand, displacement sensors read *relative* displacements well. One type is inserted into a drilled, threaded hole in the frame of a machine. Its sensing end is close to a rotating shaft, either perpendicular to the shaft axis for measuring eccentric shaft motion or parallel to the shaft for measuring axial shaft motion. If motion is not perfect, a signal proportional to relative motion between shaft and housing will result.

Since the greatest trouble with directly measuring *absolute* *D*'s is in the presence of background vibration, we are much interested in systems that permit us to largely ignore such background vibration. We will not try to measure *D* directly, but will measure velocity or acceleration and then convert those signals into displacement readings. The velocity pickup of Fig. 23.18, with minor modifications, eliminated the problems mentioned above. A coil was attached to a vibrating structure; leads were brought out to an electronic voltmeter or a recorder. A permanent magnet was held (without any motion!) close to the coil; the electrical signal generated by the coil was proportional to the relative velocity between coil and magnet, so any motion of the magnet created an error in the measurement. This system was calibrated in volts per unit of velocity.

If the magnet could be held still, the reading of the meter was proportional to actual velocity of the moving structure. Such a pickup was too bulky to be practical for much vibration measurement work. A number of firms built self-contained seismic velocity pickups, as exemplified by Fig. 23.19. In this unit, the sensing coil was attached to an arm; the arm in turn was attached to the frame by pivots and delicate springs. The coil moved relative to the frame and to a magnetic field which was supplied by Alnico permanent magnets. The entire pickup was attached to the structure being inves-

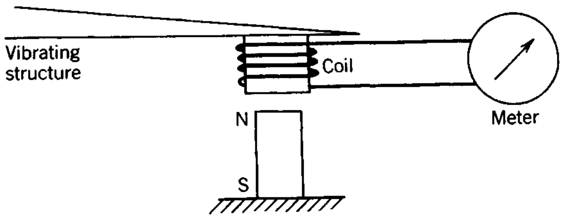

Fig. 23.18 A velocity pickup.

tigated. At the pickup's f_N of about 5 Hz, the arm and coil moved more than did the pickup frame. This unit was only used at frequencies above 10 Hz where the arm and coil were "isolated" from vibration; thus they remained stationary (seismic support). The magnetic field swept up-and-down across the coil; this generated a voltage proportional to the velocity of the pickup.

Functions of the coil and magnet may be reversed. The permanent magnet of Fig. 23.20 was seismically suspended. When the case vibration was well above resonance (operation of this unit starts at 45 Hz), the magnet remained stationary. The coil, attached to the frame, swept through the magnetic field so that a voltage was induced. The frequency range is 45–1500 Hz. Stroke was limited; the maximum D was 0.1 in. Some velocity pickups were built for lower frequencies and longer strokes. Accelerations greater than $50g$ could cause damage.

The velocity pickup shown in Fig. 23.21 was similar to the unit of Fig. 23.19. However, a length of 0.030 in. drill rod, connected to the sensing coil, passed out through the housing. This unit was normally held by hand, with the "probe" firmly touching a vibrating surface. Motion was transmitted to the coil, in which a voltage proportional to velocity was generated. This unit was very useful for hand-scanning over a vibrating surface, to locate points of maximum and minimum motion.

Velocity pickups had many advantages: they were self-contained devices requiring no external source of power—no dc or ac excitation. Because of their low internal impedance, they could be used at great distances from the readout instrument. They were most used in the frequency range 10–500 Hz. A typical sensitivity was 100 millivolts (peak) per inch per second (peak) velocity.

Velocity pickups had certain disadvantages, of course. They were generally quite large and heavy, especially as compared with some accelerometers. They could not be used at very large displacements, because of limited stroke of their moving parts. Since they will sometimes be used close to their natural frequencies, some form of damping (usually oil or eddy current) was needed; this introduced certain problems in measuring any nonsinusoidal motion.

Readout from velocity pickups was very simple, as in Fig. 23.18. The pickup sensitivity was known in terms of millivolts per unit of velocity. An electronic voltmeter reading in terms of millivolts could be interpreted in terms of velocity.

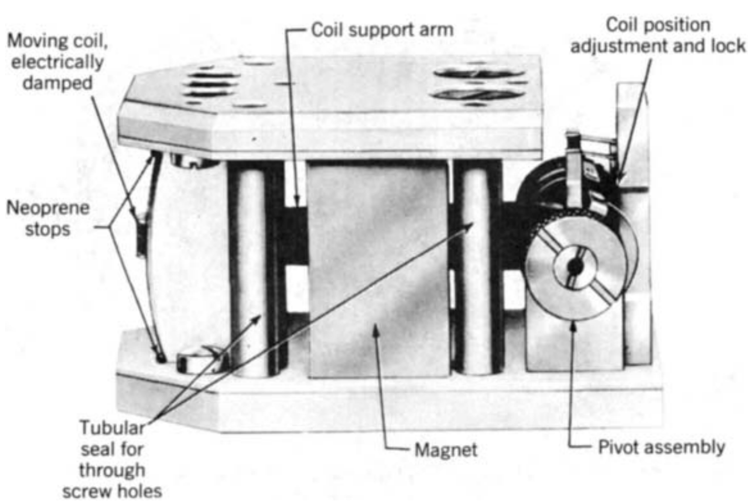

Fig. 23.19 A self-contained seismic velocity pickup. (Courtesy of Vibra-Metrics Inc.)

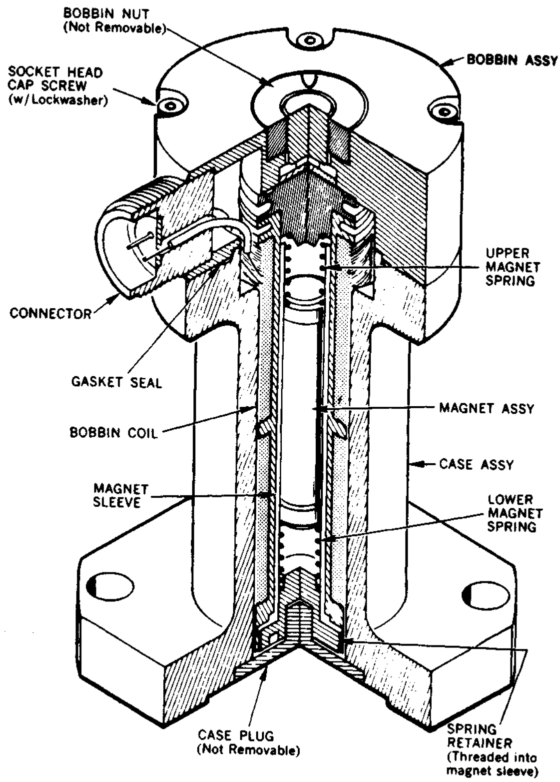

Fig. 23.20 Velocity pickup with permanent magnet suspended seismically. (Courtesy of CEC.)

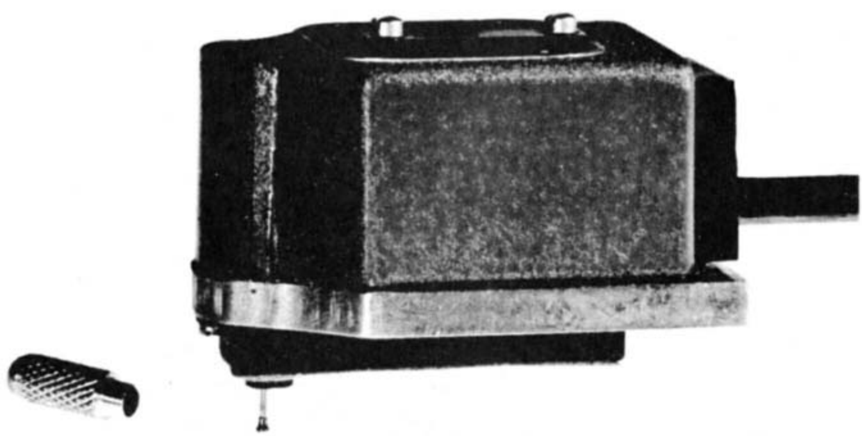

Fig. 23.21 Hand-held velocity pickup used for locating points of minimum and maximum motion.

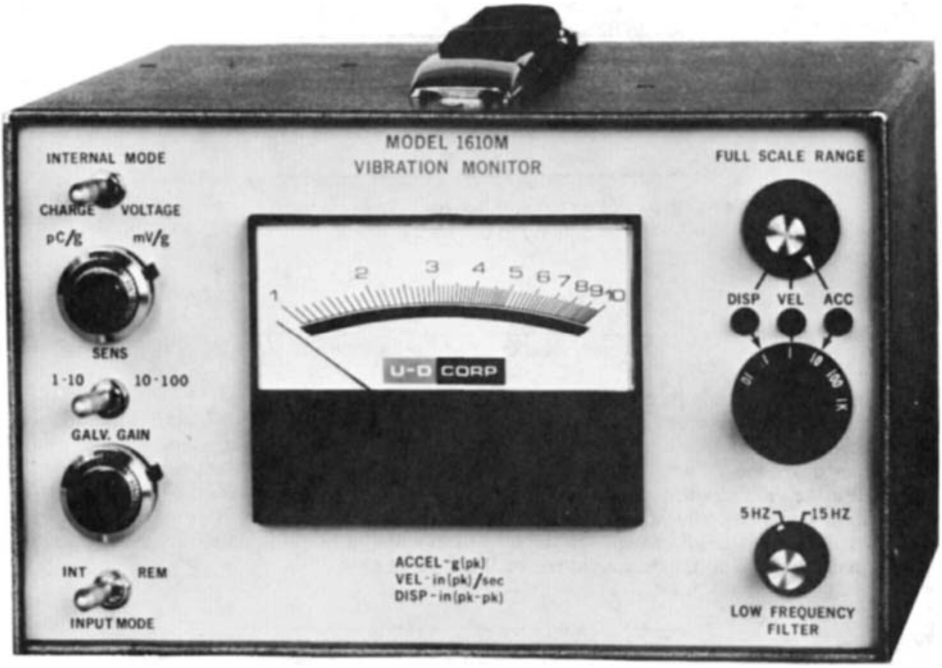

Fig. 23.22 A specialized vibration meter.

If you knew the velocity and the frequency of *sinusoidal* vibration, but wished to know the displacement or acceleration, you could calculate these. If you did not need extreme accuracy, you could use one of the cardboard vibration calculators that are available from some shaker and accelerometer manufacturers. One of the advantages of velocity pickups when used with a common electronic voltmeter: values of velocity, displacement, and/or acceleration were easily determined. This advantage was very important around 1945–1955; funds for purchase of more sophisticated vibration instruments were not readily available.

Many measurements that formerly were made with coil-and-magnet velocity pickups now employ an accelerometer packaged with an integrating network. Power is of course required.

Specialized vibration meters such as the unit in Fig. 23.22 were soon developed. When one wished to read D on a meter whose input signal comes from a velocity pickup, the meter had to contain an integrating network similar to Fig. 23.23. The velocity signal was electronically integrated to form a displacement signal, which was then measured by conventional meter circuitry. Test the circuit with a constant-voltage, variable-frequency test oscillator; the circuit integrates the signal. If frequency doubles, the output voltage should drop to ½, etc. Be sure that R is large enough and that X_c is small enough for proper integrating action; the "time constant," the product of R in ohms times C in farads, should be large compared to the period T, where T is

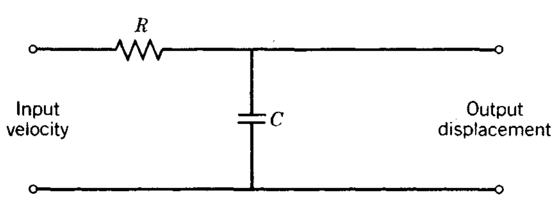

Fig. 23.23 Integrating network for reading displacements when input signals come from a velocity pickup.

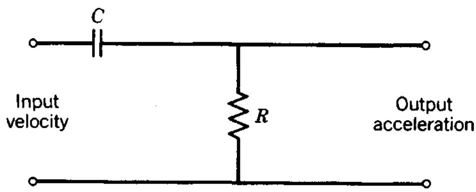

Fig. 23.24 Differentiating network for reading accelerations from signals originating in a velocity pickup.

$$T = \frac{1}{\text{lowest frequency at which integrator is used}}$$

and R should be large compared to X_c.

Most meters also provided a differentiating network, something like Fig. 23.24, so that A could be read from a signal that originated in a velocity pickup. The velocity signal was electronically differentiated to form an acceleration signal, which was then measured by conventional meter circuitry. Test the circuit with a constant-voltage, variable-frequency test oscillator; the circuit differentiated the signal. If frequency doubled, the output voltage doubled, etc. Be sure that X_c is large enough and that R is small enough for proper differentiating action; the time constant, the product of R in ohms times C in farads, should be small compared to

$$T = \frac{1}{\text{highest frequency at which differentiator is used}}$$

and R should be small compared to X_c.

Vibration meters are simply electronic voltmeters with additional features. Most provided for inputs from velocity pickups and from velocity coils built into older model shakers. Most (see Fig. 23.25) had a three-position switch marked Displacement-Velocity-Acceleration. In the Velocity position, the unit was a simple ac voltmeter. The variable-gain "normalizing" control was set for the particular pickup sensitivity being used. When such a meter was used to read displacement, an integrating network was inserted into the signal path, as in Fig. 23.25, by switching to Displacement. For reading acceleration, the signal was switched through a differentiating network, as shown, by switching to Acceleration.

All meters today provide for accelerometer inputs, as in Fig. 23.25. If the incoming signal is twice integrated, it becomes proportional to displacement. Integration and differentiation "waste" much signal; therefore preceding and subsequent stages must have considerable gain. Enough voltage and power must be developed to run the detector, the indicating meter and any recording galvanometer that may be used for a permanent record of vibration.

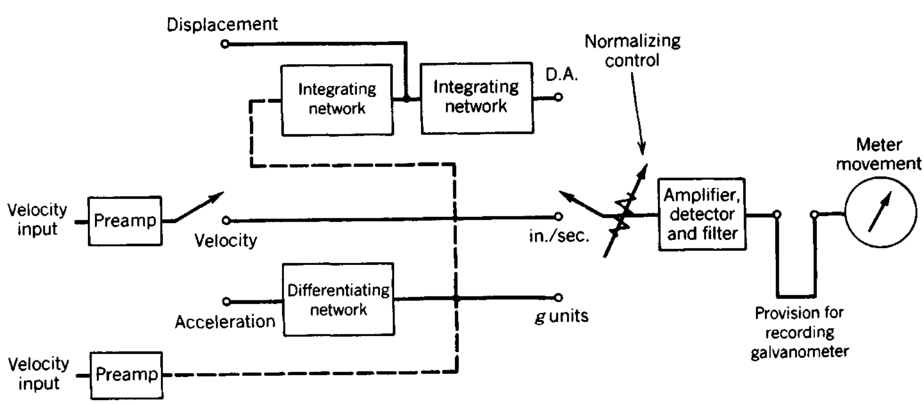

Fig. 23.25 A vibration meter circuit.

Differentiating and integrating networks are not satisfactory over many octaves of frequency. Fortunately, displacement information is usually only needed over a range such as 5–100 Hz, so integrating components are chosen for best accuracy in that region. Acceleration data are usually needed above about 50 Hz; when velocity pickups are used, differentiating components are selected for best operation from 50 to perhaps 2000 Hz. Serious inaccuracies result from use beyond the frequencies indicated in Table 23.2. Table 23.2 gives typical tolerances for the vibration meter only, not including the velocity pickup or accelerometer.

23.4 ACCELERATION MEASUREMENT

Let us now take up the subject of accelerometers—units whose instantaneous output voltage is proportional to the instantaneous value of acceleration. Most vibration and shock measurements are even more true today than in 1986 are made with accelerometers.

At high frequencies, accelerometers generate a larger signal than do velocity pickups. This may be shown by calculating the peak velocity that would exist if the peak acceleration were $1g$ at a frequency of 2000 Hz:

English units:

$$V = \frac{A}{0.0162f}$$

$$V = 0.031 \text{ in./sec (peak)}$$

SI units:

$$V = \frac{A}{0.000\,642f}$$

$$V = 0.78 \text{ mm/sec (peak)}$$

Let us assume that a velocity pickup has a sensitivity of 105 mV/in. sec. It will generate about 3.22 mV (peak) or about 2.28 mV (rms). (At 200 Hz, $1g$, the velocity would be 10 times greater, of course, and so would be the voltage.) A signal of only 2 or 3 mV rms is difficult to measure under some conditions. We will show that accelerometers built for these higher frequencies have a number of advantages; one advantage is higher output signal at higher test frequencies found in modern vibration tests and field measurements.

A reasonable sensitivity figure for a crystal accelerometer is 10 mV (peak) per g (peak); this sensitivity is relatively constant up to, say, 10,000 Hz. In the situation discussed above ($1g$ at 2000 Hz) the accelerometer would generate 10 mV (peak) or 7.07 mV rms. This is about three times as much signal as the velocity pickup generates. This advantage would double with each doubling of frequency (octave).

Accelerometers always operate *below* their natural frequencies. Thus the natural frequency must be high; the useful "flat" range is to about one-fifth of the natural frequency, where the sensitivity is about 4% higher than it is at low frequencies. A 50-kHz unit thus permits operation to 10 kHz.

How might we build an instrument that would sense acceleration? The automobile enthusiast senses acceleration by "feeling" his or her body sink into the seat cushions. As the car and seat gain a high velocity (accelerate), a deflection proportional to the intensity of acceleration is noted. His or her body has inertia and tends to keep its former velocity. If we could measure the amount of deflection, we would have a crude accelerometer. In Fig. 23.27 we see a crude accelerometer and how acceleration could be measured on a meter. A cantilever beam supports a weight W. On the top and bottom are cemented strain gages, elements that change resistance when stretched or compressed. When the structure being observed accelerates upward, the weight W, having inertia, tends to stay behind; the top side of the beam stretches while the bottom side shortens.

Table 23.2 Vibration Meter Data

Function	Range	Frequency Limits	Tolerance
Velocity	0.01–100 in./sec	5–500 Hz	±2%
Displacement	0.001–10 in. D	5–5 000 Hz	±2%
Acceleration	0.1–1 000g	10–5 000 Hz	±1%
		5–10 000 Hz	±2%
		2–25 000 Hz	±5%