6

Steam Turbines

6.1 INTRODUCTION

In a steam turbine, high-pressure steam from the boiler expands in a set of stationary blades or vanes (or nozzles). The high-velocity steam from the nozzles strikes the set of moving blades (or buckets). Here the kinetic energy of the steam is utilized to produce work on the turbine rotor. Low-pressure steam then exhausts to the condenser. There are two classical types of turbine stage designs: the impulse stage and the reaction stage.

Steam turbines can be noncondensing or condensing. In noncondensing turbines (or backpressure turbines), steam exhausts at a pressure greater than atmospheric. Steam then leaves the turbine and is utilized in other parts of the plant that use the heat of the steam for other processes. The backpressure turbines have very high efficiencies (range from 67% to 75%). A multi-stage condensing turbine is a turbine in which steam exhausts to a condenser and is condensed by air-cooled condensers. The exhaust pressure from the turbine is less than the atmospheric. In this turbine, cycle efficiency is low because a large part of the steam energy is lost in the condenser.

6.2 STEAM NOZZLES

The pressure and volume are related by the simple expression, $PV^\gamma =$ constant, for a perfect gas. Steam deviates from the laws of perfect gases. The *P-V* relationship is given by:

$$PV^n = \text{constant}$$

where:

$$n = 1.135 \text{ for saturated steam}$$

$$n = 1.3 \text{ for superheated steam}$$

For wet steam, the Zeuner relation,

$$n = \left(1.035 + \frac{x}{10}\right)$$

(where x is the initial dryness fraction of the steam) may be used.

All nozzles consist of an inlet section, a throat, and an exit. The velocity through a nozzle is a function of the pressure-differential across the nozzle. Consider a nozzle as shown in Fig. 6.1.

Assume that the flow occurs adiabatically under steady conditions. Since no work is transferred, the velocity of the fluid at the nozzle entry is usually very small and its kinetic energy is negligible compared with that at the outlet. Hence, the equation reduces to:

$$C_2 = \sqrt{\{2(h_1 - h_2)\}} \tag{6.1}$$

where h_1 and h_2 are the enthalpies at the inlet and outlet of the nozzle, respectively. As the outlet pressure decreases, the velocity increases. Eventually, a point is reached called the critical pressure ratio, where the velocity is equal to the velocity of sound in steam. Any further reduction in pressure will not produce any further increases in the velocity. The temperature, pressure, and density are called critical temperature, critical pressure, and critical

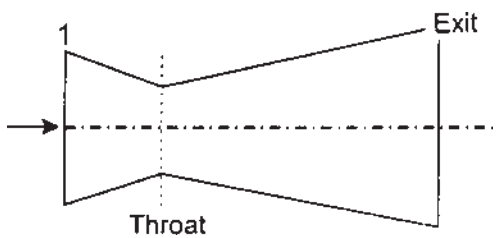

Figure 6.1 Nozzle.

density, respectively. The ratio between nozzle inlet temperature and critical temperature is given by:

$$\frac{T_1}{T_c} = \frac{2}{n+1} \tag{6.2}$$

where T_c is the critical temperature at which section $M = 1$. Assuming isentropic flow in the nozzle, the critical pressure ratio is:

$$\frac{P_1}{P_c} = \left(\frac{T_1}{T'_c}\right)^{\frac{n}{n-1}} \tag{6.3}$$

where T'_c is the temperature, which would have been reached after an isentropic expansion in the nozzle. The critical pressure ratio is approximately 0.55 for superheated steam. When the outlet pressure is designed to be higher than the critical pressure, a simple convergent nozzle may be used. In a convergent nozzle, shown in Fig. 6.2, the outlet cross-sectional area and the throat cross-sectional areas are equal. The operation of a convergent nozzle is not practical in high-pressure applications. In this case, steam tends to expand in all directions and is very turbulent. This will cause increased friction losses as the steam flows through the moving blades. To allow the steam to expand without turbulence, the convergent–divergent nozzle is used. In this type of nozzle, the area of the section from the throat to the exit gradually increases, as shown in Fig. 6.1.

The increase in area causes the steam to emerge in a uniform steady flow. The size of the throat and the length of the divergent section of every nozzle must be specifically designed for the pressure ratio for which the nozzle will be used. If a nozzle is designed to operate so that it is just choked, any other operating condition is an off-design condition. In this respect, the behavior of convergent and convergent–divergent nozzles is different. The temperature at the throat, i.e., the critical temperature, can be found from steam tables at the value of P_c and $s_c = s_1$. The critical velocity is given by the equation:

$$C_c = \sqrt{\{2(h_1 - h_c)\}} \tag{6.4}$$

where h_c is read from tables or the h–s chart at P_c and s_c.

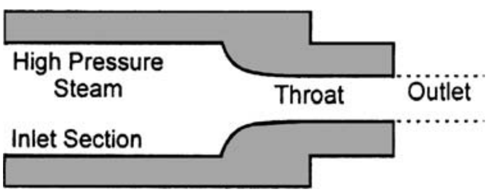

Figure 6.2 Convergent nozzle.

6.3 NOZZLE EFFICIENCY

The expansion process is irreversible due to friction between the fluid and walls of the nozzle, and friction within the fluid itself. However, it is still approximately adiabatic as shown in Fig. 6.3.

$1-2'$ is the isentropic enthalpy drop and $1-2$ is the actual enthalpy drop in the nozzle. Then the nozzle efficiency is defined as

$$\eta_n = \frac{h_1 - h_2}{h_1 - h_2{}'}$$

6.4 THE REHEAT FACTOR

Consider a multi-stage turbine as shown by the Mollier diagram, Fig. 6.4.

The reheat factor is defined by:

$$\begin{aligned} \text{R.F.} &= \frac{\text{Cumulative stage isentropic enthalpy drop}}{\text{Turbine isentropic enthalpy drop}} \\ &= \frac{\sum\left[\Delta h'\right]_{\text{stage}}}{[\Delta h']_{\text{turbine}}} \\ &= \frac{\left(h_1 - h_2^{'}\right) + \left(h_2 - h_3^{'}\right) + \left(h_3 - h_4^{'}\right)}{\left(h_1 - h_4^{''}\right)} \end{aligned} \tag{6.5}$$

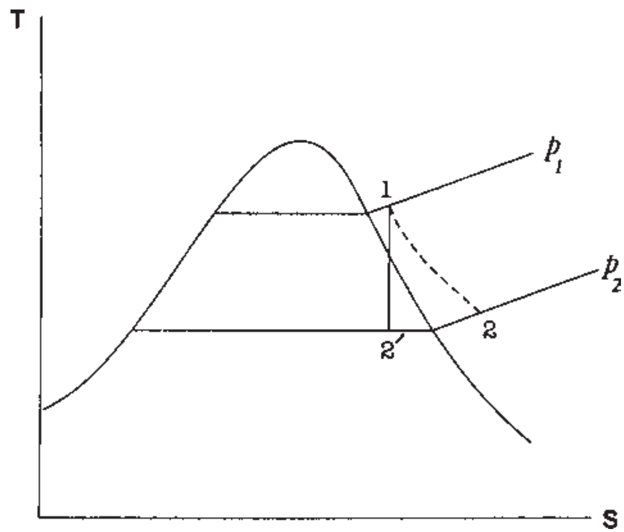

Figure 6.3 Nozzle expansion process for a vapor.

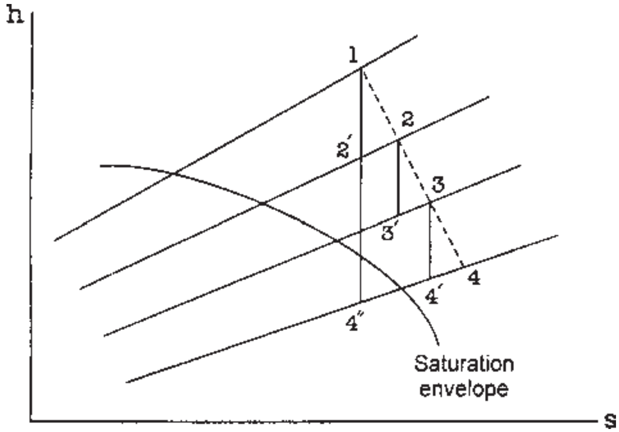

Figure 6.4 Mollier chart for a multi-stage turbine.

Since the isobars diverge, R.F. > 1.

The reheat factor may be used to relate the stage efficiency and the turbine efficiency.

Turbine isentropic efficiency is given by:

$$\eta_t = \frac{\Delta h}{\Delta h'} \tag{6.6}$$

where Δh is the actual enthalpy drop and $\Delta h'$ is the isentropic enthalpy drop. From diagram 6.4 it is clear that:

$$\Delta h = \sum [\Delta h]_{\text{stage}}$$

$$\Delta h_{1-4} = (h_1 - h_2) + (h_2 - h_3) + (h_3 - h_4)$$

if η_s (stage efficiency) is constant, then:

$$\eta_t = \frac{\sum \eta_s \left[\Delta h'\right]_{\text{stage}}}{[\Delta h']_{\text{turbine}}} = \frac{\eta_s \sum \left[\Delta h'\right]_{\text{stage}}}{[\Delta h']_{\text{turbine}}}$$

$$\text{or} \quad \eta_t = \eta_s \times (\text{R.F}). \tag{6.7}$$

Equation 6.7 indicates that the turbine efficiency is greater than the stage efficiency. The reheat factor is usually of the order of 1.03–1.04.

6.5 METASTABLE EQUILIBRIUM

As shown in Fig. 6.5, slightly superheated steam at point 1 is expanded in a convergent–divergent nozzle. Assume reversible and adiabatic processes. 1–2 is

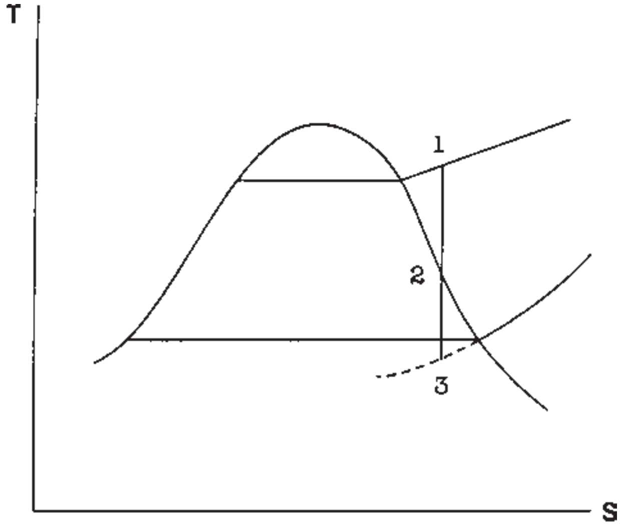

Figure 6.5 Phenomenon of supersaturation on T–S diagram.

the path followed by the steam and point 2 is on the saturated vapor line. Here, we can expect condensation to occur. But, if point 2 is reached in the divergent section of the nozzle, then condensation could not occur until point 3 is reached. At this point, condensation occurs very rapidly. Although the steam between points 2–3 is in the vapor state, the temperature is below the saturation temperature for the given pressure. This is known as the metastable state. In fact, the change of temperature and pressure in the nozzle is faster than the condensation process under such conditions. The condensation lags behind the expansion process. Steam does not condense at the saturation temperature corresponding to the pressure. Degree of undercooling is the difference between the saturation temperature corresponding to pressure at point 3 and the actual temperature of the superheated vapor at point 3. Degree of supersaturation is the actual pressure at point 3 divided by the saturation pressure corresponding to the actual temperature of the superheated vapor at point 3.

Illustrative Example 6.1: Dry saturated steam at 2 MPa enters a steam nozzle and leaves at 0.2 MPa. Find the exit velocity of the steam and dryness fraction. Assume isentropic expansion and neglect inlet velocity.

Solution:

From saturated steam tables, enthalpy of saturated vapor at 2 MPa:

$$h_1 = h_g = 2799.5\,\text{kJ/kg and entropy } s_1 = s_g = 6.3409\,\text{kJ/kg K}$$

Since the expansion is isentropic, $s_1 = s_2$: i.e., $s_1 = s_2 = 6.3409 = s_{f2} + x_2 s_{fg2}$, where x_2 is the dryness fraction after isentropic expansion, s_{f2} is the entropy of saturated liquid at 0.2 MPa, s_{fg2} is the entropy of vaporization at 0.2 MPa. Using tables:

$$x_2 = \frac{6.3409 - 1.5301}{5.5970} = 0.8595$$

Therefore,

$h_2 = h_{f2} + x_2 h_{fg2} = 504.7 + 0.8595 \times 2201.9 = 2397.233$ kJ/kg

Using the energy equation:

$$C_2 = \sqrt{\{2(h_1 - h_2)\}}$$

$$= \sqrt{\{(2) \times (1000) \times (2799.5 - 2397.233)\}}$$

or: $C_2 = 897$ m/s

Illustrative Example 6.2: Dry saturated steam is expanded in a nozzle from 1.3 MPa to 0.1 MPa. Assume friction loss in the nozzle is equal to 10% of the total enthalpy drop; calculate the mass of steam discharged when the nozzle exit diameter is 10 mm.

Solution:

Enthalpy of dry saturated steam at 1.3 MPa, using steam tables,

$$h_1 = 2787.6 \text{ kJ/kg}, \text{ and entropy } s_1 = 6.4953 \text{ kJ/kg K}.$$

Since the expansion process is isentropic, $s_1 = s_2 = s_{f2} + x_2 s_{fg2}$, hence dryness fraction after expansion:

$$x_2 = \frac{6.4953 - 1.3026}{6.0568} = 0.857$$

Now, the enthalpy at the exit:

$$h_2 = h_{f2} + x_2 h_{fg2} = 417.46 + (0.857) \times (2258)$$

$$= 2352.566 \text{ kJ/kg}$$

Therefore enthalpy drop from 1.3 MPa to 0.1 MPa

$$= h_1 - h_2 = 2787.6 - 2352.566 = 435.034 \text{ kJ/kg}$$

Actual enthalpy drop due to friction loss in the nozzle

$$= 0.90 \times 435.034 = 391.531 \text{ kJ/kg}$$

Hence, the velocity of steam at the nozzle exit:

$$C_2 = \sqrt{\{(2) \times (1000) \times (391.531)\}} = 884.908 \text{ m/s}$$

Specific volume of steam at 0.1 MPa:

$$= x_2 v_{g_2} = (0.857) \times (1.694) = 1.4517\,\mathrm{m^3/kg}$$

(since the volume of the liquid is usually negligible compared to the volume of dry saturated vapor, hence for most practical problems, $v = xv_g$)
Mass flow rate of steam at the nozzle exit:

$$= \frac{AC_2}{x_2 v_{g_2}} = \frac{(\pi) \times (0.01)^2 \times (884.908) \times (3600)}{(4) \times (1.4517)} = 172.42\,\mathrm{kg/h}.$$

Illustrative Example 6.3: Steam at 7.5 MPa and 500°C expands through an ideal nozzle to a pressure of 5 MPa. What exit area is required to accommodate a flow of 2.8 kg/s? Neglect initial velocity of steam and assume isentropic expansion in the nozzle.

Solution:

Initial conditions:

$$P_1 = 7.5\,\mathrm{MPa},\ 500^\circ\mathrm{C}$$
$$h_1 = 3404.3\,\mathrm{kJ/kg}$$
$$s_1 = 6.7598\,\mathrm{kJ/kg\ K}$$

(h_1 and s_1 from superheated steam tables)

At the exit state, $P_2 > P_c = (0.545) \times (7.5) = 4.0875\,\mathrm{MPa}$; and therefore the nozzle is convergent. State 2 is fixed by $P_2 = 5\,\mathrm{MPa}$, $s_1 = s_2 = 6.7598\,\mathrm{kJ/kg\ K}$
$T_2 = 435^\circ\mathrm{K}$, $v_2 = 0.06152\,\mathrm{m^3/kg}$, $h_2 = 3277.9\,\mathrm{kJ/kg}$ (from the superheated steam tables or the Mollier Chart).
The exit velocity:

$$C_2 = \sqrt{\{(2) \times (1000) \times (h_1 - h_2)\}}$$
$$= \sqrt{\{(2) \times (1000) \times (3404.3 - 3277.9)\}} = 502.8\,\mathrm{m/s}$$

Using the continuity equation, the exit area is

$$A_2 = \frac{mv_2}{C_2} = \frac{(2.8) \times (0.06152)}{502.8} = (3.42) \times (10^{-4})\,\mathrm{m^2}$$

Illustrative Example 6.4: Consider a convergent–divergent nozzle in which steam enters at 0.8 MPa and leaves the nozzle at 0.15 MPa. Assuming isentropic expansion and index $n = 1.135$, find the ratio of cross-sectional area, the area at the exit, and the area at the throat for choked conditions (i. e. , for maximum mass flow).

Solution:

Critical pressure for maximum mass flow is given by Fig. 6.6:

$$P_c = P_2 = P_1 \left(\frac{2}{n+1}\right)^{\frac{n}{n-1}} = 0.8\left(\frac{2}{2.135}\right)^{8.41} = 0.462\,\text{MPa}$$

From the Mollier chart:

$h_1 = 2769\,\text{kJ/kg}$
$h_2 = 2659\,\text{kJ/kg}$
$h_3 = 2452\,\text{kJ/kg}$

Enthalpy drop from 0.8 MPa to 0.15 MPa:

$$\Delta h_{1-3} = h_1 - h_3 = 2769 - 2452 = 317\,\text{kJ/kg}$$

Enthalpy drop from 0.8 MPa to 0.462 MPa:

$$\Delta h_{1-2} = h_1 - h_2 = 2769 - 2659 = 110\,\text{kJ/kg}$$

Dryness fraction: $x_2 = 0.954$
Dryness fraction: $x_3 = 0.902$

The velocity at the exit,

$$C_3 = \sqrt{\{(2)\times(1000)\times(\Delta h_{1-3})\}}$$

$$= \sqrt{\{(2)\times(1000)\times(317)\}} = 796\text{m/s}$$

The velocity at the throat

$$C_2 = \sqrt{\{(2)\times(1000)\times(\Delta h_{1-2})\}} = \sqrt{\{(2)\times(1000)\times(110)\}}$$

$$= 469\text{m/s}$$

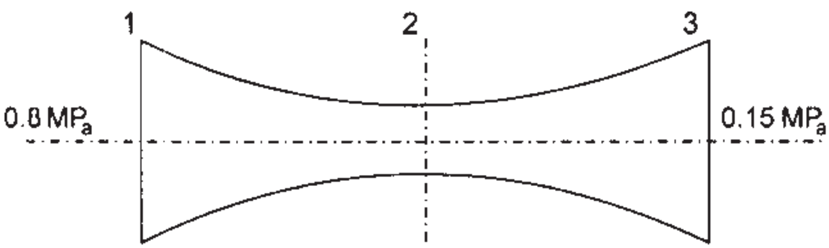

Figure 6.6 Convergent–divergent nozzle.

Mass discharged at the throat:

$$\dot{m}_2 = \frac{A_2 C_2}{x_2 v_{g_2}}$$

Mass discharged at the exit

$$\dot{m}_3 = \frac{A_3 C_3}{x_3 v_{g_3}}$$

Therefore

$$\frac{A_3 C_3}{x_3 v_{g_3}} = \frac{A_2 C_2}{x_2 v_{g_2}}$$

Hence,

$$\frac{A_3}{A_2} = \left[\frac{C_2}{C_3}\right]\left[\frac{x_3 v_{g3}}{x_2 v_{g2}}\right] = \left[\frac{469}{796}\right]\left[\frac{(0.902)(1.1593)}{(0.954)(0.4038)}\right] = 1.599$$

Illustrative Example 6.5: Dry saturated steam enters the convergent–divergent nozzle and leaves the nozzle at 0.1 MPa; the dryness fraction at the exit is 0.85. Find the supply pressure of steam. Assume isentropic expansion (see Fig. 6.7).

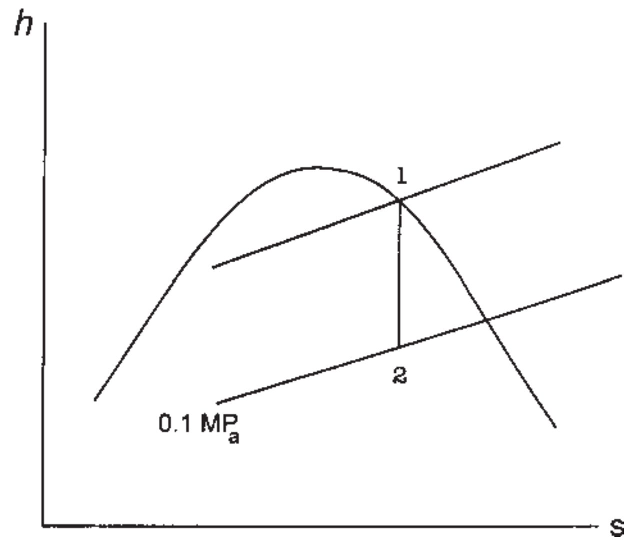

Figure 6.7 h–s diagram for Example 6.5.

Solution:

At the state point 2, the dryness fraction is 0.85 and the pressure is 0.1 MPa. This problem can be solved easily by the Mollier chart or by calculations. Enthalpy and entropy may be determined using the following equations:

$$h_2 = h_{f2} + x_2 h_{fg2} \text{ and } s_2 = s_{f2} + x_2 s_{fg2},$$

i.e.: $h_2 = 417.46 + (0.85) \times (2258) = 2336.76\,\text{kJ/kg}$

and $s_2 = 1.3026 + (0.85) \times (6.0568) = 6.451\,\text{kJ/kg K}$

Since $s_1 = s_2$, the state 1 is fixed by $s_1 = 6.451$ kJ/kg K, and point 1 is at the dry saturated line. Therefore pressure P_1 may be determined by the Mollier chart or by calculations: i.e.: $P_1 = 1.474$ MPa.

6.6 STAGE DESIGN

A turbine stage is defined as a set of stationary blades (or nozzles) followed by a set of moving blades (or buckets or rotor). Together, the two sets of blades allow the steam to perform work on the turbine rotor. This work is then transmitted to the driven load by the shaft on which the rotor assembly is carried. Two turbine stage designs in use are: the impulse stage and reaction stage. The first turbine, designated by DeLaval in 1889, was a single-stage impulse turbine, which ran at 30,000 rpm. Because of its high speed, this type of turbine has very limited applications in practice. High speeds are extremely undesirable due to high blade tip stresses and large losses due to disc friction, which cannot be avoided. In large power plants, the single-stage impulse turbine is ruled out, since alternators usually run speeds around 3000 rpm. Photographs of actual steam turbines are reproduced in Figs. 6.8–6.10.

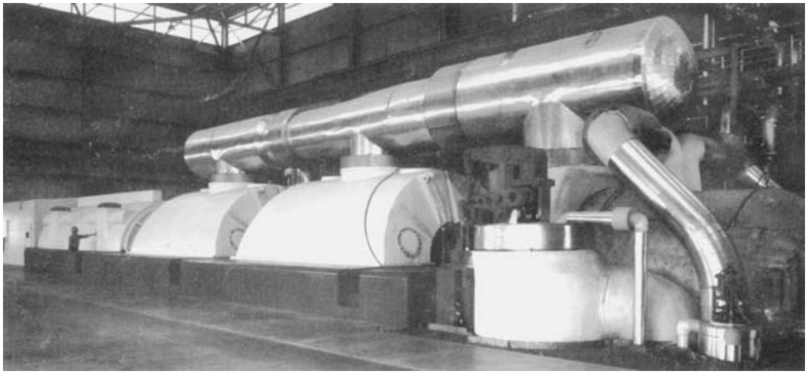

Figure 6.8 Steam turbine.

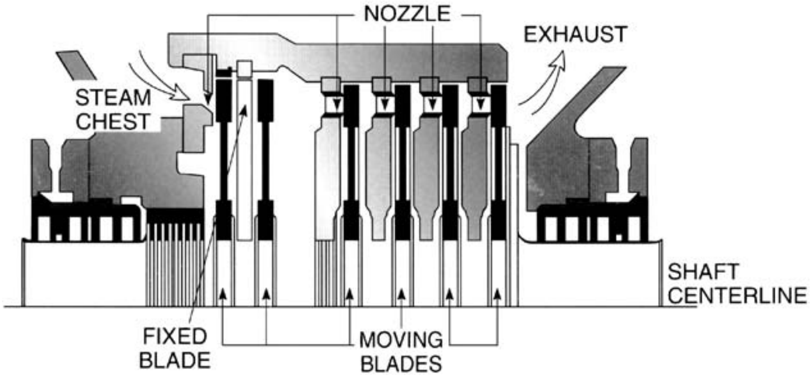

Figure 6.9 Pressure velocity-compounded impulse turbine.

6.7 IMPULSE STAGE

In the impulse stage, the total pressure drop occurs across the stationary blades (or nozzles). This pressure drop increases the velocity of the steam. However, in the reaction stage, the total pressure drop is divided equally across the stationary blades and the moving blades. The pressure drop again results in a corresponding increase in the velocity of the steam flow.

As shown in Figs. 6.10 and 6.11, the shape of the stationary blades or nozzles in both stage designs is very similar. However, a big difference exists in the shapes of the moving blades. In an impulse stage, the shape of the moving blades or buckets is like a cup. The shape of the moving blades in a reaction stage is more like that of an airfoil. These blades look very similar to the stationary blades or nozzles.

6.8 THE IMPULSE STEAM TURBINE

Most of the steam turbine plants use impulse steam turbines, whereas gas turbine plants seldom do. The general principles are the same whether steam or gas is the working substance.

As shown in Fig. 6.12, the steam supplied to a single-wheel impulse turbine expands completely in the nozzles and leaves with absolute velocity C_1 at an angle α_1, and by subtracting the blade velocity vector U, the relative velocity vector at entry to the rotor V_1 can be determined. The relative velocity V_1 makes an angle of β_1 with respect to U. The increase in value of α_1 decreases the value of the useful component, $C_1 \cos \alpha_1$ and increases the value of the axial or flow component $C_a \sin \alpha_1$. The two points of particular interest are the inlet and exit of the blades. As shown in Fig. 6.12, these velocities are V_1 and V_2, respectively.

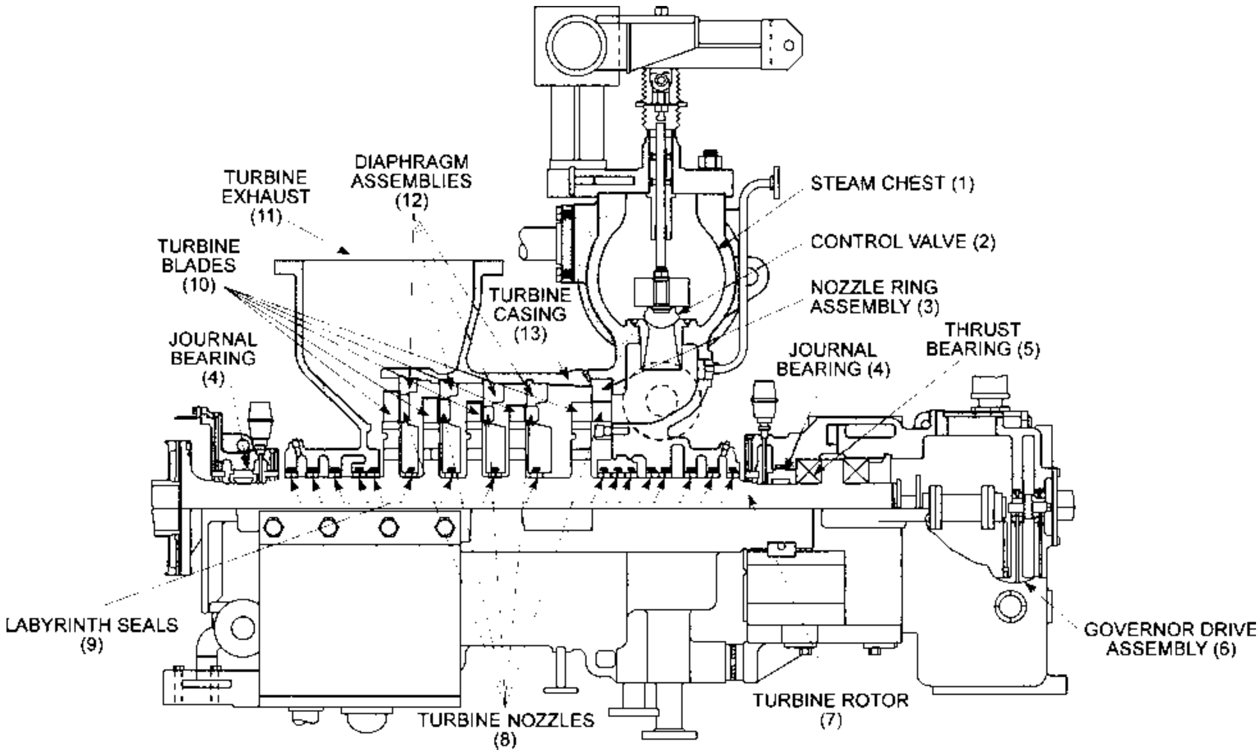

Figure 6.10 Steam turbine cross-sectional view.

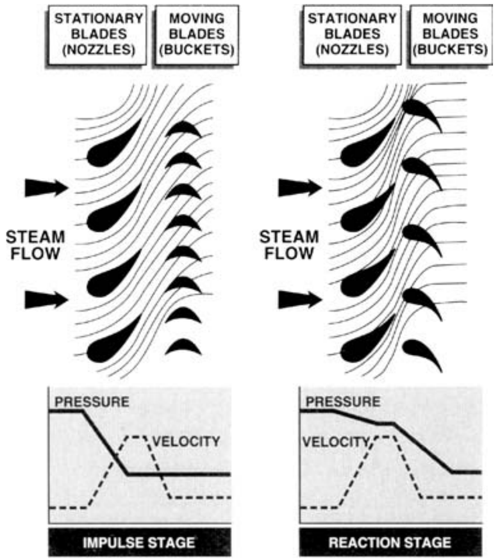

Figure 6.11 Impulse and reaction stage design.

Vectorially subtracting the blade speed results in absolute velocity C_2. The steam leaves tangentially at an angle β_2 with relative velocity V_2. Since the two velocity triangles have the same common side U, these triangles can be combined to give a single diagram as shown in Fig. 6.13.

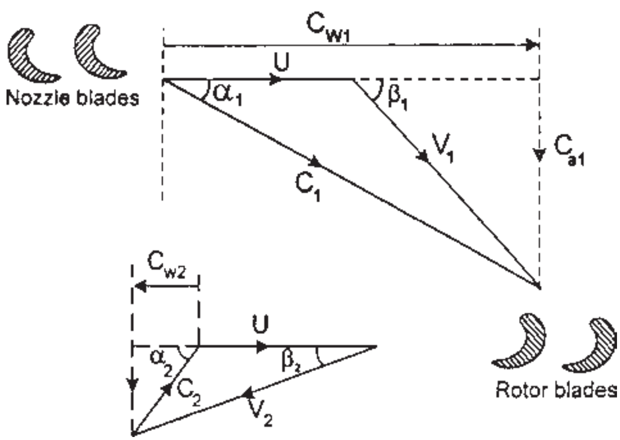

Figure 6.12 Velocity triangles for turbine stage.

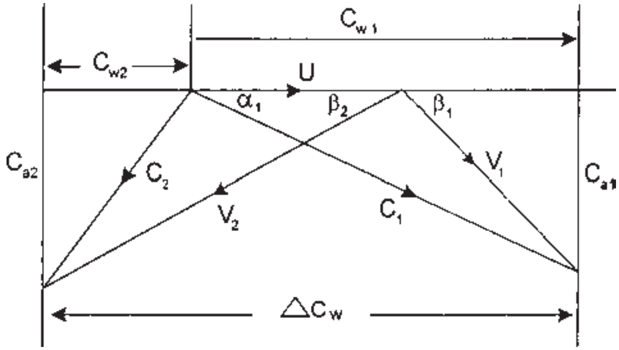

Figure 6.13 Combined velocity diagram.

If the blade is symmetrical then $\beta_1 = \beta_2$ and neglecting the friction effects of blades on the steam, $V_1 = V_2$. In the actual case, the relative velocity is reduced by friction and expressed by a blade velocity coefficient k. That is:

$$k = \frac{V_2}{V_1}$$

From Euler's equation, work done by the steam is given by:

$$W_t = U(C_{w1} + C_{w2})$$

Since C_{w2} is in the negative r direction, the work done per unit mass flow is given by:

$$W_t = U(C_{w1} + C_{w2}) \tag{6.9}$$

If $C_{a1} \neq C_{a2}$, there will be an axial thrust in the flow direction. Assume that C_a is constant. Then:

$$W_t = UC_a(\tan\alpha_1 + \tan\alpha_2) \tag{6.10}$$

$$W_t = UC_a(\tan\beta_1 + \tan\beta_2) \tag{6.11}$$

Equation (6.11) is often referred to as the diagram work per unit mass flow and hence the diagram efficiency is defined as:

$$\eta_d = \frac{\text{Diagram work done per unit mass flow}}{\text{Work available per unit mass flow}} \tag{6.12}$$

Referring to the combined diagram of Fig. 6.13: ΔC_w is the change in the velocity of whirl. Therefore:

$$\text{The driving force on the wheel} = \dot{m}C_w \tag{6.13}$$

The product of the driving force and the blade velocity gives the rate at which work is done on the wheel. From Eq. (6.13):

$$\text{Power output} = \dot{m}U\Delta C_w \tag{6.14}$$

If $C_{a1} - C_{a2} = \Delta C_a$, the axial thrust is given by:

$$\text{Axial thrust}: F_a = \dot{m}\Delta C_a \tag{6.15}$$

The maximum velocity of the steam striking the blades

$$C_1 = \sqrt{\{2(h_0 - h_1)\}} \tag{6.16}$$

where h_0 is the enthalpy at the entry to the nozzle and h_1 is the enthalpy at the nozzle exit, neglecting the velocity at the inlet to the nozzle. The energy supplied to the blades is the kinetic energy of the jet, $C_1^2/2$ and the blading efficiency or diagram efficiency:

$$\eta_d = \frac{\text{Rate of work performed per unit mass flow}}{\text{Energy supplied per unit mass of steam}}$$

$$\eta_d = (U\Delta C_w) \times \frac{2}{C_1^2} = \frac{2U\Delta C_w}{C_1^2} \tag{6.17}$$

Using the blade velocity coefficient $\left(k = \frac{V_2}{V_1}\right)$ and symmetrical blades (i.e., $\beta_1 = \beta_2$), then:

$$\Delta C_w = 2V_1 \cos\alpha_1 - U$$

Hence

$$\Delta C_w = 2(C_1 \cos\alpha_1 - U) \tag{6.18}$$

And the rate of work performed per unit mass $= 2(C_1 \cos\alpha_1 - U)U$

Therefore:

$$\eta_d = 2(C_1 \cos\alpha_1 - U)U \times \frac{2}{C_1^2}$$

$$\eta_d = \frac{4(C_1 \cos\alpha_1 - U)U}{C_1^2}$$

$$\eta_d = \frac{4U}{C_1}\left(\cos\alpha_1 - \frac{U}{C_1}\right) \tag{6.19}$$

where $\frac{U}{C_1}$ is called the blade speed ratio.

Differentiating Eq. (6.19) and equating it to zero provides the maximum diagram efficiency:

$$\frac{d(\eta_d)}{d\left(\frac{U}{C_1}\right)} = 4\cos\alpha_1 - \frac{8U}{C_1} = 0$$

or

$$\frac{U}{C_1} = \frac{\cos\alpha_1}{2} \tag{6.20}$$

i.e., maximum diagram efficiency

$$= \frac{4\cos\alpha_1}{2}\left(\cos\alpha_1 - \frac{\cos\alpha_1}{2}\right)$$

or:

$$\eta_d = \cos^2\alpha_1 \tag{6.21}$$

Substituting this value into Eq. (6.14), the power output per unit mass flow rate at the maximum diagram efficiency:

$$P = 2U^2 \tag{6.22}$$

6.9 PRESSURE COMPOUNDING (THE RATEAU TURBINE)

A Rateau-stage impulse turbine uses one row of nozzles and one row of moving blades mounted on a wheel or rotor, as shown in Fig. 6.14. The total pressure drop is divided in a series of small increments over the stages. In each stage, which consists of a nozzle and a moving blade, the steam is expanded and the kinetic energy is used in moving the rotor and useful work is obtained.

The separating walls, which carry the nozzles, are known as diaphragms. Each diaphragm and the disc onto which the diaphragm discharges its steam is known as a stage of the turbine, and the combination of stages forms a pressure compounded turbine. Rateau-stage turbines are unable to extract a large amount of energy from the steam and, therefore, have a low efficiency. Although the Rateau turbine is inefficient, its simplicity of design and construction makes it well suited for small auxiliary turbines.

6.10 VELOCITY COMPOUNDING (THE CURTIS TURBINE)

In this type of turbine, the whole of the pressure drop occurs in a single nozzle, and the steam passes through a series of blades attached to a single wheel or rotor. The Curtis stage impulse turbine is shown in Fig. 6.15.

Fixed blades between the rows of moving blades redirect the steam flow into the next row of moving blades. Because the reduction of velocity occurs over two stages for the same pressure decreases, a Curtis-stage turbine can extract

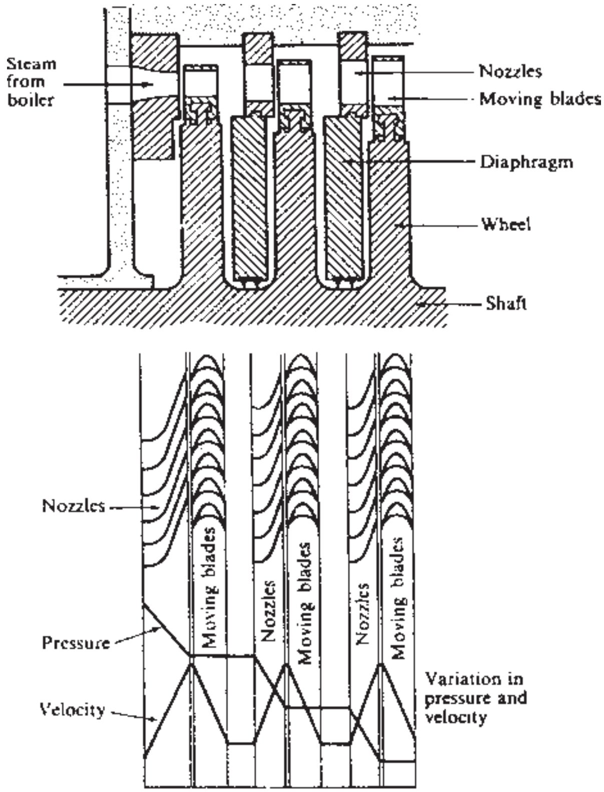

Figure 6.14 Rateau-stage impulse turbine.

more energy from the steam than a Rateau-stage turbine. As a result, a Curtis-stage turbine has a higher efficiency than a Rateau-stage turbine.

6.11 AXIAL FLOW STEAM TURBINES

Sir Charles Parsons invented the reaction steam turbine. The reaction turbine stage consists of a fixed row of blades and an equal number of moving blades fixed on a wheel. In this turbine pressure drop or expansion takes place both in the fixed blades (or nozzles) as well as in the moving blades. Because the pressure drop from inlet to exhaust is divided into many steps through use of alternate rows of fixed and moving blades, reaction turbines that have more than one stage are classified as pressure-compounded turbines. In a reaction turbine, a reactive force is produced on the moving blades when the steam increases in velocity and when the steam changes direction. Reaction turbines are normally used as

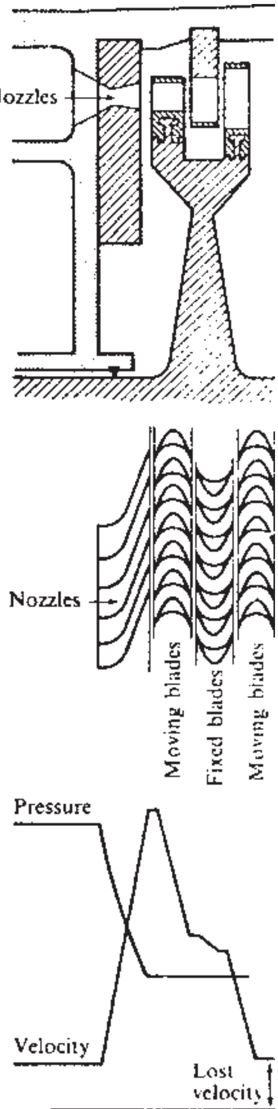

Figure 6.15 The Curtis-stage impulse turbine.

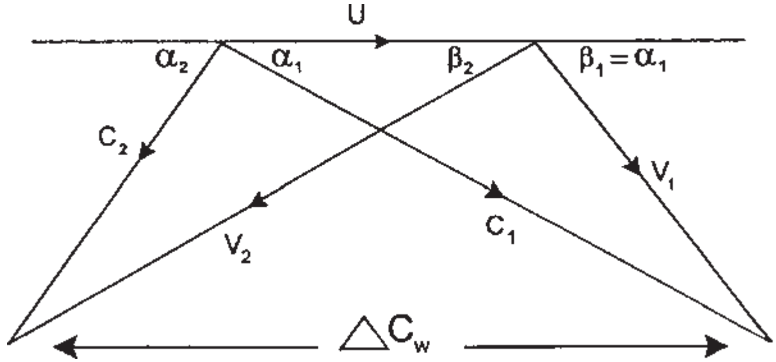

Figure 6.16 Velocity triangles for 50% reaction design.

low-pressure turbines. High-pressure reaction turbines are very costly because they must be constructed from heavy and expensive materials. For a 50% reaction, the fixed and moving blades have the same shape and, therefore, the velocity diagram is symmetrical as shown in Fig. 6.16.

6.12 DEGREE OF REACTION

The degree of reaction or reaction ratio (Λ) is a parameter that describes the relation between the energy transfer due to static pressure change and the energy transfer due to dynamic pressure change. The degree of reaction is defined as the ratio of the static pressure drop in the rotor to the static pressure drop in the stage. It is also defined as the ratio of the static enthalpy drop in the rotor to the static enthalpy drop in the stage. If h_0, h_1, and h_2 are the enthalpies at the inlet due to the fixed blades, at the entry to the moving blades and at the exit from the moving blades, respectively, then:

$$\Lambda = \frac{h_1 - h_2}{h_0 - h_2} \tag{6.23}$$

The static enthalpy at the inlet to the fixed blades in terms of stagnation enthalpy and velocity at the inlet to the fixed blades is given by

$$h_0 = h_{00} - \frac{C_0^2}{2C_p}$$

Similarly,

$$h_2 = h_{02} - \frac{C_2^2}{2C_p}$$

Substituting,

$$\Lambda = \frac{(h_1 - h_2)}{\left(h_{00} - \frac{C_0^2}{2C_p}\right) - \left(h_{02} - \frac{C_2^2}{2C_p}\right)}$$

But for a normal stage, $C_0 = C_2$ and since $h_{00} = h_{01}$ in the nozzle, then:

$$\Lambda = \frac{h_1 - h_2}{h_{01} - h_{02}} \tag{6.24}$$

We know that $h_{01\mathrm{Re}1} = h_{02\mathrm{Re}2}$. Then:

$$h_{01\mathrm{Re}1} - h_{02\mathrm{Re}2} = (h_1 - h_2) + \frac{\left(V_1^2 - V_2^2\right)}{2} = 0$$

Substituting for $(h_1 - h_2)$ in Eq. (6.24):

$$\Lambda = \frac{\left(V_2^2 - V_1^2\right)}{[2(h_{01} - h_{02})]}$$

$$\Lambda = \frac{\left(V_2^2 - V_1^2\right)}{[2U(C_{w1} + C_{w2})]} \tag{6.25}$$

Assuming the axial velocity is constant through the stage, then:

$$\Lambda = \frac{\left(V_{w2}^2 - V_{w1}^2\right)}{[2U(U + V_{w1} + V_{w2} - U)]}$$

$$\Lambda = \frac{(V_{w2} - V_{w1})(V_{w2} + V_{w1})}{[2U(V_{w1} + V_{w2})]}$$

$$\Lambda = \frac{C_a\left(\tan\beta_2 - \tan\beta_1\right)}{2U} \tag{6.26}$$

From the velocity triangles, it is seen that

$$C_{w1} = U + V_{w1}, \text{ and } C_{w2} = V_{w2} - U$$

Therefore, Eq. (6.26) can be arranged into a second form:

$$\Lambda = \frac{1}{2} + \frac{C_a}{2U}\left(\tan\beta_2 - \tan\alpha_2\right) \tag{6.27}$$

Putting $\Lambda = 0$ in Eq. (6.26), we get

$$\beta_2 = \beta_1 \text{ and } V_1 = V_2, \text{ and for } \Lambda = 0.5, \beta_2 = \alpha_1.$$

Zero Reaction Stage:

Let us first discuss the special case of zero reaction. According to the definition of reaction, when $\Lambda = 0$, Eq. (6.23) reveals that $h_1 = h_2$ and Eq. (6.26) that $\beta_1 = \beta_2$. The Mollier diagram and velocity triangles for $\Lambda = 0$ are shown in Figs. 6.17 and 6.18:

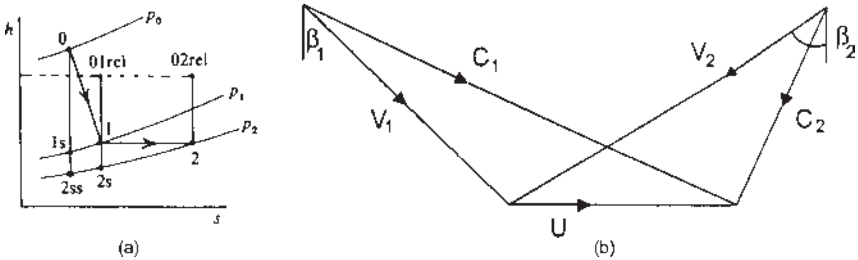

Figure 6.17 Zero reaction (a) Mollier diagram and (b) velocity diagram.

Now, $h_{01r01} = h_{02r01}$ and $h_1 = h_2$ for $\Lambda = 0$. Then, $V_1 = V_2$. In the ideal case, there is no pressure drop in the rotor, and points 1, 2 and 2s on the Mollier chart should coincide. But due to irreversibility, there is a pressure drop through the rotor. The zero reaction in the impulse stage, by definition, means there is no pressure drop through the rotor. The Mollier diagram for an impulse stage is shown in Fig. 6.18, where it can be observed that the enthalpy increases through the rotor.

From Eq. (6.23), it is clear that the reaction is negative for the impulse turbine stage when irreversibility is taken into account.

Fifty-Percent Reaction Stage

From Eq. (6.23), Fig. (6.19) for $\Lambda = 0.5$, $\alpha_1 = \beta_2$, and the velocity diagram is symmetrical. Because of symmetry, it is also clear that $\alpha_2 = \beta_1$. For $\Lambda = 1/2$, the enthalpy drop in the nozzle row equals the enthalpy drop in the rotor. That is:

$$h_0 - h_1 = h_1 - h_2$$

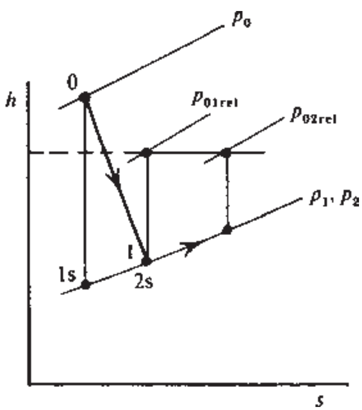

Figure 6.18 Mollier diagram for an impulse stage.

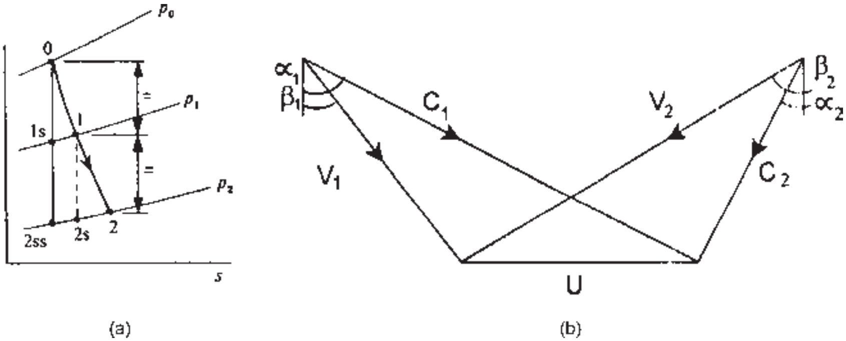

Figure 6.19 A 50% reaction stage (a) Mollier diagram and (b) velocity diagram.

Substituting $\beta_2 = \tan\alpha_2 + \dfrac{U}{C_a}$ into Eq. (6.27) gives

$$\Lambda = 1 + \frac{C_a}{2U}(\tan\alpha_2 - \tan\alpha_1) \tag{6.28}$$

Thus, when $\alpha_2 = \alpha_1$, the reaction is unity (also $C_1 = C_2$). The velocity diagram for $\Lambda = 1$ is shown in Fig. 6.20 with the same value of C_a, U, and W used for $\Lambda = 0$ and $\Lambda = \frac{1}{2}$. It is obvious that if Λ exceeds unity, then $C_1 < C_0$ (i.e., nozzle flow diffusion).

Choice of Reaction and Effect on Efficiency:

Eq. (6.24) can be rewritten as:

$$\Lambda = 1 + \frac{C_{w2} - C_{w1}}{2U}.$$

C_{w2} can be eliminated by using this equation:

$$C_{w2} = \frac{W}{U} - C_{w1},$$

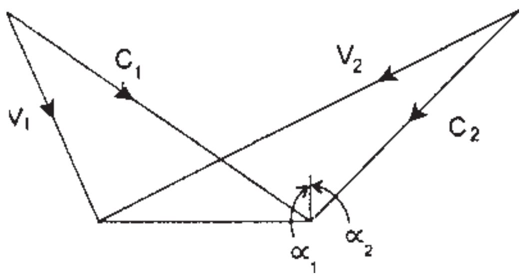

Figure 6.20 Velocity diagram for 100% reaction turbine stage.

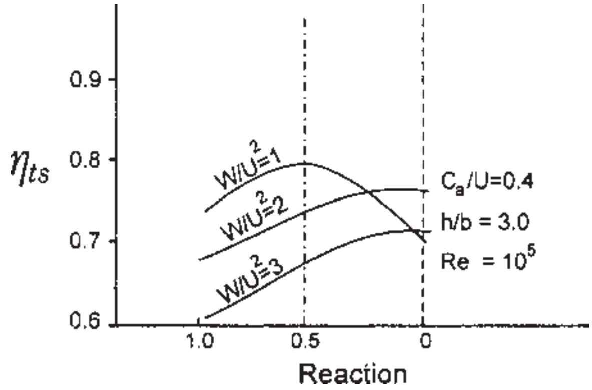

Figure 6.21 Influence of reaction on total-to-static efficiency with fixed values of stage-loading factor.

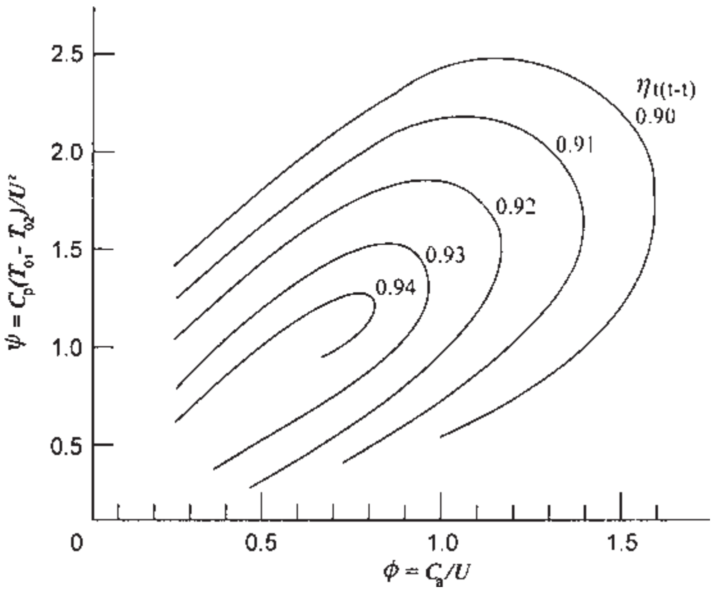

Figure 6.22 Blade loading coefficient vs. flow coefficient.

yielding:

$$\Lambda = 1 + \frac{W}{2U^2} - \frac{C_{w1}}{U} \tag{6.29}$$

In Fig. 6.21 the total-to-static efficiencies are shown plotted against the degree of reaction.

When $\frac{W}{U^2} = 2$, η_{ts} is maximum at $\Lambda = 0$. With higher loading, the optimum η_{ts} is obtained with higher reaction ratios. As shown in Fig. 6.22 for a high total-to-total efficiency, the blade-loading factor should be as small as possible, which implies the highest possible value of blade speed is consistent with blade stress limitations. It means that the total-to-static efficiency is heavily dependent upon the reaction ratio and η_{ts} can be optimized by choosing a suitable value of reaction.

6.13 BLADE HEIGHT IN AXIAL FLOW MACHINES

The continuity equation, $\dot{m} = \rho AC$, may be used to find the blade height h. The annular area of flow $= \pi Dh$. Thus, the mass flow rate through an axial flow compressor or turbine is:

$$\dot{m} = \rho \pi D h C_a \tag{6.30}$$

Blade height will increase in the direction of flow in a turbine and decrease in the direction of flow in a compressor.

Illustrative Example 6.6: The velocity of steam leaving a nozzle is 925 m/s and the nozzle angle is 20°. The blade speed is 250 m/s. The mass flow through the turbine nozzles and blading is 0.182 kg/s and the blade velocity coefficient is 0.7. Calculate the following:

1. Velocity of whirl.
2. Tangential force on blades.
3. Axial force on blades.
4. Work done on blades.
5. Efficiency of blading.
6. Inlet angle of blades for shockless inflow of steam.

Assume that the inlet and outlet blade angles are equal.

Solution:

From the data given, the velocity diagram can be constructed as shown in Fig. 6.23. The problem can be solved either graphically or by calculation.

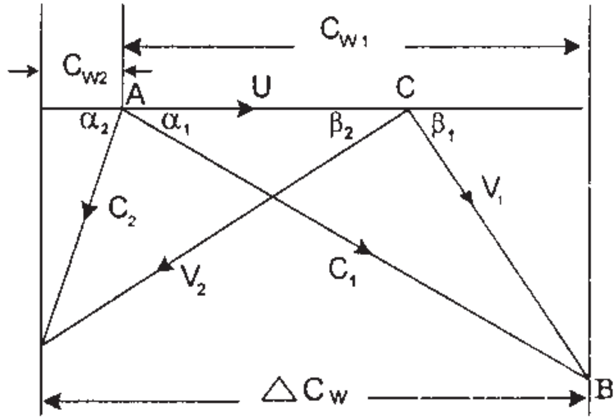

Figure 6.23 Velocity triangles for Example 6.6.

Applying the cosine rule to the $\triangle$ABC,

$$V_1^2 = U^2 + C_1^2 - 2UC_1\cos\alpha_1$$
$$= 250^2 + 925^2 - (2)\times(250)\times(925)\times\cos 20°$$

so: $V_1 = 695.35$ m/s
But,

$$k = \frac{V_2}{V_1}, \text{or} V_2 = (0.70)\times(695.35) = 487 \text{m/s}.$$

Velocity of whirl at inlet:

$$C_{w1} = C_1 cos\alpha_1 = 925\cos 20° = 869.22 \text{m/s}$$

Axial component at inlet:

$$C_{a1} = \text{BD} = C_1\sin\alpha_1 = 925\sin 20° = 316.37 \text{m/s}$$

Blade angle at inlet:

$$\tan\beta_1 = \frac{C_{a1}}{C_{w1} - U} = \frac{316.37}{619.22} = 0.511$$

Therefore, $\beta_1 = 27.06° = \beta_2 =$ outlet blade angle.

$$\cos\beta_2 = \frac{C_{w2} + U}{V_2},$$

or: $$C_{w2} = V_2\cos\beta_2 - U = 487\times\cos 27.06° - 250$$
$$= 433.69 - 250 = 183.69 \text{ m/s}$$

and: $C_{a2} = \text{FE} = (U + C_{w2})\tan\beta_2 = 433.69\tan 27.06° = 221.548$ m/s

1. Velocity of whirl at inlet, $C_{w1} = 869.22\,\text{m/s}$;
   Velocity of whirl at outlet, $C_{w2} = 183.69\,\text{m/s}$
2. Tangential force on blades
$$= \dot{m}\,(C_{w1} + C_{w2}) = (0.182)\,(1052.9) = 191.63\,\text{N}.$$
3. Axial force on blades
$$= \dot{m}(C_{a1} - C_{a2}) = (0.182)(316.37 - 221.548) = 17.26\,\text{N}$$
4. Work done on blades
$$= \text{tangential force on blades} \times \text{blade velocity}$$
$$= (191.63) \times (250)/1000 = 47.91\,\text{kW}.$$
5. Efficiency of blading $= \dfrac{\text{Work done on blades}}{\text{Kinetic energy supplied}}$
$$= \frac{47.91}{\frac{1}{2}mC_1^2} = \frac{(47.91)(2)(10^3)}{(0.182)(925^2)}$$
$$= 0.6153 \text{ or } 61.53\%$$
6. Inlet angle of blades $\beta_1 = 27.06° = \beta_2$.

Design Example 6.7: The steam velocity leaving the nozzle is 590 m/s and the nozzle angle is 20°. The blade is running at 2800 rpm and blade diameter is 1050 mm. The axial velocity at rotor outlet = 155 m/s, and the blades are symmetrical. Calculate the work done, the diagram efficiency and the blade velocity coefficient.

Solution:

Blade speed U is given by:

$$U = \frac{\pi DN}{60} = \frac{(\pi \times 1050) \times (2800)}{(1000) \times (60)} = 154\,\text{m/s}$$

The velocity diagram is shown in Fig. 6.24.
Applying the cosine rule to the triangle ABC,

$$V_1^2 = U^2 + C_1^2 - 2UC_1 \cos\alpha_1$$
$$= 154^2 + 590^2 - (2) \times (154) \times (590) \cos 20°$$

i.e. $V_1 = 448.4\,\text{m/s}$.

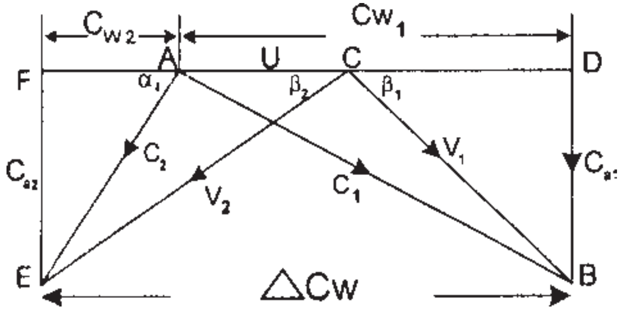

Figure 6.24 Velocity diagram for Example 6.7.

Applying the sine rule to the triangle ABC,

$$\frac{C_1}{\sin(\mathrm{ACB})} = \frac{V_1}{\sin(\alpha_1)}$$

But $\sin(\mathrm{ACB}) = \sin(180° - \beta_1) = \sin(\beta_1)$

Therefore,

$$\sin(\beta_1) = \frac{C_1 \sin(\alpha_1)}{V_1} = \frac{590 \sin(20°)}{448.4} = 0.450$$

and: $\beta_1 = 26.75°$

From triangle ABD,

$$C_{w1} = C_1 \cos(\alpha_1) = 590 \cos(20°) = 554.42\ \mathrm{m/s}$$

From triangle CEF,

$$\frac{C_{a2}}{U + C_{w2}} = \tan(\beta_2) = \tan(\beta_1) = \tan(26.75°) = 0.504$$

or: $U + C_{w2} = \dfrac{C_{a2}}{0.504} = \dfrac{155}{0.504} = 307.54$

so : $C_{w2} = 307.54 - 154 = 153.54\ \mathrm{m/s}$

Therefore,

$$\Delta C_w = C_{w1} + C_{w2} = 554.42 + 153.54 = 707.96\ \mathrm{m/s}$$

Relative velocity at the rotor outlet is:

$$V_2 = \frac{C_{a2}}{\sin(\beta_2)} = \frac{155}{\sin(26.75°)} = 344.4\ \mathrm{m/s}$$

Blade velocity coefficient is:

$$k = \frac{V_2}{V_1} = \frac{344.4}{448.4} = 0.768$$

Work done on the blades per kg/s:

$$\Delta C_{w2} U = (707.96) \times (154) \times (10^{-3}) = 109\,\text{kW}$$

The diagram efficiency is:

$$\eta_d = \frac{2U\Delta C_w}{C_1^2} = \frac{(2) \times (707.96) \times (154)}{590^2} = 0.6264$$

or, $\eta_d = 62.64\%$

Illustrative Example 6.8: In one stage of an impulse turbine the velocity of steam at the exit from the nozzle is 460 m/s, the nozzle angle is 22° and the blade angle is 33°. Find the blade speed so that the steam shall pass on without shock. Also find the stage efficiency and end thrust on the shaft, assuming velocity coefficient = 0.75, and blades are symmetrical.

Solution:

From triangle ABC (Fig. 6.25):

$$C_{w1} = C_1 \cos 22° = 460 \cos 22° = 426.5\,\text{m/s}$$

and:

$$C_{a1} = C_1 \sin 22° = 460 \sin 22° = 172.32\,\text{m/s}$$

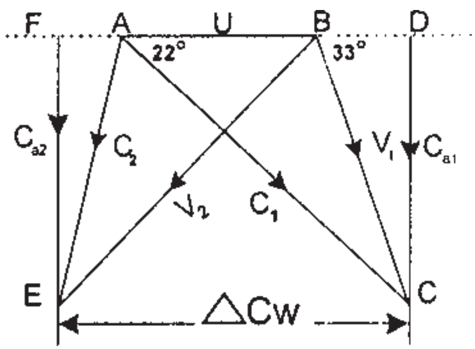

Figure 6.25 Velocity triangles for Example 6.8.

Now, from triangle BCD:

$$\mathrm{BD} = \frac{C_{a1}}{\tan(33^\circ)} = \frac{172.32}{0.649} = 265.5$$

Hence, blade speed is given by:

$$U = C_{w1} - \mathrm{BD} = 426.5 - 265.5 = 161\,\mathrm{m/s}$$

From Triangle BCD, relative velocity at blade inlet is given by:

$$V_1 = \frac{C_{a1}}{\sin(33^\circ)} = \frac{172.32}{0.545} = 316.2\,\mathrm{m/s}$$

Velocity coefficient:

$$k = \frac{V_2}{V_1}, \quad \text{or} \quad V_2 = kV_1 = (0.75) \times (316.2) = 237.2\,\mathrm{m/s}$$

From Triangle BEF,

$$\mathrm{BF} = V_2 \cos(33^\circ) = 237.2 \times \cos(33^\circ) = 198.9$$

and

$$C_{w2} = \mathrm{AF} = \mathrm{BF} - U = 198.9 - 161 = 37.9\,\mathrm{m/s}$$

$$C_{a2} = V_2 \sin(33^\circ) = 237.2 \sin(33^\circ) = 129.2\,\mathrm{m/s}$$

The change in velocity of whirl:

$$\Delta C_w = C_{w1} + C_{w2} = 426.5 + 37.9 = 464.4\,\mathrm{m/s}$$

Diagram efficiency:

$$\eta_d = \frac{2U\Delta C_w}{C_1^2} = \frac{(2) \times (464.4) \times (161)}{460^2} = 0.7067, \text{ or } 70.67\%.$$

End thrust on the shaft per unit mass flow:

$$C_{a1} - C_{a2} = 172.32 - 129.2 = 43.12\,\mathrm{N}$$

Design Example 6.9: In a Parson's turbine, the axial velocity of flow of steam is 0.5 times the mean blade speed. The outlet angle of the blade is 20°, diameter of the ring is 1.30 m and the rotational speed is 3000 rpm. Determine the inlet angles of the blades and power developed if dry saturated steam at 0.5 MPa passes through the blades where blade height is 6 cm. Neglect the effect of the blade thickness.

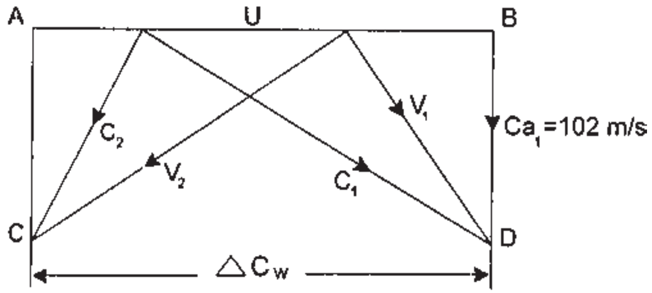

Figure 6.26 Velocity triangles for Example 6.9.

Solution:

The blade speed, $U = \dfrac{\pi DN}{60} = \dfrac{\pi \times (1.30) \times (3000)}{60} = 204\,\text{m/s}$

Velocity of flow, $C_a = (0.5) \times (204) = 102\,\text{m/s}$

Draw lines AB and CD parallel to each other Fig. 6.26 at the distance of 102 m/s, i.e., velocity of flow, $C_{a1} = 102\,\text{m/s}$.

At any point B, construct an angle $\alpha_2 = 20°$ to intersect line CD at point C. Thus, the velocity triangle at the outlet is completed. For Parson's turbine,

$$\alpha_1 = \beta_2, \quad \beta_1 = \alpha_2, \quad C_1 = V_2, \quad \text{and} \quad V_1 = C_2.$$

By measurement,

$$\Delta C_w = C_{w1} + C_{w2} = 280.26 + 76.23 = 356.5\,\text{m/s}$$

The inlet angles are 53.22°.Specific volume of vapor at 0.5 MPa, from the steam tables, is

$$v_g = 0.3749\,\text{m}^3/\text{kg}$$

Therefore the mass flow is given by:

$$\dot{m} = \frac{AC_2}{x_2 v_{g_2}} = \frac{\pi \times (1.30) \times (6) \times (102)}{(100) \times (0.3749)} = 66.7\,\text{kg/s}$$

Power developed:

$$P = \frac{\dot{m} U \Delta C_w}{1000} = \frac{(66.7) \times (356.5) \times (102)}{1000} = 2425.4\,\text{kW}$$

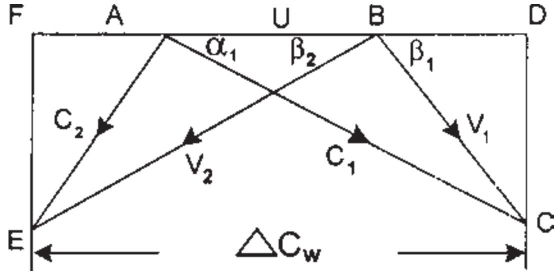

Figure 6.27 Velocity triangles for Example 6.10.

Design Example 6.10: In an impulse turbine, steam is leaving the nozzle with velocity of 950 m/s and the nozzle angle is 20°. The nozzle delivers steam at the rate of 12 kg/min. The mean blade speed is 380 m/s and the blades are symmetrical. Neglect friction losses. Calculate (1) the blade angle, (2) the tangential force on the blades, and (3) the horsepower developed.

Solution:

With the help of α_1, U and C_1, the velocity triangle at the blade inlet can be constructed easily as shown in Fig. 6.27.

Applying the cosine rule to the triangle ABC,

$$V_1^2 = U^2 + C_1^2 - 2UC_1\cos\alpha_1$$

$$= 950^2 + 380^2 - (2)\times(950)\times(380)\times\cos 20° = 607\text{m/s}$$

Now, applying the sine rule to the triangle ABC,

$$\frac{V_1}{\sin(\alpha_1)} = \frac{C_1}{\sin(180° - \beta_1)} = \frac{C_1}{\sin(\beta_1)}$$

or:

$$\sin(\beta_1) = \frac{C_1\sin(\alpha_1)}{V_1} = \frac{(950)\times(0.342)}{607} = 0.535$$

so:

$$\beta_1 = 32.36°$$

From Triangle ACD,

$$C_{w1} = C_1\cos(\alpha_1) = 950\times\cos(20°) = (950)\times(0.9397)$$

$$= 892.71\text{m/s}$$

As $\beta_1 = \beta_2$, using triangle BEF and neglecting friction loss, i.e,: $V_1 = V_2$

$$\mathrm{BF} = V_2 \cos\beta_2 = 607 \times \cos 32.36° = 512.73$$

Therefore,

$$C_{w2} = \mathrm{BF} - U = 512.73 - 380 = 132.73\,\mathrm{m/s}$$

Change in velocity of whirl:

$$\Delta C_w = C_{w1} + C_{w2} = 892.71 + 132.73 = 1025.44\,\mathrm{m/s}$$

Tangential force on blades:

$$F = \dot{m}\Delta C_w = \frac{(12) \times (1025.44)}{60} = 205\,\mathrm{N}$$

Horsepower, $P = \dot{m}U\Delta C_w = \dfrac{(12) \times (1025.44) \times (380)}{(60) \times (1000) \times (0.746)} = 104.47\,\mathrm{hp}$

Design Example 6.11: In an impulse turbine, the velocity of steam at the exit from the nozzle is 700 m/s and the nozzles are inclined at 22° to the blades, whose tips are both 34°. If the relative velocity of steam to the blade is reduced by 10% while passing through the blade ring, calculate the blade speed, end thrust on the shaft, and efficiency when the turbine develops 1600 kW.

Solution:

Velocity triangles for this problem are shown in Fig. 6.28.
From the triangle ACD,

$$C_{a1} = C_1 \sin\alpha_1 = 700 \times \sin 22° = 262.224\,\mathrm{m/s}$$

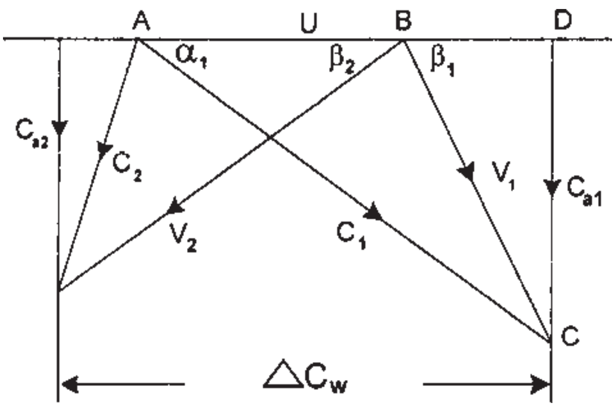

Figure 6.28 Velocity triangles for Example 6.11.

and

$$V_1 = \frac{C_{a1}}{\sin(\beta_1)} = \frac{262.224}{\sin 34°} = 469.32\,\text{m/s}$$

Whirl component of C_1 is given by

$$C_{w1} = C_1 \cos(\alpha_1) = 700 \cos(22°) = 700 \times 0.927 = 649\,\text{m/s}$$

Now, BD $= C_{w1} - U = V_1 \cos \beta_1 = (469.32) \times (0.829) = 389$

Hence, blade speed

$$U = 649 - 389 = 260\,\text{m/s}$$

Using the velocity coefficient to find V_2:

$$\text{i.e., } V_2 = (0.90) \times (469.32) = 422.39\,\text{m/s}$$

From velocity triangle BEF,

$$C_{a2} = V_2 \sin(\beta_2) = 422.39 \sin 34° = 236.2\,\text{m/s}$$

And

$$U + C_{w2} = V_2 \cos 34° = (422.39) \times (0.829) = 350.2\,\text{m/s}$$

Therefore,

$$C_{w2} = 350.2 - 260 = 90.2\,\text{m/s}$$

Then,

$$\Delta C_w = C_{w1} + C_{w2} = 649 + 90.2 = 739.2\,\text{m/s}$$

Mass flow rate is given by:

$$P = \dot{m} U \Delta C_w$$

or

$$\dot{m} = \frac{(1600) \times (1000)}{(739.2) \times (260)} = 8.325\,\text{kg/s}$$

Thrust on the shaft,

$$F = \dot{m}(C_{a1} - C_{a2}) = 8.325(262.224 - 236.2) = 216.65\,\text{N}$$

Diagram efficiency:

$$\eta_d = \frac{2U\Delta C_w}{C_1^2} = \frac{(2) \times (739.2) \times (260)}{700^2} = 0.7844, \quad \text{or} \quad 78.44\%.$$

Illustrative Example 6.12: The moving and fixed blades are identical in shape in a reaction turbine. The absolute velocity of steam leaving the fixed blade is 105 m/s, and the blade velocity is 40 m/s. The nozzle angle is 20°. Assume axial

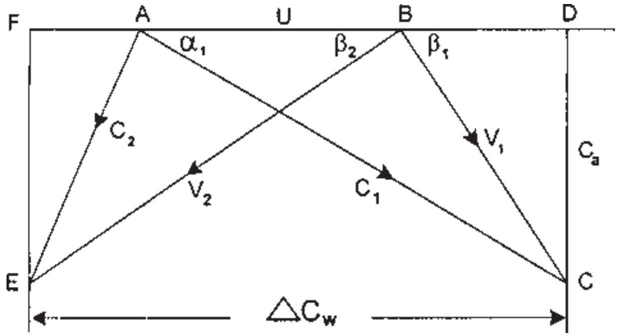

Figure 6.29 Velocity triangles for Example 6.12.

velocity is constant through the stage. Determine the horsepower developed if the steam flow rate is 2 kg/s.

Solution:

For 50% reaction turbine Fig. 6.29, $\alpha_1 = \beta_2$, and $\alpha_2 = \beta_1$.
From the velocity triangle ACD,

$$C_{w1} = C_1 \cos \alpha_1 = 105 \cos 20° = 98.67 \text{ m/s}$$

Applying cosine rule to the Triangle ABC:

$$V_1^2 = C_1^2 + U^2 - 2C_1 U \cos \alpha_1$$

so:

$$V_1 = \sqrt{105^2 + 40^2 - (2) \times (105) \times (40) \times \cos 20°} = 68.79 \text{ m/s}$$

Now,

$$\text{BD} = C_{w1} - U = V_1 \cos \beta_1 = 98.67 - 40 = 58.67$$

Hence,

$$\cos\beta_1 = \frac{58.67}{68.79} = 0.853, \quad \text{and} \quad \beta_1 = 31.47°$$

Change in the velocity of whirl is:

$$\Delta C_w = C_{w1} + C_{w2} = 98.67 + 58.67 = 157.34 \text{ m/s}$$

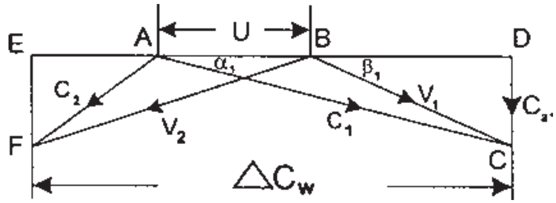

Figure 6.30 Velocity triangles for Example 6.13.

Horsepower developed is:

$$P = \dot{m}U\Delta C_w = \frac{(2) \times (157.34) \times (40)}{(0.746) \times (1000)} = 16.87 \text{ hp}$$

Illustrative Example 6.13: The inlet and outlet angles of blades of a reaction turbine are 25 and 18°, respectively. The pressure and temperature of steam at the inlet to the turbine are 5 bar and 250°C. If the steam flow rate is 10 kg/s and the rotor diameter is 0.72 m, find the blade height and power developed. The velocity of steam at the exit from the fixed blades is 90 m/s.

Solution:

Figure 6.30 shows the velocity triangles.

$$\alpha_1 = \beta_2 = 18°, \text{ and } \alpha_2 = \beta_1 = 25°$$

$$C_1 = 90 \text{ m/s}$$

From the velocity triangle,

$$C_{w1} = C_1 \cos(\alpha_1) = 90 \cos 18° = 85.6 \text{ m/s}$$

$$C_{a1} = \text{CD} = C_1 \sin \alpha_1 = 90 \sin 18° = 27.8 \text{ m/s}$$

From triangle BDC

$$\text{BD} = \frac{C_{a1}}{\sin(\beta_1)} = \frac{27.8}{\sin(25°)} = \frac{27.8}{0.423} = 65.72 \text{ m/s}$$

Hence blade velocity is given by:

$$U = C_{w1} - \text{BD} = 85.6 - 65.62 = 19.98 \text{ m/s.}$$

Applying the cosine rule,

$$V_1^2 = C_1^2 + U^2 - 2C_1U\cos\alpha_1$$

$$= 90^2 + 19.98^2 - (2) \times (90) \times (19.98) \cos 18°$$

$$V_1 = 71.27 \text{ m/s}$$

From triangle AEF,

$$C_{w2} = C_2 \cos(\alpha_2) = 71.27 \cos 25^\circ = 64.59\,\text{m/s}$$

Change in the velocity of whirl:

$$\Delta C_w = C_{w1} + C_{w2} = 85.6 + 64.59 = 150.19\,\text{m/s}$$

Power developed by the rotor:

$$P = \dot{m}U\Delta C_w = \frac{(10) \times (19.98) \times (150.19)}{1000} = 30\,\text{kW}$$

From superheated steam tables at 5 bar, 250°C, the specific volume of steam is:

$$v = 0.4744\,\text{m}^3/\text{kg}$$

Blade height is given by the volume of flow equation:

$$v = \pi D h C_a$$

where C_a is the velocity of flow and h is the blade height.Therefore,

$$0.4744 = \pi \times (0.72) \times (h) \times (27.8), \quad \text{and}$$

$$h = 0.0075\,\text{m} \ \text{ or } \ 0.75\,\text{cm}$$

Design Example 6.14: From the following data, for 50% reaction steam turbine, determine the blade height:

RPM:	440
Power developed:	5.5 MW
Steam mass flow rate:	6.8 kg/kW – h
Stage absolute pressure:	0.90 bar
Steam dryness fraction:	0.95
Exit angles of the blades:	70°

(angle measured from the axial flow direction).

The outlet relative velocity of steam is 1.2 times the mean blade speed. The ratio of the rotor hub diameter to blade height is 14.5.

Solution:

Figure 6.31 shows the velocity triangles.

From the velocity diagram,

$$V_2 = 1.2U$$

$$\begin{aligned} C_{a2} &= V_2 \cos(\beta_2) \\ &= 1.2U \cos 70° \\ &= 0.41U \text{ m/s} \end{aligned}$$

At mean diameter,

$$U = \frac{\pi DN}{60} = \frac{2\pi N(D_h + h)}{(60) \times (2)}$$

where D_h is the rotor diameter at the hub and h is the blade height. Substituting the value of U in the above equation,

$$C_{a2} = \frac{(0.41) \times (2\pi) \times (440)(14.5h + h)}{(2) \times (60)} = 146.45\,h \text{ m/s}$$

Annular area of flow is given by:

$$A = \pi h(D_h + h) = \pi h(14.5h + h)$$

or

$$A = 15.5\pi h^2$$

Specific volume of saturated steam at 0.90 bar, $v_g = 1.869 \text{ m}^3/\text{kg}$.

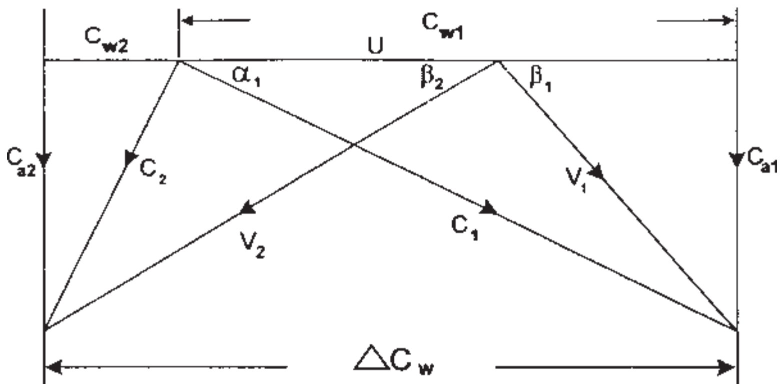

Figure 6.31 Velocity triangles for Example 6.14.

Then the specific volume of steam = (1.869) × (0.95) = 1.776 m^3/kg. The mass flow rate is given by:

$$\dot{m} = \frac{(5.5) \times (10^3) \times (6.8)}{3600} = 10.39\ \text{kg/s}$$

But,

$$\dot{m} = \frac{C_{a2}A}{v} = \frac{C_{a2}15.5\pi h^2}{v}$$

Therefore:

$$10.39 = \frac{(146.45) \times (h) \times (15.5) \times (\pi h^2)}{1.776}$$

or:

$$h^3 = 0.00259, \ \text{and}\ h = 0.137\ \text{m}$$

Design Example 6.15: From the following data for a two-row velocity compounded impulse turbine, determine the power developed and the diagram efficiency:

Blade speed:	115 m/s
Velocity of steam exiting the nozzle:	590 m/s
Nozzle efflux angle:	18°
Outlet angle from first moving blades:	37°
Blade velocity coefficient (all blades):	0.9

Solution:

Figure 6.32 shows the velocity triangles.

Graphical solution:

$U = 115$ m/s

$C_1 = 590$ m/s

$\alpha_1 = 18°$

$\beta_2 = 20°$

The velocity diagrams are drawn to scale, as shown in Fig. 6.33, and the relative velocity:

$V_1 = 482$ m/s using the velocity coefficient

$V_2 = (0.9) \times (482) = 434$ m/s

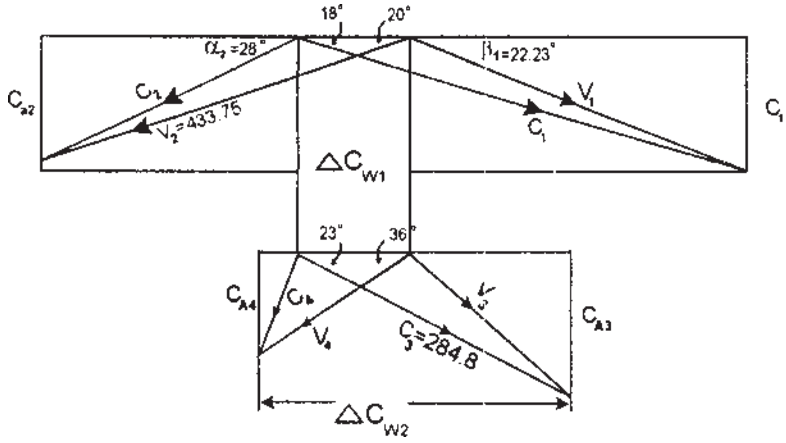

Figure 6.32 Velocity triangle for Example 6.15.

The absolute velocity at the inlet to the second row of moving blades, C_3, is equal to the velocity of steam leaving the fixed row of blades.

$$\text{i.e.,}: C_3 = kC_2 = (0.9) \times (316.4) = 284.8$$

Driving force $= \dot{m}\ \Delta C_w$

For the first row of moving blades, $\dot{m}\Delta C_{w1} = (1) \times (854) = 854\,\text{N}$.

For the second row of moving blades, $\dot{m}\Delta C_{w2} = (1) \times (281.46)$ $\text{N} = 281.46\,\text{N}$

where ΔC_{w1} and ΔC_{w2} are scaled from the velocity diagram.

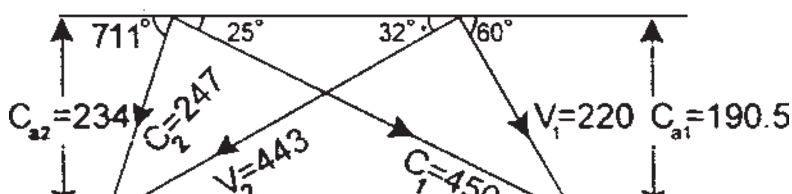

Figure 6.33 Velocity diagram for Example 6.16.

Total driving force $= 854 + 281.46 = 1135.46\,\mathrm{N}$ per kg/s

$$\text{Power} = \text{driving force} \times \text{blade velocity}$$
$$= \frac{(1135.46) \times (115)}{1000} = 130.58\,\mathrm{kW} \text{ per kg/s}$$

Energy supplied to the wheel

$$= \frac{mC_1^2}{2} = \frac{(1) \times (590^2)}{(2) \times (10^3)} = 174.05\mathrm{kW} \text{ per kg/s}$$

Therefore, the diagram efficiency is:

$$\eta_\mathrm{d} = \frac{(130.58) \times (10^3) \times (2)}{590^2} = 0.7502, \text{ or } 75.02\%$$

Maximum diagram efficiency:

$$= \cos^2 \alpha_1 = \cos^2 8° = 0.9045, \text{ or } 90.45\%$$

Axial thrust on the first row of moving blades (per kg/s):

$$= \dot{m}(C_{\mathrm{a}1} - C_{\mathrm{a}2}) = (1) \times (182.32 - 148.4) = 33.9\,\mathrm{N}$$

Axial thrust on the second row of moving blades (per kg/s):

$$= \dot{m}(C_{\mathrm{a}3} - C_{\mathrm{a}4}) = (1) \times (111.3 - 97.57) = 13.73\,\mathrm{N}$$

Total axial thrust:

$$= 33.9 + 13.73 = 47.63\,\mathrm{N} \text{ per kg/s}$$

Design Example 6.16: In a reaction stage of a steam turbine, the blade angles for the stators and rotors of each stage are: $\alpha_1 = 25°$, $\beta_1 = 60°$, $\alpha_2 = 71.1°$, $\beta_2 = 32°$. If the blade velocity is 300 m/s, and the steam flow rate is 5 kg/s, find the power developed, degree of reaction, and the axial thrust.

Solution:

Figure 6.34 shows the velocity triangles.

The velocity triangles can easily be constructed as the blade velocity and blade angles are given.From velocity triangles, work output per kg is given by:

$$W_\mathrm{t} = U(C_{\mathrm{w}1} + C_{\mathrm{w}2})$$
$$= (300) \times (450 \cos 25° + 247 \cos 71.1°)$$
$$= 14,6,354 \text{ J}$$

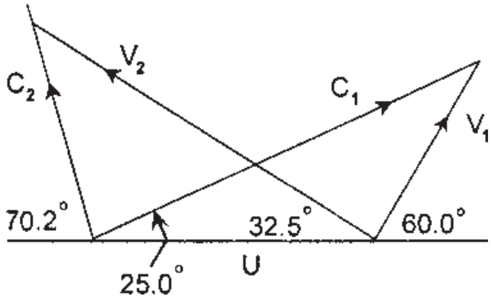

Figure 6.34 Velocity diagram for Example 6.17.

Power output:

$$\dot{m}W_t = \frac{(5) \times (1,46,354)}{1000} = 732 \text{ kW}$$

Degree of reaction is given by:

$$\Lambda = \frac{V_2^2 - V_1^2}{2 \times W_t} = \frac{443^2 - 220^2}{(2) \times (14,6,354)} = 0.5051, \quad \text{or} \quad 50.51\%$$

Axial thrust:

$$F = \dot{m}(C_{a1} - C_{a2}) = (5) \times (190.5 - 234) = -217.5 \text{ N}$$

The thrust is negative because its direction is the opposite to the fluid flow.

Design Example 6.17: Steam enters the first row of a series of stages at a static pressure of 10 bars and a static temperature of 300°C. The blade angles for the rotor and stator of each stage are: $\alpha_1 = 25°$, $\beta_1 = 60°$, $\alpha_2 = 70.2°$, $\beta_2 = 32°$. If the blade speed is 250 m/s, and the rotor efficiency is 0.94, find the degree of reaction and power developed for a 5.2 kg/s of steam flow. Also find the static pressures at the rotor inlet and exit if the stator efficiency is 0.93 and the carry-over efficiency is 0.89.

Solution:

Using the given data, the velocity triangles for the inlet and outlet are shown in Fig. 6.34. By measurement, $C_2 = 225$ m/s, $V_2 = 375$ m/s, $C_1 = 400$ m/s, $V_1 = 200$ m/s.

Work done per unit mass flow:

$$W_t = (250) \times (400 \cos 25^\circ + 225 \cos 70.2^\circ) = 1,09,685 \text{ J/kg}$$

Degree of reaction [Eq. (6.25)]

$$\Lambda = \frac{V_2^2 - V_1^2}{2 \times W_t} = \frac{375^2 - 200^2}{(2) \times (1,09,685)} = 0.4587, \quad \text{or} \quad 45.87\%$$

Power output:

$$P = \dot{m}W = \frac{(5.2) \times (1,09,685)}{1000} = 570.37 \text{ kW}$$

Isentropic static enthalpy drop in the stator:

$$\Delta h_s' = \frac{\left(C_1^2 - C_2^2\right)}{\eta_s} = \frac{\left(400^2 - (0.89) \times (225^2)\right)}{0.93}$$

$$= 1,23,595 \text{ J/kg}, \quad \text{or} \quad 123.6 \text{ kJ/kg}$$

Isentropic static enthalpy drops in the rotor:

$$\Delta h_r' = \frac{W}{\eta_r \eta_s} = \frac{1,09,685}{(0.94) \times (0.93)}$$

$$= 1,25,469 \text{ J/kg}, \quad \text{or} \quad 125.47 \text{ kJ/kg}$$

Since the state of the steam at the stage entry is given as 10 bar, 300°C, Enthalpy at nozzle exit:

$$h_1 - \left[\Delta h'\right]_{\text{stator}} = 3051.05 - 123.6 = 2927.5 \text{kJ/kg}$$

Enthalpy at rotor exit:

$$h_1 - \left[\Delta h'\right]_{\text{rotor}} = 3051.05 - 125.47 = 2925.58 \text{kJ/kg}$$

The rotor inlet and outlet conditions can be found by using the Mollier Chart.

Rotor inlet conditions: $P_1 = 7$ bar, $T_1 = 235$°C
Rotor outlet conditions: $P_2 = 5$ bar, $T_2 = 220$°C

PROBLEMS

6.1 Dry saturated steam is expanded in a steam nozzle from 1 MPa to 0.01 MPa. Calculate dryness fraction of steam at the exit and the heat drop.

(0.79, 686 kJ/kg)

6.2 Steam initially dry and at 1.5 MPa is expanded adiabatically in a nozzle to 7.5 KPa. Find the dryness fraction and velocity of steam at the exit. If the exit diameter of the nozzles is 12.5 mm, find the mass of steam discharged per hour.

(0.756, 1251.26 m/s, 0.376 kg/h)

6.3 Dry saturated steam expands isentropically in a nozzle from 2.5 MPa to 0.30 MPa. Find the dryness fraction and velocity of steam at the exit from the nozzle. Neglect the initial velocity of the steam.

(0.862, 867.68 m/s)

6.4 The nozzles receive steam at 1.75 MPa, 300°C, and exit pressure of steam is 1.05 MPa. If there are 16 nozzles, find the cross-sectional area of the exit of each nozzle for a total discharge to be 280 kg/min. Assume nozzle efficiency of 90%. If the steam has velocity of 120 m/s at the entry to the nozzles, by how much would the discharge be increased?

($1.36\,\text{cm}^2$, 33.42%)

6.5 The steam jet velocity of a turbine is 615 m/s and nozzle angle is 22°, The blade velocity coefficient = 0.70 and the blade is rotating at 3000 rpm. Assume mean blade radius = 600 mm and the axial velocity at the outlet = 160 m/s. Determine the work output per unit mass flow of steam and diagram efficiency.

(93.43 kW, 49.4%)

6.6 Steam is supplied from the nozzle with velocity 400 m/s at an angle of 20° with the direction of motion of moving blades. If the speed of the blade is 200 m/s and there is no thrust on the blades, determine the inlet and outlet blade angles, and the power developed by the turbine. Assume velocity coefficient = 0.86, and mass flow rate of steam is 14 kg/s.

(37° 50′, 45°, 31′, 1234.8 kW)

6.7 Steam expands isentropically in the reaction turbine from 4 MPa, 400°C to 0.225 MPa. The turbine efficiency is 0.84 and the nozzle angles and blade angles are 20 and 36° respectively. Assume constant axial velocity throughout the stage and the blade speed is 160 m/s. How many stages are there in the turbine?

(8 stages)

6.8 Consider one stage of an impulse turbine consisting of a converging nozzle and one ring of moving blades. The nozzles are inclined at 20° to the blades, whose tip angles are both 33°. If the velocity of the steam at the exit from the nozzle is 650 m/s, find the blade speed so that steam passes through without shock and find the diagram efficiency, neglecting losses.

(273 m/s, 88.2%)

6.9 One stage of an impulse turbine consists of a converging nozzle and one ring of moving blades. The nozzle angles are 22° and the blade angles are 35°. The velocity of steam at the exit from the nozzle is 650 m/s. If the relative velocity of steam to the blades is reduced by 14% in passing through the blade ring, find the diagram efficiency and the end thrust on the shaft when the blade ring develops 1650 kW.

(79.2%, 449 N)

6.10 The following refer to a stage of a Parson's reaction turbine:

Mean diameter of the blade ring:	92 cm
Blade speed:	3000 rpm
Inlet absolute velocity of steam:	310 m/s
Blade outlet angle:	20°
Steam flow rate:	6.9 kg/s

Determine the following: (1) blade inlet angle, (2) tangential force, and (3) power developed.

(38°, 2.66 kW, 384.7 kW)

NOTATION

C	absolute velocity, velocity of steam at nozzle exit
D	diameter
h	enthalpy, blade height
h_0	stagnation enthalpy, static enthalpy at the inlet to the fixed blades
h_1	enthalpy at the entry to the moving blades
h_2	enthalpy at the exit from the moving blades
h_{00}	stagnation enthalpy at the entry to the fixed blades
h_{01}	stagnation enthalpy at the entry to the fixed blades
h_{02}	stagnation enthalpy at the exit from the moving blade
k	blade velocity coefficient
N	rotational speed
R. F.	reheat factor
U	blade speed
V	relative velocity
α	angle with absolute velocity
β	angle with relative velocity
ΔC_w	change in the velocity of whirl
Δh	actual enthalpy drop
$\Delta h'$	isentropic enthalpy drop

η_d	diffuser efficiency
η_n	nozzle efficiency
η_s	stage efficiency
η_t	turbine efficiency
η_{ts}	total - to - static efficiency
η_{tt}	total - to - total efficiency
Λ	degree of reaction

SUFFIXES

0	inlet to fixed blades
1	inlet to moving blades
2	outlet from the moving blades
a	axial, ambient
r	radial
w	whirl