FOAM

See **WHITECAPS AND FOAM**

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FORAMINIFERA

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FORWARD PROBLEM IN NUMERICAL MODELS

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Introduction

The Forward Problem in numerical modeling describes a class of problems for which computers are used to advance in time discretized forms of a continuous set of equations of motion subject to initial and boundary conditions. Numerical models have emerged as powerful and versatile tools for addressing both fundamental and applied problems in the ocean sciences. This advance has been partly driven by increases in computing speed and memory over the past 30 years. However, advances in fundamental theories of ocean dynamics and a better description of the ocean currents as a result of a diverse and growing observational database have also helped to make numerical models both more realistic and more useful aids for understanding the basic physics of the ocean. Although many of the issues discussed in this article are relevant to interdisciplinary applications of forward models, the focus here is on using ocean models to understand the physics of the large-scale oceanic circulation.

One of the advantages of forward numerical models is that they provide dynamically consistent solutions over a wide range of parameter space and thus offer great flexibility for a variety of oceanographic problems. Forward models can be configured in quite realistic domains and subjected to complex, realistic forcing fields to produce simulations of the present-day ocean, past climates, or specific time periods or regions. However, the model physics and domain configuration can also be greatly simplified in order to address fundamental issues via process-oriented studies. The choice of model physics, numerical discretization, initial and boundary conditions, and analysis approach is strongly dependent on the application in mind.

More complete descriptions of the mean and time-dependent characteristics of the ocean circulation can provide valuable reference points for ocean models, targets towards which the scientist can aim. Discrepancies between models and data serve to identify deficiencies in the model solutions, which in turn lead to improved physics or numerics and, hopefully, a more faithful representation of the observations. The scientist can use the dynamical model to study the sensitivity of the ocean circulation to variations in model configuration, e.g., surface forcing, dissipation, or topography or to variations in model physics. Understanding how the solution depends on the fundamental parameters of the system leads to an increased understanding of the dynamics of the ocean circulation in general.

Analytic solutions to the equations of motion also provide valuable reference points for numerical models. Forward models can generally be applied to a wider range of problems than are typically accessible by purely analytic methods. However, the numerical models do not solve exactly the continuous equations of motion. They provide approximate solutions on a finite numerical grid and are generally subject to some form of truncation error, dissipation, and smoothing (discussed further below). It is often useful to configure the model such that direct comparisons with analytic solutions are possible in order to quantify the influences of the numerical method and to verify that, at least in the parameter space for which the analytic solution is valid, the model produces the correct solution. This starting point provides a useful reference for extending the model calculations into parameter space for which analytic solutions are not available. This typically involves increasing the nonlinearity of the system, introducing time dependence, and/or complexity of the domain configuration (topography, coastlines, stratification) and forcing.

This article provides an overview of the general issues relating to the use of forward numerical models for the study of the meso- to basin-scale general oceanic circulation. Although space does not permit a detailed discussion of each of the subject areas discussed below, it is intended that this introduction identify the major issues and concerns that need to be considered when using a numerical model to address a problem of interest. More detailed treatments of each of these topics can be found elsewhere in this text and in the Further reading list.

Equations of Motion

Forward models integrate discrete forms of a dynamically consistent set of equations of motion. The fundamental equations of motion for the oceanic circulation are the Navier–Stokes equations in a rotating coordinate system together with an equation of state that relates the density of the seawater to temperature, salinity, and pressure. However, it is not necessary to solve the full Navier–Stokes equations to study the large-scale, low frequency aspects of the oceanic circulation. Many general circulation models are based on the primitive equations, which form a subset of the Navier–Stokes equations by making the Boussinesq and hydrostatic approximations. The Boussinesq approximation neglects variations in the density everywhere in the momentum equations except where it is multiplied by the acceleration of gravity. This assumption is generally well satisfied in the ocean because changes in the density of sea water are much less than the density of sea water itself. The hydrostatic approximation neglects all vertical accelerations except that due to gravity. This assumption is valid as long as the horizontal scales of motion are much larger than the vertical scales of motion, and is well satisfied by the large-scale general circulation and mesoscale variability, but is violated in regions of active convection.

The horizontal momentum balance for the primitive equations is written in vector form as

$$\frac{d\vec{u}}{dt} + f\hat{k}\times\vec{u} = -\nabla P/\rho_0 + F_V \qquad [1]$$

where $\vec{u}$ is the horizontal velocity vector, $f = 2\Omega\sin\phi$ is the Coriolis term, ϕ is the latitude, P is pressure, ρ_0 is the mean density of sea water, and F_V represents horizontal and vertical subgrid-scale viscosity. The advection operator is defined as

$$\frac{d}{dt} = \frac{\partial}{\partial t} + \frac{u}{a\cos\phi}\frac{\partial}{\partial\lambda} + \frac{v}{a}\frac{\partial}{\partial\phi} + w\frac{\partial}{\partial z} \qquad [2]$$

where w is the vertical velocity, λ is longitude and a is the radius of the earth.

The vertical momentum equation is replaced by the hydrostatic approximation

$$\frac{\partial P}{\partial z} = -g\rho \qquad [3]$$

The fluid is assumed to be incompressible, so that the continuity equation reduces to

$$\nabla\cdot\vec{u} + \frac{\partial w}{\partial z} = 0 \qquad [4]$$

The density of sea water is a nonlinear function of temperature, salinity, and pressure. In most general form, the primitive equations integrate conservation equations for both temperature T and salinity S.

$$\frac{dT}{dt} = F_T \qquad [5]$$

$$\frac{dS}{dt} = F_S \qquad [6]$$

Subgridscale horizontal and vertical dissipative processes are represented as F_T and F_S. An equation of state is used to calculate density from T, S, and P. For process-oriented studies, it is often sufficient to solve for only one active tracer or, equivalently, the density,

$$\rho = \rho(T, S, P) \qquad [7]$$

The primary advantage of the primitive equations is that they are valid over essentially the entire range of scales appropriate for the large-scale, low-frequency oceanic motions. They are a good choice for climate models because they allow for spatially variable stratification, independent temperature and salinity influences on density and air–sea exchange, water mass transformations, large vertical velocities, and steep and tall topography. High resolution studies also benefit from a complete treatment of advection in regions of large Rossby number and large vertical velocities.

The main disadvantages of these equations are computational in nature. The equations admit high-frequency gravity waves, which are generally not of direct consequence for the large-scale physics of the ocean, yet can place computational constraints on the time step allowed to integrate the equations. Models based on the primitive equations must integrate four prognostic equations and one diagnostic equation, making them computationally more expensive than some lower order equations. Finally, for regional applications, the implementation of open boundary conditions is not well posed mathematically and, in practice, models are often found to be very sensitive to the details of how the boundary conditions on open boundaries (mainly on outflow and transitional points) are specified.

There are several additional subsets of dynamically consistent equations that may be derived from the primitive equations. The most common form used in forward numerical models are the quasigeostrophic equations, which are a leading order asymptotic approximation to the primitive equations for small Rossby number, $R = V/f_0L$, and small aspect ratio $\delta = D/L$, where V is a characteristic velocity scale, D and L are characteristic vertical and horizontal length scales, and f_0 is the Coriolis parameter at the central latitude. The Coriolis parameter is assumed to vary linearly with latitude y, $f = f_0 + \beta y$. The mean stratification is specified and uniform over the entire model domain. Interface displacements due to the fluid motion are assumed to be small compared to the mean layer thicknesses. The quasigeostrophic limit allows for the length scale of motion to be the same order as the oceanic mesoscale, typically 10–100 km.

Quasigeostrophic models have been most often used for process studies of the wind-driven general circulation and its low-frequency variability. Their formulation in terms of a single prognostic variable, the quasigeostrophic potential vorticity, is often an advantage from a conceptual point of view. The equations are adiabatic by design, so that spurious diapycnal mixing is not a problem. The quasigeostrophic equations are generally more efficient to integrate numerically because there is only one prognostic equation and the time step can generally be larger than for comparable resolution primitive equation models because high-frequency Kelvin and gravity waves are not supported.

There are numerous drawbacks to the quasigeostrophic equations that make them less practical for large-scale realistic modeling studies. Several of these drawbacks stem from the assumption that the mean stratification is uniform throughout the model domain. This prohibits isopycnal surfaces from outcropping or intersecting the bottom. The influences of bottom topography are represented by a vertical velocity consistent with no normal flow through the sloping bottom, however it is imposed at the mean bottom depth. It is also not possible to represent the subduction of water masses, the process by which water is advected from the near surface, where it is in turbulent contact with the atmosphere, to the stratified interior, where it is shielded from direct influence from the atmosphere. Various higher order terms, such as the advection of relative vorticity and density anomalies by the nongeostrophic velocity field, are not represented. In addition, the equations do not consider temperature and salinity independently, and the geostrophic approximation breaks down near the equator.

Discretization Issues

Forward models solve for the equations of motion on a discrete grid in space and time. There are numerous considerations that need to be taken into account in determining what form of discretization is most appropriate for the problem of interest. Most numerical methods used in ocean general circulation models are relatively simple, often relying on finite difference or spectral discretization schemes, although some models have employed finite element methods. The convergence properties of discretization techniques are well documented in numerical methods literature, and the reader is referred there for a detailed discussion. Perhaps of most critical importance to the ocean modeler is the choice of vertical discretization. As will be clear, the following issues are most relevant to the primitive equation models because they arise as a consequence of spatially variable stratification or bottom topography. There are three commonly used vertical coordinates: z-coordinate or level models; sigma-coordinate or generalized stretched coordinates; and isopycnal coordinates. The ramifications of the truncation errors associated with each of these approaches differ widely depending on the problem of interest, so it is worth some discussion on the relative advantages and disadvantages of each method.

Level models solve for the dynamic and thermodynamic variables at specified depths which are fixed in time and uniform throughout the model domain. A schematic diagram of a surface intensified density gradient over a sloping bottom in level coordinates is shown in **Figure 1**. The advantages of this approach include: its relative simplicity, an accurate treatment of the horizontal pressure gradient terms, high vertical resolution in regions of weak stratification, and well-resolved surface boundary layers. The major disadvantage of the level models is in their treatment of bottom topography. In the simplest, and most commonly used, form the bottom depth must reside at one of the level interfaces. Thus, in order to resolve small variations in bottom depth one must devote large amounts of vertical resolution throughout the model domain (even on land points). As is evident in **Figure 1(A)**, the bottom slopes on horizontal scales of several model grid points will be only coarsely represented. There are two main errors associated with an inaccurate treatment of the bottom slope. The first is a poor representation of the large-scale stretching and/or compression associated with flow across a sloping bottom, which can affect the propagation characteristics of large-scale planetary waves. The second problem arises when dense water flows down steep topography, such as is found near sills connecting marginal seas to the open ocean. Standard level models excessively mix the dense water with the ambient water as it is advected downslope, severely compromising the properties of the dense water. In the ocean interior, care must be taken to ensure that most of the mixing takes place along isopycnal surfaces.

Terrain-following or, in more general terms, stretched coordinate models solve the equations of motion on a vertical grid that is fixed in time but varies in space. A schematic diagram of a surface intensified density gradient over a sloping bottom in terrain-following coordinates is shown in **Figure 1(B)**. The advantages of this approach are: accurate representation of topographic slopes, all model grid points reside in the ocean, and high resolution of surface and bottom boundary layers. Notice the smooth representation of the bottom slope and increase in vertical resolution in shallow water. The main disadvantage of the stretched coordinate approach is in calculating the horizontal pressure gradient in regions of steep topography. The calculation of the pressure gradient is prone to errors because the hydrostatic component of the pressure between adjacent grid points that are at different depths must be subtracted before calculating the (typically much smaller) dynamically significant lateral variation in pressure. As with level models, the parameterization of lateral subgridscale mixing processes must be carefully formulated to avoid spurious mixing across isopycnal surfaces. There can

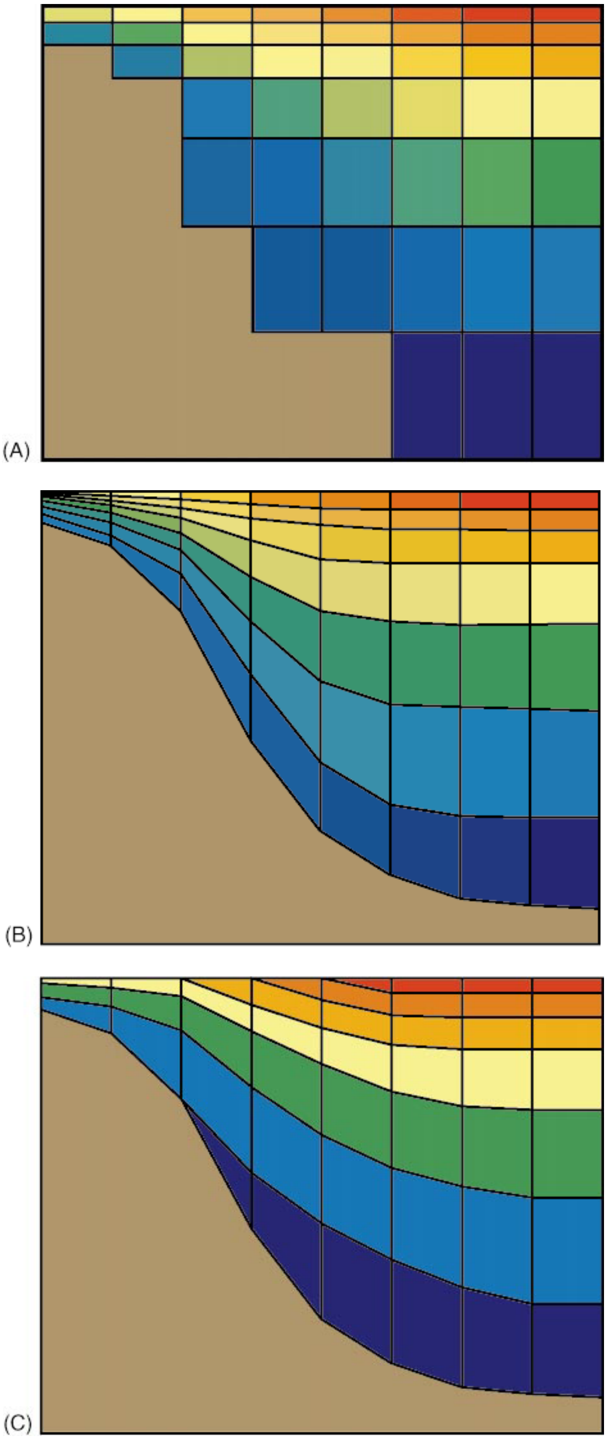

Figure 1 Schematic diagrams of a density front over a sloping bottom as represented in different model coordinate systems. Black lines denote boundaries of model grid cells, color indicates density (red light, blue dense). (A) Level coordinates; (B) terrain-following coordinates; (C) isopycnal coordinates.

also be some computational constraints that arise when stretched coordinate models are extended into very shallow water as the vertical grid spacing becomes very small.

Isopycnal coordinate models solve the equations of motion within specified density layers, which are allowed to move vertically during the course of integration. A schematic diagram of a surface intensified density gradient over a sloping bottom in isopycnal coordinates is shown in **Figure 1(C)**. Notice that the density is uniform along the model coordinate surfaces, and that these surfaces may intersect the surface and/or the bottom. This gives rise to a more discrete representation of the stratification than either level or stretched coordinate models. The advantages of this approach include: high vertical resolution in regions of large vertical density gradients; straightforward mixing along isopycnal surfaces; explicit control of diapycnal mixing; accurate treatment of lateral pressure gradients; smooth topography, and a natural framework for analysis. The primary disadvantages of the isopycnal discretization are: low resolution in weakly stratified regions (such as the mixed layer and high latitude seas); difficulty in the implementation of the nonlinear equation of state over regions of widely varying stratification such that static stability is uniformly assured; and the handling of the exchange of water of continuously varying density in the surface mixed layer with the discrete density layers in the stratified interior. Care must be taken so that the layer thicknesses can vanish in regions of isopycnal outcropping or intersection with topography. Although readily handled with higher order discretization methods, treatment of vanishing layers makes the models more computationally intensive and numerically complex.

Initial and Boundary Conditions

Forward numerical models integrate the equations of motion subject to initial and boundary conditions. Initial conditions for process-oriented studies are typically very idealized, such as a motionless, homogenous, stratified basin or statistically uniform, geostrophically balanced turbulence in a periodic basin. The initial conditions used for simulations in realistic basins can be more complex, depending on the application in mind. Long-term climate studies are often initialized with a motionless ocean of uniform temperature and salinity. Short-term basin-scale integrations are often initialized with a more realistic density field derived from climatological hydrographic databases. Although subject to much smoothing (particularly near narrow boundary currents and overflows), such renditions contain many realistic features of the large-scale general circulation. Some care should be taken so as not to introduce spurious water masses when averaging hydrographic data onto the model grid.

Forward models can also be used to produce short-term simulations or predictions of the oceanic state of a specific region and time. Although such calculations are technically feasible, they require large amounts of data to initialize the model state variables and force the models at the boundaries of the regional domain. The initialization of such models often makes use of direct *in situ* or remotely sensed observations together with statistical or dynamical extrapolation techniques to fill the regions void of data. The initial dynamic adjustment is greatly reduced if the velocity field is initialized to be in geostrophic balance with the density field. Higher order initializations can further reduce the high-frequency transients that are generated upon integration, although the effects of these transients are generally confined to the early integration period and are of most importance to short-term predictions.

Lateral boundary conditions are most straightforward at solid boundaries. Boundary conditions for the momentum equations are typically either no-slip (tangential velocity equals zero) or free-slip (no stress) and no normal flow. Tracer fluxes are generally no flux through the solid wall. Boundary conditions at the bottom are generally no normal flow for velocity and no flux for tracers. A stress proportional to $\vec{v}$ or $\vec{v}^2$ is often imposed as a vertical momentum flux at the bottom to parameterize the influences of unresolved bottom boundary layers.

Lateral boundary conditions are more problematic when the boundary of the model domain is part of the open ocean. The model prognostic variables need to be specified here but, in general, some of the information that determines the model variables on the open boundary will be controlled by information propagating from within the model domain and some will be controlled by information propagating from outside the domain. There is no general solution to this problem. Most regional models adopt a practical, rather than rigorous, approach through a combination of specifying the flow variables from 'observations' on inflow points and propagating information using simple advective equations on outflow points. Increasing the model dissipation near the open boundaries is sometimes required. The 'observations' in this case may be based on actual oceanic observations (in the case of regional simulations), climatology (basin-scale long-term integrations), or some idealized flow state (process studies). Basin-scale general circulation models often represent water mass exchanges that take place outside the model domain through a restoring term in the tracer equations that forces the model tracers towards the climatological tracer values near the boundaries. In their most simple application, however, these boundary conditions do not permit an advective tracer flux through the boundary ($\vec{v} = 0$) so that even though the tracer field may replicate the observed values, the tracer flux may be in error.

Primitive equation models explicitly integrate conservation equations for heat, salt, and momentum and thus require surface fluxes for these variables. It is commonly assumed that the flux of a property through the surface of the ocean is transported vertically away from the very thin air–sea interface through small-scale turbulent motions. Primitive equation models do not generally resolve such small-scale motions and rely on subgridscale parameterizations (see next section) to represent their effort on the large-scale flow. The vertical turbulent momentum and tracer fluxes are generally assumed to be downgradient and represented as a vertical diffusion coefficient times the mean vertical gradient of the quantity of interest. Thus, the surface flux of magnitude F for a model variable T is incorporated into the vertical diffusion term as

$$K_T \frac{\partial T}{\partial z} = F \qquad [8]$$

where K_T is a vertical diffusion coefficient.

Historically, climate models have represented the complex suite of heat flux components by simply restoring the model sea surface temperature towards a specified, spatially variable 'atmospheric' temperature with a given time scale (which itself may be a function of space and/or time). In its simplest form, the atmospheric temperature is taken to be the observed climatological sea surface temperature. This approach is simple and has the advantage that the ocean temperature will never stray too far away from the range specified by the boundary conditions. In the context of simulating the real ocean, however, the obvious drawback is that if the model ocean has the correct sea surface temperature, it also has zero heat flux, a condition that holds only over very limited regions of the ocean. Conversely, if the model ocean is being forced with the correct net surface heat flux it must have the wrong sea surface temperature. This approach also assumes that the atmosphere has an infinite heat capacity and thus can not respond thermodynamically to changes in the sea surface temperature.

A more realistic approach is to specify the surface heat flux to be the sum of the best estimate of the real net surface heat flux plus a restoring term that is proportional to the difference between the model sea surface temperature and the observed climatological sea surface temperature. An advantage of this approach is that the model is forced with surface heat fluxes consistent with the best observational estimates when the model SST agrees with climatology. This approach also does not allow the model SST to stray too far away from the specified climatology. The drawbacks include having to specify the surface heat flux (which is not well known) and enforcing at the surface the spatial scales inherent in the smoothed hydrographic climatology.

A dynamically more complex approach for climate is to include a simple active planetary boundary layer model to represent the atmosphere. In this case, the only specifications that are required are the incoming short-wave solar flux, which is reasonably well known, the temperature of the land surrounding the ocean basin, and the wind stress. The planetary boundary layer model calculates the atmospheric temperature and humidity and, with the use of bulk formulae, the net sensible, latent, and long-wave radiative heat fluxes. Although this approach is more complex than the simple restoring conditions, it has the advantages of thermodynamic consistency between the model physics and the surface heat fluxes, the spatial scales of the surface heat flux are determined by the model dynamics, and it allows the atmosphere to respond to changes in sea surface temperature.

The net fresh water flux at the surface is fundamentally different from and technically more problematic than the net heat flux. Primitive equation models integrate conservation equations for salinity (the number of grams of salt contained in 1 kg of sea water) yet the net salt flux through the sea surface is zero. The salinity is changed by exchanges of fresh water between the ocean and atmosphere. A major difference between the air–sea exchange of freshwater and the air–sea heat flux is that the freshwater flux is largely independent of salinity, the dynamically active tracer which it strongly influences. The change in salinity in the ocean that results from a net freshwater flux at the surface is typically represented as a virtual salt flux by changing the salinity of the ocean without a corresponding change in the volume of water in the ocean. Differences between the exact freshwater flux boundary condition and the virtual salt flux boundary condition are generally small. However, a serious challenge facing long-term climate integrations is to represent faithfully the surface boundary condition for salinity without allowing the model salinity to drift too far from the observed state. Most climate models parameterize the influence of the net freshwater flux at the surface by including at least a weak restoring of the sea surface salinity towards a specified spatially variable value.

Subgridscale Parameterizations

The large-scale general circulation in the ocean is influenced by processes that occur on spatial scales from less than a centimeter to thousands of kilometers. Temporal variations occur on time scales from minutes for turbulent mixing to millennia for climate variability. It is not possible now, nor in the forseeable future, to represent explicitly all of these scales in ocean models. It is also attractive, from a conceptual point of view, to represent the interplay between scales of motion through clear dynamically based theories. The important and formidable task facing ocean modelers is to parameterize effectively those processes that are not resolved by the space/time grid used in the model. The two main classes of motion that may be important to the large-scale circulation, and are often not explicitly represented in forward models, are small-scale turbulence and mesoscale eddy variability. Although a comprehensive review of the physics and parameterizations of these phenomena are beyond the scope of this article, a summary of the physical processes that need to be considered is useful.

Turbulent mixing of properties (temperature, salinity, momentum) across density surfaces takes place on spatial scales of centimeters and is driven by small-scale turbulence, shear and convective instabilities, and breaking internal waves. Although these processes take place on very small scales, they play a fundamental role in the global energy budget and are essential components of the basin- to global-scale thermohaline general circulation. Mixing across density surfaces is thought to be small over most of the ocean, however it can be intense in regions of strong boundary currents and dense water overflows, rough bottom topography, and near the ocean surface where direct atmospheric forcing is important. It will likely be a long time before these processes will be able to be resolved in large-scale general circulation models, so parameterizations of the turbulent mixing are necessary.

Intense diapycnal turbulent mixing is found in the boundary layers near the ocean surface and bottom. The planetary boundary layer near the ocean surface has received much attention from physical oceanographers because the ocean circulation is forced through this interface and because of its fundamental importance to air–sea exchange of heat, fresh water, and biogeochemically important tracers. Strong turbulent mixing is forced in the surface mixed layer as a result of shear and convective instabilities resulting from these surface fluxes. The bottom boundary layer is similar in some regards, but is primarily driven by the bottom stress with buoyancy fluxes generally negligible.

Parameterization approaches to the turbulent boundary layers generally fall into three categories. Bulk models assume that all properties are homogenized within the planetary boundary layer over a depth called the mixed layer depth. This depth is determined by a budget between energy input through the air–sea interface, turbulent dissipation, and entrainment of stratified fluid from below. These models are relatively simple and inexpensive to use, yet fail to resolve any vertical structure in the planetary boundary layer and assume that all properties mix in the same way to the same depth. Local closure models allow for vertical structure in the planetary boundary layer, yet assume that the strength of turbulent mixing is dependent only on the local properties of the fluid. A third class of planetary boundary layer models does allow for nonlocal influences on turbulent mixing.

Turbulent mixing in the ocean interior is generally much smaller than it is within the surface and bottom boundary layers. Nonetheless, water mass budgets in semi-enclosed abyssal basins indicate that substantial and important mixing must take place in the deep ocean interiors. Recent tracer release experiments and microstructure measurements suggest that elevated mixing may be found near and above regions of rough bottom topography, perhaps as a result of internal wave generation and subsequent breaking. Early subgridscale parameterizations of diapycnal mixing were very crude, and often dictated more by numerical stability constraints than by ocean physics. Downgradient diffusion is generally assumed, often with spatially uniform mixing coefficients for temperature and salinity. A few studies using a mixing coefficient that increases with decreasing stratification have been carried out, but there is still much more work to be done to understand fully the importance of diapycnal mixing distributions on the general oceanic circulation.

Moving to larger scales, the next category of motions that is likely to be important to the general circulation is the oceanic mesoscale. Mesoscale eddies are found throughout the world's oceans and are characterized by spatial scales of tens to hundreds of kilometers and time scales of tens to hundreds of days. This is much larger than the turbulent mixing scale yet still considerably smaller than the basin-scale. Variability on these space and time scales can result from many processes, such as wave radiation from distance sources, local instability of mean currents, vortex propagation from remote sources, and local atmospheric forcing. Although the oceanic mesoscale has been known to exist since the early 1960s, oceanographers still do not know in what proportion each of these generation mechanisms are important in determining the local mesoscale variability and, perhaps ever more daunting, their role in the general circulation. However, eddy-resolving modeling studies have shown that significant, large-scale mean circulations can be driven by mesoscale eddy motions so, in at least some regards, they are clearly important for the general circulation.

The diversity of generation mechanisms and complex turbulent dynamics of mesoscale eddies makes it very difficult to parameterize their influences on the large-scale circulation. Historical approaches have relied on downgradient diffusion of tracers with uniform mixing coefficients, often along the model coordinate surfaces. This approach is simple and produces smooth and stable solutions. Yet, in the case of coordinate surfaces that do not coincide with isopycnal surfaces, this approach can introduce excessive spurious diapycnal mixing, particularly in regions of strongly sloping isopycnal surfaces, such as near the western boundary current. More physically based subgridscale parameterizations of mesoscale eddies project the eddy-induced tracer fluxes along isopycnal surfaces. Note, however, that in the case where density is determined by only one tracer, there will be no effective tracer flux by the mesoscale eddies and some additional form of subgridscale mixing may be required for numerical stability.

Recent advances in the parameterization of mesoscale tracer transports have been based on the assumptions that (1) eddy tracer fluxes are primarily along isopycnal surfaces and (2) the strength of the eddy-induced tracer flux is proportional to the local gradient in isopycnal slope. This approach has several nice characteristics. First, it allows one to define a stream function for the eddy-induced velocity, thus ensuring adiabatic tracer advection and eliminating spurious diapycnal mass fluxes due to the parameterization. Second, the eddy fluxes work to relax sloping isopycnal surfaces and extract potential energy from the mean flow, crudely representing the effects of local baroclinic instability. Implementation of such parameterizations in global and basin-scale general circulation models has allowed for removal of horizontal diffusion and has resulted in much improved model simulations. Additional theories have been developed that relate the strength of the eddy-induced transport velocities to the local properties of the mean flow by making use of baroclinic instability theory. It should be pointed out that this approach makes the implicit assumption that the eddy flux divergence is proportional to the local mean flow so that it is not intended to parameterize the tracer transport by eddies that travel far from their source, either by self-propagation or advection by the mean flow.

Summary

Forward numerical models have emerged as a powerful tool in large-scale ocean circulation studies. They provide dynamically consistent solutions while allowing for much flexibility in the choice of model physics, domain configuration, and external forcing. When used together with observations, laboratory experiments, and/or theories of the ocean circulation, forward models can help extend our understanding of ocean physics into dynamically rich and complex regimes. However, the scientist must always be aware that forward models provide only approximate solutions to the equations of motion and are subject to various forms of smoothing and dissipation. Perhaps the most critical area for future development in forward models is in improving our ability to parameterize unresolved turbulent processes on scales from centimeters to the oceanic mesoscale.

See also

Elemental Distribution: Overview. Heat and Momentum Fluxes at the Sea Surface. General Circulation Models. Mesoscale Eddies. Rossby Waves. Satellite Remote Sensing of Sea Surface Temperatures.

Further Reading

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