VORTICAL MODES

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Introduction

In the late 1970s, moored measurements in the ocean found that gradient quantities such as shear and strain exhibited frequency behavior inconsistent with linear internal gravity waves. While this behavior could arise from Doppler shifting or other nonlinearities within the internal wave spectrum, it was also realized that geostrophic, or nonlinear potential vorticity-carrying, motions might be contributing to fine-scale variance. In their simplest form, these could take the form of thin layers of varying stratification with large horizontal extent and little shear associated with them, so-called 'passive fine-structure'. These perturbations would be subinertial in a water-following frame, and spread tracers much more efficiently along isopycnals (density surfaces) through stirring and shear dispersion than internal waves. The term 'vortical mode' was originally coined to refer to the zero-frequency eigenmode of the linear stratified f-plane equations associated with potential vorticity-carrying perturbations, that is, geostrophy, regardless of scale. However, the vortical mode has come to denote both linear and nonlinear subinertial (intrinsic frequencies $\omega \ll f$) ocean fine-structure with vertical wavelengths $\lambda_z < 100\,\text{m}$ which cannot be described as internal gravity waves. The term vortical mode will be used in this sense here.

In the sections below, potential vorticity is defined, its role on basin scales and mesoscales briefly described, then evidence for potential vorticity-carrying fine-structure in the ocean interior is discussed. Such evidence is indirect and inferential. Fine-scale vortical modes are expected to arise from (i) the potential enstrophy cascade of geostrophic turbulence, (ii) mixing in turbulent patches, (iii) bottom friction and eddy-shedding of flow past topography, and (iv) double diffusion. Interpretation of fine-scale observations is challenging because of the presence of fine-scale internal waves and nonlinear advection by large-scale internal waves. As a result, the spatial and spectral distributions of vortical mode shear and strain variance are still unknown.

Potential Vorticity

In a rotating buoyancy-stratified fluid, potential vorticity is:

$$\Pi \equiv q = (2\Omega + \nabla \times \mathbf{v}) \cdot \nabla h(\rho, p) \qquad [1]$$

where $2\Omega = (0,\ f\cot(\text{latitude}),\ f)$ is the planetary vorticity vector associated with Earth's rotation Ω, $f = 2|\Omega|\sin(\text{latitude})$ the Coriolis frequency, $\nabla \times \mathbf{v}$ the relative vorticity, and $h(\rho, p)$ any well-behaved function of density and pressure. It is convenient to use the buoyancy $b = -g\rho'/\rho_0$ for $h(\rho, p)$ where $\rho' = \rho(x, y, z) - \rho_0$. The vertical gradient of buoyancy $\partial b/\partial z = N^2$ is the stratification, or buoyancy frequency squared. The potential vorticity can be thought of as the dot product of the absolute vorticity vector and the stratification vector, that is, a multiplication of the stratification vector ∇b with that component of the absolute vorticity vector $2\Omega + \nabla \times \mathbf{v}$ parallel to it (**Figure 1**).

As shown by Hans Ertel in 1942, potential vorticity is conserved following a fluid parcel in the absence of irreversible processes.[1] That is, potential vorticity is invariant without forcing by wind stresses, radiation, and evaporation/precipitation at the sea surface, molecular dissipation by microscale turbulence and double diffusion in the ocean interior, or stresses and geothermal heating at the bottom. Potential vorticity perturbations are stable if $2\Omega \cdot \Pi$ is everywhere of the same sign. Relative to a background potential vorticity in a rotating stratified fluid $\Pi = f\bar{N}^2$, unstable conditions can arise from (i) fine-scale stratification perturbations $\sim O(\bar{N}^2)$ (equivalent of strain $\partial\xi/\partial z \sim O(1)$), (ii) relative vorticity $\zeta \sim O(f)$ (equivalent to vorticity Rossby numbers $R_\zeta = \zeta/f \sim O(1)$), or (iii) vertical shears

[1]In a rotating buoyancy-stratified fluid, the potential vorticity of a water parcel is conserved in the absence of dissipative processes. The potential vorticity is

$$\Pi \equiv q = (2\Omega + \nabla \times \mathbf{v}) \cdot \Delta h(\rho, p)$$

where $2\Omega = (0, f\cot(\text{latitude}), f)$ is the planetary vorticity vector associated with Earth's rotation Ω, $f = 2|\Omega|\sin(\text{latitude})$ the Coriolis frequency, $\nabla \times v$ the relative vorticity, and $h(\rho, p)$ any well-behaved function of density and pressure. The potential vorticity can be thought of as the dot product of the absolute vorticity vector and the stratification vector, that is, a multiplication of the stratification vector ∇b with that component of the absolute vorticity vector $2\Omega + \nabla \times \mathbf{v}$ parallel to it. As a conserved quantity, it is a useful dynamical tracer.

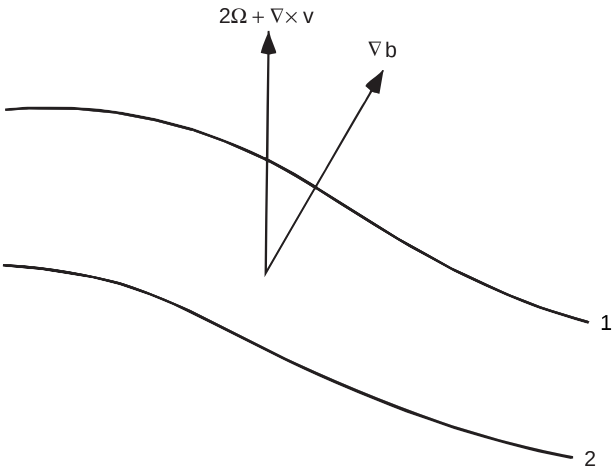

Figure 1 Schematic of potential vorticity, which is the buoyancy gradient vector ∇b times that component of the absolute vorticity $2\Omega + \nabla \times v$ vector parallel with it. When isopycnals are flat, the absolute vorticity is just the Coriolis frequency f plus the vertical relative vorticity. When isopycnals are sloped as shown, horizontal vorticities (vertical shears) also become important.

$|\partial \mathbf{v}/\partial z| \sim O(N)$ (equivalent to gradient Richardson numbers $Ri = N^2/(\partial \mathbf{v}/\partial z)^2 \sim O(1)$).

In **Figure 2**, the relationship between the ratio of available potential to horizontal kinetic energy PE/KE and scaled aspect ratio $(NH/fL)^2 = (Nk_H/fk_z)^2$ is shown for geostrophic (thick dashed) and nonzero vorticity Rossby numbers (thin dashed and dotted), as well as linear internal gravity waves (thick dotted). At low aspect ratios (large horizontal compared to vertical scales), potential exceeds kinetic energy. At high aspect ratios, kinetic energy dominates. At vorticity Rossby numbers of -1, there is little potential energy because the Coriolis and centripetal acceleration terms balance. For vorticities exceeding f of either sign ($|R_\varsigma| \gg 1$, there is excess potential energy. These relations are independent of scale.

Basin Scales

Basin scale potential vorticity structure does not fit into our definition of the vortical mode. However, the dynamics of potential vorticity anomalies should be independent of scale and much of our intuition comes from studies on these larger scales. Moreover, it is not yet known whether a spectral gap exists separating large- and small-scale potential vorticity perturbations.

On basin scales O(1000 km), baroclinic potential vorticity anomalies are linear and quasigeostrophic, associated with the large-scale gyres. Potential energy greatly exceeds kinetic energy for these very low aspect ratio motions (**Figure 2**). Potential

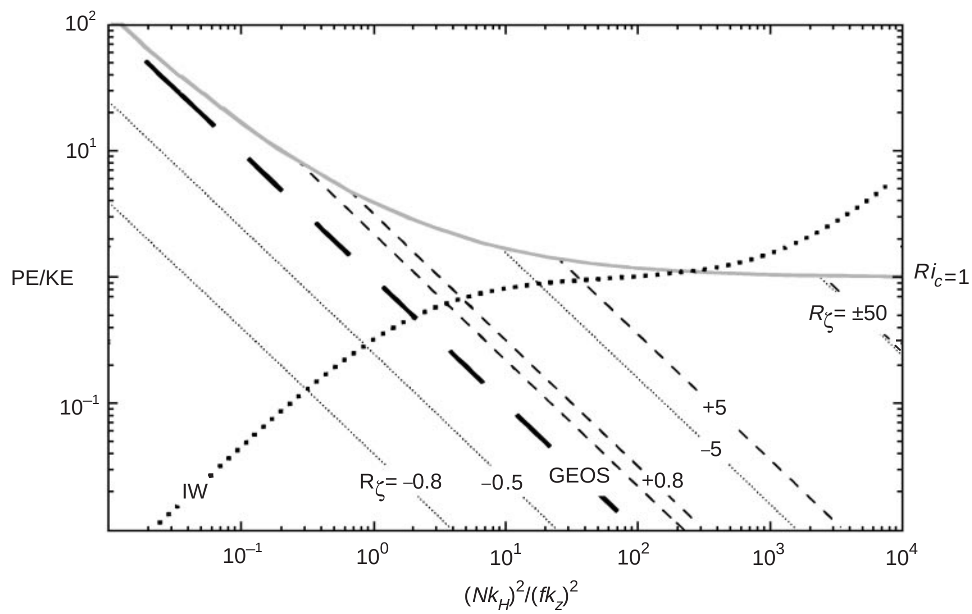

Figure 2 Dynamic diagram describing the dependence of ratio of available potential to horizontal kinetic energy $R_E = \mathrm{PE/KE}$ (energy Burger number) on scaled aspect ratio $R_L = (NH/fL)^2$ (length scale Burger number), where $H \sim k_z^{-1}$ is the vertical scale and $L \sim k_H^{-1}$ the horizontal scale. The thick dashed diagonal corresponds to geostrophy (linear vortical modes). $R_\zeta = \zeta/f$ is the vorticity Rossby number. The thin diagonals correspond to nonzero negative (dotted) and positive (dashed) vorticity Rossby number vortical modes. Nonzero Rossby number vortical modes in the domain above the $Ri_c = 1$ curve have vertical shears exceeding the buoyancy frequency N. The thick dotted curve is the relation for linear internal gravity waves for $N/f = 40$. (Reproduced with permission from Kunze and Sanford, 1993.)

vorticity can be simplified to $f(y)N^2(x, y, z)$. This 'stretching vorticity' is a powerful dynamical tracer, facilitating diagnosis of the gyre-scale circulation. Ventilated wind-driven waters can be tracked from their winter outcrop. Under the assumption that, once a water parcel enters the pycnocline, its behavior can be explained by inviscid quasigeostrophic dynamics on a β-plane, potential vorticity conservation determines the stratification along particle paths, a powerful constraint in ideal thermocline theory. Unventilated waters in shadow zones (backwaters isolated from direct atmospheric forcing) become homogenized over time. Also on these scales, long planetary Rossby waves have potential vorticity as their restoring forces.

Mesoscale

Similarly, mesoscale O(10–100 km) potential vorticity-carrying structures do not fit into our definition of the vortical mode, but are better understood. They have lower aspect ratios than basin scale anomalies. These include western boundary currents like the Gulf Stream and Kuroshio, rings, eddies, fronts, short Rossby waves, Meddies, and other submesoscale thermocline vortices. Vertical relative vorticity is often important on these scales, $\Pi \simeq (f + \nabla \times \mathbf{v})N^2$. Baroclinic and barotropic instability are means of transferring potential vorticity toward smaller scales as part of the potential enstrophy cascade of 2-D geostrophic turbulence which tends to coalesce potential vorticity into coherent vortices resembling Meddies. This 2-D upscale energy cascade will be arrested by planetary Rossby wave radiation if amplitudes are too weak to overcome the β-effect ($\beta = \partial f/\partial y$). Rossby wave radiation is unlikely to be important for vortical mode because group velocities are very small for small vertical scales.

Fine-scale

On the fine-scale O(100–1000 m), the vortical mode has been invoked on vertical scales of 1–10 m to account for (i) fine-structure contamination of internal waves in mooring measurements and (ii) scale-dependent isopycnal diffusivities from tracer release experiments that are too large to be explained by internal wave shear dispersion.

Sampling designed to minimize instrument motion contamination of internal wave measurements in the Internal Wave Experiment [e.g. (IWEX)] reveals that Eulerian frequency spectra of fine-scale fluctuations such as shear and strain are not consistent with linear internal gravity waves. Because of strong advective nonlinearity on these scales, particularly from internal wave heaving, subinertial geostrophic fine-structure would be Doppler shifted into the internal wave frequency band, $f < \omega < N$. However, Doppler shifting will also smear fine-scale internal waves across all frequencies, so it is unclear whether 'fine-structure' contamination is not just due to fine-scale internal gravity waves of different intrinsic frequencies becoming confused. Efforts to reduce Doppler shifting by examining time-series on isopycnal surfaces or with a water-following float have found signals much more compatible with linear internal wave dynamics. Lagrangian time-series have not yet been of sufficient duration to characterize subinertial variances.

Isopycnal diffusivities increase with scale so that a fine-scale patch diffuses more and more rapidly as it spreads with time. On 0.1–1.0 km scales, $0.07 \pm 0.04\,\mathrm{m^2\,s^{-1}}$ diffusivities were inferred from a North Atlantic Tracer Release Experiment (NATRE). These may be explicable from internal wave shear dispersion in which vertical turbulent diffusion is spread horizontally by vertically varying horizontal displacements. However, 1–30 km diffusivities of 1–3 $\mathrm{m^2\ s^{-1}}$ cannot be accounted for by either shear dispersion due to internal waves or persistent large-scale (100 m) vertical shears. The vortical mode has been invoked to explain the O(10 km) diffusivities. Arguing that excess fine-scale strain is associated with the vortical mode, and assuming that dominant aspect ratios for the internal wave and vortical mode fields are the same and independent of vertical wavenumber, yields quantitatively plausible horizontal diffusivities.

Generation Mechanisms

Potential vorticity can only be modified by irreversible processes, and even then remains conserved within a volume containing all the dissipation. Moreover, it cannot flux across isopycnals. This puts severe restrictions on sources for the vortical mode. Away from atmospheric forcing, potential vorticity can only be altered through molecular dissipation. Fine-scale vortical modes are expected to arise from:

1. The potential enstrophy cascade of geostrophic turbulence, including baroclinic instability, although atmospherically forced mesoscale property anomalies appear to be smoothed out in only a few months.
2. Mixing and dissipation in micro-scale turbulence patches.

3. Bottom friction and eddy-shedding of flow past topography.
4. Double-diffusive layering and interleaving.

Whether any of these mechanisms is sufficient to maintain a widespread or universal vortical mode field is unknown. The third and fourth mechanisms in particular are expected to be highly localized.

Observational Challenge

The fine-scale poses considerable observational challenges because of the presence of energetic fine-scale internal waves, and nonlinear advection by large-scale internal waves. Measurements in the wake of a seamount found potential vorticity structure on vertical wavelengths of 50–200 m and horizontal scales of O(1 km), which was attributed to shedding of bottom boundary layers or flow separation (**Figure 3**). Strong coherent vortices with horizontal scales O(1 km) have been found in a number of ocean pycnoclines, generated by either bottom-hugging flows encountering abrupt changes in topography or deep convection. However, whether a universal vortical mode spectrum exists throughout the ocean, analogous to the canonical internal wave spectrum, has not been established since it has yet to be isolated from the omnipresent internal wave field.

Four methods have been attempted to identify potential vorticity-carrying fine-structure.

1. Intrinsic frequency should be subinertial ($\omega \ll f$) for the vortical mode and superinertial for gravity waves away from boundaries where Kelvin and other bottom-trapped topographic waves, while not vortical modes, can have subinertial frequencies. For large-scale flows that experience little Doppler shifting $(\mathbf{v} \cdot \nabla)\psi$, where ψ represents (u, v, w, b), this is unambiguous from fixed Eulerian measurements. However, fine-structure with vertical wavelengths 1–10 m is strongly advected both vertically and horizontally by larger-scale internal wave flows, so that Eulerian frequency measurements such as moorings are no longer unambiguous. Lagrangian time-series are

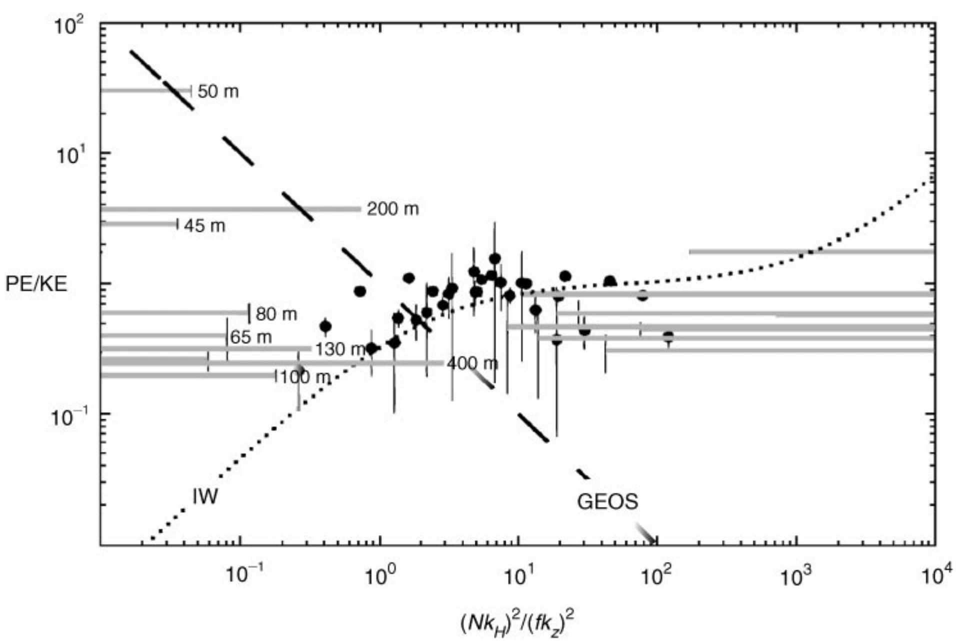

Figure 3 Energy ratios versus scaled aspect ratios (as in **Figure 2**) in the wake of a seamount. Gray bars emanating from the left axis correspond to horizontal wavelengths exceeding 8.5 km (survey averages) with vertical wavelengths marked. These intersect the geostrophic curve (thick dashed diagonal) for vertical wavelengths $\lambda_z = 50$ and 200 m, lie near the internal wave curve (thick dotted curve) for $\lambda_z = 100$, 130 and 400 m, and between the curves (corresponding to kinetic energy being dominated by near-inertial waves and potential energy by geostrophic motions) otherwise. Black dots (●) correspond to scales resolved by the survey. At lower aspect ratios, these mostly cluster near the internal wave curve. At higher aspect ratios, they fall slightly below the internal wave curve, suggesting excess horizontal kinetic energy. Gray bars emanating from the right axis denote horizontal wavelengths 0.3 km (incoherent scales) at various vertical wavelengths. Two of these intersect the internal wave curve. The remainder, with scaled aspect ratios of O(10), lie below it, again suggesting excess horizontal kinetic energy, possibly from the vortical mode. Very little energy is associated with higher aspect ratio estimates, so these may be aliased. (Reproduced with permission from Kunze and Sanford, 1993.)

necessary to identify the intrinsic frequency. Water-following measurements of shear and strain have yet to be made for sufficient duration to reliably identify the vortical mode.

2. Potential vorticity anomalies should be associated with vortical modes but not internal gravity waves (except possibly advection of background gradients – which should be small given the short timescales of internal waves). From the definition of potential vorticity in eqn [1], this requires resolving fine-scale gradients on both the vertical and horizontal, which is difficult in itself. Moreover, since gradient quantities such as relative vorticity and buoyancy gradients ∇b have blue horizontal wavenumber spectra, i.e. more variance at smaller than larger scales, sampling must be designed to filter out variance at scales smaller than those of interest.
3. The ratio of relative vorticity to horizontal divergence. For linear (geostrophic) vortical mode, vorticity greatly exceeds horizontal divergence (which vanishes in the steady geostrophic limit). For internal waves, the horizontal divergence is greater or equal to the relative vorticity. This approach has the same problems of spatial resolution as the potential vorticity method.
4. Ratio of horizontal kinetic to available potential energy (shear/strain ratio) HKE/APE as a function of dynamic length scale ratio $(fL/NH)^2$. These differ for linear internal waves and geostrophic flow (**Figure 2**). This approach also suffers potential contamination by aliasing, in this case, by larger scales.

Conclusions

Observational evidence for vortical mode finestructure in the ocean is sparse, largely indirect, and inferential. As a result, the spatial and spectral distributions of vortical mode variances are unknown. Given their potentially important role in submesoscale isopycnal stirring, the oceanic vortical mode warrants further study.

See also

Acoustics, Deep Ocean. Acoustics, Shallow Water. Dispersion and Diffusion in the Deep Ocean. Dispersion in Shallow Seas. Double-diffusive Convection. Flows in Straits and Channels. General Circulation Models. Internal Tidal Mixing. Internal Tides. Internal Waves. Meddies and Sub-surface Eddies. Overflows and Cascades. Patch Dynamics. Rossby Waves. Three-dimensional (3D) Turbulence. Tracer Release Experiments. Tracers and Large Scale Models. Upper Ocean Mixing Processes.

Further Reading

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