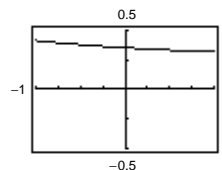


Review Exercises for Chapter 1

2. Precalculus. $L = \sqrt{(9-1)^2 + (3-1)^2} \approx 8.25$

4. x	-0.1	-0.01	-0.001	0.001	0.01	0.1
$f(x)$	0.358	0.354	0.354	0.354	0.353	0.349

$$\lim_{x \rightarrow 0} f(x) \approx 0.2$$



6. $g(x) = \frac{3x}{x-2}$

(a) $\lim_{x \rightarrow 2} g(x)$ does not exist.

(b) $\lim_{x \rightarrow 0} g(x) = 0$

8. $\lim_{x \rightarrow 9} \sqrt{x} = \sqrt{9} = 3$.

Let $\epsilon > 0$ be given. We need

$$|\sqrt{x} - 3| < \epsilon \implies |\sqrt{x} + 3| |\sqrt{x} - 3| < \epsilon |\sqrt{x} + 3|$$

$$|x - 9| < \epsilon |\sqrt{x} + 3|$$

Assuming $4 < x < 16$, you can choose $\delta = 5\epsilon$.Hence, for $0 < |x - 9| < \delta = 5\epsilon$, you have

$$|x - 9| < 5\epsilon < |\sqrt{x} + 3| \epsilon$$

$$|\sqrt{x} - 3| < \epsilon$$

$$|f(x) - L| < \epsilon$$

10. $\lim_{x \rightarrow 5} 9 = 9$. Let $\epsilon > 0$ be given. δ can be any positive number. Hence, for $0 < |x - 5| < \delta$, you have

$$|9 - 9| < \epsilon$$

$$|f(x) - L| < \epsilon$$

12. $\lim_{y \rightarrow 4} 3|y - 1| = 3|4 - 1| = 9$

14. $\lim_{t \rightarrow 3} \frac{t^2 - 9}{t - 3} = \lim_{t \rightarrow 3} (t + 3) = 6$

$$16. \lim_{x \rightarrow 0} \frac{\sqrt{4+x} - 2}{x} = \lim_{x \rightarrow 0} \frac{\sqrt{4+x} - 2}{x} \cdot \frac{\sqrt{4+x} + 2}{\sqrt{4+x} + 2}$$

$$= \lim_{x \rightarrow 0} \frac{1}{\sqrt{4+x} + 2} = \frac{1}{4}$$

$$18. \lim_{s \rightarrow 0} \frac{(1/\sqrt{1+s}) - 1}{s} = \lim_{s \rightarrow 0} \left[\frac{(1/\sqrt{1+s}) - 1}{s} \cdot \frac{(1/\sqrt{1+s}) + 1}{(1/\sqrt{1+s}) + 1} \right]$$

$$= \lim_{s \rightarrow 0} \frac{[1/(1+s)] - 1}{s[(1/\sqrt{1+s}) + 1]} = \lim_{s \rightarrow 0} \frac{-1}{(1+s)[(1/\sqrt{1+s}) + 1]} = -\frac{1}{2}$$

20. $\lim_{x \rightarrow -2} \frac{x^2 - 4}{x^3 + 8} = \lim_{x \rightarrow -2} \frac{(x+2)(x-2)}{(x+2)(x^2 - 2x + 4)}$

$$= \lim_{x \rightarrow -2} \frac{x-2}{x^2 - 2x + 4}$$

$$= -\frac{4}{12} = -\frac{1}{3}$$

22. $\lim_{x \rightarrow (\pi/4)} \frac{4x}{\tan x} = \frac{4(\pi/4)}{1} = \pi$

$$\begin{aligned}
 24. \lim_{\Delta x \rightarrow 0} \frac{\cos(\pi + \Delta x) + 1}{\Delta x} &= \lim_{\Delta x \rightarrow 0} \frac{\cos \pi \cos \Delta x - \sin \pi \sin \Delta x + 1}{\Delta x} \\
 &= \lim_{\Delta x \rightarrow 0} \left[-\frac{(\cos \Delta x - 1)}{\Delta x} \right] - \lim_{\Delta x \rightarrow 0} \left[\sin \pi \frac{\sin \Delta x}{\Delta x} \right] \\
 &= -0 - (0)(1) = 0
 \end{aligned}$$

$$26. \lim_{x \rightarrow c} [f(x) + 2g(x)] = -\frac{3}{4} + 2\left(\frac{2}{3}\right) = \frac{7}{12}$$

$$28. f(x) = \frac{1 - \sqrt[3]{x}}{x - 1}$$

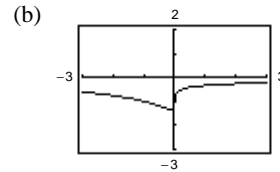
(a)

x	1.1	1.01	1.001	1.0001
$f(x)$	-0.3228	-0.3322	-0.3332	-0.3333

$$\lim_{x \rightarrow 1^+} \frac{1 - \sqrt[3]{x}}{x - 1} \approx -0.333 \quad (\text{Actual limit is } -\frac{1}{3}.)$$

(c)

$$\begin{aligned}
 \lim_{x \rightarrow 1^+} \frac{1 - \sqrt[3]{x}}{x - 1} &= \lim_{x \rightarrow 1^+} \frac{1 - \sqrt[3]{x}}{x - 1} \cdot \frac{1 + \sqrt[3]{x} + (\sqrt[3]{x})^2}{1 + \sqrt[3]{x} + (\sqrt[3]{x})^2} \\
 &= \lim_{x \rightarrow 1^+} \frac{1 - x}{(x - 1)[1 + \sqrt[3]{x} + (\sqrt[3]{x})^2]} \\
 &= \lim_{x \rightarrow 1^+} \frac{-1}{1 + \sqrt[3]{x} + (\sqrt[3]{x})^2} \\
 &= -\frac{1}{3}
 \end{aligned}$$



$$30. s(t) = 0 \Rightarrow -4.9t^2 + 200 = 0 \Rightarrow t^2 \approx 40.816 \Rightarrow t \approx 6.39 \text{ sec}$$

When $t = 6.39$, the velocity is approximately

$$\begin{aligned}
 \lim_{t \rightarrow a} \frac{s(a) - s(t)}{a - t} &= \lim_{t \rightarrow a} -4.9(a + t) \\
 &= \lim_{t \rightarrow 6.39} -4.9(6.39 + 6.39) = -62.6 \text{ m/sec.}
 \end{aligned}$$

$$32. \lim_{x \rightarrow 4} \llbracket x - 1 \rrbracket \text{ does not exist. The graph jumps from 2 to 3 at } x = 4.$$

$$34. \lim_{x \rightarrow 1^+} g(x) = 1 + 1 = 2.$$

$$36. \lim_{s \rightarrow -2} f(s) = 2$$

$$38. f(x) = \begin{cases} \frac{3x^2 - x - 2}{x - 1}, & x \neq 1 \\ 0, & x = 1 \end{cases}$$

$$\begin{aligned}
 \lim_{x \rightarrow 1} f(x) &= \lim_{x \rightarrow 1} \frac{3x^2 - x - 2}{x - 1} \\
 &= \lim_{x \rightarrow 1} (3x + 2) = 5 \neq 0
 \end{aligned}$$

Removable discontinuity at $x = 1$
Continuous on $(-\infty, 1) \cup (1, \infty)$

$$40. f(x) = \begin{cases} 5 - x, & x \leq 2 \\ 2x - 3, & x > 2 \end{cases}$$

$$\lim_{x \rightarrow 2^-} (5 - x) = 3$$

$$\lim_{x \rightarrow 2^+} (2x - 3) = 1$$

Nonremovable discontinuity at $x = 2$

Continuous on $(-\infty, 2) \cup (2, \infty)$

$$44. f(x) = \frac{x + 1}{2x + 2}$$

$$\lim_{x \rightarrow -1} \frac{x + 1}{2(x + 1)} = \frac{1}{2}$$

Removable discontinuity at $x = -1$

Continuous on $(-\infty, -1) \cup (-1, \infty)$

$$48. \lim_{x \rightarrow 1^+} (x + 1) = 2$$

$$\lim_{x \rightarrow 3^-} (x + 1) = 4$$

Find b and c so that $\lim_{x \rightarrow 1^-} (x^2 + bx + c) = 2$ and $\lim_{x \rightarrow 3^+} (x^2 + bx + c) = 4$.

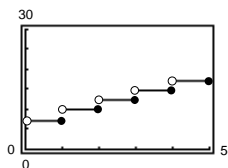
Consequently we get $1 + b + c = 2$ and $9 + 3b + c = 4$.

Solving simultaneously, $b = -3$ and $c = 4$.

$$50. C = 9.80 + 2.50[-\lceil -x \rceil - 1], \quad x > 0$$

$$= 9.80 - 2.50[\lceil -x \rceil + 1]$$

C has a nonremovable discontinuity at each integer.



$$54. h(x) = \frac{4x}{4 - x^2}$$

Vertical asymptotes at $x = 2$ and $x = -2$

$$58. \lim_{x \rightarrow (1/2)^+} \frac{x}{2x - 1} = \infty$$

$$62. \lim_{x \rightarrow -1^+} \frac{x^2 - 2x + 1}{x + 1} = \infty$$

$$66. \lim_{x \rightarrow 0^+} \frac{\sec x}{x} = \infty$$

$$42. f(x) = \sqrt{\frac{x+1}{x}} = \sqrt{1 + \frac{1}{x}}$$

$$\lim_{x \rightarrow 0^+} \sqrt{1 + \frac{1}{x}} = \infty$$

Domain: $(-\infty, -1], (0, \infty)$

Nonremovable discontinuity at $x = 0$

Continuous on $(-\infty, -1] \cup (0, \infty)$

$$46. f(x) = \tan 2x$$

Nonremovable discontinuities when

$$x = \frac{(2n + 1)\pi}{4}$$

Continuous on

$$\left(\frac{(2n - 1)\pi}{4}, \frac{(2n + 1)\pi}{4} \right)$$

for all integers n .

$$52. f(x) = \sqrt{(x - 1)x}$$

(a) Domain: $(-\infty, 0] \cup [1, \infty)$

$$(b) \lim_{x \rightarrow 0^-} f(x) = 0$$

$$(c) \lim_{x \rightarrow 1^+} f(x) = 0$$

$$56. f(x) = \csc \pi x$$

Vertical asymptote at every integer k

$$60. \lim_{x \rightarrow -1^-} \frac{x + 1}{x^4 - 1} = \lim_{x \rightarrow -1^-} \frac{1}{(x^2 + 1)(x - 1)} = -\frac{1}{4}$$

$$64. \lim_{x \rightarrow 2^-} \frac{1}{\sqrt[3]{x^2 - 4}} = -\infty$$

$$68. \lim_{x \rightarrow 0^-} \frac{\cos^2 x}{x} = -\infty$$

70. $f(x) = \frac{\tan 2x}{x}$

(a)

x	-0.1	-0.01	-0.001	0.001	0.01	0.1
$f(x)$	2.0271	2.0003	2.0000	2.0000	2.0003	2.0271

$$\lim_{x \rightarrow 0} \frac{\tan 2x}{x} = 2$$

(b) Yes, define

$$f(x) = \begin{cases} \frac{\tan 2x}{x}, & x \neq 0 \\ 2, & x = 0 \end{cases}$$

Now $f(x)$ is continuous at $x = 0$.

Problem Solving for Chapter 1

2. (a) Area $\triangle PAO = \frac{1}{2}bh = \frac{1}{2}(1)(x) = \frac{x}{2}$

Area $\triangle PBO = \frac{1}{2}bh = \frac{1}{2}(1)(y) = \frac{y}{2} = \frac{x^2}{2}$

(b) $a(x) = \frac{\text{Area } \triangle PBO}{\text{Area } \triangle PAO} = \frac{x^2/2}{x/2} = x$

x	4	2	1	0.1	0.01
Area $\triangle PAO$	2	1	1/2	1/20	1/200
Area $\triangle PBO$	8	2	1/2	1/200	1/20,000
$a(x)$	4	2	1	1/10	1/100

(c) $\lim_{x \rightarrow 0^+} a(x) = \lim_{x \rightarrow 0^+} x = 0$

6.
$$\frac{\sqrt{a+bx} - \sqrt{3}}{x} = \frac{\sqrt{a+bx} - \sqrt{3}}{x} \cdot \frac{\sqrt{a+bx} + \sqrt{3}}{\sqrt{a+bx} + \sqrt{3}}$$

$$= \frac{(a+bx) - 3}{x(\sqrt{a+bx} + \sqrt{3})}$$

Letting $a = 3$ simplifies the numerator.

Thus,

$$\lim_{x \rightarrow 0} \frac{\sqrt{3+bx} - \sqrt{3}}{x} = \lim_{x \rightarrow 0} \frac{bx}{x(\sqrt{3+bx} + \sqrt{3})}$$

$$= \lim_{x \rightarrow 0} \frac{b}{\sqrt{3+bx} + \sqrt{3}}$$

Setting $\frac{b}{\sqrt{3} + \sqrt{3}} = \sqrt{3}$, you obtain $b = 6$.

Thus, $a = 3$ and $b = 6$.

4. (a) Slope = $\frac{4 - 0}{3 - 0} = \frac{4}{3}$

(b) Slope = $-\frac{3}{4}$ Tangent line: $y - 4 = -\frac{3}{4}(x - 3)$

$$y = -\frac{3}{4}x + \frac{25}{4}$$

(c) Let $Q = (x, y) = (x, \sqrt{25 - x^2})$

$$m_x = \frac{\sqrt{25 - x^2} - 4}{x - 3}$$

(d) $\lim_{x \rightarrow 3} m_x = \lim_{x \rightarrow 3} \frac{\sqrt{25 - x^2} - 4}{x - 3} \cdot \frac{\sqrt{25 - x^2} + 4}{\sqrt{25 - x^2} + 4}$

$$= \lim_{x \rightarrow 3} \frac{25 - x^2 - 16}{(x - 3)(\sqrt{25 - x^2} + 4)}$$

$$= \lim_{x \rightarrow 3} \frac{(3 - x)(3 + x)}{(x - 3)(\sqrt{25 - x^2} + 4)}$$

$$= \lim_{x \rightarrow 3} \frac{-(3 + x)}{\sqrt{25 - x^2} + 4} = \frac{-6}{4 + 4} = -\frac{3}{4}$$

This is the slope of the tangent line at P .

8. $\lim_{x \rightarrow 0^-} f(x) = \lim_{x \rightarrow 0^-} (a^2 - 2) = a^2 - 2$

$$\lim_{x \rightarrow 0^+} f(x) = \lim_{x \rightarrow 0^+} \frac{ax}{\tan x} = a \left(\text{because } \lim_{x \rightarrow 0} \frac{\tan x}{x} = 1 \right)$$

Thus,

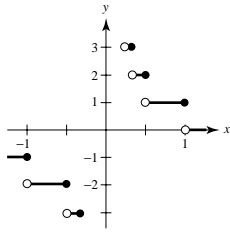
$$a^2 - 2 = a$$

$$a^2 - a - 2 = 0$$

$$(a - 2)(a + 1) = 0$$

$$a = -1, 2$$

10.



$$\begin{aligned} \text{(a) } f\left(\frac{1}{4}\right) &= \llbracket 4 \rrbracket = 4 \\ f(3) &= \left\lfloor \frac{1}{3} \right\rfloor = 0 \\ f(1) &= \llbracket 1 \rrbracket = 1 \end{aligned}$$

$$\text{(b) } \lim_{x \rightarrow 1^-} f(x) = 1$$

$$\begin{aligned} \lim_{x \rightarrow 1^+} f(x) &= 0 \\ \lim_{x \rightarrow 0^-} f(x) &= -\infty \\ \lim_{x \rightarrow 0^+} f(x) &= \infty \end{aligned}$$

(c) f is continuous for all real numbers except $x = 0, \pm 1, \pm \frac{1}{2}, \pm \frac{1}{3}, \dots$

$$12. \text{ (a) } v^2 = \frac{192,000}{r} + v_0^2 - 48$$

$$\frac{192,000}{r} = v^2 - v_0^2 + 48$$

$$r = \frac{192,000}{v - v_0^2 + 48}$$

$$\lim_{v \rightarrow 0} r = \frac{192,000}{48 - v_0^2}$$

$$\text{Let } v_0 = \sqrt{48} = 4\sqrt{3} \text{ feet/sec.}$$

$$\text{(b) } v^2 = \frac{1920}{r} + v_0^2 - 2.17$$

$$\frac{1920}{r} = v^2 - v_0^2 + 2.17$$

$$r = \frac{1920}{v^2 - v_0^2 + 2.17}$$

$$\lim_{v \rightarrow 0} r = \frac{1920}{2.17 - v_0^2}$$

$$\text{Let } v_0 = \sqrt{2.17} \text{ mi/sec } (\approx 1.47 \text{ mi/sec}).$$

$$\text{(c) } r = \frac{10,600}{v^2 - v_0^2 + 6.99}$$

$$\lim_{v \rightarrow 0} r = \frac{10,600}{6.99 - v_0^2}$$

$$\text{Let } v_0 = \sqrt{6.99} \approx 2.64 \text{ mi/sec.}$$

Since this is smaller than the escape velocity for earth, the mass is less.

14. Let $a \neq 0$ and let $\epsilon > 0$ be given. There exists $\delta_1 > 0$ such that if $0 < |x - 0| < \delta$, then $|f(x) - L| < \epsilon$.

Let $\delta = \delta_1/|a|$. Then for $0 < |x - 0| < \delta = \delta_1/|a|$, you have

$$|x| < \frac{\delta_1}{|a|}$$

$$|ax| < \delta_1$$

$$|f(ax) - L| < \epsilon.$$

As a counterexample, let $f(x) = \begin{cases} 1 & x \neq 0 \\ 2 & x = 0 \end{cases}$.

Then $\lim_{x \rightarrow 0} f(x) = 1 = L$,

but $\lim_{x \rightarrow 0} f(ax) = \lim_{x \rightarrow 0} f(0) = 2$.