

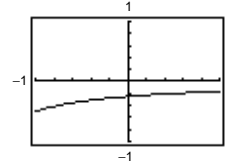
Review Exercises for Chapter 1

1. Calculus required. Using a graphing utility, you can estimate the length to be 8.3.
Or, the length is slightly longer than the distance between the two points, 8.25.

3.

| | | | | | | |
|--------|-------|-------|--------|---------|--------|-------|
| x | -0.1 | -0.01 | -0.001 | 0.001 | 0.01 | 0.1 |
| $f(x)$ | -0.26 | -0.25 | -0.250 | -0.2499 | -0.249 | -0.24 |

$$\lim_{x \rightarrow 0} f(x) \approx -0.25$$



5. $h(x) = \frac{x^2 - 2x}{x}$

(a) $\lim_{x \rightarrow 0} h(x) = -2$

(b) $\lim_{x \rightarrow -1} h(x) = -3$

7. $\lim_{x \rightarrow 1} (3 - x) = 3 - 1 = 2$

Let $\epsilon > 0$ be given. Choose $\delta = \epsilon$. Then for

$$0 < |x - 1| < \delta = \epsilon, \text{ you have}$$

$$|x - 1| < \epsilon$$

$$|1 - x| < \epsilon$$

$$|(3 - x) - 2| < \epsilon$$

$$|f(x) - L| < \epsilon$$

9. $\lim_{x \rightarrow 2} (x^2 - 3) = 1$

Let $\epsilon > 0$ be given. We need $|x^2 - 3 - 1| < \epsilon \Rightarrow |x^2 - 4| = |(x - 2)(x + 2)| < \epsilon \Rightarrow |x - 2| < \frac{1}{|x + 2|}\epsilon$.

Assuming, $1 < x < 3$, you can choose $\delta = \epsilon/5$. Hence, for $0 < |x - 2| < \delta = \epsilon/5$ you have

$$|x - 2| < \frac{\epsilon}{5} < \frac{1}{|x + 2|}\epsilon$$

$$|x - 2||x + 2| < \epsilon$$

$$|x^2 - 4| < \epsilon$$

$$|(x^2 - 3) - 1| < \epsilon$$

$$|f(x) - L| < \epsilon$$

11. $\lim_{t \rightarrow 4} \sqrt{t + 2} = \sqrt{4 + 2} = \sqrt{6} \approx 2.45$

13. $\lim_{t \rightarrow -2} \frac{t + 2}{t^2 - 4} = \lim_{t \rightarrow -2} \frac{1}{t - 2} = -\frac{1}{4}$

15. $\lim_{x \rightarrow 4} \frac{\sqrt{x} - 2}{x - 4} = \lim_{x \rightarrow 4} \frac{\sqrt{x} - 2}{(\sqrt{x} - 2)(\sqrt{x} + 2)}$
 $= \lim_{x \rightarrow 4} \frac{1}{\sqrt{x} + 2} = \frac{1}{\sqrt{4} + 2} = \frac{1}{4}$

17. $\lim_{x \rightarrow 0} \frac{[1/(x + 1)] - 1}{x} = \lim_{x \rightarrow 0} \frac{1 - (x + 1)}{x(x + 1)}$
 $= \lim_{x \rightarrow 0} \frac{-1}{x + 1} = -1$

19. $\lim_{x \rightarrow -5} \frac{x^3 + 125}{x + 5} = \lim_{x \rightarrow -5} \frac{(x + 5)(x^2 - 5x + 25)}{x + 5}$
 $= \lim_{x \rightarrow -5} (x^2 - 5x + 25)$
 $= 75$

21. $\lim_{x \rightarrow 0} \frac{1 - \cos x}{\sin x} = \lim_{x \rightarrow 0} \left(\frac{x}{\sin x} \right) \left(\frac{1 - \cos x}{x} \right) = (1)(0) = 0$

$$\begin{aligned}
 23. \lim_{\Delta x \rightarrow 0} \frac{\sin[(\pi/6) + \Delta x] - (1/2)}{\Delta x} &= \lim_{\Delta x \rightarrow 0} \frac{\sin(\pi/6) \cos \Delta x + \cos(\pi/6) \sin \Delta x - (1/2)}{\Delta x} \\
 &= \lim_{\Delta x \rightarrow 0} \frac{1}{2} \cdot \frac{(\cos \Delta x - 1)}{\Delta x} + \lim_{\Delta x \rightarrow 0} \frac{\sqrt{3}}{2} \cdot \frac{\sin \Delta x}{\Delta x} \\
 &= 0 + \frac{\sqrt{3}}{2}(1) = \frac{\sqrt{3}}{2}
 \end{aligned}$$

$$25. \lim_{x \rightarrow c} [f(x) \cdot g(x)] = \left(-\frac{3}{4}\right)\left(\frac{2}{3}\right) = -\frac{1}{2}$$

$$27. f(x) = \frac{\sqrt{2x+1} - \sqrt{3}}{x-1}$$

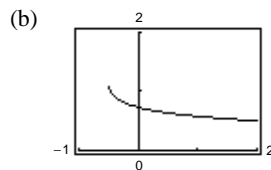
(a)

| | | | | |
|--------|--------|--------|--------|--------|
| x | 1.1 | 1.01 | 1.001 | 1.0001 |
| $f(x)$ | 0.5680 | 0.5764 | 0.5773 | 0.5773 |

$$\lim_{x \rightarrow 1^+} \frac{\sqrt{2x+1} - \sqrt{3}}{x-1} \approx 0.577 \quad (\text{Actual limit is } \sqrt{3}/3)$$

(c)

$$\begin{aligned}
 \lim_{x \rightarrow 1^+} \frac{\sqrt{2x+1} - \sqrt{3}}{x-1} &= \lim_{x \rightarrow 1^+} \frac{\sqrt{2x+1} - \sqrt{3}}{x-1} \cdot \frac{\sqrt{2x+1} + \sqrt{3}}{\sqrt{2x+1} + \sqrt{3}} \\
 &= \lim_{x \rightarrow 1^+} \frac{(2x+1) - 3}{(x-1)(\sqrt{2x+1} + \sqrt{3})} \\
 &= \lim_{x \rightarrow 1^+} \frac{2}{\sqrt{2x+1} + \sqrt{3}} \\
 &= \frac{2}{2\sqrt{3}} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}
 \end{aligned}$$



$$\begin{aligned}
 29. \lim_{t \rightarrow a} \frac{s(a) - s(t)}{a - t} &= \lim_{t \rightarrow 4} \frac{(-4.9(4)^2 + 200) - (-4.9t^2 + 200)}{4 - t} \\
 &= \lim_{t \rightarrow 4} \frac{4.9(t-4)(t+4)}{4-t} \\
 &= \lim_{t \rightarrow 4} -4.9(t+4) = -39.2 \text{ m/sec}
 \end{aligned}$$

$$31. \lim_{x \rightarrow 3^-} \frac{|x-3|}{x-3} = \lim_{x \rightarrow 3^-} \frac{-(x-3)}{x-3} = -1$$

$$33. \lim_{x \rightarrow 2} f(x) = 0$$

$$35. \lim_{t \rightarrow 1} h(t) \text{ does not exist because } \lim_{t \rightarrow 1^-} h(t) = 1 + 1 = 2 \text{ and } \lim_{t \rightarrow 1^+} h(t) = \frac{1}{2}(1 + 1) = 1.$$

$$37. f(x) = \llbracket x + 3 \rrbracket$$

$$\lim_{x \rightarrow k^+} \llbracket x + 3 \rrbracket = k + 3 \text{ where } k \text{ is an integer.}$$

$$\lim_{x \rightarrow k^-} \llbracket x + 3 \rrbracket = k + 2 \text{ where } k \text{ is an integer.}$$

Nonremovable discontinuity at each integer k

Continuous on $(k, k + 1)$ for all integers k

$$39. f(x) = \frac{3x^2 - x - 2}{x-1} = \frac{(3x+2)(x-1)}{x-1}$$

$$\lim_{x \rightarrow 1^-} f(x) = \lim_{x \rightarrow 1^-} (3x+2) = 5$$

Removable discontinuity at $x = 1$

Continuous on $(-\infty, 1) \cup (1, \infty)$

$$41. f(x) = \frac{1}{(x-2)^2}$$

$$\lim_{x \rightarrow 2} \frac{1}{(x-2)^2} = \infty$$

Nonremovable discontinuity at $x = 2$

Continuous on $(-\infty, 2) \cup (2, \infty)$

$$43. f(x) = \frac{3}{x+1}$$

$$\lim_{x \rightarrow 1^-} f(x) = -\infty$$

$$\lim_{x \rightarrow 1^+} f(x) = \infty$$

Nonremovable discontinuity at $x = -1$

Continuous on $(-\infty, -1) \cup (-1, \infty)$

45. $f(x) = \csc \frac{\pi x}{2}$

Nonremovable discontinuities at each even integer.
Continuous on

$$(2k, 2k + 2)$$

for all integers k .

49. f is continuous on $[1, 2]$. $f(1) = -1 < 0$ and $f(2) = 13 > 0$. Therefore by the Intermediate Value Theorem, there is at least one value c in $(1, 2)$ such that $2c^3 - 3 = 0$.

53. $g(x) = 1 + \frac{2}{x}$

Vertical asymptote at $x = 0$

57. $\lim_{x \rightarrow -2^-} \frac{2x^2 + x + 1}{x + 2} = -\infty$

61. $\lim_{x \rightarrow 1^-} \frac{x^2 + 2x + 1}{x - 1} = -\infty$

65. $\lim_{x \rightarrow 0^+} \frac{\sin 4x}{5x} = \lim_{x \rightarrow 0^+} \left[\frac{4(\sin 4x)}{5(4x)} \right] = \frac{4}{5}$

69. $C = \frac{80,000p}{100 - p}$, $0 \leq 0 < 100$

(a) $C(15) \approx \$14,117.65$ (b) $C(50) = \$80,000$

(c) $C(90) = \$720,000$ (d) $\lim_{p \rightarrow 100^-} \frac{80,000p}{100 - p} = \infty$

47. $f(2) = 5$

Find c so that $\lim_{x \rightarrow 2^+} (cx + 6) = 5$.

$$c(2) + 6 = 5$$

$$2c = -1$$

$$c = -\frac{1}{2}$$

51. $f(x) = \frac{x^2 - 4}{|x - 2|} = (x + 2) \left[\frac{x - 2}{|x - 2|} \right]$

(a) $\lim_{x \rightarrow 2^-} f(x) = -4$

(b) $\lim_{x \rightarrow 2^+} f(x) = 4$

(c) $\lim_{x \rightarrow 2} f(x)$ does not exist.

55. $f(x) = \frac{8}{(x - 10)^2}$

Vertical asymptote at $x = 10$

59. $\lim_{x \rightarrow -1^+} \frac{x + 1}{x^3 + 1} = \lim_{x \rightarrow -1^+} \frac{1}{x^2 - x + 1} = \frac{1}{3}$

63. $\lim_{x \rightarrow 0^+} \left(x - \frac{1}{x^3} \right) = -\infty$

67. $\lim_{x \rightarrow 0^+} \frac{\csc 2x}{x} = \lim_{x \rightarrow 0^+} \frac{1}{x \sin 2x} = \infty$

Problem Solving for Chapter 1

1. (a) Perimeter $\triangle PAO = \sqrt{x^2 + (y - 1)^2} + \sqrt{x^2 + y^2} + 1$
 $= \sqrt{x^2 + (x^2 - 1)^2} + \sqrt{x^2 + x^4} + 1$

Perimeter $\triangle PBO = \sqrt{(x - 1)^2 + y^2} + \sqrt{x^2 + y^2} + 1$
 $= \sqrt{(x - 1)^2 + x^4} + \sqrt{x^2 + x^4} + 1$

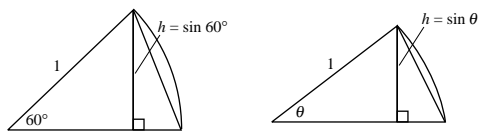
(b) $r(x) = \frac{\sqrt{x^2 + (x^2 - 1)^2} + \sqrt{x^2 + x^4} + 1}{\sqrt{(x - 1)^2 + x^4} + \sqrt{x^2 + x^4} + 1}$

| | | | | | |
|---------------------------|-------|------|------|------|-------|
| x | 4 | 2 | 1 | 0.1 | 0.01 |
| Perimeter $\triangle PAO$ | 33.02 | 9.08 | 3.41 | 2.10 | 2.01 |
| Perimeter $\triangle PBO$ | 33.77 | 9.60 | 3.41 | 2.00 | 2.00 |
| $r(x)$ | 0.98 | 0.95 | 1 | 1.05 | 1.005 |

(c) $\lim_{x \rightarrow 0^+} r(x) = \frac{1 + 0 + 1}{1 + 0 + 1} = \frac{2}{2} = 1$

3. (a) There are 6 triangles, each with a central angle of $60^\circ = \pi/3$. Hence,

$$\begin{aligned}\text{Area hexagon} &= 6 \left[\frac{1}{2}bh \right] = 6 \left[\frac{1}{2}(1) \sin \frac{\pi}{3} \right] \\ &= \frac{3\sqrt{3}}{2} \approx 2.598.\end{aligned}$$



$$\text{Error: } \pi - \frac{3\sqrt{3}}{2} \approx 0.5435.$$

- (b) There are n triangles, each with central angle of $\theta = 2\pi/n$. Hence,

$$An = n \left[\frac{1}{2}bh \right] = n \left[\frac{1}{2}(1) \sin \frac{2\pi}{n} \right] = \frac{n \sin(2\pi/n)}{2}.$$

(c)

| | | | | | |
|------|-------|----|-------|-------|-------|
| n | 6 | 12 | 24 | 48 | 96 |
| An | 2.598 | 3 | 3.106 | 3.133 | 3.139 |

- (d) As n gets larger and larger, $2\pi/n$ approaches 0.

Letting $x = 2\pi/n$,

$$An = \frac{\sin(2\pi/n)}{2/n} = \frac{\sin(2\pi/n)}{(2\pi/n)} \pi = \frac{\sin x}{x} \pi$$

which approaches $(1)\pi = \pi$.

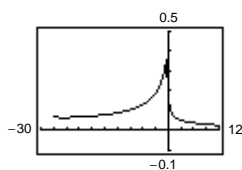
7. (a) $3 + x^{1/3} \geq 0$

$$x^{1/3} \geq -3$$

$$x \geq -27$$

Domain: $x \geq -27, x \neq 1$

- (b)



(d)
$$\begin{aligned}\lim_{x \rightarrow 1} f(x) &= \lim_{x \rightarrow 1} \frac{\sqrt{3 + x^{1/3}} - 2}{x - 1} \cdot \frac{\sqrt{3 + x^{1/3}} + 2}{\sqrt{3 + x^{1/3}} + 2} \\ &= \lim_{x \rightarrow 1} \frac{3 + x^{1/3} - 4}{(x - 1)(\sqrt{3 + x^{1/3}} + 2)} \\ &= \lim_{x \rightarrow 1} \frac{x^{1/3} - 1}{(x^{1/3} - 1)(x^{2/3} + x^{1/3} + 1)(\sqrt{3 + x^{1/3}} + 2)} \\ &= \lim_{x \rightarrow 1} \frac{1}{(x^{2/3} + x^{1/3} + 1)(\sqrt{3 + x^{1/3}} + 2)} \\ &= \frac{1}{(1 + 1 + 1)(2 + 2)} = \frac{1}{12}\end{aligned}$$

9. (a) $\lim_{x \rightarrow 2} f(x) = 3$: g_1, g_4

(b) f continuous at 2: g_1

- (c) $\lim_{x \rightarrow 2} f(x) = 3$: g_1, g_3, g_4

5. (a) Slope = $-\frac{12}{5}$

(b) Slope of tangent line is $\frac{5}{12}$.

$$y + 12 = \frac{5}{12}(x - 5)$$

$$y = \frac{5}{12}x - \frac{169}{12} \quad \text{Tangent line}$$

(c) $Q = (x, y) = (x, \sqrt{169 - x^2})$

$$m_x = \frac{-\sqrt{169 - x^2} + 12}{x - 5}$$

(d)
$$\lim_{x \rightarrow 5} m_x = \lim_{x \rightarrow 5} \frac{12 - \sqrt{169 - x^2}}{x - 5} \cdot \frac{12 + \sqrt{169 - x^2}}{12 + \sqrt{169 - x^2}}$$

$$= \lim_{x \rightarrow 5} \frac{144 - (169 - x^2)}{(x - 5)(12 + \sqrt{169 - x^2})}$$

$$= \lim_{x \rightarrow 5} \frac{x^2 - 25}{(x - 5)(12 + \sqrt{169 - x^2})}$$

$$= \lim_{x \rightarrow 5} \frac{(x + 5)}{12 + \sqrt{169 - x^2}}$$

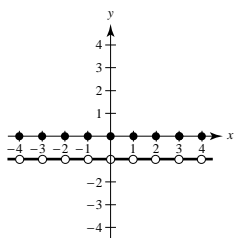
$$= \frac{10}{12 + 12} = \frac{5}{12}$$

This is the same slope as part (b).

(c)
$$\lim_{x \rightarrow -27^+} f(x) = \frac{\sqrt{3 + (-27)^{1/3}} - 2}{-27 - 1}$$

$$= \frac{-2}{-28} = \frac{1}{14} \approx 0.0714$$

11.



(a) $f(1) = \llbracket 1 \rrbracket + \llbracket -1 \rrbracket = 1 + (-1) = 0$

$f(0) = 0$

$f(\frac{1}{2}) = 0 + (-1) = -1$

$f(-2.7) = -3 + 2 = -1$

(b) $\lim_{x \rightarrow 1^-} f(x) = -1$

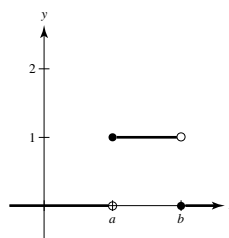
$\lim_{x \rightarrow 1^+} f(x) = -1$

$\lim_{x \rightarrow 1/2} f(x) = -1$

(c) f is continuous for all real numbers except

$x = 0, \pm 1, \pm 2, \pm 3, \dots$

13. (a)



(b) (i) $\lim_{x \rightarrow a^+} P_{a,b}(x) = 1$

(ii) $\lim_{x \rightarrow a^-} P_{a,b}(x) = 0$

(iii) $\lim_{x \rightarrow b^+} P_{a,b}(x) = 0$

(iv) $\lim_{x \rightarrow b^-} P_{a,b}(x) = 1$

(c) $P_{a,b}$ is continuous for all positive real numbers except $x = a, b$.

(d) The area under the graph of u , and above the x -axis, is 1.