CHAPTER 10
Vectors and the Geometry of Space

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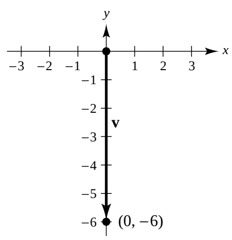
Section 10.1 Vectors in the Plane

Solutions to Even-Numbered Exercises

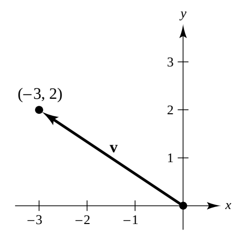
2. (a) $\mathbf{v} = \langle 3 - 3, -2 - 4 \rangle = \langle 0, -6 \rangle$

(b)



4. (a) $\mathbf{v} = \langle -1 - 2, 3 - 1 \rangle = \langle -3, 2 \rangle$

(b)



6. $\mathbf{u} = \langle 1 - (-4), 8 - 0 \rangle = \langle 5, 8 \rangle$

$\mathbf{v} = \langle 7 - 2, 7 - (-1) \rangle = \langle 5, 8 \rangle$

$\mathbf{u} = \mathbf{v}$

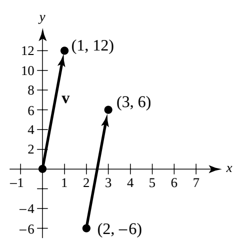
8. $\mathbf{u} = \langle 11 - (-4), -4 - (-1) \rangle = \langle 15, -3 \rangle$

$\mathbf{v} = \langle 25 - 0, 10 - 13 \rangle = \langle 15, -3 \rangle$

$\mathbf{u} = \mathbf{v}$

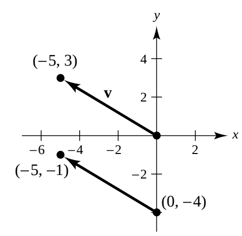
10. (b) $\mathbf{v} = \langle 3 - 2, 6 - (-6) \rangle = \langle 1, 12 \rangle$

(a) and (c).



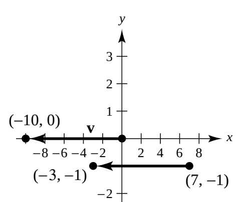
12. (b) $\mathbf{v} = \langle -5 - 0, -1 - (-4) \rangle = \langle -5, 3 \rangle$

(a) and (c).



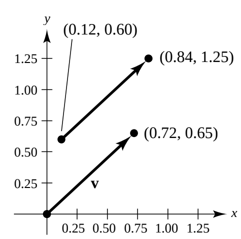
14. (b) $\mathbf{v} = \langle -3 - 7, -1 - (-1) \rangle = \langle -10, 0 \rangle$

(a) and (c).

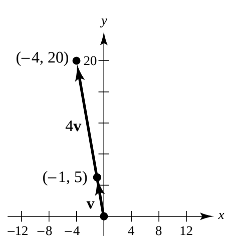


16. (b) $\mathbf{v} = \langle 0.84 - 0.12, 1.25 - 0.60 \rangle = \langle 0.72, 0.65 \rangle$

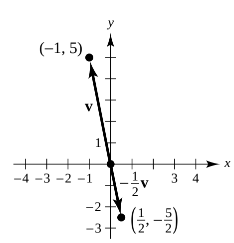
(a) and (c).



18. (a) $4\mathbf{v} = \langle -4, 20 \rangle$



(b) $-\frac{1}{2}\mathbf{v} = \left\langle \frac{1}{2}, -\frac{5}{2} \right\rangle$



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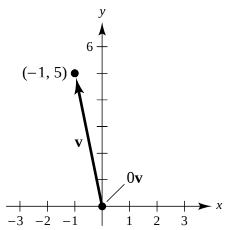
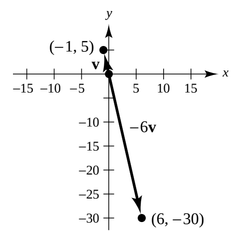
18. —CONTINUED—

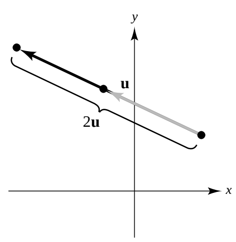
(c) $0\mathbf{v} = \langle 0, 0\rangle$



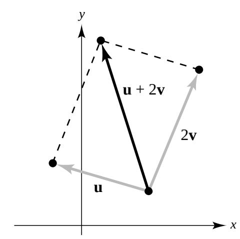
(d) $-6\mathbf{v} = \langle 6, -30\rangle$



20. Twice as long as given vector **u**.



22.



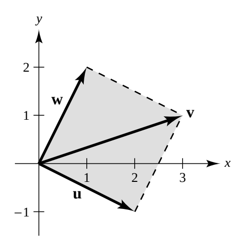
24. (a) $\frac{2}{3}\mathbf{u} = \frac{2}{3}\langle -3, -8\rangle = \left\langle -2, -\frac{16}{3}\right\rangle$

(b) $\mathbf{v} - \mathbf{u} = \langle 8, 25\rangle - \langle -3, -8\rangle = \langle 11, 33\rangle$

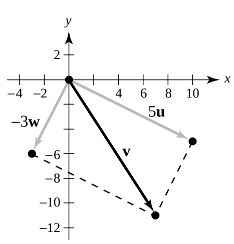
(c) $2\mathbf{u} + 5\mathbf{v} = 2\langle -3, -8\rangle + 5\langle 8, 25\rangle = \langle 34, 109\rangle$

26. $\mathbf{v} = (2\mathbf{i} - \mathbf{j}) + (\mathbf{i} + 2\mathbf{j})$

$= 3\mathbf{i} + \mathbf{j} = \langle 3, 1\rangle$



28. $\mathbf{v} = 5\mathbf{u} - 3\mathbf{w} = 5\langle 2, -1\rangle - 3\langle 1, 2\rangle = \langle 7, -11\rangle$



30. $u_1 - 3 = 4$

$u_2 - 2 = -9$

$u_1 = 7$

$u_2 = -7$

$Q = (7, -7)$

32. $\|\mathbf{v}\| = \sqrt{144 + 25} = 13$

34. $\|\mathbf{v}\| = \sqrt{100 + 9} = \sqrt{109}$

36. $\|\mathbf{v}\| = \sqrt{1 + 1} = \sqrt{2}$

38. $\|\mathbf{u}\| = \sqrt{5^2 + 15^2} = \sqrt{250} = 5\sqrt{10}$

$$\mathbf{v} = \frac{\mathbf{u}}{\|\mathbf{u}\|} = \frac{\langle 5, 15\rangle}{5\sqrt{10}} = \left\langle \frac{1}{\sqrt{10}}, \frac{3}{\sqrt{10}}\right\rangle \text{ unit vector}$$

40. $\|\mathbf{u}\| = \sqrt{(-6.2)^2 + (3.4)^2} = \sqrt{50} = 5\sqrt{2}$

$$\mathbf{v} = \frac{\mathbf{u}}{\|\mathbf{u}\|} = \frac{\langle -6.2, 3.4\rangle}{5\sqrt{2}} = \left\langle \frac{-1.24}{\sqrt{2}}, \frac{0.68}{\sqrt{2}}\right\rangle \text{ unit vector}$$

42. $\mathbf{u} = \langle 0, 1\rangle, \mathbf{v} = \langle 3, -3\rangle$

(a) $\|\mathbf{u}\| = \sqrt{0+1} = 1$

(b) $\|\mathbf{v}\| = \sqrt{9+9} = 3\sqrt{2}$

(c) $\mathbf{u} + \mathbf{v} = \langle 3, -2\rangle$

$\|\mathbf{u} + \mathbf{v}\| = \sqrt{9+4} = \sqrt{13}$

(d) $\dfrac{\mathbf{u}}{\|\mathbf{u}\|} = \langle 0, 1\rangle$

$\left\|\dfrac{\mathbf{u}}{\|\mathbf{u}\|}\right\| = 1$

(e) $\dfrac{\mathbf{v}}{\|\mathbf{v}\|} = \dfrac{1}{3\sqrt{2}}\langle 3, -3\rangle$

$\left\|\dfrac{\mathbf{v}}{\|\mathbf{v}\|}\right\| = 1$

(f) $\dfrac{\mathbf{u}+\mathbf{v}}{\|\mathbf{u}+\mathbf{v}\|} = \dfrac{1}{\sqrt{13}}\langle 3, -2\rangle$

$\left\|\dfrac{\mathbf{u}+\mathbf{v}}{\|\mathbf{u}+\mathbf{v}\|}\right\| = 1$

44. $\mathbf{u} = \langle 2, -4\rangle, \mathbf{v} = \langle 5, 5\rangle$

(a) $\|\mathbf{u}\| = \sqrt{4+16} = 2\sqrt{5}$

(b) $\|\mathbf{v}\| = \sqrt{25+25} = 5\sqrt{2}$

(c) $\mathbf{u} + \mathbf{v} = \langle 7, 1\rangle$

$\|\mathbf{u} + \mathbf{v}\| = \sqrt{49+1} = 5\sqrt{2}$

(d) $\dfrac{\mathbf{u}}{\|\mathbf{u}\|} = \dfrac{1}{2\sqrt{5}}\langle 2, -4\rangle$

$\left\|\dfrac{\mathbf{u}}{\|\mathbf{u}\|}\right\| = 1$

(e) $\dfrac{\mathbf{v}}{\|\mathbf{v}\|} = \dfrac{1}{5\sqrt{2}}\langle 5, 5\rangle$

$\left\|\dfrac{\mathbf{v}}{\|\mathbf{v}\|}\right\| = 1$

(f) $\dfrac{\mathbf{u}+\mathbf{v}}{\|\mathbf{u}+\mathbf{v}\|} = \dfrac{1}{5\sqrt{2}}\langle 7, 1\rangle$

$\left\|\dfrac{\mathbf{u}+\mathbf{v}}{\|\mathbf{u}+\mathbf{v}\|}\right\| = 1$

46. $\mathbf{u} = \langle -3, 2\rangle$

$\|\mathbf{u}\| = \sqrt{13} \approx 3.606$

$\mathbf{v} = \langle 1, -2\rangle$

$\|\mathbf{v}\| = \sqrt{5} \approx 2.236$

$\mathbf{u} + \mathbf{v} = \langle -2, 0\rangle$

$\|\mathbf{u} + \mathbf{v}\| = 2$

$\|\mathbf{u} + \mathbf{v}\| \le \|\mathbf{u}\| + \|\mathbf{v}\|$

48. $\dfrac{\mathbf{u}}{\|\mathbf{u}\|} = \dfrac{1}{\sqrt{2}}\langle -1, 1\rangle$

$4\left(\dfrac{\mathbf{u}}{\|\mathbf{u}\|}\right) = 2\sqrt{2}\langle -1, 1\rangle$

$\mathbf{v} = \left\langle -2\sqrt{2}, 2\sqrt{2}\right\rangle$

50. $\dfrac{\mathbf{u}}{\|\mathbf{u}\|} = \dfrac{1}{3}\langle 0, 3\rangle$

$3\left(\dfrac{\mathbf{u}}{\|\mathbf{u}\|}\right) = \langle 0, 3\rangle$

$\mathbf{v} = \langle 0, 3\rangle$

52. $\mathrm{v} = 5[(\cos 120°)\mathbf{i} + (\sin 120°)\mathbf{j}]$

$= -\dfrac{5}{2}\mathbf{i} + \dfrac{5\sqrt{3}}{2}\mathbf{j}$

54. $\mathbf{v} = (\cos 3.5°)\mathbf{i} + (\sin 3.5°)\mathbf{j}$

$\approx 0.9981\mathbf{i} + 0.0610\mathbf{j} = \langle 0.9981, 0.0610\rangle$

56. $\mathbf{u} = 4\mathbf{i}$

$\mathbf{v} = \mathbf{i} + \sqrt{3}\mathbf{j}$

$\mathbf{u} + \mathbf{v} = 5\mathbf{i} + \sqrt{3}\mathbf{j}$

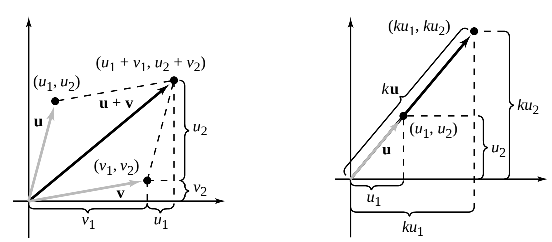
58. $\mathbf{u} = 5[\cos(-0.5)]\mathbf{i} + 5[\sin(-0.5)]\mathbf{j}$

$= 5[\cos(0.5)]\mathbf{i} - 5[\sin(0.5)]\mathbf{j}$

$\mathbf{v} = 5[\cos(0.5)]\mathbf{i} + 5[\sin(0.5)]\mathbf{j}$

$\mathbf{u} + \mathbf{v} = 10[\cos(0.5)]\mathbf{i}$

60. See page 718:



62. See Theorem 10.1, page 719.

For Exercises 64–68, $a\mathbf{u} + b\mathbf{w} = a(\mathbf{i} + 2\mathbf{j}) + b(\mathbf{i} - \mathbf{j}) = (a + b)\mathbf{i} + (2a - b)\mathbf{j}$.

64. $\mathbf{v} = 3\mathbf{j}$. Therefore, $a + b = 0$, $2a - b = 3$. Solving simultaneously, we have $a = 1$, $b = -1$.

66. $\mathbf{v} = 3\mathbf{i} + 3\mathbf{j}$. Therefore, $a + b = 3$, $2a - b = 3$. Solving simultaneously, we have $a = 2$, $b = 1$.

68. $\mathbf{v} = -\mathbf{i} + 7\mathbf{j}$. Therefore, $a + b = -1$, $2a - b = 7$. Solving simultaneously, we have $a = 2$, $b = -3$.

70. $y = x^3, y' = 3x^2 = 12$ at $x = -2$.

(a) $m = 12$. Let $\mathbf{w} = \langle 1, 12\rangle$, then

$$\frac{\mathbf{w}}{\|\mathbf{w}\|} = \pm\frac{1}{\sqrt{145}}\langle 1, 12\rangle.$$

(b) $m = -\frac{1}{12}$. Let $\mathbf{w} = \langle 12, -1\rangle$, then

$$\frac{\mathbf{w}}{\|\mathbf{w}\|} = \pm\frac{1}{\sqrt{145}}\langle 12, -1\rangle.$$

72. $f(x) = \tan x$

$f'(x) = \sec^2 x = 2$ at $x = \dfrac{\pi}{4}$.

(a) $m = 2$. Let $\mathbf{w} = \langle 1, 2\rangle$, then

$$\frac{\mathbf{w}}{\|\mathbf{w}\|} = \pm\frac{1}{\sqrt{5}}\langle 1, 2\rangle.$$

(b) $m = -\frac{1}{2}$. Let $\mathbf{w} = \langle -2, 1\rangle$, then

$$\frac{\mathbf{w}}{\|\mathbf{w}\|} = \pm\frac{1}{\sqrt{5}}\langle -2, 1\rangle.$$

74.

$$\mathbf{u} = 2\sqrt{3}\mathbf{i} + 2\mathbf{j}$$

$$\mathbf{u} + \mathbf{v} = -3\mathbf{i} + 3\sqrt{3}\mathbf{j}$$

$$\mathbf{v} = (\mathbf{u} + \mathbf{v}) - \mathbf{u} = \left(-3 - 2\sqrt{3}\right)\mathbf{i} + \left(3\sqrt{3} - 2\right)\mathbf{j}$$

76. magnitude ≈ 63.5

direction $\approx -8.26^\circ$

78. $\|\mathbf{F}_1\| = 2,\ \theta_{\mathbf{F}_1} = -10^\circ$

$\|\mathbf{F}_2\| = 4,\ \theta_{\mathbf{F}_2} = 140^\circ$

$\|\mathbf{F}_3\| = 3,\ \theta_{\mathbf{F}_3} = 200^\circ$

$\|\mathbf{R}\| = \|\mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3\| \approx 4.09$

$\theta_{\mathbf{R}} = \theta_{\mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3} \approx 163.0^\circ$

80.

$$\mathbf{F}_1 + \mathbf{F}_2 = (500\cos 30^\circ\mathbf{i} + 500\sin 30^\circ\mathbf{j}) + (200\cos(-45^\circ)\mathbf{i} + 200\sin(-45^\circ)\mathbf{j})$$

$$= \left(250\sqrt{3} + 100\sqrt{2}\right)\mathbf{i} + \left(250 - 100\sqrt{2}\right)\mathbf{j}$$

$$\|\mathbf{F}_1 + \mathbf{F}_2\| = \sqrt{\left(250\sqrt{3} + 100\sqrt{2}\right)^2 + \left(250 - 100\sqrt{2}\right)^2} \approx 584.6 \text{ lb}$$

$$\tan\theta = \frac{250 - 100\sqrt{2}}{250\sqrt{3} + 100\sqrt{2}} \implies \theta \approx 10.7^\circ$$

82.

$$\mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 = [400(\cos(-30^\circ)\mathbf{i} + \sin(-30^\circ)\mathbf{j})] + [280(\cos(45^\circ)\mathbf{i} + \sin(45^\circ)\mathbf{j})] + [350(\cos(135^\circ)\mathbf{i} + \sin(135^\circ)\mathbf{j}]$$

$$= \left[200\sqrt{3} + 140\sqrt{2} - 175\sqrt{2}\right]\mathbf{i} + \left[-200 + 140\sqrt{2} + 175\sqrt{2}\right]\mathbf{j}$$

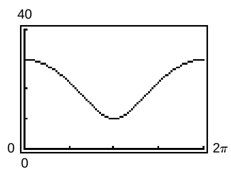
$$\|\mathbf{R}\| = \sqrt{\left(200\sqrt{3} - 35\sqrt{2}\right)^2 + \left(-200 + 315\sqrt{2}\right)^2} \approx 385.2483 \text{ newtons}$$

$$\theta_{\mathbf{R}} = \arctan\left(\frac{-200 + 315\sqrt{2}}{200\sqrt{3} - 35\sqrt{2}}\right) \approx 0.6908 \approx 39.6^\circ$$

84. $\mathbf{F}_1 = \langle 20, 0\rangle, \mathbf{F}_2 = 10\langle \cos\theta, \sin\theta\rangle$

(a) $$\begin{aligned}\|\mathbf{F}_1 + \mathbf{F}_2\| &= \|\langle 20 + 10\cos\theta, 10\sin\theta\rangle\|\\ &= \sqrt{400 + 400\cos\theta + 100\cos^2\theta + 100\sin^2\theta}\\ &= \sqrt{500 + 400\cos\theta}\end{aligned}$$

(b)



(c) The range is $10 \le \|\mathbf{F}_1 + \mathbf{F}_2\| \le 30$.

The maximum is 30, which occur at $\theta = 0$ and $\theta = 2\pi$.

The minimum is 10 at $\theta = \pi$.

(d) The minimum of the resultant is 10.

86. $\mathbf{u} = \langle 7 - 1, 5 - 2\rangle = \langle 6, 3\rangle$

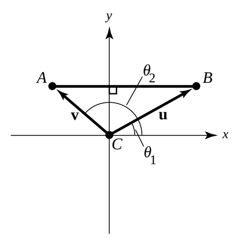
$\frac{1}{3}\mathbf{u} = \langle 2, 1\rangle$

$P_1 = (1, 2) + (2, 1) = (3, 3)$

$P_2 = (1, 2) + 2(2, 1) = (5, 4)$

88. $\theta_1 = \arctan\left(\frac{24}{20}\right) \approx 0.8761 \text{ or } 50.2°$

$\theta_2 = \arctan\left(\frac{24}{-10}\right) + \pi \approx 1.9656 \text{ or } 112.6°$



$\mathbf{u} = \|\mathbf{u}\|(\cos\theta_1\,\mathbf{i} + \sin\theta_1\,\mathbf{j})$

$\mathbf{v} = \|\mathbf{v}\|(\cos\theta_2\,\mathbf{i} + \sin\theta_2\,\mathbf{j})$

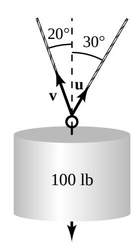
Vertical components: $\|\mathbf{u}\|\sin\theta_1 + \|\mathbf{v}\|\sin\theta_2 = 5000$

Horizontal components: $\|\mathbf{u}\|\cos\theta_1 + \|\mathbf{v}\|\cos\theta_2 = 0$

Solving this system, you obtain

$\|\mathbf{u}\| \approx 2169.4$ and $\|\mathbf{v}\| \approx 3611.2$.

90. To lift the weight vertically, the sum of the vertical components of **u** and **v** must be 100 and the sum of the horizontal components must be 0.



$\mathbf{u} = \|\mathbf{u}\|\,(\cos 60°\mathbf{i} + \sin 60°\mathbf{j})$

$\mathbf{v} = \|\mathbf{v}\|\,(\cos 110°\mathbf{i} + \sin 110°\mathbf{j})$

Thus, $\|\mathbf{u}\|\sin 60° + \|\mathbf{v}\|\sin 110° = 100$, or

$$\|\mathbf{u}\|\left(\frac{\sqrt{3}}{2}\right) + \|\mathbf{v}\|\sin 110° = 100.$$

And $\|\mathbf{u}\|\cos 60° + \|\mathbf{v}\|\cos 110° = 0$ or

$$\|\mathbf{u}\|\left(\frac{1}{2}\right) + \|\mathbf{v}\|\cos 110° = 0$$

Multiplying the last equation by $\left(\sqrt{3}\right)$ and adding to the first equation gives

$$\|\mathbf{u}\|\left(\sin 110° - \sqrt{3}\cos 110°\right) = 100 \implies \|\mathbf{v}\| \approx 65.27 \text{ lb}.$$

Then, $\|\mathbf{u}\|\left(\frac{1}{2}\right) + 65.27\cos 110° = 0$ gives

$\|\mathbf{u}\| \approx 44.65$ lb.

(a) The tension in each rope: $\|\mathbf{u}\| = 44.65$ lb, $\|\mathbf{v}\| = 65.27$ lb.

(b) Vertical components: $\|\mathbf{u}\|\sin 60° \approx 38.67$ lb.

$\|\mathbf{v}\|\sin 110° \approx 61.33$ lb.

92. $\mathbf{u} = 400\mathbf{i}$(plane)

$\mathbf{v} = 50(\cos 135°\mathbf{i} + \sin 135°\mathbf{j}) = -25\sqrt{2}\mathbf{i} + 25\sqrt{2}\mathbf{j}$ (wind)

$\mathbf{u} + \mathbf{v} = (400 - 25\sqrt{2})\mathbf{i} + 25\sqrt{2}\mathbf{j} \approx 364.64\mathbf{i} + 35.36\mathbf{j}$

$\tan\theta = \frac{35.36}{364.64} \implies \theta \approx 5.54°$

Direction North of East: $\approx$ N 84.46° E

Speed: $\approx$ 336.35 mph

94. $\|\mathbf{u}\| = \sqrt{\cos^2\theta + \sin^2\theta} = 1,$

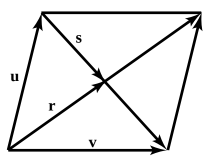
$\|\mathbf{v}\| = \sqrt{\sin^2\theta + \cos^2\theta} = 1$

96. Let $\mathbf{u}$ and $\mathbf{v}$ be the vectors that determine the parallelogram, as indicated in the figure. The two diagonals are $\mathbf{u} + \mathbf{v}$ and $\mathbf{v} - \mathbf{u}$. Therefore, $\mathbf{r} = x(\mathbf{u} + \mathbf{v})$, $\mathbf{s} = y(\mathbf{v} - \mathbf{u})$. But,

$$\mathbf{u} = \mathbf{r} - \mathbf{s}$$

$$= x(\mathbf{u} + \mathbf{v}) - y(\mathbf{v} - \mathbf{u}) = (x + y)\mathbf{u} + (x - y)\mathbf{v}.$$

Therefore, $x + y = 1$ and $x - y = 0$. Solving we have $x = y = \frac{1}{2}$.



98. The set is a circle of radius 5, centered at the origin.

$\|\mathbf{u}\| = \|\langle x, y\rangle\| = \sqrt{x^2 + y^2} = 5 \implies x^2 + y^2 = 25$

100. True

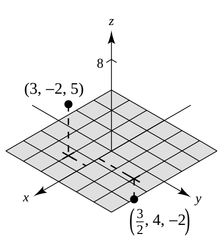
102. False

$a = b = 0$

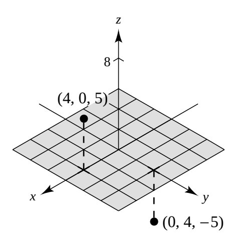
104. True

Section 10.2 Space Coordinates and Vectors in Space

2.



4.



6. $A(2, -3, -1)$

$B(-3, 1, 4)$

8. $x = 7, y = -2, z = -1$:

$(7, -2, -1)$

10. $x = 0, y = 3, z = 2$: $(0, 3, 2)$

12. The x-coordinate is 0.

14. The point is 2 units in front of the xz-plane.

16. The point is on the plane $z = -3$.

18. The point is behind the yz-plane.

20. The point is in front of the plane $x = 4$.

22. The point (x, y, z) is 4 units above the xy-plane, and above either quadrant II or IV.

24. The point could be above the xy-plane, and thus above quadrants I or III, or below the xy-plane, and thus below quadrants II or IV.

26. $$d = \sqrt{(2-(-2))^2 + (-5-3)^2 + (-2-2)^2} = \sqrt{16+64+16} = \sqrt{96} = 4\sqrt{6}$$

28. $$d = \sqrt{(4-2)^2 + (-5-2)^2 + (6-3)^2} = \sqrt{4+49+9} = \sqrt{62}$$

30. $A(5, 3, 4)$, $B(7, 1, 3)$, $C(3, 5, 3)$

$|AB| = \sqrt{4+4+1} = 3$

$|AC| = \sqrt{4+4+1} = 3$

$|BC| = \sqrt{16+16+0} = 4\sqrt{2}$

Since $|AB| = |AC|$, the triangle is isosceles.

32. $A(5, 0, 0)$, $B(0, 2, 0)$, $C(0, 0, -3)$

$|AB| = \sqrt{25+4+0} = \sqrt{29}$

$|AC| = \sqrt{25+0+9} = \sqrt{34}$

$|BC| = \sqrt{0+4+9} = \sqrt{13}$

Neither

34. The y-coordinate is changed by 3 units:

$(5, 6, 4), (7, 4, 3), (3, 8, 3)$

36. $$\left(\frac{4+8}{2}, \frac{0+8}{2}, \frac{-6+20}{2}\right) = (6, 4, 7)$$

38. Center: $(4, -1, 1)$

Radius: 5

$$(x-4)^2 + (y+1)^2 + (z-1)^2 = 25$$

$$x^2 + y^2 + z^2 - 8x + 2y - 2z - 7 = 0$$

40. Center: $(-3, 2, 4)$

$r = 3$

(tangent to yz-plane)

$$(x+3)^2 + (y-2)^2 + (z-4)^2 = 9$$

42.
$$x^2 + y^2 + z^2 + 9x - 2y + 10z + 19 = 0$$
$$\left(x^2 + 9x + \frac{81}{4}\right) + (y^2 - 2y + 1) + (z^2 + 10z + 25) = -19 + \frac{81}{4} + 1 + 25$$
$$\left(x + \frac{9}{2}\right)^2 + (y-1)^2 + (z+5)^2 = \frac{109}{4}$$

Center: $\left(-\frac{9}{2}, 1, -5\right)$

Radius: $\frac{\sqrt{109}}{2}$

44.
$$4x^2 + 4y^2 + 4z^2 - 4x - 32y + 8z + 33 = 0$$
$$x^2 + y^2 + z^2 - x - 8y + 2z + \frac{33}{4} = 0$$
$$\left(x^2 - x + \frac{1}{4}\right) + (y^2 - 8y + 16) + (z^2 + 2z + 1) = -\frac{33}{4} + \frac{1}{4} + 16 + 1$$
$$\left(x - \frac{1}{2}\right)^2 + (y-4)^2 + (z+1)^2 = 9$$

Center: $\left(\frac{1}{2}, 4, -1\right)$

Radius: 3

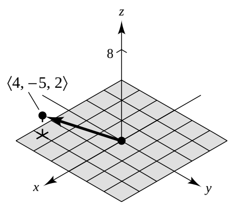
46. $$x^2 + y^2 + z^2 < 4x - 6y + 8z - 13$$
$$(x^2 - 4x + 4) + (y^2 + 6y + 9) + (z^2 - 8z + 16) < 4 + 9 + 16 - 13$$
$$(x - 2)^2 + (y + 3)^2 + (z - 4)^2 < 16$$

Interior of sphere of radius 4 centered at $(2, -3, 4)$.

48. (a) $\mathbf{v} = (4 - 0)\mathbf{i} + (0 - 5)\mathbf{j} + (3 - 1)\mathbf{k}$

$= 4\mathbf{i} - 5\mathbf{j} + 2\mathbf{k} = \langle 4, -5, 2\rangle$

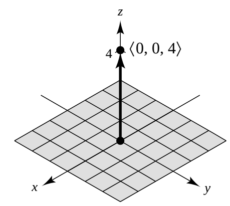
(b)



50. (a) $\mathbf{v} = (2 - 2)\mathbf{i} + (3 - 3)\mathbf{j} + (4 - 0)\mathbf{k}$

$= 4\mathbf{k} = \langle 0, 0, 4\rangle$

(b)



52. $\langle -1 - 4, 7 - (-5), -3 - 2\rangle = \langle -5, 12, -5\rangle$

$\|\langle -5, 12, -5\rangle\| = \sqrt{25 + 144 + 25} = \sqrt{194}$

Unit vector: $\dfrac{\langle -5, 12, -5\rangle}{\sqrt{194}} = \left\langle \dfrac{-5}{\sqrt{194}}, \dfrac{12}{\sqrt{194}}, \dfrac{-5}{\sqrt{194}}\right\rangle$

54. $\langle 2 - 1, 4 - (-2), -2 - 4\rangle = \langle 1, 6, -6\rangle$

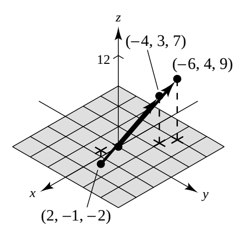
$\|\langle 1, 6, -6\rangle\| = \sqrt{1 + 36 + 36} = \sqrt{73}$

Unit vector: $\left\langle \dfrac{1}{\sqrt{73}}, \dfrac{6}{\sqrt{73}}, \dfrac{-6}{\sqrt{73}}\right\rangle$

56. (b) $\mathbf{v} = (-4 - 2)\mathbf{i} + (3 + 1)\mathbf{j} + (7 + 2)\mathbf{k}$

$= -6\mathbf{i} + 4\mathbf{j} + 9\mathbf{k} = \langle -6, 4, 9\rangle$

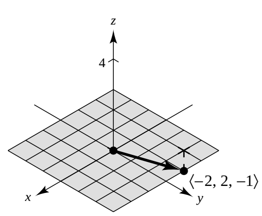
(a) and (c).



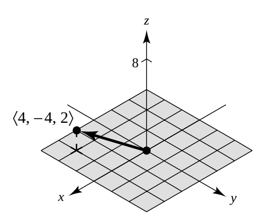
58. $(q_1, q_2, q_3) - \left(0, 2, \frac{5}{2}\right) = \left(1, -\frac{2}{3}, \frac{1}{2}\right)$

$Q = \left(1, -\frac{8}{3}, 3\right)$

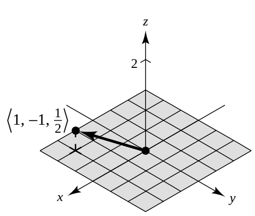
60. (a) $-\mathbf{v} = \langle -2, 2, -1\rangle$



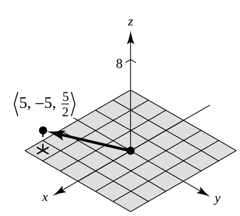
(b) $2\mathbf{v} = \langle 4, -4, 2\rangle$



(c) $\frac{1}{2}\mathbf{v} = \left\langle 1, -1, \frac{1}{2}\right\rangle$



(d) $\frac{5}{2}\mathbf{v} = \left\langle 5, -5, \frac{5}{2}\right\rangle$



62. $\mathbf{z} = \mathbf{u} - \mathbf{v} + 2\mathbf{w} = \langle 1, 2, 3\rangle - \langle 2, 2, -1\rangle + \langle 8, 0, -8\rangle = \langle 7, 0, -4\rangle$

64. $\mathbf{z} = 5\mathbf{u} - 3\mathbf{v} - \frac{1}{2}\mathbf{w} = \langle 5, 10, 15\rangle - \langle 6, 6, -3\rangle - \langle 2, 0, -2\rangle = \langle -3, 4, 20\rangle$

66. $2\mathbf{u} + \mathbf{v} - \mathbf{w} + 3\mathbf{z} = 2\langle 1, 2, 3\rangle + \langle 2, 2, -1\rangle - \langle 4, 0, -4\rangle + 3\langle z_1, z_2, z_3\rangle = \langle 0, 0, 0\rangle$

$\langle 0, 6, 9\rangle + \langle 3z_1, 3z_2, 3z_3\rangle = \langle 0, 0, 0\rangle$

$0 + 3z_1 = 0 \implies z_1 = 0$

$6 + 3z_2 = 0 \implies z_2 = -2$

$9 + 3z_3 = 0 \implies z_3 = -3$

$\mathbf{z} = \langle 0, -2, -3\rangle$

68. (b) and (d) are parallel since $-\mathbf{i} + \frac{4}{3}\mathbf{j} - \frac{3}{2}\mathbf{k} = -2\left(\frac{1}{2}\mathbf{i} - \frac{2}{3}\mathbf{j} + \frac{3}{4}\mathbf{k}\right)$ and $\frac{3}{4}\mathbf{i} - \mathbf{j} + \frac{9}{8}\mathbf{k} = \frac{3}{2}\left(\frac{1}{2}\mathbf{i} - \frac{2}{3}\mathbf{j} + \frac{3}{4}\mathbf{k}\right)$.

70. $\mathbf{z} = \langle -7, -8, 3\rangle$

(b) is parallel since $(-z)\mathbf{z} = \langle 14, 16, -6\rangle$.

72. $P(4, -2, 7), Q(-2, 0, 3), R(7, -3, 9)$

$\overrightarrow{PQ} = \langle -6, 2, -4\rangle$

$\overrightarrow{PR} = \langle 3, -1, 2\rangle$

$\langle 3, -1, 2\rangle = -\frac{1}{2}\langle -6, 2, -4\rangle$

Therefore, $\overrightarrow{PQ}$ and $\overrightarrow{PR}$ are parallel.

The points are collinear.

74. $P(0, 0, 0), Q(1, 3, -2), R(2, -6, 4)$

$\overrightarrow{PQ} = \langle 1, 3, -2\rangle$

$\overrightarrow{PR} = \langle 2, -6, 4\rangle$

Since $\overrightarrow{PQ}$ and $\overrightarrow{PR}$ are not parallel, the points are not collinear.

76. $A(1, 1, 3), B(9, -1, -2), C(11, 2, -9), D(3, 4, -4)$

$\overrightarrow{AB} = \langle 8, -2, -5\rangle$

$\overrightarrow{DC} = \langle 8, -2, -5\rangle$

$\overrightarrow{AD} = \langle 2, 3, -7\rangle$

$\overrightarrow{BC} = \langle 2, 3, -7\rangle$

Since $\overrightarrow{AB} = \overrightarrow{DC}$ and $\overrightarrow{AD} = \overrightarrow{BC}$, the given points form the vertices of a parallelogram.

78. $\|\mathbf{v}\| = \sqrt{1 + 0 + 9} = \sqrt{10}$

80. $\mathbf{v} = \langle -4, 3, 7\rangle$

$\|\mathbf{v}\| = \sqrt{16 + 9 + 49} = \sqrt{74}$

82. $\mathbf{v} = \langle 1, 3, -2\rangle$

$\|\mathbf{v}\| = \sqrt{1 + 9 + 4} = \sqrt{14}$

84. $\mathbf{u} = \langle 6, 0, 8\rangle$

$\|\mathbf{u}\| = \sqrt{36 + 0 + 64} = 10$

(a) $\dfrac{\mathbf{u}}{\|\mathbf{u}\|} = \dfrac{1}{10}\langle 6, 0, 8\rangle$

(b) $-\dfrac{\mathbf{u}}{\|\mathbf{u}\|} = -\dfrac{1}{10}\langle 6, 0, 8\rangle$

86. $\mathbf{u} = \langle 8, 0, 0\rangle$

$\|\mathbf{u}\| = 8$

(a) $\dfrac{\mathbf{u}}{\|\mathbf{u}\|} = \langle 1, 0, 0\rangle$

(b) $-\dfrac{\mathbf{u}}{\|\mathbf{u}\|} = \langle -1, 0, 0\rangle$

88. (a) $\mathbf{u} + \mathbf{v} = \langle 4, 7.5, -2\rangle$

(b) $\|\mathbf{u} + \mathbf{v}\| \approx 8.732$

(c) $\|\mathbf{u}\| \approx 5.099$

(d) $\|\mathbf{v}\| \approx 9.014$

90. $c\mathbf{u} = \langle c, 2c, 3c\rangle$

$\|c\mathbf{u}\| = \sqrt{c^2 + 4c^2 + 9c^2} = 3$

$14c^2 = 9$

$c = \pm\dfrac{3\sqrt{14}}{14}$

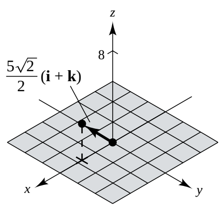
92. $\mathbf{v} = 3\dfrac{\mathbf{u}}{\|\mathbf{u}\|} = 3\left\langle \dfrac{1}{\sqrt{3}}, \dfrac{1}{\sqrt{3}}, \dfrac{1}{\sqrt{3}} \right\rangle = \left\langle \dfrac{3}{\sqrt{3}}, \dfrac{3}{\sqrt{3}}, \dfrac{3}{\sqrt{3}} \right\rangle$

94. $\mathbf{v} = \sqrt{5}\dfrac{\mathbf{u}}{\|\mathbf{u}\|} = \sqrt{5}\left\langle \dfrac{-2}{\sqrt{14}}, \dfrac{3}{\sqrt{14}}, \dfrac{1}{\sqrt{14}} \right\rangle$

$= \left\langle \dfrac{-\sqrt{70}}{7}, \dfrac{3\sqrt{70}}{14}, \dfrac{\sqrt{70}}{14} \right\rangle$

96. $\mathbf{v} = 5(\cos 45°\mathbf{i} + \sin 45°\mathbf{k}) = \dfrac{5\sqrt{2}}{2}(\mathbf{i} + \mathbf{k})$ or

$\mathbf{v} = 5(\cos 135°\mathbf{i} + \sin 135°\mathbf{k}) = \dfrac{5\sqrt{2}}{2}(-i + k)$



98. $\mathbf{v} = \langle 5, 6, -3 \rangle$

$\frac{2}{3}\mathbf{v} = \left\langle \frac{10}{3}, 4, -2 \right\rangle$

$(1, 2, 5) + \left(\frac{10}{3}, 4, -2\right) = \left(\frac{13}{3}, 6, 3\right)$

100. x_0 is directed distance to yz-plane.

y_0 is directed distance to xz-plane.

z_0 is directed distance to xy-plane.

102. $(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2 = r^2$

104. A sphere of radius 4 centered at (x_1, y_1, z_1).

$\|\mathbf{v}\| = \|\langle x - x_2, y - y_1, z - z_1 \rangle\|$

$= \sqrt{(x - x_1)^2 + (y - y_1)^2 + (z - z_1)^2} = 4$

$(x - x_1)^2 + (y - y_1)^2 + (z - z_1)^2 = 16$ sphere

106. As in Exercise 105(c), $x = a$ will be a vertical asymptote. Hence, $\lim\limits_{r_0 \to a^-} T = \infty$.

108. $550 = \|c(75\mathbf{i} - 50\mathbf{j} - 100\mathbf{k})\|$

$302{,}500 = 18{,}125c^2$

$c^2 = 16.689655$

$c \approx 4.085$

$\mathbf{F} \approx 4.085(75\mathbf{i} - 50\mathbf{j} - 100\mathbf{k})$

$\approx 306\mathbf{i} - 204\mathbf{j} - 409\mathbf{k}$

110. Let A lie on the y-axis and the wall on the x-axis. Then $A = (0, 10, 0)$, $B = (8, 0, 6)$, $C = (-10, 0, 6)$ and

$\overrightarrow{AB} = \langle 8, -10, 6 \rangle$, $\overrightarrow{AC} = \langle -10, -10, 6 \rangle$.

$\|\overrightarrow{AB}\| = 10\sqrt{2}$, $\|\overrightarrow{AC}\| = 2\sqrt{59}$

Thus, $\mathbf{F}_1 = 420\dfrac{\overrightarrow{AB}}{\|\overrightarrow{AB}\|}$, $\mathbf{F}_2 = 650\dfrac{\overrightarrow{AC}}{\|\overrightarrow{AC}\|}$

$\mathbf{F} = \mathbf{F}_1 + \mathbf{F}_2 \approx \langle 237.6, -297.0, 178.2 \rangle$

$+ \langle -423.1, -423.1, 253.9 \rangle$

$\approx \langle -185.5, -720.1, 432.1 \rangle$

$\|\mathbf{F}\| \approx 860.0$ lb

Section 10.3 The Dot Product of Two Vectors

2. $\mathbf{u} = \langle 4, 10 \rangle$, $\mathbf{v} = \langle -2, 3 \rangle$

(a) $\mathbf{u} \cdot \mathbf{v} = 4(-2) + 10(3) = 22$

(b) $\mathbf{u} \cdot \mathbf{u} = 4(4) + 10(10) = 116$

(c) $\|\mathbf{u}\|^2 = 116$

(d) $(\mathbf{u} \cdot \mathbf{v})\mathbf{v} = 22\langle -2, 3 \rangle = \langle -44, 66 \rangle$

(e) $\mathbf{u} \cdot (2\mathbf{v}) = 2(\mathbf{u} \cdot \mathbf{v}) = 2(22) = 44$

4. $\mathbf{u} = \mathbf{i}$, $\mathbf{v} = \mathbf{i}$

(a) $\mathbf{u} \cdot \mathbf{v} = 1$

(b) $\mathbf{u} \cdot \mathbf{u} = 1$

(c) $\|\mathbf{u}\|^2 = 1$

(d) $(\mathbf{u} \cdot \mathbf{v})\mathbf{v} = \mathbf{i}$

(e) $\mathbf{u} \cdot (2\mathbf{v}) = 2(\mathbf{u} \cdot \mathbf{v}) = 2$

6. $\mathbf{u} = 2\mathbf{i} + \mathbf{j} - 2\mathbf{k}, \mathbf{v} = \mathbf{i} - 3\mathbf{j} + 2\mathbf{k}$

(a) $\mathbf{u} \cdot \mathbf{v} = 2(1) + 1(-3) + (-2)(2) = -5$

(b) $\mathbf{u} \cdot \mathbf{u} = 2(2) + 1(1) + (-2)(-2) = 9$

(c) $\|\mathbf{u}\|^2 = 9$

(d) $(\mathbf{u} \cdot \mathbf{v})\mathbf{v} = -5(\mathbf{i} - 3\mathbf{j} + 2\mathbf{k}) = -5\mathbf{i} + 15\mathbf{j} - 10\mathbf{k}$

(e) $\mathbf{u} \cdot (2\mathbf{v}) = 2(\mathbf{u} \cdot \mathbf{v}) = 2(-5) = -10$

8. $\mathbf{u} = \langle 3240, 1450, 2235 \rangle$

$\mathbf{v} = \langle 2.22, 1.85, 3.25 \rangle$

Increase prices by 4%: $1.04\langle 2.22, 1.85, 3.25 \rangle$.

New total amount: $1.04(\mathbf{u} \cdot \mathbf{v}) = 1.04(17{,}139.05)$

$= \$17{,}824.61$

10. $\dfrac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{u}\|\,\|\mathbf{v}\|} = \cos\theta$

$$\mathbf{u} \cdot \mathbf{v} = (40)(25)\cos\frac{5\pi}{6} = -500\sqrt{3}$$

12. $\mathbf{u} = \langle 3, 1 \rangle, \mathbf{v} = \langle 2, -1 \rangle$

$$\cos\theta = \frac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{u}\|\,\|\mathbf{v}\|} = \frac{5}{\sqrt{10}\sqrt{5}} = \frac{1}{\sqrt{2}}$$

$$\theta = \frac{\pi}{4}$$

14.
$$\mathbf{u} = \cos\left(\frac{\pi}{6}\right)\mathbf{i} + \sin\left(\frac{\pi}{6}\right)\mathbf{j} = \frac{\sqrt{3}}{2}\mathbf{i} + \frac{1}{2}\mathbf{j}$$

$$\mathbf{v} = \cos\left(\frac{3\pi}{4}\right)\mathbf{i} + \sin\left(\frac{3\pi}{4}\right)\mathbf{j} = -\frac{\sqrt{2}}{2}\mathbf{i} + \frac{\sqrt{2}}{2}\mathbf{j}$$

$$\cos\theta = \frac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{u}\|\,\|\mathbf{v}\|}$$

$$= \frac{\sqrt{3}}{2}\left(-\frac{\sqrt{2}}{2}\right) + \frac{1}{2}\left(\frac{\sqrt{2}}{2}\right) = \frac{\sqrt{2}}{4}\left(1 - \sqrt{3}\right)$$

$$\theta = \arccos\left[\frac{\sqrt{2}}{4}\left(1 - \sqrt{3}\right)\right] = 105^\circ$$

16. $\mathbf{u} = 3\mathbf{i} + 2\mathbf{j} + \mathbf{k}, \mathbf{v} = 2\mathbf{i} - 3\mathbf{j}$

$$\cos\theta = \frac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{u}\|\,\|\mathbf{v}\|} = \frac{3(2) + 2(-3) + 0}{\|\mathbf{u}\|\,\|\mathbf{v}\|} = 0$$

$$\theta = \frac{\pi}{2}$$

18. $\mathbf{u} = 2\mathbf{i} - 3\mathbf{j} + \mathbf{k}, \mathbf{v} = \mathbf{i} - 2\mathbf{j} + \mathbf{k}$

$$\cos\theta = \frac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{u}\|\,\|\mathbf{v}\|}$$

$$= \frac{9}{\sqrt{14}\sqrt{6}} = \frac{9}{2\sqrt{21}} = \frac{3\sqrt{21}}{14}$$

$$\theta = \arccos\left(\frac{3\sqrt{21}}{14}\right) \approx 10.9^\circ$$

20. $\mathbf{u} = \langle 2, 18 \rangle, \mathbf{v} = \left\langle \frac{3}{2}, -\frac{1}{6} \right\rangle$

$\mathbf{u} \neq c\mathbf{v} \implies$ not parallel

$\mathbf{u} \cdot \mathbf{v} = 0 \implies$ orthogonal

22. $\mathbf{u} = -\frac{1}{3}(\mathbf{i} - 2\mathbf{j}), \mathbf{v} = 2\mathbf{i} - 4\mathbf{j}$

$\mathbf{u} = -\frac{1}{6}\mathbf{v} \implies$ parallel

24. $\mathbf{u} = -2\mathbf{i} + 3\mathbf{j} - \mathbf{k}, \mathbf{v} = 2\mathbf{i} + \mathbf{j} - \mathbf{k}$

$\mathbf{u} \neq c\mathbf{v} \implies$ not parallel

$\mathbf{u} \cdot \mathbf{v} = 0 \implies$ orthogonal

26. $\mathbf{u} = \langle \cos\theta, \sin\theta, -1 \rangle$,

$\mathbf{v} = \langle \sin\theta, -\cos\theta, 0 \rangle$

$\mathbf{u} \neq c\mathbf{v} \implies$ not parallel

$\mathbf{u} \cdot \mathbf{v} = 0 \implies$ orthogonal

28. $\mathbf{u} = \langle 5, 3, -1 \rangle \quad \|\mathbf{u}\| = \sqrt{35}$

$$\cos\alpha = \frac{5}{\sqrt{35}}$$

$$\cos\beta = \frac{3}{\sqrt{35}}$$

$$\cos\gamma = \frac{-1}{\sqrt{35}}$$

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = \frac{25}{35} + \frac{9}{35} + \frac{1}{35} = 1$$

30. $\mathbf{u} = \langle a, b, c \rangle, \|\mathbf{u}\| = \sqrt{a^2 + b^2 + c^2}$

$$\cos\alpha = \frac{a}{\sqrt{a^2 + b^2 + c^2}}$$

$$\cos\beta = \frac{b}{\sqrt{a^2 + b^2 + c^2}}$$

$$\cos\gamma = \frac{c}{\sqrt{a^2 + b^2 + c^2}}$$

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = \frac{a^2}{a^2 + b^2 + c^2} + \frac{b^2}{a^2 + b^2 + c^2} + \frac{c^2}{a^2 + b^2 + c^2} = 1$$

32. $\mathbf{u} = \langle -4, 3, 5 \rangle \quad \|\mathbf{u}\| = \sqrt{50} = 5\sqrt{2}$

$$\cos\alpha = \frac{-4}{5\sqrt{2}} \implies \alpha \approx 2.1721 \text{ or } 124.4°$$

$$\cos\beta = \frac{3}{5\sqrt{2}} \implies \beta \approx 1.1326 \text{ or } 64.9°$$

$$\cos\gamma = \frac{5}{5\sqrt{2}} = \frac{1}{\sqrt{2}} \implies \gamma \approx \frac{\pi}{4} \text{ or } 45°$$

34. $\mathbf{u} = \langle -2, 6, 1 \rangle \quad \|\mathbf{u}\| = \sqrt{41}$

$$\cos\alpha = \frac{-2}{\sqrt{41}} \implies \alpha \approx 1.8885 \text{ or } 108.2°$$

$$\cos\beta = \frac{6}{\sqrt{41}} \implies \alpha \approx 0.3567 \text{ or } 20.4°$$

$$\cos\gamma = \frac{1}{\sqrt{41}} \implies \alpha \approx 1.4140 \text{ or } 81.0°$$

36. $\mathbf{F}_1\text{:}\ C_1 = \dfrac{300}{\|\mathbf{F}_1\|} \approx 13.0931$

$\mathbf{F}_2\text{:}\ C_2 = \dfrac{100}{\|\mathbf{F}_2\|} \approx 6.3246$

$$\begin{aligned}\mathbf{F} &= \mathbf{F}_1 + \mathbf{F}_2 \\ &\approx 13.0931\langle -20, -10, 5\rangle + 6.3246\langle 5, 15, 0\rangle\rangle \\ &= \langle -230.239, -36.062, 65.4655\rangle\end{aligned}$$

$\|\mathbf{F}\| \approx 242.067$ lb

$$\cos\alpha \approx \frac{-230.239}{\|\mathbf{F}\|} \implies \alpha \approx 162.02°$$

$$\cos\beta \approx \frac{-36.062}{\|\mathbf{F}\|} \implies \beta \approx 98.57°$$

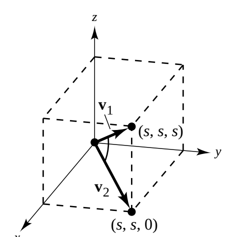
$$\cos\gamma \approx \frac{65.4655}{\|\mathbf{F}\|} \implies \gamma \approx 74.31°$$

38.

$$\begin{aligned}\mathbf{v}_1 &= \langle s, s, s\rangle \\ \|\mathbf{v}_1\| &= s\sqrt{3} \\ \mathbf{v}_2 &= \langle s, s, 0\rangle \\ \|\mathbf{v}_2\| &= s\sqrt{2}\end{aligned}$$

$$\cos\theta = \frac{s\sqrt{2}}{s\sqrt{3}} = \frac{\sqrt{6}}{3}$$

$$\theta = \arccos\frac{\sqrt{6}}{3} \approx 35.26°$$



40. $\mathbf{F}_1 = C_1(0, 10, 10).\ \|\mathbf{F}_1\| = 200 = C_1 10\sqrt{2} \implies C_1 = 10\sqrt{2}$

and $\mathbf{F}_1 = \left\langle 0, 100\sqrt{2}, 100\sqrt{2}\right\rangle$

$\mathbf{F}_2 = C_2\langle -4, -6, 10\rangle$

$\mathbf{F}_2 = C_3\langle 4, -6, 10\rangle$

$\mathbf{F} = \langle 0, 0, w\rangle$

$\mathbf{F} + \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 = 0$

$-4C_2 + 4C_3 = 0 \implies C_2 = C_3$

$$100\sqrt{2} - 6C_2 - 6C_3 = 0 \implies C_2 = C_3 = \frac{25\sqrt{2}}{3}N$$

42. $\mathbf{w}_2 = \mathbf{u} - \mathbf{w}_1 = \langle 9, 7\rangle - \langle 3, 9\rangle = \langle 6, -2\rangle$

44. $\mathbf{w}_2 = \mathbf{u} - \mathbf{w}_1 = \langle 8, 2, 0\rangle - \langle 6, 3, -3\rangle = \langle 2, -1, 3\rangle$

46. $\mathbf{u} = \langle 2, -3 \rangle, \mathbf{v} = \langle 3, 2 \rangle$

(a) $\mathbf{w}_1 = \left(\dfrac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{v}\|^2}\right)\mathbf{v} = 0\mathbf{v} = \langle 0, 0 \rangle$

(b) $\mathbf{w}_2 = \mathbf{u} - \mathbf{w}_1 = \langle 2, -3 \rangle$

48. $\mathbf{u} = \langle 1, 0, 4 \rangle, \mathbf{v} = \langle 3, 0, 2 \rangle$

(a) $\mathbf{w}_1 = \left(\dfrac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{v}\|^2}\right)\mathbf{v} = \dfrac{11}{13}\langle 3, 0, 2 \rangle = \left\langle \dfrac{33}{13}, 0, \dfrac{22}{13} \right\rangle$

(b) $\mathbf{w}_2 = \mathbf{u} - \mathbf{w}_1 = \langle 1, 0, 4 \rangle - \left\langle \dfrac{33}{13}, 0, \dfrac{22}{13} \right\rangle$

$$= \left\langle -\frac{20}{13}, 0, \frac{30}{13} \right\rangle$$

50. The vectors $\mathbf{u}$ and $\mathbf{v}$ are orthogonal if $\mathbf{u} \cdot \mathbf{v} = 0$.

The angle θ between $\mathbf{u}$ and $\mathbf{v}$ is given by

$$\cos\theta = \frac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{u}\|\,\|\mathbf{v}\|}.$$

52. (a) and (b) are defined.

54. See figure 10.29, page 739.

56. Yes, $\left\| \dfrac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{v}\|^2}\mathbf{v} \right\| = \left\| \dfrac{\mathbf{v} \cdot \mathbf{u}}{\|\mathbf{u}\|^2}\mathbf{u} \right\|$

$$|\mathbf{u} \cdot \mathbf{v}|\frac{\|\mathbf{v}\|}{\|\mathbf{v}\|^2} = |\mathbf{v} \cdot \mathbf{u}|\frac{\|\mathbf{u}\|}{\|\mathbf{u}\|^2}$$

$$\frac{1}{\|\mathbf{v}\|} = \frac{1}{\|\mathbf{u}\|}$$

$$\|\mathbf{u}\| = \|\mathbf{v}\|$$

58. (a) $\|\mathbf{u}\| = 5, \|\mathbf{v}\| \approx 8.602, \theta \approx 91.33°$

(b) $\|\mathbf{u}\| \approx 9.165, \|\mathbf{v}\| \approx 5.745, \theta = 90°$

60. (a) $\left\langle \dfrac{64}{17}, \dfrac{16}{17} \right\rangle$

(b) $\left\langle -\dfrac{21}{26}, \dfrac{63}{26}, \dfrac{42}{13} \right\rangle$

62. Because $\mathbf{u}$ appears to be a multiple of $\mathbf{v}$, the projection of $\mathbf{u}$ onto $\mathbf{v}$ is $\mathbf{u}$. Analytically,

$$\text{proj}_{\mathbf{v}}\mathbf{u} = \frac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{v}\|^2}\mathbf{v} = \frac{\langle -3, -2 \rangle \cdot \langle 6, 4 \rangle}{\langle 6, 4 \rangle \cdot \langle 6, 4 \rangle}\langle 6, 4 \rangle$$

$$= \frac{-26}{52}\langle 6, 4 \rangle = \langle -3, -2 \rangle = \mathbf{u}.$$

64. $\mathbf{u} = -8\mathbf{i} + 3\mathbf{j}$. Want $\mathbf{u} \cdot \mathbf{v} = 0$.

$\mathbf{v} = 3\mathbf{i} + 8\mathbf{j}$ and $-\mathbf{v} = -3\mathbf{i} - 8\mathbf{j}$ are orthogonal to $\mathbf{u}$.

66. $\mathbf{u} = \langle 0, -3, 6 \rangle$. Want $\mathbf{u} \cdot \mathbf{v} = 0$.

$\mathbf{v} = \langle 0, 6, 3 \rangle$ and $-\mathbf{v} = \langle 0, -6, -3 \rangle$ are orthogonal to $\mathbf{u}$.

68. $\overrightarrow{OA} = \langle 10, 5, 20 \rangle, \mathbf{v} = \langle 0, 0, 1 \rangle$

$$\text{proj}_{\mathbf{v}}\overrightarrow{OA} = \frac{20}{1^2}\langle 0, 0, 1 \rangle = \langle 0, 0, 20 \rangle$$

$\|\text{proj}_{\mathbf{v}}\overrightarrow{OA}\| = 20$

70. $\mathbf{F} = 25(\cos 20°\mathbf{i} + \sin 20°\mathbf{j})$

$\mathbf{v} = 50\mathbf{i}$

$W = \mathbf{F} \cdot \mathbf{v} = 1250 \cos 20° \approx 1174.6 \text{ ft} \cdot \text{lb}$

72. $\overrightarrow{PQ} = \langle -4, 2, 10 \rangle$

$\vec{V} = \langle -2, 3, 6 \rangle$

$W = \overrightarrow{PQ} \cdot \vec{V} = 74$

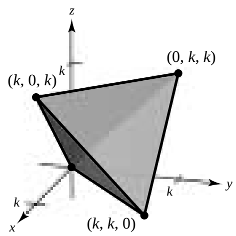
74. True

$$\mathbf{w} \cdot (\mathbf{u} + \mathbf{v}) = \mathbf{w} \cdot \mathbf{u} + \mathbf{w} \cdot \mathbf{v}$$

$$= 0 + 0 = 0 \implies \mathbf{w}$$

and $\mathbf{u} + \mathbf{v}$ are orthogonal.

76. (a)



(b) Length of each edge:

$$\sqrt{k^2 + k^2 + 0^2} = k\sqrt{2}$$

(c) $\cos\theta = \dfrac{k^2}{(k\sqrt{2})(k\sqrt{2})} = \dfrac{1}{2}$

$$\theta = \arccos\left(\frac{1}{2}\right) = 60°$$

(d) $\vec{r_1} = \langle k, k, 0\rangle - \left\langle \dfrac{k}{2}, \dfrac{k}{2}, \dfrac{k}{2}\right\rangle = \left\langle \dfrac{k}{2}, \dfrac{k}{2}, -\dfrac{k}{2}\right\rangle$

$$\vec{r_2} = \langle 0, 0, 0\rangle - \left\langle \frac{k}{2}, \frac{k}{2}, \frac{k}{2}\right\rangle = \left\langle -\frac{k}{2}, -\frac{k}{2}, -\frac{k}{2}\right\rangle$$

$$\cos\theta = \frac{-\dfrac{k^2}{4}}{\left(\dfrac{k}{2}\right)^2 \cdot 3} = -\frac{1}{3}$$

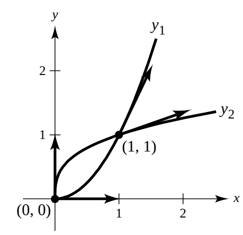
$$\theta = 109.5°$$

78. The curves $y_1 = x^2$ and $y_2 = x^{1/3}$ intersect at $(0, 0)$ and at $(1, 1)$.

At $(0, 0)$: $\langle 1, 0\rangle$ is tangent to y_1 and $\langle 0, 1\rangle$ is tangent to y_2. The angle between these vectors is 90°.

At $(1, 1)$: $(1/\sqrt{5})\langle 1, 2\rangle$ is tangent to y_1 and $(3/\sqrt{10})\langle 1, 1/3\rangle = (1/\sqrt{10})\langle 3, 1\rangle$ is tangent to y_2. To find the angle between these vectors,

$$\cos\theta = \frac{1}{\sqrt{5}}\frac{1}{\sqrt{10}}(3 + 2) = \frac{1}{\sqrt{2}} \implies \theta = 45°.$$



80.

$$\mathbf{u} \cdot \mathbf{v} = \|\mathbf{u}\|\,\|\mathbf{v}\| \cos\theta$$

$$|\mathbf{u} \cdot \mathbf{v}| = \big|\|\mathbf{u}\|\,\|\mathbf{v}\| \cos\theta\big|$$

$$= \|\mathbf{u}\|\,\|\mathbf{v}\|\,|\cos\theta|$$

$$\leq \|\mathbf{u}\|\,\|\mathbf{v}\| \text{ since } |\cos\theta| \leq 1.$$

82. Let $\mathbf{w}_1 = \text{proj}_{\mathbf{v}}\mathbf{u}$, as indicated in the figure. Because $\mathbf{w}_1$ is a scalar multiple of $\mathbf{v}$, you can write

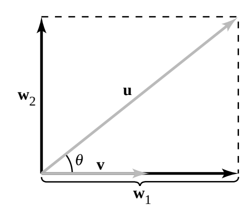
$$\mathbf{u} = \mathbf{w}_1 + \mathbf{w}_2 = c\mathbf{v} + \mathbf{w}_2.$$

Taking the dot product of both sides with $\mathbf{v}$ produces

$$\mathbf{u} \cdot \mathbf{v} = (c\mathbf{v} + \mathbf{w}_2) \cdot \mathbf{v} = c\mathbf{v} \cdot \mathbf{v} + \mathbf{w}_2 \cdot \mathbf{v}$$

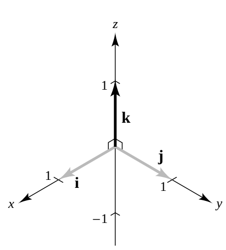
$$= c\|\mathbf{v}\|^2, \text{ since } \mathbf{w}_2 \text{ and } \mathbf{v} \text{ are orthogonol.}$$

Thus, $\mathbf{u} \cdot \mathbf{v} = c\|\mathbf{v}\|^2 \implies c = \dfrac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{v}\|^2}$ and $\mathbf{w}_1 = \text{proj}_{\mathbf{v}}\mathbf{u} = c\mathbf{v} = \dfrac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{v}\|^2}\mathbf{v}$.

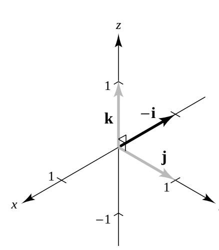


Section 10.4 The Cross Product of Two Vectors in Space

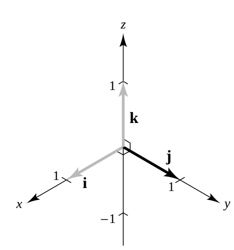
2. $\mathbf{i} \times \mathbf{j} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{vmatrix} = \mathbf{k}$



4. $\mathbf{k} \times \mathbf{j} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{vmatrix} = -\mathbf{i}$



6. $\mathbf{k} \times \mathbf{i} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{vmatrix} = \mathbf{j}$



8. (a) $\mathbf{u}\times\mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & 0 & 5 \\ 2 & 3 & -2 \end{vmatrix} = \langle -15, 16, 9\rangle$

(b) $\mathbf{v}\times\mathbf{u} = -(\mathbf{u}\times\mathbf{v}) = \langle 15, -16, -9\rangle$

(c) $\mathbf{v}\times\mathbf{v} = \mathbf{0}$

10. (a) $\mathbf{u}\times\mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & -2 & -2 \\ 1 & 5 & 1 \end{vmatrix} = \langle 8, -5, 17\rangle$

(b) $\mathbf{v}\times\mathbf{u} = -(\mathbf{u}\times\mathbf{v}) = \langle -8, 5, -17\rangle$

(c) $\mathbf{v}\times\mathbf{v} = \mathbf{0}$

12. $\mathbf{u} = \langle -1, 1, 2\rangle, \mathbf{v} = \langle 0, 1, 0\rangle$

$\mathbf{u}\times\mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -1 & 1 & 2 \\ 0 & 1 & 0 \end{vmatrix} = -2\mathbf{i} - \mathbf{k} = \langle -2, 0, -1\rangle$

$\mathbf{u}\cdot(\mathbf{u}\times\mathbf{v}) = (-1)(-2) + (1)(0) + (2)(-1)$

$= 0 \implies \mathbf{u}\perp\mathbf{u}\times\mathbf{v}$

$\mathbf{v}\cdot(\mathbf{u}\times\mathbf{v}) = (0)(-2) + (1)(0) + (0)(-1)$

$= 0 \implies \mathbf{v}\perp\mathbf{u}\times\mathbf{v}$

14. $\mathbf{u} = \langle -10, 0, 6\rangle, \mathbf{v} = \langle 7, 0, 0\rangle$

$\mathbf{u}\times\mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -10 & 0 & 6 \\ 7 & 0 & 0 \end{vmatrix} = 42\mathbf{j} = \langle 0, 42, 0\rangle$

$\mathbf{u}\cdot(\mathbf{u}\times\mathbf{v}) = (-10)(0) + (0)(42) + 6(0)$

$= 0 \implies \mathbf{u}\perp\mathbf{u}\times\mathbf{v}$

$\mathbf{v}\cdot(\mathbf{u}\times\mathbf{v}) = 7(0) + (0)(42) + (0)(0)$

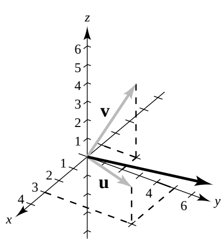
$= 0 \implies \mathbf{v}\perp\mathbf{u}\times\mathbf{v}$

16. $\mathbf{u}\times\mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 6 & 0 \\ -2 & 1 & 1 \end{vmatrix} = 6\mathbf{i} - \mathbf{j} + 13\mathbf{k}$

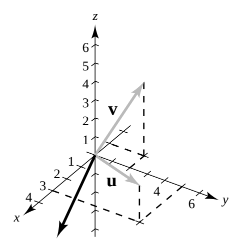
$\mathbf{u}\cdot(\mathbf{u}\times\mathbf{v}) = 1(6) + 6(-1) = 0 \implies \mathbf{u}\perp(\mathbf{u}\times\mathbf{v})$

$\mathbf{v}\cdot(\mathbf{u}\times\mathbf{v}) = -2(6) + 1(-1) + 1(13) = 0 \implies \mathbf{v}\perp(\mathbf{u}\times\mathbf{v})$

18.



20.



22. $\mathbf{u} = \langle -8, -6, 4\rangle$

$\mathbf{v} = \langle 10, -12, -2\rangle$

$\mathbf{u}\times\mathbf{v} = \langle 60, 24, 156\rangle$

$\dfrac{\mathbf{u}\times\mathbf{v}}{\|\mathbf{u}\times\mathbf{v}\|} = \dfrac{1}{36\sqrt{22}}\langle 60, 24, 156\rangle$

$= \left\langle \dfrac{5}{3\sqrt{22}}, \dfrac{2}{3\sqrt{22}}, \dfrac{13}{3\sqrt{22}}\right\rangle$

24. $\mathbf{u} = \dfrac{2}{3}\mathbf{k}$

$\mathbf{v} = \dfrac{1}{2}\mathbf{i} + 6\mathbf{k}$

$\mathbf{u}\times\mathbf{v} = \left\langle 0, \dfrac{1}{3}, 0\right\rangle$

$\dfrac{\mathbf{u}\times\mathbf{v}}{\|\mathbf{u}\times\mathbf{v}\|} = \langle 0, 1, 0\rangle$

26. (a) $\mathbf{u}\times\mathbf{v} = \langle -18, -12, 48\rangle$

$\|\mathbf{u}\times\mathbf{v}\| \approx 52.650$

(b) $\mathbf{u}\times\mathbf{v} = \langle -50, 40, -34\rangle$

$\|\mathbf{u}\times\mathbf{v}\| \approx 72.498$

28. $\mathbf{u} = \mathbf{i} + \mathbf{j} + \mathbf{k}$

$\mathbf{v} = \mathbf{j} + \mathbf{k}$

$\mathbf{u}\times\mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{vmatrix} = -\mathbf{j} + \mathbf{k}$

$A = \|\mathbf{u}\times\mathbf{v}\| = \|-\mathbf{j} + \mathbf{k}\| = \sqrt{2}$

30. $\mathbf{u} = \langle 2, -1, 0\rangle$

$\mathbf{v} = \langle -1, 2, 0\rangle$

$$\mathbf{u}\times\mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & -1 & 0 \\ -1 & 2 & 0 \end{vmatrix} = \langle 0, 0, 3\rangle$$

$A = \|\mathbf{u}\times\mathbf{v}\| = \|\langle 0, 0, 3\rangle\| = 3$

32. $A(2, -3, 1), B(6, 5, -1), C(3, -6, 4), D(7, 2, 2)$

$\overrightarrow{AB} = \langle 4, 8, -2\rangle, \overrightarrow{AC} = \langle 1, -3, 3\rangle, \overrightarrow{CD} = \langle 4, 8, -2\rangle, \overrightarrow{BD} = \langle 1, -3, 3\rangle$

Since $\overrightarrow{AB} = \overrightarrow{CD}$ and $\overrightarrow{AC} = \overrightarrow{BD}$, the figure is a parallelogram.

$\overrightarrow{AB}$ and $\overrightarrow{AC}$ are adjacent sides and

$$\overrightarrow{AB}\times\overrightarrow{AC} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 4 & 8 & -2 \\ 1 & -3 & 3 \end{vmatrix} = \langle 18, -14, -20\rangle.$$

$\text{Area} = \|\overrightarrow{AB}\times\overrightarrow{AC}\| = \sqrt{920} = 2\sqrt{230}$

34. $A(2, -3, 4), B(0, 1, 2), C(-1, 2, 0)$

$\overrightarrow{AB} = \langle -2, 4, -2\rangle, \overrightarrow{AC} = \langle -3, 5, -4\rangle$

$$\overrightarrow{AB}\times\overrightarrow{AC} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -2 & 4 & -2 \\ -3 & 5 & -4 \end{vmatrix} = -6\mathbf{i} - 2\mathbf{j} + 2\mathbf{k}$$

$A = \frac{1}{2}\|\overrightarrow{AB}\times\overrightarrow{AC}\| = \frac{1}{2}\sqrt{44} = \sqrt{11}$

36. $A(1, 2, 0), B(-2, 1, 0), C(0, 0, 0)$

$\overrightarrow{AB} = \langle -3, -1, 0\rangle, \overrightarrow{AC} = \langle -1, -2, 0\rangle$

$$\overrightarrow{AB}\times\overrightarrow{AC} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -3 & -1 & 0 \\ -1 & -2 & 0 \end{vmatrix} = 5\mathbf{k}$$

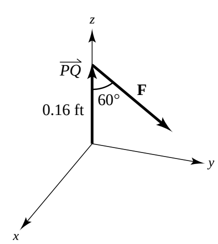
$A = \frac{1}{2}\|\overrightarrow{AB}\times\overrightarrow{AC}\| = \frac{5}{2}$

38. $\mathbf{F} = -2000(\cos 30^\circ\mathbf{j} + \sin 30^\circ\mathbf{k}) = -1000\sqrt{3}\mathbf{j} - 1000\mathbf{k}$

$\overrightarrow{PQ} = 0.16\mathbf{k}$

$$\overrightarrow{PQ}\times\mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 0.16 \\ 0 & -1000\sqrt{3} & -1000 \end{vmatrix} = 160\sqrt{3}\mathbf{i}$$

$\|\overrightarrow{PQ}\times\mathbf{F}\| = 160\sqrt{3}\text{ ft}\cdot\text{lb}$



40. (a) B is $-\frac{15}{12} = -\frac{5}{4}$ to the left of A, and one foot upwards:

$\overrightarrow{AB} = \frac{-5}{4}\mathbf{j} + \mathbf{k}$

$\mathbf{F} = -200(\cos\theta\mathbf{j} + \sin\theta\mathbf{k})$

(b) $$\overrightarrow{AB}\times\mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & -5/4 & 1 \\ 0 & -200\cos\theta & -200\sin\theta \end{vmatrix}$$

$$= (250\sin\theta + 200\cos\theta)\mathbf{i}$$

$$\|\overrightarrow{AB}\times\mathbf{F}\| = |250\sin\theta + 200\cos\theta|$$

$$= 25(10\sin\theta + 8\cos\theta)$$

(c) For $\theta = 30^\circ$,

$$\|\overrightarrow{AB}\times\mathbf{F}\| = 25\left(10\left(\frac{1}{2}\right) + 8\left(\frac{\sqrt{3}}{2}\right)\right)$$

$$= 25\left(5 + 4\sqrt{3}\right) \approx 298.2.$$

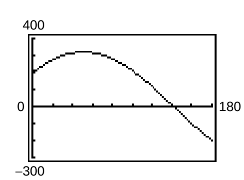
(d) If $T = \|\overrightarrow{AB}\times\mathbf{F}\|$,

$$\frac{dT}{d\theta} = 25(10\cos\theta - 8\sin\theta) = 0 \implies \tan\theta = \frac{5}{4}$$

$$\implies \theta \approx 51.34^\circ.$$

The vectors are orthogonal.

(e) The zero is $\theta \approx 141.34^\circ$, the angle making $\overrightarrow{AB}$ parallel to $\mathbf{F}$.



42. $\mathbf{u} \cdot (\mathbf{v} \times \mathbf{w}) = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} = -1$

44. $\mathbf{u} \cdot (\mathbf{v} \times \mathbf{w}) = \begin{vmatrix} 2 & 0 & 0 \\ 1 & 1 & 1 \\ 0 & 2 & 2 \end{vmatrix} = 0$

46. $\mathbf{u} \cdot (\mathbf{v} \times \mathbf{w}) = \begin{vmatrix} 1 & 3 & 1 \\ 0 & 6 & 6 \\ -4 & 0 & -4 \end{vmatrix} = -72$

$V = |\mathbf{u} \cdot (\mathbf{v} \times \mathbf{w})| = 72$

48. $\mathbf{u} = \langle 1, 1, 0 \rangle$

$\mathbf{v} = \langle 1, 0, 2 \rangle$

$\mathbf{w} = \langle 0, 1, 1 \rangle$

$\mathbf{u} \cdot (\mathbf{v} \times \mathbf{w}) = \begin{vmatrix} 1 & 1 & 0 \\ 1 & 0 & 2 \\ 0 & 1 & 1 \end{vmatrix} = -3$

$V = |\mathbf{u} \cdot (\mathbf{v} \times \mathbf{w})| = 3$

50. See Theorem 10.8, page 746.

52. Form the vectors for two sides of the triangle, and compute their cross product:

$$\langle x_2 - x_1, y_2 - y_1, z_2 - z_1 \rangle \times \langle x_3 - x_1, y_3 - y_1, z_3 - z_1 \rangle$$

54. False, let $\mathbf{u} = \langle 1, 0, 0 \rangle$, $\mathbf{v} = \langle 1, 0, 0 \rangle$, $\mathbf{w} = \langle -1, 0, 0 \rangle$.

Then,

$\mathbf{u} \times \mathbf{v} = \mathbf{u} \times \mathbf{w} = \mathbf{0}$, but $\mathbf{v} \neq \mathbf{w}$.

56. $\mathbf{u} = \langle u_1, u_2, u_3 \rangle$, $\mathbf{v} = \langle v_1, v_2, v_3 \rangle$, $\mathbf{w} = \langle w_1, w_2, w_3 \rangle$

$\mathbf{u} = u_1\mathbf{i} + u_2\mathbf{j} + u_3\mathbf{k}$

$\mathbf{v} \times \mathbf{w} = (v_2w_3 - v_3w_2)\mathbf{i} - (v_1w_3 - v_3w_1)\mathbf{j} + (v_1w_2 - v_2w_1)\mathbf{k}$

$\mathbf{u} \cdot (\mathbf{v} + \mathbf{w}) = u_1(v_2w_3 - v_3w_2) - u_2(v_1w_3 - v_3w_1) + u_3(v_1w_2 - v_2w_1) = \begin{vmatrix} u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \\ w_1 & w_2 & w_3 \end{vmatrix}$

58. $\mathbf{u} = \langle u_1, u_2, u_3 \rangle$, $\mathbf{v} = \langle v_1, v_2, v_3 \rangle$, c is a scalar.

$$\begin{aligned} (c\mathbf{u}) \times \mathbf{v} &= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ cu_1 & cu_2 & cu_3 \\ v_1 & v_2 & v_3 \end{vmatrix} \\ &= (cu_2v_3 - cu_3v_2)\mathbf{i} - (cu_1v_3 - cu_3v_1)\mathbf{j} + (cu_1v_2 - cu_2v_1)\mathbf{k} \\ &= c[(u_2v_3 - u_3v_2)\mathbf{i} - (u_1v_3 - u_3v_1)\mathbf{j} + (u_1v_2 - u_2v_1)\mathbf{k}] = c(\mathbf{u} \times \mathbf{v}) \end{aligned}$$

60. $\mathbf{u} \cdot (\mathbf{v} \times \mathbf{w}) = \begin{vmatrix} u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \\ w_1 & w_2 & w_3 \end{vmatrix}$

$(\mathbf{u} \times \mathbf{v}) \cdot \mathbf{w} = \mathbf{w} \cdot (\mathbf{u} \times \mathbf{v}) = \begin{vmatrix} w_1 & w_2 & w_3 \\ u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \end{vmatrix}$

$= w_1(u_2v_3 - v_2u_3) - w_2(u_1v_3 - v_1u_3) + w_3(u_1v_2 - v_1u_2)$

$= u_1(v_2w_3 - w_2v_3) - u_2(v_1w_3 - w_1v_3) + u_3(v_1w_2 - w_1v_2)$

$= \mathbf{u} \cdot (\mathbf{v} \times \mathbf{w})$

62. If $\mathbf{u}$ and $\mathbf{v}$ are scalar multiples of each other, $\mathbf{u} = c\mathbf{v}$ for some scalar c.

$$\mathbf{u} \times \mathbf{v} = (c\mathbf{v}) \times \mathbf{v} = c(\mathbf{v} \times \mathbf{v}) = c(\mathbf{0}) = \mathbf{0}$$

If $\mathbf{u} \times \mathbf{v} = \mathbf{0}$, then $\|\mathbf{u}\|\,\|\mathbf{v}\| \sin\theta = 0$. (Assume $\mathbf{u} \neq \mathbf{0}$, $\mathbf{v} \neq \mathbf{0}$.) Thus, $\sin\theta = 0$, $\theta = 0$, and $\mathbf{u}$ and $\mathbf{v}$ are parallel. Therefore,

$\mathbf{u} = c\mathbf{v}$ for some scalar c.

64. $\mathbf{u} = \langle a_1, b_1, c_1\rangle$, $\mathbf{v} = \langle a_2, b_2, c_2\rangle$, $\mathbf{w} = \langle a_3, b_3, c_3\rangle$

$$\mathbf{v} \times \mathbf{w} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = (b_2c_3 - b_3c_2)\mathbf{i} - (a_2c_3 - a_3c_2)\mathbf{j} + (a_2b_3 - a_3b_2)\mathbf{k}$$

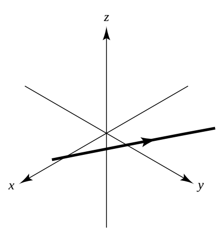
$$\mathbf{u} \times (\mathbf{v} \times \mathbf{w}) = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ a_1 & b_1 & c_1 \\ (b_2c_3 - b_3c_2) & (a_3c_2 - a_2c_3) & (a_2b_3 - a_3b_2) \end{vmatrix}$$

$$\begin{aligned}\mathbf{u} \times (\mathbf{v} \times \mathbf{w}) &= [b_1(a_2b_3 - a_3b_2) - c_1(a_3c_2 - a_2c_3)]\mathbf{i} - [a_1(a_2b_3 - a_3b_2) - c_1(b_2c_3 - b_3c_2)]\mathbf{j} + \\ &\quad [a_1(a_3c_2 - a_2c_3) - b_1(b_2c_3 - b_3c_2)]\mathbf{k} \\ &= [a_2(a_1a_3 + b_1b_3 + c_1c_3) - a_3(a_1a_2 + b_1b_2 + c_1c_2)]\mathbf{i} + \\ &\quad [b_2(a_1a_3 + b_1b_3 + c_1c_3) - b_3(a_1a_2 + b_1b_2 + c_1c_2)]\mathbf{j} + \\ &\quad [c_2(a_1a_3 + b_1b_3 + c_1c_3) - c_3(a_1a_2 + b_1b_2 + c_1c_2)]\mathbf{k} \\ &= (a_1a_3 + b_1b_3 + c_1c_3)\langle a_2, b_2, c_2\rangle - (a_1a_2 + b_1b_2 + c_1c_2)\langle a_3, b_3, c_3\rangle \\ &= (\mathbf{u}\cdot\mathbf{w})\mathbf{v} - (\mathbf{u}\cdot\mathbf{v})\mathbf{w}\end{aligned}$$

Section 10.5 Lines and Planes in Space

2. $x = 2 - 3t$, $y = 2$, $z = 1 - t$

(a)



(b) When $t = 0$ we have $P = (2, 2, 1)$. When $t = 2$ we have $Q = (-4, 2, -1)$.

$\overrightarrow{PQ} = \langle -6, 0, -2\rangle$

The components of the vector and the coefficients of t are proportional since the line is parallel to $\overrightarrow{PQ}$.

(c) $z = 0$ when $t = 1$. Thus, $x = -1$ and $y = 2$.

Point: $(-1, 2, 0)$

$x = 0$ when $t = \frac{2}{3}$. Point: $\left(0, 2, \frac{1}{3}\right)$

4. Point: $(0, 0, 0)$

Direction vector: $\mathbf{v} = \left\langle -2, \frac{5}{2}, 1\right\rangle$

Direction numbers: $-4, 5, 2$

(a) Parametric: $x = -4t$, $y = 5t$, $z = 2t$

(b) Symmetric: $\frac{x}{-4} = \frac{y}{5} = \frac{z}{2}$

6. Point: $(-3, 0, 2)$

Direction vector: $\mathbf{v} = \langle 0, 6, 3\rangle$

Direction numbers: $0, 2, 1$

(a) Parametric: $x = -3$, $y = 2t$, $z = 2 + t$

(b) Symmetric: $\frac{y}{2} = z - 2$, $x = -3$

8. Point: $(-3, 5, 4)$

Directions numbers: $3, -2, 1$

(a) Parametric: $x = -3 + 3t, y = 5 - 2t, z = 4 + t$

(b) Symmetric: $\frac{x+3}{3} = \frac{y-5}{-2} = z - 4$

10. Points: $(2, 0, 2), (1, 4, -3)$

Direction vector: $\langle 1, -4, 5 \rangle$

Direction numbers: $1, -4, 5$

(a) Parametric: $x = 2 + t, y = -4t, z = 2 + 5t$

(b) Symmetric: $x - 2 = \frac{y}{-4} = \frac{z-2}{5}$

12. Points: $(0, 0, 25), (10, 10, 0)$

Direction vector: $\langle 10, 10, -25 \rangle$

Direction numbers: $2, 2, -5$

(a) Parametric: $x = 2t, y = 2t, z = 25 - 5t$

(b) Symmetric: $\frac{x}{2} = \frac{y}{2} = \frac{z-25}{-5}$

14. Point: $(2, 3, 4)$

Direction vector: $\mathbf{v} = 3\mathbf{i} + 2\mathbf{j} - \mathbf{k}$

Direction numbers: $3, 2, -1$

Parametric: $x = 2 + 3t, y = 3 + 2t, z = 4 - t$

16. Points: $(2, 0, -3), (4, 2, -2)$

Direction vector: $\mathbf{v} = 2\mathbf{i} + 2\mathbf{j} + \mathbf{k}$

Direction numbers: $2, 2, 1$

Parametric: $x = 2 + 2t, y = 2t, z = -3 + t$

Symmetric: $\frac{x-2}{2} = \frac{y}{2} = \frac{z+3}{1}$

(a) Not on line $\left(1 \neq \frac{1}{2} \neq 1\right)$

(b) On line

(c) Not on line $\left(\frac{-3}{2} = \frac{-3}{2} \neq -1\right)$

18. L_1: $\mathbf{v} = \langle 4, -2, 3 \rangle$ $(8, -5, -9)$ on line

L_2: $\mathbf{v} = \langle 2, 1, 5 \rangle$

L_3: $\mathbf{v} = \langle -8, 4, -6 \rangle$ $(8, -5, -9)$ on line

L_4: $\mathbf{v} = \langle -2, 1, 1.5 \rangle$

L_1 and L_2 are identical.

20. By equating like variables, we have

(i) $-3t + 1 = 3s + 1$, (ii) $4t + 1 = 2s + 4$, and (iii) $2t + 4 = -s + 1$.

From (i) we have $s = -t$, and consequently from (ii), $t = \frac{1}{2}$ and from (iii), $t = -3$. The lines do not intersect.

22. Writing the equations of the lines in parametric form we have

$x = 2 - 3t \qquad y = 2 + 6t \qquad z = 3 + t$

$x = 3 + 2s \qquad y = -5 + s \qquad z = -2 + 4s.$

By equating like variables, we have $2 - 3t = 3 + 2s,\ 2 + 6t = -5 + s,\ 3 + t = -2 + 4s$. Thus, $t = -1$, $s = 1$ and the point of intersection is $(5, -4, 2)$.

$\mathbf{u} = \langle -3, 6, 1 \rangle$ (First line)

$\mathbf{v} = \langle 2, 1, 4 \rangle$ (Second line)

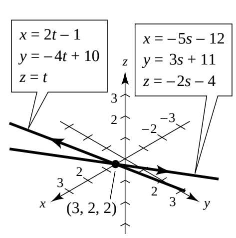
$$\cos\theta = \frac{|\mathbf{u} \cdot \mathbf{v}|}{\|\mathbf{u}\|\,\|\mathbf{v}\|} = \frac{4}{\sqrt{46}\sqrt{21}} = \frac{4}{\sqrt{966}} = \frac{2\sqrt{966}}{483}$$

24. $x = 2t - 1 \qquad x = -5s - 12$

$y = -4t + 10 \qquad y = 3s + 11$

$z = t \qquad z = -2s - 4$

Point of intersection: $(3, 2, 2)$



26. $2x + 3y + 4z = 4$

$P = (0, 0, 1), Q = (2, 0, 0), R = (3, 2, -2)$

(a) $\overrightarrow{PQ} = \langle 2, 0, -1\rangle, \overrightarrow{PR} = \langle 3, 2, -3\rangle$

(b) $\overrightarrow{PQ} \times \overrightarrow{PR} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & 0 & -1 \\ 3 & 2 & -3 \end{vmatrix} = \langle 2, 3, 4\rangle$

The components of the cross product are proportional (for this choice of P, Q, and R, they are the same) to the coefficients of the variables in the equation. The cross product is parallel to the normal vector.

28. Point: $(1, 0, -3)$

$\mathbf{n} = \mathbf{k} = \langle 0, 0, 1\rangle$

$0(x - 1) + 0(y - 0) + 1[z - (-3)] = 0$

$z + 3 = 0$

30. Point: $(0, 0, 0)$

Normal vector: $\mathbf{n} = -3\mathbf{i} + 2\mathbf{k}$

$-3(x - 0) + 0(y - 0) + 2(z - 0) = 0$

$-3x + 2z = 0$

32. Point: $(3, 2, 2)$

Normal vector: $\mathbf{v} = 4\mathbf{i} + \mathbf{j} - 3\mathbf{k}$

$4(x - 3) + (y - 2) - 3(z - 2) = 0$

$4x + y - 3z = 8$

34. Let $\mathbf{u}$ be vector from $(2, 3, -2)$ to $(3, 4, 2)$: $\langle 1, 1, 4\rangle$.

Let $\mathbf{v}$ be vector from $(2, 3, -2)$ to $(1, -1, 0)$: $\langle -1, -4, 2\rangle$.

Normal vector: $\mathbf{u} \times \mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 1 & 4 \\ -1 & -4 & 2 \end{vmatrix} = \langle 18, -6, -3\rangle$

$= -3\langle -6, 2, 1\rangle$

$-6(x - 2) + 2(y - 3) + 1(z + 2) = 0$

$-6x + 2y + z = -8$

36. $(1, 2, 3)$, Normal vector: $\mathbf{v} = \mathbf{i}$, $1(x - 1) = 0, x = 1$

38. The plane passes through the three points $(0, 0, 0)$, $(0, 1, 0)$ $(\sqrt{3}, 0, 1)$.

The vector from $(0, 0, 0)$ to $(0, 1, 0)$: $\mathbf{u} = \mathbf{j}$

The vector from $(0, 0, 0)$ to $(\sqrt{3}, 0, 1)$: $\mathbf{v} = \sqrt{3}\mathbf{i} + \mathbf{k}$

Normal vector: $\mathbf{u} \times \mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 1 & 0 \\ \sqrt{3} & 0 & 1 \end{vmatrix} = \mathbf{i} - \sqrt{3}\mathbf{k}$

$x - \sqrt{3}z = 0$

40. The direction of the line is $\mathbf{u} = 2\mathbf{i} - \mathbf{j} + \mathbf{k}$. Choose any point on the line, $[(0, 4, 0)$, for example$]$, and let $\mathbf{v}$ be the vector from $(0, 4, 0)$ to the given point $(2, 2, 1)$:

$\mathbf{v} = 2\mathbf{i} - 2\mathbf{j} + \mathbf{k}$

Normal vector: $\mathbf{u} \times \mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & -1 & 1 \\ 2 & -2 & 1 \end{vmatrix} = \mathbf{i} - 2\mathbf{k}$

$(x - 2) - 2(z - 1) = 0$

$x - 2z = 0$

42. Let $\mathbf{v}$ be the vector from $(3, 2, 1)$ to $(3, 1, -5)$:
$\mathbf{v} = -\mathbf{j} - 6\mathbf{k}$

Let $\mathbf{n}$ be the normal to the given plane: $\mathbf{n} = 6\mathbf{i} + 7\mathbf{j} + 2\mathbf{k}$

Since $\mathbf{v}$ and $\mathbf{n}$ both lie in the plane P, the normal vector to P is:

$\mathbf{v} \times \mathbf{n} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & -1 & -6 \\ 6 & 7 & 2 \end{vmatrix} = 40\mathbf{i} - 36\mathbf{j} + 6\mathbf{k}$

$= 2(20\mathbf{i} - 18\mathbf{j} + 3\mathbf{k})$

$20(x - 3) - 18(y - 2) + 3(z - 1) = 0$

$20x - 18y + 3z = 27$

44. Let $\mathbf{u} = \mathbf{k}$ and let $\mathbf{v}$ be the vector from $(4, 2, 1)$ to $(-3, 5, 7)$: $\mathbf{v} = -7\mathbf{i} + 3\mathbf{j} + 6\mathbf{k}$

Since $\mathbf{u}$ and $\mathbf{v}$ both lie in the plane P, the normal vector to P is:

$\mathbf{u} \times \mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 1 \\ -7 & 3 & 6 \end{vmatrix} = -3\mathbf{i} - 7\mathbf{j} = -(3\mathbf{i} + 7\mathbf{j})$

$3(x - 4) + 7(y - 2) = 0$

$3x + 7y = 26$

46. The normal vectors to the planes are $\mathbf{n}_1 = \langle 3, 1, -4\rangle$, $\mathbf{n}_2 = \langle -9, -3, 12\rangle$. Since $\mathbf{n}_2 = -3\mathbf{n}_1$, the planes are parallel, but not equal

48. The normal vectors to the planes are

$$\mathbf{n}_1 = 3\mathbf{i} + 2\mathbf{j} - \mathbf{k},\ \mathbf{n}_2 = \mathbf{i} - 4\mathbf{j} + 2\mathbf{k},$$

$$\cos\theta = \frac{|\mathbf{n}_1 \cdot \mathbf{n}_2|}{\|\mathbf{n}_1\|\,\|\mathbf{n}_2\|} = \frac{|3 - 8 - 2|}{\sqrt{14}\sqrt{21}} = \frac{1}{\sqrt{6}}.$$

Therefore, $\theta = \arccos\left(\frac{1}{\sqrt{6}}\right) \approx 65.9°$.

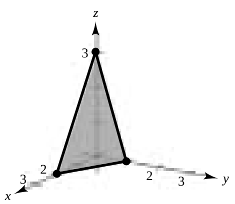
50. The normal vectors to the planes are

$$\mathbf{n}_1 = \langle 2, 0, -1\rangle, \mathbf{n}_2 = \langle 4, 1, 8\rangle,$$

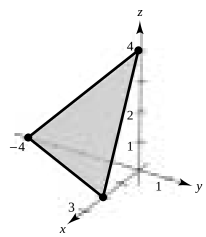
$$\cos\theta = \frac{|\mathbf{n}_1 \cdot \mathbf{n}_2|}{\|\mathbf{n}_1\|\,\|\mathbf{n}_2\|} = 0.$$

Thus, $\theta = \frac{\pi}{2}$ and the planes are orthogonal.

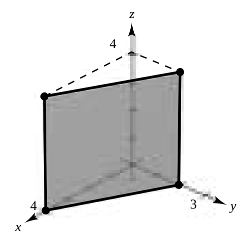
52. $3x + 6y + 2z = 6$



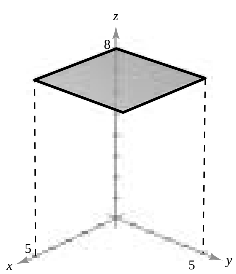
54. $2x - y + z = 4$



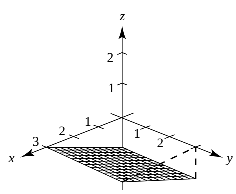
56. $x + 2y = 4$



58. $z = 8$



60. $x - 3z = 3$



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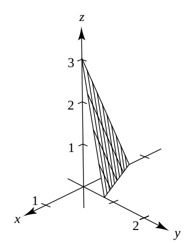
62. $2.1x - 4.7y - z + 3 = 0$



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64. P_1: $\mathbf{n} = \langle -60, 90, 30\rangle$ or $\langle -2, 3, 1\rangle$ $\quad \left(0, 0, \frac{9}{10}\right)$ on plane

P_2: $\mathbf{n} = \langle 6, -9, -3\rangle$ or $\langle -2, 3, 1\rangle$ $\quad \left(0, 0, -\frac{2}{3}\right)$ on plane

P_3: $\mathbf{n} = \langle -20, 30, 10\rangle$ or $\langle -2, 3, 1\rangle$ $\quad \left(0, 0, \frac{5}{6}\right)$ on plane

P_4: $\mathbf{n} = \langle 12, -18, 6\rangle$ or $\langle -2, 3, -1\rangle$

P_1, P_2, and P_3 are parallel.

66. If $c = 0$, $z = 0$ is xy-plane.

If $c \neq 0$, $cy + z = 0 \implies y = \frac{-1}{c}z$ is a plane parallel to x-axis and passing through the points $(0, 0, 0)$ and $(0, 1, -c)$.

68. The normals to the planes are $\mathbf{n}_1 = \langle 6, -3, 1\rangle$. and $\mathbf{n}_2 = \langle -1, 1, 5\rangle$.

The direction vector for the line is

$$\mathbf{n}_1 \times \mathbf{n}_2 = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 6 & -3 & 1 \\ -1 & 1 & 5 \end{vmatrix} = \langle -16, -31, 3\rangle.$$

Now find a point of intersection of the planes.

$$\begin{aligned} 6x - 3y + z &= 5 &\implies\quad 6x - 3y + z &= 5 \\ -x + y + 5z &= 5 &\implies\quad -6x + 6y + 30z &= 30 \\ & & 3y + 31z &= 35 \end{aligned}$$

Let $y = -9, z = 2 \implies x = -4 \implies (-4, -9, 2)$.

$$x = -4 - 16t, y = -9 - 31t, z = 2 + 3t$$

70. Writing the equation of the line in parametric form and substituting into the equation of the plane we have:

$$x = 1 + 4t,\ y = 2t,\ z = 3 + 6t$$

$$2(1 + 4t) + 3(2t) = -5,\ t = \frac{-1}{2}$$

Substituting $t = -\frac{1}{2}$ into the parametric equations for the line we have the point of intersection $(-1, -1, 0)$. The line does not lie in the plane.

72. Writing the equation of the line in parametric form and substituting into the equation of the plane we have:

$$x = 4 + 2t,\ y = -1 - 3t,\ z = -2 + 5t$$

$$5(4 + 2t) + 3(-1 - 3t) = 17,\ t = 0$$

Substituting $t = 0$ into the parametric equations for the line we have the point of intersection $(4, -1, -2)$. The line does not lie in the plane.

74. Point: $Q(0, 0, 0)$

Plane: $8x - 4y + z = 8$

Normal to plane: $\mathbf{n} = \langle 8, -4, 1\rangle$

Point in plane: $P\langle 1, 0, 0\rangle$

Vector: $\overrightarrow{PQ} = \langle -1, 0, 0\rangle$

$$D = \frac{|\overrightarrow{PQ} \cdot \mathbf{n}|}{\|\mathbf{n}\|} = \frac{|-8|}{\sqrt{81}} = \frac{8}{9}$$

76. Point: $Q(3, 2, 1)$

Plane: $x - y + 2z = 4$

Normal to plane: $\mathbf{n} = \langle 1, -1, 2\rangle$

Point in plane: $P\langle 4, 0, 0\rangle$

Vector: $\overrightarrow{PQ} = \langle -1, 2, 1\rangle$

$$D = \frac{|\overrightarrow{PQ} \cdot \mathbf{n}|}{\|\mathbf{n}\|} = \frac{|-1|}{\sqrt{6}} = \frac{1}{\sqrt{6}} = \frac{\sqrt{6}}{6}$$

78. The normal vectors to the planes are $\mathbf{n}_1 = \langle 4, -4, 9\rangle$ and $\mathbf{n}_2 = \langle 4, -4, 9\rangle$. Since $\mathbf{n}_1 = \mathbf{n}_2$, the planes are parallel. Choose a point in each plane.

$P = (-5,0, 3)$ is a point in $4x - 4y + 9z = 7$.

$Q = (0, 0, 2)$ is a point in $4x - 4y + 9z = 18$.

$$\overrightarrow{PQ} = \langle 5, 0, -1\rangle$$

$$D = \frac{|\overrightarrow{PQ} \cdot \mathbf{n}_1|}{\|\mathbf{n}_1\|} = \frac{11}{\sqrt{113}} = \frac{11\sqrt{113}}{113}$$

80. The normal vectors to the planes are $\mathbf{n}_1 = \langle 2, 0, -4\rangle$ and $\mathbf{n}_2 = \langle 2, 0, -4\rangle$. Since $\mathbf{n}_1 = \mathbf{n}_2$, the planes are parallel. Choose a point in each plane.

$P = (2, 0, 0)$ is a point in $2x - 4z = 4$. $Q = (5, 0, 0)$ is a point in $2x - 4z = 10$.

$$\overrightarrow{PQ} = \langle 3, 0, 0\rangle,\ D = \frac{|\overrightarrow{PQ} \cdot \mathbf{n}_1|}{\|\mathbf{n}_1\|} = \frac{6}{\sqrt{20}} = \frac{3\sqrt{5}}{5}$$

82. $\mathbf{u} = \langle 2, 1, 2\rangle$ is the direction vector for the line.

$P = \langle 0, -3, 2\rangle$ is a point on the line (let $t = 0$).

$$\overrightarrow{PQ} = \langle 1, 1, 2\rangle$$

$$\overrightarrow{PQ} \times \mathbf{u} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 1 & 2 \\ 2 & 1 & 2 \end{vmatrix} = \langle 0, 2, -1\rangle$$

$$D = \frac{\|\overrightarrow{PQ} \times \mathbf{u}\|}{\|\mathbf{u}\|} = \frac{\sqrt{5}}{\sqrt{9}} = \frac{\sqrt{5}}{3}$$

84. The equation of the plane containing $P(x_1, y_1, z_1)$ and having normal vector $\mathbf{n} = \langle a, b, c\rangle$ is

$$a(x - x_1) + b(y - y_1) + c(z - z_1) = 0.$$

You need $\mathbf{n}$ and P to find the equation.

86. $x = a$: plane parallel to yz-plane containing $(a, 0, 0)$

$y = b$: plane parallel to xz-plane containing $(0, b, 0)$

$z = c$: plane parallel to xy-plane containing $(0, 0, c)$

88. (a) $t\mathbf{v}$ represents a line parallel to $\mathbf{v}$.

(b) $\mathbf{u} + t\mathbf{v}$ represents a line through the terminal point of $\mathbf{u}$ parallel to $\mathbf{v}$.

(c) $s\mathbf{u} + t\mathbf{v}$ represent the plane containing $\mathbf{u}$ and $\mathbf{v}$.

90. On one side we have the points $(0, 0, 0)$, $(6, 0, 0)$, and $(-1, -1, 8)$.

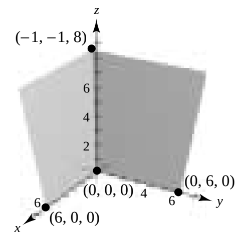
$$\mathbf{n}_1 = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 6 & 0 & 0 \\ -1 & -1 & 8 \end{vmatrix} = -48\mathbf{j} - 6\mathbf{k}$$

On the adjacent side we have the points $(0, 0, 0)$, $(0, 6, 0)$, and $(-1, -1, 8)$.

$$\mathbf{n}_2 = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 6 & 0 \\ -1 & -1 & 8 \end{vmatrix} = 48\mathbf{i} + 6\mathbf{k}$$

$$\cos\theta = \frac{|\mathbf{n}_1 \cdot \mathbf{n}_2|}{\|\mathbf{n}_1\|\,\|\mathbf{n}_2\|} = \frac{36}{2340} = \frac{1}{65}$$

$$\theta = \arccos\frac{1}{65} \approx 89.1°$$



92. False. They may be skew lines. (See Section Project)

Section 10.6 Surfaces in Space

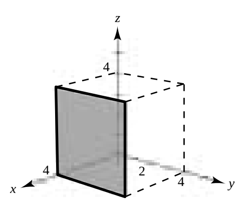
2. Hyperboloid of two sheets
Matches graph (e)

4. Elliptic cone
Matches graph (b)

6. Hyperbolic paraboloid
Matches graph (a)

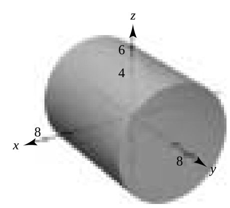
8. $x = 4$

Plane parallel to the yz-coordinate plane



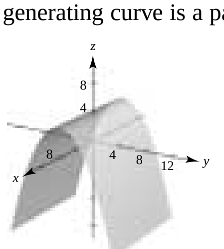
10. $x^2 + z^2 = 25$

The y-coordinate is missing so we have a cylindrical surface with rulings parallel to the y-axis. The generating curve is a circle.



12. $z = 4 - y^2$

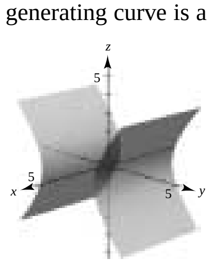
The x-coordinate is missing so we have a cylindrical surface with rulings parallel to the x-axis. The generating curve is a parabola.



14. $y^2 - z^2 = 4$

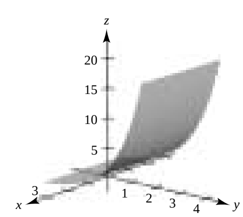
$$\frac{y^2}{4} - \frac{z^2}{4} = 1$$

The x-coordinate is missing so we have a cylindrical surface with rulings parallel to the x-axis. The generating curve is a hyperbola.



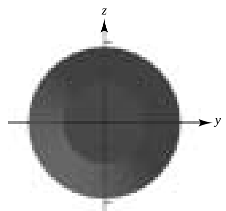
16. $z = e^y$

The x-coordinate is missing so we have a cylindrical surface with rulings parallel to the x-axis. The generating curve is the exponential curve.

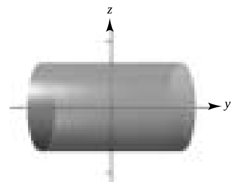


18. $y^2 + z^2 = 4$

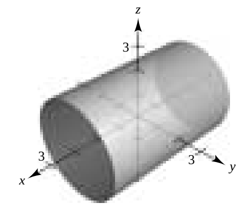
(a) From (10, 0, 0):



(b) From (0, 10, 0):



(c) From (10, 10, 10):



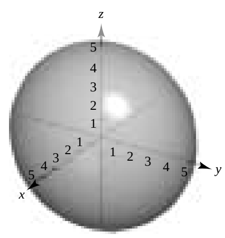
20. $\dfrac{x^2}{16} + \dfrac{y^2}{25} + \dfrac{z^2}{25} = 1$

Ellipsoid

xy-trace: $\dfrac{x^2}{16} + \dfrac{y^2}{25} = 1$ ellipse

xz-trace: $\dfrac{x^2}{16} + \dfrac{z^2}{25} = 1$ ellipse

yz-trace: $y^2 + z^2 = 25$ circle



22. $z^2 - x^2 - \dfrac{y^2}{4} = 1$

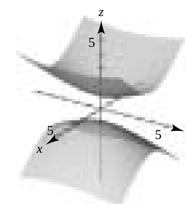
Hyperboloid of two sheets

xy-trace: none

xz-trace: $z^2 - x^2 = 1$ hyperbola

yz-trace: $z^2 - \dfrac{y^2}{4} = 1$ hyperbola

$z = \pm\sqrt{10}$: $\dfrac{x^2}{9} + \dfrac{y^2}{36} = 1$ ellipse



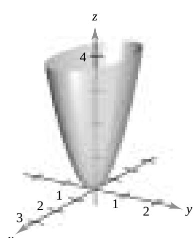
24. $z = x^2 + 4y^2$

Elliptic paraboloid

xy-trace: point (0, 0, 0)

xz-trace: $z = x^2$ parabola

yz-trace: $z = 4y^2$ parabola



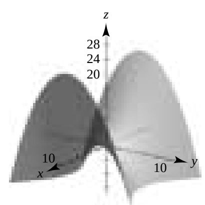
26. $3z = -y^2 + x^2$

Hyperbolic paraboloid

xy-trace: $y = \pm x$

xz-trace: $z = \frac{1}{3}x^2$

yz-trace: $z = -\frac{1}{3}y^2$



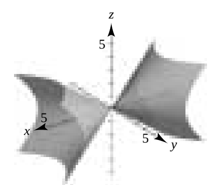
28. $x^2 = 2y^2 + 2z^2$

Elliptic Cone

xy-trace: $x = \pm\sqrt{2}y$

xz-trace: $x = \pm\sqrt{2}z$

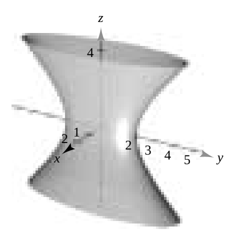
yz-trace: point: (0, 0, 0)



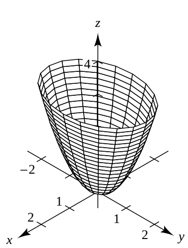
30.

$$9x^2 + y^2 - 9z^2 - 54x - 4y - 54z + 4 = 0$$
$$9(x^2 - 6x + 9) + (y^2 - 4y + 4) - 9(z^2 + 6z + 9) = 81 + 4 - 81$$
$$9(x - 3)^2 + (y - 2)^2 - 9(z + 3)^2 = 4$$
$$\frac{(x - 3)^2}{4/9} + \frac{(y - 2)^2}{4} - \frac{(z + 3)^2}{4/9} = 1$$

Hyperboloid of one sheet with center $(3, 2, -3)$.

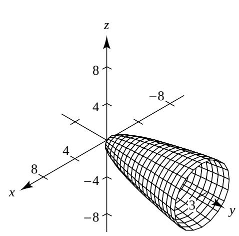


32. $z = x^2 + 0.5y^2$



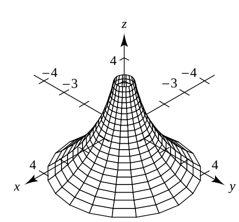
34. $z^2 = 4y - x^2$

$z = \pm\sqrt{4y - x^2}$

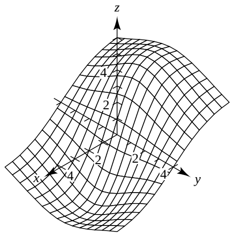


36. $x^2 + y^2 = e^{-z}$

$-\ln(x^2 + y^2) = z$

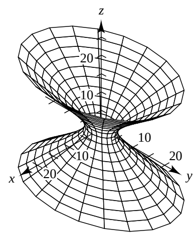


38. $z = \dfrac{-x}{8 + x^2 + y^2}$



40. $9x^2 + 4y^2 - 8z^2 = 72$

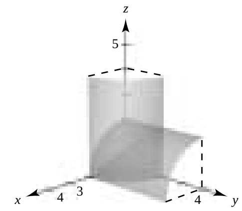
$z = \pm\sqrt{\dfrac{9}{8}x^2 + \dfrac{1}{2}y^2 - 9}$



42. $z = \sqrt{4 - x^2}$

$y = \sqrt{4 - x^2}$

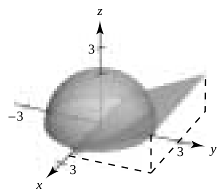
$x = 0, y = 0, z = 0$



44. $z = \sqrt{4 - x^2 - y^2}$

$y = 2z$

$z = 0$



46. $x^2 + z^2 = [r(y)]^2$ and $z = r(y) = 3y$; therefore,

$x^2 + z^2 = 9y^2.$

48. $y^2 + z^2 = [r(x)]^2$ and $z = r(x) = \dfrac{1}{2}\sqrt{4 - x^2}$; therefore,

$y^2 + z^2 = \dfrac{1}{4}(4 - x^2), x^2 + 4y^2 + 4z^2 = 4.$

50. $x^2 + y^2 = [r(z)]^2$ and $y = r(z) = e^z$; therefore,

$x^2 + y^2 = e^{2z}.$

52. $x^2 + z^2 = \cos^2 y$

Equation of generating curve:
$x = \cos y$ or $z = \cos y$

54. The trace of a surface is the intersection of the surface with a plane. You find a trace by setting one variable equal to a constant, such as $x = 0$ or $z = 2$.

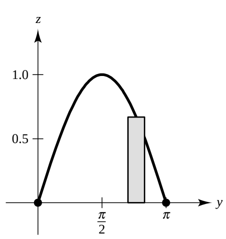
56. About x-axis: $y^2 + z^2 = [r(x)]^2$

About y-axis: $x^2 + z^2 = [r(y)]^2$

About z-axis: $x^2 + y^2 = [r(z)]^2$

58. $V = 2\pi\displaystyle\int_0^{\pi} y \sin y \, dy$

$= 2\pi\Big[\sin y - y\cos y\Big]_0^{\pi} = 2\pi^2$



60. $z = \dfrac{x^2}{2} + \dfrac{y^2}{4}$

(a) When $y = 4$ we have $z = \dfrac{x^2}{2} + 4, 4\left(\dfrac{1}{2}\right)(z - 4) = x^2.$

Focus: $\left(0, 4, \dfrac{9}{2}\right)$

(b) When $x = 2$ we have

$z = 2 + \dfrac{y^2}{4}, 4(z - 2) = y^2.$

Focus: $(2, 0, 3)$

62. If (x, y, z) is on the surface, then

$z^2 = x^2 + y^2 + (z - 4)^2$

$z^2 = x^2 + y^2 + z^2 - 8z + 16$

$8z = x^2 + y^2 + 16 \Longrightarrow z = \dfrac{x^2}{8} + \dfrac{y^2}{8} + 2$

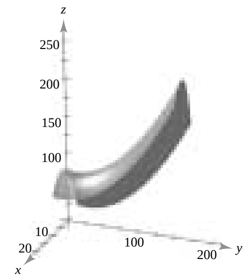
Elliptic paraboloid shifted up 2 units. Traces parallel to xy-plane are circles.

64. $z = -0.775x^2 + 0.007y^2 + 22.15x - 0.54y - 45.4$

(a)

Year	1980	1985	1990	1995	1996	1997
z	37.5	72.2	111.5	185.2	200.1	214.6
Model	37.8	72.0	112.2	185.8	204.5	214.7

(b)



(c) For y constant, the traces parallel to the xz-plane are concave downward. That is, for fixed y (public assistance), the rate of increase of z (Medicare) is decreasing with respect to x (worker's compensation).

(d) The traces parallel to the yz-plane (x constant) are concave upward. That is, for fixed x (worker's compensation), the rate of increase of z (Medicare) is increasing with respect to y (public assistance).

66. Equating twice the first equation with the second equation,

$$2x^2 + 6y^2 - 4z^2 + 4y - 8 = 2x^2 + 6y^2 - 4z^2 - 3x - 2$$
$$4y - 8 = -3x - 2$$
$$3x + 4y = 6, \text{ a plane}$$

Section 10.7 Cylindrical and Spherical Coordinates

2. $\left(4, \frac{\pi}{2}, -2\right)$, cylindrical

$x = 4 \cos \frac{\pi}{2} = 0$

$y = 4 \sin \frac{\pi}{2} = 4$

$z = -2$

$(0, 4, -2)$, rectangular

4. $\left(6, -\frac{\pi}{4}, 2\right)$, cylindrical

$x = 6 \cos\left(-\frac{\pi}{4}\right) = 3\sqrt{2}$

$y = 6 \sin\left(-\frac{\pi}{4}\right) = -3\sqrt{2}$

$z = 2$

$\left(3\sqrt{2}, -3\sqrt{2}, 2\right)$

6. $\left(1, \frac{3\pi}{2}, 1\right)$, cylindrical

$x = \cos \frac{3\pi}{2} = 0$

$y = \sin \frac{3\pi}{2} = -1$

$z = 1$

$(0, -1, 1)$, rectangular

8. $\left(2\sqrt{2}, -2\sqrt{2}, 4\right)$, rectangular

$r = \sqrt{\left(2\sqrt{2}\right)^2 + \left(-2\sqrt{2}\right)^2} = 4$

$\theta = \arctan(-1) = -\frac{\pi}{4}$

$z = 4$

$\left(4, -\frac{\pi}{4}, 4\right)$, cylindrical

10. $\left(2\sqrt{3}, -2, 6\right)$, rectangular

$r = \sqrt{12 + 4} = 4$

$\theta = \arctan\left(-\frac{1}{\sqrt{3}}\right) = \frac{5\pi}{6}$

$z = 1$

$\left(4, -\frac{\pi}{6}, 1\right)$, cylindrical

12. $(-3, 2, -1)$, rectangular

$r = \sqrt{(-3)^2 + 2^2} = \sqrt{13}$

$\theta = \arctan\left(\frac{-2}{3}\right) = -\arctan \frac{2}{3}$

$z = -1$

$\left(\sqrt{13}, -\arctan \frac{2}{3}, -1\right)$, cylindrical

14. $z = x^2 + y^2 - 2$ rectangular equation

$z = r^2 - 2$ cylindrical equation

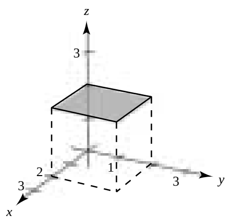
16. $x^2 + y^2 = 8x$ rectangular equation

$r^2 = 8r \cos \theta$

$r = 8 \cos \theta$ cylindrical equation

18. $z = 2$

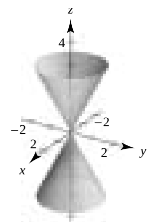
Same



20. $r = \dfrac{z}{2}$

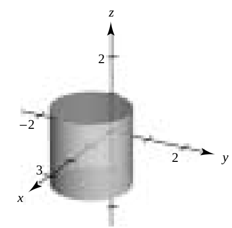
$$\sqrt{x^2 + y^2} = \frac{z}{2}$$

$$x^2 + y^2 - \frac{z^2}{4} = 0$$



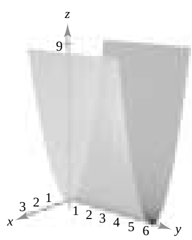
22. $r = 2\cos\theta$

$$r^2 = 2r\cos\theta$$
$$x^2 + y^2 = 2x$$
$$x^2 + y^2 - 2x = 0$$
$$(x - 1)^2 + y^2 = 1$$



24. $z = r^2\cos^2\theta$

$$z = x^2$$



26. $(1, 1, 1)$, rectangular

$$\rho = \sqrt{1^2 + 1^2 + 1^2} = \sqrt{3}$$

$$\theta = \arctan 1 = \frac{\pi}{4}$$

$$\phi = \arccos\frac{1}{\sqrt{3}}$$

$$\left(\sqrt{3}, \frac{\pi}{4}, \arccos\frac{1}{\sqrt{3}}\right), \text{ spherical}$$

28. $\left(2, 2, 4\sqrt{2}\right)$, rectangular

$$\rho = \sqrt{2^2 + 2^2 + \left(4\sqrt{2}\right)^2} = 2\sqrt{10}$$

$$\theta = \arctan 1 = \frac{\pi}{4}$$

$$\phi = \arccos\frac{2}{\sqrt{5}}$$

$$\left(2\sqrt{10}, \frac{\pi}{4}, \arccos\frac{2}{\sqrt{5}}\right), \text{ spherical}$$

30. $(-4, 0, 0)$, rectangular

$$\rho = \sqrt{(-4)^2 + 0^2 + 0^2} = 4$$
$$\theta = \pi$$

$$\phi = \arccos 0 = \frac{\pi}{2}$$

$$\left(4, \pi, \frac{\pi}{2}\right), \text{ spherical}$$

32. $\left(12, \dfrac{3\pi}{4}, \dfrac{\pi}{9}\right)$, spherical

$$x = 12\sin\frac{\pi}{9}\cos\frac{3\pi}{4} \approx -2.902$$

$$y = 12\sin\frac{\pi}{9}\sin\frac{3\pi}{4} \approx 2.902$$

$$z = 12\cos\frac{\pi}{9} \approx 11.276$$

$(-2.902, 2.902, 11.276)$, rectangular

34. $\left(9, \dfrac{\pi}{4}, \pi\right)$, spherical

$$x = 9\sin\pi\cos\frac{\pi}{4} = 0$$

$$y = 9\sin\pi\sin\frac{\pi}{4} = 0$$

$$z = 9\cos\pi = -9$$

$(0, 0, -9)$, rectangular

36. $\left(6, \pi, \dfrac{\pi}{2}\right)$, spherical

$$x = 6\sin\frac{\pi}{2}\cos\pi = -6$$

$$y = 6\sin\frac{\pi}{2}\sin\pi = 0$$

$$z = 6\cos\frac{\pi}{2} = 0$$

$(-6, 0, 0)$, rectangular

38. (a) Programs will vary.

(b) $(\rho, \theta, \phi) = (5, 1, 0.5)$

$(x, y, z) = (1.295, 2.017, 4.388)$

40. $x^2 + y^2 - 3z^2 = 0$ rectangular equation

$$x^2 + y^2 + z^2 = 4z^2$$

$$\rho^2 = 4\rho^2 \cos^2 \phi$$

$$1 = 4\cos^2 \phi$$

$$\cos \phi = \frac{1}{2}$$

$$\phi = \frac{\pi}{3} \quad \text{(cone) spherical equation}$$

42. $x = 10$ rectangular equation

$$\rho \sin \phi \cos \theta = 10$$

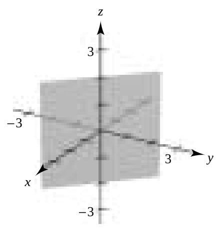
$$\rho = 10 \csc \phi \sec \theta \quad \text{spherical equation}$$

44. $\theta = \dfrac{3\pi}{4}$

$$\tan \theta = \frac{y}{x}$$

$$-1 = \frac{y}{x}$$

$$x + y = 0$$



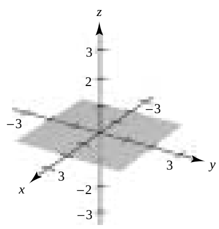
46. $\phi = \dfrac{\pi}{2}$

$$\cos \phi = \frac{z}{\sqrt{x^2 + y^2 + z^2}}$$

$$0 = \frac{z}{\sqrt{x^2 + y^2 + z^2}}$$

$$z = 0$$

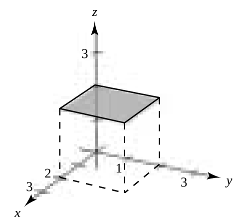
xy-plane



48. $\rho = 2 \sec \phi$

$$\rho \cos \phi = 2$$

$$z = 2$$

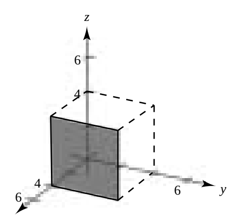


50. $\rho = 4 \csc \phi \sec \phi$

$$= \frac{4}{\sin \phi \cos \theta}$$

$$\rho \sin \phi \cos \theta = 4$$

$$x = 4$$



52. $\left(3, -\dfrac{\pi}{4}, 0\right)$, cylindrical

$$\rho = \sqrt{3^2 + 0^2} = 3$$

$$\theta = -\frac{\pi}{4}$$

$$\phi = \arccos\left(\frac{0}{9}\right) = \frac{\pi}{2}$$

$\left(3, -\dfrac{\pi}{4}, \dfrac{\pi}{2}\right)$, spherical

54. $\left(2, \dfrac{2\pi}{3}, -2\right)$, cylindrical

$$\rho = \sqrt{2^2 + (-2)^2} = 2\sqrt{2}$$

$$\theta = \frac{2\pi}{3}$$

$$\phi = \arccos\left(\frac{-1}{\sqrt{2}}\right) = \frac{3\pi}{4}$$

$\left(2\sqrt{2}, \dfrac{2\pi}{3}, \dfrac{3\pi}{4}\right)$, spherical

56. $\left(-4, \dfrac{\pi}{3}, 4\right)$, cylindrical

$$\rho = \sqrt{(-4)^2 + 4^2} = 4\sqrt{2}$$

$$\theta = \frac{\pi}{3}$$

$$\phi = \arccos \frac{1}{\sqrt{2}} = \frac{\pi}{4}$$

$\left(4\sqrt{2}, \dfrac{\pi}{3}, \dfrac{\pi}{4}\right)$, spherical

58. $\left(4, \dfrac{\pi}{2}, 3\right)$, cylindrical

$$\rho = \sqrt{4^2 + 3^2} = 5$$

$$\theta = \frac{\pi}{2}$$

$$\phi = \arccos \frac{3}{5}$$

$\left(5, \dfrac{\pi}{2}, \arccos \dfrac{3}{5}\right)$, spherical

60. $\left(4, \dfrac{\pi}{18}, \dfrac{\pi}{2}\right)$, spherical

$$r = 4 \sin \frac{\pi}{2} = 4$$

$$\theta = \frac{\pi}{18}$$

$$z = 4 \cos \frac{\pi}{2} = 0$$

$\left(4, \dfrac{\pi}{18}, 0\right)$, cylindrical

62. $\left(18, \frac{\pi}{3}, \frac{\pi}{3}\right)$, spherical

$r = \rho \sin \phi = 18 \sin \frac{\pi}{3} = 9$

$\theta = \frac{\pi}{3}$

$z = \rho \cos \phi = 18 \cos \frac{\pi}{3} = 9\sqrt{3}$

$\left(9, \frac{\pi}{3}, 9\sqrt{3}\right)$, cylindrical

64. $\left(5, -\frac{5\pi}{6}, \pi\right)$, spherical

$r = 5 \sin \pi = 0$

$\theta = -\frac{5\pi}{6}$

$z = 5 \cos \pi = -5$

$\left(0, -\frac{5\pi}{6}, -5\right)$, cylindrical

66. $\left(7, \frac{\pi}{4}, \frac{3\pi}{4}\right)$, spherical

$r = 7 \sin \frac{3\pi}{4} = \frac{7\sqrt{2}}{2}$

$\theta = \frac{\pi}{4}$

$z = 7 \cos \frac{3\pi}{4} = -\frac{7\sqrt{2}}{2}$

$\left(\frac{7\sqrt{2}}{2}, \frac{\pi}{4}, -\frac{7\sqrt{2}}{2}\right)$, cylindrical

	Rectangular	Cylindrical	Spherical
68.	$(6, -2, -3)$	$(6.325, -0.322, -3)$	$(7.000, -0.322, 2.014)$
70.	$(7.317, -6.816, 6)$	$(10, -0.75, 6)$	$(11.662, -0.750, 1.030)$
72.	$(6.115, 1.561, 4.052)$	$(6.311, 0.25, 4.052)$	$(7.5, 0.25, 1)$
74.	$(3\sqrt{2}, 3\sqrt{2}, -3)$	$(6, 0.785, -3)$	$(6.708, 0.785, 2.034)$
76.	$(0, -5, 4)$	$(5, -1.571, 4)$	$(6.403, -1.571, 0.896)$
78.	$(-1.732, 1, 3)$	$\left(-2, \frac{11\pi}{6}, 3\right)$	$(3.606, 2.618, 0.588)$
	[Note: Use the cylindrical coordinate $\left(2, \frac{5\pi}{6}, 3\right)$]		
80.	$(2.207, 7.949, -4)$	$(8.25, 1.3, -4)$	$(9.169, 1.3, 2.022)$

82. $\theta = \frac{\pi}{4}$

Plane

Matches graph (e)

84. $\phi = \frac{\pi}{4}$

Cone

Matches graph (a)

86. $\rho = 4 \sec \phi, z = \rho \cos \phi = 4$

Plane

Matches graph (b)

88. $r = a$ Cylinder with z-axis symmetry

$\theta = b$ Plane perpendicular to xy-plane

$z = c$ Plane parallel to xy-plane

90. $\rho = a$ Sphere

$\theta = b$ Vertical half-plane

$\phi = c$ Half-cone

92. $4(x^2 + y^2) = z^2$

(a) $4r^2 = z^2, 2r = z$

(b) $4(\rho^2 \sin^2 \phi \cos^2 \theta + \rho^2 \sin^2 \phi \sin^2 \theta) = \rho^2 \cos^2 \phi,$

$4 \sin^2 \phi = \cos^2 \phi, \tan^2 \phi = \frac{1}{4},$

$\tan \phi = \frac{1}{2}, \phi = \arctan \frac{1}{2}$

94. $x^2 + y^2 = z$

(a) $r^2 = z$

(b) $\rho^2 \sin^2 \phi = \rho \cos \phi, \rho \sin^2 \phi = \cos \phi,$

$\rho = \frac{\cos \phi}{\sin^2 \phi}, \rho = \csc \phi \cot \phi$

96. $x^2 + y^2 = 16$

(a) $r^2 = 16, r = 4$

(b) $\rho^2 \sin^2 \phi = 16, \rho^2 \sin^2 \phi - 16 = 0,$
$(\rho \sin \phi - 4)(\rho \sin \phi + 4) = 0, \rho = 4 \csc \phi$

98. $y = 4$

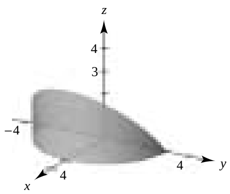
(a) $r \sin \theta = 4, r = 4 \csc \theta$

(b) $\rho \sin \phi \sin \theta = 4, \rho = 4 \csc \phi \csc \theta$

100. $-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$

$0 \le r \le 3$

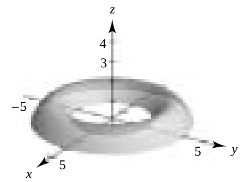
$0 \le z \le r\cos\theta$



102. $0 \le \theta \le 2\pi$

$2 \le r \le 4$

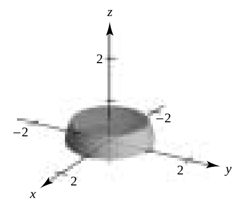
$z^2 \le -r^2 + 6r - 8$



104. $0 \le \theta \le 2\pi$

$\frac{\pi}{4} \le \phi \le \frac{\pi}{2}$

$0 \le \rho \le 1$



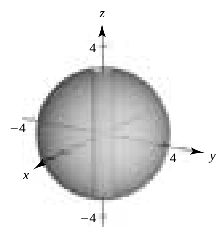
106. Cylindrical: $0.75 \le r \le 1.25, z = 8$

108. Cylindrical

$\frac{1}{2} \le r \le 3$

$0 \le \theta \le 2\pi$

$-\sqrt{9 - r^2} \le z \le \sqrt{9 - r^2}$



110. $\rho = 2\sec\phi \implies \rho\cos\phi = 2 \implies z = 2$ plane

$\rho = 4$ sphere

The intersection of the plane and the sphere is a circle.

Review Exercises for Chapter 10

2. $P = (-2, -1),\ Q = (5, -1)\ \ R = (2, 4)$

(a) $\mathbf{u} = \overrightarrow{PQ} = \langle 7, 0\rangle = 7\mathbf{i},\ \mathbf{v} = \overrightarrow{PR} = \langle 4, 5\rangle = 4\mathbf{i} + 5\mathbf{j}$

(b) $\|\mathbf{v}\| = \sqrt{4^2 + 5^2} = \sqrt{41}$

(c) $2\mathbf{u} + \mathbf{v} = 14\mathbf{i} + (4\mathbf{i} + 5\mathbf{j}) = 18\mathbf{i} + 5\mathbf{j}$

4. $\mathbf{v} = \|\mathbf{v}\|\cos\theta\,\mathbf{i} + \|\mathbf{v}\|\sin\theta\,\mathbf{j} = \frac{1}{2}\cos 225°\,\mathbf{i} + \frac{1}{2}\sin 225°\,\mathbf{j}$

$= -\frac{\sqrt{2}}{4}\mathbf{i} + \frac{\sqrt{2}}{4}\mathbf{j}$

6. (a) The length of cable *POQ* is *L*.

$\overrightarrow{OQ} = 9\mathbf{i} - y\mathbf{j}$

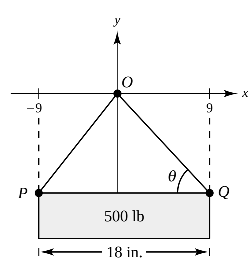
$L = 2\sqrt{9^2 + y^2} \implies \sqrt{\frac{L^2}{4} - 81} = y$

Tension: $T = c\|\overrightarrow{OQ}\| = c\sqrt{81 + y^2}$

Also,

$cy = 250 \implies T = \frac{250}{y}\sqrt{81 + y^2} \implies T = \frac{250}{\sqrt{(L^2/4) - 81}} \cdot \frac{L}{2} = \frac{250L}{\sqrt{L^2 - 324}}$

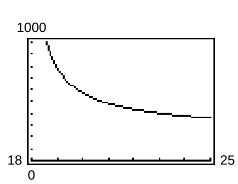
Domain: $L > 18$ inches



(b)

L	19	20	21	22	23	24	25
T	780.9	573.54	485.36	434.81	401.60	377.96	360.24

(c)



(d) The line $T = 400$ intersects the curve at

$L = 23.06$ inches.

(e) $\lim_{L\to\infty} T = 250$

The maximum tension is 250 pounds in each side of the cable since the total weight is 500 pounds.

8. $x = z = 0,\ y = -7{:}\ (0, -7, 0)$

10. Looking towards the *xy*-plane from the positive *z*-axis. The point is either in the second quadrant $(x < 0, y > 0)$ or in the fourth quadrant $(x > 0, y < 0)$. The *z*-coordinate can be any number.

12. Center: $\left(\dfrac{0+4}{2}, \dfrac{0+6}{2}, \dfrac{4+0}{2}\right) = (2, 3, 2)$

Radius: $\sqrt{(2-0)^2 + (3-0)^2 + (2-4)^2} = \sqrt{4+9+4} = \sqrt{17}$

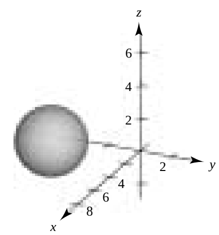
$(x-2)^2 + (y-3)^2 + (z-2)^2 = 17$

14. $(x^2 - 10x + 25) + (y^2 + 6y + 9) + (z^2 - 4z + 4) = -34 + 25 + 9 + 4$

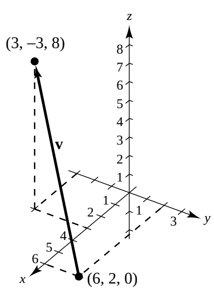
$(x-5)^2 + (y+3)^2 + (z-2)^2 = 4$

Center: $(5, -3, 2)$

Radius: 2



16. $\mathbf{v} = \langle 3-6, -3-2, 8-0\rangle = \langle -3, -5, 8\rangle$



18. $\mathbf{v} = \langle 8-5, -5+4, 5-7\rangle = \langle 3, -1, -2\rangle$

$\mathbf{w} = \langle 11-5, 6+4, 3-7\rangle = \langle 6, 10, -4\rangle$

Since **v** and **w** are not parallel, the points do not lie in a straight line.

20. $8\dfrac{\langle 6, -3, 2\rangle}{\sqrt{49}} = \dfrac{8}{7}\langle 6, -3, 2\rangle = \left\langle \dfrac{48}{7}, -\dfrac{24}{7}, \dfrac{16}{7}\right\rangle$

22. $P = (2, -1, 3),\ Q = (0, 5, 1),\ R = (5, 5, 0)$

(a) $\mathbf{u} = \overrightarrow{PQ} = \langle -2, 6, -2\rangle = -2\mathbf{i} + 6\mathbf{j} - 2\mathbf{k},$
$\mathbf{v} = \overrightarrow{PR} = \langle 3, 6, -3\rangle = 3\mathbf{i} + 6\mathbf{j} - 3\mathbf{k}$

(b) $\mathbf{u}\cdot\mathbf{v} = (-2)(3) + (6)(6) + (-2)(-3) = 36$

(c) $\mathbf{v}\cdot\mathbf{v} = 9 + 36 + 9 = 54$

24. $\mathbf{u} = \langle -4, 3, -6\rangle,\ \mathbf{v} = \langle 16, -12, 24\rangle$

Since $\mathbf{v} = -4\mathbf{u}$, the vectors are parallel.

26. $\mathbf{u} = \langle 4, -1, 5\rangle,\ \mathbf{v} = \langle 3, 2, -2\rangle$

$\mathbf{u}\cdot\mathbf{v} = 0 \implies$ is orthogonal to **v**.

$\theta = \dfrac{\pi}{2}$

28. $\mathbf{u} = \langle 1, 0, -3\rangle$

$\mathbf{v} = \langle 2, -2, 1\rangle$

$\mathbf{u}\cdot\mathbf{v} = -1$

$\|\mathbf{u}\| = \sqrt{10}$

$\|\mathbf{v}\| = 3$

$\cos\theta = \dfrac{|\mathbf{u}\cdot\mathbf{v}|}{\|\mathbf{u}\|\,\|\mathbf{v}\|} = \dfrac{1}{3\sqrt{10}}$

$\theta \approx 83.9°$

30. $W = \mathbf{F}\cdot\overrightarrow{PQ} = \|\mathbf{F}\|\,\|\overrightarrow{PQ}\|\cos\theta = (75)(8)\cos 30°$

$= 300\sqrt{3}$ ft · lb

In Exercises 32–40, $\mathbf{u} = \langle 3, -2, 1\rangle$, $\mathbf{v} = \langle 2, -4, -3\rangle$, $\mathbf{w} = \langle -1, 2, 2\rangle$.

32. $\cos\theta = \dfrac{|\mathbf{u}\cdot\mathbf{v}|}{\|\mathbf{u}\|\,\|\mathbf{v}\|} = \dfrac{11}{\sqrt{14}\sqrt{29}}$

$$\theta = \arccos\left(\frac{11}{\sqrt{14}\sqrt{29}}\right) \approx 56.9^\circ$$

34. Work $= |\mathbf{u}\cdot\mathbf{w}| = |-3 - 4 + 2| = 5$

36. $\mathbf{u}\times\mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & -2 & 1 \\ 2 & -4 & -3 \end{vmatrix} = 10\mathbf{i} + 11\mathbf{j} - 8\mathbf{k}$

$\mathbf{v}\times\mathbf{u} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & -4 & -3 \\ 3 & -2 & 1 \end{vmatrix} = -10\mathbf{i} - 11\mathbf{j} + 8\mathbf{k}$

Thus, $\mathbf{u}\times\mathbf{v} = -(\mathbf{v}\times\mathbf{u})$.

38. $\mathbf{u}\times(\mathbf{v}+\mathbf{w}) = \langle 3, -2, 1\rangle \times \langle 1, -2, -1\rangle = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & -2 & 1 \\ 1 & -2 & -1 \end{vmatrix} = 4\mathbf{i} + 4\mathbf{j} - 4\mathbf{k}$

$\mathbf{u}\times\mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & -2 & 1 \\ 2 & -4 & -3 \end{vmatrix} = 10\mathbf{i} + 11\mathbf{j} - 8\mathbf{k}$

$\mathbf{u}\times\mathbf{w} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & -2 & 1 \\ -1 & 2 & 2 \end{vmatrix} = -6\mathbf{i} - 7\mathbf{j} + 4\mathbf{k}$

$(\mathbf{u}\times\mathbf{v}) + (\mathbf{u}\times\mathbf{w}) = 4\mathbf{i} + 4\mathbf{j} - 4\mathbf{k} = \mathbf{u}\times(\mathbf{v}+\mathbf{w})$

40. Area triangle $= \dfrac{1}{2}\|\mathbf{v}\times\mathbf{w}\| = \dfrac{1}{2}\sqrt{(-2)^2 + (-1)^2} = \dfrac{\sqrt{5}}{2}$ (See Exercise 35)

42. $V = |\mathbf{u}\cdot(\mathbf{v}\times\mathbf{w})| = \begin{vmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & -1 & 2 \end{vmatrix} = 2(5) = 10$

44. Direction numbers: 1, 1, 1

(a) $x = 1 + t,\ y = 2 + t,\ z = 3 + t$

(b) $x - 1 = y - 2 = z - 3$

46. $\mathbf{u}\times\mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & -5 & 1 \\ -3 & 1 & 4 \end{vmatrix} = -21\mathbf{i} - 11\mathbf{j} - 13\mathbf{k}$

Direction numbers: 21, 11, 13

(a) $x = 21t,\ y = 1 + 11t,\ z = 4 + 13t$

(b) $\dfrac{x}{21} = \dfrac{y-1}{11} = \dfrac{z-4}{13}$

48. $P = (-3, -4, 2),\ Q = (-3, 4, 1),\ R = (1, 1, -2)$

$\overrightarrow{PQ} = \langle 0, 8, -1\rangle,\ \overrightarrow{PR} = \langle 4, 5, -4\rangle$

$\mathbf{n} = \overrightarrow{PQ}\times\overrightarrow{PR} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 8 & -1 \\ 4 & 5 & -4 \end{vmatrix} = -27\mathbf{i} - 4\mathbf{j} - 32\mathbf{k}$

$-27(x + 3) - 4(y + 4) - 32(z - 2) = 0$

$$27x + 4y + 32z = -33$$

50. The normal vectors to the planes are the same,

$\mathbf{n} = \langle 5, -3, 1 \rangle.$

Choose a point in the first plane, $P = (0, 0, 2)$. Choose a point in the second plane, $Q = (0, 0, -3)$.

$\overrightarrow{PQ} = \langle 0, 0, -5 \rangle$

$$D = \frac{|\overrightarrow{PQ} \cdot \mathbf{n}|}{\|\mathbf{n}\|} = \frac{|-5|}{\sqrt{35}} = \frac{5}{\sqrt{35}} = \frac{\sqrt{35}}{7}$$

52. $Q(-5, 1, 3)$ point

$\mathbf{u} = \langle 1, -2, -1 \rangle$ direction vector

$P = (1, 3, 5)$ point on line

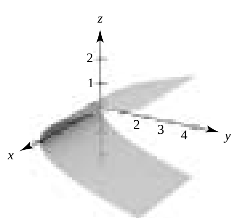
$\overrightarrow{PQ} = \langle -6, -2, -2 \rangle$

$$\overrightarrow{PQ} \times \mathbf{u} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -6 & -2 & -2 \\ 1 & -2 & -1 \end{vmatrix} = \langle -2, -8, 14 \rangle$$

$$D = \frac{\|\overrightarrow{PQ} \times \mathbf{u}\|}{\|\mathbf{u}\|} = \frac{\sqrt{264}}{\sqrt{6}} = 2\sqrt{11}$$

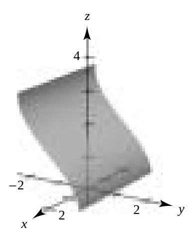
54. $y = z^2$

Since the x-coordinate is missing, we have a cylindrical surface with rulings parallel to the x-axis. The generating curve is a parabola in the yz-coordinate plane.



56. $y = \cos z$

Since the x-coordinate is missing, we have a cylindrical surface with rulings parallel to the x-axis. The generating curve is $y = \cos z$.



58. $16x^2 + 16y^2 - 9z^2 = 0$

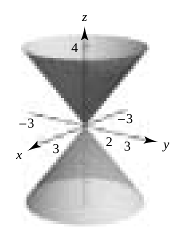
Cone

xy-trace: point $(0,0,0)$

xz-trace: $z = \pm\dfrac{4x}{3}$

yz-trace: $z = \pm\dfrac{4y}{3}$

$z = 4,\ x^2 + y^2 = 9$



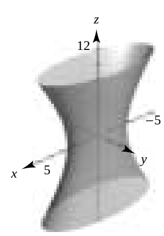
60. $\dfrac{x^2}{25} + \dfrac{y^2}{4} - \dfrac{z^2}{100} = 1$

Hyperboloid of one sheet

xy-trace: $\dfrac{x^2}{25} + \dfrac{y^2}{4} = 1$

xz-trace: $\dfrac{x^2}{25} - \dfrac{z^2}{100} = 1$

yz-trace: $\dfrac{y^2}{4} - \dfrac{z^2}{100} = 1$



62. Let $y = r(x) = 2\sqrt{x}$ and revolve the curve about the x-axis.

64. $\left(\dfrac{\sqrt{3}}{4}, \dfrac{3}{4}, \dfrac{3\sqrt{3}}{2}\right)$, rectangular

(a) $r = \sqrt{\left(\dfrac{\sqrt{3}}{4}\right)^2 + \left(\dfrac{3}{4}\right)^2} = \dfrac{\sqrt{3}}{2},\ \theta = \arctan\sqrt{3} = \dfrac{\pi}{3},\ z = \dfrac{3\sqrt{3}}{2},\ \left(\dfrac{\sqrt{3}}{2}, \dfrac{\pi}{2}, \dfrac{3\sqrt{3}}{2}\right)$, cylindrical

(b) $\rho = \sqrt{\left(\dfrac{\sqrt{3}}{4}\right)^2 + \left(\dfrac{3}{4}\right)^2 + \left(\dfrac{3\sqrt{3}}{2}\right)^2} = \dfrac{\sqrt{30}}{2},\ \theta = \dfrac{\pi}{3},\ \phi = \arccos\dfrac{3}{\sqrt{10}},\ \left(\dfrac{\sqrt{30}}{2}, \dfrac{\pi}{3}, \arccos\dfrac{3}{\sqrt{10}}\right)$, spherical