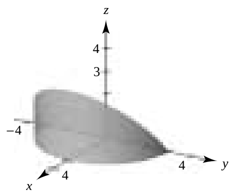
100. $-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$

$0 \le r \le 3$

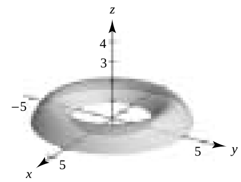
$0 \le z \le r\cos\theta$



102. $0 \le \theta \le 2\pi$

$2 \le r \le 4$

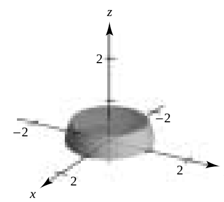
$z^2 \le -r^2 + 6r - 8$



104. $0 \le \theta \le 2\pi$

$\frac{\pi}{4} \le \phi \le \frac{\pi}{2}$

$0 \le \rho \le 1$



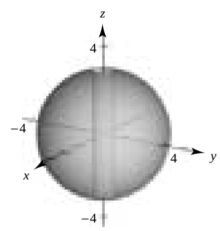
106. Cylindrical: $0.75 \le r \le 1.25, z = 8$

108. Cylindrical

$\frac{1}{2} \le r \le 3$

$0 \le \theta \le 2\pi$

$-\sqrt{9 - r^2} \le z \le \sqrt{9 - r^2}$



110. $\rho = 2\sec\phi \implies \rho\cos\phi = 2 \implies z = 2$ plane

$\rho = 4$ sphere

The intersection of the plane and the sphere is a circle.

Review Exercises for Chapter 10

2. $P = (-2, -1),\ Q = (5, -1)\ R = (2, 4)$

(a) $\mathbf{u} = \overrightarrow{PQ} = \langle 7, 0\rangle = 7\mathbf{i},\ \mathbf{v} = \overrightarrow{PR} = \langle 4, 5\rangle = 4\mathbf{i} + 5\mathbf{j}$

(b) $\|\mathbf{v}\| = \sqrt{4^2 + 5^2} = \sqrt{41}$

(c) $2\mathbf{u} + \mathbf{v} = 14\mathbf{i} + (4\mathbf{i} + 5\mathbf{j}) = 18\mathbf{i} + 5\mathbf{j}$

4. $\mathbf{v} = \|\mathbf{v}\|\cos\theta\,\mathbf{i} + \|\mathbf{v}\|\sin\theta\,\mathbf{j} = \frac{1}{2}\cos 225°\,\mathbf{i} + \frac{1}{2}\sin 225°\,\mathbf{j}$

$$= -\frac{\sqrt{2}}{4}\mathbf{i} + \frac{\sqrt{2}}{4}\mathbf{j}$$

6. (a) The length of cable POQ is L.

$$\overrightarrow{OQ} = 9\mathbf{i} - y\mathbf{j}$$

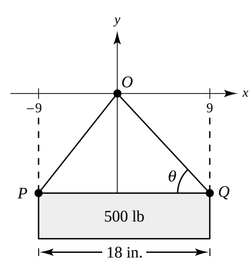
$$L = 2\sqrt{9^2 + y^2} \implies \sqrt{\frac{L^2}{4} - 81} = y$$

Tension: $T = c\|\overrightarrow{OQ}\| = c\sqrt{81 + y^2}$

Also,

$$cy = 250 \implies T = \frac{250}{y}\sqrt{81 + y^2} \implies T = \frac{250}{\sqrt{(L^2/4) - 81}} \cdot \frac{L}{2} = \frac{250L}{\sqrt{L^2 - 324}}$$

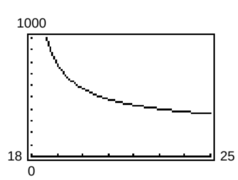
Domain: $L > 18$ inches



(b)

L	19	20	21	22	23	24	25
T	780.9	573.54	485.36	434.81	401.60	377.96	360.24

(c)



(d) The line $T = 400$ intersects the curve at

$$L = 23.06 \text{ inches.}$$

(e) $\lim_{L\to\infty} T = 250$

The maximum tension is 250 pounds in each side of the cable since the total weight is 500 pounds.

8. $x = z = 0,\ y = -7{:}\ (0, -7, 0)$

10. Looking towards the xy-plane from the positive z-axis. The point is either in the second quadrant ($x < 0, y > 0$) or in the fourth quadrant ($x > 0, y < 0$). The z-coordinate can be any number.

12. Center: $\left(\frac{0+4}{2}, \frac{0+6}{2}, \frac{4+0}{2}\right) = (2, 3, 2)$

Radius: $\sqrt{(2-0)^2 + (3-0)^2 + (2-4)^2} = \sqrt{4+9+4} = \sqrt{17}$

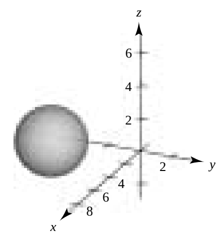
$(x-2)^2 + (y-3)^2 + (z-2)^2 = 17$

14. $(x^2 - 10x + 25) + (y^2 + 6y + 9) + (z^2 - 4z + 4) = -34 + 25 + 9 + 4$

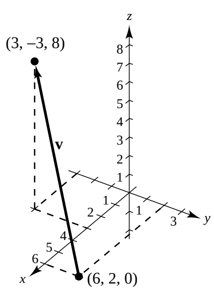
$(x-5)^2 + (y+3)^2 + (z-2)^2 = 4$

Center: $(5, -3, 2)$

Radius: 2



16. $\mathbf{v} = \langle 3-6, -3-2, 8-0\rangle = \langle -3, -5, 8\rangle$



18. $\mathbf{v} = \langle 8-5, -5+4, 5-7\rangle = \langle 3, -1, -2\rangle$

$\mathbf{w} = \langle 11-5, 6+4, 3-7\rangle = \langle 6, 10, -4\rangle$

Since $\mathbf{v}$ and $\mathbf{w}$ are not parallel, the points do not lie in a straight line.

20. $8\frac{\langle 6, -3, 2\rangle}{\sqrt{49}} = \frac{8}{7}\langle 6, -3, 2\rangle = \left\langle \frac{48}{7}, -\frac{24}{7}, \frac{16}{7}\right\rangle$

22. $P = (2, -1, 3),\ Q = (0, 5, 1),\ R = (5, 5, 0)$

(a) $\mathbf{u} = \overrightarrow{PQ} = \langle -2, 6, -2\rangle = -2\mathbf{i} + 6\mathbf{j} - 2\mathbf{k}$,
$\mathbf{v} = \overrightarrow{PR} = \langle 3, 6, -3\rangle = 3\mathbf{i} + 6\mathbf{j} - 3\mathbf{k}$

(b) $\mathbf{u} \cdot \mathbf{v} = (-2)(3) + (6)(6) + (-2)(-3) = 36$

(c) $\mathbf{v} \cdot \mathbf{v} = 9 + 36 + 9 = 54$

24. $\mathbf{u} = \langle -4, 3, -6\rangle,\ \mathbf{v} = \langle 16, -12, 24\rangle$

Since $\mathbf{v} = -4\mathbf{u}$, the vectors are parallel.

26. $\mathbf{u} = \langle 4, -1, 5\rangle,\ \mathbf{v} = \langle 3, 2, -2\rangle$

$\mathbf{u} \cdot \mathbf{v} = 0 \implies$ is orthogonal to $\mathbf{v}$.

$\theta = \frac{\pi}{2}$

28. $\mathbf{u} = \langle 1, 0, -3\rangle$

$\mathbf{v} = \langle 2, -2, 1\rangle$

$\mathbf{u} \cdot \mathbf{v} = -1$

$\|\mathbf{u}\| = \sqrt{10}$

$\|\mathbf{v}\| = 3$

$\cos\theta = \frac{|\mathbf{u} \cdot \mathbf{v}|}{\|\mathbf{u}\|\,\|\mathbf{v}\|} = \frac{1}{3\sqrt{10}}$

$\theta \approx 83.9°$

30. $W = \mathbf{F} \cdot \overrightarrow{PQ} = \|\mathbf{F}\|\,\|\overrightarrow{PQ}\|\cos\theta = (75)(8)\cos 30°$

$= 300\sqrt{3}$ ft · lb

In Exercises 32–40, u = ⟨3, −2, 1⟩, v = ⟨2, −4, −3⟩, w = ⟨−1, 2, 2⟩.

32. $\cos\theta = \dfrac{|\mathbf{u}\cdot\mathbf{v}|}{\|\mathbf{u}\|\,\|\mathbf{v}\|} = \dfrac{11}{\sqrt{14}\sqrt{29}}$

$\theta = \arccos\left(\dfrac{11}{\sqrt{14}\sqrt{29}}\right) \approx 56.9°$

34. Work $= |\mathbf{u}\cdot\mathbf{w}| = |-3-4+2| = 5$

36. $\mathbf{u}\times\mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & -2 & 1 \\ 2 & -4 & -3 \end{vmatrix} = 10\mathbf{i} + 11\mathbf{j} - 8\mathbf{k}$

$\mathbf{v}\times\mathbf{u} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & -4 & -3 \\ 3 & -2 & 1 \end{vmatrix} = -10\mathbf{i} - 11\mathbf{j} + 8\mathbf{k}$

Thus, $\mathbf{u}\times\mathbf{v} = -(\mathbf{v}\times\mathbf{u})$.

38. $\mathbf{u}\times(\mathbf{v}+\mathbf{w}) = \langle 3,-2,1\rangle \times \langle 1,-2,-1\rangle = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & -2 & 1 \\ 1 & -2 & -1 \end{vmatrix} = 4\mathbf{i} + 4\mathbf{j} - 4\mathbf{k}$

$\mathbf{u}\times\mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & -2 & 1 \\ 2 & -4 & -3 \end{vmatrix} = 10\mathbf{i} + 11\mathbf{j} - 8\mathbf{k}$

$\mathbf{u}\times\mathbf{w} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & -2 & 1 \\ -1 & 2 & 2 \end{vmatrix} = -6\mathbf{i} - 7\mathbf{j} + 4\mathbf{k}$

$(\mathbf{u}\times\mathbf{v}) + (\mathbf{u}\times\mathbf{w}) = 4\mathbf{i} + 4\mathbf{j} - 4\mathbf{k} = \mathbf{u}\times(\mathbf{v}+\mathbf{w})$

40. Area triangle $= \dfrac{1}{2}\|\mathbf{v}\times\mathbf{w}\| = \dfrac{1}{2}\sqrt{(-2)^2 + (-1)^2} = \dfrac{\sqrt{5}}{2}$ (See Exercise 35)

42. $V = |\mathbf{u}\cdot(\mathbf{v}\times\mathbf{w})| = \begin{vmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & -1 & 2 \end{vmatrix} = 2(5) = 10$

44. Direction numbers: 1, 1, 1

(a) $x = 1 + t,\ y = 2 + t,\ z = 3 + t$

(b) $x - 1 = y - 2 = z - 3$

46. $\mathbf{u}\times\mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & -5 & 1 \\ -3 & 1 & 4 \end{vmatrix} = -21\mathbf{i} - 11\mathbf{j} - 13\mathbf{k}$

Direction numbers: 21, 11, 13

(a) $x = 21t,\ y = 1 + 11t,\ z = 4 + 13t$

(b) $\dfrac{x}{21} = \dfrac{y-1}{11} = \dfrac{z-4}{13}$

48. $P = (-3,-4,2),\ Q = (-3,4,1),\ R = (1,1,-2)$

$\overrightarrow{PQ} = \langle 0, 8, -1\rangle,\ \overrightarrow{PR} = \langle 4, 5, -4\rangle$

$\mathbf{n} = \overrightarrow{PQ}\times\overrightarrow{PR} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 8 & -1 \\ 4 & 5 & -4 \end{vmatrix} = -27\mathbf{i} - 4\mathbf{j} - 32\mathbf{k}$

$-27(x+3) - 4(y+4) - 32(z-2) = 0$

$27x + 4y + 32z = -33$

50. The normal vectors to the planes are the same,

$\mathbf{n} = \langle 5, -3, 1 \rangle$.

Choose a point in the first plane, $P = (0, 0, 2)$. Choose a point in the second plane, $Q = (0, 0, -3)$.

$$\overrightarrow{PQ} = \langle 0, 0, -5 \rangle$$

$$D = \frac{|\overrightarrow{PQ} \cdot \mathbf{n}|}{\|\mathbf{n}\|} = \frac{|-5|}{\sqrt{35}} = \frac{5}{\sqrt{35}} = \frac{\sqrt{35}}{7}$$

52. $Q(-5, 1, 3)$ point

$\mathbf{u} = \langle 1, -2, -1 \rangle$ direction vector

$P = (1, 3, 5)$ point on line

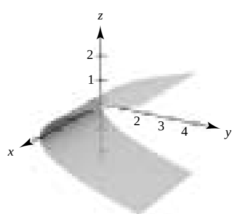
$$\overrightarrow{PQ} = \langle -6, -2, -2 \rangle$$

$$\overrightarrow{PQ} \times \mathbf{u} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -6 & -2 & -2 \\ 1 & -2 & -1 \end{vmatrix} = \langle -2, -8, 14 \rangle$$

$$D = \frac{\|\overrightarrow{PQ} \times \mathbf{u}\|}{\|\mathbf{u}\|} = \frac{\sqrt{264}}{\sqrt{6}} = 2\sqrt{11}$$

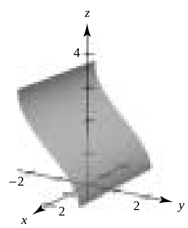
54. $y = z^2$

Since the x-coordinate is missing, we have a cylindrical surface with rulings parallel to the x-axis. The generating curve is a parabola in the yz-coordinate plane.



56. $y = \cos z$

Since the x-coordinate is missing, we have a cylindrical surface with rulings parallel to the x-axis. The generating curve is $y = \cos z$.



58. $16x^2 + 16y^2 - 9z^2 = 0$

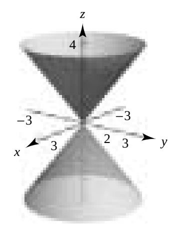
Cone

xy-trace: point $(0,0,0)$

xz-trace: $z = \pm\dfrac{4x}{3}$

yz-trace: $z = \pm\dfrac{4y}{3}$

$z = 4,\ x^2 + y^2 = 9$



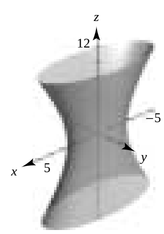
60. $\dfrac{x^2}{25} + \dfrac{y^2}{4} - \dfrac{z^2}{100} = 1$

Hyperboloid of one sheet

xy-trace: $\dfrac{x^2}{25} + \dfrac{y^2}{4} = 1$

xz-trace: $\dfrac{x^2}{25} - \dfrac{z^2}{100} = 1$

yz-trace: $\dfrac{y^2}{4} - \dfrac{z^2}{100} = 1$



62. Let $y = r(x) = 2\sqrt{x}$ and revolve the curve about the x-axis.

64. $\left(\dfrac{\sqrt{3}}{4}, \dfrac{3}{4}, \dfrac{3\sqrt{3}}{2}\right)$, rectangular

(a) $r = \sqrt{\left(\dfrac{\sqrt{3}}{4}\right)^2 + \left(\dfrac{3}{4}\right)^2} = \dfrac{\sqrt{3}}{2},\ \theta = \arctan\sqrt{3} = \dfrac{\pi}{3},\ z = \dfrac{3\sqrt{3}}{2},\ \left(\dfrac{\sqrt{3}}{2}, \dfrac{\pi}{2}, \dfrac{3\sqrt{3}}{2}\right)$, cylindrical

(b) $\rho = \sqrt{\left(\dfrac{\sqrt{3}}{4}\right)^2 + \left(\dfrac{3}{4}\right)^2 + \left(\dfrac{3\sqrt{3}}{2}\right)^2} = \dfrac{\sqrt{30}}{2},\ \theta = \dfrac{\pi}{3},\ \phi = \arccos\dfrac{3}{\sqrt{10}},\ \left(\dfrac{\sqrt{30}}{2}, \dfrac{\pi}{3}, \arccos\dfrac{3}{\sqrt{10}}\right)$, spherical

66. $\left(81, -\dfrac{5\pi}{6}, 27\sqrt{3}\right)$, cylindrical

$$\rho = \sqrt{6561 + 2187} = 54\sqrt{3}$$

$$\theta = -\frac{5\pi}{6}$$

$$\phi = \arccos\left(\frac{27\sqrt{3}}{54\sqrt{3}}\right) = \arccos\frac{1}{2} = \frac{\pi}{3}$$

$\left(54\sqrt{3}, -\dfrac{5\pi}{6}, \dfrac{\pi}{3}\right)$, spherical

68. $\left(12, -\dfrac{\pi}{2}, \dfrac{2\pi}{3}\right)$, spherical

$$r^2 = \left(12\sin\left(\frac{2\pi}{3}\right)\right)^2 \implies r = 6\sqrt{3}$$

$$\theta = -\frac{\pi}{2}$$

$$z = \rho\cos\phi = 12\cos\left(\frac{2\pi}{3}\right) = -6$$

$\left(6\sqrt{3}, -\dfrac{\pi}{2}, -6\right)$, cylindrical

70. $x^2 + y^2 + z^2 = 16$

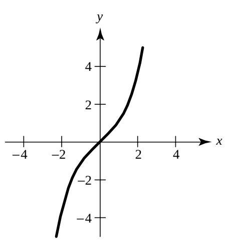
(a) Cylindrical: $r^2 + z^2 = 16$

(b) Spherical: $\rho = 4$

Problem Solving for Chapter 10

2. $f(x) = \displaystyle\int_0^x \sqrt{t^4 + 1}\, dt$

(a)



(b) $f'(x) = \sqrt{x^4 + 1}$

$f'(0) = 1 = \tan\theta$

$$\theta = \frac{\pi}{4}$$

$$\mathbf{u} = \frac{1}{\sqrt{2}}(\mathbf{i} + \mathbf{j}) = \left\langle \frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2} \right\rangle$$

(c) $\pm\left\langle \dfrac{\sqrt{2}}{2}, -\dfrac{\sqrt{2}}{2} \right\rangle$

(d) The line is $y = x$: $x = t, y = t$.

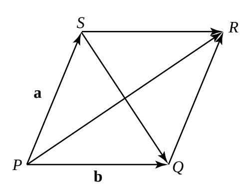
4. Label the figure as indicated.

$\overrightarrow{PR} = \mathbf{a} + \mathbf{b}$

$\overrightarrow{SQ} = \mathbf{b} - \mathbf{a}$

$(\mathbf{a} + \mathbf{b}) \cdot (\mathbf{b} - \mathbf{a}) = \|\mathbf{b}\|^2 - \|\mathbf{a}\|^2 = 0$, because

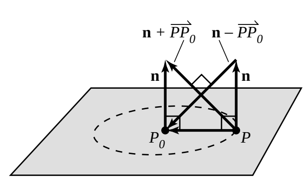
$\|\mathbf{a}\| = \|\mathbf{b}\|$ in a rhombus.



6. $(\mathbf{n} + \overrightarrow{PP_0}) \perp (\mathbf{n} - \overrightarrow{PP_0})$

Figure is a square.

Thus, $\|\overrightarrow{PP_0}\| = \|\mathbf{n}\|$ and the points P form a circle of radius $\|\mathbf{n}\|$ in the plane with center at P.



8. (a) $V = 2\int_0^r \pi(r^2 - x^2)\,dx = 2\pi\left[r^2x - \frac{x^3}{3}\right]_0^r = \frac{4}{3}\pi r^3$

(b) At height $z = d > 0$,

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{d^2}{c^2} = 1$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 - \frac{d^2}{c^2} = \frac{c^2 - d^2}{c^2}$$

$$\frac{x^2}{\frac{a^2(c^2 - d^2)}{c^2}} + \frac{y^2}{\frac{b^2(c^2 - d^2)}{c^2}} = 1.$$

$$\text{Area} = \pi\sqrt{\left(\frac{a^2(c^2 - d^2)}{c^2}\right)\left(\frac{b^2(c^2 - d^2)}{c^2}\right)} = \frac{\pi ab}{c^2}(c^2 - d^2)$$

$$V = 2\int_0^c \frac{\pi ab}{c^2}(c^2 - d^2)\,dd$$

$$= \frac{2\pi ab}{c^2}\left[c^2d - \frac{d^3}{3}\right]_0^c$$

$$= \frac{4}{3}\pi abc$$

10. (a) $r = 2\cos\theta$

Cylinder

(b) $z = r^2 \cos 2\theta$

$z^2 = x^2 - y^2$

Hyperbolic paraboloid

12. $x = -t + 3, y = \frac{1}{2}t + 1, z = 2t - 1; Q = (4, 3, s)$

(a) $\mathbf{u} = \langle -2, 1, 4\rangle$ direction vector for line

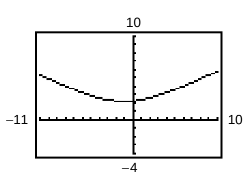
$P = (3, 1, -1)$ point on line

$\overrightarrow{PQ} = \langle 1, 2, s + 1\rangle$

$$\overrightarrow{PQ} \times \mathbf{u} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 2 & s + 1 \\ -2 & 1 & 4 \end{vmatrix} = (7 - s)\mathbf{i} + (-6 - 2s)\mathbf{j} + 5\mathbf{k}$$

$$D = \frac{\|\overrightarrow{PQ} \times \mathbf{u}\|}{\|\mathbf{u}\|} = \frac{\sqrt{(7 - s)^2 + (-6 - 2s)^2 + 25}}{\sqrt{21}}$$

(b)



The minimum is $D \approx 2.2361$ at $s = -1$.

(c) Yes, there are slant asymptotes. Using $s = x$, we have

$$D(s) = \frac{1}{\sqrt{21}}\sqrt{5x^2 + 10x + 110} = \frac{\sqrt{5}}{\sqrt{21}}\sqrt{x^2 + 2x + 22}$$

$$= \frac{\sqrt{5}}{\sqrt{21}}\sqrt{(x + 1)^2 + 21} \;\rightarrow\; \pm\sqrt{\frac{5}{21}}(x + 1)$$

$y = \pm\frac{\sqrt{105}}{21}(s + 1)$ slant asymptotes.

14. (a) The tension T is the same in each tow line.

$$6000\mathbf{i} = T(\cos 20° + \cos(-20))\mathbf{i} + T(\sin 20° + \sin(-20°))\mathbf{j}$$

$$= 2T \cos 20°\mathbf{i}$$

$$\Rightarrow T = \frac{6000}{2\cos 20°} \approx 3192.5 \text{ lbs}$$

(b) As in part (a), $6000\mathbf{i} = 2T\cos\theta$

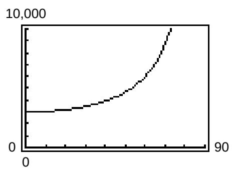
$$\Rightarrow T = \frac{3000}{\cos\theta}$$

Domain: $0 < \theta < 90°$

(c)

θ	10°	20°	30°	40°	50°	60°
T	3046.3	3192.5	3464.1	3916.2	4667.2	6000.0

(d)



(e) As θ increases, there is less force applied in the direction of motion.

16. (a) Los Angeles: $(4000, -118.24°, 55.95°)$

Rio de Janeiro: $(4000, -43.22°, 112.90°)$

(b) Los Angeles: $x = 4000 \sin 55.95° \cos(-118.24°)$

$y = 4000 \sin 55.95° \sin(-118.24°)$

$z = 4000 \cos 55.95°$

$(-1568.2, -2919.7, 2239.7)$

Rio de Janeiro: $x = (4000 \sin 112.90° \cos(-43.22°)$

$y = 4000 \sin 112.90° \sin(-43.22°)$

$z = 4000 \cos 112.90°$

$(2685.2, -2523.3, -1556.5)$

(c) $$\cos\theta = \frac{\mathbf{u}\cdot\mathbf{v}}{\|\mathbf{u}\|\,\|\mathbf{v}\|} = \frac{(-1568.2)(2685.2) + (-2919.7)(-2523.3) + (2239.7)(-1556.5)}{(4000)(4000)}$$

$\theta \approx 91.18° \approx 1.59$ radians

(d) $s = 4000(1.59) \approx 6366$ miles

—CONTINUED—

16. —CONTINUED—

(e) For Boston and Honolulu:

a. Boston: $(4000, -71.06°, 47.64°)$

Honolulu: $(4000, -157.86°, 68.69°)$

b. Boston: $x = 4000 \sin 47.64° \cos(-71.06°)$

$y = 4000 \sin 47.64° \sin(-71.06°)$

$z = 4000 \cos 47.64°$

$(959.4, -2795.7, 2695.1)$

Honolulu: $x = (4000 \sin 68.69° \cos(-157.86°)$

$y = 4000 \sin 68.69° \sin(-157.86°)$

$z = 4000 \cos 68.69°$

$(-3451.7, -1404.4, 1453.7)$

(f) $\cos \theta = \dfrac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{u}\| \|\mathbf{v}\|} = \dfrac{(959.4)(-3451.7) + (-2795.7)(-1404.4) + (2695.1)(1453.7)}{(4000)(4000)}$

$\theta \approx 73.5° \approx 1.28$ radians

(g) $s = 4000(1.28) \approx 5120$ miles

18. Assume one of $a, b, c,$ is not zero, say a. Choose a point in the first plane such as $(-d_1/a, 0, 0)$. The distance between this point and the second plane is

$$D = \frac{|a(-d_1/a) + b(0) + c(0) + d_2|}{\sqrt{a^2 + b^2 + c^2}}$$

$$= \frac{|-d_1 + d_2|}{\sqrt{a^2 + b^2 + c^2}} = \frac{|d_1 - d_2|}{\sqrt{a^2 + b^2 + c^2}}.$$

20. Essay.