CHAPTER 10
Vectors and the Geometry of Space

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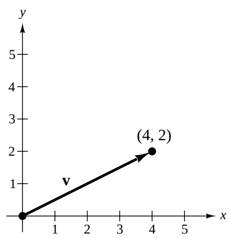
Section 10.1 Vectors in the Plane

Solutions to Odd-Numbered Exercises

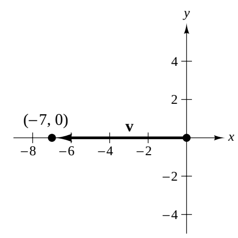
1. (a) $\mathbf{v} = \langle 5 - 1, 3 - 1 \rangle = \langle 4, 2 \rangle$

(b)



3. (a) $\mathbf{v} = \langle -4 - 3, -2 - (-2) \rangle = \langle -7, 0 \rangle$

(b)



5. $\mathbf{u} = \langle 5 - 3, 6 - 2 \rangle = \langle 2, 4 \rangle$

$\mathbf{v} = \langle 1 - (-1), 8 - 4 \rangle = \langle 2, 4 \rangle$

$\mathbf{u} = \mathbf{v}$

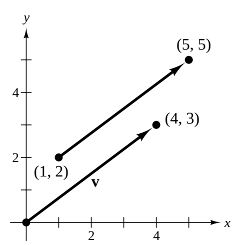
7. $\mathbf{u} = \langle 6 - 0, -2 - 3 \rangle = \langle 6, -5 \rangle$

$\mathbf{v} = \langle 9 - 3, 5 - 10 \rangle = \langle 6, -5 \rangle$

$\mathbf{u} = \mathbf{v}$

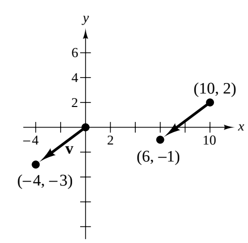
9. (b) $\mathbf{v} = \langle 5 - 1, 5 - 2 \rangle = \langle 4, 3 \rangle$

(a) and (c).



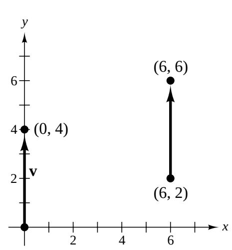
11. (b) $\mathbf{v} = \langle 6 - 10, -1 - 2 \rangle = \langle -4, -3 \rangle$

(a) and (c).



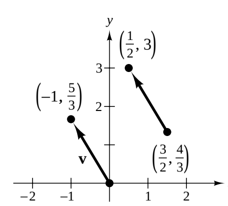
13. (b) $\mathbf{v} = \langle 6 - 6, 6 - 2 \rangle = \langle 0, 4 \rangle$

(a) and (c).

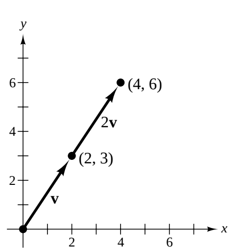


15. (b) $\mathbf{v} = \left\langle \frac{1}{2} - \frac{3}{2}, 3 - \frac{4}{3} \right\rangle = \left\langle -1, \frac{5}{3} \right\rangle$

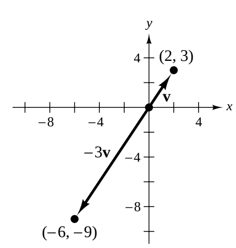
(a) and (c).



17. (a) $2\mathbf{v} = \langle 4, 6 \rangle$



(b) $-3\mathbf{v} = \langle -6, -9 \rangle$



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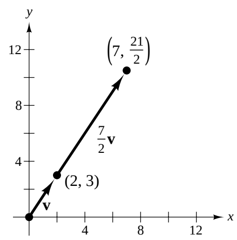
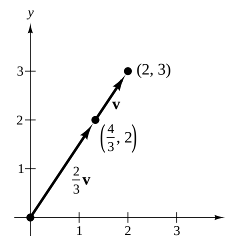
17. —CONTINUED—

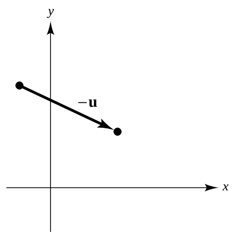
(c) $\frac{7}{2}\mathbf{v} = \left\langle 7, \frac{21}{2} \right\rangle$



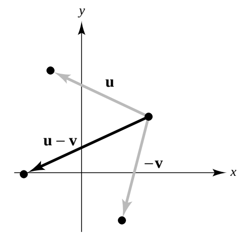
(d) $\frac{2}{3}\mathbf{v} = \left\langle \frac{4}{3}, 2 \right\rangle$



19.



21.



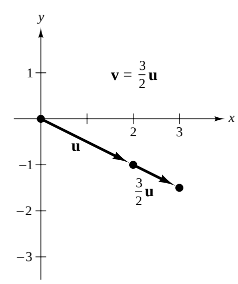
23. (a) $\frac{2}{3}\mathbf{u} = \frac{2}{3}\langle 4, 9\rangle = \left\langle \frac{8}{3}, 6 \right\rangle$

(b) $\mathbf{v} - \mathbf{u} = \langle 2, -5\rangle - \langle 4, 9\rangle = \langle -2, -14\rangle$

(c) $2\mathbf{u} + 5\mathbf{v} = 2\langle 4, 9\rangle + 5\langle 2, -5\rangle = \langle 18, -7\rangle$

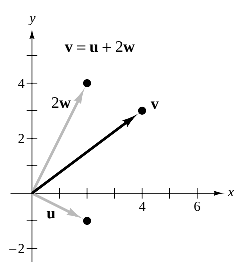
25. $\mathbf{v} = \frac{3}{2}(2\mathbf{i} - \mathbf{j}) = 3\mathbf{i} - \frac{3}{2}\mathbf{j}$

$= \left\langle 3, -\frac{3}{2} \right\rangle$



27. $\mathbf{v} = (2\mathbf{i} - \mathbf{j}) + 2(\mathbf{i} + 2\mathbf{j})$

$= 4\mathbf{i} + 3\mathbf{j} = \langle 4, 3\rangle$



29. $u_1 - 4 = -1$

$u_2 - 2 = 3$

$u_1 = 3$

$u_2 = 5$

$Q = (3, 5)$

31. $\|\mathbf{v}\| = \sqrt{16 + 9} = 5$

33. $\|\mathbf{v}\| = \sqrt{36 + 25} = \sqrt{61}$

35. $\|\mathbf{v}\| = \sqrt{0 + 16} = 4$

37. $\|\mathbf{u}\| = \sqrt{3^2 + 12^2} = \sqrt{153}$

$$\mathbf{v} = \frac{\mathbf{u}}{\|\mathbf{u}\|} = \frac{\langle 3, 12\rangle}{\sqrt{153}} = \left\langle \frac{3}{\sqrt{153}}, \frac{12}{\sqrt{153}} \right\rangle$$

$$= \left\langle \frac{\sqrt{17}}{17}, \frac{4\sqrt{17}}{17} \right\rangle \text{ unit vector}$$

39. $\|\mathbf{u}\| = \sqrt{\left(\frac{3}{2}\right)^2 + \left(\frac{5}{2}\right)^2} = \frac{\sqrt{34}}{2}$

$$\mathbf{v} = \frac{\mathbf{u}}{\|\mathbf{u}\|} = \frac{\langle (3/2), (5/2)\rangle}{\sqrt{34}/2} = \left\langle \frac{3}{\sqrt{34}}, \frac{5}{\sqrt{34}} \right\rangle$$

$$= \left\langle \frac{3\sqrt{34}}{34}, \frac{5\sqrt{34}}{34} \right\rangle \text{ unit vector}$$

41. $\|\mathbf{u}\| = \langle 1, -1\rangle, \mathbf{v} = \langle -1, 2\rangle$

(a) $\|\mathbf{u}\| = \sqrt{1+1} = \sqrt{2}$

(b) $\|\mathbf{v}\| = \sqrt{1+4} = \sqrt{5}$

(c) $\mathbf{u} + \mathbf{v} = \langle 0, 1\rangle$

$\|\mathbf{u} + \mathbf{v}\| = \sqrt{0+1} = 1$

(d) $\dfrac{\mathbf{u}}{\|\mathbf{u}\|} = \dfrac{1}{\sqrt{2}}\langle 1, -1\rangle$

$\left\|\dfrac{\mathbf{u}}{\|\mathbf{u}\|}\right\| = 1$

(e) $\dfrac{\mathbf{v}}{\|\mathbf{v}\|} = \dfrac{1}{\sqrt{5}}\langle -1, 2\rangle$

$\left\|\dfrac{\mathbf{v}}{\|\mathbf{v}\|}\right\| = 1$

(f) $\dfrac{\mathbf{u}+\mathbf{v}}{\|\mathbf{u}+\mathbf{v}\|} = \langle 0, 1\rangle$

$\left\|\dfrac{\mathbf{u}+\mathbf{v}}{\|\mathbf{u}+\mathbf{v}\|}\right\| = 1$

43. $\mathbf{u} = \left\langle 1, \dfrac{1}{2}\right\rangle, \mathbf{v} = \langle 2, 3\rangle$

(a) $\|\mathbf{u}\| = \sqrt{1 + \dfrac{1}{4}} = \dfrac{\sqrt{5}}{2}$

(b) $\|\mathbf{v}\| = \sqrt{4+9} = \sqrt{13}$

(c) $\mathbf{u} + \mathbf{v} = \left\langle 3, \dfrac{7}{2}\right\rangle$

$\|\mathbf{u} + \mathbf{v}\| = \sqrt{9 + \dfrac{49}{4}} = \dfrac{\sqrt{85}}{2}$

(d) $\dfrac{\mathbf{u}}{\|\mathbf{u}\|} = \dfrac{2}{\sqrt{5}}\left\langle 1, \dfrac{1}{2}\right\rangle$

$\left\|\dfrac{\mathbf{u}}{\|\mathbf{u}\|}\right\| = 1$

(e) $\dfrac{\mathbf{v}}{\|\mathbf{v}\|} = \dfrac{1}{\sqrt{13}}\langle 2, 3\rangle$

$\left\|\dfrac{\mathbf{v}}{\|\mathbf{v}\|}\right\| = 1$

(f) $\dfrac{\mathbf{u}+\mathbf{v}}{\|\mathbf{u}+\mathbf{v}\|} = \dfrac{2}{\sqrt{85}}\left\langle 3, \dfrac{7}{2}\right\rangle$

$\left\|\dfrac{\mathbf{u}+\mathbf{v}}{\|\mathbf{u}+\mathbf{v}\|}\right\| = 1$

45. $\mathbf{u} = \langle 2, 1\rangle$

$\|\mathbf{u}\| = \sqrt{5} \approx 2.236$

$\mathbf{v} = \langle 5, 4\rangle$

$\|\mathbf{v}\| = \sqrt{41} \approx 6.403$

$\mathbf{u} + \mathbf{v} = \langle 7, 5\rangle$

$\|\mathbf{u} + \mathbf{v}\| = \sqrt{74} \approx 8.602$

$\|\mathbf{u} + \mathbf{v}\| \le \|\mathbf{u}\| + \|\mathbf{v}\|$

47. $\dfrac{\mathbf{u}}{\|\mathbf{u}\|} = \dfrac{1}{\sqrt{2}}\langle 1, 1\rangle$

$4\left(\dfrac{\mathbf{u}}{\|\mathbf{u}\|}\right) = 2\sqrt{2}\langle 1, 1\rangle$

$\mathbf{v} = \langle 2\sqrt{2}, 2\sqrt{2}\rangle$

49. $\dfrac{\mathbf{u}}{\|\mathbf{u}\|} = \dfrac{1}{2\sqrt{3}}\langle \sqrt{3}, 3\rangle$

$2\left(\dfrac{\mathbf{u}}{\|\mathbf{u}\|}\right) = \dfrac{1}{\sqrt{3}}\langle \sqrt{3}, 3\rangle$

$\mathbf{v} = \langle 1, \sqrt{3}\rangle$

51. $\mathbf{v} = 3[(\cos 0°)\mathbf{i} + (\sin 0°)\mathbf{j}] = 3\mathbf{i} = \langle 3, 0\rangle$

53. $\mathbf{v} = 2[(\cos 150°)\mathbf{i} + (\sin 150°)\mathbf{j}]$

$= -\sqrt{3}\mathbf{i} + \mathbf{j} = \langle -\sqrt{3}, 1\rangle$

55. $\mathbf{u} = \mathbf{i}$

$\mathbf{v} = \dfrac{3\sqrt{2}}{2}\mathbf{i} + \dfrac{3\sqrt{2}}{2}\mathbf{j}$

$\mathbf{u} + \mathbf{v} = \left(\dfrac{2 + 3\sqrt{2}}{2}\right)\mathbf{i} + \dfrac{3\sqrt{2}}{2}\mathbf{j}$

57. $\mathbf{u} = 2(\cos 4)\mathbf{i} + 2(\sin 4)\mathbf{j}$

$\mathbf{v} = (\cos 2)\mathbf{i} + (\sin 2)\mathbf{j}$

$\mathbf{u} + \mathbf{v} = (2\cos 4 + \cos 2)\mathbf{i} + (2\sin 4 + \sin 2)\mathbf{j}$

59. A scalar is a real number. A vector is represented by a directed line segment. A vector has both length and direction.

61. To normalize $\mathbf{v}$, you find a unit vector $\mathbf{u}$ in the direction of $\mathbf{v}$:

$$\mathbf{u} = \frac{\mathbf{v}}{\|\mathbf{v}\|}.$$

For Exercises 63–67, $a\mathbf{u} + b\mathbf{w} = a(\mathbf{i} + 2\mathbf{j}) + b(\mathbf{i} - \mathbf{j}) = (a + b)\mathbf{i} + (2a - b)\mathbf{j}$.

63. $\mathbf{v} = 2\mathbf{i} + \mathbf{j}$. Therefore, $a + b = 2$, $2a - b = 1$. Solving simultaneously, we have $a = 1$, $b = 1$.

65. $\mathbf{v} = 3\mathbf{i}$. Therefore, $a + b = 3$, $2a - b = 0$. Solving simultaneously, we have $a = 1$, $b = 2$.

67. $\mathbf{v} = \mathbf{i} + \mathbf{j}$. Therefore, $a + b = 1$, $2a - b = 1$. Solving simultaneously, we have $a = \frac{2}{3}$, $b = \frac{1}{3}$.

69. $y = x^3$, $y' = 3x^2 = 3$ at $x = 1$.

(a) $m = 3$. Let $\mathbf{w} = \langle 1, 3\rangle$, then

$$\frac{\mathbf{w}}{\|\mathbf{w}\|} = \pm\frac{1}{\sqrt{10}}\langle 1, 3\rangle.$$

(b) $m = -\frac{1}{3}$. Let $\mathbf{w} = \langle 3, -1\rangle$, then

$$\frac{\mathbf{w}}{\|\mathbf{w}\|} = \pm\frac{1}{\sqrt{10}}\langle 3, -1\rangle.$$

71. $f(x) = \sqrt{25 - x^2}$

$$f'(x) = \frac{-x}{\sqrt{25 - x^2}} = \frac{-3}{4} \text{ at } x = 3.$$

(a) $m = -\frac{3}{4}$. Let $\mathbf{w} = \langle -4, 3\rangle$, then

$$\frac{\mathbf{w}}{\|\mathbf{w}\|} = \pm\frac{1}{5}\langle -4, 3\rangle.$$

(b) $m = \frac{4}{3}$. Let $\mathbf{w} = \langle 3, 4\rangle$, then

$$\frac{\mathbf{w}}{\|\mathbf{w}\|} = \pm\frac{1}{5}\langle 3, 4\rangle$$

73. $\mathbf{u} = \frac{\sqrt{2}}{2}\mathbf{i} + \frac{\sqrt{2}}{2}\mathbf{j}$

$\mathbf{u} + \mathbf{v} = \sqrt{2}\mathbf{j}$

$\mathbf{v} = (\mathbf{u} + \mathbf{v}) - \mathbf{u} = -\frac{\sqrt{2}}{2}\mathbf{i} + \frac{\sqrt{2}}{2}\mathbf{j}$

75. Programs will vary.

77. $\|\mathbf{F}_1\| = 2$, $\theta_{\mathbf{F}_1} = 33°$

$\|\mathbf{F}_2\| = 3$, $\theta_{\mathbf{F}_2} = -125°$

$\|\mathbf{F}_3\| = 2.5$, $\theta_{\mathbf{F}_3} = 110°$

$\|\mathbf{R}\| = \|\mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3\| \approx 1.33$

$\theta_{\mathbf{R}} = \theta_{\mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3} \approx 132.5°$

79. (a) $180(\cos 30\mathbf{i} + \sin 30\mathbf{j}) + 275\mathbf{i} = 430.88\mathbf{i} + 90\mathbf{j}$

Direction: $\alpha = \arctan\left(\frac{90}{430.88}\right) = 0.206(= 11.8°)$

Magnitude: $\sqrt{430.88^2 + 90^2} = 440.18$ newtons

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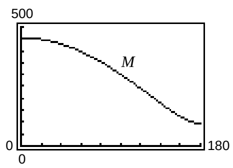
(b) $M = \sqrt{(275 + 180\cos\theta)^2 + (180\sin\theta)^2}$

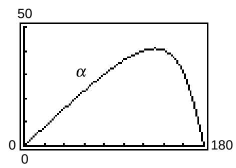
$$\alpha = \arctan\left[\frac{180\sin\theta}{275 + 180\cos\theta}\right]$$

79. —CONTINUED—

(c)

θ	0°	30°	60°	90°	120°	150°	180°
M	455	440.2	396.9	328.7	241.9	149.3	95
α	0°	11.8°	23.1°	33.2°	40.1°	37.1°	0

(d) 



(e) M decreases because the forces change from acting in the same direction to acting in the opposite direction as θ increases from 0° to 180°.

81. $\mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 = (75\cos 30°\mathbf{i} + 75\sin 30°\mathbf{j}) + (100\cos 45°\mathbf{i} + 100\sin 45°\mathbf{j}) + (125\cos 120°\mathbf{i} + 125\sin 120°\mathbf{j})$

$$= \left(\frac{75}{2}\sqrt{3} + 50\sqrt{2} - \frac{125}{2}\right)\mathbf{i} + \left(\frac{75}{2} + 50\sqrt{2} + \frac{125}{2}\sqrt{3}\right)\mathbf{j}$$

$$\|\mathbf{R}\| = \|\mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3\| \approx 228.5 \text{ lb}$$

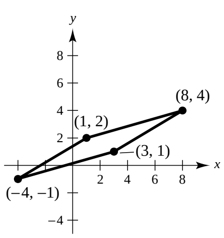
$$\theta_{\mathbf{R}} = \theta_{\mathbf{F}_1+\mathbf{F}_2+\mathbf{F}_3} \approx 71.3°$$

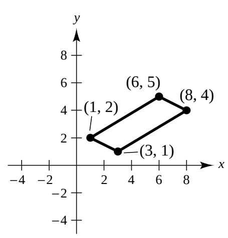
83. (a) The forces act along the same direction. $\theta = 0°$.

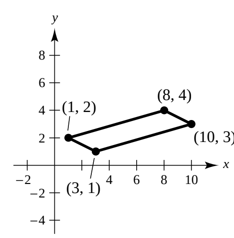
(b) The forces cancel out each other. $\theta = 180°$.

(c) No, the magnitude of the resultant can not be greater than the sum.

85. $(-4, -1)$, $(6, 5)$, $(10, 3)$







87. $\mathbf{u} = \overrightarrow{CB} = \|\mathbf{u}\|(\cos 30°\,\mathbf{i} + \sin 30°\,\mathbf{j})$

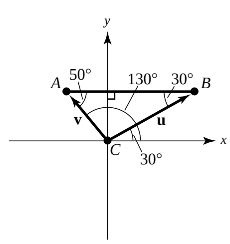
$\mathbf{v} = \overrightarrow{CA} = \|\mathbf{v}\|(\cos 130°\,\mathbf{i} + \sin 130°\,\mathbf{j})$

Vertical components: $\|\mathbf{u}\|\sin 30° + \|\mathbf{v}\|\sin 130° = 2000$

Horizontal components: $\|\mathbf{u}\|\cos 30° + \|\mathbf{v}\|\cos 130° = 0$

Solving this system, you obtain

$\|\mathbf{u}\| \approx 1305.5$ and $\|\mathbf{v}\| \approx 1758.8$.



89. Horizontal component $= \|\mathbf{v}\|\cos\theta = 1200\cos 6° \approx 1193.43$ ft/sec

Vertical component $= \|\mathbf{v}\|\sin\theta = 1200\sin 6° \approx 125.43$ ft/sec

91. $\mathbf{u} = 900[\cos 148°\,\mathbf{i} + \sin 148°\,\mathbf{j}]$

$\mathbf{v} = 100[\cos 45°\,\mathbf{i} + \sin 45°\,\mathbf{j}]$

$$\mathbf{u} + \mathbf{v} = [900\cos 148° + 100\cos 45°]\mathbf{i} + [900\sin 148° + 100\sin 45°]\mathbf{j}$$

$$\approx -692.53\,\mathbf{i} + 547.64\,\mathbf{j}$$

$$\theta \approx \arctan\left(\frac{547.64}{-692.53}\right) \approx -38.34°. \quad 38.34° \text{ North of West.}$$

$$\|\mathbf{u} + \mathbf{v}\| = \sqrt{(-692.53)^2 + (547.64)^2} \approx 882.9 \text{ km/hr.}$$

93. $\mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 = \mathbf{0}$

$-3600\mathbf{j} + T_2(\cos 35°\mathbf{i} - \sin 35°\,\mathbf{j}) + T_3(\cos 92°\mathbf{i} + \sin 92°\mathbf{j}) = 0$

$T_2 \cos 35° + T_3 \cos 92° = 0$

$-T_2 \cos 35° + T_3 \sin 92° = 3600$

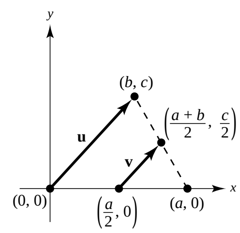
$T_2 = \dfrac{-T_3 \cos 92°}{\cos 35°} \implies \dfrac{T_3 \cos 92°}{\cos 35°}\sin 35° + T_3 \sin 92° = 3600$ and $T_3(0.97495) = 3600 \implies T_3 \approx 3692.48$

Finally, $T_2 = 157.32$

95. Let the triangle have vertices at $(0, 0)$, $(a, 0)$, and (b, c). Let $\mathbf{u}$ be the vector joining $(0, 0)$ and (b, c), as indicated in the figure. Then $\mathbf{v}$, the vector joining the midpoints, is

$$\mathbf{v} = \left(\frac{a + b}{2} - \frac{a}{2}\right)\mathbf{i} + \frac{c}{2}\mathbf{j}$$

$$= \frac{b}{2}\mathbf{i} + \frac{c}{2}\mathbf{j} = \frac{1}{2}(b\mathbf{i} + c\mathbf{j}) = \frac{1}{2}\mathbf{u}$$



97. $\mathbf{w} = \|\mathbf{u}\|\mathbf{v} + \|\mathbf{v}\|\mathbf{u}$

$$= \|\mathbf{u}\|\big[\|\mathbf{v}\|\cos\theta_{\mathbf{v}}\,\mathbf{i} + \|\mathbf{v}\|\sin\theta_{\mathbf{v}}\,\mathbf{j}\big] + \|\mathbf{v}\|\big[\|\mathbf{u}\|\cos\theta_{\mathbf{u}}\mathbf{i} + \|\mathbf{u}\|\sin\theta_{\mathbf{u}}\mathbf{j}\big] = \|\mathbf{u}\|\,\|\mathbf{v}\|\big[(\cos\theta_{\mathbf{u}} + \cos\theta_{\mathbf{v}})\mathbf{i} + (\sin\theta_{\mathbf{u}} + \sin\theta_{\mathbf{v}})\,\mathbf{j}\big]$$

$$= 2\|\mathbf{u}\|\,\|\mathbf{v}\|\left[\cos\left(\frac{\theta_{\mathbf{u}} + \theta_{\mathbf{v}}}{2}\right)\cos\left(\frac{\theta_{\mathbf{u}} - \theta_{\mathbf{v}}}{2}\right)\mathbf{i} + \sin\left(\frac{\theta_{\mathbf{u}} + \theta_{\mathbf{v}}}{2}\right)\cos\left(\frac{\theta_{\mathbf{u}} - \theta_{\mathbf{v}}}{2}\right)\mathbf{j}\right]$$

$$\tan\theta_{\mathbf{w}} = \frac{\sin\left(\frac{\theta_{\mathbf{u}} + \theta_{\mathbf{v}}}{2}\right)\cos\left(\frac{\theta_{\mathbf{u}} - \theta_{\mathbf{v}}}{2}\right)}{\cos\left(\frac{\theta_{\mathbf{u}} + \theta_{\mathbf{v}}}{2}\right)\cos\left(\frac{\theta_{\mathbf{u}} - \theta_{\mathbf{v}}}{2}\right)} = \tan\left(\frac{\theta_{\mathbf{u}} + \theta_{\mathbf{v}}}{2}\right)$$

Thus, $\theta_{\mathbf{w}} = (\theta_{\mathbf{u}} + \theta_{\mathbf{v}})/2$ and $\mathbf{w}$ bisects the angle between $\mathbf{u}$ and $\mathbf{v}$.

99. True

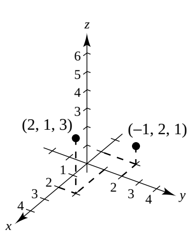
101. True

103. False

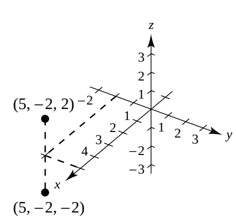
$\|a\mathbf{i} + b\mathbf{j}\| = \sqrt{2}\,|a|$

Section 10.2 Space Coordinates and Vectors in Space

1.



3.



5. $A(2, 3, 4)$

$B(-1, -2, 2)$

7. $x = -3, y = 4, z = 5$: $(-3, 4, 5)$

9. $y = z = 0, x = 10$: $(10, 0, 0)$

11. The z-coordinate is 0.

13. The point is 6 units above the xy-plane.

15. The point is on the plane parallel to the yz-plane that passes through $x = 4$.

17. The point is to the left of the xz-plane.

19. The point is on or between the planes $y = 3$ and $y = -3$.

21. The point (x, y, z) is 3 units below the xy-plane, and below either quadrant I or III.

23. The point could be above the xy-plane and thus above quadrants II or IV, or below the xy-plane, and thus below quadrants I or III.

25. $d = \sqrt{(5-0)^2 + (2-0)^2 + (6-0)^2}$

$= \sqrt{25 + 4 + 36} = \sqrt{65}$

27. $d = \sqrt{(6-1)^2 + (-2-(-2))^2 + (-2-4)^2}$

$= \sqrt{25 + 0 + 36} = \sqrt{61}$

29. $A(0, 0, 0), B(2, 2, 1), C(2, -4, 4)$

$|AB| = \sqrt{4 + 4 + 1} = 3$

$|AC| = \sqrt{4 + 16 + 16} = 6$

$|BC| = \sqrt{0 + 36 + 9} = 3\sqrt{5}$

$|BC|^2 = |AB|^2 + |AC|^2$

Right triangle

31. $A(1, -3, -2), B(5, -1, 2), C(-1, 1, 2)$

$|AB| = \sqrt{16 + 4 + 16} = 6$

$|AC| = \sqrt{4 + 16 + 16} = 6$

$|BC| = \sqrt{36 + 4 + 0} = 2\sqrt{10}$

Since $|AB| = |AC|$, the triangle is isosceles.

33. The z-coordinate is changed by 5 units:

$(0, 0, 5), (2, 2, 6), (2, -4, 9)$

35. $\left(\dfrac{5 + (-2)}{2}, \dfrac{-9 + 3}{2}, \dfrac{7 + 3}{2}\right) = \left(\dfrac{3}{2}, -3, 5\right)$

37. Center: $(0, 2, 5)$

Radius: 2

$(x-0)^2 + (y-2)^2 + (z-5)^2 = 4$

$x^2 + y^2 + z^2 - 4y - 10z + 25 = 0$

39. Center: $\dfrac{(2, 0, 0) + (0, 6, 0)}{2} = (1, 3, 0)$

Radius: $\sqrt{10}$

$(x-1)^2 + (y-3)^2 + (z-0)^2 = 10$

$x^2 + y^2 + z^2 - 2x - 6y = 0$

41.

$$x^2 + y^2 + z^2 - 2x + 6y + 8z + 1 = 0$$

$$(x^2 - 2x + 1) + (y^2 + 6y + 9) + (z^2 + 8z + 16) = -1 + 1 + 9 + 16$$

$$(x-1)^2 + (y+3)^2 + (z+4)^2 = 25$$

Center: $(1, -3, -4)$

Radius: 5

43. $9x^2 + 9y^2 + 9z^2 - 6x + 18y + 1 = 0$

$$x^2 + y^2 + z^2 - \frac{2}{3}x + 2y + \frac{1}{9} = 0$$

$$\left(x^2 - \frac{2}{3}x + \frac{1}{9}\right) + (y^2 + 2y + 1) + z^2 = -\frac{1}{9} + \frac{1}{9} + 1$$

$$\left(x - \frac{1}{3}\right)^2 + (y + 1)^2 + (z - 0)^2 = 1$$

Center: $\left(\frac{1}{3}, -1, 0\right)$

Radius: 1

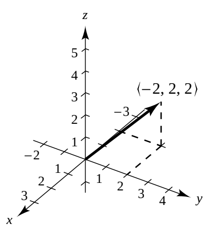
45. $x^2 + y^2 + z^2 \le 36$

Solid ball of radius 6 centered at origin.

47. (a) $\mathbf{v} = (2 - 4)\mathbf{i} + (4 - 2)\mathbf{j} + (3 - 1)\mathbf{k}$

$= -2\mathbf{i} + 2\mathbf{j} + 2\mathbf{k} = \langle -2, 2, 2\rangle$

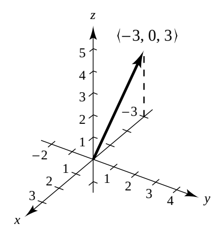
(b)



49. (a) $\mathbf{v} = (0 - 3)\mathbf{i} + (3 - 3)\mathbf{j} + (3 - 0)\mathbf{k}$

$= -3\mathbf{i} + 3\mathbf{k} = \langle -3, 0, 3\rangle$

(b)



51. $\langle 4 - 3, 1 - 2, 6 - 0\rangle = \langle 1, -1, 6\rangle$

$\|\langle 1, -1, 6\rangle\| = \sqrt{1 + 1 + 36} = \sqrt{38}$

Unit vector: $\dfrac{\langle 1, -1, 6\rangle}{\sqrt{38}} = \left\langle \dfrac{1}{\sqrt{38}}, \dfrac{-1}{\sqrt{38}}, \dfrac{6}{\sqrt{38}}\right\rangle$

53. $\langle -5 - (-4), 3 - 3, 0 - 1\rangle = \langle -1, 0, -1\rangle$

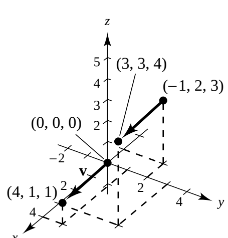
$\|\langle -1, 0, -1\rangle\| = \sqrt{1 + 1} = \sqrt{2}$

Unit vector: $\left\langle \dfrac{-1}{\sqrt{2}}, 0, \dfrac{-1}{\sqrt{2}}\right\rangle$

55. (b) $\mathbf{v} = (3 + 1)\mathbf{i} + (3 - 2)\mathbf{j} + (4 - 3)\mathbf{k}$

$= 4\mathbf{i} + \mathbf{j} + \mathbf{k} = \langle 4, 1, 1\rangle$

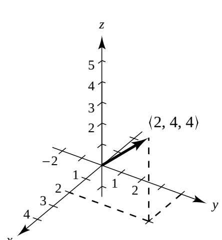
(a) and (c).



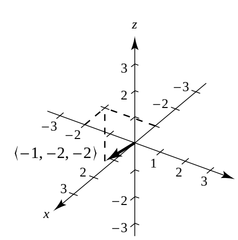
57. $(q_1, q_2, q_3) - (0, 6, 2) = (3, -5, 6)$

$Q = (3, 1, 8)$

59. (a) $2\mathbf{v} = \langle 2, 4, 4\rangle$



(b) $-\mathbf{v} = \langle -1, -2, -2\rangle$



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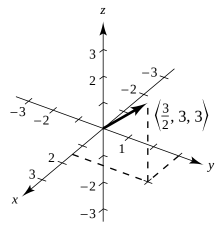
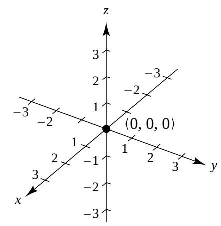
59. —CONTINUED—

(c) $\frac{3}{2}\mathbf{v} = \left\langle \frac{3}{2}, 3, 3 \right\rangle$



(d) $0\mathbf{v} = \langle 0, 0, 0 \rangle$



61. $\mathbf{z} = \mathbf{u} - \mathbf{v} = \langle 1, 2, 3 \rangle - \langle 2, 2, -1 \rangle = \langle -1, 0, 4 \rangle$

63. $\mathbf{z} = 2\mathbf{u} + 4\mathbf{v} - \mathbf{w} = \langle 2, 4, 6 \rangle + \langle 8, 8, -4 \rangle - \langle 4, 0, -4 \rangle = \langle 6, 12, 6 \rangle$

65. $2\mathbf{z} - 3\mathbf{u} = 2\langle z_1, z_2, z_3 \rangle - 3\langle 1, 2, 3 \rangle = \langle 4, 0, -4 \rangle$

$2z_1 - 3 = 4 \implies z_1 = \frac{7}{2}$

$2z_2 - 6 = 0 \implies z_2 = 3$

$2z_3 - 9 = -4 \implies z_3 = \frac{5}{2}$

$\mathbf{z} = \left\langle \frac{7}{2}, 3, \frac{5}{2} \right\rangle$

67. (a) and (b) are parallel since $\langle -6, -4, 10 \rangle = -2\langle 3, 2, -5 \rangle$ and $\left\langle 2, \frac{4}{3}, -\frac{10}{3} \right\rangle = \frac{2}{3}\langle 3, 2, -5 \rangle$.

69. $\mathbf{z} = -3\mathbf{i} + 4\mathbf{j} + 2\mathbf{k}$

(a) is parallel since $-6\mathbf{i} + 8\mathbf{j} + 4\mathbf{k} = 2\mathbf{z}$.

71. $P(0, -2, -5)$, $Q(3, 4, 4)$, $R(2, 2, 1)$

$\overrightarrow{PQ} = \langle 3, 6, 9 \rangle$

$\overrightarrow{PR} = \langle 2, 4, 6 \rangle$

$\langle 3, 6, 9 \rangle = \frac{3}{2}\langle 2, 4, 6 \rangle$

Therefore, $\overrightarrow{PQ}$ and $\overrightarrow{PR}$ are parallel. The points are collinear.

73. $P(1, 2, 4)$, $Q(2, 5, 0)$, $R(0, 1, 5)$

$\overrightarrow{PQ} = \langle 1, 3, -4 \rangle$

$\overrightarrow{PR} = \langle -1, -1, 1 \rangle$

Since $\overrightarrow{PQ}$ and $\overrightarrow{PR}$ are not parallel, the points are not collinear.

75. $A(2, 9, 1)$, $B(3, 11, 4)$, $C(0, 10, 2)$, $D(1, 12, 5)$

$\overrightarrow{AB} = \langle 1, 2, 3 \rangle$

$\overrightarrow{CD} = \langle 1, 2, 3 \rangle$

$\overrightarrow{AC} = \langle -2, 1, 1 \rangle$

$\overrightarrow{BD} = \langle -2, 1, 1 \rangle$

Since $\overrightarrow{AB} = \overrightarrow{CD}$ and $\overrightarrow{AC} = \overrightarrow{BD}$, the given points form the vertices of a parallelogram.

77. $\|\mathbf{v}\| = 0$

79. $\mathbf{v} = \langle 1, -2, -3 \rangle$

$\|\mathbf{v}\| = \sqrt{1 + 4 + 9} = \sqrt{14}$

81. $\mathbf{v} = \langle 0, 3, -5 \rangle$

$\|\mathbf{v}\| = \sqrt{0 + 9 + 25} = \sqrt{34}$

83. $\mathbf{u} = \langle 2, -1, 2 \rangle$

$\|\mathbf{u}\| = \sqrt{4 + 1 + 4} = 3$

(a) $\dfrac{\mathbf{u}}{\|\mathbf{u}\|} = \dfrac{1}{3}\langle 2, -1, 2 \rangle$

(b) $-\dfrac{\mathbf{u}}{\|\mathbf{u}\|} = -\dfrac{1}{3}\langle 2, -1, 2 \rangle$

85. $\mathbf{u} = \langle 3, 2, -5 \rangle$

$\|\mathbf{u}\| = \sqrt{9 + 4 + 25} = \sqrt{38}$

(a) $\dfrac{\mathbf{u}}{\|\mathbf{u}\|} = \dfrac{1}{\sqrt{38}}\langle 3, 2, -5 \rangle$

(b) $-\dfrac{\mathbf{u}}{\|\mathbf{u}\|} = -\dfrac{1}{\sqrt{38}}\langle 3, 2, -5 \rangle$

87. Programs will vary.

89. $c\mathbf{v} = \langle 2c, 2c, -c\rangle$

$\|c\mathbf{v}\| = \sqrt{4c^2 + 4c^2 + c^2} = 5$

$9c^2 = 25$

$c = \pm\frac{5}{3}$

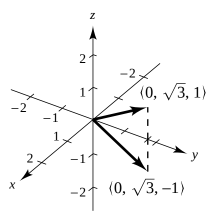
91. $\mathbf{v} = 10\frac{\mathbf{u}}{\|\mathbf{u}\|} = 10\left\langle 0, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right\rangle$

$= \left\langle 0, \frac{10}{\sqrt{2}}, \frac{10}{\sqrt{2}}\right\rangle$

93. $\mathbf{v} = \frac{3}{2}\frac{\mathbf{u}}{\|\mathbf{u}\|} = \frac{3}{2}\left\langle\frac{2}{3}, \frac{-2}{3}, \frac{1}{3}\right\rangle = \left\langle 1, -1, \frac{1}{2}\right\rangle$

95. $\mathbf{v} = 2[\cos(\pm 30°)\mathbf{j} + \sin(\pm 30°)\mathbf{k}]$

$= \sqrt{3}\mathbf{j} \pm \mathbf{k} = \langle 0, \sqrt{3}, \pm 1\rangle$

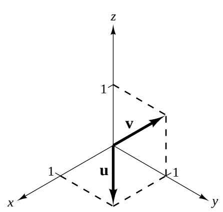


97. $\mathbf{v} = \langle -3, -6, 3\rangle$

$\frac{2}{3}\mathbf{v} = \langle -2, -4, 2\rangle$

$(4, 3, 0) + (-2, -4, 2) = (2, -1, 2)$

99. (a)



(b) $\mathbf{w} = a\mathbf{u} + b\mathbf{v} = a\mathbf{i} + (a + b)\mathbf{j} + b\mathbf{k} = \mathbf{0}$

$a = 0, a + b = 0, b = 0$

Thus, a and b are both zero.

(c) $a\mathbf{i} + (a + b)\mathbf{j} + b\mathbf{k} = \mathbf{i} + 2\mathbf{j} + \mathbf{k}$

$a = 1, b = 1$

$\mathbf{w} = \mathbf{u} + \mathbf{v}$

(d) $a\mathbf{i} + (a + b)\mathbf{j} + b\mathbf{k} = \mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$

$a = 1, a + b = 2, b = 3$

Not possible

101. $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$

103. Two nonzero vectors **u** and **v** are parallel if $\mathbf{u} = c\mathbf{v}$ for some scalar c.

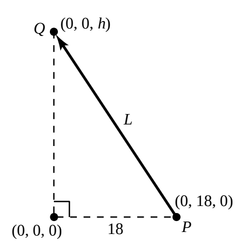
105. (a) The height of the right triangle is $h = \sqrt{L^2 - 18^2}$. The vector $\overrightarrow{PQ}$ is given by

$\overrightarrow{PQ} = \langle 0, -18, h\rangle$.

The tension vector **T** in each wire is

$\mathbf{T} = c\langle 0, -18, h\rangle$ where $ch = \frac{24}{3} = 8$.

Hence, $\mathbf{T} = \frac{8}{h}\langle 0, -18, h\rangle$ and



$$T = \|\mathbf{T}\| = \frac{8}{h}\sqrt{18^2 + h^2} = \frac{8}{\sqrt{L^2 - 18^2}}\sqrt{18^2 + (L^2 - 18^2)} = \frac{8L}{\sqrt{L^2 - 18^2}}$$

(b)

L	20	25	30	35	40	45	50
T	18.4	11.5	10	9.3	9.0	8.7	8.6

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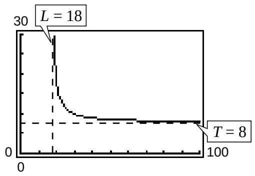
105. —CONTINUED—

(c) 

$x = 18$ is a vertical asymptote and $y = 8$ is a horizontal asymptote.

(d) $\lim_{L \to 18^+} \frac{8L}{\sqrt{L^2 - 18^2}} = \infty$

$$\lim_{L \to \infty} \frac{8L}{\sqrt{L^2 - 18^2}} = \lim_{L \to \infty} \frac{8}{\sqrt{1 - (18/L)^2}} = 8$$

(e) From the table, $T = 10$ implies $L = 30$ inches.

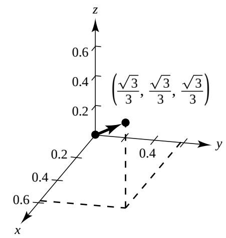
107. Let α be the angle between $\mathbf{v}$ and the coordinate axes.

$$\mathbf{v} = (\cos \alpha)\mathbf{i} + (\cos \alpha)\mathbf{j} + (\cos \alpha)\mathbf{k}$$

$$\|\mathbf{v}\| = \sqrt{3} \cos \alpha = 1$$

$$\cos \alpha = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$

$$\mathbf{v} = \frac{\sqrt{3}}{3}(\mathbf{i} + \mathbf{j} + \mathbf{k}) = \frac{\sqrt{3}}{3}\langle 1, 1, 1 \rangle$$



109. $\overrightarrow{AB} = \langle 0, 70, 115 \rangle, \mathbf{F}_1 = C_1\langle 0, 70, 115 \rangle$

$\overrightarrow{AC} = \langle -60, 0, 115 \rangle, \mathbf{F}_2 = C_2\langle -60, 0, 115 \rangle$

$\overrightarrow{AD} = \langle 45, -65, 115 \rangle, \mathbf{F}_3 = C_3\langle 45, -65, 115 \rangle$

$\mathbf{F} = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 = \langle 0, 0, 500 \rangle$

Thus:

$$\begin{aligned} -60C_2 + 45C_3 &= 0 \\ 70C_1 - 65C_3 &= 0 \\ 115(C_1 + C_2 + C_3) &= 500 \end{aligned}$$

Solving this system yields $C_1 = \frac{104}{69}$, $C_2 = \frac{28}{23}$, and $C_3 = -\frac{112}{69}$. Thus:

$\|\mathbf{F}_1\| \approx 202.919N$

$\|\mathbf{F}_2\| \approx 157.909N$

$\|\mathbf{F}_3\| \approx 226.521N$

111. $d(AP) = 2d(BP)$

$$\sqrt{x^2 + (y+1)^2 + (z-1)^2} = 2\sqrt{(x-1)^2 + (y-2)^2 + z^2}$$

$$x^2 + y^2 + z^2 + 2y - 2z + 2 = 4(x^2 + y^2 + z^2 - 2x - 4y + 5)$$

$$0 = 3x^2 + 3y^2 + 3z^2 - 8x - 18y + 2z + 18$$

$$-6 + \frac{16}{9} + 9 + \frac{1}{9} = \left(x^2 - \frac{8}{3}x + \frac{16}{9}\right) + (y^2 - 6y + 9) + \left(z^2 + \frac{2}{3}z + \frac{1}{9}\right)$$

$$\frac{44}{9} = \left(x - \frac{4}{3}\right)^2 + (y-3)^2 + \left(z + \frac{1}{3}\right)^2$$

Sphere; center: $\left(\frac{4}{3}, 3, -\frac{1}{3}\right)$, radius: $\frac{2\sqrt{11}}{3}$

Section 10.3 The Dot Product of Two Vectors

1. $\mathbf{u} = \langle 3, 4\rangle, \mathbf{v} = \langle 2, -3\rangle$

(a) $\mathbf{u}\cdot\mathbf{v} = 3(2) + 4(-3) = -6$

(b) $\mathbf{u}\cdot\mathbf{u} = 3(3) + 4(4) = 25$

(c) $\|\mathbf{u}\|^2 = 25$

(d) $(\mathbf{u}\cdot\mathbf{v})\mathbf{v} = -6\langle 2, -3\rangle = \langle -12, 18\rangle$

(e) $\mathbf{u}\cdot(2\mathbf{v}) = 2(\mathbf{u}\cdot\mathbf{v}) = 2(-6) = -12$

3. $\mathbf{u} = \langle 2, -3, 4\rangle, \mathbf{v} = \langle 0, 6, 5\rangle$

(a) $\mathbf{u}\cdot\mathbf{v} = 2(0) + (-3)(6) + (4)(5) = 2$

(b) $\mathbf{u}\cdot\mathbf{u} = 2(2) + (-3)(-3) + 4(4) = 29$

(c) $\|\mathbf{u}\|^2 = 29$

(d) $(\mathbf{u}\cdot\mathbf{v})\mathbf{v} = 2\langle 0, 6, 5\rangle = \langle 0, 12, 10\rangle$

(e) $\mathbf{u}\cdot(2\mathbf{v}) = 2(\mathbf{u}\cdot\mathbf{v}) = 2(2) = 4$

5. $\mathbf{u} = 2\mathbf{i} - \mathbf{j} + \mathbf{k}, \mathbf{v} = \mathbf{i} - \mathbf{k}$

(a) $\mathbf{u}\cdot\mathbf{v} = 2(1) + (-1)(0) + 1(-1) = 1$

(b) $\mathbf{u}\cdot\mathbf{u} = 2(2) + (-1)(-1) + (1)(1) = 6$

(c) $\|\mathbf{u}\|^2 = 6$

(d) $(\mathbf{u}\cdot\mathbf{v})\mathbf{v} = \mathbf{v} = \mathbf{i} - \mathbf{k}$

(e) $\mathbf{u}\cdot(2\mathbf{v}) = 2(\mathbf{u}\cdot\mathbf{v}) = 2$

7. $\mathbf{u} = \langle 3240, 1450, 2235\rangle$

$\mathbf{v} = \langle 2.22, 1.85, 3.25\rangle$

$\mathbf{u}\cdot\mathbf{v} = \$17{,}139.05$

This gives the total amount that the person earned on his products.

9. $\dfrac{\mathbf{u}\cdot\mathbf{v}}{\|\mathbf{u}\|\,\|\mathbf{v}\|} = \cos\theta$

$$\mathbf{u}\cdot\mathbf{v} = (8)(5)\cos\frac{\pi}{3} = 20$$

11. $\mathbf{u} = \langle 1, 1\rangle, \mathbf{v} = \langle 2, -2\rangle$

$$\cos\theta = \frac{\mathbf{u}\cdot\mathbf{v}}{\|\mathbf{u}\|\,\|\mathbf{v}\|} = \frac{0}{\sqrt{2}\sqrt{8}} = 0$$

$$\theta = \frac{\pi}{2}$$

13. $\mathbf{u} = 3\mathbf{i} + \mathbf{j}, \mathbf{v} = -2\mathbf{i} + 4\mathbf{j}$

$$\cos\theta = \frac{\mathbf{u}\cdot\mathbf{v}}{\|\mathbf{u}\|\,\|\mathbf{v}\|} = \frac{-2}{\sqrt{10}\sqrt{20}} = \frac{-1}{5\sqrt{2}}$$

$$\theta = \arccos\left(-\frac{1}{5\sqrt{2}}\right) \approx 98.1^\circ$$

15. $\mathbf{u} = \langle 1, 1, 1\rangle, \mathbf{v} = \langle 2, 1, -1\rangle$

$$\cos\theta = \frac{\mathbf{u}\cdot\mathbf{v}}{\|\mathbf{u}\|\,\|\mathbf{v}\|} = \frac{2}{\sqrt{3}\sqrt{6}} = \frac{\sqrt{2}}{3}$$

$$\theta = \arccos\frac{\sqrt{2}}{3} \approx 61.9^\circ$$

17. $\mathbf{u} = 3\mathbf{i} + 4\mathbf{j}, \mathbf{v} = -2\mathbf{j} + 3\mathbf{k}$

$$\cos\theta = \frac{\mathbf{u}\cdot\mathbf{v}}{\|\mathbf{u}\|\,\|\mathbf{v}\|} = \frac{-8}{5\sqrt{13}} = \frac{-8\sqrt{13}}{65}$$

$$\theta = \arccos\left(-\frac{8\sqrt{13}}{65}\right) \approx 116.3^\circ$$

19. $\mathbf{u} = \langle 4, 0\rangle, \mathbf{v} = \langle 1, 1\rangle$

$\mathbf{u} \neq c\mathbf{v} \implies$ not parallel

$\mathbf{u}\cdot\mathbf{v} = 4 \neq 0 \implies$ not orthogonal

Neither

21. $\mathbf{u} = \langle 4, 3\rangle, \mathbf{v} = \left\langle \frac{1}{2}, -\frac{2}{3}\right\rangle$

$\mathbf{u} \neq c\mathbf{v} \implies$ not parallel

$\mathbf{u}\cdot\mathbf{v} = 0 \implies$ orthogonal

23. $\mathbf{u} = \mathbf{j} + 6\mathbf{k}, \mathbf{v} = \mathbf{i} - 2\mathbf{j} - \mathbf{k}$

$\mathbf{u} \neq c\mathbf{v} \implies$ not parallel

$\mathbf{u}\cdot\mathbf{v} = -8 \neq 0 \implies$ not orthogonal

Neither

25. $\mathbf{u} = \langle 2, -3, 1\rangle, \mathbf{v} = \langle -1, -1, -1\rangle$

$\mathbf{u} \neq c\mathbf{v} \implies$ not parallel

$\mathbf{u}\cdot\mathbf{v} = 0 \implies$ orthogonal

27. $\mathbf{u} = \mathbf{i} + 2\mathbf{j} + 2\mathbf{k}, \|\mathbf{u}\| = 3$

$$\cos \alpha = \frac{1}{3}$$

$$\cos \beta = \frac{2}{3}$$

$$\cos \gamma = \frac{2}{3}$$

$$\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = \frac{1}{9} + \frac{4}{9} + \frac{4}{9} = 1$$

29. $\mathbf{u} = \langle 0, 6, -4\rangle, \|\mathbf{u}\| = \sqrt{52} = 2\sqrt{13}$

$$\cos \alpha = 0$$

$$\cos \beta = \frac{3}{\sqrt{13}}$$

$$\cos \gamma = -\frac{2}{\sqrt{13}}$$

$$\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 0 + \frac{9}{13} + \frac{4}{13} = 1$$

31. $\mathbf{u} = \langle 3, 2, -2\rangle \quad \|\mathbf{u}\| = \sqrt{17}$

$$\cos \alpha = \frac{3}{\sqrt{17}} \implies \alpha \approx 0.7560 \text{ or } 43.3°$$

$$\cos \beta = \frac{2}{\sqrt{17}} \implies \beta \approx 1.0644 \text{ or } 61.0°$$

$$\cos \gamma = \frac{-2}{\sqrt{17}} \implies \gamma \approx 2.0772 \text{ or } 119.0°$$

33. $\mathbf{u} = \langle -1, 5, 2\rangle \quad \|\mathbf{u}\| = \sqrt{30}$

$$\cos \alpha = \frac{-1}{\sqrt{30}} \implies \alpha \approx 1.7544 \text{ or } 100.5°$$

$$\cos \beta = \frac{5}{\sqrt{30}} \implies \beta \approx 0.4205 \text{ or } 24.1°$$

$$\cos \gamma = \frac{2}{\sqrt{30}} \implies \gamma \approx 1.1970 \text{ or } 68.6°$$

35. $\mathbf{F}_1\colon C_1 = \dfrac{50}{\|\mathbf{F}_1\|} \approx 4.3193$

$$\mathbf{F}_2\colon C_2 = \frac{80}{\|\mathbf{F}_2\|} \approx 5.4183$$

$$\begin{aligned}\mathbf{F} &= \mathbf{F}_1 + \mathbf{F}_2\\ &\approx 4.3193\langle 10, 5, 3\rangle + 5.4183\langle 12, 7, -5\rangle\\ &= \langle 108.2126, 59.5246, -14.1336\rangle\end{aligned}$$

$$\|\mathbf{F}\| \approx 124.310 \text{ lb}$$

$$\cos \alpha \approx \frac{108.2126}{\|\mathbf{F}\|} \implies \alpha \approx 29.48°$$

$$\cos \beta \approx \frac{59.5246}{\|\mathbf{F}\|} \implies \beta \approx 61.39°$$

$$\cos \gamma \approx \frac{-14.1336}{\|\mathbf{F}\|} \implies \gamma \approx 96.53°$$

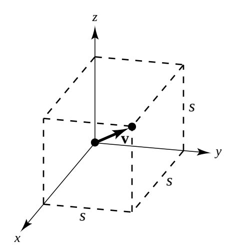
37. Let s = length of a side.

$$\mathbf{v} = \langle s, s, s\rangle$$

$$\|\mathbf{v}\| = s\sqrt{3}$$

$$\cos \alpha = \cos \beta = \cos \gamma = \frac{s}{s\sqrt{3}} = \frac{1}{\sqrt{3}}$$

$$\alpha = \beta = \gamma = \arccos\left(\frac{1}{\sqrt{3}}\right) \approx 54.7°$$



39. $\overrightarrow{OA} = \langle 0, 10, 10\rangle$

$$\cos \alpha = \frac{0}{\sqrt{0^2 + 10^2 + 10^2}} = 0 \implies \alpha = 90°$$

$$\cos \beta = \cos \gamma = \frac{10}{\sqrt{0^2 + 10^2 + 10^2}}$$

$$= \frac{1}{\sqrt{2}} \implies \beta = \gamma = 45°$$

41. $\mathbf{w}_2 = \mathbf{u} - \mathbf{w}_1 = \langle 6, 7\rangle - \langle 2, 8\rangle = \langle 4, -1\rangle$

43. $\mathbf{w}_2 = \mathbf{u} - \mathbf{w}_1 = \langle 0, 3, 3\rangle - \langle -2, 2, 2\rangle = \langle 2, 1, 1\rangle$

45. $\mathbf{u} = \langle 2, 3\rangle, \mathbf{v} = \langle 5, 1\rangle$

(a) $\mathbf{w}_1 = \left(\dfrac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{v}\|^2}\right)\mathbf{v} = \dfrac{13}{26}\langle 5, 1\rangle = \left\langle \dfrac{5}{2}, \dfrac{1}{2}\right\rangle$

(b) $\mathbf{w}_2 = \mathbf{u} - \mathbf{w}_1 = \left\langle -\dfrac{1}{2}, \dfrac{5}{2}\right\rangle$

47. $\mathbf{u} = \langle 2, 1, 2\rangle$, $\mathbf{v} = \langle 0, 3, 4\rangle$

(a) $\mathbf{w}_1 = \left(\dfrac{\mathbf{u}\cdot\mathbf{v}}{\|\mathbf{v}\|^2}\right)\mathbf{v}$

$= \dfrac{11}{25}\langle 0, 3, 4\rangle = \left\langle 0, \dfrac{33}{25}, \dfrac{44}{25}\right\rangle$

(b) $\mathbf{w}_2 = \mathbf{u} - \mathbf{w}_1 = \left\langle 2, -\dfrac{8}{25}, \dfrac{6}{25}\right\rangle$

49. $\mathbf{u}\cdot\mathbf{v} = \langle u_1, u_2, u_3\rangle \cdot \langle v_1, v_2, v_3\rangle = u_1v_1 + u_2v_2 + u_3v_3$

51. (a) Orthogonal, $\theta = \dfrac{\pi}{2}$ (b) Acute, $0 < \theta < \dfrac{\pi}{2}$ (c) Obtuse, $\dfrac{\pi}{2} < \theta < \pi$

53. See page 738. Direction cosines of $\mathbf{v} = \langle v_1, v_2, v_3\rangle$ are

$\cos\alpha = \dfrac{v_1}{\|\mathbf{v}\|}, \cos\beta = \dfrac{v_2}{\|\mathbf{v}\|}, \cos\gamma = \dfrac{v_3}{\|\mathbf{v}\|}.$

α, β, and γ are the direction angles. See Figure 10.26.

55. (a) $\left(\dfrac{\mathbf{u}\cdot\mathbf{v}}{\|\mathbf{v}\|^2}\right)\mathbf{v} = \mathbf{u} \implies \mathbf{u} = c\mathbf{v} \implies \mathbf{u}$ and $\mathbf{v}$ are parallel.

(b) $\left(\dfrac{\mathbf{u}\cdot\mathbf{v}}{\|\mathbf{v}\|^2}\right)\mathbf{v} = \mathbf{0} \implies \mathbf{u}\cdot\mathbf{v} = 0 \implies \mathbf{u}$ and $\mathbf{v}$ are orthogonal.

57. Programs will vary.

59. Programs will vary.

61. Because **u** appears to be perpendicular to **v**, the projection of **u** onto **v** is **0**. Analytically,

$$\text{proj}_{\mathbf{v}}\mathbf{u} = \frac{\mathbf{u}\cdot\mathbf{v}}{\|\mathbf{v}\|^2}\mathbf{v} = \frac{\langle 2, -3\rangle\cdot\langle 6, 4\rangle}{\|\langle 6, 4\rangle\|^2}\langle 6, 4\rangle = 0\langle 6, 4\rangle = \mathbf{0}.$$

63. $\mathbf{u} = \dfrac{1}{2}\mathbf{i} - \dfrac{2}{3}\mathbf{j}$. Want $\mathbf{u}\cdot\mathbf{v} = 0$.

$\mathbf{v} = 8\mathbf{i} + 6\mathbf{j}$ and $-\mathbf{v} = -8\mathbf{i} - 6\mathbf{j}$ are orthogonal to **u**.

65. $\mathbf{u} = \langle 3, 1, -2\rangle$. Want $\mathbf{u}\cdot\mathbf{v} = 0$.

$\mathbf{v} = \langle 0, 2, 1\rangle$ and $-\mathbf{v} = \langle 0, -2, -1\rangle$ are orthogonal to **u**.

67. (a) Gravitational Force $\mathbf{F} = -48{,}000\,\mathbf{j}$

$\mathbf{v} = \cos 10°\,\mathbf{i} + \sin 10°\,\mathbf{j}$

$\mathbf{w}_1 = \dfrac{\mathbf{F}\cdot\mathbf{v}}{\|\mathbf{v}\|^2}\mathbf{v} = (\mathbf{F}\cdot\mathbf{v})\mathbf{v} = (-48{,}000)(\sin 10°)\mathbf{v}$

$\approx -8335.1(\cos 10°\,\mathbf{i} + \sin 10°\,\mathbf{j})$

$\|\mathbf{w}_1\| \approx 8335.1$ lb

(b) $\mathbf{w}_2 = \mathbf{F} \cdot \mathbf{w}_1 = -48{,}000\,\mathbf{j} + 8335.1(\cos 10°\,\mathbf{i} + \sin 10°\,\mathbf{j})$

$= 8208.5\,\mathbf{i} - 46{,}552.6\,\mathbf{j}$

$\|\mathbf{w}_2\| \approx 47{,}270.8$ lb

69. $\mathbf{F} = 85\left(\dfrac{1}{2}\mathbf{i} + \dfrac{\sqrt{3}}{2}\mathbf{j}\right)$

$\mathbf{v} = 10\mathbf{i}$

$W = \mathbf{F}\cdot\mathbf{v} = 425$ ft · lb

71. $\overrightarrow{PQ} = \langle 4, 7, 5\rangle$

$\mathbf{v} = \langle 1, 4, 8\rangle$

$W = \overrightarrow{PQ}\cdot\mathbf{v} = 72$

73. False. Let $\mathbf{u} = \langle 2, 4\rangle$, $\mathbf{v} = \langle 1, 7\rangle$ and $\mathbf{w} = \langle 5, 5\rangle$. Then $\mathbf{u}\cdot\mathbf{v} = 2 + 28 = 30$ and $\mathbf{u}\cdot\mathbf{w} = 10 + 20 = 30$.

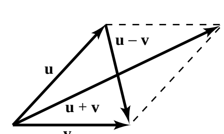
75. In a rhombus, $\|\mathbf{u}\| = \|\mathbf{v}\|$. The diagonals are $\mathbf{u} + \mathbf{v}$ and $\mathbf{u} - \mathbf{v}$.

$(\mathbf{u} + \mathbf{v})\cdot(\mathbf{u} - \mathbf{v}) = (\mathbf{u} + \mathbf{v})\cdot\mathbf{u} - (\mathbf{u} + \mathbf{v})\cdot\mathbf{v}$

$= \mathbf{u}\cdot\mathbf{u} + \mathbf{v}\cdot\mathbf{u} - \mathbf{u}\cdot\mathbf{v} - \mathbf{v}\cdot\mathbf{v}$

$= \|\mathbf{u}\|^2 - \|\mathbf{v}\|^2 = 0$

Therefore, the diagonals are orthogonal.



77. $\mathbf{u} = \langle \cos\alpha, \sin\alpha, 0\rangle, \mathbf{v} = \langle \cos\beta, \sin\beta, 0\rangle$

The angle between **u** and **v** is $\alpha - \beta$. (Assuming that $\alpha > \beta$). Also,

$$\cos(\alpha - \beta) = \frac{\mathbf{u}\cdot\mathbf{v}}{\|\mathbf{u}\|\,\|\mathbf{v}\|} = \frac{\cos\alpha\cos\beta + \sin\alpha\sin\beta}{(1)(1)} = \cos\alpha\cos\beta + \sin\alpha\sin\beta.$$

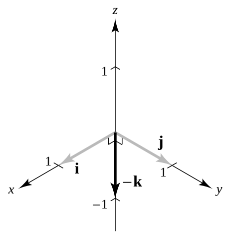
79.
$$\begin{aligned}\|\mathbf{u}-\mathbf{v}\|^2 &= (\mathbf{u}-\mathbf{v})\cdot(\mathbf{u}-\mathbf{v})\\ &= (\mathbf{u}-\mathbf{v})\cdot\mathbf{u} - (\mathbf{u}-\mathbf{v})\cdot\mathbf{v}\\ &= \mathbf{u}\cdot\mathbf{u} - \mathbf{v}\cdot\mathbf{u} - \mathbf{u}\cdot\mathbf{v} + \mathbf{v}\cdot\mathbf{v}\\ &= \|\mathbf{u}\|^2 - \mathbf{u}\cdot\mathbf{v} - \mathbf{u}\cdot\mathbf{v} + \|\mathbf{v}\|^2\\ &= \|\mathbf{u}\|^2 + \|\mathbf{v}\|^2 - 2\mathbf{u}\cdot\mathbf{v}\end{aligned}$$

81.
$$\begin{aligned}\|\mathbf{u}+\mathbf{v}\|^2 &= (\mathbf{u}+\mathbf{v})\cdot(\mathbf{u}+\mathbf{v})\\ &= (\mathbf{u}+\mathbf{v})\cdot\mathbf{u} + (\mathbf{u}+\mathbf{v})\cdot\mathbf{v}\\ &= \mathbf{u}\cdot\mathbf{u} + \mathbf{v}\cdot\mathbf{u} + \mathbf{u}\cdot\mathbf{v} + \mathbf{v}\cdot\mathbf{v}\\ &= \|\mathbf{u}\|^2 + 2\mathbf{u}\cdot\mathbf{v} + \|\mathbf{v}\|^2\\ &\le \|\mathbf{u}\|^2 + 2\|\mathbf{u}\|\,\|\mathbf{v}\| + \|\mathbf{v}\|^2 \text{ from Exercise 66}\\ &\le (\|\mathbf{u}\| + \|\mathbf{v}\|)^2\end{aligned}$$

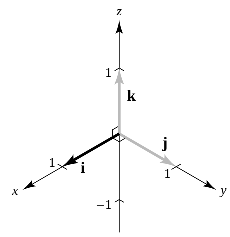
Therefore, $\|\mathbf{u}+\mathbf{v}\| \le \|\mathbf{u}\| + \|\mathbf{v}\|$.

Section 10.4 The Cross Product of Two Vectors in Space

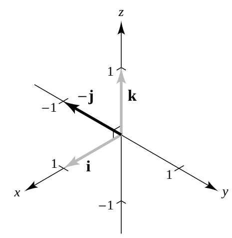
1. $\mathbf{j}\times\mathbf{i} = \begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ 0 & 1 & 0\\ 1 & 0 & 0\end{vmatrix} = -\mathbf{k}$



3. $\mathbf{j}\times\mathbf{k} = \begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ 0 & 1 & 0\\ 0 & 0 & 1\end{vmatrix} = \mathbf{i}$



5. $\mathbf{i}\times\mathbf{k} = \begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ 1 & 0 & 0\\ 0 & 0 & 1\end{vmatrix} = -\mathbf{j}$



7. (a) $\mathbf{u}\times\mathbf{v} = \begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ -2 & 3 & 4\\ 3 & 7 & 2\end{vmatrix} = \langle -22, 16, -23\rangle$

(b) $\mathbf{v}\times\mathbf{u} = -(\mathbf{u}\times\mathbf{v}) = \langle 22, -16, 23\rangle$

(c) $\mathbf{v}\times\mathbf{v} = \begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ 3 & 7 & 2\\ 3 & 7 & 2\end{vmatrix} = 0$

9. (a) $\mathbf{u}\times\mathbf{v} = \begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ 7 & 3 & 2\\ 1 & -1 & 5\end{vmatrix} = \langle 17, -33, -10\rangle$

(b) $\mathbf{v}\times\mathbf{u} = -(\mathbf{u}\times\mathbf{v}) = \langle -17, 33, 10\rangle$

(c) $\mathbf{v}\times\mathbf{v} = 0$

11. $\mathbf{u} = \langle 2, -3, 1\rangle, \mathbf{v} = \langle 1, -2, 1\rangle$

$\mathbf{u}\times\mathbf{v} = \begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ 2 & -3 & 1\\ 1 & -2 & 1\end{vmatrix} = -\mathbf{i} - \mathbf{j} - \mathbf{k} = \langle -1, -1, -1\rangle$

$\mathbf{u}\cdot(\mathbf{u}\times\mathbf{v}) = 2(-1) + (-3)(-1) + (1)(-1) = 0 \implies \mathbf{u}\perp\mathbf{u}\times\mathbf{v}$

$\mathbf{v}\cdot(\mathbf{u}\times\mathbf{v}) = 1(-1) + (-2)(-1) + (1)(-1) = 0 \implies \mathbf{v}\perp\mathbf{u}\times\mathbf{v}$

13. $\mathbf{u} = \langle 12, -3, 0\rangle$, $\mathbf{v} = \langle -2, 5, 0\rangle$

$$\mathbf{u}\times\mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 12 & -3 & 0 \\ -2 & 5 & 0 \end{vmatrix} = 54\mathbf{k} = \langle 0, 0, 54\rangle$$

$$\mathbf{u}\cdot(\mathbf{u}\times\mathbf{v}) = 12(0) + (-3)(0) + 0(54) = 0 \implies \mathbf{u}\perp\mathbf{u}\times\mathbf{v}$$

$$\mathbf{v}\cdot(\mathbf{u}\times\mathbf{v}) = -2(0) + 5(0) + 0(54) = 0 \implies \mathbf{v}\perp\mathbf{u}\times\mathbf{v}$$

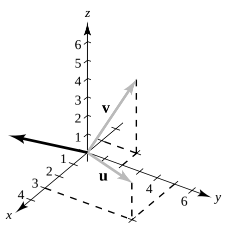
15. $\mathbf{u} = \mathbf{i} + \mathbf{j} + \mathbf{k}$, $\mathbf{v} = 2\mathbf{i} + \mathbf{j} - \mathbf{k}$

$$\mathbf{u}\times\mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 1 & 1 \\ 2 & 1 & -1 \end{vmatrix} = -2\mathbf{i} + 3\mathbf{j} - \mathbf{k} = \langle -2, 3, -1\rangle$$

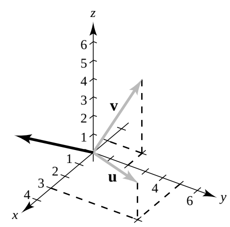
$$\mathbf{u}\cdot(\mathbf{u}\times\mathbf{v}) = 1(-2) + 1(3) + 1(-1) = 0 \implies \mathbf{u}\perp\mathbf{u}\times\mathbf{v}$$

$$\mathbf{v}\cdot(\mathbf{u}\times\mathbf{v}) = 2(-2) + 1(3) + (-1)(-1) = 0 \implies \mathbf{v}\perp\mathbf{u}\times\mathbf{v}$$

17.



19.



21. $\mathbf{u} = \langle 4, -3.5, 7\rangle$

$\mathbf{v} = \langle -1, 8, 4\rangle$

$$\mathbf{u}\times\mathbf{v} = \left\langle -70, -23, \frac{57}{2}\right\rangle$$

$$\frac{\mathbf{u}\times\mathbf{v}}{\|\mathbf{u}\times\mathbf{v}\|} = \left\langle \frac{-140}{\sqrt{24{,}965}}, \frac{-46}{\sqrt{24{,}965}}, \frac{57}{\sqrt{24{,}965}}\right\rangle$$

23. $\mathbf{u} = -3\mathbf{i} + 2\mathbf{j} - 5\mathbf{k}$

$$\mathbf{v} = \frac{1}{2}\mathbf{i} - \frac{3}{4}\mathbf{j} + \frac{1}{10}\mathbf{k}$$

$$\mathbf{u}\times\mathbf{v} = \left\langle -\frac{71}{20}, -\frac{11}{5}, \frac{5}{4}\right\rangle$$

$$\frac{\mathbf{u}\times\mathbf{v}}{\|\mathbf{u}\times\mathbf{v}\|} = \frac{20}{\sqrt{7602}}\left\langle -\frac{71}{20}, -\frac{11}{5}, \frac{5}{4}\right\rangle = \left\langle -\frac{71}{\sqrt{7602}}, -\frac{44}{\sqrt{7602}}, \frac{25}{\sqrt{7602}}\right\rangle$$

25. Programs will vary.

27. $\mathbf{u} = \mathbf{j}$

$\mathbf{v} = \mathbf{j} + \mathbf{k}$

$$\mathbf{u}\times\mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{vmatrix} = \mathbf{i}$$

$$A = \|\mathbf{u}\times\mathbf{v}\| = \|\mathbf{i}\| = 1$$

29. $\mathbf{u} = \langle 3, 2, -1\rangle$

$\mathbf{v} = \langle 1, 2, 3\rangle$

$$\mathbf{u}\times\mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & 2 & -1 \\ 1 & 2 & 3 \end{vmatrix} = \langle 8, -10, 4\rangle$$

$$A = \|\mathbf{u}\times\mathbf{v}\| = \|\langle 8, -10, 4\rangle\| = \sqrt{180} = 6\sqrt{5}$$

31. $A(1, 1, 1,)$, $B(2, 3, 4)$, $C(6, 5, 2)$, $D(7, 7, 5)$

$\overrightarrow{AB} = \langle 1, 2, 3\rangle$, $\overrightarrow{AC} = \langle 5, 4, 1\rangle$, $\overrightarrow{CD} = \langle 1, 2, 3\rangle$, $\overrightarrow{BD} = \langle 5, 4, 1\rangle$

Since $\overrightarrow{AB} = \overrightarrow{CD}$ and $\overrightarrow{AC} = \overrightarrow{BD}$, the figure is a parallelogram. $\overrightarrow{AB}$ and $\overrightarrow{AC}$ are adjacent sides and

$$\overrightarrow{AB}\times\overrightarrow{AC} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 2 & 3 \\ 5 & 4 & 1 \end{vmatrix} = -10\mathbf{i} + 14\mathbf{j} - 6\mathbf{k}.$$

$$A = \|\overrightarrow{AB}\times\overrightarrow{AC}\| = \sqrt{332} = 2\sqrt{83}$$

33. $A(0, 0, 0)$, $B(1, 2, 3)$, $C(-3, 0, 0)$

$\overrightarrow{AB} = \langle 1, 2, 3\rangle$, $\overrightarrow{AC} = \langle -3, 0, 0\rangle$

$$\overrightarrow{AB}\times\overrightarrow{AC} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 2 & 3 \\ -3 & 0 & 0 \end{vmatrix} = -9\mathbf{j} + 6\mathbf{k}$$

$$A = \frac{1}{2}\|\overrightarrow{AB}\times\overrightarrow{AC}\| = \frac{1}{2}\sqrt{117} = \frac{3}{2}\sqrt{13}$$

35. $A(2, -7, 3), B(-1, 5, 8), C(4, 6, -1)$

$\overrightarrow{AB} = \langle -3, 12, 5\rangle, \overrightarrow{AC} = \langle 2, 13, -4\rangle$

$$\overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -3 & 12 & 5 \\ 2 & 13 & -4 \end{vmatrix} = \langle -113, -2, -63\rangle$$

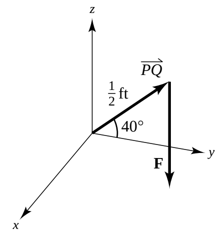
$$\text{Area} = \frac{1}{2}\|\overrightarrow{AB} \times \overrightarrow{AC}\| = \frac{1}{2}\sqrt{16{,}742}$$

37. $\mathbf{F} = -20\mathbf{k}$

$$\overrightarrow{PQ} = \frac{1}{2}(\cos 40^\circ \mathbf{j} + \sin 40^\circ \mathbf{k})$$

$$\overrightarrow{PQ} \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & \cos 40^\circ/2 & \sin 40^\circ/2 \\ 0 & 0 & -20 \end{vmatrix} = -10\cos 40^\circ \mathbf{i}$$

$$\|\overrightarrow{PQ} \times \mathbf{F}\| = 10\cos 40^\circ \approx 7.66 \text{ ft} \cdot \text{lb}$$

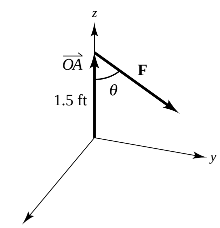


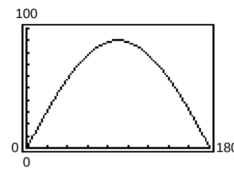
39. (a) $\overrightarrow{OA} = \frac{3}{2}\mathbf{k}$

$$\mathbf{F} = -60(\sin\theta\mathbf{j} + \cos\theta\mathbf{k})$$

$$\overrightarrow{OA} \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 3/2 \\ 0 & -60\sin\theta & -60\cos\theta \end{vmatrix} = 90\sin\theta\mathbf{i}$$

$$\|\overrightarrow{OA} \times \mathbf{F}\| = 90\sin\theta$$





(b) When $\theta = 45^\circ$: $\|\overrightarrow{OA} \times \mathbf{F}\| = 90\left(\frac{\sqrt{2}}{2}\right) = 45\sqrt{2} \approx 63.64.$

(c) Let $T = 90\sin\theta$.

$$\frac{dT}{d\theta} = 90\cos\theta = 0 \text{ when } \theta = 90^\circ.$$

This is what we expected. When $\theta = 90^\circ$ the pipe wrench is horizontal.

41. $$\mathbf{u} \cdot (\mathbf{v} \times \mathbf{w}) = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} = 1$$

43. $$\mathbf{u} \cdot (\mathbf{v} \times \mathbf{w}) = \begin{vmatrix} 2 & 0 & 1 \\ 0 & 3 & 0 \\ 0 & 0 & 1 \end{vmatrix} = 6$$

45. $$\mathbf{u} \cdot (\mathbf{v} \times \mathbf{w}) = \begin{vmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{vmatrix} = 2$$

$$V = |\mathbf{u} \cdot (\mathbf{v} \times \mathbf{w})| = 2$$

47. $\mathbf{u} = \langle 3, 0, 0\rangle$

$\mathbf{v} = \langle 0, 5, 1\rangle$

$\mathbf{w} = \langle 2, 0, 5\rangle$

$$\mathbf{u} \cdot (\mathbf{v} \times \mathbf{w}) = \begin{vmatrix} 3 & 0 & 0 \\ 0 & 5 & 1 \\ 2 & 0 & 5 \end{vmatrix} = 75$$

$$V = |\mathbf{u} \cdot (\mathbf{v} \times \mathbf{w})| = 75$$

49. $\mathbf{u} \times \mathbf{v} = \langle u_1, u_2, u_3\rangle \cdot \langle v_1, v_2, v_3\rangle = (u_2v_3 - u_3v_2)\mathbf{i} - (u_1v_3 - u_3v_1)\mathbf{j} + (u_1v_2 - u_2v_1)\mathbf{k}$

51. The magnitude of the cross product will increase by a factor of 4.

53. If the vectors are ordered pairs, then the cross product does not exist. False.

55. True

57. $\mathbf{u} = \langle u_1, u_2, u_3 \rangle$, $\mathbf{v} = \langle v_1, v_2, v_3 \rangle$, $\mathbf{w} = \langle w_1, w_2, w_3 \rangle$

$$\mathbf{u} \times (\mathbf{v} + \mathbf{w}) = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ u_1 & u_2 & u_3 \\ v_1 + w_1 & v_2 + w_2 & v_3 + w_3 \end{vmatrix}$$

$$= [u_2(v_3 + w_3) - u_3(v_2 + w_2)]\mathbf{i} - [u_1(v_3 + w_3) - u_3(v_1 + w_1)]\mathbf{j} + [u_1(v_2 + w_2) - u_2(v_1 + w_1)]\mathbf{k}$$

$$= (u_2v_3 - u_3v_2)\mathbf{i} - (u_1v_3 - u_3v_1)\mathbf{j} + (u_1v_2 - u_2v_1)\mathbf{k} + (u_2w_3 - u_3w_2)\mathbf{i} - (u_1w_3 - u_3w_1)\mathbf{j} + (u_1w_2 - u_2w_1)\mathbf{k}$$

$$= (\mathbf{u} \times \mathbf{v}) + (\mathbf{u} \times \mathbf{w})$$

59. $\mathbf{u} = \langle u_1, u_2, u_3 \rangle$

$$\mathbf{u} \times \mathbf{u} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ u_1 & u_2 & u_3 \\ u_1 & u_2 & u_3 \end{vmatrix} = (u_2u_3 - u_3u_2)\mathbf{i} - (u_1u_3 - u_3u_1)\mathbf{j} + (u_1u_2 - u_2u_1)\mathbf{k} = \mathbf{0}$$

61. $$\mathbf{u} \times \mathbf{v} = (u_2v_3 - u_3v_2)\mathbf{i} - (u_1v_3 - u_3v_1)\mathbf{j} + (u_1v_2 - u_2v_1)\mathbf{k}$$

$$(\mathbf{u} \times \mathbf{v}) \cdot \mathbf{u} = (u_2v_3 - u_3v_2)u_1 + (u_3v_1 - u_1v_3)u_2 + (u_1v_2 - u_2v_1)u_3 = 0$$

$$(\mathbf{u} \times \mathbf{v}) \cdot \mathbf{v} = (u_2v_3 - u_3v_2)v_1 + (u_3v_1 - u_1v_3)v_2 + (u_1v_2 - u_2v_1)v_3 = 0$$

Thus, $\mathbf{u} \times \mathbf{v} \perp \mathbf{u}$ and $\mathbf{u} \times \mathbf{v} \perp \mathbf{v}$.

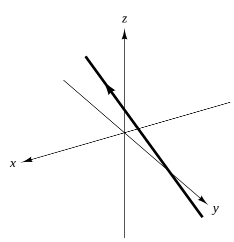
63. $\|\mathbf{u} \times \mathbf{v}\| = \|\mathbf{u}\|\,\|\mathbf{v}\| \sin\theta$

If $\mathbf{u}$ and $\mathbf{v}$ are orthogonal, $\theta = \pi/2$ and $\sin\theta = 1$. Therefore, $\|\mathbf{u} \times \mathbf{v}\| = \|\mathbf{u}\|\,\|\mathbf{v}\|$.

Section 10.5 Lines and Planes in Space

1. $x = 1 + 3t$, $y = 2 - t$, $z = 2 + 5t$

(a)



(b) When $t = 0$ we have $P = (1, 2, 2)$. When $t = 3$ we have $Q = (10, -1, 17)$.

$$\overrightarrow{PQ} = \langle 9, -3, 15 \rangle$$

The components of the vector and the coefficients of t are proportional since the line is parallel to $\overrightarrow{PQ}$.

(c) $y = 0$ when $t = 2$. Thus, $x = 7$ and $z = 12$.
Point: $(7, 0, 12)$

$x = 0$ when $t = -\frac{1}{3}$. Point: $\left(0, \frac{7}{3}, \frac{1}{3}\right)$

$z = 0$ when $t = -\frac{2}{5}$. Point: $\left(-\frac{1}{5}, \frac{12}{5}, 0\right)$

3. Point: $(0, 0, 0)$

Direction vector: $\mathbf{v} = \langle 1, 2, 3 \rangle$

Direction numbers: 1, 2, 3

(a) Parametric: $x = t$, $y = 2t$, $z = 3t$

(b) Symmetric: $x = \dfrac{y}{2} = \dfrac{z}{3}$

5. Point: $(-2, 0, 3)$

Direction vector: $\mathbf{v} = \langle 2, 4, -2 \rangle$

Direction numbers: $2, 4, -2$

(a) Parametric: $x = -2 + 2t$, $y = 4t$, $z = 3 - 2t$

(b) Symmetric: $\dfrac{x + 2}{2} = \dfrac{y}{4} = \dfrac{z - 3}{-2}$

7. Point: $(1, 0, 1)$

Direction vector: $\mathbf{v} = 3\mathbf{i} - 2\mathbf{j} + \mathbf{k}$

Direction numbers: $3, -2, 1$

(a) Parametric: $x = 1 + 3t, y = -2t, z = 1 + t$

(b) Symmetric: $\dfrac{x-1}{3} = \dfrac{y}{-2} = \dfrac{z-1}{1}$

9. Points: $(5, -3, -2), \left(\dfrac{-2}{3}, \dfrac{2}{3}, 1\right)$

Direction vector: $\mathbf{v} = \dfrac{17}{3}\mathbf{i} - \dfrac{11}{3}\mathbf{j} - 3\mathbf{k}$

Direction numbers: $17, -11, -9$

(a) Parametric: $x = 5 + 17t, y = -3 - 11t, z = -2 - 9t$

(b) Symmetric: $\dfrac{x-5}{17} = \dfrac{y+3}{-11} = \dfrac{z+2}{-9}$

11. Points: $(2, 3, 0), (10, 8, 12)$

Direction vector: $\langle 8, 5, 12 \rangle$

Direction numbers: $8, 5, 12$

(a) Parametric: $x = 2 + 8t, y = 3 + 5t, z = 12t$

(b) Symmetric: $\dfrac{x-2}{8} = \dfrac{y-3}{5} = \dfrac{z}{12}$

13. Point: $(2, 3, 4)$

Direction vector: $\mathbf{v} = \mathbf{k}$

Direction numbers: $0, 0, 1$

Parametric: $x = 2, y = 3, z = 4 + t$

15. Point: $(-2, 3, 1)$

Direction vector: $\mathbf{v} = 4\mathbf{i} - \mathbf{k}$

Direction numbers: $4, 0, -1$

Parametric: $x = -2 + 4t, y = 3, z = 1 - t$

Symmetric: $\dfrac{x+2}{4} = \dfrac{z-1}{-1}, y = 3$

(a) On line

(b) On line

(c) Not on line $(y \neq 3)$

(d) Not on line $\left(\dfrac{6+2}{4} \neq \dfrac{-2-1}{-1}\right)$

17. L_i: $\mathbf{v} = \langle -3, 2, 4 \rangle$ $(6, -2, 5)$ on line

L_2: $\mathbf{v} = \langle 6, -4, -8 \rangle$ $(6, -2, 5)$ on line

L_3: $\mathbf{v} = \langle -6, 4, 8 \rangle$ $(6, -2, 5)$ not on line

L_4: $\mathbf{v} = \langle 6, 4, -6 \rangle$ not parallel to L_1, L_2, nor L_3

Hence, L_1 and L_2 are identical.

$L_1 = L_2$ and L_3 are parallel.

19. At the point of intersection, the coordinates for one line equal the corresponding coordinates for the other line. Thus,

(i) $4t + 2 = 2s + 2$, (ii) $3 = 2s + 3$, and (iii) $-t + 1 = s + 1$.

From (ii), we find that $s = 0$ and consequently, from (iii), $t = 0$. Letting $s = t = 0$, we see that equation (i) is satisfied and therefore the two lines intersect. Substituting zero for s or for t, we obtain the point $(2, 3, 1)$.

$\mathbf{u} = 4\mathbf{i} - \mathbf{k}$ (First line)

$\mathbf{v} = 2\mathbf{i} + 2\mathbf{j} + \mathbf{k}$ (Second line)

$$\cos\theta = \frac{|\mathbf{u}\cdot\mathbf{v}|}{\|\mathbf{u}\|\,\|\mathbf{v}\|} = \frac{8-1}{\sqrt{17}\sqrt{9}} = \frac{7}{3\sqrt{17}} = \frac{7\sqrt{17}}{51}$$

21. Writing the equations of the lines in parametric form we have

$x = 3t$ $y = 2 - t$ $z = -1 + t$

$x = 1 + 4s$ $y = -2 + s$ $z = -3 - 3s.$

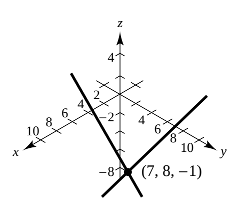
For the coordinates to be equal, $3t = 1 + 4s$ and $2 - t = -2 + s$. Solving this system yields $t = \frac{17}{7}$ and $s = \frac{11}{7}$. When using these values for s and t, the z coordinates are not equal. The lines do not intersect.

23. $x = 2t + 3$ $x = -2s + 7$

$y = 5t - 2$ $y = s + 8$

$z = -t + 1$ $z = 2s - 1$

Point of intersection: $(7, 8, -1)$



25. $4x - 3y - 6z = 6$

(a) $P = (0, 0, -1)$, $Q = (0, -2, 0)$, $R = (3, 4, -1)$

$\overrightarrow{PQ} = \langle 0, -2, 1\rangle$, $\overrightarrow{PR} = \langle 3, 4, 0\rangle$

(b) $\overrightarrow{PQ} \times \overrightarrow{PR} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & -2 & 1 \\ 3 & 4 & 0 \end{vmatrix} = \langle -4, 3, 6\rangle$

The components of the cross product are proportional to the coefficients of the variables in the equation. The cross product is parallel to the normal vector.

27. Point: $(2, 1, 2)$

$\mathbf{n} = \mathbf{i} = \langle 1, 0, 0\rangle$

$1(x - 2) + 0(y - 1) + 0(z - 2) = 0$

$x - 2 = 0$

29. Point: $(3, 2, 2)$

Normal vector: $\mathbf{n} = 2\mathbf{i} + 3\mathbf{j} - \mathbf{k}$

$2(x - 3) + 3(y - 2) - 1(z - 2) = 0$

$2x + 3y - z = 10$

31. Point: $(0, 0, 6)$

Normal vector: $\mathbf{n} = -\mathbf{i} + \mathbf{j} - 2\mathbf{k}$

$-1(x - 0) + 1(y - 0) - 2(z - 6) = 0$

$-x + y - 2z + 12 = 0$

$x - y + 2z = 12$

33. Let $\mathbf{u}$ be the vector from $(0, 0, 0)$ to $(1, 2, 3)$:

$\mathbf{u} = \mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$

Let $\mathbf{v}$ be the vector from $(0, 0, 0)$ to $(-2, 3, 3)$:

$\mathbf{v} = -2\mathbf{i} + 3\mathbf{j} + 3\mathbf{k}$

Normal vector: $\mathbf{u} \times \mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 2 & 3 \\ -2 & 3 & 3 \end{vmatrix}$

$= -3\mathbf{i} + (-9)\mathbf{j} + 7\mathbf{k}$

$-3(x - 0) - 9(y - 0) + 7(z - 0) = 0$

$3x + 9y - 7z = 0$

35. Let $\mathbf{u}$ be the vector from $(1, 2, 3)$ to $(3, 2, 1)$: $\mathbf{u} = 2\mathbf{i} - 2\mathbf{k}$

Let $\mathbf{v}$ be the vector from $(1, 2, 3)$ to $(-1, -2, 2)$: $\mathbf{v} = -2\mathbf{i} - 4\mathbf{j} - \mathbf{k}$

Normal vector: $\left(\frac{1}{2}\mathbf{u}\right) \times (-\mathbf{v}) = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 0 & -1 \\ 2 & 4 & 1 \end{vmatrix} = 4\mathbf{i} - 3\mathbf{j} + 4\mathbf{k}$

$4(x - 1) - 3(y - 2) + 4(z - 3) = 0$

$4x - 3y + 4z = 10$

37. $(1, 2, 3)$, Normal vector: $\mathbf{v} = \mathbf{k}$, $1(z - 3) = 0$, $z = 3$

39. The direction vectors for the lines are $\mathbf{u} = -2\mathbf{i} + \mathbf{j} + \mathbf{k}$, $\mathbf{v} = -3\mathbf{i} + 4\mathbf{j} - \mathbf{k}$.

Normal vector: $\mathbf{u} \times \mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -2 & 1 & 1 \\ -3 & 4 & -1 \end{vmatrix} = -5(\mathbf{i} + \mathbf{j} + \mathbf{k})$

Point of intersection of the lines: $(-1, 5, 1)$

$(x + 1) + (y - 5) + (z - 1) = 0$

$x + y + z = 5$

41. Let $\mathbf{v}$ be the vector from $(-1, 1, -1)$ to $(2, 2, 1)$: $\mathbf{v} = 3\mathbf{i} + \mathbf{j} + 2\mathbf{k}$

Let $\mathbf{n}$ be a vector normal to the plane $2x - 3y + z = 3$: $\mathbf{n} = 2\mathbf{i} - 3\mathbf{j} + \mathbf{k}$

Since v and n both lie in the plane p, the normal vector to p is

$\mathbf{v} \times \mathbf{n} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & 1 & 2 \\ 2 & -3 & 1 \end{vmatrix} = 7\mathbf{i} + \mathbf{j} - 11\mathbf{k}$

$7(x - 2) + 1(y - 2) - 11(z - 1) = 0$

$7x + y - 11z = 5$

43. Let $\mathbf{u} = \mathbf{i}$ and let $\mathbf{v}$ be the vector from $(1, -2, -1)$ to $(2, 5, 6)$: $\mathbf{v} = \mathbf{i} + 7\mathbf{j} + 7\mathbf{k}$

Since $\mathbf{u}$ and $\mathbf{v}$ both lie in the plane P, the normal vector to P is:

$$\mathbf{u} \times \mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 0 & 0 \\ 1 & 7 & 7 \end{vmatrix} = -7\mathbf{j} + 7\mathbf{k} = -7(\mathbf{j} - \mathbf{k})$$

$$[y - (-2)] - [z - (-1)] = 0$$

$$y - z = -1$$

45. The normal vectors to the planes are

$$\mathbf{n}_1 = \langle 5, -3, 1\rangle, \mathbf{n}_2 = \langle 1, 4, 7\rangle, \cos\theta = \frac{|\mathbf{n}_1 \cdot \mathbf{n}_2|}{\|\mathbf{n}_1\|\,\|\mathbf{n}_2\|} = 0.$$

Thus, $\theta = \pi/2$ and the planes are orthogonal.

47. The normal vectors to the planes are

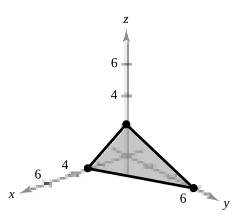
$$\mathbf{n}_1 = \mathbf{i} - 3\mathbf{j} + 6\mathbf{k},\ \mathbf{n}_2 = 5\mathbf{i} + \mathbf{j} - \mathbf{k},$$

$$\cos\theta = \frac{|\mathbf{n}_1 \cdot \mathbf{n}_2|}{\|\mathbf{n}_1\|\,\|\mathbf{n}_2\|} = \frac{|5 - 3 - 6|}{\sqrt{46}\sqrt{27}} = \frac{4\sqrt{138}}{414}.$$

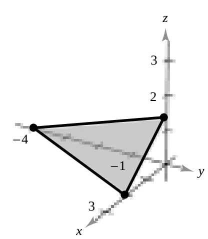
Therefore, $\theta = \arccos\left(\frac{4\sqrt{138}}{414}\right) \approx 83.5°$.

49. The normal vectors to the planes are $\mathbf{n}_1 = \langle 1, -5, -1\rangle$ and $\mathbf{n}_2 = \langle 5, -25, -5\rangle$. Since $\mathbf{n}_2 = 5\mathbf{n}_1$, the planes are parallel, but not equal.

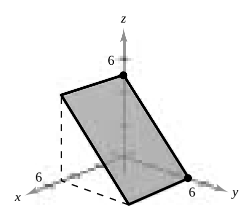
51. $4x + 2y + 6z = 12$



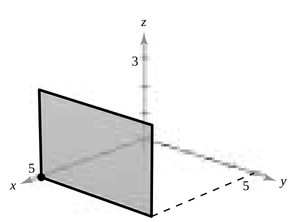
53. $2x - y + 3z = 4$



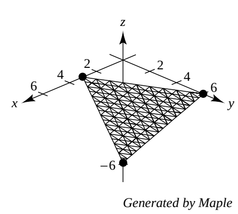
55. $y + z = 5$



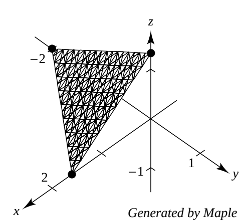
57. $x = 5$



59. $2x + y - z = 6$



61. $-5x + 4y - 6z + 8 = 0$



63. P_1: $\mathbf{n} = \langle 3, -2, 5\rangle$ $(1, -1, 1)$ on plane

P_2: $\mathbf{n} = \langle -6, 4, -10\rangle$ $(1, -1, 1)$ not on plane

P_3: $\mathbf{n} = \langle -3, 2, 5\rangle$

P_4: $\mathbf{n} = \langle 75, -50, 125\rangle$ $(1, -1, 1)$ on plane

P_1 and P_4 are identical.

$P_1 = P_4$ is parallel to P_2.

65. Each plane passes through the points

$$(c, 0, 0), (0, c, 0), \text{ and } (0, 0, c).$$

67. The normals to the planes are $\mathbf{n}_1 = 3\mathbf{i} + 2\mathbf{j} - \mathbf{k}$ and $\mathbf{n}_2 = \mathbf{i} - 4\mathbf{j} + 2\mathbf{k}$. The direction vector for the line is

$$\mathbf{n}_2 \times \mathbf{n}_1 = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & -4 & 2 \\ 3 & 2 & -1 \end{vmatrix} = 7(\mathbf{j} + 2\mathbf{k}).$$

Now find a point of intersection of the planes.

$$\begin{aligned} 6x + 4y - 2y &= 14 \\ x - 4y + 2z &= 0 \\ 7x \qquad\qquad &= 14 \\ x &= 2 \end{aligned}$$

Substituting 2 for x in the second equation, we have $-4y + 2z = -2$ or $z = 2y - 1$. Letting $y = 1$, a point of intersection is $(2, 1, 1)$.

$$x = 2, y = 1 + t, z = 1 + 2t$$

69. Writing the equation of the line in parametric form and substituting into the equation of the plane we have:

$$x = \frac{1}{2} + t,\ y = \frac{-3}{2} - t,\ z = -1 + 2t$$

$$2\left(\frac{1}{2} + t\right) - 2\left(\frac{-3}{2} - t\right) + (-1 + 2t) = 12,\ t = \frac{3}{2}$$

Substituting $t = 3/2$ into the parametric equations for the line we have the point of intersection $(2, -3, 2)$. The line does not lie in the plane.

71. Writing the equation of the line in parametric form and substituting into the equation of the plane we have:

$$x = 1 + 3t,\ y = -1 - 2t,\ z = 3 + t$$

$$2(1 + 3t) + 3(-1 - 2t) = 10,\ -1 = 10, \text{ contradiction}$$

Therefore, the line does not intersect the plane.

73. Point: $Q(0, 0, 0)$

Plane: $2x + 3y + z - 12 = 0$

Normal to plane: $\mathbf{n} = \langle 2, 3, 1 \rangle$

Point in plane: $P(6, 0, 0)$

Vector $\overrightarrow{PQ} = \langle -6, 0\ 0 \rangle$

$$D = \frac{|\overrightarrow{PQ} \cdot \mathbf{n}|}{\|\mathbf{n}\|} = \frac{|-12|}{\sqrt{14}} = \frac{6\sqrt{14}}{7}$$

75. Point: $Q(2, 8, 4)$

Plane: $2x + y + z = 5$

Normal to plane: $\mathbf{n} = \langle 2, 1, 1 \rangle$

Point in plane: $P\langle 0, 0, 5 \rangle$

Vector: $\overrightarrow{PQ} = \langle 2, 8, -1 \rangle$

$$D = \frac{|\overrightarrow{PQ} \cdot \mathbf{n}|}{\|\mathbf{n}\|} = \frac{11}{\sqrt{6}} = \frac{11\sqrt{6}}{6}$$

77. The normal vectors to the planes are $\mathbf{n}_1 = \langle 1, -3, 4 \rangle$ and $\mathbf{n}_2 = \langle 1, -3, 4 \rangle$. Since $\mathbf{n}_1 = \mathbf{n}_2$, the planes are parallel. Choose a point in each plane.

$P = (10, 0, 0)$ is a point in $x - 3y + 4z = 10$.
$Q = (6, 0, 0)$ is a point in $x - 3y + 4z = 6$.

$$\overrightarrow{PQ} = \langle -4, 0, 0 \rangle, D = \frac{|\overrightarrow{PQ} \cdot \mathbf{n}_1|}{\|\mathbf{n}_1\|} = \frac{4}{\sqrt{26}} = \frac{2\sqrt{26}}{13}$$

79. The normal vectors to the planes are $\mathbf{n}_1 = \langle -3, 6, 7 \rangle$ and $\mathbf{n}_2 = \langle 6, -12, -14 \rangle$. Since $\mathbf{n}_2 = -2\mathbf{n}_1$, the planes are parallel. Choose a point in each plane.

$P = (0, -1, 1)$ is a point in $-3x + 6y + 7z = 1$.

$Q = \left(\frac{25}{6}, 0, 0\right)$ is a point in $6x - 12y - 14z = 25$.

$$\overrightarrow{PQ} = \left\langle \frac{25}{6}, 1, -1 \right\rangle$$

$$D = \frac{|\overrightarrow{PQ} \cdot \mathbf{n}_1|}{\|\mathbf{n}_1\|} = \frac{|-27/2|}{\sqrt{94}} = \frac{27}{2\sqrt{94}} = \frac{27\sqrt{94}}{188}$$

81. $\mathbf{u} = \langle 4, 0, -1 \rangle$ is the direction vector for the line. $Q(1, 5, -2)$ is the given point, and $P(-2, 3, 1)$ is on the line. Hence, $\overrightarrow{PQ} = \langle 3, 2, -3 \rangle$ and

$$\overrightarrow{PQ} \times \mathbf{u} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & 2 & -3 \\ 4 & 0 & -1 \end{vmatrix} = \langle -2, -9, -8 \rangle$$

$$D = \frac{\|\overrightarrow{PQ} \times \mathbf{u}\|}{\|\mathbf{u}\|} = \frac{\sqrt{149}}{\sqrt{17}} = \frac{\sqrt{2533}}{17}$$

83. The parametric equations of a line L parallel to $\mathbf{v} = \langle a, b, c, \rangle$ and passing through the point $P(x_1, y_1, z_1)$ are

$$x = x_1 + at, y = y_1 + bt, z = z_1 + ct.$$

The symmetric equations are

$$\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c}.$$

85. Solve the two linear equations representing the planes to find two points of intersection. Then find the line determined by the two points.

87. (a) Sphere

$(x - 3)^2 + (y + 2)^2 + (z - 5)^2 = 16$

$x^2 + y^2 + z^2 - 6x + 4y - 10z + 22 = 0$

(b) Parallel planes

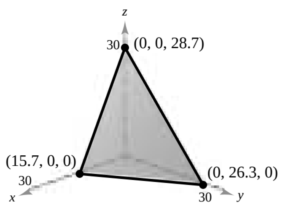
$4x - 3y + z = 10 \pm 4\|\mathbf{n}\| = 10 \pm 4\sqrt{26}$

89. (a) $z = 28.7 - 1.83x - 1.09y$

Year	1980	1985	1990	1994	1995	1996	1997
z (approx.)	16.16	14.23	9.81	8.60	8.42	8.27	8.23

(b) An increase in x or y will cause a decrease in z. In fact, any increase in two variables will cause a decrease in the third.

(c)



91. True

Section 10.6 Surfaces in Space

1. Ellipsoid

Matches graph (c)

3. Hyperboloid of one sheet

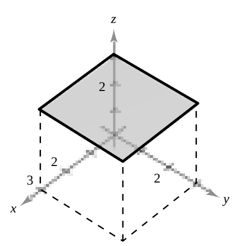
Matches graph (f)

5. Elliptic paraboloid

Matches graph (d)

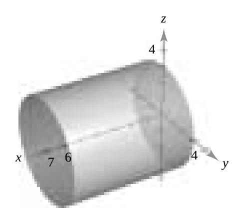
7. $z = 3$

Plane parallel to the xy-coordinate plane



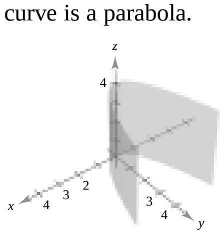
9. $y^2 + z^2 = 9$

The x-coordinate is missing so we have a cylindrical surface with rulings parallel to the x-axis. The generating curve is a circle.



11. $y = x^2$

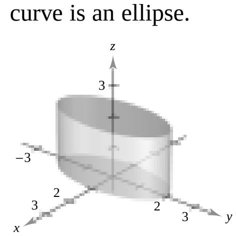
The z-coordinate is missing so we have a cylindrical surface with rulings parallel to the z-axis. The generating curve is a parabola.



13. $4x^2 + y^2 = 4$

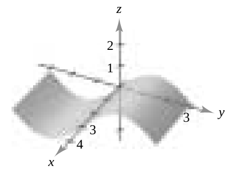
$$\frac{x^2}{1} + \frac{y^2}{4} = 1$$

The z-coordinate is missing so we have a cylindrical surface with rulings parallel to the z-axis. The generating curve is an ellipse.



15. $z = \sin y$

The x-coordinate is missing so we have a cylindrical surface with rulings parallel to the x-axis. The generating curve is the sine curve.



17. $x = x^2 + y^2$

(a) You are viewing the paraboloid from the x-axis: $(20, 0, 0)$

(b) You are viewing the paraboloid from above, but not on the z-axis: $(10, 10, 20)$

(c) You are viewing the paraboloid from the z-axis: $(0, 0, 20)$

(d) You are viewing the paraboloid from the y-axis: $(0, 20, 0)$

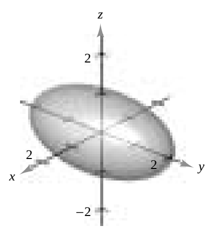
19. $\dfrac{x^2}{1} + \dfrac{y^2}{4} + \dfrac{z^2}{1} = 1$

Ellipsoid

xy-trace: $\dfrac{x^2}{1} + \dfrac{y^2}{4} = 1$ ellipse

xz-trace: $x^2 + z^2 = 1$ circle

yz-trace: $\dfrac{y^2}{4} + \dfrac{z^2}{1} = 1$ ellipse



21. $16x^2 - y^2 + 16z^2 = 4$

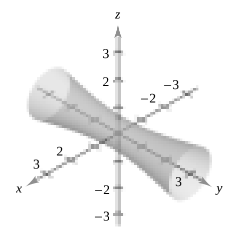
$4x^2 - \dfrac{y^2}{4} + 4z^2 = 1$

Hyperboloid on one sheet

xy-trace: $4x^2 - \dfrac{y^2}{4} = 1$ hyperbola

xz-trace: $4(x^2 + z^2) = 1$ circle

yz-trace: $\dfrac{-y^2}{4} + 4z^2 = 1$ hyperbola



23. $x^2 - y + z^2 = 0$

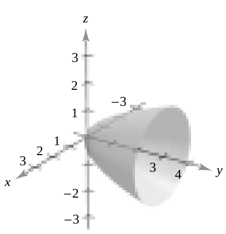
Elliptic paraboloid

xy-trace: $y = x^2$

xz-trace: $x^2 + z^2 = 0$, point $(0, 0, 0)$

yz-trace: $y = z^2$

$y = 1$: $x^2 + z^2 = 1$



25. $x^2 - y^2 + z = 0$

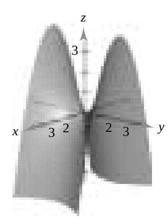
Hyperbolic paraboloid

xy-trace: $y = \pm x$

xz-trace: $z = -x^2$

yz-trace: $z = y^2$

$y = \pm 1$: $z = 1 - x^2$



27. $z^2 = x^2 + \dfrac{y^2}{4}$

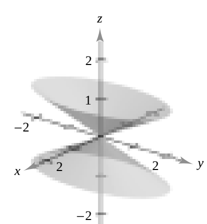
Elliptic Cone

xy-trace: point $(0, 0, 0)$

xz-trace: $z = \pm x$

yz-trace: $z = \dfrac{\pm 1}{2}y$

$z = \pm 1$: $x^2 + \dfrac{y^2}{4} = 1$



29.

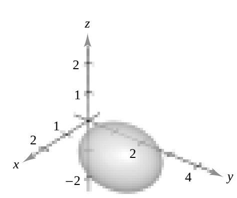
$$16x^2 + 9y^2 + 16z^2 - 32x - 36y + 36 = 0$$

$$16(x^2 - 2x + 1) + 9(y^2 - 4y + 4) + 16z^2 = -36 + 16 + 36$$

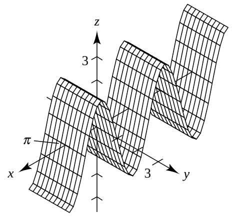
$$16(x - 1)^2 + 9(y - 2)^2 + 16z^2 = 16$$

$$\frac{(x - 1)^2}{1} + \frac{(y - 2)^2}{16/9} + \frac{z^2}{1} = 1$$

Ellipsoid with center $(1, 2, 0)$.

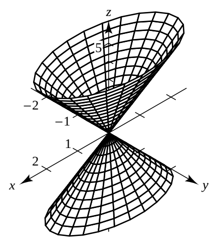


31. $z = 2\sin x$



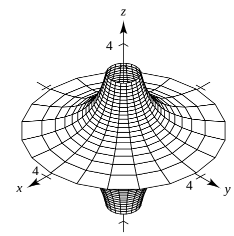
33. $z^2 = x^2 + 4y^2$

$z = \pm\sqrt{x^2 + 4y^2}$

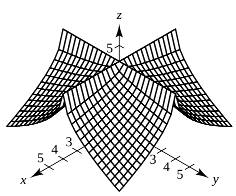


35. $x^2 + y^2 = \left(\dfrac{2}{z}\right)^2$

$y = \pm\sqrt{\dfrac{4}{z^2} - x^2}$

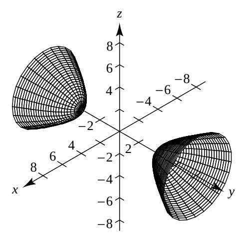


37. $z = 4 - \sqrt{|xy|}$



39. $4x^2 - y^2 + 4z^2 = -16$

$z = \pm\sqrt{\dfrac{y^2}{4} - x^2 - 4}$

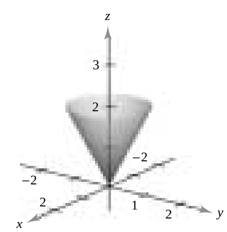


41. $z = 2\sqrt{x^2 + y^2}$

$z = 2$

$2\sqrt{x^2 + y^2} = 2$

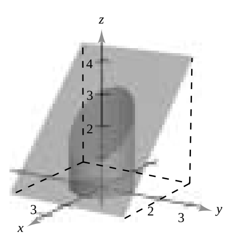
$x^2 + y^2 = 1$



43. $x^2 + y^2 = 1$

$x + z = 2$

$z = 0$



45. $x^2 + z^2 = [r(y)]^2$ and $z = r(y) = \pm 2\sqrt{y}$; therefore,

$x^2 + z^2 = 4y.$

47. $x^2 + y^2 = [r(z)]^2$ and $y = r(z) = \dfrac{z}{2}$; therefore,

$x^2 + y^2 = \dfrac{z^2}{4}, 4x^2 + 4y^2 = z^2.$

49. $y^2 + z^2 = [r(x)]^2$ and $y = r(x) = \dfrac{2}{x}$; therefore,

$y^2 + z^2 = \left(\dfrac{2}{x}\right)^2, y^2 + z^2 = \dfrac{4}{x^2}.$

51. $x^2 + y^2 - 2z = 0$

$x^2 + y^2 = \left(\sqrt{2z}\right)^2$

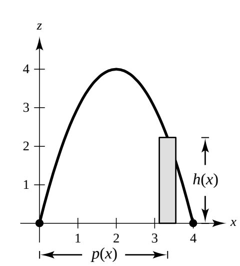
Equation of generating curve: $y = \sqrt{2z}$ or $x = \sqrt{2z}$

53. Let C be a curve in a plane and let L be a line not in a parallel plane. The set of all lines parallel to L and intersecting C is called a cylinder.

55. See pages 765 and 766.

57. $V = 2\pi\displaystyle\int_0^4 x(4x - x^2)\,dx$

$= 2\pi\left[\dfrac{4x^3}{3} - \dfrac{x^4}{4}\right]_0^4 = \dfrac{128\pi}{3}$



59. $z = \frac{x^2}{2} + \frac{y^2}{4}$

(a) When $z = 2$ we have $2 = \frac{x^2}{2} + \frac{y^2}{4}$, or $1 = \frac{x^2}{4} + \frac{y^2}{8}$

Major axis: $2\sqrt{8} = 4\sqrt{2}$

Minor axis: $2\sqrt{4} = 4$

$c^2 = a^2 - b^2, c^2 = 4, c = 2$

Foci: $(0, \pm 2, 2)$

(b) When $z = 8$ we have $8 = \frac{x^2}{2} + \frac{y^2}{4}$, or $1 = \frac{x^2}{16} + \frac{y^2}{32}$.

Major axis: $2\sqrt{32} = 8\sqrt{2}$

Minor axis: $2\sqrt{16} = 8$

$c^2 = 32 - 16 = 16, c = 4$

Foci: $(0, \pm 4, 8)$

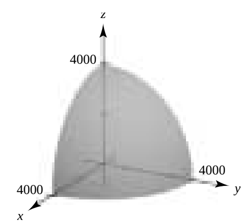
61. If (x, y, z) is on the surface, then

$$(y + 2)^2 = x^2 + (y - 2)^2 + z^2$$
$$y^2 + 4y + 4 = x^2 + y^2 - 4y + 4 + z^2$$
$$x^2 + z^2 = 8y$$

Elliptic paraboloid

Traces parallel to xz-plane are circles.

63. $\frac{x^2}{3963^2} + \frac{y^2}{3963^2} + \frac{z^2}{3942^2} = 1$



65. $z = \frac{y^2}{b^2} - \frac{x^2}{a^2}, z = bx + ay$

$$bx + ay = \frac{y^2}{b^2} - \frac{x^2}{a^2}$$
$$\frac{1}{a^2}\left(x^2 + a^2bx + \frac{a^4b^2}{4}\right) = \frac{1}{b^2}\left(y^2 - ab^2y + \frac{a^2b^4}{4}\right)$$
$$\frac{\left(x + \frac{a^2b}{2}\right)^2}{a^2} = \frac{\left(y - \frac{ab^2}{2}\right)^2}{b^2}$$
$$y = \pm\frac{b}{a}\left(x + \frac{a^2b}{2}\right) + \frac{ab^2}{2}$$

Letting $x = at$, you obtain the two intersecting lines $x = at, y = -bt, z = 0$ and $x = at, y = bt + ab^2$ $z = 2abt + a^2b^2$.

67. The Klein bottle *does not* have both an "inside" and an "outside." It is formed by inserting the small open end through the side of the bottle and making it contiguous with the top of the bottle.

Section 10.7 Cylindrical and Spherical Coordinates

1. $(5, 0, 2)$, cylindrical

$x = 5\cos 0 = 5$

$y = 5\sin 0 = 0$

$z = 2$

$(5, 0, 2)$, rectangular

3. $\left(2, \frac{\pi}{3}, 2\right)$, cylindrical

$x = 2\cos\frac{\pi}{3} = 1$

$y = 2\sin\frac{\pi}{3} = \sqrt{3}$

$z = 2$

$\left(1, \sqrt{3}, 2\right)$, rectangular

5. $\left(4, \frac{7\pi}{6}, 3\right)$, cylindrical

$x = 4\cos\frac{7\pi}{6} = -2\sqrt{3}$

$y = 4\sin\frac{7\pi}{6} = -2$

$z = 3$

$\left(-2\sqrt{3}, -2, 3\right)$, rectangular

7. $(0, 5, 1)$, rectangular

$r = \sqrt{(0)^2 + (5)^2} = 5$

$\theta = \arctan \frac{5}{0} = \frac{\pi}{2}$

$z = 1$

$\left(5, \frac{\pi}{2}, 1\right)$, cylindrical

9. $\left(1, \sqrt{3}, 4\right)$, rectangular

$r = \sqrt{1^2 + \left(\sqrt{3}\right)^2} = 2$

$\theta = \arctan \sqrt{3} = \frac{\pi}{3}$

$z = 4$

$\left(2, \frac{\pi}{3}, 4\right)$, cylindrical

11. $(2, -2, -4)$, rectangular

$r = \sqrt{2^2 + (-2)^2} = 2\sqrt{2}$

$\theta = \arctan(-1) = -\frac{\pi}{4}$

$z = -4$

$\left(2\sqrt{2}, \frac{-\pi}{4}, -4\right)$, cylindrical

13. $x^2 + y^2 + z^2 = 10$ rectangular equation

$r^2 + z^2 = 10$ cylindrical equation

15. $y = x^2$ rectangular equation

$r \sin \theta = (r \cos \theta)^2$

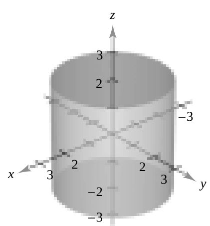
$\sin \theta = r \cos^2 \theta$

$r = \sec \theta \cdot \tan \theta$ cylindrical equation

17. $r = 2$

$\sqrt{x^2 + y^2} = 2$

$x^2 + y^2 = 4$



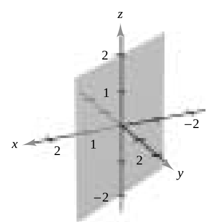
19. $\theta = \frac{\pi}{6}$

$\tan \frac{\pi}{6} = \frac{y}{x}$

$\frac{1}{\sqrt{3}} = \frac{y}{x}$

$x = \sqrt{3}y$

$x - \sqrt{3}y = 0$



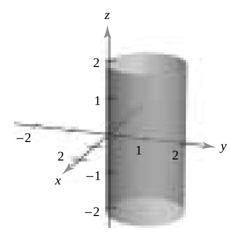
21. $r = 2 \sin \theta$

$r^2 = 2r \sin \theta$

$x^2 + y^2 = 2y$

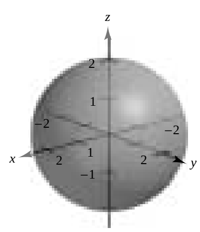
$x^2 + y^2 - 2y = 0$

$x^2 + (y - 1)^2 = 1$



23. $r^2 + z^2 = 4$

$x^2 + y^2 + z^2 = 4$



25. $(4, 0, 0)$, rectangular

$\rho = \sqrt{4^2 + 0^2 + 0^2} = 4$

$\theta = \arctan 0 = 0$

$\phi = \arccos 0 = \frac{\pi}{2}$

$\left(4, 0, \frac{\pi}{2}\right)$, spherical

27. $\left(-2, 2\sqrt{3}, 4\right)$, rectangular

$\rho = \sqrt{(-2)^2 + \left(2\sqrt{3}\right)^2 + 4^2} = 4\sqrt{2}$

$\theta = \arctan\left(-\sqrt{3}\right) = \frac{2\pi}{3}$

$\phi = \arccos \frac{1}{\sqrt{2}} = \frac{\pi}{4}$

$\left(4\sqrt{2}, \frac{2\pi}{3}, \frac{\pi}{4}\right)$, spherical

29. $\left(\sqrt{3}, 1, 2\sqrt{3}\right)$, rectangular

$\rho = \sqrt{3 + 1 + 12} = 4$

$\theta = \arctan \frac{1}{\sqrt{3}} = \frac{\pi}{6}$

$\phi = \arccos \frac{\sqrt{3}}{2} = \frac{\pi}{6}$

$\left(4, \frac{\pi}{6}, \frac{\pi}{6}\right)$, spherical

31. $\left(4, \frac{\pi}{6}, \frac{\pi}{4}\right)$, spherical

$x = 4 \sin \frac{\pi}{4} \cos \frac{\pi}{6} = \sqrt{6}$

$y = 4 \sin \frac{\pi}{4} \sin \frac{\pi}{6} = \sqrt{2}$

$z = 4 \cos \frac{\pi}{4} = 2\sqrt{2}$

$\left(\sqrt{6}, \sqrt{2}, 2\sqrt{2}\right)$, rectangular

33. $\left(12, \frac{-\pi}{4}, 0\right)$, spherical

$x = 12 \sin 0 \cos\left(\frac{-\pi}{4}\right) = 0$

$y = 12 \sin 0 \sin\left(\frac{-\pi}{4}\right) = 0$

$z = 12 \cos 0 = 12$

$(0, 0, 12)$, rectangular

35. $\left(5, \frac{\pi}{4}, \frac{3\pi}{4}\right)$, spherical

$x = 5 \sin \frac{3\pi}{4} \cos \frac{\pi}{4} = \frac{5}{2}$

$y = 5 \sin \frac{3\pi}{4} \sin \frac{\pi}{4} = \frac{5}{2}$

$z = 5 \cos \frac{3\pi}{4} = -\frac{5\sqrt{2}}{2}$

$\left(\frac{5}{2}, \frac{5}{2}, -\frac{5\sqrt{2}}{2}\right)$, rectangular

37. (a) Programs will vary.

(b) $(x, y, z) = (3, -4, 2)$

$(\rho, \theta, \phi) = (5.385, -0.927, 1.190)$

39. $x^2 + y^2 + z^2 = 36$ rectangular equation

$\rho^2 = 36$ spherical equation

41. $x^2 + y^2 = 9$ rectangular equation

$\rho^2 \sin^2 \phi \cos^2 \theta + \rho^2 \sin^2 \phi \sin^2 \theta = 9$

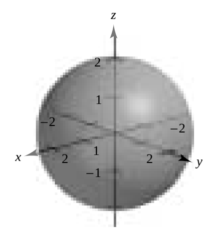
$\rho^2 \sin^2 \phi = 9$

$\rho \sin \phi = 3$

$\rho = 3 \csc \phi$ spherical equation

43. $\rho = 2$

$x^2 + y^2 + z^2 = 4$



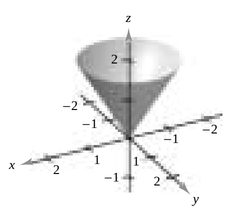
45. $\phi = \frac{\pi}{6}$

$\cos \phi = \frac{z}{\sqrt{x^2 + y^2 + z^2}}$

$\frac{\sqrt{3}}{2} = \frac{z}{\sqrt{x^2 + y^2 + z^2}}$

$\frac{3}{4} = \frac{z^2}{x^2 + y^2 + z^2}$

$3x^2 + 3y^2 - z^2 = 0$

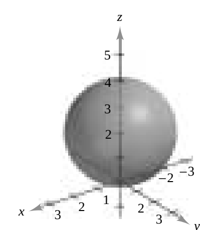


47. $\rho = 4 \cos \phi$

$\sqrt{x^2 + y^2 + z^2} = \frac{4z}{\sqrt{x^2 + y^2 + z^2}}$

$x^2 + y^2 + z^2 - 4z = 0$

$x^2 + y^2 + (z - 2)^2 = 4$

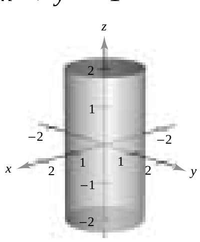


49. $\rho = \csc \phi$

$\rho \sin \phi = 1$

$\sqrt{x^2 + y^2} = 1$

$x^2 + y^2 = 1$



51. $\left(4, \frac{\pi}{4}, 0\right)$, cylindrical

$\rho = \sqrt{4^2 + 0^2} = 4$

$\theta = \frac{\pi}{4}$

$\phi = \arccos 0 = \frac{\pi}{2}$

$\left(4, \frac{\pi}{4}, \frac{\pi}{2}\right)$, spherical

53. $\left(4, \frac{\pi}{2}, 4\right)$, cylindrical

$\rho = \sqrt{4^2 + 4^2} = 4\sqrt{2}$

$\theta = \frac{\pi}{2}$

$\phi = \arccos\left(\frac{4}{4\sqrt{2}}\right) = \frac{\pi}{4}$

$\left(4\sqrt{2}, \frac{\pi}{2}, \frac{\pi}{4}\right)$, spherical

55. $\left(4, \dfrac{-\pi}{6}, 6\right)$, cylindrical

$\rho = \sqrt{4^2 + 6^2} = 2\sqrt{13}$

$\theta = \dfrac{-\pi}{6}$

$\phi = \arccos \dfrac{3}{\sqrt{13}}$

$\left(2\sqrt{13}, \dfrac{-\pi}{6}, \arccos \dfrac{3}{\sqrt{13}}\right)$, spherical

57. $(12, \pi, 5)$, cylindrical

$\rho = \sqrt{12^2 + 5^2} = 13$

$\theta = \pi$

$\phi = \arccos \dfrac{5}{13}$

$\left(13, \pi, \arccos \dfrac{5}{13}\right)$, spherical

59. $\left(10, \dfrac{\pi}{6}, \dfrac{\pi}{2}\right)$, spherical

$r = 10 \sin \dfrac{\pi}{2} = 10$

$\theta = \dfrac{\pi}{6}$

$z = 10 \cos \dfrac{\pi}{2} = 0$

$\left(10, \dfrac{\pi}{6}, 0\right)$, cylindrical

61. $\left(36, \pi, \dfrac{\pi}{2}\right)$, spherical

$r = \rho \sin \phi = 36 \sin \dfrac{\pi}{2} = 36$

$\theta = \pi$

$z = \rho \cos \phi = 36 \cos \dfrac{\pi}{2} = 0$

$(36, \pi, 0)$, cylindrical

63. $\left(6, -\dfrac{\pi}{6}, \dfrac{\pi}{3}\right)$, spherical

$r = 6 \sin \dfrac{\pi}{3} = 3\sqrt{3}$

$\theta = -\dfrac{\pi}{6}$

$z = 6 \cos \dfrac{\pi}{3} = 3$

$\left(3\sqrt{3}, -\dfrac{\pi}{6}, 3\right)$, cylindrical

65. $\left(8, \dfrac{7\pi}{6}, \dfrac{\pi}{6}\right)$, spherical

$r = 8 \sin \dfrac{\pi}{6} = 4$

$\theta = \dfrac{7\pi}{6}$

$z = 8 \cos \dfrac{\pi}{6} = \dfrac{8\sqrt{3}}{2}$

$\left(4, \dfrac{7\pi}{6}, 4\sqrt{3}\right)$, cylindrical

	Rectangular	Cylindrical	Spherical
67.	(4, 6, 3)	(7.211, 0.983, 3)	(7.810, 0.983, 1.177)
69.	(4.698, 1.710, 8)	$\left(5, \dfrac{\pi}{9}, 8\right)$	(9.434, 0.349, 0.559)
71.	(−7.071, 12.247, 14.142)	(14.142, 2.094, 14.142)	$\left(20, \dfrac{2\pi}{3}, \dfrac{\pi}{4}\right)$
73.	(3, −2, 2)	(3.606, −0.588, 2)	(4.123, −0.588, 1.064)
75.	$\left(\dfrac{5}{2}, \dfrac{4}{3}, \dfrac{-3}{2}\right)$	(2.833, 0.490, −1.5)	(3.206, 0.490, 2.058)
77.	(−3.536, 3.536, −5)	$\left(5, \dfrac{3\pi}{4}, -5\right)$	(7.071, 2.356, 2.356)
79.	(2.804, −2.095, 6)	(−3.5, 2.5, 6)	(6.946, 5.642, 0.528)

[Note: Use the cylindrical coordinates (3.5, 5.642, 6)]

81. $r = 5$

Cylinder

Matches graph (d)

83. $\rho = 5$

Sphere

Matches graph (c)

85. $r^2 = z, x^2 + y^2 = z$

Paraboloid

Matches graph (f)

87. Rectangular to cylindrical: $r^2 = x^2 + y^2$

$\tan \theta = \dfrac{y}{x}$

$z = z$

Cylindrical to rectangular: $x = r \cos \theta$

$y = r \sin \theta$

$z = z$

89. Rectangular to spherical: $\rho^2 = x^2 + y^2 + z^2$

$\tan \theta = \dfrac{y}{x}$

$\phi = \arccos\left(\dfrac{z}{\sqrt{x^2 + y^2 + z^2}}\right)$

Spherical to rectangular: $x = \rho \sin \phi \cos \theta$

$y = \rho \sin \phi \sin \theta$

$z = \rho \cos \phi$

91. $x^2 + y^2 + z^2 = 16$

(a) $r^2 + z^2 = 16$

(b) $\rho^2 = 16, \rho = 4$

93. $x^2 + y^2 + z^2 - 2z = 0$

(a) $r^2 + z^2 - 2z = 0, r^2 + (z - 1)^2 = 1$

(b) $\rho^2 - 2\rho \cos \phi = 0, \rho(\rho - 2 \cos \phi) = 0,$

$\rho = 2 \cos \phi$

95. $x^2 + y^2 = 4y$

(a) $r^2 = 4r \sin \theta, \; r = 4 \sin \theta$

(b) $\rho^2 \sin^2 \phi = 4\rho \sin \phi \sin \theta,$

$\rho \sin \phi(\rho \sin \phi - 4 \sin \theta) = 0,$

$\rho = \dfrac{4 \sin \theta}{\sin \phi}, \; \rho = 4 \sin \theta \csc \phi$

97. $x^2 - y^2 = 9$

(a) $r^2 \cos^2 \theta - r^2 \sin^2 \theta = 9,$

$r^2 = \dfrac{9}{\cos^2 \theta - \sin^2 \theta}$

(b) $\rho^2 \sin^2 \phi \cos^2 \theta - \rho^2 \sin^2 \phi \sin^2 \theta = 9,$

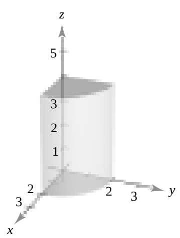
$\rho^2 \sin^2 \phi = \dfrac{9}{\cos^2 \theta - \sin^2 \theta},$

$\rho^2 = \dfrac{9 \csc^2 \phi}{\cos^2 \theta - \sin^2 \theta}$

99. $0 \le \theta \le \dfrac{\pi}{2}$

$0 \le r \le 2$

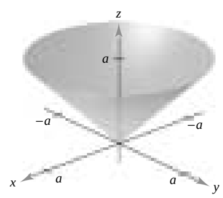
$0 \le z \le 4$



101. $0 \le \theta \le 2\pi$

$0 \le r \le a$

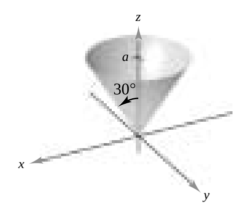
$r \le z \le a$



103. $0 \le \theta \le 2\pi$

$0 \le \phi \le \dfrac{\pi}{6}$

$0 \le \rho \le a \sec \phi$

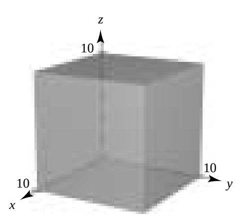


105. Rectangular

$0 \le x \le 10$

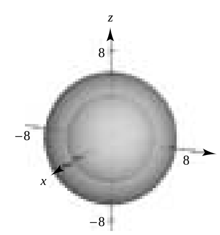
$0 \le y \le 10$

$0 \le z \le 10$



107. Spherical

$4 \le \rho \le 6$



109. $z = \sin \theta, r = 1$

$z = \dfrac{y}{r} = \dfrac{y}{1} = y$

The curve of intersection is the ellipse formed by the intersection of the plane $z = y$ and the cylinder $r = 1$.

Review Exercises for Chapter 10

1. $P = (1, 2), \; Q = (4, 1), \; R = (5, 4)$

(a) $\mathbf{u} = \overrightarrow{PQ} = \langle 3, -1 \rangle = 3\mathbf{i} - \mathbf{j},$

$\mathbf{v} = \overrightarrow{PR} = \langle 4, 2 \rangle = 4\mathbf{i} + 2\mathbf{j}$

(b) $\|\mathbf{v}\| = \sqrt{4^2 + 2^2} = 2\sqrt{5}$

(c) $2\mathbf{u} + \mathbf{v} = \langle 6, -2 \rangle + \langle 4, 2 \rangle = \langle 10, 0 \rangle = 10\mathbf{i}$

3. $\mathbf{v} = \|\mathbf{v}\| \cos \theta \, \mathbf{i} + \|\mathbf{v}\| \sin \theta \, \mathbf{j} = 8 \cos 120^\circ \, \mathbf{i} + 8 \sin 120^\circ \, \mathbf{j}$

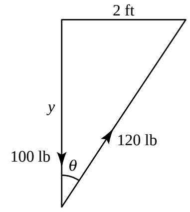
$= -4\mathbf{i} + 4\sqrt{3}\mathbf{j}$

5. $120 \cos \theta = 100$

$$\theta = \arccos\left(\frac{5}{6}\right)$$

$$\tan \theta = \frac{2}{y} \Rightarrow y = \frac{2}{\tan \theta}$$

$$y = \frac{2}{\tan[\arccos(5/6)]} = \frac{2}{\sqrt{11}/5} = \frac{10}{\sqrt{11}} \approx 3.015 \text{ ft}$$



7. $z = 0,\ y = 4,\ x = -5\text{: } (-5, 4, 0)$

9. Looking down from the positive x-axis towards the yz-plane, the point is either in the first quadrant $(y > 0, z > 0)$ or in the third quadrant $(y < 0, z < 0)$. The x-coordinate can be any number.

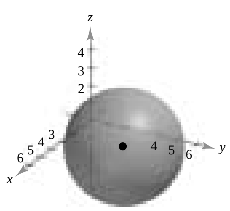
11. $(x - 3)^2 + (y + 2)^2 + (z - 6)^2 = \left(\frac{15}{2}\right)^2$

13. $(x^2 - 4x + 4) + (y^2 - 6y + 9) + z^2 = -4 + 4 + 9$

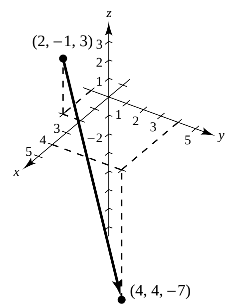
$(x - 2)^2 + (y - 3)^2 + z^2 = 9$

Center: $(2, 3, 0)$

Radius: 3



15. $\mathbf{v} = \langle 4 - 2, 4 + 1, -7 - 3 \rangle = \langle 2, 5, -10 \rangle$



17. $\mathbf{v} = \langle -1 - 3, 6 - 4, 9 + 1 \rangle = \langle -4, 2, 10 \rangle$

$\mathbf{w} = \langle 5 - 3, 3 - 4, -6 + 1 \rangle = \langle 2, -1, -5 \rangle$

Since $-2\mathbf{w} = \mathbf{v}$, the points lie in a straight line.

19. Unit vector: $\dfrac{\mathbf{u}}{\|\mathbf{u}\|} = \dfrac{\langle 2, 3, 5 \rangle}{\sqrt{38}} = \left\langle \dfrac{2}{\sqrt{38}}, \dfrac{3}{\sqrt{38}}, \dfrac{5}{\sqrt{38}} \right\rangle$

21. $P = (5, 0, 0),\ Q = (4, 4, 0),\ R = (2, 0, 6)$

(a) $\mathbf{u} = \overrightarrow{PQ} = \langle -1, 4, 0 \rangle = -\mathbf{i} + 4\mathbf{j}$,

$\mathbf{v} = \overrightarrow{PR} = \langle -3, 0, 6 \rangle = -3\mathbf{i} + 6\mathbf{k}$

(b) $\mathbf{u} \cdot \mathbf{v} = (-1)(-3) + 4(0) + 0(6) = 3$

(c) $\mathbf{v} \cdot \mathbf{v} = 9 + 36 = 45$

23. $\mathbf{u} = \langle 7, -2, 3 \rangle,\ \mathbf{v} = \langle -1, 4, 5 \rangle$

Since $\mathbf{u} \cdot \mathbf{v} = 0$, the vectors are orthogonal.

25. $\mathbf{u} = 5\left(\cos \frac{3\pi}{4}\mathbf{i} + \sin \frac{3\pi}{4}\mathbf{j}\right) = \frac{5\sqrt{2}}{2}[-\mathbf{i} + \mathbf{j}]$

$\mathbf{v} = 2\left(\cos \frac{2\pi}{3}\mathbf{i} + \sin \frac{2\pi}{3}\mathbf{j}\right) = -\mathbf{i} + \sqrt{3}\mathbf{j}$

$\mathbf{u} \cdot \mathbf{v} = \frac{5\sqrt{2}}{2}\left(1 + \sqrt{3}\right)$

$\|\mathbf{u}\| = 5$

$\|\mathbf{v}\| = 2$

$$\cos \theta = \frac{|\mathbf{u} \cdot \mathbf{v}|}{\|\mathbf{u}\|\,\|\mathbf{v}\|} = \frac{\left(5\sqrt{2}/2\right)\left(1 + \sqrt{3}\right)}{5(2)} = \frac{\sqrt{2} + \sqrt{6}}{4}$$

$$\theta = \arccos \frac{\sqrt{2} + \sqrt{6}}{4} = 15^\circ$$

27. $\mathbf{u} = \langle 10, -5, 15 \rangle,\ \mathbf{v} = \langle -2, 1, -3 \rangle$

$\mathbf{u} = -5\mathbf{v} \Rightarrow \mathbf{u}$ is parallel to $\mathbf{v}$ and in the opposite direction.

$\theta = \pi$

29. There are many correct answers. For example: $v = \pm\langle 6, -5, 0\rangle$.

In Exercises 31–39, $\mathbf{u} = \langle 3, -2, 1\rangle$, $\mathbf{v} = \langle 2, -4, -3\rangle$, $\mathbf{w} = \langle -1, 2, 2\rangle$.

31. $\mathbf{u}\cdot\mathbf{u} = 3(3) + (-2)(-2) + (1)(1)$

$= 14 = \left(\sqrt{14}\right)^2 = \|\mathbf{u}\|^2$

33. $\text{proj}_{\mathbf{u}}\mathbf{w} = \left(\dfrac{\mathbf{u}\cdot\mathbf{w}}{\|\mathbf{u}\|^2}\right)\mathbf{u}$

$= -\dfrac{5}{14}\langle 3, -2, 1\rangle$

$= \left\langle -\dfrac{15}{14}, \dfrac{10}{14}, -\dfrac{5}{14}\right\rangle$

$= \left\langle -\dfrac{15}{14}, \dfrac{5}{7}, -\dfrac{5}{14}\right\rangle$

35. $\mathbf{n} = \mathbf{v}\times\mathbf{w} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & -4 & -3 \\ -1 & 2 & 2 \end{vmatrix} = -2\mathbf{i} - \mathbf{j}$

$\|\mathbf{n}\| = \sqrt{5}$

$\dfrac{\mathbf{n}}{\|\mathbf{n}\|} = \dfrac{1}{\sqrt{5}}(-2\mathbf{i} - \mathbf{j})$

37. $V = |\mathbf{u}\cdot(\mathbf{v}\times\mathbf{w})|$

$= |\langle 3, -2, 1\rangle \cdot \langle -2, -1, 0\rangle| = |-4| = 4$

39. Area parallelogram $= \|\mathbf{u}\times\mathbf{v}\| = \sqrt{10^2 + 11^2 + (-8)^2}$ (See Exercises 36, 38)

$= \sqrt{285}$

41. $\mathbf{F} = c(\cos 20°\mathbf{j} + \sin 20°\mathbf{k})$

$\overrightarrow{PQ} = 2\mathbf{k}$

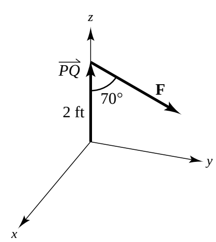
$\overrightarrow{PQ}\times\mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 2 \\ 0 & c\cos 20° & c\sin 20° \end{vmatrix} = -2c\cos 20°\mathbf{i}$

$200 = \|\overrightarrow{PQ}\times\mathbf{F}\| = 2c\cos 20°$

$c = \dfrac{100}{\cos 20°}$

$\mathbf{F} = \dfrac{100}{\cos 20°}(\cos 20°\mathbf{j} + \sin 20°\mathbf{k}) = 100(\mathbf{j} + \tan 20°\mathbf{k})$

$\|\mathbf{F}\| = 100\sqrt{1 + \tan^2 20°} = 100\sec 20° \approx 106.4$ lb



43. $\mathbf{v} = \mathbf{j}$

(a) $x = 1,\ y = 2 + t,\ z = 3$

(b) None

45. $3x - 3y - 7z = -4,\ x - y + 2z = 3$

Solving simultaneously, we have $z = 1$. Substituting $z = 1$ into the second equation we have $y = x - 1$. Substituting for x in this equation we obtain two points on the line of intersection, $(0, -1, 1)$, $(1, 0, 1)$. The direction vector of the line of intersection is $\mathbf{v} = \mathbf{i} + \mathbf{j}$.

(a) $x = t,\ y = -1 + t,\ z = 1$

(b) $x = y + 1,\ z = 1$

47. The two lines are parallel as they have the same direction numbers, $-2, 1, 1$. Therefore, a vector parallel to the plane is $\mathbf{v} = -2\mathbf{i} + \mathbf{j} + \mathbf{k}$. A point on the first line is $(1, 0, -1)$ and a point on the second line is $(-1, 1, 2)$. The vector $\mathbf{u} = 2\mathbf{i} - \mathbf{j} - 3\mathbf{k}$ connecting these two points is also parallel to the plane. Therefore, a normal to the plane is

$$\mathbf{v} \times \mathbf{u} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -2 & 1 & 1 \\ 2 & -1 & -3 \end{vmatrix}$$

$$= -2\mathbf{i} - 4\mathbf{j} = -2(\mathbf{i} + 2\mathbf{j}).$$

Equation of the plane: $(x - 1) + 2y = 0$

$$x + 2y = 1$$

49. $Q = (1, 0, 2)$

$2x - 3y + 6z = 6$

A point P on the plane is $(3, 0, 0)$.

$\overrightarrow{PQ} = \langle -2, 0, 2 \rangle$

$\mathbf{n} = \langle 2, -3, 6 \rangle$

$$D = \frac{|\overrightarrow{PQ} \cdot \mathbf{n}|}{\|\mathbf{n}\|} = \frac{8}{7}$$

51. $Q(3, -2, 4)$ point

$P(5, 0, 0)$ point on plane

$\mathbf{n} = \langle 2, -5, 1 \rangle$ normal to plane

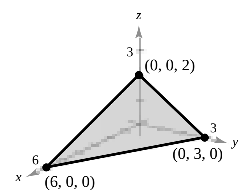
$\overrightarrow{PQ} = \langle -2, -2, 4 \rangle$

$$D = \frac{|\overrightarrow{PQ} \cdot \mathbf{n}|}{\|\mathbf{n}\|} = \frac{10}{\sqrt{30}} = \frac{\sqrt{30}}{3}$$

53. $x + 2y + 3z = 6$

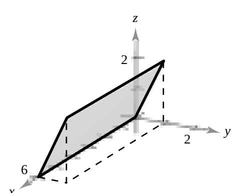
Plane

Intercepts: $(6, 0, 0)$, $(0, 3, 0)$, $(0, 0, 2)$



55. $y = \dfrac{1}{2}z$

Plane with rulings parallel to the x-axis



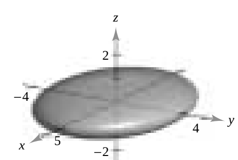
57. $\dfrac{x^2}{16} + \dfrac{y^2}{9} + z^2 = 1$

Ellipsoid

xy-trace: $\dfrac{x^2}{16} + \dfrac{y^2}{9} = 1$

xz-trace: $\dfrac{x^2}{16} + z^2 = 1$

yz-trace: $\dfrac{y^2}{9} + z^2 = 1$



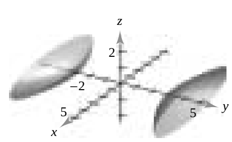
59. $\dfrac{x^2}{16} - \dfrac{y^2}{9} + z^2 = -1$

Hyperboloid of two sheets

xy-trace: $\dfrac{y^2}{4} - \dfrac{x^2}{16} = 1$

xz-trace: None

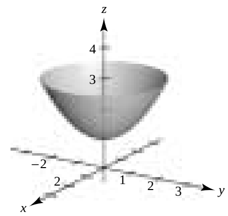
yz-trace: $\dfrac{y^2}{9} - z^2 = 1$



61. (a)

$$x^2 + y^2 = [r(z)]^2 = \left[\sqrt{2(z-1)}\right]^2$$

$$x^2 + y^2 - 2z + 2 = 0$$

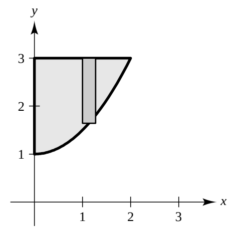


(b) $V = 2\pi\int_0^2 x\left[3 - \left(\frac{1}{2}x^2 + 1\right)\right] dx$

$= 2\pi\int_0^2 \left(2x - \frac{1}{2}x^3\right) dx$

$= 2\pi\left[x^2 - \frac{x^4}{8}\right]_0^2$

$= 4\pi \approx 12.6 \text{ cm}^3$

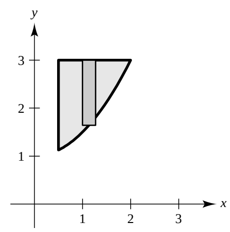


(c) $V = 2\pi\int_{1/2}^2 x\left[3 - \left(\frac{1}{2}x^2 + 1\right)\right] dx$

$= 2\pi\int_{1/2}^2 \left(2x - \frac{1}{2}x^3\right) dx$

$= 2\pi\left[x^2 - \frac{x^4}{8}\right]_{1/2}^2$

$= 4\pi - \frac{31\pi}{64} = \frac{225\pi}{64} \approx 11.04 \text{ cm}^3$



63. $\left(-2\sqrt{2}, 2\sqrt{2}, 2\right)$, rectangular

(a) $r = \sqrt{\left(-2\sqrt{2}\right)^2 + \left(2\sqrt{2}\right)^2} = 4,\ \theta = \arctan(-1) = \frac{3\pi}{4},\ z = 2,\ \left(4, \frac{3\pi}{4}, 2\right)$, cylindrical

(b) $\rho = \sqrt{\left(-2\sqrt{2}\right)^2 + \left(2\sqrt{2}\right)^2 + (2)^2} = 2\sqrt{5},\ \theta = \frac{3\pi}{4},\ \phi = \arccos\frac{2}{2\sqrt{5}} = \arccos\frac{1}{\sqrt{5}},\ \left(2\sqrt{5}, \frac{3\pi}{4}, \arccos\frac{\sqrt{5}}{5}\right)$, spherical

65. $\left(100, -\frac{\pi}{6}, 50\right)$, cylindrical

$\rho = \sqrt{100^2 + 50^2} = 50\sqrt{5}$

$\theta = -\frac{\pi}{6}$

$\phi = \arccos\left(\frac{50}{50\sqrt{5}}\right) = \arccos\left(\frac{1}{\sqrt{5}}\right) \approx 63.4°$

$\left(50\sqrt{5}, -\frac{\pi}{6}, 63.4°\right)$, spherical

67. $\left(25, -\frac{\pi}{4}, \frac{3\pi}{4}\right)$, spherical

$r^2 = \left(25\sin\left(\frac{3\pi}{4}\right)\right)^2 \implies r = 25\frac{\sqrt{2}}{2}$

$\theta = -\frac{\pi}{4}$

$z = \rho\cos\phi - 25\cos\frac{3\pi}{4} = -25\frac{\sqrt{2}}{2}$

$\left(25\frac{\sqrt{2}}{2}, -\frac{\pi}{4}, -\frac{25\sqrt{2}}{2}\right)$, cylindrical

69. $x^2 - y^2 = 2z$

(a) Cylindrical: $r^2\cos^2\theta - r^2\sin^2\theta = 2z,\ r^2\cos 2\theta = 2z$

(b) Spherical: $\rho^2\sin^2\phi\cos^2\theta - \rho^2\sin^2\phi\sin^2\theta = 2\rho\cos\phi,\ \rho\sin^2\phi\cos 2\theta - 2\cos\phi = 0,\ \rho = 2\sec 2\theta\cos\phi\csc^2\phi$