91. $x^2 + y^2 + z^2 = 16$

(a) $r^2 + z^2 = 16$

(b) $\rho^2 = 16, \rho = 4$

93. $x^2 + y^2 + z^2 - 2z = 0$

(a) $r^2 + z^2 - 2z = 0, r^2 + (z - 1)^2 = 1$

(b) $\rho^2 - 2\rho\cos\phi = 0, \rho(\rho - 2\cos\phi) = 0,$

$\rho = 2\cos\phi$

95. $x^2 + y^2 = 4y$

(a) $r^2 = 4r\sin\theta,\ r = 4\sin\theta$

(b) $\rho^2\sin^2\phi = 4\rho\sin\phi\sin\theta,$

$\rho\sin\phi(\rho\sin\phi - 4\sin\theta) = 0,$

$\rho = \dfrac{4\sin\theta}{\sin\phi},\ \rho = 4\sin\theta\csc\phi$

97. $x^2 - y^2 = 9$

(a) $r^2\cos^2\theta - r^2\sin^2\theta = 9,$

$r^2 = \dfrac{9}{\cos^2\theta - \sin^2\theta}$

(b) $\rho^2\sin^2\phi\cos^2\theta - \rho^2\sin^2\phi\sin^2\theta = 9,$

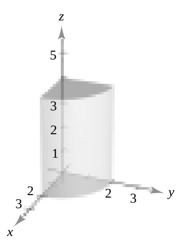
$\rho^2\sin^2\phi = \dfrac{9}{\cos^2\theta - \sin^2\theta},$

$\rho^2 = \dfrac{9\csc^2\phi}{\cos^2\theta - \sin^2\theta}$

99. $0 \le \theta \le \dfrac{\pi}{2}$

$0 \le r \le 2$

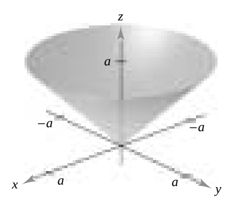
$0 \le z \le 4$



101. $0 \le \theta \le 2\pi$

$0 \le r \le a$

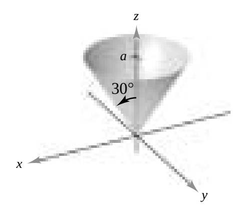
$r \le z \le a$



103. $0 \le \theta \le 2\pi$

$0 \le \phi \le \dfrac{\pi}{6}$

$0 \le \rho \le a\sec\phi$

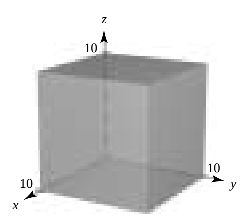


105. Rectangular

$0 \le x \le 10$

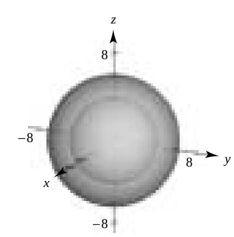
$0 \le y \le 10$

$0 \le z \le 10$



107. Spherical

$4 \le \rho \le 6$



109. $z = \sin\theta, r = 1$

$z = \dfrac{y}{r} = \dfrac{y}{1} = y$

The curve of intersection is the ellipse formed by the intersection of the plane $z = y$ and the cylinder $r = 1$.

Review Exercises for Chapter 10

1. $P = (1, 2),\ Q = (4, 1),\ R = (5, 4)$

(a) $\mathbf{u} = \overrightarrow{PQ} = \langle 3, -1\rangle = 3\mathbf{i} - \mathbf{j},$

$\mathbf{v} = \overrightarrow{PR} = \langle 4, 2\rangle = 4\mathbf{i} + 2\mathbf{j}$

(b) $\|\mathbf{v}\| = \sqrt{4^2 + 2^2} = 2\sqrt{5}$

(c) $2\mathbf{u} + \mathbf{v} = \langle 6, -2\rangle + \langle 4, 2\rangle = \langle 10, 0\rangle = 10\mathbf{i}$

3. $\mathbf{v} = \|\mathbf{v}\|\cos\theta\,\mathbf{i} + \|\mathbf{v}\|\sin\theta\,\mathbf{j} = 8\cos 120^\circ\,\mathbf{i} + 8\sin 120^\circ\,\mathbf{j}$

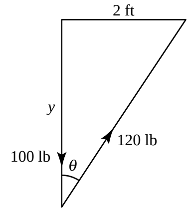
$= -4\mathbf{i} + 4\sqrt{3}\mathbf{j}$

5. $120 \cos \theta = 100$

$$\theta = \arccos\left(\frac{5}{6}\right)$$

$$\tan \theta = \frac{2}{y} \Rightarrow y = \frac{2}{\tan \theta}$$

$$y = \frac{2}{\tan[\arccos(5/6)]} = \frac{2}{\sqrt{11}/5} = \frac{10}{\sqrt{11}} \approx 3.015 \text{ ft}$$



7. $z = 0,\ y = 4,\ x = -5:\ (-5, 4, 0)$

9. Looking down from the positive x-axis towards the yz-plane, the point is either in the first quadrant $(y > 0, z > 0)$ or in the third quadrant $(y < 0, z < 0)$. The x-coordinate can be any number.

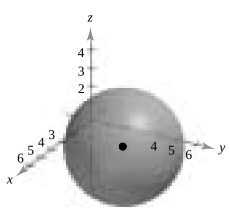
11. $(x - 3)^2 + (y + 2)^2 + (z - 6)^2 = \left(\frac{15}{2}\right)^2$

13. $(x^2 - 4x + 4) + (y^2 - 6y + 9) + z^2 = -4 + 4 + 9$

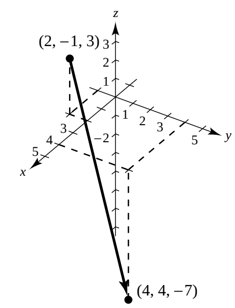
$(x - 2)^2 + (y - 3)^2 + z^2 = 9$

Center: $(2, 3, 0)$

Radius: 3



15. $\mathbf{v} = \langle 4 - 2, 4 + 1, -7 - 3\rangle = \langle 2, 5, -10\rangle$



17. $\mathbf{v} = \langle -1 - 3, 6 - 4, 9 + 1\rangle = \langle -4, 2, 10\rangle$

$\mathbf{w} = \langle 5 - 3, 3 - 4, -6 + 1\rangle = \langle 2, -1, -5\rangle$

Since $-2\mathbf{w} = \mathbf{v}$, the points lie in a straight line.

19. Unit vector: $\dfrac{\mathbf{u}}{\|\mathbf{u}\|} = \dfrac{\langle 2, 3, 5\rangle}{\sqrt{38}} = \left\langle \dfrac{2}{\sqrt{38}}, \dfrac{3}{\sqrt{38}}, \dfrac{5}{\sqrt{38}}\right\rangle$

21. $P = (5, 0, 0),\ Q = (4, 4, 0),\ R = (2, 0, 6)$

(a) $\mathbf{u} = \overrightarrow{PQ} = \langle -1, 4, 0\rangle = -\mathbf{i} + 4\mathbf{j}$,

$\mathbf{v} = \overrightarrow{PR} = \langle -3, 0, 6\rangle = -3\mathbf{i} + 6\mathbf{k}$

(b) $\mathbf{u} \cdot \mathbf{v} = (-1)(-3) + 4(0) + 0(6) = 3$

(c) $\mathbf{v} \cdot \mathbf{v} = 9 + 36 = 45$

23. $\mathbf{u} = \langle 7, -2, 3\rangle,\ \mathbf{v} = \langle -1, 4, 5\rangle$

Since $\mathbf{u} \cdot \mathbf{v} = 0$, the vectors are orthogonal.

25. $\mathbf{u} = 5\left(\cos \frac{3\pi}{4}\mathbf{i} + \sin \frac{3\pi}{4}\mathbf{j}\right) = \frac{5\sqrt{2}}{2}[-\mathbf{i} + \mathbf{j}]$

$\mathbf{v} = 2\left(\cos \frac{2\pi}{3}\mathbf{i} + \sin \frac{2\pi}{3}\mathbf{j}\right) = -\mathbf{i} + \sqrt{3}\mathbf{j}$

$\mathbf{u} \cdot \mathbf{v} = \frac{5\sqrt{2}}{2}(1 + \sqrt{3})$

$\|\mathbf{u}\| = 5$

$\|\mathbf{v}\| = 2$

$$\cos \theta = \frac{|\mathbf{u} \cdot \mathbf{v}|}{\|\mathbf{u}\|\,\|\mathbf{v}\|} = \frac{(5\sqrt{2}/2)(1 + \sqrt{3})}{5(2)} = \frac{\sqrt{2} + \sqrt{6}}{4}$$

$$\theta = \arccos \frac{\sqrt{2} + \sqrt{6}}{4} = 15°$$

27. $\mathbf{u} = \langle 10, -5, 15\rangle,\ \mathbf{v} = \langle -2, 1, -3\rangle$

$\mathbf{u} = -5\mathbf{v} \Rightarrow \mathbf{u}$ is parallel to $\mathbf{v}$ and in the opposite direction.

$\theta = \pi$

29. There are many correct answers. For example: $v = \pm\langle 6, -5, 0\rangle$.

In Exercises 31–39, $\mathbf{u} = \langle 3, -2, 1\rangle$, $\mathbf{v} = \langle 2, -4, -3\rangle$, $\mathbf{w} = \langle -1, 2, 2\rangle$.

31. $$\mathbf{u}\cdot\mathbf{u} = 3(3) + (-2)(-2) + (1)(1) = 14 = \left(\sqrt{14}\right)^2 = \|\mathbf{u}\|^2$$

33. $$\text{proj}_{\mathbf{u}}\mathbf{w} = \left(\frac{\mathbf{u}\cdot\mathbf{w}}{\|\mathbf{u}\|^2}\right)\mathbf{u} = -\frac{5}{14}\langle 3, -2, 1\rangle = \left\langle -\frac{15}{14}, \frac{10}{14}, -\frac{5}{14}\right\rangle = \left\langle -\frac{15}{14}, \frac{5}{7}, -\frac{5}{14}\right\rangle$$

35. $$\mathbf{n} = \mathbf{v}\times\mathbf{w} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & -4 & -3 \\ -1 & 2 & 2 \end{vmatrix} = -2\mathbf{i} - \mathbf{j}$$

$$\|\mathbf{n}\| = \sqrt{5}$$

$$\frac{\mathbf{n}}{\|\mathbf{n}\|} = \frac{1}{\sqrt{5}}(-2\mathbf{i} - \mathbf{j})$$

37. $$V = |\mathbf{u}\cdot(\mathbf{v}\times\mathbf{w})| = |\langle 3, -2, 1\rangle \cdot \langle -2, -1, 0\rangle| = |-4| = 4$$

39. $$\text{Area parallelogram} = \|\mathbf{u}\times\mathbf{v}\| = \sqrt{10^2 + 11^2 + (-8)^2} \text{ (See Exercises 36, 38)} = \sqrt{285}$$

41. $$\mathbf{F} = c(\cos 20^\circ\mathbf{j} + \sin 20^\circ\mathbf{k})$$

$$\overrightarrow{PQ} = 2\mathbf{k}$$

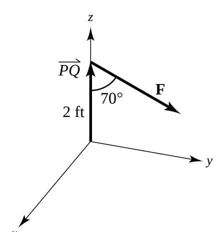
$$\overrightarrow{PQ}\times\mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 2 \\ 0 & c\cos 20^\circ & c\sin 20^\circ \end{vmatrix} = -2c\cos 20^\circ\mathbf{i}$$

$$200 = \|\overrightarrow{PQ}\times\mathbf{F}\| = 2c\cos 20^\circ$$

$$c = \frac{100}{\cos 20^\circ}$$

$$\mathbf{F} = \frac{100}{\cos 20^\circ}(\cos 20^\circ\mathbf{j} + \sin 20^\circ\mathbf{k}) = 100(\mathbf{j} + \tan 20^\circ\mathbf{k})$$

$$\|\mathbf{F}\| = 100\sqrt{1 + \tan^2 20^\circ} = 100\sec 20^\circ \approx 106.4 \text{ lb}$$



43. $\mathbf{v} = \mathbf{j}$

(a) $x = 1,\ y = 2 + t,\ z = 3$

(b) None

45. $3x - 3y - 7z = -4,\ x - y + 2z = 3$

Solving simultaneously, we have $z = 1$. Substituting $z = 1$ into the second equation we have $y = x - 1$. Substituting for x in this equation we obtain two points on the line of intersection, $(0, -1, 1)$, $(1, 0, 1)$. The direction vector of the line of intersection is $\mathbf{v} = \mathbf{i} + \mathbf{j}$.

(a) $x = t,\ y = -1 + t,\ z = 1$

(b) $x = y + 1,\ z = 1$

47. The two lines are parallel as they have the same direction numbers, $-2, 1, 1$. Therefore, a vector parallel to the plane is $\mathbf{v} = -2\mathbf{i} + \mathbf{j} + \mathbf{k}$. A point on the first line is $(1, 0, -1)$ and a point on the second line is $(-1, 1, 2)$. The vector $\mathbf{u} = 2\mathbf{i} - \mathbf{j} - 3\mathbf{k}$ connecting these two points is also parallel to the plane. Therefore, a normal to the plane is

$$\mathbf{v} \times \mathbf{u} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -2 & 1 & 1 \\ 2 & -1 & -3 \end{vmatrix}$$

$$= -2\mathbf{i} - 4\mathbf{j} = -2(\mathbf{i} + 2\mathbf{j}).$$

Equation of the plane: $(x - 1) + 2y = 0$

$$x + 2y = 1$$

49. $Q = (1, 0, 2)$

$2x - 3y + 6z = 6$

A point P on the plane is $(3, 0, 0)$.

$\overrightarrow{PQ} = \langle -2, 0, 2 \rangle$

$\mathbf{n} = \langle 2, -3, 6 \rangle$

$$D = \frac{|\overrightarrow{PQ} \cdot \mathbf{n}|}{\|\mathbf{n}\|} = \frac{8}{7}$$

51. $Q(3, -2, 4)$ point

$P(5, 0, 0)$ point on plane

$\mathbf{n} = \langle 2, -5, 1 \rangle$ normal to plane

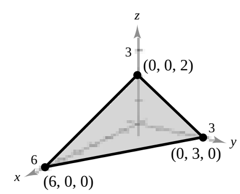
$\overrightarrow{PQ} = \langle -2, -2, 4 \rangle$

$$D = \frac{|\overrightarrow{PQ} \cdot \mathbf{n}|}{\|\mathbf{n}\|} = \frac{10}{\sqrt{30}} = \frac{\sqrt{30}}{3}$$

53. $x + 2y + 3z = 6$

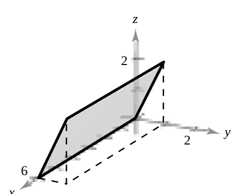
Plane

Intercepts: $(6, 0, 0)$, $(0, 3, 0)$, $(0, 0, 2)$



55. $y = \frac{1}{2}z$

Plane with rulings parallel to the x-axis



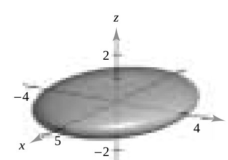
57. $\frac{x^2}{16} + \frac{y^2}{9} + z^2 = 1$

Ellipsoid

xy-trace: $\frac{x^2}{16} + \frac{y^2}{9} = 1$

xz-trace: $\frac{x^2}{16} + z^2 = 1$

yz-trace: $\frac{y^2}{9} + z^2 = 1$



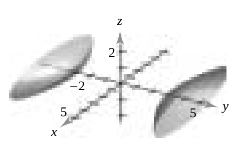
59. $\frac{x^2}{16} - \frac{y^2}{9} + z^2 = -1$

Hyperboloid of two sheets

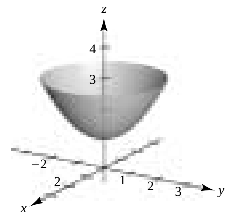
xy-trace: $\frac{y^2}{4} - \frac{x^2}{16} = 1$

xz-trace: None

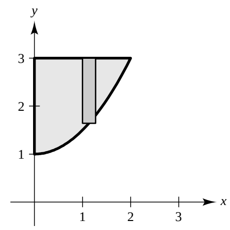
yz-trace: $\frac{y^2}{9} - z^2 = 1$



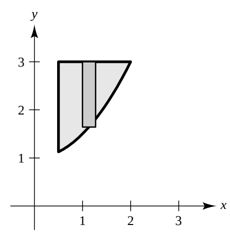
61. (a) $$x^2 + y^2 = [r(z)]^2 = \left[\sqrt{2(z-1)}\right]^2$$
$$x^2 + y^2 - 2z + 2 = 0$$



(b) $$V = 2\pi\int_0^2 x\left[3 - \left(\frac{1}{2}x^2 + 1\right)\right]dx = 2\pi\int_0^2 \left(2x - \frac{1}{2}x^3\right)dx = 2\pi\left[x^2 - \frac{x^4}{8}\right]_0^2 = 4\pi \approx 12.6 \text{ cm}^3$$



(c) $$V = 2\pi\int_{1/2}^2 x\left[3 - \left(\frac{1}{2}x^2 + 1\right)\right]dx = 2\pi\int_{1/2}^2 \left(2x - \frac{1}{2}x^3\right)dx = 2\pi\left[x^2 - \frac{x^4}{8}\right]_{1/2}^2 = 4\pi - \frac{31\pi}{64} = \frac{225\pi}{64} \approx 11.04 \text{ cm}^3$$



63. $\left(-2\sqrt{2}, 2\sqrt{2}, 2\right)$, rectangular

(a) $r = \sqrt{\left(-2\sqrt{2}\right)^2 + \left(2\sqrt{2}\right)^2} = 4$, $\theta = \arctan(-1) = \frac{3\pi}{4}$, $z = 2$, $\left(4, \frac{3\pi}{4}, 2\right)$, cylindrical

(b) $\rho = \sqrt{\left(-2\sqrt{2}\right)^2 + \left(2\sqrt{2}\right)^2 + (2)^2} = 2\sqrt{5}$, $\theta = \frac{3\pi}{4}$, $\phi = \arccos\frac{2}{2\sqrt{5}} = \arccos\frac{1}{\sqrt{5}}$, $\left(2\sqrt{5}, \frac{3\pi}{4}, \arccos\frac{\sqrt{5}}{5}\right)$, spherical

65. $\left(100, -\frac{\pi}{6}, 50\right)$, cylindrical

$$\rho = \sqrt{100^2 + 50^2} = 50\sqrt{5}$$
$$\theta = -\frac{\pi}{6}$$
$$\phi = \arccos\left(\frac{50}{50\sqrt{5}}\right) = \arccos\left(\frac{1}{\sqrt{5}}\right) \approx 63.4°$$

$\left(50\sqrt{5}, -\frac{\pi}{6}, 63.4°\right)$, spherical

67. $\left(25, -\frac{\pi}{4}, \frac{3\pi}{4}\right)$, spherical

$$r^2 = \left(25\sin\left(\frac{3\pi}{4}\right)\right)^2 \implies r = 25\frac{\sqrt{2}}{2}$$
$$\theta = -\frac{\pi}{4}$$
$$z = \rho\cos\phi - 25\cos\frac{3\pi}{4} = -25\frac{\sqrt{2}}{2}$$

$\left(25\frac{\sqrt{2}}{2}, -\frac{\pi}{4}, -\frac{25\sqrt{2}}{2}\right)$, cylindrical

69. $x^2 - y^2 = 2z$

(a) Cylindrical: $r^2\cos^2\theta - r^2\sin^2\theta = 2z$, $r^2\cos 2\theta = 2z$

(b) Spherical: $\rho^2\sin^2\phi\cos^2\theta - \rho^2\sin^2\phi\sin^2\theta = 2\rho\cos\phi$, $\rho\sin^2\phi\cos 2\theta - 2\cos\phi = 0$, $\rho = 2\sec 2\theta\cos\phi\csc^2\phi$

Problem Solving for Chapter 10

1. $\mathbf{a} + \mathbf{b} + \mathbf{c} = \mathbf{0}$

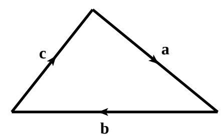
$\mathbf{b} \times (\mathbf{a} + \mathbf{b} + \mathbf{c}) = \mathbf{0}$

$(\mathbf{b} \times \mathbf{a}) + (\mathbf{b} \times \mathbf{c}) = \mathbf{0}$

$\|\mathbf{a} \times \mathbf{b}\| = \|\mathbf{b} \times \mathbf{c}\|$

$\|\mathbf{b} \times \mathbf{c}\| = \|\mathbf{b}\|\,\|\mathbf{c}\| \sin A$

$\|\mathbf{a} \times \mathbf{b}\| = \|\mathbf{a}\|\,\|\mathbf{b}\| \sin C$



Then,

$$\frac{\sin A}{\|\mathbf{a}\|} = \frac{\|\mathbf{b} \times \mathbf{c}\|}{\|\mathbf{a}\|\,\|\mathbf{b}\|\,\|\mathbf{c}\|} = \frac{\|\mathbf{a} \times \mathbf{b}\|}{\|\mathbf{a}\|\,\|\mathbf{b}\|\,\|\mathbf{c}\|} = \frac{\sin C}{\|\mathbf{c}\|}.$$

The other case, $\dfrac{\sin A}{\|\mathbf{a}\|} = \dfrac{\sin B}{\|\mathbf{b}\|}$ is similar.

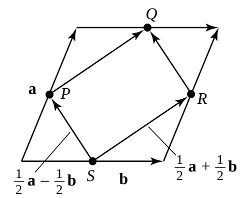
3. Label the figure as indicated.

From the figure, you see that

$$\overrightarrow{SP} = \frac{1}{2}\mathbf{a} - \frac{1}{2}\mathbf{b} = \overrightarrow{RQ} \text{ and}$$

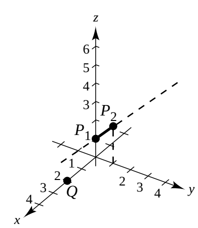
$$\overrightarrow{SR} = \frac{1}{2}\mathbf{a} + \frac{1}{2}\mathbf{b} = \overrightarrow{PQ}.$$

Since $\overrightarrow{SP} = \overrightarrow{RQ}$ and $\overrightarrow{SR} = \overrightarrow{PQ}$, $PSRQ$ is a parallelogram.



5. (a) $\mathbf{u} = \langle 0, 1, 1\rangle$ direction vector of line determined by P_1 and P_2.

$$D = \frac{\|\overrightarrow{P_1Q} \times \mathbf{u}\|}{\|\mathbf{u}\|} = \frac{\|\langle 2, 0, -1\rangle \times \langle 0, 1, 1\rangle\|}{\sqrt{2}} = \frac{\|\langle 1, -2, 2\rangle\|}{\sqrt{2}} = \frac{3}{\sqrt{2}} = \frac{3\sqrt{2}}{2}$$



(b) The shortest distance to the line **segment** is $\|P_1Q\| = \|\langle 2, 0, -1\rangle\| = \sqrt{5}$.

7. (a) $V = \pi\displaystyle\int_0^1 \left(\sqrt{2}\right)^2 dz = \left[\pi\frac{z^2}{2}\right]_0^1 = \frac{1}{2}\pi$

Note: $\frac{1}{2}$(base)(altitude) $= \frac{1}{2}\pi(1) = \frac{1}{2}\pi$

(b) $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = z$: (slice at $z = c$)

$$\frac{x^2}{\left(\sqrt{c}a\right)^2} + \frac{y^2}{\left(\sqrt{c}b\right)^2} = 1$$

At $z = c$, figure is ellipse of area

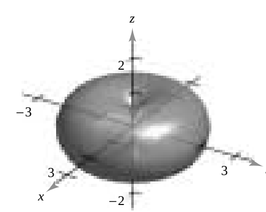
$$\pi\left(\sqrt{c}a\right)\left(\sqrt{c}b\right) = \pi abc.$$

$$V = \int_0^k \pi abc \cdot dc = \left[\frac{\pi abc^2}{2}\right]_0^k = \frac{\pi abk^2}{2}$$

(c) $V = \frac{1}{2}(\pi abk)k = \frac{1}{2}$(base)(height)

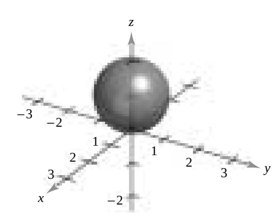
9. (a) $\rho = 2\sin\phi$

Torus



(b) $\rho = 2\cos\phi$

Sphere



11. From Exercise 64, Section 10.4, $(\mathbf{u} \times \mathbf{v}) \times (\mathbf{w} \times \mathbf{z}) = [(\mathbf{u} \times \mathbf{v}) \cdot \mathbf{z}]\mathbf{w} - [(\mathbf{u} \times \mathbf{v}) \cdot \mathbf{w}]\mathbf{z}$.

13. (a) $\mathbf{u} = \|\mathbf{u}\|(\cos 0\,\mathbf{i} + \sin 0\,\mathbf{j}) = \|\mathbf{u}\|\mathbf{i}$

Downward force $\mathbf{w} = -\mathbf{j}$

$$\mathbf{T} = \|\mathbf{T}\|(\cos(90° + \theta)\mathbf{i} + \sin(90° + \theta)\mathbf{j})$$
$$= \|\mathbf{T}\|(-\sin\theta\,\mathbf{i} + \cos\theta\,\mathbf{j})$$
$$\mathbf{0} = \mathbf{u} + \mathbf{w} + \mathbf{T} = \|\mathbf{u}\|\mathbf{i} - \mathbf{j} + \|\mathbf{T}\|(-\sin\theta\,\mathbf{i} + \cos\theta\,\mathbf{j})$$
$$\|\mathbf{u}\| = \sin\theta\,\|\mathbf{T}\|$$
$$1 = \cos\theta\,\|\mathbf{T}\|$$

If $\theta = 30°$, $\|\mathbf{u}\| = (1/2)\|\mathbf{T}\|$ and $1 = \left(\sqrt{3}/2\right)\|\mathbf{T}\|$

$$\Rightarrow \|\mathbf{T}\| = \frac{2}{\sqrt{3}} \approx 1.1547 \text{ lb}$$

and

$$\|\mathbf{u}\| = \frac{1}{2}\left(\frac{2}{\sqrt{3}}\right) \approx 0.5774 \text{ lb}$$

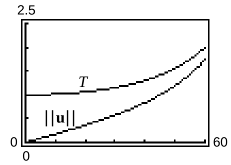
(b) From part (a), $\|\mathbf{u}\| = \tan\theta$ and $\|\mathbf{T}\| = \sec\theta$.

Domain: $0 \le \theta \le 90°$

(c)

θ	0°	10°	20°	30°	40°	50°	60°
T	1	1.0154	1.0642	1.1547	1.3054	1.5557	2
$\|\mathbf{u}\|$	0	0.1763	0.3640	0.5774	0.8391	1.1918	1.7321

(d)



(e) Both are increasing functions.

(f) $\lim_{\theta\to\pi/2^-} T = \infty$ and $\lim_{\theta\to\pi/2^-} \|\mathbf{u}\| = \infty$.

15. Let $\theta = \alpha - \beta$, the angle between $\mathbf{u}$ and $\mathbf{v}$. Then

$$\sin(\alpha - \beta) = \frac{\|\mathbf{u}\times\mathbf{v}\|}{\|\mathbf{u}\|\,\|\mathbf{v}\|} = \frac{\|\mathbf{v}\times\mathbf{u}\|}{\|\mathbf{u}\|\,\|\mathbf{v}\|}.$$

For $\mathbf{u} = \langle\cos\alpha, \sin\alpha, 0\rangle$ and $\mathbf{v} = \langle\cos\beta, \sin\beta, 0\rangle$, $\|\mathbf{u}\| = \|\mathbf{v}\| = 1$ and

$$\mathbf{v}\times\mathbf{u} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \cos\beta & \sin\beta & 0 \\ \cos\alpha & \sin\alpha & 0 \end{vmatrix} = (\sin\alpha\cos\beta - \cos\alpha\sin\beta)\mathbf{k}.$$

Thus, $\sin(\alpha - \beta) = \|\mathbf{v}\times\mathbf{u}\| = \sin\alpha\cos\beta - \cos\alpha\sin\beta$.

17. From Theorem 10.13 and Theorem 10.7 (6) we have

$$D = \frac{|\overrightarrow{PQ}\cdot\mathbf{n}|}{\|\mathbf{n}\|}$$
$$= \frac{|\mathbf{w}\cdot(\mathbf{u}\times\mathbf{v})|}{\|\mathbf{u}\times\mathbf{v}\|} = \frac{|(\mathbf{u}\times\mathbf{v})\cdot\mathbf{w}|}{\|\mathbf{u}\times\mathbf{v}\|} = \frac{|\mathbf{u}\cdot(\mathbf{v}\times\mathbf{w})|}{\|\mathbf{u}\times\mathbf{v}\|}.$$

19. a_1, b_1, c_1, and a_2, b_2, c_2 are two sets of direction numbers for the same line. The line is parallel to both $\mathbf{u} = a_1\mathbf{i} + b_1\mathbf{j} + c_1\mathbf{k}$ and $\mathbf{v} = a_2\mathbf{i} + b_2\mathbf{j} + c_2\mathbf{k}$. Therefore, $\mathbf{u}$ and $\mathbf{v}$ are parallel, and there exists a scalar d such that $\mathbf{u} = d\mathbf{v}$, $a_1\mathbf{i} + b_1\mathbf{j} + c_1\mathbf{k} = d(a_2\mathbf{i} + b_2\mathbf{j} + c_2\mathbf{k})$, $a_1 = a_2 d$, $b_1 = b_2 d$, $c_1 = c_2 d$.