22. $\mathbf{F}(x, y, z) = -z\mathbf{i} + y\mathbf{k}$

$S: x^2 + y^2 = 1$

$$\mathbf{curl\ F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ -z & 0 & y \end{vmatrix} = \mathbf{i} - \mathbf{j}$$

Letting $\mathbf{N} = \mathbf{k}$, $\mathbf{curl\ F} \cdot \mathbf{N} = 0$ and $\displaystyle\int_S\!\!\int (\mathbf{curl\ F}) \cdot \mathbf{N}\, dS = 0$.

24. **curl F** measures the rotational tendency.

See page 1084.

26. $f(x, y, z) = xyz,\ g(x, y, z) = z,\ S: z = \sqrt{4 - x^2 - y^2}$

(a) $\nabla g(x, y, z) = \mathbf{k}$

$f(x, y, z)\nabla g(x, y, z) = xyz\mathbf{k}$

$\mathbf{r}(t) = 2\cos t\,\mathbf{i} + 2\sin t\,\mathbf{j} + 0\mathbf{k},\quad 0 \le t \le 2\pi$

$$\int_C [f(x, y, z)\nabla g(x, y, z)] \cdot d\mathbf{r} = 0$$

(b) $\nabla f(x, y, z) = yz\mathbf{i} + xz\mathbf{j} + xy\mathbf{k}$

$\nabla g(x, y, z) = \mathbf{k}$

$$\nabla f \times \nabla g = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ yz & xz & xy \\ 0 & 0 & 1 \end{vmatrix} = xz\mathbf{i} - yz\mathbf{j}$$

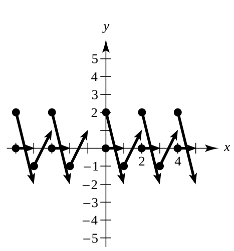
$$\mathbf{N} = \frac{x}{\sqrt{4 - x^2 - y^2}}\mathbf{i} + \frac{y}{\sqrt{4 - x^2 - y^2}}\mathbf{j} + \mathbf{k}$$

$$dS = \sqrt{1 + \left(\frac{-x}{\sqrt{4 - x^2 - y^2}}\right)^2 + \left(\frac{-y}{\sqrt{4 - x^2 - y^2}}\right)^2}\, dA = \frac{2}{\sqrt{4 - x^2 - y^2}}\, dA$$

$$\begin{aligned} \int_S\!\!\int [\nabla f(x, y, z) \times \nabla g(x, y, z)] \cdot \mathbf{N}\, dS &= \int_S\!\!\int \left[\frac{x^2 z}{\sqrt{4 - x^2 - y^2}} - \frac{y^2 z}{\sqrt{4 - x^2 - y^2}}\right] \frac{2}{\sqrt{4 - x^2 - y^2}} dA \\ &= \int_S\!\!\int \frac{2(x^2 - y^2)}{\sqrt{4 - x^2 - y^2}}\, dA \\ &= \int_0^2 \int_0^{2\pi} \frac{2r^2(\cos^2\theta - \sin^2\theta)}{\sqrt{4 - r^2}} r\, d\theta\, dr \\ &= \int_0^2 \left[\frac{2r^3}{\sqrt{4 - r^2}}\left(\frac{1}{2}\sin 2\theta\right)\right]_0^{2\pi} dr = 0 \end{aligned}$$

Review Exercises for Chapter 14

2. $\mathbf{F}(x, y) = \mathbf{i} - 2y\mathbf{j}$



4. $f(x, y, z) = x^2 e^{yz}$

$$\begin{aligned} \mathbf{F}(x, y, z) &= 2xe^{yz}\,\mathbf{i} + x^2 z e^{yz}\,\mathbf{j} + x^2 y e^{yz}\,\mathbf{k} \\ &= xe^{yz}(2\mathbf{i} + xz\mathbf{j} + xy\mathbf{k}) \end{aligned}$$

6. Since $\partial M/\partial y = -1/x^2 = \partial N/\partial x$, **F** is conservative. From $M = \partial U/\partial x = -y/x^2$ and $N = \partial U/\partial y = 1/x$, partial integration yields $U = (y/x) + h(y)$ and $U = (y/x) + g(x)$ which suggests that $U(x, y) = (y/x) + C$.

8. Since $\partial M/\partial y = -6y^2 \sin 2x = \partial N/\partial x$, **F** is conservative. From $M = \partial U/\partial x = -2y^3 \sin 2x$ and $N = \partial U/\partial y = 3y^2(1 + \cos 2x)$, we obtain $U = y^3 \cos 2x + h(y)$ and $U = y^3(1 + \cos 2x) + g(x)$ which suggests that $h(y) = y^3$, $g(x) = C$, and $U(x, y) = y^3(1 + \cos 2x) + C$.

10. Since

$$\frac{\partial M}{\partial y} = 4x = \frac{\partial N}{\partial x},$$

$$\frac{\partial M}{\partial z} = 2z = \frac{\partial P}{\partial x},$$

$$\frac{\partial N}{\partial z} = 6y \neq \frac{\partial P}{\partial y}$$

F is not conservative.

12. Since

$$\frac{\partial M}{\partial y} = \sin z = \frac{\partial N}{\partial x}, \quad \frac{\partial M}{\partial z} = y \cos z \neq \frac{\partial P}{\partial x},$$

F is not conservative.

14. Since $\mathbf{F} = xy^2\mathbf{j} - zx^2\mathbf{k}$;

(a) $\text{div}\,\mathbf{F} = 2xy - x^2$

(b) $\textbf{curl}\,\mathbf{F} = 2xz\,\mathbf{j} + y^2\mathbf{k}$

16. Since $\mathbf{F} = (3x - y)\mathbf{i} + (y - 2z)\mathbf{j} + (z - 3x)\mathbf{k}$:

(a) $\text{div}\,\mathbf{F} = 3 + 1 + 1 = 5$

(b) $\textbf{curl}\,\mathbf{F} = 2\mathbf{i} + 3\mathbf{j} + \mathbf{k}$

18. Since $\mathbf{F} = (x^2 - y)\mathbf{i} - (x + \sin^2 y)\mathbf{j}$:

(a) $\text{div}\,\mathbf{F} = 2x - 2 \sin y \cos y$

(b) $\textbf{curl}\,\mathbf{F} = \mathbf{0}$

20. Since $\mathbf{F} = \dfrac{z}{x}\mathbf{i} + \dfrac{z}{y}\mathbf{j} + z^2\mathbf{k}$:

(a) $\text{div}\,\mathbf{F} = -\dfrac{z}{x^2} - \dfrac{z}{y^2} + 2z = z\left(2 - \dfrac{1}{x^2} - \dfrac{1}{y^2}\right)$

(b) $\textbf{curl}\,\mathbf{F} = -\dfrac{1}{y}\mathbf{i} + \dfrac{1}{x}\mathbf{j}$

22. (a) Let $x = 5t$, $y = 4t$, $0 \le t \le 1$, then $ds = \sqrt{41}\,dt$.

$$\int_C xy\,ds = \int_0^1 20t^2\sqrt{41}\,dt = \frac{20\sqrt{41}}{3}$$

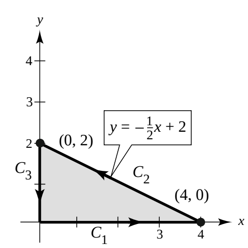
(b) C_1: $x = t$, $y = 0$, $0 \le t \le 4$, $ds = dt$

C_2: $x = 4 - 4t$, $y = 2t$, $0 \le t \le 1$, $ds = 2\sqrt{5}\,dt$

C_3: $x = 0$, $y = 2 - t$, $0 \le t \le 2$, $ds = dt$

Therefore, $\displaystyle\int_C xy\,ds = \int_0^4 0\,dt = \int_0^1 (8t - 8t^2)2\sqrt{5}\,dt + \int_0^2 0\,dt$

$$= 16\sqrt{5}\left[\frac{t^2}{2} - \frac{t^3}{3}\right]_0^1 = \frac{8\sqrt{5}}{3}.$$



24. $x = t - \sin t$, $y = 1 - \cos t$, $0 \le t \le 2\pi$, $\dfrac{dx}{dt} = 1 - \cos t$, $\dfrac{dy}{dt} = \sin t$

$$\int_C x\,ds = \int_0^{2\pi} (t - \sin t)\sqrt{(1 - \cos t)^2 + (\sin t)^2}\,dt = \int_0^{2\pi} (t - \sin t)\sqrt{2 - 2\cos t}\,dt$$

$$= \sqrt{2}\int_0^{2\pi}\left[t\sqrt{1 - \cos t} - \sin t\sqrt{1 - \cos t}\right]dt = \sqrt{2}\left[-\frac{2}{3}(1 - \cos t)^{3/2}\right]_0^{2\pi} + \sqrt{2}\int_0^{2\pi} t\sqrt{1 - \cos t}\,dt$$

$$= \sqrt{2}\int_0^{2\pi} t\sqrt{1 - \cos t}\,dt$$

$$= 8\pi$$

26. $x = \cos t + t \sin t,\ y = \sin t - t \sin t,\ 0 \le t \le \frac{\pi}{2},\ dx = t \cos t\, dt,\ dy = (\cos t - t \cos t - \sin t)\, dt$

$$\int_C (2x - y)\, dx + (x + 3y)\, dy = \int_0^{\pi/2} \left[\sin t \cos t(5t^2 - 6t + 2) + \cos^2 t(t + 1) + \sin^2 t(2t - 3)\right] dt \approx 1.01$$

28. $\mathbf{r}(t) = t\mathbf{i} + t^2\mathbf{j} + t^{3/2}\mathbf{k},\ 0 \le t \le 4$

$x'(t) = 1,\ y'(t) = 2t,\ z'(t) = \frac{3}{2}t^{1/2}$

$$\int_C (x^2 + y^2 + z^2)\, ds = \int_0^4 (t^2 + t^4 + t^3)\sqrt{1 + 4t^2 + \frac{9}{4}t}\ dt \approx 2080.59$$

30. $f(x, y) = 12 - x - y$

C: $y = x^2$ from $(0, 0)$ to $(2, 4)$

$\mathbf{r}(t) = t\mathbf{i} + t^2\mathbf{j},\ 0 \le t \le 2$

$\mathbf{r}'(t) = \mathbf{i} + 2t\mathbf{j}$

$\|\mathbf{r}'(t)\| = \sqrt{1 + 4t^2}$

Lateral surface area:

$$\int_C f(x, y)\, ds = \int_0^2 (12 - t - t^2)\sqrt{1 + 4t^2}\, dt \approx 41.532$$

32. $d\mathbf{r} = [(-4 \sin t)\mathbf{i} + 3 \cos t\,\mathbf{j}]\, dt$

$\mathbf{F} = (4 \cos t - 3 \sin t)\mathbf{i} + (4 \cos t + 3 \sin t)\mathbf{j},\ 0 \le t \le 2\pi$

$$\int_C \mathbf{F} \cdot d\mathbf{r} = \int_0^{2\pi} (12 - 7 \sin t \cos t)\, dt = \left[12t - \frac{7 \sin^2 t}{2}\right]_0^{2\pi} = 24\pi$$

34. $x = 2 - t,\ y = 2 - t,\ z = \sqrt{4t - t^2},\ 0 \le t \le 2$

$$d\mathbf{r} = \left[-\mathbf{i} - \mathbf{j} + \frac{2 - t}{\sqrt{4t - t^2}}\mathbf{k}\right] dt$$

$$\mathbf{F} = \left(4 - 2t - \sqrt{4t - t^2}\right)\mathbf{i} + \left(\sqrt{4t - t^2} - 2 + t\right)\mathbf{j} + 0\mathbf{k}$$

$$\int_C \mathbf{F} \cdot d\mathbf{r} = \int_0^2 (t - 2)\, dt = \left[\frac{t^2}{2} - 2t\right]_0^2 = -2$$

36. Let $x = 2 \sin t,\ y = -2 \cos t,\ z = 4 \sin^2 t,\ 0 \le t \le \pi$.

$d\mathbf{r} = [(2 \cos t)\mathbf{i} + (2 \sin t)\mathbf{j} + (8 \sin t \cos t)\mathbf{k}]\, dt$

$\mathbf{F} = 0\mathbf{i} + 4\mathbf{j} + (2 \sin t)\mathbf{k}$

$$\int_C \mathbf{F} \cdot d\mathbf{r} = \int_0^{\pi} (8 \sin t + 16 \sin^2 t \cos t)\, dt = \left[-8 \cos t + \frac{16}{3} \sin^3 t\right]_0^{\pi} = 16$$

38. $\int_C \mathbf{F} \cdot d\mathbf{r} = \int_C (2x - y)\, dx + (2y - x)\, dy$

$\mathbf{r}(t) = (2 \cos t + 2t \sin t)\mathbf{i} + (2 \sin t - 2t \cos t)\mathbf{j},\ 0 \le t \le \pi$

$$\int_C \mathbf{F} \cdot d\mathbf{r} = 4\pi^2 + 4\pi$$

40. $\mathbf{r}(t) = 10 \sin t\mathbf{i} + 10 \cos t\mathbf{j} + \dfrac{2000/5280}{\pi/2} t\mathbf{k}, \quad 0 \le t \le \dfrac{\pi}{2}$

$= 10 \sin t\mathbf{i} + 10 \cos t\mathbf{j} + \dfrac{25}{33\pi} t\mathbf{k}$

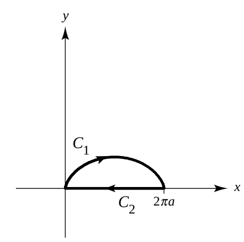
$\mathbf{F} = 20\mathbf{k}$

$d\mathbf{r} = \left(10 \cos t\mathbf{i} - 10 \sin t\mathbf{j} + \dfrac{25}{33\pi}\mathbf{k}\right)$

$\displaystyle\int_C \mathbf{F} \cdot d\mathbf{r} = \int_0^{\pi/2} \frac{500}{33\pi}\, dt = \frac{250}{33}$ mi · ton

42. $\displaystyle\int_C y\, dx + x\, dy + \frac{1}{z}\, dz = \Big[xy + \ln|z|\Big]_{(0,0,1)}^{(4,4,4)} = 16 + \ln 4$

44. $x = a(\theta - \sin\theta),\ y = a(1 - \cos\theta),\ 0 \le \theta \le 2\pi$



(a) $\displaystyle A = \frac{1}{2}\int_C x\, dy - y\, dx.$

Since these equations orient the curve backwards, we will use

$$A = \frac{1}{2}\int (y\, dx - x\, dy)$$

$$= \frac{1}{2}\int_0^{2\pi} \left[a^2(1 - \cos\theta)(1 - \cos\theta) - a^2(\theta - \sin\theta)(\sin\theta)\right] d\theta + \frac{1}{2}\int_0^{2\pi a} (0 - 0)\, d\theta$$

$$= \frac{a^2}{2}\int_0^{2\pi} \left[1 - 2\cos\theta + \cos^2\theta - \theta\sin\theta + \sin^2\theta\right] d\theta$$

$$= \frac{a^2}{2}\int_0^{2\pi} (2 - 2\cos\theta - \theta\sin\theta)\, d\theta = \frac{a^2}{2}(6\pi) = 3\pi a^2.$$

(b) By symmetry, $\bar{x} = \pi a$. From Section 14.4,

$$\bar{y} = -\frac{1}{2A}\int_C y^2\, dx = \frac{1}{2A}\int_0^{2\pi} a^3(1 - \cos\theta)^2(1 - \cos\theta)\, d\theta = \frac{1}{2(3\pi a^2)} a^3(5\pi) = \frac{5}{6}a$$

46. $\displaystyle\int_C xy\, dx + (x^2 + y^2)\, dy = \int_0^2\int_0^2 (2x - x)\, dy\, dx$

$\displaystyle= \int_0^2 2x\, dx = 4$

48. $\displaystyle\int_C (x^2 - y^2)\, dx + 2xy\, dy = \int_{-a}^{a}\int_{-\sqrt{a^2-x^2}}^{\sqrt{a^2-x^2}} 4y\, dy\, dx$

$\displaystyle= \int_{-a}^{a} 0\, dx = 0$

50. $\displaystyle\int_C y^2\, dx + x^{4/3}\, dy = \int_{-1}^{1}\int_{-(1-x^{2/3})^{3/2}}^{(1-x^{2/3})^{3/2}} \left(\frac{4}{3}x^{1/3} - 2y\right) dy\, dx$

$\displaystyle= \int_{-1}^{1} \left[\frac{4}{3}x^{1/3}y - y^2\right]_{-(1-x^{2/3})^{3/2}}^{(1-x^{2/3})^{3/2}} dx$

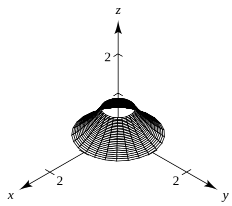
$\displaystyle= \int_{-1}^{1} \frac{8}{3}x^{1/3}(1 - x^{2/3})^{3/2}\, dx$

$\displaystyle= \left[-\frac{8}{7}x^{2/3}(1 - x^{2/3})^{5/2} - \frac{16}{35}(1 - x^{2/3})^{5/2}\right]_{-1}^{1}$

$= 0$

52. $\mathbf{r}(u, v) = e^{-u/4}\cos v\,\mathbf{i} + e^{-u/4}\sin v\,\mathbf{j} + \frac{u}{6}\mathbf{k}$

$0 \le u \le 4, \quad 0 \le v \le 2\pi$



54. S: $\mathbf{r}(u, v) = (u + v)\mathbf{i} + (u - v)\mathbf{j} + \sin v\,\mathbf{k}, \quad 0 \le u \le 2, 0 \le v \le \pi$

$\mathbf{r}_u(u, v) = \mathbf{i} + \mathbf{j}$

$\mathbf{r}_u(u, v) = \mathbf{i} - \mathbf{j} + \cos v\,\mathbf{k}$

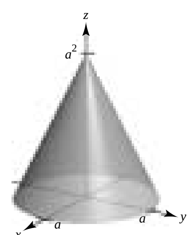
$$\mathbf{r}_u \times \mathbf{r}_v = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 1 & 0 \\ 1 & -1 & \cos v \end{vmatrix} = \cos v\,\mathbf{i} - \cos v\,\mathbf{j} - 2\mathbf{k}$$

$$\|\mathbf{r}_u \times \mathbf{r}_v\| = \sqrt{2\cos^2 v + 4}$$

$$\iint_S z\,dS = \int_0^{\pi}\int_0^2 \sin v\sqrt{2\cos^2 v + 4}\,du\,dv = 2\left[\sqrt{6} + \sqrt{2}\ln\left(\frac{\sqrt{6} + \sqrt{2}}{\sqrt{6} - \sqrt{2}}\right)\right]$$

56. (a) $z = a\left(a - \sqrt{x^2 + y^2}\right), 0 \le z \le a^2$

$z = 0 \Longrightarrow x^2 + y^2 = a^2$



(b) S: $g(x, y) = z = a^2 - a\sqrt{x^2 + y^2}$

$\rho(x, y) = k\sqrt{x^2 + y^2}$

$$m = \iint_S e(x, y, z)\,dS$$

$$= \iint_R k\sqrt{x^2 + y^2}\,\sqrt{1 + g_x{}^2 + g_y{}^2}\,dA$$

$$= k\iint_R \sqrt{x^2 + y^2}\,\sqrt{1 + \frac{a^2x^2}{x^2 + y^2} + \frac{a^2y^2}{x^2 + y^2}}\,dA$$

$$= k\iint_R \sqrt{a^2 + 1}\left(\sqrt{x^2 + y^2}\right)dA$$

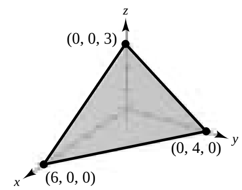
$$= k\sqrt{a^2 + 1}\int_0^{2\pi}\int_0^a r^2\,dr\,d\theta$$

$$= k\sqrt{a^2 + 1}\int_0^{2\pi}\frac{a^3}{3}\,d\theta$$

$$= \frac{2}{3}k\sqrt{a^2 + 1}\,a^3\pi$$

58. $\mathbf{F}(x, y, z) = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$

Q: solid region bounded by the coordinate planes and the plane $2x + 3y + 4z = 12$



Surface Integral: There are four surfaces for this solid.

$$z = 0 \qquad \mathbf{N} = -\mathbf{k}, \qquad \mathbf{F}\cdot\mathbf{N} = -z, \qquad \iint_{S_1} 0\,dS = 0$$

$$y = 0, \qquad \mathbf{N} = -\mathbf{j}, \qquad \mathbf{F}\cdot\mathbf{N} = -y, \qquad \iint_{S_2} 0\,dS = 0$$

$$x = 0, \qquad \mathbf{N} = -\mathbf{i}, \qquad \mathbf{F}\cdot\mathbf{N} = -x, \qquad \iint_{S_3} 0\,dS = 0$$

$$2x + 3y + 4z = 12, \mathbf{N} = \frac{2\mathbf{i} + 3\mathbf{j} + 4\mathbf{k}}{\sqrt{29}}, dS = \sqrt{1 + \left(\frac{1}{4}\right) + \left(\frac{9}{16}\right)}dA = \frac{\sqrt{29}}{4}\,dA$$

$$\iint_{S_4} \mathbf{N}\cdot\mathbf{F}\,dS = \frac{1}{4}\iint_R (2x + 3y + 4z)\,dy\,dx$$

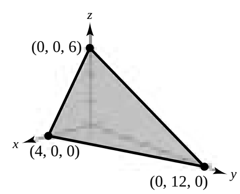
$$= \frac{1}{4}\int_0^6\int_0^{(12-2x)/3} 12\,dy\,dx = 3\int_0^6\left(4 - \frac{2x}{3}\right)dx = 3\left[4x - \frac{x^2}{3}\right]_0^6 = 36$$

Triple Integral: Since div $\mathbf{F} = 3$, the Divergence Theorem yields.

$$\iiint_Q \text{div}\,\mathbf{F}\,dV = \iiint_Q 3\,dV = 3(\text{Volume of solid}) = 3\left[\frac{1}{3}(\text{Area of base})(\text{Height})\right] = \frac{1}{2}(6)(4)(3) = 36.$$

60. $\mathbf{F}(x, y, z) = (x - z)\mathbf{i} + (y - z)\mathbf{j} + x^2\mathbf{k}$

S: first octant portion of the plane $3x + y + 2z = 12$



Line Integral:

$$C_1: y = 0, \qquad dy = 0, \qquad z = \frac{12 - 3x}{2}, \qquad dz = -\frac{3}{2}\,dx$$

$$C_2: x = 0, \qquad dx = 0, \qquad z = \frac{12 - y}{2}, \qquad dz = -\frac{1}{2}\,dy$$

$$C_3: z = 0, \qquad dz = 0, \qquad y = 12 - 3x, \qquad dy = -3\,dx$$

$$\int_C \mathbf{F}\cdot d\mathbf{r} = \int_C (x - z)\,dx + (y - z)\,dy + x^2\,dz$$

$$= \int_{C_1}\left[x - \frac{12 - 3x}{2} + x^2\left(-\frac{3}{2}\right)\right]dx + \int_{C_2}\left[y - \frac{12 - y}{2}\right]dy + \int_{C_3}[x + (12 - 3x)(-3)]\,dx$$

$$= \int_4^0\left(-\frac{3}{2}x^2 + \frac{5}{2}x - 6\right)dx + \int_0^{12}\left(\frac{3}{2}y - 6\right)dy + \int_0^4 (10x - 36)\,dx = 8$$

Double Integral: $G(x, y, z) = \dfrac{12 - 3x - y}{2} - z$

$$\nabla G(x, y, z) = -\frac{3}{2}\mathbf{i} - \frac{1}{2}\mathbf{j} - \mathbf{k}$$

$$\mathbf{curl\ F} = \mathbf{i} - (2x + 1)\mathbf{j}$$

$$\iint_S (\mathbf{curl\ F})\cdot\mathbf{N}\,dS = \int_0^4\int_0^{12-3x}(x - 1)\,dy\,dx = \int_0^4 (-3x^2 + 15x - 12)\,dx = 8$$

Problem Solving for Chapter 14

2. (a) $z = \sqrt{1 - x^2 - y^2}, \dfrac{\partial z}{\partial x} = \dfrac{-x}{\sqrt{1 - x^2 - y^2}}, \dfrac{\partial z}{\partial y} = \dfrac{-y}{\sqrt{1 - x^2 - y^2}}$

$$dS = \sqrt{\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2 + 1}\, dA = \frac{1}{\sqrt{1 - x^2 - y^2}}$$

$$\nabla T = \frac{-25}{(x^2 + y^2 + z^2)^{3/2}}(x\mathbf{i} + y\mathbf{i} + z\mathbf{k}) = -25(x\mathbf{i} + y\mathbf{j} + z\mathbf{k})$$

$$\mathbf{N} = \frac{-\dfrac{\partial z}{\partial x}\mathbf{i} - \dfrac{\partial z}{\partial y}\mathbf{j} + \mathbf{k}}{\sqrt{\left(\dfrac{\partial z}{\partial x}\right)^2 + \left(\dfrac{\partial z}{\partial y}\right)^2 + 1}}$$

$$= \left(\frac{x}{\sqrt{1 - x^2 - y^2}}\,\mathbf{i} + \frac{y}{\sqrt{1 - x^2 - y^2}}\,\mathbf{j} + \mathbf{k}\right)\sqrt{1 - x^2 - y^2}$$

$$= x\mathbf{i} + y\mathbf{j} + \sqrt{1 - x^2 - y^2}\,\mathbf{k} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$$

$$\text{Flux} = \iint_S -k\nabla T \cdot N\, dS$$

$$= k\iint_R 25(x\mathbf{i} + y\mathbf{j} + z\mathbf{k}) \cdot (x\mathbf{i} + y\mathbf{j} + z\mathbf{k})\frac{1}{\sqrt{1 - x^2 - y^2}}\, dA$$

$$= k\iint_R \frac{25}{\sqrt{1 - x^2 - y^2}}\, dA$$

$$= 25k\int_0^{2\pi}\int_0^1 \frac{1}{\sqrt{1 - r^2}}\, r\, dr\, d\theta = 50\pi k$$

(b) $\mathbf{r}(u, v) = \langle \sin u \cos v, \sin u \sin v, \cos u\rangle$

$\mathbf{r_u} = \langle \cos u \cos v, \cos u \sin v, -\sin u\rangle$

$\mathbf{r_v} = \langle -\sin u \sin v, \sin u \cos v, 0\rangle$

$\mathbf{r_u} \times \mathbf{r_v} = \langle \sin^2 u \cos v, \sin^2 u \sin v, \sin u \cos u \sin^2 v + \sin u \cos u \cos^2 v\rangle$

$\|\mathbf{r_u} \times \mathbf{r_v}\| = \sin u$

$$\text{Flux} = 25k\int_0^{2\pi}\int_0^{\pi/2} \sin u\, du\, dv = 50\pi k$$

4. $\mathbf{r}(t) = \left\langle \dfrac{t^2}{2}, t, \dfrac{2\sqrt{2}t^{3/2}}{3}\right\rangle$

$\mathbf{r}'(t) = \left\langle t, 1, \sqrt{2}t^{1/2}\right\rangle, \|\mathbf{r}'(t)\| = t + 1$

$$\rho\, ds = \frac{1}{1 + t}(t + 1)\, dt = 1$$

$$I_y = \int_C (x^2 + z^2)\rho\, ds = \int_0^1 \left(\frac{t^4}{4} + \frac{8}{9}t^3\right) dt = \frac{49}{180}$$

$$I_x = \int_C (y^2 + z^2)\rho\, ds = \int_0^1 \left(t^2 + \frac{8}{9}t^3\right) dt = \frac{5}{9}$$

$$I_z = \int_C (x^2 + y^2)\rho\, ds = \int_0^1 \left(\frac{t^4}{4} + t^2\right) dt = \frac{23}{60}$$

6. $\frac{1}{2}\int_C x\,dy - y\,dx = 2\int_0^{\pi/2}\left[\frac{1}{2}\sin 2t\cos t - \sin t\cos 2t\right]dt = 2\left(\frac{2}{3}\right)$

Hence, the area is $4/3$.

8. $F(x, y) = 3x^2y^2\mathbf{i} + 2x^3y\mathbf{j}$ is conservative.

$f(x, y) = x^3y^2$ potential function.

Work $= f(2, 4) - f(1, 1) = 8(16) - 1 = 127$

10. Area $= \pi ab$

$\mathbf{r}(t) = a\cos t\,\mathbf{i} + b\sin t\,\mathbf{j},\ 0 \le t \le 2\pi$

$\mathbf{r}'(t) = -a\sin t\,\mathbf{i} + b\cos t\,\mathbf{j}$

$$\mathbf{F} = -\frac{1}{2}b\sin t\,\mathbf{i} + \frac{1}{2}a\cos t\,\mathbf{j}$$

$\mathbf{F}\cdot d\mathbf{r} = \left[\frac{1}{2}ab\sin^2 t + \frac{1}{2}ab\cos^2 t\right]dt = \frac{1}{2}ab$

$W = \int_0^{2\pi}\mathbf{F}\cdot d\mathbf{r} = \frac{1}{2}ab(2\pi) = \pi ab$

Same as area.