Review Exercises for Chapter 2

2. $f(x) = \dfrac{x+1}{x-1}$

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x} = \lim_{\Delta x \to 0} \frac{\dfrac{x+\Delta x+1}{x+\Delta x-1} - \dfrac{x+1}{x-1}}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{(x+\Delta x+1)(x-1) - (x+\Delta x-1)(x+1)}{\Delta x(x+\Delta x-1)(x-1)}$$

$$= \lim_{\Delta x \to 0} \frac{(x^2 + x\Delta x + x - x - \Delta x - 1) - (x^2 + x\Delta x - x + x + \Delta x - 1)}{\Delta x(x+\Delta x-1)(x-1)}$$

$$= \lim_{\Delta x \to 0} \frac{-2\Delta x}{\Delta x(x+\Delta x-1)(x-1)} = \lim_{\Delta x \to 0} \frac{-2}{(x+\Delta x-1)(x-1)} = \frac{-2}{(x-1)^2}$$

4. $f(x) = \dfrac{2}{x}$

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\dfrac{2}{x+\Delta x} - \dfrac{2}{x}}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{2x - (2x + 2\Delta x)}{\Delta x(x+\Delta x)x}$$

$$= \lim_{\Delta x \to 0} \frac{-2\Delta x}{\Delta x(x+\Delta x)x}$$

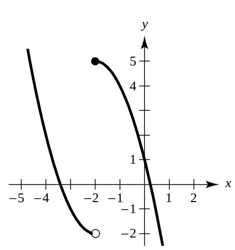
$$= \lim_{\Delta x \to 0} \frac{-2}{(x+\Delta x)x} = \frac{-2}{x^2}$$

6. f is differentiable for all $x \neq -3$.

8. $f(x) = \begin{cases} x^2 + 4x + 2, & \text{if } x < -2 \\ 1 - 4x - x^2, & \text{if } x \geq -2 \end{cases}$

(a) Nonremovable discontinuity at $x = -2$.

(b) Not differentiable at $x = -2$ because the function is discontinuous there.



10. Using the limit defintion, you obtain $h'(x) = \dfrac{3}{8} - 4x$.

At $x = -2$, $h'(-2) = \dfrac{3}{8} - 4(-2) = \dfrac{67}{8}$.

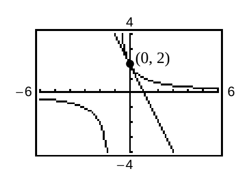
12. (a) Using the limit definition, $f'(x) = \dfrac{-2}{(x+1)^2}$.

At $x = 0$, $f'(0) = -2$. The tangent line is

$$y - 2 = -2(x - 0)$$

$$y = -2x + 2$$

(b)



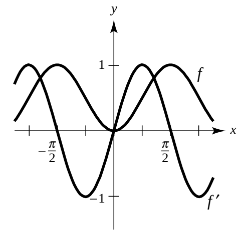
14. $f'(2) = \lim_{x\to2} \frac{f(x) - f(2)}{x - 2}$

$= \lim_{x\to2} \frac{\frac{1}{x+1} - \frac{1}{3}}{x - 2}$

$= \lim_{x\to2} \frac{3 - x - 1}{(x - 2)(x + 1)3}$

$= \lim_{x\to2} \frac{-1}{(x + 1)3} = \frac{-1}{9}$

16.



18. $y = -12$

$y' = 0$

20. $g(x) = x^{12}$

$g'(x) = 12x^{11}$

22. $f(t) = -8t^5$

$f'(t) = -40t^4$

24. $g(s) = 4s^4 - 5s^2$

$g'(s) = 16s^3 - 10s$

26. $f(x) = x^{1/2} - x^{-1/2}$

$f'(x) = \frac{1}{2}x^{-1/2} + \frac{1}{2}x^{-3/2} = \frac{x + 1}{2x^{3/2}}$

28. $h(x) = \frac{2}{9}x^{-2}$

$h'(x) = \frac{-4}{9}x^{-3} = \frac{-4}{9x^3}$

30. $g(\alpha) = 4\cos\alpha + 6$

$g'(\alpha) = -4\sin\alpha$

32. $g(\alpha) = \frac{5\sin\alpha}{3} - 2\alpha$

$g'(\alpha) = \frac{5\cos\alpha}{3} - 2$

34. $s = -16t^2 + s_0$

First ball:

$-16t^2 + 100 = 0$

$t = \sqrt{\frac{100}{16}} = \frac{10}{4} = 2.5$ seconds to hit ground

Second ball:

$-16t^2 + 75 = 0$

$t^2 = \sqrt{\frac{75}{16}} = \frac{5\sqrt{3}}{4} \approx 2.165$ seconds to hit ground

Since the second ball was released one second after the first ball, the first ball will hit the ground first. The second ball will hit the ground $3.165 - 2.5 = 0.665$ second later.

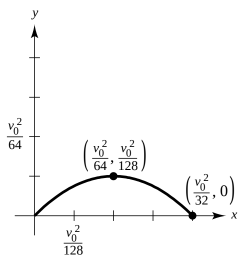
36. $s(t) = -16t^2 + 14{,}400 = 0$

$16t^2 = 14{,}400$

$t = 30$ sec

Since 600 mph $= \frac{1}{6}$ mi/sec, in 30 seconds the bomb will move horizontally $\left(\frac{1}{6}\right)(30) = 5$ miles.

38.



(a) $y = x - \dfrac{32}{v_0^2}x^2 = x\left(1 - \dfrac{32}{v_0^2}x\right)$

$= 0$ if $x = 0$ or $x = \dfrac{v_0^2}{32}$.

Projectile strikes the ground when $x = v_0^2/32$.

Projectile reaches its maximum height at $x = v_0^2/64$. (one-half the distance)

(b) $y' = 1 - \dfrac{64}{v_0^2}x$

When $x = \dfrac{v_0^2}{64}$, $y' = 1 - \dfrac{64}{v_0^2}\left(\dfrac{v_0^2}{64}\right) = 0$.

(c) $y = x - \dfrac{32}{v_0^2}x^2 = x\left(1 - \dfrac{32}{v_0^2}x\right) = 0$

when $x = 0$ and $x = x_0^2/32$. Therefore, the range is $x = v_0^2/32$. When the initial velocity is doubled the range is

$$x = \frac{(2v_0)^2}{32} = \frac{4v_0^2}{32}$$

or four times the initial range. From part (a), the maximum height occurs when $x = v_0^2/64$. The maximum height is

$$y\left(\frac{v_0^2}{64}\right) = \frac{v_0^2}{64} - \frac{32}{v_0^2}\left(\frac{v_0^2}{64}\right)^2 = \frac{v_0^2}{64} - \frac{v_0^2}{128} = \frac{v_0^2}{128}.$$

If the initial velocity is doubled, the maximum height is

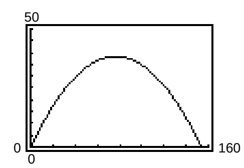
$$y\left[\frac{(2v_0)^2}{64}\right] = \frac{(2v_0)^2}{128} = 4\left(\frac{v_0^2}{128}\right)$$

or four times the original maximum height.

(d) $v_0 = 70$ ft/sec

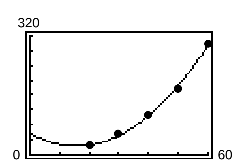
Range: $x = \dfrac{v_0^2}{32} = \dfrac{(70)^2}{32} = 153.125$ ft

Maximum height: $y = \dfrac{v_0^2}{128} = \dfrac{(70)^2}{128} \approx 38.28$ ft

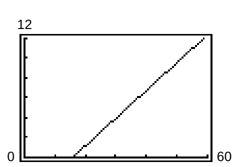


40. (a) $y = 0.14x^2 - 4.43x + 58.4$

(b)



(c)



(d) If $x = 65$, $y \approx 362$ feet.

(e) As the speed increases, the stopping distance increases at an increasing rate.

42. $g(x) = (x^3 - 3x)(x + 2)$

$$g'(x) = (x^3 - 3x)(1) + (x + 2)(3x^2 - 3)$$
$$= x^3 - 3x + 3x^3 + 6x^2 - 3x - 6$$
$$= 4x^3 + 6x^2 - 6x - 6$$

44. $f(t) = t^3 \cos t$

$$f'(t) = t^3(-\sin t) + \cos t(3t^2)$$
$$= -t^3 \sin t + 3t^2 \cos t$$

46. $f(x) = \dfrac{x + 1}{x - 1}$

$$f'(x) = \frac{(x - 1)(1) - (x + 1)(1)}{(x - 1)^2}$$
$$= \frac{-2}{(x - 1)^2}$$

48. $f(x) = \dfrac{6x - 5}{x^2 + 1}$

$$f'(x) = \frac{(x^2 + 1)(6) - (6x - 5)(2x)}{(x^2 + 1)^2}$$
$$= \frac{2(3 + 5x - 3x^2)}{(x^2 + 1)^2}$$

50. $f(x) = 9(3x^2 - 2x)^{-1}$

$$f'(x) = -9(3x^2 - 2x)^{-2}(6x - 2) = \frac{18(1 - 3x)}{(3x^2 - 2x)^2}$$

52. $y = \dfrac{\sin x}{x^2}$

$$y' = \frac{(x^2)\cos x - (\sin x)(2x)}{x^4} = \frac{x \cos x - 2 \sin x}{x^3}$$

54. $y = 2x - x^2 \tan x$

$$y' = 2 - x^2 \sec^2 x - 2x \tan x$$

56. $y = \dfrac{1 + \sin x}{1 - \sin x}$

$$y' = \frac{(1 - \sin x)\cos x - (1 + \sin x)(-\cos x)}{(1 - \sin x)^2}$$
$$= \frac{2 \cos x}{(1 - \sin x)^2}$$

58. $v(t) = 36 - t^2,\ 0 \le t \le 6$

$$a(t) = v'(t) = -2t$$
$$v(4) = 36 - 16 = 20 \text{ m/sec}$$
$$a(4) = -8 \text{ m/sec}$$

60. $f(x) = 12x^{1/4}$

$$f'(x) = 3x^{-3/4}$$
$$f''(x) = \frac{-9}{4}x^{-7/4} = \frac{-9}{4x^{7/4}}$$

62. $h(t) = 4 \sin t - 5 \cos t$

$$h'(t) = 4 \cos t + 5 \sin t$$
$$h''(t) = -4 \sin t + 5 \cos t$$

64. $y = \dfrac{(10 - \cos x)}{x}$

$$xy + \cos x = 10$$
$$xy' + y - \sin x = 0$$
$$xy' = \sin x - y$$
$$xy' + y = (\sin x - y) + y = \sin x$$

66. $f(x) = (x^2 - 1)^{1/3}$

$$f'(x) = \frac{1}{3}(x^2 - 1)^{-2/3}(2x)$$
$$= \frac{2x}{3(x^2 - 1)^{2/3}}$$

68. $f(x) = \left(x^2 + \dfrac{1}{x}\right)^5$

$$f'(x) = 5\left(x^2 + \frac{1}{x}\right)^4\left(2x - \frac{1}{x^2}\right)$$

70. $h(\theta) = \dfrac{\theta}{(1-\theta)^3}$

$$h'(\theta) = \frac{(1-\theta)^3 - \theta[3(1-\theta)^2(-1)]}{(1-\theta)^6}$$

$$= \frac{(1-\theta)^2(1-\theta+3\theta)}{(1-\theta)^6} = \frac{2\theta+1}{(1-\theta)^4}$$

72. $y = 1 - \cos 2x + 2\cos^2 x$

$$y' = 2\sin 2x - 4\cos x \sin x$$

$$= 2[2\sin x\cos x] - 4\sin x\cos x$$

$$= 0$$

74. $y = \csc 3x + \cot 3x$

$$y' = -3\csc 3x\cot 3x - 3\csc^2 3x$$

$$= -3\csc 3x(\cot 3x + \csc 3x)$$

76. $y = \dfrac{\sec^7 x}{7} - \dfrac{\sec^5 x}{5}$

$$y' = \sec^6 x(\sec x\tan x) - \sec^4 x(\sec x\tan x)$$

$$= \sec^5 x\tan x(\sec^2 x - 1)$$

$$= \sec^5 x\tan^3 x$$

78. $f(x) = \dfrac{3x}{\sqrt{x^2+1}}$

$$f'(x) = \frac{3(x^2+1)^{1/2} - 3x\frac{1}{2}(x^2+1)^{-1/2}(2x)}{x^2+1}$$

$$= \frac{3(x^2+1) - 3x^2}{(x^2+1)^{3/2}}$$

$$= \frac{3}{(x^2+1)^{3/2}}$$

80. $y = \dfrac{\cos(x-1)}{x-1}$

$$y' = \frac{-(x-1)\sin(x-1) - \cos(x-1)(1)}{(x-1)^2}$$

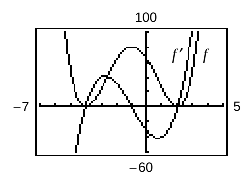
$$= -\frac{1}{(x-1)^2}[(x-1)\sin(x-1) + \cos(x-1)]$$

82. $f(x) = [(x-2)(x+4)]^2 = (x^2+2x-8)^2$

$$f'(x) = 4(x^3+3x^2-6x-8)$$

$$= 4(x-2)(x+1)(x+4)$$

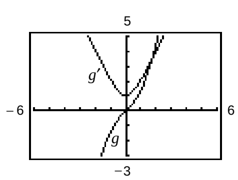
The zeros of f' correspond to the points on the graph of f where the tangent line is horizontal.



84. $g(x) = x(x^2+1)^{1/2}$

$$g'(x) = \frac{2x^2+1}{\sqrt{x^2+1}}$$

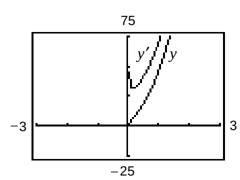
g' does not equal zero for any value of x. The graph of g has no horizontal tangent lines.



86. $y = \sqrt{3x}(x+2)^3$

$$y' = \frac{3(x+2)^2(7x+2)}{2\sqrt{3x}}$$

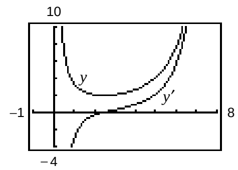
y' does not equal zero for any x in the domain. The graph has no horizontal tangent lines.



88. $y = 2\csc^3\left(\sqrt{x}\right)$

$y' = -\dfrac{3}{\sqrt{x}}\csc^3\sqrt{x}\cot\sqrt{x}$

The zero of y' corresponds to the point on the graph of y where the tangent line is horizontal.



90. $y = x^{-1} + \tan x$

$y' = -x^{-2} + \sec^2 x$

$y'' = 2x^{-3} + 2\sec x(\sec x\tan x)$

$= \dfrac{2}{x^3} + 2\sec^2 x\tan x$

92. $y = \sin^2 x$

$y' = 2\sin x\cos x = \sin 2x$

$y'' = 2\cos 2x$

94. $g(x) = \dfrac{6x - 5}{x^2 + 1}$

$g'(x) = \dfrac{2(-3x^2 + 5x + 3)}{(x^2 + 1)^2}$

$g''(x) = \dfrac{2(6x^3 - 15x^2 - 18x + 5)}{(x^2 + 1)^3}$

96. $h(x) = x\sqrt{x^2 - 1}$

$h'(x) = \dfrac{2x^2 - 1}{\sqrt{x^2 - 1}}$

$h''(x) = \dfrac{x(2x^2 - 3)}{(x^2 - 1)^{3/2}}$

98. $v = \sqrt{2gh} = \sqrt{2(32)h} = 8\sqrt{h}$

$\dfrac{dv}{dh} = \dfrac{4}{\sqrt{h}}$

(a) When $h = 9$, $\dfrac{dv}{dh} = \dfrac{4}{3}$ ft/sec.

(b) When $h = 4$, $\dfrac{dv}{dh} = 2$ ft/sec.

100. $x^2 + 9y^2 - 4x + 3y = 0$

$2x + 18yy' - 4 + 3y' = 0$

$3(6y + 1)y' = 4 - 2x$

$y' = \dfrac{4 - 2x}{3(6y + 1)}$

102. $y^2 = x^3 - x^2y + xy - y^2$

$0 = x^3 - x^2y + xy - 2y^2$

$0 = 3x^2 - x^2y' - 2xy + xy' + y - 4yy'$

$(x^2 - x + 4y)y' = 3x^2 - 2xy + y$

$y' = \dfrac{3x^2 - 2xy + y}{x^2 - x + 4y}$

104. $\cos(x + y) = x$

$-(1 + y')\sin(x + y) = 1$

$-y'\sin(x + y) = 1 + \sin(x + y)$

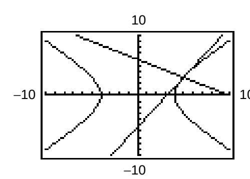
$y' = -\dfrac{1 + \sin(x + y)}{\sin(x + y)}$

$= -\csc(x + 1) - 1$

106. $x^2 - y^2 = 16$

$2x - 2yy' = 0$

$y' = \dfrac{x}{y}$



At (5, 3): $y' = \dfrac{5}{3}$

Tangent line: $y - 3 = \dfrac{5}{3}(x - 5)$

$5x - 3y - 16 = 0$

Normal line: $y - 3 = -\dfrac{3}{5}(x - 5)$

$3x + 5y - 30 = 0$

108. Surface area $= A = 6x^2$, x length of edge.

$\dfrac{dx}{dt} = 5$

$\dfrac{da}{dt} = 12x\dfrac{dx}{dt} = 12(4.5)(5) = 270\ \text{cm}^2/\text{sec}$

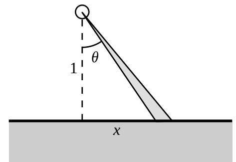
110. $\tan \theta = x$

$$\frac{d\theta}{dt} = 3(2\pi) \text{ rad/min}$$

$$\sec^2 \theta\left(\frac{d\theta}{dt}\right) = \frac{dx}{dt}$$

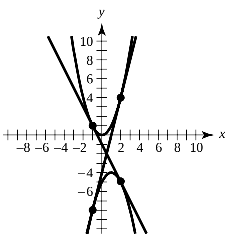
$$\frac{dx}{dt} = (\tan^2 \theta + 1)(6\pi) = 6\pi(x^2 + 1)$$

When $x = \frac{1}{2}$, $\frac{dx}{dt} = 6\pi\left(\frac{1}{4} + 1\right) = \frac{15\pi}{2}$ km/min $= 450\pi$ km/hr.



Problem Solving for Chapter 2

2.



Let (a, a^2) and $(b, -b^2 + 2b - 5)$ be the points of tangency.

For $y = x^2$, $y' = 2x$ and for $y = -x^2 + 2x - 5$, $y' = -2x + 2$.

Thus, $2a = -2b + 2 \Rightarrow a + b = 1$, or $a = 1 - b$. Furthermore, the slope of the common tangent line is

$$\frac{a^2 - (-b^2 + 2b - 5)}{a - b} = \frac{(1 - b)^2 + b^2 - 2b + 5}{(1 - b) - b} = -2b + 2$$

$$\Rightarrow \frac{1 - 2b + b^2 + b^2 - 2b + 5}{1 - 2b} = -2b + 2$$

$$\Rightarrow 2b^2 - 4b + 6 = 4b^2 - 6b + 2$$

$$\Rightarrow 2b^2 - 2b - 4 = 0$$

$$\Rightarrow b^2 - b - 2 = 0$$

$$\Rightarrow (b - 2)(b + 1) = 0$$

$$b = 2, -1$$

For $b = 2$, $a = 1 - b = -1$ and the points of tangency are $(-1, 1)$, $(2, -5)$. The tangent line has slope -2:

$$y - 1 = -2(x = 1) \Rightarrow y = -2x - 1$$

For $b = -1$, $a = 1 - b = 2$ and the points of tangency are $(2, 4)$ and $(-1, -8)$. The tangent line has slope 4:

$$y - 4 = 4(x - 2) \Rightarrow y = 4x - 4.$$

4. (a) $y = x^2$, $y' = 2x$. Slope $= 4$ at $(2, 4)$.

Tangent line: $y - 4 = 4(x - 2)$

$$y = 4x - 4$$

(b) Slope of normal line: $-\frac{1}{4}$.

Normal line: $y - 4 = -\frac{1}{4}(x - 2)$

$$y = -\frac{1}{4}x + \frac{9}{2}$$

$y = -\frac{1}{4}x + \frac{9}{2} = x^2 \Rightarrow 4x^2 + x - 18 = 0 \Rightarrow (4x + 9)(x - 2) = 0$

$x = 2, -\frac{9}{4}$. Second intersection point: $\left(-\frac{9}{4}, \frac{81}{16}\right)$

(c) Tangent line: $y = 0$

Normal line: $x = 0$

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4. —CONTINUED—

(d) Let (a, a^2), $a \neq 0$, be a point on the parabola $y = x^2$. Tangent line at (a, a^2) is $y = 2a(x - a) + a^2$. Normal line at (a, a^2) is $y = -\frac{1}{2a}(x - a) + a^2$. To find points of intersection, solve

$$x^2 = -\frac{1}{2a}(x - a) + a^2$$

$$x^2 + \frac{1}{2a}x = a^2 + \frac{1}{2}$$

$$x^2 + \frac{1}{2a}x + \frac{1}{16a^2} = a^2 + \frac{1}{2} + \frac{1}{16a^2}$$

$$\left(x + \frac{1}{4a}\right)^2 = \left(a + \frac{1}{4a}\right)^2$$

$$x + \frac{1}{4a} = \pm\left(a + \frac{1}{4a}\right)$$

$$x + \frac{1}{4a} = a + \frac{1}{4a} \Rightarrow x = a \text{ (Point of tangency)}$$

$$x + \frac{1}{4a} = -\left(a + \frac{1}{4a}\right) \Rightarrow x = -a - \frac{1}{2a} = -\frac{2a^2 + 1}{2a}$$

The normal line intersects a second time at $x = -\frac{2a^2 + 1}{2a}$.

6. $f(x) = a + b \cos cx$

$f'(x) = -bc \sin cx$

At $(0, 1)$: $a + b = 1$ — Equation 1

At $\left(\frac{\pi}{4}, \frac{3}{2}\right)$: $a + b \cos\left(\frac{c\pi}{4}\right) = \frac{3}{2}$ — Equation 2

$-bc \sin\left(\frac{c\pi}{4}\right) = 1$ — Equation 3

From Equation 1, $a = 1 - b$. Equation 2 becomes $(1 - b) + b \cos\left(\frac{c\pi}{4}\right) = \frac{3}{2} \Rightarrow -b + b \cos \frac{c\pi}{4} = \frac{1}{2}$

From Equation 3, $b = \dfrac{-1}{c \sin\left(\frac{c\pi}{4}\right)}$. Thus $\dfrac{1}{c \sin\left(\frac{c\pi}{4}\right)} + \dfrac{-1}{c \sin\left(\frac{c\pi}{4}\right)} \cos\left(\frac{c\pi}{4}\right) = \frac{1}{2}$

$$1 - \cos\left(\frac{c\pi}{4}\right) = \frac{1}{2} c \sin\left(\frac{c\pi}{4}\right)$$

Graphing the equation $g(c) = \frac{1}{2} c \sin\left(\frac{c\,\pi}{4}\right) + \cos\left(\frac{c\,\pi}{4}\right) - 1$, you see that many values of c will work.

One answer: $c = 2$, $b = -\frac{1}{2}$, $a = \frac{3}{2} \Rightarrow f(x) = \frac{3}{2} - \frac{1}{2} \cos 2x$

8. (a) $b^2y^2 = x^3(a - x);\ a, b > 0$

$$y^2 = \frac{x^3(a - x)}{b^2}$$

Graph $y_1 = \dfrac{\sqrt{x^3(a - x)}}{b}$ and $y_2 = -\dfrac{\sqrt{x^3(a - x)}}{b}$

(b) a determines the x-intercept on the right: $(a, 0)$.

b affects the height.

(c) Differentiating implicitly.

$$2b^2yy' = 3x^2(a - x) - x^3 = 3ax^2 - 4x^3$$

$$y' = \frac{(3ax^2 - 4x^3)}{2b^2y} = 0$$

$$\Longrightarrow 3ax^2 = 4x^3$$

$$3a = 4x$$

$$x = \frac{3a}{4}.$$

$$b^2y^2 = \left(\frac{3a}{4}\right)^3\left(a - \frac{3a}{4}\right) = \frac{27a^3}{64}\left(\frac{1}{4}a\right)$$

$$y^2 = \frac{27a^4}{256b^2} \Longrightarrow y = \pm\frac{3\sqrt{3}a^2}{16b}$$

Two points: $\left(\dfrac{3a}{4}, \dfrac{3\sqrt{3}a^2}{16b}\right), \left(\dfrac{3a}{4}, \dfrac{-3\sqrt{3}a^2}{16b}\right)$

10. (a) $y = x^{1/3} \Longrightarrow \dfrac{dy}{dt} = \dfrac{1}{3}x^{-2/3}\dfrac{dx}{dt}$

$$1 = \frac{1}{3}(8)^{-2/3}\frac{dx}{dt}$$

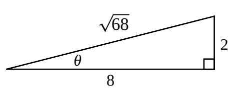
$$\frac{dx}{dt} = 12 \text{ cm/sec}$$

(b) $D = \sqrt{x^2 + y^2} \Longrightarrow \dfrac{dD}{dt} = \dfrac{1}{2}(x^2 + y^2)\left(2x\dfrac{dx}{dt} + 2y\dfrac{dy}{dt}\right)$

$$= \frac{x\dfrac{dx}{dt} + y\dfrac{dy}{dt}}{\sqrt{x^2 + y^2}}$$

$$= \frac{8(12) + 2(1)}{\sqrt{64 + 4}} = \frac{98}{\sqrt{68}} = \frac{49}{\sqrt{17}} \text{ cm/sec.}$$

(c) $\tan\theta = \dfrac{y}{x} \Longrightarrow \sec^2\theta \cdot \dfrac{d\theta}{dt} = \dfrac{x\dfrac{dy}{dt} - y\dfrac{dx}{dt}}{x^2}$



From the triangle, $\sec\theta = \dfrac{\sqrt{68}}{8}$. Hence $\dfrac{d\theta}{dt} = \dfrac{8(1) - 2(12)}{64\left(\dfrac{68}{64}\right)} = \dfrac{-16}{68} = \dfrac{-4}{17}$ rad/sec

12. $E'(x) = \lim_{\Delta x \to 0} \frac{E(x + \Delta x) - E(x)}{\Delta x}$

$= \lim_{\Delta x \to 0} \frac{E(x)E(\Delta x) - E(x)}{\Delta x}$

$= \lim_{\Delta x \to 0} E(x)\left(\frac{E(\Delta x) - 1}{\Delta x}\right)$

$= E(x) \lim_{\Delta x \to 0} \frac{E(\Delta x) - 1}{\Delta x}$

But, $E'(0) = \lim_{\Delta x \to 0} \frac{E(\Delta x) - E(0)}{\Delta x} = \lim_{\Delta x \to 0} \frac{E(\Delta x) - 1}{\Delta x} = 1.$

Thus, $E'(x) = E(x)E'(0) = E(x)$ exists for all x.

For example: $E(x) = e^x$.

14. (a) $v(t) = -\frac{27}{5}t + 27$ ft/sec

$a(t) = -\frac{27}{5}$ ft/sec^2

(b) $v(t) = -\frac{27}{5}t + 27 = 0 \Longrightarrow \frac{27}{5}t = 27 \Longrightarrow t = 5$ seconds

$S(5) = -\frac{27}{10}(5)^2 + 27(5) + 6 = 73.5$ feet

(c) The acceleration due to gravity on Earth is greater in magnitude than that on the moon.