51. $x^2 + y^2 = 25$; acceleration of the top of the ladder $= \dfrac{d^2y}{dt^2}$

First derivative: $2x\dfrac{dx}{dt} + 2y\dfrac{dy}{dt} = 0$

$$x\frac{dx}{dt} + y\frac{dy}{dt} = 0$$

Second derivative: $x\dfrac{d^2x}{dt^2} + \dfrac{dx}{dt}\cdot\dfrac{dx}{dt} + y\dfrac{d^2y}{dt^2} + \dfrac{dy}{dt}\cdot\dfrac{dy}{dt} = 0$

$$\frac{d^2y}{dt^2} = \left(\frac{1}{y}\right)\left[-x\frac{d^2x}{dt^2} - \left(\frac{dx}{dt}\right)^2 - \left(\frac{dy}{dt}\right)^2\right]$$

When $x = 7, y = 24, \dfrac{dy}{dt} = -\dfrac{7}{12}$, and $\dfrac{dx}{dt} = 2$ (see Exercise 27). Since $\dfrac{dx}{dt}$ is constant, $\dfrac{d^2x}{dt^2} = 0$.

$$\frac{d^2y}{dt^2} = \frac{1}{24}\left[-7(0) - (2)^2 - \left(-\frac{7}{12}\right)^2\right] = \frac{1}{24}\left[-4 - \frac{49}{144}\right] = \frac{1}{24}\left[-\frac{625}{144}\right] \approx -0.1808 \text{ ft/sec}^2$$

53. (a) Using a graphing utility, you obtain $m(s) = -0.881s^2 + 29.10s - 206.2$

(b) $\dfrac{dm}{dt} = \dfrac{dm}{ds}\dfrac{ds}{dt} = (-1.762s + 29.10)\dfrac{ds}{dt}$

(c) If $t = s$ (1995), then $s = 15.5$ and $\dfrac{ds}{dt} = 1.2$.

Thus, $\dfrac{dm}{dt} = (-1.762(15.5) + 29.10)(1.2) \approx 2.15$ million.

Review Exercises for Chapter 2

1. $f(x) = x^2 - 2x + 3$

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{[(x + \Delta x)^2 - 2(x + \Delta x) + 3] - [x^2 - 2x + 3]}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{(x^2 + 2x(\Delta x) + (\Delta x)^2 - 2x - 2(\Delta x) + 3) - (x^2 - 2x + 3)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{2x(\Delta x) + (\Delta x)^2 - 2(\Delta x)}{\Delta x} = \lim_{\Delta x \to 0} (2x + \Delta x - 2) = 2x - 2$$

3. $f(x) = \sqrt{x} + 1$

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{(\sqrt{x + \Delta x} + 1) - (\sqrt{x} + 1)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\sqrt{x + \Delta x} - \sqrt{x}}{\Delta x} \cdot \frac{\sqrt{x + \Delta x} + \sqrt{x}}{\sqrt{x + \Delta x} + \sqrt{x}}$$

$$= \lim_{\Delta x \to 0} \frac{(x + \Delta x) - x}{\Delta x(\sqrt{x + \Delta x} + \sqrt{x})}$$

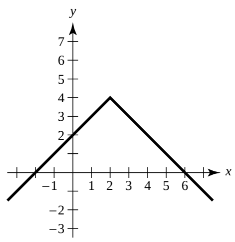
$$= \lim_{\Delta x \to 0} \frac{1}{\sqrt{x + \Delta x} + \sqrt{x}} = \frac{1}{2\sqrt{x}}$$

5. f is differentiable for all $x \neq -1$.

7. $f(x) = 4 - |x - 2|$

(a) Continuous at $x = 2$.

(b) Not differentiable at $x = 2$ because of the sharp turn in the graph.



9. Using the limit definition, you obtain $g'(x) = \frac{4}{3}x - \frac{1}{6}$.

At $x = -1$, $g'(-1) = -\frac{4}{3} - \frac{1}{6} = \frac{-3}{2}$

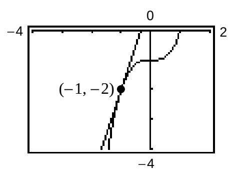
11. (a) Using the limit defintion, $f'(x) = 3x^2$.

At $x = -1$, $f'(-1) = 3$. The tangent line is

$$y - (-2) = 3(x - (-1))$$

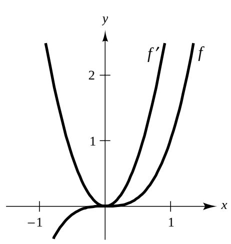
$$y = 3x + 1$$

(b)



13.
$$g'(2) = \lim_{x\to 2} \frac{g(x) - g(2)}{x - 2}$$
$$= \lim_{x\to 2} \frac{x^2(x - 1) - 4}{x - 2}$$
$$= \lim_{x\to 2} \frac{x^3 - x^2 - 4}{x - 2}$$
$$= \lim_{x\to 2} \frac{(x - 2)(x^2 + x + 2)}{x - 2}$$
$$= \lim_{x\to 2} (x^2 + x + 2) = 8$$

15.



17. $y = 25$

$y' = 0$

19. $f(x) = x^8$

$f'(x) = 8x^7$

21. $h(t) = 3t^4$

$h'(t) = 12t^3$

23. $f(x) = x^3 - 3x^2$

$f'(x) = 3x^2 - 6x = 3x(x - 2)$

25. $h(x) = 6\sqrt{x} + 3\sqrt[3]{x} = 6x^{1/2} + 3x^{1/3}$

$$h'(x) = 3x^{-1/2} + x^{-2/3} = \frac{3}{\sqrt{x}} + \frac{1}{\sqrt[3]{x^2}}$$

27. $g(t) = \frac{2}{3}t^{-2}$

$$g'(x) = \frac{-4}{3}t^{-3} = \frac{-4}{3t^3}$$

29. $f(\theta) = 2\theta - 3\sin\theta$

$f'(\theta) = 2 - 3\cos\theta$

31. $f(\theta) = 3\cos\theta - \frac{\sin\theta}{4}$

$$f'(\theta) = -3\sin\theta - \frac{\cos\theta}{4}$$

33. $F = 200\sqrt{T}$

$F'(t) = \dfrac{100}{\sqrt{T}}$

(a) When $T = 4$, $F'(4) = 50$ vibrations/sec/lb.

(b) When $T = 9$, $F'(9) = 33\frac{1}{3}$ vibrations/sec/lb.

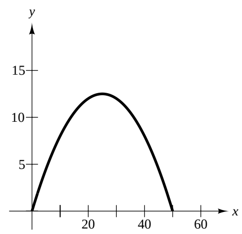
35. $s(t) = -16t^2 + s_0$

$s(9.2) = -16(9.2)^2 + s_0 = 0$

$s_0 = 1354.24$

The building is approximately 1354 feet high (or 415 m).

37. (a)



Total horizontal distance: 50

(b) $0 = x - 0.02x^2$

$0 = x\left(1 - \dfrac{x}{50}\right)$ implies $x = 50$.

(c) Ball reaches maximum height when $x = 25$.

(d) $y = x - 0.02x^2$

$y' = 1 - 0.04x$

$y'(0) = 1$

$y'(10) = 0.6$

$y'(25) = 0$

$y'(30) = -0.2$

$y'(50) = -1$

(e) $y'(25) = 0$

39. $x(t) = t^2 - 3t + 2 = (t - 2)(t - 1)$

(a) $v(t) = x'(t) = 2t - 3$

$a(t) = v'(t) = 2$

(c) $v(t) = 0$ for $t = \frac{3}{2}$.

$x = \left(\frac{3}{2} - 2\right)\left(\frac{3}{2} - 1\right) = \left(-\frac{1}{2}\right)\left(\frac{1}{2}\right) = -\frac{1}{4}$

(b) $v(t) < 0$ for $t < \frac{3}{2}$.

(d) $x(t) = 0$ for $t = 1, 2$.

$|v(1)| = |2(1) - 3| = 1$

$|v(2)| = |2(2) - 3| = 1$

The speed is 1 when the position is 0.

41. $f(x) = (3x^2 + 7)(x^2 - 2x + 3)$

$f'(x) = (3x^2 + 7)(2x - 2) + (x^2 - 2x + 3)(6x)$

$= 2(6x^3 - 9x^2 + 16x - 7)$

43. $h(x) = \sqrt{x}\sin x = x^{1/2}\sin x$

$h'(x) = \dfrac{1}{2\sqrt{x}}\sin x + \sqrt{x}\cos x$

45. $f(x) = 2x - x^{-2}$

$f'(x) = 2 + 2x^{-3} = 2\left(1 + \dfrac{1}{x^3}\right)$

$= \dfrac{2(x^3 + 1)}{x^3}$

47. $f(x) = \dfrac{x^2 + x - 1}{x^2 - 1}$

$f'(x) = \dfrac{(x^2 - 1)(2x + 1) - (x^2 + x - 1)(2x)}{(x^2 - 1)^2}$

$= \dfrac{-(x^2 + 1)}{(x^2 - 1)^2}$

49. $f(x) = (4 - 3x^2)^{-1}$

$f'(x) = -(4 - 3x^2)^{-2}(-6x) = \dfrac{6x}{(4 - 3x^2)^2}$

51. $y = \dfrac{x^2}{\cos x}$

$y' = \dfrac{\cos x\,(2x) - x^2(-\sin x)}{\cos^2 x} = \dfrac{2x\cos x + x^2\sin x}{\cos^2 x}$

53. $y = 3x^2 \sec x$

$y' = 3x^2 \sec x \tan x + 6x \sec x$

55. $y = -x\tan x$

$y' = -x\sec^2 x - \tan x$

57. $y = x\cos x - \sin x$

$y' = -x\sin x + \cos x - \cos x = -x\sin x$

59. $g(t) = t^3 - 3t + 2$

$g'(t) = 3t^2 - 3$

$g''(t) = 6t$

61. $f(\theta) = 3\tan\theta$

$f'(\theta) = 3\sec^2\theta$

$f''(\theta) = 6\sec\theta(\sec\theta\tan\theta) = 6\sec^2\theta\tan\theta$

63. $y = 2\sin x + 3\cos x$

$y' = 2\cos x - 3\sin x$

$y'' = -2\sin x - 3\cos x$

$y'' + y = -(2\sin x + 3\cos x) + (2\sin x + 3\cos x)$

$= 0$

65. $f(x) = (1 - x^3)^{1/2}$

$f'(x) = \frac{1}{2}(1 - x^3)^{-1/2}(-3x^2)$

$= -\frac{3x^2}{2\sqrt{1 - x^3}}$

67. $h(x) = \left(\frac{x - 3}{x^2 + 1}\right)^2$

$h'(x) = 2\left(\frac{x - 3}{x^2 + 1}\right)\left(\frac{(x^2 + 1)(1) - (x - 3)(2x)}{(x^2 + 1)^2}\right)$

$= \frac{2(x - 3)(-x^2 + 6x + 1)}{(x^2 + 1)^3}$

69. $f(s) = (s^2 - 1)^{5/2}(s^3 + 5)$

$f'(s) = (s^2 - 1)^{5/2}(3s^2) + (s^3 + 5)\left(\frac{5}{2}\right)(s^2 - 1)^{3/2}(2s)$

$= s(s^2 - 1)^{3/2}[3s(s^2 - 1) + 5(s^3 + 5)]$

$= s(s^2 - 1)^{3/2}(8s^3 - 3s + 25)$

71. $y = 3\cos(3x + 1)$

$y' = -9\sin(3x + 1)$

73. $y = \frac{1}{2}\csc 2x$

$y' = \frac{1}{2}(-\csc 2x\cot 2x)(2)$

$= -\csc 2x\cot 2x$

75. $y = \frac{x}{2} - \frac{\sin 2x}{4}$

$y' = \frac{1}{2} - \frac{1}{4}\cos 2x(2)$

$= \frac{1}{2}(1 - \cos 2x) = \sin^2 x$

77. $y = \frac{2}{3}\sin^{3/2} x - \frac{2}{7}\sin^{7/2} x$

$y' = \sin^{1/2} x\cos x - \sin^{5/2} x\cos x$

$= (\cos x)\sqrt{\sin x}(1 - \sin^2 x)$

$= (\cos^3 x)\sqrt{\sin x}$

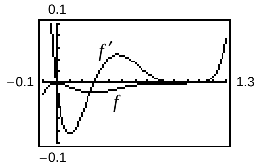
79. $y = \frac{\sin \pi x}{x + 2}$

$y' = \frac{(x + 2)\pi\cos\pi x - \sin\pi x}{(x + 2)^2}$

81. $f(t) = t^2(t - 1)^5$

$f'(t) = t(t - 1)^4(7t - 2)$

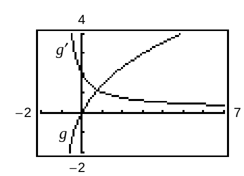
The zeros of f' correspond to the points on the graph of f where the tangent line is horizontal.



83. $g(x) = 2x(x + 1)^{-1/2}$

$g'(x) = \frac{x + 2}{(x + 1)^{3/2}}$

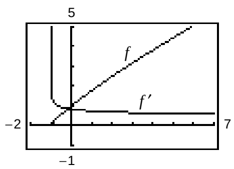
g' does not equal zero for any value of x in the domain. The graph of g has no horizontal tangent lines.



85. $f(t) = (t+1)^{1/2}(t+1)^{1/3} = (t+1)^{5/6}$

$$f'(t) = \frac{5}{6(t+1)^{1/6}}$$

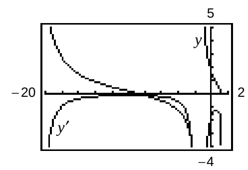
f' does not equal zero for any x in the domain. The graph of f has no horizontal tangent lines.



87. $y = \tan\sqrt{1-x}$

$$y' = -\frac{\sec^2\sqrt{1-x}}{2\sqrt{1-x}}$$

y' does not equal zero for any x in the domain. The graph has no horizontal tangent lines.



89. $y = 2x^2 + \sin 2x$

$y' = 4x + 2\cos 2x$

$y'' = 4 - 4\sin 2x$

91. $f(x) = \cot x$

$f'(x) = -\csc^2 x$

$f'' = -2\csc x(-\csc x \cdot \cot x)$

$= 2\csc^2 x \cot x$

93. $f(t) = \dfrac{t}{(1-t)^2}$

$$f'(t) = \frac{t+1}{(1-t)^3}$$

$$f''(t) = \frac{2(t+2)}{(1-t)^4}$$

95. $g(\theta) = \tan 3\theta - \sin(\theta - 1)$

$g'(\theta) = 3\sec^2 3\theta - \cos(\theta - 1)$

$g''(\theta) = 18\sec^2 3\theta \tan 3\theta + \sin(\theta - 1)$

97. $T = 700(t^2 + 4t + 10)^{-1}$

$$T' = \frac{-1400(t+2)}{(t^2+4t+10)^2}$$

(a) When $t = 1$,

$$T' = \frac{-1400(1+2)}{(1+4+10)^2} \approx -18.667 \text{ deg/hr.}$$

(b) When $t = 3$,

$$T' = \frac{-1400(3+2)}{(9+12+10)^2} \approx -7.284 \text{ deg/hr.}$$

(c) When $t = 5$,

$$T' = \frac{-1400(5+2)}{(25+30+10)^2} \approx -3.240 \text{ deg/hr.}$$

(d) When $t = 10$,

$$T' = \frac{-1400(10+2)}{(100+40+10)^2} \approx -0.747 \text{ deg/hr.}$$

99.

$$x^2 + 3xy + y^3 = 10$$

$$2x + 3xy' + 3y + 3y^2y' = 0$$

$$3(x + y^2)y' = -(2x + 3y)$$

$$y' = \frac{-(2x+3y)}{3(x+y^2)}$$

101.

$$y\sqrt{x} - x\sqrt{y} = 16$$

$$y\left(\frac{1}{2}x^{-1/2}\right) + x^{1/2}y' - x\left(\frac{1}{2}y^{-1/2}y'\right) - y^{1/2} = 0$$

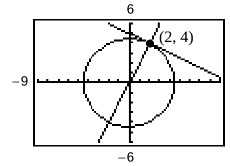
$$\left(\sqrt{x} - \frac{x}{2\sqrt{y}}\right)y' = \sqrt{y} - \frac{y}{2\sqrt{x}}$$

$$\frac{2\sqrt{xy} - x}{2\sqrt{y}}y' = \frac{2\sqrt{xy} - y}{2\sqrt{x}}$$

$$y' = \frac{2\sqrt{xy} - y}{2\sqrt{x}} \cdot \frac{2\sqrt{y}}{2\sqrt{xy} - x} = \frac{2y\sqrt{x} - y\sqrt{y}}{2x\sqrt{y} - x\sqrt{x}}$$

103. $$x \sin y = y \cos x$$
$$(x \cos y)y' + \sin y = -y \sin x + y' \cos x$$
$$y'(x \cos y - \cos x) = -y \sin x - \sin y$$
$$y' = \frac{y \sin x + \sin y}{\cos x - x \cos y}$$

105. $x^2 + y^2 = 20$



$$2x + 2yy' = 0$$
$$y' = -\frac{x}{y}$$
At $(2, 4)$: $y' = -\frac{1}{2}$

Tangent line: $y - 4 = -\frac{1}{2}(x - 2)$
$$x + 2y - 10 = 0$$
Normal line: $y - 4 = 2(x - 2)$
$$2x - y = 0$$

107. $y = \sqrt{x}$

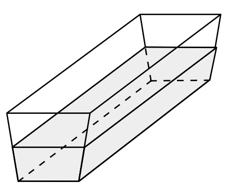
$$\frac{dy}{dt} = 2 \text{ units/sec}$$
$$\frac{dy}{dt} = \frac{1}{2\sqrt{x}}\frac{dx}{dt} \Longrightarrow \frac{dx}{dt} = 2\sqrt{x}\frac{dy}{dt} = 4\sqrt{x}$$

(a) When $x = \frac{1}{2}$, $\frac{dx}{dt} = 2\sqrt{2}$ units/sec.

(b) When $x = 1$, $\frac{dx}{dt} = 4$ units/sec.

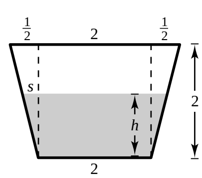
(c) When $x = 4$, $\frac{dx}{dt} = 8$ units/sec.

109. $\frac{s}{h} = \frac{1/2}{2}$



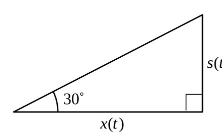
$$s = \frac{1}{4}h$$
$$\frac{dV}{dt} = 1$$

Width of water at depth h:
$$w = 2 + 2s = 2 + 2\left(\frac{1}{4}h\right) = \frac{4 + h}{2}$$
$$V = \frac{5}{2}\left(2 + \frac{4 + h}{2}\right)h = \frac{5}{4}(8 + h)h$$
$$\frac{dV}{dt} = \frac{5}{2}(4 + h)\frac{dh}{dt}$$
$$\frac{dh}{dt} = \frac{2(dV/dt)}{5(4 + h)}$$



When $h = 1$, $\frac{dh}{dt} = \frac{2}{25}$ m/min.

111. $s(t) = 60 - 4.9t^2$

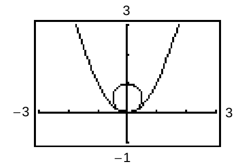


$$s'(t) = -9.8t$$
$$s = 35 = 60 - 4.9t^2$$
$$4.9t^2 = 25$$
$$t = \frac{5}{\sqrt{4.9}}$$
$$\tan 30° = \frac{1}{\sqrt{3}} = \frac{s(t)}{x(t)}$$
$$x(t) = \sqrt{3}s(t)$$
$$\frac{dx}{dt} = \sqrt{3}\frac{ds}{dt} = \sqrt{3}(-9.8)\frac{5}{\sqrt{4.9}}$$
$$\approx -38.34 \text{ m/sec}$$

Problem Solving for Chapter 2

1. (a) $x^2 + (y - r)^2 = r^2$ Circle

$x^2 = y$ Parabola



Substituting,

$$(y - r)^2 = r^2 - y$$
$$y^2 - 2ry + r^2 = r^2 - y$$
$$y^2 - 2ry + y = 0$$
$$y(y - 2r + 1) = 0$$

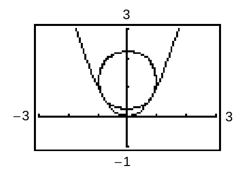
Since you want only one solution, let $1 - 2r = 0 \Rightarrow r = \frac{1}{2}$

Graph $y = x^2$ and $x^2 + \left(y - \frac{1}{2}\right)^2 = \frac{1}{4}$

(b) Let (x, y) be a point of tangency: $x^2 + (y - b)^2 = 1 \Rightarrow 2x + 2(y - b)y' = 0 \Rightarrow y' = \dfrac{x}{b - y}$ (circle).

$y = x^2 \Rightarrow y' = 2x$ (parabola). Equating,

$$2x = \frac{x}{b - y}$$
$$2(b - y) = 1$$
$$b - y = \frac{1}{2} \Rightarrow b = y + \frac{1}{2}$$



Also, $x^2 + (y - b)^2 = 1$ and $y = x^2$ imply

$$y + (y - b)^2 = 1 \Rightarrow y + \left[y - \left(y + \frac{1}{2}\right)\right] = 1 \Rightarrow y - \frac{1}{2} = 1 \Rightarrow y = \frac{3}{4} \text{ and } b = \frac{5}{4}.$$

Center: $\left(0, \dfrac{5}{4}\right)$

Graph $y = x^2$ and $x^2 + \left(y - \dfrac{5}{4}\right)^2 = 1$

3. (a) $f(x) = \cos x$ $\quad P_1(x) = a_0 + a_1x$

$f(0) = 1$ $\quad P_1(0) = a_0 \Rightarrow a_0 = 1$

$f'(0) = 0$ $\quad P'_1(0) = a_1 \Rightarrow a_1 = 0$

$P_1(x) = 1$

(b) $f(x) = \cos x$ $\quad P_2(x) = a_0 + a_1x + a_2x^2$

$f(0) = 1$ $\quad P_2(0) = a_0 \Rightarrow a_0 = 1$

$f'(0) = 0$ $\quad P'_2(0) = a_1 \Rightarrow a_1 = 0$

$f''(0) = -1$ $\quad P''_2(0) = 2a_2 \Rightarrow a_2 = -\frac{1}{2}$

$P_2(x) = 1 - \frac{1}{2}x^2$

(c)

x	-1.0	-0.1	-0.001	0	0.001	0.1	1.0
$\cos x$	0.5403	0.9950	≈ 1	1	≈ 1	0.9950	0.5403
$P_2(x)$	0.5	0.9950	≈ 1	1	≈ 1	0.9950	0.5

$P_2(x)$ is a good approximation of $f(x) = \cos x$ when x is near 0.

(d) $f(x) = \sin x$ $\quad P_3(x) = a_0 + a_1x + a_2x^2 + a_3x^3$

$f(0) = 0$ $\quad P_3(0) = a_0 \Rightarrow a_0 = 0$

$f'(0) = 1$ $\quad P'_3(0) = a_1 \Rightarrow a_1 = 1$

$f''(0) = 0$ $\quad P''_3(0) = 2a_2 \Rightarrow a_2 = 0$

$f'''(0) = -1$ $\quad P'''_3(0) = 6a_3 \Rightarrow a_3 = -\frac{1}{6}$

$P_3(x) = x - \frac{1}{6}x^3$

5. Let $p(x) = Ax^3 + Bx^2 + Cx + D$

$p'(x) = 3Ax^2 + 2Bx + C$

At $(1, 1)$: $A + B + C + D = 1$ Equation 1

$3A + 2B + C = 14$ Equation 2

At $(-1, -3)$: $-A + B - C + D = -3$ Equation 3

$3A - 2B + C = -2$ Equation 4

Adding Equations 1 and 3: $2B + 2D = -2$

Subtracting Equations 1 and 3: $2A + 2C = 4$

Adding Equations 2 and 4: $6A + 2C = 12$

Subtracting Equations 2 and 4: $4B = 16$

Hence, $B = 4$ and $D = \frac{1}{2}(-2 - 2B) = -5$

Subtracting $2A + 2C = 4$ and $6A + 2C = 12$, you obtain $4A = 8 \Longrightarrow A = 2$. Finally, $C = \frac{1}{2}(4 - 2A) = 0$

Thus, $p(x) = 2x^3 + 4x^2 - 5$.

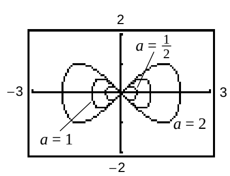
7. (a) $x^4 = a^2x^2 - a^2y^2$

$a^2y^2 = a^2x^2 - x^4$

$$y = \frac{\pm\sqrt{a^2x^2 - x^4}}{a}$$

Graph: $y_1 = \dfrac{\sqrt{a^2x^2 - x^4}}{a}$ and $y_2 = -\dfrac{\sqrt{a^2x^2 - x^4}}{a}$

(b)



$(\pm a, 0)$ are the x-intercepts, along with $(0, 0)$.

(c) Differentiating implicitly,

$4x^3 = 2a^2x - 2a^2yy'$

$$y' = \frac{2a^2x - 4x^3}{2a^2y} = \frac{x(a^2 - 2x^2)}{a^2y} = 0 \Longrightarrow 2x^2 = a^2 \Longrightarrow x = \frac{\pm a}{\sqrt{2}}.$$

$$\left(\frac{a^2}{2}\right)^2 = a^2\left(\frac{a^2}{2}\right) - a^2y^2$$

$$\frac{a^4}{4} = \frac{a^4}{2} - a^2y^2$$

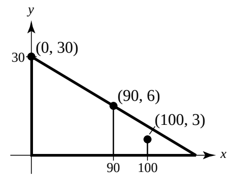
$$a^2y^2 = \frac{a^4}{4}$$

$$y^2 = \frac{a^2}{4}$$

$$y = \pm\frac{a}{2}$$

Four points: $\left(\frac{a}{\sqrt{2}}, \frac{a}{2}\right), \left(\frac{a}{\sqrt{2}}, -\frac{a}{2}\right), \left(-\frac{a}{\sqrt{2}}, \frac{a}{2}\right), \left(\frac{-a}{\sqrt{2}}, -\frac{a}{2}\right)$

9. (a)



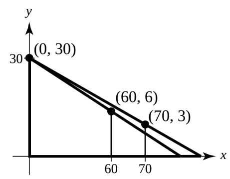
Not drawn to scale

Line determined by $(0, 30)$ and $(90, 6)$:

$$y - 30 = \frac{30 - 6}{0 - 90}(x - 0) = -\frac{24}{90}x = -\frac{4}{15}x \Longrightarrow y = -\frac{4}{15}x + 30$$

When $x = 100$, $y = \frac{-4}{15}(100) + 30 = \frac{10}{3} > 3 \Longrightarrow$ Shadow determined by man.

(b)



Not drawn to scale

Line determined by $(0, 30)$ and $(60, 6)$:

$$y - 30 = \frac{30 - 6}{0 - 60}(x - 0) = -\frac{2}{5}x \Longrightarrow y = -\frac{2}{5}x + 30$$

When $x = 70$, $y = \frac{-2}{5}(70) + 30 = 2 < 3 \Longrightarrow$ Shadow determined by child.

(c) Need $(0, 30)$, $(d, 6)$, $(d + 10, 3)$ collinear.

$$\frac{30 - 6}{0 - d} = \frac{6 - 3}{d - (d + 10)} \Longrightarrow \frac{24}{d} = \frac{3}{10} \Longrightarrow d = 80 \text{ feet}$$

(d) Let y be the length of the street light to the tip of the shadow. We know that $\frac{dx}{dt} = -5$.

For $x > 80$, the shadow is determined by the man.

$$\frac{y}{30} = \frac{y - x}{6} \Longrightarrow y = \frac{5}{4}x \text{ and } \frac{dy}{dt} = \frac{5}{4}\frac{dx}{dt} = \frac{-25}{4}.$$

For $x < 80$, the shadow is determined by the child.

$$\frac{y}{30} = \frac{y - x - 10}{3} \Longrightarrow y = \frac{10}{9}x + \frac{100}{9} \text{ and } \frac{dy}{dt} = \frac{10}{9}\frac{dx}{dt} = \frac{-50}{9}.$$

Therefore,

$$\frac{dy}{dt} = \begin{cases} \frac{-25}{4} & x > 80 \\ \frac{-50}{9} & 0 < x < 80 \end{cases}$$

$\frac{dy}{dt}$ is not continuous at $x = 80$.

11. $L'(x) = \lim_{\Delta x \to 0} \frac{L(x + \Delta x) - L(x)}{\Delta x}$

$$= \lim_{\Delta x \to 0} \frac{L(x) + L(\Delta x) - L(x)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{L(\Delta x)}{\Delta x}$$

Also, $L'(0) = \lim_{\Delta x \to 0} \frac{L(\Delta x) - L(0)}{\Delta x}$

But, $L(0) = 0$ because $L(0) = L(0 + 0) = L(0) + L(0) \Longrightarrow L(0) = 0$.

Thus, $L'(x) = L'(0)$, for all x.

The graph of L is a line through the origin of slope $L'(0)$.

13. (a)

z (degrees)	0.1	0.01	0.0001
$\dfrac{\sin z}{z}$	0.0174524	0.0174533	0.0174533

(b) $\lim_{z\to 0} \dfrac{\sin z}{z} \approx 0.0174533$

In fact, $\lim_{z\to 0} \dfrac{\sin z}{z} = \dfrac{\pi}{180}$

(c)
$$\frac{d}{dz}(\sin z) = \lim_{\Delta z \to 0} \frac{\sin(z + \Delta z) - \sin z}{\Delta z}$$
$$= \lim_{\Delta z \to 0} \frac{\sin z \cdot \cos \Delta z + \sin \Delta z \cdot \cos z - \sin z}{\Delta z}$$
$$= \lim_{\Delta z \to 0} \left[\sin z\left(\frac{\cos \Delta z - 1}{\Delta z}\right)\right] + \lim_{\Delta z \to 0} \left[\cos z\left(\frac{\sin \Delta z}{\Delta z}\right)\right]$$
$$= \sin z(0) + \cos z\left(\frac{\pi}{180}\right) = \frac{\pi}{180}\cos z$$

(d) $S(90) = \sin\left(\dfrac{\pi}{180}90\right) = \sin\dfrac{\pi}{2} = 1;\ C(180) = \cos\left(\dfrac{\pi}{180}180\right) = -1$

$$\frac{d}{dz}S(z) = \frac{d}{dz}\sin(cz) = c \cdot \cos(cz) = \frac{\pi}{180}C(z)$$

(e) The formulas for the derivatives are more complicated in degrees.

15. $j(t) = a'(t)$

(a) $j(t)$ is the rate of change of the acceleration.

(b) From Exercise 102 in Section 2.3,

$s(t) = -8.25t^2 + 66t$

$v(t) = -16.5t + 66$

$a(t) = -16.5$

$a'(t) = j(t) = 0$