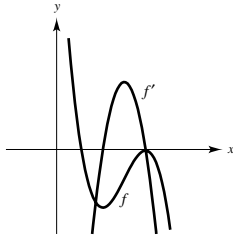


Review Exercises for Chapter 4

2.



$$\begin{aligned} 6. \int \frac{x^3 - 2x^2 + 1}{x^2} dx &= \int (x - 2 + x^{-2}) dx \\ &= \frac{1}{2}x^2 - 2x - \frac{1}{x} + C \end{aligned}$$

$$10. f''(x) = 6(x - 1)$$

$$f'(x) = \int 6(x - 1) dx = 3(x - 1)^2 + C_1$$

Since the slope of the tangent line at (2, 1) is 3, it follows that $f'(2) = 3 + C_1 = 3$ when $C_1 = 0$.

$$f'(x) = 3(x - 1)^2$$

$$f(x) = \int 3(x - 1)^2 dx = (x - 1)^3 + C_2$$

$$f(2) = 1 + C_2 = 1 \text{ when } C_2 = 0.$$

$$f(x) = (x - 1)^3$$

$$14. a(t) = -9.8 \text{ m/sec}^2$$

$$v(t) = -9.8t + v_0 = -9.8t + 40$$

$$s(t) = -4.9t^2 + 40t \quad (s(0) = 0)$$

$$(a) v(t) = -9.8t + 40 = 0 \text{ when } t = \frac{40}{9.8} \approx 4.08 \text{ sec.}$$

$$(b) s(4.08) \approx 81.63 \text{ m}$$

$$(c) v(t) = -9.8t + 40 = 20 \text{ when } t = \frac{20}{9.8} \approx 2.04 \text{ sec.}$$

$$(d) s(2.04) \approx 61.2 \text{ m}$$

$$4. u = 3x$$

$$du = 3 dx$$

$$\int \frac{2}{\sqrt[3]{3x}} dx = \frac{2}{3} \int (3x)^{-1/3} (3) dx = (3x)^{2/3} + C$$

$$8. \int (5 \cos x - 2 \sec^2 x) dx = 5 \sin x - 2 \tan x + C$$

$$12. 45 \text{ mph} = 66 \text{ ft/sec}$$

$$30 \text{ mph} = 44 \text{ ft/sec}$$

$$a(t) = -a$$

$$v(t) = -at + 66 \text{ since } v(0) = 66 \text{ ft/sec.}$$

$$s(t) = -\frac{a}{2}t^2 + 66t \text{ since } s(0) = 0.$$

Solving the system

$$v(t) = -at + 66 = 44$$

$$s(t) = -\frac{a}{2}t^2 + 66t = 264$$

we obtain $t = 24/5$ and $a = 55/12$. We now solve $-(55/12)t + 66 = 0$ and get $t = 72/5$. Thus,

$$s\left(\frac{72}{5}\right) = -\frac{55/12}{2}\left(\frac{72}{5}\right)^2 + 66\left(\frac{72}{5}\right) \approx 475.2 \text{ ft.}$$

Stopping distance from 30 mph to rest is

$$475.2 - 264 = 211.2 \text{ ft.}$$

16. $x_1 = 2, x_2 = -1, x_3 = 5, x_4 = 3, x_5 = 7$

(a) $\frac{1}{5} \sum_{i=1}^5 x_i = \frac{1}{5}(2 - 1 + 5 + 3 + 7) = \frac{16}{5}$

(b) $\sum_{i=1}^5 \frac{1}{x_i} = \frac{1}{2} - 1 + \frac{1}{5} + \frac{1}{3} + \frac{1}{7} = \frac{37}{210}$

(c) $\sum_{i=1}^5 (2x_i - x_i^2) = [2(2) - (2)^2] + [2(-1) - (-1)^2] + [2(5) - (5)^2] + [2(3) - (3)^2] + [2(7) - (7)^2] = -56$

(d) $\sum_{i=2}^5 (x_i - x_{i-1}) = (-1 - 2) + [5 - (-1)] + (3 - 5) + (7 - 3) = 5$

18. $y = 9 - \frac{1}{4}x^2, \Delta x = 1, n = 4$

$$S(4) = 1 \left[\left(9 - \frac{1}{4}(4)\right) + \left(9 - \frac{1}{4}(9)\right) + \left(9 - \frac{1}{4}(16)\right) + 9 - \frac{1}{4}(25) \right]$$

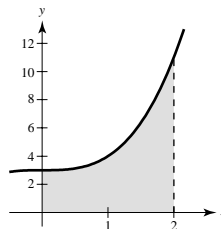
$$\approx 22.5$$

$$s(4) = 1 \left[\left(9 - \frac{1}{4}(9)\right) + \left(9 - \frac{1}{4}(16)\right) + \left(9 - \frac{1}{4}(25)\right) + (9 - 9) \right]$$

$$\approx 14.5$$

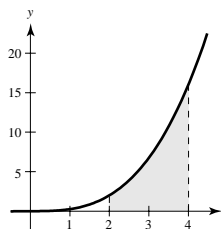
20. $y = x^2 + 3, \Delta x = \frac{2}{n}$ right endpoints

$$\begin{aligned} \text{Area} &= \lim_{n \rightarrow \infty} \sum_{i=1}^n f(ci) \Delta x \\ &= \lim_{n \rightarrow \infty} \sum_{i=1}^n \left[\left(\frac{2i}{n}\right)^2 + 3 \right] \left(\frac{2}{n}\right) \\ &= \lim_{n \rightarrow \infty} \frac{2}{n} \sum_{i=1}^n \left[\frac{4i^2}{n^2} + 3 \right] \\ &= \lim_{n \rightarrow \infty} \frac{2}{n} \left[\frac{4}{n^2} \frac{n(n+1)(2n+1)}{6} + 3n \right] \\ &= \lim_{n \rightarrow \infty} \left[\frac{4}{3} \frac{(n+1)(2n+1)}{n^2} + 6 \right] = \frac{8}{3} + 6 = \frac{26}{3} \end{aligned}$$



22. $y = \frac{1}{4}x^3, \Delta x = \frac{2}{n}$

$$\begin{aligned} \text{Area} &= \lim_{n \rightarrow \infty} \sum_{i=1}^n f(ci) \Delta x \\ &= \lim_{n \rightarrow \infty} \sum_{i=1}^n \frac{1}{4} \left(2 + \frac{2i}{n}\right)^3 \left(\frac{2}{n}\right) \\ &= \lim_{n \rightarrow \infty} \frac{1}{2n} \sum_{i=1}^n \left[8 + \frac{24i}{n} + \frac{24i^2}{n^2} + \frac{8i^3}{n^3} \right] \\ &= \lim_{n \rightarrow \infty} \frac{4}{n} \sum_{i=1}^n \left[1 + \frac{3i}{n} + \frac{3i^2}{n^2} + \frac{i^3}{n^3} \right] \\ &= \lim_{n \rightarrow \infty} \frac{4}{n} \left[n + \frac{3n(n+1)}{2} + \frac{3n(n+1)(2n+1)}{6} + \frac{1}{n^3} \frac{n^2(n+1)^2}{4} \right] \\ &= 4 + 6 + 4 + 1 = 15 \end{aligned}$$



$$24. (a) S = m\left(\frac{b}{4}\right)\left(\frac{b}{4}\right) + m\left(\frac{2b}{4}\right)\left(\frac{b}{4}\right) + m\left(\frac{3b}{4}\right)\left(\frac{b}{4}\right) + m\left(\frac{4b}{4}\right)\left(\frac{b}{4}\right) = \frac{mb^2}{16}(1 + 2 + 3 + 4) = \frac{5mb^2}{8}$$

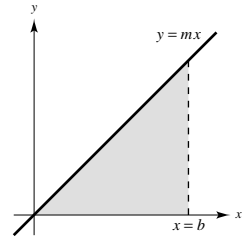
$$s = m(0)\left(\frac{b}{4}\right) + m\left(\frac{b}{4}\right)\left(\frac{b}{4}\right) + m\left(\frac{2b}{4}\right)\left(\frac{b}{4}\right) + m\left(\frac{3b}{4}\right)\left(\frac{b}{4}\right) = \frac{mb^2}{16}(1 + 2 + 3) = \frac{3mb^2}{8}$$

$$(b) S(n) = \sum_{i=1}^n f\left(\frac{bi}{n}\right)\left(\frac{b}{n}\right) = \sum_{i=1}^n \left(\frac{mbi}{n}\right)\left(\frac{b}{n}\right) = m\left(\frac{b}{n}\right)^2 \sum_{i=1}^n i = \frac{mb^2}{n^2} \left(\frac{n(n+1)}{2}\right) = \frac{mb^2(n+1)}{2n}$$

$$s(n) = \sum_{i=0}^{n-1} f\left(\frac{bi}{n}\right)\left(\frac{b}{n}\right) = \sum_{i=0}^{n-1} m\left(\frac{bi}{n}\right)\left(\frac{b}{n}\right) = m\left(\frac{b}{n}\right)^2 \sum_{i=0}^{n-1} i = \frac{mb^2}{n^2} \left(\frac{(n-1)n}{2}\right) = \frac{mb^2(n-1)}{2n}$$

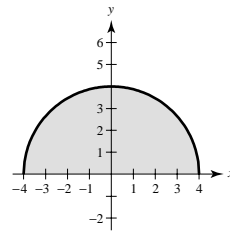
$$(c) \text{Area} = \lim_{n \rightarrow \infty} \frac{mb^2(n+1)}{2n} = \lim_{n \rightarrow \infty} \frac{mb^2(n-1)}{2n} = \frac{1}{2}mb^2 = \frac{1}{2}(b)(mb) = \frac{1}{2}(\text{base})(\text{height})$$

$$(d) \int_0^b mx \, dx = \left[\frac{1}{2}mx^2 \right]_0^b = \frac{1}{2}mb^2$$



$$26. \lim_{\|\Delta\| \rightarrow \infty} \sum_{i=1}^n 3ci(9 - ci^2) \Delta xi = \int_1^3 3x(9 - x^2) \, dx$$

28.



$$\int_{-4}^4 \sqrt{16 - x^2} \, dx = \frac{1}{2} \pi(4)^2 = 8\pi \quad (\text{semicircle})$$

$$30. (a) \int_0^6 f(x) \, dx = \int_0^3 f(x) \, dx + \int_3^6 f(x) \, dx = 4 + (-1) = 3$$

$$(b) \int_6^3 f(x) \, dx = -\int_3^6 f(x) \, dx = -(-1) = 1$$

$$(c) \int_4^4 f(x) \, dx = 0$$

$$(d) \int_3^6 -10f(x) \, dx = -10 \int_3^6 f(x) \, dx = -10(-1) = 10$$

$$32. \int_1^3 \frac{12}{x^3} \, dx = \left[\frac{12x^{-2}}{-2} \right]_1^3 = \left[\frac{-6}{x^2} \right]_1^3 = \frac{-6}{9} + 6 = \frac{16}{3}, (d)$$

$$34. \int_{-1}^1 (t^2 + 2) \, dt = \left[\frac{t^3}{3} + 2t \right]_{-1}^1 = \frac{14}{3}$$

$$36. \int_{-2}^2 (x^4 + 2x^2 - 5) \, dx = \left[\frac{x^5}{5} + \frac{2x^3}{3} - 5x \right]_{-2}^2$$

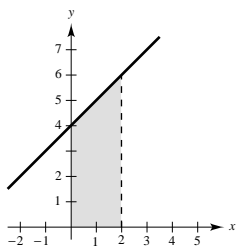
$$= \left(\frac{32}{5} + \frac{16}{3} - 10 \right) - \left(-\frac{32}{5} - \frac{16}{3} + 10 \right)$$

$$= \frac{52}{15}$$

$$38. \int_1^2 \left(\frac{1}{x^2} - \frac{1}{x^3} \right) \, dx = \int_1^2 (x^{-2} - x^{-3}) \, dx = \left[-\frac{1}{x} + \frac{1}{2x^2} \right]_1^2 = \left(-\frac{1}{2} + \frac{1}{8} \right) - \left(-1 + \frac{1}{2} \right) = \frac{1}{8}$$

$$40. \int_{-\pi/4}^{\pi/4} \sec^2 t \, dt = \left[\tan t \right]_{-\pi/4}^{\pi/4} = 1 - (-1) = 2$$

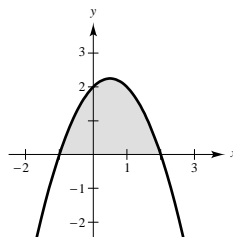
$$42. \int_0^2 (x + 4) dx = \left[\frac{x^2}{2} + 4x \right]_0^2 = 10$$



$$44. \int_{-1}^2 (-x^2 + x + 2) dx = \left[-\frac{x^3}{3} + \frac{x^2}{2} + 2x \right]_{-1}^2$$

$$= \left(-\frac{8}{3} + 2 + 4 \right) - \left(\frac{1}{3} + \frac{1}{2} - 2 \right)$$

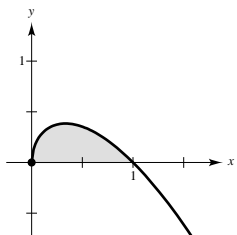
$$= \frac{10}{3} + \frac{7}{6} = \frac{9}{2}$$



$$46. \int_0^1 \sqrt{x}(1-x) dx = \int_0^1 (x^{1/2} - x^{3/2}) dx$$

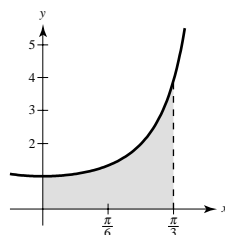
$$= \left[\frac{2}{3}x^{3/2} - \frac{2}{5}x^{5/2} \right]_0^1$$

$$= \frac{2}{3} - \frac{2}{5} = \frac{4}{15}$$



$$48. \text{Area} = \int_0^{\pi/3} \sec^2 x dx$$

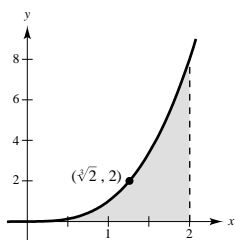
$$= \tan x \Big|_0^{\pi/3} = \sqrt{3}$$



$$50. \frac{1}{2} \int_0^2 x^3 dx = \left[\frac{x^4}{8} \right]_0^2 = 2$$

$$x^3 = 2$$

$$x = \sqrt[3]{2}$$



$$52. F'(x) = \frac{1}{x^2}$$

$$54. F'(x) = \csc^2 x$$

$$56. \int \left(x + \frac{1}{x} \right)^2 dx = \int (x^2 + 2 + x^{-2}) dx$$

$$= \frac{x^3}{3} + 2x - \frac{1}{x} + C$$

58. $u = x^3 + 3, du = 3x^2 dx$

$$\int x^2 \sqrt{x^3 + 3} dx = \frac{1}{3} \int (x^3 + 3)^{1/2} 3x^2 dx = \frac{2}{9} (x^3 + 3)^{3/2} + C$$

60. $u = x^2 + 6x - 5, du = (2x + 6) dx$

$$\int \frac{x + 3}{(x^2 + 6x - 5)^2} dx = \frac{1}{2} \int \frac{2x + 6}{(x^2 + 6x - 5)^2} dx = \frac{-1}{2} (x^2 + 6x - 5)^{-1} + C = \frac{-1}{2(x^2 + 6x - 5)} + C$$

62. $\int x \sin 3x^2 dx = \frac{1}{6} \int (\sin 3x^2)(6x) dx = -\frac{1}{6} \cos 3x^2 + C$

64. $\int \frac{\cos x}{\sqrt{\sin x}} dx = \int (\sin x)^{-1/2} \cos x dx = 2(\sin x)^{1/2} + C = 2\sqrt{\sin x} + C$

66. $\int \sec 2x \tan 2x dx = \frac{1}{2} \int (\sec 2x \tan 2x)(2) dx = \frac{1}{2} \sec 2x + C$

68. $\int \cot^4 \alpha \csc^2 \alpha d\alpha = -\int (\cot \alpha)^4 (-\csc^2 \alpha) d\alpha = -\frac{1}{5} \cot^5 \alpha + C$

70. $\int_0^1 x^2(x^3 + 1)^3 dx = \frac{1}{3} \int_0^1 (x^3 + 1)^3 (3x^2) dx = \frac{1}{12} [(x^3 + 1)^4]_0^1 = \frac{1}{12} (16 - 1) = \frac{5}{4}$

72. $\int_3^6 \frac{x}{3\sqrt{x^2 - 8}} dx = \frac{1}{6} \int_3^6 (x^2 - 8)^{-1/2} (2x) dx = \left[\frac{1}{3} (x^2 - 8)^{1/2} \right]_3^6 = \frac{1}{3} (2\sqrt{7} - 1)$

74. $u = x + 1, x = u - 1, dx = du$

When $x = -1, u = 0$. When $x = 0, u = 1$.

$$\begin{aligned} 2\pi \int_{-1}^0 x^2 \sqrt{x+1} dx &= 2\pi \int_0^1 (u-1)^2 \sqrt{u} du \\ &= 2\pi \int_0^1 (u^{5/2} - 2u^{3/2} + u^{1/2}) du = 2\pi \left[\frac{2}{7} u^{7/2} - \frac{4}{5} u^{5/2} + \frac{2}{3} u^{3/2} \right]_0^1 = \frac{32\pi}{105} \end{aligned}$$

76. $\int_{-\pi/4}^{\pi/4} \sin 2x dx = 0$ since $\sin 2x$ is an odd function.

78. $u = 1 - x, x = 1 - u, dx = -du$

When $x = a, u = 1 - a$. When $x = b, u = 1 - b$.

$$\begin{aligned} P_{a,b} &= \int_a^b \frac{1155}{32} x^3 (1-x)^{3/2} dx = \frac{1155}{32} \int_{1-a}^{1-b} -(1-u)^3 u^{3/2} du \\ &= \frac{1155}{32} \int_{1-a}^{1-b} (u^{9/2} - 3u^{7/2} + 3u^{5/2} - u^{3/2}) du = \frac{1155}{32} \left[\frac{2}{11} u^{11/2} - \frac{2}{3} u^{9/2} + \frac{6}{7} u^{7/2} - \frac{2}{5} u^{5/2} \right]_{1-a}^{1-b} \\ &= \frac{1155}{32} \left[\frac{2u^{5/2}}{1155} (105u^3 - 385u^2 + 495u - 231) \right]_{1-a}^{1-b} = \left[\frac{u^{5/2}}{16} (105u^3 - 385u^2 + 495u - 231) \right]_{1-a}^{1-b} \end{aligned}$$

(a) $P_{0,0.25} = \left[\frac{u^{5/2}}{16} (105u^3 - 385u^2 + 495u - 231) \right]_1^{0.75} \approx 0.025 = 2.5\%$

(b) $P_{0.5,1} = \left[\frac{u^{5/2}}{16} (105u^3 - 385u^2 + 495u - 231) \right]_{0.5}^0 \approx 0.736 = 73.6\%$

$$80. \int_0^2 1.75 \sin \frac{\pi t}{2} dt = -\frac{2}{\pi} \left[1.75 \cos \frac{\pi t}{2} \right]_0^2 = -\frac{2}{\pi} (1.75)(-1 - 1) = \frac{7}{\pi} \approx 2.2282 \text{ liters}$$

Increase is

$$\frac{7}{\pi} - \frac{5.1}{\pi} = \frac{1.9}{\pi} \approx 0.6048 \text{ liters.}$$

$$82. \text{ Trapezoidal Rule } (n = 4): \int_0^1 \frac{x^{3/2}}{3 - x^2} dx \approx \frac{1}{8} \left[0 + \frac{2(1/4)^{3/2}}{3 - (1/4)^2} + \frac{2(1/2)^{3/2}}{3 - (1/2)^2} + \frac{2(3/4)^{3/2}}{3 - (3/4)^2} + \frac{1}{2} \right] \approx 0.172$$

$$\text{Simpson's Rule } (n = 4): \int_0^1 \frac{x^{3/2}}{3 - x^2} dx \approx \frac{1}{12} \left[0 + \frac{4(1/4)^{3/2}}{3 - (1/4)^2} + \frac{2(1/2)^{3/2}}{3 - (1/2)^2} + \frac{4(3/4)^{3/2}}{3 - (3/4)^2} + \frac{1}{2} \right] \approx 0.166$$

Graphing utility: 0.166

$$84. \text{ Trapezoidal Rule } (n = 4): \int_0^{\pi} \sqrt{1 + \sin^2 x} dx \approx 3.820$$

Simpson's Rule ($n = 4$): 3.820

Graphing utility: 3.820

Problem Solving for Chapter 4

$$2. (a) F(x) = \int_2^x \sin t^2 dt$$

x	0	1.0	1.5	1.9	2.0	2.1	2.5	3.0	4.0	5.0
$F(x)$	-0.8048	-0.4945	-0.0265	0.0611	0	-0.0867	-0.3743	-0.0312	-0.0576	-0.2769

$$(b) G(x) = \frac{1}{x-2} \int_2^x \sin t^2 dt$$

x	1.9	1.95	1.99	2.01	2.05	2.1
$G(x)$	-0.6106	-0.6873	-0.7436	-0.7697	-0.8174	-0.8671

$$\lim_{x \rightarrow 2} G(x) \approx -0.75$$

$$\begin{aligned} (c) F'(2) &= \lim_{x \rightarrow 2} \frac{F(x) - F(2)}{x - 2} \\ &= \lim_{x \rightarrow 2} \frac{1}{x - 2} \int_2^x \sin t^2 dt \\ &= \lim_{x \rightarrow 2} G(x) \end{aligned}$$

$$\text{Since } F'(x) = \sin x^2, F'(2) = \sin 4 = \lim_{x \rightarrow 2} G(x).$$

(Note: $\sin 4 \approx -0.7568$)

4. Let d be the distance traversed and a be the uniform acceleration.

We can assume that $v(0) = 0$ and $s(0) = 0$. Then

$$a(t) = a$$

$$v(t) = at$$

$$s(t) = \frac{1}{2}at^2.$$

$$s(t) = d \text{ when } t = \sqrt{\frac{2d}{a}}.$$

$$\text{The highest speed is } v = a\sqrt{\frac{2d}{a}} = \sqrt{2ad}.$$

The lowest speed is $v = 0$.

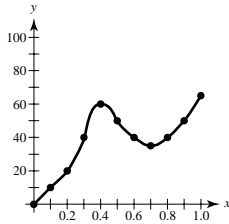
$$\text{The mean speed is } \frac{1}{2}(\sqrt{2ad} + 0) = \sqrt{\frac{ad}{2}}.$$

The time necessary to traverse the distance d at the mean speed is

$$t = \frac{d}{\sqrt{ad/2}} = \sqrt{\frac{2d}{a}}$$

which is the same as the time calculated above.

6. (a)



- (b) v is increasing (positive acceleration) on $(0, 0.4)$ and $(0.7, 1.0)$.

(c) Average acceleration = $\frac{v(0.4) - v(0)}{0.4 - 0} = \frac{60 - 0}{0.4} = 150 \text{ mi/hr}^2$

- (d) This integral is the total distance traveled in miles.

$$\int_0^1 v(t) dt \approx \frac{1}{10}[0 + 2(20) + 2(60) + 2(40) + 2(40) + 65] = \frac{385}{10} = 38.5 \text{ miles}$$

- (e) One approximation is

$$a(0.8) \approx \frac{v(0.9) - v(0.8)}{0.9 - 0.8} = \frac{50 - 40}{0.1} = 100 \text{ mi/hr}^2$$

(other answers possible)

$$8. \int_0^x f(t)(x-t) dt = \int_0^x xf(t) dt - \int_0^x tf(t) dt = x \int_0^x f(t) dt - \int_0^x tf(t) dt$$

$$\text{Thus, } \frac{d}{dx} \int_0^x f(t)(x-t) dt = xf(x) + \int_0^x f(t) dt - xf(x) = \int_0^x f(t) dt$$

Differentiating the other integral,

$$\frac{d}{dx} \int_0^x \left(\int_0^x f(v) dv \right) dt = \int_0^x f(v) dv.$$

Thus, the two original integrals have equal derivatives,

$$\int_0^x f(t)(x-t) dt = \int_0^x \left(\int_0^t f(v) dv \right) dt + C$$

Letting $x = 0$, we see that $C = 0$.

10. Consider $\int_0^1 \sqrt{x} dx = \frac{2}{3}x^{3/2} \Big|_0^1 = \frac{2}{3}$. The corresponding

Riemann Sum using right-hand endpoints is

$$\begin{aligned} S(n) &= \frac{1}{n} \left[\sqrt{\frac{1}{n}} + \sqrt{\frac{2}{n}} + \cdots + \sqrt{\frac{n}{n}} \right] \\ &= \frac{1}{n^{3/2}} [\sqrt{1} + \sqrt{2} + \cdots + \sqrt{n}] \end{aligned}$$

$$\text{Thus, } \lim_{n \rightarrow \infty} \frac{\sqrt{1} + \sqrt{2} + \cdots + \sqrt{n}}{n^{3/2}} = \frac{2}{3}.$$

$$\begin{aligned} 12. \text{ (a) Area} &= \int_{-3}^3 (9 - x^2) dx = 2 \int_0^3 (9 - x^2) dx \\ &= 2 \left[9x - \frac{x^3}{3} \right]_0^3 \\ &= 2[27 - 9] = 36 \end{aligned}$$

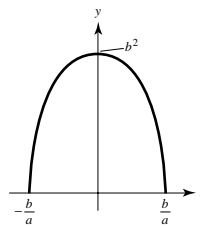
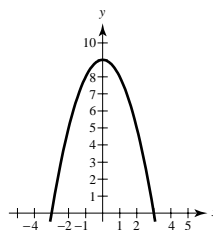
$$\text{(b) Base} = 6, \text{ height} = 9. \text{ Area} = \frac{2}{3}bh = \frac{2}{3}(6)(9) = 36.$$

(c) Let the parabola be given by $y = b^2 - a^2x^2$, $a, b > 0$.

$$\begin{aligned} \text{Area} &= 2 \int_0^{b/a} (b^2 - a^2x^2) dx \\ &= 2 \left[b^2x - a^2 \frac{x^3}{3} \right]_0^{b/a} \\ &= 2 \left[b^2 \left(\frac{b}{a} \right) - \frac{a^2}{3} \left(\frac{b}{a} \right)^3 \right] \\ &= 2 \left[\frac{b^3}{a} - \frac{1}{3} \frac{b^3}{a} \right] = \frac{4}{3} \frac{b^3}{a} \end{aligned}$$

$$\text{Base} = \frac{2b}{a}, \text{ height} = b^2$$

$$\text{Archimedes' Formula: Area} = \frac{2}{3} \left(\frac{2b}{a} \right) (b^2) = \frac{4}{3} \frac{b^3}{a}$$



14. (a) $(1 + i)^3 = 1 + 3i + 3i^2 + i^3 \Rightarrow (1 + i)^3 - i^3 = 3i^2 + 3i + 1$

(b) $3i^2 + 3i + 1 = (i + 1)^3 - i^3$

$$\begin{aligned} \sum_{i=1}^n (3i^2 + 3i + 1) &= \sum_{i=1}^n [(i + 1)^3 - i^3] \\ &= (2^3 - 1^3) + (3^3 - 2^3) + \cdots + [(n + 1)^3 - n^3] \\ &= (n + 1)^3 - 1 \end{aligned}$$

Hence, $(n + 1)^3 = \sum_{i=1}^n (3i^2 + 3i + 1) + 1$

(c) $(n + 1)^3 - 1 = \sum_{i=1}^n (3i^2 + 3i + 1) = \sum_{i=1}^n 3i^2 + \frac{3(n)(n + 1)}{2} + n$

$$\begin{aligned} \Rightarrow \sum_{i=1}^n 3i^2 &= n^3 + 3n^2 + 3n - \frac{3n(n + 1)}{2} - n \\ &= \frac{2n^3 + 6n^2 + 6n - 3n^2 - 3n - 2n}{2} \\ &= \frac{2n^3 + 3n^2 + n}{2} \\ &= \frac{n(n + 1)(2n + 1)}{2} \\ \Rightarrow \sum_{i=1}^n i^2 &= \frac{n(n + 1)(2n + 1)}{6} \end{aligned}$$

16. (a) $C = 0.1 \int_8^{20} \left[12 \sin \frac{\pi(t - 8)}{12} \right] dt = \left[-\frac{14.4}{\pi} \cos \frac{\pi(t - 8)}{12} \right]_8^{20} = \frac{-14.4}{\pi} (-1 - 1) \approx \9.17

(b) $C = 0.1 \int_{10}^{18} \left[12 \sin \frac{\pi(t - 8)}{12} - 6 \right] dt = \left[-\frac{14.4}{\pi} \cos \frac{\pi(t - 8)}{12} - 0.6t \right]_{10}^{18}$
 $= \left[\frac{-14.4}{\pi} \left(\frac{-\sqrt{3}}{2} \right) - 10.8 \right] - \left[\frac{-14.4}{\pi} \left(\frac{\sqrt{3}}{2} \right) - 6 \right] \approx \3.14

Savings $\approx 9.17 - 3.14 = \$6.03$.

18. (a) Let $A = \int_0^b \frac{f(x)}{f(x) + f(b - x)} dx$.

Let $u = b - x, du = -dx$.

$$\begin{aligned} A &= \int_b^0 \frac{f(b - u)}{f(b - u) + f(u)} (-du) \\ &= \int_0^b \frac{f(b - u)}{f(b - u) + f(u)} du \\ &= \int_0^b \frac{f(b - x)}{f(b - x) + f(x)} dx \end{aligned}$$

Then,

$$\begin{aligned} 2A &= \int_0^b \frac{f(x)}{f(x) + f(b - x)} dx + \int_0^b \frac{f(b - x)}{f(b - x) + f(x)} dx \\ &= \int_0^b 1 dx = b. \end{aligned}$$

Thus, $A = \frac{b}{2}$.

(b) $b = 1 \Rightarrow \int_0^1 \frac{\sin x}{\sin(1 - x) + \sin x} dx = \frac{1}{2}$