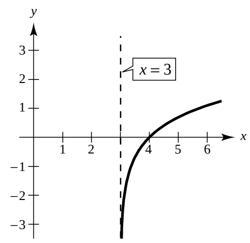
Review Exercises for Chapter 5

2. $f(x) = \ln(x - 3)$

Horizontal shift 3 units to the right

Vertical asymptote: $x = 3$



4. $\ln[(x^2 + 1)(x - 1)] = \ln(x^2 + 1) + \ln(x - 1)$

6. $3[\ln x - 2\ln(x^2 + 1)] + 2\ln 5 = 3\ln x - 6\ln(x^2 + 1) + \ln 5^2 = \ln x^3 - \ln(x^2 + 1)^6 + \ln 25 = \ln\left[\dfrac{25x^3}{(x^2 + 1)^6}\right]$

8.
$$\ln x + \ln(x - 3) = 0$$
$$\ln x(x - 3) = 0$$
$$x(x - 3) = e^0$$
$$x^2 - 3x - 1 = 0$$
$$x = \frac{3 \pm \sqrt{13}}{2}$$
$$x = \frac{3 + \sqrt{13}}{2} \text{ only since } \frac{3 - \sqrt{13}}{2} < 0.$$

10. $h(x) = \ln\dfrac{x(x - 1)}{x - 2} = \ln x + \ln(x - 1) - \ln(x - 2)$

$$h'(x) = \frac{1}{x} + \frac{1}{x - 1} - \frac{1}{x - 2} = \frac{x^2 - 4x + 2}{x^3 - 3x^2 + 2x}$$

12. $f(x) = \ln[x(x^2 - 2)^{2/3}] = \ln x + \dfrac{2}{3}\ln(x^2 - 2)$

$$f'(x) = \frac{1}{x} + \frac{2}{3}\left(\frac{2x}{x^2 - 2}\right) = \frac{7x^2 - 6}{3x^3 - 6x}$$

14. $y = \dfrac{1}{b^2}[a + bx - a\ln(a + bx)]$

$$\frac{dy}{dx} = \frac{1}{b^2}\left(b - \frac{ab}{a + bx}\right) = \frac{x}{a + bx}$$

16.
$$y = -\frac{1}{ax} + \frac{b}{a^2}\ln\frac{a + bx}{x}$$
$$= -\frac{1}{ax} + \frac{b}{a^2}[\ln(a + bx) - \ln x]$$
$$\frac{dy}{dx} = -\frac{1}{a}\left(-\frac{1}{x^2}\right) + \frac{b}{a^2}\left[\frac{b}{a + bx} - \frac{1}{x}\right]$$
$$= \frac{1}{ax^2} + \frac{b}{a^2}\left[\frac{-a}{x(a + bx)}\right] = \frac{1}{ax^2} - \frac{b}{ax(a + bx)}$$
$$= \frac{(a + bx) - bx}{ax^2(a + bx)} = \frac{1}{x^2(a + bx)}$$

18. $u = x^2 - 1,\ du = 2x\,dx$

$$\int \frac{x}{x^2 - 1}\,dx = \frac{1}{2}\int \frac{2x}{x^2 - 1}\,dx = \frac{1}{2}\ln|x^2 - 1| + C$$

20. $u = \ln x,\ du = \dfrac{1}{x}\,dx$

$$\int \frac{\ln\sqrt{x}}{x}\,dx = \frac{1}{2}\int (\ln x)\left(\frac{1}{x}\right)dx = \frac{1}{4}(\ln x)^2 + C$$

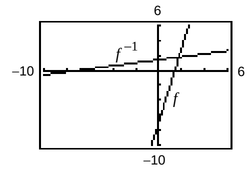
22. $\displaystyle\int_1^e \frac{\ln x}{x}\,dx = \int_1^e (\ln x)^1\left(\frac{1}{x}\right)dx = \left[\frac{1}{2}(\ln x)^2\right]_1^e = \frac{1}{2}$

24.
$$\int_0^{\pi/4} \tan\left(\frac{\pi}{4} - x\right)dx = \left[\ln\left|\cos\left(\frac{\pi}{4} - x\right)\right|\right]_0^{\pi/4}$$
$$= 0 - \ln\left(\frac{1}{\sqrt{2}}\right) = \frac{1}{2}\ln 2$$

26. (a)

$$f(x) = 5x - 7$$
$$y = 5x - 7$$
$$\frac{y+7}{5} = x$$
$$\frac{x+7}{5} = y$$
$$f^{-1}(x) = \frac{x+7}{5}$$

(b)



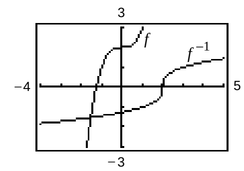
(c) $f^{-1}(f(x)) = f^{-1}(5x - 7) = \dfrac{(5x-7)+7}{5} = x$

$$f(f^{-1}(x)) = f\left(\frac{x+7}{5}\right) = 5\left(\frac{x+7}{5}\right) - 7 = x$$

28. (a)

$$f(x) = x^3 + 2$$
$$y = x^3 + 2$$
$$\sqrt[3]{y-2} = x$$
$$\sqrt[3]{x-2} = y$$
$$f^{-1}(x) = \sqrt[3]{x-2}$$

(b)



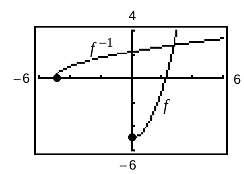
(c) $f^{-1}(f(x)) = f^{-1}(x^3 + 2) = \sqrt[3]{(x^3+2)-2} = x$

$$f(f^{-1}(x)) = f\left(\sqrt[3]{x-2}\right) = \left(\sqrt[3]{x-2}\right)^3 + 2 = x$$

30. (a)

$$f(x) = x^2 - 5, x \geq 0$$
$$y = x^2 - 5$$
$$\sqrt{y+5} = x$$
$$\sqrt{x+5} = y$$
$$f^{-1}(x) = \sqrt{x+5}$$

(b)



(c) $f^{-1}(f(x)) = f^{-1}(x^2 - 5) = \sqrt{(x^2-5)+5} = x$ for $x \geq 0$.

$$f(f^{-1}(x)) = f\left(\sqrt{x+5}\right) = \left(\sqrt{x+5}\right)^2 - 5 = x$$

32.

$$f(x) = x\sqrt{x-3}$$
$$f(4) = 4$$
$$f'(x) = \sqrt{x-3} + \frac{1}{2}x(x-3)^{-1/2}$$
$$f'(4) = 1 + 2 = 3$$
$$(f^{-1})'(4) = \frac{1}{f'(4)} = \frac{1}{3}$$

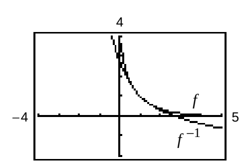
34.

$$f(x) = \ln x$$
$$f^{-1}(x) = e^x$$
$$(f^{-1})'(x) = e^x$$
$$(f^{-1})'(0) = e^0 = 1$$

36. (a)

$$f(x) = e^{1-x}$$
$$y = e^{1-x}$$
$$\ln y = 1 - x$$
$$x = 1 - \ln y$$
$$y = 1 - \ln x$$
$$f^{-1}(x) = 1 - \ln x$$

(b)

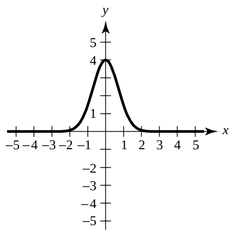


(c) $f^{-1}(f(x)) = f^{-1}\left(e^{1-x}\right) = 1 - \ln(e^{1-x})$

$$= 1 - (1 - x) = x$$

$$f(f^{-1}(x)) = f(1 - \ln x) = e^{1-(1-\ln x)} = e^{\ln x} = x$$

38. $y = 4e^{-x^2}$



40. $g(x) = \ln\left(\dfrac{e^x}{1 + e^x}\right)$

$$= \ln e^x - \ln(1 + e^x) = x - \ln(1 + e^x)$$

$$g'(x) = 1 - \frac{e^x}{1 + e^x} = \frac{1}{1 + e^x}$$

42. $h(z) = e^{-z^2/2}$

$$h'(z) = -ze^{-z^2/2}$$

44. $y = 3e^{-3/t}$

$$y' = 3e^{-3/t}(3t^{-2}) = \frac{9e^{-3/t}}{t^2}$$

46. $f(\theta) = \dfrac{1}{2}e^{\sin 2\theta}$

$$f'(\theta) = \cos 2\theta\, e^{\sin 2\theta}$$

48. $\cos x^2 = xe^y$

$$-2x \sin x^2 = xe^y \frac{dy}{dx} + e^y$$

$$\frac{dy}{dx} = -\frac{2x \sin x^2 + e^y}{xe^y}$$

50. Let $u = \dfrac{1}{x}$, $du = \dfrac{-1}{x^2}\,dx$.

$$\int \frac{e^{1/x}}{x^2}\,dx = -\int e^{1/x}\left(-\frac{1}{x^2}\right)dx = -e^{1/x} + C$$

52. Let $u = e^{2x} + e^{-2x}$, $du = (2e^{2x} - e^{-2x})\,dx$.

$$\int \frac{e^{2x} - e^{-2x}}{e^{2x} + e^{-2x}}\,dx = \frac{1}{2}\int \frac{2e^{2x} - 2e^{-2x}}{e^{2x} + e^{-2x}}\,dx$$

$$= \frac{1}{2}\ln(e^{2x} + e^{-2x}) + C$$

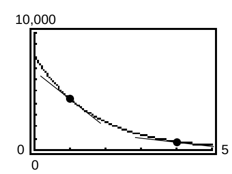
54. Let $u = x^3 + 1$, $du = 3x^2\,dx$.

$$\int x^2 e^{x^3+1}\,dx = \frac{1}{3}\int e^{x^3+1}(3x^2)\,dx = \frac{1}{3}e^{x^3+1} + C$$

56. $\displaystyle\int \frac{e^{2x}}{e^{2x} + 1}\,dx = \frac{1}{2}\int \frac{1}{e^{2x} + 1}2e^{2x}\,dx$

$$= \frac{1}{2}\ln(e^{2x} + 1) + C$$

58. (a), (c)



(b) $V = 8000e^{-0.6t},\ 0 \le t \le 5$

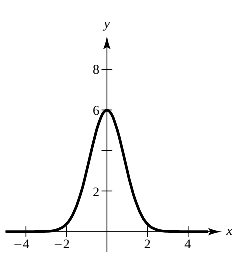
$$V'(t) = -4800e^{-0.6t}$$

$V'(1) = -2634.3$ dollars/year

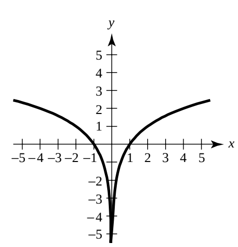
$V'(4) = -435.4$ dollars/year

60. $\displaystyle Area = \int_0^2 2e^{-x}\,dx = \Big[-2e^{-x}\Big]_0^2 = -2e^{-2} + 2 = 2 - \frac{2}{e^2} \approx 1.729$

62. $g(x) = 6(2^{-x^2})$



64. $y = \log_4 x^2$



66. $f(x) = 4^x e^x$

$f'(x) = 4^x e^x + (\ln 4)4^x e^x = 4^x e^x(1 + \ln 4)$

68. $y = x(4^{-x})$

$y' = 4^{-x} - x \cdot 4^{-x} \ln 4$

70. $h(x) = \log_5 \dfrac{x}{x-1} = \log_5 x - \log_5(x-1)$

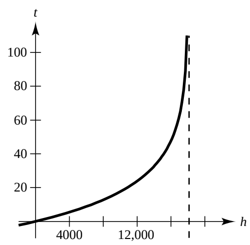
$$h'(x) = \frac{1}{\ln 5}\left[\frac{1}{x} - \frac{1}{x-1}\right] = \frac{1}{\ln 5}\left[\frac{-1}{x(x-1)}\right]$$

72. $\displaystyle\int \frac{2^{-1/t}}{t^2}\,dt = \frac{1}{\ln 2}2^{-1/t} + C$

74. $t = 50 \log_{10}\left(\dfrac{18{,}000}{18{,}000 - h}\right)$

(a) Domain: $0 \le h < 18{,}000$

(b)



Vertical asymptote: $h = 18{,}000$

(c) $t = 50 \log_{10}\left(\dfrac{18{,}000}{18{,}000 - h}\right)$

$$10^{t/50} = \frac{18{,}000}{18{,}000 - h}$$

$$18{,}000 - h = 18{,}000(10^{-t/50})$$

$$h = 18{,}000(1 - 10^{-t/50})$$

As $h \to 18{,}000$, $t \to \infty$.

(d) $t = 50 \log_{10} 18{,}000 - 50 \log_{10}(18{,}000 - h)$

$$\frac{dt}{dh} = \frac{50}{(\ln 10)(18{,}000 - h)}$$

$$\frac{d^2t}{dh^2} = \frac{50}{(\ln 10)(18{,}000 - h)^2}$$

No critical numbers

As t increases, the rate of change of the altitude is increasing.

76. $2P = Pe^{10r}$

$2 = e^{10r}$

$\ln 2 = 10r$

$r = \dfrac{\ln 2}{10} \approx 6.93\%$

78. $y = 5\left(\dfrac{1}{2}\right)^{t/1620}$

$y(600) = 5\left(\dfrac{1}{2}\right)^{600/1620} \approx 3.868$ grams

80. (a) $\dfrac{dy}{ds} = -0.012y,\ s > 50$

$$\frac{-1}{0.012}\int \frac{dy}{y} = \int ds$$

$$\frac{-1}{0.012}\ln y = s + C_1$$

$$y = Ce^{-0.012s}$$

When $s = 50$, $y = 28 = Ce^{-0.012(50)} \implies C = 28e^{0.6}$

$y = 28e^{0.6 - 0.012s},\ s > 50$

(b)

Speed(s)	50	55	60	65	70
Miles per Gallon (y)	28	26.4	24.8	23.4	22.0

82. $\dfrac{dy}{dx} = \dfrac{e^{-2x}}{1 + e^{-2x}}$

$$\int dy = \int \frac{e^{-2x}}{1 + e^{-2x}}\,dx = -\frac{1}{2}\int \frac{-2e^{-2x}}{1 + e^{-2x}}\,dx$$

$$y = -\frac{1}{2}\ln(1 + e^{-2x}) + C$$

84. $y' - e^y \sin x = 0$

$$\frac{dy}{dx} = e^y \sin x$$

$$\int e^{-y}\,dy = \int \sin x\,dx$$

$$-e^{-y} = -\cos x + C_1$$

$$e^y = \frac{1}{\cos x + C} \qquad (C = -C_1)$$

$$y = \ln\left|\frac{1}{\cos x + C}\right| = -\ln|\cos x + C|$$

86. $\dfrac{dy}{dx} = \dfrac{3(x + y)}{x}$ (homogeneous differential equation)

$$3(x + y)\,dx - x\,dy = 0$$

Let $y = vx$, $dy = x\,dv + v\,dx$.

$$3(x + vx)\,dx - x(x\,dv + v\,dx) = 0$$

$$(3x + 2vx)\,dx - x^2\,dv = 0$$

$$(3 + 2v)\,dx = x\,dv$$

$$\int \frac{1}{x}\,dx = \int \frac{1}{3 + 2v}\,dv$$

$$\ln|x| = \frac{1}{2}\ln|3 + 2v| + C_1 = \ln(3 + 2v)^{1/2} + \ln C_2$$

$$x = C_2(3 + 2v)^{1/2}$$

$$x^2 = C(3 + 2v) = C\left(3 + 2\left(\frac{y}{x}\right)\right)$$

$$x^3 = C(3x + 2y) = 3Cx + 2Cy$$

$$y = \frac{x^3 - 3Cx}{2C}$$

88. $\dfrac{dv}{dt} = kv - 9.8$

(a) $$\int \frac{dv}{kv - 9.8} = \int dt$$

$$\frac{1}{k}\ln|kv - 9.8| = t + C_1$$

$$\ln|kv - 9.8| = kt + C_2$$

$$kv - 9.8 = e^{kt + C_2} = C_3e^{kt}$$

$$v = \frac{1}{k}\left[9.8 + C_3e^{kt}\right]$$

At $t = 0$, $\quad v_0 = \dfrac{1}{k}(9.8 + C_3) \implies C_3 = kv_0 - 9.8$

$$v = \frac{1}{k}[9.8 + (kv_0 - 9.8)e^{kt}]$$

Note that $k < 0$ since the object is moving downward.

(b) $\displaystyle\lim_{t\to\infty} v(t) = \frac{9.8}{k}$

(c) $$s(t) = \int \frac{1}{k}[9.8 + (kv_0 - 9.8)e^{kt}]\,dt$$

$$= \frac{1}{k}\left[9.8t + \frac{1}{k}(kv_0 - 9.8)e^{kt}\right] + C$$

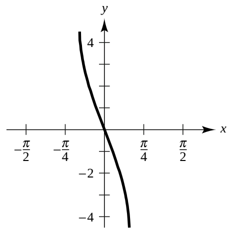
$$= \frac{9.8t}{k} + \frac{1}{k^2}(kv_0 - 9.8)e^{kt} + C$$

$$s(0) = \frac{1}{k^2}(kv_0 - 9.8) + C \implies C = s_0 - \frac{1}{k^2}(kv_0 - 9.8)$$

$$s(t) = \frac{9.8t}{k} + \frac{1}{k^2}(kv_0 - 9.8)e^{kt} + s_0 - \frac{1}{k^2}(kv_0 - 9.8)$$

$$= \frac{9.8t}{k} + \frac{1}{k^2}(kv_0 - 9.8)(e^{kt} - 1) + s_0$$

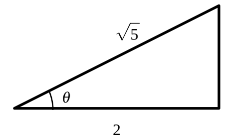
90. $h(x) = -3\arcsin(2x)$



92. (a) Let $\theta = \text{arccot } 2$

$\cot\theta = 2$

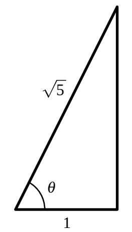
$\tan(\text{arccot } 2) = \tan\theta = \dfrac{1}{2}.$



(b) Let $\theta = \text{arcsec } \sqrt{5}$

$\sec\theta = \sqrt{5}$

$\cos\left(\text{arcsec } \sqrt{5}\right) = \cos\theta = \dfrac{1}{\sqrt{5}}.$



94. $y = \arctan(x^2 - 1)$

$$y' = \frac{2x}{1 + (x^2 - 1)^2} = \frac{2x}{x^4 - 2x^2 + 2}$$

96. $y = \dfrac{1}{2}\arctan e^{2x}$

$$y' = \frac{1}{2}\left(\frac{1}{1 + e^{4x}}\right)(2e^{2x}) = \frac{e^{2x}}{1 + e^{4x}}$$

98. $y = \sqrt{x^2 - 4} - 2\,\text{arcsec}\,\dfrac{x}{2},\ 2 < x < 4$

$$y' = \frac{x}{\sqrt{x^2 - 4}} - \frac{1}{\left(|x|/2\right)\sqrt{(x/2)^2 - 1}} = \frac{x}{\sqrt{x^2 - 4}} - \frac{4}{|x|\sqrt{x^2 - 4}} = \frac{x^2 - 4}{|x|\sqrt{x^2 - 4}} = \frac{\sqrt{x^2 - 4}}{x}$$

100. Let $u = 5x,\ du = 5\,dx$.

$$\int \frac{1}{3 + 25x^2}\,dx = \frac{1}{5}\int \frac{1}{\left(\sqrt{3}\right)^2 + (5x)^2}(5)\,dx = \frac{1}{5\sqrt{3}}\arctan\frac{5x}{\sqrt{3}} + C$$

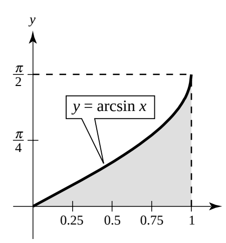
102. $$\int \frac{1}{16 + x^2}\,dx = \frac{1}{4}\arctan\frac{x}{4} + C$$

104. $$\int \frac{4 - x}{\sqrt{4 - x^2}}\,dx = 4\int \frac{1}{\sqrt{4 - x^2}}\,dx + \frac{1}{2}\int (4 - x^2)^{-1/2}(-2x)\,dx = 4\arcsin\frac{x}{2} + \sqrt{4 - x^2} + C$$

106. Let $u = \arcsin x,\ du = \dfrac{1}{\sqrt{1 - x^2}}\,dx$.

$$\int \frac{\arcsin x}{\sqrt{1 - x^2}}\,dx = \frac{1}{2}(\arcsin x)^2 + C$$

108.



Since the area of region A is $1\left(\displaystyle\int_0^{\pi/2} \sin y\,dy\right)$,

the shaded area is $\displaystyle\int_0^1 \arcsin x\,dx = \frac{\pi}{2} - 1 \approx 0.571.$

110. $y = x\tanh^{-1} 2x$

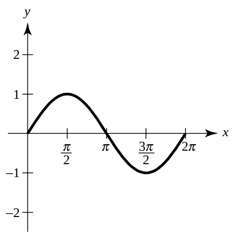
$$y' = x\left(\frac{2}{1 - 4x^2}\right) + \tanh^{-1} 2x = \frac{2x}{1 - 4x^2} + \tanh^{-1} 2x$$

112. Let $u = x^3,\ du = 3x^2\,dx$.

$$\int x^2(\text{sech } x^3)^2\,dx = \frac{1}{3}\int (\text{sech } x^3)^2(3x^2)\,dx = \frac{1}{3}\tanh x^3 + C$$

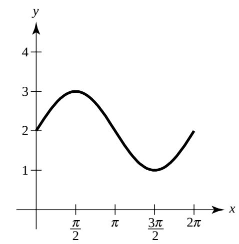
Problem Solving for Chapter 5

2. (a)



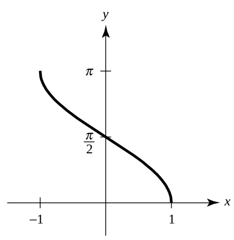
$$\int_0^{\pi} \sin x\, dx = -\int_{\pi}^{2\pi} \sin x\, dx \implies \int_0^{2\pi} \sin x\, dx = 0$$

(b)



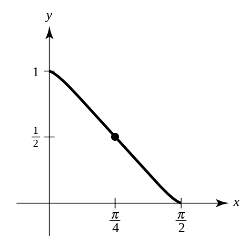
$$\int_0^{2\pi} (\sin x + 2)\, dx = 2(2\pi) = 4\pi$$

(c)



$$\int_{-1}^{1} \arccos x\, dx = 2\left(\frac{\pi}{2}\right) = \pi$$

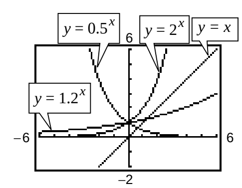
(d)



$y = \dfrac{1}{1 + (\tan x)^{\sqrt{2}}}$ is symmetric with respect to the point $\left(\dfrac{\pi}{4}, \dfrac{1}{2}\right)$. $\displaystyle\int_0^{\pi/2} \frac{1}{1 + (\tan x)^{\sqrt{2}}}\, dx = \frac{\pi}{2}\left(\frac{1}{2}\right) = \frac{\pi}{4}$

4. $y = 0.5^x$ and $y = 1.2^x$ intersect $y = x$.

$y = 2^x$ does not intersect $y = x$.



Suppose $y = x$ is tangent to $y = a^x$ at (x, y).

$a^x = x \implies a = x^{1/x}$.

$y' = a^x \ln a = 1 \implies x \ln x^{1/x} = 1 \implies \ln x = 1 \implies x = e, a = e^{1/e}$

For $0 < a \le e^{1/e} \approx 1.445$, the curve $y = a^x$ intersects $y = x$.

6. (a) $y = f(x) = \arcsin x$

$\sin y = x$

$$\text{Area } A = \int_{\pi/6}^{\pi/4} \sin y \cdot dy = -\cos y\Big]_{\pi/6}^{\pi/4} = -\frac{\sqrt{2}}{2} + \frac{\sqrt{3}}{2} = \frac{\sqrt{3} - \sqrt{2}}{2} \approx 0.1589$$

$$\text{Area } B = \left(\frac{1}{2}\right)\left(\frac{\pi}{6}\right) = \frac{\pi}{12} \approx 0.2618$$

(b)
$$\int_{1/2}^{\sqrt{2}/2} \arcsin x\, dx = \text{Area}(C) = \left(\frac{\pi}{4}\right)\left(\frac{\sqrt{2}}{2}\right) - A - B$$
$$= \frac{\pi\sqrt{2}}{8} - \frac{\sqrt{3} - \sqrt{2}}{2} - \frac{\pi}{12}$$
$$= \pi\left(\frac{\sqrt{2}}{8} - \frac{1}{12}\right) + \frac{\sqrt{2} - \sqrt{3}}{2} \approx 0.1346$$

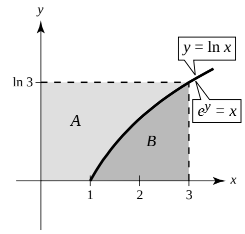
—CONTINUED—

6. —CONTINUED—

(c) $\text{Area } A = \int_0^{\ln 3} e^y\,dy$

$= e^y\Big]_0^{\ln 3} = 3 - 1 = 2$

$\text{Area } B = \int_1^3 \ln x\,dx = 3(\ln 3) - A = 3\ln 3 - 2 = \ln 27 - 2 \approx 1.2958$



(d) $\tan y = x$

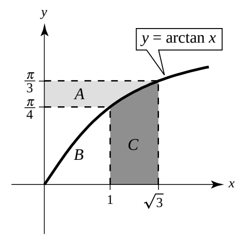
$\text{Area } A = \int_{\pi/4}^{\pi/3} \tan y\,dy$

$= -\ln|\cos y|\Big]_{\pi/4}^{\pi/3}$

$= -\ln\frac{1}{2} + \ln\frac{\sqrt{2}}{2} = \ln\sqrt{2} = \frac{1}{2}\ln 2$

$\text{Area } C = \int_1^{\sqrt{3}} \arctan x\,dx = \left(\frac{\pi}{3}\right)\left(\sqrt{3}\right) - \frac{1}{2}\ln 2 - \left(\frac{\pi}{4}\right)(1)$

$= \frac{\pi}{12}\left(4\sqrt{3} - 3\right) - \frac{1}{2}\ln 2 \approx 0.6818$



8. $y = e^x$

$y' = e^x$

$y - b = e^a(x - a)$

$y = e^a x - ae^a + b$ Tangent line

If $y = 0$,

$e^a x = ae^a - b$

$bx = ab - b \quad (b = e^a)$

$x = a - 1$

$c = a - 1$

Thus, $a - c = a - (a - 1) = 1$.

10. Let $u = \tan x$, $du = \sec^2 x\,dx$

$\text{Area} = \int_0^{\pi/4} \frac{1}{\sin^2 x + 4\cos^2 x}\,dx = \int_0^{\pi/4} \frac{\sec^2 x}{\tan^2 x + 4}\,dx$

$= \int_0^1 \frac{du}{u^2 + 4}$

$= \left[\frac{1}{2}\arctan\left(\frac{u}{2}\right)\right]_0^1$

$= \frac{1}{2}\arctan\left(\frac{1}{2}\right)$

12. (a) $\frac{dy}{dt} = y(1 - y),\ y(0) = \frac{1}{4}$

$\int\left(\frac{1}{y} + \frac{1}{1 - y}\right)dy = \int dt$

$\ln|y| - \ln|1 - y| = t + C$

$\ln\left|\frac{y}{1 - y}\right| = t + C$

$\frac{y}{1 - y} = e^{t + C} = C_1 e^t$

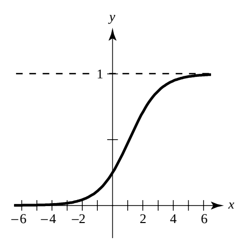
$y = C_1 e^t - yC_1 e^t$

$y = \frac{C_1 e^t}{1 + C_1 e^t} = \frac{1}{1 + C_2 e^{-t}}$

$y(0) = \frac{1}{4} = \frac{1}{1 + C_2} \implies C_2 = 3$

Hence, $y = \frac{1}{1 + 3e^{-t}}$.

(b)



$\frac{dy}{dt} = y(1 - y) = y - y^2$

$\frac{d^2y}{dt^2} = y'' = y' - 2yy' \implies y'' = 0 \text{ for } y = \frac{1}{2}$

$\frac{d^2y}{dt^2} > 0 \text{ if } 0 < y < \frac{1}{2} \text{ and } \frac{d^2y}{dt^2} < 0 \text{ if } \frac{1}{2} < y < 1.$

Thus, the rate of growth is maximum at $y = \frac{1}{2}$, the point of inflection.

—CONTINUED—

12. —CONTINUED—

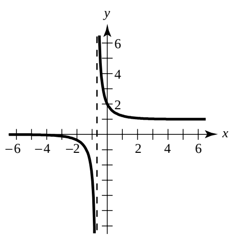
(c) $y' = y(1 - y), y(0) - 2$

As before, $y = \dfrac{1}{1 + C_2 e^{-t}}$

$y(0) = 2 = \dfrac{1}{1 + C_2} \Rightarrow C_2 = -\dfrac{1}{2}$

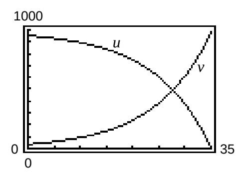
Thus, $y = \dfrac{1}{1 - \frac{1}{2}e^{-t}} = \dfrac{2}{2 - e^{-t}}$.

The graph is different:



14. (a) $u = 985.93 - \left(985.93 - \dfrac{(120{,}000)(0.095)}{12}\right)\left(1 + \dfrac{0.095}{12}\right)^{12t}$

$v = \left(985.93 - \dfrac{(120{,}000)(0.095)}{12}\right)\left(1 + \dfrac{0.095}{12}\right)^{12t}$



(b) The larger part goes for interest. The curves intersect when $t \approx 27.7$ years.

(c) The slopes are negatives of each other. Analytically,

$$u = 985.93 - v \quad \Rightarrow \frac{du}{dt} = -\frac{dv}{dt}$$

$u'(15) = -v'(15) = -14.06.$

(d) $t = 12.7$ years

Again, the larger part goes for interest.