PART II

CHAPTER 6
Applications of Integration

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Applications of Integration

Section 6.1 Area of a Region Between Two Curves

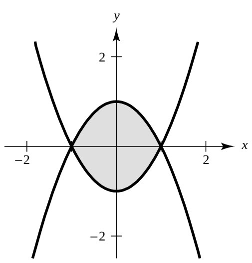
Solutions to Even-Numbered Exercises

2. $A = \int_{-2}^{2} [(2x + 5) - (x^2 + 2x + 1)]\,dx = \int_{-2}^{2} (-x^2 + 4)\,dx$

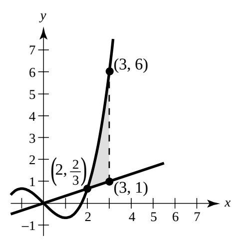
4. $A = \int_{0}^{1} (x^2 - x^3)\,dx$

6. $A = 2\int_{0}^{1} [(x - 1)^3 - (x - 1)]\,dx$

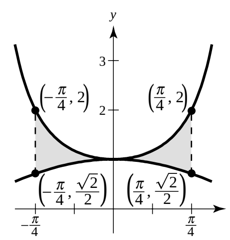
8. $\int_{-1}^{1} [(1 - x^2) - (x^2 - 1)]\,dx$



10. $\int_{2}^{3}\left[\left(\frac{x^3}{3} - x\right) - \frac{x}{3}\right]dx$



12. $\int_{-\pi/4}^{\pi/4} (\sec^2 x - \cos x)\,dx$

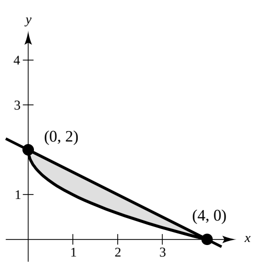


14. $f(x) = 2 - \frac{1}{2}x$

$g(x) = 2 - \sqrt{x}$

$A \approx 1$

Matches (a)

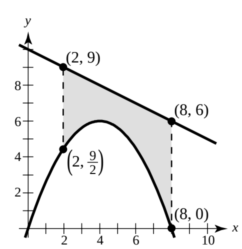


16. $A = \int_{2}^{8}\left[\left(10 - \frac{1}{2}x\right) - \left(-\frac{3}{8}x(x - 8)\right)\right]dx$

$= \int_{2}^{8}\left(\frac{3}{8}x^2 - \frac{7}{2}x + 10\right)dx$

$= \left[\frac{x^3}{8} - \frac{7x^2}{4} + 10x\right]_2^8$

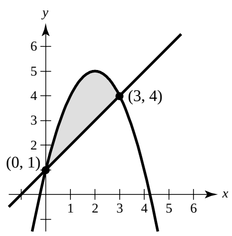
$= (64 - 112 + 80) - (1 - 7 + 20) = 18$



18. The points of intersection are given by

$$-x^2 + 4x + 1 = x + 1$$
$$-x^2 + 3x = 0$$
$$x^2 = 3x \text{ when } x = 0, 3$$

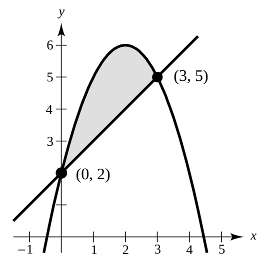
$$A = \int_0^3 [(-x^2 + 4x + 1) - (x + 1)]\,dx$$
$$= \int_0^3 (-x^2 + 3x)\,dx$$
$$= \left[-\frac{x^3}{3} + \frac{3x^2}{2}\right]_0^3$$
$$= -9 + \frac{27}{2} = \frac{9}{2}$$



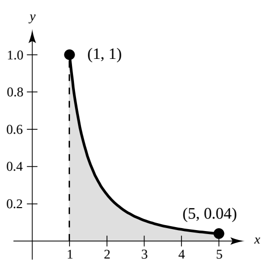
20. The points of intersection are given by:

$$-x^2 + 4x + 2 = x + 2$$
$$x(3 - x) = 0 \quad \text{when} \quad x = 0, 3$$

$$A = \int_0^3 [f(x) - g(x)]\,dx$$
$$= \int_0^3 [(-x^2 + 4x + 2) - (x + 2)]\,dx$$
$$= \int_0^3 (-x^2 + 3x)\,dx = \left[\frac{-x^3}{3} + \frac{3}{2}x^2\right]_0^3 = \frac{9}{2}$$



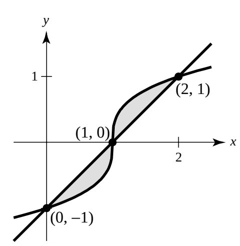
22. $A = \int_1^5 \left(\frac{1}{x^2} - 0\right) dx = \left[-\frac{1}{x}\right]_1^5 = \frac{4}{5}$



24. The points of intersection are given by

$$\sqrt[3]{x - 1} = x - 1$$
$$x - 1 = (x - 1)^3 = x^3 - 3x^2 + 3x - 1$$
$$x^3 - 3x^2 + 2x = 0$$
$$x(x^2 - 3x + 2) = 0$$
$$x(x - 2)(x - 1) = 0 \implies x = 0, 1, 2$$

$$A = 2\int_0^1 \left[(x - 1) - \sqrt[3]{x - 1}\right] dx$$
$$= 2\left[\frac{x^2}{2} - x - \frac{3}{4}(x - 1)^{4/3}\right]_0^1$$
$$= 2\left[\left(\frac{1}{2} - 1 - 0\right) - \left(-\frac{3}{4}\right)\right] = \frac{1}{2}$$



26. The points of intersection are given by:

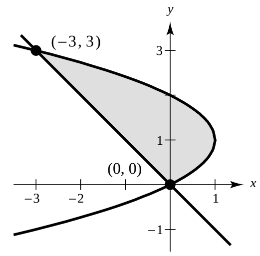
$$2y - y^2 = -y$$

$$y(y-3) = 0 \quad \text{when} \quad y = 0, 3$$

$$A = \int_0^3 [f(y) - g(y)]\,dy$$

$$= \int_0^3 [(2y - y^2) - (-y)]\,dy$$

$$= \int_0^3 (3y - y^2)\,dy = \left[\frac{3}{2}y^2 - \frac{1}{3}y^3\right]_0^3 = \frac{9}{2}$$

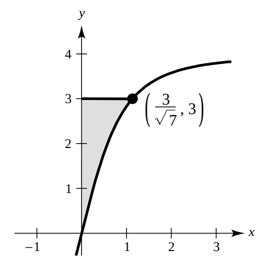


28. $$A = \int_0^3 [f(y) - g(y)]\,dy$$

$$= \int_0^3 \left[\frac{y}{\sqrt{16 - y^2}} - 0\right] dy$$

$$= -\frac{1}{2}\int_0^3 (16 - y^2)^{-1/2}(-2y)\,dy$$

$$= \left[-\sqrt{16 - y^2}\right]_0^3 = 4 - \sqrt{7} \approx 1.354$$

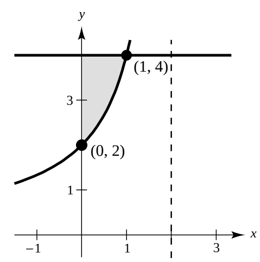


30. $$A = \int_0^1 \left(4 - \frac{4}{2 - x}\right) dx$$

$$= \left[4x + 4\ln|2 - x|\right]_0^1$$

$$= 4 - 4\ln 2$$

$$\approx 1.227$$



32. The point of intersection is given by:

$$x^3 - 2x + 1 = -2x$$

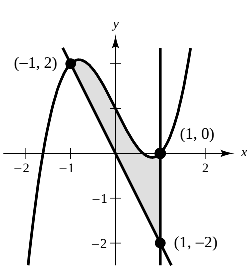
$$x^3 + 1 = 0 \quad \text{when} \quad x = -1$$

$$A = \int_{-1}^1 [f(x) - g(x)]\,dx$$

$$= \int_{-1}^1 [(x^3 - 2x + 1) - (-2x)]\,dx$$

$$= \int_{-1}^1 (x^3 + 1)\,dx = \left[\frac{x^4}{4} + x\right]_{-1}^1 = 2$$

Numerical Approximation: 2.0



34. The points of intersection are given by:

$$x^4 - 2x^2 = 2x^2$$

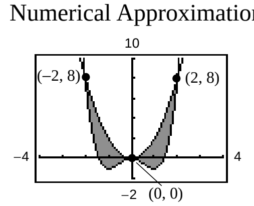
$$x^2(x^2 - 4) = 0 \quad \text{when} \quad x = 0, \pm 2$$

$$A = 2\int_0^2 [2x^2 - (x^4 - 2x^2)]\,dx$$

$$= 2\int_0^2 (4x^2 - x^4)\,dx$$

$$= 2\left[\frac{4x^3}{3} - \frac{x^5}{5}\right]_0^2 = \frac{128}{15}$$

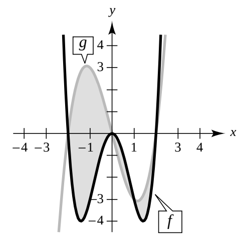
Numerical Approximation: 8.533



36. $f(x) = x^4 - 4x^2,\quad g(x) = x^3 - 4x$

The points of intersection are given by:

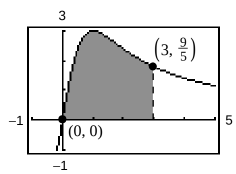
$$x^4 - 4x^2 = x^3 - 4x$$
$$x^4 - x^3 - 4x^2 + 4x = 0$$
$$x(x-1)(x+2)(x-2) = 0 \quad \text{when} \quad x = -2, 0, 1, 2$$



$$A = \int_{-2}^{0} [(x^3 - 4x) - (x^4 - 4x^2)]dx + \int_{0}^{1} [(x^4 - 4x^2) - (x^3 - 4x)]dx + \int_{1}^{2} [(x^3 - 4x) - (x^4 - 4x^2)]dx$$
$$= \frac{248}{30} + \frac{37}{60} + \frac{53}{60} = \frac{293}{30}$$

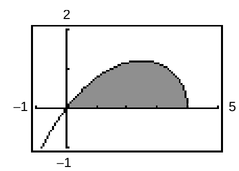
Numerical Approximation: $8.267 + 0.617 + 0.883 \approx 9.767$

38. $A = \int_0^3 \left[\frac{6x}{x^2+1} - 0\right] dx$
$$= \left[3\ln(x^2+1)\right]_0^3$$
$$= 3\ln 10$$
$$\approx 6.908$$

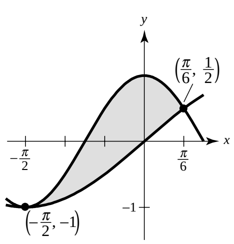


Numerical Approximation: 6.908

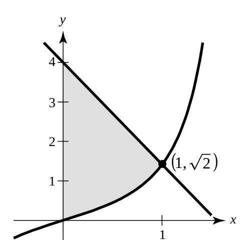
40. $A = \int_0^4 x\sqrt{\frac{4-x}{4+x}}\, dx \approx 3.434$



42. $A = \int_{-\pi/2}^{\pi/6} (\cos 2x - \sin x)\, dx$
$$= \left[\frac{1}{2}\sin 2x + \cos x\right]_{-\pi/2}^{\pi/6}$$
$$= \left(\frac{\sqrt{3}}{4} + \frac{\sqrt{3}}{2}\right) - (0) = \frac{3\sqrt{3}}{4} \approx 1.299$$

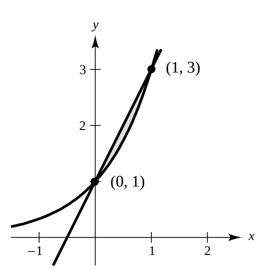


44. $A = \int_0^1 \left[\left(\sqrt{2} - 4\right)x + 4 - \sec\frac{\pi x}{4}\tan\frac{\pi x}{4}\right] dx$
$$= \left[\frac{\sqrt{2}-4}{2}x^2 + 4x - \frac{4}{\pi}\sec\frac{\pi x}{4}\right]_0^1$$
$$= \left(\frac{\sqrt{2}-4}{2} + 4 - \frac{4}{\pi}\sqrt{2}\right) - \left(-\frac{4}{\pi}\right)$$
$$= \frac{\sqrt{2}}{2} + 2 + \frac{4}{\pi}\left(1 - \sqrt{2}\right) \approx 2.1797$$

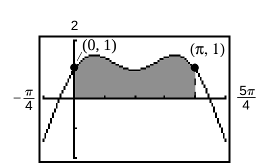


46. From the graph we see that f and g intersect twice at $x = 0$ and $x = 1$.

$$A = \int_0^1 [g(x) - f(x)]\, dx$$
$$= \int_0^1 [(2x+1) - 3^x]\, dx$$
$$= \left[x^2 + x - \frac{1}{\ln 3}(3^x)\right]_0^1$$
$$= 2\left(1 - \frac{1}{\ln 3}\right) \approx 0.180$$

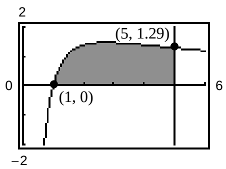


48. $A = \int_0^\pi [(2\sin x + \cos 2x) - 0]\, dx$
$$= \left[-2\cos x + \frac{1}{2}\sin 2x\right]_0^\pi = 4$$



50. $A = \int_1^5 \left[\frac{4 \ln x}{x} - 0\right] dx$

$= \left[2(\ln x)^2\right]_1^5 = 2(\ln 5)^2 \approx 5.181$

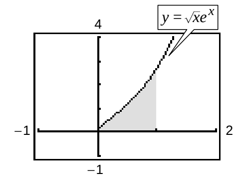


52. (a) $y = \sqrt{x}\, e^x,\ y = 0,\ x = 0,\ x = 1$

(b) $A = \int_0^1 \sqrt{x}\, e^x \, dx.$

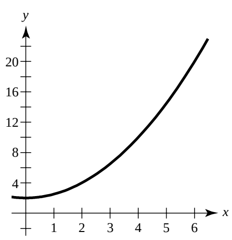
No, it cannot be evaluated by hand.

(c) 1.2556

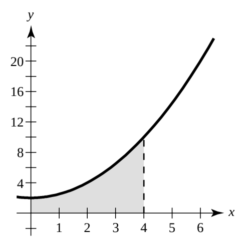


54. $F(x) = \int_0^x \left(\frac{1}{2}t^2 + 2\right) dt = \left[\frac{1}{6}t^3 + 2t\right]_0^x = \frac{x^3}{6} + 2x$

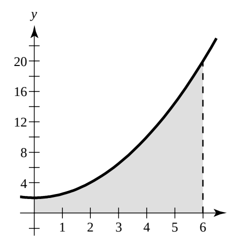
(a) $F(0) = 0$



(b) $F(4) = \frac{4^3}{6} + 2(4) = \frac{56}{3}$

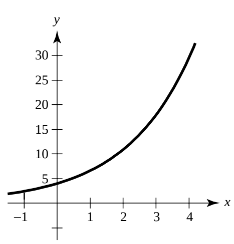


(c) $F(6) = 36 + 12 = 48$

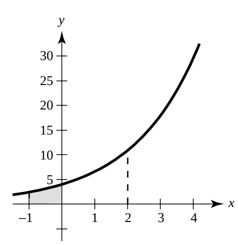


56. $F(y) = \int_{-1}^{y} 4e^{x/2} \, dx = \left[8e^{x/2}\right]_{-1}^{y} = 8e^{y/2} - 8e^{-1/2}$

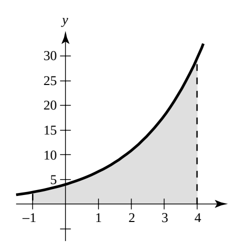
(a) $F(-1) = 0$



(b) $F(0) = 8 - 8e^{-1/2} \approx 3.1478$



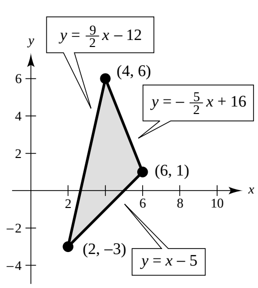
(c) $F(4) = 8e^2 - 8e^{-1/2} \approx 54.2602$



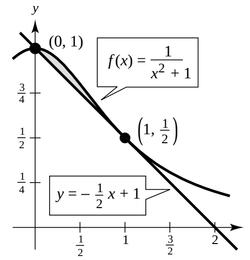
58. $A = \int_2^4 \left[\left(\frac{9}{2}x - 12\right) - (x - 5)\right] dx + \int_4^6 \left[\left(-\frac{5}{2}x + 16\right) - (x - 5)\right] dx$

$= \int_2^4 \left(\frac{7}{2}x - 7\right) dx + \int_4^6 \left(-\frac{7}{2}x + 21\right) dx$

$= \left[\frac{7}{4}x^2 - 7x\right]_2^4 + \left[-\frac{7}{4}x^2 + 21x\right]_4^6 = 7 + 7 = 14$



60. $f(x) = \dfrac{1}{x^2 + 1}$



$f'(x) = -\dfrac{2x}{(x^2 + 1)^2}$

At $\left(1, \dfrac{1}{2}\right)$, $f'(1) = -\dfrac{1}{2}$.

Tangent line:

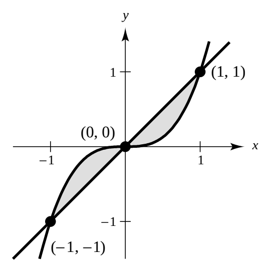
$$y - \frac{1}{2} = -\frac{1}{2}(x - 1) \text{ or } y = -\frac{1}{2}x + 1$$

The tangent line intersects $f(x) = \dfrac{1}{x^2 + 1}$ at $x = 0$.

$$A = \int_0^1 \left[\frac{1}{x^2 + 1} - \left(-\frac{1}{2}x + 1\right)\right] dx = \left[\arctan x + \frac{x^2}{4} - x\right]_0^1 = \frac{\pi - 3}{4} \approx 0.0354$$

62. Answers will vary. See page 417.

64. $x^3 \geq x$ on $[-1, 0]$



$x^3 \leq x$ on $[0, 1]$

Both functions symmetric to origin

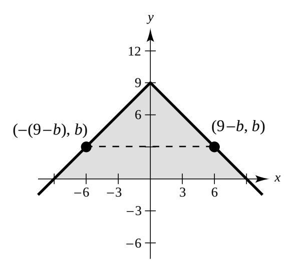
$$\int_{-1}^{0} (x^3 - x)\, dx = -\int_0^1 (x^3 - x)\, dx.$$

Thus, $\displaystyle\int_{-1}^{1} (x^3 - x)\, dx = 0$.

$$A = 2\int_0^1 (x - x^3)\, dx = 2\left[\frac{x^2}{2} - \frac{x^4}{4}\right]_0^1 = \frac{1}{2}$$

66. Proposal 2 is better, since the cummulative deficit (the area under the curve) is less.

68. $A = 2\displaystyle\int_0^9 (9 - x)\, dx = 2\left[9x - \frac{x^2}{2}\right]_0^9 = 81$



$$2\int_0^{9-b} [(9 - x) - b]\, dx = \frac{81}{2}$$

$$2\int_0^{9-b} [(9 - b) - x]\, dx = \frac{81}{2}$$

$$2\left[(9 - b)x - \frac{x^2}{2}\right]_0^{9-b} = \frac{81}{2}$$

$$(9 - b)(9 - b) = \frac{81}{2}$$

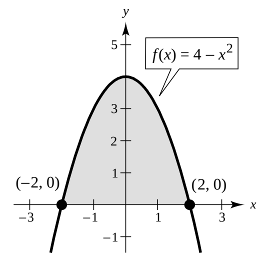
$$9 - b = \frac{9}{\sqrt{2}}$$

$$b = 9 - \frac{9}{\sqrt{2}} \approx 2.636$$

70. $\lim_{\|\Delta\| \to 0} \sum_{i=1}^{n} (4 - x_i^2)\, \Delta x$

where $x_i = -2 + \dfrac{4i}{n}$ and $\Delta x = \dfrac{4}{n}$ is the same as

$$\int_{-2}^{2} (4 - x^2)\, dx = \left[4x - \frac{x^3}{3}\right]_{-2}^{2} = \frac{32}{3}.$$



72. $\displaystyle\int_0^5 [(7.21 + 0.26t + 0.02t^2) - (7.21 + 0.1t + 0.01t^2)]\, dt = \int_0^5 (0.01t^2 + 0.16t)\, dt$

$$= \left[\frac{0.01t^3}{3} + \frac{0.16t^2}{2}\right]_0^5$$

$$= \frac{29}{12} \text{ billion} \approx \$2.417 \text{ billion}$$

74. 5% : $P_1 = 893{,}000\, e^{(0.05)t}$

$3\frac{1}{2}$%: $P_2 = 893{,}000\, e^{(0.035)t}$

Difference in profits over 5 years:

$$\int_0^5 [893{,}000e^{0.05t} - 893{,}000e^{0.035t}]\, dt = 893{,}000\left[\frac{e^{0.05t}}{0.05} - \frac{e^{0.035t}}{0.035}\right]_0^5$$

$$\approx 893{,}000[(25.6805 - 34.0356) - (20 - 28.5714)]$$

$$\approx 893{,}000(0.2163) \approx \$193{,}156$$

Note: Using a graphing utility you obtain $193,183.

76. The curves intersect at the point where the slope of y_2 equals that of y_1, 1.

$$y_2 = 0.08x^2 + k \implies y'_2 = 0.16x = 1 \implies x = \frac{1}{.16} = 6.25$$

(a) The value of k is given by

$$y_1 = y_2$$

$$6.25 = (0.08)(6.25)^2 + k$$

$$k = 3.125.$$

(b) $\displaystyle \text{Area} = 2\int_0^{6.25} (y_2 - y_1)\, dx$

$$= 2\int_0^{6.25} (0.08x^2 + 3.125 - x)\, dx$$

$$= 2\left[\frac{0.08x^3}{3} + 3.125x - \frac{x^2}{2}\right]_0^{6.25}$$

$$= 2(6.510417) \approx 13.02083$$

78. (a) $A \approx 6.031 - 2\left[\pi\left(\frac{1}{16}\right)^2\right] - 2\left[\pi\left(\frac{1}{8}\right)^2\right] \approx 5.908$

(b) $V = 2A \approx 2(5.908) \approx 11.816 \text{ m}^3$

(c) $5000V \approx 5000(11.816) = 59{,}082$ pounds

80. True

Section 6.2 Volume: The Disk Method

2. $V = \pi \int_0^2 (4 - x^2)^2 \, dx = \pi \int_0^2 (x^4 - 8x^2 + 16) \, dx = \pi \left[\frac{x^5}{5} - \frac{8x^3}{3} + 16x \right]_0^2 = \frac{256\pi}{15}$

4. $V = \pi \int_0^3 \left(\sqrt{9 - x^2}\right)^2 dx = \pi \int_0^3 (9 - x^2) \, dx$

$= \pi \left[9x - \frac{x^3}{3} \right]_0^3 = 18\pi$

6. $2 = 4 - \frac{x^2}{4}$

$8 = 16 - x^2$

$x^2 = 8$

$x = \pm 2\sqrt{2}$

$V = \pi \int_{-2\sqrt{2}}^{2\sqrt{2}} \left[\left(4 - \frac{x^2}{4}\right)^2 - (2)^2 \right] dx$

$= 2\pi \int_0^{2\sqrt{2}} \left[\frac{x^4}{16} - 2x^2 + 12 \right] dx$

$= 2\pi \left[\frac{x^5}{80} - \frac{2x^3}{3} + 12x \right]_0^{2\sqrt{2}}$

$= 2\pi \left[\frac{128\sqrt{2}}{80} - \frac{32\sqrt{2}}{3} + 24\sqrt{2} \right]$

$= \frac{448\sqrt{2}}{15}\pi \approx 132.69$

8. $y = \sqrt{16 - x^2} \implies x = \sqrt{16 - y^2}$

$V = \pi \int_0^4 \left(\sqrt{16 - y^2}\right)^2 dy = \pi \int_0^4 (16 - y^2) \, dy$

$= \pi \left[16y - \frac{y^3}{3} \right]_0^4 = \frac{128\pi}{3}$

10. $V = \pi \int_1^4 (-y^2 + 4y)^2 \, dy = \pi \int_1^4 (y^4 - 8y^3 + 16y^2) \, dy$

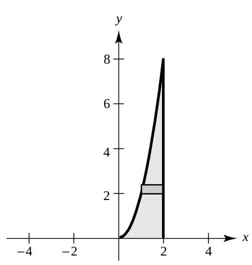
$= \pi \left[\frac{y^5}{5} - 2y^4 + \frac{16y^3}{3} \right]_1^4$

$= \frac{459\pi}{15} = \frac{153\pi}{5}$

12. $y = 2x^2,\ y = 0,\ x = 2$

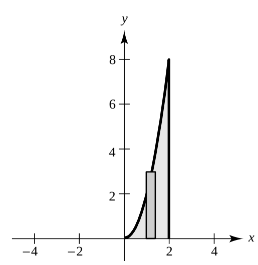
(a) $R(y) = 2,\ r(y) = \sqrt{y/2}$

$V = \pi \int_0^8 \left(4 - \frac{y}{2}\right) dy = \pi \left[4y - \frac{y^2}{4} \right]_0^8 = 16\pi$



(b) $R(x) = 2x^2,\ r(x) = 0$

$V = \pi \int_0^2 4x^4 \, dx = \pi \left[\frac{4x^5}{5} \right]_0^2 = \frac{128\pi}{5}$

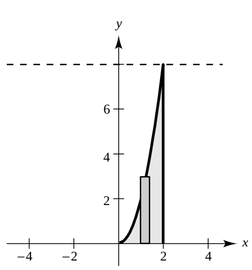


(c) $R(x) = 8,\ r(x) = 8 - 2x^2$

$V = \pi \int_0^2 [64 - (64 - 32x^2 + 4x^4)] \, dx$

$= \pi \int_0^2 (32x^2 - 4x^4) \, dx = 4\pi \int_0^2 (8x^2 - x^4) \, dx$

$= 4\pi \left[\frac{8}{3}x^3 - \frac{1}{5}x^5 \right]_0^2 = \frac{896\pi}{15}$

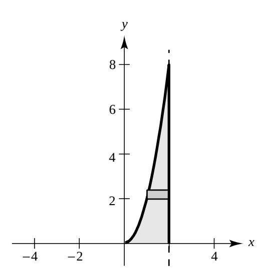


(d) $R(y) = 2 - \sqrt{y/2},\ r(y) = 0$

$V = \pi \int_0^8 \left(2 - \sqrt{\frac{y}{2}}\right)^2 dy$

$= \pi \int_0^8 \left(4 - 4\sqrt{\frac{y}{2}} + \frac{y}{2}\right) dy$

$= \pi \left[4y - \frac{4\sqrt{2}}{3} y^{3/2} + \frac{y^2}{4} \right]_0^8 = \frac{16\pi}{3}$



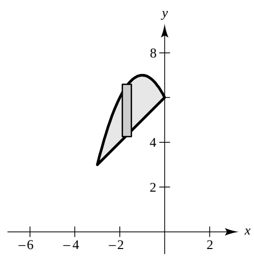
14. $y = 6 - 2x - x^2$, $y = x + 6$ intersect at $(-3, 3)$ and $(0, 6)$.

(a) $R(x) = 6 - 2x - x^2$, $r(x) = x + 6$

$$V = \pi\int_{-3}^{0}\left[(6 - 2x - x^2)^2 - (x + 6)^2\right] dx$$

$$= \pi\int_{-3}^{0}(x^4 + 4x^3 - 9x^2 - 36x)\, dx$$

$$= \pi\left[\frac{1}{5}x^5 + x^4 - 3x^3 - 18x^2\right]_{-3}^{0} = \frac{243\pi}{5}$$

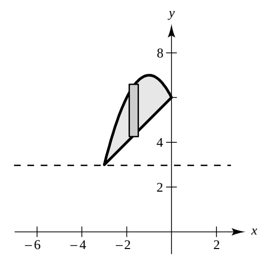


(b) $R(x) = (6 - 2x - x^2) - 3$, $r(x) = (x + 6) - 3$

$$V = \pi\int_{-3}^{0}\left[(3 - 2x - x^2)^2 - (x + 3)^2\right] dx$$

$$= \pi\int_{-3}^{0}(x^4 + 4x^3 - 3x^2 - 18x)\, dx$$

$$= \pi\left[\frac{1}{5}x^5 + x^4 - x^3 - 9x^2\right]_{-3}^{0} = \frac{108\pi}{5}$$



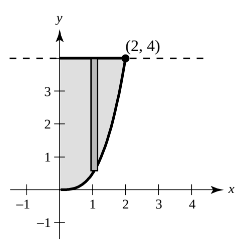
16. $R(x) = 4 - \dfrac{x^3}{2}$, $r(x) = 0$

$$V = \pi\int_{0}^{2}\left(4 - \frac{x^3}{2}\right)^2 dx$$

$$= \pi\int_{0}^{2}\left[16 - 4x^3 + \frac{x^6}{4}\right] dx$$

$$= \pi\left[16x - x^4 + \frac{x^7}{28}\right]_{0}^{2}$$

$$= \pi\left[32 - 16 + \frac{128}{28}\right] = \frac{144}{7}\pi$$



18. $R(x) = 4$, $r(x) = 4 - \sec x$

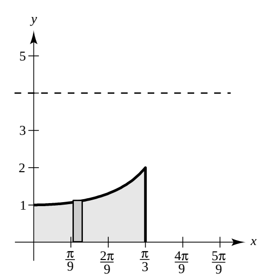
$$V = \pi\int_{0}^{\pi/3}\left[(4)^2 - (4 - \sec x)^2\right] dx$$

$$= \pi\int_{0}^{\pi/3}(8\sec x - \sec^2 x)\, dx$$

$$= \pi\left[8\ln|\sec x + \tan x| - \tan x\right]_{0}^{\pi/3}$$

$$= \pi\left[\left(8\ln|2 + \sqrt{3}| - \sqrt{3}\right) - \left(8\ln|1 + 0| - 0\right)\right]$$

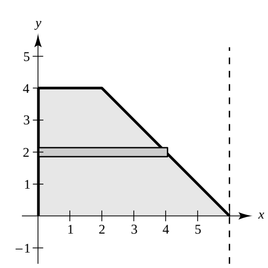
$$= \pi\left[8\ln\left(2 + \sqrt{3}\right) - \sqrt{3}\right] \approx 27.66$$



20. $R(y) = 6$, $r(y) = 6 - (6 - y) = y$

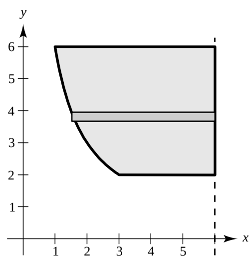
$$V = \pi\int_{0}^{4}\left[(6)^2 - (y)^2\right] dy$$

$$= \pi\left[36y - \frac{y^3}{3}\right]_{0}^{4} = \frac{368\pi}{3}$$



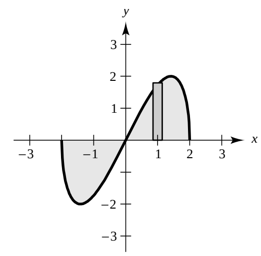
22. $R(y) = 6 - \dfrac{6}{y}, \; r(y) = 0$

$$V = \pi\int_2^6 \left(6 - \frac{6}{y}\right)^2 dy$$
$$= 36\pi\int_2^6 \left(1 - \frac{2}{y} + \frac{1}{y^2}\right) dy$$
$$= 36\pi\left[y - 2\ln|y| - \frac{1}{y}\right]_2^6$$
$$= 36\pi\left[\left(\frac{35}{6} - 2\ln 6\right) - \left(\frac{3}{2} - 2\ln 2\right)\right]$$
$$= 36\pi\left(\frac{13}{3} + 2\ln\frac{1}{3}\right) = 12\pi(13 - 6\ln 3) \approx 241.59$$



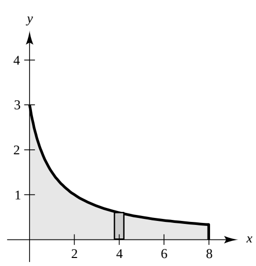
24. $R(x) = x\sqrt{4 - x^2}, \quad r(x) = 0$

$$V = 2\pi\int_0^2 \left[x\sqrt{4 - x^2}\right]^2 dx$$
$$= 2\pi\int_0^2 (4x^2 - x^4)\, dx$$
$$= 2\pi\left[\frac{4x^3}{3} - \frac{x^5}{5}\right]_0^2$$
$$= \frac{128\pi}{15}$$



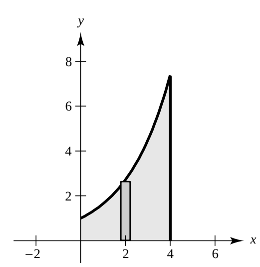
26. $R(x) = \dfrac{3}{x + 1}, \; r(x) = 0$

$$V = \pi\int_0^8 \left(\frac{3}{x + 1}\right)^2 dx$$
$$= 9\pi\int_0^8 (x + 1)^{-2}\, dx$$
$$= 9\pi\left[-\frac{1}{x + 1}\right]_0^8 = 8\pi$$



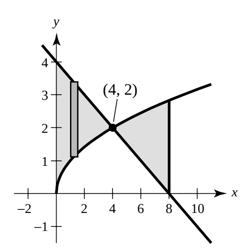
28. $R(x) = e^{x/2}, \; r(x) = 0$

$$V = \pi\int_0^4 (e^{x/2})^2\, dx$$
$$= \pi\int_0^4 e^x\, dx$$
$$= \Big[\pi e^x\Big]_0^4$$
$$= \pi(e^4 - 1) \approx 168.38$$



30. $V = \pi\displaystyle\int_0^4 \left[\left(4 - \frac{1}{2}x\right)^2 - \left(\sqrt{x}\right)^2\right] dx + \pi\int_4^8 \left[\left(\sqrt{x}\right)^2 - \left(4 - \frac{1}{2}x\right)^2\right] dx$

$$= \pi\int_0^4 \left(\frac{x^2}{4} - 5x + 16\right) dx + \pi\int_4^8 \left(-\frac{x^2}{4} + 5x - 16\right) dx$$
$$= \pi\left[\frac{x^3}{12} - \frac{5x^2}{2} + 16x\right]_0^4 + \pi\left[-\frac{x^3}{12} + \frac{5x^2}{2} - 16x\right]_4^8$$
$$= \frac{88}{3}\pi + \frac{56}{3}\pi = 48\pi$$



32. $y = 9 - x^2,\ y = 0,\ x = 2,\ x = 3$

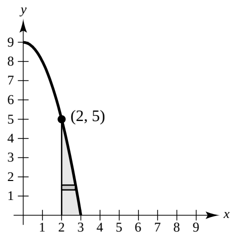
$x = \sqrt{9 - y}$

$$V = \pi\int_0^5 \left[\left(\sqrt{9-y}\right)^2 - 2^2\right] dy$$

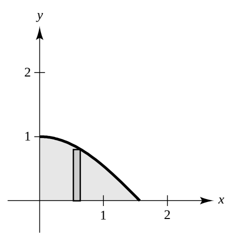
$$= \pi\int_0^5 (5 - y)\, dy$$

$$= \pi\left[5y - \frac{y^2}{2}\right]_0^5$$

$$= \pi\left(25 - \frac{25}{2}\right) = \frac{25\pi}{2}$$



34. $V = \pi\int_0^{\pi/2} [\cos x]^2\, dx \approx 2.4674$

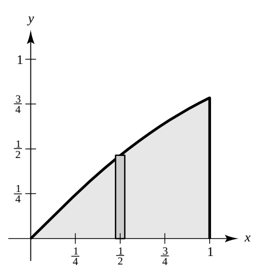


36. $V = \pi\int_1^3 [\ln x]^2\, dx \approx 3.2332$

38. $V = \pi\int_0^5 [2 \arctan(0.2x)]^2\, dx \approx 15.4115$

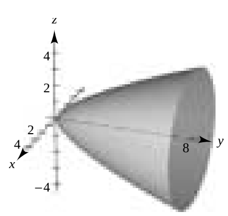
40. $A \approx \frac{3}{4}$

Matches (b)

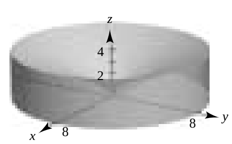


42. $V = \int_a^b A(x)\, dx$ or $V = \int_c^d A(y)\, dy$

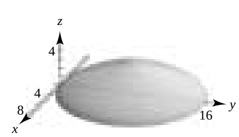
44. (a)



(b)



(c)



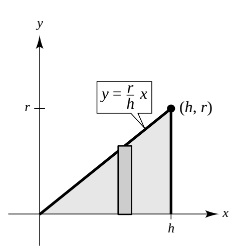
$a < c < b.$

46. $R(x) = \dfrac{r}{h}x,\ r(x) = 0$

$$V = \pi\int_0^h \frac{r^2}{h^2}x^2\, dx$$

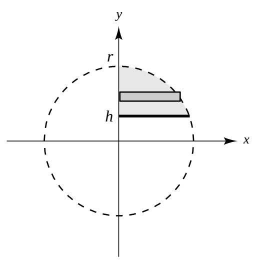
$$= \left[\frac{r^2\pi}{3h^2}x^3\right]_0^h$$

$$= \frac{r^2\pi}{3h^2}h^3 = \frac{1}{3}\pi r^2 h$$



48. $x = \sqrt{r^2 - y^2},\ R(y) = \sqrt{r^2 - y^2},\ r(y) = 0$

$$V = \pi\int_h^r \left(\sqrt{r^2 - y^2}\right)^2 dy$$
$$= \pi\int_h^r (r^2 - y^2)\,dy$$
$$= \pi\left[r^2y - \frac{y^3}{3}\right]_h^r$$
$$= \pi\left[\left(r^3 - \frac{r^3}{3}\right) - \left(r^2h - \frac{h^3}{3}\right)\right]$$
$$= \pi\left(\frac{2r^3}{3} - r^2h + \frac{h^3}{3}\right)$$
$$= \frac{\pi}{3}(2r^3 - 3r^2h + h^3)$$



50. (a) $V = \pi\int_0^4 \left(\sqrt{x}\right)^2 dx = \pi\int_0^4 x\,dx = \left[\frac{\pi x^2}{2}\right]_0^4 = 8\pi$

Let $0 < c < 4$ and set

$$\pi\int_0^c x\,dx = \left[\frac{\pi x^2}{2}\right]_0^c = \frac{\pi c^2}{2} = 4\pi.$$
$$c^2 = 8$$
$$c = \sqrt{8} = 2\sqrt{2}$$

Thus, when $x = 2\sqrt{2}$, the solid is divided into two parts of equal volume.

(b) Set $\pi\int_0^c x\,dx = \frac{8\pi}{3}$ (one third of the volume). Then

$$\frac{\pi c^2}{2} = \frac{8\pi}{3},\ c^2 = \frac{16}{3},\ c = \frac{4}{\sqrt{3}} = \frac{4\sqrt{3}}{3}.$$

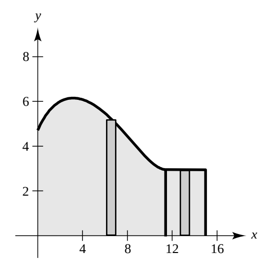
To find the other value, set $\pi\int_0^d x\,dx = \frac{16\pi}{3}$ (two thirds of the volume). Then

$$\frac{\pi d^2}{2} = \frac{16\pi}{3},\ d^2 = \frac{32}{3},\ d = \frac{\sqrt{32}}{\sqrt{3}} = \frac{4\sqrt{6}}{3}.$$

The x-values that divide the solid into three parts of equal volume are $x = \left(4\sqrt{3}\right)/3$ and $x = \left(4\sqrt{6}\right)/3$.

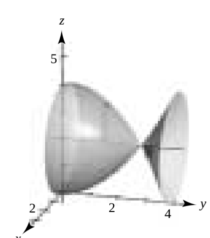
52. $y = \begin{cases} \sqrt{0.1x^3 - 2.2x^2 + 10.9x + 22.2}, & 0 \le x \le 11.5 \\ 2.95, & 11.5 < x \le 15 \end{cases}$

$$V = \pi\int_0^{11.5} \left(\sqrt{0.1x^3 - 2.2x^2 + 10.9x + 22.2}\right)^2 dx + \pi\int_{11.5}^{15} 2.95^2\,dx$$
$$= \pi\left[\frac{0.1x^4}{4} - \frac{2.2x^3}{3} + \frac{10.9x^2}{2} + 22.2x\right]_0^{11.5} + \pi\left[2.95^2x\right]_{11.5}^{15}$$
$$\approx 1031.9016 \text{ cubic centimeters}$$



54. (a) First find where $y = b$ intersects the parabola:

$$b = 4 - \frac{x^2}{4}$$
$$x^2 = 16 - 4b = 4(4 - b)$$
$$x = 2\sqrt{4 - b}$$
$$V = \int_0^{2\sqrt{4-b}} \pi\left[4 - \frac{x^2}{4} - b\right]^2 dx + \int_{2\sqrt{4-b}}^4 \pi\left[b - 4 + \frac{x^2}{4}\right]^2 dx$$
$$= \int_0^4 \pi\left[4 - \frac{x^2}{4} - b\right]^2 dx$$
$$= \pi\int_0^4 \left[\frac{x^4}{16} - 2x^2 + \frac{bx^2}{2} + b^2 - 8b + 16\right] dx$$
$$= \pi\left[\frac{x^5}{80} - \frac{2x^3}{3} + \frac{bx^3}{6} + b^2x - 8bx + 16x\right]_0^4$$
$$= \pi\left[\frac{64}{5} - \frac{128}{3} + \frac{32}{3}b + 4b^2 - 32b + 64\right] = \pi\left[4b^2 - \frac{64}{3}b + \frac{512}{15}\right]$$



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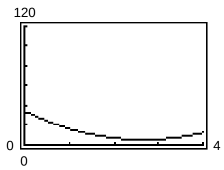
54. —CONTINUED—

(b) graph of $V(b) = \pi\left[4b^2 - \frac{64}{3}b + \frac{512}{15}\right]$



Minimum Volume is 17.87 for $b = 2.67$

(c) $V'(b) = \pi\left[8b - \frac{64}{3}\right] = 0 \Rightarrow b = \frac{64/3}{8} = \frac{8}{3} = 2\frac{2}{3}$

$V''(b) = 8\pi > 0 \Rightarrow b = \frac{8}{3}$ is a relative minimum.

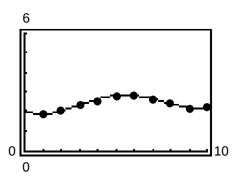
56. (a) $V = \int_0^{10} \pi[f(x)]^2\,dx$

Simpson's Rule: $b - a = 10 - 0 = 10, \quad n = 10$

$$V \approx \frac{\pi}{3}[(2.1)^2 + 4(1.9)^2 + 2(2.1)^2 + 4(2.35)^2 + 2(2.6)^2 + 4(2.85)^2 + 2(2.9)^2 + 4(2.7)^2 + 2(2.45)^2 + 4(2.2)^2 + (2.3)^2]$$

$$\approx \frac{\pi}{3}[178.405] \approx 186.83 \text{ cm}^3$$

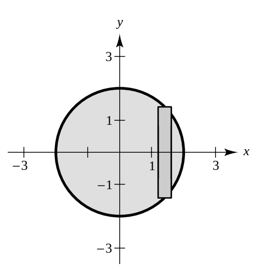
(b) $f(x) = 0.00249x^4 - 0.0529x^3 + 0.3314x^2 - 0.4999x + 2.112$



(c) $V \approx \int_0^{10} \pi f(x)^2\,dx \approx 186.35 \text{ cm}^3$

58. $V = \frac{1}{2}(10)(2)(3) = 30 \text{ m}^3$

60.

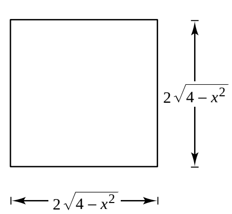


Base of Cross Section $= 2\sqrt{4 - x^2}$

(a) $A(x) = b^2 = \left(2\sqrt{4 - x^2}\right)^2$

$$V = \int_{-2}^{2} 4(4 - x^2)\,dx$$

$$= 4\left[4x - \frac{x^3}{3}\right]_{-2}^{2} = \frac{128}{3}$$

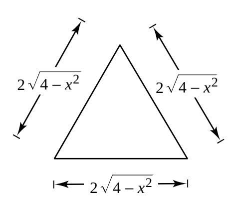


(b) $A(x) = \frac{1}{2}bh = \frac{1}{2}\left(2\sqrt{4 - x^2}\right)\left(\sqrt{3}\sqrt{4 - x^2}\right)$

$$= \sqrt{3}\,(4 - x^2)$$

$$V = \sqrt{3}\int_{-2}^{2} (4 - x^2)\,dx$$

$$= \sqrt{3}\left[4x - \frac{x^3}{3}\right]_{-2}^{2} = \frac{32\sqrt{3}}{3}$$



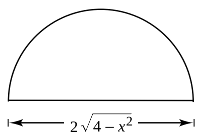
—CONTINUED—

60. —CONTINUED—

(c) $A(x) = \frac{1}{2}\pi r^2$

$= \frac{\pi}{2}\left(\sqrt{4 - x^2}\right)^2 = \frac{\pi}{2}(4 - x^2)$

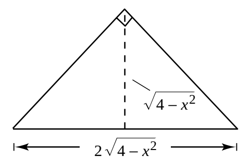
$V = \frac{\pi}{2}\int_{-2}^{2}(4 - x^2)\,dx = \frac{\pi}{2}\left[4x - \frac{x^3}{3}\right]_{-2}^{2} = \frac{16\pi}{3}$



(d) $A(x) = \frac{1}{2}bh$

$= \frac{1}{2}\left(2\sqrt{4 - x^2}\right)\left(\sqrt{4 - x^2}\right) = 4 - x^2$

$V = \int_{-2}^{2}(4 - x^2)\,dx = \left[4x - \frac{x^3}{3}\right]_{-2}^{2} = \frac{32}{3}$



62. The cross sections are squares. By symmetry, we can set up an integral for an eighth of the volume and multiply by 8.

$A(y) = b^2 = \left(\sqrt{r^2 - y^2}\right)^2$

$V = 8\int_0^r (r^2 - y^2)\,dy$

$= 8\left[r^2 y - \frac{1}{3}y^3\right]_0^r$

$= \frac{16}{3}r^3$

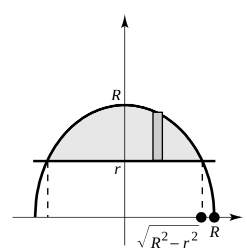
64. $V = \pi\int_{-\sqrt{R^2-r^2}}^{\sqrt{R^2-r^2}}\left[\left(\sqrt{R^2 - x^2}\right)^2 - r^2\right]dx$

$= 2\pi\int_0^{\sqrt{R^2-r^2}}(R^2 - r^2 - x^2)\,dx$

$= 2\pi\left[(R^2 - r^2)x - \frac{x^3}{3}\right]_0^{\sqrt{R^2-r^2}}$

$= 2\pi\left[(R^2 - r^2)^{3/2} - \frac{(R^2 - r^2)^{3/2}}{3}\right]$

$= \frac{4}{3}\pi(R^2 - r^2)^{3/2}$



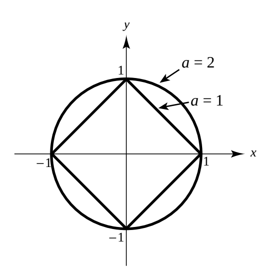
66. (a) When $a = 1$: $|x| + |y| = 1$ represents a square.

When $a = 2$: $|x|^2 + |y|^2 = 1$ represents a circle.

(b) $|y| = (1 - |x|^a)^{1/a}$

$A = 2\int_{-1}^{1}(1 - |x|^a)^{1/a}\,dx = 4\int_0^1 (1 - x^a)^{1/a}\,dx$

To approximate the volume of the solid, form n slices, each of whose area is approximated by the integral above. Then sum the volumes of these n slices.



Section 6.3 Volume: The Shell Method

2. $p(x) = x$

$h(x) = 1 - x$

$$V = 2\pi\int_0^1 x(1 - x)\,dx$$

$$= 2\pi\int_0^1 (x - x^2)\,dx = 2\pi\left[\frac{x^2}{2} - \frac{x^3}{3}\right]_0^1 = \frac{\pi}{3}$$

4. $p(x) = x$

$h(x) = 8 - (x^2 + 4) = 4 - x^2$

$$V = 2\pi\int_0^2 x(4 - x^2)\,dx$$

$$= 2\pi\int_0^2 (4x - x^3)\,dx$$

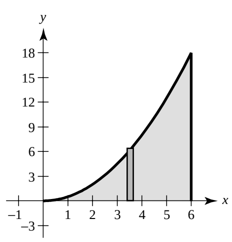
$$= 2\pi\left[2x^2 - \frac{x^4}{4}\right]_0^2 = 8\pi$$

6. $p(x) = x$

$h(x) = \frac{1}{2}x^2$

$$V = 2\pi\int_0^6 \frac{1}{2}x^3\,dx$$

$$= \left[\pi\frac{x^4}{4}\right]_0^6 = 324\pi$$

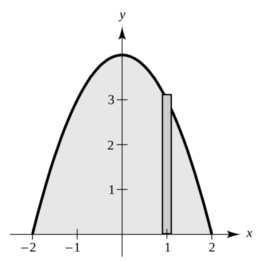


8. $p(x) = x$

$h(x) = 4 - x^2$

$$V = 2\pi\int_0^2 (4x - x^3)\,dx$$

$$= 2\pi\left[2x^2 - \frac{1}{4}x^4\right]_0^2 = 8\pi$$



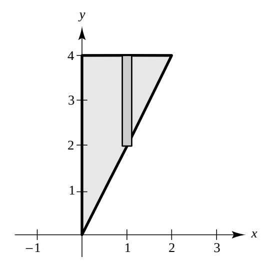
10. $p(x) = x$

$h(x) = 4 - 2x$

$$V = 2\pi\int_0^2 x(4 - 2x)\,dx$$

$$= 2\pi\int_0^2 (4x - 2x^2)\,dx$$

$$= 2\pi\left[2x^2 - \frac{2}{3}x^3\right]_0^2 = \frac{16\pi}{3}$$

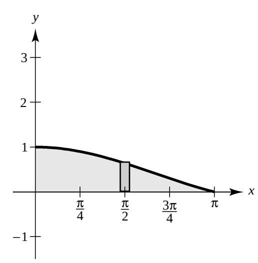


12. $p(x) = x$

$h(x) = \frac{\sin x}{x}$

$$V = 2\pi\int_0^\pi x\left[\frac{\sin x}{x}\right]dx$$

$$= 2\pi\int_0^\pi \sin x\,dx = \left[-2\pi\cos x\right]_0^\pi = 4\pi$$



14. $p(y) = -y \quad (p(y) \ge 0 \text{ on } [-2, 0])$

$h(y) = 4 - (2 - y) = 2 + y$

$$V = 2\pi\int_{-2}^0 (-y)(2 + y)\,dy$$

$$= 2\pi\int_{-2}^0 (-2y - y^2)\,dy$$

$$= 2\pi\left[-y^2 - \frac{y^3}{3}\right]_{-2}^0 = \frac{8\pi}{3}$$

16. $p(y) = y$

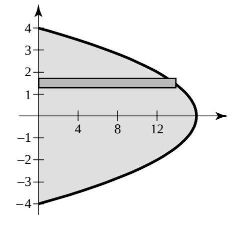
$h(y) = 16 - y^2$

$$V = 2\pi\int_0^4 y(16 - y^2)\,dy$$

$$= 2\pi\int_0^4 (16y - y^3)\,dy$$

$$= 2\pi\left[8y^2 - \frac{y^4}{4}\right]_0^4$$

$$= 2\pi[128 - 64] = 128\pi$$



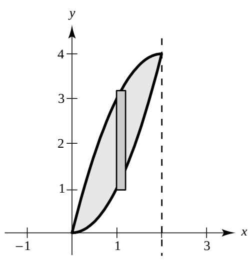
18. $p(x) = 2 - x$

$h(x) = 4x - x^2 - x^2 = 4x - 2x^2$

$$V = 2\pi \int_0^2 (2 - x)(4x - 2x^2)\,dx$$

$$= 2\pi \int_0^2 (8x - 8x^2 + 2x^3)\,dx$$

$$= 2\pi\left[4x^2 - \frac{8}{3}x^3 + \frac{1}{2}x^4\right]_0^2 = \frac{16\pi}{3}$$



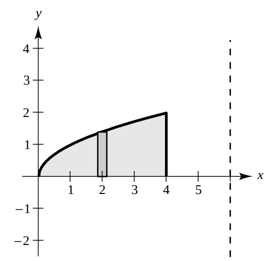
20. $p(x) = 6 - x$

$h(x) = \sqrt{x}$

$$V = 2\pi \int_0^4 (6 - x)\sqrt{x}\,dx$$

$$= 2\pi \int_0^4 (6x^{1/2} - x^{3/2})\,dx$$

$$= 2\pi\left[4x^{3/2} - \frac{2}{5}x^{5/2}\right]_0^4 = \frac{192\pi}{5}$$



22. (a) **Disk**

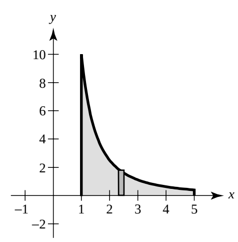
$$R(x) = \frac{10}{x^2},\ r(x) = 0$$

$$V = \pi\int_1^5 \left(\frac{10}{x^2}\right)^2 dx$$

$$= 100\pi\int_1^5 x^{-4}\,dx$$

$$= 100\pi\left[\frac{x^{-3}}{-3}\right]_1^5$$

$$= -\frac{100\pi}{3}\left[\frac{1}{125} - 1\right] = \frac{496}{15}\pi$$



(b) **Shell**

$$R(x) = x,\ r(x) = 0$$

$$V = 2\pi\int_1^5 x\left(\frac{10}{x^2}\right) dx$$

$$= 20\pi\int_1^5 \frac{1}{x}\,dx$$

$$= 20\pi\Big[\ln|x|\Big]_1^5 = 20\pi \ln 5$$

(c) **Disk**

$$R(x) = 10,\ r(x) = 10 - \frac{10}{x^2}$$

$$V = \pi\int_1^5 \left[10^2 - \left(10 - \frac{10}{x^2}\right)^2\right] dx$$

$$= \pi\left[\frac{100}{3x^3} - \frac{200}{x}\right]_1^5 = \frac{1904}{15}\pi$$

24. (a) **Disk**

$$R(x) = (a^{2/3} - x^{2/3})^{3/2}$$

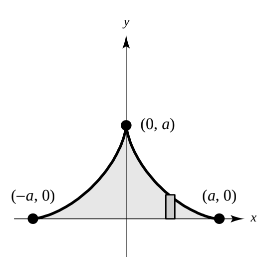
$$r(x) = 0$$

$$V = \pi\int_{-a}^{a} (a^{2/3} - x^{2/3})^3\,dx$$

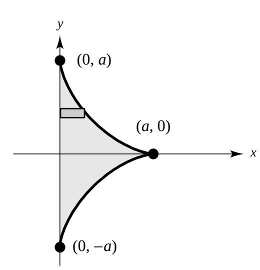
$$= 2\pi\int_0^a (a^2 - 3a^{4/3}x^{2/3} + 3a^{2/3}x^{4/3} - x^2)\,dx$$

$$= 2\pi\left[a^2x - \frac{9}{5}a^{4/3}x^{5/3} + \frac{9}{7}a^{2/3}x^{7/3} - \frac{1}{3}x^3\right]_0^a$$

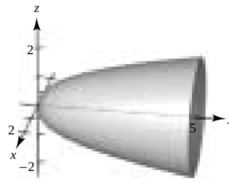
$$= 2\pi\left(a^3 - \frac{9}{5}a^3 + \frac{9}{7}a^3 - \frac{1}{3}a^3\right) = \frac{32\pi a^3}{105}$$



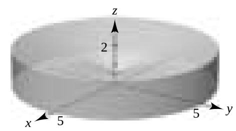
(b) Same as part a by symmetry



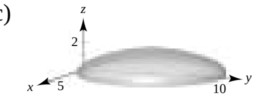
26. (a)



(b)



(c)



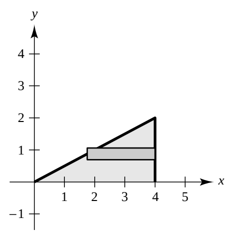
$a < c < b$

28. $2\pi \int_0^4 x\left(\frac{x}{2}\right) dx$

represents the volume of the solid generated by revolving the region bounded by $y = x/2$, $y = 0$, and $x = 4$ about the y-axis by using the Shell Method.

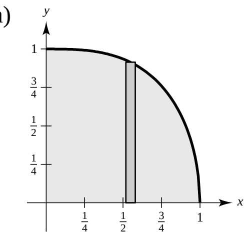
$$\pi \int_0^2 [16 - (2y)^2]\, dy = \pi \int_0^2 [(4)^2 - (2y)^2]\, dy$$

represents this same volume by using the Disk Method.



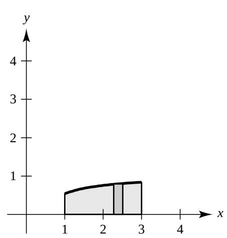
Disk Method

30. (a)



(b) $V = 2\pi \int_0^1 x\sqrt{1 - x^3}\, dx \approx 2.3222$

32. (a)

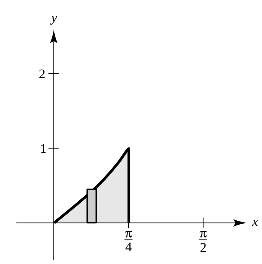


(b) $V = 2\pi \int_1^3 \frac{2x}{1 + e^{1/x}}\, dx \approx 19.0162$

34. $y = \tan x,\ y = 0,\ x = 0,\ x = \frac{\pi}{4}$

Volume ≈ 1

Matches (e)



36. Total volume of the hemisphere is $\frac{1}{2}\left(\frac{4}{3}\right)\pi r^3 = \frac{2}{3}\pi(3)^3 = 18\pi$. By the Shell Method, $p(x) = x$, $h(x) = \sqrt{9 - x^2}$. Find x_0 such that

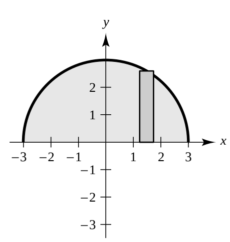
$$6\pi = 2\pi \int_0^{x_0} x\sqrt{9 - x^2}\, dx$$

$$6 = -\int_0^{x_0} (9 - x^2)^{1/2}(-2x)\, dx$$

$$= \left[-\frac{2}{3}(9 - x^2)^{3/2}\right]_0^{x_0} = 18 - \frac{2}{3}(9 - x_0^2)^{3/2}$$

$$(9 - x_0^2)^{3/2} = 18$$

$$x_0 = \sqrt{9 - 18^{2/3}} \approx 1.460.$$



Diameter: $2\sqrt{9 - 18^{2/3}} \approx 2.920$

38. $V = 4\pi \int_{-r}^{r} (R - x)\sqrt{r^2 - x^2}\,dx$

$= 4\pi R \int_{-r}^{r} \sqrt{r^2 - x^2}\,dx - 4\pi \int_{-r}^{r} x\sqrt{r^2 - x^2}\,dx$

$= 4\pi R\left(\frac{\pi r^2}{2}\right) + \left[2\pi\left(\frac{2}{3}\right)(r^2 - x^2)^{3/2}\right]_{-r}^{r}$

$= 2\pi^2 r^2 R$

40. (a) Area region $= \int_0^b [ab^n - ax^n]\,dx$

$= \left[ab^n x - a\frac{x^{n+1}}{n+1}\right]_0^b$

$= ab^{n+1} - a\frac{b^{n+1}}{n+1}$

$= ab^{n+1}\left(1 - \frac{1}{n+1}\right) = ab^{n+1}\left(\frac{n}{n+1}\right)$

$R_1(n) = \frac{ab^{n+1}\left(\frac{n}{n+1}\right)}{(ab^n)b} = \frac{n}{n+1}$

(b) $\lim_{n\to\infty} R_1(n) = \lim_{n\to\infty} \frac{n}{n+1} = 1$

$\lim_{n\to\infty} (ab^n)b = \infty$

(c) **Disk Method:**

$V = 2\pi\int_0^b x(ab^n - ax^n)\,dx$

$= 2\pi a\int_0^b (xb^n - x^{n+1})\,dx$

$= 2\pi a\left[\frac{b^n}{2}x^2 - \frac{x^{n+2}}{n+2}\right]_0^b$

$= 2\pi a\left[\frac{b^{n+2}}{2} - \frac{b^{n+2}}{n+2}\right] = \pi ab^{n+2}\left(\frac{n}{n+2}\right)$

$R_2(n) = \frac{\pi ab^{n+2}\left(\frac{n}{n+2}\right)}{(\pi b^2)(ab^n)} = \left(\frac{n}{n+2}\right)$

(d) $\lim_{n\to\infty} R_2(n) = \lim_{n\to\infty} \left(\frac{n}{n+2}\right) = 1$

$\lim_{n\to\infty} (\pi b^2)(ab^n) = \infty$

(e) As $n \to \infty$, the graph approaches the line $x = 1$.

42. (a) $V = 2\pi \int_0^4 x f(x)\,dx$

$= \frac{2\pi(40)}{3(4)}[0 + 4(10)(45) + 2(20)(40) + 4(30)(20) + 0]$

$= \frac{20\pi}{3}[5800] \approx 121{,}475$ cubic feet

(b) Top line: $y - 50 = \frac{40 - 50}{20 - 0}(x - 0) = -\frac{1}{2}x \implies y = -\frac{1}{2}x + 50$

Bottom line: $y - 40 = \frac{0 - 40}{40 - 20}(x - 20) = -2(x - 20) \implies y = -2x + 80$

$V = 2\pi\int_0^{20} x\left(-\frac{1}{2}x + 50\right)dx + 2\pi\int_{20}^{40} x(-2x + 80)\,dx$

$= 2\pi\int_0^{20}\left(-\frac{1}{2}x^2 + 50x\right)dx + 2\pi\int_{20}^{40}(-2x^2 + 80x)\,dx$

$= 2\pi\left[-\frac{x^3}{6} + 25x^2\right]_0^{20} + 2\pi\left[-\frac{2x^3}{3} + 40x^2\right]_{20}^{40}$

$= 2\pi\left[\frac{26{,}000}{3}\right] + 2\pi\left[\frac{32{,}000}{3}\right]$

$\approx 121{,}475$ cubic feet

(Note that Simpson's Rule is exact for this problem.)

Section 6.4 Arc Length and Surfaces of Revolution

2. $(1, 2), (7, 10)$

(a) $d = \sqrt{(7-1)^2 + (10-2)^2} = 10$

(b) $y = \frac{4}{3}x + \frac{2}{3}$

$y' = \frac{4}{3}$

$s = \int_1^7 \sqrt{1 + \left(\frac{4}{3}\right)^2}\,dx = \left[\frac{5}{3}x\right]_1^7 = 10$

4. $y = 2x^{3/2} + 3$

$y' = 3x^{1/2}, [0, 9]$

$s = \int_0^9 \sqrt{1 + 9x}\,dx$

$= \left[\frac{2}{27}(1 + 9x)^{3/2}\right]_0^9$

$= \frac{2}{27}(82^{3/2} - 1) \approx 54.929$

6. $y = \frac{3}{2}x^{2/3} + 4$

$y' = x^{-1/3}, [1, 27]$

$s = \int_1^{27} \sqrt{1 + \left(\frac{1}{x^{1/3}}\right)^2}\,dx$

$= \int_1^{27} \sqrt{\frac{x^{2/3} + 1}{x^{2/3}}}\,dx$

$= \frac{3}{2}\int_1^{27} \sqrt{x^{2/3} + 1}\left(\frac{2}{3x^{1/3}}\right)dx$

$= \left[\frac{3}{2} \cdot \frac{2}{3}(x^{2/3} + 1)^{3/2}\right]_1^{27}$

$= 10^{3/2} - 2^{3/2} \approx 28.794$

8. $y = \frac{x^5}{10} + \frac{1}{6x^3}$

$y' = \frac{1}{2}x^4 - \frac{1}{2x^4}$

$1 + (y')^2 = \left(\frac{1}{2}x^4 + \frac{1}{2x^4}\right)^2, [1, 2]$

$s = \int_a^b \sqrt{1 + (y')^2}\,dx$

$= \int_1^2 \sqrt{\left(\frac{1}{2}x^4 + \frac{1}{2x^4}\right)^2}\,dx$

$= \int_1^2 \left(\frac{1}{2}x^4 + \frac{1}{2x^4}\right)dx$

$= \left[\frac{1}{10}x^5 - \frac{1}{6x^3}\right]_1^2 = \frac{779}{240} \approx 3.246$

10. $y = \frac{1}{2}(e^x + e^{-x})$

$y' = \frac{1}{2}(e^x - e^{-x}), [0, 2]$

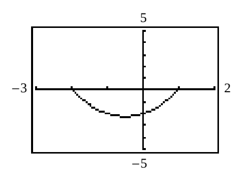
$1 + (y')^2 = \left[\frac{1}{2}(e^x + e^{-x})\right]^2, [0, 2]$

$s = \int_0^2 \sqrt{\left[\frac{1}{2}(e^x + e^{-x})\right]^2}\,dx$

$= \frac{1}{2}\int_0^2 (e^x + e^{-x})\,dx$

$= \frac{1}{2}\left[e^x - e^{-x}\right]_0^2 = \frac{1}{2}\left(e^2 - \frac{1}{e^2}\right) \approx 3.627$

12. (a) $y = x^2 + x - 2, -2 \le x \le 1$



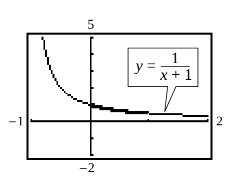
(b) $y' = 2x + 1$

$1 + (y')^2 = 1 + 4x^2 + 4x + 1$

$L = \int_{-2}^1 \sqrt{2 + 4x + 4x^2}\,dx$

(c) $L \approx 5.653$

14. (a) $y = \frac{1}{1+x}, 0 \le x \le 1$



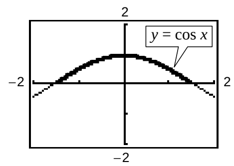
(b) $y' = -\frac{1}{(1+x)^2}$

$1 + (y')^2 = 1 + \frac{1}{(1+x)^4}$

$L = \int_0^1 \sqrt{1 + \frac{1}{(1+x)^4}}\,dx$

(c) $L \approx 1.132$

16. (a) $y = \cos x, -\frac{\pi}{2} \le x \le \frac{\pi}{2}$



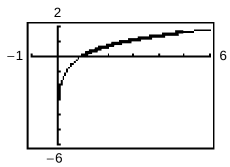
(b) $y' = -\sin x$

$1 + (y')^2 = 1 + \sin^2 x$

$L = \int_{-\pi/2}^{\pi/2} \sqrt{1 + \sin^2 x}\, dx$

(c) 3.820

18. (a) $y = \ln x, 1 \le x \le 5$



(b) $y' = \frac{1}{x}$

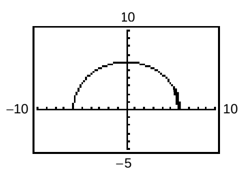
$1 + (y')^2 = 1 + \frac{1}{x^2}$

$L = \int_1^5 \sqrt{1 + \frac{1}{x^2}}\, dx$

(c) $L \approx 4.367$

20. (a) $x = \sqrt{36 - y^2}, 0 \le y \le 3$

$y = \sqrt{36 - x^2}, 3\sqrt{3} \le x \le 6$



(b) $\frac{dx}{dy} = \frac{1}{2}(36 - y^2)^{-1/2}(-2y)$

$= \frac{-y}{\sqrt{36 - y^2}}$

$L = \int_0^3 \sqrt{1 + \frac{y^2}{36 - y^2}}\, dy$

$= \int_0^3 \frac{6}{\sqrt{36 - y^2}}\, dy$

(c) $L \approx 3.142$ (π!)

Alternatively, you can convert to a function of x.

$y = \sqrt{36 - x^2}$

$y' = \frac{dy}{dx} = -\frac{x}{\sqrt{36 - x^2}}$

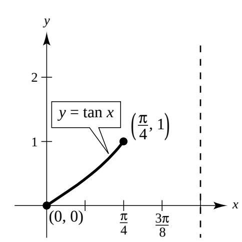
$L = \int_{3\sqrt{3}}^{6} \sqrt{1 + \frac{x^2}{36 - x^2}}\, dx = \int_{3\sqrt{3}}^{6} \frac{6}{\sqrt{36 - x^2}}\, dx$

Although this integral is undefined at $x = 0$, a graphing utility still gives $L \approx 3.142$.

22. $\int_0^{\pi/4} \sqrt{1 + \left[\frac{d}{dx}(\tan x)\right]^2}\, dx$

$s \approx 1$

Matches (e)



24. $f(x) = (x^2 - 4)^2, [0, 4]$

(a) $d = \sqrt{(4 - 0)^2 + (144 - 16)^2} \approx 128.062$

(b) $d = \sqrt{(1 - 0)^2 + (9 - 16)^2} + \sqrt{(2 - 1)^2 + (0 - 9)^2} + \sqrt{(3 - 2)^2 + (25 - 0)^2} + \sqrt{(4 - 3)^2 + (144 - 25)^2}$

≈ 160.151

(c) $s = \int_0^4 \sqrt{1 + [4x(x^2 - 4)]^2}\, dx \approx 159.087$

(d) 160.287

26. Let $y = \ln x$, $1 \le x \le e$, $y' = \dfrac{1}{x}$ and $L_1 = \displaystyle\int_1^e \sqrt{1 + \frac{1}{x^2}}\, dx$.

Equivalently, $x = e^y$, $0 \le y \le 1$, $\dfrac{dx}{dy} = e^y$, and $L_2 = \displaystyle\int_0^1 \sqrt{1 + e^{2y}}\, dy = \int_0^1 \sqrt{1 + e^{2x}}\, dx$.

Numerically, both integrals yield $L = 2.0035$

28.

$$y = 31 - 10(e^{x/20} + e^{-x/20})$$

$$y' = -\frac{1}{2}(e^{x/20} - e^{-x/20})$$

$$1 + (y')^2 = 1 + \frac{1}{4}(e^{x/10} - 2 + e^{-x/10}) = \left[\frac{1}{2}(e^{x/20} + e^{-x/20})\right]^2$$

$$s = \int_{-20}^{20} \sqrt{\left[\frac{1}{2}(e^{x/20} + e^{-x/20})\right]^2}\, dx$$

$$= \frac{1}{2}\int_{-20}^{20} (e^{x/20} + e^{-x/20})\, dx = \Big[10(e^{x/20} - e^{-x/20})\Big]_{-20}^{20} = 20\left(e - \frac{1}{e}\right) \approx 47 \text{ ft}$$

Thus, there are $100(47) = 4700$ square feet of roofing on the barn.

30. $y = 693.8597 - 68.7672 \cosh 0.0100333x$

$y' = -0.6899619478 \sinh 0.0100333x$

$$s = \int_{-299.2239}^{299.2239} \sqrt{1 + (-0.6899619478 \sinh 0.0100333x)^2}\, dx \approx 1480$$

(Use Simpson's Rule with $n = 100$ or a graphing utility.)

32.

$$y = \sqrt{25 - x^2}$$

$$y' = \frac{-x}{\sqrt{25 - x^2}}$$

$$1 + (y')^2 = \frac{25}{25 - x^2}$$

$$s = \int_{-3}^{4} \sqrt{\frac{25}{25 - x^2}}\, dx$$

$$= \int_{-3}^{4} \frac{5}{\sqrt{25 - x^2}}\, dx$$

$$= \left[5 \arcsin \frac{x}{5}\right]_{-3}^{4}$$

$$= 5\left[\arcsin \frac{4}{5} - \arcsin\left(-\frac{3}{5}\right)\right] \approx 7.8540$$

$$\frac{1}{4}[2\pi(5)] \approx 7.8540 = s$$

34. $y = 2\sqrt{x}$

$$y' = \frac{1}{\sqrt{x}},\ [4, 9]$$

$$S = 2\pi\int_4^9 2\sqrt{x}\sqrt{1 + \frac{1}{x}}\, dx$$

$$= 4\pi\int_4^9 \sqrt{x + 1}\, dx$$

$$= \frac{8}{3}\pi(x + 1)^{3/2}\Big]_4^9$$

$$= \frac{8\pi}{3}(10^{3/2} - 5^{3/2}) \approx 171.258$$

36.
$$y = \frac{x}{2}$$
$$y' = \frac{1}{2}$$
$$1 + (y')^2 = \frac{5}{4}, [0, 6]$$
$$S = 2\pi \int_0^6 \frac{x}{2}\sqrt{\frac{5}{4}}\, dx$$
$$= \left[\frac{2\pi\sqrt{5}}{8}x^2\right]_0^6 = 9\sqrt{5}\,\pi$$

38. $y = 9 - x^2, [0, 3]$
$$y' = -2x$$
$$S = 2\pi\int_0^3 x\sqrt{1 + 4x^2}\, dx$$
$$= \frac{\pi}{4}\int_0^3 (1 + 4x^2)^{1/2}(8x)\, dx$$
$$= \left[\frac{\pi}{6}(1 + 4x^2)^{3/2}\right]_0^3$$
$$= \frac{\pi}{6}(37^{3/2} - 1) \approx 117.319$$

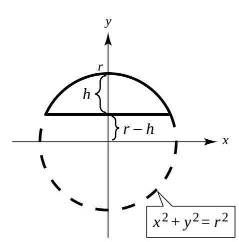
40.
$$y = \ln x$$
$$y' = \frac{1}{x}$$
$$1 + (y')^2 = \frac{x^2 + 1}{x^2}, [1, e]$$
$$S = 2\pi\int_1^e x\sqrt{\frac{x^2 + 1}{x^2}}\, dx$$
$$= 2\pi\int_1^e \sqrt{x^2 + 1}\, dx \approx 22.943$$

42. The precalculus formula is the distance formula between two points. The representative element is
$$\sqrt{(\Delta x_i)^2 + (\Delta y_i)^2} = \sqrt{1 + \left(\frac{\Delta y_i}{\Delta x_i}\right)^2}\,\Delta xi.$$

44. The surface of revolution given by f_1 will be larger. $r(x)$ is larger for f_1.

46. $y = \sqrt{r^2 - x^2}$
$$y' = \frac{-x}{\sqrt{r^2 - x^2}}$$
$$1 + (y')^2 = \frac{r^2}{r^2 - x^2}$$
$$S = 2\pi\int_{-r}^{r} \sqrt{r^2 - x^2}\sqrt{\frac{r^2}{r^2 - x^2}}\, dx$$
$$= 2\pi\int_{-r}^{r} r\, dx = \Big[2\pi r x\Big]_{-r}^{r} = 4\pi r^2$$

48. From Exercise 47 we have:
$$S = 2\pi\int_0^a \frac{rx}{\sqrt{r^2 - x^2}}\, dx$$
$$= -r\pi\int_0^a \frac{-2x\, dx}{\sqrt{r^2 - x^2}}$$
$$= \left[-2r\pi\sqrt{r^2 - x^2}\right]_0^a$$
$$= 2r^2\pi - 2r\pi\sqrt{r^2 - a^2}$$
$$= 2r\pi\left(r - \sqrt{r^2 - a^2}\right)$$
$$= 2\pi r h \text{ (where } h \text{ is the height of the zone)}$$



50. (a) We approximate the volume by summing 6 disks of thickness 3 and circumference C_i equal to the average of the given circumferences:

$$V \approx \sum_{i=1}^{6} \pi r_i^2(3) = \sum_{i=1}^{6} \pi\left(\frac{C_i}{2\pi}\right)^2(3) = \frac{3}{4\pi}\sum_{i=1}^{6} C_i^2$$

$$= \frac{3}{4\pi}\left[\left(\frac{50 + 65.5}{2}\right)^2 + \left(\frac{65.5 + 70}{2}\right)^2 + \left(\frac{70 + 66}{2}\right)^2 + \left(\frac{66 + 58}{2}\right)^2 + \left(\frac{58 + 51}{2}\right)^2 + \left(\frac{51 + 48}{2}\right)^2\right]$$

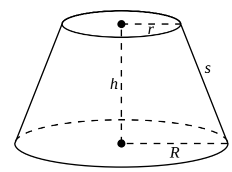
$$= \frac{3}{4\pi}[57.75^2 + 67.75^2 + 68^2 + 62^2 + 54.5^2 + 49.5^2]$$

$$= \frac{3}{4\pi}[21813.625] = 5207.62 \text{ cubic inches}$$

(b) The lateral surface area of a frustum of a right circular cone is $\pi s(R + r)$. For the first frustum,

$$S_1 \approx \pi\left[3^2 + \left(\frac{65.5 - 50}{2\pi}\right)^2\right]^{1/2}\left[\frac{50}{2\pi} + \frac{65.5}{2\pi}\right]$$

$$= \left(\frac{50 + 65.5}{2}\right)\left[9 + \left(\frac{65.5 - 50}{2\pi}\right)^2\right]^{1/2}.$$



Adding the six frustums together,

$$S \approx \left(\frac{50 + 65.5}{2}\right)\left[9 + \left(\frac{15.5}{2\pi}\right)^2\right]^{1/2} + \left(\frac{65.5 + 70}{2}\right)\left[9 + \left(\frac{4.5}{2\pi}\right)^2\right]^{1/2} +$$

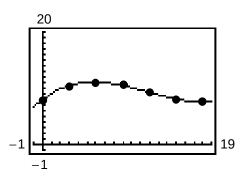
$$\left(\frac{70 + 66}{2}\right)\left[9 + \left(\frac{4}{2\pi}\right)^2\right]^{1/2} + \left(\frac{66 + 58}{2}\right)\left[9 + \left(\frac{8}{2\pi}\right)^2\right]^{1/2} +$$

$$\left(\frac{58 + 51}{2}\right)\left[9 + \left(\frac{7}{2\pi}\right)^2\right]^{1/2} + \left(\frac{51 + 48}{2}\right)\left[9 + \left(\frac{3}{2\pi}\right)^2\right]^{1/2}$$

$$\approx 224.30 + 208.96 + 208.54 + 202.06 + 174.41 + 150.37$$

$$= 1168.64$$

(c) $r = 0.00401y^3 - 0.1416y^2 + 1.232y + 7.943$



(d) $V = \int_0^{18} \pi r^2\,dy \approx 5275.9$ cubic inches

$$S = \int_0^{18} 2\pi r(y)\sqrt{1 + r'(y)^2}\,dy$$

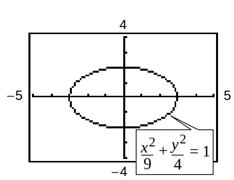
$$\approx 1179.5 \text{ square inches}$$

52. Individual project, see Exercise 50, 51.

54. (a) $\dfrac{x^2}{9} + \dfrac{y^2}{4} = 1$

Ellipse: $y_1 = 2\sqrt{1 - \dfrac{x^2}{9}}$

$y_2 = -2\sqrt{1 - \dfrac{x^2}{9}}$



(b) $y = 2\sqrt{1 - \dfrac{x^2}{9}}, 0 \le x \le 3$

$$y' = 2\left(\frac{1}{2}\right)\left(1 - \frac{x^2}{9}\right)^{-1/2}\left(\frac{-2x}{9}\right)$$

$$= \frac{-2x}{9\sqrt{1 - \frac{x^2}{9}}} = \frac{-2x}{3\sqrt{9 - x^2}}$$

$$L = \int_0^3 \sqrt{1 + \frac{4x^2}{81 - 9x^2}}\,dx$$

(c) You cannot evaluate this definite integral, since the integrand is not defined at $x = 3$. Simpson's Rule will not work for the same reason. Also, the integrand does not have an elementary antiderivative.

56. Essay

Section 6.5 Work

2. $W = Fd = (2800)(4) = 11{,}200 \text{ ft} \cdot \text{lb}$

4. $W = Fd = [9(2000)]\left[\frac{1}{2}(5280)\right] = 47{,}520{,}000 \text{ ft} \cdot \text{lb}$

6. $W = \int_a^b F(x)\,dx$ is the work done by a force F moving an object along a straight line from $x = a$ to $x = b$.

8. (a) $W = \int_0^9 6\,dx = 54 \text{ ft} \cdot \text{lbs}$

(b) $W = \int_0^7 20\,dx + \int_7^9 (-10x + 90)\,dx = 140 + 20$

$= 160 \text{ ft} \cdot \text{lbs}$

(c) $W = \int_0^9 \frac{1}{27}x^2\,dx = \frac{x^3}{81}\Big]_0^9 = 9 \text{ ft} \cdot \text{lbs}$

(d) $W = \int_0^9 \sqrt{x}\,dx = \frac{2}{3}x^{3/2}\Big]_0^9 = \frac{2}{3}(27) = 18 \text{ ft} \cdot \text{lbs}$

10. $W = \int_0^{10} \frac{5}{4}x\,dx = \left[\frac{5}{8}x^2\right]_6^{10}$

$= 40 \text{ in} \cdot \text{lb} \approx 3.33 \text{ ft} \cdot \text{lb}$

12. $F(x) = kx$

$800 = k(70) \implies k = \frac{80}{7}$

$W = \int_0^{70} F(x)\,dx = \int_0^{70} \frac{80}{7}x\,dx = \frac{40x^2}{7}\Big]_0^{70}$

$= 28000 \text{ n} \cdot \text{cm} = 280 \text{ Nm}$

14. $F(x) = kx$

$15 = k(1) = k$

$W = 2\int_0^4 15x\,dx = \left[15x^2\right]_0^4$

$= 240 \text{ ft} \cdot \text{lb}$

16. $W = 7.5 = \int_0^{1/6} kx\,dx = \frac{kx^2}{2}\Big]_0^{1/6} = \frac{k}{72} \implies k = 540$

$W = \int_{1/6}^{5/24} 540x\,dx = 270x^2\Big]_{1/6}^{5/24} = 4.21875 \text{ ft} \cdot \text{lbs}$

18. $W = \int_{4000}^{h} \frac{80{,}000{,}000}{x^2}\,dx = \left[-\frac{80{,}000{,}000}{x}\right]_{4000}^{h}$

$= \frac{-80{,}000{,}000}{h} + 20{,}000$

$\lim_{h\to\infty} W = 20{,}000 \text{ mi/ton} \approx 2.1 \times 10^{11} \text{ ft} \cdot \text{lb}$

20. Weight on surface of moon: $\frac{1}{6}(12) = 2$ tons

Weight varies inversely as the square of distance from the center of the moon. Therefore,

$F(x) = \frac{k}{x^2}$

$2 = \frac{k}{(1100)^2}$

$k = 2.42 \times 10^6$

$W = \int_{1100}^{1150} \frac{2.42 \times 10^6}{x^2}\,dx = \left[\frac{-2.42 \times 10^6}{x}\right]_{1100}^{1150} = 2.42 \times 10^6\left(\frac{1}{1100} - \frac{1}{1150}\right)$

$\approx 95.652 \text{ mi} \cdot \text{ton} \approx 1.01 \times 10^9 \text{ ft} \cdot \text{lb}$

22. The bottom half had to be pumped a greater distance then the top half.

24. Volume of disk: $4\pi\,\Delta y$

Weight of disk: $9800(4\pi)\,\Delta y$

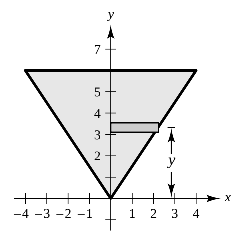
Distance the disk of water is moved: y

$$W = \int_{10}^{12} y(9800)(4\pi)\,dy = 39{,}200\pi\left[\frac{y^2}{2}\right]_{10}^{12}$$

$$= 39{,}200\pi(22)$$

$$= 862{,}400\pi \text{ newton–meters}$$

26. Volume of disk: $\pi\left(\frac{2}{3}y\right)^2\,\Delta y$



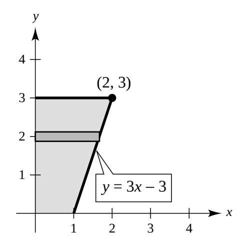
Weight of disk: $62.4\pi\left(\frac{2}{3}y\right)^2\,\Delta y$

Distance: y

(a) $W = \frac{4}{9}(62.4)\pi\int_0^2 y^3\,dy = \left[\frac{4}{9}(62.4)\pi\left(\frac{1}{4}y^4\right)\right]_0^2 \approx 110.9\pi \text{ ft} \cdot \text{lb}$

(b) $W = \frac{4}{9}(62.4)\pi\int_4^6 y^3\,dy = \left[\frac{4}{9}(62.4)\pi\left(\frac{1}{4}y^4\right)\right]_4^6 \approx 7210.7\pi \text{ ft} \cdot \text{lb}$

28. Volume of each layer: $\frac{y+3}{3}(3)\,\Delta y = (y+3)\,\Delta y$



Weight of each layer: $55.6(y+3)\,\Delta y$

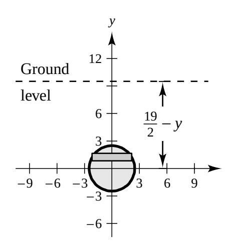
Distance: $6 - y$

$$W = \int_0^3 55.6(6-y)(y+3)\,dy = 55.6\int_0^3 (18 + 3y - y^2)\,dy$$

$$= 55.6\left[18y + \frac{3y^2}{2} - \frac{y^3}{3}\right]_0^3$$

$$= 3252.6 \text{ ft} \cdot \text{lb}$$

30. Volume of layer: $V = 12(2)\sqrt{(25/4) - y^2}\,\Delta y$



Weight of layer: $W = 42(24)\sqrt{(25/4) - y^2}\,\Delta y$

Distance: $\frac{19}{2} - y$

$$W = \int_{-2.5}^{2.5} 42(24)\sqrt{\frac{25}{4} - y^2}\left(\frac{19}{2} - y\right)dy$$

$$= 1008\left[\frac{19}{2}\int_{-2.5}^{2.5}\sqrt{\frac{25}{4} - y^2}\,dy + \int_{-2.5}^{2.5}\sqrt{\frac{25}{4} - y^2}(-y)\,dy\right]$$

The second integral is zero since the integrand is odd and the limits of integration are symmetric to the origin. The first integral represents the area of a semicircle of radius $\frac{5}{2}$. Thus, the work is

$$W = 1008\left(\frac{19}{2}\right)\pi\left(\frac{5}{2}\right)^2\left(\frac{1}{2}\right) = 29{,}925\pi \text{ ft} \cdot \text{lb} \approx 94{,}012.16 \text{ ft} \cdot \text{lb}.$$

32. The lower 10 feet of chain are raised 5 feet with a constant force.

$$W_1 = 3(10)5 = 150 \text{ ft} \cdot \text{lb}$$

The top 5 feet will be raised with variable force.

Weight of section: $3\,\Delta y$

Distance: $5 - y$

$$W_2 = 3\int_0^5 (5 - y)\,dy = \left[-\frac{3}{2}(5 - y)^2\right]_0^5 = \frac{75}{2} \text{ ft} \cdot \text{lb}$$

$$W = W_1 + W_2 = 150 + \frac{75}{2} = \frac{375}{2} \text{ ft} \cdot \text{lb}$$

34. The work required to lift the chain is 337.5 ft · lb (from Exercise 31). The work required to lift the 500-pound load is $W = (500)(15) = 7500$. The work required to lift the chain with a 100-pound load attached is

$$W = 337.5 + 7500 = 7837.5 \text{ ft} \cdot \text{lbs}$$

36. $W = 3\displaystyle\int_0^6 (12 - 2y)\,dy = \left[-\frac{3}{4}(12 - 2y)^2\right]_0^6 = \frac{3}{4}(12)^2 = 108 \text{ ft} \cdot \text{lb}$

38. Work to pull up the ball: $W_1 = 500(40) = 20{,}000 \text{ ft} \cdot \text{lb}$

Work to pull up the cable: force is variable

Weight per section: $1\,\Delta y$

Distance: $40 - x$

$$W_2 = \int_0^{40} (40 - x)\,dx = \left[-\frac{1}{2}(40 - x)^2\right]_0^{40}$$

$$= 800 \text{ ft} \cdot \text{lb}$$

$$W = W_1 + W_2 = 20{,}000 + 800 = 20{,}800 \text{ ft} \cdot \text{lb}$$

40. $p = \dfrac{k}{V}$

$$2500 = \frac{k}{1} \implies k = 2500$$

$$W = \int_1^3 \frac{2500}{V}\,dV = \Big[2500 \ln V\Big]_1^3$$

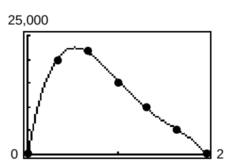
$$= 2500 \ln 3 \approx 2746.53 \text{ ft} \cdot \text{lb}$$

42. (a) $W = FD = (8000\pi)(2) = 16{,}000\pi \text{ ft} \cdot \text{lbs}$

(b) $W \approx \dfrac{2 - 0}{3(6)}[0 + 4(20{,}000) + 2(22{,}000) + 4(15{,}000) + 2(10{,}000) + 4(5000) + 0]$

$\approx 24{,}88.889 \text{ ft} \cdot \text{lb}$

(c) $F(x) = -16{,}261.36x^4 + 85{,}295.45x^3 - 157{,}738.64x^2 + 104{,}386.36x - 32.4675$



(d) $F(x) = 0$ when $x \approx 0.524$ feet. $F(x)$ is a maximum when $x \approx 0.524$ feet.

(e) $W = \displaystyle\int_0^2 F(x)\,dx \approx 25{,}180.5 \text{ ft} \cdot \text{lbs}$

44. $W = \displaystyle\int_0^4 \left(\frac{e^{x^2} - 1}{100}\right)dx \approx 11{,}494 \text{ ft} \cdot \text{lb}$

46. $W = \displaystyle\int_0^2 1000 \sinh x\,dx \approx 2762.2 \text{ ft} \cdot \text{lb}$

Section 6.6 Moments, Centers of Mass, and Centroids

2. $\bar{x} = \dfrac{7(-3) + 4(-2) + 3(5) + 8(6)}{7 + 4 + 3 + 8} = \dfrac{17}{11}$

4. $\bar{x} = \dfrac{12(-6) + 1(-4) + 6(-2) + 3(0) + 11(8)}{12 + 1 + 6 + 3 + 11} = 0$

6. The center of mass is translated k units as well.

8. $200x = 550(5 - x)$ (Person on left)

$200x = 2750 - 550x$

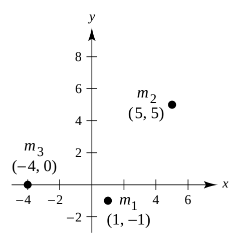
$750x = 2750$

$x = 3\frac{2}{3}$ feet

10. $\bar{x} = \dfrac{10(1) + 2(5) + 5(-4)}{10 + 2 + 5} = 0$

$\bar{y} = \dfrac{10(-1) + 2(5) + 5(0)}{10 + 2 + 5} = 0$

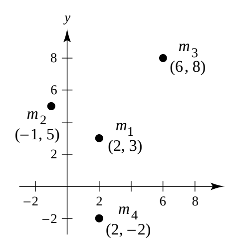
$(\bar{x}, \bar{y}) = (0, 0)$



12. $\bar{x} = \dfrac{12(2) + 6(-1) + \frac{15}{2}(6) + 15(2)}{12 + 6 + \frac{15}{2} + 15} = \dfrac{93}{40.5} = \dfrac{62}{27}$

$\bar{y} = \dfrac{12(3) + 6(5) + \frac{15}{2}(8) + 15(-2)}{12 + 6 + \frac{15}{2} + 15} = \dfrac{96}{40.5} = \dfrac{64}{27}$

$(\bar{x}, \bar{y}) = \left(\dfrac{62}{27}, \dfrac{64}{27}\right)$



14. $m = \rho\displaystyle\int_0^2 \frac{1}{2}x^2\,dx = \left[\rho\frac{x^3}{6}\right]_0^2 = \frac{4}{3}\rho$

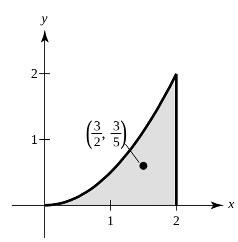
$M_x = \rho\displaystyle\int_0^2 \frac{1}{2}\left(\frac{1}{2}x^2\right)\left(\frac{1}{2}x^2\right)dx = \frac{\rho}{8}\int_0^2 x^4\,dx = \left[\frac{\rho}{40}x^5\right]_0^2 = \frac{32}{40}\rho = \frac{4}{5}\rho$

$\bar{y} = \dfrac{M_x}{m} = \dfrac{\frac{4}{5}\rho}{\frac{4}{3}\rho} = \dfrac{3}{5}$

$M_y = \rho\displaystyle\int_0^2 x\left(\frac{1}{2}x^2\right)dx = \frac{1}{2}\rho\int_0^2 x^3\,dx = \left[\frac{\rho}{8}x^4\right]_0^2 = 2\rho$

$\bar{x} = \dfrac{M_y}{m} = \dfrac{2\rho}{\frac{4}{3}\rho} = \dfrac{3}{2}$

$(\bar{x}, \bar{y}) = \left(\dfrac{3}{2}, \dfrac{3}{5}\right)$



16. $m = \rho \int_0^1 \left(\sqrt{x} - x\right) dx = \rho\left[\frac{2}{3}x^{3/2} - \frac{x^2}{2}\right]_0^1 = \frac{\rho}{6}$

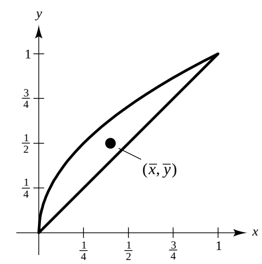
$M_x = \rho \int_0^1 \frac{\left(\sqrt{x} + x\right)}{2}\left(\sqrt{x} - x\right) dx = \frac{\rho}{2}\int_0^1 (x - x^2)\, dx = \frac{\rho}{2}\left[\frac{x^2}{2} - \frac{x^3}{3}\right]_0^1 = \frac{\rho}{12}$

$\bar{y} = \frac{M_x}{m} = \frac{\rho}{12}\left(\frac{6}{\rho}\right) = \frac{1}{2}$

$M_y = \rho \int_0^1 x\left(\sqrt{x} - x\right) dx = \rho \int_0^1 (x^{3/2} - x^2)\, dx = \rho\left[\frac{2}{5}x^{5/2} - \frac{x^3}{3}\right]_0^1 = \frac{\rho}{15}$

$\bar{x} = \frac{M_y}{m} = \frac{\rho}{15}\left(\frac{6}{\rho}\right) = \frac{2}{5}$

$(\bar{x}, \bar{y}) = \left(\frac{2}{5}, \frac{1}{2}\right)$



18. $m = \rho\int_0^9 \left[\left(\sqrt{x} + 1\right) - \left(\frac{1}{3}x + 1\right)\right] dx = \rho\int_0^9 \left(\sqrt{x} - \frac{1}{3}x\right) dx$

$= \rho\left[\frac{2}{3}x^{3/2} - \frac{x^2}{6}\right]_0^9 = \rho\left(18 - \frac{27}{2}\right) = \frac{9}{2}\rho$

$M_x = \rho\int_0^9 \frac{\sqrt{x} + 1 + \frac{1}{3}x + 1}{2}\left(\sqrt{x} + 1 - \frac{1}{3}x - 1\right) dx = \frac{\rho}{2}\int_0^9 \left(\sqrt{x} + \frac{1}{3}x + 2\right)\left(\sqrt{x} - \frac{1}{3}x\right) dx$

$= \frac{\rho}{2}\int_0^9 \left(x - \frac{1}{3}x^{3/2} + \frac{1}{3}x^{3/2} - \frac{1}{9}x^2 + 2\sqrt{x} - \frac{2}{3}x\right) dx = \frac{\rho}{2}\int_0^9 \left(\frac{1}{3}x - \frac{1}{9}x^2 + 2\sqrt{x}\right) dx$

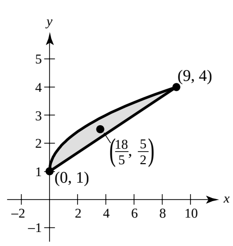
$= \frac{\rho}{2}\left[\frac{x^2}{6} - \frac{x^3}{27} + \frac{4}{3}x^{3/2}\right]_0^9 = \frac{\rho}{2}\left[\frac{27}{2} - 27 + 36\right] = \frac{45}{3}\rho$

$M_y = \rho\int_0^9 x\left[\sqrt{x} + 1 - \frac{1}{3}x - 1\right] dx = \rho\int_0^9 \left(x^{3/2} - \frac{1}{3}x^2\right) dx = \rho\left[\frac{2}{5}x^{5/2} - \frac{1}{9}x^3\right]_0^9$

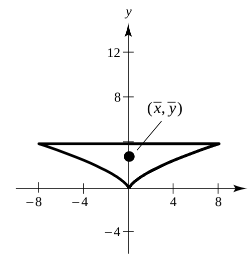
$= \rho\left[\frac{486}{5} - 81\right] = \frac{81}{5}\rho$

$\bar{x} = \frac{M_y}{m} = \frac{\frac{81}{5}\rho}{\frac{9}{2}\rho} = \frac{18}{5}; \bar{y} = \frac{M_x}{m} = \frac{\frac{45}{4}\rho}{\frac{9}{2}\rho} = \frac{5}{2}$

$(\bar{x}, \bar{y}) = \left(\frac{18}{5}, \frac{5}{2}\right)$



20. $m = 2\rho \int_0^8 (4 - x^{2/3})\, dx = 2\rho \left[4x - \frac{3}{5}x^{5/3}\right]_0^8 = \frac{128\rho}{5}$



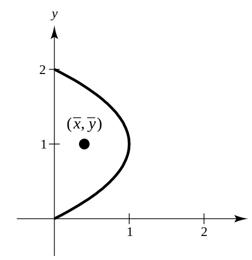
By symmetry, M_y and $\bar{x} = 0$.

$$M_x = 2\rho \int_0^8 \left(\frac{4 + x^{2/3}}{2}\right)(4 - x^{2/3})\, dx = \rho\left[16x - \frac{3}{7}x^{7/3}\right]_0^8 = \frac{512\rho}{7}$$

$$\bar{y} = \frac{512\rho}{7}\left(\frac{5}{128\rho}\right) = \frac{20}{7}$$

$$(\bar{x}, \bar{y}) = \left(0, \frac{20}{7}\right)$$

22. $m = \rho \int_0^2 (2y - y^2)\, dy = \rho\left[y^2 - \frac{y^3}{3}\right]_0^2 = \frac{4\rho}{3}$



$$M_y = \rho \int_0^2 \left(\frac{2y - y^2}{2}\right)(2y - y^2)\, dy = \frac{\rho}{2}\left[\frac{4y^3}{3} - y^4 + \frac{y^5}{5}\right]_0^2 = \frac{8\rho}{15}$$

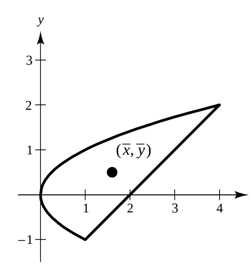
$$\bar{x} = \frac{M_y}{m} = \frac{8\rho}{15}\left(\frac{3}{4\rho}\right) = \frac{2}{5}$$

$$M_x = \rho \int_0^2 y(2y - y^2)\, dy = \rho\left[\frac{2y^3}{3} - \frac{y^4}{4}\right]_0^2 = \frac{4\rho}{3}$$

$$\bar{y} = \frac{M_x}{m} = \frac{4\rho}{3}\left(\frac{3}{4\rho}\right) = 1$$

$$(\bar{x}, \bar{y}) = \left(\frac{2}{5}, 1\right)$$

24. $m = \rho \int_{-1}^2 [(y + 2) - y^2]\, dy = \rho\left[\frac{y^2}{2} + 2y - \frac{y^3}{3}\right]_{-1}^2 = \frac{9\rho}{2}$



$$M_y = \rho \int_{-1}^2 \frac{[(y + 2) + y^2]}{2}[(y + 2) - y^2]\, dy$$

$$= \frac{\rho}{2}\int_{-1}^2 [(y + 2)^2 - y^4]\, dy = \frac{\rho}{2}\left[\frac{(y + 2)^3}{3} - \frac{y^5}{5}\right]_{-1}^2 = \frac{36\rho}{5}$$

$$\bar{x} = \frac{M_y}{m} = \frac{36\rho}{5}\left(\frac{2}{9\rho}\right) = \frac{8}{5}$$

$$M_x = \rho \int_{-1}^2 y[(y + 2) - y^2]\, dy$$

$$= \rho \int_{-1}^2 (2y + y^2 - y^3)\, dy = \rho\left[y^2 + \frac{y^3}{3} - \frac{y^4}{4}\right]_{-1}^2 = \frac{9\rho}{4}$$

$$\bar{y} = \frac{M_x}{m} = \frac{9\rho}{4}\left(\frac{2}{9\rho}\right) = \frac{1}{2}$$

$$(\bar{x}, \bar{y}) = \left(\frac{8}{5}, \frac{1}{2}\right)$$

26. $A = \int_1^4 \frac{1}{x}\, dx = \Big[\ln|x|\Big]_1^4 = \ln 4$

$$M_x = \frac{1}{2}\int_1^4 \frac{1}{x^2}\, dx = \left[\frac{1}{2}\left(-\frac{1}{x}\right)\right]_1^4 = \left(-\frac{1}{8} + \frac{1}{2}\right) = \frac{3}{8}$$

$$M_y = \int_1^4 x\left(\frac{1}{x}\right) dx = \Big[x\Big]_1^4 = 3$$

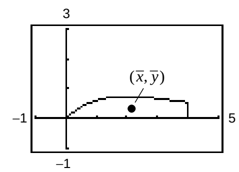
28. $A = \int_{-2}^{2} -(x^2 - 4)\,dx = 2\int_{0}^{2} (4 - x^2)\,dx = \left[8x - \frac{2x^3}{3}\right]_0^2 = 16 - \frac{16}{3} = \frac{32}{3}$

$$M_x = \frac{1}{2}\int_{-2}^{2} (x^2 - 4)(4 - x^2)\,dx = -\frac{1}{2}\int_{-2}^{2} (x^4 - 8x^2 + 16)\,dx$$

$$= -\frac{1}{2}\left[\frac{x^5}{5} - \frac{8x^3}{3} + 16x\right]_{-2}^{2} = -\left[\frac{32}{5} - \frac{64}{3} + 32\right] = -\frac{256}{15}$$

$M_y = 0$ by symmetry.

30. $m = \rho\int_0^4 xe^{-x/2}\,dx \approx 2.3760\rho$



$$M_x = \rho\int_0^4 \left(\frac{xe^{-x/2}}{2}\right)(xe^{-x/2})\,dx = \frac{\rho}{2}\int_0^4 x^2e^{-x}\,dx \approx 0.7619\rho$$

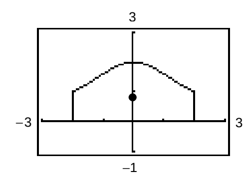
$$M_y = \rho\int_0^4 x^2e^{-x/2}\,dx \approx 5.1732\rho$$

$$\bar{x} = \frac{M_y}{m} \approx 2.2$$

$$\bar{y} = \frac{M_x}{m} \approx 0.3$$

Therefore, the centroid is (2.2, 0.3).

32. $m = \rho\int_{-2}^{2} \frac{8}{x^2 + 4}\,dx \approx 6.2832\rho$

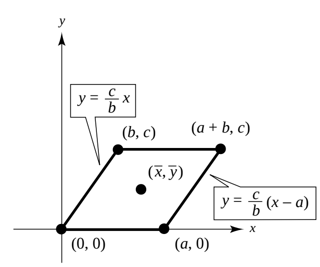


$$M_x = \rho\int_{-2}^{2} \frac{1}{2}\left(\frac{8}{x^2 + 4}\right)\left(\frac{8}{x^2 + 4}\right)dx = 32\rho\int_{-2}^{2} \frac{1}{(x^2 + 4)^2}\,dx \approx 5.14149\rho$$

$$\bar{y} = \frac{M_x}{m} \approx 0.8$$

$\bar{x} = 0$ by symmetry. Therefore, the centroid is (0, 0.8).

34. $A = bh = ac$



$$\frac{1}{A} = \frac{1}{ac}$$

$$\bar{x} = \frac{1}{ac}\frac{1}{2}\int_0^c \left[\left(\frac{b}{c}y + a\right)^2 - \left(\frac{b}{c}y\right)^2\right]dy$$

$$= \frac{1}{2ac}\int_0^c \left(\frac{2ab}{c}y + a^2\right)dy$$

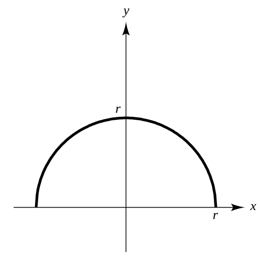
$$= \frac{1}{2ac}\left[\frac{ab}{c}y^2 + a^2y\right]_0^c = \frac{1}{2ac}[abc + a^2c] = \frac{1}{2}(b + a)$$

$$\bar{y} = \frac{1}{ac}\int_0^c y\left[\left(\frac{b}{c}y + a\right) - \left(\frac{b}{c}y\right)\right]dy = \left[\frac{1}{c}\frac{y^2}{2}\right]_0^c = \frac{c}{2}$$

$$(\bar{x}, \bar{y}) = \left(\frac{b + a}{2}, \frac{c}{2}\right)$$

This is the point of intersection of the diagonals.

36. $\bar{x} = 0$ by symmetry



$$A = \frac{1}{2}\pi r^2$$

$$\frac{1}{A} = \frac{2}{\pi r^2}$$

$$\bar{y} = \frac{2}{\pi r^2}\frac{1}{2}\int_{-r}^{r}\left(\sqrt{r^2 - x^2}\right)^2 dx$$

$$= \frac{1}{\pi r^2}\left[r^2 x - \frac{x^3}{3}\right]_{-r}^{r} = \frac{1}{\pi r^2}\left[\frac{4r^3}{3}\right] = \frac{4r}{3\pi}$$

$$(\bar{x}, \bar{y}) = \left(0, \frac{4r}{3\pi}\right)$$

38. $$A = \int_0^1 [1 - (2x - x^2)]\, dx = \frac{1}{3}$$

$$\frac{1}{A} = 3$$

$$\bar{x} = 3\int_0^1 x[1 - (2x - x^2)]\, dx = 3\int_0^1 [x - 2x^2 + x^3]\, dx = 3\left[\frac{x^2}{2} - \frac{2}{3}x^3 + \frac{x^4}{4}\right]_0^1 = \frac{1}{4}$$

$$\bar{y} = 3\int_0^1 \frac{[1 + (2x - x^2)]}{2}[1 - (2x - x^2)]\, dx = \frac{3}{2}\int_0^1 [1 - (2x - x^2)^2]\, dx$$

$$= \frac{3}{2}\int_0^1 [1 - 4x^2 + 4x^3 - x^4]\, dx = \frac{3}{2}\left[x - \frac{4}{3}x^3 + x^4 - \frac{x^5}{5}\right]_0^1 = \frac{7}{10}$$

$$(\bar{x}, \bar{y}) = \left(\frac{1}{4}, \frac{7}{10}\right)$$

40. (a) $M_y = 0$ by symmetry

$$M_y = \int_{-\sqrt[2n]{b}}^{\sqrt[2n]{b}} x(b - x^{2n})\, dx = 0$$

because $bx - x^{2n+1}$ is an odd function.

(b) $\bar{y} > \frac{b}{2}$ because there is more area above $y = \frac{b}{2}$ than below.

(c) $$M_x = \int_{-\sqrt[2n]{b}}^{\sqrt[2n]{b}} \frac{(b + x^{2n})(b - x^{2n})}{2}\, dx = \int_{-\sqrt[2n]{b}}^{\sqrt[2n]{b}} \frac{1}{2}(b^2 - x^{4n})\, dx$$

$$= \frac{1}{2}\left(b^2 x - \frac{x^{4n+1}}{4n+1}\right)\Bigg]_{-\sqrt[2n]{b}}^{\sqrt[2n]{b}}$$

$$= b^2 b^{1/2n} - \frac{b^{(4n+1)/2n}}{4n+1} = \frac{4n}{4n+1}b^{(4n+1)/2n}$$

$$A = \int_{-\sqrt[2n]{b}}^{\sqrt[2n]{b}} (b - x^{2n})\, dx = 2\left[bx - \frac{x^{2n+1}}{2n+1}\right]_0^{\sqrt[2n]{b}}$$

$$= 2\left[b \cdot b^{1/2n} - \frac{b^{(2n+1)/2n}}{2n+1}\right] = \frac{4n}{2n+1}b^{(2n+1)/2n}$$

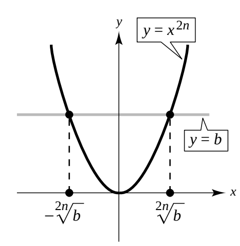
$$\bar{y} = \frac{M_x}{A} = \frac{4n\, b^{(4n+1)/2n}/(4n+1)}{4n\, b^{(24n+1)/2n}/(2n+1)} = \frac{2n+1}{4n+1}b$$

(d)

n	1	2	3	4
$\bar{y}$	$\frac{3}{5}b$	$\frac{5}{9}b$	$\frac{7}{13}b$	$\frac{9}{17}b$

(e) $$\lim_{n\to\infty} \bar{y} = \lim_{n\to\infty} \frac{2n+1}{4n+1}b = \frac{1}{2}b$$

(f) As $n \to \infty$, the figure gets narrower.



42. Let $f(x)$ be the top curve, given by $l + d$. The bottom curve is $d(x)$.

x	0	0.5	1.0	1.5	2.0
f	2.0	1.93	1.73	1.32	0
d	0.50	0.48	0.43	0.33	0

(a) $$\text{Area} = 2\int_0^2 [f(x) - d(x)]\,dx$$

$$\approx 2\frac{2}{3(4)}[1.50 + 4(1.45) + 2(1.30) + 4(.99) + 0]$$

$$= \frac{1}{3}[13.86] = 4.62$$

$$M_x = \int_{-2}^{2} \frac{f(x) + d(x)}{2}(f(x) - d(x))\,dx$$

$$= \int_0^2 [f(x)^2 - d(x)^2]\,dx$$

$$= \frac{2}{3(4)}[3.75 + 4(3.4945) + 2(2.808) + 4(1.6335) + 0]$$

$$= \frac{1}{6}[29.878] = 4.9797$$

$$\bar{y} = \frac{M_x}{A} = \frac{4.9797}{4.62} = 1.078$$

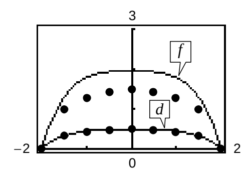
$$(\bar{x}, \bar{y}) = (0, 1.078)$$

(b) $f(x) = -0.1061x^4 - 0.06126x^2 + 1.9527$

$d(x) = -0.02648x^4 - 0.01497x^2 + .4862$

(c) $$\bar{y} = \frac{M_x}{A} \approx \frac{4.9133}{4.59998} = 1.068$$

$$(\bar{x}, \bar{y}) = (0, 1.068)$$



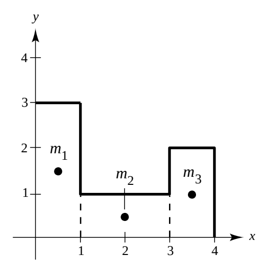
44. Centroids of the given regions: $\left(\frac{1}{2}, \frac{3}{2}\right)$, $\left(2, \frac{1}{2}\right)$, and $\left(\frac{7}{2}, 1\right)$

Area: $A = 3 + 2 + 2 = 7$

$$\bar{x} = \frac{3(1/2) + 2(2) + 2(7/2)}{7} = \frac{25/2}{7} = \frac{25}{14}$$

$$\bar{y} = \frac{3(3/2) + 2(1/2) + 2(1)}{7} = \frac{15/2}{7} = \frac{15}{14}$$

$$(\bar{x}, \bar{y}) = \left(\frac{25}{14}, \frac{15}{14}\right)$$



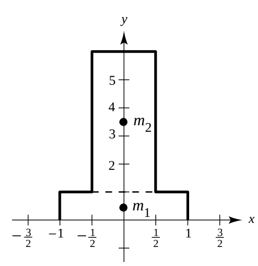
46. $m_1 = \frac{7}{8}(2) = \frac{7}{4}$, $P_1 = \left(0, \frac{7}{16}\right)$

$m_2 = \frac{7}{8}\left(6 - \frac{7}{8}\right) = \frac{287}{64}$, $P_2 = \left(0, \frac{55}{16}\right)$

By symmetry, $\bar{x} = 0$.

$$\bar{y} = \frac{(7/4)(7/16) + (287/64)(55/16)}{(7/4) + (287/64)} = \frac{16{,}569}{6384} = \frac{5523}{2128}$$

$$(\bar{x}, \bar{y}) = \left(0, \frac{5523}{2128}\right) \approx (0, 2.595)$$



48. Centroids of the given regions: $(3, 0)$ and $(1, 0)$

Mass: $8 + \pi$

$$\bar{y} = 0$$

$$\bar{x} = \frac{8(1) + \pi(3)}{8 + \pi} = \frac{8 + 3\pi}{8 + \pi}$$

$$(\bar{x}, \bar{y}) = \left(\frac{8 + 3\pi}{8 + \pi}, 0\right) \approx (1.56, 0)$$

50. $V = 2\pi rA = 2\pi(3)(4\pi) = 24\pi^2$

52. $A = \int_2^6 2\sqrt{x - 2}\, dx = \frac{4}{3}(x - 2)^{3/2}\Big]_2^6 = \frac{32}{3}$

$$M_y = \int_2^6 (x)2\sqrt{x - 2}\, dx = 2\int_2^6 x\sqrt{x - 2}\, dx$$

Let $u = x - 2$, $x = u + 2$, $du = dx$:

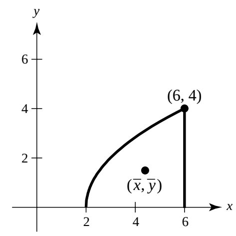
$$M_y = 2\int_0^4 (u + 2)\sqrt{u}\, du = 2\int_0^4 (u^{3/2} + 2u^{1/2})\, du = 2\left[\frac{2}{5}u^{5/2} + \frac{4}{3}u^{3/2}\right]_0^4$$

$$= 2\left[\frac{64}{5} + \frac{32}{3}\right] = \frac{704}{15}$$

$$\bar{x} = \frac{M_y}{A} = \frac{704/15}{32/3} = \frac{22}{5}$$

$$r = \bar{x} = \frac{22}{5}$$

$$V = 2\pi rA = 2\pi\left(\frac{22}{5}\right)\left(\frac{32}{3}\right) = \frac{1408\pi}{15} \approx 294.89$$



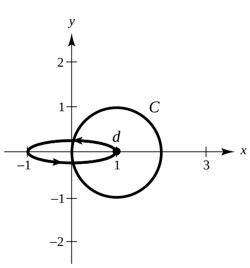
54. A planar lamina is a thin flat plate of constant density. The center of mass $(\bar{x}, \bar{y})$ is the balancing point on the lamina.

56. Let R be a region in a plane and let L be a line such that L does not intersect the interior of R. If r is the distance between the centroid of R and L, then the volume V of the solid of revolution formed by revolving R about L is

$$V = 2\pi rA$$

where A is the area of R.

58. The centroid of the circle is $(1, 0)$. The distance traveled by the centroid is 2π. The arc length of the circle is also 2π. Therefore, $S = (2\pi)(2\pi) = 4\pi^2$.



Section 6.7 Fluid Pressure and Fluid Force

2. $F = PA = [62.4(5)](16) = 4992 \text{ lb}$

4. $F = 62.4(h + 4)(48) - (62.4)(h)(48)$

$= 62.4(4)(48) = 11{,}980.8 \text{ lb}$

6. $h(y) = 3 - y$

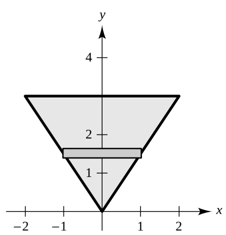
$L(y) = \frac{4}{3}y$

$F = 62.4 \int_0^3 (3 - y)\left(\frac{4}{3}y\right) dy$

$= \frac{4}{3}(62.4) \int_0^3 (3y - y^2)\, dy$

$= \frac{4}{3}(62.4)\left[\frac{3y^2}{2} - \frac{y^3}{3}\right]_0^3 = 374.4 \text{ lb}$

Force is one-third that of Exercise 5.

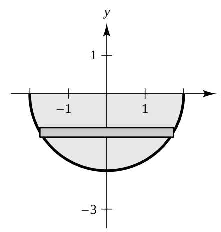


8. $h(y) = -y$

$L(y) = 2\sqrt{4 - y^2}$

$F = 62.4 \int_{-2}^0 (-y)(2)\sqrt{4 - y^2}\, dy$

$= \left[62.4\left(\frac{2}{3}\right)(4 - y^2)^{3/2}\right]_{-2}^0 = 332.8 \text{ lb}$



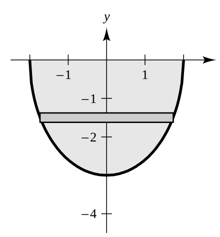
10. $h(y) = -y$

$L(y) = \frac{4}{3}\sqrt{9 - y^2}$

$F = 62.4 \int_{-3}^0 (-y)\frac{4}{3}\sqrt{9 - y^2}\, dy$

$= 62.4\left(\frac{2}{3}\right) \int_{-3}^0 (9 - y^2)^{1/2}(-2y)\, dy$

$= \left[62.4\left(\frac{4}{9}\right)(9 - y^2)^{3/2}\right]_{-3}^0 = 748.8 \text{ lb}$



12. $h(y) = \left(1 + 3\sqrt{2}\right) - y$

$L_1(y) = 2y$ (lower part)

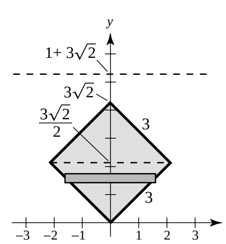
$L_2(y) = 2\left(3\sqrt{2} - y\right)$ (upper part)

$F = 2(9800)\left[\int_0^{3\sqrt{2}/2} \left(1 + 3\sqrt{2} - y\right)y\, dy + \int_{3\sqrt{2}/2}^{3\sqrt{2}} \left(1 + 3\sqrt{2} - y\right)\left(3\sqrt{2} - y\right) dy\right]$

$= 19{,}600\left[\left[\frac{y^2}{2} - 3\sqrt{2}y - \frac{y^3}{3}\right]_0^{3\sqrt{2}/2} + \left[3\sqrt{2}y + 18y + \frac{y^3}{3} - \frac{6\sqrt{2} + 1}{2}y\right]_{3\sqrt{2}/2}^{3\sqrt{2}}\right]$

$= 19{,}600\left[\frac{9\left(2\sqrt{2} + 1\right)}{4} + \frac{9\left(\sqrt{2} + 1\right)}{4}\right]$

$= 44{,}100\left(3\sqrt{2} + 2\right)$ Newtons

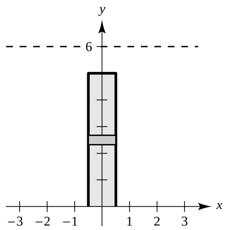


14. $h(y) = 6 - y$

$L(y) = 1$

$$F = 9800 \int_0^5 1(6 - y)\, dy$$

$$= 9800\left[6y - \frac{y^2}{2}\right]_0^5 = 171{,}500 \text{ Newtons}$$



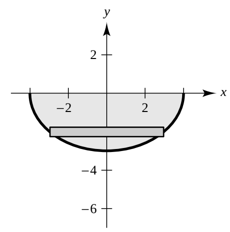
16. $h(y) = -y$

$$L(y) = 2\left(\frac{4}{3}\sqrt{9 - y^2}\right)$$

$$F = 140.7 \int_{-3}^{0} (-y)(2)\left(\frac{4}{3}\sqrt{9 - y^2}\right) dy$$

$$= \frac{(140.7)(4)}{3} \int_{-3}^{0} \sqrt{9 - y^2}\,(-2y)\, dy$$

$$= \left[\frac{(140.7)(4)}{3}\left(\frac{2}{3}\right)(9 - y^2)^{3/2}\right]_{-3}^{0} = 3376.8 \text{ lb}$$



18. $h(y) = -y$

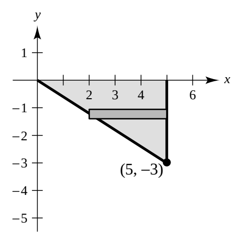
$$L(y) = 5 + \frac{5}{3}y$$

$$F = 140.7 \int_{-3}^{0} (-y)\left(5 + \frac{5}{3}y\right) dy$$

$$= 140.7 \int_{-3}^{0} \left(-5y - \frac{5}{3}y^2\right) dy$$

$$= 140.7\left[-\frac{5}{2}y^2 - \frac{5}{9}y^3\right]_{-3}^{0}$$

$$= 140.7\left[\frac{45}{2} - 15\right] = 1055.25 \text{ lb}$$



20. $h(y) = \dfrac{3}{2} - y$

$$L(y) = 2\left(\frac{1}{2}\right)\sqrt{9 - 4y^2}$$

$$F = 42 \int_{-3/2}^{3/2} \left(\frac{3}{2} - y\right)\sqrt{9 - 4y^2}\, dy = 63 \int_{-3/2}^{3/2} \sqrt{9 - 4y^2}\, dy + \frac{21}{4} \int_{-3/2}^{3/2} \sqrt{9 - 4y^2}\,(-8y)\, dy$$

The second integral is zero since it is an odd function and the limits of integration are symmetric to the origin. The first integral is twice the area of a semicircle of radius $\frac{3}{2}$.

$$\left(\sqrt{9 - 4y^2} = 2\sqrt{(9/4) - y^2}\right)$$

Thus, the force is $63\left(\frac{9}{4}\pi\right) = 141.75\pi \approx 445.32$ lb.

22. (a) $F = wk\pi r^2 = (62.4)(7)(\pi 2^2) = 1747.2\pi$ lbs

(b) $F = wk\pi r^2 = (62.4)(5)(\pi 3^2) = 2808\pi$ lbs

24. (a) $F = wkhb = (62.4)\left(\dfrac{11}{2}\right)(3)(5) = 5148$ lbs

(b) $F = wkhb = (62.4)\left(\dfrac{17}{5}\right)(5)(10) = 10{,}608$ lbs

26. From Exercise 21:

$$F = 64(15)\pi\left(\frac{1}{2}\right)^2 \approx 753.98 \text{ lb}$$

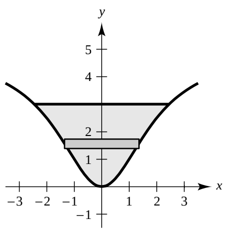
28. $h(y) = 3 - y$

Solving $y = 5x^2/(x^2 + 4)$ for x, you obtain $x = \sqrt{4y/(5 - y)}$.

$$L(y) = 2\sqrt{\frac{4y}{5 - y}}$$

$$F = 62.4(2)\int_0^3 (3 - y)\sqrt{\frac{4y}{5 - y}}\,dy$$

$$= 2(124.8)\int_0^3 (3 - y)\sqrt{\frac{y}{5 - y}}\,dy \approx 546.265 \text{ lb}$$

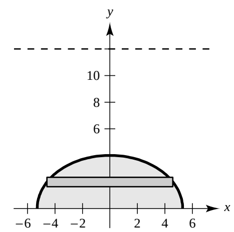


30. $h(y) = 12 - y$

$$L(y) = 2\frac{\sqrt{7(16 - y^2)}}{2} = \sqrt{7(16 - y^2)}$$

$$F = 62.4\int_0^4 (12 - y)\sqrt{7(16 - y^2)}\,dy$$

$$= 62.4\sqrt{7}\int_0^4 (12 - y)\sqrt{16 - y^2}\,dy \approx 21373.7 \text{ lb}$$



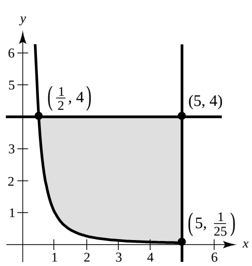
32. Fluid pressure is the force per unit of area exerted by a fluid over the surface of a body.

34. The left window experiences the greater fluid force because its centroid is lower.

Review Exercises for Chapter 6

2. $A = \displaystyle\int_{1/2}^5 \left(4 - \frac{1}{x^2}\right) dx$

$$= \left[4x + \frac{1}{x}\right]_{1/2}^5 = \frac{81}{5}$$

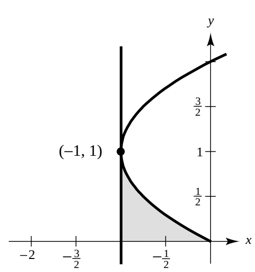


4. $A = \displaystyle\int_0^1 [(y^2 - 2y) - (-1)]\,dy$

$$= \int_0^1 (y^2 - 2y + 1)\,dy$$

$$= \int_0^1 (y - 1)^2\,dy$$

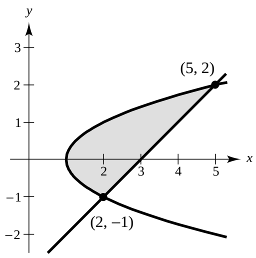
$$= \left[\frac{(y - 1)^3}{3}\right]_0^1 = \frac{1}{3}$$



6. $A = \int_{-1}^{2} [(y + 3) - (y^2 + 1)]\, dy$

$= \int_{-1}^{2} (2 + y - y^2)\, dy$

$= \left[2y + \frac{1}{2}y^2 - \frac{1}{3}y^3\right]_{-1}^{2} = \frac{9}{2}$

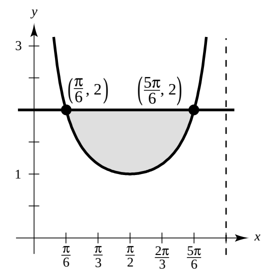


8. $A = 2\int_{\pi/6}^{\pi/2} (2 - \csc x)\, dx$

$= 2\Big[2x - \ln|\csc x - \cot x|\Big]_{\pi/6}^{\pi/2}$

$= 2\left([\pi - 0] - \left[\frac{\pi}{3} - \ln\left(2 - \sqrt{3}\right)\right]\right)$

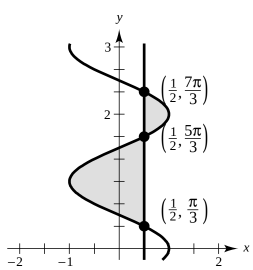
$= 2\left[\frac{2\pi}{3} + \ln\left(2 - \sqrt{3}\right)\right] \approx 1.555$



10. $A = \int_{\pi/3}^{5\pi/3} \left(\frac{1}{2} - \cos y\right) dy + \int_{5\pi/3}^{7\pi/3} \left(\cos y - \frac{1}{2}\right) dy$

$= \left[\frac{y}{2} - \sin y\right]_{\pi/3}^{5\pi/3} + \left[\sin y - \frac{y}{2}\right]_{5\pi/3}^{7\pi/3}$

$= \frac{\pi}{3} + 2\sqrt{3}$



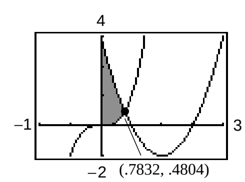
12. Point of intersection is given by:

$x^3 - x^2 + 4x - 3 = 0 \implies x \approx 0.783.$

$A \approx \int_{0}^{0.783} (3 - 4x + x^2 - x^3)\, dx$

$= \left[3x - 2x^2 + \frac{1}{3}x^3 - \frac{1}{4}x^4\right]_{0}^{0.783}$

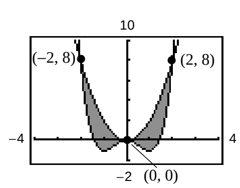
≈ 1.189



14. $A = 2\int_{0}^{2} [2x^2 - (x^4 - 2x^2)]\, dx$

$= 2\int_{0}^{2} (4x^2 - x^4)\, dx$

$= 2\left[\frac{4}{3}x^3 - \frac{1}{5}x^5\right]_{0}^{2} = \frac{128}{15} \approx 8.5333$



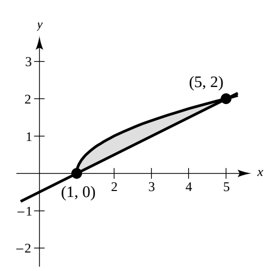
16. $y = \sqrt{x - 1} \implies x = y^2 + 1$

$y = \frac{x - 1}{2} \implies x = 2y + 1$

$A = \int_{0}^{2} [(2y + 1) - (y^2 + 1)]\, dy$

$= \int_{1}^{5} \left[\sqrt{x - 1} - \frac{x - 1}{2}\right] dx$

$= \left[\frac{2}{3}(x - 1)^{3/2} - \frac{1}{4}(x - 1)^2\right]_{1}^{5} = \frac{4}{3}$

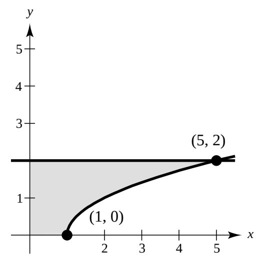


18. $A = \int_0^1 2\,dx + \int_1^5 \left[2 - \sqrt{x-1}\right] dx$

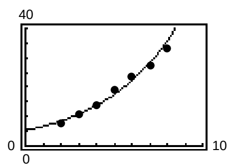
$x = y^2 + 1$

$A = \int_0^2 (y^2 + 1)\,dy$

$= \left[\frac{1}{3}y^3 + y\right]_0^2 = \frac{14}{3}$



20. (a) $R_1(t) = 5.2834(1.2701)^t = 5.2834\,e^{0.2391t}$

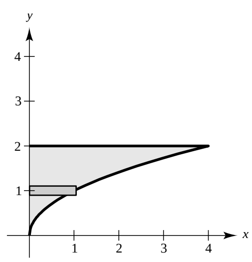


(b) $R_2(t) = 10 + 5.28\,e^{0.2t}$

Difference $= \int_{10}^{15} \left[R_1(t) - R_2(t)\right] dt \approx 171.25$ billion dollars

22. (a) **Shell**

$V = 2\pi \int_0^2 y^3\,dy = \left[\frac{\pi}{2}y^4\right]_0^2 = 8\pi$

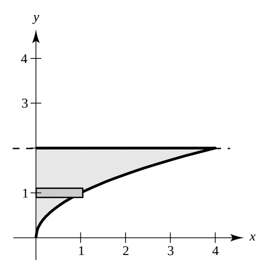


(b) **Shell**

$V = 2\pi \int_0^2 (2 - y)y^2\,dy$

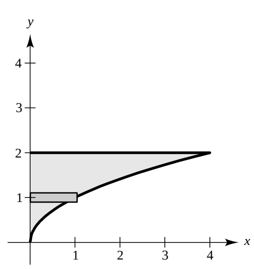
$= 2\pi \int_0^2 (2y^2 - y^3)\,dy$

$= 2\pi\left[\frac{2}{3}y^3 - \frac{1}{4}y^4\right]_0^2 = \frac{8\pi}{3}$



(c) **Disk**

$V = \pi \int_0^2 y^4\,dy = \left[\frac{\pi}{5}y^5\right]_0^2 = \frac{32\pi}{5}$

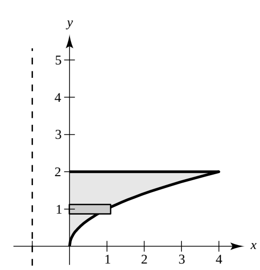


(d) **Disk**

$V = \pi \int_0^2 \left[(y^2 + 1)^2 - 1^2\right] dy$

$= \pi \int_0^2 (y^4 + 2y^2)\,dy$

$= \pi\left[\frac{1}{5}y^5 + \frac{2}{3}y^3\right]_0^2 = \frac{176\pi}{15}$

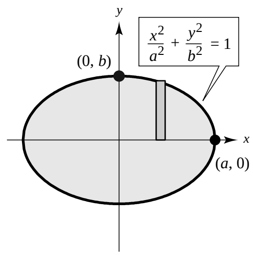


24. (a) **Shell**

$$V = 4\pi \int_0^a (x)\frac{b}{a}\sqrt{a^2 - x^2}\,dx$$

$$= \frac{-2\pi b}{a}\int_0^a (a^2 - x^2)^{1/2}(-2x)\,dx$$

$$= \left[\frac{-4\pi b}{3a}(a^2 - x^2)^{3/2}\right]_0^a = \frac{4}{3}\pi a^2 b$$

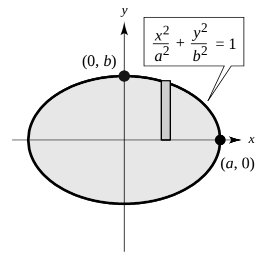


(b) **Disk**

$$V = 2\pi \int_0^a \frac{b^2}{a^2}(a^2 - x^2)\,dx$$

$$= \frac{2\pi b^2}{a^2}\left[a^2x - \frac{1}{3}x^3\right]_0^a$$

$$= \frac{4}{3}\pi a b^2$$



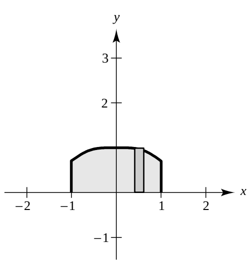
26. Disk

$$V = 2\pi \int_0^1 \left[\frac{1}{\sqrt{1 + x^2}}\right]^2 dx$$

$$= \left[2\pi \arctan x\right]_0^1$$

$$= 2\pi\left(\frac{\pi}{4} - 0\right)$$

$$= \frac{\pi^2}{2}$$

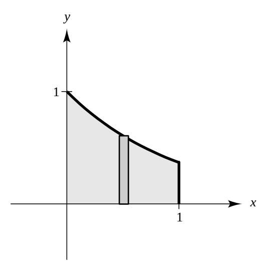


28. Disk

$$V = \pi \int_0^1 (e^{-x})^2\,dx$$

$$= \pi \int_0^1 e^{-2x}\,dx = \left[-\frac{\pi}{2}e^{-2x}\right]_0^1$$

$$= \left(\frac{-\pi}{2e^2} + \frac{\pi}{2}\right) = \frac{\pi}{2}\left(1 - \frac{1}{e^2}\right)$$

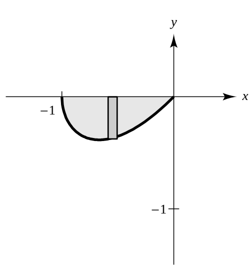


30. (a) **Disk**

$$V = \pi \int_{-1}^0 x^2(x + 1)\,dx$$

$$= \pi \int_{-1}^0 (x^3 + x^2)\,dx$$

$$= \pi\left[\frac{x^4}{4} + \frac{x^3}{3}\right]_{-1}^0 = \frac{\pi}{12}$$



(b) **Shell**

$$u = \sqrt{x + 1}$$

$$x = u^2 - 1$$

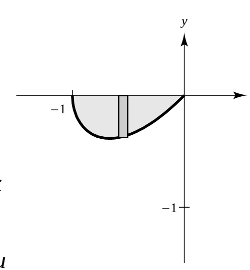
$$dx = 2u\,du$$

$$V = 2\pi \int_{-1}^0 x^2\sqrt{x + 1}\,dx$$

$$= 4\pi \int_0^1 (u^2 - 1)^2 u^2 du$$

$$= 4\pi \int_0^1 (u^6 - 2u^4 + u^2)\,du$$

$$= 4\pi\left[\frac{1}{7}u^7 - \frac{2}{5}u^5 + \frac{1}{3}u^3\right]_0^1 = \frac{32\pi}{105}$$



32. $A(x) = \frac{1}{2}bh = \frac{1}{2}\left(2\sqrt{a^2 - x^2}\right)\left(\sqrt{3}\sqrt{a^2 - x^2}\right)$

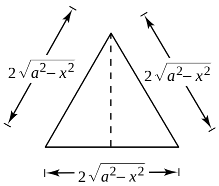
$= \sqrt{3}(a^2 - x^2)$

$$V = \sqrt{3}\int_{-a}^{a} (a^2 - x^2)\,dx = \sqrt{3}\left[a^2x - \frac{x^3}{3}\right]_{-a}^{a}$$

$$= \sqrt{3}\left(\frac{4a^3}{3}\right)$$

Since $\left(4\sqrt{3}\,a^3\right)/3 = 10$, we have $a^3 = \left(5\sqrt{3}\right)/2$. Thus,

$$a = \sqrt[3]{\frac{5\sqrt{3}}{2}} \approx 1.630 \text{ meters.}$$



34. $y = \frac{x^3}{6} + \frac{1}{2x}$

$$y' = \frac{1}{2}x^2 - \frac{1}{2x^2}$$

$$1 + (y')^2 = \left(\frac{1}{2}x^2 + \frac{1}{2x^2}\right)^2$$

$$s = \int_1^3 \left(\frac{1}{2}x^2 + \frac{1}{2x^2}\right)dx = \left[\frac{1}{6}x^3 - \frac{1}{2x}\right]_1^3 = \frac{14}{3}$$

36. Since $f(x) = \tan x$ has $f'(x) = \sec^2 x$, this integral represents the length of the graph of $\tan x$ from $x = 0$ to $x = \pi/4$. This length is a little over 1 unit. Answers (b).

38. $y = 2\sqrt{x}$

$$y' = \frac{1}{\sqrt{x}}$$

$$1 + (y')^2 = 1 + \frac{1}{x} = \frac{x+1}{x}$$

$$S = 2\pi\int_0^3 2\sqrt{x}\sqrt{\frac{x+1}{x}}\,dx = 4\pi\int_0^3 \sqrt{x+1}\,dx$$

$$= 4\pi\left[\left(\frac{2}{3}\right)(x+1)^{3/2}\right]_0^3 = \frac{56\pi}{3}$$

40. $F = kx$

$$50 = k(9) \implies k = \frac{50}{9}$$

$$F = \frac{50}{9}x$$

$$W = \int_0^9 \frac{50}{9}x\,dx = \left[\frac{25}{9}x^2\right]_0^9$$

$$= 225 \text{ in} \cdot \text{lb} = 18.75 \text{ ft} \cdot \text{lb}$$

42. We know that

$$\frac{dV}{dt} = \frac{4 \text{ gal/min} - 12 \text{ gal/min}}{7.481 \text{ gal/ft}^3} = -\frac{8}{7.481} \text{ ft}^3\text{/min}$$

$$V = \pi r^2 h = \pi\left(\frac{1}{9}\right)h$$

$$\frac{dV}{dt} = \frac{\pi}{9}\left(\frac{dh}{dt}\right)$$

$$\frac{dh}{dt} = \frac{9}{\pi}\left(\frac{dV}{dt}\right) = \frac{9}{\pi}\left(-\frac{8}{7.481}\right) \approx -3.064 \text{ ft/min.}$$

Depth of water: $-3.064t + 150$

Time to drain well: $t = \frac{150}{3.064} \approx 49$ minutes

$(49)(12) = 588$ gallons pumped

Volume of water pumped in Exercise 41: 391.7 gallons

$$\frac{391.7}{52\pi} = \frac{588}{x\pi}$$

$$x = \frac{588(52)}{391.7} \approx 78$$

Work $\approx 78\pi$ ft $\cdot$ ton

44. (a) Weight of section of cable: $4\,\Delta x$

Distance: $200 - x$

$$W = 4\int_0^{200} (200 - x)\,dx = \left[-2(200 - x)^2\right]_0^{200} = 80{,}000 \text{ ft} \cdot \text{lb} = 40 \text{ ft} \cdot \text{ton}$$

(b) Work to move 300 pounds 200 feet vertically: $200(300) = 60{,}000 \text{ ft} \cdot \text{lb} = 30 \text{ ft} \cdot \text{ton}$

Total work = work for drawing up the cable + work of lifting the load

$= 40 \text{ ft} \cdot \text{ton} + 30 \text{ ft} \cdot \text{ton} = 70 \text{ ft} \cdot \text{ton}$

46. $W = \int_a^b F(x)\,dx$

$$F(x) = \begin{cases} -(2/9)x + 6, & 0 \le x \le 9 \\ -(4/3)x + 16, & 9 \le x \le 12 \end{cases}$$

$$W = \int_0^9 \left(-\frac{2}{9}x + 6\right)dx + \int_9^{12}\left(-\frac{4}{3}x + 16\right)dx$$

$$= \left[-\frac{1}{9}x^2 + 6x\right]_0^9 + \left[-\frac{2}{3}x^2 + 16x\right]_9^{12}$$

$$= (-9 + 54) + (-96 + 192 + 54 - 144) = 51 \text{ ft} \cdot \text{lbs}$$

48. $A = \int_{-1}^3 \left[(2x + 3) - x^2\right]dx = \left[x^2 + 3x - \frac{1}{3}x^3\right]_{-1}^3 = \frac{32}{3}$

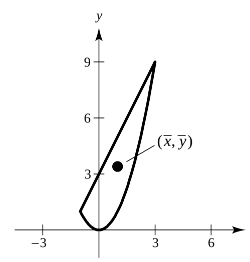
$$\frac{1}{A} = \frac{3}{32}$$

$$\bar{x} = \frac{3}{32}\int_{-1}^3 x(2x + 3 - x^2)\,dx = \frac{3}{32}\int_{-1}^3 (3x + 2x^2 - x^3)\,dx = \frac{3}{32}\left[\frac{3}{2}x^2 + \frac{2}{3}x^3 - \frac{1}{4}x^4\right]_{-1}^3 = 1$$

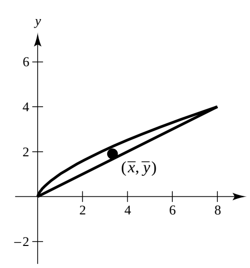
$$\bar{y} = \left(\frac{3}{32}\right)\frac{1}{2}\int_{-1}^3 \left[(2x + 3)^2 - x^4\right]dx = \frac{3}{64}\int_{-1}^3 (9 + 12x + 4x^2 - x^4)\,dx$$

$$= \frac{3}{64}\left[9x + 6x^2 + \frac{4}{3}x^3 - \frac{1}{5}x^5\right]_{-1}^3 = \frac{17}{5}$$

$$(\bar{x}, \bar{y}) = \left(1, \frac{17}{5}\right)$$



50. $A = \int_0^8 \left(x^{2/3} - \frac{1}{2}x\right)dx = \left[\frac{3}{5}x^{5/3} - \frac{1}{4}x^2\right]_0^8 = \frac{16}{5}$



$$\frac{1}{A} = \frac{5}{16}$$

$$\bar{x} = \frac{5}{16}\int_0^8 x\left(x^{2/3} - \frac{1}{2}x\right)dx$$

$$= \;= \frac{5}{16}\left[\frac{3}{8}x^{8/3} - \frac{1}{6}x^3\right]_0^8 = \frac{10}{3}$$

$$\bar{y} = \left(\frac{5}{16}\right)\frac{1}{2}\int_0^8 \left(x^{4/3} - \frac{1}{4}x^2\right)dx$$

$$= \frac{1}{2}\left(\frac{5}{16}\right)\left[\frac{3}{7}x^{7/3} - \frac{1}{12}x^3\right]_0^8 = \frac{40}{21}$$

$$(\bar{x}, \bar{y}) = \left(\frac{10}{3}, \frac{40}{21}\right)$$