26. From Exercise 21:

$$F = 64(15)\pi\left(\frac{1}{2}\right)^2 \approx 753.98 \text{ lb}$$

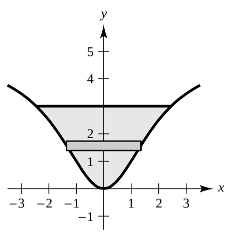
28. $h(y) = 3 - y$

Solving $y = 5x^2/(x^2 + 4)$ for x, you obtain $x = \sqrt{4y/(5 - y)}$.

$$L(y) = 2\sqrt{\frac{4y}{5 - y}}$$

$$F = 62.4(2)\int_0^3 (3 - y)\sqrt{\frac{4y}{5 - y}}\,dy$$

$$= 2(124.8)\int_0^3 (3 - y)\sqrt{\frac{y}{5 - y}}\,dy \approx 546.265 \text{ lb}$$

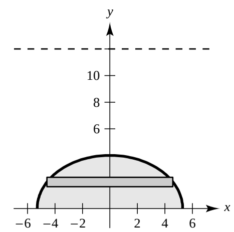


30. $h(y) = 12 - y$

$$L(y) = 2\frac{\sqrt{7(16 - y^2)}}{2} = \sqrt{7(16 - y^2)}$$

$$F = 62.4\int_0^4 (12 - y)\sqrt{7(16 - y^2)}\,dy$$

$$= 62.4\sqrt{7}\int_0^4 (12 - y)\sqrt{16 - y^2}\,dy \approx 21373.7 \text{ lb}$$



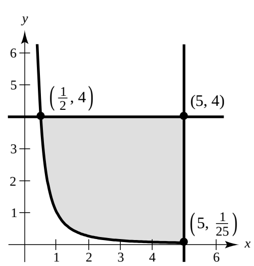
32. Fluid pressure is the force per unit of area exerted by a fluid over the surface of a body.

34. The left window experiences the greater fluid force because its centroid is lower.

Review Exercises for Chapter 6

2. $A = \int_{1/2}^5 \left(4 - \frac{1}{x^2}\right) dx$

$$= \left[4x + \frac{1}{x}\right]_{1/2}^5 = \frac{81}{5}$$

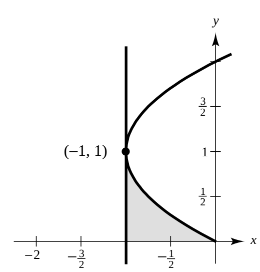


4. $A = \int_0^1 \left[(y^2 - 2y) - (-1)\right] dy$

$$= \int_0^1 (y^2 - 2y + 1)\,dy$$

$$= \int_0^1 (y - 1)^2\,dy$$

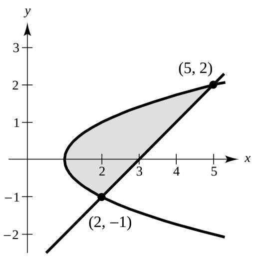
$$= \left[\frac{(y - 1)^3}{3}\right]_0^1 = \frac{1}{3}$$



6. $A = \int_{-1}^{2} [(y + 3) - (y^2 + 1)]\, dy$

$= \int_{-1}^{2} (2 + y - y^2)\, dy$

$= \left[2y + \frac{1}{2}y^2 - \frac{1}{3}y^3\right]_{-1}^{2} = \frac{9}{2}$

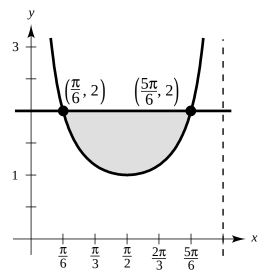


8. $A = 2\int_{\pi/6}^{\pi/2} (2 - \csc x)\, dx$

$= 2\Big[2x - \ln|\csc x - \cot x|\Big]_{\pi/6}^{\pi/2}$

$= 2\left([\pi - 0] - \left[\frac{\pi}{3} - \ln\left(2 - \sqrt{3}\right)\right]\right)$

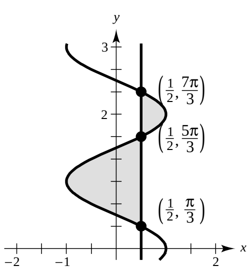
$= 2\left[\frac{2\pi}{3} + \ln\left(2 - \sqrt{3}\right)\right] \approx 1.555$



10. $A = \int_{\pi/3}^{5\pi/3} \left(\frac{1}{2} - \cos y\right) dy + \int_{5\pi/3}^{7\pi/3} \left(\cos y - \frac{1}{2}\right) dy$

$= \left[\frac{y}{2} - \sin y\right]_{\pi/3}^{5\pi/3} + \left[\sin y - \frac{y}{2}\right]_{5\pi/3}^{7\pi/3}$

$= \frac{\pi}{3} + 2\sqrt{3}$



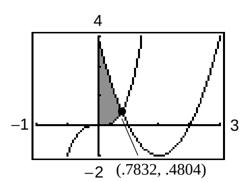
12. Point of intersection is given by:

$x^3 - x^2 + 4x - 3 = 0 \implies x \approx 0.783.$

$A \approx \int_{0}^{0.783} (3 - 4x + x^2 - x^3)\, dx$

$= \left[3x - 2x^2 + \frac{1}{3}x^3 - \frac{1}{4}x^4\right]_{0}^{0.783}$

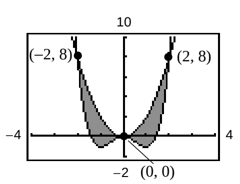
≈ 1.189



14. $A = 2\int_{0}^{2} [2x^2 - (x^4 - 2x^2)]\, dx$

$= 2\int_{0}^{2} (4x^2 - x^4)\, dx$

$= 2\left[\frac{4}{3}x^3 - \frac{1}{5}x^5\right]_{0}^{2} = \frac{128}{15} \approx 8.5333$



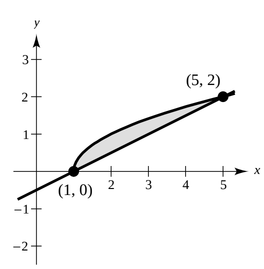
16. $y = \sqrt{x - 1} \implies x = y^2 + 1$

$y = \frac{x - 1}{2} \implies x = 2y + 1$

$A = \int_{0}^{2} [(2y + 1) - (y^2 + 1)]\, dy$

$= \int_{1}^{5} \left[\sqrt{x - 1} - \frac{x - 1}{2}\right] dx$

$= \left[\frac{2}{3}(x - 1)^{3/2} - \frac{1}{4}(x - 1)^2\right]_{1}^{5} = \frac{4}{3}$

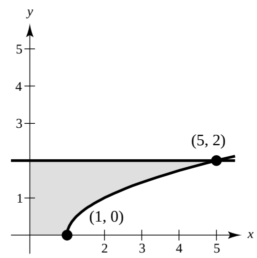


18. $A = \int_0^1 2\,dx + \int_1^5 \left[2 - \sqrt{x-1}\right] dx$

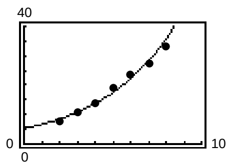
$x = y^2 + 1$

$A = \int_0^2 (y^2 + 1)\,dy$

$= \left[\frac{1}{3}y^3 + y\right]_0^2 = \frac{14}{3}$



20. (a) $R_1(t) = 5.2834(1.2701)^t = 5.2834\,e^{0.2391t}$

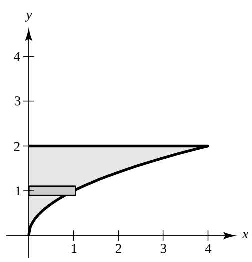


(b) $R_2(t) = 10 + 5.28\,e^{0.2t}$

$\text{Difference} = \int_{10}^{15} [\mathrm{R}_1(t) - \mathrm{R}_2(t)]\,dt \approx 171.25$ billion dollars

22. (a) **Shell**

$V = 2\pi \int_0^2 y^3\,dy = \left[\frac{\pi}{2}y^4\right]_0^2 = 8\pi$

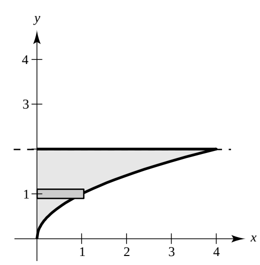


(b) **Shell**

$V = 2\pi \int_0^2 (2 - y)y^2\,dy$

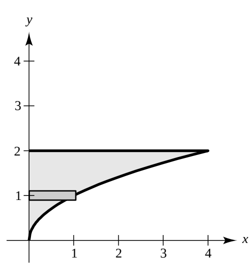
$= 2\pi \int_0^2 (2y^2 - y^3)\,dy$

$= 2\pi\left[\frac{2}{3}y^3 - \frac{1}{4}y^4\right]_0^2 = \frac{8\pi}{3}$



(c) **Disk**

$V = \pi \int_0^2 y^4\,dy = \left[\frac{\pi}{5}y^5\right]_0^2 = \frac{32\pi}{5}$

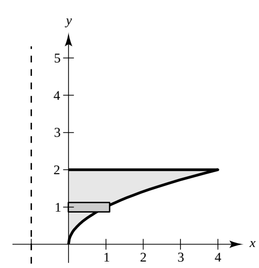


(d) **Disk**

$V = \pi \int_0^2 \left[(y^2 + 1)^2 - 1^2\right] dy$

$= \pi \int_0^2 (y^4 + 2y^2)\,dy$

$= \pi\left[\frac{1}{5}y^5 + \frac{2}{3}y^3\right]_0^2 = \frac{176\pi}{15}$

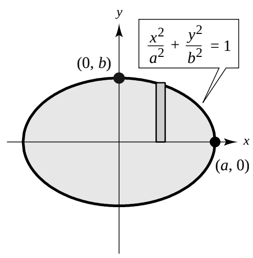


24. (a) **Shell**

$$V = 4\pi\int_0^a (x)\frac{b}{a}\sqrt{a^2 - x^2}\,dx$$

$$= \frac{-2\pi b}{a}\int_0^a (a^2 - x^2)^{1/2}(-2x)\,dx$$

$$= \left[\frac{-4\pi b}{3a}(a^2 - x^2)^{3/2}\right]_0^a = \frac{4}{3}\pi a^2 b$$

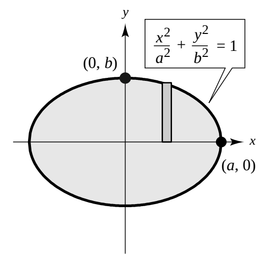


(b) **Disk**

$$V = 2\pi\int_0^a \frac{b^2}{a^2}(a^2 - x^2)\,dx$$

$$= \frac{2\pi b^2}{a^2}\left[a^2x - \frac{1}{3}x^3\right]_0^a$$

$$= \frac{4}{3}\pi ab^2$$



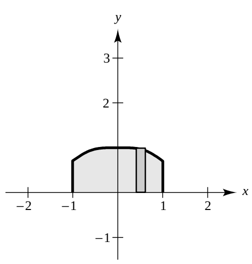
26. Disk

$$V = 2\pi\int_0^1 \left[\frac{1}{\sqrt{1 + x^2}}\right]^2 dx$$

$$= \left[2\pi\arctan x\right]_0^1$$

$$= 2\pi\left(\frac{\pi}{4} - 0\right)$$

$$= \frac{\pi^2}{2}$$

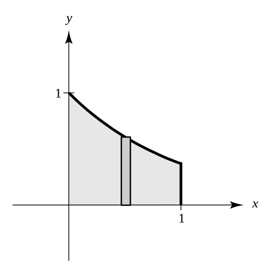


28. Disk

$$V = \pi\int_0^1 (e^{-x})^2\,dx$$

$$= \pi\int_0^1 e^{-2x}\,dx = \left[-\frac{\pi}{2}e^{-2x}\right]_0^1$$

$$= \left(\frac{-\pi}{2e^2} + \frac{\pi}{2}\right) = \frac{\pi}{2}\left(1 - \frac{1}{e^2}\right)$$

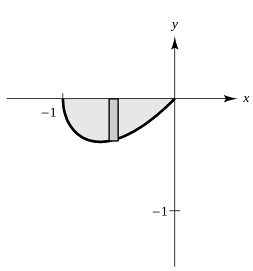


30. (a) **Disk**

$$V = \pi\int_{-1}^0 x^2(x + 1)\,dx$$

$$= \pi\int_{-1}^0 (x^3 + x^2)\,dx$$

$$= \pi\left[\frac{x^4}{4} + \frac{x^3}{3}\right]_{-1}^0 = \frac{\pi}{12}$$



(b) **Shell**

$$u = \sqrt{x + 1}$$

$$x = u^2 - 1$$

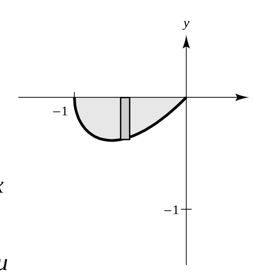
$$dx = 2u\,du$$

$$V = 2\pi\int_{-1}^0 x^2\sqrt{x + 1}\,dx$$

$$= 4\pi\int_0^1 (u^2 - 1)^2u^2du$$

$$= 4\pi\int_0^1 (u^6 - 2u^4 + u^2)\,du$$

$$= 4\pi\left[\frac{1}{7}u^7 - \frac{2}{5}u^5 + \frac{1}{3}u^3\right]_0^1 = \frac{32\pi}{105}$$



32. $A(x) = \frac{1}{2}bh = \frac{1}{2}\left(2\sqrt{a^2 - x^2}\right)\left(\sqrt{3}\sqrt{a^2 - x^2}\right)$

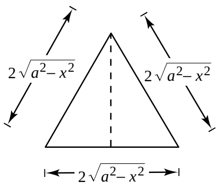
$= \sqrt{3}(a^2 - x^2)$

$$V = \sqrt{3}\int_{-a}^{a}(a^2 - x^2)\,dx = \sqrt{3}\left[a^2x - \frac{x^3}{3}\right]_{-a}^{a}$$

$= \sqrt{3}\left(\frac{4a^3}{3}\right)$

Since $\left(4\sqrt{3}\,a^3\right)/3 = 10$, we have $a^3 = \left(5\sqrt{3}\right)/2$. Thus,

$a = \sqrt[3]{\frac{5\sqrt{3}}{2}} \approx 1.630$ meters.



34. $y = \frac{x^3}{6} + \frac{1}{2x}$

$y' = \frac{1}{2}x^2 - \frac{1}{2x^2}$

$1 + (y')^2 = \left(\frac{1}{2}x^2 + \frac{1}{2x^2}\right)^2$

$$s = \int_1^3 \left(\frac{1}{2}x^2 + \frac{1}{2x^2}\right)dx = \left[\frac{1}{6}x^3 - \frac{1}{2x}\right]_1^3 = \frac{14}{3}$$

36. Since $f(x) = \tan x$ has $f'(x) = \sec^2 x$, this integral represents the length of the graph of $\tan x$ from $x = 0$ to $x = \pi/4$. This length is a little over 1 unit. Answers (b).

38. $y = 2\sqrt{x}$

$y' = \frac{1}{\sqrt{x}}$

$1 + (y')^2 = 1 + \frac{1}{x} = \frac{x + 1}{x}$

$$S = 2\pi\int_0^3 2\sqrt{x}\sqrt{\frac{x + 1}{x}}\,dx = 4\pi\int_0^3 \sqrt{x + 1}\,dx$$

$$= 4\pi\left[\left(\frac{2}{3}\right)(x + 1)^{3/2}\right]_0^3 = \frac{56\pi}{3}$$

40. $F = kx$

$50 = k(9) \implies k = \frac{50}{9}$

$F = \frac{50}{9}x$

$$W = \int_0^9 \frac{50}{9}x\,dx = \left[\frac{25}{9}x^2\right]_0^9$$

$= 225 \text{ in} \cdot \text{lb} = 18.75 \text{ ft} \cdot \text{lb}$

42. We know that

$$\frac{dV}{dt} = \frac{4 \text{ gal/min} - 12 \text{ gal/min}}{7.481 \text{ gal/ft}^3} = -\frac{8}{7.481}\text{ ft}^3\text{/min}$$

$V = \pi r^2 h = \pi\left(\frac{1}{9}\right)h$

$\frac{dV}{dt} = \frac{\pi}{9}\left(\frac{dh}{dt}\right)$

$$\frac{dh}{dt} = \frac{9}{\pi}\left(\frac{dV}{dt}\right) = \frac{9}{\pi}\left(-\frac{8}{7.481}\right) \approx -3.064 \text{ ft/min.}$$

Depth of water: $-3.064t + 150$

Time to drain well: $t = \frac{150}{3.064} \approx 49$ minutes

$(49)(12) = 588$ gallons pumped

Volume of water pumped in Exercise 41: 391.7 gallons

$\frac{391.7}{52\pi} = \frac{588}{x\pi}$

$x = \frac{588(52)}{391.7} \approx 78$

Work $\approx 78\pi$ ft $\cdot$ ton

44. (a) Weight of section of cable: $4\,\Delta x$

Distance: $200 - x$

$$W = 4\int_0^{200} (200 - x)\,dx = \left[-2(200 - x)^2\right]_0^{200} = 80{,}000 \text{ ft} \cdot \text{lb} = 40 \text{ ft} \cdot \text{ton}$$

(b) Work to move 300 pounds 200 feet vertically: $200(300) = 60{,}000 \text{ ft} \cdot \text{lb} = 30 \text{ ft} \cdot \text{ton}$

Total work = work for drawing up the cable + work of lifting the load

$= 40 \text{ ft} \cdot \text{ton} + 30 \text{ ft} \cdot \text{ton} = 70 \text{ ft} \cdot \text{ton}$

46. $W = \int_a^b F(x)\,dx$

$$F(x) = \begin{cases} -(2/9)x + 6, & 0 \le x \le 9 \\ -(4/3)x + 16, & 9 \le x \le 12 \end{cases}$$

$$W = \int_0^9 \left(-\frac{2}{9}x + 6\right) dx + \int_9^{12} \left(-\frac{4}{3}x + 16\right) dx$$

$$= \left[-\frac{1}{9}x^2 + 6x\right]_0^9 + \left[-\frac{2}{3}x^2 + 16x\right]_9^{12}$$

$$= (-9 + 54) + (-96 + 192 + 54 - 144) = 51 \text{ ft} \cdot \text{lbs}$$

48. $A = \int_{-1}^3 [(2x + 3) - x^2]\,dx = \left[x^2 + 3x - \frac{1}{3}x^3\right]_{-1}^3 = \frac{32}{3}$

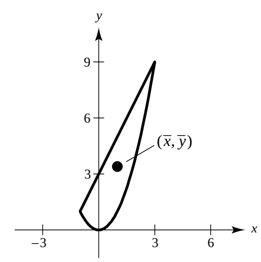
$$\frac{1}{A} = \frac{3}{32}$$

$$\bar{x} = \frac{3}{32}\int_{-1}^3 x(2x + 3 - x^2)\,dx = \frac{3}{32}\int_{-1}^3 (3x + 2x^2 - x^3)\,dx = \frac{3}{32}\left[\frac{3}{2}x^2 + \frac{2}{3}x^3 - \frac{1}{4}x^4\right]_{-1}^3 = 1$$

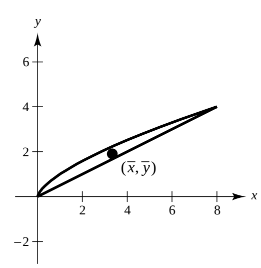
$$\bar{y} = \left(\frac{3}{32}\right)\frac{1}{2}\int_{-1}^3 [(2x + 3)^2 - x^4]\,dx = \frac{3}{64}\int_{-1}^3 (9 + 12x + 4x^2 - x^4)\,dx$$

$$= \frac{3}{64}\left[9x + 6x^2 + \frac{4}{3}x^3 - \frac{1}{5}x^5\right]_{-1}^3 = \frac{17}{5}$$

$$(\bar{x}, \bar{y}) = \left(1, \frac{17}{5}\right)$$



50. $A = \int_0^8 \left(x^{2/3} - \frac{1}{2}x\right) dx = \left[\frac{3}{5}x^{5/3} - \frac{1}{4}x^2\right]_0^8 = \frac{16}{5}$



$$\frac{1}{A} = \frac{5}{16}$$

$$\bar{x} = \frac{5}{16}\int_0^8 x\left(x^{2/3} - \frac{1}{2}x\right) dx$$

$$= \ = \frac{5}{16}\left[\frac{3}{8}x^{8/3} - \frac{1}{6}x^3\right]_0^8 = \frac{10}{3}$$

$$\bar{y} = \left(\frac{5}{16}\right)\frac{1}{2}\int_0^8 \left(x^{4/3} - \frac{1}{4}x^2\right) dx$$

$$= \frac{1}{2}\left(\frac{5}{16}\right)\left[\frac{3}{7}x^{7/3} - \frac{1}{12}x^3\right]_0^8 = \frac{40}{21}$$

$$(\bar{x}, \bar{y}) = \left(\frac{10}{3}, \frac{40}{21}\right)$$

52. Wall at shallow end:

$$F = 62.4\int_0^5 y(20)\,dy = \left[(1248)\frac{y^2}{2}\right]_0^5 = 15{,}600 \text{ lb}$$

Wall at deep end:

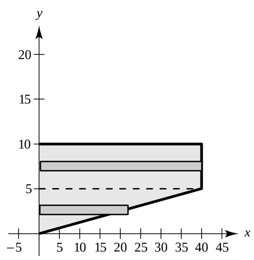
$$F = 62.4\int_0^{10} y(20)\,dy = \left[(624)y^2\right]_0^{10} = 62{,}400 \text{ lb}$$

Side wall:

$$F_1 = 62.4\int_0^5 y(40)\,dy = \left[(1248)y^2\right]_0^5 = 31{,}200 \text{ lb}$$

$$F_2 = 62.4\int_0^5 (10 - y)8y\,dy = 62.4\int_0^5 (80y - 8y^2)\,dy$$

$$F = F_1 + F_2 = 72{,}800 \text{ lb}$$



54. $F = 62.4(16\pi)5 = 4992\pi$ lb

Problem Solving for Chapter 6

2. $R = \int_0^1 x(1 - x)\,dx = \left[\frac{x^2}{2} - \frac{x^3}{3}\right]_0^1 = \frac{1}{2} - \frac{1}{3} = \frac{1}{6}$

Let (c, mc) be the intersection of the line and the parabola.

Then, $mc = c(1 - c) \implies m = 1 - c$ or $c = 1 - m$.

$$\frac{1}{2}\left(\frac{1}{6}\right) = \int_0^{1-m} (x - x^2 - mx)\,dx$$

$$\frac{1}{12} = \left[\frac{x^2}{2} - \frac{x^3}{3} - m\frac{x^2}{2}\right]_0^{1-m}$$

$$= \frac{(1 - m)^2}{2} - \frac{(1 - m)^3}{3} - m\frac{(1 - m)^2}{2}$$

$$1 = 6(1 - m)^2 - 4(1 - m)^3 - 6m(1 - m)^2$$

$$= (1 - m)^2(6 - 4(1 - m) - 6m)$$

$$= (1 - m)^2(2 - 2m)$$

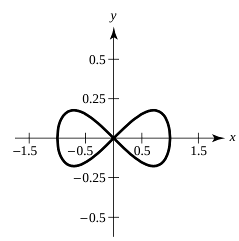
$$\frac{1}{2} = (1 - m)^3$$

$$\left(\frac{1}{2}\right)^{1/3} = 1 - m$$

$$m = 1 - \left(\frac{1}{2}\right)^{1/3} \approx 0.2063$$

4. $8y^2 = x^2(1 - x^2)$

$$y = \pm\frac{|x|\sqrt{1 - x^2}}{2\sqrt{2}}$$



For $x > 0$, $y' = \dfrac{1 - 2x^2}{2\sqrt{2}\sqrt{1 - x^2}}$

$$S = 2(2\pi)\int_0^1 x\sqrt{1 + \left(\frac{1 - 2x^2}{2\sqrt{2}\sqrt{1 - x^2}}\right)^2}\,dx$$

$$= \frac{5\sqrt{2}\pi}{3}$$

6. By the Theorem of Pappus,

$$V = 2\pi r A = 2\pi\left[d + \frac{1}{2}\sqrt{w^2 + l^2}\right]lw$$

8. $f'(x)^2 = e^x$

$$f'(x) = e^{x/2}$$
$$f(x) = 2e^{x/2} + C$$
$$f(0) = 0 \implies C = -2$$
$$f(x) = 2e^{x/2} - 2$$

10. Let ρ_f be the density of the fluid and ρ_0 the density of the iceberg. The buoyant force is

$$F = \rho_f g\int_{-h}^{0} A(y)\,dy$$

where $A(y)$ is a typical cross section and g is the acceleration due to gravity. The weight of the object is

$$W = \rho_0 g\int_{-h}^{L-h} A(y)\,dy.$$

$$F = W$$

$$\rho_f g\int_{-h}^{0} A(y)\,dy = \rho_0 g\int_{-h}^{L-h} A(y)\,dy$$

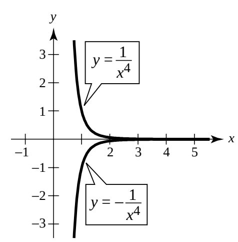
$$\frac{\rho_0}{\rho_f} = \frac{\text{submerged volume}}{\text{total volume}} = \frac{0.92 \times 10^3}{1.03 \times 10^3} = 0.893 \text{ or } 89.3\%$$

12. (a) $\bar{y} = 0$ by symmetry

$$M_y = 2\int_1^6 x\frac{1}{x^4}\,dx = 2\int_1^6 \frac{1}{x^3}\,dx = \frac{35}{36}$$

$$m = 2\int_1^6 \frac{1}{x^4}\,dx = \frac{215}{324}$$

$$\bar{x} = \frac{35/36}{215/324} = \frac{63}{43} \qquad (\bar{x}, \bar{y}) = \left(\frac{63}{43}, 0\right)$$



(b) $M_y = 2\int_1^b \frac{1}{x^3}\,dx = \frac{b^2 - 1}{b^2}$

$$m = 2\int_1^b \frac{1}{x^4}\,dx = \frac{2(b^3 - 1)}{3b^3}$$

$$\bar{x} = \frac{(b^2 - 1)/b^2}{2(b^3 - 1)/3b^3} = \frac{3b(b + 1)}{2(b^2 + b + 1)} \qquad (\bar{x}, \bar{y}) = \left(\frac{3b(b + 1)}{2(b^2 + b + 1)}, 0\right)$$

$$\lim_{b\to\infty} \bar{x} = \frac{3}{2} \qquad (\bar{x}, \bar{y}) = \left(\frac{3}{2}, 0\right)$$

14. (a) Trapezoidal: Area $\approx \frac{160}{2(8)}[0 + 2(50) + 2(54) + 2(82) + 2(82) + 2(73) + 2(75) + 2(80) + 0] = 9920$ sq ft

(b) Simpson's: Area $\approx \frac{160}{3(8)}[0 + 4(50) + 2(54) + 4(82) + 2(82) + 4(73) + 2(75) + 4(80) + 0] = 10{,}413\frac{1}{3}$ sq ft

16. Point of equilibrium: $1000 - 0.4x^2 = 42x$

$$x = 20,\ p = 840$$

$(P_0, x_0) = (840, 20)$

Consumer surplus $= \int_0^{20} [(1000 - 0.4x^2) - 840]\,dx = 2133.33$

Producer surplus $= \int_0^{20} [840 - 42x]\,dx = 8400$