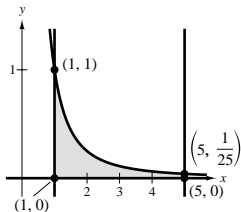


Review Exercises for Chapter 6

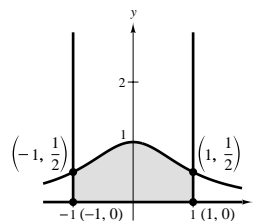
$$1. A = \int_1^5 \frac{1}{x^2} dx = \left[-\frac{1}{x} \right]_1^5 = \frac{4}{5}$$



$$3. A = \int_{-1}^1 \frac{1}{x^2 + 1} dx$$

$$= \left[\arctan x \right]_{-1}^1$$

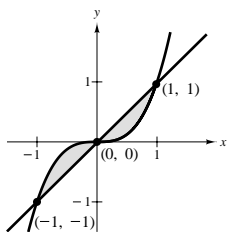
$$= \frac{\pi}{4} - \left(-\frac{\pi}{4} \right) = \frac{\pi}{2}$$



$$5. A = 2 \int_0^1 (x - x^3) dx$$

$$= 2 \left[\frac{1}{2}x^2 - \frac{1}{4}x^4 \right]_0^1$$

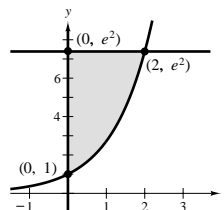
$$= \frac{1}{2}$$



$$7. A = \int_0^2 (e^2 - e^x) dx$$

$$= \left[xe^2 - e^x \right]_0^2$$

$$= e^2 + 1$$



$$9. A = \int_{\pi/4}^{5\pi/4} (\sin x - \cos x) dx$$

$$= \left[-\cos x - \sin x \right]_{\pi/4}^{5\pi/4}$$

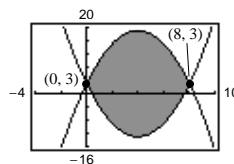
$$= \left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} \right) - \left(-\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} \right)$$

$$= \frac{4}{\sqrt{2}} = 2\sqrt{2}$$

$$11. A = \int_0^8 [(3 + 8x - x^2) - (x^2 - 8x + 3)] dx$$

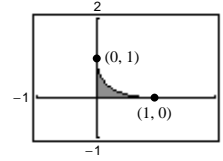
$$= \int_0^8 (16x - 2x^2) dx$$

$$= \left[8x^2 - \frac{2}{3}x^3 \right]_0^8 = \frac{512}{3} \approx 170.667$$



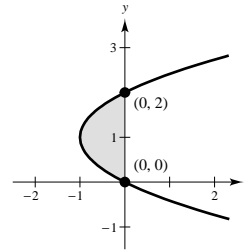
$$13. y = (1 - \sqrt{x})^2$$

$$\begin{aligned} A &= \int_0^1 (1 - \sqrt{x})^2 dx \\ &= \int_0^1 (1 - 2x^{1/2} + x) dx \\ &= \left[x - \frac{4}{3}x^{3/2} + \frac{1}{2}x^2 \right]_0^1 = \frac{1}{6} \approx 0.1667 \end{aligned}$$



$$15. x = y^2 - 2y \Rightarrow x + 1 = (y - 1)^2 \Rightarrow y = 1 \pm \sqrt{x + 1}$$

$$\begin{aligned} A &= \int_{-1}^0 [(1 + \sqrt{x+1}) - (1 - \sqrt{x+1})] dx = \int_{-1}^0 2\sqrt{x+1} dx \\ A &= \int_0^2 [0 - (y^2 - 2y)] dy = \int_0^2 (2y - y^2) dy = \left[y^2 - \frac{1}{3}y^3 \right]_0^2 = \frac{4}{3} \end{aligned}$$

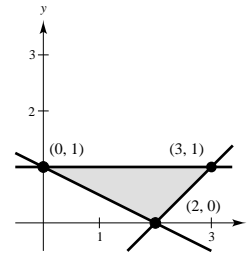


$$\begin{aligned} 17. A &= \int_0^2 \left[1 - \left(1 - \frac{x}{2} \right) \right] dx + \int_2^3 [1 - (x - 2)] dx \\ &= \int_0^2 \frac{x}{2} dx + \int_2^3 (3 - x) dx \end{aligned}$$

$$y = 1 - \frac{x}{2} \Rightarrow x = 2 - 2y$$

$$y = x - 2 \Rightarrow x = y + 2, y = 1$$

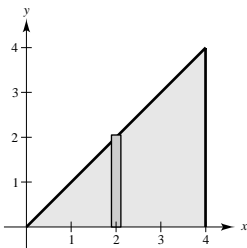
$$\begin{aligned} A &= \int_0^1 [(y + 2) - (2 - 2y)] dy \\ &= \int_0^1 3y dy = \left[\frac{3}{2}y^2 \right]_0^1 = \frac{3}{2} \end{aligned}$$



19. Job 1 is better. The salary for Job 1 is greater than the salary for Job 2 for all the years except the first and 10th years.

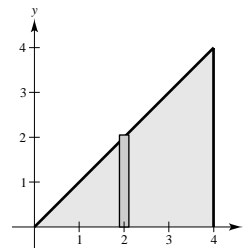
21. (a) Disk

$$V = \pi \int_0^4 x^2 dx = \left[\frac{\pi x^3}{3} \right]_0^4 = \frac{64\pi}{3}$$



(b) Shell

$$V = 2\pi \int_0^4 x^2 dx = \left[\frac{2\pi}{3}x^3 \right]_0^4 = \frac{128\pi}{3}$$

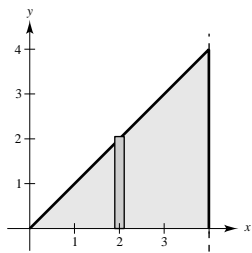


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21. —CONTINUED—

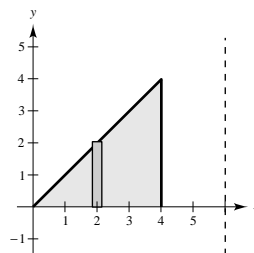
(c) Shell

$$\begin{aligned} V &= 2\pi \int_0^4 (4-x)x \, dx \\ &= 2\pi \int_0^4 (4x - x^2) \, dx \\ &= 2\pi \left[2x^2 - \frac{x^3}{3} \right]_0^4 = \frac{64\pi}{3} \end{aligned}$$



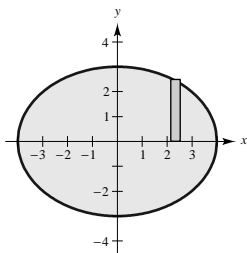
(d) Shell

$$\begin{aligned} V &= 2\pi \int_0^4 (6-x)x \, dx \\ &= 2\pi \int_0^4 (6x - x^2) \, dx \\ &= 2\pi \left[3x^2 - \frac{1}{3}x^3 \right]_0^4 = \frac{160\pi}{3} \end{aligned}$$



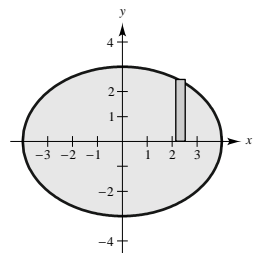
23. (a) Shell

$$\begin{aligned} V &= 4\pi \int_0^4 x \left(\frac{3}{4} \right) \sqrt{16-x^2} \, dx \\ &= \left[3\pi \left(-\frac{1}{2} \right) \left(\frac{2}{3} \right) (16-x^2)^{3/2} \right]_0^4 = 64\pi \end{aligned}$$



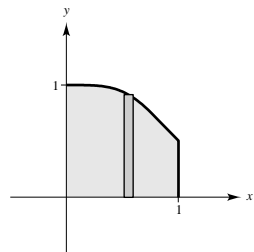
(b) Disk

$$\begin{aligned} V &= 2\pi \int_0^4 \left[\frac{3}{4} \sqrt{16-x^2} \right]^2 dx \\ &= \frac{9\pi}{8} \left[16x - \frac{x^3}{3} \right]_0^4 = 48\pi \end{aligned}$$



25. Shell

$$\begin{aligned} V &= 2\pi \int_0^1 \frac{x}{x^4+1} \, dx \\ &= \pi \int_0^1 \frac{(2x)}{(x^2)^2+1} \, dx \\ &= \left[\pi \arctan(x^2) \right]_0^1 \\ &= \pi \left[\frac{\pi}{4} - 0 \right] = \frac{\pi^2}{4} \end{aligned}$$



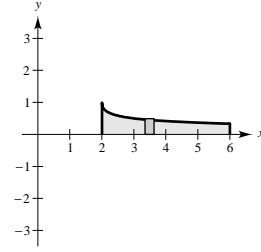
27. Shell

$$u = \sqrt{x-2}$$

$$x = u^2 + 2$$

$$dx = 2u \, du$$

$$\begin{aligned} V &= 2\pi \int_2^6 \frac{x}{1 + \sqrt{x-2}} dx = 4\pi \int_0^2 \frac{(u^2 + 2)u}{1 + u} du \\ &= 4\pi \int_0^2 \frac{u^3 + 2u}{1 + u} du = 4\pi \int_0^2 \left(u^2 - u + 3 - \frac{3}{1+u} \right) du \\ &= 4\pi \left[\frac{1}{3}u^3 - \frac{1}{2}u^2 + 3u - 3 \ln(1+u) \right]_0^2 = \frac{4\pi}{3}(20 - 9 \ln 3) \approx 42.359 \end{aligned}$$



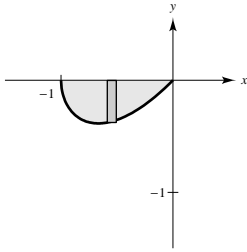
29. Since $y \leq 0$, $A = -\int_{-1}^0 x\sqrt{x+1} \, dx$.

$$u = x + 1$$

$$x = u - 1$$

$$dx = du$$

$$\begin{aligned} A &= -\int_0^1 (u-1)\sqrt{u} \, du = -\int_0^1 (u^{3/2} - u^{1/2}) \, du \\ &= -\left[\frac{2}{5}u^{5/2} - \frac{2}{3}u^{3/2} \right]_0^1 = \frac{4}{15} \end{aligned}$$



33. $f(x) = \frac{4}{5}x^{5/4}$

$$f'(x) = x^{1/4}$$

$$1 + [f'(x)]^2 = 1 + \sqrt{x}$$

$$u = 1 + \sqrt{x}$$

$$x = (u-1)^2$$

$$dx = 2(u-1) \, du$$

$$\begin{aligned} s &= \int_0^4 \sqrt{1 + \sqrt{x}} \, dx = 2 \int_1^3 \sqrt{u}(u-1) \, du \\ &= 2 \int_1^3 (u^{3/2} - u^{1/2}) \, du \\ &= 2 \left[\frac{2}{5}u^{5/2} - \frac{2}{3}u^{3/2} \right]_1^3 = \frac{4}{15} \left[u^{3/2}(3u-5) \right]_1^3 \\ &= \frac{8}{15}(1 + 6\sqrt{3}) \approx 6.076 \end{aligned}$$

31. From Exercise 23(a) we have: $V = 64\pi \text{ ft}^3$

$$\frac{1}{4}V = 16\pi$$

Disk: $\pi \int_{-3}^{y_0} \frac{16}{9}(9-y^2) \, dy = 16\pi$

$$\frac{1}{9} \int_{-3}^{y_0} (9-y^2) \, dy = 1$$

$$\left[9y - \frac{1}{3}y^3 \right]_{-3}^{y_0} = 9$$

$$\left(9y_0 - \frac{1}{3}y_0^3 \right) - (-27 + 9) = 9$$

$$y_0^3 - 27y_0 - 27 = 0$$

By Newton's Method, $y_0 \approx -1.042$ and the depth of the gasoline is $3 - 1.042 = 1.958$ ft.

35. $y = 300 \cosh\left(\frac{x}{2000}\right) - 280, -2000 \leq x \leq 2000$

$$y' = \frac{3}{20} \sinh\left(\frac{x}{2000}\right)$$

$$s = \int_{-2000}^{2000} \sqrt{1 + \left[\frac{3}{20} \sinh\left(\frac{x}{2000}\right) \right]^2} \, dx$$

$$= \frac{1}{20} \int_{-2000}^{2000} \sqrt{400 + 9 \sinh^2\left(\frac{x}{2000}\right)} \, dx$$

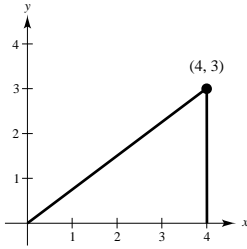
≈ 4018.2 ft (by Simpson's Rule or graphing utility)

$$37. y = \frac{3}{4}x$$

$$y' = \frac{3}{4}$$

$$1 + (y')^2 = \frac{25}{16}$$

$$S = 2\pi \int_0^4 \left(\frac{3}{4}x\right) \sqrt{\frac{25}{16}} dx = \left[\left(\frac{15\pi}{8}\right)\frac{x^2}{2}\right]_0^4 = 15\pi$$



$$39. F = kx$$

$$4 = k(1)$$

$$F = 4x$$

$$W = \int_0^5 4x dx = \left[2x^2\right]_0^5 = 50 \text{ in} \cdot \text{lb} \approx 4.167 \text{ ft} \cdot \text{lb}$$

$$41. \text{Volume of disk: } \pi\left(\frac{1}{3}\right)^2 \Delta y$$

$$\text{Weight of disk: } 62.4\pi\left(\frac{1}{3}\right)^2 \Delta y$$

$$\text{Distance: } 175 - y$$

$$W = \frac{62.4\pi}{9} \int_0^{150} (175 - y) dy = \frac{62.4\pi}{9} \left[175y - \frac{y^2}{2}\right]_0^{150} = 104,000\pi \text{ ft} \cdot \text{lb} \approx 163.4 \text{ ft} \cdot \text{ton}$$

$$43. \text{Weight of section of chain: } 5 \Delta x$$

$$\text{Distance moved: } 10 - x$$

$$W = 5 \int_0^{10} (10 - x) dx = \left[-\frac{5}{2}(10 - x)^2\right]_0^{10} = 250 \text{ ft} \cdot \text{lb}$$

$$45. W = \int_a^b F(x) dx$$

$$80 = \int_0^4 ax^2 dx = \left[\frac{ax^3}{3}\right]_0^4 = \frac{64}{3}a$$

$$a = \frac{3(80)}{64} = \frac{15}{4} = 3.75$$

$$47. A = \int_0^a (\sqrt{a} - \sqrt{x})^2 dx = \int_0^a (a - 2\sqrt{a}x^{1/2} + x) dx = \left[ax - \frac{4}{3}\sqrt{a}x^{3/2} + \frac{1}{2}x^2\right]_0^a = \frac{a^2}{6}$$

$$\frac{1}{A} = \frac{6}{a^2}$$

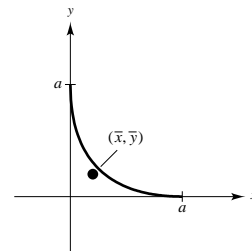
$$\bar{x} = \frac{6}{a^2} \int_0^a x(\sqrt{a} - \sqrt{x})^2 dx = \frac{6}{a^2} \int_0^a (ax - 2\sqrt{a}x^{3/2} + x^2) dx$$

$$\bar{y} = \left(\frac{6}{a^2}\right) \frac{1}{2} \int_0^a (\sqrt{a} - \sqrt{x})^4 dx$$

$$= \frac{3}{a^2} \int_0^a (a^2 - 4a^{3/2}x^{1/2} + 6ax - 4a^{1/2}x^{3/2} + x^2) dx$$

$$= \frac{3}{a^2} \left[a^2x - \frac{8}{3}a^{3/2}x^{3/2} + 3ax^2 - \frac{8}{5}a^{1/2}x^{5/2} + \frac{1}{3}x^3 \right]_0^a = \frac{a}{5}$$

$$(\bar{x}, \bar{y}) = \left(\frac{a}{5}, \frac{a}{5}\right)$$



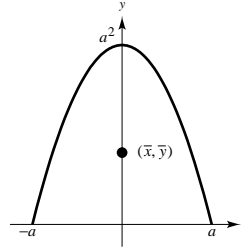
49. By symmetry, $x = 0$.

$$A = 2 \int_0^1 (a^2 - x^2) dx = 2 \left[a^2x - \frac{x^3}{3} \right]_0^1 = \frac{4a^3}{3}$$

$$\frac{1}{A} = \frac{3}{4a^3}$$

$$\begin{aligned} \bar{y} &= \left(\frac{3}{4a^3} \right) \frac{1}{2} \int_{-a}^a (a^2 - x^2)^2 dx \\ &= \frac{6}{8a^3} \int_0^a (a^4 - 2a^2x^2 + x^4) dx \\ &= \frac{6}{8a^3} \left[a^4x - \frac{2a^2}{3}x^3 + \frac{1}{5}x^5 \right]_0^a \\ &= \frac{6}{8a^3} \left(a^5 - \frac{2}{3}a^5 + \frac{1}{5}a^5 \right) = \frac{2a^2}{5} \end{aligned}$$

$$(\bar{x}, \bar{y}) = \left(0, \frac{2a^2}{5} \right)$$



51. $\bar{y} = 0$ by symmetry

For the trapezoid:

$$m = [(4)(6) - (1)(6)]\rho = 18\rho$$

$$\begin{aligned} M_y &= \rho \int_0^6 x \left[\left(\frac{1}{6}x + 1 \right) - \left(-\frac{1}{6}x - 1 \right) \right] dx \\ &= \rho \int_0^6 \left(\frac{1}{3}x^2 + 2x \right) dx = \rho \left[\frac{x^3}{9} + x^2 \right]_0^6 = 60\rho \end{aligned}$$

For the semicircle:

$$m = \left(\frac{1}{2} \right) (\pi)(2)^2 \rho = 2\pi\rho$$

$$M_y = \rho \int_6^8 x \left[\sqrt{4 - (x-6)^2} - \left(-\sqrt{4 - (x-6)^2} \right) \right] dx = 2\rho \int_6^8 x \sqrt{4 - (x-6)^2} dx$$

Let $u = x - 6$, then $x = u + 6$ and $dx = du$. When $x = 6$, $u = 0$. When $x = 8$, $u = 2$.

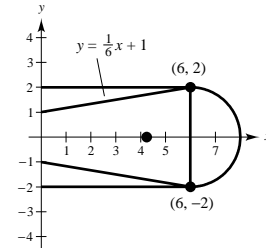
$$\begin{aligned} M_y &= 2\rho \int_0^2 (u+6) \sqrt{4-u^2} du = 2\rho \int_0^2 u \sqrt{4-u^2} du + 12\rho \int_0^2 \sqrt{4-u^2} du \\ &= 2\rho \left[\left(-\frac{1}{2} \right) \left(\frac{2}{3} \right) (4-u^2)^{3/2} \right]_0^2 + 12\rho \left[\frac{\pi(2)^2}{4} \right] = \frac{16\rho}{3} + 12\pi\rho = \frac{4\rho(4+9\pi)}{3} \end{aligned}$$

Thus, we have:

$$\bar{x}(18\rho + 2\pi\rho) = 60\rho + \frac{4\rho(4+9\pi)}{3}$$

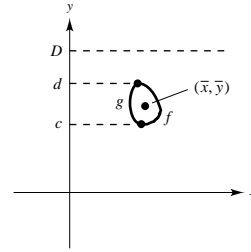
$$\bar{x} = \frac{180\rho + 4\rho(4+9\pi)}{3} \cdot \frac{1}{2\rho(9+\pi)} = \frac{2(9\pi+49)}{3(\pi+9)}$$

The centroid of the blade is $\left(\frac{2(9\pi+49)}{3(\pi+9)}, 0 \right)$.



53. Let D = surface of liquid; ρ = weight per cubic volume.

$$\begin{aligned}
 F &= \rho \int_c^d (D - y)[f(y) - g(y)] dy \\
 &= \rho \left[\int_c^d D[f(y) - g(y)] dy - \int_c^d y[f(y) - g(y)] dy \right] \\
 &= \rho \left[\int_c^d [f(y) - g(y)] dy \right] \left[D - \frac{\int_c^d y[f(y) - g(y)] dy}{\int_c^d [f(y) - g(y)] dy} \right] \\
 &= \rho(\text{Area})(D - \bar{y}) \\
 &= \rho(\text{Area})(\text{depth of centroid})
 \end{aligned}$$



Problem Solving for Chapter 6

1. $T = \frac{1}{2}c(c^2) = \frac{1}{2}c^3$

$$R = \int_0^c (cx - x^2) dx = \left[\frac{cx^2}{2} - \frac{x^3}{3} \right]_0^c = \frac{c^3}{2} - \frac{c^3}{3} = \frac{c^3}{6}$$

$$\lim_{c \rightarrow 0^+} \frac{T}{R} = \lim_{c \rightarrow 0^+} \frac{\frac{1}{2}c^3}{\frac{1}{6}c^3} = 3$$

3. (a) $\frac{1}{2}V = \int_0^1 [\pi(2 + \sqrt{1 - y^2})^2 - \pi(2 - \sqrt{1 - y^2})^2] dy$

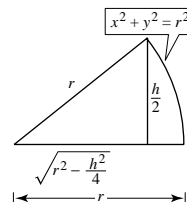
$$\begin{aligned}
 &= \pi \int_0^1 [(4 + 4\sqrt{1 - y^2} + (1 - y^2)) - (4 - 4\sqrt{1 - y^2} + (1 - y^2))] dy \\
 &= 8\pi \int_0^1 \sqrt{1 - y^2} dy \quad (\text{Integral represents } 1/4 \text{ (area of circle)}) \\
 &= 8\pi \left(\frac{\pi}{4} \right) = 2\pi^2 \Rightarrow V = 4\pi^2
 \end{aligned}$$

(b) $(x - R)^2 + y^2 = r^2 \Rightarrow x = R \pm \sqrt{r^2 - y^2}$

$$\begin{aligned}
 \frac{1}{2}V &= \int_0^r [\pi(R + \sqrt{r^2 - y^2})^2 - \pi(R - \sqrt{r^2 - y^2})^2] dy \\
 &= \pi \int_0^r 4R\sqrt{r^2 - y^2} dy \\
 &= \pi(4R) \frac{1}{4} \pi r^2 - \pi^2 r^2 R \\
 V &= 2\pi^2 r^2 R
 \end{aligned}$$

5. $V = 2(2\pi) \int_{\sqrt{r^2 - (h^2/4)}}^r x \sqrt{r^2 - x^2} dx$

$$\begin{aligned}
 &= -2\pi \left[\frac{2}{3}(r^2 - x^2)^{3/2} \right]_{\sqrt{r^2 - (h^2/4)}}^r \\
 &= \frac{-4\pi}{3} \left[-\frac{h^3}{8} \right] = \frac{\pi h^3}{6} \text{ which does not depend on } r!
 \end{aligned}$$



7. (a) Tangent at A: $y = x^3, y' = 3x^2$

$$y - 1 = 3(x - 1)$$

$$y = 3x - 2$$

To find point B: $x^3 = 3x - 2$

$$x^3 - 3x + 2 = 0$$

$$(x - 1)^2(x + 2) = 0 \Rightarrow B = (-2, -8)$$

Tangent at B: $y = x^3, y' = 3x^2$

$$y + 8 = 12(x + 2)$$

$$y = 12x + 16$$

To find point C: $x^3 = 12x + 16$

$$x^3 - 12x - 16 = 0$$

$$(x + 2)^2(x - 4) = 0 \Rightarrow C = (4, 64)$$

$$\text{Area of } R = \int_{-2}^1 (x^3 - 3x + 2) dx = \frac{27}{4}$$

$$\text{Area of } S = \int_{-2}^4 (12x + 16 - x^3) dx = 108$$

$$\text{Area of } S = 16(\text{area of } R) \quad \left[\frac{\text{area } S}{\text{area } R} = 16 \right]$$

(b) Tangent at $A(a, a^3)$: $y - a^3 = 3a^2(x - a)$

$$y = 3a^2x - 2a^3$$

To find point B: $x^3 - 3a^2x + 2a^3 = 0$

$$(x - a)^2(x + 2a) = 0 \Rightarrow$$

$$B = (-2a, -8a^3)$$

Tangent at B: $y + 8a^3 = 12a^2(x + 2a)$

$$y = 12a^2x + 16a^3$$

To find point C: $x^3 - 12a^2x - 16a^3 = 0$

$$(x + 2a)^2(x - 4a) = 0 \Rightarrow$$

$$C = (4a, 64a^3)$$

$$\text{Area of } R = \int_{-2a}^a [x^3 - 3a^2x + 2a^3] dx = \frac{27}{4}a^4$$

$$\text{Area of } S = \int_{-2a}^{4a} [12a^2x + 16a^3 - x^3] dx = 108a^4$$

$$\text{Area of } S = 16(\text{area of } R)$$

9. $s(x) = \int_{\alpha}^x \sqrt{1 + f'(t)^2} dt$

(a) $s'(x) = \frac{ds}{dx} = \sqrt{1 + f'(x)^2}$

(b) $ds = \sqrt{1 + f'(x)^2} dx$

$$(ds)^2 = [1 + f'(x)^2](dx)^2 = \left[1 + \left(\frac{dy}{dx} \right)^2 \right] (dx)^2 = (dx)^2 + (dy)^2$$

(c) $s(x) = \int_1^x \sqrt{1 + \left(\frac{3}{2}t^{1/2} \right)^2} dt = \int_1^x \sqrt{1 + \frac{9}{4}t} dt$

(d) $s(2) = \int_1^2 \sqrt{1 + \frac{9}{4}t} dt = \left[\frac{8}{27} \left(1 + \frac{9}{4}t \right)^{3/2} \right]_1^2 = \frac{22}{27}\sqrt{22} - \frac{13}{27}\sqrt{13} \approx 2.0858$

This is the length of the curve $y = x^{3/2}$ from $x = 1$ to $x = 2$.

11. (a) $\bar{y} = 0$ by symmetry

$$M_y = \int_1^6 x \left(\frac{1}{x^3} - \left(-\frac{1}{x^3} \right) \right) dx = \int_1^6 \frac{2}{x^2} dx = \left[-2\frac{1}{x} \right]_1^6 = \frac{5}{3}$$

$$m = 2 \int_1^6 \frac{1}{x^3} dx = \left[-\frac{1}{x^2} \right]_1^6 = \frac{35}{36}$$

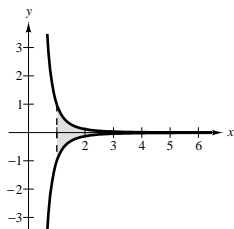
$$\bar{x} = \frac{5/3}{35/36} = \frac{12}{7} \quad (\bar{x}, \bar{y}) = \left(\frac{12}{7}, 0 \right)$$

(b) $m = 2 \int_1^b \frac{1}{x^3} dx = \frac{b^2 - 1}{b^2}$

$$M_y = 2 \int_1^b \frac{1}{x^2} dx = \frac{2(b - 1)}{b}$$

$$\bar{x} = \frac{2(b - 1)/b}{(b^2 - 1)/b^2} = \frac{2b}{b + 1} \quad (\bar{x}, \bar{y}) = \left(\frac{2b}{b + 1}, 0 \right)$$

(c) $\lim_{b \rightarrow \infty} \bar{x} = \lim_{b \rightarrow \infty} \frac{2b}{b + 1} = 2 \quad (\bar{x}, \bar{y}) = (2, 0)$



13. (a) $W = \text{area} = 2 + 4 + 6 = 12$

(b) $W = \text{area} = 3 + (1 + 1) + 2 + \frac{1}{2} = 7\frac{1}{2}$

17. (a) Wall at shallow end

From Exercise 22: $F = 62.4(2)(4)(20) = 9984 \text{ lb}$

(b) Wall at deep end

From Exercise 22: $F = 62.4(4)(8)(20) = 39,936 \text{ lb}$

(c) Side wall

From Exercise 22: $F_1 = 62.4(2)(4)(40) = 19,968 \text{ lb}$

$$\begin{aligned} F_2 &= 62.4 \int_0^4 (8 - y)(10y) dy \\ &= 624 \int_0^4 (8y - y^2) dy = 624 \left[4y^2 - \frac{y^3}{3} \right]_0^4 \\ &= 26,624 \text{ lb} \end{aligned}$$

Total force: $F_1 + F_2 = 46,592 \text{ lb}$

15. Point of equilibrium: $50 - 0.5x = 0.125x$

$$x = 80, p = 10$$

$$(P_0, x_0) = (10, 80)$$

$$\text{Consumer surplus} = \int_0^{80} [(50 - 0.5x) - 10] dx = 1600$$

$$\text{Producer surplus} = \int_0^{80} [10 - 0.125x] dx = 400$$

