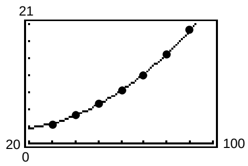
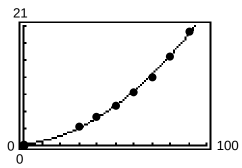
14. (a) $t = 0.00271s^2 - 0.0529s + 2.671$

(b)



(c) The curve levels off for $s < 20$.

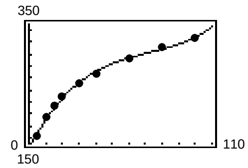
(d) $t = 0.002s^2 + 0.0346s + 0.183$



The model is better for low speeds.

16. (a) $T = 2.9856 \times 10^{-4} p^3 - 0.0641 p^2 + 5.2826p + 143.1$

(b)



(c) For $T = 300°\text{F}$, $p \approx 68.29$ pounds per square inch.

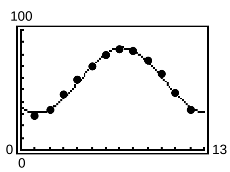
(d) The model is based on data up to 100 pounds per square inch.

18. (a) $H(t) = 84.4 + 4.28 \sin\left(\frac{\pi t}{6} + 3.86\right)$

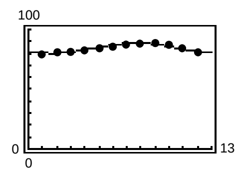
One model is

$$C(t) = 58 + 27 \sin\left(\frac{\pi t}{6} + 4.1\right).$$

(b)



(c)



(d) The average in Honolulu is 84.4.

The average in Chicago is 58.

(e) The period is 12 months (1 year).

(f) Chicago has greater variability $(27 > 4.28)$.

20. Answers will vary.

Review Exercises for Chapter P

2. $y = (x - 1)(x - 3)$

$x = 0 \Longrightarrow y = (0 - 1)(0 - 3) = 3 \Longrightarrow (0, 3)$ $\quad$ y-intercept

$y = 0 \Longrightarrow 0 = (x - 1)(x - 3) \Longrightarrow x = 1, 3 \Longrightarrow (1, 0), (3, 0)$ $\quad$ x-intercepts

4. $xy = 4$

$x = 0$ and $y = 0$ are both impossible. No intercepts.

6. Symmetric with respect to y-axis since

$$y = (-x)^4 - (-x)^2 + 3$$

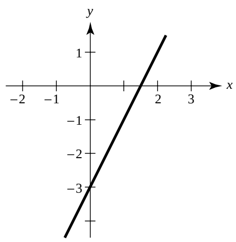
$$y = x^4 - x^2 + 3.$$

8. $4x - 2y = 6$

$$y = 2x - 3$$

Slope: 2

y-intercept: -3



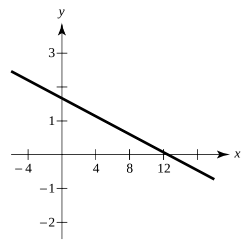
10. $0.02x + 0.15y = 0.25$

$$2x + 15y = 25$$

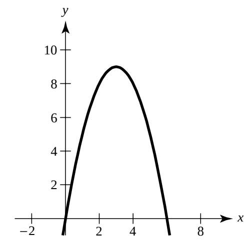
$$y = -\frac{2}{15}x + \frac{5}{3}$$

Slope: $-\frac{2}{15}$

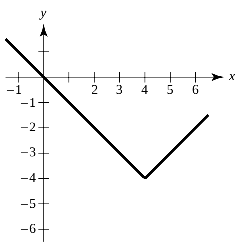
y-intercept: $\frac{5}{3}$



12. $y = x(6 - x)$



14. $y = |x - 4| - 4$



16. $y = 8\sqrt[3]{x - 6}$

Xmin = -40
Xmax = 40
Xscl = 10
Ymin = -40
Ymax = 40
Yscl = 10

18.

$$y = x + 1$$
$$(x + 1) - x^2 = 7$$
$$0 = x^2 - x + 6$$

No real solution
No points of intersection
The graphs of $y = x + 1$ and $y = x^2 + 7$ do not intersect.

20. $y = kx^3$

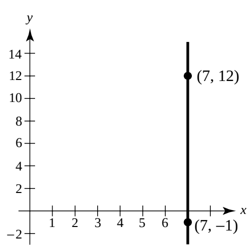
(a) $4 = k(1)^3 \Longrightarrow k = 4$ and $y = 4x^3$

(b) $1 = k(-2)^3 \Longrightarrow k = -\frac{1}{8}$ and $y = -\frac{1}{8}x^3$

(c) $0 = k(0)^3 \Longrightarrow$ any k will do!

(d) $-1 = k(-1)^3 \Longrightarrow k = 1 \Longrightarrow y = x^3$

22.



The line is vertical and has no slope.

24.

$$\frac{3 - (-1)}{-3 - t} = \frac{3 - 6}{-3 - 8}$$

$$\frac{4}{-3 - t} = \frac{-3}{-11}$$

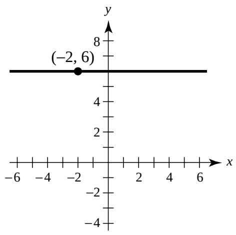
$$-44 = 9 + 3t$$

$$-53 = 3t$$

$$t = -\frac{53}{3}$$

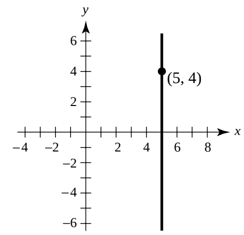
26. $y - 6 = 0(x - (-2))$

$y = 6$ Horizontal line



28. m is undefined. Line is vertical.

$x = 5$



30. (a) $y - 3 = -\frac{2}{3}(x - 1)$

$3y - 9 = -2x + 2$

$2x + 3y - 11 = 0$

(b) Slope of perpendicular line is 1.

$y - 3 = 1(x - 1)$

$y = x + 2$

$0 = x - y + 2$

(c) $m = \frac{4 - 3}{2 - 1} = 1$

$y - 3 = 1(x - 1)$

$y = x + 2$

$0 = x - y + 2$

(d) $y = 3$

$y - 3 = 0$

32. (a) $C = 9.25t + 13.50t + 36{,}500$

$= 22.75t + 36{,}500$

(b) $R = 30t$

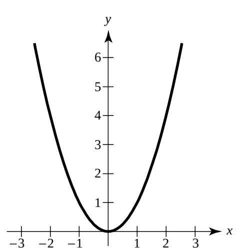
(c) $30t = 22.75t + 36{,}500$

$7.25t = 36{,}000$

$t \approx 5034.48$ hours to break even.

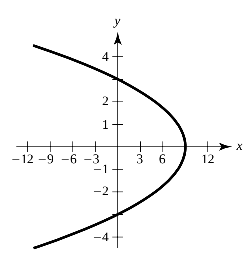
34. $x^2 - y = 0$

Function of x since there is one value for y for each x.



36. $x = 9 - y^2$

Not a function of x since there are two values of y for some x.



38. (a) $f(-4) = (-4)^2 + 2 = 18$ (because $-4 < 0$)

(b) $f(0) = |0 - 2| = 2$

(c) $f(1) = |1 - 2| = 1$

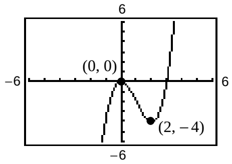
40. $f(x) = 1 - x^2$ and $g(x) = 2x + 1$

(a) $f(x) - g(x) = (1 - x^2) - (2x + 1) = -x^2 - 2x$

(b) $f(x)g(x) = (1 - x^2)(2x + 1) = -2x^3 - x^2 + 2x + 1$

(c) $g(f(x)) = g(1 - x^2) = 2(1 - x^2) + 1 = 3 - 2x^2$

42. $f(x) = x^3 - 3x^2$



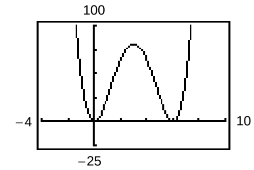
(a) The graph of g is obtained from f by a vertical shift down 1 unit, followed by a reflection in the x-axis:

$$g(x) = -[f(x) - 1]$$
$$= -x^3 + 3x^2 + 1$$

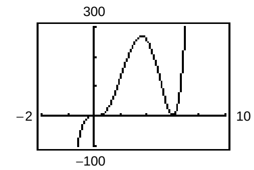
(b) The graph of g is obtained from f by a vertical shift upwards of 1 and a horizontal shift of 2 to the right.

$$g(x) = f(x - 2) + 1$$
$$= (x - 2)^3 - 3(x - 2)^2 + 1$$

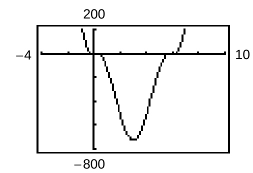
44. (a) $f(x) = x^2(x - 6)^2$



(b) $g(x) = x^3(x - 6)^2$



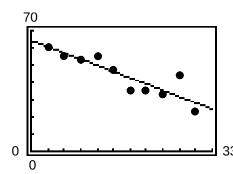
(c) $h(x) = x^3(x - 6)^3$



46. For company (a) the profit rose rapidly for the first year, and then leveled off. For the second company (b), the profit dropped, and then rose again later.

48. (a) $y = -1.204x + 64.2667$

(b)



(c) The data point $(27, 44)$ is probably an error. Without this point, the new model is

$$y = -1.4344x + 66.4387.$$

Problem Solving for Chapter P

2. Let $y = mx + 1$ be a tangent line to the circle from the point $(0, 1)$. Then

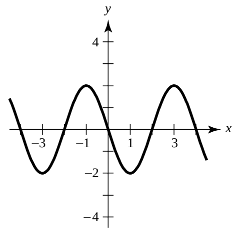
$$x^2 + (y + 1)^2 = 1$$
$$x^2 + (mx + 1 + 1)^2 = 1$$
$$(m^2 + 1)x^2 + 4mx + 3 = 0$$

Setting the discriminant $b^2 - 4ac$ equal to zero,

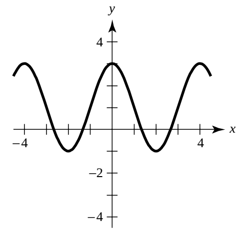
$$16m^2 - 4(m^2 + 1)(3) = 0$$
$$16m^2 - 12m^2 = 12$$
$$4m^2 = 12$$
$$m = \pm\sqrt{3}$$

Tangent lines: $y = \sqrt{3}x + 1$ and $y = -\sqrt{3}x + 1$.

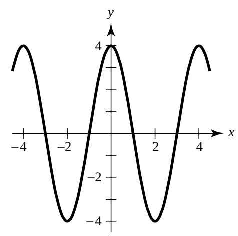
4. (a) $f(x + 1)$



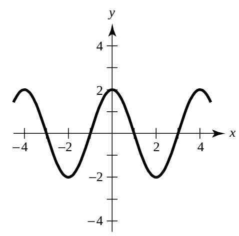
(b) $f(x) + 1$



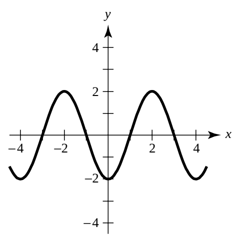
(c) $2f(x)$



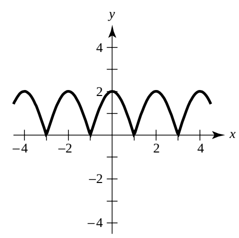
(d) $f(-x)$



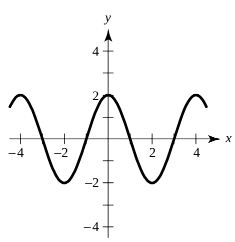
(e) $-f(x)$



(f) $|f(x)|$



(g) $f(|x|)$

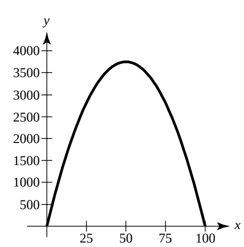


6. (a) $4y + 3x = 300 \implies y = \dfrac{300 - 3x}{4}$

$$A(x) = x(2y) = x\left(\frac{300 - 3x}{2}\right) = \frac{-3x^2 + 300x}{2}$$

Domain: $0 < x < 100$

(b)



Maximum of 3750 ft^2 at $x = 50$ ft, $y = 37.5$ ft.

(c) $A(x) = -\dfrac{3}{2}(x^2 - 100x)$

$$= -\frac{3}{2}(x^2 - 100x + 2500) + 3750$$

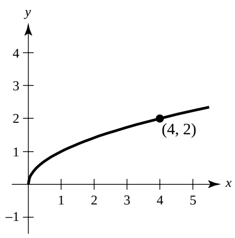
$$= -\frac{3}{2}(x - 50)^2 + 3750$$

$A(50) = 3750$ square feet is the maximum area, where $x = 50$ ft and $y = 37.5$ ft.

8. Let d be the distance from the starting point to the beach.

$$\text{Average velocity} = \frac{\text{distance}}{\text{time}}$$
$$= \frac{2d}{\frac{d}{120} + \frac{d}{60}}$$
$$= \frac{2}{\frac{1}{120} + \frac{1}{60}}$$
$$= 80 \text{ km/hr}$$

10.



(a) Slope $= \frac{3 - 2}{9 - 4} = \frac{1}{5}$. Slope of tangent line is greater than $\frac{1}{5}$.

(b) Slope $= \frac{2 - 1}{4 - 1} = \frac{1}{3}$. Slope of tangent line is less than $\frac{1}{3}$.

(c) Slope $= \frac{2.1 - 2}{4.1 - 4} = \frac{10}{41}$. Slope of tangent line is greater than $\frac{10}{41}$.

(d) Slope $= \frac{f(4 + h) - f(4)}{(4 + h) - 4}$

$$= \frac{\sqrt{4 + h} - 2}{h}$$

(e) $$\frac{\sqrt{4 + h} - 2}{h} = \frac{\sqrt{4 + h} - 2}{h} \cdot \frac{\sqrt{4 + h} + 2}{\sqrt{4 + h} + 2}$$
$$= \frac{(4 + h) - 4}{h\left(\sqrt{4 + h} + 2\right)}$$
$$= \frac{1}{\sqrt{4 + h} + 2}, h \neq 0$$

As h gets closer to 0, the slope gets closer to $\frac{1}{4}$. The slope is $\frac{1}{4}$ at the point $(4, 2)$.

12. (a) $$\frac{I}{\sqrt{x^2 + y^2}} = \frac{kI}{\sqrt{(x - 4)^2 + y^2}}$$

$$(x - 4)^2 + y^2 = k^2(x^2 + y^2)$$

$$(k^2 - 1)x^2 + (k^2 - 1)y^2 + 8x = 16$$

If $k = 1$, then $x = 2$ is a vertical line. So, assume $k^2 - 1 \neq 0$. Then

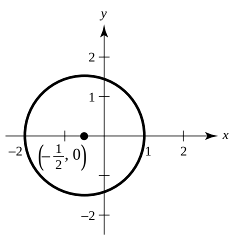
$$x^2 + y^2 + \frac{8x}{k^2 - 1} = \frac{16}{k^2 - 1}$$

$$\left(x + \frac{4}{k^2 - 1}\right)^2 + y^2 = \frac{16}{k^2 - 1} + \frac{16}{(k^2 - 1)^2}$$

$$\left(x + \frac{4}{k^2 - 1}\right)^2 + y^2 = \left(\frac{4k}{k^2 - 1}\right)^2, \text{ Circle}$$

(b) If $k = 3$, $\left(x + \frac{1}{2}\right)^2 + y^2 = \left(\frac{3}{2}\right)^2$

(c) For large k, the center of the circle is near $(0, 0)$, and the radius becomes smaller.



14. $f(x) = y = \dfrac{1}{1 - x}$

(a) Domain: all $x \neq 1$

Range: all $y \neq 0$

(b) $$f(f(x)) = f\left(\frac{1}{1 - x}\right) = \frac{1}{1 - \left(\frac{1}{1 - x}\right)} = \frac{1}{\frac{1 - x - 1}{1 - x}} = \frac{1 - x}{-x} = \frac{x - 1}{x}$$

Domain: all $x \neq 0, 1$

(c) $$f(f(f(x))) = f\left(\frac{x - 1}{x}\right) = \frac{1}{1 - \left(\frac{x - 1}{x}\right)} = \frac{1}{\frac{1}{x}} = x$$

Domain: all $x \neq 0, 1$

(d) The graph is not a line. It has holes at $(0, 0)$ and $(1, 1)$.