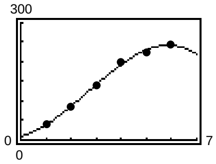
15. (a) $y = -1.81x^3 + 14.58x^2 + 16.39x + 10$

(b)



(c) If $x = 4.5$, $y \approx 214$ horsepower.

17. (a) Yes, y is a function of t. At each time t, there is one and only one displacement y.

(b) The amplitude is approximately

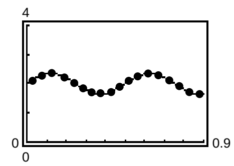
$$(2.35 - 1.65)/2 = 0.35.$$

The period is approximately

$$2(0.375 - 0.125) = 0.5.$$

(c) One model is $y = 0.35 \sin(4\pi t) + 2$.

(d)



19. Answers will vary.

Review Exercises for Chapter P

1. $y = 2x - 3$

$x = 0 \Longrightarrow y = 2(0) - 3 = -3 \Longrightarrow (0, -3)$ y-intercept

$y = 0 \Longrightarrow 0 = 2x - 3 \Longrightarrow x = \frac{3}{2} \Longrightarrow \left(\frac{3}{2}, 0\right)$ x-intercept

3. $y = \dfrac{x - 1}{x - 2}$

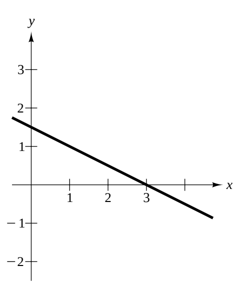
$x = 0 \Longrightarrow y = \dfrac{0 - 1}{0 - 2} = \dfrac{1}{2} \Longrightarrow \left(0, \dfrac{1}{2}\right)$ y-intercept

$y = 0 \Longrightarrow 0 = \dfrac{x - 1}{x - 2} \Longrightarrow x = 1 \Longrightarrow (1, 0)$ x-intercept

5. Symmetric with respect to y-axis since

$$(-x)^2y - (-x)^2 + 4y = 0$$
$$x^2y - x^2 + 4y = 0.$$

7. $y = -\frac{1}{2}x + \frac{3}{2}$



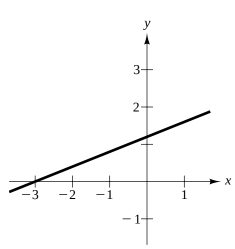
9. $-\frac{1}{3}x + \frac{5}{6}y = 1$

$-\frac{2}{5}x + y = \frac{6}{5}$

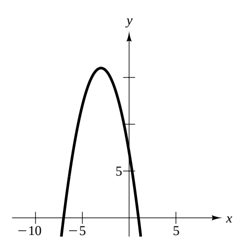
$y = \frac{2}{5}x + \frac{6}{5}$

Slope: $\frac{2}{5}$

y-intercept: $\frac{6}{5}$

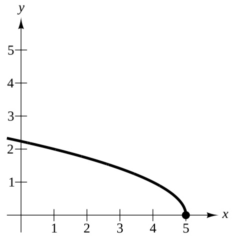


11. $y = 7 - 6x - x^2$



13. $y = \sqrt{5 - x}$

Domain: $(-\infty, 5]$



15. $y = 4x^2 - 25$

Xmin = -5
Xmax = 5
Xscl = 1
Ymin = -30
Ymax = 10
Yscl = 5

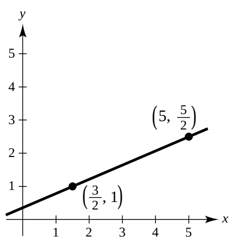
17.
$$\begin{aligned} 3x - 4y &= 8 \\ 4x + 4y &= 20 \\ \hline 7x \quad &= 28 \\ x &= 4 \\ y &= 1 \end{aligned}$$

Point: $(4, 1)$

19. You need factors $(x + 2)$ and $(x - 2)$. Multiply by x to obtain origin symmetry

$$\begin{aligned} y &= x(x + 2)(x - 2). \\ &= x^3 - 4x. \end{aligned}$$

21.

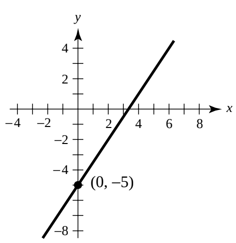


$$\text{Slope} = \frac{(5/2) - 1}{5 - (3/2)} = \frac{3/2}{7/2} = \frac{3}{7}$$

23.
$$\begin{aligned} \frac{1 - t}{1 - 0} &= \frac{1 - 5}{1 - (-2)} \\ 1 - t &= -\frac{4}{3} \\ t &= \frac{7}{3} \end{aligned}$$

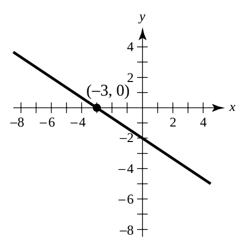
25.
$$\begin{aligned} y - (-5) &= \tfrac{3}{2}(x - 0) \\ y &= \tfrac{3}{2}x - 5 \end{aligned}$$

$2y - 3x + 10 = 0$



27.
$$\begin{aligned} y - 0 &= -\tfrac{2}{3}(x - (-3)) \\ y &= -\tfrac{2}{3}x - 2 \end{aligned}$$

$3y + 2x + 6 = 0$



29. (a) $y - 4 = \frac{7}{16}(x + 2)$

$16y - 64 = 7x + 14$

$0 = 7x - 16y + 78$

(b) Slope of line is $\frac{5}{3}$.

$y - 4 = \frac{5}{3}(x + 2)$

$3y - 12 = 5x + 10$

$0 = 5x - 3y + 22$

(c) $m = \frac{4 - 0}{-2 - 0} = -2$

$y = -2x$

$2x + y = 0$

(d) $x = -2$

$x + 2 = 0$

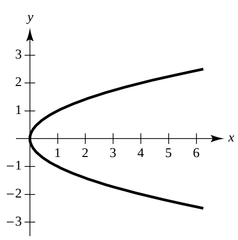
31. The slope is -850. $V = -850t + 12{,}500$.

$V(3) = -850(3) + 12{,}500 = \$9950$

33. $x - y^2 = 0$

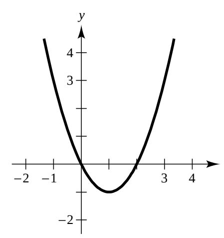
$y = \pm\sqrt{x}$

Not a function of x since there are two values of y for some x.



35. $y = x^2 - 2x$

Function of x since there is one value of y for each x.



37. $f(x) = \frac{1}{x}$

(a) $f(0)$ does not exist.

(b) $\frac{f(1 + \Delta x) - f(1)}{\Delta x} = \frac{\frac{1}{1 + \Delta x} - \frac{1}{1}}{\Delta x} = \frac{1 - 1 - \Delta x}{(1 + \Delta x)\Delta x}$

$= \frac{-1}{1 + \Delta x}, \Delta x \neq -1, 0$

39. (a) Domain: $36 - x^2 \geq 0 \Rightarrow -6 \leq x \leq 6$ or $[-6, 6]$

Range: $[0, 6]$

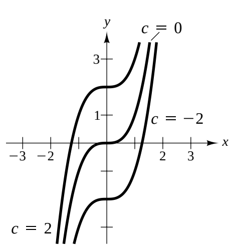
(b) Domain: all $x \neq 5$ or $(-\infty, 5), (5, \infty)$

Range: all $y \neq 0$ or $(-\infty, 0), (0, \infty)$

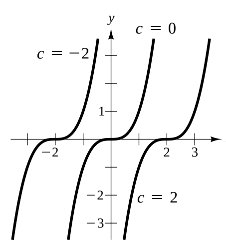
(c) Domain: all x or $(-\infty, \infty)$

Range: all y or $(-\infty, \infty)$

41. (a) $f(x) = x^3 + c, c = -2, 0, 2$



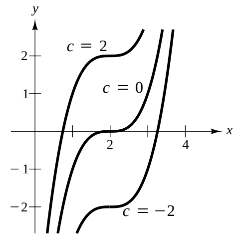
(b) $f(x) = (x - c)^3, c = -2, 0, 2$



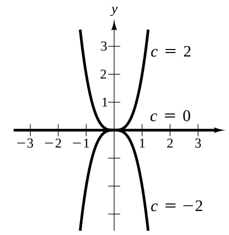
—CONTINUED—

41. —CONTINUED—

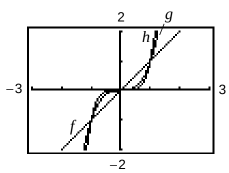
(c) $f(x) = (x - 2)^3 + c$, $c = -2, 0, 2$



(d) $f(x) = cx^3$, $c = -2, 0, 2$

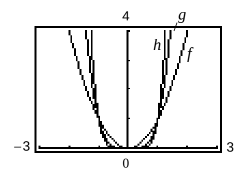


43. (a) Odd powers: $f(x) = x$, $g(x) = x^3$, $h(x) = x^5$



The graphs of f, g, and h all rise to the right and fall to the left. As the degree increases, the graph rises and falls more steeply. All three graphs pass through the points $(0, 0)$, $(1, 1)$, and $(-1, -1)$.

Even powers: $f(x) = x^2$, $g(x) = x^4$, $h(x) = x^6$

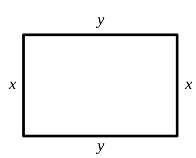


The graphs of f, g, and h all rise to the left and to the right. As the degree increases, the graph rises more steeply. All three graphs pass through the points $(0, 0)$, $(1, 1)$, and $(-1, 1)$.

(b) $y = x^7$ will look like $h(x) = x^5$, but rise and fall even more steeply.

$y = x^8$ will look like $h(x) = x^6$, but rise even more steeply.

45. (a)

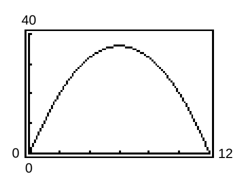


$$2x + 2y = 24$$

$$y = 12 - x$$

$$A = xy = x(12 - x) = 12x - x^2$$

(b) Domain: $0 < x < 12$



(c) Maximum area is $A = 36$. In general, the maximum area is attained when the rectangle is a square. In this case, $x = 6$.

47. (a) 3 (cubic), negative leading coefficient

(b) 4 (quartic), positive leading coefficient

(c) 2 (quadratic), negative leading coefficient

(d) 5, positive leading coefficient

49. (a) Yes, y is a function of t. At each time t, there is one and only one displacement y.

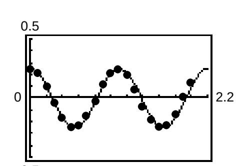
(b) The amplitude is approximately

$$(0.25 - (-0.25))/2 = 0.25.$$

The period is approximately 1.1.

(c) One model is $y = \frac{1}{4}\cos\left(\frac{2\pi}{1.1}t\right) \approx \frac{1}{4}\cos(5.7t)$

(d)



Problem Solving for Chapter P

1. (a)

$$x^2 - 6x + y^2 - 8y = 0$$
$$(x^2 - 6x + 9) + (y^2 - 8y + 16) = 9 + 16$$
$$(x - 3)^2 + (y - 4)^2 = 25$$

Center: $(3, 4)$ Radius: 5

(b) Slope of line from $(0, 0)$ to $(3, 4)$ is $\frac{4}{3}$ Slope of tangent line is $-\frac{3}{4}$ Hence,

$$y - 0 = -\frac{3}{4}(x - 0) \Rightarrow y = -\frac{3}{4}x \quad \text{Tangent line}$$

(c) Slope of line from $(6, 0)$ to $(3, 4)$ is $\frac{4 - 0}{3 - 6} = -\frac{4}{3}$.

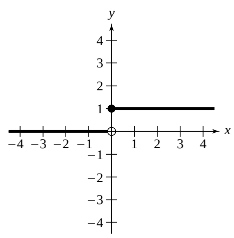
Slope of tangent line is $\frac{3}{4}$. Hence,

$$y - 0 = \frac{3}{4}(x - 6) \Rightarrow y = \frac{3}{4}x - \frac{9}{2} \quad \text{Tangent line}$$

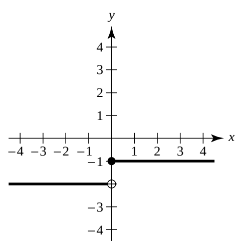
(d)
$$-\frac{3}{4}x = \frac{3}{4}x - \frac{9}{2}$$
$$\frac{3}{2}x = \frac{9}{2}$$
$$x = 3$$

Intersection: $\left(3, -\frac{9}{4}\right)$

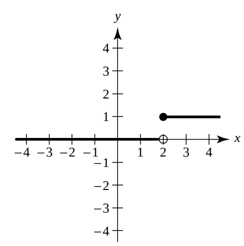
3. $H(x) = \begin{cases} 1 & x \geq 0 \\ 0 & x < 0 \end{cases}$



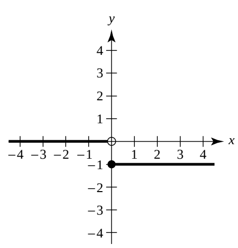
(a) $H(x) - 2$



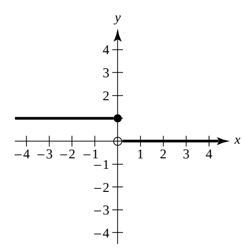
(b) $H(x - 2)$



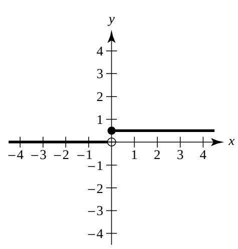
(c) $-H(x)$



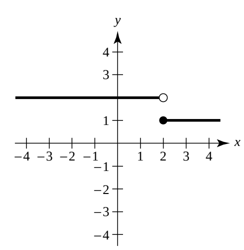
(d) $H(-x)$



(e) $\frac{1}{2}H(x)$



(f) $-H(x - 2) + 2$

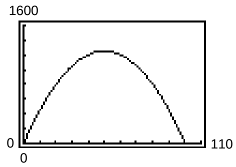


5. (a) $x + 2y = 100 \Rightarrow y = \dfrac{100 - x}{2}$

$A(x) = xy = x\left(\dfrac{100 - x}{2}\right) = -\dfrac{x^2}{2} + 50x$

Domain: $0 < x < 100$

(b)



Maximum of 1250 m^2 at $x = 50$ m, $y = 25$ m.

(c) $A(x) = -\dfrac{1}{2}(x^2 - 100x)$

$= -\dfrac{1}{2}(x^2 - 100x + 2500) + 1250$

$= -\dfrac{1}{2}(x - 50)^2 + 1250$

$A(50) = 1250$ m^2 is the maximum. $x = 50$ m, $y = 25$ m.

7. The length of the trip in the water is $\sqrt{2^2 + x^2}$, and the length of the trip over land is $\sqrt{1 + (3 - x)^2}$. Hence, the total time is

$$T = \frac{\sqrt{4 + x^2}}{2} + \frac{\sqrt{1 + (3 - x)^2}}{4} \text{ hours.}$$

9. (a) Slope $= \dfrac{9 - 4}{3 - 2} = 5$. Slope of tangent line is less than 5.

(b) Slope $= \dfrac{4 - 1}{2 - 1} = 3$. Slope of tangent line is greater than 3.

(c) Slope $= \dfrac{4.41 - 4}{2.1 - 2} = 4.1$. Slope of tangent line is less than 4.1.

(d) Slope $= \dfrac{f(2 + h) - f(2)}{(2 + h) - 2}$

$= \dfrac{(2 + h)^2 - 4}{h}$

$= \dfrac{4h + h^2}{h}$

$= 4 + h, h \neq 0$

(e) Letting h get closer and closer to 0, the slope approaches 4. Hence, the slope at $(2, 4)$ is 4.

11. (a) At $x = 1$ and $x = -3$ the sounds are equal.

(b) $\dfrac{I}{\sqrt{x^2 + y^2}} = \dfrac{2I}{\sqrt{(x - 3)^2 + y^2}}$

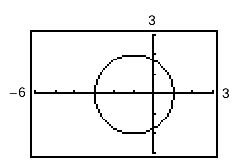
$(x - 3)^2 + y^2 = 4(x^2 + y^2)$

$3x^2 + 3y^2 + 6x = 9$

$x^2 + 2x + y^2 = 3$

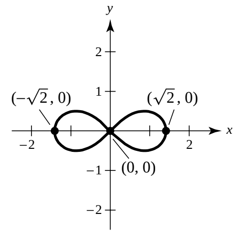
$(x + 1)^2 + y^2 = 4$

Circle of radius 2 centered at $(-1, 0)$



13.

$$d_1 d_2 = 1$$
$$[(x+1)^2 + y^2][(x-1)^2 + y^2] = 1$$
$$(x+1)^2(x-1)^2 + y^2[(x+1)^2 + (x-1)^2] + y^4 = 1$$
$$(x^2-1)^2 + y^2[2x^2+2] + y^4 = 1$$
$$x^4 - 2x^2 + 1 + 2x^2y^2 + 2y^2 + y^4 = 1$$
$$(x^4 + 2x^2y^2 + y^4) - 2x^2 + 2y^2 = 0$$
$$(x^2+y^2)^2 = 2(x^2 - y^2)$$



Let $y = 0$. Then $x^4 = 2x^2 \implies x = 0$ or $x^2 = 2$.

Thus, $(0, 0)$, $(\sqrt{2}, 0)$ and $(-\sqrt{2}, 0)$ are on the curve.