# INSTRUCTOR'S RESOURCE MANUAL

Steve Rigdon
SOUTHERN ILLINOIS UNIVERSITY - EDWARDSVILLE

# CALCULUS

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## 1.1 Concepts Review

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2. $\sqrt{2};\pi$
3. real
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1. $4-2(8-11)+6=4-2(-3)+6$
$=4+6+6=16$

2. $3[2-4(7-12)]=3[2-4(-5)]$
$=3[2+20]=3(22)=66$

3. $-4[5(-3+12-4)+2(13-7)]$
$=-4[5(5)+2(6)]=-4[25+12]$
$=-4(37)=-148$

4. $5[-1(7+12-16)+4]+2$
$=5[-1(3)+4]+2=5(-3+4)+2$
$=5(1)+2=5+2=7$

5. $\frac{5}{7}-\frac{1}{13}=\frac{65}{91}-\frac{7}{91}=\frac{58}{91}$

6. $\frac{3}{4-7}+\frac{3}{21}-\frac{1}{6}=\frac{3}{-3}+\frac{3}{21}-\frac{1}{6}$
$=-\frac{42}{42}+\frac{6}{42}-\frac{7}{42}=-\frac{43}{42}$

7. $\frac{1}{3}\left[\frac{1}{2}\left(\frac{1}{4}-\frac{1}{3}\right)+\frac{1}{6}\right]=\frac{1}{3}\left[\frac{1}{2}\left(\frac{3-4}{12}\right)+\frac{1}{6}\right]$
$=\frac{1}{3}\left[\frac{1}{2}\left(-\frac{1}{12}\right)+\frac{1}{6}\right]$
$=\frac{1}{3}\left[-\frac{1}{24}+\frac{4}{24}\right]$
$=\frac{1}{3}\left(\frac{3}{24}\right)=\frac{1}{24}$

8. $-\frac{1}{3}\left[\frac{2}{5}-\frac{1}{2}\left(\frac{1}{3}-\frac{1}{5}\right)\right]$
$=-\frac{1}{3\left[\frac{2}{5}-\frac{1}{2}\left(\frac{5}{15}-\frac{3}{15}\right)\right]}$
$=-\frac{1}{3}\left[\frac{2}{5}-\frac{1}{2}\left(\frac{2}{15}\right)\right]=-\frac{1}{3}\left[\frac{2}{5}-\frac{1}{15}\right]$
$=-\frac{1}{3}\left(\frac{6}{15}-\frac{1}{15}\right)=-\frac{1}{3}\left(\frac{5}{15}\right)=-\frac{1}{9}$

9. $\frac{14}{21}\left(\frac{2}{5-\frac{1}{3}}\right)^2=\frac{14}{21}\left(\frac{2}{\frac{14}{3}}\right)^2=\frac{14}{21}\left(\frac{6}{14}\right)^2$
$=\frac{14}{21}\left(\frac{3}{7}\right)^2=\frac{2}{3}\left(\frac{9}{49}\right)=\frac{6}{49}$

10. $\frac{\left(\frac{2}{7}-5\right)}{\left(1-\frac{1}{7}\right)}=\frac{\left(\frac{2}{7}-\frac{35}{7}\right)}{\left(\frac{7}{7}-\frac{1}{7}\right)}=\frac{\left(-\frac{33}{7}\right)}{\left(\frac{6}{7}\right)}=-\frac{33}{6}=-\frac{11}{2}$

11. $\frac{\frac{11}{7}-\frac{12}{21}}{\frac{11}{7}+\frac{12}{21}}=\frac{\frac{11}{7}-\frac{4}{7}}{\frac{11}{7}+\frac{4}{7}}=\frac{\frac{7}{7}}{\frac{15}{7}}=\frac{7}{15}$

12. $\frac{\frac{1}{2}-\frac{3}{4}+\frac{7}{8}}{\frac{1}{2}+\frac{3}{4}-\frac{7}{8}}=\frac{\frac{4}{8}-\frac{6}{8}+\frac{7}{8}}{\frac{4}{8}+\frac{6}{8}-\frac{7}{8}}=\frac{\frac{5}{8}}{\frac{3}{8}}=\frac{5}{3}$

13. $1-\frac{1}{1+\frac{1}{2}}=1-\frac{1}{\frac{3}{2}}=1-\frac{2}{3}=\frac{3}{3}-\frac{2}{3}=\frac{1}{3}$

14. $2+\frac{3}{1+\frac{5}{2}}=2+\frac{3}{\frac{2}{2}-\frac{5}{2}}=2+\frac{3}{\frac{7}{2}}$
$=2+\frac{6}{7}=\frac{14}{7}+\frac{6}{7}=\frac{20}{7}$

15. $\left(\sqrt{5}+\sqrt{3}\right)\left(\sqrt{5}-\sqrt{3}\right)=\left(\sqrt{5}\right)^2-\left(\sqrt{3}\right)^2$
$=5-3=2$

16. $\left(\sqrt{5}-\sqrt{3}\right)^2=\left(\sqrt{5}\right)^2-2\left(\sqrt{5}\right)\left(\sqrt{3}\right)+\left(\sqrt{3}\right)^2$
$=5-2\sqrt{15}+3=8-2\sqrt{15}$

17. $3\sqrt{2}\left(\sqrt{2}-\sqrt{8}\right)=3\sqrt{4}-3\sqrt{16}$
$=3\cdot 2-3\cdot 4$
$=6-12=-6$

18. $2\sqrt[3]{4}\left[\sqrt[3]{2}+\sqrt[3]{16}\right]=2\sqrt[3]{8}+2\sqrt[3]{64}$
$=2\cdot 2+2\cdot 4$
$=4+8=12$

19. $\left(\frac{7}{4}+\frac{1}{2}\right)^{-2}=\left(\frac{7}{4}+\frac{2}{4}\right)^{-2}=\left(\frac{9}{4}\right)^{-2}$
$=\frac{1}{\left(\frac{9}{4}\right)^2}=\frac{1}{\frac{81}{16}}=\frac{16}{81}$

20. $\left(\frac{1}{\sqrt{2}}-\frac{5}{2\sqrt{2}}\right)^{-2}=\left(\frac{2}{2\sqrt{2}}-\frac{5}{2\sqrt{2}}\right)^{-2}$
$=\left(-\frac{3}{2\sqrt{2}}\right)^{-2}$
$=\frac{1}{\left(-\frac{3}{2\sqrt{2}}\right)^2}=\frac{1}{\frac{9}{8}}$
$=\frac{8}{9}$

21. $(3x-4)(x+1)=3x^2+3x-4x-4$
$=3x^2-x-4$

22. $(2x-3)^2=(2x-3)(2x-3)$
$=4x^2-6x-6x+9$
$=4x^2-12x+9$

23. $(3x-9)(2x+1)=6x^2+3x-18x-9$
$=6x^2-15x-9$

24. $(4x-11)(3x-7)=12x^2-28x-33x+77$
$=12x^2-61x+77$

25. $(3t^2-t+1)^2$
$=(3t^2-t+1)(3t^2-t+1)$
$=9t^4-3t^3+3t^2-3t^3+t^2-t+3t^2-t+1$
$=9t^4-6t^3+7t^2-2t+1$

26. $(2t+3)^3=(2t+3)(2t+3)(2t+3)$
$=(4t^2+12t+9)(2t+3)$
$=8t^3+12t^2+24t^2+36t+18t+27$
$=8t^3+36t^2+54t+27$

27. $\frac{x^2-4}{x-2}=\frac{(x-2)(x+2)}{x-2}=x+2$

28. $\frac{x^2-x-6}{x-3}=\frac{(x-3)(x+2)}{(x-3)}=x+2$

29. $\frac{t^2-4t-21}{t+3}=\frac{(t+3)(t-7)}{t+3}=t-7$

30. $\frac{2x-2x^2}{x^3-2x^2+x}=\frac{2x(1-x)}{x(x^2-2x+1)}$
$=\frac{-2x(x-1)}{x(x-1)(x-1)}$
$=-\frac{2}{x-1}$

31. $\frac{12}{x^2+2x}+\frac{4}{x}+\frac{2}{x+2}$
$=\frac{12}{x(x+2)}+\frac{4(x+2)}{x(x+2)}+\frac{2x}{x(x+2)}$
$=\frac{12+4x+8+2x}{x(x+2)}$
$=\frac{6x+20}{x(x+2)}$
$=\frac{2(3x+10)}{x(x+2)}$

32. $\frac{2}{6y-2}+\frac{y}{9y^2-1}-\frac{2y+1}{1-3y}$
$=\frac{2}{2(3y-1)}+\frac{y}{(3y+1)(3y-1)}+\frac{2y+1}{3y-1}$
$=\frac{2(3y+1)}{2(3y+1)(3y-1)}+\frac{2y}{2(3y+1)(3y-1)}$
$+\frac{(2y+1)(3y+1)}{2(3y+1)(3y-1)}$
$=\frac{6y+2+2y+12y^2+10y+2}{2(3y+1)(3y-1)}$

$$= \frac{12y^2 + 18y + 4}{2(3y+1)(3y-1)}$$
$$= \frac{2(6y^2 + 9y + 2)}{2(3y+1)(3y-1)}$$
$$= \frac{6y^2 + 9y + 2}{(3y+1)(3y-1)}$$

**33.** $\frac{t^2 + t - 12}{x^2 - 1} \cdot \frac{x^2 - 6x - 7}{8t - t^2 - 15}$
$$= \frac{(t+4)(t-3)(x-7)(x+1)}{-(x+1)(x-1)(t-3)(t-5)}$$
$$= -\frac{(t+4)(x-7)}{(x-1)(t-5)}$$

**34.** $\frac{\frac{x}{x-3} - \frac{2}{x^2-4x+3}}{\frac{5}{x-1} + \frac{5}{x+3}} = \frac{\frac{x}{x-3} - \frac{2}{(x-3)(x-1)}}{\frac{5}{x-1} + \frac{5}{x-3}}$
$$= \frac{x(x-1) - 2}{5(x-3) + 5(x-1)}$$
$$= \frac{x^2 - x - 2}{5x - 15 + 5x - 5}$$
$$= \frac{(x-2)(x+1)}{10(x-2)} = \frac{x+1}{10}$$

**35.** **a.** $0 \cdot 0 = 0$ **b.** $\frac{0}{0}$ is undefined.

**c.** $\frac{0}{17} = 0$ **d.** $\frac{3}{0}$ is undefined.

**e.** $0^5 = 0$ **f.** $17^0 = 1$

**36.** If $\frac{0}{0} = a$, then $0 = 0 \cdot a$, but this is meaningless because $a$ could be any real number. No single value satisfies $\frac{0}{0} = a$.

**37.** **a.** $-3 < -7$; False **b.** $-1 > -17$; True

**c.** $-3 < -\frac{22}{7}; -\frac{21}{7} < -\frac{22}{7}$; False

**d.** $-5 > -\sqrt{26}; -\sqrt{25} > -\sqrt{26}$; True

**e.** $\frac{6}{7} < \frac{34}{39}; \frac{234}{273} < \frac{238}{273}$; True

**f.** $-\frac{5}{7} < -\frac{44}{59}; -\frac{295}{413} < -\frac{308}{413}$; False

**38.** **a.** $a < b; a^2 < ab$ and $ab < b^2$, so $a^2 < b^2$

**b.** $a < b;\ \frac{a}{b} < 1;\ \frac{1}{b} < \frac{1}{a};\ \frac{1}{a} > \frac{1}{b}$

**39.** $a < b$; $2a < a + b$ and $a + b < 2b$, so $2a < a + b < 2b$; $a < \frac{a+b}{2} < b$

**40.** **a.** is false if $a < 0$

**b.** is always true

**c.** is always true

**d.** is false

**41.** **a.** True; If $x$ is positive, then $x^2$ is positive.

**b.** False; Take $x = -2$. Then $x^2 > 0$ but $x < 0$.

**c.** False; Take $x = \frac{1}{2}$. Then $x^2 = \frac{1}{4} < x$.

**d.** True; $n = 2$ is even and prime.

**e.** True; Let $x$ be any number. Take $y = x^2 + 1$. Then $y > x^2$.

**f.** False; There is no number larger than every $x^2$.

**g.** True; $1/n$ can be made arbitrarily close to 0.

**h.** True $2^{-n}$ can be made arbitrarily close to 0.

**42.** **a.** If $n$ is odd, then there is an integer $k$ such that $n = 2k + 1$. Then $n^2 = (2k+1)^2 = 4k^2 + 4k + 1 = 2(2k^2 + 2k) + 1$

b. Prove the contrapositive. Suppose $n$ is even. Then there is an integer $k$ such that $n = 2k$. Then

$n^2 = (2k)^2 = 4k^2 = 2(2k^2)$

Thus $n^2$ is even.

c. Parts (a) and (b) prove that $n$ is odd if and only if $n^2$ is odd.

43. a. $243 = 3\cdot 3\cdot 3\cdot 3\cdot 3$

b. $127 = 1\cdot 127$

c. $5100 = 2\cdot 2550 = 2\cdot 2\cdot 1275 = 2\cdot 2\cdot 3\cdot 425$
$= 2\cdot 2\cdot 3\cdot 5\cdot 85 = 2\cdot 2\cdot 3\cdot 5\cdot 5\cdot 17$

d. $346 = 2\cdot 173$

44. Let $A = b\cdot c^2\cdot d^3$; then $A^2 = b^2\cdot c^4\cdot d^6$, so the square of the number is the product of primes which occur an even number of times

45. $\sqrt{2} = \frac{p}{q}; 2 = \frac{p^2}{q^2}; 2q^2 = p^2$; Since the prime factors of $p^2$ must occur an even number of times, $2q^2$ would not be valid and $\frac{p}{q} = \sqrt{2}$ must be irrational.

46. $\sqrt{3} = \frac{p}{q}$; $3 = \frac{p^2}{q^2}$; $3q^2 = p^2$; Since the prime factors of $p^2$ must occur an even number of times, $3q^2$ would not be valid and $\frac{p}{q} = \sqrt{3}$ must be irrational.

47. Let $a$, $b$, $p$, and $q$ be natural numbers, so $\frac{a}{b}$ and $\frac{p}{q}$ are rational. $\frac{a}{b} + \frac{p}{q} = \frac{aq+bp}{bq}$ This sum is the quotient of natural numbers, so it is also rational.

48. Let $a$ be an irrational number and $p$ and $q$ be natural numbers. $a\cdot\frac{p}{q} = \frac{ap}{q}$. Since the numerator is not a natural number, the product is irrational.

49. a. $-\sqrt{9} = -3$; rational

b. $0.375 = \frac{3}{8}$; rational

c. $1 - \sqrt{2}$; irrational

d. $(1+\sqrt{3})^2 = 1 + 2\sqrt{3} + 3 = 4 + 2\sqrt{3}$; irrational

e. $(3\sqrt{2})(5\sqrt{2}) = 15\sqrt{4} = 30$; rational

e. $5\sqrt{2}$; irrational

50. The sum of two irrational numbers is not always irrational. If the numbers are additive inverses, the sum is 0, which is rational.

51. If $m$ were a perfect square, its prime factors would occur even numbers of times. If $m$ is not a perfect square, some factors will occur an odd number of times and $\sqrt{m}$ will be irrational.

52. $\sqrt{6} + \sqrt{3} \approx 4.18154$ is irrational.

53. $\sqrt{2} - \sqrt{3} + \sqrt{6} \approx 2.13165$ is irrational.

54. $\log_{10} 5 \approx 0.69897$ is irrational

55. a. Converse: If I get an A in this course, then I do all of the homework.
Contrapositive: If I don't get an A in this course, then I don't do all of the homework.

b. Converse: If $x$ is an integer, then $x$ is a real number.
Contrapositive: If $x$ is not an integer, then $x$ is not a real number.

c. Converse: If $\Delta ABC$ is an isosceles triangle, then $\Delta ABC$ is an equilateral triangle.
Contrapositive: If $\Delta ABC$ is not an isosceles triangle, then $\Delta ABC$ is not an equilateral triangle.

## 1.2 Concepts Review

1. 0.333..... (3s repeat); 0.200... (0s repeat); 3.14159...

2. rational

3. rational; irrational

4. real

## Problem Set 1.2

1. $$\begin{array}{r} .0833 \\ 12\overline{)1.0000} \\ \underline{96} \\ 40 \\ \underline{36} \\ 40 \end{array}$$

2. $$\begin{array}{r} .285714 \\ 7\overline{)2.000000} \\ \underline{1\ 4} \\ 60 \\ \underline{56} \\ 40 \\ \underline{35} \\ 50 \\ \underline{49} \\ 10 \\ \underline{7} \\ 30 \\ \underline{28} \\ 2 \end{array}$$

3. $$\begin{array}{r} .846153 \\ 13\overline{)11.000000} \\ \underline{10\ 4} \\ 60 \\ \underline{52} \\ 80 \\ \underline{78} \\ 20 \\ \underline{13} \\ 70 \\ \underline{65} \\ 50 \\ \underline{39} \\ 11 \end{array}$$

4. $$\begin{array}{r} .294117 \\ 17\overline{)5.000000} \\ \underline{3\ 4} \\ 160 \\ \underline{153} \\ 70 \\ \underline{68} \\ 20 \\ \underline{17} \\ 30 \\ \underline{17} \\ 130 \\ \underline{119} \\ 11 \end{array}$$

5. $$\begin{array}{r} 3.66 \\ 3\overline{)11.00} \\ \underline{9} \\ 20 \\ \underline{18} \\ 20 \\ \underline{18} \\ 2 \end{array}$$

6. 
$$\begin{array}{r} .846153 \\ 13\overline{)11.000000} \\ \underline{10\ 4} \\ 60 \\ \underline{52} \\ 80 \\ \underline{78} \\ 20 \\ \underline{13} \\ 70 \\ \underline{65} \\ 50 \\ \underline{39} \\ 11 \end{array}$$

7. $x = 0.123123123...$
$$\begin{array}{r} 1000x = 123.123123... \\ \underline{x = 0.123123...} \\ 999x = 123 \\ x = \frac{123}{999} = \frac{41}{333} \end{array}$$

8. $x = 0.217171717\ldots$
$$\begin{array}{r} 1000x = 217.171717... \\ \underline{10x = 2.171717...} \\ 990x = 215 \\ x = \frac{215}{990} = \frac{43}{198} \end{array}$$

9. $x = 2.56565656...$
$$\begin{array}{r} 100x = 256.565656... \\ \underline{x = 2.565656...} \\ 99x = 254 \\ x = \frac{254}{99} \end{array}$$

10. $x = 3.929292\ldots$
$$\begin{array}{r} 100x = 392.929292... \\ \underline{x = 3.929292...} \\ 99x = 389 \\ x = \frac{389}{99} \end{array}$$

11. $x = 0.199999...$
$$\begin{array}{r} 100x = 19.99999... \\ \underline{10x = 1.99999...} \\ 90x = 18 \\ x = \frac{18}{90} = \frac{1}{5} \end{array}$$

12. $x = 0.399999\ldots$
$$\begin{array}{r} 100x = 39.99999... \\ \underline{10x = 3.99999...} \\ 90x = 36 \\ x = \frac{36}{90} = \frac{2}{5} \end{array}$$

13. Those rational numbers that can be expressed by a terminating decimal followed by zeros.

14. $\frac{p}{q} = p\left(\frac{1}{q}\right)$, so we only need to look at $\frac{1}{q}$. If $q = 2^n \cdot 5^m$, then $\frac{1}{q} = \left(\frac{1}{2}\right)^n \cdot \left(\frac{1}{5}\right)^m = (0.5)^n (0.2)^m$. The product of any number of terminating decimals is also a terminating decimal, so $(0.5)^n$ and $(0.2)^m$, and hence their product, $\frac{1}{q}$, is a terminating decimal. Thus $\frac{p}{q}$ has a terminating decimal expansion.

15. Answers will vary. Possible answer: 0.000001, $\frac{1}{\pi^{12}} \approx 0.0000010819...$

16. Smallest positive integer: 1; There is no smallest positive rational or irrational number.

17. Answers will vary. Possible answer: 3.14159101001...

18. There is no real number between 0.9999... (repeating 9's) and 1. 0.9999... and 1 represent the *same* real number.

19. Irrational

20. Answers will vary. Possible answers: $-\pi$ and $\pi$, $-\sqrt{2}$ and $\sqrt{2}$

21. $(\sqrt{3}+1)^3 \approx 20.39230485$

22. $\left(\sqrt{2}-\sqrt{3}\right)^4 \approx 0.0102051443$

23. $\sqrt[4]{1.123}-\sqrt[3]{1.09} \approx 0.00028307388$

24. $(3.1415)^{-1/2} \approx 0.5641979034$

25. $\dfrac{\sqrt{130}-\sqrt{5}}{3^{1.2}-3} \approx 12.43322783$

26. $\dfrac{(0.00121)\left(5.23\times10^{-3}\right)}{6.16\times10^{-4}} \approx 0.0102732143$

27. $\sqrt{8.9\pi^2+1}-3\pi \approx 0.000691744752$

28. $\sqrt[4]{(6\pi^2-2)\pi} \approx 3.661591807$

29. Let $a$ and $b$ be real numbers with $a < b$. Let $n$ be a natural number that satisfies $1/n < b-a$. Let $S = \{k : k/n > b\}$. Since a nonempty set of integers that is bounded below contains a least element, there is a $k_0 \in S$ such that $k_0/n > b$ but $(k_0-1)/n \le b$. Then

$$\frac{k_0-1}{n} = \frac{k_0}{n} - \frac{1}{n} > b - \frac{1}{n} > a$$

Thus, $a < \frac{k_0-1}{n} \le b$. If $\frac{k_0-1}{n} < b$, then choose $r = \frac{k_0-1}{n}$. Otherwise, choose $r = \frac{k_0-2}{n}$.

30. Answers will vary. Possible answer: $\approx 120$ in$^3$

31. $r = 4000 \text{ mi} \times 5280\dfrac{\text{ft}}{\text{mi}} = 21{,}120{,}000 \text{ ft}$

equator $= 2\pi r = 2\pi(21{,}120{,}000) \approx 132{,}700{,}874$ ft

32. Answers will vary. Possible answer:

$$70\frac{\text{beats}}{\text{min}}\times 60\frac{\text{min}}{\text{hr}}\times 24\frac{\text{hr}}{\text{day}}\times 365\frac{\text{day}}{\text{year}}\times 20 \text{ yr}$$

$= 735{,}840{,}000$ beats

33. $V = \pi r^2 h = \pi\left(\dfrac{16}{2}\cdot 12\right)^2 (270\cdot 12)$

$\approx 93{,}807{,}453.98$ in.$^3$

volume of one board foot (in inches):
1 3 12 3 12 = 144 in.$^3$
number of board feet:

$\dfrac{93{,}807{,}453.98}{144} \approx 651{,}441$ board ft

34. $V = \pi(8.004)^2(270) - \pi(8)^2(270) \approx 54.3$ in.$^3$

35. a. At $x = 2\pi$: 286.866542

b. At $x = 2.15$: 9.16925

c. At $x = 2.71828$: 16.34874967

d. At $x = 1.1$: 4.292

36. $x^4 - 3x^3 + 4x^2 + 6x - 10$

$= (x^3 - 3x^2 + 4x + 6)x - 10$

$= \left[(x^2 - 3x + 4)x + 6\right]x - 10$

$= \left[((x-3)x+4)x+6\right]x - 10$

a. At $x = 1; -2$

b. At $x = \pi; 52.71823452$

c. At $x = 14.2; 32{,}950.5856$

d. At $x = 1.2157; 0.0000269681$

37. a. −2 b. −2

c. $x = 2.4444...;$

$10x = 24.4444...$

$x = 2.4444...$

$9x = 22$

$x = \dfrac{22}{9}$

d. 1

e. $n = 1: x = 0,\ n = 2: x = \dfrac{3}{2},\ n = 3: x = -\dfrac{2}{3},$

$n = 4: x = \dfrac{5}{4}$

The upper bound is $\dfrac{3}{2}$.

f. $\sqrt{2}$

38. a. Answers will vary. Possible answer: An example is $S = \{x : x^2 < 5, x \text{ a rational number}\}$. Here the least upper bound is $\sqrt{5}$, which is real but irrational.

b. True

## 1.3 Concepts Review

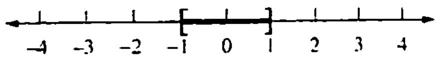
1. interval; intervals

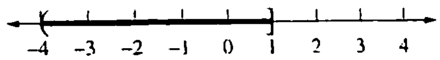
2. $[-1,5); (-\infty, 2]$

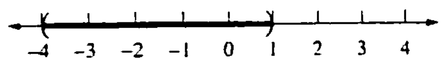
3. $b > 0; b < 0$

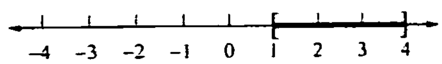
4. $-5, -4, 3$

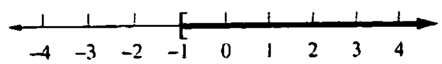
## Problem Set 1.3

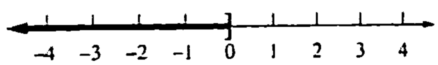
1. a. 

   b. 

   c. 

   d. 

   e. 

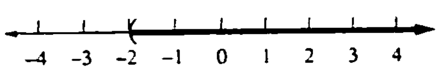
   f. 

2. a. $(2, 7)$ b. $[-3, 4)$

   c. $(-\infty, 2]$ d. $[-1, 3]$

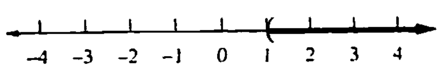
3. $x - 7 < 2x - 5$

   $-2 < x; (-2, \infty)$

4. $3x - 5 < 4x - 6$

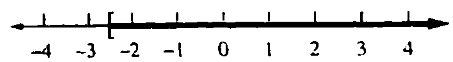
   $1 < x; (1, \infty)$

5. $7x - 2 \le 9x + 3$

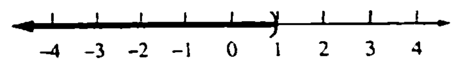
   $-5 \le 2x$

   $x \ge -\frac{5}{2}; \left[-\frac{5}{2}, \infty\right)$

6. $5x - 3 > 6x - 4$

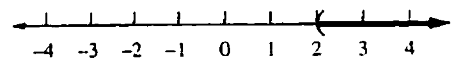
   $1 > x; (-\infty, 1)$

7. $10x + 1 > 8x + 5$

   $2x > 4$

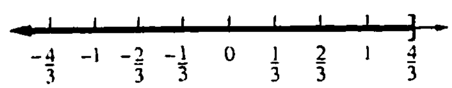
   $x > 2; (2, \infty)$

8. $-2x + 5 \ge 4x - 3$

   $8 > 6x$

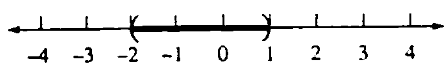
   $x \le \frac{4}{3}; \left(-\infty, \frac{4}{3}\right)$

9. $-4 < 3x + 2 < 5$

   $-6 < 3x < 3$

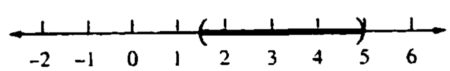
   $-2 < x < 1; (-2, -1)$

10. $-3 < 4x - 9 < 11$

    $6 < 4x < 20$

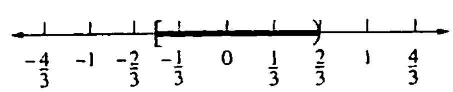
    $\frac{3}{2} < x < 5; \left(\frac{3}{2}, 5\right)$

11. $-3 < 1 - 6x \le 4$

    $-4 < -6x \le 3$

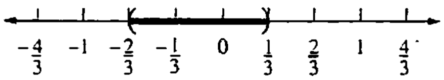
    $\frac{2}{3} > x \ge -\frac{1}{2}; \left[-\frac{1}{2}, \frac{2}{3}\right)$

12. $4 < 5 - 3x < 7$

$-1 < -3x < 2$

$\frac{1}{3} > x > -\frac{2}{3};\ \left(-\frac{2}{3}, \frac{1}{3}\right)$

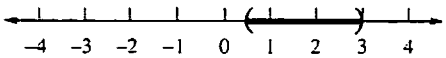


13. $2 + 3x < 5x + 1 < 16$

$2 + 3x < 5x + 1$ and $5x + 1 < 16$

$1 < 2x$ and $5x < 15$

$x > \frac{1}{2}$ and $x < 3;\ \left(\frac{1}{2}, 3\right)$

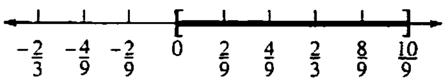


14. $2x - 4 \le 6 - 7x \le x + 6$

$2x - 4 \le -7x$ and $6 - 7x \le 3x + 6$

$9x \le 10$ and $10x \ge 0$

$x \le \frac{10}{9}$ and $x \ge 0;\left[0, \frac{10}{9}\right]$



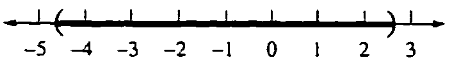
15. $x^2 + 2x - 12 < 0;$

$$x = \frac{-2 \pm \sqrt{(2)^2 - 4(1)(-12)}}{2(1)} = \frac{-2 \pm \sqrt{52}}{2}$$

$$= -1 \pm \sqrt{13}$$

$\left[x - \left(-1 + \sqrt{13}\right)\right]\left[x - \left(-1 - \sqrt{13}\right)\right] < 0;$

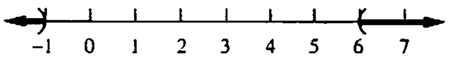
$\left(-1 - \sqrt{13}, -1 + \sqrt{13}\right)$



16. $x^2 - 5x - 6 > 0$

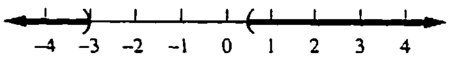
$(x + 1)(x - 6) > 0;$

$(-\infty, 1) \cup (6, \infty)$



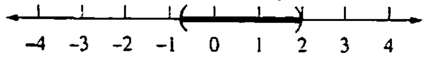
17. $2x^2 + 5x - 3 > 0;\ (2x - 1)(x + 3) > 0;$

$(-\infty, -3) \cup \left(\frac{1}{2}, \infty\right)$

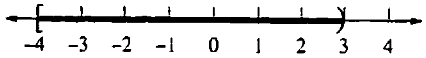


18. $4x^2 - 5x - 6 < 0$

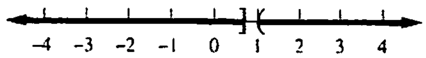
$(4x + 3)(x - 2) < 0;\left(-\frac{3}{4}, 2\right)$



19. $\frac{x + 4}{x - 3} \le 0;\ [-4, 3)$



20. $\frac{3x - 2}{x - 1} \ge 0;\left(-\infty, \frac{2}{3}\right] \cup (1, \infty)$

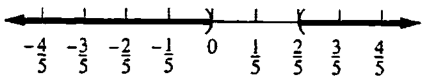


21. $\frac{2}{x} < 5$

$\frac{2}{x} - 5 < 0$

$\frac{2 - 5x}{x} < 0;$

$(-\infty, 0) \cup \left(\frac{2}{5}, \infty\right)$

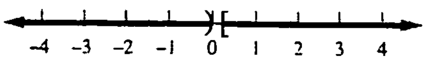


22. $\frac{7}{4x} \le 7$

$\frac{7}{4x} - 7 \le 0$

$\frac{7 - 28x}{4x} \le 0;$

$(-\infty, 0) \cup \left[\frac{1}{4}, \infty\right)$

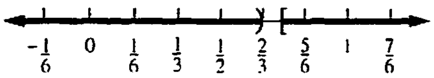


**23.** $$\frac{1}{3x-2} \le 4$$

$$\frac{1}{3x-2} - 4 \le 0$$

$$\frac{1-4(3x-2)}{3x-2} \le 0$$

$$\frac{9-12x}{3x-2} \le 0; \left(-\infty, \frac{2}{3}\right) \cup \left[\frac{3}{4}, \infty\right)$$

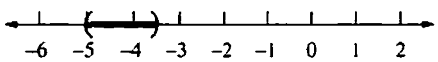



**24.** $$\frac{3}{x+5} > 2$$

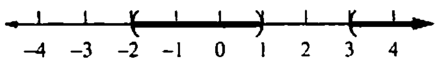
$$\frac{3}{x+5} - 2 > 0$$

$$\frac{3-2(x+5)}{x+5} > 0$$

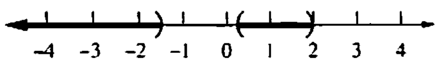
$$\frac{-2x-7}{x+5} > 0; \left(-5, -\frac{7}{2}\right)$$



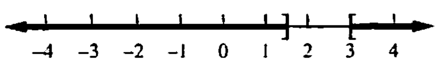

**25.** $(x+2)(x-1)(x-3) > 0; (-2,1) \cup (3,8)$




**26.** $(2x+3)(3x-1)(x-2) < 0; \left(-\infty, -\frac{3}{2}\right) \cup \left(\frac{1}{3}, 2\right)$

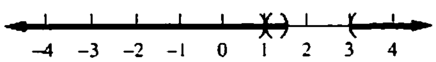



**27.** $(2x-3)(x-1)^2(x-3) \ge 0; \left(-\infty, \frac{3}{2}\right] \cup [3, \infty)$




**28.** $(2x-3)(x-1)^2(x-3) > 0;$

$(-\infty, 1) \cup \left(1, \frac{3}{2}\right) \cup (3, \infty)$

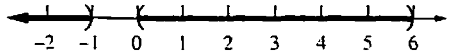



**29.** $x^3 - 5x^2 - 6x < 0$

$x(x^2 - 5x - 6) < 0$

$x(x+1)(x-6) < 0;$

$(-\infty, -1) \cup (0, 6)$

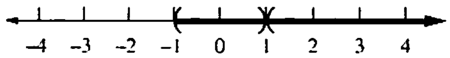



**30.** $x^3 - x^2 - x + 1 > 0$

$(x^2 - 1)(x - 1) > 0$

$(x+1)(x-1)^2 > 0;$

$(-1, 1) \cup (1, \infty)$




**31. a.** $3x + 7 > 1$ and $2x + 1 < 3$
$3x > -6$ and $2x < 2$
$x > -2$ and $x < 1$; $(-2, 1)$

**b.** $3x + 7 > 1$ and $2x + 1 > -4$
$3x > -6$ and $2x > -5$
$x > -2$ and $x > -\frac{5}{2}$; $(-2, \infty)$

**c.** $3x + 7 > 1$ and $2x + 1 < -4$
$x > -2$ and $x < -\frac{5}{2}$; $\varnothing$

**32. a.** $2x - 7 > 1$
$\{4 < x\}$ or $2x + 1 < 3$
$2x > 8$ or $2x < 2$; $x > 4$ or $x < 1$;
$(-\infty, 1) \cup (4, \infty)$

**b.** $2x - 7 \le 1$
$\{x \le 4\}$ or $2x + 1 < 3$
$x \le 4$ or $x < 1$; $(-\infty, 4]$

**c.** $2x - 7 \le 1$
$\{x \le 4\}$ or $2x + 1 > 3$
$x \le 4$ or $x > 1$; $(-\infty, \infty)$

**33. a.** $(x+1)(x^2 + 2x - 7) \ge x^2 - 1$
$x^3 + 3x^2 - 5x - 7 \ge x^2 - 1$
$x^3 + 2x^2 - 5x - 6 \ge 0$
$(x+3)(x+1)(x-2) \ge 0$
$[-3, -1] \cup [2, \infty)$

**b.** $x^4 - 2x^2 \geq 8$

$x^4 - 2x^2 - 8 \geq 0$

$(x^2 - 4)(x^2 + 2) \geq 0$

$(x^2 + 2)(x + 2)(x - 2) \geq 0$

$(-\infty, 12] \cup [2, \infty)$

**c.** $(x^2 + 1)^2 - 7(x^2 + 1) + 10 < 0$

$[(x^2 + 1) - 5][(x^2 + 1) - 2] < 0$

$(x^2 - 4)(x^2 - 1) < 0$

$(x + 2)(x + 1)(x - 1)(x - 2) < 0$

$(-2, -1) \cup (1, 2)$

**34.** Suppose $x > 0$. If we divide both sides of the inequality $1 > 0$ by $x$, we obtain $1/x > 0$. To prove the converse, divide both sides of the equation $1 > 0$ by $1/x$. This gives $\frac{1}{1/x} > \frac{0}{1/x}$, which is equivalent to $x > 0$.

**35.** **a.** $1.99 < \frac{1}{x} < 2.01$

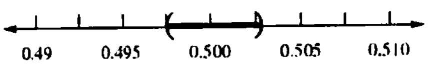
$1.99x < 1 < 2.01x$

$1.99x < 1$ and $1 < 2.01x$

$x < \frac{1}{1.99}$ and $x > \frac{1}{2.01}$

$\frac{1}{2.01} < x < \frac{1}{1.99}$

$\left(\frac{1}{2.01}, \frac{1}{1.99}\right)$




**b.** $2.99 < \frac{1}{x+2} < 3.01$

$2.99(x + 2) < 1 < 3.01(x + 2)$

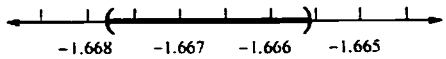
$2.99x + 5.98 < 1$ and

$1 < 3.01x + 6.02$

$x < \frac{-4.98}{2.99}$ and $x > \frac{-5.02}{3.01}$

$-\frac{5.02}{3.01} < x < -\frac{4.98}{2.99}$

$\left(-\frac{5.02}{3.01}, -\frac{4.98}{2.99}\right)$




**c.** $3 - \varepsilon < \frac{1}{x+2} < 3 + \varepsilon$

$(3 - \varepsilon)(x + 2) < 1 < (3 + \varepsilon)(x + 2)$

$(3 - \varepsilon)x + (3 - \varepsilon)2 < 1 < (3 + \varepsilon)x + (3 + \varepsilon)2$

$x < \frac{1 - 2(3 - \varepsilon)}{3 - \varepsilon}$ and $x > \frac{1 - 2(3 + \varepsilon)}{3 + \varepsilon}$

$\left(\frac{1 - 2(3 + \varepsilon)}{3 + \varepsilon}, \frac{1 - 2(3 - \varepsilon)}{3 - \varepsilon}\right)$

**36.** $1 + x + x^2 + x^3 + \ldots + x^{99} \leq 0$;

$(-\infty, -1)$

**37.** $\frac{1}{R} \leq \frac{1}{10} + \frac{1}{20} + \frac{1}{30}$

$\frac{1}{R} \leq \frac{6 + 3 + 2}{60}$

$\frac{1}{R} \leq \frac{11}{60}$

$R \geq \frac{60}{11}$

$\frac{1}{R} \geq \frac{1}{20} + \frac{1}{30} + \frac{1}{40}$

$\frac{1}{R} \geq \frac{6 + 4 + 3}{120}$

$\frac{1}{R} \geq \frac{13}{120}$

$R \leq \frac{120}{13}$

$\frac{60}{11} \leq R \leq \frac{120}{13}$

## 1.4 Concepts Review

1. $-1$; 5

2. $|a+b| \le |a| + |b|$

3. $b, c$

4. $\frac{0.2}{5} = 0.04$

## Problem Set 1.4

1. $|x+2| < 1$;
$-1 < x+2 < 1$
$-3 < x < -1$
$(-3, -1)$

2. $|x-2| \ge 5$;
$x-2 \le -5$ or $x-2 \ge 5$
$x \le -3$ or $x \ge 7$
$(-\infty, -3] \cup [7, \infty)$

3. $|2x-1| > 2$;
$2x-1 < -2$ or $2x-1 > 2$
$2x < -1$ or $2x > 3$;
$x < -\frac{1}{2}$ or $x > \frac{3}{2}$, $\left(-\infty, -\frac{1}{2}\right) \cup \left(\frac{3}{2}, \infty\right)$

4. $|4x+5| \le 10$;
$-10 \le 4x+5 \le 10$
$-15 \le 4x \le 5$
$-\frac{15}{4} \le x \le \frac{5}{4}$; $\left[-\frac{15}{4}, \frac{5}{4}\right]$

5. $\left|\frac{x}{4}+1\right| < 1$
$-1 < \frac{x}{4}+1 < 1$
$-2 < \frac{x}{4} < 0$;
$-8 < x < 0$; $(-8, 0)$

6. $\left|\frac{2x}{7}-5\right| \ge 7$
$\frac{2x}{7}-5 \le -7$ or $\frac{2x}{7}-5 \ge 7$
$\frac{2x}{7} \le -2$ or $\frac{2x}{7} \ge 12$
$x \le -7$ or $x \ge 42$;
$(-\infty, -7] \cup [42, \infty)$

7. $|2x-7| > 3$;
$2x-7 < -3$ or $2x-7 > 3$
$2x < 4$ or $2x > 10$
$x < 2$ or $x > 5$; $(-\infty, 2) \cup (5, \infty)$

8. $|5x-6| > 1$;
$5x-6 < -1$ or $5x-6 > 1$
$5x < 5$ or $5x > 7$
$x < 1$ or $x > \frac{7}{5}$; $(-\infty, 1) \cup \left(\frac{7}{5}, \infty\right)$

9. $|4x+2| \ge 10$;
$4x+2 \le -10$ or $4x+2 \ge 10$
$4x \le -12$ or $4x \ge 8$
$x \le -3$ or $x \ge 2$
$(-\infty, -3] \cup [2, \infty)$

10. $\left|\frac{x}{2}+7\right| \ge 2$;
$\frac{x}{2}+7 \le -2$ or $\frac{x}{2}+7 \ge 2$
$\frac{x}{2} \le -9$ or $\frac{x}{2} \ge -5$
$x \le -18$ or $x \ge -10$
$(-\infty, -18] \cup [-10, \infty)$

11. $\left|2+\frac{5}{x}\right| > 1$;
$2+\frac{5}{x} < -1$ or $2+\frac{5}{x} > 1$
$3+\frac{5}{x} < 0$ or $1+\frac{5}{x} > 0$
$\frac{3x+5}{x} < 0$ or $\frac{x+5}{x} > 0$;
$(-\infty, -5) \cup \left(-\frac{5}{3}, 0\right) \cup (0, \infty)$

12. $\left|\frac{1}{x}-3\right| > 6$;
$\frac{1}{x}-3 < -6$ or $\frac{1}{x}-3 > 6$
$\frac{1}{x}+3 < 0$ or $\frac{1}{x}-9 > 0$
$\frac{1+3x}{x} < 0$ or $\frac{1-9x}{x} > 0$;
$\left(\frac{-1}{3}, 0\right) \cup \left(0, \frac{1}{9}\right)$

13. $x^2-3x-4\ge 0;$

$$x=\frac{3\pm\sqrt{(-3)^2-4(1)(-4)}}{2(1)}=\frac{3\pm 5}{2}=-1,4$$

$(x+1)(x-4)=0;(-\infty,-1]\cup[4,\infty)$

14. $x^2-4x+4\le 0;\ x=\dfrac{4\pm\sqrt{(-4)^2-4(1)(4)}}{2(1)}=2$

$(x-2)(x-2)\le 0; x=2$

15. $3x^2+17x-6>0;$

$$x=\frac{-17\pm\sqrt{(17)^2-4(3)(-6)}}{2(3)}=\frac{-17\pm 19}{6}=-6,\frac{1}{3}$$

$(3x-1)(x+6)>0;\ (-\infty,-6)\cup\left(\frac{1}{3},\infty\right)$

16. $14x^2+11x-15\le 0;$

$$x=\frac{-11\pm\sqrt{(11)^2-4(14)(-15)}}{2(14)}=\frac{-11\pm 31}{28}$$

$x=-\frac{3}{2},\frac{5}{7}$

$\left(x+\frac{3}{2}\right)\left(x-\frac{5}{7}\right)\le 0;\left[-\frac{3}{2},\frac{5}{7}\right]$

17. $|x-3|<0.5\Leftrightarrow 5|x-3|<5(0.5)\Leftrightarrow|5x-15|<2.5$

18. $|x+2|<0.3\Leftrightarrow 4|x+2|<4(0.3)\Leftrightarrow|4x+18|<1.2$

19. $|x-2|<\frac{\varepsilon}{6}\Leftrightarrow 6|x-2|<\varepsilon\Leftrightarrow|6x-12|<\varepsilon$

20. $|x+4|<\frac{\varepsilon}{2}\Leftrightarrow 2|x+4|<\varepsilon\Leftrightarrow|2x+8|<\varepsilon$

21. $|3x-15|<\varepsilon\Leftrightarrow|3(x-5)|<\varepsilon$

$\Leftrightarrow 3|x-5|<\varepsilon$

$\Leftrightarrow|x-5|<\frac{\varepsilon}{3};\delta=\frac{\varepsilon}{3}$

22. $|4x-8|<\varepsilon\Leftrightarrow|4(x-2)|<\varepsilon$

$\Leftrightarrow 4|x-2|<\varepsilon$

$\Leftrightarrow|x-2|<\frac{\varepsilon}{4};\delta=\frac{\varepsilon}{4}$

23. $|6x+36|<\varepsilon\Leftrightarrow|6(x+6)|<\varepsilon$

$\Leftrightarrow 6|x+6|<\varepsilon$

$\Leftrightarrow|x+6|<\frac{\varepsilon}{6};\delta=\frac{\varepsilon}{6}$

24. $|5x+25|<\varepsilon\Leftrightarrow|5(x+5)|<\varepsilon$

$\Leftrightarrow 5|x+5|<\varepsilon$

$\Leftrightarrow|x+5|<\frac{\varepsilon}{5};\delta=\frac{\varepsilon}{5}$

25. $C=\pi d$

$|C-10|\le 0.02$

$|\pi d-10|\le 0.02$

$\left|\pi\left(d-\frac{10}{\pi}\right)\right|\le 0.02$

$\left|d-\frac{10}{\pi}\right|\le\frac{0.02}{\pi}\approx 0.0064$

We must measure the diameter to an accuracy of 0.0064 in.

26. $|C-50|\le 1.5,\left|\frac{5}{9}(F-32)-50\right|\le 1.5;$

$\frac{5}{9}|(F-32)-90|\le 1.5$

$|F-122|\le 2.7$

We are allowed an error of $2.7^\circ$ F.

27. $|x-1|<2|x-3|$

$|x-1|<|2x-6|$

$(x-1)^2<(2x-6)^2$

$x^2-2x+1<4x^2-24x+36$

$3x^2-22x+35>0$

$(3x-7)(x-5)>0;$

$\left(-\infty,\frac{7}{3}\right)\cup(5,\infty)$

28. $|2x-1|\ge|x+1|$

$(2x-1)^2\ge(x+1)^2$

$4x^2-4x+1\ge x^2+2x+1$

$3x^2-6x\ge 0$

$3x(x-2)\ge 0$

$(-\infty,0]\cup[2,\infty)$

29.
$$2|2x-3|<|x+10|$$
$$|4x-6|<|x+10|$$
$$(4x-6)^2<(x+10)^2$$
$$16x^2-48x+36<x^2+20x+100$$
$$15x^2-68x-64<0$$
$$(5x+4)(3x-16)<0;$$
$$\left(-\frac{4}{5},\frac{16}{3}\right)$$

30.
$$|3x-1|<2|x+6|$$
$$|3x-1|<|2x+12|$$
$$(3x-1)^2<(2x-12)^2$$
$$9x^2-6x+1<4x^2+48x+144$$
$$5x^2-54x-143<0$$
$$(5x+11)(x-13)<0;$$
$$\left(-\frac{11}{5},13\right)$$

31. $|x|<|y| \Rightarrow |x||x| \le |x||y|$ and $|x||y|<|y||y|$ Order property: $x<y \Leftrightarrow xz<yz$ when $z$ is positive.

$\Rightarrow |x|^2<|y|^2$ Transitivity

$\Rightarrow x^2<y^2$ $\left(|x|^2=x^2\right)$

Conversely,

$x^2<y^2 \Rightarrow |x|^2<|y|^2$ $\left(x^2=|x|^2\right)$

$\Rightarrow |x|^2-|y|^2<0$ Subtract $|y|^2$ from each side.

$\Rightarrow (|x|-|y|)(|x|+|y|)<0$ Factor the difference of two squares.

$\Rightarrow |x|-|y|<0$ This is the only factor that can be negative.

$\Rightarrow |x|<|y|$ Add $|y|$ to each side.

32. $0<a<b \Rightarrow a=\left(\sqrt{a}\right)^2$ *and* $b=\left(\sqrt{b}\right)^2$, so $\left(\sqrt{a}\right)^2<\left(\sqrt{b}\right)^2$, and, by Problem 31, $\left|\sqrt{a}\right|<\left|\sqrt{b}\right| \Rightarrow \sqrt{a}<\sqrt{b}$

33. **a.** $|a-b|=|a+(-b)|\le|a|+|-b|=|a|+|b|$

**b.** $|a-b|\ge\big||a|-|b|\big|\ge|a|-|b|$ Use Property 4 of absolute values.

**c.** $|a+b+c|=|(a+b)+c|\le|a+b|+|c|$
$\le|a|+|b|+|c|$

34.
$$\left|\frac{1}{x^2+3}-\frac{1}{|x|+2}\right|=\left|\frac{1}{x^2+3}+\left(-\frac{1}{|x|+2}\right)\right|$$
$$\le\left|\frac{1}{x^2+3}\right|+\left|-\frac{1}{|x|+2}\right|$$
$$=\left|\frac{1}{x^2+3}\right|+\left|\frac{1}{|x|+2}\right|$$
$$=\frac{1}{x^2+3}+\frac{1}{|x|+2}$$

by the Triangular Inequality, and since $x^2+3>0,\ |x|+2>0 \Rightarrow \frac{1}{x^2+3}>0, \frac{1}{|x|+2}>0.$

$x^2+3\ge3$ and $|x|+2\ge2$, so

$\frac{1}{x^2+3}\le\frac{1}{3}$ and $\frac{1}{|x|+2}\le\frac{1}{2}$, thus,

$$\frac{1}{x^2+3}+\frac{1}{|x|+2}\le\frac{1}{3}+\frac{1}{2}$$

35. $\left|\frac{x-2}{x^2+9}\right| = \left|\frac{x+(-2)}{x^2+9}\right|$

$\left|\frac{x-2}{x^2+9}\right| \le \left|\frac{x}{x^2+9}\right| + \left|\frac{-2}{x^2+9}\right|$

$\left|\frac{x-2}{x^2+9}\right| \le \frac{|x|}{x^2+9} + \frac{2}{x^2+9} = \frac{|x|+2}{x^2+9}$

Since $x^2+9 \ge 9, \frac{1}{x^2+9} \le \frac{1}{9}$

$\frac{|x|+2}{x^2+9} \le \frac{|x|+2}{9}$

$\left|\frac{x-2}{x^2+9}\right| \le \frac{|x|+2}{9}$

36. $|x| \le 2 \Rightarrow \left|x^2+2x+7\right| \le \left|x^2\right| + |2x| + 7$

$\le 4+4+7 = 15$

and $\left|x^2+1\right| \ge 1$ so $\frac{1}{x^2+1} \le 1$.

Thus, $\left|\frac{x^2+2x+7}{x^2+1}\right| = \left|x^2+2x+7\right|\left|\frac{1}{x^2+1}\right|$

$\le 15 \cdot 1 = 15$

37. $\left|x^4 + \frac{1}{2}x^3 + \frac{1}{4}x^2 + \frac{1}{8}x + \frac{1}{16}\right|$

$\le \left|x^4\right| + \frac{1}{2}\left|x^3\right| + \frac{1}{4}\left|x^2\right| + \frac{1}{8}|x| + \frac{1}{16}$

$\le 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16}$ since $|x| \le 1$.

So $\left|x^4 + \frac{1}{2}x^3 + \frac{1}{4}x^2 + \frac{1}{8}x + \frac{1}{16}\right| \le 1.9375 < 2$.

38. a. $x < x^2$

$x - x^2 < 0$

$x(1-x) < 0$

$x < 0$ or $x > 1$

b. $x^2 < x$

$x^2 - x < 0$

$x(x-1) < 0$

$0 < x < 1$

39. $a \ne 0 \Rightarrow$

$0 \le \left(a - \frac{1}{a}\right)^2 = a^2 - 2 + \frac{1}{a^2}$

so, $2 \le a^2 + \frac{1}{a^2}$ or $a^2 + \frac{1}{a^2} \ge 2$

40. $a < b$

$a + a < a + b$ and $a + b < b + b$

$2a < a + b < 2b$

$a < \frac{a+b}{2} < b$

41. $0 < a < b$

$a^2 < ab$ and $ab < b^2$

$a^2 < ab < b^2$

$a < \sqrt{ab} < b$

42. $\sqrt{ab} \le \frac{1}{2}(a+b) \Leftrightarrow ab \le \frac{1}{4}\left(a^2+2ab+b^2\right)$

$\Leftrightarrow 0 \le \frac{1}{4}a^2 - \frac{1}{2}ab + \frac{1}{4}b^2 = \frac{1}{4}\left(a^2-2ab+b^2\right)$

$\Leftrightarrow 0 \le \frac{1}{4}(a-b)^2$ which is always true.

43. For a rectangle the area is $ab$, while for a square the area is $a^2 = \left(\frac{a+b}{2}\right)^2$. From Problem 42, $\sqrt{ab} \le \frac{1}{2}(a+b) \Leftrightarrow ab \le \left(\frac{a+b}{2}\right)^2$ so the square has the largest area.

44. $A = \pi r^2; A = 4\pi(10)^2 = 400\pi$

$\left|4\pi r^2 - 400\pi\right| < 0.01$

$4\pi\left|r^2 - 100\right| < 0.01$

$\left|r^2 - 100\right| < \frac{0.01}{4\pi}$

$-\frac{0.01}{4\pi} < r^2 - 100 < \frac{0.01}{4\pi}$

$\sqrt{100 - \frac{0.01}{4\pi}} < r < \sqrt{100 + \frac{0.01}{4\pi}}$

$\delta \approx 0.00004$ in

## 1.5 Concepts Review

1. II; IV

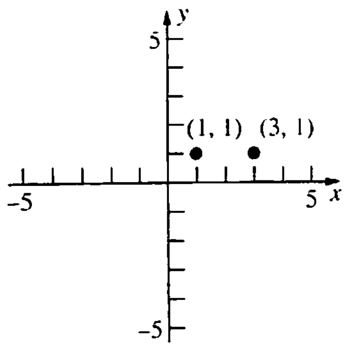
2. $\sqrt{(x+2)^2+(y-3)^2}$

3. $(x+4)^2+(y-2)^2=25$

4. $\left(\frac{-2+5}{2},\frac{3+7}{2}\right)=(1.5,5)$

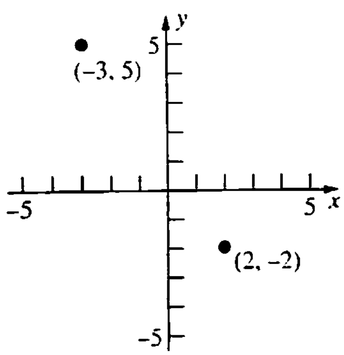
## Problem Set 1.5

1.



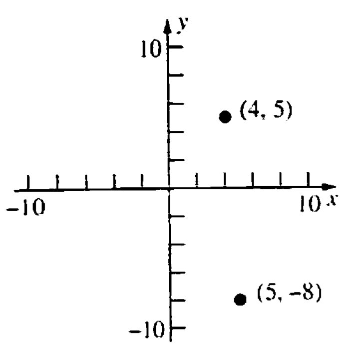

$d=\sqrt{(3-1)^2+(1-1)^2}=\sqrt{4}=2$

2.



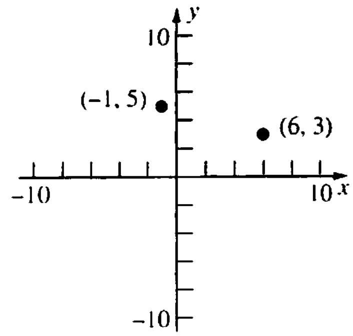

$d=\sqrt{(-3-2)^2+(5+2)^2}=\sqrt{74}\approx 8.60$

3.



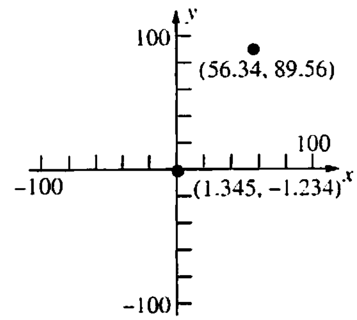

$d=\sqrt{(4-5)^2+(5+8)^2}=\sqrt{170}\approx 13.04$

4.



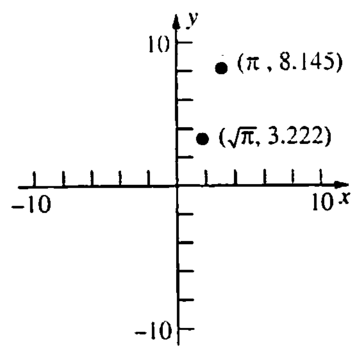

$d=\sqrt{(-1-6)^2+(5-3)^2}=\sqrt{49+4}=\sqrt{53}$
$\approx 7.28$

5.




$d=\sqrt{(1.345-56.34)^2+(-1.234-89.56)^2}$
$\approx 106.151$

6.




$d=\sqrt{(\sqrt{\pi}-\pi)^2+(3.222-8.145)^2}\approx 5.110$

7. $d_1=\sqrt{(5+2)^2+(3-4)^2}=\sqrt{49+1}=\sqrt{50}$

$d_2=\sqrt{(5-10)^2+(3-8)^2}=\sqrt{25+25}=\sqrt{50}$

$d_3=\sqrt{(-2-10)^2+(4-8)^2}=\sqrt{144+16}=\sqrt{160}$

$d_1=d_2$ so the triangle is isosceles.

8. $a=\sqrt{(2-4)^2+(-4-0)^2}=\sqrt{4+16}=\sqrt{20}$

$b=\sqrt{(4-8)^2+(0+2)^2}=\sqrt{16+4}=\sqrt{20}$

$c=\sqrt{(2-8)^2+(-4+2)^2}=\sqrt{36+4}=\sqrt{40}$

$a^2 + b^2 = c^2$, so the triangle is a right triangle.

9. (–1, –1), (–1, 3); (7, –1), (7, 3); (1, 1), (5, 1)

10. $\sqrt{(x-3)^2+(0-1)^2} = \sqrt{(x-6)^2+(0-4)^2}$;

$$x^2 - 6x + 10 = x^2 - 12x + 52$$
$$6x = 42$$
$$x = 7 \Rightarrow (7, 0)$$

11. $\left(\frac{-2+4}{2}, \frac{-2+3}{2}\right) = \left(1, \frac{1}{2}\right)$;

$$d = \sqrt{(1+2)^2 + \left(\frac{1}{2}-3\right)^2} = \sqrt{9+\frac{25}{4}} \approx 3.91$$

12. midpoint of $AB = \left(\frac{1+2}{2}, \frac{3+6}{2}\right) = \left(\frac{3}{2}, \frac{9}{2}\right)$

midpoint of $CD = \left(\frac{4+3}{2}, \frac{7+4}{2}\right) = \left(\frac{7}{2}, \frac{11}{2}\right)$

$$d = \sqrt{\left(\frac{3}{2}-\frac{7}{2}\right)^2 + \left(\frac{9}{2}-\frac{11}{2}\right)^2}$$
$$= \sqrt{4+1} = \sqrt{5} \approx 2/24$$

13. $(x-1)^2 + (y-1)^2 = 1$

14. $(x+2)^2 + (y-3)^2 = 4^2$

$(x+2)^2 + (y-3)^2 = 16$

15. $(x-2)^2 + (y+1)^2 = r^2$

$(5-2)^2 + (3+1)^2 = r^2$

$r^2 = 9 + 16 = 25$

$(x-2)^2 + (y+1)^2 = 25$

16. $(x-4)^2 + (y-3)^2 = r^2$

$(6-4)^2 + (2-3)^2 = r^2$

$r^2 = 4 + 1 = 5$

$(x-4)^2 + (y-3)^2 = 5$

17. center $= \left(\frac{1+3}{2}, \frac{3+7}{2}\right) = (2, 5)$

radius $= \frac{1}{2}\sqrt{(1-3)^2 + (3-7)^2} = \frac{1}{2}\sqrt{4+16}$

$= \frac{1}{2}\sqrt{20} = \sqrt{5}$

$(x-2)^2 + (y-5)^2 = 5$

18. Since the circle is tangent to the $x$-axis, $r = 4$.

$(x-3)^2 + (y-4)^2 = 16$

19. Substitute $x = \frac{1}{4}$ into the equation and solve for $y$.

$\left(-\frac{3}{4}\right)^2 + (y-1)^2 = 1$

$(y-1)^2 = \frac{7}{16}$

$y - 1 = \pm\frac{\sqrt{7}}{4}$

$y = 1 \pm \frac{\sqrt{7}}{4}$

20. Substitute $y = 1$ into the equation and solve for $x$.

$(x-1)^2 + (0)^2 = 1$

$x - 1 = \pm 1$

$x = 0, 2$

21. $x^2 + 2x + 10 + y^2 - 6y - 10 = 0$

$$x^2 + 2x + y^2 - 6y = 0$$
$$(x^2 + 2x + 1) + (y^2 - 6y + 9) = 1 + 9$$
$$(x+1)^2 + (y-3)^2 = 10$$

center $= (-1, 3)$; radius $= \sqrt{10}$

22. $x^2 + y^2 - 6y = 36$

$$x^2 + (y^2 - 6y + 9) = 16 + 9$$
$$x^2 + (y-3)^2 = 25$$

center $= (0, 3)$; radius $= \sqrt{5}$

23. $x^2 + y^2 - 12x + 35 = 0$

$$x^2 - 12x + y^2 = -35$$
$$(x^2 - 12x + 36) + y^2 = -35 + 36$$
$$(x-6)^2 + y^2 = 1$$

center $= (6, 0)$; radius $= 1$

24. $x^2 + y^2 - 10x + 10y = 0$

$$(x^2 - 10x + 25) + (y^2 + 10y + 25) = 25 + 25$$
$$(x-5)^2 + (y+5)^2 = 50$$

center $= (5, -5)$; radius $= \sqrt{50} = 5\sqrt{2}$

25. $4x^2 + 16x + 15 + 4y^2 + 6y = 0$

$$4(x^2 + 4x + 4) + 4\left(y^2 + \frac{3}{2}y + \frac{9}{16}\right) = -15 + 16 + \frac{9}{4}$$

$$4(x+2)^2+4\left(y+\frac{3}{4}\right)^2=\frac{13}{4}$$

$$(x+2)^2+\left(y+\frac{3}{4}\right)^2=\frac{13}{16}$$

$$\text{center}=\left(-2,-\frac{3}{4}\right);\ \text{radius}=\frac{\sqrt{13}}{4}$$

**26.** $$4x^2+16x+\frac{105}{16}+4y^2+3y=0$$

$$4(^2+4x+4)+4\left(y^2+\frac{3}{4}y+\frac{9}{64}\right)$$
$$=-\frac{105}{16}+16+\frac{9}{16}$$

$$4(x+2)^2+4\left(y+\frac{3}{8}\right)^2=10$$

$$(x+2)^2+\left(y+\frac{3}{8}\right)^2=\frac{5}{2}$$

$$\text{center}=\left(-2,-\frac{3}{8}\right);\ \text{radius}=\sqrt{\frac{5}{2}}=\frac{\sqrt{10}}{2}$$

**27.** $$\text{center:}\left(\frac{2+6}{2},\frac{-1+3}{2}\right)=(4,1)$$

$$\text{midpoint}=\left(\frac{2+6}{2},\frac{3+3}{2}\right)=(4,3)$$

$$\text{inscribed circle: radius}=\sqrt{(4-4)^2+(1-3)^2}$$
$$=\sqrt{4}=2$$

$$(x-4)^2+(y-1)^2=4$$

circumscribed circle:

$$\text{radius}=\sqrt{(4-2)^2+(1-3)^2}=\sqrt{8}$$

$$(x-4)^2+(y-1)^2=8$$

**28.** The radius of each circle is $\sqrt{16}=4$. The centers are $(1,-2)$ and $(-9,10)$. The length of the belt is the sum of half the circumference of the first circle, half the circumference of the second circle, and twice the distance between their centers.

$$L=\frac{1}{2}\cdot 2\pi(4)+\frac{1}{2}\cdot 2\pi(4)+2\sqrt{(1+9)^2+(-2-10)^2}$$
$$=8\pi+2\sqrt{100+144}$$
$$\approx 56.37$$

**29.** $$AC=\sqrt{AB^2+BC^2}=\sqrt{(214)^2+(179)^2}$$
$$=\sqrt{77{,}837}\approx 278.99$$

Cost by truck = 3.71(214 + 179) = \$1458.03

Cost by plane = 4.82(279) = \$1344.78; cheaper by plane.

**30.** Distance running = 6 mi

$$\text{Distance swimming}=\sqrt{(10-6)^2+\left(\frac{1}{2}\right)^2}$$
$$=\sqrt{16+\frac{1}{4}}=\sqrt{16.25}$$

$$\text{Time}=\frac{\text{Distance8}}{\text{Rate}}=\frac{6}{8}+\frac{\sqrt{16.25}}{3}\approx 2.09\text{ hr}$$

**31.** Put the vertex of the right angle at the origin with the other vertices at $(a, 0)$ and $(0, b)$. The midpoint of the hypotenuse is $\left(\frac{a}{2},\frac{b}{2}\right)$. The distances from the vertices are

$$\sqrt{\left(a-\frac{a}{2}\right)^2+\left(0-\frac{b}{2}\right)^2}=\sqrt{\frac{a^2}{4}+\frac{b^2}{4}}$$
$$=\frac{1}{2}\sqrt{a^2+b^2},$$

$$\sqrt{\left(0-\frac{a}{2}\right)^2+\left(b-\frac{b}{2}\right)^2}=\sqrt{\frac{a^2}{4}+\frac{b^2}{4}}$$
$$=\frac{1}{2}\sqrt{a^2+b^2},\text{ and}$$

$$\sqrt{\left(0-\frac{a}{2}\right)^2+\left(0-\frac{b}{2}\right)^2}=\sqrt{\frac{a^2}{4}+\frac{b^2}{4}}$$
$$=\frac{1}{2}\sqrt{a^2+b^2},$$

which are all the same.

**32.** From Problem 31, the midpoint of the hypotenuse, $(4,3,)$, is equidistant from the vertices. This is the center of the circle. The radius is $\sqrt{16+9}=5$. The equation of the circle is

$$(x-4)^2+(y-3)^2=25.$$

**33.** $$x^2+y^2-4x-2y-11=0$$

$$(x^2-4x+4)+(y^2-2y+1)=11+4+1$$

$$(x-2)^2+(y-1)^2=16$$

$$x^2+y^2+20x-12y+72=0$$

$$(x^2+20x+100)+(y^2-12y+36)$$
$$=-72+100+36$$

$$(x+10)^2+(y-6)^2=64$$

center of first circle: (2, 1)

center of second circle: (–10, 6)

$$d=\sqrt{(2+10)^2+(1-6)^2}=\sqrt{144+25}$$
$$=\sqrt{169}=13$$

However, the radii only sum to 4 + 8 = 12, so the circles must not intersect if the distance between their centers is 13.

**34.** $x^2+ax+y^2+by+c=0$

$$\left(x^2+ax+\frac{a^2}{4}\right)+\left(y^2+by+\frac{b^2}{4}\right)=-c+\frac{a^2}{4}+\frac{b^2}{4}$$

$$\left(x+\frac{a}{2}\right)^2+\left(y+\frac{b}{2}\right)^2=\frac{a^2+b^2-4c}{4}$$

$$\frac{a^2+b^2-4c}{4}>0\Rightarrow a^2+b^2>4c$$

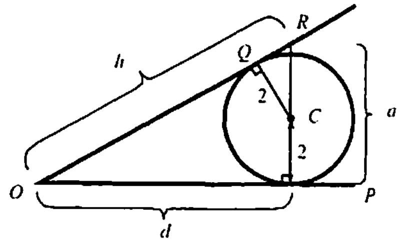
**35.** Label the points $C$, $P$, $Q$, and $R$ as shown in the figure below. Let $d=|OP|$, $h=|OQ|$, and $a=|PR|$. Triangles $\Delta OPR$ and $\Delta CQR$ are similar because each contains a right angle and they share angle $\angle QRC$. For an angle of $30^\circ$, $d/h=\sqrt{3}/2$. Thus, using a property of similar triangles,

$$|QC|/|RC|=\sqrt{3}/2$$
$$\frac{2}{a-2}=\frac{\sqrt{3}}{2}$$
$$a=2+4/\sqrt{3}$$

Thus, $h=2a=2(2+4\sqrt{3})=4(1+2\sqrt{3})$.

By the Pythagorean Theorem, we have

$$d=\sqrt{h^2-a^2}=\sqrt{3}a=2\sqrt{3}+4\approx 7.464$$




**36.** The equations of the two circles are

$$(x-R)^2+(y-R)^2=R^2$$
$$(x-r)^2+(y-r)^2=r^2$$

Let $(a,a)$ denote the point where the two circles touch. This point must satisfy

$$(a-R)^2+(a-R)^2=R^2$$
$$(a-R)^2=\frac{R^2}{2}$$
$$a=\left(1\pm\frac{\sqrt{2}}{2}\right)R$$

Since $a<R$, $a=\left(1-\frac{\sqrt{2}}{2}\right)R.$

At the same time, the point where the two circles touch must satisfy

$$(a-r)^2+(a-r)^2=r^2$$
$$a=\left(1\pm\frac{\sqrt{2}}{2}\right)r$$

Since $a>r$, $a=\left(1+\frac{\sqrt{2}}{2}\right)r.$

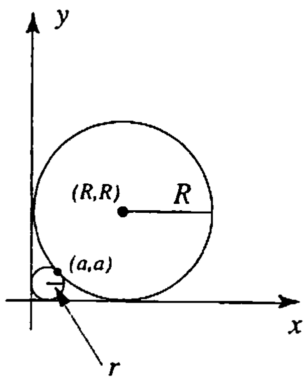
Equating the two expressions for $a$ yields

$$\left(1-\frac{\sqrt{2}}{2}\right)R=\left(1+\frac{\sqrt{2}}{2}\right)r$$

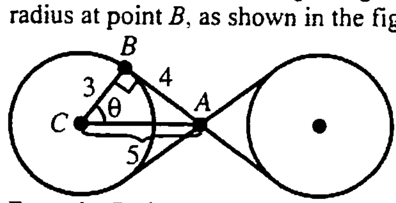
$$r=\frac{1-\frac{\sqrt{2}}{2}}{1+\frac{\sqrt{2}}{2}}R=\frac{\left(1-\frac{\sqrt{2}}{2}\right)^2}{\left(1+\frac{\sqrt{2}}{2}\right)\left(1-\frac{\sqrt{2}}{2}\right)}R$$

$$r=\frac{1-\sqrt{2}+\frac{1}{2}}{1-\frac{1}{2}}R$$

$$r=(3-2\sqrt{2})R\approx 0.1716R$$




**37.** The centers of the circles are $\sqrt{(10-2)^2+(8-2)^2}=\sqrt{100}=10$ units apart, so the belts cross at a point 5 units from each center. The belt makes a right angle with the radius at point $B$, as shown in the figure.




From the Pythagorean Theorem, the length of the belt from $A$ to $B$ is $\sqrt{5^2-3^2}=\sqrt{16}=4$.

$\sin\theta=\frac{4}{5}\Rightarrow\theta\approx 0.93$ radians. The situation on the lower half of the wheel is identical, and the two wheels are identical, so the length of the belt around each wheel is $3(2\pi-1.86)\approx 13.3$ units. The length of the belt is $2(13.3)+4(4)\approx 42.6$ units.

**38.** $2\sqrt{(x-1)^2+(y-1)^2}=\sqrt{(x-3)^2+(y-4)^2}$

$4(x^2-2x+1+y^2-2y+1)$

$=x^2-6x+9+y^2-8y+16$

$3x^2-2x+3y^2=9+16-4-4;$

$3x^2-2x+3y^2=17; x^2-\frac{2}{3}x+y^2=\frac{17}{3};$

$\left(x^2-\frac{2}{3}x+\frac{1}{9}\right)+y^2=\frac{17}{3}+\frac{1}{9}$

$\left(x-\frac{1}{3}\right)^2+y^2=\frac{52}{9}$

center: $\left(\frac{1}{3},0\right)$; radius: $\left(\frac{\sqrt{52}}{3}\right)$

**39.** Let $a$, $b$, and $c$ be the lengths of the sides of the right triangle, with $c$ the length of the hypotenuse. Then the Pythagorean Theorem says that $a^2+b^2=c^2$

Thus, $\frac{\pi a^2}{8}+\frac{\pi b^2}{8}=\frac{\pi c^2}{8}$ or

$\frac{1}{2}\pi\left(\frac{a}{2}\right)^2+\frac{1}{2}\pi\left(\frac{b}{2}\right)^2=\frac{1}{2}\pi\left(\frac{c}{2}\right)^2$

$\frac{1}{2}\pi\left(\frac{x}{2}\right)^2$ is the area of a semicircle with diameter $x$, so the circles on the legs of the triangle have total area equal to the area of the semicircle on the hypotenuse.

From $a^2+b^2=c^2$,

$\frac{\sqrt{3}}{4}a^2+\frac{\sqrt{3}}{4}b^2=\frac{\sqrt{3}}{4}c^2$

$\frac{\sqrt{3}}{4}x^2$ is the area of an equilateral triangle with sides of length $x$, so the equilateral triangles on the legs of the right triangle have total area equal to the area of the equilateral triangle on the hypotenuse of the right triangle.

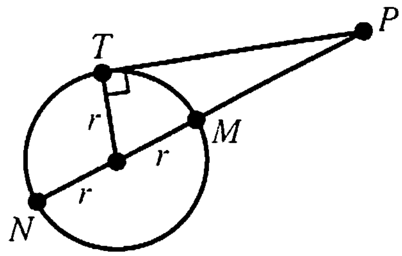
**40.** See the figure below. The angle at $T$ is a right angle, so the Pythagorean Theorem gives

$(PM+r)^2=(PT)^2+r^2$

$\Leftrightarrow (PM)^2+2rPM+r^2=(PT)^2+r^2$

$\Leftrightarrow PM(PM+2r)=(PT)^2$

$PM+2r=PN$ so this gives $(PM)(PN)=(PT)^2$




**41.** The lengths $A$, $B$, and $C$ are the same as the corresponding distances between the centers of the circles:

$A=\sqrt{(-2)^2+(8)^2}=\sqrt{68}\approx 8.2$

$B=\sqrt{(6)^2+(8)^2}=\sqrt{100}=10$

$C=\sqrt{(8)^2+(0)^2}=\sqrt{64}=8$

Each circle has radius 2, so the part of the belt around the wheels is

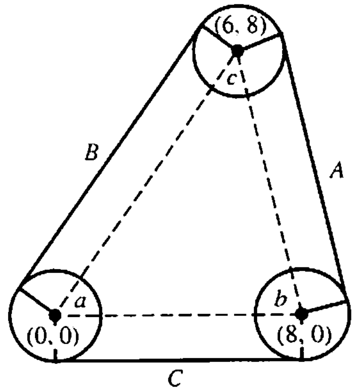
$2(2\pi-a-\pi)+2(2\pi-b-\pi)+2(2\pi-c-\pi)$

$=2[3\pi-(a+b+c)]=2(2\pi)=4\pi$

Since $a+b+c=\pi$, the sum of the angles of a triangle.

The length of the belt is $\approx 8.2+10+8+4\pi$

$\approx 38.8$ units.




**42.** In Problems 28 and 41, the curved portions of the belt have total length $2\pi r$. The lengths of the straight portions will be the same as the lengths of the sides. The belt will have length $2\pi r+d_1+d_2+\ldots+d_n$.

## 1.6 Concepts Review

1. $\dfrac{(d-b)}{(c-a)}$

2. 0

3. $y = mx + b;\ x = k$

4. $Ax + By + C = 0$

## Problem Set 1.6

1. $\dfrac{2-1}{2-1} = 1$

2. $\dfrac{7-5}{4-3} = 1$

3. $\dfrac{-6-3}{-5-2} = \dfrac{9}{7}$

4. $\dfrac{-6+4}{0-2} = 1$

5. $\dfrac{5-0}{0-3} = -\dfrac{5}{3}$

6. $\dfrac{6-0}{0+6} = 1$

7. $\dfrac{-3.456-5.678}{7.654+1.234} \approx -1.028$

8. $\dfrac{\sqrt{2}-\sqrt{3}}{1.642-\pi} \approx 0.212$

9. $y-2 = -1(x-2)$
$y-2 = -x+2$
$x+y-4 = 0$

10. $y-4 = -1(x-3)$
$y-4 = -x+3$
$x+y-7 = 0$

11. $y = 2x+3$
$2x-y+3 = 0$

12. $y = 0x+5$
$0x+y-5 = 0$

13. $m = \dfrac{8-3}{4-2} = \dfrac{5}{2};$
$y-3 = \dfrac{5}{2}(x-2)$
$2y-6 = 5x-10$
$5x-2y-4 = 0$

14. $m = \dfrac{2-1}{8-4} = \dfrac{1}{4};$
$y-1 = \dfrac{1}{4}(x-4)$
$4y-4 = x-4$
$x-4y+0 = 0$

15. $m = \dfrac{5+3}{2-2}$: undefined; $x + 0y - 2 = 0$

16. $x = -5;\ x+0y+5 = 0$

17. $3y = -2x + 1;\ y = -\dfrac{2}{3}x + \dfrac{1}{3};$ slope $= -\dfrac{2}{3};$
$y$-intercept $= \dfrac{1}{3}$

18. $-4y = 5x-6$
$y = -\dfrac{5}{4}x + \dfrac{3}{2}$
slope $= -\dfrac{5}{4}$; $y$-intercept $= \dfrac{3}{2}$

19. $6-2y = 10x-2$
$-2y = 10x-8$
$y = -5x+4;$
slope $= -5$; $y$-intercept $= 4$

20. $4x+5y = -20$
$5y = -4x-20$
$y = -\dfrac{4}{5}x-4$
slope $= -\dfrac{4}{5}$; $y$-intercept $= \dfrac{3}{2}$

21. a. $m = 2;$
$y+3 = 2(x-3)$
$y = 2x-9$

b. $m = -\dfrac{1}{2};$
$y+3 = -\dfrac{1}{2}(x-3)$
$y = -\dfrac{1}{2}x - \dfrac{3}{2}$

c. $2x+3y = 6$
$3y = -2x+6$
$y = -\dfrac{2}{3}x+2;$
$m = -\dfrac{2}{3};$
$y+3 = -\dfrac{2}{3}(x-3)$
$y = -\dfrac{2}{3}x-1$

d. $m = \frac{3}{2}$;

$$y + 3 = \frac{3}{2}(x - 3)$$

$$y = \frac{3}{2}x - \frac{15}{2}$$

e. $m = \frac{-1-2}{3+1} = -\frac{3}{4}$;

$$y + 3 = -\frac{3}{4}(x - 3)$$

$$y = -\frac{3}{4}x - \frac{3}{4}$$

f. $x = 3$ g. $y = -3$

22. a. $3x + cy = 5$

$$3(3) + c(1) = 5$$

$$c = -4$$

b. $c = 0$

c. $2x + y = -1$

$$y = -2x - 1$$

$m = -2$;

$$3x + cy = 5$$

$$cy = -3x + 5$$

$$y = -\frac{3}{c}x + \frac{5}{c}$$

$$-2 = -\frac{3}{c}$$

$$c = \frac{3}{2}$$

d. $c$ must be the same as the coefficient of $x$, so $c = 3$.

e. $y - 2 = 3(x + 3)$;

perpendicular slope $= -\frac{1}{3}$;

$$-\frac{1}{3} = -\frac{3}{c}$$

$$c = 9$$

23. $m = \frac{3}{2}$;

$$y + 1 = \frac{3}{2}(x + 2)$$

$$y = \frac{3}{2}x + 2$$

24. a. $m = 2$;

$$kx - 3y = 10$$

$$-3y = -kx + 10$$

$$y = \frac{k}{3}x - \frac{10}{3}$$

$$\frac{k}{3} = 2; k = 6$$

b. $m = -\frac{1}{2}$;

$$\frac{k}{3} = -\frac{1}{2}$$

$$k = -\frac{3}{2}$$

c. $2x + 3y = 6$

$$3y = -2x + 6$$

$$y = -\frac{2}{3}x + 2;$$

$$m = \frac{3}{2};\ \frac{k}{3} = \frac{3}{2};\ k = \frac{9}{2}$$

25. $y = 3(3) - 1 = 8$; (3, 9) is above the line.

26. $(a, 0), (0, b)$; $m = \frac{b-0}{0-a} = -\frac{b}{a}$

$$y = -\frac{b}{a}x + b;\ \frac{bx}{a} + y = b; \frac{x}{a} + \frac{y}{b} = 1$$

27. $2x + 3y = 4$

$$-3x + y = 5$$

$$\begin{aligned} 2x + 3y &= 4 \\ 9x - 3y &= -15 \\ \hline 11x \quad &= -11 \end{aligned}$$

$$x = -1$$

$$-3(-1) + y = 5$$

$$y = 2$$

Point of intersection: $(-1, 2)$

$$3y = -2x + 4$$

$$y = -\frac{2}{3}x + \frac{4}{3}$$

$$m = \frac{3}{2}$$

$$y - 2 = \frac{3}{2}(x + 1)$$

$$y = \frac{3}{2}x + \frac{7}{2}$$

**28.** $4x-5y=8$
$2x+y=-10$

$$\begin{aligned} 4x-5y&=8\\ -4x-2y&=20\\ \hline -7y&=28\\ y&=-4 \end{aligned}$$

$4x-5(-4)=8$
$4x=-12$
$x=-3$

Point of intersection: $(-3,-4)$;
$4x-5y=8$
$-5y=-4x+8$
$y=\frac{4}{5}x-\frac{8}{5}$
$m=-\frac{5}{4}$
$y+4=-\frac{5}{4}(x+3)$
$y=-\frac{5}{4}x-\frac{31}{4}$

**29.** $3x-4y=5$
$2x+3y=9$

$$\begin{aligned} 9x-12y&=15\\ 8x+12y&=36\\ \hline 17x\quad&=51\\ x&=3 \end{aligned}$$

$3(3)-4y=5$
$-4y=-4$
$y=1$

Point of intersection: $(3, 1)$; $3x-4y=5$;
$-4y=-3x+5$
$y=\frac{3}{4}x-\frac{5}{4}$
$m=-\frac{4}{3}$
$y-1=-\frac{4}{3}(x-3)$
$y=-\frac{4}{3}x+5$

**30.** $5x-2y=5$
$2x+3y=6$

$$\begin{aligned} 15x-6y&=15\\ 4x+6y&=12\\ \hline 19x\quad&=27\\ x&=\frac{27}{19} \end{aligned}$$

$2\left(\frac{27}{19}\right)+3y=6$
$3y=\frac{60}{19}$
$y=\frac{20}{19}$

Point of intersection: $\left(\frac{27}{19},\frac{20}{19}\right)$;
$5x-2y=5$
$-2y=-5x+5$
$y=\frac{5}{2}x-\frac{5}{2}$
$m=-\frac{2}{5}$
$y-\frac{20}{19}=-\frac{2}{5}\left(x-\frac{27}{19}\right)$
$y=-\frac{2}{5}x+\frac{54}{95}+\frac{20}{19}$
$y=-\frac{2}{5}x+\frac{154}{95}$

**31.** $A=3, B=4, C=-6$
$$d=\frac{|3(-3)+4(2)+(-6)|}{\sqrt{(3)^2+(4)^2}}=\frac{7}{5}$$

**32.** $$d=\frac{|2(4)-2(-1)+4|}{\sqrt{(2)^2+(2)^2}}=\frac{14}{\sqrt{8}}=\frac{7\sqrt{2}}{2}$$

**33.** $A=12, B=-5, C=1$
$$d=\frac{|12(-2)-5(-1)+1|}{\sqrt{(12)^2+(-5)^2}}=\frac{18}{13}$$

**34.** $A=2, B=-1, C=-5$
$$d=\frac{|2(3)-1(-1)-5|}{\sqrt{(2)^2+(-1)^2}}=\frac{2}{\sqrt{5}}=\frac{2\sqrt{5}}{5}$$

35. $2x+4(0)=5$

$$x=\frac{5}{2}$$

$$d=\frac{\left|2\left(\frac{5}{2}\right)+4(0)-7\right|}{\sqrt{(2)^2+(4)^2}}=\frac{2}{\sqrt{20}}=\frac{\sqrt{5}}{5}$$

36. $7(0)-5y=-1$

$$y=\frac{1}{5}$$

$$d=\frac{\left|7(0)-5\left(\frac{1}{5}\right)-6\right|}{\sqrt{(7)^2+(-5)^2}}=\frac{7}{\sqrt{74}}=\frac{7\sqrt{74}}{74}$$

37. $120{,}000(0.08)=9600$; $V=120{,}000-9600t$

38. Slope $=-9600$; The bulldozer depreciates at \$9600 per year.

39. (0, 700,000), (10, 820,000)

$$m=\frac{820{,}000-700{,}000}{10-0}=12{,}000$$

$N=12{,}000n+700{,}000$

At $n=25$: $N=12{,}000(25)+700{,}000=1{,}000{,}000$

40. (0, 80,000), (20, 2000)

$$m=\frac{2000-80{,}000}{20-0}=-3900$$

$V=-3900n+80{,}000$

41. a. When $x=0$, $P=-2000$, which indicates that the company loses money if no items are sold.

b. Slope = 450; this is the amount of profit gained with the sale of each item.

42. a. Slope $=0.75$; this is the amount of money added to the cost with each item produced.

b. When $x=0$, $C=200$. This is the **fixed cost**, that is, the cost to produce zero units..

43. If $(x_0, y_0)$ is on both lines, then $2x_0-y_0+4=0$ and $x_0+3y_0-6=0$ so $2x_0-y_0+4+k(x_0+3y_0-6)=0+0\cdot k=0$, which means that $(x_0, y_0)$ is on the line $2x-y+4+k(x+3y-6)=0$ regardless of the value of $k$.

44.

$$\frac{x}{a}+\frac{y}{a}=1$$

$$\frac{2}{a}+\frac{3}{a}=1$$

$$\frac{5}{a}=1$$

$$a=5$$

$$\frac{x}{5}+\frac{y}{5}=1$$

$$x+y-5=0$$

45. $m=\frac{-2-3}{1+2}=-\frac{5}{3}$; $m=\frac{3}{5}$; passes through

$$\left(\frac{-2+1}{2},\frac{3-2}{2}\right)=\left(-\frac{1}{2},\frac{1}{2}\right)$$

$$y-\frac{1}{2}=\frac{3}{5}\left(x+\frac{1}{2}\right)$$

$$y=\frac{3}{5}x+\frac{4}{5}$$

46. $m=\frac{0-4}{2-0}=-2$; $m=\frac{1}{2}$; passes through

$$\left(\frac{0+2}{2},\frac{4+0}{2}\right)=(1,2)$$

$$y-2=\frac{1}{2}(x-1)$$

$$y=\frac{1}{2}x+\frac{3}{2}$$

$m=\frac{6-0}{4-2}=3$; $m=-\frac{1}{3}$; passes through

$$\left(\frac{2+4}{2},\frac{0+6}{2}\right)=(3,3)$$

$$y-3=-\frac{1}{3}(x-3)$$

$$y=-\frac{1}{3}x+4$$

$$\frac{1}{2}x+\frac{3}{2}=-\frac{1}{3}x+4$$

$$\frac{5}{6}x=\frac{5}{2}$$

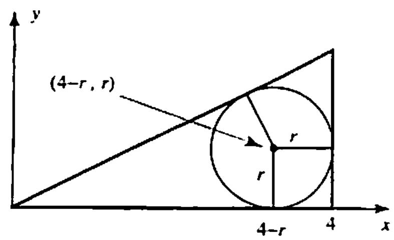
$$x=3$$

$$y=\frac{1}{2}(3)+\frac{3}{2}=3$$

center = (3, 3)

47. Let the origin be at the vertex as shown in the figure below. The center of the circle is then $(4-r, r)$, so it has equation $(x-(4-r))^2+(y-r)^2=r^2$. Along the side of

length 5, the $y$-coordinate is always $\frac{3}{4}$ times the $x$-coordinate. Thus, we need to find the value of $r$ for which there is exactly one $x$-solution to $(x-4+r)^2+(y-r)^2=r^2$. Solving for $x$ in this equation gives $x=\frac{4}{25}\left(16-r\pm\sqrt{24\left(-r^2+7r-6\right)}\right)$. There is exactly one solution when $-r^2+7r-6=0$, that is, when $r=1$ or $r=6$. The root $r=6$ is extraneous. Thus, the largest circle that can be inscribed in this triangle has radius $r=1$.




**48.** The line tangent to the circle at $(a,b)$ will be perpendicular to the line through $(a,b)$ and the center of the circle, which is $(0,0)$. The line through $(a,b)$ and $(0,0)$ has slope

$$m=\frac{0-b}{0-a}=\frac{b}{a};ax+by=r^2\Rightarrow y=-\frac{a}{b}x+\frac{r^2}{b}$$

so $ax+by=r^2$ has slope $-\frac{a}{b}$ and is perpendicular to the line through $(a,b)$ and $(0,0)$, so it is tangent to the circle at $(a,b)$.

**49.** $12a+0b=36$

$a=3$

$3^2+b^2=36$

$b=\pm 3\sqrt{3}$

$3x-3\sqrt{3}y=36$

$x-\sqrt{3}y=12$

$3x+3\sqrt{3}y=36$

$x+\sqrt{3}y=12$

**50.** $A=m, B=-1, C=B-b; (0,0)$

$$d=\frac{|m(0)-1(0)+B-b|}{\sqrt{m^2+(-1)^2}}=\frac{|B-b|}{\sqrt{m^2+1}}$$

**51.** The midpoint of the side from (0, 0) to $(a, 0)$ is $\left(\frac{0+a}{2},\frac{0+0}{2}\right)=\left(\frac{a}{2},0\right)$

The midpoint of the side from (0, 0) to $(b, c)$ is $\left(\frac{0+b}{2},\frac{0+c}{2}\right)=\left(\frac{b}{2},\frac{c}{2}\right)$

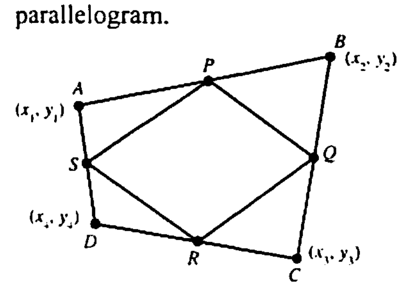
$$m_1=\frac{c-0}{b-a}=\frac{c}{b-a}$$

$$m_2=\frac{\frac{c}{2}-0}{\frac{b}{2}-\frac{a}{2}}=\frac{c}{b-a}; m_1=m_2$$

**52.** See the figure below. The midpoints of the sides are

$P\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$, $Q\left(\frac{x_2+x_3}{2},\frac{y_2+y_3}{2}\right)$, $R\left(\frac{x_3+x_4}{2},\frac{y_3+y_4}{2}\right)$, and $S\left(\frac{x_1+x_4}{2},\frac{y_1+y_4}{2}\right)$. The slope of $PS$ is

$$\frac{\frac{1}{2}[y_1+y_4-(y_1+y_2)]}{\frac{1}{2}[x_1+x_4-(x_1+x_2)]}=\frac{y_4-y_2}{x_4-x_2}.$$ The slope of $QR$ is $$\frac{\frac{1}{2}[y_3+y_4-(y_2+y_3)]}{\frac{1}{2}[x_3+x_4-(x_2+x_3)]}=\frac{y_4-y_2}{x_4-x_2}.$$ Thus $PS$ and $QR$ are parallel. The slopes of $SR$ and $PQ$ are both $\frac{y_3-y_1}{x_3-x_1}$, so $PQRS$ is a parallelogram.




**53.** $x^2+(y-6)^2=25$; passes through (3, 2)

tangent line: $3x-4y=1$

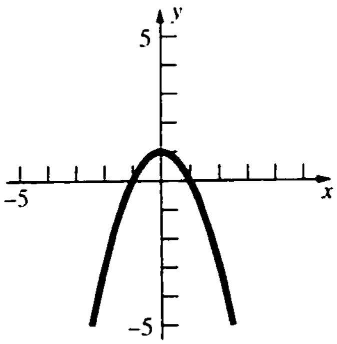
The dirt hits the wall at $y=8$.

## 1.7 Concepts Review

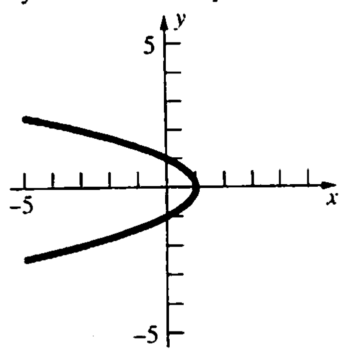
1. $y$-axis
2. $(4,-2)$
3. 8; –2, 1, 4
4. line; parabola

## Problem Set 1.7

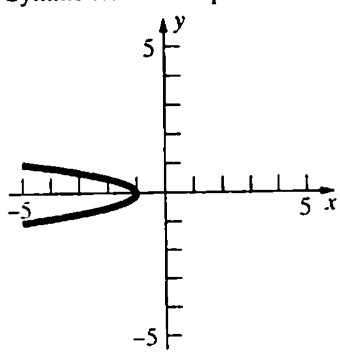
1. $y = -x^2 + 1$; $y$-intercept = 1; $y = (1 + x)(1 - x)$; $x$-intercepts = –1, 1
Symmetric with respect to the $y$-axis



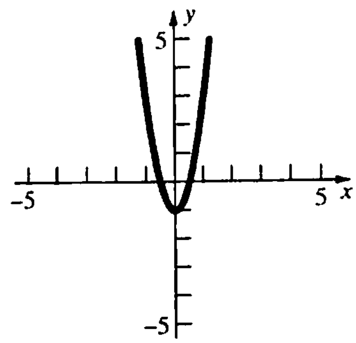

2. $x = -y^2 + 1$; $y$-intercepts = –1, 1;
$x$-intercept = 1
Symmetric with respect to the $x$-axis.



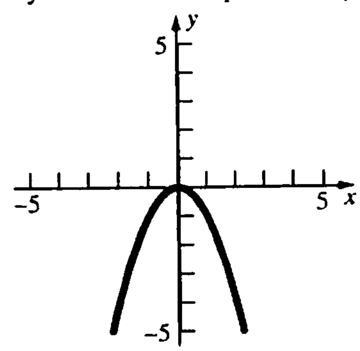

3. $x = -4y^2 - 1$; $x$-intercept = –1
Symmetric with respect to the $x$-axis



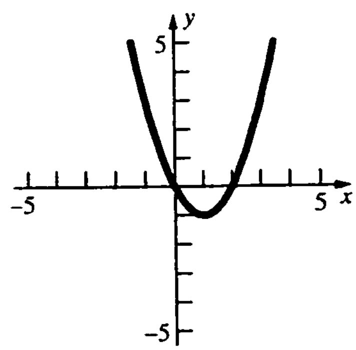

4. $y = 4x^2 - 1$; $y$-intercept = –1
$y = (2x+1)(2x-1)$; $x$-intercepts $= -\frac{1}{2}, \frac{1}{2}$
Symmetric with respect to the $y$-axis.




5. $x^2 + y = 0$; $y = -x^2$
$x$-intercept = 0, $y$-intercept = 0
Symmetric with respect to the $y$-axis

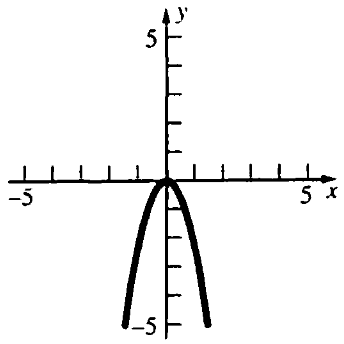



6. $y = x^2 - 2x$; $y$-intercept = 0
$y = x(2-x)$; $x$-intercepts = 0, 2

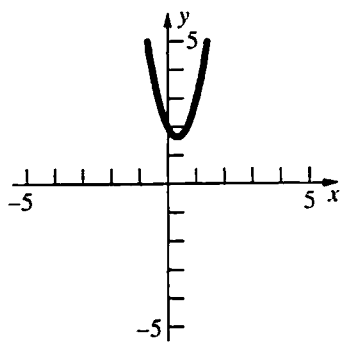


7. $7x^2 + 3y = 0$; $3y = -7x^2$; $y = -\frac{7}{3}x^2$

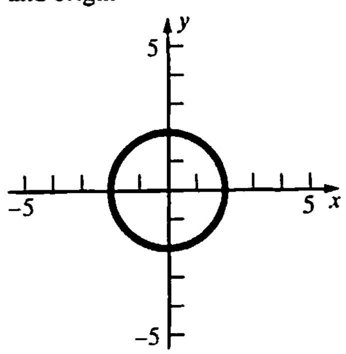
$x$-intercept = 0, $y$-intercept = 0
Symmetric with respect to the $y$-axis



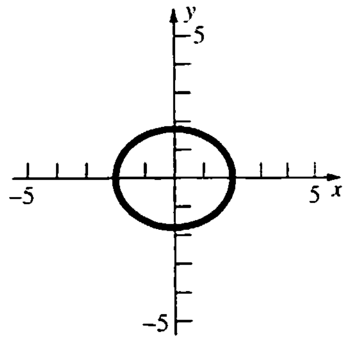
8. $y = 3x^2 - 2x + 2$; $y$-intercept = 2



9. $x^2 + y^2 = 4$
$x$-intercepts = -2, 2; $y$-intercepts = -2, 2
Symmetric with respect to the $x$-axis, $y$-axis, and origin

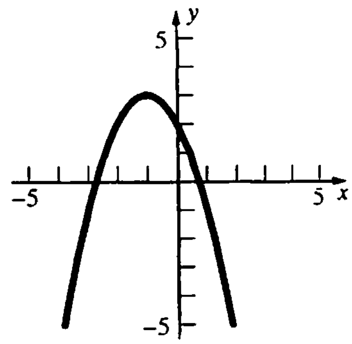


10. $3x^2 + 4y^2 = 12$; $y$-intercepts = $-\sqrt{3}, \sqrt{3}$
$x$-intercepts = $-2, 2$
Symmetric with respect to the $x$-axis, $y$-axis, and origin

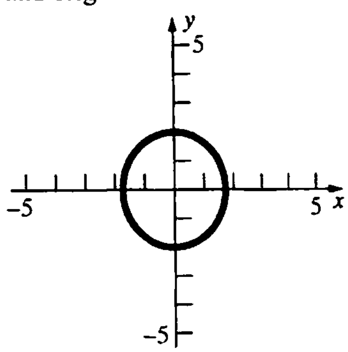


11. $y = -x^2 - 2x + 2$: $y$-intercept = 2

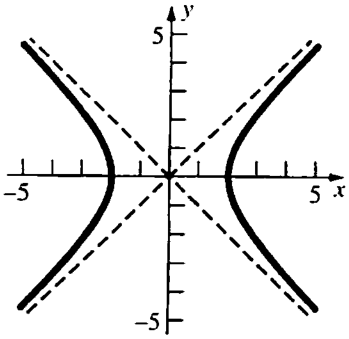
$$x\text{-intercepts} = \frac{2 \pm \sqrt{4+8}}{-2} = \frac{2 \pm 2\sqrt{3}}{-2} = -1 \pm \sqrt{3}$$



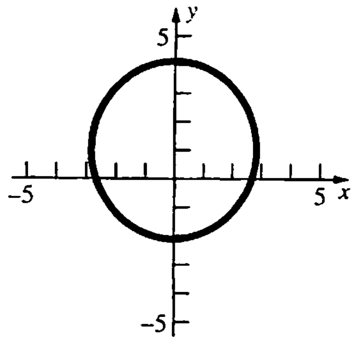
12. $4x^2 + 3y^2 = 12$; $y$-intercepts = $-2, 2$
$x$-intercepts $= -\sqrt{3}, \sqrt{3}$
Symmetric with respect to the $x$-axis, $y$-axis, and origin



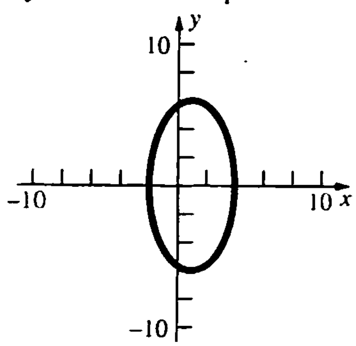
13. $x^2 - y^2 = 4$
$x$-intercept = -2, 2
Symmetric with respect to the $x$-axis, $y$-axis, and origin



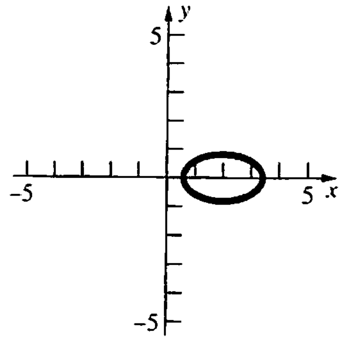

14. $x^2 + (y-1)^2 = 9$; $y$-intercepts = −2, 4
$x$-intercepts = $-2\sqrt{2}, 2\sqrt{2}$
Symmetric with respect to the $y$-axis



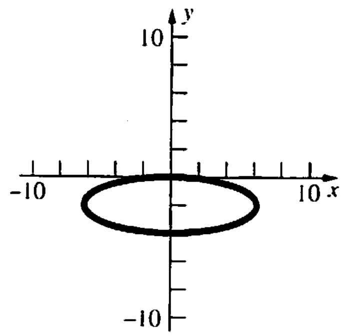

15. $4(x-1)^2 + y^2 = 36$;
$y$-intercepts $= \pm\sqrt{32} = \pm 4\sqrt{2}$
$x$-intercepts = −2, 4
Symmetric with respect to the $x$-axis



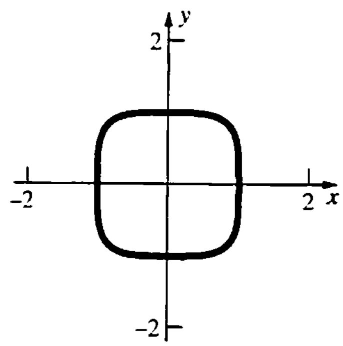

16. $x^2 - 4x + 3y^2 = -2$
$x$-intercepts = $2 \pm \sqrt{2}$
Symmetric with respect to the $x$-axis



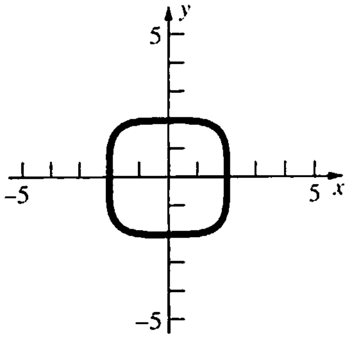

17. $x^2 + 9(y+2)^2 = 36$; $y$-intercepts = −4, 0
$x$-intercept = 0
Symmetric with respect to the $y$-axis



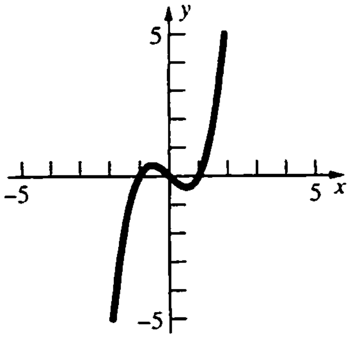

18. $x^4 + y^4 = 1$; $y$-intercepts = −1, 1
$x$-intercepts = −1, 1
Symmetric with respect to the $x$-axis, $y$-axis, and origin



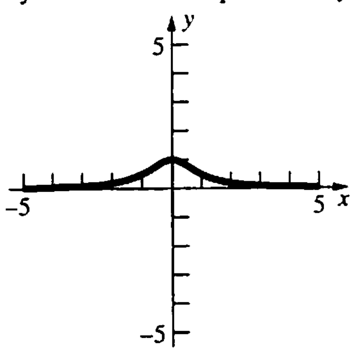
**19.** $x^4 + y^4 = 16$; $y$-intercepts $= -2, 2$
$x$-intercepts $= -2, 2$
Symmetric with respect to the $y$-axis, $x$-axis and origin



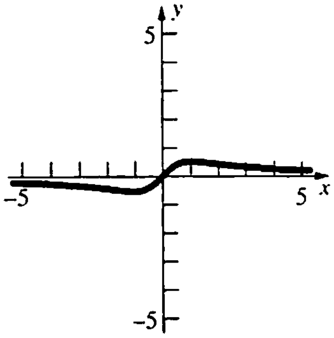

**20.** $y = x^3 - x$; $y$-intercepts = 0;
$y = x(x^2 - 1) = x(x + 1)(x - 1)$;
$x$-intercepts $= -1, 0, 1$
Symmetric with respect to the origin




**21.** $y = \dfrac{1}{x^2 + 1}$; $y$-intercept = 1
Symmetric with respect to the $y$-axis




**22.** $y = \dfrac{x}{x^2 + 1}$; $y$-intercept = 0
$x$-intercept = 0
Symmetric with respect to the origin



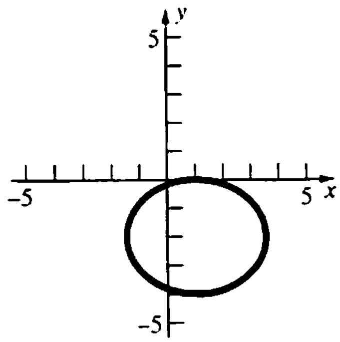

**23.** $2x^2 - 4x + 3y^2 + 12y = -2$

$2(x^2 - 2x + 1) + 3(y^2 + 4y + 4) = -2 + 2 + 12$

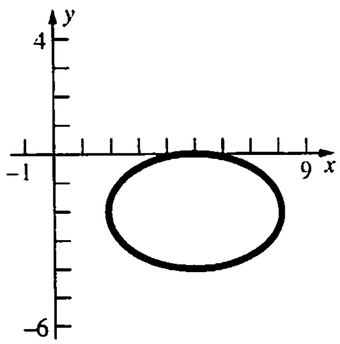
$2(x - 1)^2 + 3(y + 2)^2 = 12$

$y$-intercepts $= -2 \pm \dfrac{\sqrt{30}}{3}$

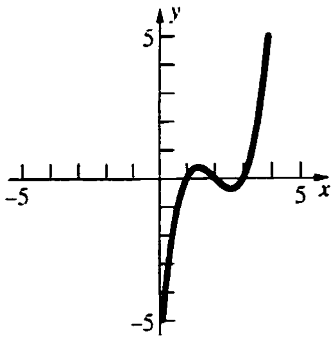
$x$-intercept = 1



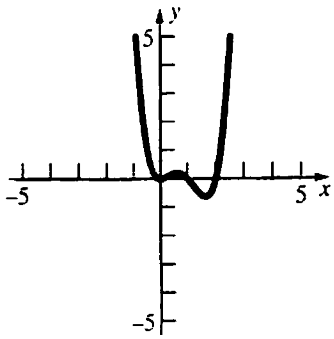

**24.** $4(x - 5)^2 + 9(y + 2)^2 = 36$; $x$-intercept = 5



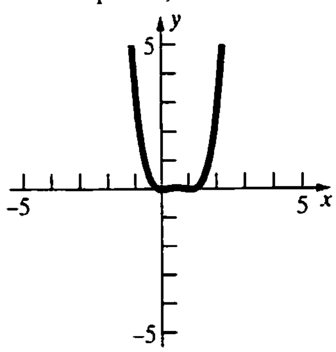
**25.** $y = (x - 1)(x - 2)(x - 3)$; $y$-intercept = –6
$x$-intercepts = 1, 2, 3



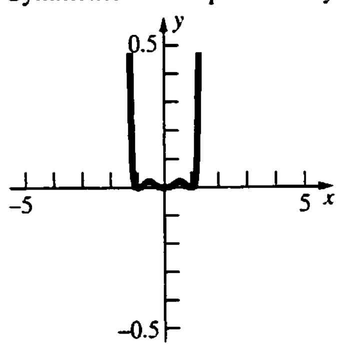

**26.** $y = x^2(x - 1)(x - 2)$; $y$-intercept = 0
$x$-intercepts = 0, 1, 2



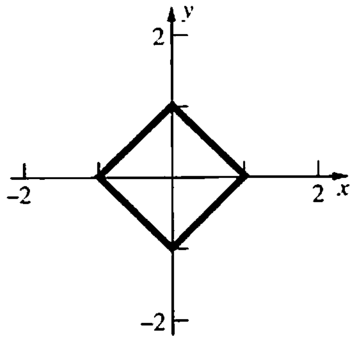

**27.** $y = x^2(x-1)^2$; $y$-intercept = 0
$x$-intercepts = 0, 1



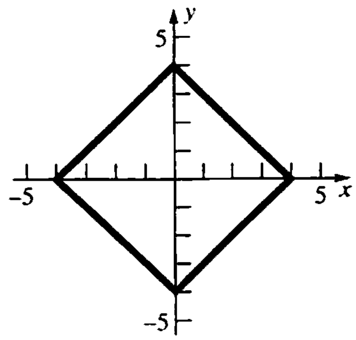

**28.** $y = x^4(x-1)^4(x+1)^4$; $y$-intercept = 0
$x$-intercepts = –1, 0, 1
Symmetric with respect to the $y$-axis




**29.** $|x| + |y| = 1$; $y$-intercepts = –1, 1;
$x$-intercepts = –1, 1
Symmetric with respect to the $x$-axis, $y$-axis and origin

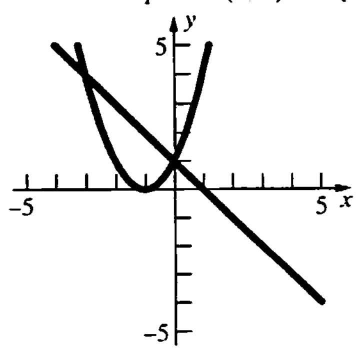



**30.** $|x| + |y| = 4$; $y$-intercepts = –4, 4;
$x$-intercepts = –4, 4
Symmetric with respect to the $x$-axis, $y$-axis and origin




$$-x + 1 = (x + 1)^2$$
$$-x + 1 = x^2 + 2x + 1$$

**31.** $x^2 + 3x = 0$
$x(x + 3) = 0$
$x = 0, -3$
Intersection points: (0, 1) and (–3, 4)

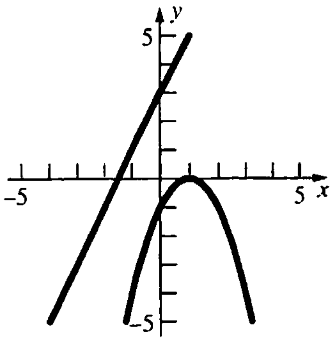


**32.** $2x+3=-(x-1)^2$

$2x-3=-x^2+2x-1$

$x^2+4=0$

No points of intersection



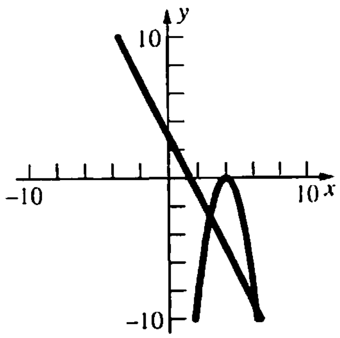

**33.** $-2x+3=-2(x-4)^2$

$-2x+3=-2x^2+16x-32$

$2x^2-18x+35=0$

$$x=\frac{18\pm\sqrt{324-280}}{4}=\frac{18\pm 2\sqrt{11}}{4}=\frac{9\pm\sqrt{11}}{2};$$

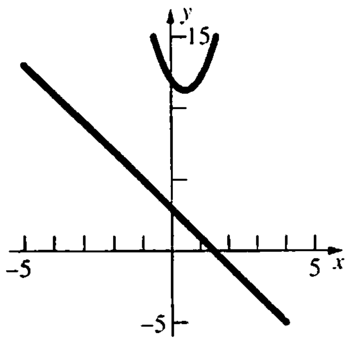
Intersection points: $\left(\frac{9-\sqrt{11}}{2},-6+\sqrt{11}\right)$, $\left(\frac{9+\sqrt{11}}{2},-6-\sqrt{11}\right)$




**34.** $-2x+3=-3x^2-3x+12$

$3x^2-x+9=0$

No points of intersection

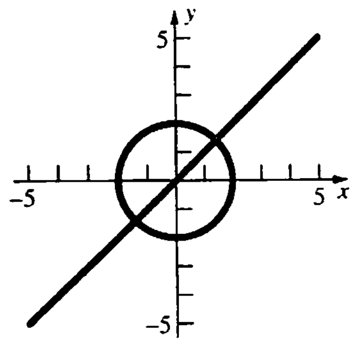



$x^2+x^2=4$

**35.** $x^2=2$

$x=\pm\sqrt{2}$

Intersection points: $\left(-\sqrt{2},-\sqrt{2}\right),\left(\sqrt{2},\sqrt{2}\right)$




**36.** $2x^2+3(x-1)^2=12$

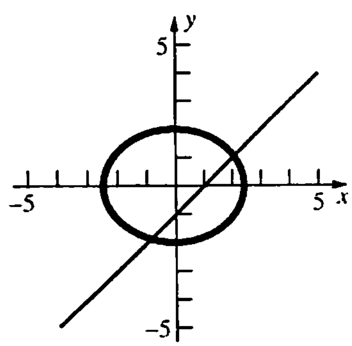
$2x^2+3x^2-6x+3=12$

$5x^2-6x-9=0$

$$x=\frac{6\pm\sqrt{36+180}}{10}=\frac{6\pm 6\sqrt{6}}{10}=\frac{3\pm 3\sqrt{6}}{5}$$

Intersection points:

$\left(\frac{3-3\sqrt{6}}{5},\frac{-2-3\sqrt{6}}{5}\right),\left(\frac{3+3\sqrt{6}}{5},\frac{-2+3\sqrt{6}}{5}\right)$



37.

$y = 3x + 1$

$x^2 + 2x + (3x+1)^2 = 15$

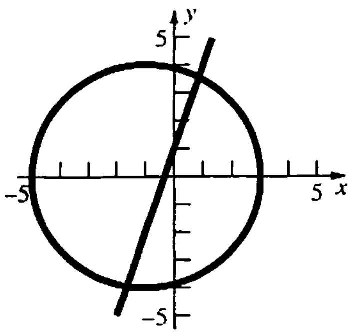
$x^2 + 2x + 9x^2 + 6x + 1 = 15$

$10x^2 + 8x - 14 = 0$

$2(5x^2 + 4x - 7) = 0$

$x \approx -1.65, 0.85$

Intersection points: (−1.65, −3.95) and (0.85, 3.55)



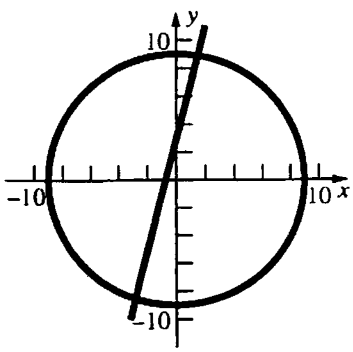

38. $x^2 + (4x+3)^2 = 81$

$x^2 + 16x^2 + 24x + 9 = 81$

$17x^2 + 24x - 72 = 0$

$x \approx -2.88, 1.47$

Intersection points: $(-2.88, -8.52), (1.47, 8.88)$




39. a. $y = x^2$; (2)

b. $ax^3 + bx^2 + cx + d$, with $a > 0$: (1)

c. $ax^3 + bx^2 + cx + d$, with $a < 0$: (3)

d. $y = ax^3$, with $a > 0$: (4)

40. $x^2 + y^2 = 13; (-2,-3), (-2,3), (2,-3), (2,3)$

$d_1 = \sqrt{(2+2)^2 + (-3+3)^2} = 4$

$d_2 = \sqrt{(2+2)^2 + (-3-3)^2} = \sqrt{52} = 2\sqrt{13}$

$d_3 = \sqrt{(2-2)^2 + (3+3)^2} = 6$

Three such distances

41. $x^2 + 2x + y^2 - 2y = 20$; $\left(-2, 1+\sqrt{21}\right)$, $\left(-2, 1-\sqrt{21}\right), \left(2, 1+\sqrt{13}\right), \left(2, 1-\sqrt{13}\right)$

$d_1 = \sqrt{(-2-2)^2 + \left[1+\sqrt{21} - \left(1+\sqrt{13}\right)\right]^2}$

$= \sqrt{16 + \left(\sqrt{21} - \sqrt{13}\right)^2}$

$= \sqrt{50 - 2\sqrt{273}} \approx 4.12$

$d_2 = \sqrt{(-2-2)^2 + \left[1+\sqrt{21} - \left(1-\sqrt{13}\right)\right]^2}$

$= \sqrt{16 + \left(\sqrt{21} + \sqrt{13}\right)^2}$

$= \sqrt{50 + 2\sqrt{273}} \approx 9.11$

$d_3 = \sqrt{(-2+2)^2 + \left[1+\sqrt{21} - \left(1-\sqrt{21}\right)\right]^2}$

$= \sqrt{0 + \left(\sqrt{21} + \sqrt{21}\right)^2} = \sqrt{\left(2\sqrt{21}\right)^2} = 2\sqrt{21} \approx 9.17$

$d_4 = \sqrt{(-2-2)^2 + \left[1-\sqrt{21} - (1+\sqrt{13})\right]^2}$

$= \sqrt{16 + \left(-\sqrt{21} - \sqrt{13}\right)^2} = \sqrt{50 + 2\sqrt{273}} \approx 9.11$

$d_5 = \sqrt{(-2-2)^2 + \left[1-\sqrt{21} - \left(1-\sqrt{13}\right)\right]^2}$

$= \sqrt{16 + \left(\sqrt{13} - \sqrt{21}\right)^2} = \sqrt{50 - 2\sqrt{273}} \approx 4.12$

$d_6 = \sqrt{(2-2)^2 + \left[1+\sqrt{13} - \left(1-\sqrt{13}\right)\right]^2}$

$= \sqrt{0 + \left(\sqrt{13} + \sqrt{13}\right)^2} = \sqrt{\left(2\sqrt{13}\right)^2} = 2\sqrt{13} \approx 7.21$

Four such distances ( $d_2 = d_4$ and $d_1 = d_5$ ).

## 1.8 Chapter Review

### Concepts Test

1. False: $p$ and $q$ must be integers.

2. True: $\frac{p_1}{q_1}-\frac{p_2}{q_2}=\frac{p_1q_2-p_2q_1}{q_1q_2}$; since $p_1, q_1, p_2$, and $q_2$ are integers, so are $p_1q_2-p_2q_1$ and $q_1q_2$.

3. False: If the numbers are opposites ($-\pi$ and $\pi$) then the sum is 0, which is rational.

4. True: Between any two distinct real numbers there are both a rational and an irrational number.

5. False: 0.999... is equal to 1.

6. True: $\left(a^m\right)^n=\left(a^n\right)^m=a^{mn}$

7. False: $(a\cdot b)\cdot c=a^{bc}; a\cdot(b\cdot c)=a^{b^c}$

8. True: Since $x\le y\le z$ and $x\ge z, x=y=z$

9. True: If $x$ was not 0, then $\varepsilon=\frac{|x|}{2}$ would be a positive number less than $|x|$.

10. True: $y-x=-(x-y)$ so $(x-y)(y-x)=(x-y)(-1)(x-y)$ $=(-1)(x-y)^2$. $(x-y)^2\ge 0$ for all $x$ and $y$, so $-(x-y)^2\le 0$.

11. True: $a<b<0; a<b; \frac{a}{b}>1; \frac{1}{b}<\frac{1}{a}$

12. True: $[a,b]$ and $[b,c]$ share point $b$ in common.

13. True: If $(a, b)$ and $(c, d)$ share a point then $c<b$ so they share the infinitely many points between $b$ and $c$.

14. True: $\sqrt{x^2}=|x|=-x$ if $x<0$.

15. False: If $x=0$, the number has no sign.

16. False: This only holds if $x$ and $y$ are greater than 0.

17. True: $|x|<|y|\Leftrightarrow|x|^4<|y|^4$
$|x|^4=x^4$ and $|y|^4=y^4$, so $x^4<y^4$.

18. True: $|x+y|=-(x+y)$
$=-x+(-y)=|x|+|y|$

19. True: If $r=0$, then
$\frac{1}{1+|r|}=\frac{1}{1-r}=\frac{1}{1-|r|}=1.$
For any $r$, $1+|r|\ge 1-|r|$.
Since $|r|<1, 1-|r|>0$ so
$\frac{1}{1+|r|}\le\frac{1}{1-|r|}$; also, $-1<r<1$.

If $-1<r<0$, then $|r|=-r$ and $1-r=1+|r|$, so
$\frac{1}{1+|r|}=\frac{1}{1-r}\le\frac{1}{1-|r|}$.
If $0<r<1$, then $|r|=r$ and $1-r=1-|r|$, so
$\frac{1}{1+|r|}\le\frac{1}{1-r}=\frac{1}{1-|r|}$.

20. True: If $|r|>1$, then $1-|r|<0$. Thus, since $1+|r|\ge 1-|r|$, $\frac{1}{1-|r|}\le\frac{1}{1+|r|}$. If $r>1$, $|r|=r$, and $1-r=1-|r|$, so
$\frac{1}{1-|r|}=\frac{1}{1-r}\le\frac{1}{1+|r|}$.
If $r<-1, |r|=-r$ and $1-r=1+|r|$,
so $\frac{1}{1-|r|}\le\frac{1}{1-r}=\frac{1}{1+|r|}$.

21. True: If $x$ and $y$ are the same sign, then $\left||x|-|y|\right|=|x-y|$. $|x-y|\le|x+y|$ when $x$ and $y$ are the same sign, so $\left||x|-|y|\right|\le|x+y|$. If $x$ and $y$ have opposite signs then either $\left||x|-|y|\right|=|x-(-y)|=|x+y|$ $(x>0, y<0)$ or $\left||x|-|y|\right|=|-x-y|=|x+y|$ $(x<0, y>0)$. In either case

$||x|-|y|| = |x+y|$.
If either $x = 0$ or $y = 0$, the inequality is easily seen to be true.

**22.** True: If $y$ is positive, then $x = \sqrt{y}$ satisfies

$$x^2 = \left(\sqrt{y}\right)^2 = y.$$

**23.** True: For every real number $y$, whether it is positive, zero, or negative, the cube root $x = \sqrt[3]{y}$ satisfies $x^3 = \left(\sqrt[3]{y}\right)^3 = y$

**24.** True: For example $x^2 \le 0$ has solution [0].

**25.** True:

$$x^2 + ax + y^2 + y = 0$$

$$x^2 + ax + \frac{a^2}{4} + y^2 + y + \frac{1}{4} = \frac{a^2}{4} + \frac{1}{4}$$

$$\left(x + \frac{a}{2}\right)^2 + \left(y + \frac{1}{2}\right)^2 = \frac{a^2+1}{4}$$

is a circle for all values of $a$.

**26.** False: If $x = b = 0$ and $c < 0$, the equation does not represent a circle.

**27.** True;

$$y - b = \frac{3}{4}(x - a)$$

$$y = \frac{3}{4}x - \frac{3a}{4} + b;$$

If $x = a + 4$:

$$y = \frac{3}{4}(a+4) - \frac{3a}{4} + b$$

$$= \frac{3a}{4} + 3 - \frac{3a}{4} + b = b + 3$$

**28.** True: If the points are on the same line, they have equal slope. Then the reciprocals of the slopes are also equal.

**29.** True: If $ab > 0$, $a$ and $b$ have the same sign, so $(a, b)$ is in either the first or third quadrant.

**30.** True: Let $x = \varepsilon/2$. If $\varepsilon > 0$, then $x \cdot 0$ and $x < \varepsilon$.

**31.** True: If $ab = 0$, $a$ or $b$ is 0, so $(a, b)$ lies on the $x$-axis or the $y$-axis. If $a = b = 0$, $(a, b)$ is the origin.

**32.** True: $y_1 = y_2$, so $(x_1, y_1)$ and $(x_2, y_2)$ are on the same horizontal line.

**33.** True:

$$d = \sqrt{[(a+b)-(a-b)]^2 + (a-a)^2}$$

$$= \sqrt{(2b)^2} = |2b|$$

**34.** False: The equation of a vertical line cannot be written in point-slope form.

**35.** True: This is the general linear equation.

**36.** True: Two non-vertical lines are parallel if and only if they have the same slope.

**37.** False: The slopes of perpendicular lines are negative reciprocals.

**38.** True: If $a$ and $b$ are rational and $(a,0),(0,b)$ are the intercepts, the slope is $-\frac{b}{a}$ which is rational.

**39.** False:

$$ax + y = c \Rightarrow y = -ax + c$$

$$ax - y = c \Rightarrow y = ax - c$$

$$(a)(-a) \ne -1.$$

**40.** True: The equation is $(3+2m)x + (6m-2)y + 4 - 2m = 0$ which is the equation of a straight line unless $3 + 2m$ and $6m - 2$ are both 0, and there is no real number $m$ such that $3 + 2m = 0$ and $6m - 2 = 0$.

## Sample Test Problems

**1. a.** $\left(n + \frac{1}{n}\right)^n; \left(1 + \frac{1}{1}\right)^1 = 2; \left(2 + \frac{1}{2}\right)^2 = \frac{25}{4};$

$$\left(-2 + \frac{1}{-2}\right)^{-2} = \frac{4}{25}$$

**b.** $(n^2 - n + 1)^2; \left[(1)^2 - (1) + 1\right]^2 = 1;$

$$\left[(2)^2 - (2) + 1\right]^2 = 9;$$

$$\left[(-2)^2 - (-2) + 1\right]^2 = 49$$

**c.** $4^{3/n}; 4^{3/1} = 64; 4^{3/2} = 8; 4^{-3/2} = \frac{1}{8}$

d. $\sqrt[n]{\left|\frac{1}{n}\right|}; \sqrt[1]{\left|\frac{1}{1}\right|}=1; \sqrt{\left|\frac{1}{2}\right|}=\frac{1}{\sqrt{2}}=\frac{\sqrt{2}}{2};$

$\sqrt[-2]{\left|\frac{1}{-2}\right|}=\sqrt{2}$

2. a. $\left(1+\frac{1}{m}+\frac{1}{n}\right)\left(1-\frac{1}{m}+\frac{1}{n}\right)^{-1}=\frac{1+\frac{1}{m}+\frac{1}{n}}{1-\frac{1}{m}+\frac{1}{n}}$

$=\frac{mn+n+m}{mn-n+m}$

b. $\frac{\frac{2}{x+1}-\frac{x}{x^2-x-2}}{\frac{3}{x+1}-\frac{2}{x-2}}=\frac{\frac{2}{x+1}-\frac{x}{(x-2)(x+1)}}{\frac{3}{x+1}-\frac{2}{x-2}}$

$=\frac{2(x-2)-x}{3(x-2)-2(x+1)}$

$=\frac{x-4}{x-8}$

c. $\frac{(t^3-1)}{t-1}=\frac{(t-1)(t^2+t+1)}{t-1}=t^2+t+1$

3. Let $a$, $b$, $c$, and $d$ be integers.

$\frac{\frac{a}{b}+\frac{c}{d}}{2}=\frac{a}{2b}+\frac{c}{2d}=\frac{ad+bc}{2bd}$ which is rational.

4. $x=4.1282828\ldots$

$1000x=4128.282828\ldots$

$10x=41.282828\ldots$

$990x=4807$

$x=\frac{4807}{990}$

5. Answers will vary. Possible answer:

$\sqrt{\frac{13}{50}}\approx 0.50990\ldots$

6. $\frac{\left(\sqrt[3]{8.15\times 10^4}-1.32\right)^2}{3.24}\approx 545.39$

7. $\left(\pi-\sqrt{2.0}\right)^{2.5}-\sqrt[3]{2.0}\approx 2.66$

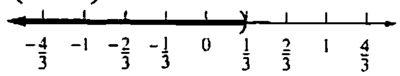
8. $\sin^2(2.45)+\cos^2(2.40)-1.00\approx -0.0495$

9. $1-3x>0$

$3x<1$

$x<\frac{1}{3}$

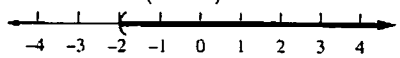
$\left(-\infty,\frac{1}{3}\right)$




10. $6x+3>2x-5$

$4x>-8$

$x>-2;(-2,\infty)$

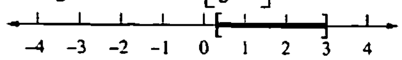



11. $3-2x\le 4x+1\le 2x+7$

$3-2x\le 4x+1$ and $4x+1\le 2x+7$

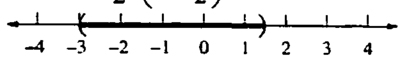
$6x\ge 2$ and $2x\ge 6$

$x\ge\frac{1}{3}$ and $x\le 3$; $\left[\frac{1}{3},3\right]$




12. $2x^2+5x-3<0;(2x-1)(x+3)<0;$

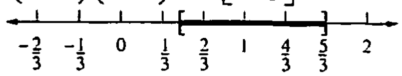
$-3<x<\frac{1}{2};\left(-3,\frac{1}{2}\right)$



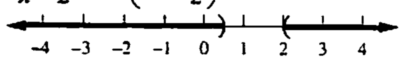

13. $21t^2-44t+12\le -3; 21t^2-44t+15\le 0;$

$t=\frac{44\pm\sqrt{44^2-4(21)(15)}}{2(21)}=\frac{44\pm 26}{42}=\frac{3}{7},\frac{5}{3}$

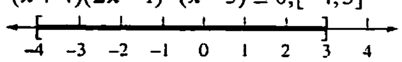
$\left(t-\frac{3}{7}\right)\left(t-\frac{5}{3}\right)\le 0;\left[\frac{3}{7},\frac{5}{3}\right]$




14. $\frac{2x-1}{x-2}>0;\left(-\infty,\frac{1}{2}\right)\cup(2,\infty)$

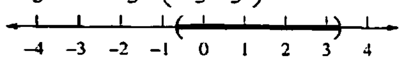



15. $(x+4)(2x-1)^2(x-3)\le 0;[-4,3]$




16. $|3x-4|<6;-6<3x-4<6;-2<3x<10;$

$-\frac{2}{3}<x<\frac{10}{3};\left(-\frac{2}{3},\frac{10}{3}\right)$



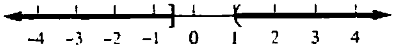
17. $\frac{3}{1-x} \le 2$

$\frac{3}{1-x} - 2 \le 0$

$\frac{3-2(1-x)}{1-x} \le 0$

$\frac{2x+1}{1-x} \le 0;$

$\left(-\infty, -\frac{1}{2}\right] \cup (1, \infty)$



18. $|12-3x| \ge |x|$

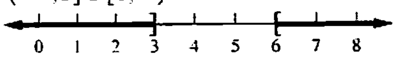
$(12-3x)^2 \ge x^2$

$144 - 72x + 9x^2 \ge x^2$

$8x^2 - 72x + 144 \ge 0$

$8(x-3)(x-6) \ge 0$

$(-\infty, 3] \cup [6, \infty)$



19. For example, if $x = -2$, $|-(-2)| = 2 \ne -2$

$|-x| \ne x$ for any $x < 0$

20. If $|-x| = x$, then $|x| = x$.

$x \ge 0$

21. $|t-5| = |-(5-t)| = |5-t|$

If $|5-t| = 5-t$, then $5 - t \ge 0$.

$t \le 5$

22. $|t-a| = |-(a-t)| = |a-t|$

If $|a-t| = a-t$, then $a - t \ge 0$.

$t \le a$

23. If $|x| \le 2$, then

$0 \le |2x^2 + 3x + 2| \le |2x^2| + |3x| + 2 \le 8 + 6 + 2 = 16$

also $|x^2 + 2| \ge 2$ so $\frac{1}{|x^2+2|} \le \frac{1}{2}$. Thus

$\left|\frac{2x^2+3x+2}{x^2+2}\right| = |2x^2+3x+2|\left|\frac{1}{x^2+2}\right| \le 16\left(\frac{1}{2}\right)$

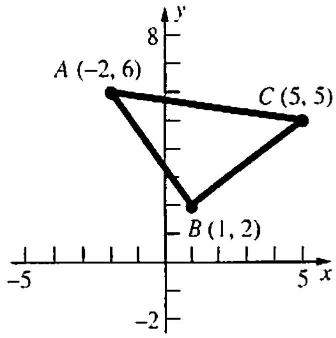
$= 8$

24. **a.** The distance between $x$ and 5 is 3.

**b.** The distance between $x$ and $-1$ is less than or equal to 2.

**c.** The distance between $x$ and $a$ is greater than $b$.

25.




$d(A, B) = \sqrt{(1+2)^2 + (2-6)^2} = \sqrt{9+16} = 5$

$d(B, C) = \sqrt{(5-1)^2 + (5-2)^2} = \sqrt{16+9} = 5$

$d(A, C) = \sqrt{(5+2)^2 + (5-6)^2} = \sqrt{49+1} = \sqrt{50}$

$= 5\sqrt{2}$

$(AB)^2 + (BC)^2 = (AC)^2$, so $\Delta ABC$ is a right triangle.

26. midpoint: $\left(\frac{1+7}{2}, \frac{2+8}{2}\right) = (4, 5)$

$d = \sqrt{(4-3)^2 + (5+6)^2} = \sqrt{1+121} = \sqrt{122}$

27. center $= \left(\frac{2+10}{2}, \frac{0+4}{2}\right) = (6, 2)$

radius $= \frac{1}{2}\sqrt{(10-2)^2 + (4-0)^2} = \frac{1}{2}\sqrt{64+16}$

$= 2\sqrt{5}$

circle: $(x-6)^2 + (y-2)^2 = 20$

28. $x^2 + y^2 - 8x + 6y = 0$

$x^2 - 8x + 16 + y^2 + 6y + 9 = 16 + 9$

$(x-4)^2 + (y+3)^2 = 25;$

center $= (4, -3)$, radius $= 5$

$x^2 - 2x + y^2 + 2y = 2$

29. $x^2 - 2x + 1 + y^2 + 2y + 1 = 2 + 1 + 1$

$(x-1)^2 + (y+1)^2 = 4$

center $= (1, -1)$

$x^2 + 6x + y^2 - 4y = -7$

$x^2 + 6x + 9 + y^2 - 4y + 4 = -7 + 9 + 4$

$(x+3)^2 + (y-2)^2 = 6$

center $= (-3, 2)$

$d = \sqrt{(-3-1)^2 + (2+1)^2} = \sqrt{16+9} = 5$

**30. a.** $3x + 2y = 6$

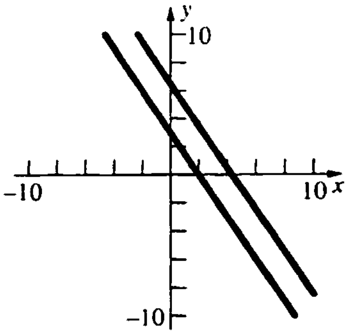
$2y = -3x + 6$

$y = -\frac{3}{2}x + 3$

$m = -\frac{3}{2}$

$y - 2 = -\frac{3}{2}(x - 3)$

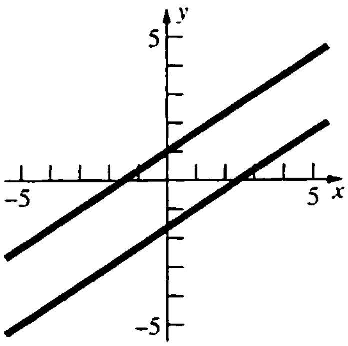
$y = -\frac{3}{2}x + \frac{13}{2}$



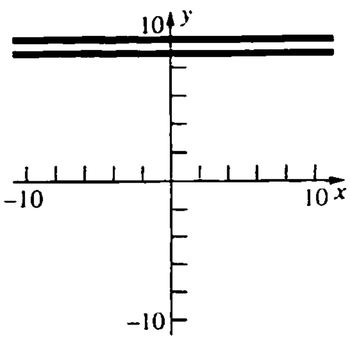

**b.** $m = \frac{2}{3};$

$y + 1 = \frac{2}{3}(x - 1)$

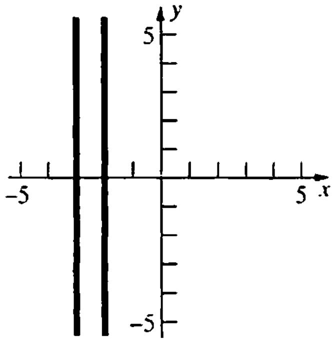
$y = \frac{2}{3}x - \frac{5}{3}$




**c.** $y = 9$




**d.** $x = -3$




**31. a.** $m = \frac{3-1}{7+2} = \frac{2}{9};$

$y - 1 = \frac{2}{9}(x + 2)$

$y = \frac{2}{9}x + \frac{13}{9}$

**b.** $3x - 2y = 5$

$-2y = -3x + 5$

$y = \frac{3}{2}x - \frac{5}{2};$

$m = \frac{3}{2}$

$y - 1 = \frac{3}{2}(x + 2)$

$y = \frac{3}{2}x + 4$

**c.** $3x + 4y = 9$

$4y = -3x + 9;$

$y = -\frac{3}{4}x + \frac{9}{4};\ m = \frac{4}{3}$

$y - 1 = \frac{4}{3}(x + 2)$

$y = \frac{4}{3}x + \frac{11}{3}$

**d.** $x = -2$

**e.** contains (–2, 1) and (0, 3); $m = \frac{3-1}{0+2};$

$y = x + 3$

**32.** $m_1 = \frac{3+1}{5-2} = \frac{4}{3}; m_2 = \frac{11-3}{11-5} = \frac{8}{6} = \frac{4}{3};$

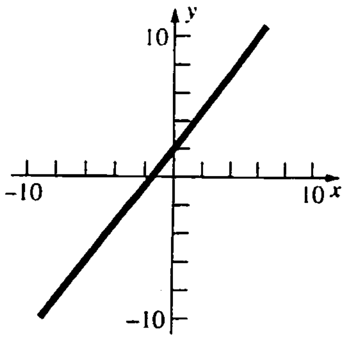
$m_3 = \frac{11+1}{11-2} = \frac{12}{9} = \frac{4}{3}$

$m_1 = m_2 = m_3$, so the points lie on the same line.

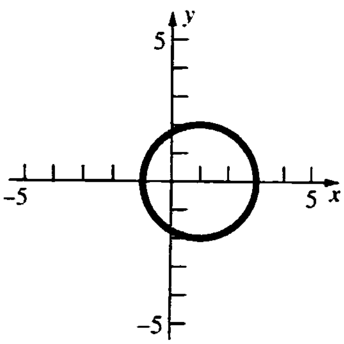
33. The figure is a cubic with respect to $y$. The equation is **(b)** $x = y^3$.

34. The figure is a quadratic, opening downward, with a negative $y$-intercept. The equation is **(c)** $y = ax^2 + bx + c$, with $a < 0$, $b > 0$, and $c < 0$.

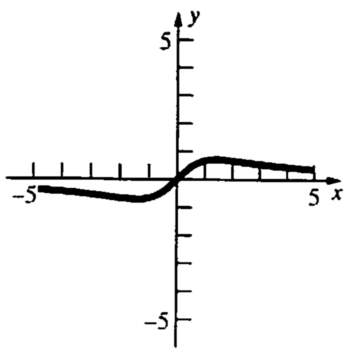
35.



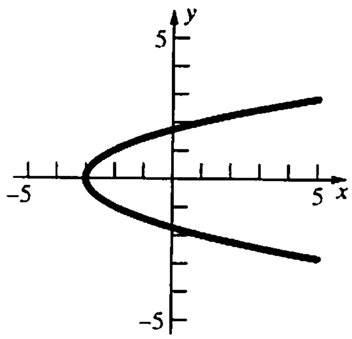

36.
$$x^2 - 2x + y^2 = 3$$
$$x^2 - 2x + 1 + y^2 = 4$$
$$(x-1)^2 + y^2 = 4$$




37.




38.




39. $y = x^2 - 2x + 4$ and $y - x = 4$;
$$x + 4 = x^2 - 2x + 4$$
$$x^2 - 3x = 0$$
$$x(x-3) = 0$$
points of intersection: (0, 4) and (3, 7)

40. $4x - y = 2$
$$y = 4x - 2;$$
$$m = -\frac{1}{4}$$
contains $(a, 0), (0, b)$;
$$\frac{ab}{2} = 8$$
$$ab = 16$$
$$b = \frac{16}{a}$$
$$\frac{b-0}{0-a} = -\frac{b}{a} = -\frac{1}{4};$$
$$a = 4b$$
$$a = 4\left(\frac{16}{a}\right)$$
$$a^2 = 64$$
$$a = 8$$
$$b = \frac{16}{8} = 2; y = -\frac{1}{4}x + 2$$