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Complete Mathematics for Cambridge Secondary 1

1

Deborah Barton

Oxford excellence for Cambridge Secondary 1

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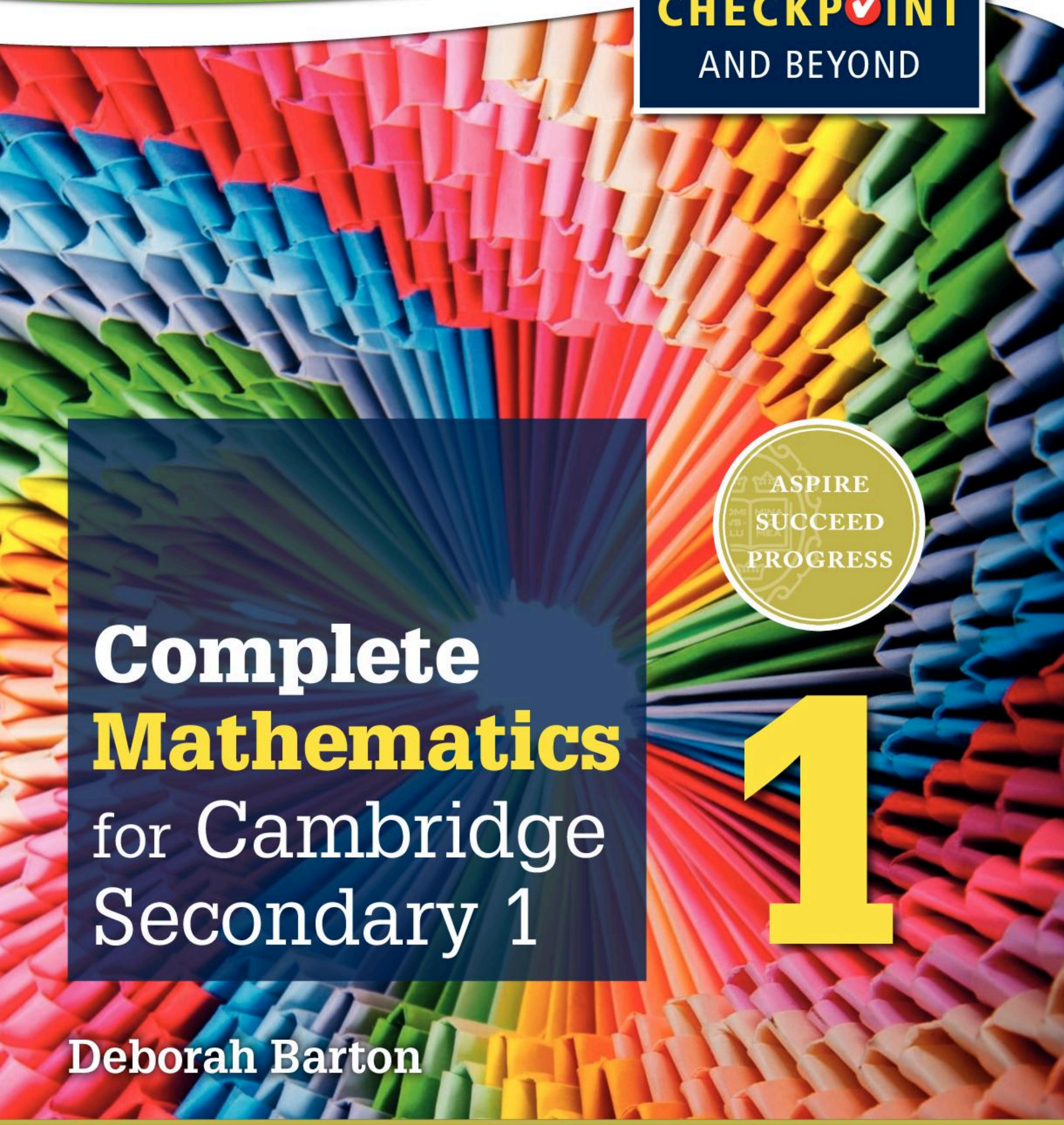

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Contents

About this book

This book follows the Cambridge Secondary 1 Mathematics curriculum framework for Cambridge International Examinations in preparation for the Checkpoint assessments. It has been written by a highly experienced teacher, examiner and author .

This book is part of a series of nine books. There are three student textbooks each covering stages 7, 8 and 9 and three homework books written to closely match the textbooks, as well as a teacher book for each stage.

The books are carefully balanced between all the content areas in the framework: number, algebra, geometry, measure and handling data. Some of the questions in the exercises and the investigations within the book are underpinned by the final framework area: problem solving, providing a structure for the application of mathematical skills.

Features of the book:

- **Objectives** – from the Cambridge Secondary 1 framework.
- **What's the point?** – providing rationale for inclusion of topics in a real–world setting.
- **Chapter Check in** – to asses whether the student has the required prior knowledge.
- **Notes and worked examples** – in a clear style using accessible English and culturally appropriate material.
- **Exercises** – carefully designed to gradually increase in difficulty providing plenty of practice of techniques.
- **Considerable variation in question style** – encouraging deeper thinking and learning, including open questions.
- **Comprehensive practice** – plenty of initial questions practice followed by varied questions for stretch, challenge, cross–over between topics and links to the real world with questions set in context.
- **Extension questions** – providing stretch and challenge for students
  - questions with a box e.g. $\boxed{1}$ provide challenge for the average student
  - questions with a filled box e.g. 1 provide extra challenge for more able students.
- **Technology boxes** – direct to websites for review material, fun games and challenges to enhance learning.
- **Investigation and puzzle boxes** – providing extra fun, challenge and interest.
- **Full colour with modern artwork** – pleasing to the eye, more interesting to look at, drawing the attention of the reader.
- **Consolidation examples and exercises** – providing review material on the chapter.
- **Summary and Check out** – providing a quick review of chapter's key points aiding revision, enabling you to to assess progress.
- **Review exercises** – provided every six chapters with mixed questions covering all topics.
- **Bonus chapter** – the work from Chapter 19 is not in the Cambridge Secondary 1 Mathematics curriculum. It is in the Cambridge IGCSE® curriculum and is included to stretch and challenge more able students.

A note from the author:

If you don't already love maths as much as I do, I hope that after working through this book you will enjoy it more. Maths is more than just learning concepts and applying them. It isn't just about right and wrong answers. It is a wonderful subject full of challenges, puzzles and beautiful proofs. Studying maths develops your analysis and problem-solving skills and improves your logical thinking - all important skills in the workplace.

Be a responsible learner – if you don't understand something, ask or look it up. Be determined and courageous. Keep trying without giving up when things go wrong. No one needs to be 'bad at maths'. Anyone can improve with hard work and practice in just the same way sports men and women improve their skills through training. If you are finding work too easy, say. Look for challenges, then maths will never be boring.

Most of all, enjoy the book. Do the 'training', enjoy the challenges and have fun!

Deborah Barton

1 Number and calculation 1

Objectives

- Consolidate the rapid recall of number facts, including positive integer complements to 100, multiplication facts to 10 × 10 and associated division facts.
- Interpret decimal notation and place value; multiply and divide whole numbers and decimals by 10, 100 or 1000.
- Order decimals including measurements, changing these to the same units.
- Round whole numbers to the nearest 10, 100 or 1000 and decimals, including measurements, to the nearest whole number or one decimal place.
- Recognise negative numbers as positions on a number line and order, add and subtract positive and negative integers in context.
- Use the laws of arithmetic and inverse operations to simplify calculations with whole numbers.
- Use the order of operations, including brackets, to work out simple calculations.

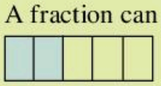
What's the point?

How many? Who has more? How much? Questions such as these led early man to develop number systems. Today, numbers are everywhere, from banks, to supermarkets, to airports.

Before you start

You should know ...

1 A fraction can be shown by a picture:



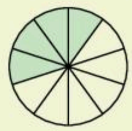
$\frac{2}{5}$ is shaded

$\frac{3}{10}$ is shaded

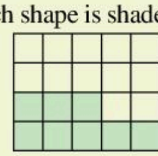
Check in

1 **a** What fraction of each shape is shaded?

i



ii



b Draw shapes to represent

i $\frac{2}{10}$ **ii** $\frac{16}{100}$

2 Your multiplication tables up to 10 × 10.

3 What is the value of the digit 4 in these numbers?

a 94 **b** 481 **c** 4012

a four
b four hundred
c four thousand

2 Write down the answers to:

a 2 × 7 **b** 3 × 6
c 8 × 4 **d** 5 × 4
e 9 × 7 **f** 9 × 8

3 Write down the value of the 4 in:

a 24 **b** 42
c 402 **d** 645
e 4132 **f** 49 206
g 14 873

1.1 Number facts

An important skill in mathematics is to be able to do calculations with numbers without a calculator. We can use **complements** to help us. Addition and subtraction are inverse operations. Sometimes a subtraction problem can be turned into an addition problem to make it easier and faster to do.

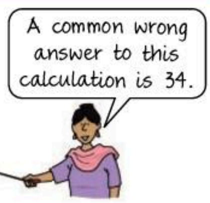
EXAMPLE 1

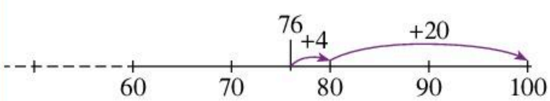
100 − 76 = □

If we change it to

76 + □ = 100

We can then use



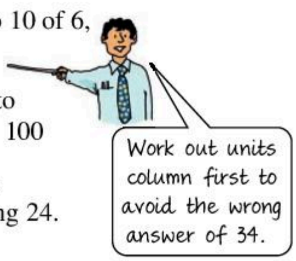


4 is the complement to 10 of 6, 76 + 4 = 80.

20 is the complement to 100 of 80, 80 + 20 = 100

Adding 4, then adding 20 is the same as adding 24.

76 + 24 = 100



Exercise 1A

Work out all of the following without a calculator. You could race a friend to see who is faster and more accurate.

1 **a** 39 + □ = 100 **b** 62 + □ = 100
c 41 + □ = 100 **d** 100 − 28
e 100 − 34 **f** 100 − 16
g 100 − 82

2 **a** 5 + 14 **b** 17 + 5 **c** 30 − 16
d 27 − 19 **e** 68 + 23 **f** 58 + 34
g 93 − 48 **h** 76 − 28

3 **a** \$22 + □ = \$100 **b** 43m + □ = 100m
c \$14 + □ = \$100 **d** \$100 − \$55
e 100cm − 91cm **f** 100kg − 4kg

4 Match the complements to 100 to each other. The first is done for you.

17	69
41	59
68	42
27	83
58	32
31	73

You can use the same method to do complements to 1000.

5 **a** 240 + □ = 1000 **b** 76 + □ = 1000
c 318 + □ = 1000 **d** 1000 − 180
e 1000 − 840 **f** 1000 − 293
g 1000 − 96 **h** 1000 − 544

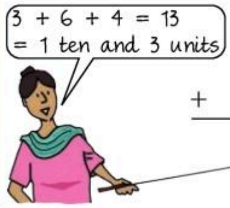
1.2 Adding and subtracting numbers

Sometimes you may want to use column addition and subtraction if calculations are harder.

To add numbers you must line up their place values.

EXAMPLE 2

What is 53 + 6 + 204?



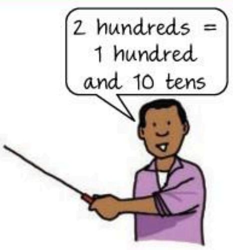
	H	T	U
		5	3
			6
+	2	0_1	4
	2	6	3

Subtraction is done in a similar way.

EXAMPLE 3

Work out 205 − 34.

	H	T	U
	$^{1}2$	$^{1}0$	5
−		3	4
	1	7	1



Exercise 1B

1 Add these numbers:

a 13 + 27 + 46
b 162 + 39
c 615 + 34 + 143
d 1068 + 39 + 7 + 214

2 Work out:

a 125 − 29 **b** 269 − 158
c 463 − 258 **d** 452 − 168
e 227 − 132 **f** 1101 − 990

3 At Market School there are 93 students in Form 1, 105 in Form 2, 87 in Form 3, 79 in Form 4 and 81 in Form 5. How many students are there altogether?

4 Orji wants to buy a new bicycle for $1000. He has saved $824. How much more does he need?

5 In 3 test cricket tournaments in 2010 Sachin Tendulkar scored 214 runs, 203 runs and 146 runs. How many runs did he score altogether?

6 18 years ago Chris was 5 years old. How old will he be in 16 years' time?

1.3 Multiplication and associated division facts

It is very important to know the multiplication tables up to 10 × 10. This will help you with many harder calculations.

The multiplication grid below looks like a lot of facts to remember. It can be made easier.

×	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
1	1	2	3	4	5	6	7	8	9	10
2	2	4	6	8	10	12	14	16	18	20
3	3	6	9	12	15	18	21	24	27	30
4	4	8	12	16	20	24	28	32	36	40
5	5	10	15	20	25	30	35	40	45	50
6	6	12	18	24	30	36	42	48	54	60
7	7	14	21	28	35	42	49	56	63	70
8	8	16	24	32	40	48	56	64	72	80
9	9	18	27	36	45	54	63	72	81	90
10	10	20	30	40	50	60	70	80	90	100

The following facts will help you:

- The order you multiply doesn't matter.
  7 × 8 is the same as 8 × 7. This halves the number of facts to learn.
- Multiplying a number by 1 doesn't change the number.
  e.g. 4 × 1 = 4
- Multiplying a number by 10 means you can just write a zero after the number.
  e.g. 7 × 10 = 70
- Multiplying by 2 is the same as doubling the number.

Now there are fewer multiplication facts to learn:

×	1	2	3	4	5	6	7	8	9	10
1										
2										
3			9							
4			12	16						
5			15	20	25					
6			18	24	30	36				
7			21	28	35	42	49			
8			24	32	40	48	56	64		
9			27	36	45	54	63	72	81	
10										

There are some other hints that can help if you find your multiplication tables difficult.

4's made easy:

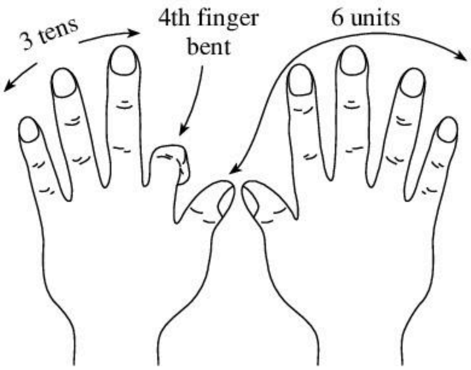
- Double a number and then double it again.

5's made easy:

- Multiply a number by 10 then halve it.
  For example, for 8×5, do $8 \times 10 = 80$,
  so $8 \times 5 = 40$ (half of 80).

9's made easy:

- Hold your hands in front of you with your fingers spread out.
- For 9×4 bend your fourth finger from the left down. (9×7 would be the seventh finger etc.)
- You have 3 fingers in front of the bent finger (the tens) and 6 after the bent finger (the units).
  So $9 \times 4 = 36$.



$9 \times 4 = 36$

Now there are very few to learn. Usually $3 \times 3 = 9$ is done well. The red numbers in the table are the ones that many students find harder. There are only nine red numbers!

×	3	6	7	8
3	9			
6	18	36		
7	21	42	49	
8	24	48	56	64

Once you have learned your multiplication facts you need to remember that multiplication and division are inverse operations. Sometimes a division problem can be turned into a multiplication problem to make it easier and faster to do.

$56 \div 8 = \square$ can be changed to: $\square \times 8 = 56$.
Then you can use the multiplication tables in reverse.
$7 \times 8 = 56$

Exercise 1C

1 Copy and complete this mixed-up tables grid as fast as you can.

×	4	6	9	2	7	3	5	8
7								
2								
6								
8								
3								
5								
9								
4								

2 Work out:

a 4×8 **b** $63 \div 9$ **c** 4×7
d $36 \div 4$ **e** 3×6 **f** $21 \div 3$
g 8×3 **h** $48 \div 6$ **i** 6×6
j 7×7 **k** $42 \div 7$ **l** $64 \div 8$

3 Jane earns \$9 an hour. If she works for 9 hours how much will she earn?

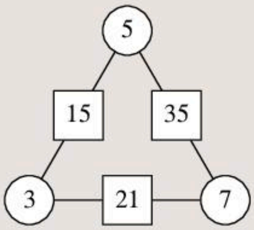
4 If Wayan shares \$45 between his 5 children, how much do they each get?

5 Learning more multiplication facts can speed up your working. Try these (they go up to 12×12.) Work out:

a 11×9 **b** $96 \div 12$ **c** $88 \div 8$
d 12×7 **e** 11×11 **f** $132 \div 11$
g 12×12 **h** $120 \div 12$

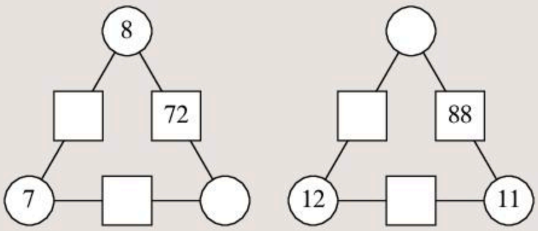
6 A teacher organising a school trip has 108 students to split into 12 equal groups. How many students will there be in each group?

7

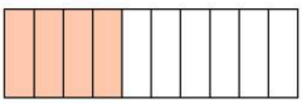


In this example, the numbers in the squares are the product of the numbers in the circles.

Find the missing numbers from the diagrams below:



1.4 Decimals



$\frac{4}{10}$ of the rectangle is shaded.

Tenths can be written a shorter way:

$\frac{4}{10}$ = 0 . 4

no units

decimal point

4 tenths

0.4 is the **decimal form** for the fraction $\frac{4}{10}$.

The · is called the **decimal point**.

It separates the whole numbers from the tenths.

EXAMPLE 4

Write as decimals

a $2\frac{3}{10}$ **b** $6\frac{7}{10}$

a $2\frac{3}{10} = 2.3$

b $6\frac{7}{10} = 6.7$

Exercise 1D

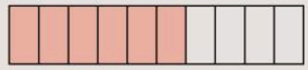
1 Write these fractions in decimal form.

a $\frac{3}{10}$ **b** $\frac{6}{10}$ **c** $\frac{9}{10}$ **d** $1\frac{2}{10}$

e $2\frac{8}{10}$ **f** $3\frac{4}{10}$ **g** $12\frac{1}{10}$ **h** $132\frac{5}{10}$

2 Write the part of each shape that is shaded – first as a fraction, then as a decimal:

a



b

3 Write in order of size, smallest first.

a 0.3, 0.6, 0.2, 0.8

b 0.9, 0.5, 1, 0

c 2.1, 0.6, 0.9, 1.8

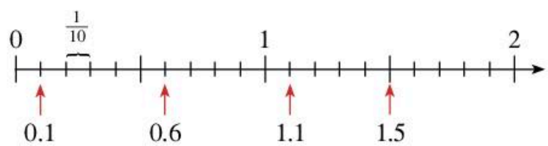
$\boxed{4}$ What number is $\frac{1}{10}$ more than

a 0.3 **b** 0.7 **c** 0.9

d 1 **e** 2.9 **f** 6?

Representing decimals on a number line

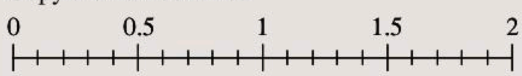
This number line has been divided into tenths.



The distance between each division is *one* tenth.

Exercise 1E

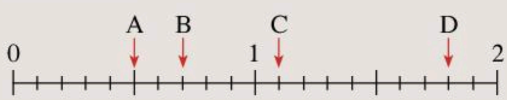
1 Copy the number line:



Show these numbers on your number line.

a 0.3 **b** 1.3 **c** 1.9

2 Write as decimals the letters marked on the number line.

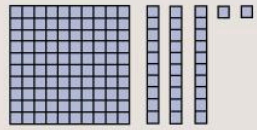


3 What is the temperature, in °C, on these thermometers?

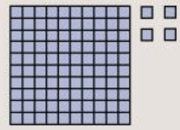
a

b

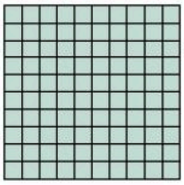
e



f



Two decimal places

If  represents 1 unit

then  represents $\frac{1}{10}$ (one tenth)

and  represents $\frac{1}{100}$ (one hundredth)

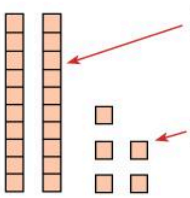
Hundredths can be written a shorter way as decimals:

$\frac{1}{100}$ = 0 . 0 1

1 hundredth
no units
no tenths
decimal point

EXAMPLE 5

Show the decimal 0.25 using diagrams



$\frac{2}{10} = 0.2$

$\frac{5}{100} = 0.05$

0.25

0.25

Exercise 1F

1 Use diagrams to show these decimals.

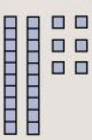
a 0.2 **b** 0.07 **c** 0.12
d 0.36 **e** 1.2 **f** 1.23

2 What decimals do these pictures represent?

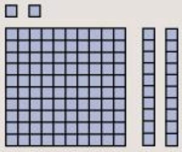
a

b

c



d



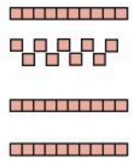
Diagrams are very useful when comparing decimals.

EXAMPLE 6

Which is the larger decimal 0.19 or 0.2?

$0.19 = \frac{1}{10} + \frac{9}{100}$

$0.2 = \frac{2}{10}$



The rectangles show that 0.2 is larger.

Exercise 1G

1 Which is larger?

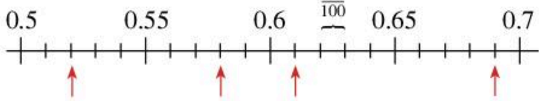
a 0.4 or 0.39 **b** 0.06 or 0.1
c 1.2 or 1.13 **d** 1.02 or 1.3
e 2 or 1.64

2 Write down four numbers that are

a bigger than 3 but less than 4
b bigger than 3.4 but less than 3.6
c bigger than 0.6 but less than 0.7

Representing hundredths on a number line

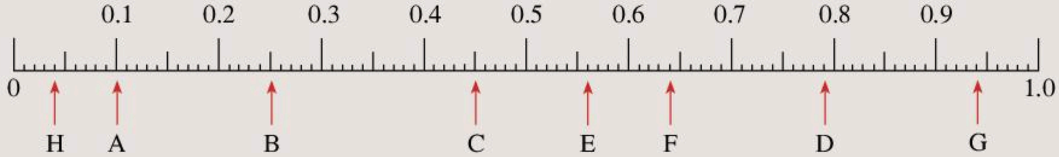
You can divide a number line into hundredths to show two place decimals.
The number line between 0.5 and 0.7:



The distance between each division is one hundredth.

Exercise 1H

1 Write down the positions of each of the letters on this number line:



2 Copy this number line:

On your number line show the points:

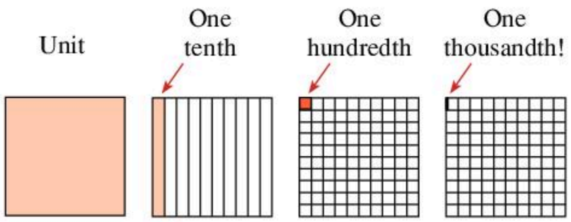
A = 0.2, B = 0.25, C = 0.45, D = 0.40, E = 0.30, F = 0.05

Thousandths

Each column heading is one tenth of the one to its left:

$$\text{Th} \xrightarrow{\div 10} \text{H} \xrightarrow{\div 10} \text{T} \xrightarrow{\div 10} \text{U}$$

What happens if you divide the units column by 10?



$$1 \xrightarrow{\div 10} \tfrac{1}{10} \xrightarrow{\div 10} \tfrac{1}{100} \xrightarrow{\div 10} \tfrac{1}{1000}$$

The column to the right of $\frac{1}{100}$ column is $\frac{1}{1000}$ column.

Thousandths can be written as decimals:

$\frac{1}{1000}$ = 0 . 0 0 1

no units, no tenths, no hundredths, 1 thousandth

1.5 Decimals and place value

Column headings are often used to show place value.

For example:

The number 384.615 written with column headings is

H	T	U	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$
3	8	4	6	1	5

EXAMPLE 7

Give the value of each underlined digit.

a $0.\underline{2}7$ **b** $6.13\underline{4}$ **c** $\underline{2}8.1$

a Two tenths or $\frac{2}{10}$ or 0.2

b Four thousandths or $\frac{4}{1000}$ or 0.004

c Two tens or 20

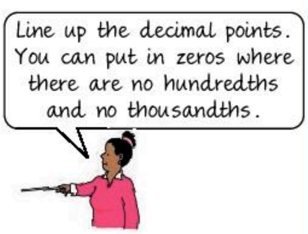
You can use place value to help you write decimals in order of size.

EXAMPLE 8

Write these numbers in order of size, smallest first

6.3, 6.304, 6.24, 6.242, 6.2, 6.34

6.3 → 6.300
6.304 → 6.304
6.24 → 6.240
6.242 → 6.242
6.2 → 6.200
6.34 → 6.340



In order, smallest first, this is: 6.2, 6.24, 6.242, 6.3, 6.304, 6.34

Exercise 1I

1 Give the value of each underlined digit.
- **a** $\underline{1}3.3$ **b** $6\underline{2}.4$ **c** $12.8\underline{4}$
- **d** $0.03\underline{2}$ **e** $1.24\underline{5}$ **f** $13.\underline{4}6$
- **g** $11.8\underline{0}4$

2 Write these numbers in order of size, smallest first.
- **a** 0.3, 0.02, 0.4, 0.006
- **b** 3, 30, 0.3, 0.003
- **c** 36, 3.6, 0.36, 0.036

3 Write down four numbers that lie between:
- **a** 3.14 and 3.15
- **b** 0.68 and 0.69

4 The batting averages of five international batsmen are shown in this table:

Kallis	54.66
Tendulkar	54.72
Gambhir	56.00
Hussey	53.04
Ponting	55.67

Put them in order, starting with the highest average.

5 Write these numbers in order of size, smallest first
- **a** 12.5, 14, 6.35, 10.2, 8.323, 0.099
- **b** 2.44, 2.5, 2.501, 2.41, 2.412, 2.4
- **c** 0.4, 0.04, 0.44, 0.444, 0.404, 0.044

6 Write these numbers in order of size, largest first
- **a** 1, 0.9, 1.19, 0.119, 1.119, 1.1
- **b** 7.703, 7.7, 7.65, 7.651, 7.61, 7.73
- **c** 12.1, 12.101, 12.03, 12.042, 12.01, 12.11

Decimals and money

Money is written using decimals.

For example:
2 dollars and 25 cents is written \$2.25

39 cents is written \$0.39

3 dollars and 7 cents is written \$3.07

We usually don't write zeros at the end of decimals. 1.2 means 1 ten and 2 tenths. This can be written as 1.20 (as there are no hundredths). When you are talking about money you need to have two numbers after the decimal point. An answer of 1 dollar and 20 cents on a calculator will be 1.2, but you must write it as \$1.20 not \$1.2.

Exercise 1J

1 **a** How many cents are there in a dollar?
- **b** Write 1 cent as a fraction of 1 dollar.
- **c** Write this fraction as a decimal.

2 **a** Write 15 cents as a fraction of a dollar.
- **b** Write this fraction as a decimal.

3 Write in dollars using the decimal point:
- **a** 1 dollar 10 cents
- **b** 25 cents
- **c** 3 dollars 55 cents
- **d** 5 dollars 5 cents
- **e** 9 cents
- **f** 100 dollars 5 cents
- **g** 54 dollars 13 cents
- **h** 1 dollar 50 cents

1.6 Decimals and your calculator

Decimal calculations are done very easily on a calculator. To display a decimal you must use the $\boxed{\cdot}$ button.

Exercise 1K

1 Use your calculator to work out:
- **a** 10×8.4 **b** 10×1.07
- **c** 10×6.045 **d** 100×8.4
- **e** 100×1.07 **f** 100×6.045
- **g** 1000×8.4 **h** 1000×1.07
- **i** 1000×6.045

2 What happens when you multiply by 10, 100 or 1000?

3 Use your calculator to work out:
- **a** $7.6 \div 10$ **b** $26.1 \div 10$
- **c** $48 \div 10$ **d** $523 \div 10$

4 Use your calculator to work out:
- **a** $7.6 \div 100$ **b** $26.1 \div 100$
- **c** $48 \div 100$ **d** $523 \div 100$

5 Use your calculator to work out:
- **a** $7.6 \div 1000$ **b** $26.1 \div 1000$
- **c** $48 \div 1000$ **d** $523 \div 1000$

6 What happens when you divide by 10, 100 or 1000?

7 Do these multiplications, without using a calculator.

a	7.45×10	**b**	8.9×10
c	0.34×100	**d**	3.04×1000
e	0.06×10	**f**	1.006×100
g	18.4×1000	**h**	21.63×10

8 Do these divisions without using a calculator.

a	$7.8 \div 10$	**b**	$9.2 \div 100$
c	$27.3 \div 100$	**d**	$59 \div 1000$
e	$947 \div 10$	**f**	$31.5 \div 100$
g	$2 \div 1000$	**h**	$1.3 \div 1000$
i	$37 \div 10$		

1.7 Multiplying and dividing decimals by powers of ten

Look at these results:

$8.4 \times 1 = 8.4$
$8.4 \times 10 = 84$
$8.4 \times 100 = 840$
$8.4 \times 1000 = 8400$

You should have noticed that when a decimal is **multiplied** by 10, each number in the decimal moves one place to the **left**.

In 8.4 the 8 is in the units column

$8.4 \times 10 = 84$

In this calculation the 8 moves one place to the left and is now in the tens column.

When **multiplied** by 100 each number moves **two** places to the **left**.

When multiplied by 1000 each number moves **three** places to the **left**.

Look at these results.
$7.6 \div 1 = 7.6$
$7.6 \div 10 = 0.76$
$7.6 \div 100 = 0.076$
$7.6 \div 1000 = 0.0076$

Notice here that when a decimal is **divided** by 10 each number in the decimal moves **one** place to the **right**.

When **divided** by 100 each number moves **two** places to the right. When **divided** by 1000 each number moves **three** places to the right.

EXAMPLE 9

Work out:

a 83.75×10 **b** $83.75 \div 100$
c 8.375×1000

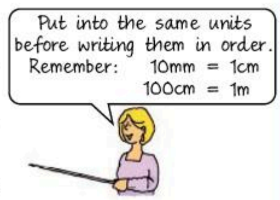
a $83.75 \times 10 = 837.5$
b $83.75 \div 100 = 0.8375$
c $83.75 \times 1000 = 83750$

Sometimes you will be asked to write numbers in order when they have different units. If there are units given, change all numbers to the same unit before ordering them.

EXAMPLE 10

Write these numbers in order of size, smallest first:
1 m, 119 cm, 0.99 m, 1.2 m, 98 cm, 112 mm

$1\text{ m} \times 100 \rightarrow 100\text{ cm}$
$119\text{ cm} \rightarrow 119\text{ cm}$
$0.99\text{ m} \times 100 \rightarrow 99\text{ cm}$
$1.2\text{ m} \times 100 \rightarrow 120\text{ cm}$
$98\text{ cm} \rightarrow 98\text{ cm}$
$112\text{ mm} \div 10 \rightarrow 11.2\text{ cm}$



In order, smallest first, this is: 112mm, 98cm, 0.99m, 1m, 119cm, 1.2m

Exercise 1L

1 Work out without a calculator:

a	6.35×10	**b**	3×100
c	2.6×1000	**d**	71.4×10
e	8.2×100	**f**	1.89×1000
g	0.318×10	**h**	0.34×100
i	0.771×1000	**j**	76×10
k	6.125×100	**l**	4×1000

2 Multiply each of the numbers below by
i 10
ii 100
iii 1000

a 9.843 **b** 16 **c** 0.14

3 Work out without a calculator:

a	$8.1 \div 10$	**b**	$2 \div 100$
c	$8.1 \div 1000$	**d**	$53.7 \div 10$
e	$1.5 \div 100$	**f**	$0.83 \div 1000$
g	$0.014 \div 10$	**h**	$0.341 \div 100$
i	$5.762 \div 1000$	**j**	$176 \div 10$
k	$7.165 \div 100$	**l**	$7 \div 1000$

4 Divide each of the numbers below by
 i 10
 ii 100
 iii 1000
 a 46.2 b 7.08 c 314

5 Write in cents:
 a \$6.42 b \$19.06 c \$247.11

6 Write in dollars:
 a 45 cents b 6 cents c 137 cents

7 Copy and complete:
 a $1.6 \times \square = 16$
 b $\square \times 1000 = 2300$
 c $\square \div 100 = 0.442$
 d $8.9 \div \square = 0.089$
 e $0.09 \times \square = 90$
 f $0.6 \times \square = 6$
 g $1.8 \div \square = 0.18$
 h $921.9 \div \square = 0.9219$
 i $83.8 \times \square = 8380$
 j $\square \times 100 = 614.4$
 k $\square \div 1000 = 0.071$
 l $25.12 \div \square = 2.512$

8 Eisa has made a mistake in his working.
He says these numbers are in order starting with the smallest.
2.7 cm, 2.71 mm, 2.83 m, 3 cm
What mistake has he made?

9 Copy and complete:
 a 4 km = □ m
 b 2.3 kg = □ g
 c 230 ml = □ litre
 d 0.8 m = □ cm
 e 22 g = □ kg
 f 26 mm = □ cm
 g 0.04 litres = □ ml
 h 350 m = □ km
 i 920 cm = □ mm

Remember:
10 mm = 1 cm
100 cm = 1 m
1000 m = 1 km
1000 g = 1 kg
1000 ml = 1 litre

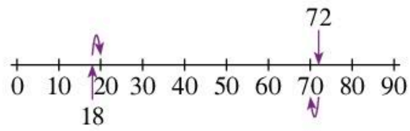
10 Write these numbers in order of size, smallest first:
 a 8810 m, 8.9 km, 8901 m, 8.821 km, 8812 m, 8.8 km
 b 10.8 kg, 10801 g, 10.79 kg, 10792 g, 10781 g, 10.81 kg
 c 0.66 litre, 0.6 litre, 0.06 litre, 666 ml, 0.606 litre, 66 ml
 d 4.1 cm, 401 mm, 4.01 cm, 401 cm, 4.01 mm, 4.1 mm, 0.04 m

1.8 Rounding

Often you do not need to work with exact numbers. For example, if 13 284 people attended a cricket match, you can say about 10 000 people attended. Such an **approximation** is called **rounding**.

You can round numbers to the nearest 10, 100, 1000 etc.

Look at the number line below.



It shows multiples of 10.

Notice that 18 is closest to 20.
Notice that 72 is closest to 70.

You can write

 18 is 20 to the nearest 10

 72 is 70 to the nearest 10

When you round to the nearest 10 the last digit must be 0

18 has been rounded up to 20.

72 has been rounded down to 70.

In the case of a number ending in 5, for example 65, it is usual to round it up, so

 65 is 70 to the nearest 10

Rounding to the nearest 100 is done in a similar manner.

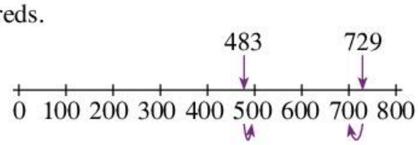
EXAMPLE 11

Round

a 729 **b** 483

to the nearest hundred.

Show the numbers on a number line marked in hundreds.



From the number line, you can see
483 is 500 to the nearest 100
729 is 700 to the nearest 100

When you round to the nearest 100 the last two digits must be 0

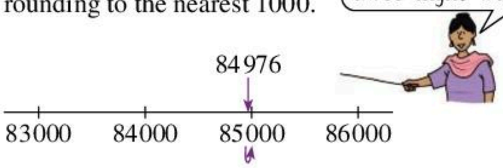
Rounding is commonly used by the media.

At Eden Gardens cricket ground in Kolkata, India, 84 976 people went to watch a cricket match. A newspaper reported

85 000 people watched Kolkata Knight Riders' victory.

The paper has made the figure of 84 976 more manageable by rounding to the nearest 1000.

When you round to the nearest 1000 the last three digits must be 0



Exercise 1M

1 **a** Draw a number line from 0 to 100 marked in tens.
b Show the following numbers on your line:
i 8 **ii** 27 **iii** 42 **iv** 63 **v** 89
c Round each of the numbers to the nearest ten.

2 **a** Copy the number line below.

250 260 270 280 290 300 310 320

b Show these numbers on your line:
i 283 **ii** 255 **iii** 292 **iv** 306 **v** 319
c Round each of these numbers to the nearest 10.

3 **a** Draw a number line from 100 to 1000 marked in hundreds.
b Show these numbers on your line:
i 164 **ii** 375 **iii** 604 **iv** 429 **v** 781
c Round each of these numbers to the nearest 100.

4 **a** Copy the number line below.

1800 1900 2000 2100 2200 2300 2400

b Show these numbers on your line:
i 2003 **ii** 1917 **iii** 2189
iv 2362 **v** 2240
c Round each of these numbers to the nearest 100.

5 By drawing a suitable number line, round each of these numbers to the nearest 1000.
a 1402 **b** 3812 **c** 5617
d 949 **e** 33 609 **f** 45 113

6 Round these numbers to the nearest 10:
a 76 **b** 105 **c** 846 **d** 1048
e 6142 **f** 7624 **g** 11 306 **h** 12953

7 Round the numbers in Question **6** to the nearest 100.

8 Round the numbers in Question **6** to the nearest 1000.

9 Round the answers to these calculations to the nearest 100. You may use your calculator.
a 27×146 **b** 136×97 **c** 812×124

10 Ethan has made some mistakes in his homework. Which questions are wrong? Write the correct answers for any he has got wrong.
a 436 789 to the nearest 1000 is 44 000
b 2354 to the nearest 10 is 2350
c 712 350 to the nearest 100 is 712 300

11

In his test match career to December 2011, Rahul Dravid has scored 12752 runs. How many runs is this to the nearest
a ten **b** hundred **c** thousand?

12 649 is the largest whole number that when rounded to the nearest hundred gives 600. What is the smallest such number?

13 Hind has rounded the number of people in her year group to the nearest 10. She says there are 230 in her year group. What is
a the smallest possible number of students in her year group
b the largest possible number of students in her year group?

Rounding decimals

We can use a similar idea to round to the nearest whole number or to one decimal place.

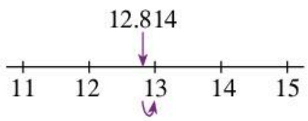
EXAMPLE 12

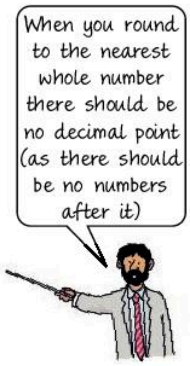
Round 12.814 to

a the nearest whole number

b one decimal place

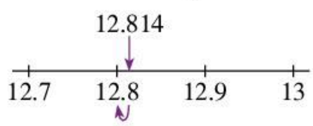
a Show the numbers on a number line marked in whole numbers.

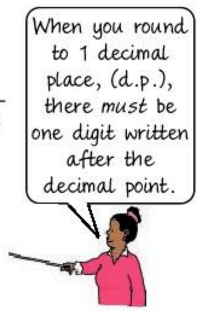




From the number line, you can see 12.814 is 13 to the nearest whole number

b Show the numbers on a number line marked in decimals showing tenths.





From the number line, you can see 12.814 is 12.8 to 1 d.p.

Rounding without a number line

To round to 1 decimal place you only need to look at the second decimal place (the number in the hundredths column). Use this rule:

- If the second decimal place is less than 5, leave the first decimal place as it is and don't write any more numbers after it. If it is 5 or more, you must round up the digit in the first decimal place.

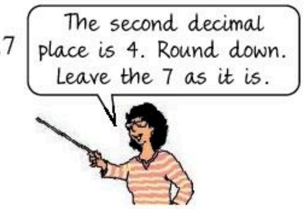
EXAMPLE 13

Round to 1 decimal place

a 6.741 **b** 8.35 **c** 7.968

a $6.7\underline{4}1$

6.741 to 1 d.p. is 6.7



b $8.3\underline{5}$

8.35 to 1 d.p. is 8.4

c $7.9\underline{6}8$

7.968 to 1 d.p. is 8.0

In part **c** of Example 13, when you increase the digit 9 by adding 0.1 you get 8.0. You must leave the 0 in. Don't just write 8 as this would be rounded to the nearest whole number instead of to 1 d.p.

Exercise 1N

1 Round to the nearest whole number:

a 12.356 **b** 4.8 **c** 11.096
d 0.467 **e** 0.5

2 Round to 1 decimal place:

a 0.45 **b** 2.148 **c** 0.05
d 6.249 **e** 32.092

3 Round to **i** the nearest whole number and **ii** to 1 d.p.

a 0.6721 **b** 4.349 **c** 6.53213
d 41.283 **e** 0.05345

4 Round the following measurements to
i the nearest whole number
ii to 1 d.p.

a 4.75 km **b** 2.32 litres **c** 17.814 kg
d 23.15 cm **e** 403.447 tonnes

5 Maahes has rounded a number. He says: "When I round my number to 1 d.p it is 5.5 and it is 5 to the nearest whole number." What could Maahes's number be?

Estimation

Have you ever gone to buy things and found you didn't have enough money?

You can avoid this if you are able to make good **estimates**. When you round to the nearest ten, hundred, etc. you are making a good estimate. You can estimate the answer to a calculation by rounding the numbers first.

EXAMPLE 14

Estimate the result of

a $76 + 296 + 82$

b 69×84

---

a 76 is 100 (to nearest 100)
296 is 300 (to nearest 100)
82 is 100 (to nearest 100)
So $76 + 296 + 82$ is approximately
$100 + 300 + 100 = 500$

b 69 is 70 (to nearest 10)
84 is 80 (to nearest 10)
So 69×84 is approximately $70 \times 80 = 5600$

Exercise 10

1 Make good estimates for these calculations:

a $58 + 94 + 86$ **b** $213 + 789$

c $931 - 286$ **d** $1124 - 919$

e 29×104 **f** $814 \div 124$

2 The distance around a race track is 1.85 km. If a car drives 35 laps around the race track, estimate how far it has travelled in total.

3 A shirt costs \$79.55. Alan wishes to buy 14 such shirts. He has \$1000. Does he have enough money?

4

A motorbike is priced at \$847.50. Johnson has \$5000. How many motorbikes could he buy?

TECHNOLOGY

For further lessons and tests on rounding and estimations visit the website

www.aaaknow.com/est.htm

See if you can beat the clock when you play the 'Countdown' game or '20 Questions'.
Take care! Some of these are challenging!

Using a calculator

A calculator makes arithmetic easy and can save a great deal of time. However, unless you are careful it is still possible to make mistakes.

One way of reducing mistakes when you use your calculator is to make a good guess at the answer first. Your calculator answer should be similar to your guess.

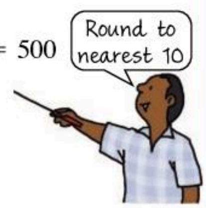
EXAMPLE 15

What is 52×8?

A good guess is about $50 \times 10 = 500$

$\boxed{5}\ \boxed{2}\ \boxed{\times}\ \boxed{8}\ \boxed{=}\ 416$

The answer 416 is fairly close to the guess of 500. It is unlikely that a mistake has been made.



Exercise 1P

Copy and complete the table. For each question make a sensible guess and then use your calculator.

Problem	Guess	Calculator
29×4		
$216 - 82$		
$76 + 42 + 95$		
$256 \div 8$		
96×98		
$966 \div 14$		
$4611 \div 87$		
103×37		
$2520 \div 96$		

INVESTIGATION

a Write down a three-figure number, for example 458

b Repeat the figures to get, for example, 458 458

c Divide this number by 13, then divide your answer by 11 and finally divide that answer by 7.

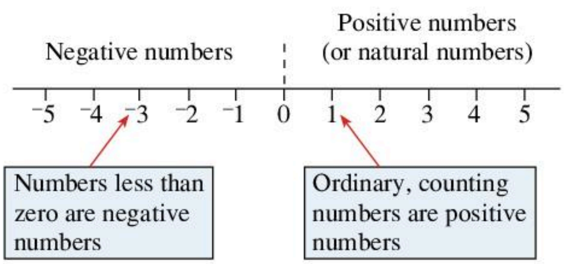
d What is your answer? What do you notice?

e Repeat with another three-figure number. What happens? Does this always happen? Why?

1.9 Negative numbers

Numbers less than zero are called **negative numbers**.

You can see them on this number line:



Numbers to the right on the number line are always larger than those on the left.

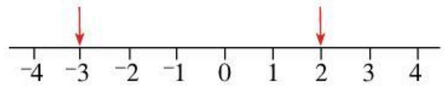
Integers are whole numbers that can be positive, negative or zero

EXAMPLE 16

Which is greater

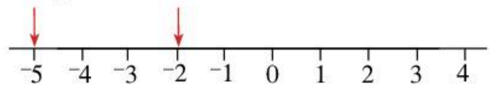
a 2 or $^{-}3$? **b** $^{-}2$ or $^{-}5$?

a



On the number line 2 is to the right of $^{-}3$, so 2 is greater than $^{-}3$

b



$^{-}2$ is to the right of $^{-}5$, so $^{-}2$ is greater than $^{-}5$

Exercise 1Q

1 Which of these statements are true?

a 5 is greater than 3
b 2 is greater than $^{-}1$
c $^{-}3$ is greater than 4
d $^{-}2$ is less than 3
e $^{-}3$ is less than $^{-}4$
f $^{-}9$ is less than $^{-}3$

2 Copy and complete by filling in the gap. The first is done for you.

a 10 is greater than 8 **b** $^{-}6$ ________ 8
c $^{-}5$ ________ 0 **d** 0 ________ 2
e 0 ________ $^{-}1$ **f** 2 ________ $^{-}2$
g $^{-}5$ ________ $^{-}1$ **h** $^{-}6$ ________ $^{-}11$

3 Pick the greater integer from each pair.

a 16, 200 **b** $^{-}17$, 86
c $^{-}30$, 25 **d** $^{-}57$, $^{-}70$
e $^{-}250$, 1 **f** $^{-}100$, $^{-}5$

4 Write each set of numbers in order of size, smallest first.

a 4, $^{-}3$, $^{-}2$
b 2, 4, $^{-}2$, $^{-}4$
c 0, $^{-}3$, 5, $^{-}2$, 4
d $^{-}3$, $^{-}2$, $^{-}5$, $^{-}6$
e $^{-}17$, 23, 5, $^{-}9$, $^{-}4$
f $^{-}8$, $^{-}10$, $^{-}6$, 4, $^{-}2$, 6

5 6 is one greater than 5. Write down the number that is one greater than:

a 4 **b** 0 **c** $^{-}3$ **d** $^{-}5$
e $^{-}7$ **f** $^{-}10$ **g** $^{-}12$ **h** $^{-}2$

6 6 is one less than 7. Write down the number that is one less than:

a 4 **b** 0 **c** $^{-}3$ **d** $^{-}5$

e $^{-}7$ **f** $^{-}10$ **g** $^{-}12$ **h** $^{-}29$

7 Copy and complete these sequences:

a 3, 2, 1, 0, _, _, _

b 5, 3, 1, _, _, _

c $^{-}4$, $^{-}2$, 0, _, _, _

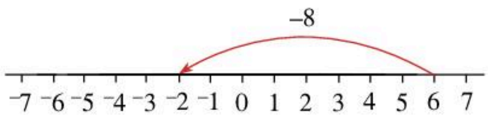
d $^{-}14$, $^{-}11$, $^{-}8$, _, _, _

e $^{-}19$, $^{-}14$, $^{-}9$, _, _, _

You can subtract larger numbers from smaller numbers using a number line.

EXAMPLE 17

Work out $6 - 8$.

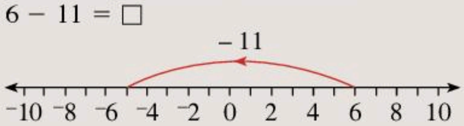


Start at 6 and move 8 steps back.
You will arrive at $^{-}2$.
That is, $6 - 8 = {}^{-}2$

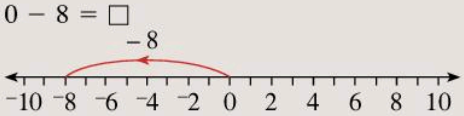
Exercise 1R

1 Copy and complete these subtractions:

a $6 - 11 = \square$



b $0 - 8 = \square$



2 On squared paper draw number lines and arrows to show these subtractions.
Write down the answer for each one.

a $4 - 5$ **b** $6 - 9$ **c** $1 - 3$

d $0 - 4$ **e** $6 - 8$ **f** $2 - 5$

3 Copy and complete these subtractions:

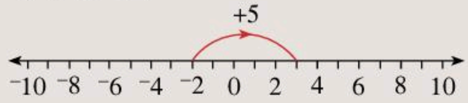
a $7 - 8 = \square$ **b** $1 - 6 = \square$

c $0 - 9 = \square$ **d** $5 - 6 = \square$

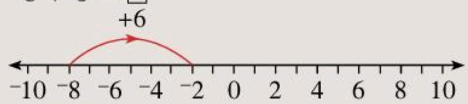
e $3 - 11 = \square$ **f** $4 - 8 = \square$

4 Copy and complete these additions:

a $^{-}2 + 5 = \square$



b $^{-}8 + 6 = \square$



5 Draw number lines to show these additions:

a $^{-}3 + 5$ **b** $0 + 6$

c $^{-}5 + 8$ **d** $^{-}9 + 2$

e $^{-}9 + 12$ **f** $^{-}8 + 14$

6 Copy and complete:

a $^{-}3 + 4 = \square$ **b** $^{-}5 + 3 = \square$

c $^{-}8 + 3 = \square$ **d** $^{-}12 + 4 = \square$

e $^{-}9 + 13 = \square$ **f** $^{-}17 + 21 = \square$

1.10 Negative numbers and addition

You will need squared paper.

You may have noticed that the order that you add numbers does not matter.

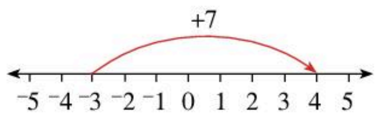
For example,

$$4 + 3 = 3 + 4$$

You can use this idea to help you add negative numbers.

To work out $7 + -3$

$7 + -3 = -3 + 7 = 4$

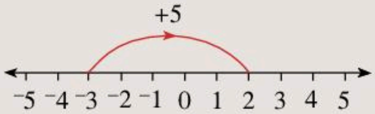


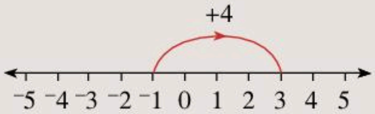
Exercise 1S

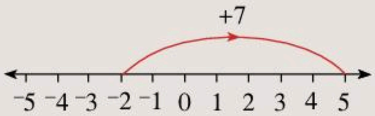
1 Write these additions in another way by changing the order of adding.

a $5 + {}^{-}3$ **b** $4 + {}^{-}1$ **c** $7 + {}^{-}2$

2 Look at the additions shown on these number lines.







Using the drawings to help you, copy and complete:

a $5 + {}^{-}3 = {}^{-}3 + 5 = \square$
b $4 + {}^{-}1 = \square + \square = \square$
c $7 + {}^{-}2 = \square + \square = \square$

3 Work out:

a $4 + {}^{-}2$ **b** $5 + {}^{-}4$
c $7 + {}^{-}5$ **d** $3 + {}^{-}5$
e $6 + {}^{-}8$ **f** $9 + {}^{-}12$

4 Copy and complete:

a $5 - 3 = \square$ **b** $4 - 1 = \square$
c $7 - 2 = \square$

5 The answers for Questions **2** and **4** are shown below.

$5 + {}^{-}3 = 2$ $5 - 3 = 2$
$4 + {}^{-}1 = 3$ $4 - 1 = 3$
$7 + {}^{-}2 = 5$ $7 - 2 = 5$

What do you notice by comparing both columns? Can you see a quick way to find the answer, when you add a negative number to a positive number?

6 Use the quick way from Question **5** to work out:

a $5 + {}^{-}2$ **b** $3 + {}^{-}1$
c $7 + {}^{-}4$ **d** ${}^{-}3 + 8$
e ${}^{-}6 + 9$ **f** ${}^{-}2 + 10$

7 Copy the sentence, and choose words from those in the brackets to complete it:
"Adding a negative number gives the same answer as ...... a ...... number."
(multiplying, larger, positive, subtracting, smaller, negative)

8 Work out:

a ${}^{-}10 + 12$ **b** ${}^{-}6 + 4$
c ${}^{-}1 + 8$ **d** ${}^{-}9 + 0$
e $5 + {}^{-}10$ **f** $7 + {}^{-}11$
g ${}^{-}15 + 10$ **h** ${}^{-}30 + 14$

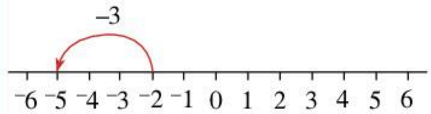
You can use number lines to add two negative numbers.

EXAMPLE 18

Work out: ${}^{-}2 + {}^{-}3$

$${}^{-}2 + {}^{-}3 = {}^{-}2 - 3$$

First, draw a number line to help.



So ${}^{-}2 + {}^{-}3 = {}^{-}2 - 3 = {}^{-}5$.

Exercise 1T

1 With the help of a number line, work out:

a ${}^{-}3 + {}^{-}2$ **b** ${}^{-}1 + {}^{-}4$
c ${}^{-}2 + {}^{-}2$ **d** ${}^{-}5 + {}^{-}2$
e ${}^{-}3 + {}^{-}5$ **f** ${}^{-}4 + {}^{-}3$
g ${}^{-}6 + {}^{-}5$ **h** ${}^{-}7 + {}^{-}6$
i ${}^{-}8 + {}^{-}6$ **j** ${}^{-}9 + {}^{-}9$

2 Look at your completed additions in Question **1**. Can you see a quick way to find the answer, when you add two negative numbers?

3 Work out:

a ${}^{-}6 + {}^{-}7$ **b** ${}^{-}10 + {}^{-}10$
c ${}^{-}7 + {}^{-}7$ **d** ${}^{-}9 + {}^{-}3$
e ${}^{-}2 + {}^{-}10$ **f** ${}^{-}15 + {}^{-}6$

4 Find the answer:

a ${}^{-}1 + {}^{-}19$ **b** ${}^{-}16 + {}^{-}10$
c ${}^{-}30 + {}^{-}30$ **d** ${}^{-}37 + {}^{-}63$
e ${}^{-}18 + {}^{-}22$ **f** ${}^{-}26 + {}^{-}99$

5 Copy and complete:

a $3 + \square = 5$ **b** $3 + \square = 3$
c $3 + \square = 1$ **d** $\square + 3 = 1$
e ${}^{-}3 + \square = 1$ **f** ${}^{-}3 + \square = {}^{-}1$
g $\square + {}^{-}3 = 5$ **h** $\square + {}^{-}3 = {}^{-}3$
i ${}^{-}5 + \square = {}^{-}2$ **j** $5 + \square = 2$

6 Work out:

a $12 + {}^{-}18 + 12$
b $4 + {}^{-}6 + 3$
c ${}^{-}20 + 10 + 35$
d $14 + {}^{-}20 + 30$
e $20 + {}^{-}5 + {}^{-}1$
f ${}^{-}12 + 19 + {}^{-}3$
g $8 + {}^{-}14 + 9$
h ${}^{-}11 + 6 + {}^{-}5$
i ${}^{-}11 + {}^{-}5 + 6$
j ${}^{-}6 + 11 + {}^{-}5$

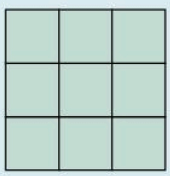
7 The temperature in New York was $^{-}5$°C. It rose by 9°C. What was the new temperature?

8 The sum of two numbers is $^{-}5$. What could the numbers be?
Find as many solutions as you can.
Compare your answers with a friend.
Who has more solutions?

9 Work out:

a $^{-}6 + 10 + {}^{-}7$
b $^{-}5 + {}^{-}4 + 6$
c $^{-}9 + {}^{-}6 + {}^{-}3$
d $11 + {}^{-}7 + {}^{-}3$
e $10 + {}^{-}4 + 13 + {}^{-}9$
f $16 + {}^{-}3 + {}^{-}18 + 3$
g $^{-}8 + {}^{-}17 + 4 + 6$

INVESTIGATION

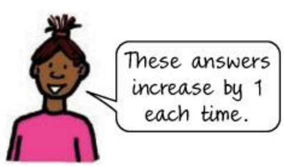


Copy and complete the magic square with the numbers $^{-}10$, $^{-}8$, $^{-}6$, $^{-}4$, $^{-}2$, 0, 2, 4, 6 so that all the rows, columns and diagonals add to the same number.

1.11 Subtracting negative numbers

Look at these subtractions:

$4 - 4 = 0$
$4 - 3 = 1$
$4 - 2 = 2$
$4 - 1 = 3$
$4 - 0 = 4$
$4 - {}^{-}1 = \square$



What do you think $\square$ should be? The answers are increasing by one so $\square = 5$.

That is,

$4 - {}^{-}1 = 5$

- This shows that **subtracting a negative number** is the same as **adding a positive number**.

EXAMPLE 19

Work out

a $6 - {}^{-}3$ **b** $^{-}4 - {}^{-}2$

a $6 - {}^{-}3 = 6 + 3$
$= 9$

b $^{-}4 - {}^{-}2 = {}^{-}4 + 2$
$= {}^{-}2$

You can use the same idea to work out harder calculations.

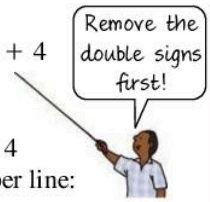
EXAMPLE 20

Work out

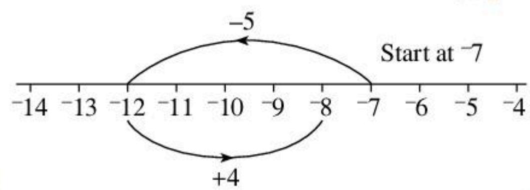
a $^{-}6 - {}^{-}3 + 4$ **b** $^{-}7 + {}^{-}5 - {}^{-}4$

a $^{-}6 - {}^{-}3 + 4 = {}^{-}6 + 3 + 4$
$= {}^{-}6 + 7$
$= 1$

b $^{-}7 + {}^{-}5 - {}^{-}4 = {}^{-}7 - 5 + 4$
$= {}^{-}7 - 1$
$= {}^{-}8$



Notice that in part **b** $^{-}7 - 5 + 4$ can be worked out on the number line:



Exercise 1U

1 Copy and complete.
The first one has been done for you.

a $10 - {}^{-}7 = 10 + 7 = 17$
b $3 - {}^{-}6 = 3 + \square = \square$
c $^{-}2 - {}^{-}2 = {}^{-}2 + \square = \square$
d $0 - {}^{-}6 = 0 + \square = \square$
e $^{-}5 - {}^{-}3 = {}^{-}5 + \square = \square$

2 Work out:

a $4 - {}^{-}3$ **b** $10 - {}^{-}2$
c $5 - {}^{-}2$ **d** $11 - {}^{-}1$
e $3 - {}^{-}8$ **f** $2 - {}^{-}4$
g $^{-}3 - {}^{-}4$ **h** $^{-}5 - {}^{-}6$
i $^{-}6 - {}^{-}9$ **j** $^{-}13 - {}^{-}14$
k $17 - {}^{-}13$ **l** $^{-}5 - {}^{-}41$
m $^{-}34 - {}^{-}28$ **n** $^{-}16 - {}^{-}29$

3 Find the answer:

a $6 + {}^-2 + 3$ **b** $6 - {}^-2 + 3$
c $6 + {}^-3 - 2$ **d** $6 - 4 - {}^-2$
e $8 - {}^-3 + 1$ **f** $1 - 6 + {}^-7$
g $8 - {}^-8 - 8$ **h** $3 + {}^-3 - 3$

4 Find the answer:

a $4 + {}^-4 - {}^-7$ **b** $8 + {}^-5 - {}^-3$
c $4 + {}^-1 - 5$ **d** ${}^-3 + {}^-3 + {}^-3$
e ${}^-2 - {}^-2 + 2$ **f** ${}^-6 + {}^-5 - 3$
g ${}^-9 + 7 + {}^-6$ **h** ${}^-12 - {}^-13 - 14$
i ${}^-21 + {}^-6 + 19$ **j** $16 - {}^-17 - 23$

5 Find numbers that make these subtraction tables work.

a

−	Second number	
First number	1	4
	2	5

b

−	Second number	
First number	4	3
	5	4

TECHNOLOGY

Review what you have learnt about adding and subtracting integers. Visit

www.onlinemathlearning.com

and follow the links to 'Arithmetic', 'Adding Integers' and 'Subtracting Integers'.

Study the examples and watch the videos!

INVESTIGATION

a Find two numbers such that when you subtract one from the other you get $^-4$.
b Can you find any other such pairs of numbers?
c Can you find a rule for finding other pairs of numbers?

How do you multiply and divide negative numbers? Work through the Technology box to become an expert.

TECHNOLOGY

Learn more about working with negative numbers.

Check out

www.purplemath.com

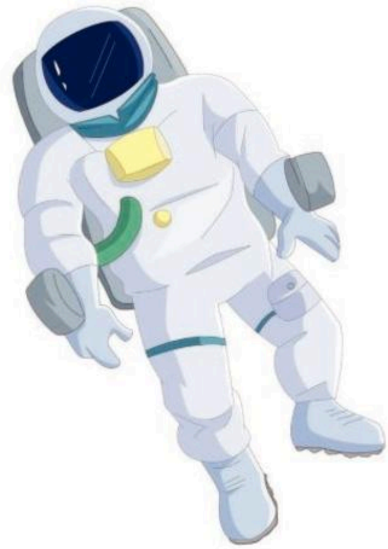
and

www.coolmath.com/prealgebra

Go through the lessons carefully.

There's much to learn!

1.12 Some ways we use negative numbers



Time

The time when something is due to happen, like the launching of a space ship, is often called **time zero**. Times before zero are counted as negative, times after it are counted as positive.

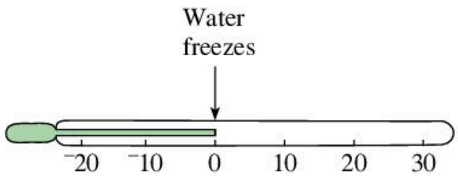
Sea Level

Sea level is zero metres. Heights above sea level are positive and you could think of depths below sea level as negative.

Temperature

The **temperature** of something tells us how hot or cold it is.

It is measured using a thermometer. The temperature of ice is 0 °C.



Thermometer

The C is short for Celsius (pronounced Sell-see-us) who was a famous scientist. 0 °C is read as *zero degrees Celsius.*

There are many things which are colder than ice. These things have negative temperatures. In many places, the temperature of the air is always colder than 0 °C.

Money

If you have $400 in the bank you are in **credit** by $400. Money going into your account is credit. You can use positive numbers to represent credit. Money coming out of your account is **debit**. You can use negative numbers to represent debit.

Exercise 1V

1 This is a timetable for launching a space ship

Time	Hour	Things to do
8.00 am	$^{-}4$ h	Check weather report
9.00 am	$^{-}3$ h	Fuelling
10.00 am	$^{-}2$ h	Final check
11.00 am	$^{-}1$ h	Count-down begins
12 noon	0 h	Launch
1.00 pm	1 h	Firing final rocket
2.00 pm	2 h	Into orbit
3.00 pm	3 h	Docking with space laboratory

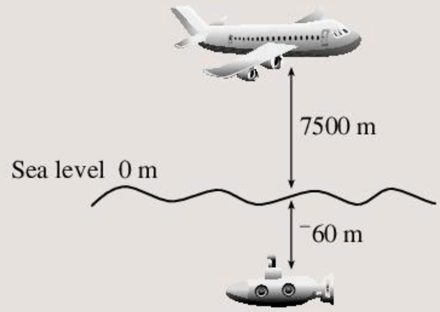
a The first check of the space ship is done at $^{-}6$h. What time is that?

b On launch day, the astronaut sleeps until $^{-}8$h. How many hours are there between the astronaut waking up and docking with the space laboratory?

c The astronaut must be dressed in a space suit $2\frac{1}{2}$ hours before launching. How would you record that on the timetable?

d $1\frac{3}{4}$ hours after fuelling, the astronaut is locked into the space capsule. How would you show that on the timetable?

2 An aeroplane is flying at a height of 7500 metres above sea level. A submarine is directly below it at a depth of 60 metres below sea level. What is the distance between the aeroplane and the submarine?



3 Do you think that this picture shows a place that is colder than 0 °C?
Why do you think so?

4 The temperature in Alaska on a certain day was 2 °C. Find the temperature the next day if it

a rose by 4 °C

b fell by 4 °C

5 Find the new temperature after:

a a rise of 4 °C from $^{-}1$ °C

b a rise of 6 °C from $^{-}10$ °C

c a rise of 1 °C from $^{-}9$ °C

d a fall of 2 °C from 3 °C

e a fall of 4 °C from 3 °C

f a fall of 7 °C from $^{-}2$ °C.

6 The temperature in Toronto was $^{-}5$ °C on Wednesday. It fell by 3 °C on Thursday. What was Thursday's temperature?

7 Liquid mercury freezes at $^{-}39$ °C and boils at 357 °C. What is the temperature difference between these states?

8 At a temperature of $^-183$°C oxygen becomes a liquid. If its temperature is reduced by another 31°C it freezes. What is the freezing point of liquid oxygen?

9 At a weather station in the Arctic the temperature was recorded as $^-23$°C. Two hours later it had fallen by 8°C.

a What was the new temperature?

b Four hours after the first readings the temperature was $^-41$°C. By how much had the temperature fallen in four hours?

10 The table shows the average temperature of the air each month last year, in a North American city.

Month	Temperature
January	$^-4$ °C
February	$^-11$ C
March	2 °C
April	15 °C
May	27 °C
June	31 °C
July	31 °C
August	33 °C
September	25 °C
October	17 °C
November	9 °C
December	$^-1$ °C

a What was the temperature difference between the hottest and coldest months?

b Did the temperature rise between January and February? By how many degrees did it change?

c How many degrees did the temperature fall between November and December?

d How many degrees did the temperature rise between February and March?

11 This is a copy of part of Steven's bank statement. Find the values **a**, **b**, **c** and **d**.

Date	Description	Money out	Money in	Balance
				20.00
9 Oct	Insurance payment	245.00		$^-225.00$
11 Oct	Salary		1250.00	**a**
11 Oct	Visa bill	1070.00		**b**
15 Oct	Refund		42.00	**c**
16 Oct	Hotel bill	180.00		**d**

12 This is a copy of part of Kanika's bank statement. Find the values **a**, **b**, **c** and **d**.

Date	Description	Money out	Money in	Balance
				580.00
9 Oct	Rent	600.00		$^-20.00$
11 Oct	Electricity bill	45.00		**a**
11 Oct	Salary		**b**	1380.00
15 Oct	Credit card bill	**c**		$^-20.00$
16 Oct	Refund		230.00	**d**

Exercise 1W – mixed questions

1 Write in order of size, smallest first:

a $^-3$, 0, 7, 8, $^-6$

b 14, $^-1$, 2, $^-8$, 12

c $^-28$, 17, 46, $^-33$, 12

d $^-6$, 104, $^-99$, $^-81$, $^-63$

2 Copy and complete the sequences:

a 8, 5, 2, _, _, _

b $^-3$, $^-5$, $^-7$, _, _, _

c $^-9$, $^-2$, 5, _, _, _

d 5, 1, $^-3$, _, _, _

3 The table shows the temperature in five cities on one day in December.

City	Temperature (°C)
Sharm El Sheikh	25
New York	$^-3$
Abu Dhabi	28
Toronto	$^-11$
London	0

a How much hotter was it in Sharm El Sheikh than New York?

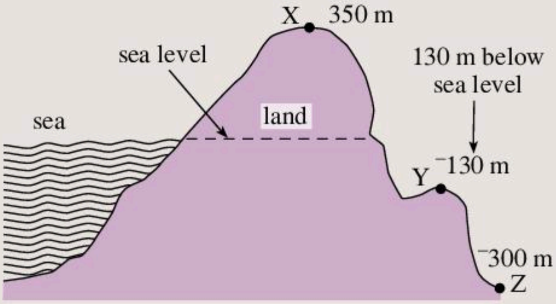
b Which city was the coldest on that day?

c How much colder was Toronto than London?

d Between which two cities was the temperature difference the greatest?

e Write down the cities in order of temperature beginning with the hottest.

4 The drawing shows three villages, X, Y and Z. The dotted line has been continued from the sea, to show sea level. X is above sea level. Y and Z are below sea level.



What is the difference in height between:

a Y and Z **b** X and Y
c X and Z?

5 The highest place in the world where you could sit is on top of Mount Everest. It is 8840 m high. The lowest place where you could sit is on the shore of the Dead Sea. It is $^{-}393$ m high. What is the difference between these heights?

6 Work out:

a $3 + 4$ **b** $3 + {}^{-}4$
c ${}^{-}3 + 4$ **d** ${}^{-}3 + {}^{-}4$
e $6 + {}^{-}10$ **f** ${}^{-}6 + {}^{-}10$
g $7 + {}^{-}9$ **h** ${}^{-}3 + {}^{-}9$
i ${}^{-}13 + {}^{-}14$ **j** $2 + {}^{-}18$

7 Work out:

a $3 - {}^{-}4$ **b** $7 - {}^{-}8$
c $6 - {}^{-}12$ **d** $4 - {}^{-}11$
e $11 - {}^{-}13$ **f** $12 - {}^{-}2$
g ${}^{-}9 - {}^{-}7$ **h** ${}^{-}18 - {}^{-}14$

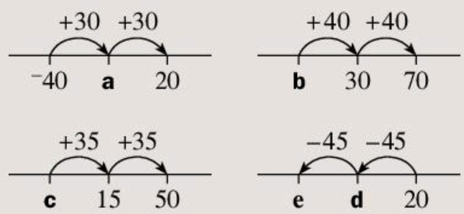
8 Copy and complete:

a $5 + \square = 3$
b ${}^{-}5 + \square = {}^{-}3$
c $\square + 2 = {}^{-}7$
d ${}^{-}3 - \square = {}^{-}9$
e $4 + \square - 3 = {}^{-}5$
f ${}^{-}6 + {}^{-}8 + \square = {}^{-}11$
g $\square + \square = {}^{-}7$

9 Try to add negative numbers on your calculator. You will need to use the $\boxed{+/-}$ key to show a negative number. Use your calculator to work out:

a $6 + {}^{-}3$
b $10 + {}^{-}12$
c ${}^{-}3 + {}^{-}6$
d ${}^{-}12 + {}^{-}10$
e ${}^{-}3 + 5 + 8$
f ${}^{-}9 + {}^{-}3 + {}^{-}6$
g ${}^{-}114 + {}^{-}56$
h $63 + {}^{-}14 + {}^{-}29$

10 In the following parts of number lines what do the letters stand for?



INVESTIGATION

Investigate what happens when you use your calculator to:

a subtract negative numbers
b multiply negative numbers
c divide negative numbers.

Can you find any rules?

ACTIVITY

A	$^{-}12$	$^{-}11$	$^{-}10$	$^{-}9$	$^{-}8$	$^{-}7$	$^{-}6$	$^{-}5$	$^{-}4$	$^{-}3$	$^{-}2$	$^{-}1$	0	1	2	3	4	5	6	7	8	9	10	11	12	B

Here is a game for two players, A and B.

You will need a rectangle of cardboard, about 3 cm wide and 30 cm long.

You will also need a counter (a button or bean will do) and two dice (different colours if possible).

First, make the game board from your cardboard, as shown in the drawing above.

If your dice are different colours, choose one of them to be negative. If they are both the same colour, mark each face of one dice with a negative sign, using a crayon or coloured pencil.

You are now ready to begin. Decide whether A or B will go first.

The rules

1. The counter is put on the space below 0.
2. The players take turns to throw the two dice. The total score for the throw is the scores on the two dice added together.
3. If the total score is positive, move the counter right. If it is negative, move the counter left. See the box on the right for an example.
4. The game is over when the counter lands on or beyond the 12 or $^{-}12$ space. If the counter lands on 12, player A wins. If it lands on $^{-}12$ player B wins.

Player	Scores on dice	Total score	Result
B	6 and $^{-}2$	4	Move 4 places right to 4
A	$^{-}6$ and 1	$^{-}5$	Move 5 places left to $^{-}1$
B	$^{-}3$ and 3	0	Stay at $^{-}1$

1.13 Laws of arithmetic and inverse operations

You may be aware of the laws of arithmetic and use them all the time. You may not know the names of the laws you are using. Most useful to you are:

- The **commutative law**: When adding two numbers or multiplying two numbers the order of doing this doesn't matter.

  For example:

  $$3 + 2 = 2 + 3 = 5$$

  and $$2 \times 5 = 5 \times 2 = 10$$

- The **associative law**: When adding three or more numbers you can add any pair of numbers first. When multiplying three or more numbers you can multiply any pair of numbers first.

  For example:

  $3 + 2 + 4 = 5 + 4 = 9$ (adding the 3 and 2 first)

  or $3 + 2 + 4 = 3 + 6 = 9$ (adding the 2 and 4 first)

  $2 \times 5 \times 3 = 10 \times 3 = 30$ (multiplying the 2 and 5 first)

  or $2 \times 5 \times 3 = 2 \times 15 = 30$ (multiplying the 5 and 3 first)

- The **distributive law**: When a sum (or difference) is being multiplied by a number, each number in the sum (or difference) can be multiplied by the number first then these products are added (or subtracted). It is the same with division.

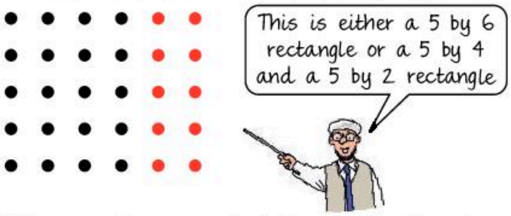
  For example:

  $5 \times (4 + 2) = 5 \times 6 = 30$ This is the sum $(4 + 2)$ being multiplied by the number 5

  and $5 \times 4 + 5 \times 2 = 20 + 10 = 30$

  This is each number in the sum, 4 and 2, being multiplied by the number 5 first then these answers are added.

This can be shown in the following diagram:



When you know a calculation can be done in any order, you can find the easiest way to do it.

EXAMPLE 21

Work out

a $26 + 97 + 74$

b $23 \times 25 \times 2 \times 2$

c $25 \times (100 + 4)$

---

a To do $26 + 97 + 74$, if you add complements to 100 together first it makes the calculation much easier. You can change the order to:

$26 + 74 + 97 = 197$ which makes it easy and quick to do in your head

b To do $23 \times 25 \times 2 \times 2$, look for the easiest order to do this in. Rather than doing the 23×25 first, do $25 \times 2 \times 2$ as this makes 100, multiplying by 100 is easier

$23 \times 25 \times 2 \times 2 = 23 \times 25 \times 4 = 23 \times 100 = 2300$

c $25 \times (100 + 4)$ if we do the $100 + 4$ first then 25×104 involves doing long multiplication. It is easier to do:

$25 \times 100 + 25 \times 4 = 2500 + 100 = 2600$

You probably use inverse operations all the time without being aware that they are called inverse operations.

- **Inverse operations:**

  Multiplying and dividing are inverses of each other.
  If $4 \times 3 = 12$ then $12 \div 4 = 3$ and $12 \div 3 = 4$
  Adding and subtracting are inverses of each other.
  If $2 + 5 = 7$ then $7 - 5 = 2$ and $7 - 2 = 5$

Other inverse operations are squaring and square rooting (you will learn about these later).

You can use laws of arithmetic and inverse operations to make calculations easier or to check your work.

EXAMPLE 22

a If $245 \times 19 = 4655$ what is

i $4655 \div 19$ **ii** $4655 \div 245$?

b Complete:

i $72 + \square = 104$ **ii** $\square \div 15 = 8$

---

a **i** $4655 \div 19 = 245$
as $\div 19$ is the inverse of $\times 19$

ii $4655 \div 245 = 19$
as $\div 245$ is the inverse of $\times 245$

b **i** $72 + \square = 104$

The inverse of $+ 72$ is $- 72$

$104 - 72 = 32$

Check by adding: $32 + 72 = 104$

ii $\square \div 15 = 8$

The inverse of $\div 15$ is $\times 15$

$8 \times 15 = 120$

Check by dividing: $120 \div 15 = 8$

Exercise 1X

1 Copy and complete:

a $18 \times 3 = 3 \times \square = 54$

b $19 + 48 + 81 = 19 + \square + \square = \square$

c $17 \times (2 + 10) = 17 \times 2 + 17 \times \square = 34 + \square = \square$

d $21 \times (10 + 3) = 21 \times \square + 21 \times \square = \square + \square = \square$

e $22 \times (4 + 10) = \square \times \square + \square = \square + \square = \square$

f $328 \times 27 = 8856$ so $8856 \div 27 = \square$ and $8856 \div \square = \square$

g $986 + 458 = 1444$ so $1444 \square 986 = 458$ and $1444 \square 458 = 986$

2 Use inverse operations to copy and complete:

a $\square + 53 = 186$ **b** $\square \div 12 = 7$

c $\square - 54 = 228$ **d** $\square \times 9 = 108$

3 Use the laws of arithmetic to help make these calculations easier:

a $83 + 48 + 17$

b $2 \times 25 \times 19 \times 2$

c $24 \times (10 + 2)$

d $37 + 42 + 63 + 58 + 19$

e $25 \times (100 - 10)$

f $2 \times 17 \times 4 \times 5 \times 25$

4 If $87084 \div 246 = 354$ what is

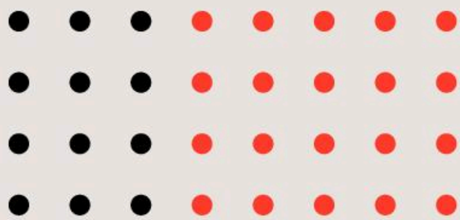
a 354×246 **b** $87084 \div 354$?

5 Atahalne was working on his homework. He wrote $462 - 287 = 185$.
His teacher said he was wrong and that he could see his answer was wrong by doing an addition sum. What sum should he do to check his answer? $\square + \square = \square$

6 Show how you use the distributive law to work out $(60 - 12) \div 3$

7 Draw dots arranged in a rectangle to show why the distributive law works. Use these numbers for your example:
$6 \times (2 + 5) = 6 \times 2 + 6 \times 5 = 42$

8 This diagram shows how the distributive law works.



The diagram above represents the distributive law, fill in the correct numbers for this diagram:

$\square \times (\square + \square) = \square \times \square + \square \times \square = \square$

1.14 Order of operations

There are **rules of arithmetic** to help make sure that everyone completes calculations the same way.

Kade does the calculation $2 + 4 \times 5$,
she says the answer is 30.
Setiawan says,

"That is wrong. I did it on my calculator and the answer is 22."

Kade has made a common mistake. She has worked from left to right doing the calculation in the order it appears. (Kade does $2 + 4 = 6$ first then 6×5).

This is wrong because of the rules of arithmetic which tell us the order to do mathematical operations in.

The **order of operations** (BIDMAS) tells us that in calculations we do:

Brackets first	• Operations in brackets are completed first.
Then Indices	• Numbers raised to a power (index) are done next (you will learn about this later).
Then Division and Multiplication	• Divisions and multiplications are completed next, the order you do these doesn't matter.
Then Addition and Subtraction	• Additions and subtractions are completed next, the order you do these doesn't matter.

If we look again at Kade's question:

$2 + 4 \times 5 =$
$2 + 20 = 22$

The best way to set out these calculations is to work down the page a stage at a time as shown in the following examples:

EXAMPLE 23

Work out

a $14 - 2 \times (5 + 1)$ **b** $(10 - 3) + 20 \div 4$

a $14 - 2 \times (5 + 1)$ BIDMAS
$= 14 - 2 \times 6$ Brackets first
$= 14 - 12$ Then Multiplication
$= 2$ Then Subtraction

b $(10 - 3) + 20 \div 4$ BIDMAS
$= 7 + 20 \div 4$ Brackets first
$= 7 + 5$ Then Division
$= 12$ Then Addition

Exercise 1Y

1 Work out:

a $8 + 2 \times 10$ **b** $16 - 12 \div 4$
c $4 \times 4 + 3$ **d** $7 + 15 \div 5$
e $15 - 3 \times 5$ **f** $21 \div 7 - 1$
g $^{-}3 + 4 \times 7$ **h** $^{-}10 - 8 \times 5$
i $5 \times 9 + {}^{-}100$

2 Work out:

a $17 - 3 \times 5 + 2$
b $3 \times 5 + 2 \times 4$
c $9 \times 2 - 21 \div 3$
d $40 \div 4 + 9 \times 6$
e $15 - 24 \div 3 + 12$
f $2 - 12 \div 2 - 1$
g $15 - 2 \times 3 \times 2$
h $^{-}6 + 9 \times 2 \div 3$
i $^{-}8 - 4 \times 8 + 35$

3 Work out:

a $(8 - 3) \times (3 + 2)$
b $(27 + 33) \div (25 - 19)$

c $5 + (7 - 4) \times 4$
d $26 - 10 \times (9 - 7)$
e $(11 + 9) \div 5 + 13$
f $35 \div (17 - 12) + 2$
g $2 \times (3 + 1) \times 2$
h $-2 + (8 - 5) \times 5$
i $-10 - (3 + 2) \times 4$

4 Write brackets in these to make them correct:
a $6 + 4 \times 10 = 100$
b $3 + 12 \div 2 + 1 = 7$
c $100 - 10 \times 6 - 4 = 80$

5 When working out the answer to $15 - 2 \times 4 + 2$, Maahes has made a mistake.

$15 - 2 \times 4 + 2$	
$= 15 - 8 + 2$	Multiplication first
$= 15 - 10$	Then Addition
$= 5$	Then Subtraction

What mistake has he made?

6 Work out:
a $7 + (9 - 4) \times 4 - (9 + 5) \div 7$
b $^{-}5 - (4 - 5) - 3 \times 2 + {}^{-}4$
c $(10 - 4 \times 2) + (2 \times 2 + 3) \times (10 - 2 \times 4)$
d $29 - ((3 + 9) \div 2) \times 4$
e $3 + 8 \times (15 - (10 - 3))$

Consolidation

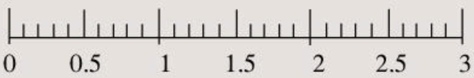
Exercise 1

1 Write these numbers as decimals.

a $\frac{3}{10}$ **b** $\frac{7}{10}$ **c** $\frac{49}{100}$

d $1\frac{6}{10}$ **e** $13\frac{9}{10}$

2 Copy the number line and place on these decimals.

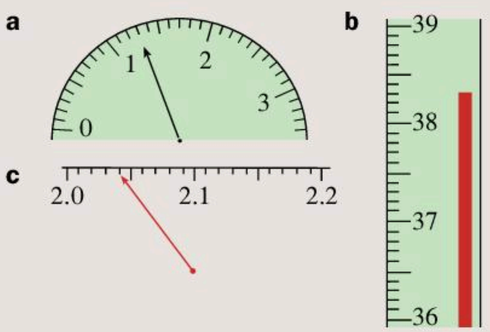


a 0.3 **b** 1.2 **c** 0.6
d 2.9 **e** 1 **f** 2.0
g 0.45 **h** 1.65

3 Find the largest of these pairs of numbers.
a 0.2, 0.14 **b** 0.32, 0.4 **c** 0.1, 0.03

4 Write these numbers in order of size, smallest first.
a 0.3, 0.17, 0.2, 3 **b** 1.6, 0.07, 15, 0.8
c 0.48, 0.5, 1, 8.02

5 Read the value on each scale.



6 The times of eight athletes in the women's 200 m race at the Olympic games were

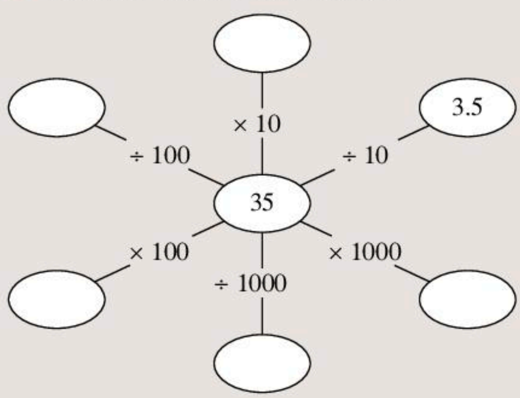
Lane 1	Kim Gevaert	Belgium	22.84 sec
Lane 2	Ivet Lalova	Bulgaria	22.57
Lane 3	Allyson Felix	USA	22.18
Lane 4	Veronica Campbell	Jamaica	22.05
Lane 5	Abiodun Oyepitan	Britain	22.87
Lane 6	Aleen Bailey	Jamaica	22.42
Lane 7	Muna Lee	USA	22.87
Lane 8	Debbie Ferguson	Bahamas	22.3

a Who won the race?
b Who came last?
c How much faster was Campbell than Felix?
d The World record in this race was held by Florence Griffith Joyner at 21.34 seconds. How far outside this time was the third place runner?

7 Work out:

a 2.3×10 **b** 1.7×1000
c $81.2 \div 100$ **d** $1.4 \div 100$
e 0.03×1000 **f** 0.7×10
g $2.6 \div 10$ **h** $152.3 \div 1000$
i 42.3×100 **j** 9.142×100
k $32 \div 1000$ **l** $75.64 \div 10$

8 Copy and complete this diagram:



9 Copy and complete:

a $1.4 \times \square = 14$ **b** $3.81 \times \square = 3810$
c $\square \div 100 = 2.41$ **d** $12.7 \div \square = 0.0127$
e $\square \times 100 = 2$ **f** $0.61 \div \square = 0.061$

10 Round these numbers as shown:

a 985 to nearest 10
b 1.746 to 1 d.p.
c 34526 to nearest 100
d 234814 to nearest 1000
e 0.0975 to 1 d.p.
f 14.7 to nearest 10
g 145676 to nearest 1000
h 4.05 to nearest whole number
i 58.361 to nearest whole number
j 22567 to nearest 100

11 Draw number lines to show and work out these additions.

a $^{-}2 + 5$ **b** $^{-}4 + 6$ **c** $3 + {}^{-}5$
d $2 + {}^{-}6$ **e** $^{-}2 + {}^{-}3$ **f** $^{-}6 + {}^{-}5$
g $^{-}7 + 2$ **h** $^{-}9 + 4$ **i** $^{-}4 + {}^{-}4$
j $8 + {}^{-}4$

12 Use a number line to work out these subtractions.

a $5 - 2$ **b** $6 - 3$ **c** $^{-}5 - 3$
d $^{-}4 - 6$ **e** $2 - 6$ **f** $8 - 1$
g $^{-}3 - 8$ **h** $7 - 12$ **i** $2 - 9$
j $^{-}2 - 9$

13 Work out:

a $^{-}5 + {}^{-}6$ **b** $^{-}11 + {}^{-}15$
c $11 - 16$ **d** $23 - 32$
e $^{-}5 + {}^{-}4 + 2$ **f** $6 + {}^{-}8 - 3$
g $7 - 9 + {}^{-}2$ **h** $^{-}13 + 17 - 28$
i $214 + {}^{-}385$

14 Calculate:

a $3 - {}^{-}2$ **b** $5 - {}^{-}3$
c $^{-}5 - {}^{-}3$ **d** $3 - {}^{-}9$
e $^{-}7 - {}^{-}6$ **f** $^{-}6 - {}^{-}7$
g $^{-}13 - 8$ **h** $^{-}3 - 13$
i $14 - {}^{-}7$ **j** $^{-}7 - {}^{-}14$

15 Work out:

a $5 + {}^{-}3 - 2$ **b** $^{-}3 - 3 - 2$
c $3 - {}^{-}3 - 2$ **d** $^{-}6 - 4 - {}^{-}4$
e $^{-}4 + 6 - 8$ **f** $7 - {}^{-}6 + 4$
g $^{-}2 + 6 - {}^{-}4$ **h** $^{-}3 - {}^{-}4 - 9$
i $14 - 17 - 11$ **j** $8 + {}^{-}9 - {}^{-}4$
k $^{-}8 - 7 + {}^{-}6$ **l** $^{-}23 - 41 - {}^{-}19$

16 The temperatures in four towns on 1st December were as follows:

Town	Temperature (°C)
Cairo	23
Moscow	$^{-}14$
Calgary	$^{-}5$
Abuja	31

a What was the coldest town?
b How much hotter is Abuja than Calgary?
c How much colder is Moscow than Calgary?
d Write the towns in temperature order, starting with the hottest.

17 Using the data in Question **16**, find the temperature on 2nd December if the:

a temperature in Cairo fell by 3°C.
b temperature in Calgary fell by 6°C.
c temperature in Moscow rose by 2°C.

18 Copy and complete:

a $14 \times 9 = 9 \times \square = 126$
b $34 + 59 + 66 = 34 + \square + \square = \square$
c $29 \times (2 + 10) = 29 \times \square + 29 \times \square = \square + \square = \square$
d $1035 \div 23 = 45$ so $45 \times 23 = \square$ and $1035 \div \square = \square$
e $511 - 327 = 184$ so $184 + \square = 511$ and $511 \square \square = 327$

19 Use inverse operations to copy and complete:

a $\square + 87 = 143$ **b** $\square \div 12 = 9$
c $\square - 82 = 241$ **d** $\square \times 8 = 96$

20 Use the laws of arithmetic to help make these calculations easier.

a $58 + 39 + 42$ **b** $25 \times 2 \times 48 \times 2$
c $21 \times (100 + 3)$

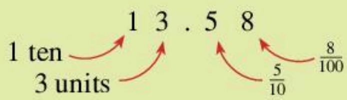
21 Work out:

a $9 + 3 \times 5$ **b** $20 - 15 \div 5$
c $^{-}3 - 4 \times 6$ **d** $14 - 7 + 1$
e $2 \times 7 + 33 \div 3$ **f** $15 - 16 \div 2 - 10$
g $^{-}4 + 5 \times 9 \div 3$ **h** $12 + (7 - 5) \times 10$
i $(48 + 52) \div (28 - 3)$

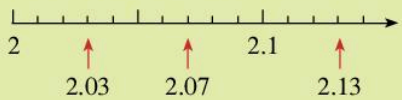
Summary

You should know …

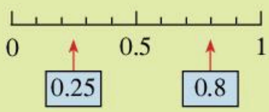
1 A decimal is a way of writing a number using place values of tenths, hundredths etc.



2 A decimal can be shown on a number line.



3 You can use a number line to compare the size of two decimals.



0.25 is smaller than 0.8

4 When multiplying by powers of 10 all digits in the number move left depending how many zeros there are:

$2.41 \times 10 = 24.1$

$2.41 \times 100 = 241$

$2.41 \times 1000 = 2410$

5 When dividing by powers of 10 all digits in the number move right depending how many zeros there are:

$321.7 \div 10 = 32.17$

$321.7 \div 100 = 3.217$

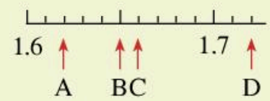
$321.7 \div 1000 = 0.3217$

Check out

1 Write the value of the underlined digit.

a $0.\underline{6}8$ **b** $3\underline{2}.61$
c $0.0\underline{7}$ **d** $\underline{4}03.128$
c $1.30\underline{2}$

2 What numbers are represented by the letters A, B, C, D on the number line?



3 For each number pair, write the larger number.

a 0.3, 0.5 **b** 3, 1.6
c 0.3, 0.29 **d** 0.93, 1

4 Work out:

a 8.6×10
b 2.6×1000
c 12.5×100
d 1.045×100
e 0.07×1000
f 0.3×10

5 Work out:

a $4.6 \div 10$
b $232.6 \div 1000$
c $91.3 \div 100$
d $5.3 \div 100$
e $12 \div 1000$
f $15.21 \div 10$

6 Numbers can be rounded by using the following rule: if the next digit is 5 or more round up, if it is 4 or less round down.

Round 48 653 to nearest hundred:
The figure in the tens column is 5 so round up.
48 6<u>5</u>3 to the nearest hundred is 48 700

Round 12.432 to 1 d.p.:

The figure in the second decimal place is 3 so round down.
12.432 to 1 d.p. is 12.4

6 Round these numbers as shown:

a 431 to nearest 10

b 2.489 to 1 d.p.

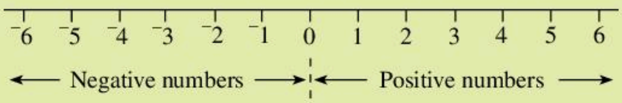
c 92 146 to nearest 100

d 12.975 to nearest whole number

e 574 516 to nearest 1000

---

7 Negative numbers are numbers less than zero.



Numbers to the right of another number are always greater.
For example:

$^-1$ is greater than $^-2$
3 is greater than $^-5$

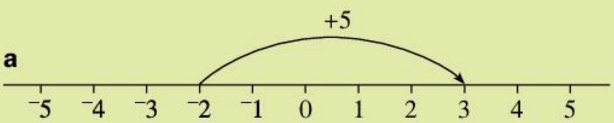
7 Write down the smaller number in each of these pairs of numbers:

a $6, {}^-3$ **b** $^-2, 4$
c $8, {}^-9$ **d** $^-3, {}^-5$

---

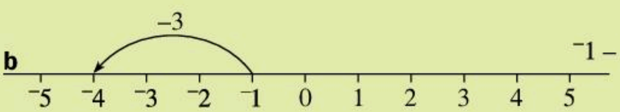
8 A number line can help you add and subtract numbers.
For example:

a



$^-2 + 5 = 3$

b



$^-1 - 3 = {}^-4$

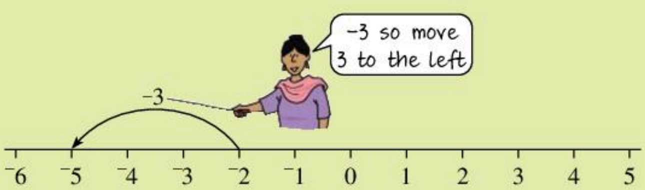
8 Use a number line to work out:

a $^-3 + 4$
b $^-4 - 2$
c $^-6 + 2$
d $3 - 5$
e $4 - 7$

---

9 Adding a negative number is the same as subtracting a positive number.
For example:

$^-2 + {}^-3 = {}^-2 - 3 = {}^-5$



9 Work out:

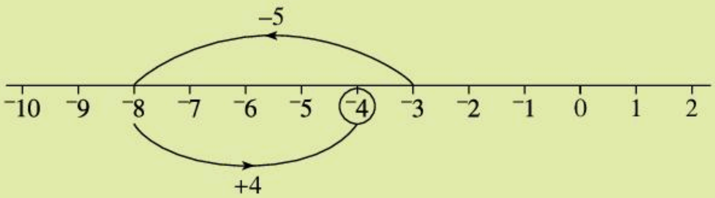
a $2 + {}^-3$
b $3 + {}^-2$
c $^-3 + {}^-2$
d $^-5 + {}^-6$
e $3 + {}^-4 + {}^-5$
f $^-4 + 3 + {}^-5$

10 a Subtracting a negative number is the same as adding a positive number,
For example:

$6 - {}^-2 = 6 + 2 \quad = 8$
${}^-5 - {}^-4 = {}^-5 + 4 = {}^-1$

b More complicated additions and subtractions are done in the same way:
For example:

${}^-3 + {}^-5 - {}^-4 = {}^-3 - 5 + 4$
$= {}^-8 + 4$
$= {}^-4$



10 Calculate:

a $3 - {}^-2$
b $2 - {}^-3$
c ${}^-4 - {}^-5$
d ${}^-3 - {}^-7$
e $3 + {}^-2 - {}^-4$
f ${}^-4 + {}^-3 - {}^-5$

11

- The **commutative law** says when adding or multiplying the order of doing this doesn't matter.
  e.g. $2 \times 8 = 8 \times 2 = 16$
- The **associative law** says when adding or multiplying three or more numbers you can do any pair of numbers first.
  e.g. $1 + 7 + 3 = 8 + 3 = 11$
  or $= 1 + 10 = 11$
- The **distributive law** says when a sum is being multiplied by a number, each number in the sum can be multiplied by the number first then these products are added.
  e.g. $2 \times (7 + 3) = 2 \times 10 = 20$
  or $2 \times 7 + 2 \times 3 = 14 + 6 = 20$
- Multiplying and dividing are inverses of each other.
  If $5 \times 8 = 40$ then $40 \div 5 = 8$ and $40 \div 8 = 5$
- Adding and subtracting are inverses of each other.
  If $20 + 15 = 35$ then $35 - 15 = 20$ and $35 - 20 = 15$

11 Copy and complete:

a 14×8
$= 8 \times \square = 112$
b $63 + 12 + 37$
$= 63 + \square + \square = \square$
c $18 \times (3 + 10)$
$= 18 \times \square + 18 \times \square$
$= \square + \square = \square$
d $812 \times 13 = 10556$
so $10556 \div 13 = \square$
and $10556 \div \square = \square$
e $2173 - 496 = 1677$
so $1677 \square 496 = 2173$
and $2173 \square \square = 496$

12 BIDMAS tells you the order you should do operations:
Brackets first
Then Indices
Then Division and Multiplication
Then Addition and Subtraction

$20 - 2 \times 4 - (10 - 3)$
$= 20 - 2 \times 4 - 7$ Brackets first
$= 20 - 8 - 7$ Then Multiplication
$= 5$ Then Subtraction

12 Work out:

a $19 - 3 \times 5$
b $10 + 16 \div 4$
c $3 \times 5 + 121 \div 11$
d $-14 + (8 - 3) \times 15$
e $(21 + 19) \div 5 + 4$
f $21 - 7 \times 3 + 1$

2 Expressions

Objectives

- Use letters to represent unknown numbers or variables; know the meanings of the words term, expression and equation.
- Construct simple algebraic expressions by using letters to represent numbers.
- Simplify linear expressions, e.g. collect like terms; multiply a constant over a bracket.
- Know that algebraic operations follow the same order as arithmetic operations.

What's the point?

The use of symbols or letters for numbers helps to describe relationships among variables. For example, the speed (v) of a race car is related to the time (t) it takes to travel a particular distance (d) by $v = d \div t$.

Before you start

You should know ...

1 The basics of algebra:

- $a + a + a = 3 \times a$ or $3a$ for short. No need for the multiplication symbol when letters are used. This is called simplifying.
- $a \times 5 = 5a$ (write the number first)
- $a \times b = ab$ for short
- $b \times 3 \times a = 3ab$ for short (number first, then letters in alphabetical order)

Check in

1 **a** Write in a shorter way:

i $4 \times p$
ii $t \times 3$
iii $h \times k$
iv $a \times b \times c$
v $2 \times 4m$
vi $7y \times 5$
vii $a \times 2 \times b$
viii $3n \times 4u$
ix $4t \times 6r$

- $3 \times 5a = 15a$
(multiply numbers together, then write in front of the letter)
- $2p \times 3q = 2 \times p \times 3 \times q$
(multiply numbers first)
$2 \times 3 \times p \times q = 6pq$

b Simplify:
i $p + p + p + p + p$
ii $G + G$
iii $b + b + b - b$
iv $m + m - m - m$

c 3x can be written as $3 \times x$ or in full as $x + x + x$. Write in full:
i $4m$ **ii** $5y$

2 How to add and subtract with negative numbers
For example:
$^{-}2 + 5 = 3$
$4 + {}^{-}7 = 4 - 7 = {}^{-}3$
$^{-}6 - {}^{-}1 = {}^{-}6 + 1 = {}^{-}5$

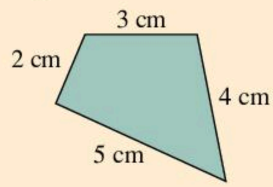
2 Work out:
a $^{-}8 + 10$
b $4 - 12$
c $3 + {}^{-}9$
d $^{-}3 - 4$
e $^{-}7 - {}^{-}5$
f $^{-}1 + {}^{-}8$

3 The area of a rectangle is length $\times$ width.

3 What is the area of a rectangle of length 12 cm and width 8 cm?

4 The perimeter of a shape is the distance around it.

4 Find the perimeter of the figure.



2.1 Expressions

In maths we try to make things simpler by writing as few words as possible. For example, these two sentences can be written in a shorter way:

3 apples and 7 bananas = $3a + 7b$

6 apples and 12 bananas = $6a + 12b$

We have used the letter a to represent apples, the letter b to represent bananas and the + symbol to replace the word 'and'. The process of using letters to represent unknown numbers or variables is **algebra**.

- A **constant** is a symbol which always means the same thing. For example 9 and 7 are constants.
- A **variable** is a symbol which can represent different numbers.

EXAMPLE 1

a Write in a shorter way: The length of a piece of string with 3 cm cut off it.
b Write in a shorter way: The total number of cakes baked if I bake two cakes for each of my friends and 5 spare cakes.
c What sentence could go with: $4h$?

a Use the letter s to represent the unknown variable length of the string in cm.
$s - 3$ represents the length when the constant 3cm is cut off.
b Let f represent the unknown number of friends.
$2 \times f$ or $2f$ represents 2 cakes for each of my friends.

$2f + 5$ represents 2 cakes for each of my friends and 5 spare cakes.

c If h represented the unknown number of horses in my field and $4h$ means $4 \times h$ which could represent the number of legs on all my horses.

Exercise 2A

1 Write in a shorter way:

- **a** The length of a piece of string with 4cm cut off. Let s represent the unknown length of string in cm.
- **b** The total number of apples I am going to buy if I buy 2 apples for each of my horses and three spare apples. Use h to represent the unknown number of horses.
- **c** The number of text books needed in my class if the students are sharing text books in pairs. Let n represent the unknown number of children.

2 A pencil costs m cents. A pen costs n cents. Match each expression with the correct amount in cents. The first is done for you.

	$4 - 4n$
The total cost of 4 pencils	$4m$
The total cost of 4 pens and 4 pencils	
	$4n$
How much more 4 pens cost than 4 pencils	$4n - 4m$
	$4m - 4n$
The change from \$4 in cents when you buy 4 pens	$400 - 4n$
	$4n + m$

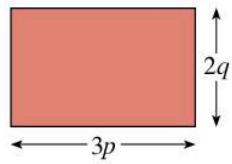
3 Write a sentence to go with the following:

a $p + 5$ **b** $3t$
c $7 - m$ **d** $3f + 1$

We can also work out areas of rectangles and perimeters of shapes using algebra. We do this when we have unknown side lengths.

EXAMPLE 2

Find the area of the rectangle.

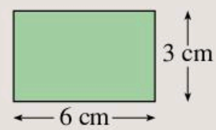


$$\begin{aligned}\text{Area} &= \text{length} \times \text{width}\\ &= 3p \times 2q\\ &= 3 \times p \times 2 \times q\\ &= 3 \times 2 \times p \times q\\ &= 6\,pq\end{aligned}$$

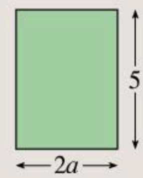
Exercise 2B

1 Find the areas of the following rectangles.

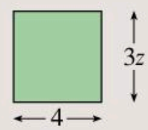
a



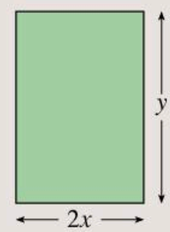
b



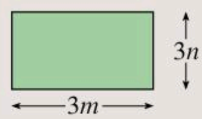
c



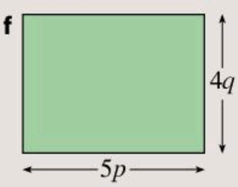
d



e

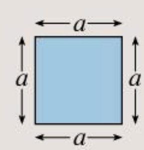


f

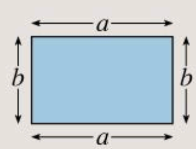


2 Find the perimeter of the following shapes.

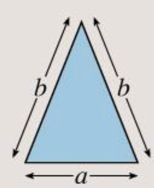
a



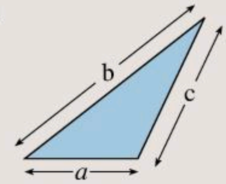
b



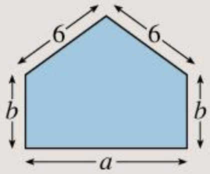
c



d



e



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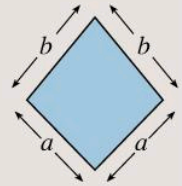


An **algebraic expression** is one which contains some letters instead of numbers.

You have been finding expressions when writing things in a shorter way.

For example $x + y$, $4p$, and $2m + 3n - 3t$ are expressions.

Exercise 2C covers working with expressions and how expressions change when you change units (e.g. from weeks to days).

EXAMPLE 3

How many days are there in:

a 4 weeks
b 9 weeks
c x weeks
d $5x$ weeks?

---

a In one week there are 7 days
In 4 weeks there are $7 \times 4 = 28$ days

b In 9 weeks there are $7 \times 9 = 63$ days
c In x weeks there are $7 \times x = 7x$ days
d In $5x$ weeks there are $7 \times 5x = 35x$ days

Exercise 2C

1 How many millimetres are there in
a 1 cm **b** 6 cm **c** x cm?

2 How many seconds are there in
a 1 minute **b** 5 minutes
c x minutes?

3 How many legs have
a 1 person **b** 7 people
c x people?

4 How many cents are there in
a 4 dollars **b** 15 dollars
c x dollars **d** $3x$ dollars?

5 If one cricket team has 11 players, how many players are there in
a 2 cricket teams
b 5 cricket teams
c y teams
d $4y$ cricket teams?

6 How many centimetres are there in
a 6 metres **b** 16 metres
c x metres **d** $8x$ metres?

7 How many days are there in
a 2 years **b** 10 years
c y years **d** $5y$ years?

8 A week has 7 days. So in x weeks and 3 days there are $7x + 3$ days.
In the same way, write an expression for the number of
a days in x weeks and 4 days
b days in y weeks -5 days
c days in $2y$ weeks
d months in x years -3 months
e months in $2x$ years $+5$ months
f months in $3x$ years -11 months
g cm in z metres -30 centimetres
h cm in z metres $+40$ centimetres.

9 **a** Adam is 7 years old. How old will he be in 3 years' time? How did you get your answer?
b Waluyo is y years old. How old will he be in three years' time?
c Johnny is p years old. How old will he be in 5 years' time?
d Lintang is m years old. How old will she be in n years' time?

10 **a** Sohan got \$5 from his father and \$8 from his mother. How much did he get altogether? How did you get your answer?
b Avtar got \$10 from her father and \$$d$ from her mother. How much did she get altogether?

11 **a** A car can hold 5 people. How many people can fit in 6 such cars? How did you get your answer?
b How many people could you fit into y cars?
c If a bus holds 48 people, how many people can fit in z buses?

12 **a** Steve has \$10. He wishes to share it equally between his 5 friends. How much does each friend get? How did you get your answer?
b June has \$$d$. She wishes to share it equally between 3 friends. How much does each friend get?

13 **a** Ambrose has \$14. Peter has \$6 less than Ambrose. How many dollars has Peter? How did you get your answer?
b How many dollars do Ambrose and Peter have altogether? How did you get your answer?
c Len has \$$x$. Tom has 10 dollars less than Len. How many dollars has Tom?
d How many dollars have they both together?

2.2 Simplifying

Parts of an expression are called **terms**. $7a$ is an expression with one term and $5x - 2y$ is an expression with two terms. We separate terms with + or − signs. The starting term is positive if it has no sign in front of it.

This is an algebraic expression with three terms:

$$3a + 2b - 5c$$

These are the terms.

Like terms are terms which have the same letter or letters. They can be collected together by adding or subtracting. We can only collect together like terms.

For example, $6x$, $^{-}2x$, x, $\frac{2}{3}x$ are all like terms because they all contain the letter x and only the letter x.

EXAMPLE 4

Underline the like terms from the lists. Include the sign on the left if there is one.

a $4x, 3, {}^{-}x, 2y, 3xy, 5x$
b $3xy, 4m, {}^{-}xy, 2, 5yx$

a $\underline{4x}, 3, \underline{{}^{-}x}, 2y, 3xy, \underline{5x}$: these are all like terms as they all have the letter x only
b $\underline{3xy}, 4m, \underline{{}^{-}xy}, 2, \underline{5yx}$ (since $5yx$ can be written as $5xy$): these are all like terms as they all contain xy.

Exercise 2D

1 Copy these lists and underline like terms:
a $4p, 5y, 4x, 2y, 6xy, {}^{-}y$
b $t, 4t^2, {}^{-}t, \frac{3}{4}t, 7st, 5, 400t$
c $7x, 3, 40xy, {}^{-}yx, 8y, 5xy$
d $x, 7, 2y, 8, 4p, {}^{-}1, 0.2$

2 Match these terms into pairs of like terms to find the odd one out:
ab cab bc ac acb cb ba

3 Jasdeep says these are all like terms:

$3r, -r, 100r, 2R, \frac{1}{2}r$

Jasdeep is wrong. Which term is the odd one out?

4 Copy the boxes below. Tick (✓) which pairs are like terms

Hint: If you don't know what x^2 means look ahead to Chapter 4. x^2p means $x \times x \times p$

$4x^2p$, $0.4x^2p$	$4x^2p$, $3px^2$
$3y^2x$, $5x^2y$	$3x^2y^3$, $4y^2x^3$

In an expression, you can collect like terms:

$$6x + 2y + x + 3y$$

These are like terms — These are like terms

$$= 7x + 5y$$

so $6x + 2y + x + 3y = 7x + 5y$

To **simplify** an algebraic expression you have to collect like terms.

EXAMPLE 5

Simplify:

When rearranging the order, the sign to the left of each term stays with the term.

a $9t - 7t + 2t$
b $5a + 2b - 3a + 6b - a - b$
c $2x + 4y - 6 - 5y - x + 3$

a $9t - 7t + 2t = 4t$
b $5a + 2b - 3a + 6b - a - b$
$= 5a - 3a - a + 2b + 6b - b$
$= a + 7b$
c $2x + 4y - 6 - 5y - x + 3$
$= 2x - x + 4y - 5y - 6 + 3$
$= x - y - 3$

Numbers are also like terms

An **equation** is different from an expression. An equation contains an equals sign. The equals sign shows that the expressions either side of it are equal to each other.

$x + 1 = 2$ or $40 = 3x + 5$ are examples of equations.

You will learn more about equations in Chapter 8.

Exercise 2E

1 Simplify:

a $3p + 2p + 5p$
b $5s + 8s - 4s$
c $15t - 3t + 8t$
d $6l + 14l - 9l$
e $4m + m - 8m$

2 Rearrange these expressions by putting like terms together, and simplify.

a $2a + b + a - 3b$
b $x + 3y + 2x - 4y$
c $f + 2g - 3f + g + f - 5$

3 Simplify:

a $4a + 2b + 3a + 4b$
b $a + b + 2a - b$
c $c + 3d - 2c + 2d$
d $2p - 3q - p + 4q$
e $-3x + 5y + 5x - 3y$
f $4f + 2g + h + 2f - g$
g $r + 4s - 2t - r - 3s + t$
h $m + 4n - 4p + 2n + p - 2m$
i $5x + 2y + 3 - 2x - y - 2$
j $14 - 4x + 3y - 2 + 6x - 2y$

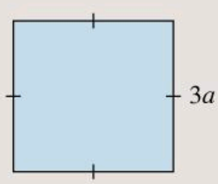
4 From the boxes below write down which of these are *equations* rather than expressions:

5 Copy out the boxes below. When these expressions are simplified tick (✓) which of them are equal:

6 Add together *tpm*, *mtp* and *ptm*

7 Write an expression for the perimeter of these shapes (the distance around the edge). Simplify your expression where possible

a



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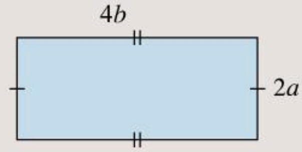

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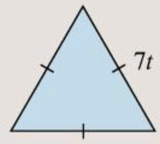
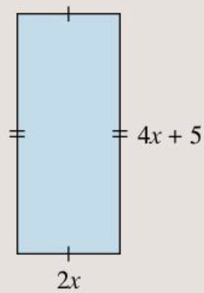
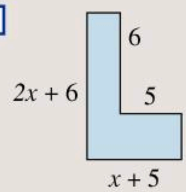


d



e



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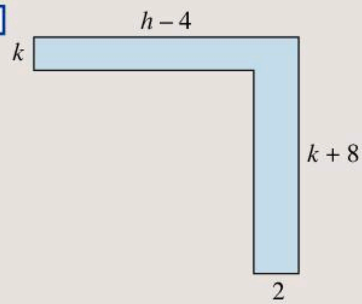
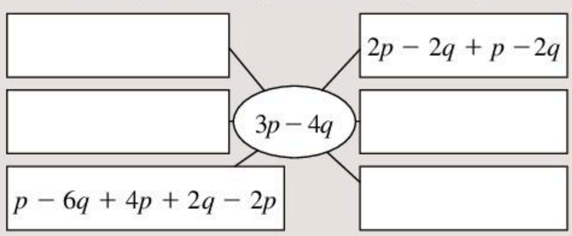


8 Copy and complete this diagram with four more equivalent expressions to $3p - 4q$.



9 Copy and complete:

a $6r + \square - 2r + 4s = 4r + 7s$
b $\square - 2q - 2p - q = 2p - 3q$
c $2m - 4w - \square - \square = {}^{-}4m - 7w$
d $20x + 15y - 17x + \square - \square = 6x + y$

10 Maraim has made some mistakes in her homework. Which are wrong? What should the answers be?

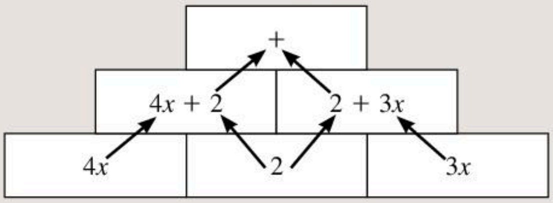
a $12 + 2x = 14x$

b $4p - 3p = 1$

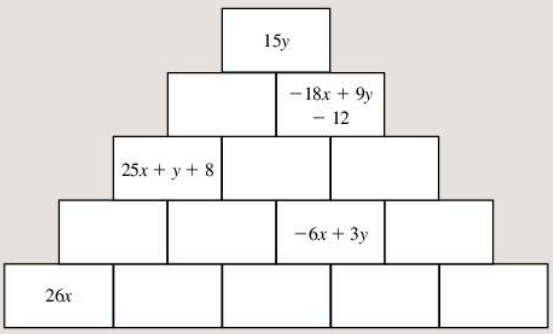
c $10x - 2y + 5x = 10x - 5x + 2y$
$= 5x + 2y$

d $4 + 3p - 2 = 3p + 4 - 2 = 3p + 2$

11 **a** To complete the pyramid shown, each block is found by adding the two blocks below it. The second row of this pyramid has been completed for you. What goes in the top block? Simplify your answer.



b Fill in the missing blocks in this pyramid. (**Hint:** Simplify all expressions and put the terms in alphabetical order with spare numbers at the end e.g. $2x + 3y + 4$, not $3y + 4 + 2x$)



2.3 Expanding brackets

In Chapter 1 you learned about the distributive law.

The **distributive law** says that when a sum is being multiplied by a number, each number in the sum can be multiplied by the number first, then these products are added.

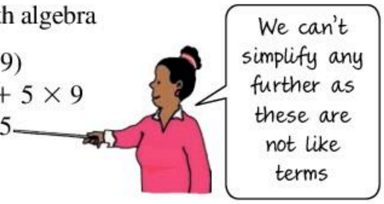
e.g. $4 \times (7 + 3) = 4 \times 10 = 40$
or $4 \times 7 + 4 \times 3 = 28 + 12 = 40$

This also applies when finding a difference

e.g. $2 \times (7 - 3) = 2 \times 4 = 8$
or $2 \times 7 - 2 \times 3 = 14 - 6 = 8$

This also applies with algebra

e.g. $5 \times (x + 9)$
$= 5 \times x + 5 \times 9$
$= 5x + 45$



Usually $5 \times (x + 9)$ is written without the multiplication sign as $5(x + 9)$

The process of multiplying out the brackets is called **expanding**.

EXAMPLE 6

Expand the brackets.

a $5(6x - 3y)$

b $4(3p - q + 5)$

c $^{-}2(4y - 8)$

a $5(6x - 3y) = 5 \times 6x - 5 \times 3y$
$= 30x - 15y$

b $4(3p - q + 5) = 4 \times 3p - 4 \times q + 4 \times 5$
$= 12p - 4q + 20$

c $^{-}2(4y - 8)$

If $2(4y - 8) = 2 \times 4y - 2 \times 8 = 8y - 16$

then $^{-}2(4y - 8) = {}^{-}8y + 16$

The negative number outside the brackets changes the sign of every term inside the brackets

In Example 6, part **c** can be completed a different way. If you are happy multiplying with negative numbers then:

$^{-}2(4y - 8) = {}^{-}2 \times 4y - {}^{-}2 \times 8$
$= {}^{-}8y - {}^{-}16$
$= {}^{-}8y + 16$

If you are going to do it this way do not forget that multiplying by a negative number reverses the sign of the number you are multiplying. A positive number becomes negative and a negative becomes positive.

Exercise 2F

1 Expand the brackets.

a	$3 \times (p + q)$	**b**	$5 \times (p - q)$
c	$4 \times (l + m)$	**d**	$3(x - y)$
e	$6 \times (r + s)$	**f**	$5 \times (l - m)$
g	$8(p + 5q)$	**h**	$9(2x + y)$
i	$3(2x + 5)$	**j**	$5(3y - 2)$
k	$7(p + 3q)$	**l**	$4(2u - 5v)$

2 Expand the brackets:

a	$5(2x + y - 7)$	**b**	$4(5m - 8t + 1)$
c	$11(1 - x + 4y)$	**d**	$8(4m - 3 + 9n)$

3 Expand the brackets:

a	$3(2x + 4)$	**b**	$^-3(2x + 4)$
c	$7(10t - 1)$	**d**	$^-7(10t - 1)$
e	$10(11p - 3)$	**f**	$^-10(11p - 3)$
g	$4(8 - 2x + y)$	**h**	$^-4(8 - 2x + y)$

4 Copy and complete:

a $6(10m - \square) = 60m - 6$
b $\square(4x + 3) = 8x + 6$
c $\square(8x + 3) = 56x + 21$
d $-5(\square x - 4) = -25x + 20$

In Chapter 1 you learned about the order of operations and how they apply to numbers:

Brackets first
Then Indices
Then Division and Multiplication
Then Addition and Subtraction

BIDMAS tells you the order in which you should do operations.

The same rules apply to algebra.

EXAMPLE 7

Simplify **a** $4x + 2 \times 3x + 8x$
b $4t + 5(3p + 2t) - 4p$
c $6(2x - y) - 3(5x + 2y)$

a $4x + 2 \times 3x + 8x$
$= 4x + 6x + 8x$
$= 18x$
Multiplication first. Then Addition to simplify

b $4t + 5(3p + 2t) - 4p$
$= 4t + 15p + 10t - 4p$
$= 14t + 11p$
Expand Brackets first. Then Addition and Subtraction to simplify

c $6(2x - y) - 3(5x + 2y)$
$= 12x - 6y - 15x - 6y$
$= {^-3x} - 12y$
Expand Brackets first. Then Subtraction to simplify

Watch out for this red minus sign. Most people forget to change *both* signs when a negative number is outside the brackets!

Exercise 2G

1 Simplify:

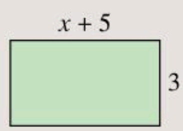
a $3(x + y) + 7(x + y)$
b $6(p + q) + 5(p + q)$
c $5(r + 2s) + 3(r + s)$
d $3(2r - 2s) + 4(r + 3s)$
e $7(q - 4p) + 6(2q + 5p)$
f $4(3x - 2y) + 6(3y - x)$
g $7(3 - 2z) + 3(z - 2)$
h $14 + 3(y + 4) - 2y$
i $7p + 6(4 - 2p) - 7$
j $3(x + 2) - 4 + 4(x - y)$
k $\frac{1}{7}(49x + 7) + \frac{1}{8}(64x + 8)$
l $\frac{1}{2}(6x + 2) + \frac{1}{5}(25x - 10)$

2 Simplify

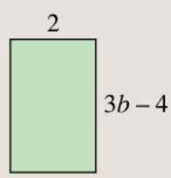
a $4(2x + y) - 3(5x + 4y)$
b $8 - (x - 4) - 2x + 3(5 - x)$
c $2w + 7 \times 8w - 50w$
d $3(4x - 5y) - 2(7x - 3y)$
e $10(3p - 8t) - 5(2p - 5t)$
f $6(2m - 5p + q) - 5(5m + 2p - 3q)$
g $g - (h - 2g) + 6 \times 3h - 5g$
h $7r + 10 - 2(2r + 3s - 5t) + 2(4s - t) + 5$

3 Write an expression for the area of these shapes, expanding and simplifying where necessary.

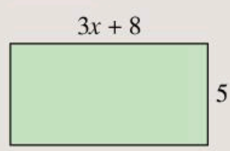
a



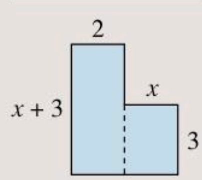
b



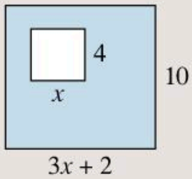
c



d



e



Write an expression for the blue shaded area.

4 Pair up equivalent expressions to find the odd one out.

$3 - 2(4x + 5)$

$4 - 3(2 - 7x) + 30$

Exercise 2H – mixed questions

1 Simplify:

a $3x + 7x + 5x$

b $9x + 3y + y - 2x$

c $10n + 8 + 2m - 3n - 1$

d $5x \times 3y$

e $3m + 6 \times 9m - 40m$

f $2 + 8p - 2 \times 5p + 20p - 6$

2 Expand and simplify where necessary

a $4(1 + 3w)$

b $^{-}5(2x - 3)$

c $3(4t - 5s) + 4(8t + s)$

d $6(y - 4) - 2(3y + 1)$

e $15 + 4(x - 8) + 3x$

f $1 - (4x - 3) + 2x$

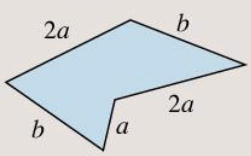
3 Write an expression for the number of

a cents in $\$d + p$ cents

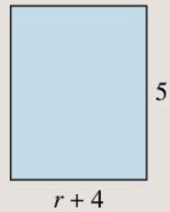
b days in $3x$ weeks $+ 5$ days

c metres in k kilometres $+ s$ metres.

4 **a** Write an expression for the perimeter of this shape:



b Write an expression for the area of this rectangle.



5 A man walks $2z$ km on the first day, 8 km on the second day and $(z + 3)$ km on the third day. How far does he walk in three days?

6 In a test Matt got 15 more marks than Nana, who got x marks.

a How many marks did Matt get?

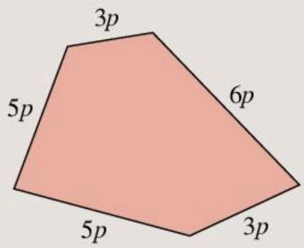
Percy got y marks more than Matt.

b How many marks did Percy get?

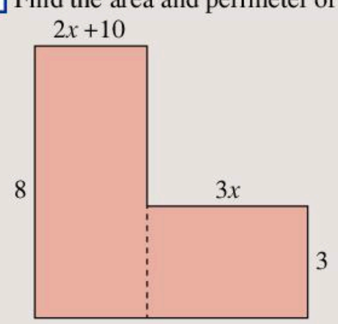
c How many marks did the three have altogether?

7 Lorne buys 5 books at x dollars each, and y books at 7 dollars each.
How much does she spend altogether?

8 Write an expression for the perimeter of this shape:



9 Find the area and perimeter of this shape:



Consolidation

Example 1

Simplify the algebraic expressions

a $3a + b + b - a + 4b$ **b** $6a \times 4b$

a $3a + b + b - a + 4b$
$= 3a + 2b - a + 4b$
$= 3a - a + 2b + 4b$
$= 2a + 6b$

b $6a \times 4b$
$= 6 \times a \times 4 \times b$
$= 24 \times a \times b$
$= 24ab$

Example 2

How many hours are there in

a x days **b** p weeks q days?

a There are 24 hours in 1 day so there are $24 \times x$ hours $= 24x$ hours in x days.

b Number of hours in a day $= 24$
Number of hours in a week $= 24 \times 7$
$= 168$
Hours in p weeks q days $= 168 \times p + 24 \times q$
$= 168p + 24q$

Example 3

Write expressions to represent these situations.

a Rhoda scored 17 marks on her first maths test and x marks on her second test.

b The perimeter of a rectangle of length 5 and width y.

a $17 + x$

b Perimeter is the distance around the outside of a shape
$= 5 + y + 5 + y$
$= 10 + 2y$

Example 4

a Expand $3(4x - 5)$

b Expand and simplify $3(4x + y) - 5(5x - 2y)$

a $3(4x - 5) = 3 \times 4x - 3 \times 5 = 12x - 15$

b $3(4x + y) - 5(5x - 2y)$
$= 12x + 3y - 25x + 10y$ Using BIDMAS expand Brackets first
$= {}^{-}13x + 13y$ Then Addition and Subtraction to simplify in any order

Exercise 2

1 Simplify:

a $b + b - a + a + b$

b $3a - 2b + 4a + 6b$

c $6a \times 12b$

2 How many minutes in

a 3 hours **b** x hours

c 1 day **d** y days

e 2 days x hours **f** y days 14 hours

g y days x hours?

3 What is the total value of

a 6 \$5 notes

b x \$5 notes

c y \$20 notes

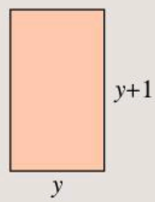
d y \$20 notes and 6 \$5 notes

e x \$5 notes and y \$20 notes?

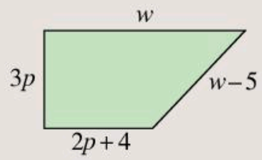
4 Write down the perimeter of these shapes.

a

b

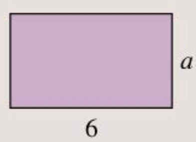


c

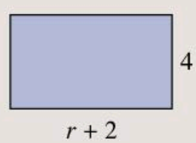


5 Write down the area of these rectangles.

a



b



6 The bus fare from the coast to the city is \$9 for adults and \$5 for children.

a What is the bus fare for 3 adults and 2 children?

b What is the bus fare for d adults and c children?

7 Expand and simplify where necessary

a $3(4p + 1)$

b $-6(3T + 5)$

c $4(3x - 2y) + 4(x + 4y)$

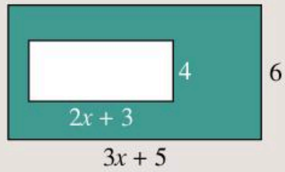
d $7(2m - 3) - 9(y - 1)$

e $10 + 2(R - 7) - 3R$

f $40 - 4(8n + 6) - 2n$

g $2(3c - 4b + d) - 3(3c + 2b - d)$

8 Write an expression for the green shaded area:



Summary

You should know …

1 You can add, subtract and multiply symbols.
For example:

$$x + x + x + y + y = 3 \times x + 2 \times y = 3x + 2y$$

2 You can write an algebraic expression to describe a situation.
For example:

John is 4 years old. In d years time, John will be $4 + d$ years old.

3 An algebraic **expression** is one which contains some letters instead of numbers.
For example:

$3x + 2y$, $4m$, and $5p + 23q - 3$

Parts of an expression are called **terms**.
For example:

$3x + 2y$, is an expression with two terms.

Like terms contain the same letters
For example:

$3p$, $-2p$ and p are like terms

An **equation** contains an equals sign to show that the expressions either side of it equal to each other.
For example:

$3x + 2 = 4$

Check out

1 Simplify

a $2a + 3a - 2b$

b $5x \times 2y$

2 a Mary has \$$2x$ and Lilly has 7 dollars more. How much money does Lilly have?

b Susie shares t cakes among her four friends.
How many cakes does each child receive?

3 a How many terms are there in these expressions?

i $2p - 4t + 5$

ii $0.6x$

b Are the following expressions or equations?

i $4b - 2$

ii $8x = 3$

iii $4A + W$

c Copy this list and underline the like terms:

$3t$, $2m$, $-t$, $5x$, $\frac{1}{2}t$, 7, $400t$, k, $5T$

4 You can expand brackets and use the order of operations with algebra.
For example:

$$\begin{aligned}&10 - 3(2x - 4) + 3x\\ &= 10 - 6x + 12 + 3x\\ &= 22 - 3x\end{aligned}$$

4 Expand

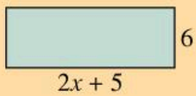
a $10(4r - 3)$

b $^{-}3(1 - 4x)$

c $2(5t - 4f) + 4(2t + 3f)$

d $20 - 3(5W + 2) - 4W$

5 You can work out the area and perimeter of shapes with unknown side lengths.
For example:



The area is length $\times$ width $= 6 \times (2x + 5)$

$6(2x + 5) = 6 \times 2x + 6 \times 5 = 12x + 30$

The perimeter is the distance around the outside

$= 6 + 2x + 5 + 6 + 2x + 5 = 4x + 22$

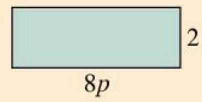
5 Work out

i the area and

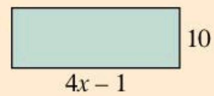
ii the perimeter of

these rectangles:

a



b



3 Shapes and constructions

Objectives

- Identify, describe, visualise and draw 2D shapes in different orientations.
- Use the notation and labelling conventions for points, lines, angles and shapes.
- Name and identify side, angle and symmetry properties of special quadrilaterals and triangles, and regular polygons with 5, 6 and 8 sides.
- Recognise and describe common solids and some of their properties, e.g. the number of faces, edges and vertices.
- Use a ruler, set square and protractor to:
  - measure and draw straight lines to the nearest millimetre
  - measure and draw acute, obtuse and reflex angles to the nearest degree
  - draw parallel and perpendicular lines
  - construct a triangle given two sides and the included angle (SAS) or two angles and the included side (ASA)
  - construct squares and rectangles
  - construct regular polygons, given a side and the internal angle.

What's the point?

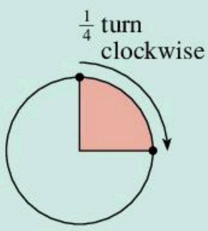
Wherever you look you will see angles and shapes. Your classroom may have a rectangular board, the hands of a clock make angles, food comes in boxes. Architects are just one group of professionals who use angles to design buildings of various shapes.

Before you start

You should know ...

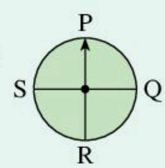
1 About whole turns, half turns, quarter turns, three-quarter turns, clockwise and anticlockwise.

For example:

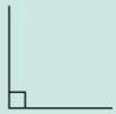


Check in

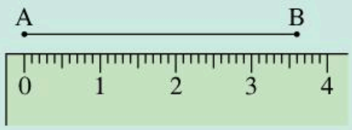
1 **a** Copy and complete:
After turning a half turn clockwise from P, the arrow will point to □.



2 A right angle looks like this:



3 How to measure lines:



The line AB = 3.6 cm

4 Parallel lines are always the same distance apart.

b Copy and complete the table:

After turning clockwise from P	The arrow will point to
i a quarter turn	
ii a whole turn	
iii a three-quarter turn	

2 Which of these shapes contain right angles?

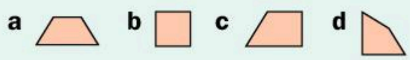
3 a Measure these lines:

i

ii

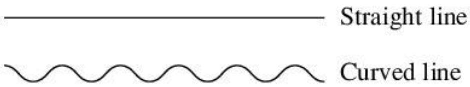
b Draw a line 6.2 cm long.

4 Which of these shapes have parallel lines?



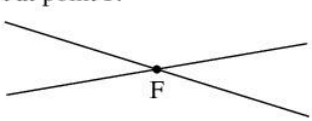
3.1 Lines and angles

Lines can either be straight or curved.



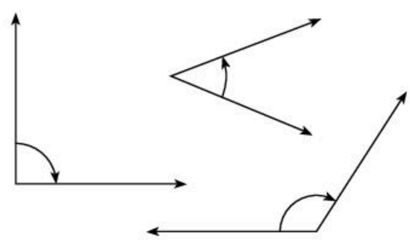
Two straight lines intersect in a **point**.

We usually name points with capital letters. These two lines intersect at point F.



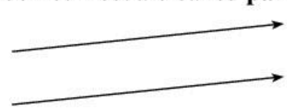
A straight line that extends from a point is called a **ray**, or often just a line.

Two rays and the point where they meet form an **angle**.



- An **angle** is a measure of turn.

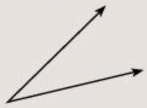
Two rays that do not meet are called **parallel** rays.



Exercise 3A

1 Look at each pair of rays below.
Is an angle formed? Explain.

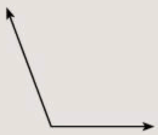
a



b

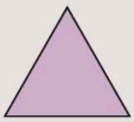
c

d

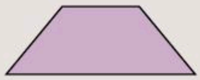


2 Write down the number of angles you can see inside each of these shapes.

a

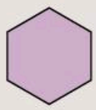


b



c

d



e

f

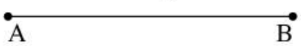
3 Write a list of five objects, from school and home, that contain angles.

Naming angles

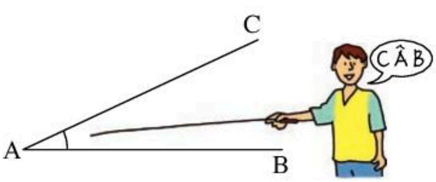
The usual way to name a line is to use the letters at the end points of the line.

A line joining two such points is called a **line segment**.

The picture shows the line segment AB.

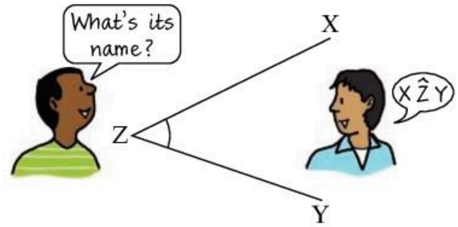


In the drawing below, the line segments AB and AC meet at the point A. An angle is formed.
A small curve is drawn between the lines to show the angle we mean.



The angle is called *angle* BAC, or *angle* CAB.
The letter at the point always goes in the middle.
Angle BAC is usually written in a short way as BÂC, or ∠BAC

In the drawing below, the lines XZ and YZ meet. The angle we are looking at is XẐY, or just Ẑ



Labelling shapes

To label a shape we use the points around the outside.
This rectangle is called shape ABCD.

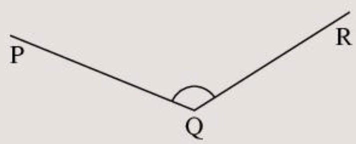
List the points in order.

Exercise 3B

1 Which of these are correct names for the angle below?

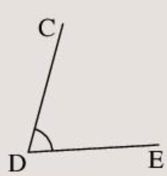
a QR̂P **b** PQ̂R **c** RP̂Q

d RQ̂P **e** PR̂Q

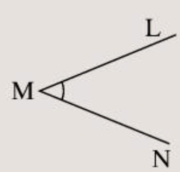


2 Write down the names of the lines that meet and the name of the marked angle in each diagram.

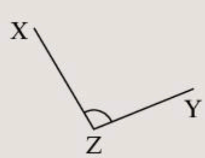
a



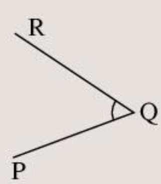
b



c



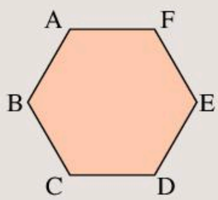
d



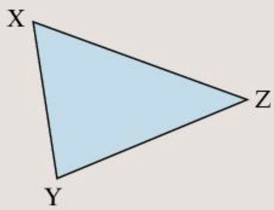
3 Label these shapes using the points:

a

b



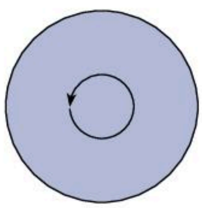
c



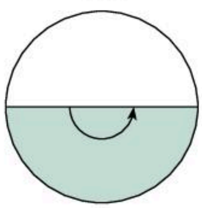
Some special angles

You can classify angles by the size of the turn:

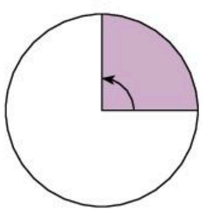
A complete turn



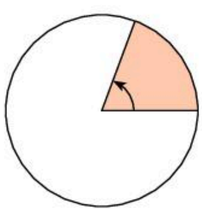
A $\frac{1}{2}$ turn is a **straight angle**



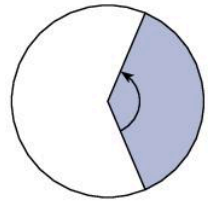
A $\frac{1}{4}$ turn is a **right angle**



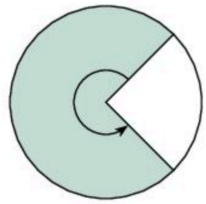
Less than a $\frac{1}{4}$ turn is an **acute angle**



Between a $\frac{1}{4}$ and a $\frac{1}{2}$ turn is an **obtuse angle**



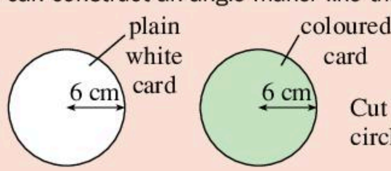
More than a $\frac{1}{2}$ turn but less than a whole turn is a **reflex angle**



ACTIVITY

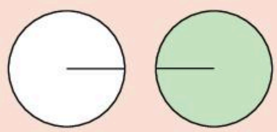
You can construct an angle maker like this:

1



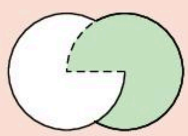
Cut out two circles.

2



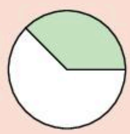
Cut a slit from the outside to the centre of each circle.

3



Slide the circles together so the centres meet.

4

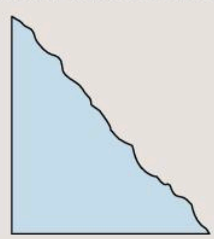


Rotate your angle marker to show different angles. Name each angle you make.

Exercise 3C

1 What is the name of the angle formed at the corner of this page?

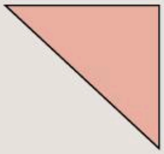
2 Tear off a corner from a sheet of paper.



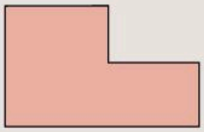
Use your square corner or right angle to find the number of right angles inside these shapes.

a

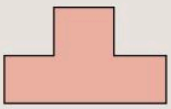
b



c



d



3 Draw shapes that have:

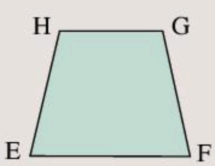
- **a** two right angles
- **b** three right angles
- **c** four right angles
- **d** five right angles
- **e** six right angles.

4 When two lines meet to form a right angle, they are called **perpendicular** lines.
Look at the shapes. Identify the lines that are

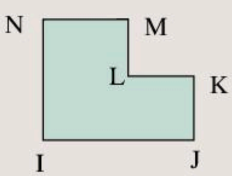
- **a** perpendicular
- **b** parallel.

i

ii



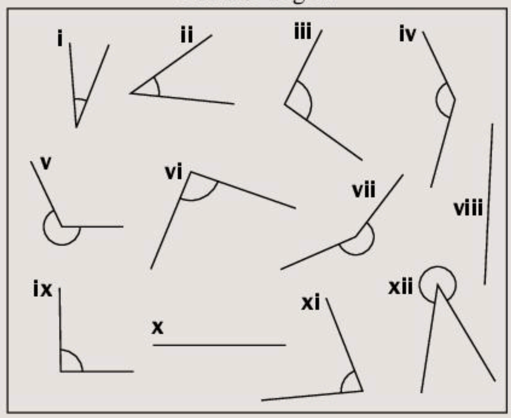
iii



5 Look at the set of angles below.

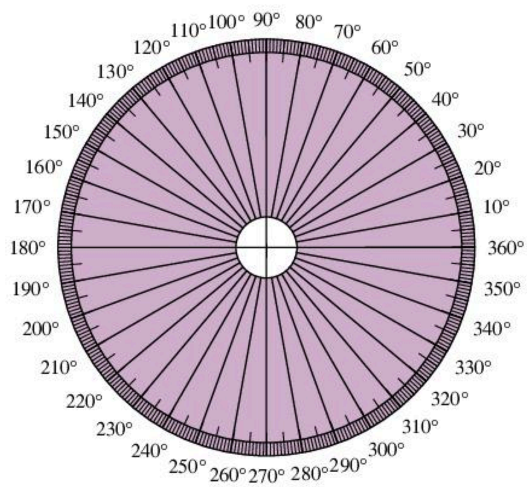
- **a** Which angles are right angles?
- **b** Which angles are straight angles?
- **c** Which angles are acute angles?
- **d** Which angles are obtuse angles?
- **e** Which angles are reflex angles?

A set of angles

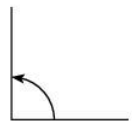


3.2 Measuring angles

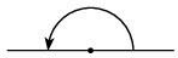
About 3000 years ago, the Babylonians thought of a good way to measure angles.
They divided a complete turn into 360 equal parts because they thought there were 360 days in a year.



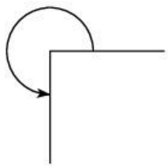
Each of these equal parts is now called a **degree**.
$\frac{1}{360}$ of a complete turn is a degree.
One degree is written as 1°.



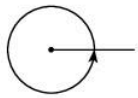
a quarter turn = 90°



a half turn = 180°



three-quarter turn = 270°



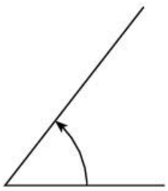
one whole turn = 360°

Dividing a complete turn into 360 equal parts helps you make good estimates of angle size.
You should estimate the size of an angle before measuring.

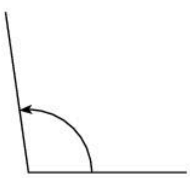
EXAMPLE 1

Estimate the size of these two angles.

a



b



a This is less than a $\frac{1}{4}$-turn or 90°.
It is about 50°.

b This is more than 90°, but not much more.
It is about 100°.

TECHNOLOGY

Forgotten all about angles? Have a go at the Kung Fu angles game in The Maths Zone at the website

www.woodlands-junior.kent.sch.uk

Start at level 1. Level 3 is for experts!

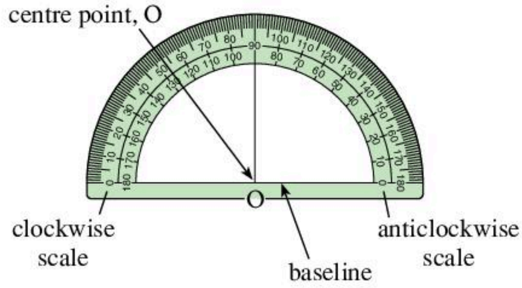
If you prefer something easier, try the Alien Angles game at

www.mathplayground.com

(Click on the 'Estimating Angles' link on the home page.)

Measuring angles accurately — using a protractor

To measure angles accurately, we use an instrument with the degrees marked on it called a **protractor**.

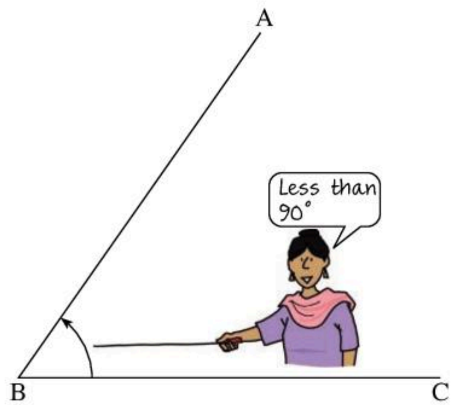


There are two scales; both read from 0 to 180°.

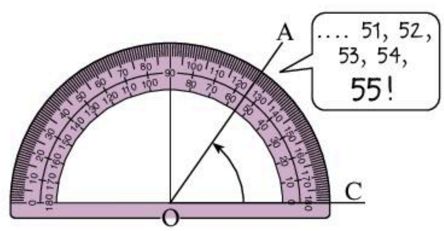
To decide which scale to read, check if the angle is larger or smaller than 90°.

EXAMPLE 2

Measure the angle ABC.

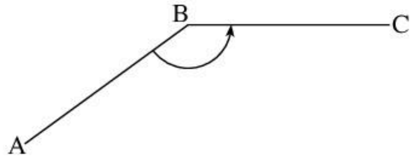


Put the protractor on the angle as shown. Make sure that the centre of the protractor, O, lies on the point B of the angle and that the baseline lies on BC.

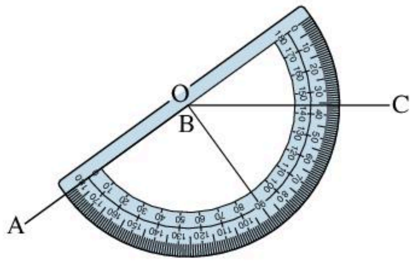


A$\hat{B}$C is less than 90°, so we can use the anticlockwise scale.
A$\hat{B}$C = 55°

The position of the angle does not matter:



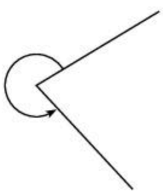
Just place the baseline on AB and make sure O lies on the corner of the angle B.



A$\hat{B}$C is larger than 90°, so we use the anticlockwise scale.
A$\hat{B}$C = 144°
(A clockwise scale reading would be 36°. This cannot be correct as the angle is larger than 90°.)

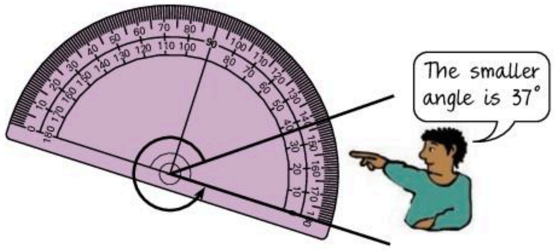
Angles larger than 180°

This angle is larger than 180°.

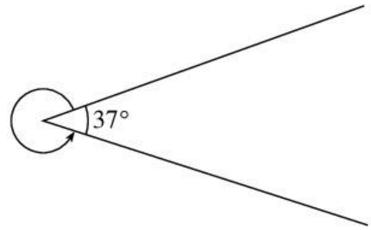


- An angle larger than 180° but smaller than 360° is called a **reflex** angle.

Your protractor cannot measure such a large angle. Instead you first measure the smaller angle.



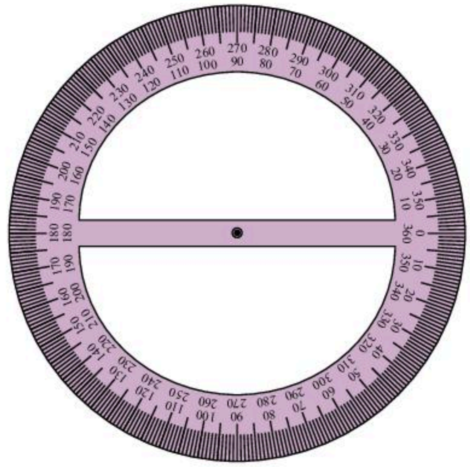
The two angles must make a complete turn or 360°.



The larger angle is then 360° − 37° = 323°.

An angle larger than 180° can easily be drawn by splitting it up.

Measuring reflex angles is easier if you have a full 360° protractor like this one:

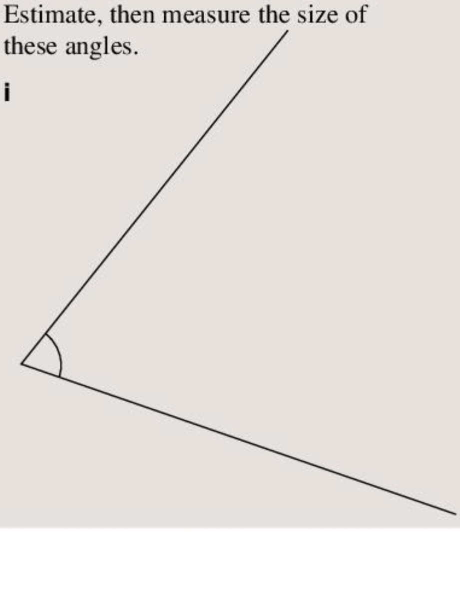


You can use the same method as for angles less than 180°. Place the centre on the point and make sure 0 lies on one of the lines. Then use either the clockwise or anticlockwise scale just as you did before.

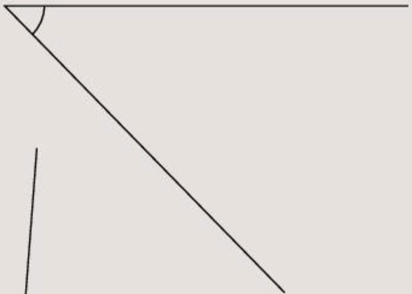
Exercise 3D

1 **a** Estimate, then measure the size of these angles.

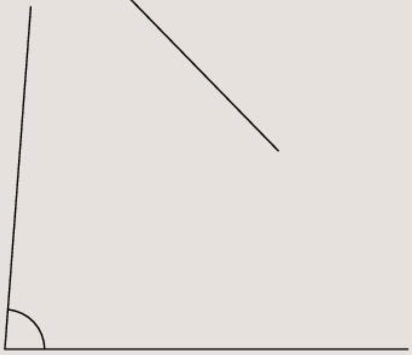
i



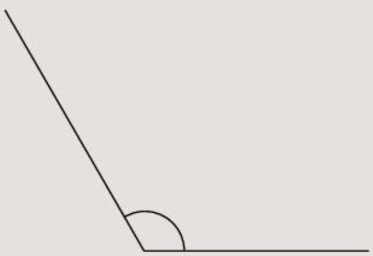
ii



iii



iv

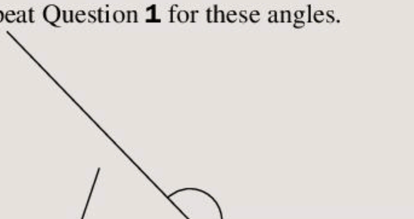


b Copy and complete the table for the angles in part **a**.

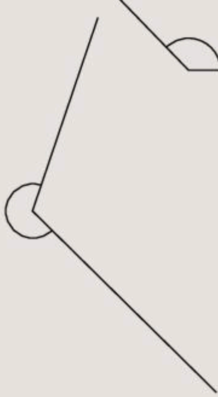
Angle	Estimate	Actual size
i		
ii		
iii		
iv		

2 Repeat Question **1** for these angles.

i

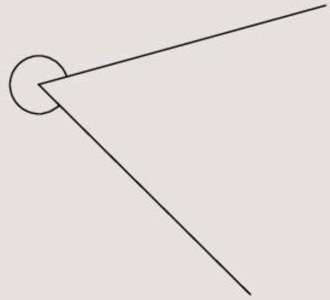


ii



iii

iv

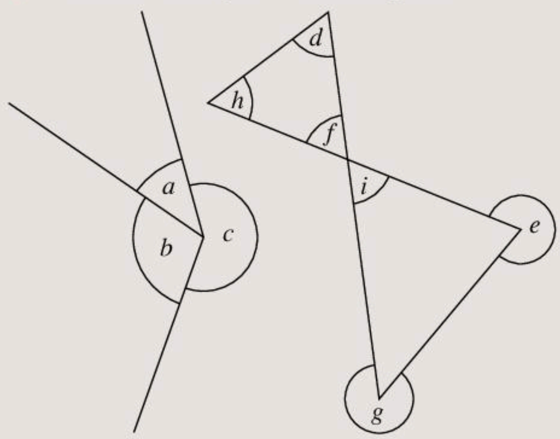


3 Using a ruler and pencil, draw an angle you think is:

a 10° **b** 30° **c** 60°
d 80° **e** 105° **f** 130°
g 145° **h** 280° **i** 315°

Now measure your angles. Did you guess well?

4 Measure the angles in these diagrams:



5 From the angles you measured in Question **4**:

a Look at angles *a*, *b* and *c*. Do you know what they should add up to? What do yours add up to?

b Look at angles *d*, *h* and *f*. Do you know what they should add up to? What do yours add up to?

c Look at angles *i* and *f*. What do you notice?

3.3 Drawing angles

A protractor can be used to draw angles as well as to measure them. You will need a protractor in this section, and a ruler.

Drawing an angle accurately is almost the reverse of measuring an angle.

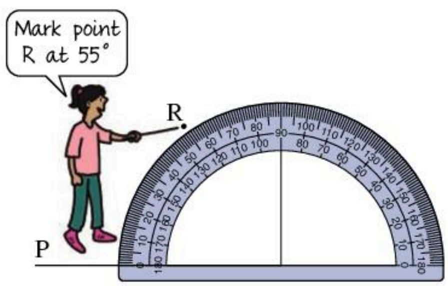
EXAMPLE 3

Draw the angle PQR = 55°.

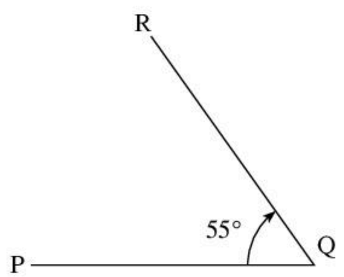
The drawing of angle PQR is made up of the two lines PQ and QR which meet at Q.

First draw the line PQ.

Place your protractor with its base line on PQ and its centre on Q, as shown:



Start at 0°, on the clockwise scale, and move around until 55° is reached. Mark that point R.



Finally join the point R to Q with your ruler.

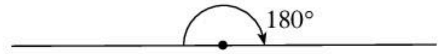
EXAMPLE 4

Draw accurately an angle of 235°.

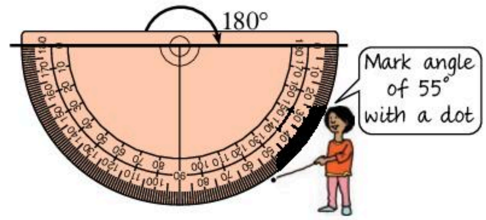
First split the angle up into a straight angle and its remainder.

235° = 180° + 55°

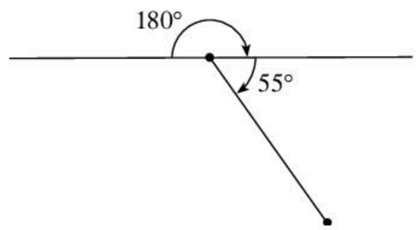
The 180° angle is easy to draw.
Mark a dot on the line to position your protractor.



Now add on an angle of 55°.



Remove your protractor and join your two dots.

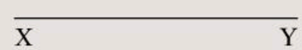


A total of 235°

You could also use your full 360° protractor to draw the angle 235°.

Exercise 3E

1 **a** Draw the line XY, as shown below.



b Now use a protractor to draw an angle $Z\hat{X}Y$ of 55°.
This time, where should you put the centre point O of the protractor?
Should you use the clockwise or anticlockwise scale?

2 For each of the following angles, decide whether it is acute, obtuse or reflex, then draw it.

a 60° **b** 78° **c** 335°
d 90° **e** 120° **f** 244°

3 Repeat Question **2** for:

a 177° **b** 10° **c** 212°
d 136° **e** 94° **f** 300°

4 You need to be able to draw lines accurately. Draw these lines with a sharp pencil:

a **i** 4.2 cm **ii** 3.7 cm
iii 2.1 cm **iv** 6.9 cm

b Ask someone to measure your lines to see how accurate you were.

5 Measure these lines to the nearest mm:

a ____________

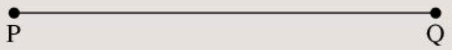
b ____________________

c ____________________________

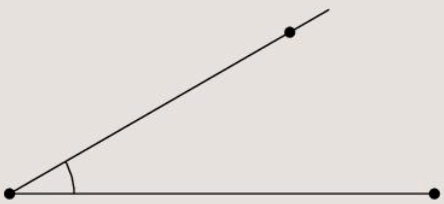
6 Without using a ruler, draw two points you think are 10cm apart. Now measure the distance between them with a ruler. Were you nearly right? If not, try again.

7 Repeat Question **6** for a distance of:
a 4.5 cm **b** 6.2 cm

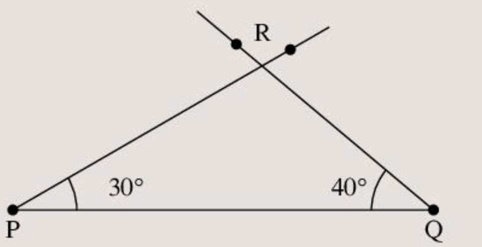
8 **a** Draw the line PQ = $5\frac{1}{2}$cm, as shown below.



b Draw a 30° angle at P.
Your drawing should look like this:



c At Q, draw a 40° angle.
Your drawing should now look like this:



d Label the point where the two lines meet R.

You have drawn the triangle PQR with

PQ = $5\frac{1}{2}$ cm, $\hat{P}$ = 30° and $\hat{Q}$ = 40°.

9 Draw these triangles in the same way as you did in Question **8**.

a Triangle PQR with PQ = 6 cm, $\hat{P}$ = 35° and $\hat{Q}$ = 50°.

b Triangle XYZ with XY = 6 cm, $\hat{X}$ = 60° and $\hat{Y}$ = 60°.

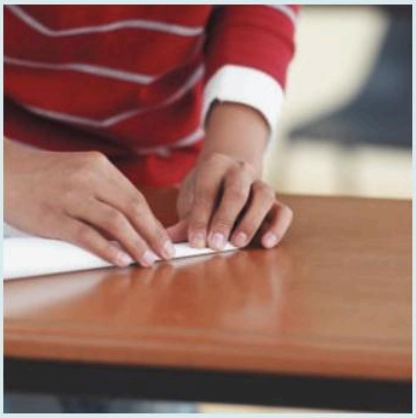
c Triangle ABC with AB = 4 cm, $\hat{A}$ = 90° and $\hat{B}$ = 37°.

INVESTIGATION

Using a rectangular piece of paper can you make an angle of

a 45°

b 60°?

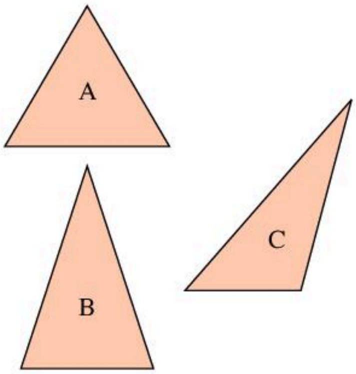


What other angles can you make?

3.4 Looking at triangles

You will need squared paper, a ruler and a protractor.

Measure the sides of each of these triangles.
Write down the measurements.



Triangle A has all its sides equal in length. It is called an **equilateral** triangle.

Triangle B has two sides equal. It is an **isosceles** triangle.

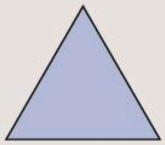
In triangle C there are no equal sides.

It is a **scalene** triangle.

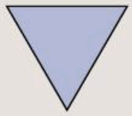
Exercise 3F

1 Here are two triangles.

i



ii



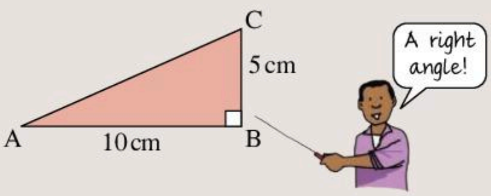
For both triangles:

a Measure the length of each side. What sort of triangle is it?

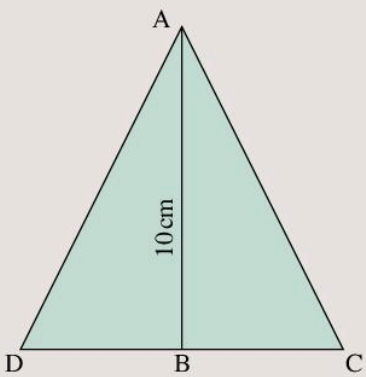
b Now measure each angle. What do you notice?

2 Write down two properties of an equilateral triangle.

3 **a** Cut out two triangles like the one shown below:



b Tape them together carefully along the 10 cm long side.



c Fold the triangle along AB. What do you notice about AD and AC? What type of triangle is ACD?

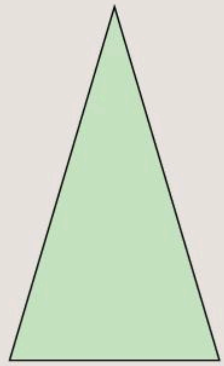
d Fold the triangle along AB again. What do you notice about $A\hat{C}B$ and $A\hat{D}B$?

4 For both of the following triangles:

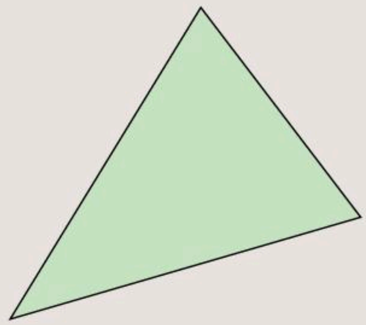
a Measure the sides. What sort of triangle is it?

b Now measure the angles with a protractor. What do you notice?

i



ii



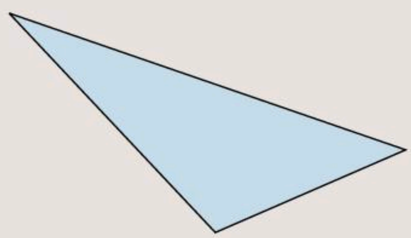
5 Write down the properties you have discovered for an isosceles triangle.

6 For both of the following triangles:

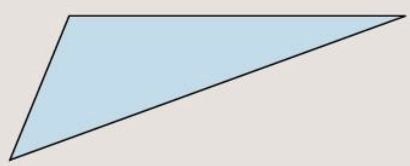
i Measure the edges. What sort of triangle is it?

ii Measure the angles. Are any of them equal?

a

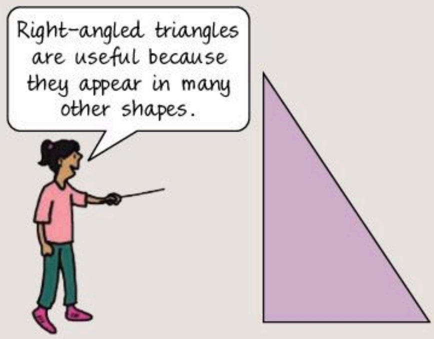


b

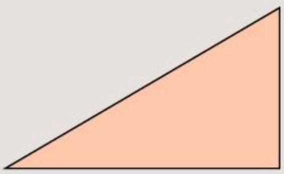


7 Write down two properties of a scalene triangle.

8 **a** Look at the triangle below.
It is called a right-angled triangle.
Can you see why?
b Measure its angles.
c Did you find one of those angles was a right angle?

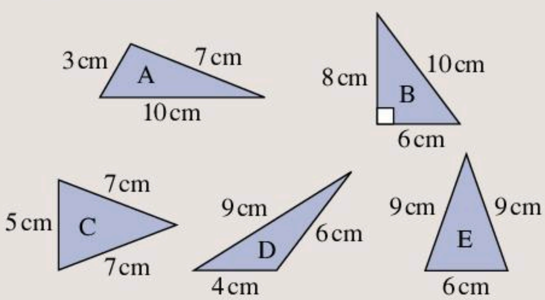


9 **a** Make two copies on card of this right-angled triangle.



b Cut them out.
c How many different shapes can you make using your two right-angled triangles?
d Make a list of all the things you notice about each shape.

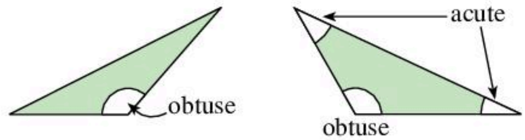
10 Which of these triangles are:
a right-angled
b isosceles
c scalene?



Triangles classified by angles

Obtuse-angled triangles have one obtuse angle and two acute angles.

For example:

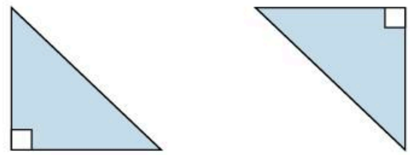


Acute-angled triangles have all three angles acute.

For example:

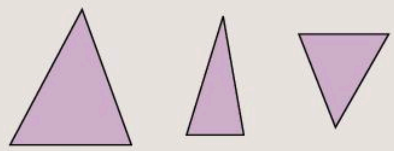
Right-angled triangles have one angle of 90°.

For example:



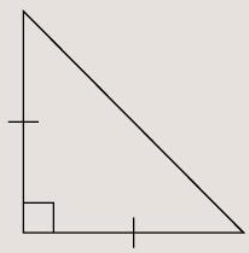
Exercise 3G

1 These are acute-angled triangles. What kind of angle is every angle in each triangle?



a **b** **c**

2 Look at this triangle:



a What do the two dashes on the edges mean?
b What does the little square in the bottom left corner mean?
c Using your answers to parts **a** and **b**, what is the name of this triangle?

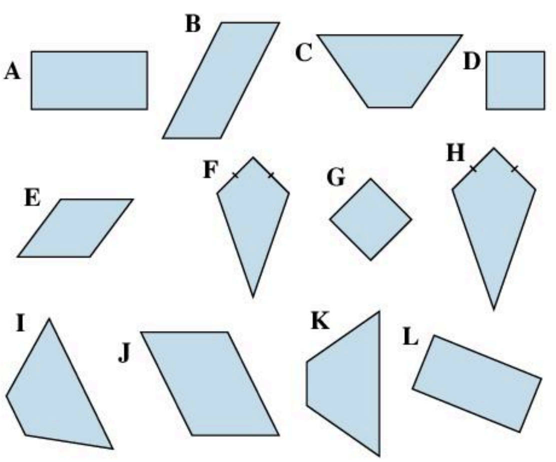
3 Copy and complete this table using ticks (✓) and crosses (×). The first row has been done for you.

		Type of triangle		
Triangle	Angles	Scalene	Right-Angled	Isosceles
ABC	40°, 100°, 40°	×	×	✓
DEF	40°, 90°, 50°			
RST	45°, 90°, 45°			
XYZ	45°, 80°, 55°			

4 Copy the following statements completing them with 'All', 'Some' or 'No'.

- **a** ... right-angled triangles are obtuse-angled triangles.
- **b** ... isosceles triangles are right-angled triangles.
- **c** ... equilateral triangles are isosceles triangles.
- **d** ... right-angled triangles are equilateral triangles.
- **e** ... acute-angled triangles are equilateral triangles.

3.5 Looking at quadrilaterals



Look at the shapes in the set above. They all have four sides. Four-sided shapes are called **quadrilaterals**.

For the next exercise you will need 12 card cut-outs of the shapes shown in the set above. Ask your teacher for a photocopy from the Teacher Book.

Exercise 3H

1 a Divide your set of quadrilaterals into two groups.
b How did you sort them?
c Sort your set using a different method.
d How did you sort them this time?

2 a Sort your quadrilaterals into three groups.
b How did you sort them?

3 Which of the shapes have four square corners?

4 Which shapes have just one pair of parallel sides?

5 Which shapes have two pairs of parallel sides?

6 Which shapes have two pairs of parallel sides and four square corners?

7 Which shapes have four sides all equal in length?

8 What is similar about shapes A, B, E and J?

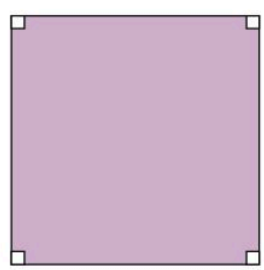
9 What is similar about shapes C, K and I?

10 What is similar about shapes F and H?

In Exercise 3H you should have found that some quadrilaterals have special properties.

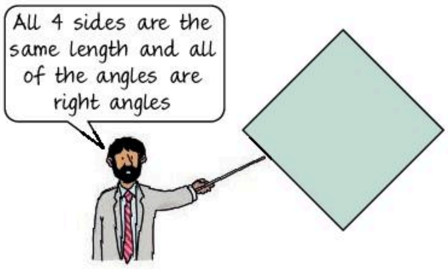
Rectangles and squares

A rectangle has four right angles.
So does a square.



How are they different?

Be careful. This is still a square!



Many people call it something different when the bottom edge is at an angle like this rather than horizontal.

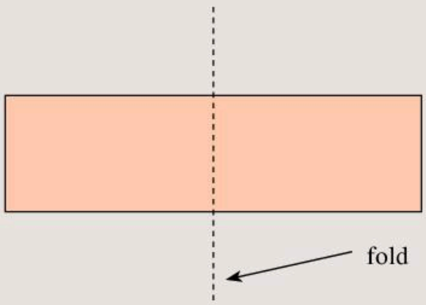
Exercise 3I

1 **a** Name five things that are rectangular in shape.

b Name five things that are square.

2 **a** Draw three different-sized rectangles and cut them out.

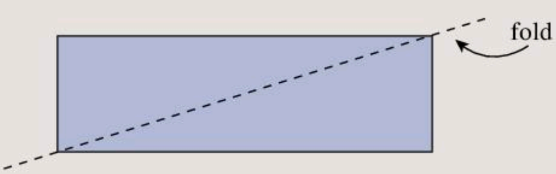
b Fold one of your rectangles down the middle.



What do you notice?

c Repeat for your other rectangles. What do you notice?

d Fold your rectangles along their diagonal.



What do you notice?

3 Repeat Question **2** for three different-sized squares.

4 **a** Measure both diagonals of one of your squares. What do you notice?

b Measure the angles between the diagonals of one of your squares. What do you notice?

5 Repeat Question **4** for one of your rectangles.

6 **a** Write down three properties of a rectangle.

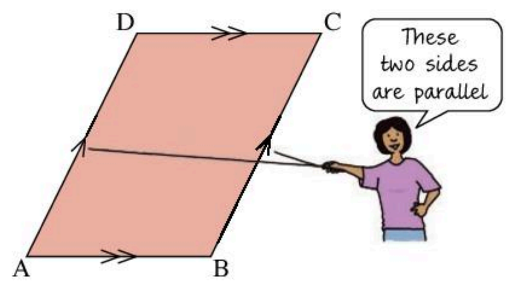
b Write down three properties of a square.

c How does a square differ from a rectangle?

7 Is it true to say that all rectangles are squares? Or all squares are rectangles? Or some squares are rectangles?

Parallelograms and rhombi

A **parallelogram** is a four-sided shape or quadrilateral with opposite sides parallel.

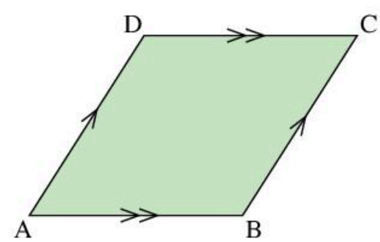


In the diagram, notice

- line segment AB is parallel to DC and the same length
- line segment AD is parallel to BC and the same length.

A **rhombus** is a special type of parallelogram.

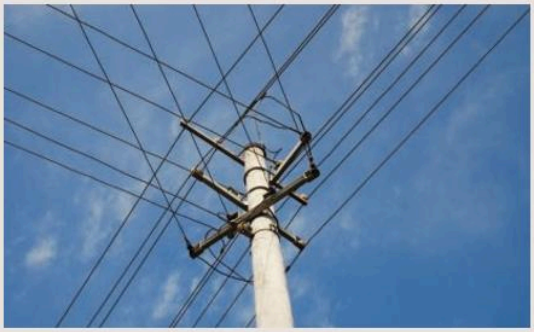
Rhombi is plural for rhombus.



It has all four sides equal in length.

Exercise 3J

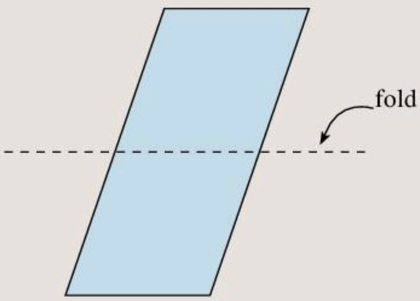
1 What type of quadrilateral do you see in the photograph?



2 Give some other examples where you may see parallelograms in or out of the classroom.

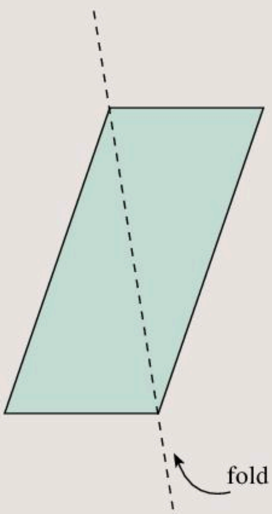
3 **a** Draw three different-sized parallelograms and cut them out.

b Fold your parallelograms down the middle.



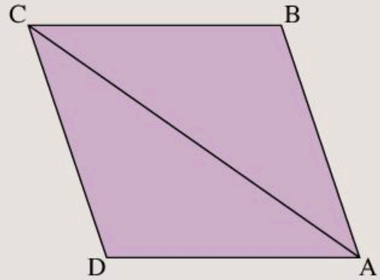
What do you notice?

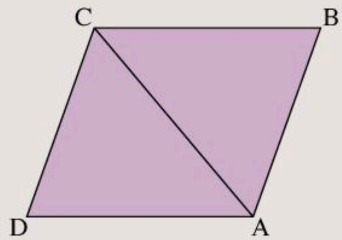
c Fold the parallelograms along a diagonal.



What do you notice?

4 **a** Look at these parallelograms.





In each case measure the angles

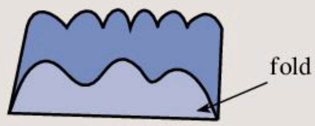
i $A\hat{C}B$, $D\hat{A}C$

ii $B\hat{A}C$, $D\hat{C}A$

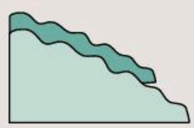
with your protractor.

b What do you notice?

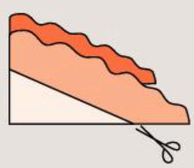
5 **a** Take a piece of paper and fold it in two.



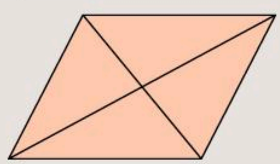
Fold it in two again.



Then cut off a corner ...



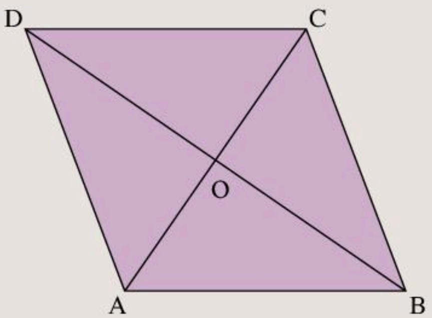
and open it out.



b What shape is it?

6 Use your protractor to measure each of the angles you made in Question **5**. What do you notice?

7



In the rhombus ABCD above:

a Measure the length of the diagonals AC and BD.

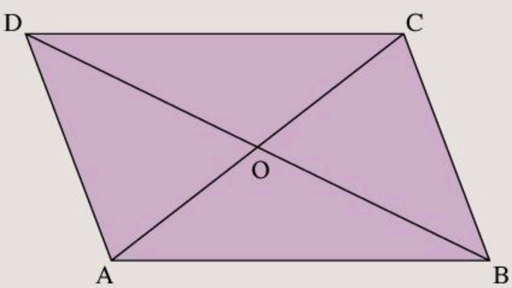
b If the diagonals meet at O, what can you say about

i AO and CO

ii BO and DO?

c Measure the angles AÔB and DÔC. What do you notice?

8 Repeat Question **7** for the parallelogram below.



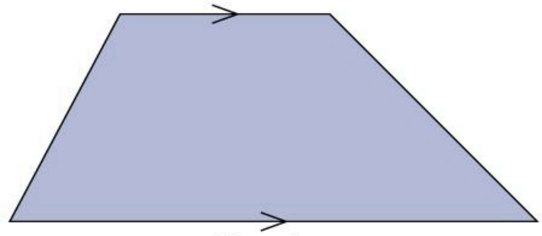
9 Copy the table and complete with ticks (✓) where appropriate.

Properties	Parallelogram	Rhombus
Opposite sides parallel		
Opposite sides equal		
All sides equal		
Opposite angles equal		
Diagonals equal		
Diagonals bisect each other		
Diagonals meet at 90°		

Other quadrilaterals

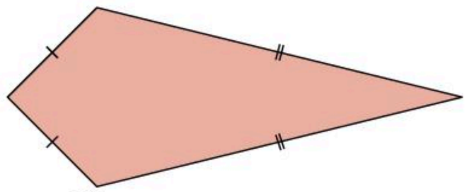
Two other special quadrilaterals are the **trapezium** and the **kite**.

A **trapezium** has just one pair of parallel sides.



Trapezium

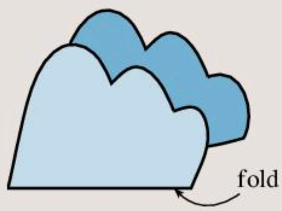
A **kite** has two pairs of adjacent sides that are equal in length.



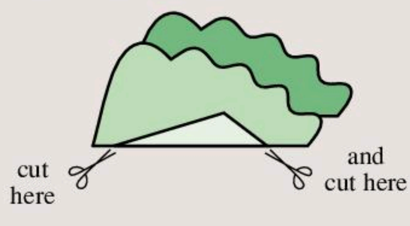
Kite

Exercise 3K

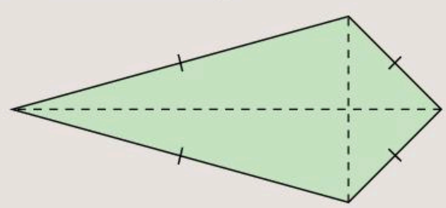
1 **a** Take a piece of paper and fold it in two.



b With a pair of scissors make two cuts across the fold line.



c Unfold the triangle.



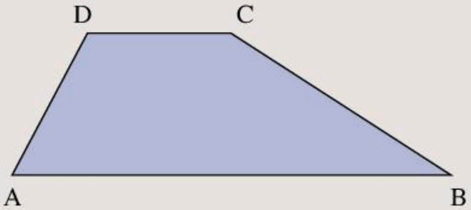
You should get a kite.

2 Take the kite you made in Question **1**.

- **a** Measure each of the sides.
- **b** Measure each of the diagonals.
- **c** Do the diagonals cut each other into two equal lengths?
- **d** Do the diagonals bisect each other at right angles?

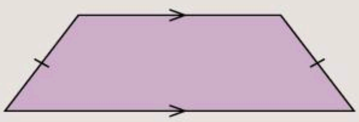
3 Make some more kites and repeat Question **2**. Do you get similar answers?

4



- **a** Measure each of the angles in the trapezium ABCD with your protractor.
- **b** Measure the length of each side of trapezium ABCD.
- **c** Did you notice anything special?

5 Look at this trapezium:



- **a** What do the two dashes on the side edges mean?
- **b** What is the name of this trapezium?

TECHNOLOGY

Need to review all this work about quadrilaterals? Visit

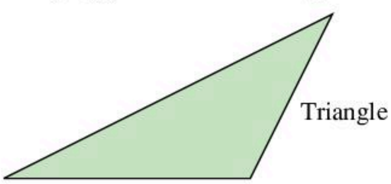
www.mathsisfun.com/geometry

Then try the Quadrilateral Quest in the Shapes section at www.woodlands-junior.kent.sch.uk/maths

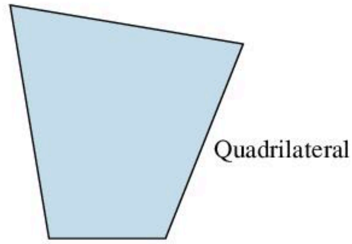
3.6 Polygons

A **polygon** is a closed shape with 3 or more straight sides.

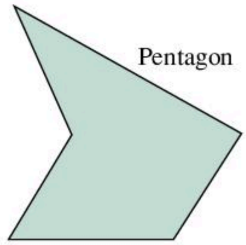
A three-sided polygon is called a triangle.



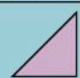
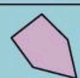
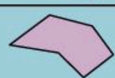
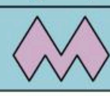
A four-sided polygon is called a quadrilateral.



A five-sided polygon is called a pentagon.

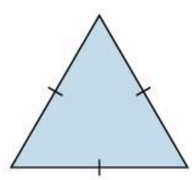


The easiest way to sort polygons is to check how many sides they have. See the table below.

Shape	Number of sides	Name of shape
	3	**Tri**angle
	4	**Quad**rilateral
	5	**Pent**agon
	6	**Hex**agon
	7	**Hept**agon
	8	**Oct**agon
	9	**Non**agon
	10	**Dec**agon

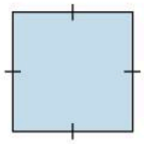
A **regular polygon** is one where all the angles and sides are the same length.

An equilateral triangle is a regular triangle:



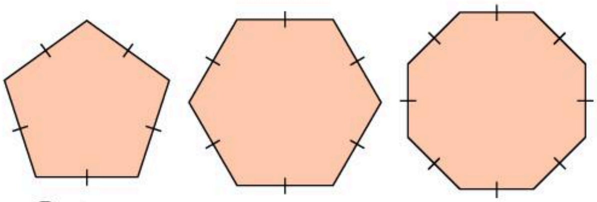
All the angles in an equilateral triangle are 60 degrees.

A square is a regular quadrilateral:



All the angles in a square are 90 degrees.

Three regular polygons you are going to focus on are:



For Exercise 3L you will need cut-outs of an equilateral triangle and a regular pentagon, a regular hexagon and a regular octagon. Ask your teacher for a photocopy of this resource from the Teacher Book.

Exercise 3L

1 Using your pentagon, hexagon and octagon can you fold them in half exactly? How many times can you do this for each shape?

2 Using your pentagon, hexagon and octagon can you turn them around so that they fit back on themselves exactly? How many times can you do this for each shape? (It might help to label the top of each with a 'T' so you don't lose count).

3 Repeat Questions **1** and **2** for your equilateral triangle.

4 Measure the angles in your pentagon, hexagon and octagon. Copy and complete the sentences filling in the correct number:

- **a** All of the angles in a regular pentagon are…….degrees
- **b** All of the angles in a regular hexagon are……. degrees
- **c** All of the angles in a regular octagon are……. degrees
- **d** Check the answers to **a**, **b** and **c** with your teacher. Were you accurate in your measuring? Correct these sentences in your book if you were not.

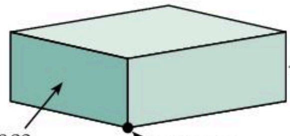
5 Using a protractor and ruler and the method described in Question **8** of Exercise 3E, draw an equilateral triangle with side lengths of 4cm.

Questions **1**, **2** and **3** from Exercise 3L are all about symmetry properties of polygons. You will learn more about this in Chapter 15.

All polygons are flat. They are called **two-dimensional** (2D) shapes. A solid shape is **three-dimensional** (3D).

3.7 Solid shapes

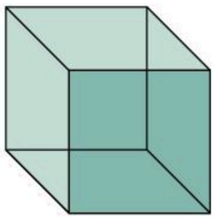
In 3D shapes each side is called a **face**. Two faces meet at an **edge**, and edges meet to form a **vertex** (or corner). The plural of vertex is **vertices**.



The next section is about properties of solid shapes to do with edges, vertices and faces.

The cube

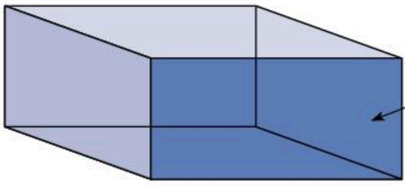
A **cube** has all square faces:



Sugar cubes, chicken stock cubes and dice are all examples of cubes.

The cuboid

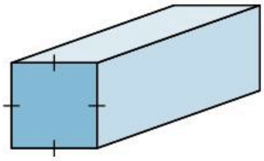
Box-like solids are called **cuboids**. Their six faces are rectangular.



A shoe box is an example of a cuboid.

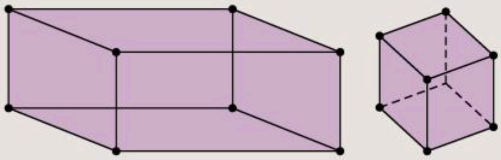
A cuboid can have square faces.

If a cuboid has 6 square faces it is a cube, if it has 2 square faces it is a cuboid:



Exercise 3M

1



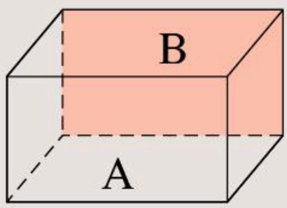
• = vertex
— = edge

How many vertices has a

a cuboid
b cube?

2 **a** How many edges does a cuboid have?
b How many edges does a cube have?

3 In the cuboid, the front face has been marked A. The face directly behind it (shaded in the diagram) has been marked B.



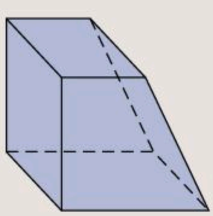
A and B are **opposite** faces.
How many pairs of opposite faces does a cuboid have?

4 Using a cuboid-shaped box (for example a shoe box), mark one pair of opposite faces A and B, as in the drawing for Question **3**. Mark the second pair C and D. Mark the third pair E and F.
Carefully cut out the faces. Fit face A on top of face B. What do you notice?
Repeat for the other two pairs of faces.

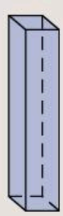
5 Make a list of all the properties you have discovered for a cuboid and a cube.

6 Which of these shapes are cuboids? Give reasons.

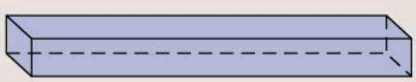
a



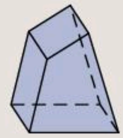
b



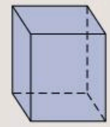
c



d



e



7 Are all cubes cuboids?
Are all cuboids cubes?
Give reasons for your answers.

The cylinder

These objects are cylinders.

Exercise 3N

1 Make a list of five other objects which are cylinders.

2 Look at the evaporated-milk tin.
a How many flat faces has it?
b How many curved faces has it?
c Are the flat faces opposite each other?

3 Repeat Question **2** for each of the cylinders. What can you say about the number of faces of a cylinder?

4 How many vertices does a cylinder have?

5 Choose a cylinder.
- **a** How many edges has the cylinder?
- **b** Are the edges straight or curved?
- **c** What shape are the edges?

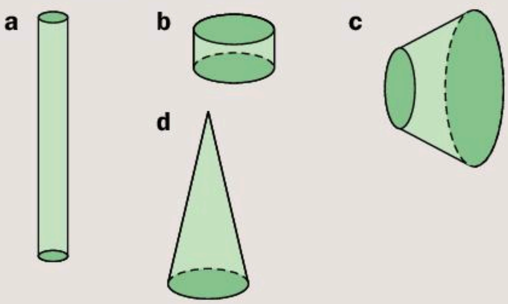
6 Mark a 'T' (for 'top') on one of the flat faces of a cylinder. Mark a 'B' (for 'bottom') on the other. Stand the cylinder on face B. Draw the outline of the face. What shape is it?

7 Now place face T of the cylinder in Question **6** on the outline for face B. Does it fit? What can you say about the two faces of a cylinder?

8 Repeat Questions **6** and **7** for the other cylinders in your set. Do you get the same answers each time?

9 Write down all the properties you have discovered for a cylinder.

10 Look at each shape below. Is it a cylinder? Give reasons for your answer.

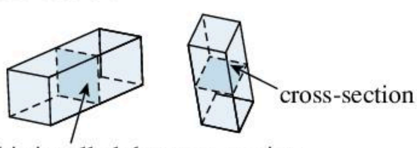


11 Look at a cylinder and a cuboid. Can you see any way in which they are alike?

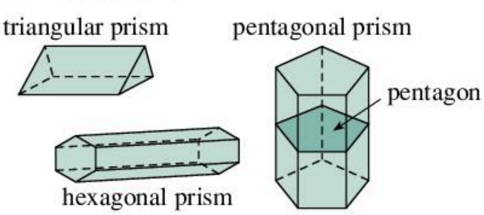
Prisms

Cuboids are special kinds of **prisms**.

- A **prism** is a solid shape with constant cross-section.

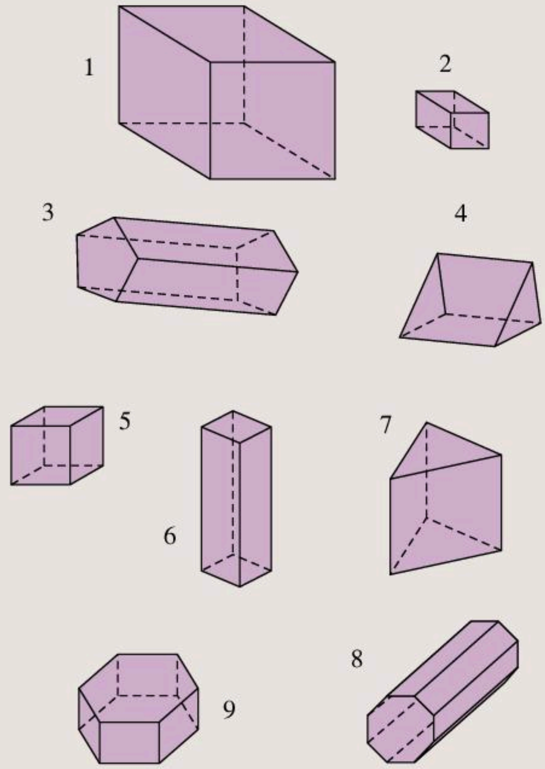


- You can identify and name a prism by the shape of its cross-section:



Exercise 30

1 Here are some prisms.



Which are
- **a** cuboids?
- **b** triangular prisms?
- **c** other prisms?

2 Are all cubes prisms? Are all prisms cubes? Give reasons for your answers.

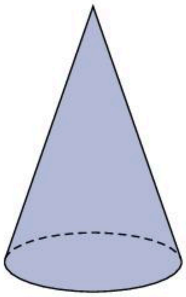
3 **a** Copy and complete the table for the prisms in Question **1**.

Shape	Number of vertices	Number of faces	Number of edges
1. Cuboid	8	6	
2.			
3.			
9.			

b Is there any relationship between the numbers of vertices, faces and edges?

The cone

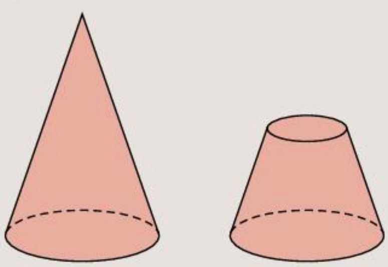
A **cone** looks like this:



Exercise 3P

1. How many edges does a cone have?
2. Stand a cone on a sheet of paper. Carefully draw the outline of the edge. What shape is the outline?
3. **a** How many curved faces does a cone have?
   b How many flat faces has it?
   c How many vertices?
4. Make a list of the properties you have discovered for a cone.
5. Look at a cone and a cylinder.
   a In what ways are they alike?
   b In what ways are they different?
6. Make three different sketches of a cone.
7. A cone with the top cut off is called a **truncated cone**.

   Here is a sketch of a cone and a truncated cone.

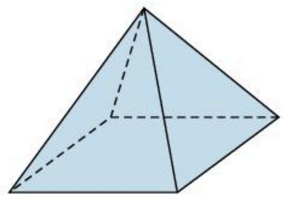
   

   a Write a list of three objects, which are cones.
   b Write a list of three objects, which are truncated cones.

The pyramid

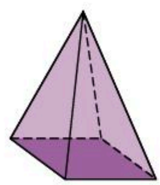
The pyramids at Giza in Egypt were built some 4500 years ago. They are, of course, shaped like pyramids.

This shape is a **pyramid**.

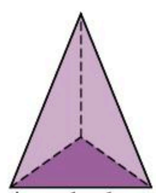


The base of this pyramid is a square.
It is called a **square-based pyramid**.

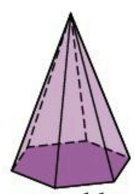
Here are some sketches of pyramids.
The base of each has been shaded.



square-based pyramid



triangular-based pyramid

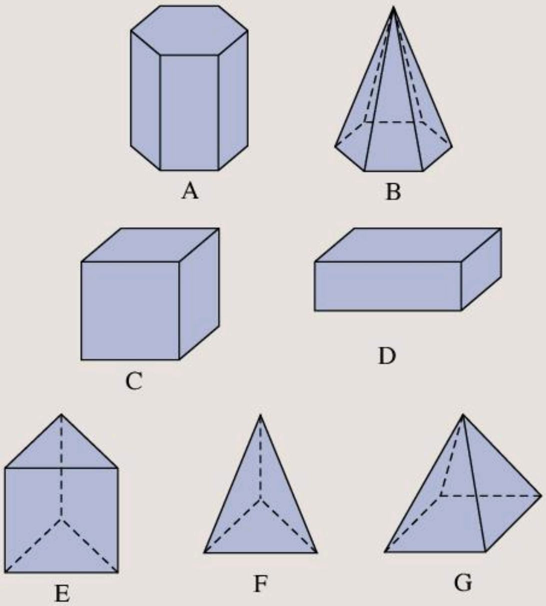


hexagonal-based pyramid

- You can name a pyramid by the shape of its base. A triangular based pyramid is also called a **tetrahedron**.

Exercise 3Q

1 Which of these shapes are
 a pyramids
 b prisms?



2 Which shapes in Question **1** have
 a more than 6 faces
 b more than 5 vertices
 c less than 12 edges
 d exactly 6 vertices
 e more than one triangular face
 f square faces?

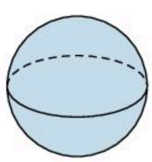
3 **a** Copy and complete the table for the 3D shapes in Question **1**.

Shape	Number of vertices	Number of faces	Number of edges
A		8	
G			

 b Is there any relationship between the numbers of vertices, faces and edges?

The sphere

- A **sphere** looks like a ball:



Exercise 3R

1 **a** Find a ball. Look at its outline. What shape is the outline?
 b Turn it around. Look at it again. Does the outline always look the same?

2 Make a sketch of a ball.

3 Look again at the ball.
 a How many flat surfaces has it?
 b How many curved surfaces has it?
 c How many edges?
 d How many vertices?

4 Write down the properties of spheres you have discovered.

5 Make a list of five objects that are spheres.

Properties of solids

The table summarises the properties of the different solids:

Solid	Faces	Edges	Vertices
Cube	6	12	8
Cuboid	6	12	8
Cylinder	3	2	0
Triangular prism	5	9	6
Cone	2	1	1
Square-based pyramid	5	8	5
Sphere	1	0	0

Three-dimensional shapes with flat faces that are all polygons are called **polyhedron**, or **polyhedra** for plural. A **regular polyhedron** has identical regular polygons as faces.

A cube is an example of a regular polyhedron as all of its faces are regular identical squares.

A cuboid is a polyhedron but not regular.

A cylinder is not a polyhedron as it has curved faces and faces that are not polygons.

Exercise 3S

1 Write down the name of any solid which
 a has a flat face
 b has a curved face
 c has a flat face and a curved face
 d has a pair of equal and opposite faces.

2 Write down the name of the shape of each object.
- **a** A piece of chalk
- **b** An orange
- **c** A drainpipe
- **d** A ten-cent coin
- **e** This book
- **f** A broom handle
- **g** A drinking glass
- **h** A globe
- **i** A candle
- **j** A football

3 What shape is the object?
- **a** The 'nose' of a rocket
- **b** A tomato
- **c** A church steeple
- **d** A record
- **e** A bicycle pump
- **f** A steel drum
- **g** The sharp end of a pencil
- **h** A new pencil, before it is sharpened

4 Draw pictures to show a shape that has
- **a** one vertex, one edge and two faces
- **b** six square faces
- **c** one curved face and no vertices
- **d** four vertices, four faces and six edges
- **e** one curved face and two edges.

Can you name these shapes?

5 Here is a list of solid shapes

Tetrahedron | Cone | Sphere | Cuboid

Hexagonal prism | Cylinder | Cube | Square based pyramid

- **a** Which of these solids are polyhedra?
- **b** Which of these solids are regular polyhedra?

3.8 Constructions

You will need a protractor and a pair of compasses as well as a ruler.

Some things to remember about constructions

- **i** The pencil you use should be sharp.
- **ii** Don't press the pencil too heavily on the paper.
- **iii** Measure and draw lengths and angles carefully.

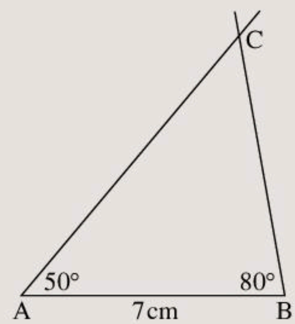
Given two angles and the included side (ASA)

You can draw triangles if you know two angles and the side length between them.

Exercise 3T

1 **a** Mark two points 7 centimetres apart. Join them with a straight line. Call the line AB.

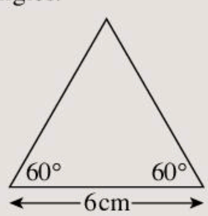
b At A, draw an angle of 50°. At B, draw an angle of 80°. Continue the lines to intersect at the point C, making a triangle ABC, as in the sketch.



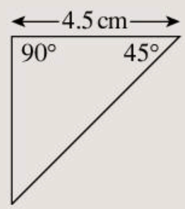
c Measure AC and BC. What sort of triangle have you drawn?

2 Make an accurate drawing of each of these triangles.

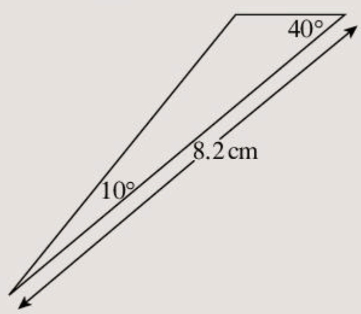
a



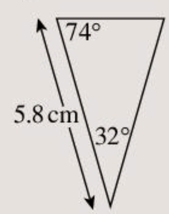
b



c



d



3 **a** Measure the unmarked angles in Question **2**.

b Name the type of each triangle you drew.

4 Construct these triangles:

a Triangle ABC where AB = 6.8 cm, $\angle BAC = 55°$ and $\angle ABC = 40°$

b Triangle DEF where DE = 7.4 cm, $\angle EDF = 110°$ and $\angle DEF = 42°$

Using compasses

To draw other sorts of triangles you will need a pair of compasses.

First you need to know how to draw a line accurately using compasses.

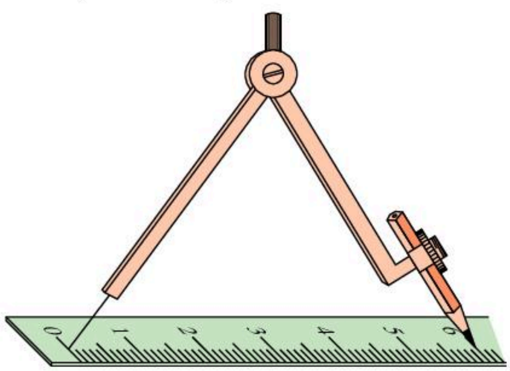
EXAMPLE 5

Draw the line AB exactly 6 cm long.

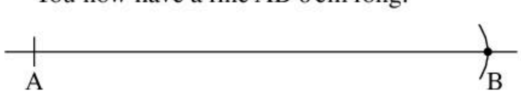
a First draw a line longer than 6 cm, say about 7.5 cm.

b Next, mark a point A near one end.

c Open the compasses to exactly 6 cm on your ruler, as in the diagram below.



d An arc is part of a circle. Put the point of the compasses at A, and draw a small arc to cut the line.
Call the point where the arc intersects the line B.
You now have a line AB 6 cm long.



Given two sides and the included angle (SAS)

You can construct a triangle if you know two side lengths and the angle between them.

Exercise 3U

1 Using compasses, draw the line:

a MN, 6 cm **b** PQ, 6.5 cm

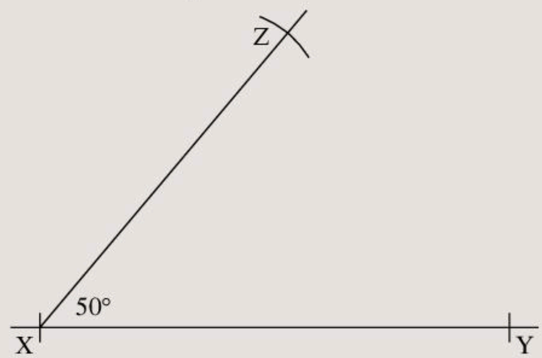
c ST, 5.8 cm **d** CD, 7.2 cm

e AB, 8.6 cm **f** GH, 7.9 cm

2 **a** Using compasses, draw the line XY, 6.2 cm.

b At X, draw an angle of 50° using your protractor.

c Open the compasses to 5.1 cm. With the point of the compasses at X, draw an arc 5.1 cm along the second arm of the angle, as in the drawing below.
Call the point Z.

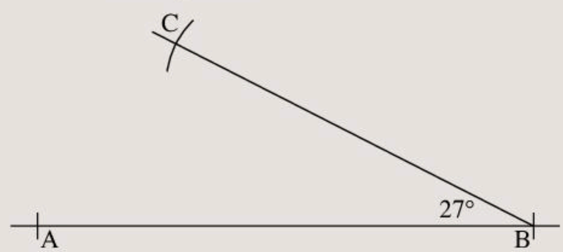


d Join YZ. What sort of triangle have you constructed?

3 **a** Draw the line AB, 7.3 cm.

b At B, draw an angle of 27°.

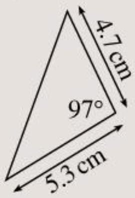
c With the point of the compasses at B, mark a point C, 5.9 cm from B, as in the drawing below.



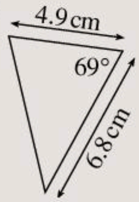
d Join AC. What sort of triangle have you constructed?

4 In Questions **2** and **3**, you could draw a triangle when you knew just two sides and the angle between them. Use the same method to make an accurate drawing of each of the triangles below.

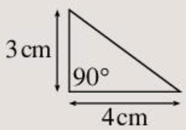
a



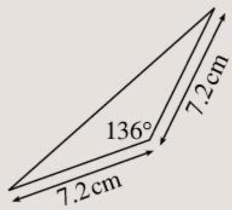
b



c



d



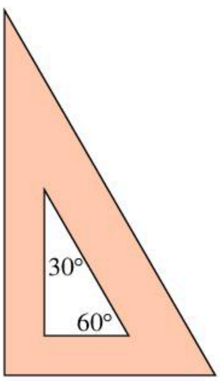
5 Construct these triangles:

a Triangle ABC where AB = 7.9 cm, AC = 9.5 cm and ∠BAC = 45°

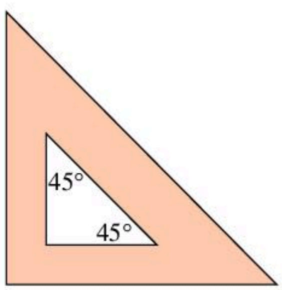
b Triangle DEF where DE = 6.7 cm, DF = 7 cm and ∠EDF = 71°

Using a set square

There are two types of **set squares** and they are named according to the angles.



60-30 set square



45 set square

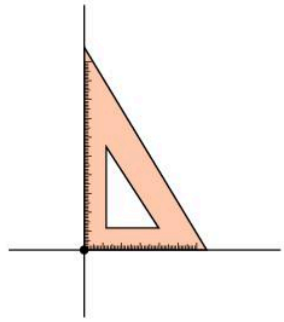
As seen earlier in this chapter, lines that are at right angles to each other are said to be **perpendicular lines**.

A **vertical line** is perpendicular to the **horizontal**, whereas **perpendicular lines** can be drawn in any position.

You can use a set square to draw perpendicular lines.

EXAMPLE 6

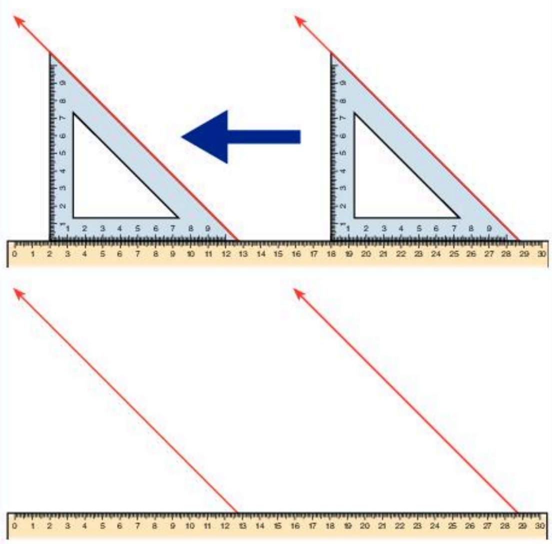
- Draw a line
- Place a set square on that line
- Draw a perpendicular line



You can also use a set square to draw parallel lines.

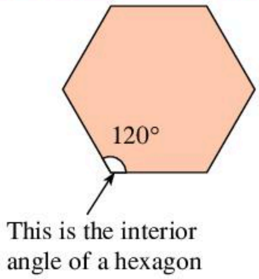
EXAMPLE 7

- Place an edge of the set square against a ruler and draw a line along one of the other edges.
- Slide the set square into a new position while keeping the ruler fixed exactly at the same position.
- Draw a line along the same edge that was used before to create the parallel lines.



Now that you can draw parallel and perpendicular lines using your set square you can draw squares, rectangles and parallelograms.

Constructing other regular polygons

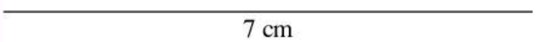


All the interior angles or a regular hexagon are the same. They are all 120°. All the sides are the same length. We can use this to help us draw a regular hexagon.

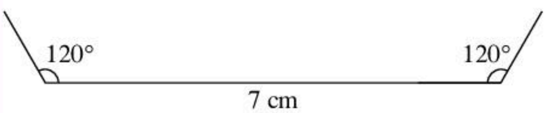
EXAMPLE 8

Draw a regular hexagon with side length 7 cm.

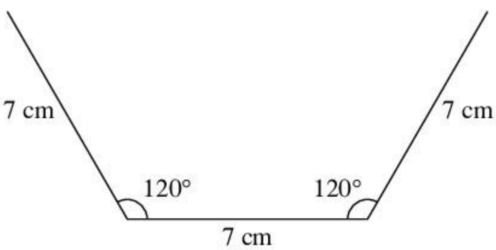
- Draw a base line of 7 cm



- Place your protractor on this line and draw an angle of 120° at each end of the line



- Extend these lines at 120° to 7cm long



- Turn your paper around and repeat this for each edge until you have drawn a hexagon

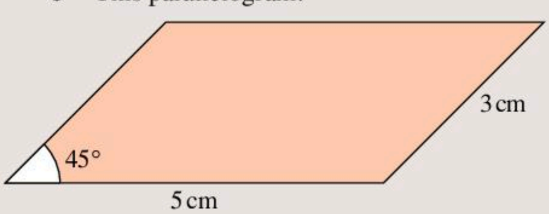
Exercise 3V

1 Using a ruler and a set square construct these shapes:

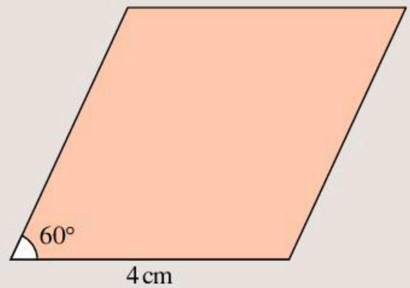
a A square with sides of 4 cm.

b A rectangle measuring 3 cm by 5.7 cm.

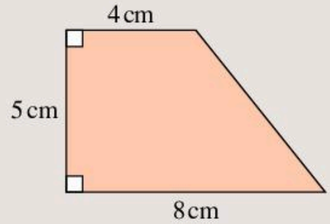
c This parallelogram.



d This rhombus.



e This trapezium.



2 Using a ruler and protractor, construct a regular octagon with side length 6 cm (the interior angle of a regular octagon is 135°).

3 **a** Using a ruler and protractor, construct a regular polygon with side length 7 cm and interior angle of 108°.

b What is the name of this polygon?

TECHNOLOGY

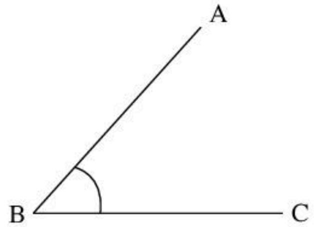
The method for drawing parallel lines with a set square (from Example 7) has a problem. You are limited with the angles you can make with your set squares. To learn how to construct parallel lines using a pair of compasses and a ruler go to the website
www.mathopenref.com
and look in the Constructions section.

Consolidation

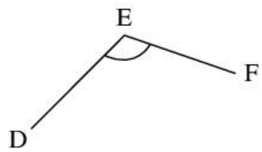
Example 1

a Estimate the size of the marked angles.

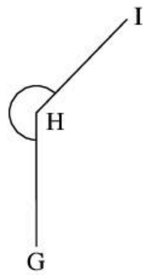
i



ii



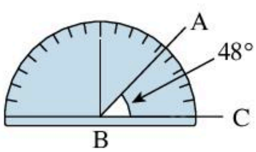
iii



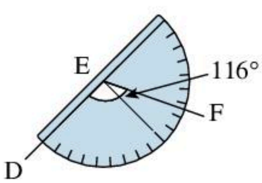
b Measure these angles with a protractor.

a **i** AB̂C is less than 90°, so estimate 40°.
ii DÊF is less than 180°, so estimate 120°.
iii GĤI is more than 180°, so estimate 22°.

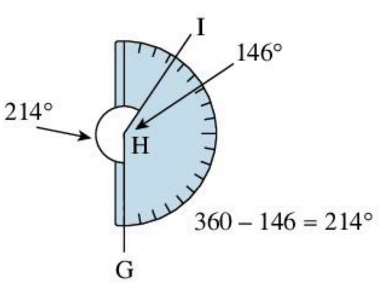
b **i**



ii



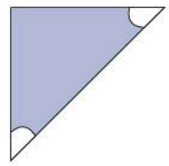
iii



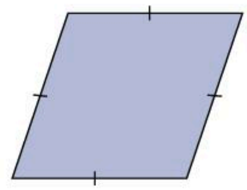
Example 2

Describe these triangles and quadrilaterals in terms of their properties.

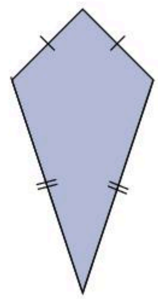
a



b



c



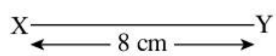
a The triangle has two equal acute angles. It is isosceles.
b The quadrilateral has two pairs of parallel sides and all sides equal. It is a rhombus.
c The quadrilateral has two pairs of adjacent sides equal in length. It is a kite.

Example 3

Construct the triangle XYZ with

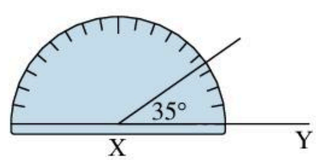
XY = 8cm, $Z\hat{X}Y = 35°$ and $Z\hat{Y}X = 65°$

1

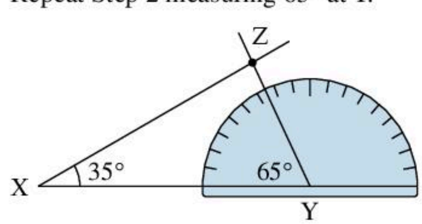


Draw base line XY 8 cm long.

2 Put the centre of your protractor at X then measure 35° at X.



3 Repeat Step 2 measuring 65° at Y.



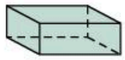
4 Put point Z where these lines meet.

Example 4

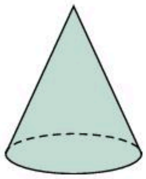
What are the names of these shapes?

a

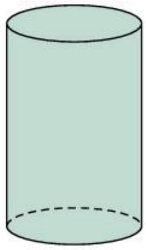
b



c



d



a Truncated cone
b Cuboid
c Cone
d Cylinder

Example 5

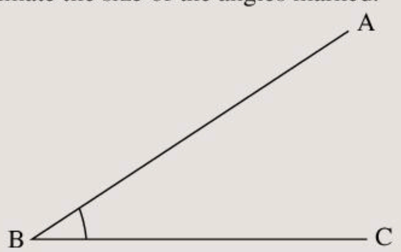
Write down three properties of
a a cube
b a cylinder

a	A cube has	**i**	6 square faces
		ii	8 vertices
		iii	12 edges
b	A cylinder has	**i**	2 flat faces
		ii	1 curved face
		iii	2 edges

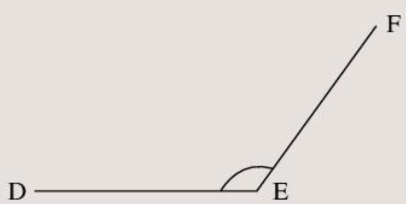
Exercise 3

1 **a** Estimate the size of the angles marked.

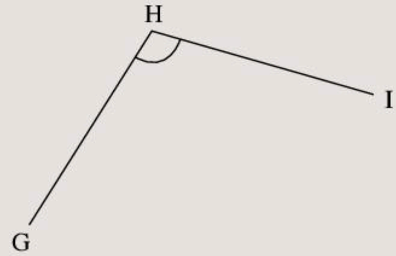
i



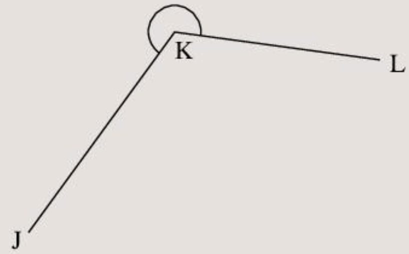
ii



iii



iv



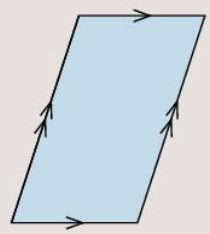
b Measure the angles from part **a** accurately with a protractor.
c How good were your estimates?

2 Use your protractor to draw angles of
a 30° **b** 37° **c** 52°
d 100° **e** 169° **f** 340°

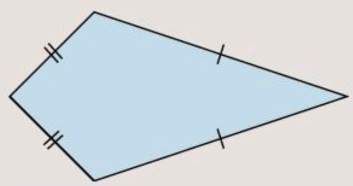
3 Name these shapes.

a

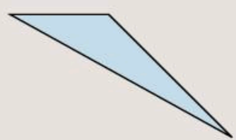
b



c



d

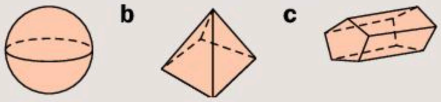


e

4 Construct these triangles:
a ABC, AB = 7 cm, $B\hat{A}C$ = 40, $A\hat{B}C$ = 60
b JKL, JK = 7.5 cm, $L\hat{J}K$ = 60°, JL = 6 cm

5 What are the names of these shapes?

a **b** **c**



6 Make sketches of these shapes:

a cylinder **b** cone
c triangular prism **d** sphere
e truncated cone **f** truncated pyramid

7 Sketch and name a shape that has

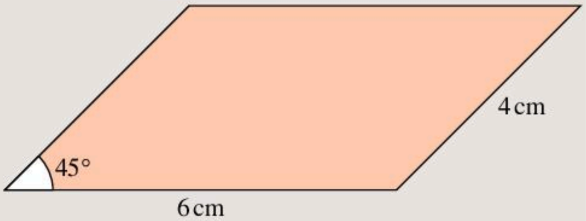
a 2 curved edges **b** 2 flat faces
c 8 vertices **d** 1 curved surface
e 4 triangular faces **f** 9 edges

8 Write down the properties of a

a cuboid **b** square-based prism
c truncated cone **d** triangular prism
e square-based pyramid
f hexagonal-based pyramid

9 Using a ruler and a set square, construct these shapes:

a A rectangle measuring 4 cm by 6.8 cm
b This parallelogram:

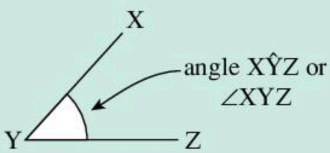


10 Using a ruler and protractor construct a regular hexagon with side length 7 cm (the interior angle of a regular hexagon is 120°)

Summary

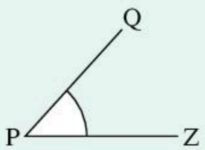
You should know …

1 You can use letters to name an angle:

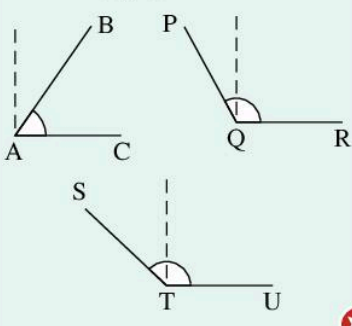


Check out

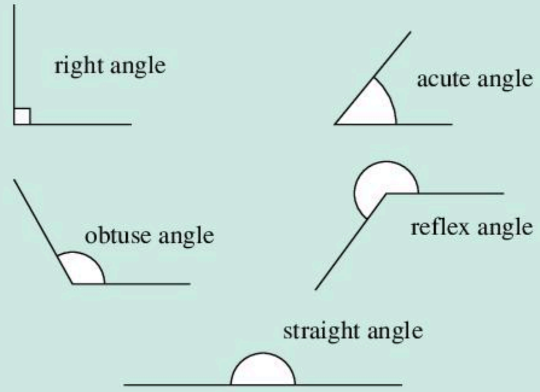
1 a Name this angle.



b Which is the largest angle?



2 You can classify angles by the amount of turn:

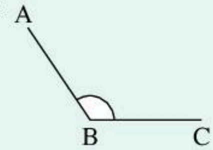


3 You can draw an angle using a protractor.

4 An angle can be measured in degrees, or ° for short. You use a protractor to measure angles in degrees.

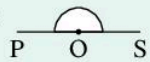
2 Classify these angles:

a

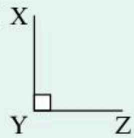


b

c



d

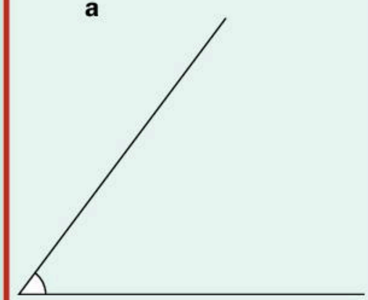


3 Draw the following angles using a protractor.

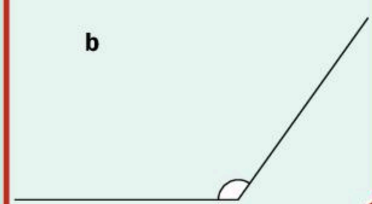
a 60° **b** 80°
c 120° **d** 160°
e 200° **f** 270°
g 180° **h** 150°

4 Use your protractor to measure these angles:

a

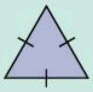


b

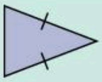


5 Triangles can be classified according to their sides:

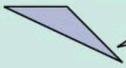
a An equilateral triangle



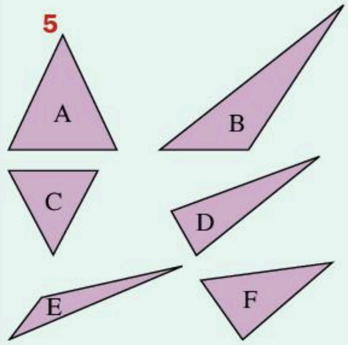
b An isosceles triangle



c A scalene triangle



5



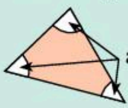
a Which triangles are equilateral?

b Which triangles are isosceles?

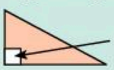
c Which triangles are scalene?

6 Triangles can also be classified by angles:

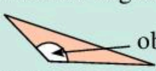
a acute-angled triangle



b right-angled triangle



c obtuse-angled triangle

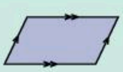


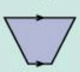
6 a Draw a right-angled triangle.

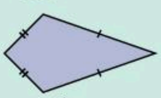
b Draw a right-angled isosceles triangle.

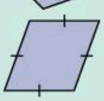
c Draw an obtuse-angled isosceles triangle.

7 Quadrilaterals can be sorted according to their properties:

a  A parallelogram has two pairs of parallel sides.

b  A trapezium has one pair of parallel sides.

c  A kite has two pairs of adjacent sides equal in length.

d  A rhombus has two pairs of parallel sides and all sides equal in length.

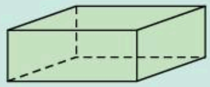
7 a Write down four properties of a square.

b Which quadrilateral has diagonals that

i are perpendicular

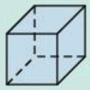
ii bisect each other?

8 A cuboid is a solid with six rectangular faces:



Properties: faces are rectangular
opposite edges are parallel and equal
it has 6 faces, 12 edges, 8 vertices

A cube is a special cuboid with six square faces:



8 Write down two examples of

a a cuboid

b a cube.

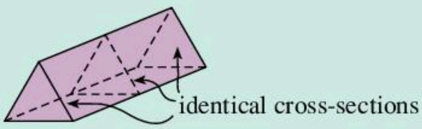
c How are cubes and cuboids the same?

d How are they different?

---

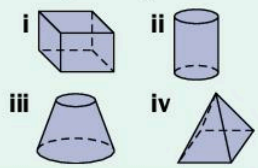
9 A cylinder is a solid with a circular cross-section:

A prism is a solid with a constant cross-section:



9 a Write down two examples of a cylinder.

b Which of these shapes are prisms?



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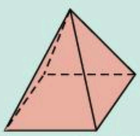
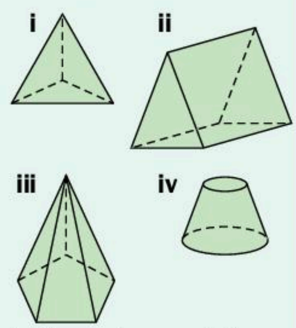
10 You can name a pyramid by the shape of its base:



square-based pyramid

10 a Which of these shapes are pyramids?



b For each pyramid in part **a**, give its full name.

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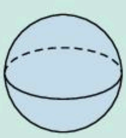
11 A sphere looks like a ball:



11 How many faces, edges and corners does a sphere have?

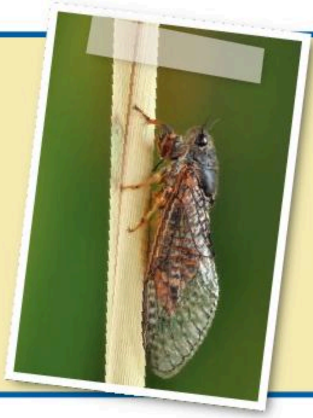
4 Number and calculation 2

Objectives

- Recognise multiples, factors, common factors, primes (all less than 100), making use of simple tests of divisibility; find the lowest common multiple in simple cases; use the 'sieve' for generating primes developed by Eratosthenes.
- Recognise squares of whole numbers to at least 20×20 and the corresponding square roots; use the notation 7^2 and $\sqrt{49}$.
- Use known facts and place value to multiply and divide two-digit numbers by a single-digit number, e.g. 45×6, $96 \div 6$.
- Know and apply tests of divisibility by 2, 3, 5, 6, 8, 9, 10 and 100.
- Know when to round up or down after division when the context requires a whole-number answer.

What's the point?

Cicadas live in the ground for a long time. Some species emerge after 13 or 17 years. These are both prime numbers. By emerging at these times, it makes it harder for predators with a shorter life cycle to adapt and kill them. Prime numbers help more cicadas to survive.



Before you start

You should know ...

1 Your multiplication and division facts from Chapter 1

Check in

1 **a** 7×8 **b** $63 \div 9$
c 6×7 **d** $36 \div 9$
e 8×8 **f** $48 \div 6$

4.1 Multiples and factors

Multiples

- The **multiples** of a number are all the numbers from its times table.

  For example, the multiples of 3 are 3, 6, 9, 12, …

Exercise 4A

1 **a** Write down the first six multiples of five.
b Write down the first six multiples of seven.

2 **a** What is the eighth multiple of six?
b What is the fifth multiple of twelve?
c What is the tenth multiple of nine?

3 Copy and complete:
a 4, 8, 12, 16, 20, □, □
b 9, 18, 27, 36, 45, □, □
c 12, 24, 36, 48, 60, □, □
d 16, 32, 48, 64, 80, □, □
e 63, 70, 77, 84, 91, □, □

4 Write down two numbers that are multiples of
a 2 and 3
b 3 and 5
c 4 and 6

5 How can you tell if a number is a multiple of 5?

6 What patterns can you find in
a multiples of 9 **b** multiples of 6?

INVESTIGATION

Look at the number grid below. It has ten columns and ten rows. The multiples of 3 have been coloured.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

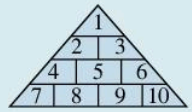
a Describe the pattern made.
b What happens if you use a number grid with only 5 columns?

1	2	3	4	5
6	7	8	9	10
11	12	13	14	15
16				

or 3 columns?

1	2	3
4	5	6
7	8	9
10	11	

Make different number grids and colour the multiples of 3. Do they all make patterns?
c What happens if you colour the multiples of
i 4
ii 5?
Investigate.
d What if you coloured multiples on a triangular grid?



Factors

There are two ways of putting six counters in rows:

1 row of 6 $= 1 \times 6$

2 rows of 3 $= 2 \times 3$

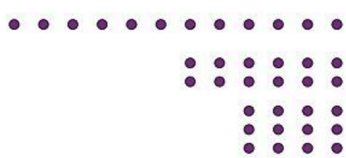
The numbers 1 and 6, and 2 and 3 are called factors of 6.

- The **factors** of a number are all the whole numbers that divide into it.
  For example, 1, 2, 3 and 6 all divide into 6, so they are all factors of 6.

EXAMPLE 1

Using counters, find all the factors of 12.

We can arrange a set of 12 counters in several ways like this:



1×12

2×6

3×4

There are no other row arrangements, so the factors of 12 are 1, 12, 2, 6, 3, 4.

Factors can also be found without using the counters, by dividing.

EXAMPLE 2

Find the factors of 30.

First divide by 1
$30 \div 1 = 30$
1 and 30 are factors because $1 \times 30 = 30$
Then divide by 2
$30 \div 2 = 15$
2 and 15 are factors because $2 \times 15 = 30$
Then divide by 3
$30 \div 3 = 10$
3 and 10 are factors because $3 \times 10 = 30$
If you divide by 4 you don't get a whole number, so 4 is not a factor.
Then divide by 5
$30 \times 5 = 6$
5 and 6 are factors because $5 \times 6 = 30$
The next number to divide by is 6, but you already have this as a factor so you know you have found all the factors of 30.
Finally, list the factors in order:
The factors of 30 are 1, 2, 3, 5, 6, 10, 15 and 30.

Dividing by each number in turn makes sure you get all factors without missing any out.

Exercise 4B

1 Find all the factors of
a 3 **b** 8 **c** 15
d 20 **e** 9

Write down how many different factors each number has.

2 Copy and complete the table started below, for the numbers 1 to 20.

Number	Factors	Number of different factors
1	1	1
2	1, 2	2
3		
4		
5		
6	1, 2, 3, 6	4
7		

3 a For the number 7 there is only one arrangement that can be made from 7 counters.

● ● ● ● ● ● ● 1×7

What other numbers between 2 and 20 can be arranged in only one row?

b How many factors do these numbers have?

4 Find three numbers bigger than 20 that have only two factors.

5 Which number has just one factor?

INVESTIGATION

Which number between 1 and 100 has the most factors?

Prime numbers

In Question **3** of Exercise 4B you should have found that 2, 3, 5, 7, 9, 11, 13, 17 and 19 can be arranged in only one row.
They all have just two factors.

- A number with exactly two different factors is called a **prime number**.

EXAMPLE 3

Find two prime numbers.

23 is a prime number.
It has exactly two factors: 1 and 23.
2 is a prime number.
It has exactly two factors: 1 and 2.

- A number which has more than two different factors is called a **composite number**.

EXAMPLE 4

Find a composite number.

12 is a composite number.
It has 6 factors: 1, 2, 3, 4, 6, 12.

- The number 1 is special.

1 is not a prime number (as it doesn't have two factors).
1 is not a composite number (as it only has one factor).

Exercise 4C

1 **a** **i** What are the factors of 35?
ii Is 35 a prime number?
b **i** What are the factors of 37?
ii Is 37 a prime number?

2 **a** Which numbers between 20 and 50 have just two factors?
b Write down all the prime numbers from 1 to 50.

3 Copy and complete these sentences:
a A number which has only two different factors, itself and 1, is called a … number.
b A number which has three or more different factors is called a … number.

4 The following grid shows all the numbers from 1 to 100. Make a larger copy of the grid on squared paper (or ask your teacher for a copy of this grid from the Teacher Book).

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

The **sieve of Eratosthenes** is a simple, ancient method for finding all prime numbers up to a specified integer. Instructions for how to use it are below.
a The first number in the grid is 1, cross it out as you know it is not prime.
b The next number in the grid is 2, circle it as you know it is prime.
c Cross out all of the rest of the multiples of 2 as these are composite numbers.
d The next number in the grid is 3, circle it as you know it is prime.
e Cross out the rest of the multiples of 3 as these are composite numbers.
f The next number (not already crossed out) is 5, circle it as you know it is prime.
g Cross out the rest of the multiples of 5 as these are composite numbers.
h The next number (not already crossed out) is 7, circle it as you know it is prime.
i Cross out the rest of the multiples of 7 as these are composite numbers.
j You should find that the rest of the numbers in your grid, that are not crossed out, can be circled. They are prime. (You could continue this method by circling 11 and crossing out all multiples of 11 etc. and extend your grid beyond 100 if you want.)

5 List all the prime numbers between 1 and 100.

6 **a** Look at your grid for Question **4**. How many times does a pair of primes occur together? Write down these primes.
b How many times do three primes occur together? Can you explain why?
c Apart from 2, what digits do all other primes end in?

7 3 and 5 differ by two and are both prime numbers.
a What is the next such pair?
b How many such pairs are there between 0 and 100?

8 **a** Is 613 a prime number?
How did you find out?
b Find out whether 4999 is a prime number.
c What about 30 031?

9 What is the biggest prime number currently known? (You may use the internet to find out.)

INVESTIGATION

The number 8 can be written as the sum of two prime numbers, 3 and 5:

$3 + 5 = 8$

The number 9 is the sum of the primes 2 and 7:

$2 + 7 = 9$

a Can all the numbers between 5 and 20 be written as the sum of two primes?
b Copy and complete the table where possible.

Number	Two primes equal to number
5	2 + 3
6	3 + 3
7	
8	

c Can any number be written as the sum of two primes?
Which ones can? Which ones cannot?
Any rules?

Prime factors

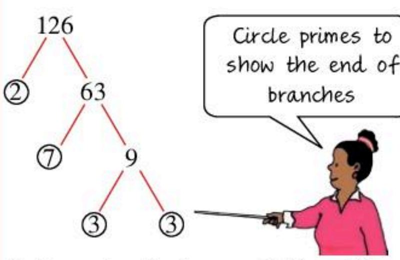
The factors of a number that are also prime numbers are called the **prime factors** of that number.

For example, the prime factors of 15 are 3 and 5.

One of the easiest ways of finding the prime factors of a number is to use a factor tree.

EXAMPLE 5

Find the prime factors of 126.

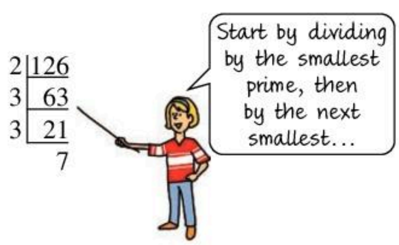


So the prime factors are 2, 3 and 7

You can also find the prime factors of a number by repeatedly dividing it by primes.

EXAMPLE 6

Find the prime factors of 126.



Prime factors are 2, 3 and 7.

Exercise 4D

1 Use a factor tree to find the prime factors of

a 12	**b** 16	**c** 24	**d** 36
e 46	**f** 48	**g** 64	**h** 65
i 72	**j** 84	**k** 136	**l** 196

2 Check your answers to Question **1** using the repeated division method.

3 A number has 2 and 3 as its prime factors. What are the five smallest values it could take?

4 What is the smallest number with
a two different prime factors
b three different prime factors?

5 What is the smallest number greater than 144 with four different prime factors?

Highest Common Factor (HCF) and Lowest Common Multiple (LCM)

Look at the factors of 20 and 30.

Factors of 20 = **1**, **2**, 4, **5**, **10**, 20
Factors of 30 = **1**, **2**, 3, **5**, 6, **10**, 15, 30

The **common factors** of 20 and 30 are 1, 2, 5, 10.

The **highest common factor** (HCF) is 10.

- The highest common factor of two (or more) numbers is the common factor that has the greatest value.

To find the **lowest common multiple** (LCM) of two numbers, you need to look at their multiples.

For example,
Multiples of 6 = 6, 12, 18, **24**, 30, 36, 42, **48**, ...
Multiples of 8 = 8, 16, **24**, 32, 40, **48**, 54, ...

The common multiples of 6 and 8 are 24, 48, ...
The LCM of 6 and 8 is 24.

- The lowest common multiple of two (or more) numbers is the smallest possible number into which all of them will divide.

Exercise 4E

1 **a** Write down all the factors of these number pairs:

i 12, 8	**ii** 18, 24	**iii** 36, 60
iv 48, 60	**v** 55, 80	**vi** 144, 108

b Find the common factors of each number pair.
c Find the HCF of each number pair.

2 **a** Write down the first eight multiples of these number pairs:
i 3, 5 **ii** 2, 7 **iii** 8, 12
iv 9, 12 **v** 12, 18 **vi** 8, 24
b Find the LCM of each number pair.

3 Find the HCF of these numbers:
a 21, 28, 42 **b** 12, 8, 32
c 36, 60, 84

4 Find the LCM of these numbers:
a 4, 5, 10 **b** 6, 9, 12 **c** 2, 4, 8

5 Think about the following scenario:

Cicadas emerge from the ground every 13 years and a cicada predator has a life cycle of 3 years, but is only large enough to kill a cicada during its third year of life. The predator is large enough to catch the cicada now and will next be large enough in 39 years.

Cicada emerges every 13 years: 13, 26, **39**, 52

The predator can catch when mature in the 3rd year of life: 3, 6, 9, 12, 15, 18, 21, 24, 27, 30, 33, 36, **39**, 42

a Copy and complete this sentence using 'HCF' or 'LCM' in the gap:
The cicada scenario is a real life example of using ……
b How many years after that will it be before the next predator will be large enough to kill the emerging cicadas?
c Imagine a predator had a 4-year life cycle but is only large enough to kill a cicada during its fourth year of life. If this predator is large enough to catch cicadas now, when would it be able to catch cicadas next?

6 One athlete runs around a track in 65 seconds. The second athlete takes 70 seconds. If they both start together
a when will the first 'lap' the second?
b how many laps will the first have completed when he 'laps' the second?

7 The HCF of two numbers is 12. What could these two numbers be? How many answers can you get?

TECHNOLOGY

Practise your skill at finding prime factors using a factor tree by visiting

www.mathgoodies.com/games

Need more practice or help? Visit

www.math.com

and click on the lessons dealing with factoring in the Pre-Algebra section.

4.2 Divisibility tests

Numbers which end in 0, 2, 4, 6 or 8 are even numbers. They are all divisible by 2.

Numbers which end in 1, 3, 5, 7 or 9 are odd numbers.

For example, 84, 206, 1118 are all even numbers and are divisible by 2 without remainder:

$$84 \div 2 = 42$$
$$206 \div 2 = 103$$
$$1118 \div 2 = 559$$

Complete Exercise 4F to find other divisibility rules.

Exercise 4F

1 **a** Write down twelve multiples of 10.
b What do you notice about these numbers?
c What is the rule for divisibility by 10?

2 **a** Write down twelve multiples of 5.
b What do you notice about these numbers?
c What is the rule for divisibility by 5?

3 **a** Write down twelve multiples of 4.
b Divide these numbers by 2. What do you notice about your results?
c What is the rule for divisibility by 4?

4 **a** Copy and complete the table:

Multiple of 3	Digit sum (single digit)
3	3
6	6
9	9
12	$1 + 2 = 3$
15	$1 + 5 = 6$
18	$1 + 8 = 9$
⋮	⋮
90	$9 + 0 = 9$

b What do you notice about the sum of the digits for multiples of 3?

c Write down five 4-digit numbers that are multiples of 3. What is the sum of their digits?

d What do you think is the rule for deciding if a number is divisible by 3?

5 Repeat Question **4** but this time look at multiples of 9.

6 $2 \times 3 = 6$
Use this fact to find the rule for divisibility by 6.

7 Look at multiples for other numbers. What divisibility rules can you find?

If you completed Exercise 4F carefully, you should have found these divisibility rules or tests:

Divisibility test	
$\div$ 2	Number ends in an even number
$\div$ 3	Sum of digits is a multiple of 3
$\div$ 4	Last two digits divisible by 4
$\div$ 5	Number ends in 5 or 0
$\div$ 6	Any even number with digit sum a multiple of 3
$\div$ 7	No test!
$\div$ 8	The number when halved has its last two digits divisible by 4
$\div$ 9	Sum of digits is a multiple of 9
$\div$ 10	Number ends in 0
$\div$ 100	Number ends in 00

Divisibility tests can speed up calculations.

EXAMPLE 7

Is 237 a prime number?

237 is odd, it is not divisible by 2.
237 has digit sum: $2 + 3 + 7 = 12$, which is a multiple of 3.

So 237 is divisible by 3 and is not prime.

Exercise 4G

1 Use divisibility tests to find out if
- **a** 1275 is divisible by 5
- **b** 3141 is divisible by 9
- **c** 21 648 is divisible by 8
- **d** 43 572 is divisible by 6
- **e** 38 520 is divisible by 10
- **f** 512 617 is divisible by 3
- **g** 48020 is divisible by 100
- **h** 418 is divisible by 2.

2 Check your answers to Question **1** by seeing if the numbers have remainders when you divide.

3 Use the divisibility tests to find out which of these numbers are prime.

a 81 **b** 97 **c** 67
d 117 **e** 111 **f** 127

4 Use the divisibility tests to help you complete factor trees for

a 448 **b** 729 **c** 6345
d 1024 **e** 12 640 **f** 72 144

INVESTIGATION

When you multiply a number ending in 5 by another number ending in 5 your answer also ends in 5.

For example

15 × 2**5** = 37**5**

What other endings have a similar property?

4.3 Squares and square roots

- To **square** a number you multiply it by itself.

  We can use **indices** to write it more easily.

  For example,

  $4 \times 4 = 4^2 = 16$
  $12 \times 12 = 12^2 = 144$
  $2.5 \times 2.5 = 2.5^2 = 6.25$
  $16 \times 16 = 16^2 = 256$

  3^2 is read as 'three squared'.
  x^2 is read as 'x squared'.

Square numbers come from squaring an integer (whole number). A square number can be shown as a picture of dots arranged in a square shape:

$2^2 = 4$ (2 rows by 2 columns)

$4^2 = 16$ (4 rows by 4 columns)

There is a special squaring key on your calculator, look for $\boxed{x^2}$

To work out 16^2 key in $\boxed{1}$ $\boxed{6}$ $\boxed{x^2}$ $\boxed{}$

You should get the answer 256.

Exercise 4H

1 Work these out without a calculator:

a 6^2 **b** 10^2 **c** 8^2
d 11^2 **e** 7^2 **f** 1^2

2 Work these out (you may use a calculator):

a 17^2 **b** 14^2 **c** 19^2 **d** 18^2

3 Copy and complete this list of square numbers:

$1^2 = 1 \times 1 = 1$
$2^2 = 2 \times 2 = 4$
$3^2 = 3 \times 3 = 9$
$4^2 = 4 \times 4 = 16$
$5^2 = 5 \times 5 = 25$
$6^2 = 6 \times 6 = 36$
$7^2 = 7 \times 7 = 49$
$\vdots \quad \vdots \quad \vdots$
$20^2 = 20 \times 20 = 400$

4 Copy and complete:

a $\square^2 = 36$ **b** $\square^2 = 100$
c $\square^2 = 225$ **d** $\square^2 = 324$

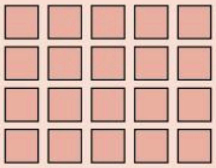
ACTIVITY

Square pairs memory game

Cut out twenty pieces of card, all of the same size.
Write on ten pieces of card the calculations
$11^2, 12^2, 13^2, \ldots, 19^2, 20^2$
Write on ten pieces of card the matching answers
121, 144, 169, …., 361, 400

(Or you can ask your teacher for a copy of these cards from the Teacher Book.)

Shuffle the twenty cards together and deal them face down on the table in a 4 by 5 arrangement like this:

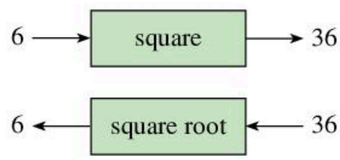


Play in a group with your friends. This is a memory game, the aim is to learn your square numbers between 11^2 and 20^2. This is also a memory test, to see if you can remember where cards are. Take it in turns to turn two cards face up to see if they are a pair. If they are not a pair, e.g. 11^2 and 400, turn them face down again in the place where they came from. Then it is the next player's turn. If they are a pair e.g. 11^2 and 121, you keep them and have another go. Keep going until there are no cards left. The winner is the person with the most pairs.

Square roots

The inverse of the square of a number is called its **square root** ($\sqrt{\ }$).

For example, as a flow chart:



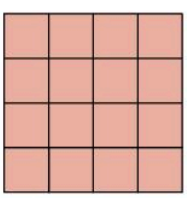
That is $\sqrt{36} = 6$

Square root means 'what number multiplied by itself makes this number'?

$\sqrt{25}$ means '? × ? = 25' (where both numbers are the same).

$5 \times 5 = 25$ so $\sqrt{25} = 5$

Square root also means 'what is the side length of a square when you know the area'?



There are 16 small squares in the picture, so the area is 16. $\sqrt{16}$ means what is the side length of this square?

The side length is 4 so $\sqrt{16} = 4$

Exercise 4I

1 Find the square root of each of these numbers:
a 49 **b** 81 **c** 121 **d** 169

2 In each list find the square number and work out its square root.
a 35, 36, 39, 42, 48
b 39, 49, 69, 89, 99
c 63, 64, 65, 74, 75

3 Find all the numbers less than 400 which have whole number square roots.

4 Copy and complete:
a $\sqrt{64} = \square$ **b** $\sqrt{1} = \square$
c $\sqrt{100} = \square$ **d** $\sqrt{400} = \square$
e $\sqrt{225} = \square$ **f** $\sqrt{4} = \square$

5 Work out:
a $\sqrt{289}$ **b** $\sqrt{196}$ **c** $\sqrt{1600}$
d $\sqrt{3600}$ **e** $\sqrt{8100}$ **f** $\sqrt{324}$
g $\sqrt{361}$ **h** $\sqrt{6400}$ **i** $\sqrt{900}$

Using a calculator

You can use the $\boxed{\sqrt{}}$ button on your calculator to find the square root of a number.

EXAMPLE 8

Find the square root of 25.

Press

$\boxed{2}\ \boxed{5}\ \boxed{\sqrt{}}$

to get $\sqrt{25} = 5$.

On more modern calculators you press the $\boxed{\sqrt{}}$ key first.

Press

$\boxed{\sqrt{}}\ \boxed{2}\ \boxed{5}$

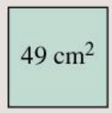
Exercise 4J

1 Using your calculator, find:
a $\sqrt{81}$ **b** $\sqrt{144}$
c $\sqrt{169}$ **d** $\sqrt{256}$

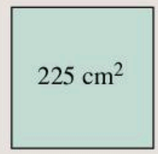
2 Work out:
a $\sqrt{49} + \sqrt{900}$ **b** $\sqrt{100} \times \sqrt{64}$
c $\sqrt{100} \div \sqrt{25}$ **d** $\sqrt{144} - \sqrt{81}$

3 Find the edge length of a square with the area shown:

a

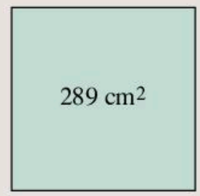


b



c

d



4 Copy and complete, using words and symbols from the list below:
a 9^2 is a __ way of writing __ __ __.
b The small raised __ is called the __ or the __.
c The symbol __ stands for the words __ __ __.
d The __ __ on a __ is pressed to find the __ of a number.

1. short	6. the square root
2. x^2 key	7. nine times nine
3. square	8. index
4. calculator	9. ^{2}
5. power	10. $\sqrt{}$

5 To work out the square root of a fraction you square root the top and bottom separately.

$$\sqrt{\frac{4}{25}} = \frac{\sqrt{4}}{\sqrt{25}} = \frac{2}{5}$$

Using this method work out:
a $\sqrt{\frac{9}{25}}$ **b** $\sqrt{\frac{49}{100}}$ **c** $\sqrt{\frac{36}{4}}$
d can you see a different way to do part **c**?

6 To work out $\sqrt{0.25}$ without a calculator you can write the decimal as a fraction. (The numbers in the fraction should be square numbers.)

So $\sqrt{0.25} = \sqrt{\frac{25}{100}}$ then square root the top and bottom separately.

$$\sqrt{\frac{25}{100}} = \frac{\sqrt{25}}{\sqrt{100}} = \frac{5}{10} = 0.5$$

Using this method work out:

a $\sqrt{0.81}$ **b** $\sqrt{0.16}$ **c** $\sqrt{0.04}$

4.4 Multiplying and dividing with two digit numbers

Multiplying

To multiply by 5 you multiply by 10 then halve your answer.

To multiply by 4 you double your number, then double again.

To multiply by 8 you double your number, then double again, then double again.

EXAMPLE 9

Work out

a 18×5 **b** 33×4 **c** 23×8

a $18 \times 5 = \frac{18 \times 10}{2} = \frac{180}{2} = 90$

b $33 \times 4 = 33 \times 2 \times 2 = 66 \times 2 = 132$

c $23 \times 8 = 23 \times 2 \times 2 \times 2 = 46 \times 2 \times 2$
$= 92 \times 2 = 184$

To multiply by other numbers you may want to use the distributive law. You learned about this in Chapter 1.

EXAMPLE 10

Work out 7×18

Using the distributive law

$7 \times 18 = 7 \times (10 + 8)$

$= 7 \times 10 + 7 \times 8 = 70 + 56 = 126$

Some people prefer to set out their working like this:

$$\begin{array}{rrr} & \mathbf{1} & \mathbf{8} \\ \times & & \mathbf{7} \\ \hline \mathbf{1} & \mathbf{2} & \mathbf{6} \\ & \cancel{5} & \end{array}$$

This comes from:

$$\begin{array}{rrr} & \mathbf{1} & \mathbf{8} \\ \times & & \mathbf{7} \\ \hline \mathbf{1} & \mathbf{2} & \mathbf{6} \\ & \cancel{5} & \end{array}$$

7 × 8 = 56, carry the 5 into

7 × 1 (ten) = 7 (tens) then add on the 5 (tens) from 56 to get 12 (tens)

Exercise 4K

1 Using the method from Example 9 work out:

a 24×5 **b** 32×5 **c** 17×5
d 41×4 **e** 27×4 **f** 16×4
g 14×8 **h** 31×8

2 Using either method from Example 10 work out:

a 36×7 **b** 74×6 **c** 53×8
d 68×9 **e** 98×3 **f** 88×6
g 89×5 **i** 76×4

3 If I buy 23 packets of sweets each containing 6 sweets, how many sweets do I have altogether?

4 A bus can travel 5 km on one litre of diesel. If Ahmed puts 52 litres of diesel in his bus how far can he travel?

We can use the distributive law to carry out **long multiplication** sums. This is multiplying 2 (or more) digits by 2 (or more) digits:

Work out 24×18

Using the distributive law: $24 \times 18 = 24 \times (10 + 8)$
$= 24 \times 10 + 24 \times 8$

Using the distributive law again: $24 \times 10 = 20 \times 10 + 4 \times 10$
$+ 24 \times 8 = 20 \times 8 + 4 \times 8$
$200 + 40 + 160 + 32 = 432$

Set out like this it is difficult to follow. Some people set this out using a grid. You partition 24 into 20 and 4 and partition 18 into 10 and 8:

×	20	4
10	**200**	**40**
8	**160**	**32**

Then add up **200** + **40** + **160** + **32** = 432

Some people prefer to set out their working like this:

$$\begin{array}{ccc} & 2 & 4 \\ \times & 1 & 8 \\ \hline & & 2 \\ & {}_{3} & \\ \hline \end{array}$$

$$\begin{array}{ccc} & 2 & 4 \\ \times & 1 & 8 \\ \hline 1 & 9 & 2 \\ & {}_{3} & \\ \hline \end{array}$$

8 × 2 (tens) = 16 (tens)

$$\begin{array}{ccc} & 2 & 4 \\ \times & 1 & 8 \\ \hline 1 & 9 & 2 \\ & {}_{3} & 0 \\ \hline \end{array}$$

Put in a 0 to show you are multiplying by 10

$$\begin{array}{ccc} & 2 & 4 \\ \times & 1 & 8 \\ \hline 1 & 9 & 2 \\ & 4^{3} & 0 \\ \hline \end{array}$$

Now do 1 (ten) × 4

$$\begin{array}{ccc} & 2 & 4 \\ \times & 1 & 8 \\ \hline 1 & 9 & 2 \\ 2 & 4^{3} & 0 \\ \hline \end{array}$$

Now do 1 (ten) ×

$$\begin{array}{ccc} & 2 & 4 \\ \times & 1 & 8 \\ \hline 1 & 9 & 2 \\ 2 & 4^{3} & 0 \\ \hline \end{array}$$

This row is 24 × 8

$$\begin{array}{ccc} & 2 & 4 \\ \times & 1 & 8 \\ \hline 1 & 9 & 2 \\ 2 & 4^{3} & 0 \\ \hline \end{array}$$

This row is 24 × 10

$$\begin{array}{cccc} & & 2 & 4 \\ & \times & 1 & 8 \\ \hline & 1 & 9 & 2 \\ + & 2 & 4^{\cancel{3}} & 0 \\ \hline & 4 & 3 & 2 \\ \hline & {}_{\cancel{4}} & & \end{array}$$

Now add these rows

Exercise 4L

1 Work out:

a 24 × 15 **b** 52 × 26

c 41 × 53 **d** 71 × 38

2 Work out:

a 316 × 23 **b** 235 × 64

c 108 × 18 **d** 345 × 28

3 How many minutes are there in a day?

4 A school has 36 forms each with 24 students in. How many students are there in the school?

5 A farmer buys food for his animals in 45kg bags. If he buys 23 bags in a week, what is the total mass of the food bought

a in a week

b in a year?

Factors can be used to work out other multiplications quickly.

EXAMPLE 11

What is 15 × 28?

$$\begin{aligned} 15 \times 28 &= 15 \times 2 \times 14 \\ &= 30 \times 14 \\ &= 10 \times 3 \times 14 \\ &= 10 \times 42 \\ &= 420 \end{aligned}$$

You can also use the doubling/halving method. When multiplying, if you double one number and halve the other you get the same answer:

EXAMPLE 12

Work out 14×16

$= 28 \times 8$
$= 56 \times 4$
$= 112 \times 2 = 224$

This is not always obvious. Look out for $\times$ 16, $\times$ 32 etc. as you can keep halving these until you get to 2.

Exercise 4M

1 Do you think the method used in Example 11 is quicker than long multiplication?

2 Use the method from Example 11 to work out

a 5×62 **b** 14×15
c 15×54 **d** 25×82
e 35×16 **f** 55×42
g 88×65 **h** 75×124

3 Use your knowledge of factors to find a quick way of working out $45 \times 65 \times 16$.

4 Using the doubling and halving method in example 12 to work out
a 13×16 **b** 18×32 **c** 17×160

Division

The multiplication 7×6 is really a repeated addition.

$7 \times 6 = 6 + 6 + 6 + 6 + 6 + 6 + 6$

In the same way, division can be thought of as a repeated subtraction.

For example $18 \div 6 = 3$ can be written as:

18	
−6	1 subtraction
12	
−6	1 subtraction
6	
−6	1 subtraction
0	3 subtractions in total

6 can be subtracted from 18 three times until nothing remains.

You can speed up the process by subtracting multiples of the number you subtract.

EXAMPLE 13

Work out $32 \div 4$

4	32		*Note:*
	− 20	5 fours	$5 \times 4 = 20$
	12		
	− 12	3 fours	$3 \times 4 = 12$
	0	8 fours	$8 \times 4 = 32$

8 fours can be subtracted from 32
So we can write the answer as:
$32 \div 4 = 8$

Example 13 could also be worked as:

4	32		
	− 16	4 fours	$4 \times 4 = 16$
	16		
	− 16	4 fours	$4 \times 4 = 16$
	0	8 fours	$8 \times 4 = 32$

Exercise 4N

1 Use repeated subtraction to work out

a $6 \div 2$ **b** $12 \div 4$
c $24 \div 6$ **d** $30 \div 5$
e $36 \div 18$ **f** $56 \div 14$
g $72 \div 24$ **h** $96 \div 16$

2 Work out Example 13 another way.

3 Use the method of Example 13 to work out

a $22 \div 2$ **b** $64 \div 4$
c $84 \div 6$ **d** $104 \div 4$
e $80 \div 5$ **f** $135 \div 5$
g $165 \div 15$ **h** $168 \div 14$

Question **3 f** can be worked out in many ways. Here are two:

i

5	135	
	− 50	10 fives
	85	
	− 50	10 fives
	35	
	− 35	7 fives
	0	27 fives

ii

5	135	
	− 100	20 fives
	35	
	− 35	7 fives
	0	27 fives

The method in **ii** is the shortest. For large numbers you will have to subtract multiples of 10 and 100.

EXAMPLE 14

Work out $938 \div 14$

14	938		*Note:*
	-700	50 fourteens	$50 \times 14 = 700$
	238		
	-140	10 fourteens	$10 \times 14 = 140$
	98		
	-98	7 fourteens	$7 \times 14 = 98$
	0	67	$67 \times 14 = 938$

So $938 \div 14 = 67$

Some people prefer to use a different method for **long division** which can be quicker.

EXAMPLE 15

Work out $8448 \div 24$

$1 \times 24 = 24$
$2 \times 24 = 48$
$3 \times 24 = 72$
$4 \times 24 = 96$

Write the first few multiples of 24 down the side of your page (you may need to extend this list). Do this easily by repeatedly adding 24.

$24\overline{)8448}$

Since 24 is a 2 digit number it won't go into 8 so look at the first two digits of 8448.

$1 \times 24 = 24$
$2 \times 24 = 48$
$3 \times 24 = 72$
$4 \times 24 = 96$

$$\begin{array}{r} 03 \\ 24\overline{)8448} \\ 72 \end{array}$$

72 is the largest multiple of 24, smaller than 84. $72 = 3 \times 24$. Write the 3 above the line and the 72 below the 84.

$$\begin{array}{r} 3 \\ 24\overline{)8448} \\ \underline{72} \\ 124 \end{array}$$

Work out the remainder by subtracting 72 from 84. This gives us 12. Bring down the 4 to make 124.

$1 \times 24 = 24$
$2 \times 24 = 48$
$3 \times 24 = 72$
$4 \times 24 = 96$
$5 \times 24 = 120$
$6 \times 24 = 144$

We are now dividing into 124. We need to extend our list of multiples of 24.

$$\begin{array}{r} 35 \\ 24\overline{)8448} \\ \underline{72} \\ 124 \\ 120 \end{array}$$

120 is the largest multiple of 24, smaller than 124. $120 = 5 \times 24$. Write the 5 above the line and the 120 below the 124.

$$\begin{array}{r} 35 \\ 24\overline{)8448} \\ \underline{72} \\ 124 \\ \underline{120} \\ 48 \end{array}$$

Work out the remainder by subtracting 120 from 124. This gives us 4. Bring down the 8 to make 48.

$1 \times 24 = 24$
$2 \times 24 = 48$
$3 \times 24 = 72$

We are now dividing into 48.

$$\begin{array}{r} 352 \\ 24\overline{)8448} \\ \underline{72} \\ 124 \\ \underline{120} \\ 48 \\ 48 \end{array}$$

48 is a multiple of 24, which is 2×24, write the 2 above the line and the 48 below the 48.

$$\begin{array}{r} 352 \\ 24\overline{)8448} \\ \underline{72} \\ 124 \\ \underline{120} \\ 48 \\ \underline{48} \\ 00 \end{array}$$

In this case there is no remainder when you subtract, since $48 - 48 = 0$.

So $8448 \div 24 = 352$

When dividing sometimes you must give whole number answers, even if there are remainders. This will happen in a certain context. You need to know when to round up and when to round down.

EXAMPLE 16

A box can hold 25 sweets, how many boxes do you need to pack 434 sweets?

$$\begin{array}{r} 17 \;\text{rem } 9 \\ 25\overline{)434} \\ \underline{25} \\ 184 \\ \underline{175} \\ 9 \end{array}$$

There will be 17 full boxes, but the last 9 sweets need to be in a box. Although the answer is closer to 17 than 18 we need to round up to 18 boxes otherwise some sweets will be left unboxed.

EXAMPLE 17

Making a pillow case requires 54 cm of fabric.
If I have 9 metres of fabric how many pillow cases can I make?

First convert 9 metres into 900 centimetres as units must be the same before dividing.

$$\begin{array}{r} 16 \quad \text{rem } 36 \\ 54\overline{)900} \\ \underline{54} \\ 360 \\ \underline{324} \\ 36 \end{array}$$

I can make 16 pillow cases. In this case the answer is closer to 17 than 16, however we need to round down to 16 pillow cases since we don't have enough fabric for the 17^{th} pillow case.

Questions like those in Examples 16 and 17 are easier to answer correctly when you are not using a calculator. When a sum like 900 ÷ 54, from Example 17, is done on a calculator the answer of 16.666666666 is seen on the display. Students will often round this up to 17 without thinking of the context of the question.

Do the next exercise without a calculator to practise your long division skills.

Exercise 40

1 Use the method from Example 14 to work out:

a 216 ÷ 12 **b** 256 ÷ 8
c 238 ÷ 14 **d** 299 ÷ 13

2 Use the method from Example 15 to work out:

a 675 ÷ 15 **b** 1558 ÷ 19
c 1344 ÷ 21 **d** 1767 ÷ 31
e 1938 ÷ 19 **f** 2834 ÷ 26

3 Work out these divisions:

a 512 ÷ 13
b 609 ÷ 14
c 435 ÷ 17
d 932 ÷ 15
e 1244 ÷ 16
f 1847 ÷ 24
g 2089 ÷ 22
h 2345 ÷ 36
i 6148 ÷ 45
j 7426 ÷ 43

4 An egg box holds 12 eggs. How many boxes are needed to pack 8596 eggs?

5 All 623 students and their 36 teachers are going on a school trip. How many buses are needed if each bus holds 23 people?

6 A car can travel 12 km on one litre of petrol. How many litres of petrol would you need for a journey of 208 km?

7 Alex shares \$7502 equally between 31 people. How much do they each receive?

8 A theatre has 1232 seats. The seats are arranged in rows of 44 seats. How many rows are there?

9 On a school trip there are 252 children and 18 adults. If each adult supervises the same number of children, how many children will each adult supervise?

10 72 sweets are to be shared between 13 children. How many whole sweets will each child receive?

11 A piece of wood 238 cm long is cut into smaller pieces each 18 cm long, how many smaller pieces can be cut from the larger one?

Consolidation

Example 1

a Write down the first three multiples of 7.

b What are the factors of 42?

---

a Multiples of 7 $= 1 \times 7, 2 \times 7, 3 \times 7$ etc.
First three multiples of 7 = 7, 14, 21

b Factors of 42 = 1, 2, 3, 6, 7, 14, 21, 42

Example 2

Find the HCF and the LCM of 16 and 12.

Factors of 12 are: **1**, **2**, 3, **4**, 6 ,12
Factors of 16 are: **1**, **2**, **4**, 8 ,16

Common factors are: 1, 2, **4**. The HCF is 4.

Multiples of 12 are: 12, 24, 36, **48**, 60, 72, 84, **96**,….
Multiples of 16 are: 16, 32, **48**, 64, 80, **96**,…

Common multiples are: **48**, 96,…. The LCM is 48.

Example 3

Is 153 a prime number?
To be a prime number 153 should have exactly two factors, 1 and 153.
If you can find any other factor then it is not prime.
Since there are no even prime numbers except 2 there is no need to test for divisibility of 2, 4, 6, 8 or 10.
No prime numbers end in 5 (except 5).
There is no test for divisibility by 7 (you could just divide by 7 to see what you get).
So the easiest tests are for divisibility are for 3 or 9.
Add the digits of 153: $1 + 5 + 3 = 9$, so this is divisible by 3 and 9, therefore not a prime number.

Example 4

Find 7^2 and $\sqrt{81}$
$7^2 = 7 \times 7 = 49$
$\sqrt{81}$ means 'what times itself makes 81?'
$9^2 = 81$ so $\sqrt{81} = 9$

Example 5

Work out 39×45

$$
\begin{array}{rrrrl}
 & & 3 & 9 & \\
 & \times & 4 & 5 & \\
\hline
 & 1 & 9_4 & 5 & \leftarrow \text{This row is } 39 \times 5 \\
+ \; 1 & 5_3 & 6 & 0 & \leftarrow \text{This row is } 39 \times 40 \\
\hline
1 & 7_1 & 5 & 5 &
\end{array}
$$

Example 6

Use repeated subtraction or otherwise to calculate

a $132 \div 4$ **b** $259 \div 17$

a

$$
\begin{array}{rr}
4\overline{)132} & \\
-\;40 & 10 \\
\hline
92 & \\
-\;40 & 10 \\
\hline
52 & \\
-\;40 & 10 \\
\hline
12 & \\
12 & \underline{3} \\
\hline
0 & 33
\end{array}
$$

b

$$
\begin{array}{r}
1 \\
17\overline{)259} \\
\underline{17} \\
89
\end{array}
$$

1 × 17 = 17
25 − 17 = 8
remainder.
Bring down the 9

$$
\begin{array}{r}
15 \\
17\overline{)259} \\
\underline{17} \\
89 \\
85
\end{array}
$$

$$
\begin{array}{r}
15 \text{ rem } 4 \\
17\overline{)259} \\
\underline{17} \\
89 \\
\underline{85} \\
4
\end{array}
$$

5 × 17 = 85
89 − 85 = 4
remainder.
So 259 ÷ 17 =

So, $132 \div 4 = 33$

Check: $33 \times 4 = 132$

So, $259 \div 17 = 15$ r 4

Check: $15 \times 17 + 4 = 259$

Exercise 4

1 Write down the first four multiples of
a 6 **b** 13 **c** 23 **d** 37 **e** 48

2 List all the factors of
a 24 **b** 36 **c** 27 **d** 54 **e** 112
f 96 **g** 108 **h** 144 **i** 256 **j** 1024

3 Find the HCF and the LCM of these pairs:
a 16, 8 **b** 14, 7 **c** 24, 20 **d** 18, 20, 12
e 16, 24 **f** 20, 25, 30 **g** 25, 30
h 36, 42 **i** 54, 28 **j** 72, 40, 20

4 Are these prime numbers?
a 141 **b** 163 **c** 121 **d** 191 **e** 119

5 Work out:
a 17×13 **b** 33×14 **c** 36×24
d 18×29 **e** 54×13 **f** 27×27
g 47×38 **h** 342×37

6 Calculate:

a	448 ÷ 32	**b**	164 ÷ 41
c	270 ÷ 15	**d**	208 ÷ 13
e	182 ÷ 13	**f**	244 ÷ 14
g	258 ÷ 15	**h**	456 ÷ 19

Check your answers with your teacher.

7 Work out:

a 5^2 **b** $\sqrt{36}$ **c** 10^2

d $\sqrt{169}$ **e** $\sqrt{1}$ **f** 11^2

8 Work out the side length of this square:

Area
144 cm^2

9 A piece of cloth 222 m long is cut to make curtains. If each curtain uses 8 m of cloth, how many curtains can be made?

10 There are 624 students and 27 staff at the King Stevens school. The school wants to go on a day trip to Seaview Bay.

a How many people will go on the trip?

b How many buses should the school hire if a bus can hold 31 people?

c If the cost of transporting a busload of people is \$225, what will be the total cost of the trip?

d How much should each person pay?

11 \$8304 profit for a company is to be shared equally between the 24 workers as a bonus payment. How much do they each receive?

12 Look at the numbers in the box:

36	15
10	14
7	49

Copy and complete these statements using numbers from the box:

a …. is a multiple of 6 and ….. is a factor of 40

b ….. is a prime number and ….. is a square number

13 Here are six digit cards.

$\boxed{1}$ $\boxed{2}$ $\boxed{3}$ $\boxed{4}$ $\boxed{5}$ $\boxed{6}$

Copy and complete these statements using each digit card once:

☐☐ is a prime number

☐☐ is a square number

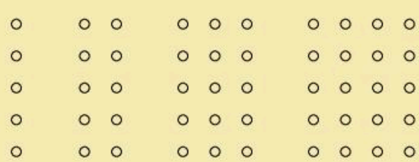
☐☐ is a multiple of 16

Summary

You should know …

1 How to find multiples.

For example:
The first four multiples of 5 are



5 10 15 20

Check out

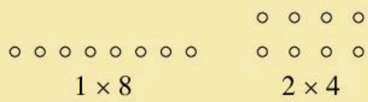
1 a Write down the first five multiples of 3.

b Write down the first five multiples of 11.

2 How to find factors.

For example:

Factors of 8



Factors of 8 are 1, 8, 2, 4.

2 Find all the factors of

a 14

b 24

---

3 A prime number has exactly two different factors, one and itself.

A number which has more than two different factors is called a composite number.

3 State whether these are prime or composite numbers:

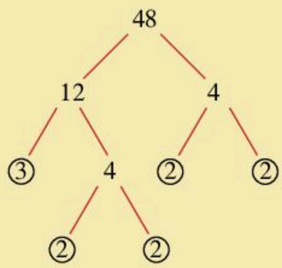
a 7 **b** 27

c 23 **d** 51

---

4 How to find prime factors of a number.

For example:



Prime factors are 2 and 3.

4 By drawing factor trees, find the prime factors of

a 24 **b** 72

c 60 **d** 252

---

5 How to find HCFs and LCMs.

For example:

Factors of 10 are: **1**, 2, **5**, 10

Factors of 15 are: **1**, 3, **5**, 15

Common factors of 10 and 15 are: 1 and **5** the HCF is 5.

Multiples of 10 are: 10, 20, **30**, 40, 50, **60**,….

Multiples of 15 are: 15, **30**, 45, **60**,…

Common multiples of 10 and 15 are: **30**, 60,…. the LCM is 30.

5 Find the HCF and the LCM of

a 16, 20

b 18, 27

c 36, 40

d 15, 35, 20

6

The divisibility tests	
$\div 2$	Number ends in an even number
$\div 3$	Sum of digits is a multiple of 3
$\div 4$	Last two digits divisible by 4
$\div 5$	Number ends in 5 or 0
$\div 6$	Any even number with digit sum a multiple of 3
$\div 8$	The number when halved has its last two digits divisible by 4
$\div 9$	Sum of digits is a multiple of 9
$\div 10$	Number ends in 0
$\div 100$	Number ends in 00

6 Use divisibility tests to see if

a 256 is divisible by 3
b 508 is divisible by 4
c 2538 is divisible by 6
d 1306 is divisible by 9.

7 Squares of whole numbers to 20^2.

For example:
$18^2 = 18 \times 18 = 324$

7 Work out:

a 6^2 **b** 12^2
c 14^2 **d** 17^2

8 How to find the square root of a number.

For example:
$\sqrt{64} = 8$ because $8^2 = 64$

8 Find the square root of

a 121 **a** 169
c 225 **d** 81

9 How to do long multiplication.

For example:

$$\begin{array}{cccc}
 & & \mathbf{3} & \mathbf{7} \\
 & \times & \mathbf{2} & \mathbf{5} \\
\hline
 & \mathbf{1} & \mathbf{8} & \mathbf{5} \\
 & & \cancel{3} & \\
+ & \mathbf{7} & \mathbf{4} & \mathbf{0} \\
 & \cancel{1} & & \\
\hline
 & \mathbf{9} & \mathbf{2} & \mathbf{5} \\
\hline
 & \cancel{1} & &
\end{array}$$

← This row is 37×5

← This row is 37×20

Add the 2 rows above

9 Work out:

a 35×24
b 49×23
c 78×52
d 124×67
e A school has 48 classes of 26 children, how many children are there altogether?

10 Long division is really repeated subtraction.

For example: $182 \div 13$

$$\begin{array}{rr|r}
13 & 182 & \\
 & -\ 130 & 10 \\
\hline
 & 52 & \\
 & -\ 52 & \underline{4} \\
\hline
 & 0 & 14
\end{array}$$

Note:
$10 \times 13 = 130$

$4 \times 13 = 52$
$14 \times 14 = 182$

or

$$\begin{array}{r}
14 \\
13\overline{)182} \\
\underline{13} \\
52 \\
\underline{52} \\
00
\end{array}$$

$182 \div 13 = 14$

10 Work out:

a $308 \div 22$
b $416 \div 16$
c $650 \div 15$
d $1442 \div 14$
e Metal pipes measuring 38 cm need to be cut from a larger piece of metal pipe measuring 5 m long. How many shorter pipes can be made?

5 Length, mass and capacity

Objectives

- Choose suitable units of measurement to estimate, measure, calculate and solve problems in everyday contexts.
- Know abbreviations for and relationships between metric units; convert between:
  – kilometres (km), metres (m), centimetres (cm), millimetres (mm)
  – tonnes (t), kilograms (kg) and grams (g)
  – litres (l) and millilitres (ml)
- Read the scales on a range of analogue and digital measuring instruments.

What's the point?

You ask questions such as 'How far?', 'How heavy?', 'How long?' on a daily basis. Masons and carpenters are groups of people whose livelihood depends on good measures.

Before you start

You should know ...

1 How to multiply and divide by 10, 100 and 1000.

For example:

To multiply by 10:

$4.27 \times 10 = 42.7$

decimal point moves one place to the right

or move every digit one place left

To divide by 100:

$320.6 \div 100 = 3.206$

decimal point moves two places to the left

or move every digit two places right

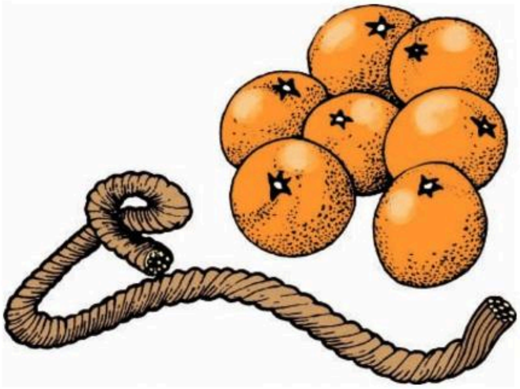
Check in

1 Find:

a 3.7×10
b 4.3×100
c 18×1000
d $2.33 \div 10$
e $1424 \div 100$
f $1424 \div 1000$

5.1 Length

Units of length

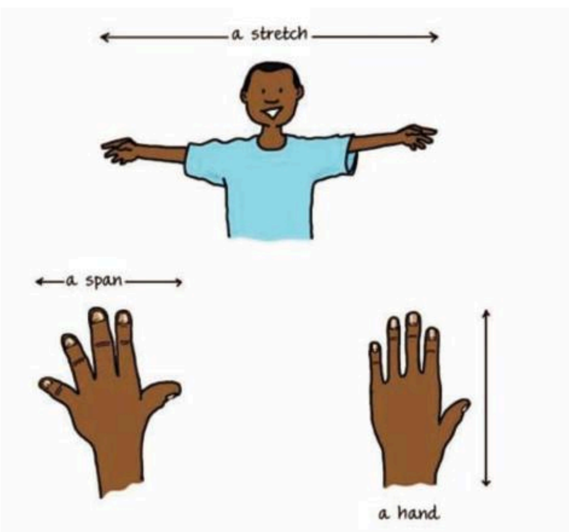


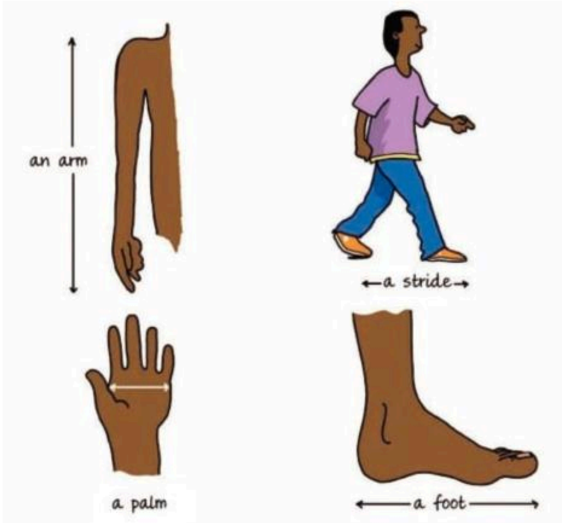
To answer the question 'How many oranges are there?' you can count. There are 7 oranges.

To answer the question 'How long is the rope?' you cannot count. You must **measure**.

To measure the rope you must first choose a unit of length.

Long ago, people used many different units for measuring the length of things.
For example:



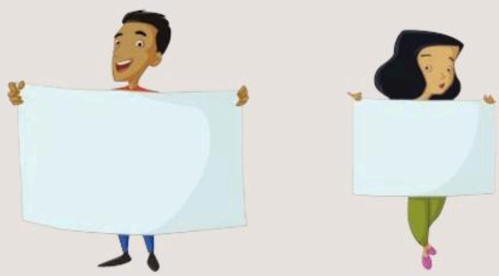


Exercise 5A

1 Which of the units above would be suitable for finding:

- **a** the width and length of a field
- **b** the distance round your classroom
- **c** the height of your best friend
- **d** the length of a page in this book?

2 Deo and Jess sell the same cloth for \$6 a stretch. From which of them would you buy cloth? Why?



Deo Jess

3 Look again at the units based on body parts. Do you think that using them might cause problems? Why?

4 Write down four units of length that are used today.

SI units

The units based on body parts were not satisfactory to use, because not everyone's hands, arms, feet, and strides are the same.
You need to use standard units.

More and more countries are using standard units based on the French-invented metric system, called the **Système international d'unités**, or **SI** for short.

The metre

The SI unit of length is the **metre**, **m**.
A good estimate of a metre is one long stride.

Exercise 5B

You will need a metre rule.

1 Use your metre rule to measure the
  a height of your classroom door
  b width of your classroom
  c length of your classroom.

2 Write down the name of an object that is about
  a 1 m long
  b 2 m long
  c 5 m long.

3 Practise taking strides 1 m long. Now measure the length and width of your classroom by striding around the walls. Do you think this is an accurate way of measuring?

4 **a** Work with a friend. Choose five different places in your school.
Estimate (guess) the distance to them from your classroom door.
  b Now check your estimate using a metre rule.
  c Copy and complete the table:

Distance from classroom	Estimated distance	Actual distance
1.		
2.		
3.		
4.		
5.		

ACTIVITY

Work with a group of friends. You will need a watch and a metre rule.

a Estimate the time it would take to walk 10 m, 20 m, 30 m, 40 m and 50 m.
b Using the metre rule to measure the distances and a watch to measure time, check your estimates in **a**.
c Copy and complete the table:

Distance	Estimated distance	Actual time
10 m		
20 m		
30 m		
40 m		
50 m		

The centimetre and millimetre

The metre is a fairly large unit of length.

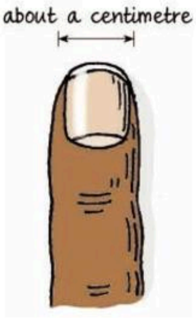
To measure shorter lengths, we use

- the **centimetre**, $\frac{1}{100}$ of a metre
- the **millimetre**, $\frac{1}{1000}$ of a metre or $\frac{1}{10}$ of a centimetre

Here is a line 1 centimetre long: ______
Here is a line 1 millimetre long: -

The abbreviation **cm** is used for centimetre.
The abbreviation **mm** is used for millimetre.

A rough estimate of a centimetre is the width of your smallest finger.



- $100\,\text{cm} = 1\,\text{m}$
  $1\,\text{cm} = \frac{1}{100}\,\text{m} = 0.01\,\text{m}$
- $10\,\text{mm} = 1\,\text{cm}$
  $1\,\text{mm} = \frac{1}{10}\,\text{cm} = 0.1\,\text{cm}$
  $1\,\text{mm} = \frac{1}{1000}\,\text{m} = 0.001\,\text{m}$
  $1000\,\text{mm} = 1\,\text{m}$

Exercise 5C

1 Use your smallest finger to estimate the length of:
  a your exercise book
  b your pencil

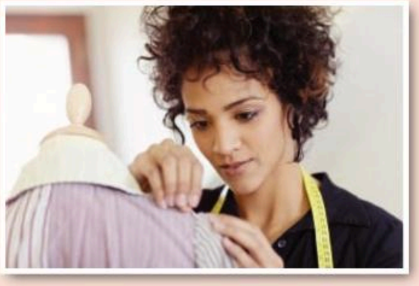
2 Estimate the length of each line.
  a ________________________________
  b ______________
  c __________________________________
  d _____
  e ________

3 Now measure each line in Question **2**. Were your estimates good ones?

4 **a** Estimate the height of this book using cm.
  b Now measure its height using cm and mm. Was your estimate a good one?

5 Repeat Question **4** for the width of the book.

ACTIVITY



Interview a dressmaker, a seamstress or someone from another profession who uses measurements daily.

- Find out what units of measurement they use.
- What do they use to make measurements?
- What measurements do they take?
- Write up and present your findings to the class.

The kilometre

For great distances, a metre is too small a unit to use. Instead, we use the **kilometre**, **km**.

A kilometre is roughly the distance you can walk in 15 minutes.

- $1000\,\text{m} = 1\,\text{km}$
  $1\,\text{m} = \frac{1}{1000}\,\text{km} = 0.001\,\text{km}$

Exercise 5D

1 **a** Write down five places that are about 15 minutes' walk from your school.
  b How could you check that these places are really 1 km away?

2 **a** How long does it take you to walk to school?
  b Estimate the distance you walk.

3 Estimate the distance from your school to the nearest post office. How would you measure the distance?

4 Write down the names of four cities or towns in your country. Now find out the distances between them in kilometres.

Relationships between metric units

To change lengths in metres to centimetres, or metres to kilometres, you need to remember that

$1000\,\text{m} = 1\,\text{km}$
$100\,\text{cm} = 1\,\text{m}$
$10\,\text{mm} = 1\,\text{cm}$

Sometimes you multiply, sometimes you divide.
This diagram can help you decide:

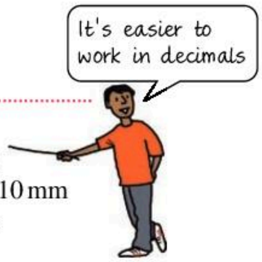
units getting bigger,
fewer of them, so divide

$$\text{mm} \underset{\times 10}{\overset{\div 10}{\rightleftarrows}} \text{cm} \underset{\times 100}{\overset{\div 100}{\rightleftarrows}} \text{m} \underset{\times 1000}{\overset{\div 1000}{\rightleftarrows}} \text{km}$$

units getting smaller,
more of them, so multiply

EXAMPLE 1

Change
a 6 cm 5 mm to mm
b 5.6 km to m



a 1 cm = 10 mm
so 6 cm 5 mm = 6.5 cm
= 6.5 × 10 mm
= 65 mm

b 1 km = 1000 m
So 5.6 km = 5.6 × 1000 m
= 5600 m

EXAMPLE 2

Change 6314 m to km.

1000 m = 1 km
So 6314 m = 6314 ÷ 1000 km
= 6.314 km

Exercise 5E

1 Change to mm:
a 3 cm **b** 16 cm
c 3 cm 2 mm **d** 5.7 cm

2 Change to cm:
a 40 mm **b** 120 mm
c 58 mm **d** 92 mm

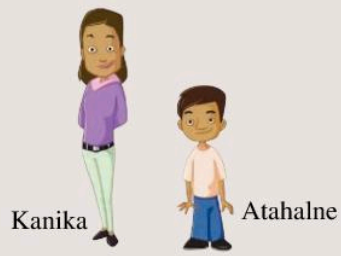
3 Write these distances in metres:
a 1 km **b** 5 km
c 3.6 km **d** 4.12 km

4 Write these distances in kilometres:
a 3000 m **b** 12 000 m
c 500 m **d** 1680 m

5 The table gives the height of five students.
Copy and complete the table:

Student	Height m and cm	Height m	Height cm
James	1 m 42 cm		
Anthony		1.56 m	
Garth			145 cm
Albert		1.25 m	
Andy	1 m 70 cm		

6 Atahalne is 1 m 36 cm tall.
Kanika is 18 cm taller.
How tall is Kanika in metres?



7 Seta went on a tour of her island. The table shows how far she walked each day.

Day	Mon	Tue	Wed	Thur	Fri
Km travelled	19.5	38.7	24.8	30.1	35.9

a Find the total number of km she walked.
b Express the answer to part **a** in m.

(**Hint:** If you are not sure how to add and subtract decimals, look ahead to Chapter 10. Alternatively, you can change units before adding or subtracting.)

8 Change to mm:
a 0.07 m **b** 0.6 cm
c 0.004 m **d** 0.38 cm
e 0.14 m **f** 0.5 m

9 Change to m:
a 270 cm **b** 4300 cm
c 0.65 km **d** 0.2 km
e 34 km **f** 42 cm
g 3 cm **h** 0.345 km

10 Which distance is greater:
a 2000 m or 1 km 571 m
b 196 mm or 32 cm 7 mm
c 3 km or 30 000 cm?

11 Find the value of the following:
a 73.9 cm + 36 mm (answer in cm)
b 3.61 m + 58.7 cm (answer in m)
c 2531 m + 793 m − 1.7 km (answer in m)
d 1.818 km − 972 m (answer in m)

12 I have to walk 2.3 km to school. I walk some of the way with my friend. I walk for 835 m to meet my friend. Then we walk for a further 1.2 km together. How far do I still need to walk?

13 A plank of wood is 4.3 m long. It has the following lengths cut from it:
49 cm, 583 mm, 76 cm, 1.2 m.
What length remains in
a mm **b** cm?

5.2 Mass

Units of mass

Long ago, people measured things like rice in handfuls or bowlfuls.

Exercise 5F

1 Mrs Addy and Mrs Armstrong both sell sugar by the bowlful.

Mrs Addy   Mrs Armstrong

They each charge 40 cents for a bowlful. From which of them would you buy your sugar? Why?

2 Do you think that measuring in handfuls could cause problems? Why?

3 Look again at the units shown above. Could you use either of these for measuring meat?

Handfuls and bowlfuls are not satisfactory units for measuring quantities. Bowls come in all different shapes and sizes.

You need to use standard sizes so that you know how much you are getting.

First you need to understand the difference between mass and weight.

Mass and weight

Many people might ask 'What is your weight?'
But it is more correct to ask 'What is your mass?'
This is because weight can change from place to place.
It depends on how far you are from the centre of the earth.

- The **mass** of an object is always the same.

An astronaut who weighs 70 kilograms on earth will weigh almost nothing at all when he is in space.
He will have to be tied to his spaceship to prevent him floating away.

But the astronaut himself does not change.
He is still the same person. The quantity of him is the same. This quantity is his mass.
You should use the word 'mass' instead of 'weight'.

The SI unit of mass – the kilogram

The SI unit of mass is the **kilogram** or **kg**.
It is the mass of a small block of metal kept in a laboratory near Paris.

- The mass of a bag of sugar is a kilogram.

Exercise 5G

You will need a 1 kg mass, some oranges, grapefruit and bananas (you can use other fruits if you don't have these).

1 Lift up the 1 kg mass. Feel how heavy it is.
  a Write down five objects that are heavier than 1 kg .
  b Write down five objects that are lighter than 1 kg .

2 Work with a group of friends.

a Compare the masses of a grapefruit, an orange and a banana with the 1 kg mass.

b How many oranges are needed to make 1 kg? How many bananas? How many grapefruits?

3 **a** Find out your own mass in kilograms from a set of scales.

b Find out if you are above average or below average mass for your age.

4 Here are some approximate masses of everyday objects:

A large pineapple	about 1 kg
7 or 8 medium-sized bananas	about 1 kg
5 or 6 large oranges	about 1 kg
An average 12-year-old boy	about 38 kg
A small car	about 750 kg

Estimate the mass in kilograms of:

a a grown man **b** a newborn baby

c a bicycle **d** a cat

e a cow.

Find out if your estimates are correct – you can use the internet.

The gram

The mass of a biscuit is much much less than 1 kg. For such small masses we need a smaller unit.
We use the **gram**, **g**.

- 1000 g = 1 kg

  $1\text{ g} = \frac{1}{1000}\text{ kg} = 0.001\text{ kg}$

Exercise 5H

You will need a balance and 1 g, 10 g and 100 g masses.

1 Lift up the 1 g, 10 g and 100 g masses in turn.

a Write down three objects whose masses are less than 10 g.

b Write down three objects whose masses are less than 100 g but more than 10 g.

2 You will need your pencil, ruler, exercise book, textbook and geometry set. Lift up each object and compare their masses.

a Write down the objects in order of mass, smallest first.

b Guess the masses of each object.

3 **a** Find the masses of the objects in Question **2** using a balance. Ask your science teacher to help you.

b Copy and complete the table:

Object	Estimated mass	Actual mass
Pencil		
Ruler		
Geometry set		
Exercise book		
Textbook		

c Which was your best estimate?

4 Here are some approximate masses:

A postcard	2 g
A new pencil	3 g
A large egg	60 g
A small loaf of bread	400 g

Estimate the mass of:

a a ball-point pen

b 1 ten cent coin

c a teaspoon

d a knitting needle

e a letter.

5 Change to g:

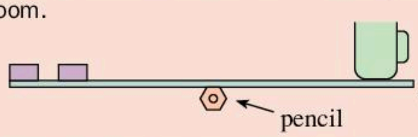
a 4 kg **b** 0.32 kg

c 0.07 kg **d** 0.002 kg

6 A pile of rubbish has a mass of 17 kg.
If 8450 g of rubbish is removed, what is the mass of the rubbish left?

ACTIVITY

A very simple balance can be made using your ruler, a pencil and some masses from your science room.



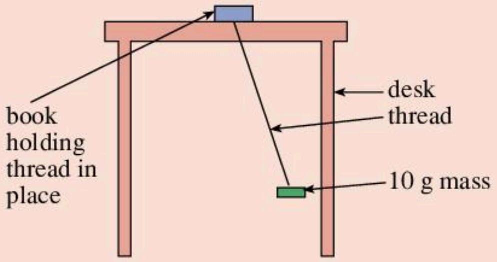
The ruler must first balance on the pencil on its own.

a Work with a friend and use the balance to find the masses of five objects in your school bag.

b Find out how a spring balance works.
In what ways is a spring balance different from the one you made in part **a**?

c Ask your science teacher for a spring balance.
Use it to check your answers to part **a**.

Work with a group of friends. You will need a metre of thread, a watch and some standard masses. Tie a 10 g mass to a metre of thread and attach the thread to your desk so it can swing freely.



- **a** Push the mass slightly, the thread will swing from side to side.
- **b** How long does it take to make 20 swings?
- **c** Remove the 10 g mass and put on a 50 g mass. How long does it take to make 20 swings?
- **d** Copy and complete the table:

Mass	Time for 20 swings
10 g	
50 g	
100 g	
200 g	

- **e** What do you notice?
- **f** Repeat, but this time keep the 10 g mass on the thread. Use only 50 cm of thread and find the time for 20 swings.
- **g** Use 30 cm of thread, and find the time for 20 swings.
- **h** Copy and complete the table:

Length of thread	Time for 20 swings
1 cm	
50 cm	
30 cm	
10 cm	

- **i** What do you notice?
- **j** Estimate the time for 20 swings if only 20 cm of thread were used.

The tonne

For very large masses, a larger unit is used.
It is the **tonne**, **t**.

- 1 t = 1000 kg

Here are some approximate masses:

Car, about 1 t

Elephant, about 5 t

Exercise 5I

1 Write down five objects with masses greater than 1 t. Try to find out their masses.

2 Copy and complete:

1 kg = □ g 1 t = □ kg
1 g = □ kg 1 kg = □ t

3 Change to kg :

a	4 t	**b**	3.7 t
c	4200 g	**d**	320 g
e	0.72 t	**f**	0.004 t
g	3 000 000 g	**h**	25 g

(**Hint:** If you are not sure how to add and subtract decimals look ahead to Chapter 10 – or you can change units before adding or subtracting.)

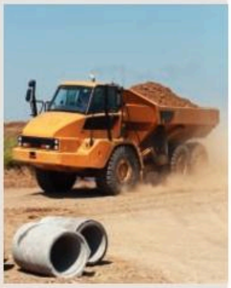
4 Add these masses and give the answer in
i grams **ii** kilograms.

a 972 g, 83 g, 523 g
b 323 g, 1.521 kg, 97 g
c 2 t, 731 kg, 432 g

5 Give your answers in the most appropriate units.
a 734 g + 88 g − 236.7 g
b 396 g + 7 kg 86 g − 3 kg 746 g
c 45.87 g × 27

6 One tin of biscuits has a mass of 0.32 kg.
What is the mass of 7 of these tins in
a kg
b g?

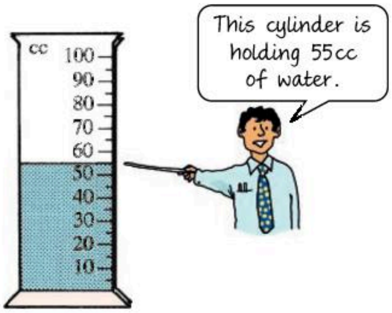
7 A truck full of sand has a mass of 3.2 t.
What is the mass of the truck if the mass of the sand is 1875 kg?
Give your answer in
a tonnes
b kilograms.



8 Merlene buys a piece of meat with a mass of 4 kg 65 g.
Express the mass in
a grams **b** kilograms.

5.3 Capacity

Liquids do not have a fixed shape, but you can still measure their volume.
Scientists often use a measuring cylinder to measure the volume of liquids.



cc is short for 'cubic centimetre' – that is 55 cc = 55 cm³.

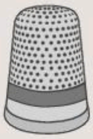
Exercise 5J

1 Collect five bottles which usually contain liquid.
a Estimate their volume.
b Fill your containers with water. Use your measuring cylinder to find their actual volume.
c Copy and complete the table:

Bottle	Estimated volume (cm³)	Actual volume (cm³)
1		
2		
3		
4		
5		

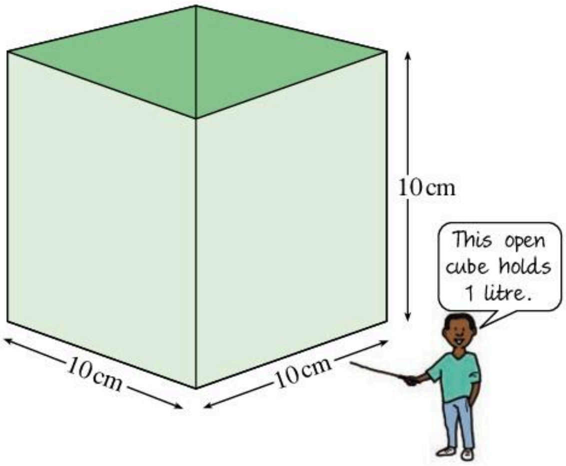
2 **a** Estimate the volume of a bucket of water.
b Could you use a measuring cylinder to find its actual volume? Why might it be difficult?

3 **a** Estimate the volume of a thimble of water.
b Could you use a measuring cylinder to find the actual volume of the thimble? Why might it be difficult?



- The amount of liquid a container can hold is the **capacity** of the container.

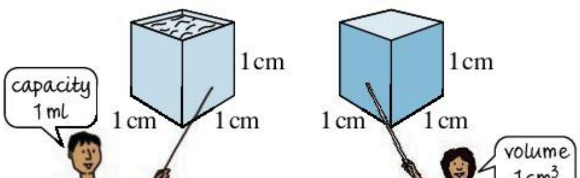
A litre is the volume of liquid that can be held in an open cube of edge 10 cm.



The volume of this cube $= 10\,\text{cm} \times 10\,\text{cm} \times 10\,\text{cm}$
$= 1000\,\text{cm}^3$

That is **1 litre = 1000 cm³**

Another unit for measuring small volumes of liquid is the **millilitre**. It is written in short as **ml**.
It is the amount of liquid that can be held in a cubic centimetre.



$1\,\text{cm}^3 = 1\,\text{ml}$
$1000\,\text{ml} = 1\,\text{litre}$

ACTIVITY

Using thick card, make a net of a cube with side 10 cm. Fold it to make an open cube and fix it together.
Line your cube with a plastic bag and fill it with water. The volume of water in your cube is 1 litre.

Use your cube to find the capacity of a bucket.

Exercise 5K

1 **a** Guess the volume, in ml of:
  i a teaspoonful of medicine
  ii a small bottle of ink
  iii a cupful of coffee
  iv a full cola bottle
  v a raindrop.

  b Find the exact volumes of as many of these as you can.

2 Bottles and tins of liquid often have the volume of the contents written on the labels. Find five examples.

3 Change these to ml:
  a 4.9 litres **b** 0.23 litre
  c 0.4 litre **d** 0.005 litre
  e 0.034 litre

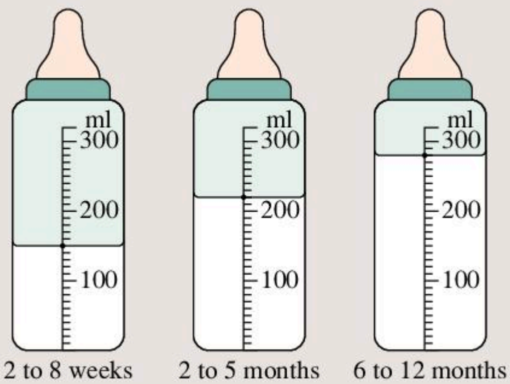
4 Change these to l:
  a 2400 ml **b** 350 ml
  c 17000 ml **d** 32 ml
  e 7 ml

5 **a** What is the total volume of Judy's Punch in
  i litres
  ii millilitres?

  b A glass holds 100 ml. How many glasses can Judy fill from her punch bowl?

Judy's Punch
5 litres lemonade
1 litre of pineapple juice
500 ml of watermelon juice

6



These bottles show the volumes of feed for babies of different ages.
  a Write the volume of feed in each bottle.
  b Joan is 8 months old and has 4 feeds per day. Write, in litres, the total volume of feed Joan has in one week.
  c Ryan is 4 months old and has 6 feeds per day. Write, in litres and millilitres, the total volume of feed Ryan has in one week.

7 **a** Estimate how many cups of liquid you drink every day.
  b How many litres is that?

8 For each person find how many days the medicine will last.

ACTIVITY

Can you think of a way to measure the volume of air you breathe out in one breath?

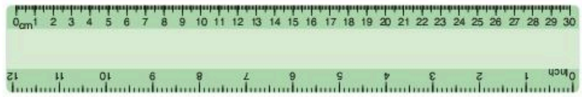
5.4 Reading scales

It is important to be able to read scales accurately. For example, a nurse needs to be able to read the scale on a needle to inject the correct amount of medicine.

There are many different scales to measure lots of different quantities such as weighing scales, rulers, thermometers, clocks and measuring cylinders.

To read a scale accurately you need to:

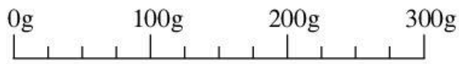
- know what each division on the scale stands for
- make sure the scale is level and not at an angle
- make sure you are looking from the correct angle (not from above or below)
- make sure you are reading the correct units (some scales have more than one unit on them – for example a ruler in cm and inches, like the one below.)



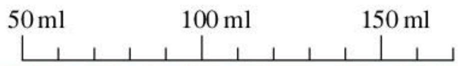
EXAMPLE 3

What do the small divisions on these scales stand for?

a



b



a There are 4 gaps between 0 and 100

$100 \div 4 = 25$

Each small division is worth 25 g

b There are 5 gaps between 100 and 150

$$\frac{150 - 100}{5} = 10$$

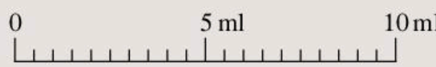
Each small division is worth 10 ml

Exercise 5L

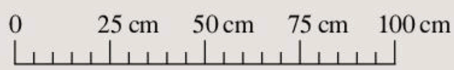
1 Work out what each of the small divisions are worth on these scales:

a

b

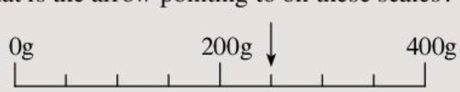


c



2 What is the arrow pointing to on these scales?

a



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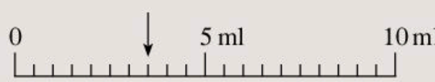

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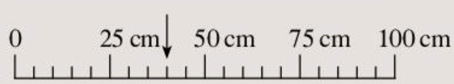
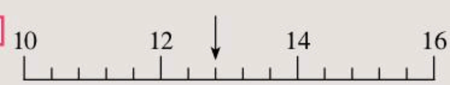


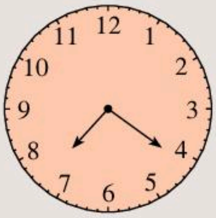
d

e

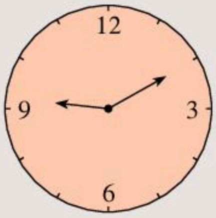


3 What is the time on these clocks?

a



b



c

d From these three clocks can you tell if it is morning or afternoon? Explain your answer.

4 What is the temperature on this thermometer in °C?

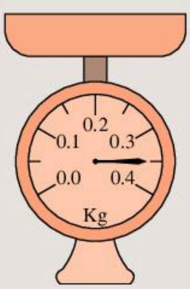
5 What is the mass in

i kg

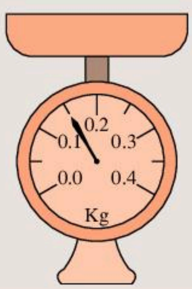
ii grams

shown on these scales?

a



b

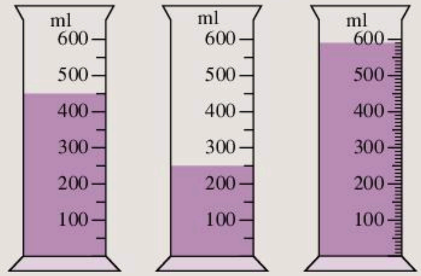


6 What is the capacity in

a ml

b litres

shown on these measuring cylinders?



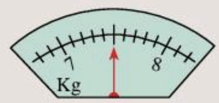
7 What is the mass in

i kg

ii grams

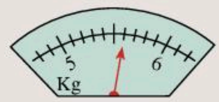
on these scales when the masses written below them are added or subtracted?

a



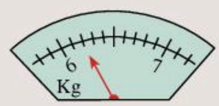
Subtract 800 g

b



Add 700 g

c



Subtract 600 g

8 This thermometer is showing my temperature when healthy. When I was ill I was 4 °C hotter than this. What was my temperature when I was ill?

Consolidation

Example 1

Which unit would you use to measure the

a width of your classroom
b length of your little finger
c distance from London to New York?

a Your classroom is a few big strides long, so use metres.
b Your little finger is much smaller than a stride, so use centimetres.
c London to New York is a great distance, so use kilometres.

Example 2

Convert

a 35 mm to cm
b 3.2 km to m

a $10\,\text{mm} = 1\,\text{cm}$
so $35\,\text{mm} = 35 \div 10\,\text{cm}$
$= 3.5\,\text{cm}$

b $1000\,\text{m} = 1\,\text{km}$
so $3.2\,\text{km} = 3.2 \times 1000\,\text{m}$
$= 3200\,\text{m}$

Example 3

Estimate the mass of

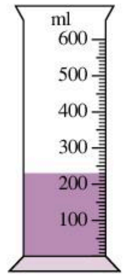
a a pen in grams; in kg
b a chair in kg; in grams

a A pen is very light. Its mass could be 25 g.
$1000\,\text{g} = 1\,\text{kg}$
so $25\,\text{g} = 25 \div 1000\,\text{kg}$
$= 0.025\,\text{kg}$

b A chair is quite heavy. Its mass could be 25 kg.
$1000\,\text{g} = 1\,\text{kg}$
so $25\,\text{kg} = 25 \times 1000\,\text{g}$
$= 25\,000\,\text{g}$

Example 4

I add the water from this measuring cylinder to 4.1 litres of water.
How much water do I now have altogether?



Make the units the same

4.1 litres is 4100 ml

This scale says 230 ml

$4100 + 230 = 4330\,\text{ml}$

or 4.33 litres

Exercise 5

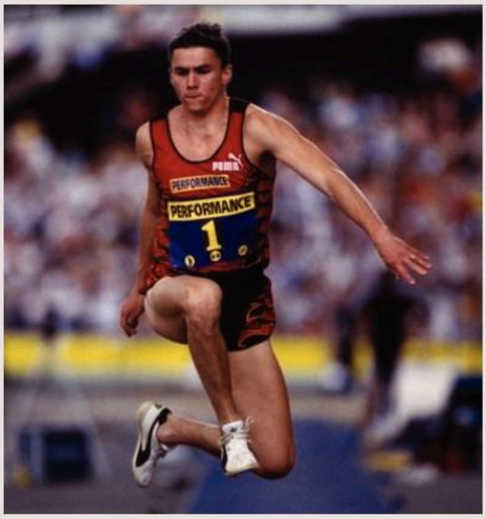
1 Which unit would you use to measure
a your height
b length of your foot
c height of your classroom
d width of a fly's wing
e length of a cricket pitch
f distance from Wellington, New Zealand to Jakarta, Indonesia?

2 Estimate the mass of these objects using appropriate units.
a a pencil **b** a cup
c a football **d** a table

3 Convert
a 23 mm to cm **b** 4 cm to mm
c 3 km to m **d** 2 kg to g
e 3.1 kg to g **f** 800 g to kg
g 5200 ml to litres **h** 43000 kg to t

4 Which metric unit would you choose to measure:
a the mass of a rhinoceros
b the mass of your maths teacher
c the mass of a pencil
d the mass of this book
e the mass of your desk
f the mass of Nigeria's crop?

5 In Athletics, triple jump records are held for a long time. Jonathan Edwards from Great Britain set the men's triple jump world record in 1995 with a distance of 18.29 m.



a How far is this in centimetres?

b Renjith Maheshwary set the Indian national triple jump record in 2010 with a jump of 17.07 m. How much further would he need to jump to beat the world record?

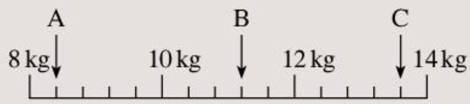
6 A fish currently 3 cm long grows 6 mm per year.

a How long will it be in:

i 2 years' time **ii** 5 years' time?

b Use a ruler to find the age of the fish shown below.

7 **a** What do the arrows point to on this scale? Give your answers in kg.



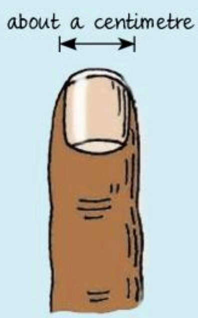
b Write your answers to part **a** in grams.

Summary

You should know ...

1 The SI unit of length is the metre (m).
Smaller units used are centimetres (cm) or millimetres (mm).

1 m is approximately one long stride.



1 cm is approximately the width of your smallest finger.

A larger unit used is the kilometre (km).

2 10 mm = 1 cm
100 cm = 1 m

Check out

1 What unit would be most convenient to measure

a the width of a football field

b the width of this book

c the distance between Paris and Berlin?

2 Express

a 3 m in cm

b 270 m in km.

3 The SI unit of mass is the kilogram (kg).
A smaller unit used is the gram (g).
A larger unit used is the tonne (t).

1 kg is approximately the mass of a large pineapple.

A pencil's mass is about 3 g.

3 What unit would be most convenient to measure

a the mass of a bicycle
b the mass of a paint brush
c the mass of a bus?

---

4 $1000\,g = 1\,kg$
$1000\,kg = 1\,t$

4 **a** Add together 2 t, 57 kg and 321 g. Give your answer in grams.
b Find $33.5\,g \times 32$
Give your answer in
i g
ii kg.

---

5 We use litres (l) for capacity
A smaller unit used is millilitres (ml)

5 What unit would be most convenient to measure

a the amount of liquid a spoon holds
b the amount of liquid a bath holds?

---

6 $1000\,ml = 1$ litre

6 Express

a 5600 ml in litres
b 0.04 litres in ml

6 Representing information

Objectives

- Decide which data would be relevant to an enquiry and collect and organise the data.
- Design and use a data collection sheet or questionnaire for a simple survey.
- Find the mode (or modal class for grouped data), median and range.
- Construct and use frequency tables to gather discrete data, grouped where appropriate in equal class intervals.
- Calculate the mean, including from a simple frequency table.

What's the point?

What is your favourite TV programme? Which foods do you prefer? Which soap powder do you think washes whitest? To answer such questions advertisers and market researchers carry out surveys.

Before you start

You should know ...

1 How to use the order of operations:
For example:

$2 + 3 + 3 + 8 \div 4 = 10$ BIDMAS tells us to do the division first then the addition.

This is different to:
$(2 + 3 + 3 + 8) \div 4 = 4$ Here BIDMAS tells us to do the brackets first then the division.

2 That a long line acts like brackets.
For example:

$\frac{2 + 3 + 3 + 8}{4} = 4$ Do this in the same way as:

$(2 + 3 + 3 + 8) \div 4$

Check in

1 Work out:
a $3 + 3 + 4 + 5 + 10 \div 5$
b $(3 + 3 + 4 + 5 + 10) \div 5$

2 Work out:
a $\frac{6 + 8 + 9 + 11 + 16}{5}$
b $\frac{12 + 13 + 14 + 17}{4}$

6.1 Collecting data

You wish to find out from your classmates which of these dishes is their favourite:

Chicken curry
Roast Lamb
Pizza
Kebab

How would you obtain the information?
You could:

- ask each classmate individually
- pass around a sheet with the dishes listed and ask each classmate to tick one of them
- ask for a show of hands.

Whichever method is used, **data** or information is being collected.

Data you have collected yourself in this way is called **primary data**. If someone else has collected the data (e.g. data from the internet) this is **secondary data**. If you have collected primary data you know how reliable the data is, secondary data may be unreliable. The advantage of secondary data is that it is quicker to collect as someone else has done all the work. This chapter looks at ways of collecting and organising primary data.

The data about favourite foods is **discrete data**.

Discrete data can only take on definite values.

For example:

- shoe sizes – size 1, size 2 etc.
- number of family members – 3, 4, 5 etc.

A useful way of collecting and recording discrete data is to keep a tally.

Look at this **frequency table**:

Item	Tally	Frequency
Chicken curry	||||	4
Roast Lamb	𝍸 ||	7
Pizza	𝍸 𝍸 𝍸 |	16
Kebab	𝍸 𝍸 |||	13

𝍸 represents 5 classmates.

The frequency column gives the total of the tally marks.

EXAMPLE 1

Represent the following numbers using tally marks.

a 2 **b** 9 **c** 15 **d** 23

a 2 — ||
b 9 — 𝍸 ||||
c 15 — 𝍸 𝍸 𝍸
d 23 — 𝍸 𝍸 𝍸 𝍸 |||

If you have a lot of data with a big range, organising the data into tables like these would not make it much clearer.

For example, you could construct a frequency table to gather the following data (this is the number of seconds people can hold their breath for):

10	24	16	37	36	32	34
21	18	26	22	35	31	31
26	16	31	11	40	13	42

However, looking at each value separately, you can see the frequency table below isn't very helpful – and it is not even finished!

Frequency table:

Number of seconds	Frequency
10	1
11	1
12	0
13	1
14	0
15	0
16	2
.	.
.	.
.	.

In these cases it is useful to use a **grouped frequency table**.

EXAMPLE 2

Construct a grouped frequency table to gather the following data (this is the number of seconds people can hold their breath for):

10	24	16	37	36	32	34
21	18	26	22	35	31	31
26	16	31	11	40	13	42

Grouped frequency table:

Number of seconds	Tally	Frequency
10 – 19	𝍸 |	6
20 – 29	𝍸	5
30 – 39	𝍸 |||	8
40 – 49	||	2

When you are constructing a grouped frequency table, make sure that the groups do not overlap.

You couldn't have these groups:
10 – 20
20 – 30,

as 20 could go in either group.

Exercise 6A

1 State the numbers represented by each of the following tallies.

a |||
b 𝍸 𝍸 𝍸 ||||
c 𝍸 𝍸
d 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 ||

2 The heights of a group of children are measured in cm.

139	138	142	136	142
142	136	138	139	141
142	136	136	139	143
136	143	138	141	142
141	142	142	138	142
138	142	141	136	139

Copy and complete the frequency table:

Height (cm)	Tally marks	Frequency
136		
138		
139		
141		
142		
143		

3 Construct a frequency table to gather the following data on numbers of students late to school each day for one month:

11	16	11	12	15	14	13
12	14	12	13	13	11	14
14	13	14	15	14	12	15

4 These are the ages of people at a party:

2	13	4	17	36	64	45
18	41	51	18	18	17	15
65	28	16	18	27	21	18
17	19	37	44	5	18	67

Copy and complete this grouped frequency table:

Age	Tally	Frequency
0 – 9		
10 – 19		
20 – 29		
30 – 39		
40 – 49		
50 – 59		
60 – 69		

5 Construct a grouped frequency table to gather the following data on marks in a maths test:

41	61	70	52	67	48
59	59	88	60	94	64
53	77	44	83	56	98
74	66	77	57	43	68

6 Find out the number of students who were absent from your class each day last month. Use tally marks to represent the information in a frequency table.

7 Collect information from your classmates about their favourite game out of: football, cricket, baseball, basketball and hockey.
Record the information in a frequency table.

8 Go to a safe place where you can see traffic passing. Use tallies to record the number of each type of vehicle (car, lorry, bus, bicycle, taxi, motorbike) that passes by during 30 minutes.
Record the information in a frequency table.

Relevant data

Before you start collecting data and putting it into frequency tables, it is important to decide which data would be relevant to the survey.

Exercise 6B

1 The teachers at Westfield High School are concerned about the safety of their students due to the amount of traffic around the school at the start and end of the school day.

Students were asked what they thought teachers should find out about. Their suggestions were:

- How many students walked to school?
- How many students are there in the school?
- How many children were in each car?
- When did most students arrive at school?
- What colour cars did people drive?
- How many students came by bus?
- How many students lived a long way away?
- Did parents drive all the time or just when weather was bad?
- Which parent drove the child to school?
- How many car parking spaces were there in the school car park?
- Where was the nearest zebra crossing?
- Why did the parents drive to school?
- How many people went home for dinner?
- How long did it take students to travel to school?
- Was there another street nearby which had less traffic?
- Did people have bicycles?

Discuss their suggestions.

a Are they all relevant? Which are not?
b Which are most relevant?
c If you could ask only 5 questions, which would you choose and why?
d Can you think of any more?

2 Discuss each of the following. What would be relevant when finding out about:

a the litter problem in a school
b town recycling habits
c people's views on a new sports centre in the town
d community use of the school buildings and grounds?

3 Ella was asked to find out people's views on school uniform.
She decided to think about these questions:

1 Are you male or female?
2 How old are you?
3 Do you wear school uniform?
4 Do you like wearing school uniform? Why?
5 How do you travel to school?
6 Why is wearing school uniform bad?
7 Why is wearing school uniform good?
8 What is your teacher's name?

a Which of these are irrelevant?
b If you were only allowed to ask 3 questions from the list above, which 3 would you choose and why?
c Ella's friend Katy said that "What is your teacher's name?" wasn't relevant. Do you agree with Katy or not? Why?

Data collection sheets

Once you have decided which data is relevant and which is not then you need to think about how to collect it. You could make a **data collection sheet**, which is a sheet with various headings to be filled in.

Data collection sheets are useful when conducting **interviews**. Long questions are not necessary because the person conducting the interview explains the questions and the possible answer choices.

Data collection sheets can also be used for logging data for example in a weather or traffic survey.

The advantage of a data collection sheet is that you can quickly gather information straight into easy to read tables and you only need to print out one sheet of paper. The disadvantage is there is a limit to how much detail or information you can fit in the table.

EXAMPLE 3

Design a data collection sheet to find out about boys and girls test results in maths and the amount of television they watch:

Male or female	Maths test mark	Hours of television watched per week

EXAMPLE 4

Design a data collection sheet to see what the weather conditions are.

Date	Time	Temperature	Weather conditions

Questionnaires

You may prefer to do a survey by sending out a **questionnaire**, which is a printed list of questions. These are useful because lots of people can fill them out at the same time and you can collect more data. You can also post these to different areas. People have time to think carefully about their responses.

There are disadvantages to questionnaires. Many people don't fill them in. There is no one to explain any questions that are not understood. Printing, posting and interpreting lots of questionnaires can be time consuming and costly.

Questionnaire design is very important. A good questionnaire is more likely to be returned and more likely to be answered honestly. Also you will be able to easily use the data collected from the questionnaire.

How to write a good questionnaire:

• Be polite. Explain the purpose of the questionnaire	e.g. "I am trying to find out how people feel about the new library."
• Keep it short.	People will not want to spend a long time filling in questionnaires. Don't ask irrelevant questions.
• Keep questions simple and closed (this means give a choice of answers). Include a time frame if necessary.	e.g. How much television do you watch per week on average: 0–1hrs, 2–3hrs, 4–5hrs, more than 5 hours? Without a choice of answers, if you just wrote the open question "how much television do you watch?" the answer could be "lots" which is not helpful. The time frame is also important, as "how much television do you watch?" could mean per day, per week or per month etc.
• Make sure the answer choices do not overlap and that they cover all options.	e.g. If you wrote "Please tick your age group": ☐ 0–24, ☐ 24–30, ☐ 31–38, ☐ 40–60 ☐ 61 or more, someone who is 24 has two boxes they could tick and someone who is 39 has no box to tick.
• Keep personal questions tactful	Some people are happier to tick a box that their age group fits in rather than give their exact age.
• No leading questions	Asking "do you agree that......?" will produce biased answers
• Do a pilot survey	This means print out just a few copies of your questionnaire to give to a few people to fill in to make sure everything is clear and understood. Errors can be corrected before printing out lots.

Exercise 6C

1 Design a data collection sheet to help you compare the pulse rates of boys and girls.

2 Design a data collection sheet to find out what sort of television programmes different age groups and different sexes prefer.

3 Design a data collection sheet to find out about a subject of your choice.

4 These questions were all found on questionnaires.There is something wrong with them. For each question:
 i Say what is wrong with the question (in many cases there is more than one thing wrong with the question).
 ii Write a better version.
  a How much television do you watch?
  b Which is your favourite drink?
  Tea ☐ Coffee ☐ Water ☐
  c Do you agree that we get too much homework?
  d What sort of films do you like to watch?
  e Maths is my favourite subject, is it yours?
  f Why don't you walk to school?
  g Rate these from 1 to 5: cinema, television, going out with friends, playing computer games.
  h How often do you go swimming?
  never ☐ once ☐ twice ☐
  three times or more ☐
  i How much do you earn?
  j How much sleep did you get last night? Please tick.
  Less than average ☐
  About average ☐
  More than average ☐
  k Do you think maths is:
  brilliant ☐ quite good ☐

5 Write five bad questions of your own. Give them to the person sitting next to you to find out what is wrong with them.

6 Your head teacher wants to know what improvements can be made to your school. Write five possible questions she could write on her questionnaire.

7 Write five possible questions for a questionnaire to find out how your friends like to spend their free time, if they think there is enough to do in your area and what improvements they would make.

8 A company wants to find out what people think about their advertising campaign. Write five possible questions for their questionnaire.

9 Design a questionnaire to find out about a subject of your choice.

6.2 Averages and range

It is often useful to describe the data with a single value. An **average** is a single value that describes a data set.

There are three types of average:
- **mean** **mode** **median**

The one to use depends on the circumstances.

The mean

This is what many people mean by the word 'average'.

To find the mean of a data set, add all the values and divide by the number of values.

For example, in a cricket series, the batsman scored 107, 94, 106, 291, 14, 14 in six test innings.

Batsman's mean score

$$= \frac{\text{sum of scores}}{\text{number of scores}}$$

$$= \frac{107 + 94 + 106 + 291 + 14 + 14}{6} = \frac{626}{6}$$

$$= 104.3$$

Exercise 6D

1 Find the mean of these sets of data:
 a 1, 3, 5, 7, 9, 11, 13, 15
 b 24, 21, 20, 25, 21, 27
 c 6.5, 7.2, 4.1, 3.8, 9.4, 8.7, 5.3, 6.9, 7.2, 8.5

2 Aldie rolled a die four times. His scores were

4, 1, 5, 2

What was his mean score?

3 Aaron picked six pea pods. The numbers of peas in the pods were

7, 8, 10, 5, 6, 6

 a How many peas did he get altogether?
 b What was the mean number of peas in the pods?

4 Nailah bought five packets of sweets. The numbers of sweets in each packet were

28, 32, 29, 31, 30

What is the mean number of sweets in a packet?

5 The Super Stationery Company does a control check of its boxes of paperclips. The numbers of paperclips in eight boxes were found to be 48, 52, 52, 51, 49, 47, 53, 48

What is the mean number of paperclips per box?

6 The mean number of sweets in a packet is 24 sweets. If there are 3 packets how many sweets are there in total?

7 The mean of five numbers is 5. If four of the numbers are 4, 3, 7 and 2, what is the missing number?

8 The mean of seven numbers is 7. When another number is added the mean is still 7. What is the extra number? Choose from the following:

a 7 **b** 8 **c** $\frac{8}{7}$ **d** 0 **e** $\frac{7}{8}$

9 The mean height of five students is 158 cm. If another student is added to this group the mean height is then 158.4 cm. What is the height of the sixth student?

The mode and median

The **mode** is the data value that occurs most often.

Asking for the **modal** value means the same as "what is the mode?"

EXAMPLE 5

One morning, ten pairs of shoes were sold at Better Fit Shoe Shop. The sizes were:
6, 7, 7, 5, 8, 9, 8, 9, 9, 5
What was the modal size?
2 size 5
1 size 6
2 size 7
2 size 8
3 size 9 were sold.
Size 9 is the most common. The modal size is 9.

The **median** is the middle value of a data set.

To find the median you need to put the data in order of size.

EXAMPLE 6

Find the median of

a Test scores 3, 8, 4, 5, 9
b Temperatures 16°, 23°, 20°, 18°, 17°, 25°

a Test scores in size order are
3, 4, 5, 8, 9
The middle value is 5.
The median test score is 5.
b Temperatures in size order are
16°, 17°, 18°, 20°, 23°, 25°
The two middle values are 18° and 20°.
The median is half way between the two: 19°.
Hence median temperature is 19°.

Exercise 6E

1 Find the median and mode of these sets of data:
a 72, 68, 65, 70, 75, 79, 73, 70, 82
b 24, 21, 20, 25, 21, 27
c 2, 5, 5, 6, 9, 5, 4, 6, 7

2 The heights, in centimetres, of five flowers were

12 cm, 7 cm, 6 cm, 11 cm, 11 cm

a What is the modal height?
b What is the median height?

3 The shoe sizes of 8 students were

8, 6, 4, 6, 6, 7, 9, 10

a What is the modal size?
b What is the median size?
c Which of the two results would be more useful for the manager of a shoe store? Why?

4 The temperatures at Auckland Airport in New Zealand on six days were

21°C, 23°C, 19°C, 25°C, 25°C, 24°C

a What was the median temperature in that period?
b What was the modal temperature?

ACTIVITY

- Take some body measurements for some boys and girls, for example weight, arm length, shoe size.
- Find the mean, mode and median for boys and girls.
- Present your results to the class.

The range

- The difference between the lowest and highest number in a set is called the **range**. The range gives you an idea of how spread out the numbers are.

EXAMPLE 7

Find the range of the following data:

2, 11, 7, 3, 2, 8, 4

The range is the highest number 11, minus the lowest number 2.
Range $= 11 - 2 = 9$

Exercise 6F

1 Find the range of these sets of data:
a 72, 68, 65, 70, 75, 79, 73, 70, 82
b 24, 21, 20, 25, 21, 27
c 2, 5, 5, 6, 9, 5, 4, 6, 7

2 The weekly wages paid in an office are \$900, \$1200, \$1750, \$2500 and \$3200.
Find the range of the wages.

3 Errol Hendry is a farmer. He rears chickens for both eggs and meat. He sells some of his chickens at market.

Below is a record of his chicken sales over a period of eleven years.

Year	Chickens sold
2001	2100
2002	1800
2003	2300
2004	2000
2005	2400
2006	1700
2007	2300
2008	2200
2009	2500
2010	2300
2011	1500

What was the range of annual chicken sales for Errol?

4 Find the range of sentence length for the first 40 sentences of a book given below:
8, 17, 7, 3, 8, 13, 11, 79, 22, 4, 10, 16, 40, 6, 2, 16, 7, 11, 14, 3, 12, 9, 4, 11, 11, 23, 7, 51, 61, 43, 21, 5, 34, 14, 56, 48, 10, 12, 15, 20

Working with frequency tables

A television repair shop received the following number of calls per day over a period of 20 days:

6	6	3	2	7	4	2	4	7	6
3	4	2	6	2	6	5	3	7	5

To work out the mean number of calls per day you would need to add these numbers and divide by 20.

(There is no need to put the numbers in order before adding them. This has only been done to help with the next part of the explanation.)

Mean =

$$\frac{2+2+2+3+3+3+4+4+4+5+5+6+6+6+6+6+7+7+7}{20} = \frac{90}{20} = 4.5$$

This would take a while to work out. There is a faster way:

there are four 2s, three 3s, three 4s, two 5s, five 6s and three 7s so we can speed things up a little by doing:

$$\text{Mean} = \frac{4 \times 2 + 3 \times 3 + 3 \times 4 + 2 \times 5 + 5 \times 6 + 3 \times 7}{20} = \frac{90}{20} = 4.5$$

You can do this more clearly in a table:

Data value	Frequency	Data value × frequency
2	4	$2 \times 4 = 8$
3	3	$3 \times 3 = 9$
4	3	$4 \times 3 = 12$
5	2	$5 \times 2 = 10$
6	5	$6 \times 5 = 30$
7	3	$7 \times 3 = 21$
Total	20	90

$$\text{Mean} = \frac{\textit{Total (data value} \times \textit{frequency)}}{\textit{Total frequency}} = \frac{90}{20} = 4.5$$

This is an important method to use when the frequencies are really high!

Exercise 6G

1 The number of peanuts found in 42 pods is given in this frequency table. Copy and complete it to find the mean.

Peanuts per pod	Frequency	Peanuts × Frequency
1	2	
2	9	18
3	22	
4	9	
Total		

2 Rangi counted the number of people in 50 cars that passed her on the road into the town. Here is her record:

1, 1, 1, 1, 1, 2, 1, 1, 2, 1, 1, 1, 1, 1, 4, 2, 1,
1, 3, 1, 1, 1, 1, 1, 2, 2, 1, 1, 1, 2, 1, 1, 3, 1,
1, 1, 2, 1, 1, 2, 1, 1, 2, 1, 1, 3, 1, 1, 1, 2

Construct a frequency table for these results and use it to find the mean number of people per car.

3 Use a frequency table to calculate the mean of this set of twenty numbers:

6, 2, 1, 5, 3, 1, 2, 4, 4, 3, 4, 5, 6, 2, 1, 1, 3, 2, 2, 5

4 The table shows the scores of 30 students in a test.

Score	1	2	3	4	5	6	7	8	9	10
Number of students	2	0	3	2	3	5	8	4	2	1

Find the mean score for the 30 students.

5 The table shows the ages of 20 students who entered a music competition.

Age in years	12	13	14	15	16	17
Number of students	4	4	5	3	3	1

Find the mean age of the entrants.

6 Jasper keeps hens. The frequency table shows the number of eggs he gets per day.

Number of eggs	14	15	16	17	18	19	20	21	22	23
Number of days	1	0	2	4	6	3	4	4	3	1

Calculate the mean egg yield per day.

Mode or modal class from frequency tables

If data is in a frequency table, to find the mode you look for the highest frequency:

Score	2	3	4	5	6	7
Number of students	4	3	3	2	5	3

The highest frequency is 5, the data value with the highest frequency is 6.

The mode is 6.

If your data is in a grouped frequency table, you cannot find the mode as you do not know what the individual data values are. Instead you find the **modal class** which is the class with the highest frequency.

EXAMPLE 8

The ages of 50 people in a village are:

Age	0–9	10–19	20–29	30–39	40–49
Frequency	12	9	7	7	6
Age	50–59	60–69	70–79	80–89	90–99
Frequency	4	3	1	1	0

What is the modal class?
The age group with the highest frequency is 0–9 years.
The modal class is 0–9 years.

Exercise 6H

1 Find the modal number of peanuts per pod from Question **1** in Exercise 6G.

2 Find the modal score from Question **4** in Exercise 6G.

3 Find the modal age in years from Question **5** in Exercise 6G.

4 Find the modal number of eggs per day from Question **6** in Exercise 6G.

5 The marks in a test of 70 students were:

Marks	0–9	10–19	20–29	30–39	40–49
Frequency	2	5	10	13	21
Marks	50–59	60–69	70–79	80–89	90–99
Frequency	6	6	3	2	2

What is the modal class?

6 The masses of 100 school children were:

Mass (kg)	31–35	36–40	41–45	46–50
Frequency	6	8	22	31
Mass (kg)	51–55	56–60	61–65	66–70
Frequency	12	11	5	5

What is the modal class?

7 A biologist measures the lengths of 190 leaves:

Length (cm)	0–1.9	2–3.9	4–5.9
Frequency	3	33	62
Length (cm)	6–7.9	8–9.9	10–11.9
Frequency	49	36	7

What is the modal class?

TECHNOLOGY

Need more practice? Visit the Statistics and probability section at

www.bbc.co.uk/schools/gcsebitesize/maths

to learn more about mode, mean and median and how to calculate them. Make sure you do the activities and questions!

Consolidation

Example 1

The heights of 30 children in centimetres are:

128 143 162 152 147 143 137 129 145 152
132 137 141 146 149 153 151 148 147 161
126 133 142 146 138 139 156 151 149 143

Construct a frequency table using the groups 125–129, 130–134 etc. What is the modal class?

Height	125–129	130–134	135–139	140–144	145–149	150–154	155–159	160–164
Tally	|||	||	||||	~~||||~~	~~||||~~ |||	~~||||~~	|	||
Frequency	3	2	4	5	8	5	1	2

The modal class is 145–149.

Example 2

Design a data collection sheet to find out if height and shoe size are related.

Height (cm)	Shoe size

Example 3

The shoe sizes of ten boys are:

3, 7, 4, 6, 7, 6, 5, 3, 8, 6

Find the:

a mean
b mode
c median
shoe size
d range

---

a Mean $= \dfrac{3+7+4+6+7+6+5+3+8+6}{10}$

$= \dfrac{55}{10} = 5.5$

b Mode = most frequently occurring number

= 6 (occurs 3 times)

c To find the median write in size order:

$3, 3, 4, 5, \underbrace{6, 6}, 6, 7, 7, 8$

There are two middle numbers; 6 and 6,

so the median is $\dfrac{6+6}{2} = \dfrac{12}{2} = 6$.

d range = highest – smallest $= 8 - 3 = 5$

Example 4

What is wrong with these questions:

1 *Do you agree that eating sweets and chocolate is bad for you?*

This is a leading question and it is open. It would be better to ask:

Is eating sweets and chocolate bad for you? Please tick.

☐ yes ☐ no ☐ don't know

2 *How often do you visit a fast food restaurant?*

This question gives no time frame or response boxes. It would be better to ask:

Approximately how many times do you eat at a fast food restaurant per month? Please tick.

☐ 1–2 ☐ 3–4 ☐ 5–6 ☐ 7–8
☐ more than 8 ☐ I don't eat fast food

3 *How many fizzy drinks do you have during the day? Please tick.*

☐ 1–2 ☐ 3–4 ☐ more than 4

This question doesn't cover all options in the response boxes. There is nothing to tick if you don't drink fizzy drinks. It would be better to ask:

How many fizzy drinks do you have during the day?

Please tick. ☐ 0 ☐ 1–2 ☐ 3–4 ☐ more than 4

Exercise 6

1 The weights of each of the students in Form 101 were recorded in kilograms as follows:

36, 34, 42, 53, 52, 45, 36, 47, 38, 50
47, 35, 39, 47, 44, 43, 51, 60, 46, 49
52, 38, 42, 43, 53, 41, 53, 61, 47, 50

a Construct a frequency table using the groups 30–34, 35–39 etc.
b What is the modal class?

2 The ages of the students in Form 101 are:

11, 12, 12, 11, 11, 13, 12, 12, 13, 10,
14, 12, 13, 13, 11, 12, 12, 13, 13, 14,
11, 11, 12, 14, 13, 12, 13, 10, 14, 13

a Construct a frequency table to show these ages.
b Find the mean using the frequency table.
c What is the mode?

3 Find the mean, mode, median and range of these sets of numbers.
a 7, 3, 5, 4, 6, 2, 5
b 54, 32, 56, 48, 32, 56
c 12, 13, 11, 14, 13, 10, 8, 7, 13, 12

4 The number of tickets bought per person for a show are shown in the table.

No. of tickets bought	Frequency
1	13
2	42
3	37
4	25
5	3

Calculate the mean number of tickets bought per person for the show.

5 Design a data collection sheet to find out the difference between how much time boys and girls spend doing homework.

6 What is wrong with these questions? In each case write an improved question:
a How much time do you spend on sport?
☐ 0–10 ☐ 11–20 ☐ 21–30
☐ more than 30
b How old are you?
c I prefer swimming to running, do you?
d How much of the Olympics did you watch on television?
e How much time do you spend swimming a month? ☐ 0–2 hours ☐ 2–4 hours
☐ more than 4 hours.

7 Sian does a survey about the time that people spend reading and watching television.
She asks:
1 How much do you read?
2 Do you agree that people watch too much television?
a Explain why each question is not suitable
b Rewrite these questions to improve them

8 

The mean length for four snakes in a zoo is 51.4 cm. The lengths for three of the snakes are 48.0 cm, 52.2 cm and 55.3 cm.
a What is the total length of the four snakes?
b How long is the fourth snake?
c A fifth snake of length 53.4 cm is added to the show case. What is the mean length of the five snakes?

9 The mean mass of Tom, Mick and Harry is 51 kg. Tom weighs 47.5 kg and Mick weighs 52 kg.
a What is the total mass of the three boys?
b How much does Harry weigh?

Summary

You should know …

1 A tally helps you count items. A frequency table can be used to show data clearly.
For example:
The votes for favourite food in a class

Singer	Tally	Frequency
Sandwiches	𝍸 |	6
Pizza	𝍸 𝍸 ||	12
Fajitas	|||	3
Curry	𝍸 ||||	9

2 Data should be grouped in a frequency table when there is a lot of data with a big range.
For example:
11–20, 21–30, 31–40 etc.

3 How to describe data using a single value called an **average**. There are three types of average: **mean**, **mode** and **median**.
For example:
What is the mean, mode and median of 4, 6, 2, 10, 2?

$$\text{Mean} = \frac{4 + 6 + 2 + 10 + 2}{5} = \frac{24}{5} = 4.8$$

Mode = most common value = 2
Values in order are 2, 2, 4, 6, 10
Median = middle value = 4

Check out

1 State the numbers represented by the following tallies:

a ||||
b 𝍸 𝍸 ||
c 𝍸 𝍸 𝍸

2 Construct a grouped frequency table to gather the following data (these are the ages of people at a party):

11 23 15 38 34 31 36
24 19 21 22 47 35 32
53 17 34 15 49 13 61
64 23 54 17 28 33 34

3 Find the mean, mode and median of these test scores.

a 6, 6, 10, 8, 4
b 17, 14, 13, 12, 13, 18
c Find the mean and mode for this data:

Data value	Frequency
4	3
5	2
6	5

d Find the modal class for this data:

Group	Frequency
10–19	4
20–29	7
30–39	5

4 The range tells us how spread out the numbers are.
For example:
The range of 23, 11, 54, 26, 41 is the highest – the smallest
$54 - 11 = 43$
The range is 43

4 Work out the range for these:
a 72, 68, 65, 70, 75, 70, 82
b 24, 21, 20, 25, 21, 27
c 2, 5, 6, 9, 5, 4, 6

5 Data collection sheets and questionnaires can be used to collect data. When designing questionnaires:
- Be polite. Explain the purpose of the questionnaire.
- Keep it short.
- Give a choice of answers. Include a time frame if necessary.
- Make sure the answer choices do not overlap and that they cover all options.
- Keep personal questions tactful.
- No leading questions.
- Do a pilot survey.

5 Design
a a data collection sheet
b a questionnaire
to collect data about your friends on how they spend their spare time and how much spare time they have.

Review A

1 Calculate:
a $21 + 24 + 241$ **b** $296 - 187$
c $2141 - 719$ **d** $1052 + 89 + 45$

2 Kristy spends about the same amount each day on lunch. Last week she spent
\$3.25, \$3.08, \$3.63, \$2.95 and \$3.12.

Write these amounts in order, from the smallest to the largest.

3 Write the first six multiples of these numbers:
a 3 **b** 5 **c** 8
d 13 **e** 15 **f** 35

4 Choose from the words below to describe these:
a $2x$ **b** $3x + 8 = 2x$ **c** $4t + 8m$

Expression Term Equation

5 Draw the following angles:
a 42° **b** 134° **c** 195°

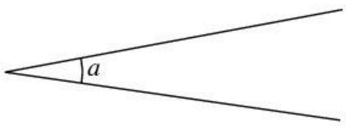
6 Design a questionnaire to find out how much pocket money students get and what they spend it on.

7 Calculate:
a 26×53 **b** 34×68 **c** 54×86 **d** 84×239

8 **a** A passenger bus holds 46 people.
How many people can fit into 17 buses?
b How many buses are needed for 1058 passengers?

9 Measure the lettered angles shown in each of the following:

a



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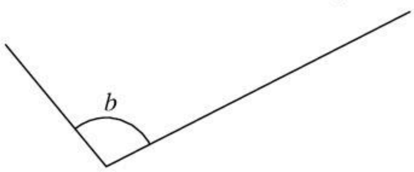
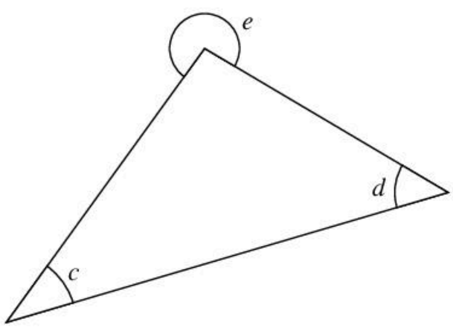


c



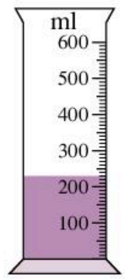
10 Write these numbers in order, largest first.
a 94.36, 94.5, 94.63, 95.071
b 10.9, 11.092, 10.84, 10.892, 11.01
c 42.393, 43.2, 43.05, 42.811

11 Round these numbers as shown:
a 2345 to the nearest 100
b 755 to the nearest 10
c 23.6573 to the nearest whole number
d 23500 to the nearest 1000

12 Expand:
a $3(2t + 1)$
b $5(4T + 3)$
c $10(7y + 6c)$

13 Are the following numbers divisible by 6?
a 252 **b** 290 **c** 726

14 I add the water from this measuring cylinder to 2.7 litres of water. How much water do I now have altogether?



15 A cube has 6 faces and 8 vertices. Name a solid that has 8 faces and 6 vertices.

16 Eggs are sold in boxes of six. How many boxes are needed to pack 838 eggs?

17 **a** Mr Shillingford drives 32 kilometres to and from work each day. How many kilometres does he drive in 28 working days?
b How many days does it take him to travel 2208 kilometres?

18 Write down:
a the number of days in w weeks
b the number of hours in d days
c the number of minutes in h hours.

19 List the factors of these numbers and state whether the numbers are prime or composite.
a 6 **b** 23 **c** 15
d 30 **e** 39 **f** 48

20 Work out:

a $1 + 2 \times 5$ **b** $10 - 16 \div 8$

c $5 \times 2 - 35 \div 5$ **d** $30 - 2 \times 5 \times 2$

e $4 + (8 - 2) \times 5$ **f** $^{-}20 - (2 + 1) \times 4$

21 Construct triangle ABC where AB = 6.4 cm, AC = 5.2 cm and $\angle$BAC = 50°

22 Design a data collection sheet to find out information about shoe sizes and clothes sizes of boys and girls in your class.

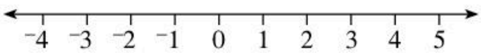
23 Change to cm:

a 2.7 m **b** 4300 mm

c 0.002 km **d** 0.3 m

24 Calculate:

a 7^2 **b** $\sqrt{81}$ **c** 13^2 **d** $\sqrt{144}$

25 

Use the number line above to help you to find:

a $2 - 5$ **b** $3 - 7$

c $^{-}3 + 4$ **d** $^{-}4 + 2$

e $^{-}1 - 3$ **f** $^{-}2 + 6$

26 24 students in a maths test scored the following marks.

7, 9, 3, 8, 7, 2, 9, 7, 6, 2, 7, 4, 5, 1, 3, 7, 8, 5, 2, 7, 5, 3, 7, 8

a Copy and complete the frequency table started below:

Mark	Tally	Frequency
1		
2		
3		

b Which mark is the mode?

c What is the median score?

d What is the mean score?

e What is the range?

27 Copy and complete:

a $3 + \square = 19$ **b** $19 + \square = 3$

c $^{-}4 + \square = 6$ **d** $^{-}4 - \square = {}^{-}9$

e $\square + {}^{-}3 = {}^{-}8$ **f** $\square - 9 = {}^{-}7$

g $\square - 4 = {}^{-}5$ **h** $\square + {}^{-}15 = {}^{-}10$

i $\square + \square + \square = {}^{-}12$ **j** $\square + \square = {}^{-}18$

28 Simplify:

a $x + y + 2x$

b $a + 2a + b + a$

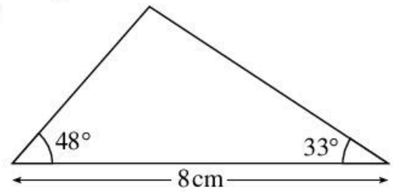
c $p + q + q + q + p$

d $3s + 2t + s$

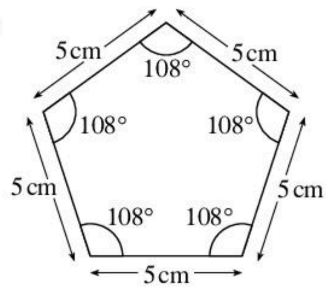
29 16 sheets of my writing paper have a mass of 70 g. Does a box of 500 sheets have a mass of more or less than 2 kg?

30 Use your ruler and protractor to draw accurately the shapes below.

a



b



31 Find the HCF of:

a 14 and 63 **b** 18 and 42

c 72 and 64 **d** 84, 32 and 48

32 What units would you use to measure:

a the height of a tree

b the length of your hair

c the weight of a banana

d the weight of your school bag

e the distance from your school to New York

f the amount of liquid you drank today?

33 Simplify:

a $2x + y - x$

b $3a + 2b - 2a$

c $2p + q - p + q - p$

d $3s - 2t + t + s + 4t$

34 **a** The mean of four marks is 73%. If three of the marks are 52%, 61% and 89%, what is the fourth mark?

b If the mean of four marks is 62% and when a fifth is added the mean becomes 68%, what is the fifth mark?

35 Write these numbers to one decimal place.

a 14.321 **b** 6.467

c 0.355 **d** 1.7241

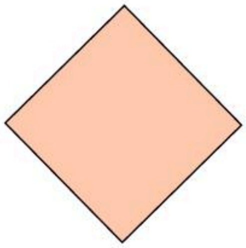
e 7.096 **f** 0.00345

36 **a** I am a factor of 12. I am a prime number. I am not factor of 20. What number am I?

b I am a multiple of 3. I am also a multiple of 4. The sum of my digits is a prime number. I have 2 digits. What number am I?

c I am a square number. I am a factor of 36. I am also a factor of 18. I am not 1. What number am I?

37 What is the name of the shape below?



38 For this data:

Age	Frequency
10	5
11	12
12	18
13	15

Find **a** the mean age **b** the modal age

39 Change to ml:

a 4 litres **b** 0.23 litres **c** 0.007 litres

40 The table below shows how the temperature in London varied during one night in January.

Time	Frequency
9.00 pm	5°C
10.00 pm	2°C
11.00 pm	1°C
12.00 midnight	$^{-}1$°C
1.00 am	$^{-}5$°C
2.00 am	$^{-}5$°C
3.00 am	$^{-}6$°C
4.00 am	$^{-}1$°C
5.00 am	3°C
6.00 am	5°C

a At what time was the coldest temperature recorded?

b Between which hours was the greatest fall in temperature?

c Between which hours was the greatest rise in temperature?

d By how much did the temperature change between 9.00 pm and 6.00 am?

41 Find the LCM of:

a 9 and 15 **b** 24 and 36

c 26 and 78 **d** 5, 12 and 20

42 Write out these lengths in order starting with the longest:

a 23 mm, 2 cm, 3 cm 2 mm

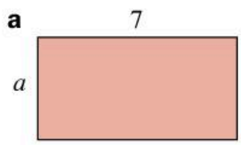
b 20 cm, 700 m, 2 km

c 6 m, 500 cm, 4 km

d 6000 m, 5000 cm, 4 km

43 Write down the area and the perimeter of:

a



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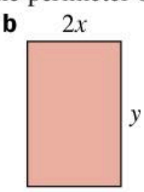


44 Write down the answer:

a $5 - 17$ **b** $^{-}6 + 19$

c $^{-}6 - 4$ **d** $^{-}5 + {}^{-}7$

e $^{-}15 + {}^{-}23$ **f** $49 + {}^{-}53$

g $7 - {}^{-}2$ **h** $^{-}4 - {}^{-}3$

45 Copy and complete:

a $2.3 \times \square = 23$ **b** $\square \times 100 = 350$

c $\square \div 100 = 0.54$ **d** $8.9 \div \square = 0.089$

e $0.05 \times \square = 50$ **f** $0.2 \times \square = 20$

46 What is the modal class for this data?

Group	Frequency
0–9	7
10–19	10
20–29	13
30–39	8

47 Change to kg:

a 4000 g **b** 0.32 t **c** 700g **d** 3.4 t

48 Calculate:

a 14^2 **b** $\sqrt{324}$ **c** 19^2 **d** $\sqrt{225}$

49 My age is x years and my son's age is y years. Write down an equation that states:

a The total of our ages is 72.

b I am twice as old as my son.

50 Calculate:

a $135 \div 9$ **b** $224 \div 7$

c $722 \div 19$ **d** $805 \div 23$

e $364 \div 14$ **f** $476 \div 28$

51 Expand:

a $3(2m - 4t + 7)$

b $-3(10x - 3)$

c $10(x - 2y) - 2(5x + 2y)$

d $10t + 3(5p + 2t) - p$

52 2, 8, 2, 6, 9, 12, 2, 2, 1, 6

For this data find the

a mean **b** median **c** mode **d** range

53 Are the following numbers divisible by 9?

a 259 **b** 351 **c** 432

54 A car has a mass of 0.9 t. What is its total mass when carrying four passengers with a mass of 80 kg each?

55 What is the largest prime number less than 100?

56 **a** What is the LCM of 18 and 12?

b What is the smallest number that can be divided exactly by 8, 13 and 24?

c What is the smallest number which has remainder 3 when divided by 5, 7, and 8?

57 Which, if any, of the following solids are prisms?

a cube **b** tetrahedron

c cylinder **d** sphere

58 A prism has equal triangles at each end. Sketch the solid and write down its number of

a edges **b** faces **c** vertices.

59 Write down a pair of lengths that are the same, one from box A, one from box B.

A = $\boxed{\text{3 cm, 32 mm, 0.32 m, 250 m}}$

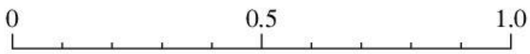
B = $\boxed{\text{0.25 km, 3.2 cm, 0.03 m, 32 cm}}$

60 Write an equation that states:

a Sarah has \$$a$ and Ruth has \$$b$ and they have \$10 together.

b Eli has \$$c$ and Tom has \$$d$ but Eli has \$10 more than Tom.

61 Draw a number line with divisions like this:



On your number line mark, as accurately as you can, the positions of the numbers:

a 0.3 **b** 0.8 **c** 0.15

d 0.75 **e** 0.43 **f** 0.06

62 Write down two numbers which are:

a less than $^{-}1$ **b** more than $^{-}3$

7 Fractions

Objectives

- Calculate simple fractions of quantities.
- Simplify fractions by cancelling common factors and identify equivalent fractions; change an improper fraction to a mixed number, and vice versa.
- Know that in any division where the dividend is not a multiple of the divisor there will be a remainder, e.g. $157 \div 25 = 6$ remainder 7. The remainder can be expressed as a fraction of the divisor, e.g. $157 \div 25 = 6\frac{7}{25}$.
- Add and subtract two simple fractions, e.g. $\frac{1}{8} + \frac{9}{8}$, $\frac{11}{12} - \frac{5}{6}$; find fractions of quantities (whole number answers); multiply a fraction by an integer.

What's the point?

Fractions are used frequently in everyday life. How often do you say things such as 'I'd like half a glass', or 'Two-thirds of the class were sick'? Cooks use fractions all the time in their recipes: 'Take $2\frac{1}{2}$ cups of flour and mix in $\frac{1}{4}$ teaspoonful of salt ...'

Before you start

You should know ...

1 A **fraction** is the amount of each part when something is divided into equal sized parts.

For example:

One half $(\frac{1}{2})$
Half an orange

One quarter $(\frac{1}{4})$
One quarter of a pizza

Check in

1 **a** Janis shared a grapefruit equally between her sister and herself. How much grapefruit did each get?

b Alan got half marks in a test marked out of ten.
What was Alan's score?

2 How to use numerals to describe fractions:

this shows how many parts you have — 2 is the numerator

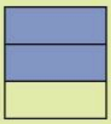
$$\frac{2}{3}$$

this shows how many parts there are altogether — 3 is the denominator

3 How to use words to describe fractions:

$\frac{2}{3}$ is two-thirds.

4 How to describe what fraction of a diagram is shaded:



$\frac{2}{3}$ of a rectangle is shaded (two-thirds)

2 Write down the numerator and denominator of these fractions.

a $\frac{3}{4}$

b $\frac{2}{3}$

c $\frac{5}{7}$

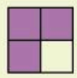
d $\frac{2}{10}$

3 Write these fractions as numerals.

a three-quarters
b two-fifths
c five-sixths

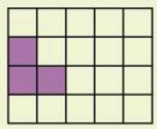
4 What fraction of each shape is shaded?

a

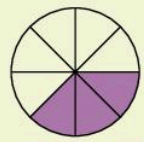


b

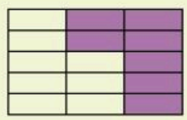
c



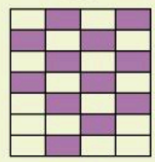
d



e



f



7.1 Calculating fractions

Fractions of a whole

To find a half $(\frac{1}{2})$ of an amount you divide by 2.

To find a third $(\frac{1}{3})$ of a whole amount you divide by 3, and so on ...

To find $\frac{1}{3}$ of 12 you need to divide 12 into 3 equal parts.

So $\frac{1}{3}$ of $12 = 12 \div 3 = 4$

EXAMPLE 1

What is $\frac{1}{5}$ of \$30?

This means that \$30 is divided into 5 equal parts, so

$\frac{1}{5}$ of \$30 = \$30 ÷ 5 = \$6

Exercise 7A

1 Work out

a $\frac{1}{2}$ of 10 **b** $\frac{1}{4}$ of 20

c $\frac{1}{3}$ of 9 **d** $\frac{1}{5}$ of 15

e $\frac{1}{12}$ of 60 **f** $\frac{1}{15}$ of 90

2 What is $\frac{1}{3}$ of \$27?

3 Alicia's class has 36 pupils. One-third are girls. How many are boys?

4 Akinni earns \$1600 each month. He saves one-quarter of it. How much does he spend each year?

Other fractions of a whole amount can also be worked out in a similar manner.

EXAMPLE 2

What is $\frac{3}{4}$ of 36?

$\frac{1}{4}$ of 36 = 36 ÷ 4 = 9

$\frac{3}{4}$ of 36 is three times this, so

$\frac{3}{4}$ of 36 = 3 × 9 = 27

A picture shows Example 2 more clearly.

$\frac{1}{4}$ of 36 • • • • • • • • •
$\frac{1}{4}$ of 36 • • • • • • • • • $\frac{3}{4}$ of 36 = 27
$\frac{1}{4}$ of 36 • • • • • • • • •
$\frac{1}{4}$ of 36 • • • • • • • • •

Exercise 7B

1 What is:

a $\frac{1}{8}$ of 24 **b** $\frac{3}{8}$ of 24

c $\frac{1}{5}$ of 35 **d** $\frac{4}{5}$ of 35

e $\frac{1}{7}$ of 42 **f** $\frac{6}{7}$ of 42

g $\frac{1}{9}$ of 63 **h** $\frac{5}{9}$ of 63

i $\frac{1}{12}$ of 60 **j** $\frac{7}{12}$ of 60?

2 Copy out the table below. Draw lines to show the calculations that have the same answer. One has been done for you.

$\frac{1}{4}$ of 28	$\frac{3}{7}$ of 49
$\frac{2}{3}$ of 27	$\frac{3}{5}$ of 30
$\frac{5}{6}$ of 12	$\frac{1}{5}$ of 35
$\frac{3}{8}$ of 56	$\frac{5}{12}$ of 24

3 Find

a $\frac{2}{3}$ of 18 centimetres

b $\frac{3}{4}$ of 24 students

c $\frac{7}{10}$ of 80 fishing boats

d $\frac{5}{8}$ of 64 lemons

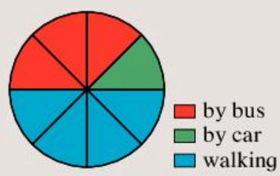
e $\frac{3}{11}$ of 121 exercises

f $\frac{5}{8}$ of 96 minutes

g $\frac{3}{5}$ of 75 pencils

h $\frac{5}{6}$ of 42 bananas

4 **a** The circle represents the students of Form 2B, and shows how they get to school every day.



What fraction of the class comes to school

i by bus
ii on foot
iii by car?

b There are 48 students in Form 2B. Work out the *number* of students who come to school:

i by bus
ii on foot
iii by car.

5 The diagram represents the passengers on a flight between Indonesia and the UK. The shaded part represents the female passengers.

a What fraction does each square represent?
b What fraction of the passengers are female?
c There are 100 passengers on the plane. How many passengers does each square represent?
d How many of the passengers are female?

6 Two brands of soap powder, Cleano and Super Wash, are both sold for $9.95.

In a sales promotion:
- $\frac{1}{3}$ box of Cleano (normal weight 720 g) is given away free
- $\frac{1}{4}$ box of Super Wash (normal weight 800 g) is given away free

Which brand of soap powder is the better buy during the promotion?

7 Find:

a $\frac{3}{11}$ of 132 **b** $\frac{6}{13}$ of 169
c $\frac{5}{12}$ of 144 **d** $\frac{3}{13}$ of 169
e $\frac{1}{5}$ of $2.40 **f** $\frac{3}{5}$ of $8.35
g $\frac{2}{7}$ of $8.82 **h** $\frac{3}{8}$ of $36.48

(**Hint:** in parts **e** to **h** you may want to change the currency from dollars to cents so that you can work with whole numbers).

8 Fill in the missing numbers:

a $\frac{1}{2}$ of $40 = \frac{1}{4}$ of $\square$
b $\frac{3}{4}$ of $200 = \frac{1}{3}$ of $\square$
c $\frac{1}{3}$ of $\square = \frac{2}{3}$ of $\square$
d There are many possible answers to part **c**. Find some more answer pairs and write down the connection between the pairs of answers.

9 Aakesh earns $280 a week working at the cricket ground. $\frac{1}{4}$ of his earnings is spent on rent. $\frac{1}{5}$ of his earnings is taken in tax. He spends $\frac{3}{8}$ of his earnings on going out, clothes and food. He saves the rest.

a How much is Aakesh's rent?
b How much does Aakesh pay in tax?
c How much does Aakesh spend on going out, clothes and food?
d How much does Aakesh save?
e What fraction of his earnings does Aakesh save per week?

10 Calculate:

a $\frac{2}{5}$ of 4 metres (giving your answer in cm)
b $\frac{3}{10}$ of 12 cm (giving your answer in mm)
c $\frac{5}{6}$ of 1 day (giving your answer in hours)
d $\frac{7}{8}$ of 3 kg (giving your answer in g)

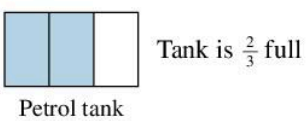
The whole from fractions

Sometimes you know the amount a fraction represents, but you need to find the size of the whole.

EXAMPLE 3

The petrol tank in Henry's car holds 40 litres when it is $\frac{2}{3}$ full.

How much petrol does the tank hold when full?



The shaded part ($\frac{2}{3}$) represents 40 litres.

So each part represents $40 \div 2 = 20$ litres.

There are 3 parts, so the tanks holds $20 \times 3 = 60$ litres.

Exercise 7C

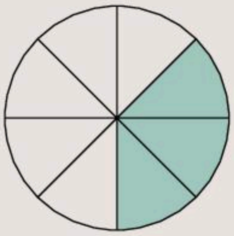
1 Find how much these petrol tanks hold if

a $\frac{1}{3}$ of one tank holds 20 litres

b $\frac{1}{4}$ of one tank holds 16 litres

c $\frac{2}{3}$ of one tank holds 24 litres.

2 The circle represents Form 1A.

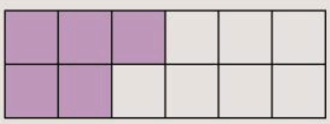


a What fraction has been shaded?

b The shaded part represents 15 students. How many students are represented by one part?

c How many equal parts are there? How many students are in the class?

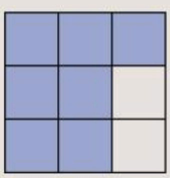
3 This drawing represents the teachers in a large school. The shaded part represents 30 teachers.



a How many teachers are represented by one part?

b How many teachers are in the school?

4 In the drawing the shaded part represents 49 litres.



How many litres are there altogether?

5 Find the answer to Question **4**, if the shaded part represents

a 7 litres

b 21 litres

c 35 litres.

6 What are the missing numbers?

a $\frac{1}{4}$ of $\square = 15$

b $\frac{3}{5}$ of $\square = 18$

c $\frac{2}{3}$ of $\square = 60$

d $\frac{1}{\square}$ of $80 = 16$

e $\frac{2}{\square}$ of $15 = 10$

f $\frac{3}{\square}$ of $80 = 24$

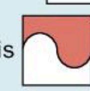
g $\frac{\square}{3}$ of $90 = 30$

h $\frac{\square}{4}$ of $20 = 15$

i $\frac{\square}{8}$ of $16 = 14$

INVESTIGATION

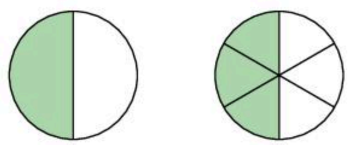
You can show $\frac{1}{2}$ on a square like this 

or like this  or even like this .

How many different ways can you find of dividing up a square into two halves?

7.2 Equivalent fractions

In each of these circles one half is shaded.



$\frac{1}{2}$ $\frac{3}{6}$

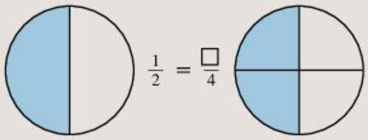
The same fraction is shaded so $\frac{1}{2} = \frac{3}{6}$.
$\frac{1}{2}$ and $\frac{3}{6}$ are **equivalent fractions**.

- Equivalent fractions show the same fraction, but use different numbers.

Exercise 7D

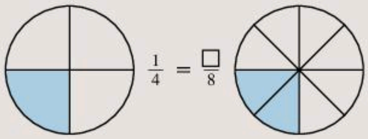
1 The diagrams show pairs of equivalent fractions. Use them to find the missing numbers.

a



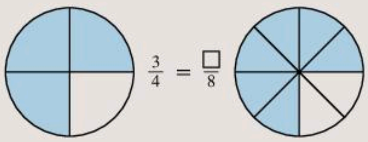
$\frac{1}{2} = \frac{\square}{4}$

b



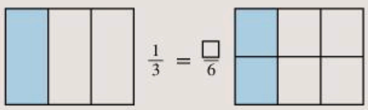
$\frac{1}{4} = \frac{\square}{8}$

c



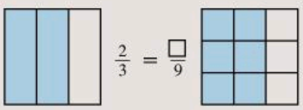
$\frac{3}{4} = \frac{\square}{8}$

d



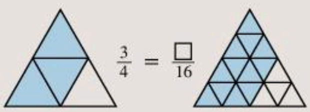
$\frac{1}{3} = \frac{\square}{6}$

e



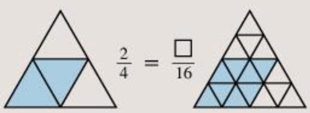
$\frac{2}{3} = \frac{\square}{9}$

f



$\frac{3}{4} = \frac{\square}{16}$

g



$\frac{2}{4} = \frac{\square}{16}$

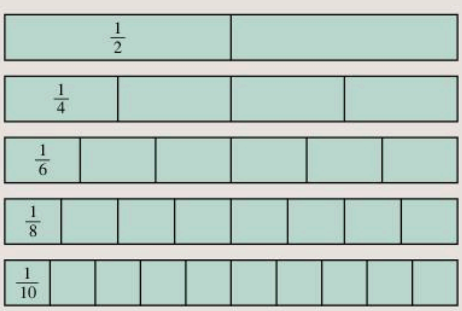
2 On squared paper, draw rectangles to show that

a $\frac{1}{2} = \frac{2}{4}$ **b** $\frac{1}{3} = \frac{2}{6}$

c $\frac{3}{5} = \frac{6}{10}$ **d** $\frac{3}{4} = \frac{6}{8}$

e $\frac{4}{5} = \frac{8}{10}$ **f** $\frac{5}{6} = \frac{10}{12}$

3



a The drawing shows five strips of paper. Are all the strips equal in length?

b The first strip is divided into halves. The second strip is divided into quarters. The third strip is divided into sixths, and so on.

i How many halves are in the first strip?

ii How many quarters are in the second strip?

iii How many sixths are in the third strip?

iv How many eighths are in the fourth strip?

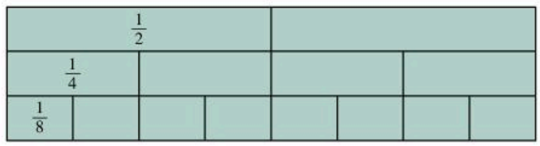
v How many tenths are in the fifth strip?

c Copy and complete:

$$\frac{1}{2} = \frac{\square}{4} = \frac{\square}{6} = \frac{\square}{8} = \frac{\square}{10}$$

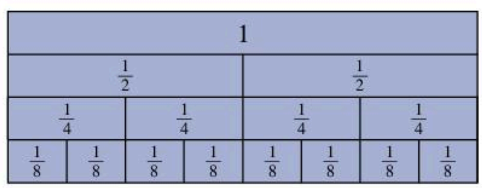
Fraction walls

You can put the strips from Question **3** of Exercise 7D together to get a fraction wall.



Fraction walls can help you find equivalent fractions.

EXAMPLE 4

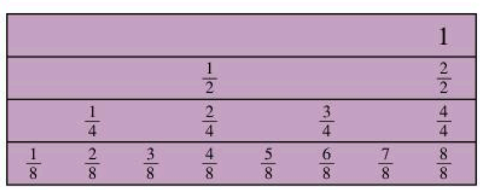


Use the fraction wall to

a write down a fraction equal to $\frac{3}{4}$

b write down two fractions greater than $\frac{1}{2}$.

The fraction wall can also be shown as:

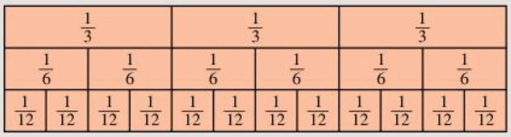


a You can see that $\frac{3}{4} = \frac{6}{8}$

b All the fractions to the right of $\frac{1}{2}$, that is $\frac{5}{8}, \frac{3}{4}, \frac{7}{8}$ etc. are greater than $\frac{1}{2}$.

Exercise 7E

1



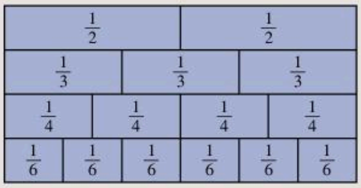
Use the fraction wall to write down:

a two fractions equal to $\frac{2}{3}$

b two fractions equal to $\frac{2}{6}$

c two fractions greater than $\frac{3}{6}$.

2



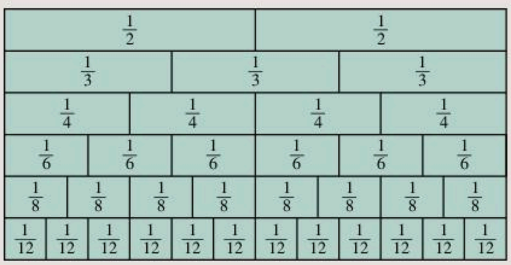
Use the fraction wall to write down:

a two fractions equal to $\frac{1}{2}$

b one fraction equal to $\frac{1}{3}$

c three fractions less than $\frac{3}{4}$.

3



Use the fraction wall to copy and complete:

a $\frac{1}{3} = \frac{\square}{6} = \frac{\square}{12}$

b $\frac{1}{4} = \frac{\square}{8} = \frac{\square}{12}$

c $\frac{3}{4} = \frac{\square}{8} = \frac{\square}{12}$

d $\frac{3}{6} = \frac{\square}{8} = \frac{\square}{12}$

e $\frac{9}{12} = \frac{\square}{8} = \frac{\square}{4}$

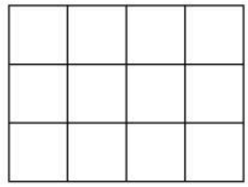
f $\frac{4}{12} = \frac{\square}{6} = \frac{\square}{3}$

Fractions of shapes

Another way of showing equivalent fractions is to shade parts of a shape.

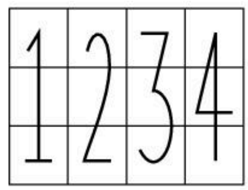
EXAMPLE 5

a Shade $\frac{3}{4}$ of the shape.

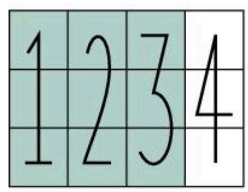


b Use your diagram to find a fraction equivalent to $\frac{3}{4}$.

a First, divide the shape into four equal parts, that is, into quarters.



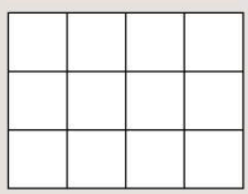
You want three of these quarters, $\frac{3}{4}$



b 9 of the 12 small squares are shaded.
So $\frac{3}{4} = \frac{9}{12}$

Exercise 7F

1 **a** Copy this shape on to squared paper.



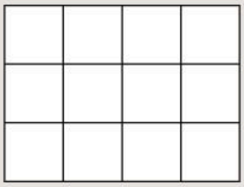
b What fraction of the whole rectangle is each small square?

c Divide the shape into three equal parts. What fraction of the whole rectangle is each of these parts?

d Look at one of the parts. How many small squares does it contain?

e Copy and complete: $\frac{1}{3} = \frac{\square}{12}$

2 **a** Copy this shape on to squared paper.

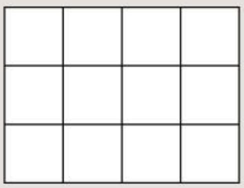


b Divide it into six equal parts.

c Shade $\frac{4}{6}$ of the shape.

d Copy and complete: $\frac{4}{6} = \frac{\square}{12}$.

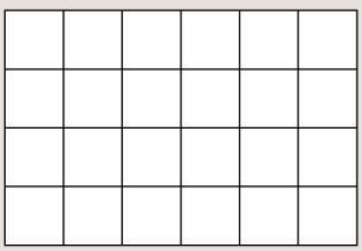
3 **a** Copy this shape on to squared paper.



b Shade $\frac{5}{6}$ of the shape.

c Copy and complete: $\frac{5}{6} = \frac{\square}{12}$.

4 **a** Copy the shape on to squared paper.



b Shade $\frac{2}{3}$ of the shape.

5 Repeat Question **4** for each of these fractions:

a $\frac{4}{6}$ **b** $\frac{6}{12}$ **c** $\frac{3}{8}$

d $\frac{6}{8}$ **e** $\frac{3}{4}$

6 Use your shaded shapes in Questions **4** and **5** to copy and complete:

a $\frac{2}{3} = \frac{\square}{24}$ **b** $\frac{4}{6} = \frac{\square}{24}$

c $\frac{6}{12} = \frac{\square}{24}$ **d** $\frac{3}{8} = \frac{\square}{24}$

e $\frac{6}{8} = \frac{\square}{24}$ **f** $\frac{3}{4} = \frac{\square}{24}$

Calculating equivalent fractions

There is a quicker way to find equivalent fractions than by drawing a picture. The Investigation will help you find out how.

INVESTIGATION

The fractions $\frac{1}{3}, \frac{2}{6}, \frac{3}{9}, \frac{4}{12}$ are equivalent fractions.

$\frac{1}{3} = \frac{2}{6} = \frac{3}{9} = \frac{4}{12}$

a Can you see how the fraction $\frac{2}{6}$ can be found from the fraction $\frac{1}{3}$?

b Can you see how the fraction $\frac{3}{9}$ can be found from the fraction $\frac{1}{3}$?

c Can you see how the fraction $\frac{4}{12}$ can be found from the fraction $\frac{1}{3}$?

d Can you write any other fractions equal to $\frac{1}{3}$?

You can write an equivalent for any fraction by multiplying its numerator and denominator by the same number.

$$\frac{1}{3} \overset{\times 2}{\underset{\times 2}{=}} \frac{2}{6} \qquad \frac{1}{3} \overset{\times 3}{\underset{\times 3}{=}} \frac{3}{9} \qquad \frac{1}{3} \overset{\times 4}{\underset{\times 4}{=}} \frac{4}{12}$$

EXAMPLE 6

Copy and complete:

$$\frac{3}{4} = \frac{\square}{24}$$

$$\frac{3}{4} \underset{\times 6}{=} \frac{\square}{24}$$

24 is 4 multiplied by 6. So we multiply both numerator and denominator by 6.

$$\frac{3}{4} \overset{\times 6}{\underset{\times 6}{=}} \frac{18}{24}$$

Exercise 7G

1 Copy and complete:

a $\frac{1}{2} = \frac{\square}{4}$ **b** $\frac{2}{3} = \frac{\square}{6}$ **c** $\frac{3}{4} = \frac{\square}{12}$

d $\frac{3}{8} = \frac{\square}{24}$ **e** $\frac{5}{6} = \frac{\square}{24}$ **f** $\frac{5}{8} = \frac{\square}{32}$

g $\frac{4}{7} = \frac{\square}{14}$ **h** $\frac{3}{11} = \frac{\square}{55}$ **i** $\frac{6}{13} = \frac{\square}{52}$

2 You can work out a problem like

$$\frac{12}{36} = \frac{2}{\square}$$

by dividing instead of multiplying:

2 is 12 divided by 6, so we divide both numerator and denominator by 6.

$$\frac{12}{36} \underset{\div 6}{\overset{\div 6}{=}} \frac{2}{6}$$

Use the method above to complete:

$$\frac{6}{15} = \frac{\square}{5}$$

3 Copy and complete:

a $\frac{4}{8} = \frac{\square}{2}$ **b** $\frac{12}{24} = \frac{\square}{8}$ **c** $\frac{6}{9} = \frac{\square}{3}$

d $\frac{5}{15} = \frac{\square}{3}$ **e** $\frac{35}{40} = \frac{7}{\square}$ **f** $\frac{32}{40} = \frac{8}{\square}$

g $\frac{56}{84} = \frac{\square}{12}$ **h** $\frac{54}{63} = \frac{6}{\square}$ **i** $\frac{65}{91} = \frac{5}{\square}$

4 Copy and complete:

a $\frac{40}{100} = \frac{20}{\square} = \frac{\square}{25} = \frac{2}{5}$

b $\frac{75}{90} = \frac{\square}{60} = \frac{25}{\square} = \frac{5}{\square}$

c $\frac{72}{84} = \frac{\square}{42} = \frac{\square}{14} = \frac{6}{\square}$

d $\frac{81}{189} = \frac{\square}{63} = \frac{9}{\square} = \frac{3}{\square}$

e $\frac{105}{135} = \frac{\square}{27} = \frac{7}{\square}$

5 From the following list write down all the fractions that are equivalent to $\frac{2}{5}$

$\frac{4}{15}$, $\frac{20}{50}$, $\frac{18}{40}$, $\frac{8}{20}$, $\frac{10}{25}$, $\frac{14}{30}$

Simplifying fractions

A fraction is in its **simplest form** when its numerator and denominator have no common factor (other than 1).

The fraction $\frac{2}{5}$ is in its simplest form (2 and 5 have no common factors).

- To write a fraction in its simplest form you have to divide both parts of the fraction by their highest common factor.

For example, the HCF of 6 and 15 is 3, so

$$\frac{6}{15} \underset{\div 3}{\overset{\div 3}{=}} \frac{2}{5}$$

Dividing numerators and denominators of fractions by common factors is known as **cancelling**.

EXAMPLE 6

Write $\frac{36}{48}$ in its simplest form.

The HCF of 36 and 48 is 12.
So

$$\frac{36}{48} \underset{\div 12}{\overset{\div 12}{=}} \frac{3}{4}$$

or you can simplify in steps:

$$\frac{36}{48} \underset{\div 4}{\overset{\div 4}{=}} \frac{9}{12} \underset{\div 3}{\overset{\div 3}{=}} \frac{3}{4}$$

Exercise 7H

1 Write each fraction in its simplest form:

a $\frac{3}{9}$ **b** $\frac{3}{12}$

c $\frac{8}{20}$ **d** $\frac{15}{21}$

e $\frac{24}{36}$ **f** $\frac{17}{51}$

g $\frac{28}{84}$ **h** $\frac{42}{54}$

i $\frac{48}{88}$ **j** $\frac{6}{58}$

2 Which fraction does not belong to the set of equivalent fractions?

a $\left\{\frac{1}{4}, \frac{3}{12}, \frac{12}{48}, \frac{20}{72}, \frac{23}{92}\right\}$

b $\left\{\frac{5}{55}, \frac{11}{121}, \frac{8}{98}, \frac{7}{77}, \frac{1}{11}\right\}$

c $\left\{\frac{2}{9}, \frac{14}{45}, \frac{6}{27}, \frac{18}{81}, \frac{40}{180}\right\}$

3 Explain why $\frac{195}{234}$ in its simplest form is not $\frac{65}{78}$.

4 Write these fractions in their simplest form:

a $\frac{504}{616}$ **b** $\frac{96}{216}$

c $\frac{576}{1444}$ **d** $\frac{441}{567}$

5 Express each of your answers as fractions in their simplest form.

a The towns of Hampton, Belles and Croft have populations of 700, 252 and 1568 respectively.
What fraction of their total population lives in
i Hampton **ii** Belles?

b Dr Williams earned \$6350 last month. He paid \$1550 in tax and \$2450 for medical equipment.
What fraction of his earnings went on
i tax **ii** medical equipment?

6 Look at the fractions $\frac{2}{3}$ and $\frac{3}{4}$.

a Can you tell quickly which is larger?

b Copy and complete:

$\frac{2}{3} = \frac{\square}{12}, \quad \frac{3}{4} = \frac{\square}{12}$

c Now can you tell which fraction is larger?

7 Atin says that $\frac{1}{3}$ is larger than $\frac{3}{8}$. Explain why Atin is wrong.

8 Which of these fractions is nearer to 1: $\frac{2}{3}$ or $\frac{5}{8}$? Explain your answer.

9 Find a fraction that is smaller than $\frac{11}{24}$ but larger than $\frac{5}{16}$.

10 Express each of your answers as fractions in their simplest form. What fraction of

a \$1 is 20c

b 4 metres is 400 mm

c 1 kg is 350 g

d 2 litres is 400 ml

e 1 week is 48 hours

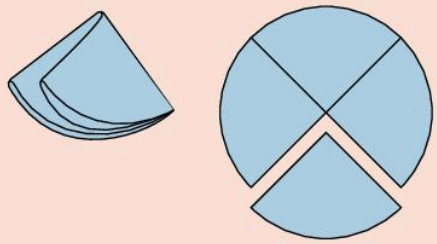
f 3 tonnes is 800 kg?

7.3 Fractions greater than 1

ACTIVITY

You will need paper and scissors. Use something circular, like the rim of a cup, to draw six identical circles.

Cut out each circle carefully. Fold it in half, then fold it in half again. Cut along the fold lines. This will give you quarter-circles.



Take seven of your quarter-circles.

Put them together to make complete circles.

How many complete circles can you make?
What fraction of a circle is left over?

Keep your quarter-circles, you will need them again.

In the Activity, you should have made one complete circle, and had $\frac{3}{4}$ of a circle left over.

1 and $\frac{3}{4}$ is written $1\frac{3}{4}$.

- $1\frac{3}{4}$ is called a **mixed number** because it has both a whole number part and a fraction part.

The seven quarter-circles can also be written as the fraction $\frac{7}{4}$.

$$\frac{7}{4} = 1\frac{3}{4}$$

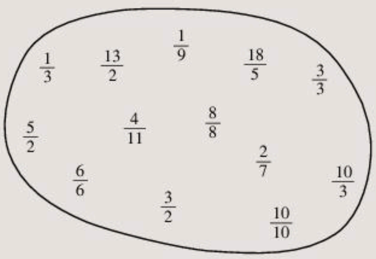
- Fractions like $\frac{7}{4}$ where the numerator is larger than the denominator, are called **improper fractions**. (Sometimes they are called 'top-heavy' fractions.)
- Fractions like $\frac{3}{4}$, where the numerator is smaller than the denominator, are called **proper fractions**.

Exercise 7I

1 Copy and complete the table. Use your quarter-circles to help you.

Number of quarter-circles	As a fraction	As a mixed or whole number
1	$\frac{1}{4}$	–
2	$\frac{2}{4}$	–
3	$\frac{3}{4}$	–
4	$\frac{4}{4}$	1
5	$\frac{5}{4}$	$1\frac{1}{4}$
6	$\frac{6}{4}$	
7	$\frac{7}{4}$	$1\frac{3}{4}$
8		
9		
10		
11		
12		

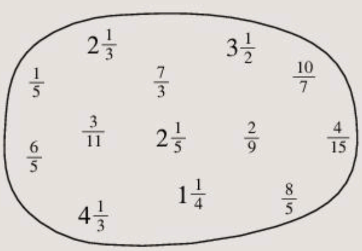
2 Here is a set of fractions:



From this set, write down which are

a proper fractions,
b improper fractions,
c fractions equal to 1.

3 Here is a set of fractions and mixed numbers:



From this set, write down which are

a proper fractions,
b improper fractions,
c mixed numbers.

4 You can write the improper fraction $\frac{7}{6}$ as

$$\frac{7}{6} = \frac{6+1}{6} = \frac{6}{6} + \frac{1}{6} = 1 + \frac{1}{6} = 1\frac{1}{6}$$

Use this method to write $\frac{17}{6}$ as a mixed number.

5 Use the method given in Question **4** to change each fraction to a mixed number.

a $\frac{5}{3}$ **b** $\frac{7}{3}$ **c** $\frac{15}{9}$

d $\frac{17}{4}$ **e** $\frac{28}{11}$

Changing between improper fractions and mixed numbers

- To change an improper fraction to a mixed number divide the numerator by the denominator and write the remainder as a fraction over the denominator.
- To change a mixed number to an improper fraction multiply the whole number part by the denominator and add the numerator.

EXAMPLE 8

Change these into mixed numbers:

a $\frac{8}{5}$

b $\frac{18}{7}$

a $\frac{8}{5}$ means $8 \div 5 = 1$ remainder 3

$= 1\frac{3}{5}$ (the denominator is still 5)

b $\frac{18}{7}$ means $18 \div 7 = 2$ remainder 4

$= 2\frac{4}{7}$ (the denominator is still 7)

Change these into improper fractions:

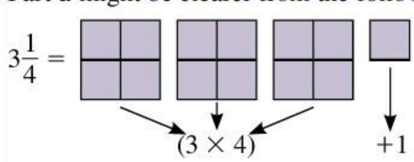
c $1\frac{5}{7}$

d $3\frac{1}{4}$

c $(1 \times 7) + 5 = 12$, so $1\frac{5}{7} = \frac{12}{7}$

d $(3 \times 4) + 1 = 13$ so, $3\frac{1}{4} = \frac{13}{4}$

Part **d** might be clearer from the following picture:



The three whole ones are each four quarters, (12 quarters altogether), plus one more quarter, which makes 13 quarters.

Exercise 7J

1 Change these improper fractions into mixed numbers:

a $\frac{17}{7}$ **b** $\frac{16}{5}$ **c** $\frac{28}{3}$

d $\frac{29}{4}$ **e** $\frac{37}{6}$ **f** $\frac{46}{9}$

g $\frac{91}{11}$ **h** $\frac{87}{10}$ **i** $\frac{121}{12}$

j $\frac{153}{20}$ **k** $\frac{84}{12}$ **l** $\frac{145}{13}$

2 Change these mixed numbers into improper fractions:

a $2\frac{1}{2}$ **b** $3\frac{1}{4}$ **c** $4\frac{2}{5}$

d $4\frac{7}{8}$ **e** $5\frac{2}{3}$ **f** $6\frac{5}{11}$

g $5\frac{9}{10}$ **h** $4\frac{7}{9}$ **i** $6\frac{5}{12}$

j $7\frac{9}{20}$ **k** $11\frac{7}{12}$ **l** $12\frac{12}{13}$

3 Copy and complete:

a $281 \div 10 = 28$ remainder $\square$

or as a mixed number $28\frac{\square}{10}$

b $167 \div 20 = \square$ remainder 7

or as a mixed number $\square\,\frac{7}{20}$

4 Work out the following leaving your answers as mixed numbers:

a $163 \div 10$
b $407 \div 50$
c $128 \div 25$
d $145 \div 12$
e $128 \div 11$
f $163 \div 75$
g $241 \div 24$
h $170 \div 13$

5 Nailah works out the answer to $168 \div 10$ to be $16\frac{8}{10}$.

The teacher says, "that is not the answer that I have, but there is nothing wrong with your working so far." What do you think the teacher means? What do you think the teacher's answer is?

TECHNOLOGY

You can get good practice with fractions by playing games.

The Woodlands Junior School website hosts some excellent fraction games.

Visit www.woodlands-junior.kent.sch.uk/maths and see how good you are!

7.4 Adding fractions

Adding fractions with the same denominators

Exercise 7K shows you how to add fractions with the same denominator.

Exercise 7K

You will need the quarter-circles from the Activity on page 141.

1 **a** Take one of your quarter-circles.
Now add to it two more quarter-circles.
How many have you taken altogether?

b Copy and complete:

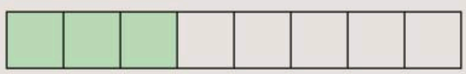
$$\frac{1}{4} + \frac{2}{4} = \frac{\square}{4}$$

2 **a** Copy the rectangle on to squared paper.

$\frac{3}{8}$ of it is shaded.

Now shade a further $\frac{2}{8}$ of it.

What fraction is shaded altogether?



b Copy and complete:

$$\frac{3}{8} + \frac{2}{8} = \frac{\square}{8}$$

3 Draw rectangles on squared paper, and use shading to find the answers to the additions:

a $\frac{3}{10} + \frac{5}{10}$

b $\frac{1}{12} + \frac{4}{12}$

c $\frac{3}{16} + \frac{5}{16} + \frac{6}{16}$

4 Work out

a $\frac{7}{10} + \frac{2}{10}$ **b** $\frac{1}{6} + \frac{5}{6}$

c $\frac{5}{9} + \frac{3}{9}$ **d** $\frac{11}{20} + \frac{8}{20}$

e $\frac{9}{32} + \frac{6}{32}$ **f** $\frac{18}{50} + \frac{25}{50}$

5 Copy and complete:

To add fractions with the same ………., you add the ………. and put the answer over the original ………..

6 Gurdeep had a packet of sweets. He ate two thirds of them and gave one third to his sister. How many sweets were left?

7 From the list below, choose pairs of fractions that add up to make 1:

$\frac{2}{9}$ $\frac{5}{9}$ $\frac{3}{8}$ $\frac{4}{9}$ $\frac{1}{8}$ $\frac{7}{8}$ $\frac{7}{9}$ $\frac{5}{8}$

8 Calculate

a $\frac{3}{4} + \frac{3}{4}$ **b** $\frac{2}{3} + \frac{2}{3}$

c $\frac{5}{6} + \frac{1}{6}$ **d** $\frac{7}{12} + \frac{9}{12}$

e $\frac{3}{8} + \frac{5}{8} + \frac{1}{8}$ **f** $\frac{15}{16} + \frac{7}{16} + \frac{5}{16}$

9 **a** Write down two fractions that add up to make $\frac{7}{8}$.

b Write down three fractions that add up to make $\frac{7}{8}$.

10 Write down two fractions that add up to make $1\frac{1}{8}$.

11 Find the value of the letters in the following:

a $\frac{a}{5} + \frac{2}{5} = \frac{4}{5}$ **b** $\frac{5}{6} + \frac{b}{6} = 1\frac{1}{6}$

c $\frac{c}{8} + \frac{7}{8} = 1\frac{1}{2}$ **d** $\frac{7}{9} + \frac{d}{9} = 1\frac{2}{3}$

Adding fractions with different denominators

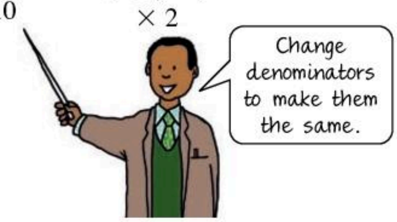
- To add fractions with different denominators you must first use equivalent fractions to make the denominators the same.

EXAMPLE 9

a $\frac{3}{5} + \frac{1}{10}$

$= \frac{6}{10} + \frac{1}{10}$ $\left(\frac{3}{5} \underset{\times 2}{\overset{\times 2}{=}} \frac{6}{10}\right)$

$= \frac{7}{10}$



b $\frac{3}{4} + \frac{2}{3}$

$= \frac{9}{12} + \frac{8}{12}$ $\left(\frac{3}{4} \underset{\times 3}{\overset{\times 3}{=}} \frac{9}{12} \text{ and } \frac{2}{3} \underset{\times 4}{\overset{\times 4}{=}} \frac{8}{12}\right)$

$= \frac{17}{12} = 1\frac{5}{12}$

(12 is the lowest common multiple of 4 and 3)

Exercise 7L

1 Copy and complete:

a $\frac{3}{5} + \frac{3}{10} = \frac{\square}{10} + \frac{3}{10} = \frac{\square}{10}$

b $\frac{1}{6} + \frac{2}{3} = \frac{1}{6} + \frac{\square}{6} = \frac{\square}{6}$

c $\frac{3}{4} + \frac{1}{8} = \frac{\square}{8} + \frac{1}{8} = \frac{\square}{8}$

d $\frac{5}{6} + \frac{1}{12} = \frac{\square}{12} + \frac{1}{12} = \frac{\square}{12}$

e $\frac{3}{8} + \frac{5}{16} = \frac{\square}{16} + \frac{5}{16} = \frac{\square}{16}$

2 Add these fractions.
Give each answer in its simplest form.

a $\frac{3}{10} + \frac{1}{5}$ **b** $\frac{2}{5} + \frac{1}{10}$

c $\frac{1}{5} + \frac{7}{10}$ **d** $\frac{1}{6} + \frac{5}{12}$

e $\frac{5}{6} + \frac{1}{12}$ **f** $\frac{2}{9} + \frac{2}{3}$

g $\frac{1}{4} + \frac{7}{16}$ **h** $\frac{5}{21} + \frac{3}{7}$

i $\frac{5}{9} + \frac{1}{3}$ **j** $\frac{5}{9} + \frac{5}{18}$

3 Copy and complete:

a $\frac{2}{3} + \frac{1}{2} = \frac{\square}{6} + \frac{\square}{6} = \frac{\square}{6}$

b $\frac{4}{5} + \frac{3}{8} = \frac{\square}{40} + \frac{\square}{40} = \frac{\square}{40}$

c $\frac{3}{8} + \frac{9}{12} = \frac{\square}{24} + \frac{\square}{24} = \frac{\square}{24}$

4 First find the LCM of the denominators, then do the addition shown.
Write the answer in its simplest form.

a $\frac{1}{3} + \frac{1}{2}$ b $\frac{1}{5} + \frac{5}{8}$

c $\frac{5}{8} + \frac{5}{12}$ d $\frac{1}{9} + \frac{5}{6}$

e $\frac{3}{5} + \frac{2}{7}$ f $\frac{4}{5} + \frac{2}{9}$

g $\frac{1}{4} + \frac{5}{7}$ h $\frac{1}{3} + \frac{6}{11}$

i $\frac{5}{12} + \frac{5}{9}$

5 a One-third of the students at Columbus School play cricket. Another quarter play football.
What fraction play either cricket or football?

b If the school has 240 students, how many play either cricket or football?

6 Add these fractions. Give each answer in its simplest form.

a $\frac{2}{3} + \frac{3}{4} + \frac{1}{2}$ b $\frac{1}{4} + \frac{2}{5} + \frac{1}{2}$

c $\frac{1}{2} + \frac{3}{4} + \frac{1}{3}$

7 Write down two fractions, with different denominators, that add up to make $\frac{5}{8}$.

8 Abeke has made a mistake in her homework, she has written:

$$\frac{1}{4} + \frac{3}{8} = \frac{1}{3}$$

The mistake she has made is one of the most common mistakes made when adding fractions. Can you work out what she has done?
(**Hint:** the fraction $\frac{1}{3}$ has been simplified.)

9 Copy and complete:

a $\frac{1}{4} + \square = \frac{3}{8}$ b $\square + \frac{2}{5} = \frac{7}{10}$

c $\frac{2}{3} + \square = 1\frac{1}{24}$

10 Put these fractions into groups of three so that each group has a total of 1.

$\frac{1}{3}$ $\frac{1}{5}$ $\frac{2}{5}$ $\frac{3}{4}$ $\frac{5}{12}$ $\frac{3}{10}$ $\frac{1}{2}$ $\frac{1}{20}$ $\frac{1}{12}$ $\frac{13}{24}$ $\frac{3}{10}$ $\frac{1}{8}$

INVESTIGATION

Complete the calculations below:

$\frac{1}{2} + \frac{1}{4} = \frac{3}{4}$

$\frac{1}{2} + \frac{1}{4} + \frac{1}{8} = \square$

$\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} = \square$

$\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \ldots.$

If you continue this pattern what happens to the total?

7.5 Subtracting fractions

To subtract fractions with the same denominators, you subtract the numerators.

EXAMPLE 10

Find: $\frac{7}{12} - \frac{5}{12}$

$$\frac{7}{12} - \frac{5}{12} = \frac{2}{12} \underset{\div 2}{\overset{\div 2}{=}} \frac{1}{6}$$

When the denominators are different you have to make them the same using equivalent fractions.

EXAMPLE 11

Work out: $\frac{3}{4} - \frac{2}{3}$

The denominators are 4 and 3
The LCM of 4 and 3 is 12

$$\frac{3}{4} - \frac{2}{3} \qquad \left(\frac{3}{4} \underset{\times 3}{\overset{\times 3}{=}} \frac{9}{12} \text{ and } \frac{2}{3} \underset{\times 4}{\overset{\times 4}{=}} \frac{8}{12}\right)$$

$$= \frac{9}{12} - \frac{8}{12} = \frac{1}{12}$$

Exercise 7M

1 Copy and complete:

a $\frac{3}{5} - \frac{3}{10} = \frac{\square}{10} - \frac{3}{10} = \frac{\square}{10}$

b $\frac{2}{3} - \frac{1}{6} = \frac{\square}{6} - \frac{1}{6} = \frac{\square}{6}$

c $\frac{3}{4} - \frac{3}{8} = \frac{\square}{8} - \frac{3}{8} = \frac{\square}{8}$

d $\frac{5}{6} - \frac{5}{12} = \frac{\square}{12} - \frac{5}{12} = \frac{\square}{12}$

e $\frac{5}{8} - \frac{3}{16} = \frac{\square}{16} - \frac{3}{16} = \frac{\square}{16}$

2 Subtract these fractions.
Give each answer in its simplest form.

a $\frac{4}{5} - \frac{3}{10}$ **b** $\frac{3}{4} - \frac{1}{8}$

c $\frac{2}{3} - \frac{4}{9}$ **d** $\frac{5}{8} - \frac{1}{4}$

e $\frac{2}{3} - \frac{1}{6}$ **f** $\frac{7}{8} - \frac{3}{4}$

g $\frac{3}{4} - \frac{5}{12}$ **h** $\frac{5}{6} - \frac{1}{3}$

i $\frac{2}{5} - \frac{3}{20}$ **j** $\frac{7}{10} - \frac{2}{5}$

3 **a** Find the LCM of 3 and 4.

b Find equivalent fractions for $\frac{1}{3}$ and $\frac{3}{4}$ using the LCM as the denominator.

c Complete the subtraction: $\frac{3}{4} - \frac{1}{3}$

4 Copy and complete:

a $\frac{1}{2} - \frac{1}{3} = \frac{\square}{6} - \frac{\square}{6} = \frac{\square}{6}$

b $\frac{2}{3} - \frac{1}{2} = \frac{\square}{6} - \frac{\square}{6} = \frac{\square}{6}$

c $\frac{1}{3} - \frac{1}{4} = \frac{\square}{12} - \frac{\square}{12} = \frac{\square}{12}$

d $\frac{3}{4} - \frac{2}{3} = \frac{\square}{12} - \frac{\square}{12} = \frac{\square}{12}$

e $\frac{5}{6} - \frac{3}{4} = \frac{\square}{12} - \frac{\square}{12} = \frac{\square}{12}$

5 Find the LCM of the denominators, then do the subtraction shown.

a $\frac{1}{2} - \frac{1}{3}$ **b** $\frac{1}{4} - \frac{1}{8}$

c $\frac{1}{12} - \frac{1}{18}$ **d** $\frac{2}{3} - \frac{1}{2}$

e $\frac{5}{8} - \frac{3}{10}$ **f** $\frac{5}{6} - \frac{1}{4}$

g $\frac{5}{8} - \frac{5}{12}$ **h** $\frac{4}{5} - \frac{3}{8}$

i $\frac{5}{6} - \frac{2}{7}$

6 Leroy and Jake bought a pizza. Leroy ate $\frac{2}{3}$ of it. Jake ate $\frac{1}{5}$ of it. What fraction was left over?

7 Yafeu was reading about the Museum of Egyptian Antiquities in Cairo. He read $\frac{3}{8}$ of his book one day and $\frac{2}{5}$ the next day. How much was there left to read?

8 At an international meeting in Wellington, $\frac{3}{4}$ of the people attending were from New Zealand. $\frac{1}{5}$ of the people were from Indonesia, and the rest were from Australia. What fraction of the people attending were from Australia?

9 Work out and simplify where possible:

a $1 - \frac{2}{3}$

(**Hint**: 1 can be written as $\frac{3}{3}$)

b $\frac{3}{4} + \frac{1}{4} - \frac{1}{2}$ **c** $\frac{5}{8} + \frac{1}{4} - \frac{1}{5}$

d $\frac{11}{12} - \frac{7}{9} + \frac{1}{2}$ **e** $\frac{3}{4} - \frac{1}{5} + \frac{1}{10}$

10 If $\frac{1}{x} - \frac{1}{y} = \frac{1}{4}$ find the values of x and y.

11 Find the value of $\otimes$ if

$$\frac{\otimes}{\otimes} - \frac{\otimes}{6} = \frac{\otimes}{12}$$

INVESTIGATION

A unit fraction is a fraction with a numerator of 1, for example:

$\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{17}, \frac{1}{26}$

The unit fraction $\frac{1}{2}$ can be written as the sum of two other different unit fractions:

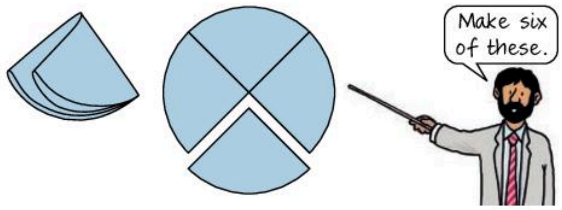
$\frac{1}{2} = \frac{1}{3} + \frac{1}{6}$

Which other unit fractions can you write as sums of different unit fractions?

Are there any rules?

7.6 Multiplying fractions

You will need 24 identical quarter-circles, made from paper or thin card.



b Look at the last column in your table. Can you see a pattern?

c What numbers multiplied together give the numerator in the second fraction?

d What do you notice about the denominator in both fractions?

e Can you see a way of multiplying any fraction by an integer?

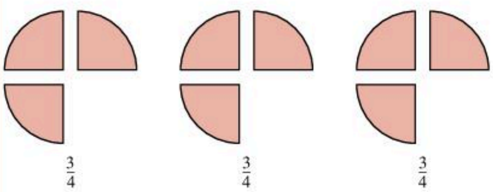
Multiplying fractions by integers

- To multiply a fraction by an integer you multiply the numerator by the integer.

EXAMPLE 12

What is $3 \times \frac{3}{4}$?

Take three $\frac{3}{4}$-circles:



There are nine $\frac{1}{4}$-circles $= \frac{9}{4}$, so $3 \times \frac{3}{4} = \frac{9}{4} = 2\frac{1}{4}$

EXAMPLE 13

Find **a** $\frac{3}{5} \times 4$ **b** $1\frac{3}{4} \times 4$ **c** $2 \times 5\frac{1}{3}$

a $\frac{3}{5} \times 4 = \frac{3 \times 4}{5} = \frac{12}{5} = 2\frac{2}{5}$

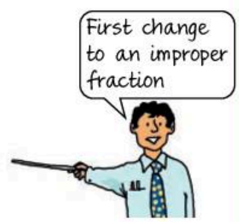
b $1\frac{3}{4} \times 4 = \frac{7}{4} \times 4$

$= \frac{28}{4} = 7$

c $2 \times 5\frac{1}{3} = 2 \times \frac{16}{3}$

$= \frac{32}{3}$

$= 10\frac{2}{3}$



Exercise 7N

1 **a** Take two $\frac{3}{4}$-circles. How many quarter-circles have you used?

b Copy and complete: $2 \times \frac{3}{4} = \frac{\square}{4}$

2 **a** Make up four $\frac{3}{4}$-circles. How many quarters have you used?

b Copy and complete: $4 \times \frac{3}{4} = \frac{\square}{4}$

3 Use your $\frac{1}{4}$-circles to make up $\frac{3}{4}$-circles.

a Copy and complete the table below:

Number of $\frac{3}{4}$-circles made	Number of $\frac{1}{4}$-circles used	The multiplication
1	3	$1 \times \frac{3}{4} = \frac{3}{4}$
2	6	$2 \times \frac{3}{4} = \frac{6}{4}$
3	9	$3 \times \frac{3}{4} = \frac{9}{4}$
4		
5		
6		
7		
8		

Exercise 7O

1 Using the method from Example 13, work out:

a $\frac{1}{3} \times 4$ **b** $\frac{1}{4} \times 7$ **c** $\frac{1}{5} \times 9$

d $7 \times \frac{1}{3}$ **e** $\frac{1}{5} \times 6$ **f** $10 \times \frac{1}{8}$

g $\frac{1}{10} \times 12$ **h** $5 \times \frac{2}{3}$ **i** $7 \times \frac{3}{8}$

2 Work out:

a $\frac{1}{6} \times 13$ **b** $4 \times \frac{5}{8}$ **c** $\frac{4}{9} \times 3$

d $\frac{3}{11} \times 8$ **e** $6 \times \frac{7}{10}$ **f** $10 \times \frac{5}{12}$

g $\frac{2}{5} \times 14$ **h** $20 \times \frac{7}{12}$ **i** $\frac{4}{5} \times 72$

j $96 \times \frac{4}{17}$

3 Work out:

a $1\frac{2}{3} \times 6$ **b** $7 \times 2\frac{1}{6}$

c $5 \times 3\frac{1}{4}$ **d** $2\frac{1}{2} \times 8$

e $6\frac{1}{9} \times 12$ **f** $14 \times 5\frac{7}{8}$

g $1\frac{2}{7} \times 5$ **h** $2 \times 4\frac{3}{8}$

i $5\frac{1}{3} \times 4$ **j** $2 \times 7\frac{1}{8}$

k $2\frac{3}{5} \times {}^{-}3$ **l** ${}^{-}3 \times 1\frac{4}{15}$

4 Akila studies for $\frac{3}{4}$ hour each day.

How long does she study each week?

5 A box holds $1\frac{1}{2}$ kg of nails. What weight of nails do seven such boxes hold?

6

A kilogram is $2\frac{1}{5}$ pounds. A baby weighs 6 kg. What is 6 kg in pounds?

7 What is the area of a rectangular garden that is $6\frac{1}{4}$ m long and 5 m wide?

(**Remember:**
Area of a rectangle = length × width.)

8 The population of Delhi is approximately 18 million. If the population increases by $\frac{2}{7}$ in the next 10 years, what will the population be?

9 Work out:

a ${}^{-}4\frac{2}{7} \times {}^{-}5$ **b** ${}^{-}2 \times {}^{-}5\frac{3}{8}$

10 Two-thirds of the children in a class are girls. Of these, $\frac{1}{4}$ wear glasses. If there are 24 children in the class, how many girls wear glasses?

11 In the village of Lowcroft there are 630 people. Two-thirds of the villagers are under 16 and $\frac{4}{7}$ of these are girls.

a How many girls are under 16 in Lowcroft?

b How many boys are under 16?

7.7 Applying order of operations rules to fractions questions

In Chapter 1 you learned about the order of operations using BIDMAS to help you.

Brackets first
Then Indices
Then Division and Multiplication
Then Addition and Subtraction

This also applies to questions with fractions in them.

EXAMPLE 14

Work out: **a** $7 \times \left(\frac{3}{5} - \frac{1}{4}\right)$ **b** $\frac{1}{3} + 2 \times \frac{3}{11}$

a $7 \times \left(\frac{3}{5} - \frac{1}{4}\right)$

$= 7 \times \left(\frac{12}{20} - \frac{5}{20}\right)$

$= 7 \times \frac{7}{20} = \frac{49}{20} = 2\frac{9}{20}$

b $\frac{1}{3} + 2 \times \frac{3}{11}$

$= \frac{1}{3} + \frac{6}{11} = \frac{11}{33} + \frac{18}{33} = \frac{29}{33}$

Exercise 7P

1 Work out:

a $2 \times \left(\frac{3}{4} + \frac{1}{3}\right)$

b $\left(\frac{5}{8} - \frac{1}{4}\right) \times 6$

c $3 \times \left(\frac{7}{8} + \frac{2}{5}\right)$

2 Work out:

a $\frac{7}{8} - 5 \times \frac{1}{6}$

b $\frac{2}{3} + 3 \times \frac{3}{10}$

c $\frac{9}{10} - 2 \times \frac{3}{8}$

3 Work out:

a $\frac{7}{8} + 3 \times \frac{1}{4} + \frac{1}{2}$

b $\frac{3}{4} + \frac{1}{5} \times 4 - \frac{1}{2}$

c $\left(\frac{3}{5} - \frac{1}{20}\right) \times \left(\frac{2}{3} + \frac{4}{5} + \frac{8}{15}\right)$

INVESTIGATION

Scientific calculators usually have a fraction key which looks like:

[a b/c] or 

This key can be used to simplify fractions and carry out calculations. It will allow you to convert fractions to decimals and some calculators will convert mixed numbers to improper fractions and improper fractions to mixed numbers.

Some examples are:

To simplify $\frac{25}{35}$ key in

[2] [5] [a b/c] [3] [5] [=]

You should see an answer of $\frac{5}{7}$.

To do $7 \times \frac{3}{14}$ key in

[7] [×] [3]  [1] [4] [=]

You should see an answer of $\frac{3}{2}$ or $1\frac{1}{2}$ or 1.5.
(Some calculators can be set up so that the answer will be an improper fraction, a mixed number or a decimal – see what yours can do.)

Your calculator will add and subtract fractions too. There are many different makes of calculator. Find out how to do calculations with fractions on your calculator. You may need to look at the instructions book if you have it, or you can ask your teacher. Repeat some of the questions from the previous exercises using your calculator.

TECHNOLOGY

Need to review?

You can find a complete course on fractions at the website

www.aaaknow.com/fra.htm

Simply click on the section or sections that you need help on. There are also plenty of practice materials and opportunities to race against the clock!

Another good site is

www.coolmath.com/prealgebra

Have fun!

7.8 Problem solving

Solving problems involving fractions is very much like solving problems involving whole numbers.

The stages are similar:

- Read the problem carefully.
- Work out what you are being asked to do.
- Draw a diagram or make a table or look for a pattern.
- Answer the problem.
  Check your answer makes sense.

Exercise 7Q – mixed questions

1 How do you spend a school day?

a Write down all the things you do each school day: at school, sleeping, playing and so on.

b Next to each activity put the number of hours you spend doing it.

c Make sure your total for all activities is 24 hours.

d Using a suitable key, copy and complete the rectangle below representing your day. You will need to make it larger.

☐ represents 1 hour

e What activity takes up the largest fraction of your school day?

2 Repeat Question **1** for a day during the school holiday.

3 **a** How many lessons do you have each week?

b Write down all the subjects you study at school.

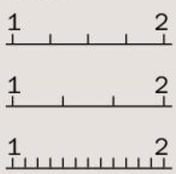
c Next to each subject write down the number of lessons in that subject that you attend each week.

d Make sure your total in part **c** is the same as your answer to part **a**.

e What fraction of your school week is spent on each subject?

f Draw a diagram with a key to show this information.

4 Here are three ways of dividing the number line between 1 and 2:



a Copy the lines and mark the positions of the numbers:

$1\frac{1}{2}$, $1\frac{3}{4}$, $1\frac{2}{3}$, $1\frac{7}{12}$, $1\frac{5}{12}$

Use the positions of the numbers to put them in order starting with the smallest.

b Draw two lines to help you decide which is greater, $1\frac{13}{20}$ or $1\frac{5}{8}$.

5 Draw a number line like this.

Mark in the approximate positions of the numbers $\frac{1}{2}$, $\frac{1}{3}$, $\frac{3}{4}$, $\frac{2}{5}$

6 Peter scored 14 out of 20 in a Maths test. He scored 20 out of 30 in an English test. Which was the better mark?

7 Mr Cenac saves $\frac{2}{7}$ of what he earns. What fraction of his earnings does he spend?

8

Aisha buys 28 apples.

Her sister Afra takes half of them and her friend Patsy eats $\frac{1}{7}$ of the remainder.

How many apples does Aisha have left?

9 When Albert had sold one-third of his chickens he had 18 chickens left. How many chickens did he have to start with?

10 **a** $\frac{1}{4}$ of Denise's weekly wage is spent on rent and $\frac{1}{3}$ on food. What fraction does she have left?

b If she has \$60 left over, what was her weekly wage?

11 A length of string is 35 cm long. It is cut into six equal pieces. How long is each piece?

12 A barrel holds 50 litres.

I have $3\frac{3}{5}$ barrels. How many litres will they hold altogether?

13 A plank of wood is $1\frac{1}{2}$ cm thick.

How thick is a pile of 25 planks?

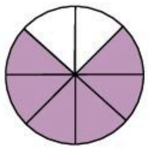
14 A bag of flour weighs $50\frac{2}{3}$ kg.

What is the weight of 10 bags?

Consolidation

Example 1

What fraction of the shape is shaded?



6 parts out of 8 equal parts are shaded, so $\frac{6}{8}$ is shaded, which simplifies to $\frac{3}{4}$

Example 2

John has \$16 and spends $\frac{1}{4}$ of it on books.

How much does he have left?

$$\frac{1}{4} \times 16 = \frac{1}{4} \times \frac{16}{1} = \frac{16}{4} = 4$$

$$16 - 4 = 12$$

John has \$12 left.

Example 3

Complete this equivalent fraction: $\frac{3}{5} = \frac{\square}{20}$

$$\overset{\times 4}{\frac{3}{5} = \frac{12}{20}}_{\times 4}$$

Example 4

Write $\frac{32}{60}$ in its simplest form.

$$\frac{32}{60} = \frac{32 \div 2}{60 \div 2} = \frac{16}{30} = \frac{16 \div 2}{30 \div 2} = \frac{8}{15}$$

Example 5

a $\frac{17}{5}$ change this improper fraction to a mixed number

b $3\frac{5}{6}$ change this mixed number to an improper fraction

a $\frac{17}{5}$ means $17 \div 5 = 3$ remainder $2 = 3\frac{2}{5}$

b $(3 \times 6) + 5 = 23$, so $3\frac{5}{6} = \frac{23}{6}$

Example 6

Work out:

a $\frac{4}{9} + \frac{1}{3}$ **b** $\frac{3}{4} - \frac{2}{5}$

a $\frac{4}{9} + \frac{1}{3} = \frac{4}{9} + \frac{3}{9} = \frac{7}{9}$ (LCM of 9 and 3 is 9)

b $\frac{3}{4} - \frac{2}{5} = \frac{15}{20} - \frac{8}{20} = \frac{7}{20}$ (LCM of 4 and 5 is 20)

Exercise 7

1 Write down the fraction of the shape that is shaded, in its simplest form.

2 Calculate:

a $\frac{9}{10}$ of \$40 **b** $\frac{3}{4}$ of 28 litres

c $\frac{7}{20}$ of 140 students **d** $\frac{2}{3}$ of 66 days

3 Copy and complete these equivalent fractions:

a $\frac{3}{4} = \frac{\square}{12}$ **b** $\frac{2}{5} = \frac{\square}{30}$

c $\frac{6}{10} = \frac{\square}{100}$ **d** $\frac{4}{25} = \frac{16}{\square}$

4 Simplify the following fractions:

a $\frac{15}{18}$ **b** $\frac{35}{60}$

c $\frac{65}{104}$ **d** $\frac{121}{132}$

5 **a** Change these improper fractions to mixed numbers:

i $\frac{18}{7}$ **ii** $\frac{39}{13}$

iii $\frac{96}{5}$ **iv** $\frac{82}{15}$

b Change these mixed numbers to improper fractions:

i $2\frac{3}{5}$ **ii** $9\frac{2}{11}$

iii $4\frac{13}{25}$ **iv** $10\frac{11}{15}$

6 Calculate:

a $\frac{3}{4} + \frac{3}{4}$ **b** $\frac{3}{4} + \frac{2}{5}$

c $\frac{3}{5} - \frac{1}{3}$ **d** $\frac{7}{8} - \frac{3}{5}$

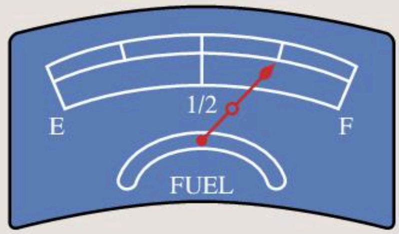
7 There are 260 girls and 240 boys at Western High School. Calculate how many students each statement represents.

- **a** $\frac{2}{3}$ of the boys come to school by bike.
- **b** $\frac{3}{10}$ of the girls eat school dinners.
- **c** All of the boys and $\frac{1}{5}$ of the girls wear ties.
- **d** $\frac{3}{10}$ of the girls and $\frac{4}{10}$ of the boys play on a school team.
- **e** $\frac{11}{100}$ of the students are left-handed.

8 Janice walked at a speed of $4\frac{1}{5}$ km per hour. How far did she walk in 3 hours?

(**Remember:** distance = speed × time).

9 The drawing shows the fuel gauge in Ray's car. When full the fuel tank holds 120 litres.



- **a** How much fuel does the tank hold now?
- **b** After travelling 80 km, Ray noticed his tank was half full. How much fuel did he use?

Summary

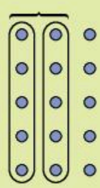
You should know …

1 You can find a fraction of an amount by first dividing.

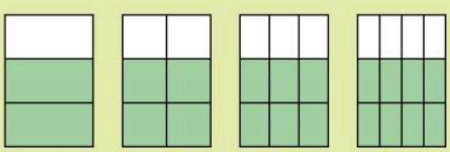
For example: to find $\frac{2}{3}$ of 15

$\frac{1}{3}$ of 15 = 15 ÷ 3 = 5

So $\frac{2}{3}$ of 15 = 2 × 5 = 10



2 Equivalent fractions show the same fraction using different numbers:



$\frac{2}{3} = \frac{4}{6}$ $\frac{6}{9} = \frac{8}{12}$

You can simplify some fractions by dividing:

$$\frac{48}{72} \xrightarrow{\div 4} \frac{12}{18} \xrightarrow{\div 6} \frac{2}{3}$$

$$\frac{48}{72} = \frac{12}{18} = \frac{2}{3}$$

2 and 3 have no common factors so $\frac{2}{3}$ is in its simplest form

Check out

1 Work out

- **a** $\frac{1}{3}$ of 30
- **b** $\frac{3}{4}$ of 28
- **c** Gemma drank $\frac{2}{3}$ of a bottle of lemonade. The bottle held 360 ml. How much did she drink?

2 Copy and complete:

- **a** $\frac{3}{4} = \frac{\square}{12}$
- **b** $\frac{2}{3} = \frac{\square}{24}$
- **c** Write these fractions in their simplest form:
  - **i** $\frac{16}{24}$
  - **ii** $\frac{25}{45}$

3 A mixed number is made up of a whole number and a fraction.
For example: $2\frac{3}{4}$

An improper fraction has a numerator bigger than its denominator.
For example: $\frac{7}{4}$

The improper fraction $\frac{13}{5} = 2$ remainder $3 = 2\frac{3}{5}$ (a mixed number).

The mixed number $3\frac{2}{3} = \frac{3 \times 3 + 2}{3} = \frac{11}{3}$ (an improper fraction).

3 a Write as mixed numbers:

i $\frac{11}{3}$

ii $\frac{17}{5}$

b Write as improper fractions:

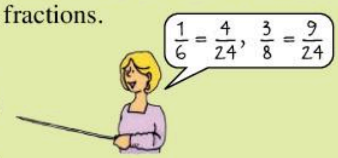
i $2\frac{1}{2}$

ii $4\frac{2}{5}$

4 You can add and subtract fractions when they have the same denominator.
When the denominators are different you have to make them the same using equivalent fractions.

For example:

$$\frac{1}{6} + \frac{3}{8} = \frac{4}{24} + \frac{9}{24} = \frac{13}{24}$$



The LCM of 6 and 8 is 24.

4 Add these fractions:

a $\frac{1}{6} + \frac{5}{6}$

b $\frac{2}{5} + \frac{1}{10}$

c Sandra ate half a cake. Tessa ate one-third of it. What fraction was left?

5 How to multiply a fraction by an integer.

For example:

$$6 \times \frac{3}{4} = \frac{6 \times 3}{4} = \frac{18}{4} = 4\frac{2}{4} = 4\frac{1}{2}$$

5 a Calculate:

i $3 \times \frac{1}{2}$

ii $6 \times \frac{2}{3}$

iii $\frac{3}{4} \times 8$

b Kathy ate $\frac{3}{4}$ of a box of cornflakes. The box holds 152 g. What mass of cornflakes remains?

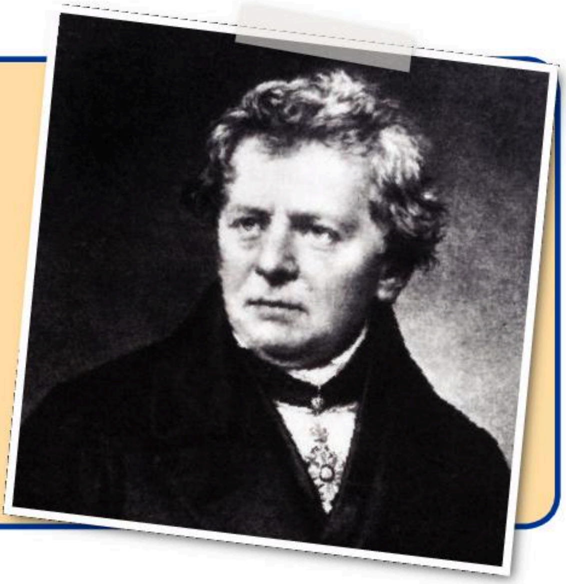
8 Equations and formulae

Objectives

- Substitute positive integers into simple linear expressions or formulae.
- Derive and use simple formulae, e.g. to change hours to minutes.
- Construct and solve simple linear equations with integer coefficients (with the unknown on one side only), e.g. $2x = 8$, $3x + 5 = 14$, $9 - 2x = 7$.

What's the point?

Voltage, current, and resistance are mathematically related to each other. You can't work with electricity without all three of these being considered. Ohm's law (named after nineteenth-century German physicist Georg Ohm) is a formula describing the relationship between voltage, current, and resistance. Ohm's law is: $V = IR$, where V = voltage, in volts, I = current, in amps, and R = resistance, in ohms.



Before you start

You should know ...

1 How to use letters to represent unknown numbers or variables.

For example:

Write in a shorter way: The length in centimetres of a piece of string with 5 cm cut off.
Let s be the length of the string before anything was cut off.
So the new string length is $s - 5$.

Check in

1 Write in a shorter way: The total number of biscuits I need to buy if each of my friends has 2 biscuits and I have 3 spare biscuits. Use f to stand for the number of friends.

2 The order you should do operations (BIDMAS). Brackets first then Indices then Division and Multiplication then Addition and Subtraction. $10 - 2 \times 4$ BIDMAS: Multiplication first … $10 - 8 = 2$ … then Subtraction	**2** Work out: **a** $15 - 3 \times 4$ **b** $1 + 20 \div 5$ **c** $3 \times 10 + 100 \div 10$

8.1 Substitution into expressions

In Chapter 2 you learned about expressions such as $2x$, $3x + 5$, $4y + 5t$.

When you know the values of the letters, you can find the value of an expression.

You substitute the values into the expression.

EXAMPLE 1

If $x = 2$ and $y = 3$, find the value of

a $5x - 2y$ **b** $4y(1 + x)$

a $5x - 2y = 5 \times x - 2 \times y$
$= 5 \times 2 - 2 \times 3$
$= 10 - 6$
$= 4$

b $4y(1 + x) = 4 \times y \times (1 + x)$
$= 4 \times 3 \times (1 + 2)$
$= 4 \times 3 \times 3$
$= 36$

Exercise 8A

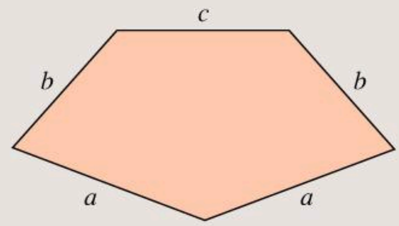
1 If $x = 4$, find the value of:
a $2x$ **b** $3x - 2$
c $7x + 1$ **d** $3(x + 5)$

2 If $p = 2$ and $q = 3$, find the value of:
a $5p - 2q$ **b** $3p + q$
c $7p - 2q$ **d** $4pq$

3 If $m = 2$, $n = 3$ and $p = 5$, find the value of:
a $m + n - p$ **b** $2m - n + 3p$
c $n(m + p)$ **d** $5p - 10m - 3n$
e $6m(p + n)$ **f** $4mp - 10n$
g $mp - mn$ **h** $4(m + n + p)$
i $-p - n - m$ **j** $11mn - 8p$
k $\frac{mnp}{3}$ **l** $\frac{7m + 2n}{5}$

4 Copy these expressions into your book:
$\boxed{x + y}$ $\boxed{xy}$ $\boxed{x + 3}$ $\boxed{4y - 6}$
Tick ✓ which ones have the same value when $x = 1$ and $y = 2$.

5 **a** Write down the perimeter, P, of this shape. (**Remember:** the perimeter is the distance around the edges.)



b Find the value of P if $a = 5$ cm, $b = 3$ cm and $c = 4$ cm.

8.2 Formulae

In Chapter 2 you learned about expressions, terms, equations and variables. A **formula** describes a relationship between variables. **Formulae** is the plural of formula. If you know the values for all but one variable, you can use substitution into the formula to work out the value of the remaining variable. For example, $P = 2L + 2W$ is a formula. You would need to know the values to substitute for L and W to work out the value of P.

Making up formulae of your own

Formulae can represent all sorts of situations.

To **derive** a formula means to use known information to write a formula connecting variables. This is what you did in Question **5 a** of Exercise 8A.

EXAMPLE 2

A tin contains P peas. There are N tins of peas. Derive a formula giving the total number of peas, T, in the tins.

Number of peas in one tin $= P$
Number of peas in two tins $= 2 \times P$
so Number of peas in N tins $= N \times P$
so $T = N \times P$ or $T = NP$

EXAMPLE 3

Derive a formula to convert US dollars ($) into euros (€) if $1 is worth €1.40.

Decide what letters you are going to use, e.g. let $D =$ number of dollars, and $E =$ number of euros.

Number of euros for 1 dollar $=$ €1.40
Number of euros for 2 dollars $= 2 \times$ €1.40
Number of euros for D dollars $= D \times$ €1.40
So $E = D \times 1.40$ or $E = 1.4D$

Exercise 8B

1 A class has B boys and G girls. The class has S students altogether. Write down a formula for the total number of students in the class, starting $S = \ldots$

2 A glass of water has a total mass of M grams. The glass has mass G grams. Write down a formula giving the mass of the water, W.

3 Amandeep scored H marks in a History test and G marks in a Geography test. Her total score in both tests was T marks. Write down a formula, starting $T = \ldots$, and involving H and G.

4 M minibuses carry a total of T people. The number of people that one minibus holds is P. Write down a formula, starting $P = \ldots$, and involving M and T.

5 **a** Find the cost, C, of M mathematics books which cost $7 each.
b Find the cost, K, of M mathematics books which cost $$A$ each.

6 **a** Find the cost, C, of P pencils at 40 cents each and E erasers at 25 cents each.
b Find the cost, K, of P pencils at A cents each and E erasers at B cents each.

7 B boys and G girls travel home by bus. The fare is $2 each.
a Find the total cost, C, of the fares.
b What is the total cost if the fare was D dollars?

8 A bucket holds W litres of water. John pours X litres of water away. Find how many litres, L, are left in the bucket.

9 Derive a formula to convert D dollars ($) into I Indian rupees (₹) if $1 is worth ₹50.

10 Derive a formula to convert H hours into M minutes.

11 Derive a formula to convert k kilometres into m metres.

12 Tony sells A papers on Monday, B on Tuesday, C on Wednesday, D on Thursday, E on Friday and F on Saturday.
a How many papers, P, does he sell altogether?
b If each paper costs Y cents, how much money, M, does he bring in each week?

13 A man walks at n kilometres per hour for 6 hours.
a How far does he walk?
b If he walks at m kilometres per hour for another 3 hours, write down the total distance, D, that he travels.

14 The cost of a knife is x dollars, a fork y dollars and a spoon z dollars. What is the cost, C, of a complete set of cutlery for
a 4 people **b** P people?

15 Joanne buys M mangoes and G grapefruits at the market. Mangoes cost 45 cents, grapefruits 25 cents. Joanne paid a total of T dollars.

a How much did Joanne pay for her mangoes, in dollars?

b How much did Joanne pay for her grapefruits, in dollars?

c Write down a formula for T.

You can substitute values into derived formulae to work out missing values.

EXAMPLE 4

Derive a formula to convert D dollars (\$) into E Egyptian pounds (EGP) using the exchange rate \$1 = 6 EGP. Use your formula to convert \$350 into Egyptian pounds.

Number of Egyptian pounds for 1 dollar = 6
Number of Egyptian pounds for 2 dollars
= 2 × 6
Number of Egyptian pounds for D dollars
= $D \times 6$

So $E = 6D$

When $D = 350$,

$E = 6D$
$= 6 \times 350$
$= 2100$ EGP

Exercise 8C

1 Derive a formula to convert D dollars (\$) into I Indonesian rupiahs (IDR) using the exchange rate \$1 = 8900 IDR. Use your formula to convert \$420 into Indonesian rupiahs.

2 Using your formula from Question **5 b** of Exercise 8B, find the cost of books, K, in dollars (\$), if $M = 40$ and $A = 15$.

3 Derive a formula to convert the number of days, d, into hours, h. Use this formula to work out the number of hours in 3 days.

4 Use your formula from Question **9** of Exercise 8B to convert \$50 into Indian rupees.

5 The international phone call charges for a hotel are a \$2 connection fee and \$1 for each minute of the call. Work out the formula for the total cost, T, of an international phone call lasting m minutes. Use this formula to work out the total cost of an international phone call lasting 8 minutes.

6 Derive a formula to convert the number of litres, L, into millilitres, m. Use your formula to convert 2.4 litres into ml.

7 An electrician needs 8 sockets for a room.

a Which of the following is the formula that gives the total number of sockets, s, needed for r rooms?

$s = 8 + r \quad s = 8r \quad s = 8 - r \quad s = 8 \div r$

b Use your formula to calculate the number of sockets needed for 7 rooms.

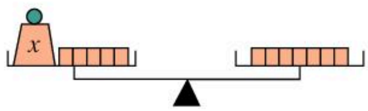
8.3 Solving equations

An **equation** is a statement using algebra that contains an equals sign.
For example: $b + 2 = 5$ is an equation.

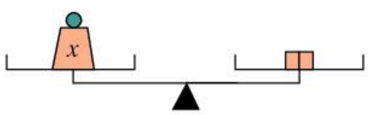
You can solve an equation using the balance method.

EXAMPLE 5

Find x if $x + 5 = 7$.
Think of the equation as a balance:



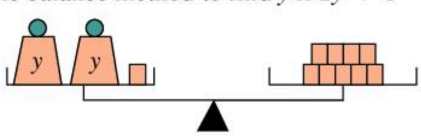
Take 5 from each side (-5).



The scales balance so $x = 2$.

EXAMPLE 6

Use the balance method to find y if $2y + 1 = 9$.



Take 1 from each side.

To make the scales balance $2y$ must be 8.
So $y = 4$.

A good way to set out a working for Example 6 is like this:

$$\begin{array}{rcl} & 2y + 1 = 9 & \\ -1] & 2y = 9 & [-1 \\ \div 2] & y = 4 & [\div 2 \end{array}$$

Solving an equation is finding the value that makes both sides of the equals sign the same as each other (equal to each other). You can check by substituting the value back into the equation:

$$2y + 1 = 9$$
$$2 \times y + 1 = 9$$

substitute $y = 4$:

$$2 \times 4 + 1 = 9$$
$$8 + 1 = 9$$
$$9 = 9$$

The left of the equals sign is the same as the right, so $y = 4$ is the correct solution.

Providing you keep the equation balanced by doing the same to both sides you can solve harder equations the same way. Terms as well as constants can be added to both sides (you will see this in Example 7).

EXAMPLE 7

Solve $17 - 2x = 7$.

$$\begin{array}{rcl} & 17 - 2x = 7 & \\ +2x] & 17 = 7 + 2x & [+2x \\ -7] & 10 = 2x & [-7 \\ \div 2] & 5 = x & [\div 2 \\ \text{or} & x = 5 & \end{array}$$

You can make your xs positive by adding the term 2x to both sides.

You want to end up with the x on its own. It doesn't matter which side of the equals sign it is on.

Exercise 8D

Use the balance method to solve these equations:

1 $a + 6 = 9$

2 $b + 3 = 11$

3 $6 = c + 5$

4 $8 + d = 12$

5 $2e + 1 = 11$

6 $3f + 4 = 19$

7 $2 + 5g = 17$

8 $h + 7 = 11$

9 $10 = 3i + 1$

10 $10 + 3j = 13$

11 $k + 7 = 7$

12 $2l + 1 = 5$

13 $9 = 3 + 2m$

14 $25 - 3n = 10$

15 $6o = 60$

16 $p - 1 = 5$

17 $1 = q - 2$

18 $8 = 18 - 5r$

19 $25 = 5s$

20 $10 - 4t = {}^{-}2$

21 $3u - 1 = 14$

22 $56 = 8v$

23 $3w - 4 = 17$

24 $15 = 4x - 1$

$\boxed{\textbf{25}}$ $12 = 5y - 8 - y$ (**Hint**: Simplify first)

$\boxed{\textbf{26}}$ $5z + 1 + 6z = 122$ (**Hint**: Simplify first)

You solve equations using inverse operations. To solve $3x + 2 = 17$, the first step is to do the inverse of $+2$, which is -2. Then you do the inverse of $\times 3$, which is $\div 3$. Do these inverse operations to both sides of the equals sign.

EXAMPLE 8

Solve **a** $\frac{m}{5} = 10$ **b** $\frac{t}{2} + 1 = 5$

a

$$\begin{array}{rcl} & \frac{m}{5} = 10 & \\ \times 5] & m = 50 & [\times 5 \end{array}$$

The inverse of ÷5 is ×5. Do this to both sides.

b

$$\begin{array}{rcl} & \frac{t}{2} + 1 = 5 & \\ -1] & \frac{t}{2} = 4 & [-1 \\ \times 2] & t = 8 & [\times 2 \end{array}$$

The inverse of +1 is −1

The inverse of ÷2 is ×2

Exercise 8E

Some of the work in this exercise is extension work as it contains equations without integer coefficients and multiplying or dividing with negative numbers is required.
Solve these equations:

1 $\frac{x}{3} = 3$ **2** $\frac{p}{4} + 1 = 3$

3 $\frac{T}{5} - 3 = 2$ **4** $\frac{t}{2} = 2$

5 $12 = \frac{g}{11} + 10$ **6** $7 = \frac{y}{6}$

7 $12 = \frac{x}{12}$ **8** $11 - 3y = {}^{-}1$

9 $20 = {}^{-}2m$ **10** ${}^{-}3x = 15$

Equations can be used to solve simple word problems.

EXAMPLE 9

Kisha and Kevin have \$20 altogether. Kisha has \$6. How much has Kevin?

Let k be Kevin's amount, then

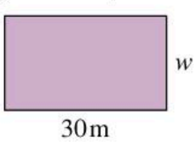
$$k + 6 = 20$$
$$-6] \quad k = 14 \quad [-6$$

Kevin has \$14.

Notice that you need to substitute a letter for the unknown and then derive an equation.

EXAMPLE 10

The perimeter of a rectangular field is 80 m. If the field's length is 30 m, what is its width?



Let width be w. Perimeter is distance around the outside.

$$w + 30 + w + 30 = 80$$
$$2w + 60 = 80$$
$$-60] \quad 2w = 20 \quad [-60$$
$$\div 2] \quad w = 10 \quad [\div 2$$

Width of field is 10 m.

You will be learning more about formulae for area and perimeter in Chapter 17.

Exercise 8F

For each question in this exercise, write down an equation then solve it.

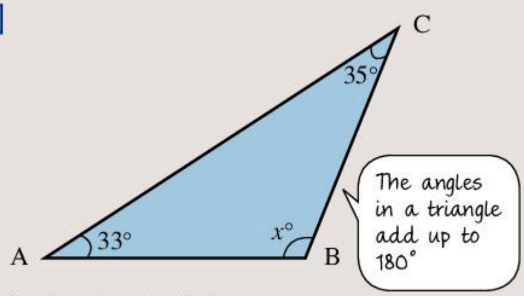
1 Three times a certain number is 78. What is the number?

2 Two less than a given number is 29. What is the number?

3 Alison and Anya have 36 mangoes between them. Alison has twice as many mangoes as Anya. How many mangoes does Anya have?

4 **a** When x is doubled the result is 24.
b When 14 is added to x the result is 30.
c When x is divided by 5 the result is 20.
d When x is doubled and 3 is added the result is 27.

5



In the triangle ABC
a Write down an equation connecting the three angles.
b Solve the equation to find the value of x.

6 Stacy has 6 more oranges than Susan. Altogether they have 74 oranges. How many oranges does Stacy have?

7 The area of a rectangle is 24 cm^2. What is its length if its width is 4 cm?

8 The perimeter of a rectangle is 45 cm. What is its width if its length is 12 cm?

9 The sum of \$90 is shared between Pete and Petra so that Petra gets \$15 more than Pete. How much does Pete receive?

10 In Bayview village there are 12 more women than men. If the village's population is 240, how many villagers are men?

Consolidation

Example 1

If $t = 8$, $r = 3$ and $v = 2$, find the value of:

a $t + 2r$ **b** $v(t + r)$

a $t + 2r = t + 2 \times r$
$= 8 + 2 \times 3$ use BIDMAS
$= 8 + 6$
$= 14$

b $v(t + r) = v \times (t + r)$
$= 2 \times (8 + 3)$ use BIDMAS
$= 2 \times 11$
$= 22$

Example 2

Derive a formula to convert E metres into I millimetres. Use the formula to work out the number of millimetres in 3.2 metres.

In 1 metre there are 1000 millimetres
In 2 metres there are 2×1000 millimetres
In E metres there are $E \times 1000$ millimetres
so $I = 1000E$
When $E = 3.2$ m
$I = 1000E$
$= 1000 \times 3.2$
$= 3200$ mm

Example 3

Solve:

a $3x - 2 = 7$ **b** $10 - 2x = 2$

a $3x - 2 = 7$
+2] $3x = 9$ [+2
÷3] $x = 3$ [÷3

b $10 - 2x = 2$
+2x] $10 = 2 + 2x$ [+2x
−2] $8 = 2x$ [−2
÷2] $4 = x$ [÷2
or $x = 4$

Exercise 8

1 If $a = 10$ and $b = 3$, find the value of:
a $a + 2b$ **b** $2ab$ **c** $a(2 + b)$

2 Derive a formula to convert P British pounds (£) into D New Zealand dollars (NZD), using the exchange rate £1 = 2 NZD. Use the formula to work out how many New Zealand dollars you get for £125.

3 Find the value of T when $x = 5$ and $m = 6$.
a $T = 3x - m$ **b** $T = 3mx + 4$
c $T = \dfrac{m + x}{m - x}$ **d** $T = 3x + 4m$

4 I am x years old now. In 5 years' time my age will be y years. Write down which of the formulae below gives my age in 5 years' time.

$\boxed{y = 5 - x}$ $\boxed{y = x + 5}$ $\boxed{y = 5x}$ $\boxed{y = x - 5}$

5 If $x = 4$, $y = 1$ and $z = 3$, find the value of B when
a $B = x + 2y - z$
b $B = 3x - y + 4z$
c $B = x(y + z)$
d $B = 2z(x + y)$
e $B = 10xz - 120y$
f $B = zx - xy$
g $B = 4xyz$
h $B = \dfrac{2x + 4y}{z}$

6 Solve:
a $7x = 35$
b $v + 3 = 19$
c $4 + m = 7$
d $10 - f = 3$
e $17 = 4 + e$
f $7x = 56$
g $3a + 4 = 10$
h $81 = 9h$
i $\dfrac{p}{5} = 6$
j $3y - 4 = 17$
k $9 = 8w - 15$
l $35 - 7w = 14$

7 A spring is 30 cm long and extends by 5 cm for each 100 g weight hung on it.
a What is the spring's length when 300 g is hung from it?
b What is the spring's length, z, when n 100 g weights are hung from it?
c What is the spring's length when $n = 4$?

8 Ann walks for 2 hours at p km/hr.

a How far does she walk?

b If she walks at q km/hr for 3 more hours, what is the total distance she travels?

c What is her average speed?
(**Hint:** Average speed = total distance ÷ total time.)

Summary

You should know …

1 When you change letters for numbers this is substitution.
Always follow BIDMAS rules when substituting.

For example:

If $x = 3$ and $y = 4$, the value of

$$\begin{aligned} x + 5y &= x + 5 \times y \\ &= 3 + 5 \times 4 \qquad \text{use BIDMAS} \\ &= 3 + 20 \\ &= 23 \end{aligned}$$

2 A formula is an equation relating different quantities.

For example:

$$A = l \times w$$

is the formula for the area A of a rectangle, with l = length and w = width.

3 You can use the idea of a balance to help you solve linear equations:

$$\begin{aligned} &4x - 5 = 19 \\ +5]\quad &4x = 24 \quad [+5 \\ \div 4]\quad &4 = 6 \quad [\div 4 \end{aligned}$$

Check out

1 If $r = 2$ and $t = 5$, find the value of:

a $3t - r$

b $3tr$

c $4r(t - 3)$

d $5r + 6t$

2 What is the cost C of a cartons of juice at p dollars a carton, and b bottles of milk at q dollars a bottle?

3 Solve these equations.

a $3x + 7 = 19$

b $1 = 10 - 3p$

c $40 = 8y$

9 Geometry

Objectives

- Estimate the size of acute, obtuse and reflex angles to the nearest 10°.
- Start to recognise the angular connections between parallel lines, perpendicular lines and transversals.
- Calculate the sum of angles at a point, on a straight line and in a triangle, and prove that vertically opposite angles are equal; derive and use the property that the angle sum of a quadrilateral is 360°.
- Solve simple geometrical problems by using side and angle properties to identify equal lengths or calculate unknown angles, and explain reasoning.
- Read and plot coordinates of points determined by geometric information in all four quadrants.

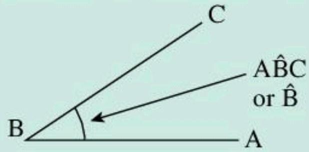
What's the point?

Angles are important in the design of many things. For example, reducing the angle made by the wing of a plane with its fuselage can reduce drag and enable the plane to fly at high speeds.

Before you start

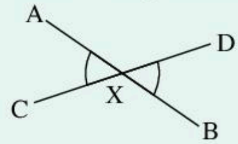
You should know ...

1 How to use letters to name an angle.

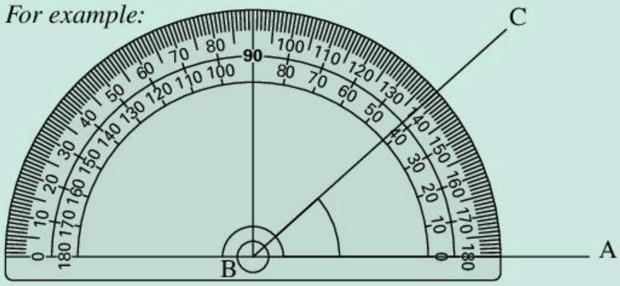


Check in

1 Name the angles marked.

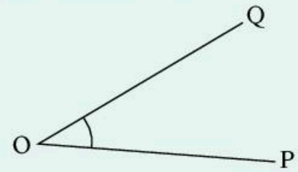


2 How to draw and measure angles.
For example:



As A$\hat{B}$C is less than 90°, you read the anticlockwise scale.
A$\hat{B}$C = 42°.

2 **a** Measure P$\hat{O}$Q.

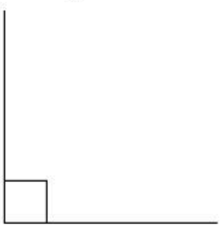


b Draw an angle of 160°.
c Draw an angle of 345°.

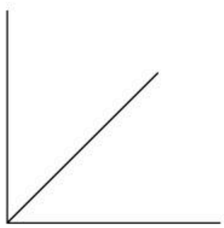
9.1 Relationships between angles

In Chapter 3 you looked at rough estimates for the size of angles before you measured them. You should be able to estimate an angle to within 10°.

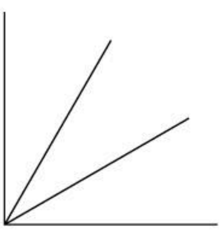
To help:



A right angle is 90°.



Half a right angle is 45°.



A third of a right angle is 30°.

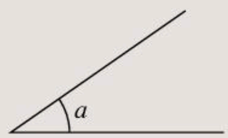
You can use other fractions of a right angle.

In Exercise 9A you will discover some facts about relationships between angles.

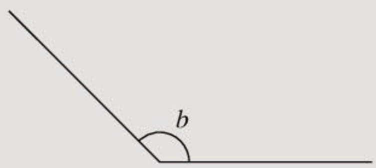
Exercise 9A

1 Estimate the size of the following angles. Measure them and see if your estimate was within 10° of the actual size.

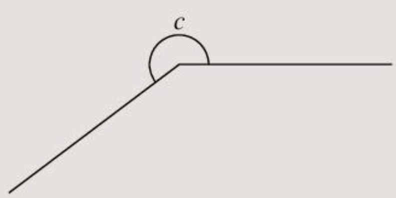
a



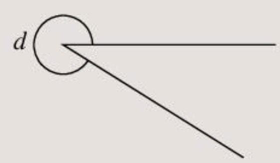
b



c



d

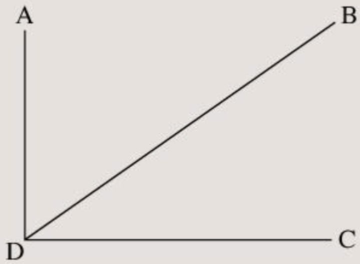


2 From the diagram, use your protractor to copy and complete:

a $A\hat{D}B = \square$

b $B\hat{D}C = \square$

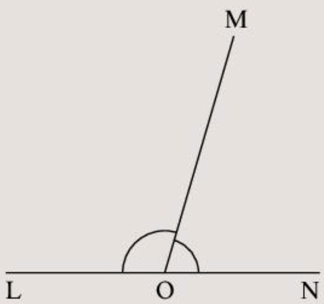
c $A\hat{D}B + B\hat{D}C = \square$



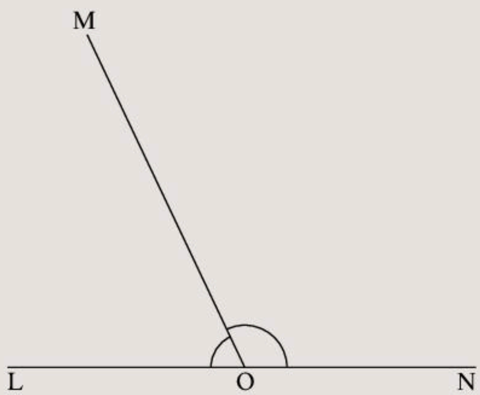
3 From the diagram, use your protractor to copy and complete:

a $M\hat{O}N = \square$ **b** $L\hat{O}M = \square$

c $M\hat{O}N + L\hat{O}M = \square$



4 **a** Repeat Question **3** for this diagram:



b Do you agree that: $M\hat{O}N + L\hat{O}M = 180°$ in this diagram? Why should this be true?

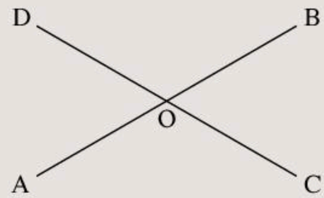
5 From the diagram, use your protractor to copy and complete:

a $A\hat{O}D = \square$

b $B\hat{O}C = \square$

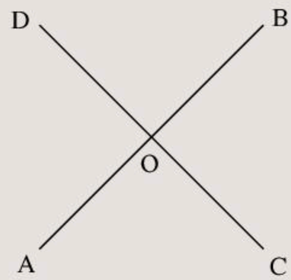
c $D\hat{O}B = \square$

d $A\hat{O}C = \square$



What do you notice about your answers?

6 Repeat Question **5** for this diagram:



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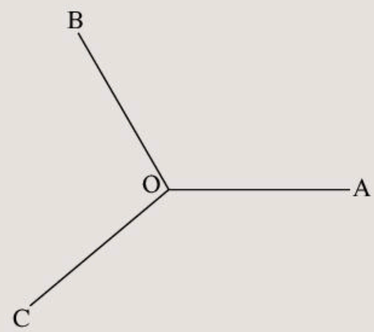


Use your protractor to copy and complete:

a $A\hat{O}B = \square$

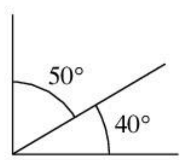
b $A\hat{O}C = \square$

c $C\hat{O}B = \square$

d $A\hat{O}B + A\hat{O}C + C\hat{O}B = \square$

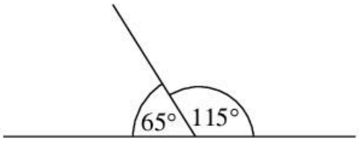
In Exercise 9A, you should have found that:

1 A **corner** measures 90°.



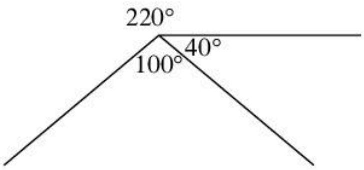
Angles that add up to 90° are called **complementary**.

2 A **straight angle** measures 180°.



Angles that add up to 180° are called **supplementary**.

3 **Angles at a point** add up to 360°.



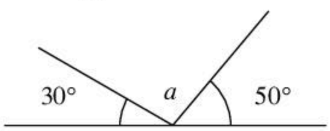
4 **Vertically opposite** angles are equal.

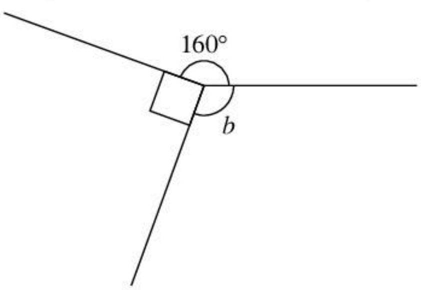
Calculating angles

In the next exercise you will be calculating angles instead of measuring them. Often in exams, questions will say 'not to scale'. These angles cannot be measured. If the diagram is not to scale, an angle that looks like an acute angle should be less than 90° when you calculate it. So you can use common sense to help you.

EXAMPLE 1

Without measuring, write down the missing angles:





As the angles 30°, a, 50° form a straight angle they add up to 180°.

That is $30° + a + 50° = 180°$

$a = 100°$

As the angles 90°, 160°, b are at a point they add up to 360°.

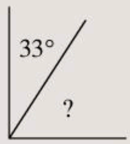
That is $90° + 160° + b = 360°$

$b = 110°$

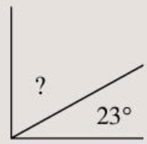
Exercise 9B

1 Without measuring, find the missing complementary angles.

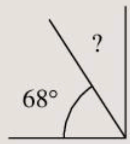
a



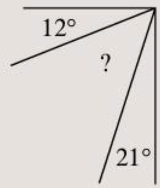
b



c

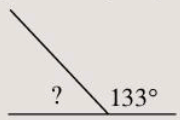


d



2 Without measuring, find the missing supplementary angles.

a



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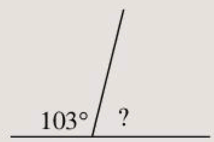

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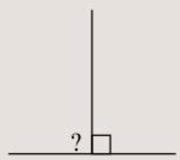

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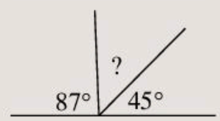

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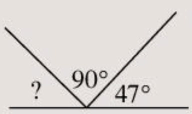
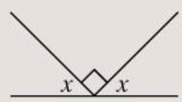
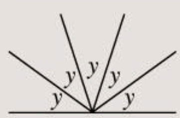


f

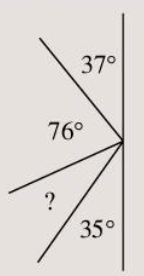


g

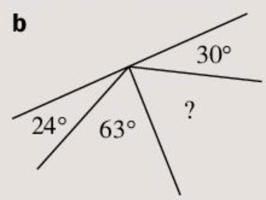


3 Without measuring, find the missing angles.

a

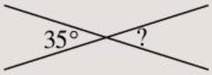


b

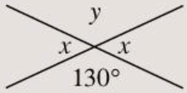


4 Without measuring, find the missing angles.

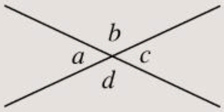
a



b



5 To prove that vertically opposite angles are equal, copy and complete:



$a + b = 180°$ as they are …

$c + b = 180°$ as they are …

Therefore $a = c$ (as the same number must be added to b to make 180°).

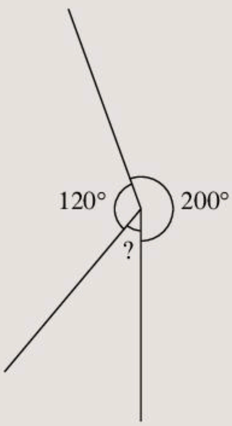
$b + c = 180°$ as they are …

$d + c =$…

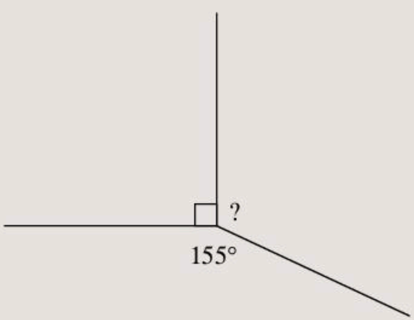
Therefore $b =$ …

6 Without measuring, find the missing angles.

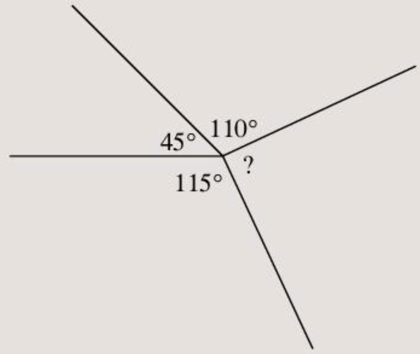
a



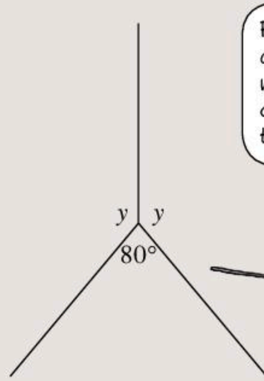
b

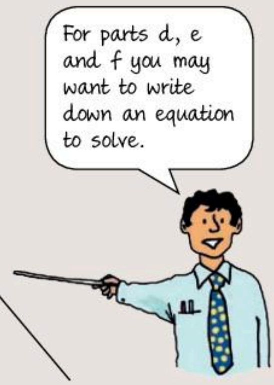


c

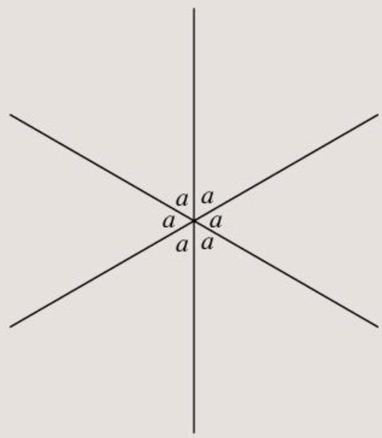


d

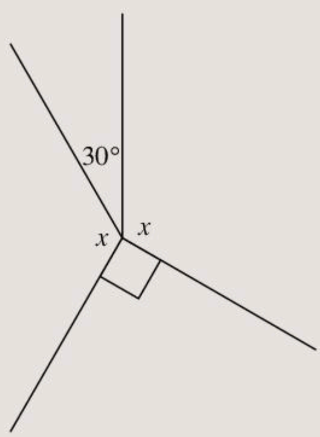




e



f



Angles in triangles and quadrilaterals

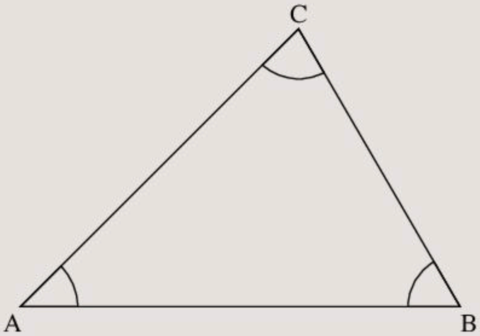
A triangle has 3 sides.

A quadrilateral has 4 sides.

Exercise 9C will help you find the sum of the angles in any triangle or quadrilateral.

Exercise 9C

1 **a** Use your protractor to measure the angles in the triangle.



b Copy and complete:

$$\begin{aligned} \hat{A} &= \square \\ \hat{B} &= \square \\ \hat{C} &= \square \\ \hline \hat{A} + \hat{B} + \hat{C} &= \square \end{aligned}$$

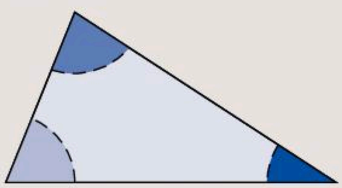
2 **a** Draw two more triangles of your own. Repeat Question **1** for each of them.

b Did you find that

$\hat{A} + \hat{B} + \hat{C} = 180°$?

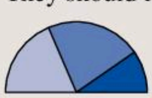
3 **a** With a ruler and pencil, draw a triangle. Cut it out very carefully.

b Now colour or shade each angle differently.



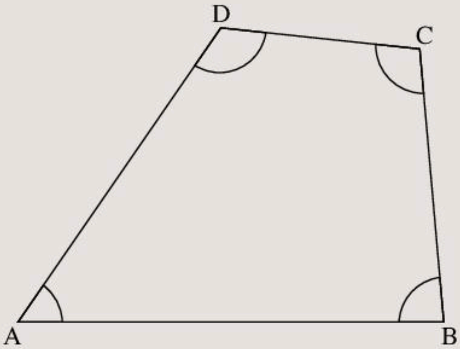
Cut off the angles along the dotted lines.

c Carefully fit the cut-out angles together. They should look like this:



d What angle is formed by the three angles?

4 **a** In the four-sided figure ABCD, measure each of the angles.



b Copy and complete:

$$\begin{aligned} \hat{A} &= \square \\ \hat{B} &= \square \\ \hat{C} &= \square \\ \hat{D} &= \square \\ \hline \hat{A} + \hat{B} + \hat{C} + \hat{D} &= \square \end{aligned}$$

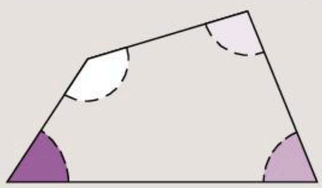
5 **a** Repeat Question **4** for two different four-sided figures of your own.

b Did you find that

$\hat{A} + \hat{B} + \hat{C} + \hat{D} = 360°$?

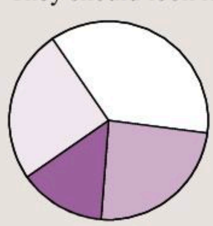
6 **a** With a ruler and pencil, draw accurately a four-sided figure. Cut it out carefully.

b Now colour or shade each angle differently.



Cut off the angles along the dotted lines.

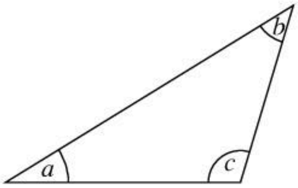
c Carefully fit the cut-out angles together. They should look like this:



d What angle is formed by the four angles?

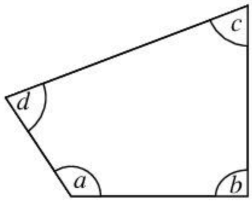
In Exercise 9C you should have found the following results:

1 The angle sum in a triangle is 180°



$$a + b + c = 180°$$

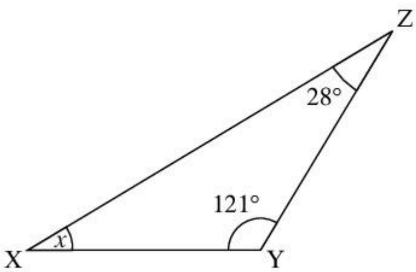
2 The angle sum in a quadrilateral is 360°



$$a + b + c + d = 360°$$

EXAMPLE 2

Find the angle x in triangle XYZ.



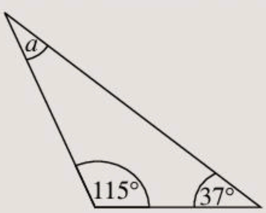
Since angle sum in a triangle is 180°

$$\begin{aligned} x + 121° + 28° &= 180° \\ x + 149° &= 180° \\ x &= 31° \end{aligned}$$

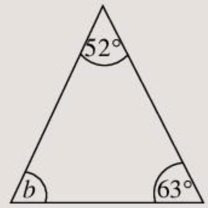
Exercise 9D

1 Find the missing angles in these triangles.

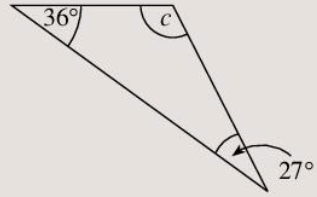
a



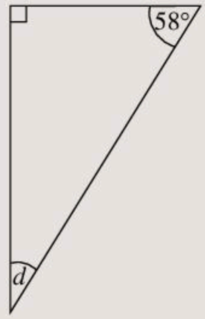
b



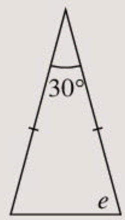
c



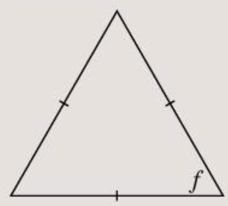
d



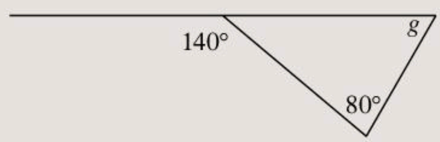
e



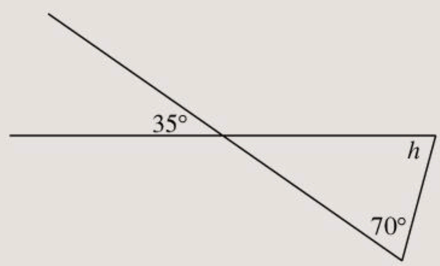
f



g

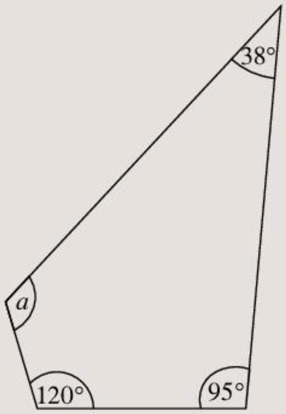


h

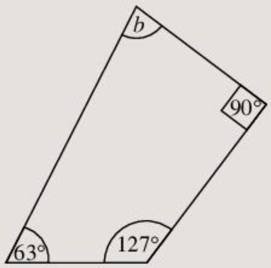


2 Find the missing angles in these quadrilaterals.

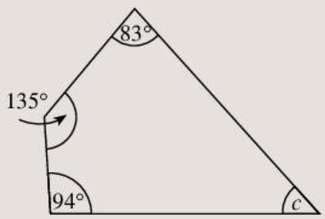
a



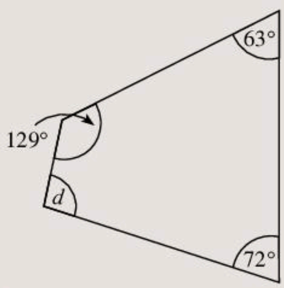
b



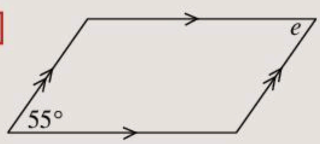
c



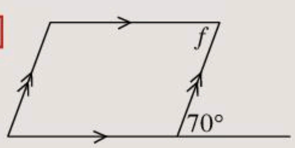
d



e



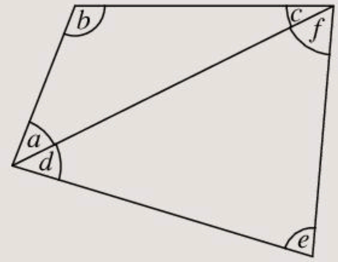
f



g Find the other two missing angles in Question **2 e**.

3 Copy and complete:

A quadrilateral can be divided into two triangles by drawing a diagonal.

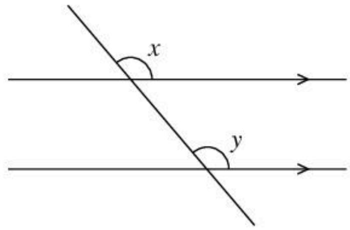


$a + b + c = 180°$ because …
$d + \square + \square = 180°$ because …
so $a + b + c + d + e + f = \square$

Therefore the angle sum of a quadrilateral is 360°.

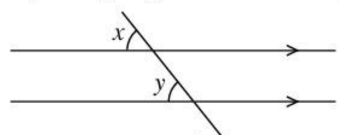
Corresponding angles

In the diagram a line crosses two parallel lines. This line is called a **transversal**. Measure with your protractor the angles x and y.



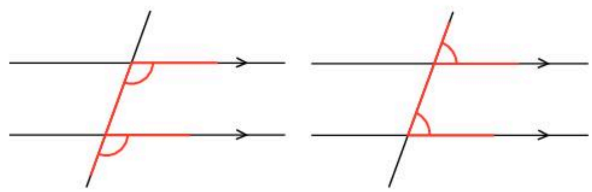
You should have found the angles were equal. These angles are called **corresponding** angles (because they are on the corresponding side of the transversal.)

- When a line crosses two parallel lines the corresponding angles formed are equal.



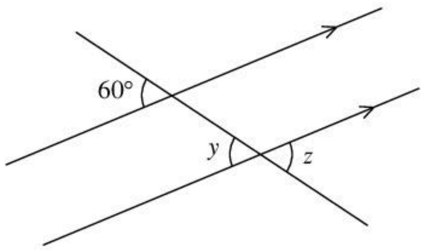
$\hat{x} = \hat{y}$

Look for an 'F' shape in the diagram. The corresponding angles are inside the F shape. The F can be upside down or backwards.



EXAMPLE 3

Find $\hat{y}$ and $\hat{z}$, giving reasons.

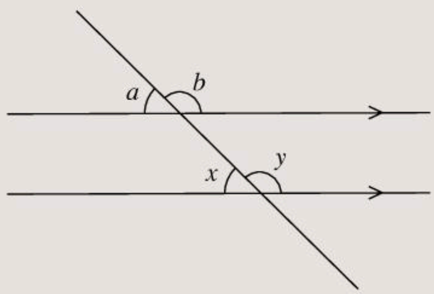


$\hat{y} = 60°$ (corresponding angles)
$\hat{z} = \hat{y}$ (vertically opposite angles)
so $\hat{z} = 60°$

Exercise 9E

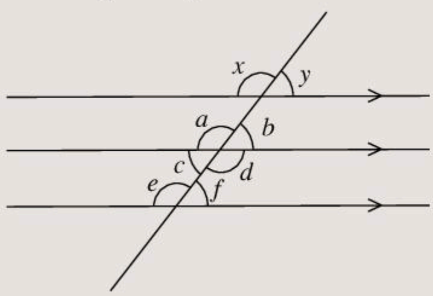
1 In the diagram, which angle is:

a the corresponding angle to $\hat{x}$
b the corresponding angle to $\hat{y}$?



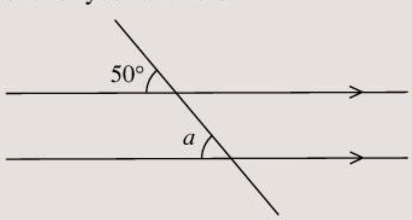
2 Which of these angles:

a correspond to $\hat{x}$
b correspond to $\hat{y}$?

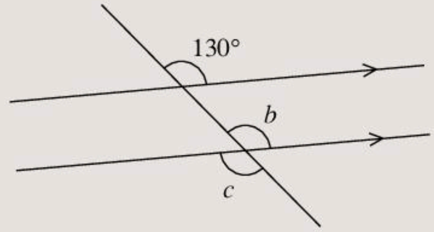


3 Find the angles marked by letters, giving reasons for your answers.

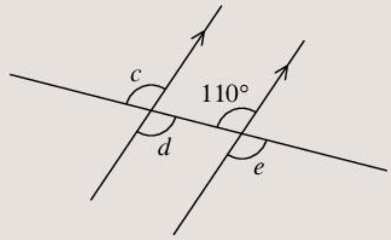
a



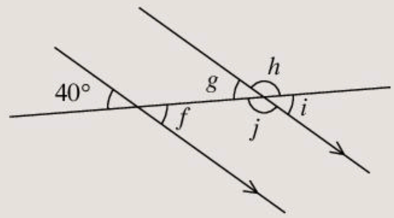
b



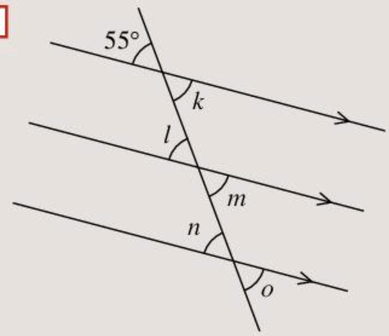
c



d

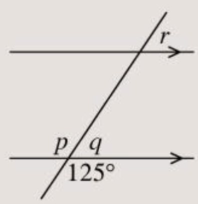


e

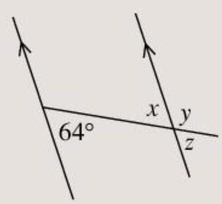


4 Without measuring, write down the value of each letter.

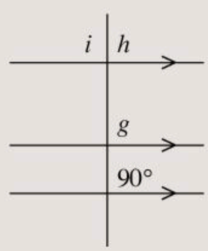
a



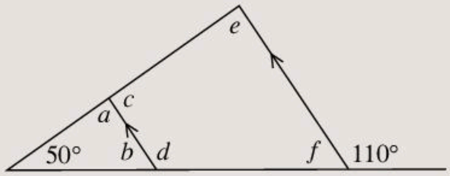
b



c

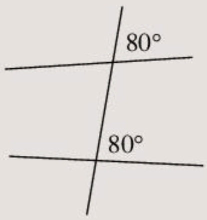


5 Calculate each of the unknown angles.

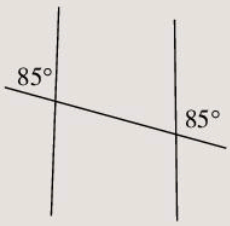


6 These diagrams have not been drawn properly. In which of them should the pair of lines be drawn parallel?

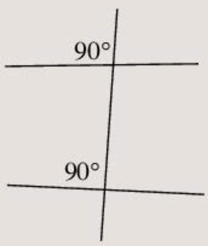
a



b



c



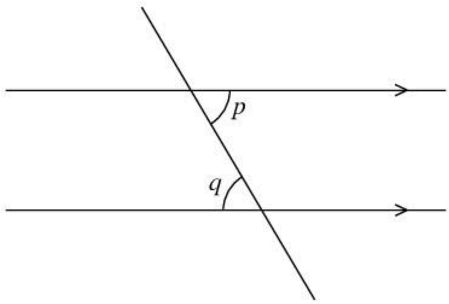
Copy and complete:

d When a line crosses a pair of parallel lines, corresponding angles are . . .

e When a line crosses a pair of lines, and corresponding angles are equal, the pair of lines must be . . .

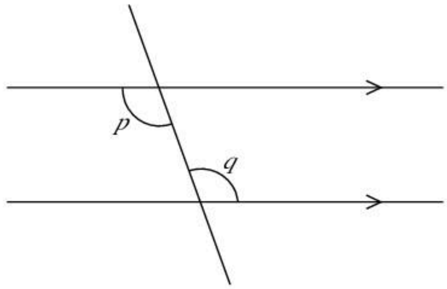
Alternate angles

Measure with your protractor the angles p and q.



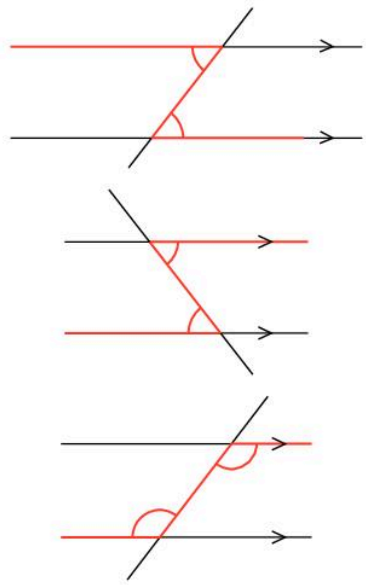
You should have found that the angles p and q are equal. These angles are called **alternate** angles. (Because they are on alternate sides of the transversal.)

- When a line crosses two parallel lines the alternate angles formed are equal.



$$\hat{p} = \hat{q}$$

Look for a 'Z' shape in the diagram. The alternate angles are inside the Z shape. The Z can be backwards and can be used to find obtuse angles.

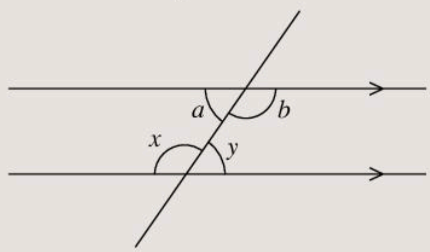


Exercise 9F

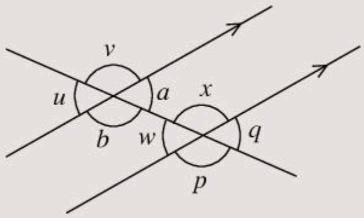
1 In the diagram which angle is:

a the alternate angle to $\hat{a}$

b the alternate angle to $\hat{b}$?



2 Look at this diagram:



Which of the angles are

a alternate to $\hat{a}$

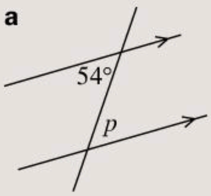
b corresponding to $\hat{a}$

c vertically opposite to $\hat{a}$?

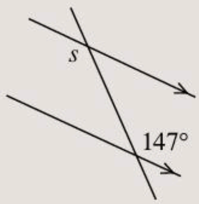
3 Repeat Question **2** for angle b.

4 Write down the size of each angle marked by a letter.

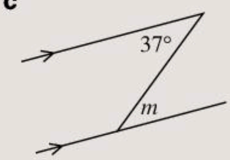
a



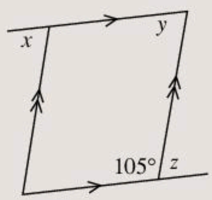
b



c



d



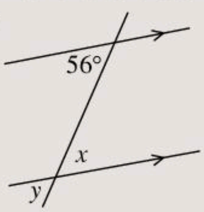
5 For Question **4 d**, copy and complete:

a $\hat{x}$ and $\hat{y}$ are . . . angles.

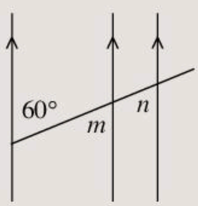
b $\hat{y}$ and $\hat{z}$ are . . . angles.

6 Find the value of each letter, giving a reason why.

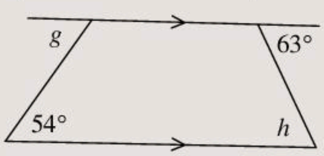
a



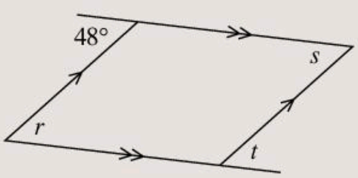
b



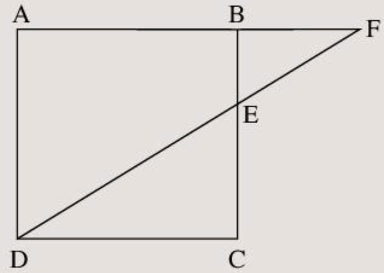
c



d



7 In the diagram, ABCD is a square. DF is a line cutting BC at E.



Copy and complete:

a AB is parallel to . . .

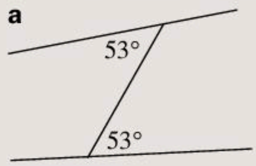
b AD is parallel to . . .

c A$\hat{F}$D and F$\hat{D}$C are equal because they are . . . angles.

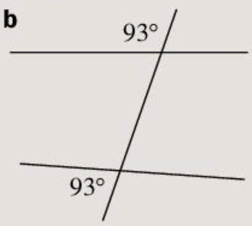
d A$\hat{D}$F and B$\hat{E}$F are equal because they are . . . angles.

8 These diagrams have not been drawn properly. In which of them should two lines be drawn parallel?
Give reasons for your answers.

a

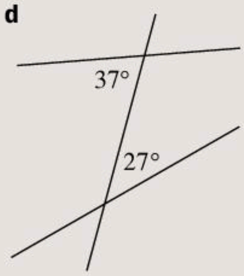


b

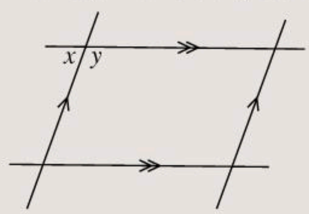


c

d



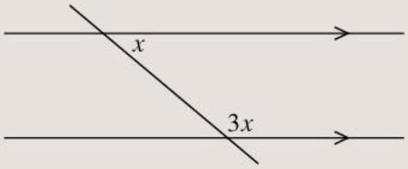
9 Copy the diagram. Mark an x in all the angles that are equal to $\hat{x}$.
Mark a y in all the angles that equal $\hat{y}$.



Copy and complete:

$\hat{x} + \hat{y} = \square°$

10 Find x.

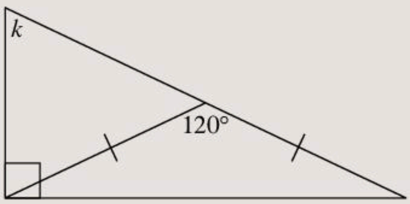


11 Adanya measures the four angles of a quadrilateral as:

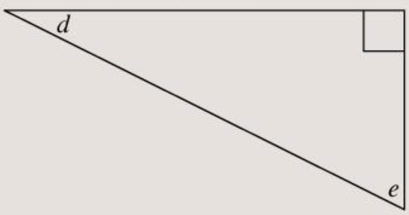
140° 63° 126° 32°

Gill says she has made a mistake.
Is Gill correct? Explain your answer.

12 Work out angle k.



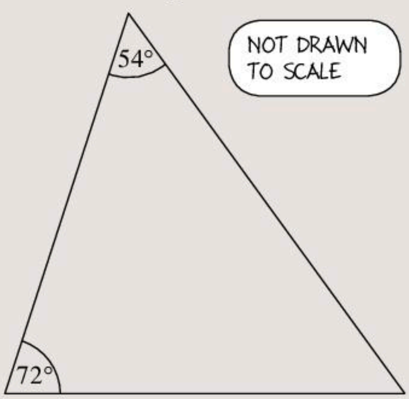
13 Here is a right-angled triangle:



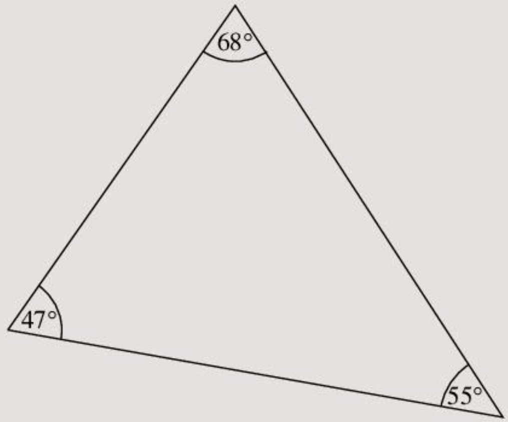
Choose two of the following angles to be used for $\hat{d}$ and $\hat{e}$.

43° 37° 27° 53° 67° 13°

14 What sort of triangle is this?

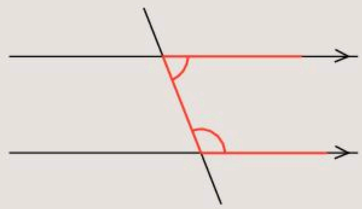


15 One of the angles in this triangle is measured wrongly. How can you tell?



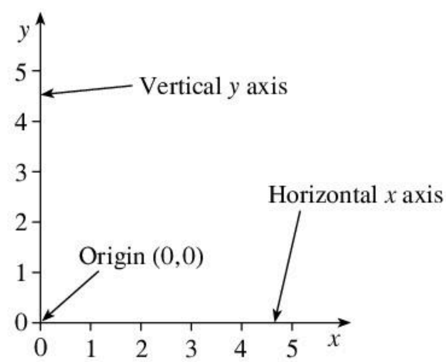
16 Aakil says that if you know one angle inside a parallelogram you know all the rest. Is he right? Explain your answer.

17 The angles marked in the diagram below are supplementary. This means they add up to 180°.



How can you prove this?
(Use any of these: *vertically opposite angles*, *corresponding angles*, *alternate angles* or *angles on a straight line*.)

9.2 Coordinates

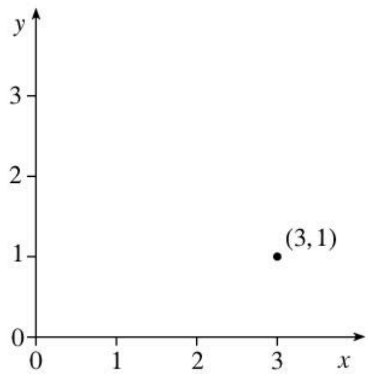


The **origin** has coordinates (0,0). It is where the **axes** intersect.

To describe the position of a point in relation to the origin you need **coordinates** (x, y).

The first number tells you how far to move along in a horizontal direction. The second number tells you how far to move up in a vertical direction.

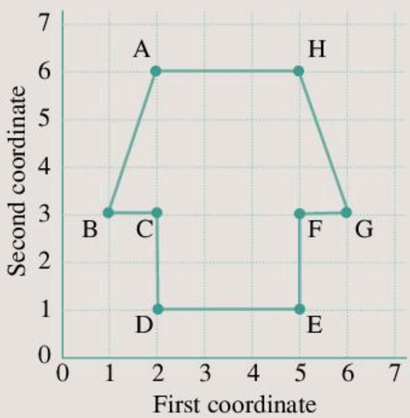
- Coordinates tell you exactly where a point is on a graph. The coordinates $(3, 1)$ represent the point 3 across and then 1 up from where the axes meet.



Exercise 9G

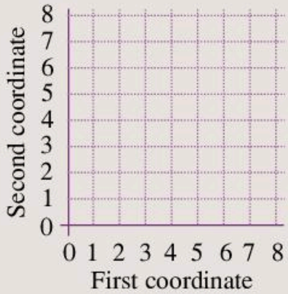
1 Are the coordinates $(7, 3)$ the same as $(3, 7)$? Explain.

2



Write down the coordinates of all the points marked on the graph.

3 **i** On squared paper, make a larger copy of the graph started below.



ii On your copy, plot the following points, using a coloured pencil.

$(1, 3)$ $(1, 5)$ $(2, 4)$ $(3, 2)$ $(3, 3)$
$(3, 6)$ $(3, 7)$ $(4, 2)$ $(4, 3)$ $(4, 6)$
$(4, 7)$ $(6, 3)$ $(6, 4)$ $(6, 5)$ $(6, 6)$
$(7, 4)$

iii Use a ruler and a different coloured pencil to join these points:

a $(1, 3)$ to $(1, 5)$	**b** $(1, 3)$ to $(2, 4)$
c $(1, 5)$ to $(2, 4)$	**d** $(2, 4)$ to $(3, 3)$
e $(3, 3)$ to $(6, 3)$	**f** $(6, 3)$ to $(7, 4)$
g $(7, 4)$ to $(6, 6)$	**h** $(6, 6)$ to $(3, 6)$
i $(3, 6)$ to $(2, 4)$	**j** $(6, 4)$ to $(7, 4)$
k $(4, 3)$ to $(4, 2)$	**l** $(4, 2)$ to $(3, 2)$
m $(3, 2)$ to $(4, 3)$	**n** $(4, 6)$ to $(4, 7)$
o $(4, 7)$ to $(3, 7)$	**p** $(3, 7)$ to $(4, 6)$

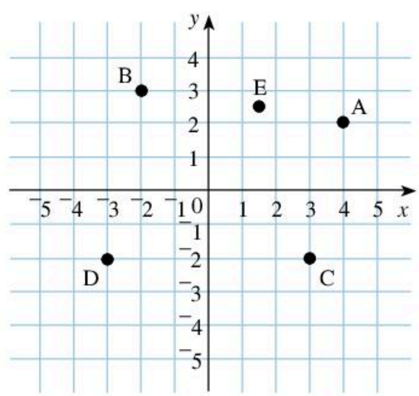
What shape have you drawn?

Negative coordinates

You can extend graphs to include negative numbers and decimals.

A negative x-coordinate means move left.

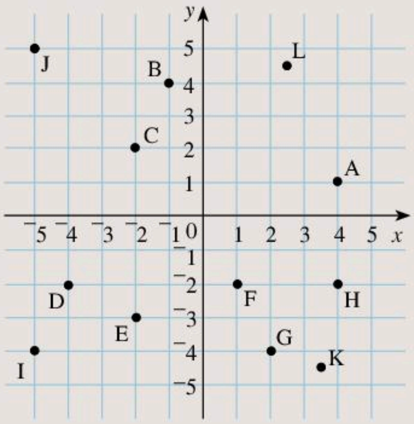
A negative y-coordinate means move down.



The coordinates of A are $(4, 2)$
B are $(^-2, 3)$
C are $(3, ^-2)$
D are $(^-3, ^-2)$
E are $(1.5, 2.5)$

Exercise 9H

1 Write down the coordinates of the points A–L marked on the graph.



2 **a** Make a copy of the axes in Question **1**.
Mark and label these points:
A $(3, 2)$ B $(^-3, 2)$
C $(^-3, ^-2)$ D $(3, ^-2)$

b Join the points A to B, B to C, C to D and D to A. What shape have you drawn?

3 Draw axes on squared paper.
On the x-axis, show the numbers

$$\{^-6, ^-5, \ldots, 5, 6\}$$

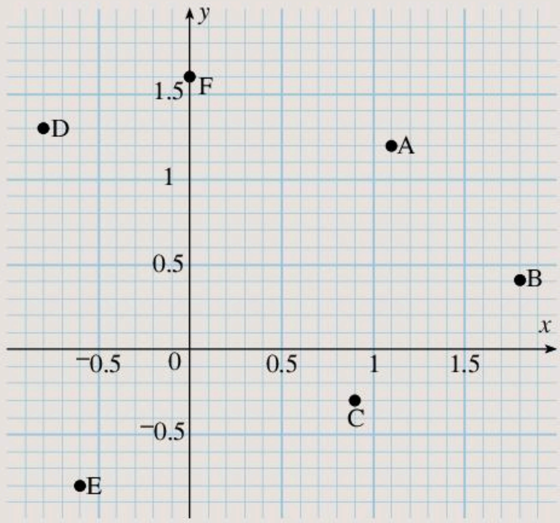
On the y-axis, show the numbers

$$\{^-8, ^-7, \ldots, 7, 8\}$$

Mark these points on your graph:
$(^-3, ^-8)$, $(^-3, ^-7)$, $(^-5, ^-5)$, $(^-5, 4)$
$(^-4, 4)$, $(^-4, ^-1)$, $(^-3, ^-1)$, $(^-3, 5)$
$(^-2, 5)$, $(^-2, ^-1)$, $(^-1, ^-1)$, $(^-1, 7)$
$(0, 7)$, $(0, ^-1)$, $(1, ^-1)$, $(1, 6)$
$(2, 6)$, $(2, ^-3)$, $(3, ^-3)$, $(4, ^-1)$
$(5, ^-1)$, $(2, ^-7)$, $(1, ^-7)$, $(1, ^-8)$

Join each point to the next one in the list.
What shape have you made?

4 Write down the coordinates of the points A–F marked on the grid.



5 Using graph paper and choosing a sensible scale, plot these points:
V $(^-0.9, 0.6)$ W $(0.2, 0)$ X $(0.5, ^-0.5)$
Y $(1.3, 2.2)$ Z $(^-1.4, ^-0.9)$

6 Draw a grid with x- and y-axes from $^-8$ to 8.

a Plot the points A $(^-2, 4)$, B $(3, 6)$, C $(5, 1)$.

b Plot the point D such that ABCD is a square. What are the coordinates of D?

c Draw in the diagonals of the square. What are the coordinates of the point of intersection of the diagonals?

d Plot the points E $(^-4, ^-1)$, F $(^-6, ^-7)$, G $(^-2, ^-7)$.

e What is shape EFG?

f Plot the point H at $(^-4, ^-7)$.

g Draw a line from E to H.

h What can you say about the line EH?

TECHNOLOGY

To revise what you have learned about angles go to the website

www.bbc.co.uk/schools/ks3bitesize/maths

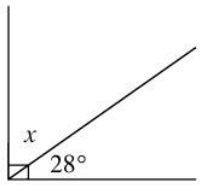
Look at the activity in the 'Angles' section of 'Shape and space'. Try the 'Revise' and 'Test' sections too.

Consolidation

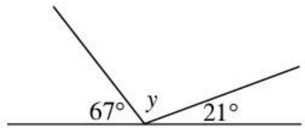
Example 1

Without measuring, find the missing angles.

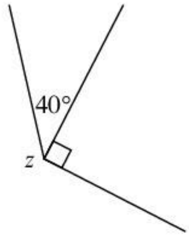
a



b



c



a The angles form a right angle.
Hence, $x + 28° = 90°$
$x = 90° - 28°$
$x = 62°$

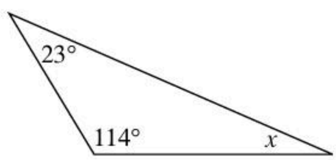
b The angles form a straight line.
Hence, $67° + y + 21° = 180°$
$88° + y = 180°$
$y = 180° - 88°$
$y = 92°$

c The angles meet at a point.
Hence, $z + 40° + 90° = 360°$
$z + 130° = 360°$
$z = 230°$

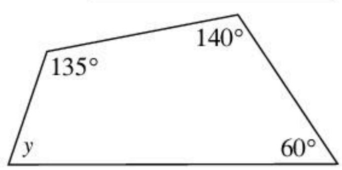
Example 2

Find the missing angles.

a



b



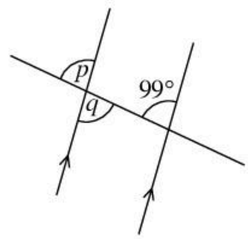
Solutions:

a Angles in a triangle sum to 180°.
$23° + 114° + x = 180°$
$137° + x = 180°$
$x = 180° - 137°$
$x = 43°$

b Angles in quadrilateral sum to 360°.
$y + 135° + 140° + 60° = 360°$
$y + 335° = 360°$
$y = 360° - 335°$
$y = 25°$

Example 3

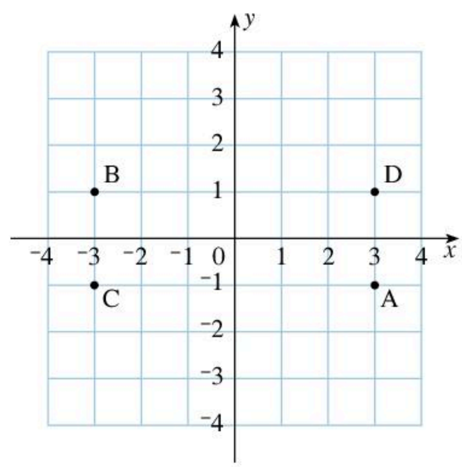
Find the angles marked by the letters.



$p = 99°$ (corresponding angles)
$q = 99°$ (vertically opposite angle to p or alternate angle to 99°).

Example 4

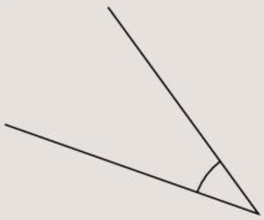
Plot the points A $(3, {}^-1)$, B $({}^-3, 1)$, C $({}^-3, {}^-1)$, D $(3, 1)$ on a graph.



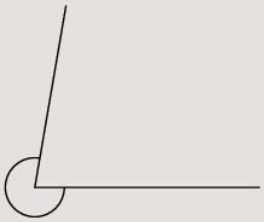
Exercise 9

1 Estimate, then measure the size of these angles:

a



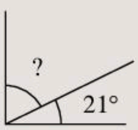
b



c

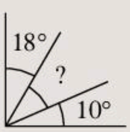
2 Without measuring, find the missing angles:

a

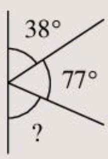


b

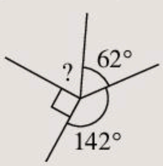
c



d

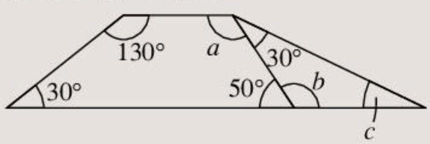


e

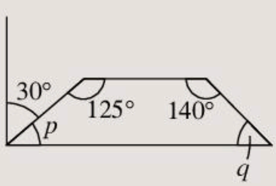


3 Find the missing angles:

a



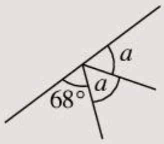
b



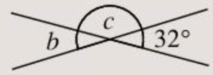
NOT TO SCALE

4 Find the angles marked by the letters:

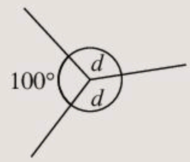
a



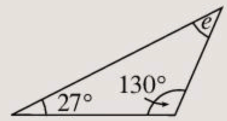
b



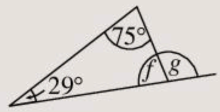
c



d

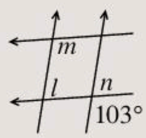


e



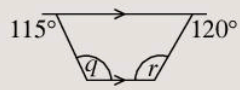
f

g



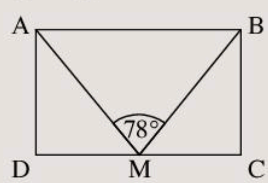
h

i



NOT TO SCALE

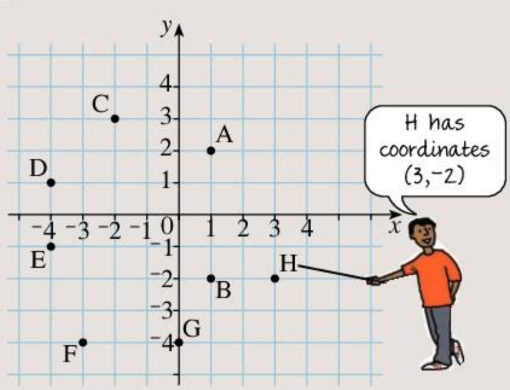
5 The design of an envelope ABCD is shown in the diagram. ABM is the flap and M is the mid-point of DC.



Assist in the construction of the envelope by calculating:

a angle ABM **b** angle AMD

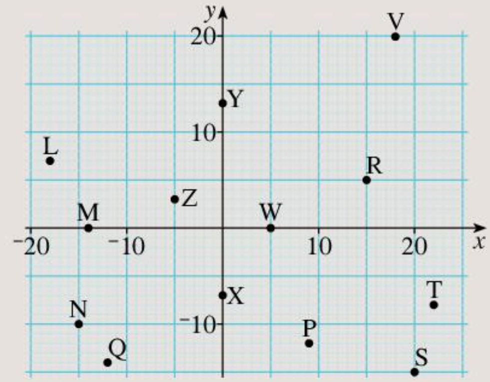
6 Write down the coordinates of these lettered points.



7 Draw x- and y-axes and plot the points:
A $(^-4, 2)$, B $(^-1, ^-2)$, C $(3, ^-2)$, D $(3, 1)$, E $(0, 5)$, F $(^-1, 3)$, G $(^-1, 1)$ and H $(^-1, 0)$

- **a** What type of quadrilaterals are CDFH, ABDE and GBCD?
- **b** What sort of triangles are BDF, EFD and AFE?
- **c** Which two triangles are identical but in different positions?

8

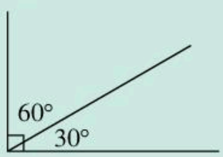


Name the points with the following coordinates:
$(15, 5)$, $(18, 20)$, $(20, ^-15)$, $(9, ^-12)$, $(5, 0)$, $(0, 13)$, $(^-12, ^-14)$, $(^-14, 0)$, $(^-18, 7)$, $(^-15, ^-10)$, $(0, ^-7)$, $(^-5, 3)$, $(22, ^-8)$

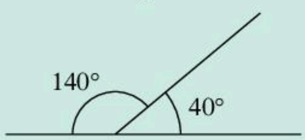
Summary

You should know …

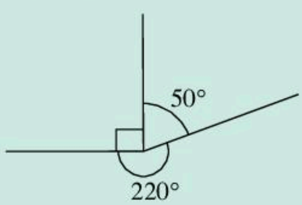
1 Complementary angles add to 90°.



Supplementary angles on a straight line add to 180°.



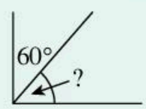
Angles at a point add to 360°.



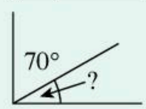
Check out

1 Find the missing angles.

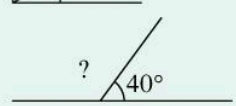
a



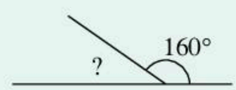
b



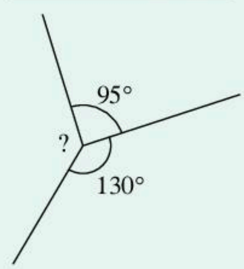
c



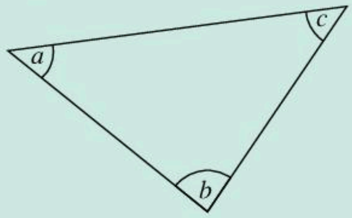
d



e

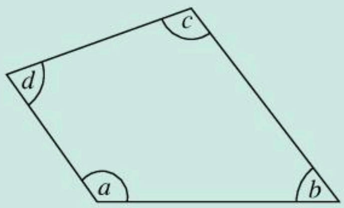


2 Angle sum in a triangle is 180°.



$$a + b + c = 180°$$

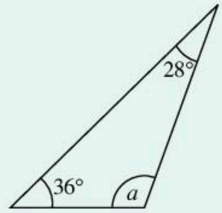
Angle sum in a quadrilateral is 360°.



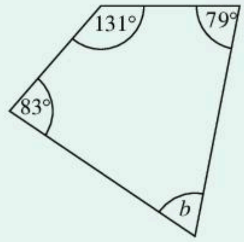
$$a + b + c + d = 360°$$

2 Find the marked angles.

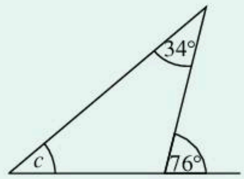
a



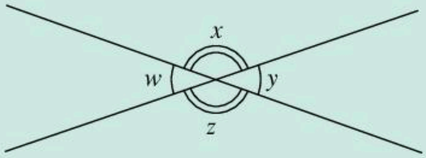
b



c



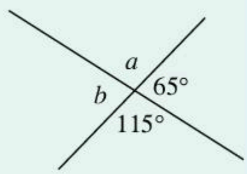
3 Vertically opposite angles are equal.



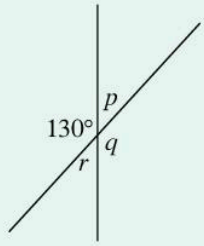
$$\hat{w} = \hat{y}$$
$$\hat{x} = \hat{z}$$

3 Give the value of the lettered angles.

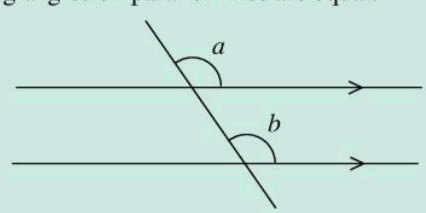
a



b

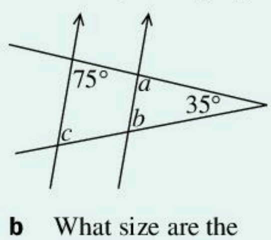


4 Corresponding angles on parallel lines are equal.



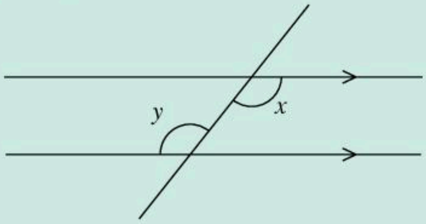
$$\hat{a} = \hat{b}$$

4 **a** Which angles in the diagram are corresponding angles?



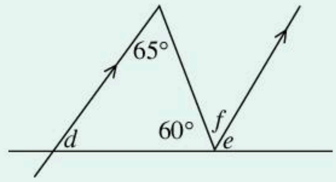
b What size are the angles a, b and c?

5 Alternate angles on parallel lines are equal.



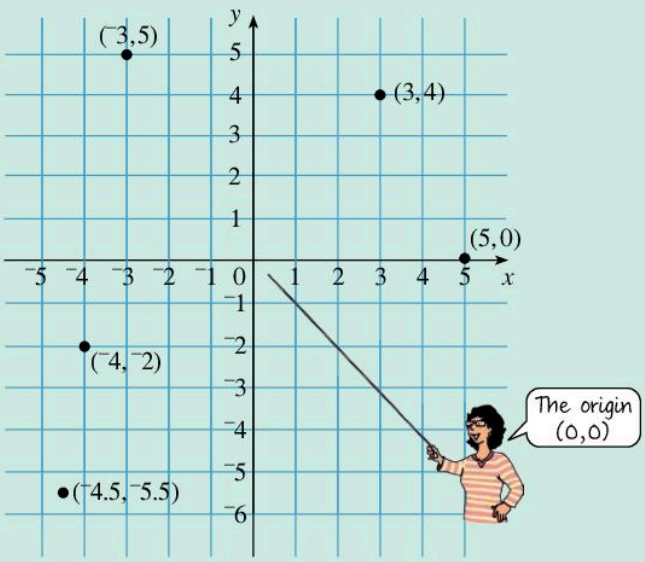
$$\hat{x} = \hat{y}$$

5 **a** Which angles in the diagram are alternate?

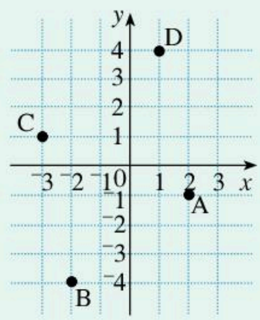


b What size are the angles d, e and f?

6 You can show coordinates on a graph.



6 **a** Write down the coordinates of the points A, B, C and D on the graph.



b What shape is ABCD?

c On suitable axes, plot the points:
$W(3, {}^-1)$, $X({}^-3, {}^-5)$
$Y({}^-5, 3)$, $Z({}^-1, 0)$

10 Fractions and decimals

Objectives

- Recognise the equivalence of simple fractions and decimals.
- Convert terminating decimals to fractions e.g. $0.23 = \frac{23}{100}$.
- Compare two fractions by using a calculator to convert the fractions to decimals, e.g. $\frac{3}{5}$ and $\frac{13}{20}$.
- Add and subtract integers and decimals, including numbers with different numbers of decimal places.
- Use known facts and place value to multiply simple decimals by one-digit numbers, e.g. 0.8×6.
- Use the laws of arithmetic and inverse operations to simplify calculations with decimals.
- Multiply and divide decimals with one or two decimal places by single-digit numbers, e.g. 13.7×8, $4.35 \div 5$.

What's the point?

Decimals are used to make accurate measurements. When Usain Bolt broke world records at the 2009 World Athletics Championships in Berlin his times were measured to two decimal places.

Before you start

You should know ...

1 Equivalent fractions are equal fractions.
For example:

$$\frac{3}{4} = \frac{6}{8} = \frac{9}{12} = \frac{12}{16}$$

2 How to add fractions.
For example:

$$\frac{3}{5} + \frac{1}{10} = \frac{6}{10} + \frac{1}{10} = \frac{7}{10}$$

Check in

1 Copy and complete:

a $\frac{2}{5} = \frac{\square}{10}$

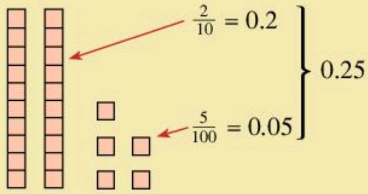
b $\frac{9}{10} = \frac{\square}{100}$

2 Work out:

a $\frac{3}{10} + \frac{5}{10}$

b $\frac{3}{10} + \frac{7}{100}$

3 How to represent decimals using diagrams.
For example:



4 A decimal is a way of writing a number using place values of tenths, hundredths etc.

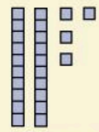
1 2 . 5 7
1 ten
2 units
$\frac{5}{10}$
$\frac{7}{100}$

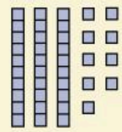
5 How to multiply and divide decimals by 10, 100 or 1000.
For example:
$0.34 \times 10 = 3.4$
$42.1 \div 1000 = 0.0421$

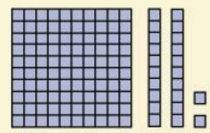
3 What decimals do these pictures represent?

a


b


c


d


e


4 Write the value of the underlined digit.
a $0.68\underline{2}$ **b** $3\underline{2}0.61$
c $0.0\underline{7}3$ **d** $\underline{4}0.18$
e $15.\underline{3}$

5 Work out:
a 2.03×10
b $0.3 \div 10$
c 1000×0.04
d $2.6 \div 100$
e 100×3.8
f $53.2 \div 1000$

10.1 Equivalence of fractions and decimals

Fractions and decimals are a way of writing parts of a whole number. You need to be able to convert between fractions and decimals.

Writing decimals as fractions

EXAMPLE 1

Write 0.63 as a fraction.

0.63 =

U	$\frac{1}{10}$	$\frac{1}{100}$
0	6	3

$$\text{So } 0.63 = \frac{6}{10} + \frac{3}{100}$$
$$= \frac{60}{100} + \frac{3}{100} \left(\frac{6}{10} = \frac{60}{100}\right)$$
$$= \frac{63}{100}$$

In the same way:

0.236 =

U	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$
0	2	3	6

$$= \frac{2}{10} + \frac{3}{100} + \frac{6}{1000}$$
$$= \frac{200}{1000} + \frac{30}{1000} + \frac{6}{1000}$$
$$= \frac{236}{1000}$$

Exercise 10A

1 Using the method from Example 1 write these decimals as fractions:

a 0.5 **b** 0.7 **c** 0.15
d 0.34 **e** 0.72 **f** 0.699
g 0.708 **h** 0.094

2 Compare each decimal in Question **1** with its corresponding fraction.

a If there is one digit after the decimal point, what is the denominator of the fraction?

b If there are two digits after the decimal point, what is the denominator of the fraction?

c If there are three digits after the decimal point, what is the denominator of the fraction?

d Can you see a quick way to write any decimal as a fraction? What is it?

3 Write as a fraction:

a 0.3 **b** 0.8
c 0.15 **d** 0.37

4 Write as a fraction:

a 0.9 **b** 0.4 **c** 0.14
d 0.46 **e** 0.07 **f** 0.05
g 0.04 **h** 0.149 **i** 0.237
j 0.097 **k** 0.062 **l** 0.003

5 Write as a mixed number:

a 1.7 **b** 2.5 **c** 3.54
d 9.27 **e** 5.114 **f** 11.438
g 8.005 **h** 21.009 **i** 9.422
j 14.803 **k** 16.002 **l** 15.077

6 Make up five decimals of your own, and change them to fractions.

7 $0.4 = \frac{4}{10} = \frac{2}{5}$

In the same way, write each decimal as a fraction in its simplest form.

a 0.2 **b** 0.6 **c** 0.8
d 0.5 **e** 0.25 **f** 0.75
g 0.125 **h** 0.375 **i** 0.625

Writing fractions as decimals

To convert some fractions into decimals the easiest way is to use equivalent fractions. If you can make the denominator 10, 100 or 1000 they are easy to convert to decimals. If it is not easy to make the denominator 10, 100 or 1000 then using a calculator may be better.

EXAMPLE 2

Write as decimals:

a $\frac{1}{2}$ **b** $\frac{3}{4}$ **c** $\frac{17}{250}$ **d** $\frac{3}{8}$ **e** $\frac{5}{32}$

a $\frac{1}{2} = \frac{5}{10} = 0.5$ **b** $\frac{3}{4} = \frac{75}{100} = 0.75$

c $\frac{17}{250} = \frac{68}{1000} = 0.068$ **d** $\frac{3}{8} = \frac{375}{1000} = 0.375$

e For $\frac{5}{32}$ it is not possible to make the denominator 10, 100 or 1000, so do $5 \div 32$ on your calculator to get 0.15625.

Exercise 10B

1 Write as a decimal:

a $\frac{1}{5}$ **b** $\frac{3}{5}$ **c** $\frac{9}{10}$ **d** $\frac{13}{20}$
e $\frac{7}{25}$ **f** $\frac{14}{25}$ **g** $\frac{3}{50}$ **h** $\frac{41}{50}$

2 Write as a decimal:

a $\frac{5}{8}$ **b** $\frac{7}{250}$ **c** $\frac{19}{125}$ **d** $\frac{7}{8}$

3 Use your calculator to write each fraction as a decimal.

a $\frac{3}{16}$ **b** $\frac{11}{32}$ **c** $\frac{249}{2000}$
d $\frac{13}{64}$ **e** $\frac{7}{9}$ **f** $\frac{2}{3}$

4 What did you notice about your answers to Question **3**, parts **e** and **f**?

5 Write as a decimal:

a $9\frac{7}{10}$ **b** $5\frac{3}{4}$ **c** $20\frac{3}{5}$
d $2\frac{3}{100}$ **e** $37\frac{12}{25}$

6 Make up five fractions of your own and change them to decimals.

7 Which of these are 10 times bigger than $\frac{1}{25}$?

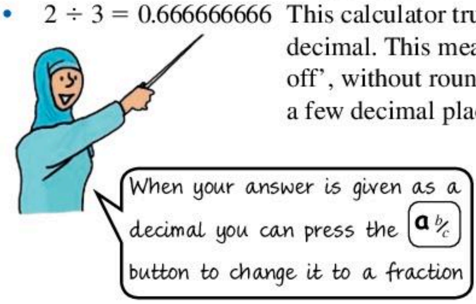
0.04 $\frac{2}{5}$ $\frac{1}{250}$ 2.5 0.4 $\frac{10}{250}$

In Questions **3e** and **3f** of Exercise 10B, you should have found your answers were **recurring decimals**. Recurring decimals go on forever, repeating the same digits. We use dots to show the digits that repeat.

- $\frac{5}{9} = 0.555555555\ldots = 0.\dot{5}$ The dot above the 5 shows that the 5 repeats.
- $\frac{4}{33} = 0.12121212\ldots = 0.\dot{1}\dot{2}$ The dots above the 1 and 2 show that these digits repeat.
- $\frac{9}{37} = 0.243243243\ldots = 0.\dot{2}4\dot{3}$ The dots above the 2 and 3 show that all the digits between and including 2 and 3 repeat.

If you work out $2 \div 3$ on different calculators you may see a variety of answers. It is important that you realise they all mean the same:

- $2 \div 3 = \frac{2}{3}$ Your calculator is set up to give answers as fractions. To convert to decimals look for the S⇔D key. Or you can change the set-up so that answers are given as decimals.
- $2 \div 3 = 0.666666666$ This calculator truncates the decimal. This means 'cuts it off', without rounding, after a few decimal places.



- $2 \div 3 = 0.666666667$ The last digit in the display is rounded on this calculator.
- $2 \div 3 = 0.\dot{6}$ Some calculators can show recurring decimals using the dot notation. Be careful as it is easy to miss the dot!

The other sort of decimals are terminating decimals. Terminating decimals are when the decimal stops after a certain number of decimal places.

0.3, 0.1255 and 0.3241452 are terminating decimals.

INVESTIGATION

Change the following fractions to recurring decimals using your calculator.

$$\frac{1}{9} \quad \frac{5}{9} \quad \frac{7}{9}$$

Can you see a pattern?

Without a calculator, change $\frac{8}{9}$ to a decimal.

What about $\frac{2}{3}$?

(**Hint:** Change this to ninths using equivalent fractions.)

Change the following fractions to recurring decimals using your calculator.

$$\frac{13}{99} \quad \frac{25}{99} \quad \frac{7}{99}$$

Can you see a pattern?

Without a calculator, change $\frac{14}{99}$ to a decimal.

What about $\frac{4}{33}$?

(**Hint:** Change this to ninety-ninths using equivalent fractions.)

Investigate the following set of fractions by converting them to decimals.

$$\frac{121}{999} \quad \frac{245}{999} \quad \frac{14}{999} \quad \frac{2}{999} \quad \frac{121}{333}$$

Comparing fractions

You can compare fractions with different denominators to see which one is larger. In Chapter 7 you did this by looking at equivalent fractions. Now you will be comparing fractions by changing them to decimals.

EXAMPLE 3

Which fraction is larger:

a $\frac{3}{5}$ or $\frac{13}{20}$ **b** $\frac{3}{16}$ or $\frac{5}{32}$?

a $\frac{3}{5} = \frac{6}{10} = 0.6$

$\frac{13}{20} = \frac{65}{100} = 0.65$

Show both decimals with two decimal places to compare them better: 0.60 is smaller than 0.65 so the larger fraction is $\frac{13}{20}$.

b $\frac{3}{16} = 3 \div 16 = 0.1875$

$\frac{5}{32} = 5 \div 32 = 0.15625$

So $\frac{3}{16}$ is larger.

Exercise 10C

You are allowed to use your calculator only for Questions **3** and **4** in this exercise.

1 By converting to decimals, which of these fractions is larger?

a $\frac{3}{4}$ or $\frac{7}{10}$

b $\frac{3}{5}$ or $\frac{61}{100}$

c $\frac{9}{20}$ or $\frac{2}{5}$

2 By converting to decimals, which of these fractions is smaller?

a $\frac{3}{8}$ or $\frac{3}{10}$

b $\frac{3}{50}$ or $\frac{1}{20}$

c $\frac{2}{25}$ or $\frac{39}{500}$

3 By converting to decimals, which of these fractions is larger?

a $\frac{29}{32}$ or $\frac{181}{200}$

b $\frac{7}{16}$ or $\frac{4}{9}$

c $\frac{5}{16}$ or $\frac{393}{1250}$

4 Choose five pairs of fractions of your own, with different denominators. For each pair, write which fraction is larger.

5 From the list below, write down which of these fractions is bigger than $\frac{3}{4}$.

0.83 $\frac{73}{100}$ $\frac{4}{5}$ $\frac{7}{10}$ 0.69 $\frac{17}{20}$ 0.712

6 Write these fractions in order of size, smallest to largest:

$\frac{7}{20}$ $\frac{21}{100}$ $\frac{1}{5}$ $\frac{3}{10}$ $\frac{1}{4}$

7 Pick any *three* numbers from 1, 2, 3, 4, 5, 6, 7, 8 and 9.

Use your numbers to form as many fractions as you can with one digit in the numerator and one digit in the denominator (no repeats). You can have improper fractions. Change all your fractions to decimals.

$\boxed{8}$ Repeat Question 7, this time using any *four* numbers from 1, 2, 3, 4, 5, 6, 7, 8 and 9.

10.2 Adding and subtracting decimals

Adding decimals

Adding decimal numbers is similar to adding whole numbers. It is a good idea to use column headings.

EXAMPLE 4

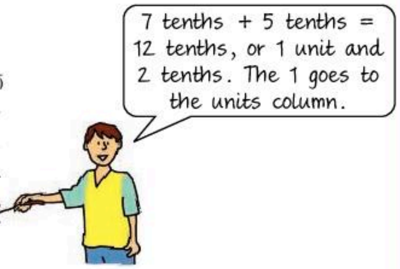
Find

a 3.7 + 1.5

b 2.4 + 3.18

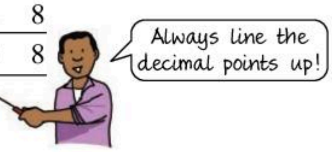
a

$$\begin{array}{rcl} \text{U} & & \frac{1}{10} \\ 3 & . & 7 \\ +1 & . & 5 \\ \hline 5 & . & 2 \\ \hline {\scriptstyle 1} & & \end{array}$$



b

$$\begin{array}{rccc} \text{U} & & \frac{1}{10} & \frac{1}{100} \\ 2 & . & 4 & \\ +3 & . & 1 & 8 \\ \hline 5 & . & 5 & 8 \\ \hline \end{array}$$



When adding decimals remember to put the decimal points one below the other. This makes sure that you don't add tenths to hundredths or units to tenths by mistake.

Exercise 10D

1 Work out:

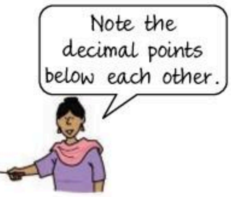
a	3.7 + 1.2	**b**	2.6 + 1.4
c	9.1 + 0.8	**d**	4.3 + 5.8
e	2.3 + 8.7	**f**	3.1 + 1.62
g	0.03 + 3.2	**h**	4 + 0.05
i	6 + 0.4	**j**	3.96 + 1.5

2 Study this example:

What is $5.2 + 6 + 1.86$?

Remember that $6 = 6.0$
Write the sum vertically:

$$\begin{array}{r} 5.2 \\ 6.0 \\ +\ 1.86 \\ \hline 13.06 \\ \hline \end{array}$$

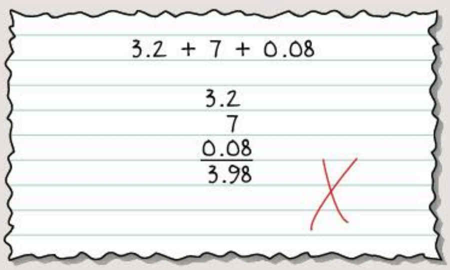


Use this method to work out $7.5 + 9 + 3.79$

3 Calculate:

a $3 + 0.6 + 0.24$
b $0.8 + 4 + 3.62$
c $17.3 + 200 + 0.08$
d $6 + 4.8 + 0.34$
e $0.01 + 0.1 + 10$

4 Look at this part of a page from Ann's exercise book:



a What was Ann's mistake?
b Work out the addition correctly.

5 Bernelle buys an exercise book for \$1.50, a pen for \$3 and a ruler for \$1.75. How much did she spend altogether?

6 The times of four boys in a team for a 4×100-metre relay race were:

Abdul	12.6	seconds
Wayne	13	seconds
Kenroy	12.4	seconds
Antoine	12.03	seconds

a Who ran the fastest leg?
b What was the team's total time for the race?

7 This example shows how you can add decimals by first changing them to fractions:

$$\begin{aligned} 1.87 + 2.79 &= 1 + \tfrac{87}{100} + 2 + \tfrac{79}{100} \\ &= 3 + \tfrac{166}{100} \\ &= 3 + 1\tfrac{66}{100} \\ &= 4\tfrac{66}{100} \\ &= 4.66 \end{aligned}$$

Use this method to calculate:

a $3.6 + 1.4$ **b** $6.2 + 1.24$
c $4.91 + 1.62$ **d** $7.28 + 3.79$

Subtracting decimals

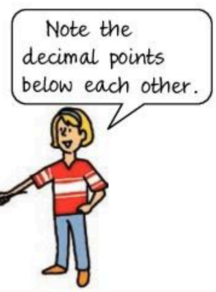
Subtraction of decimals is exactly the same as subtraction of whole numbers, except that the decimal point is included.

EXAMPLE 5

What is $3 - 1.4$?

Remember $3 = 3.0$. You write the difference vertically:

$$\begin{array}{r} {}^{2}\cancel{3}.{}^{1}0 \\ -\ 1.4 \\ \hline 1.6 \\ \hline \end{array}$$



Study the next example carefully.

EXAMPLE 6

What is $2.4 - 1.86$?

Remember $2.4 = 2.40$, so

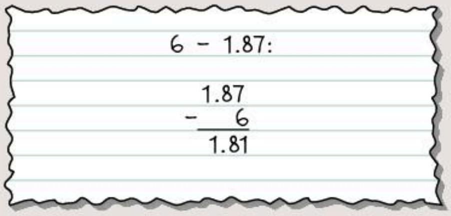
$$\begin{array}{r} {}^{1}\cancel{2}\,.\,{}^{13}\cancel{4}\,{}^{1}0 \\ -\,1\,.\,8\,6 \\ \hline 0\,.\,5\,4 \\ \hline \end{array}$$

Exercise 10E

1 Work out:

a $2.8 - 1.6$ **b** $2.8 - 1.9$
c $3 - 0.4$ **d** $2 - 1.7$
e $13.6 - 9.4$ **f** $2.9 - 0.03$
g $7 - 0.25$ **h** $4.63 - 3$
i $2.9 - 1.46$ **j** $100 - 6.8$

2 In his exercise book, Wayan wrote:



a What were Wayan's mistakes?
b If you were Wayan's teacher, how would you explain the subtraction to him?
c Work out $6 - 1.87$ correctly.

3 Rosa had a \$20 bill. She bought a packet of biscuits for \$2.95. How much change was she given?

4 The times of the first three girls in a 50-metre swimming trial were:

Lirtang	40.52 seconds
Arti	41.6 seconds
Merpati	38.71 seconds

a Who came first in the trial?
b How many seconds was the third-placed swimmer behind the winner?

5

In a 100-metre race shown on the television the commentator said this about the top three runners:

… Roberts finishes 2 hundreths of a second behind Davis in a time of 10.41 seconds...

… Johnson's winning time of 10.3 seconds is only 5 hundredths of a second outside the record…

a List the runners from first to third and their times.
b What is the record time?
c Copy and complete:
'Johnson beat Davis by … seconds'

6 Work out:

a $13.5 + {}^{-}7.82$
b $5.6 - {}^{-}3.28$
c ${}^{-}7.43 + 2.919$
d $1.8 + 3.72 + 4.899$
e $4.2 - 2.865$
f $3.8 + 1.23 - 4.199$
g $2 - 3.4$
h $4.01 - 2.6 - 3.4$
i ${}^{-}0.35 + 2.1 - 0.43$

10.3 Multiplying and dividing decimals

Multiplying decimals

You can see how to multiply decimals by using fractions.

For example:

$$0.8 \times 6 = \frac{8}{10} \times 6 = \frac{48}{10} = 4.8$$

$8 \times 6 = 48$

0.8 is 10 times smaller than 8 — so 4.8 is 10 times smaller than 48

$0.8 \times 6 = 4.8$

So if there is one decimal place in the product there must be one decimal place in the answer.

$$0.\underline{8} \times 6 = 4.\underline{8}$$

both have one decimal place

EXAMPLE 7

Work out:

a 0.03×5
b 0.002×4

a $0.\underline{03} \times 5$ — There are 2 decimal places (2 d.p.) in the product.

$3 \times 5 = 15$ — Do the calculation without decimal places.

$0.\underline{03} \times 5 = 0.\underline{15}$ — There must be 2 d.p. in the answer.

b $0.\underline{002} \times 4$ There are 3 d.p. in the product.

$2 \times 4 = 8$ Do the calculation without decimal places.

$0.\underline{002} \times 4 = 0.\underline{008}$ There must be 3 d.p. in the answer.

Watch out for sums like $0.\underline{4} \times 5$. There is 1 d.p. in the product. $4 \times 5 = 20$ so $0.\underline{4} \times 5 = 2.\underline{0}$. This has 1 d.p. in the answer but is the same as 2. We do not need to write zeros at the end of decimals as they do not affect place value. So $0.4 \times 5 = 2$.

Laws of arithmetic and inverse operations

In Chapter 1 you learnt how to apply the laws of arithmetic and inverse operations to integers. Look back at the definitions for the commutative law, the associative law and the distributive law. They also apply to decimals:

- The commutative law: $2 \times 0.6 = 0.6 \times 2 = 1.2$
- The associative law:
  $3.8 + 2.4 + 0.6 = 6.2 + 0.6 = 6.8$
  (adding 3.8 and 2.4 first)
  or $3.8 + 2.4 + 0.6 = 3.8 + 3 = 6.8$
  (adding 2.4 and 0.6 first)
  The second way is easier here, as $2.4 + 0.6$ is a little easier than $3.8 + 2.4$.
- The distributive law:
  $0.5 \times (3.4 + 2.6) = 0.5 \times 3.4 + 0.5 \times 2.6 = 1.7 + 1.3 = 3$
  (multiplying both numbers by 0.5 before adding)
- $0.5 \times (3.4 + 2.6) = 0.5 \times 6 = 3$
  (doing $3.4 + 2.6$ first is easier here)
- Inverse operations:
  if $4 \times 0.03 = 0.12$ then $0.12 \div 4 = 0.03$ and $0.12 \div 0.03 = 4$

Exercise 10F

1 Work out:

a 0.7×3 **b** 2×0.9 **c** 0.05×5
d 9×0.6 **e** 7×0.04 **f** 0.08×4
g 5×0.8 **h** 9×0.09 **i** 0.3×3
j 2×0.02

2 If $7 \times 0.02 = 0.14$, copy and complete:

a $0.14 \div 7 = \square$ **b** $0.14 \div 0.02 = \square$

3 If $0.05 \times 6 = 0.3$, copy and complete:

a $0.3 \div \square = 6$ **b** $0.3 \div \square = 0.05$

4 Using the associative law work out:

a $0.497 + 1.18 + 1.82$
b $0.7 \times 0.4 \times 5$

The associative law means you can change the order of doing these.

5 Work out $0.3 \times (2.1 + 5.9)$.

6 Work out the cost, in dollars, of:

a 8 carrots at \$0.07 per carrot
b 7 pens at \$0.09 per pen
c 3 sheets of paper at \$0.04 per sheet.

7 What is the cost of 0.3 kg of butter, if butter is sold for \$8 per kilogram?

8 Work out:

a 0.12×3 **b** 2×0.31 **c** 0.11×5

9 Work out:

a 0.004×3 **b** $^{-}2 \times 0.07$
c 5×0.003 **d** $0.04 \div 2$
e $4.2 - \square = 2.53$ **f** $5.1 - \square = {}^{-}7.23$

Multiplying harder decimals

Multiplication of decimals by whole numbers is similar to multiplying whole numbers by whole numbers.

EXAMPLE 8

Work out 12.43×5.

$$\begin{array}{r} 1\ 2\ 4\ 3 \\ \times \quad\quad\ 5 \\ \hline 6\ 2\ 1\ 5 \\ \hline {}_{1\ 2\ 1} \end{array}$$

Do the calculation without decimal places.

There are 2 d.p. in the product so there must be 2 d.p. in the answer.

$12.\underline{43} \times 5 = 62.\underline{15}$

It is always worth checking your answer by estimating to see if it is sensible.

In Example 8, the calculation 12.43×5 is roughly $12 \times 5 = 60$, so an answer of 62.15 is sensible. This can also help you get the decimal point in the correct place.

Exercise 10G

1 Given that $5436 \times 8 = 43\,488$, find:

a 543.6×8 **b** 54.36×8

2 Work out:

a 12.72×3 **b** 31.9×2
c 11.05×4 **d** 5×0.64
e 2×14.34 **f** 52.37×5
g 6×123.8 **h** 9×87.94
i 12.34×8 **j** 7×46.87

3 What is the cost of:

- **a** 8 pens at \$3.15 per pen
- **b** 6 mobile phones at \$47.25 per mobile phone
- **c** 7 erasers at \$1.32 per eraser
- **d** 3 computers at \$634.95 per computer?

4 What is the price of 3 kg of cheese, if cheese is sold for \$24.05 per kilogram?

5 A car averages 8.45 km on 1 litre of petrol. How far will it travel if the car's tank has 9 litres of petrol in it?

6 Merpati was doing her homework. For the sum 19.28 × 4, she wrote:

$$\begin{array}{r} 1\ 9\ 2\ 8 \\ \times\qquad 4 \\ \hline 8\ 6\ 1\ 2 \\ \hline {\scriptstyle 4\ 1\ 3\ } \end{array} \quad \text{So } 19.28 \times 4 = 861.2$$

Merpati's friend Budi said she had made 2 mistakes. One mistake he spotted straight away, the other was harder to find. What mistakes did she make? Which mistake do you think Budi spotted straight away?

7 Copy and complete:

- **a** $4.8 \times \square = 14.4$
- **b** $6.12 \times \square = 18.36$
- **c** $5.28 \times \square = 42.24$

8 The product of two numbers is 0.12. Write down five pairs of numbers for which this is true.

9 The product of two numbers is 4.2. What are the two numbers if their sum is 4.4?

Dividing decimals

Division of decimals by whole numbers is similar to dividing whole numbers by whole numbers.

EXAMPLE 9

Calculate 5.4 ÷ 4

$$\begin{array}{r} 1.35 \\ 4\overline{)5.40} \\ \underline{4} \\ 1\,4 \\ \underline{1\,2} \\ 20 \\ 20 \end{array}$$

You could also work this out using a 'short' division method.

$$\begin{array}{r} 1.\ 3\ 5 \\ 4\overline{)5.\,{}^{1}4\,{}^{2}0} \end{array}$$

Exercise 10H

1 Without using a calculator, work out:

- **a** 2.4 ÷ 6
- **b** 3.6 ÷ 4
- **c** 0.28 ÷ 7
- **d** 0.72 ÷ 9
- **e** 2.56 ÷ 8
- **f** 0.128 ÷ 8
- **g** 3.75 ÷ 5
- **h** 0.064 ÷ 4
- **i** 3.69 ÷ 9
- **j** 0.27 ÷ 5

2 Work out:

- **a** 7.45 ÷ 5
- **b** 0.71 ÷ 5
- **c** 14.41 ÷ 8
- **d** 0.726 ÷ 4
- **e** 36.4 ÷ 8
- **f** 1.78 ÷ 4

3 A 5 kg fish sells for \$86.40.
What is the price of the fish per kilogram?

4 A man walks 17.4 km in 4 hours.
What is his speed?

(**Hint**: Speed = distance ÷ time)

5 The cost of 8 handkerchiefs is \$29.52.
What is the cost of 1 handkerchief?

6 A 3 kg tub of soap powder sells for \$15.57.
What is the cost of 1 kg of soap powder?

7 Six boys shared \$53.40 equally among themselves.
How much money did each boy receive?

8 The result when dividing one number by another is 0.8. Write down five pairs of numbers for which this is true.

Consolidation

Example 1

Change these fractions to decimals:

$\frac{4}{5} = \frac{8}{10} = 0.8$

$7\frac{1}{4} = 7\frac{25}{100} = 7.25$

Example 2

Write these numbers in order of size, smallest first:

$\frac{17}{25}, 0.7, \frac{3}{5}, 0.65$

Change them all to decimals

$\frac{17}{25} = 0.68$ and $\frac{3}{5} = 0.6$

In order of size they are $\frac{3}{5}, 0.65, \frac{17}{25}, 0.7$

Example 3

Work out $5 - 3.34$

$$\begin{array}{rcrr} \overset{4}{\cancel{5}} & . & \overset{9}{{}^{1}\cancel{0}} & {}^{1}0 \\ -3 & . & 3 & 4 \\ \hline 1 & . & 6 & 6 \\ \hline \end{array}$$

Example 4

Work out 13.25×6

Do the sum 1325×6

$$\begin{array}{rrrr} 1 & 3 & 2 & 5 \\ \times & & & 6 \\ \hline 7 & 9 & 5 & 0 \\ \hline {}_{1} & {}_{1} & {}_{3} & \end{array}$$

$1325 \times 6 = 7950$

There are two decimal places in the product so there should be two decimal places in the answer.

So $13.\underline{25} \times 6 = 79.\underline{50} = 7.95$

Exercise 10

1 Write these numbers as decimals. (You may use a calculator for parts **g**, **h** and **i**.)

a $\frac{3}{5}$ **b** $\frac{1}{8}$ **c** $\frac{7}{50}$

d $2\frac{1}{5}$ **e** $3\frac{3}{4}$ **f** $\frac{181}{250}$

g $\frac{3}{16}$ **h** $\frac{13}{32}$ **i** $\frac{7}{12}$

2 Find the largest of each pair of numbers. (You may use a calculator for part **c**.)

a $\frac{1}{4}$ or $\frac{3}{10}$ **b** $\frac{11}{20}$ or $\frac{3}{5}$ **c** $\frac{9}{16}$ or $\frac{5}{9}$

3 Write these numbers in order of size, smallest first:

a $\frac{2}{5}, \frac{3}{8}, \frac{3}{10}, \frac{7}{20}$ **b** $\frac{2}{25}, 0.07, \frac{3}{40}, 0.6$

c $\frac{3}{20}, 0.2, \frac{7}{50}, 0.16$

4 Write these decimals as fractions.

a 0.5 **b** 0.87 **c** 0.015 **d** 0.07

e 0.375 **f** 0.004 **g** 1.42 **h** 3.8

5 Work out:

a $0.64 + 2.32$

b $3.74 + 8.9$

c $10 - 5.72$

d $14.6 - 9.35$

e $31.34 + 7.124 + 8.4$

6 Work out:

a 0.6×2 **b** 3×0.4

c 0.08×5 **d** 14.6×9

e 12.34×7 **f** 7.68×4

g 5×1.2 **h** $3.72 \div 4$

i $127.5 \div 3$ **j** $3.17 \div 5$

7 On Wednesday, Anton bought a tie for \$15 and a shirt for \$47.99. He then went to the shop and bought a packet of sweets for \$0.15 and a lollipop for \$0.75.

a How much did he spend on Wednesday in total?

b Anton left his house on Wednesday morning with a \$100 bill in his pocket. On Wednesday night, how much did he have left?

8 What is the cost of:

a 6 books costing \$8.95 per book

b 9 stamps costing \$0.74 per stamp

c 8 magazines costing \$4.35 per magazine

d 5 song tracks downloaded from the internet if it costs \$0.98 per song track?

9 A father shares \$127.50 equally between his 5 children. How much do they each receive?

10 Adanya goes out with \$20. She spends \$3.95 at the newsagents and \$12.37 at the supermarket. How much change does she have?

11 The total bill for 6 people in a restaurant is \$266.22. How much does each person pay if they share the bill equally?

12 Diane goes to Mackeprang's hairdressers. She spends \$64.89 on a haircut and \$23.45 on her taxi fare. How much change does she have from \$100?

13 Copy the numbers. Draw lines to show the equivalent numbers. One has been done for you.

$\frac{1}{4}$	0.625
$\frac{2}{9}$	0.016
$\frac{5}{8}$	0.16
$\frac{4}{25}$	0.25
$\frac{16}{1000}$	$0.\dot{2}$

Summary

You should know ...

1 A fraction can be written as a decimal.

$\frac{58}{100} =$

U	$\frac{1}{10}$	$\frac{1}{100}$
0	5	8

$= 0.58$

2 A decimal can be written as a fraction.

For example: $0.74 = \frac{7}{10} + \frac{4}{100} = \frac{70}{100} + \frac{4}{100} = \frac{74}{100} = \frac{37}{50}$

3 To add or subtract decimals, write the numbers with the place values and decimal points lined up.

For example: $26 - 0.4$

$$\begin{array}{r} 26.0 \\ -\ \ 0.4 \\ \hline 25.6 \\ \hline \end{array}$$

4 How to multiply and divide decimals.

For example: 0.62×4

$0.\underline{62} \times 4$ has 2 decimal places.

Since $62 \times 4 = 248$

$0.62 \times 4 \quad = 2.\underline{48}$ also has 2 decimal places.

For example: $7.36 \div 4$

$$4\overline{)7.^{3}3^{1}6}\quad = 1.84$$

Check out

1 Write these fractions as decimals:

a $\frac{3}{10}$ **b** $\frac{6}{100}$

c $2\frac{5}{10}$ **d** $13\frac{7}{100}$

e $9\frac{17}{100}$ **f** $\frac{63}{1000}$

2 Write these decimals as fractions:

a 0.75 **b** 0.8

c 0.3 **d** 2.25

e 13.68 **f** 0.045

3 Work out:

a $7 + 0.3 + 0.06$

b $6.3 + 7.78$

c $10 - 0.3$

4 Work out:

a 6.2×7

b 2.14×8

c $7.6 \div 4$

d $14.43 \div 3$

11 Time and rates of change

Objectives

- Know the relationships between units of time; understand and use the 12-hour and 24-hour clock systems; interpret timetables; calculate time intervals.
- Draw and interpret graphs in real-life contexts involving more than one stage, e.g. travel graphs.

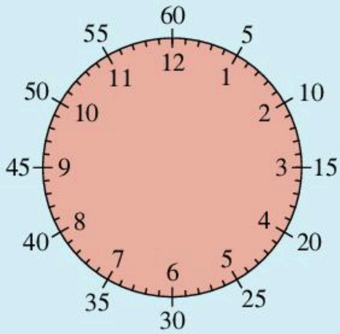
What's the point?

Twenty-four-hour clock notation is sometimes referred to as **military time**. Military operations are sometimes coordinated between countries and may happen at any time of the day or night. If an operation is to happen at 21:00 hours rather than 9 o'clock there is no confusion as to whether it is in the morning or the evening. The twenty-four-hour clock is also used in the practice of medicine, so that, when documenting patient care, there is no confusion about when events happen.

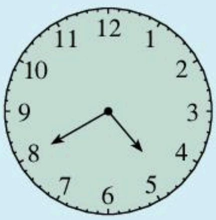
Before you start

You should know …

1 How to tell the time.
To read clock faces, you need to remember that the face is divided into 60 minutes as well as 12 hours, as shown below.



For example:

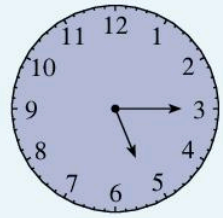


The longer minute hand points to the 8, that is 40 minutes past the hour. The shorter hour hand points between the 4 and the 5, so the time is 4.40.

Check in

1 What time is shown on these four clocks?

a i



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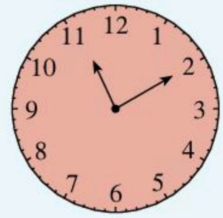
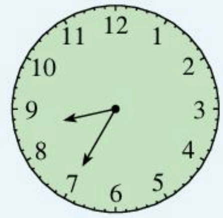


iii



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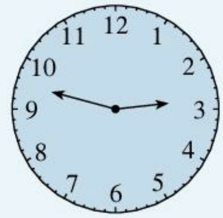


b Draw clock faces to show these times:

i 3.30 **ii** 8.45
iii 11.05 **iv** 3.54

11.1 Time

In the past, people measured time with sand timers or candles.

Today, time is measured most accurately using atomic clocks.

- The SI unit of time is the **second, s**.

A second is now defined in terms of the vibrations of a caesium atom.

Larger units of time and their relationships are shown in the table.

Unit of time	Relationship
Minute (min)	60 s = 1 min
Hour (h)	60 min = 1 h
Day	24 h = 1 day
Week	7 days = 1 week
Year	52 weeks = 1 year

Exercise 11A

1 Work with a friend. Shut your eyes and tell your friend when you think a minute has passed. Get your friend to time you.

How good was your estimate?

2 Estimate the time it takes to:
- a walk a kilometre
- b count to twenty
- c wash up the lunch plates
- d drink a glass of water.

3
- a How many seconds make an hour?
- b How many minutes make a day?
- c How many hours make a week?

4
- a How many days are there in 5 weeks?
- b How many hours are there in 2 weeks 5 days?
- c How many days are there in 1 year 3 weeks?

Most time is now recorded on digital clocks or watches.

The time on this watch is 9.37 or 37 minutes past 9 o'clock.

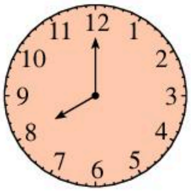
ACTIVITY

Recovery times

- Using a watch with a second hand or a stop clock, find how many times your pulse beats each minute.
- Run about 100 m as fast as you can.
- Check your pulse rate now.
- Check your pulse rate every minute until it gets back to what it was before you did your run.
- How long was this?

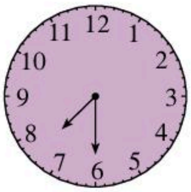
24-hour clock

On the digital watch and the clocks at the beginning of this chapter it is not possible to say whether it is morning or afternoon.



8 o'clock in the morning is represented by 8.00 am on the 12-hour clock, 08:00 on the 24-hour clock.

This time is read as 'zero eight hundred hours'.



7.30 in the evening is represented by 7.30 pm on the 12-hour clock, 19:30 on the 24-hour clock.

This time is read as 'nineteen thirty'.

- The 24-hour clock uses all 24 hours in a day to give the time.
  The first two figures give the number of hours after midnight, and the last two figures give the number of minutes past the hour.

EXAMPLE 1

a Write 8.15 pm in 24-hour clock time.
b Write 02:54 in am/pm time.

---

a 8 pm is 8 + 12 = 20 hours after midnight
so 8.15 pm is 20:15
b 02:00 is 2 am
So 02:54 is 2.54 am

Exercise 11B

1 Write in 24-hour clock time:

a	4 am	**b**	8.30 am	**c**	11 am
d	1 pm	**e**	3.30 pm	**f**	8.45 pm
g	9.25 pm	**h**	7.53 pm	**i**	1.52 am

2 Write in am/pm time:

a	03:00	**b**	05:00	**c**	12:00
d	15:00	**e**	19:00	**f**	23:00
g	01:05	**h**	04:25	**i**	21:55

It is easier to find time intervals with the 24-hour clock than the 12-hour clock.

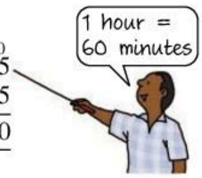
EXAMPLE 2

a A football match kicked off at 15:45 and ended at 17:25. How long did the game last?
b How long is it from 21:30 on Tuesday to 03:15 on Wednesday?

---

a

$$\text{Time taken} = \begin{array}{r} {}^{6}\ \ {}^{60} \\ 1\cancel{7}:25 \\ -\ 15:45 \\ \hline 1:40 \\ \hline \end{array}$$



The game lasted 1 hr 40 min.

b

$$\begin{array}{r} {}^{3}\ \ {}^{60} \\ 2\cancel{4}:00 \\ -\ 21:30 \\ \hline 2:30 \\ \hline \end{array} \qquad \begin{array}{r} 02:30 \\ +\ 03:15 \\ \hline 05:45 \\ \hline \end{array}$$

time to midnight

time from midnight

Time span = 5 hr 45 min

Exercise 11C

1 How long is it from:

a	06:00 to 07:15	**b**	22:00 to 23:50
c	09:20 to 13:40	**d**	03:15 to 16:28
e	12:25 to 15:10	**f**	19:32 to 22:17
g	18:40 to 07:15	**h**	21:35 to 02:05?

2 Air New Zealand flies from Wellington, New Zealand at 16:30 and arrives in Jakarta, Indonesia at 02:30 the next day. How long is the flight?

3 Write the times you begin each of today's classes in 24-hour clock time.

4 Look at the LIAT flight timetable below:

Dominica	Arr.	06:30	16:50
	Dep.	06:40	17:00
Martinique	Arr.	07:00	17:20
	Dep.	07:10	17:30
St Lucia	Arr.	07:35	17:55
	Dep.	07:45	18:05
Barbados	Arr.	08:25	18:45

a What time does the first flight leave Dominica?
b When does the second flight arrive in St Lucia?
c What is the flight time from Martinique to St Lucia, ignoring stopover times?
d What is the flight time from Dominica to Barbados including stopover times?
e Rewrite the timetable using the 12-hour clock.

5 The 'Late Show' begins at 22:45 and ends at 01:05. How long is the show?

6 How long is it from
a 9.30 am to 2.15 pm
b 11.40 am to 7.05 pm
c 3.50 am to 9.10 pm
d 8.50 pm to 3.20 am the next day
e 9.35 pm to 2.55 am the next day?

7 Passengers should arrive $2\frac{1}{2}$ hours before travelling on international airline flights. What time should I arrive if my flight leaves at
a 15:30 **b** 06:20?

8 A cyclist started a journey at 08:15. She rode for 1 h 25 min, rested for 25 min and then rode for another 1 h 55 min. At what time did she reach her destination?

9 Look at the Egyptian train timetable below.

	Train	A	B	C	D	E	F
Cairo	depart	06:15	10:55	13:45	14:40	19:45	23:35
Port Said	arrive	10:15	15:00	18:00	19:05	23:50	

a Kanika catches train C from Cairo to Port Said. How long is her journey time?
b Which is the slowest train?
c Which is the fastest train?
d Two trains take the same amount of time for the journey. Which two trains are these?
e Amenhotep needs to arrive in Port Said for a meeting at 4 pm. What is the latest train he can catch?
f Train F is the overnight train. It takes 4 hours and 20 minutes to travel from Cairo to Port Said. What is the missing time in the timetable?

TECHNOLOGY

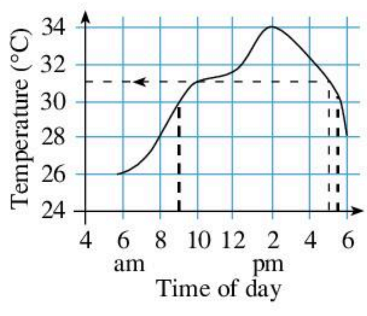
- Compare the 12-hour clock with the 24-hour clock by visiting this website and looking at the Teachers Toolkit in the 'Tools' section: www.crickweb.co.uk/ks2numeracy.html
- Did you know that when it is 7.00 am in Los Angeles it is 3.00 pm in London? Visit www.timeanddate.com/worldclock to see how time changes around the world.

11.2 Real-life graphs

Graphs are often used to show numerical information. You need to be able to read information from graphs and draw them.

EXAMPLE 3

The temperature on 15th September is given by the graph:

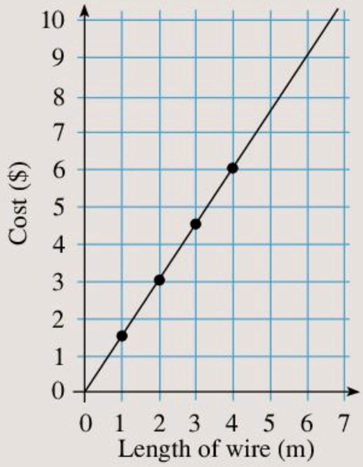


a What was the temperature at 8 am?
b What was the temperature at 5 pm?
c When was the temperature 30°C?

a At 8 am the temperature was 28°C.
b At 5 pm the temperature was 31°C, (see dotted lines on graph).
c The temperature was 30°C at 9 am and again at 5.30 pm (see thick dotted lines).

Exercise 11D

1 The graph shows the cost of lengths of electric wire.



a How much does 4 m of wire cost?

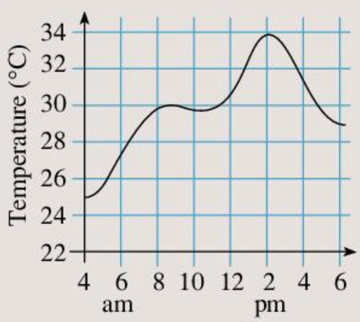
b How much wire can be bought for $7?

c How much does 4.5 m of wire cost?

d Mr Lyn spent $9 on wire. How much did he buy?

e Copy and complete the statement: Wire costs $. . . per metre.

2 The graph shows the temperature on 3rd June.



a What was the temperature at 6 am?

b What was the maximum temperature?

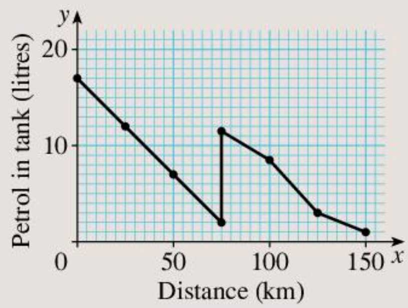
c When was the first time the temperature reached 30°C?

d What was the temperature at 6 pm?

e What was the lowest temperature recorded?

f At what times was the temperature 29°C?

3 The graph shows the amount of petrol in a car every 25 km.

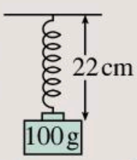


a How much petrol was in the tank after:
i 25 km **ii** 45 km **iii** 108 km?

b How far had the car travelled when the amount of petrol in the tank was:
i 17 litres
ii 14 litres
iii 1 litre?

c When was the car refilled with petrol? How much petrol was put in the tank?

4 A spring is stretched by hanging masses from its end. The lengths of the spring for different masses are given in the table.



Mass (g)	0	20	36	52	84	100
Length (cm)	7	10	12.4	14.8	19.6	22

a Draw a graph to show this information. Use a scale of 1 cm to represent 10 g on the horizontal axis and 1 cm to represent 2 cm on the vertical axis.

b From your graph find the length of the spring when the mass is:
i 26 g **ii** 63 g

c From your graph find the mass which would cause the spring to have a length of:
i 16 cm **ii** 21.6 cm

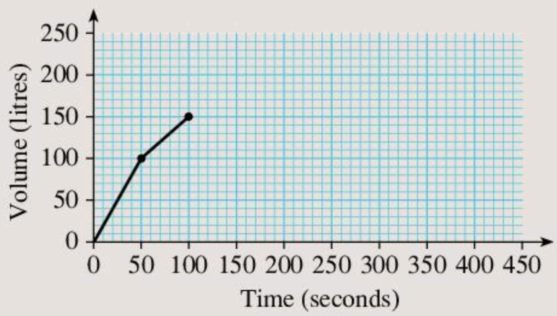
5 A baby was 4 kg when he was born. His mass over the next 7 weeks is shown in the table.

Week	0	1	2	3	4	5	6	7
Mass (kg)	4	4.4	4.8	5.2	5.6	6	6.4	6.8

Draw a graph and use it to state approximately how many days old the baby was when his mass was 5 kg.

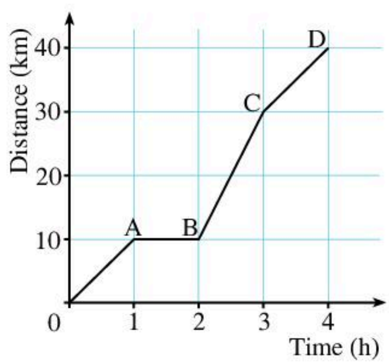
6 Copy and complete the graph below to show what happened to the volume of water in the bath.

- I turned both taps on for 50 seconds until there were 100 litres of water in the bath.
- I turned the cold tap off but left the hot tap running for another 50 seconds until there were 150 litres of water in the bath.
- I climbed into the bath and the water level rose by 70 litres.
- I stayed in the bath for 5 minutes.
- I climbed out of the bath.
- I let the water run out of the bath, which took 40 seconds.



11.3 Travel graphs

A distance–time graph shows how far you travel in a given time.



This distance–time graph shows the distance travelled by a cyclist over a four-hour period.

After the first hour she had travelled 10 km.
For the next hour she rested (AB).
During the third hour she rode a further 20 km (BC) and in the fourth hour another 10 km (CD).

If you know how far someone has travelled in a given time, you can calculate their speed from the formula:

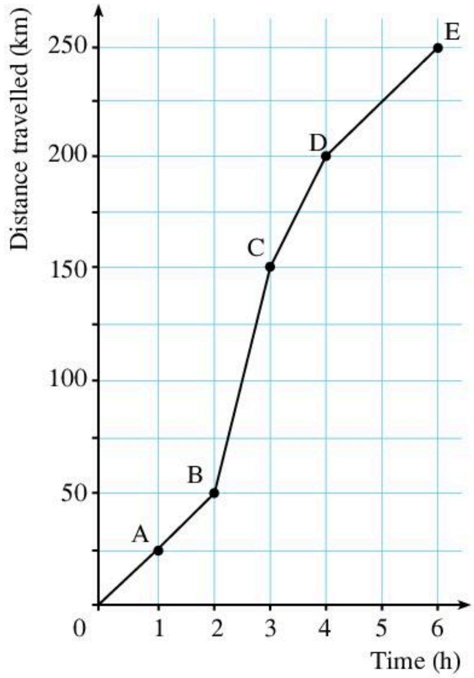
$$\text{speed} = \frac{\text{distance travelled}}{\text{time taken}}$$

This formula only tells you the **average** speed for the journey. It does not tell you the actual speed at any instant.

During the third hour the cyclist's speed was 20 km/h and in the fourth hour 10 km/h.

EXAMPLE 4

Find the average speed during the fifth and sixth hours of the car journey shown below.



The fifth and sixth hours are shown on the graph by the line DE. The car travels 50 km in these 2 hours. The average speed is $\frac{50}{2} = 25$ km/h. (This is the gradient of line DE. You will learn more about gradient later.)

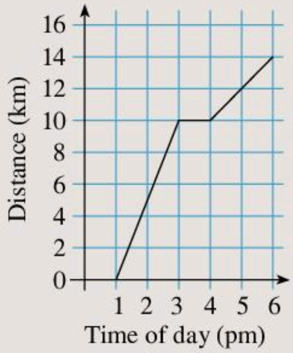
Exercise 11E

1 Using the graph in Example 4, find the average speed during:

a the first hour
b the second hour
c the third hour
d the fourth hour.

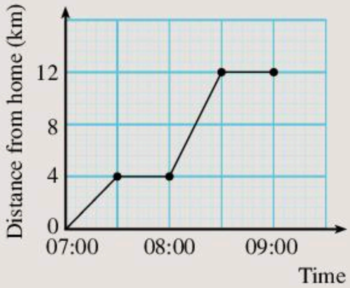
During which hour of the journey was the average speed greatest? How can you tell this from the graph?

2 The graph shows the distance a man walks from his home one afternoon.



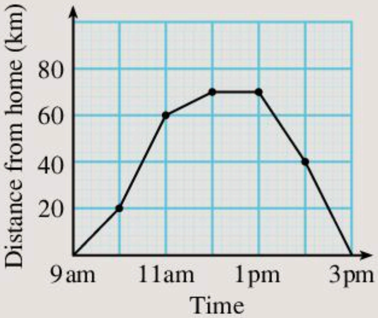
- **a** What time did he leave home?
- **b** How far had he travelled at 2 pm?
- **c** What did he do between 3 pm and 4 pm?
- **d** When was he 13 km from home?
- **e** Between which times was he walking fastest?

3 The graph shows Martin's journey to school one day.



- **a** How long did it take him to reach school?
- **b** What was his average speed during the first half-hour?
- **c** What happened between 07:30 and 08:00?
- **d** What was his average speed during the last half-hour of his journey?
- **e** How far does Martin live from school?

4 The graph shows Benji's journey by car from home to work and back one day.



- **a** What time did he reach his work?
- **b** How long did he stay at his work meeting?
- **c** What was his average speed between 11 am and 12 noon?
- **d** When was the car travelling the fastest?
- **e** How far did he travel between 10 am and 11 am?
- **f** How far was it from home to work?
- **g** How long did the whole journey take?

5 A train leaves a station at 08:45. It travels for 1 hour at 100 km/h and then stops for 15 minutes. It then travels a further 200 km which takes 2 hours. Then it stops for 15 minutes. The return journey is non-stop and takes 3 hours.

- **a** Draw the travel graph to show this journey.
- **b** Calculate the average speed for the return journey

6 Abeke goes on a bicycle ride. She leaves home at 10:00. She travels 5 km in half an hour, then the next 10 km takes half an hour. She stops for a rest for half an hour. Then she cycles 10 more kilometres in 1 hour. She stops for an hour for lunch, then cycles non-stop, returning home at 16:00.

- **a** Draw the travel graph to show this journey.
- **b** Calculate Abeke's average speed for the first half an hour of her journey.

Consolidation

Example 1

What is:

a 7.40 pm in 24-hour clock time

b 13:21 in 12-hour clock time?

a 7 pm is 7 + 12 = 19 hours after midnight so 7.40 pm is 19:40.

b 13:00 is 13 hours after midnight or 13 − 12 = 1 hour after midday so 13:21 is 1.21 pm.

Example 2

How long is it from:

a 9.20 am to 2.45 pm

b 08:50 to 13:05?

a 2:45 pm is 14:45 in 24-hour clock time. (Time intervals are easier in 24-hour clock time.)

$$\begin{array}{r} 14:45 \\ -\ 9:20 \\ \hline 5:25 \\ \hline \end{array}$$

Time span = 5 hours 25 minutes

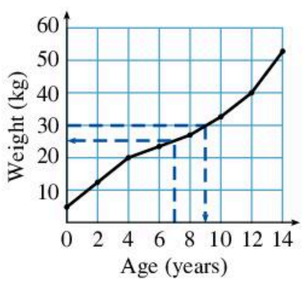
b

$$\begin{array}{r} 1\overset{2}{\cancel{3}}:\overset{60}{05} \\ -\ 8:50 \\ \hline 4:15 \\ \hline \end{array}$$

Time span = 4 hours 15 minutes

Example 3

The graph shows Ken's weight in kilograms at different birthdays.



a What was Ken's weight when he was 7?

b When did Ken weigh 30 kg?

a At 7 Ken weighed 25 kg.

b He weighed 30 kg when he was 9 years old.

Exercise 11

1 **a** Write in 24-hour clock time:

i 3.15 am **ii** 6.25 pm **iii** 10.40 pm

b Write in 12-hour clock time:

i 20:35 **ii** 07:55 **iii** 23:10

2 Egypt Air has three flights daily leaving Cairo, Egypt for Abuja, Nigera. The timetable is shown below.

Flight No.	From	Dep.	To	Arr.
520	CAI	00:40	ABV	05:50
522	CAI	08:55	ABV	14:05
424	CAI	17:30	ABV	22:40

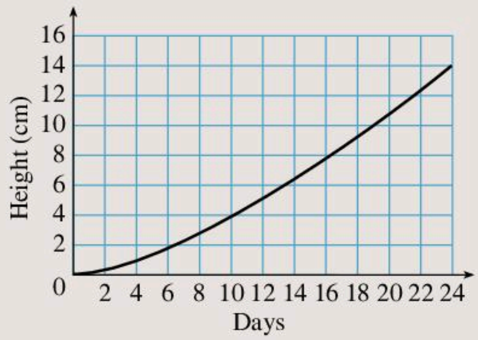
a Rewrite the timetable using the 12-hour clock.

b What is the flight time for each flight?

3 Passengers should arrive $2\frac{1}{2}$ hours before travelling on international airline flights. What time should I arrive if my flight leaves at:

a 14:25 **b** 08:05?

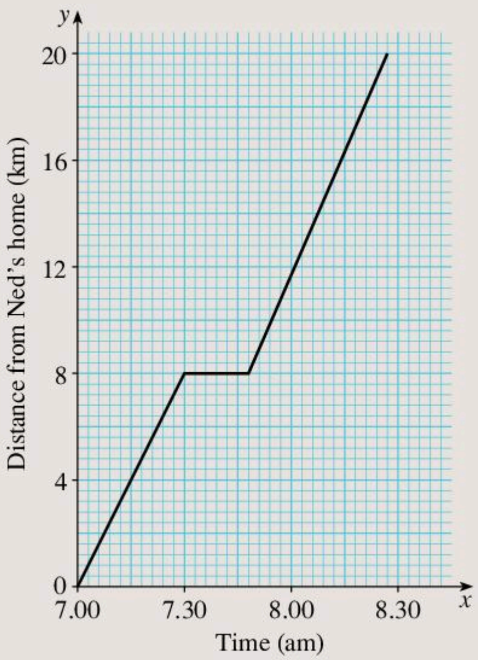
4 The graph shows the height of a seedling in centimetres over a period of 24 days.



a What is the height of the seedling 12 days after planting?

b When was the seedling 12 cm tall?

5 The graph shows Ned's journey from home to school. On the way he stops at Michael's house.



a What time did Ned leave home?
b How far is Michael's house from Ned's home?
c How long does Ned wait for Michael?
d At what time do they arrive at school?
e What was Ned's average speed during the first half-hour of his journey?

6 The cost for taxi journeys of different lengths is given in the table.

Hire charge ($)	5	11	6.50	14
Distance travelled (km)	2	6	3	8

a Draw a graph to show the information.
b What would be the cost of a journey of 5 km?

7 Mike leaves home at 11 am to cycle to his uncle's house 30 km away. In the first hour it is mostly uphill so he cycles only 10 km. He cycles the last 20 km in 1 hour. Mike stops at his uncle's house for an hour and a half. He needs to get home by 3 pm so puts his bike in the back of his uncle's car and his uncle drives him home. The journey home takes 25 minutes.

a Draw a distance–time graph for this journey.
b Mike needed to be home by 3 pm. Was he early or late? By how much?
c What was Mike's average speed during
i the first hour of his journey
ii the second hour of his journey?
d What was the average speed for the car journey home?

Summary

You should know …

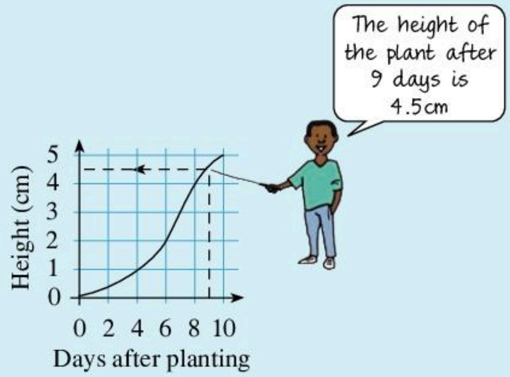
1 How to work with 24-hour and 12-hour time.

For example:
To write 6.53 am in 24-hour clock time, put 0 at the front and leave out the am.
So the time is 06:53
To write 19:36 in 12-hour clock time, subtract 12 from the hours – and remember to add the pm!
So the time is 7.36 pm

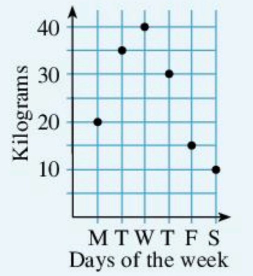
Check out

1 **a** Change these times to 24-hour clock time:
i 4.23 am
ii 5.30 pm
iii 7.22 pm
b Change these times to 12-hour clock time:
i 21:00
ii 09:31
iii 16:15

2 How to read information from graphs.
For example:



2



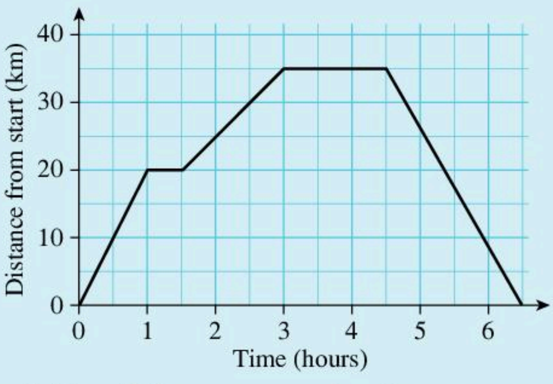
The graph shows the amount of peas that Mr Kelly picked in one week.

a How many kilograms of peas were picked on Tuesday?

b On which day were most peas picked?

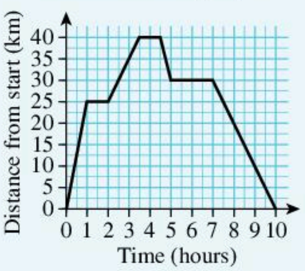
c How many kilograms of peas were picked in the entire week?

3 How to draw and interpret travel graphs.
For example:
Describe what is happening in this distance–time graph.



During the first hour 20 km were covered, so the average speed was 20 km/h.
Then there was a rest for half an hour. The next part of the journey was 15 km long and took 1.5 hours. The speed was 10 km/h. Then there was a rest for one and a half hours. The return journey was 35 km long and took 2 hours, at a speed of 17.5 km/h.

3 Describe what is happening in this distance–time graph:



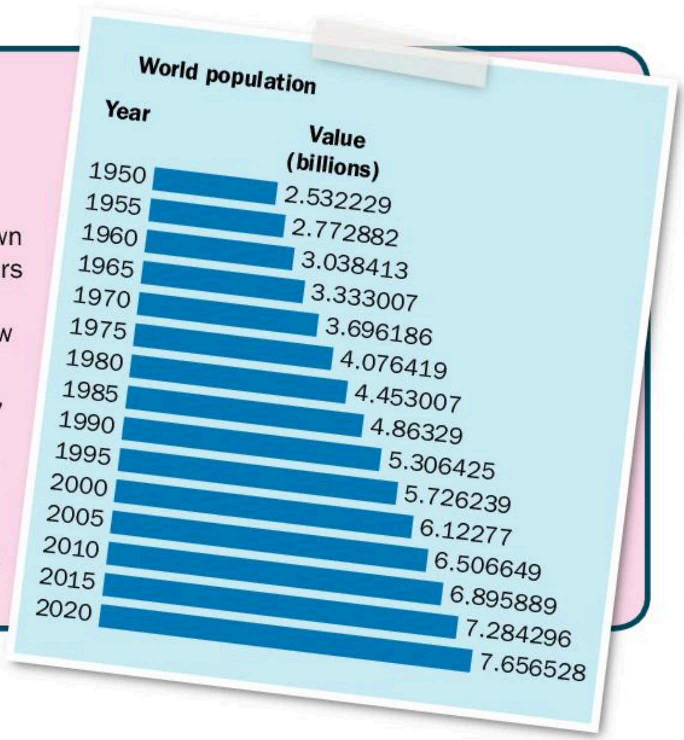
12 Presenting data and interpreting results

Objectives

- Draw and interpret:
  - pictograms
  - bar–line graphs and bar charts
  - simple pie charts
  - frequency diagrams for grouped discrete data.
- Draw conclusions based on the shape of graphs and simple statistics.
- Compare two simple distributions using the range and the mode, median or mean.

What's the point?

Graphs help organize information. A graph is a visual way to compare data. It is often easier to see patterns in data when a graph is drawn rather than looking at a list of numbers or words. Graphs are used in many areas, for example in politics (to show voting patterns across the country), in business (showing profit and loss), in medicine (tracking a patient's temperature). Journalists use graphs a lot in the news, on television and in newspapers. They use graphs to demonstrate something in a way that is interesting or nice to look at.



Before you start

You should know …

1 How to collect data using a frequency diagram and tallies.
For example:
The data about how many people live in 18 houses is:
4, 4, 5, 4, 2, 3, 2, 4, 2, 4, 3, 3, 4, 3, 5, 3, 4, 4
This can be shown more clearly in a frequency table:

Number in house	Tally	Frequency
2	|||	3
3	~~||||~~	5
4	~~||||~~ |||	8
5	||	2

2 How to work out averages and the range.
For example:
The mean, median, mode and range of 5, 3, 11, 8 and 3 can be worked out like this:

$$\text{Mean} = \frac{5 + 3 + 11 + 8 + 3}{5} = \frac{30}{5} = 6$$

The values, in order, are 3, 3, 5, 8, 11.
Median = middle number = 5
Mode = most common value = 3
Range = largest number − smallest number = 11 − 3 = 8

3 How to find fractions of an amount.
For example:

$$\frac{2}{3} \text{ of } 15 = 2 \times \frac{1}{3} \text{ of } 15 = 2 \times 5 = 10$$

Check in

1 This is the number of people who live in 25 houses:

4, 2, 2, 5, 4, 3, 5, 2, 4, 2, 4, 4, 2, 3, 3, 4, 3, 5, 3, 4, 4, 5, 5, 4, 4

Copy and complete the frequency table for the data.

Number in house	Tally	Frequency
2		
3		
4		
5		

2 Find the mean, median, mode and range for the following data:

a 9, 3, 10, 4, 5, 9, 2
b 8, 4, 5, 6, 8, 5, 4, 8

3 Find

a $\frac{1}{4}$ of 12

b $\frac{2}{3}$ of 24

c $\frac{3}{5}$ of 60

d $\frac{4}{7}$ of 56

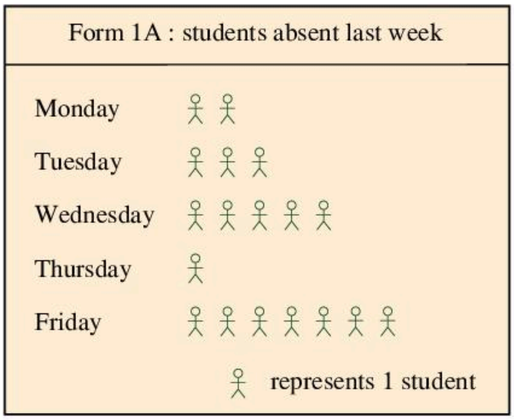
When people collect information, they try to show it in away that everyone will understand easily.

12.1 Pictograms

The table shows the students absent in David's class at Portsmouth Secondary School last week.

Day	Mon	Tue	Wed	Thur	Fri
Number absent	2	3	5	1	7

On what day were most students absent?
The chart that follows shows the information from the table.



This chart is called a **pictogram**.

- A pictogram uses pictures or drawings to represent discrete data.

Exercise 12A

1 The table shows the heights of students in Form 1C of Portsmouth Secondary School.

Height (cm)	Frequency
136	6
138	5
139	4
141	4
142	9
143	2

Show the information on a pictogram.

2 The table shows the favourite sports of the students in Form 1B of Portsmouth Secondary School.

Netball	Football	Cricket	Volleyball
13	10	19	9

Show the information on a pictogram.

EXAMPLE 1

Using the pictogram above, make a table to show how many cakes were sold at a bakery each day last week.

Day	Mon	Tues	Wed	Thur	Fri	Sat
Cakes sold	35	7	28	42	35	56

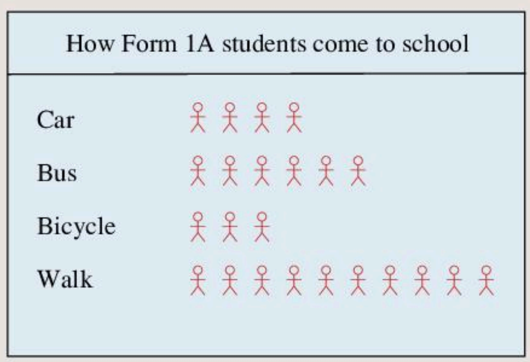
A drawing of 1 cake represents 7 cakes.
A scale of 1 to 7 has been used.

- A **scale** tells you the number of items each symbol represents.

Every pictogram must have a scale.

Exercise 12B

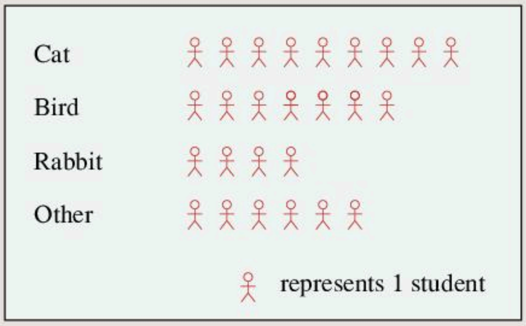
1 The pictogram shows how the students in Form 1A of Portsmouth Secondary School travel to school every day. The pictogram is not complete because it does not say what each 웃 represents.



a 20 students in Form 1A travel by car every day. What scale has been used in the pictogram?

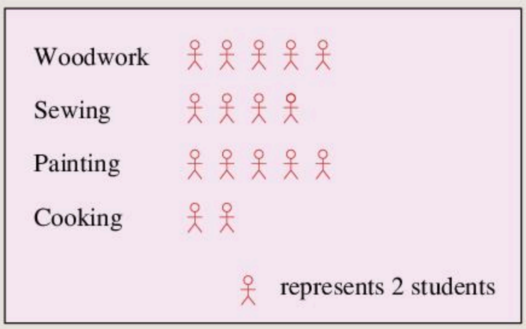
  b What should be written at the bottom?
  c Write down the number of students that walk to school, cycle to school, and go to school by bus every day.

2 The pictogram shows the pets kept by the students of Form 2 at the Little Valley School.



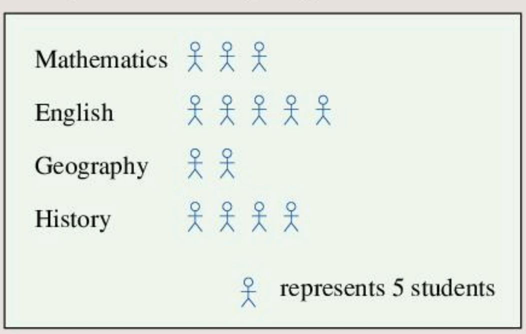
  a How many students keep a bird as a pet?
  b How many students were in the survey?
  c What fraction of the students keep a rabbit as a pet?

3 The activities chosen by Form 3 students are shown in the pictogram.



  a How many students chose painting?
  b Which was the least popular activity?
  c How many students were surveyed?

4 The favourite subjects of some students are represented in the pictogram:



  a How many students like History the most?
  b Which is the most popular subject?
  c Which is the least popular?
  d How many students were surveyed?

5 Here are the numbers of students in the first two years at a school.

Form	1A	1B	1C	2A	2B	2C
Number of students	40	35	45	45	40	40

  a How many students could a represent?
  b Make a pictogram for the table.

6 In a basketball tournament for a special trophy, the scores were:

Team	Score
A	44 goals
B	32 goals
C	56 goals
D	48 goals
E	52 goals

  a What object could you use to represent goals in the pictogram?
  b What scale could you use?
  c Draw a pictogram to show the scores.

7 Tim is a fisherman. His weekly catch, over six weeks, is shown.

Week	1st	2nd	3rd	4th	5th	6th
Kilograms (kg) of fish caught	150	140	130	110	150	130

  a What symbol would you use to represent his catch?
  b Draw a pictogram to show the information.
  c Can you think of another way you could show the information?

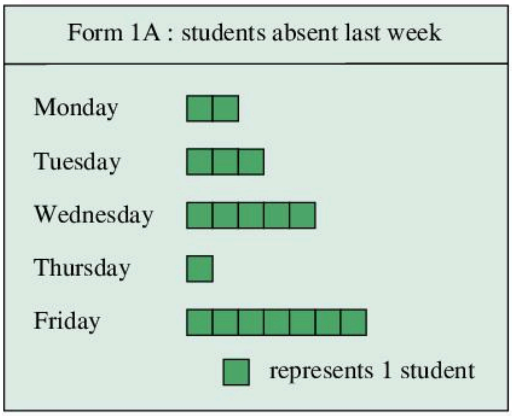
12.2 Bar charts

Sometimes it takes a lot of time to draw the pictures for a pictogram. A **bar chart** is quicker to draw.
In a bar chart, a small square is used to represent the object, instead of a picture. Bar charts are sometimes called **bar graphs** or **block graphs**.

Using the data about students absent from David's class again:

Day	Mon	Tue	Wed	Thurs	Fri
Number absent	2	3	5	1	7

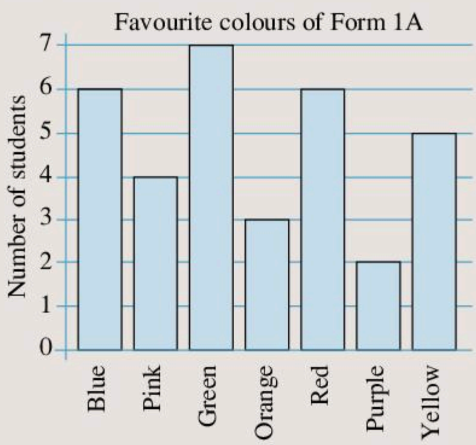
The bar chart below uses a square to represent each student in the table.



- A bar chart uses bars of different lengths or heights to represent data.
- Bar charts can be drawn horizontally or vertically. We usually use a scaled axis instead of a key. Bars in a bar chart should not touch each other.

Exercise 12C

1 The bar chart shows the favourite colours of the students in Form 1A.



a Which colour is the most popular? How can you tell?

b Which colour is the least popular? How can you tell?

c The same number of students prefer two different colours. What are the colours?

d How many students were questioned?

e List the colours in order of popularity, starting with the most popular.

f Copy and complete the table below for the information in the bar chart.

Favourite colour	Blue	Pink	Green				
Number of students							

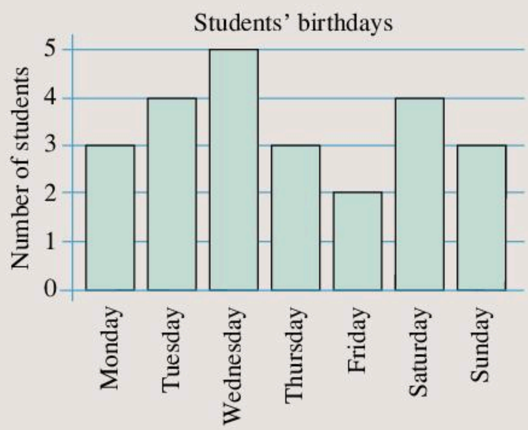
2 The table shows the subjects that students in Form 1C like the least.

Subject	Number of students
English	6
Spanish	7
Maths	10
History	5
Geography	6
P.E.	5
Science	3

a On squared paper, draw a bar chart, like the one in Question **1**, to show this information.

b Which subject is the most unpopular?

3 The bar chart shows the day of the week on which some students have their birthdays this year.



Use the bar chart to answer:

a How many students have birthdays on a Saturday this year?

b How many students have birthdays on a Sunday?

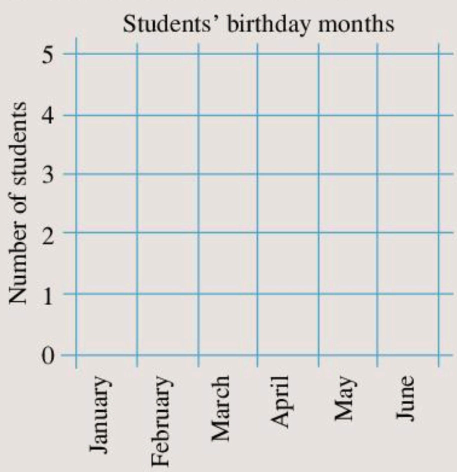
c Which day has the most students' birthdays this year?

4 **a** Find out the birthday month of each of the students in your class. Fill in a table like this one:

Birthday month	Number of students
January	
February	
March	

b Which month is the most popular for your class?

c On squared paper, make a larger copy of the bar chart started below, to show all the months of the year. On your copy, fill in the information from your table.



5 Form 3A were asked to choose their favourite radio programme. Their options were: *Record Request*, *Meet the Stars*, *Story Time*, *Sport Special* and *Music Review*.
The table below shows how they chose.

Favourite programme	Number of students
Record Request	7
Meet the Stars	5
Story Time	7
Sport Special	9
Music Review	8

a Which progamme was the most popular?

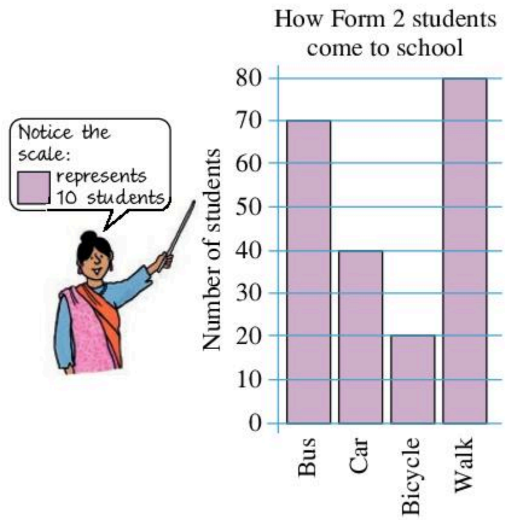
b Use squared paper to show the information in the table as a bar chart.

EXAMPLE 2

The table shows how the Form 2 students in Radley High School come to school every day.

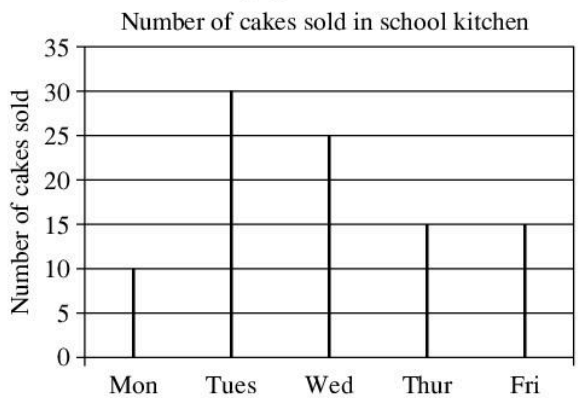
Means of transport	Number of students
Bus	70
Car	40
Bicycle	20
Walk	80

Draw a bar chart to show this information.



Each square represents 10 students.

Data can also be drawn as a **bar–line graph** instead of a bar chart. A bar–line graph looks like this:



In a bar–line graph there are no blocks to represent frequency. Instead the height of the line shows the frequency.

Bar–line graphs are useful when you have a lot of bars to draw as lines take up less space.

Exercise 12D

1 The occupations of mothers of students at Little Rock School are shown in the table.

Occupation	Number of mothers
Doctor	70
Teacher	100
Office worker	120
Lawyer	80
Stay-at-home mother	90

What scale should you use to show the data on a bar chart?

2 The table shows the shoe sizes of the Form 1C students in Radley High School.

Shoe size	Number of students
1	6
$1\frac{1}{2}$	12
2	15
$2\frac{1}{2}$	6
3	3
$3\frac{1}{2}$	3

a How many students could you represent by each square in a bar chart?

b On squared paper, draw a bar chart to show the information in the table.

c What scale did you use in your bar chart for the number of students?

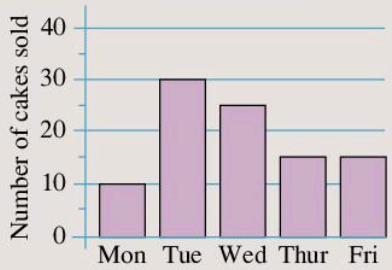
3 The table shows the favourite sports of the students of Form 2 in Radley High School.

Favourite Sport	Number of students
Rounders	35
Football	60
Netball	10
Cricket	50
Athletics	25
Basketball	30

a On squared paper draw a bar–line graph to show this information.

b Which sport is the most popular?

4 The bar chart shows the number of cakes sold in the Radley High School kitchen each day.



a How many cakes were sold on Monday?

b How many cakes were sold on Friday?

c How many cakes were sold during the week?

d If each cake costs 10 cents, how much money was taken in the week?

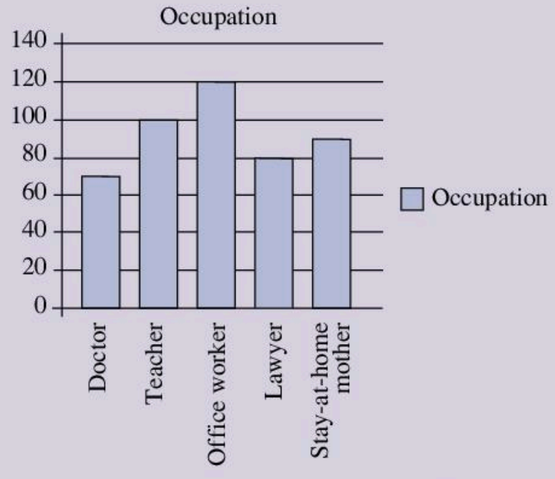
TECHNOLOGY

You can draw bar charts using Microsoft Word.

Use the data from Question **1** of Exercise 12D.

On the Word menu click on **Insert** then **Chart**.

On the pop-up spreadsheet type the data from Question **1**.



Repeat for other questions in Exercise 12D.

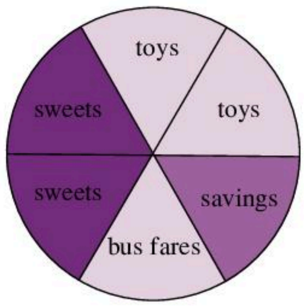
12.3 Pie charts

Another way of showing information is on a **pie chart**.

EXAMPLE 3

The circle below is a pie chart. It shows how a boy spends $12 pocket money.

Work out how much he spends on each item and how much he saves.



The circle represents the total amount of $12.
It is divided into six equal **sectors**.
So each sector is worth $\frac{1}{6}$ of $12 = $2.
He spends 2 × $2 = $4 on sweets.
He spends 2 × $2 = $4 on toys.
He spends 1 × $2 = $2 on bus fares.
He saves $2.

- A pie chart uses a circle divided into sectors to represent discrete data.

The size of each sector represents the frequency.
The angle of the sector shows the fraction of the total.
Pie charts should either have their sectors labelled or have a key so that it is clear what each sector represents.

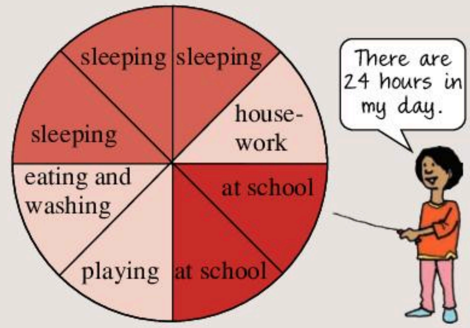
Exercise 12E

1 This pie chart shows how a girl spends $16.

a How many sectors are in the pie chart?
b How much is each sector worth?
c Copy and complete the table.

Where money goes	Sweets	Savings	Books
Money spent ($)			

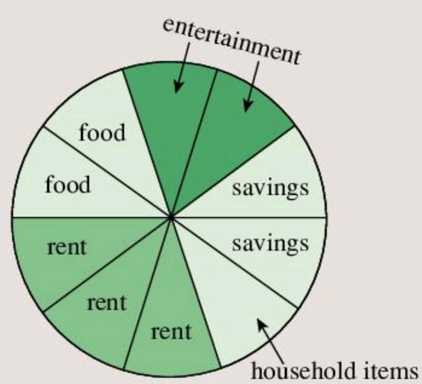
2 This pie chart shows how Susan spends her day.



a How many equal sectors are in the pie chart?
b How many hours does each sector represent?
c Copy and complete:

Activity	Sleeping	House-work	School	Eating and washing	Playing
No. of hours					

3 **a** This pie chart shows how Mr Williams spends his weekly wage of $500.



i How many sectors are in the pie chart?
ii How much is each sector worth?

b Copy and complete the table to show how Mr Williams spends his weekly wage of \$500.

Where money goes	Savings	Rent	Food	Household items	Entertainment
Money spent (\$)					

c Draw a bar chart using the data from the table.

d Draw a pictogram using the data from the table.

Drawing pie charts

You will need a protractor and a pair of compasses.

EXAMPLE 4

The table shows how 80 students travel to school.

Means of transport	Walk	Bus	Car	Bicycle
Number of students	40	20	10	10

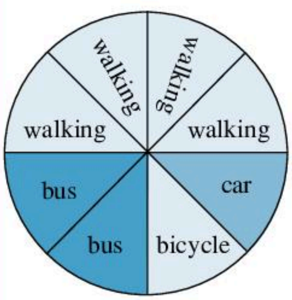
Draw a pie chart to show this information.

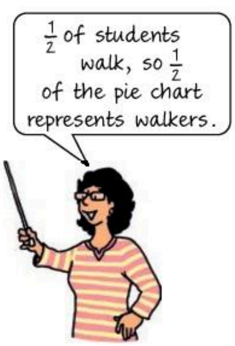
Fraction of students walking $= \frac{40}{80} = \frac{4}{8} = \frac{1}{2}$

Fraction of students coming by bus $= \frac{20}{80} = \frac{2}{8} = \frac{1}{4}$

Fraction of students coming by car $= \frac{10}{80} = \frac{1}{8}$

Fraction of students coming by bicycle $= \frac{10}{80} = \frac{1}{8}$

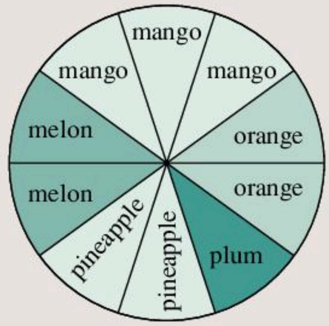




The whole circle represents the 80 students. Divide the circle into 8 equal sectors. Each sector represents $80 \div 8 = 10$ students. The angle of each sector is $\frac{360}{8} = 45°$.

Exercise 12F

1 The pie chart shows the favourite fruits of some students.



a How many equal sectors are in the pie chart?

b What is the favourite fruit?

c Which fruit is liked least?

d What fraction of the students chose **i** mango **ii** plum?

150 students were questioned.

e How many students are represented by each sector?

f How many students preferred **i** pineapple **ii** melon **iii** orange?

2 The table shows the number of letters a school receives each day.

Day	Mon	Tues	Wed	Thur	Fri
Number of letters	5	10	15	5	5

a How many letters does the school receive during the week?

b What fraction of letters are received on **i** Monday **ii** Tuesday **iii** Wednesday?

c Into how many sectors should you divide a circle in order to draw a pie chart for the information in the table?

d How many letters will be represented by each sector?

e How many sectors will represent the number of letters received on Wednesday?

f Draw a pie chart to show this information.

3 The table shows the number of cars sold each year by Cheap motors.

Year	2008	2009	2010	2011	2012
Number of cars sold	100	200	200	300	400

a How many cars were sold between 2008 and 2012?

b What fraction of the cars were sold in **i** 2008 **ii** 2011 **iii** 2012?

c Into how many sectors should you divide the circle in order to draw a pie chart?

d How many cars will each sector represent?

e How many sectors will represent the number of cars sold in 2011?

f Draw a pie chart to show this information.

4 At a café the owner kept a list of drinks ordered over two days. The results are shown in the table.

Drinks sold	Day	
	Friday	Saturday
Coffee	48	72
Tea	16	48
Hot chocolate	16	24
Lemonade	32	72
Fruit juice	16	24

Draw two pie charts (one for each day) to show this information.

5 The table shows the results of 30 hockey matches during one season.

Hockey results	
Win	10
Loss	15
Draw	5

a Draw a pie chart to show these results.

b Draw a bar chart to show the same information.

c Draw a pictogram to show the same information.

d Which method of showing the information is the easiest to read? Why?

6 **a** Which method of showing data do you prefer? Explain why.

b When may it be better to use
i a bar chart
ii a pictogram
iii a pie chart
to show data?

12.4 Frequency diagrams for grouped discrete data

Another name for a bar chart is a **frequency diagram**, which is a graph or chart with frequencies shown on the axis. You can use a frequency diagram for grouped discrete data.

50 students took a geography test. Here are their marks:

35, 37, 38, 43, 45,
47, 47, 48, 49, 51,
53, 54, 54, 54, 55,
56, 57, 57, 57, 59,
59, 60, 60, 62, 63,
64, 64, 65, 66, 66,
67, 68, 68, 69, 69,
69, 70, 71, 72, 72,
72, 74, 76, 77, 78,
78, 81, 81, 84, 84

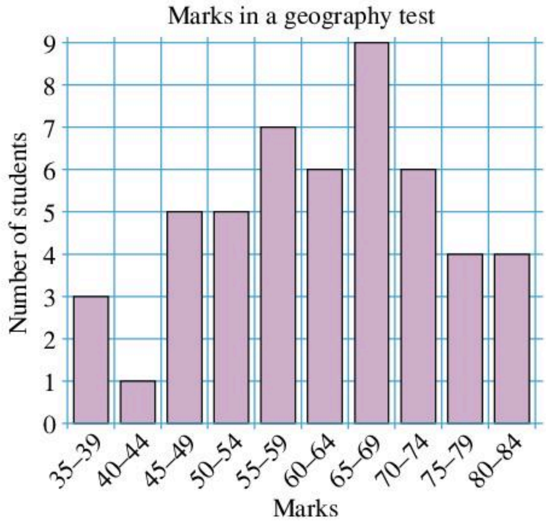
The lowest mark is 35
The highest mark is 84
The frequency table would be really long:

Mark	35	36	37	82	83	84
Number of students	1	0	1	0	0	2

It would be difficult to draw a bar chart for this data as there would be so many bars of height 0, 1 or 2. The bar chart would not show much. Instead the data can be grouped like this:

Mark	35–39	40–44	45–49	50–54	55–59	60–64	65–69	70–74	75–79	80–84
Number of students	3	1	5	5	7	6	9	6	4	4

The frequency diagram for this grouped data would look like this:

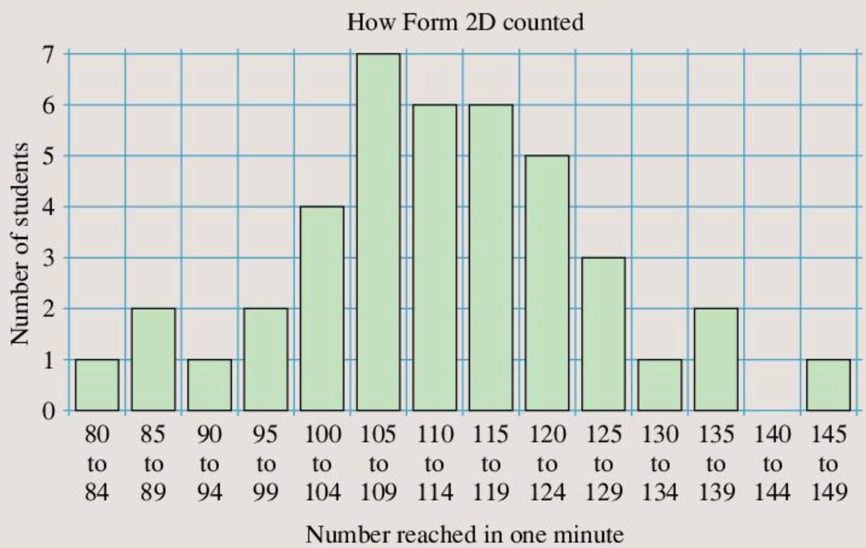


Exercise 12G

1 Use the frequency diagram above to answer the following questions (if possible).

- **a** How many students took the test?
- **b** How many students scored between 59 and 65?
- **c** How many students scored 70 or more?
- **d** How many students scored 50?
- **e** How many students failed, if the pass mark was 50?

2



Mrs Singh asked Form 2D to count quietly: *one*, *two*, *three* . . .
After exactly one minute, she asked them to stop. She wrote down the number that each student had reached.
The results are shown in the chart.

Answer the following questions:

- **a** How many students are in Form 2D?
- **b** Why do the groups of numbers start at 80 instead of 0?
- **c** How many students counted to one of the numbers 105, 106, 107, 108 or 109?
- **d** How many students counted to between 124 and 130?
- **e** How many students counted to between 94 and 100?
- **f** Can you tell how many students counted to 131?
- **g** Can you tell whether anyone counted to 149?
- **h** How many students counted to 143?

3 Redraw the bar chart from Question **2**, but this time group the *Number reached* into tens, using groups 80–89, 90–99, and so on.

TECHNOLOGY

Collect data in your class to find, for example:

- Favourite games of students
- Transportation used to reach school
- Latecomers over a 5-day period
- Favourite TV programmes.

In each case, separate the results for boys and girls.

Enter the data collected as tables in a spreadsheet.

For example:

	A	B	C
1		Boys	Girls
2	Cricket	5	2
3	Football	8	3
4	Swimming	4	10

Highlight your table and select the bar chart.

Notice how the chart allows you to compare girls' answers with boys' answers.

Copy your charts into a word processing program and make brief notes to explain your results.

12.5 Using statistics

When you take an examination, the mark you get is very important to *you*.

To the examiners, your mark is not so important. Instead they think about the overall *pattern* of marks. This tells them a great deal. For example, the pattern of results in an IGCSE exam tells the examiners how good the schools are in different countries.

Exercise 12H

1 A group of 31 candidates took exams in English and Mathematics. The list of results, in alphabetical order of candidates, is given in the table below.

English	Mathematics
73	74
38	8
48	50
57	64
84	58
40	58
49	53
58	90
12	2
44	38
50	49
60	78
21	17
45	43
53	54
64	82

English	Mathematics
25	32
45	48
54	49
66	98
30	28
46	9
55	63
66	86
33	35
47	21
56	69
70	13
38	35
47	34
56	67

a Using intervals 0–19, 20–39, etc., draw a frequency table for the marks in each subject.

b Draw frequency diagrams to show the marks in each subject.

c Describe all the differences you notice between the two frequency diagrams.

d Do you think these differences have anything to do with the differences between English and Mathematics?

2 **a** For each subject in Question **1**, write down the marks in order and find the median.

b What is the range of marks for each subject?

c Use your answers to parts **a** and **b** to compare the marks in the two subjects.

3 For the students in your class, find the results for the last English and Maths tests (ask your English and Maths teachers.) Repeat Questions **1** and **2** for these results.

4 40 students measured their pulse rates before and then immediately after jumping up and down for two minutes. The results, in beats per minute, are recorded in the table at the bottom of the page.

- **a** Draw a frequency table for the pulse rates *before* the exercise. Use the intervals 71–75, 76–80, 81–85, etc.
  Draw a bar chart of the results.
- **b** Draw a frequency table for the pulse rates *after* the exercise. Use the intervals 111–115, 116–120, 121–125, etc.
  Draw a bar chart of the results.
- **c** Compare your two bar charts. Are they alike in any way?
- **d** Write down the median and the range of the pulse rates before exercise.
- **e** Find the median and range of the pulse rates after exercise.
- **f** Compare the median before and after exercise.
- **g** Compare the range before and after exercise.

Before	After
71	117
72	124
72	138
73	123
74	118
74	122
74	119
75	118
75	127
76	121
76	118
76	143
77	127
77	119
77	120
78	120
78	129
78	129
79	126
79	112

Before	After
79	126
80	125
80	129
81	133
82	124
82	130
83	130
83	135
84	140
85	120
85	134
86	136
87	130
88	130
89	138
89	116
92	149
92	123
95	150
99	124

5 Two athletes keep a record of their times in their last ten 400-metre races. They work out the mean and range for these data.

	Athlete 1	Athlete 2
Mean time (seconds)	48.25	48.35
Range of times	3.2	1.1

- **a** Which athlete is more consistent?
- **b** Which athlete would you pick for your squad? Why?

6 A shoe shop kept a record of the number of pairs and sizes of shoes sold for men and women during one day. The data is shown below.

Shoe size	Men	Women
3	0	6
4	1	10
5	5	18
6	10	15
7	12	8
8	14	1
9	6	1
10	1	0

- **a** What was the modal shoe size for women?
- **b** What was the range of shoe sizes for women?
- **c** What was the modal shoe size for men?
- **d** What was the range of shoe sizes for men?
- **e** Compare men and women's shoe sizes using your answers to parts **a–d**.

7 The scores for two cricketers for a test series were:

Player A 33, 40, 40, 35, 2, 0, 114, 101, 3, 2
Player B 71, 26, 15, 33, 65, 17, 35, 40, 42, 26

The data above can be summarised as follows:

	Mean	Median	Mode	Range
Player A	37	34	40	114
Player B	37	34	26	56

James said, "I would pick Player A for my team because Player A has a higher modal score."
Ishwar said, "I would pick Player B for my team because they are more consistent, as they have a lower range."
Sohan said, "I would pick Player A for my team as Player B has never scored above 71."
Joshua said, "I think both players are just as good because they both have the same mean score."
Whose opinion do you agree with and why?

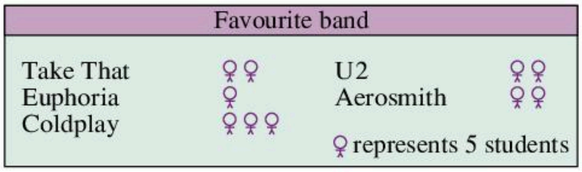
Consolidation

Example 1

This table shows the favourite bands of a group of 50 students.

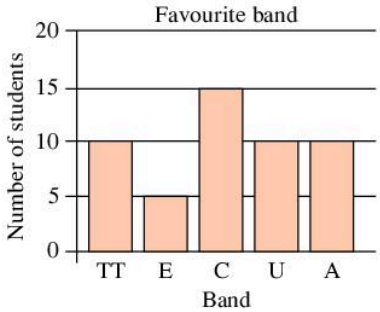
Band	Take That	Euphoria	Coldplay	U2	Aerosmith
Votes	10	5	15	10	10

Show this as a pictogram.



Example 2

Use the data in Example 1 to draw a bar chart showing this information.



Example 3

Using the data in Example 1, draw a pie chart to show this information.

There are 50 students.

Fraction liking Take That $= \frac{10}{50} = \frac{2}{10}$

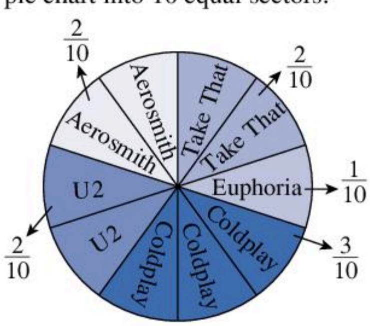
Fraction liking Euphoria $= \frac{5}{50} = \frac{1}{10}$

Fraction liking Coldplay $= \frac{15}{50} = \frac{3}{10}$

Fraction liking U2 $= \frac{10}{50} = \frac{2}{10}$

Fraction liking Aerosmith $= \frac{10}{50} = \frac{2}{10}$

Split your pie chart into 10 equal sectors.



Example 4

The marks in a maths test for five girls and five boys are shown in the table below. By working out the mean and range for girls and boys, compare their performances in the test.

Girls	71	63	58	44	39
Boys	78	18	55	61	38

Mean for girls $= \frac{71 + 63 + 58 + 44 + 39}{5} = 55$

Range for girls $= 71 - 39 = 32$

Mean for boys $= \frac{78 + 18 + 55 + 61 + 38}{5} = 50$

Range for boys $= 78 - 18 = 60$

Girls' results were better on average as they have a higher mean. Girls' results were more consistent as they have a smaller range. This comparison is not very reliable as it is only based on ten students.

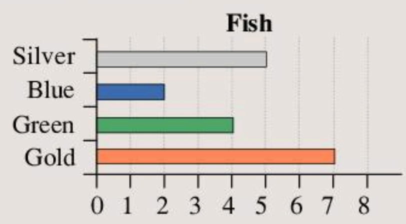
Exercise 12

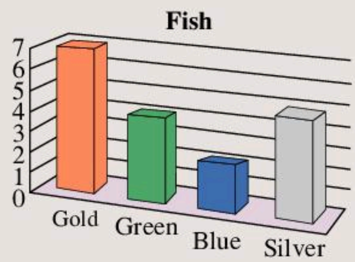
1 The table shows the colours of fish in Grace's aquarium.

Colour	Gold	Green	Blue	Silver
Number	7	4	2	5

The charts on the next page all represent the data from the table.

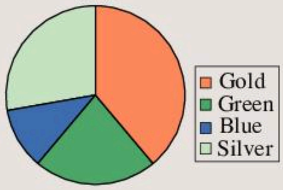
Write a paragraph comparing the charts. Explain which features of the data each chart shows most clearly, and comment on how easy each one is to read.



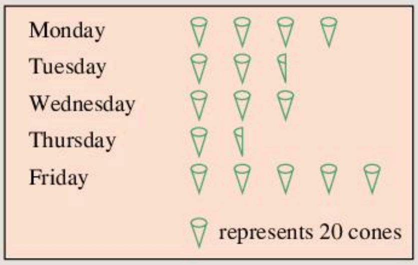


Fish

Fish

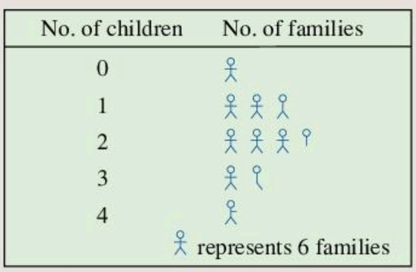


2 The pictogram shows the number of ice cream cones sold in a canteen during the first week in March.



a How many cones were sold on Tuesday?
b How many cones were sold altogether during the week?
c If the cost of a cone was \$3, how much money did the canteen make during the week?

3 The pictogram shows the number of children in 56 families.



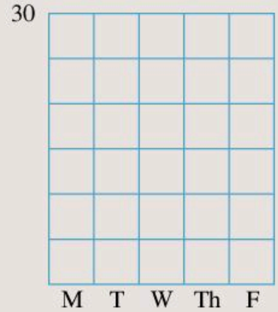
In how many families were there
a 4 children **b** 3 children **c** 2 children?

4 Miss Carty kept a record of the number of students in her class each day for one week.

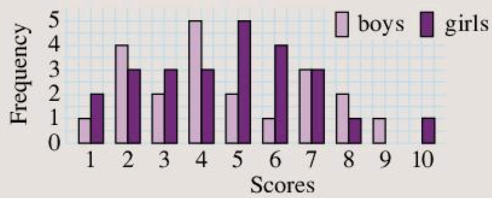
The results are shown in the table.

Day	Mon	Tues	Wed	Thur	Fri
Number present	27	24	30	25	16

Copy and complete the bar chart below.



5 The results of a class test taken by boys and girls are shown on the bar chart.



a How many
i boys **ii** girls
were in the class?
b If the pass mark is 5, how many
i boys **ii** girls
passed the test?

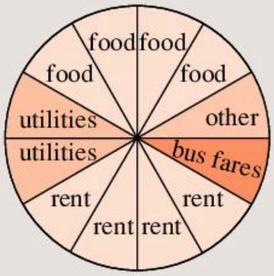
6 Every 400 g of dried fish contains about 150 g of water, 50 g of fat, 150 g of protein and 50 g of other substances.
a To represent this information, draw a pie chart divided into 8 equal sectors.
b How many grams does each sector represent?
c How many sectors will represent the amount of protein in dried fish?

7 The table shows a tally of the types of vehicles involved in serious accidents on a busy road.

Vehicle	Taxi	Car	Bus	Truck	Other
Frequency	~~||||~~ ||	~~||||~~	|||	|	||

a Write the frequencies as numbers instead of tallies.
b Which type of vehicle had the least number of accidents?
c How many accidents were there altogether?
d Which two types of vehicle account for $\frac{2}{3}$ of the total number of accidents?
e Draw a bar–line graph to show this information.

8 The pie chart shows how Mrs Peters spends her weekly housekeeping money.



The amount that she spends on rent is $100. Find:

a The amount represented by each sector.

b The amount spent on

i utilities

ii bus fares

c The total weekly housekeeping money.

9 Zoologists discovered two unknown types of ape deep in the South American jungle. To compare them they took 100 adult apes of each type and measured their heights. Type A had a mean height of 176 cm with a range of 20 cm. Type B had a mean height of 158 cm with a range of 10 cm.

a Sketch two bar charts showing how the results might have looked.

b What can you say about the two different types of ape? (Use the means and ranges in your comparison.)

TECHNOLOGY

Visit the Graphs Index at the website www.mathsisfun.com/data

Make different charts and compare them.

Which representation do you prefer? Why?

Summary

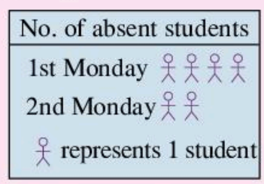
You should know ...

1 A pictogram uses pictures or drawings to represent data.
For example:

2 A scale can be used when numbers are large.

Check out

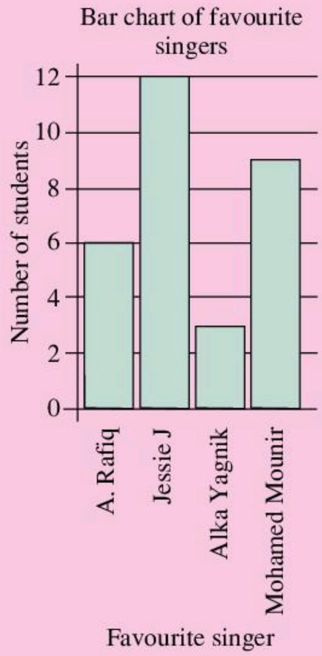
1 The diagram below shows part of a pictogram.



How many students were absent on the first Monday?

2 If represents 50 students, how many symbols would you use to represent 250 students?

3 A bar chart uses the height of bars to represent frequencies.
For example:



4 A pie chart is a circle divided into sectors.
The size of the sector represents the number of items.
For example:

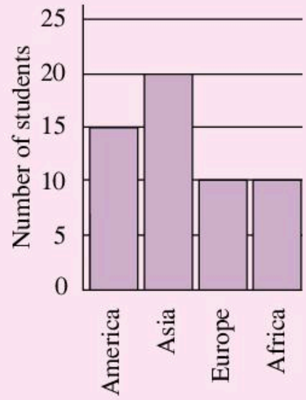
5 How to compare two distributions.
For example:

Batting averages for two cricketers for a test series are summarised as follows.

	Mean	Median	Mode	Range
Player A	35	27	41	35
Player B	38	26	23	47

Player B has a higher mean batting average. Although Player A has a higher median and mode, the mean is the more significant average in this example. Player A is more consistent, with a lower range.

3 The bar chart below shows places visited by Tobias High School students during the holidays.



a How many students visited
i Africa
ii Asia?

b How many students took part in the survey?

4 A survey showed the ways in which the students in Form 1T travelled to school.

Bus	Car	Walk	Cycle
20	8	8	4

Draw a pie chart to show this information.

5 Compare these two cricketers' batting averages:

	Player C	Player D
Mean	47	32
Median	21	30
Mode	8	33
Range	68	12

Review B

1 Copy and complete:

a $\frac{3}{4} = \frac{6}{\square} = \frac{\square}{16} = \frac{21}{\square} = \frac{63}{\square}$

b $\frac{4}{7} = \frac{\square}{21} = \frac{24}{\square} = \frac{48}{\square} = \frac{\square}{63}$

c $\frac{12}{18} = \frac{\square}{21} = \frac{10}{\square} = \frac{28}{\square} = \frac{\square}{63}$

d $\frac{\square}{6} = \frac{25}{\square} = \frac{20}{\square} = \frac{\square}{42} = \frac{60}{72}$

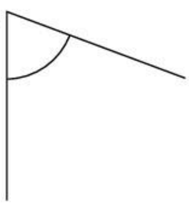
2 **a** Find the mass, M, of n tins of paint each of mass m kg.

b Find the cost c of each book if the total cost of b books is T.

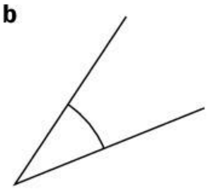
c Find A, the area of card left if a square of edge p cm is cut out from a square card of edge q cm.

3 Say whether each angle is: acute, right-angled, obtuse or reflex.

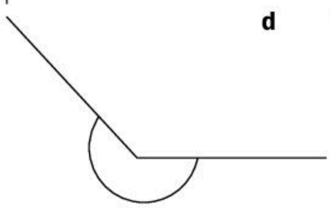
a



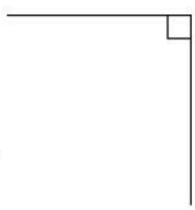
b



c



d



4 Estimate the size of each angle in Question **3**. Use a protractor to check your estimates.

5 Change to decimals:

a $\frac{3}{5}$ **b** $\frac{9}{20}$ **c** $\frac{19}{20}$

d $\frac{2}{25}$ **e** $\frac{13}{100}$ **f** $\frac{3}{100}$

g $\frac{33}{50}$ **h** $\frac{3}{250}$

6 What time is shown on a digital watch when it is:

a midday

b twenty past four in the morning

c twenty to three in the morning

d five to six in the morning

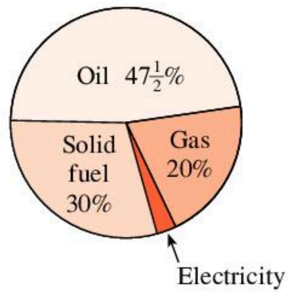
e three o'clock in the afternoon

f quarter to seven in the evening

g twenty-five minutes before midnight?

h Write all of these times as 24-hour clock times.

7 The estimated world primary energy production is shown on this pie chart.



a What percentage of the total energy is represented by electricity?

b What is the size of the angle of the sector representing electricity?

c Calculate the sizes of the angles of the other sectors.

8 Copy and complete:

a $\frac{\square}{5}$ of $90 = 36$ **b** $\frac{3}{4}$ of $60 = \frac{1}{5}$ of $\square$

c $\frac{3}{8}$ of $\square = 27$

9 Arrange these fractions in order of size, putting the smallest first:

a $\frac{1}{3}, \frac{2}{9}, \frac{5}{18}$ **b** $\frac{3}{4}, \frac{2}{3}, \frac{7}{12}$

c $\frac{9}{15}, \frac{7}{12}, \frac{13}{18}$

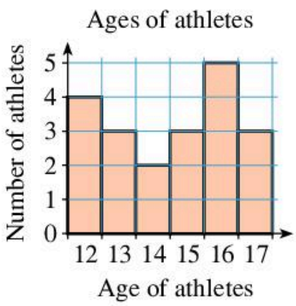
10 Show these points on a coordinate graph.
$(^-2,^-2)$, $(0,^-3)$, $(1,^-3)$, $(3,^-2)$, $(1,^-1)$, $(0,^-1)$.
Join them in order and then to $(^-2,^-2)$.
What shape have you made?
Mark in any lines of symmetry.
(See Chapter 17 if you need help with lines of symmetry.)

11 Write as a fraction in its simplest form.

a 0.8 **b** 0.2 **c** 0.25

d 0.15 **e** 0.125 **f** 0.37

12 Look at the chart below:



a How many athletes are shown?
b Which age is the **mode**?
c Which is the **median** age?
d Find the **mean** age of the athletes.

13 A water container when a quarter full holds 320 litres. How much does the full container hold?

14 Derive a formula to convert D dollars (USD) into I Indonesian rupiahs (IDR) using the exchange rate 1 USD = 9000 IDR. Use your formula to convert 250 USD into Indonesian rupiahs.

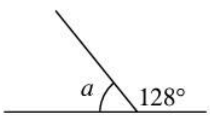
15 Write down the greater number:

a $\frac{1}{2}$, 0.2 **b** $\frac{1}{4}$, 0.4
c $\frac{1}{5}$, 0.5 **d** $\frac{3}{4}$, 0.34

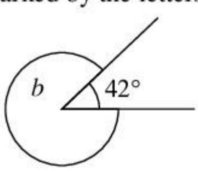
16 Through how many degrees does the minute hand of a clock turn between
a 3 o'clock and 3.15 pm
b 3.15 pm and 3.25 pm
c 3.25 pm and 4.05 pm?

17 Find the size of the angles marked by the letters.

a



b



18 Jan, Jim and Joy agreed to meet at the local cinema. Jan arrived 20 minutes before the film started, Jim arrived 5 minutes after the film started and Joy was in her seat with 3 minutes to spare.
By how many minutes did
a Jan arrive before Jim
b Jim arrive after Joy
c Jan have to wait before Joy arrived?

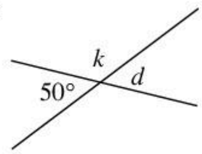
If they all left the cinema 1 hour 47 minutes after the film started how long did each person spend in the cinema?

19 Write these fractions in their simplest form:

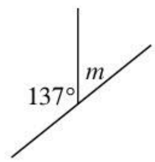
a $\frac{32}{56}$ **b** $\frac{56}{6}$ **c** $\frac{114}{12}$
d $\frac{96}{112}$ **e** $4\frac{16}{28}$ **f** $12\frac{108}{144}$

20 Write down, without measuring, the size of the angles marked with letters. Give a reason for each answer.

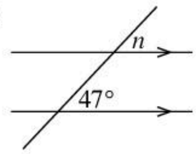
a



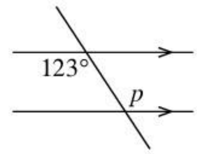
b



c



d



21 If $m = 3$, $n = 5$ and $p = 7$, find the value of:
a $m + n - p$
b $4m - n + 2p$
c $m(p - n)$
d $10p - 2m - 4n$

22 Giselle spends $\frac{1}{3}$ of her day in bed, $\frac{1}{8}$ in front of the television, $\frac{1}{4}$ at school, and $\frac{1}{12}$ eating.
What fraction of her day is left?

23 What change would you get from $10.00 if you spent:
a $3.50
b $3.05
c $0.35
d $0.03?

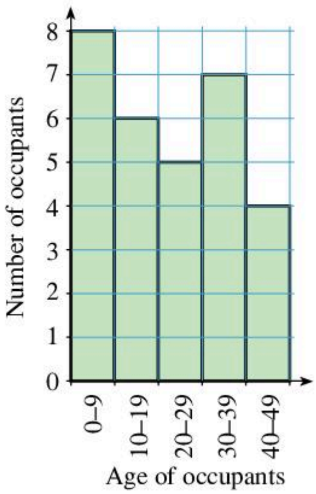
24 It is possible to travel from London to Paris by train, boat and train, or aeroplane. This is part of a timetable:

London (depart)	Paris (arrive)
0804	1807
0850	1702
1430	2228
2040	0625
2255	0915

a Write the timetable using the 12-hour clock.
b Which journeys involve travelling at night?
c Calculate the time taken for each journey.
d Which is the shortest journey?

25 Look at the chart below:

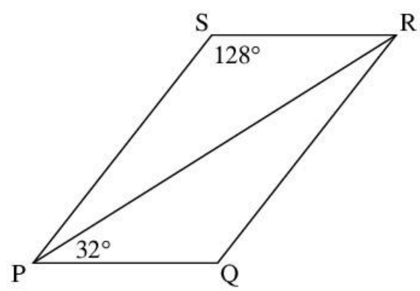
Bar chart to show the ages of people in a block of flats



a Which age group is the **mode**?
b How many occupants are 20 or older?
c How many occupants are 29 or younger?

26 A writer can type 6 pages in $\frac{3}{4}$ hour.
How long will it take to type 72 pages?

27 PQRS is a parallelogram.



$R\hat{P}Q = 32°$ and $P\hat{S}R = 128°$
Find, giving reasons:
a $P\hat{R}S$ **b** $S\hat{P}R$ **c** $P\hat{Q}R$

28 Calculate:

a $3.2 + 4.81$ **b** $3.28 + 4.82$
c $3.62 - 1.725$ **d** $3.6 - 1.06$
e $8 - 0.32$ **f** 4×0.35
g 2.8×4 **h** $8.4 \div 7$
i $14.4 \div 1.2$ **j** $1.56 \div 3$

29 Jayceline walks at $6\frac{3}{4}$ km/hr.
How far can she walk in 8 hours?

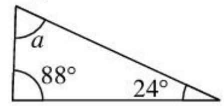
30 Solve:

a $4x = 44$ **b** $3x + 2 = 38$
c $12t - 12 = 12$ **d** $16 + 5p = 36$

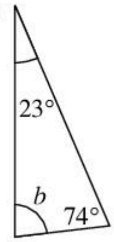
31 Are the following statements true or false?
a There are 60 seconds in an hour.
b 00:24 is 12.24 pm in 12-hour clock time.
c Between 4 am and 2 pm there are 10 hours.
d In 24-hour clock time 3 pm is 13:00.
e There are 52 weeks in a year.
f There are 24 hours in a week.
g 00:13 is 13 minutes past midnight in 12-hour clock time.
h Between 21:00 and 04:30 the next day there are six and a half hours.
i 7.15 pm is 07:15 in 24-hour clock time.

32 Find the angles marked by the letters in these triangles.

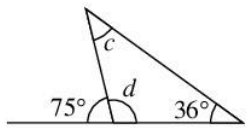
a



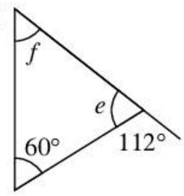
b



c



d



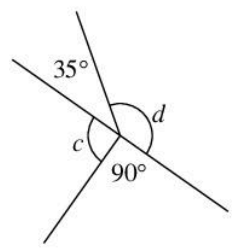
33 The estimated percentage of total world energy production in 2012 is given in this table:

Region	%
North America	30
Asia	$22\frac{1}{2}$
Russia	$22\frac{1}{2}$
Europe	15
Africa	5
South America	5

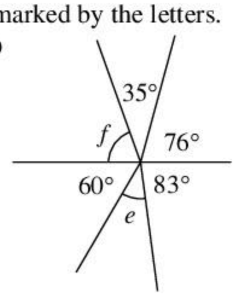
Construct a pie chart to show the information.

34 Find the size of the angles marked by the letters.

a



b



35 Here is a game you can try on a calculator or on a piece of paper. Put a number in the calculator, say:

This number is made up of 5 digits.
Can you eliminate it by subtracting 5 numbers?
Try some more.

36 Change these improper fractions to mixed numbers:

a $\frac{23}{4}$ **b** $\frac{19}{3}$ **c** $\frac{72}{15}$ **d** $\frac{171}{13}$

37 Solve:

a $4x = 0$ **b** $8 - x = 3$
c $^{-}30 = 5y$ **d** $3n - 12 = 15$
e $20 - 8x = {}^{-}4$

38 Test scores were recorded for two different classes. The results can be summarised as follows:

	Mean	Median	Mode	Range
Form 7L	54	48	67	35
Form 7M	63	59	24	48

The teacher of Form 7L said her students were better. The teacher of Form 7M thought her students were better.
Which form do you think was the best and why?
Which is the most useful average when thinking about test scores? Why?

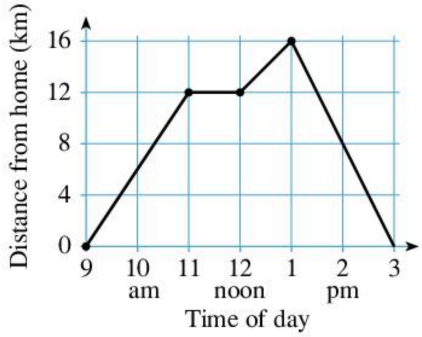
39 Ahmad shares 40 oranges between his 6 children. How many oranges do they get each? Write your answer as a mixed number.

40 A (1, 1), B (0, 3), C $(^{-}4, 1)$, D $(^{-}3, ^{-}1)$ are the four vertices of a parallelogram.
Draw the parallelogram.
Which of the following points are inside the parallelogram?

a $(^{-}2, 0)$ **b** $(2, 2)$ **c** $(^{-}3, 0)$
d $(^{-}2, ^{-}3)$ **e** $(0, 1)$ **f** $(0, 0)$

41 Look at the graph below for a walker.

a What time did he reach home?
b What was his furthest distance from home?
c When did he stop for a rest?
d When was he walking fastest? How do you know?



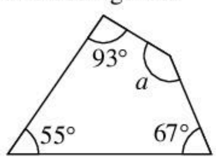
42 The total number of cell phones in use in Japan is given by this table:

Year	1990	1995	2000	2005	2010
Cell phones (millions)	35	50	80	100	135

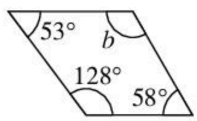
Design a pictogram to represent this information.

43 Find the angles marked by letters in these four-sided figures.

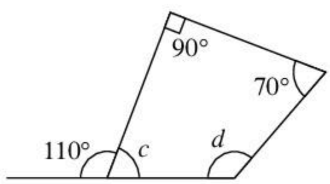
a



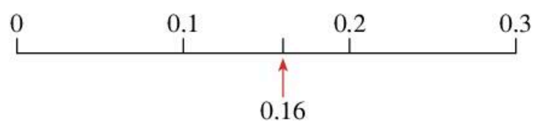
b



c



44 Is $\frac{1}{6}$ the same as 0.16?
This number line shows 0.16:



Find 0.16×6. Is it less than 1?
Find 0.17×6.Is it more than 1?
Is $\frac{1}{6}$ nearer 0.16 or 0.17?

45 The height of a plant after w weeks of growth following germination is given in this table:

w (weeks)	2	4	6	8	10	12
Height (cm)	5	10	20	40	60	70

Plot a graph of height against w and use your graph to estimate

a the height after 9 weeks

b how long it takes to reach a height of 35 cm.

46 Change these mixed numbers to improper fractions:

a $7\frac{2}{3}$ **b** $3\frac{3}{4}$ **c** $8\frac{7}{20}$ **d** $3\frac{11}{15}$

47 Isis and Nashwa have 24 oranges between them. Nashwa has twice as many oranges as Isis. Construct an equation to show this, then solve the equation.

48 The following data shows the weights of 30 people, in kilograms:

52	91	61	76	67	84	83	70	66	75
65	82	80	54	75	60	70	71	82	79
71	89	92	74	76	84	60	73	58	92

Weight (kg)	Tally	Frequency
50–59		
60–69		
70–79		
80–89		
90–99		

a Copy and complete the table.

b Draw the frequency diagram for the grouped data.

49 James eats $\frac{2}{3}$ of a pizza. Isabella eats $\frac{1}{4}$ of the same pizza. How much of the pizza have they eaten altogether? How much is left?

50 Copy out the equations below. Draw lines to show the equations that have the same solution. One has been done for you.

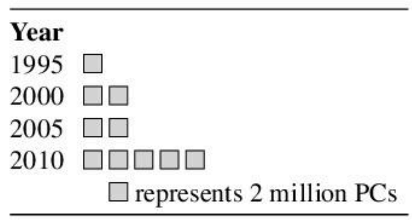
$4x = 4$	$22 = 11x$
$5x + 3 = 53$	$3x + 5 = 8$
$25 - 9x = 7$	$10 - 3x = {}^-5$
$6 + 4x = 26$	$7x - 62 = 8$

51 Write down the number that is:

a $\frac{1}{2}$ of 0.1 **b** $\frac{1}{4}$ of 0.2

c $\frac{1}{2}$ of 0.4 **d** $\frac{1}{10}$ of 0.25

52 This pictogram shows the number of PCs sold in India, to the nearest 2 million.



a How many PCs are shown for 2005?

b In 2005 there were 4.5 million PCs sold, but only 4 million is shown in the pictogram. Show a way of improving the representation.

13 Fractions, decimals and percentages

Objectives

- Recognise the equivalence of simple fractions, decimals and percentages.
- Understand percentage as the number of parts in every 100; use fractions and percentages to describe parts of shapes, quantities and measures.
- Calculate simple percentages of quantities (whole number answers), e.g. 20% of 50 kg, and express a smaller quantity as a fraction or percentage of a larger one.
- Use percentages to represent and compare different quantities.

What's the point?

We use money on a daily basis to buy and sell things. Finding how much you can save when a percentage discount is offered is important if you don't want to end up with no money!

Before you start

You should know ...

1 How to simplify fractions.

$$\frac{4}{10} \overset{\div 2}{=} \frac{2}{5} \quad (\div 2)$$

2 How to find fractions of a number.
For example:

$$\frac{1}{3} \text{ of } 12 = 12 \div 3 = 4$$

$$\frac{2}{3} \text{ of } 12 = 2 \times \frac{1}{3} \text{ of } 12 = 2 \times 4 = 8$$

Check in

1 Simplify:

a $\frac{6}{10}$ **b** $\frac{6}{9}$ **c** $\frac{8}{12}$

d $\frac{18}{24}$ **e** $\frac{12}{36}$ **f** $\frac{26}{39}$

2 Work out:

a $\frac{1}{5}$ of 10 **b** $\frac{2}{5}$ of 10

c $\frac{4}{25}$ of 50 **d** $\frac{3}{4}$ of 64

e $\frac{14}{20}$ of 80 **f** $\frac{10}{25}$ of 75

3 How to write a fraction as a decimal.
For example:

$$\frac{3}{10} = 3 \div 10 = 0.3$$

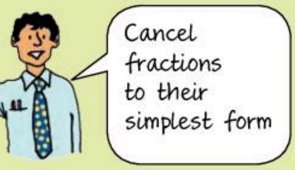
$$\frac{47}{100} = 47 \div 100 = 0.47$$

$$\frac{2}{5} \overset{\times 2}{=} \frac{4}{10} = 4 \div 10 = 0.4$$

4 How to write a decimal as a fraction.
For example:

$$0.8 = \frac{8}{10} = \frac{4}{5}$$

$$0.24 = \frac{24}{100} = \frac{6}{25}$$



3 Write as decimals:

a $\frac{2}{10}$ **b** $\frac{7}{10}$

c $\frac{16}{100}$ **d** $\frac{82}{100}$

e $\frac{3}{5}$ **f** $\frac{3}{4}$

4 Write as fractions in their simplest form:

a 0.7 **b** 0.13

c 0.2 **d** 0.25

e 0.48 **f** 0.017

13.1 Understanding percentages

Often you will go into a store and see signs like this:

The sign means that the price of the TV has been reduced by 10%.

The symbol '%' means 'per cent' or 'out of 100'.
For example:

In a bag of 100 tomatoes, 6 are bad.
$\frac{6}{100}$ or 6 per cent (6%) of the tomatoes are bad.

EXAMPLE 1

Mrs Brown the fruit vendor bought 200 oranges. She found 8 of the oranges were bad. Find the percentage that were bad.

$$\text{Fraction of bad oranges} = \frac{8}{200} = \frac{8 \div 2}{200 \div 2} = \frac{4}{100}$$

4 oranges in 100 were bad.
That is, 4% were bad.

Exercise 13A

1 Write using the symbol %.
a 5 per cent **b** 14 per cent

2 Write using the symbol %.
a $\frac{15}{100}$ **b** $\frac{25}{100}$

$\boxed{3}$ **a** Jan scored 67 out of 100 in a maths exam. What percentage is this?
b Her sister got 16 out of 20 in a science test. Who got the higher percentage?

4 Mrs Brown kept a record of the bad oranges she received from the fruit farmer. Using the method from Example 1, copy and complete the table.

Oranges bought from Mr Johnson in 2012

Month	Number bought	Number bad	Fraction bad	Percentage bad
Jan	300	36	$\frac{36}{300} = \frac{12}{100}$	12
Feb	500	40	$\frac{40}{500} = \frac{8}{100}$	8
March	600	54	$\frac{54}{600} = \frac{\square}{100}$	
April	600	42		
May	800	40		
June	900	45		
July	1200	72		
Aug	1500	75		
Sept	1000	70		
Oct	800	56		
Nov	700	63		
Dec	500	55		

5 What do you think these mean?

a 100% cotton

b 50% nylon
50% cotton

c **Messi still not 100% fit**

d

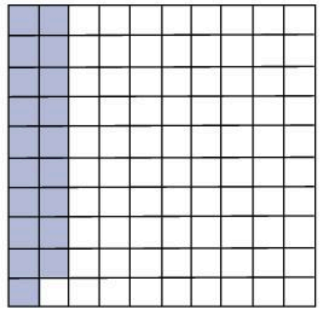

13.2 Fractions, decimals and percentages

You will need squared paper.

You can show percentages, like fractions, on diagrams.

EXAMPLE 2

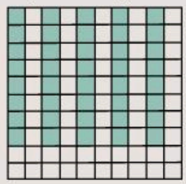
What percentage of the square is shaded?

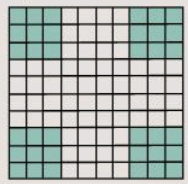


19 out of 100 squares are shaded. That is, $\frac{19}{100}$ or 19%.

Exercise 13B

1 What percentage of each shape is shaded?

a


b


2 Draw pictures on your squared paper to show:

a 70% **b** 64%

3 1% is the same as $\frac{1}{100}$.

Use this idea to copy and complete:

a 5% is the same as ...

b 39% is the same as ...

c 81% is the same as ...

4 22% = $\frac{22}{100}$ as a per cent fraction. Write these as per cent fractions.

a 34% **b** 11% **c** 9%

5 Write as a percentage:

a $\frac{15}{100}$ **b** $\frac{25}{100}$ **c** $\frac{17}{100}$

Changing fractions to percentages

It is easy to write a fraction with a denominator of 100 as a percentage.

For example:

$$\frac{23}{100} = 23\%$$

You can change any fraction to a percentage by writing it with a denominator of 100.

EXAMPLE 3

Write the fraction $\frac{6}{25}$ as a percentage.

$$\underset{\times 4}{\overset{\times 4}{\frac{6}{25} = \frac{24}{100}}} = 24\%$$

Another way of changing a fraction to a percentage is to multiply by 100%. Remember $100\% = \frac{100}{100} = 1$

EXAMPLE 4

Write the fraction $\frac{28}{80}$ as a percentage.

$$\begin{aligned}\frac{28}{80} &= \frac{28}{80} \times 100\% \\ &= \frac{28}{\cancel{80}_{8}} \times \cancel{100}^{10}\% \\ &= \frac{\cancel{28}^{7}}{\cancel{8}_{2}} \times 10\% \\ &= \frac{70}{2}\% = 35\%\end{aligned}$$

Example 4 can also be done another way.

Cancel the fraction:

$$\underset{\div 4}{\overset{\div 4}{\frac{28}{80} = \frac{7}{20}}}$$

Then make the denominator 100:

$$\underset{\times 5}{\overset{\times 5}{\frac{7}{20} = \frac{35}{100}}} = 35\%$$

Exercise 13C

1 Copy and complete, using the method in Example 3.

a $\frac{1}{4} = \frac{\square}{100} = \square\%$ **b** $\frac{1}{2} = \frac{\square}{100} = \square\%$

c $\frac{3}{4} = \frac{\square}{100} = \square\%$ **d** $\frac{7}{10} = \frac{\square}{100} = \square\%$

e $\frac{7}{20} = \frac{\square}{100} = \square\%$

2 Write as a percentage:

a $\frac{3}{5}$ **b** $\frac{4}{5}$ **c** $\frac{11}{20}$

d $\frac{9}{15}$ **e** $\frac{3}{50}$

3 Write as a percentage:

a $\frac{32}{80}$ **b** $\frac{15}{75}$ **c** $\frac{28}{40}$

d $\frac{48}{60}$ **e** $\frac{22}{55}$ **f** $\frac{17}{68}$

g $\frac{120}{150}$ **h** $\frac{108}{144}$

4 John scored 6 out of 10 in a mathematics test.

a Write his score as a fraction.

b Write his score as a percentage.

5 David scored 13 out of 20 in an English test.

a Write his score as a fraction.

b Write his score as a percentage.

6 **a** Janice scored 56 out of 80 in a science exam. What is her score as a percentage?

b Janice scored 49 out of 70 in a French paper. In which exam did she get a higher percentage?

7 Change each of these test scores to percentages:

a $\frac{3}{5}$

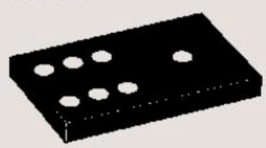
b 37 out of 50

c $\frac{41}{50}$

d 131 out of 200

8 Six children in a class of forty were absent from school. What percentage were absent?

9 Earl and Eric won the last 6 of their 8 domino matches.



Rennick and Raymond won the last 14 of their 20 domino matches.
Which pair won the higher percentage of matches?

10 Out of the 200 students at a school, 120 were present when the register was taken today, 50 were late and 30 were absent. Write down the percentage

a present **b** late **c** absent.

11 Copy and complete the table below showing the number of girls at Queens School.

Year	No. of girls	Total pupils	% girls
7	75	150	
8	90	120	
9	58	100	
10	66	120	
11	50	80	

Which form has the greatest percentage of girls?

12 At a driving centre in Cairo last week, 45 people took a driving test. The results are shown below.

	Number tested	Number passed
Men	25	13
Women	20	14

Find the percentage of

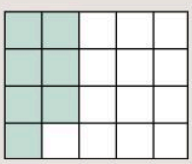
a men who passed
b women who passed
c men who failed
d women who failed
e people who passed.

13 What

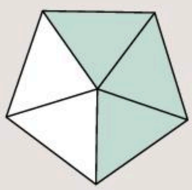
i fraction
ii percentage

of these shapes is shaded?

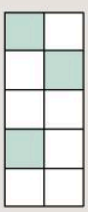
a



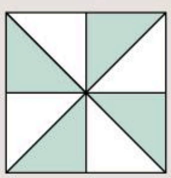
b



c



d



14 Write as a percentage:

a 13 km out of 25 km have been travelled on a journey.
b The distance still left to travel in part **a**.
c 40 g out of 200 g in a recipe is sugar.
d 420 cm^2 out of a 600 cm^2 magazine page is advertising space.
e 20 cents out of each dollar is paid in tax.
f 900 ml out of 1 litre of fruit juice is water.
g 3120 cm^3 out of 4000 cm^3 of gas in a balloon is nitrogen.

Expressing decimals as percentages

To write decimals as percentages you can first write your decimal as a fraction with denominator 100.

EXAMPLE 5

Write **a** 0.35 **b** 0.3 as percentages.

a $0.35 = \frac{35}{100} = 35\%$

b $0.3 = \frac{3}{10} = \frac{30}{100} = 30\%$

A quicker way to change a decimal to a percentage is to multiply by 100.

EXAMPLE 6

Write 0.316 as a percentage.

$0.316 = 0.316 \times 100\%$
$= 31.6\%$

Exercise 13D

1 Study Example 5, then copy and complete the table below.

Decimal	Per cent fraction	%
0.5	$\frac{50}{100}$	50%
0.68	$\frac{68}{100}$	68%
0.9		
0.75		
0.25		
0.03		

2 Look at the table in Question **1**. Can you see a quick way of changing a decimal to a percentage?

3 Using the pattern you discovered in Question **2**, write each decimal as a percentage.

a 0.8 **b** 0.41 **c** 0.97
d 0.06 **e** 0.01

4 Write each of these decimals as a percentage:

a 0.1 **b** 0.134 **c** 0.025
d 0.796 **e** 0.0234 **f** 0.8175

Expressing percentages as decimals

Percentages are easily turned back to decimals.

EXAMPLE 7

Write 45% as a decimal.

$45\% = \frac{45}{100} = 0.45$

Exercise 13E

1 Copy and complete the table:

%	Per cent fraction	Decimal
32	$\frac{32}{100}$	0.32
5	$\frac{5}{100}$	0.05
12	$\frac{12}{100}$	
8		
10		
16		
39		
64		

2 Look at the table you completed in Question **1**.

a Compare the numbers in the first and third columns.

b How many places do the digits move?

3 Copy and complete the table:

Fraction	Decimal	%
$\frac{2}{5}$		
	0.8	
		70
$\frac{3}{4}$		
	0.65	
		85
$\frac{6}{25}$		
	0.06	
		4

4 Write each percentage as a decimal:

a 19% **b** 29% **c** 66%

d 3% **e** 2% **f** 79%

g 1%

5 Look at the fractions $\frac{36}{40}$ and $\frac{51}{60}$.

a Can you tell quickly which is the larger fraction?

b Now express each fraction as a percentage. Which one is larger?

6 Which of these is larger?

a $\frac{1}{4}$ or 26%

b 0.6 or 62%

c $\frac{9}{20}$ or 40%

7 Which of these is smaller?

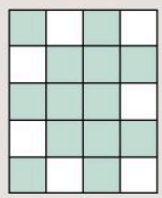
a $\frac{300}{500}$ or 65%

b 0.04 or 39%

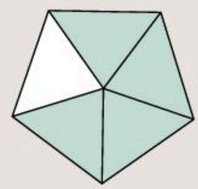
c $\frac{4}{25}$ or 15%

8 Which of these diagrams have more than 60% shaded?

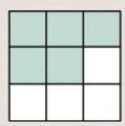
a



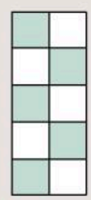
b



c



d



9 Copy and complete the table:

Fraction	Decimal	%
$\frac{1}{8}$		
	0.375	
		62.5
$\frac{3}{16}$		
		43.75
$\frac{2}{3}$		
$\frac{3}{7}$		

10 Write these numbers in order of size, smallest first.

$\frac{9}{20}$, 42%, $\frac{11}{25}$, 0.46, $\frac{43}{100}$

11 Jane's doctor has recommended a diet low in fat. She wants to buy the cheese with the least amount of fat. Here are the packets:

Jane always buys the 400 g cheese because it has only 144 g of fat, which is lower than the 204 g of fat in the other cheese. Is she doing the right thing? Why?

TECHNOLOGY

Review online what you have learnt. Visit 'How to work with fractions' at

www.mathsisfun.com/numbers

Then have a go at the conversion game at

www.nrich.maths.org/1249

Can you score 400?

13.3 Finding percentages of amounts

Percentages are often used in everyday transactions. For example,

Discount 5%   Tax 15%

In each case you have to find the percentage of an amount to find the discount or the tax.

EXAMPLE 8

Find 75% of 20 oranges.

$$75\% \text{ of } 20 = \frac{75}{100} \text{ of } 20 = \frac{75}{100} \times 20$$
$$= \frac{3}{4} \times 20 = 3 \times \frac{1}{4} \times 20$$
$$= 3 \times 5 = 15$$

So 75% of 20 oranges is 15 oranges.

EXAMPLE 9

A 10% cash discount is given on a table priced at \$1450.

How much is the discount?

$$\text{Discount} = 10\% \text{ of } \$1450 = \frac{10}{100} \times \$1450$$
$$= \frac{1}{10} \times \$1450 = \$145$$

Exercise 13F

1 Work out:

- **a** 15% of 100 cm **b** 15% of 60 kg
- **c** 20% of \$35 **d** 35% of 80 eggs
- **e** 50% of 120 tigers **f** 50% of 72 mangoes
- **g** 60% of 40 fish **h** 70% of 50 litres.

2 Find these percentages:

- **a** 50% of \$50 **b** 50% of \$64
- **c** 25% of \$64 **d** 75% of \$64
- **e** 100% of \$64 **f** 100% of \$40
- **g** 50% of \$40 **h** 75% of \$32

3 There are 25 children in Class 2A and 20% of them wear glasses.
How many wear glasses?

4 There are 60 members of the All Stars club. 50% prefer cricket, 30% football and the rest basketball. How many prefer each game?

5

- **a** Find the cash discount for the
  i blender **ii** stove.
- **b** How much would a cash customer pay for the
  i blender **ii** stove?

6 Of 80 drivers stopped by police, 20% did not have licences, 30% had defective lights and 5% failed to observe a stop sign. Calculate the number of drivers who committed each offence.

7 The fuel tank of a lorry holds 80 litres when it is 40% full. How much does the tank hold when it is full?

Consolidation

Example 1

Write:

a $\frac{3}{8}$ as a percentage
b 0.75 as percentage
c 65% as a fraction

---

a $\frac{3}{8} = \frac{3}{8} \times 100\% = \frac{300}{8}\%$
$= 37.5\%$

b $0.75 = 0.75 \times 100\%$
$= 75\%$

c $65\% = \frac{65}{100} = \frac{13}{20}$

Example 2

Find:

a 5% of \$40 **b** 15% of \$8

---

a 5% of \$40

$= \frac{5}{10\cancel{0}} \times 4\cancel{0}$

$= \frac{20}{10}$

$= \$2$

b 15% of \$8

$= \frac{\cancel{15}^{\,3}}{\cancel{100}_{\,20}} \times 8$

$= \frac{24}{20}$

$= \frac{12}{10}$

$= \$1.20$

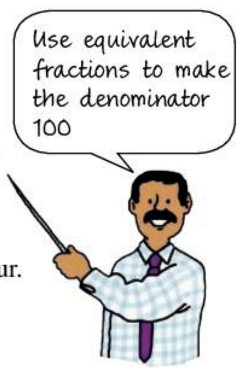
Example 3

A 400 g loaf of wholemeal bread contains 228 g of wholemeal flour. An 800 g loaf of white bread contains 472 g of white flour. Which loaf contains a higher percentage of flour?

Wholemeal bread:

Write as a fraction

$$\frac{228}{400} \overset{\div 4}{=} \frac{57}{100}$$



57% of the wholemeal loaf is flour.

White bread:

$$\frac{472}{800} \overset{\div 8}{=} \frac{59}{100}$$

59% of the white loaf is flour.

The white loaf contains a higher percentage of flour.

Exercise 13

1 Change these test scores into percentages:
a 7 out of 10
b 36 out of 80
c 105 out of 150

2 Write as a percentage.
a 72 km have been travelled out of a journey of 200 km.
b The distance still left to travel in part **a**.
c A score of $\frac{66}{150}$ in a test.
d 120 g out of 400 g in a recipe is flour.

3 Copy and complete the table:

Percentage	Fraction	Decimal
38%		
	$\frac{4}{5}$	
		0.02

4 Find:
a 10% of \$50 **b** 20% of 10 cm
c 50% of 8 km **d** 5% of \$60
e 15% of \$80 **f** 1% of 50 litres
g 3% of 500 m **h** 8% of 25 g

5 Part of James Khan's annual school report is shown; it was sent to his parents last July. It shows his marks in the tests at the end of each term.

SCHOOL REPORT

NAME: James Khan

Subject	Marks 1st term	Marks 2nd term	Marks 3rd term
English	$\frac{135}{150}$	$\frac{43}{50}$	$\frac{70}{100}$
Spanish	$\frac{38}{40}$	$\frac{89}{100}$	$\frac{166}{200}$
Science	$\frac{68}{100}$	$\frac{72}{100}$	$\frac{47}{100}$
History	$\frac{108}{150}$	$\frac{96}{120}$	$\frac{84}{100}$

Each mark is shown as a fraction of the full marks for the subject.

- **a** Can you tell quickly whether James's History marks improved over the year?
- **b** Can you tell quickly which of the subjects he got the highest mark in during the first term?
- **c** Do you think this is a good way to show the marks? Why?

6 Copy the school report in Question **5**, but this time show all the marks as percentages.

7 Look at the report you completed in Question **6**.

- **a** Did James's History marks improve over the year?
- **b** In the first term, which of his marks was highest?
- **c** In which subject and which term did he get his highest mark?

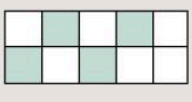
8 Could you answer the parts of Question **7** quickly? Do you think percentages are a good way to show test marks? Why?

9 Which of these is larger?

- **a** $\frac{3}{5}$ or 65%
- **b** 0.4 or 41%

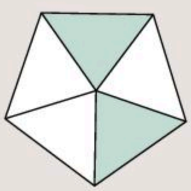
10 Which of these diagrams has 40% shaded?

a

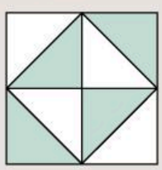


b

c



d



11 Write these numbers in order of size, smallest first.

$\frac{11}{20}$, 53%, $\frac{13}{25}$, 0.51, $\frac{54}{100}$

12 Paul's doctor has recommended a diet low in salt. When he eats pizza he usually has 3 slices (about 200 g) with a salad. Which pizza should he buy? Explain why, using percentages.

13 Copy out the text below. Draw lines to show the calculations that have the same answer. One has been done for you.

$\frac{3}{4}$ of 32	$\frac{2}{13}$ of 169
$\frac{2}{3}$ of 39	30% of 80
40% of 75	20% of 280
35% of 160	$\frac{5}{12}$ of 72

Summary

You should know ...

1 'Per cent' means 'out of a hundred'.

For example:

$\frac{25}{100}$ can be written as 25%

2 How to write percentages as fractions or decimals.

For example:

$$36\% = \frac{36}{100} = 0.36$$

$$\frac{3}{5} = \frac{3}{5} \times 100\% = 60\% = 0.6$$

3 How to find a percentage of an amount.

For example:

15% of 80 oranges

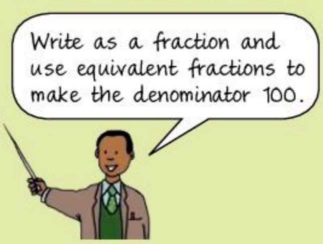
$$\begin{aligned} &= \frac{15}{100} \times 80 \\ &= \frac{3}{20} \times 80 \\ &= 3 \times \frac{1}{20} \times 80 \\ &= 3 \times 4 \\ &= 12 \text{ oranges} \end{aligned}$$

4 How to express a smaller quantity as a fraction or percentage of a larger quantity.

For example:

900 g of peas contain 36 g of sugar. What percentage of the peas is sugar?

$$\frac{36}{900} \underset{\div 9}{\overset{\div 9}{=}} \frac{4}{100}$$



So 4% of the peas is sugar.

Check out

1 **a** David got 85 out of 100 in a test. What is this as a percentage?

b 90% of the class were present for their exam. What percentage were absent?

2 Copy and complete:

Fraction	Decimal	Percentage
$\frac{4}{5}$		
	0.45	
		70%
$\frac{5}{8}$		

3 **a** Find:

i 10% of \$600

ii 15% of \$20

b A radio priced at \$95 is reduced by 20% in a sale. How much was taken off the price?

4 A 400 g pizza contains 12 g of sugar, 36 g of fat, 4 g of salt, 48 g of protein and 140 g of carbohydrate. What percentage of the pizza is

a sugar

b fat

c salt

d protein

e carbohydrate?

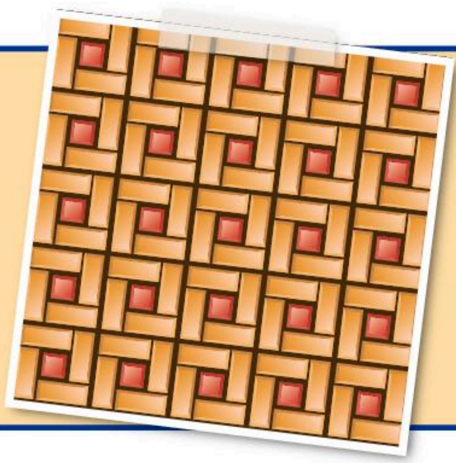
14 Sequences, functions and graphs

Objectives

- Generate sequences from spatial patterns and describe the general term in simple cases.
- Generate terms of an integer sequence and find a term given its position in the sequence; find simple term-to-term rules.
- Represent simple functions using words, symbols and mappings.
- Generate coordinate pairs that satisfy a linear equation, where y is given explicitly in terms of x; plot the corresponding graphs; recognise straight-line graphs parallel to the x- or y-axis.

What's the point?

Sequences occur in many different places, for example in nature, computer programming, science, engineering and in design. A designer may want to decorate a kitchen or bathroom with tiles. He would need to know how the sequence of tiles continues and how many of each different-coloured tile he needs.

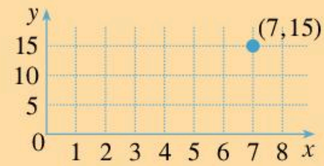


Before you start

You should know ...

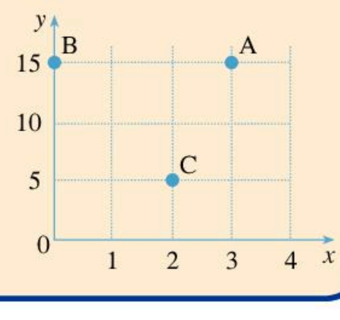
1 How to write and plot the coordinates of a point.
For example:
In the graph below the coordinates of the point shown are (7, 15).

Coordinates are also sometimes called coordinate pairs or ordered pairs.



Check in

1 From the graph, write down the coordinates of A, B and C.

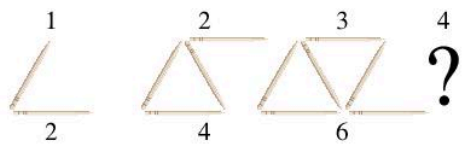


2 How to add and subtract with negative numbers. *For example:* $2 - 5 = {}^{-}3$ ${}^{-}2 + 8 = 6$	**2** Work out: **a** $7 - 9$ **b** ${}^{-}7 + 20$ **c** ${}^{-}3 - 5$ **d** ${}^{-}14 + 10$

14.1 Looking for patterns

EXAMPLE 1

Look at the pattern made from toothpicks:



The 1st pattern has 2 toothpicks.
The 2nd pattern has 4 toothpicks.
The 3rd pattern has 6 toothpicks.

You can make a table:

Pattern number	Number of toothpicks
1	2
2	4
3	6

The number of toothpicks is related to the pattern number.

a How many toothpicks are needed to make the 4th pattern?
b How many to make the 10th pattern?
c How is the number of toothpicks related to the pattern number?

a $4 \times 2 = 8$
b $10 \times 2 = 20$
c The number of toothpicks is double the pattern number.

Exercise 14A

1 Look at this pattern:

Pattern Number	1	2	3	4	5
					?
Number of dots	2	3	4	5	

a How many dots will make the 5th pattern?
b How many dots will make the 20th pattern?
c How is the pattern number related to the number of dots?

2 Repeat Question **1** for each of the following patterns:

a

Pattern Number	1	2	3	4	5
					?
Number of dots	3	6	9	12	

b

Pattern Number	1	2	3	4	5
					?
Number of dots	3	4	5	6	

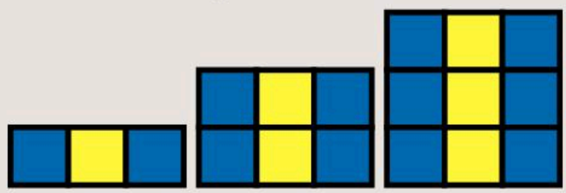
c

Pattern Number	1	2	3	4	5
					?
Number of dots	0	1	2	3	

d

Pattern Number	1	2	3	4	5
					?
Number of dots	3	5	7	9	

3 Look at the diagrams:

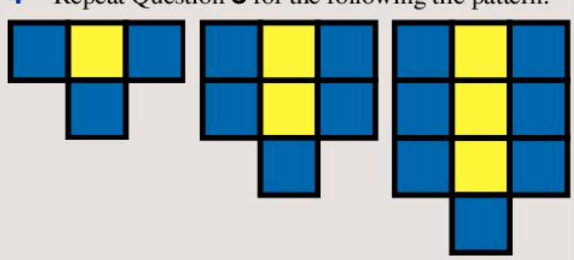


a Draw the next two diagrams.
b Copy and complete the table:

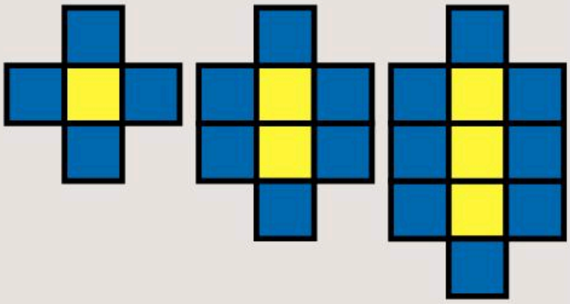
Number of yellow tiles	1	2	3	4	5
Number of blue tiles					

c Describe the pattern linking the number of blue tiles to the number of yellow tiles.
d How many blue tiles are there if there are 10 yellow tiles?

4 Repeat Question **3** for the following tile pattern:



5 Repeat Question **3** for the following tile pattern:



6

a Draw the next two diagrams in the sequence.

b Copy and complete the table:

Number of black counters	1	2	3	4	5
Number of green counters					

c Describe the relationship between the number of black counters and the number of green counters.

d How many green counters are there when there are 100 black counters?

7 Repeat Question **6** for this pattern:

8 Repeat Question **6** for this pattern:

9 Draw some diagrams of your own with green and black counters (make sure there is one black counter in the first diagram, two in the second diagram, and so on). Look for the patterns in your diagrams.

10 Think of your work from Chapter 2 on expressions and Chapter 8 on formulae.

a For each of Questions **3** to **5**, write a formula for the number of blue tiles b, if there are n yellow tiles. Start by writing an expression for the number of blue tiles there will be if there are n yellow tiles.

b For each of Questions **6** to **8**, write a formula for the number of green counters g, if there are n black counters. Start by writing an expression for the number of green counters there will be if there are n black counters.

14.2 Number sequences

The numbers 5, 8, 11, 14, 17, 20, . . . follow a pattern.

You can add 3 to each number to get the next number ($5 + 3 = 8$, $8 + 3 = 11$, and so on).

- A **sequence** is a set of numbers that follows a pattern.
- Each number in a sequence is called a **term**. For example, the third term in the sequence 5, 8, 11, 14, 17, 20, . . . is 11.
- A **term-to-term rule** describes how to get from one term to the next. For the sequence 5, 8, 11, 14, 17, 20, . . . the term-to-term rule is add 3.

By continuing the pattern we can find terms further along in the sequence. For the sequence above we can find the tenth term by continuing to add 3 until we have 10 numbers:

5, 8, 11, 14, 17, 20, 23, 26, 29, 32

So the tenth term is 32.

Exercise 14B

1 Write down the next two terms in the sequence:

a 6, 12, 18, 24, 30, . . .
b 2, 4, 6, 8, 10, . . .
c 10, 30, 50, 70, 90, . . .
d 40, 36, 32, 28, 24, . . .
e 6, 9, 12, 15, 18, . . .
f 5, 11, 17, 23, 29, . . .
g 20, 18, 16, 14, 12, . . .
h 3, 2, 1, 0, $^{-}1$, $^{-}2$, . . .
i $^{-}40$, $^{-}30$, $^{-}20$, $^{-}10$, 0, . . .
j 10, 8, 6, 4, 2, . . .

2 For each of the sequences in Question **1** write down the term-to-term rule.

3 For each of the sequences in Question **1** write down the tenth term.

4 A number is added to each term in the sequence below, to give the next term. The number added may be the same each time or it may be different. If different, there is still a pattern.
Find the pattern, then write down the next three terms of each sequence.

- **a** 1, 6, 11, 16, 21, 26, . . .
- **b** 1, 2, 4, 7, 11, 16, . . .
- **c** 1, 2, 5, 10, 17, 26, . . .
- **d** 1, 11, 31, 61, 101, 151, . . .
- **e** 1, 4, 10, 19, 31, 46, . . .
- **f** 0, 1, 5, 14, 30, 55, . . .

5 What are the missing numbers in these sequences?

- **a** 22, 28, 34, □, 46, 52, □, 64, . . .
- **b** □, 2, 7, 12, 17, □, 27, . . .
- **c** $^{-}3$, □, 1, □, □, 7, 9, 11, 13, 15, . . .
- **d** 4, □, □, $^{-}2$, $^{-}4$, $^{-}6$, $^{-}8$, □, . . .

6 Write down the first five terms of these sequences:

- **a** the first term is 7, the term-to-term rule is add 4
- **b** the first term is 20, the term-to-term rule is subtract 5
- **c** the first term is 1, the term-to-term rule is multiply by 4
- **d** the first term is 64, the term-to-term rule is divide by 2
- **e** the fourth term is 9, the term-to-term rule is add 2
- **f** the third term is 70, the term-to-term rule is subtract 10
- **g** the second term is 4, the term-to-term rule is multiply by 2
- **h** the fifth term is 6, the term-to-term rule is add 3.

7 Sometimes the pattern for a sequence is found by multiplying or dividing. Write down the next two terms and describe the term-to-term rule for these sequences:

- **a** 1, 2, 4, 8, 16, . . .
- **b** 5, 10, 20, 40, 80, . . .
- **c** 1, 10, 100, 1000, 10 000, . . .
- **d** 192, 96, 48, 24, 12, . . .

8 Write down the next two terms, the term-to-term rule and the tenth term of this sequence:
19 683, 6561, 2187, 729, 243, . . .

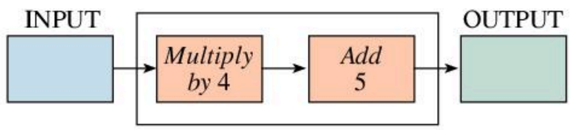
9 Copy and complete this sequence:
1, □, □, 10, □, . . .
Is there more than one way to do this?

10 The cards below can be used to create two different sequences. One of the sequences goes up in eights. Describe the term-to-term rule of the other sequence.

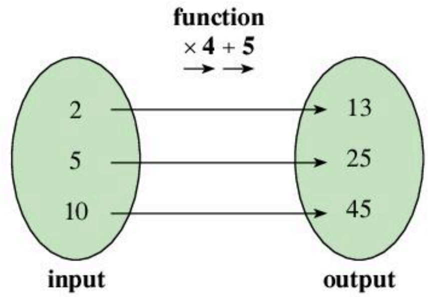
55	10	18	23	14	16
8	31	15	12	39	47

14.3 Functions

The drawing below shows a **function machine**.
A function machine is a type of flow chart.



A number entered into the function machine is called the **input**. When the function machine uses the **function**, e.g. 'multiply by 4 then add 5', it produces an **output**. If the input is 2 then the output is 13 because $2 \times 4 + 5 = 13$. If the input is 10 the output is 45, and so on. We can use a **mapping diagram** like this one to show which outputs go with which inputs:

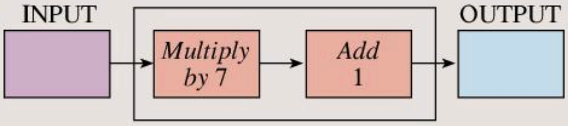


The same thing can be shown in a table:

Input	Output
2	13
5	25
10	45

Exercise 14C

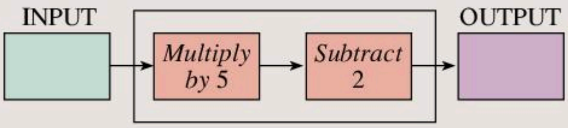
1



Use the function machine to fill in the table.

Input	Output
1	
2	
3	
4	
5	

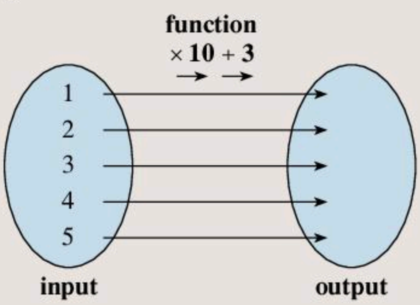
2



Use the function machine to fill in the table.

Input	Output
1	
2	
3	
4	
5	

3 Copy and complete the mapping diagram using the function shown.



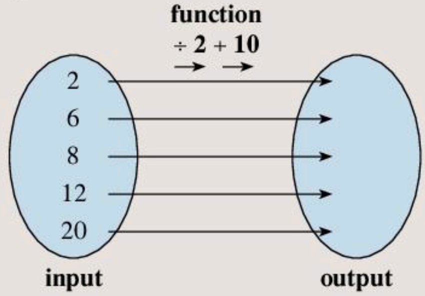
4 Look at the sequence of outputs for Question **1**. What is the term-to-term rule for the sequence of outputs?

5 Look at the sequence of outputs for Question **2**. What is the term-to-term rule for the sequence of outputs?

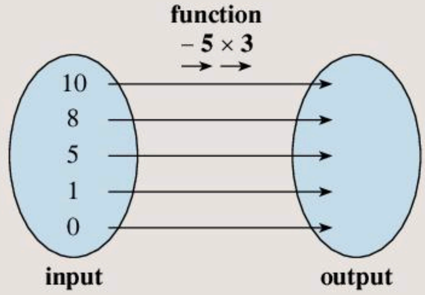
6 Look at the sequence of outputs for Question **3**. What is the term-to-term rule for the sequence of outputs?

7 Copy and complete these mapping diagrams using the functions shown.

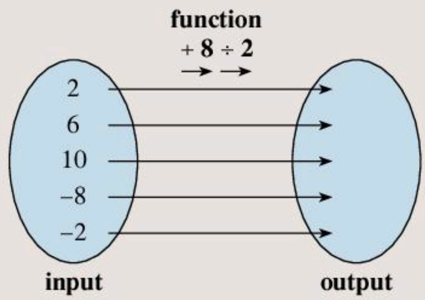
a



b



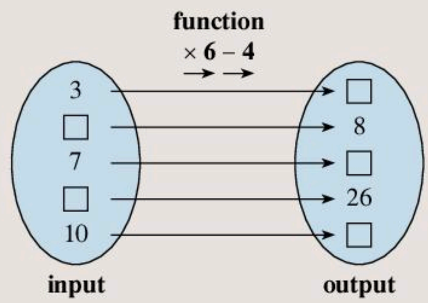
c



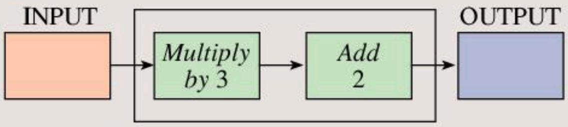
8 Using your answers to Questions **4**, **5** and **6**, which part of the function machine relates to the term-by-term rule for the sequence of outputs?

9 In Questions **1**, **2** and **3**, if the input is n, what is the output?

10 Copy and complete the mapping diagram using the function shown.



11 Look at the function machine:



a Do you agree that 5, 11, 14, 20, 32 and 302 are output numbers for your machine?

b What are the input numbers that make the outputs listed in part **a**?

12 Look at this table:

Input	1	2	3	4	5	6	7	8	9	10
Output	8	14	20	26	32	38	44	50	56	62

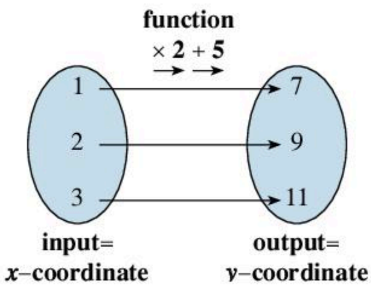
Draw the function machine that goes with this table.

14.4 Graphs of linear functions

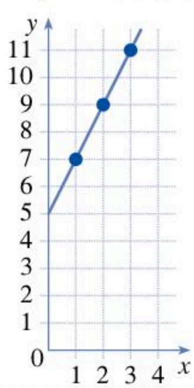
Mappings and functions can be shown as graphs. A **linear function** is a function that produces a straight line when the graph is drawn. We use the x-coordinate as the input. The y-coordinate is the output.

EXAMPLE 2

Draw the graph representing the linear function $\times 2 + 5$ if the x-coordinate is the input and the y-coordinate is the output.

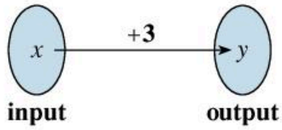


We need to plot the points (1, 7), (2, 9), and (3, 11).



In Example 2, all of the points are joined using a straight line. This is because all of the points between $x = 1$ and $x = 3$ follow the same rule. For example, if you had $x = 1.5$ as your input, the output would be $y = 1.5 \times 2 + 5 = 8$, which also lies on the same line. The line represents *all* the mappings of the linear function.

When we want to represent a linear function as a graph we can write it as a **linear equation**. For example, the linear equation $y = x + 3$ represents the mapping:



This can also be written as $x \rightarrow x + 3$

The equation $y = x + 3$ means that to find y-coordinates you apply the function '+ 3' to the x-coordinates.

Drawing a table is often quicker than drawing a mapping diagram.

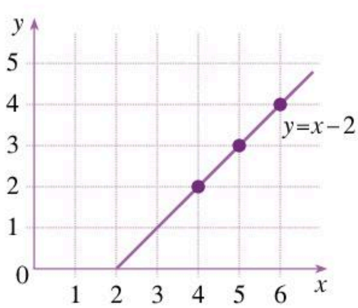
EXAMPLE 3

Draw the graph of $y = x - 2$

This means that to find the y-coordinates you need to subtract 2 from the x-coordinates:

x	y
4	$4 - 2 = 2$
5	$5 - 2 = 3$
6	$6 - 2 = 4$

Then plot the points (4, 2), (5, 3) and (6, 4).



Join these points with a straight line (which can be extended either way to go on forever). Extend the line to fill your graph.

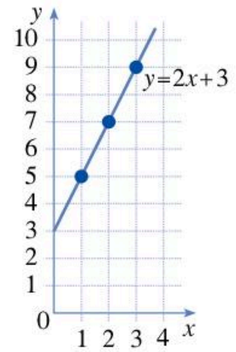
EXAMPLE 4

Draw the graph of $y = 2x + 3$

This means to find the y-coordinates you need to multiply the x-coordinates by 2 then add 3. So use the function $\times\ 2 + 3$:

x	y
1	$1 \times 2 + 3 = 5$
2	$2 \times 2 + 3 = 7$
3	$3 \times 2 + 3 = 9$

Then plot the points (1,5), (2,7) and (3,9) and join them with a straight line.



Exercise 14D

1 Copy and complete the sentences using the cards below (cards can be used more than once).

multiply	add	3	−1	subtract	halve

a When $y = 10x$, to find the y-coordinates you . . . the x-coordinates by 10.

b When $y = x - 4$, to find the y-coordinates you . . . 4 from the x-coordinates.

c When $y = 3x - 5$, to find the y-coordinates you . . . the x-coordinates by . . . then you . . . 5.

d When $y = \frac{1}{2}x + 10$, to find the y-coordinates you . . . the x-coordinates then you . . . 10.

e When $y = {}^{-}x$, to find the y-coordinates you multiply the x-coordinates by . . . .

f When $y = 10 - x$, to find the y-coordinates you . . . the x-coordinates from 10.

To complete Questions **2** to **4**, draw coordinate axes for 0 to 4 on the x-axis and 0 to 10 on the y-axis.

2 Copy and complete the table. Draw the straight line graph for $y = x + 2$

x	y
1	$1 + 2 = 3$
2	$\square + 2 = \square$
3	$\square + 2 = \square$

3 Copy and complete the table. Draw the straight line graph for $y = 3x$

x	y
1	$1 \times 3 = 3$
2	$\square \times 3 = \square$
3	$\square \times 3 = \square$

4 Copy and complete the table. Draw the straight line graph for $y = 2x - 1$

x	y
1	$1 \times 2 - 1 = \square$
2	$\square \times 2 - 1 = \square$
3	$\square \times 2 - 1 = \square$

5 To draw a straight line graph you only need to plot 2 points. Why is it a good idea actually to plot 3 or more points?

6 The equation of a straight line graph is $y = 4x - 1$. Copy and complete these coordinate pairs by using the given x-coordinate to calculate the missing y-coordinate.

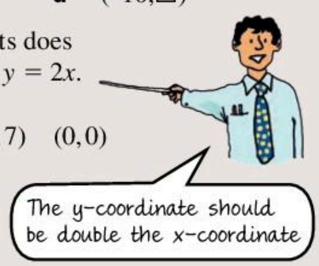
a $(2,\square)$ **b** $(1,\square)$
c $(3,\square)$ **d** $({}^{-}1,\square)$

7 The equation of a straight line graph is $y = \frac{1}{2}x + 4$. Copy and complete these coordinate pairs:

a $(2,\square)$ **b** $(4,\square)$
c $(0,\square)$ **d** $({}^{-}10,\square)$

8 One of these points does not lie on the line $y = 2x$. Which is it?

(1, 2) (4, 8) (3, 7) (0, 0)



9 Which of these points lie on the line $y = 5x - 10$?

$(1, {}^{-}5)$ (2, 0) (4, 5) (5, 10)

10 Draw the line $y = 2x - 3$ by choosing three suitable x-coordinates and working out the corresponding y-coordinates.

11 Copy and complete the table for $y = 3x + 5$. Draw the line $y = 3x + 5$, choosing suitable axes.

x	$^{-}1$	0	1	2
y				11

12 Copy and complete the table for $y = {}^{-}2x + 3$. Draw the line $y = {}^{-}2x + 3$, choosing suitable axes.

x	0	1	2	3
y		1		

13 Copy and complete the table for $y = 5 - x$. Draw the line $y = 5 - x$, choosing suitable axes.

x	0	1	2	3
y		4		

14

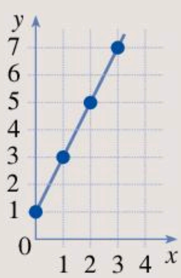
a Complete the coordinate pairs using the line graph above.

i (0,□) **ii** (1,□)
iii (2,□) **iv** (3,□)

b What is the pattern connecting the y-coordinate to the x-coordinate?

c What is the equation of the line?

15



a Complete the coordinate pairs using the line graph above.

i (0,□) **ii** (1,□)
iii (2,□) **iv** (3,□)

b What is the pattern connecting the y-coordinate to the x-coordinate?

c What is the equation of the line?

Horizontal and vertical lines

Some equations do not have the letter x in them.

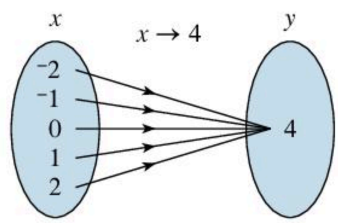
What does the graph of an equation such as $y = 4$ look like?

EXAMPLE 5

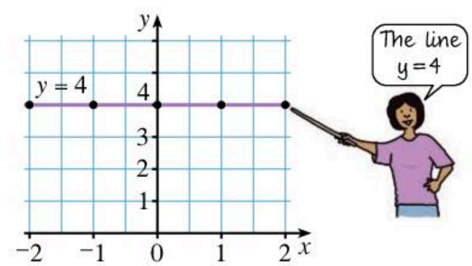
Draw a graph of the equation $y = 4$.

The equation $y = 4$ describes the mapping $x \rightarrow 4$.

A diagram of this mapping is:

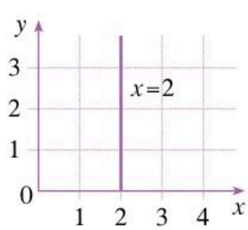


So a graph of the equation $y = 4$ is:



The equation $y = 4$ means that the y-coordinate is always 4, no matter what the x-coordinate is. Any equations in the form $y =$ 'a number' are horizontal lines.

Vertical lines are described by $x =$ 'a number'. For example, $x = 2$ means that the x-coordinate is always 2, no matter what the y-coordinate is. So the line $x = 2$ looks like this:



Exercise 14E

1 **a** Using $x = {}^{-}2$, $x = 0$ and $x = 2$, name three points that are on the line:

  i $y = 2$

  ii $y = {}^{-}1$

  iii $y = 6$

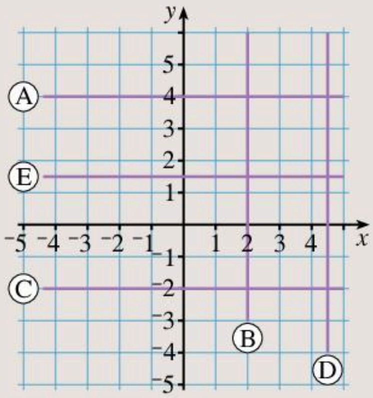
b Draw the graphs of the equations in part **a**.

2 Plot the coordinates $(3, {}^{-}2)$, $(3, {}^{-}1)$ $(3,0)$, $(3,1)$, $(3,2)$ on a pair of axes. What is the equation of the line through these points?

3 Complete the coordinate pairs $(\square, {}^{-}1)$, $(\square, 0)$, $(\square, 1)$ and $(\square, 2)$, then draw graphs for:

a $x = 4$ **b** $x = {}^{-}3$ **c** $x = 1\frac{1}{2}$

4



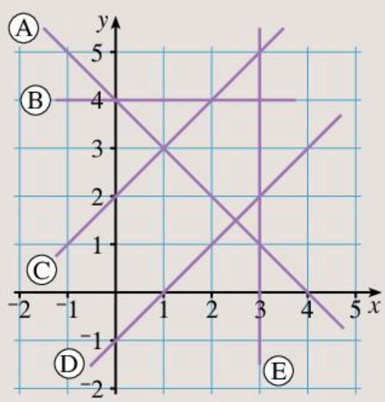
Write down the equations of the lines A, B, C, D and E.

5 Which line in the graph has the equation:

a $y = 4$ **b** $x = 3$

c $y = x + 2$ **d** $y = x - 1$

e $y = 4 - x$?



6 The following four sets of points lie on straight lines:

a $(3,2)$, $({}^{-}2,{}^{-}3)$, $(0,{}^{-}1)$, $(1,0)$

b $({}^{-}8,{}^{-}10)$, $(0,6)$, $(1,8)$, $(3,12)$

c $(2,0)$, $(0,{}^{-}1)$, $(4,1)$, $({}^{-}4,{}^{-}3)$

d $(0,{}^{-}8)$, $(1,{}^{-}4)$, $(2,0)$, $(4,8)$

Which of these equations represents which line?

$y = 2x + 6$ $y = x - 1$

$y = 4x - 8$ $y = \frac{1}{2}x - 1$

7 Write down an equation for each set of ordered pairs:

a $(0,0)$, $(1,2)$, $(2,4)$, $(3,6)$, $(4,8)$

b $(0,1)$, $(1,3)$, $(2,5)$, $(3,7)$, $(4,9)$

c $(0,{}^{-}1)$, $(1,1)$, $(2,3)$, $(3,5)$, $(4,7)$

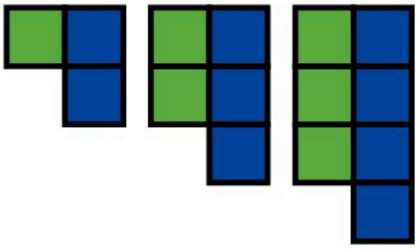
d $(0,{}^{-}4)$, $(1,{}^{-}3)$, $(2,{}^{-}2)$, $(3,{}^{-}1)$, $(4,0)$

e $(0,{}^{-}2)$, $(1,0)$, $(2,2)$, $(3,4)$, $(4,6)$

Consolidation

Example 1

a Draw the next two diagrams in the pattern.

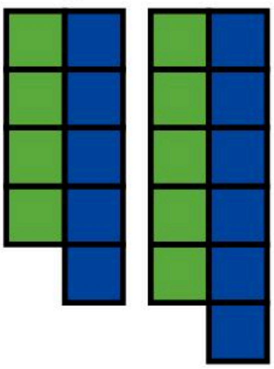


b Draw a table showing the relationship between the number of blue tiles and the number of green tiles.

c Describe the relationship between the number of blue tiles and the number of green tiles.

d How many blue tiles will there be if there are 50 green tiles?

a The next two diagrams are:



b

Number of green tiles	1	2	3	4	5
Number of blue tiles	2	3	4	5	6

c The number of blue tiles is always one more than the number of green tiles.

d If there are 50 green tiles there will be 51 blue tiles.

Example 2

a Describe the term-to-term rule in the sequence 7, 16, 25, 34, 43, . . .

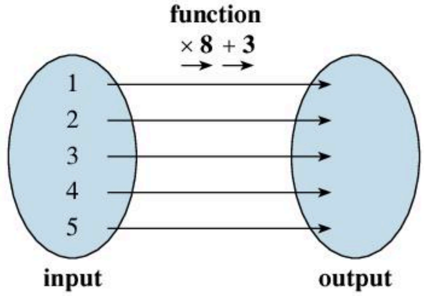
b What is the tenth term of this sequence?

a The term-to-term rule is add 9.

b Continuing the sequence to the tenth term, you get 7, 16, 25, 34, 43, 52, 61, 70, 79, 88, ...
So the tenth term is 88.

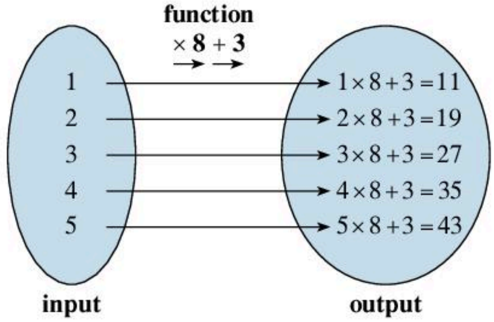
Example 3

a Copy and complete the mapping diagram using the function shown.

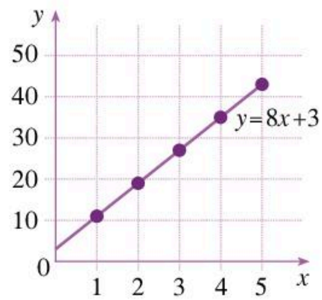


b Use your answer to part **a** to draw the line $y = 8x + 3$

a



b Using the input from part **a** as x and the output as y, the coordinates (or ordered pairs) are (1, 11), (2, 19), (3, 27), (4, 35) and (5, 43).
The graph of $y = 8x + 3$ is:



Exercise 14

1

a Draw the next two diagrams in the sequence.

b Copy and complete the table:

Number of black counters	1	2	3	4	5
Number of green counters					

c Describe the relationship between the number of black counters and the number of green counters.

d How many green counters are there when there are 100 black counters?

2 Write down the next two terms in these sequences.

a 8, 18, 28, 38, 48, . . .

b $^{-}5$, 1, 7, 13, 19, . . .

c 12, 8, 4, 0, $^{-}4$, . . .

d 1, 2, 4, 8, 16, . . .

3 For each of the sequences in Question **2**, write down the term-to-term rule and the tenth term.

4 If the first term in a sequence is 3 and the term-to-term rule is add 6, write down the first five terms.

5 Find the missing term in these sequences

a 12, 18, 24, □, 36, . . .

b $^{-}2$, □, $^{-}6$, $^{-}8$, $^{-}10$, . . .

c □, 40, 20, 10, 5, . . .

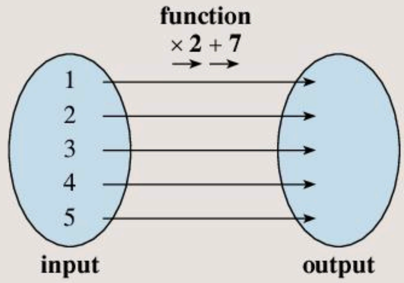
d $^{-}29$, $^{-}22$, □, $^{-}8$, $^{-}1$, . . .

6 If the third term is 10 and the term-to-term rule is subtract 2, write down the first five terms of the sequence.

7 Copy and complete the table using the function machine $+ 3 \times 4$.

Input	Output
1	
2	
3	
4	
5	

8 **a** Copy and complete the mapping diagram using the function shown.



b Use your answer to part **a** to draw the line $y = 2x + 7$

9 Copy and complete the table for each equation, then draw the line.

x	0	1	2	3
y				

a $y = 2x + 2$

b $y = 5x$

c $y = x$

d $y = 5 + x$

e $y = {}^{-}x$

f $y = 10 - x$

10 Copy and complete the table for $y = \frac{1}{2}x + 1$. Draw the line.

x	0	2	4	6
y				

11 **a** Draw axes for $^{-}5$ to 5 on the y- and x-axes.

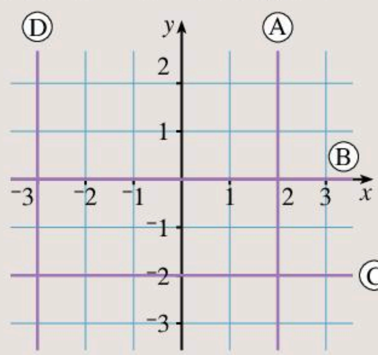
b Draw these lines on your axes.

i $y = 4$ **ii** $x = 3$

iii $y = {}^{-}1$ **iv** $x = {}^{-}2$

c The four lines intersect at the four vertices of a quadrilateral. What is the shape of this quadrilateral?

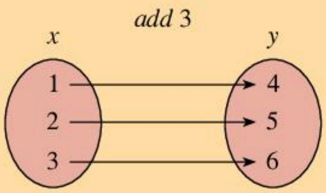
12 Name the four lines in the diagram.



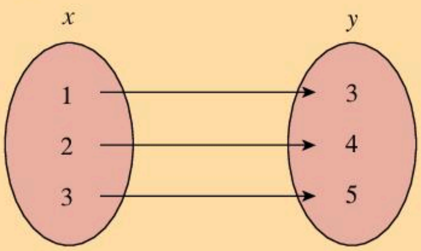
Summary

You should know …

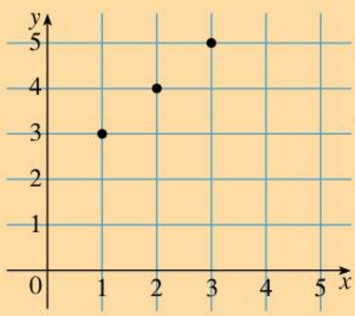
1 A mapping diagram can be used to show outputs of functions.
For example:



2 Mappings can be represented as ordered pairs, or coordinates.
For example:
In the mapping diagram



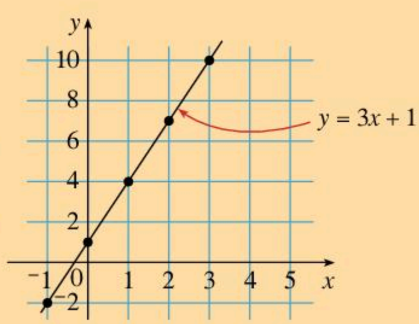
the pairs are (1, 3), (2, 4), (3, 5). These pairs can be shown on a graph.



3 How to draw the graph of an equation using a table of values.
For example:

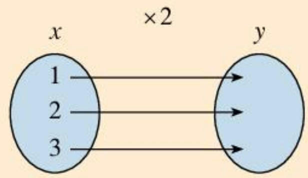
$y = 3x + 1$

x	$^{-}1$	0	1	2	3
y	$^{-}2$	1	4	7	10



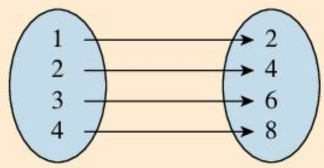
Check out

1 Copy and complete this mapping diagram:

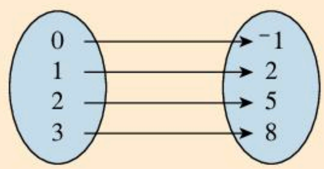


2 For each mapping diagram, write down the ordered pairs and plot them on a graph.

a



b



3 Draw graphs of

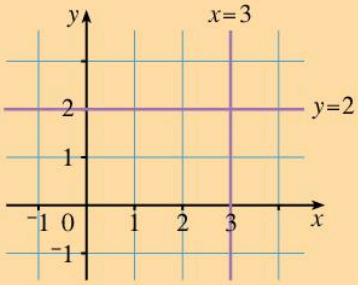
a $y = 2x + 5$

b $y = 3x - 2$

by copying and completing the table of values in each case.

x	$^{-}1$	0	1	2
y				

4 Equations of the form y = 'a number' are horizontal lines. Vertical lines are described by x = 'a number'.
For example:
$x = 3$ means that that the x-coordinate is always 3, $y = 2$ means that that the y-coordinate is always 2. The lines $x = 3$ and $y = 2$ look like this:



4 Draw these lines:

a $y = 1$
b $y = {}^{-}3$
c $x = 4$
d $x = 0$

5 A sequence is a list of numbers following a pattern. The term-to-term rule describes how to get from one term in the sequence to the next.
For example:

8, 11, 14, 17, 20, . . .

For this sequence, the term-to-term rule is add 3.

5 Write down the term-to-term rule and next two terms for these sequences.

a 7, 18, 29, 40, 51, . . .
b 10, 4, ⁻2, ⁻8, ⁻14, . . .
c 3, 6, 12, 24, 48, . . .

15 Symmetry and transformations

Objectives

- Recognise line and rotational symmetry in 2D shapes and patterns; draw lines of symmetry and complete patterns with two lines of symmetry; identify the order of rotational symmetry.
- Transform 2D points and shapes by
  – reflection in a given line
  – rotation about a given point
  – translation.
- Know that shapes remain congruent after reflection, rotation and translation.

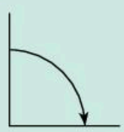
What's the point?

Symmetry occurs everywhere in nature from the radial symmetry of a starfish to the reflectional symmetry of a crab's shell. Studies even suggest that we are more attracted to people with symmetrical features!

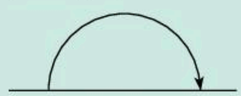
Before you start

You should know ...

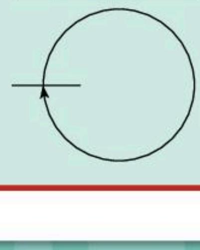
A turn of 90° is a quarter-turn:



A turn of 180° is a half-turn:



A turn of 360° is a full-turn:



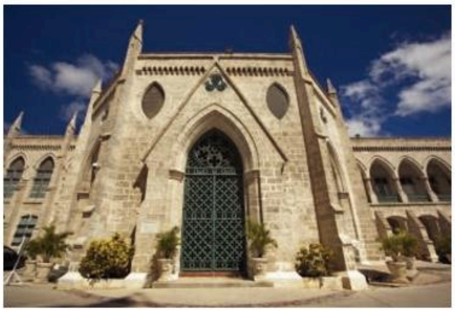
Check in

How many 90° turns are in

a a quarter-turn
b a half-turn
c a full-turn?

15.1 Symmetry

Symmetry is all around us. It occurs in nature and in the physical world.

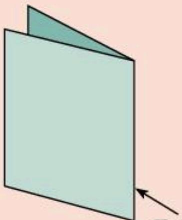


To investigate the idea of symmetry, you will need some paper, some tracing paper and a pair of scissors.

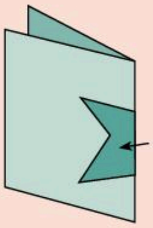
ACTIVITY

Fold a piece of paper in two.

A



B



Draw a shape on the folded paper, starting and finishing at the fold line, as in diagram B.

C

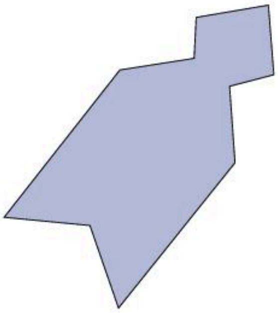
Keeping the paper folded, cut out the shape, as in diagram C. Open out your shape, as in diagram D.

D

Draw a line along the fold line with a pencil. What do you notice about the parts on each side of the fold line?

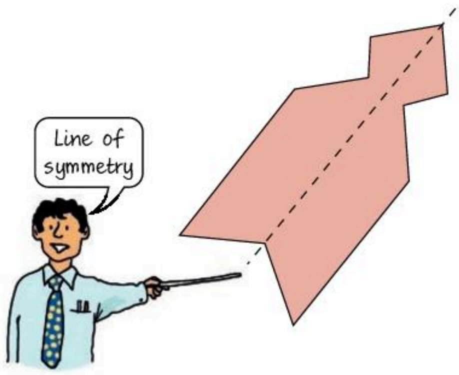
Repeat the activity for different shapes.

This shape has been cut from folded paper. Trace the shape, cut out the tracing and find the fold line.



Notice that one half of the shape fits exactly over the other. We say the shape is **symmetrical**. The shapes you cut out in the activity are also symmetrical.

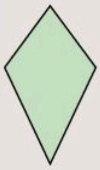
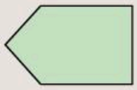
The fold line is called a **line of symmetry**.



A line of symmetry is also called a **mirror line**.

Exercise 15A

1 Copy these shapes, drawing the line of symmetry where possible.

a

b

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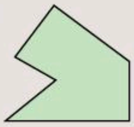
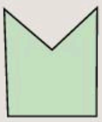
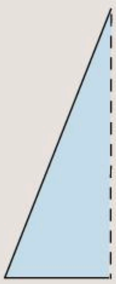
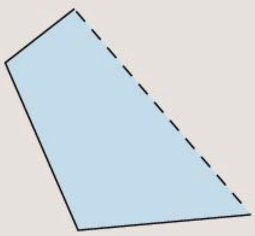

d

e

f


2 These drawings show halves of shapes. In each, the dotted line is a line of symmetry. Trace and complete each shape.

a

b

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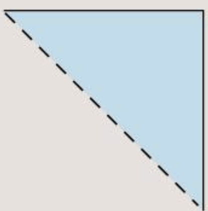
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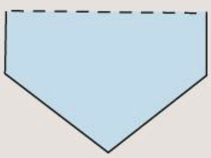
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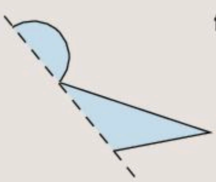
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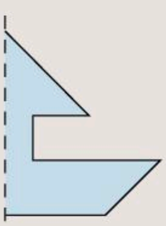
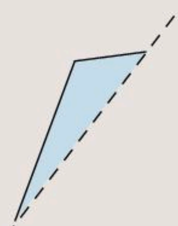
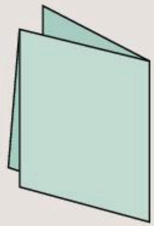
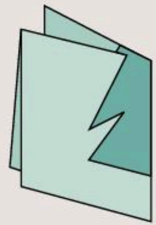
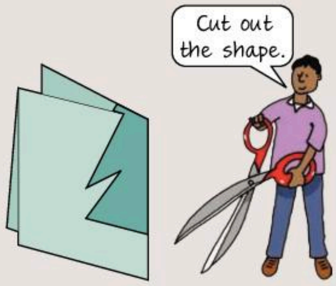
g

h


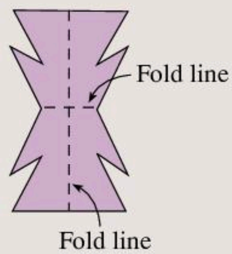
3 Fold a piece of paper, then fold it again, as in diagram A.

A

B


Draw a shape at the corner as in diagram B.

C


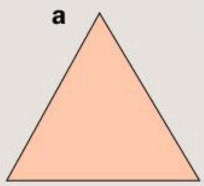
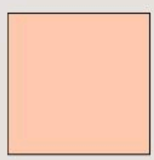
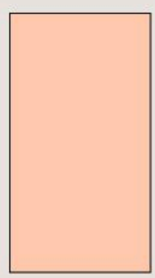
Cut out the shape at the corner.
Open out your shape as in diagram D.

D


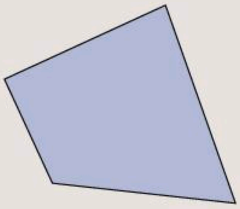
How many fold lines does your shape have?
How many lines of symmetry?

4 Repeat Question **3** for three different shapes.

5 Trace each shape. Cut out your tracing. By folding, find a line of symmetry, and mark it with a pencil. There may be more than one line. See how many you can find.

a

b

c


6 Repeat Question **5** for this shape. Is there a line of symmetry?



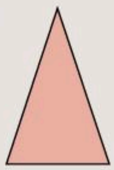
7 Copy and complete the table:

Shape	Number of lines of symmetry
Kite	
Trapezium	
Rhombus	
Square	
Parallelogram	

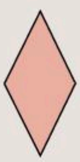
8 Copy the shapes and mark all the lines of symmetry. How many lines of symmetry does each shape have?

a

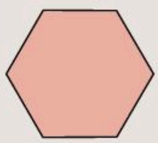
b



c



d



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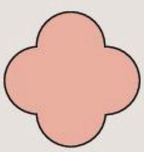

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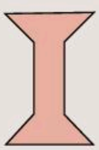
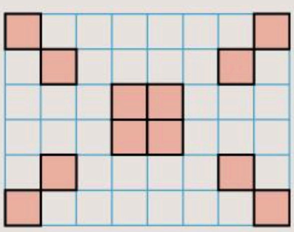


g



9 **a** Write the alphabet in capital letters: A B C D E F . . .
b Which letters have one line of symmetry?
c Which letters have two lines of symmetry?

ACTIVITY

Make a collection of pictures or photographs which show symmetry in nature.
Display these in your class.
Visit this website for some ideas:
www.misterteacher.com

INVESTIGATION

Can you draw a triangle with

a no lines of symmetry
b one line of symmetry
c two lines of symmetry
d three lines of symmetry?

Repeat for a quadrilateral.
What about a five-sided shape?

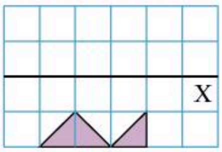
15.2 Reflection

When you look in a mirror you see your reflection.

Shapes can also be reflected in a mirror.
The shape is the **object** and its reflection is the **image**.

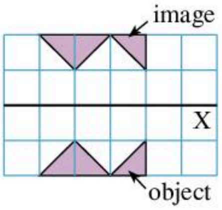
EXAMPLE 1

Reflect the shape in the line X.



object

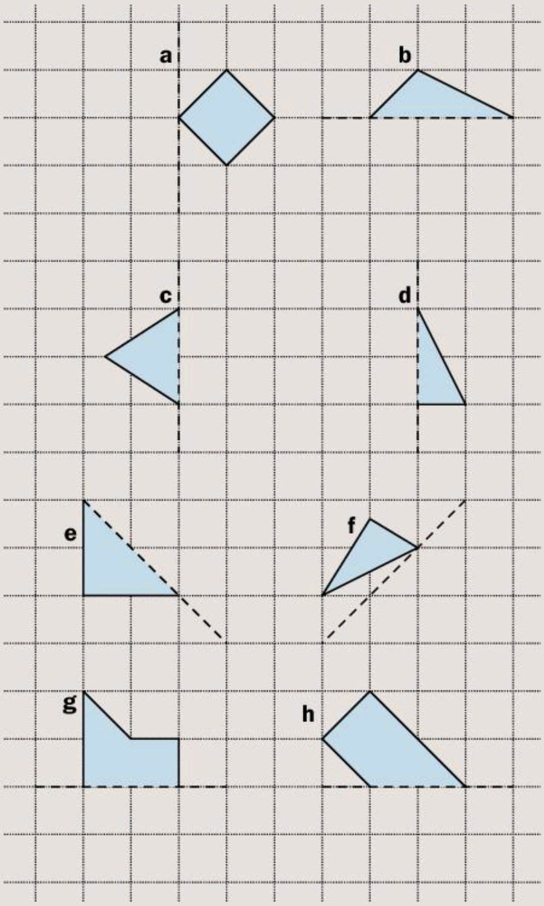
On reflection:



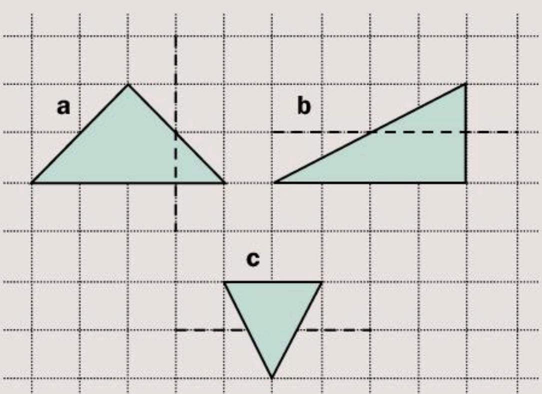
Notice that the image is the same distance from the mirror line as the object.

Exercise 15B

1 Copy the shapes and the mirror lines (indicated by dotted lines) onto squared paper. Draw the reflection of each shape.

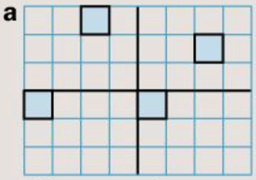


2 Reflect each of the following shapes in the mirror line indicated by a dotted line.

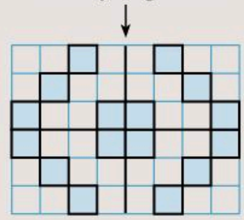


3 Copy and complete these diagrams using both mirror lines to reflect the shaded squares. The first one has been done for you.

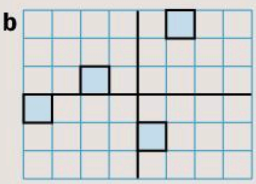
a



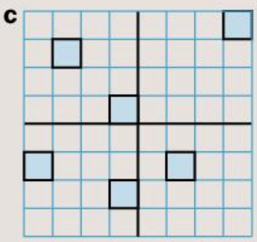
When you reflect every square in both lines, check you get this



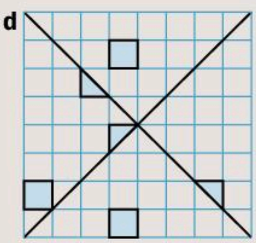
b



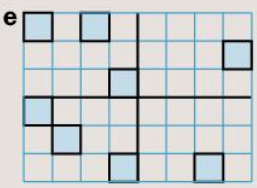
c



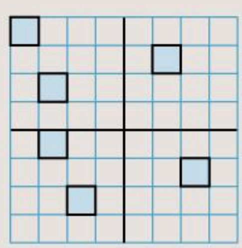
d



e

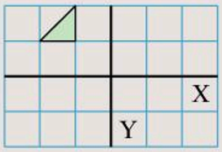


4 Tamir says that, since 6 squares are shaded in the diagram, when all the reflections are drawn in there will be 24 squares shaded in. Leilah disagrees and says she thinks only 16 squares will be shaded in. Who is right? What mistake had the other person made?

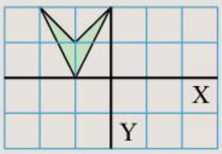


5 Copy these diagrams. Reflect each shape first in the line X and then in the line Y.

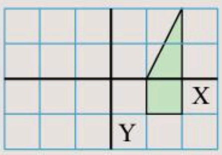
a



b



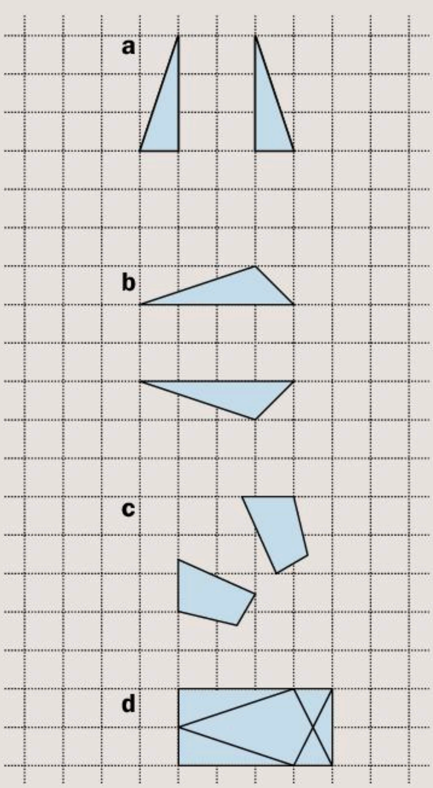
c



6 **a** Repeat Question **5** but this time reflect the shape first in the line Y and then in the line X.

b Does it matter which is done first?

7 Each diagram below shows both the object and image. Copy the diagrams and draw in the position of the mirror line.

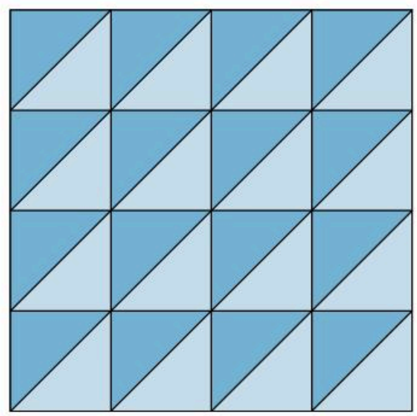


Reflection is a type of **transformation**. A transformation is a way of moving a shape, often on a coordinate grid. When a shape has been reflected, the original object and the image are **congruent** to each other.

Congruent means exactly the same shape and size. Reflecting a shape does not affect the lengths of its sides or size of its angles, only which way round the shape faces. Reflection 'flips' the shape over.

You will be learning about two other transformations in this chapter. These are translation and rotation. In Book 2 you will learn about another transformation, **enlargement**. In Cambridge IGCSE® Maths you will also learn about two further transformations: shears and stretches.

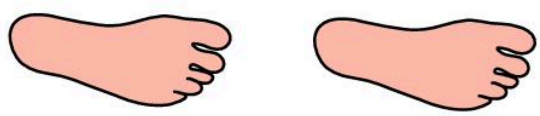
15.3 Translation



The triangular tiles shown here are congruent to each other.

Notice that the tiling pattern or **tessellation** was made simply by sliding the dark and light blue triangular tiles to new positions.

This sliding movement is called a **translation**. A translation does not change an object's size, shape or **direction**.



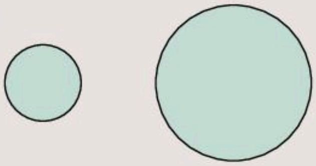
The footprint has been translated to a new position.

Exercise 15C

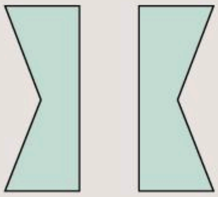
1 Which of these transformations show a translation?

a

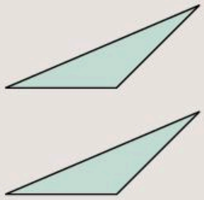
b



c



d

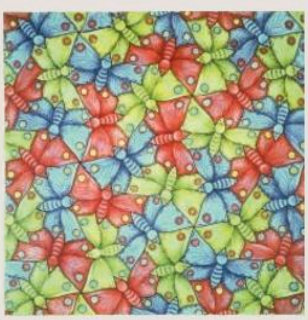


2 Which of these pictures show translations?

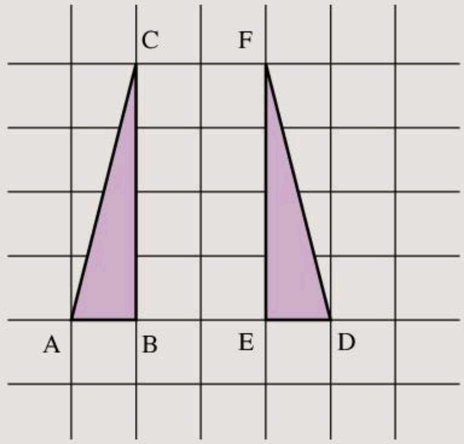
a

b

c



3

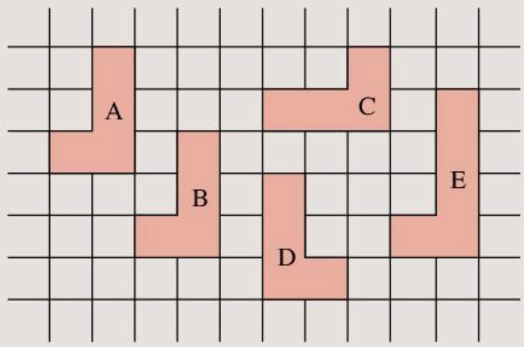


Look at the triangles ABC and DEF.

a Is triangle ABC congruent to triangle DEF?

b Is triangle DEF the image of triangle ABC under a translation? Give reasons for your answer.

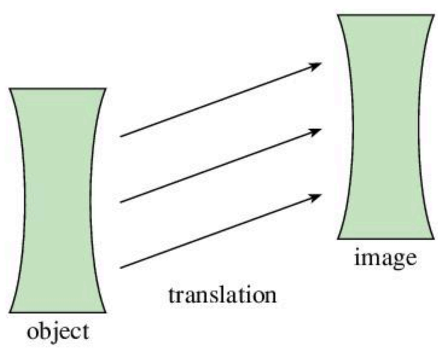
4



Look at the shapes A, B, C, D and E.

a Which of the shapes are congruent to shape A?

b Shape B is a translated image. Which shape or shapes could be the object?



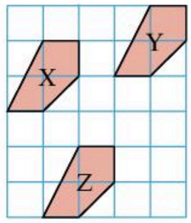
A translation can be described as a movement to the left or right followed by a movement up or down.

EXAMPLE 2

Describe the translation that takes

a shape X → shape Y

b shape Y → shape Z



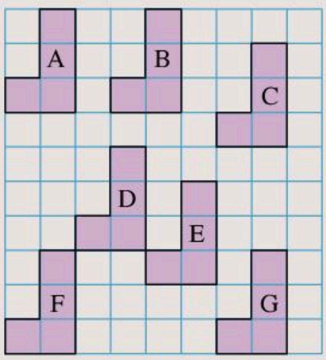
a X to Y: 3 right, 1 up

b Y to Z: 2 left, 4 down

To help you count, choose one corner of the object shape and then count to the corresponding corner on the image shape.

Exercise 15D

1 Describe these translations.



a A → B **b** B → C

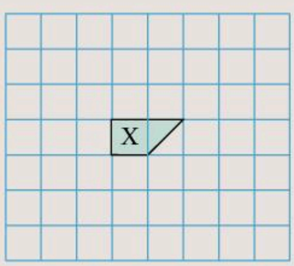
c A → C **d** C → A

e D → E **f** E → C

g F → E **h** F → G

i G → A **j** G → C

2



On squared paper, draw the image of the shape X after a translation:

a 2 right, 1 up

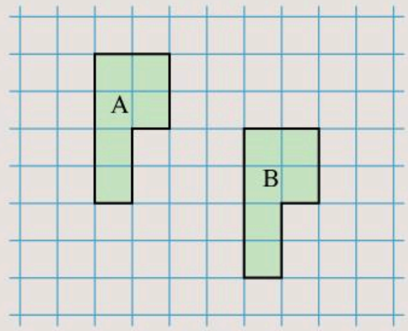
b 1 left, 1 up

c 3 right, 1 down

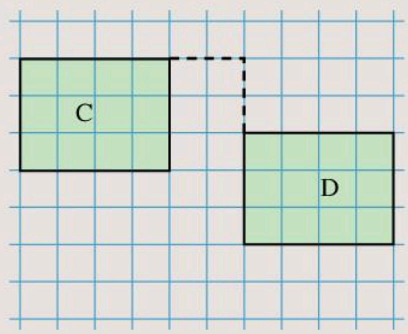
d 2 left, 3 down

e 3 left, 2 down.

3 Khalid wrote this solution: 'The translation from B to A is 4 left and 2 up.' Is he correct? If he is wrong, write down what the answer should be.



4 Habibah wrote this solution: 'The translation from C to D is 2 right and 2 down.' Is she correct? If she is wrong, write down what the answer should be.



15.4 Rotation

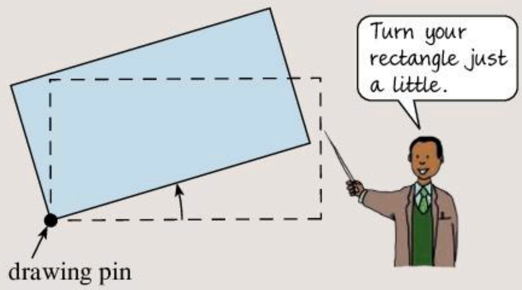
You will need some card, a drawing pin, tracing paper, a pair of scissors and some squared paper.

Exercise 15E

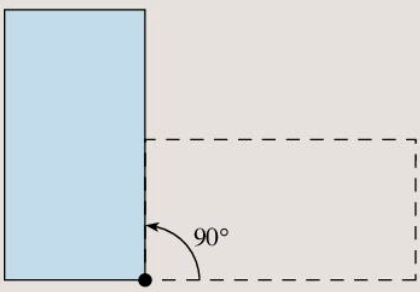
1 **a** Copy this rectangle onto a piece of card.

Cut it out and put a black dot at the bottom left-hand corner.

b Place your rectangle on a page in your exercise book and draw round it. Stick a pin through the black dot. Rotate your rectangle just a little. Draw round it.



c Now turn your rectangle through a $\frac{1}{4}$-turn (90°) about your black dot. Draw its new outline.



2 Turn the rectangle from Question **1** through:

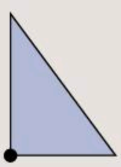
a $\frac{1}{2}$-turn (180°)

b $\frac{3}{4}$-turn (270°)

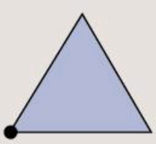
Draw around it before and after each turn.

3 Repeat Questions **1** and **2** for these shapes:

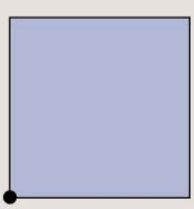
a



b

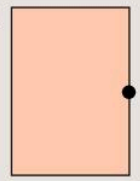


c

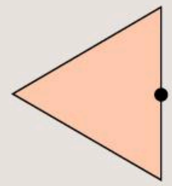


4 Draw each shape and its image after a $\frac{1}{2}$-turn rotation about the black dot.

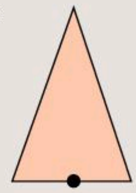
a



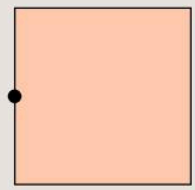
b



c



d



TECHNOLOGY

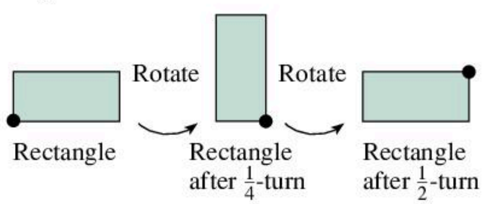
Learn more about rotation by watching the instructive video lesson at

http://lgfl.skoool.co.uk

Click on the Key Stage 3 tab and select the 'Rotational Transformations' video in the Maths section. You will see how to construct a shape after rotation through an angle.

Rotational symmetry

A shape which fits into the same position more than once when rotated through 360° has rotational symmetry.



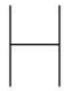
The rectangle above fits exactly on to the rectangle rotated through a $\frac{1}{2}$-turn.

So the rectangle has rotational symmetry.

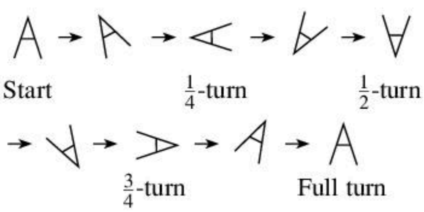
EXAMPLE 3

Which of these letters has rotational symmetry?

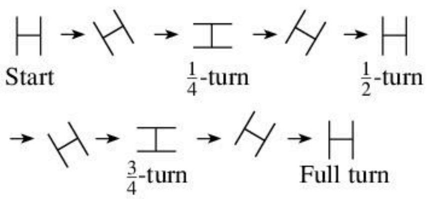
a 

b 

a Trace the letter A on tracing paper.
Rotate it; these are some of its positions:



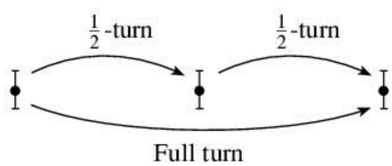
The letter A can fit exactly onto its start position only after a full turn. It has no rotational symmetry.

b 

The letter H can fit exactly onto itself after a $\frac{1}{2}$-turn and after a full turn. So it has rotational symmetry.

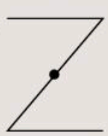
The number of times a shape fits onto itself in a complete turn is the **order** of the rotational symmetry.

The letter H has rotational symmetry of order 2, so does the letter I:

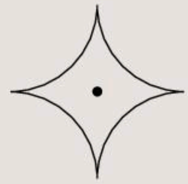


Exercise 15F

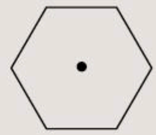
1 Trace these shapes and find their order of rotational symmetry.

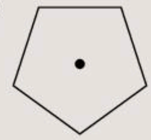
a 

b 

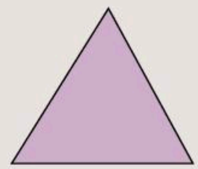
c 

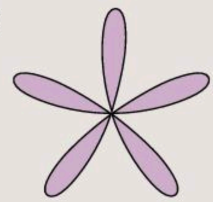
d 

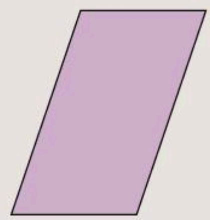
e 

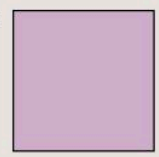
f 

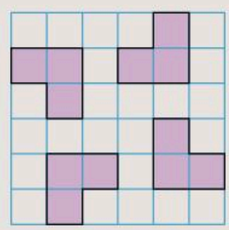
2 Find the order of rotational symmetry of each shape.

a 

b 

c 

d 

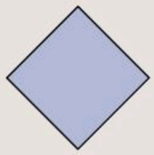
e 

3 Which of these letters possess rotational symmetry?

A B C D E
F G H I J K
L M N O P
Q R S T U
V W X Y Z

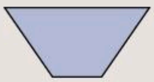
4 Which of these shapes have rotational symmetry? State the order of rotational symmetry where there is one.

a

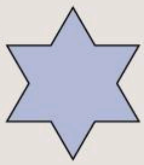


b

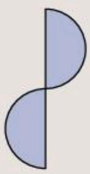
c



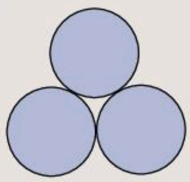
d



e



f



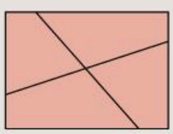
5 Which of these shapes have

a rotational symmetry only

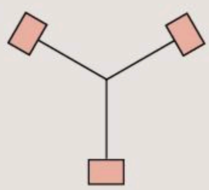
b line symmetry only

c rotational and line symmetry?

i

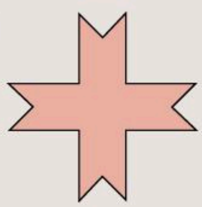


ii



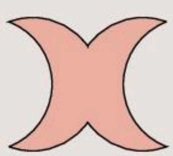
iii

iv



v

vi



Rotation about a given point

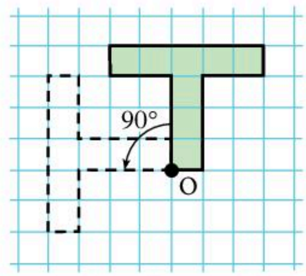
If you rotate a shape about a given point, this point is called the **centre of rotation**. In Question **1** of Exercise 15E, the point where you put your drawing pin was the centre of rotation.

Shapes can be rotated clockwise or anticlockwise, and through different angles.

You will need tracing paper, squared paper and a ruler.

EXAMPLE 4

Rotate the T-shape 90° anticlockwise about O.



The diagram above shows a T-shape and its image after rotation.
The centre of rotation is O.
The angle of rotation is 90° anticlockwise.

- Rotation always takes place about a single point in a clockwise or anticlockwise direction.

Notice that the size and shape of the object are unchanged after a rotation. Both object and image are said to be congruent to each other.

Exercise 15G

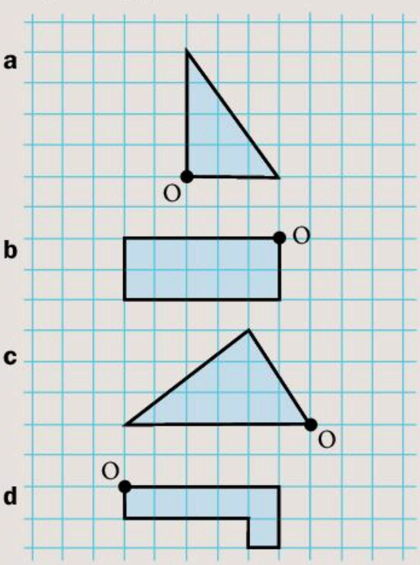
1 Copy each shape shown on squared paper. Draw the image formed by rotating the shape anticlockwise through 90° about O. Use a tracing to help you.

a

b

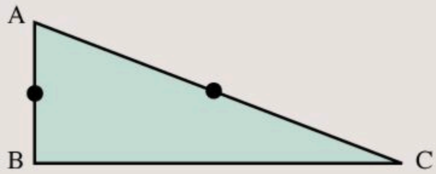
c

d



2 Repeat Question **1** but this time rotate the shapes through 180° about O.

3 ABC is a right-angled triangle.



a Trace the triangle.

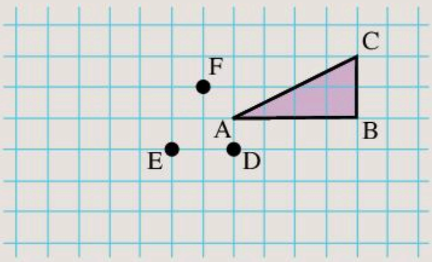
b Use your tracing to help you draw the triangle and its image after a $\frac{1}{2}$-turn rotation about

i the midpoint of AB

ii the midpoint of AC

c Does it matter whether the rotation is clockwise or anticlockwise?

4



The centre of rotation does not need to touch the shape.

Copy the diagram. Draw the image formed by rotating triangle ABC

a 90° clockwise about D

b 90° anticlockwise about E

c 180° clockwise about F

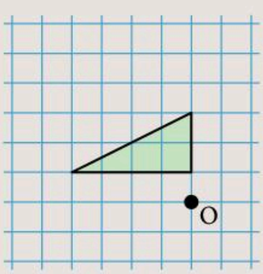
d 90° clockwise about E

e 270° clockwise about D

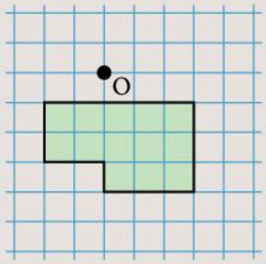
(You may find this easier with tracing paper. You can draw the answers to parts **a**, **b** and **c** all on one diagram. Make a second copy of the diagram for the answers to parts **d** and **e**.)

5 Copy the diagrams below. In each diagram, draw the image formed by rotating the object:

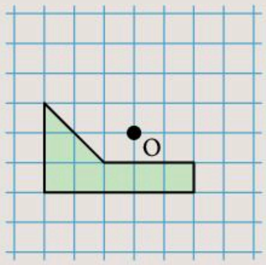
a 90° clockwise about O



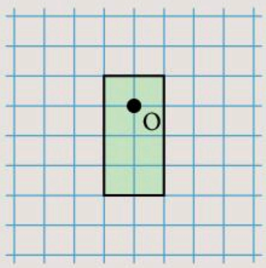
b 90° anticlockwise about O



c 180° clockwise about O



d 270° clockwise about O



INVESTIGATION

a Can you draw a triangle with order of rotational symmetry

i 2 **ii** 3?

b What quadrilaterals can you draw with rotational symmetry of order

i 2 **ii** 3 **iii** 4?

c How about five-sided shapes?

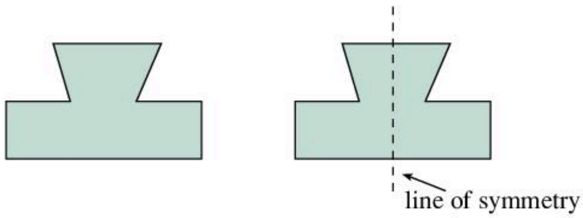
TECHNOLOGY

- Conduct an internet search for pictures and photographs that illustrate line symmetry and rotational symmetry.
- Make a booklet on symmetry by pasting the photographs into a word processing program and giving brief descriptions of each picture.
- Visit Adrian Bruce's web pages on symmetry at www.adrianbruce.com for further ideas.

Consolidation

Example 1

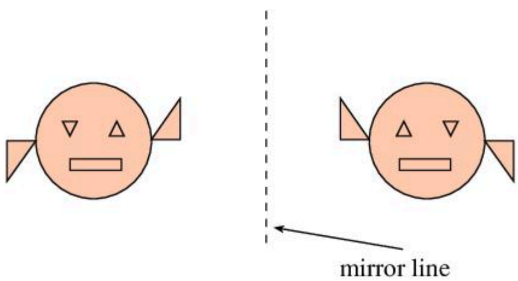
How many lines of symmetry does this shape have?



It has one line of symmetry only.

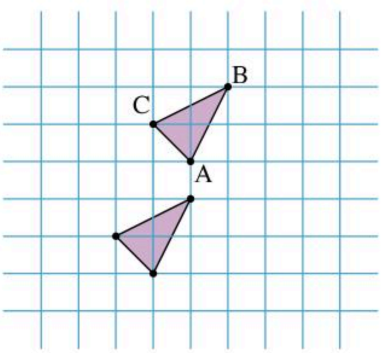
Example 2

Draw the image of the shape in the mirror line.



Example 3

What is the image of triangle ABC when translated 1 unit left and 3 units down?

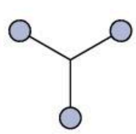


Example 4

What is the order of rotational symmetry of these shapes?

a

b



a Turn the shape through 90° about the centre.

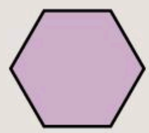
Turn it through another 90° about the centre and the shape is same as the original.
The shape has rotational symmetry of order 2.

b Rotate the shape through 120° about the centre and the image fits exactly on the original shape. Rotate it through another 120° and the image fits again. The shape has rotational symmetry of order 3.

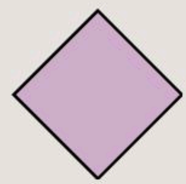
Exercise 15

1 **a** How many lines of symmetry do these shapes have?

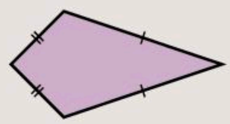
i



ii

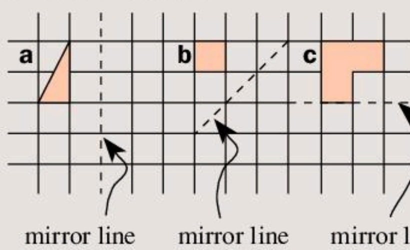


iii



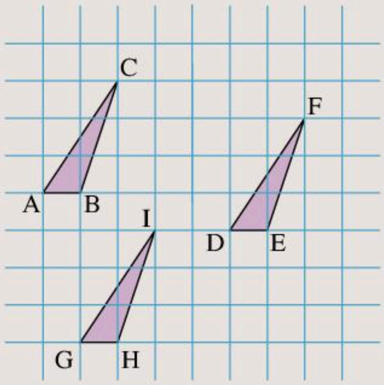
b Draw four different shapes with exactly two lines of symmetry.

2 Copy the diagram. Reflect each shape in the mirror line shown.

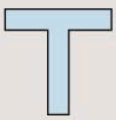


3 In the graph, describe the translations that send:

a ABC → DEF
b DEF → ABC
c GHI → DEF
d ABC → GHI



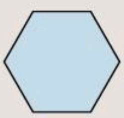
4 What is the order of rotational symmetry of these shapes?

a


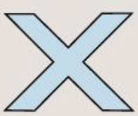
b


c


d


e


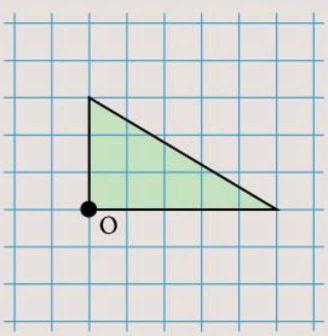
f


g


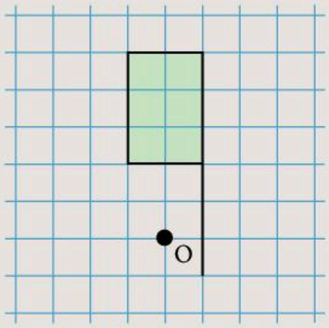
h


5 Copy the diagrams below. In each diagram, draw the image formed by rotating the object:

a 90° clockwise about O



b 90° anticlockwise about O



6 Draw two different shapes with:

a order of rotational symmetry 2
b order of rotational symmetry 3
c order of rotational symmetry 5
d 2 lines of symmetry
e 3 lines of symmetry
f 5 lines of symmetry

7 Copy and complete the table for regular shapes:

Regular shape	Order of rotational symmetry	Number of lines of symmetry
Pentagon		
Triangle		
Octagon		
Quadrilateral		
Hexagon		

8 **a** Identify three items, buildings or pictures that have reflectional symmetry.
Make a sketch of each showing clearly the line(s) of symmetry.

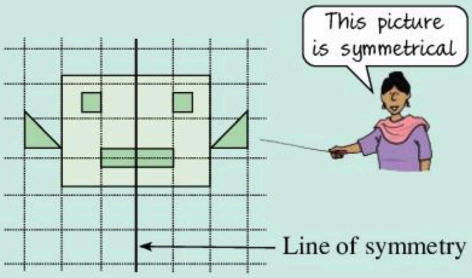
b Identify three items or pictures that have rotational symmetry.
Make a sketch of each and state the order of rotational symmetry.

Summary

You should know ...

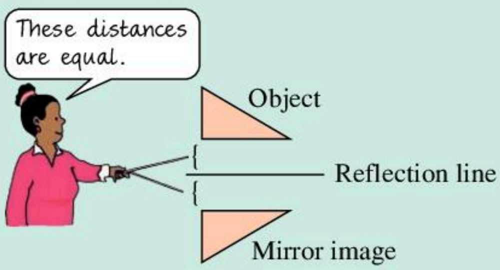
1 When a shape is folded so that one half fits exactly over the other half, the fold line is called a line of symmetry or mirror line.

For example:



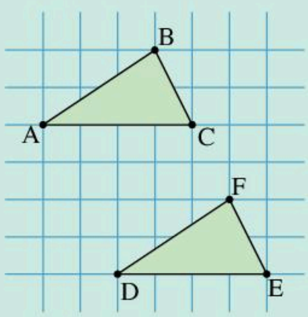
2 A shape can be reflected in a line.

For example:



3 How to find the image of a shape after a translation.

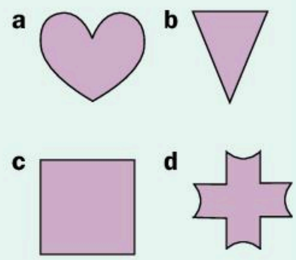
For example:



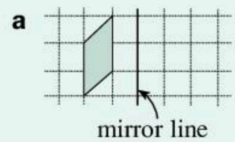
The image of ABC after a translation 2 right, 4 down is DEF.

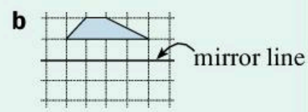
Check out

1 Copy these shapes. Draw in the lines of symmetry.

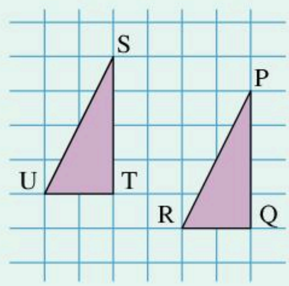


2 Draw the image of the shape after reflection in the mirror line.





3 Describe the translation of PQR to STU.



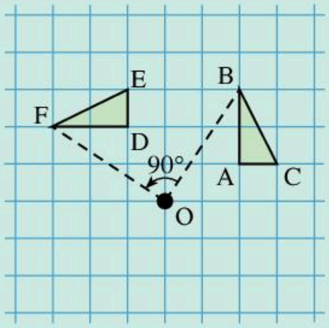
4 A shape has rotational symmetry if, when turned less than 360°, it fits onto itself. The order of rotational symmetry is the number of times the shape fits onto itself in a complete turn.

For example:

The order of rotational symmetry is 2.

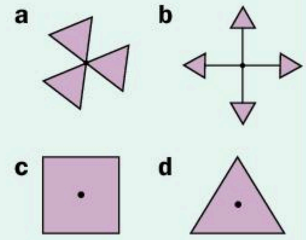
5 How to find the image of a shape after a rotation.

For example:

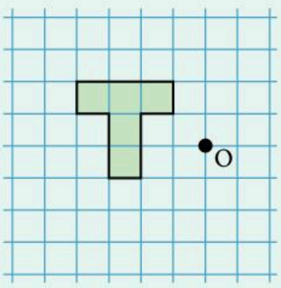


The image of triangle ABC after a rotation of 90° anticlockwise about centre of rotation O is DEF.

4 Find the order of rotational symmetry of these shapes.



5 Rotate the T-shape 90° clockwise about centre of rotation O.



16 Ratio and proportion

Objectives

- Use ratio notation, simplify ratios and divide a quantity into two parts in a given ratio.
- Recognise the relationship between ratio and proportion.
- Use direct proportion in context; solve simple problems involving ratio and direct proportion.

What's the point?

Earthquakes and hurricanes are common in many parts of the world. To withstand such forces of nature, buildings have to be constructed with solid foundations. Builders need to use suitable ratios of sand to cement to ensure structures are solid.

Before you start

You should know …

1 Equivalent fractions are different ways of showing the same fraction.
For example:

$$\frac{2}{3} = \frac{4}{6} = \frac{6}{9} = \frac{8}{12}$$

2 Fractions can be simplified by dividing the numerator and the denominator by the same number.
For example:

$$\frac{18}{24} \overset{\div 3}{=} \frac{6}{8} \overset{\div 2}{=} \frac{3}{4}$$

Check in

1 Write down two fractions equivalent to

a $\frac{3}{4}$ **b** $\frac{4}{5}$

2 Simplify:

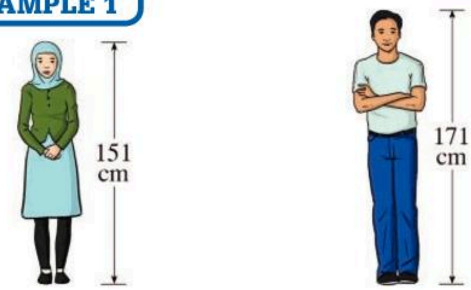
a $\frac{12}{36}$ **b** $\frac{20}{25}$

c $\frac{85}{100}$ **d** $\frac{16}{48}$

16.1 Making comparisons

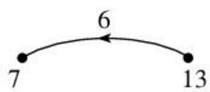
One way of comparing two quantities is to say how much larger or how much smaller one is than the other.

EXAMPLE 1



Dev *is 20 cm taller than* his friend Aisha.

EXAMPLE 2



7 *is 6 less than* 13

Exercise 16A

1 Use both of the phrases *is greater than* and *is less than* to compare each pair of numbers.

a 50, 5 **b** 40, 8
c 100, 25 **d** 1000, 25

2 Using one of the phrases *is greater than*, *is less than* or *is equal to,* complete the statements:

a The number of boys in my class . . . the number of girls.
b The number of exercise books I have brought to school . . . the number of text books.

3 Rewrite the statements in Question **2** using another phrase where possible.

Larger than, *smaller than*, *taller than*, *less than*, and *greater than* are only a few of the ways of comparing two objects or sets of objects. Another way is to use a **ratio**.

What is a ratio?

Look at this group of children:

You could compare the number of boys and girls in the group by saying:

For *two* boys there are *three* girls.

or

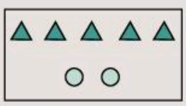
For *three* girls there are two boys.

As a ratio the number of boys compared to the number of girls can be written as 2 : 3. The ratio of girls to boys is 3 : 2.

- A **ratio** compares the size of two quantities.

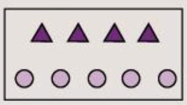
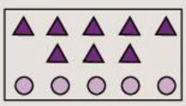
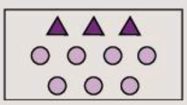
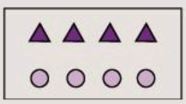
Exercise 16B

1 Compare the number of triangles and circles, then copy and complete the statement:



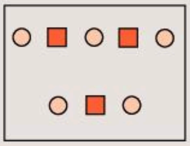
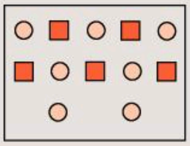
a For . . . triangles there are . . . circles.
b For . . . circles there are . . . triangles.

2 Repeat Question **1** for the diagrams below.

a

b

c

d


3 For each diagram, use a ratio to compare:

a the number of small squares to the number of circles
b the number of circles to the number of small squares.

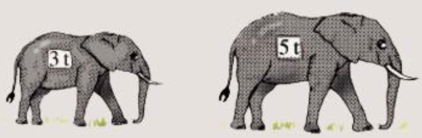
i

ii


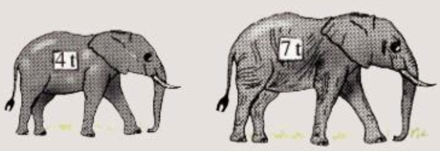
4 Each ratio below compares the number of black circles to the number of white circles. Draw a group of black circles and white circles to show the ratio.
a 3 : 2 **b** 1 : 5 **c** 7 : 4 **d** 5 : 9

5 For the students in your class, use a ratio to compare
- **a** the number of boys to the number of girls
- **b** the number of girls to the number of boys
- **c** the number with long hair to the number with short hair
- **d** the number wearing glasses to the number without glasses.

6 Measure each pair of lines in centimetres. Use a ratio to compare the longer line to the shorter line.
a
b
c

7 Write a ratio to compare the shorter line to the longer line, for each pair of lines in Question **6**.

8 Use a ratio to compare the mass of the first elephant with the mass of the second.
a


b


16.2 Simplifying ratios

The ratio of girls to boys is 2 : 4. You can write this as a fraction.

$$\frac{\text{number of girls}}{\text{number of boys}} = \frac{2}{4}$$

The ratio 2 : 4 is linked to the fraction $\frac{2}{4}$.

Equivalent ratios

The ratios 1 : 3, 2 : 6 and 3 : 9 are called **equivalent ratios**.

They are all linked to the fraction $\frac{1}{3}$, since $\frac{1}{3} = \frac{2}{6} = \frac{3}{9}$.

1 : 5, 2 : 10 and 3 : 15 are also equivalent ratios. They are all linked to the fraction $\frac{1}{5}$.

Exercise 16C

1 **a** Copy and complete:

i Number of bats : number of balls = □ : □

ii $\frac{\text{number of bats}}{\text{number of balls}} = \frac{\square}{\square}$

b Which fraction is the ratio linked to?

2 **a** Copy and complete:

i Number of bats : number of balls = □ : □

ii $\frac{\text{number of bats}}{\text{number of balls}} = \frac{\square}{\square}$

b Which fraction is the ratio linked to?

3 **a** Copy and complete:

i Number of bats : number of balls = □ : □

ii $\frac{\text{number of bats}}{\text{number of balls}} = \frac{\square}{\square}$

b Which fraction is the ratio linked to?

4 **a** For the drawings in Questions **1** to **3**, do you agree that number of balls = 3 × number of bats?

b Do you agree that the ratios
1 : 3 2 : 6 3 : 9
all show the same comparison?

c Do you agree that the ratios are all linked to the same fraction?

5 Copy and complete these equivalent ratios:

a	$1:2$,	$3:6$,	$5:\square$	
b	$\square:3$,	$4:12$,	$5:15$,	$10:\square$
c	$1:5$,	$3:\square$,	$5:\square$	
d	$1:\square$,	$2:16$,	$3:\square$	
e	$1:\square$,	$2:5$,	$4:\square$,	$7:\square$

Ratios can easily be simplified by dividing both sides of the ratio by the same number.

EXAMPLE 3

Simplify the ratio $9:108$.

Divide both sides by 9.
$9:108$ is equivalent to **1 : 12**

To write a ratio in its simplest form you divide by the HCF of the two numbers.

EXAMPLE 4

What is the simplest form of the ratio $100:30$?

Find the HCF of the two numbers.
It is 10.
Divide both numbers by 10.
$100:30$ is equivalent to **10 : 3**

Exercise 16D

1 Write each ratio in its simplest form:

a	$10:2$	**b**	$2:10$	**c**	$9:3$
d	$3:9$	**e**	$16:20$	**f**	$20:16$
g	$12:11$	**h**	$300:50$	**i**	$40:56$
j	$16:80$	**k**	$144:12$	**l**	$54:90$

2 Which pairs of ratios are equivalent?

a	$1:5, 4:20$	**b**	$1:3, 4:16$
c	$4:1, 12:6$	**d**	$12:2, 18:3$
e	$2:3, 24:12$	**f**	$10:4, 30:12$

3 Draw three different rectangles, with longer side : shorter side $= 3:2$.

4 In what ratio are the sides of
- **a** a square
- **b** an equilateral triangle
- **c** a rhombus?

Fraction ratios can also be simplified. You need first to ensure that the fractions have a common denominator.

EXAMPLE 5

Simplify

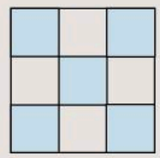
a $\frac{2}{3}:\frac{5}{6}$ **b** $\frac{3}{4}:\frac{2}{3}$

a $\frac{2}{3}:\frac{5}{6} = \frac{4}{6}:\frac{5}{6}$
$= 4:5$

b $\frac{3}{4}:\frac{2}{3} = \frac{9}{12}:\frac{8}{12}$
$= 9:8$

Exercise 16E

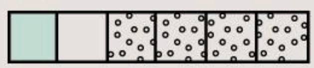
1



- **a** What fraction of the squares is shaded?
- **b** What fraction is white?
- **c** Write a ratio to compare the shaded fraction to the white fraction.
- **d** Write a ratio to compare the *number* of shaded squares to the *number* of white squares.
- **e** Copy and complete:

$\frac{5}{9}:\frac{4}{9} = 5:\square$

2



- **a** What fraction of the squares is dotted?
- **b** What fraction is shaded?
- **c** Write a ratio to compare the shaded fraction to the dotted fraction.
- **d** Write a ratio comparing the *number* of squares that are shaded to the *number* that are dotted.
- **e** Copy and complete:

$\frac{1}{6}:\frac{2}{3} = \frac{1}{6}:\frac{\square}{6} = 1:\square$

3 Write each ratio in its simplest form.

a	$\frac{2}{5}:\frac{3}{5}$	**b**	$\frac{3}{8}:\frac{5}{8}$	**c**	$\frac{1}{6}:\frac{5}{6}$
d	$\frac{7}{12}:\frac{5}{12}$	**e**	$\frac{9}{15}:\frac{6}{15}$	**f**	$\frac{5}{6}:\frac{1}{2}$

4 By first writing the fractions with a common denominator, give each ratio in its simplest form.

a $\frac{4}{5}:\frac{9}{20}$ **b** $\frac{7}{12}:\frac{5}{6}$ **c** $\frac{1}{2}:\frac{3}{8}$

d $\frac{4}{5}:\frac{3}{4}$ **e** $\frac{2}{3}:\frac{3}{8}$ **f** $\frac{3}{7}:\frac{4}{5}$

16.3 Proportion

How would you share this cake between Ivie and Naya, in the ratio 2 : 3?

Ivie will get 2 parts and Naya 3 parts. That is, they will have to share the cake into 2 + 3 = 5 parts.

EXAMPLE 6

Share $1 between Clara and Hope in the ratio 4 : 1.

For every 1 part Hope gets, Clara gets 4 parts.
4 + 1 = 5

You must divide $1 into 5 equal parts.

So 1 part is 20 cents.
Hope gets **20 cents**.
Clara gets **80 cents**.

In Example 6 you worked out the proportion of $1 that Clara and Hope each receive.

Sometimes you will be given one of the proportions and you will have to work out the other proportion or the total amount.

EXAMPLE 7

The ratio of boys to girls in Year 7 is 3 : 2.
There are 60 boys in Year 7.
How many girls are there?

For every 3 boys there are 2 girls.

So for every 1 boy there are $\frac{2}{3}$ girls.

For 60 boys there are $60 \times \frac{2}{3}$ girls
$= 40$ girls.

You can also do Example 7 another way, using equivalent ratios:

	boys	:	girls	
	3	:	2	
$\times 20$	60	:	$\square$	$\times 20$
	60	:	40	

This is an example of **direct proportion**. This means that as one quantity increases the other increases at the same rate.

If an average builder can lay 700 bricks in a day, two builders will lay 1400. If you double the number of builders, you double the number of bricks laid. If you have 4 builders, 2800 bricks will be laid, and so on. The same method works for decreasing at the same rate.

If 10 pick-up trucks can carry 6 tonnes of gravel, how much gravel can 5 pick-up trucks carry?

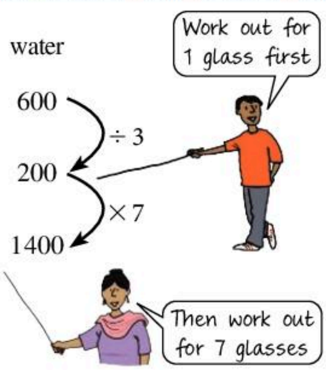
If you halve the amount of trucks you halve the amount of gravel.

So 5 pick-up trucks can carry 3 tonnes.

EXAMPLE 8

3 glasses of orange squash are made with 600 ml of water.
How much water is needed to make 7 glasses of squash?

	glasses	:	water	
$\div 3$	3	:	600	$\div 3$
$\times 7$	1	:	200	$\times 7$
	7	:	1400	

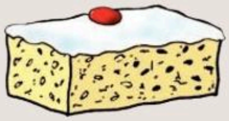


So 7 glasses of squash need 1400 ml of water.

In Example 8, the direct proportion question was solved using the method of equivalent ratios and a process called the unitary method (finding the result for 1 first).

Exercise 16F

1 A cake is shared between Dian and Lastri, in the ratio 2 : 5.



a If Dian gets 2 parts, how many parts should Lastri get?
b How would you cut up the cake?

2 Bessie and Jason share this rice in the ratio 5 : 3.

a If Jason gets 3 parts, how many parts should Bessie get?
b Do you agree that the rice should be divided into 8 equal parts?
c How many kilograms does Bessie get?

3 a How would you share these apples between Ali and Padma, in the ratio 1 : 3?

b How many apples would Ali get?
c How many apples would Padma get?

4 Share \$1 between Clara and Hope, in the ratio:
a 3 : 1 **b** 1 : 1 **c** 9 : 1
d 7 : 3 **e** 17 : 3 **f** 4 : 21

5 Share \$5 between Ben and James, in the ratio:
a 3 : 1 **b** 9 : 1 **c** 17 : 3

6 Share 30 cows between Farmer X and Farmer Y, in the ratio:
a 2 : 1 **b** 1 : 1 **c** 4 : 1
d 7 : 3 **e** 4 : 11 **f** 3 : 27

7 Share the following between X and Y in the given ratio.
a 20 apples, in the ratio 3 : 7
b 15 camels, in the ratio 2 : 3
c \$60, in the ratio 1 : 4
d \$5.50, in the ratio 4 : 1
e 3 metres of cloth, in the ratio 3 : 7
f 1 metre of rope, in the ratio 2 : 3
g 2 kilograms of honey, in the ratio 1 : 7
h 2 tonnes of sand, in the ratio 4 : 1
i 17.5 metres of ribbon, in the ratio 3 : 7
j 15.6 kilograms of sugar, in the ratio 9 : 4

8 The ratio of boys to girls at the Valley School is 3 : 2. The school has 800 students.
a How many boys are at the school?
b How many girls?

9 The ratio of sand to cement to mix concrete is 4 : 1. How many wheelbarrows of sand does Sam need to make 25 barrows of concrete mix?

10 The ratio of men to women in the Abuja Sports club is 4 : 3. Use the method shown in Example 7 to find out how many women there are, if there are:
a 12 men **b** 40 men **c** 36 men
d 108 men **e** 400 men **f** 144 men

11 a Number of boys : number of girls = 2 : 1 in Woodstock School. How many boys are there in the school, if the number of girls is
i 25 **ii** 200 **iii** 75 **iv** 80?
b How many girls are there in Woodstock School, if the number of boys is
i 50 **ii** 200 **iii** 150 **iv** 80?

12 4 pens cost \$2.80.
a How much does 1 pen cost?
b How much do 3 pens cost?

13 If 3 books cost \$9, how much do 5 books cost?

14 A worker takes 8 minutes to make 2 circuit boards. How long would it take her to make 1 circuit board?

15 In a fruit shop, the ratio of oranges to lemons is 5 : 3. If there are 39 lemons, how many oranges are there?

16 Three coaches can carry 156 passengers. How many coaches would be needed for 364 passengers?

17 In a hall, the ratio of chairs to tables is 7 : 2. If there are 18 tables, how many chairs are there?

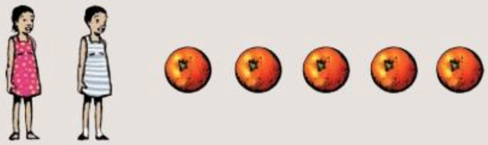
18 5 bags of sweets contain 70 sweets.
a How many sweets will there be in 7 bags?
b How many bags will there be if there are 154 sweets in total?

TECHNOLOGY

Need another look? Visit
www.onlinemathlearning.com
and follow the links to Arithmetic, Proportions, Ratios.
Go over the examples and watch the video!

Exercise 16G – mixed questions

1 Copy and complete:



Number of girls : number of oranges
= 2 : □ or 1 : □

2 Number of maths students : number of chemistry students = 25 : 2
How many students study chemistry, if the number who study maths is

a 25 **b** 125 **c** 250?

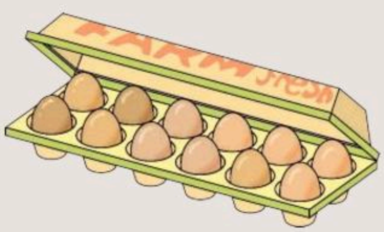
3 For the ratio in Question **2**, how many students study maths if the number who study chemistry is

a 24 **b** 30 **c** 78?

4 For a certain type of concrete, the ratio of cement powder to sand is 3 : 5.
How much cement powder is needed to make the concrete mix if the mass of sand is

a 10 kg **b** 30 kg **c** 45 kg?

5



The cost of 12 eggs is \$4.80.

a What is the cost of one egg?
b What is the cost of 9 eggs?

6 Twenty books cost \$360. Find the cost of 15 books.

7

Benny's supermarket sells 25 tea bags of Tim's Tea for \$2.25 and 30 bags for \$3.30.

a Work out the cost of one tea bag for each pack.
b Which pack is the better buy?

8 30 m of electric cable is sold for \$36.
How much does 25 m cost?

9 A loaf of bread for four people needs 440 g of flour.

How much flour will be needed to make a similar loaf for five people?

10 A 180 ml tube of 'Super Fresh' toothpaste retails for \$3.06.

What should a 270 ml tube retail for?
(Assume the same price per millilitre.)

11 A 'six-pack' of soft drinks costs \$7.20.
What is the cost of

a 4 soft drinks **b** 7 soft drinks?

12 A box of 13 floor tiles costs \$58.50.
Find the cost of

a 26 tiles **b** 20 tiles.

13 The scale on a plan is 1 : 2500.

a How long is a wall which is 3 cm in length on the plan?
b What length would be represented on the plan for a window the actual length of which is 2 m?

14 The recipe for pumpkin soup is shown below.

For two people, use:	
500ml	water
400g	pumpkin
30g	butter
10g	seasoning

a How much butter is needed if five people are to be served?

b A cook uses 35 g of seasoning to make the soup. How much pumpkin will he need?

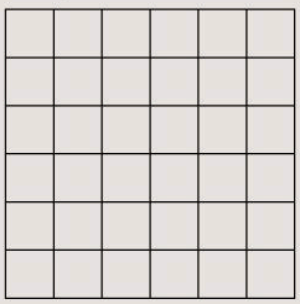
15 At Western School there are 375 boys, 325 girls and 35 teachers.

a What is ratio of boys to girls?

b What is the student–teacher ratio?

c If three teachers left and only two were replaced, what would be the new student–teacher ratio?

16 Copy the diagram.

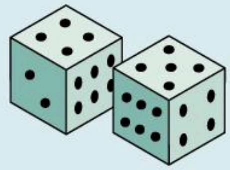


Shade some of the squares so that the ratio of shaded to unshaded squares is 2 : 7

17 Find two numbers in the ratio 3 : 7 and with a difference of 48.

INVESTIGATION

Two 6-sided dice are thrown. The ratio of the smallest score to the largest score is simplified then written down.



For example, if the first dice scored 6 and the second dice scored 4, you would write down 2 : 3, because smallest score : largest score = 4 : 6, which can then be simplified to 2 : 3.

If the first roll was 6 and the second roll was 6 you would write down 1:1.

Write down all the possible ratios you could get.

Which four ratios are you most likely to get?

Consolidation

Example 1

Write as a ratio in its simplest form:

a $5:30$ **b** $144:36$

a Divide by 5:

$5:30 = 5 \div 5 : 30 \div 5$
$= 1:6$

b Divide by 12, then 3:

$144:36 = 144 \div 12 : 36 \div 12$
$= 12:3$
$= 12 \div 3 : 3 \div 3$
$= 4:1$

Example 2

Share \$20 in the ratio

a $4:1$ **b** $7:3$

a There are $4 + 1 = 5$ parts
Each part $=$ \$20 $\div 5 =$ \$4
So 4 parts $= 4 \times$ \$4 $=$ \$16
That is, $4:1 =$ \$16 : \$4

b There are $7 + 3 = 10$ parts
Each part $=$ \$20 $\div 10 =$ \$2
So 7 parts $= 7 \times$ \$2 $=$ \$14
3 parts $= 3 \times$ \$2 $=$ \$6
That is, $7:3 =$ \$14 : \$6

Example 3

A baker uses 1800 grams of flour to make 3 loaves of bread.
How much flour will he need to use if he wants to make 5 loaves?

For 1 loaf the baker would need $1800\,\text{g} \div 3 = 600\,\text{g}$ of flour.

For 5 loaves the baker will need $600\,\text{g} \times 5 = 3000\,\text{g}$ of flour.

Exercise 16

1 Write these ratios in their simplest form:

a $2:20$ **b** $35:40$ **c** $85:17$
d $125:625$ **e** $192:56$ **f** $56:72$
g $108:68$ **h** $144:78$ **i** $195:125$
j $256:72$

2 Share the following between Anton and Dannisha:

a 20 marbles in the ratio $3:2$
b 36 biscuits in the ratio $4:5$
c 9 pencils in the ratio $1:2$
d \$72 in the ratio $7:5$
e 400 oranges in the ratio $3:17$

3 The ratio of boys to girls at a football match is $4:1$.

a How many boys are at the match if the number of girls is:
i 15 **ii** 25 **iii** 62
iv 100 **v** 214?

b How many girls are at the match if the total number of children there is:
i 200 **ii** 125 **iii** 460
iv 1025 **v** 985?

4 A necklace has black and silver beads in the ratio $7:3$.

a If the necklace has 63 silver beads, how many black beads will it have?
b If the necklace has 63 black beads, how many silver beads will it have?

5 The student : teacher ratio at a school is $15:1$.

a How many students are there if there is a total of 320 students and staff?
b What would be the new student : teacher ratio if two more teachers joined the school?

6 A cake for 6 people requires 1 cup of sugar, 2 cups of flour, $\frac{1}{2}$ cup of butter and 3 eggs.

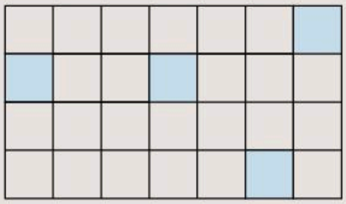
a What would be the ingredients for a cake for 8 people?
b How many eggs should be used in a cake for 13 people?

7 A fruit juice is made of 2 parts pineapple juice to 3 parts orange juice to 4 parts passion fruit juice.

a Diane wishes to make 18 litres of fruit juice. How much
i pineapple
ii orange
iii passion fruit
juice does she need?

b If Diane has 5 litres of pineapple juice, how many litres of
i orange juice
ii passion fruit juice
will she need to make the fruit juice?

8 Look at the diagram.



How many more squares need to be shaded so that the ratio of shaded to unshaded squares is 2 : 5?

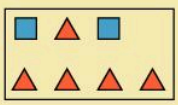
9 Find two numbers in the ratio 5 : 3 and with a sum of 144.

Summary

You should know ...

1 How to compare one thing to another using a ratio.

For example:



The ratio of triangles to squares is 5 : 2.

2 How to simplify a ratio by dividing both sides of the ratio by the HCF of the two numbers.

For example:

The ratio 8 : 12 = 2 : 3 (divide both sides by 4)

3 How to share something in a given ratio.

For example:

Share \$30 in the ratio 2 : 3

There are 2 + 3 = 5 parts

Each part is worth \$30 ÷ 5 = \$6,

so 2 parts are worth 2 × \$6 = \$12

and 3 parts are worth 3 × \$6 = \$18

4 How to increase or decrease amounts in direct proportion.

For example:

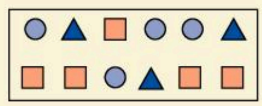
If 3 bags of sweets contain 72 sweets altogether, how many sweets are there in 4 bags?

In 1 bag there are 72 ÷ 3 = 24 sweets

In 4 bags there are 24 × 4 = 96 sweets

Check out

1



What is the ratio of

a triangles to circles

b squares to triangles

c circles to squares?

2 Write each ratio in its simplest form.

a 6 : 12 **b** 28 : 7

c 24 : 9 **d** 40 : 15

3 Share \$50 in the ratio

a 4 : 1 **b** 7 : 3

c 1 : 9 **d** 17 : 3

4 If 5 pencils cost \$1.35, how much do

a 10 pencils cost

b 3 pencils cost

c 34 pencils cost?

17 Area, perimeter and volume

Objectives

- Know the abbreviations for and relationships between square metres (m^2), square centimetres (cm^2) and square millimetres (mm^2).
- Derive and use the formula for the volume of a cuboid.
- Derive and use formulae for the area and perimeter of a rectangle; calculate the perimeters and areas of compound shapes made from rectangles.
- Calculate the surface area of cubes and cuboids from their nets.

What's the point?

How big is your bedroom? How much carpet would you need to buy to put on your bedroom floor? What about your whole house? To answer such questions you need to be able to measure the area of shapes.

Before you start

You should know ...

1 How to multiply and divide decimals by 10, 100 or 1000.
For example:
$100 \times 0.34 = 34$
$42.1 \div 10 = 4.21$

Check in

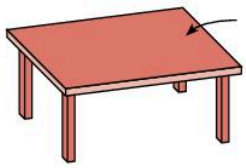
1 Work out:
- **a** 1.3×100
- **b** $0.3 \div 10$
- **c** 1000×0.07
- **d** $2640 \div 100$
- **e** 100×2.7
- **f** $13.8 \div 1000$

2 The relationship between metres, centimetres and millimetres.

1 m = 100 cm
1 cm = 10 mm
1 m = 1000 mm

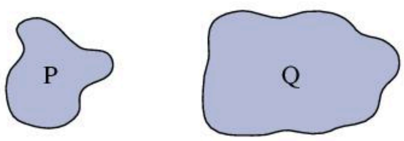
2 **a** Write in centimetres:
i 6 m **ii** 2.4 m
b Write in millimetres:
i 7 cm **ii** 7.3 cm
iii 2 m **iv** 2.6 m

17.1 What is area?



The top of a desk is a **surface**. The front of a blackboard and the floor of a room are also surfaces.

- The size of a surface is called its **area**.



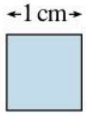
The area of surface Q is bigger than the area of surface P.

17.2 Some units of area

You will need centimetre-squared paper.

The square centimetre

Squares are used to measure area. A square centimetre is a square with each side 1 cm long.
Here it is:



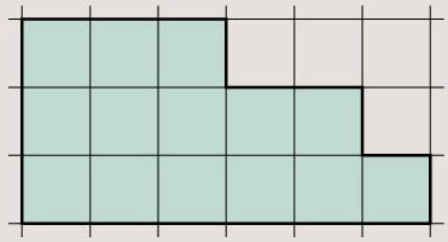
Area = 1 square centimetre
or 1 cm^2

This square has an area of **one square centimetre**. Centimetre-squared paper is covered with squares like this.

Exercise 17A

1 Write in a short way:
a 5 square centimetres
b 16 square centimetres
c 420 square centimetres.

2 The shape below is drawn on centimetre-squared paper.

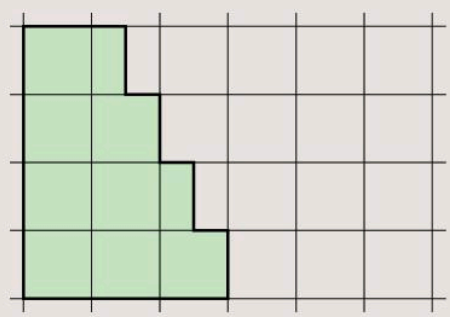


a What is the area of each square?
b How many squares are in the shaded shape?
c What is the area of the shaded shape?

3 Write down the area, in square centimetres, of each of the shaded shapes below.

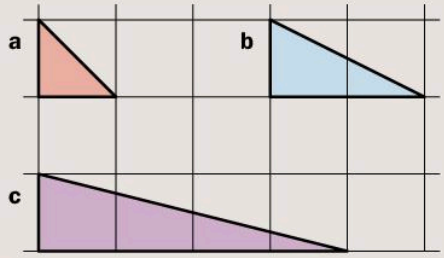
a

b

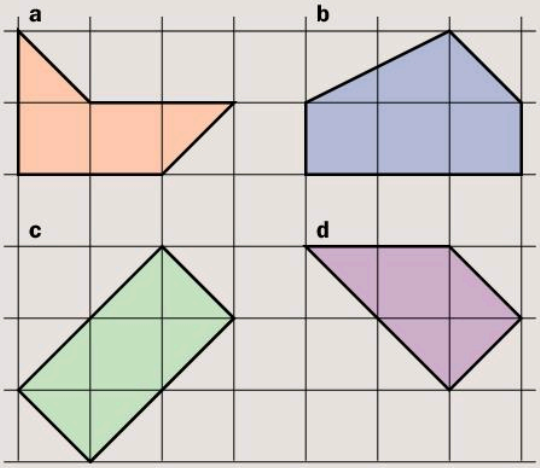


4 Draw accurately two shapes each with an area of 4 square centimetres.

5 Look at the shaded triangles below. Can you find their areas?



6 Find the area of each of the shapes below:

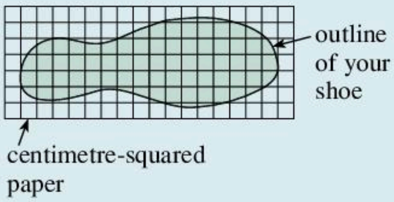


7 Which shapes in Question **6** have the same area?

INVESTIGATION

Big foot!

Put your shoe on a sheet of centimetre-squared paper.



a Draw an outline around your shoe. Find the approximate area of your foot.

b Compare your foot size with other class members. Who has the biggest foot?

c Repeat parts **a** and **b** to find the biggest hand.

d Do people with big feet have big hands?

The square millimetre

Look at the drawings below of a mosquito's wing and a square centimetre.

Mosquito's wing

$1\,cm^2$

The area of the mosquito's wing is less than $1\,cm^2$

You can measure small areas using the square millimetre (mm^2).

1 square millimetre

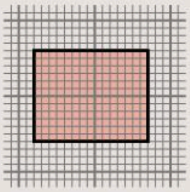
A square millimetre is a square with side 1 mm.

Below is a square centimetre divided into 100 square millimetres (10 rows with 10 square millimetres in each row).

$1\,cm^2 = 100\,mm^2$

Exercise 17B

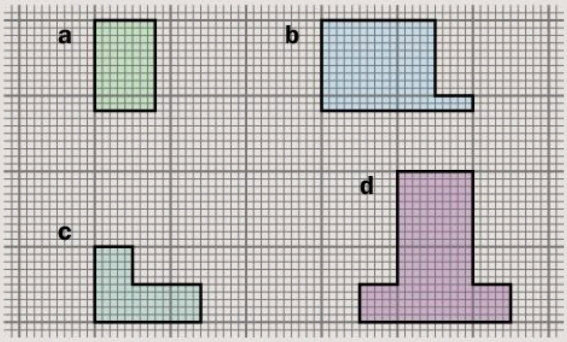
1 The shape below is drawn on cm/mm graph paper. Each small square is $1\,mm^2$.



a How many small squares are in the shape?

b What is the area of the shape?

2 Find the area of each of these shapes, in mm^2.



3 Draw accurately shapes with area:
- a $300\,mm^2$
- b $450\,mm^2$

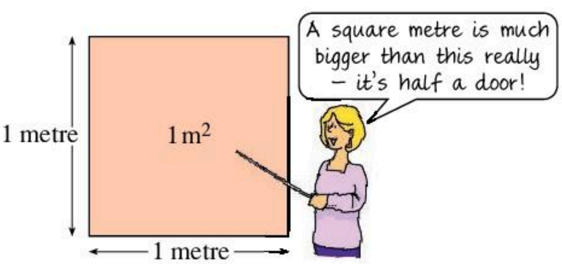
4 Change these measurements to mm^2:
- a $1\,cm^2$
- b $3\,cm^2$
- c $4.2\,cm^2$
- d $0.5\,cm^2$

5 Change these measurements to cm^2:
- a $100\,mm^2$
- b $600\,mm^2$
- c $80\,mm^2$
- d $730\,mm^2$

The square metre

The square centimetre is too small to measure large areas like the area of a football stadium.

For larger areas, the square metre (m^2) is used.



A square metre is a square with side 1 m. It is about half the size of a door.

1 square metre can be split into 100 rows with 100 square centimetres in each row.

$1\,m^2 = 10\,000\,cm^2$

One square metre can also be split into 1000 rows with 1000 square millimetres in each row.

$1\,m^2 = 1\,000\,000\,mm^2$

Exercise 17C

1 Draw a square with side length 1 metre on the classroom board.
What is the area of your square?

2 Estimate the area, in m^2, of:
- a your classroom ceiling
- b a football field
- c the classroom window.

3 Which unit would you choose for measuring the area of:
- a the classroom floor
- b your smallest fingernail
- c this page
- d the school playground
- e the top of your desk
- f a shirt button?

4 Change these measurements to m^2:
- a $20\,000\,cm^2$
- b $5\,000\,000\,mm^2$
- c $30\,000\,cm^2$
- d $7\,000\,000\,mm^2$

5 Change these measurements to cm^2:
- a $4\,m^2$
- b $3\,m^2$
- c $2.6\,m^2$
- d $0.2\,m^2$

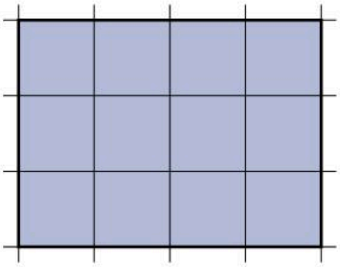
6 Change these measurements to mm^2:
- a $9\,m^2$
- b $3\,m^2$
- c $2.7\,m^2$
- d $0.05\,m^2$

17.3 Areas of rectangles

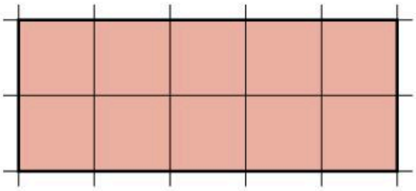
Areas of rectangles

These rectangles are drawn on centimetre-squared paper. Count squares to find the area of each. Then copy and complete the table.

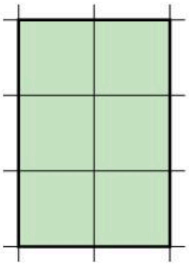
a



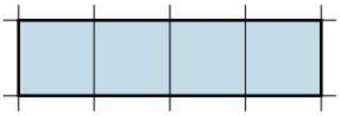
b



c



d



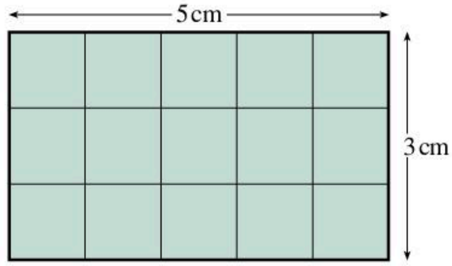
	Area	Length	Width	Length × Width
a	12	4	3	12
b				
c				
d				

Compare the first column and the last column of the completed table. What do you notice?
Can you see a quick way to find the area of a rectangle without counting all the squares?

- The area of a rectangle = length × width

EXAMPLE 1

Find the area of the rectangle below.

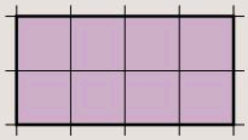


The length of the rectangle = 5 cm
The width of the rectangle = 3 cm
Area of recangle = length × width
= 5 cm × 3 cm
= $15\,\text{cm}^2$

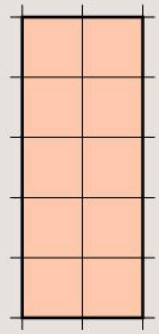
Exercise 17D

1 Find the area of each rectangle when drawn on centimetre-squared paper:

a

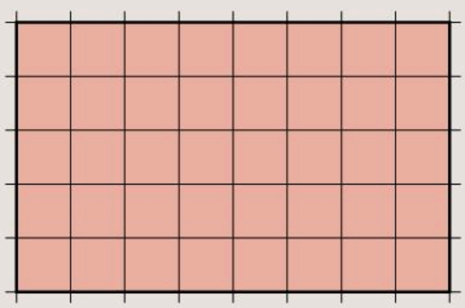


b



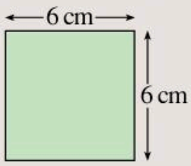
c

d

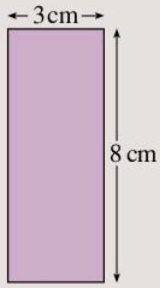


2 Find the area of each rectangle.

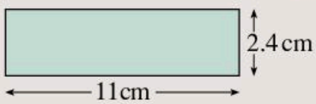
a



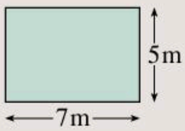
b



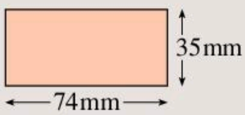
c



d



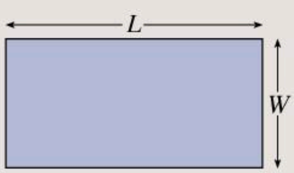
e



3 Copy and complete the table for rectangle measurements.

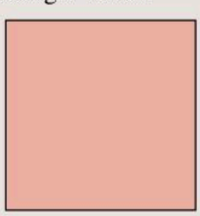
Length (in cm)	Width (in cm)	Area (in cm^2)
9	7	
6	3.5	
6		24
	4	36
	10	110
12		144

4 The rectangle below has length L and width W.



Write down its area, A, in terms of L and W.

5 Look at the rectangle below.



a Measure, with your ruler, its length and width.
b What do you notice about the length and width?
c What is the special name for this sort of rectangle?
d What is its area?

6 Find the area of the square with each side

a 3 cm **b** 5 cm
c 4 mm **d** 10 m

7 Fill in the missing measurements:

Rectangle	Length (cm)	Width (cm)	Area (cm^2)
Picture	25	20	
Envelope	25		150
Book	30		600
Poster	50		1200

8 Draw as many rectangles as you can with area 24 cm^2.

9 A square carpet has an area of 36 m^2. What is the length of one side?

10 A school has two football fields. One is 115 m long and 60 m wide and the other is 105 m long and 85 m wide.
Which football field has the larger area and by how much?

11 A rectangle is 40 mm long and 30 mm wide. Find its area in

a square millimetres
b square centimetres.

12 A carpenter wants to make a rectangular table with an area of 1.8 m^2. The width must be 90 cm. How long should he make it?

13 The floor of a room which measures 3 m by 4 m is to be covered with tiles. The tiles are 20 cm square. How many tiles are needed?

14 A rectangular photograph measures 15 cm by 12 cm. It is fixed on to a rectangular piece of cardboard which measures 24 cm by 30 cm. What area of the cardboard is not covered by the photograph?

15 Mrs Ali decided that she needed 15 m^2 of curtain material. She was shown some materials in three different widths:

a 90 cm **b** 1.2 m **c** 1.5 m

What length of each width would she have to buy?

17.4 Perimeters of rectangles

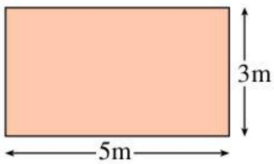
A farmer has a rectangular field. He wants to put a fence all the way around the outside edges of the field. To know how much fencing he needs to buy, he needs to know the **perimeter** of his field.

The perimeter is the total distance around the edges of a shape.

Perimeter can be measured in mm, cm or m. Be careful not to use mm^2, cm^2 or m^2, which are the units for area.

EXAMPLE 2

Find the perimeter of the rectangle below.

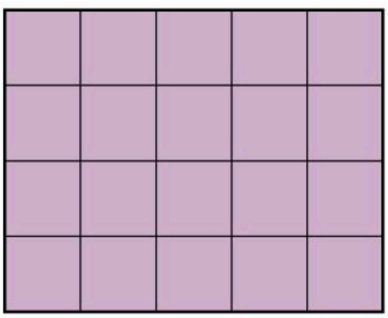


Imagine walking around the outside edge of this shape. Add up side lengths as you go:

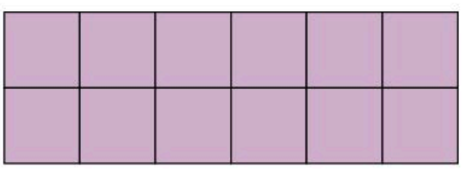
$3\,m + 5\,m + 3\,m + 5\,m = 16\,m$

Find the perimeters of the rectangles below, when drawn on centimetre-squared paper. Then copy and complete the table.

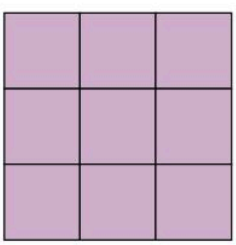
a



b



c



d

	Perimeter	Width	Length	(Width + Length) × 2
a	18	4	5	$(4 + 5) \times 2 = 18$
b				
c				
d				

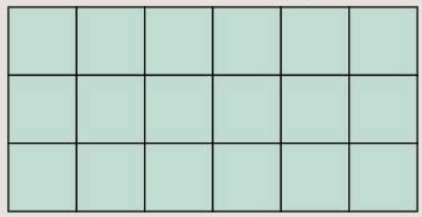
What do you notice? Is there a slightly quicker way to find the perimeter, without adding all four side lengths? Is there an even quicker way for a square?

- The perimeter of a rectangle = (width + length) × 2
- The perimeter of a square = side length × 4

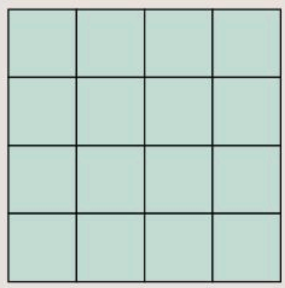
Exercise 17E

1 Find the perimeter of each rectangle, when drawn on centimetre-squared paper.

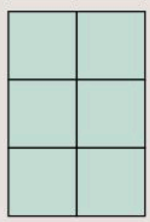
a



b



c



d

2 Find the perimeter of each rectangle from Question **2** of Exercise 17D.

3 Copy and complete the table for rectangles.

Width (cm)	Length (cm)	Perimeter (cm)
10	15	
4.5	12	
8		36
	12	42
12		56

4 The rectangle below has length L and width W.

Write down its perimeter, P, in terms of L and W.

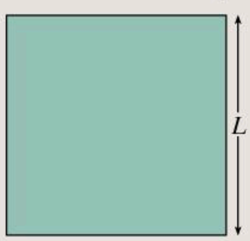
5 Draw as many rectangles as you can with a perimeter of 24 cm.

6 What is the perimeter of a square with side length 8 m?

7 Find the perimeters of the rectangles measuring:

a 18 cm by 5 cm
b 10 mm by 6 mm
c 25 m by 4.5 m
d 70 cm by 3 m

8 The square below has side length L.



Write down its perimeter, P, in terms of L.

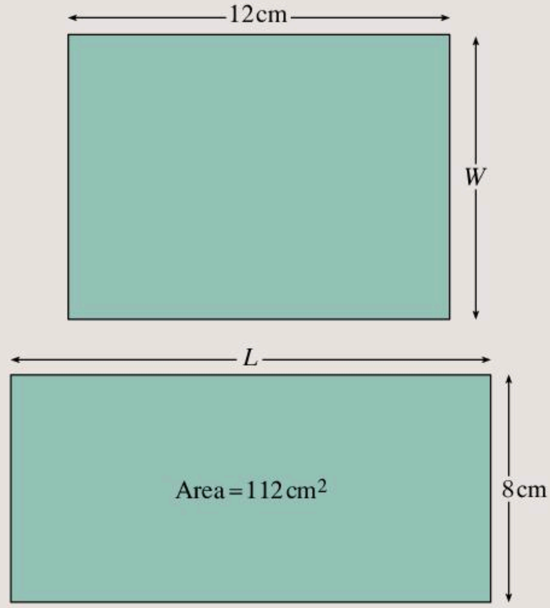
9 A square rug has a perimeter of 64 cm. Baasim says the side length of the square is 8 cm, Devaj thinks it is 16 cm. Who is correct?

10 Daryl was working out the perimeter of a rectangle measuring 4 m by 20 cm.
This was his working:

The perimeter of a rectangle
= (width + length) × 2
Perimeter = (4 + 20) × 2
= 24 × 2
= 48

What are the mistakes Daryl made? Write down what the correct working should be.

11 These two rectangles have the same perimeter. What are the missing side lengths?



12 Parvati said she thought that the formula perimeter of a rectangle = (width + length) × 2 was wrong and that the formula should be:
perimeter of a rectangle
= width × 2 + length × 2.
Milana said she thought that they both gave the same answer.

Who is correct? Explain your answer.

13 A rectangle has a perimeter of 20 cm. What is the width and length of the rectangle if its area is

a 21 cm^2
b 24 cm^2
c 9 cm^2?

14 Are there any other possible areas for a rectangle with a perimeter of 20 cm, with whole-number side lengths, that are not given in Question **13**? Is 24 cm^2 the greatest possible area? Is 9 cm^2 the smallest possible area? Give reasons for your answer.

15 A rectangular piece of card measures 430 mm by 310 mm. It is being cut up to make rectangular business cards measuring 8 cm by 5 cm.

a What is the maximum number of business cards that can be made?

If you make the maximum number of business cards:

b What area of the card have you used?
c What area of card is left over?

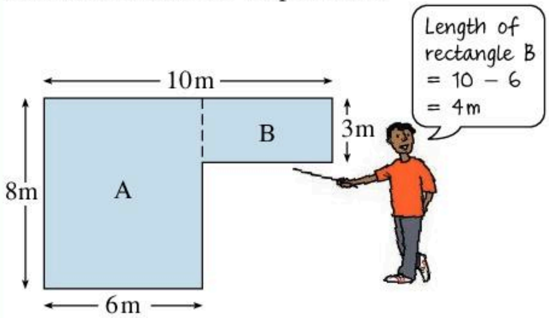
17.5 Compound shapes

A **compound shape** (or composite shape) is made up of two or more simpler shapes. To work out the area of a compound shape, work out the areas of the simpler shapes, then add or subtract them.

Areas of some shapes can be found by dividing them up into rectangles.

EXAMPLE 3

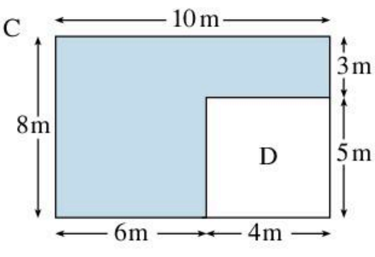
Find the area of the L-shaped room.



The shape can be divided into two rectangles A and B.

$$\begin{aligned}\text{Area of rectangle A} &= 8\,\text{m} \times 6\,\text{m}\\ &= 48\,\text{m}^2\\ \text{Area of rectangle B} &= 4\,\text{m} \times 3\,\text{m}\\ &= 12\,\text{m}^2\\ \text{Area of room} = \text{A} + \text{B} &= 48\,\text{m}^2 + 12\,\text{m}^2\\ &= 60\,\text{m}^2\end{aligned}$$

Example 3 can be done another way.



Area of large rectangle, C = $10\,\text{m} \times 8\,\text{m} = 80\,\text{m}^2$

Area of small rectangle, D = $4\,\text{m} \times 5\,\text{m} = 20\,\text{m}^2$

Then subtract the area of the smaller rectangle, D, from the larger rectangle to get the area of the L-shaped room.

$$\begin{aligned}\text{Area of room} = \text{C} - \text{D} &= 80\,\text{m}^2 - 20\,\text{m}^2\\ &= 60\,\text{m}^2\end{aligned}$$

If you need to find the perimeter of a compound shape, you need to add up all the edges around the outside. A common error is to forget to add on unlabelled edges.

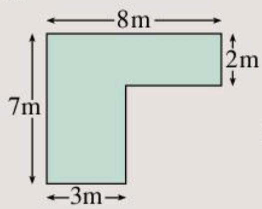
The perimeter of the shape from Example 3 is $10\,\text{m} + 3\,\text{m} + 4\,\text{m} + 5\,\text{m} + 6\,\text{m} + 8\,\text{m} = 36\,\text{m}$.

You will notice that the perimeter of the L-shaped room from Example 3 is the same as the perimeter of the large rectangle (rectangle C in the diagram). This does not always happen. Can you see why it is true in this case?

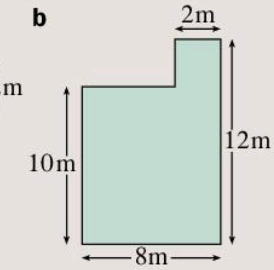
Exercise 17F

1 Find the areas of the shapes using rectangles:

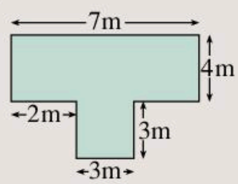
a



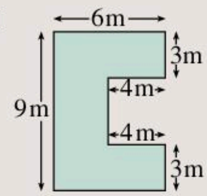
b



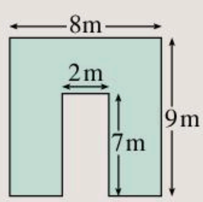
c



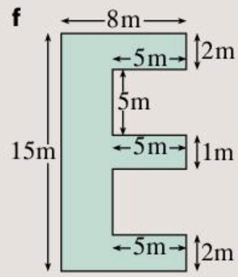
d



e

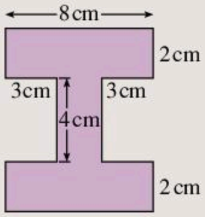


f

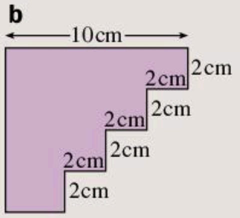


2 Find the area of each of these shapes:

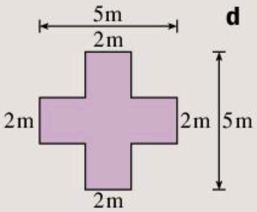
a



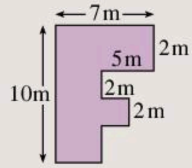
b



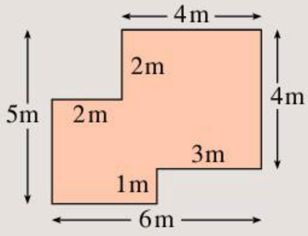
c



d



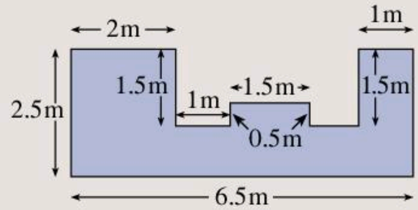
3 Calculate the area of this shape:



4 Find the perimeters of the shapes in Question **1**.

5 Find the perimeters of the shapes in Question **2**.

6 This is the plan of the paved patio outside Mr Ramchand's house. Find its area.



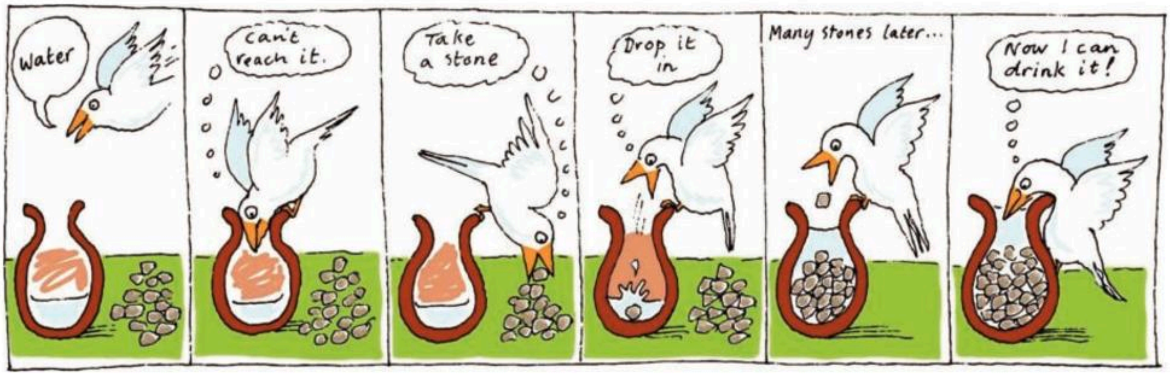
TECHNOLOGY

Visit the Area and Perimeter section at the site www.woodlands-junior.kent.sch.uk/maths and try

i Area Explorer
ii Perimeter Explorer
iii Compare Area and Perimeter

for further practice.

17.6 What is volume?



The cartoon shows how Supercrow got water from a jug.

a Was there *more* water in the jug after Supercrow had dropped in the stones?

b Why did the water rise to the top of the jug?

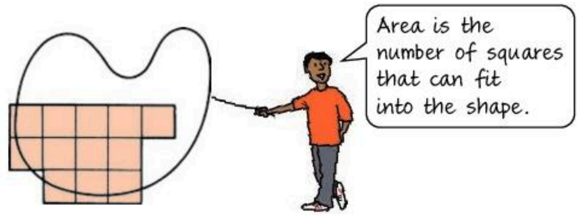
c If Supercrow kept on adding stones, what would happen to the water?

The water rose in the jug because the stones took up space in the bottom of the jug.

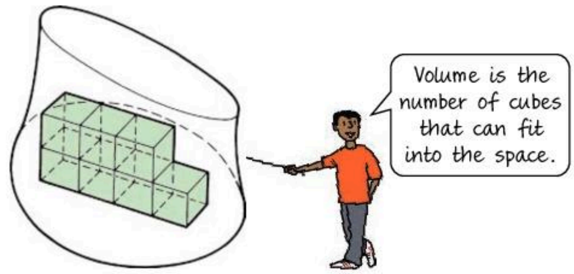
- The amount of space that something takes up is called its **volume**.

The volume of your body is much larger than the volume of this book. But it is much smaller than the volume of your classroom.

You can find the area of a surface by covering it with squares and counting them.

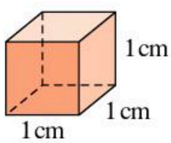


In the same way you can find the volume of a space by filling it with cubes and counting them.



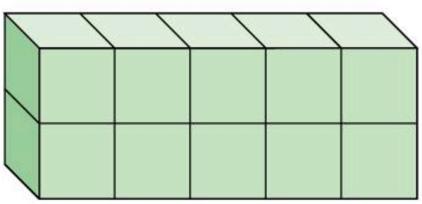
The cubic centimetre

To make sure everyone uses the same cubes for measuring volume, special sizes were chosen.
One of these is the cube with each edge 1 centimetre.



It has a volume of **1 cubic centimetre**. This is usually written as **1 cm³**.
(Sometimes you will see **1 cc**.)

This cuboid is made up of 10 cubes of edge 1 cm.

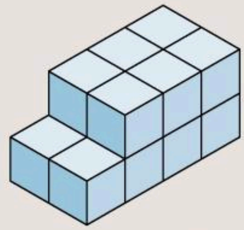


It has a volume of $10\,\text{cm}^3$.

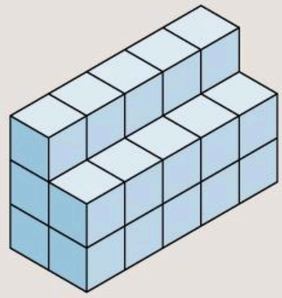
Exercise 17G

1 These shapes are made of centimetre cubes. Write down the volume of each shape.

a

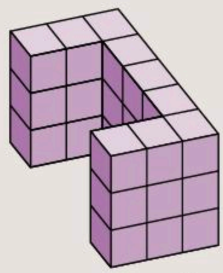


b

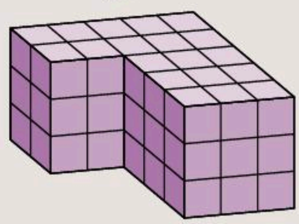


2 These shapes are made of centimetre cubes. Write down the volume of each shape.

a



b



3 Make a guess at the volume, in cm^3, of:

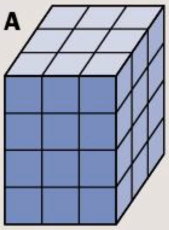
a a pencil case

b a calculator

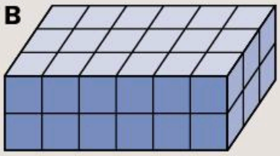
c an egg

Compare your answers with those of your friends.

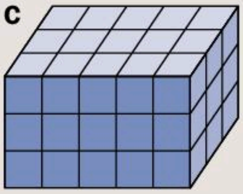
4 A



B



C



a Copy and complete the table.

	Number of cubes long L	Number of cubes wide W	Number of cubes high H	L × W × H	Total number of cubes
A	3	3	4	36	
B	6				
C					

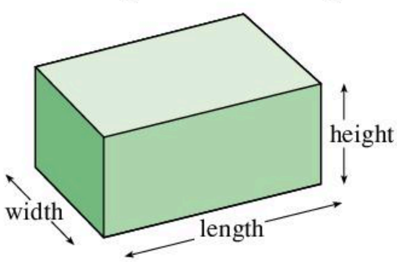
b What do you notice about the last two columns?

5 Jean found a small cardboard box 6 cm long, 4 cm wide and 3 cm high.
How many centimetre cubes could she pack into the box?

17.7 Volume of cuboids

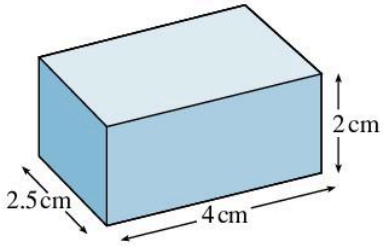
- The volume of a cuboid, V, can be worked out using:

$$V = \text{length} \times \text{width} \times \text{height}$$



EXAMPLE 4

Find the volume of the cuboid shown.

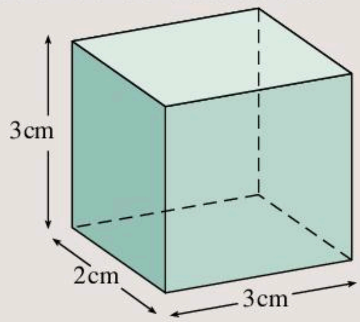


Volume of cuboid
$= \text{length} \times \text{width} \times \text{height}$
$= 4\,\text{cm} \times 2.5\,\text{cm} \times 2\,\text{cm}$
$= 10\,\text{cm}^2 \times 2\,\text{cm}$
$= 20\,\text{cm}^3$

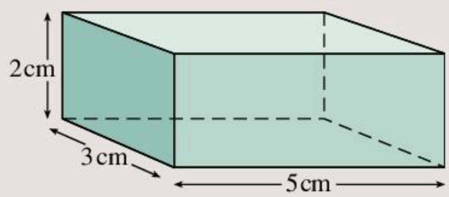
Exercise 17H

1 Find the volume of these cuboids.

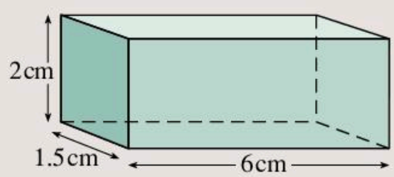
a



b



c



2 When a cuboid is measured in centimetres, its volume is given by:

$$l\,\text{cm} \times w\,\text{cm} \times h\,\text{cm} = V\,\text{cm}^3$$

Can you explain why the raised 3 is used in $V\,\text{cm}^3$?

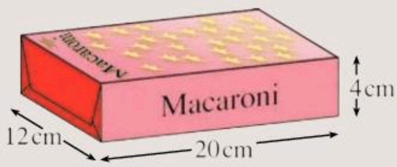
3 Copy and complete the table for cuboids:

Length (cm)	4	5	6.5	4.5
Width (cm)	2	2	3	2.5
Height (cm)	1	3	4	3
Volume (cm^3)				

4 Copy and complete the table for cuboids:

V cm^3	60	25	30	64
l cm	3	5		8
w cm	4		1	2
h cm		1	3	

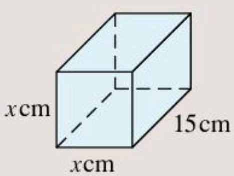
5



A box of macaroni measures 12 cm by 20 cm by 4 cm.
What is the volume of the box?

6 A cuboid measures 50 cm by 1 m by 40 cm. What is its volume?

7 The cuboid below has a volume of 2160 cm^3. It has a square cross-section. What is the side length x?



8 a Think of two things with volumes too great to measure in cm^3.
b Think of two things with volumes too small to measure in cm^3.

9 a The cubic centimetre is based on a unit of length. What unit is this?
b Write down a metric unit of length that is larger than the centimetre.
c Write down a metric unit of length that is smaller than the centimetre.
d Do you think there could be units of volume based on these? Try to imagine them.

10 A box of pins has a volume of 24 cm^3.
a Draw and show the measurements of two boxes with this volume.
b David made a box that was 1 cm long, 1 cm wide and 24 cm high. Why is this not a good design?

TECHNOLOGY

Find out more about how to find the volume of common solids by visiting the Geometry Lessons at www.mathguide.com/lessons

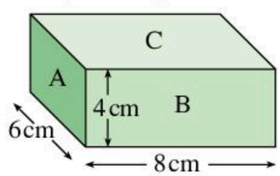
Try the tests (quizmasters) at the end of each lesson!

17.8 Surface area

The area of the total surface of a solid is called its **surface area**. For example, the surface area of a box is the sum of the areas of its six rectangular faces.

EXAMPLE 5

What is the surface area of a cuboid with dimensions 8 cm by 6 cm by 4 cm?



The cuboid has six faces.

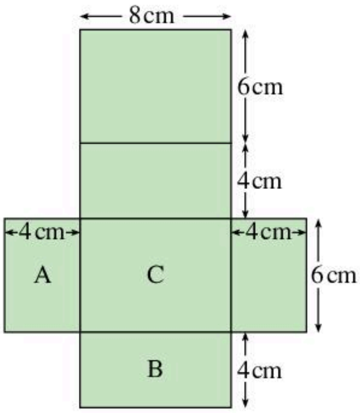
Area side A = length × width
= 6 cm × 4 cm
= 24 cm^2

Area side B = 4 cm × 8 cm = 32 cm^2

Area side C = 8 cm × 6 cm = 48 cm^2

Surface area of cuboid
$= 2 \times A + 2 \times B + 2 \times C$
$= 2 \times 24\,cm^2 + 2 \times 32\,cm^2 + 2 \times 48\,cm^2$
$= 48\,cm^2 + 64\,cm^2 + 96\,cm^2$
$= 208\,cm^2$

To help work out the surface area of a cuboid, you can draw a net of the cuboid and calculate the area of the net. The net of the cuboid from Example 5 looks like this:



The working out for the surface area is exactly the same:

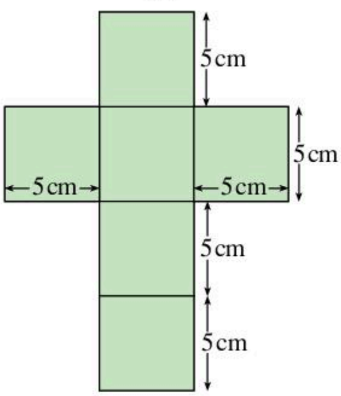
Surface area of cuboid

$= \text{surface area of net}$
$= 2 \times A + 2 \times B + 2 \times C$
$= 2 \times 24\,\text{cm}^2 + 2 \times 32\,\text{cm}^2 + 2 \times 48\,\text{cm}^2$
$= 48\,\text{cm}^2 + 64\,\text{cm}^2 + 96\,\text{cm}^2$
$= 208\,\text{cm}^2$

Note that the units for surface area are cm^2.

EXAMPLE 6

What is the surface area of a cube with side length 5 cm? Draw a net to help you.



There are 6 faces on a cube, all of them squares.

The area of one face is $5\,\text{cm} \times 5\,\text{cm} = 25\,\text{cm}^2$
So the total surface area

$= 6 \times 25\,\text{cm}^2$
$= 150\,\text{cm}^2$

Exercise 17I

1 **a** What is the surface area of a cube with side 1 cm?

b What is the surface area of a cube with side 3 cm?

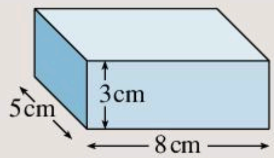
2 The face of a cube has area $24\,\text{cm}^2$. What is the surface area of the cube?

3 **a** Find the total surface area of cubes with sides 4 cm and 8 cm.

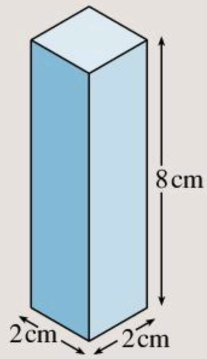
b What happens to the surface area if the edge length of a cube is doubled?

4 Find the surface area of these cuboids.

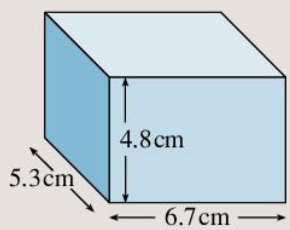
a



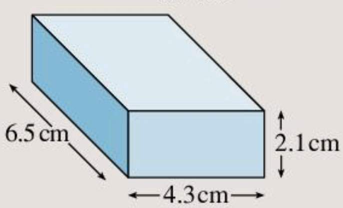
b



c



d



5

a With a ruler, measure, the dimensions of a DVD case.

b Calculate the total surface area of your DVD case.

6

a Collect three different cereal boxes.
b Measure their dimensions.
c Calculate the total surface area of each.
d Which box can be made using the least amount of card?

7 A cube has surface area $486\,\text{cm}^2$. What is its side length?

8 A room has a rectangular floor 5.5 m long and 4.5 m wide. The room is 2.5 m in height.
a What is the area of wall space in the room?
b How much paint is required to paint the walls with two coats if 1 litre of paint can cover $11.25\,\text{m}^2$?

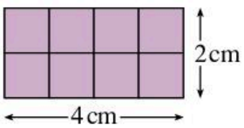
9 **a** Find the total surface area of a cuboid with dimensions $4\,\text{cm} \times 3\,\text{cm} \times 2\,\text{cm}$.
b If the length of the cube, 4 cm, is doubled, what is the new surface area of the cuboid?
c If both length 4 cm and width 3 cm are doubled, what is the new surface area?
d How does the surface area change from the original if all three side lengths are doubled?

Consolidation

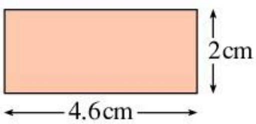
Example 1

Find the area of these rectangles.

a



b



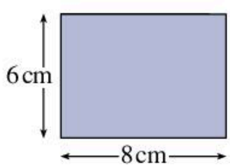
a Area of rectangle = length × width
$= 4\,\text{cm} \times 2\,\text{cm}$
$= 8\,\text{cm}^2$

b Area of rectangle = length × width
$= 4.6\,\text{cm} \times 2\,\text{cm}$
$= 9.2\,\text{cm}^2$

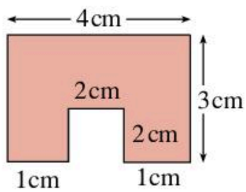
Example 2

Find the perimeter of these shapes.

a



b

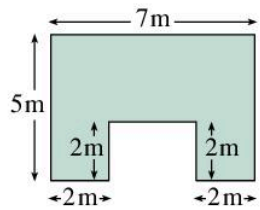


a Perimeter = (width + length) × 2
$= (6\,\text{cm} + 8\,\text{cm}) \times 2$
$= 28\,\text{cm}$

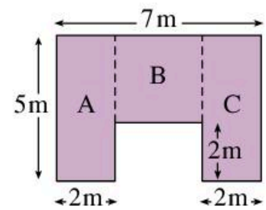
b Perimeter = distance around shape
$= 4\,\text{cm} + 3\,\text{cm} + 1\,\text{cm} + 2\,\text{cm}$
$+ 2\,\text{cm} + 2\,\text{cm} + 1\,\text{cm} + 3\,\text{cm}$
$= 18\,\text{cm}$

Example 3

Find the area of this shape.

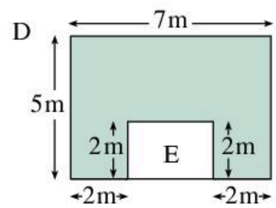


First divide the shape into rectangles.



Notice that the length of rectangle
$B = 7\,\text{m} - 2\,\text{m} - 2\,\text{m} = 3\,\text{m}$
and width of rectangle $B = 5\,\text{m} - 2\,\text{m} = 3\,\text{m}$
Area of rectangle $A = 5\,\text{m} \times 2\,\text{m} = 10\,\text{m}^2$
Area of rectangle $B = 3\,\text{m} \times 3\,\text{m} = 9\,\text{m}^2$
Area of rectangle $C = 5\,\text{m} \times 2\,\text{m} = 10\,\text{m}^2$
Area of shape $= 10\,\text{m}^2 + 9\,\text{m}^2 + 10\,\text{m}^2 = 29\,\text{m}^2$

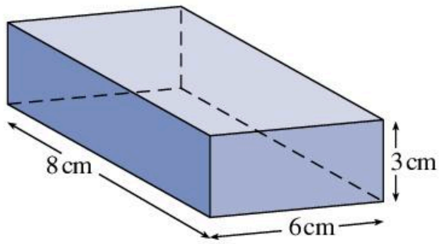
Alternative method:



Length of small rectangle, $E = 7\,\text{m} - 2\,\text{m} - 2\,\text{m} = 3\,\text{m}$
Area of large rectangle, $D = 7\,\text{m} \times 5\,\text{m} = 35\,\text{m}^2$
Area of rectangle $E = 3\,\text{m} \times 2\,\text{m} = 6\,\text{m}^2$
Area of shape $= 35\,\text{m}^2 - 6\,\text{m}^2 = 29\,\text{m}^2$

Example 4

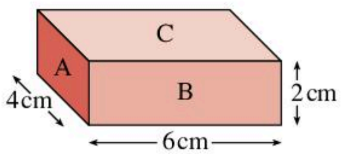
Find the volume of this cuboid.



Volume of cuboid $= \text{length} \times \text{width} \times \text{height}$
$= 8\,\text{cm} \times 6\,\text{cm} \times 3\,\text{cm}$
$= 144\,\text{cm}^3$

Example 5

Find the surface area of:



Area of A $= 4\,\text{cm} \times 2\,\text{cm} = 8\,\text{cm}^2$
Area of B $= 6\,\text{cm} \times 2\,\text{cm} = 12\,\text{cm}^2$
Area of C $= 6\,\text{cm} \times 4\,\text{cm} = 24\,\text{cm}^2$

Total surface area $= 2 \times \text{A} + 2 \times \text{B} + 2 \times \text{C}$
$= 2 \times 8\,\text{cm}^2 + 2 \times 12\,\text{cm}^2 + 2 \times 24\,\text{cm}^2$
$= 16\,\text{cm}^2 + 24\,\text{cm}^2 + 48\,\text{cm}^2$
$= 88\,\text{cm}^2$

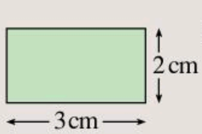
Exercise 17

1 Which metric units of area would you use to measure the area of

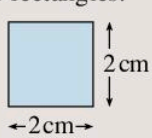
a your classroom door **b** your hand
c a computer screen **d** a cat's paw
e New Zealand **f** a blade of grass?

2 Calculate the area of these rectangles:

a



b



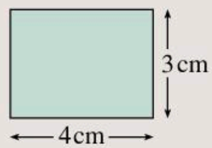
3 A rectangular room is 7 m wide. If its area is $35\,\text{m}^2$, what is the room's length?

4 Draw five different shapes with area

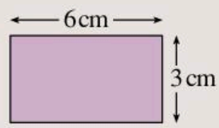
a $6\,\text{cm}^2$ **b** $8\,\text{cm}^2$ **c** $10\,\text{cm}^2$
d $28\,\text{mm}^2$

5 Find the perimeter of these rectangles:

a

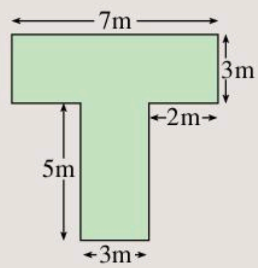


b

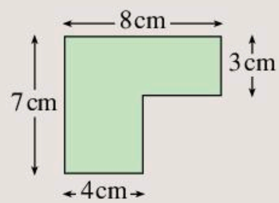


6 Find the area of these shapes:

a

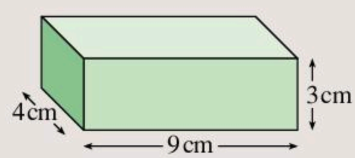


b



7 Find the perimeter of the shapes in Question **6**.

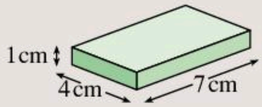
8 Calculate the total surface area of this cuboid:



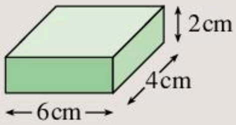
9 What is the total surface area of a cube with side length 10 cm?

10 What is the volume of these cuboids?

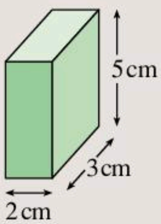
a



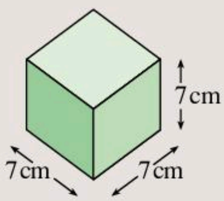
b



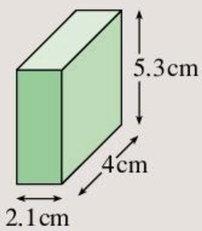
c



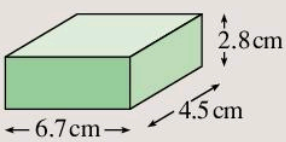
d



e



f



11 A double page of 'The Chronicle' newspaper is 56 cm wide and 43 cm deep. Find:

a the area of a single page

b the total area of 32 pages.

12 The wall of a shower in a bathroom is 2 m high and 1.5 m wide.

a What is the area of the wall, in m^2?

b What is the area of the wall, in cm^2?

c The bathroom tiles are square, with a side length of 15 cm.
What is the area of a tile, in cm^2?

d How many tiles would be needed to tile the shower wall?

e What would be the cost of tiling the wall, if the price of one tile is $3.15?

13 A store room has a rectangular base 3.2 m by 4.1 m, with walls 2.4 m tall. On one wall there is a door 2.1 m high and 0.9 m wide.

a What is the area of the door?

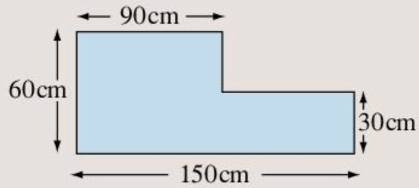
b What is the total surface area of the four walls?

c Paint costs $12.25 per litre.

i How many litres of paint are required to paint the walls if 1 litre can cover $24\,m^2$?

ii What will be the cost of painting two coats on the walls?

14 Ashton wishes to tile the area above his kitchen sink.
The area is shown in the diagram.



a What is the area that he has to tile?

b Kitchen tiles are square with length of side 15 cm.

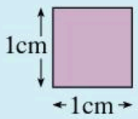
i What is the area of a kitchen tile?

ii How many tiles are needed to tile the area above Ashton's kitchen sink?

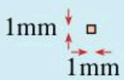
Summary

You should know ...

1 Standard units of area are:



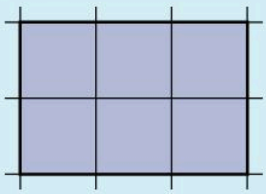
One square metre $= 1\,m^2$
One square centimetre $= 1\,cm^2$



One square millimetre $= 1\,mm^2$
$100\ mm^2 = 1\,cm^2$

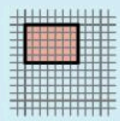
2 You can find the area of a shape by counting the number of square-centimetres or square millimetres in it.
For example:

a



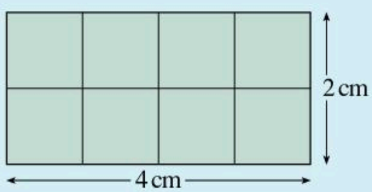
Area = $6cm^2$

b



Area = $35\,mm^2$

3 The area of a rectangle, A, is given by A = length × width
For example:



Area of rectangle = length × width
= 4cm × 2cm
= $8cm^2$

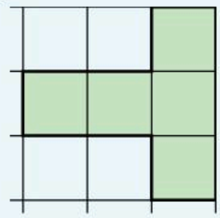
4 The distance around the edge of a shape is called its perimeter.

Check out

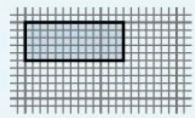
1 Which unit of area would you use to find the area of
a your fingernail
b an envelope
c a cricket pitch
d a fly's wing?

2 Find the area of these shapes.

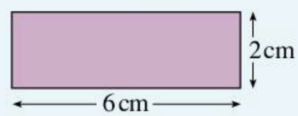
a



b

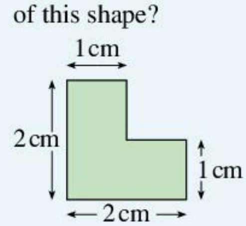


3 **a** Find the area of the rectangle below.



b What is the area of a football pitch 80m long and 50m wide?

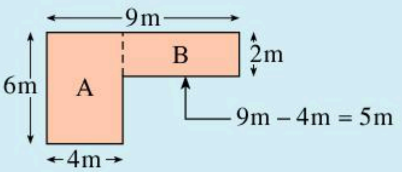
4 What is the perimeter of this shape?



5 To find the area of a compound shape, you need to divide it into simple shapes.

For example:

The area of this L-shaped room:

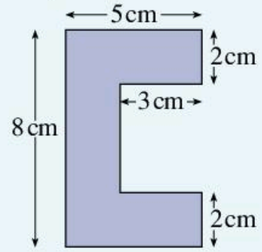


Area of rectangle A = 6cm × 4cm = 24 m^2

Area of rectangle B = 5cm × 2cm = 10 m^2

Area of room = 24 m^2 + 10 m^2 = 34 m^2

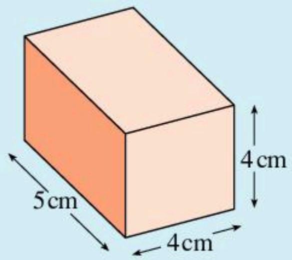
5 Find the area of this shape.



6 The volume of a cuboid is given by the formula

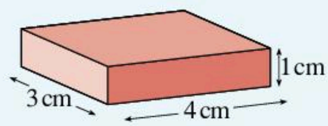
volume of cuboid = length × width × height

For example:



Volume of cuboid = 4cm × 5cm × 4cm
= 80 cm^3

6 **a** What is the volume of this box?

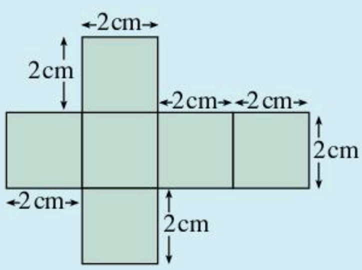


b Find the volume of a sugar cube that has sides 1.5cm long.

7 The area of the total surface of a solid is called its surface area.

For example:

The surface area of a cube with side length 2cm. There are 6 faces, all squares:



Area of each face = 2cm × 2cm = 4 cm^2

Total surface area = 6 × 4 cm^2 = 24 cm^2

7 Find the total surface area of a

a cuboid with length 7m, width 6m and height 4m

b cube with side length 11cm

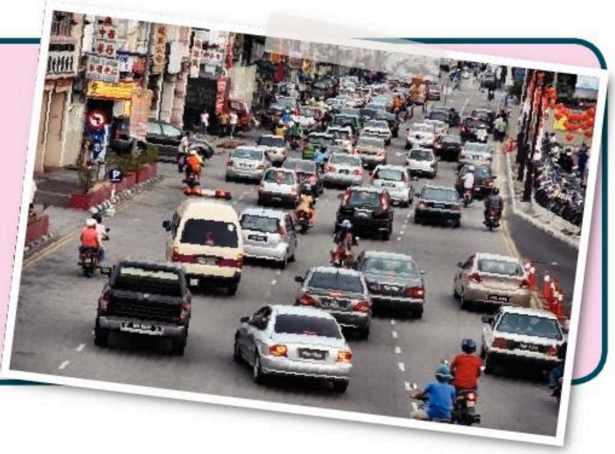
18 Probability

Objectives

- Use the language of probability to describe and interpret results involving likelihood and chance.
- Use experimental data to estimate probabilities.
- Understand and use the probability scale from 0 to 1.
- Find probabilities based on equally likely outcomes in simple contexts.
- Identify all the possible mutually exclusive outcomes of a single event.
- Compare experimental and theoretical probabilities in simple contexts.

What's the point?

What is the likelihood that you will be caught in traffic on your way to school? Busy traffic on the road would make your journey time longer and could decrease your chances of arriving on time!



Before you start

You should know ...

1 About simple equivalent fractions and decimals.
For example:

$$0.5 = \frac{1}{2} \text{ and } \frac{3}{10} = 0.3$$

2 About simple equivalent percentages and decimals.
For example:

$0.5 = 50\%$ and $24\% = 0.24$

Check in

1 **a** Write $\frac{3}{4}$ as a decimal.
b Write 0.25 as a simplified fraction.

2 **a** Write 0.4 as a percentage.
b Write 75% as a decimal.

18.1 Language of probability

Consider these questions:

- Should you take a raincoat to school today?
- Who will win the test match?
- Should I buy that newspaper?

However you answer there is a **chance** that you will be right. There is also a **risk** that you may be wrong.

In mathematics, **probability** is used to describe the likelihood of an event occurring.

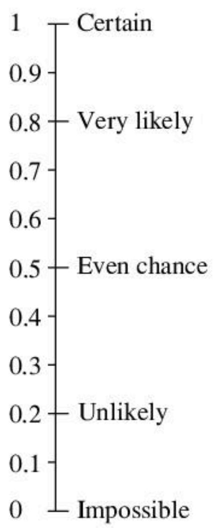
For example:
There is a high probability that you will put on your school clothes before you go to school.

There is a low probability that it will rain in the desert tomorrow.

Mathematically, the probability of an event is given as a number between 0 and 1. Probability can therefore be written as a fraction or a decimal.

A probability can also be written as a percentage. When percentages are used, the probability scale goes from 0 to 100.

In this chapter, fractions and decimals will be used.



An event that is **certain** is assigned a probability of 1. The probability of an **impossible** event is 0.

For example:

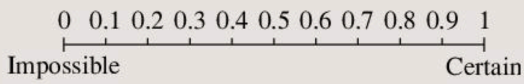
Event	Likelihood	Probability
I will live to be 200	Impossible	0
I will die someday	Certain	1

Exercise 18A

1 How likely are these events?
  a I will play cricket for my country
  b I will pass my next maths test
  c I will sleep tonight.

2 Write down three events that you think
  a are impossible
  b are certain
  c have an even chance of happening
  d are very likely to happen
  e are very unlikely to happen.

3



On a copy of the probability line above, insert the following events:
  a I will walk home tonight
  b It will rain today
  c School will end at 10.00 am today
  d At least 50 children will be at school today.

4 Assign a probability between 0 and 1 to each event.
  a I will grow 4 cm taller today
  b The maths lesson will last two hours today
  c I will be 16 next year
  d My next class will be art.

18.2 Experimental probability

Probability can be estimated through experiments.

In such cases you can define the probability of a successful outcome, P(S), as

$$P(S) = \frac{\text{number of successful outcomes}}{\text{total number of outcomes}}$$

For example:

You throw a coin 30 times and obtain 16 heads and 14 tails.
The experimental probability of getting a head, P(H) is

$$P(H) = \frac{\text{number of heads}}{\text{number of throws}} = \frac{16}{30} = \frac{8}{15}$$

Many probabilities are, in fact, found through surveys or statistics.

EXAMPLE 1

The number of pins in 30 boxes is shown in the table:

Number of pins	46	47	48	49	50	51
Number of boxes	2	4	10	8	4	2

What is the probability that a box holds
a 48 pins **b** more than 49 pins?

a $\text{P}(48 \text{ pins}) = \dfrac{\text{number of boxes with 48}}{\text{total number of boxes}}$

$= \dfrac{10}{30} = \dfrac{1}{3}$

b P(more than 49 pins)

$= \dfrac{\text{number of boxes with more than 49 pins}}{\text{total number of boxes}}$

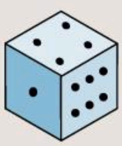
$= \dfrac{4+2}{30} = \dfrac{6}{30} = \dfrac{1}{5}$

Exercise 18B

You will need a coin and a dice.

1 Toss a coin 20 times. Record the number of times heads comes up.
For what fraction of the total number of throws was the outcome
a heads **b** tails?

2 Throw a dice 50 times.



a Copy and complete the table:

Score	1	2	3	4	5	6
Frequency						

b What fraction of throws are
i 2s **ii** even numbers
iii odd numbers **iv** 8s?

3 Petra throws a coin 50 times and gets these results:

Heads	Tails
23	27

Do you think the coin was fair? Explain.

4 The scores of 100 students on a test were:

Score	Number of students
0–9	1
10–19	2
20–29	9
30–39	9
40–49	12
50–59	24
60–69	15
70–79	18
80–89	8
90–99	2

What is the probability that a student picked at random scored
a 40–49 **b** less than 40
c 50–59 **d** 80 or more?

5 A shoe store sells shoes of the following sizes during the course of one week:

Shoe size	2	3	4	5	6	7	8	9	10	11	12
Number sold	4	4	3	6	10	14	33	24	16	10	4

a What is the probability that a customer bought shoes with
i size 6
ii size 5 or less
iii size 10 or more?

b Why is such data useful for the shoe store's manager?

6 The number of patients seen at a doctor's surgery one day is shown below by age group.

Age	0–19	20–29	30–39	40–49	50–59	60+
Patients	12	23	14	7	10	14

a What fraction of patients are in the 20–29 age group?
b Given a patient, what is the probability that the person is 50 years old or older?

TECHNOLOGY

Learn more about probability by visiting the Probability section at www.onlinemathlearning.com

18.3 Theoretical probability

It is usually impractical to carry out experiments to find the probability of events. Instead you can calculate what might be expected to occur. This is called the **theoretical probability**; it is given by the simple formula

$$\text{P(success)} = \frac{\text{number of possible successes}}{\text{number of possible outcomes}}$$

For example, when rolling an ordinary dice there are six **possible outcomes**: 1, 2, 3, 4, 5 or 6. Each of these are **equally likely outcomes** if the dice is fair. They are also **mutually exclusive** outcomes, which means they cannot happen at the same time.

If a success is 'getting an even number' then there are three **possible successes**: 2, 4, or 6. So the theoretical probability of getting an even number is shown as:

$$\text{P(even)} = \frac{3}{2} = \frac{1}{2}$$

The theoretical probability of getting heads when you toss a coin is $\frac{1}{2}$, so if you tossed 20 coins you would expect half of them to be heads.

Look back at Question **1** of Exercise 18B, where you did this experiment. Did you get 10 heads? How many of your friends got 10 heads? Because probability is based on chance, what should happen in theory doesn't necessarily happen in reality.

EXAMPLE 2

A bag contains 3 white beads and 2 black beads. What is the probability that a bead picked at random from the bag is black?

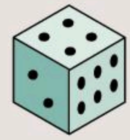
$$\text{P(black bead)} = \frac{\text{number of black beads}}{\text{number of beads}}$$
$$= \frac{2}{5}$$

Exercise 18C

1 A bag contains 6 yellow and 4 green marbles. What is the probability that a marble picked at random is

a yellow **b** not yellow?

2



A six-faced dice is thrown. What is the probability of throwing:

a a 4 **b** a 5 **c** a 3
d 4 or more **e** 2 or less **f** not a 6
g a 7 **h** less than 7?

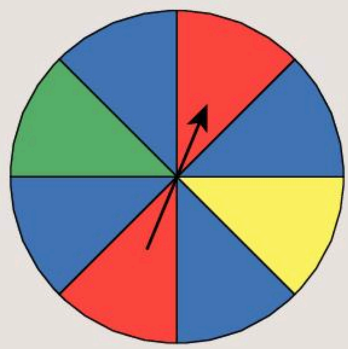
3 25 red cards numbered 1 to 25 and 25 blue cards numbered 1 to 25 are mixed together. Here are *some* of the cards:

12	7	16	19

What is the probability that a card picked at random is

a a 10
b a red
c not a 4
d not red
e a number which is a multiple of 2
f a red 11?

4



a For the spinner shown above, write down all the possible mutually exclusive outcomes.

b Using the spinner above, what is the probability of getting

i a blue
ii a red
iii a yellow
iv a white
v a yellow or green?

c Is it true that mutually exclusive outcomes all have the same probability of happening?

5 The 26 letters of the alphabet are written on tiles and placed inside a bag.

One tile is drawn at random. What is the probability of

a picking the letter W
b not picking the letter E
c picking a vowel
d picking any of the letters D, S or B
e picking a consonant
f picking a letter from the word MATHS?

6 A computer program is written to produce a number from 1 to 25 at random.

a What is the probability that the number produced is
  i 7
  ii greater than 20
  iii a multiple of 5
  iv a prime number
  v not a multiple of 7?

b Draw a probability scale. Mark your answers to part **a** on the scale.

7 When rolling a fair dice, each of the six scores, 1–6, are equally likely.

a What is the theoretical probability of each score on a dice?
b Roll a dice 60 times. Write down the experimental probabilities for each score.
c How many 2s would you expect to get? Did you get this many 2s? What about the other scores?
d Compare your experimental probabilities to the theoretical probabilities. Were they the same?
e Compare your friend's experimental probabilities to the theoretical probabilities.

8 James decided to carry out an experiment. He ripped up a piece of paper into 10 pieces. He wrote the numbers 1 to 10 on the pieces, turned them all face down on his desk and mixed them up. Then he picked a piece of paper at random, wrote down the number and put the piece of paper back on his desk. He repeated this 20 times.

a What is the theoretical probability of James picking a 9?
b Carry out this experiment yourself. What was the experimental probability of picking a 9? Compare the experimental probabilities for the other scores.
c Was this experiment fair when you did it? Can you think of any reason it might not be fair?

9 A bag has 6 green, 4 red, 5 yellow and 15 blue marbles.
A marble is picked from the bag. What is the probability that the marble is

a red **b** blue
c not red **d** not blue
e red or blue **f** neither red nor blue?

10 Tennis players toss a coin to decide who will serve first in a match. One player loses the toss five times in a row. What is the probability that he will lose it next time? Explain.

11 Draw a spinner with the following probabilities:

Colour	Red	Yellow	Blue	Green
Probability	$\frac{1}{4}$	$\frac{1}{6}$	$\frac{5}{12}$	$\frac{1}{6}$

12 Baasim was trying to draw a spinner that his friend Govinda said had the following probabilities:

Colour	Red	Yellow	Blue
Probability of colour	$\frac{2}{5}$	$\frac{1}{2}$	$\frac{1}{5}$

It is not possible to draw this spinner. Why not?

13 A coin is thrown three times. What is the probability that

a you throw three heads
b you throw only two heads
c you throw only one head
d you throw no heads?

Consolidation

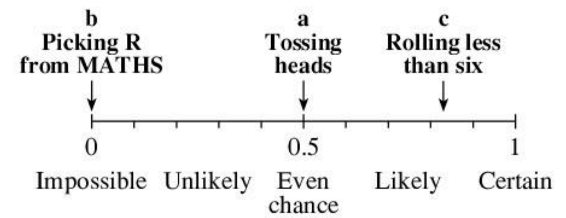
Example 1

Use the probability number scale to show the probability of

a tossing a coin and getting heads

b randomly picking the letter R from the word MATHS

c rolling a dice and scoring less than six.



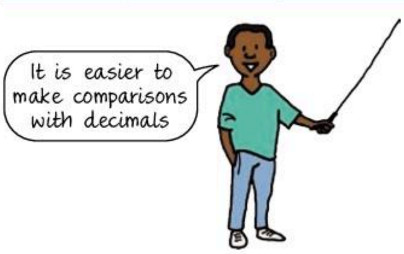
Example 2

The table below shows the outcomes when a dice is rolled 60 times.

Score	1	2	3	4	5	6
Number of times scored	12	22	6	8	10	2

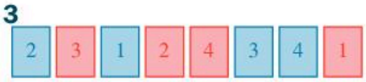
Compare the experimental probabilities with the theoretical probabilities. Do you think this dice is fair?

Score	**Number of times scored**	**Experimental probabilities (to 2 d.p.)**	**Theoretical probabilities (to 2 d.p.)**
1	12	$\frac{12}{60} = 0.20$	$\frac{1}{6} = 0.17$
2	22	$\frac{22}{60} = 0.37$	$\frac{1}{6} = 0.17$
3	6	$\frac{6}{60} = 0.10$	$\frac{1}{6} = 0.17$
4	8	$\frac{8}{60} = 0.13$	$\frac{1}{6} = 0.17$
5	10	$\frac{10}{60} = 0.17$	$\frac{1}{6} = 0.17$
6	2	$\frac{2}{60} = 0.03$	$\frac{1}{6} = 0.17$



This dice could be biased, as the experimental probabilities of getting a 2 or a 6 are very different from the theoretical probabilities.

Example 3



A card is drawn at random from the set above. What is the probability that the card is

a a blue card

b a 2

a $P(\text{Blue}) = \dfrac{\text{number of blue cards}}{\text{number of cards}} = \dfrac{4}{8} = \dfrac{1}{2}$

b $P(2) = \dfrac{\text{number of 2s}}{\text{number of cards}} = \dfrac{2}{8} = \dfrac{1}{4}$

Exercise 18

1 Draw a probability scale to show the probability of

a drawing a red marble from a jar containing 20 red marbles

b randomly choosing the letter T from the word BEST

c rolling a dice and getting an even number.

2 The Meadow Swim Club conducted a survey of the ages of its members.

Age	0–19	20–39	40–59	60–79
Frequency	96	61	36	7

What is the probability that a member selected at random is in the age group:

a 20–39 **b** 60–79?

3 Toss a coin 10 times. Compare the experimental probability of getting heads with the theoretical probability.

4 A bag contains 6 red, 3 white, 4 yellow and 7 orange marbles. What is the probability that a marble drawn at random is

a red

b yellow

c not red

d white or orange

e neither red nor white?

5 The letters of the word CHANCE are placed in a bag. One letter is chosen at random. What is the probability that the letter is

a H **b** C **c** P
d a vowel **e** not C?

6 Sally picks a bead out of a bag without looking. She writes down the colour then replaces the bead. She does this 100 times and picks out a blue bead 17 times. Estimate the probability of picking a blue bead.

Summary

You should know ...

1 Some events are more likely to happen than others. In mathematics, the likelihood of an event happening is described by a number between 0 and 1.

This number is the probability of the event.

Impossible Unlikely As likely as not Very likely Certain

0 0.2 0.5 0.9 1

2 How to find the probability of an event by performing an experiment.
The occurrence of a particular event is called a successful outcome.

$$\text{P(success)} = \frac{\text{number of successful outcomes}}{\text{total number of outcomes}}$$

For example:
A coin was thrown 100 times and 52 heads occurred, so

$$\text{P(heads)} = \frac{52}{100} = \frac{13}{25}$$

is the experimental probability.

3 How to find the probability of simple events without performing experiments.

For example:
If a coin is tossed, the probability of getting heads is

$$\text{P(heads)} = \frac{\text{number of successful outcomes}}{\text{total number of outcomes}}$$

$$= \frac{1}{2}$$

This is the theoretical probability.

Check out

1 Write down two events which are

a impossible
b certain
c very likely to happen
d unlikely to happen.

2 The shoe sizes of 20 customers at a shoe store were:

Size	4	5	6	7	8	9	10
Frequency	1	2	3	3	5	4	2

What is the probability that a customer chosen at random

a has a size 8 shoe
b has a shoe size less than 6?

3 A letter is chosen at random from the word CHECKER. What is the probability that the letter is:

a an H
b a vowel
c not a C?

Review C

1 Write as a percentage:

a 20 in 100 **b** 20 in 200

c 20 in 500 **d** 20 in 1000

2 The following ratios are equivalent.
Copy and complete:

a 1 : 3, 2 : □, □ : 12

b 5 : 1, □ : 2, 25 : □

c 2 : 7, 4 : □, □ : 35

d 6 : 5, □ : 45, 24 : □

3 Draw a coordinate graph to show these points:

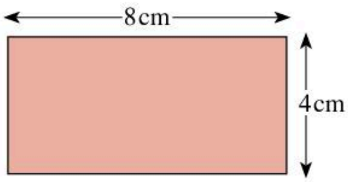
a (2, 1), (3, 2), (4, 3), (5, 4)

b (1, 5), (2, 4), (3, 3), (6, 0)

c (0, 4), (3, 4), (4, 4), (6, 4)

In each case, what can you say about the points?

4 Here is a rectangle with a perimeter of 24 cm.

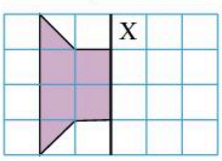


a Draw two different rectangles each with a perimeter of 24 cm.

b Calculate the area of each rectangle.

5 Copy these diagrams and reflect each shape in the mirror line, X.

a



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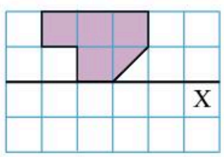
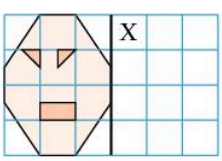
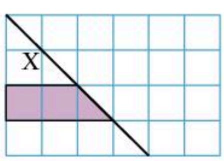


c



d

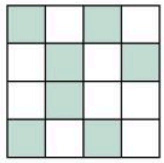


6 In a survey, 100 shoppers were asked what their favourite fruit was. The results were:

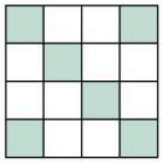
Oranges	28	Apples	12
Bananas	26	Pineapples	34

Write out the popularity of each fruit as a percentage.

7 **i**



ii



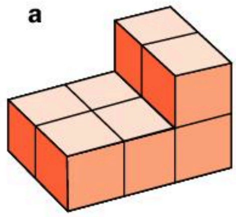
a For each diagram above, use a ratio to compare the number of white squares to the number of shaded squares.

b Simplify your ratio for part **a ii**.

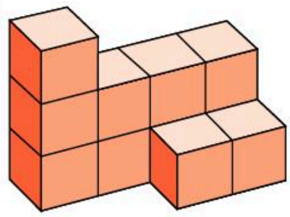
8 Each of these shapes is made of 1 cm^3 cubes.

Write down the volume of each shape.

a



b



9

a Draw the next two diagrams in the sequence.

b Copy and complete the table:

Number of black counters	1	2	3	4	5
Number of green counters					

c Describe the relationship between the number of black counters and the number of green counters.

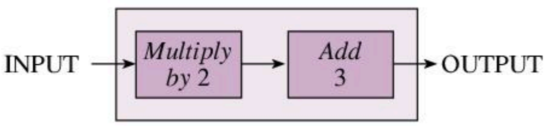
d How many green counters are there when there are 100 black counters?

10 Change these fractions to percentages:

a $\frac{1}{4}$ **b** $\frac{3}{10}$ **c** $\frac{6}{25}$ **d** $\frac{17}{20}$

e $\frac{64}{80}$ **f** $\frac{12}{75}$ **g** $\frac{80}{125}$ **h** $\frac{36}{60}$

11 The diagram shows a function machine.



Find the output when the input is

a 5 **b** 7

c 10 **d** x

12 Look at the capital letters:

A B C D E
F G H I J K
L M N O P
Q R S T U
V W X Y Z

Which of these letters have

a no lines of symmetry
b one line of symmetry
c two lines of symmetry
d more than two lines of symmetry?

13 A bag contains 6 red, 4 green and 8 yellow marbles. A marble is drawn at random from the bag. What is the probability that the marble is

a red
b yellow
c green
d not green?

14 Calculate:

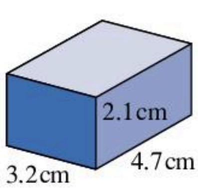
a 10% of 200 men
b 5% of 60 kg
c 75% of 140 car drivers
d 32% of 5000 ml
e 35% of 2400 cm

15 The ratio of boys to girls in Class 4W is 5 : 3.

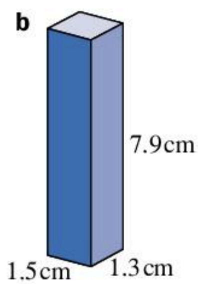
a Find how many girls there are, if there are 15 boys.
b Find how many boys there are, if there are 18 girls.
c Find how many boys and girls there are, if altogether there are
  i 32 students **ii** 48 students

16 Use your calculator to find the volume of each cuboid below.

a



b



17 Find the surface area of each cuboid in Question **16**.

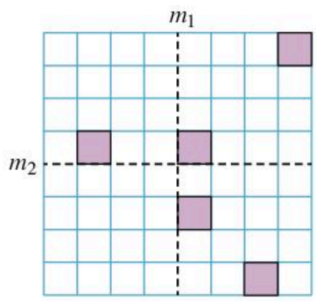
18 Copy and complete this table:

Fraction	%	Decimal
$\frac{1}{4}$		
$\frac{1}{5}$		
	75	
		0.4
	16	
	8	
		0.8
		0.04
	100	
$\frac{13}{20}$		

19 Draw a function machine that, given an input x, produces an output

a $2x$ **b** $3x + 1$ **c** $4x + 5$

20 Copy the diagram below.



Reflect each square in both mirror lines m_1 and m_2.

21 360 sheep are to be shared between Jo and Jim in a given ratio. How many sheep will each get if the ratio is:

a 7 : 3 **b** 16 : 20 **c** 1 : 8
d 7 : 5 **e** 35 : 37 **f** 5 : 13?

22 A rectangle is 3 cm long and 2 cm wide.

a What is its area, in cm^2?
b What is its length and breadth, in mm?
c What is its area, in mm^2?

23 The sides of a rectangle are 10 cm and 4.3 cm long.

a What are the side lengths, in mm?
b What is the area, in mm^2?
c What is the area, in cm^2?

24 Write as percentages:

a 80 g out of 400 g in a recipe is sugar
b 3 out of a group of 15 people wear glasses
c 33 km out of 75 km have been travelled
d 51 cm of a 60 cm length of wood remains, after some has been cut off.

25 Copy and complete the table for each equation, then draw the line.

x	0	1	2	3
y				

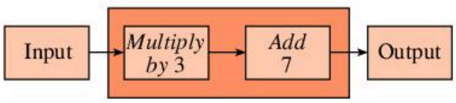
a $y = 2x - 4$ **b** $y = 4x$
c $y = x + 5$ **d** $y = 5 - x$

26 I pick a ball from a bag, replace it and then pick another. I keep doing this until I have chosen 40 balls. If I picked out 12 yellow balls, estimate the probability of not picking out a yellow ball.

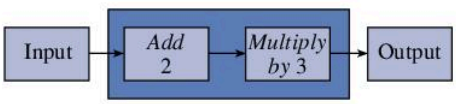
27 How much do you save if a shop offers
a 10% off a \$175 dress?
b 20% off a \$34 tie?

28 Draw mapping diagrams to show these number machines, using $x = 0$, 1, 2 and 3 as inputs.

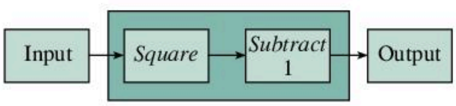
a



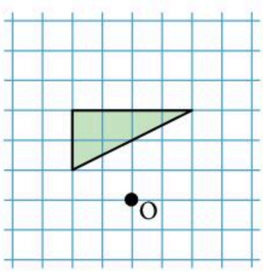
b



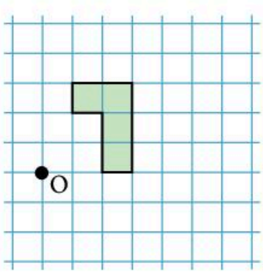
c



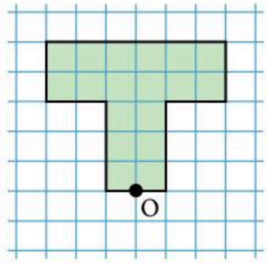
29 Copy the diagrams below. In each diagram, draw the image formed by rotating the object:
a 90° clockwise about O



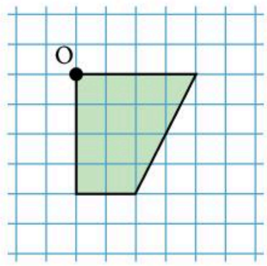
b 90° anticlockwise about O



c 180° clockwise about O



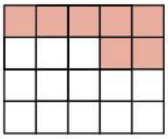
d 90° clockwise about O



30 Share the following between Ann and Jane in the given ratio:
a 12 apples, in the ratio 1 : 5
b 18 oranges, in the ratio 8 : 1
c 42 bananas, in the ratio 4 : 3
d 45 figs, in the ratio 4 : 5
e 16 records, in the ratio 5 : 3
f 84 CDs, in the ratio 13 : 15
g 16.8 metres of cloth, in the ratio 3 : 1
h 25.6 kg of rice, in the ratio 7 : 9
i 0.63 litres of juice, in the ratio 5 : 4
j \$3328, in the ratio 6 : 7

31 Copy and complete the table for cuboids:

	Length	Width	Height	Volume
a	5 cm	4 cm	3 cm	
b	7 cm	6 cm	9 cm	
c	13 cm	8 cm	2.5 cm	
d	12 cm	9 cm	3.1 cm	
e	7 cm		5 cm	280 cm^3
f	12 cm	4 cm		360 cm^3

32 

Which of these statements are true?

a 7% is shaded **b** 0.65 is unshaded

c $\frac{7}{13}$ is shaded **d** 35% is shaded

e $\frac{13}{20}$ is unshaded **f** 65% is shaded

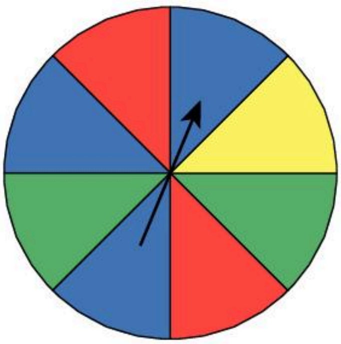
33 Write down the term-to-term rule and next two terms of these sequences:

a 3, 16, 29, 42, 55, . . .

b 9, 3, $^-3$, $^-9$, $^-15$, . . .

c 5, 10, 20, 40, 80, . . .

34



For this spinner, what is the probability of getting

a blue **b** red

c yellow **d** white

e red or green?

35 Half of $\frac{1}{4}$ is $\frac{1}{8}$. What is $\frac{1}{4}$ as a percentage?

Do you agree that $\frac{1}{8}$ must be $12\frac{1}{2}\%$?

Complete the following table:

Fraction	%
$\frac{1}{8}$	$12\frac{1}{2}$
$\frac{1}{4}$	25
$\frac{3}{8}$	
$\frac{1}{2}$	
$\frac{5}{8}$	
$\frac{3}{4}$	
$\frac{7}{8}$	
1	

You can change percentages containing fractions like this:

$$17\tfrac{1}{2}\% = 17\tfrac{1}{2} \text{ in } 100 = 35 \text{ in } 200 = \frac{35}{200} = \frac{7}{40}$$

Now try these, changing the percentages to fractions and cancelling:

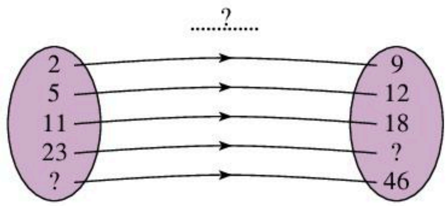
a $27\frac{1}{2}\%$ **b** $33\frac{1}{3}\%$ **c** $6\frac{1}{4}\%$

36 Write down for each mapping diagram:

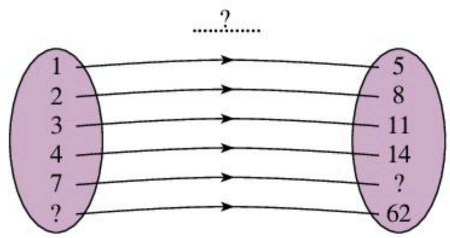
i the function

ii the missing numbers.

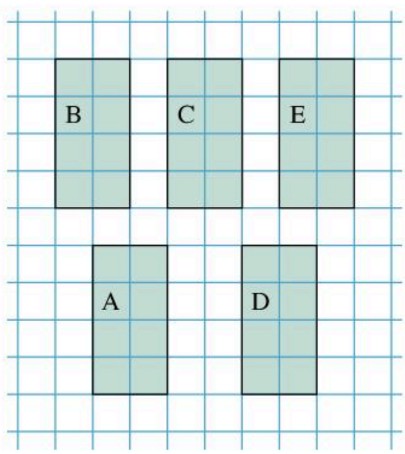
a



b



37



Describe the translation from

a A to C

b B to C

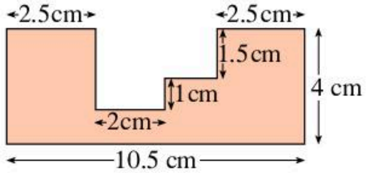
c E to D

d D to A

e C to D

38 The cross-section of a metal casting is shown below. Find

a its perimeter **b** its area.



39 Draw axes and add the lines:

a $y = 1$
b $y = 5$
c $x = 2$
d $x = 4$

Where the lines intersect they form a rectangle. Is the rectangle:

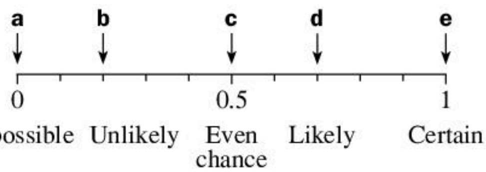
landscape ☐ or portrait ☐ ?

40 A baker uses 900 grams of flour to make 6 cakes.

a How much flour will she need use if she wants to make 5 cakes?

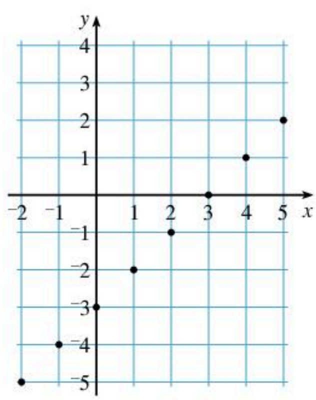
b How many cakes can she make with 1650 grams of flour?

41



Copy the probability scale. Label each arrow **a–e** with one event.

42 Look at the axes below.



a Write down the coordinates of each of the eight marked points.

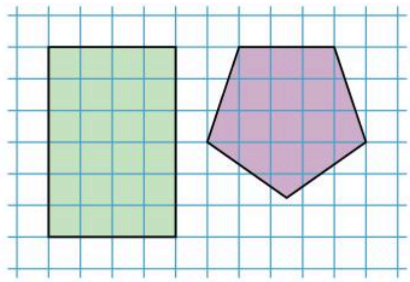
b Draw a function machine for these points.

c Write down the equation for the line passing through these points.

d What number is output from your machine if $^{-}6$ is input?

e What number must be input to your machine if $^{-}6$ is output?

43 Trace the shapes below. For each shape, draw in the lines of symmetry and write down the order of rotational symmetry.



44 **a** Copy and complete the table for cubes.

Edge length (cm)	1	2	3	4	5	6
Volume (cm^3)						

b Use your results for part **a** to write down the sum of

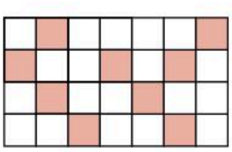
i the first two volumes
ii the first three volumes
iii the first four volumes
iv the first five volumes.

c What do you notice about your answers for part **b**?

45 Find the missing terms in these sequences:

a ☐, 14, 20, 26, ☐, 38, . . .
b $^{-}7$, ☐, $^{-}11$, $^{-}13$, $^{-}15$, ☐, . . .
c ☐, 2, 4, 8, 16, ☐, . . .
d The first term is 7 and the term-to-term rule is add 8. What is the fourth term?
e The third term is 50 and the term-to-term rule is subtract 5. What are the sixth term and the first term?

46



Look at the diagram. How many more squares need to be shaded so that the ratio of shaded to unshaded is 3 : 4?

47 Write down a definition of each of these terms:

a theoretical probability
b experimental probability
c mutually exclusive outcomes.

19 Sets and Venn diagrams

Objectives

- Understand the mathematical meaning of the word 'set'.
- Learn how ideas of sets and their notation assist in sorting and classification.
- Discover what a Venn diagram is, and how to draw and use one.
- Use Venn diagrams to find HCFs and LCMs.

The work in this chapter is not in the Cambridge Secondary 1 Mathematics curriculum framework. Sets and Venn diagrams are not in the Checkpoint tests. They are in the Cambridge IGCSE® maths curriculum and are here for you to try if you have completed the work from the other chapters.

What's the point?

Sets are used in many areas of mathematics, so much so that they are sometimes called the building blocks of the subject. Basic ideas about sets are used by many scientists to assist in sorting. Biologists use such ideas when they try to classify the 34 000 species of spider, which they sort into some 100 families!

Before you start

You should know ...

1 The even numbers are 2, 4, 6, 8, 10, . . .
The odd numbers are 1, 3, 5, 7, 9, . . .

Check in

1 17, 28, 116, 215, 7, 30
Which of these numbers are
a odd
b even?

2 The multiples of a number are the same as its multiplication table.
For example: The multiples of 3 are 3, 6, 9, 12, . . .

3 The factors of a number are the numbers that divide exactly into it.
For example: The factors of 6 are 1, 2, 3 and 6.

4 A prime number has exactly two factors: itself and one.
A composite number has more than two factors.

2 6, 15, 60, 103, 95
Which of these numbers are multiples of 5?

3 List all the factors of 12.

4 11, 15, 8, 20, 13
Which of these numbers are **a** prime **b** composite?

19.1 Sets and their members

- A **set** is a collection of things with a common feature.
  For example, a set of 4-legged animals.
- Each thing in a set is called a **member** of the set.
  For example, a camel is a member of the set of 4-legged animals.

Exercise 19A

1 Look at the drawings above. Write down what you see in each picture.

2 Look around your classroom. Write down five sets of things you can see.

3 Look at the drawings again.
How many members are in
a the set of ice creams
b the set of footballs
c the set of parrots
d the set of palm trees?

19.2 How to describe a set

To save time writing 'the set of' you can use curly brackets.
For example:
the set of ice creams = {ice creams}
or,
the set of whole numbers from 1 to 10
$= \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

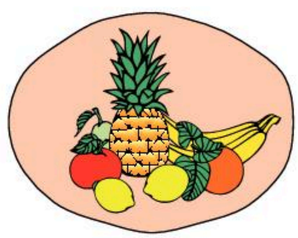
Sometimes there are too many members in a set to list them all. You can still use curly brackets:

The set of even numbers between 4 and 20000
={even numbers between 4 and 20000}

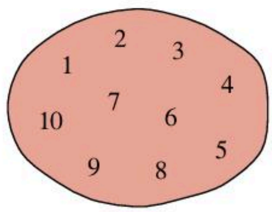
The set of students in Anil's school
={students in Anil's school}

To show that things belong to a set, you can draw a loop around them.

So a set of fruit could be shown as:



The set of whole numbers from 1 to 10 could be shown as:



EXAMPLE 2

Describe these sets in your own words:

a {Alice, Ann, Alison, Anya}

b {35, 42, 49, 56, 63}

a A set of girls' names beginning with A.

b The set of multiples of 7 from 35 to 63.

Exercise 19B

1 Using brackets, list the set of

a months of the year

b whole numbers between 10 and 20

c even numbers between 10 and 14

d odd numbers between 25 and 35.

2 Using a loop, show the set of

a names of people in your family

b days of the week

c subjects you study in school

d odd numbers between 2 and 10

e even numbers between 80 and 100.

3 For each set in Questions **1** and **2**, write down the number of members it has.

4 Describe each of these sets in your own words:

a {James, John, Judith, Jennifer, Joseph}

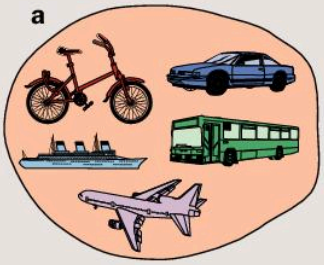
b {sandal, slipper, boot, shoe, clog}

c {3, 6, 9, 12, 15}

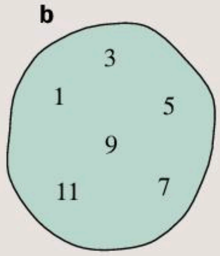
d {121, 132, 143, 154, 165}

5 Describe each of these sets in words:

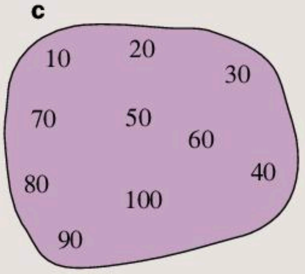
a



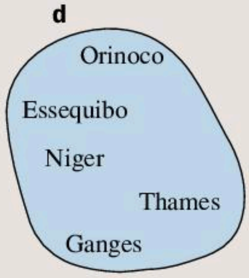
b



c



d



Using symbols

Look at the set {1, 2, 3, 4, 5}.
2 is a member of this set.
You can write this in a short way:

$2 \in \{1, 2, 3, 4, 5\}$.

- The symbol $\in$ means **is a member of**, belongs to or is an element of.

Look at the set of odd numbers between 0 and 10: {1, 3, 5, 7, 9}.

2 is not a member of this set.

You can write this:
$2 \notin \{1, 3, 5, 7, 9\}$.

- The symbol $\notin$ means **is not a member of**, does not belong to or is not an element of.

The empty set

Some sets have no members.
For example:
{people with three heads}
or
{whole numbers between 1 and 2}.

- A set with no members is an **empty set**. The symbol for any empty set is $\varnothing$.

Exercise 19C

1 Write in words:

a $3 \in \{1, 2, 3\}$

b blue $\in$ {blue, green, yellow, red, orange}

c cricket $\in$ {cricket, football, tennis, badminton, volleyball}

d $10 \in \{10, 20, 30, 40, 50\}$

e $121 \in \{121, 132, 143, 154, 165\}$

f $73 \notin \{10, 20, 30, 40, 50, 60, 70, 80, 90, 100\}$

g $W \notin \{X, Y, Z\}$

2 Write using brackets and the symbol $\in$ or $\notin$.

a 36 is a member of {6, 16, 26, 36}.

b A is a member of {A, B, C}.

c 16 is one of the even numbers 10, 12, 14, 16, 18, 20.

d 10 is not a member of {1, 2, 3, 4, 5, 6, 7, 8, 9}.

e F is not a vowel.

3 Using brackets, list the members of each set.

- **a** {numbers between 1 and 101 that end in 0}
- **b** {numbers between 5 and 80 that are divisible by 3}
- **c** {numbers between 12 and 17 that are divisible by 9}

4 Using brackets, write these sets in a different way.

- **a** {0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20}
- **b** {1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25, 27, 29, 31}
- **c** {3, 6, 9, 12, 15, 18, 21, 24, 27, 30, 33, 36, 39, 42, 45, 48, 51}

5 Which of these are empty sets?

- **a** {spiders that have twenty-four legs}
- **b** {whole numbers between 21 and 22}
- **c** {numbers ending in 1 that are divisible by 11}
- **d** {numbers between 0 and 500 that are divisible by 10}
- **e** {years with 370 days}
- **f** {boys' names beginning with A}
- **g** {days of the week beginning with X}
- **h** {rivers that run to the sea}
- **i** {months of the year ending with W}
- **j** {human beings with wings}

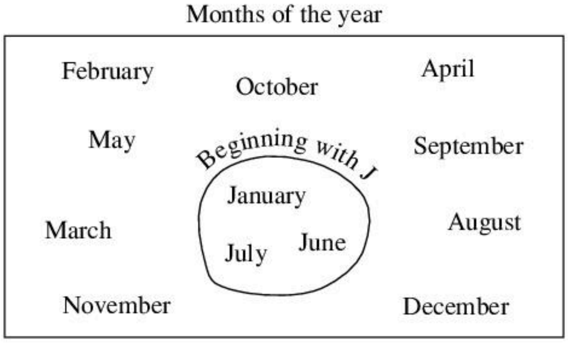
6 Make up five sets of your own.
List each one, using brackets, then describe it in your own words.

7 Describe five empty sets of your own.

19.3 Venn diagrams

A labelled diagram like this is called a **Venn diagram**.
It is named after the English mathematician John Venn.

Months of the year

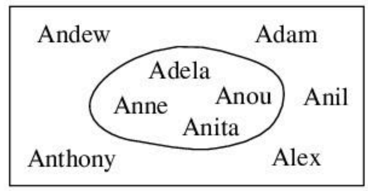


The rectangular box contains the whole set.

- The whole set is usually called the **universal set**. The symbol used for the universal set is ξ.
- The smaller loop shows a **subset** of the universal set.
  All members of a subset are also members of the universal set.

EXAMPLE 2

A Venn diagram has been started below.



- **a** Name the universal set and the subset.
- **b** List, using curly brackets:
  - **i** the universal set
  - **ii** the subset.

---

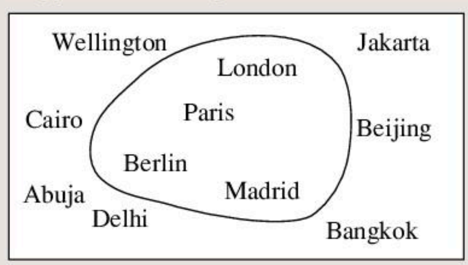
a The universal set could be called 'Names beginning with A' and the subset 'Girls' names beginning with A'.

b **i** The universal set is {Andrew, Adam, Anil, Anthony, Alex, Adela, Anou, Anne, Anita}

ii The subset is {Adela, Anou, Anne, Anita}

Exercise 19D

1 Copy this Venn diagram:



On your drawing label the universal set and the subset with their names.

2 From Question **1**, list, using brackets

- **a** the universal set
- **b** the subset.

3 Using $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ as the universal set, draw a Venn diagram showing the subset of even numbers.
Label your diagram.

4 For the universal set $\{$whole numbers from 1 to 20$\}$ draw separate Venn diagrams to show each of these subsets:

a multiples of 3
b multiples of 4
c multiples of 5
d factors of 20.

Which of these four subsets has fewest members?

5 A grid that shows information is called an **information matrix**.
Stephen asked eight of his friends about how they like to spend time. The information he found is shown in the information matrix.
✓ means *likes*. ✗ means *dislikes*
For example, Michael likes swimming but dislikes reading.

Name	Swimming	Cricket	Movies	Reading	Drawing
Hugh	✓	✗	✗	✓	✗
Mohamed	✓	✗	✗	✓	✗
Donald	✗	✓	✓	✓	✗
Michael	✓	✓	✓	✗	✓
Greg	✗	✓	✗	✗	✓
Morris	✓	✓	✗	✓	✓
Winston	✗	✓	✓	✗	✗
Lawrence	✓	✗	✓	✗	✗

Using {eight of Stephen's friends} as the universal set, draw separate Venn diagrams showing the subsets of his friends

a who like swimming
b who like cricket
c who like reading and movies
d who like reading and drawing
e who like movies and drawing
f whose names begin with K
g who are male.

Label your diagrams.

The subset symbol

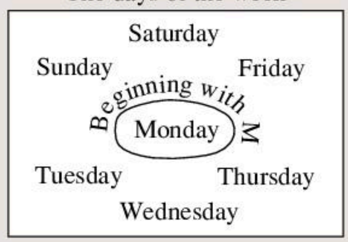
There is a quick way to show a subset of a set, without drawing a Venn diagram. You use the symbol $\subset$.

- The symbol $\subset$ means **is a proper subset of**.
  For example:
  $\{3, 6, 9\} \subset \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

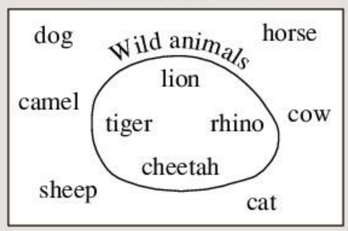
Exercise 19E

1 Using symbols, describe each Venn diagram.
The first one can be described as
{Monday} $\subset$ {Monday, Tuesday, Wednesday, Thursday, Friday, Saturday, Sunday}

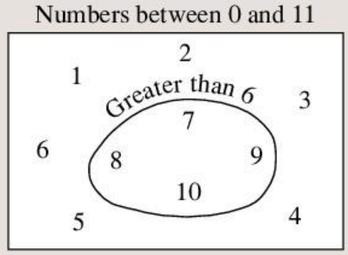
a The days of the week



b A set of animals



c Numbers between 0 and 11



2 Show, with labels, these sets on Venn diagrams.

a $\{1, 3, 5, 7, 9\} \subset \{1, 2, 3, 5, 6, 7, 8, 9, 10\}$
b $\{2, 4, 6, 8, 10\} \subset \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$
c $\{3, 6, 9\} \subset \{3, 6, 9, 12, 15, 18, 21, 24, 27\}$

19.4 Intersection of sets

This set shows people whose photos appeared in today's *Daily News*.

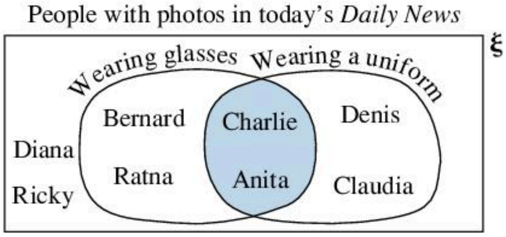
Two subsets of the universal set are:

{people wearing glasses}
= {Anita, Ratna, Charlie, Bernard}

and
{people wearing a uniform}
= {Claudia, Anita, Denis, Charlie}

Anita and Charlie are in both subsets. This can be shown on a Venn diagram:

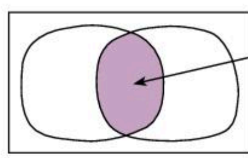
People with photos in today's *Daily News*



Anita and Charlie are both wearing glasses **and** wearing a uniform.

They make up the **intersection** of the two subsets.

- The intersection of two sets is made up of those members that are common to both sets.



Notice that the names Diana and Ricky are written inside the universal set box but are not in either subset (or loop). This is because they don't wear glasses and they are not wearing a uniform.

Exercise 19F

1 This is the set of names of Harry's friends.

Names of Harry's friends

Billy	Ratna	Regina
Rose	David	Michael
Roland	Ramsingh	Robert
Shirley	Raquel	Rocky
Janet		

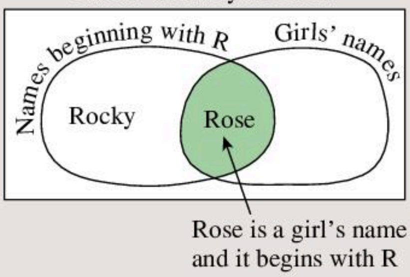
Using brackets, list the subset of these names that

a are girls' names **b** begin with R.

2 **a** In Question **1**, which names are girls' names *and* begin with R?

b Copy and complete the Venn diagram:

Names of Harry's friends



3 In Question **2**, are the names in the shaded region of the Venn diagram in part **b** the same as the names in your answer to part **a**?

4 The matrix below gives information about ten students in Form 1.
✓ means *yes*, ✗ means *no*

Name	Likes maths	Can swim	Owns a bicycle
Deo	✓	✓	✗
Wendy	✓	✗	✓
Ali	✓	✗	✗
Ramsingh	✗	✓	✓
Lorna	✗	✗	✗
Addie	✓	✗	✗
Bob	✗	✓	✓
Shirley	✓	✗	✓
Janice	✓	✓	✗
Rani	✗	✓	✗

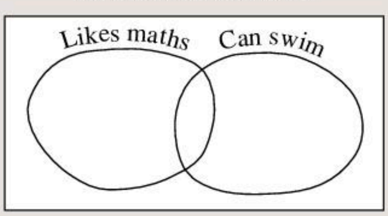
List the subset of students who

a like maths **b** can swim.

c Which students like maths *and* can swim?

d Copy and complete the Venn diagram. Remember to use all ten names.

Ten students in Form 1



e Lorna's name appears outside both small loops in the Venn diagram you drew for part **d**. What can you say about Lorna?

5 **a** Draw a Venn diagram using the students in the matrix in Question **4** showing

i the subset of students who can swim

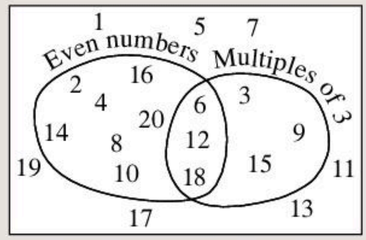
ii the subset of students who own a bicycle.

b Which students are in the intersection of the subsets?

c Which students *neither* own a bicycle *nor* can swim?

d Which students own a bicycle but cannot swim?

6 Whole numbers from 1 to 20



From the Venn diagram, list the subset of numbers from 1 to 20 which are

a even

b multiples of 3

c even *and* multiples of 3

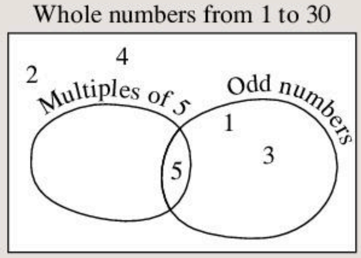
d even *but not* multiples of 3

e multiples of 3 *but not* even

f *neither* even *nor* multiples of 3.

7 **a** Using {whole numbers from 1 to 30} as the universal set, copy and complete the Venn diagram:

Whole numbers from 1 to 30



b Write down the subset of numbers from 1 to 30 which are

i odd

ii multiples of 5

iii odd *and* multiples of 5

iv odd *but not* multiples of 5

v multiples of 5 *but not* odd

vi *neither* odd *nor* multiples of 5.

8 **a** Using the numbers from 2 to 25 as the universal set, show on a Venn diagram the subset of prime numbers and the subset of composite numbers.

b Which numbers are prime *and* composite?

c Which numbers are *neither* prime *nor* composite?

19.5 Common factors, common multiples

Set notation and Venn diagrams can give you another way of finding common factors and multiples.

EXAMPLE 3

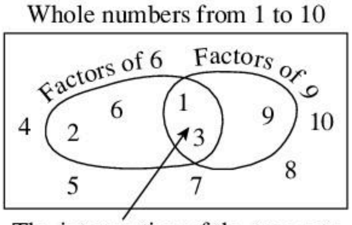
Draw a Venn diagram to show the factors of 6 and 9.

Factors of 6 $= \{1, 2, 3, 6\}$
Factors of 9 $= \{1, 3, 9\}$

Two of the factors, 1 and 3, are in both sets.
1 and 3 are **common factors** of 6 and 9.

A Venn diagram shows this well:

Whole numbers from 1 to 10



The intersection of the two sets shows the common factors.

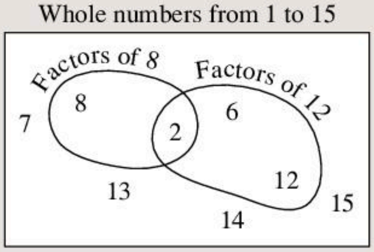
Exercise 19G

1 **a** Write down the set of factors of 8.

b Write down the set of factors of 12.

c Copy and complete the Venn diagram:

Whole numbers from 1 to 15



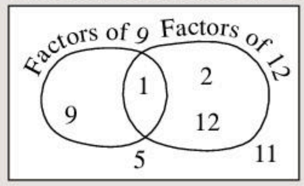
d Write down from the Venn diagram the set of common factors of 8 and 12.

2 **a** Write down the factors of 9.

b Write down the factors of 12.

c Copy and complete the Venn diagram:

Whole numbers from 1 to 12



d Write down the common factors of 9 and 12.

3 Use Venn diagrams to find the common factors of

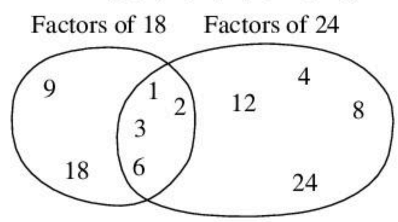
a 10 and 15	**b** 4 and 6
c 6 and 8	**d** 8 and 10
e 6 and 14	**f** 12 and 15.

It is now a short step to find the highest common factor (HCF) of two numbers using a Venn diagram.

EXAMPLE 4

Use a Venn diagram to find the HCF of 18 and 24.

Factors of 18 $= \{1, 2, 3, 6, 9, 18\}$
Factors of 24 $= \{1, 2, 3, 4, 6, 8, 12, 24\}$



Common factors of 18 and 24 $= \{1, 2, 3, 6\}$
So the HCF of 18 and 24 is 6.

Exercise 19H

1 Use the Venn diagrams you drew in Question **3** of Exercise 19G to find the HCF of

a 10 and 15	**b** 4 and 6
c 6 and 8	**d** 8 and 10
e 6 and 14	**f** 12 and 15.

2 Use Venn diagrams to find the HCF of

a 6 and 15 **b** 16 and 20
c 12 and 16 **d** 24 and 30
e 35 and 25 **f** 28 and 42
g 56 and 48 **h** 36 and 54.

3 List the elements of

a {factors of 24}
b {factors of 36}

Draw a Venn diagram to illustrate these sets and hence state the HCF of 24 and 36.

Common multiples

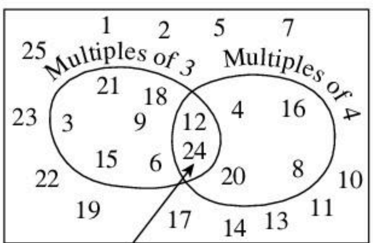
The set of the first eight multiples of 3
= {3, 6, 9, 12, 15, 18, 21, 24}

The set of the first six multiples of 4
= {4, 8, 12, 16, 20, 24}

12 and 24 belong to both sets.
12 and 24 are **common multiples** of 3 and 4.

A Venn diagram shows this:

Whole numbers from 1 to 25



The intersection of the sets shows the common multiples.

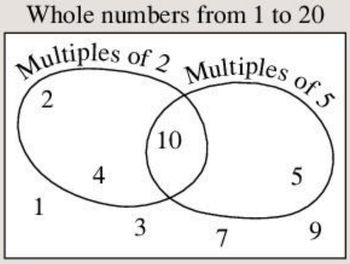
Exercise 19I

1 **a** Write down the set of the first ten multiples of 2.

b Write down the set of the first four multiples of 5.

c Copy and complete the Venn diagram:

Whole numbers from 1 to 20



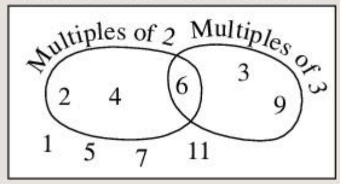
d Write down the set of common multiples of 2 and 5.

2 **a** Write down the first six multiples of 2.

b Write down the first six multiples of 3.

c Copy and complete the Venn diagram:

Whole numbers from 1 to 20



d Write down the set of common multiples of 2 and 3.

3 Use Venn diagrams to find the common multiples of

a 3 and 5 **b** 3 and 6
c 2 and 7 **d** 4 and 5
e 2 and 8 **f** 6 and 4.

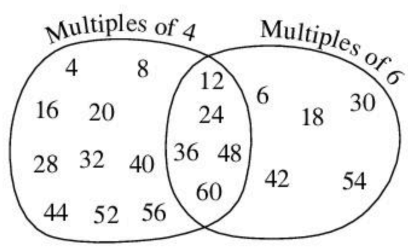
A Venn diagram can show the lowest common multiple (LCM) of a pair of numbers.

EXAMPLE 5

Use a Venn diagram to find the LCM of 4 and 6.

The set of multiples of 4
= {4, 8, 12, 16, 20, 24, 28, 32, 36, 40, 44, 48, 52, 56, 60, . . .}

The set of multiples of 6
= {6, 12, 18, 24, 30, 36, 42, 48, 54, 60, . . .}



Common multiples of 4 and 6
= {12, 24, 36, 48, 60, . . .}

So LCM of 4 and 6 is 12.

Exercise 19J

1 Use the Venn diagrams you drew in Question **3** of Exercise 19I to find the LCM of

a 3 and 5 **b** 3 and 6
c 2 and 7 **d** 4 and 5
e 2 and 8 **f** 6 and 4.

2 Use Venn diagrams to find the LCM of

a 2 and 3 **b** 4 and 8
c 3 and 4 **d** 2 and 4
e 2 and 9 **f** 5 and 10
g 5 and 7 **h** 8 and 12.

Exercise 19K – mixed questions

1 Copy and complete using the correct symbol, $\in$ or $\subset$ in the space:

a physics ... {science subjects}
b {April} ... {months of the year}
c {knife, fork, spoon} ... {kitchen utensils}
d 15 ... {multiples of 5}
e {3, 6, 12} ... {factors of 24}

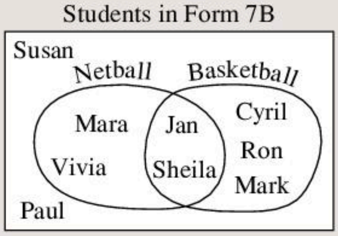
2 Which of these are empty sets?

a {student in your class who like mathematics}
b {people over 150 years old}
c {students in your class with three legs}
d {prime numbers less than 20}
e {odd numbers with a factor of 2}

3 If $a \in \{3, 6, 9, 12\}$ and $a \in \{1, 2, 3, 4, 5\}$ find the value of a.

4 The Venn diagram shows the sports played by some students in Form 7B.

Students in Form 7B



a List, using brackets, the set of students who
  i play basketball
  ii play netball, but *not* basketball
  iii play *neither* netball *nor* basketball
  iv play *both* netball *and* basketball.

b How many students play either basketball or netball or both?

5 **a** Copy and complete:
The intersection of {factors of 6} and {multiples of 2 less than 10} is ...

b Draw a Venn diagram to show the information in part **a**.

6 **a** List the factors of 6.
b List the factors of 24.
c Draw a Venn diagram to show that {factors of 6} $\subset$ {factors of 24}.

7 Use Venn diagrams to illustrate the HCF and the LCM of

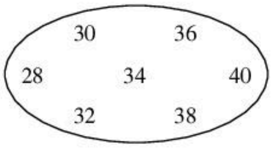
a 6 and 16
b 12 and 21
c 9 and 15.

Consolidation

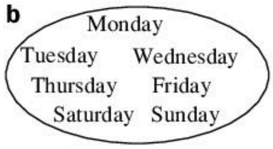
Example 1

Describe the sets.

a



b



The set can be written as {28, 30, 32, 34, 36, 38, 40}
That is, the set of even numbers from 28 to 40.

The set can be written as {Monday, Tuesday, Wednesday, Thursday, Friday, Saturday, Sunday.}
That is, the set of days of the week.

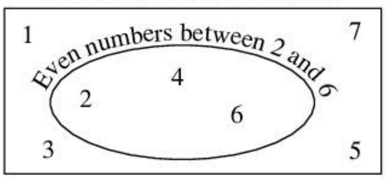
Example 2

Draw a Venn diagram to show

$$\{2, 4, 6\} \subset \{1, 2, 3, 4, 5, 6, 7\}$$

The symbol $\subset$ means **is a subset of**.
The Venn diagram to show the two sets is:

Numbers between 1 and 7

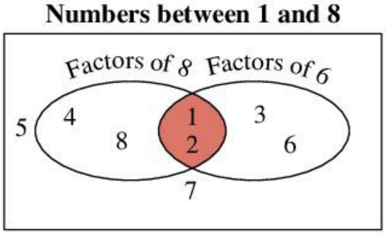


Example 3

Draw a Venn diagram to show the factors of 8 and the factors of 6.
Write down the intersection of the two sets.

Factors of 8 = {1, 2, 4, 8}
Factors of 6 = {1, 2, 3, 6}
The Venn diagram is:

Numbers between 1 and 8

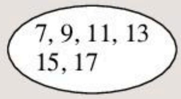


The intersection of the two sets is {1, 2}.

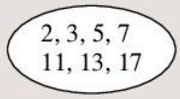
Exercise 19

1 Describe these sets:

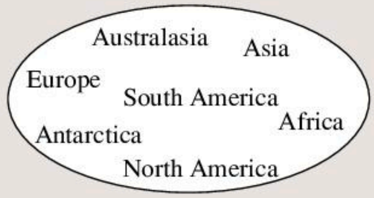
a



b



c



2 Write down five different empty sets.

3 Draw Venn diagrams to show
- **a** $\{1, 3, 5\} \subset \{1, 2, 3, 4, 5, 6\}$
- **b** $\{10, 20, 30\} \subset \{5, 10, 15, 20, 25, 30\}$
- **c** {Saturday, Sunday} $\subset$ {Days of the week}

4 Draw Venn diagrams to show
- **a** factors of 4 and factors of 6
- **b** factors of 12 and factors of 24
- **c** factors of 9 and factors of 24
- **d** factors of 36 and factors of 15.

Write down the intersection of each of the pairs of sets.

5 a Using {whole numbers from 1 to 20} as the universal set, draw a Venn diagram to show {multiples of 3} and {odd numbers}.

b Write down the subset of numbers from 1 to 20 which are
- **i** multiples of 3
- **ii** odd
- **iii** odd and multiples of 3
- **iv** odd but not multiples of 3
- **v** even and multiples of 3
- **vi** neither odd nor multiples of 3.

6 a Draw a Venn diagram to show factors of 16 and factors of 24.

b List the members of the intersection of the two sets.

c What are the common factors of 16 and 24?

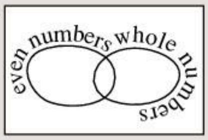
d What is the highest common factor (HCF) of 16 and 24?

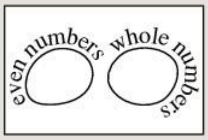
7 Use the method you used in Question **6** to find the HCF of

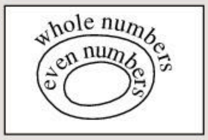
a 12 and 18 **b** 36 and 27 **c** 36 and 60.

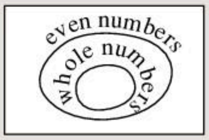
8 Use a similar method to find the LCM of
a 3 and 5 **b** 2 and 6 **c** 7 and 15.

9 Which diagram illustrates that
{even numbers} $\subset$ {whole numbers}?

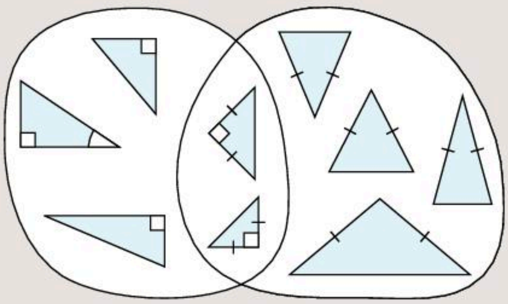
a


b


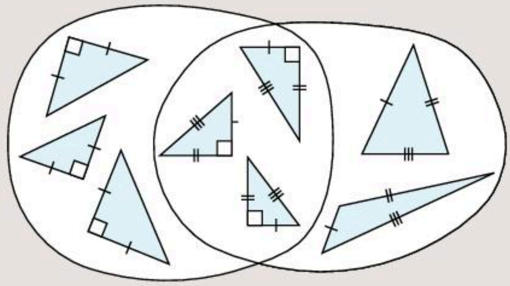
c


d


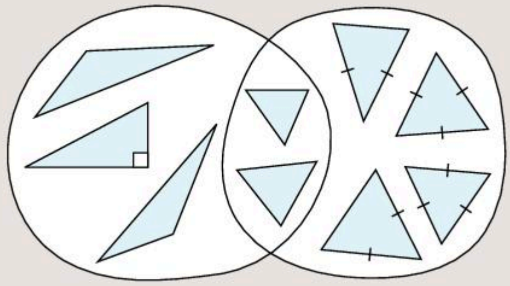
10 Label the sets in the diagram.



11 Label the sets in the diagram.



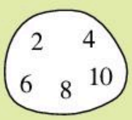
12 Label the sets in the diagram.



Summary

You should know ...

1 A set is a collection of things with a common feature.
For example:
The set of even numbers between 1 and 11.



or {2, 4, 6, 8, 10}

2 The symbol $\in$ means 'is a member of'.

The symbol $\notin$ means 'is not a member of'.

For example:
$r \in \{n, o, p, q, r, s\}$

A set with no members is an empty set, $\varnothing$.

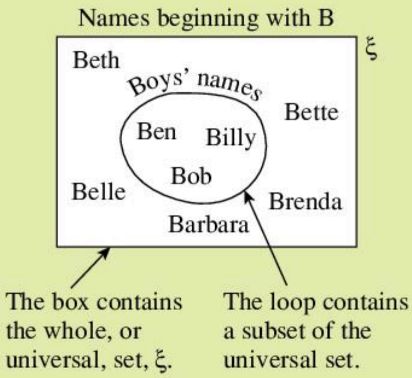
For example:
The set of girls with three heads = $\varnothing$

Check out

1 **a** Use a loop to show the set of odd numbers between 6 and 16.
b Using brackets list the set of days of the week beginning with T.

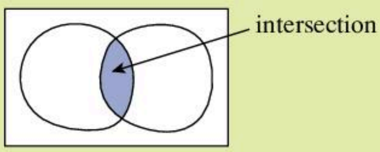
2 **a** Rewrite the statement 'cod is a member of the set of fish' using the $\in$ symbol.
b Rewrite the statement 'there are no birds with two heads' using the symbol $\varnothing$.

3 This is a Venn diagram:



In symbols,
{Billy, Ben, Bob} ⊂ {names beginning with B}
⊂ means 'is a proper subset of'.

4 The intersection of two sets consists of members common to both sets.



5 The highest common factor (HCF) is the largest factor common to two or more numbers.

For example:
The HCF of 12 and 20 is 4.

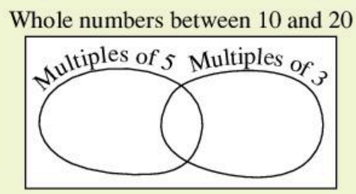
The lowest common multiple (LCM) is the smallest multiple common to two or more numbers.

For example:
The LCM of 3 and 5 is 15.

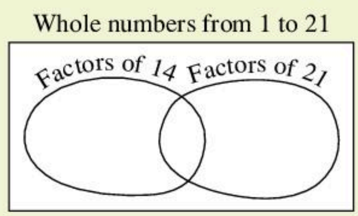
3 a Using {1, 2, 3, … 10} as the universal set, draw a Venn diagram to show the subset of prime numbers.

b Rewrite the statement 'Mary and Liz are girls' names' using the symbol ⊂.

4 Copy and complete this Venn diagram:



5 a Copy and complete the Venn diagram:



b Write down the HCF of 14 and 21.

c Draw a Venn diagram showing

i universal set = {whole numbers less than 25}

ii a subset of the multiples of 4

iii a subset of the multiples of 6.

d Find the LCM of 4 and 6.

Index

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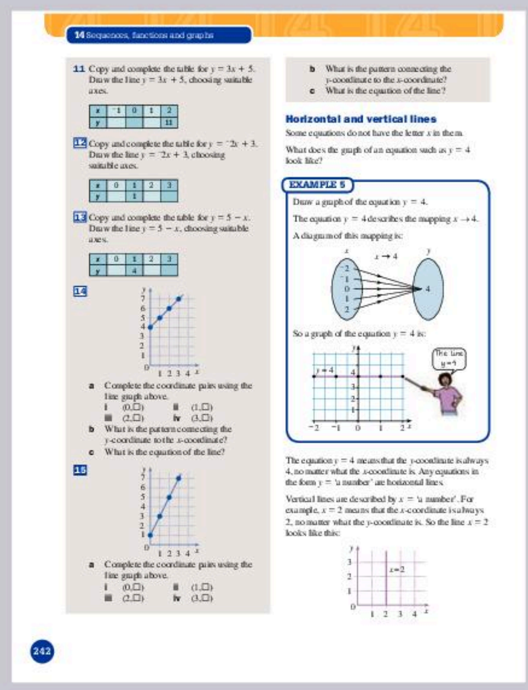

14 Sequences, functions and graphs

11 Copy and complete the table for $y = 3x + 5$. Draw the line $y = 3x + 5$, choosing suitable axes.

x	$^-1$	0	1	2
y				11

12 Copy and complete the table for $y = {}^-2x + 3$. Draw the line $y = {}^-2x + 3$, choosing suitable axes.

x	0	1	2	3
y		1		

13 Copy and complete the table for $y = 5 - x$. Draw the line $y = 5 - x$, choosing suitable axes.

x	0	1	2	3
y		4		

14

a Complete the coordinate pairs using the line graph above.
i (0,□) ii (1,□)
iii (2,□) iv (3,□)

b What is the pattern connecting the y-coordinate to the x-coordinate?

c What is the equation of the line?

15

a Complete the coordinate pairs using the line graph above.
i (0,□) ii (1,□)
iii (2,□) iv (3,□)

b What is the pattern connecting the y-coordinate to the x-coordinate?

c What is the equation of the line?

Horizontal and vertical lines

Some equations do not have the letter x in them.

What does the graph of an equation such as $y = 4$ look like?

EXAMPLE 5

Draw a graph of the equation $y = 4$.

The equation $y = 4$ describes the mapping $x \to 4$.

A diagram of this mapping is:

So a graph of the equation $y = 4$ is:

The line $y = 4$

The equation $y = 4$ means that the y-coordinate is always 4, no matter what the x-coordinate is. Any equations in the form $y =$ 'a number' are horizontal lines.

Vertical lines are described by $x =$ 'a number'. For example, $x = 2$ means that the x-coordinate is always 2, no matter what the y-coordinate is. So the line $x = 2$ looks like this:

242

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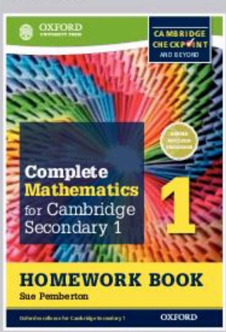

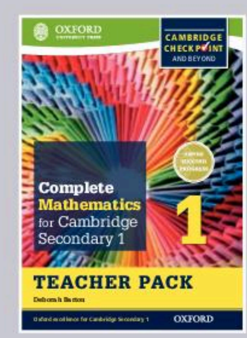

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